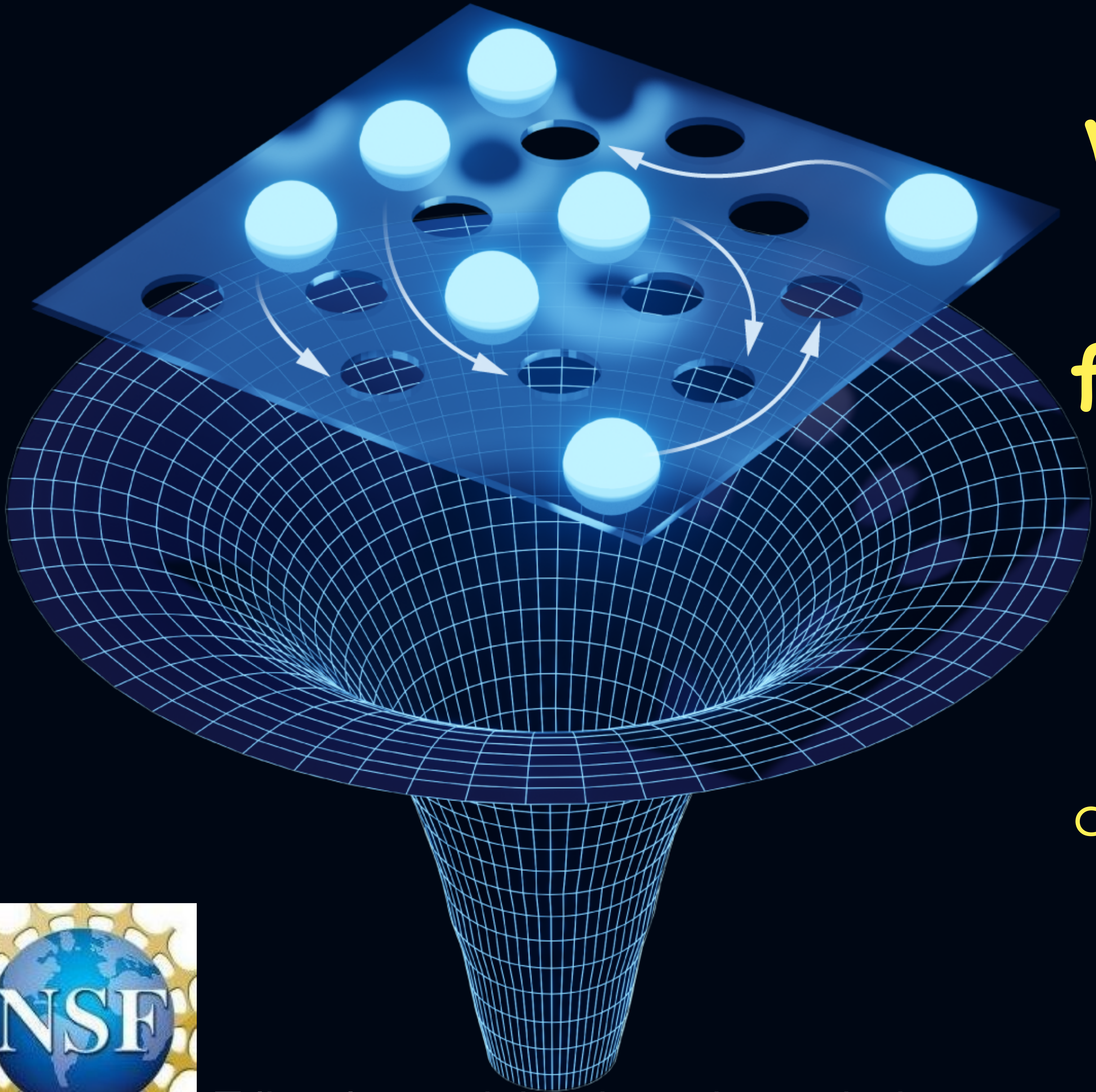


When nature entangles millions of particles: from quantum materials to black holes

Carnegie Mellon University
March 20, 2022

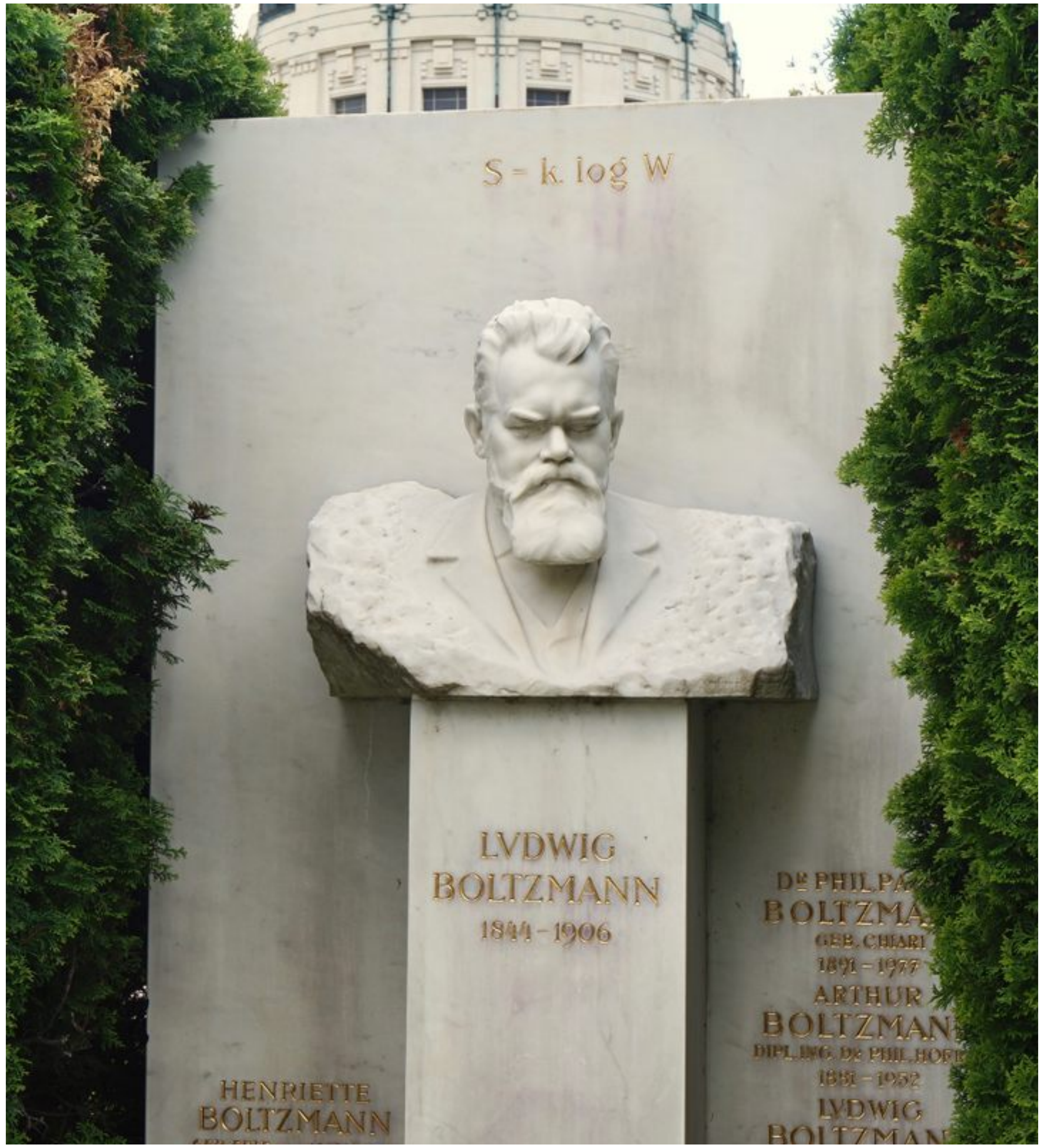
Subir Sachdev

PHYSICS
VE RI TAS
HARVARD

NSF

Talk online: sachdev.physics.harvard.edu

Foundations
by
Boltzmann

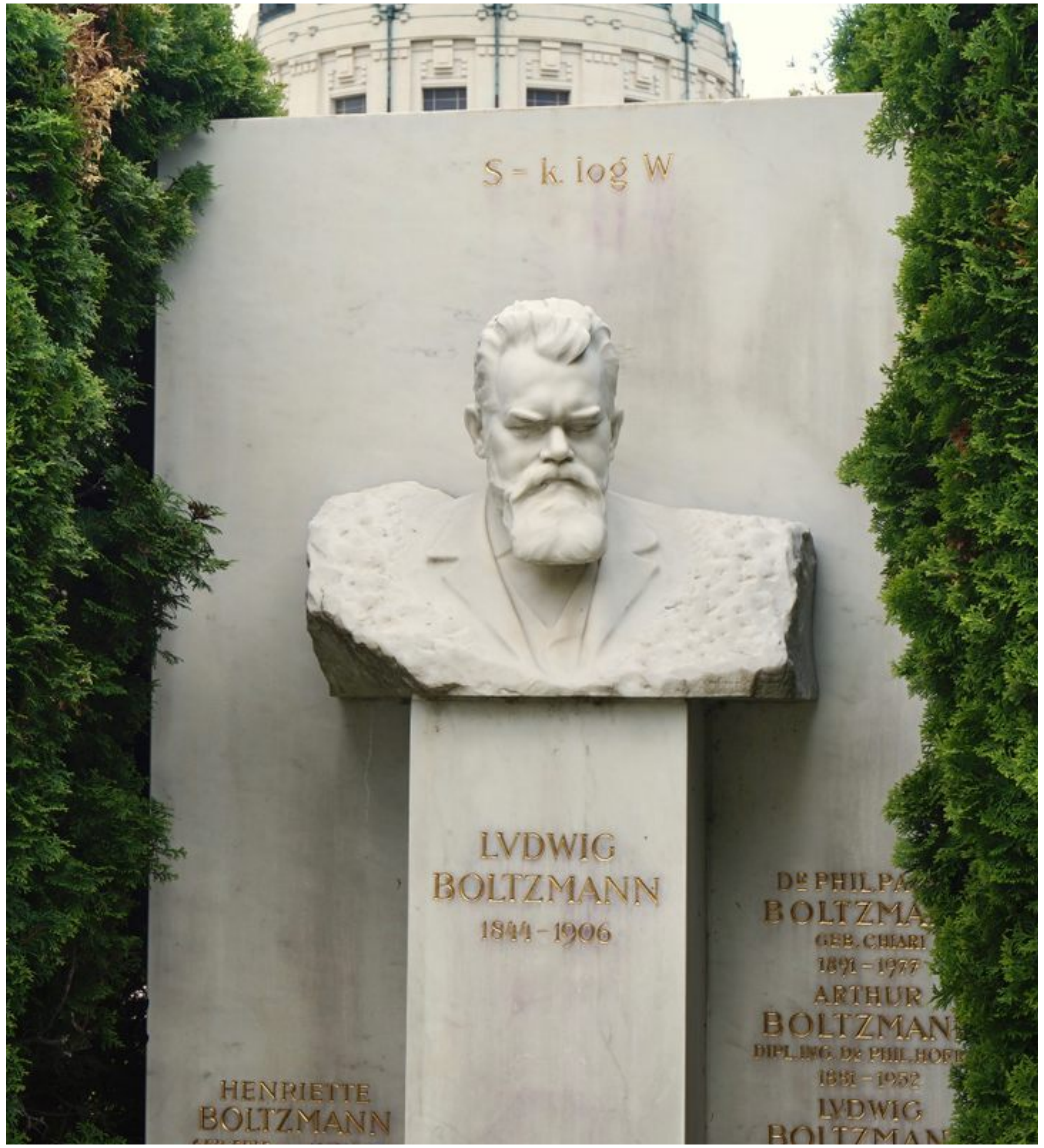


Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

Statistical interpretation of entropy (1870)

$$S = k_B \log W$$

Density of quantum states $D(E) = \exp(S(E)/k_B)$



Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

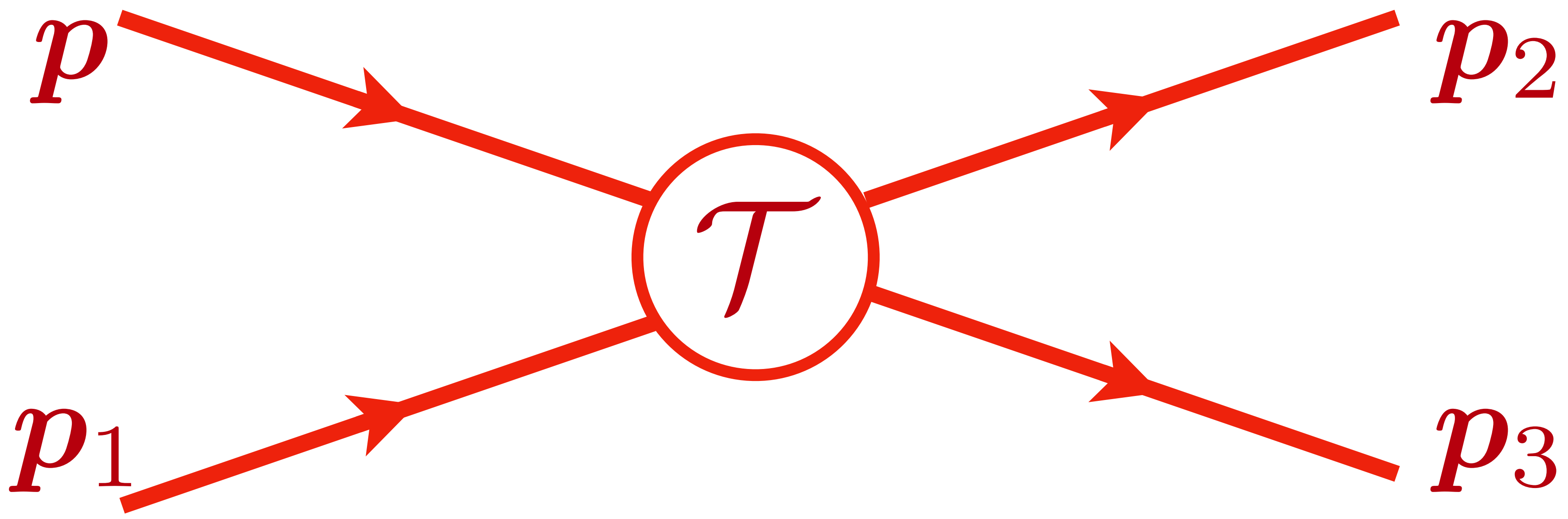
Boltzmann equation (1872)
Dilute classical gas

Molecular chaos: successive collisions are statistically independent

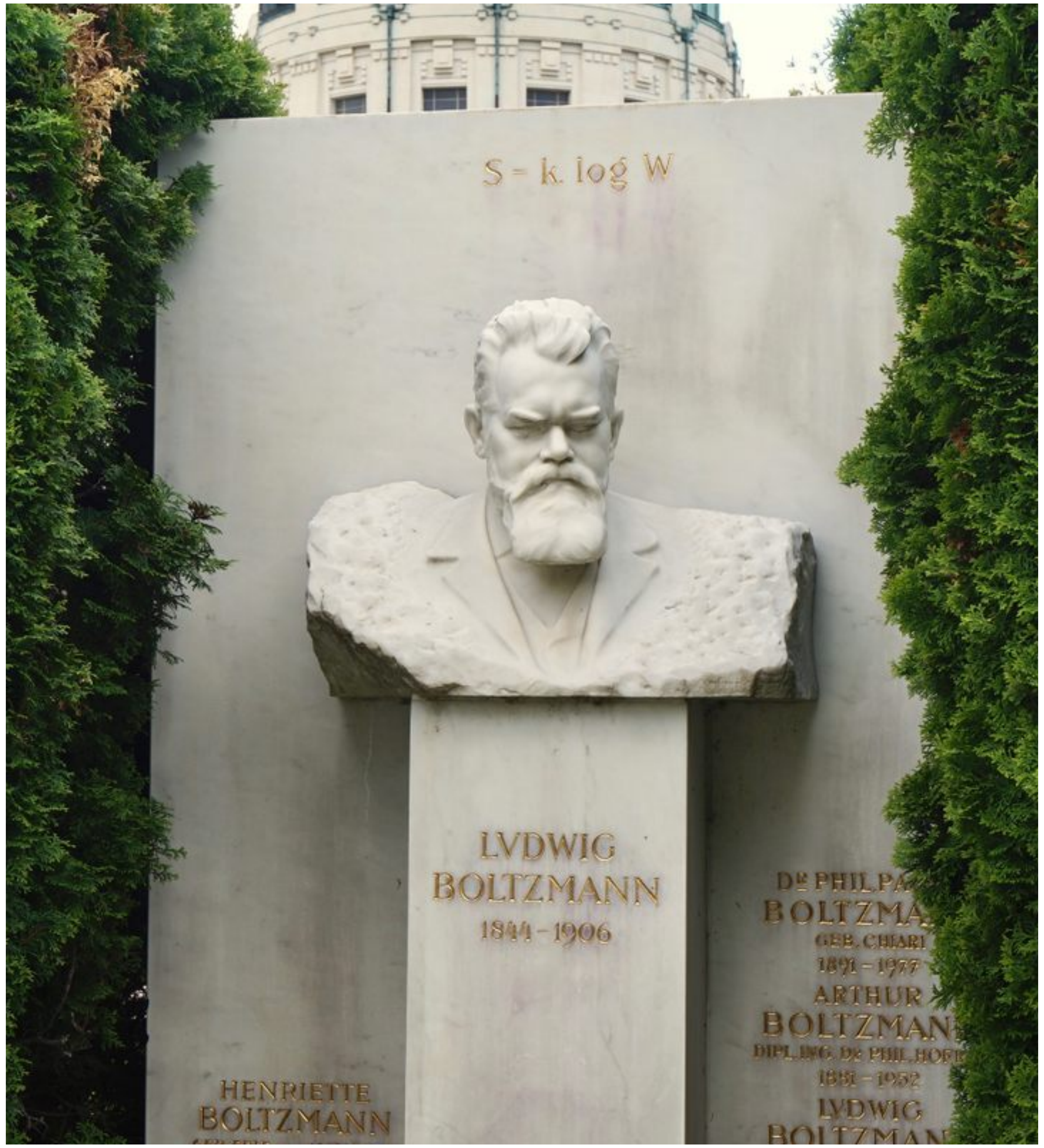
$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f_{\boldsymbol{p}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} =$$

$$-2\pi \int_{\boldsymbol{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\boldsymbol{p}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3}) \delta(\boldsymbol{p} + \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3)$$

$$\times \left[f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} \right]$$



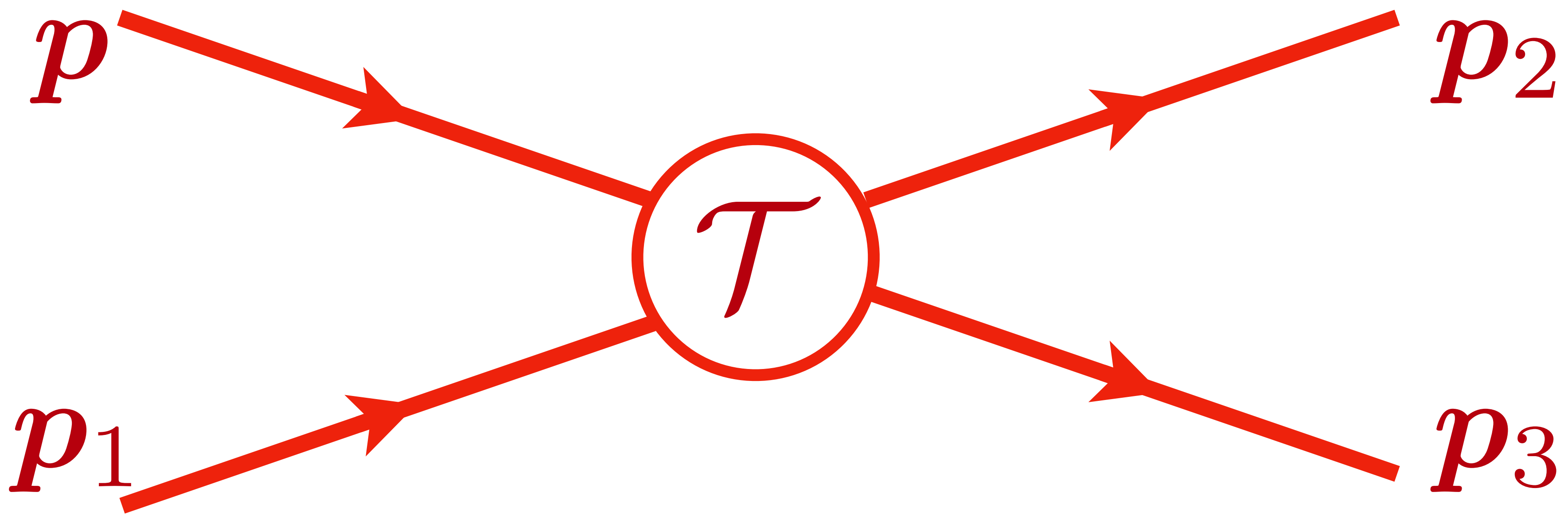
Quantum Boltzmann equation (Landau)
Dense gas of electrons



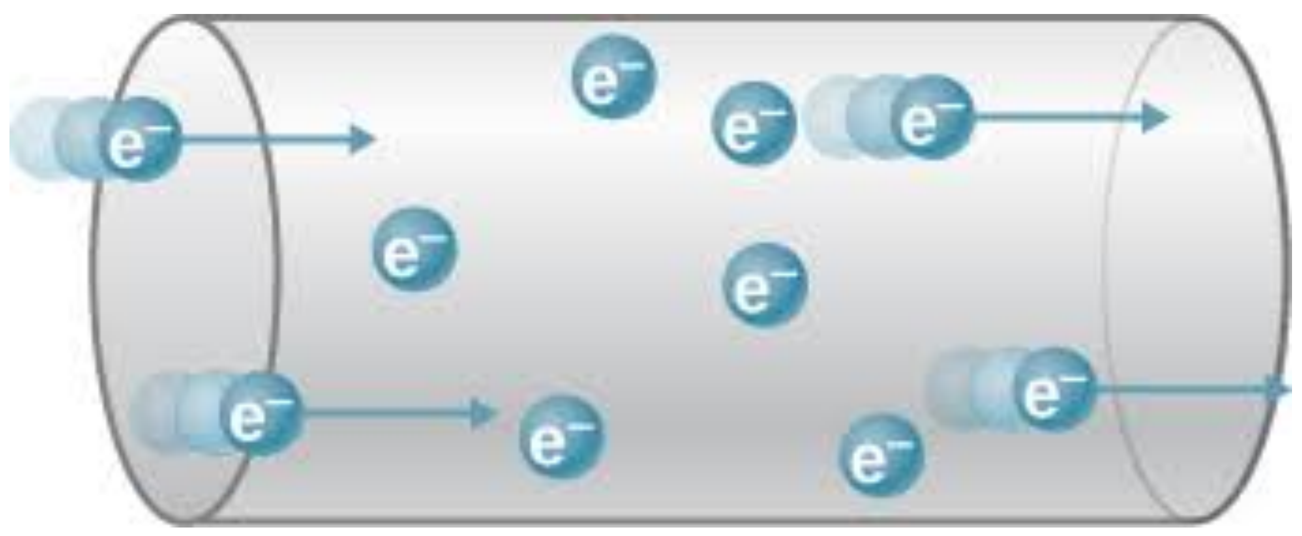
Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

Neglects quantum interference (entanglement)
between successive collisions

$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f_{\boldsymbol{p}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} =$$
$$- 2\pi \int_{\boldsymbol{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\boldsymbol{p}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3}) \delta(\boldsymbol{p} + \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3)$$
$$\times \left[f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} (1 - f_{\boldsymbol{p}_2})(1 - f_{\boldsymbol{p}_3}) - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} (1 - f_{\boldsymbol{p}})(1 - f_{\boldsymbol{p}_1}) \right]$$



Current flow with electrons in Copper



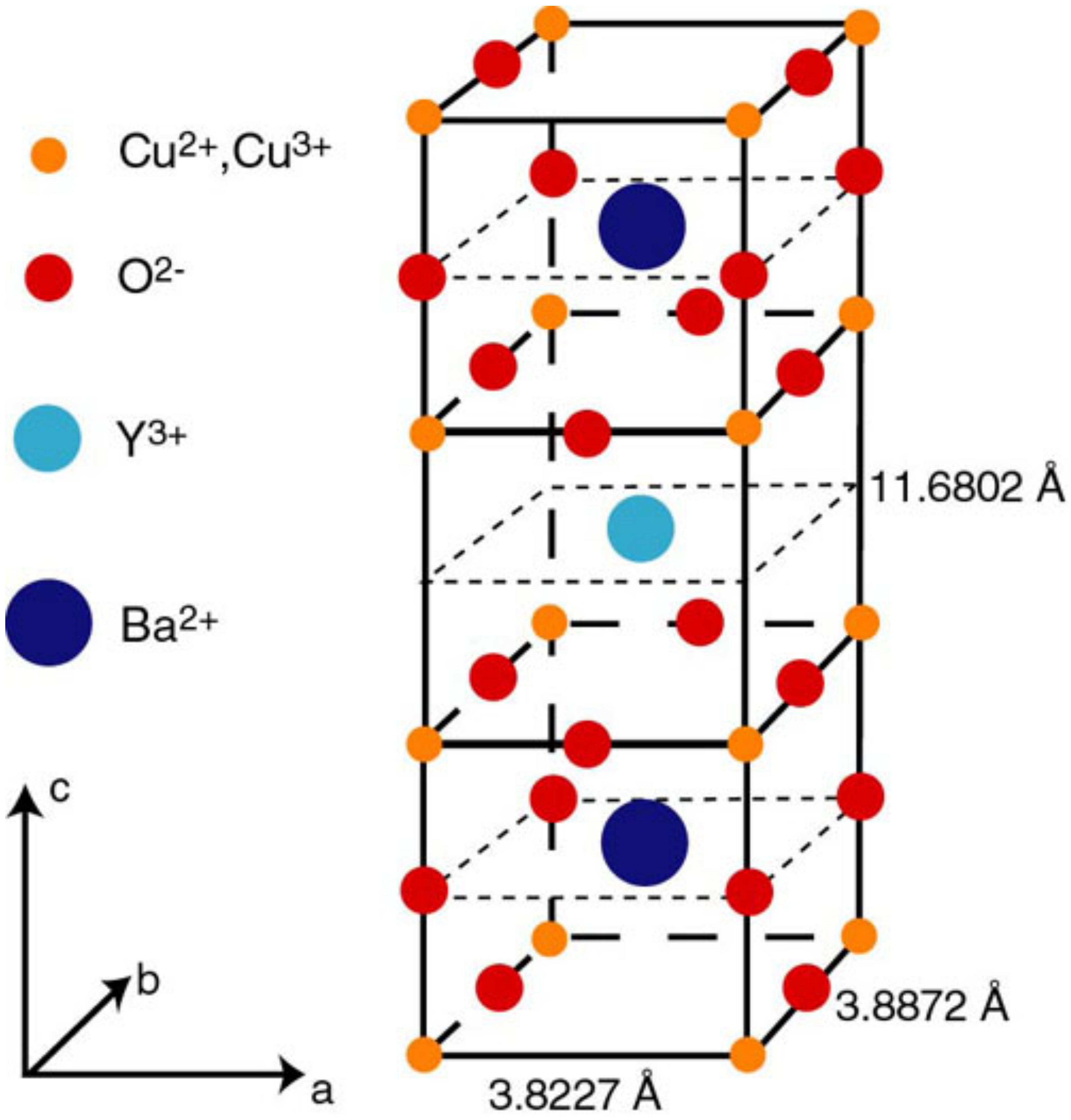
Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/T^2$, resistivity $\rho(T) = \rho(0) + AT^2$

The time τ is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

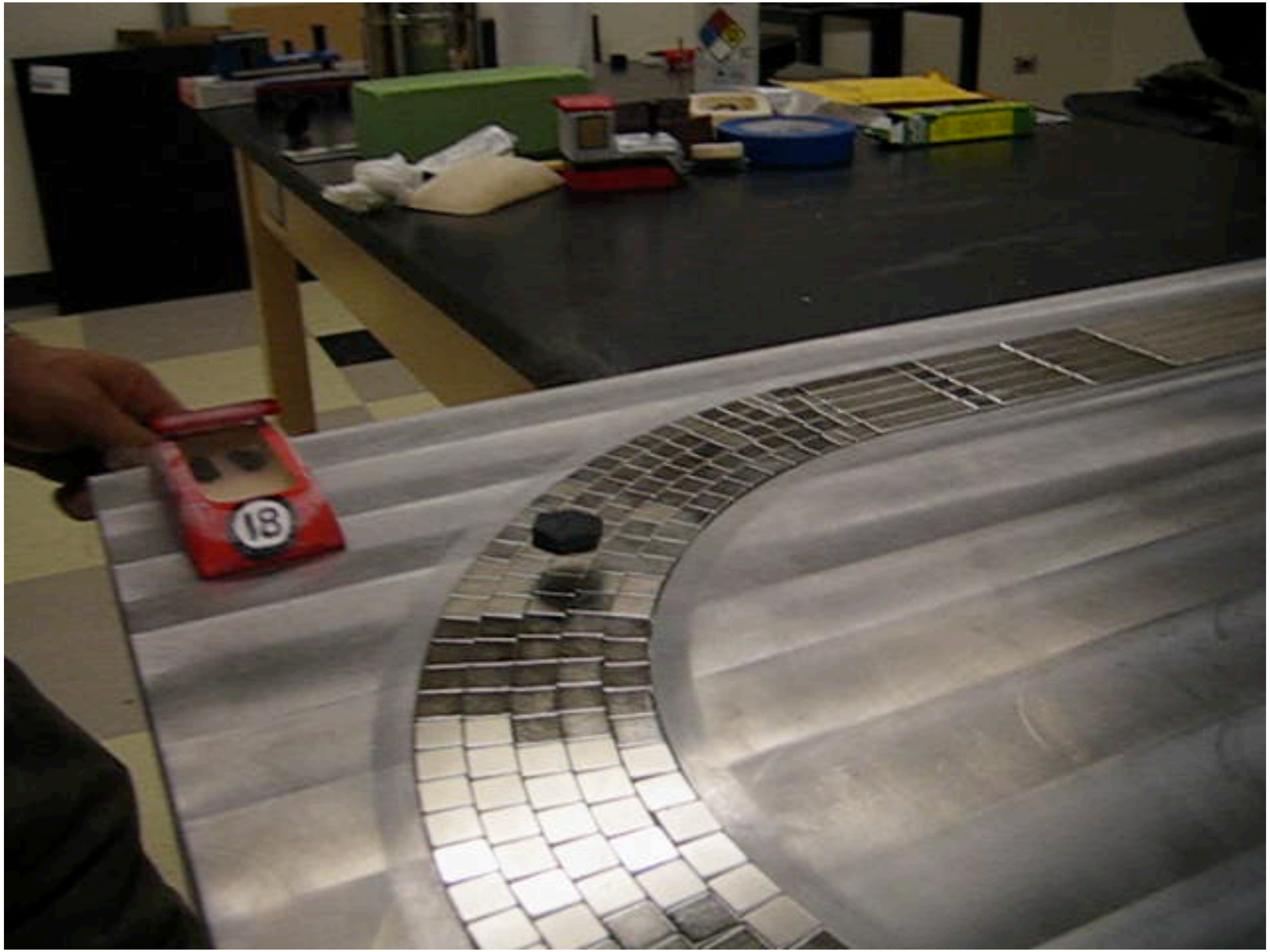
The long scattering time implies that individual electrons are well-defined.

The motion of electrons is 'ballistic' or 'integrable'
up to the long time τ, after which it is chaotic.

Cuprate
high temperature
superconductors



$YBa_2Cu_3O_{6+x}$



Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

HTS Magnets: Enabling Technology

A new high temperature superconductor (HTS) recently reached industrial maturity: Rare Earth Barium Copper Oxide (REBCO). CFS is using HTS and has built its first-of-its-kind high-field large-bore superconducting magnet. HTS magnets will allow for smaller, faster, and less expensive tokamaks using the science developed on Alcator C-Mod and other tokamaks.

The surest path to limitless, clean, fusion energy

Surest

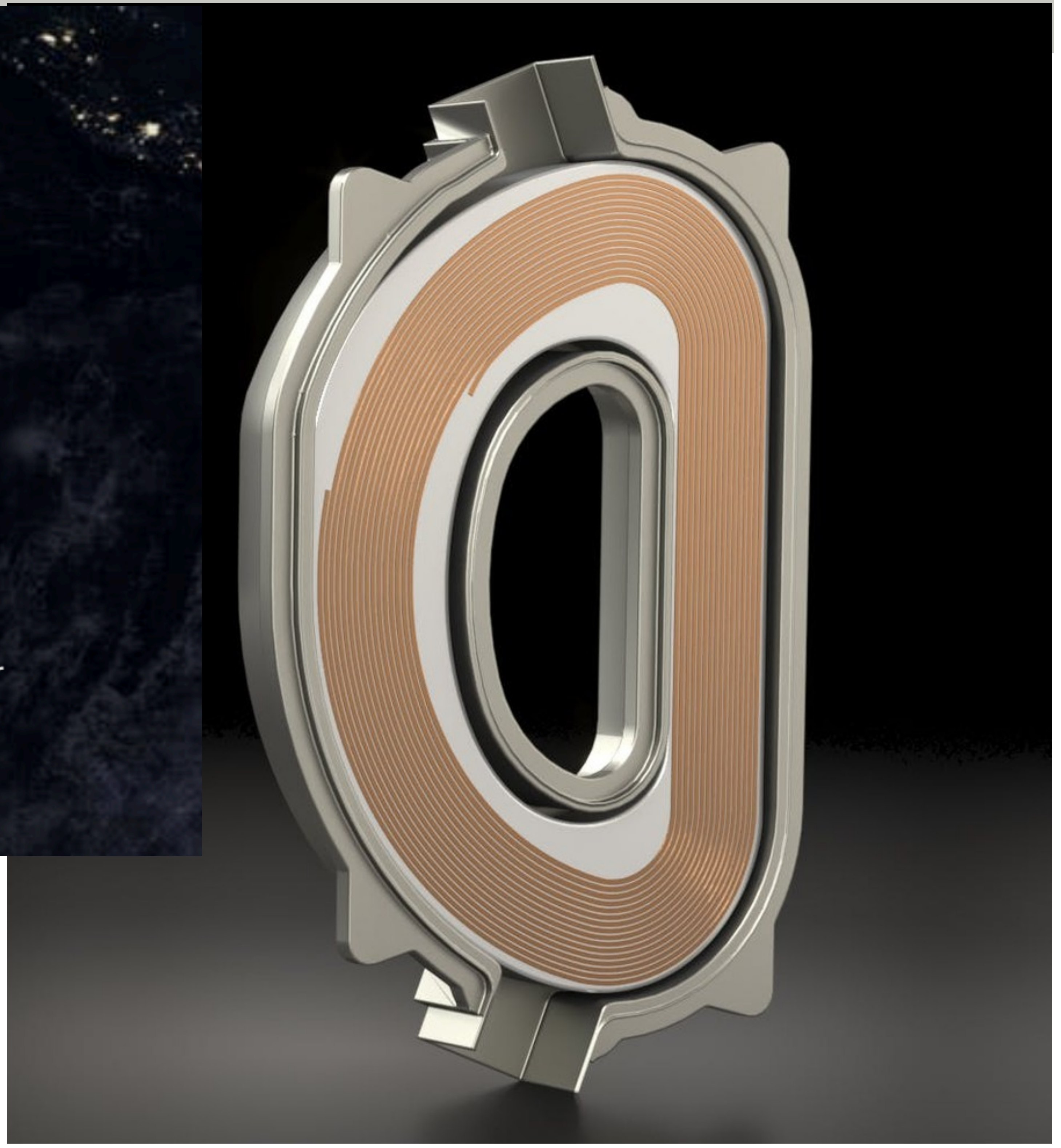
The fastest path to commercial fusion energy combining proven science with revolutionary magnet technology.

Limitless

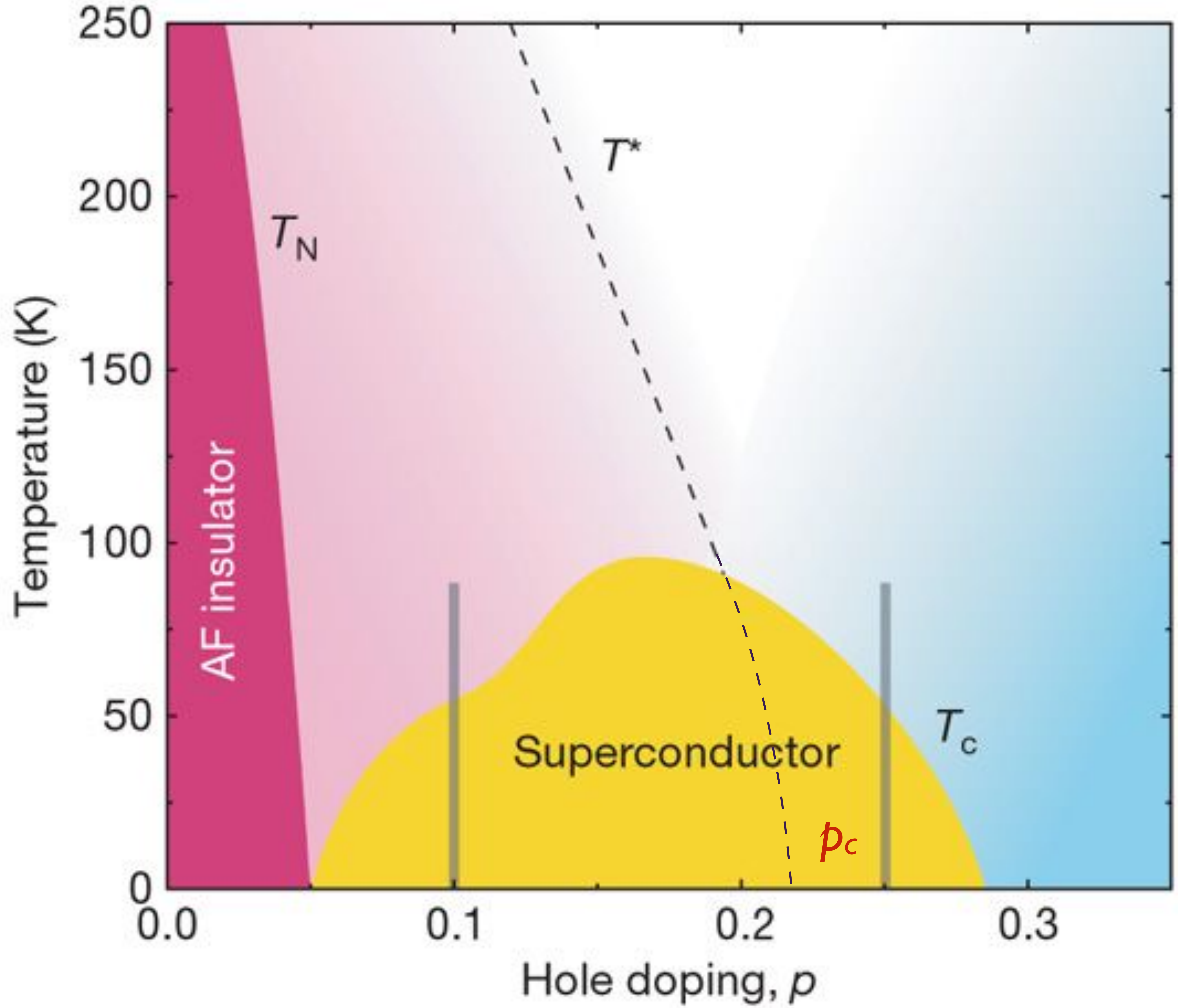
One glass of water will provide enough fusion fuel for one person's lifetime.

Clean

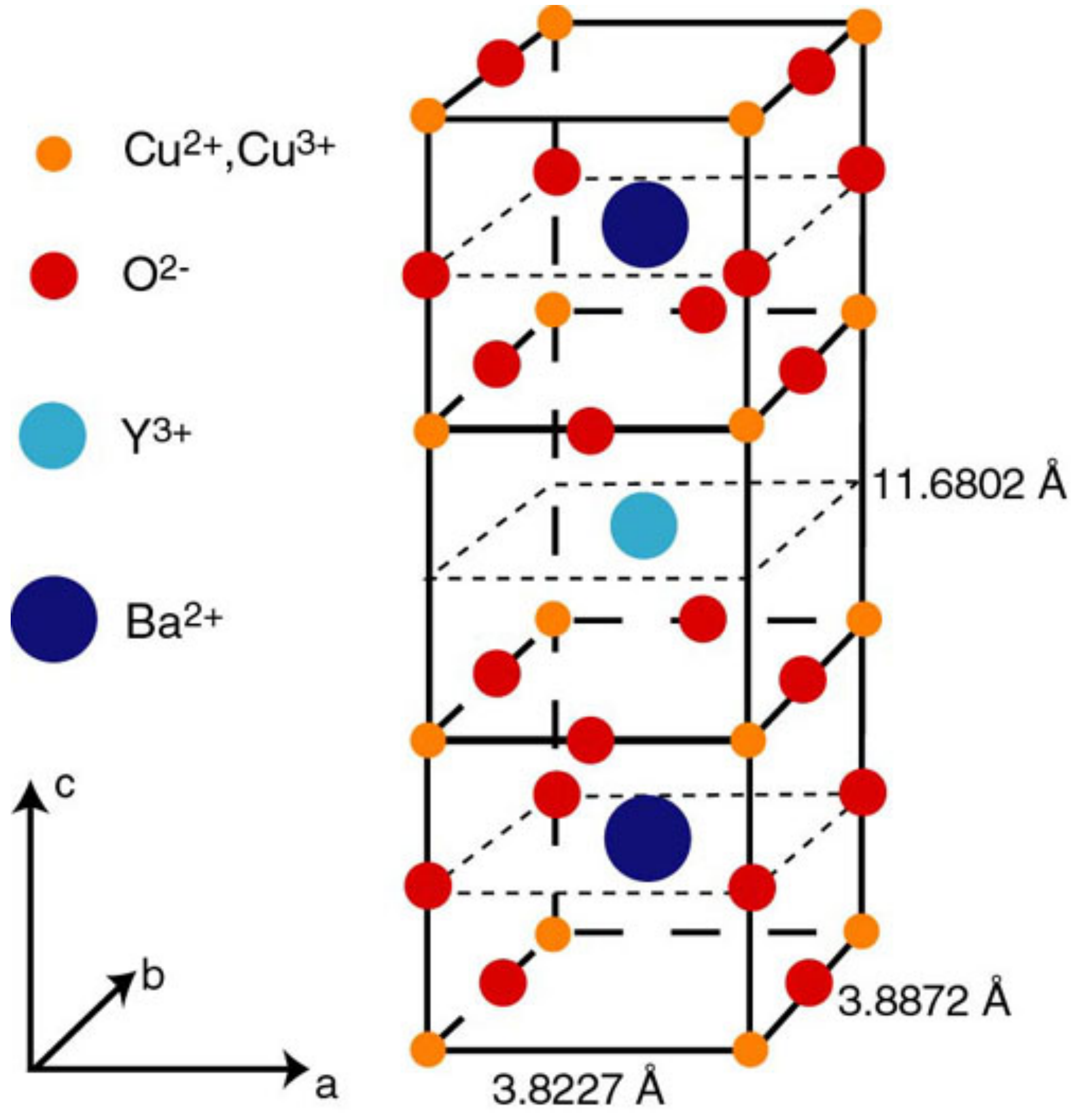
A new source of clean energy to meet our growing energy demands and combat climate change.



Commonwealth Fusion Systems

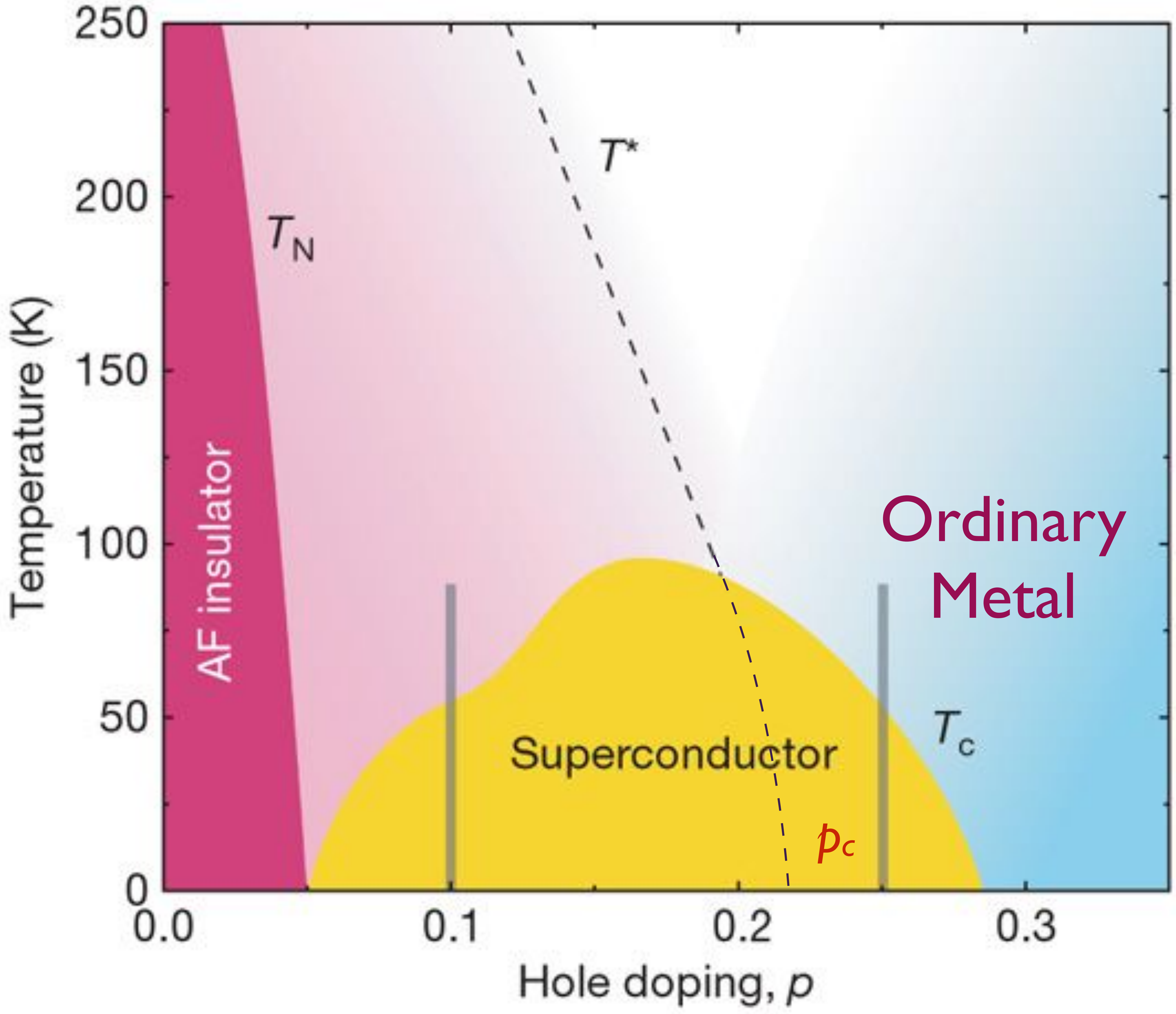


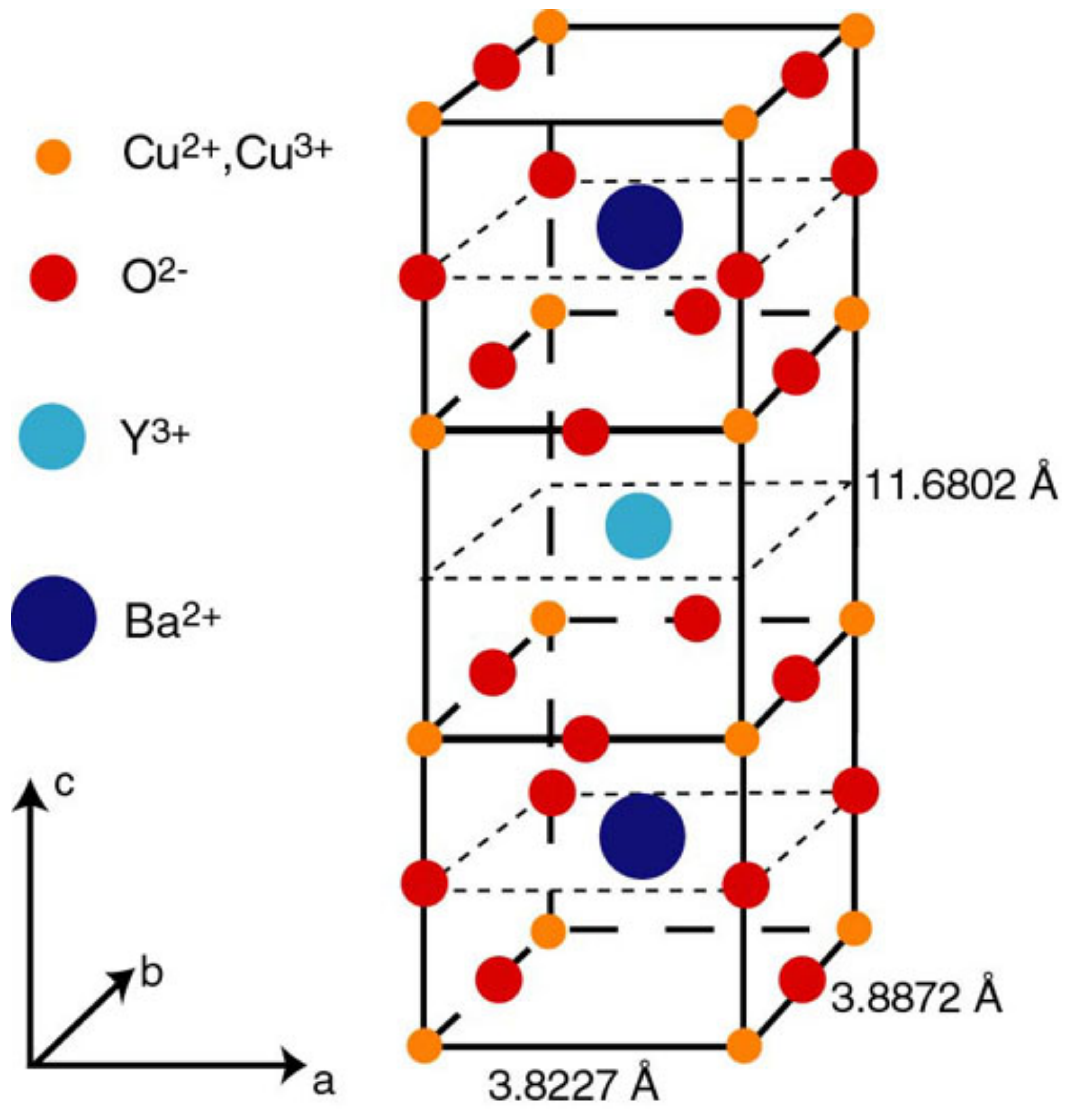
250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Superconductor
T_c
p_c



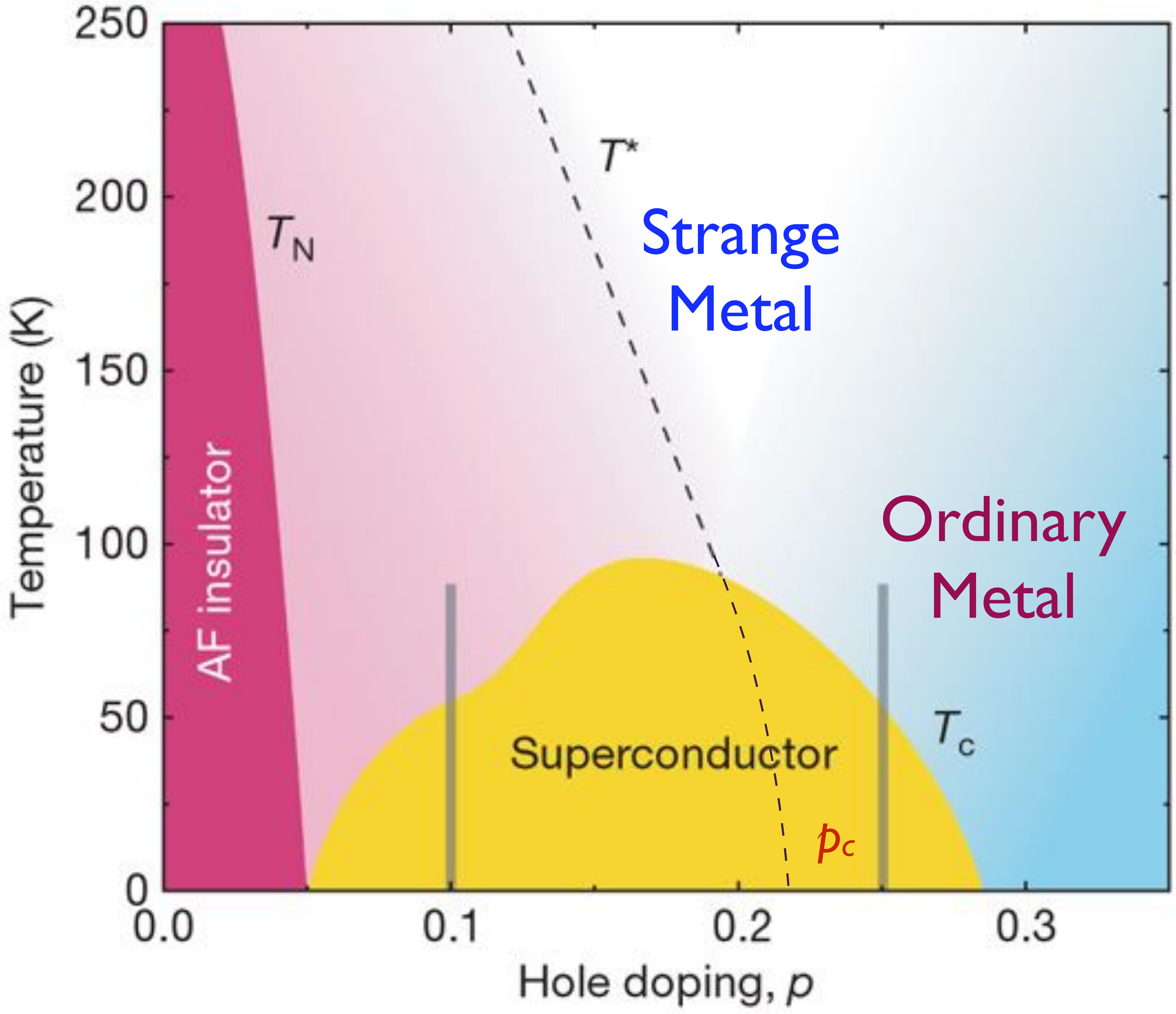
Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Ordinary Metal
Superconductor
T_c
p_c


Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
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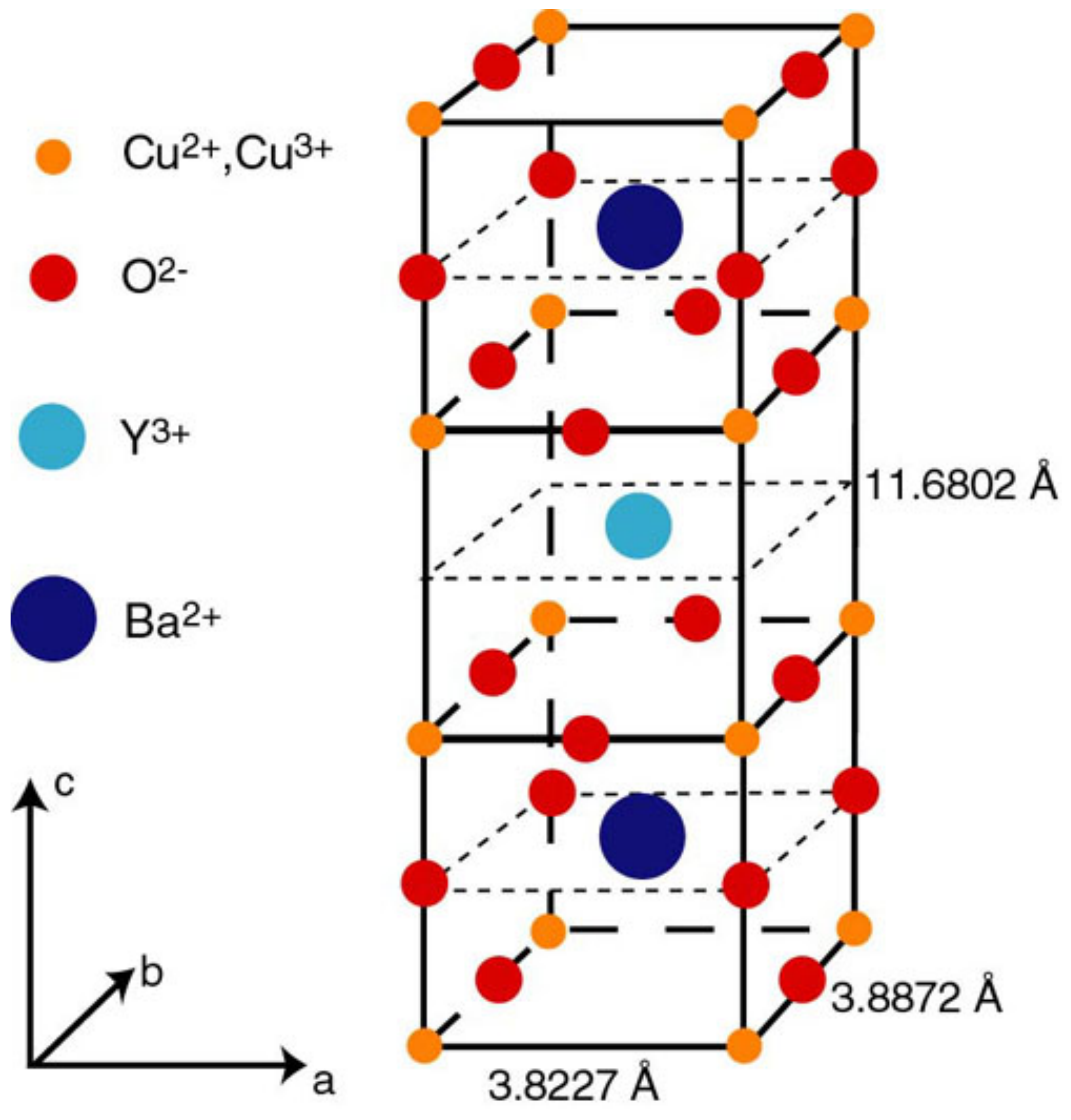
Hole doping, p
AF insulator
T_N
T*
Strange Metal
Ordinary Metal
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
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11.6802 Å
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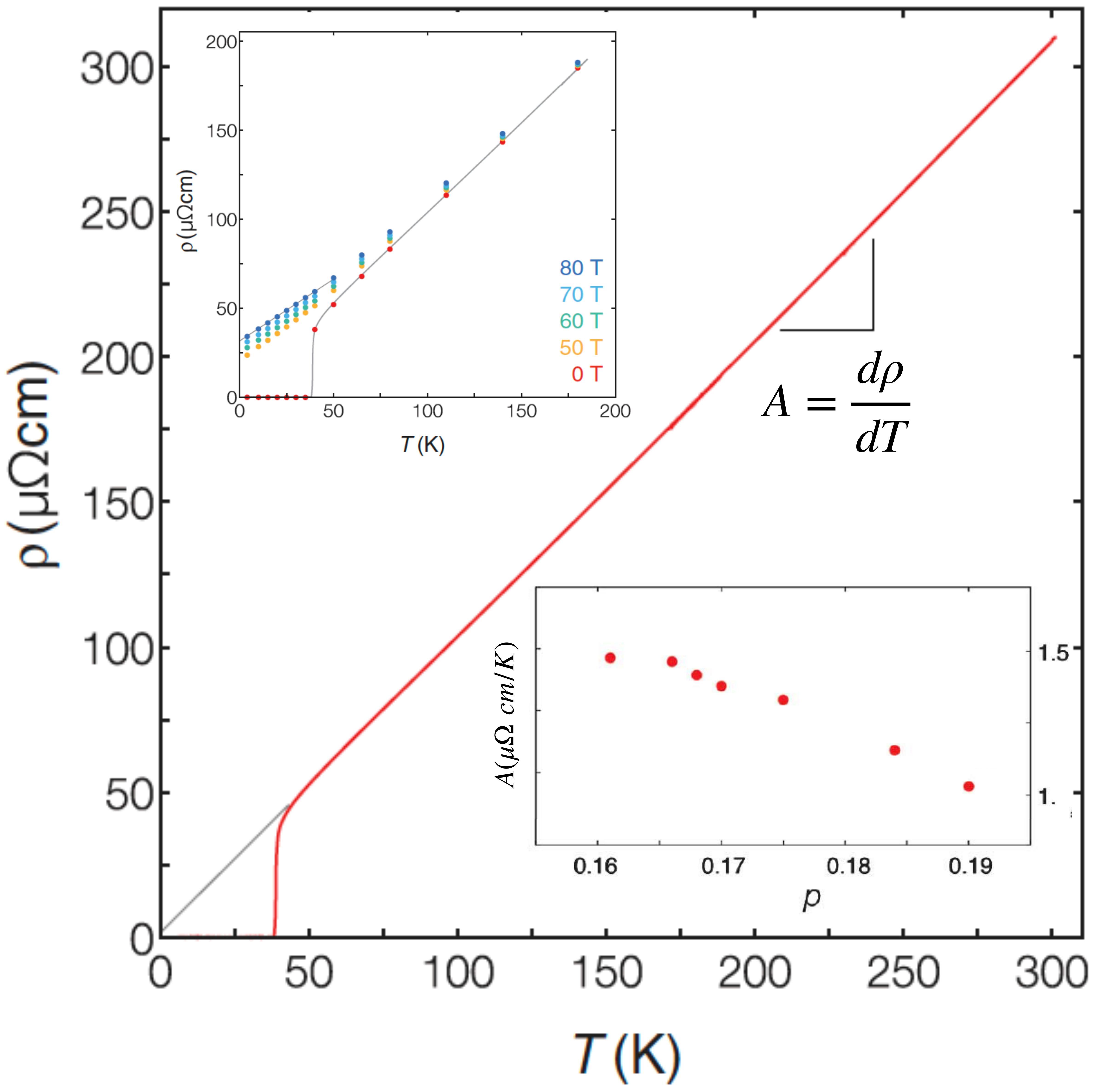
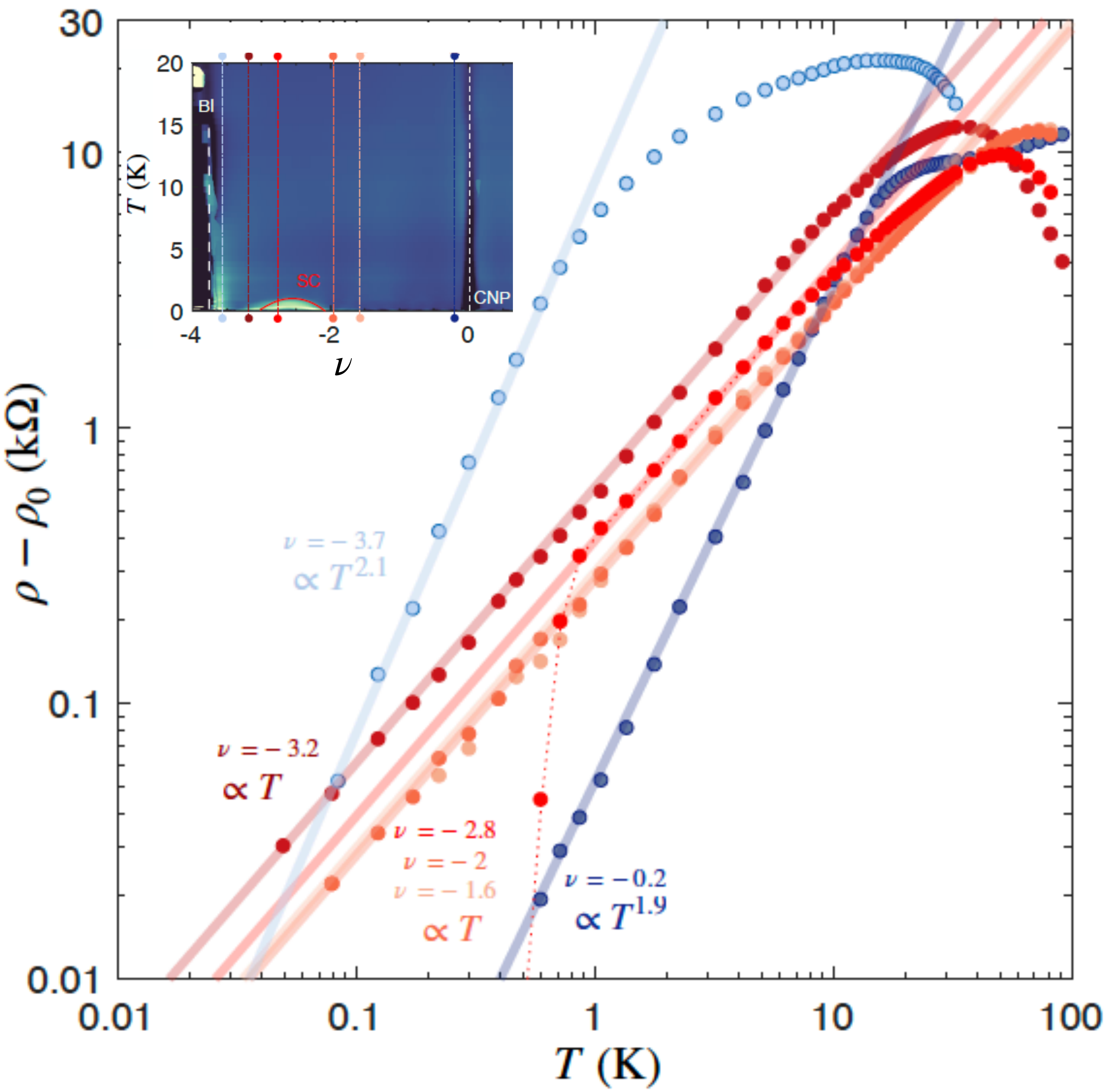


