

Thermal Hall effect in cuprates and spin liquids

CIFAR - Quantum Materials Program Meeting
Zoom, May 11-13, 2020

Subir Sachdev

Talk online: sachdev.physics.harvard.edu



Enhanced thermal Hall effect in the square lattice Neel state,
Nature Physics **15**, 1290 (2019)

Gauge theories for the thermal Hall effect, arXiv:2002.01947

Unquantized thermal Hall effect in quantum spin liquids
with spinon Fermi surfaces, arXiv:2005.02396



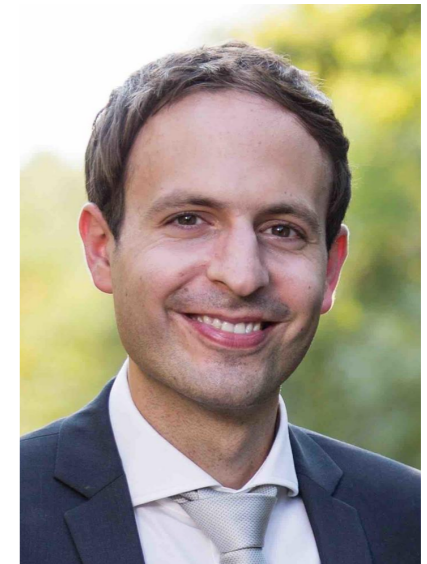
Shubhayu
Chatterjee



Haoyu Guo



Rhine
Samajdar



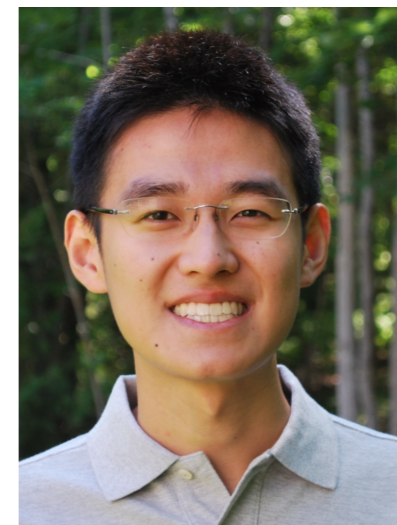
Mathias
Scheurer



Yanting
Teng



Cenke
Xu



Yunchao
Zhang

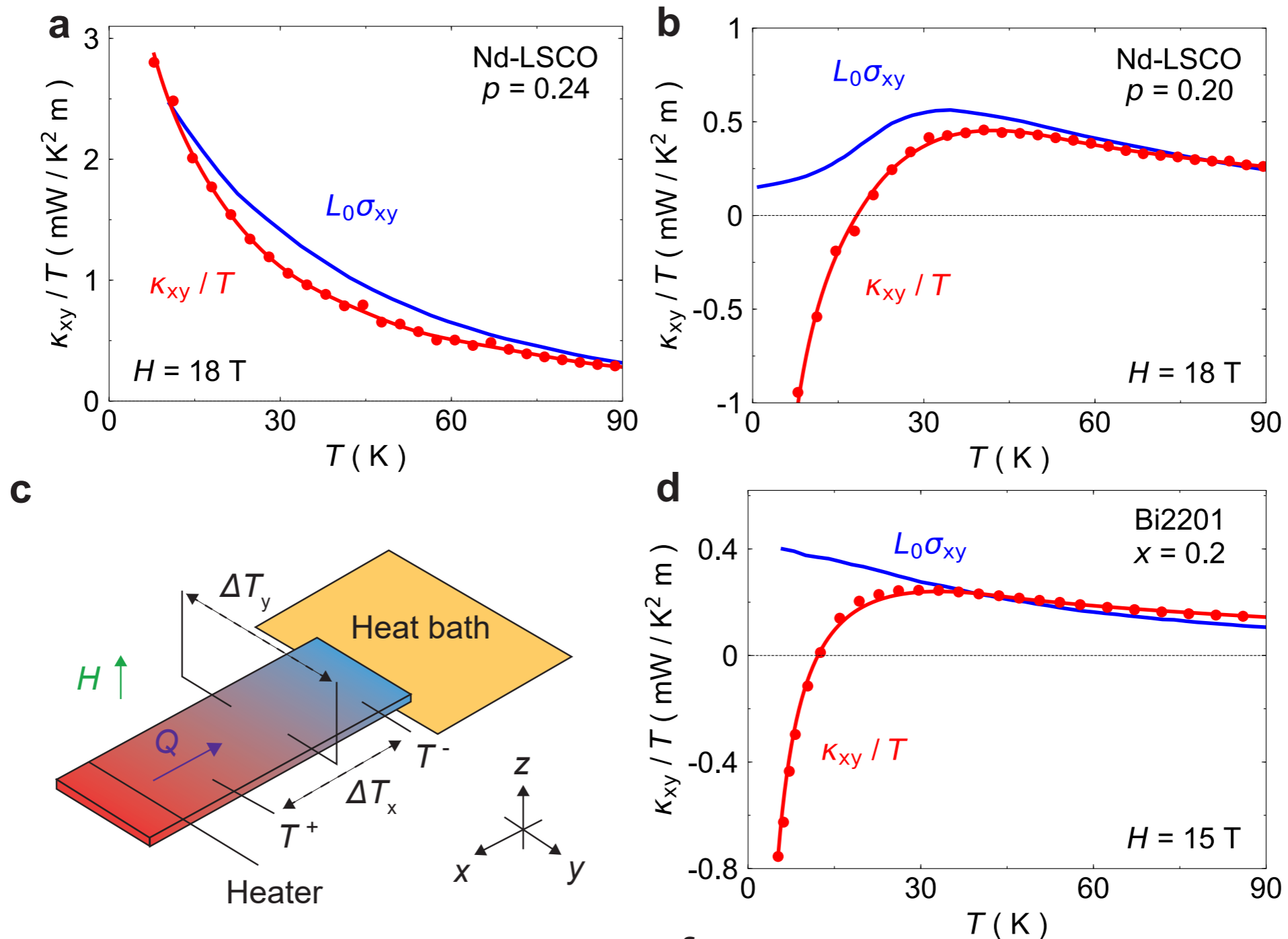
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cuprate pseudogap

2. Thermal Hall in the Neel state
enhanced by a proximate
abelian chiral spin liquid

3. Thermal Hall across a
non-Abelian spin liquid to
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Giant thermal Hall conductivity in the pseudogap phase of cuprate superconductors

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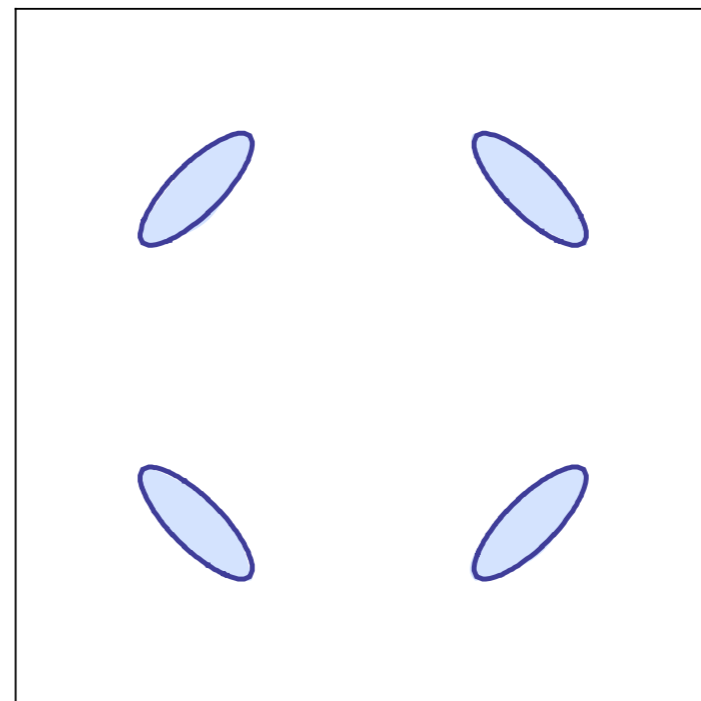
Model for the pseudogap

Fermionic ‘chargons’ of density δ in hole pockets.

The fermions carry electromagnetic gauge charge $+e$, and charges $p = \pm 1$ under an emergent U(1) gauge field.

v is a valley index, v_{dis} is an impurity potential.

$$\mathcal{L}_f = \sum_{v=1,2} \sum_{p=\pm 1} f_{pv}^\dagger \left(\frac{\partial}{\partial \tau} - \mu - ipa_\tau - \frac{(\nabla - ip\mathbf{a} - ie\mathbf{A}_{\text{em}})^2}{2m^*} \right) f_{pv} + v_{\text{dis}}(\mathbf{r}) f_{pv}^\dagger f_{pv}$$



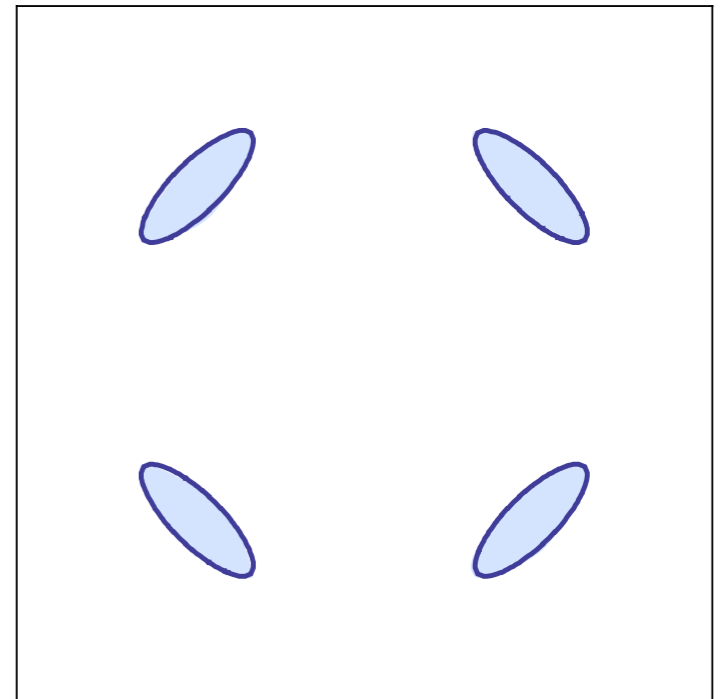
Thermal Hall conductivity

$$\mathcal{L}_f = \sum_{v=1,2} \sum_{p=\pm 1} f_{pv}^\dagger \left(\frac{\partial}{\partial \tau} - \mu - ipa_\tau - \frac{(\nabla - ip\mathbf{a} - ie\mathbf{A}_{\text{em}})^2}{2m^*} \right) f_{pv} + v_{\text{dis}}(\mathbf{r}) f_{pv}^\dagger f_{pv}$$

Leading order fermionic contribution is that implied by the Wiedemann-Franz law.

$$\sigma_{xy} = \left(\frac{\delta e^2 \tau}{m^*} \right) \omega_c \mathcal{T}$$

$$\kappa_{xy}^0 = \frac{\pi^2 T}{3} \left(\frac{k_B}{e} \right)^2 \sigma_{xy}$$



Thermal Hall conductivity

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Integrating out the fermions leads to an effective action for the emergent U(1) gauge field

$$\mathcal{S}_a = \int d^2x d\tau \left[\frac{K_1(\mathbf{x})}{2} (\nabla \times \mathbf{a})^2 + \frac{K_2(\mathbf{x})}{2} (\nabla a_\tau - \partial_\tau \mathbf{a})^2 \right. \\ \left. - \frac{i\sigma_{xy}(\mathbf{x})}{2e^2} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \right] + \int \frac{d^2k d\omega}{8\pi^3} \gamma_k |\omega| [\mathbf{a}^T(k, \omega)]^2$$

Thermal Hall conductivity

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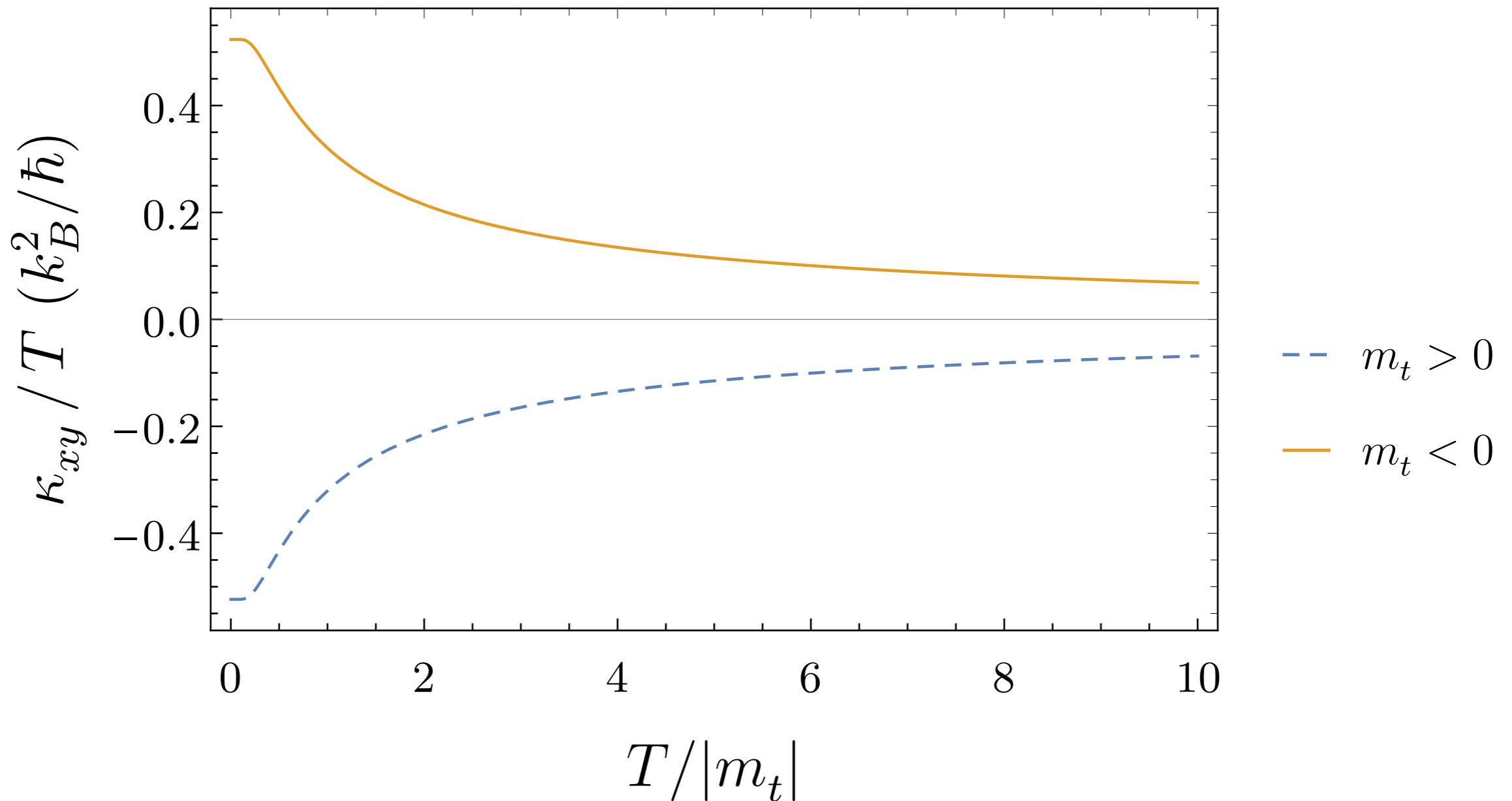
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The gauge field contributes a thermal Hall conductivity, κ_{xy}^1 , which has the *opposite sign* from the Wiedemann-Franz term determined from σ_{xy} .

