

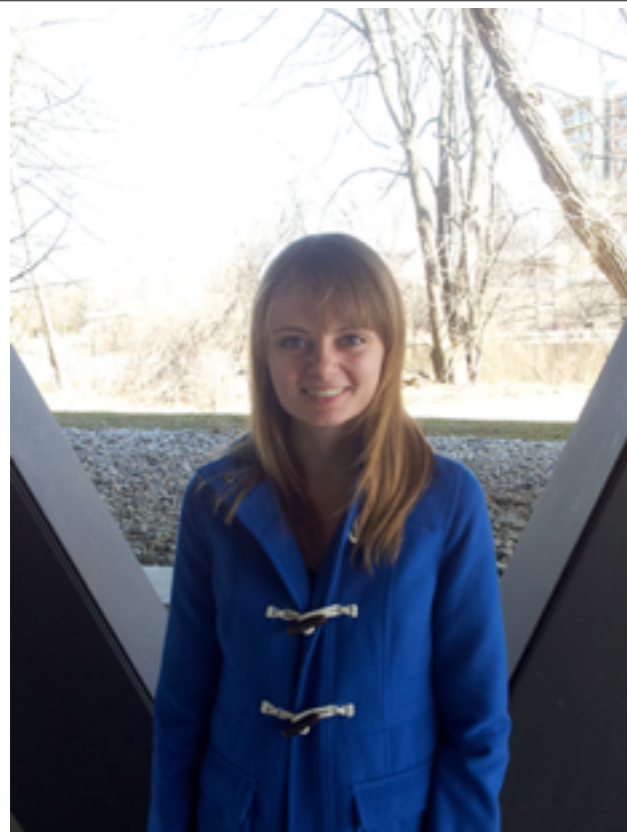
# Angular fluctuations of a multi-component order describe the pseudogap regime of the cuprate superconductors

CIFAR meeting  
Vancouver, October 17, 2013

Subir Sachdev

Talk online: [sachdev.physics.harvard.edu](http://sachdev.physics.harvard.edu)





Lauren  
Hayward



Roger Melko



David  
Hawthorn



Jay Deep Sau



Erez Berg



Max Metlitski



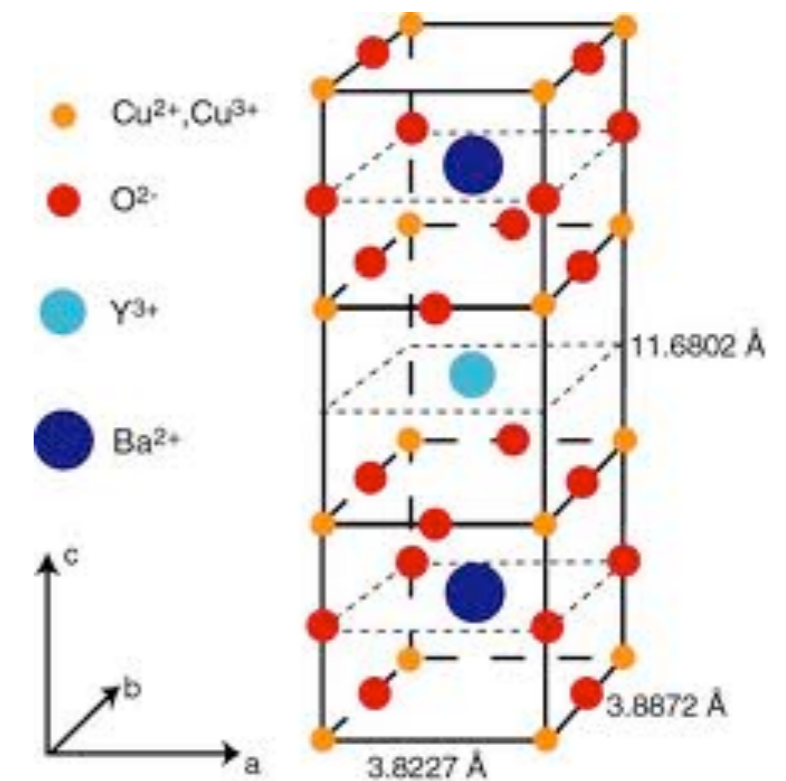
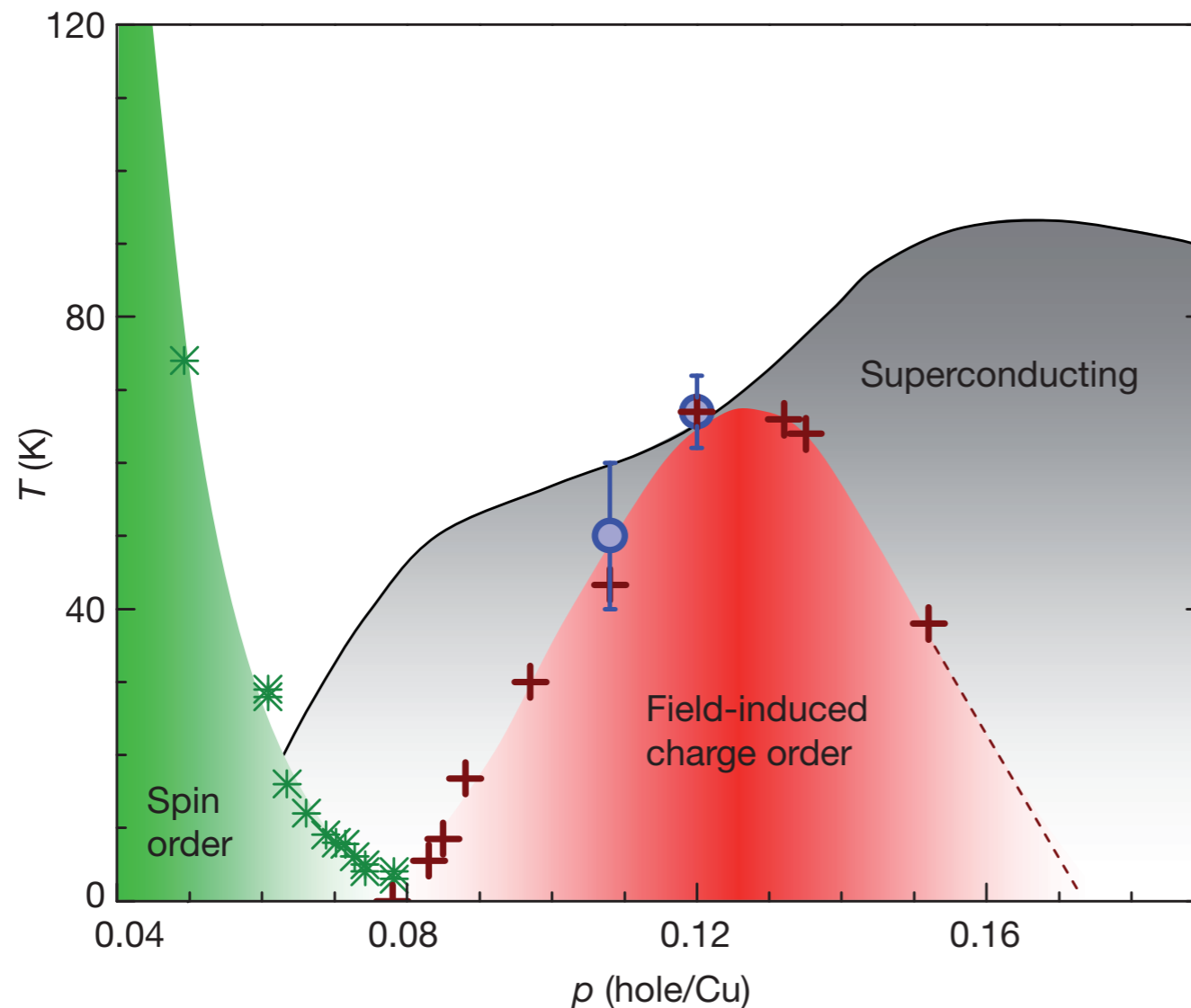
Rolando  
La Placa



# Magnetic-field-induced charge-stripe order in the high-temperature superconductor $\text{YBa}_2\text{Cu}_3\text{O}_y$

Tao Wu<sup>1</sup>, Hadrien Mayaffre<sup>1</sup>, Steffen Krämer<sup>1</sup>, Mladen Horvatić<sup>1</sup>, Claude Berthier<sup>1</sup>, W. N. Hardy<sup>2,3</sup>, Ruixing Liang<sup>2,3</sup>, D. A. Bonn<sup>2,3</sup> & Marc-Henri Julien<sup>1</sup>

8 SEPTEMBER 2011 | VOL 477 | NATURE | 191

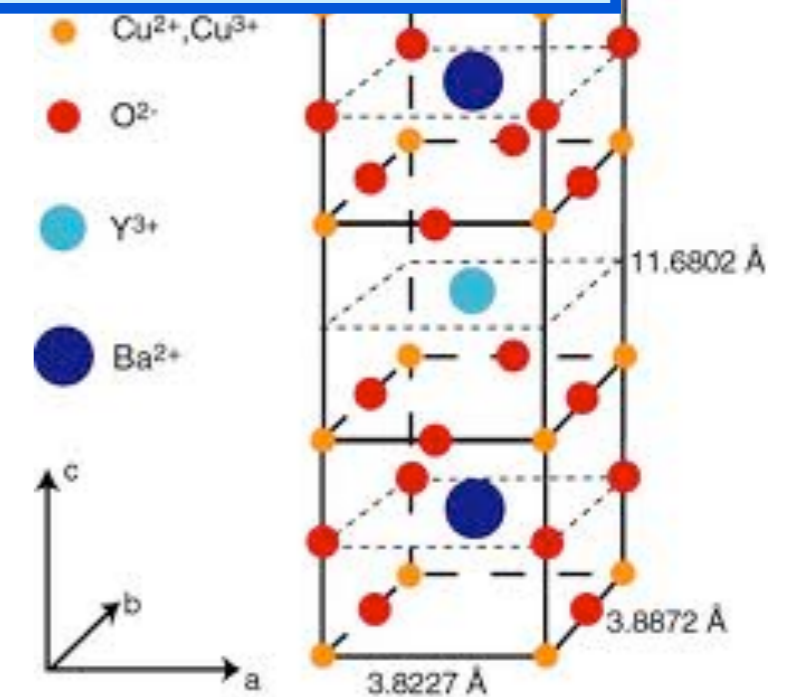
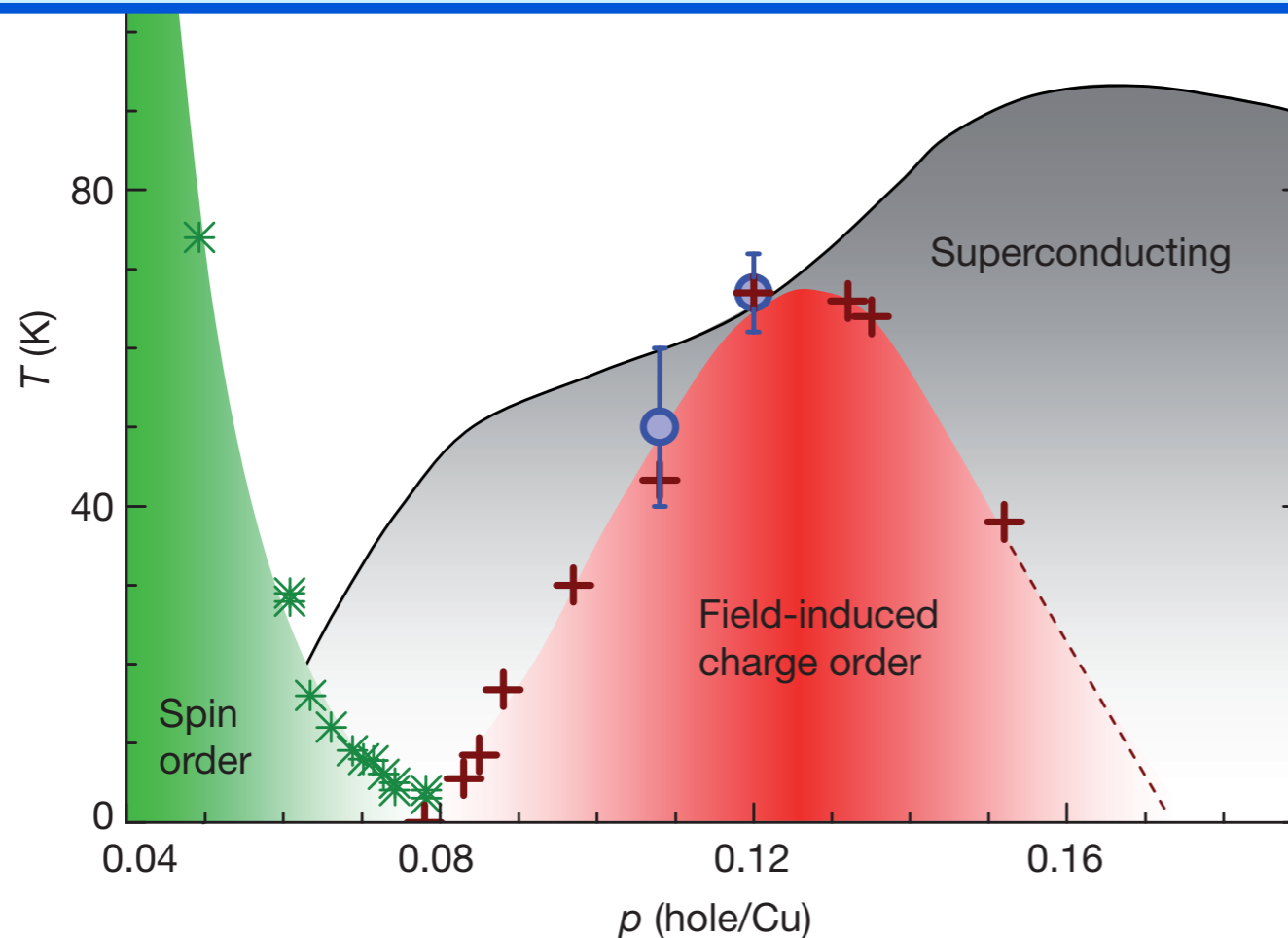


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Clear separation of regions of spin and charge order is an important simplifying feature of YBCO. Focus on the region where only charge order is seen.



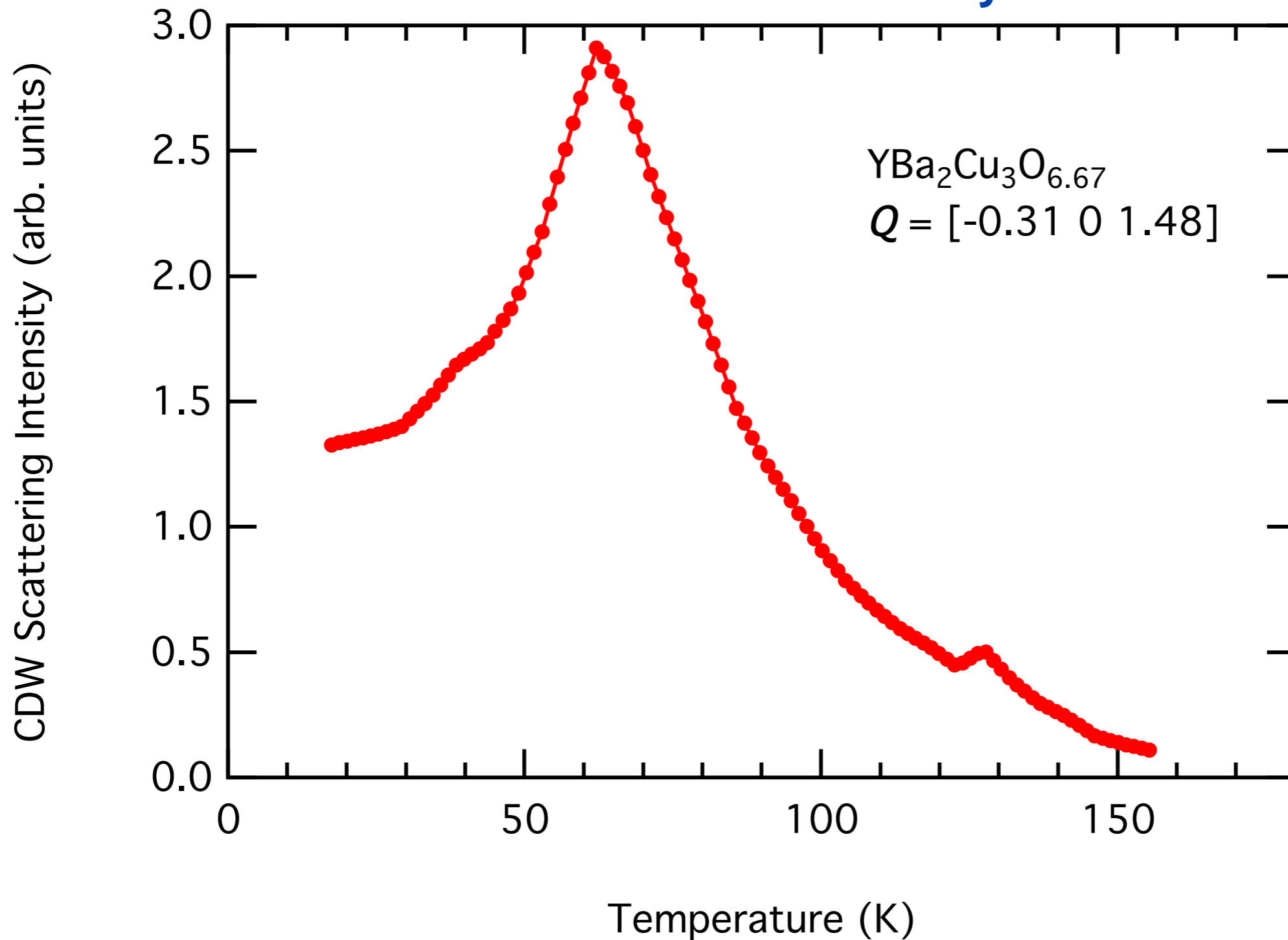


FIG. 1: The temperature dependence of the CDW scattering intensity at  $\mathbf{Q} = [-0.31 \ 0 \ 1.48]$  in  $\text{YBa}_2\text{Cu}_3\text{O}_{6.67}$  measured by resonant x-ray scattering in Ref. [4]. This sample has  $T_c \approx 65.5\text{K}$ .

