

Quantum entanglement and the phases of matter

University of Cincinnati
March 30, 2012

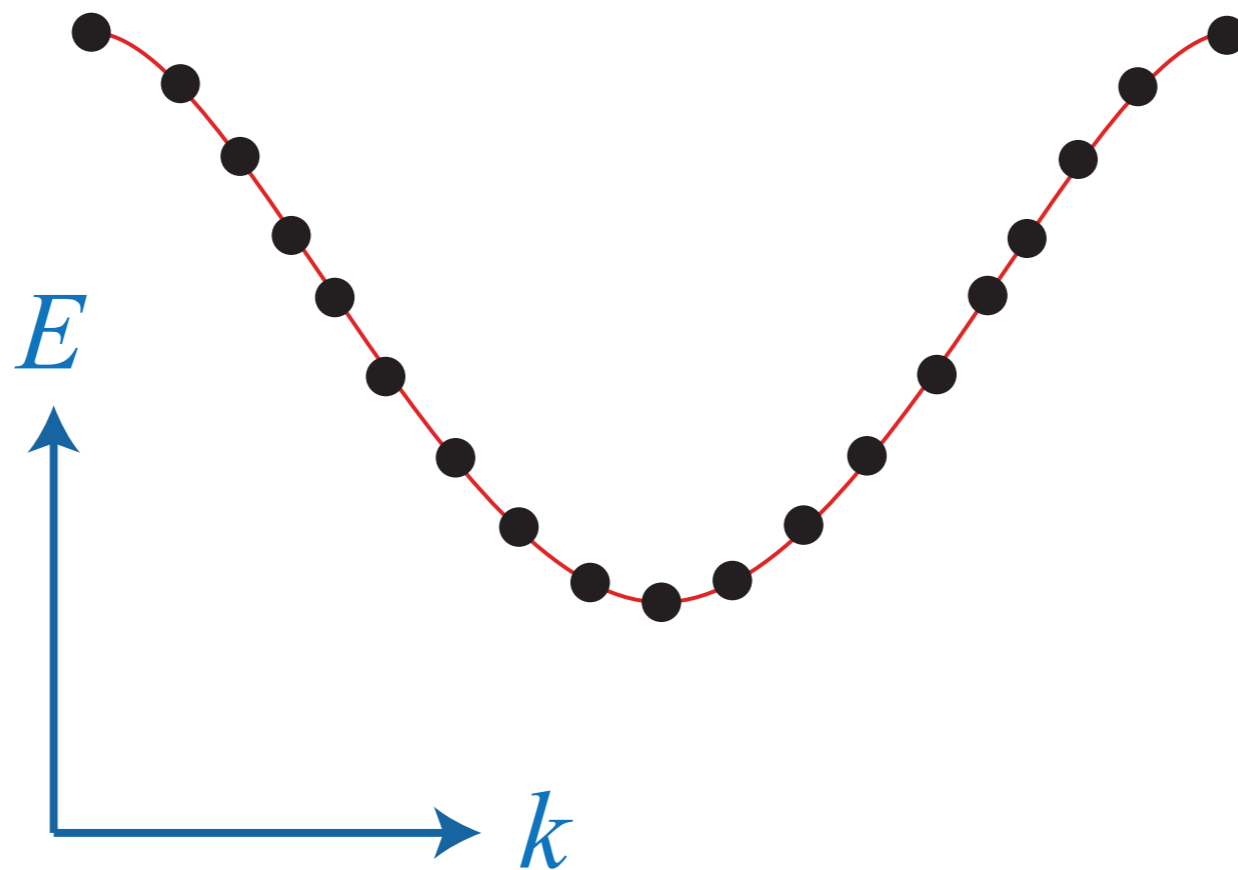
sachdev.physics.harvard.edu



Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically
connected to independent electron states

Sommerfeld-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states

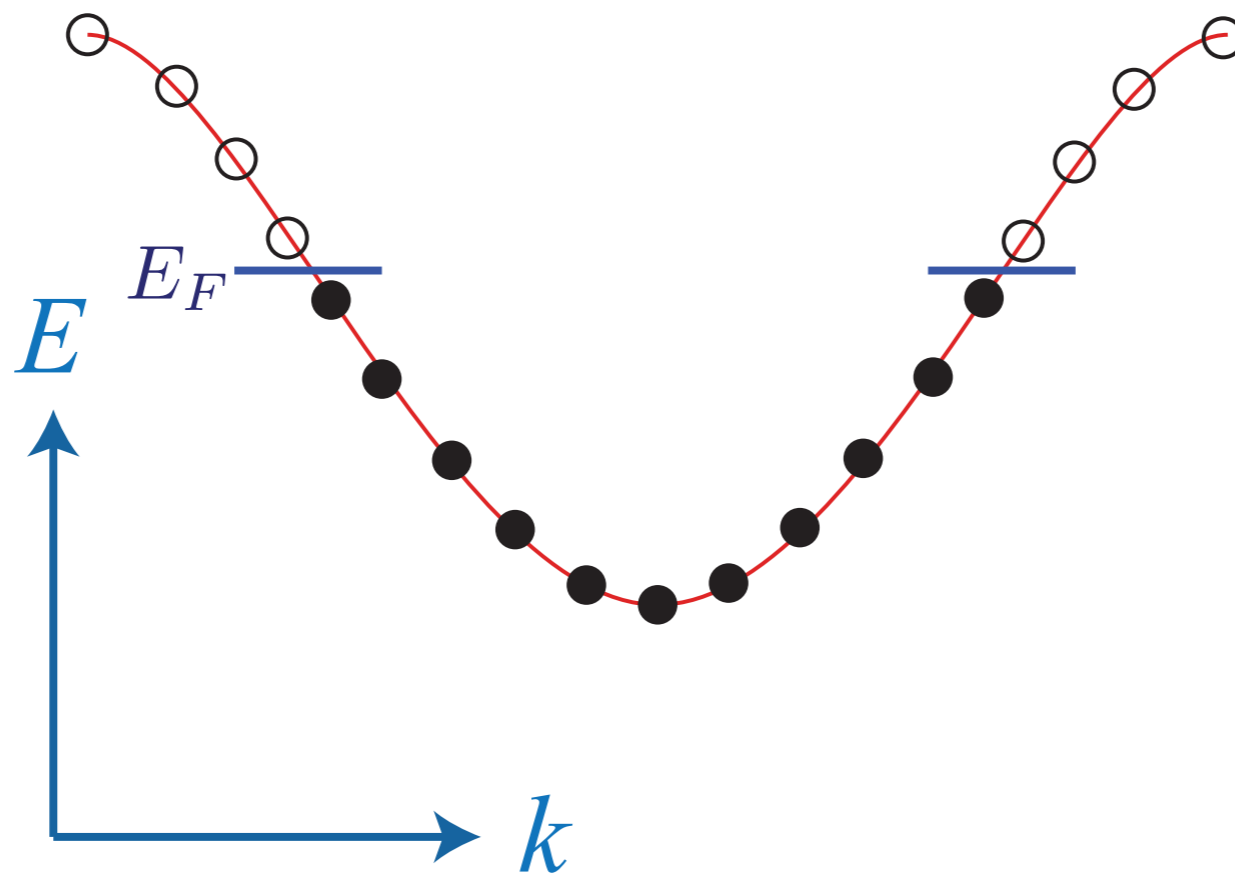
Band insulators



An even number of electrons per unit cell

Sommerfeld-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states

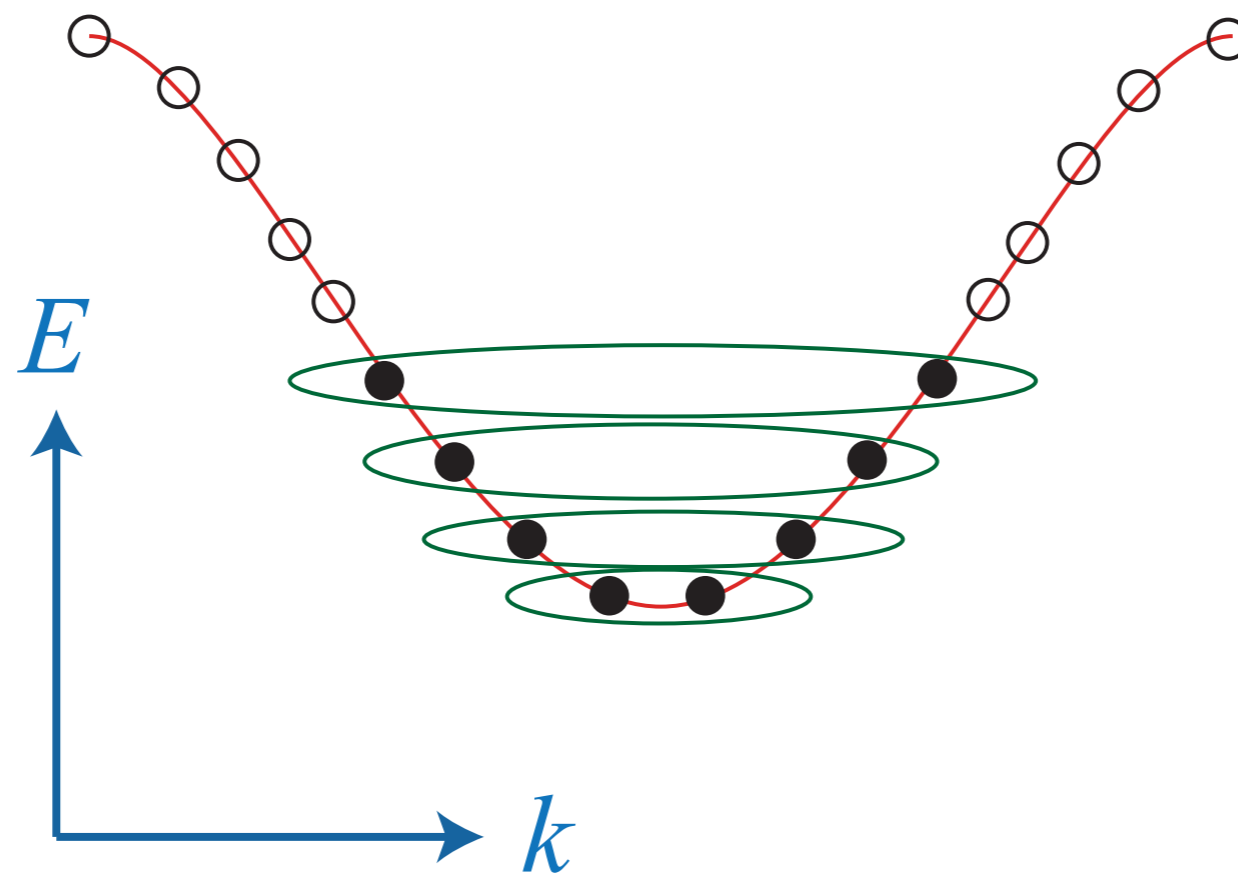
Metals



An odd number of electrons per unit cell

Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
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Superconductors



**Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically
connected to independent electron states**

Modern phases of quantum matter

Not adiabatically connected to independent electron
states:

many-particle quantum entanglement

**Quantum
superposition and
entanglement**

Quantum Entanglement: quantum superposition with more than one particle

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

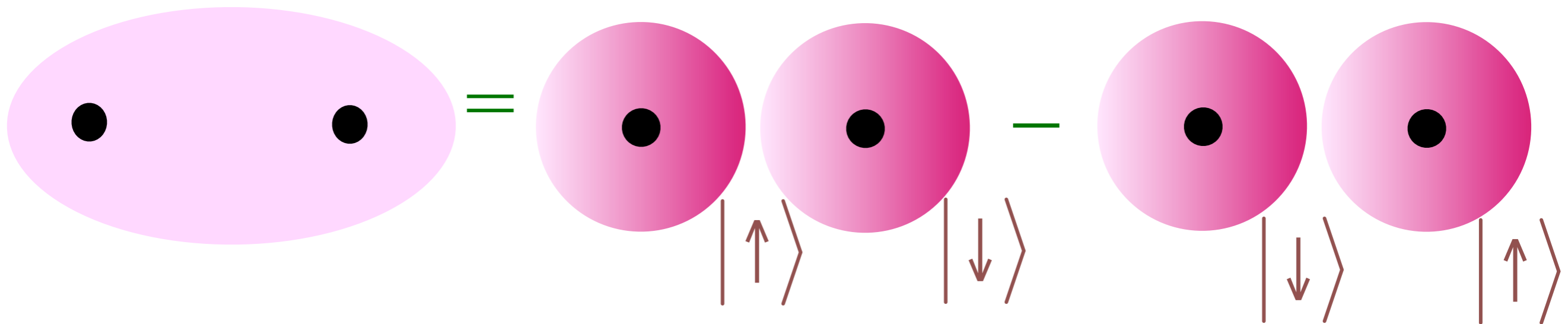


Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:



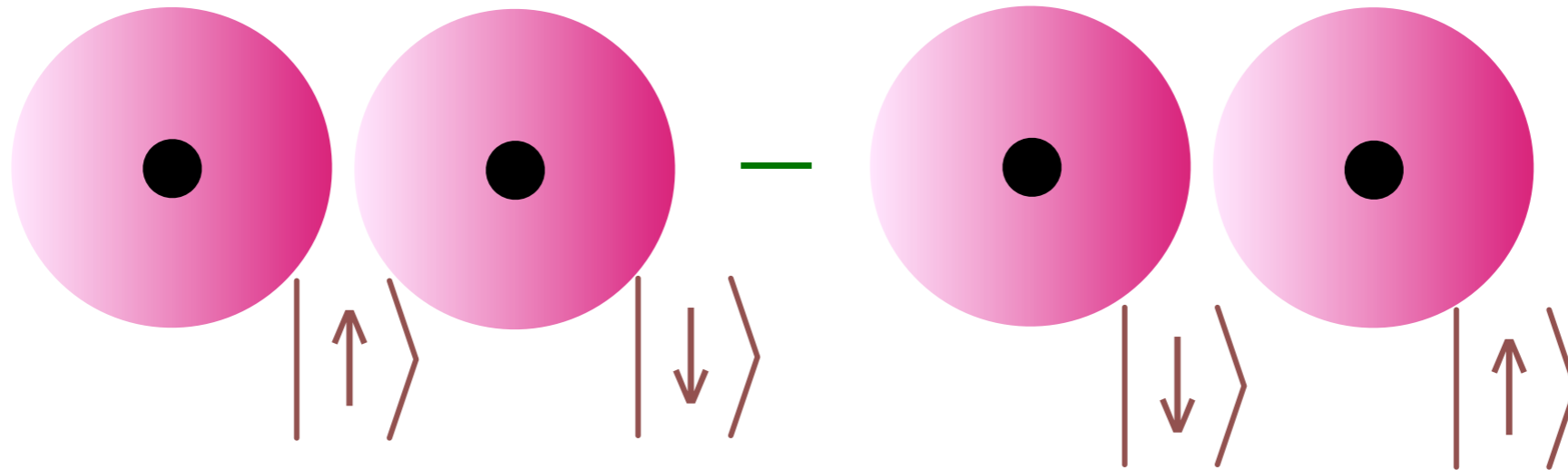
Hydrogen molecule:



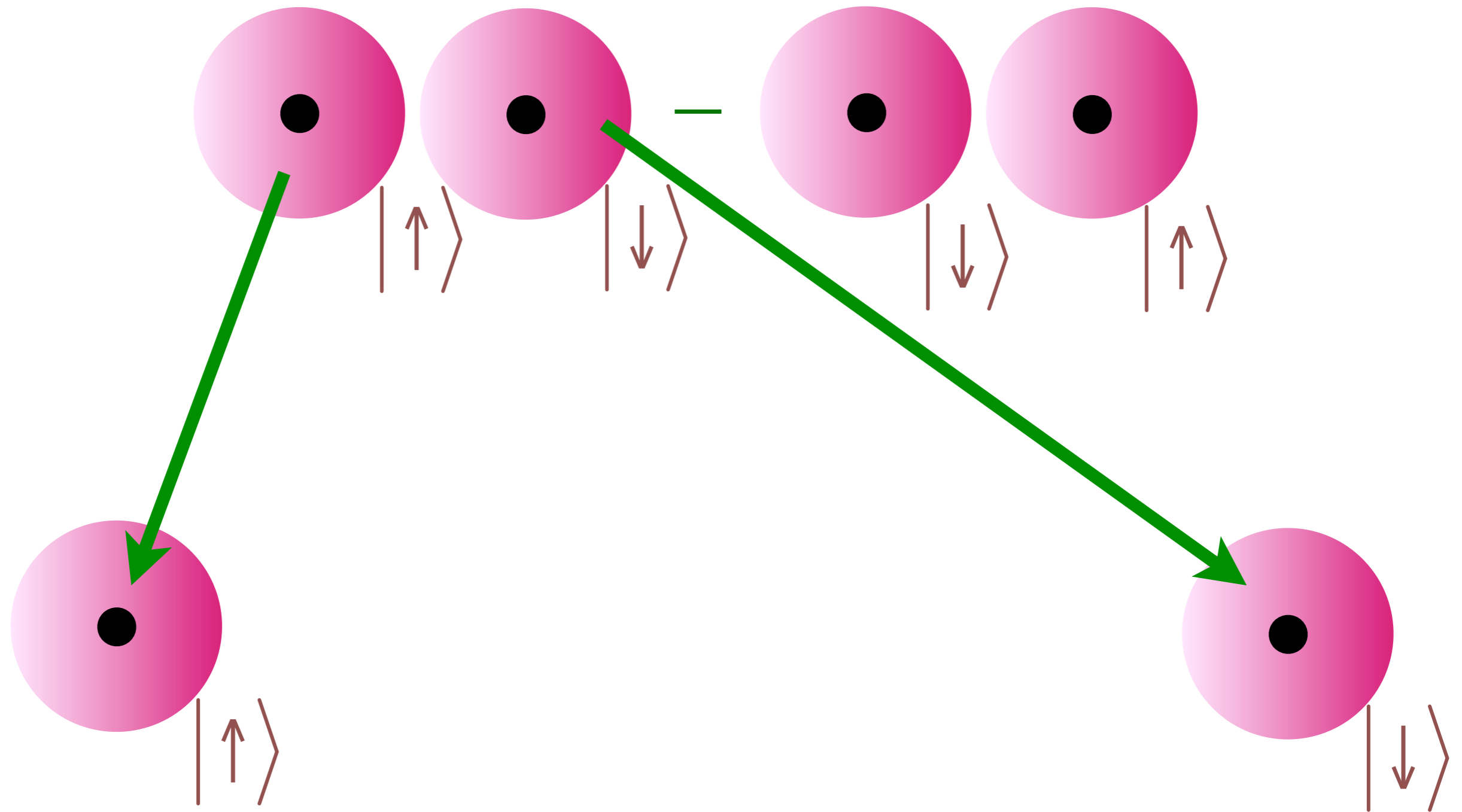
$$= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Superposition of two electron states leads to non-local
correlations between spins

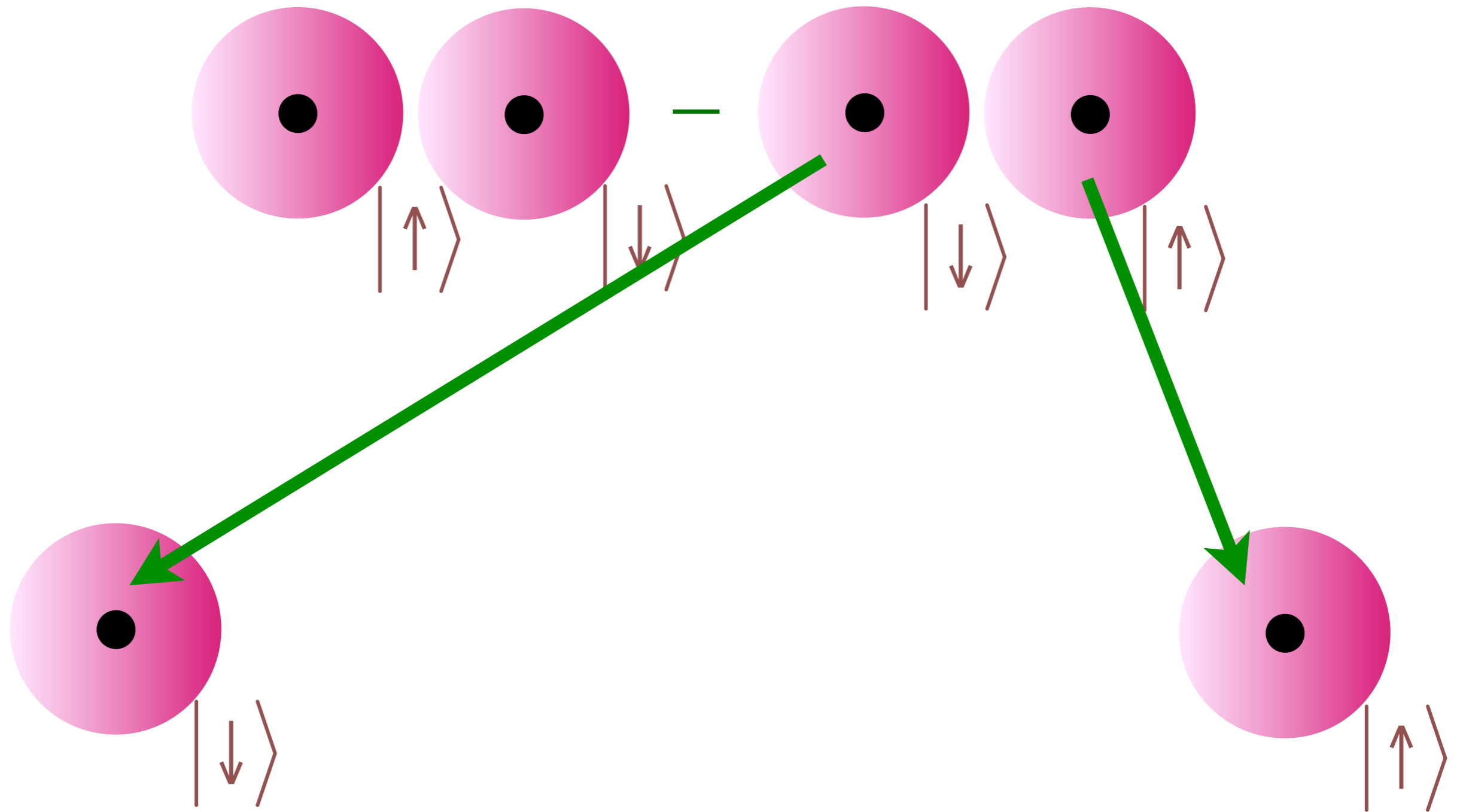
Quantum Entanglement: quantum superposition with more than one particle



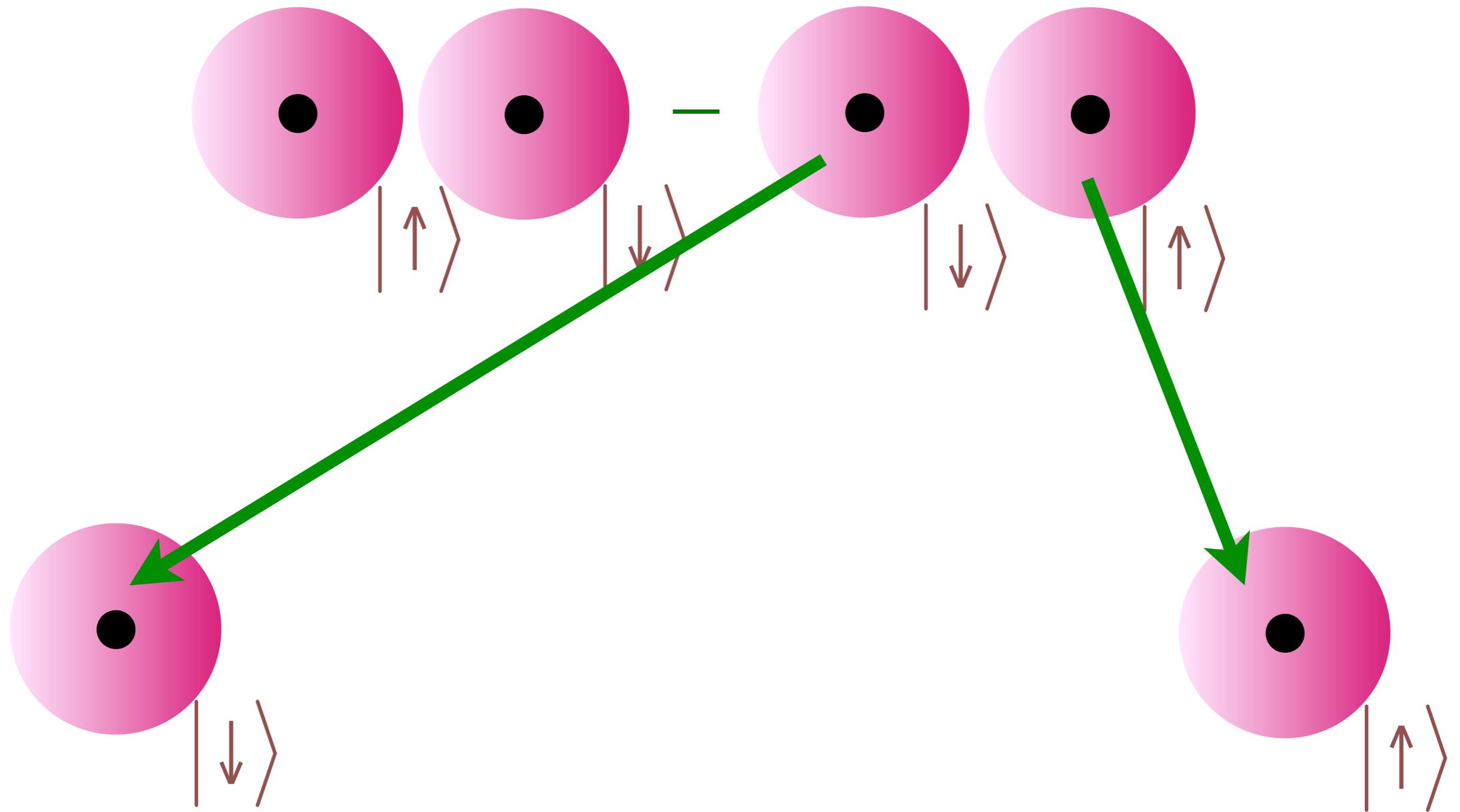
Quantum Entanglement: quantum superposition with more than one particle



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Einstein-Podolsky-Rosen “paradox”: Non-local correlations between observations arbitrarily far apart

**Quantum
superposition and
entanglement**

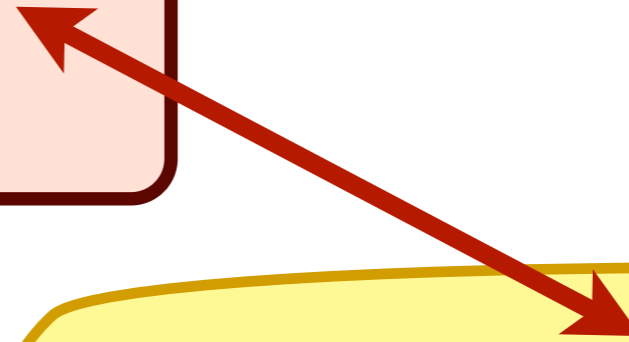
**Quantum
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String theory

**Quantum critical
points of electrons
in crystals**

Black holes

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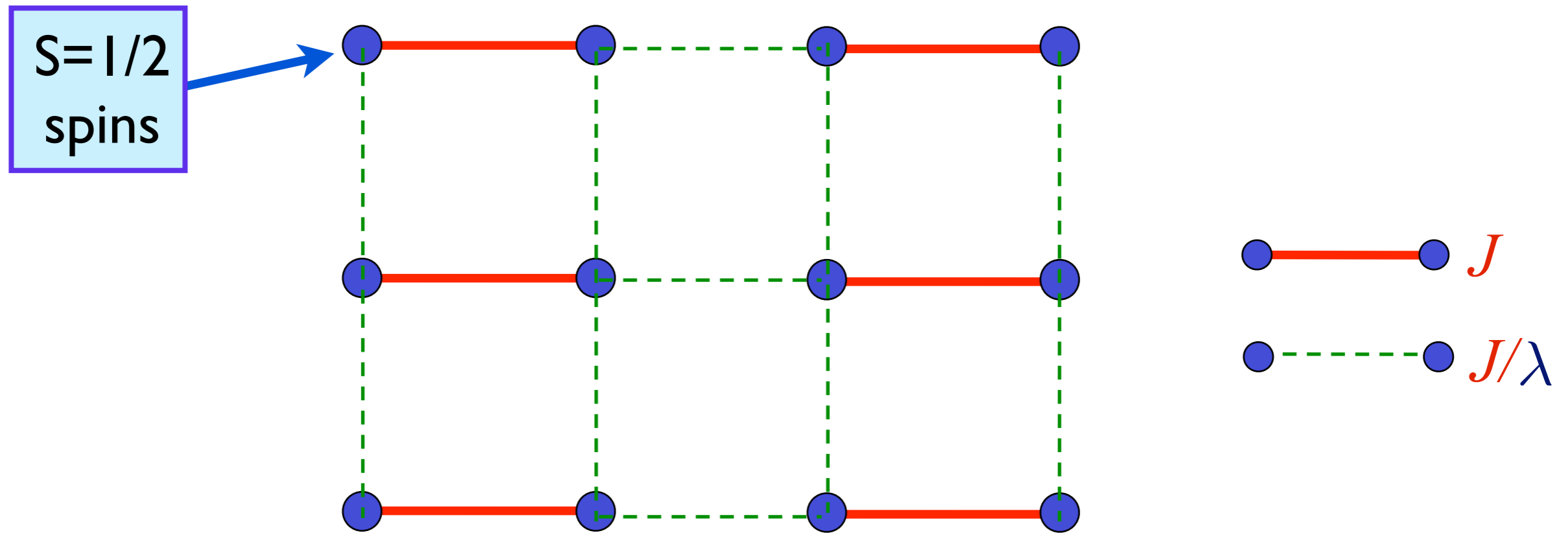
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Spinning electrons localized on a square lattice

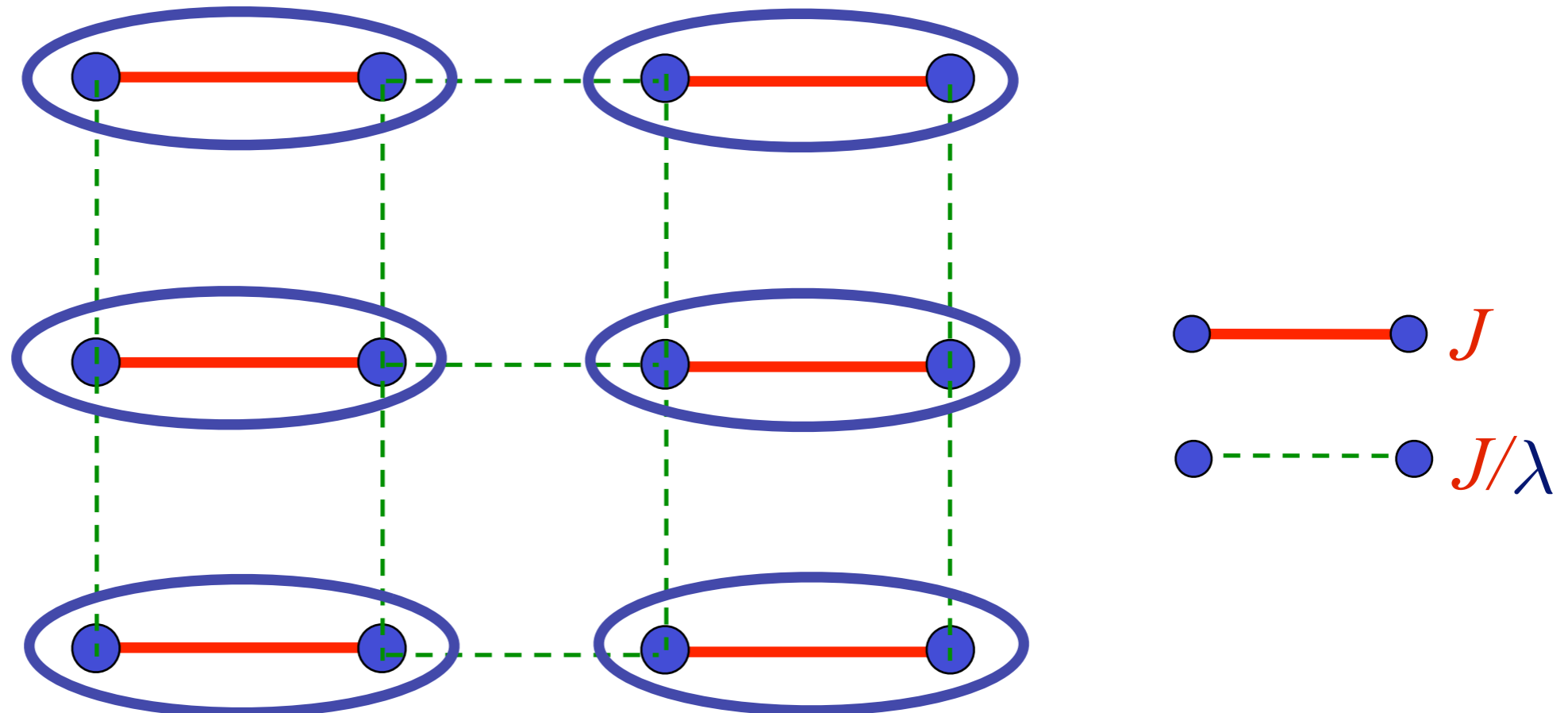
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Examine ground state as a function of λ

Spinning electrons localized on a square lattice

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

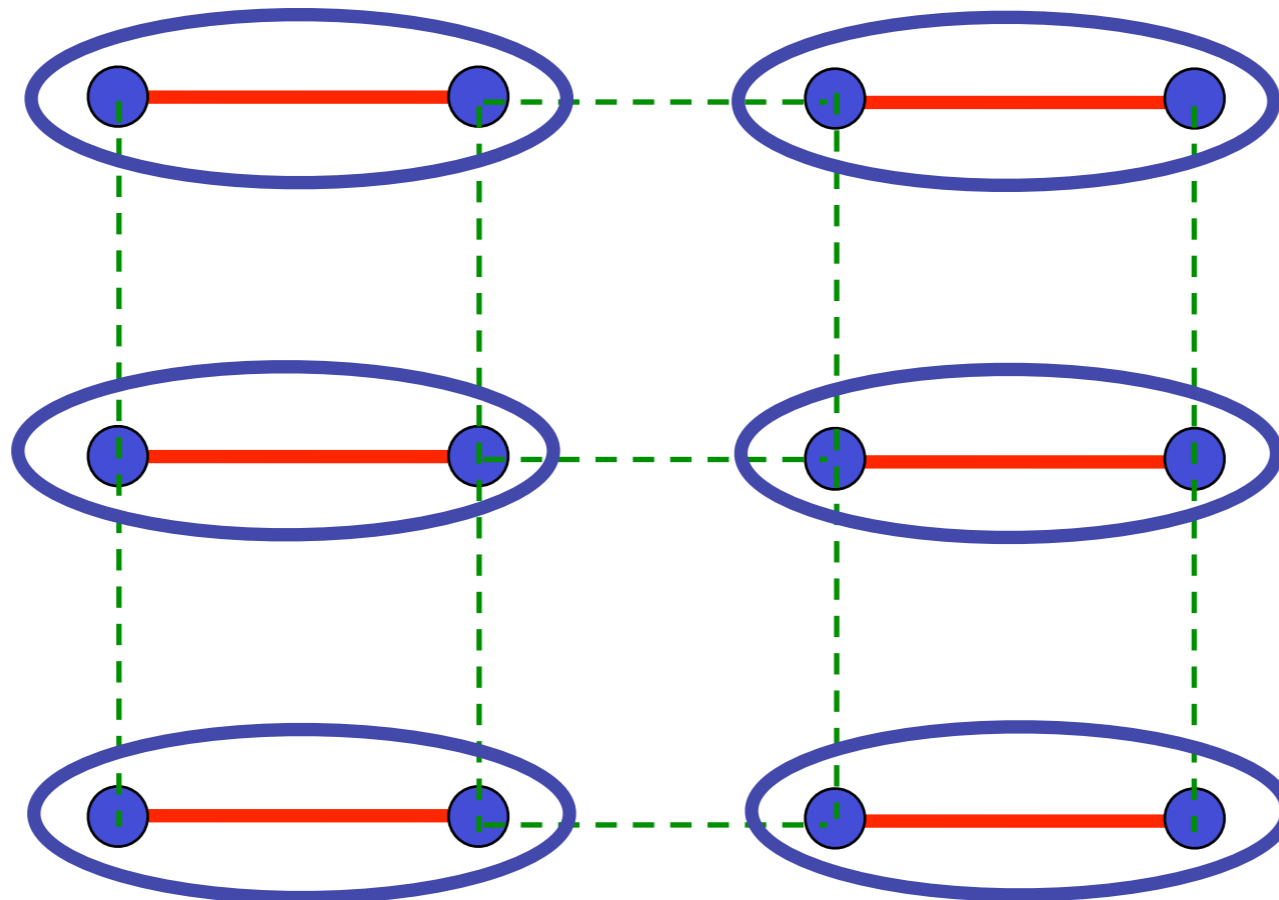


$$\text{Valence bond singlet} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

At large λ ground state is a “quantum paramagnet” with spins locked in valence bond singlets

Spinning electrons localized on a square lattice

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

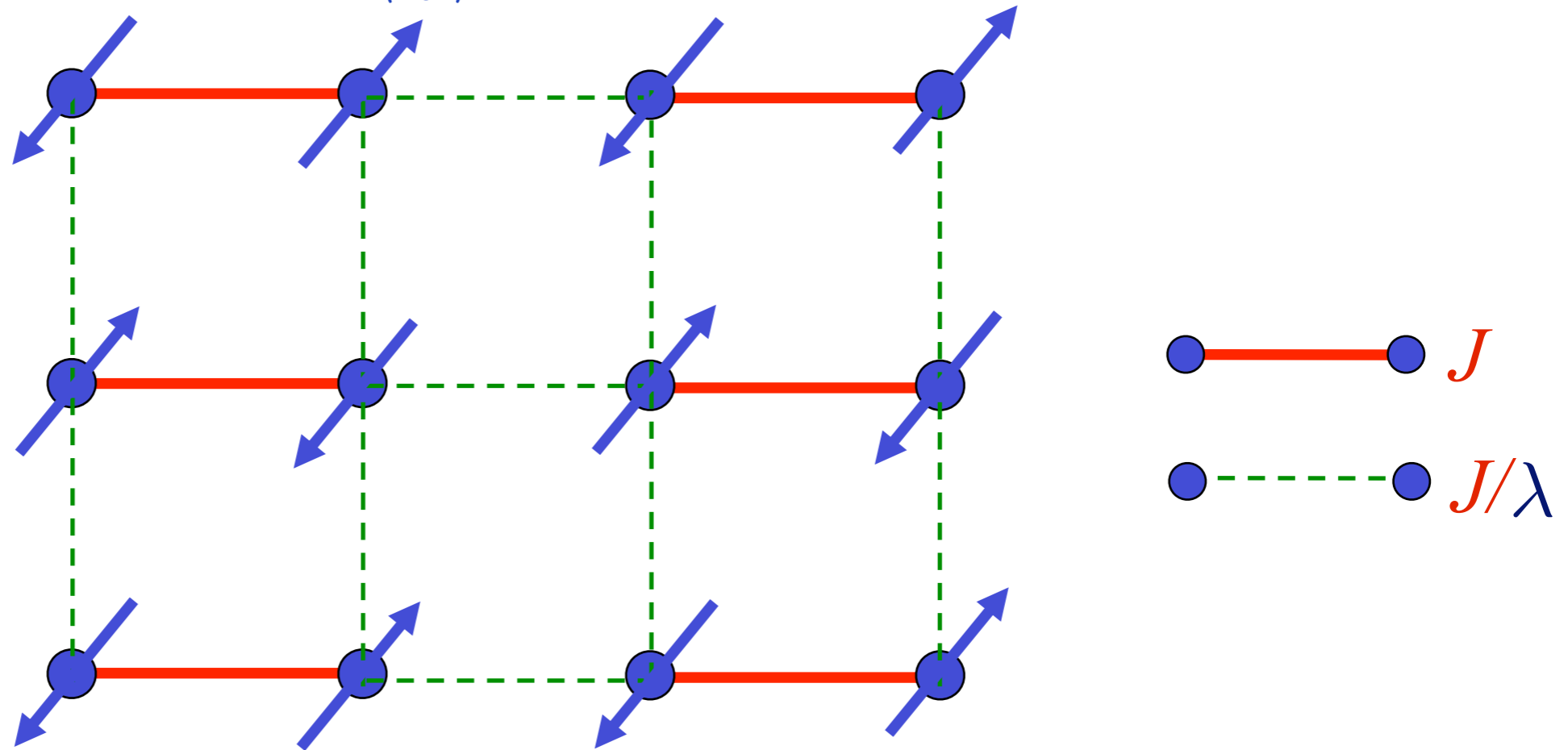


$$\text{[Diagram of a pair of spins in a blue oval]} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Nearest-neighbor spins are “entangled” with each other.
Can be separated into an Einstein-Podolsky-Rosen (EPR) pair.