Thermal Hall conductivity

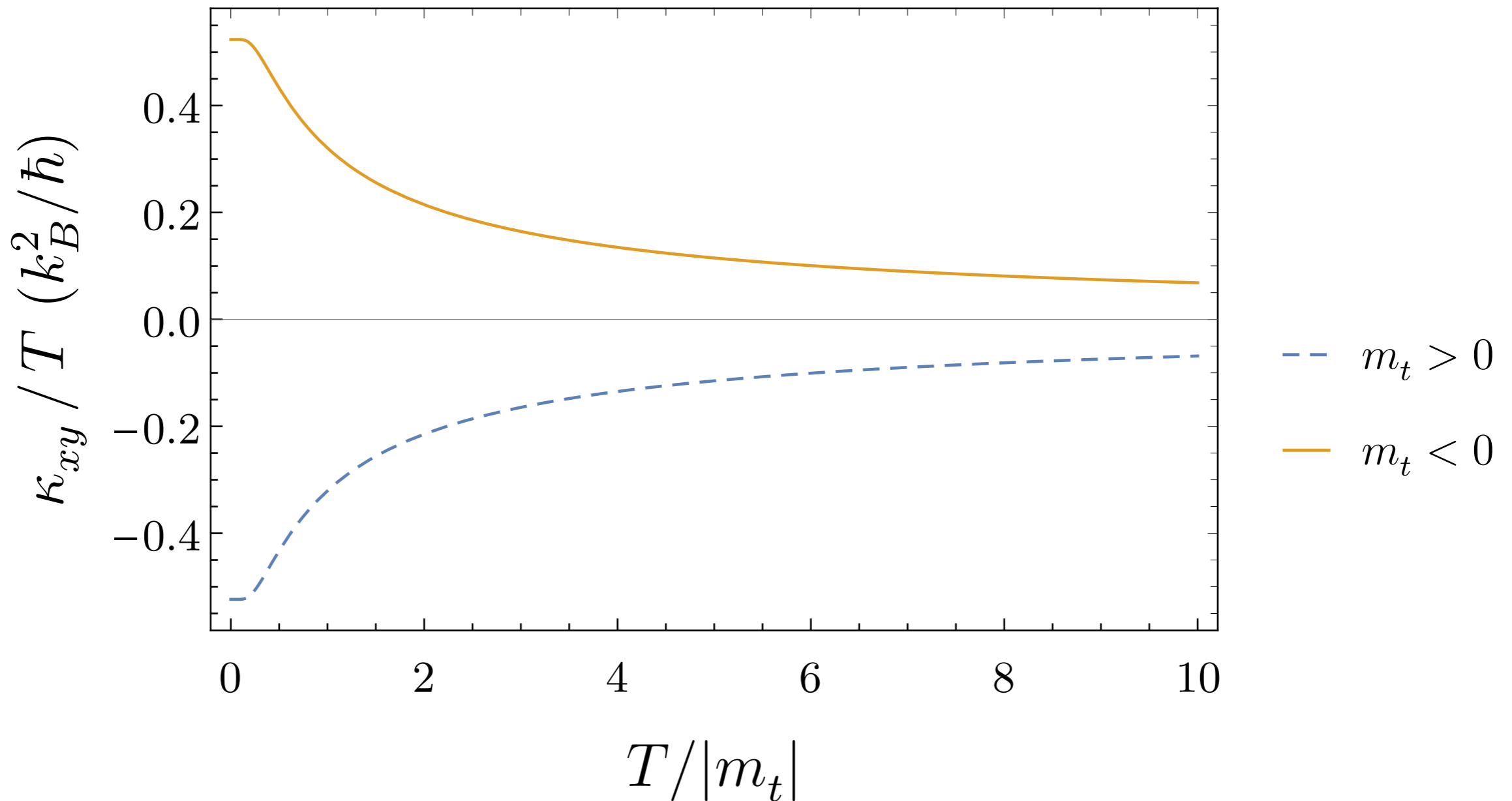
$$m_t \propto \sigma_{xy}$$



The gauge field contribution $|\kappa_{xy}^1|$ is bounded by $(\pi/6)(k_B^2 T / \hbar)$.

Thermal Hall conductivity

$$m_t \propto \sigma_{xy}$$



Needed: Phonon assisted emergent-photon transport ?

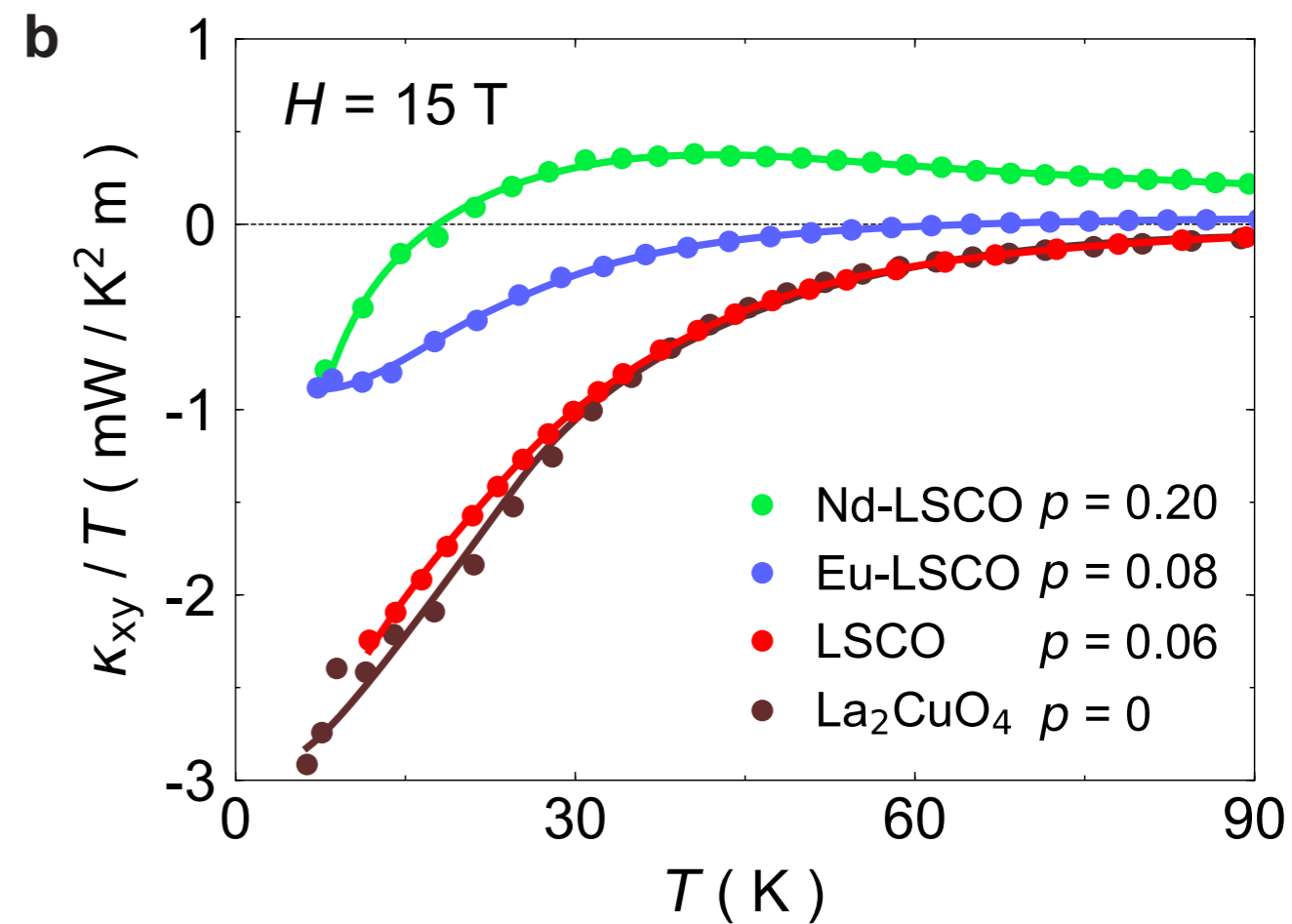
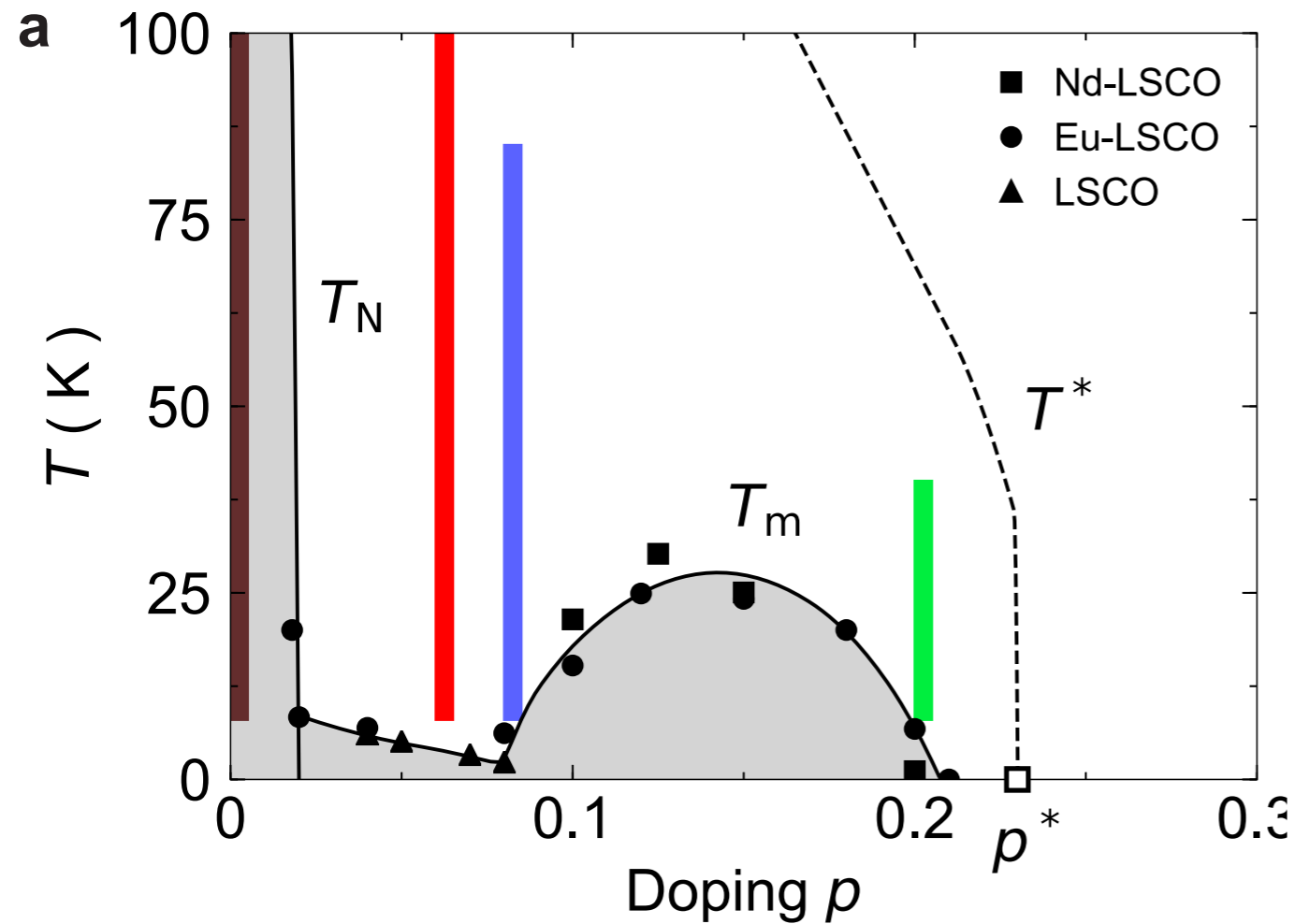
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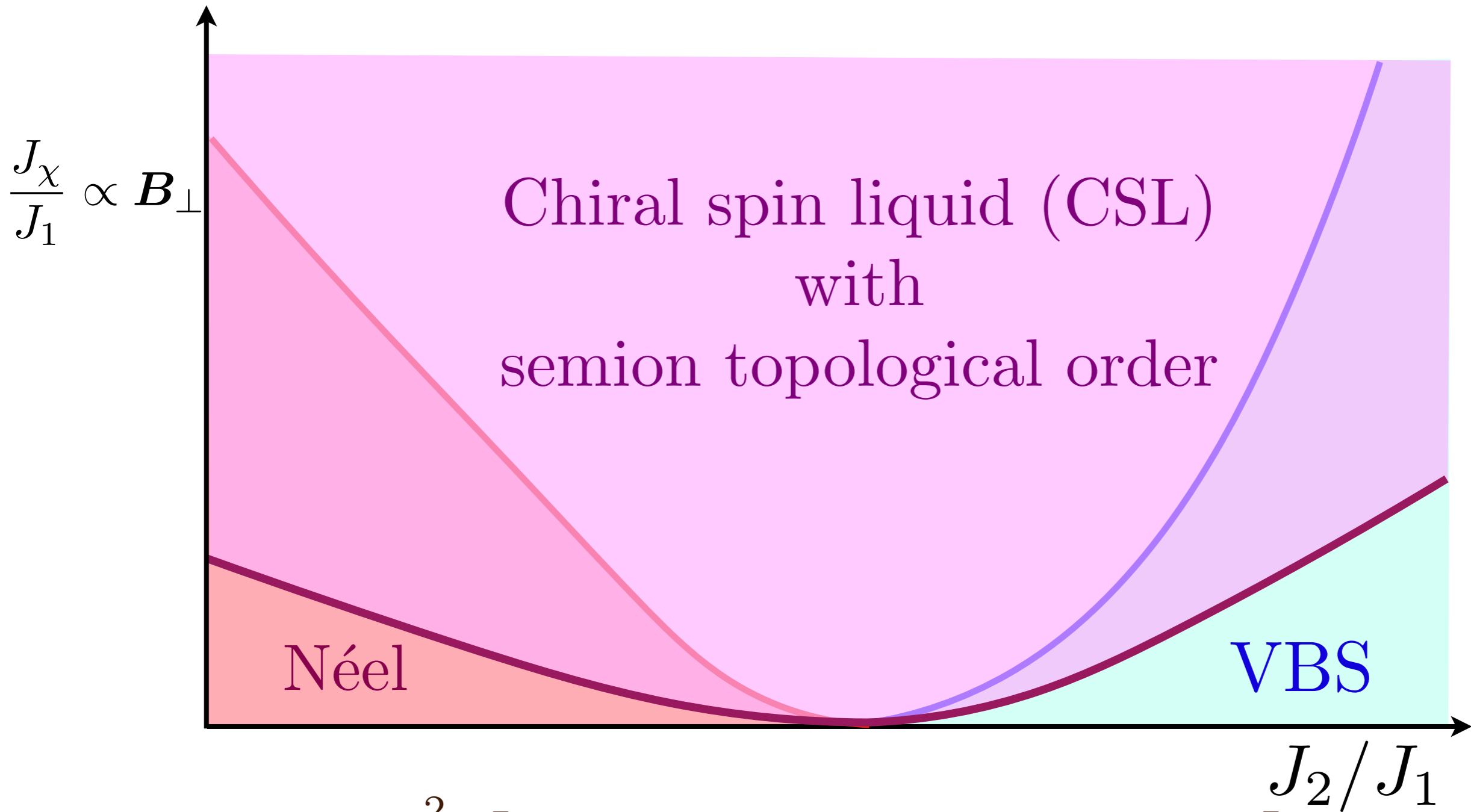
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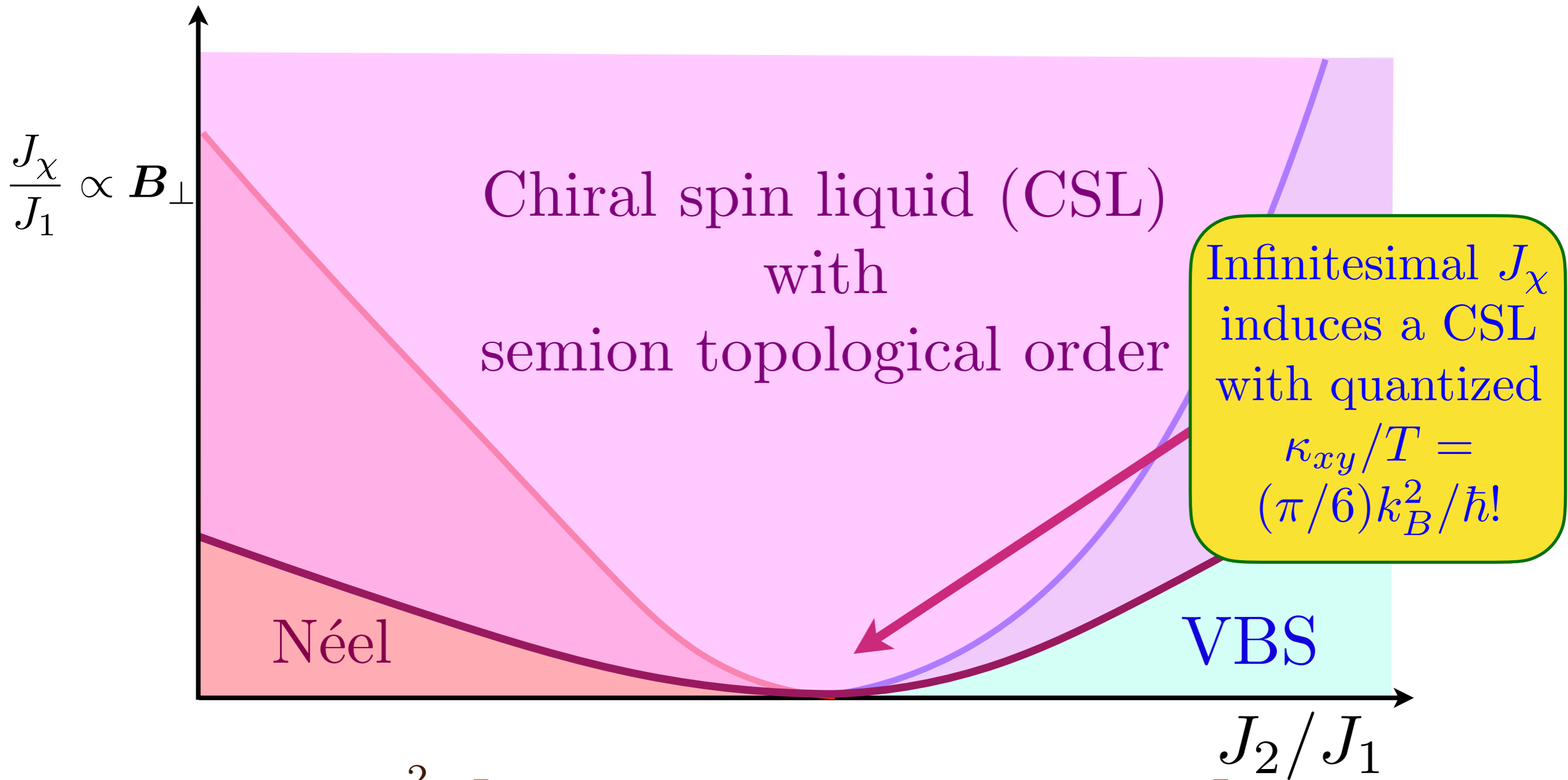
$$H = \sum_{\text{n.n.}} J_1 \mathbf{S}_i \cdot \mathbf{S}_j + \sum_{\text{n.n.n.}} J_2 \mathbf{S}_i \cdot \mathbf{S}_j + J_\chi \sum_{\Delta} \mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_k)$$



