Thermal Hall effect in the pseudogap
+
Bilocal field theory for optimal doping

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PHYSICS

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Giant thermal Hall conductivity in the pseudogap phase of cuprate superconductors

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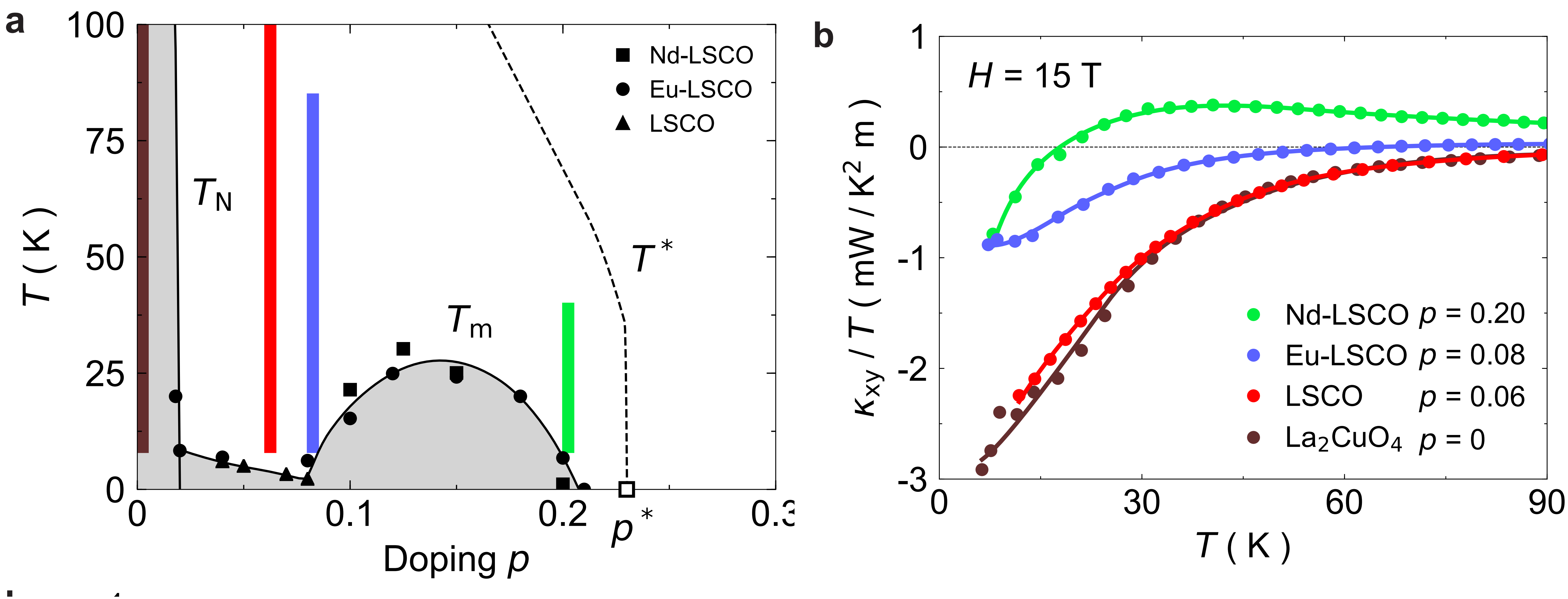

