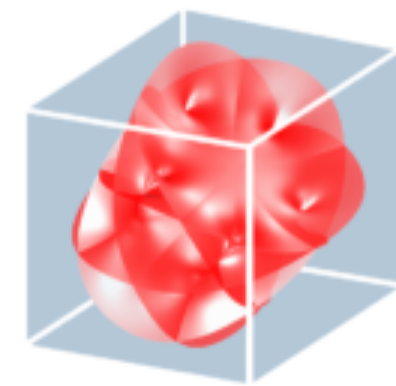


Metal-to-metal quantum phase transitions not described by symmetry breaking orders II

Strongly Correlated Quantum Materials
and High Temperature Superconductors



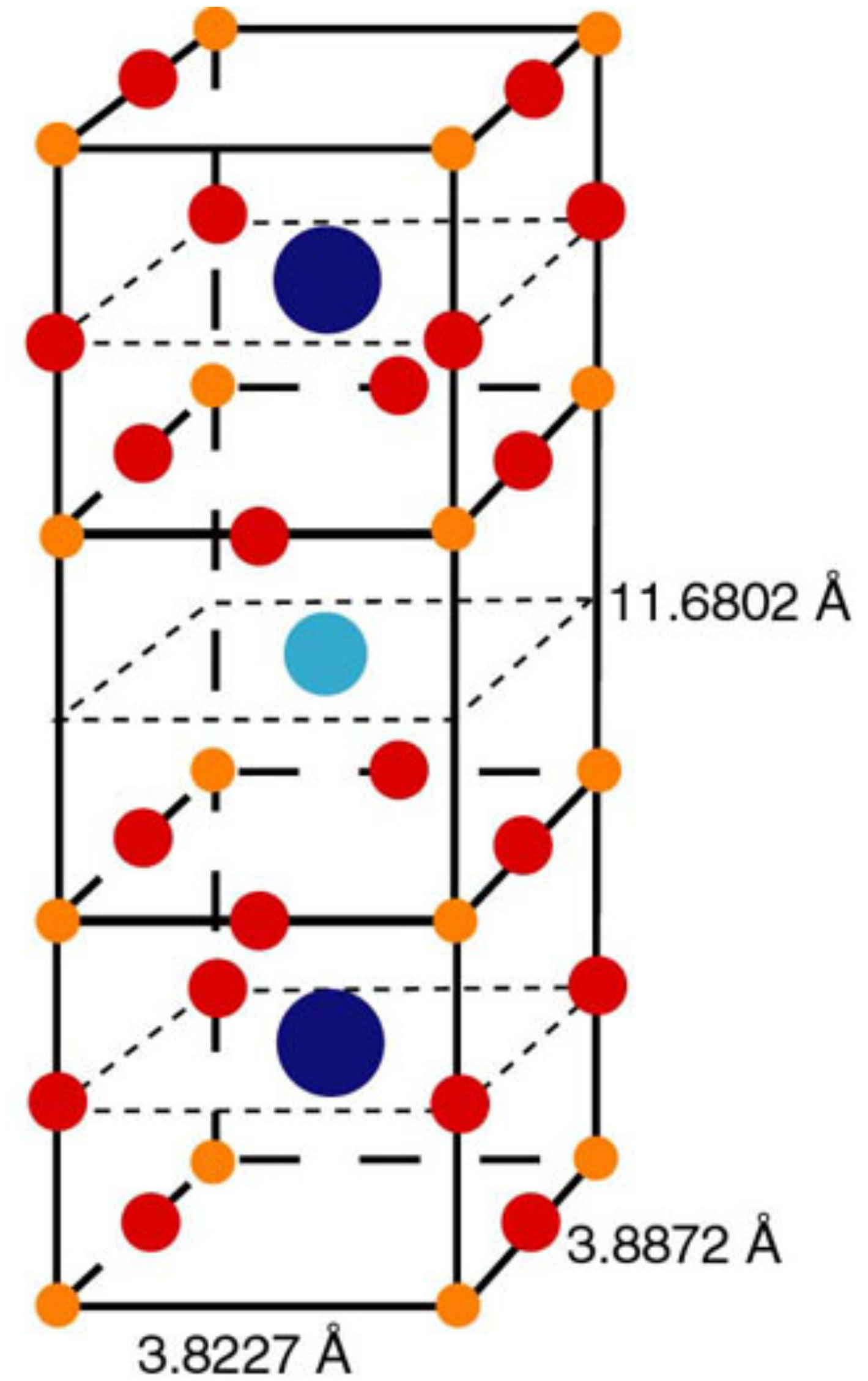
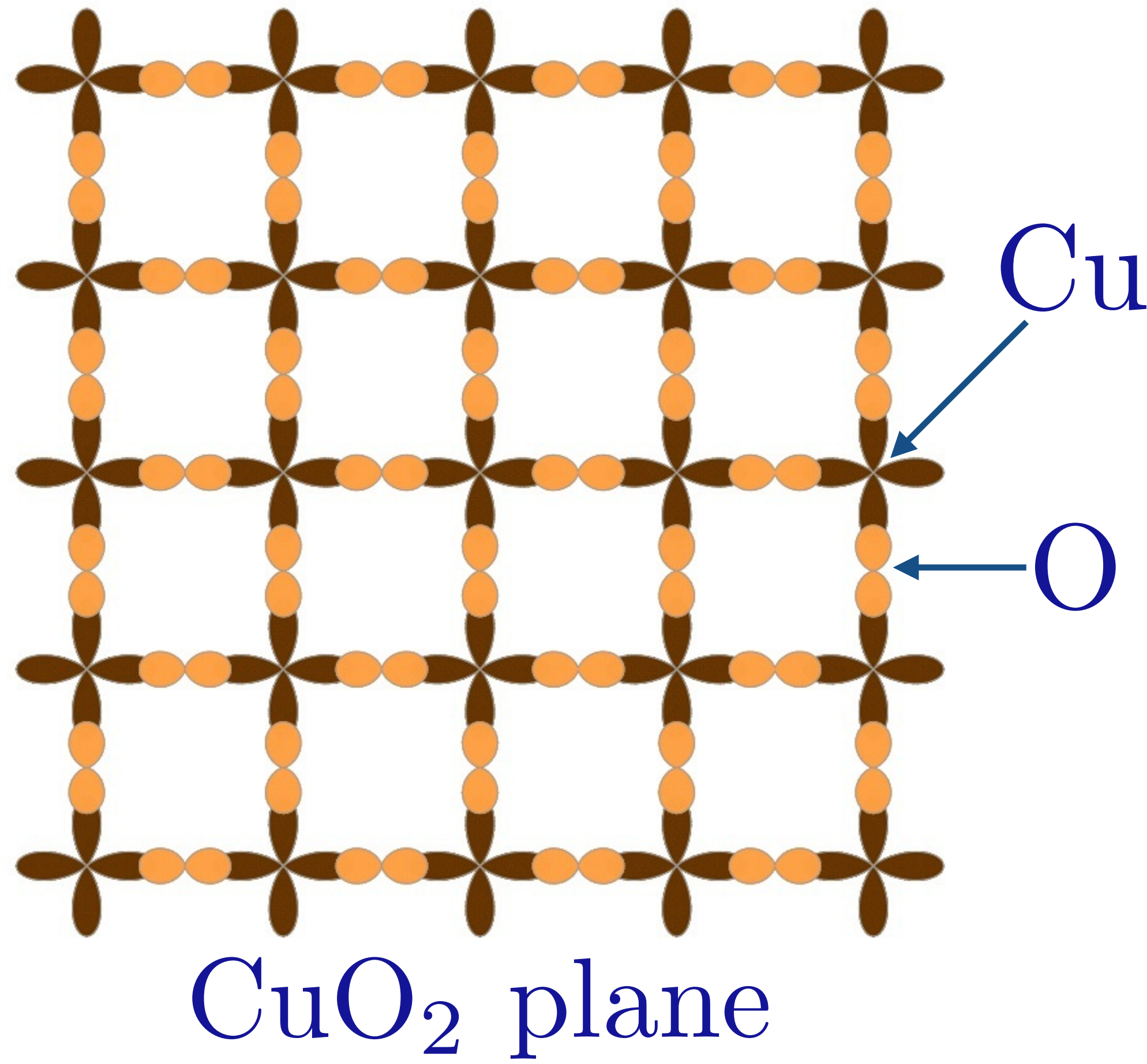
HARVARD UNIVERSITY
CENTER OF MATHEMATICAL
SCIENCES AND APPLICATIONS

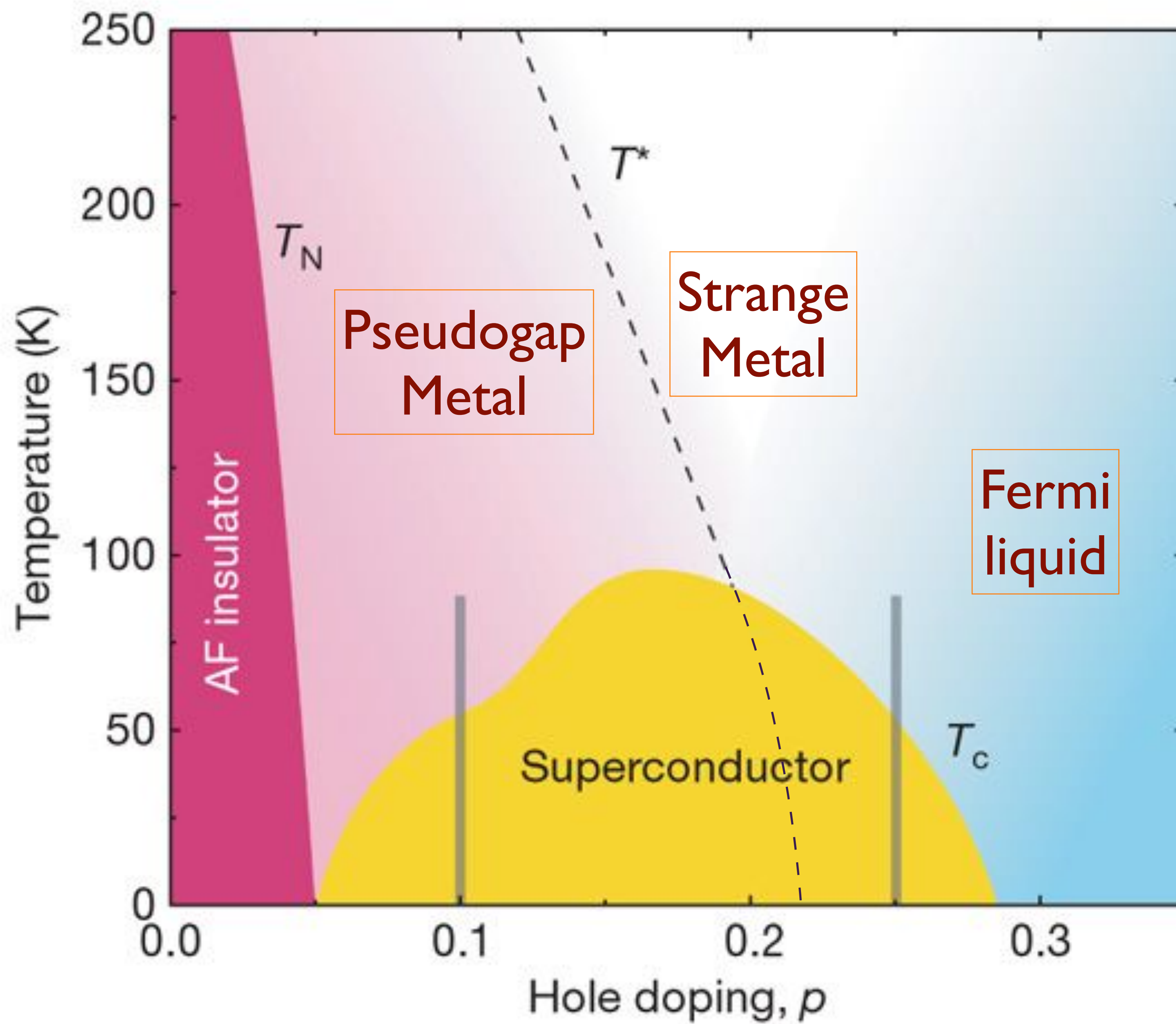
September 2, 2020
Subir Sachdev

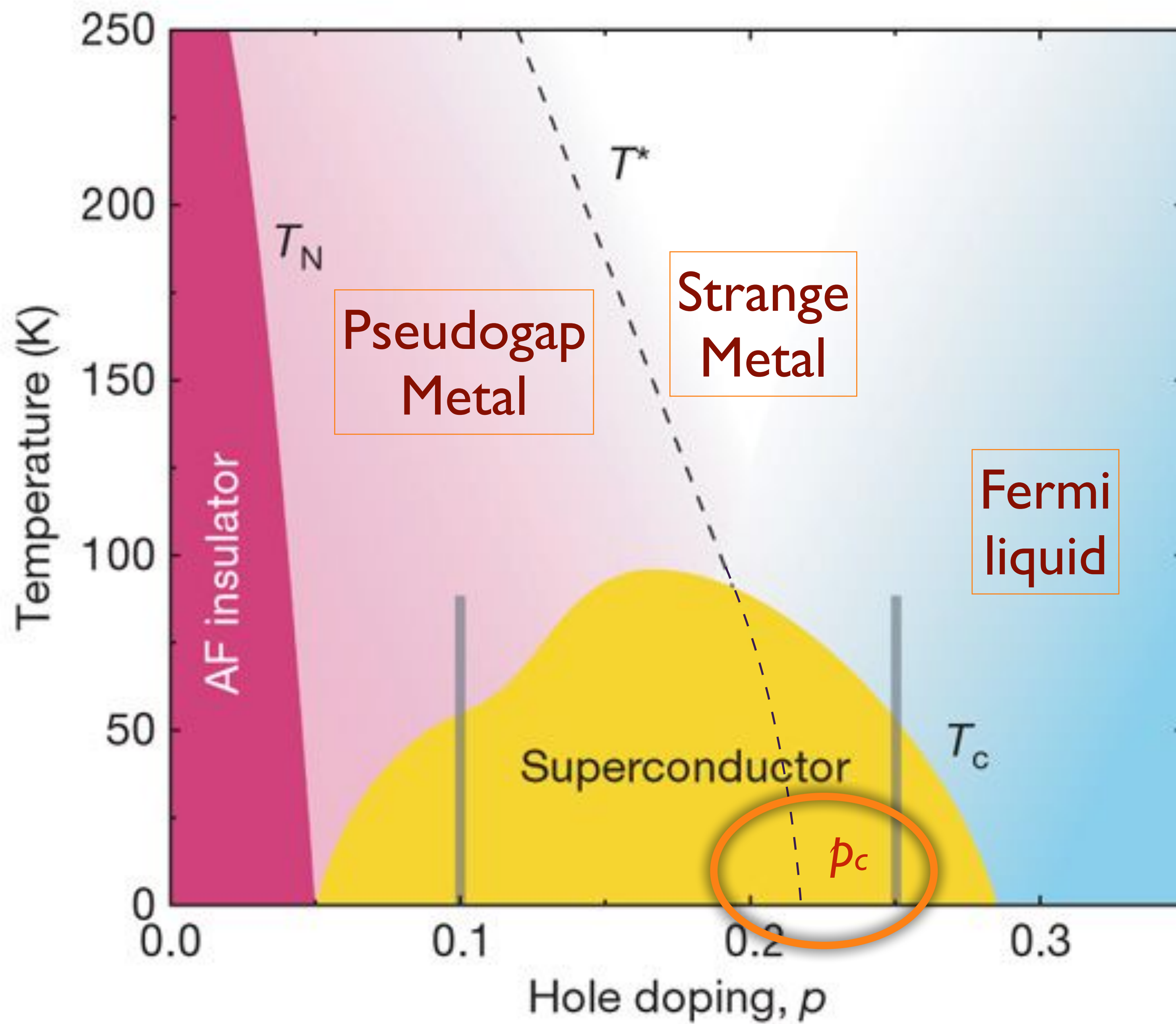


Talk online: sachdev.physics.harvard.edu

High temperature superconductors



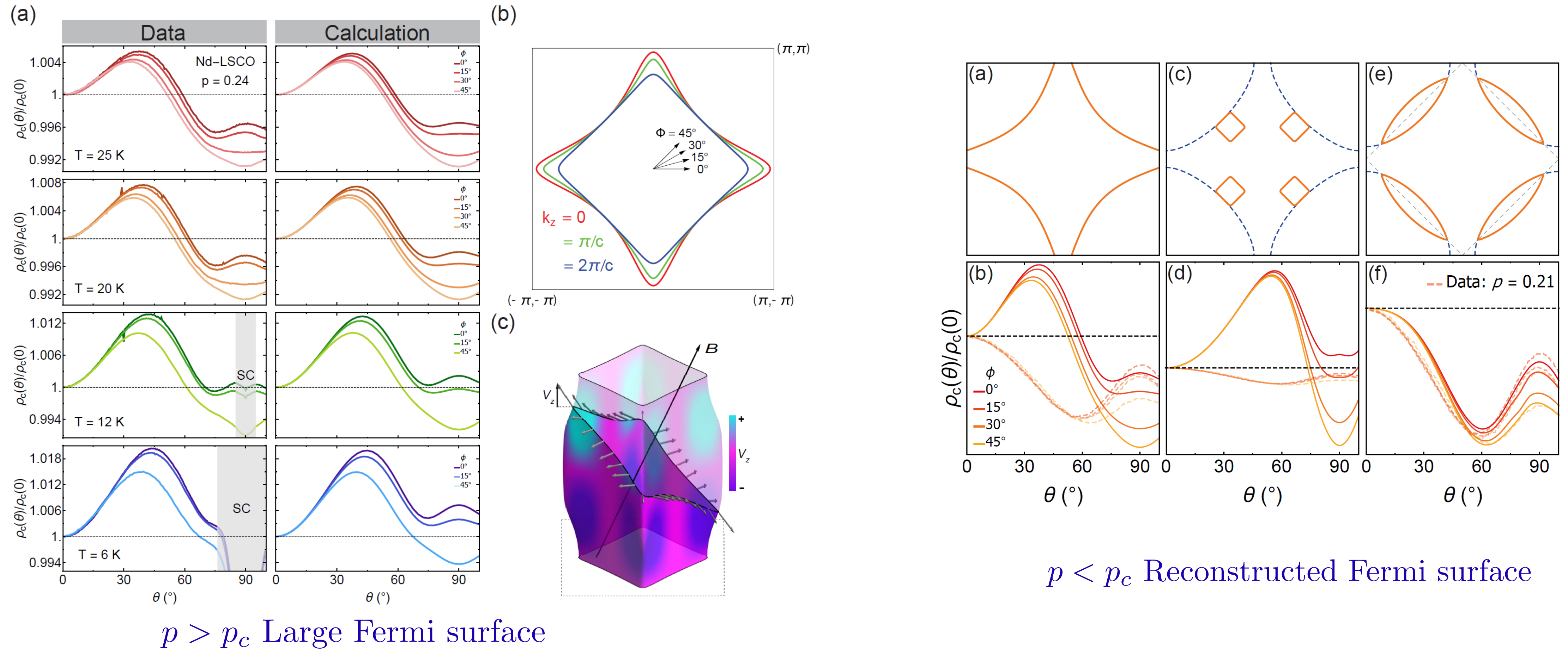




Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, arXiv:2004.01725

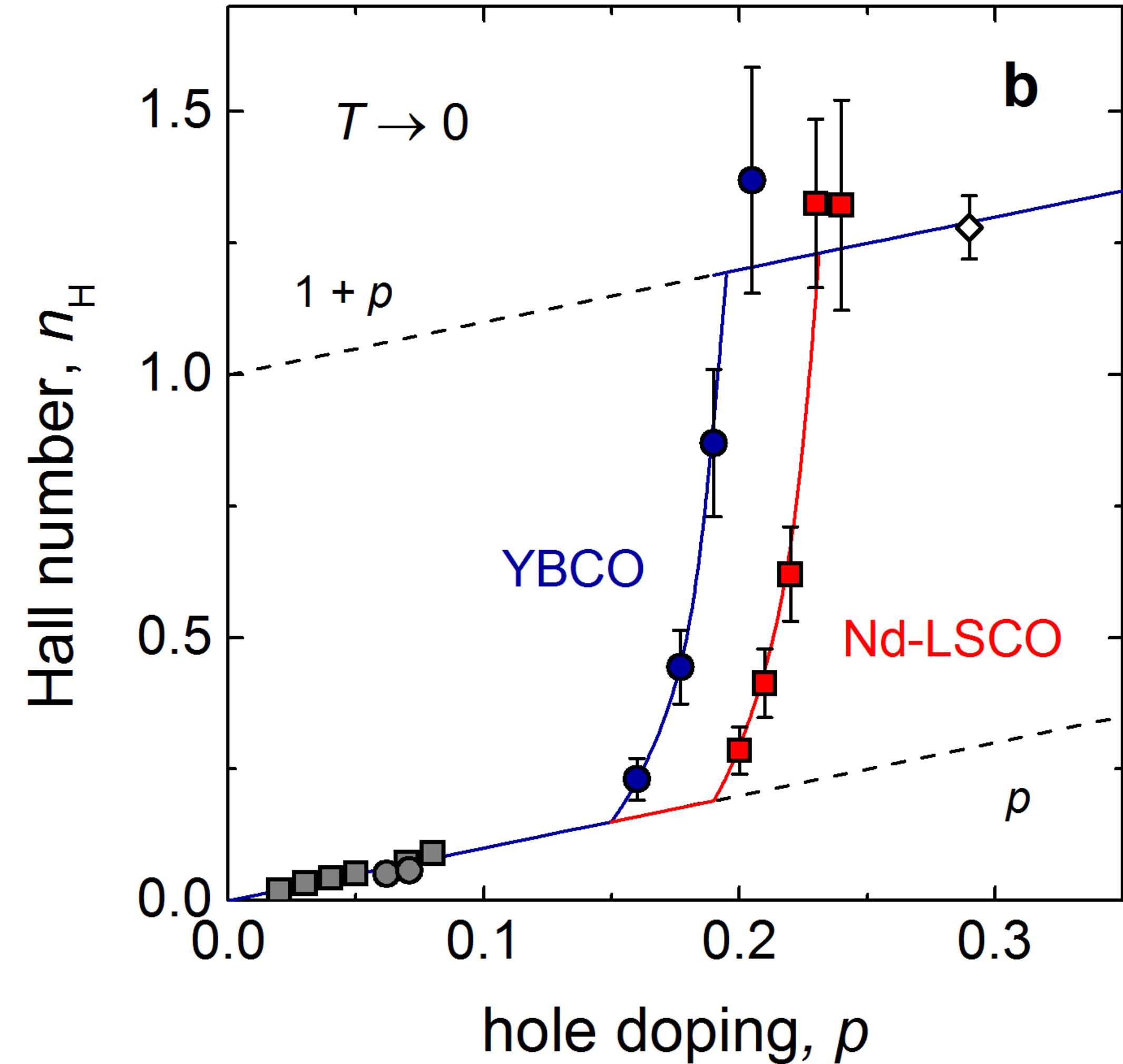
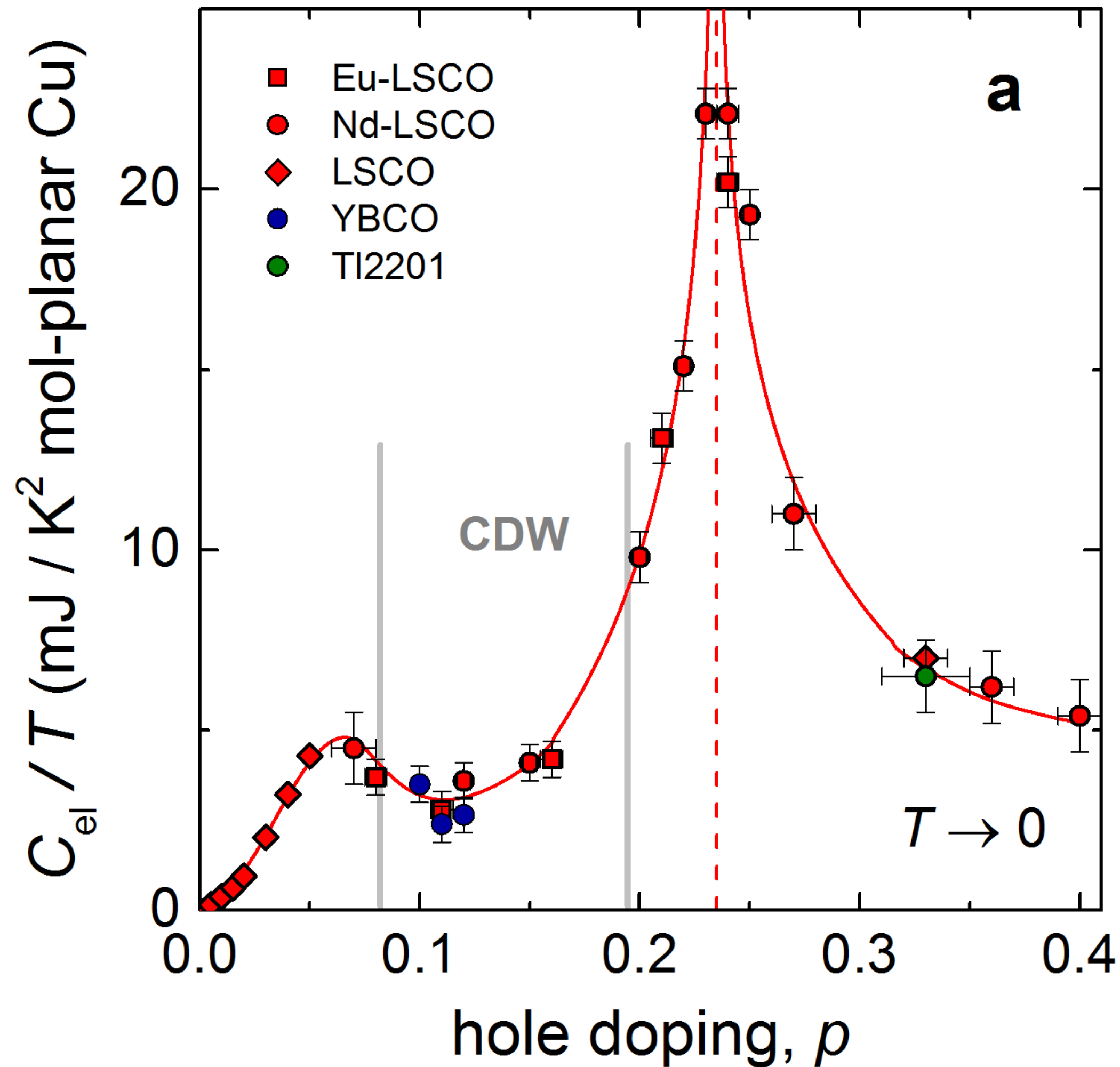
We use angle-dependent magnetoresistance (ADMR) to measure the Fermi surface of the cuprate $\text{La}_{1.6-x}\text{Nd}_{0.4}\text{Sr}_x\text{CuO}_4$. Above the critical doping p^* — outside of the pseudogap phase — we find a Fermi surface that is in quantitative agreement with angle-resolved photoemission. Below p^* , however, the ADMR is qualitatively different, revealing a clear change in Fermi surface topology. We find that our data is most consistent with a Fermi surface that has been reconstructed by a $Q = (\pi, \pi)$ wavevector. While static $Q = (\pi, \pi)$ antiferromagnetism is not found at these dopings, our results suggest that this wavevector is a fundamental organizing principle of the pseudogap phase.



Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

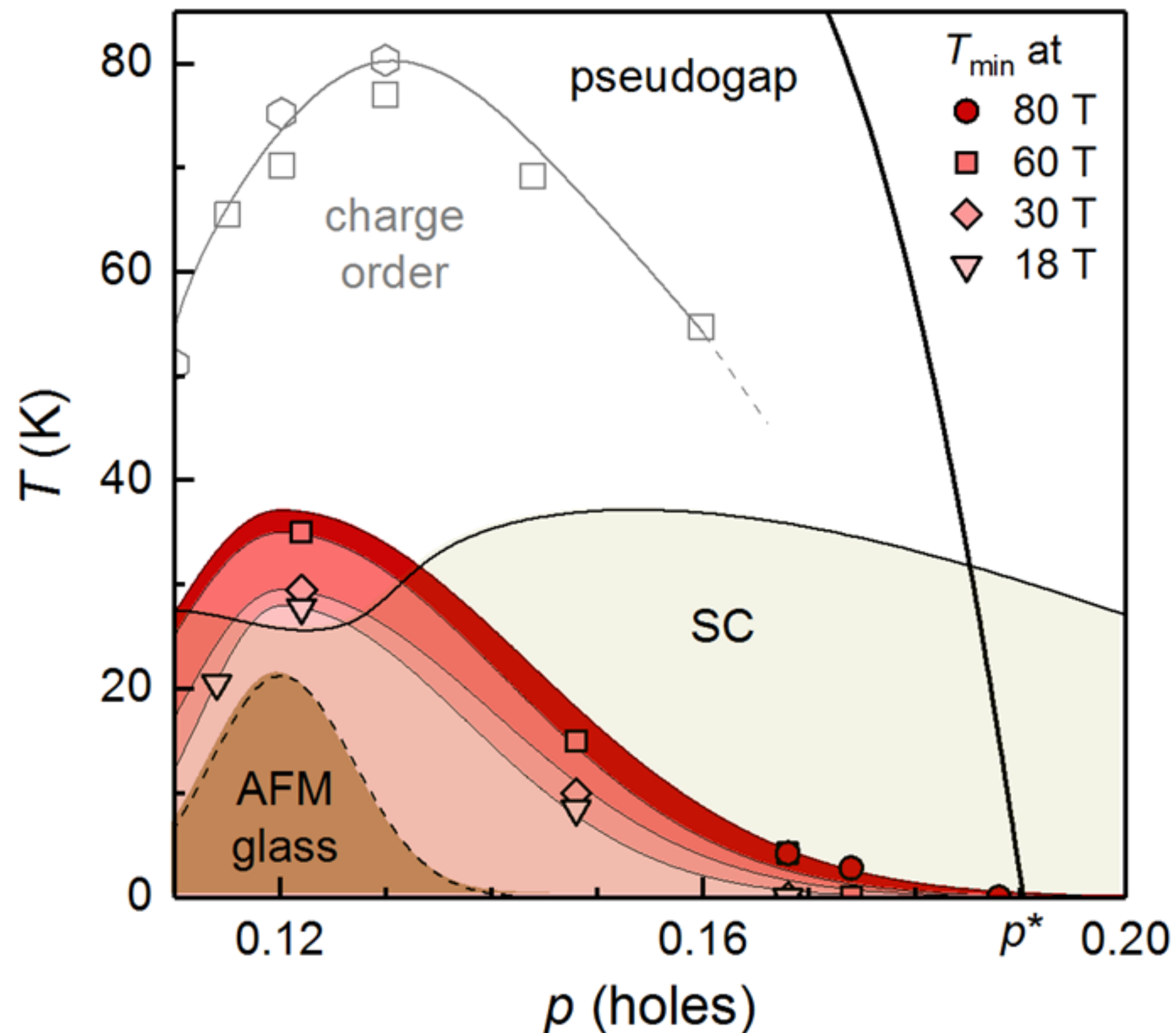
Cyril Proust and Louis Taillefer, Annual Review Condensed Matter Physics **10**, 409 (2019)



Hidden magnetism at the pseudogap critical point of a high temperature superconductor

Nature Physics doi: 10.1038/s41567-020-0950-5

Mehdi Frachet^{1†}, Igor Vinograd^{1†}, Rui Zhou^{1,2}, Siham Benhabib¹, Shangfei Wu¹, Hadrien Mayaffre¹, Steffen Krämer¹, Sanath K. Ramakrishna³, Arneil P. Reyes³, Jérôme Debray⁴, Tohru Kurosawa⁵, Naoki Momono⁶, Migaku Oda⁵, Seiki Komiya⁷, Shimpei Ono⁷, Masafumi Horio⁸, Johan Chang⁸, Cyril Proust¹, David LeBoeuf^{1*}, Marc-Henri Julien^{1*}



Quasi-static magnetism in the pseudogap state of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$.

Temperature – doping phase diagram representing T_{min} , the temperature of the minimum in the sound velocity, at different fields. Since superconductivity precludes the observation of T_{min} in zero-field, the dashed line (brown area) represents the extrapolated $T_{min}(B=0)$. While not exactly equal to the freezing temperature T_f (see Fig. 2), T_{min} is closely tied to T_f and so is expected to have the same doping dependence, including a peak around $p = 0.12$ in zero/low fields (ref. 2). Onset temperatures of charge order are from ref. 33 (squares) and 35 (hexagons).

1. Metal-metal transition in the Kondo Lattice

2. Metal-metal transition in a one-band model

A. FL model of the pseudogap*

B. Ancilla qubits and ghost Fermi surfaces

3. Random J model (insulator)

RG analysis and exact exponent

4. Random t - J model (metals)

Numerics, RG analysis and exact exponents

Why random and all-to-all couplings ?

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- Analytic and numeric progress is possible, and we can compare their results !