$$\mathcal{S}_f = \int d^2r d\tau \sum_{\alpha=1}^2 \left[\bar{f}_\alpha \gamma^\mu (\partial_\mu - iA_\mu) f_\alpha + m_\chi \bar{f}_\alpha f_\alpha \right]$$

Quantum critical theory + J_χ

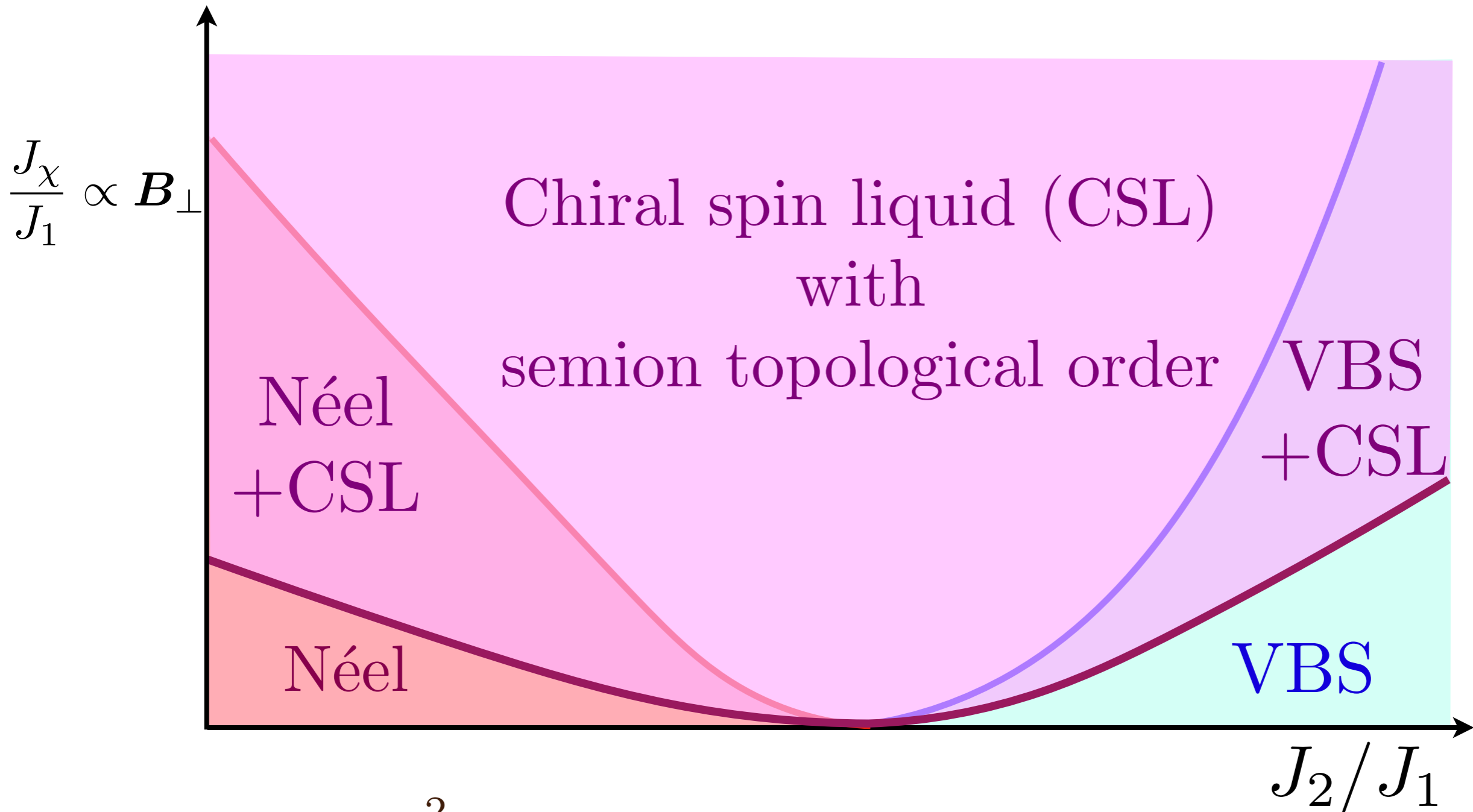
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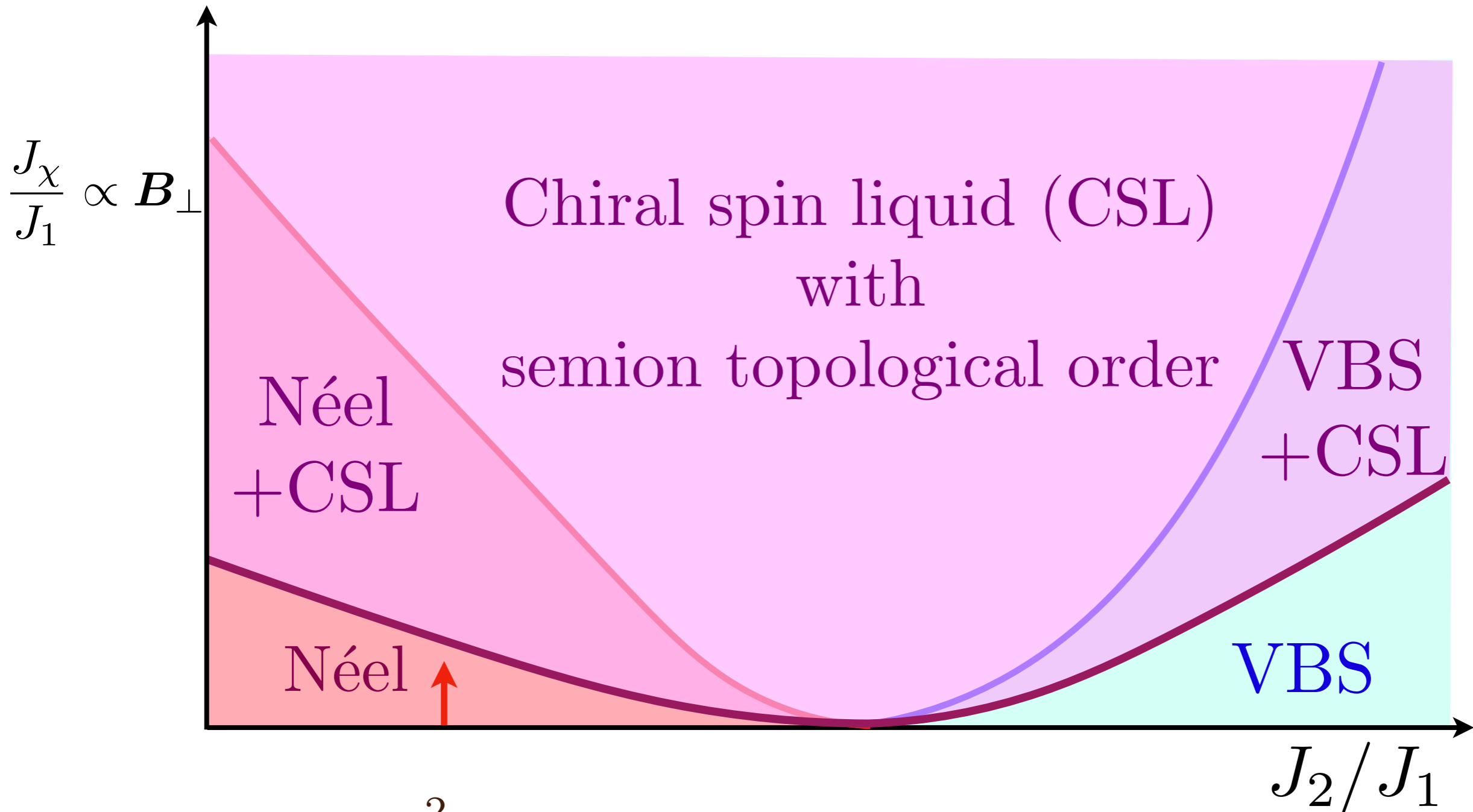
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Quantum critical theory + J_χ + Néel order