Spinning electrons localized on a square lattice

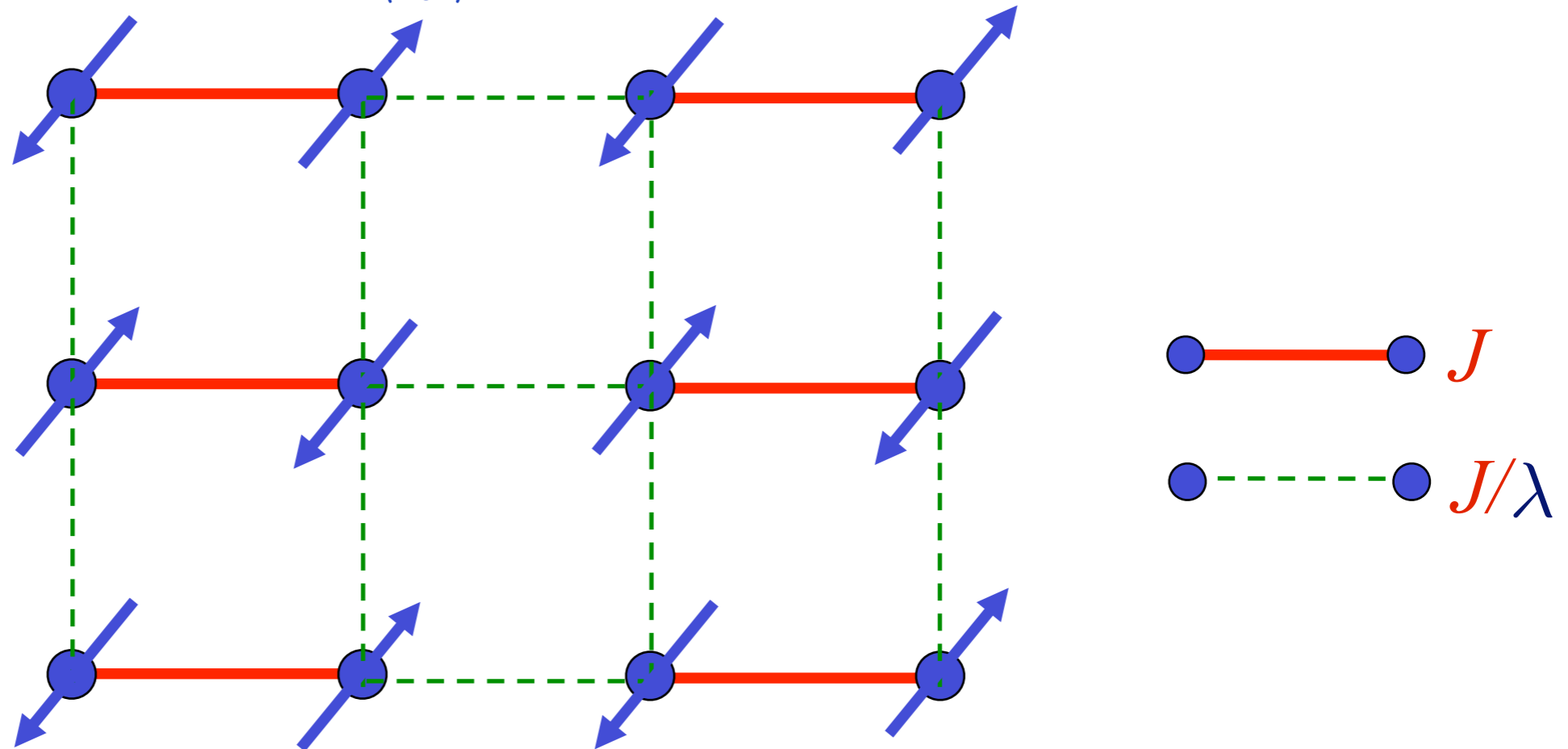
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



For $\lambda \approx 1$, the ground state has antiferromagnetic (“Néel”) order, and the spins align in a checkerboard pattern

Spinning electrons localized on a square lattice

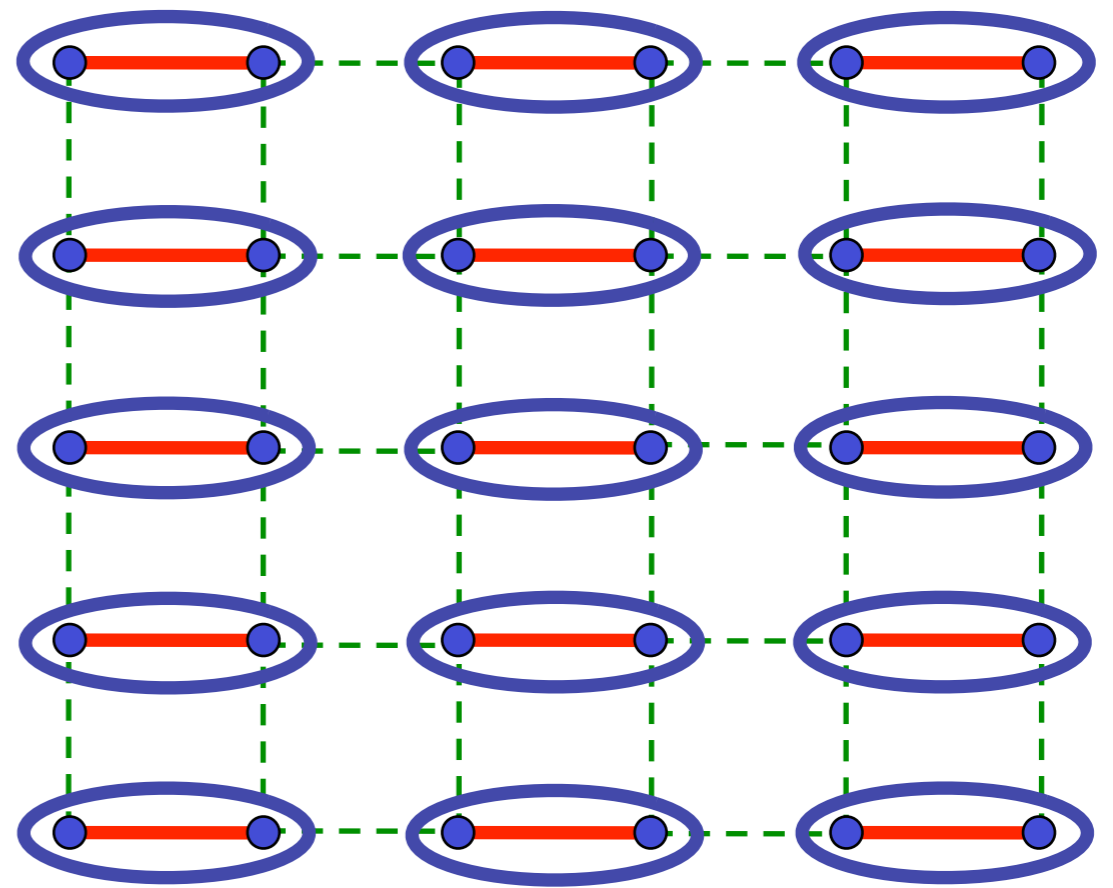
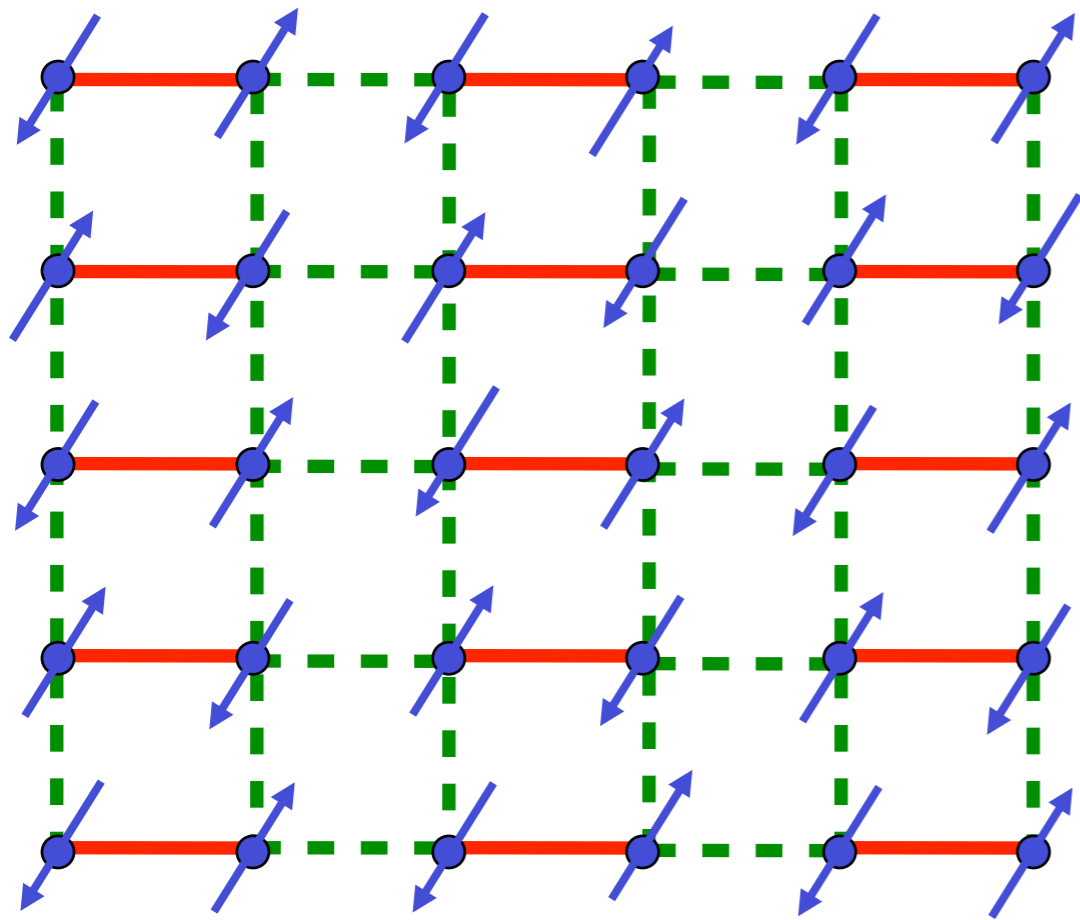
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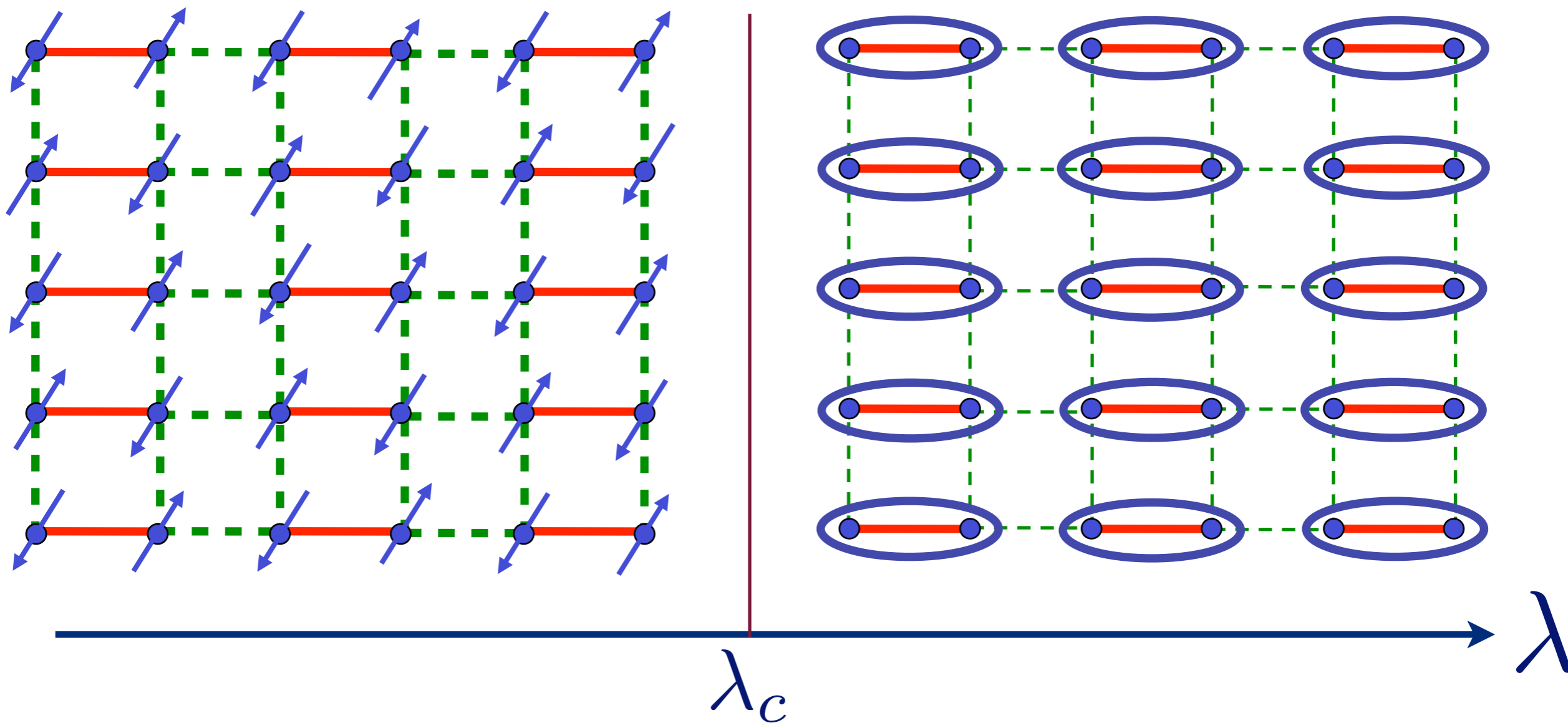
For $\lambda \approx 1$, the ground state has antiferromagnetic (“Néel”) order,
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No EPR pairs

$$\text{Diagram of two blue dots connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



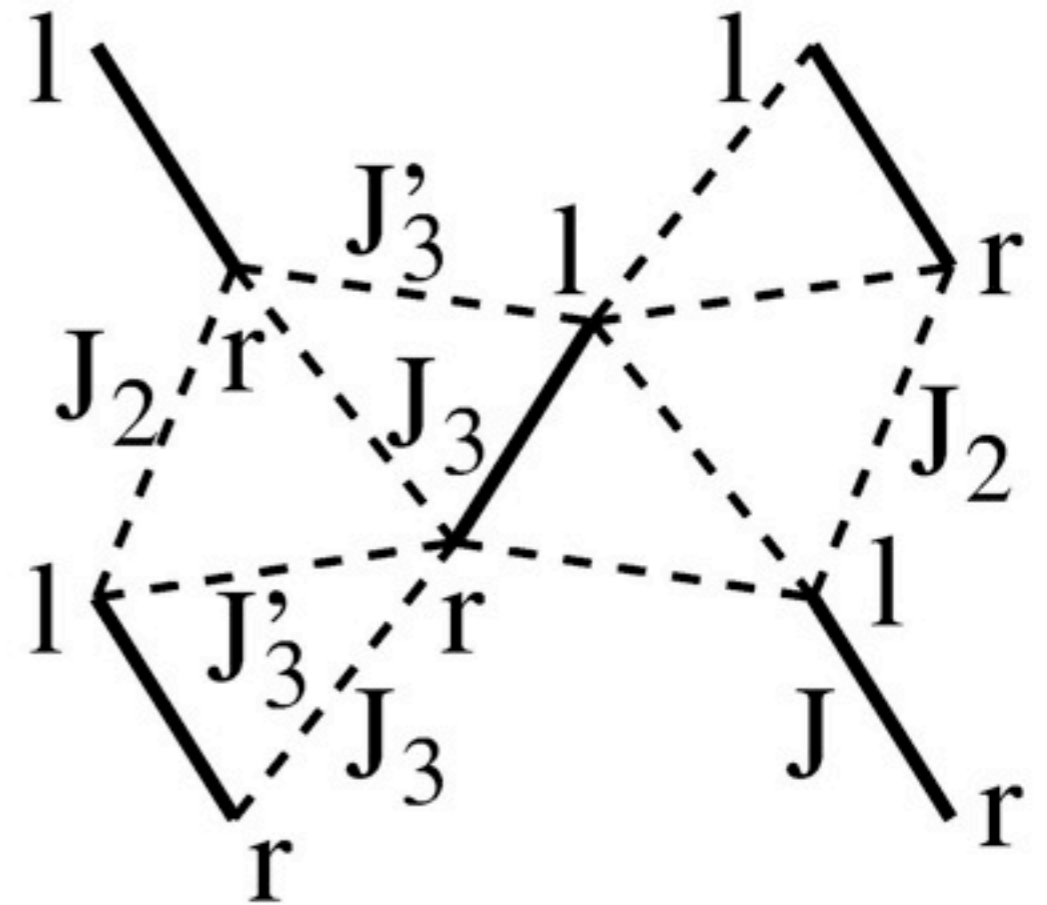
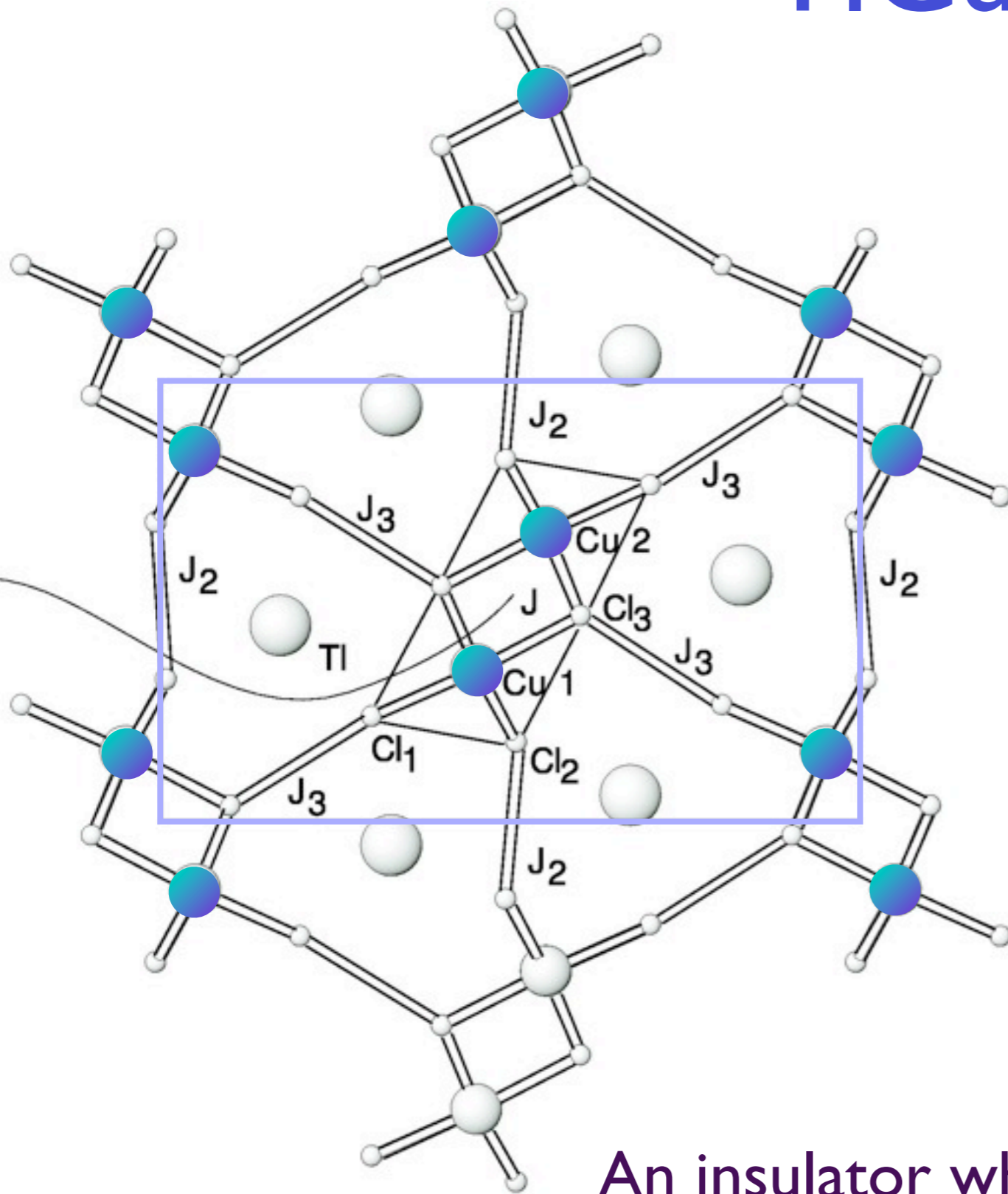
$$\text{Diagram of two blue spheres connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



Pressure in TlCuCl_3

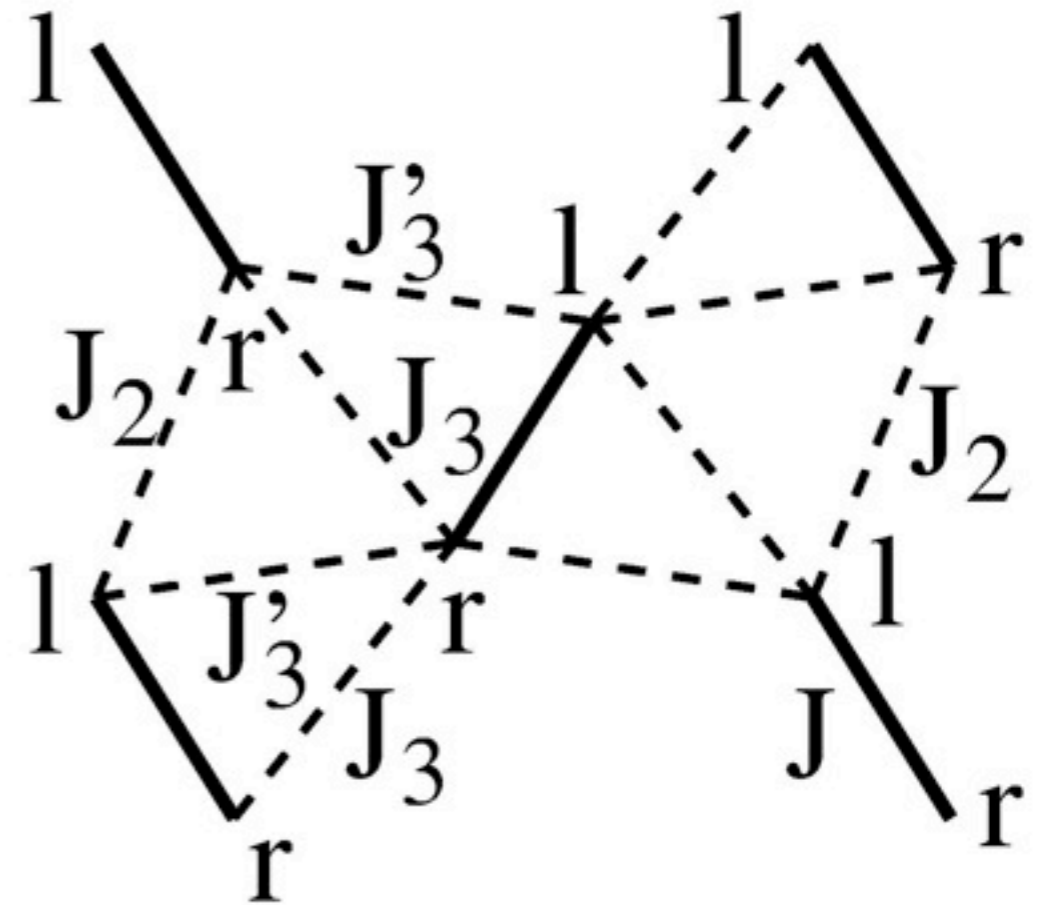
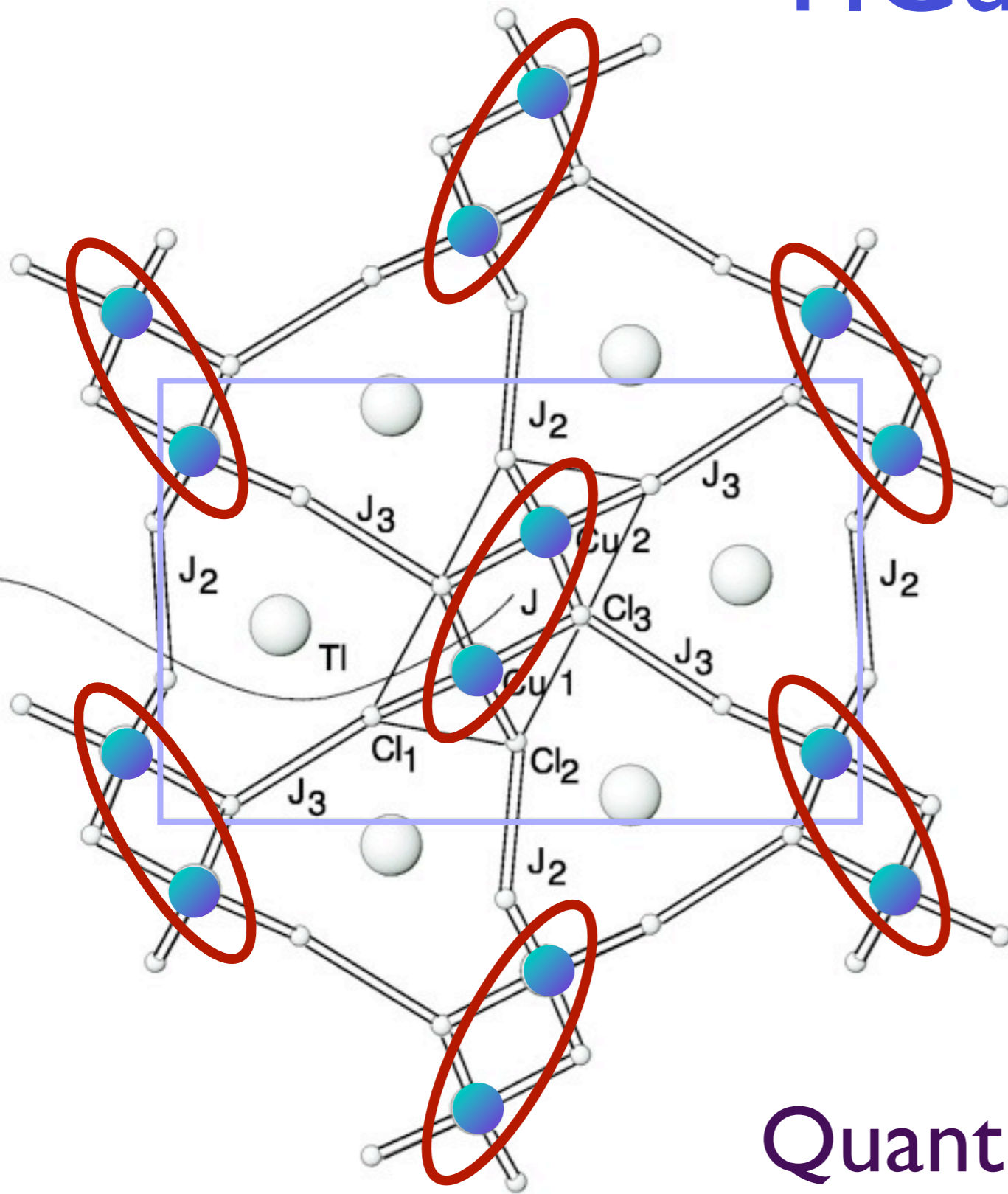
A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka,
Journal of the Physical Society of Japan, **73**, 1446 (2004).

TlCuCl₃



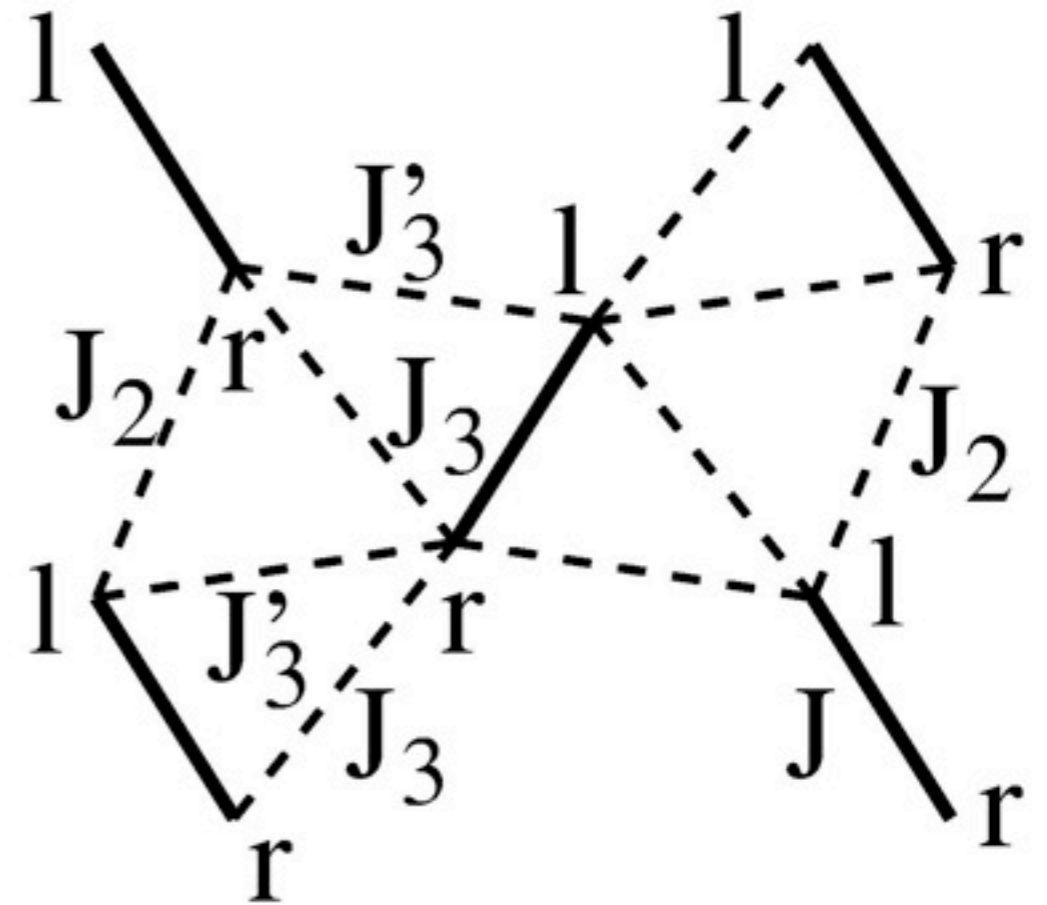
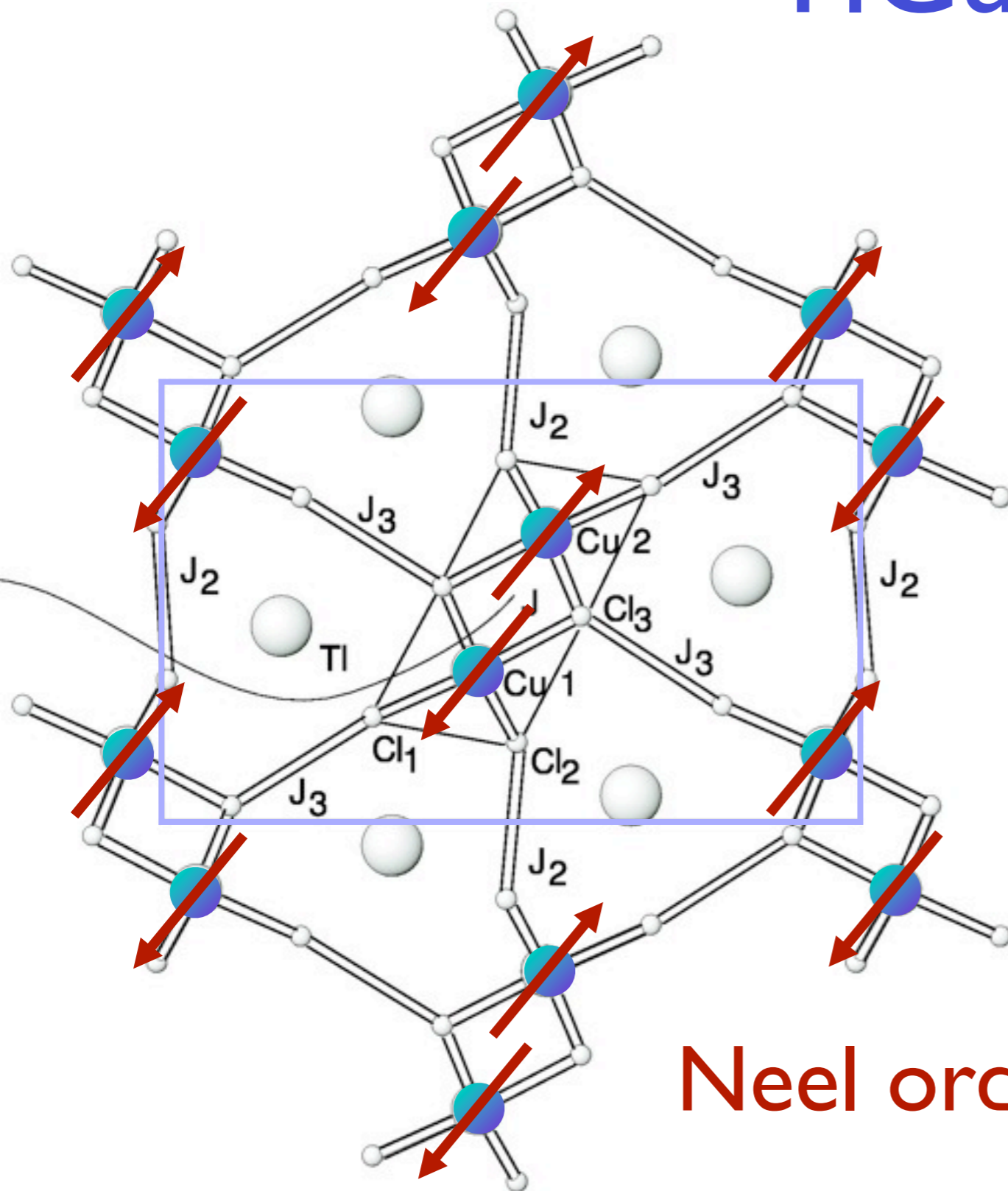
An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

TlCuCl₃



Quantum paramagnet at
ambient pressure

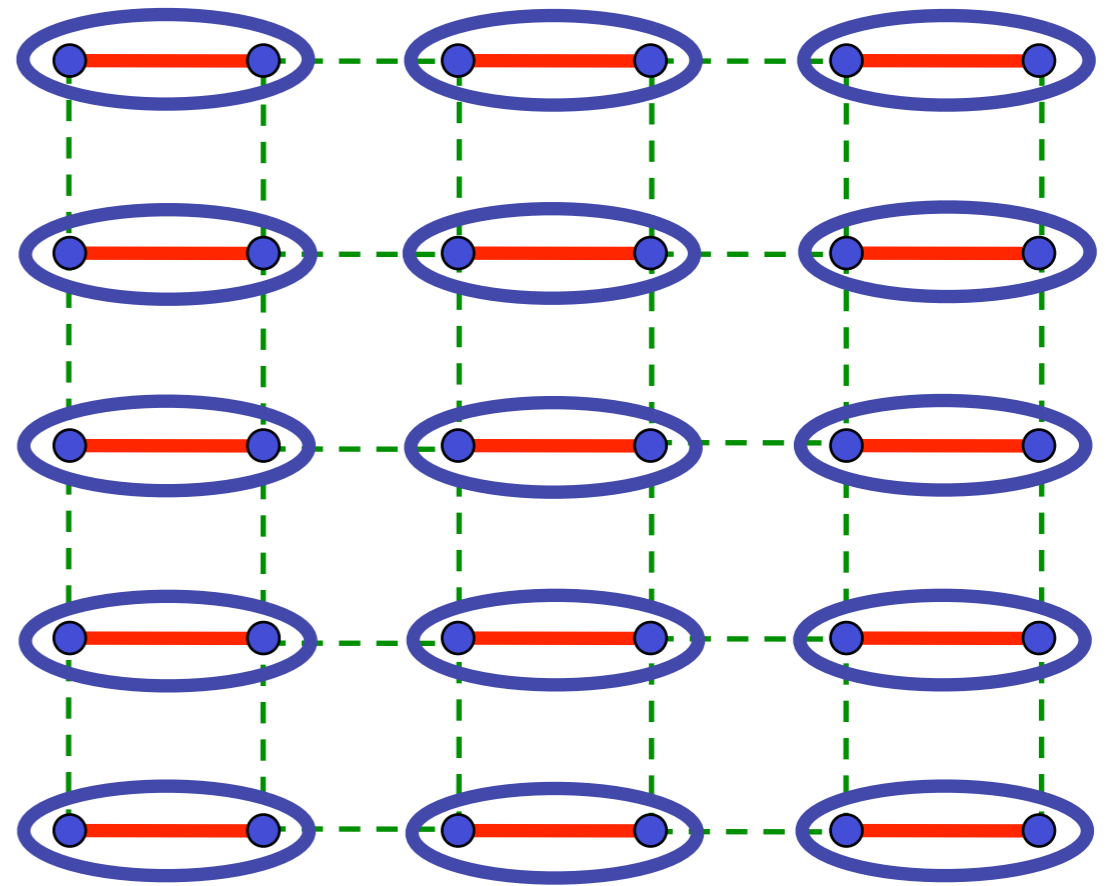
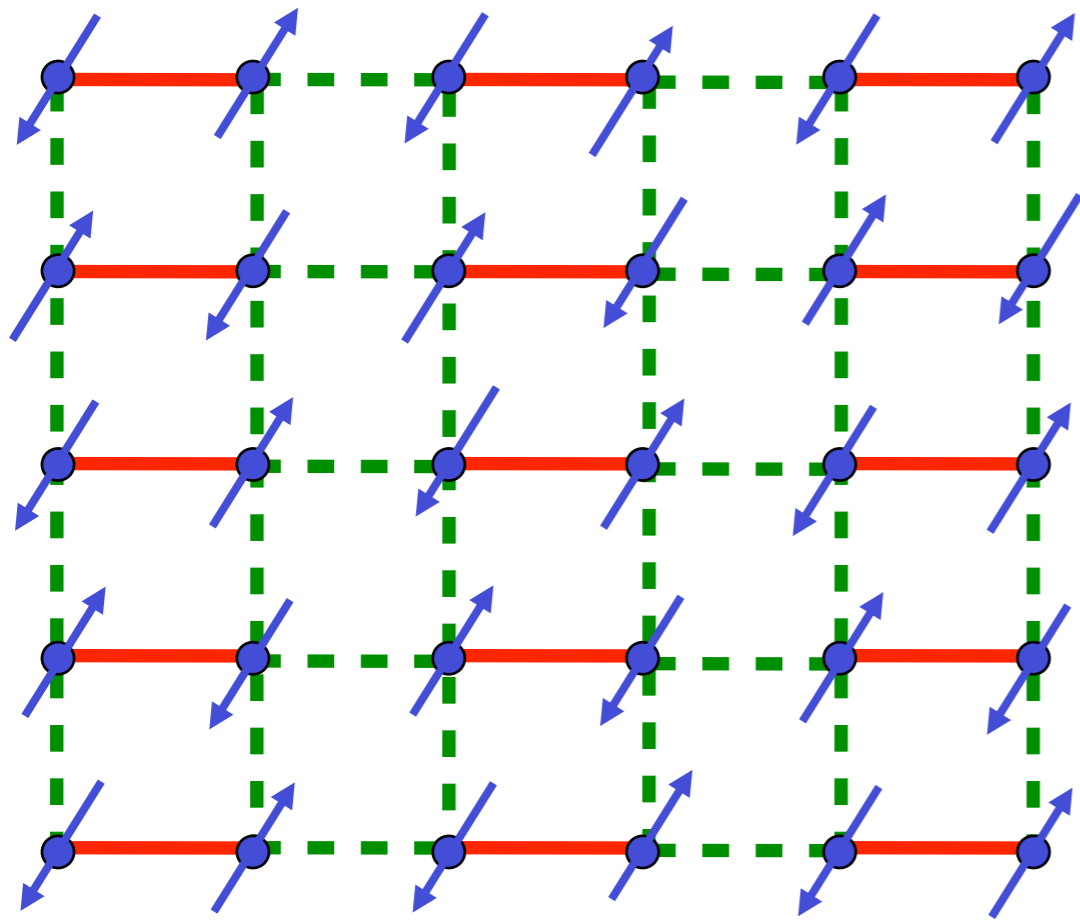
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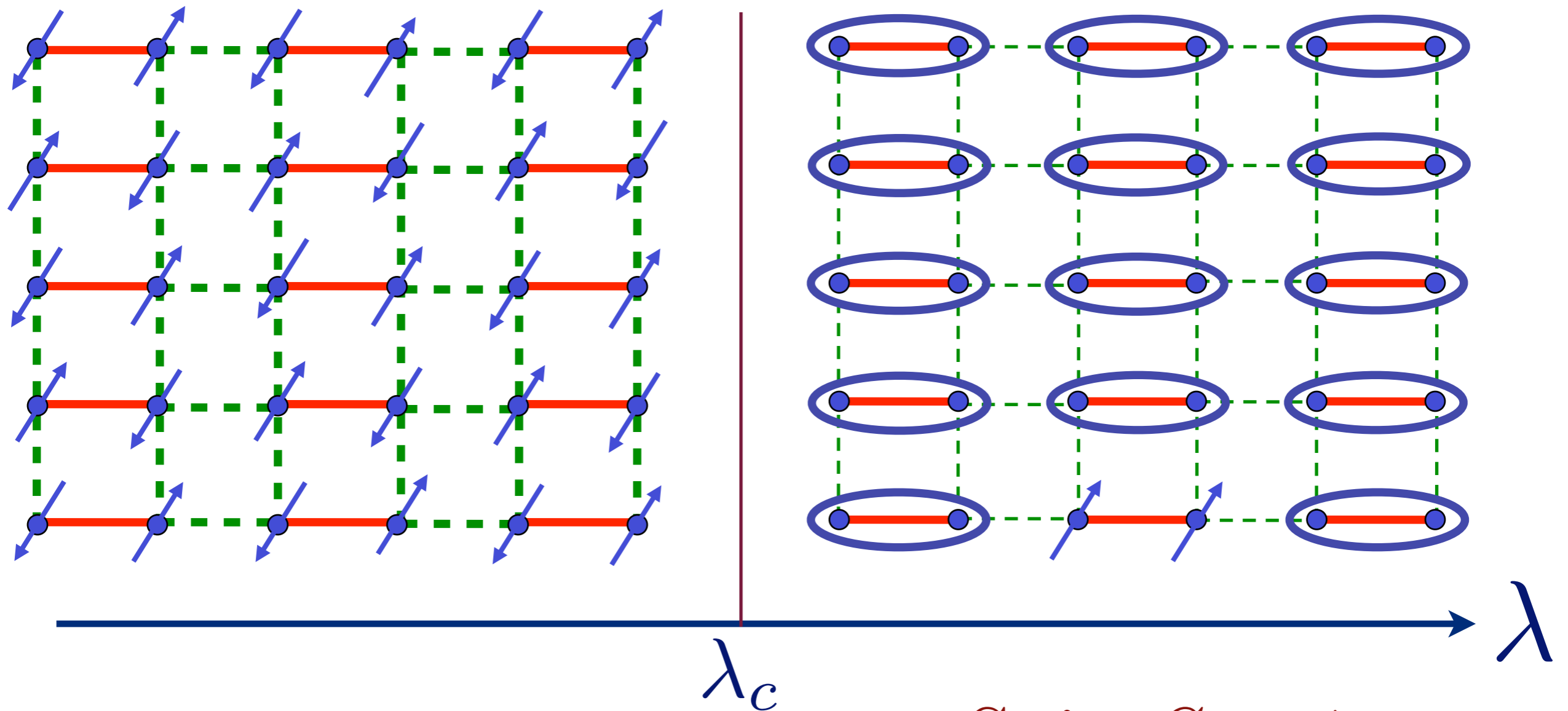
Neel order under pressure

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).