# Competing orders in thermally fluctuating superconductors in two dimensions

Subir Sachdev

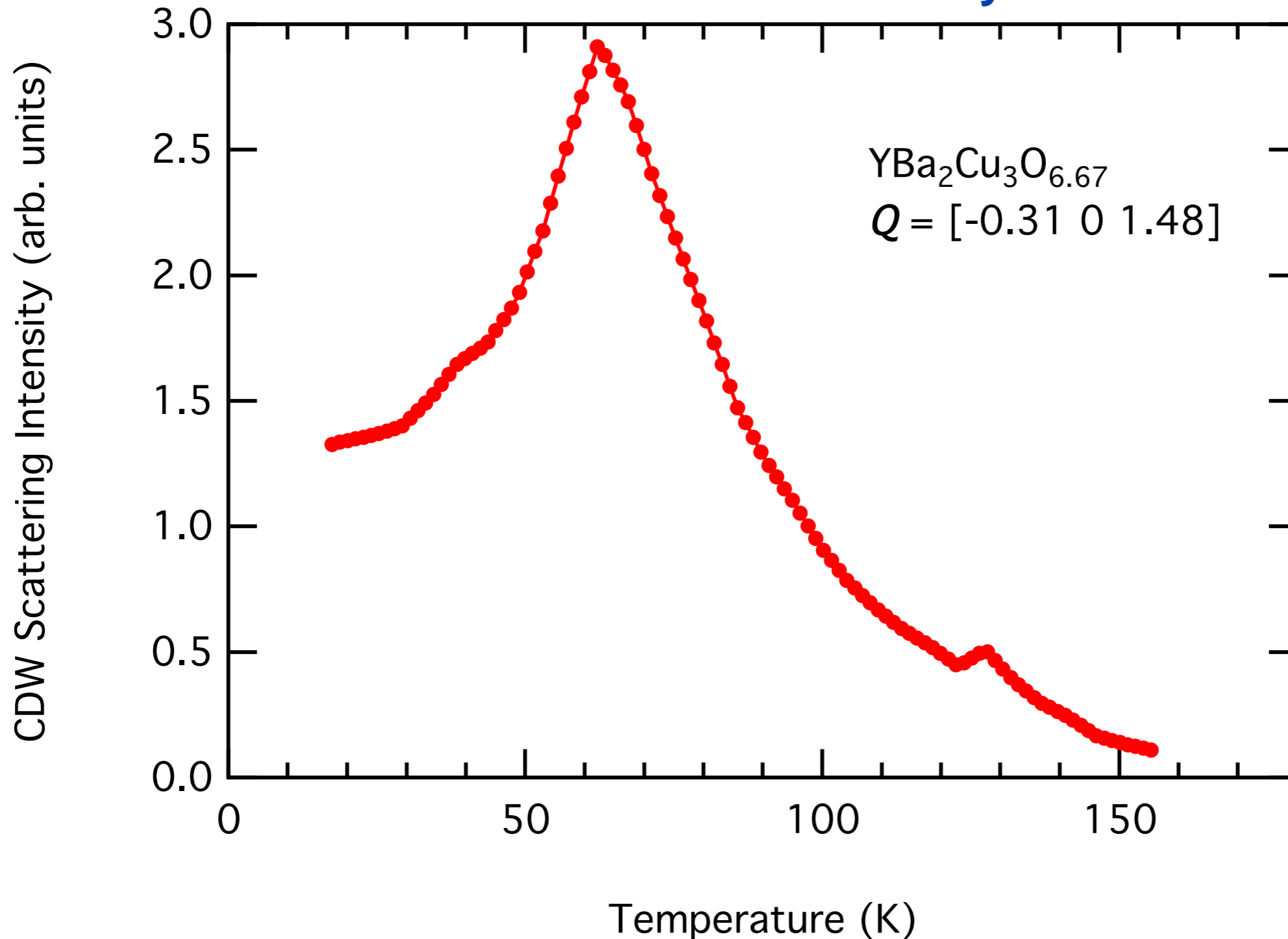
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(Received 6 August 2003; revised manuscript received 24 November 2003; published 6 April 2004)

We extend recent low-temperature analyses of competing orders in the cuprate superconductors to the **pseudogap regime** where all orders are fluctuating. A universal continuum limit of a classical Ginzburg-Landau functional is used to characterize fluctuations of the superconducting order: this describes the crossover from Gaussian fluctuations at high temperatures to the vortex-binding physics near the onset of global phase coherence. These fluctuations induce affiliated corrections in the correlations of other orders, and in particular, in the different realizations of charge order. Implications for scanning tunneling spectroscopy and neutron-scattering experiments are noted: **there may be a regime of temperatures near the onset of superconductivity where the charge order is enhanced with increasing temperatures.**



At high  $T$ , onset is unlike an arrested ordering transition,  
or precursor critical fluctuations.  
At low  $T$ , “competing order” effect is remarkably large.

Key idea: analogy with the onset of antiferromagnetism in the *insulator*  $La_2CuO_4$

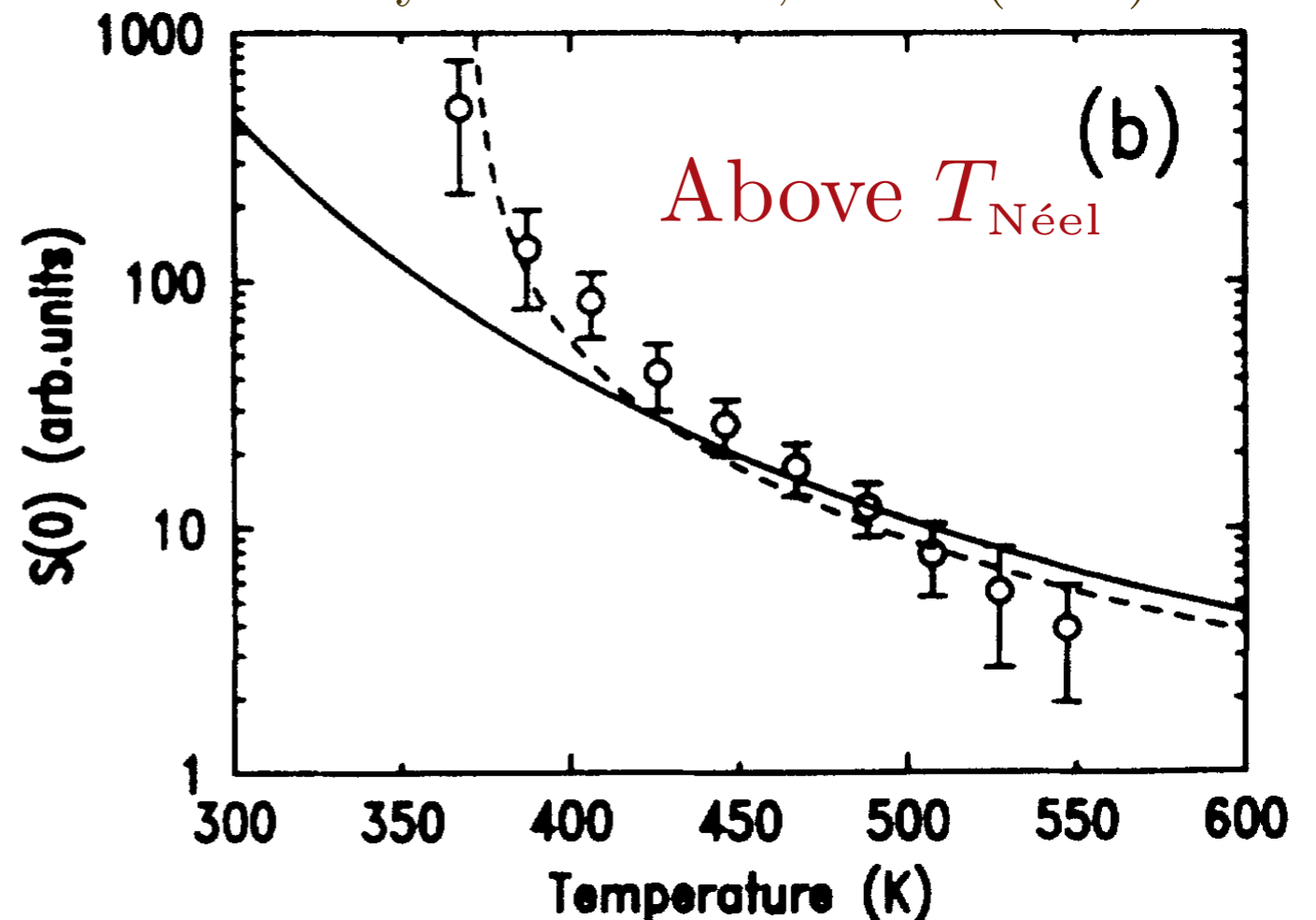
Gradual onset of intensity over a wide range of  $T$  is a consequence of angular thermal fluctuations of an order parameter with 3 or more components in 2 spatial dimensions

Polyakov, 1975

Chakravarty, Halperin, Nelson 1989

$$T_{\text{Néel}} = 325\text{K}$$

B. Keimer *et al.*,  
Phys. Rev. B **46**, 14034 (1992).



# Multi-component order parameter

Superconducting order  $\Psi(\mathbf{r})$ :

$$\langle c_{i\alpha}^\dagger c_{j\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \left[ \sum_{\mathbf{k}} \Delta_S(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] \Psi \left( (\mathbf{r}_i + \mathbf{r}_j)/2 \right)$$

Charge/bond order  $\Phi_{x,y}(\mathbf{r})$  at wavevectors  $\mathbf{Q}_{x,y}$ :

$$\begin{aligned} \langle c_{i\alpha}^\dagger c_{j\beta} \rangle &= \delta_{\alpha\beta} \left[ \sum_{\mathbf{k}} P_{\mathbf{Q}_x}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}_x\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_x \left( (\mathbf{r}_i + \mathbf{r}_j)/2 \right) \\ &\quad + \delta_{\alpha\beta} \left[ \sum_{\mathbf{k}} P_{\mathbf{Q}_y}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}_y\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_y \left( (\mathbf{r}_i + \mathbf{r}_j)/2 \right) \end{aligned}$$

# Multi-component order parameter

Symmetries:

Charge conservation ( $O(2)$ ),  $x$  translations ( $O(2)$ ),  $y$  translations ( $O(2)$ ),  $x \leftrightarrow y$  ( $\mathbb{Z}_2$ ), inversion, time-reversal.

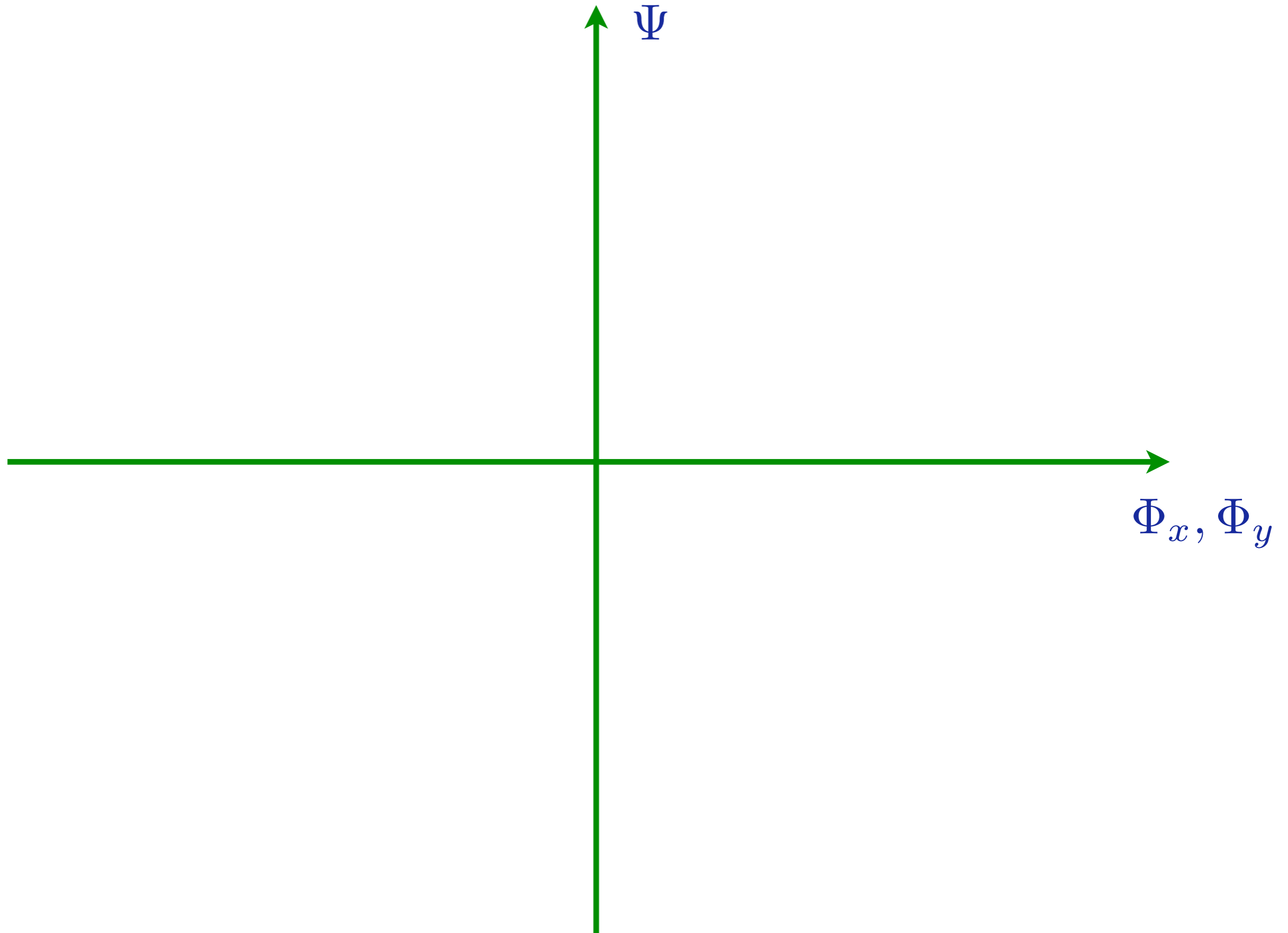
Landau-Ginzburg free energy:

$$F = \int d^2r \left[ |\nabla\Psi|^2 + s_1|\Psi|^2 + u_1|\Psi|^4 + |\nabla\Phi_x|^2 + |\nabla\Phi_y|^2 + s_2(|\Phi_x|^2 + |\Phi_y|^2) + u_2(|\Phi_x|^2 + |\Phi_y|^2)^2 + w(|\Phi_x|^4 + |\Phi_y|^4) + v|\Psi|^2(|\Phi_x|^2 + |\Phi_y|^2) \right]$$