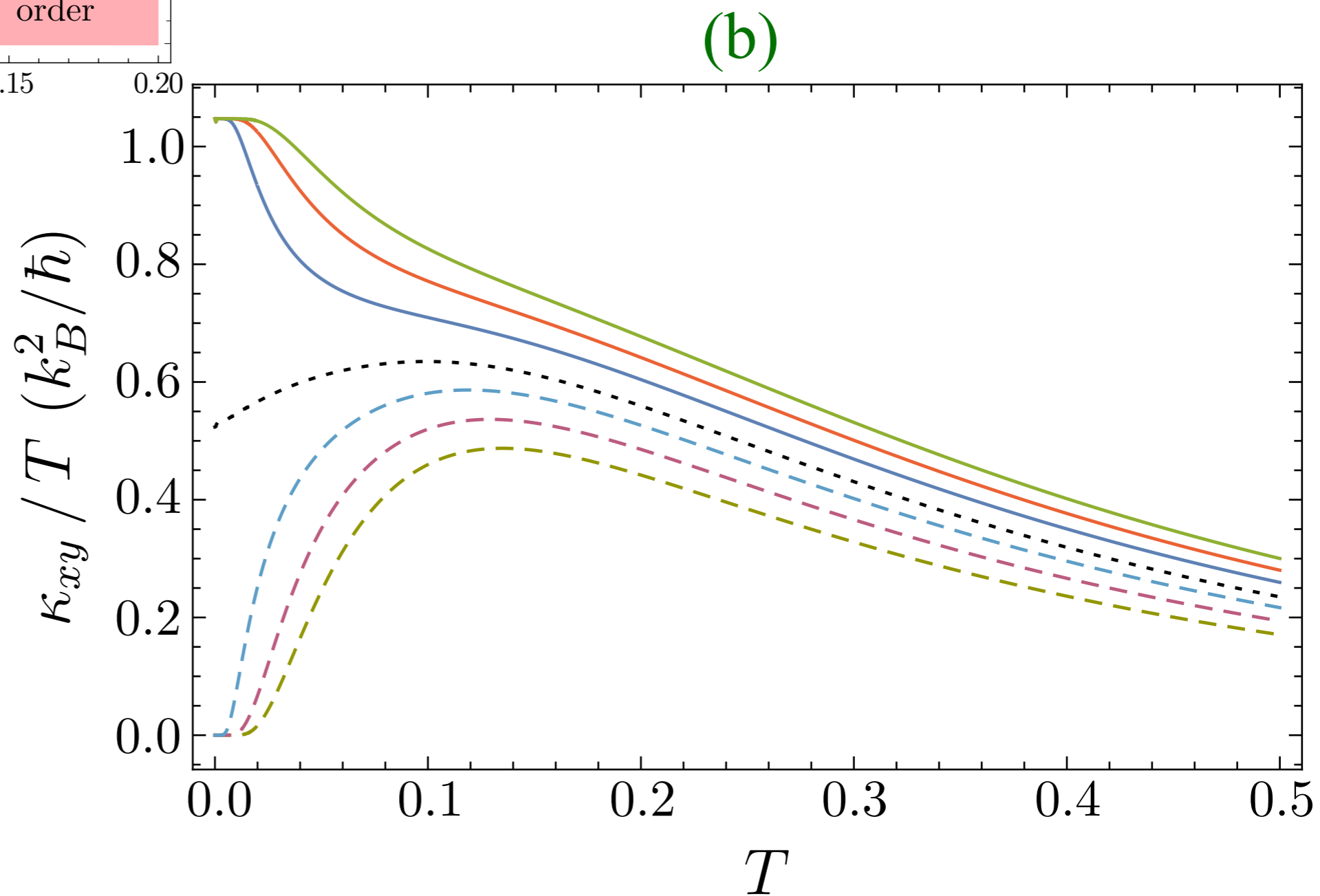
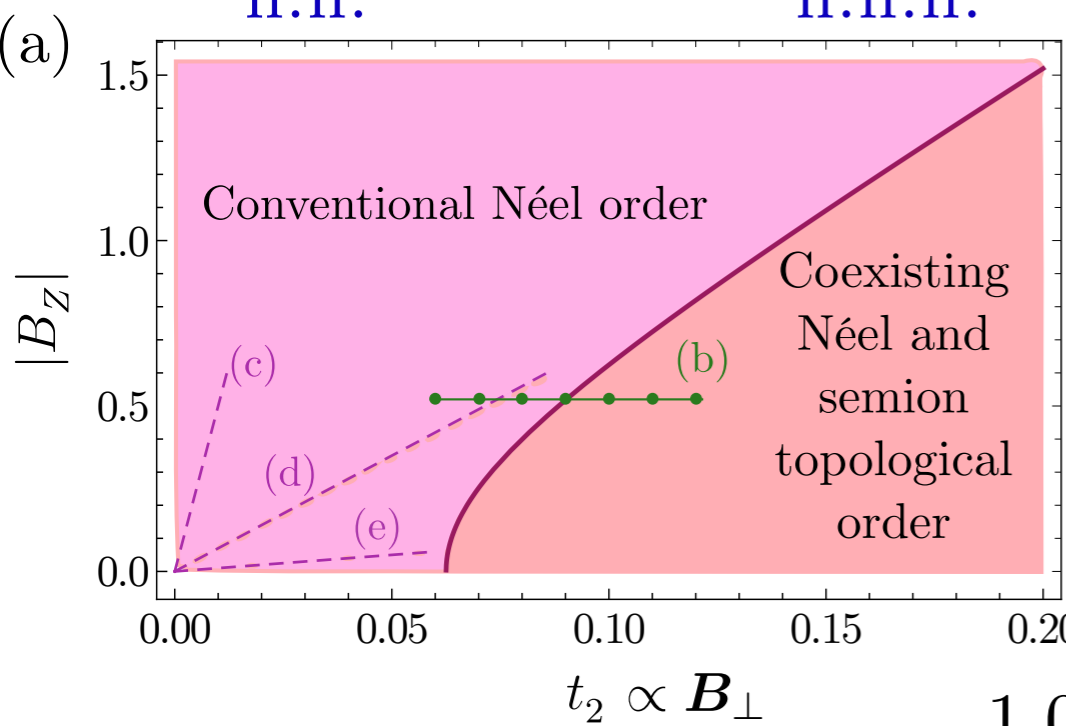
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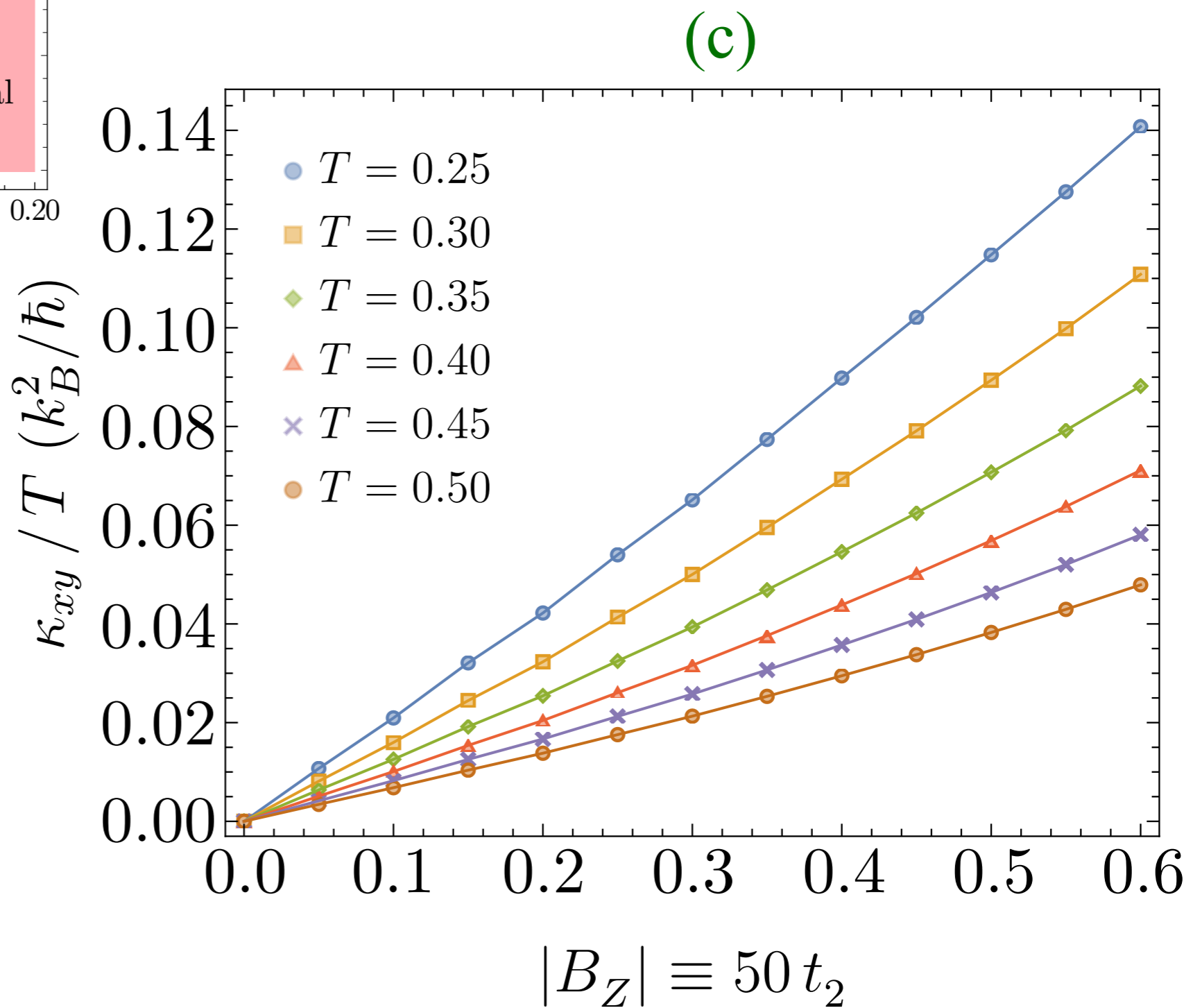
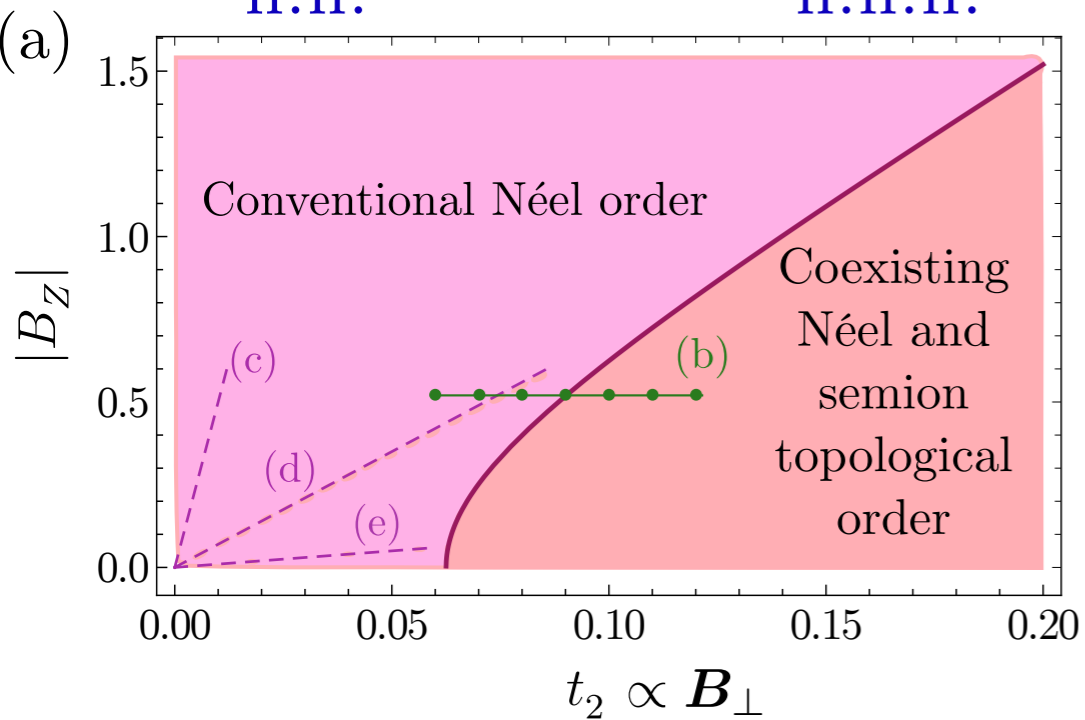
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Quantum critical theory + J_χ + Néel order