Random t - J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j$$

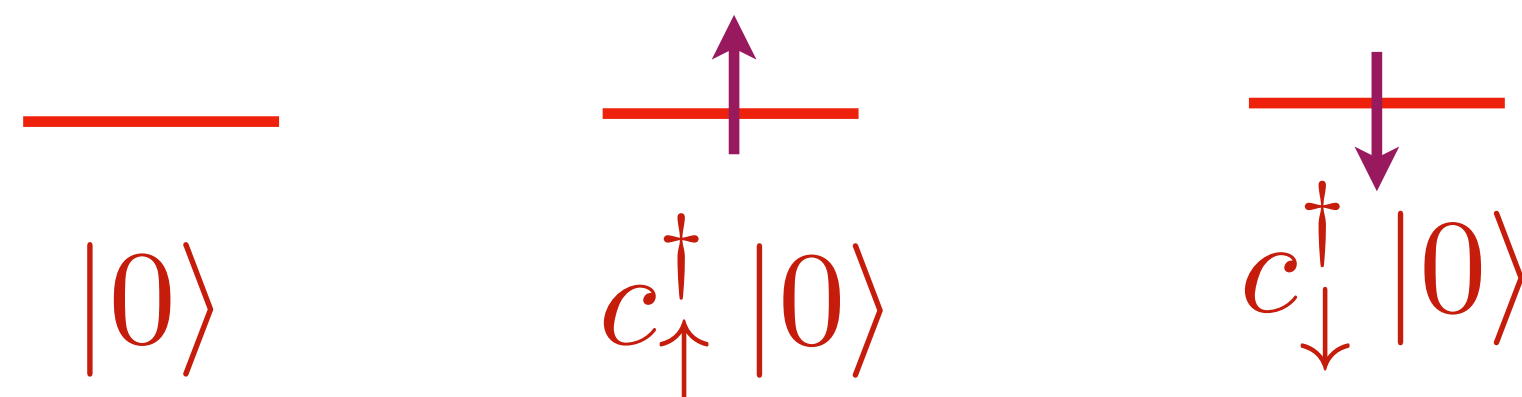
We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^\dagger\} = \delta_{ij} \delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^\dagger c_{i\alpha} \leq 1, \quad \frac{1}{N} \sum_{i\alpha} c_{i\alpha}^\dagger c_{i\alpha} = 1 - p$$

$$J_{ij} \text{ random, } \overline{J_{ij}} = 0, \quad \overline{J_{ij}^2} = J^2$$

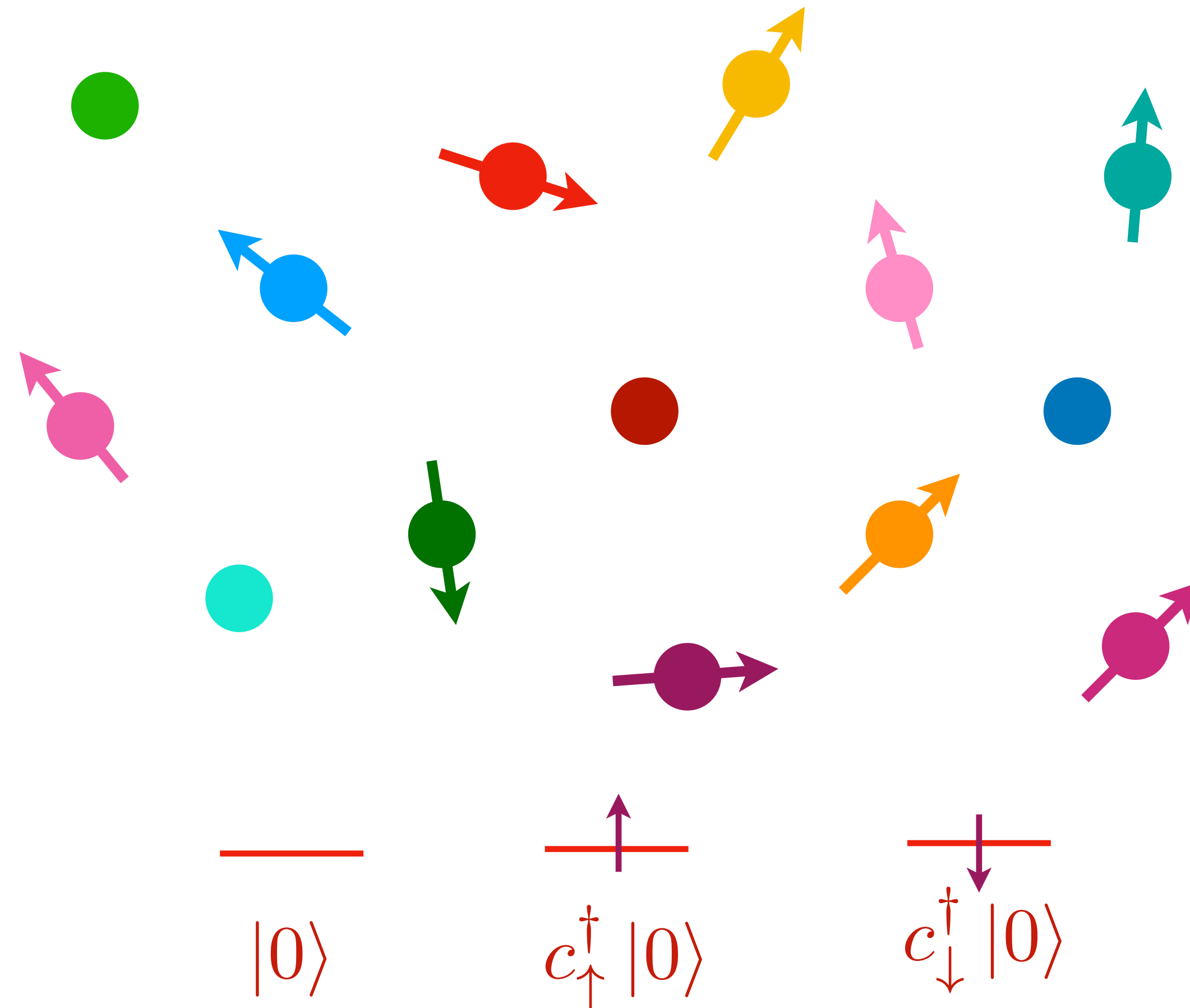
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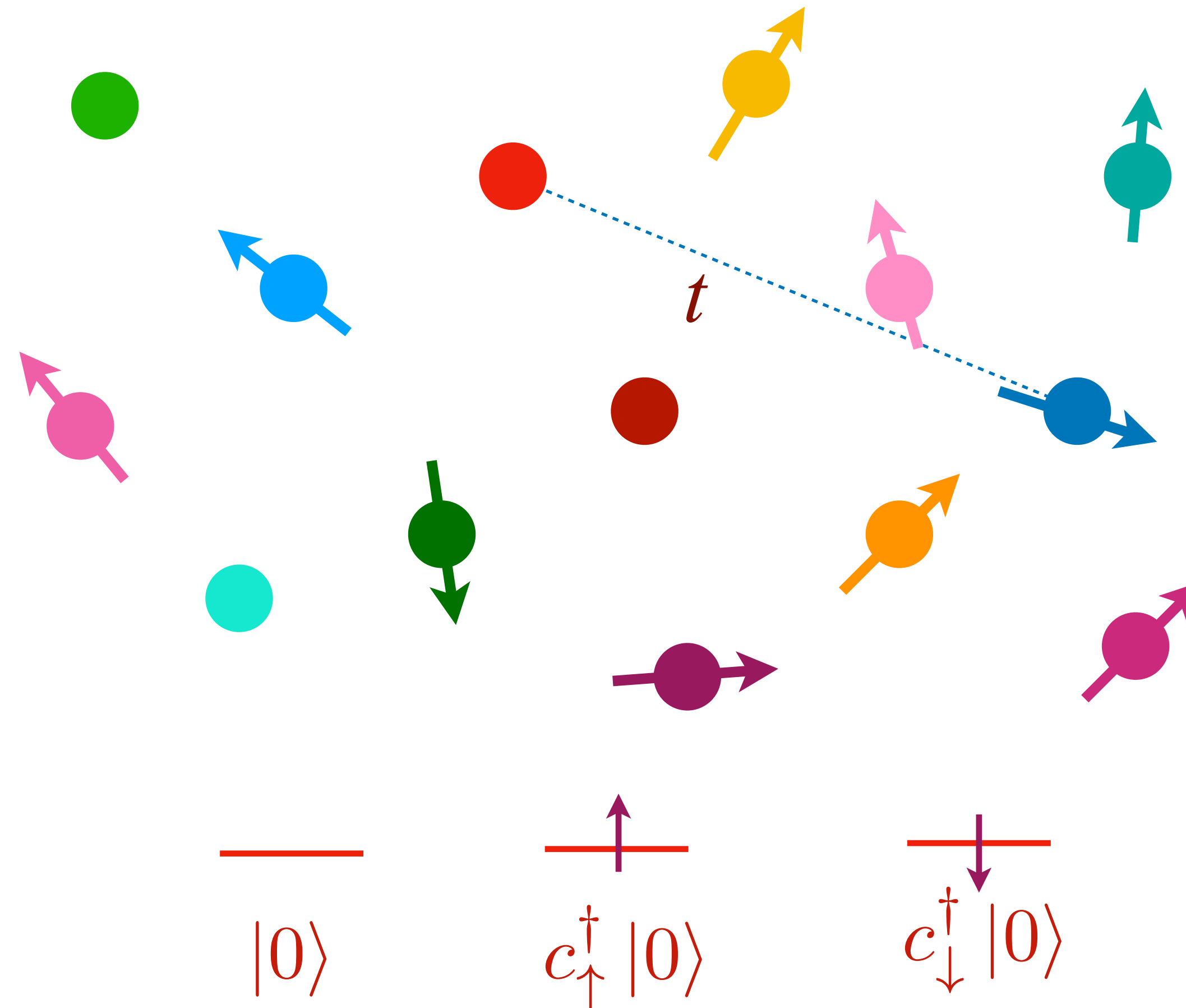
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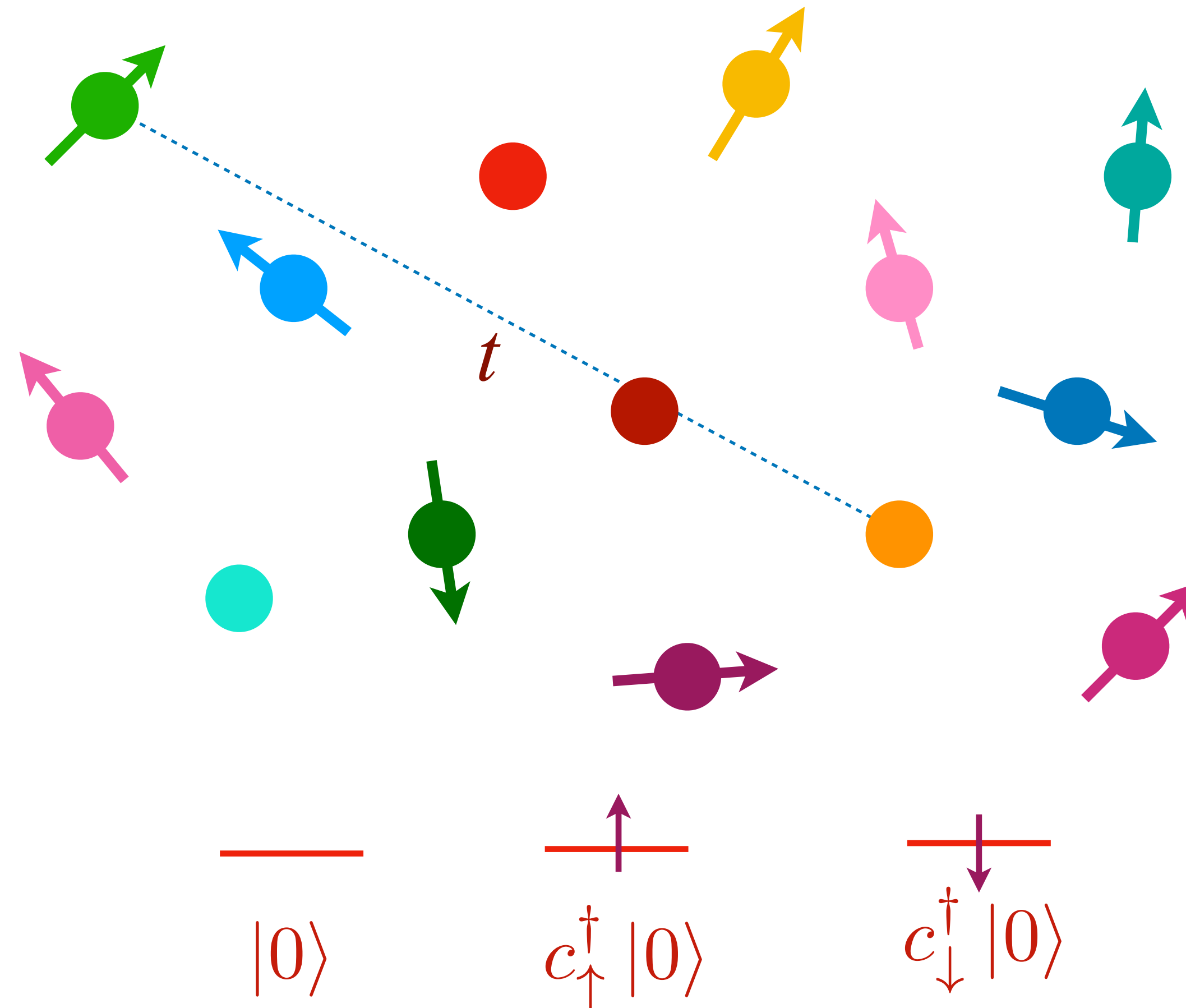
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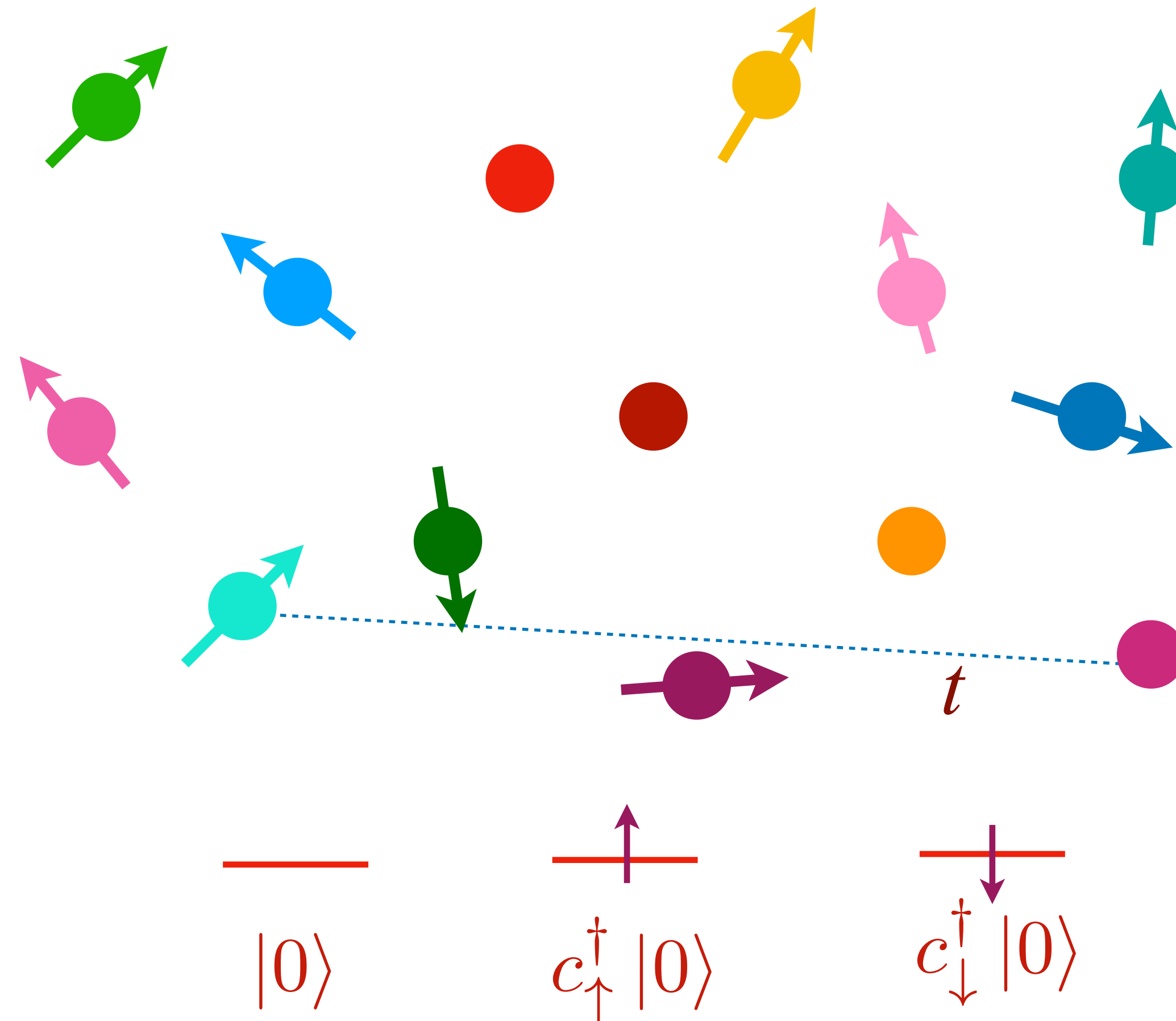
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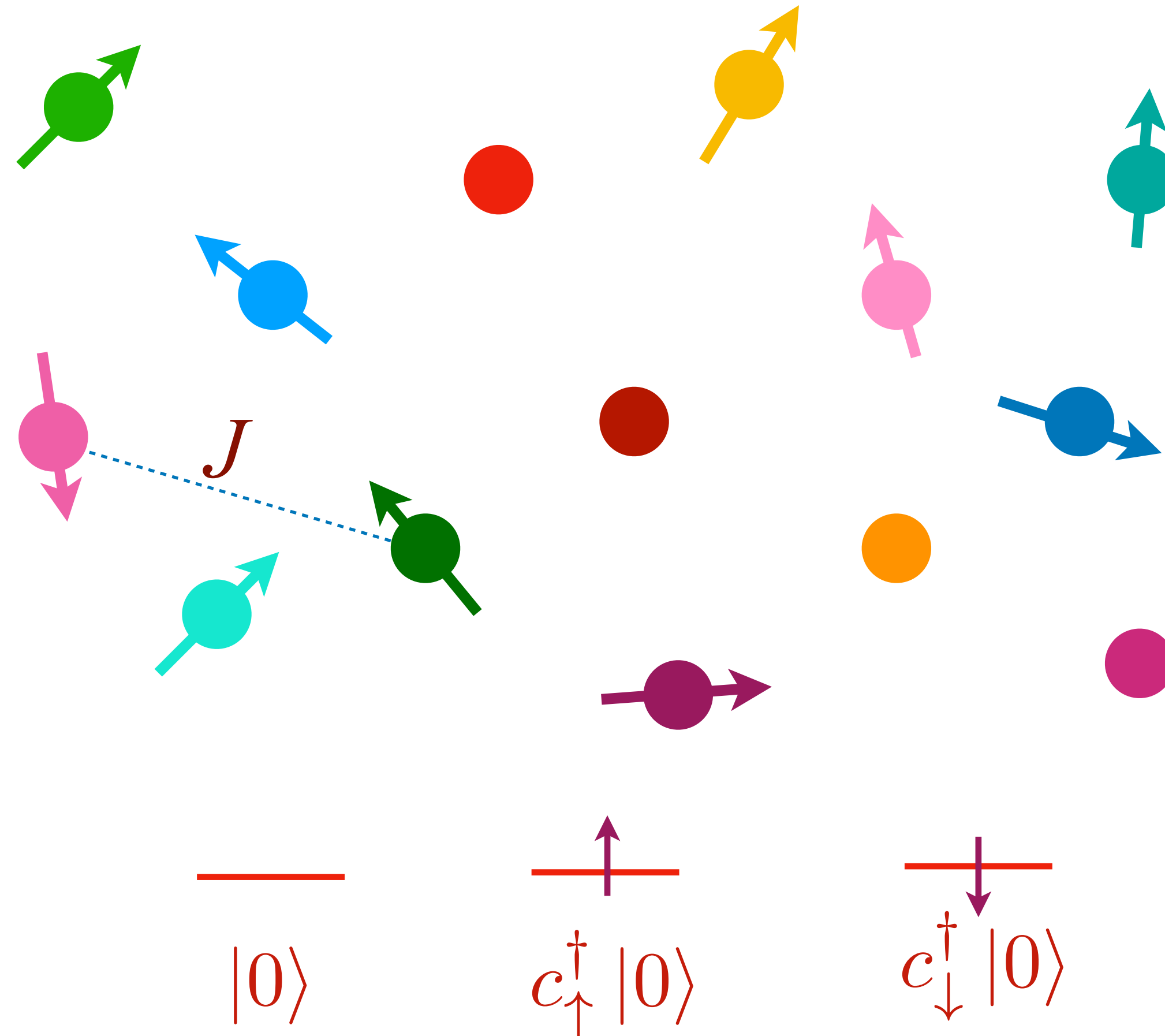
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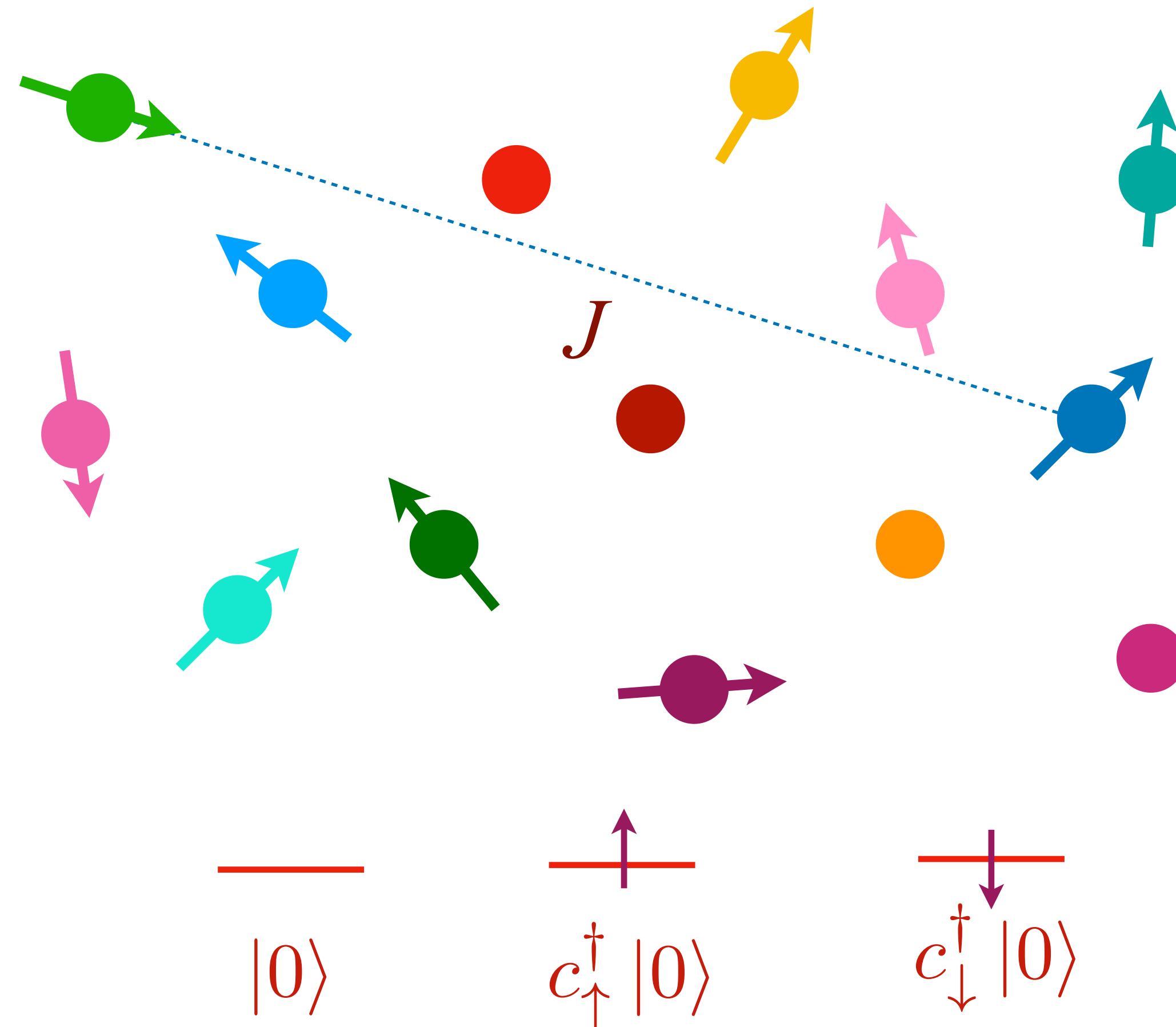
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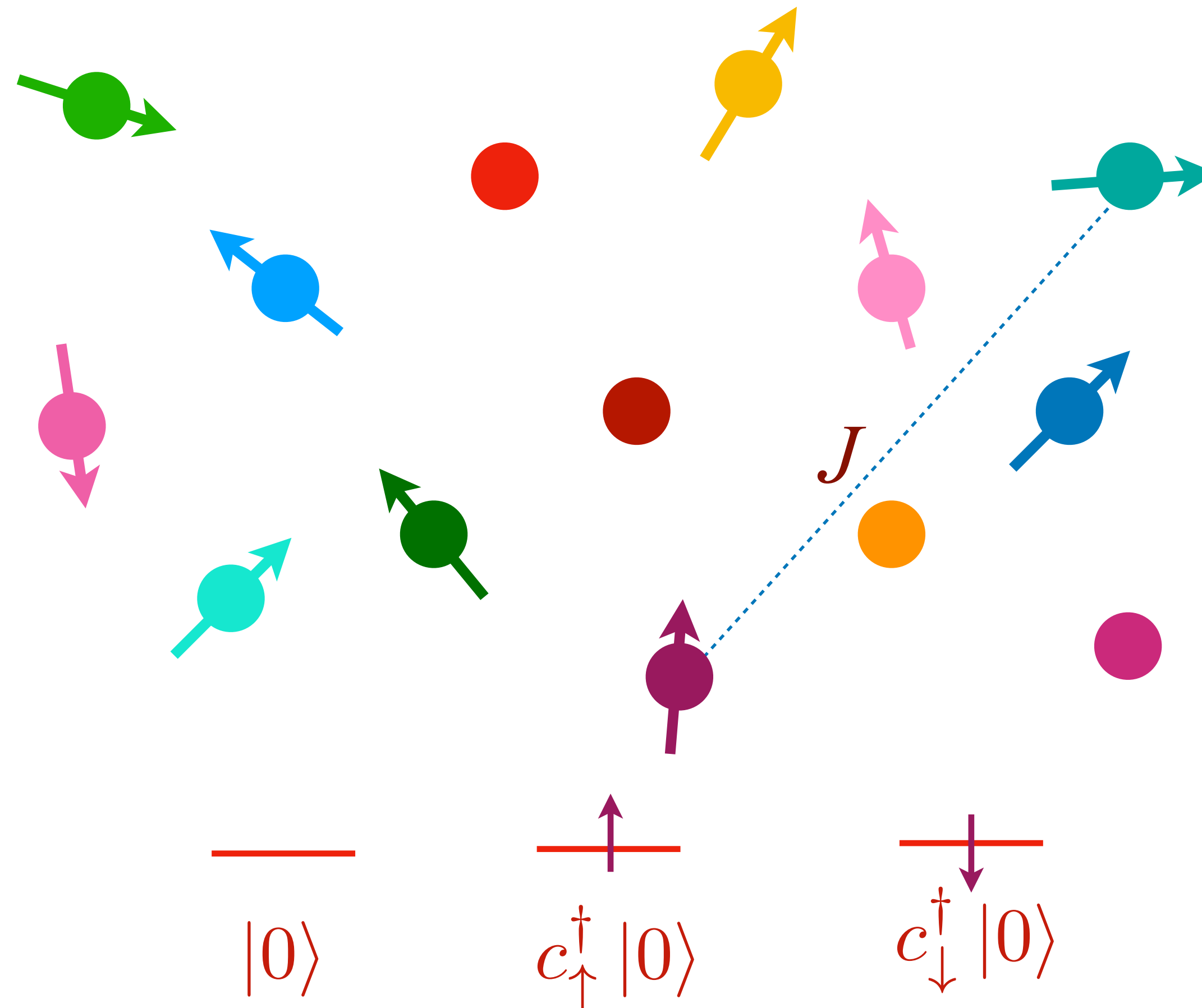
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1. Random J model (insulator)

RG analysis and exact exponent

2. SYK criticality

Time reparameterizations and spectral functions

3. Random t - J model (metals)

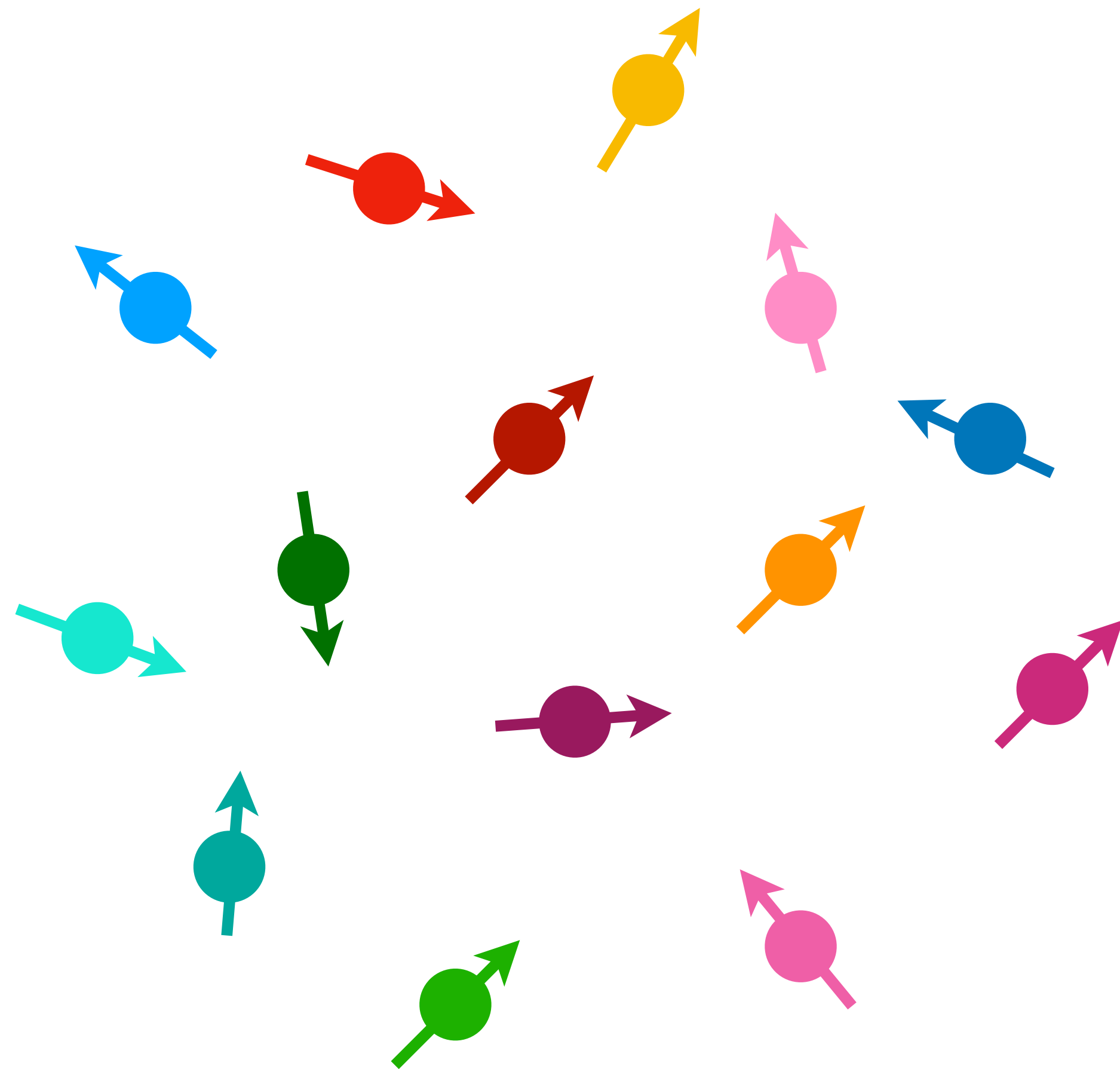
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QMC and exact diagonalization

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Fermionic spinons

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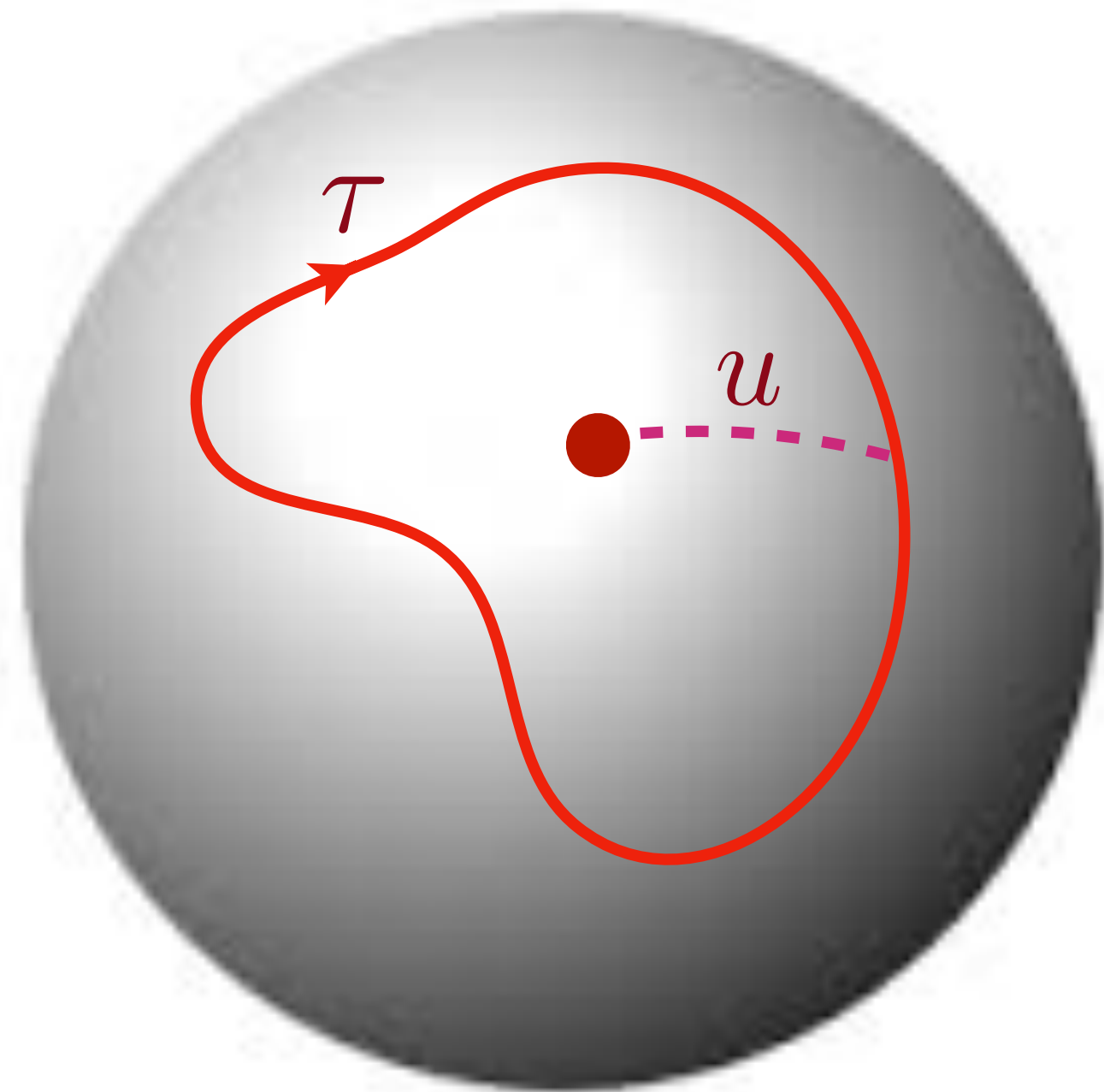
Bosonic spinons

Random J model (insulator)

$$\mathcal{Z} = \int \mathcal{D}\vec{S}(\tau) \delta(\vec{S}^2 - 1) e^{-\mathcal{S}_B - \mathcal{S}_J}$$

$$\mathcal{S}_B = \frac{i}{2} \int_0^1 du \int d\tau \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial \tau} \times \frac{\partial \vec{S}}{\partial u} \right)$$

$$\mathcal{S}_J = -\frac{J^2}{2} \int d\tau d\tau' Q(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau').$$



Random J model (insulator)

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From this action we compute

$$\bar{Q}(\tau - \tau') = \frac{1}{3} \left\langle \vec{S}(\tau) \cdot \vec{S}(\tau') \right\rangle_{\mathcal{Z}}$$

and then impose the self-consistency condition

$$Q(\tau) = \bar{Q}(\tau).$$

Random J model (insulator):RG

We assume a power-law decay

$$Q(\tau) \sim \frac{\gamma^2}{|\tau|^\alpha}.$$

Ignore the self-consistency condition for now. We decouple the $\vec{S}(\tau) \cdot \vec{S}(0)$ interaction by introducing a bosonic (ϕ_a , $a = 1 \dots 3$) bath. Then the problem reduces to the Hamiltonian

$$H_{\text{imp}} = \gamma S_a \phi_a(0) + \frac{1}{2} \int d^d x \left[\pi_a^2 + (\partial_x \phi_a)^2 \right]$$

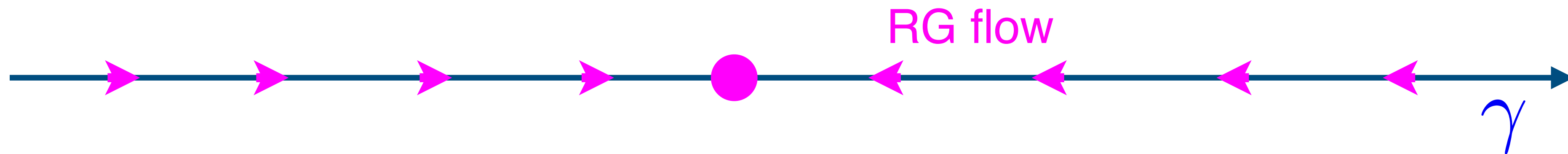
where π_a is canonically conjugate to the field ϕ_a , and $\phi_a(0) \equiv \phi_a(x=0)$. We identify $Q(\tau)$ with temporal correlator of $\phi_a(0)$, and then we need $\alpha = d - 1$.

Random J model (insulator):RG

- The β -function of γ can be computed order-by-order in $\epsilon = 2 - \alpha$

$$\frac{d\gamma}{d\ell} = \epsilon \frac{\gamma}{2} - \gamma^3.$$

- There is an attractive fixed point at $\gamma = \gamma^* = \mathcal{O}(\sqrt{\epsilon})$.
- Because of the quantized Berry phase (Wess-Zumino-Witten) term, the renormalization of the coupling γ is given only by the wavefunction renormalization. We can then prove that at this fixed point $\overline{Q}(\tau) \sim 1/|\tau|^{2-\alpha}$ to all orders in ϵ .