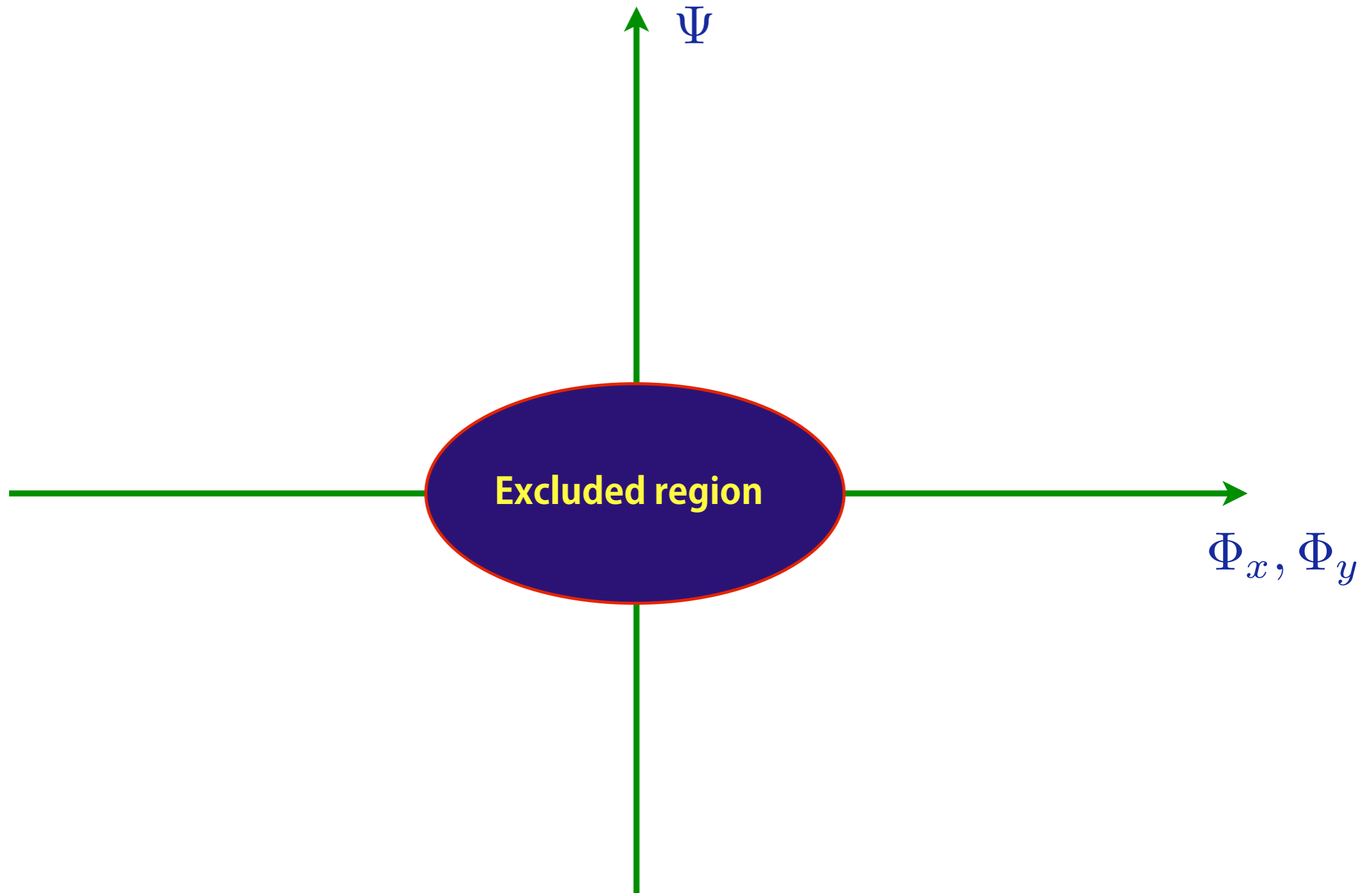
Competing orders:

**Needed: a theory for large  $v$**

# Multi-component order parameter



# Multi-component order parameter

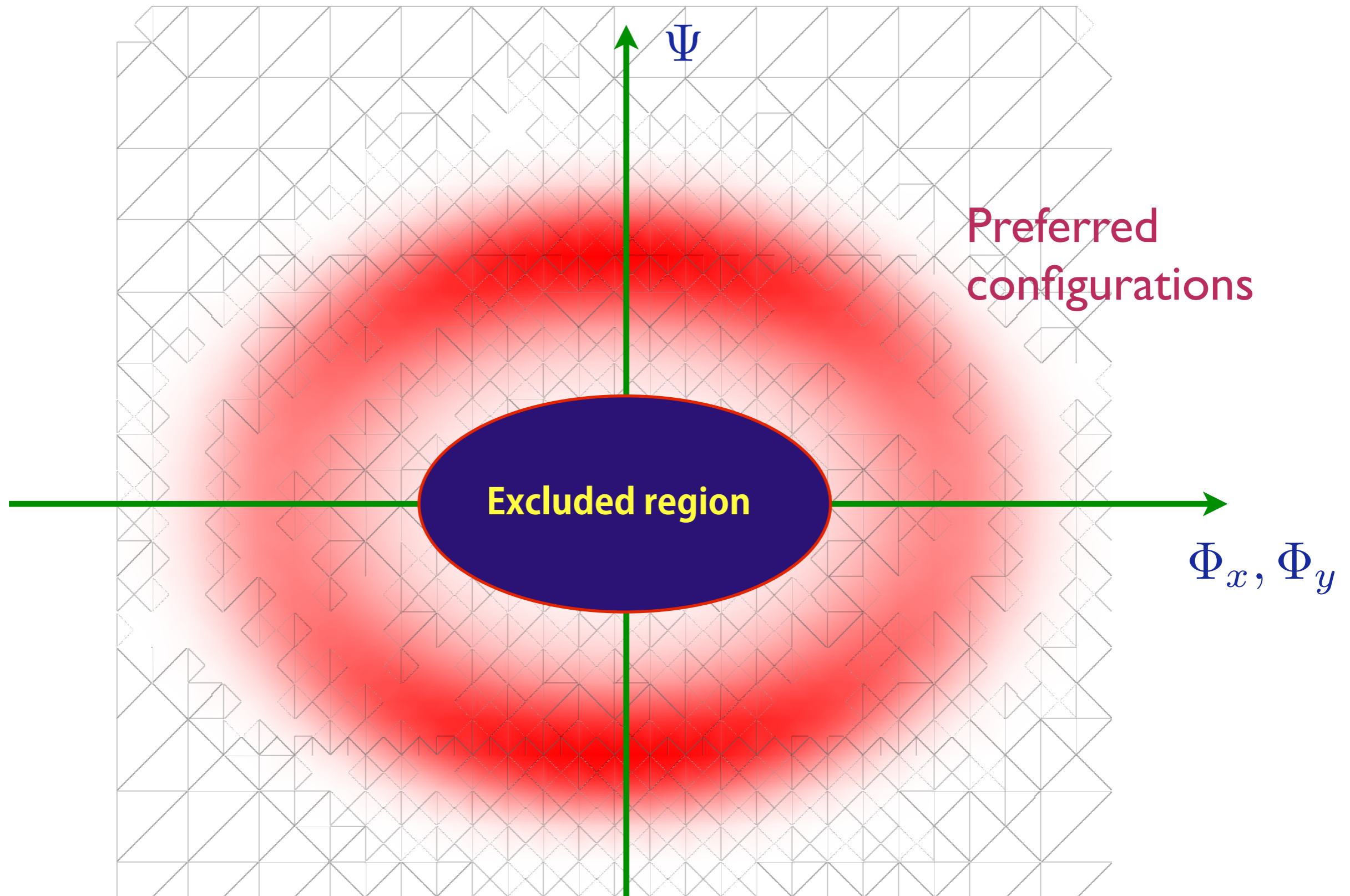


Support from theory of antiferromagnetic quantum criticality

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

K. B. Efetov, H. Meier, and C. Pepin, *Nature Physics* **9**, 442 (2013)

# Multi-component order parameter

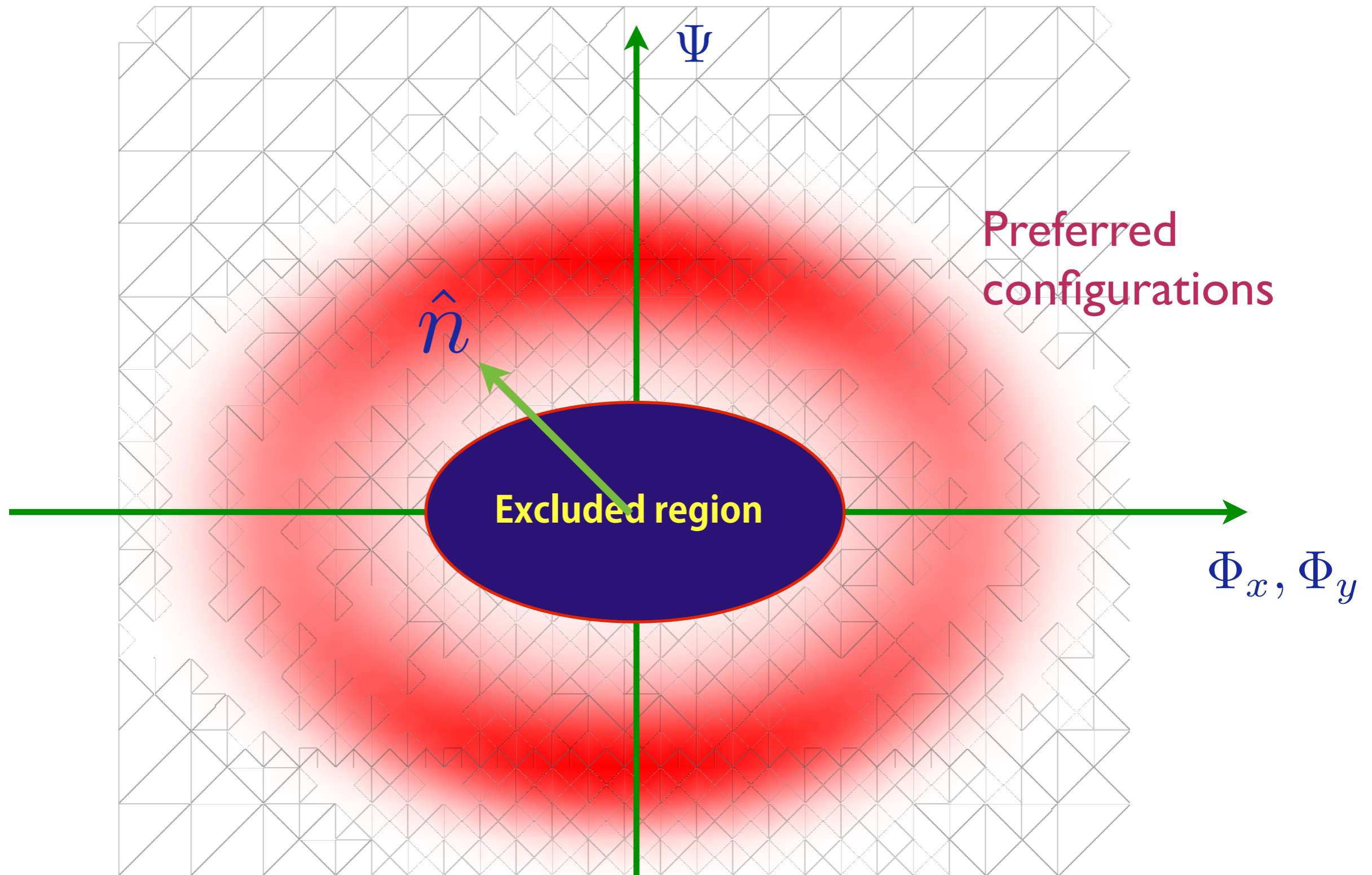


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# Multi-component order parameter



Label order parameter by a  
6-component unit vector  $n_\alpha$  with  $\sum_\alpha n_\alpha^2 = 1$

## O(6) non-linear sigma model

$$\mathcal{Z} = \int \mathcal{D}n_\alpha(\mathbf{r}) \delta \left( \sum_{\alpha=1}^6 n_\alpha^2(\mathbf{r}) - 1 \right) \exp \left( - \frac{\rho_s}{2T} \int d^2r \left[ \sum_{\alpha=1}^2 (\nabla n_\alpha)^2 + \lambda \sum_{\alpha=3}^6 (\nabla n_\alpha)^2 + g \sum_{\alpha=3}^6 n_\alpha^2 + w \left[ (n_3^2 + n_4^2)^2 + (n_5^2 + n_6^2)^2 \right] \right] \right).$$

where  $\Psi \propto n_1 + in_2$ ,  $\Phi_x \propto n_3 + in_4$ ,  $\Phi_y \propto n_5 + in_6$ .

Describes  $O(6) \Rightarrow O(2) \times O(2) \times O(2) \rtimes \mathbb{Z}_2$ . The coupling  $g$  determines the anisotropy between superconductivity and charge order.

Solve by cluster Monte Carlo and  $1/N$  expansion.

L. E. Hayward, D. G. Hawthorn, R. G. Melko, and S. Sachdev, arXiv:1309.6639

## O(6) non-linear sigma model

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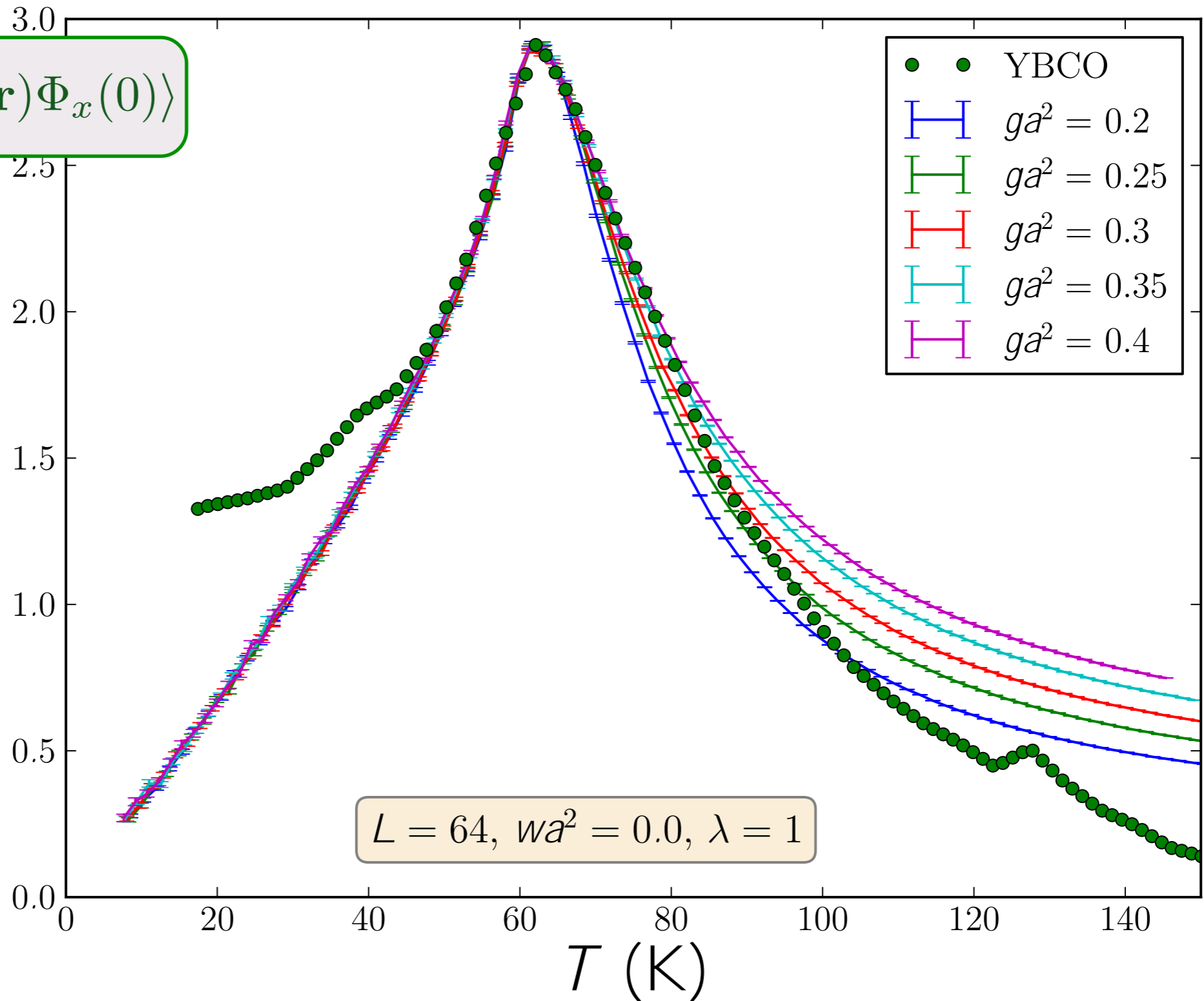
Solve by cluster Monte Carlo and  $1/N$  expansion.