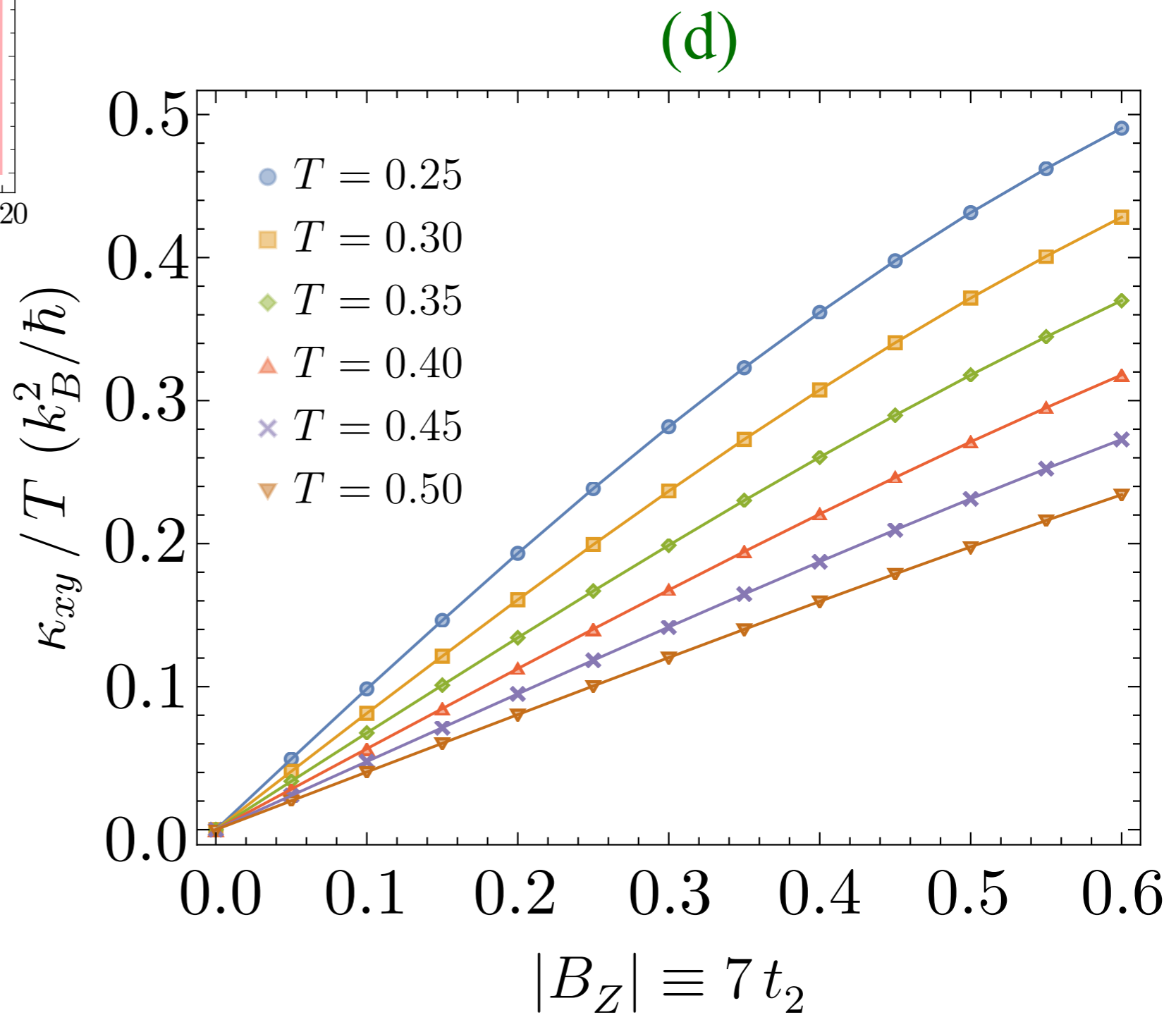
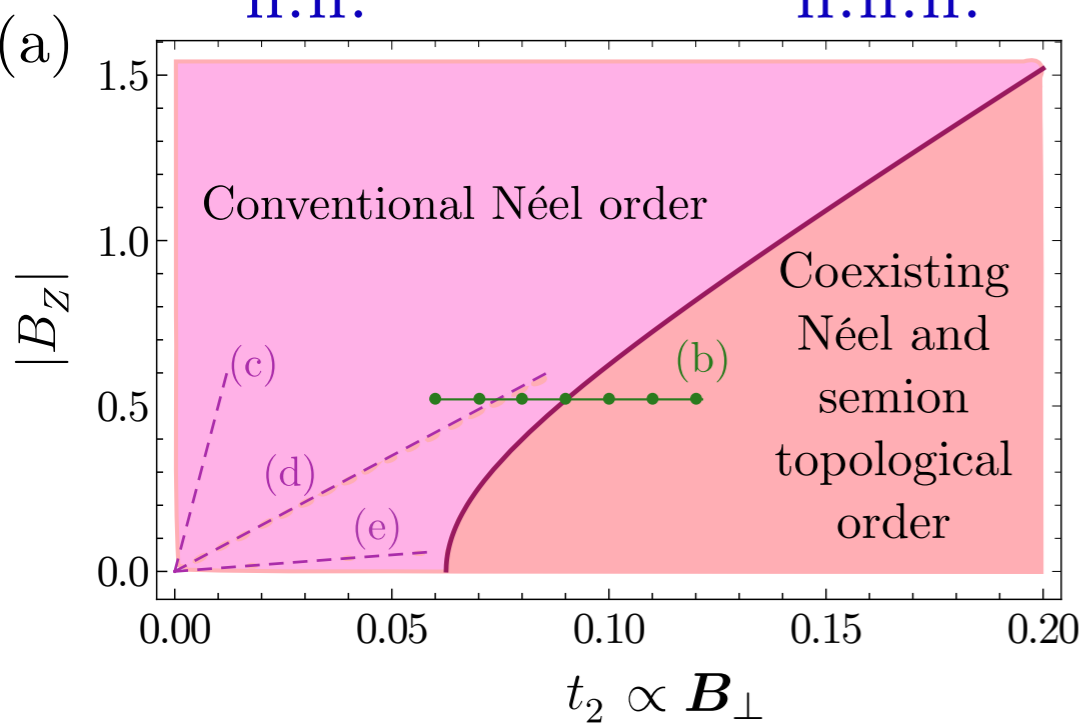
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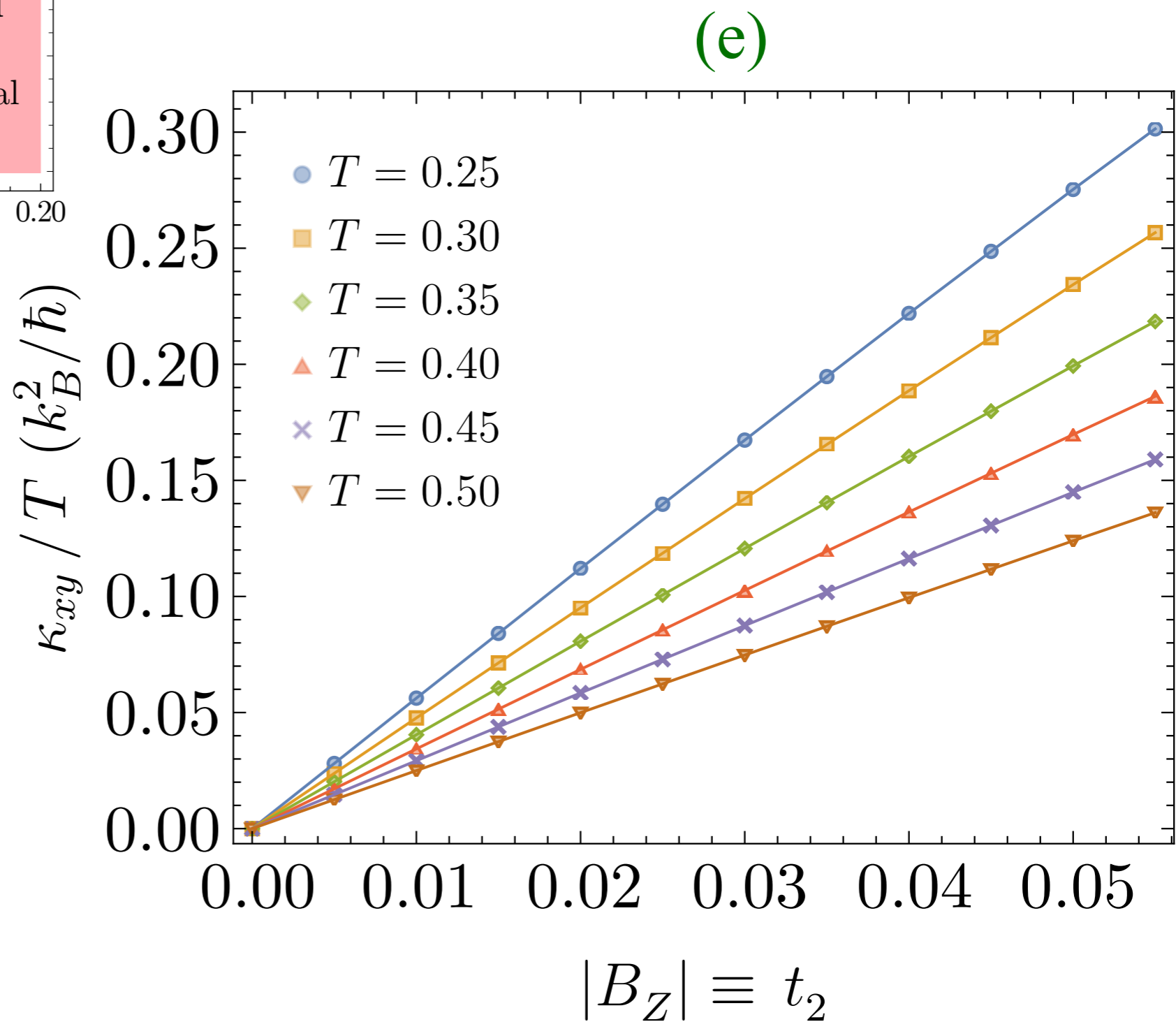
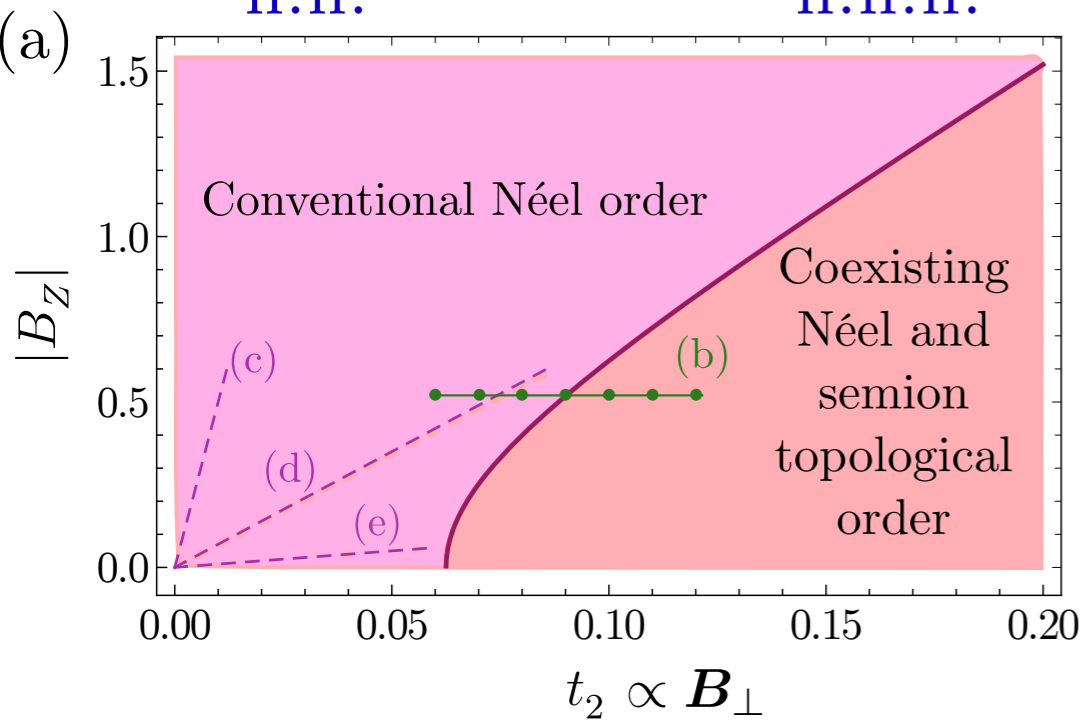
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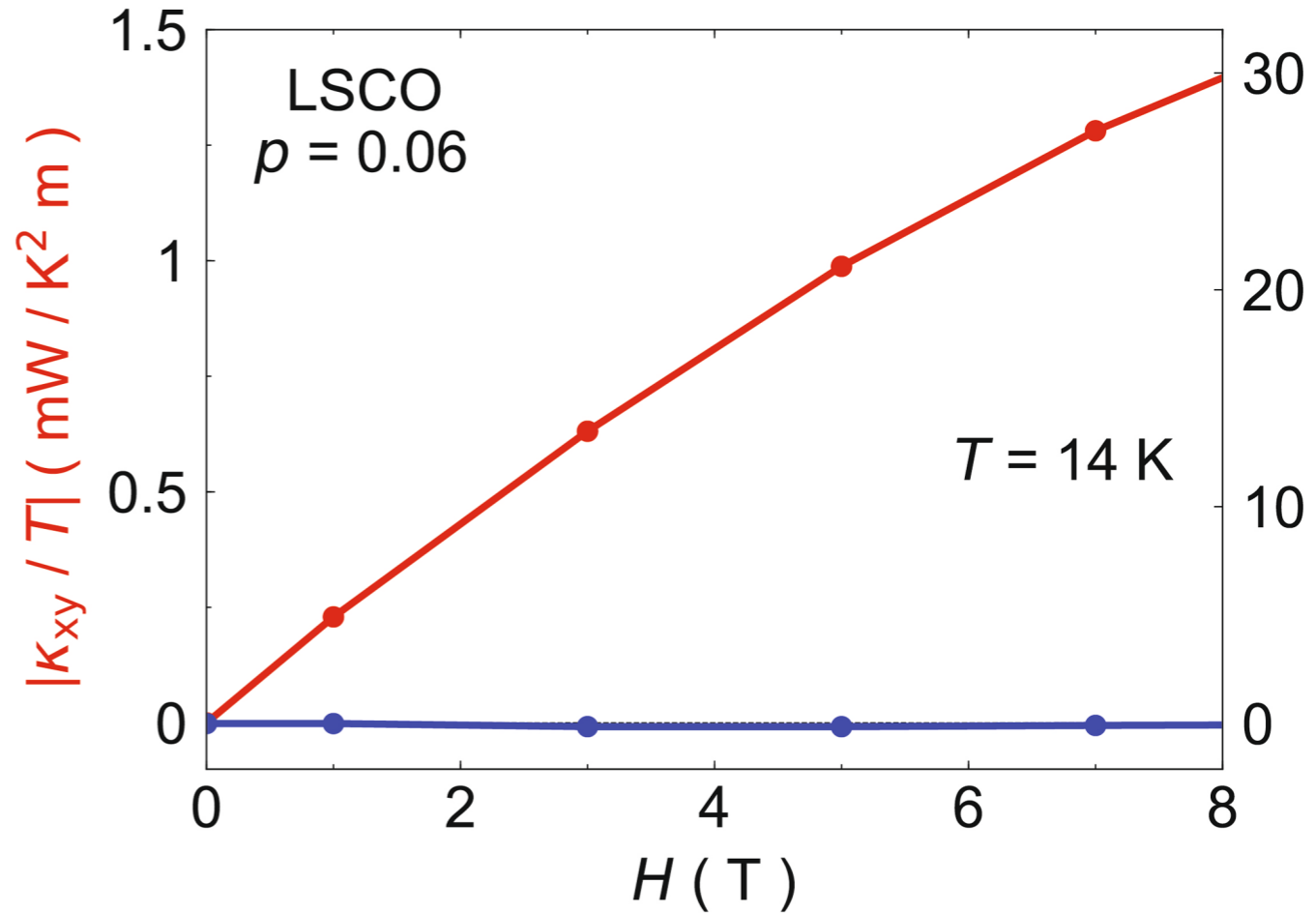
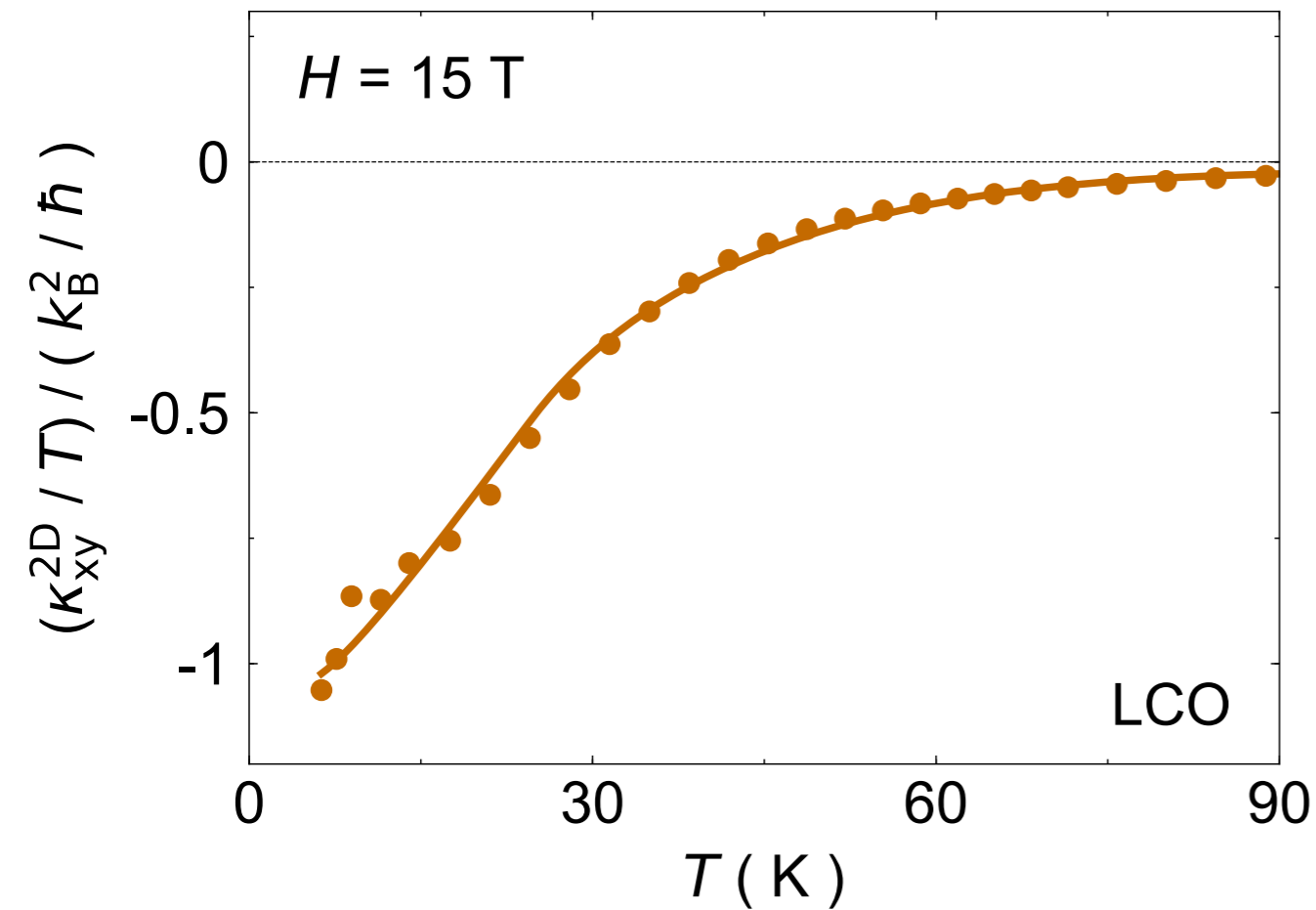