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This exponent can be understood from a SYK theory of spinons f , with $G_f \sim 1/\sqrt{\tau}$.

to all orders in ϵ .

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

M. Vojta, C. Buragohain, and S. Sachdev, PRB **61**, 15152 (2000)

S. Sachdev, Physica C **357**, 78 (2001)

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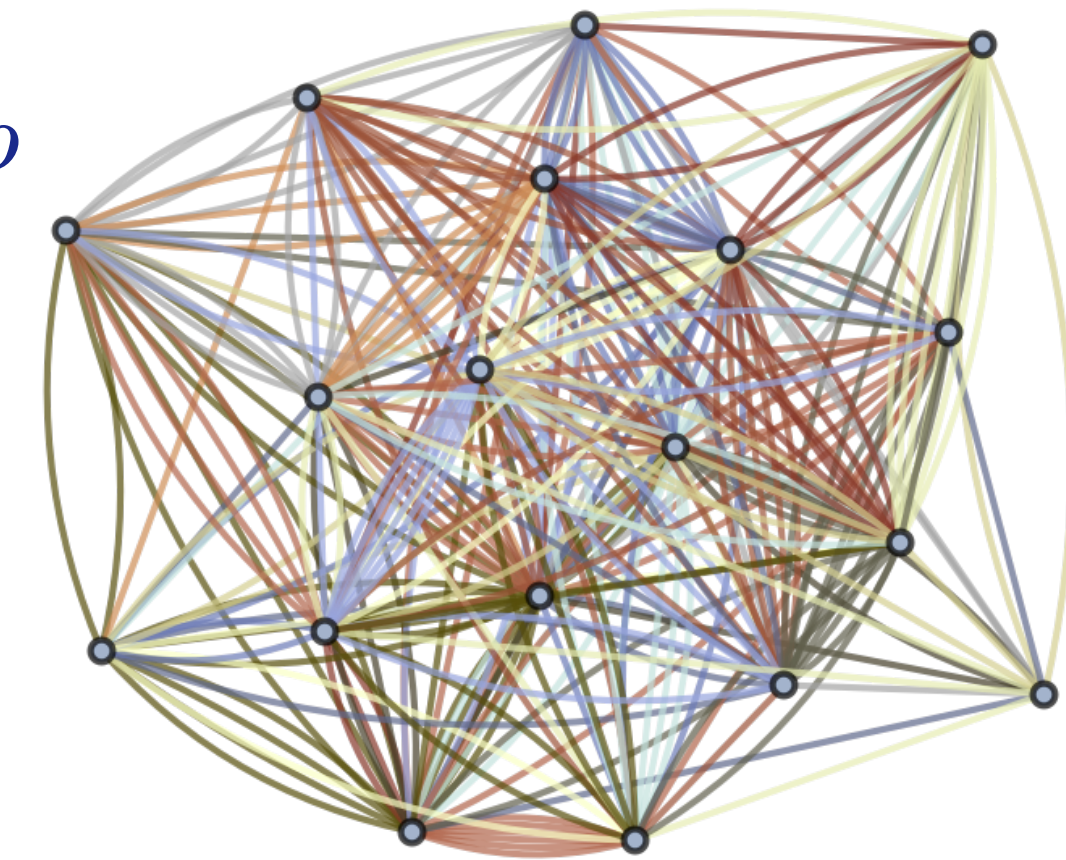
QMC and exact diagonalization

The complex SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{a,b,c,d=1}^N U_{ab;cd} c_a^\dagger c_b^\dagger c_c c_d - \mu \sum_a c_a^\dagger c_a$$

$$c_a c_b + c_b c_a = 0 \quad , \quad c_a c_b^\dagger + c_b^\dagger c_a = \delta_{ab}$$

$$Q = \frac{1}{N} \sum_a c_a^\dagger c_a$$

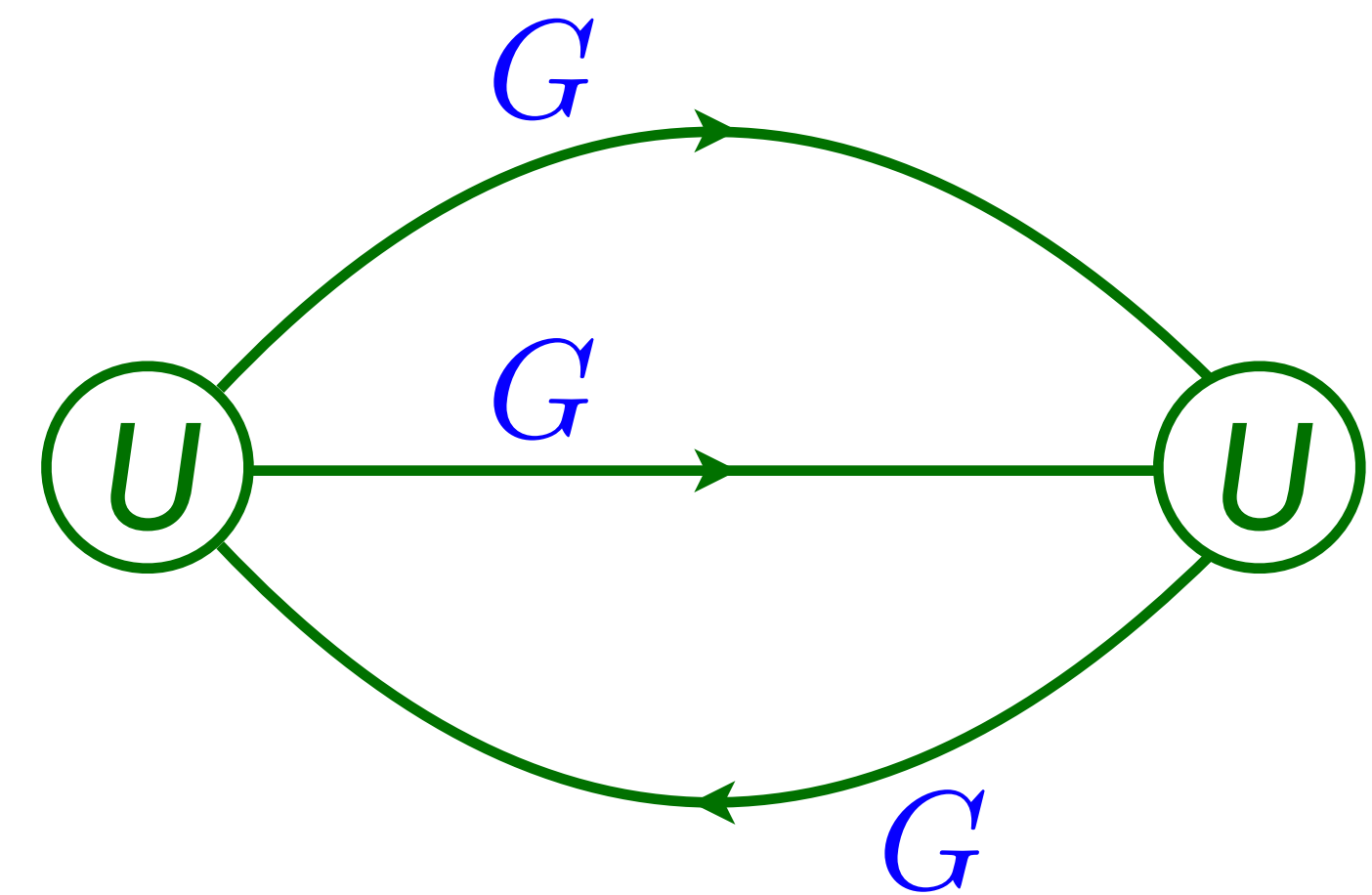


$U_{ab;cd}$ are independent random variables

with $\overline{U_{ab;cd}} = 0$ and $\overline{|U_{ab;cd}|^2} = U^2$

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

$\Sigma =$



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

Random J model (insulator): $SU(M)$ symmetry

Express the spin operator in terms of fermionic spinons $\vec{S} = (1/2)f_\alpha^\dagger \vec{\sigma}_{\alpha\beta} f_\beta$, and let $\alpha = 1 \dots M$. The fermions obey the constraint

$$\sum_{\alpha=1}^M f_\alpha^\dagger f_\alpha = \frac{M}{2}$$

In the large M limit we obtain the SYK equations for the spinon Green's function G and self energy Σ ; similar results apply for bosonic spinons.

$$G(i\omega) = \frac{1}{i\omega - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

The solution is

$$G(\tau) \sim \frac{\text{sgn}(\tau)}{\sqrt{|\tau|}} \quad , \quad \langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|}$$

SYK criticality

Key properties

1. The ground state is 'critical' and there are no quasiparticles.

S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

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A. Kitaev, KITP talk (2015)

J. Maldacena and D. Stanford, PRD **94**, 106002 (2016)

A. Kitaev and J. Suh, JHEP 183 (2018)

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A. Georges, O. Parcollet,
and S. Sachdev, PRB **63**,
134406 (2001)

$$\lim_{T \rightarrow 0} \lim_{N \rightarrow \infty} \frac{S}{N} = \mathcal{S}_0(\mathcal{Q}) \neq 0$$

This entropy is not due to an exponentially large ground degeneracy. Instead, it reflects an exponentially small many-body level spacing $\sim e^{-N\mathcal{S}_0}$ down to the ground state.

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4. Dynamic scaling of equilibrium and non-equilibrium properties in a universal time $\sim \hbar/(k_B T)$

SYK criticality

Key properties

5. The leading low temperature behavior of many observables is controlled by a time reparameterization soft mode. The action for this soft mode is controlled by an emergent $SL(2, \mathbb{R})$ symmetry. Specifically, the entropy is $S(T)/N = \mathcal{S}_0(Q) + \gamma T$, where γ is proportional to the coefficient of the Schwarzian.

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6. Spin correlations in the random J model decay as

$$\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|}$$

S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

Time reparameterization soft mode

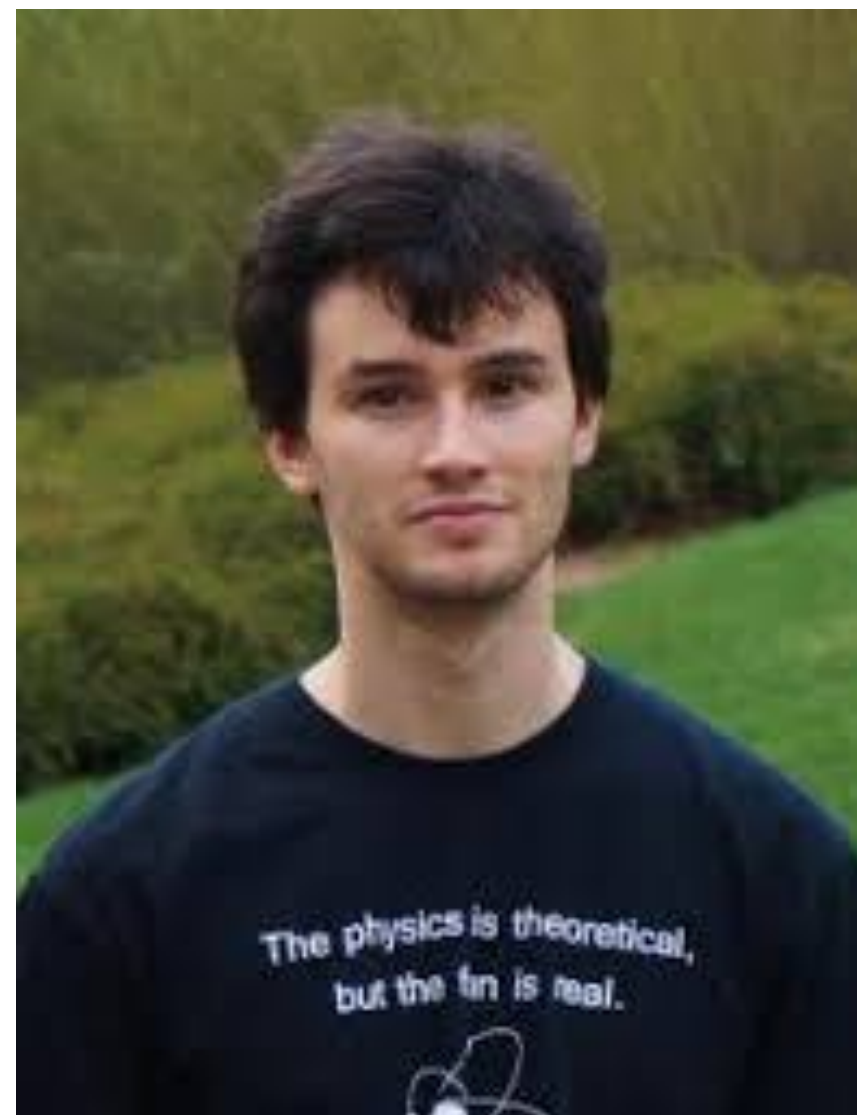
The leading corrections to the critical **spinon** Green's function arise from the time reparameterization soft mode, and these take the form

$$\langle f_\alpha(\tau) f_\alpha^\dagger(0) \rangle \sim \left[\frac{\pi T}{\sin(\pi T \tau)} \right]^{1/2} \left(1 + \alpha_G \frac{T}{J} \Phi_{\text{non-conformal}}(T\tau) \right)$$

where $\Phi_{\text{non-conformal}}(T\tau)$ is a computable (in the large M limit) scaling function, and α_G is universally proportional to the co-efficient α_S of the Schwarzian action for the time reparameterization mode.



Maria Tikhanovskaya



Grigory Tarnopolsky



Haoyu Guo

Random J model (insulator): $SU(M)$ symmetry

The local dynamic spin susceptibility, $\chi_L(i\omega_n) = \int_0^{1/T} d\tau \langle \vec{S}_i(\tau) \vec{S}_i(0) \rangle e^{i\omega_n \tau}$, obeys

$$\text{Im}\chi_L(\omega) \propto \tanh\left(\frac{\omega}{2T}\right)$$

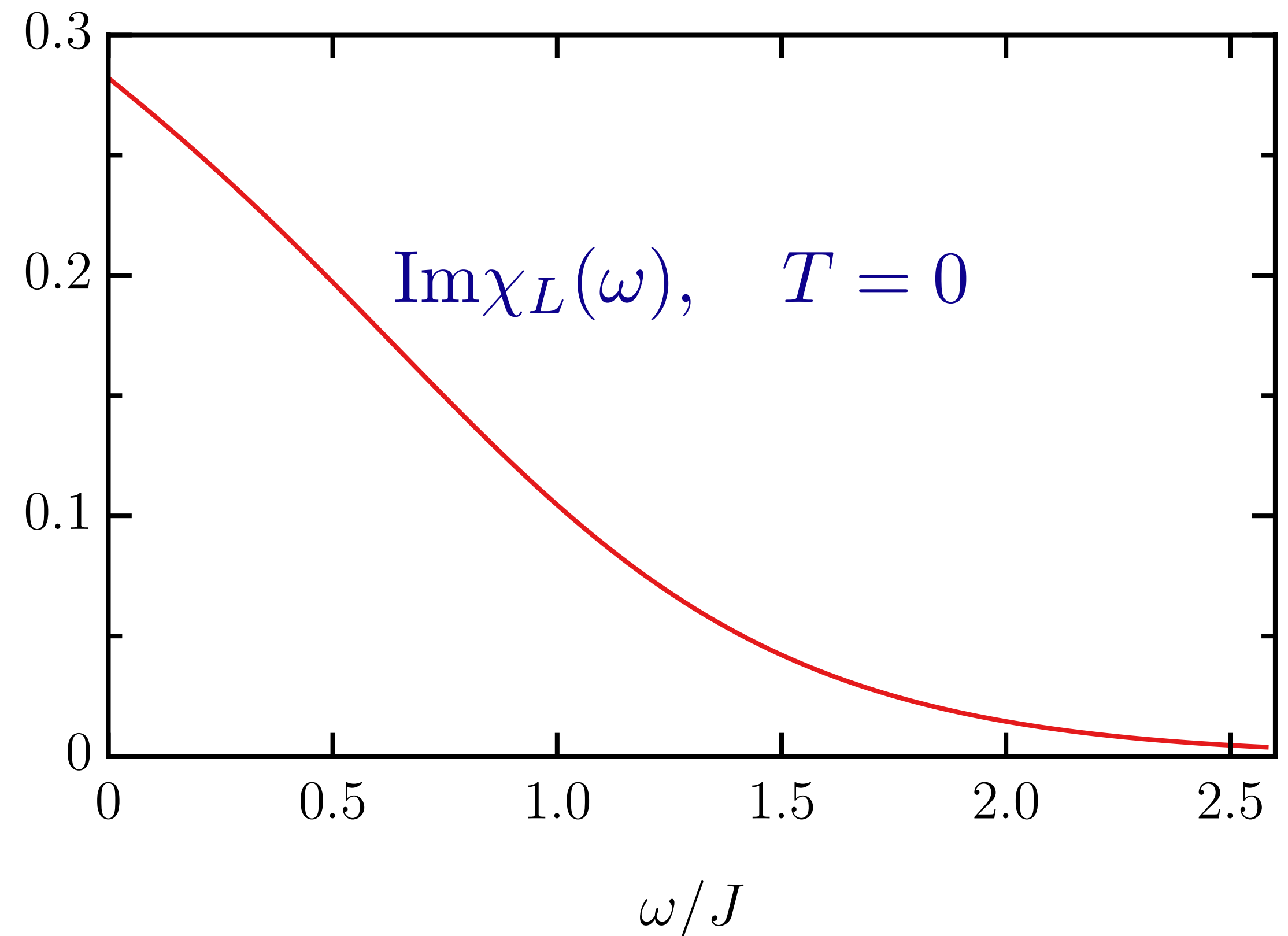
A. Georges and O. Parcollet, PRB **59**, 5341 (1999)

Random J model (insulator): $SU(M)$ symmetry

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$$\text{Im}\chi_L(\omega) \propto \tanh\left(\frac{\omega}{2T}\right) \left[1 - \alpha_S \omega \tanh\left(\frac{\omega}{2T}\right) + \dots \right]$$

From the time reparameterization soft-mode



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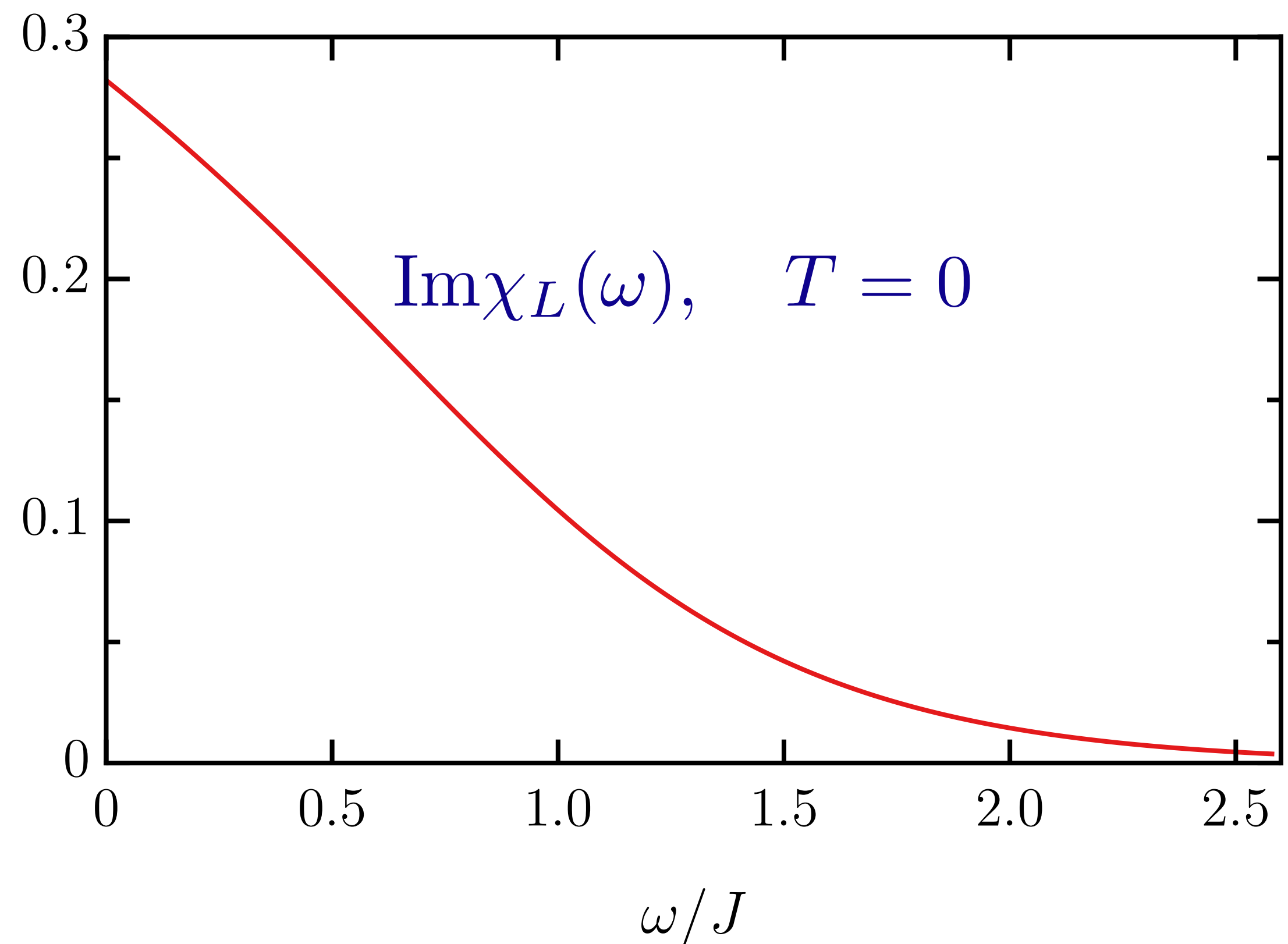
$$\text{Im}\chi_L(\omega) \propto \tanh\left(\frac{\omega}{2T}\right) \left[1 - \alpha_S \omega \tanh\left(\frac{\omega}{2T}\right) + \dots \right]$$

The ratio α_S/γ is a universal number, which we have computed in the large M limit

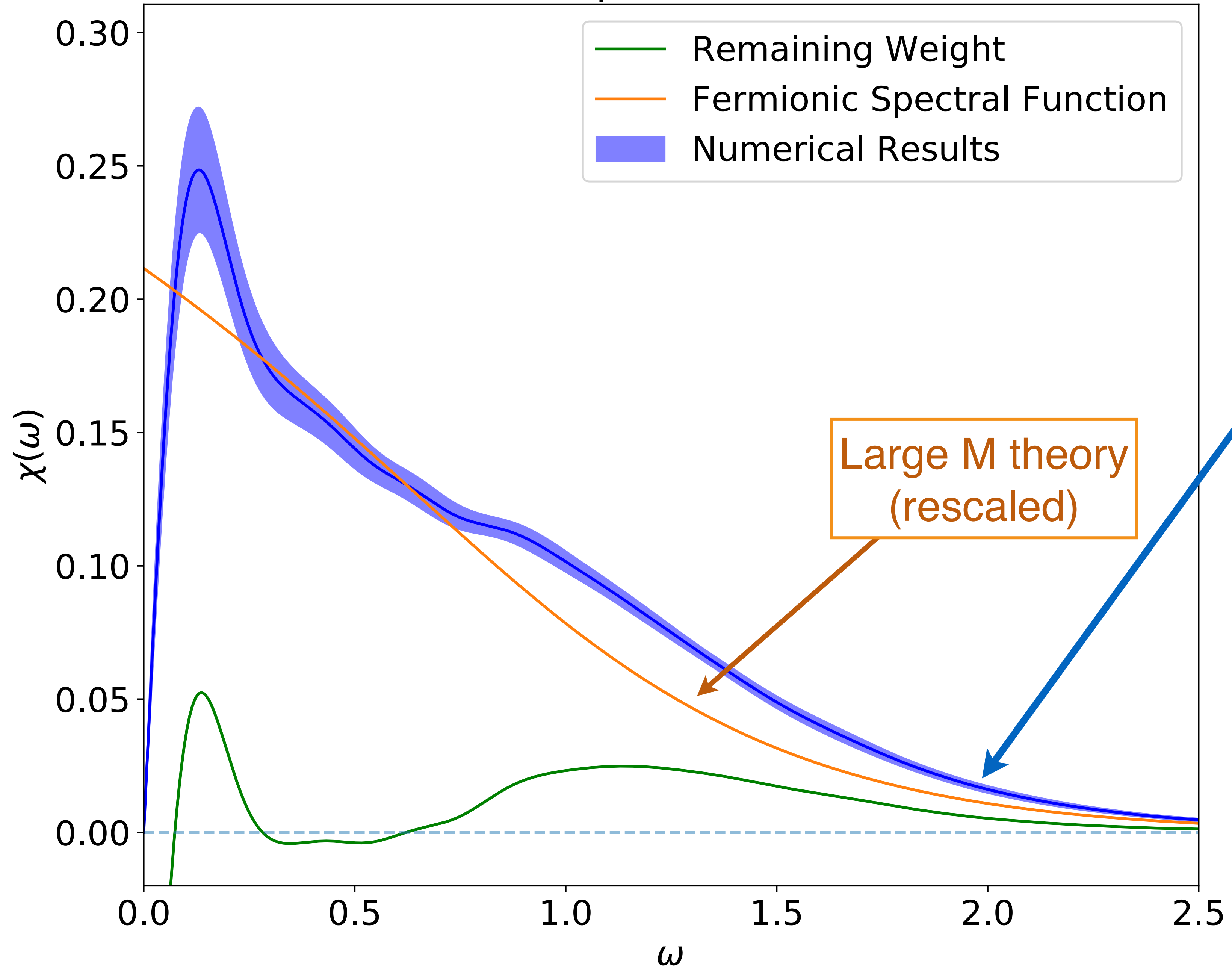
$$\frac{\alpha_S}{\gamma} = \frac{24}{\pi(2 + 3\pi)}, \quad M \rightarrow \infty.$$

(Recall the specific heat is $C = \gamma T$ per site and spin component.)

From the time reparameterization soft-mode



$p=0.00$



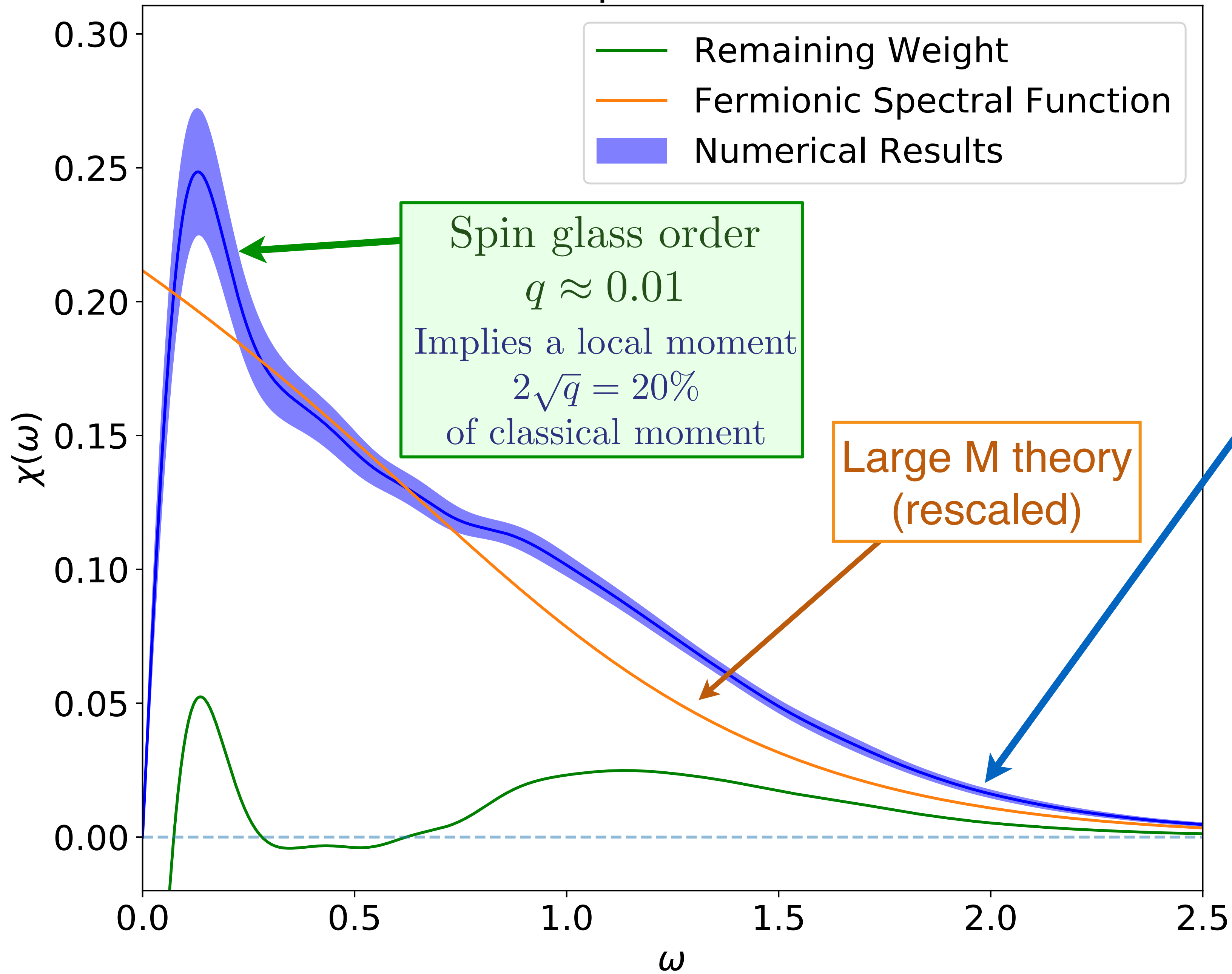
- Remaining Weight
- Fermionic Spectral Function
- Numerical Results

Large M theory
(rescaled)

Exact diagonalization
results of SU(2) random magnet
L.Arrachea and M.J. Rozenberg,
PRB **65**, 224430 (2002)
(recomputed by
Henry Shackleton)

Incoherent contribution
similar to SY spin liquid !

$p=0.00$



1. Random J model (insulator)

RG analysis and exact exponent

2. SYK criticality

Time reparameterizations and spectral functions

3. Random t - J model (metals)

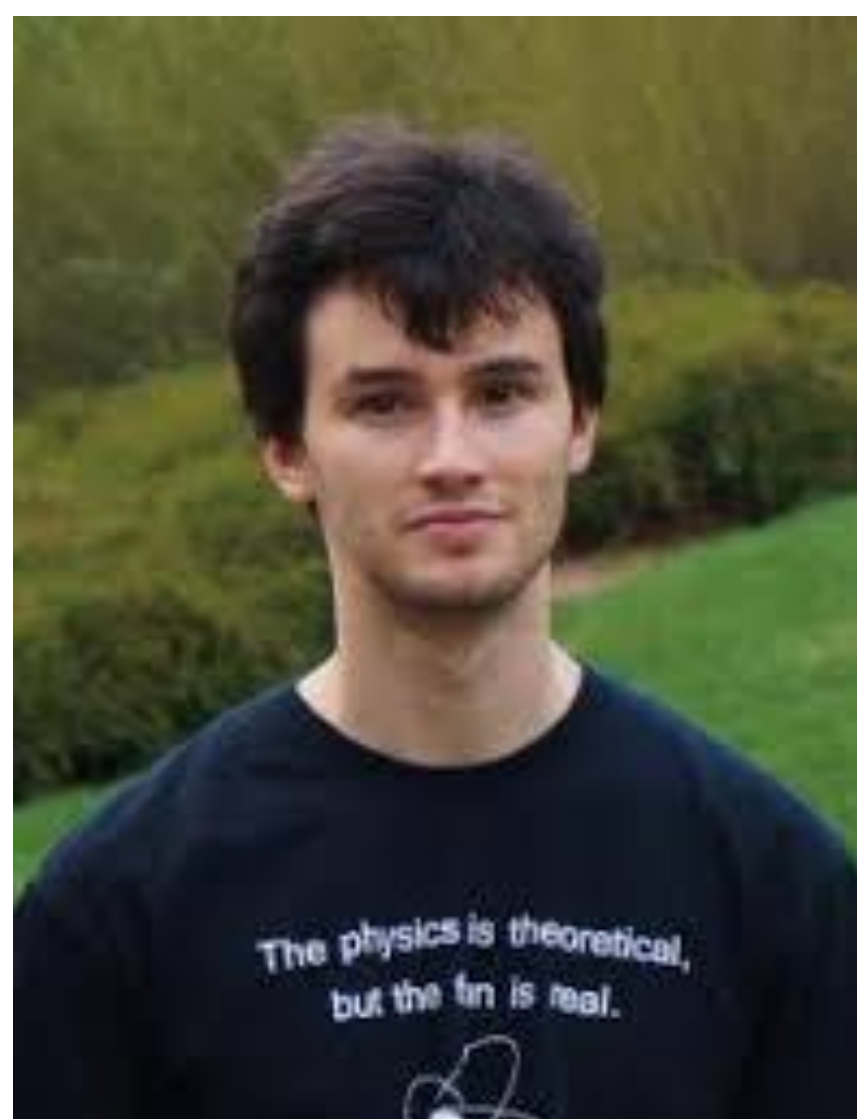
RG analysis and exact exponents

4. Numerical results

QMC and exact diagonalization



Darshan Joshi



Grigory Tarnopolsky

Physical Review X
10, 021033 (2020)



Chenyuan Li

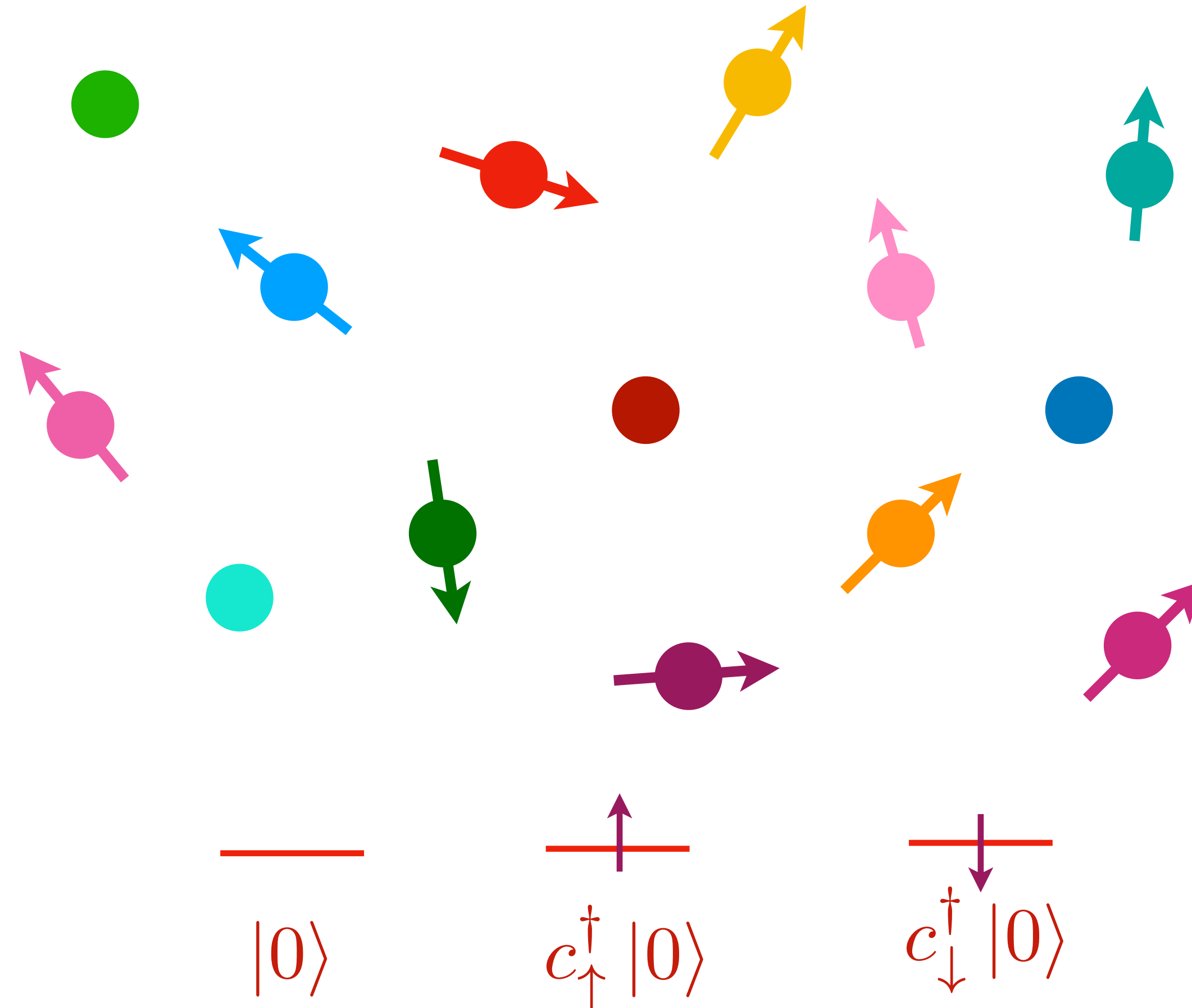


Antoine Georges

Random t - J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.