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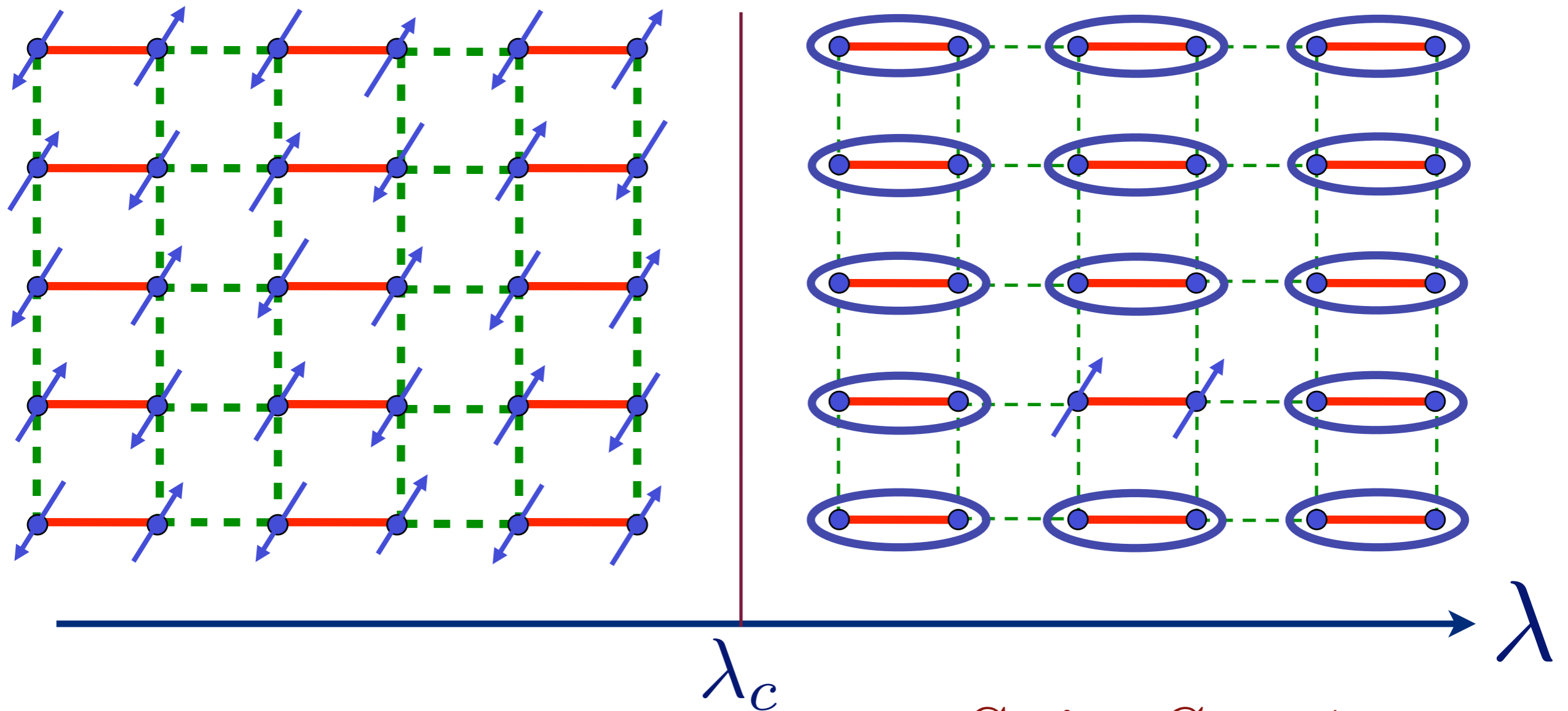


Excitation spectrum in the paramagnetic phase



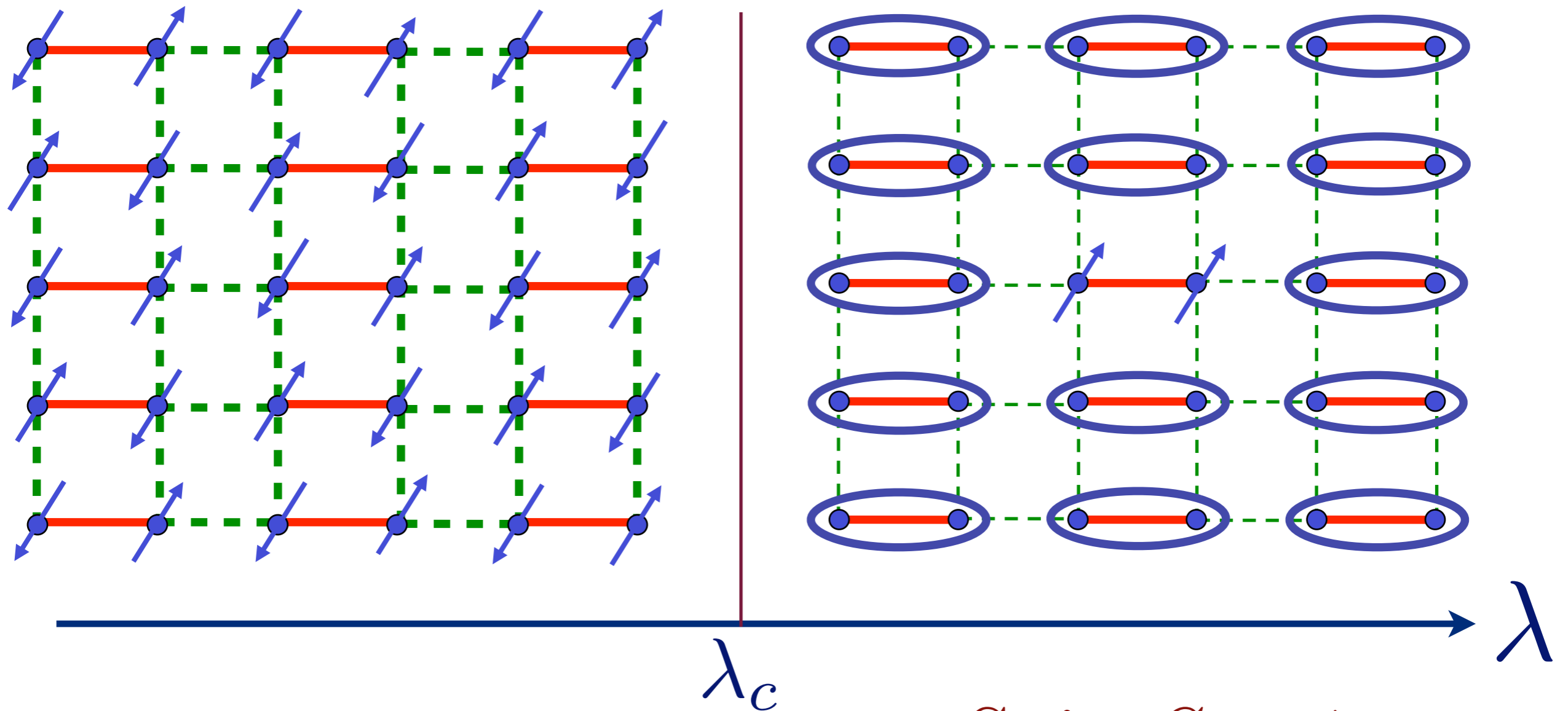
Spin $S = 1$
“triplon”

Excitation spectrum in the paramagnetic phase



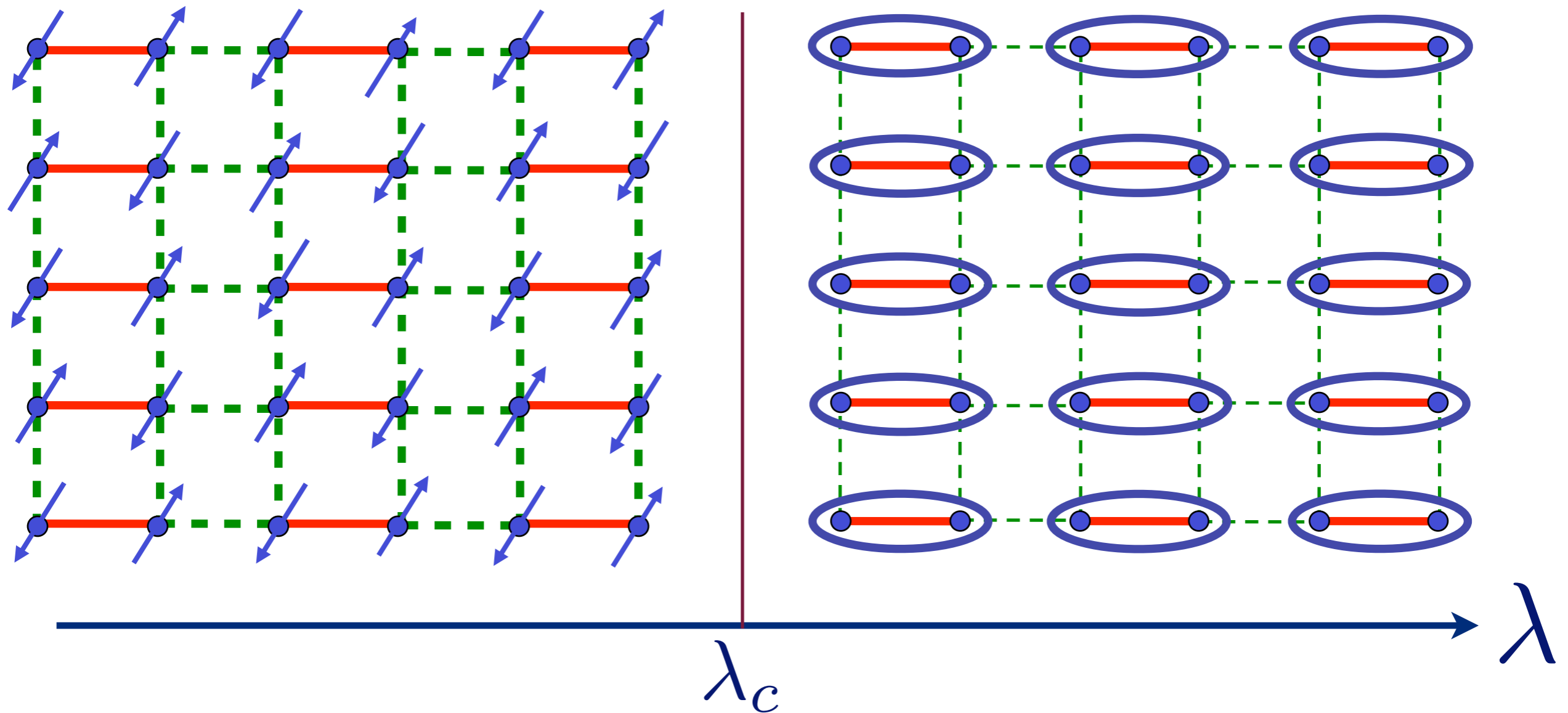
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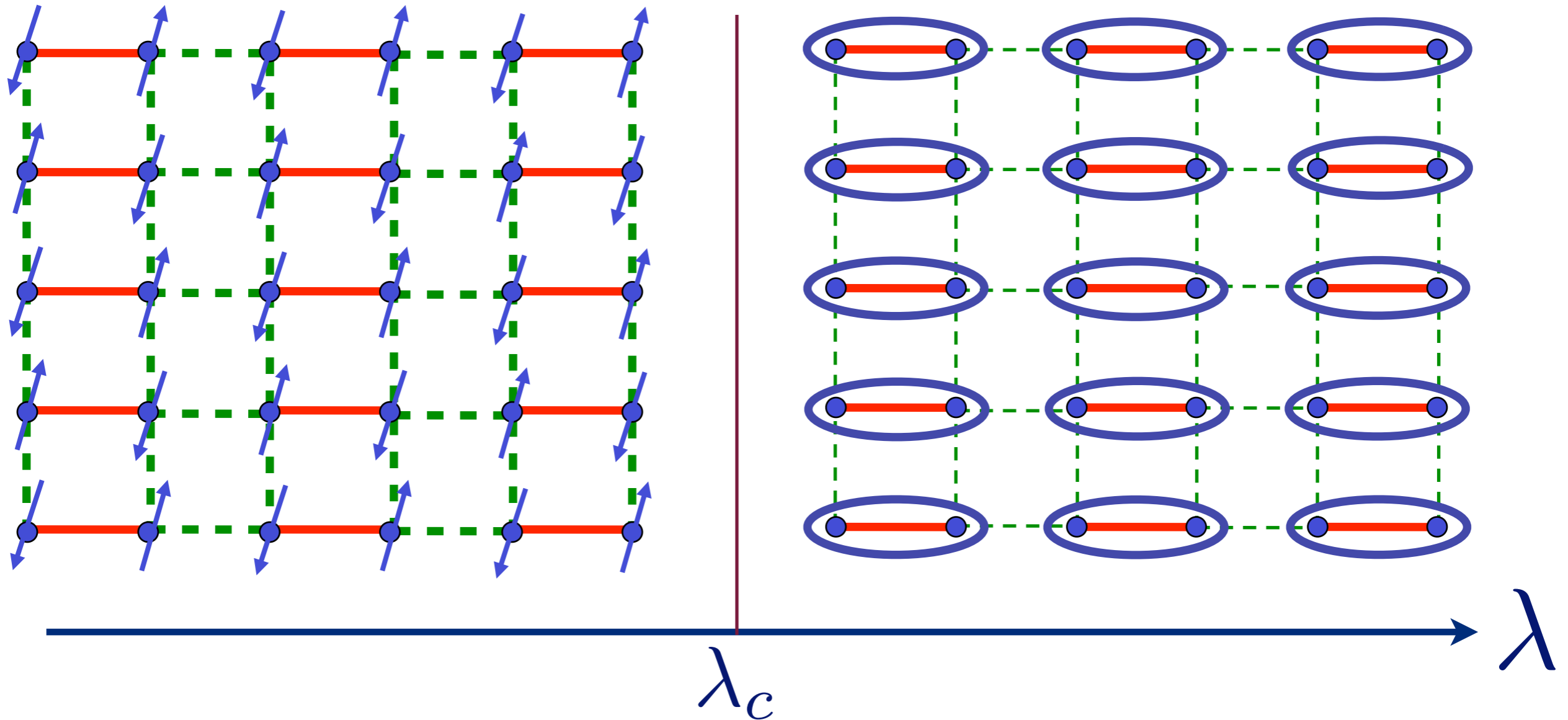
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Excitation spectrum in the Néel phase



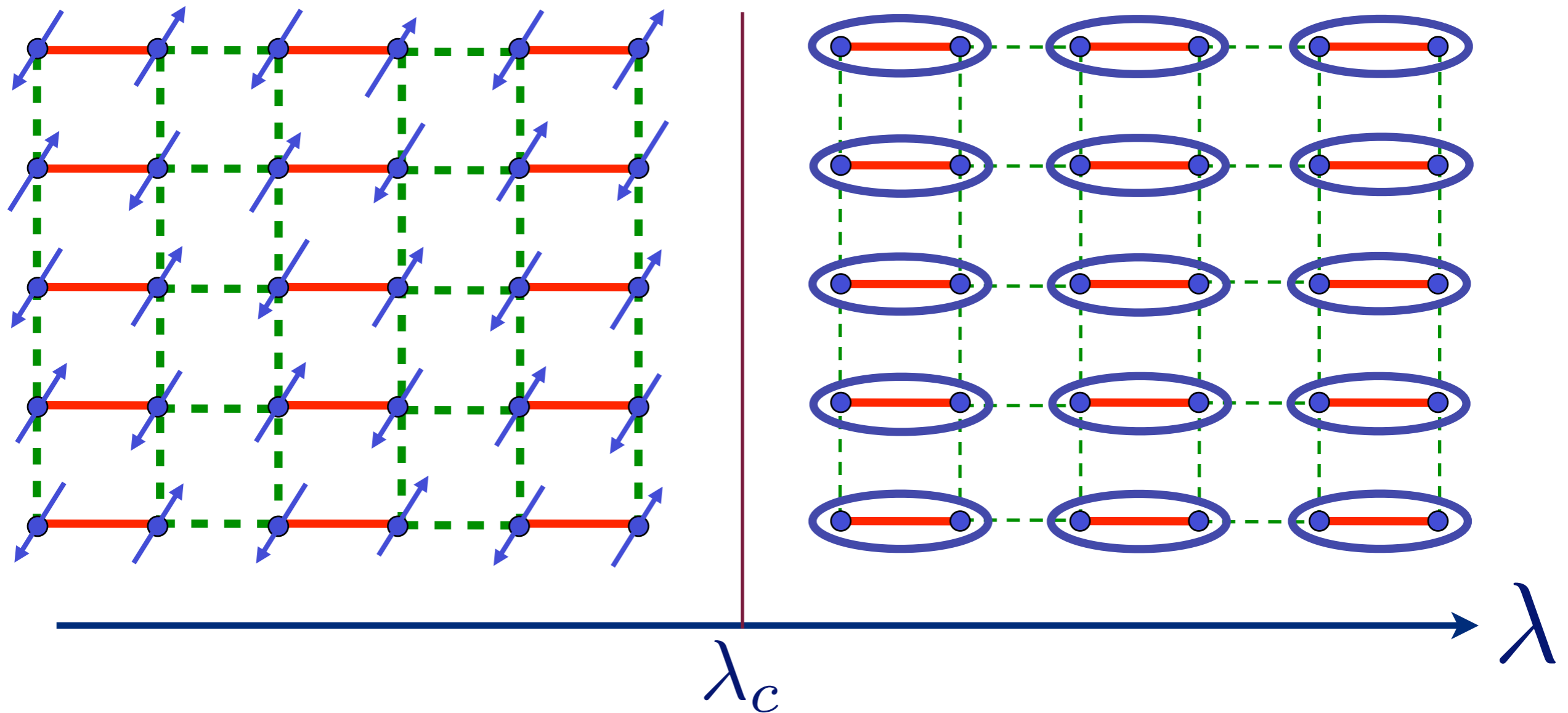
Spin waves

Excitation spectrum in the Néel phase



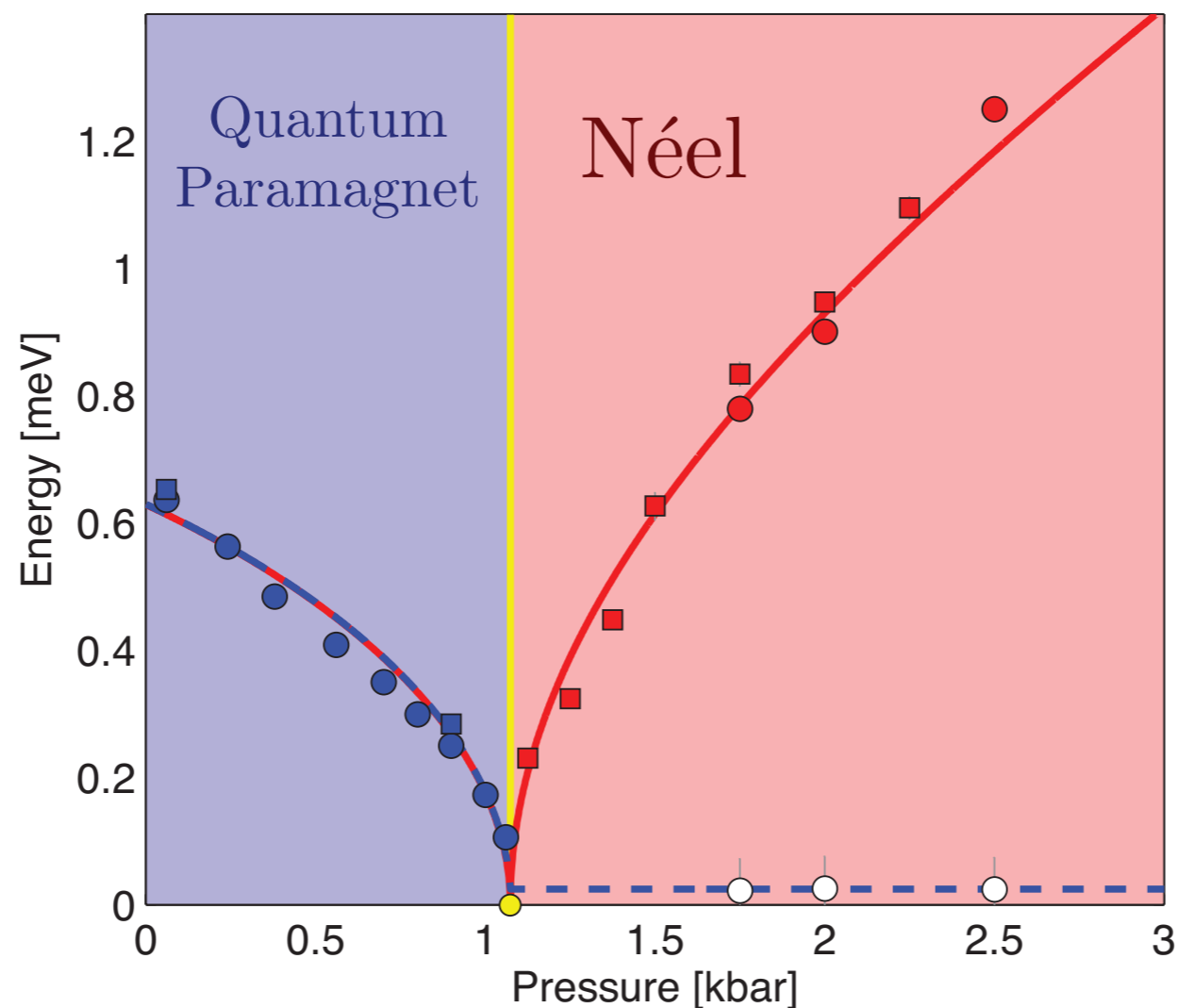
Spin waves

Excitation spectrum in the Néel phase



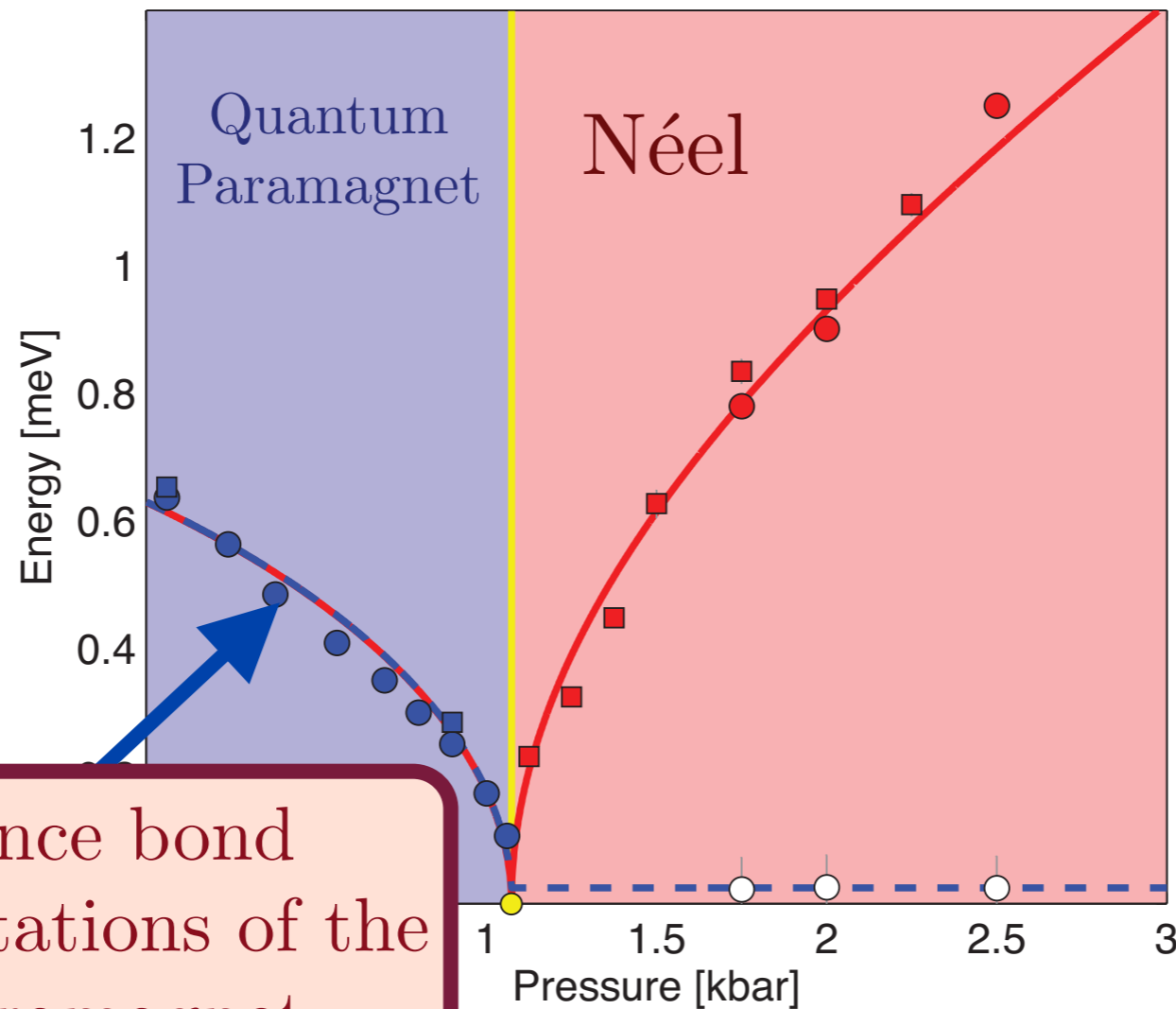
Spin waves

Excitations of TlCuCl_3 with varying pressure



Christian Ruedg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorro, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

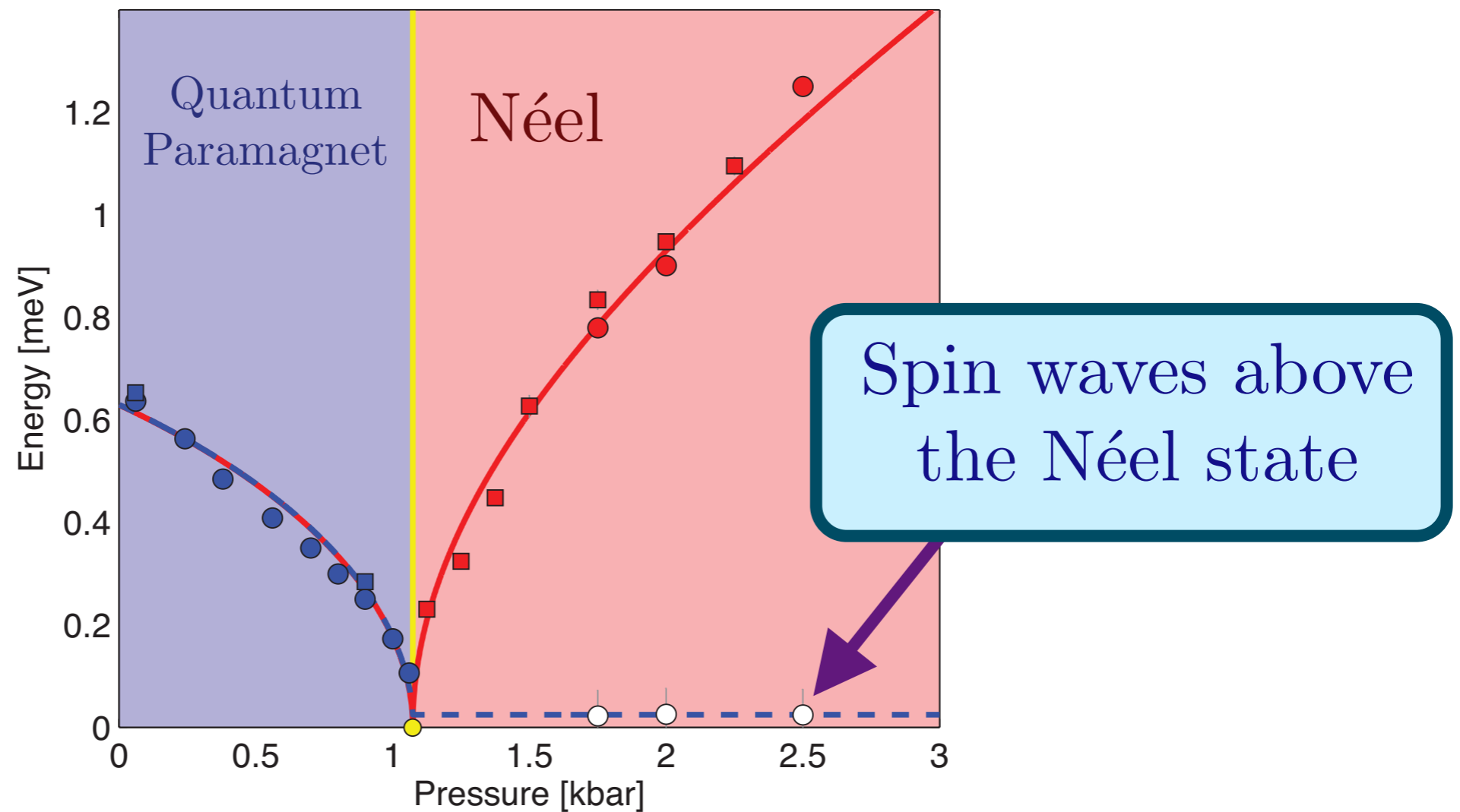
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Broken valence bond (“triplon”) excitations of the quantum paramagnet

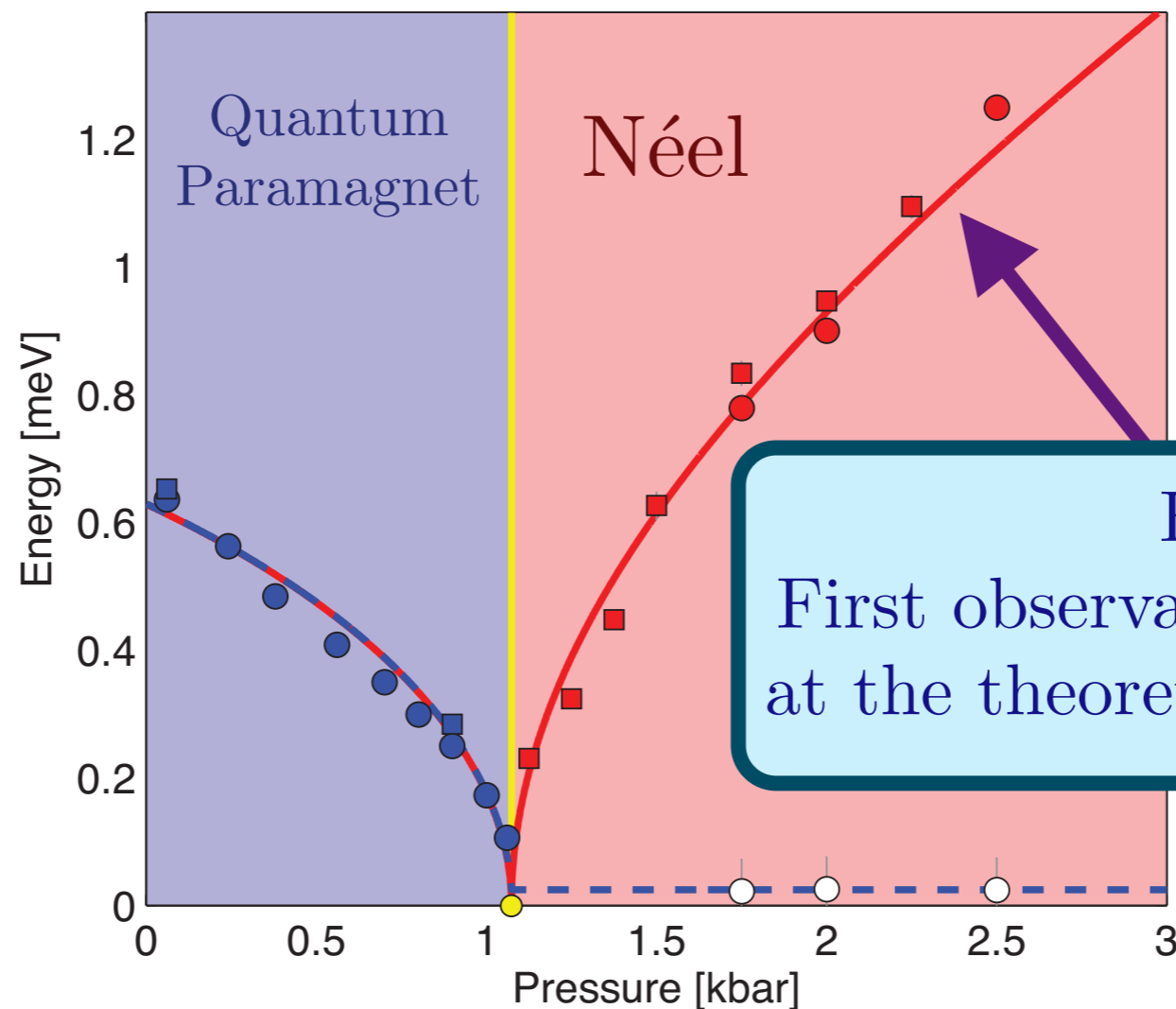
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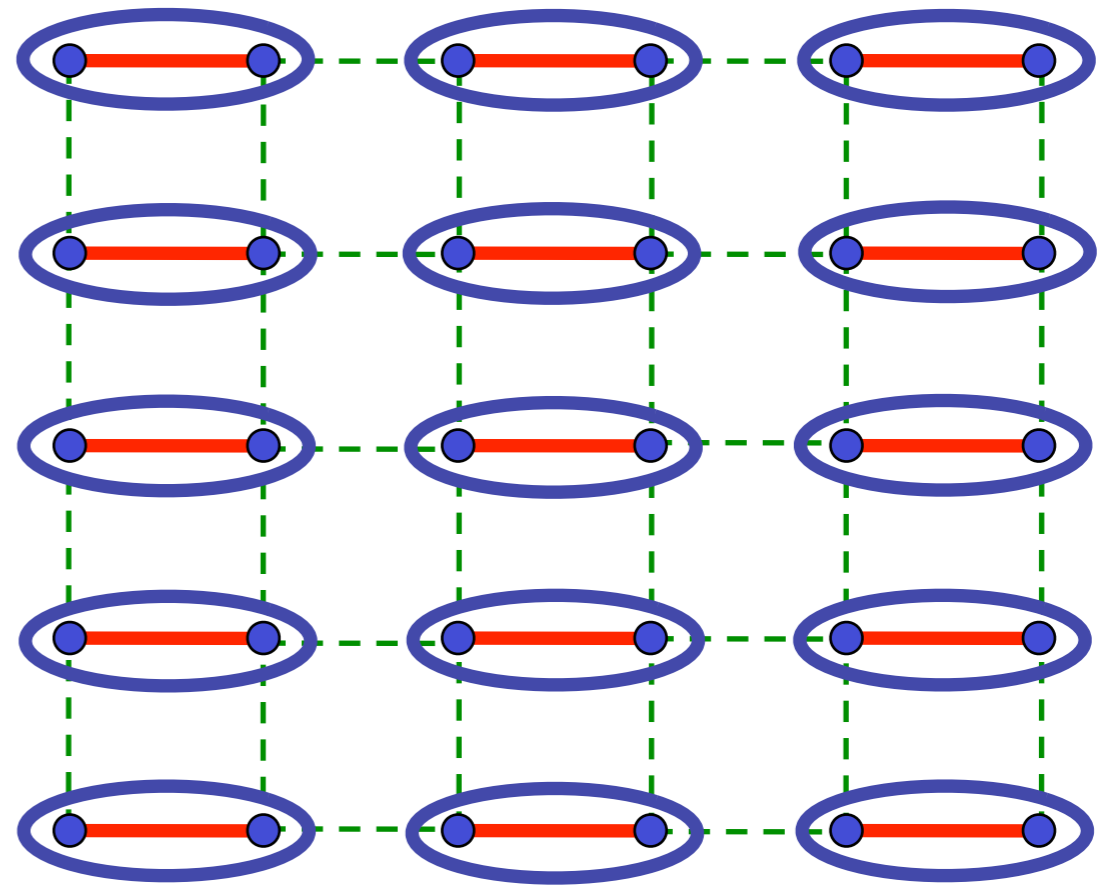
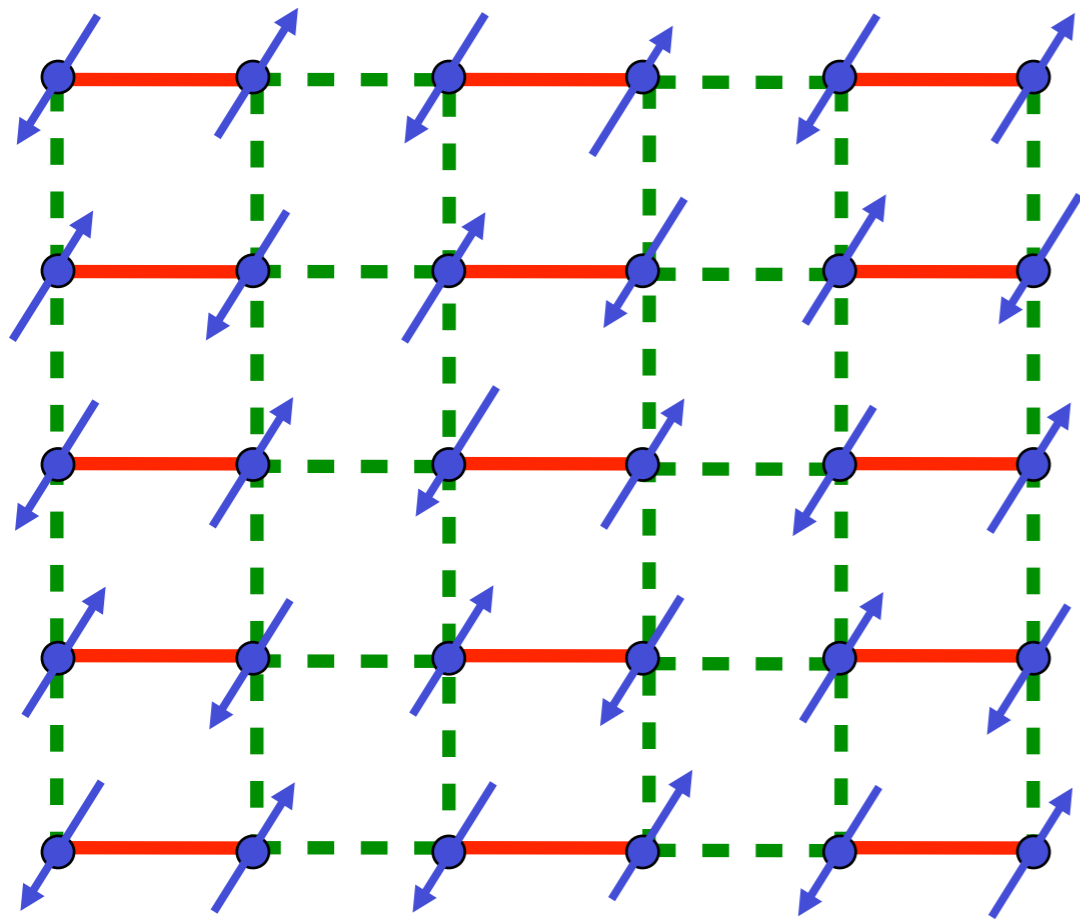
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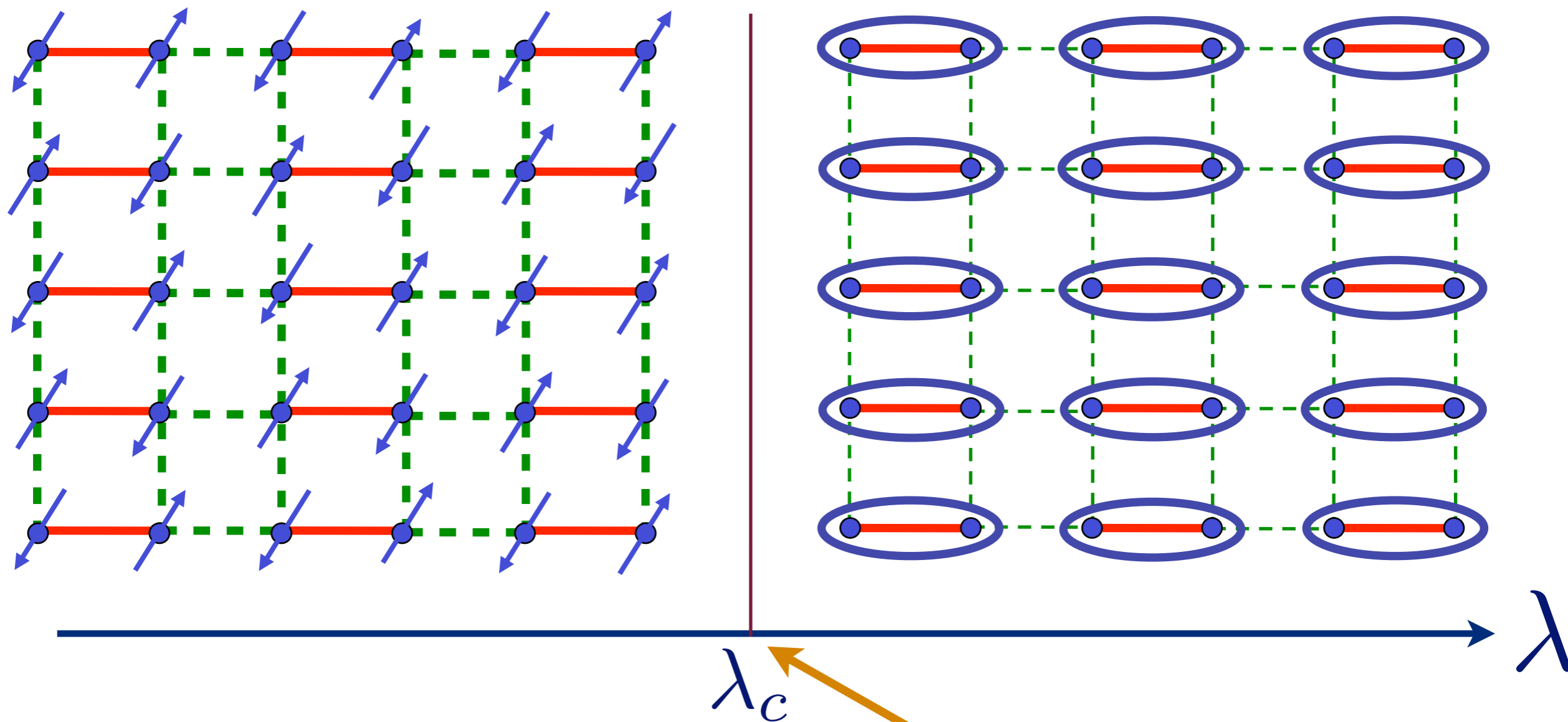
S. Sachdev,
arXiv:0901.4103

Christian Rüegg, Bruce Normand, Masahige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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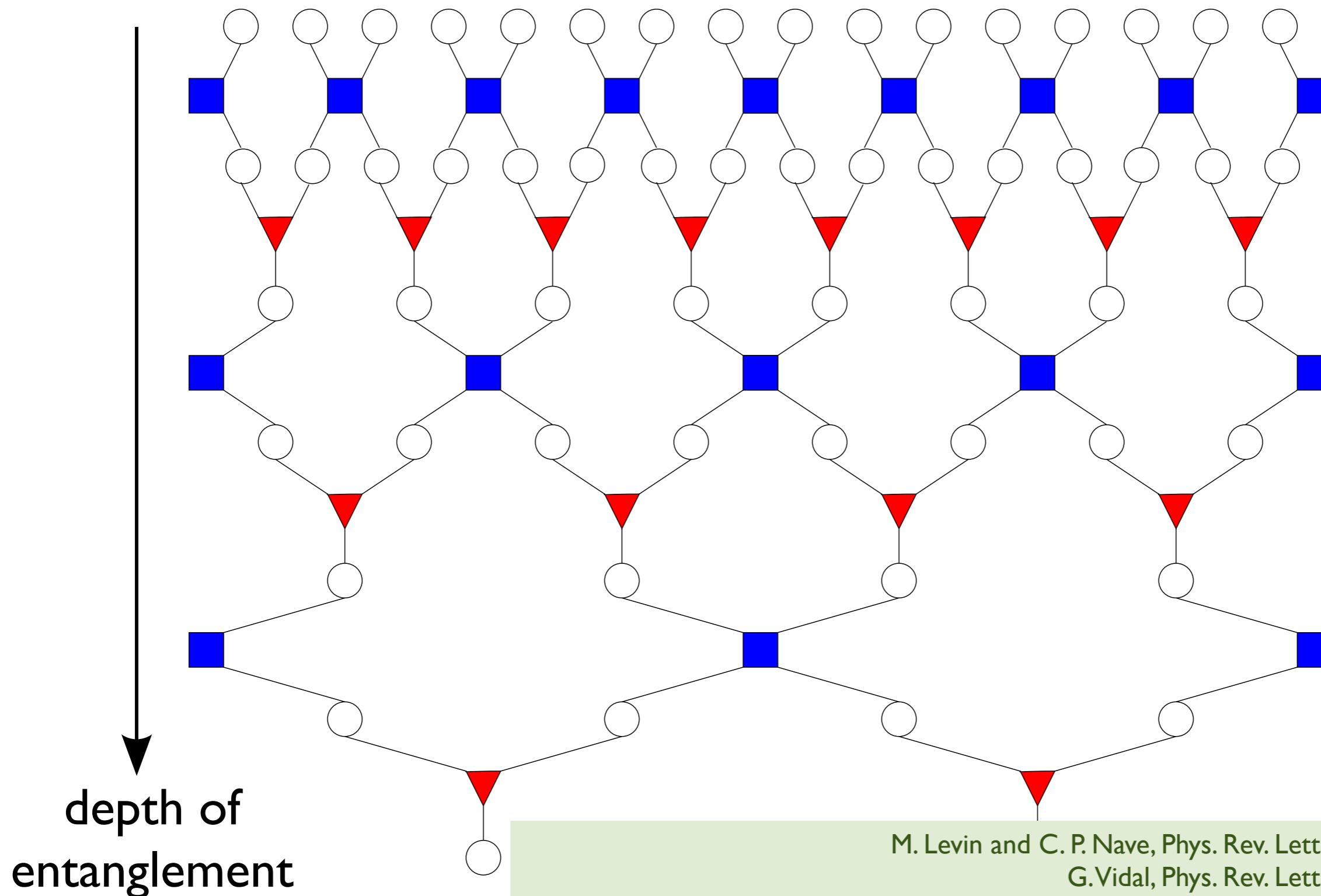
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Quantum critical point with non-local entanglement in spin wavefunction

Tensor network representation of entanglement at quantum critical point

D -dimensional
space



M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)

Characteristics of quantum critical point

- Long-range entanglement

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- The low energy excitations are described by a theory which has the same structure as Einstein's theory of special relativity, but with the spin-wave velocity playing the role of the velocity of light.