LSCO: Giraldo-Gallo et al. 2018

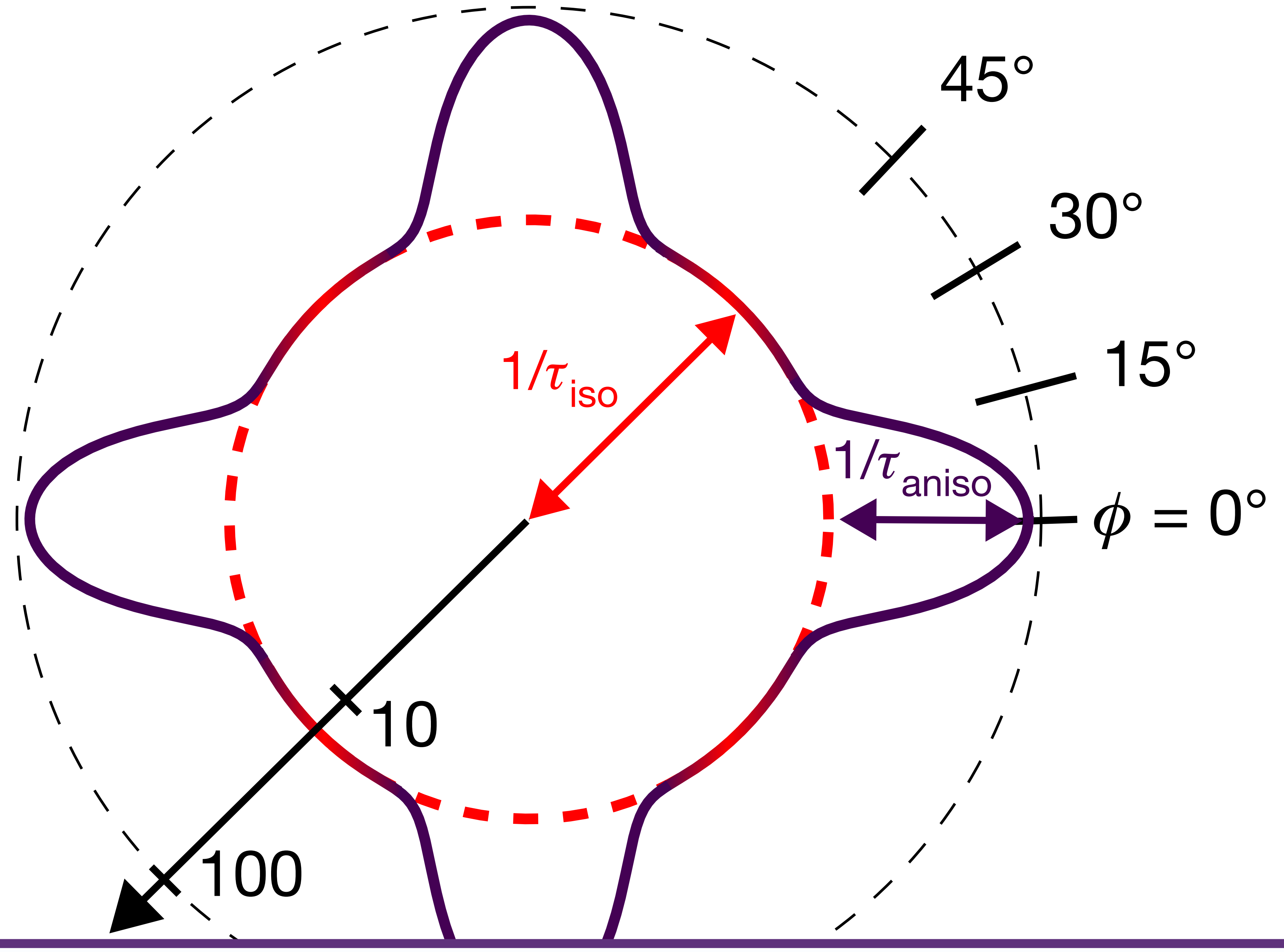


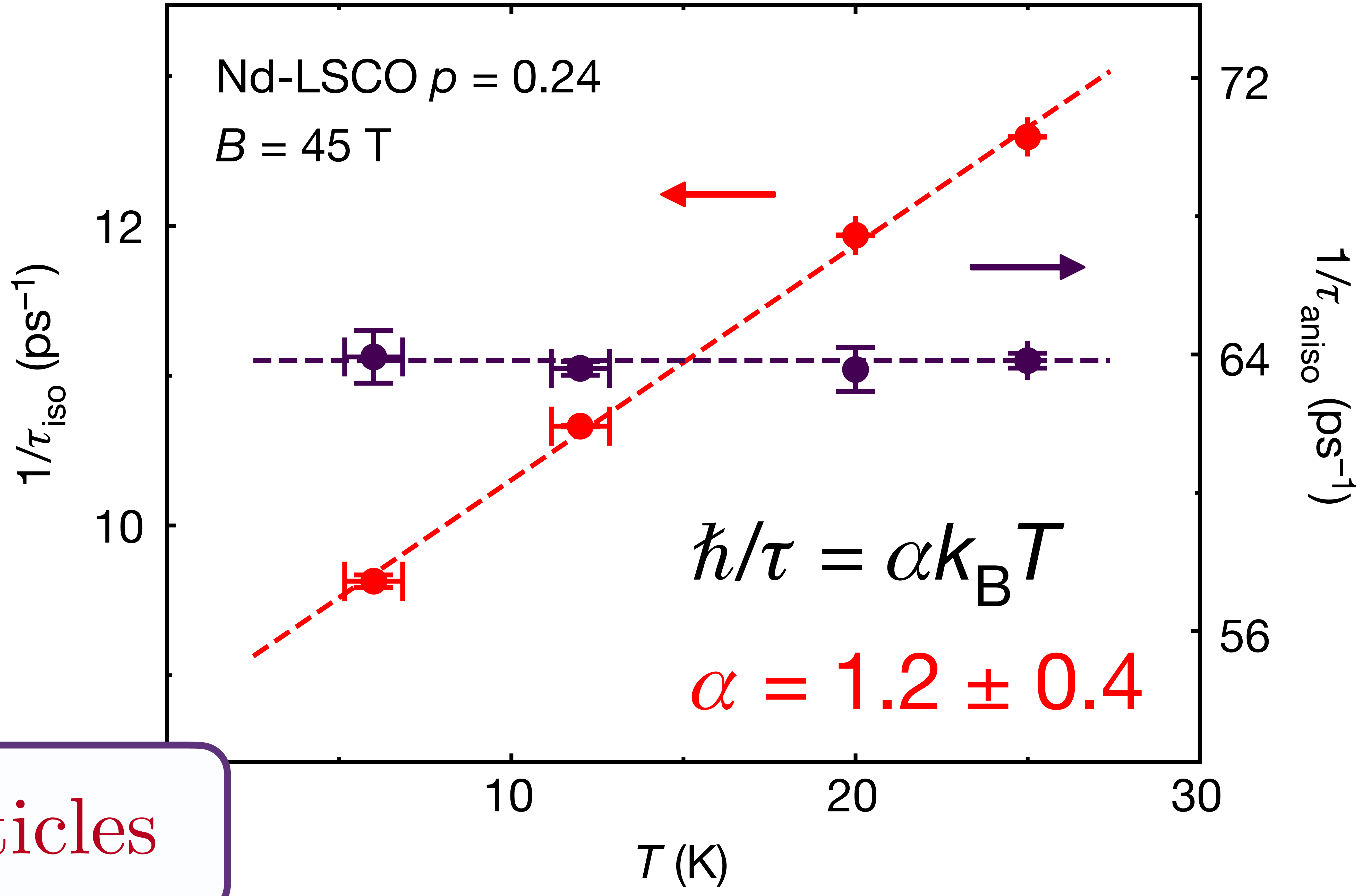
MATBG: Jaoui et al. 2021

Linear-in temperature resistivity from an isotropic Planckian scattering rate

Nature **595**, 667-672 (2021)

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw





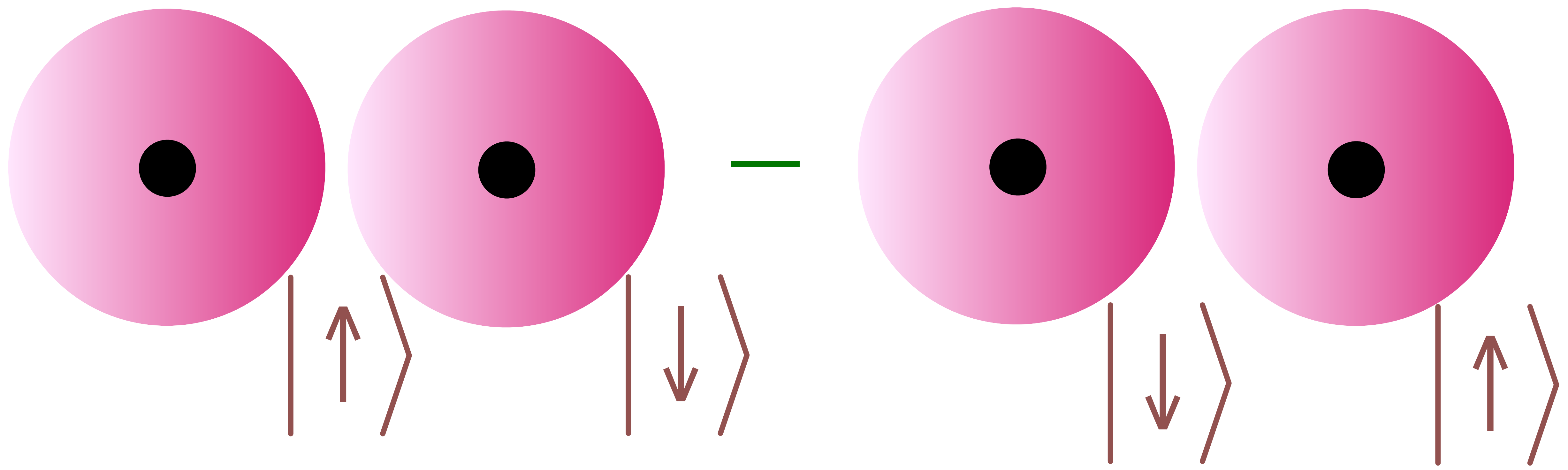
Current flow without quasiparticles

No Boltzmann-Landau quasiparticle description $\Rightarrow$
Many particle quantum entanglement
from quantum interference between “collisions”

Sachdev-Ye-Kitaev Model

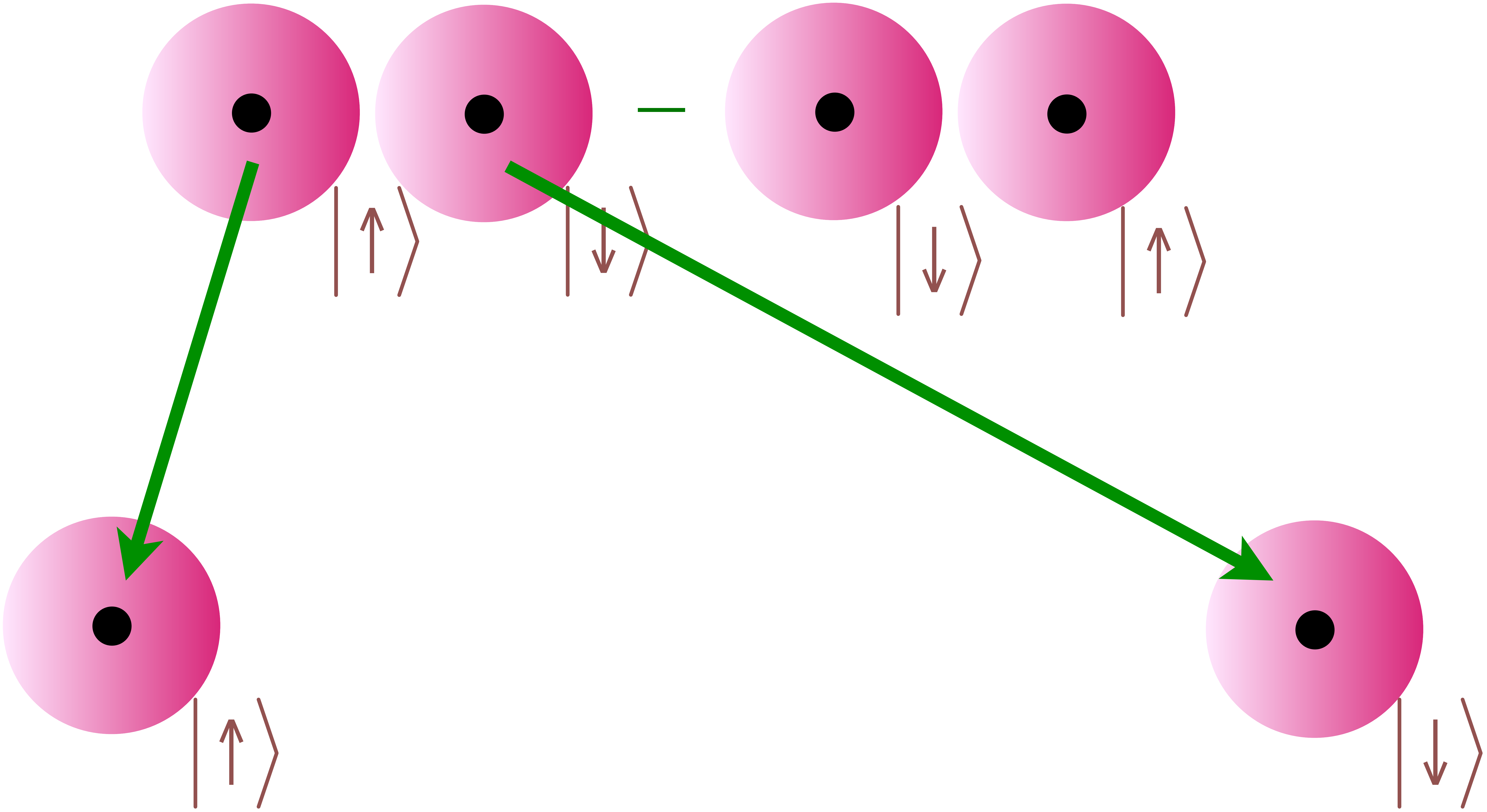
Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



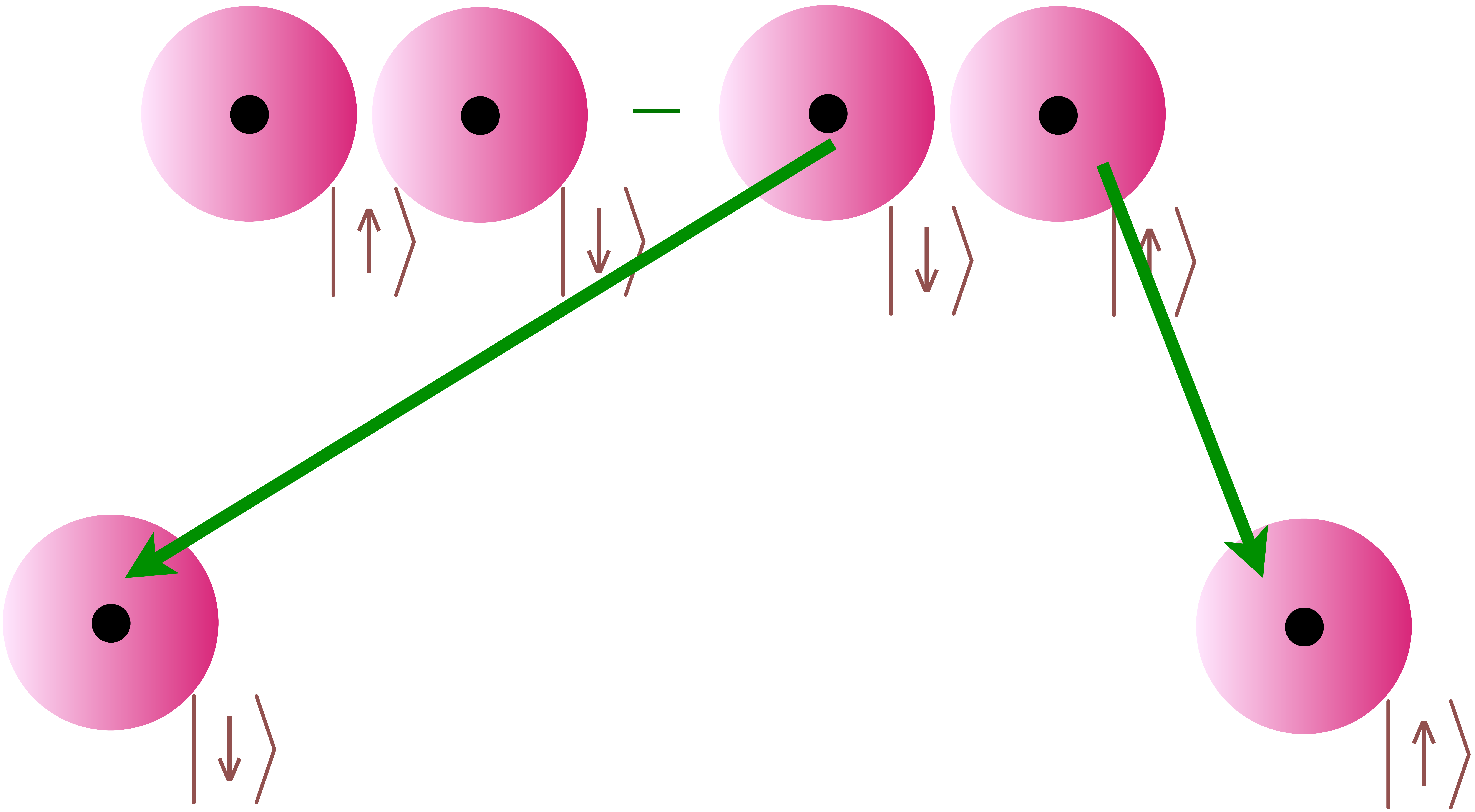
Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



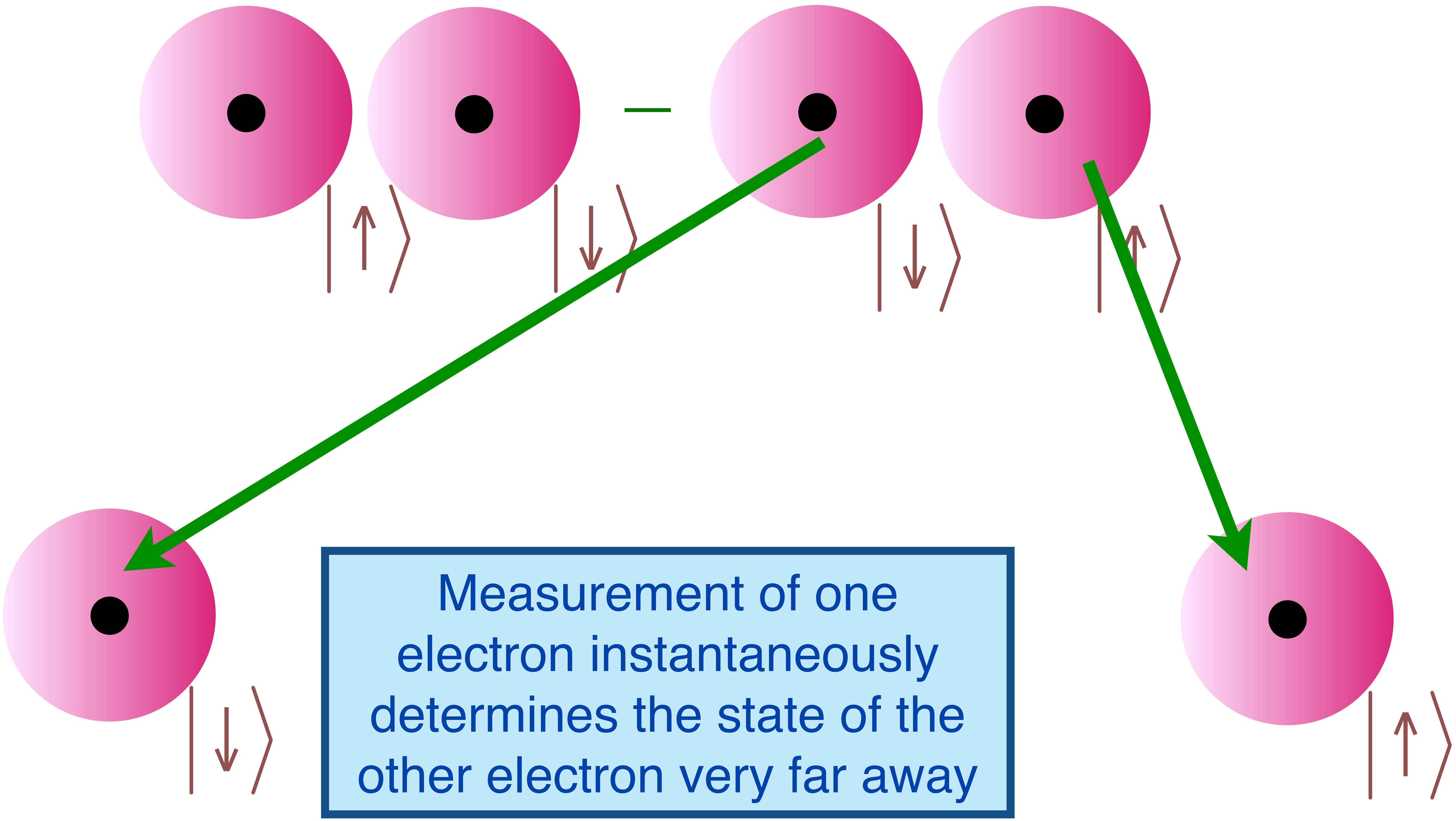
Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



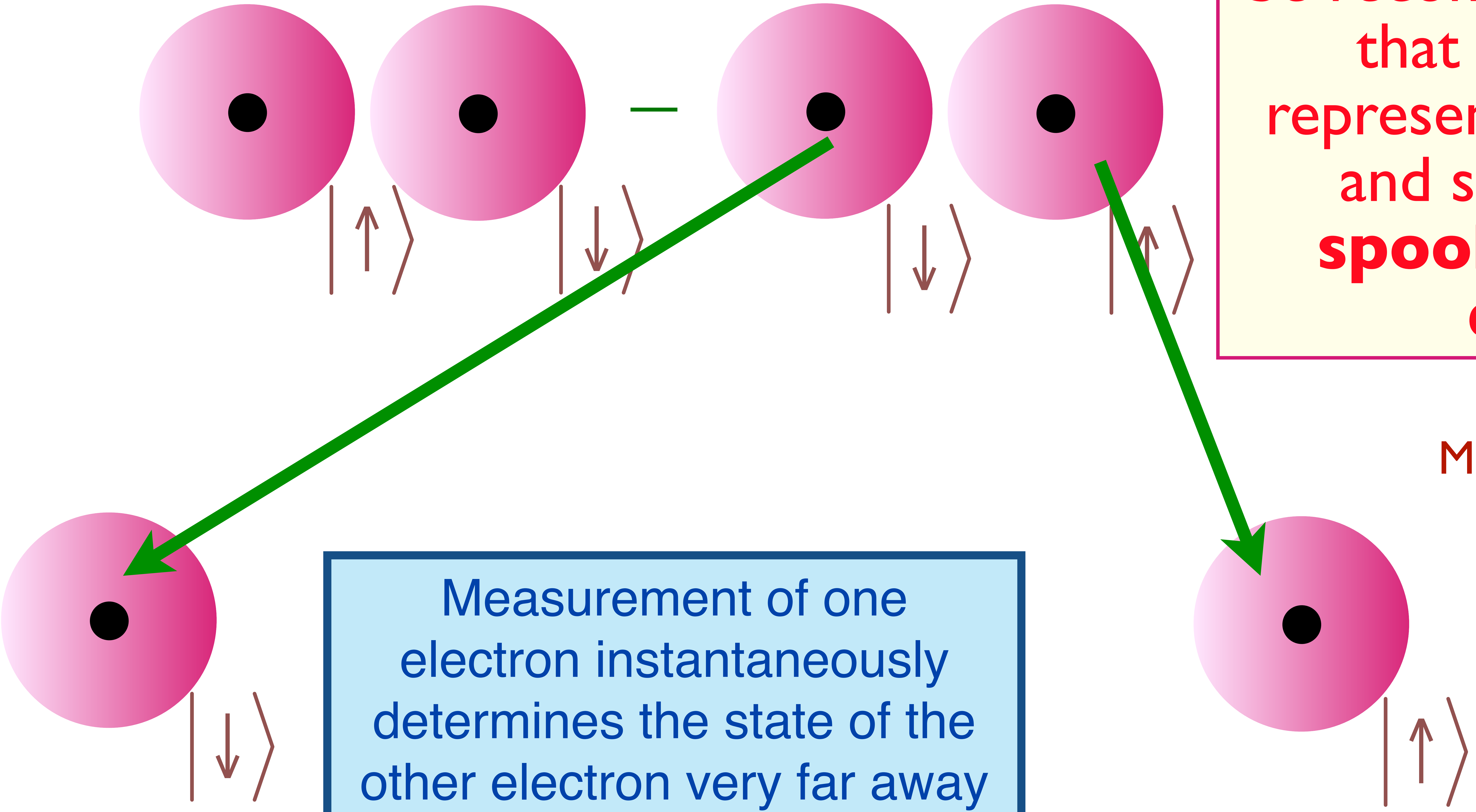
Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



Measurement of one electron instantaneously determines the state of the other electron very far away

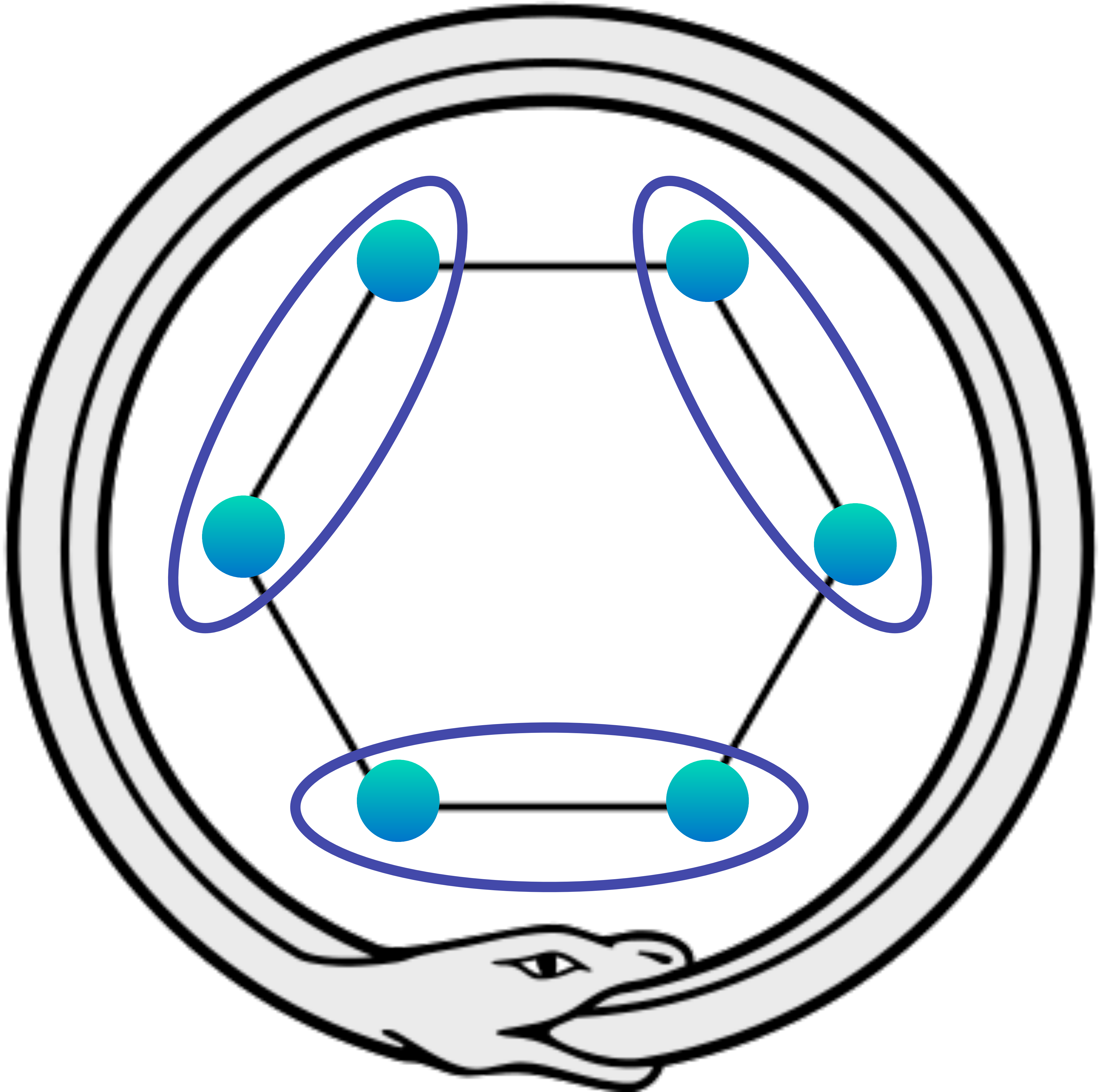
> I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from **spooky actions at distance**

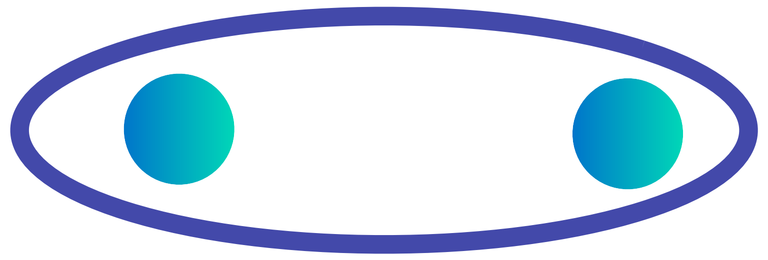
Albert Einstein to Max Born, 3 March 1947

How about quantum entanglement of $3, 4, 5, \ldots \infty$ particles?

Kekule's spooky dream (1865)

Here Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*



 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

*Wikipedia

Kekule's spooky dream (1865)

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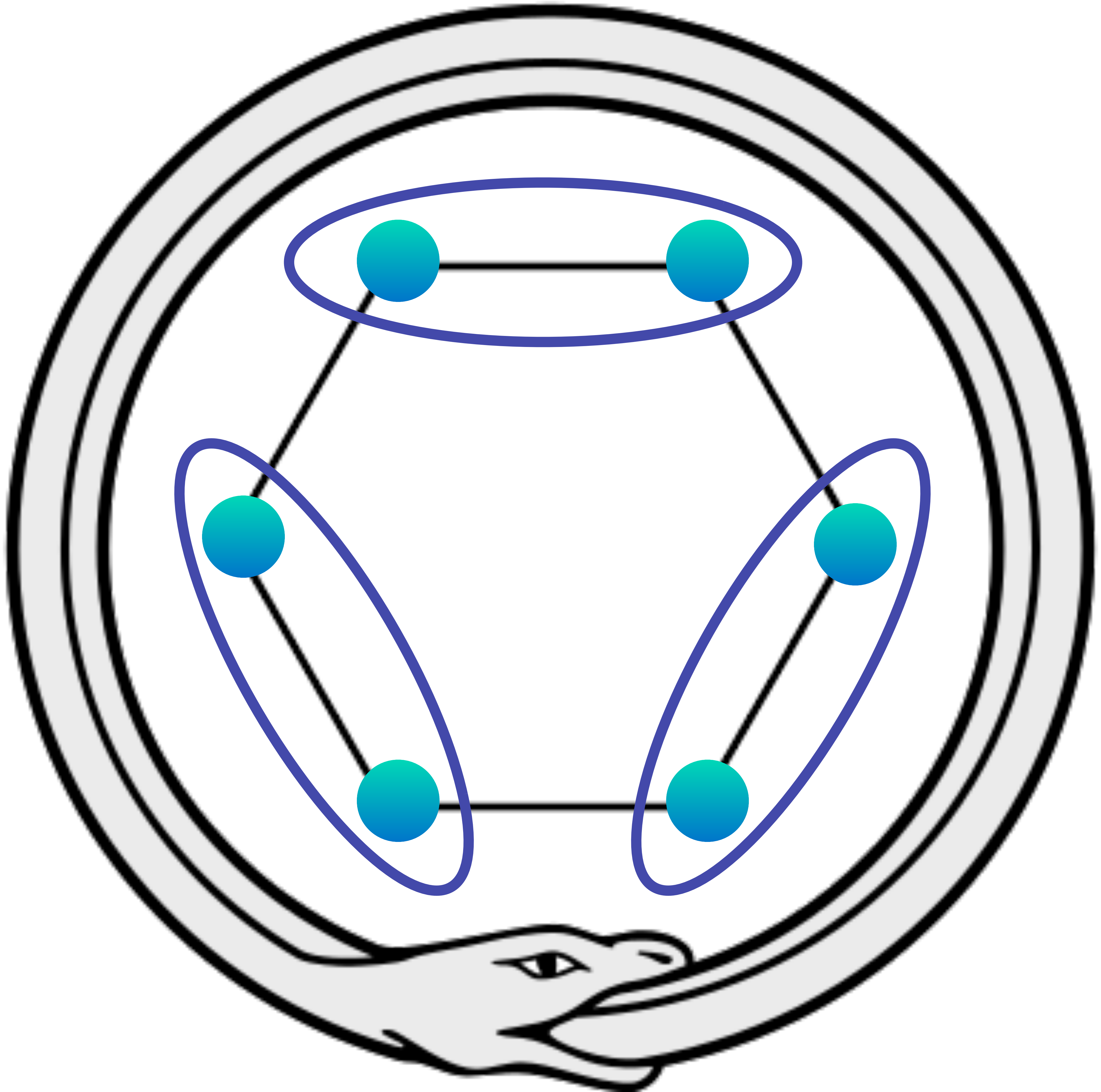

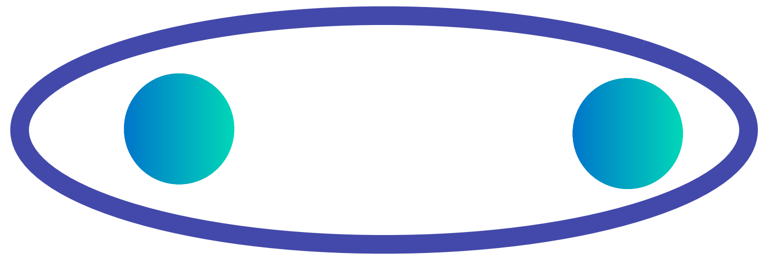

$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

*Wikipedia

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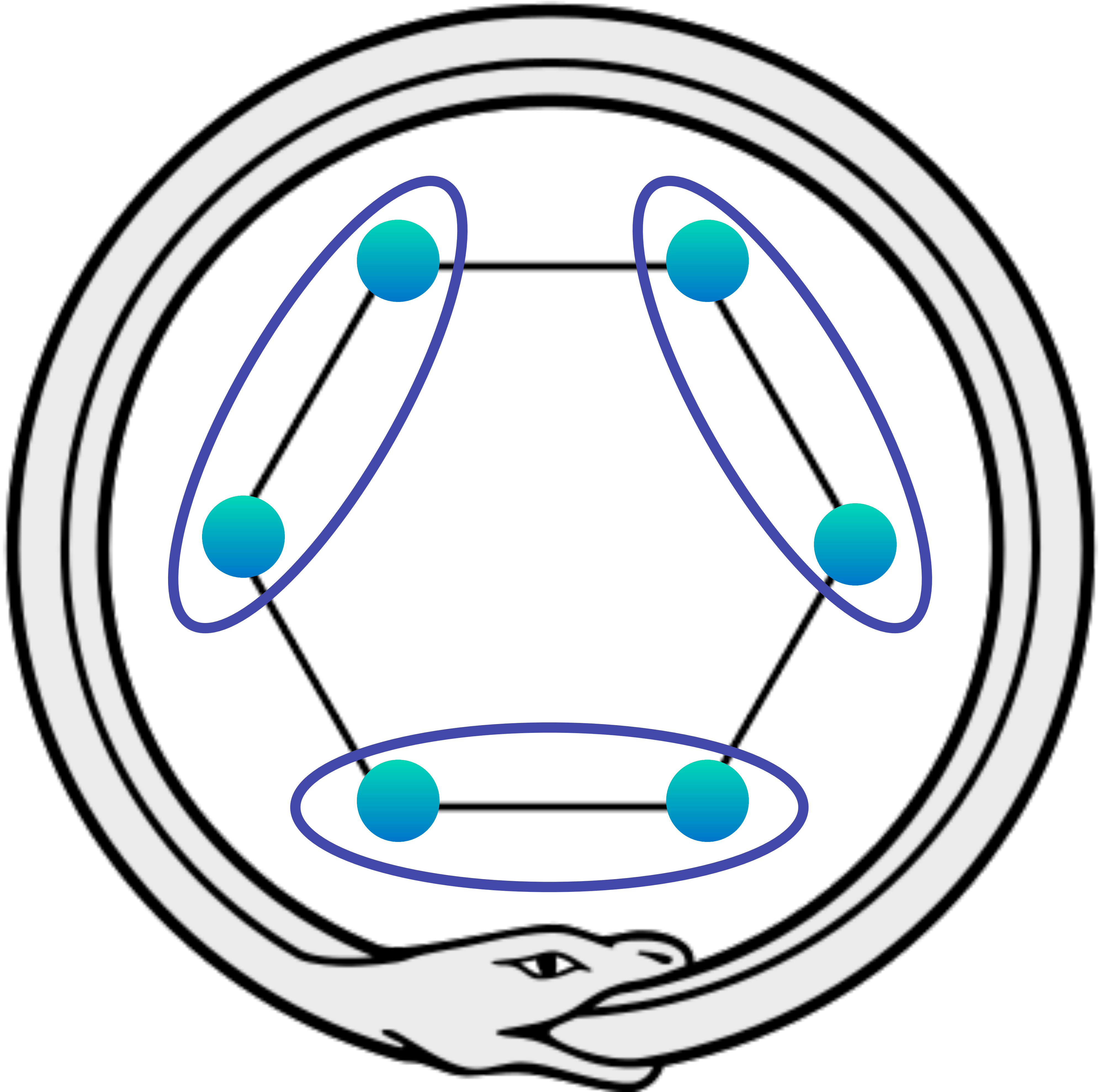
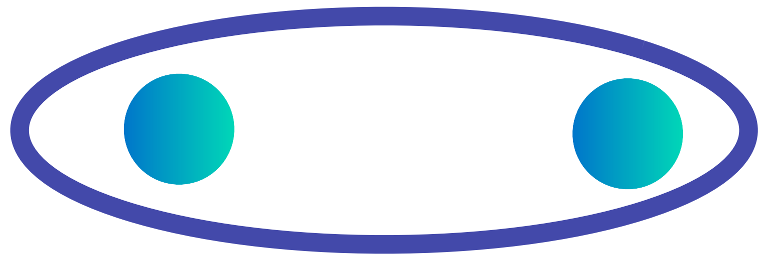
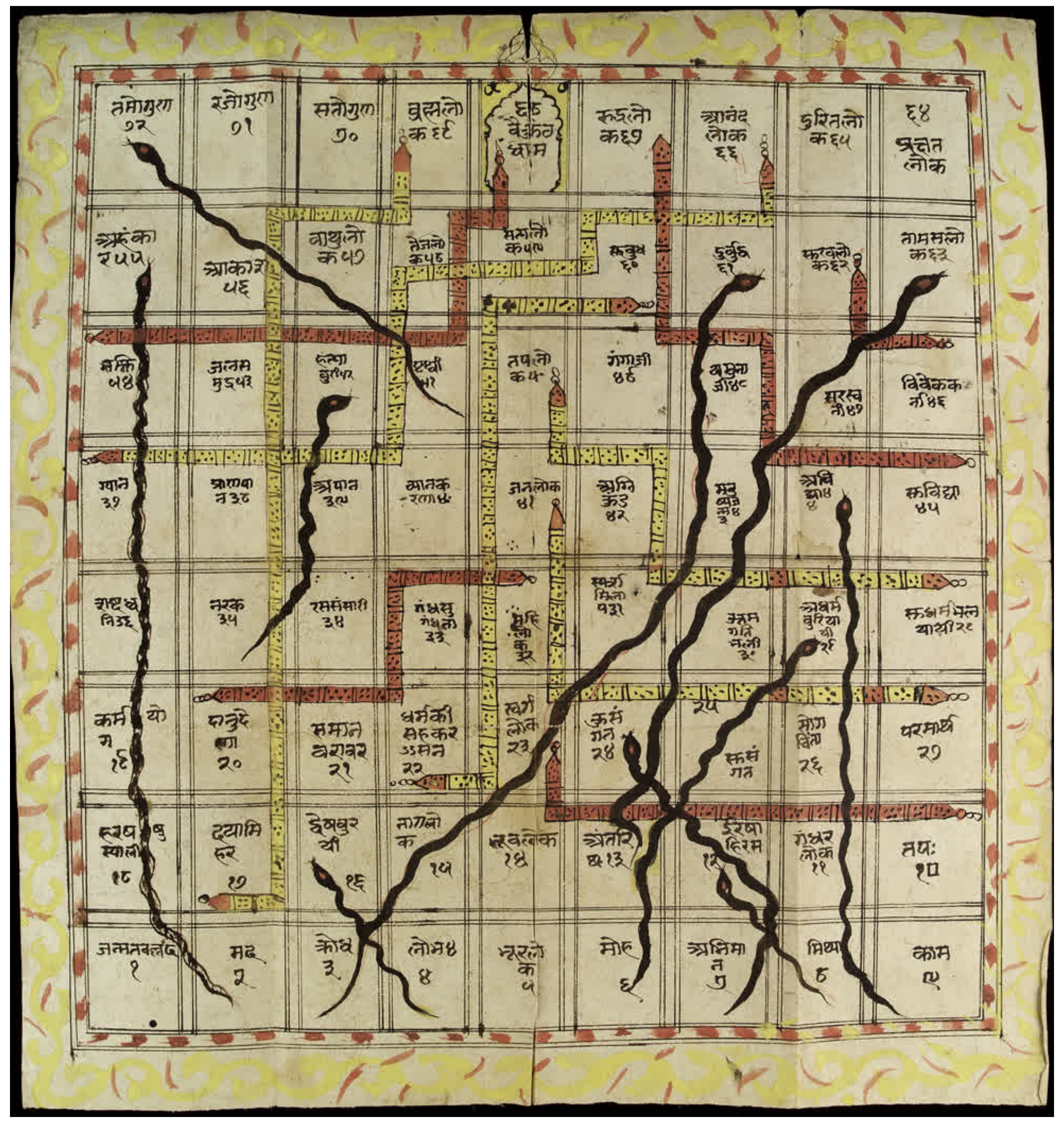




$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

*Wikipedia

My spooky dream*:
Ancient Indian game of
Snakes and Ladders



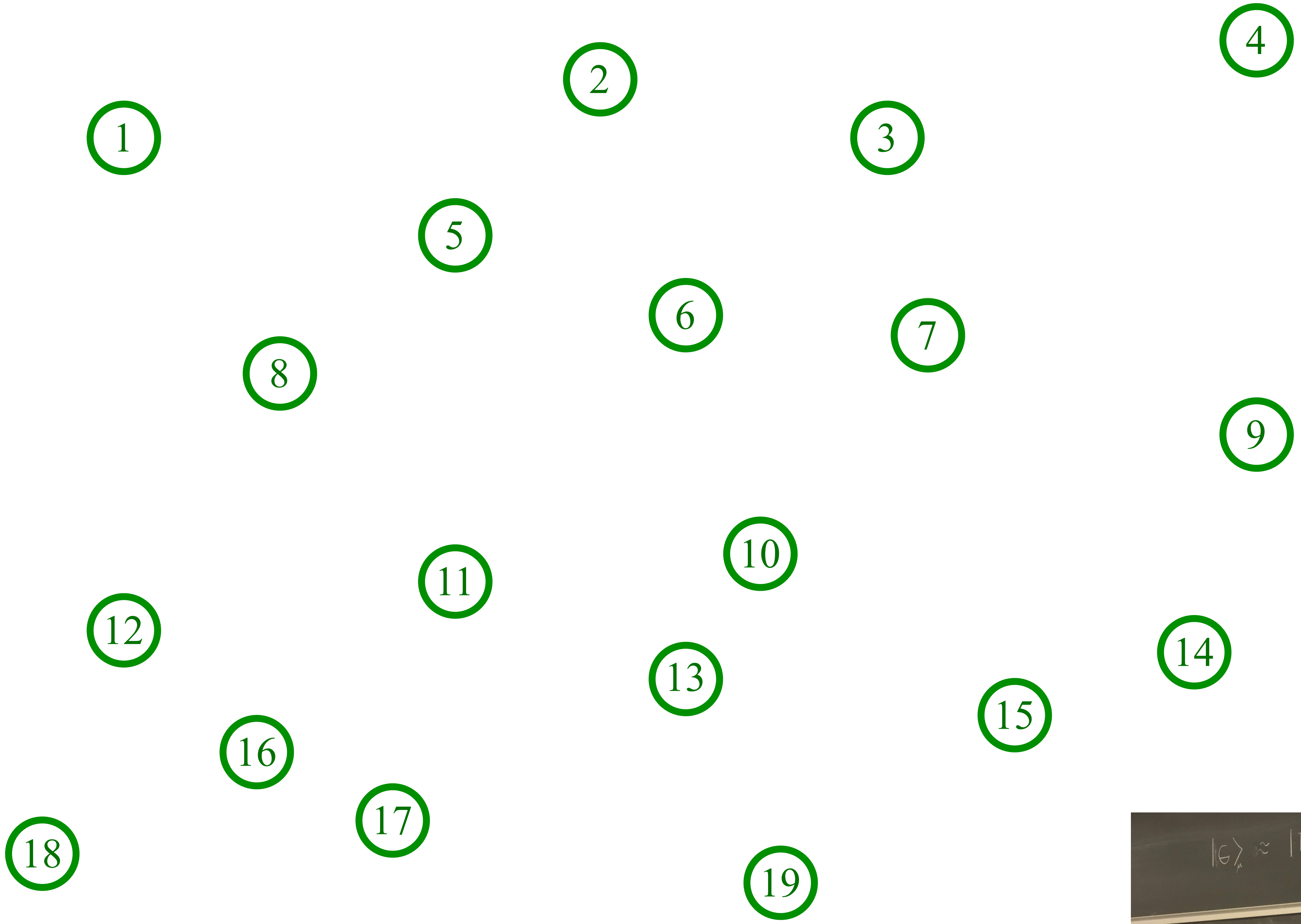
A solvable model of multi-particle entanglement which accounts for quantum interference between successive collisions:

leading to a metal with no particle-like excitations

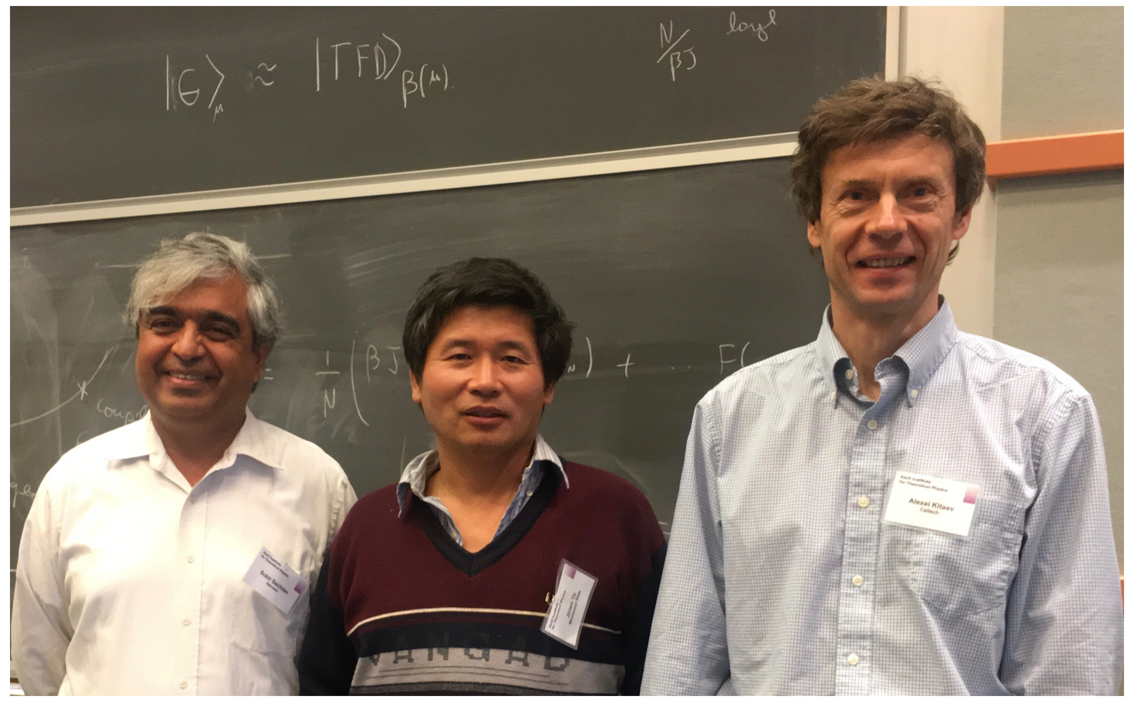
*Not true

The SYK model

Sachdev, Ye (1993); Kitaev (2015)

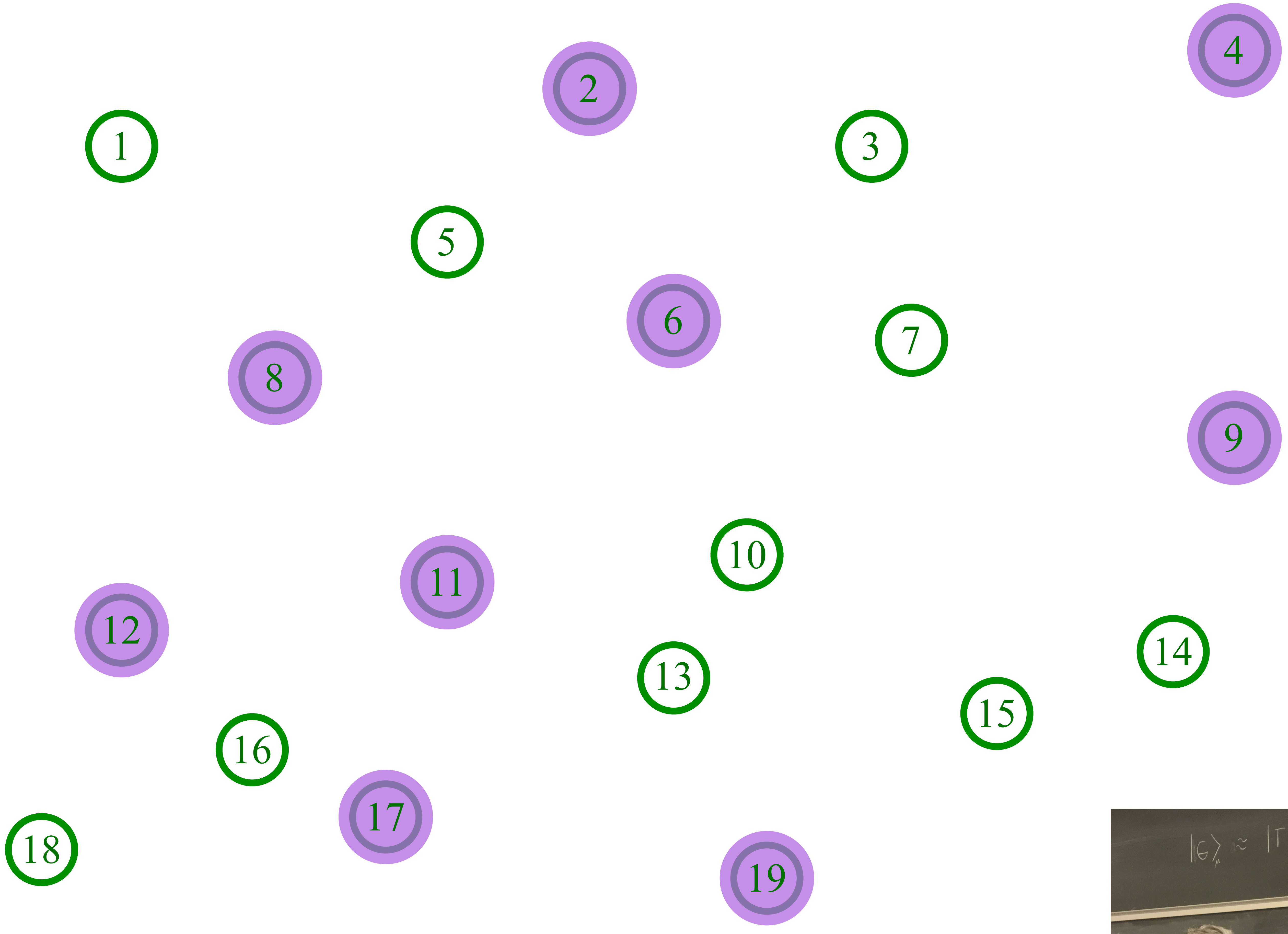


Pick a set of random positions

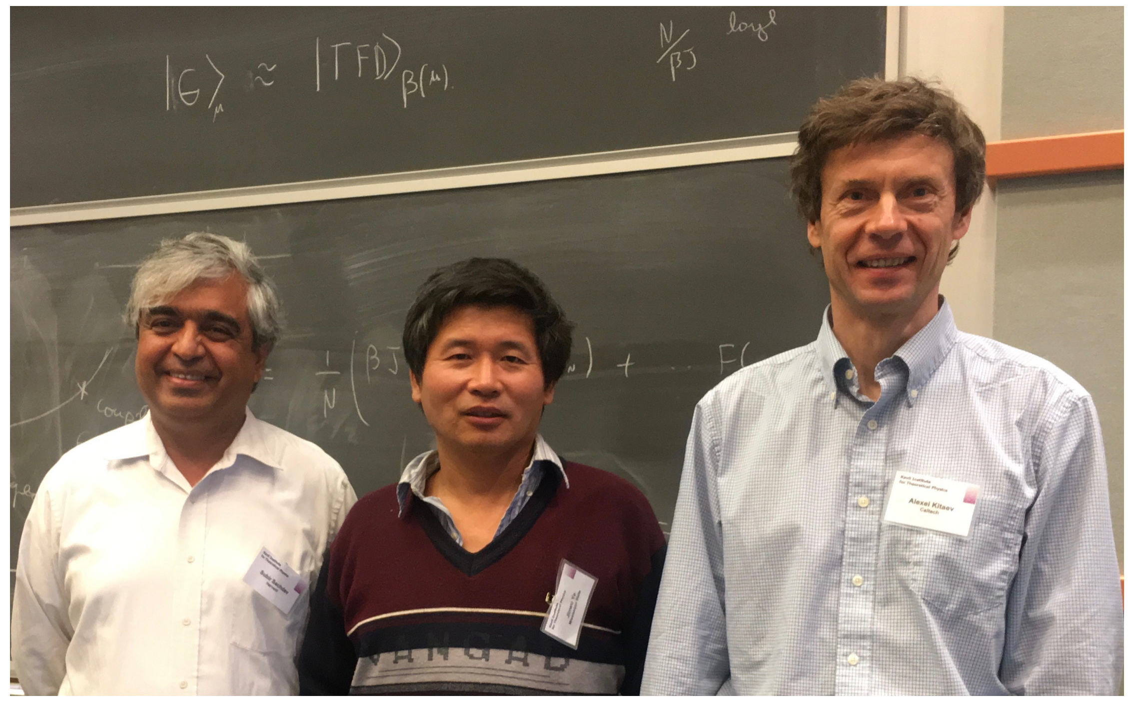


The SYK model

Sachdev, Ye (1993); Kitaev (2015)