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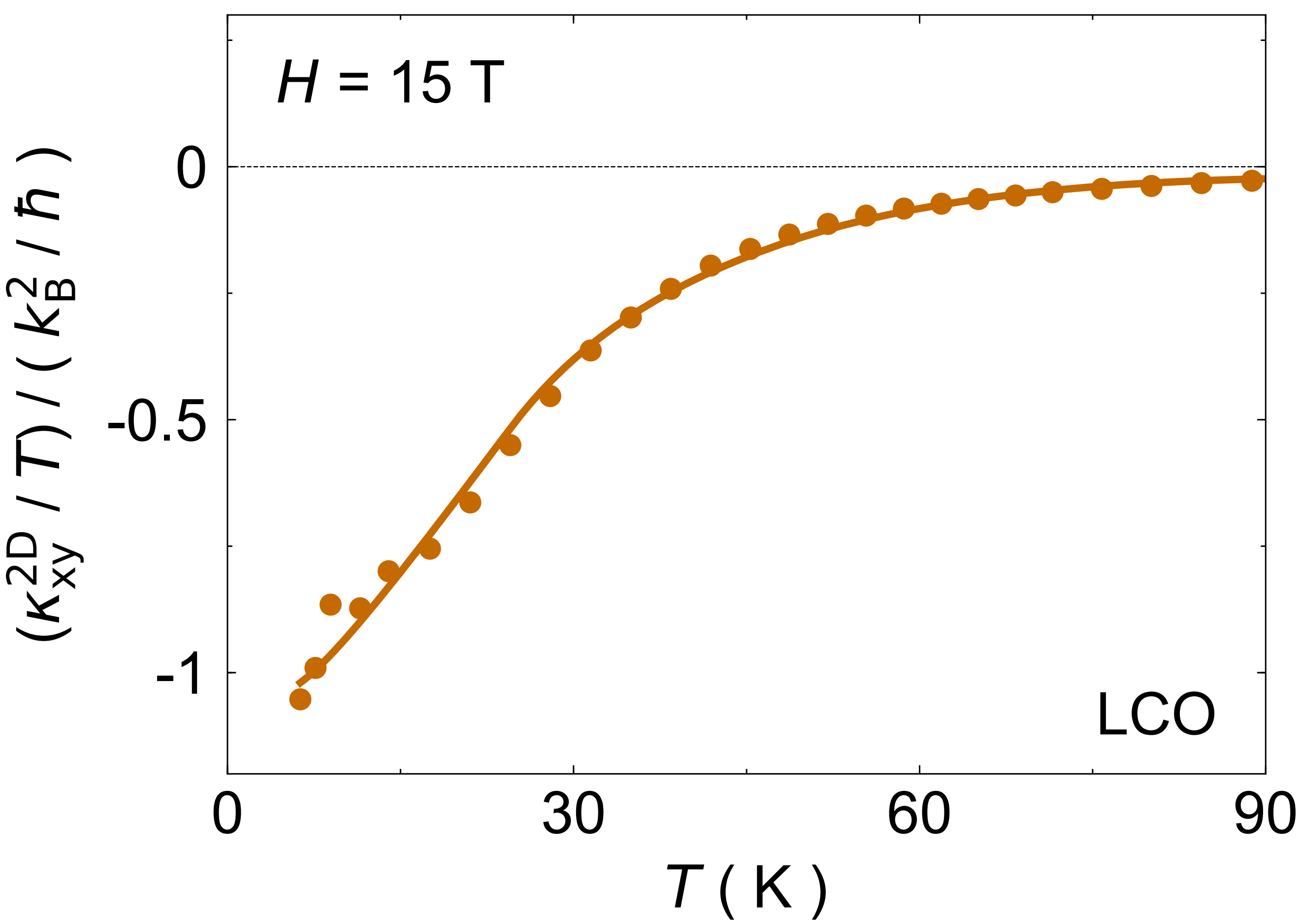