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String theory

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points of electrons
in crystals**

Black holes

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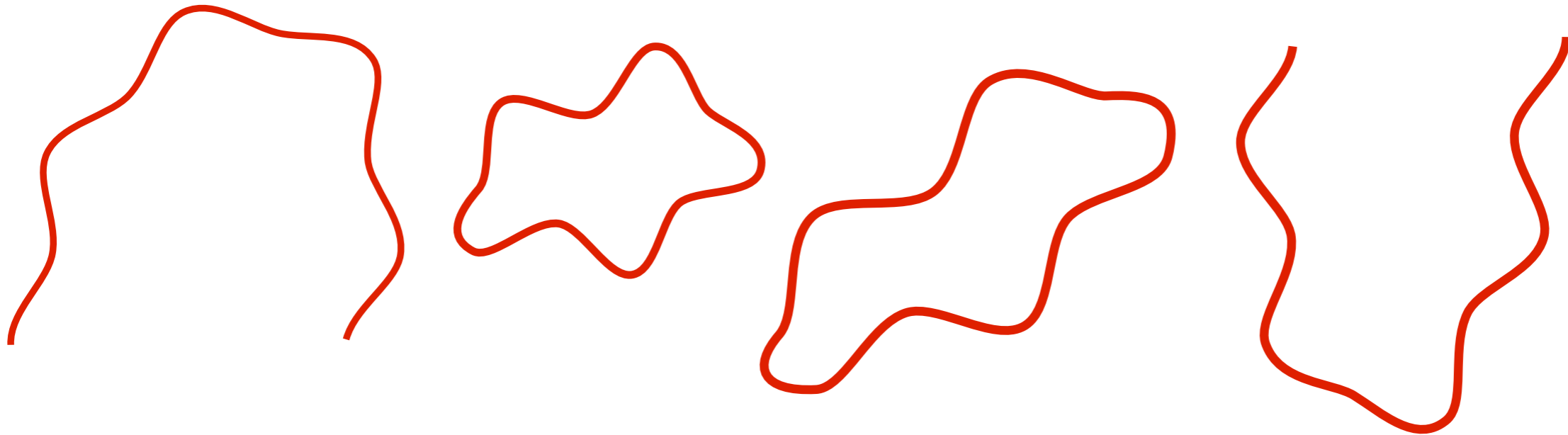
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String theory

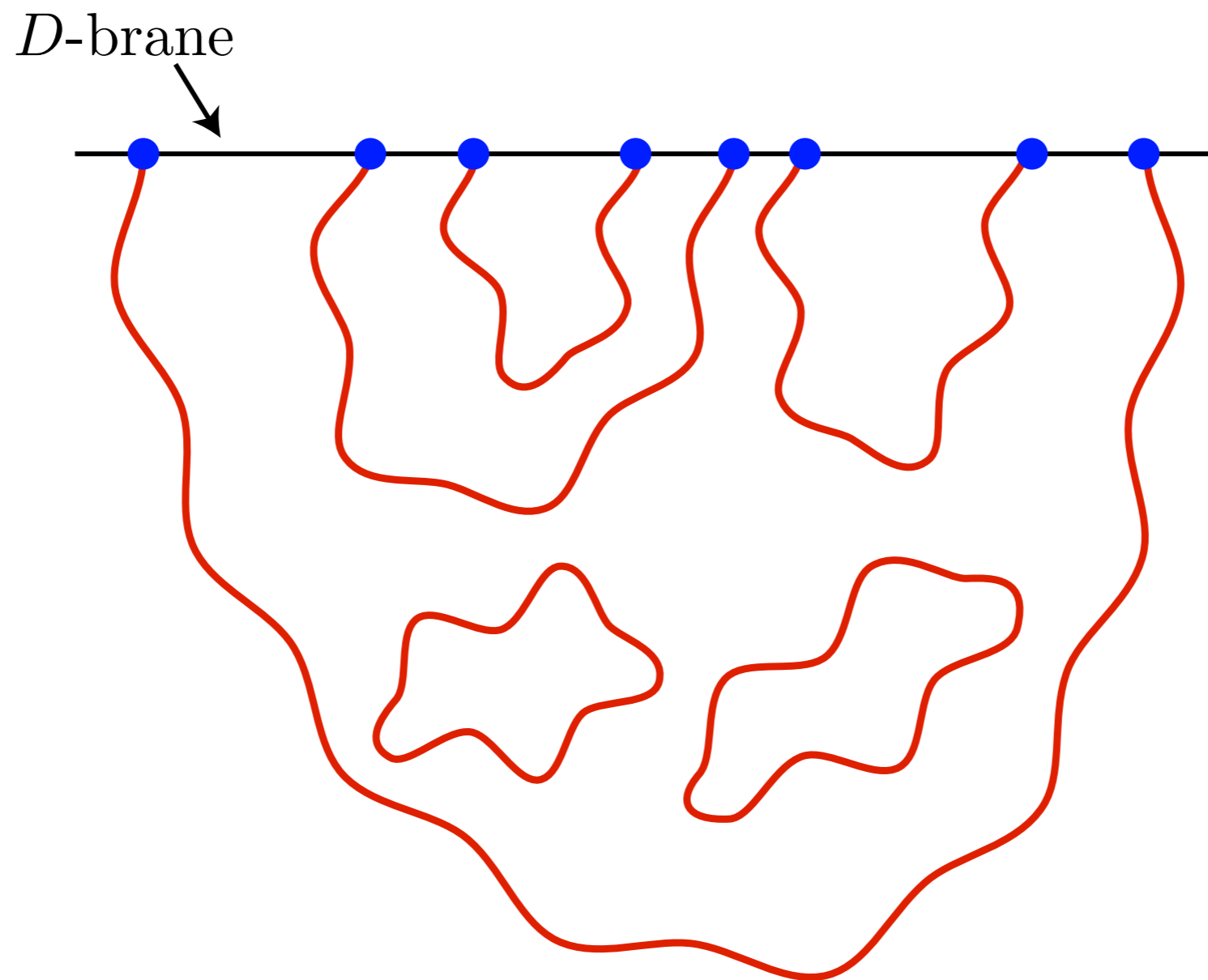
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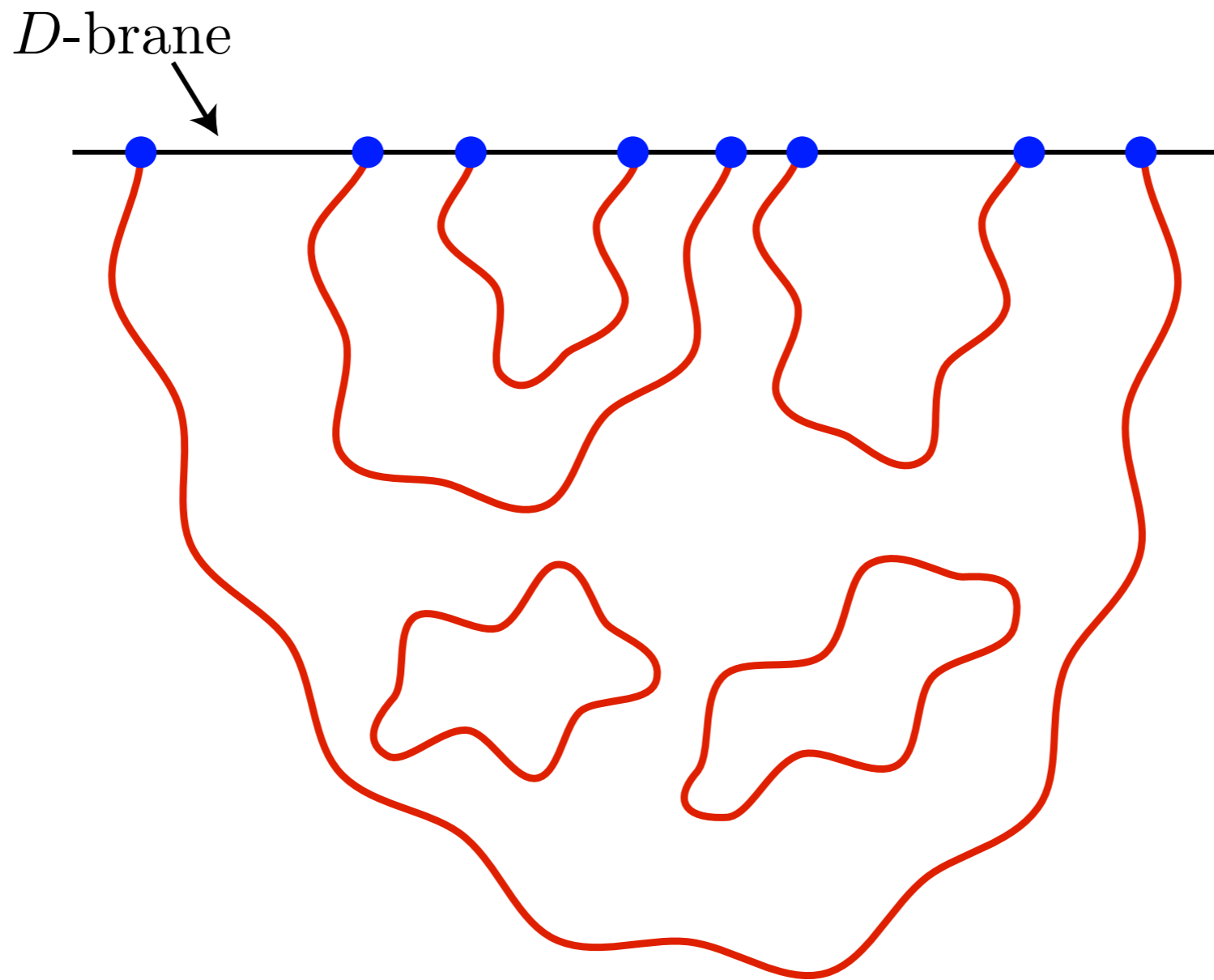
String theory



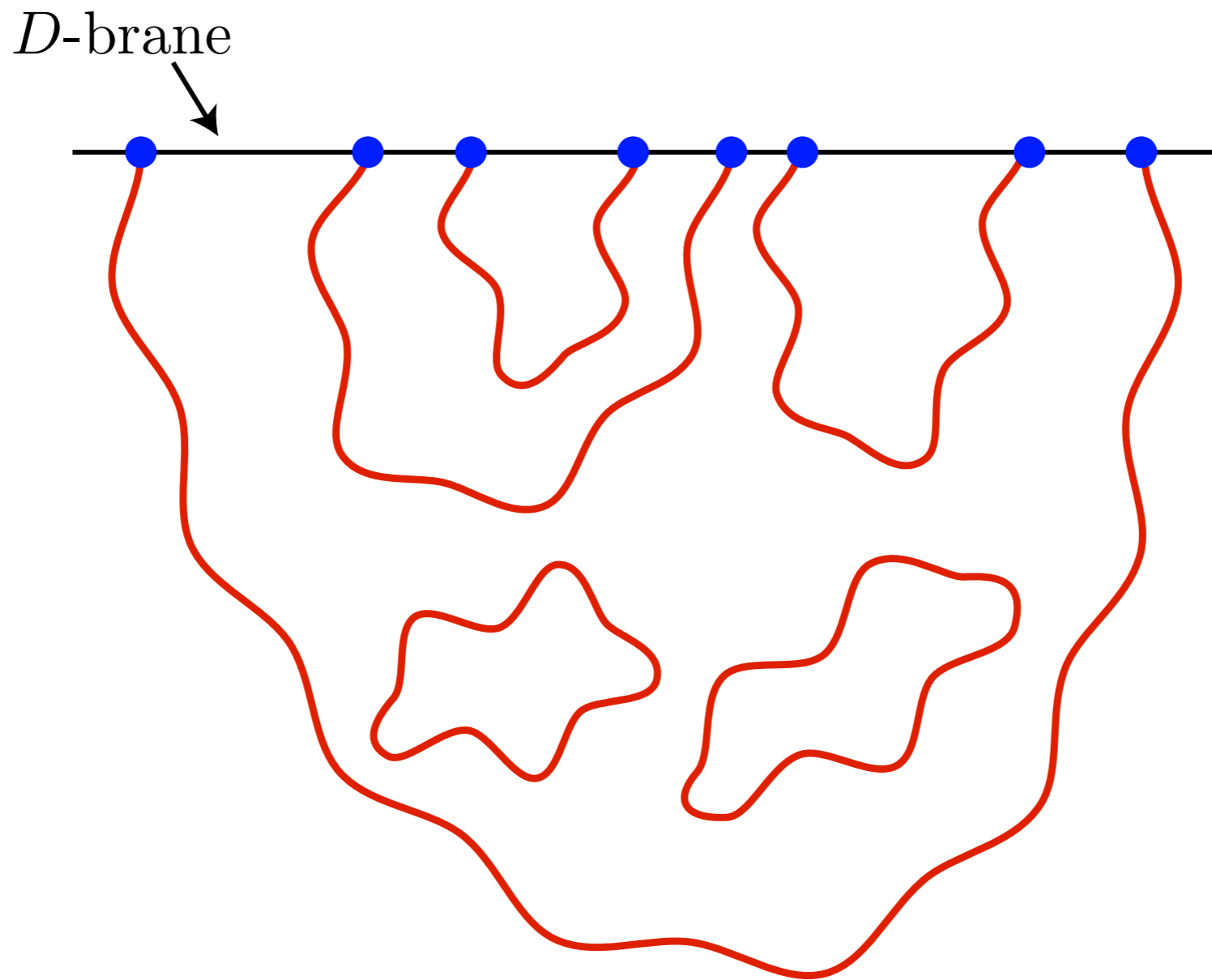
- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks ...



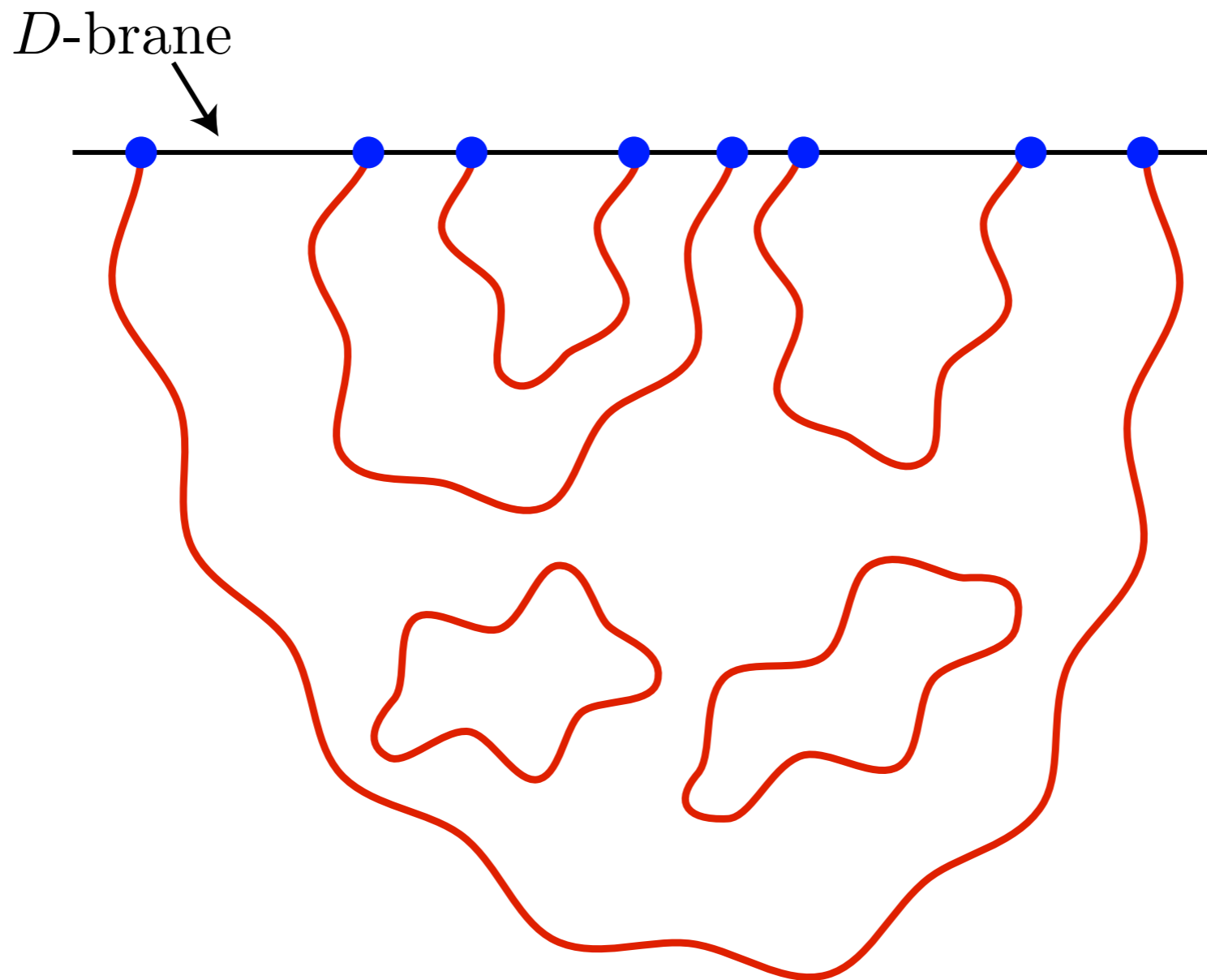
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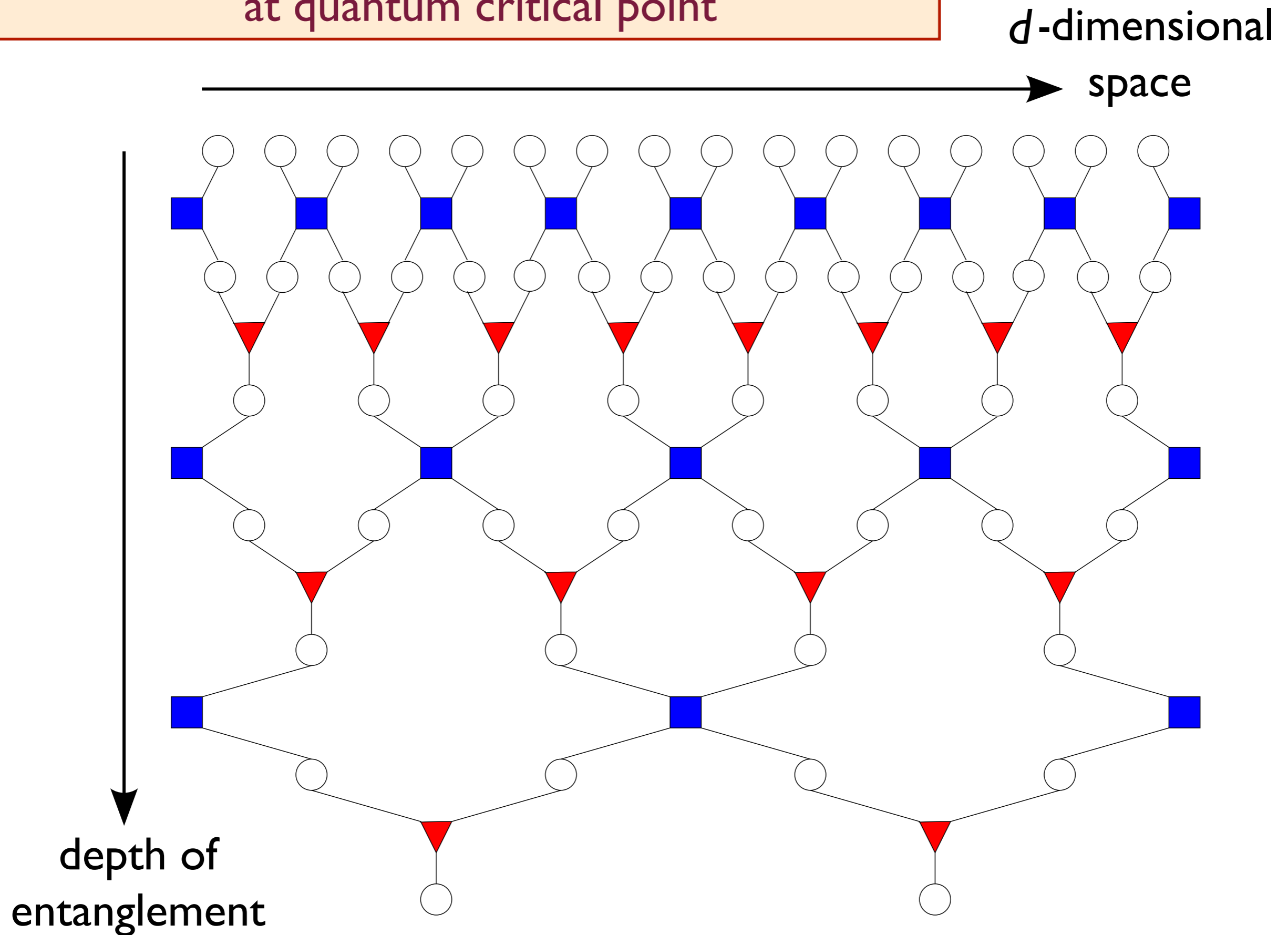


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- In $d = 2$, we obtain strongly-interacting **CFT3s**. These are “dual” to string theory on anti-de Sitter space: **AdS4**.



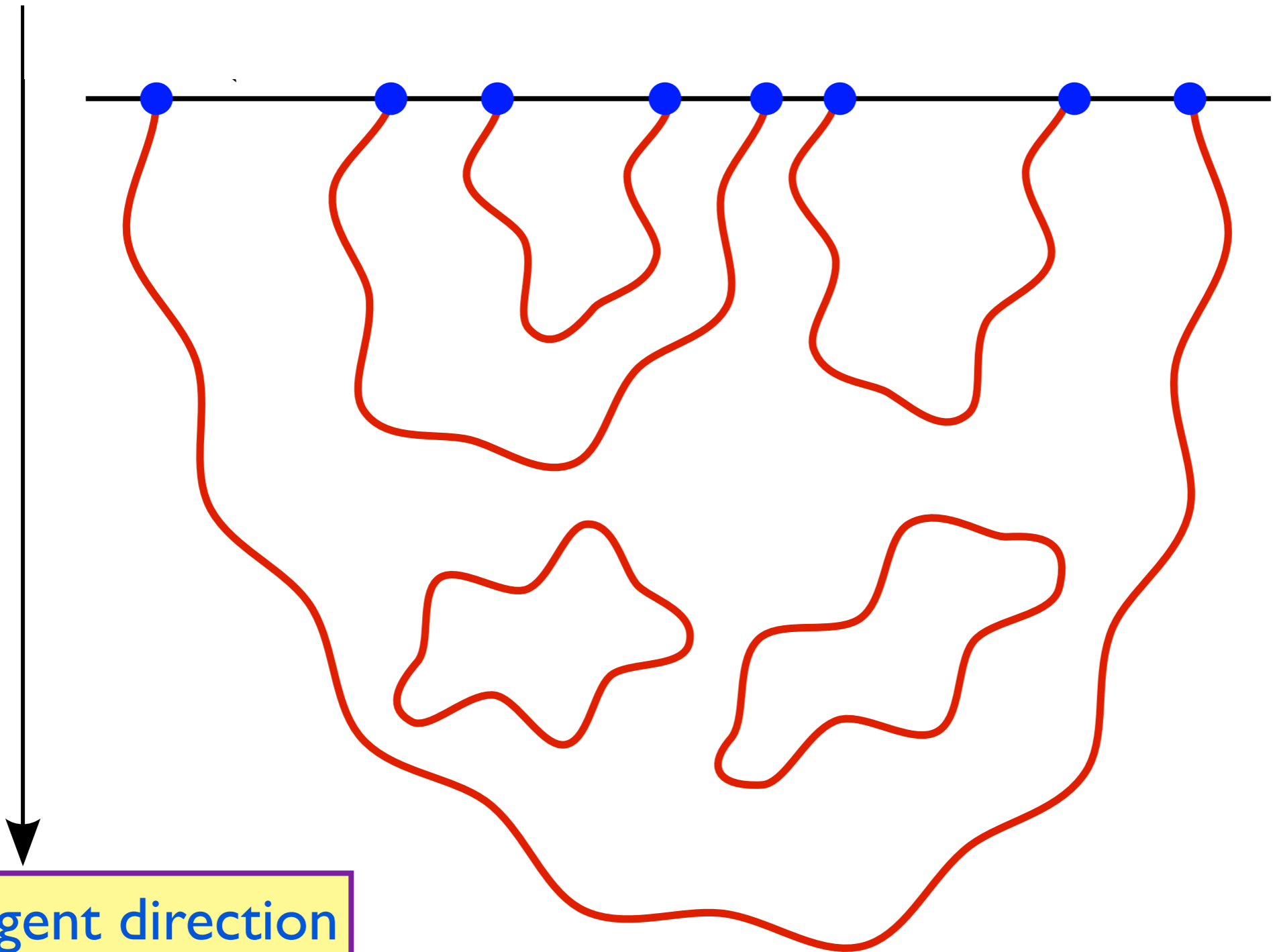
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Tensor network representation of entanglement at quantum critical point



String theory near
a D-brane

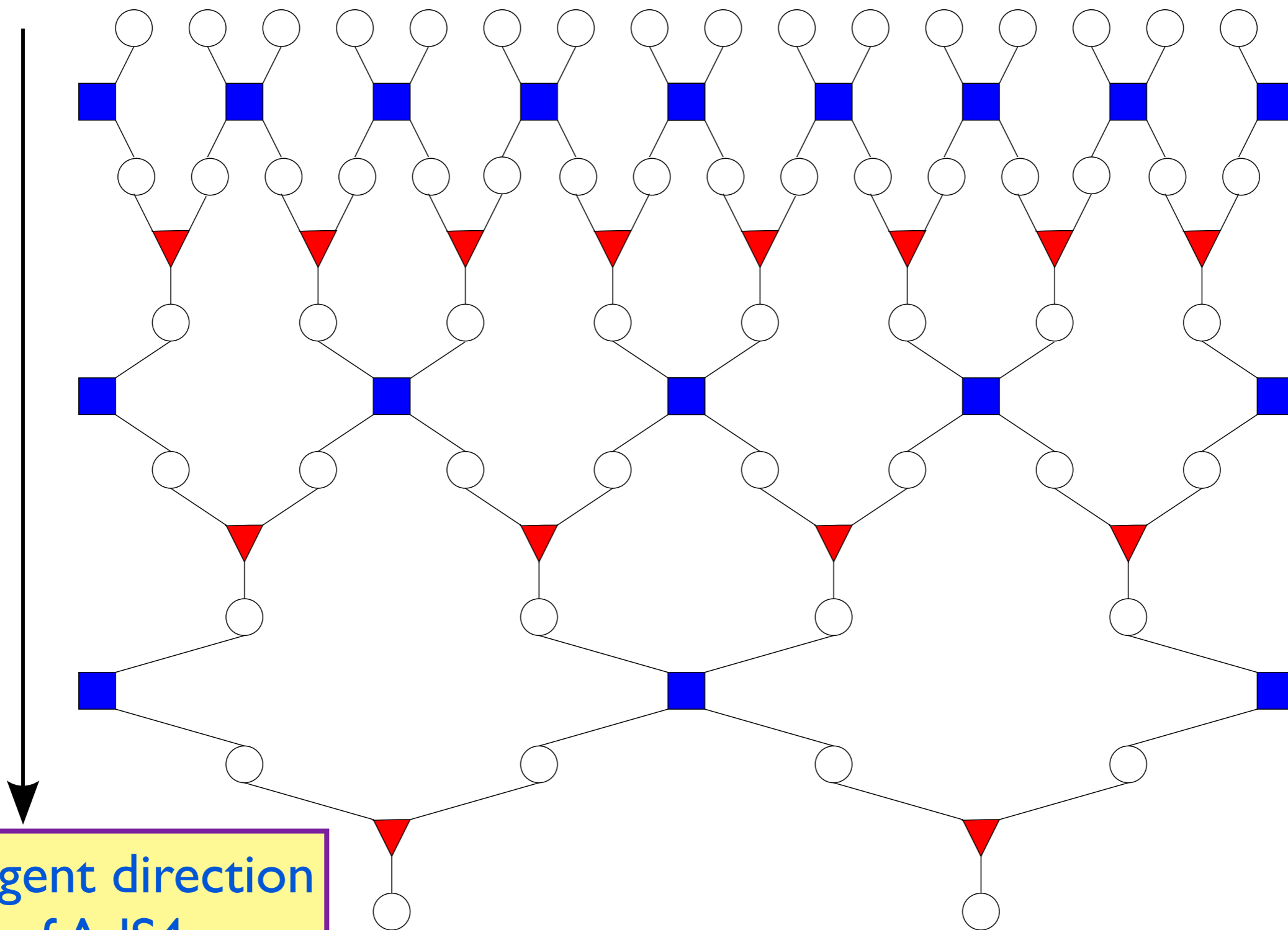
d -dimensional
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Emergent direction
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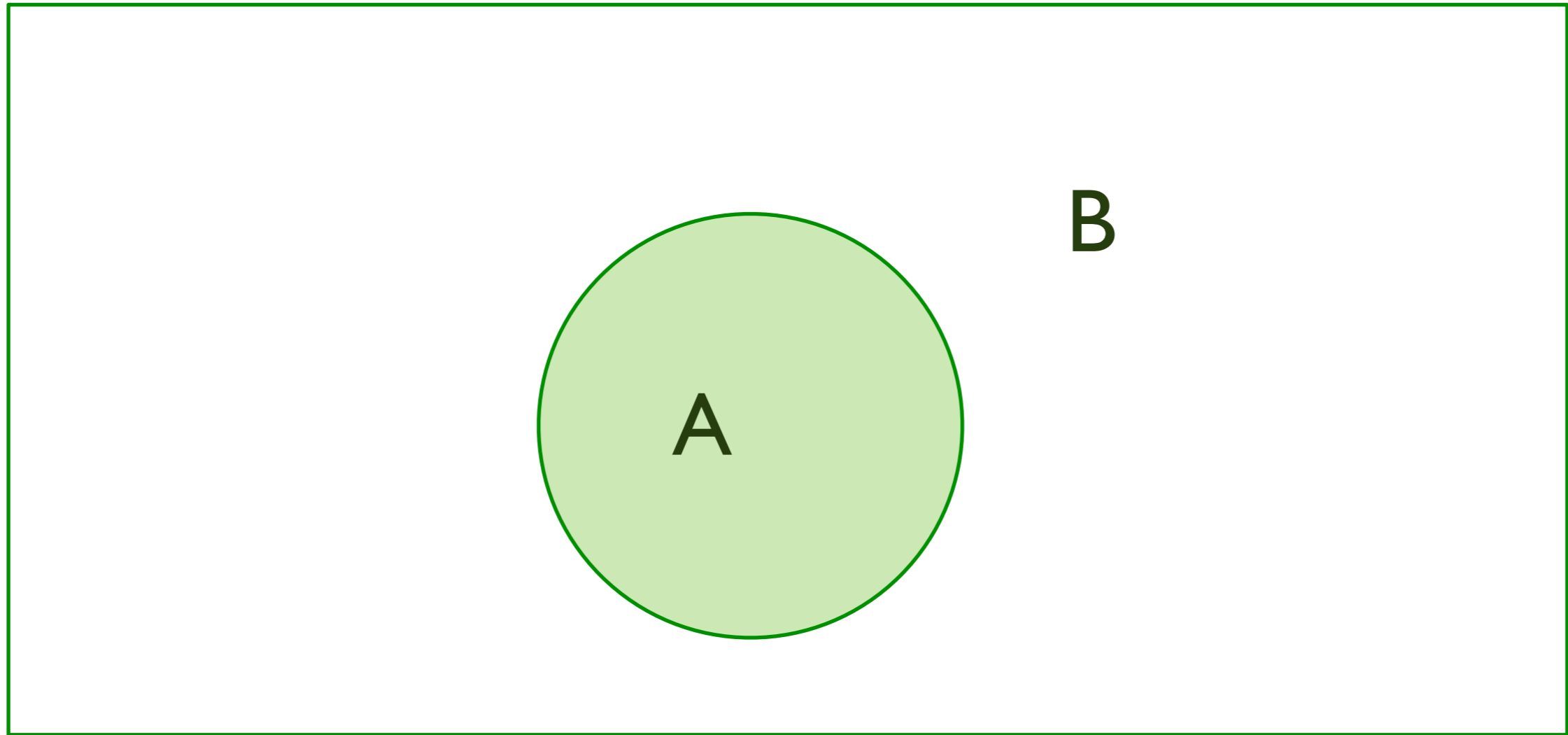
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Emergent direction
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Brian Swingle, arXiv:0905.1317

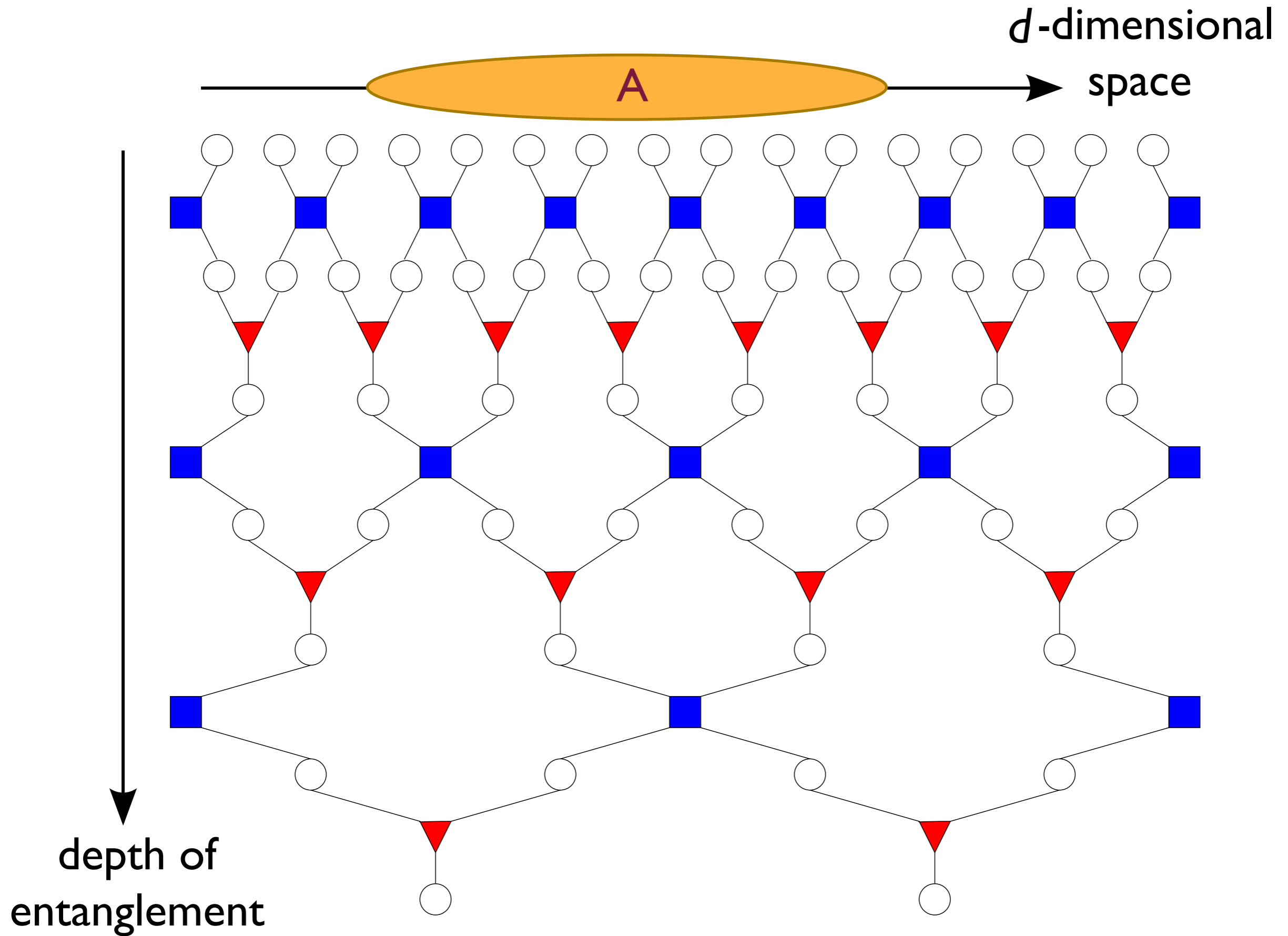
Entanglement entropy



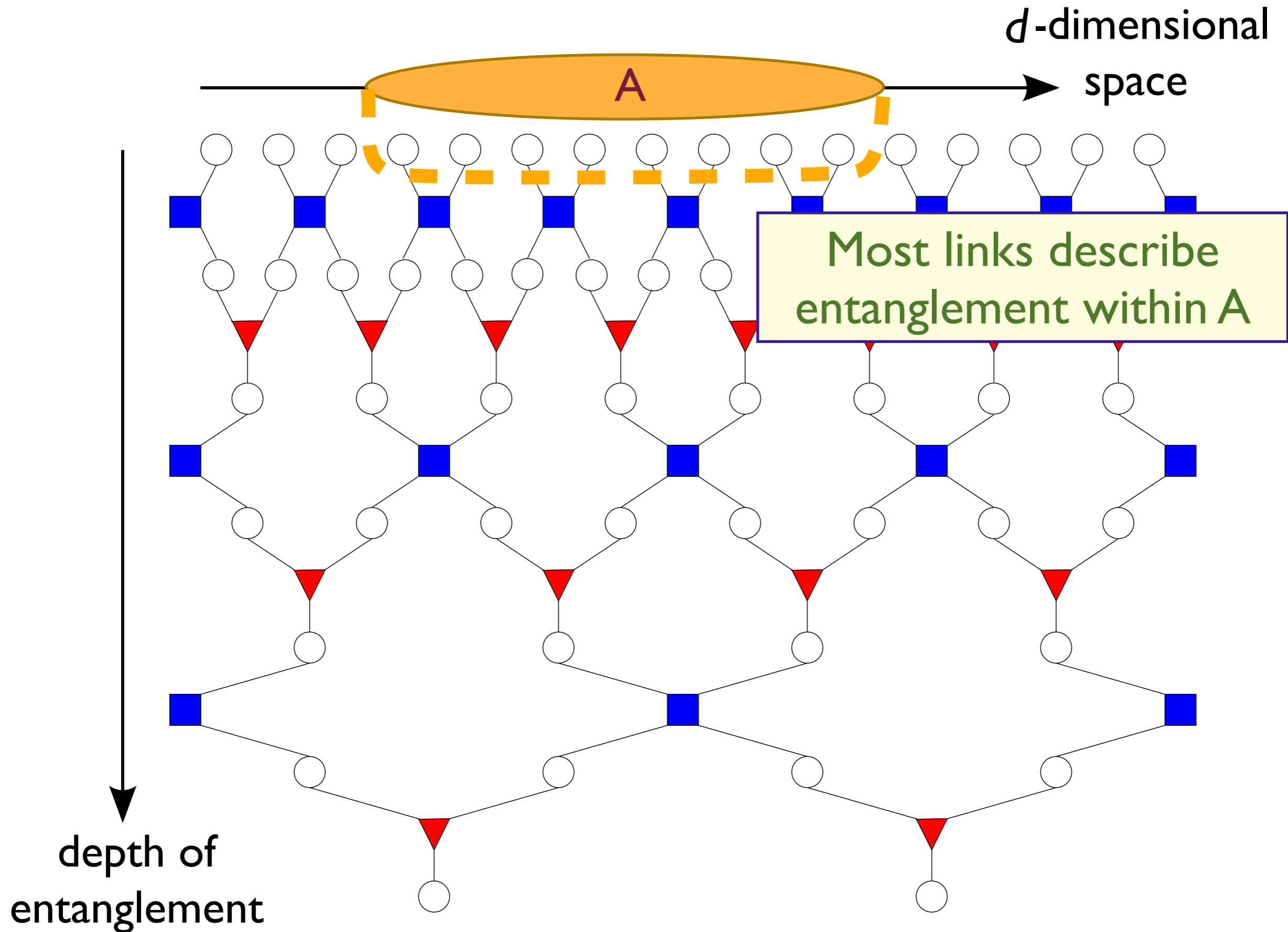
Measure strength of quantum entanglement of region A with region B .