Random t-J model (metal)

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Each site has 3 states which we map to the ‘*superspin*’ space of a boson b (the holon) and a fermion f_α (the spinon):

$$\begin{array}{ccc} \text{—} & \text{—} \uparrow & \text{—} \downarrow \\ b^\dagger |v\rangle & f_\uparrow^\dagger |v\rangle & f_\downarrow^\dagger |v\rangle \end{array}$$

$$\begin{aligned} c_\alpha &= f_\alpha b^\dagger \\ \vec{S} &= \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \end{aligned}$$

$$f_\alpha^\dagger f_\alpha + b^\dagger b = 1$$

$$\text{U(1) gauge invariance,} \quad b \rightarrow b e^{i\phi}, \quad f_\alpha \rightarrow f_\alpha e^{i\phi}$$

The physical electron (c_α) and spin (\vec{S}) operators are rotations in this SU(1|2) superspin space.

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$$\text{SU}(1|2) \equiv \text{SU}(2|1)$$

The physical electron (c_α) and spin (\vec{S}) operators are rotations in this SU(2|1) superspin space.

Random t-J model (metal)

$$\mathcal{Z} = \int \mathcal{D}\mathcal{P}(\tau) e^{-\mathcal{S}_B - \mathcal{S}_{tJ}}$$

$$\mathcal{S}_B = i \int_0^1 du \int d\tau \text{Tr} (\mathcal{P} \partial_\tau \mathcal{P} \partial_u \mathcal{P})$$

$$\begin{aligned} \mathcal{S}_{tJ} = & \int d\tau d\tau' \text{Tr} (\mathcal{P}(\tau) \mathcal{Q}(\tau - \tau') \mathcal{P}(\tau')) \\ & + \int d\tau \text{Tr} (s_0 \mathcal{P}(\tau)) . \end{aligned}$$

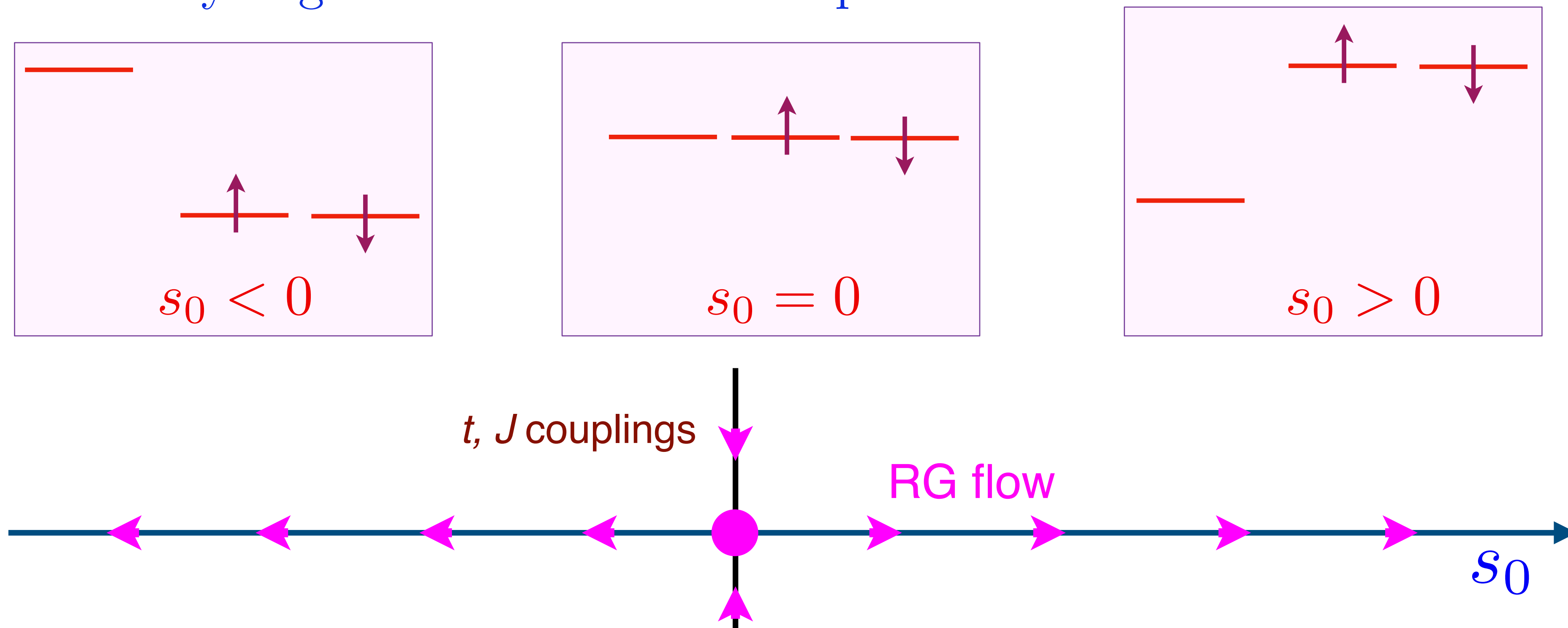
Path integral over a superspin $\mathcal{P}(\tau)$ with a self-consistent self-interaction $\mathcal{Q}(\tau)$ and a ‘Zeeman superfield’ s_0 .

Random t-*J* model (metal):RG

- The RG analysis is very similar to that for the *J* model, except that the SU(2) spin is replaced by a $SU(1|2) \cong SU(2|1)$ superspin.

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- The Wess-Zumino-Witten term in superspace now ensures the exact exponents at the fixed point

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|} \quad , \quad \left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle \sim \frac{1}{\tau} .$$

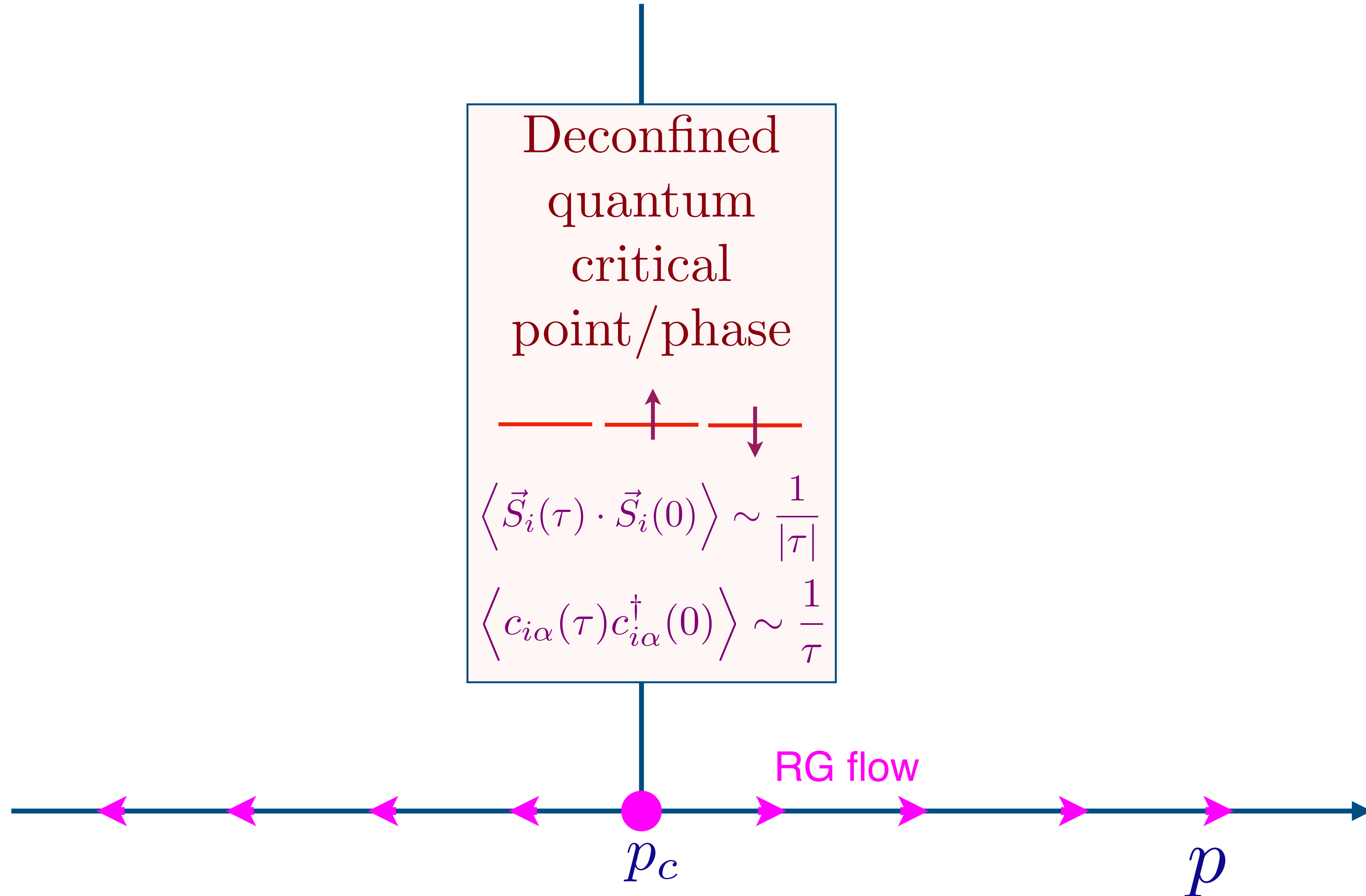
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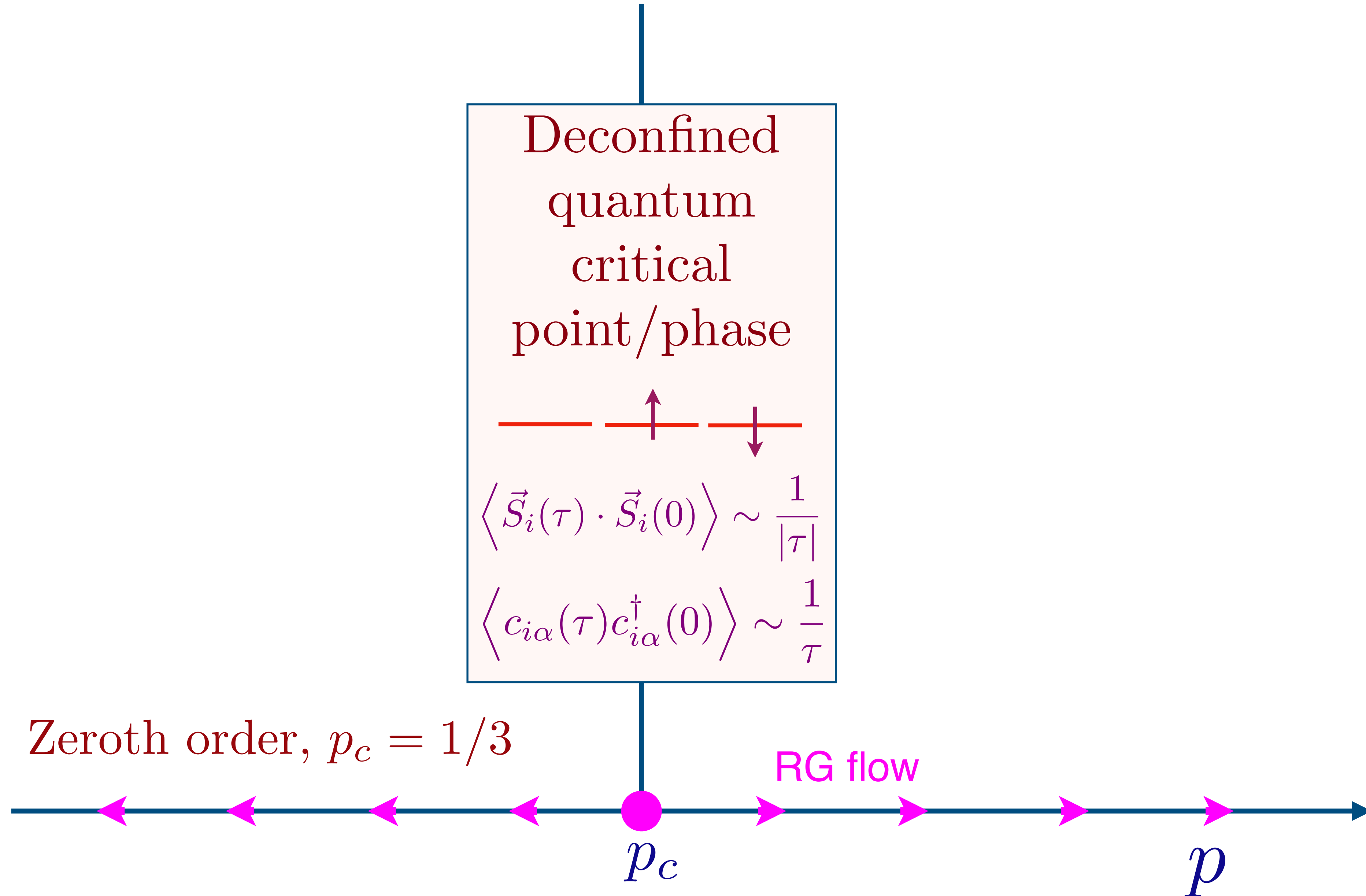
Square of spinon correlator \rightarrow $\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|}$, $\langle c_\alpha(\tau) c_\alpha^\dagger(0) \rangle \sim \frac{1}{\tau}$ \leftarrow Product of spinon and holon correlators

- These exponents do not have a quasiparticle interpretation. However, they can be understood (in a large M limit of a SYK-like model with $SU(M)$ symmetry) by *fractionalization* of the electron into a spinon and holon, each of which decay as $1/\sqrt{\tau}$.

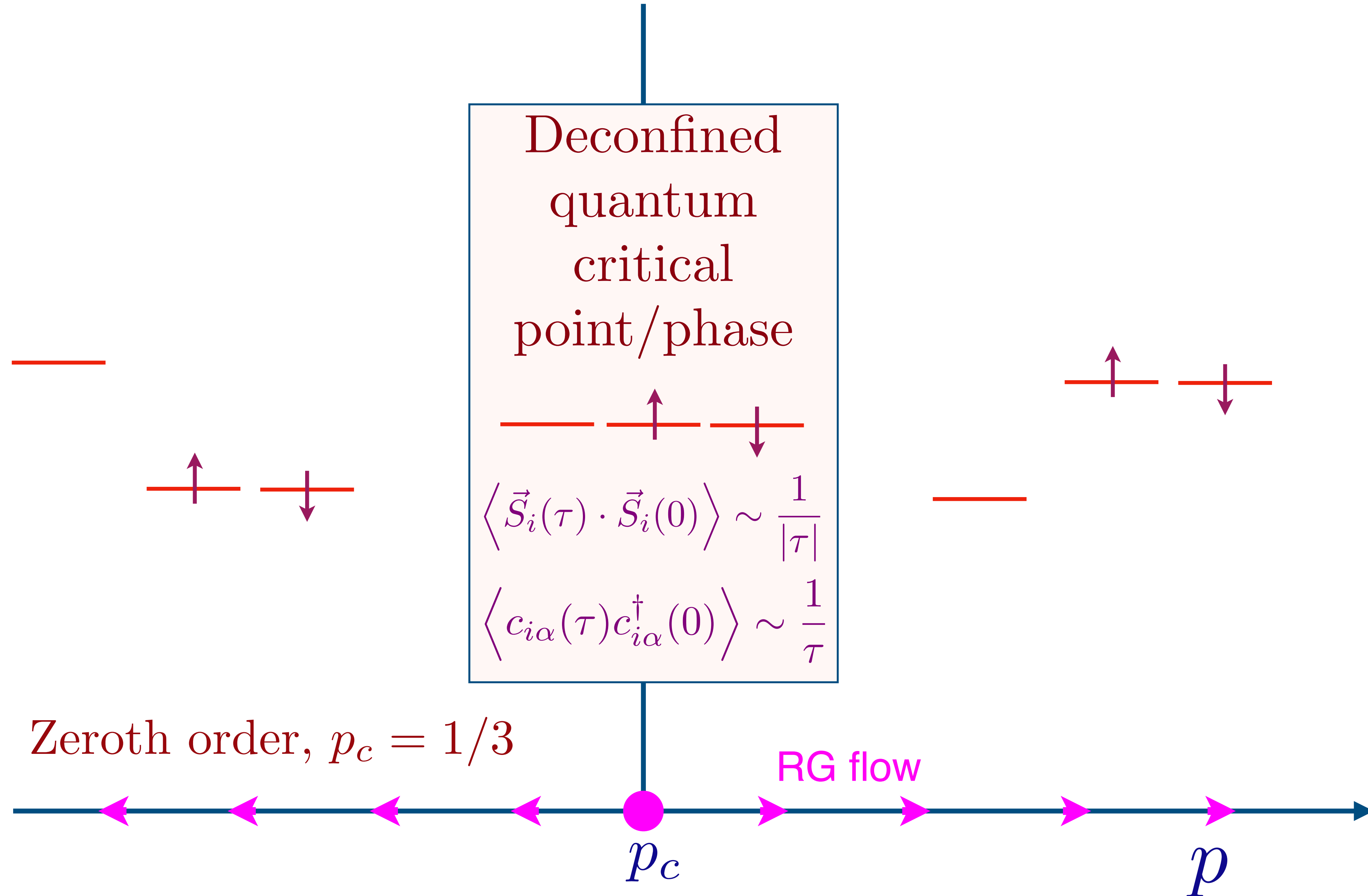
t - J phase diagram: RG using *either* $SU(2|1)$ or $SU(1|2)$



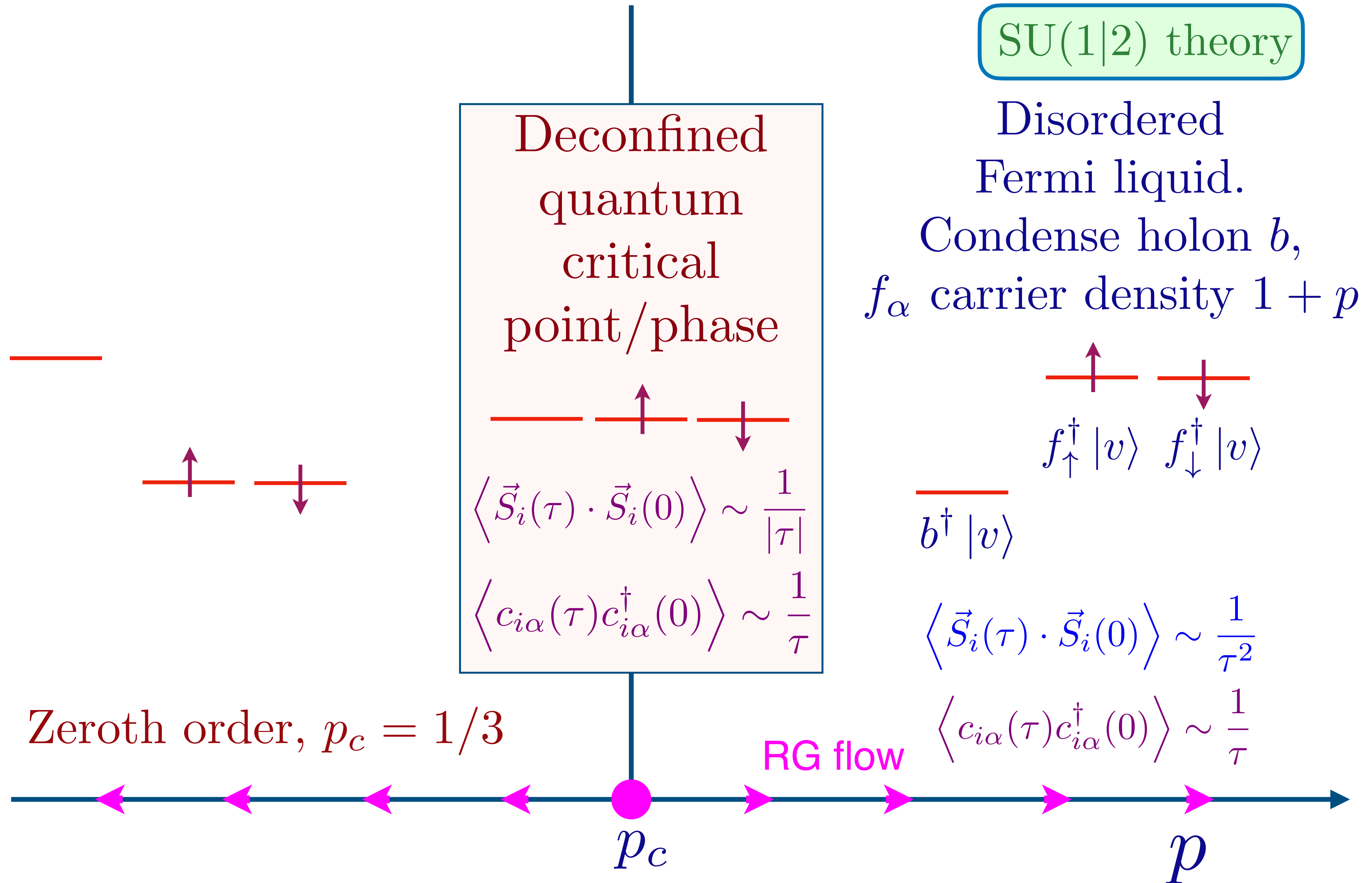
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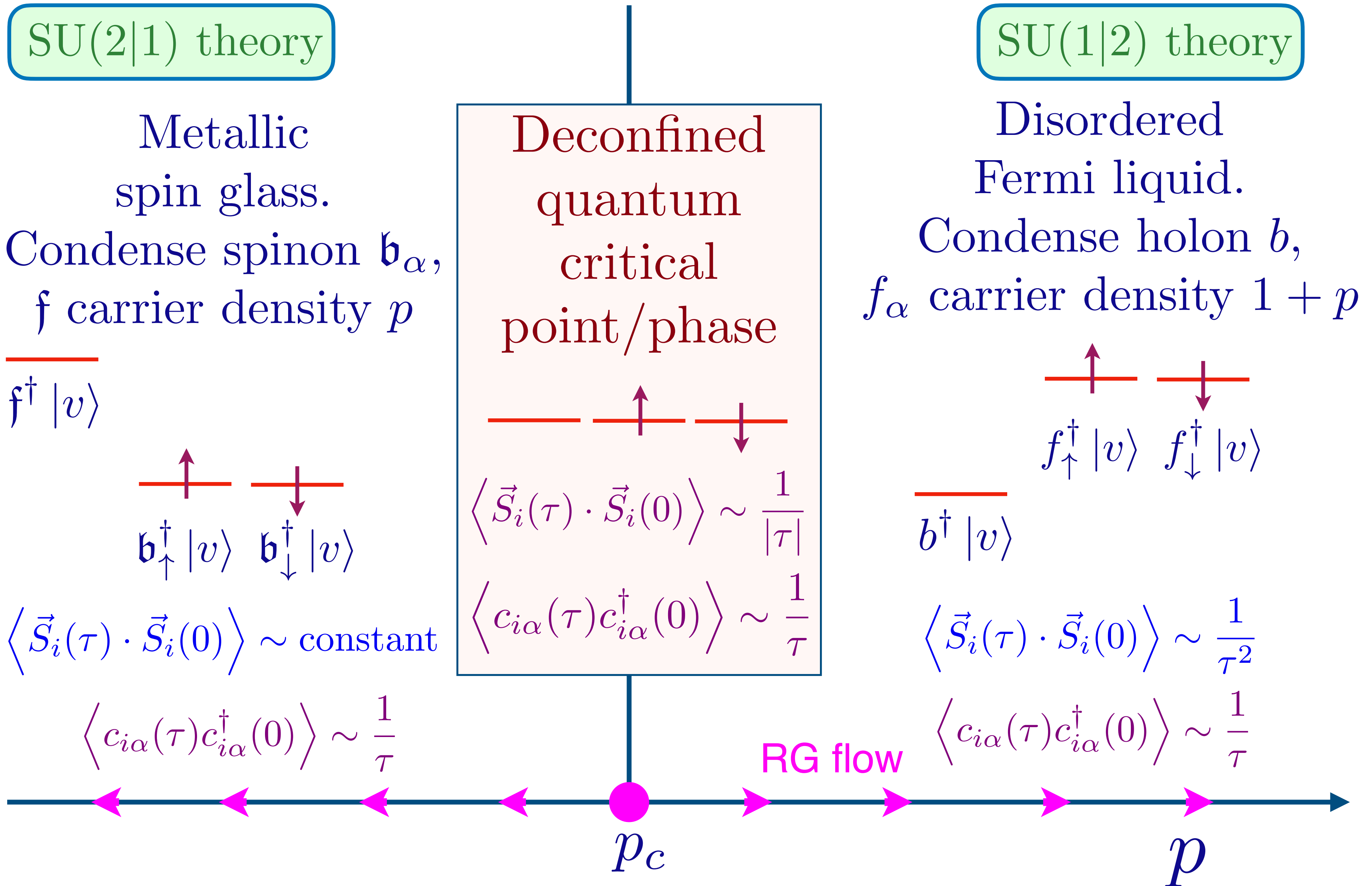
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t - J phase diagram: RG using *either* $SU(2|1)$ or $SU(1|2)$



At the critical point/phase of the t - J model, the Fermi liquid-like behavior of the electron Green's function

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

leads to a non-zero *residual resistivity*, $\rho(0) \neq 0$.

However, the critical state is *not* a Fermi liquid, as indicated by the slow decay of the spin correlations

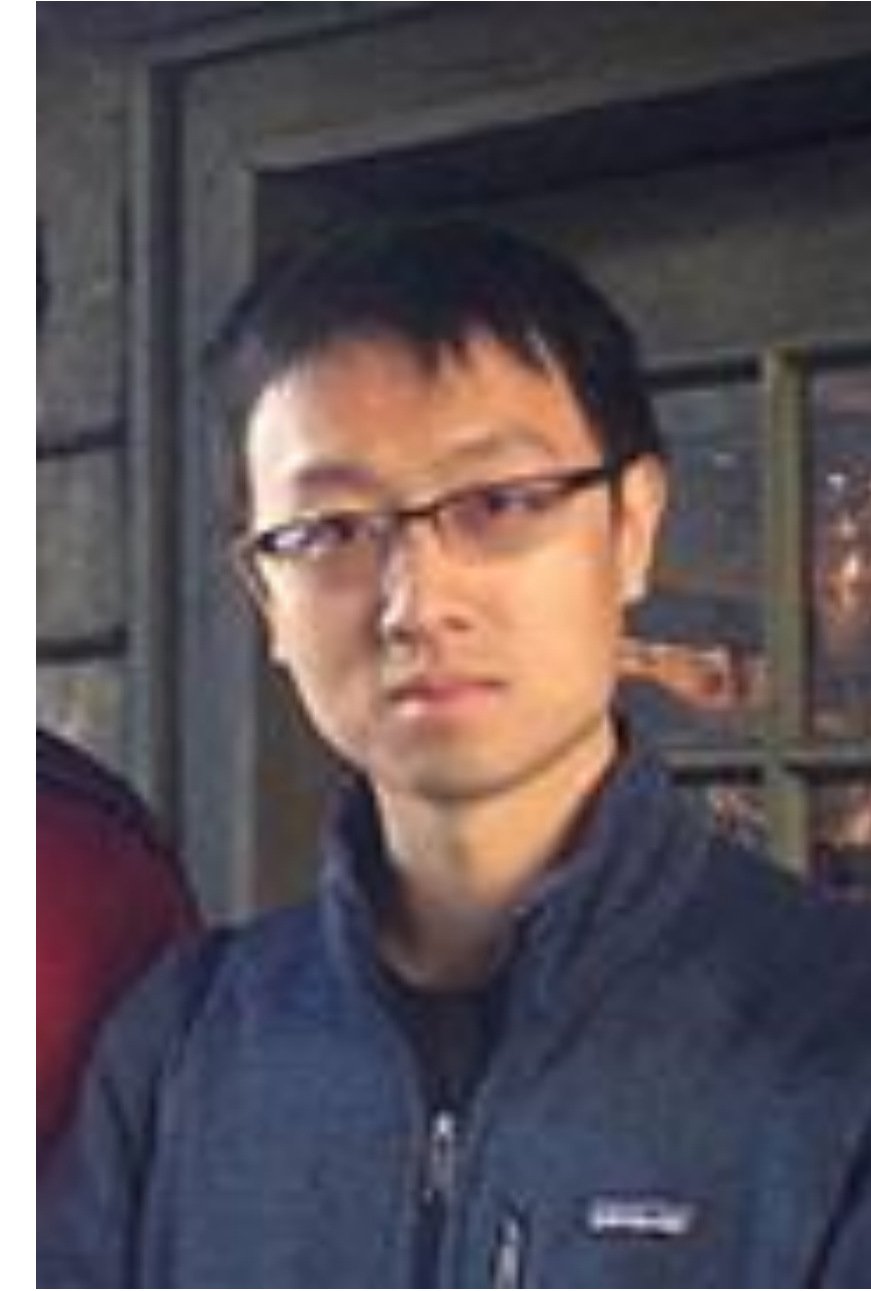
$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

Moreover, in a Fermi liquid, we expect $\rho(T) - \rho(0) \sim T^2$, which also does not hold here.



Haoyu Guo

Annals of Physics,
418, 168202 (2020)



Yingfeu Gu

Time reparameterization soft mode

The leading corrections to the critical **spinon** Green's function arise from the time reparameterization soft mode, and these take the form

$$\langle f_\alpha(\tau) f_\alpha^\dagger(0) \rangle \sim \left[\frac{\pi T}{\sin(\pi T \tau)} \right]^{1/2} \left(1 + \alpha_G \frac{T}{J} \Phi_{\text{non-conformal}}(T\tau) \right)$$

where $\Phi_{\text{non-conformal}}(T\tau)$ is a computable (in the large M limit) scaling function, and α_G is universally proportional to the co-efficient α_S of the Schwarizan action for the time reparameterization mode.

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Time reparameterization soft mode

Computing the resistivity from this Green's function via the Kubo formula, we find

$$\rho(T) = \rho(0) \left(1 + 8\alpha_G \frac{T}{J} + \dots \right)$$

Time reparameterization soft mode

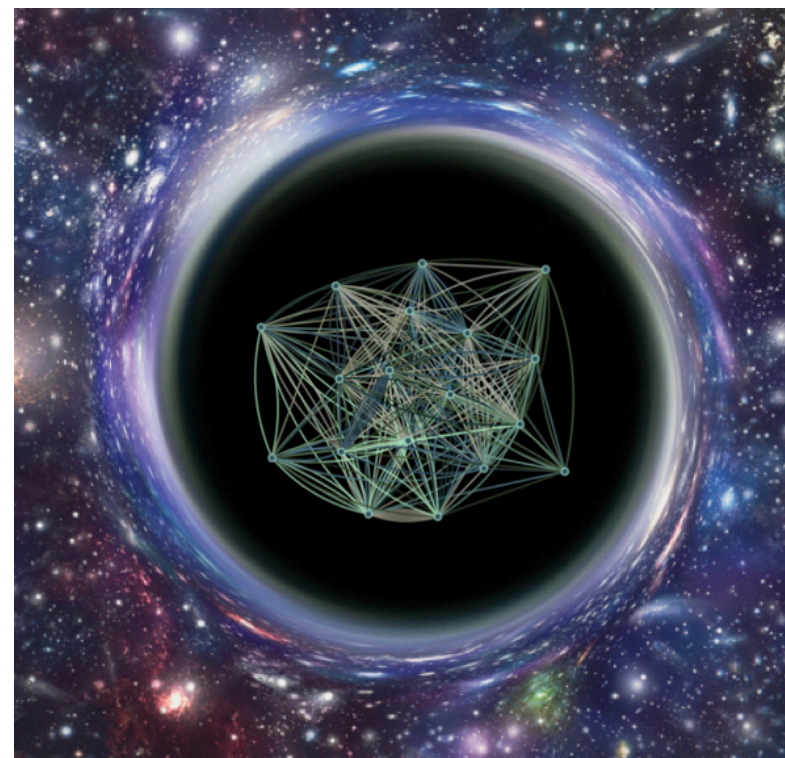
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In general, an operator with scaling dimension h , yield a resistivity $\rho(T) \sim T^{h-1}$. In the large M solution of the t - J model, we do find an operator with $1 < h < 2$, but with a smaller prefactor.

Random t - J model

- Proposed phase diagram of random t - J model captures key characteristics of cuprates.
- Critical electron Green's function $G_c(\tau) \sim 1/\tau$ and local spin correlator $Q(\tau) \sim 1/|\tau|$.
- Can be interpreted in terms of fractionalization with spinon and holon correlators $\sim 1/\sqrt{\tau}$ (deconfined criticality).
- Linear-in- T resistivity down to $T = 0$ at the critical point from the time reparameterization soft mode (but there could be other singular modes).
- Carrier density p for $p < p_c$, and $1 + p$ for $p > p_c$.
- Extensive zero temperature entropy $\lim_{T \rightarrow 0} \lim_{N \rightarrow \infty} S/N > 0$. Related to entropy of extremal black holes.