L. E. Hayward, D. G. Hawthorn, R. G. Melko, and S. Sachdev, arXiv:1309.6639

# Comparison of Monte Carlo with experiments

$$S_{\Phi_x} = \int d^2r \langle \Phi_x(\mathbf{r}) \Phi_x(0) \rangle$$

Charge order  
structure  
factor  $S_{\Phi_x}$



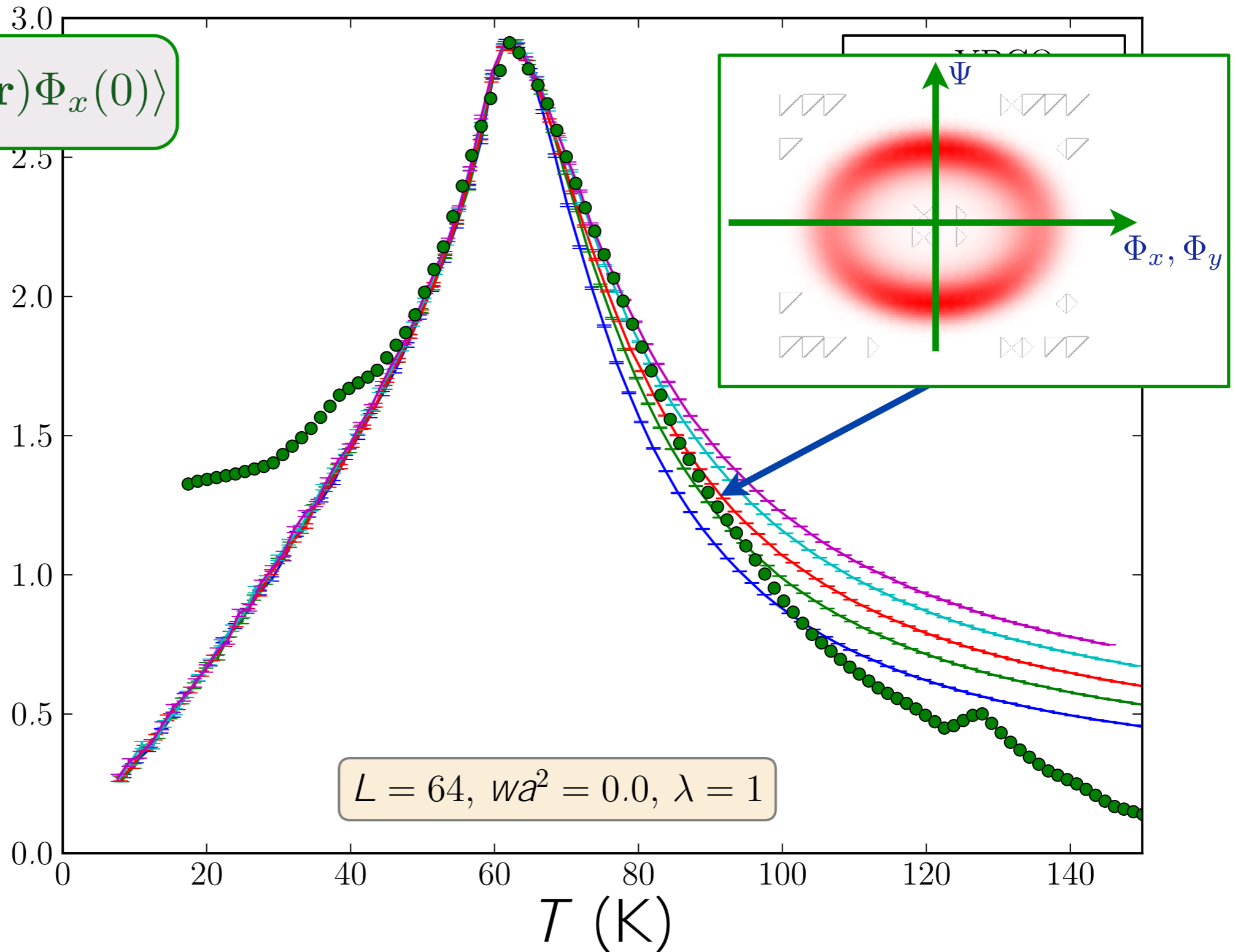
For  $ga^2 = 0.30$  and  $wa^2 = 0.0$  we have  $\rho_s = 160\text{K}$ .  
The height was also rescaled to make the peak heights match.

L. E. Hayward, D. G. Hawthorn, R. G. Melko, and S. Sachdev, arXiv:1309.6639

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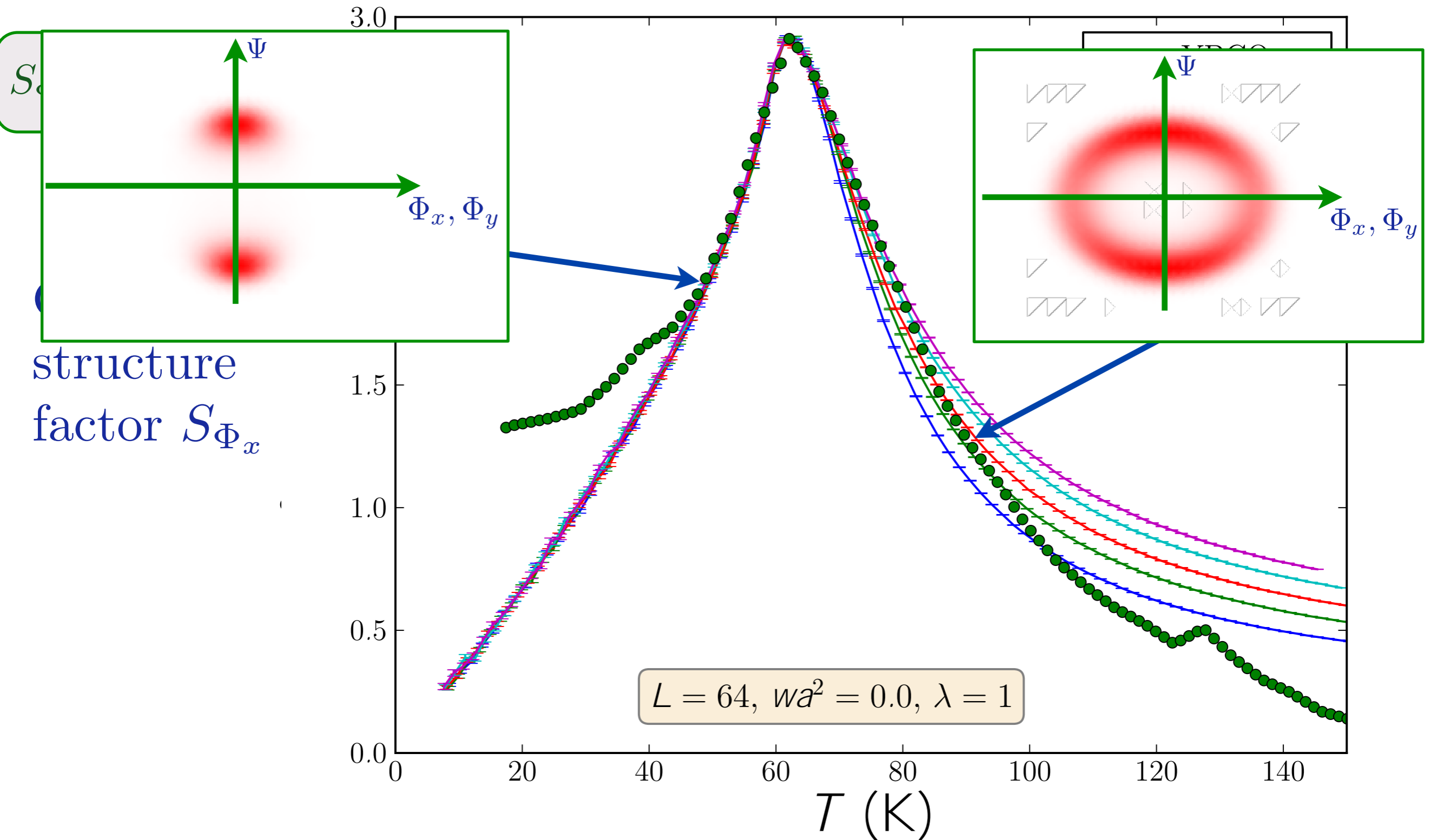
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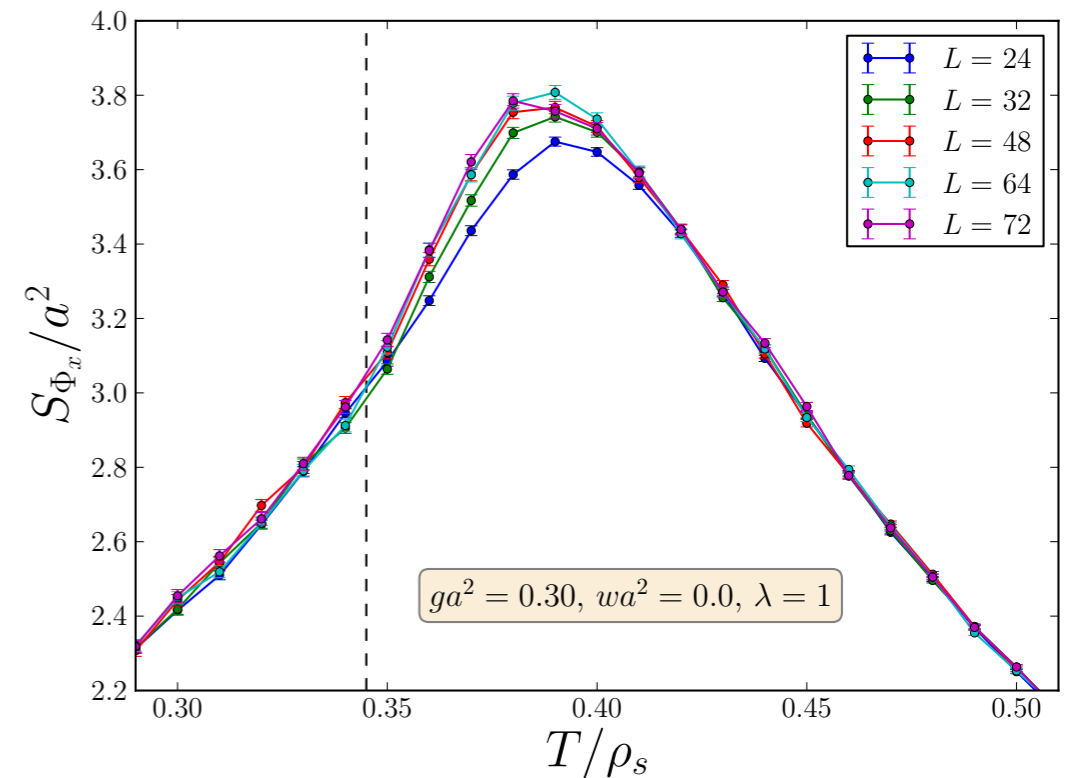
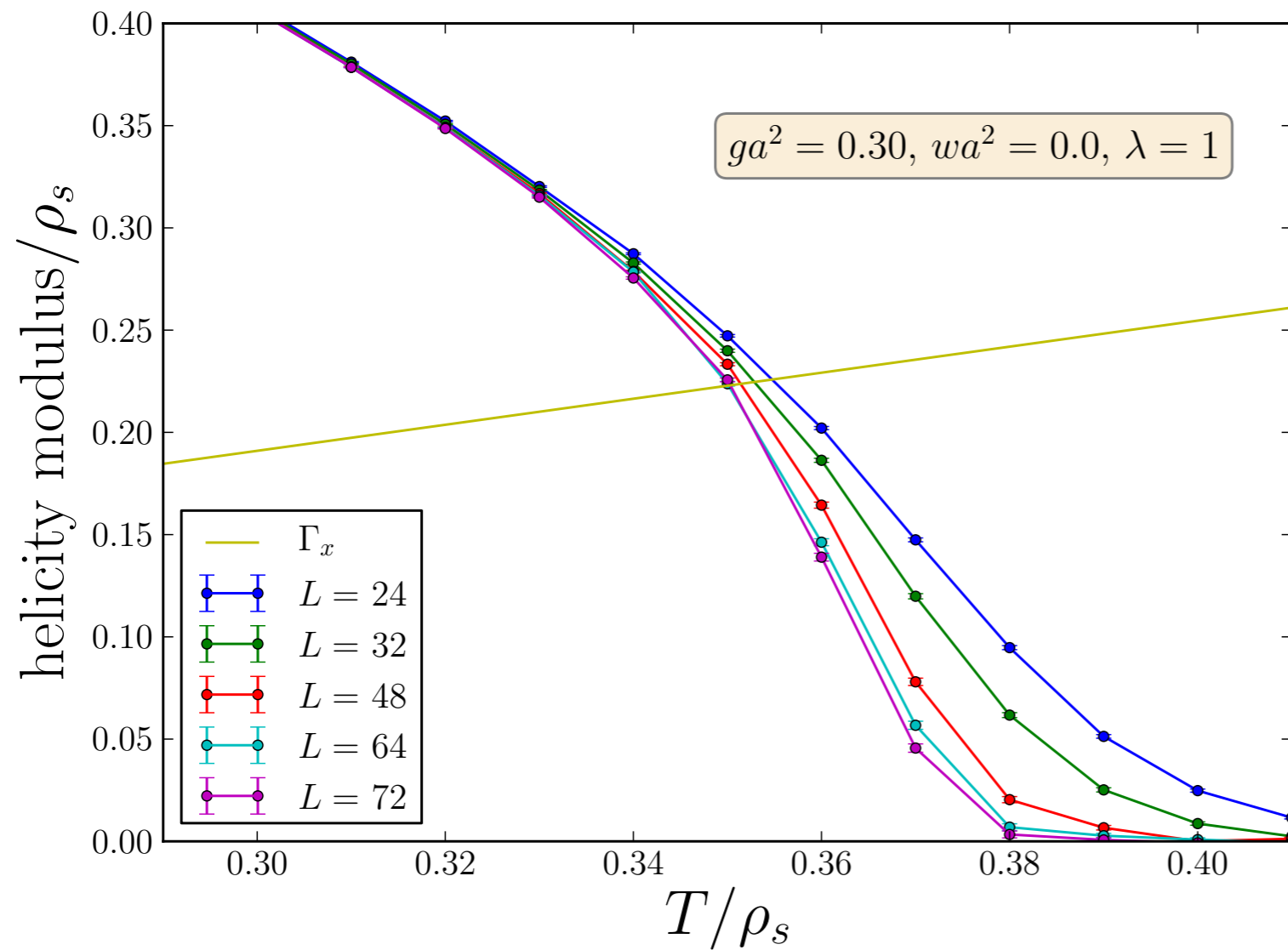
# Comparison of Monte Carlo with experiments



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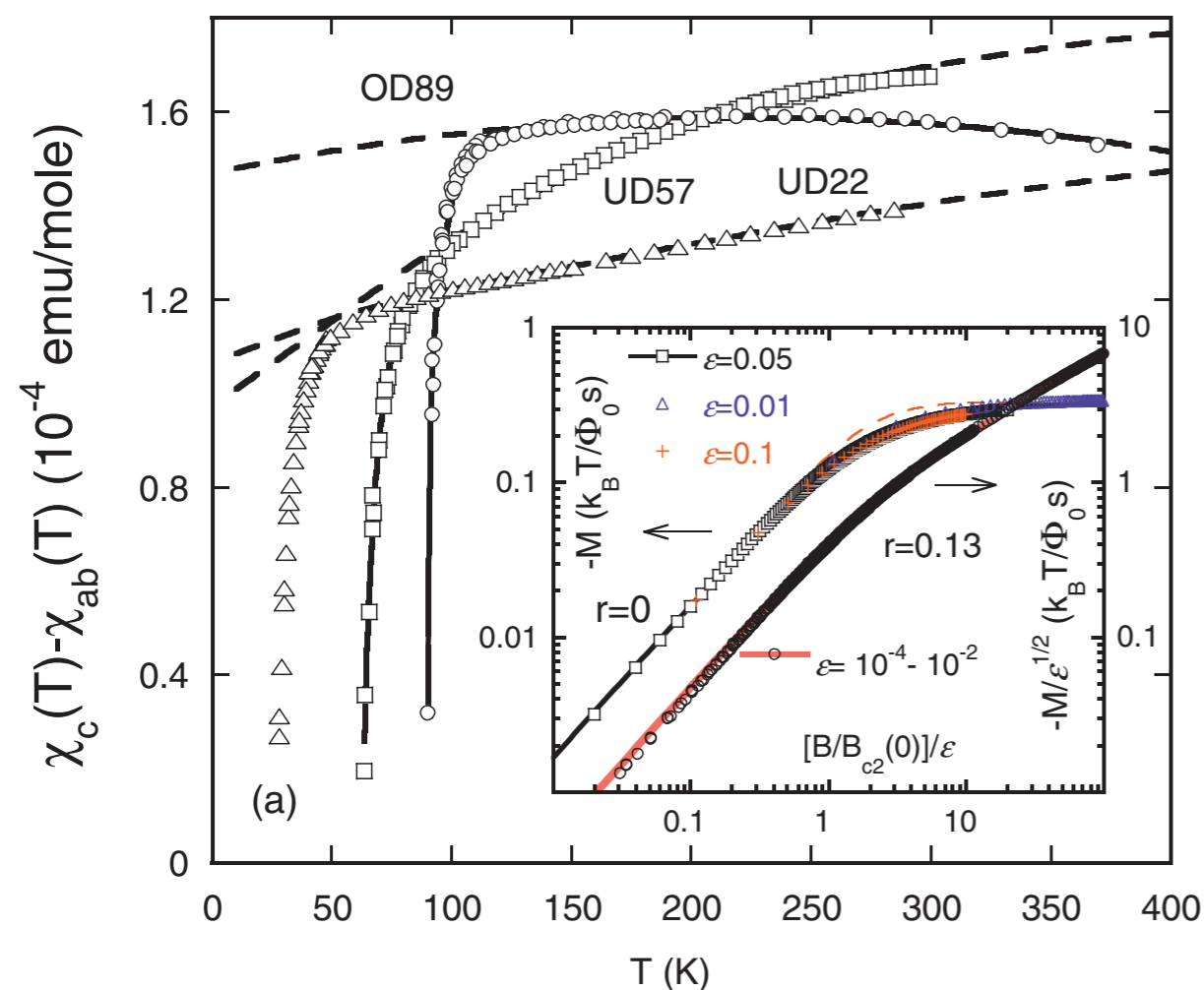
# Onset of superconductivity in Monte Carlo



L. E. Hayward, D. G. Hawthorn,  
R. G. Melko, and S. Sachdev, arXiv:1309.6639

## Other experiments in the pseudogap

- The *same* set of parameters used to describe X-ray scattering, also predict the strength of superconducting fluctuations above  $T_c$ . Indeed  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$  shows significant fluctuation diamagnetism over the same range of temperatures. (S. Chatterjee et al, in progress).