$\rho_A = \text{Tr}_B \rho =$ density matrix of region A
Entanglement entropy $S_{EE} = -\text{Tr}(\rho_A \ln \rho_A)$

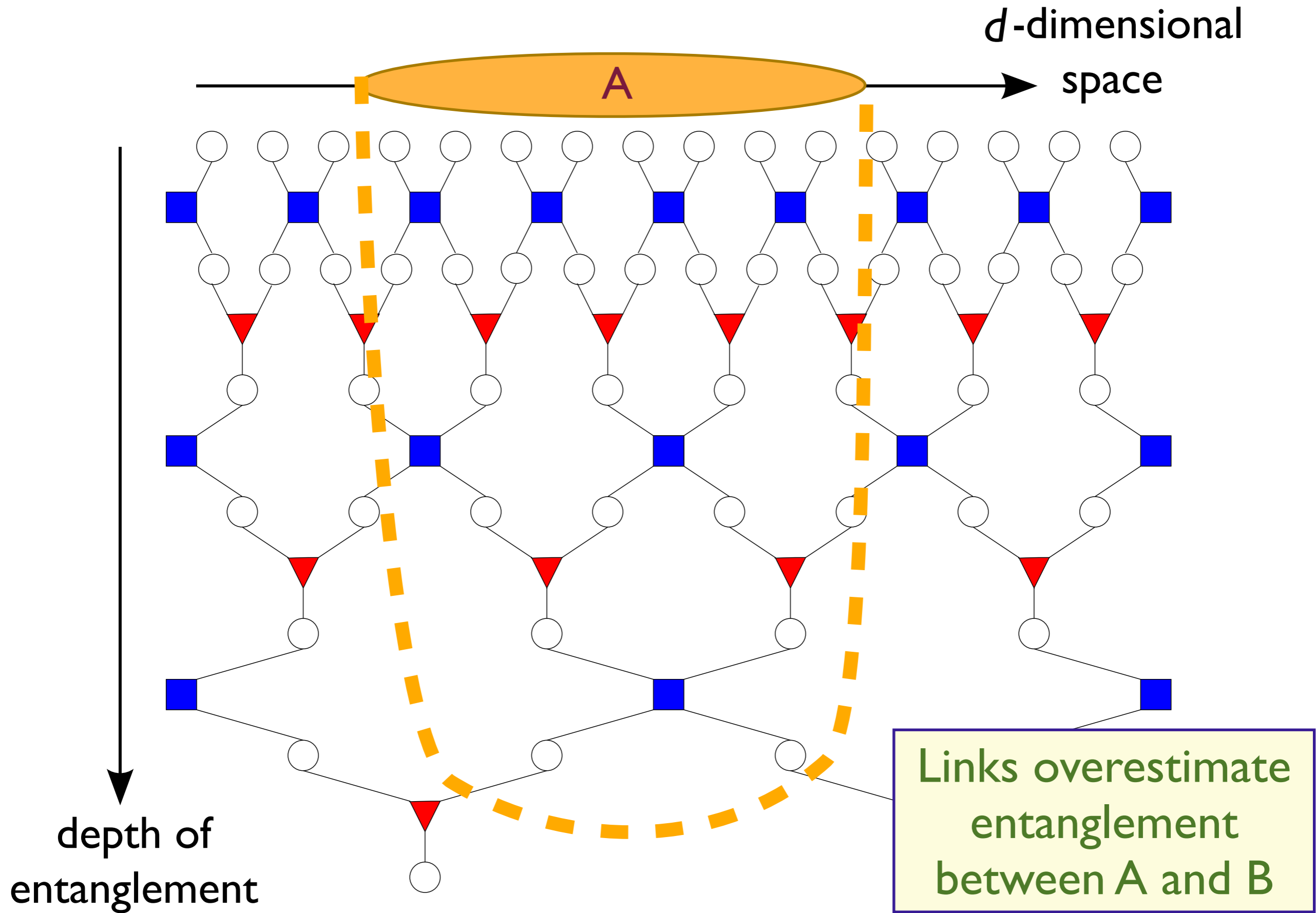
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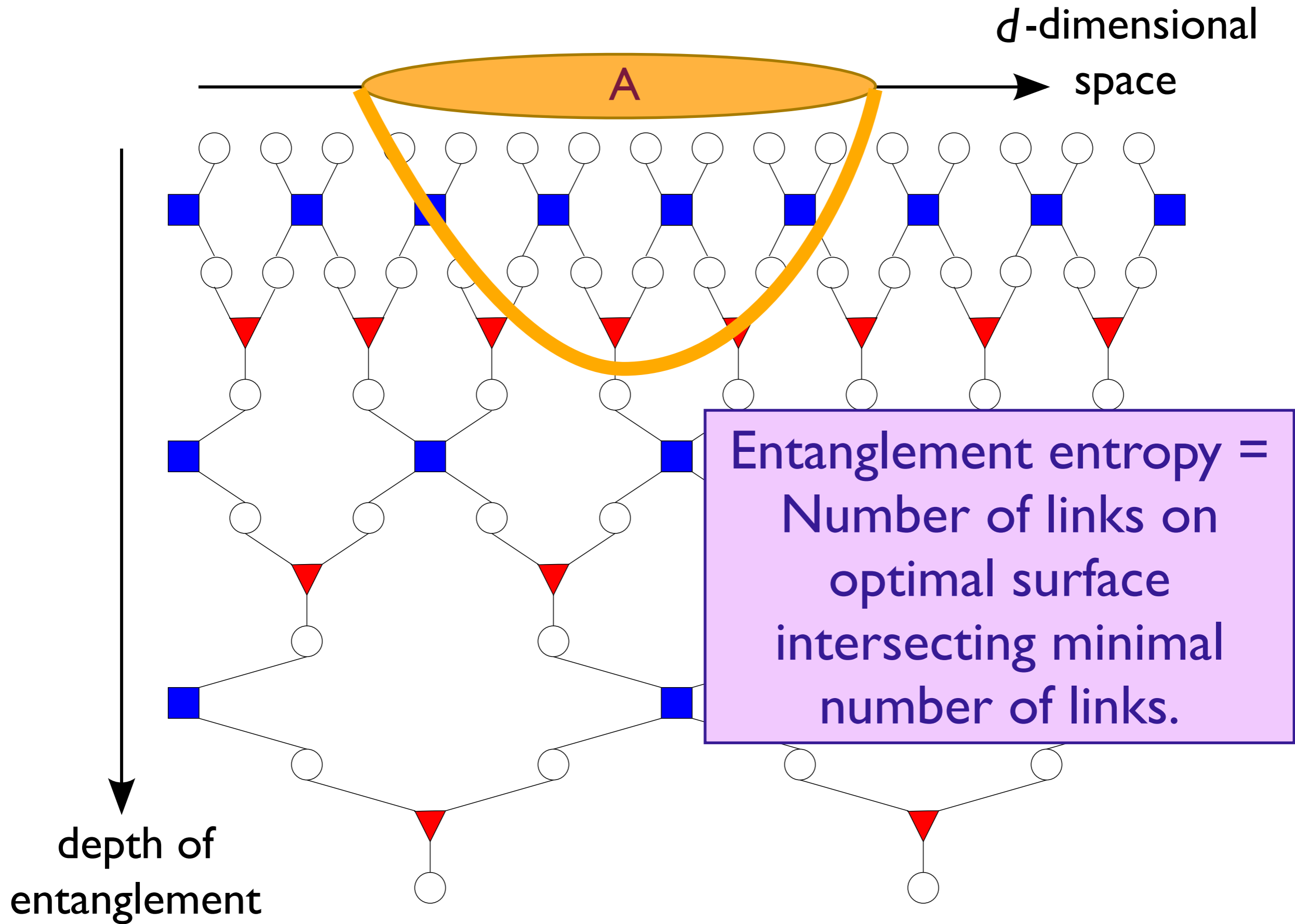
Entanglement entropy



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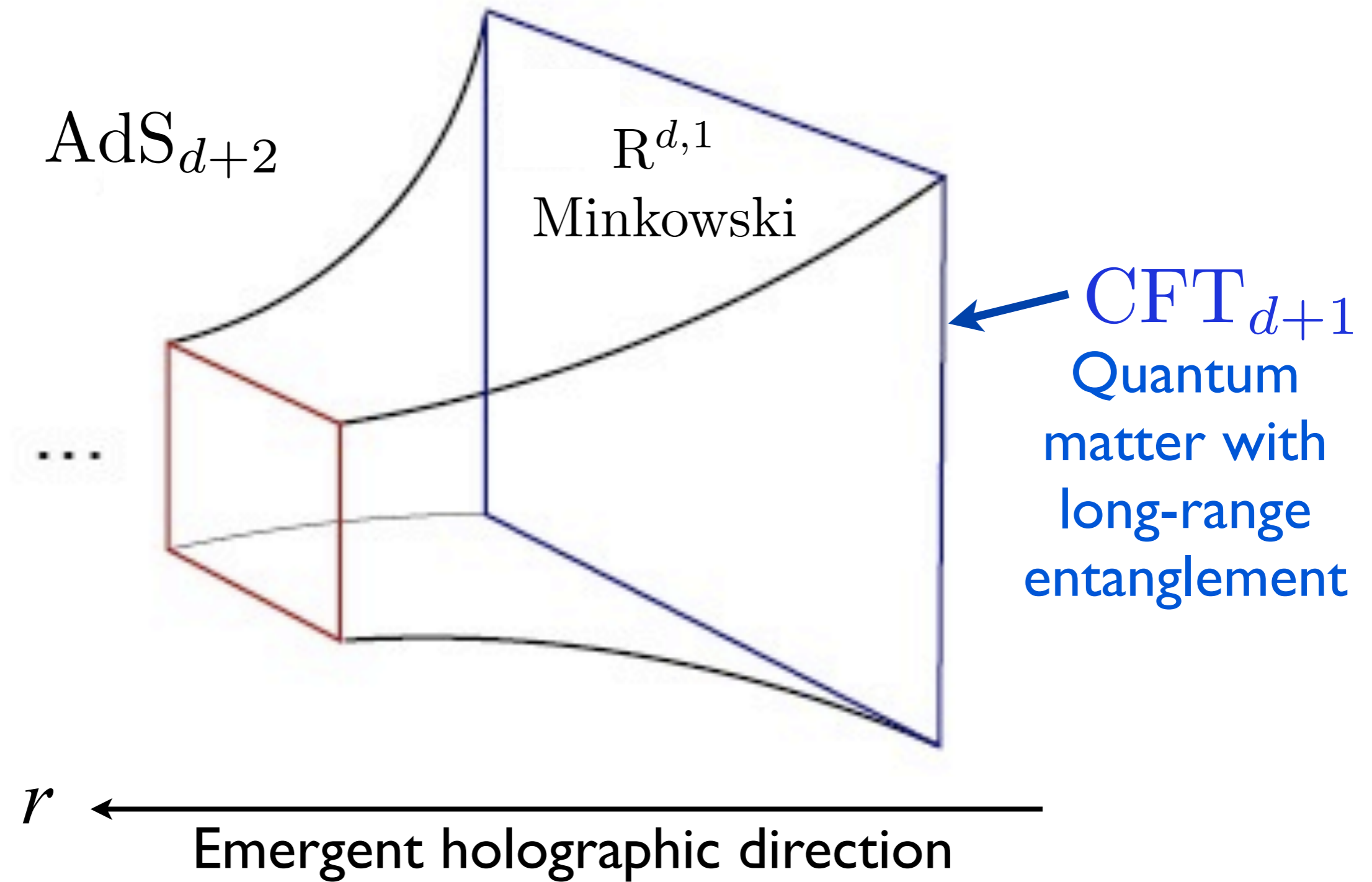
Entanglement entropy

The entanglement entropy of a region A on the boundary equals the minimal area of a surface in the higher-dimensional space whose boundary co-incides with that of A .

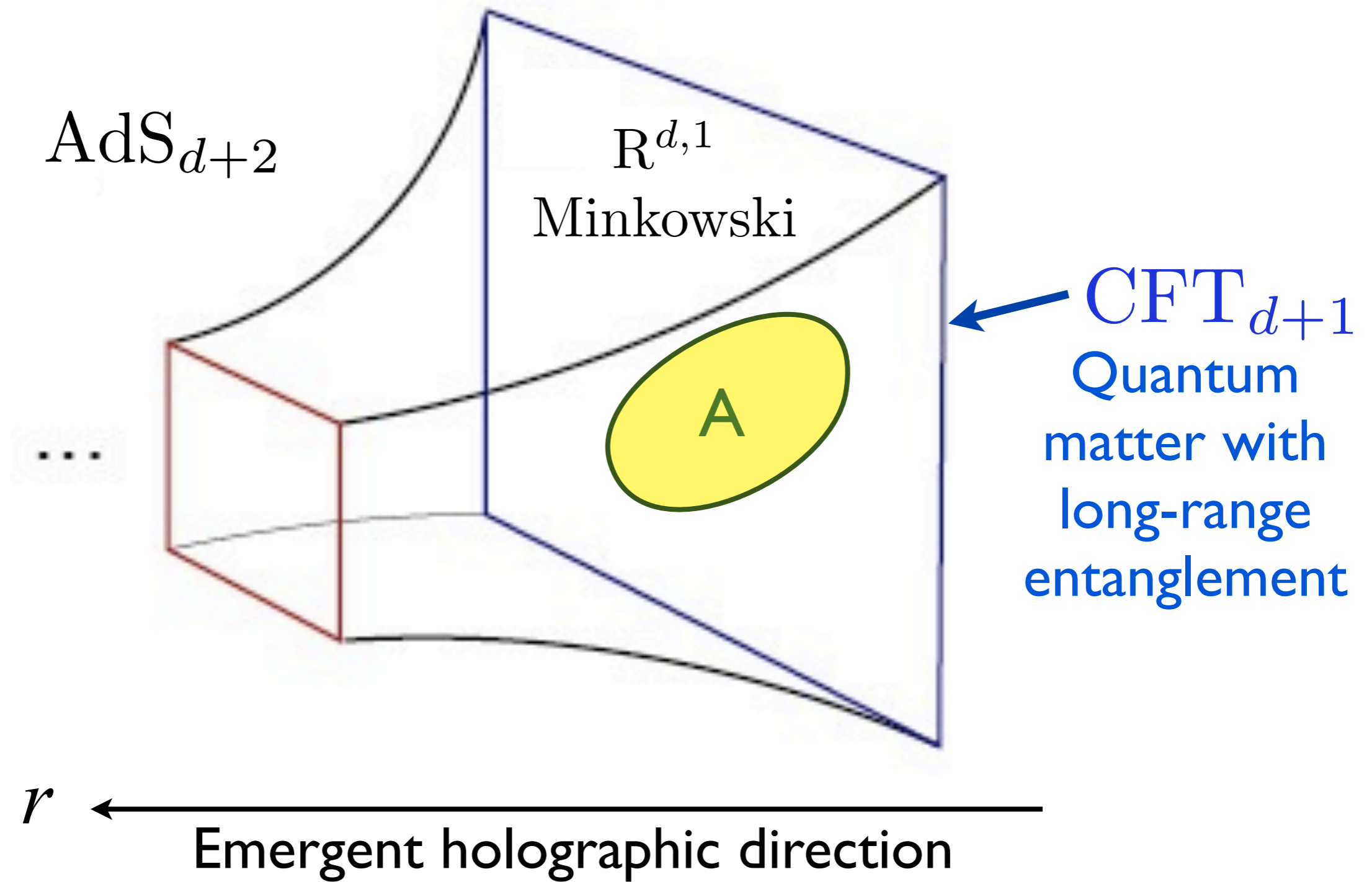
This can be seen both the string and tensor-network pictures

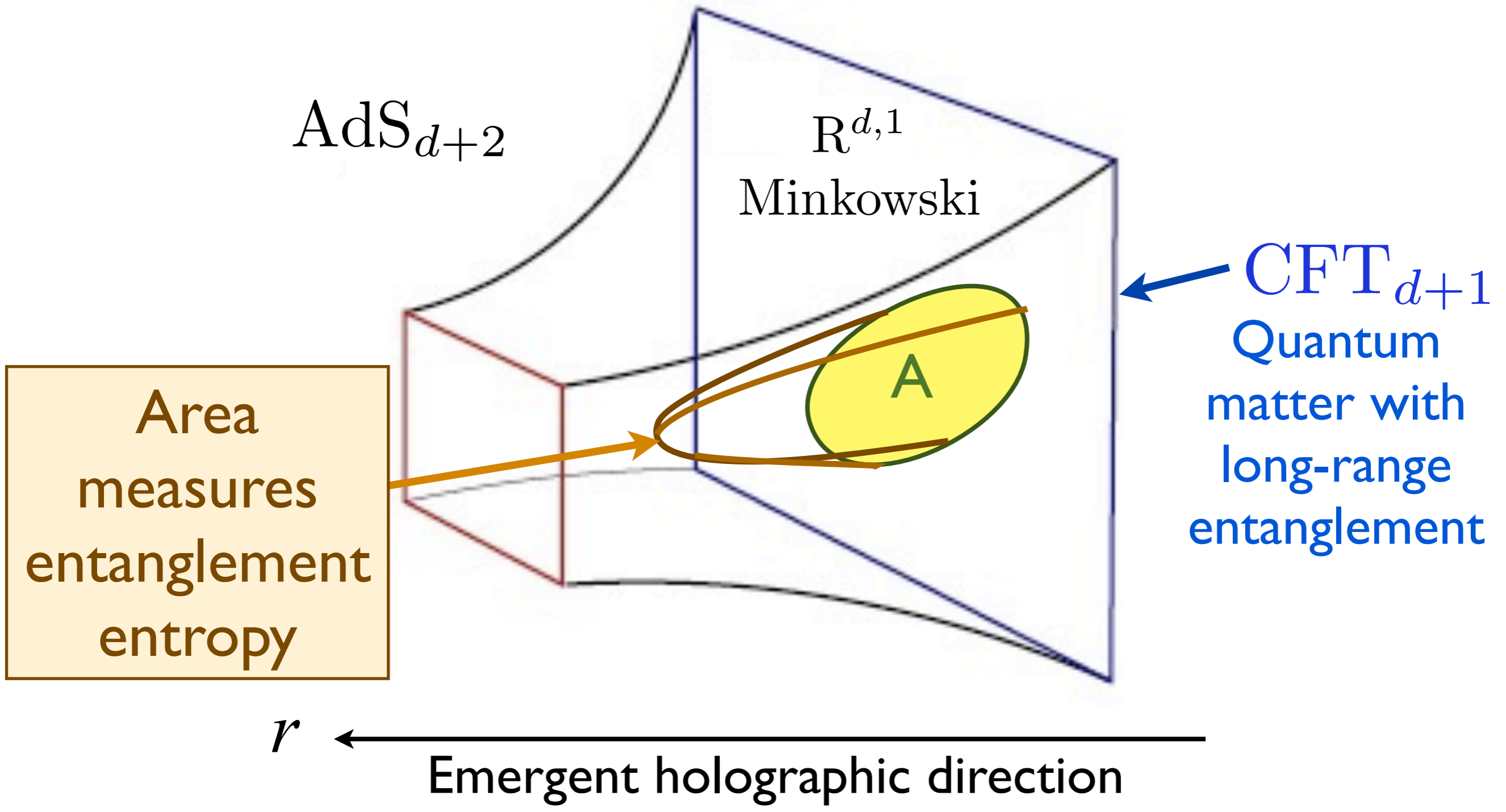
S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Brian Swingle, arXiv:0905.1317



J. McGreevy, arXiv0909.0518





S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

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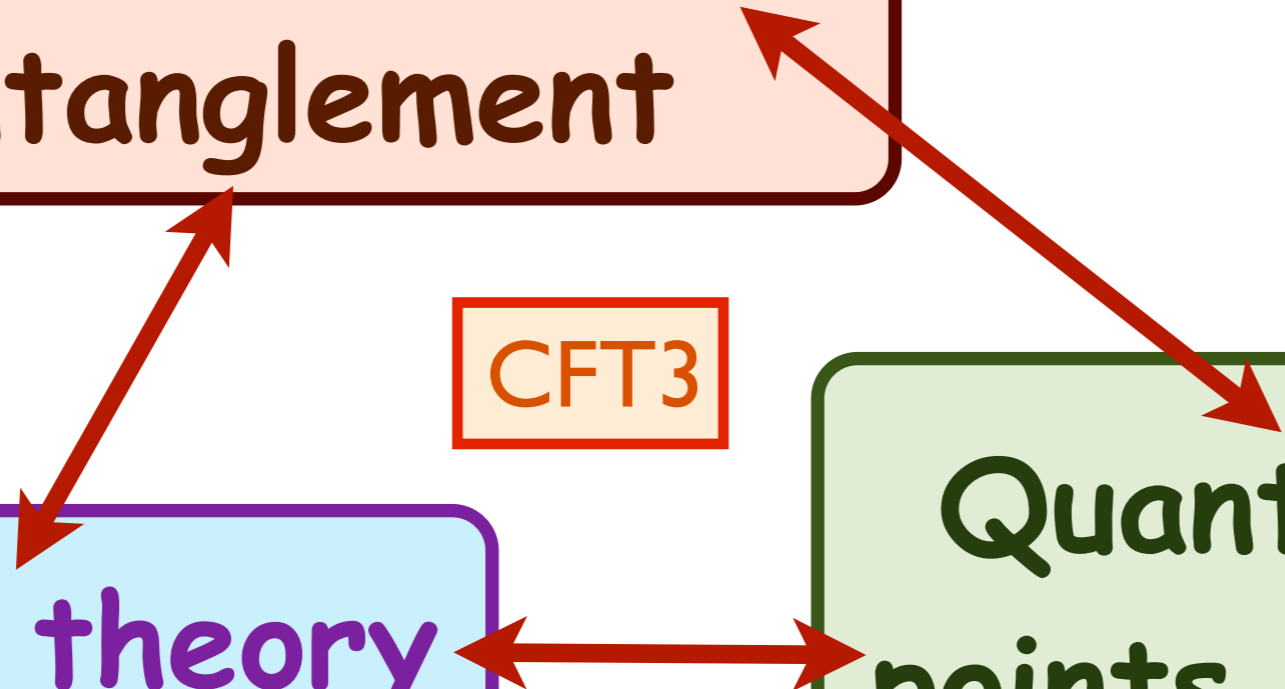
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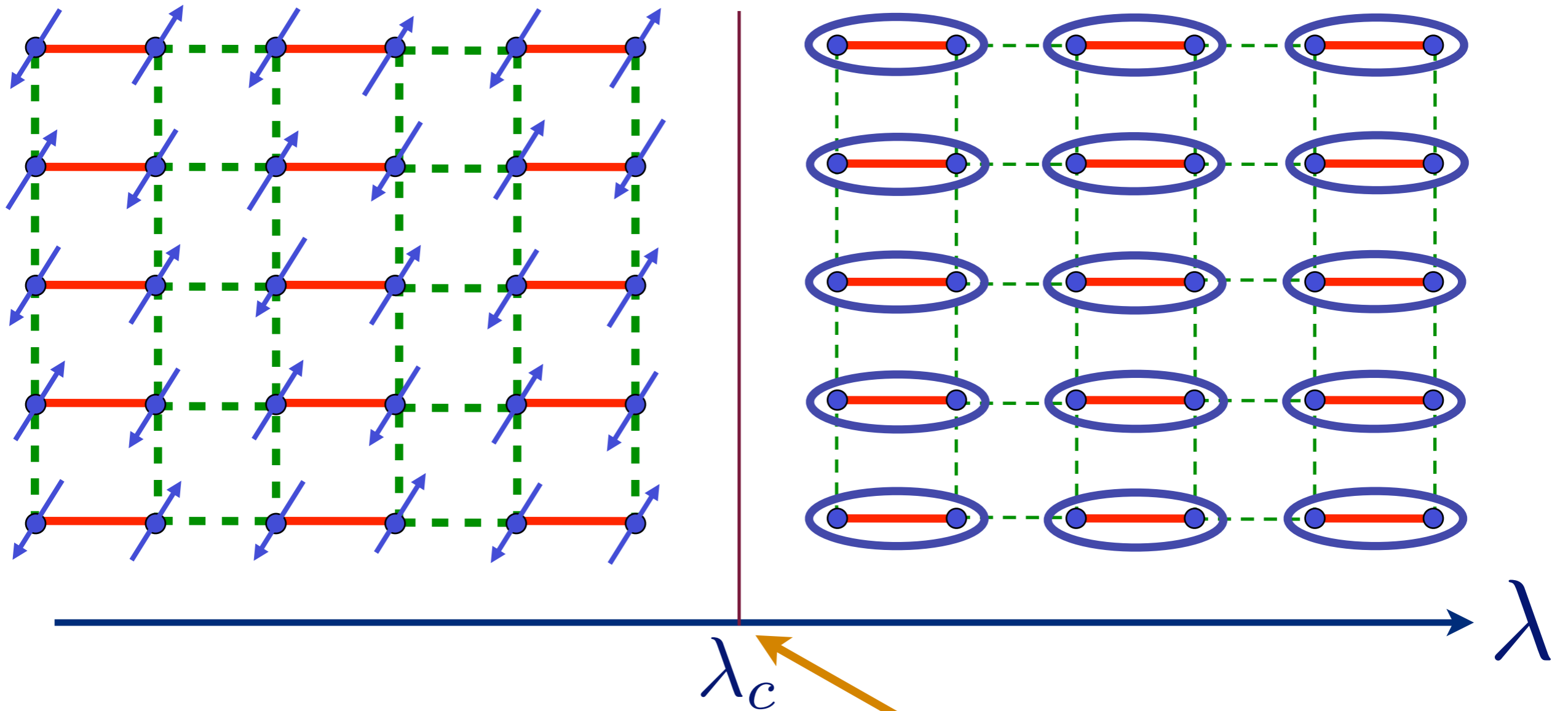
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String theory

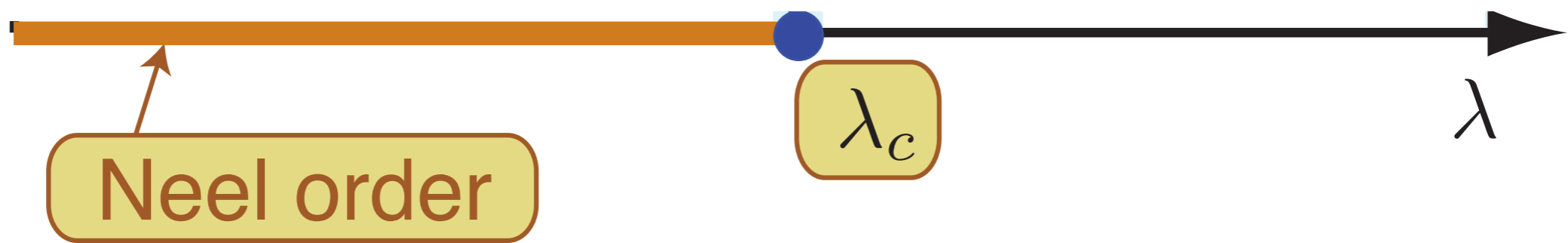
**Quantum critical
points of electrons
in crystals**

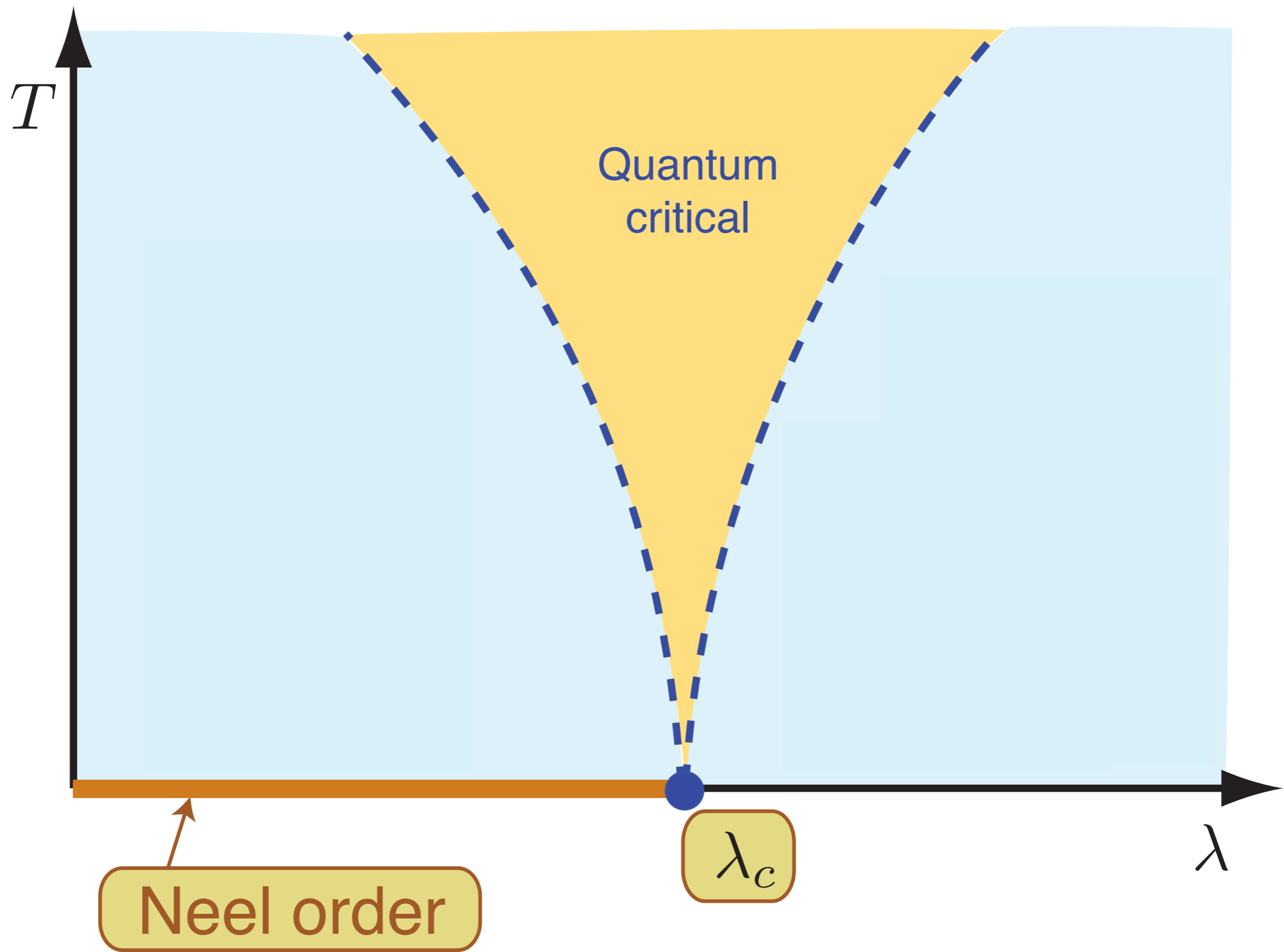
Black holes

$$\text{Diagram of two blue dots connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

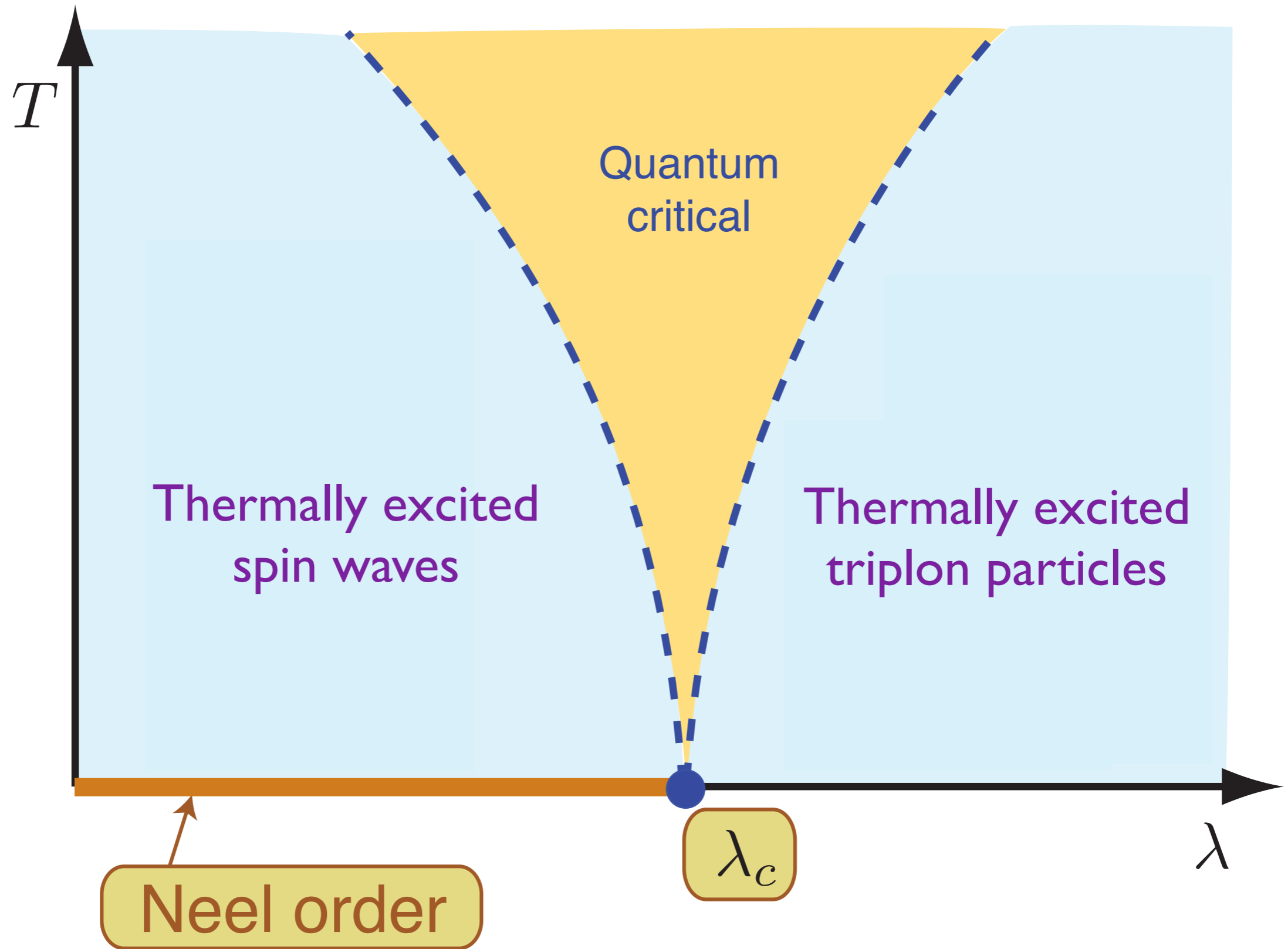


Quantum critical point with non-local entanglement in spin wavefunction

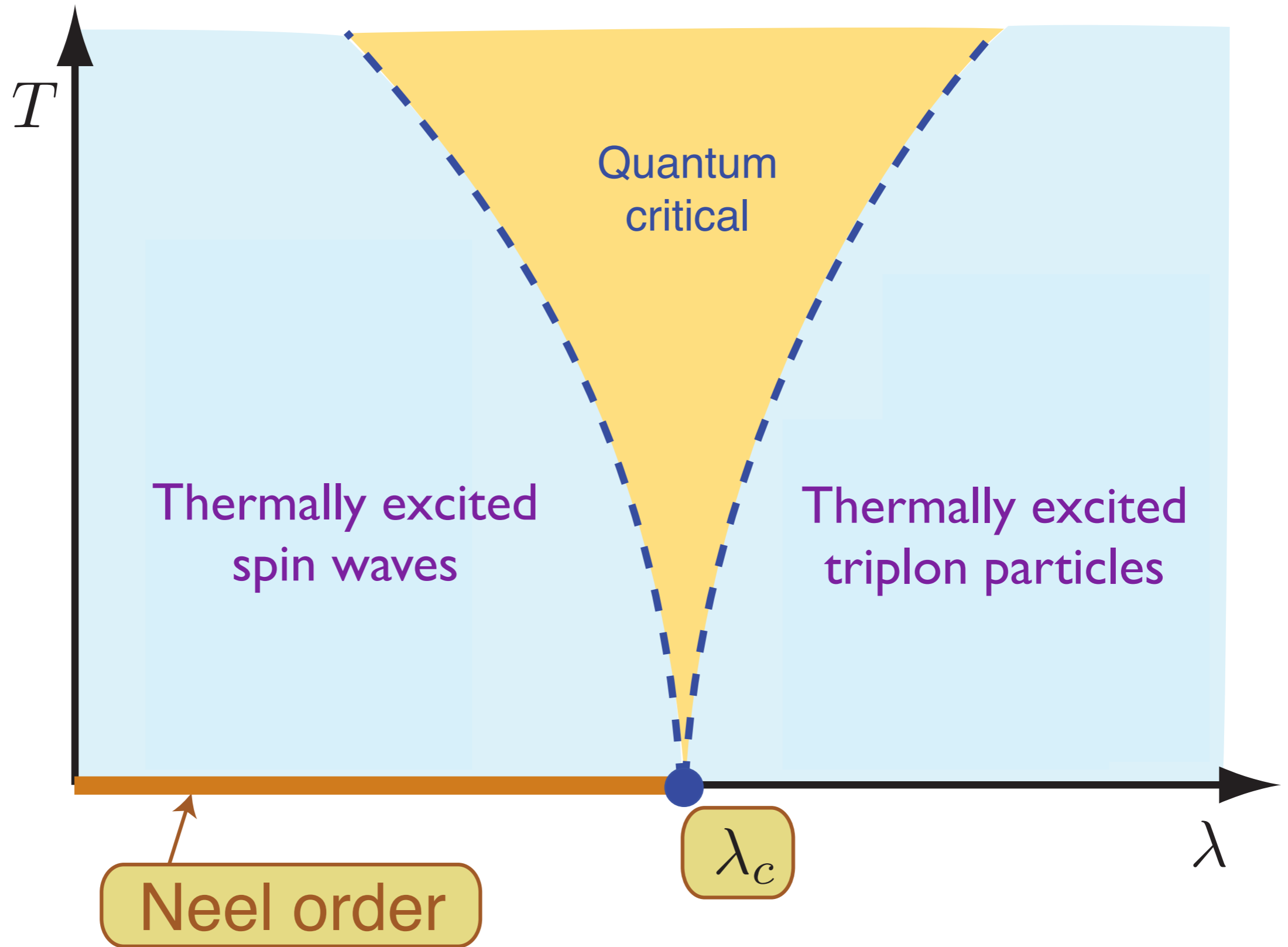




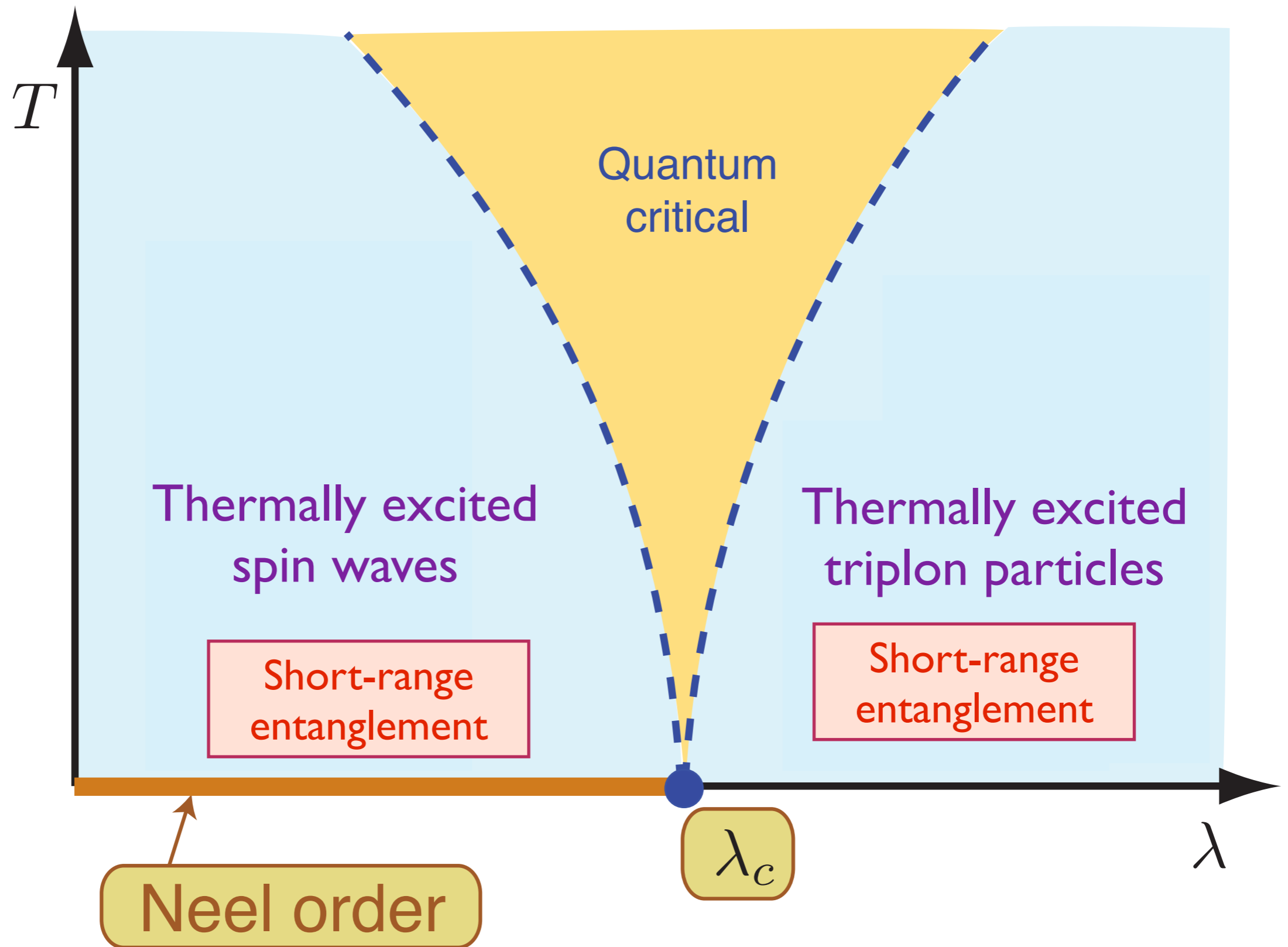
S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).