1. Random J model (insulator)

RG analysis and exact exponent

2. SYK criticality

Time reparameterizations and spectral functions

3. Random t - J model (metals)

RG analysis and exact exponents

4. Numerical results

QMC and exact diagonalization

Random t - J - U_H model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^N n_{i\uparrow} n_{i\downarrow}$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha},$$

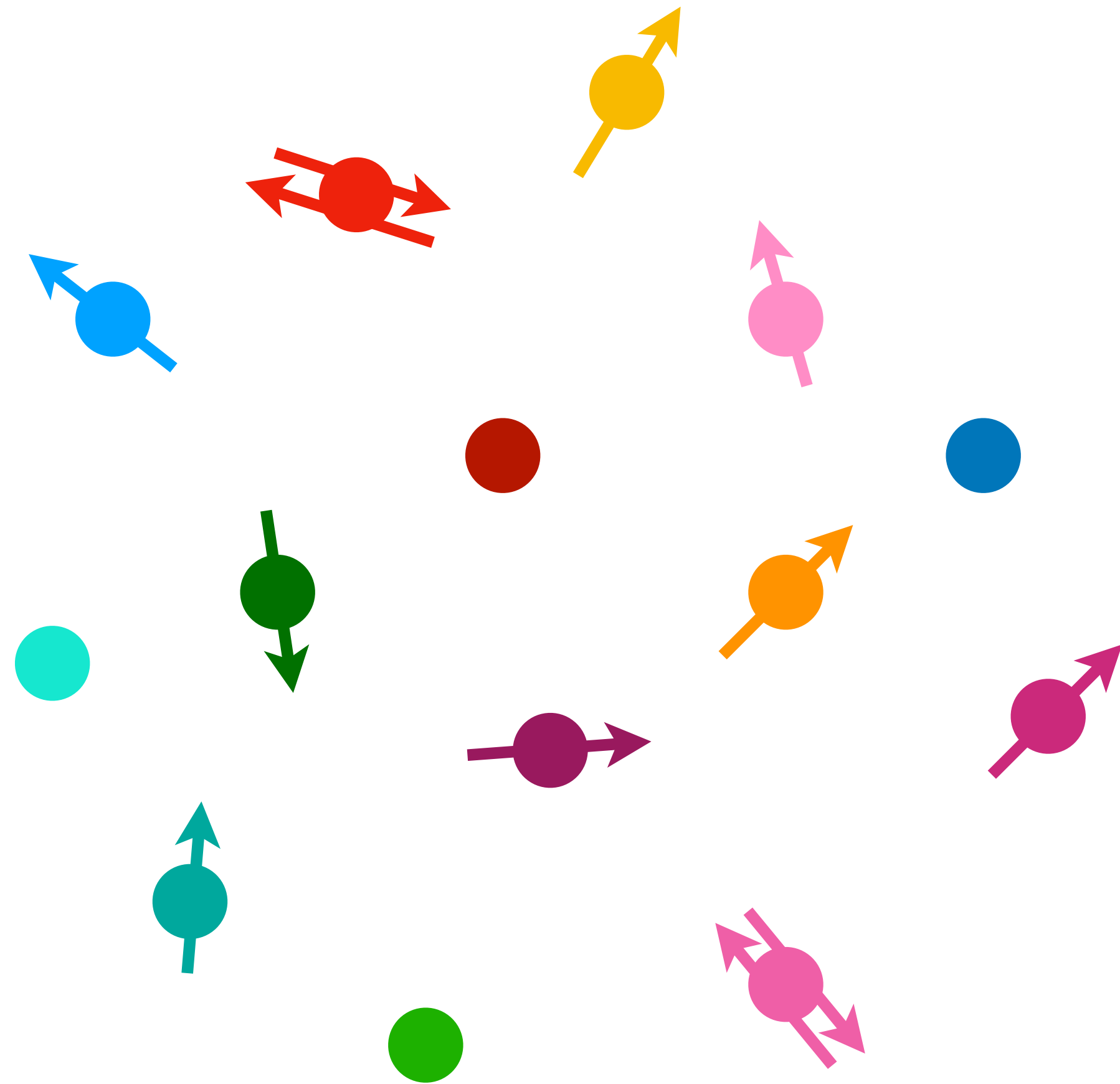
$$J_{ij} \text{ random, } \overline{J_{ij}} = 0, \overline{J_{ij}^2} = J^2$$

$$t_{ij} \text{ random, } \overline{t_{ij}} = 0, \overline{t_{ij}^2} = t^2$$

$$U_H > 0 \text{ non-random}$$

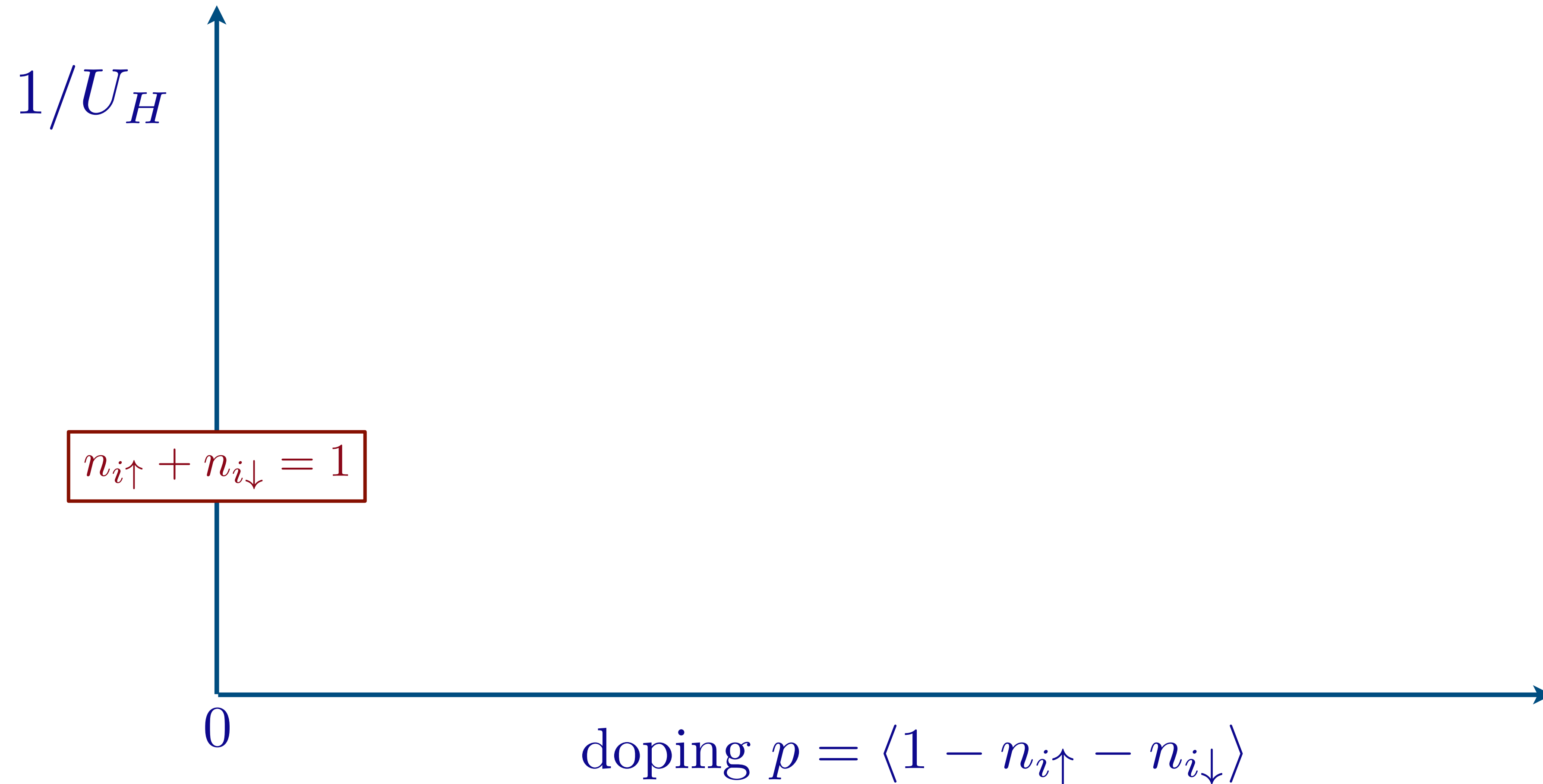
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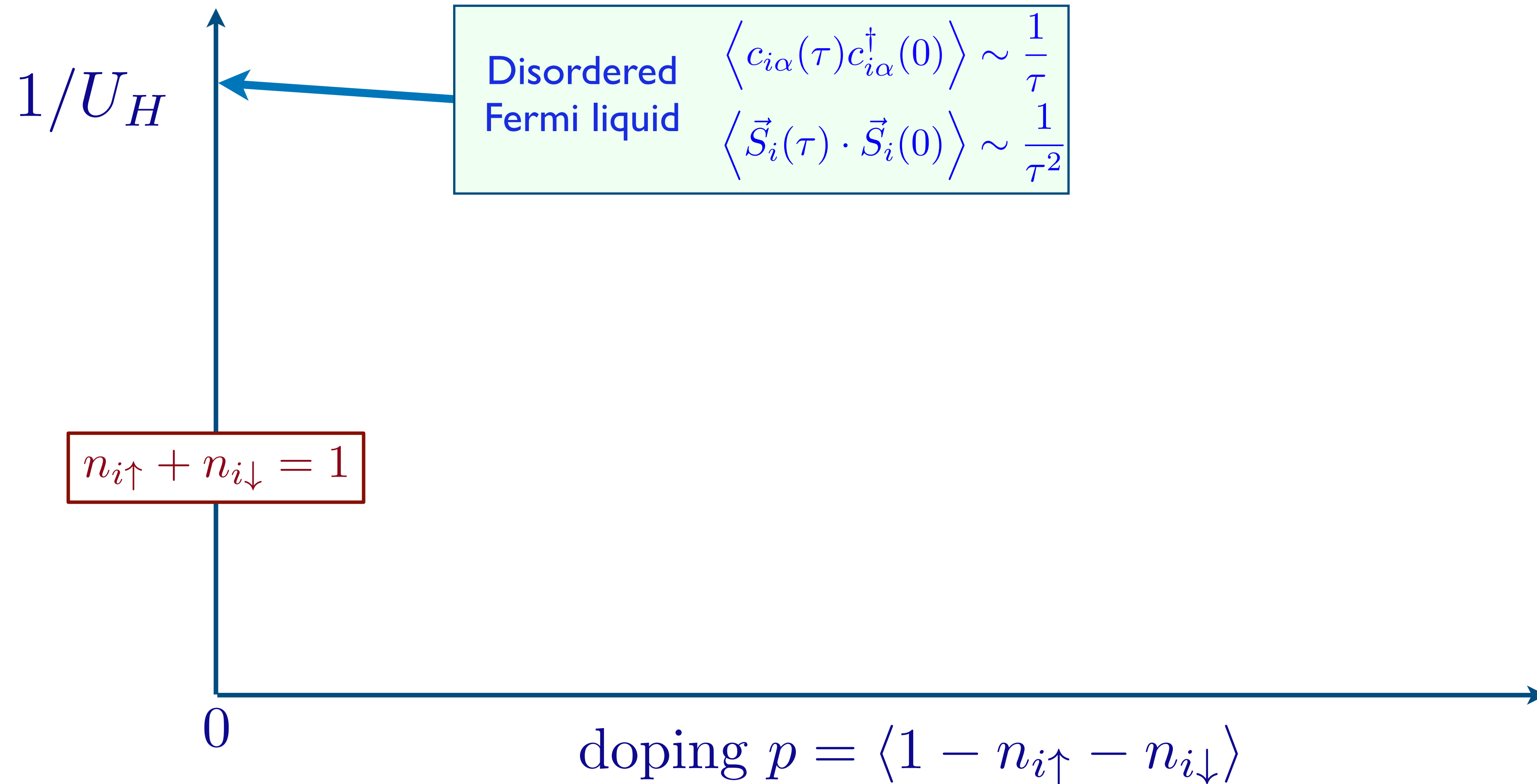
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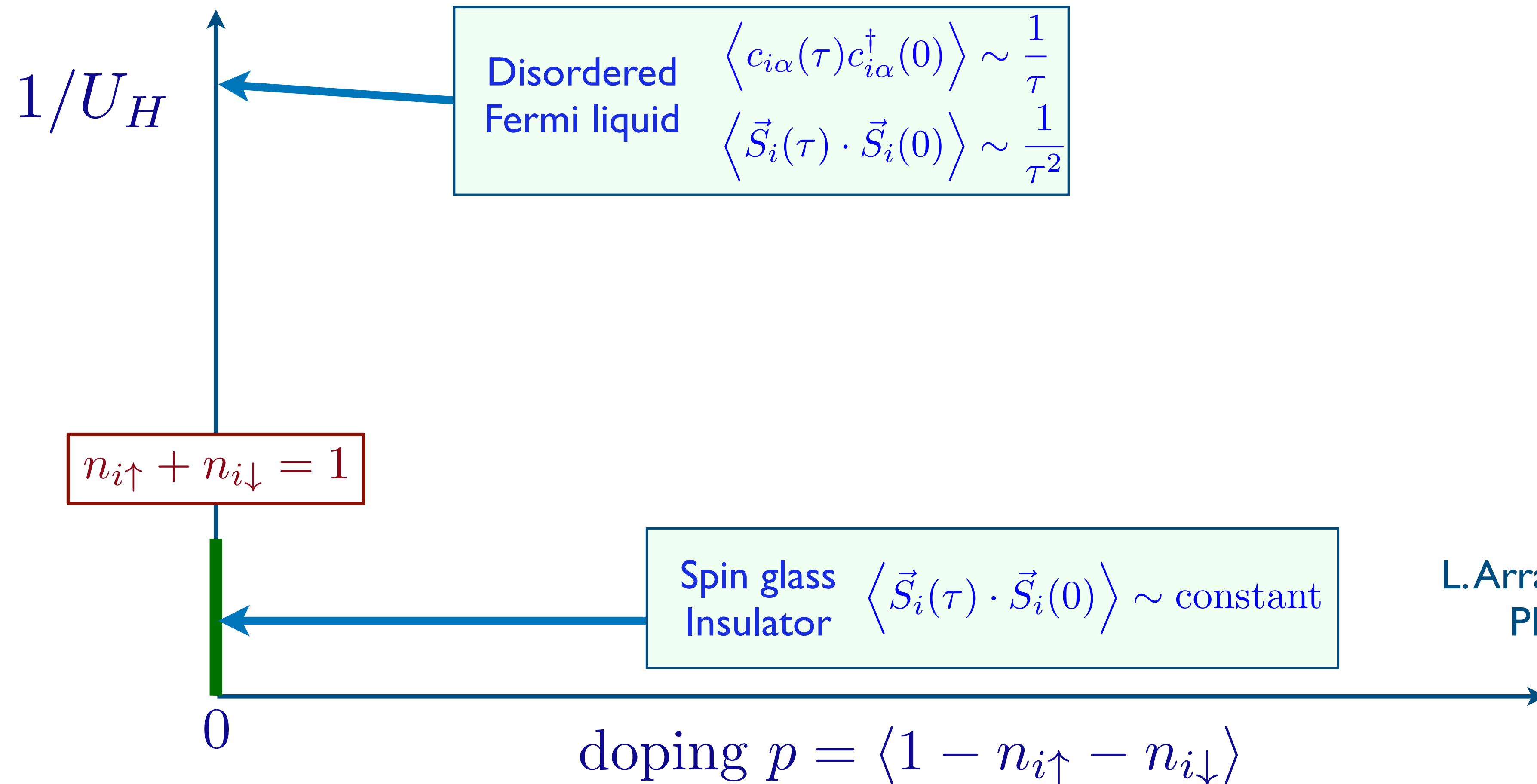
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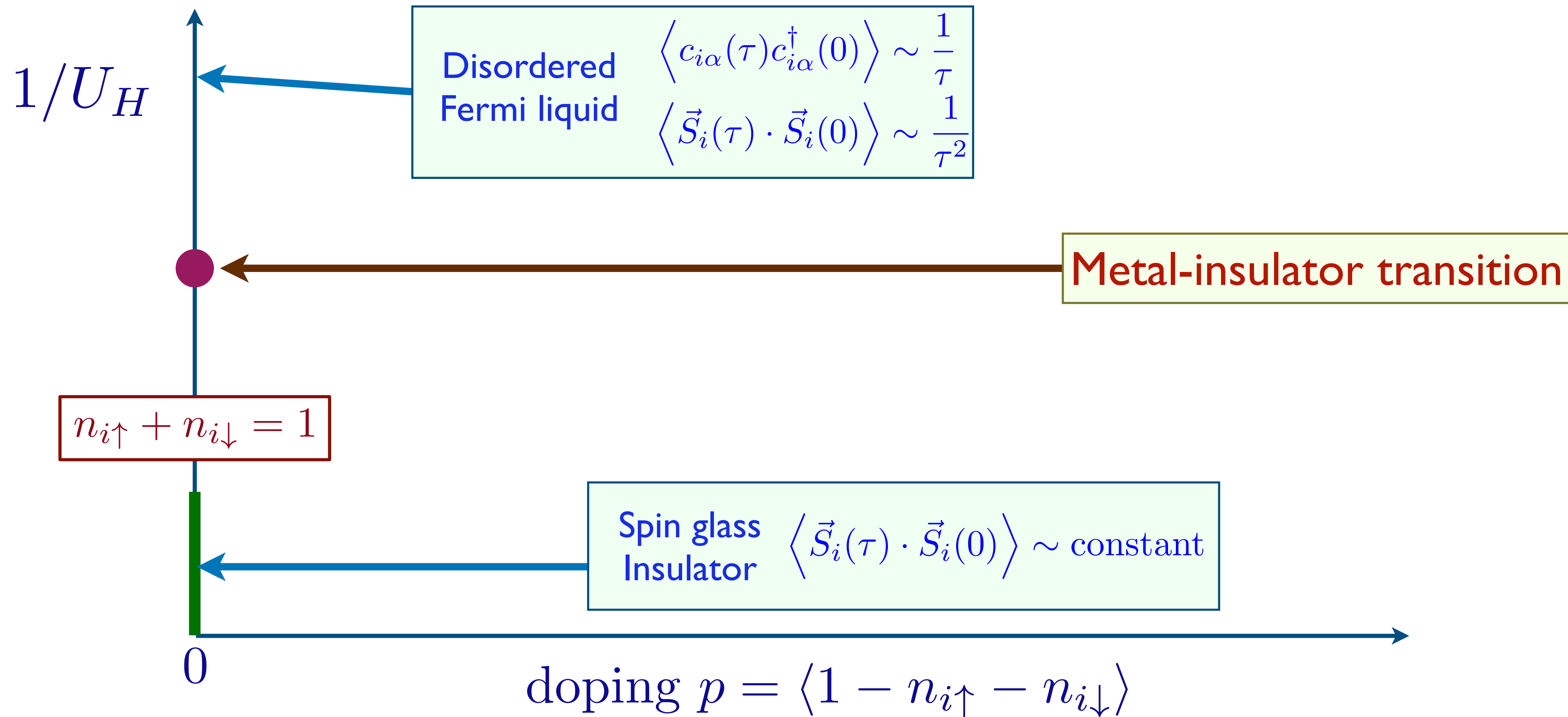
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L. Arrachea and M. J. Rozenberg,
PRB **65**, 224430 (2002)

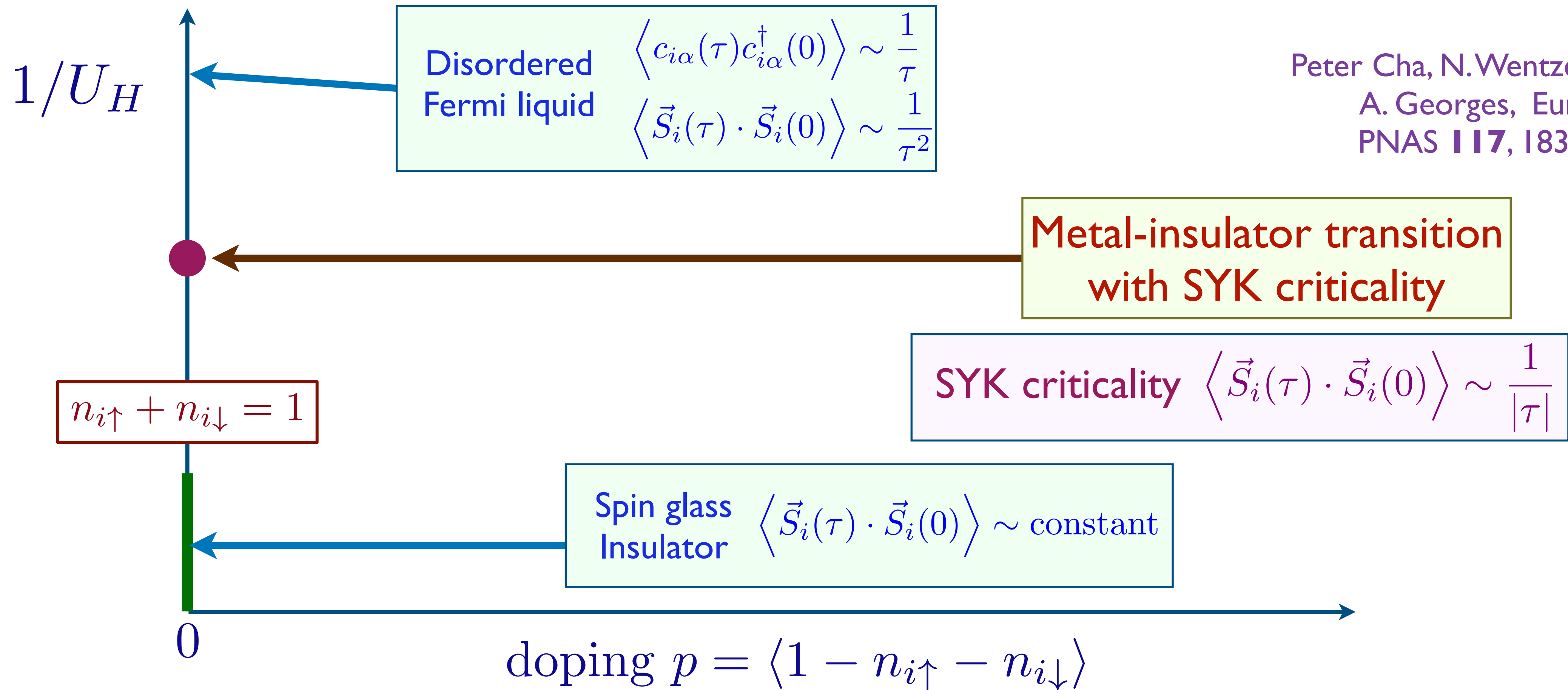
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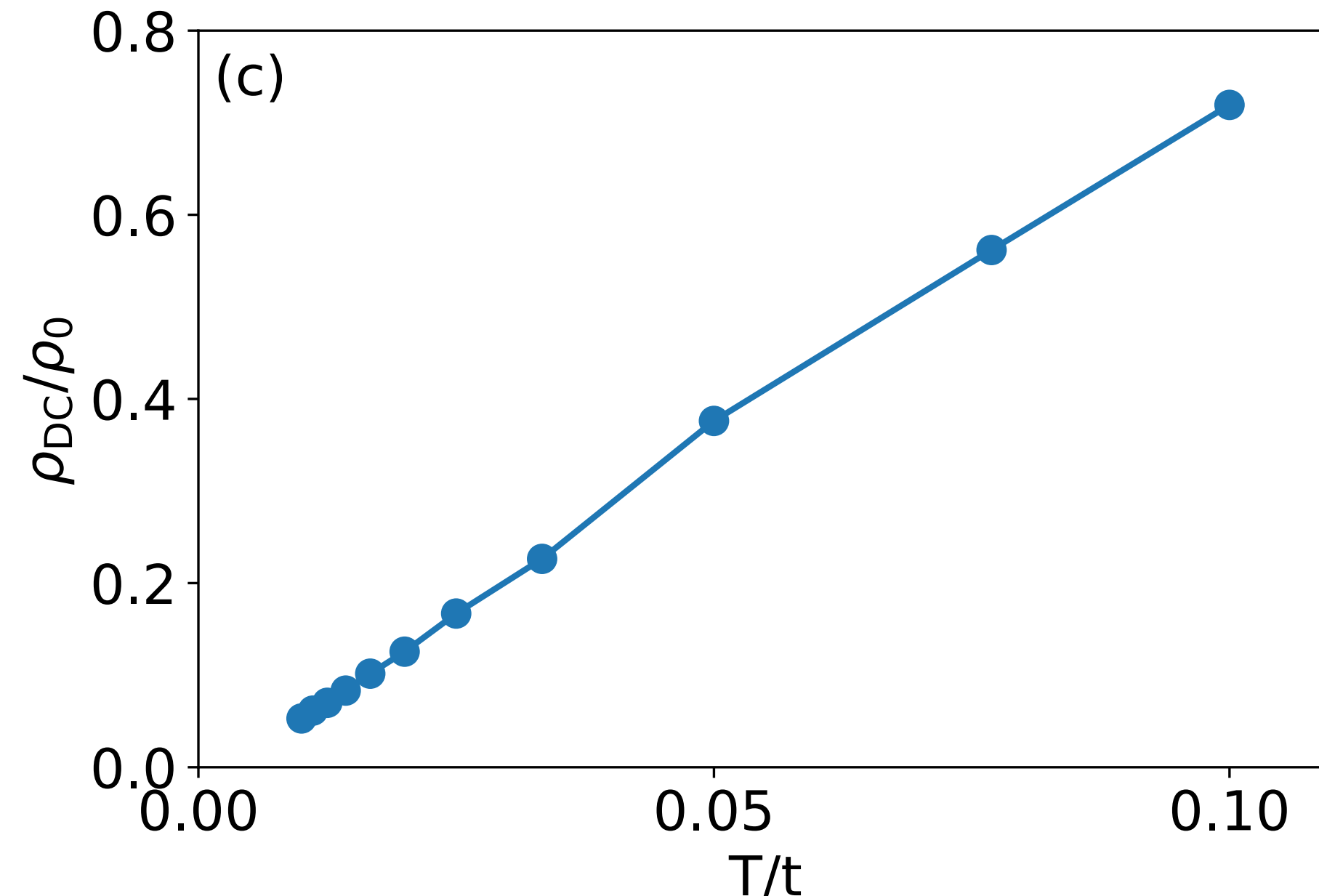
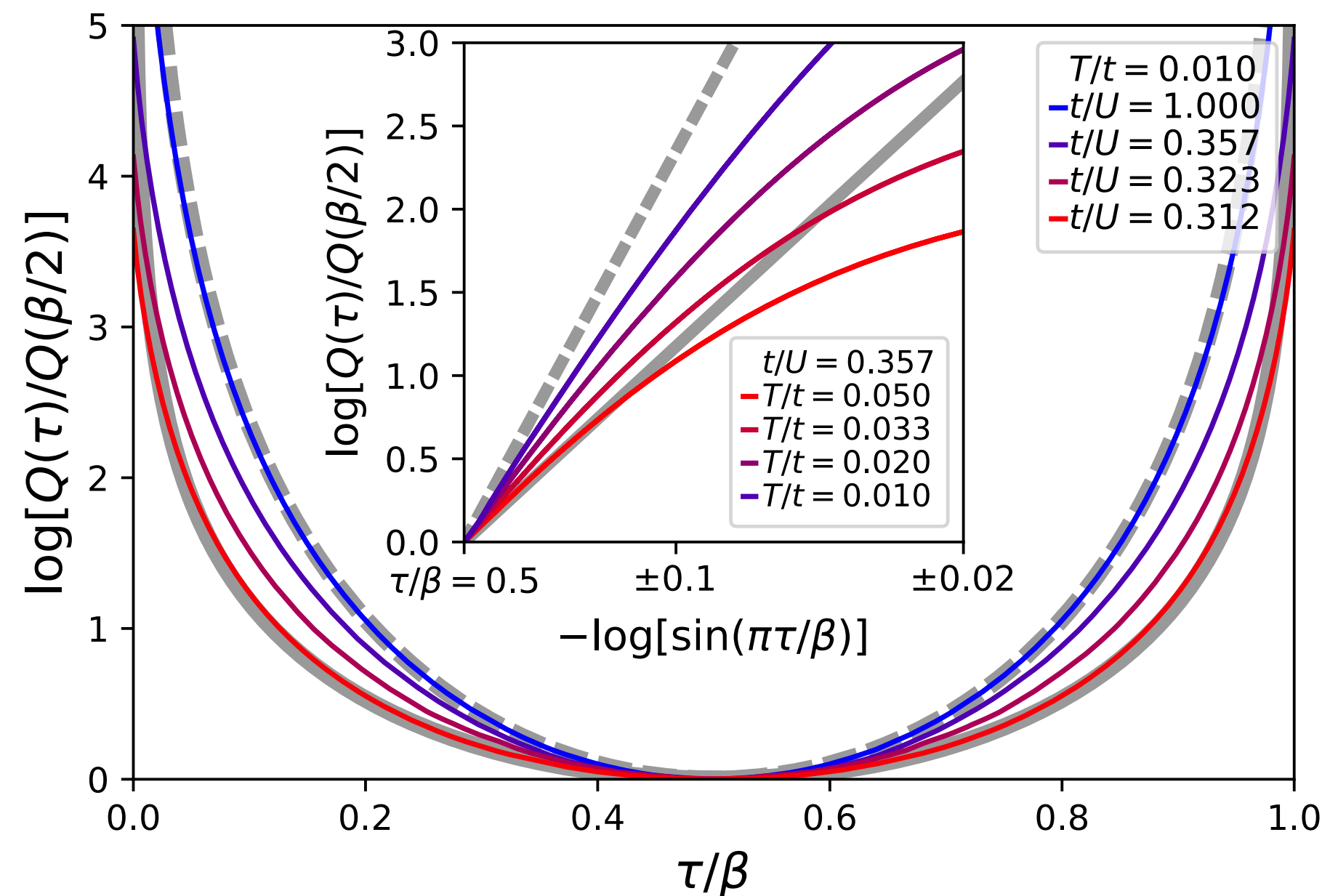
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Peter Cha, N. Wentzell, O. Parcollet,
 A. Georges, Eun-Ah Kim,
 PNAS **117**, 18341 (2020)

Linear resistivity and Sachdev-Ye-Kitaev (SYK) spin liquid behavior in a quantum critical metal with spin-1/2 fermions



Critical spin correlations:

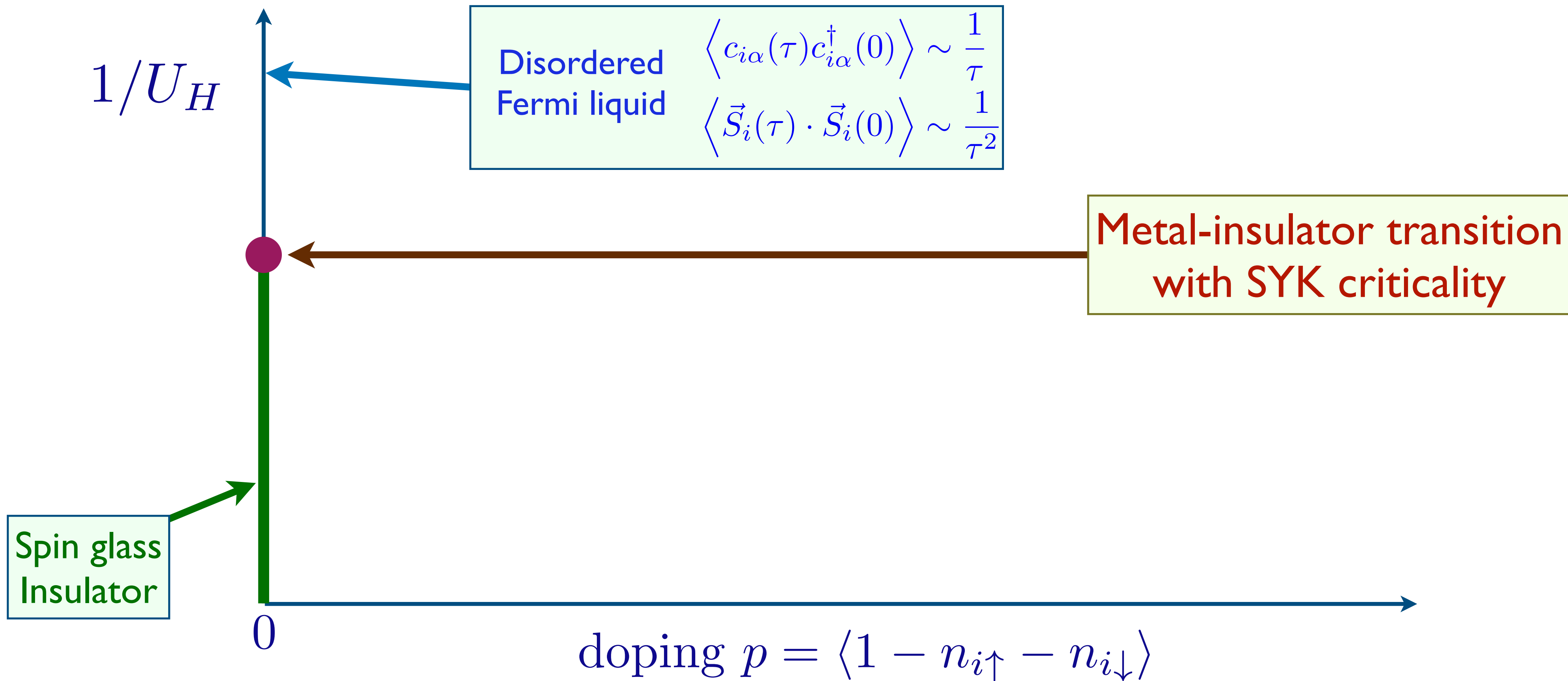
$$\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|}$$

Resistivity $\rho \sim T$ to the lowest T at the critical point

Onset of insulating gap and spin glass order co-occur.