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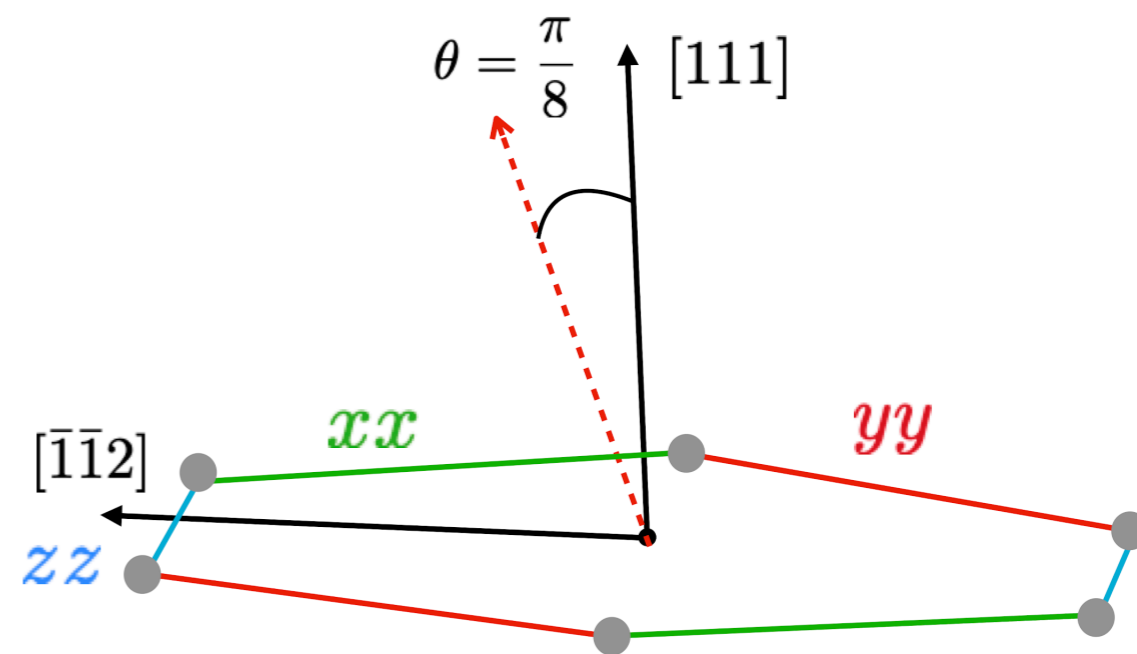
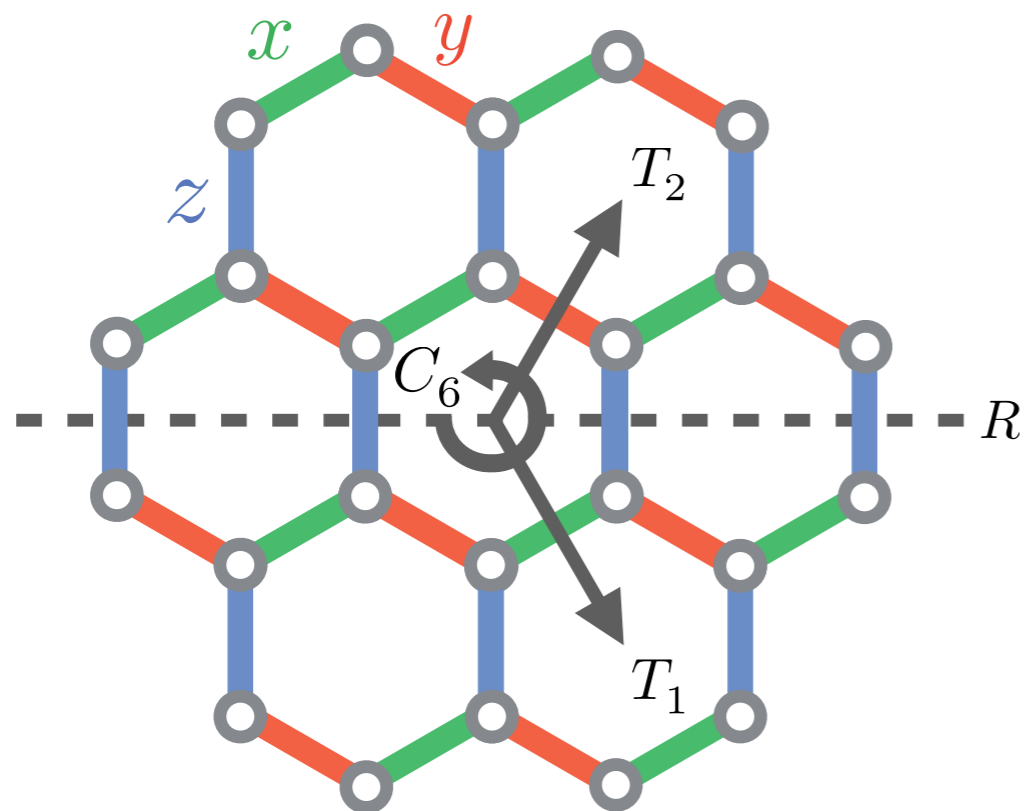
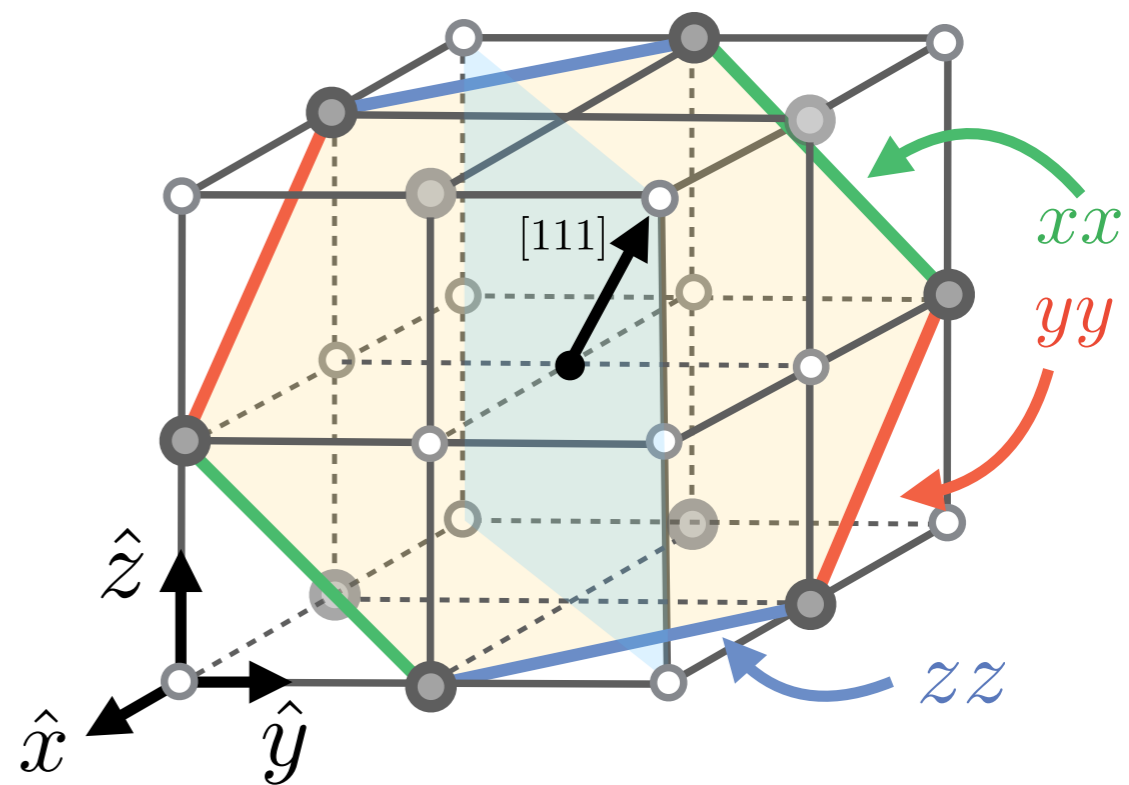
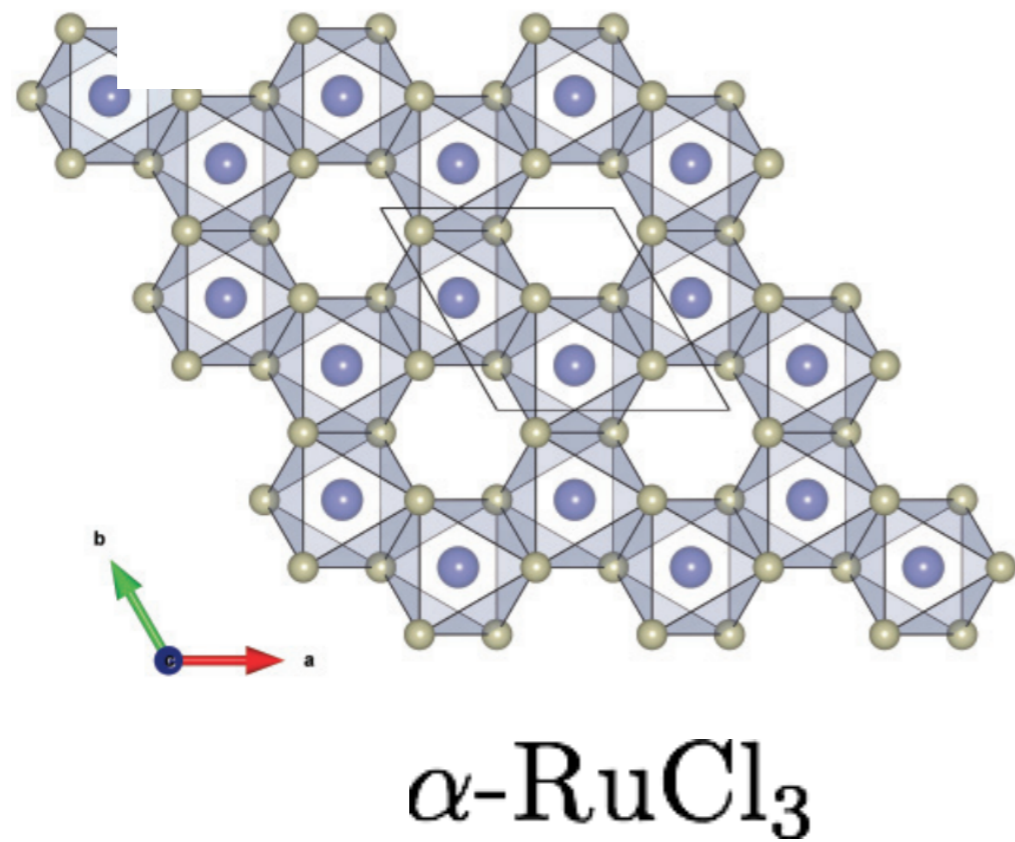
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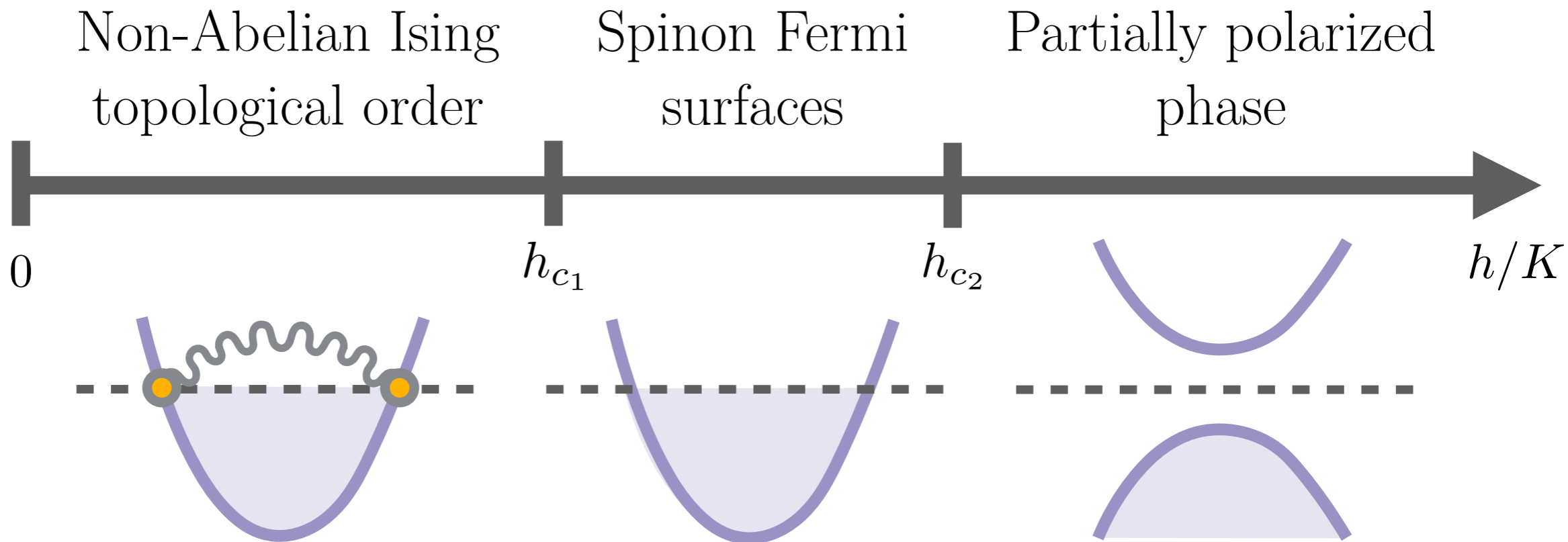
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Gapless \mathbb{Z}_2 QSL
(Kitaev B phase)

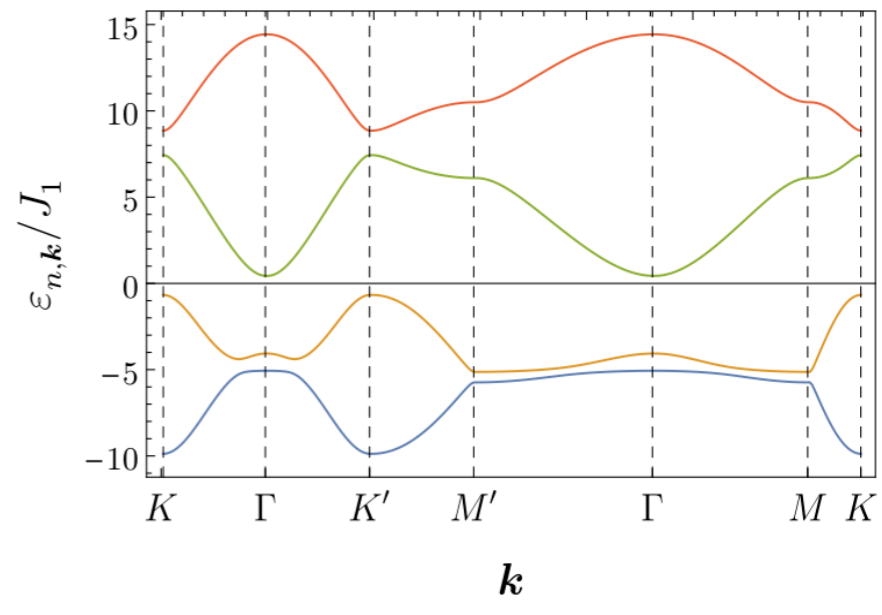
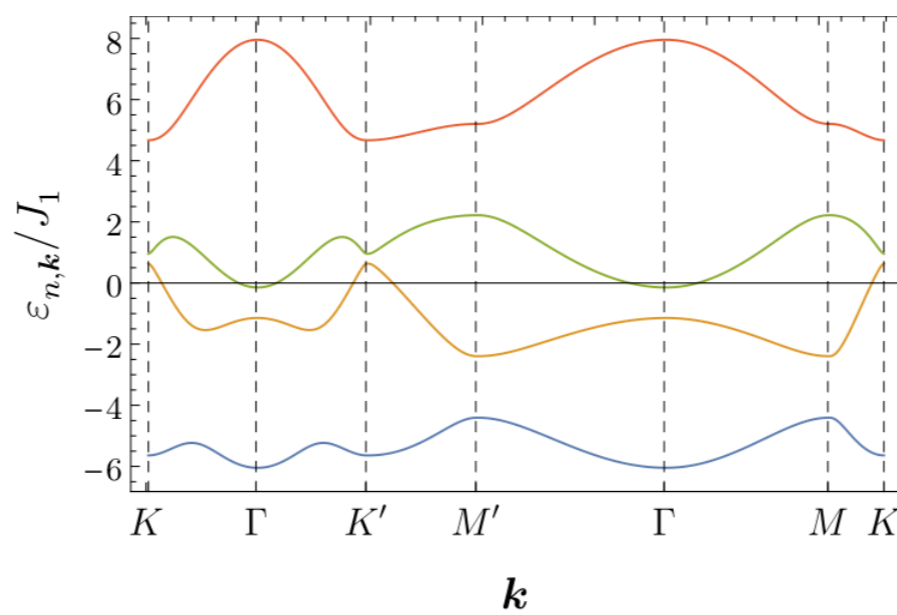
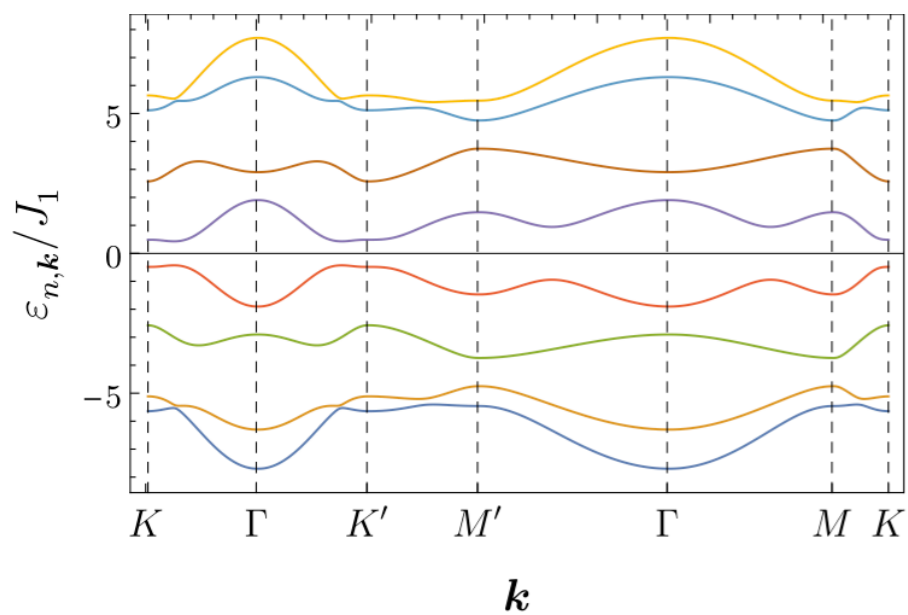
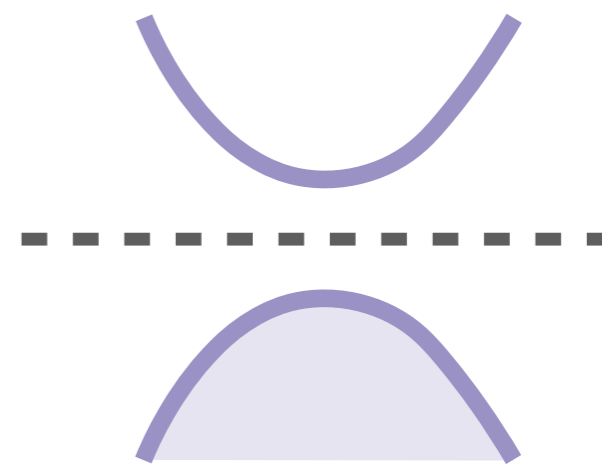
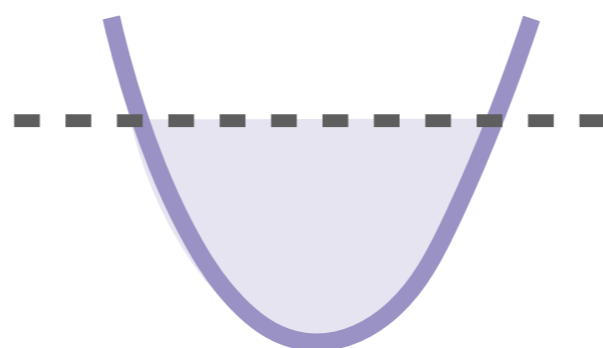
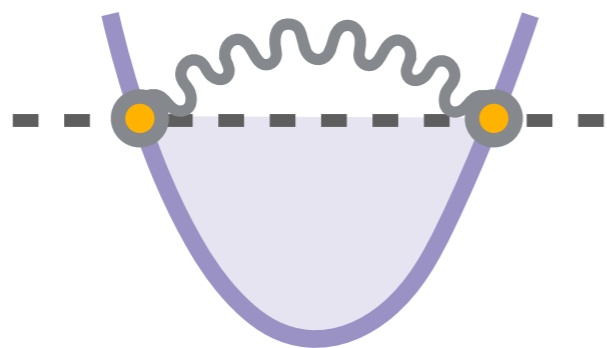


Gapless Z_2 QSL
(Kitaev B phase)