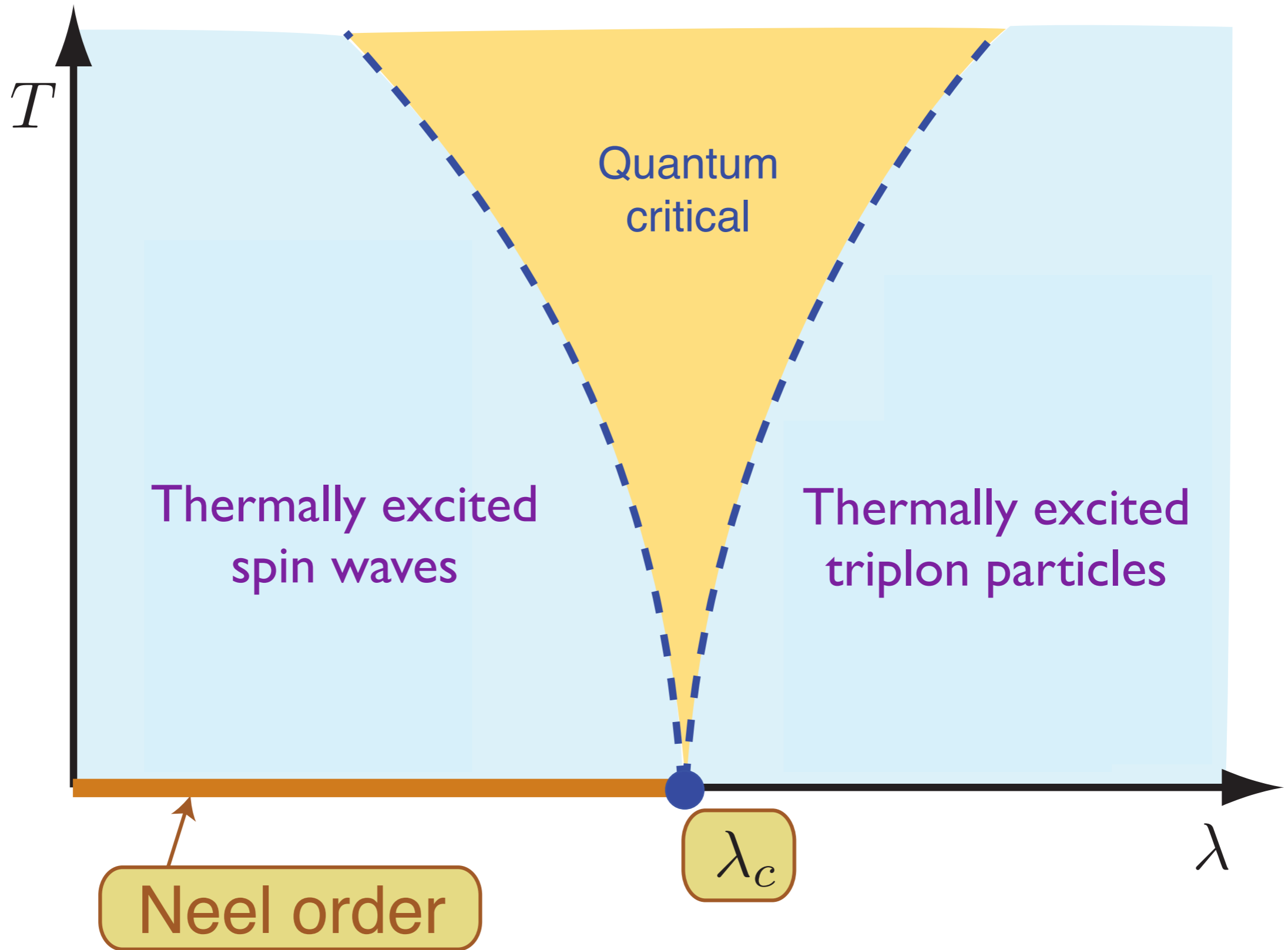
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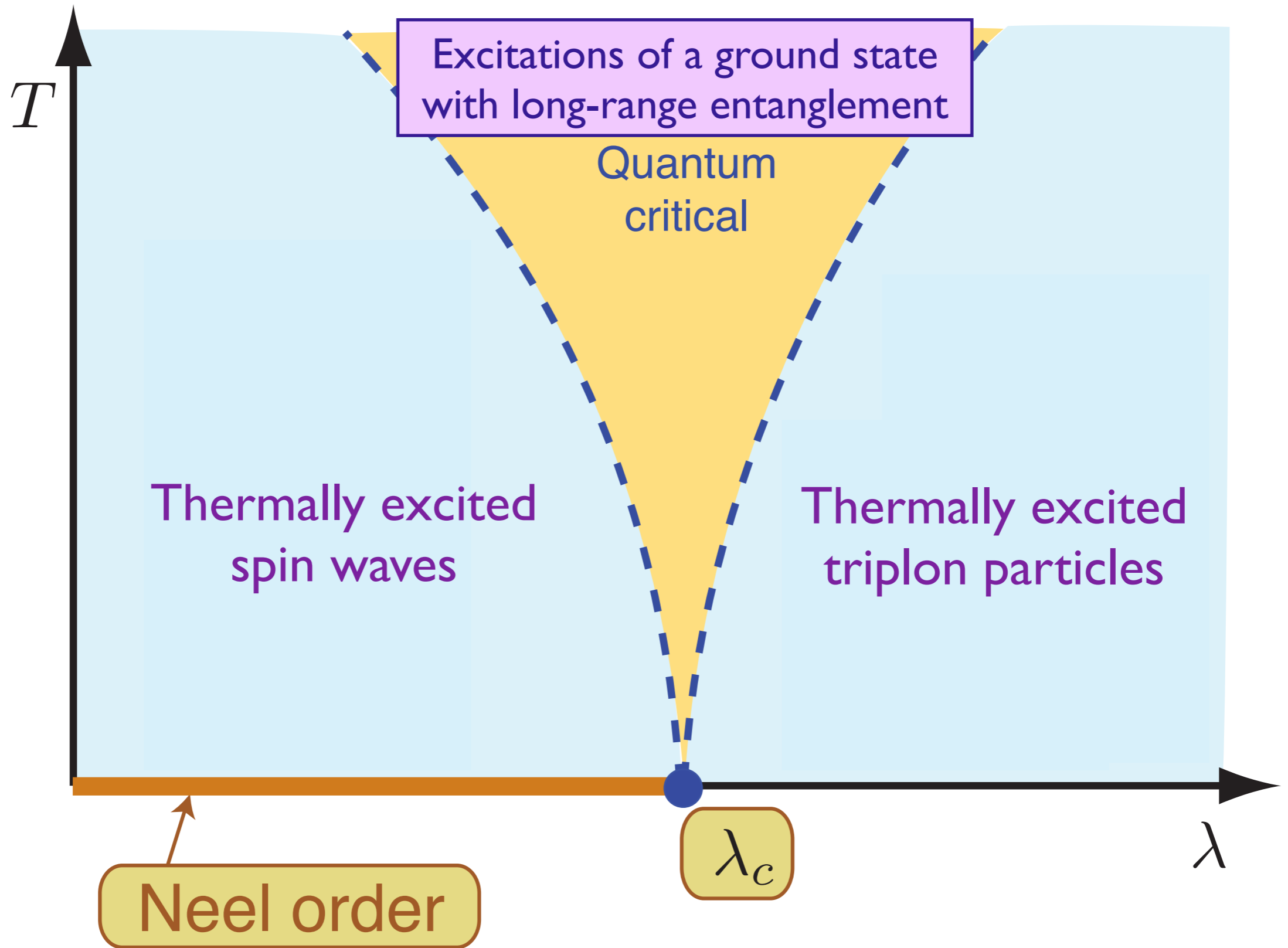
S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
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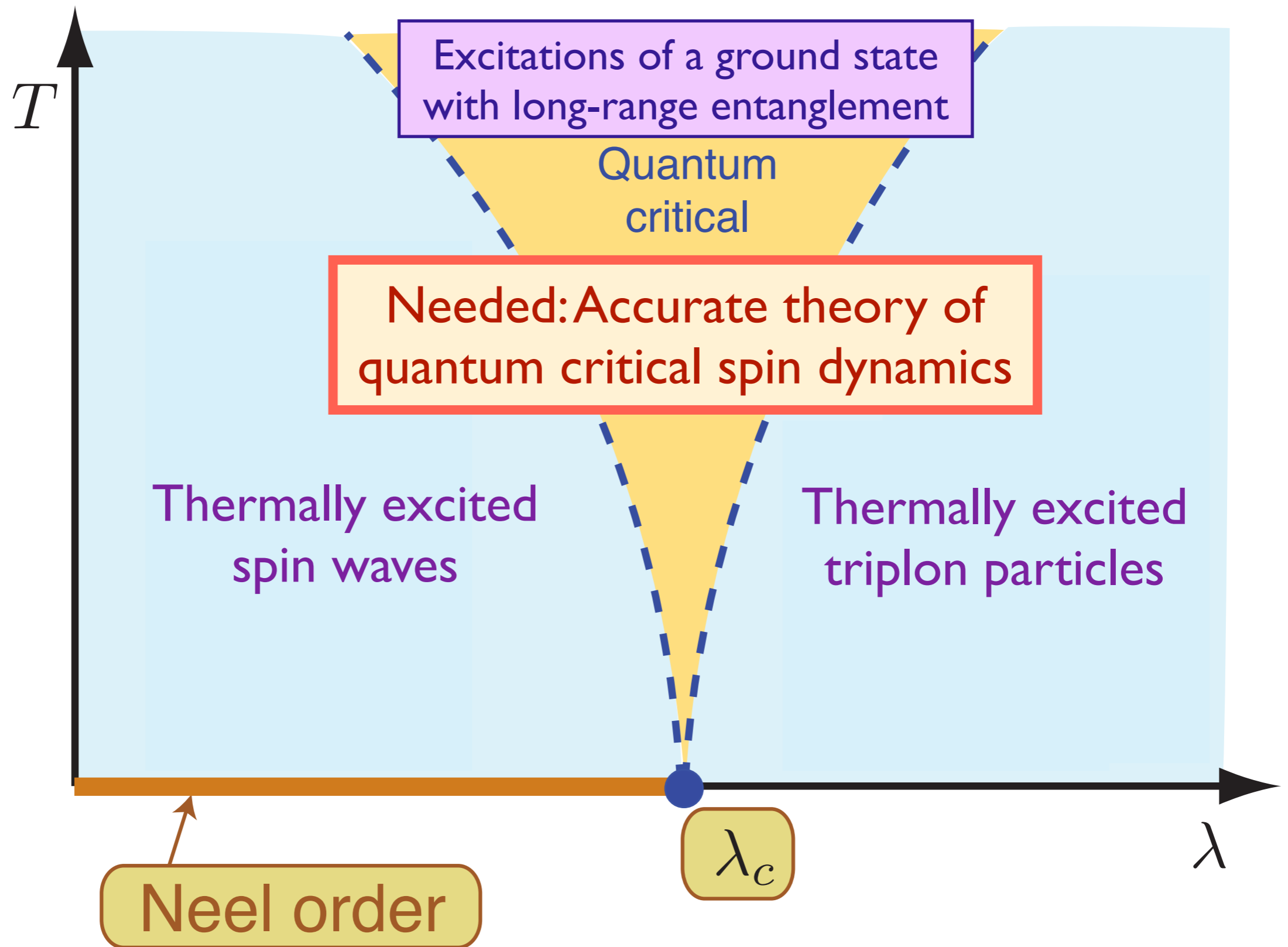


S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).

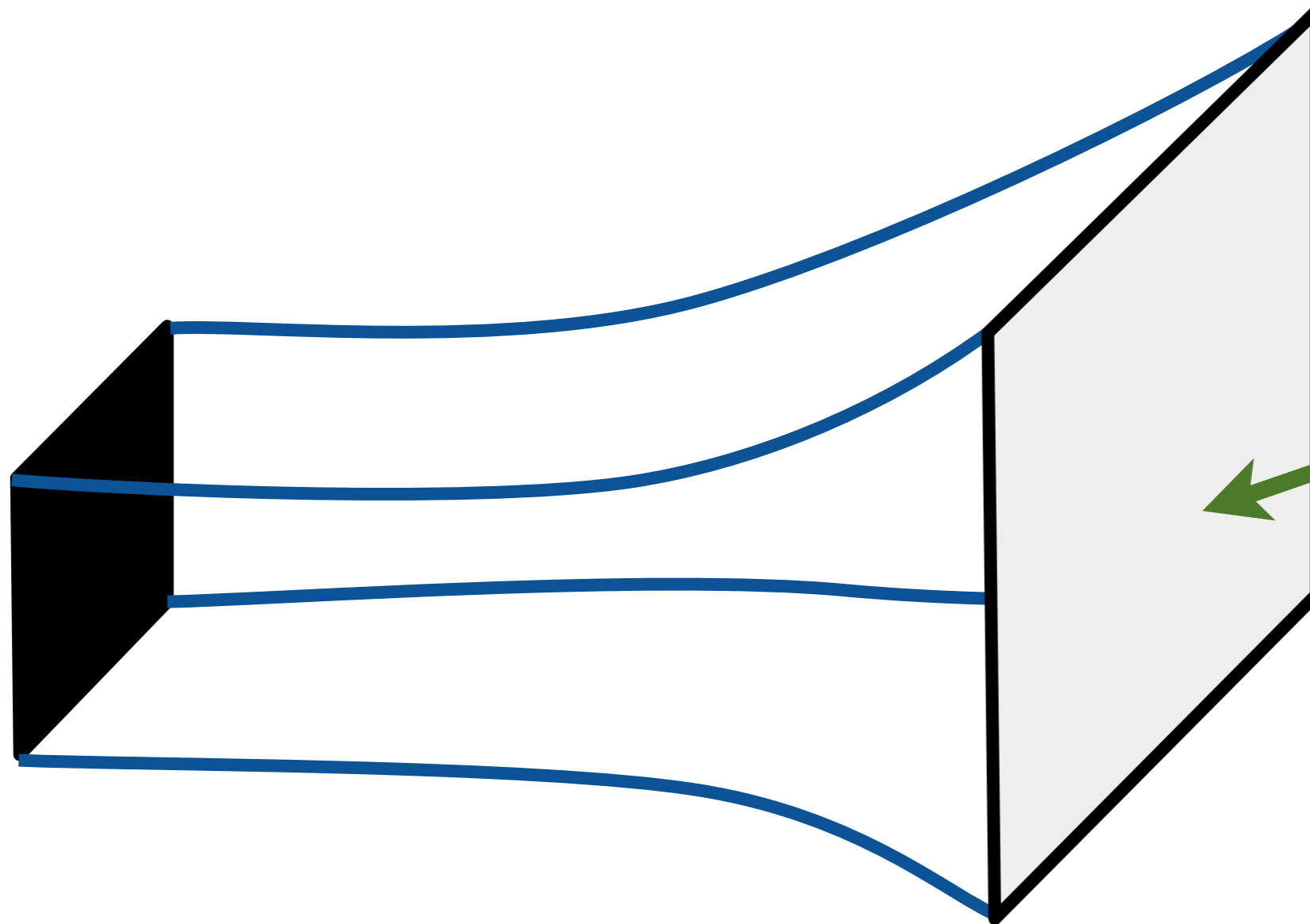


S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).



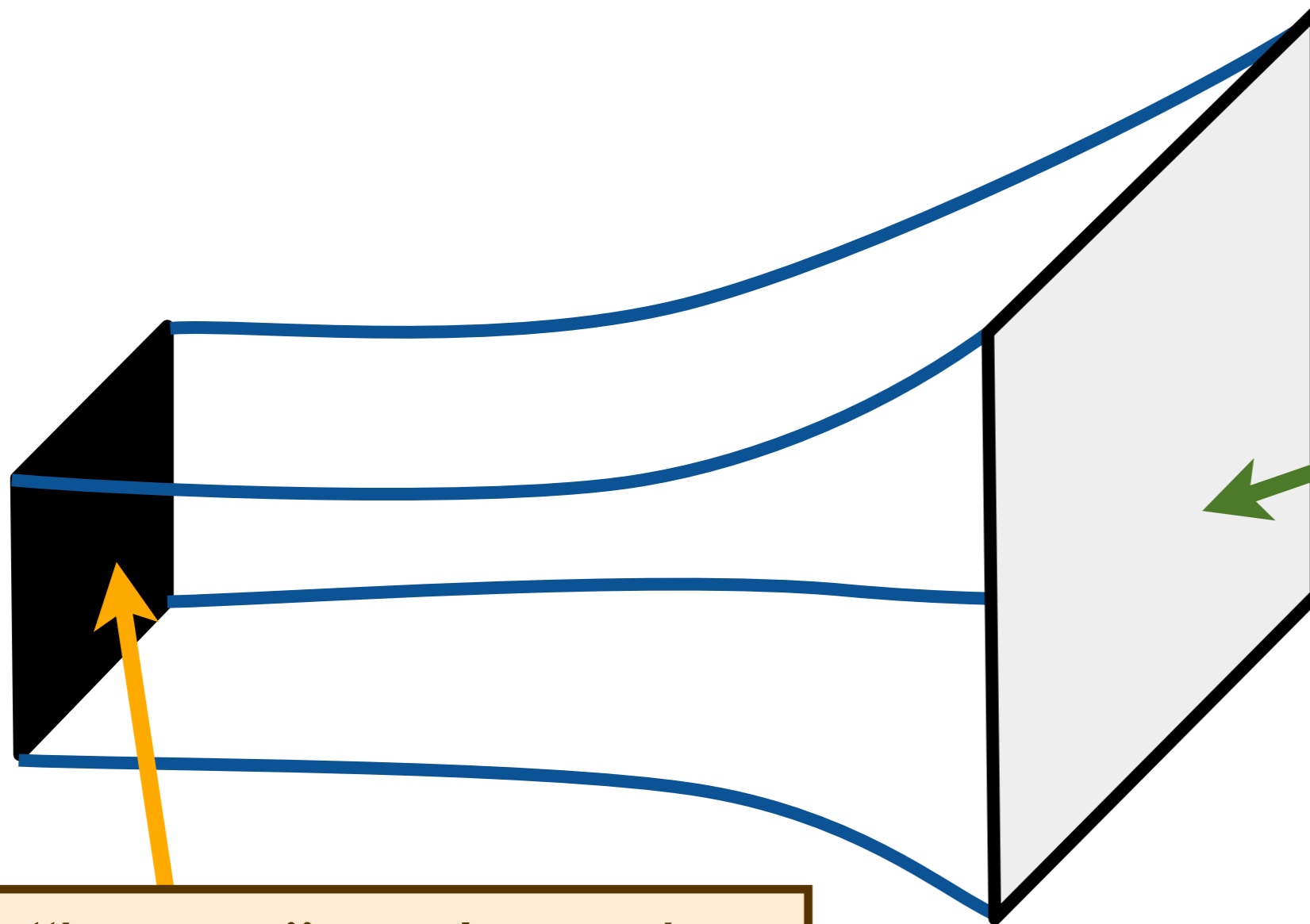


String theory at non-zero temperatures



A 2+1 dimensional system at its quantum critical point

String theory at non-zero temperatures

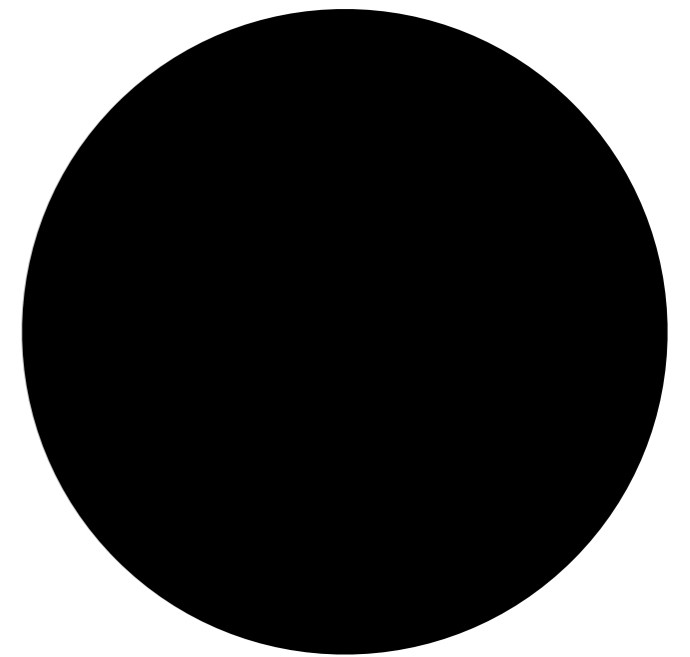


A "horizon", similar to the surface of a black hole !

A 2+1 dimensional system at its quantum critical point

Black Holes

Objects so massive that light is gravitationally bound to them.

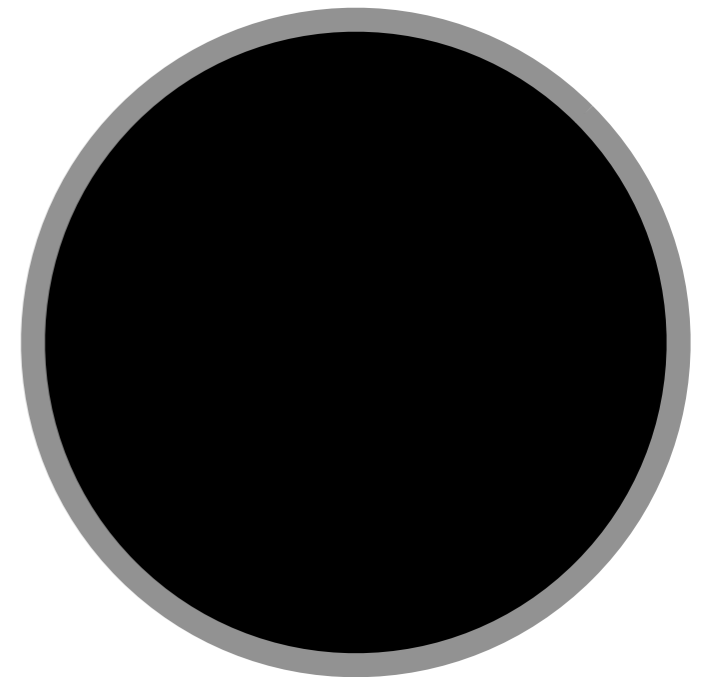


Black Holes

Objects so massive that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole **horizon** is disconnected from the rest of the universe.

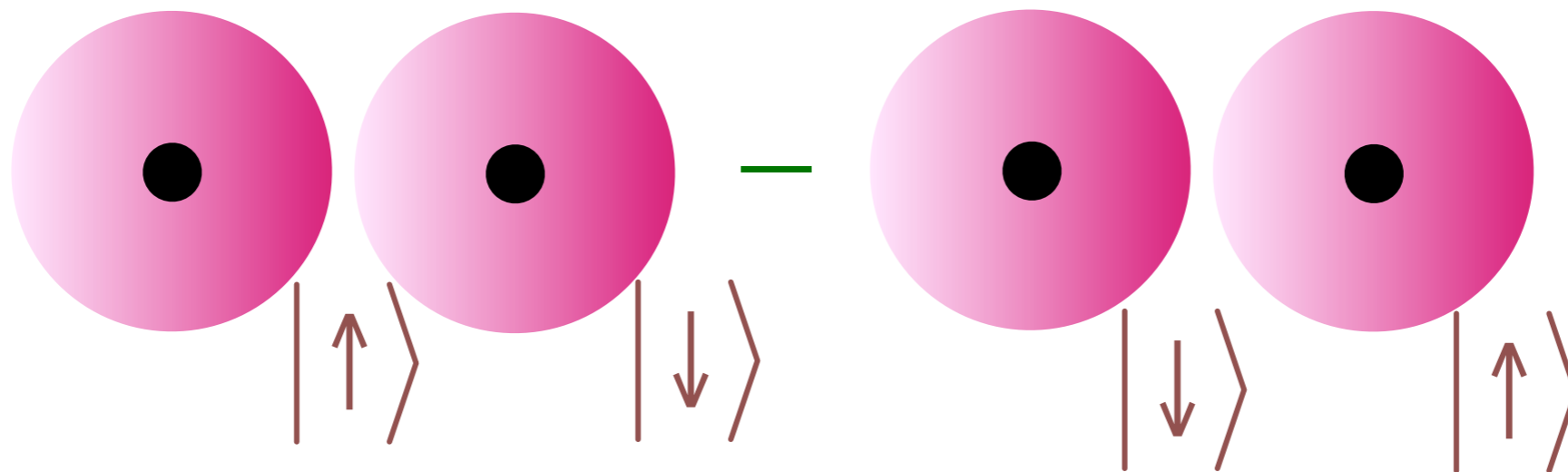
$$\text{Horizon radius } R = \frac{2GM}{c^2}$$



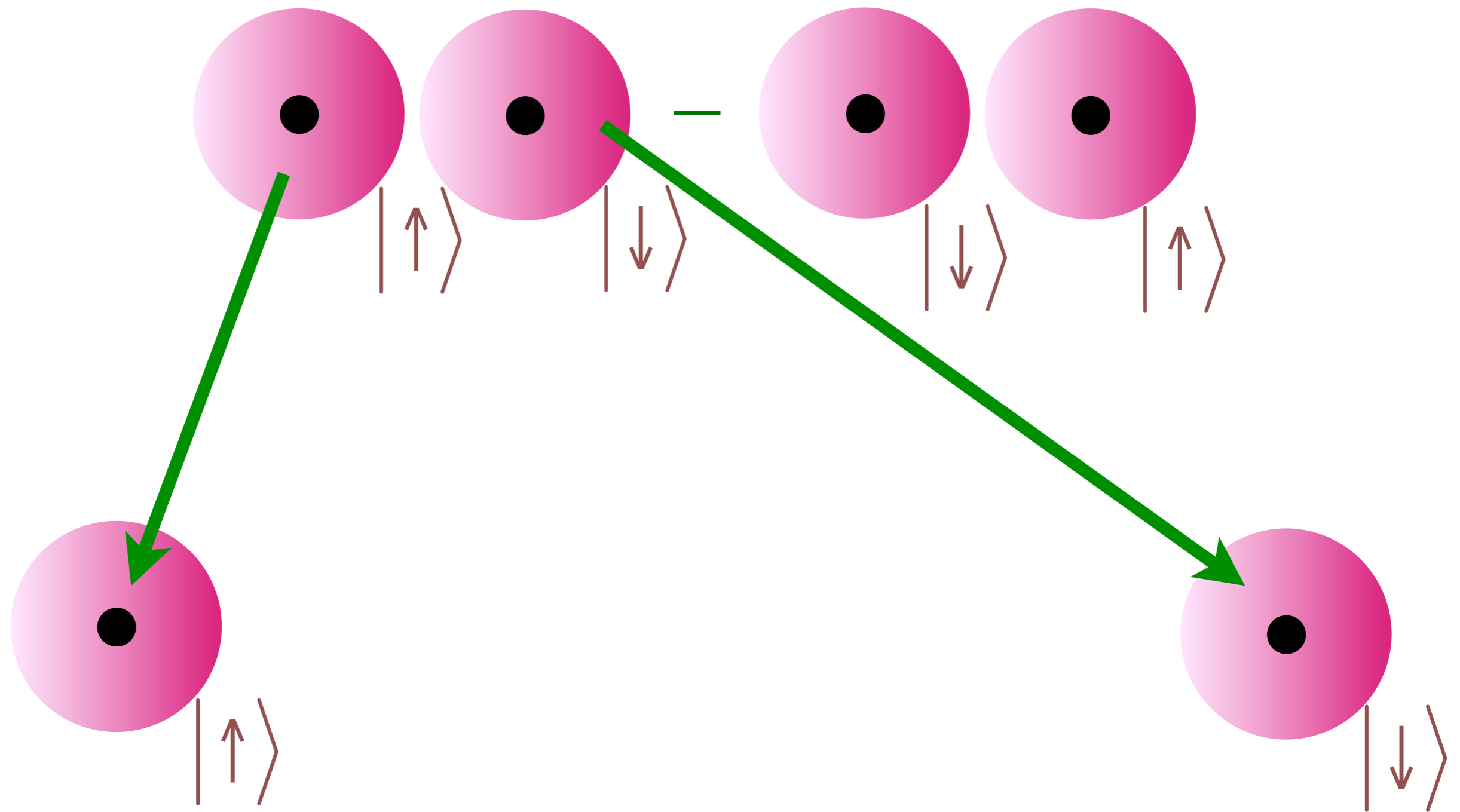
Black Holes + Quantum theory

Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions

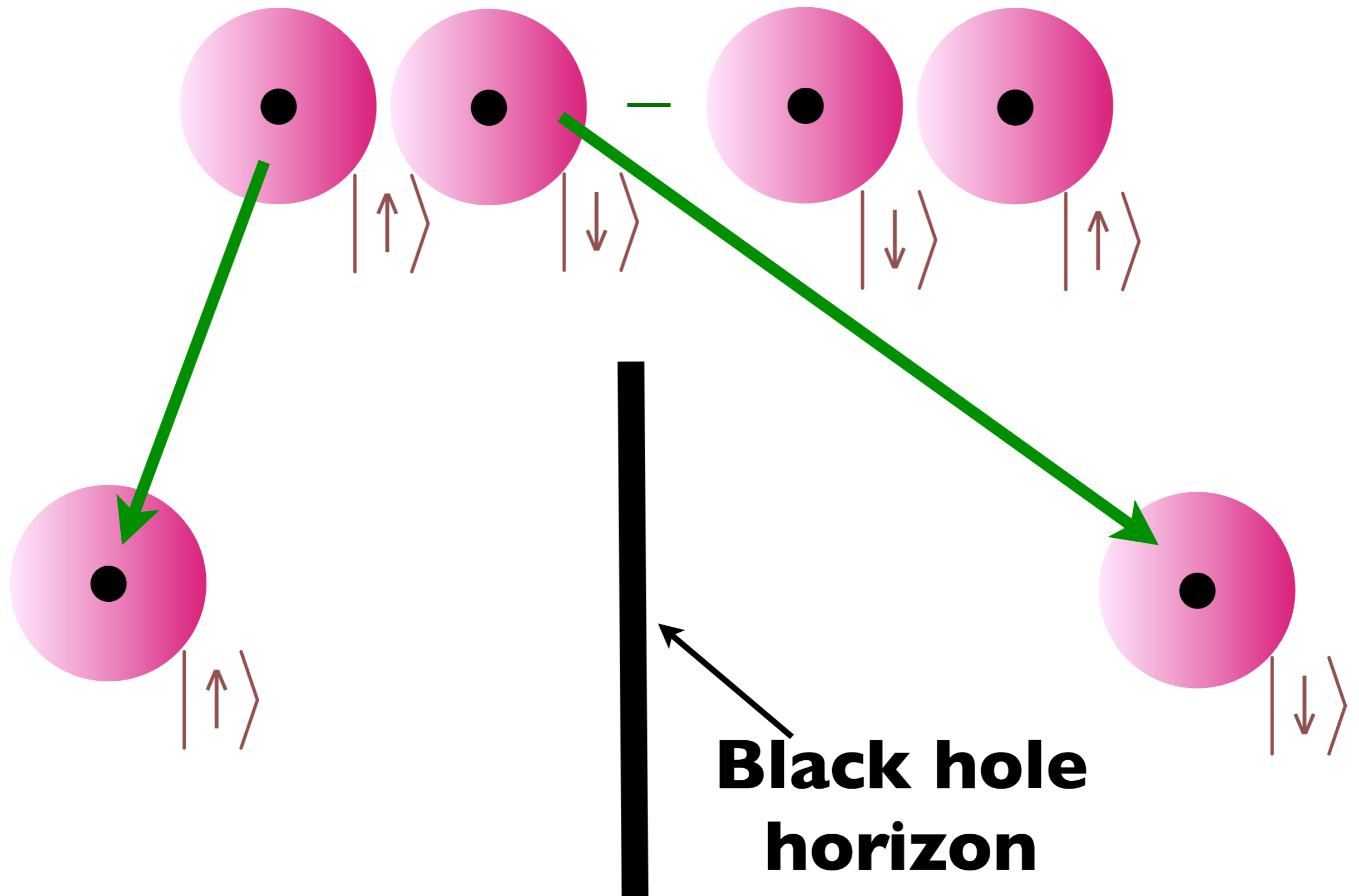
Quantum Entanglement across a black hole horizon



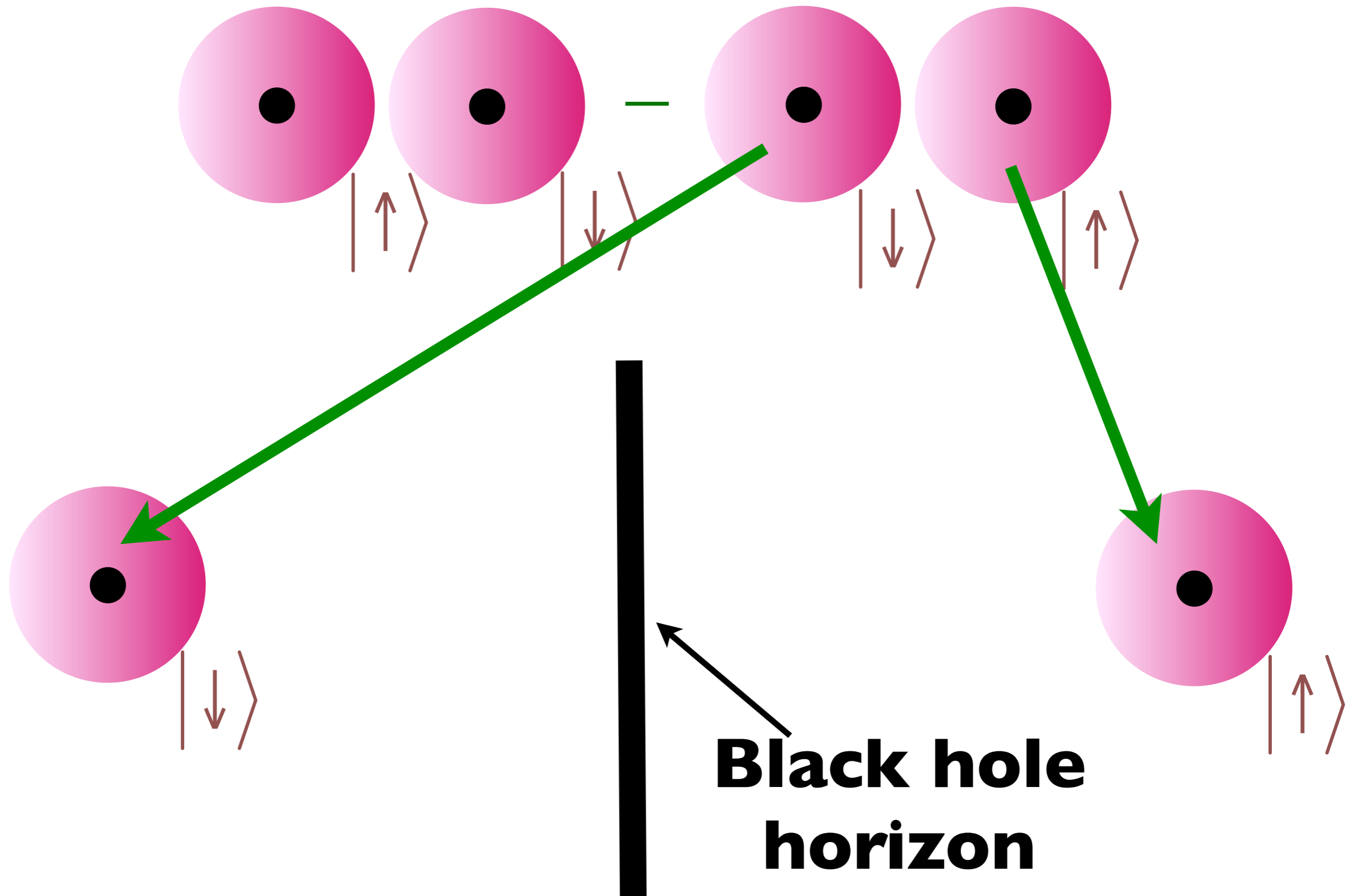
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

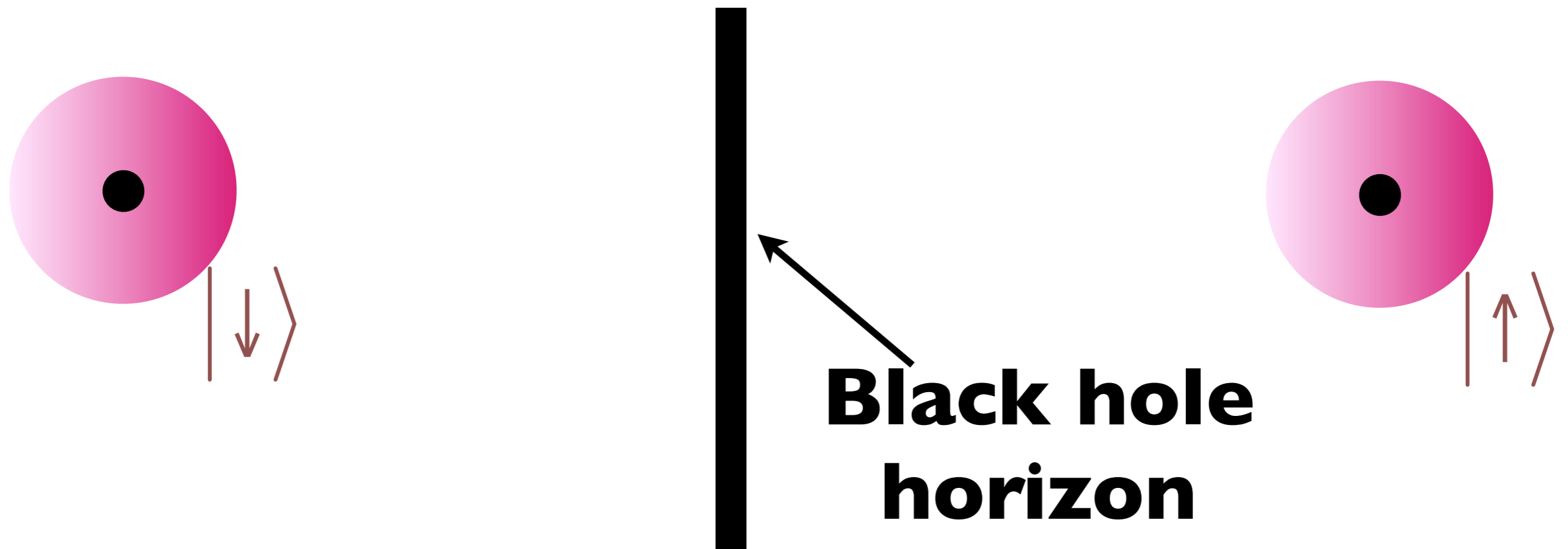


Quantum Entanglement across a black hole horizon



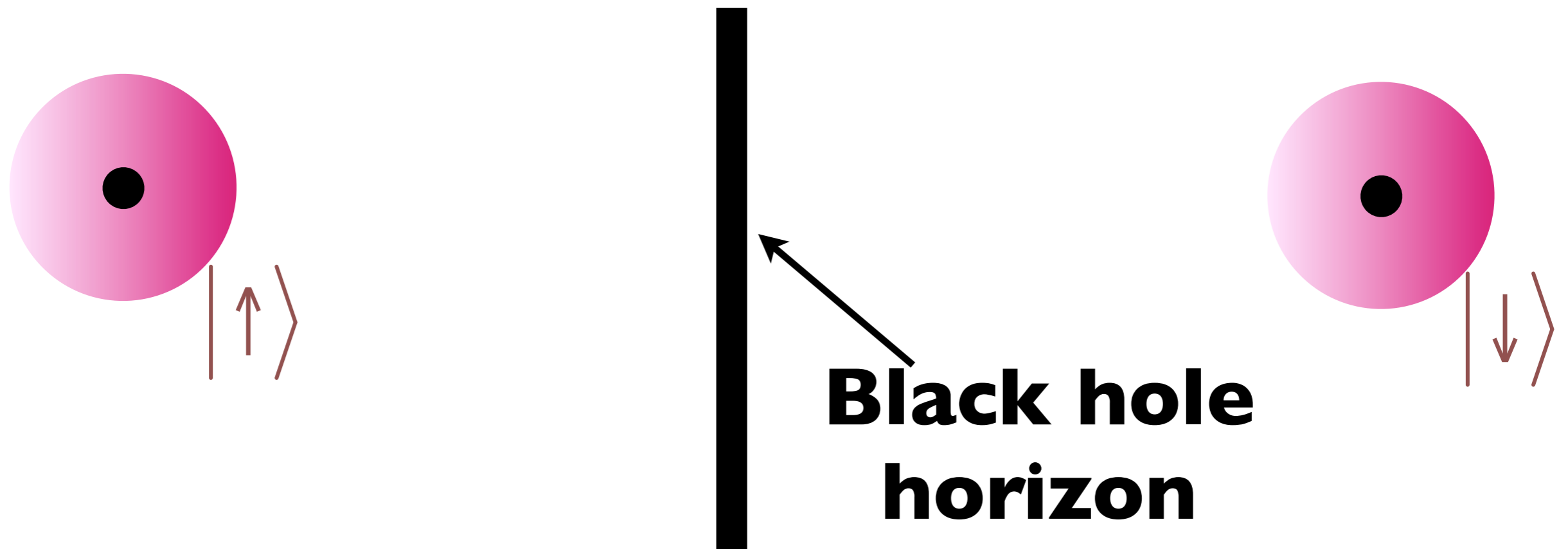
Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole