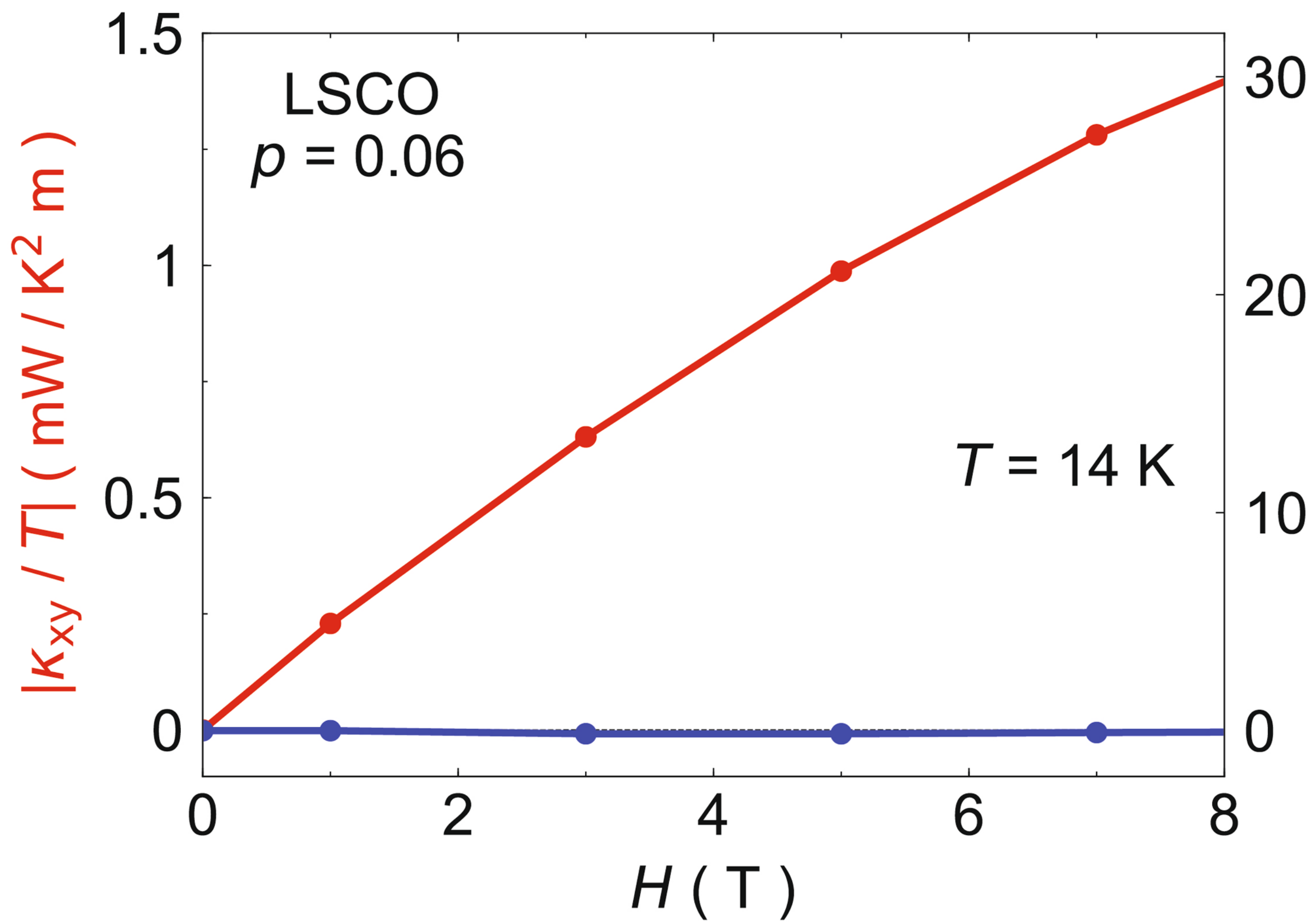
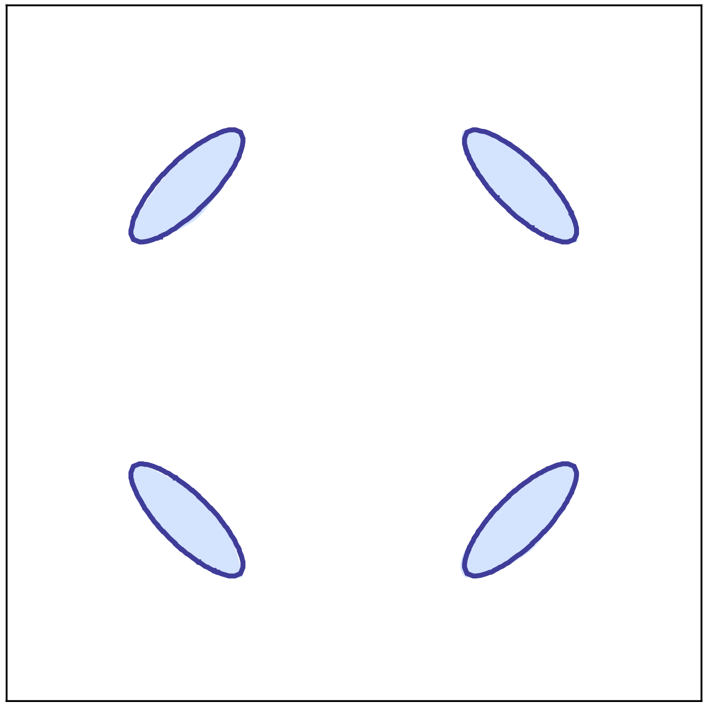