PHYSICAL REVIEW B **88**, 060505(R) (2013)

I. Kokanović,<sup>1,2,\*</sup> D. J. Hills,<sup>1</sup> M. L. Sutherland,<sup>1</sup> R. Liang,<sup>3</sup> and J. R. Cooper<sup>1</sup>

**Diamagnetism of  $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$  crystals above  $T_c$ : Evidence for Gaussian fluctuations**

# Pseudospin symmetry of the exchange interaction

$$H_{tJ} = - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

with  $\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}$  is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$D_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow}^\dagger \end{pmatrix}, \quad D_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c_{i\uparrow}^\dagger \end{pmatrix}$$

Then we can write

$$H_J = \frac{1}{8} \sum_{i<j} J_{ij} \left( D_{i\alpha a}^\dagger \vec{\sigma}_{\alpha\beta} D_{i\beta a} \right) \cdot \left( D_{j\gamma b}^\dagger \vec{\sigma}_{\gamma\delta} D_{j\delta b} \right)$$

where  $a, b$  are the Nambu indices. This form makes explicit the symmetry under *independent* SU(2) pseudospin transformations on each site

$$D_{i\alpha a} \rightarrow U_{i,ab} D_{i\alpha b}$$

This pseudospin (gauge) symmetry is important in classifying spin liquid ground states of  $H_J$ . It is fully broken by the electron hopping  $t_{ij}$  but does have remnant consequences in doped spin liquid states.

- I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988)
- E. Dagotto, E. Fradkin, and A. Moreo, Phys. Rev. B **38**, 2926 (1988)
- P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006)

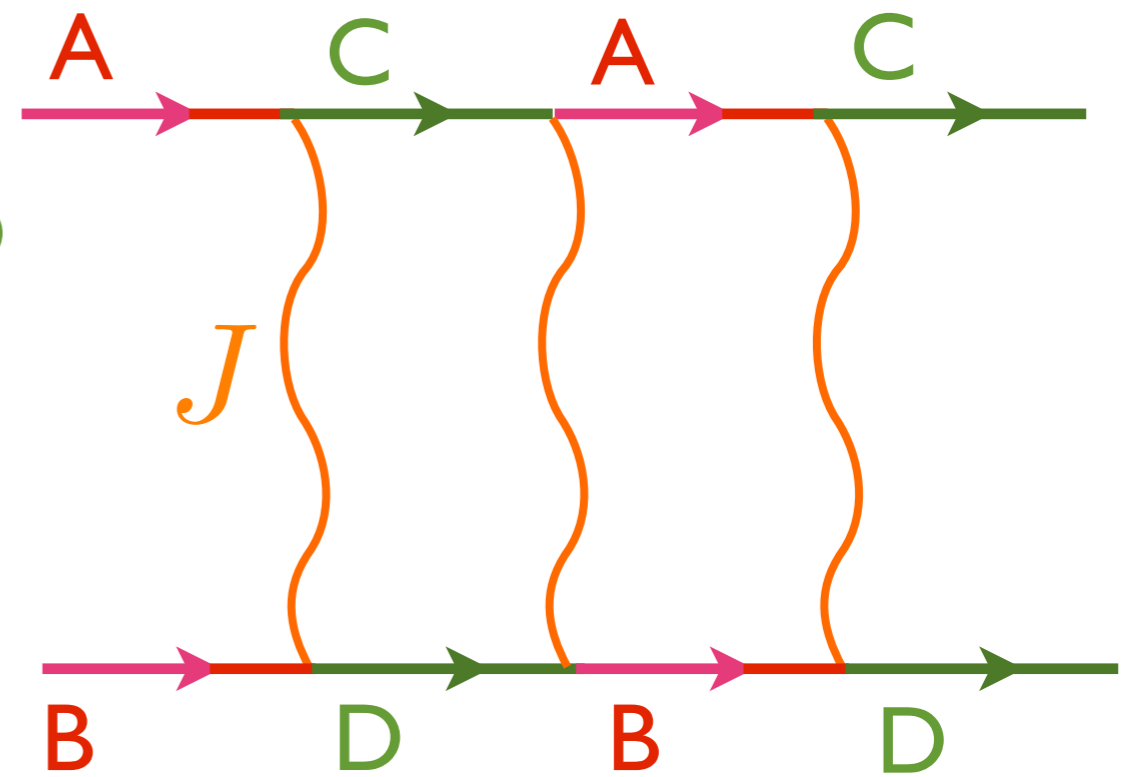
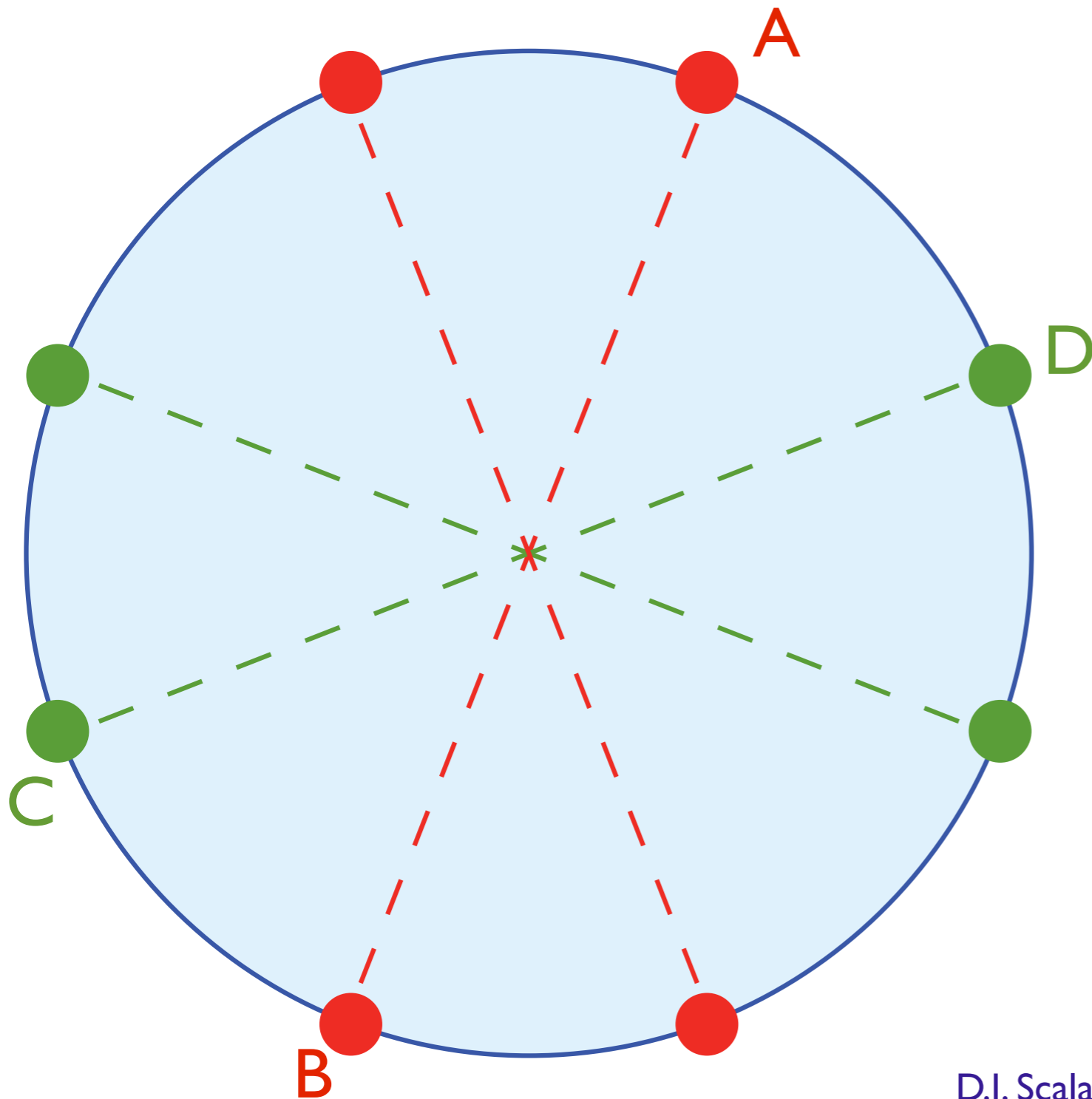
## Pseudospin symmetry of the exchange interaction

We will find important consequences of the pseudospin symmetry in ordinary metals with antiferromagnetic correlations.

$$H_{tJ} = - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

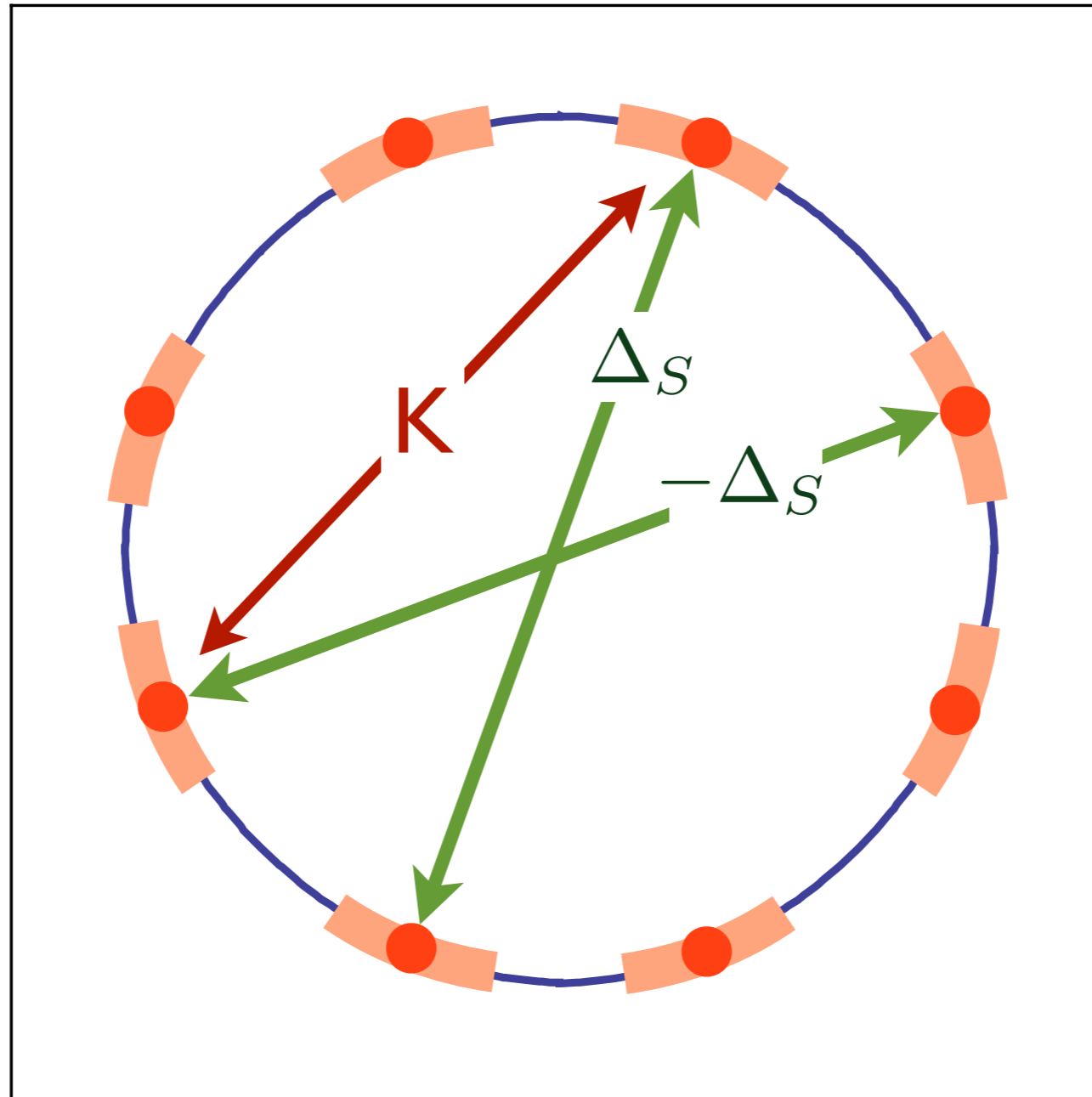
In Fourier space, the exchange interaction  $J(\mathbf{q})$  is maximum near  $\mathbf{q} = \mathbf{K} \equiv (\pi, \pi)$ . So it is most effective near points on the Fermi surface which are separated by  $\mathbf{K}$ , the so-called “hot spots”. Exchange interactions near the hot spots are expected to lead to  $d$ -wave superconductivity at low temperatures.

# Pairing “glue” from antiferromagnetic fluctuations near “hot spots” on the Fermi surface

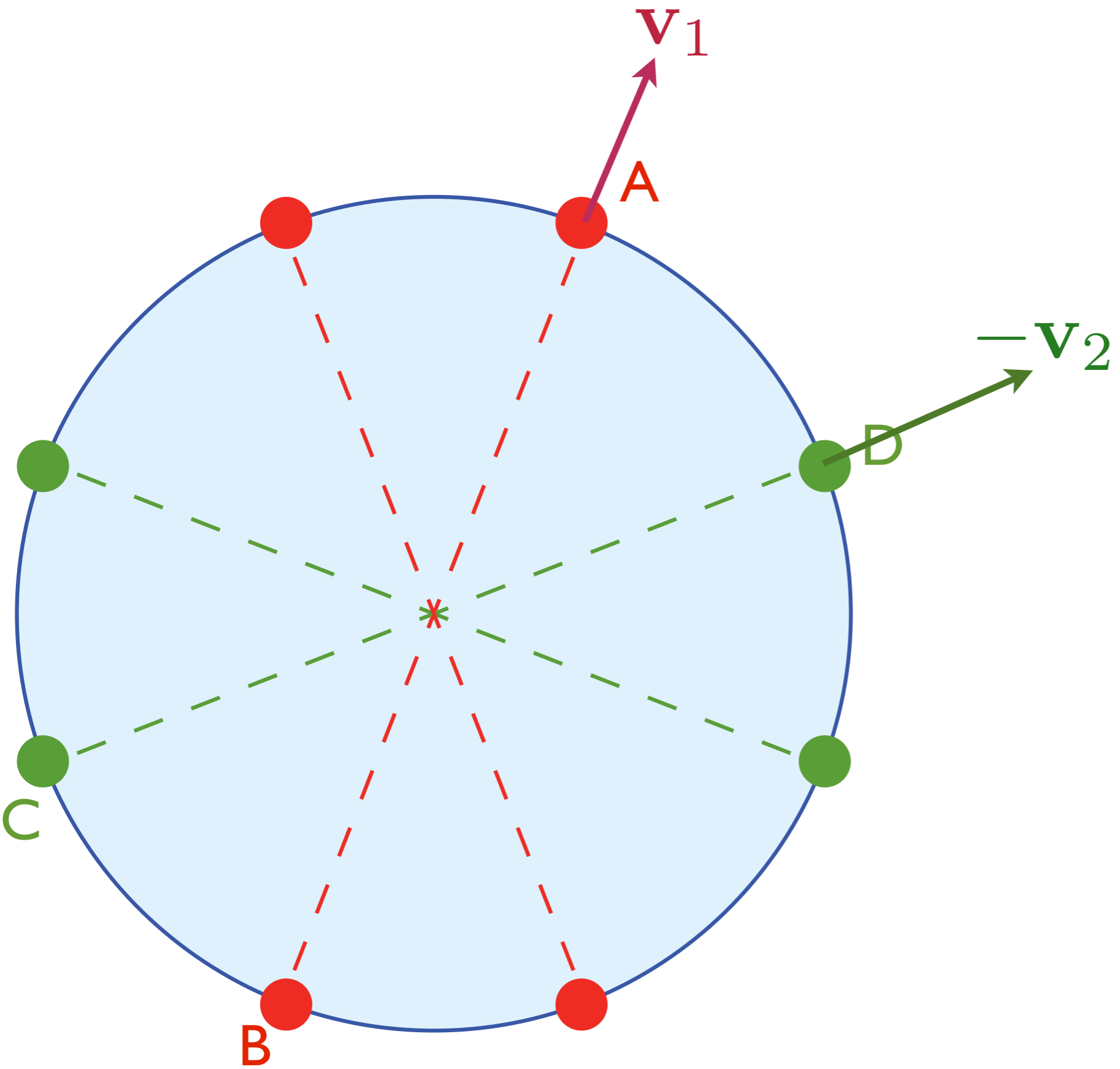


- V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)  
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)  
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)  
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)  
E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012)

$$\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y)$$



**d-wave superconductor: particle-particle pairing  
at and near hot spots, with  
sign-changing pairing amplitude**



# Hamiltonian at and near hot spots

$$\begin{aligned}
 H = \sum_{\mathbf{k}} & \left[ \mathbf{v}_1 \cdot \mathbf{k} \right] c_{A\alpha}^\dagger(\mathbf{k}) c_{A\alpha}(\mathbf{k}) \\
 & + \left[ \mathbf{v}_2 \cdot \mathbf{k} \right] c_{C\alpha}^\dagger(\mathbf{k}) c_{C\alpha}(\mathbf{k}) \\
 & + \left[ -\mathbf{v}_1 \cdot \mathbf{k} \right] c_{B\alpha}^\dagger(\mathbf{k}) c_{B\alpha}(\mathbf{k}) \\
 & + \left[ -\mathbf{v}_2 \cdot \mathbf{k} \right] c_{D\alpha}^\dagger(\mathbf{k}) c_{D\alpha}(\mathbf{k}) \\
 & + \int d^2x \left[ -J \left( c_{A\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{C\beta} + c_{C\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{A\beta} \right) \right. \\
 & \quad \left. \cdot \left( c_{B\gamma}^\dagger \vec{\sigma}_{\gamma\delta} c_{D\delta} + c_{D\gamma}^\dagger \vec{\sigma}_{\gamma\delta} c_{B\delta} \right) \right]
 \end{aligned}$$

]

# Hamiltonian at and near hot spots

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 & \quad \left. \cdot \left( c_{B\gamma}^\dagger \vec{\sigma}_{\gamma\delta} c_{D\delta} + c_{D\gamma}^\dagger \vec{\sigma}_{\gamma\delta} c_{B\delta} \right) \right]
 \end{aligned}$$

This Hamiltonian  
has an exact  
 $SU(2) \times SU(2)$   
pseudospin  
symmetry !