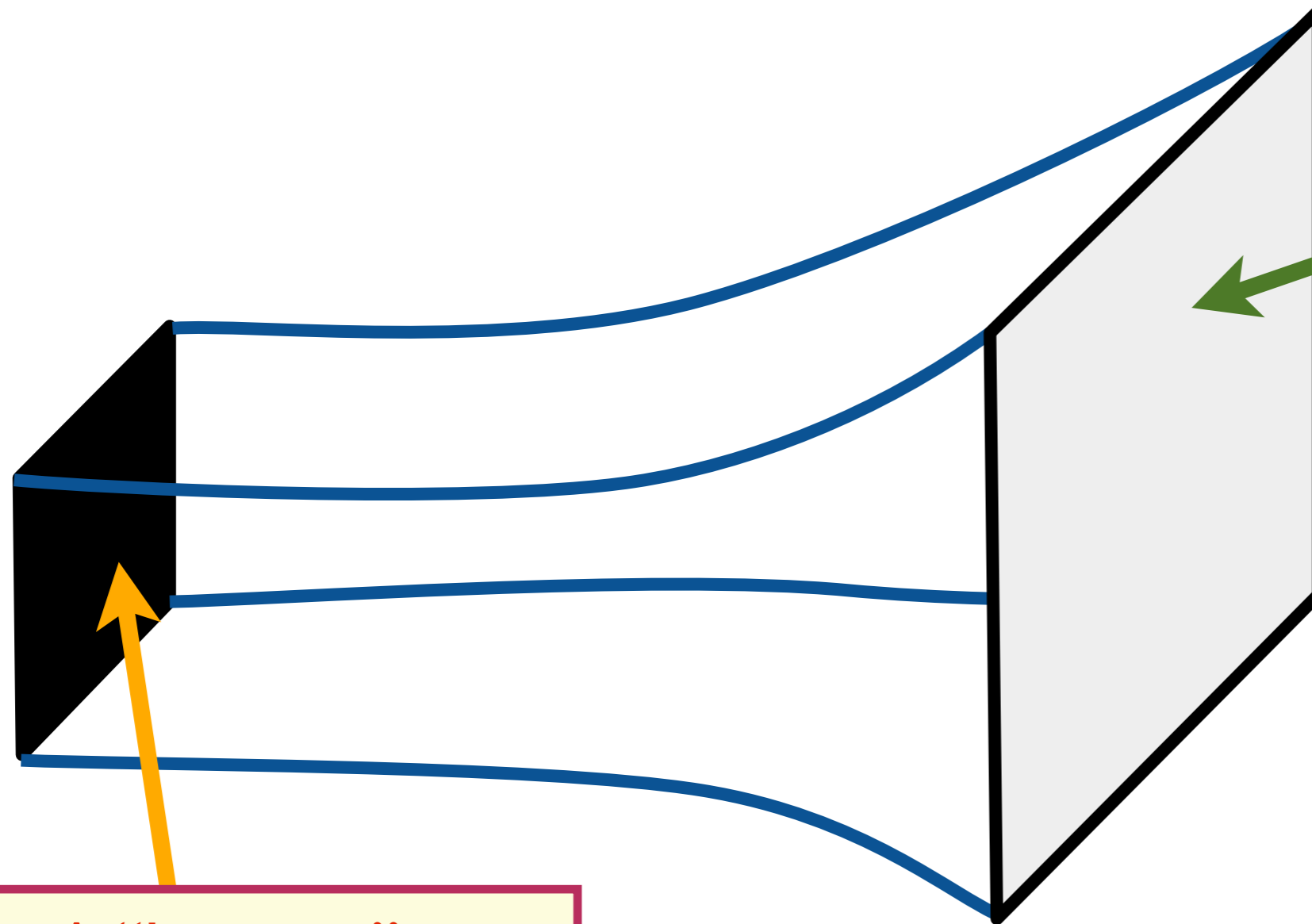


Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole

This entanglement leads to a black hole temperature (the Hawking temperature) and a black hole entropy (the Bekenstein entropy)

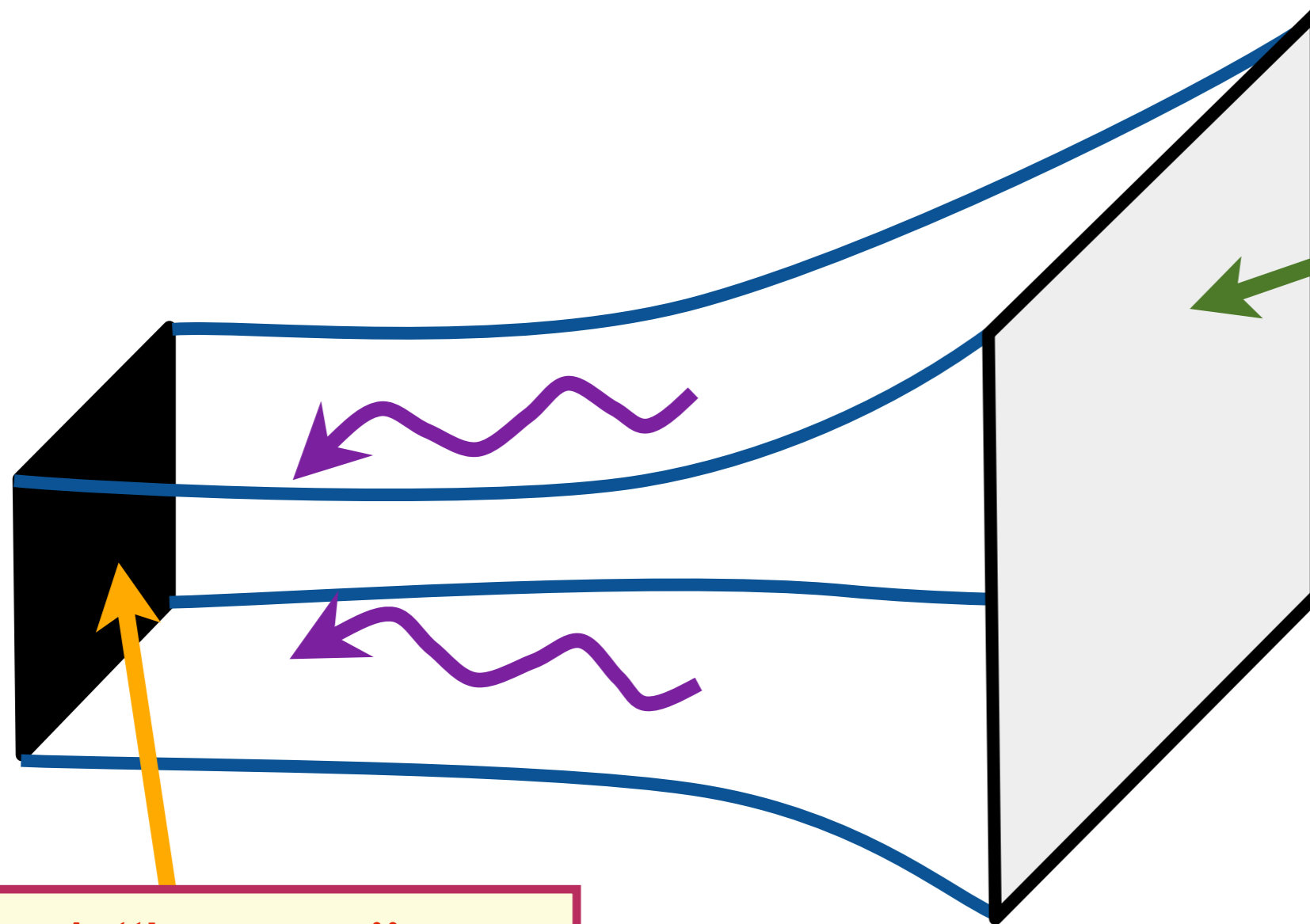
String theory at non-zero temperatures



A 2+1
dimensional
system at its
quantum
critical point

A “horizon”,
whose temperature
and entropy equal
those of the quantum
critical point

String theory at non-zero temperatures

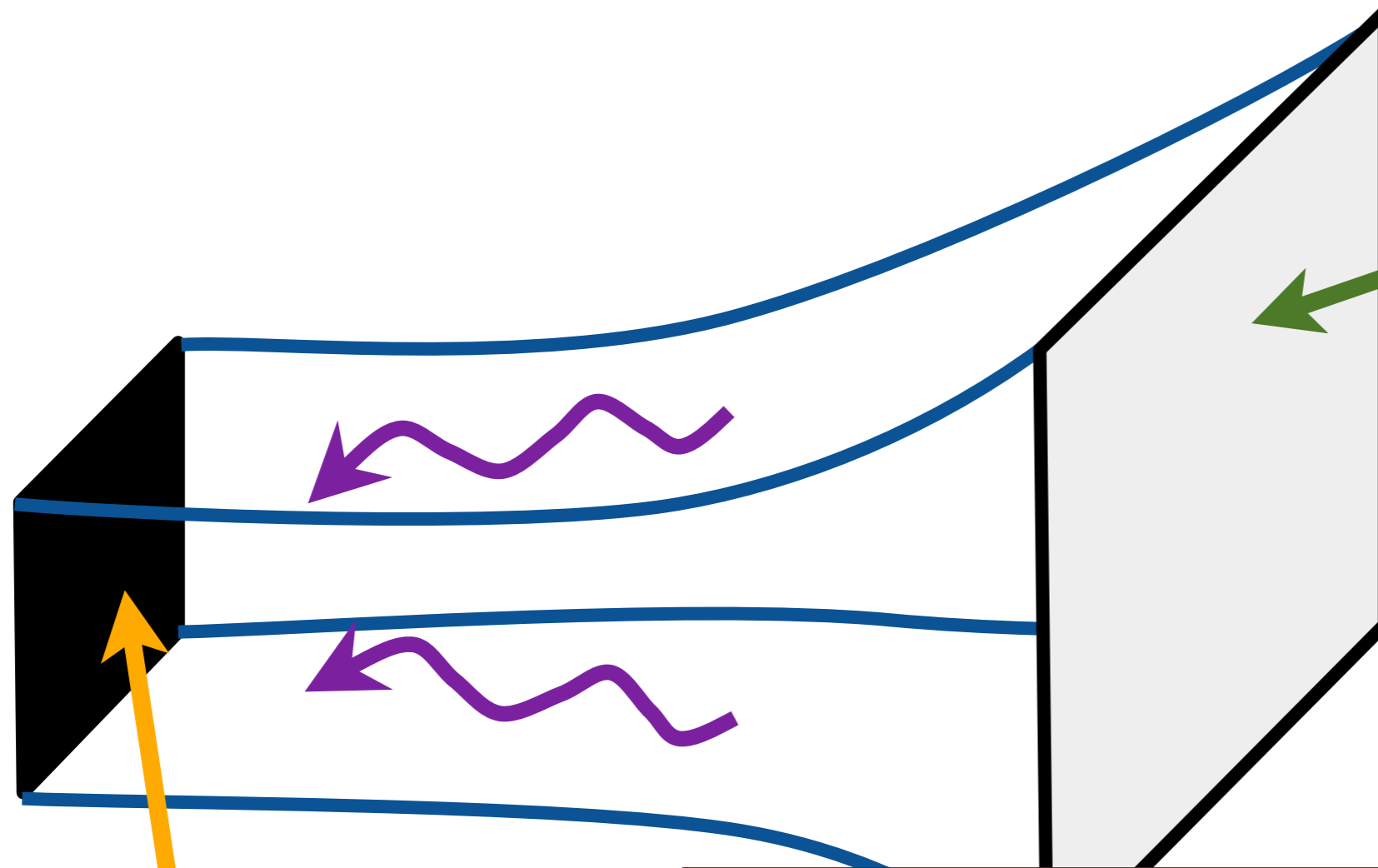


A “horizon”,
whose temperature
and entropy equal
those of the quantum
critical point

Friction of quantum
criticality = waves
falling into black brane

A 2+1
dimensional
system at its
quantum
critical point

String theory at non-zero temperatures



A 2+1 dimensional system at its quantum critical point

A “horizon”, whose temperature and entropy equal those of the quantum critical point

An (extended) Einstein-Maxwell provides successful description of dynamics of quantum critical points at non-zero temperatures (where no other methods apply)

**Quantum
superposition and
entanglement**

String theory

**Quantum critical
points of electrons
in crystals**

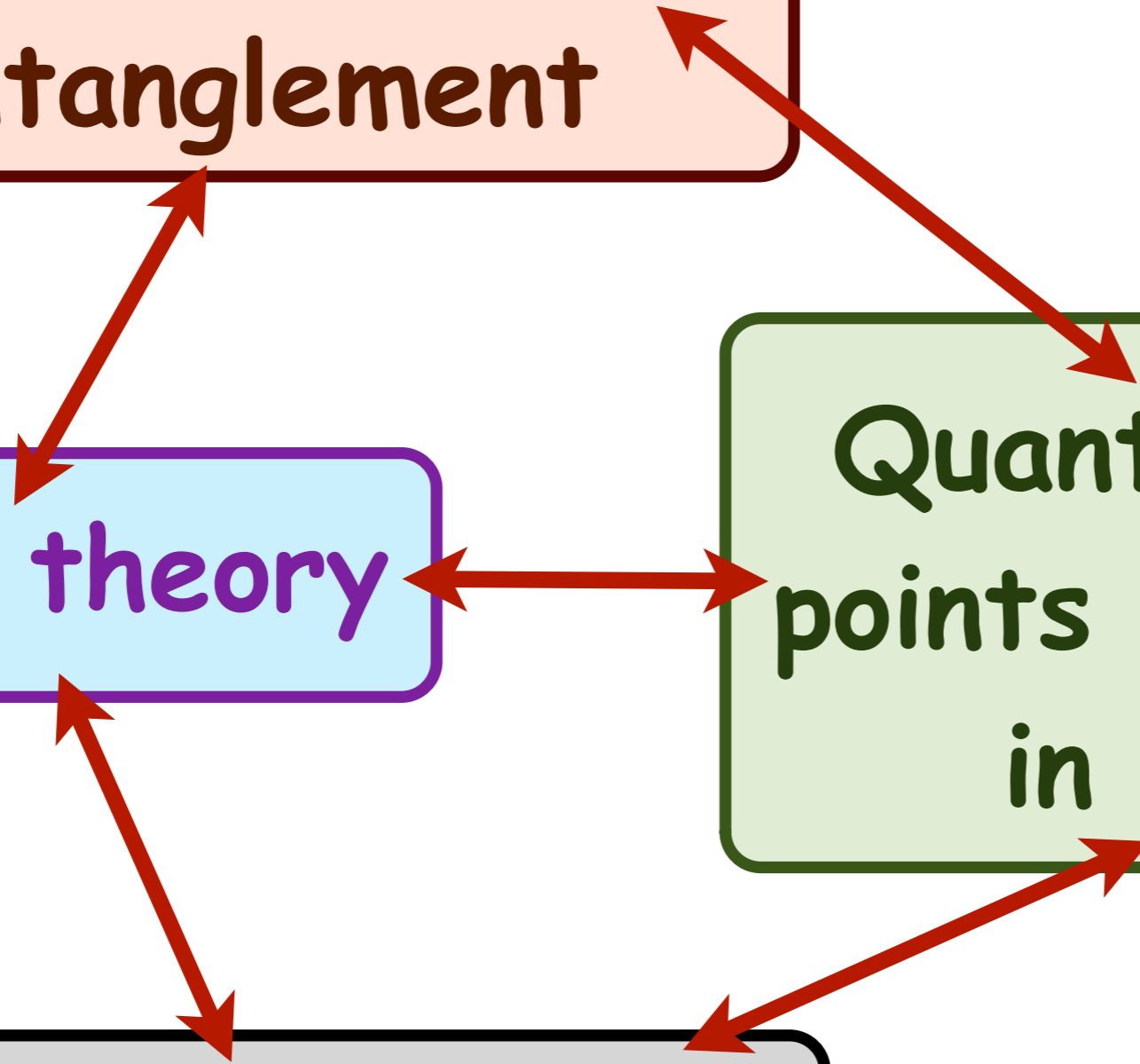
Black holes

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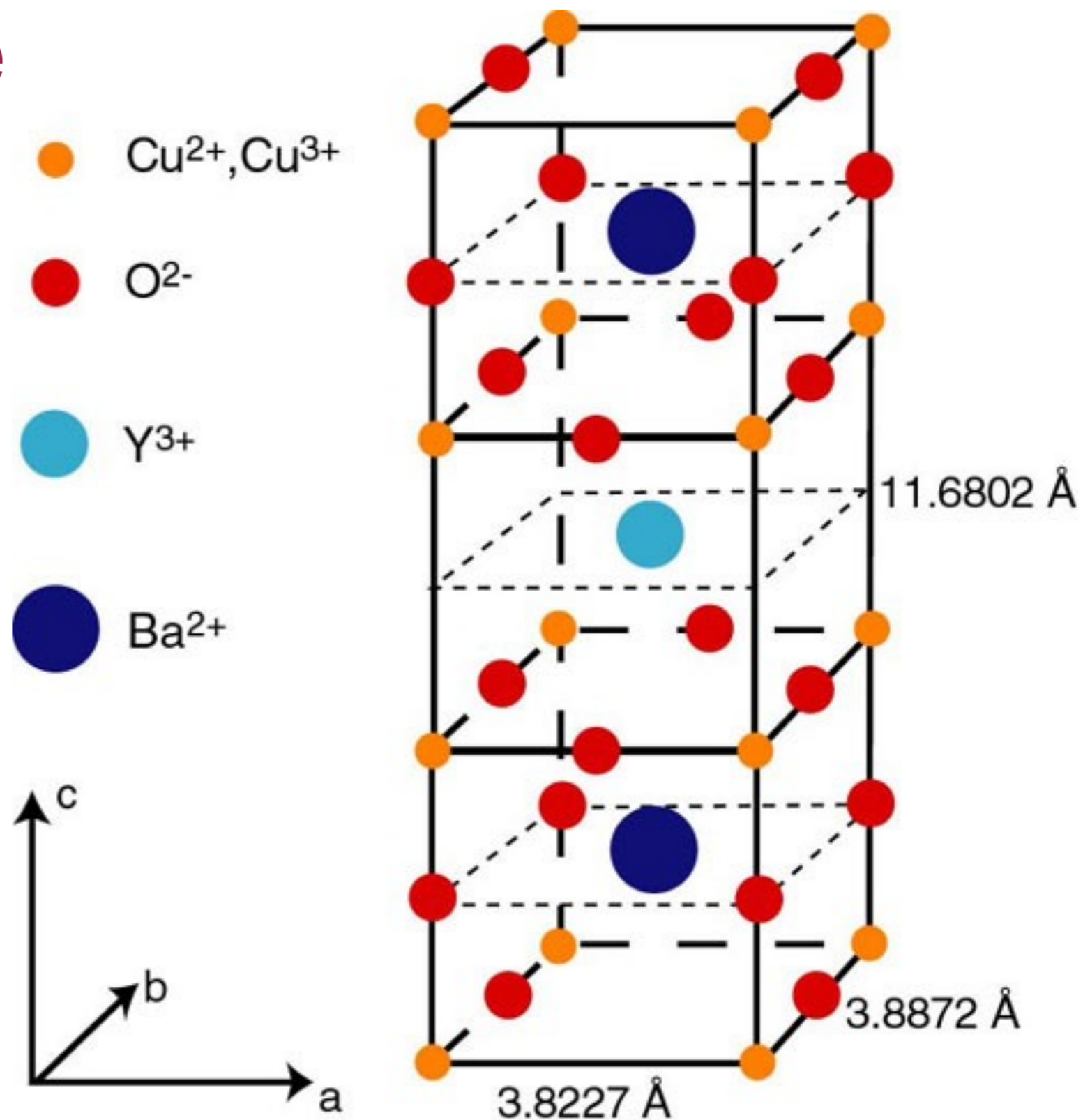
Black holes



**Metals, "strange metals", and
high temperature
superconductors**

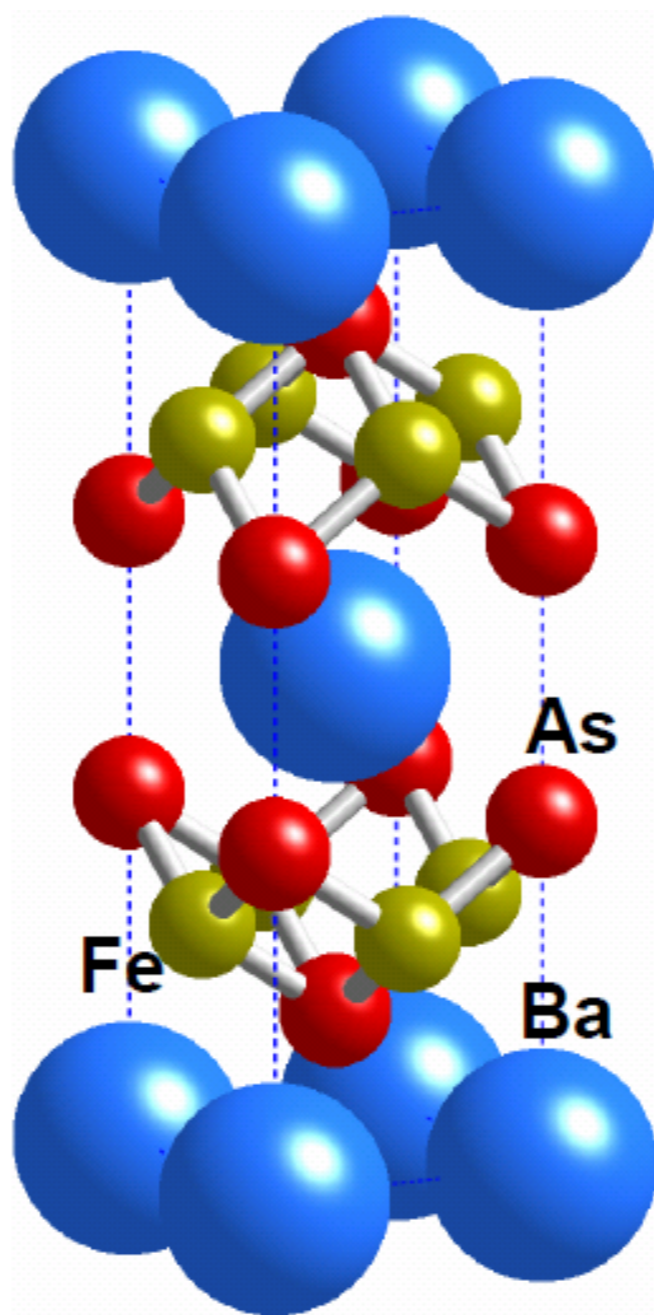
**Insights from gravitational
"duals"**

High temperature superconductors

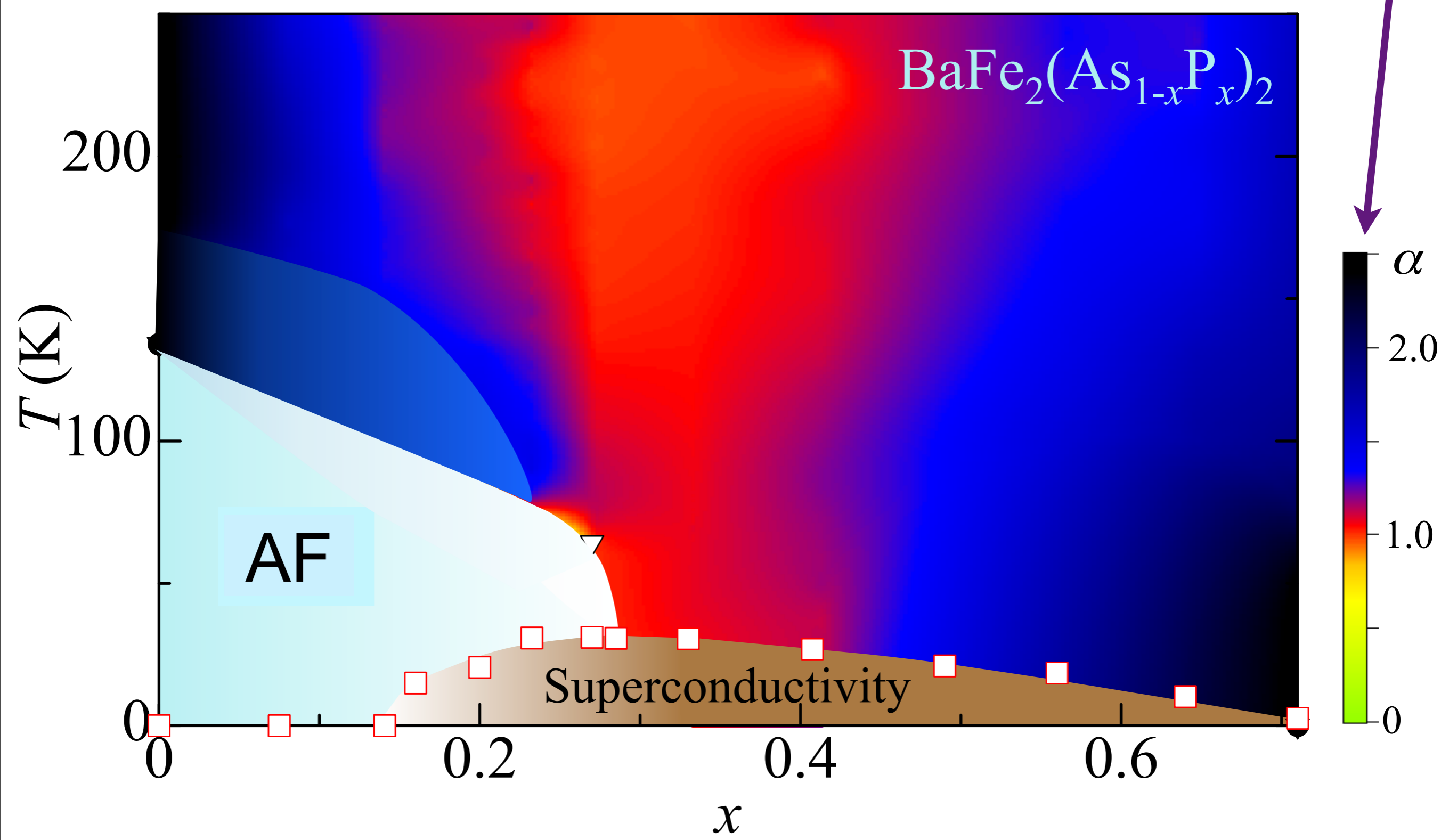


Iron pnictides:

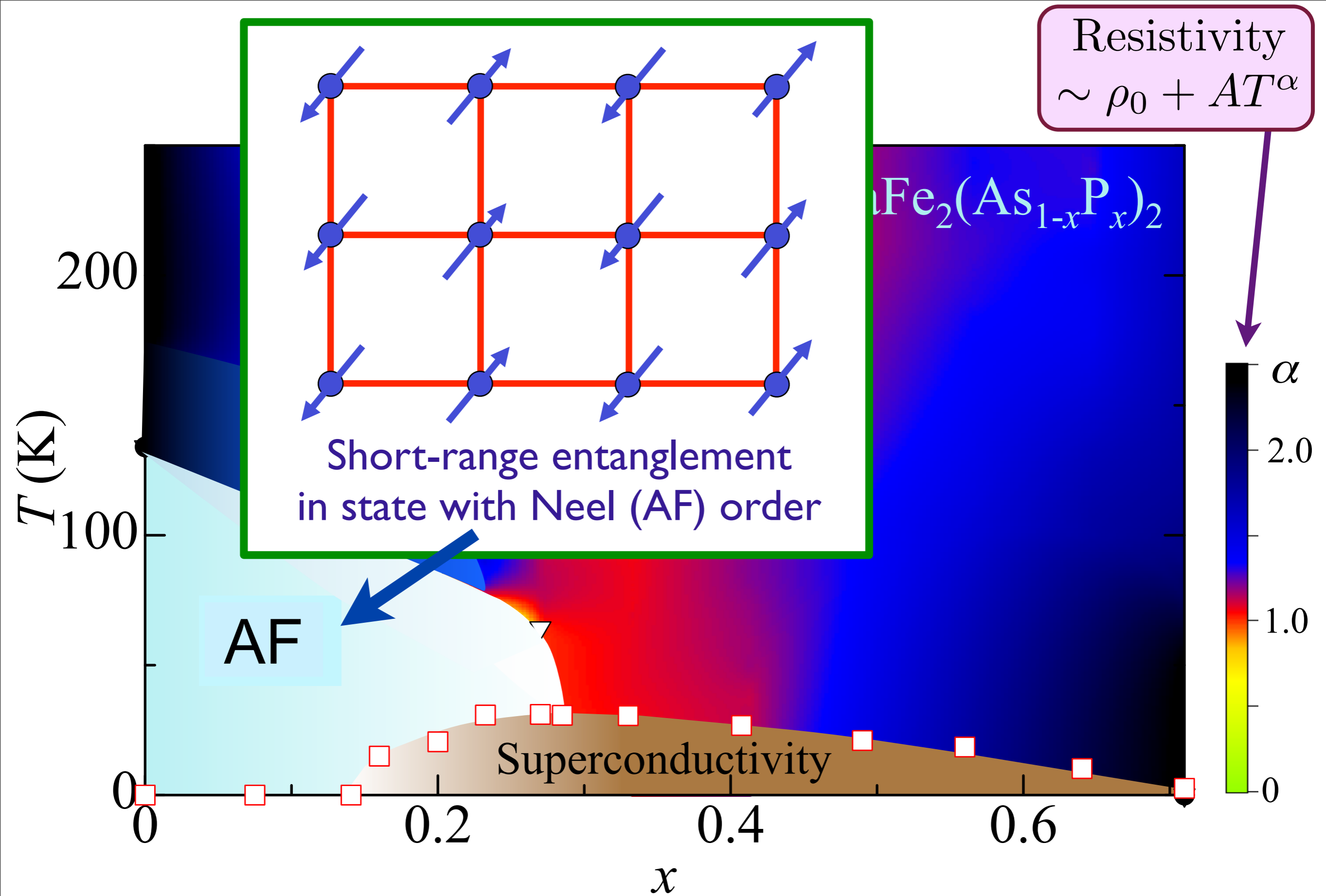
a new class of high temperature superconductors



Resistivity
 $\sim \rho_0 + AT^\alpha$

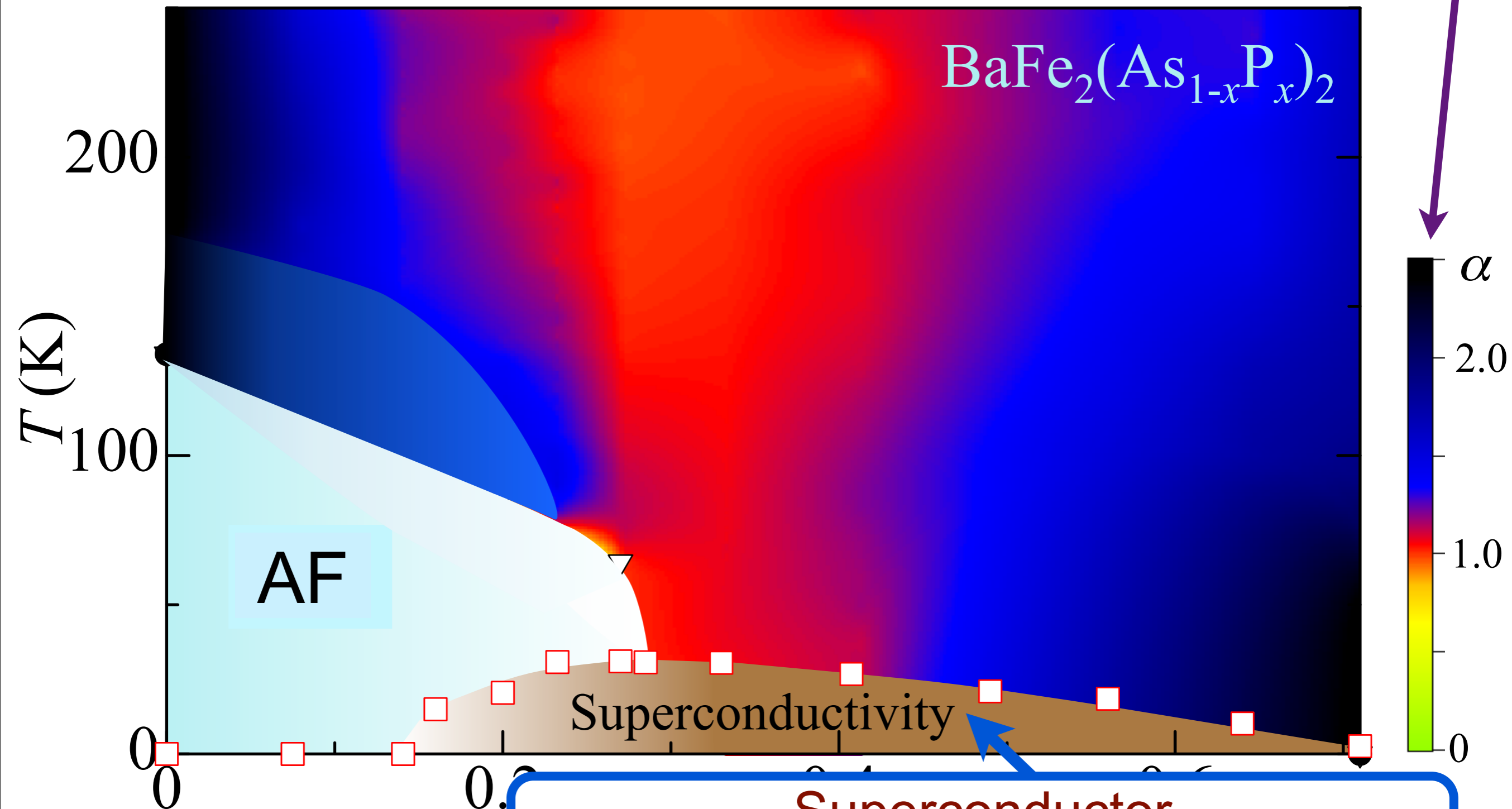


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



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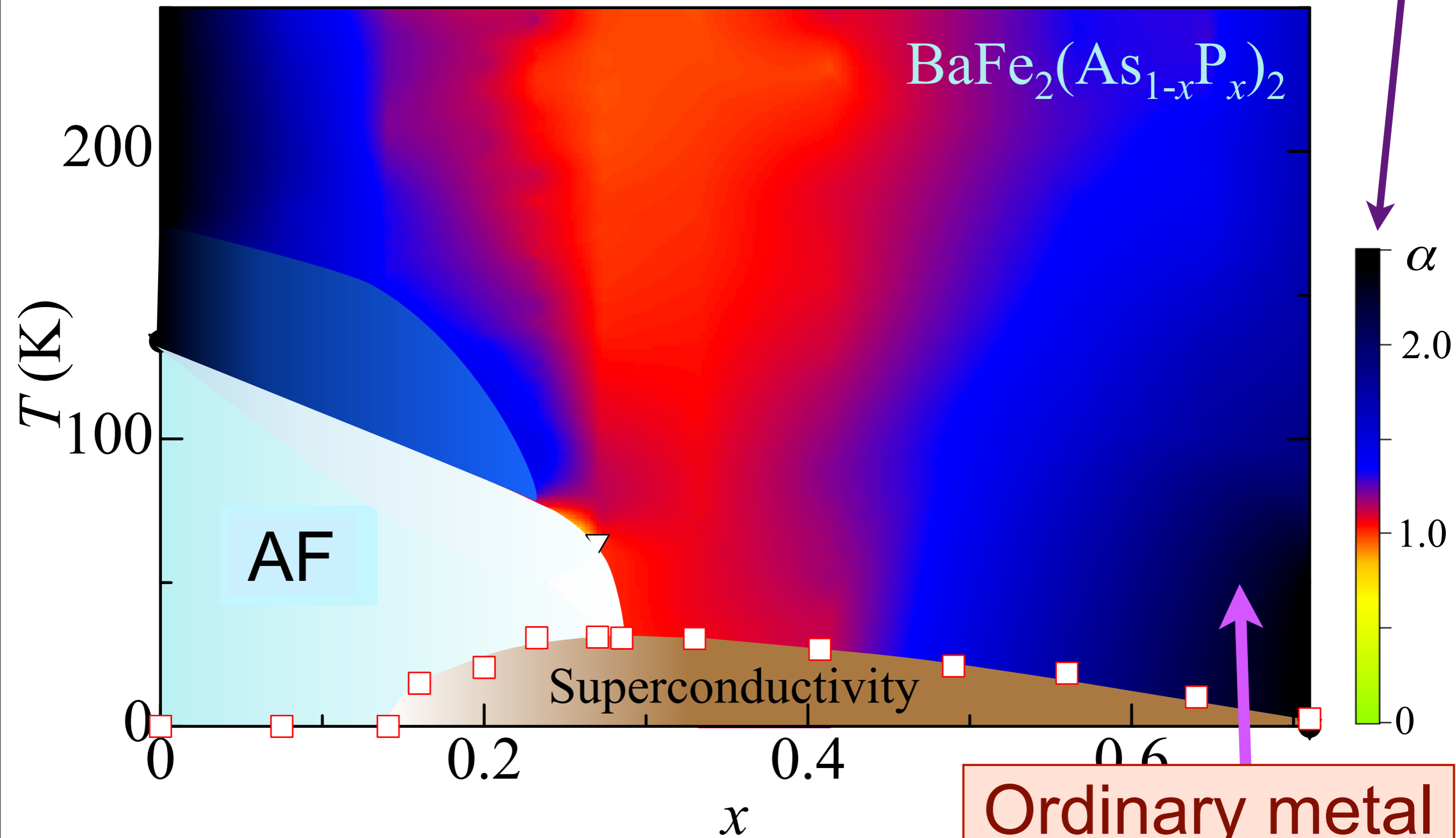
Resistivity
 $\sim \rho_0 + AT^\alpha$



Superconductor
Bose condensate of pairs of electrons
Short-range entanglement

S. Kasahara, T. Shiba
H. Ike

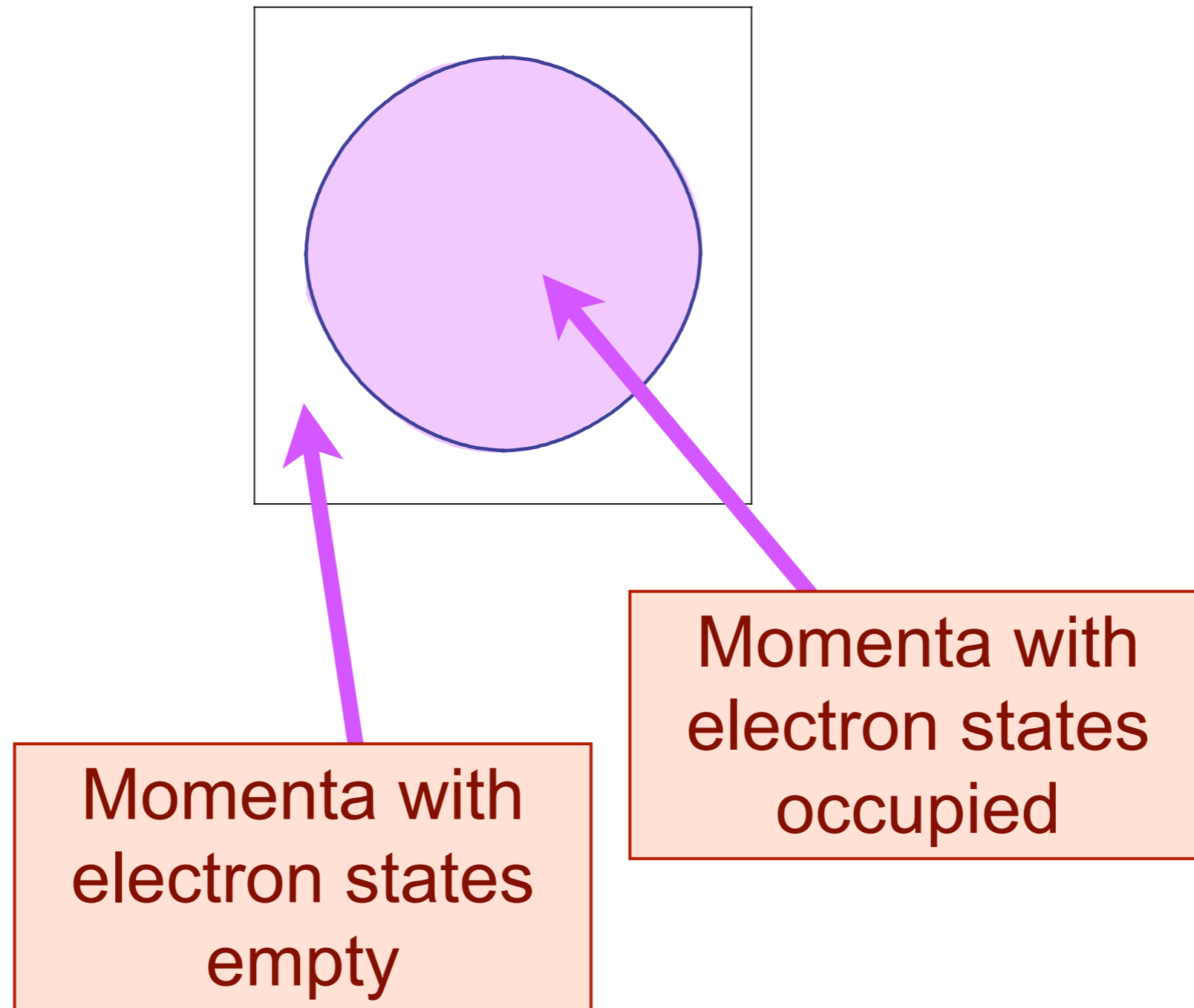
Resistivity
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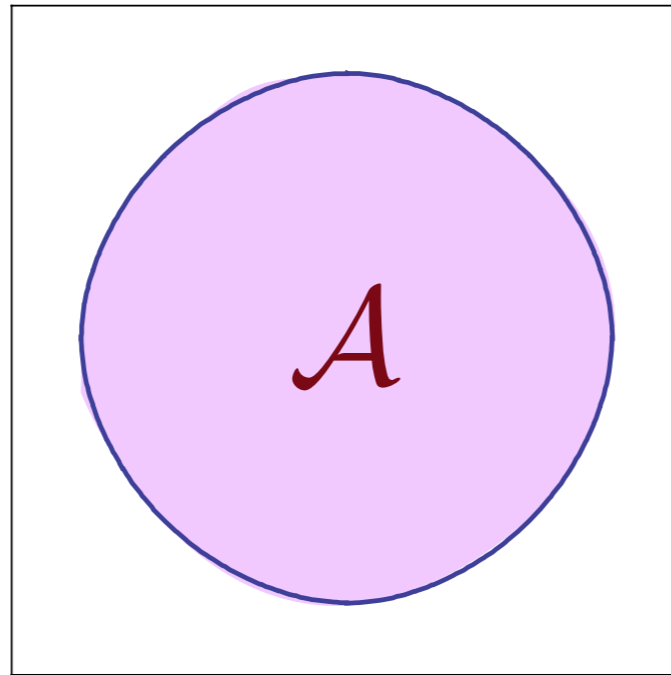
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. O.
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Ma
Physical Review B **81**, 184519 (2010)

Ordinary metal
(Fermi liquid)

Sommerfeld-Bloch theory of ordinary metals



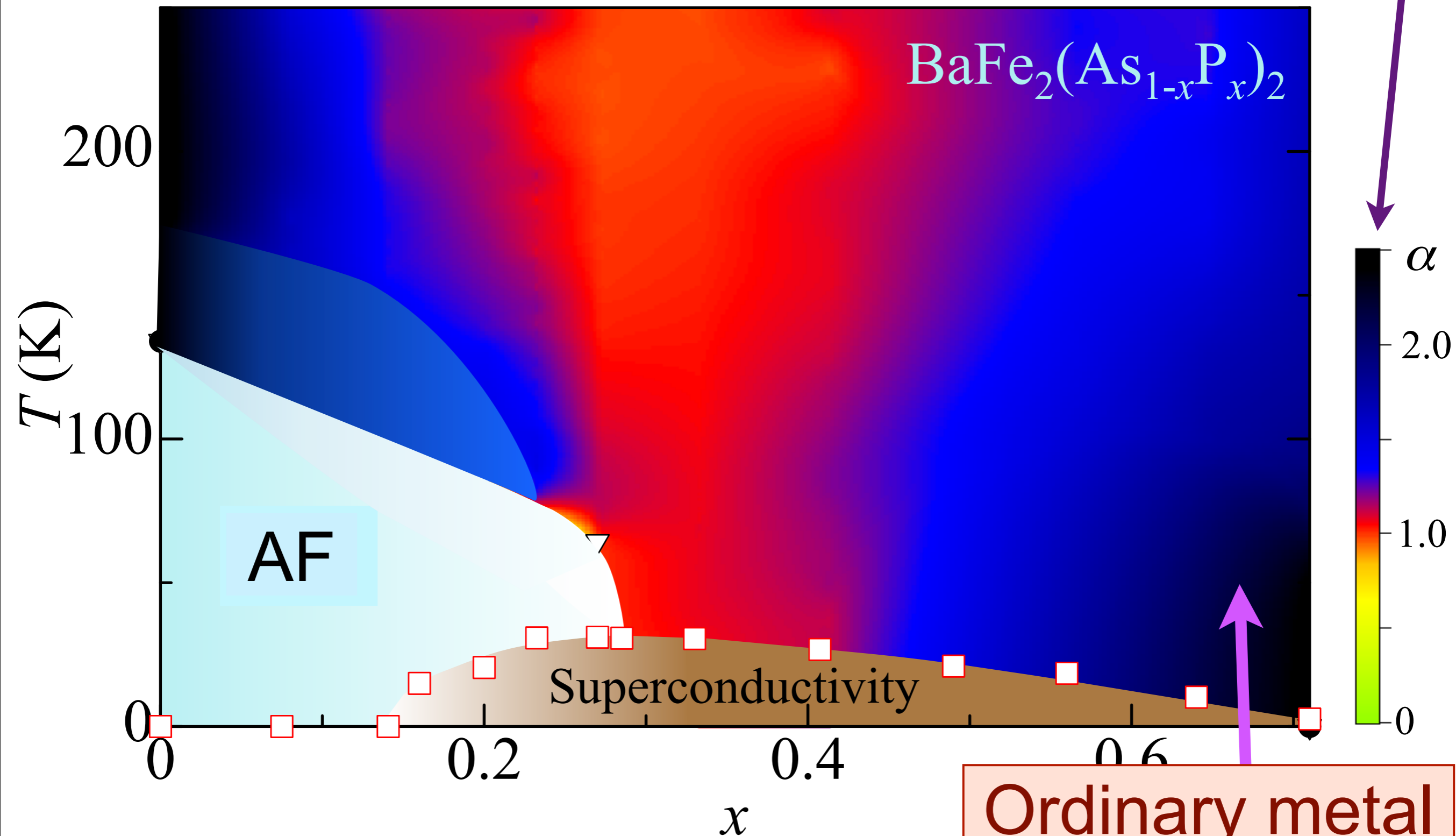
Sommerfeld-Bloch theory of ordinary metals



**Key feature of the theory:
the Fermi surface**

- Area enclosed by the Fermi surface $\mathcal{A} = Q$,
the electron density
- Excitations near the Fermi surface are responsible for the familiar properties of ordinary metals, such as resistivity $\sim T^2$.