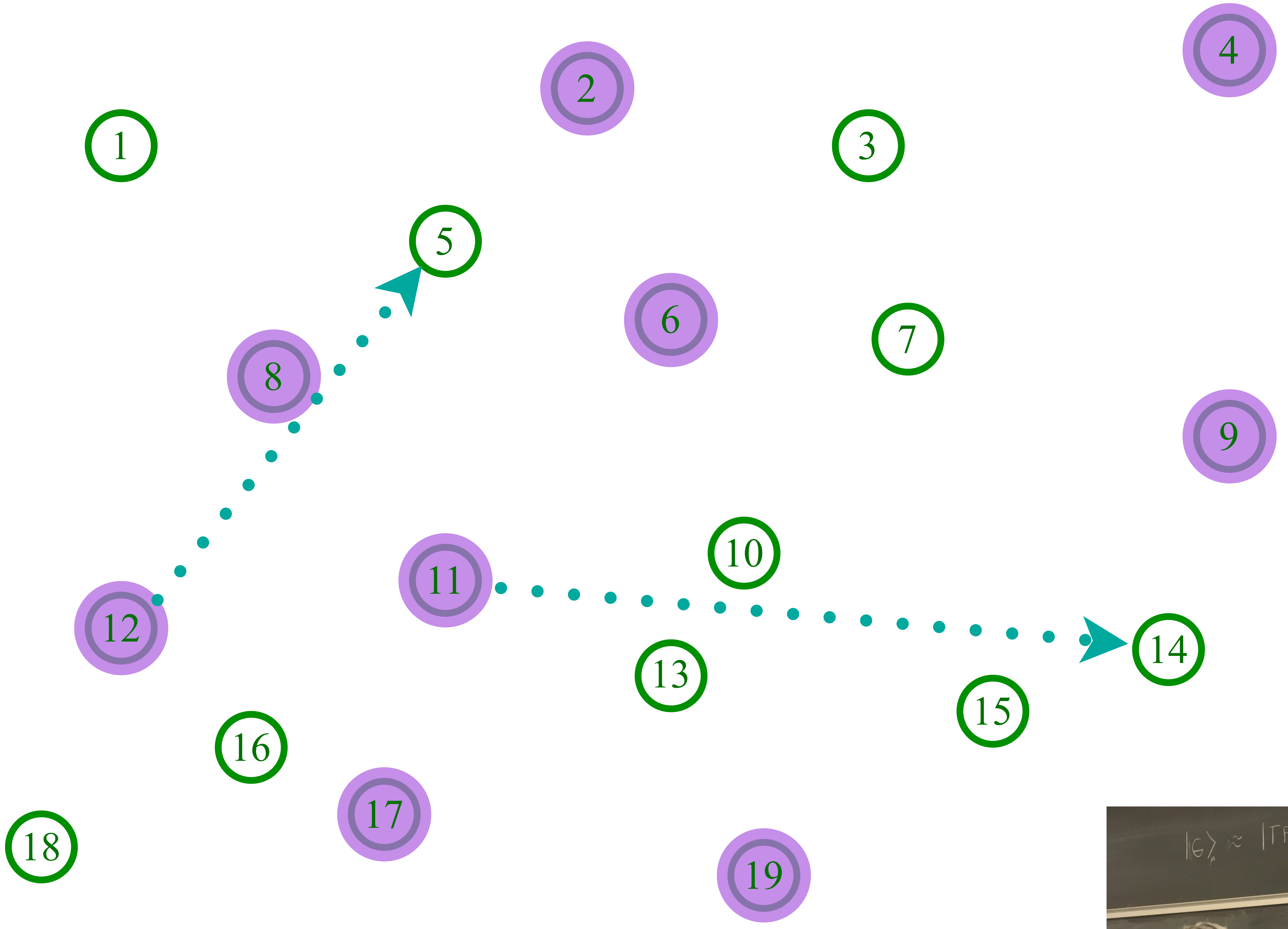
Place electrons randomly on some sites



The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



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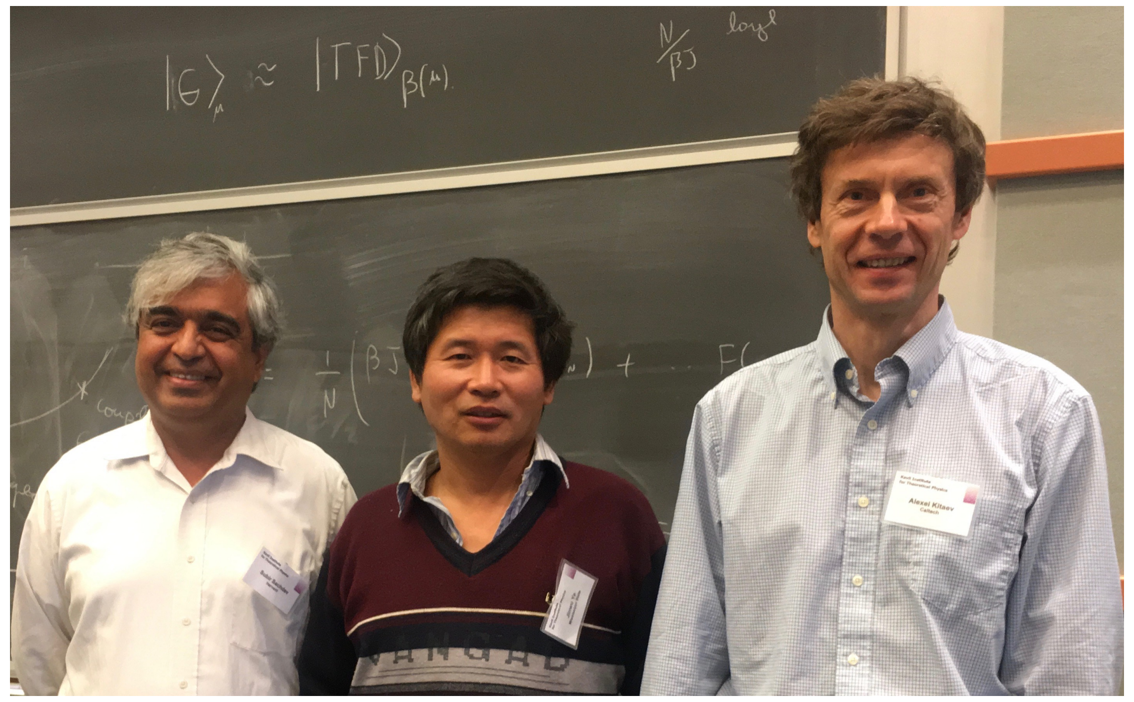
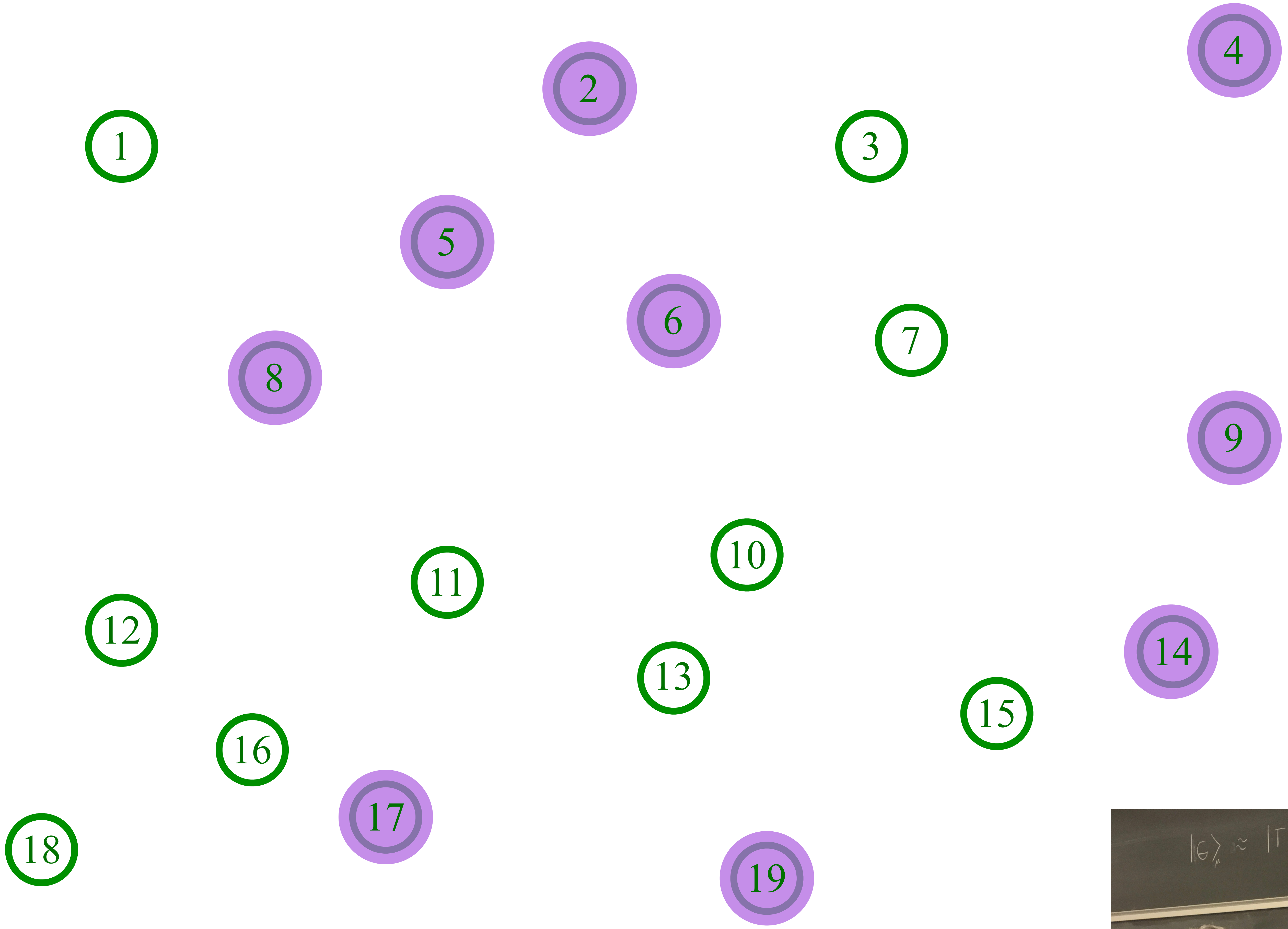


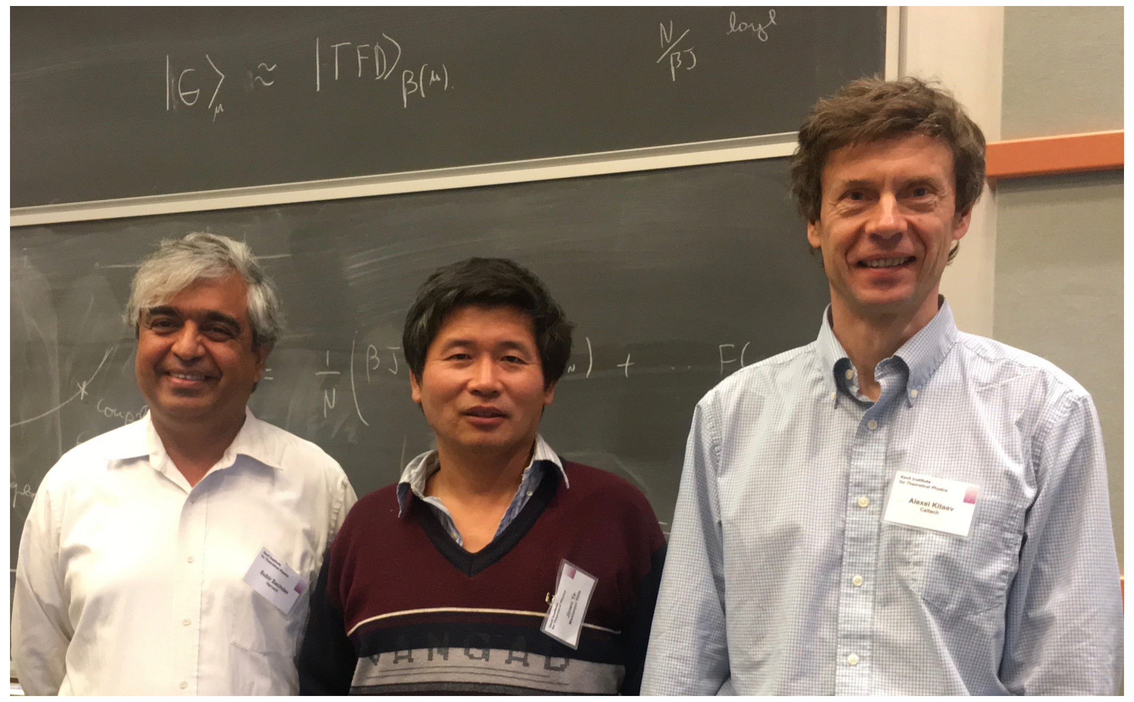
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



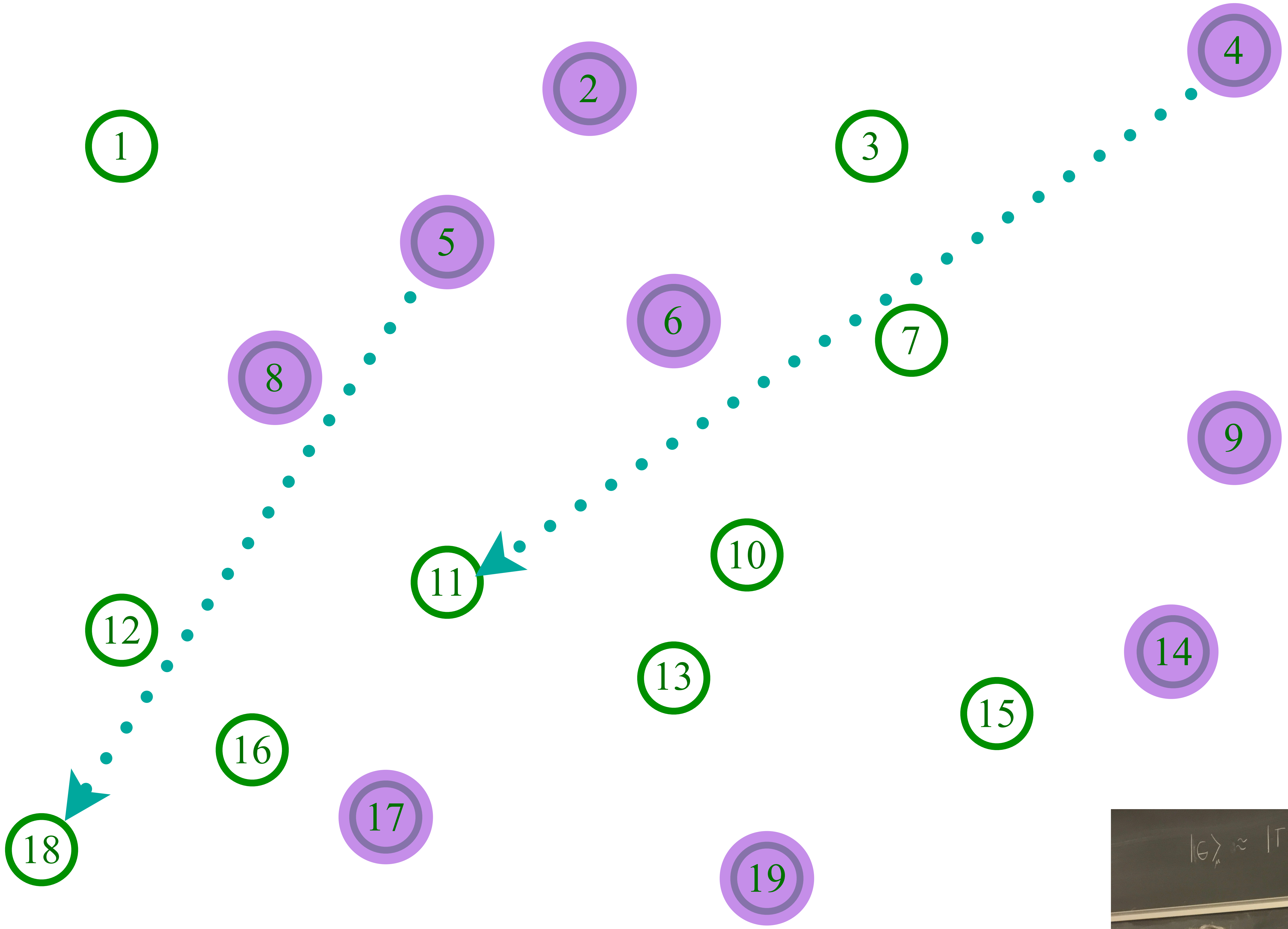
Entangle electrons pairwise randomly



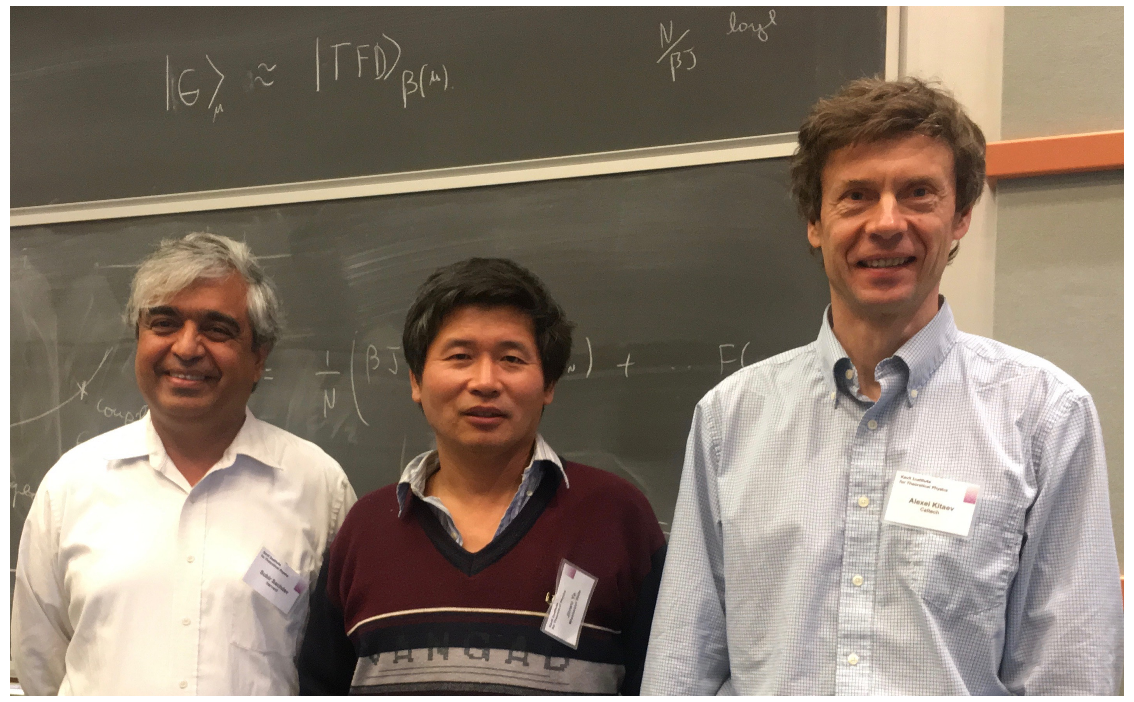
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



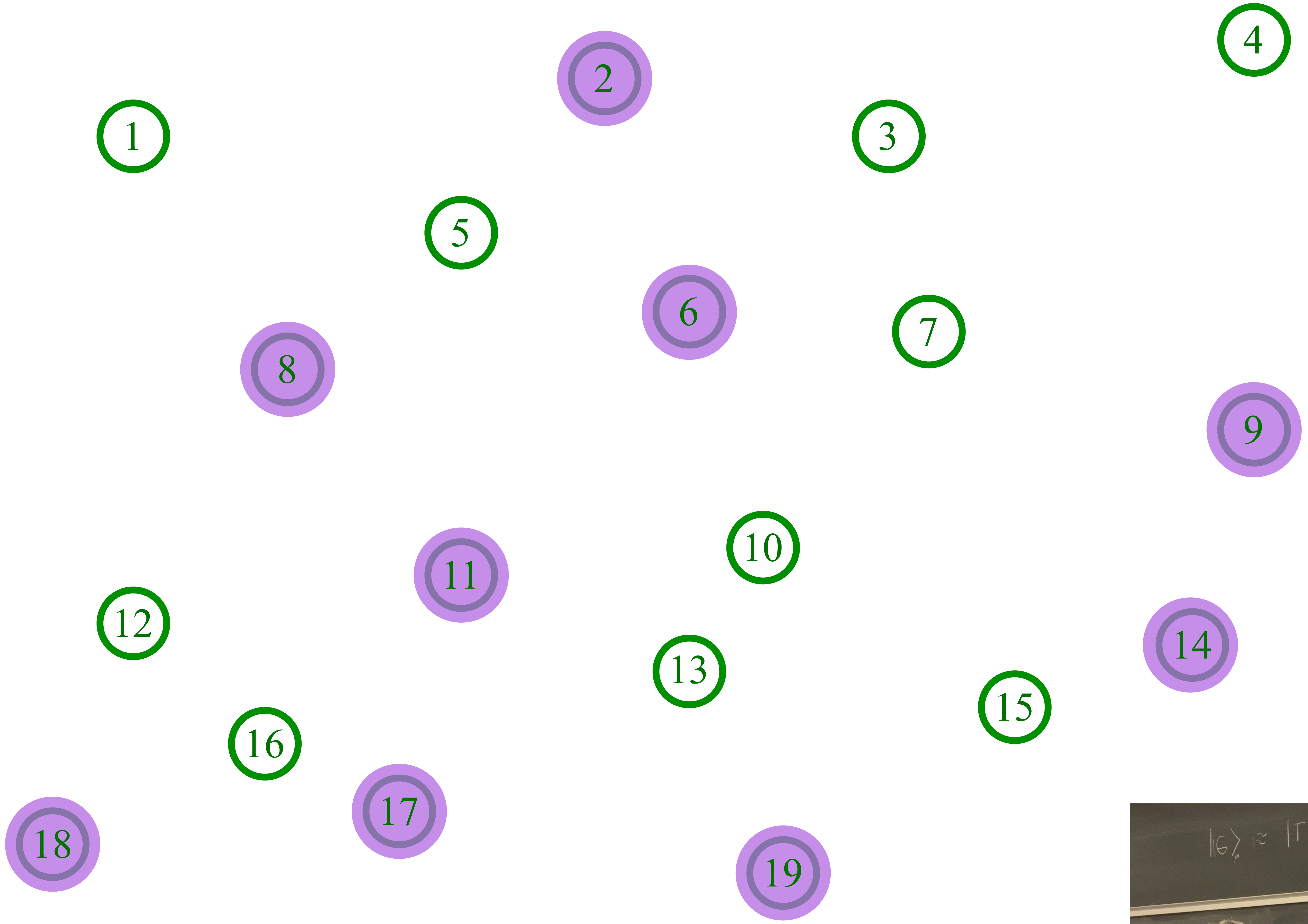
Entangle electrons pairwise randomly



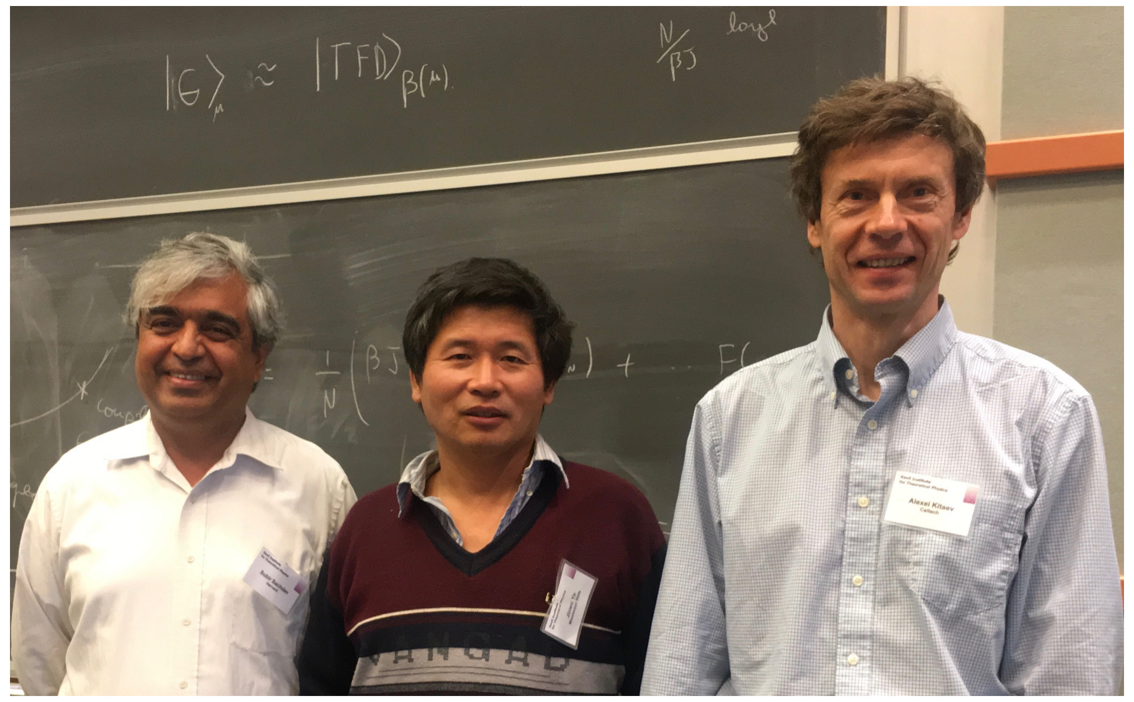
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{4,5;11,18}$



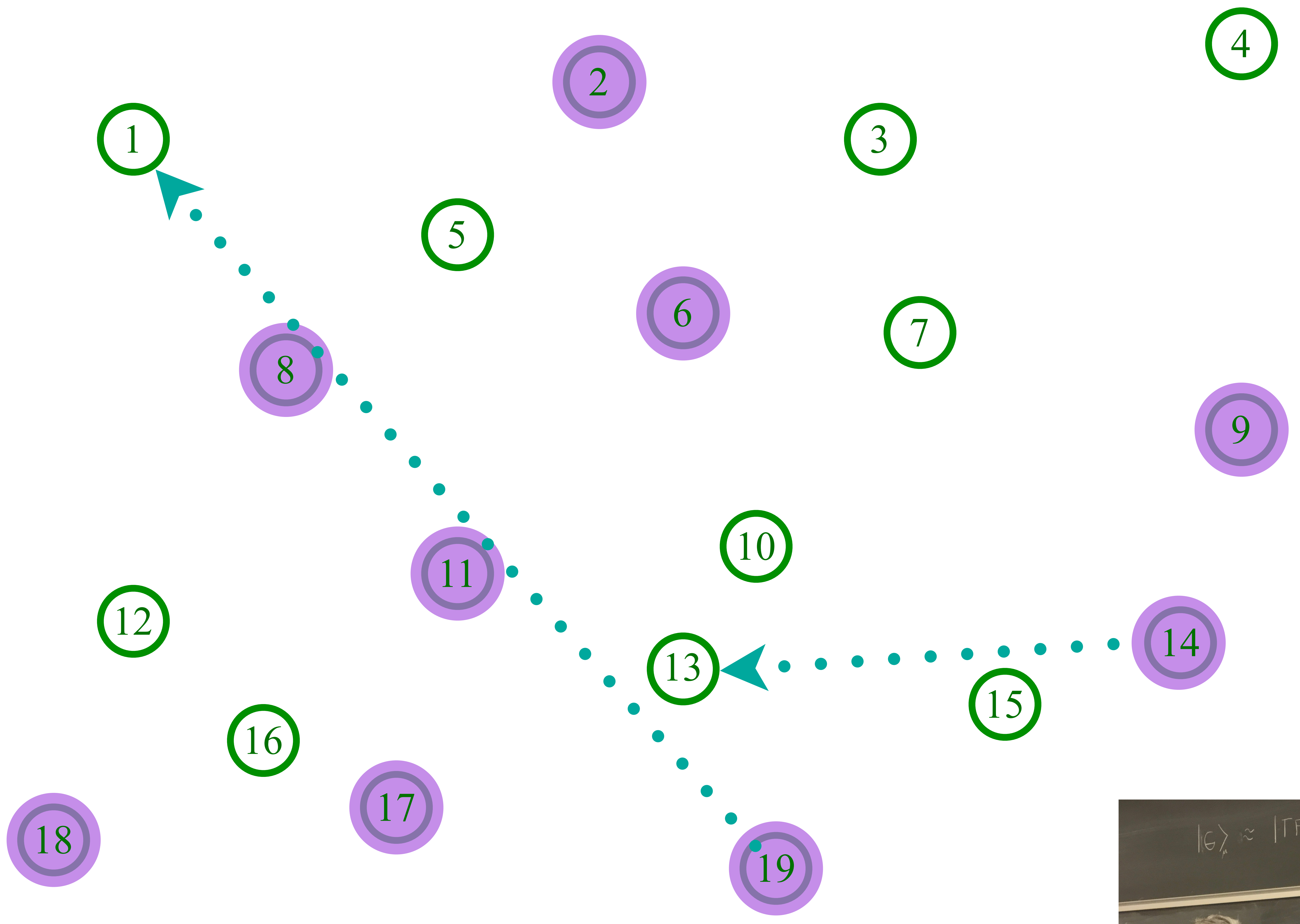
Entangle electrons pairwise randomly



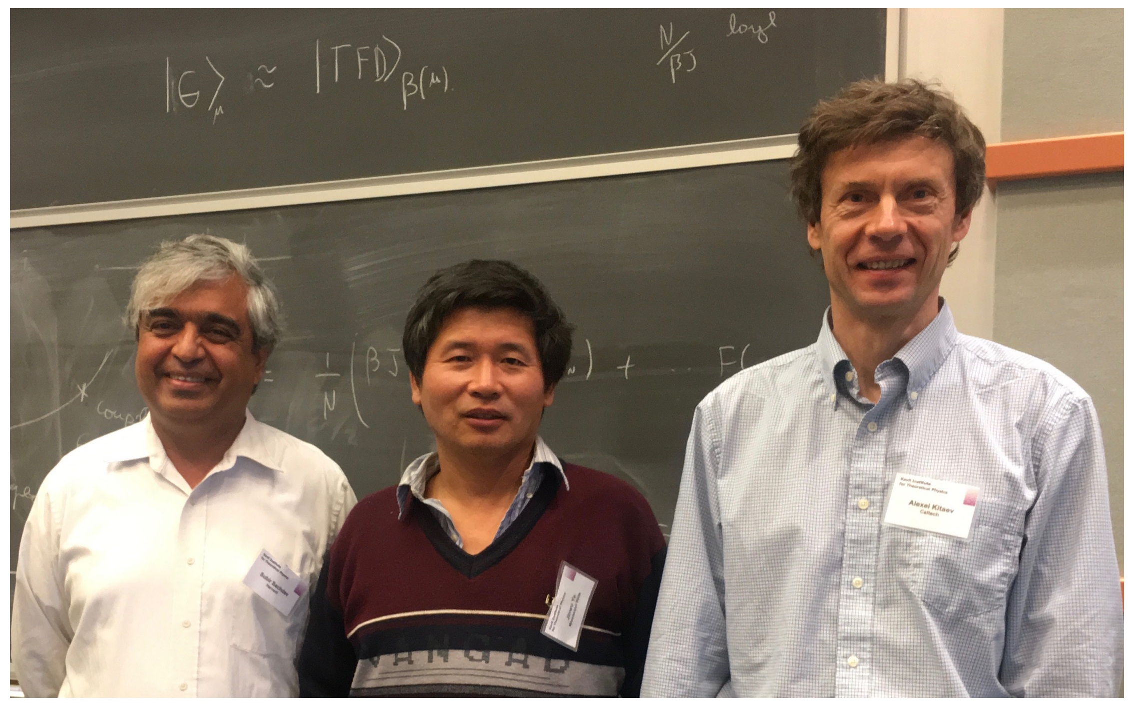
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



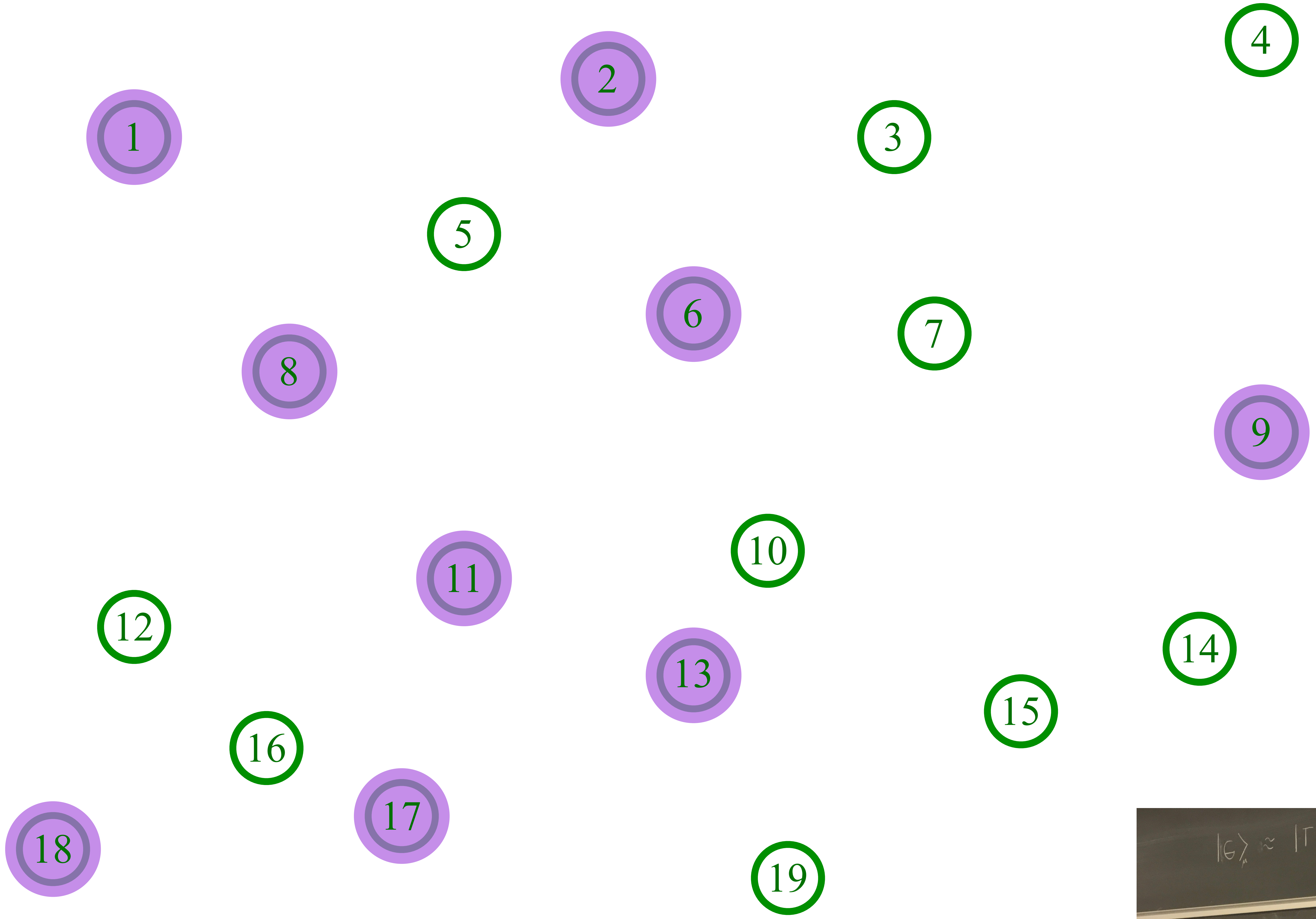
Entangle electrons pairwise randomly



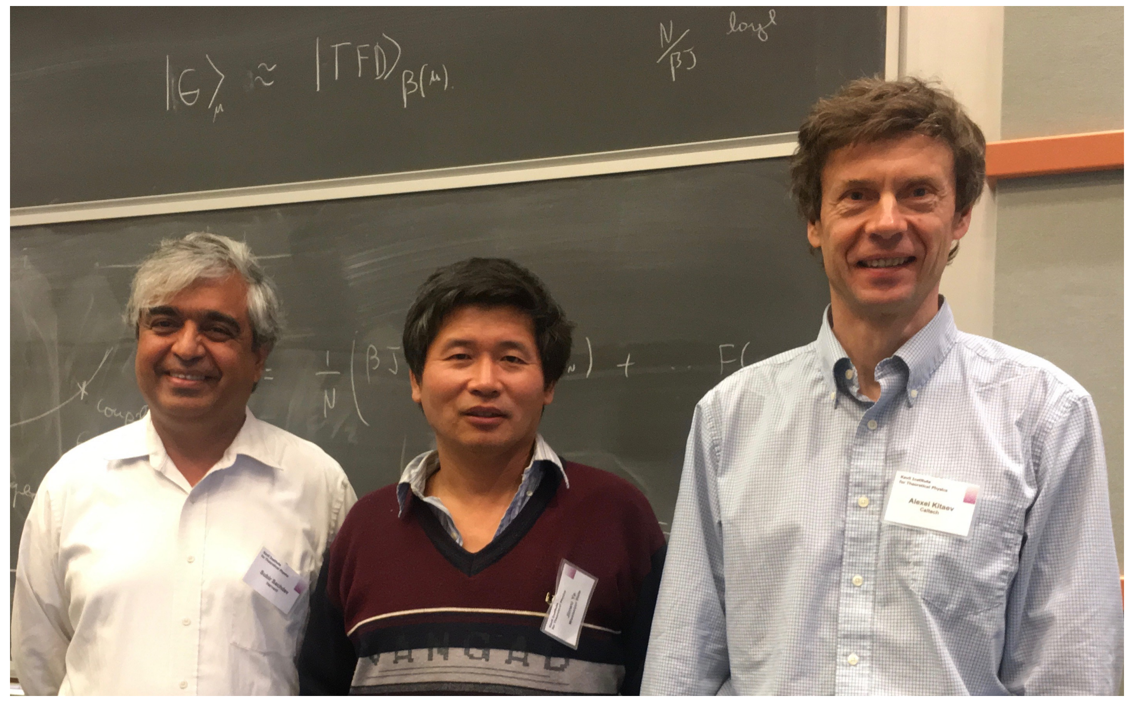
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



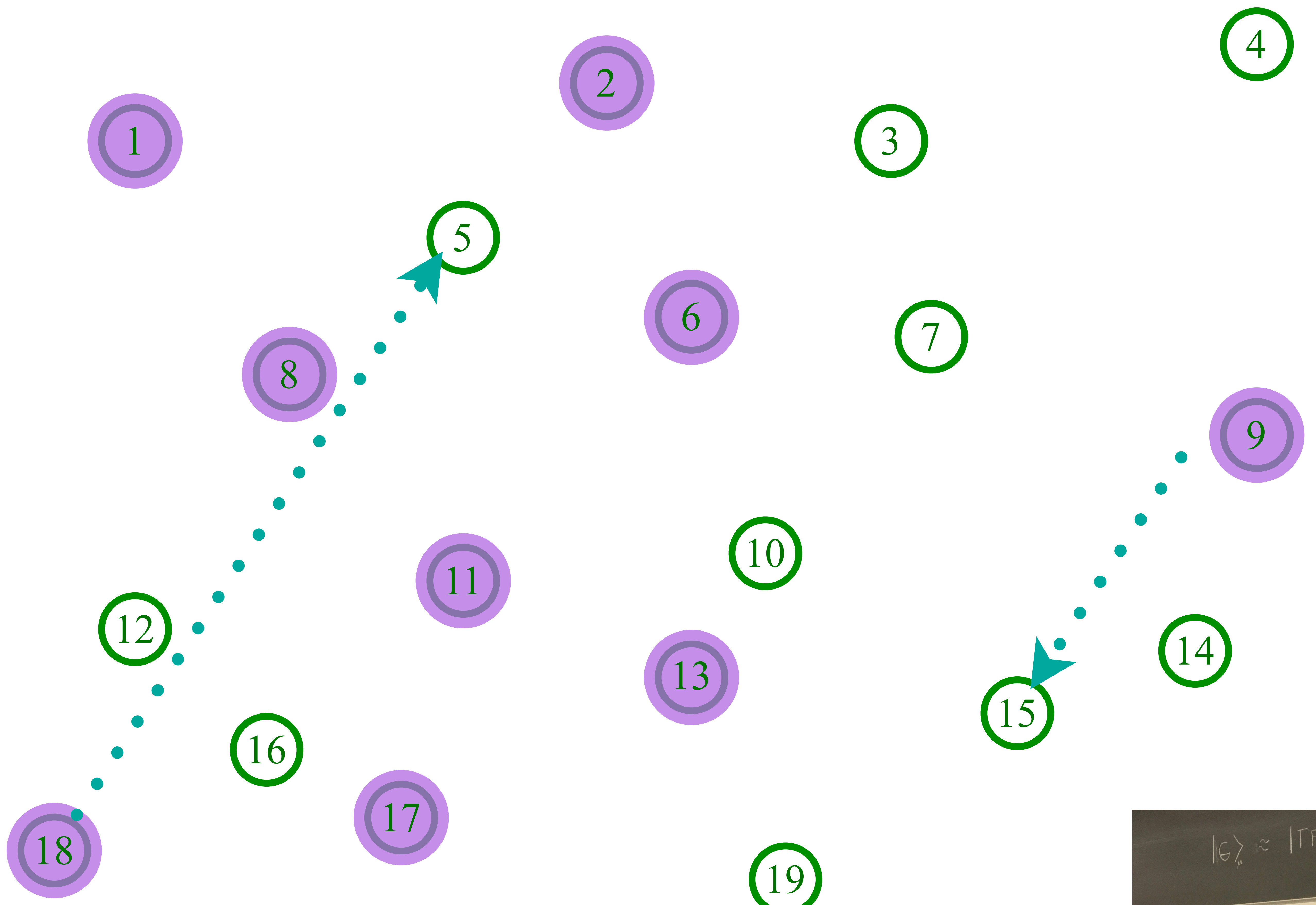
Entangle electrons pairwise randomly



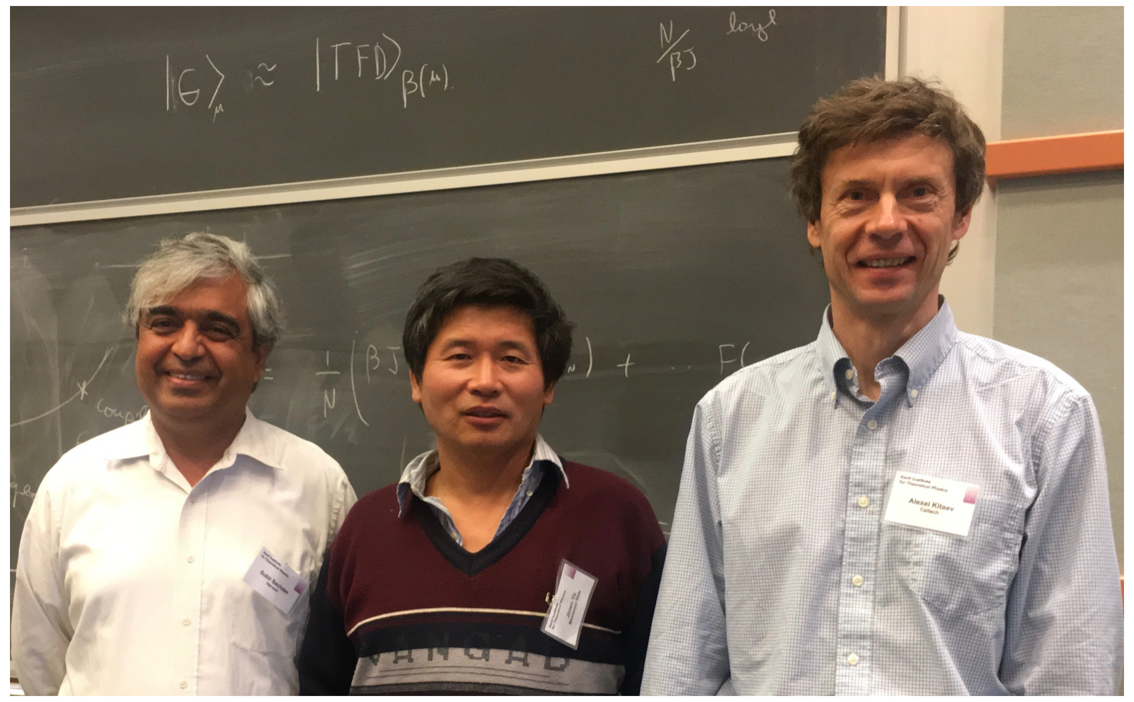
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



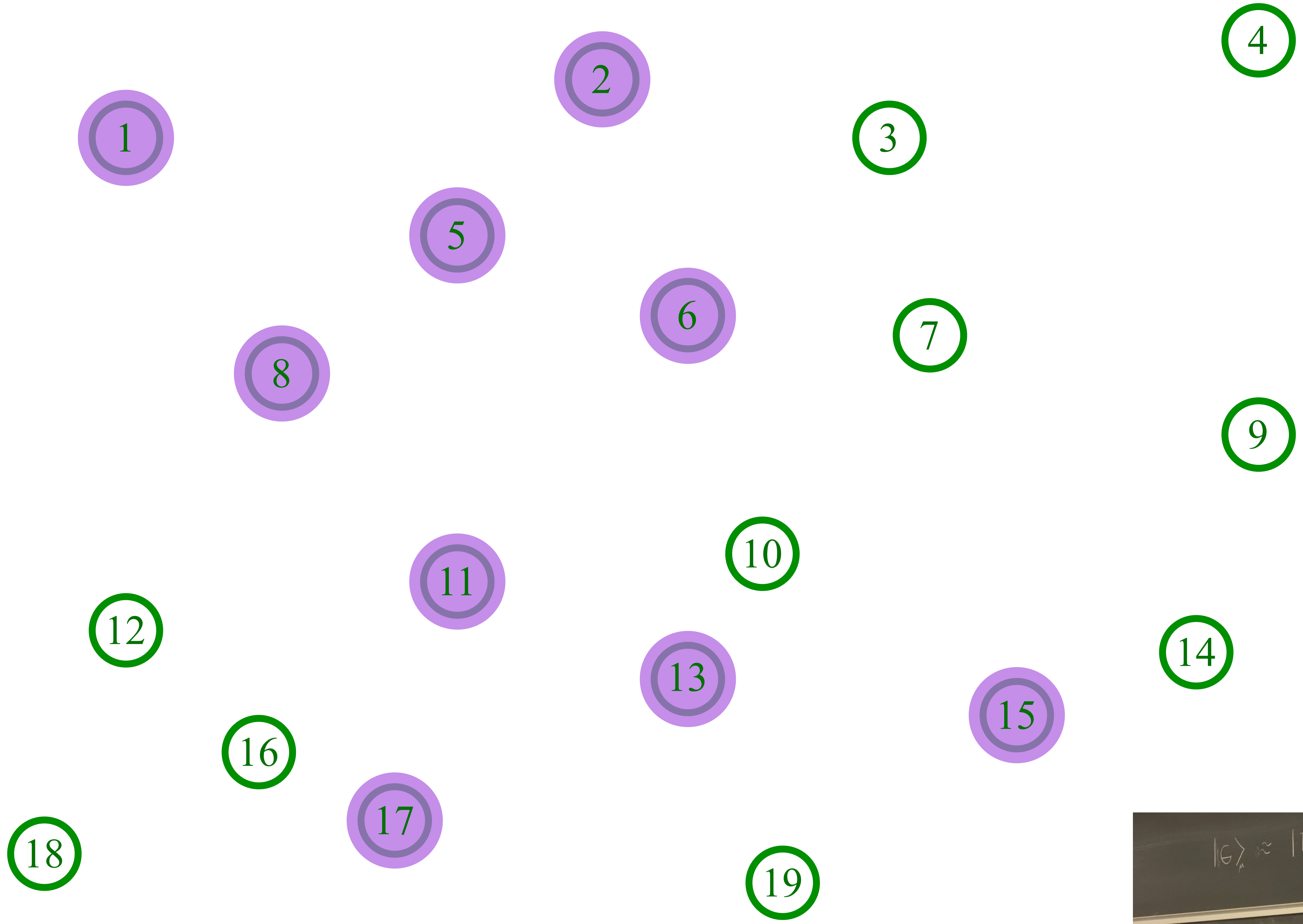
Entangle electrons pairwise randomly



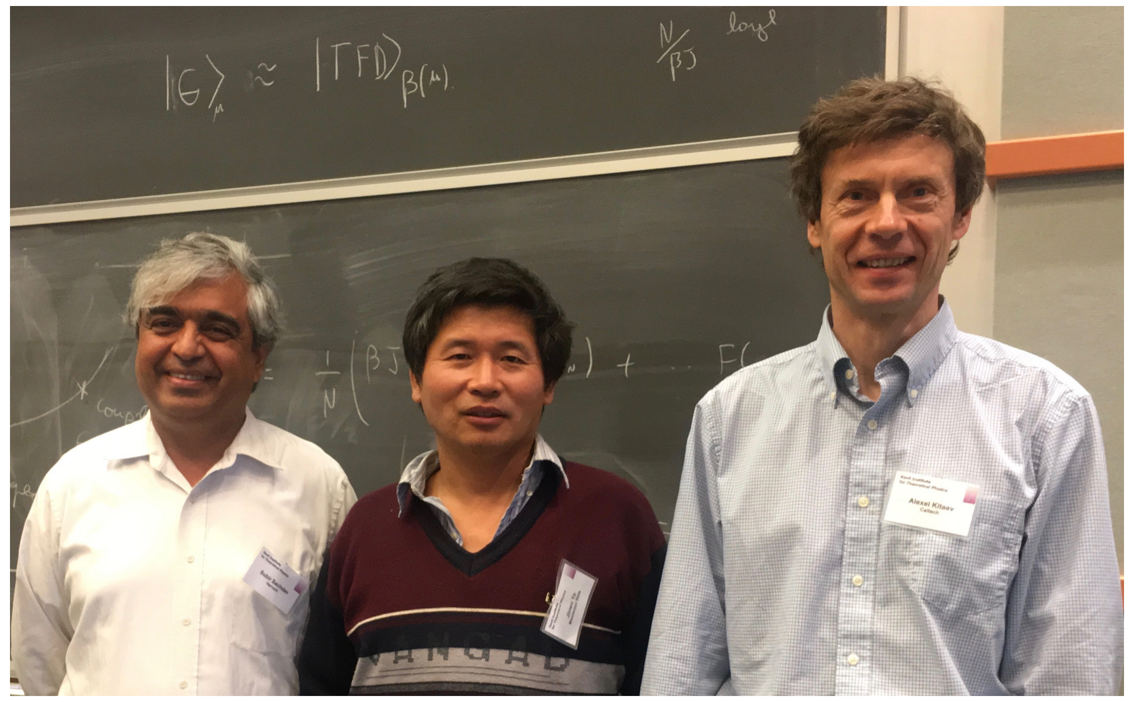
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



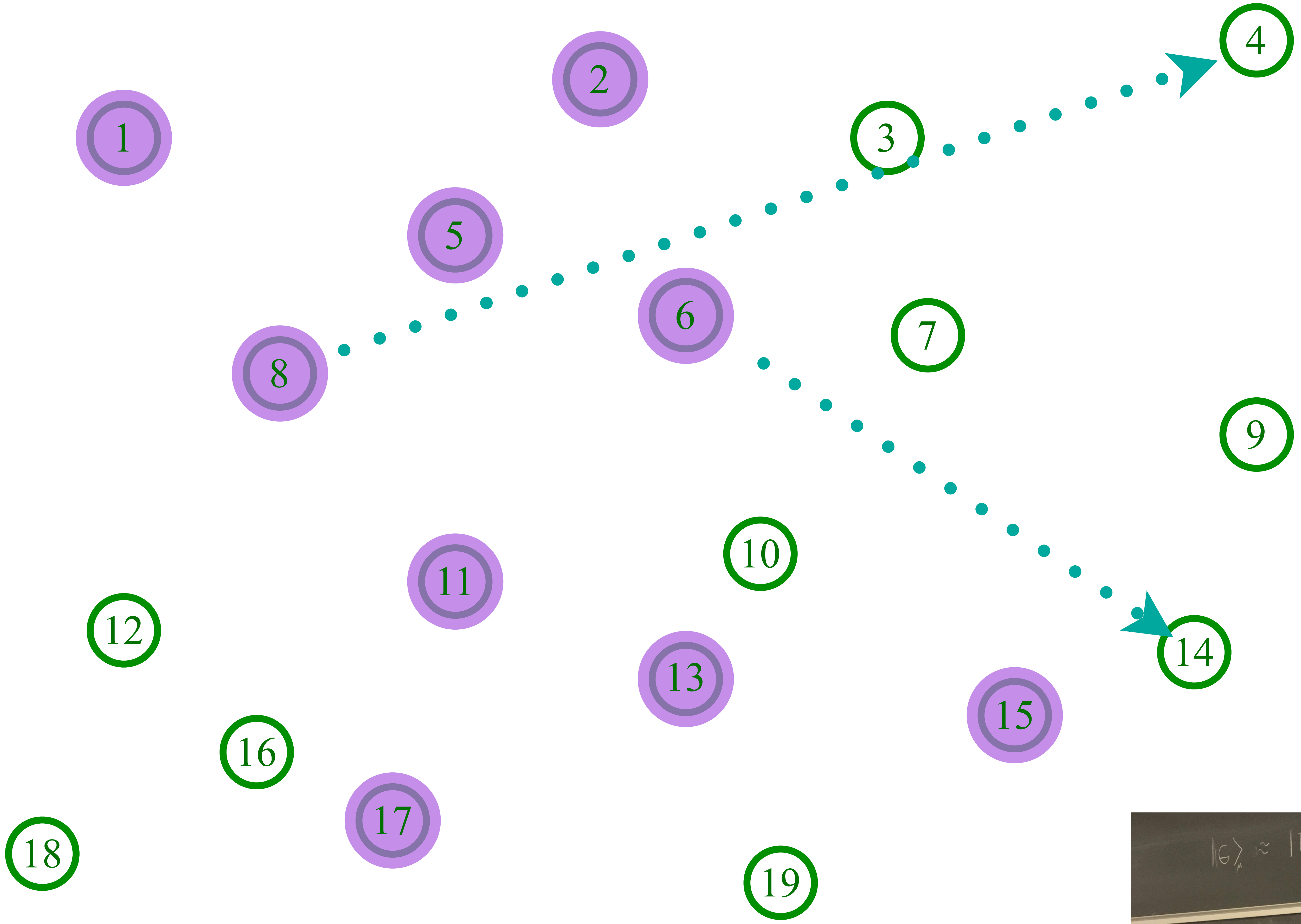
Entangle electrons pairwise randomly

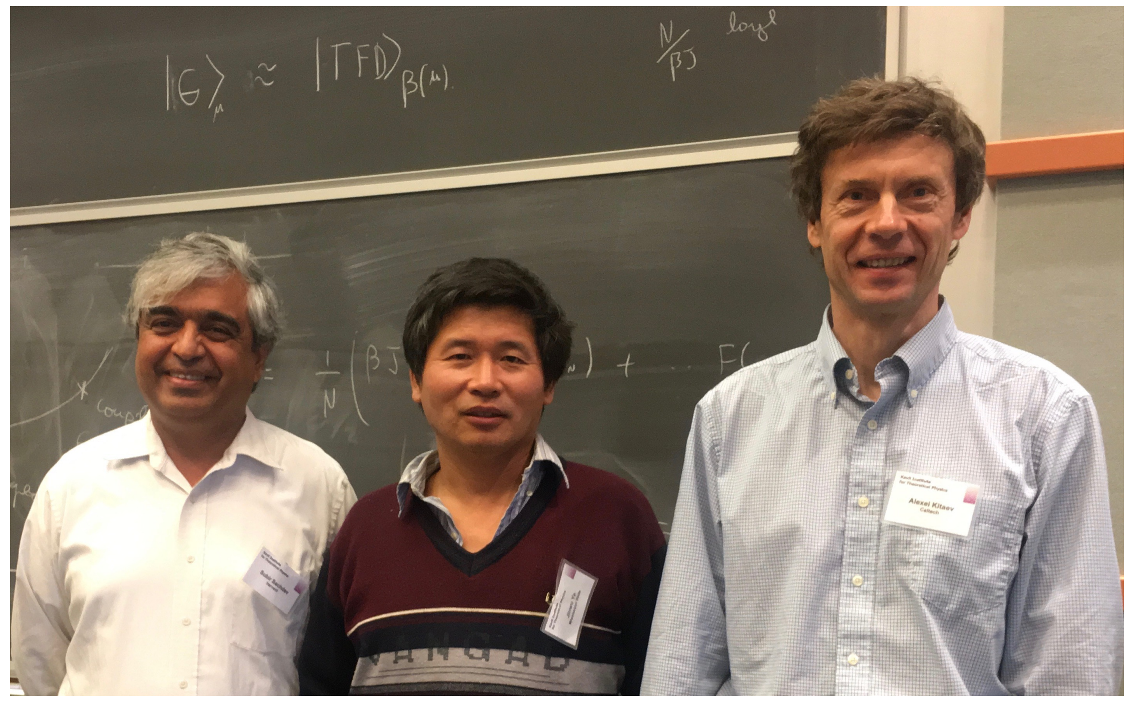


The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



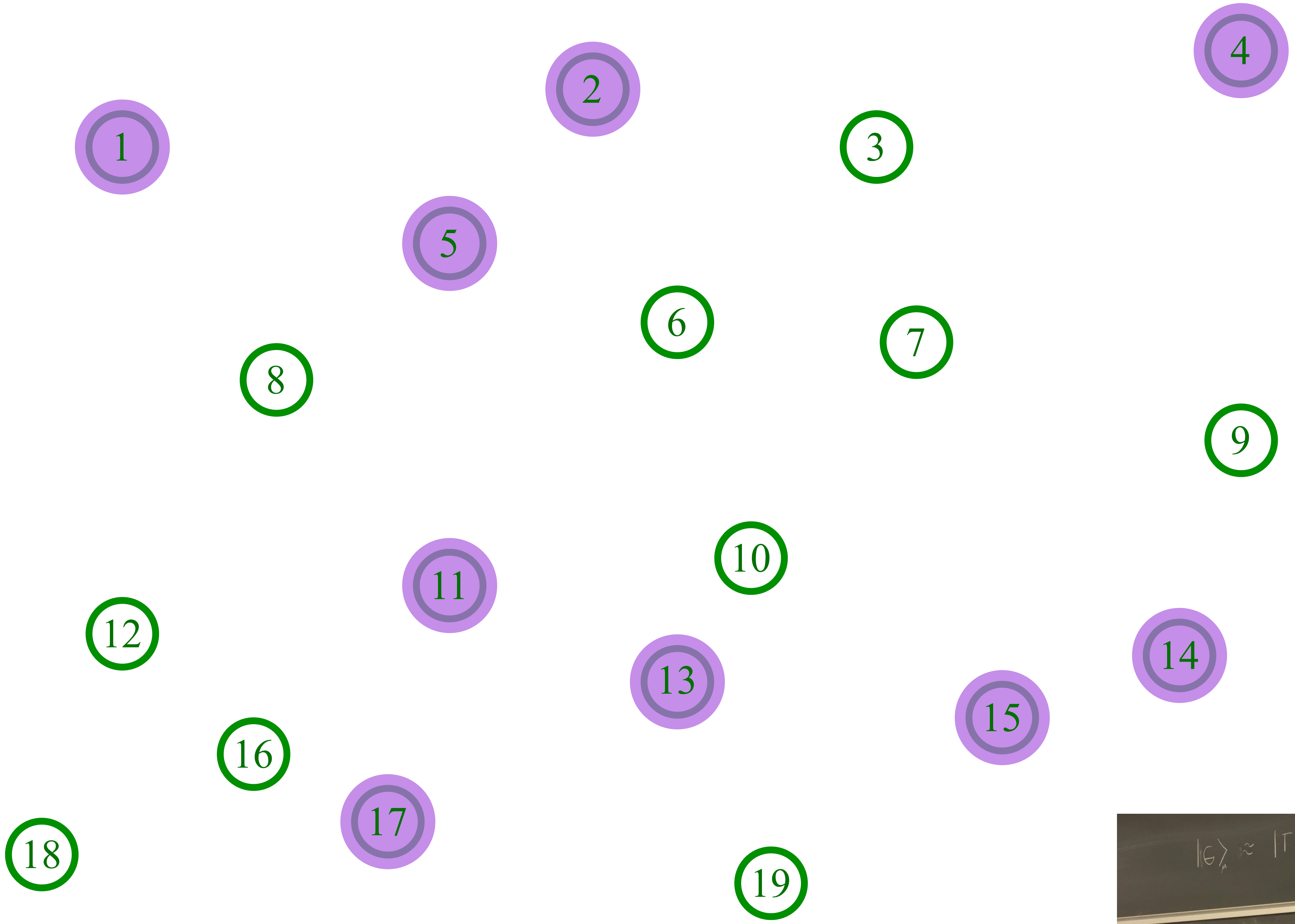


Entangle electrons pairwise randomly

The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



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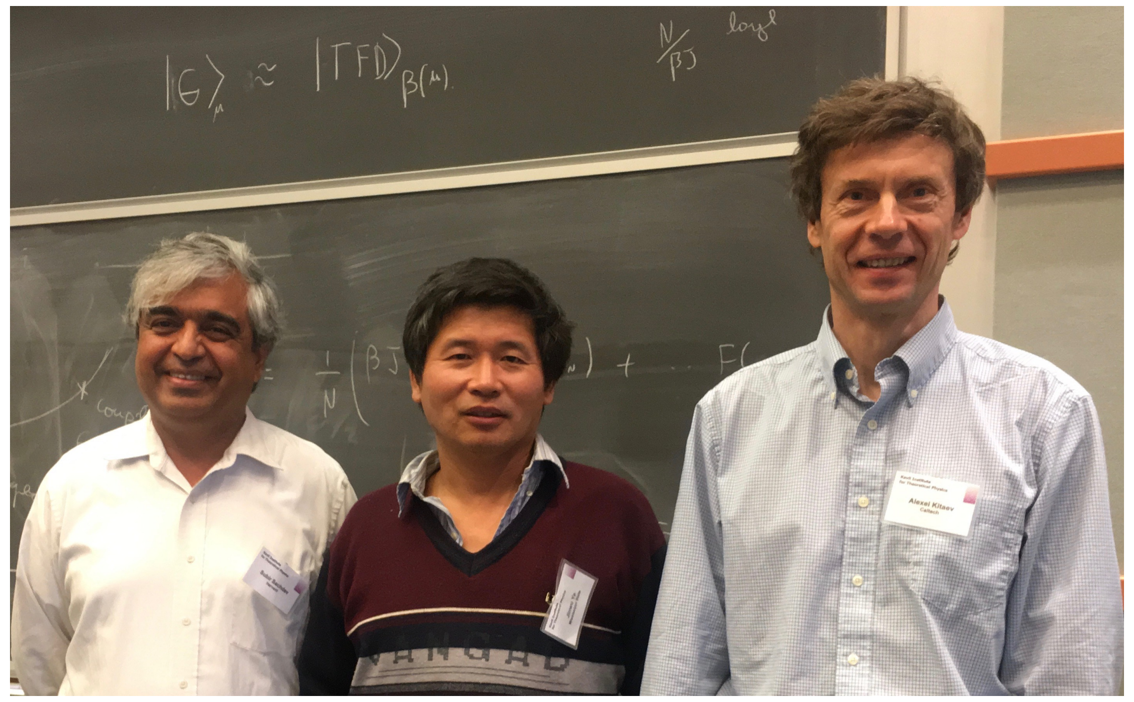