Non-Abelian Ising
topological order

Spinon Fermi
surfaces

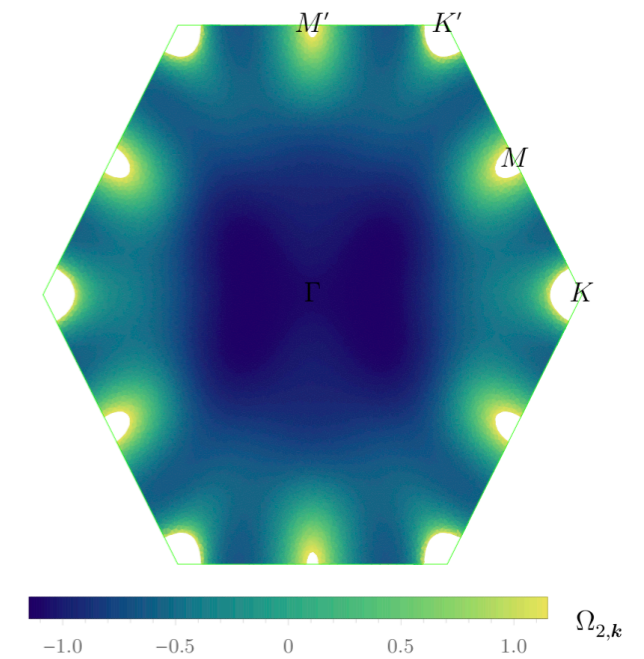
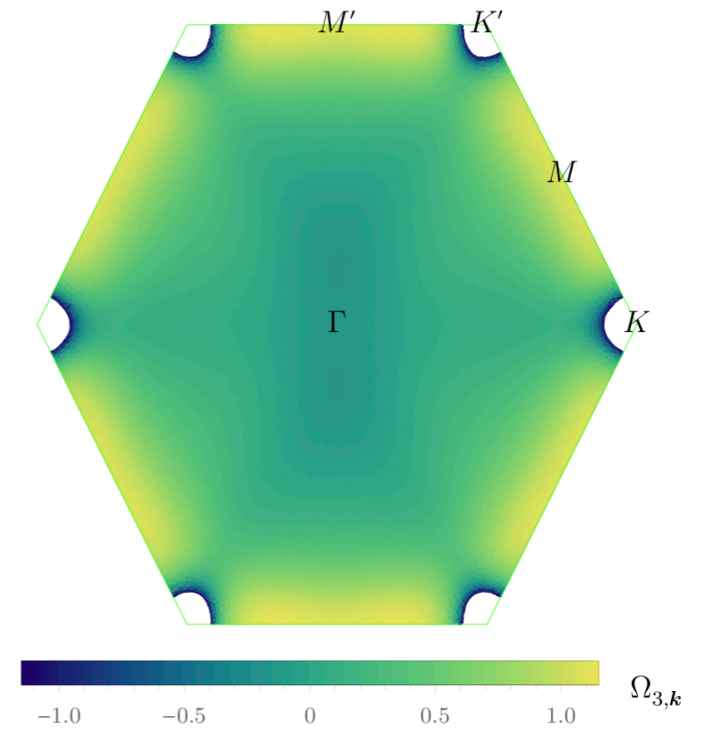
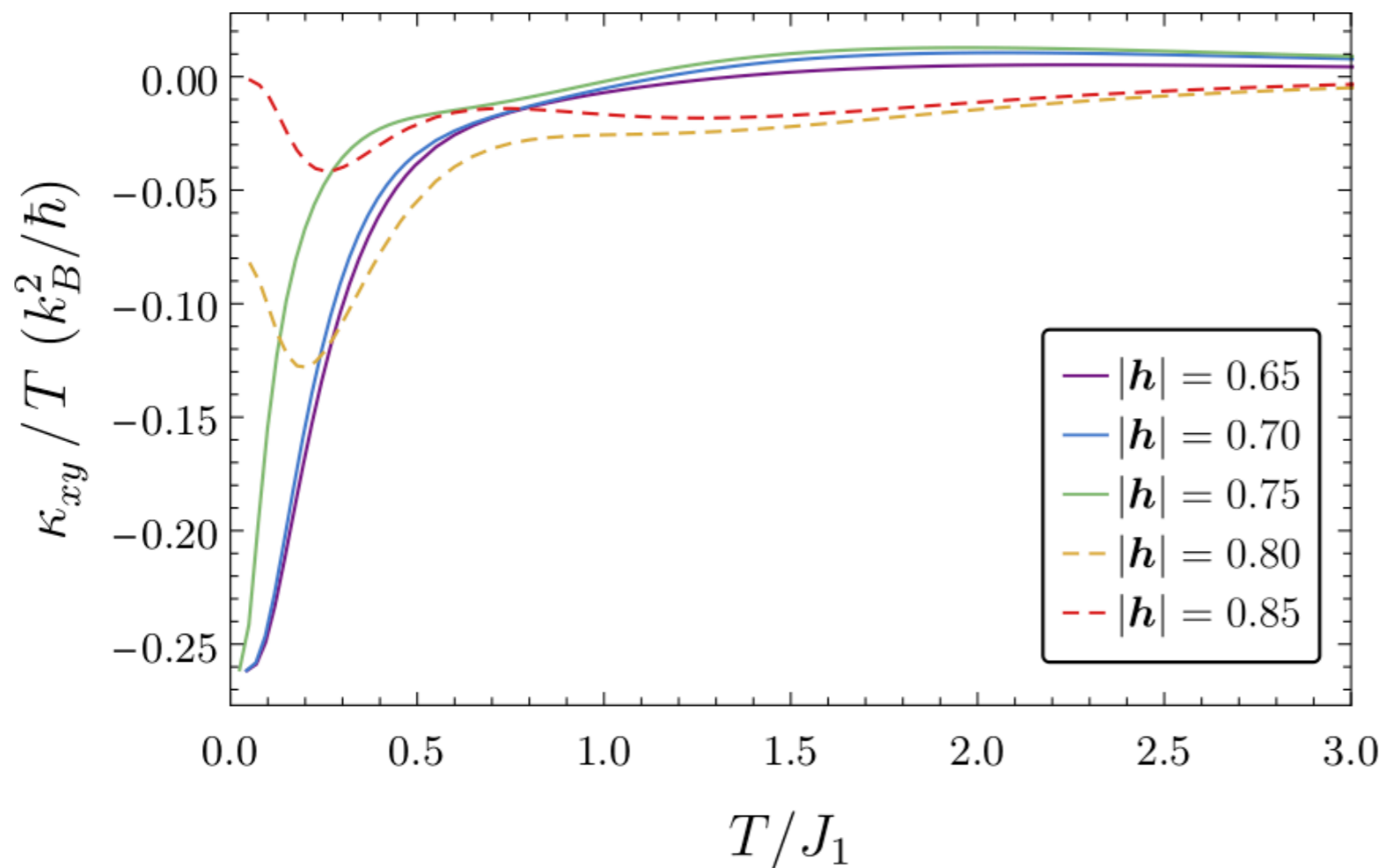
Partially polarized
phase



Spinon dispersions

Temperature dependence for fixed field

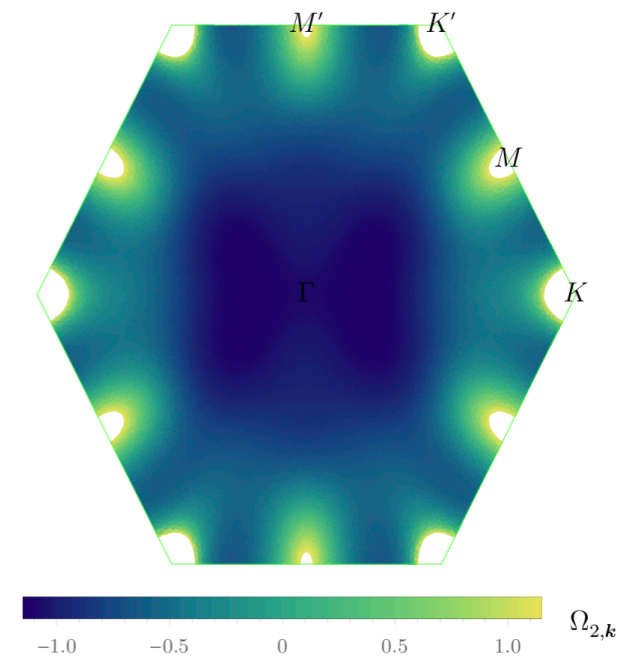
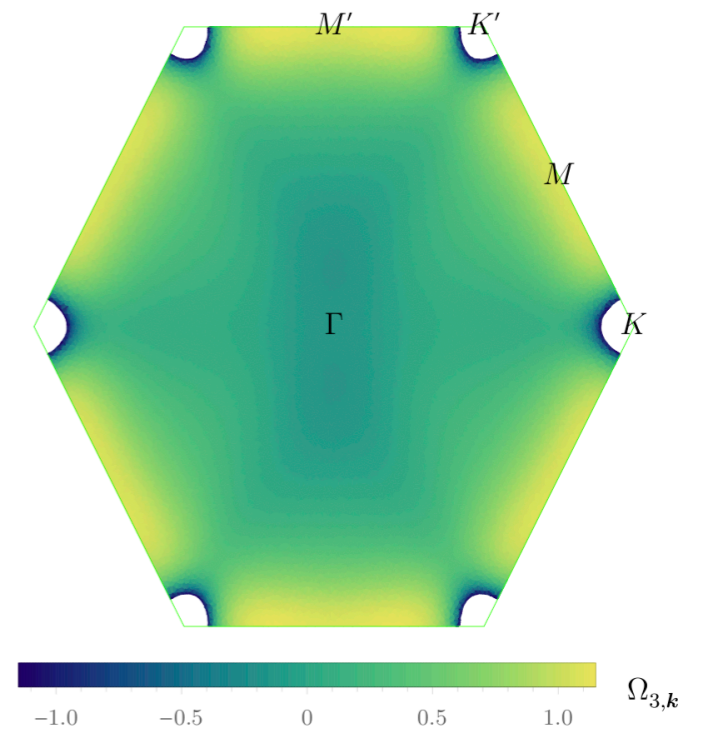
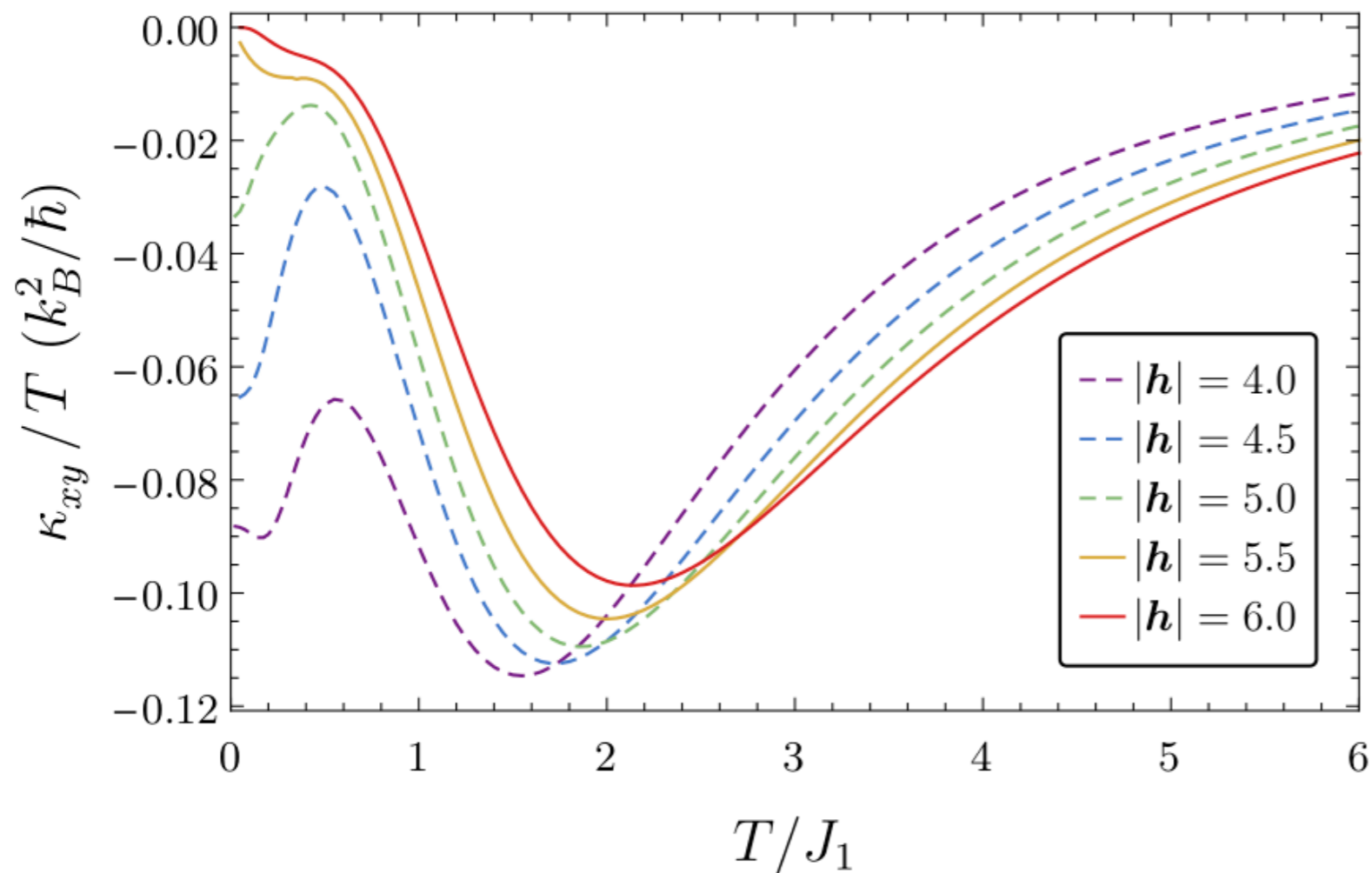
Ising Topological Order (ITO, solid)
to Spinon Fermi surface (SFS, dashed)



Spinon Berry
curvatures

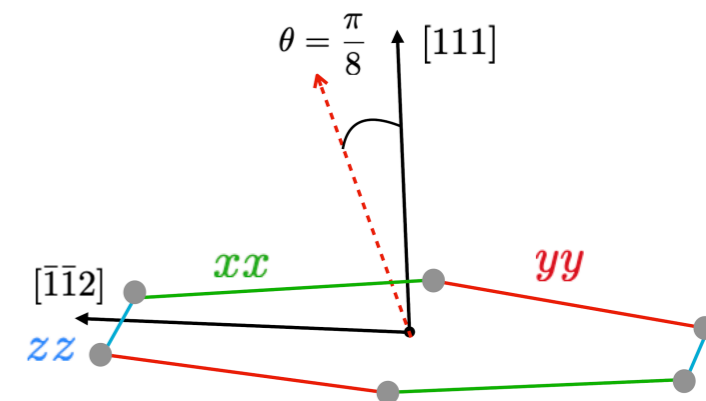
Temperature dependence for fixed field

Spinon Fermi surface (SFS, dashed)
to Partially polarized (solid)