M.A. Metlitski and S. Sachdev,  
*Phys. Rev. B* **85**, 075127 (2010)

# Hamiltonian at and *near* hot spots

$$\begin{aligned}
 H = \sum_{\mathbf{k}} & \left[ \mathbf{v}_1 \cdot \mathbf{k} + \alpha(\mathbf{v}_1 \times \mathbf{k})^2 \right] c_{A\alpha}^\dagger(\mathbf{k}) c_{A\alpha}(\mathbf{k}) \\
 & + \left[ \mathbf{v}_2 \cdot \mathbf{k} + \alpha(\mathbf{v}_2 \times \mathbf{k})^2 \right] c_{C\alpha}^\dagger(\mathbf{k}) c_{C\alpha}(\mathbf{k}) \\
 & + \left[ -\mathbf{v}_1 \cdot \mathbf{k} + \alpha(\mathbf{v}_1 \times \mathbf{k})^2 \right] c_{B\alpha}^\dagger(\mathbf{k}) c_{B\alpha}(\mathbf{k}) \\
 & + \left[ -\mathbf{v}_2 \cdot \mathbf{k} + \alpha(\mathbf{v}_2 \times \mathbf{k})^2 \right] c_{D\alpha}^\dagger(\mathbf{k}) c_{D\alpha}(\mathbf{k}) \\
 & + \int d^2x \left[ -J \left( c_{A\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{C\beta} + c_{C\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{A\beta} \right) \right. \\
 & \quad \cdot \left( c_{B\gamma}^\dagger \vec{\sigma}_{\gamma\delta} c_{D\delta} + c_{D\gamma}^\dagger \vec{\sigma}_{\gamma\delta} c_{B\delta} \right) \\
 & \quad \left. -V \left( c_{A\alpha}^\dagger c_{C\alpha} + c_{C\alpha}^\dagger c_{A\alpha} \right) \left( c_{B\beta}^\dagger c_{D\beta} + c_{D\beta}^\dagger c_{B\beta} \right) \right]
 \end{aligned}$$

Fermi surface curvature; breaks pseudospin symmetry

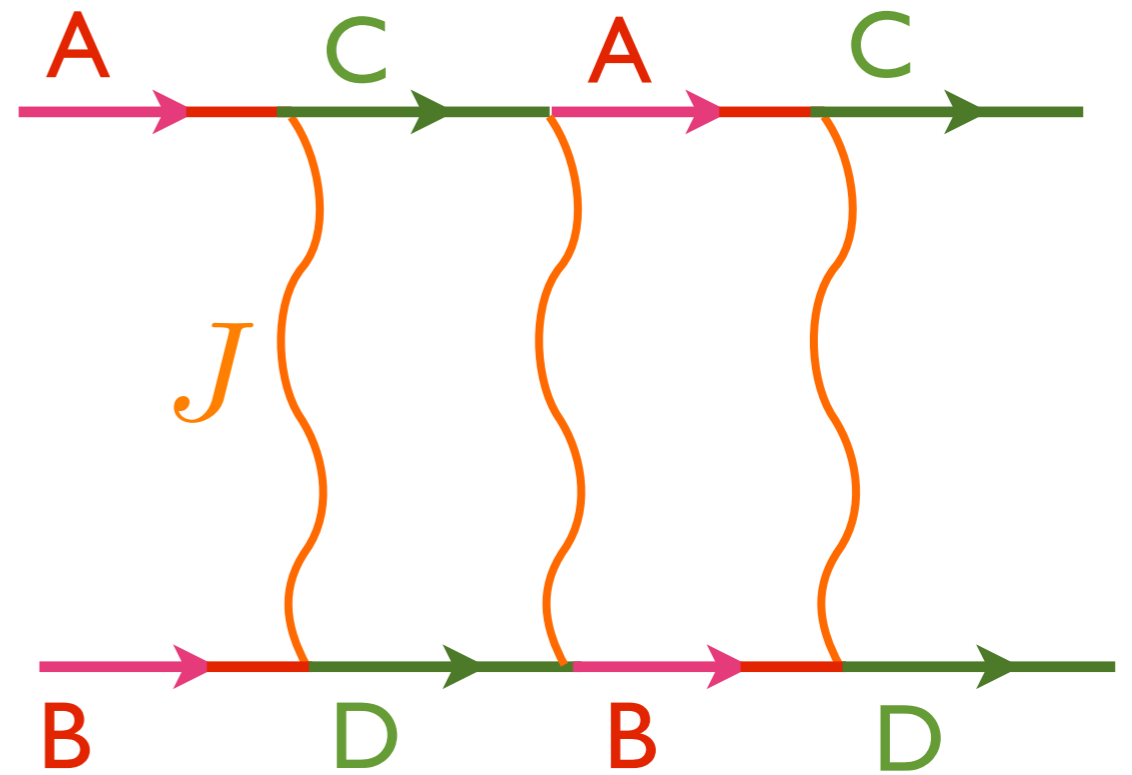
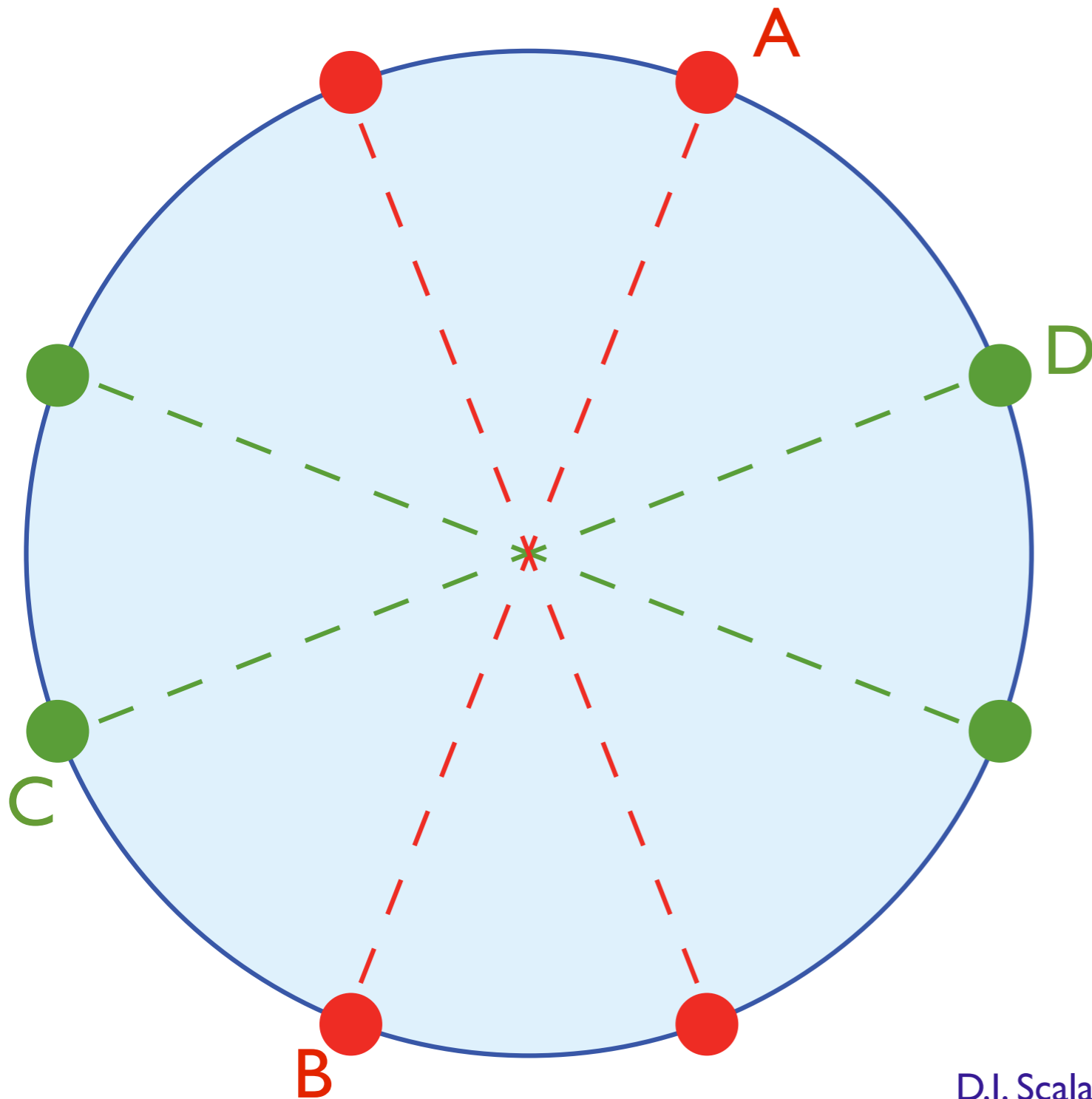
Coulomb repulsion; breaks pseudospin symmetry

Perform standard Hartree-Fock-BCS factorizations into

$$\begin{aligned}
 \Delta_S &= \langle \varepsilon_{\alpha\beta} \Psi_{A\alpha} \Psi_{B\beta} \rangle = - \langle \varepsilon_{\alpha\beta} \Psi_{C\alpha} \Psi_{D\beta} \rangle \\
 P_Q &= \langle \Psi_{A\alpha}^\dagger \Psi_{B\alpha} \rangle = - \langle \Psi_{C\alpha}^\dagger \Psi_{D\alpha} \rangle
 \end{aligned}$$

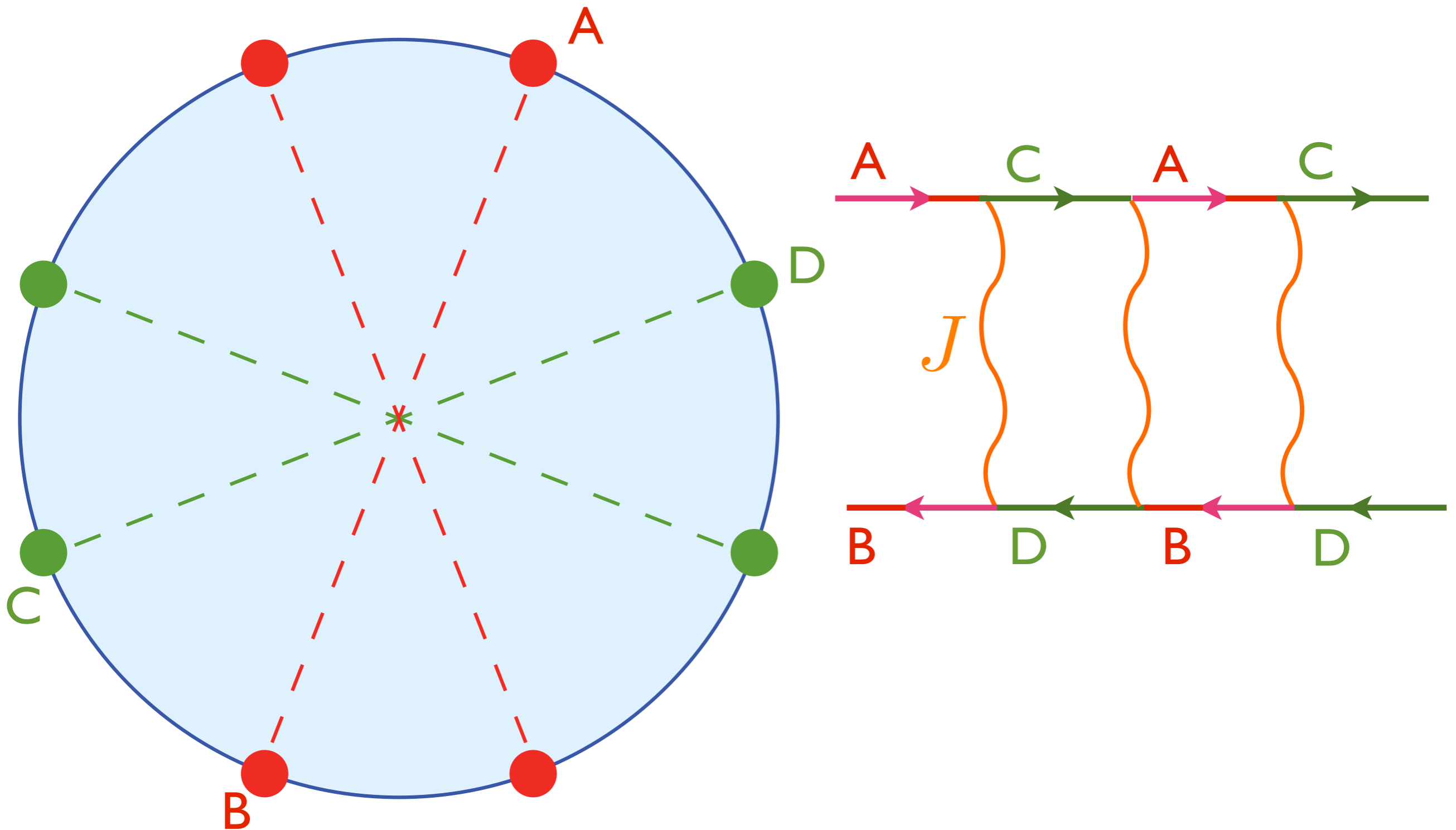
With pseudospin symmetry, energy depends only on  $|\Delta_S|^2 + |P_Q|^2$ .