The SYK model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

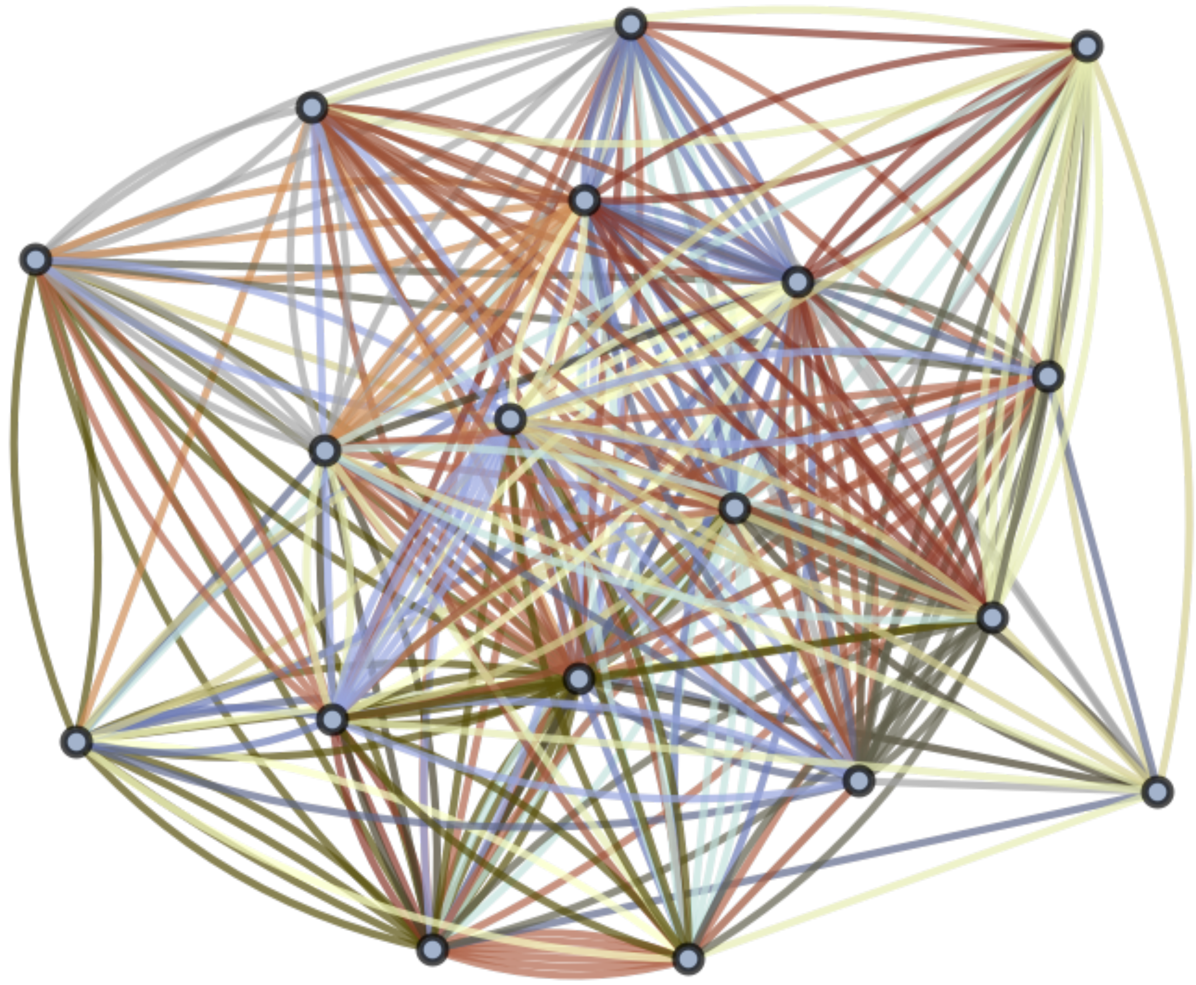
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

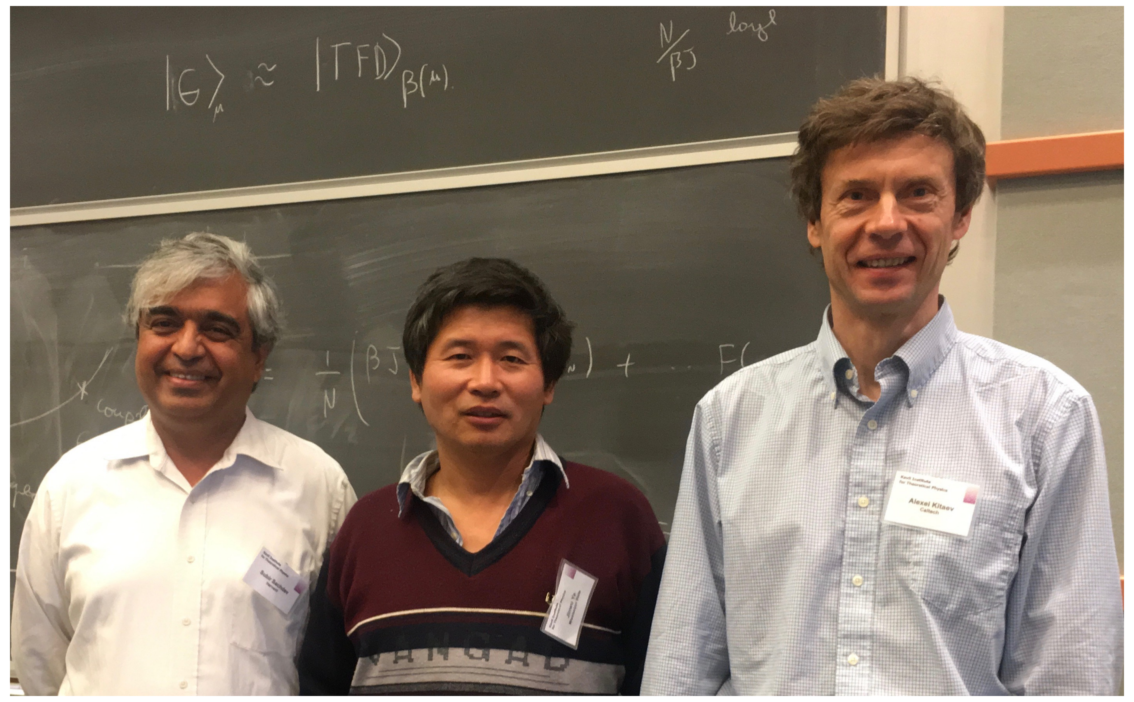
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha \, ; \quad [\mathcal{H}, \mathcal{Q}] = 0 \, ; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

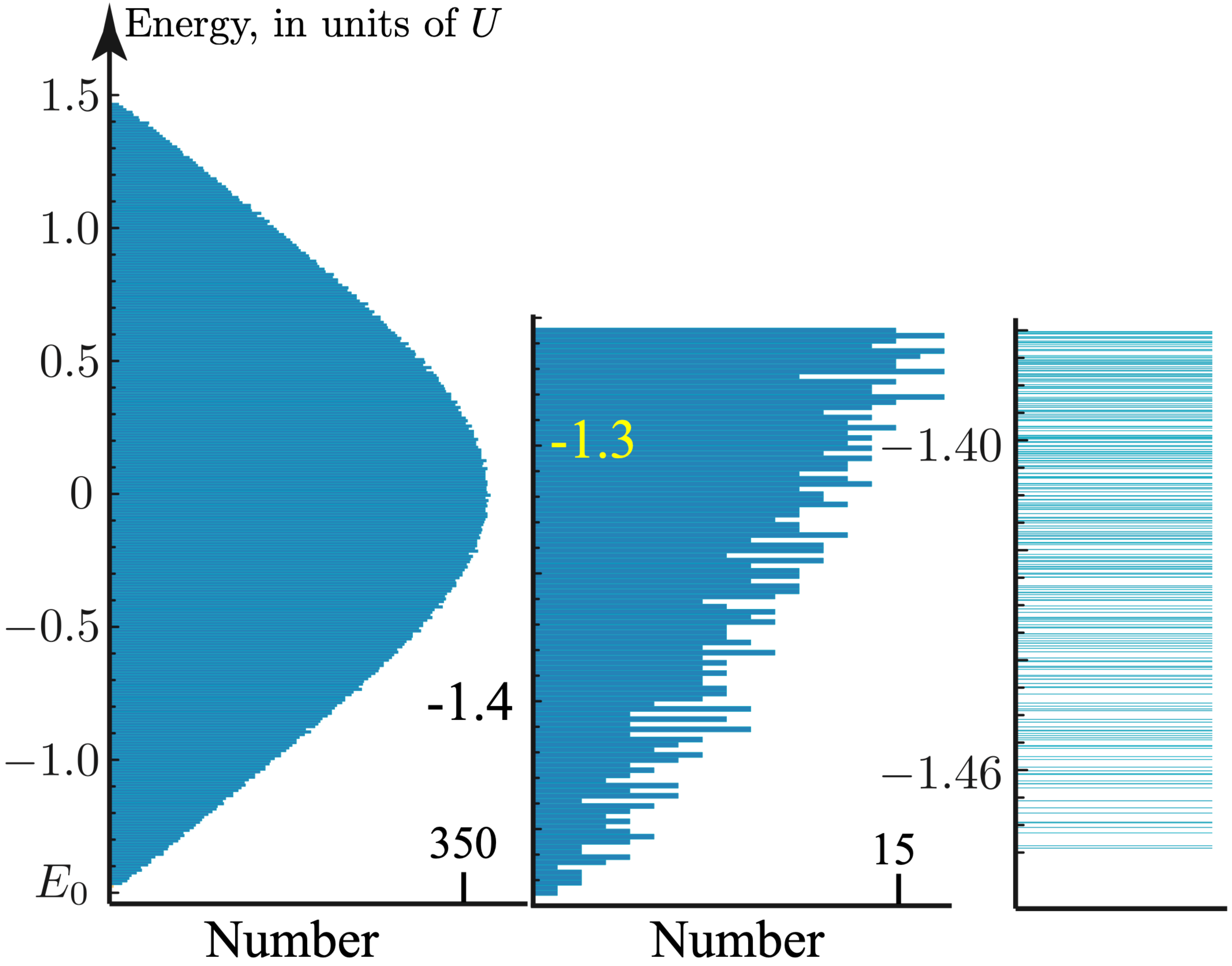
A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)





Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



Complex SYK model

(Numerics: G. Tarnopolsky)

Many-body density of states

Boltzmann

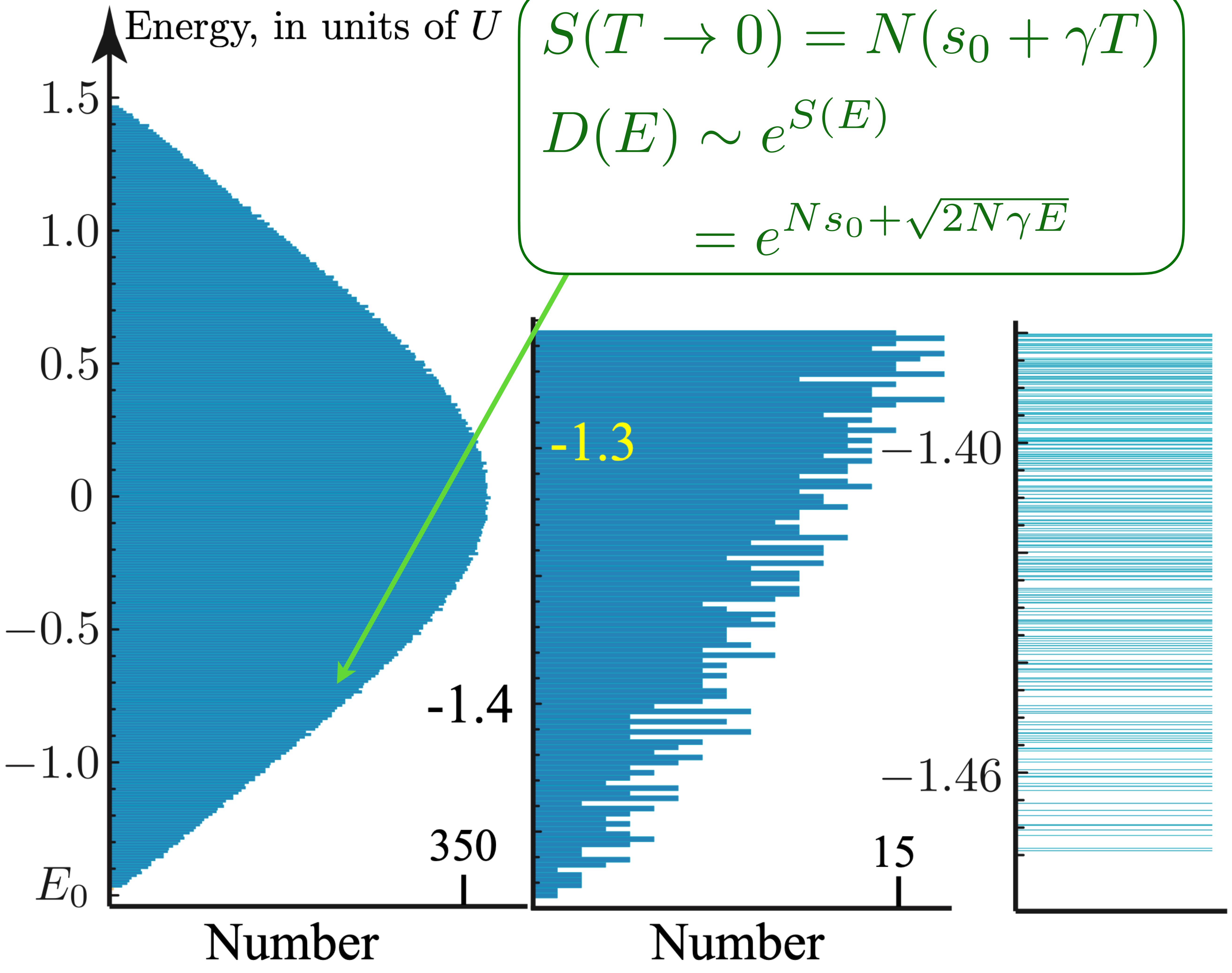
$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

$$S(T \to 0) = N(s_0 + \gamma T)$$
$$D(E) \sim e^{S(E)}$$
$$= e^{N s_0 + \sqrt{2N\gamma E}}$$

At $\mathcal{Q} = 1/2$

$$s_0 = \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4}$$
$$= 0.46484769917\ldots.$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)



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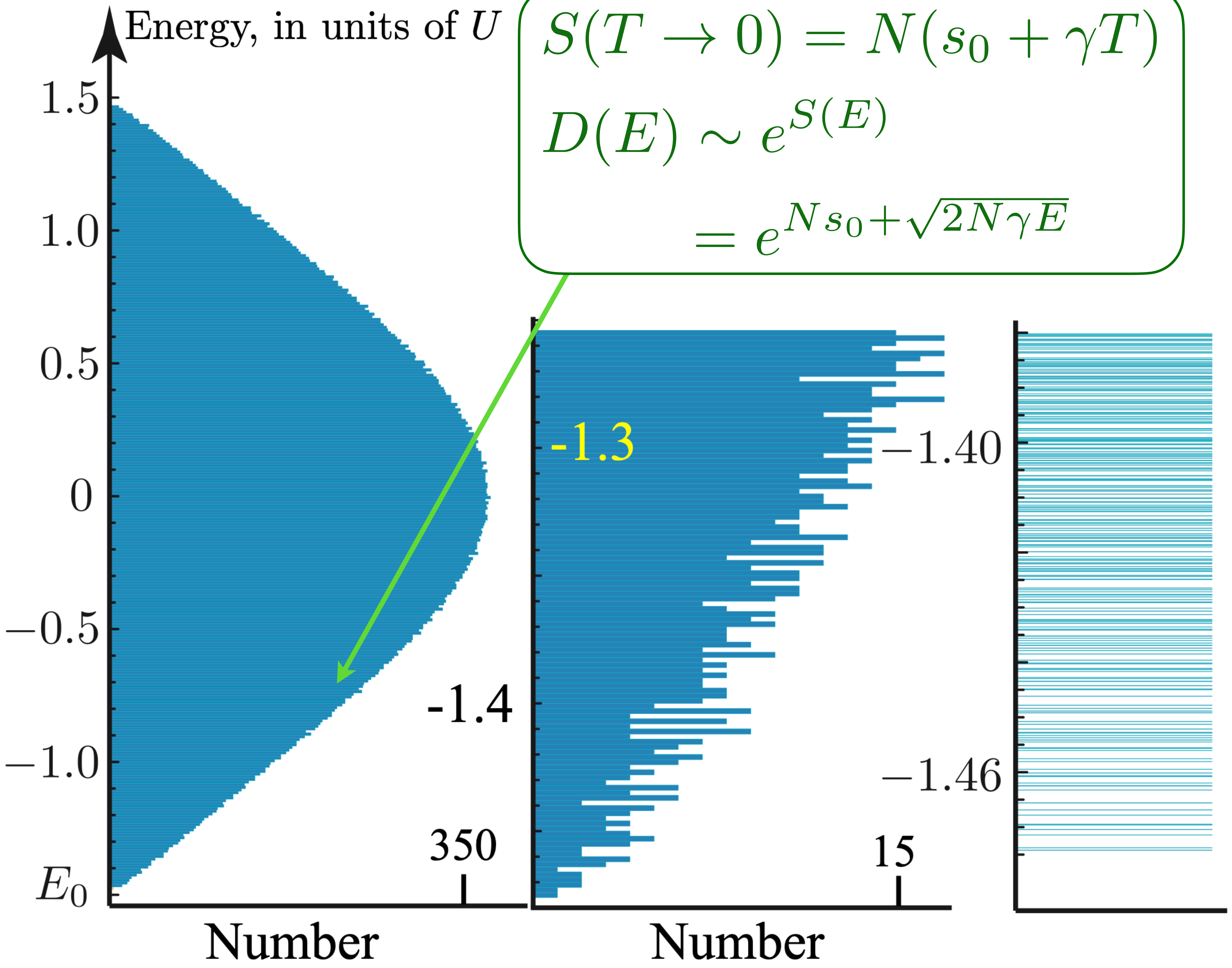

Energy level spacing $\sim e^{-N s_0}$!

No quasiparticle decomposition: wavefunctions change chaotically from one state to the next.

Complex SYK model

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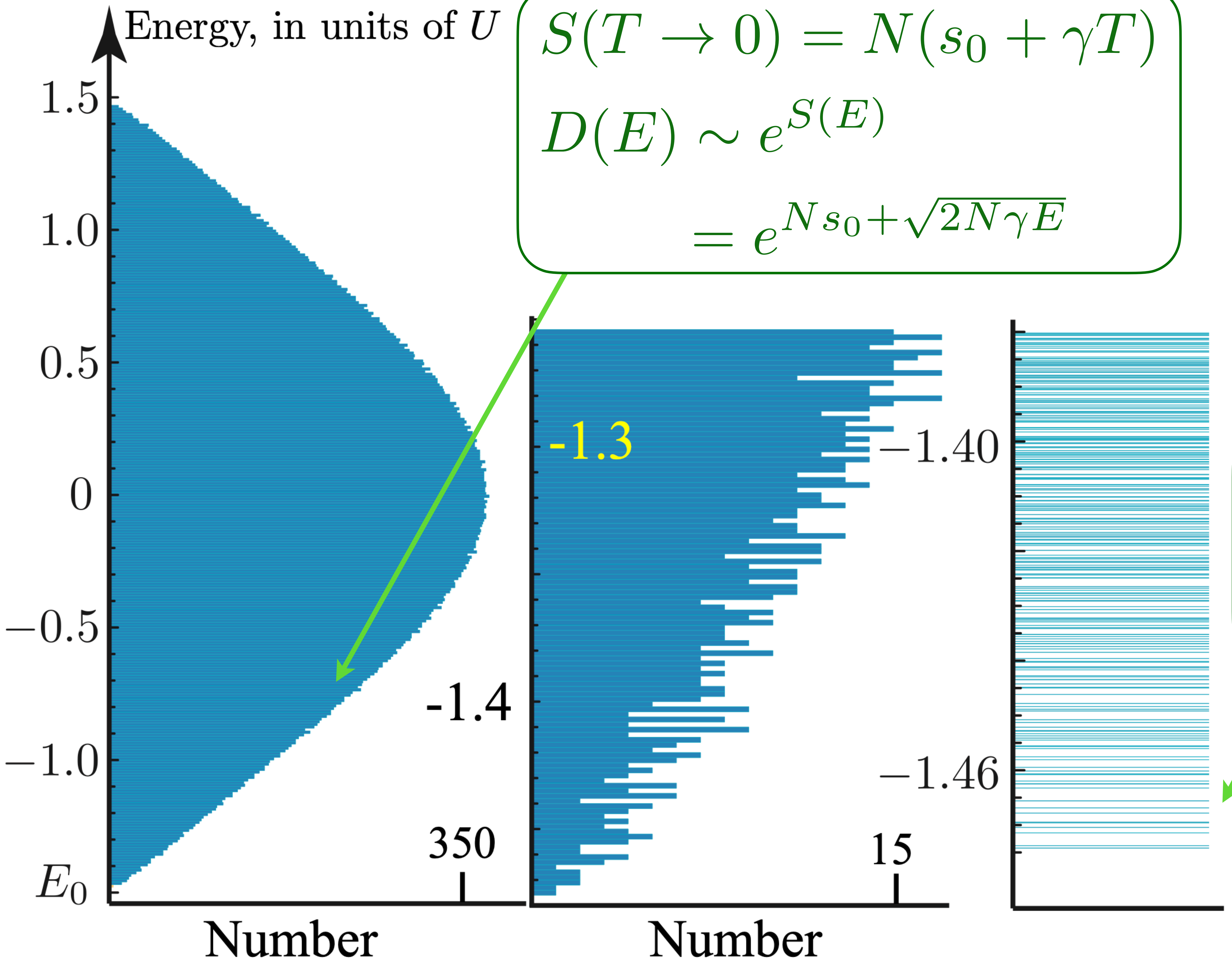

$$D(E) \sim N^{-1} \exp(N s_0) \sinh(\sqrt{2N\gamma E})$$

Complex SYK model

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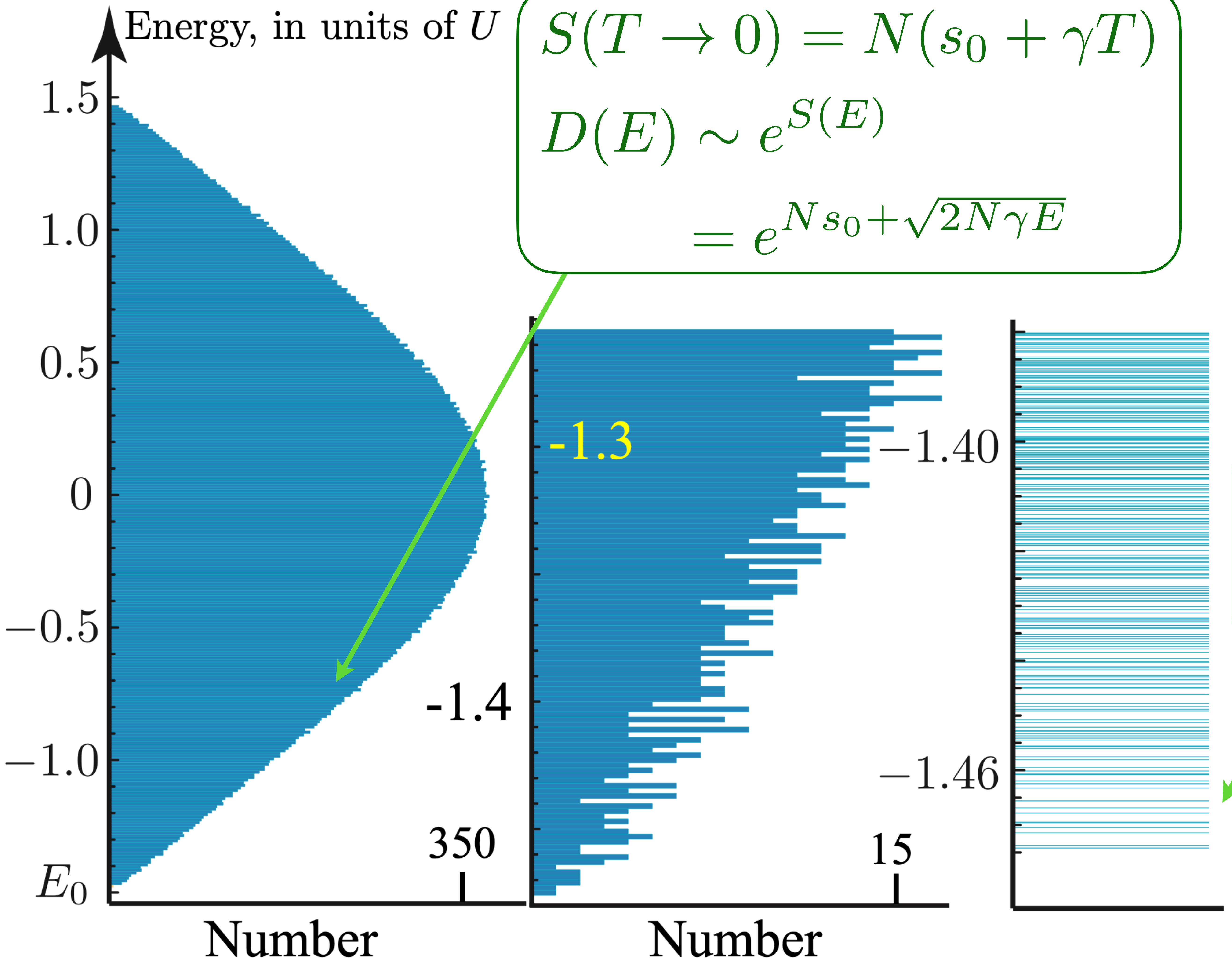

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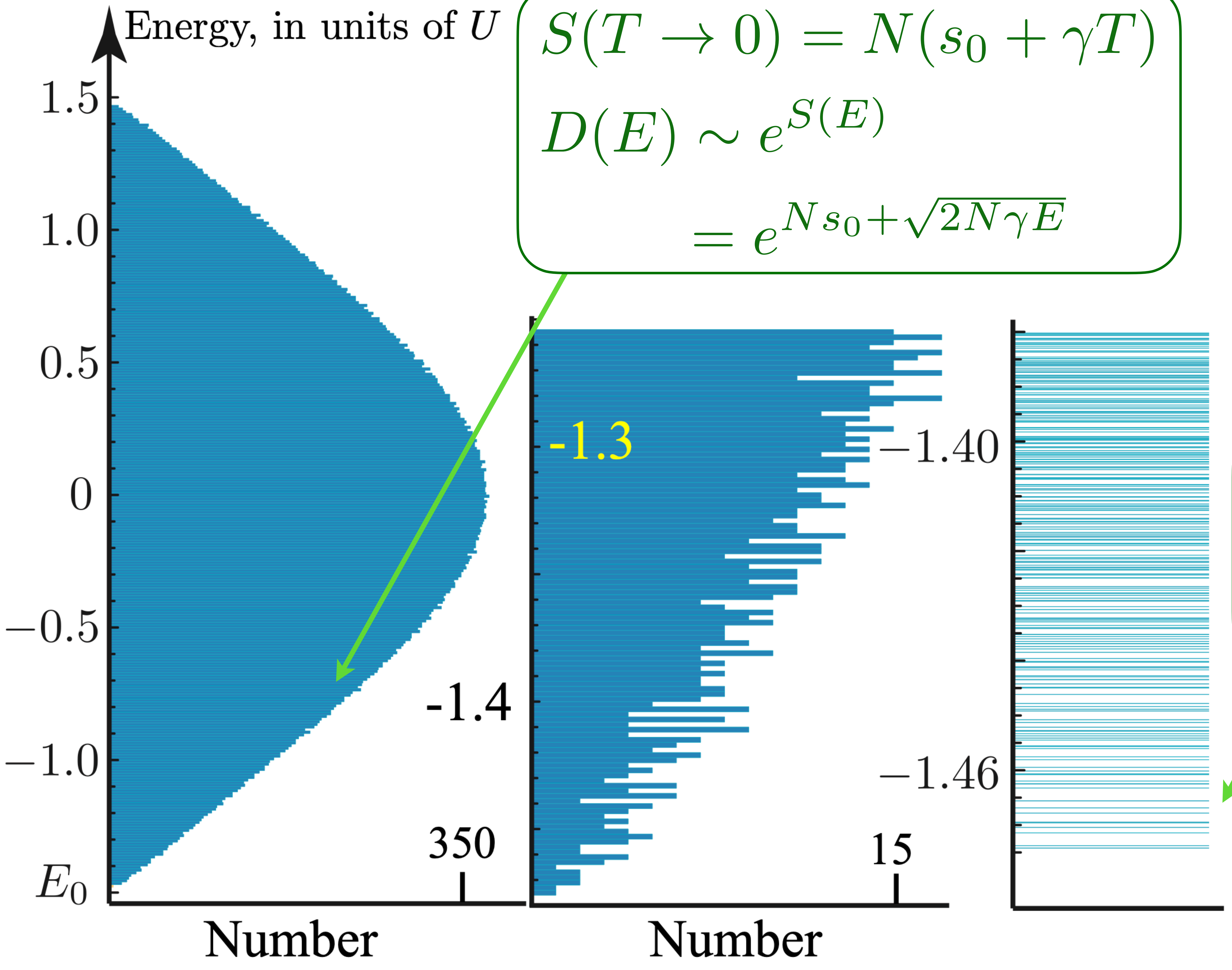

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J. S. Cotler et al., JHEP 05 (2017) 118

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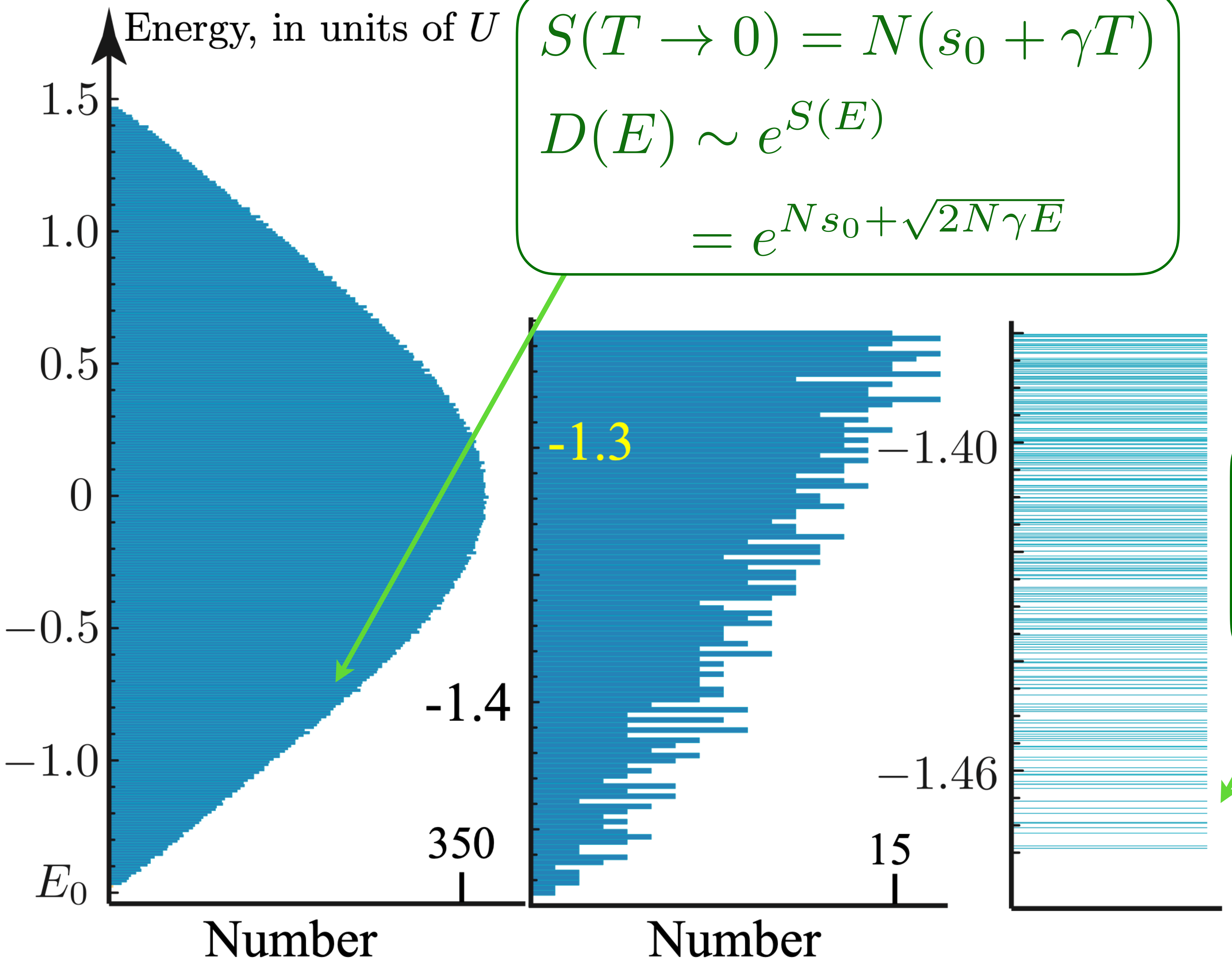

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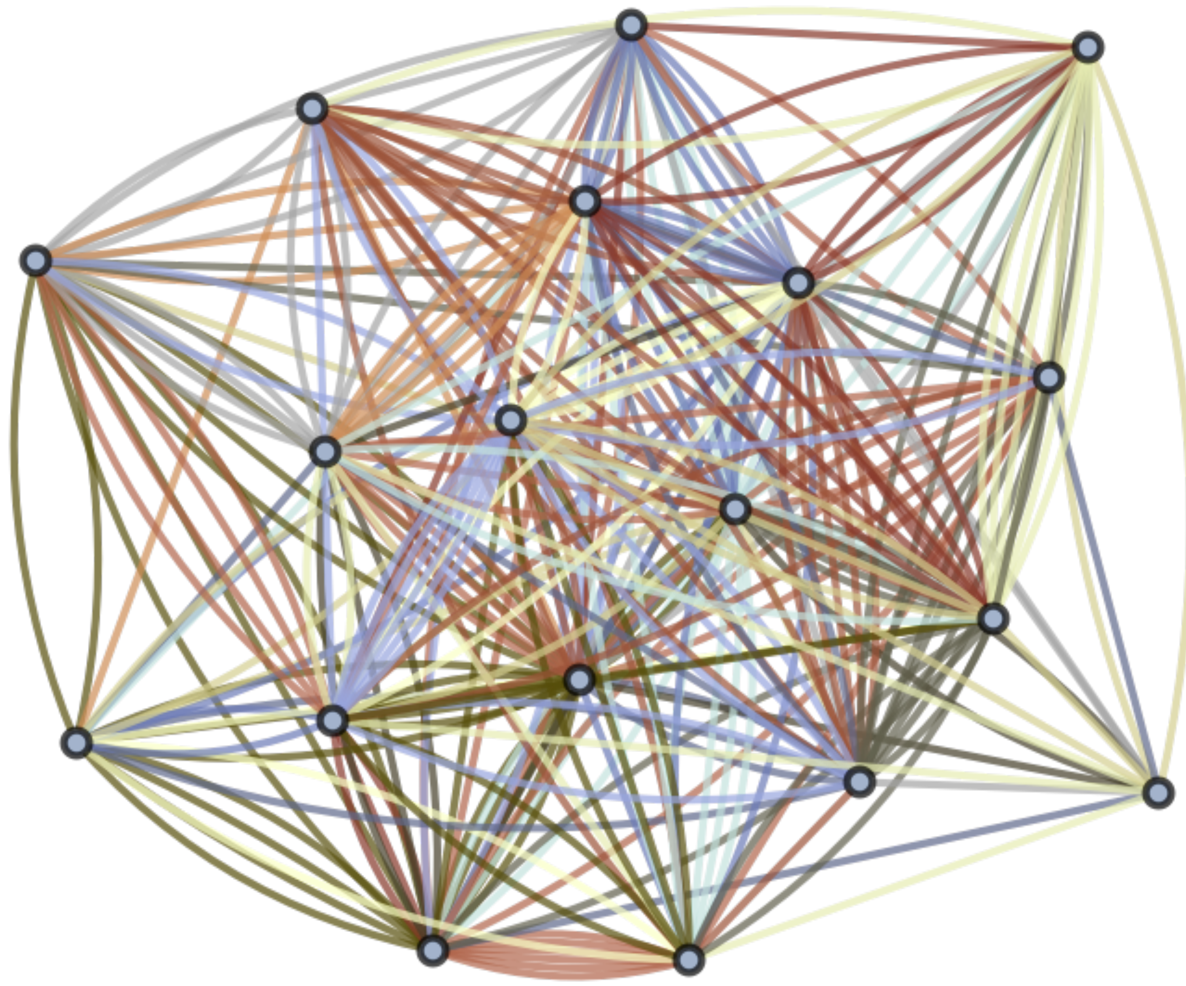
J. S. Cotler et al., JHEP 05 (2017) 118

Yingfei Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, JHEP 02 (2020) 157

(Numerics: G. Tarnopolsky)

Complex SYK model

The Sachdev-Ye-Kitaev (SYK) model



The disorder-averaged partition is given by a 'G-Σ' theory:

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1,\tau_2)\mathcal{D}\Sigma(\tau_1,\tau_2)\exp(-NI)$$

$$I = \ln\det\left[\delta(\tau_1-\tau_2)(\partial_{\tau_1}+\mu) - \Sigma(\tau_1,\tau_2)\right]$$

$$+\int d\tau_1 d\tau_2 \left[\Sigma(\tau_1,\tau_2)G(\tau_2,\tau_1) + (U^2/2)G^2(\tau_2,\tau_1)G^2(\tau_1,\tau_2)\right]$$

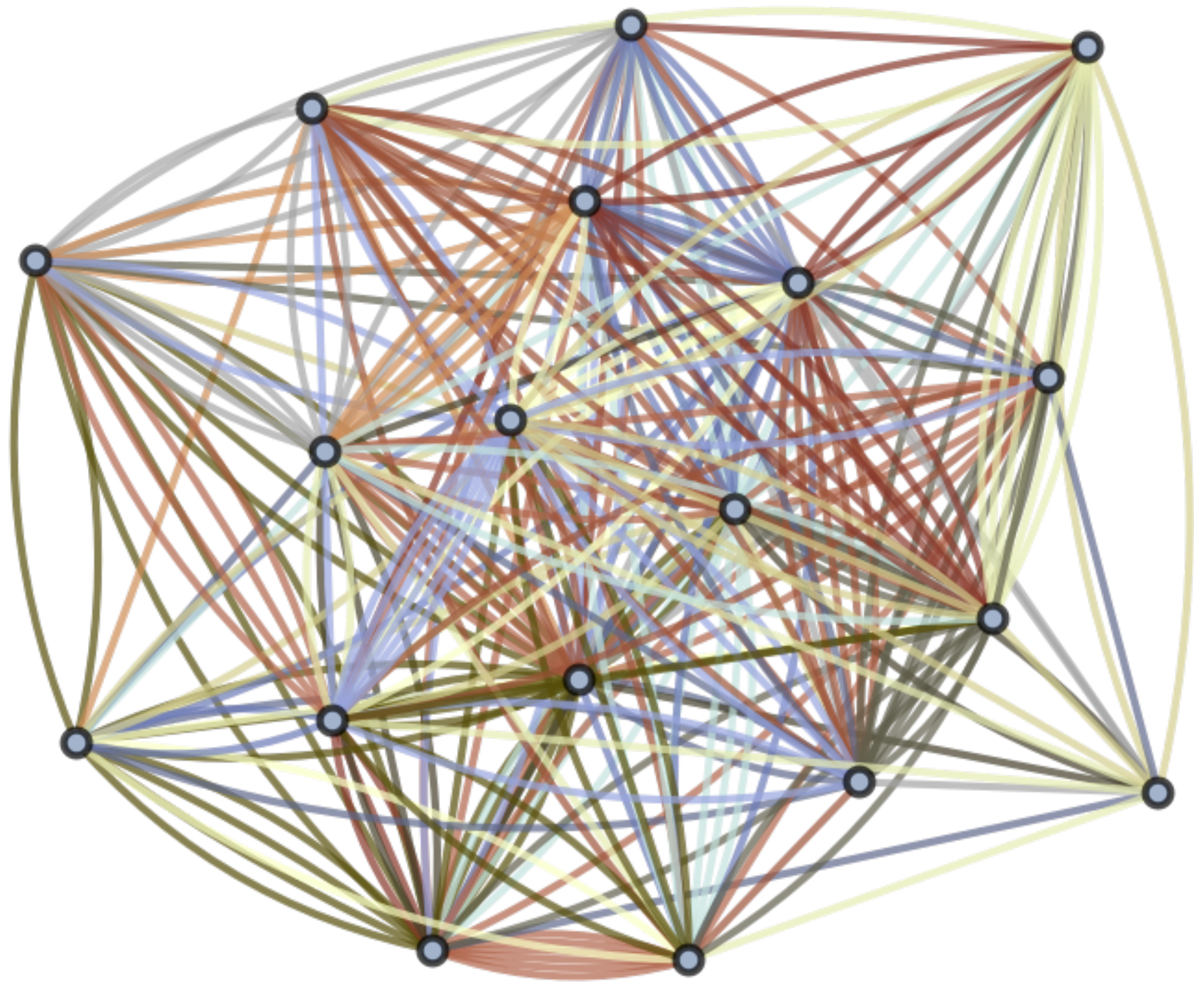
Saddle-point equations for $G(\tau_1-\tau_2)$ and $\Sigma(\tau_1-\tau_2)$:

$$G(i\omega) = \frac{1}{i\omega+\mu-\Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2G^2(\tau)G(-\tau)$$

$$G(\tau=0^-) = \mathcal{Q}.$$

The Sachdev-Ye-Kitaev (SYK) model

Time reparameterization symmetry



At frequencies $\ll U$, the time derivative in the determinant is less important, and without it the path integral is invariant under
time reparametrization and gauge transformations

$$\tau = f(\sigma)$$

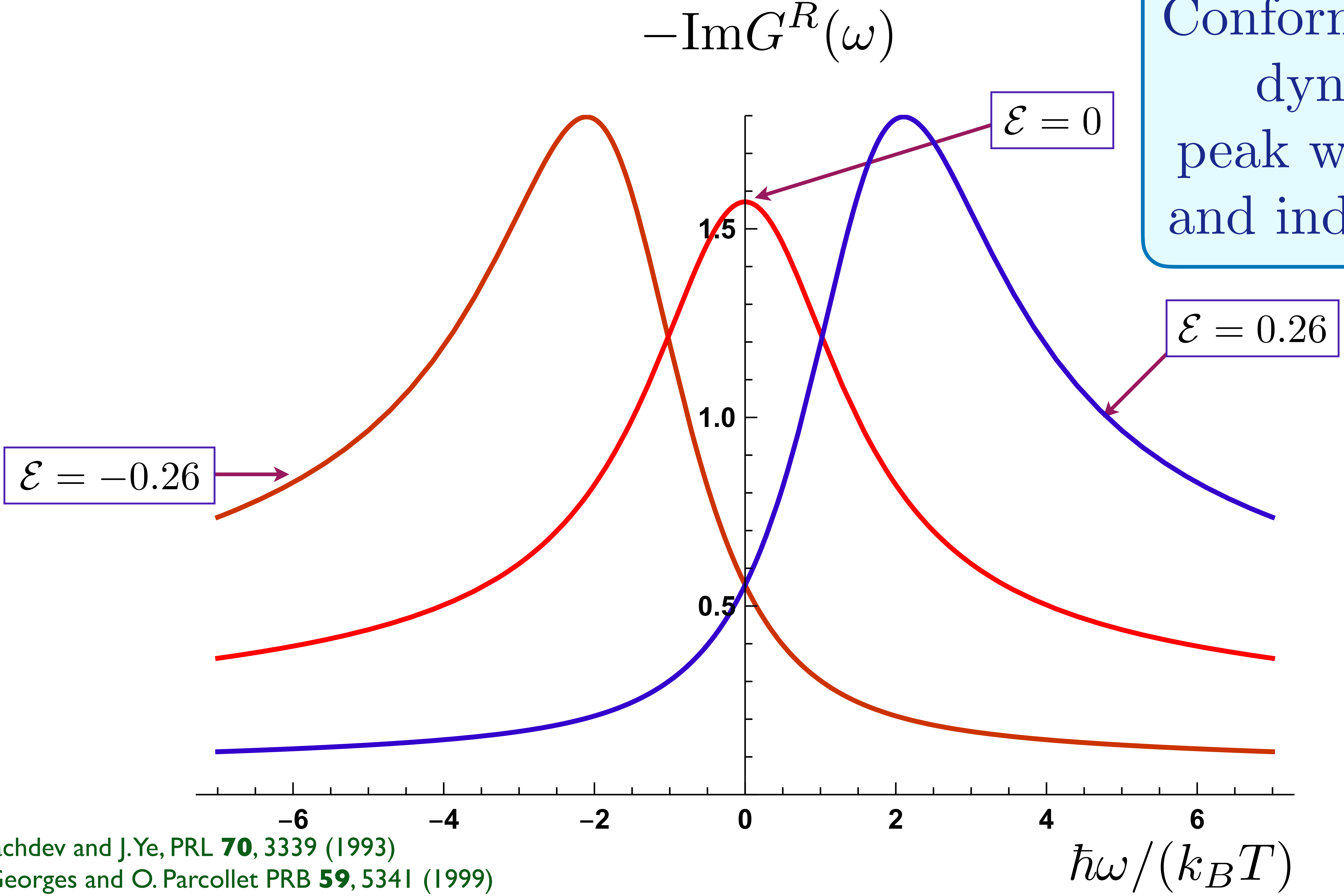
$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \, e^{-i\phi(\sigma_1) + i\phi(\sigma_2)} \, \widetilde{G}(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \, e^{-i\phi(\sigma_1) + i\phi(\sigma_2)} \, \widetilde{\Sigma}(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $\phi(\sigma)$ are arbitrary functions.

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, 2015
S. Sachdev, PRX **5,** 041025 (2015)

The SYK model



Conformal 'Planckian' dynamics with peak width $\sim k_B T/\hbar$ and independent of U

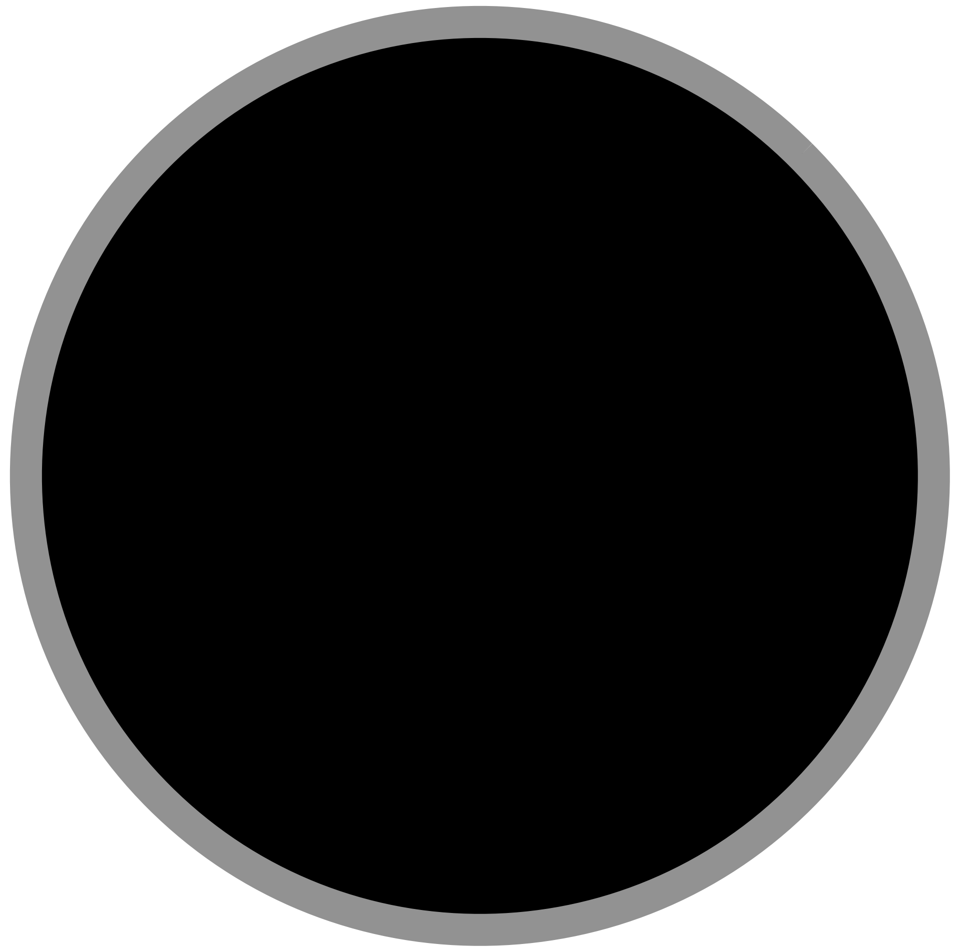
S. Sachdev and J. Ye, PRL **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

Quantum black holes

Black Holes

Objects so dense that light is gravitationally bound to them.

Horizon radius $R = \dfrac{2GM}{c^2}$

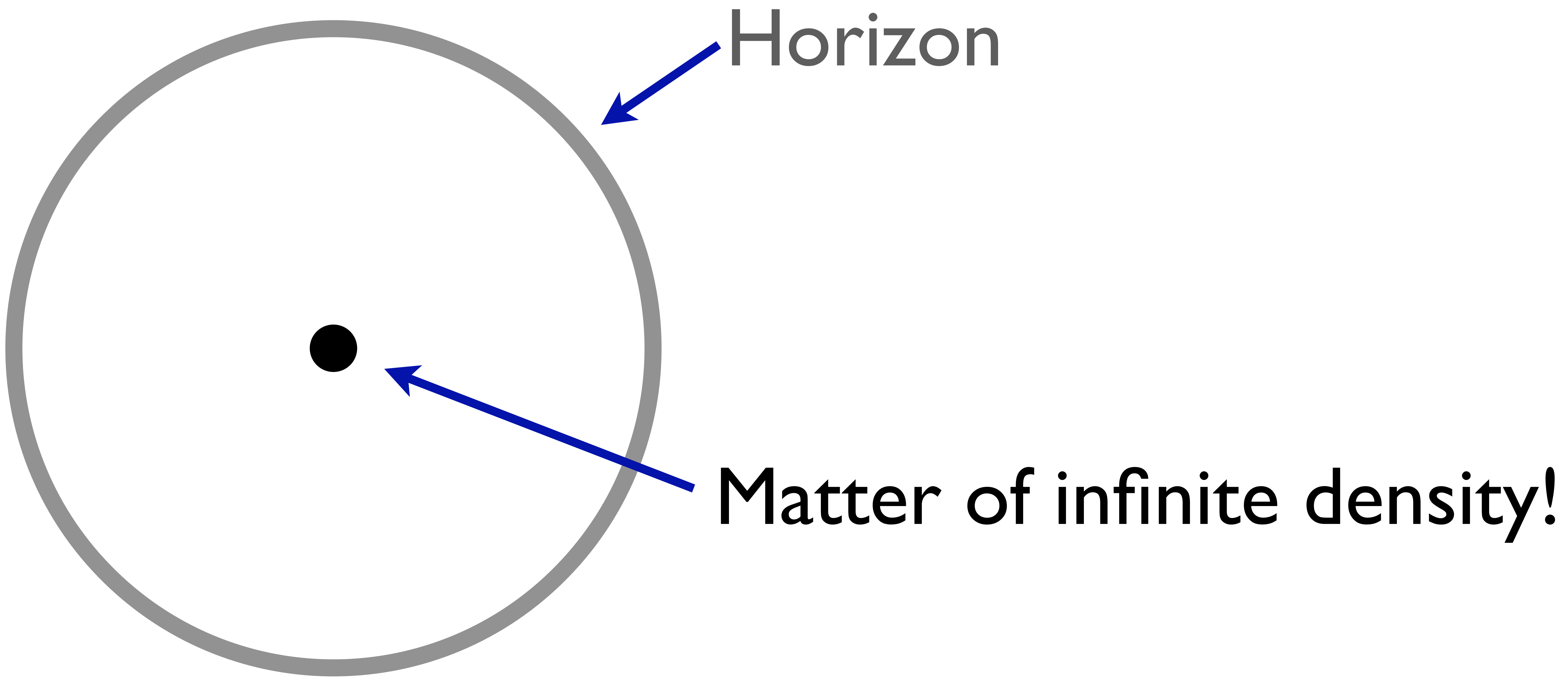


G Newton's constant, c velocity of light, M mass of black hole

For $M =$ earth's mass, $R \approx 9\,mm$!

What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



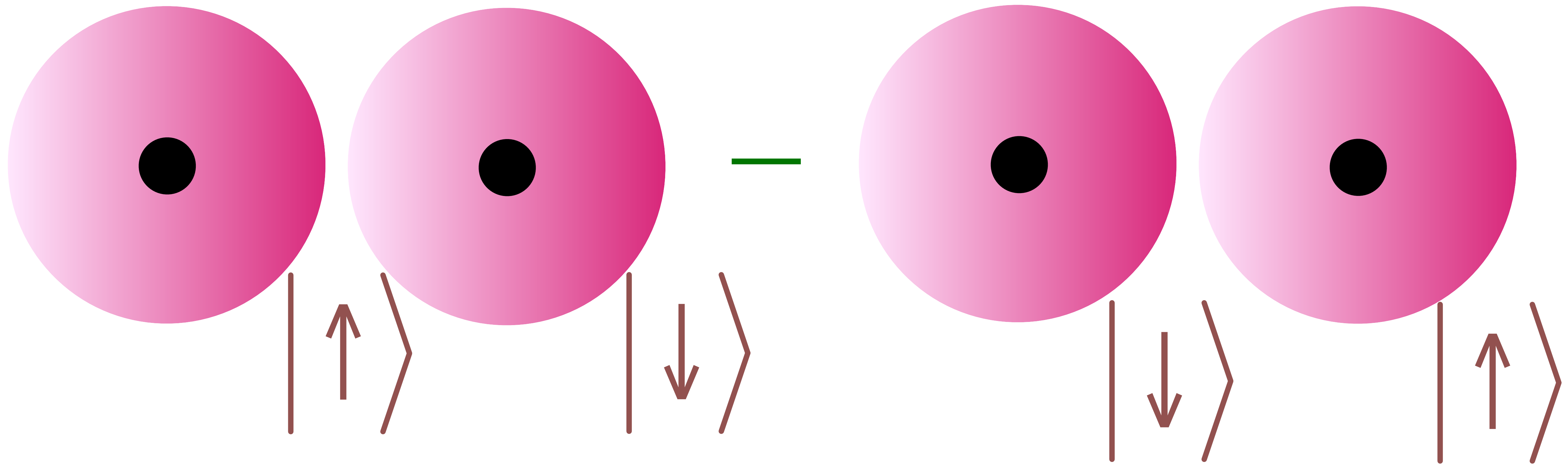
What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.

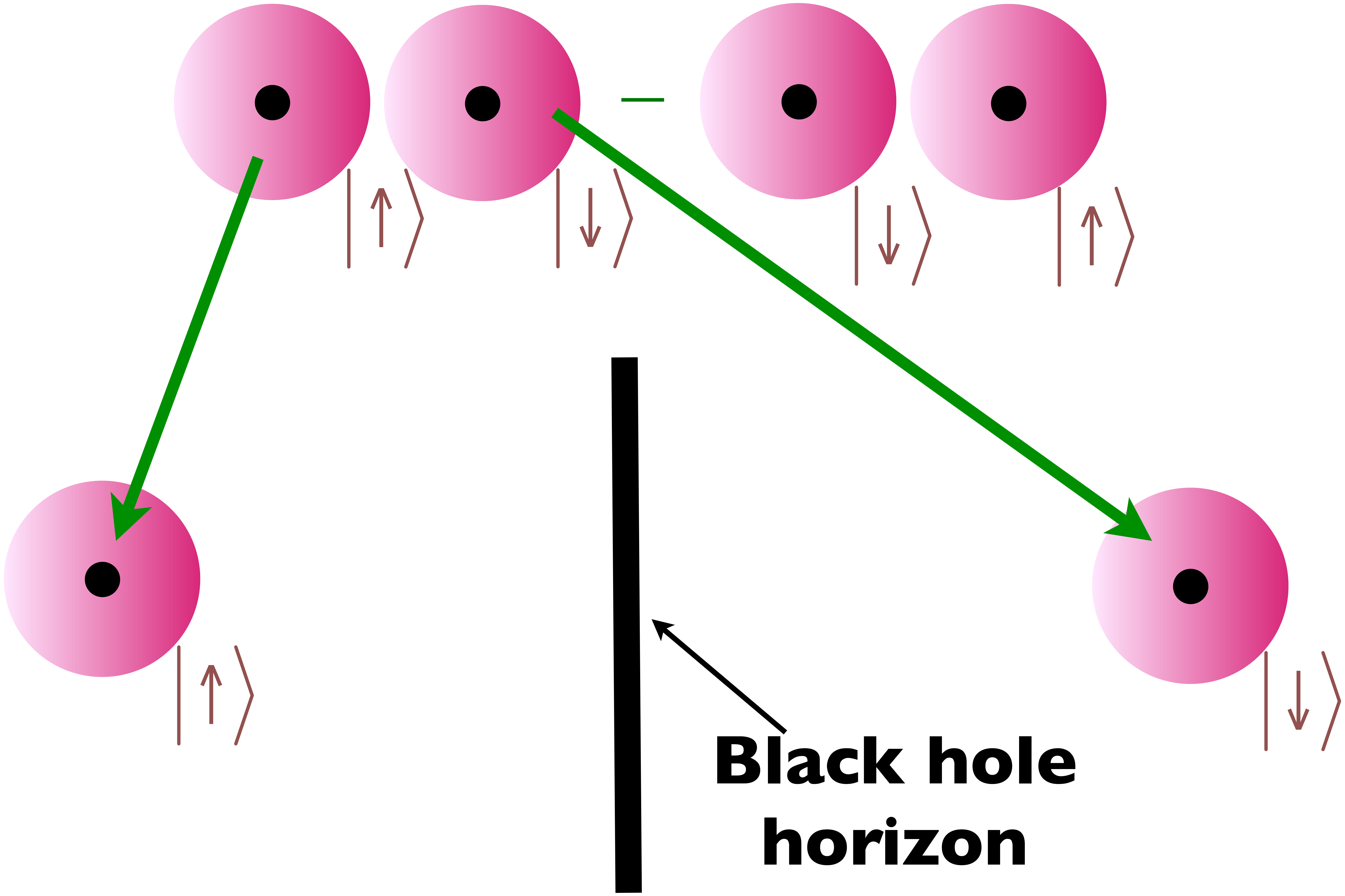
This singularity convinced many early on that black holes were unphysical solutions of Einstein's equations, and did not exist in our universe.

In any case, it was clear that quantum theory should be applied to the collapsed matter, but no one knew how to.