Model for the pseudogap

Fermionic 'chargons' of density δ in hole pockets.
The fermions carry electromagnetic gauge charge $+e$,
and charges $p = \pm 1$ under an emergent U(1) gauge field.
v is a valley index, $v_{\rm dis}$ is an impurity potential.

$$\mathcal{L}_f = \sum_{v=1,2} \sum_{p=\pm 1} f_{pv}^\dagger \left(\frac{\partial}{\partial \tau} - \mu - i p a_\tau - \frac{(\boldsymbol{\nabla} - i p \boldsymbol{a} - i e \boldsymbol{A}_{\rm em})^2}{2m^*} \right) f_{pv} + v_{\rm dis}(\boldsymbol{r}) f_{pv}^\dagger f_{pv}$$



Thermal Hall conductivity

$$\mathcal{L}_f = \sum_{v=1,2} \sum_{p=\pm 1} f_{pv}^{\dagger} \left(\frac{\partial}{\partial \tau} - \mu - ipa_\tau - \frac{(\boldsymbol{\nabla} - ip\boldsymbol{a} - ie\boldsymbol{A}_{\rm em})^2}{2m^*} \right) f_{pv}$$
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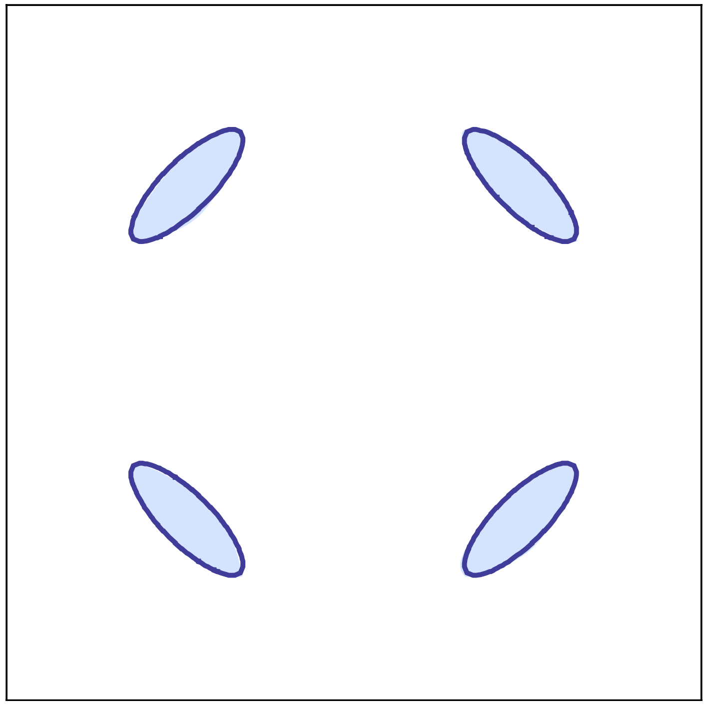

Leading order fermionic contribution is that implied by the Wiedemann-Franz law.

$$\sigma_{xy} = \left(\frac{\delta e^2 \tau}{m^*} \right) \omega_c \tau$$

$$\kappa_{xy}^0 = \frac{\pi^2 T}{3} \left(\frac{k_B}{e} \right)^2 \sigma_{xy}$$

Thermal Hall conductivity

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$$+ v_{\rm dis}(\boldsymbol{r}) f_{pv}^\dagger f_{pv}$$

Integrating out the fermions leads to an effective action for the emergent U(1) gauge field

$$\mathcal{S}_a = \int d^2x d\tau \left[\frac{K_1(\boldsymbol{x})}{2} (\boldsymbol{\nabla} \times \boldsymbol{a})^2 + \frac{K_2(\boldsymbol{x})}{2} (\boldsymbol{\nabla} a_\tau - \partial_\tau \boldsymbol{a})^2 \right.$$
$$\left. - \frac{i\sigma_{xy}(\boldsymbol{x})}{2e^2} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \right] + \int \frac{d^2 k d\omega}{8\pi^3} \gamma_k |\omega| \left[\boldsymbol{a}^T(k,\omega) \right]^2$$