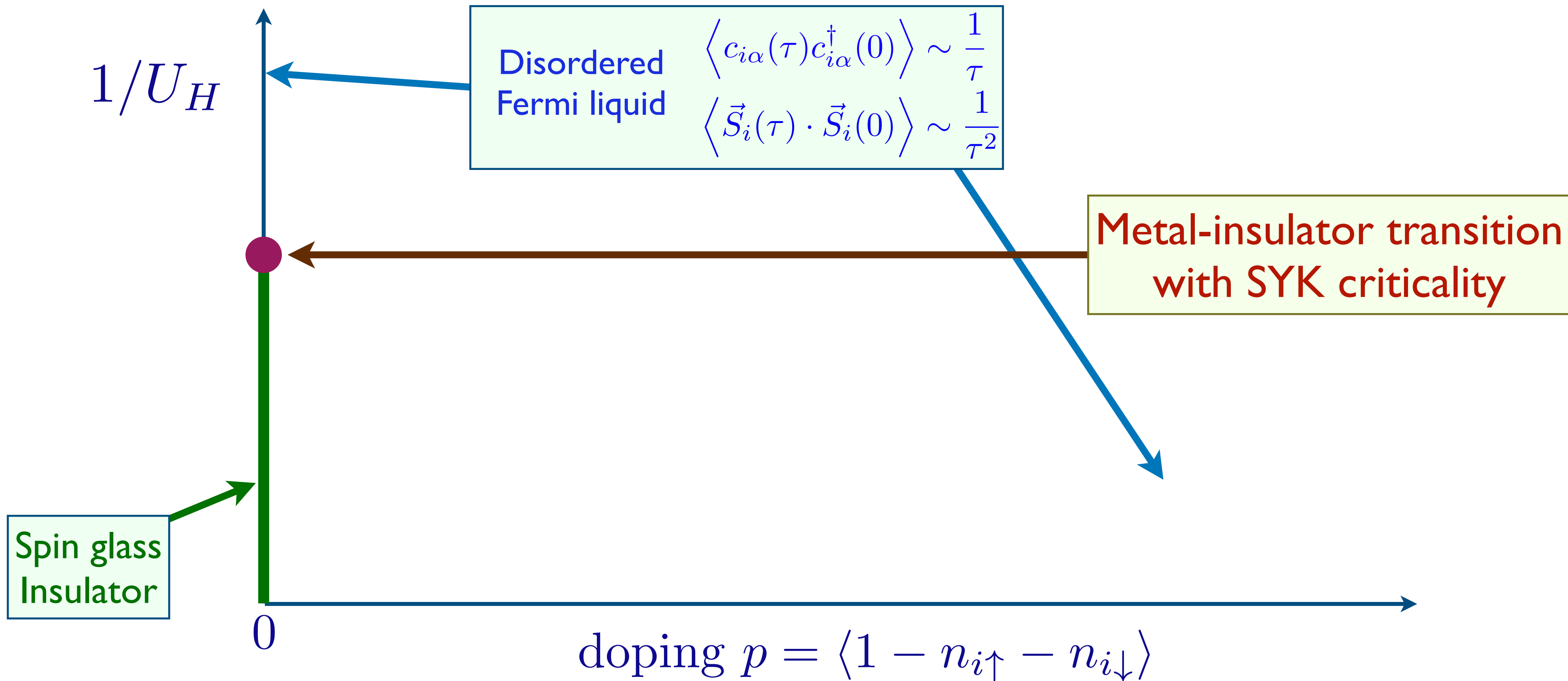
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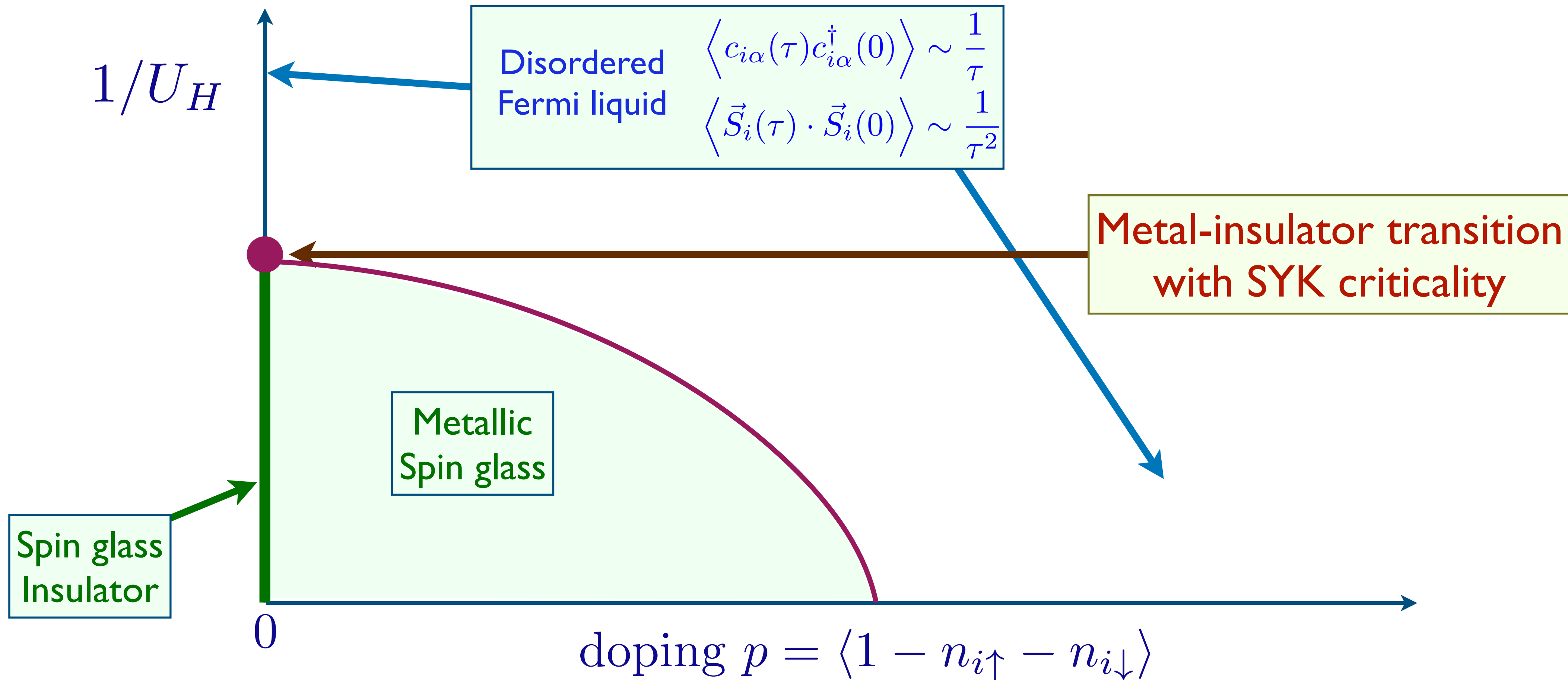
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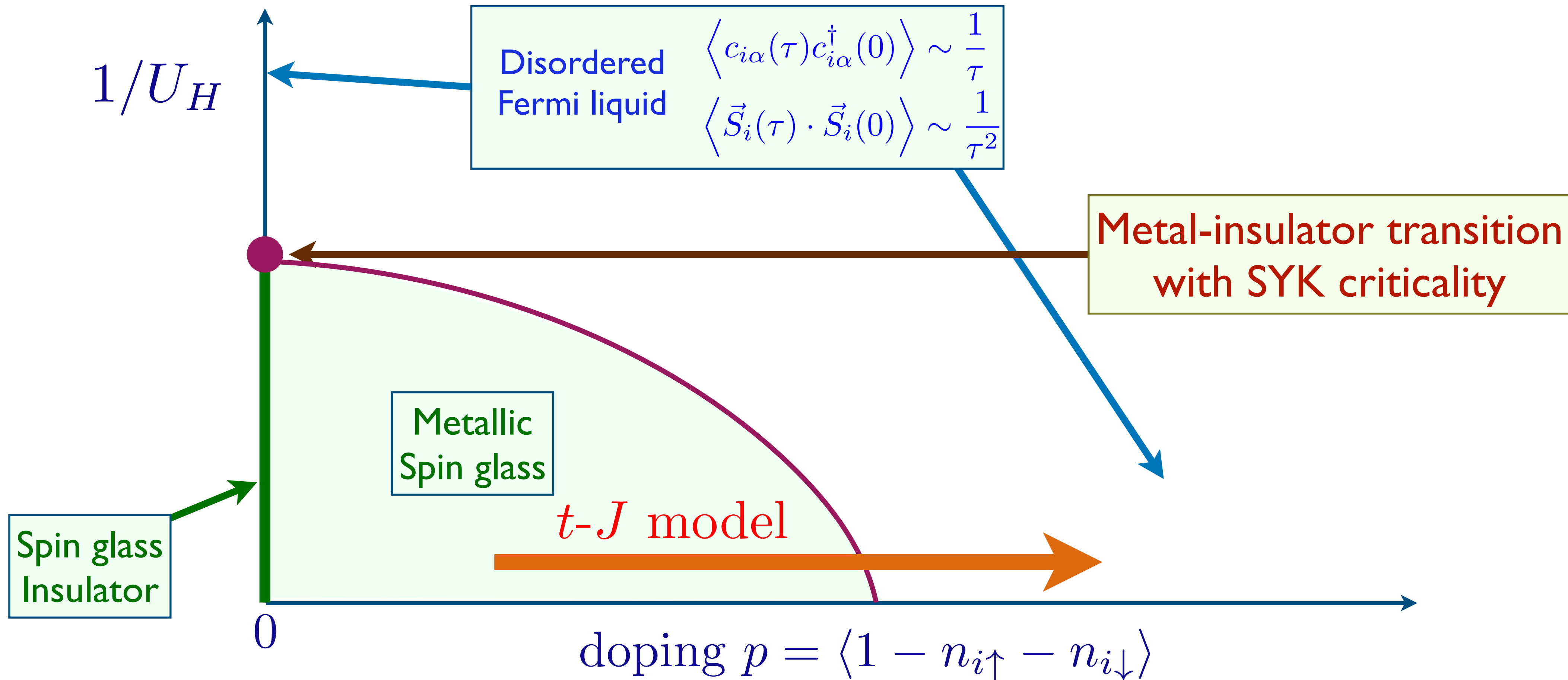
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Henry Shackleton



Alexander Wietek

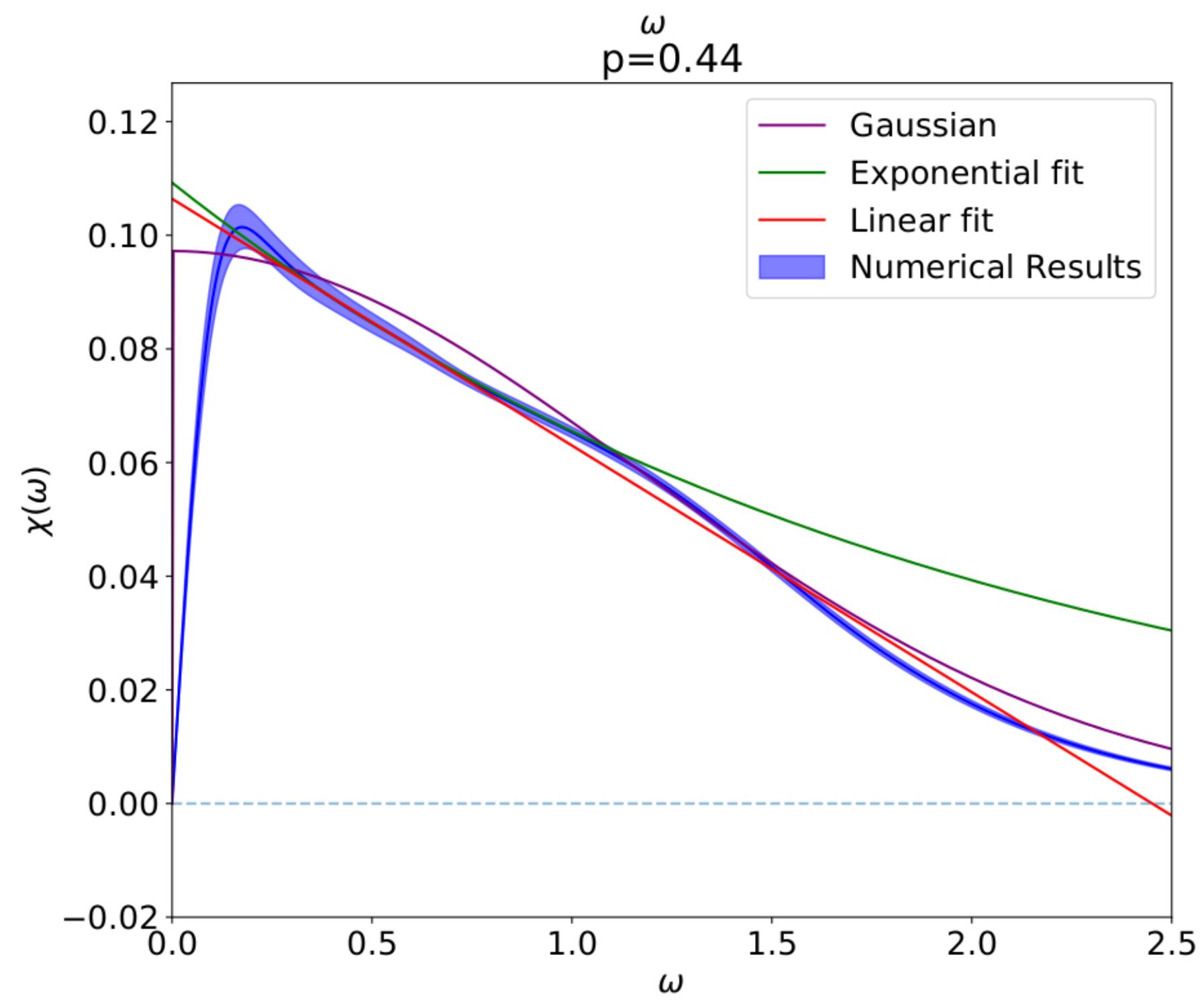
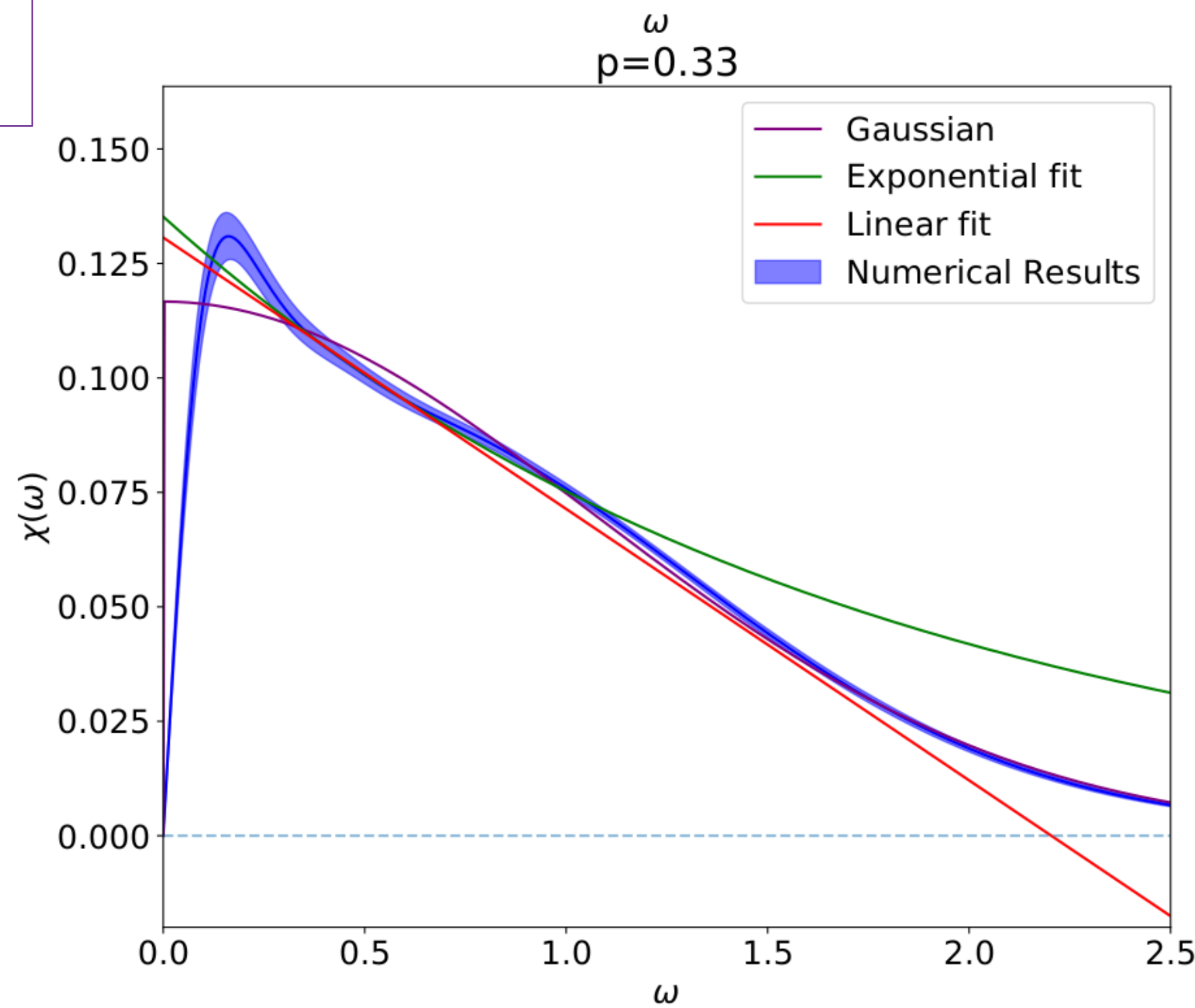
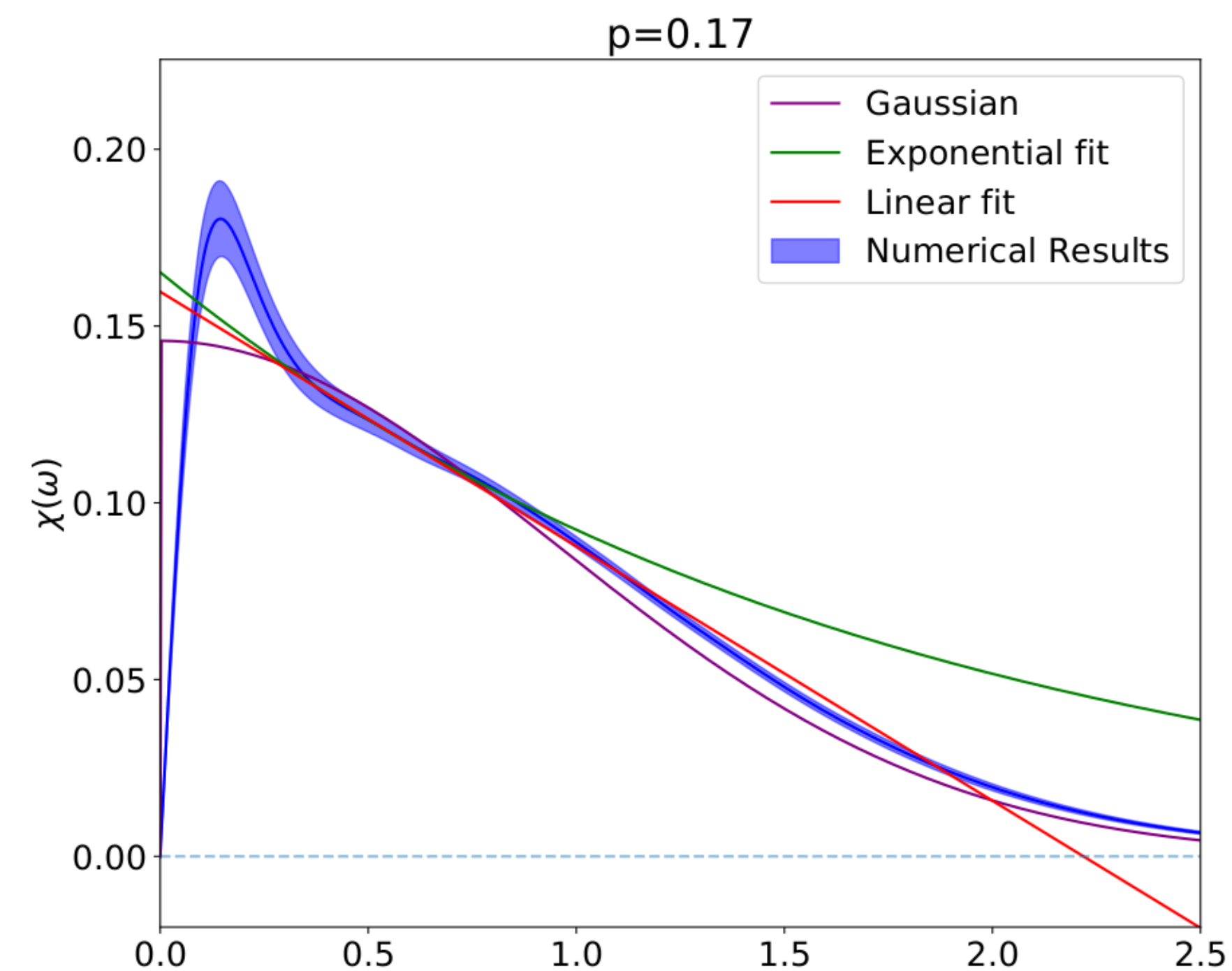
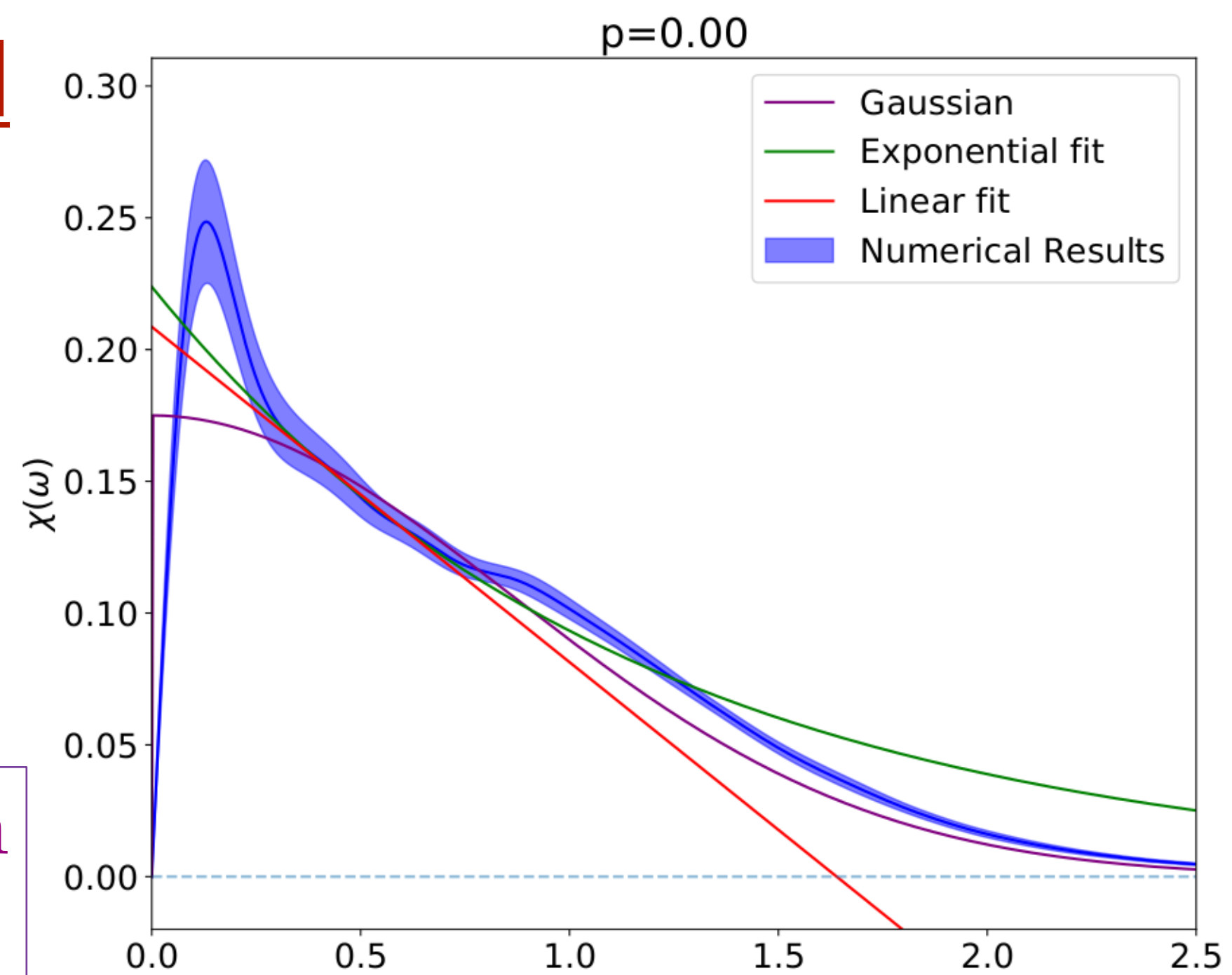


Antoine Georges

Random t - j mod

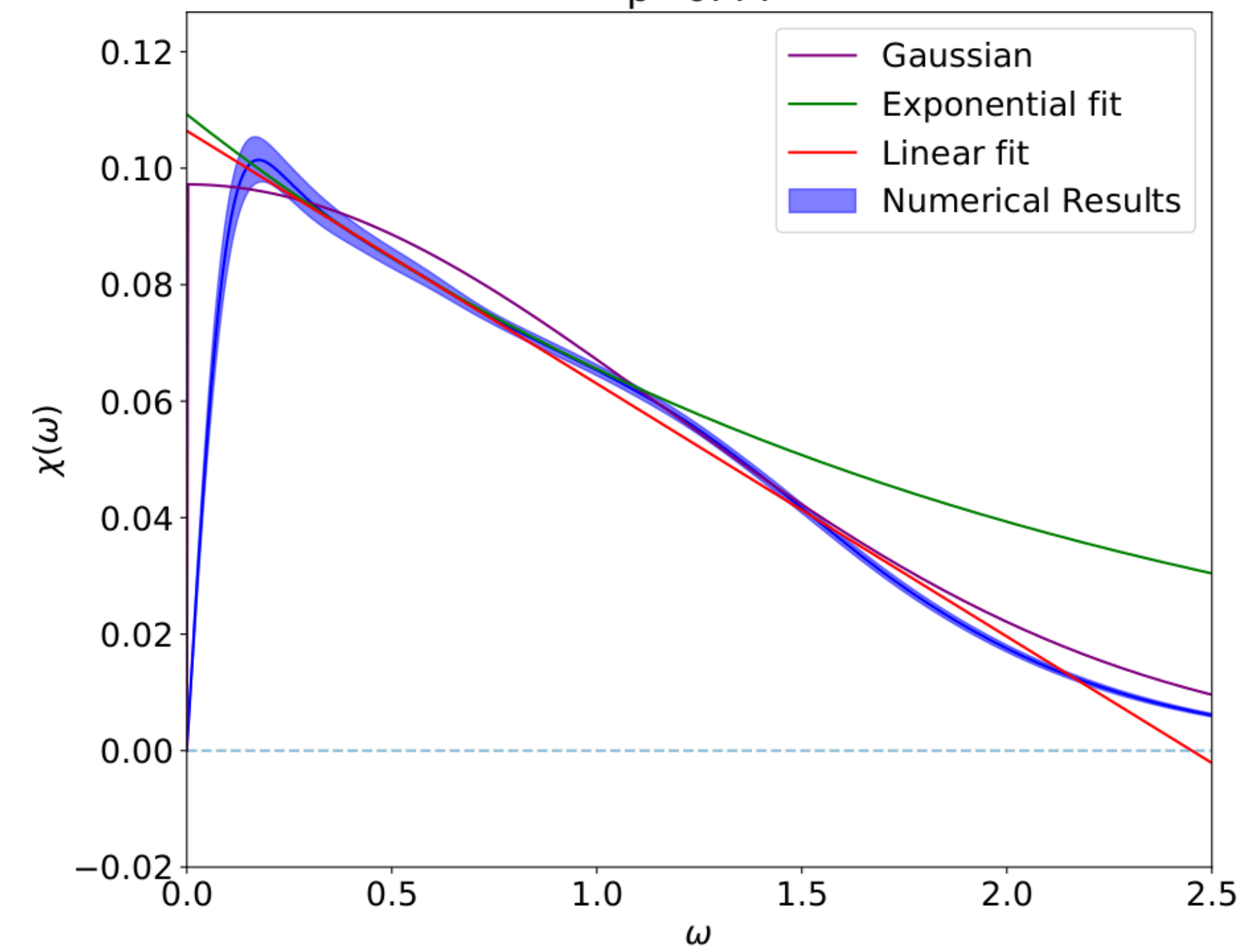
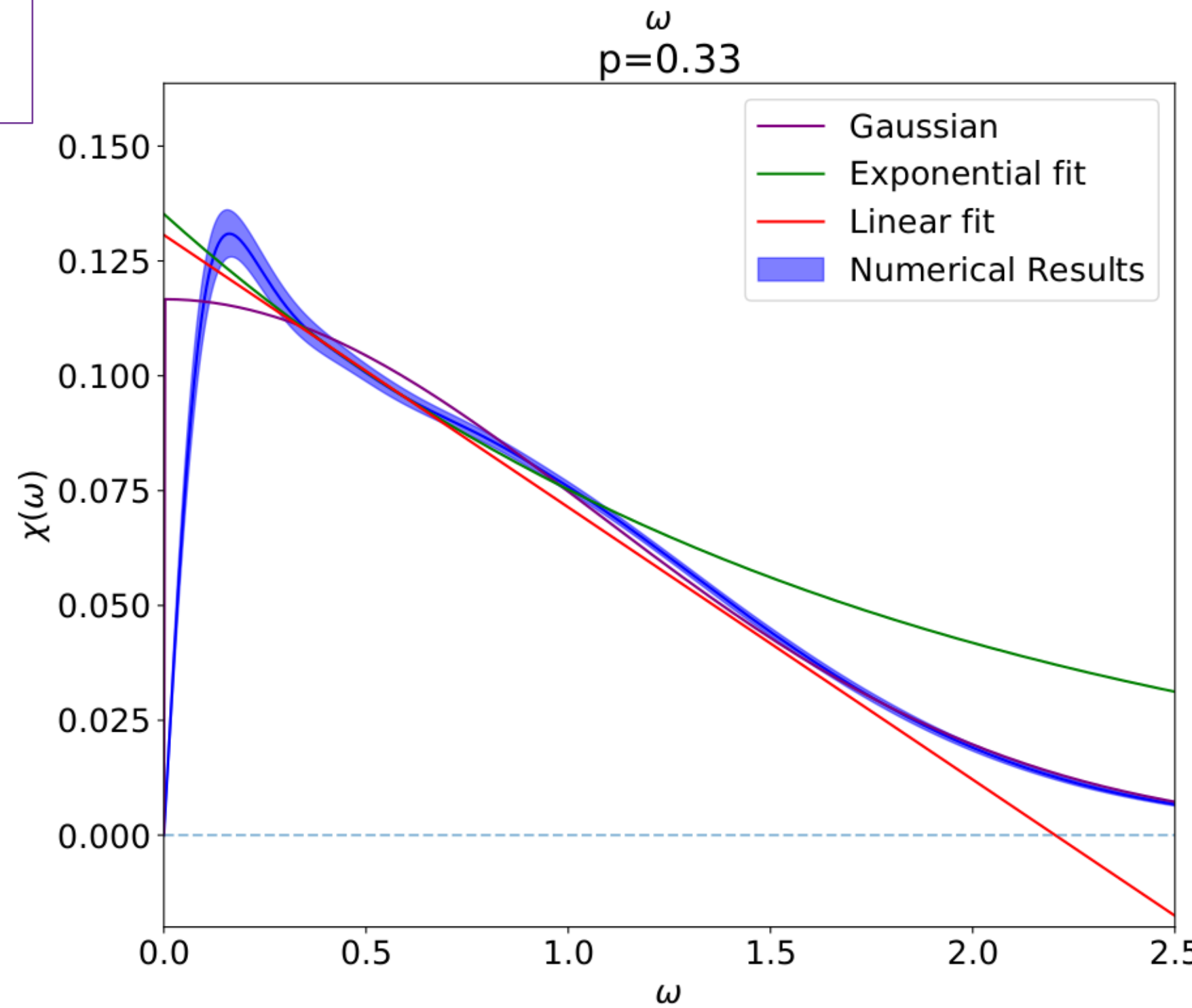
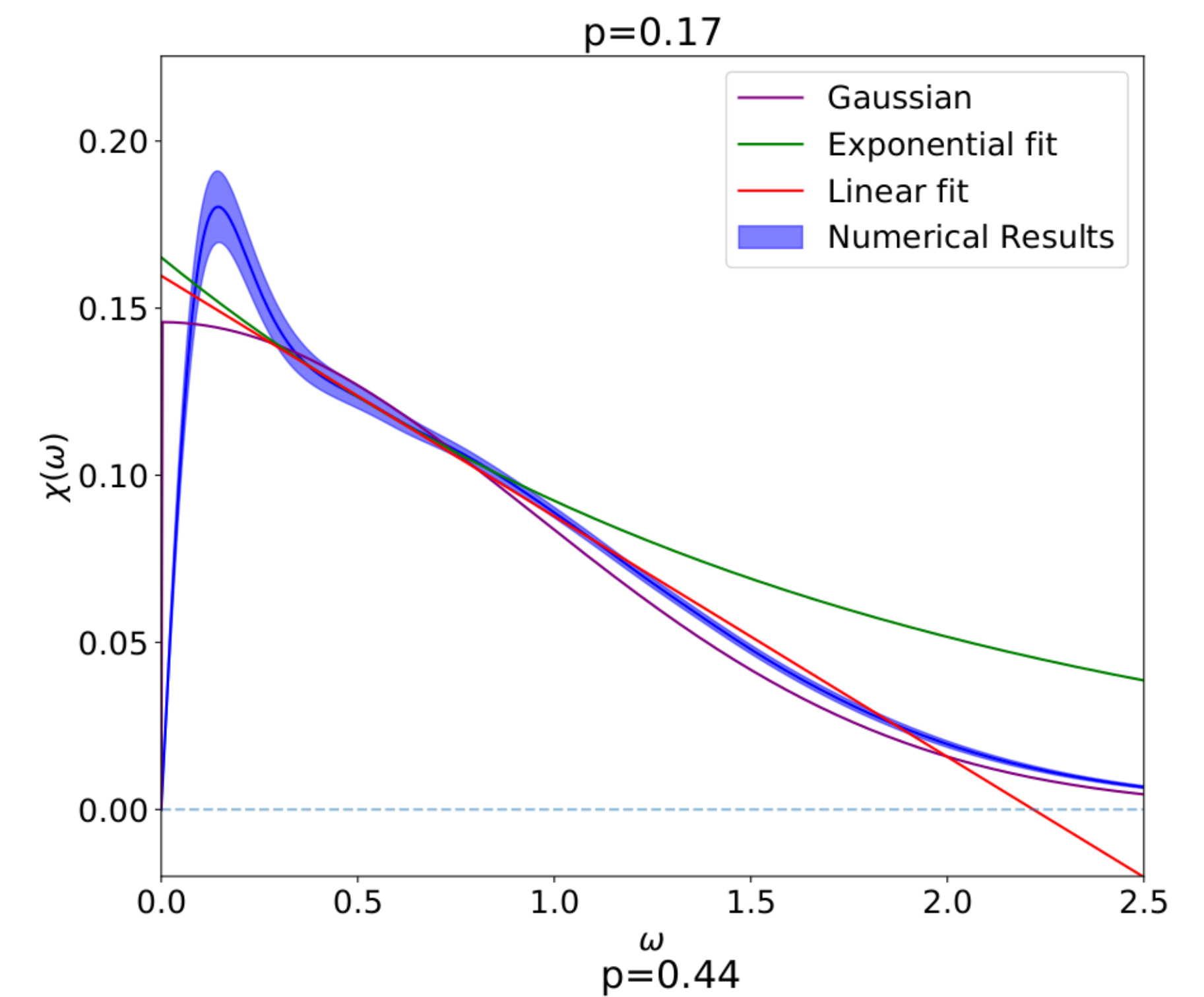
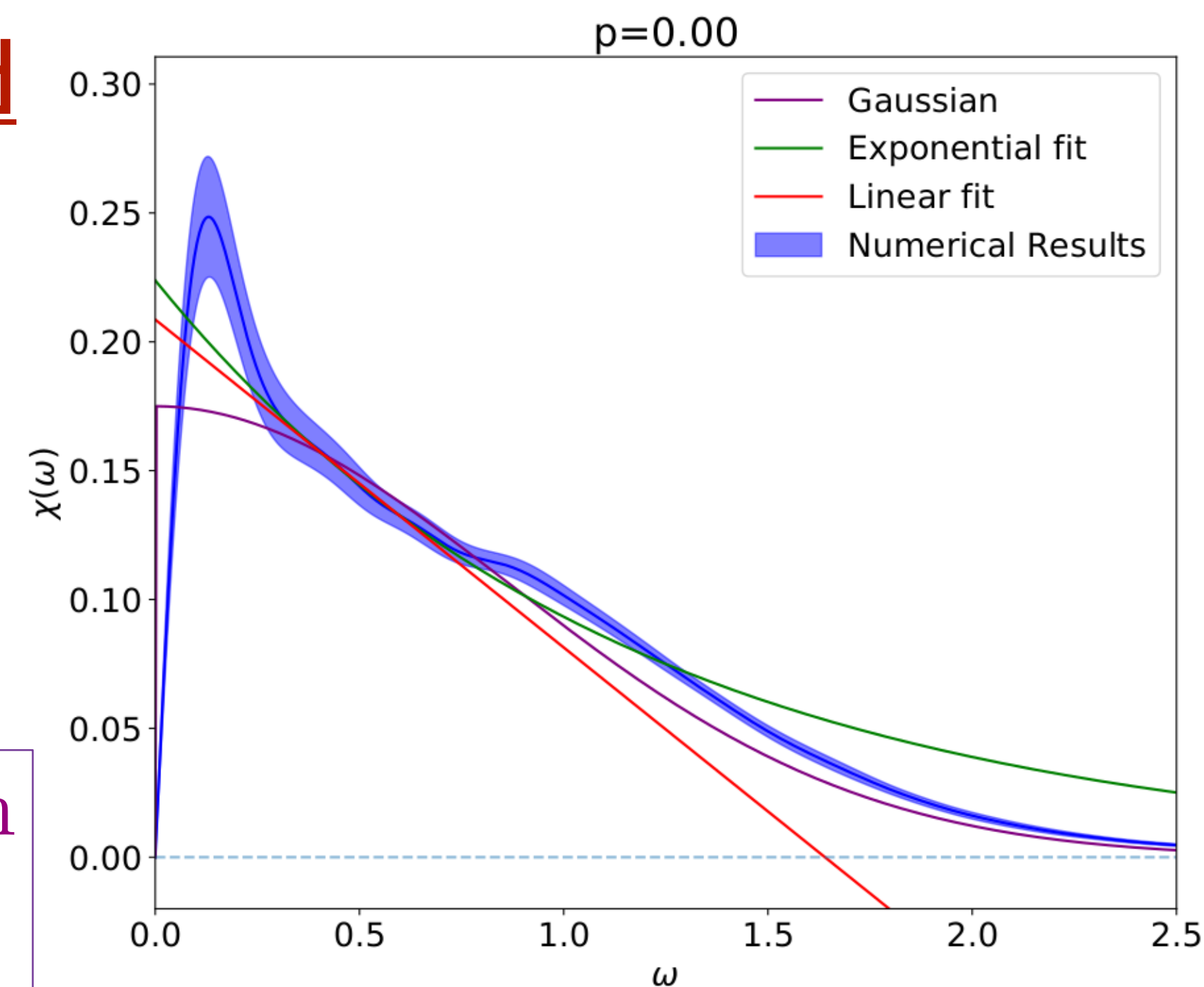
Exact diagonalization of clusters with N finite