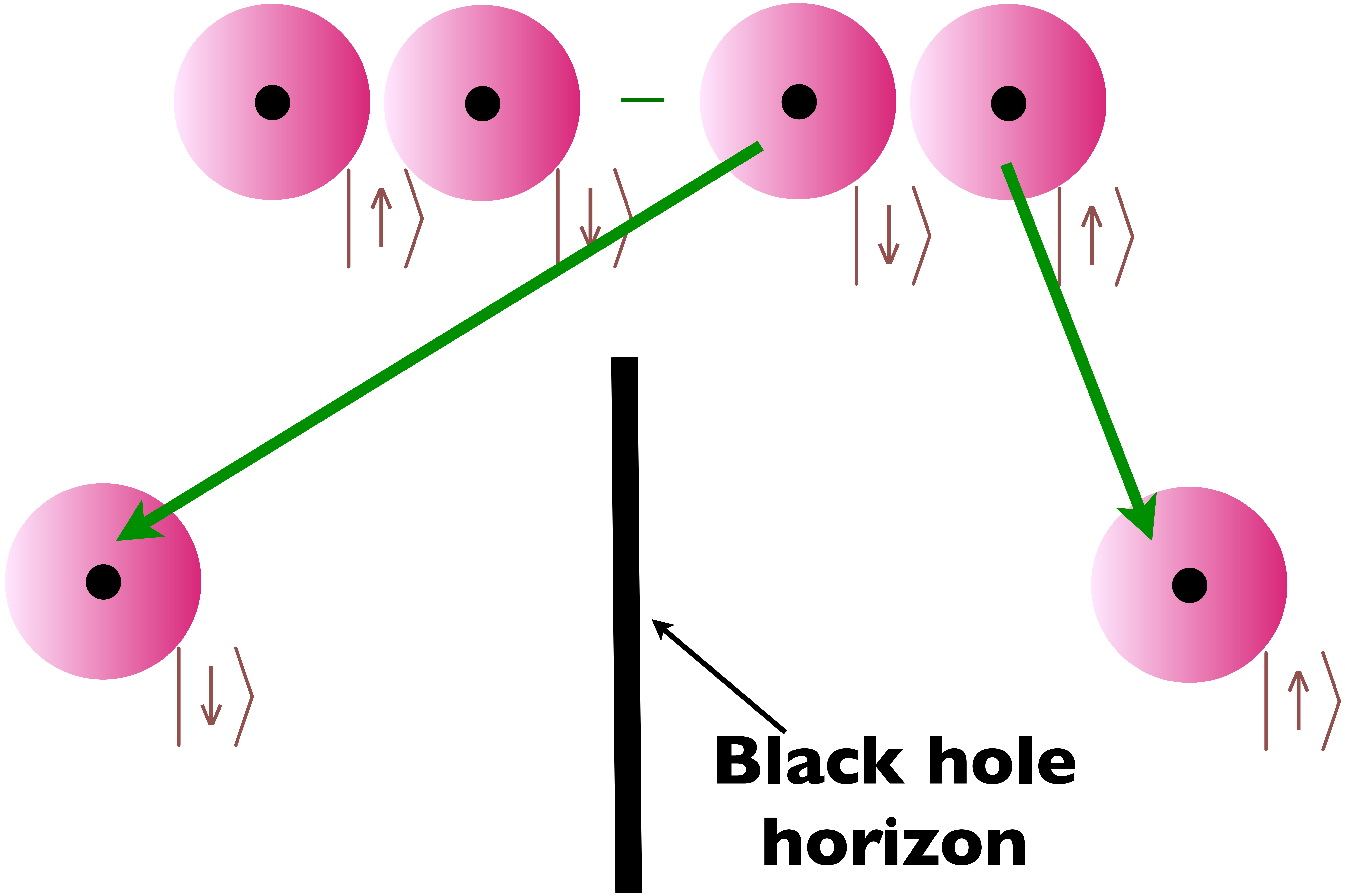
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon



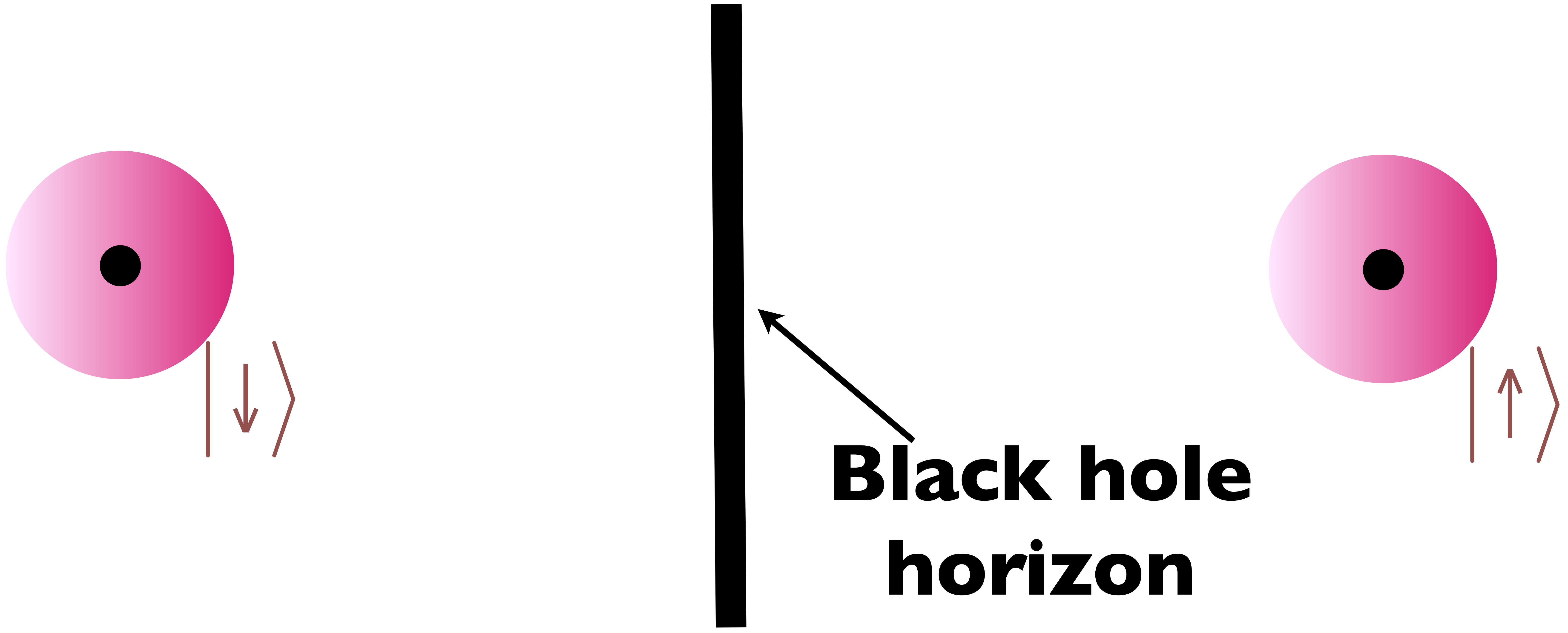
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

Hawking (1975) used other arguments to show that black hole horizons have a temperature

(The entanglement reasoning: to an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.)

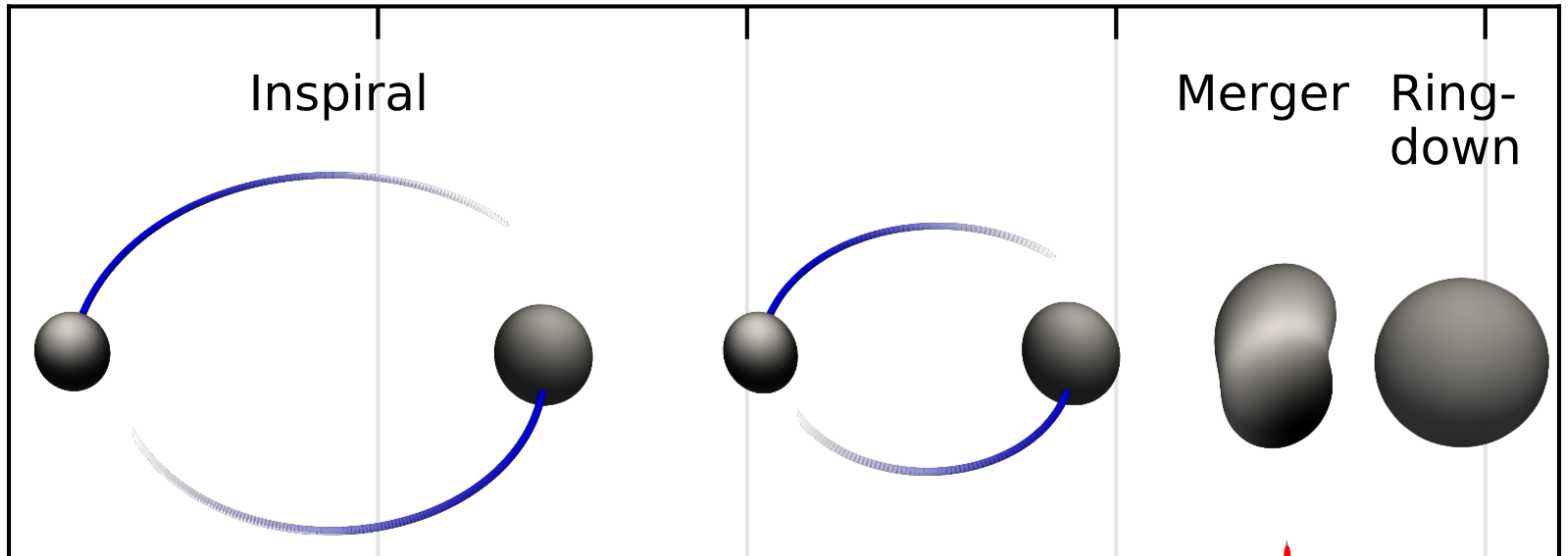


Quantum black holes

- Black holes have an entropy and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.

- The entropy is proportional to their surface area. $S = A k_B c^3/(4G\hbar)$.

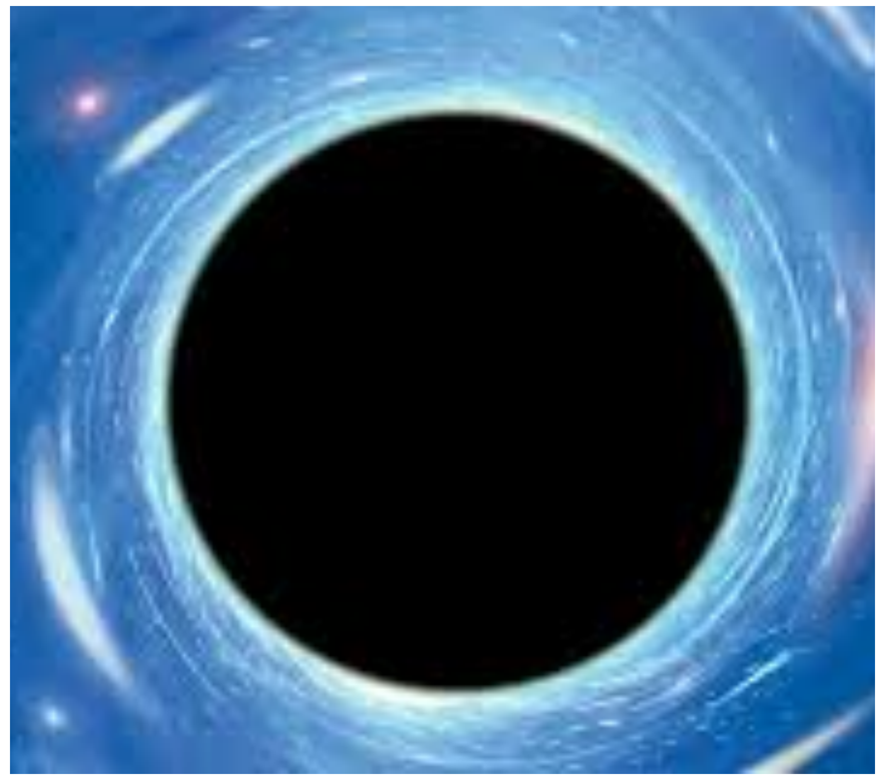
- They relax to thermal equilibrium in a time $\sim 8\pi GM/c^3 = \hbar/(k_B T_H)$ which is Planckian!

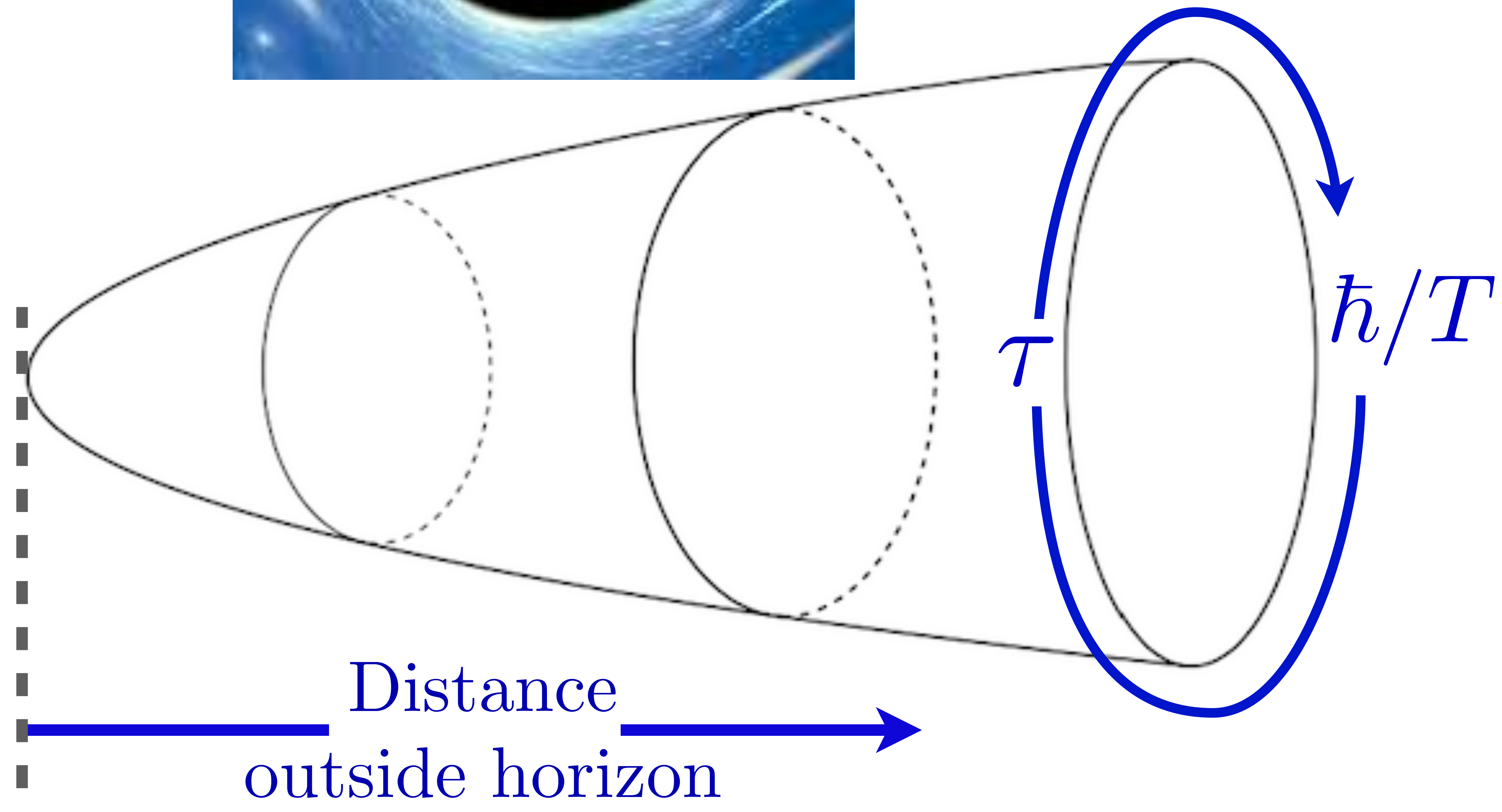


J. D. Bekenstein, PRD **7**, 2333 (1973)
S.W. Hawking, Nature **248**, 30 (1974)
C.V. Vishveshwara, Nature **227**, 936 (1970)

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\mathcal{Z}(\mathcal{Q},T) = \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu}, A_\mu]\right)$$





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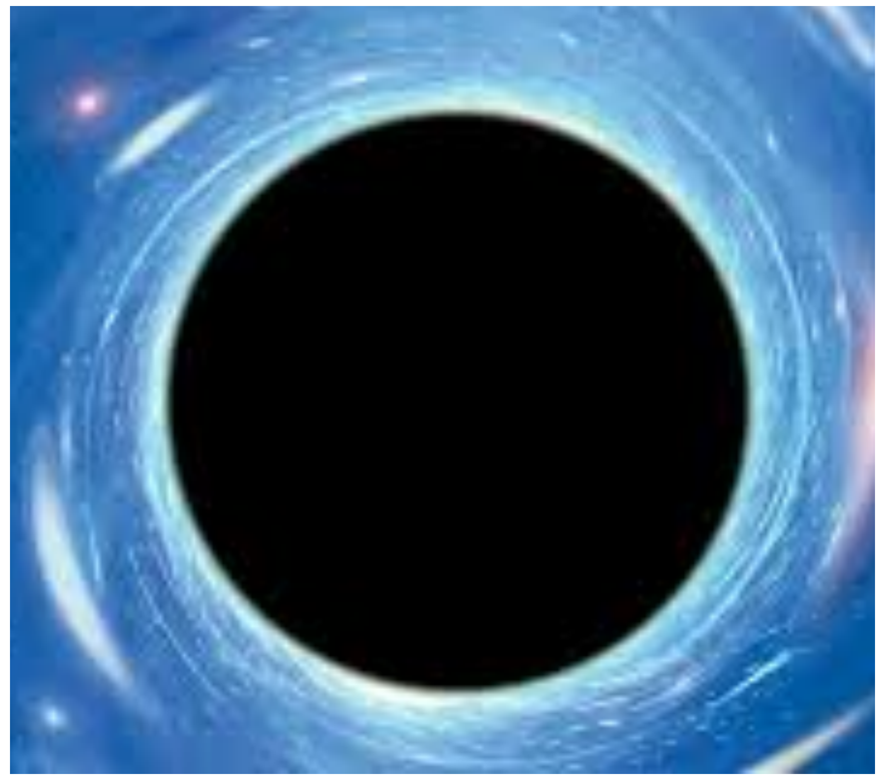
$$= \exp\left(S_{BH}\right) \times \left(\quad ....????... \quad \right)$$

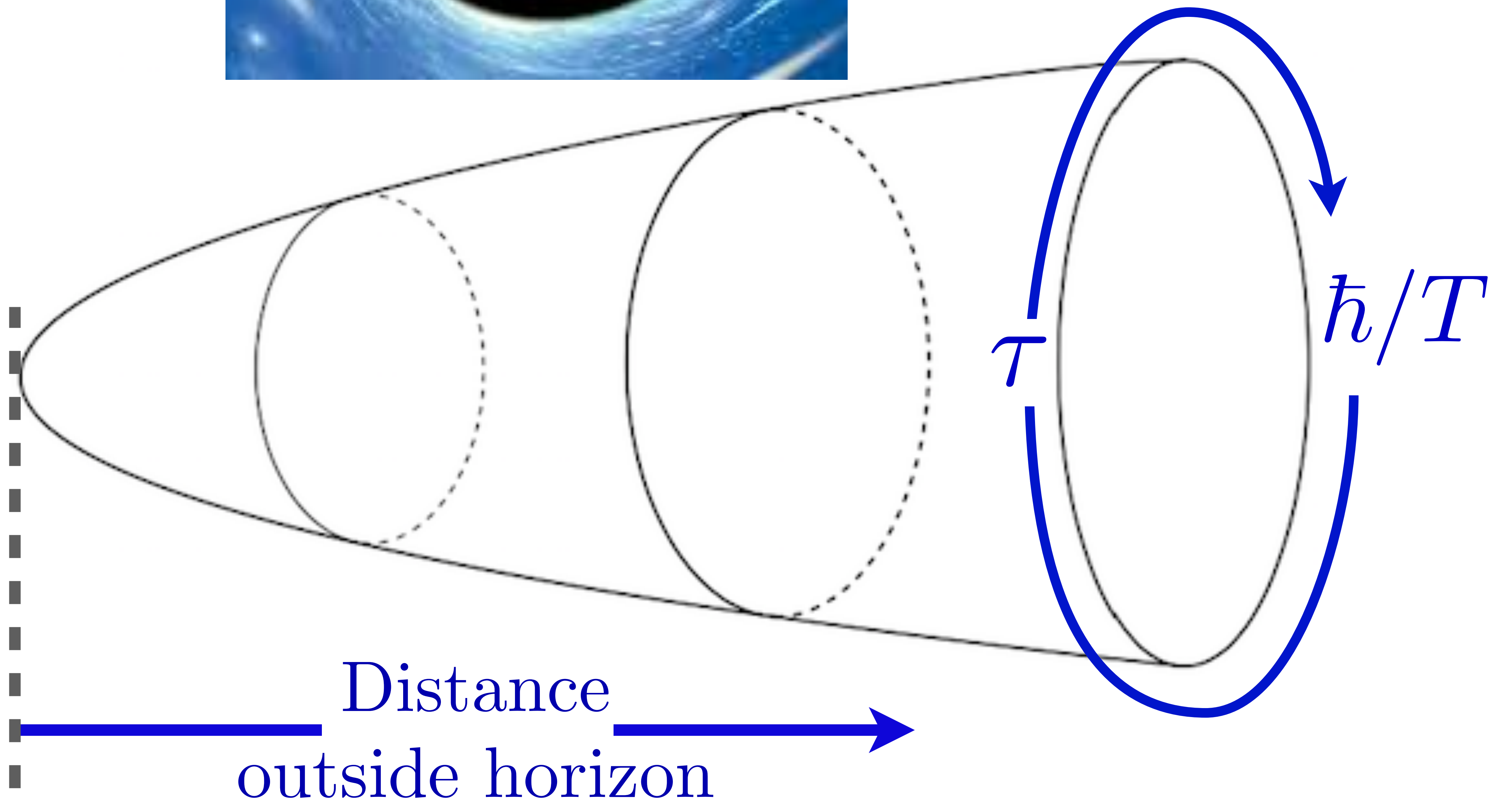
Gibbons, Hawking (1977)
Chambin, Emparan, Johnson, Myers (1999)



$$S_{BH}(T \to 0, \mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1 + \frac{2(\pi A_0)^{1/2}T}{\hbar c}\right)$$

$A_0 = 2G\mathcal{Q}^2/c^4$ is the area of the charged black hole horizon at $T = 0$.

Obtained from the saddle-point of the gravity path integral in the imaginary time spacetime outside the black hole.



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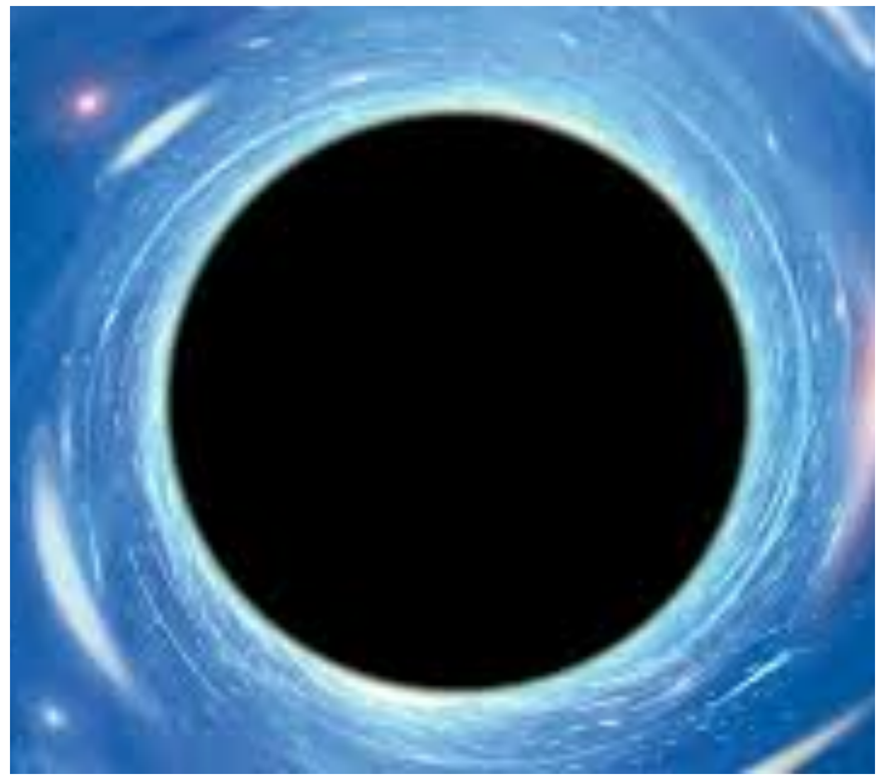

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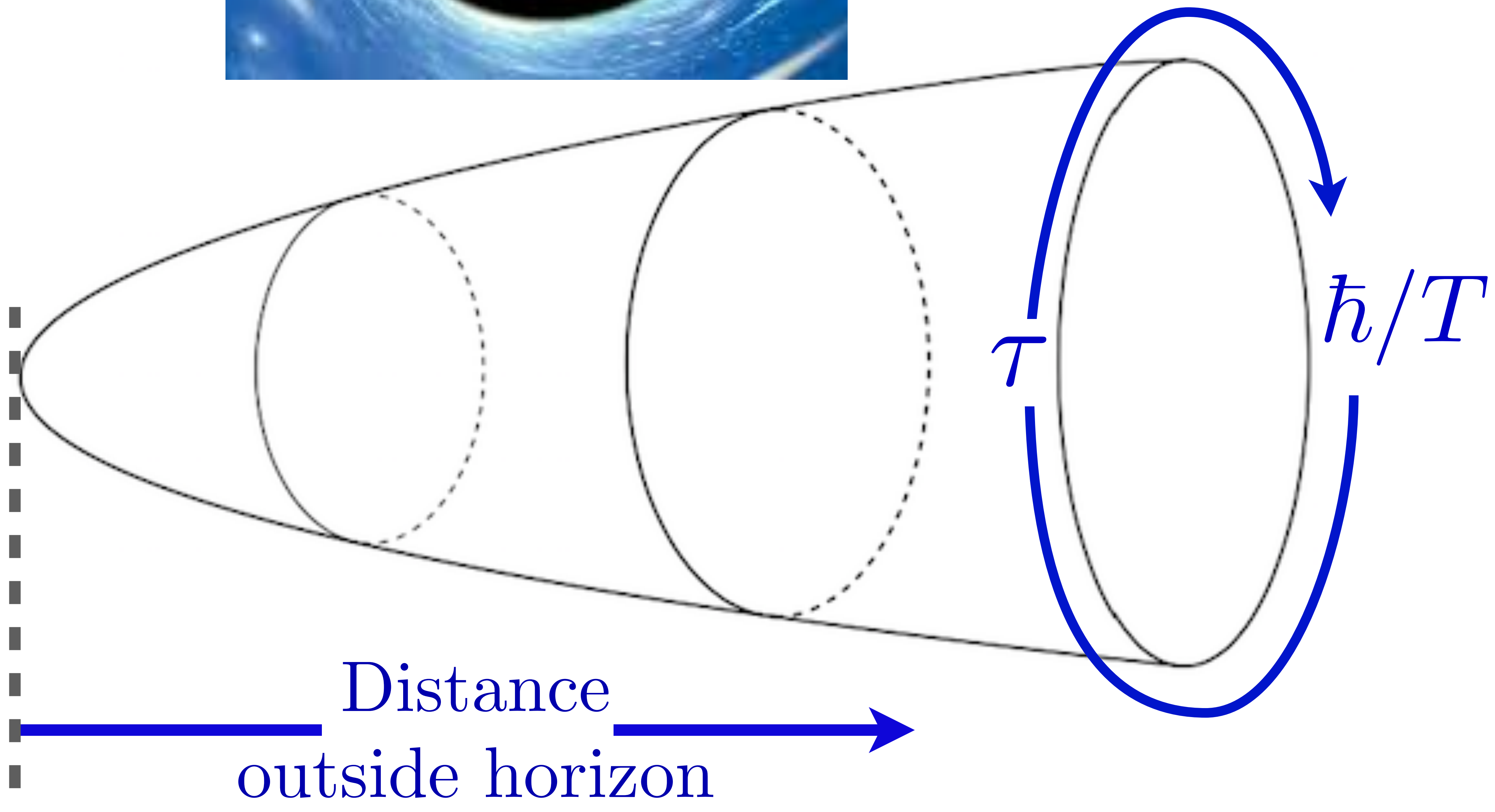


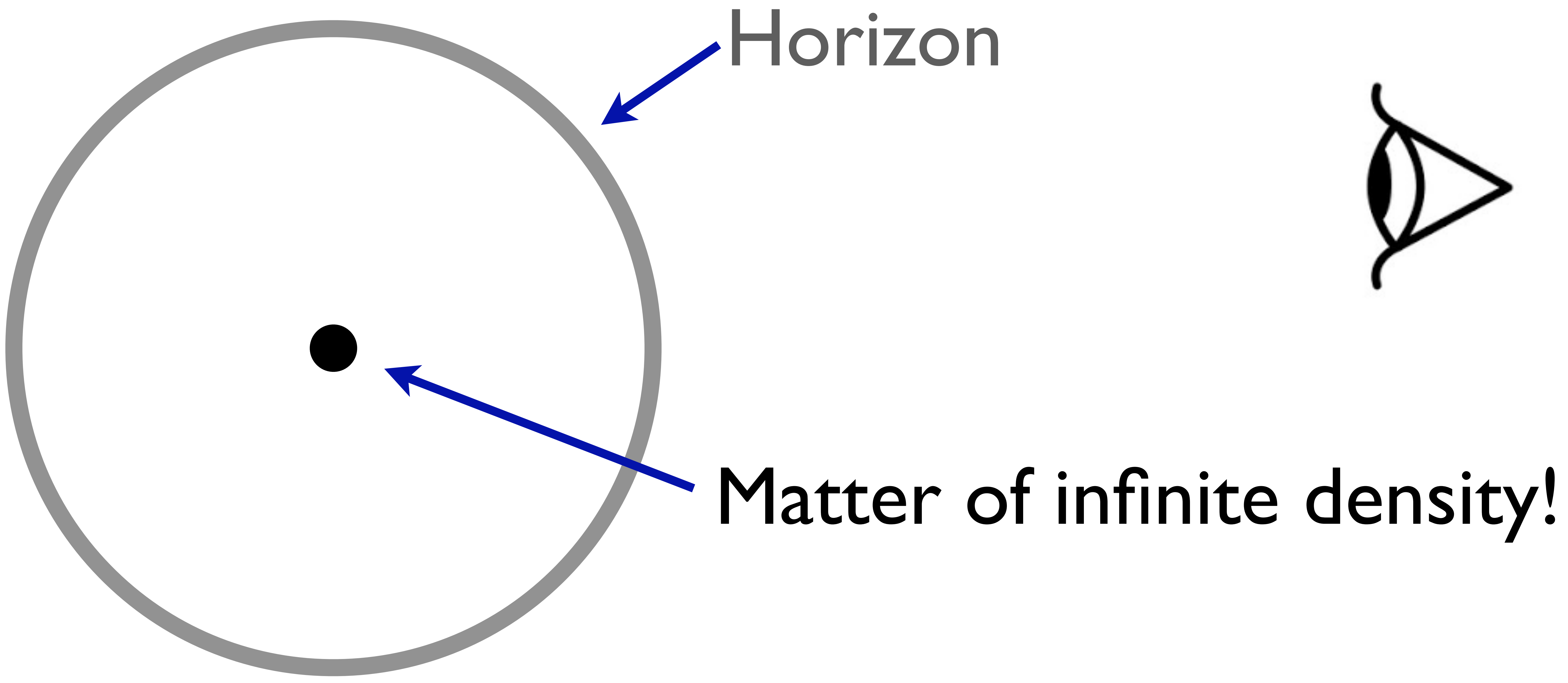
Note the similarity to the large *N* entropy of the SYK model ! (along with other similarities)

Sachdev PRL 2010

Quantum Black Holes

Hawking obtained the black hole entropy by semiclassical computations for an observer *outside* the black hole horizon.

This allowed Hawking to avoid the contradictions associated with the singularity at the center of the black hole.



Quantum Black Holes

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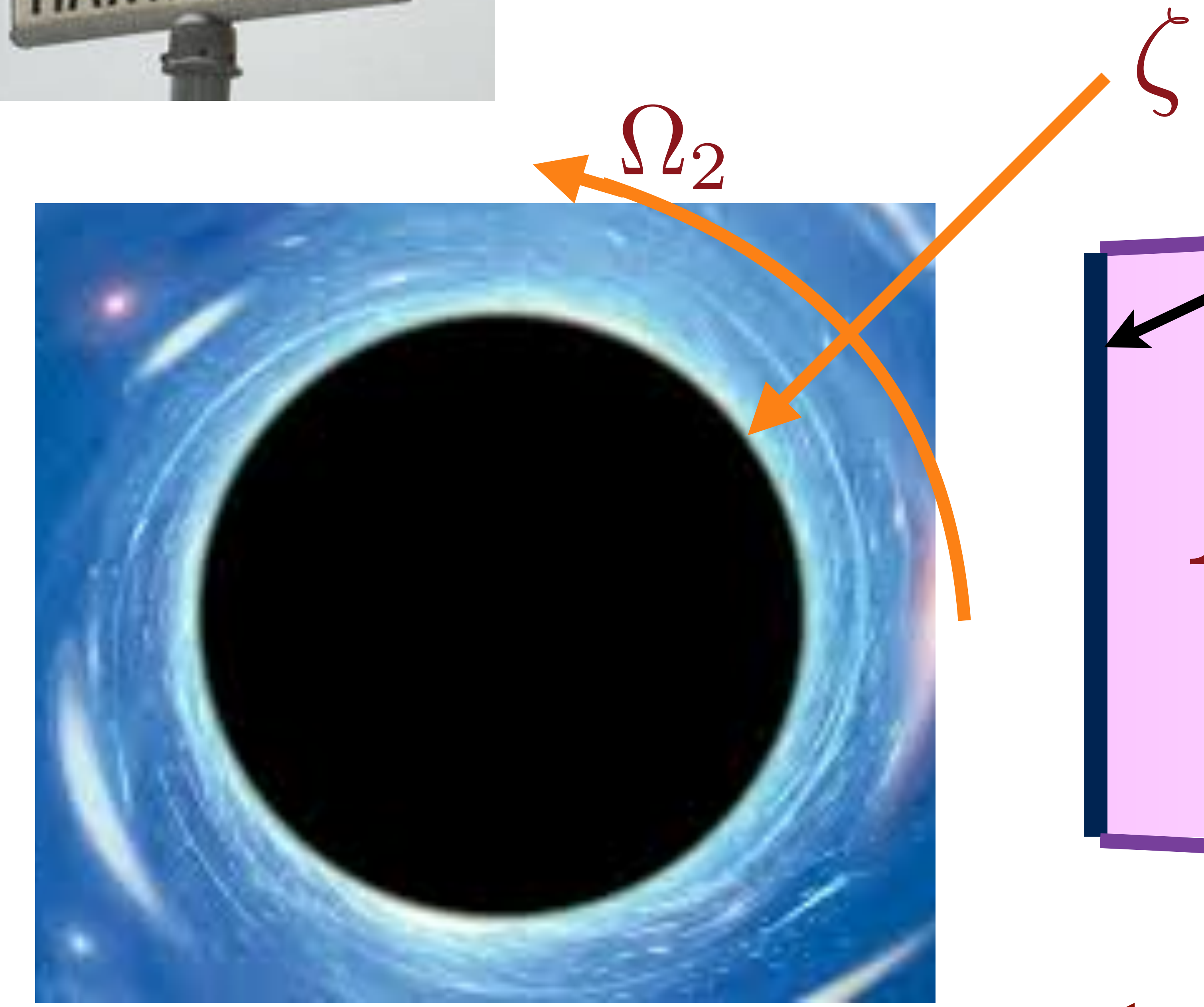
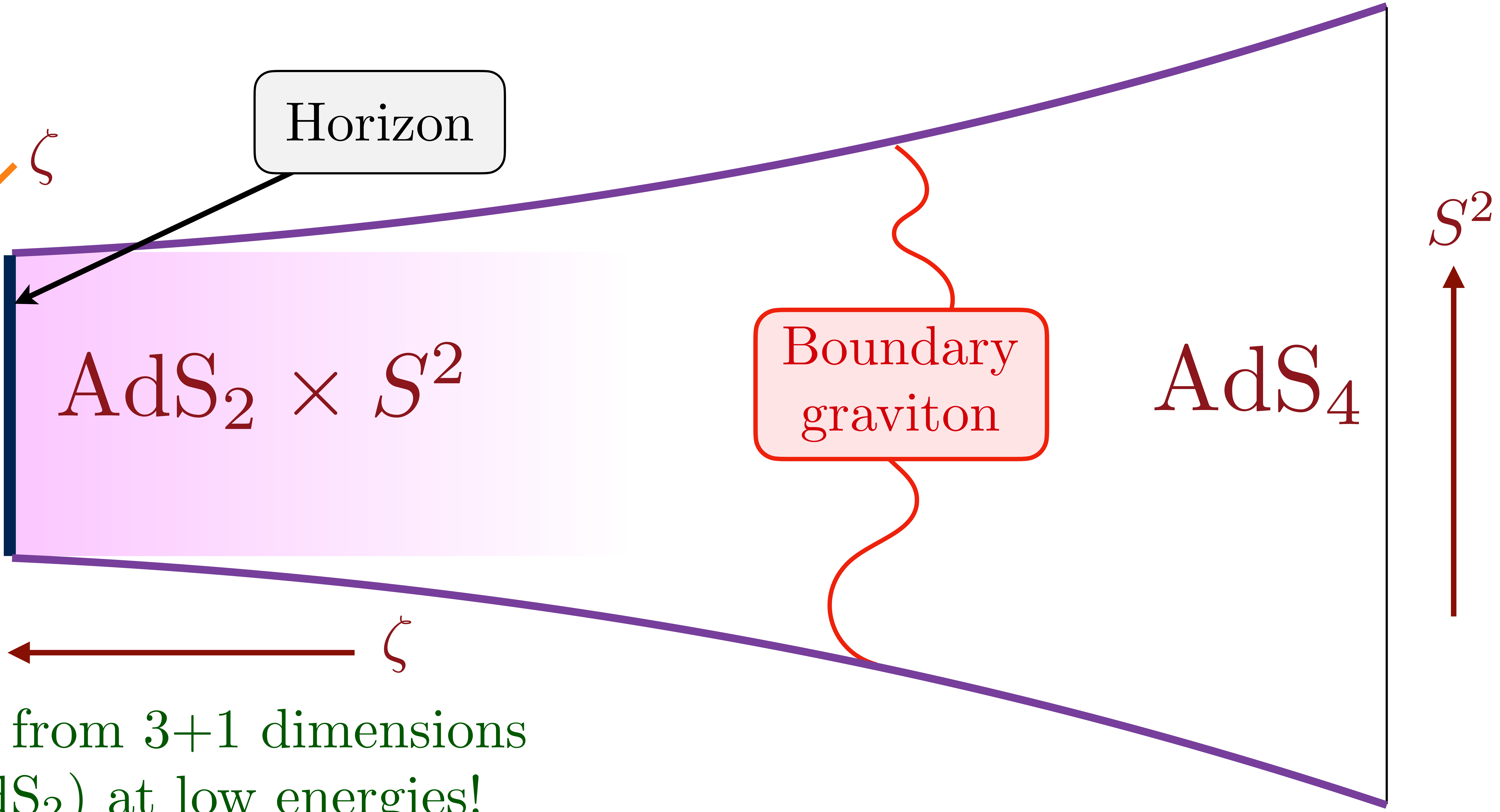
Can we find a quantum theory for the collapsed matter at the center of the black hole, whose *density of quantum states* matches the Bekenstein-Hawking entropy, in accordance with Boltzmann's principles of statistical mechanics ?

From the SYK model to a quantum theory of charged black holes

Reissner-Nordstrom black hole of Einstein-Maxwell theory





Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!
The isometry group of AdS_2 is the 0+1 dimensional conformal group SL(2,R).

Faulkner, Liu, McGreevy, Vegh 2009; Sachdev 2010

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Maldacena, Stanford, Yang (2016)

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$$= \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau)\exp\left(-\frac{1}{\hbar} I_{\text{SYK}}[\text{time reparameterizations } f(\tau), \text{ phase rotations } \phi(\tau)]\right)$$

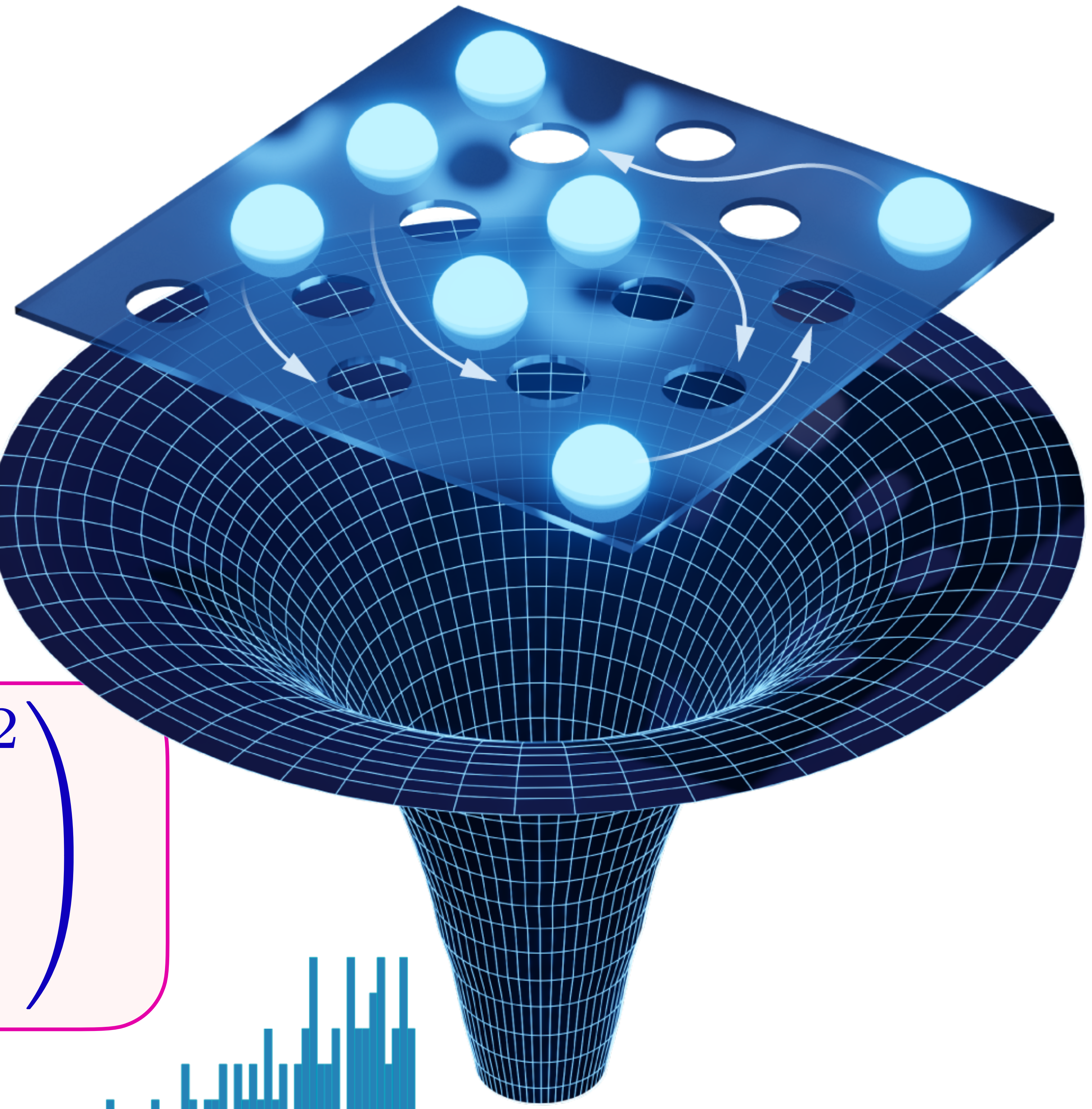
Saddle-point:

$$S_{BH}(T\to 0,\mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1 + \frac{2(\pi A_0)^{1/2}T}{\hbar c} + \ldots\right)$$

$A_0 = 2G\mathcal{Q}^2/c^4$ is the area of the charged black hole horizon at $T=0$.

Sachdev (2010); Kitaev (2015); Sachdev (2015); Bagrets, Altland, Kamenev (2016); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Sachdev (2019); Iliesiu, Turaci (2020)

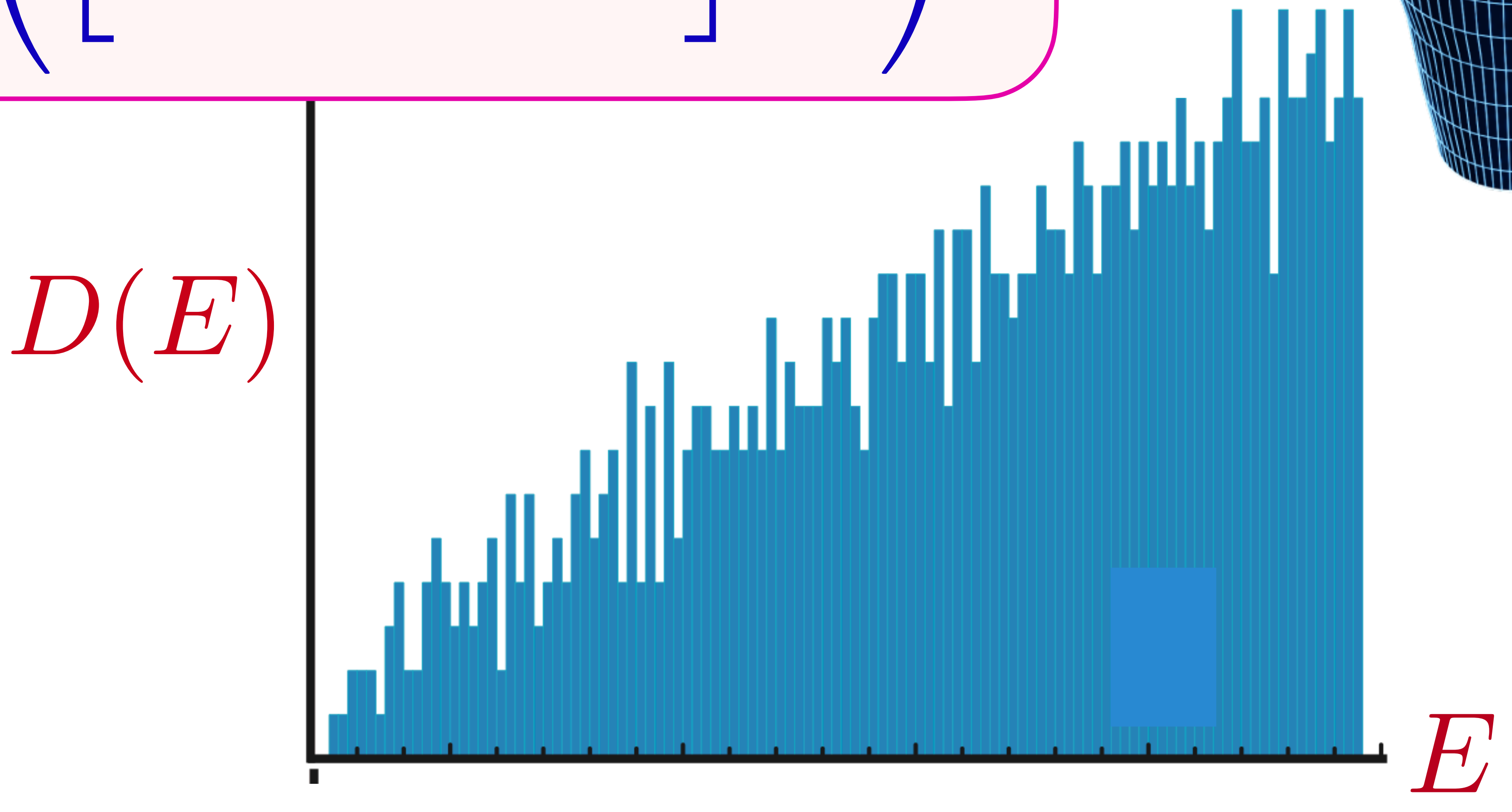
Quantum simulation of charged black holes by the SYK model



- For generic charged black holes in 3+1 dimensions, the SYK model yields, in terms of $A_0 = 2GQ^2/c^4$ the horizon area at $T = 0$:

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

There is no degeneracy, but an exponentially small level spacing down to the ground state.



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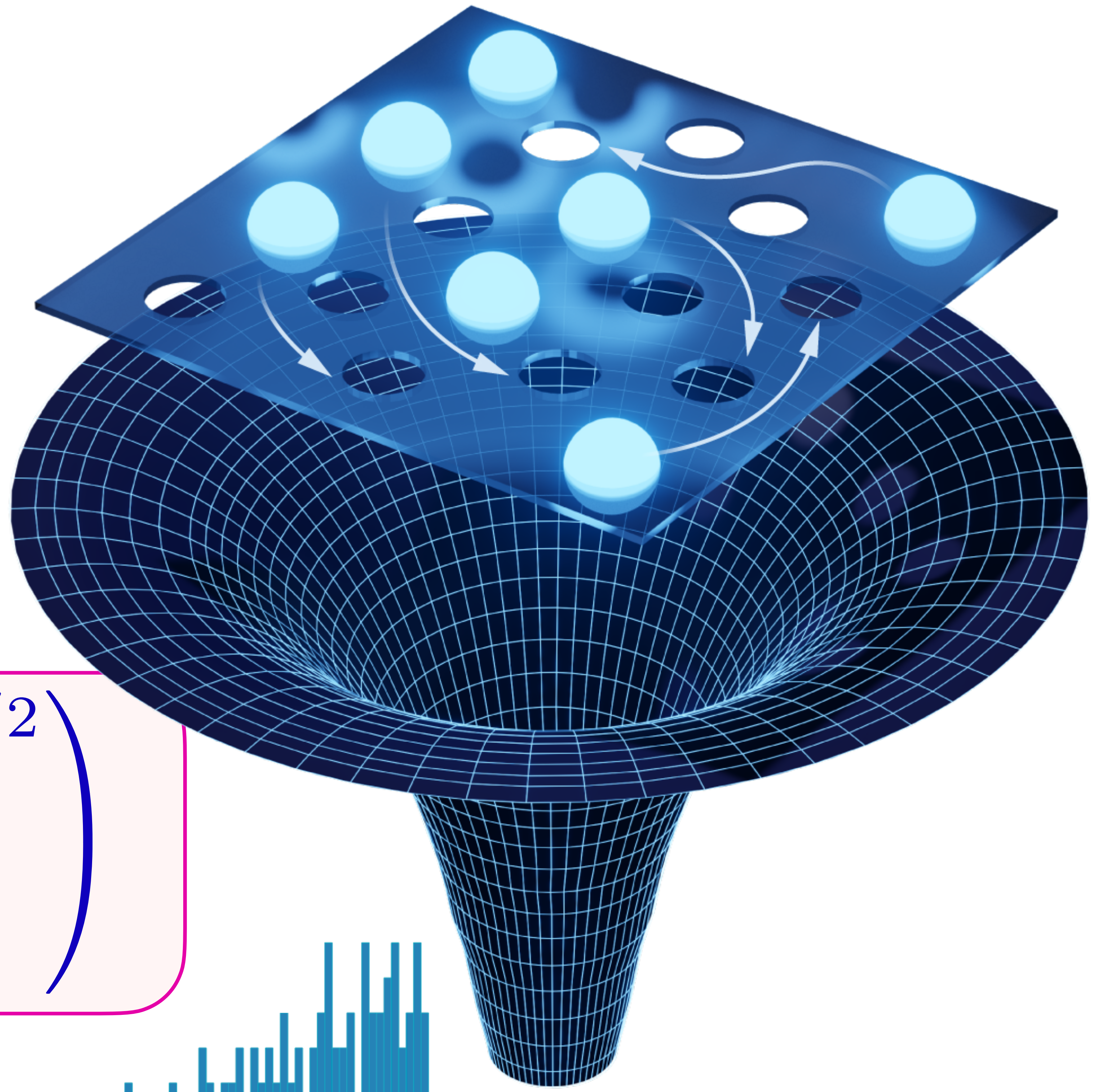

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Bekenstein-Hawking

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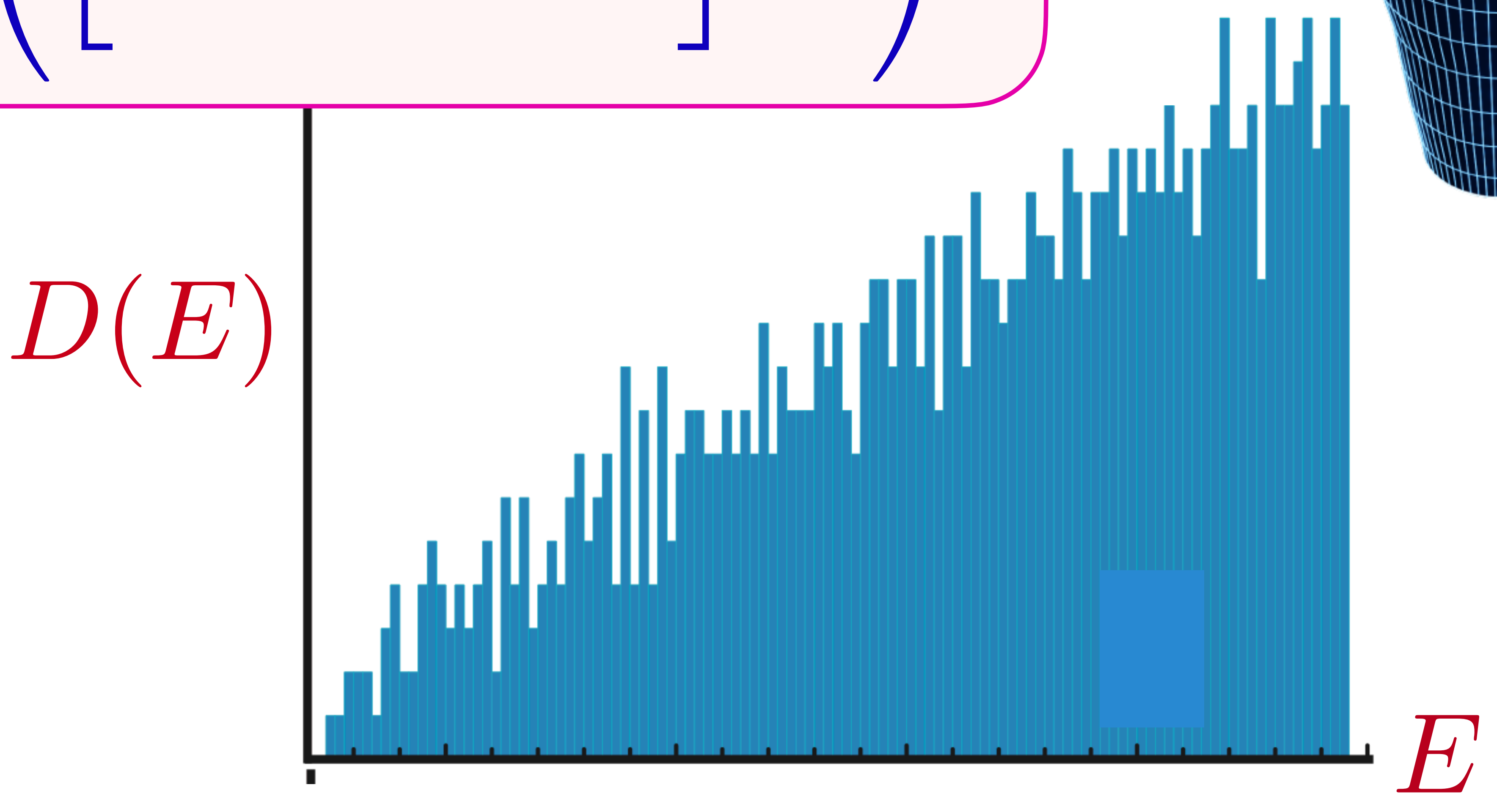

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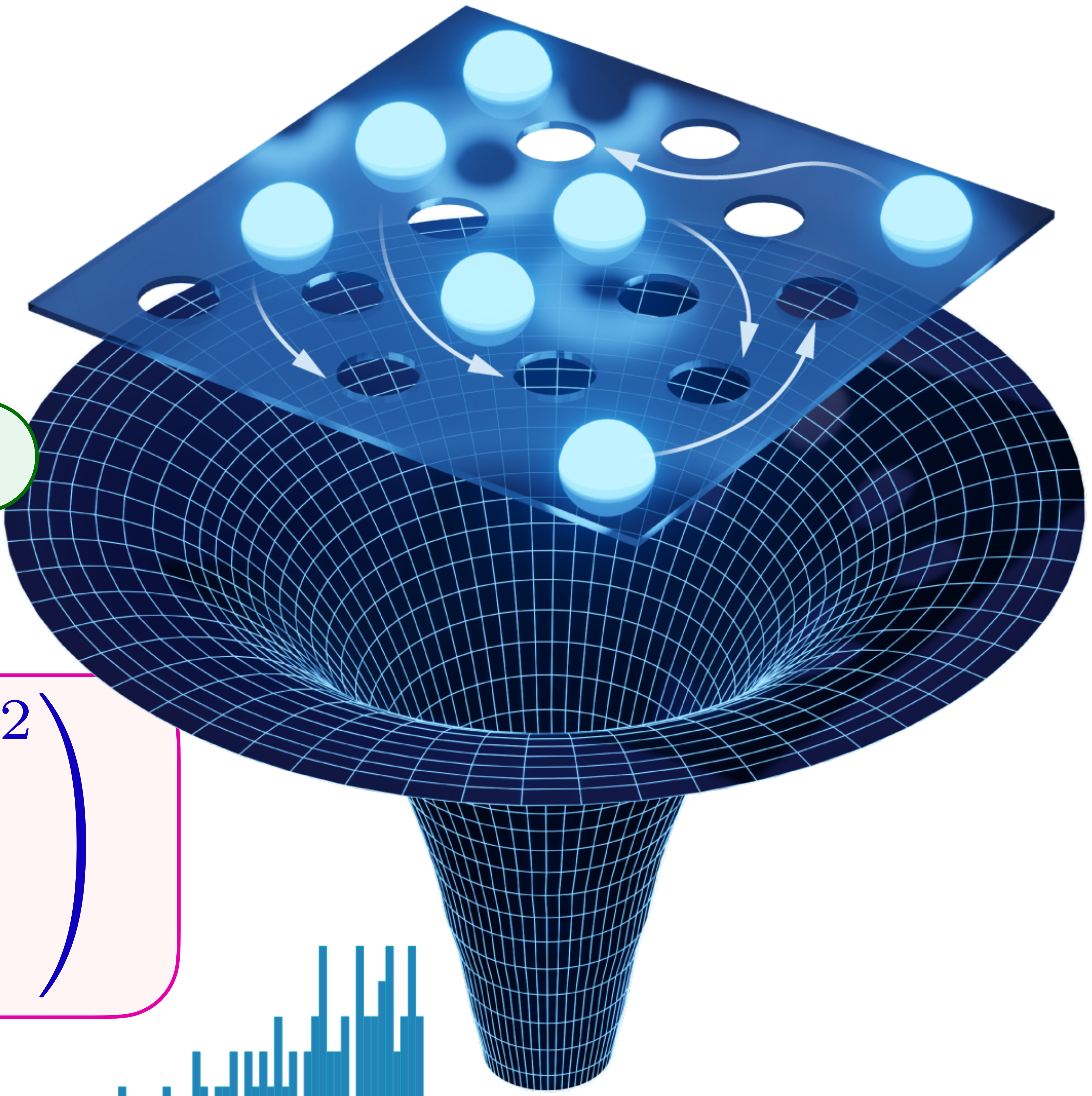

Bekenstein-Hawking

Developments from the SYK model

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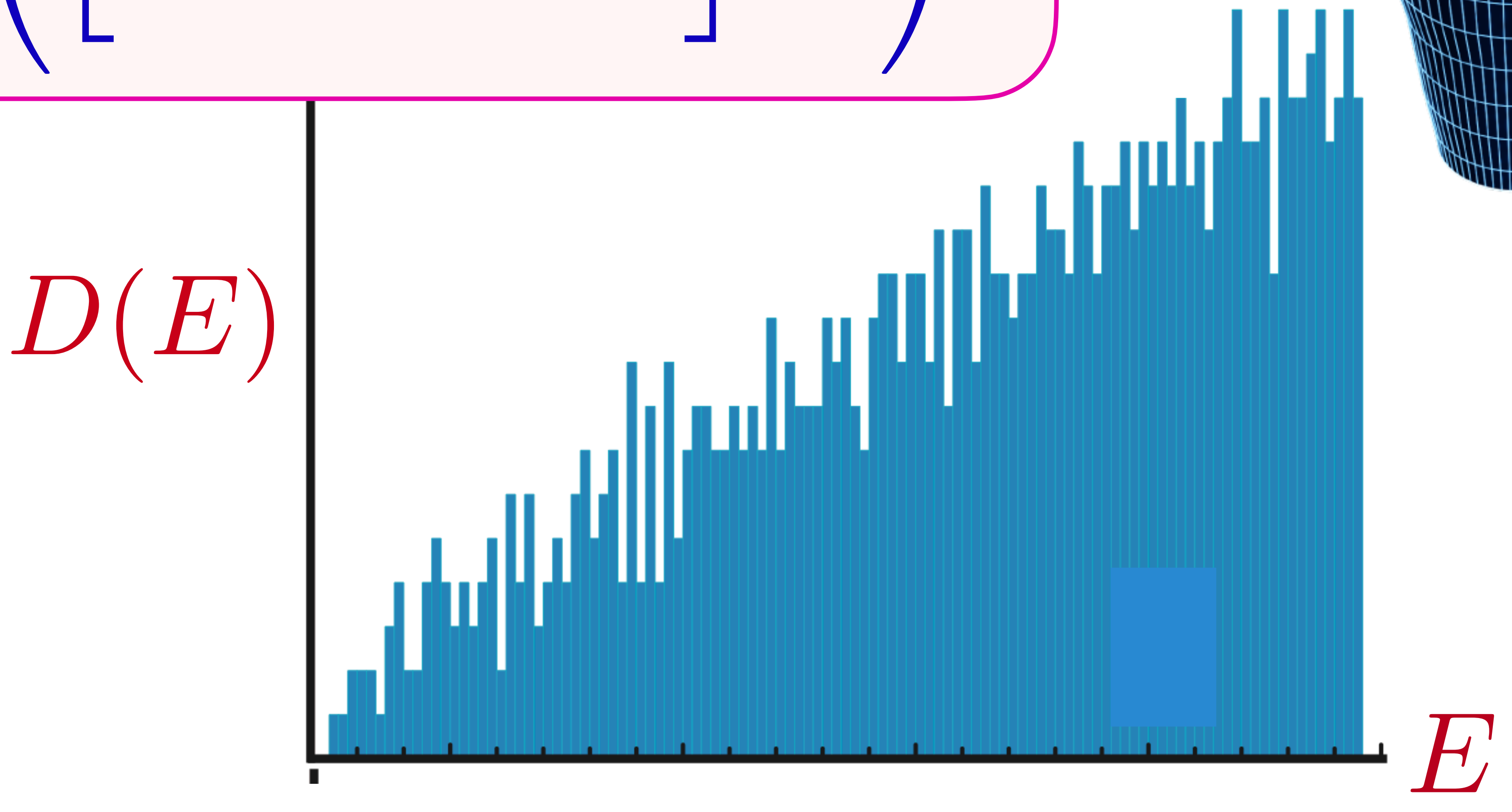