Thermal Hall conductivity

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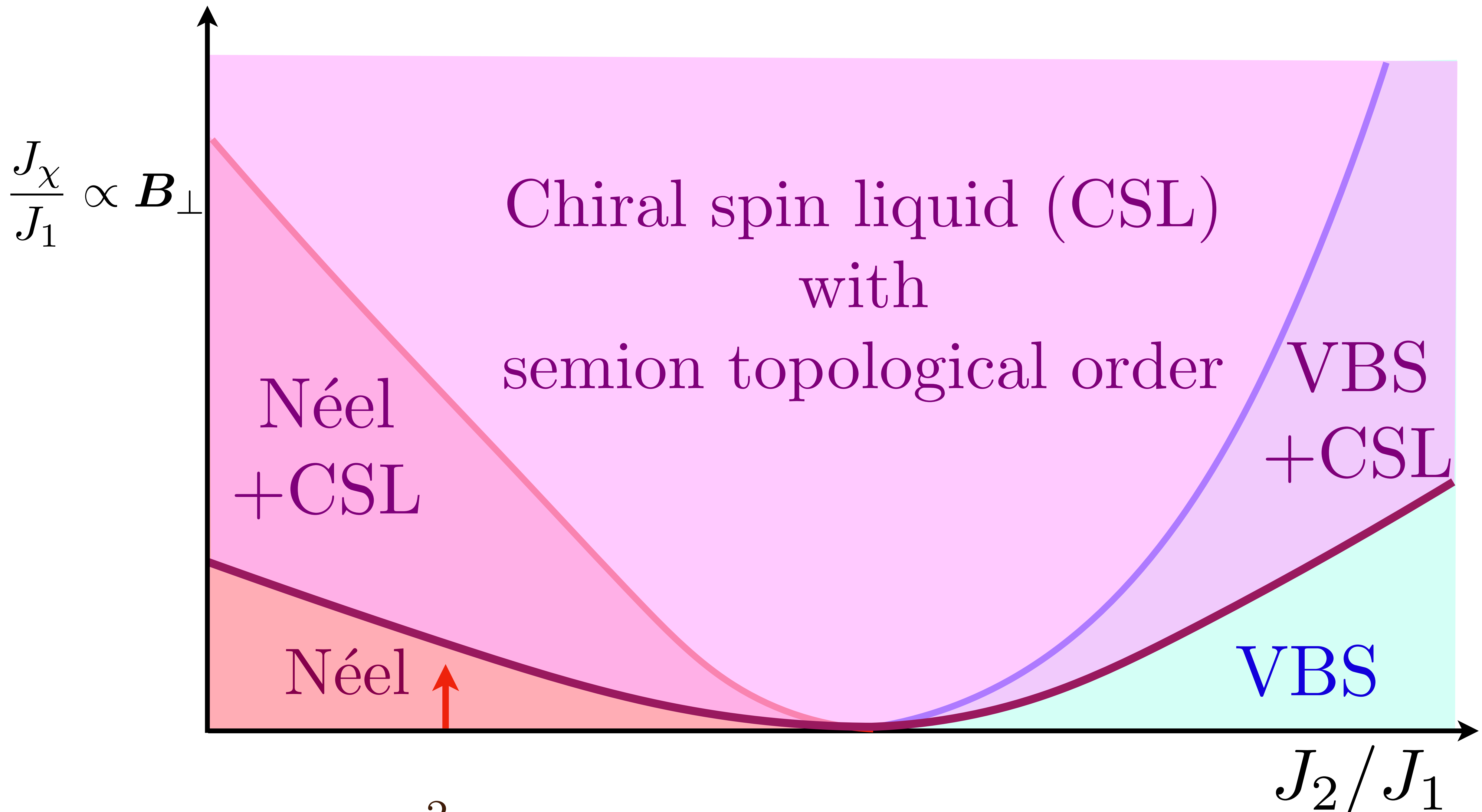
The gauge field contributes a thermal Hall conductivity, κ^1_{xy}, which has the *opposite sign* from the Wiedemann-Franz term determined from σ_{xy}.