Local dynamic spin susceptibility $\text{Im}\chi_L(\omega)$



Random t - J mod

Exact diagonalization of clusters with N finite

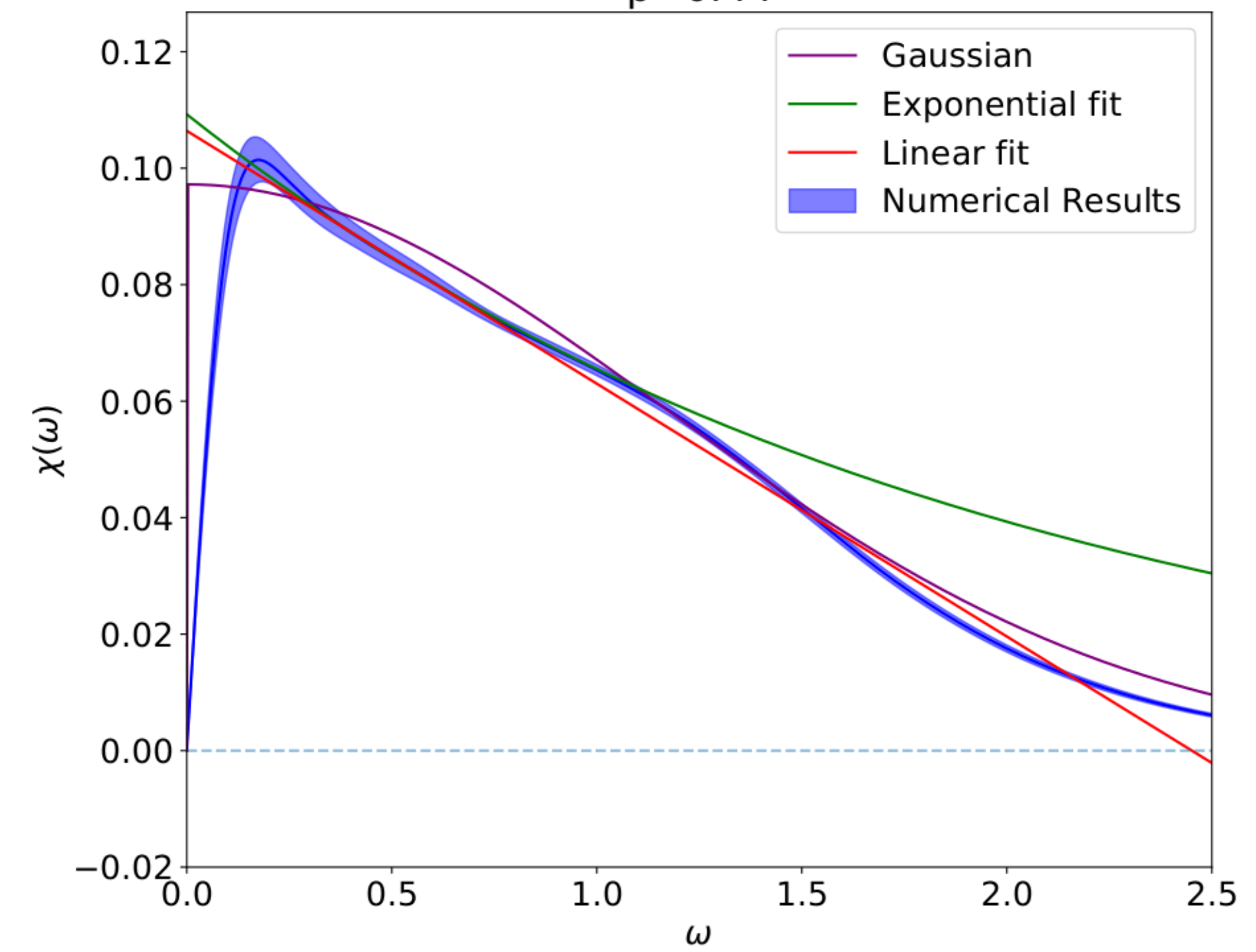
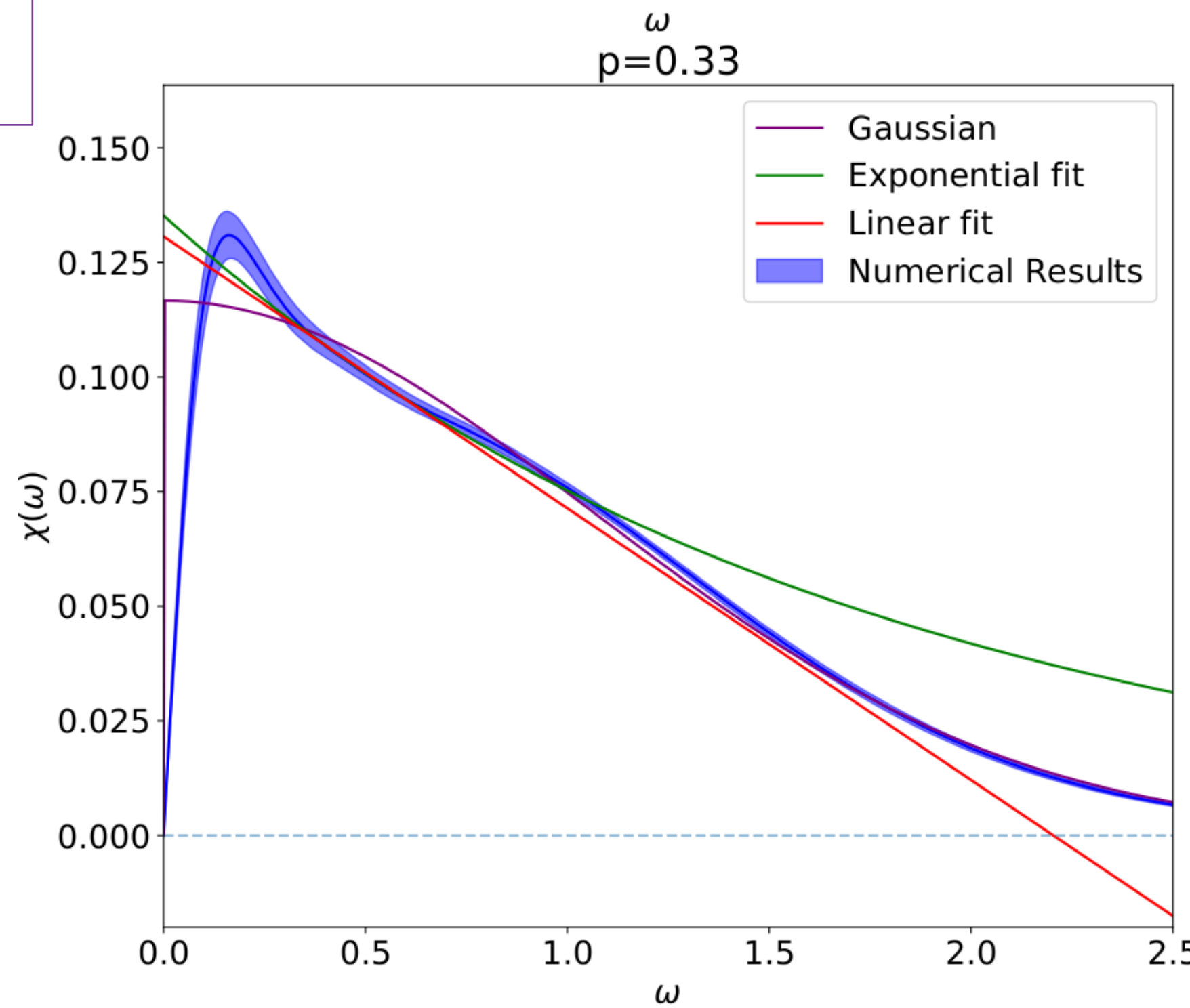
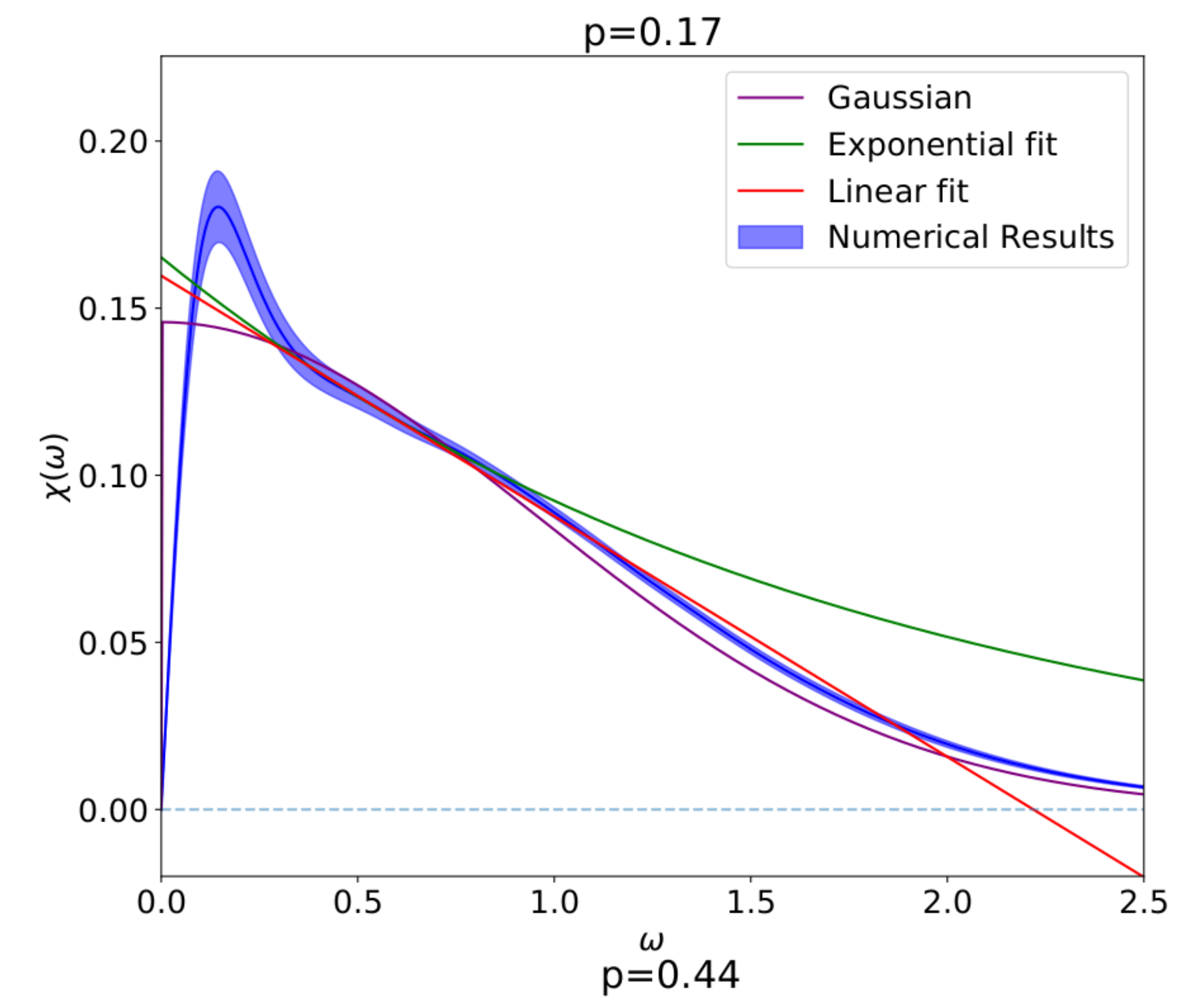
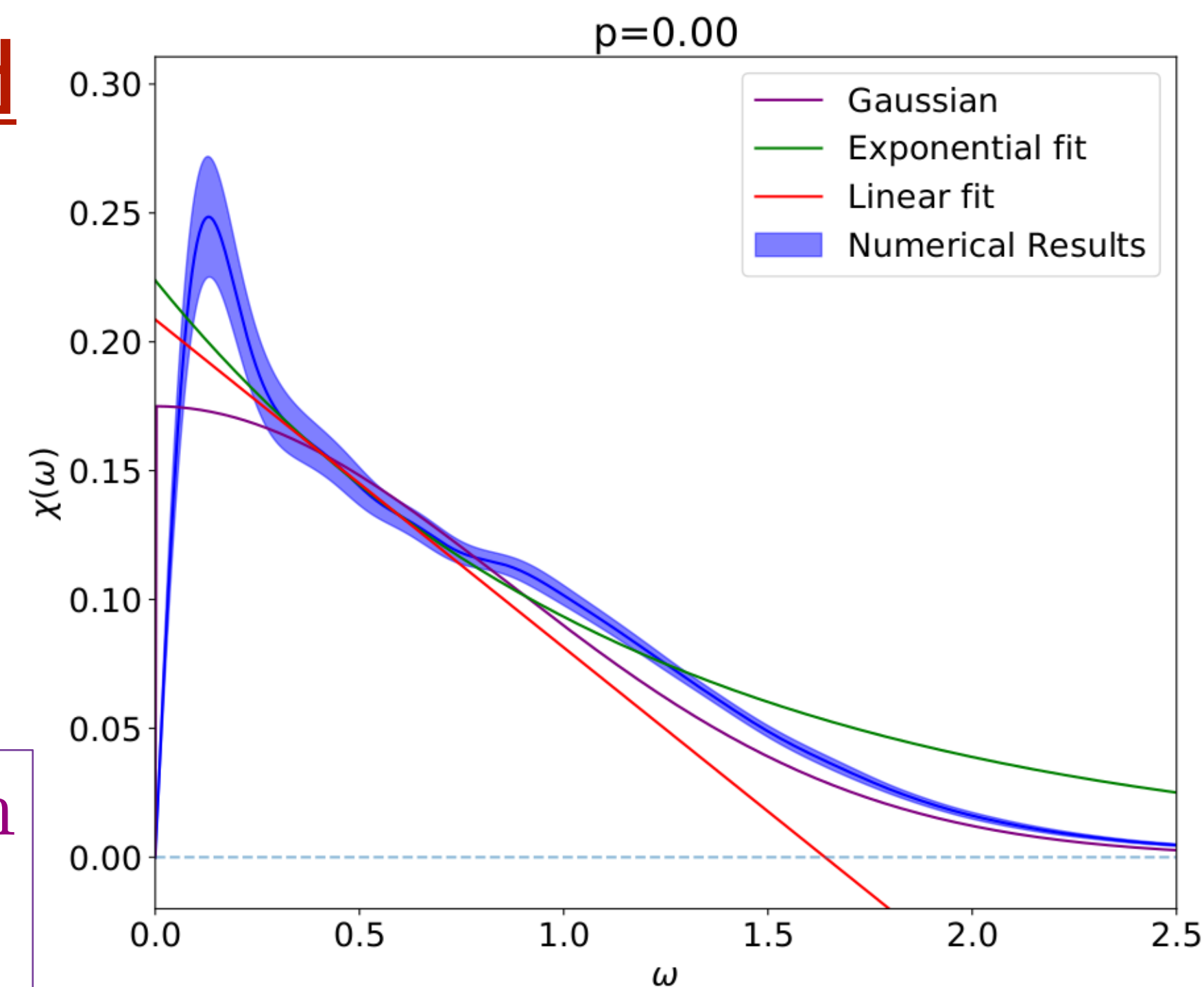


Local dynamic spin susceptibility $\text{Im}\chi_L(\omega)$

Incoherent contribution similar to SY spin liquid !

Random t - J mod

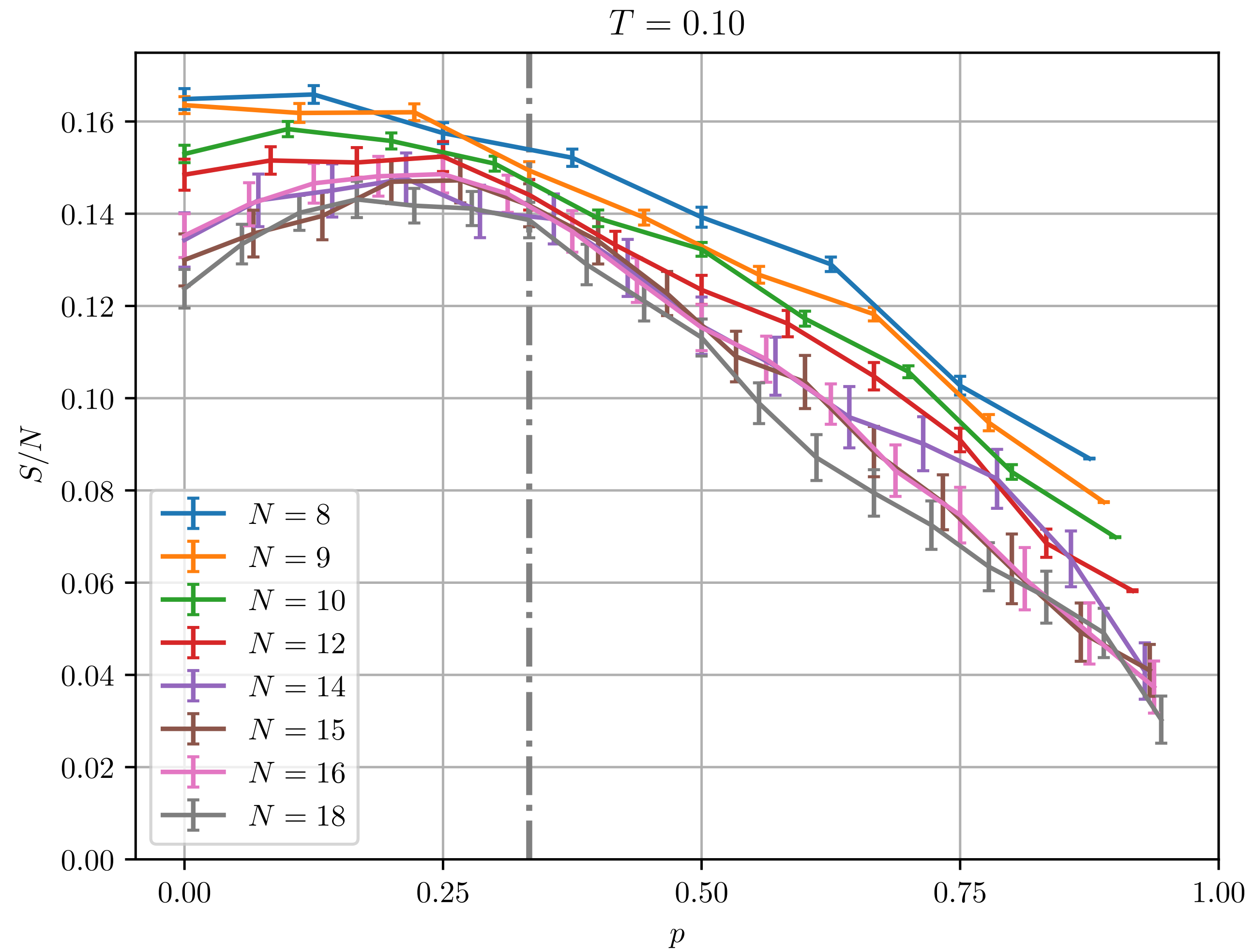
Exact diagonalization of clusters with N finite



Local dynamic spin susceptibility $\text{Im}\chi_L(\omega)$

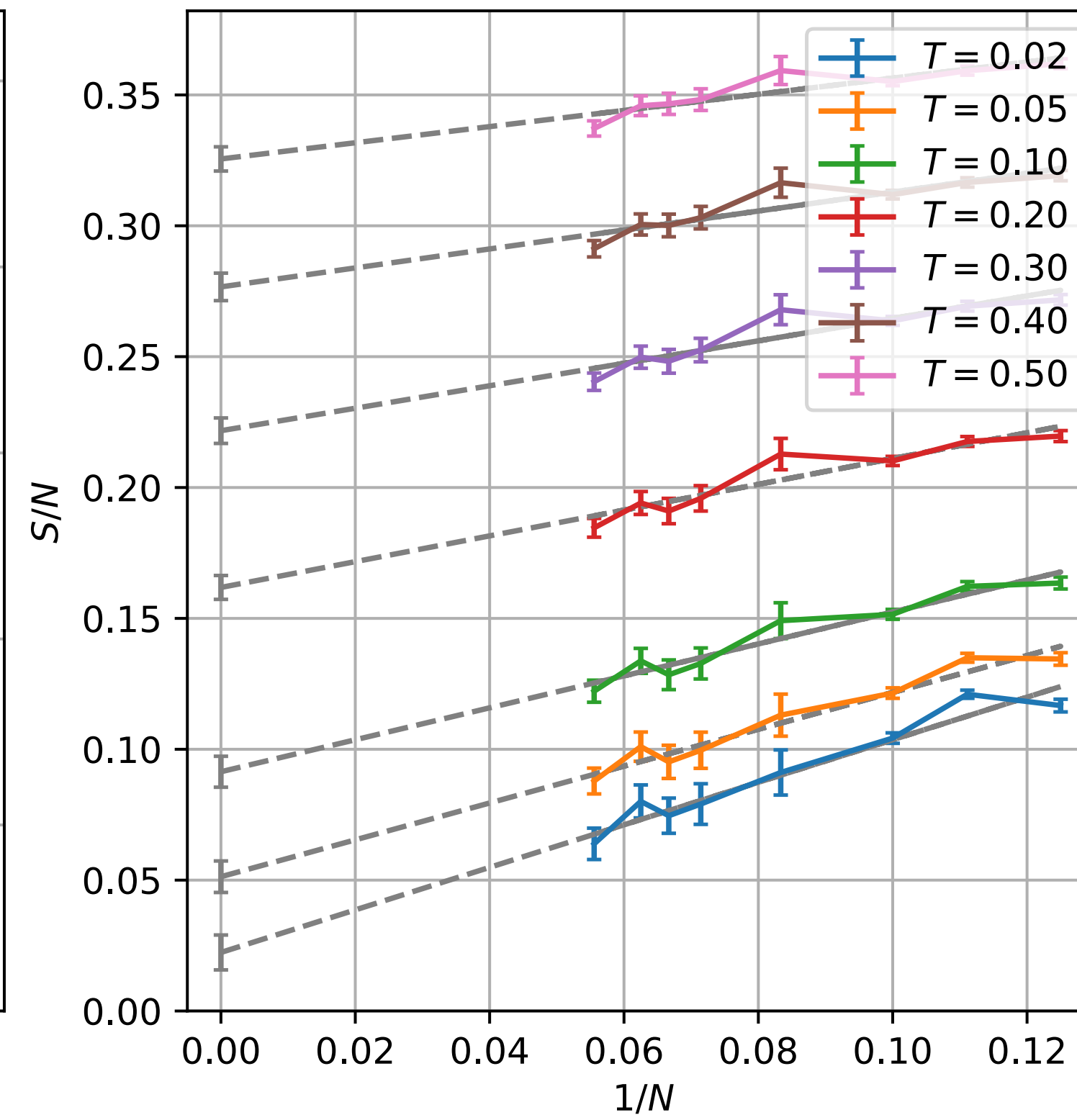
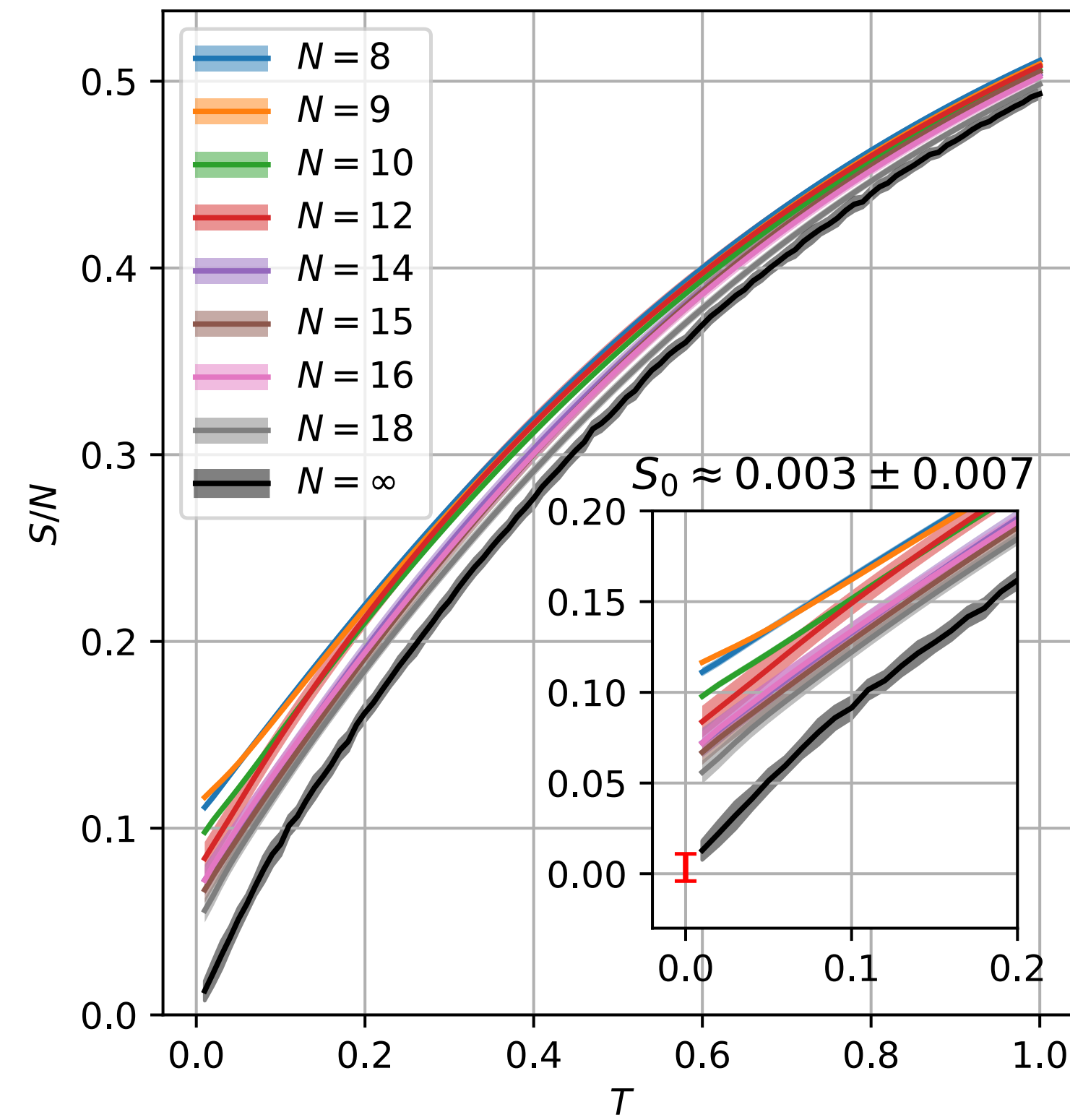
Remaining low frequency weight integrates to the spin glass order parameter q

Maximum entropy shifts at lower temperature



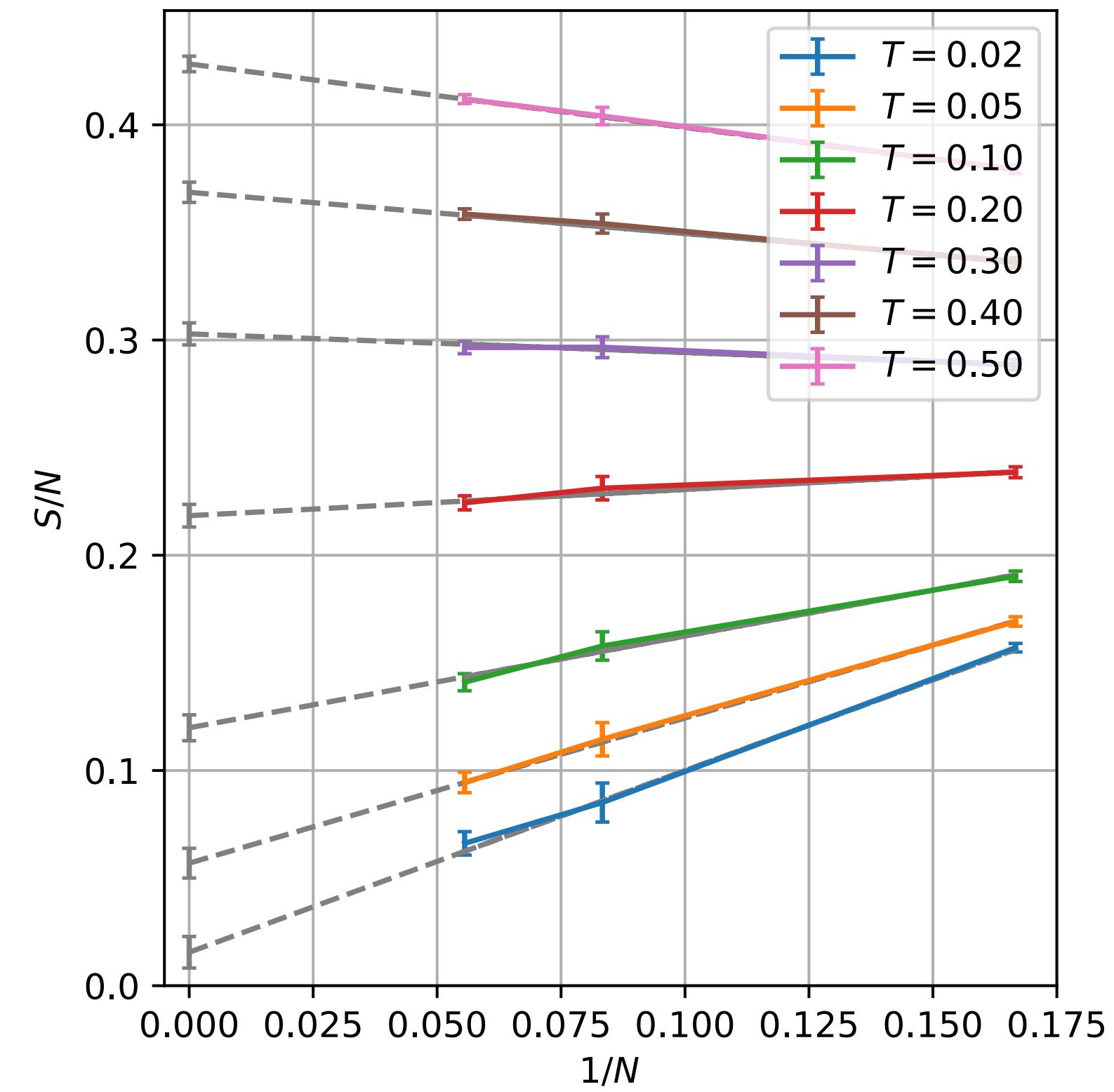
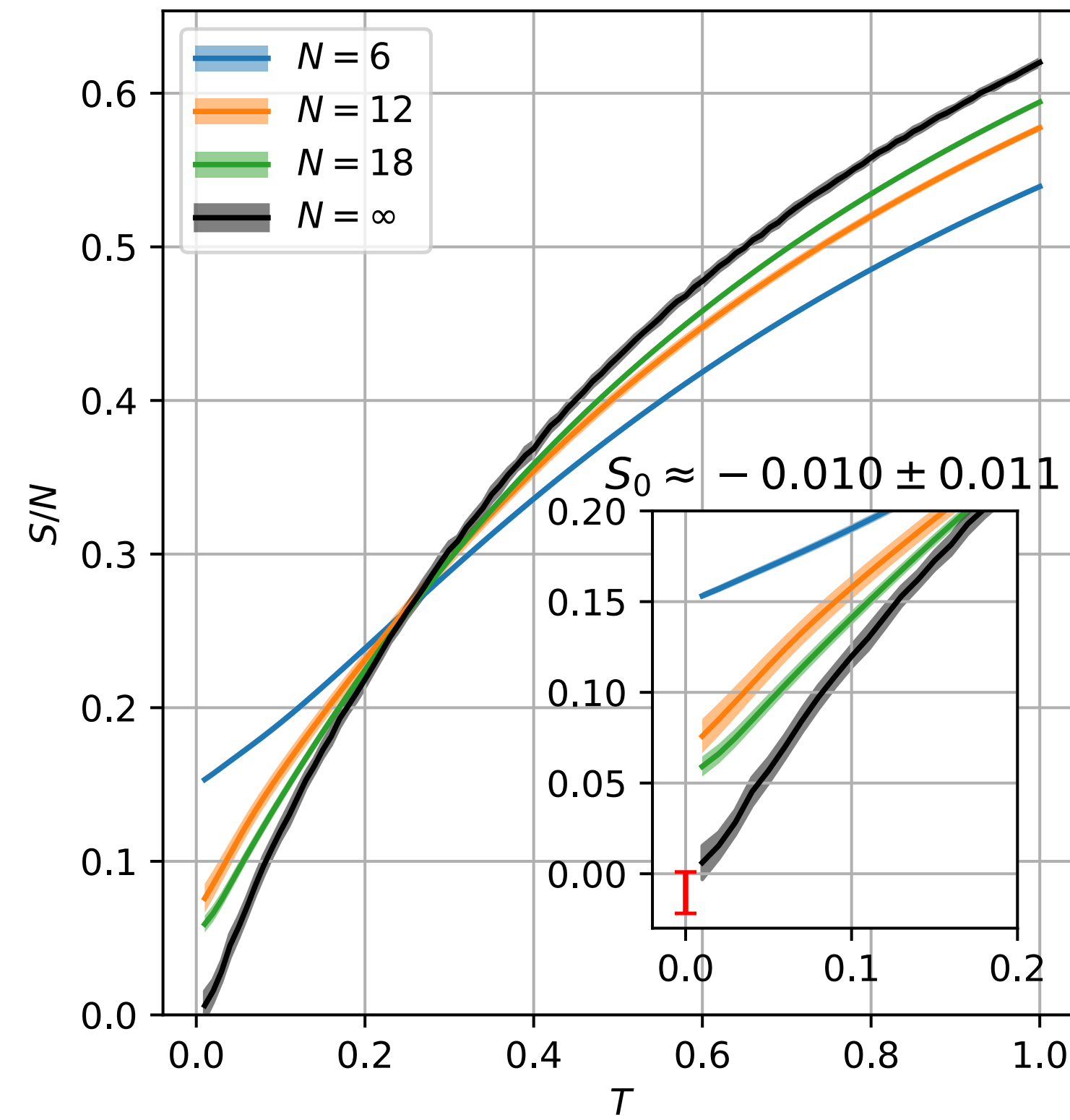
Large-N extrapolation of entropy density