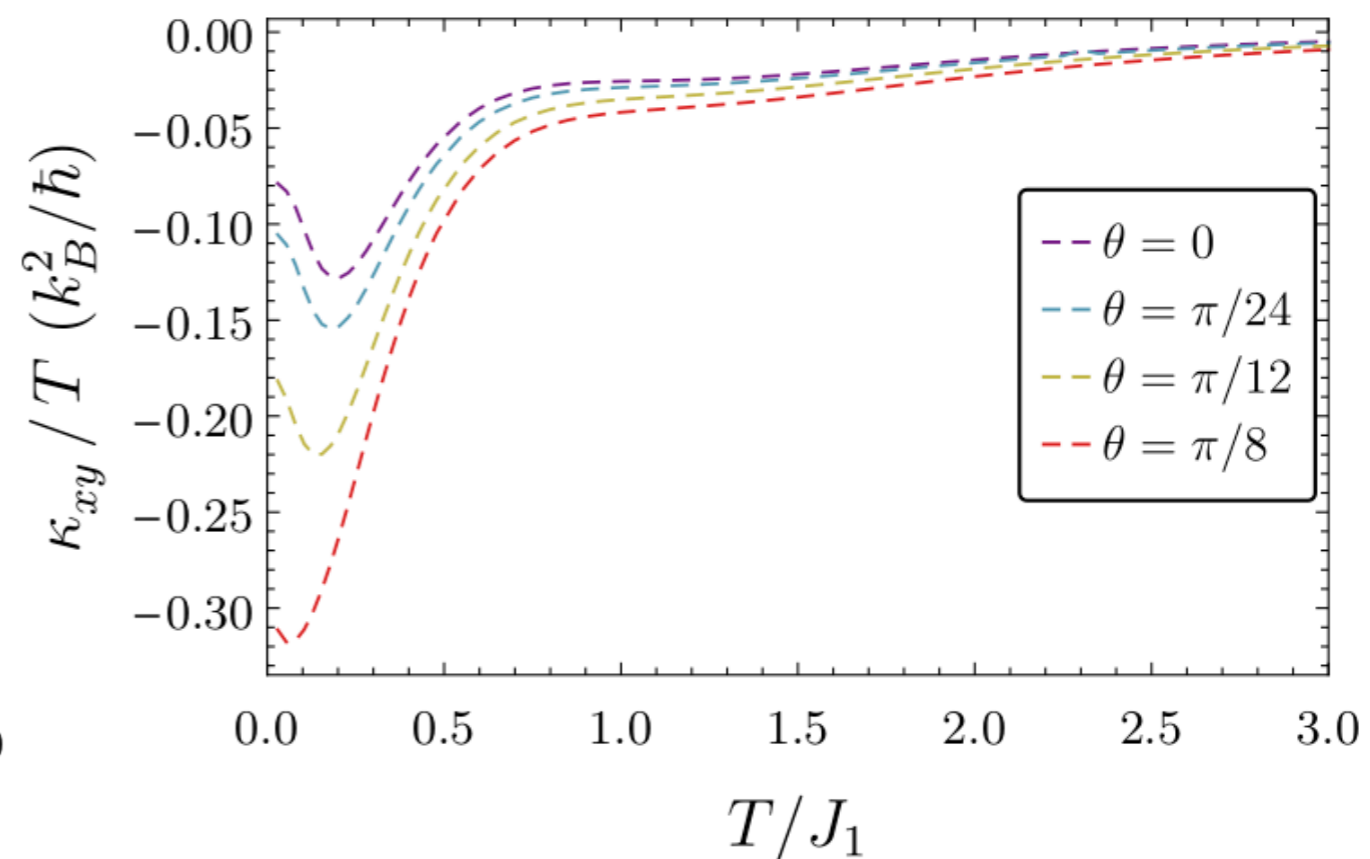
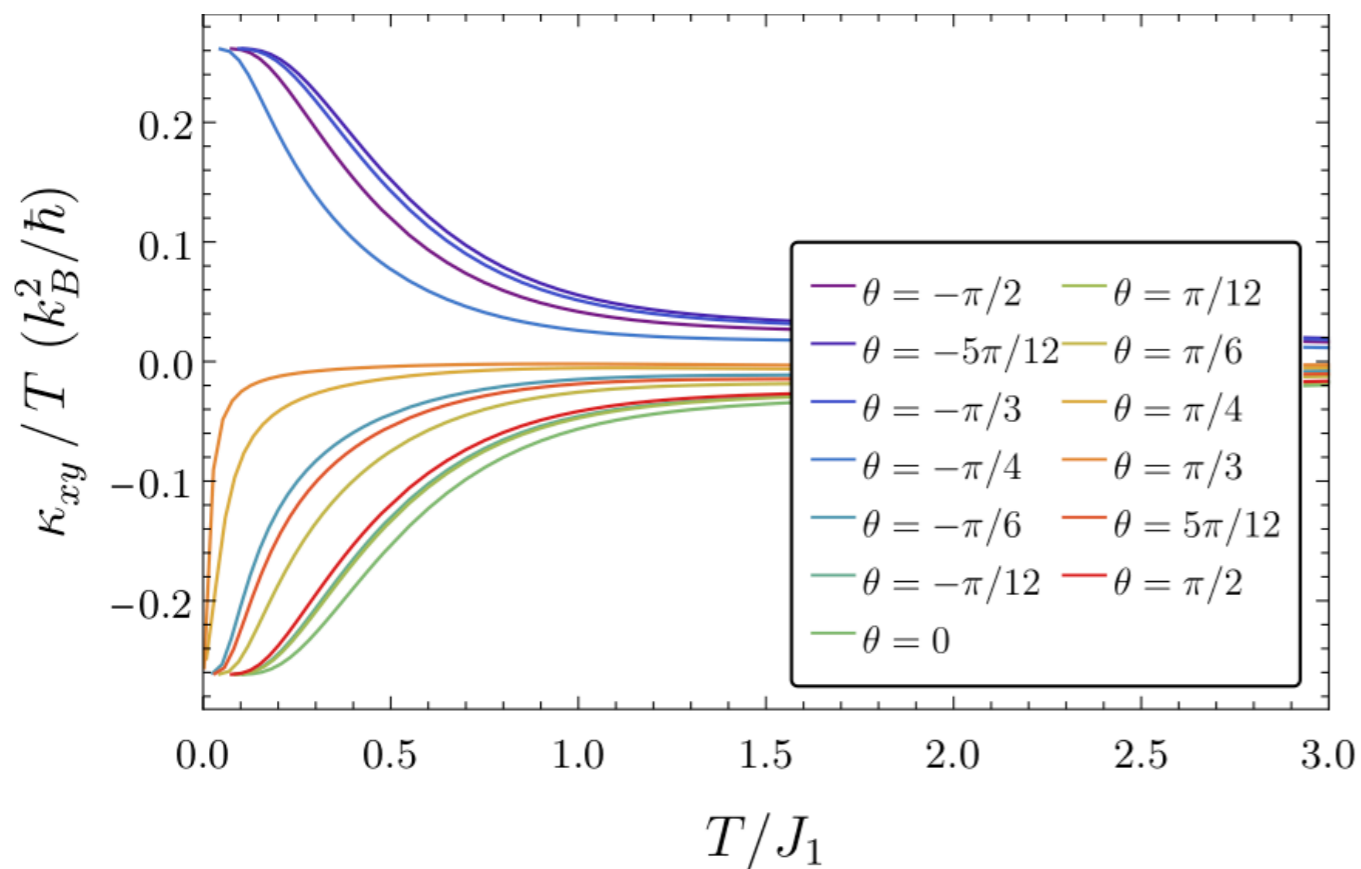
Spinon Berry
curvatures

Rotating magnetic field

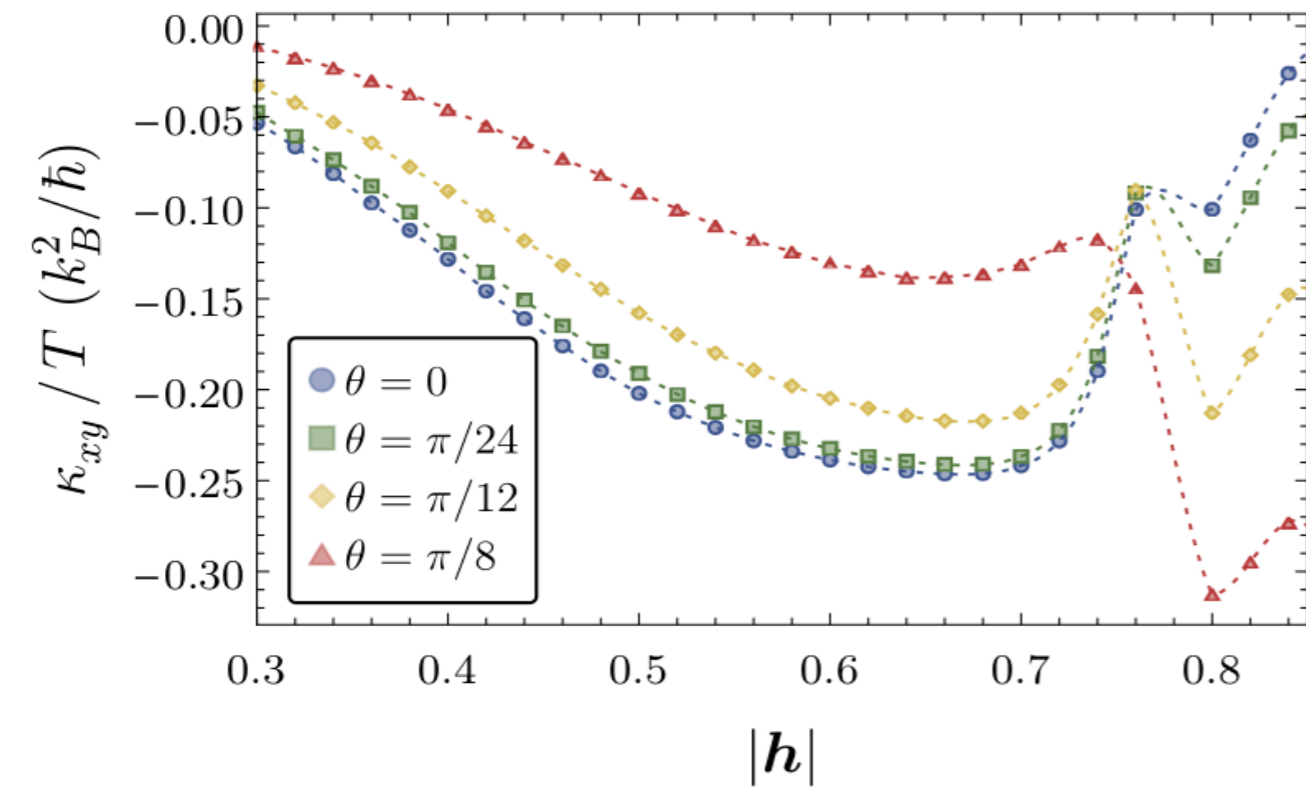
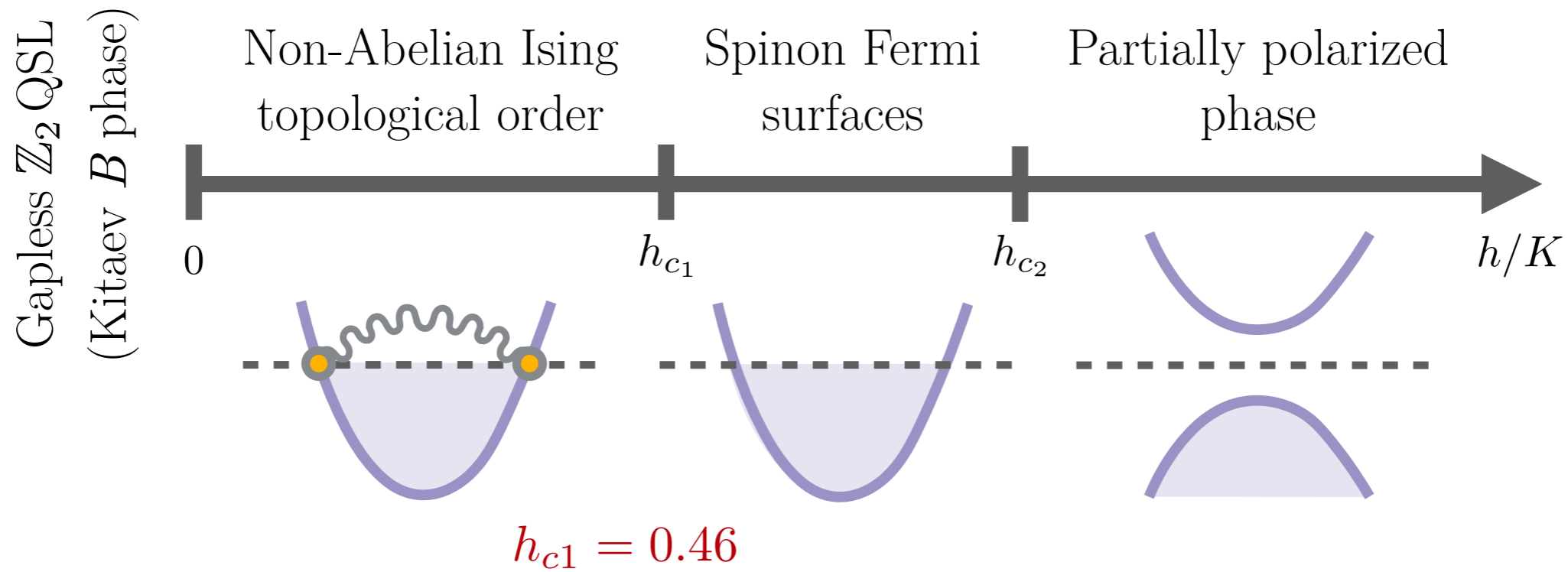


ITO

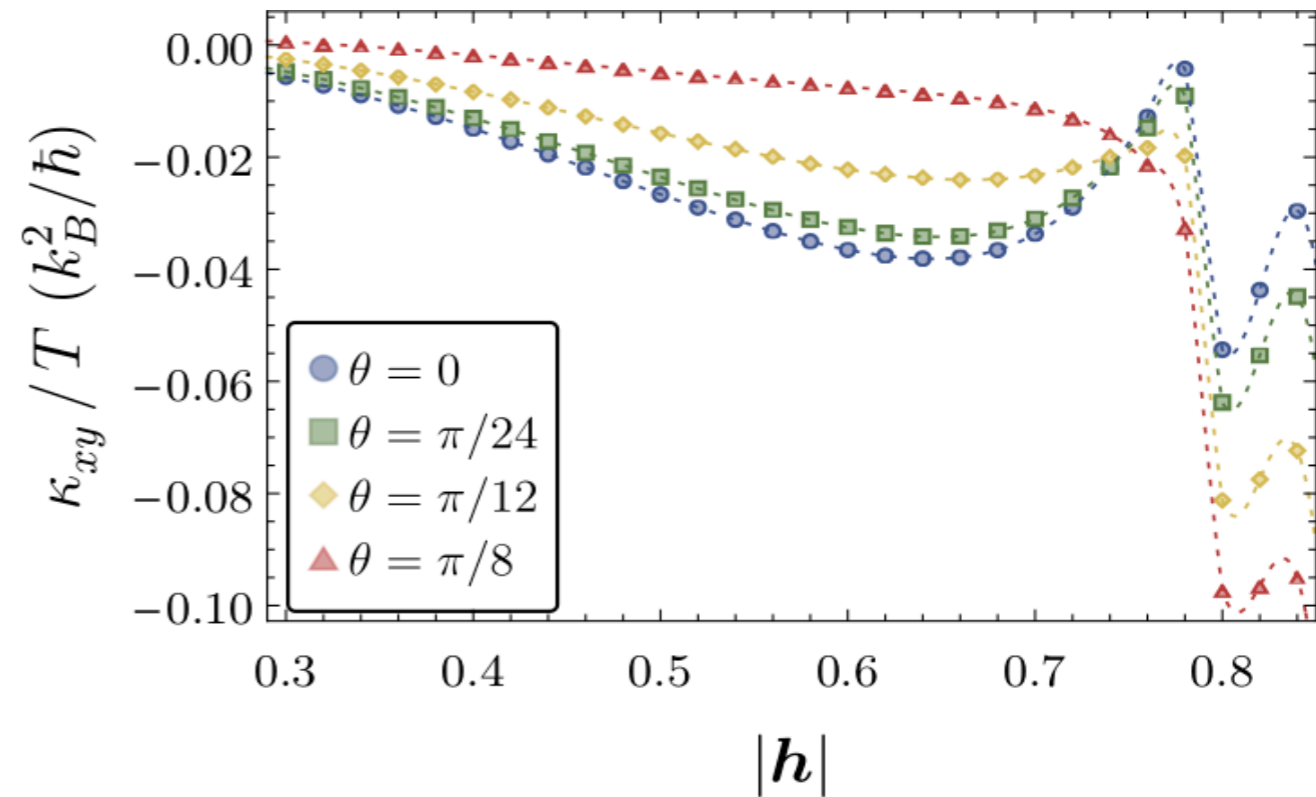
SFS



Field dependence for fixed temperatures



$$T/J_1 = 0.1$$



$$T/J_1 = 0.5$$