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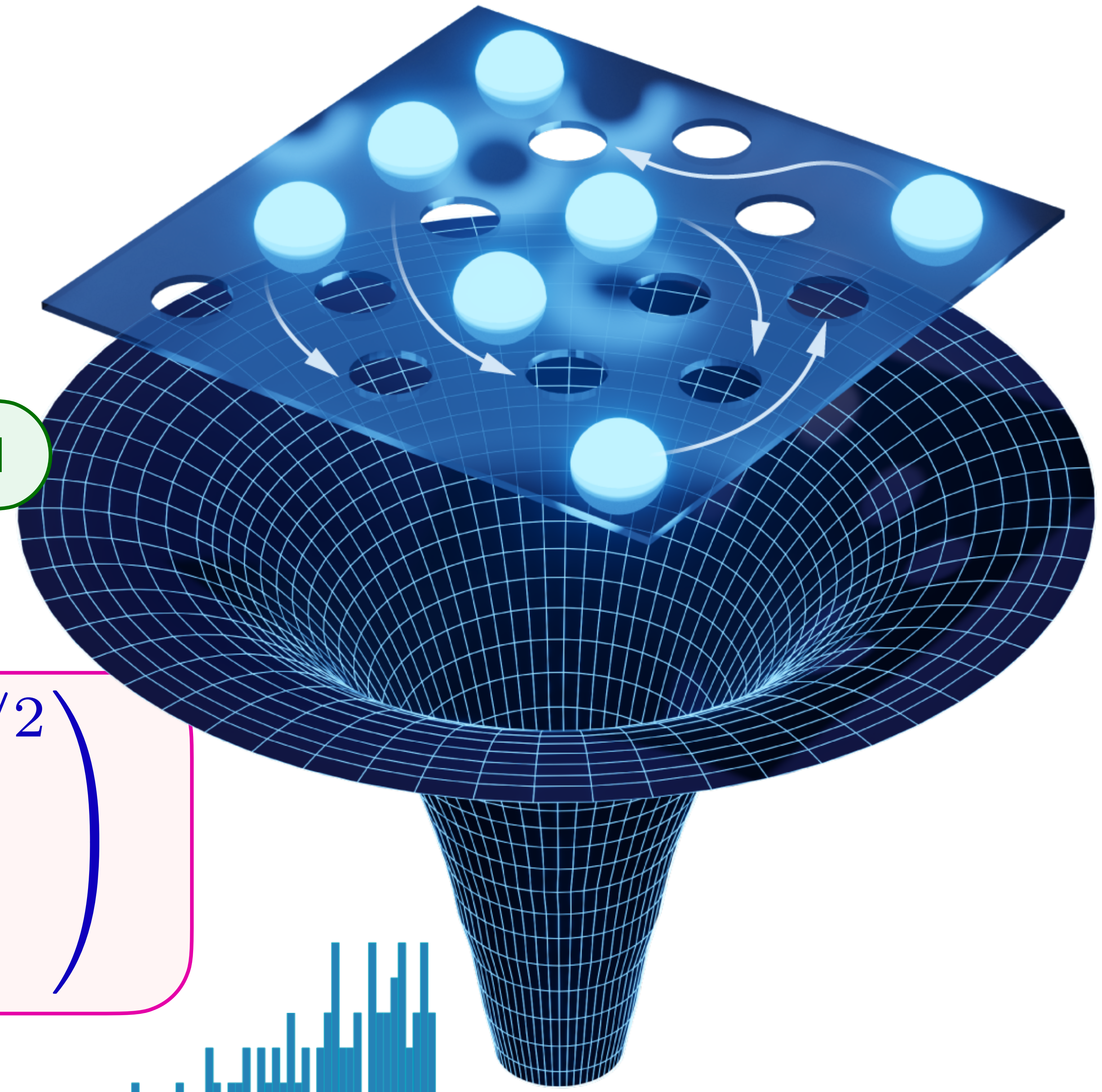

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Iliesiu, Murthy, Turiaci (2022) — Bekenstein-Hawking — Developments from the SYK model

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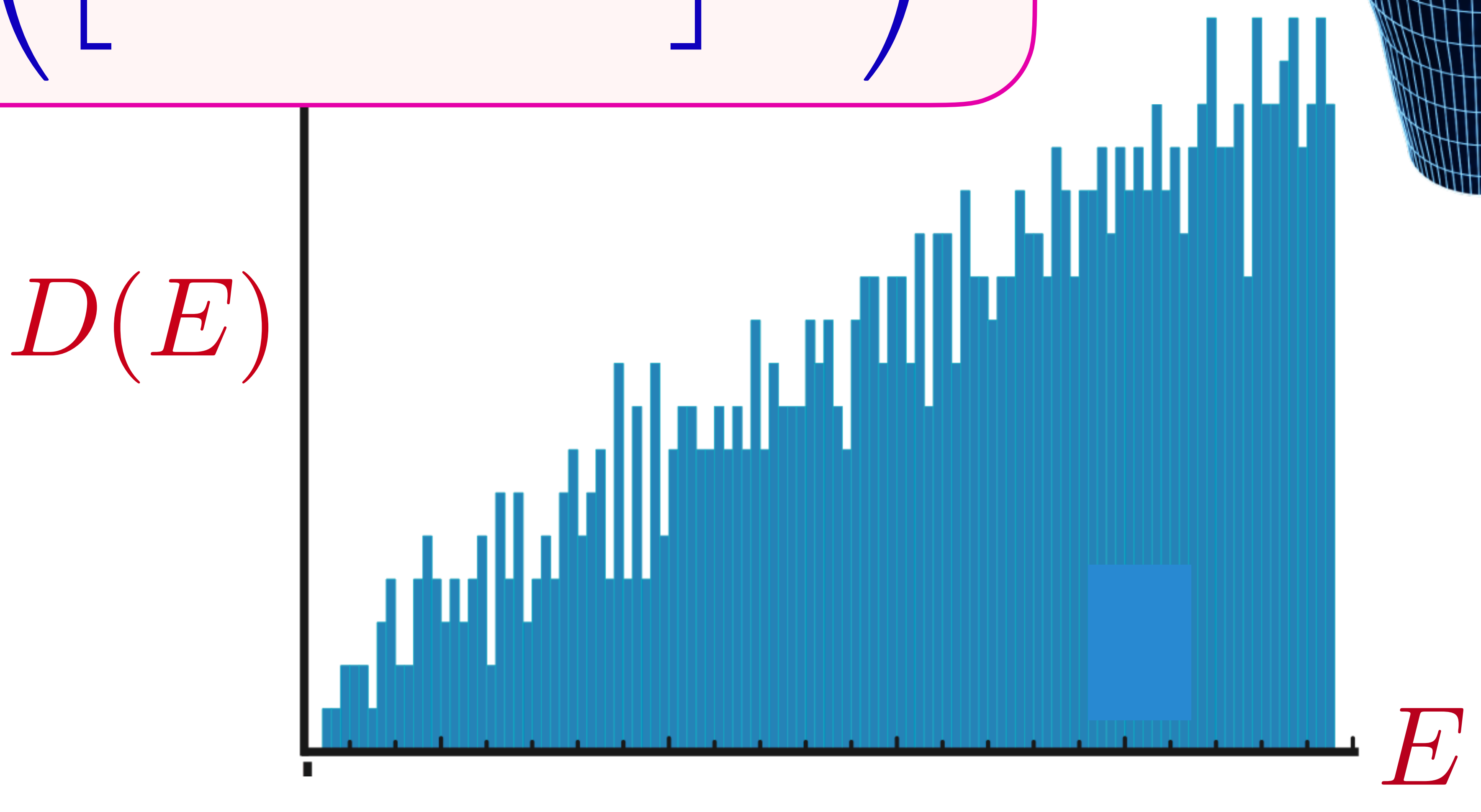


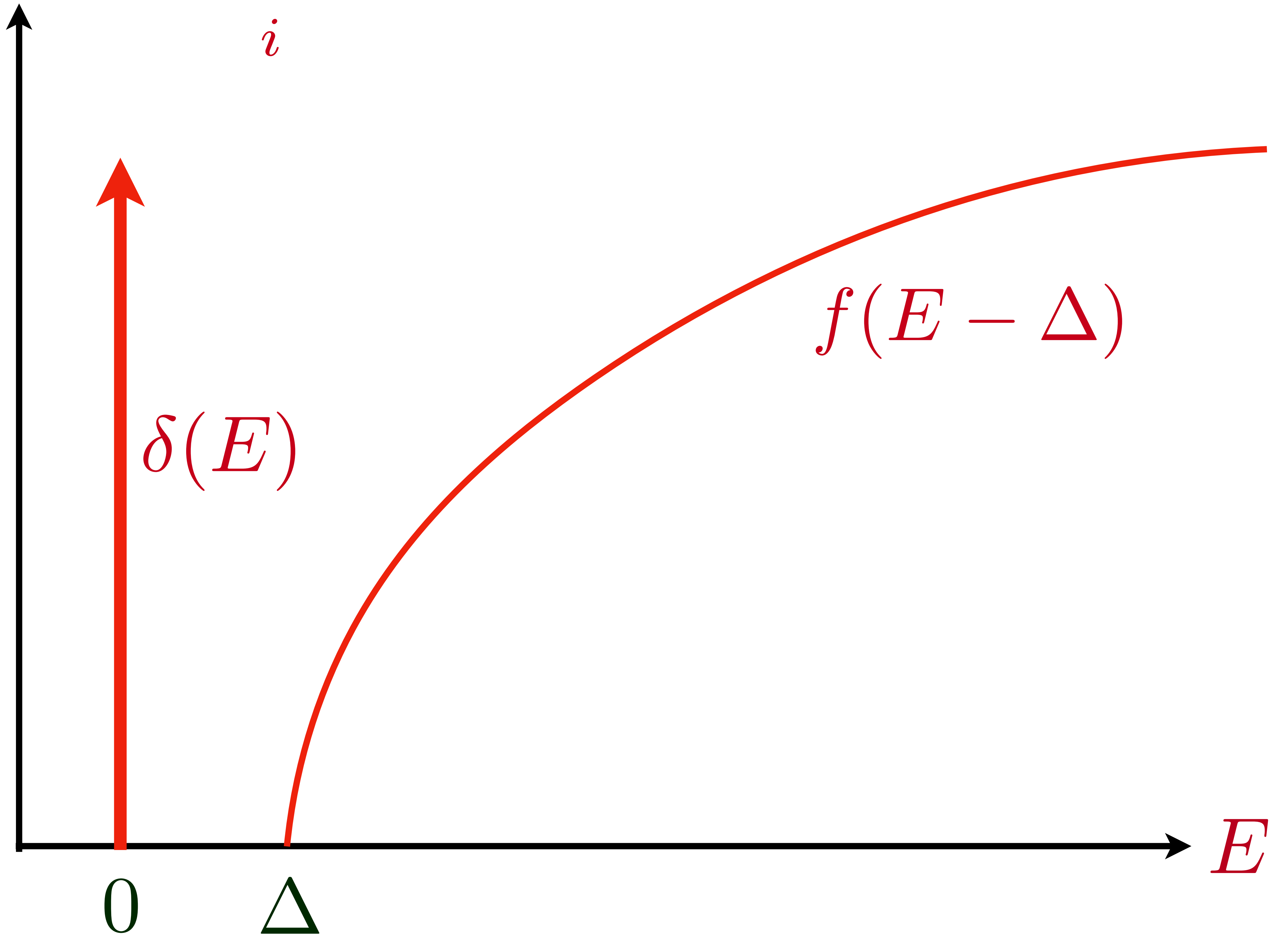
Strong theory of charged black holes

- With sufficient low energy supersymmetry, string theory yields:

$$D(E) = \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\delta(E) + \theta(E-\Delta) f(E-\Delta) + \ldots$$

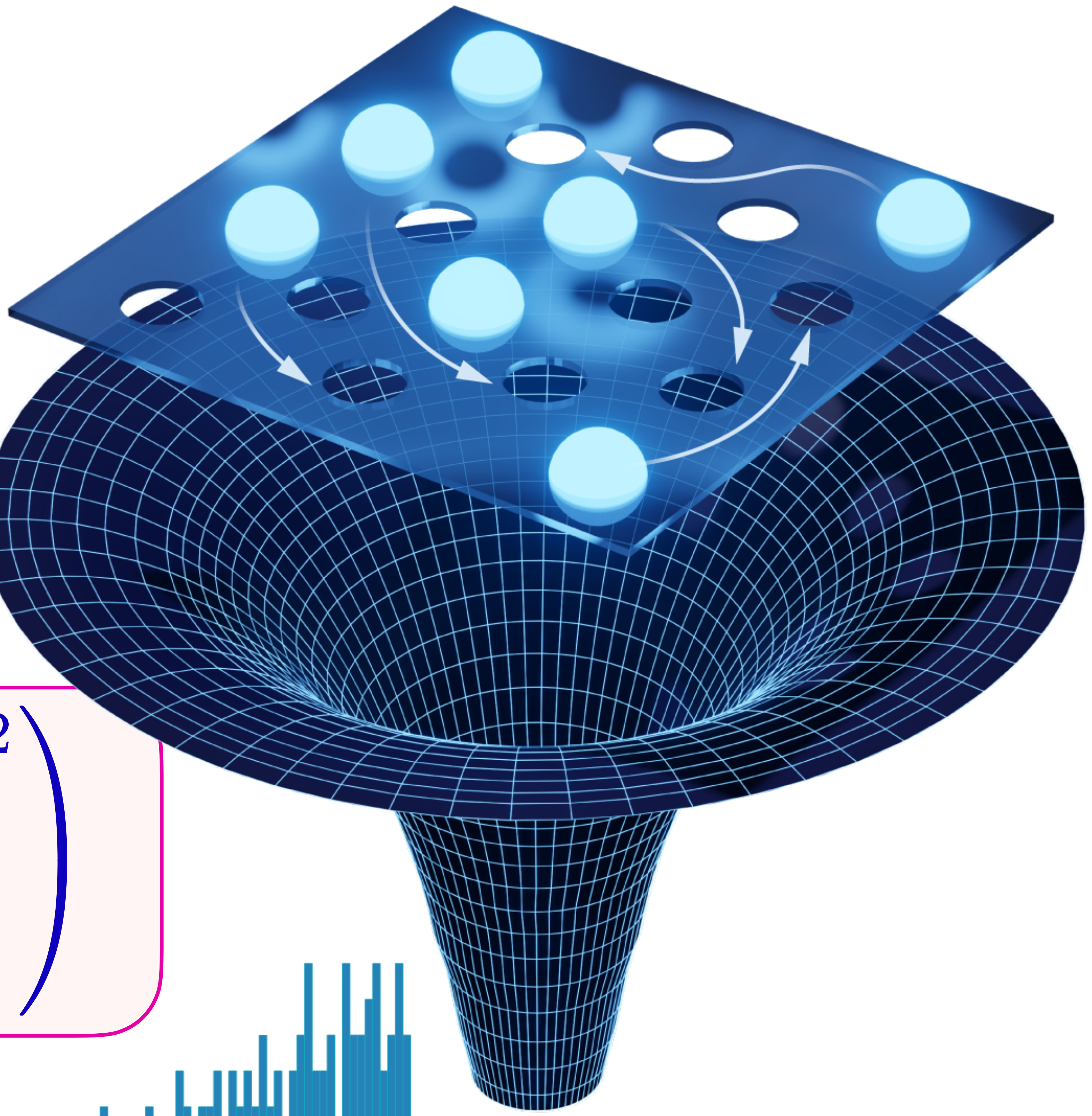
There are exponentially many degenerate BPS ground states, and an energy gap Δ above the ground state.

$$D(E) = \sum_i \delta(E - E_i)$$



M. Heydeman, L.V. Iliesiu, G. J. Turiaci, and W. Zhao, 2020
L.V. Iliesiu, S. Murthy, G. J. Turiaci, 2022

Quantum simulation of charged black holes by the SYK model



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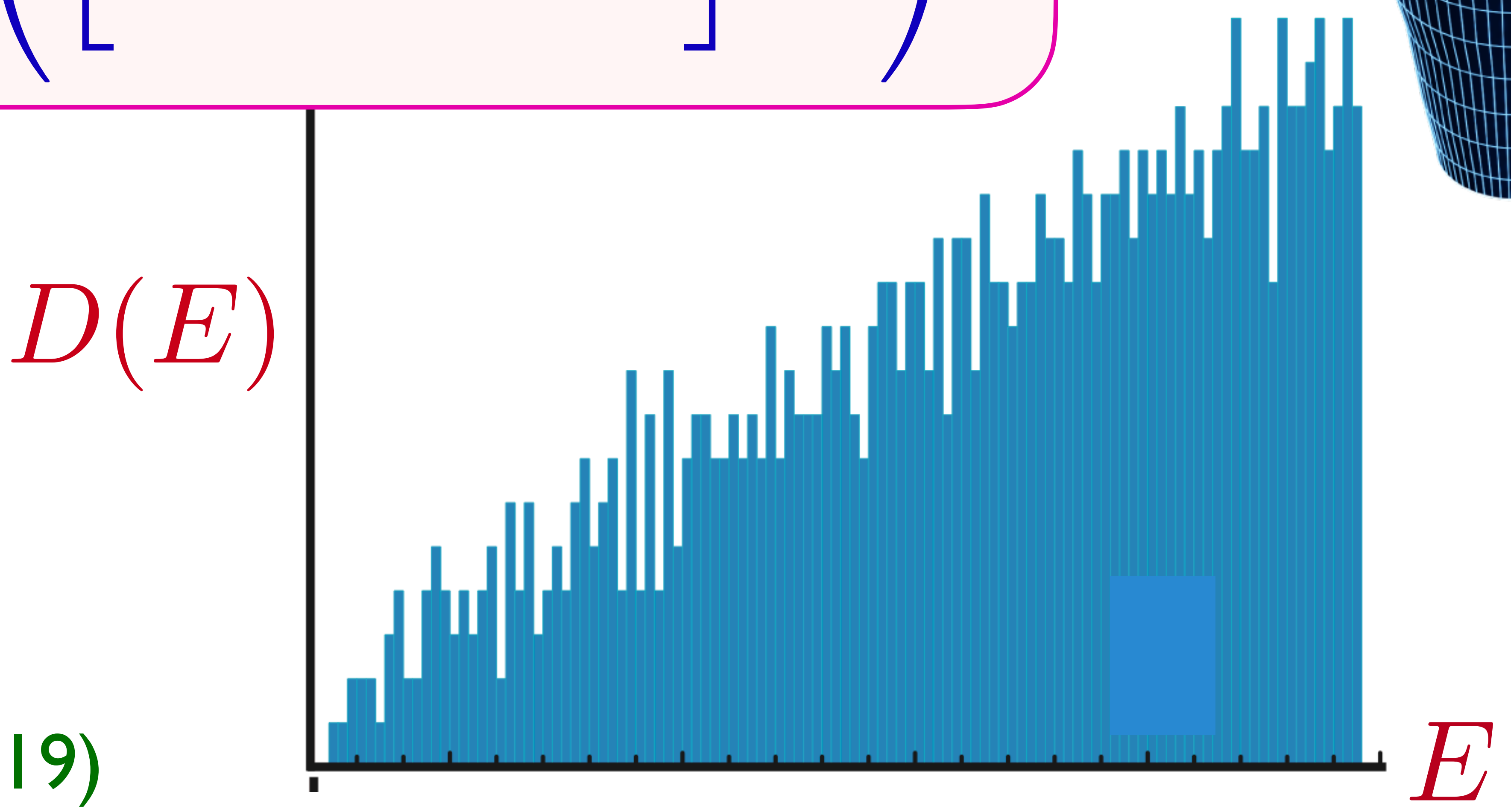
- 'Wormhole' contributions to this quantum simulation have led to an understanding of the Page curve of entanglement entropy of evaporating black holes.

Saad, Shenker, Stanford (2019)



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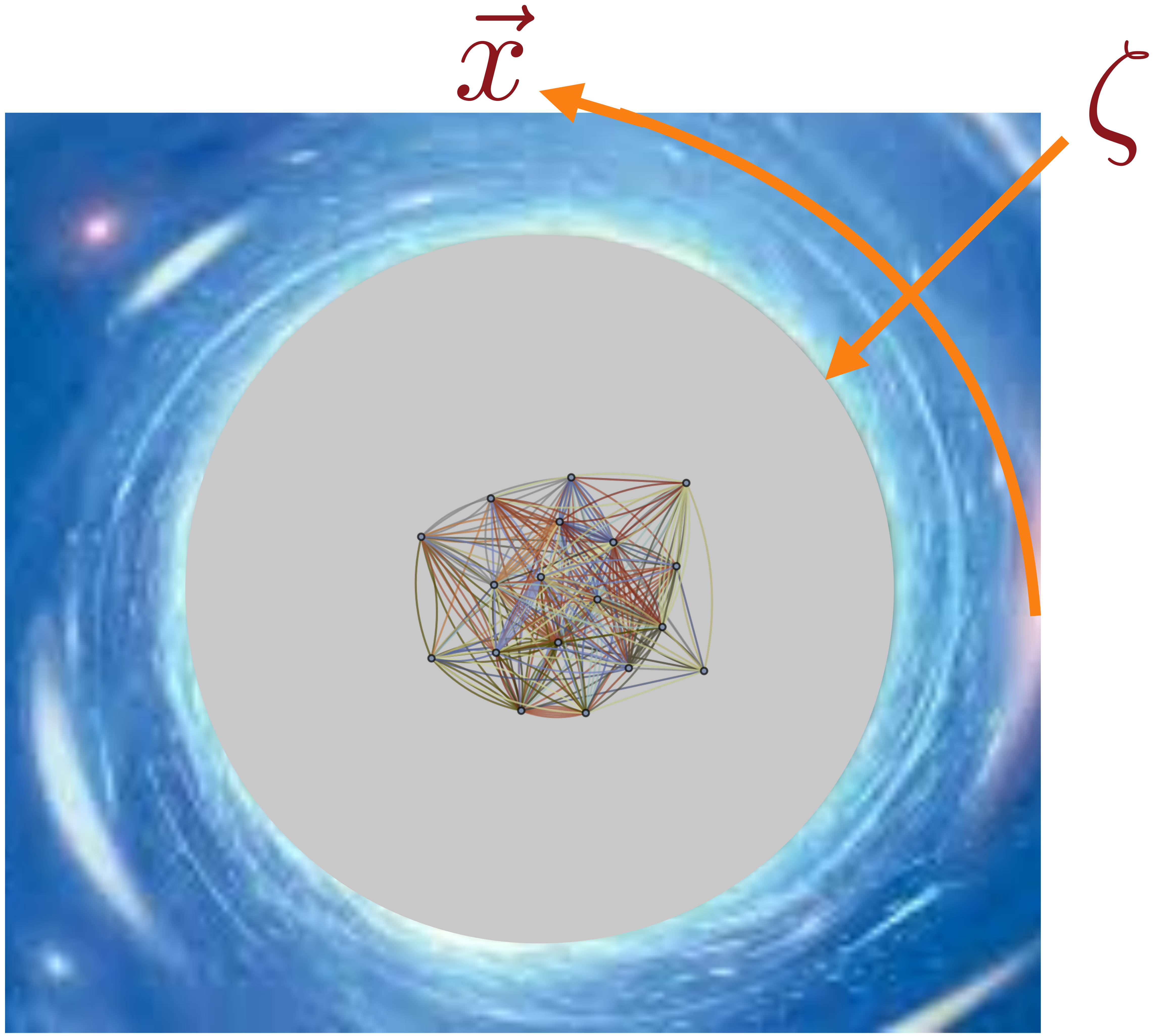


The SYK provides the needed realization of the black hole interior, and its density of quantum states matches gravitational entropy computations for charged black holes !

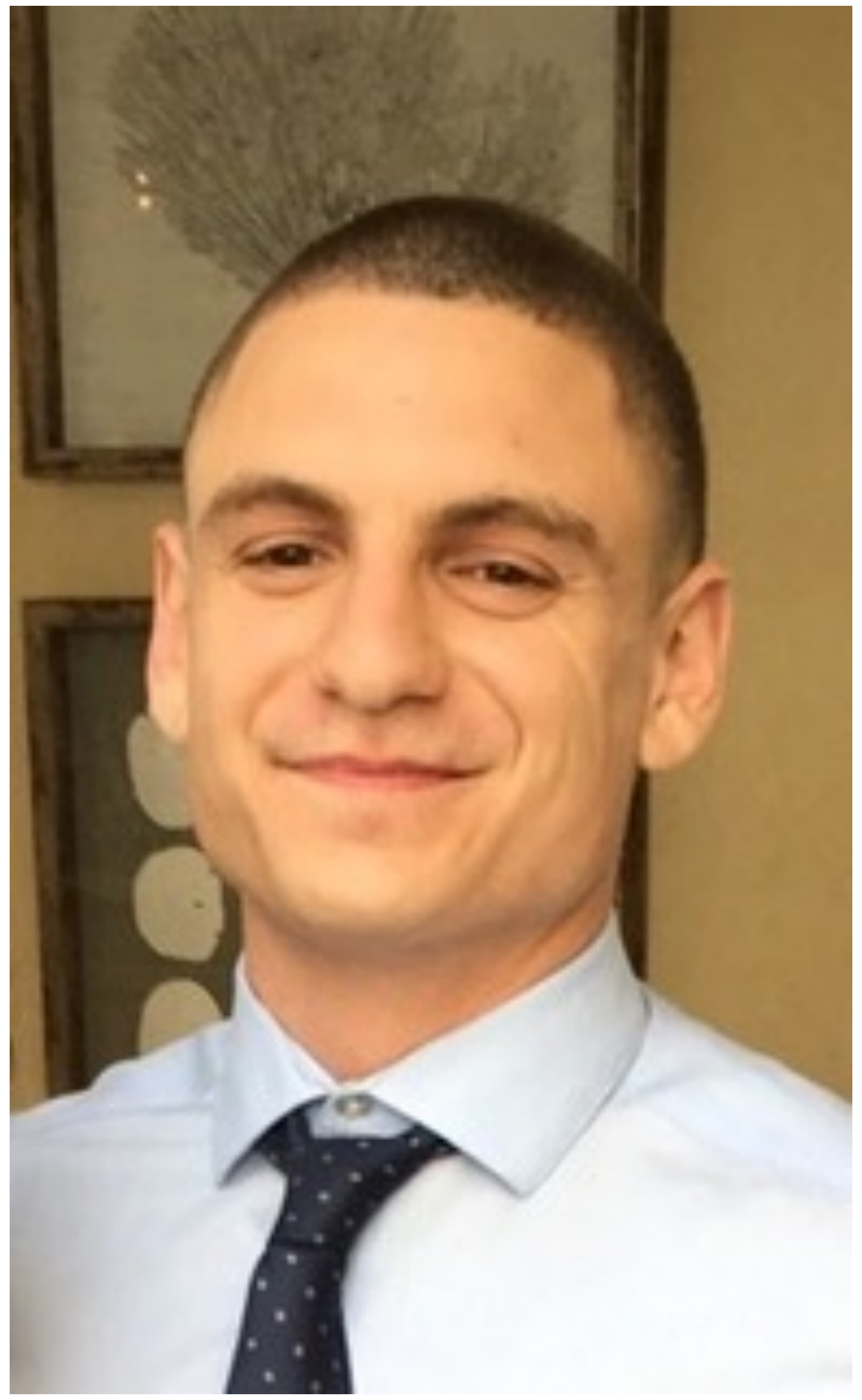
From the SYK model to a universal theory of strange metals

Aavishkar Patel

Flatiron Institute, NYC

Haoyu Guo

Harvard

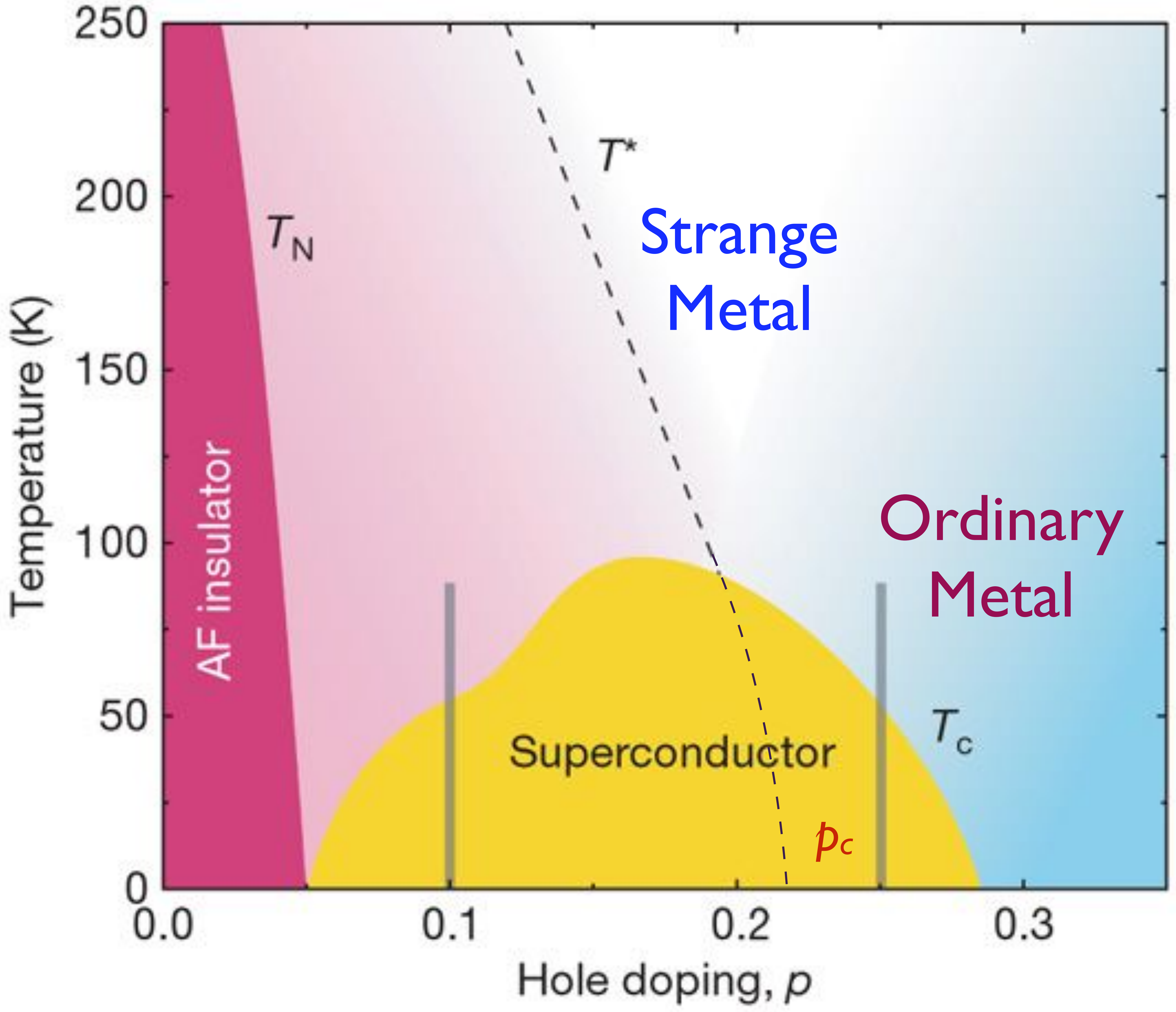


Ilya Esterlis

Harvard ➔ Wisconsin

Science to appear, arXiv: 2103.08615, 2203.04990, 2207.08841

E. E. Aldape, T. Cookmeyer, Aavishkar A. Patel, and Ehud Altman, arXiv:2012.00763



250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
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Hole doping, p
AF insulator
T_N
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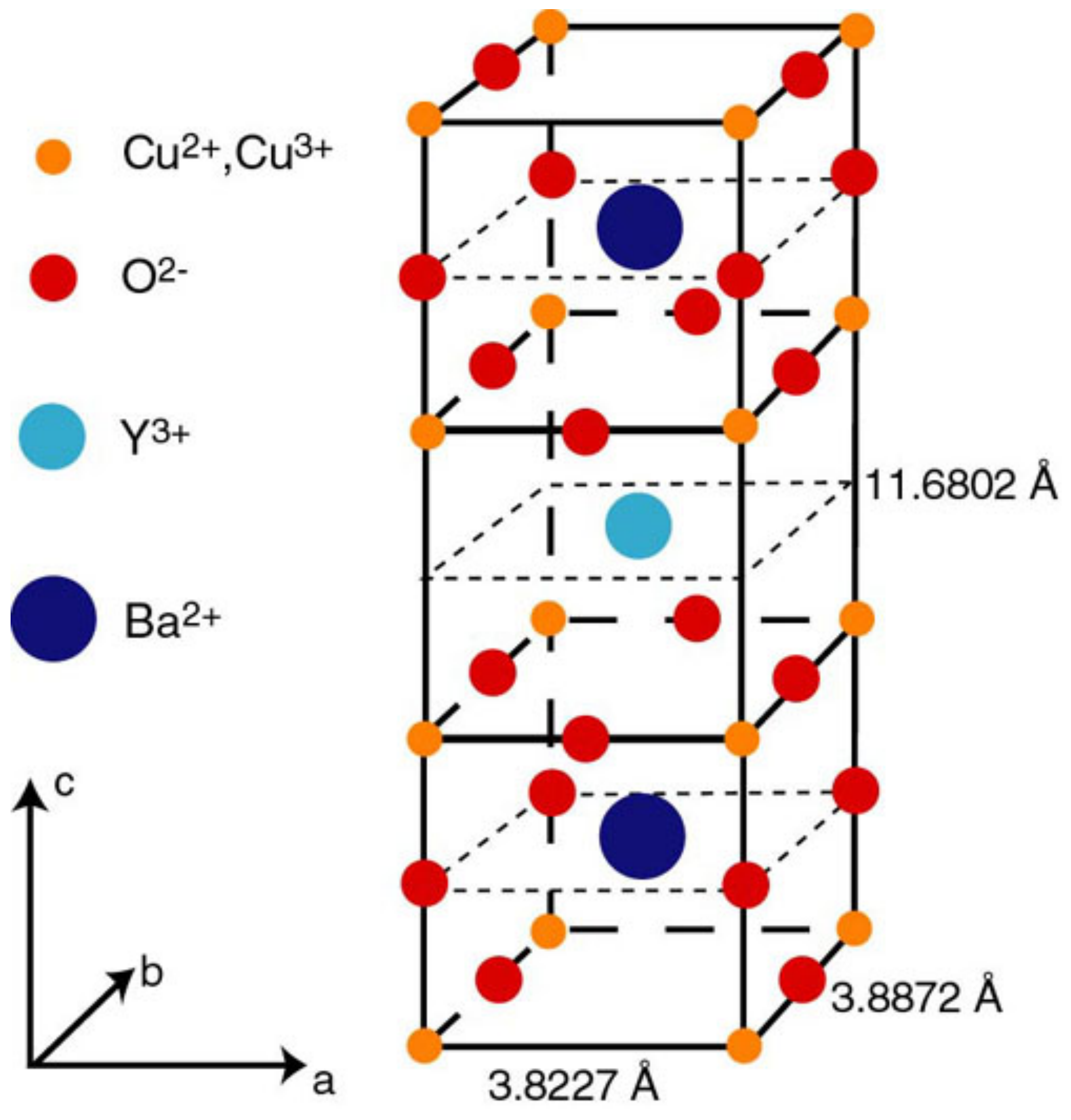

Cu2+,Cu3+
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11.6802 Å
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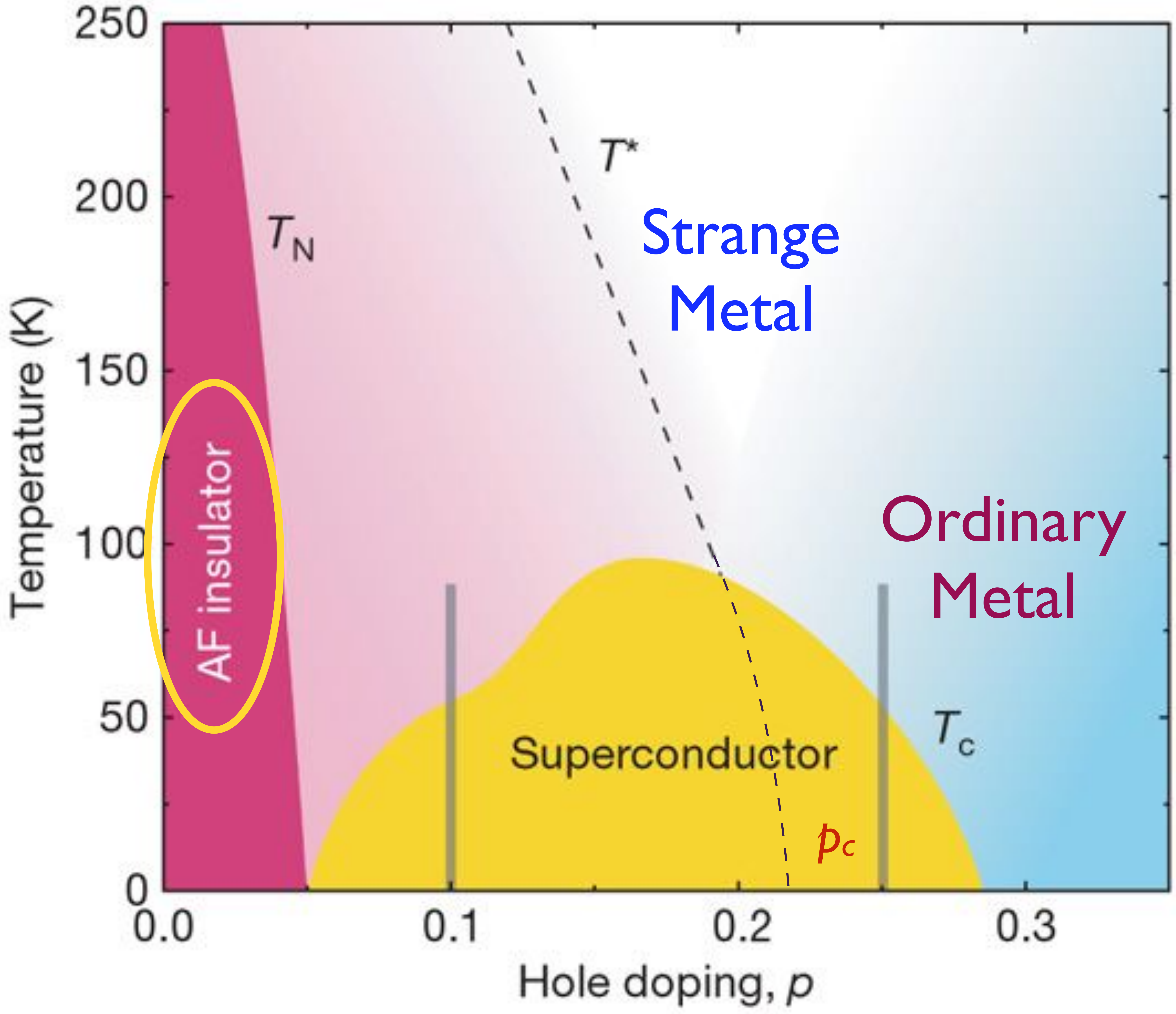
$$YBa_2Cu_3O_{6+x}$$


250
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0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
T*
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Strange
Metal
AF insulator
Ordinary
Metal
Superconductor
T_c
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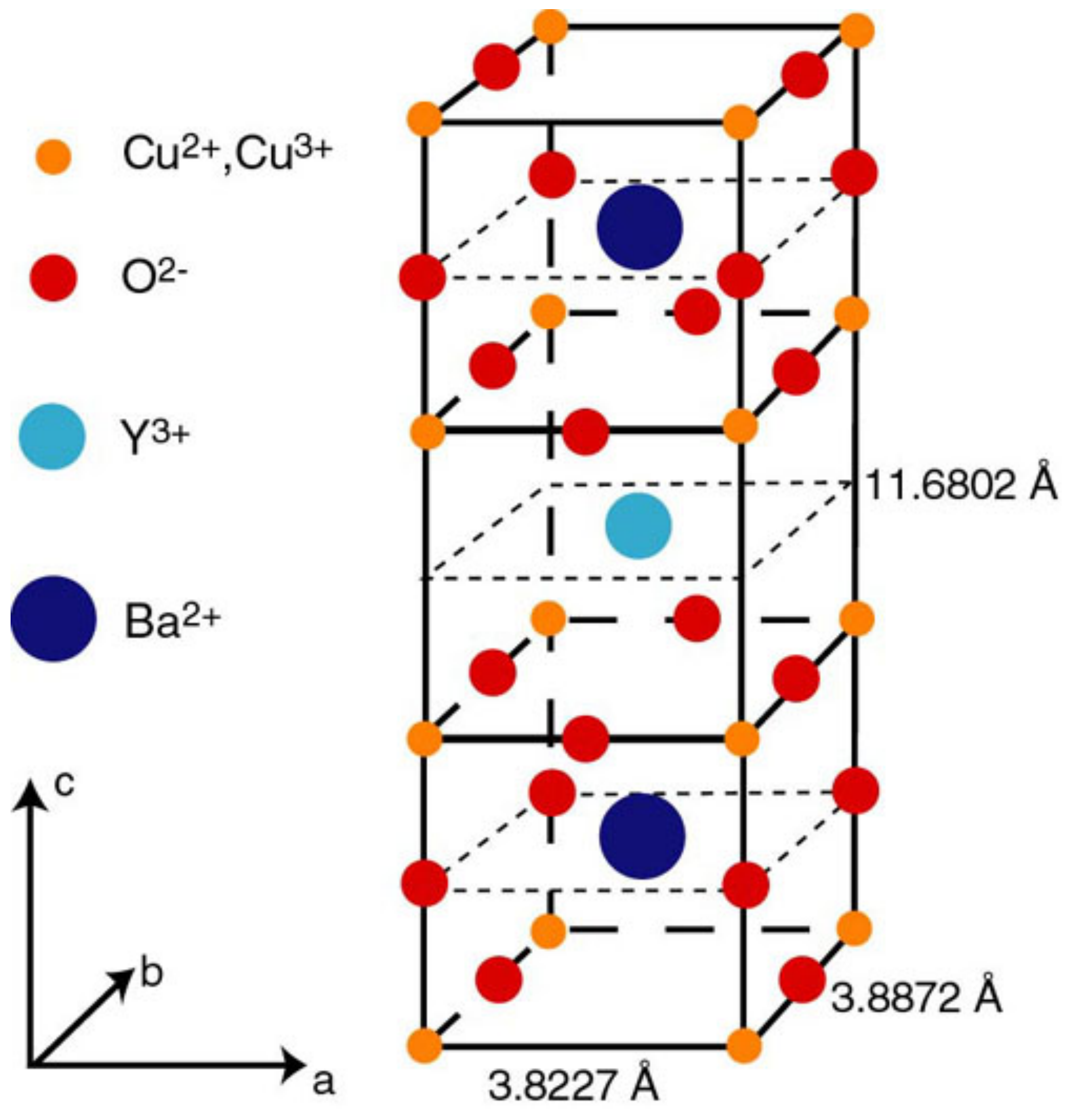
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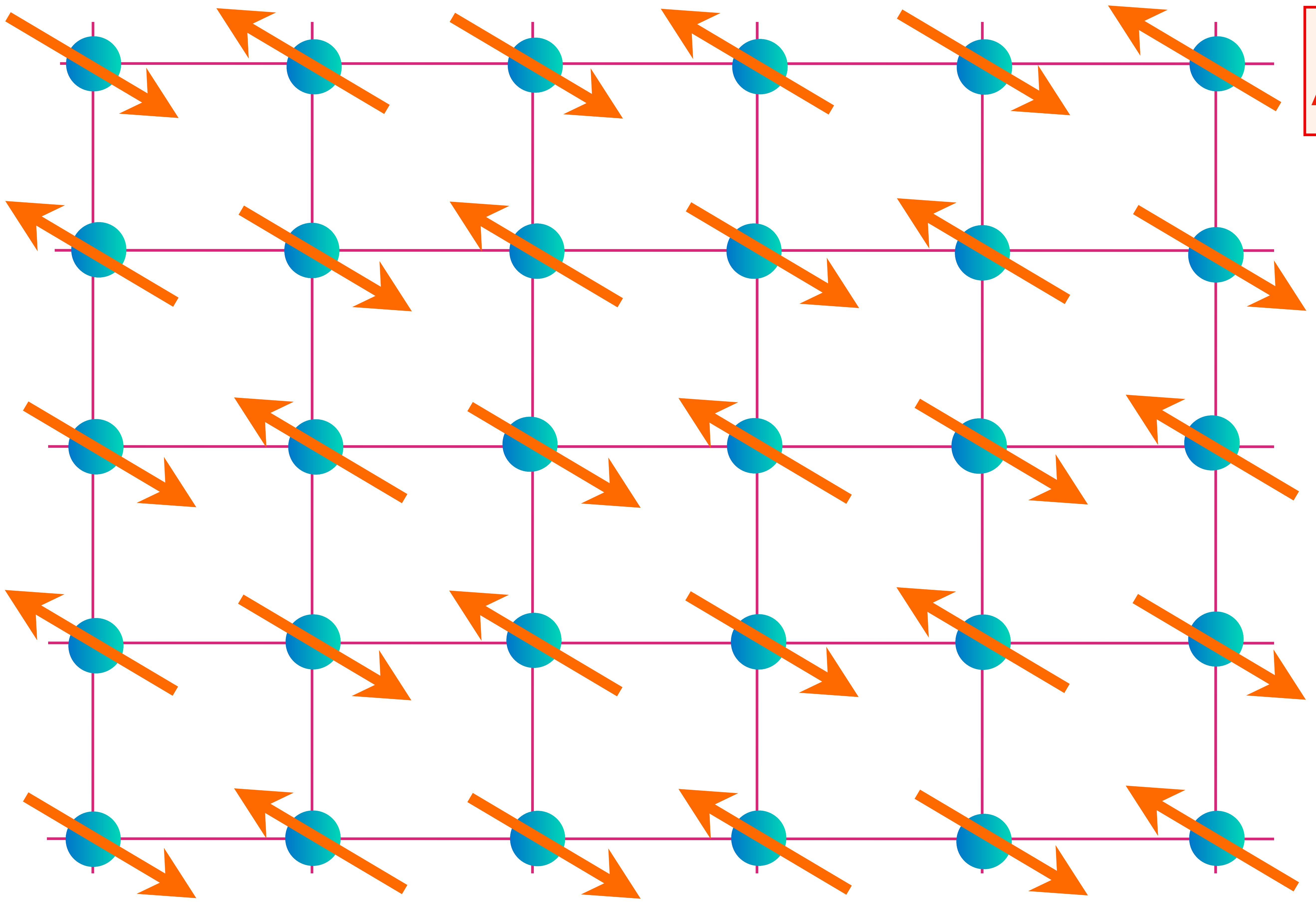
The dance of electrons on Cu atoms in YBCO



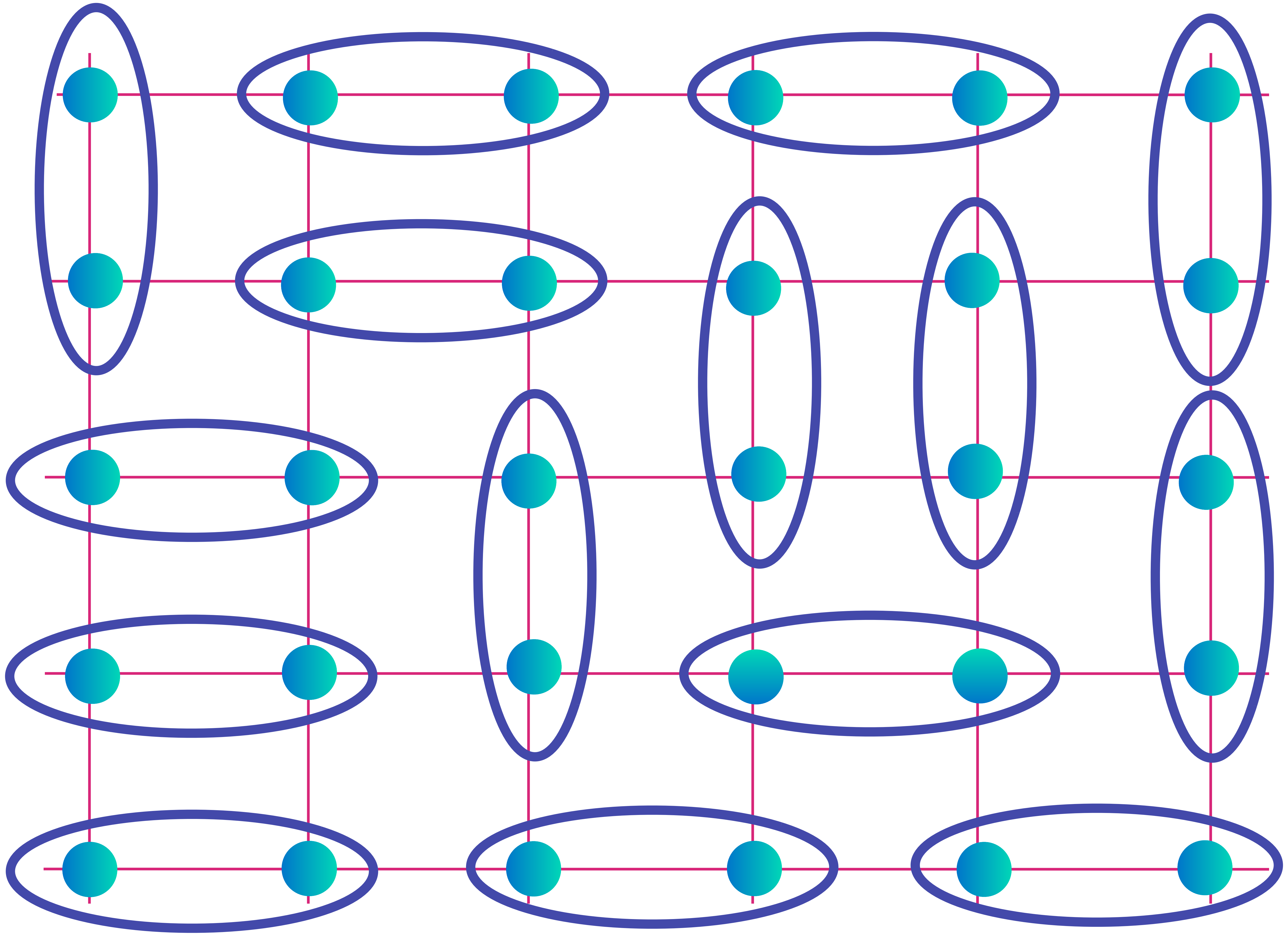
Antiferromagnetism

All nearest-neighbor pairs of electrons have opposite spins

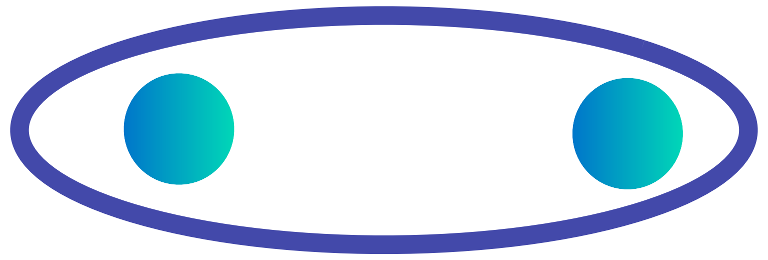
The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)

Spin liquid



Electrons form entangled pairs, and the pairs entangle across the entire sample



$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

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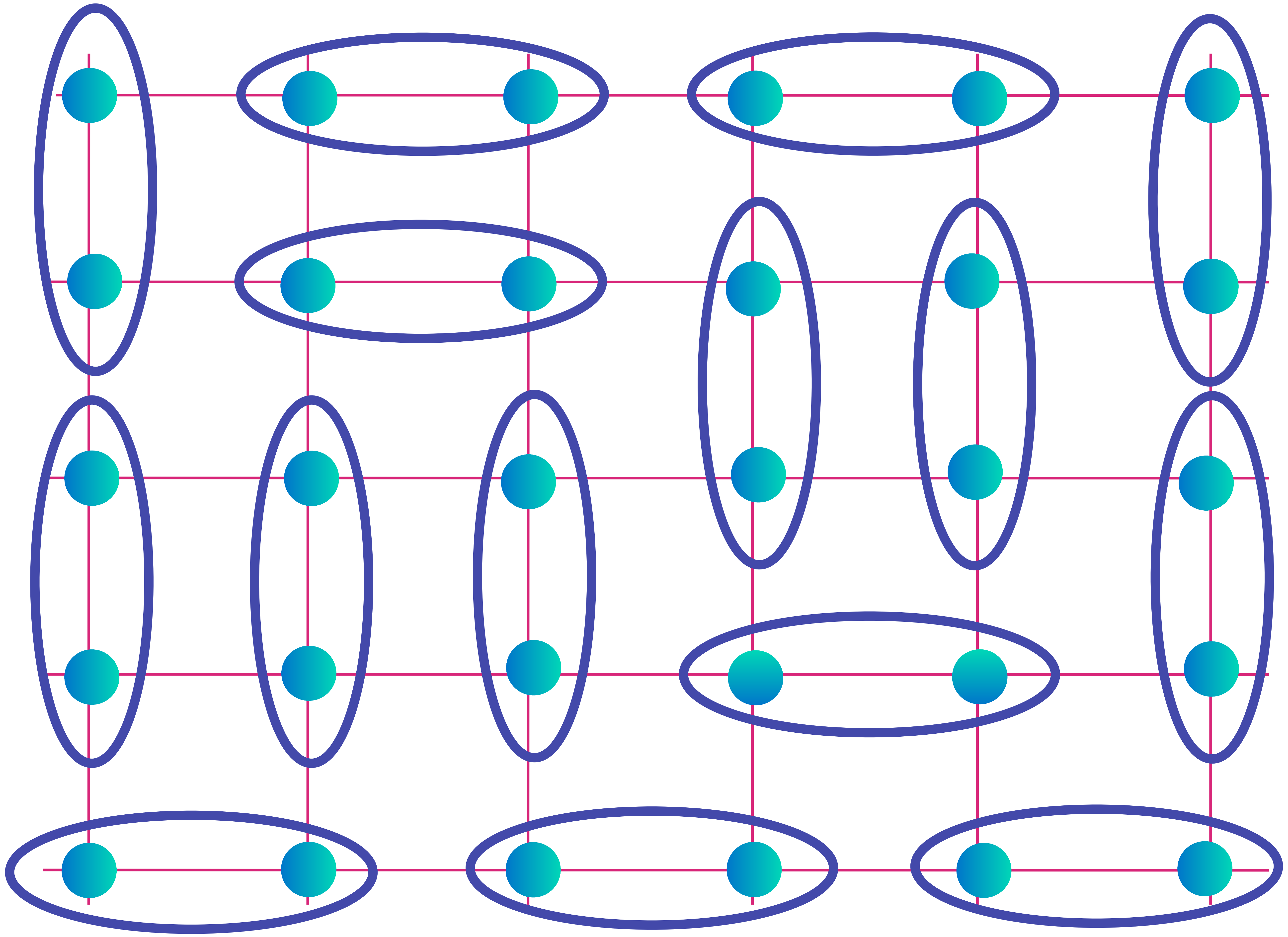

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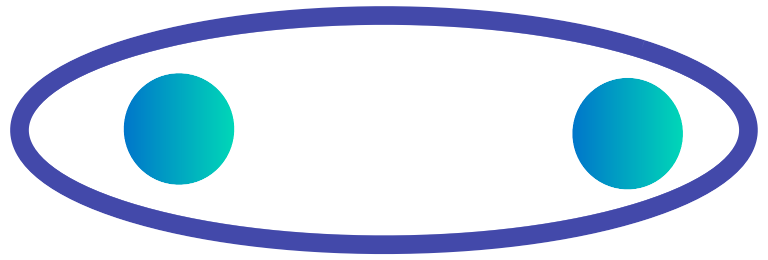 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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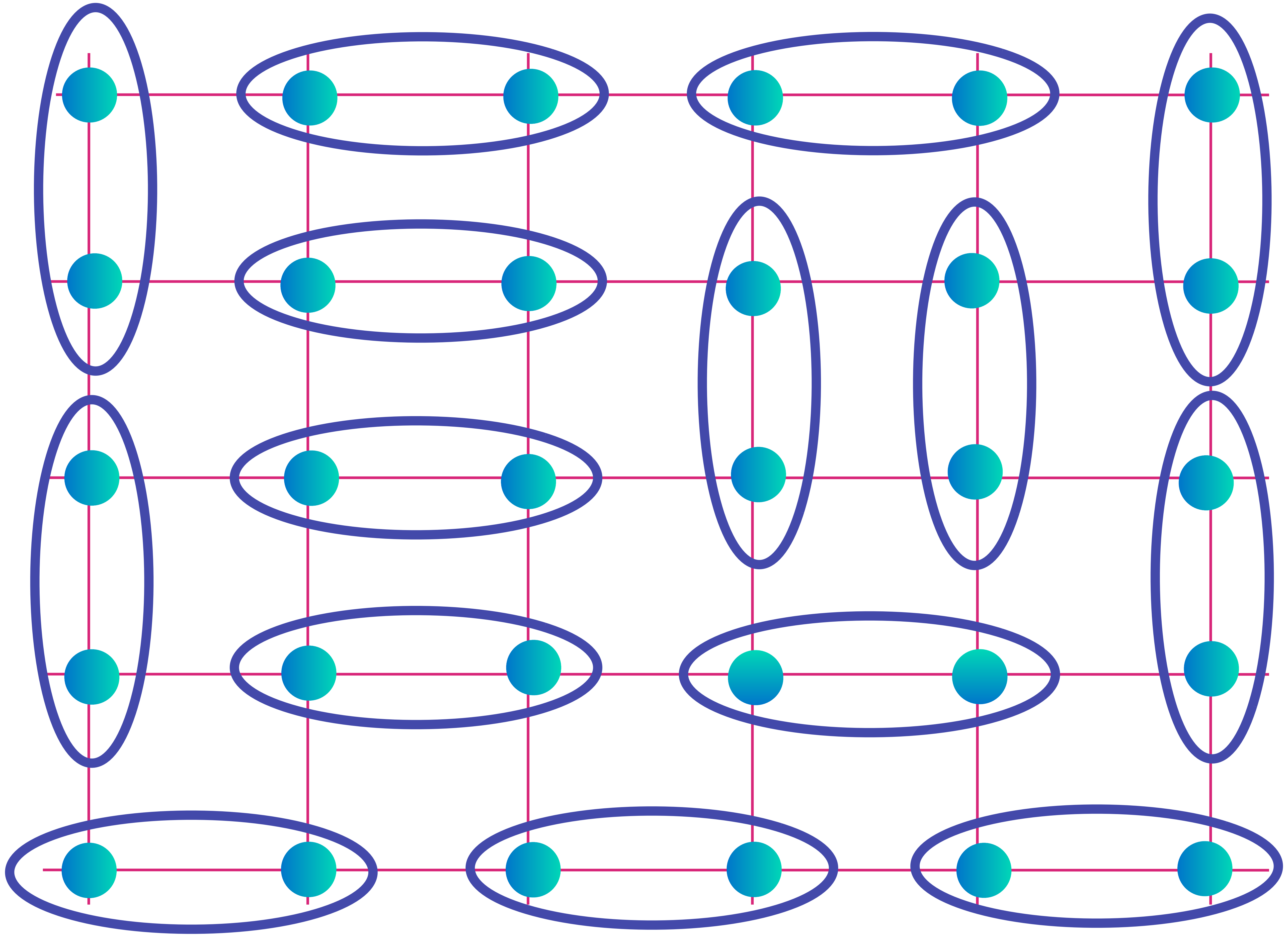

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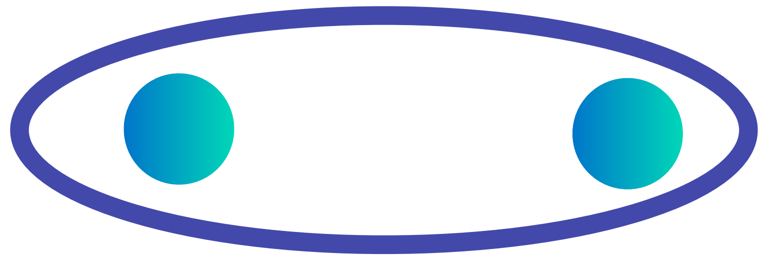 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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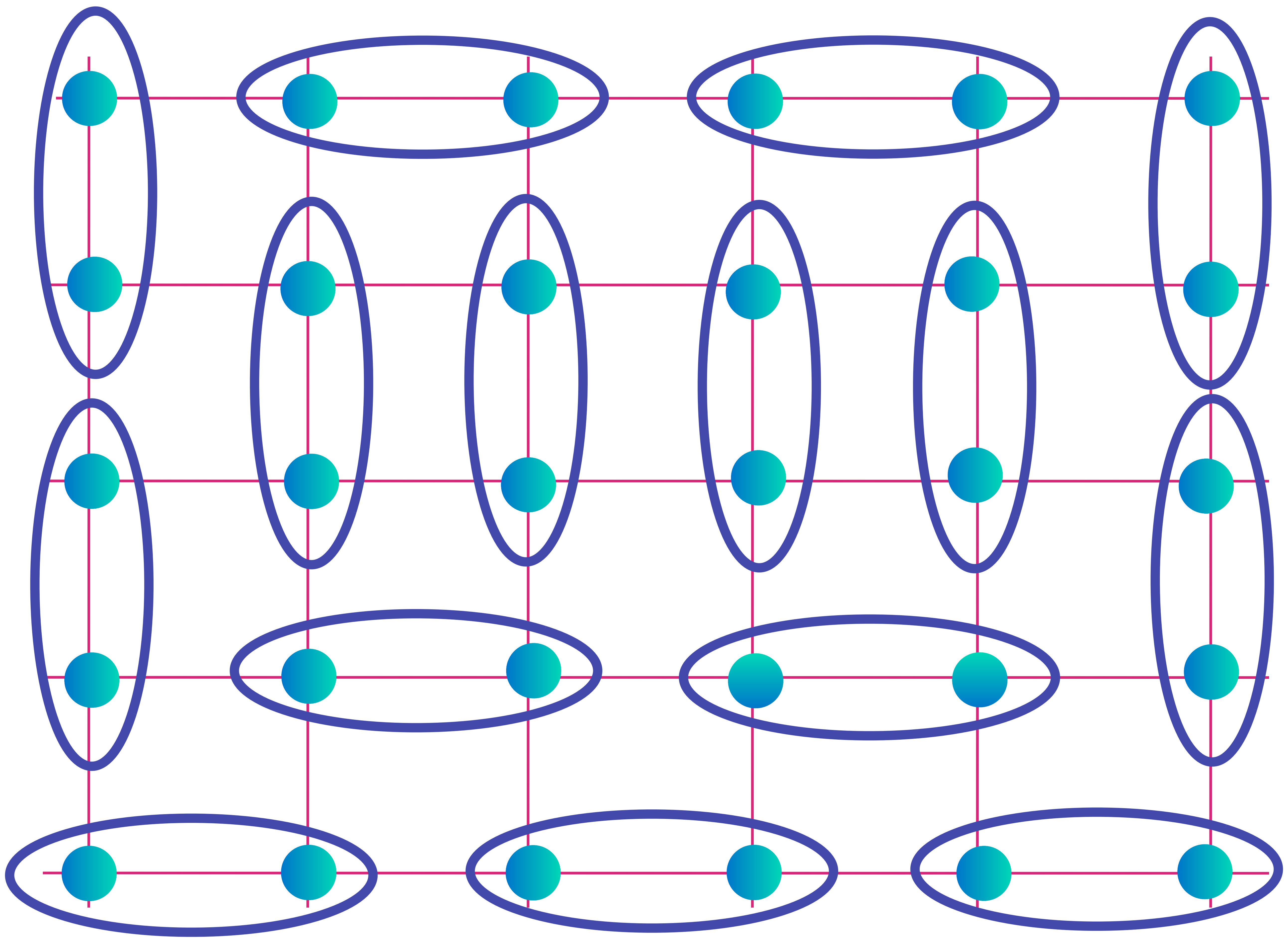