Entropy, $p = 0$



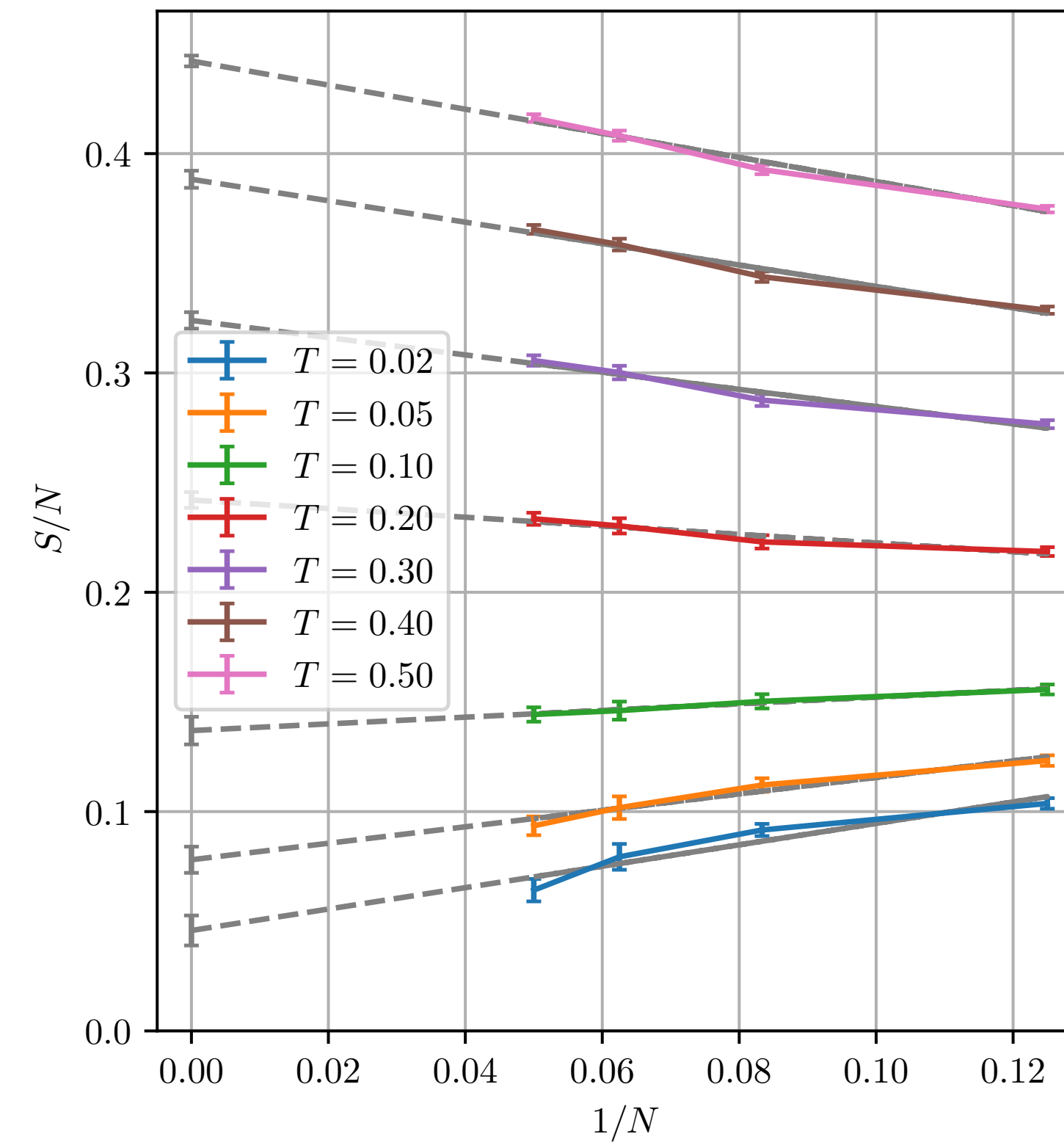
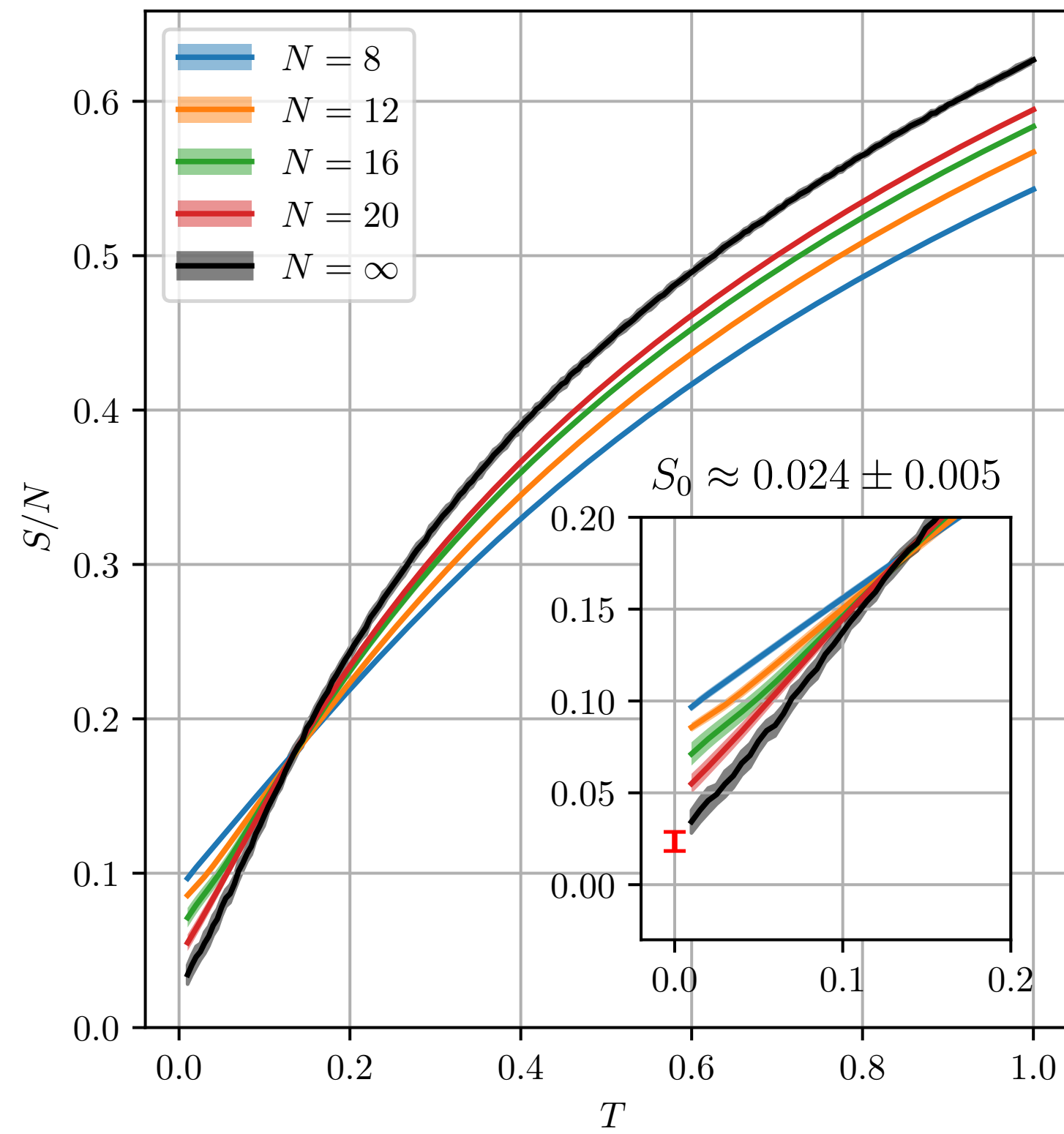
Large-N extrapolation of entropy density

Entropy, $p = 1/6$



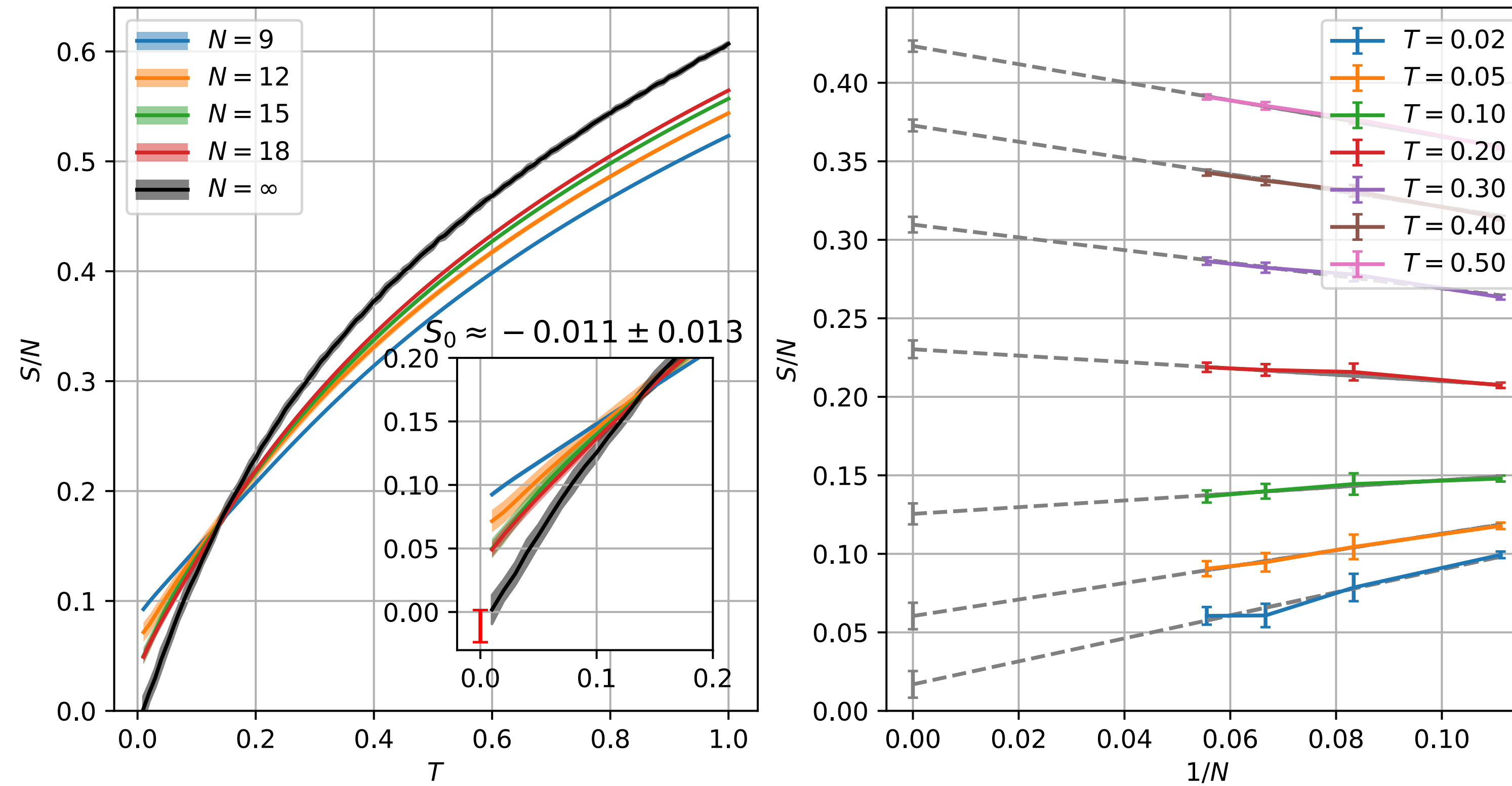
Large-N extrapolation of entropy density

$$p = 1/4$$



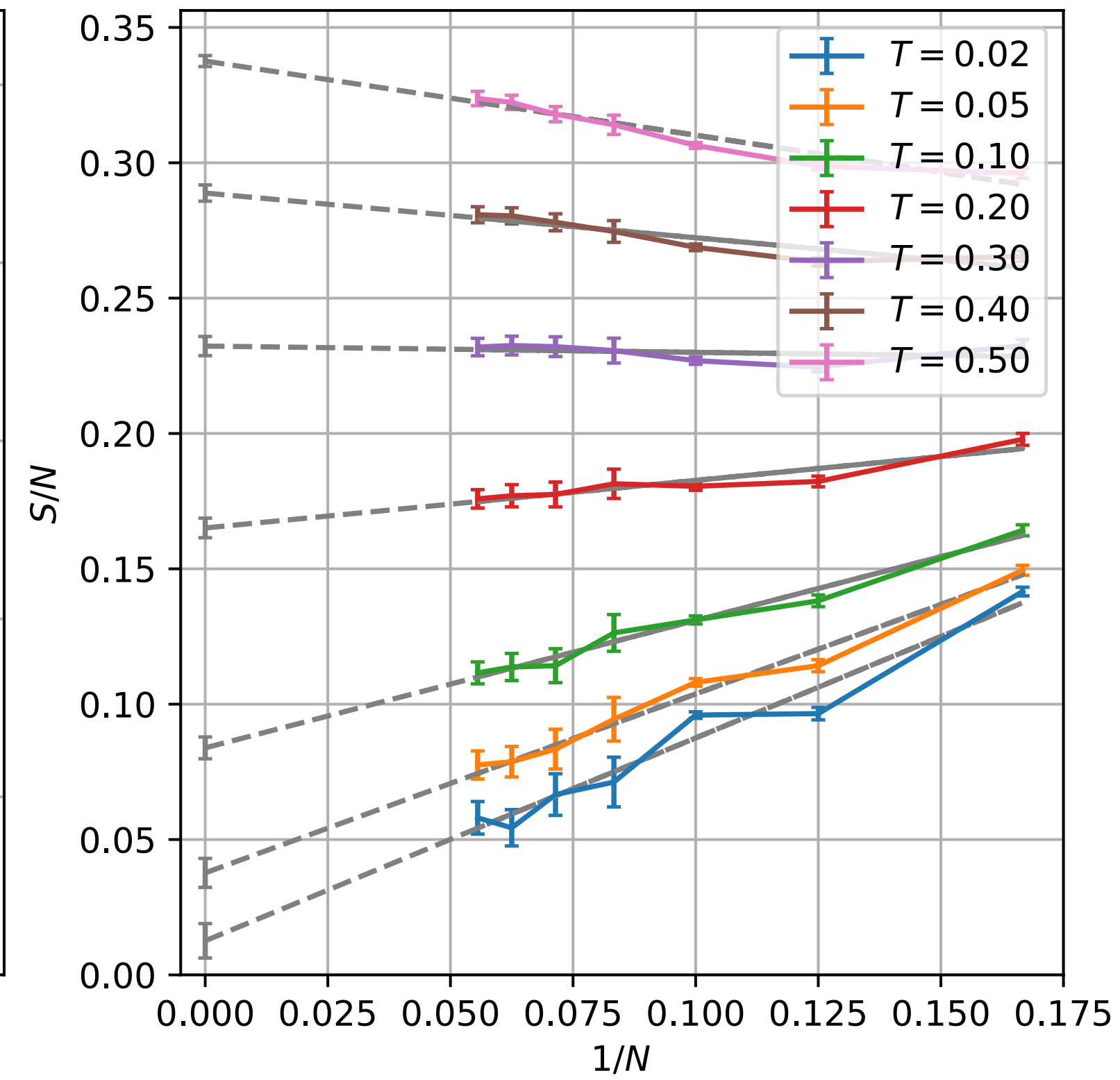
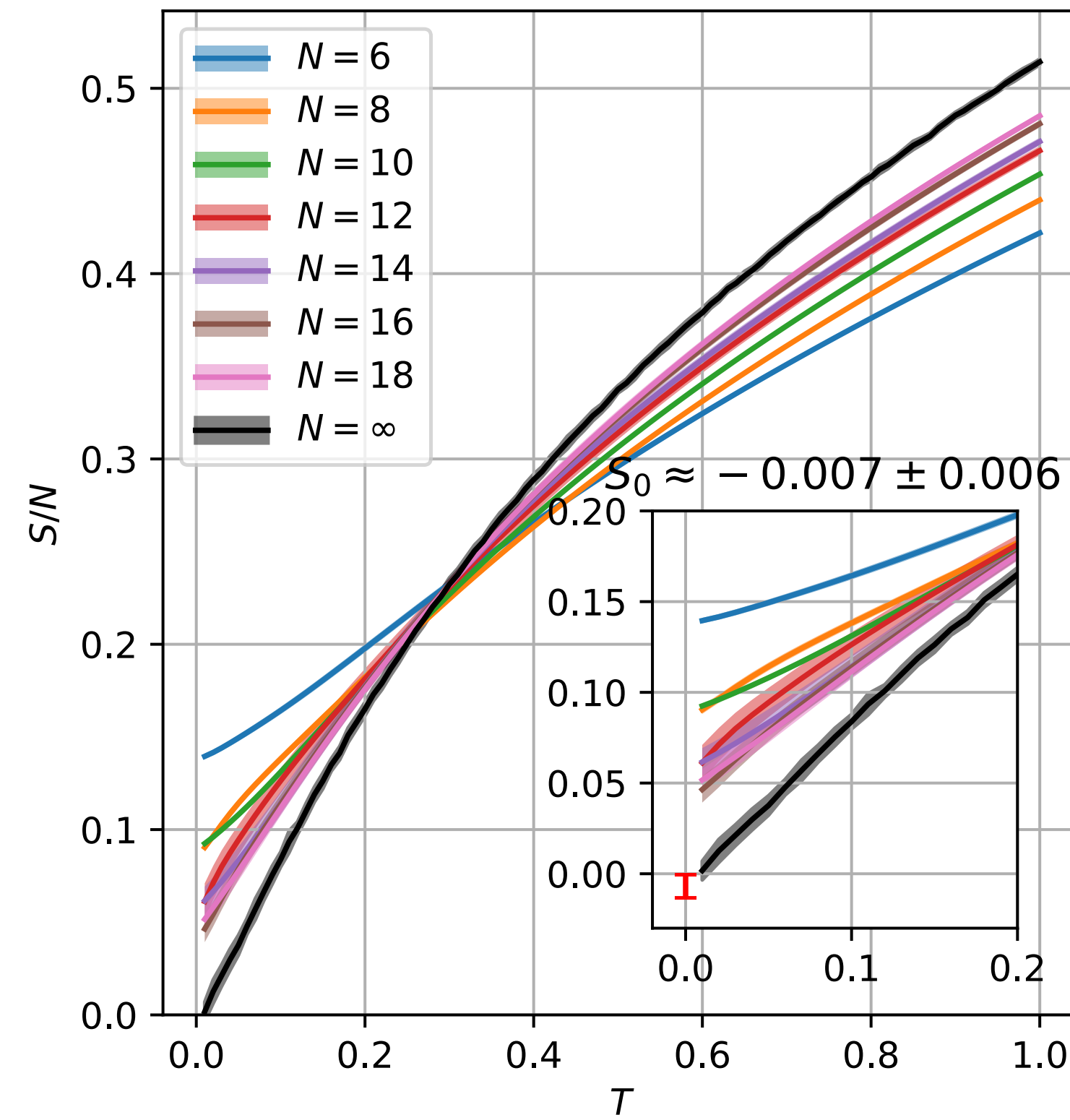
Large-N extrapolation of entropy density

Entropy, $p = 1/3$



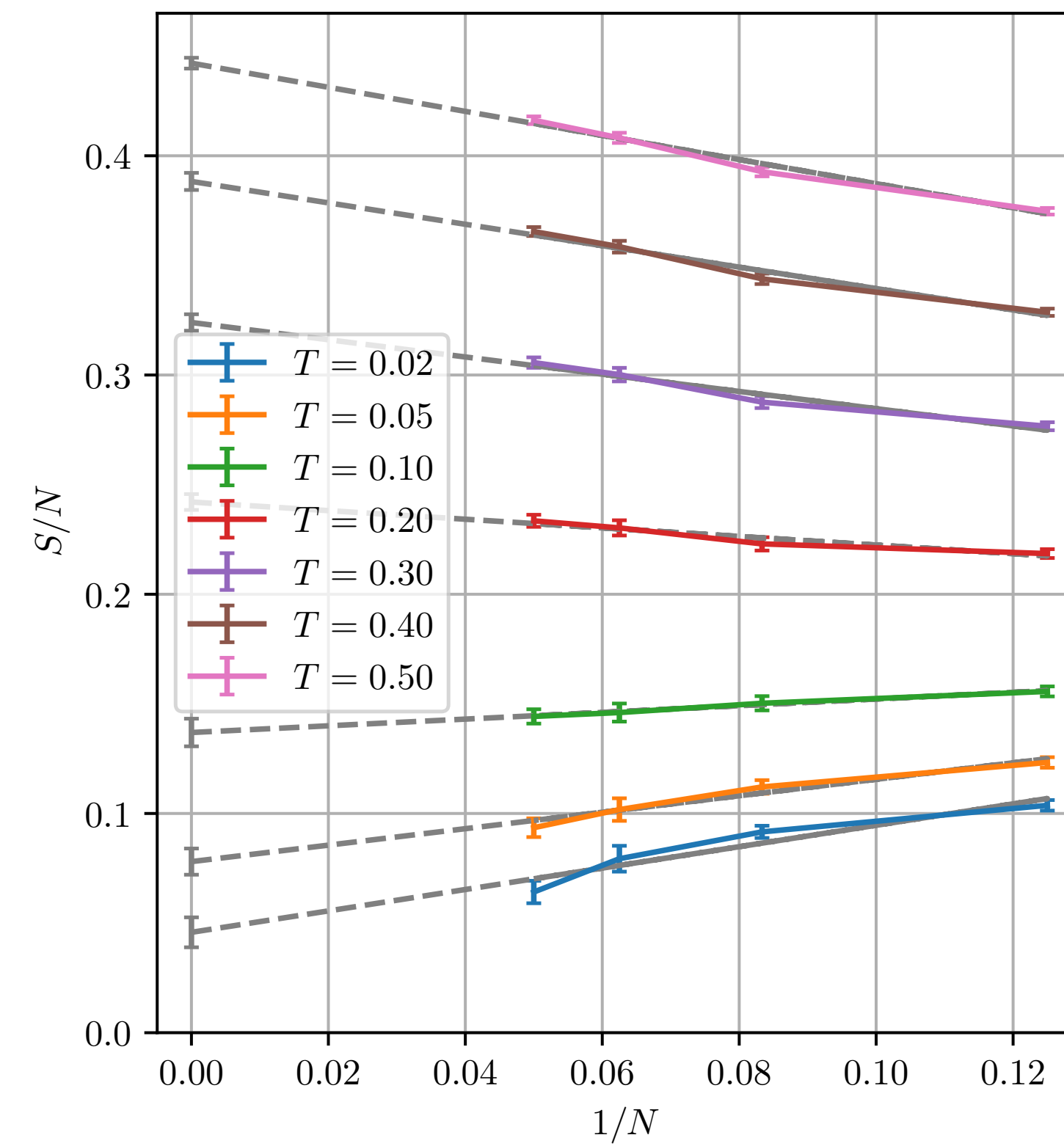
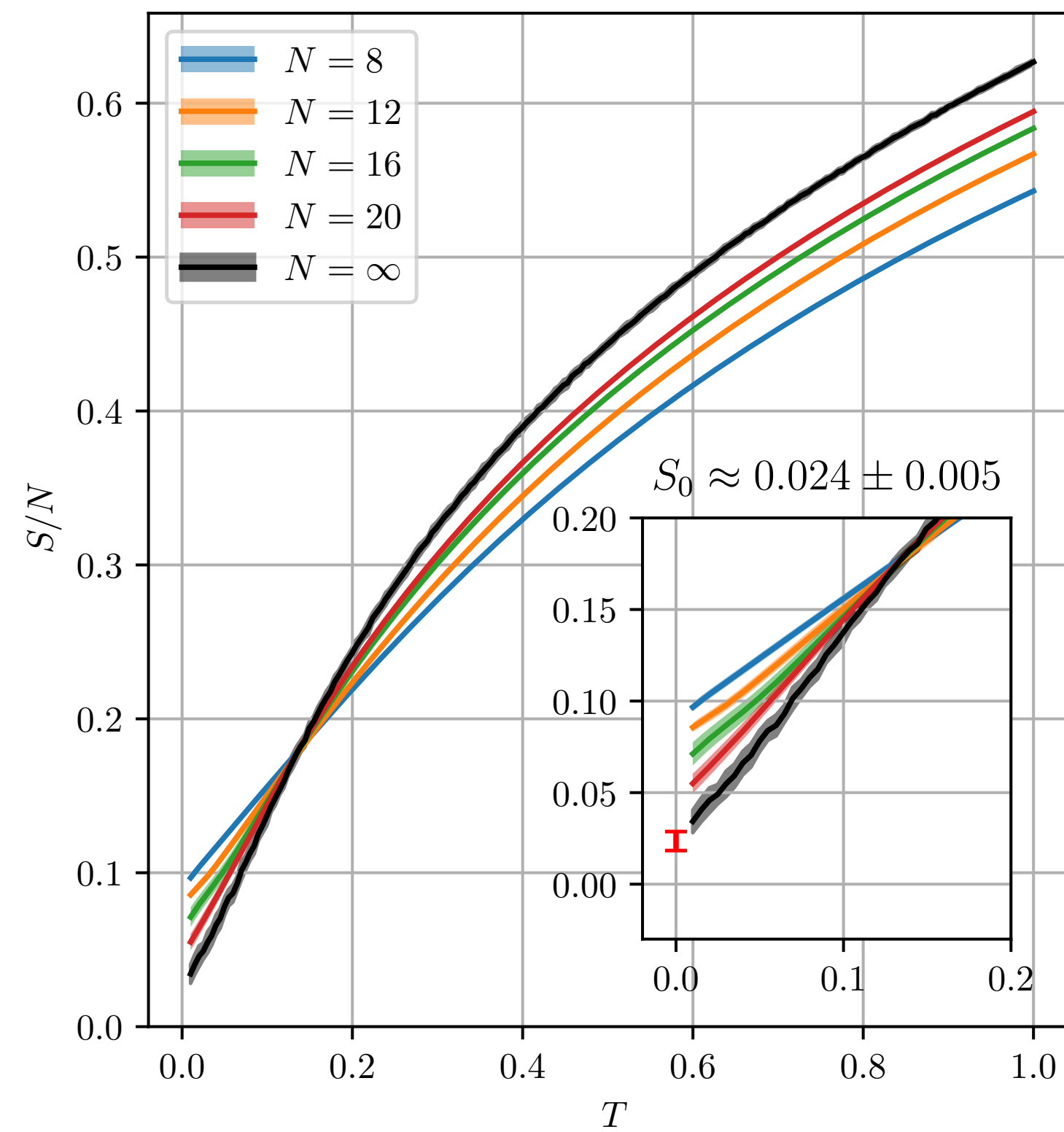
Large-N extrapolation of entropy density

Entropy, $p = 1/2$



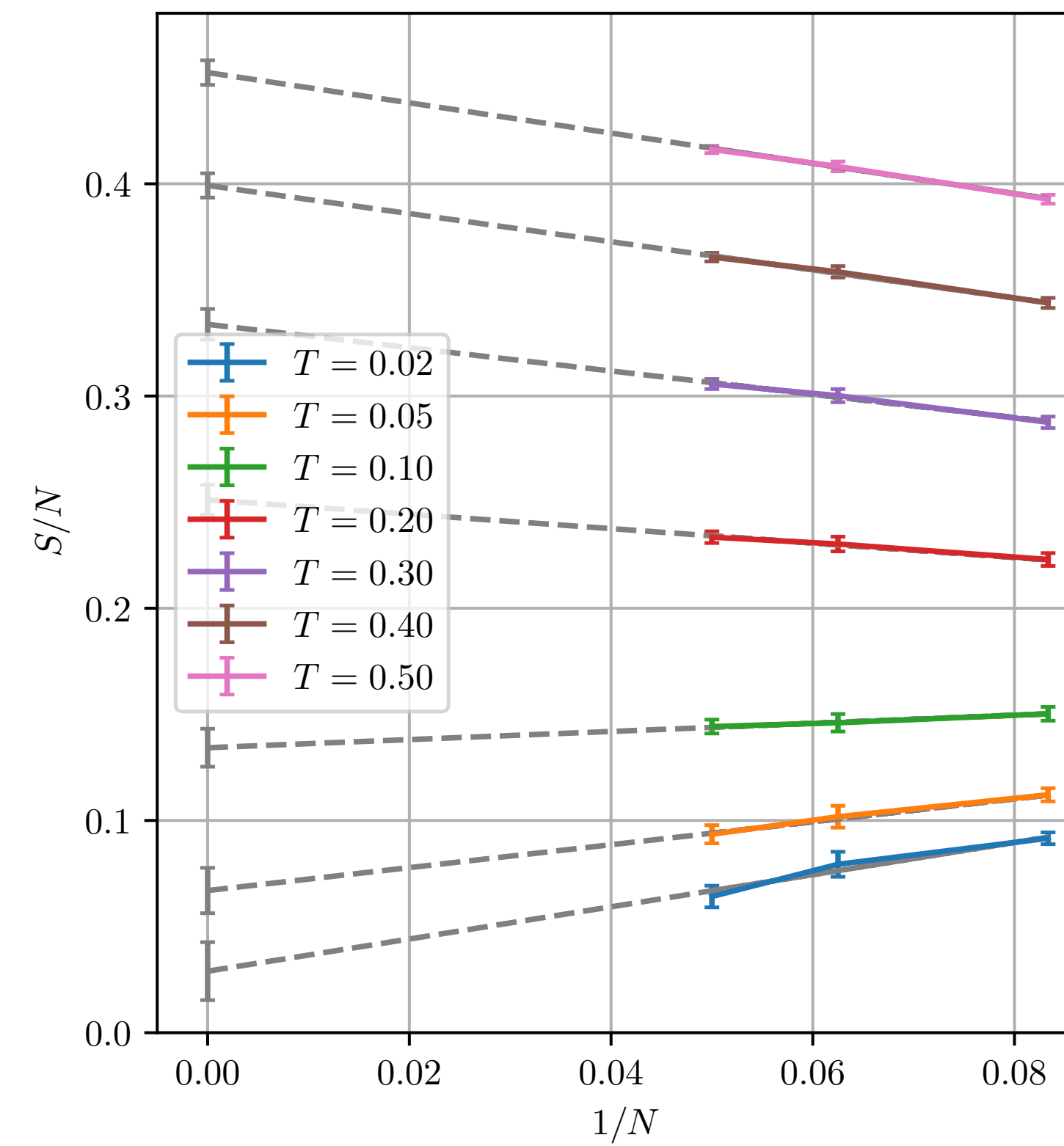
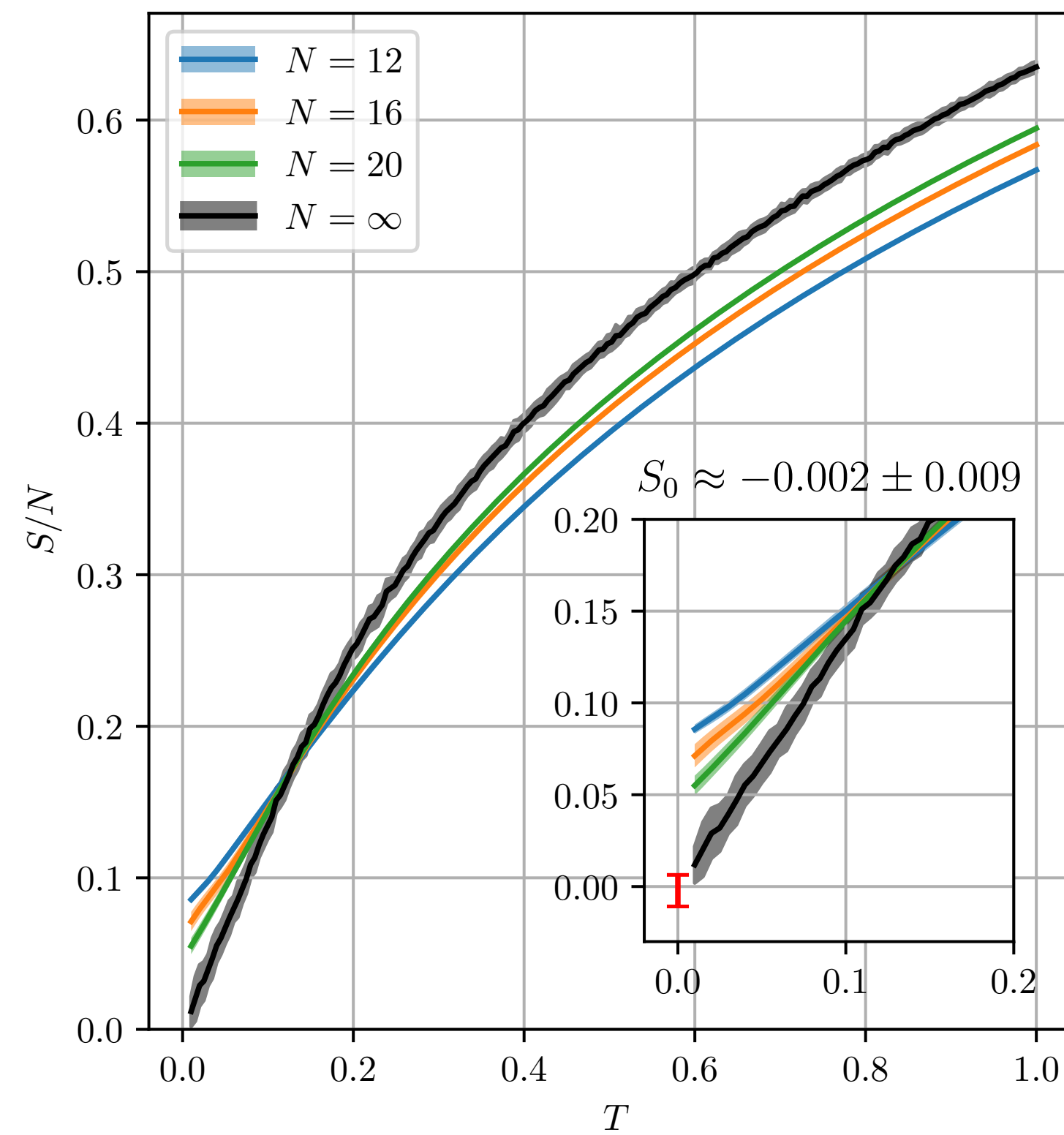
Non-zero s_0 at $p = 1/4$ is extrapolation-dependent

$p = 1/4$



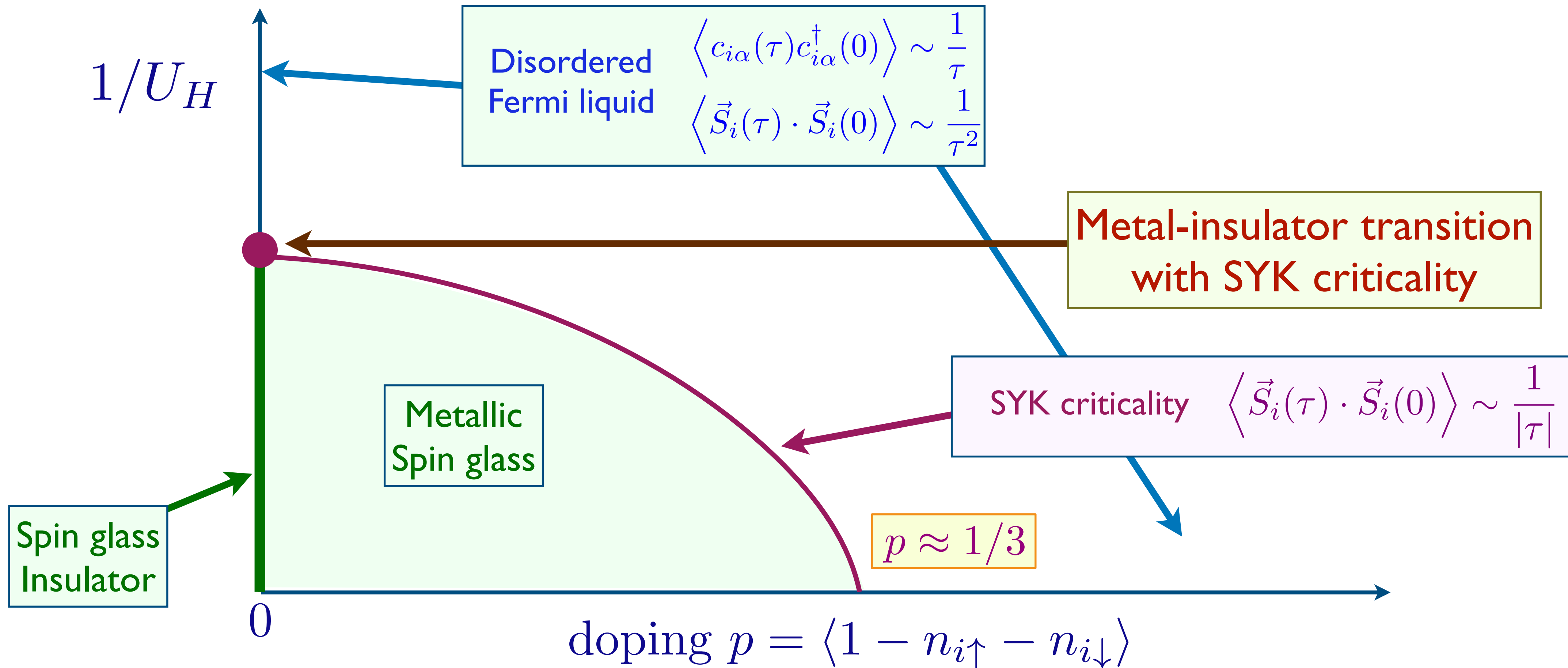
Non-zero s_0 at $p = 1/4$ is extrapolation-dependent

$p = 1/4$



Random t - J - U_H model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^N n_{i\uparrow} n_{i\downarrow}$$



Metal-metal quantum phase transitions

The ancilla qubit approach for non-random t - J models, and the random t - J model, have in common

- A metal-metal quantum phase transition with a change in carrier density from p to $1 + p$.
- Fractionalization of the electron in the critical regime
- Unexpectedly large low T entropy near the critical point (from ghost fermions, or the SYK black hole entropy).