$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$



$$\mathcal{S}_f = \int d^2 r d\tau \sum_{\alpha=1}^{2} \left[\overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha + m_\chi \overline{f}_\alpha f_\alpha + m_N \overline{f}_\alpha \Gamma f_\alpha \right]$$

R. Samajdar, M. S. Scheurer, S. Chatterjee, Haoyu Guo, Cenke Xu, and S. Sachdev,
Nature Physics (2019), arXiv:1903.01992

$\Phi^a \Rightarrow$ Incommensurate antiferromagnetism

$\langle \Phi^a \rangle \neq 0$

Metal with electron and hole pockets

Antiferromagnetic metal

$\langle H \rangle \neq 0$
$\langle R \rangle \neq 0$

$\langle \Phi^a \rangle = 0$

Metal with electron and hole pockets

Higgs phase of SU(2) gauge theory
CDW, Ising-nematic, and/or
Z_2 topological order

$\langle H \rangle \neq 0$
$\langle R \rangle = 0$

$\langle \Phi^a \rangle = 0$

Metal with large Fermi surface

Confining phase of SU(2) gauge theory

$\langle H \rangle = 0$
$\langle R \rangle \neq 0$

p

S. Sachdev, H. Scammell, M. Scheurer, G. Tarnopolsky, PRB **99**, 054516 (2019)

Gauge theory of fluctuating SDW coupled to large Fermi surface

SU(2) gauge theory for Higgs field $\mathcal{H}_\ell^a$
$a = 1, 2, 3$ is a SU(2) gauge index
$\ell = 1 \ldots N_h$ is a flavor index.

$$\mathcal{S} = \int d^2x d\tau \left[\frac{1}{2} \left(\partial_\mu \mathcal{H}_\ell^a - \epsilon_{abc} A_\mu^b \mathcal{H}_\ell^c \right)^2 + \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + V(\mathcal{H}_\ell^a) \right]$$
$$+\mathcal{S}_f - \text{coupling to electrons with large Fermi surface}$$

Gauge theory of fluctuating SDW coupled to large Fermi surface

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$+\mathcal{S}_f$–coupling to electrons with large Fermi surface

$$\mathcal{S}_f = -\frac{1}{2N_h} \int d^2x d\tau d\tau' \mathcal{H}_\ell^a(x,\tau) \mathcal{H}_m^a(x,\tau) J_f(\tau - \tau') \mathcal{H}_\ell^b(x,\tau') \mathcal{H}_m^b(x,\tau')$$

Gauge-invariant order parameters at x, τ couple to gauge invariant order parameters at x, τ' with $J_f(\tau) \sim 1/\tau^2$ via Fermi surface excitations.

This coupling is irrelevant by naive power-counting.

Gauge theory of fluctuating SDW coupled to large Fermi surface

In the large N_h limit, we are required to decouple $\mathcal{S}_f$ by introducing a bilocal field $C_{ab}(x, \tau, \tau')$

$$\mathcal{S}_f = \int d^2x d\tau d\tau' \left[\frac{N_h}{2} \frac{\left[C_{ab}(x,\tau,\tau')\right]^2}{J_f(\tau - \tau')} - C_{ab}(x,\tau,\tau')\mathcal{H}_\ell^a(x,\tau)\mathcal{H}_\ell^b(x,\tau') \right]$$

At the large N_h saddle point, we have $C_{ab}(x, \tau, \tau') = \delta_{ab} C(\tau - \tau')$. Saddle point equations show that C_{ab} displays strong scaling, and $\mathcal{S}_f$ is not irrelevant.

Bilocal quantum field theory

The path integral for the SYK model is a bilocal field theory

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1, \tau_1)\mathcal{D}\Sigma(\tau_1, \tau_2)e^{-NS[G,\Sigma]}$$

for a known action $S[G, \Sigma]$. The saddle point, $G_s(\tau_1 - \tau_2)$, $\Sigma_s(\tau_1 - \tau_2)$, depends only on time differences, and obeys Planckian ω/T scaling. The fluctuations

$$\begin{aligned} G(\tau_1, \tau_2) &= G_s(\tau_1 - \tau_2) + \delta G(\tau_1, \tau_2) \\ \Sigma(\tau_1, \tau_2) &= \Sigma_s(\tau_1 - \tau_2) + \delta \Sigma(\tau_1, \tau_2) \end{aligned}$$

require bilocal fields.

Similar remarks apply to other random quantum systems, and to DMFT.