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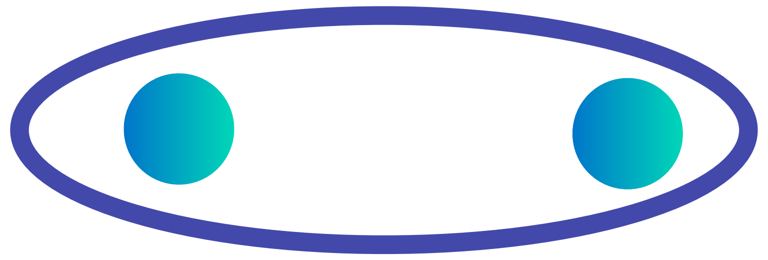

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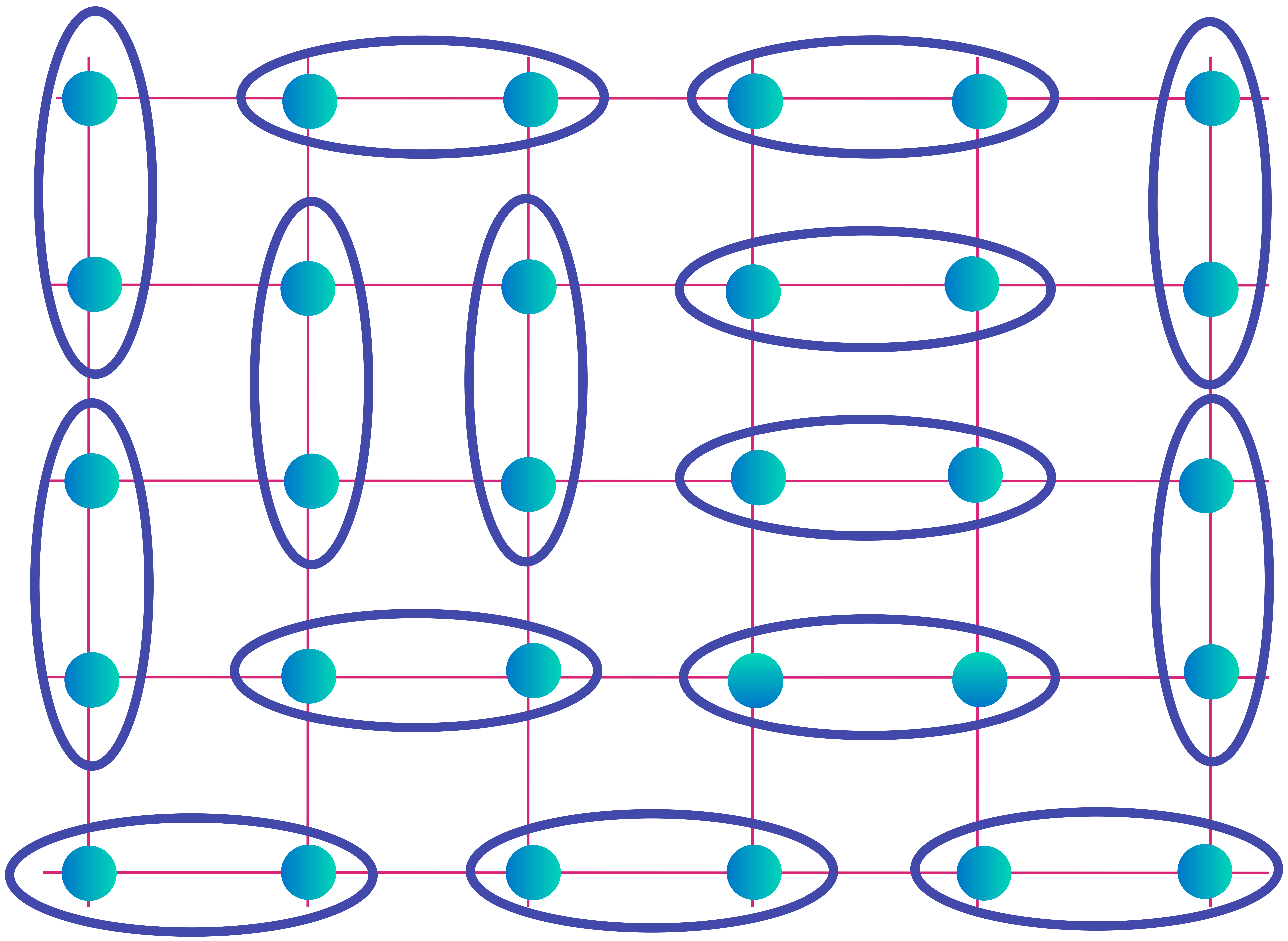

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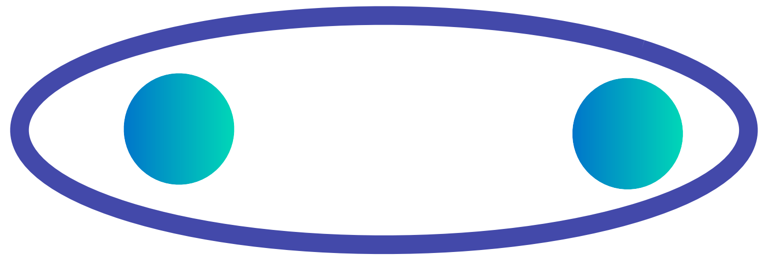

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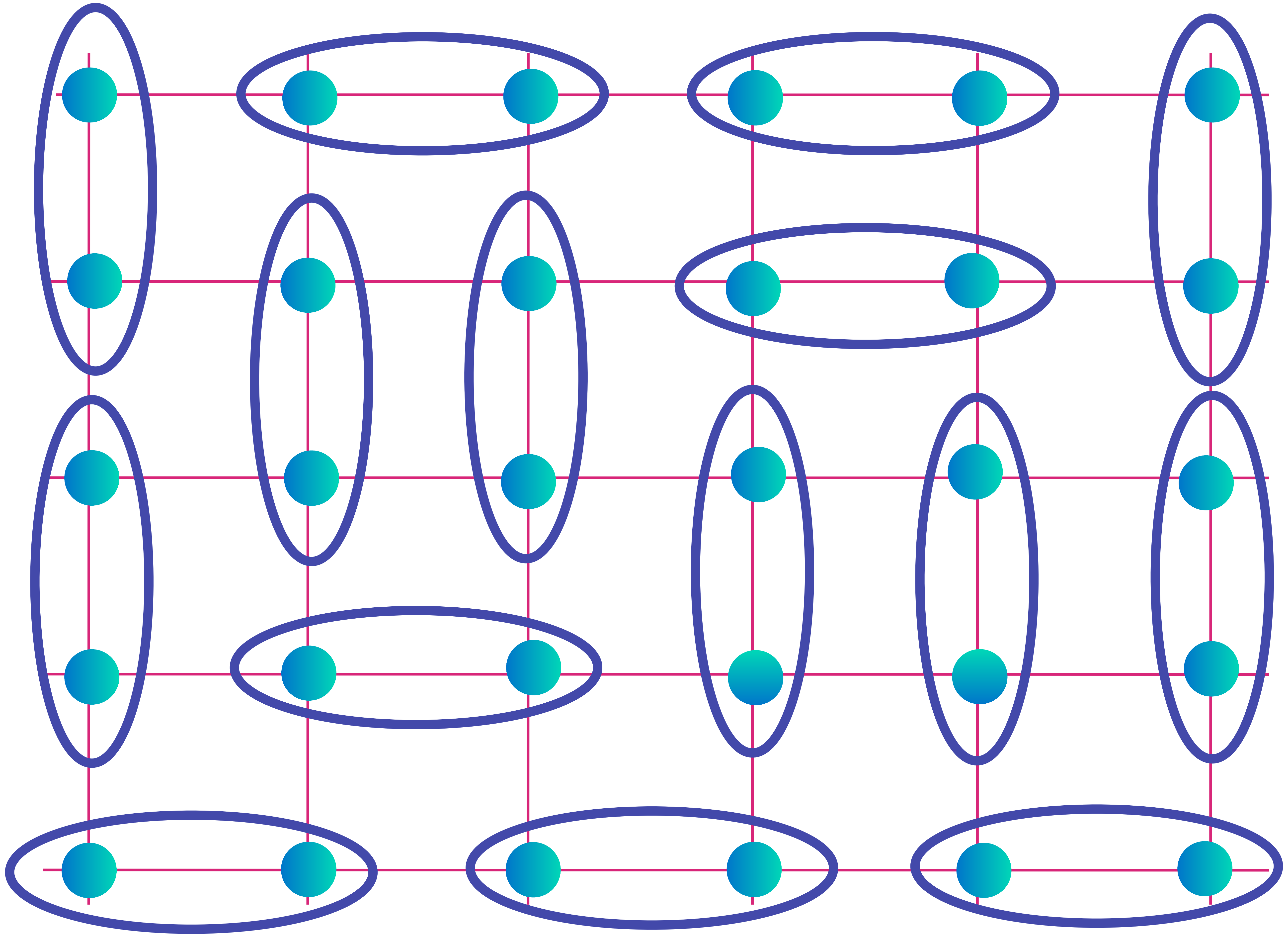

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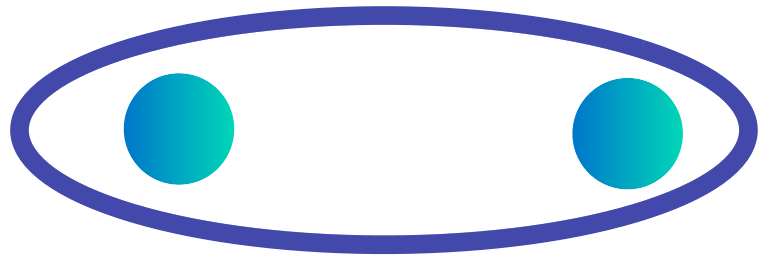 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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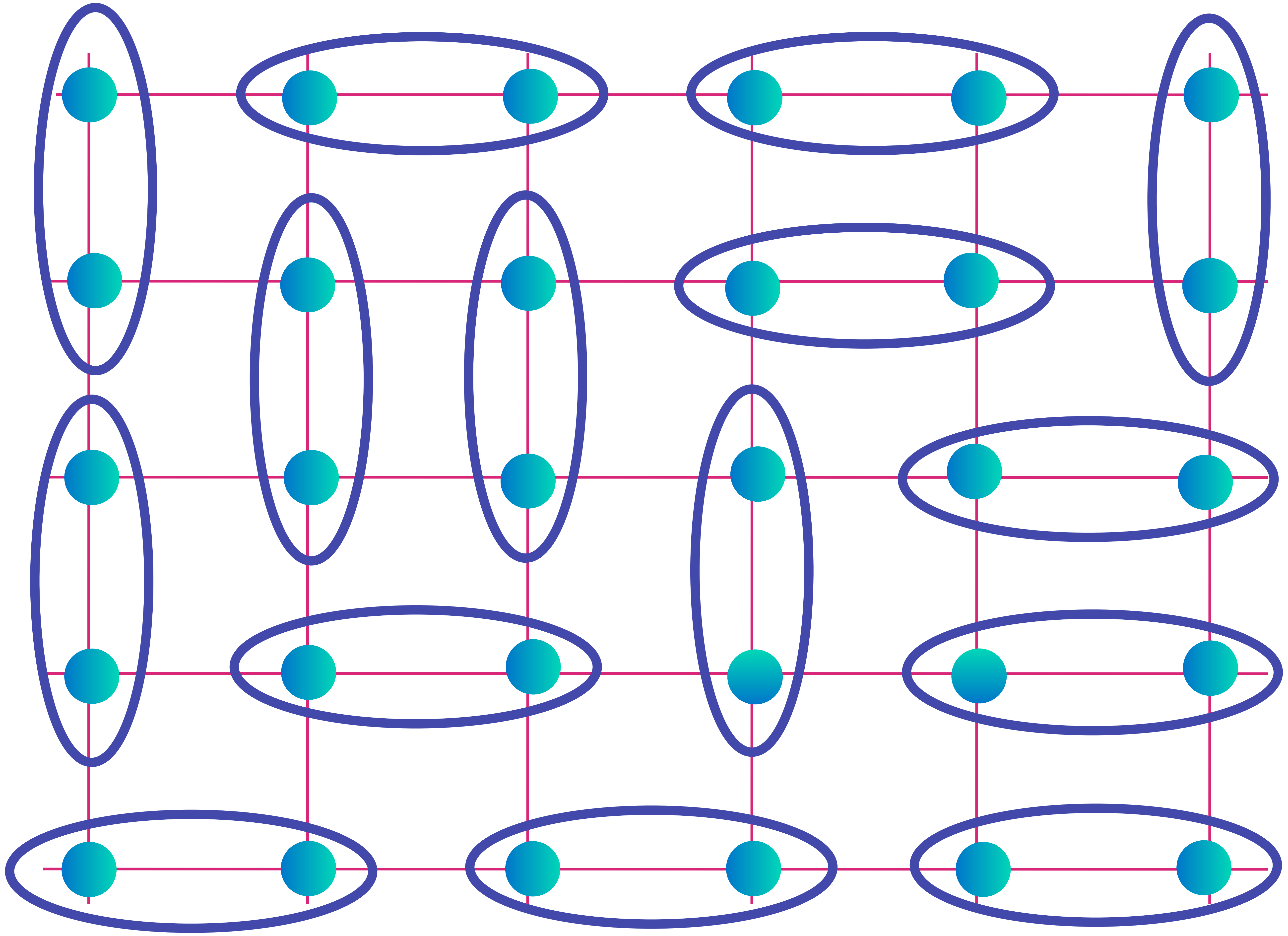

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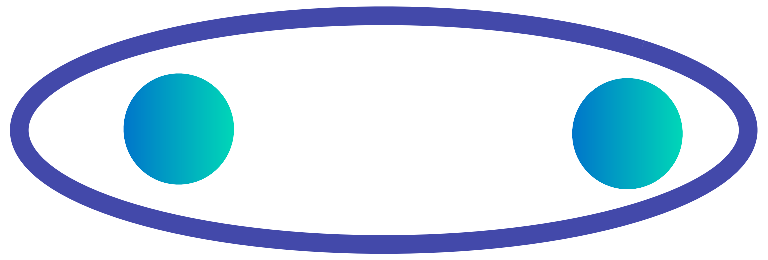

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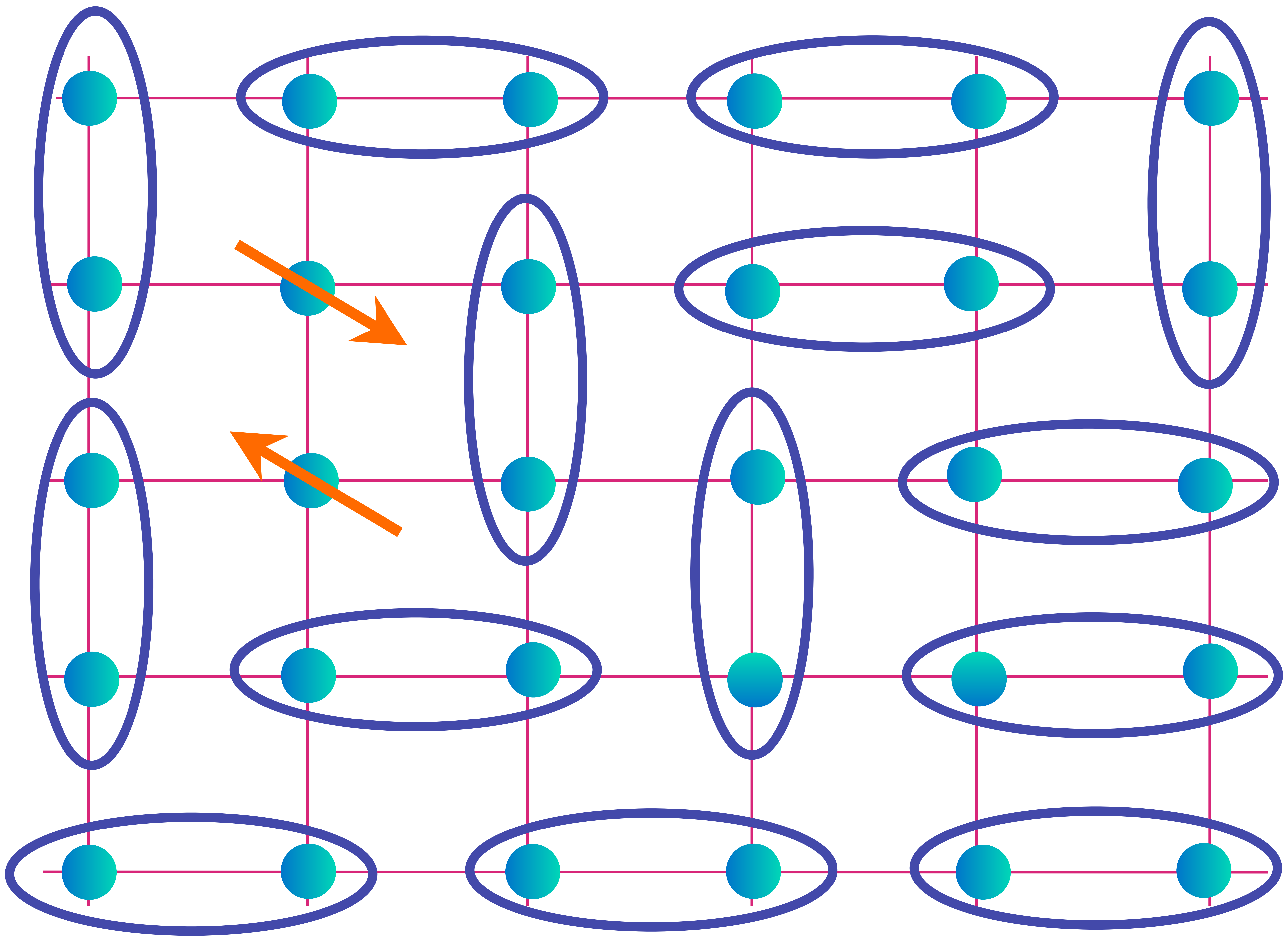
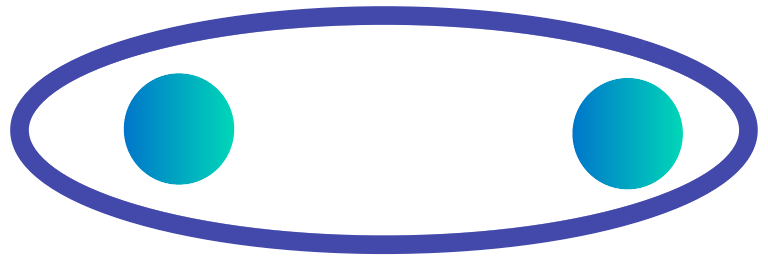
Kivelson (1987)



Fractionalization

A spinon:
an emergent
particle which
carries spin 1/2
but no charge



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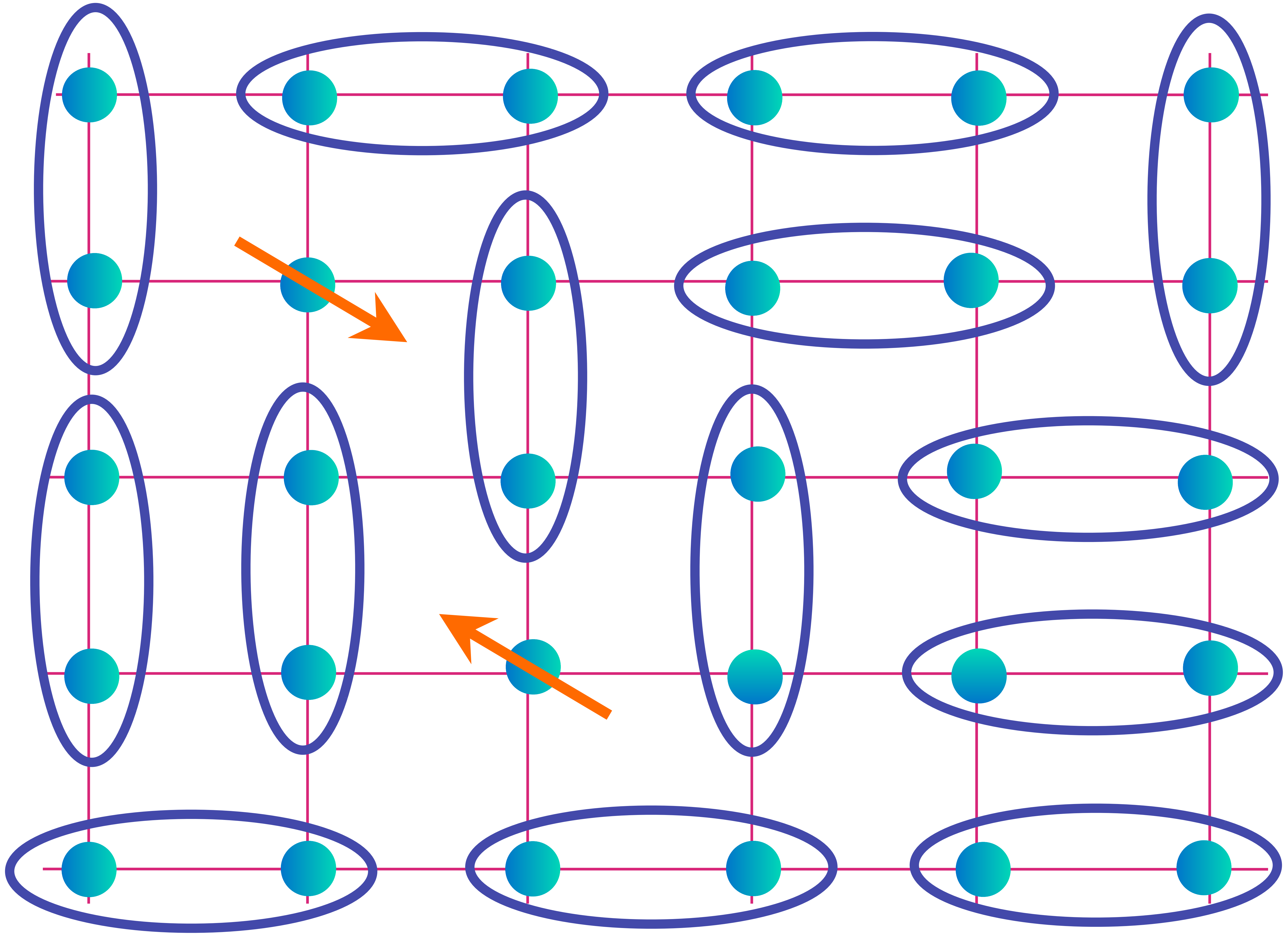

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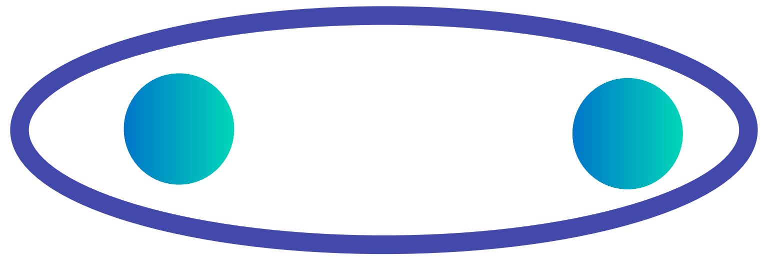 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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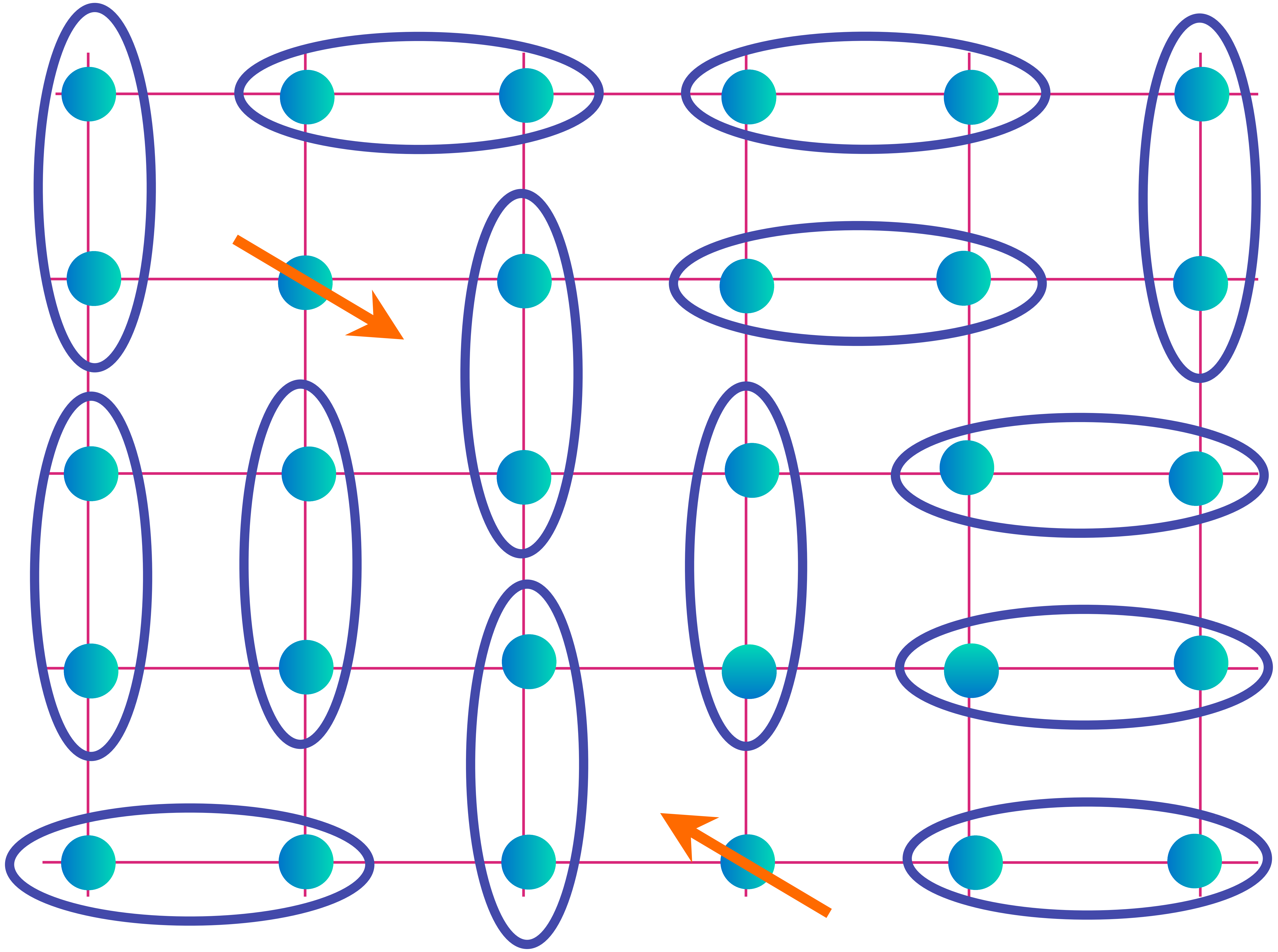

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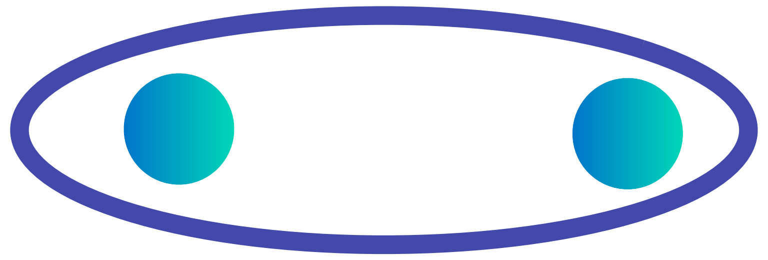

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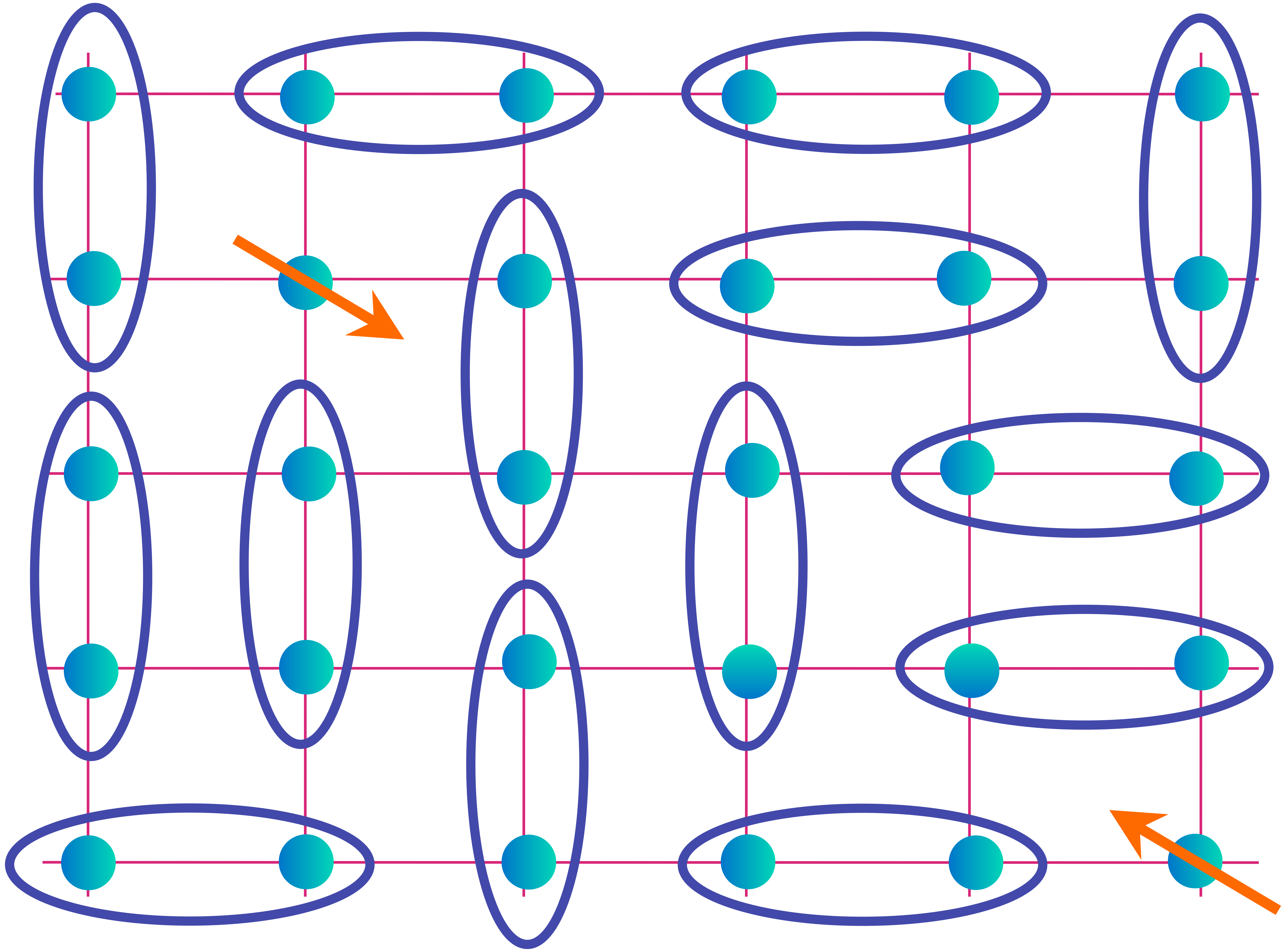

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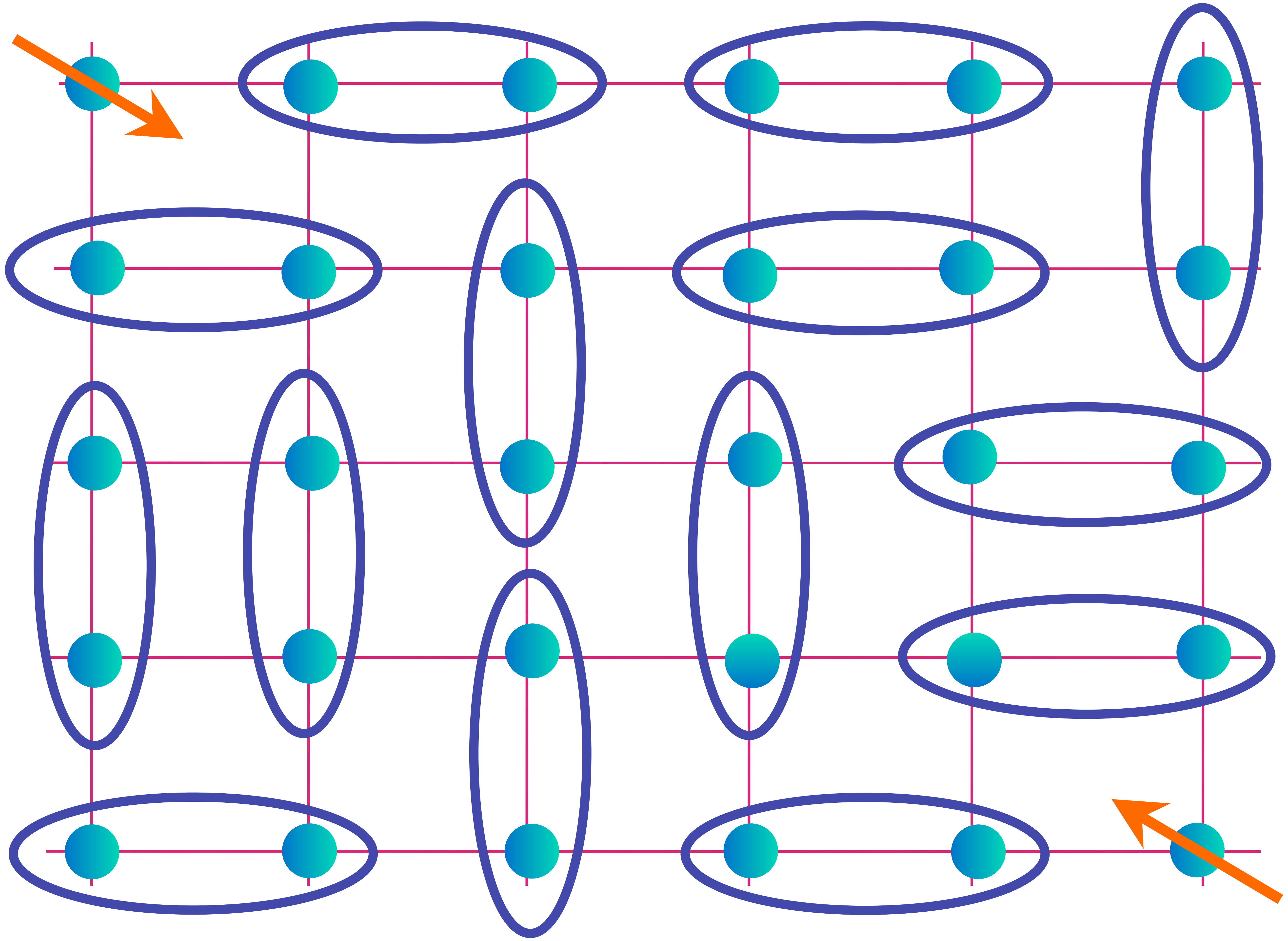

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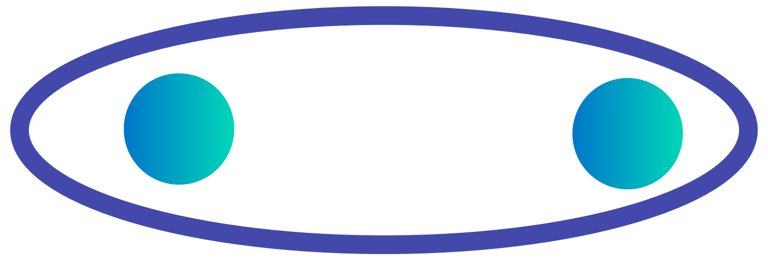

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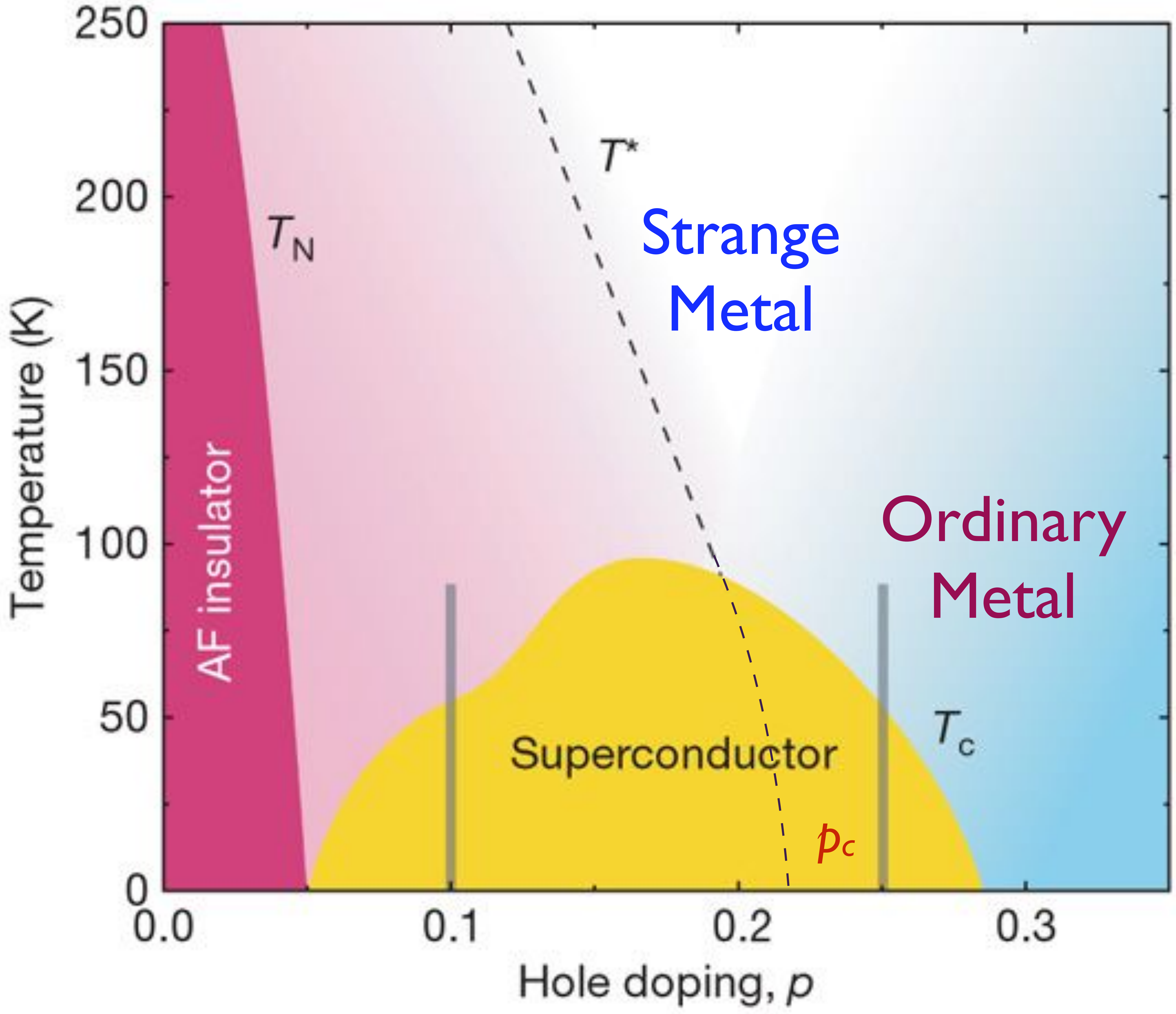
Temperature (K)
Hole doping, p
250
200
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0
0.0
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0.2
0.3
AF insulator
T_N
T*
Strange Metal
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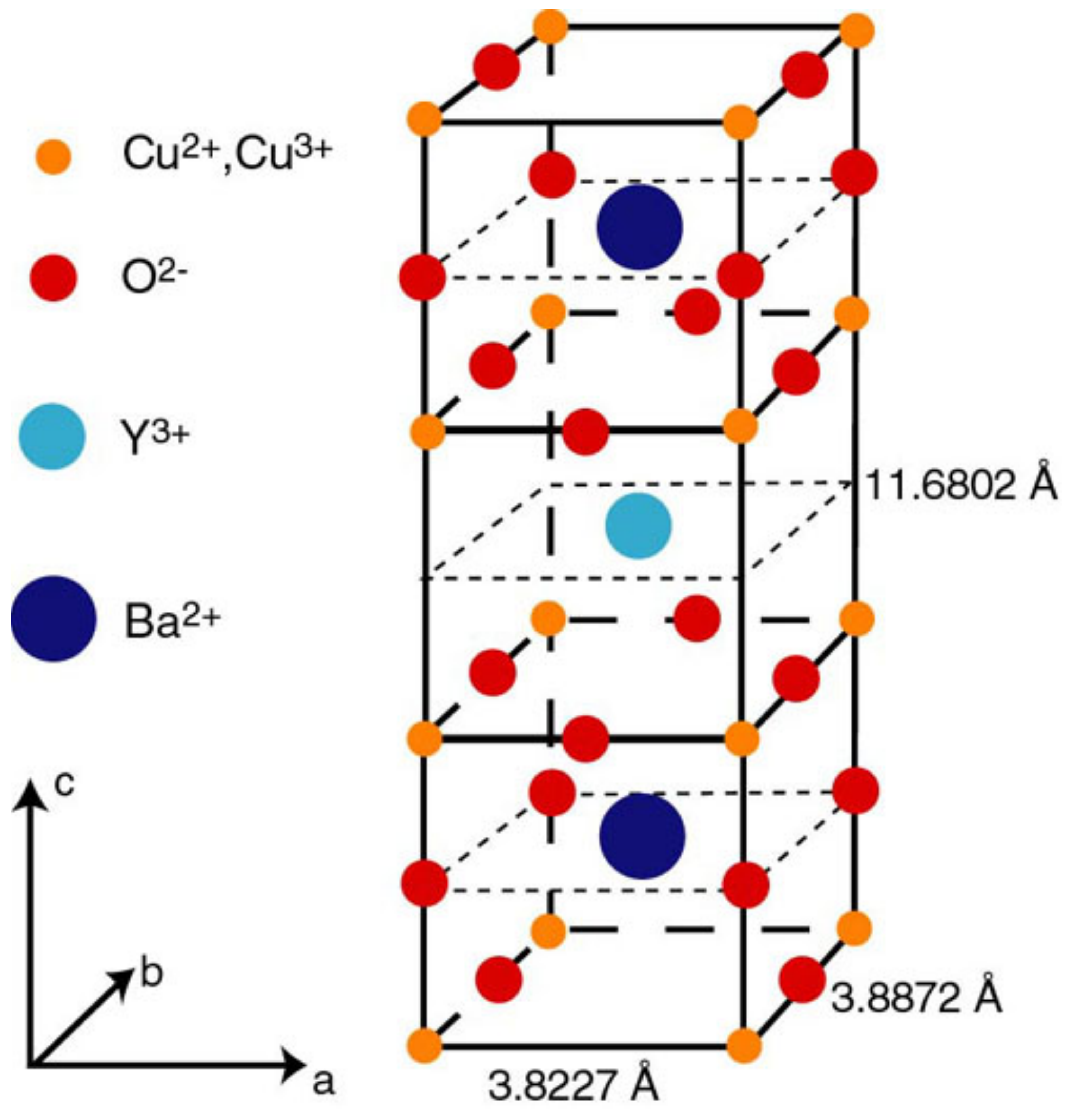
Superconductor
T_c
p_c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O2-
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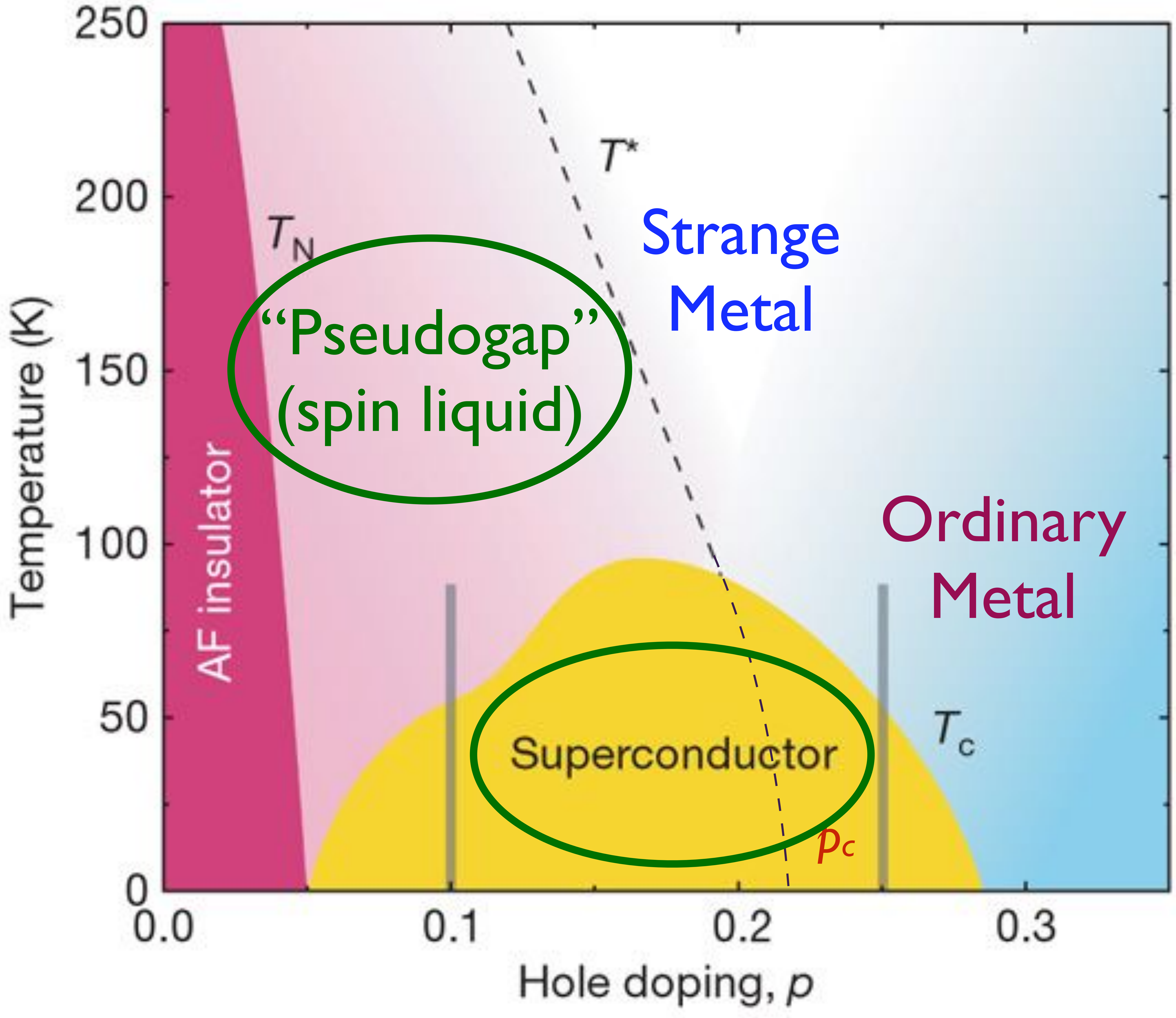
$YBa_2Cu_3O_{6+x}$



250
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Temperature (K)
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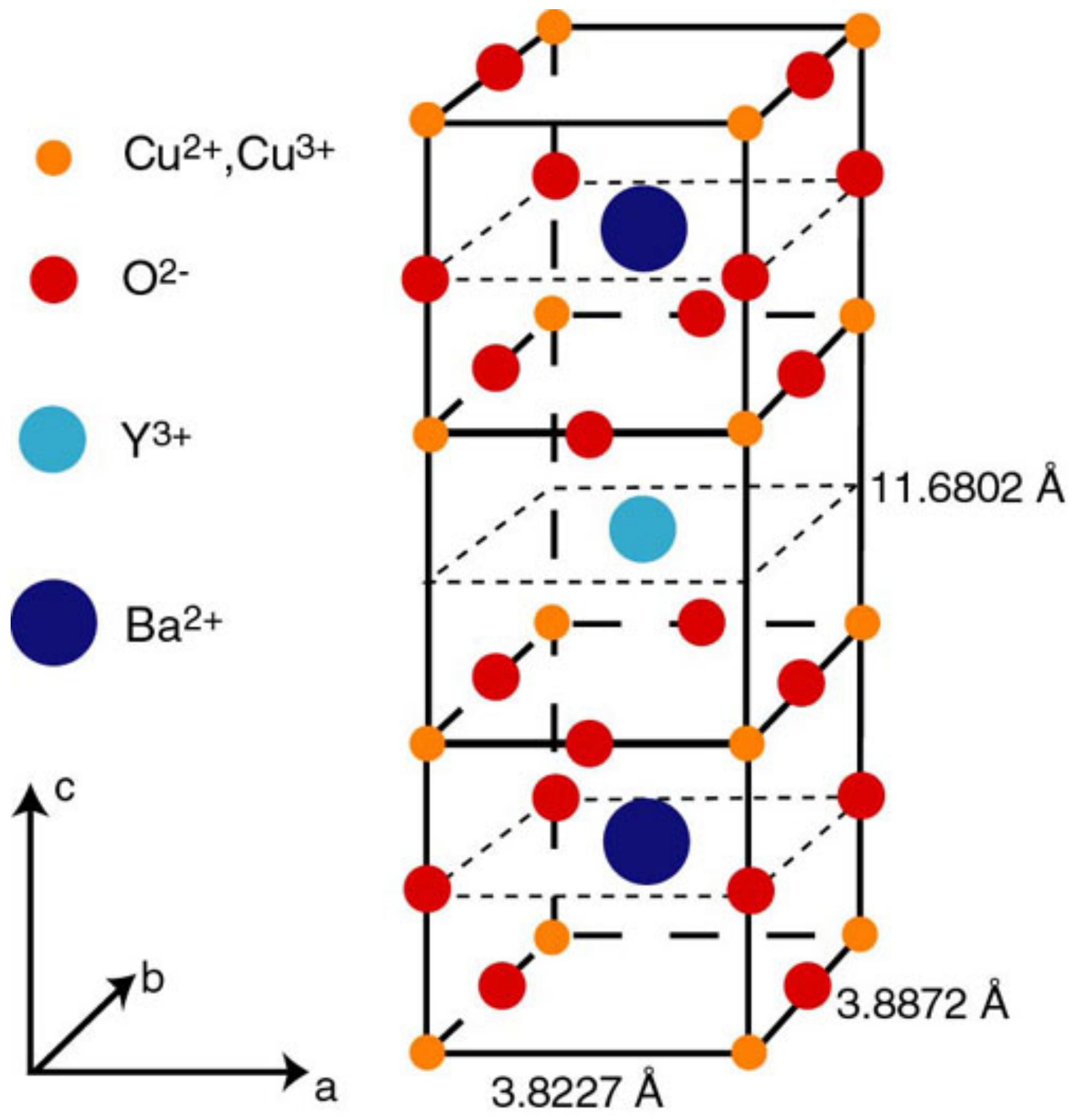

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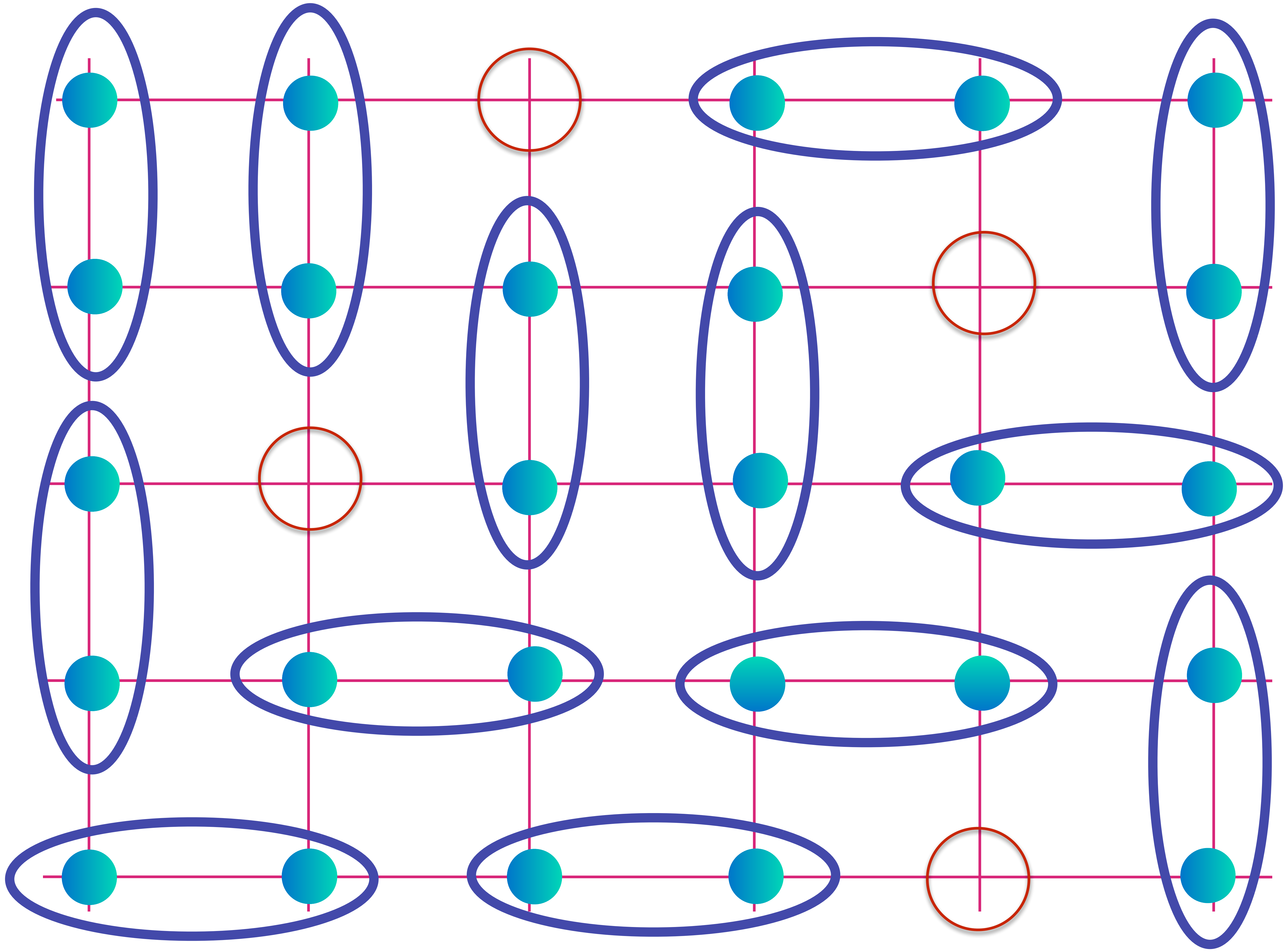
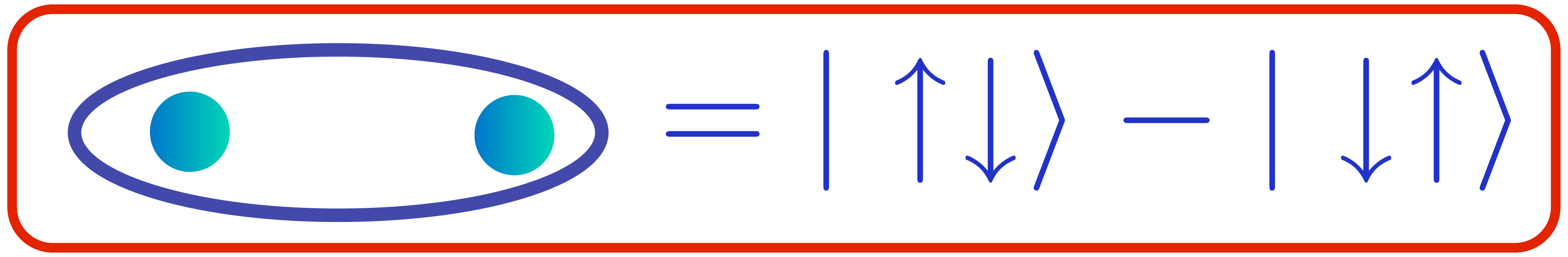


Pseudogap

Small density of mobile "holes" in a sea of entangled electron pairs



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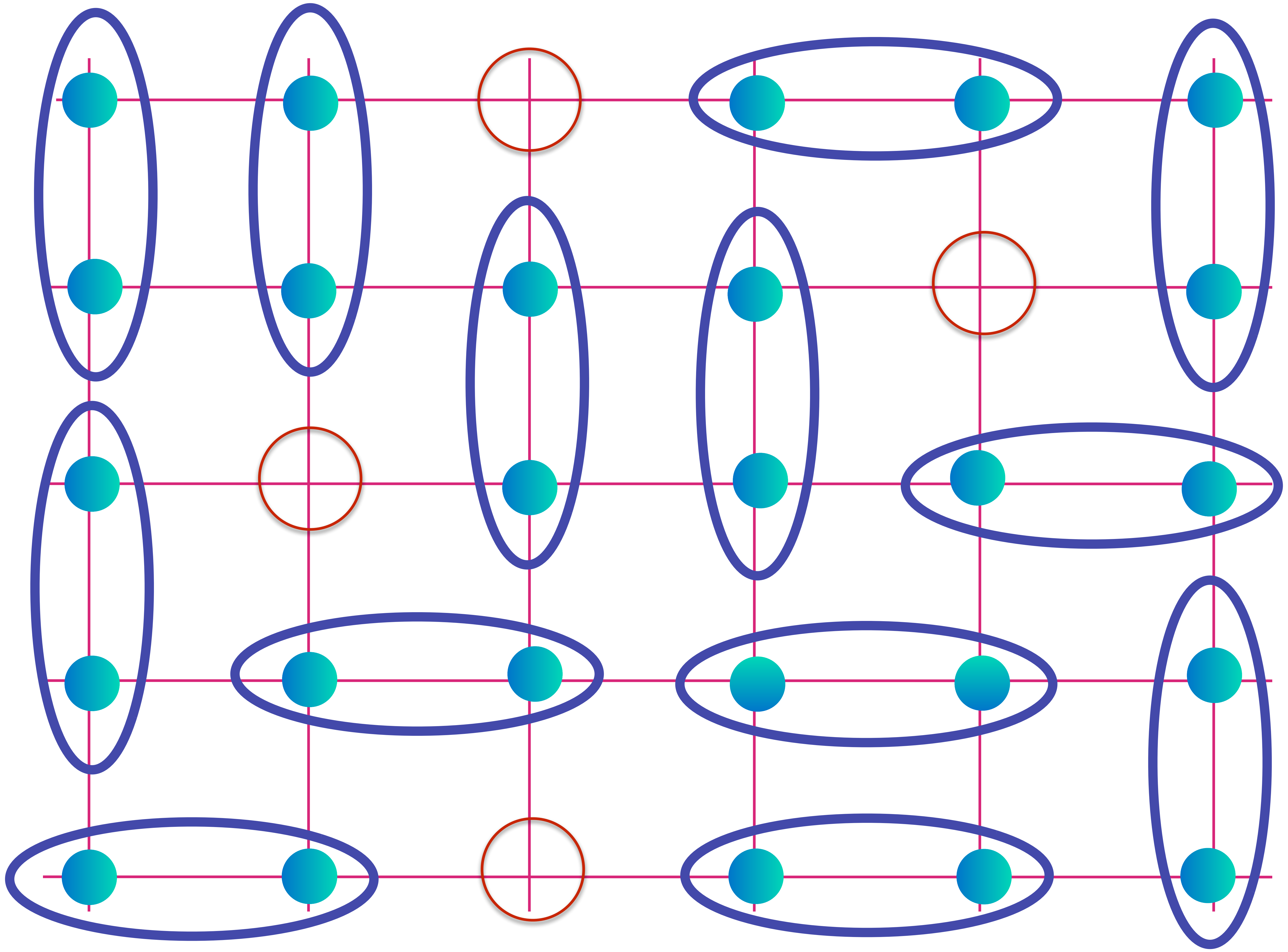