# Pairing “glue” from antiferromagnetic fluctuations



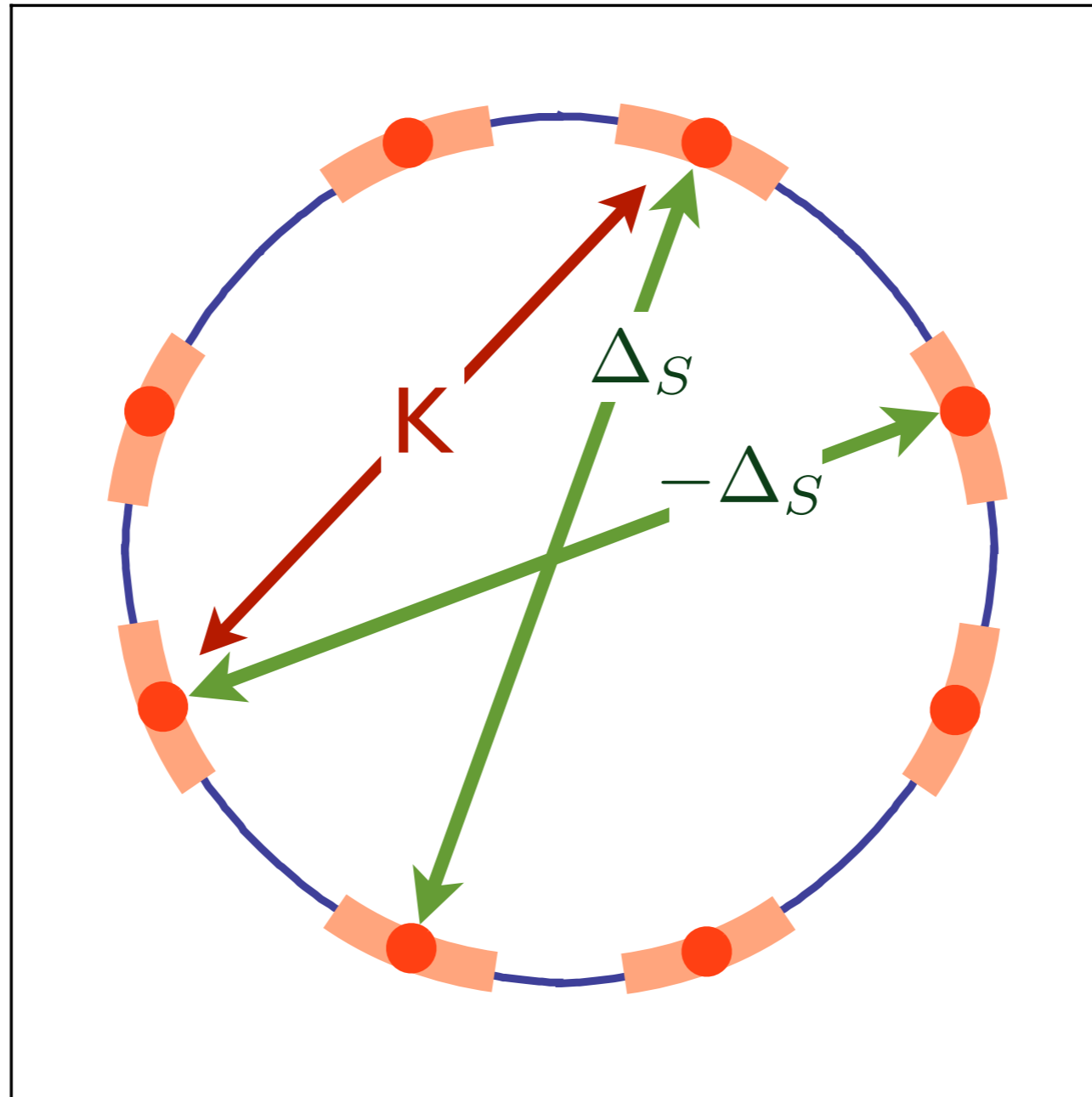
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E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012)

# Same “glue” leads to particle-hole pairing !



M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

$$\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y)$$

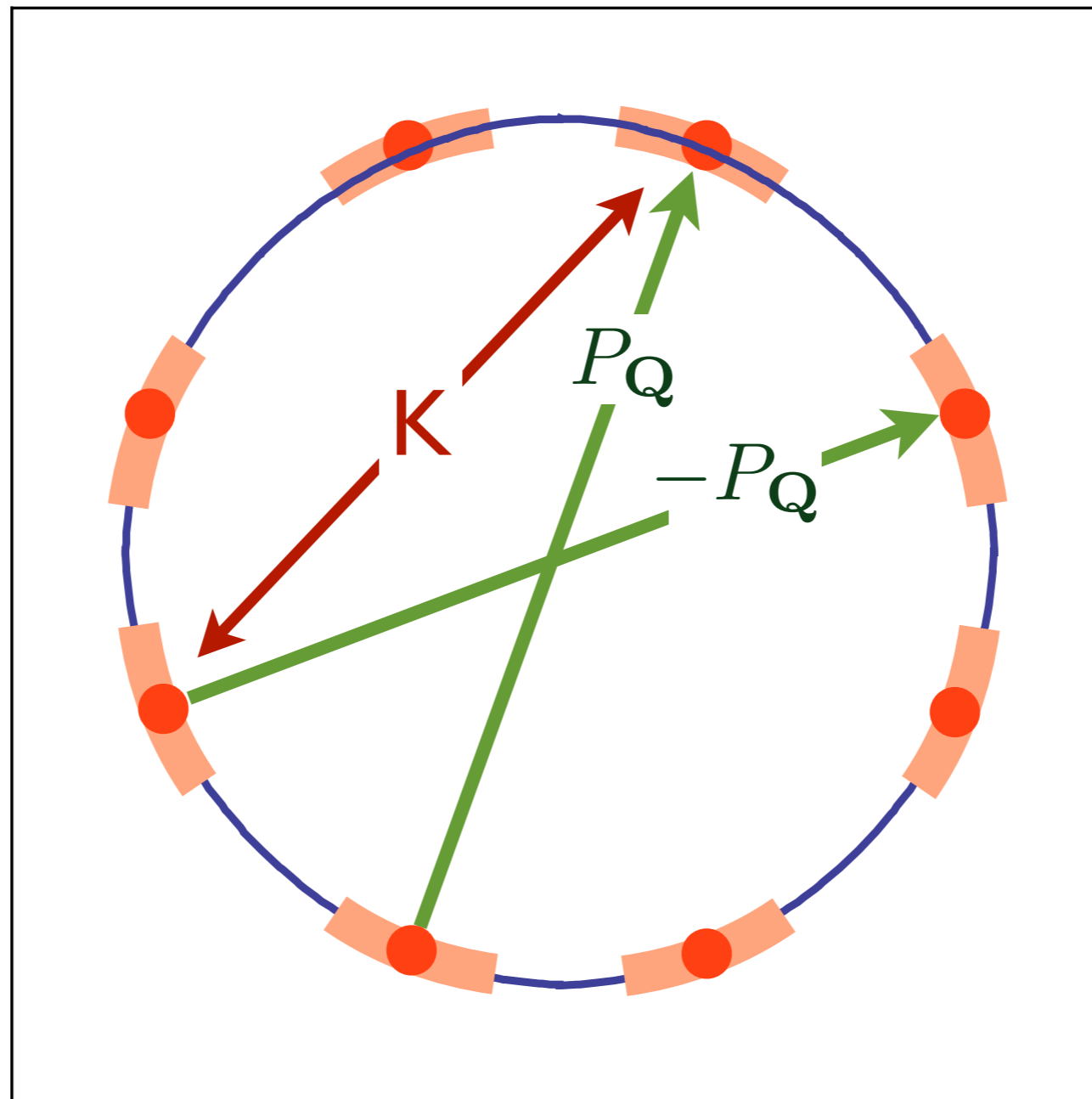


**d-wave superconductor: particle-particle pairing  
at and near hot spots, with  
sign-changing pairing amplitude**

$$\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \rangle = P_{\mathbf{Q}}(\cos k_x - \cos k_y)$$

After  
pseudospin  
rotation on  
*half* the  
hot-spots

M.A. Metlitski and  
S. Sachdev,  
*Phys. Rev. B* **85**,  
075127 (2010)

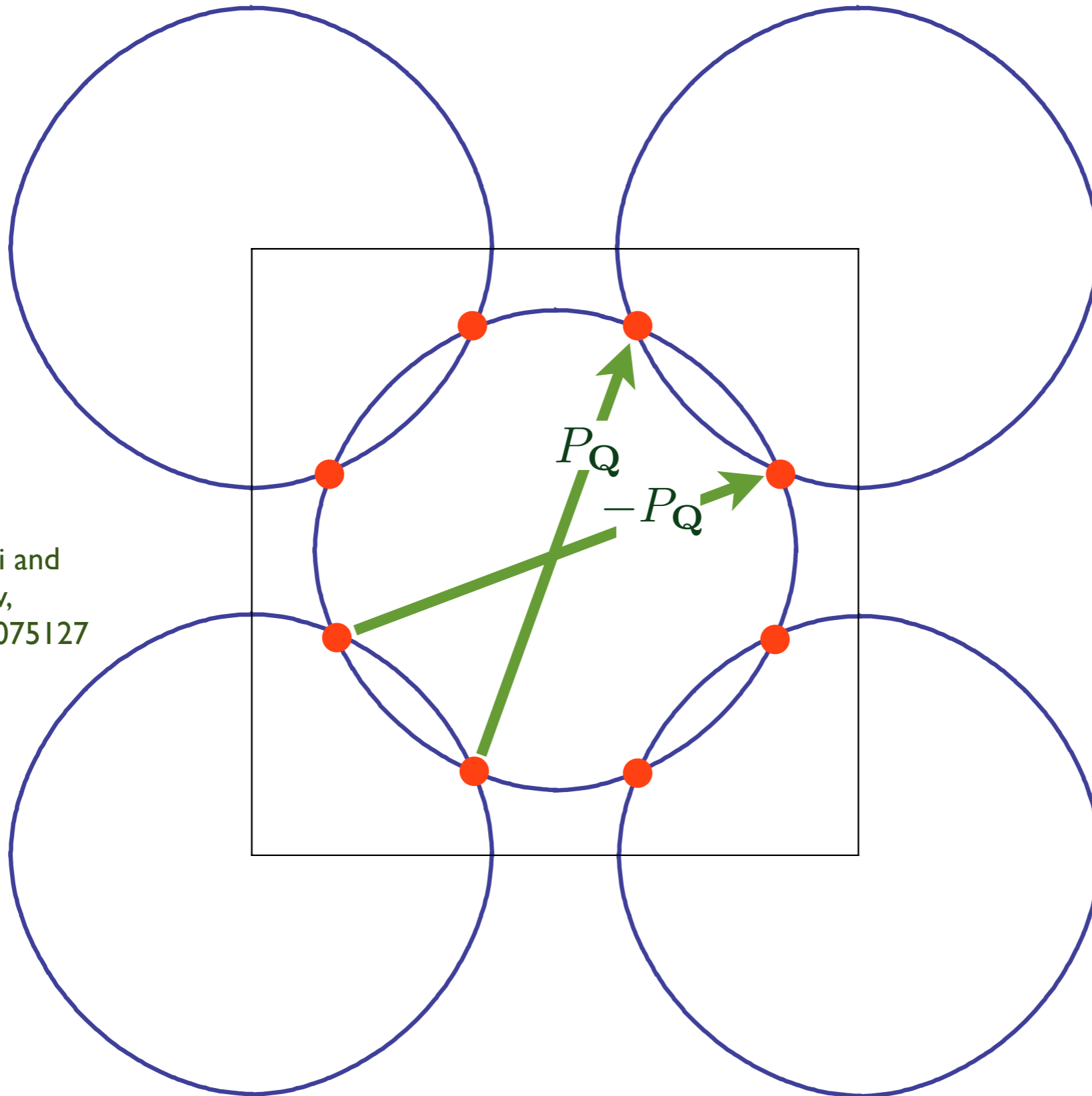


$\mathbf{Q}$  is ' $2k_F$ '  
wavevector

Incommensurate d-wave bond order:  
particle-hole pairing at and near hot spots, with  
sign-changing pairing amplitude

# Incommensurate $d$ -wave bond order

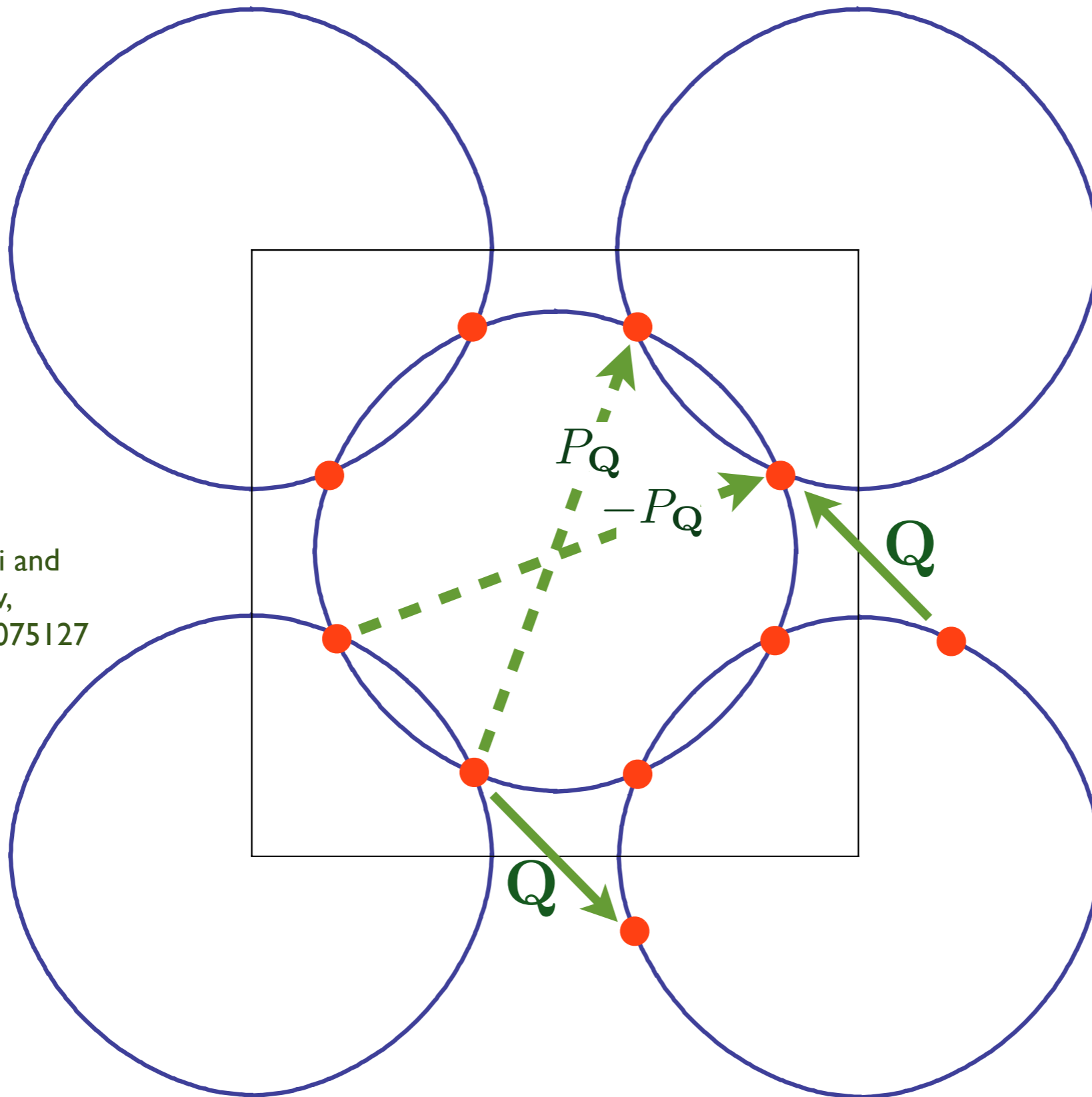
M.A. Metlitski and  
S. Sachdev,  
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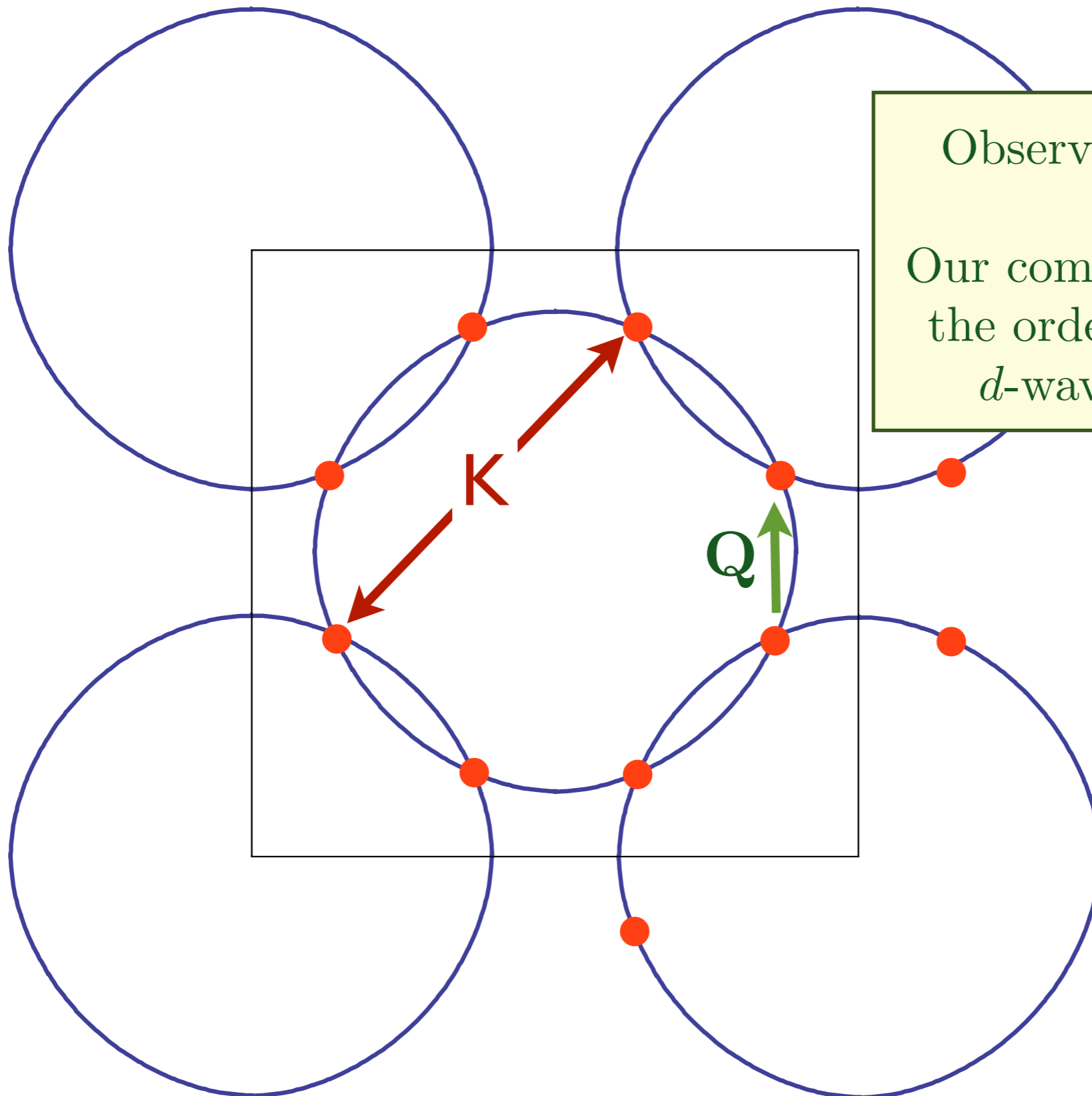
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# Incommensurate $d$ -wave bond order



Observed low  $T$  ordering.