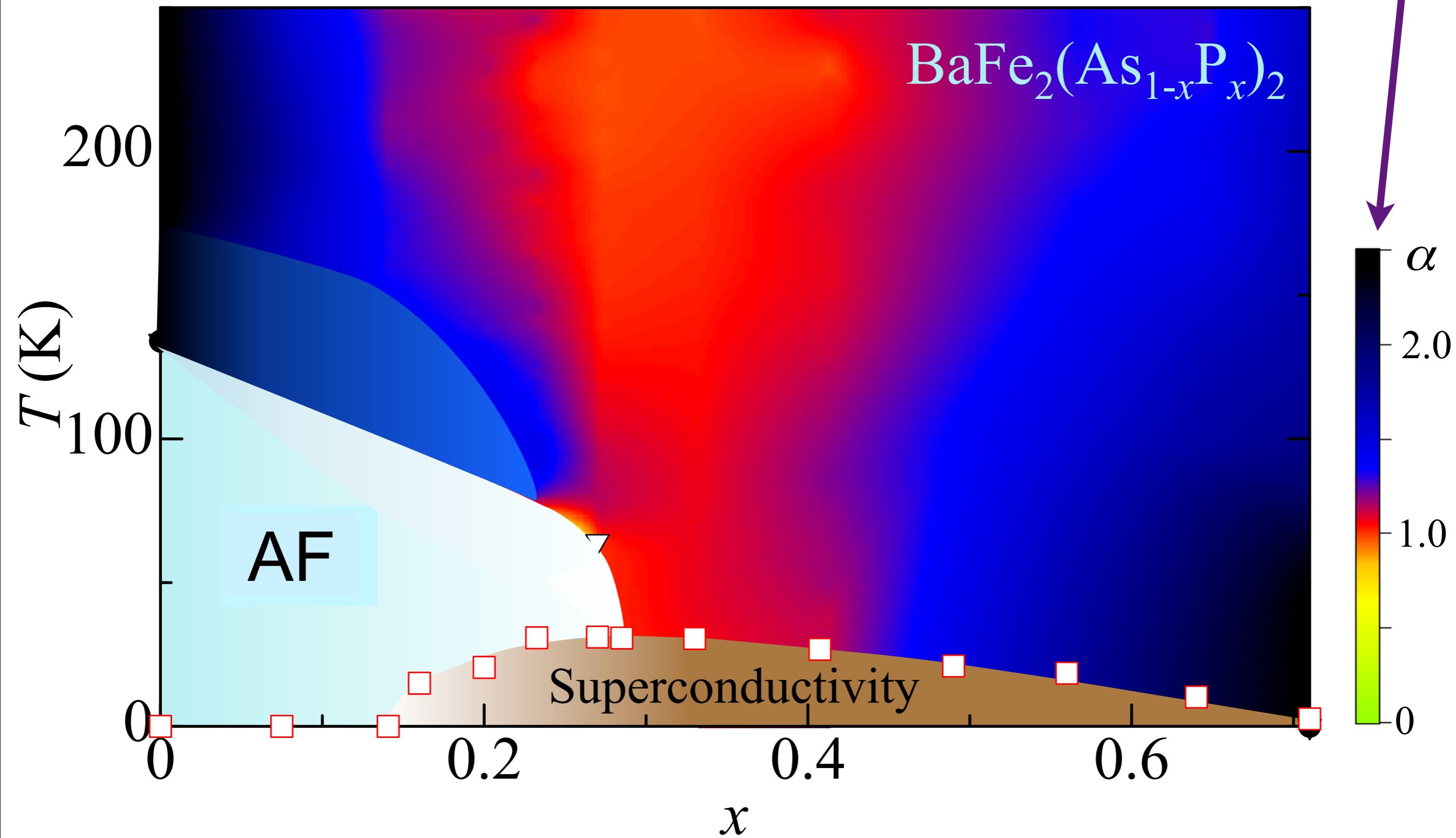
Resistivity
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S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. O.
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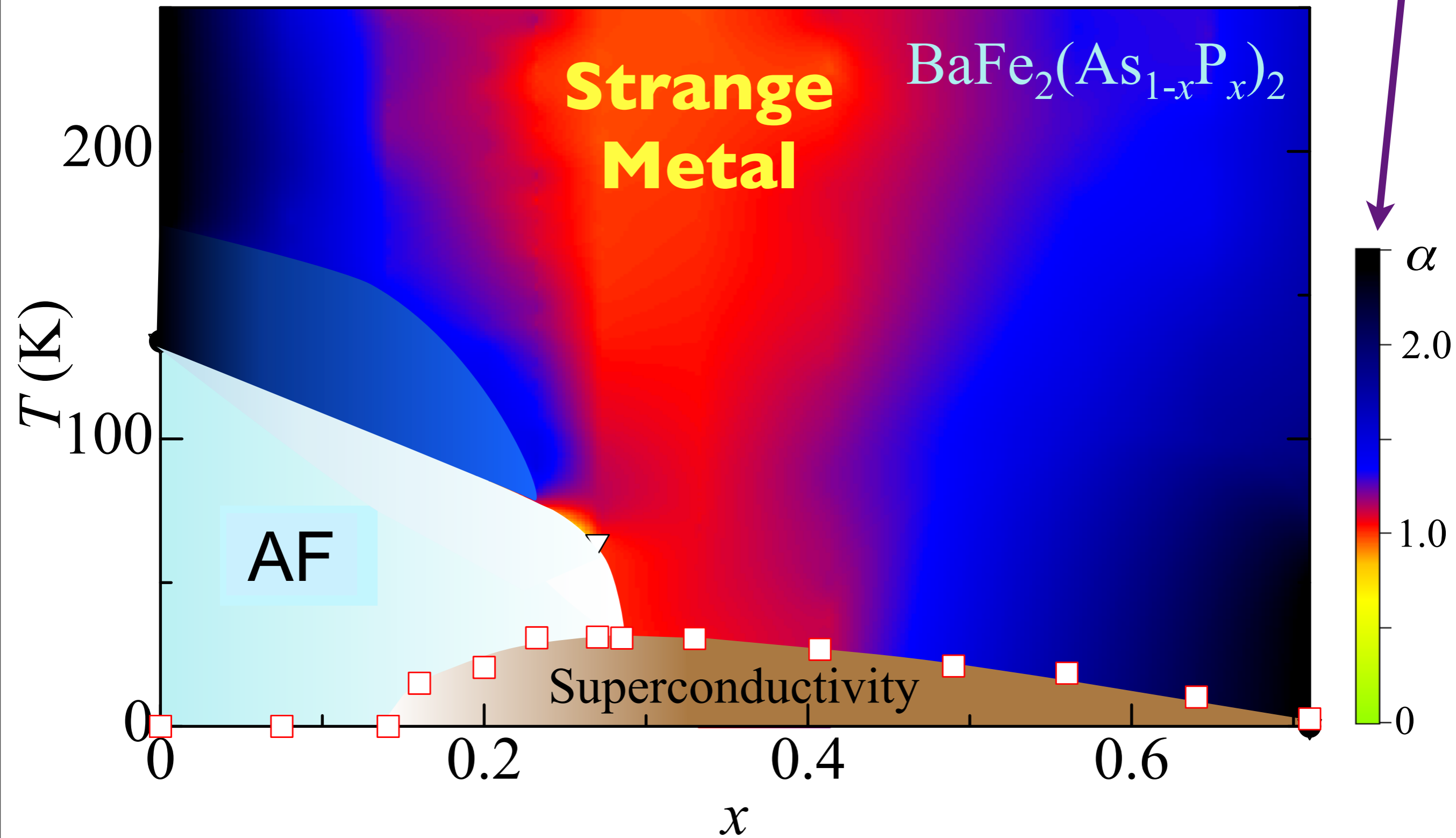
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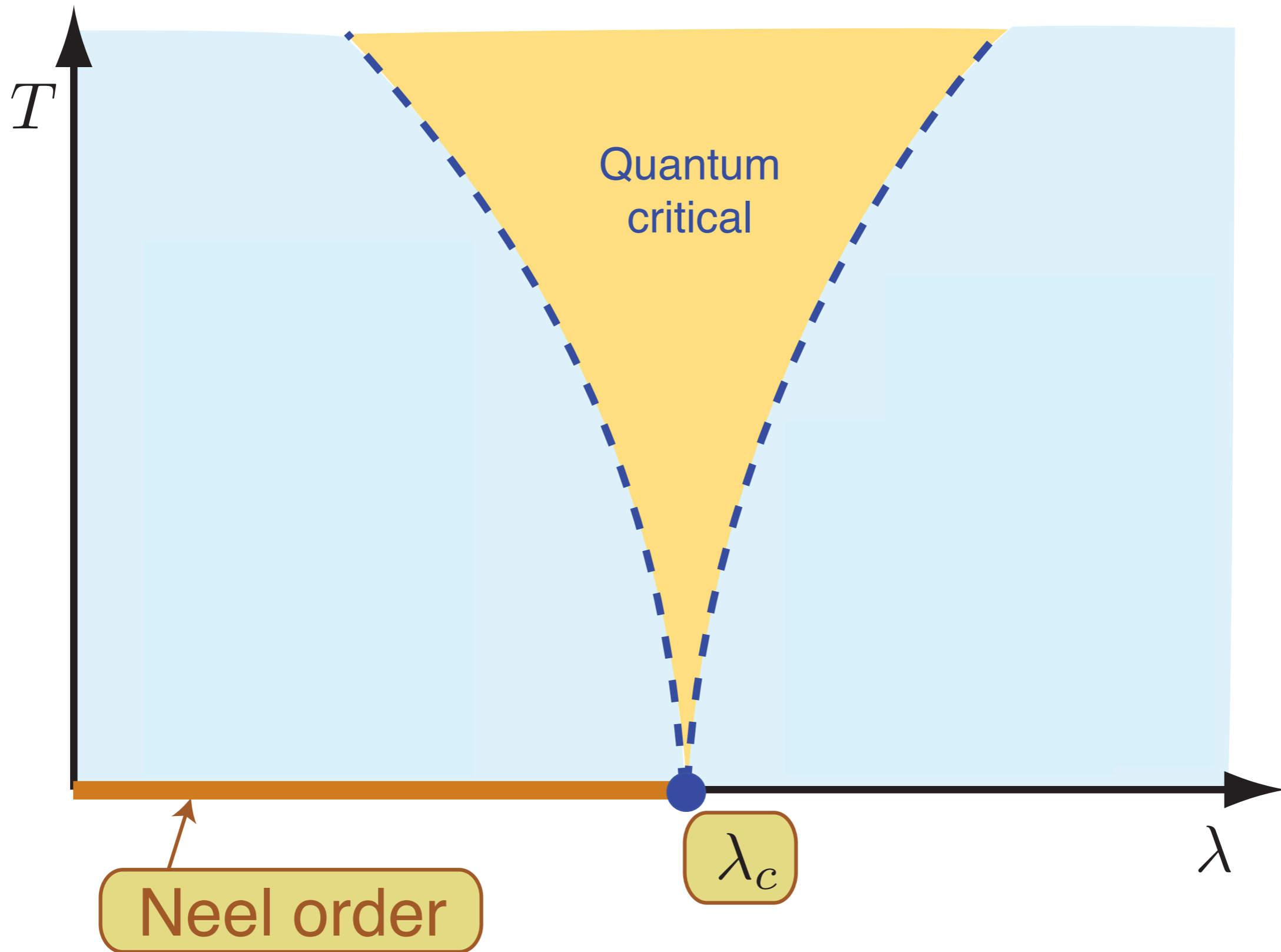
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H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,
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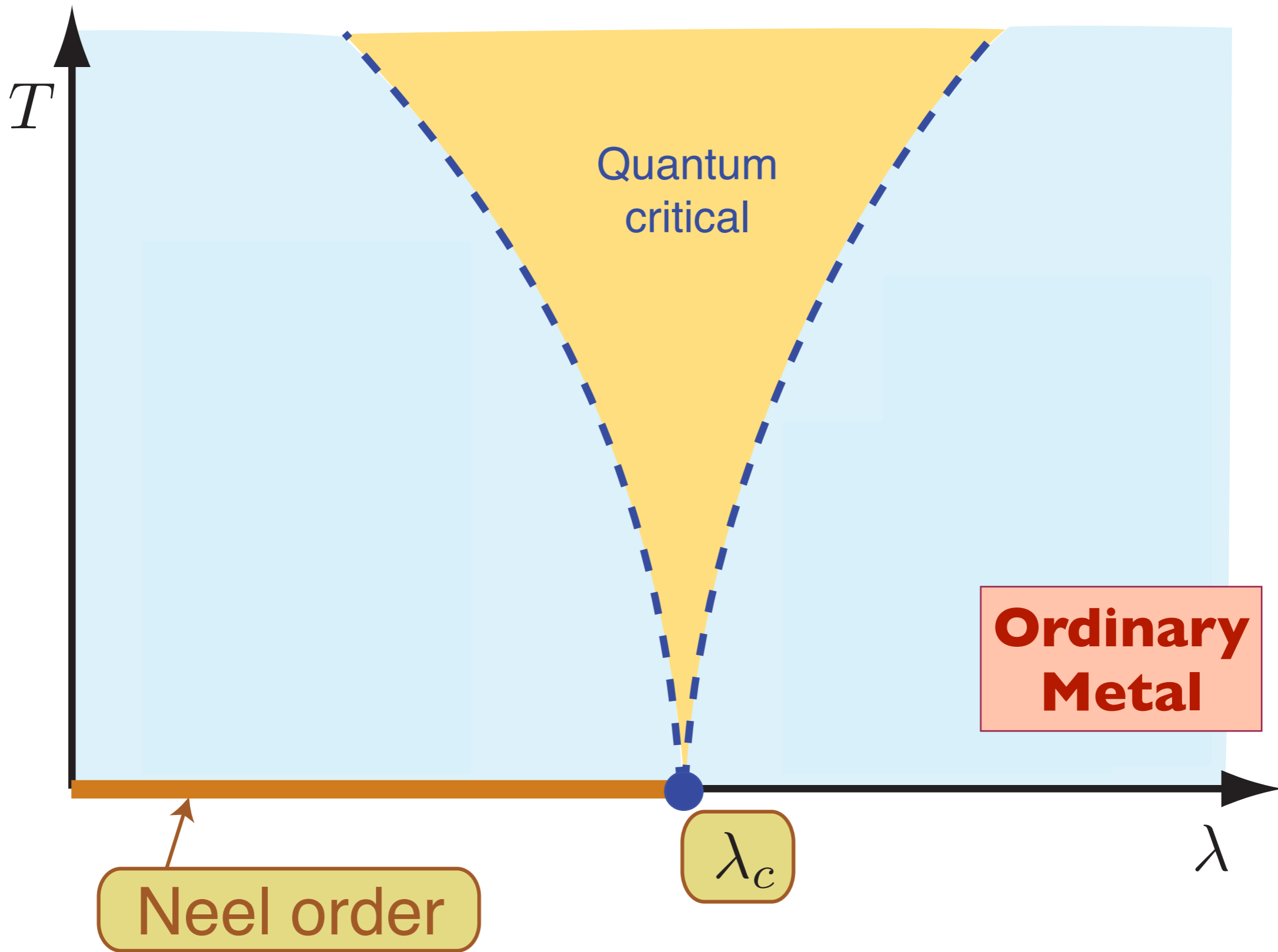
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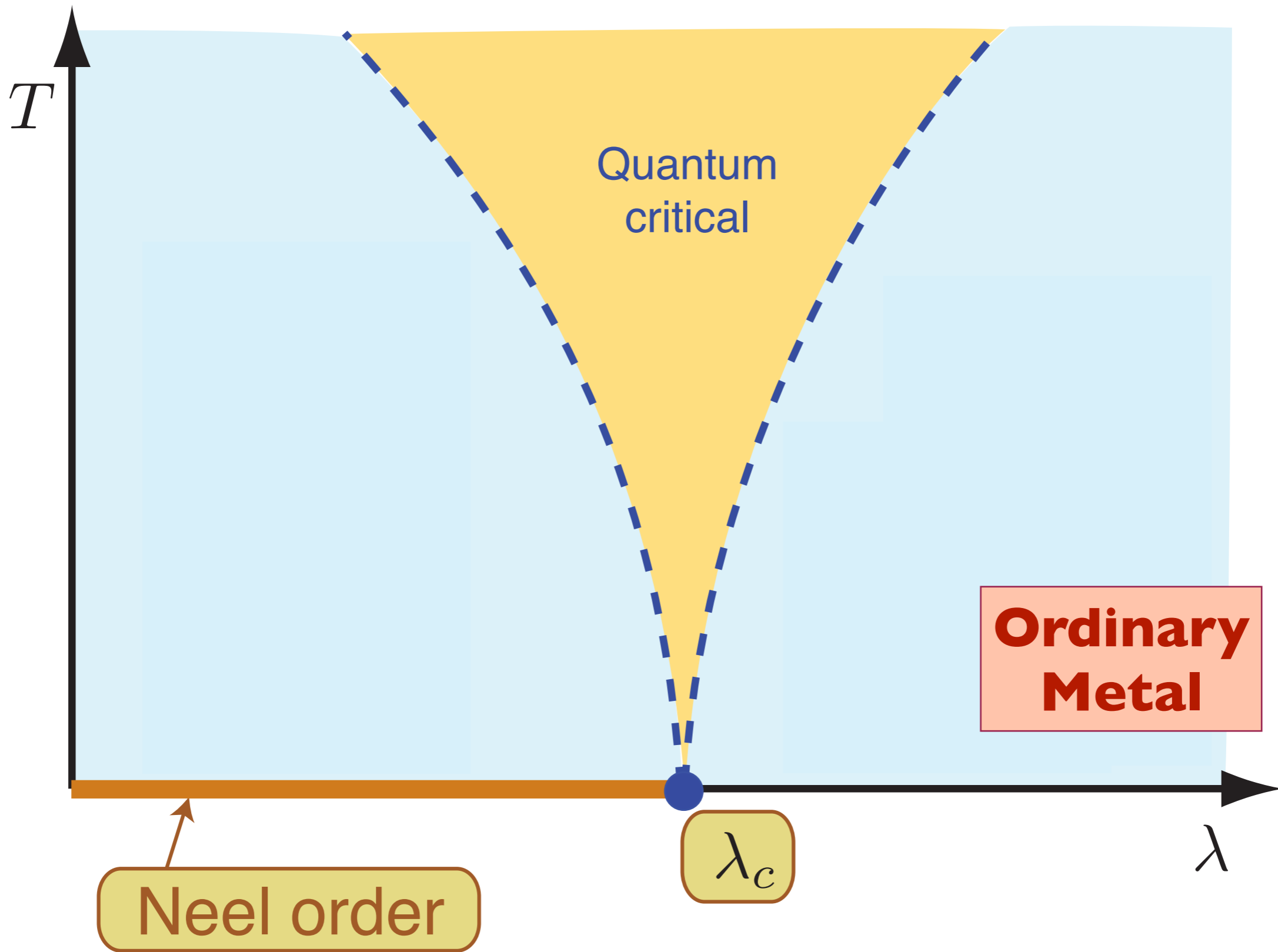


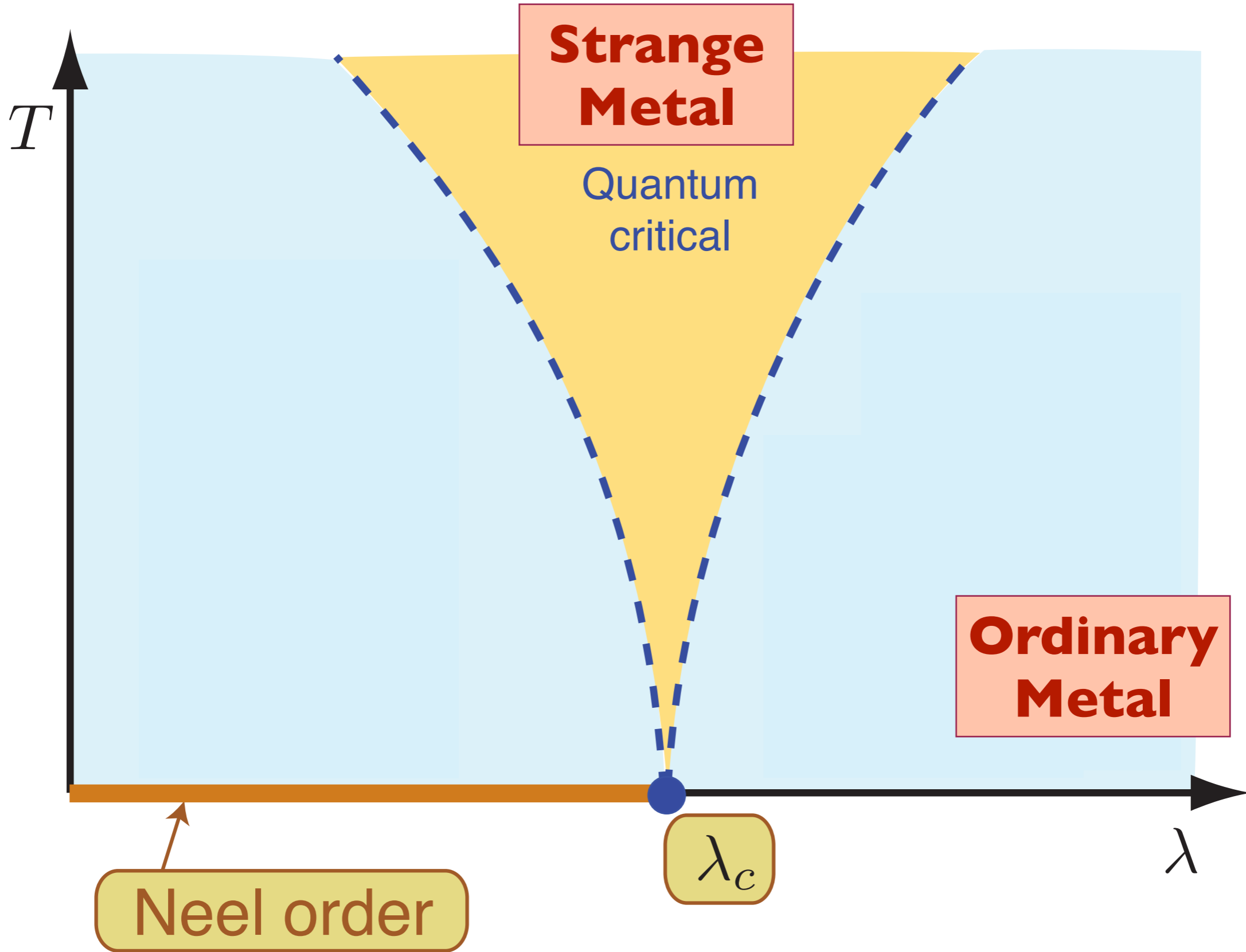
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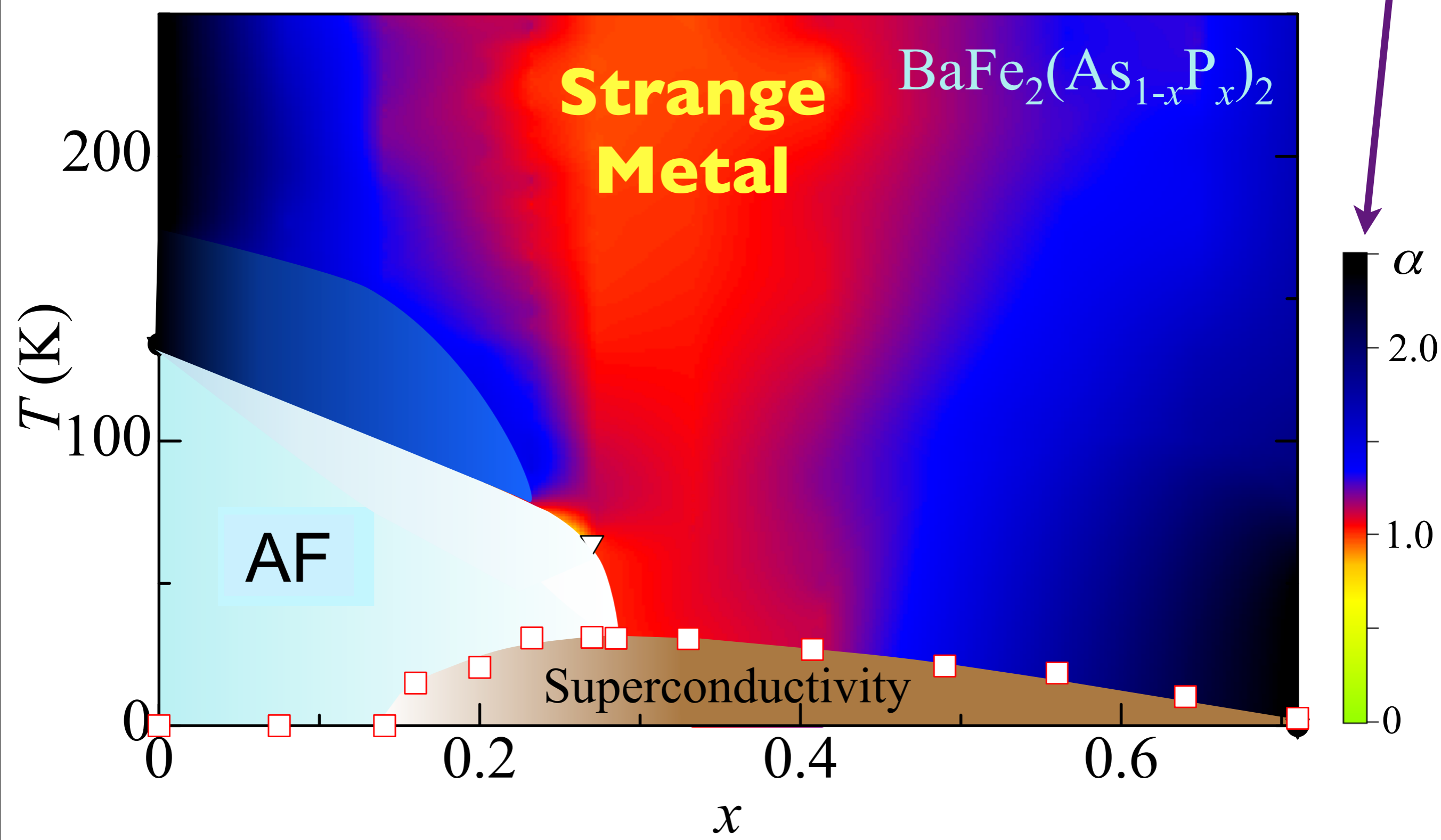






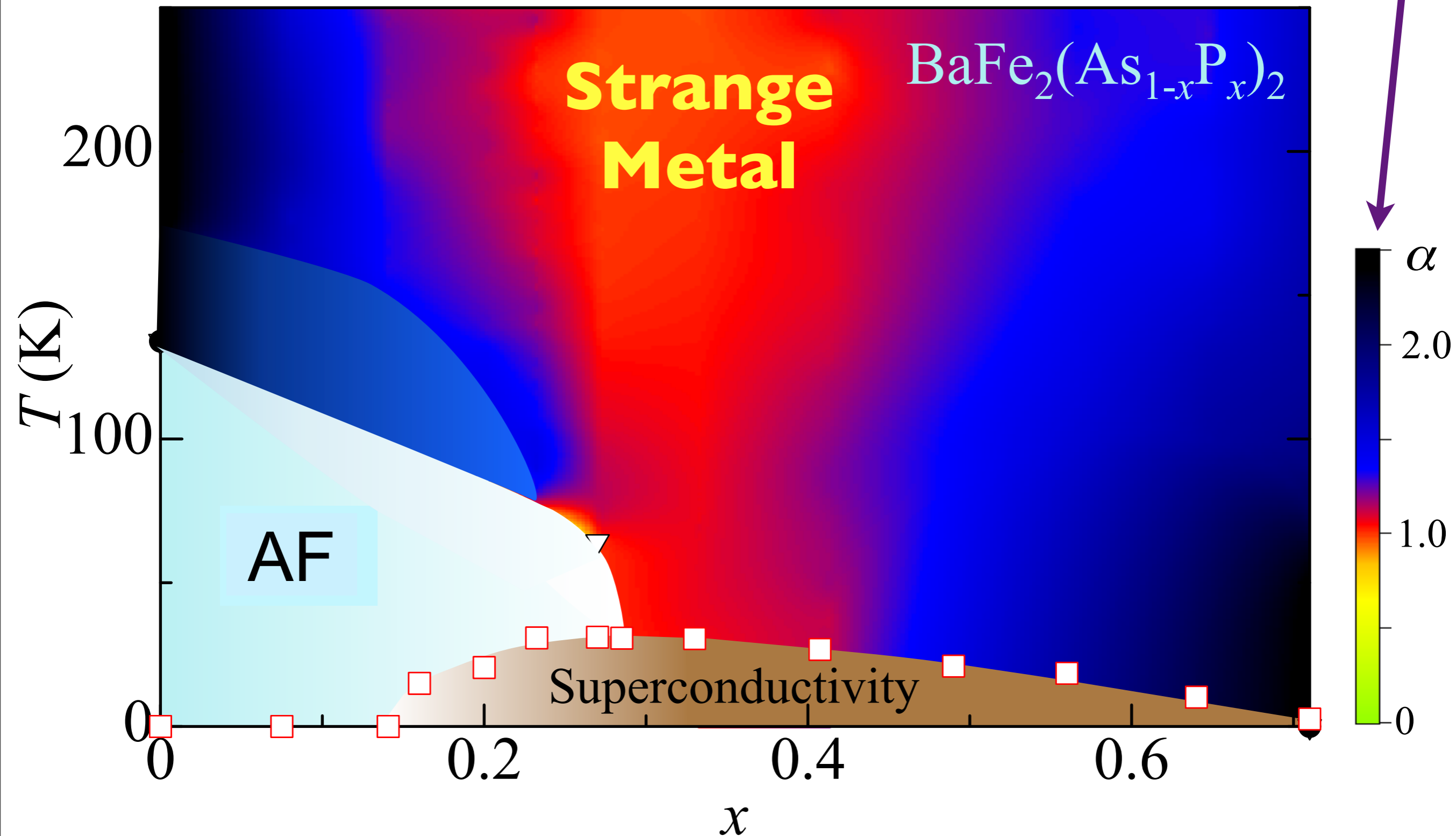


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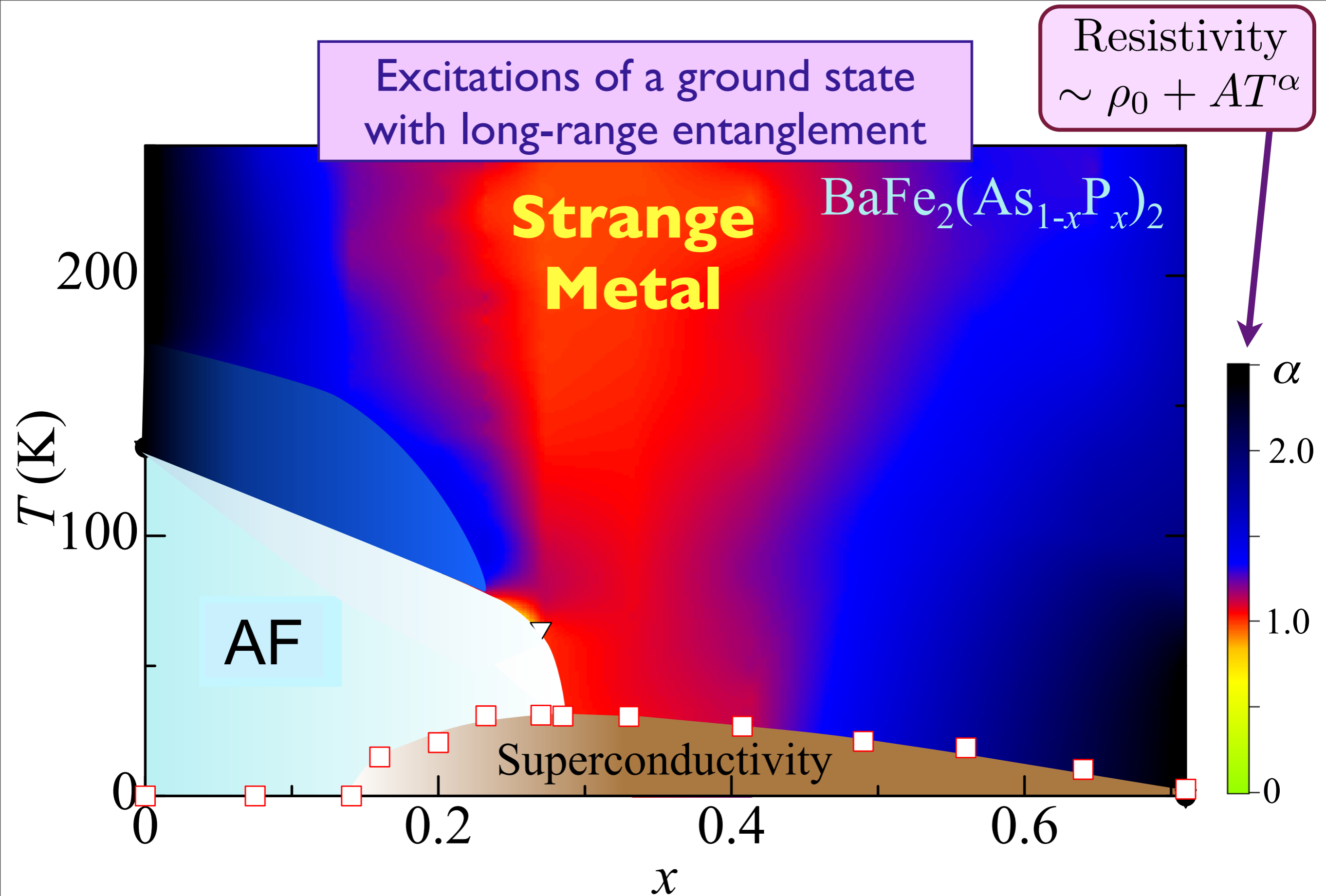


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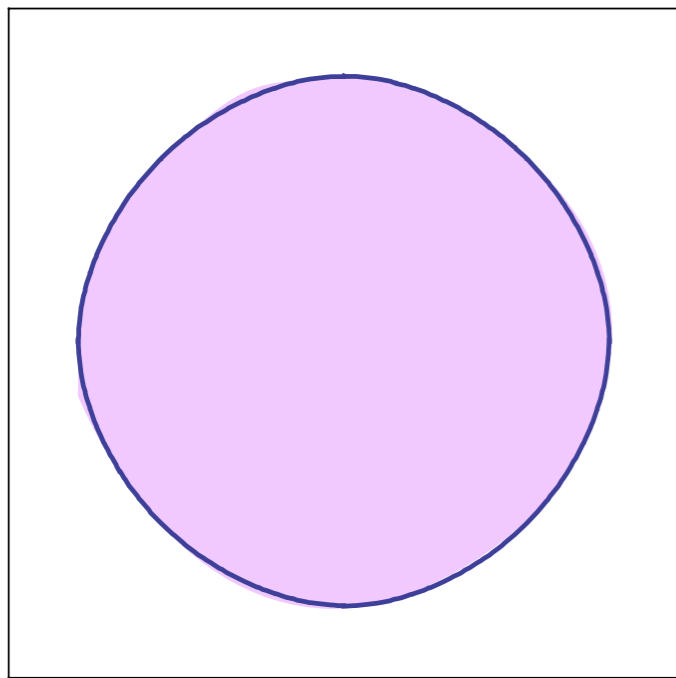
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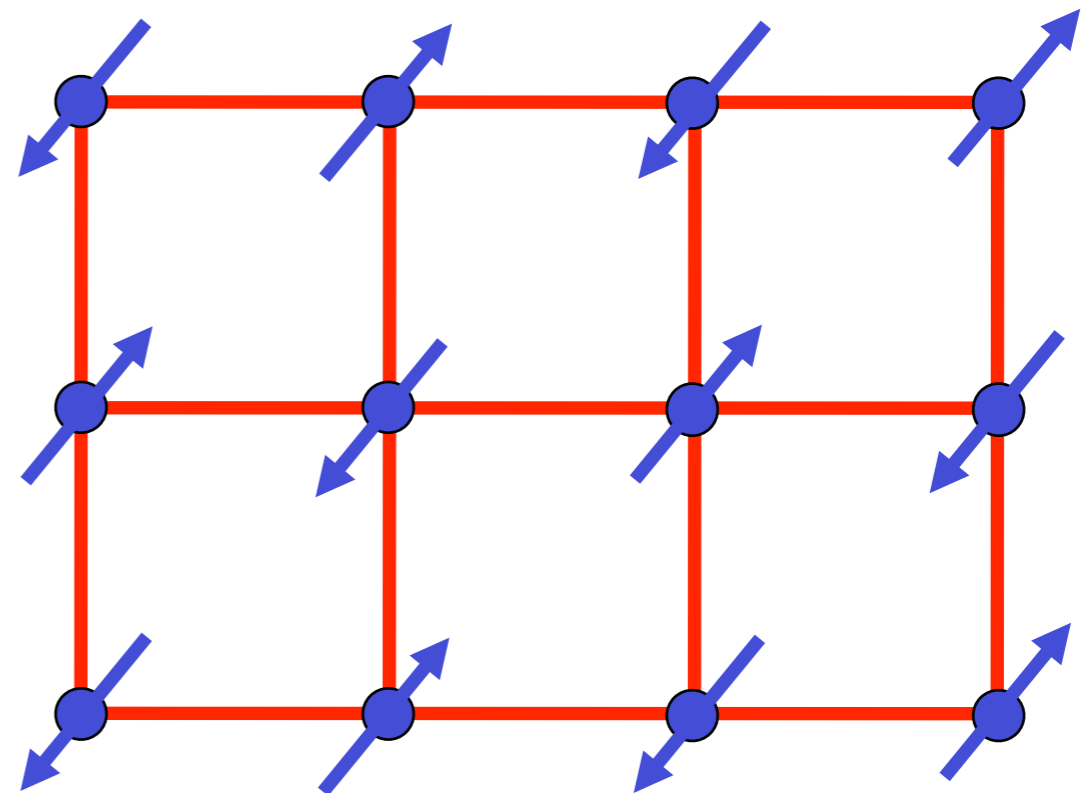
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Key (difficult) problem:

Describe quantum critical points and phases of systems with Fermi surfaces leading to metals with novel types of long-range entanglement



+



Challenge to string theory:

Describe quantum critical points
and phases of metals

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Describe quantum critical points
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Can we obtain gravitational theories
of superconductors and
ordinary Sommerfeld-Bloch metals ?

Challenge to string theory:

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and phases of metals

Can we obtain gravitational theories
of superconductors and
ordinary Sommerfeld-Bloch metals ?

Yes

T. Nishioka, S. Ryu, and T. Takayanagi, JHEP **1003**, 131 (2010)

G.T. Horowitz and B. Way, JHEP **1011**, 011 (2010)

S. Sachdev, Physical Review D **84**, 066009 (2011)

Challenge to string theory:

Describe quantum critical points
and phases of metals

Do the “holographic” gravitational theories
also yield metals distinct from
ordinary Sommerfeld-Bloch metals ?

Challenge to string theory:

Describe quantum critical points
and phases of metals

Do the “holographic” gravitational theories
also yield metals distinct from
ordinary Sommerfeld-Bloch metals ?

Yes, lots of them, with
many “strange” properties !

Challenge to string theory:

Describe quantum critical points
and phases of metals

How do we discard artifacts, and choose the
holographic theories applicable to condensed matter physics ?

Challenge to string theory:

Describe quantum critical points
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How do we discard artifacts, and choose the
holographic theories applicable to condensed matter physics ?

The simplest example of a
“strange metal” is realized by
fermions with a Fermi surface
coupled to an Abelian
or non-Abelian gauge field.

Challenge to string theory:

Describe quantum critical points
and phases of metals

How do we discard artifacts, and choose the
holographic theories applicable to condensed matter physics ?

Choose the theories with the
proper entropy density

Checks: these theories also have the
proper entanglement entropy and
Fermi surface size !

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Conclusions

Phases of matter with long-range quantum entanglement are prominent in numerous modern materials.

Conclusions

Simplest examples of long-range entanglement are at quantum-critical points of insulating antiferromagnets

Conclusions

More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory

Conclusions

String theory and gravity in emergent dimensions offer a remarkable new approach to describing states with long-range quantum entanglement.

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String theory and gravity in emergent dimensions offer a remarkable new approach to describing states with long-range quantum entanglement.

Much recent progress offers hope of a holographic description of “strange metals”