Pseudogap

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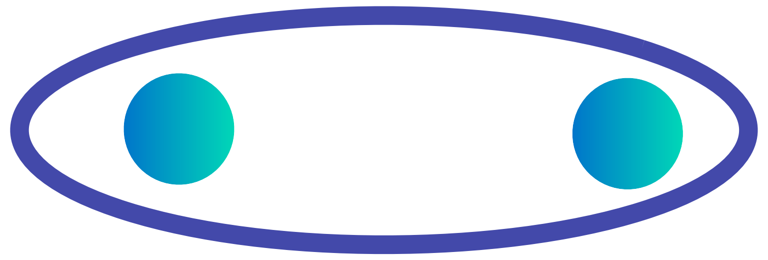
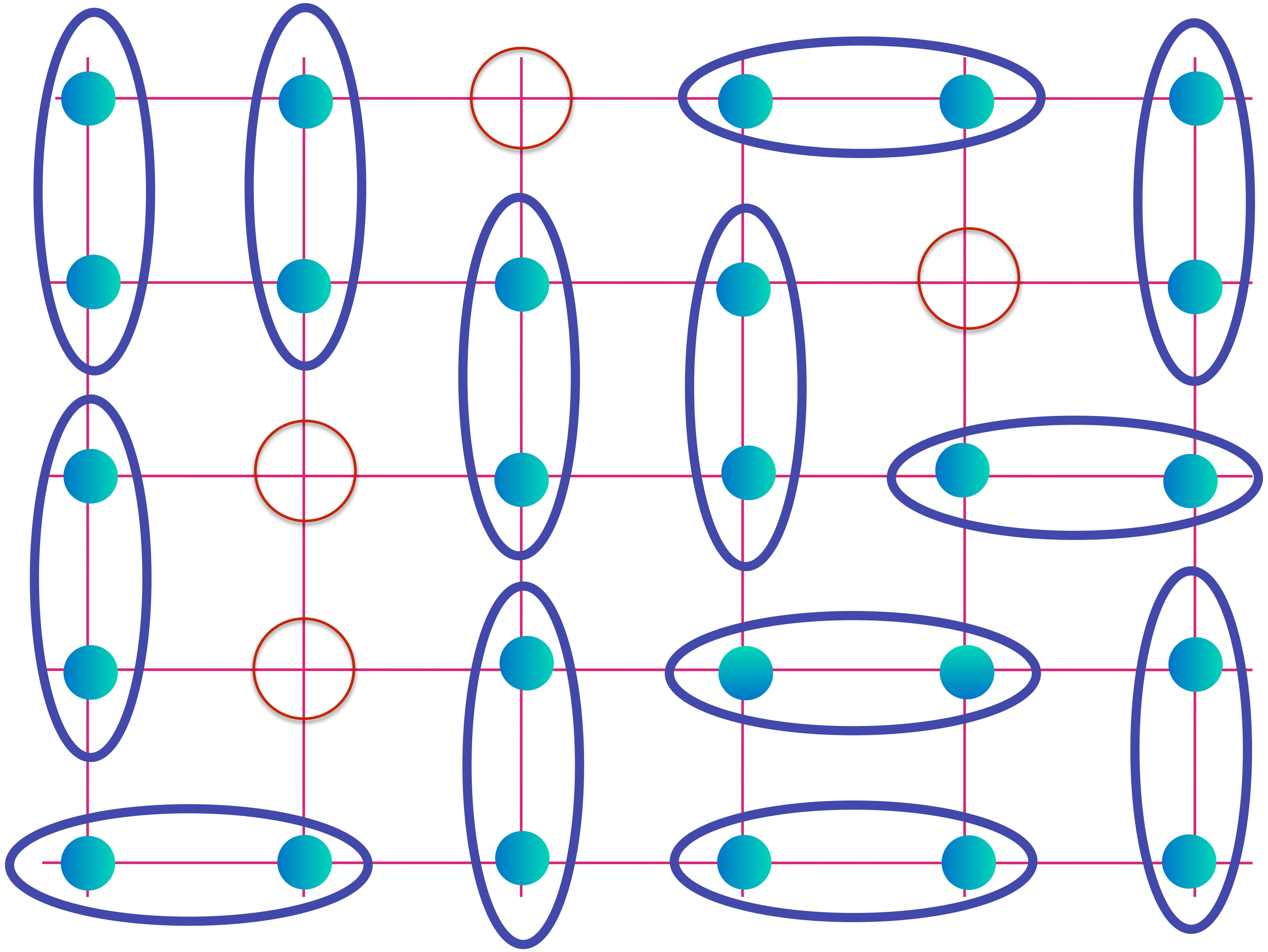
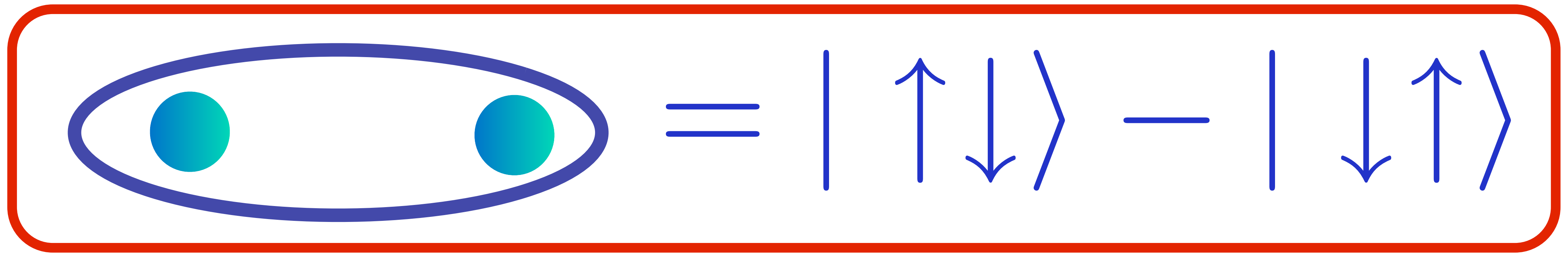
 $= | \uparrow\downarrow \rangle - | \downarrow\uparrow \rangle$

The dance of electrons on Cu atoms in YBCO





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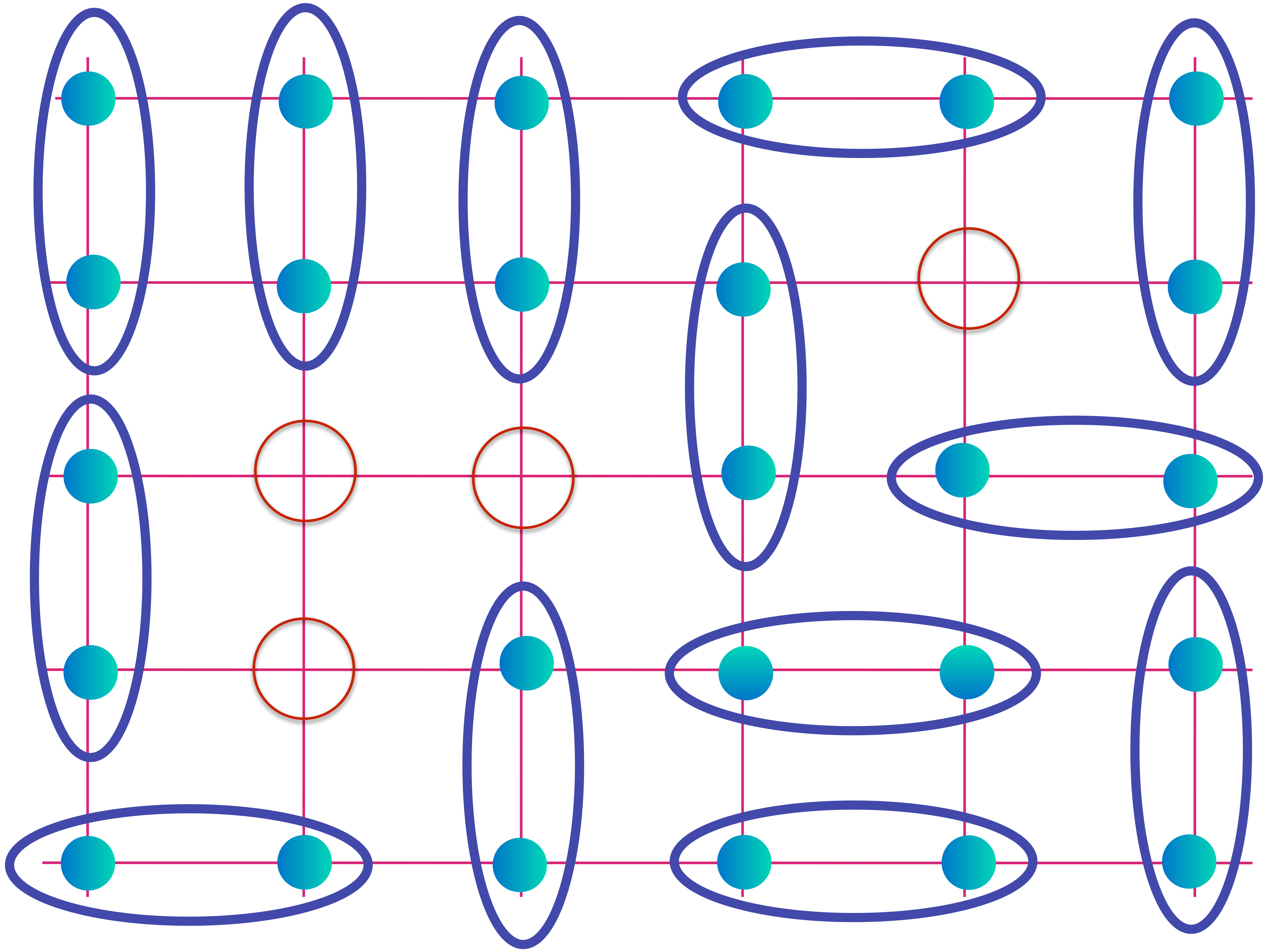

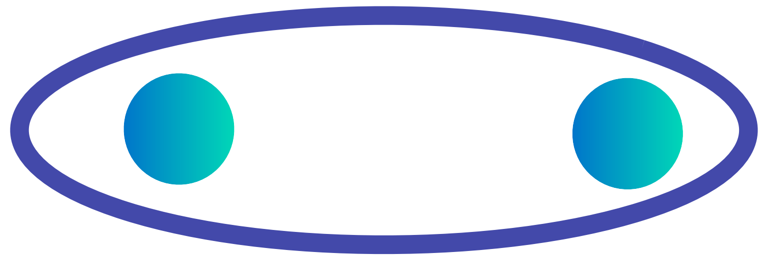

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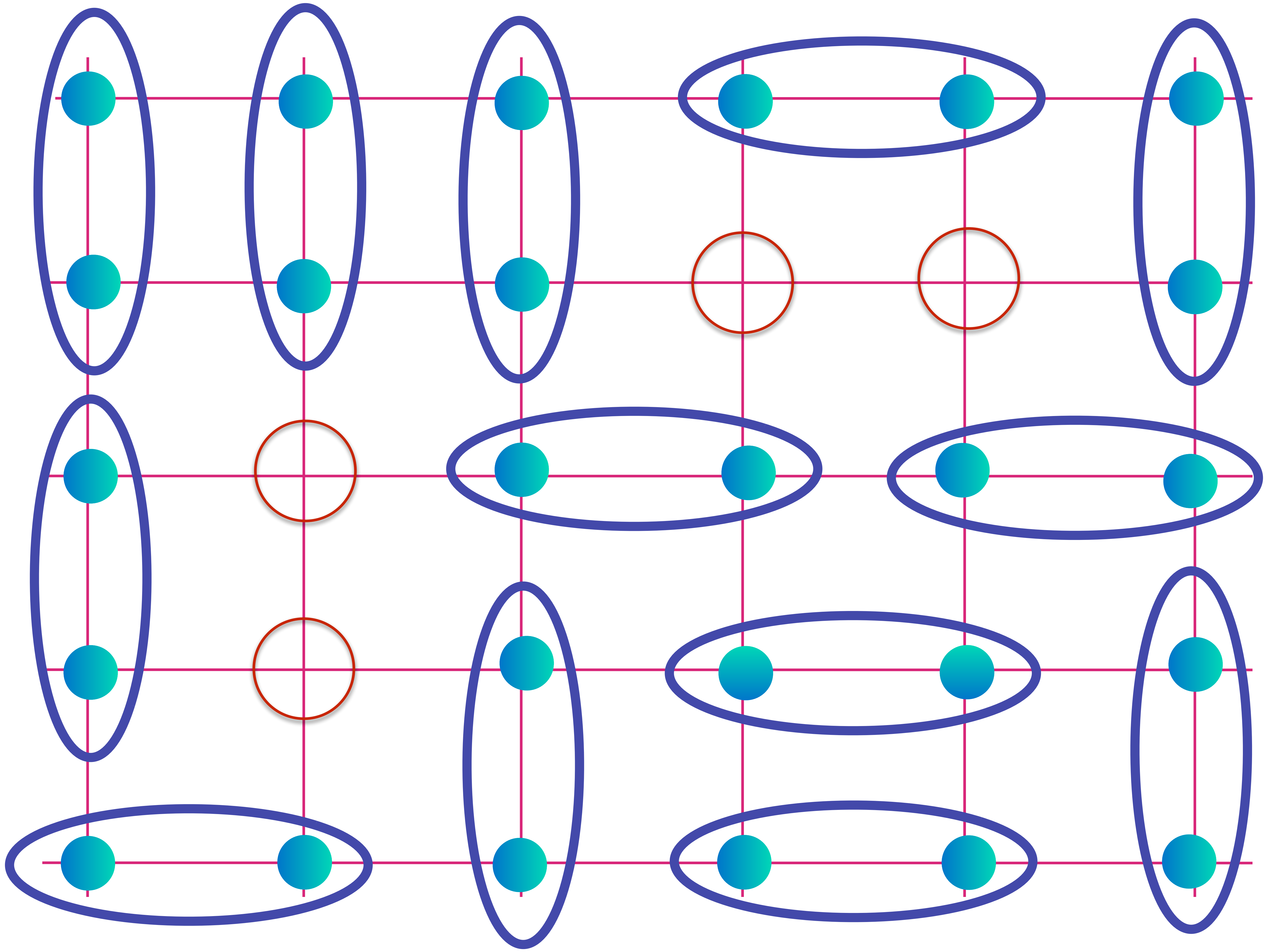

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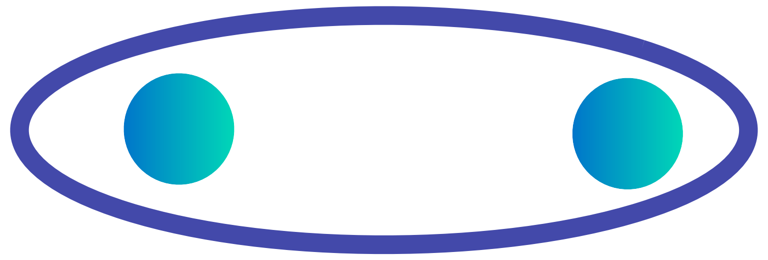
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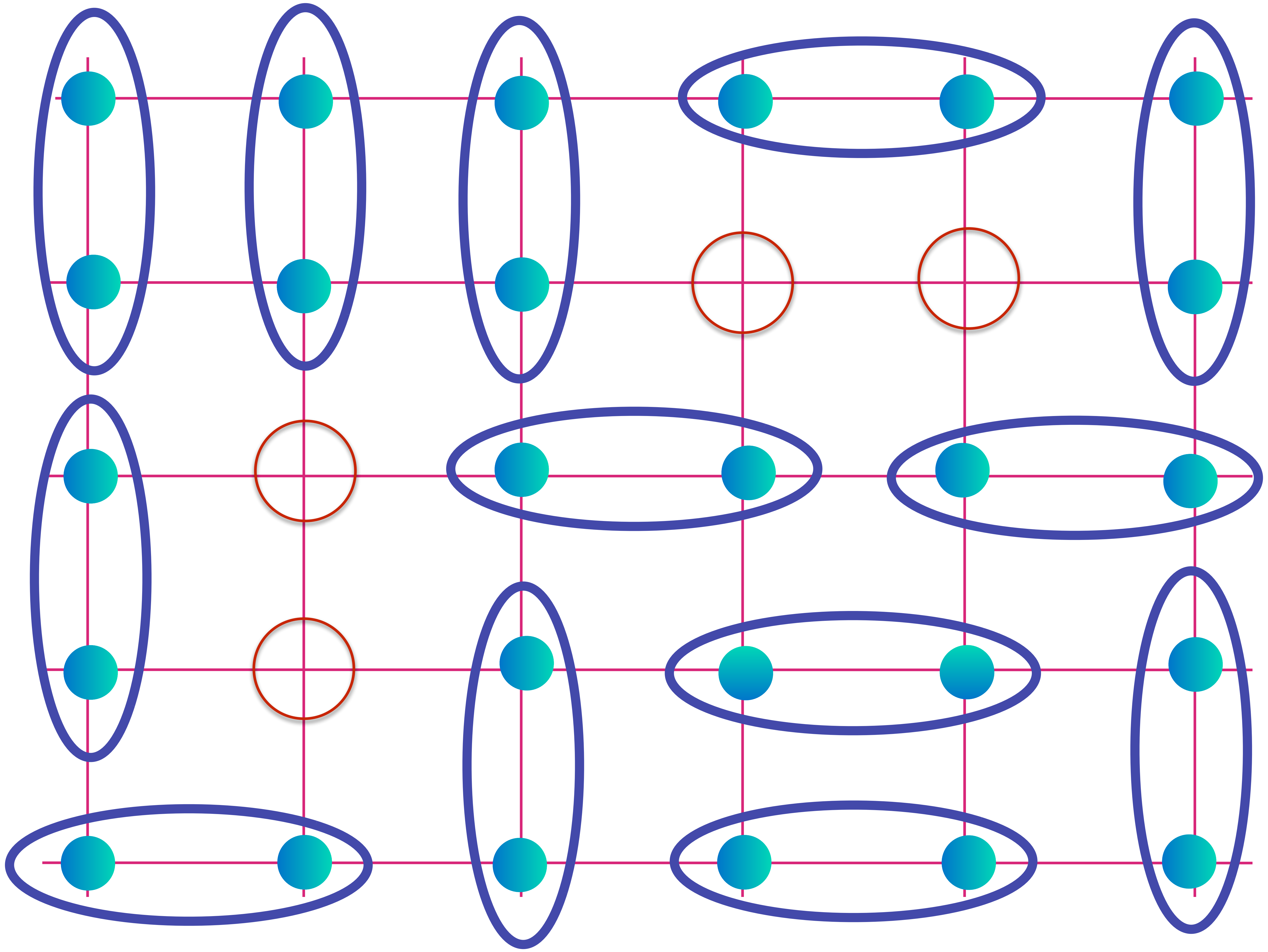
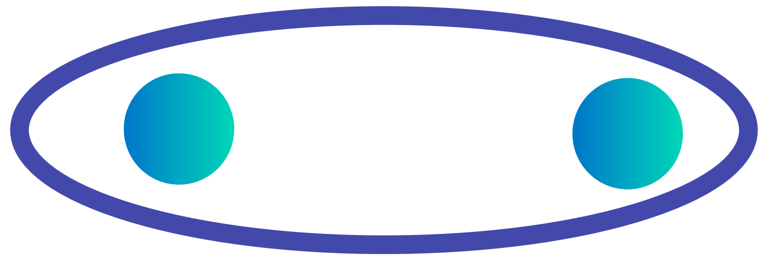


Superconductivity

Mobile entangled electron pairs



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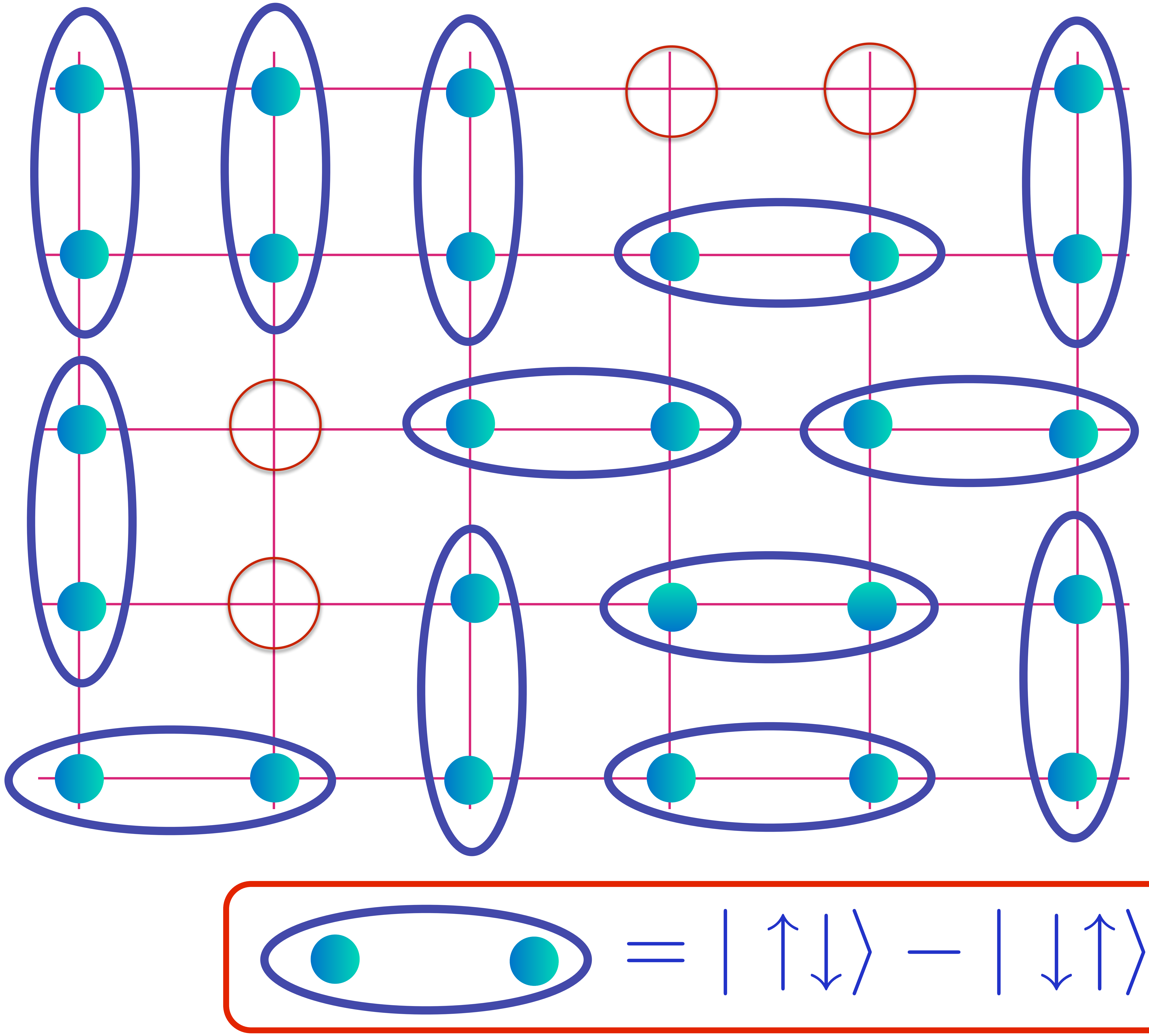

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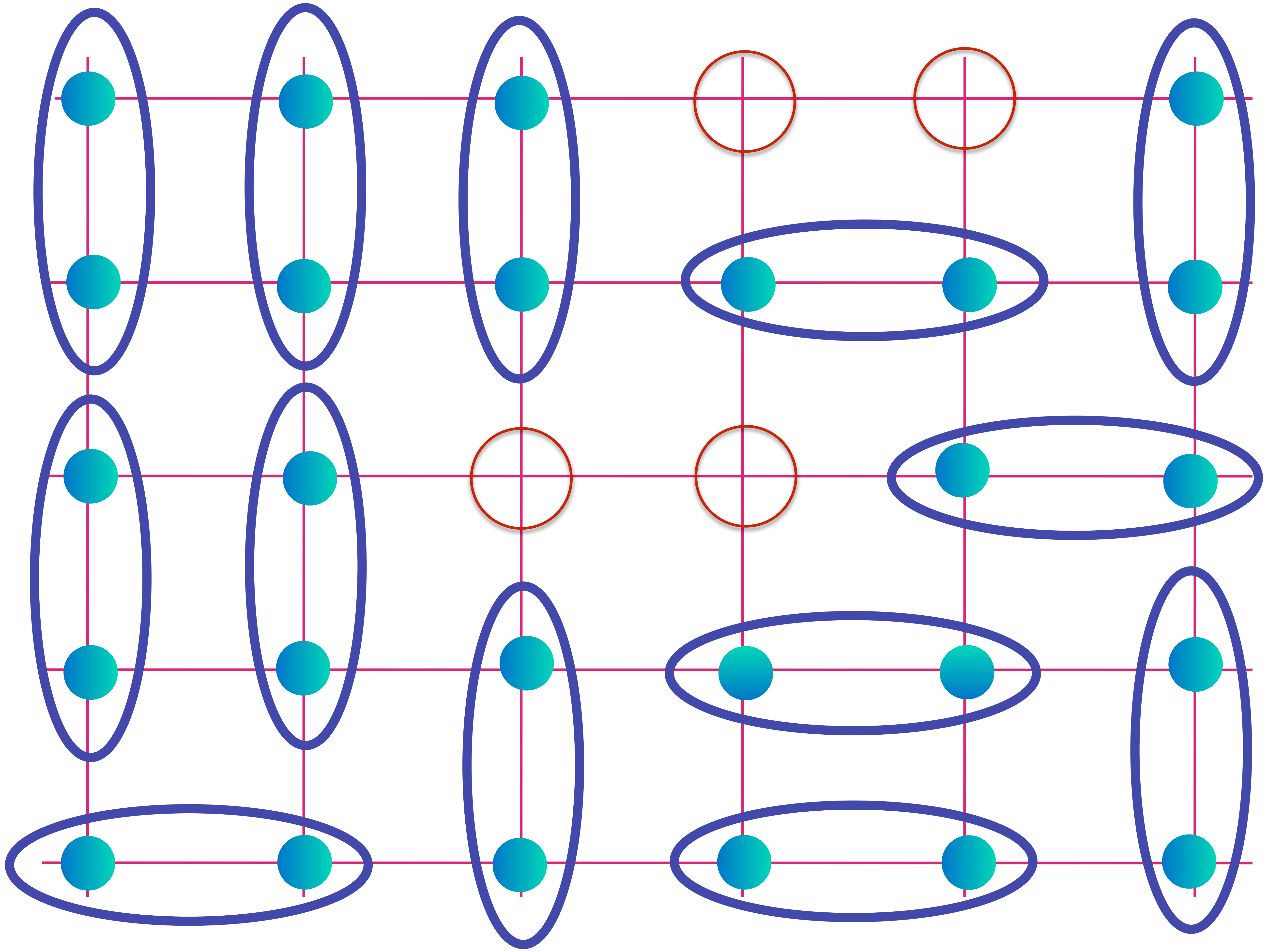

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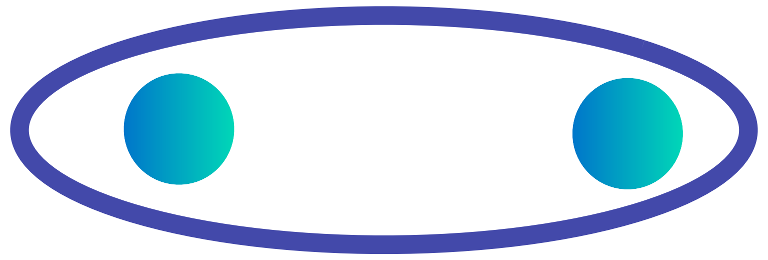

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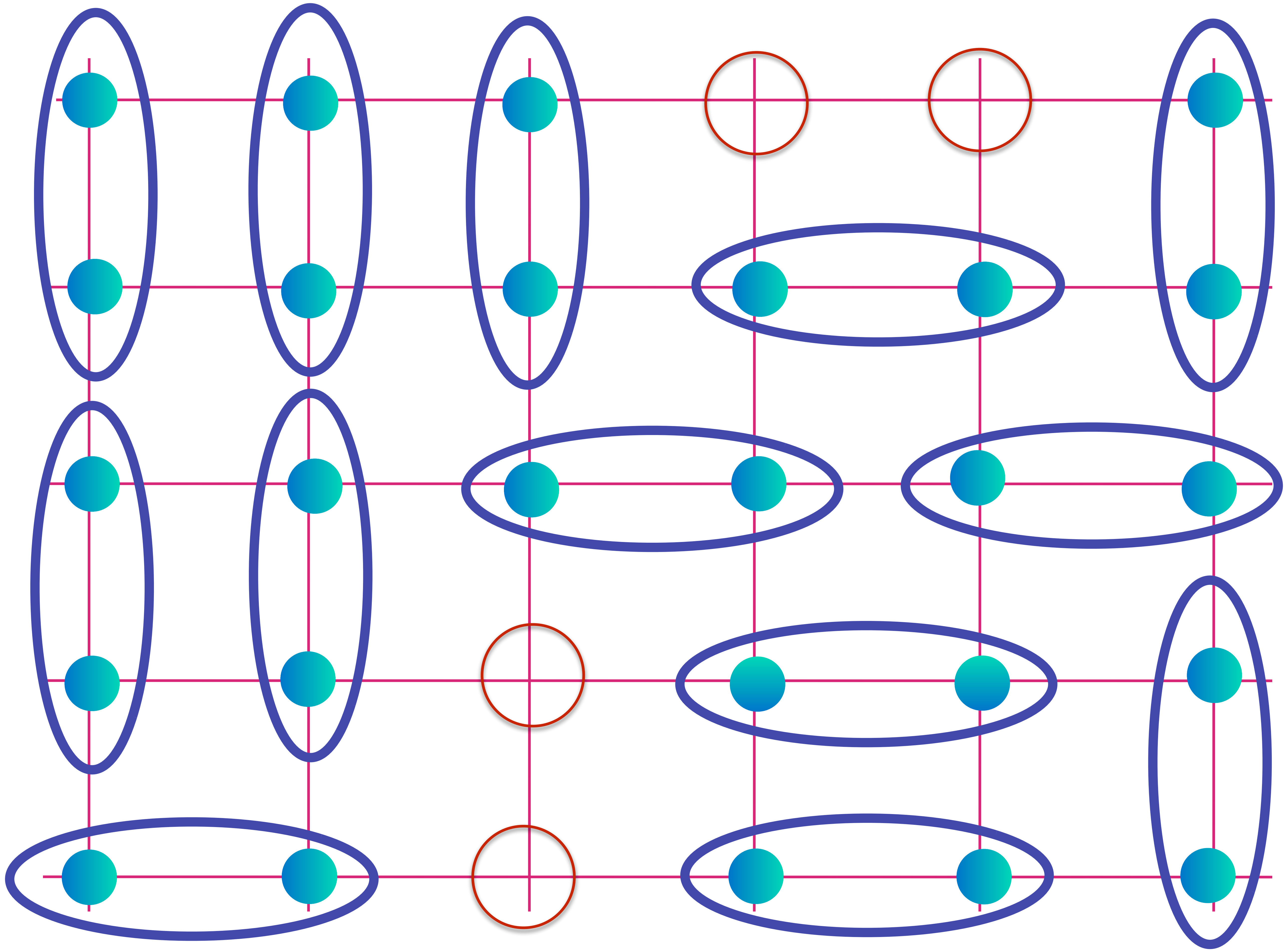

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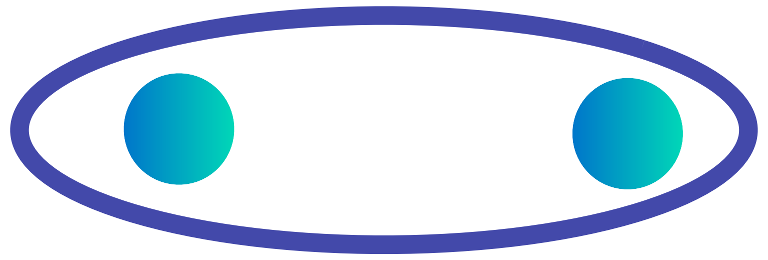 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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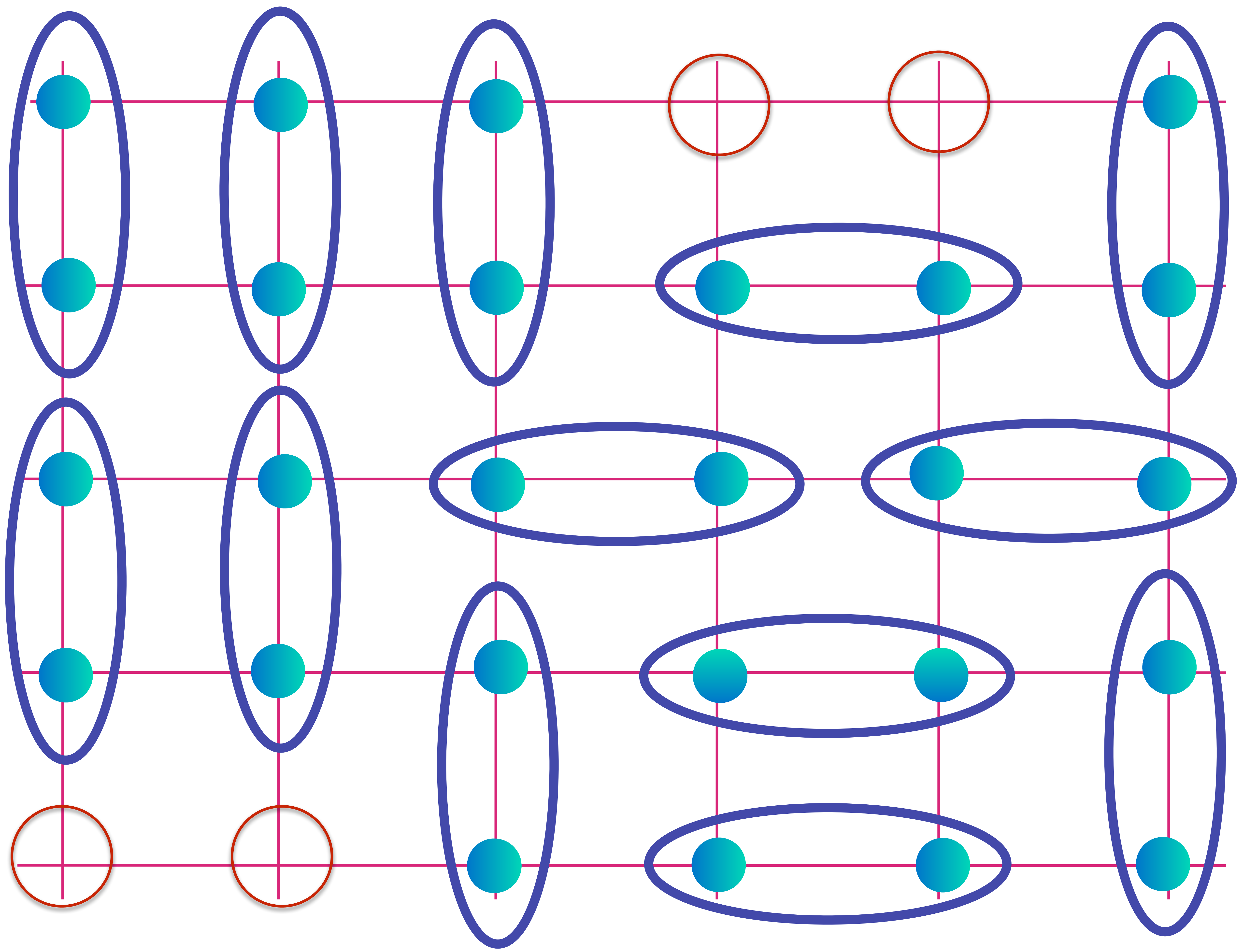

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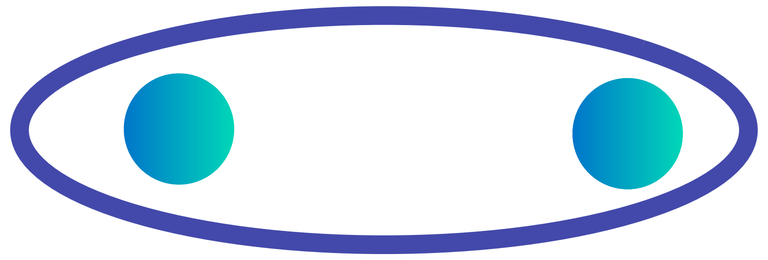

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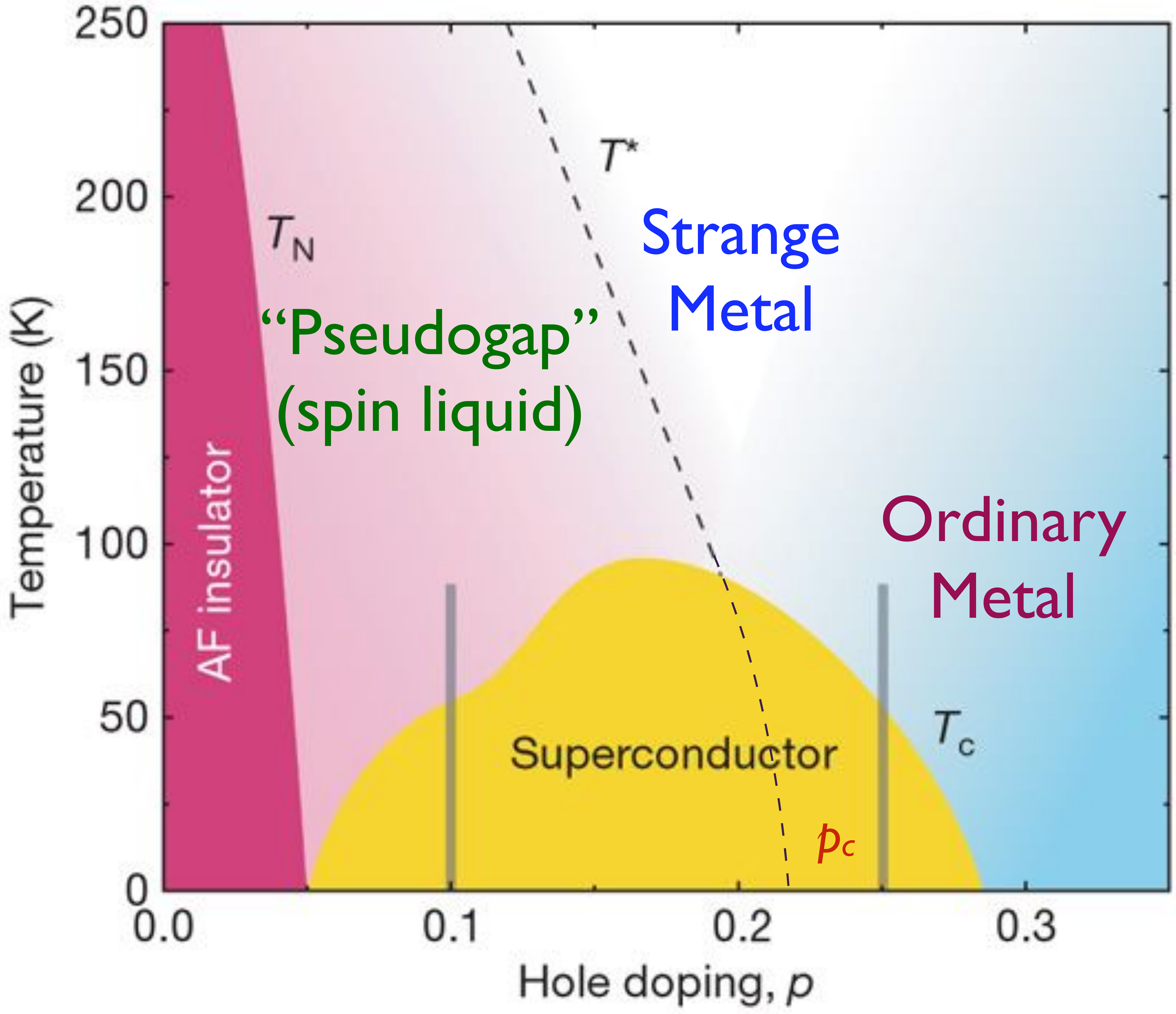
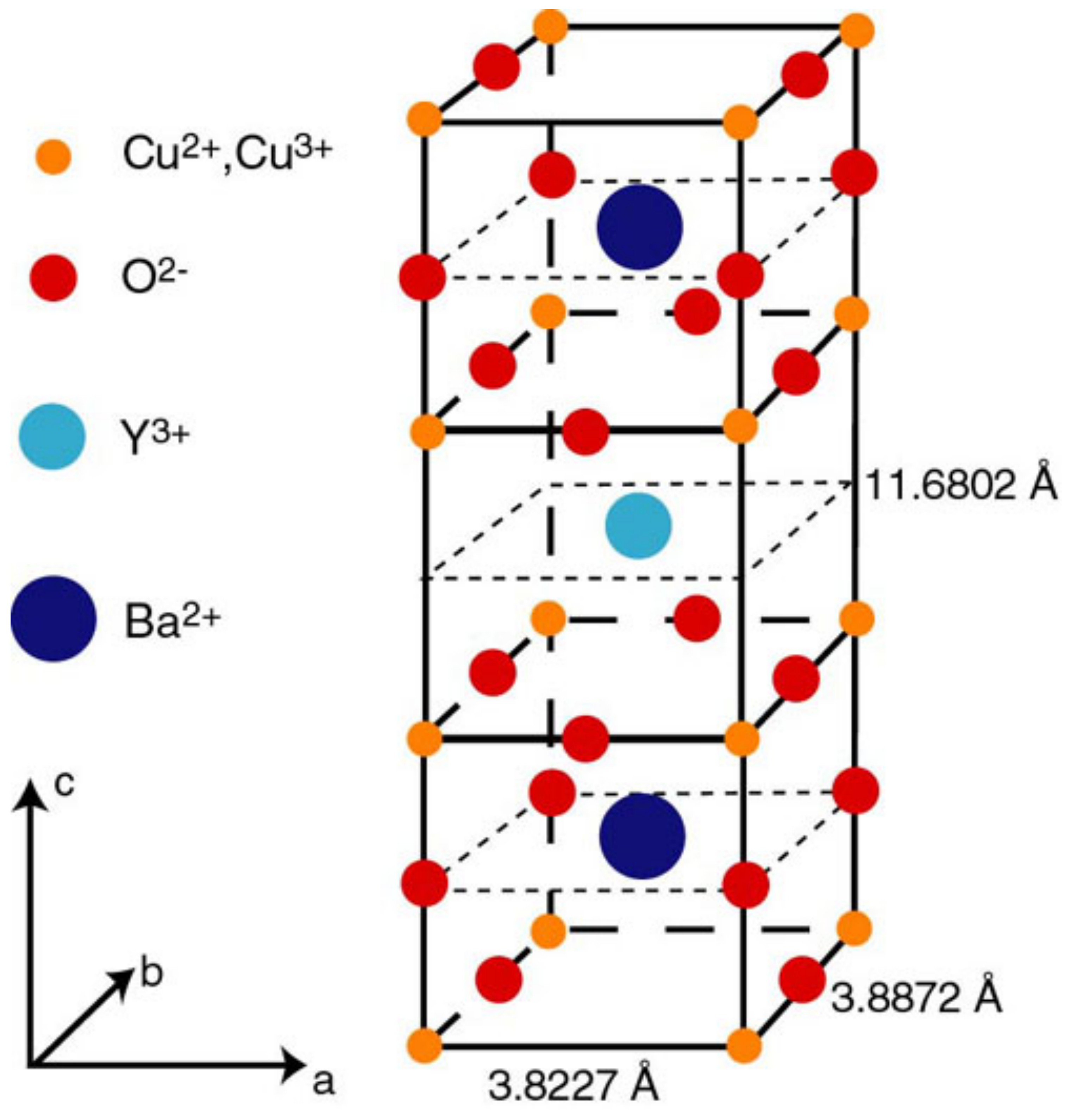
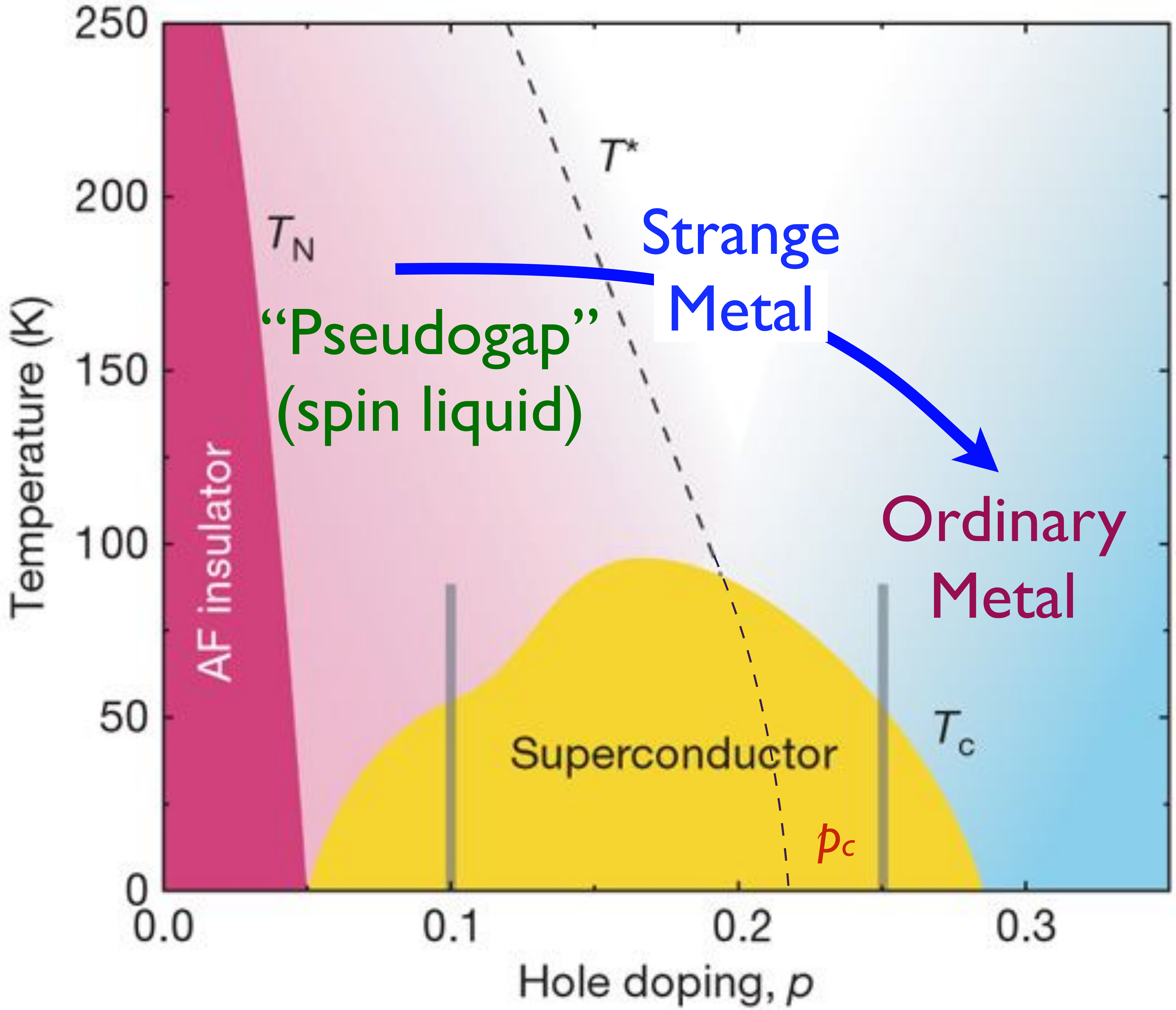


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
"Pseudogap"
(spin liquid)
Strange
Metal
Ordinary
Metal
Superconductor
T_c
p_c



Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

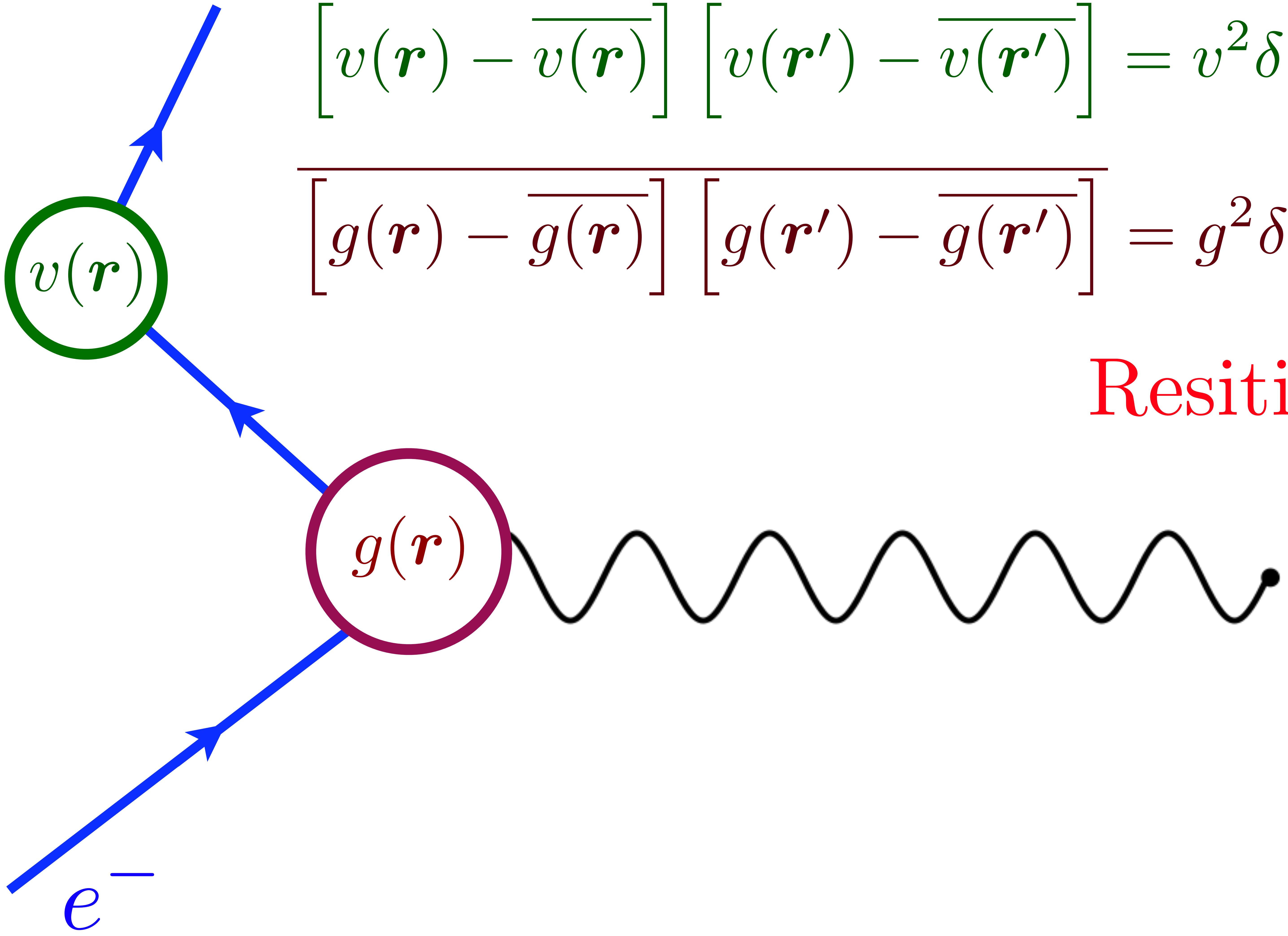
$YBa_2Cu_3O_{6+x}$



Strange Metal

Arises from the evolution of quantum entanglement between the spin liquid and the ordinary metal

The dance of electrons on Cu atoms in YBCO

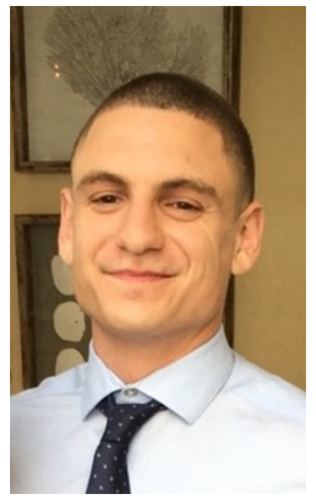


$$\overline{\left[v(\boldsymbol{r}) - \overline{v(\boldsymbol{r})}\right]\left[v(\boldsymbol{r}') - \overline{v(\boldsymbol{r}')}\right]} = v^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$$

$$\overline{\left[g(\boldsymbol{r}) - \overline{g(\boldsymbol{r})}\right]\left[g(\boldsymbol{r}') - \overline{g(\boldsymbol{r}')}\right]} = g^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$$

Random interactions as in SYK model

Resitivity $\rho(T) \sim v^2 + g^2 T$



Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv: 2203.04990, *Science*, to appear

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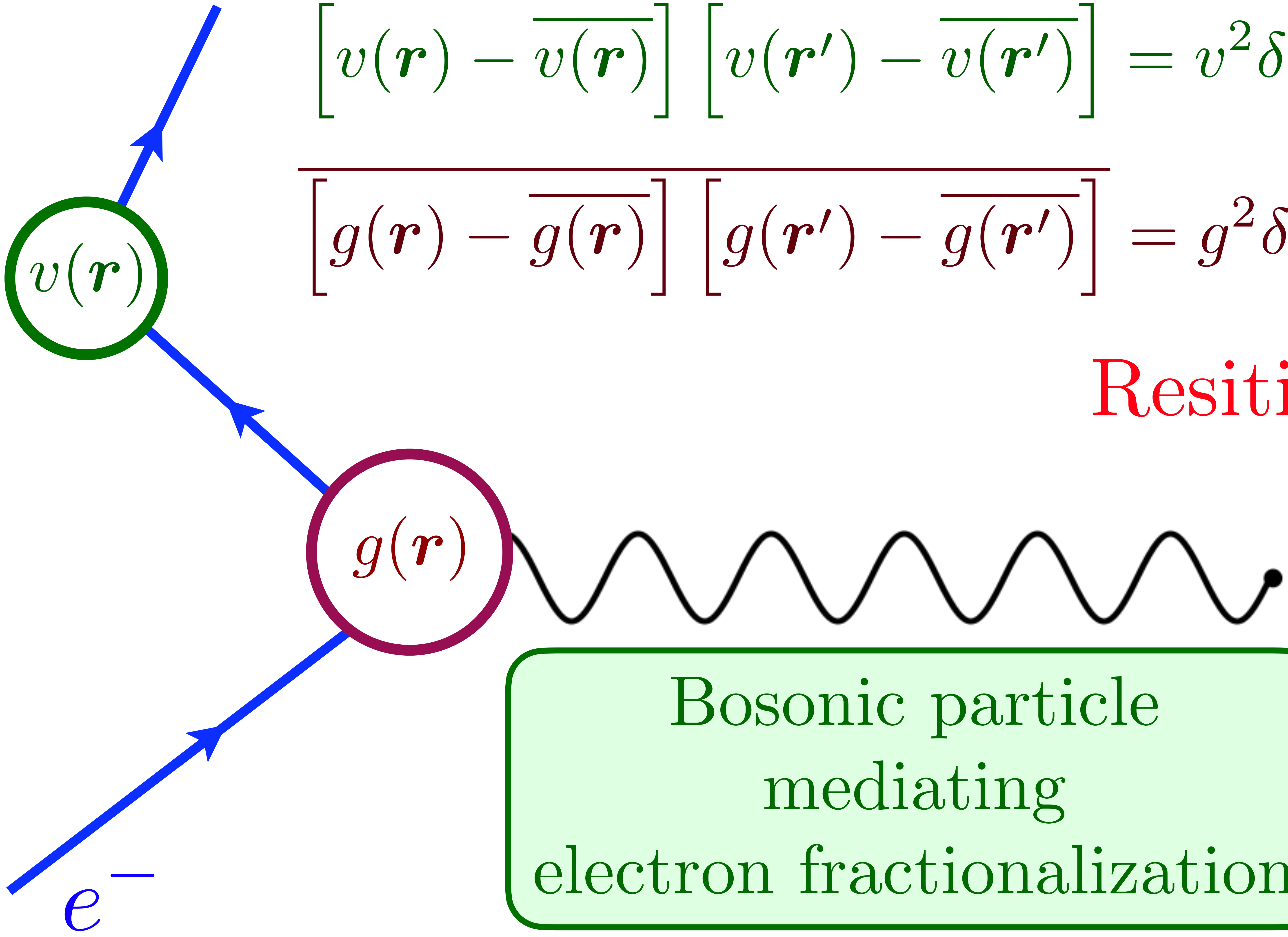

$$\overline{\left[v(\boldsymbol{r}) - \overline{v(\boldsymbol{r})}\right]\left[v(\boldsymbol{r}') - \overline{v(\boldsymbol{r}')}\right]} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$$

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Bosonic particle mediating electron fractionalization

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv: 2203.04990, *Science*, to appear

Properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
   and $\rho(T) < h/e^2$ (in $d = 2$).
   Metals with $\rho(T) > h/e^2$ are bad metals.

2. Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

3. Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \dfrac{m^*_{\rm trans}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\rm trans}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon……A. Georges, arXiv:2205.04030

4. Photoemission: nearly "marginal Fermi liquid" electron spectral density:

$$\text{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2 \quad ; \quad \frac{1}{\tau(\omega)} \sim |\omega| \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

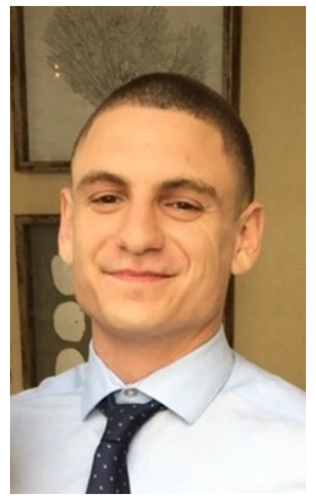
T.J. Reber….D. Dessau, Nature Communications **10**, 5737 (2019)

Summary

- SYK: a solvable toy model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

- Toy SYK model captures the correct universal low energy quantum theory of charged black holes, and provides a Hamiltonian realization of black hole microstates.



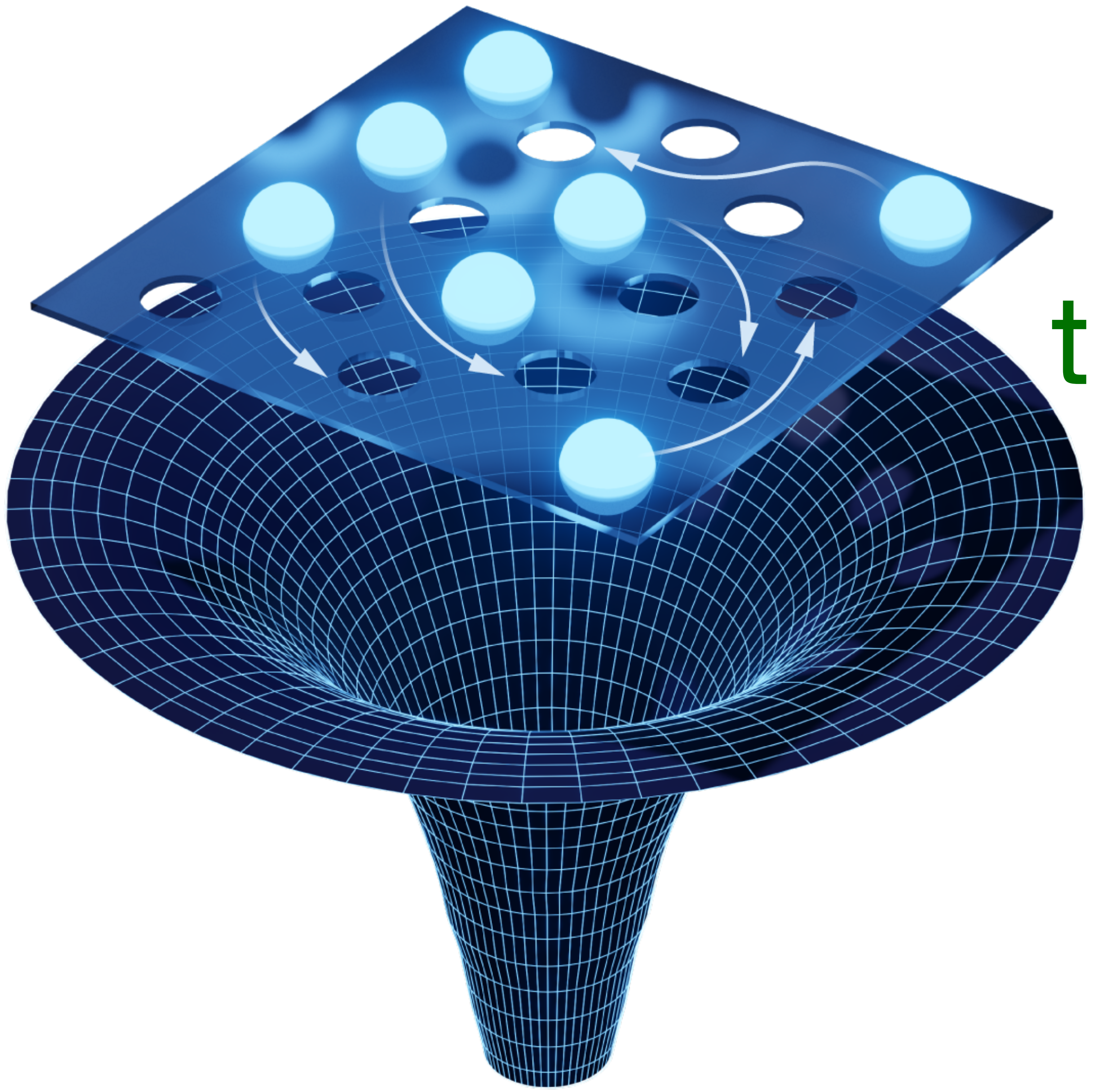


- Linear-T resistivity, $T \ln(1/T)$ specific heat, $\sim 1/\omega$ optical conductivity, and marginal Fermi liquid electron spectrum *all* arise from a SYK-like model with spatially random interactions in a two-dimensional quantum-critical metal.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S., Science to appear, arXiv: 2203.

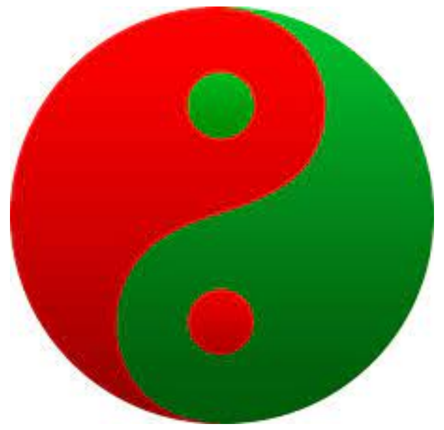
The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles



In one set of variables, it helps describe the ***strange*** electrical properties of YBCO

Sachdev, Ye (1993)



In a *dual* set of variables it describes ***charged black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)

The many faces of multi-particle entanglement

- Absence of quasiparticles, as in the SYK model and the strange metal
- Fractionalization and new emergent particles, as in spin liquids.
- Higher temperature superconductivity (?)
- A quantum theory of the interior of a black hole.