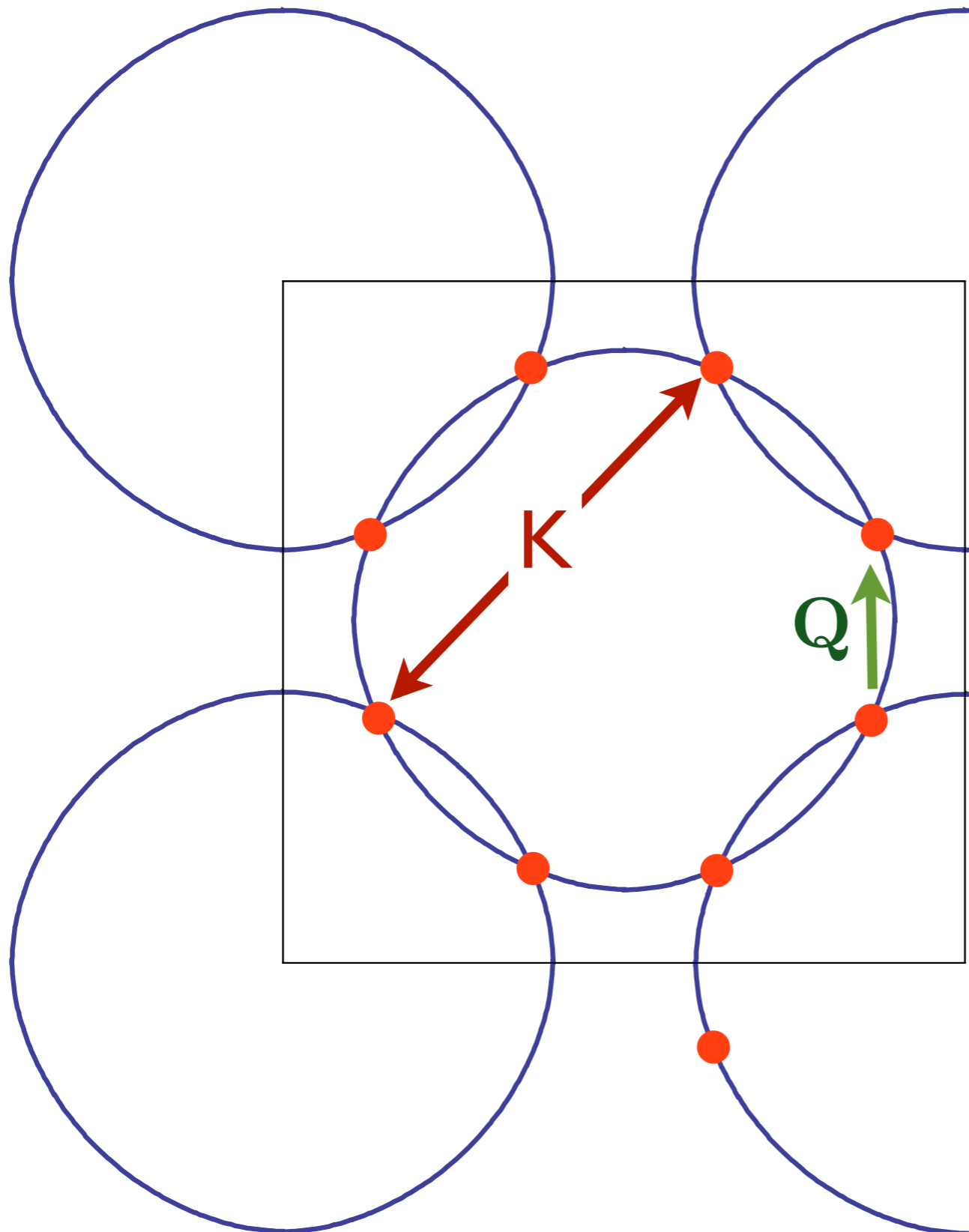
Our computations show that the order is predominantly  $d$ -wave also at this  $Q$ .

S. Sachdev and R. La Placa, Physical Review Letters **111**, 027202 (2013)

M. Vojta and S. Sachdev, Physical Review Letters **83**, 3916 (1999)

M. Vojta and O. Rosch, Physical Review B **77**, 094504 (2008)

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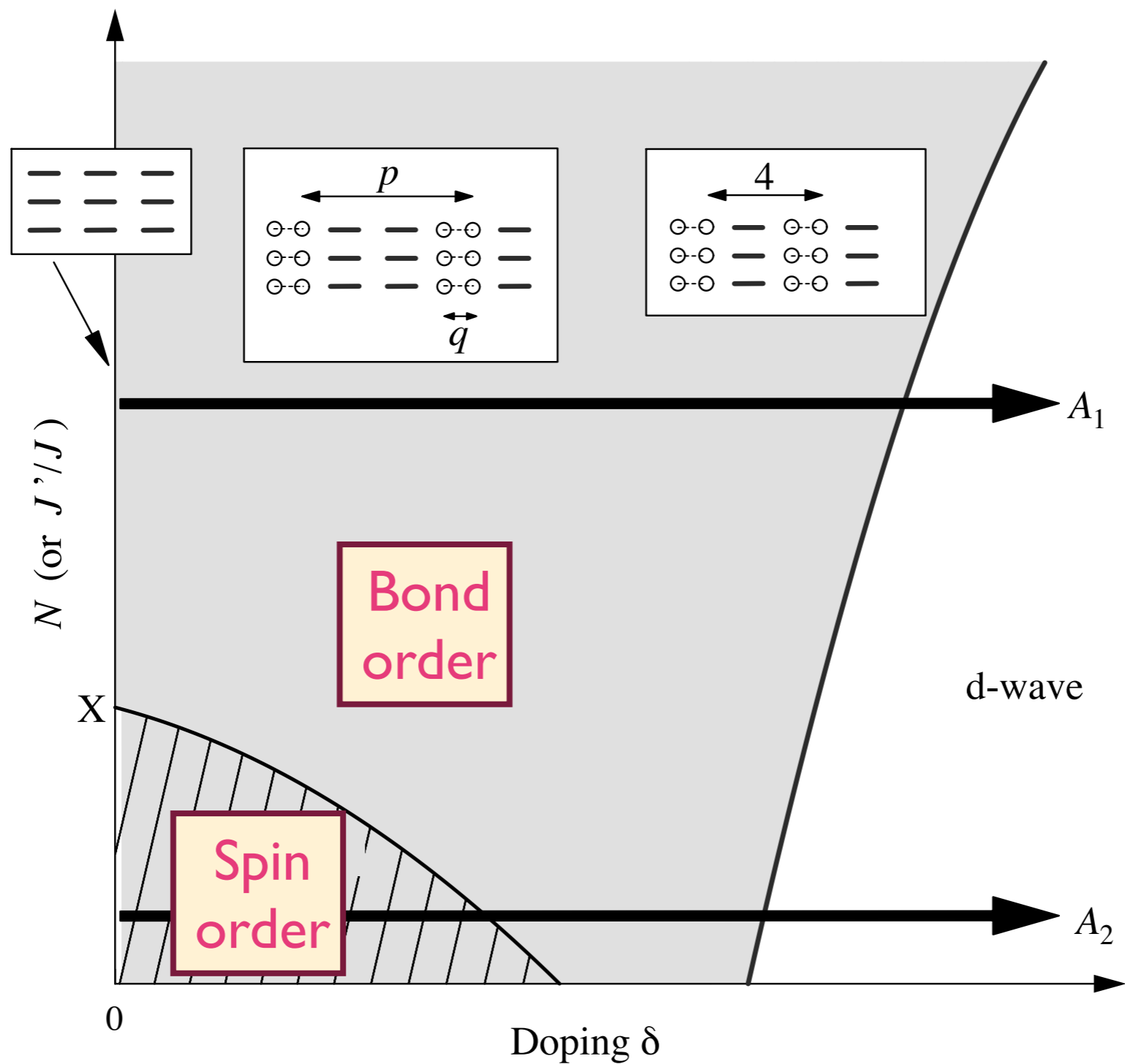
This  $\mathbf{Q}$  is preferred in computations of bond order which include large on-site  $U$ . The bond-ordering is driven primarily by spin correlations (as in a valence bond solid), while charge density correlations are suppressed by factors of  $\sim t/U$ .

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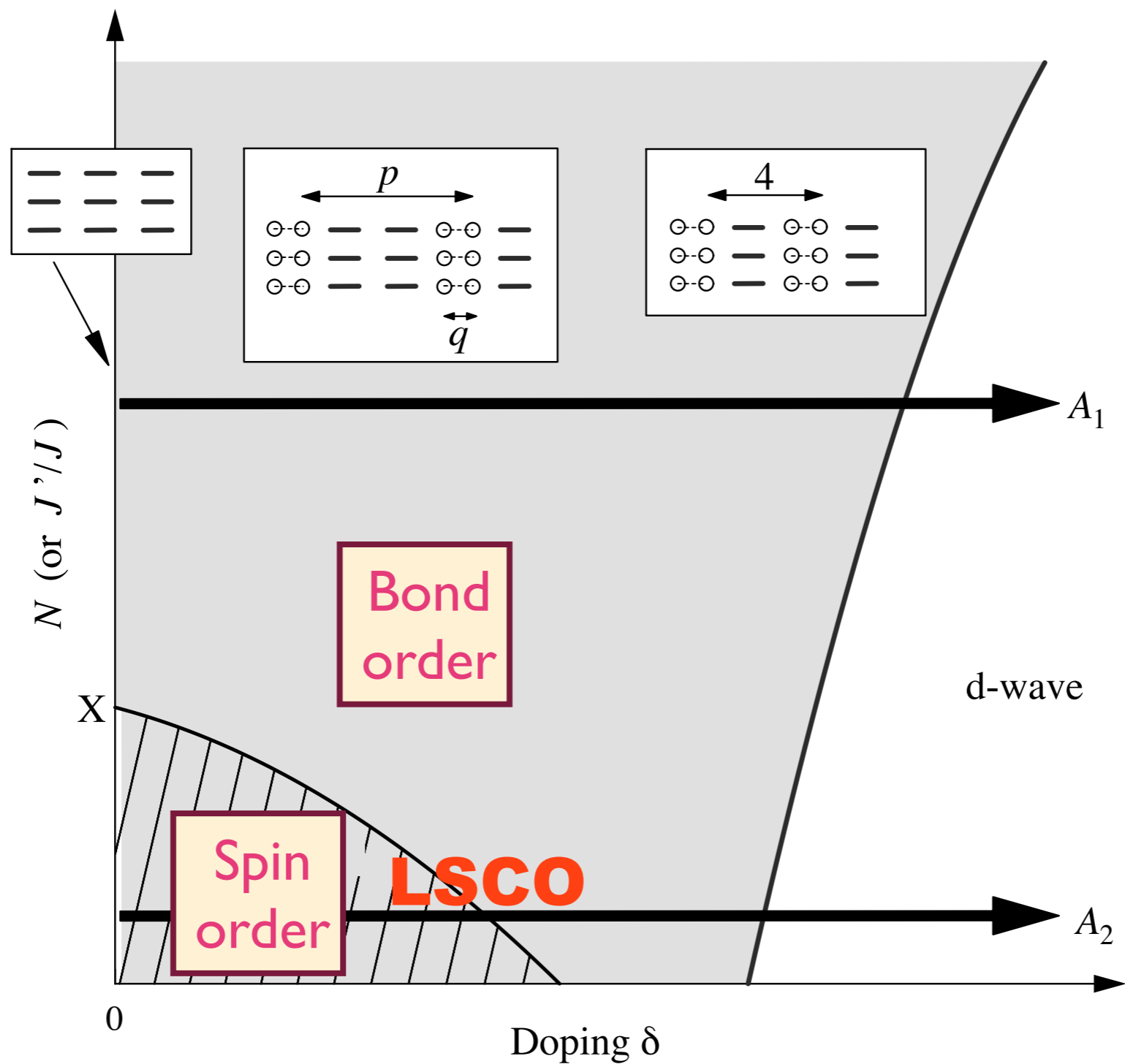
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# Phase diagram of doped antiferromagnets



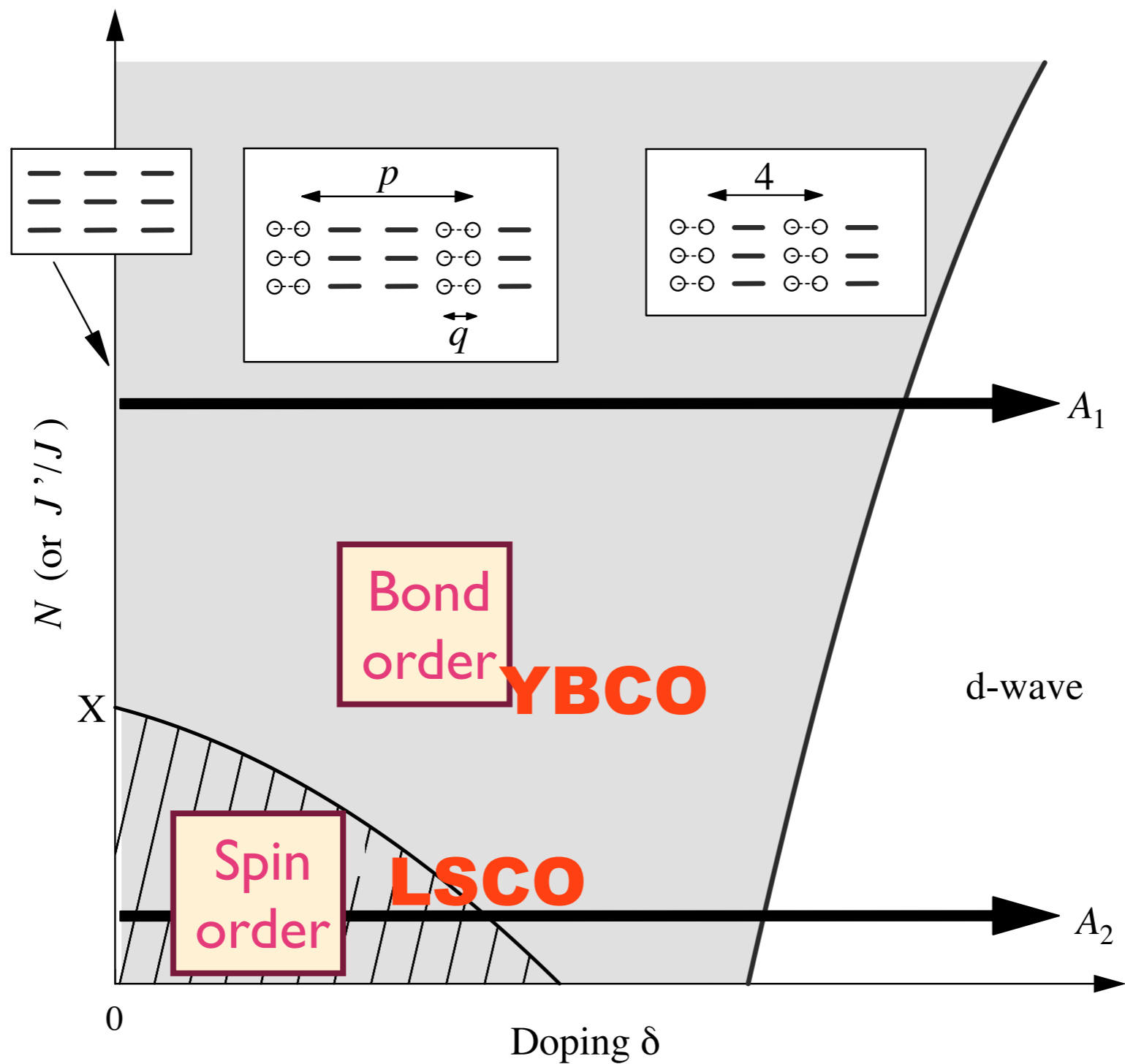
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- The pseudospin partner of  $d$ -wave superconductivity is an incommensurate  $d$ -wave bond order
- These orders form a pseudospin doublet, whose fluctuations lead to the “pseudogap” phase, described by the angular fluctuations of an order parameter with 6 real components.