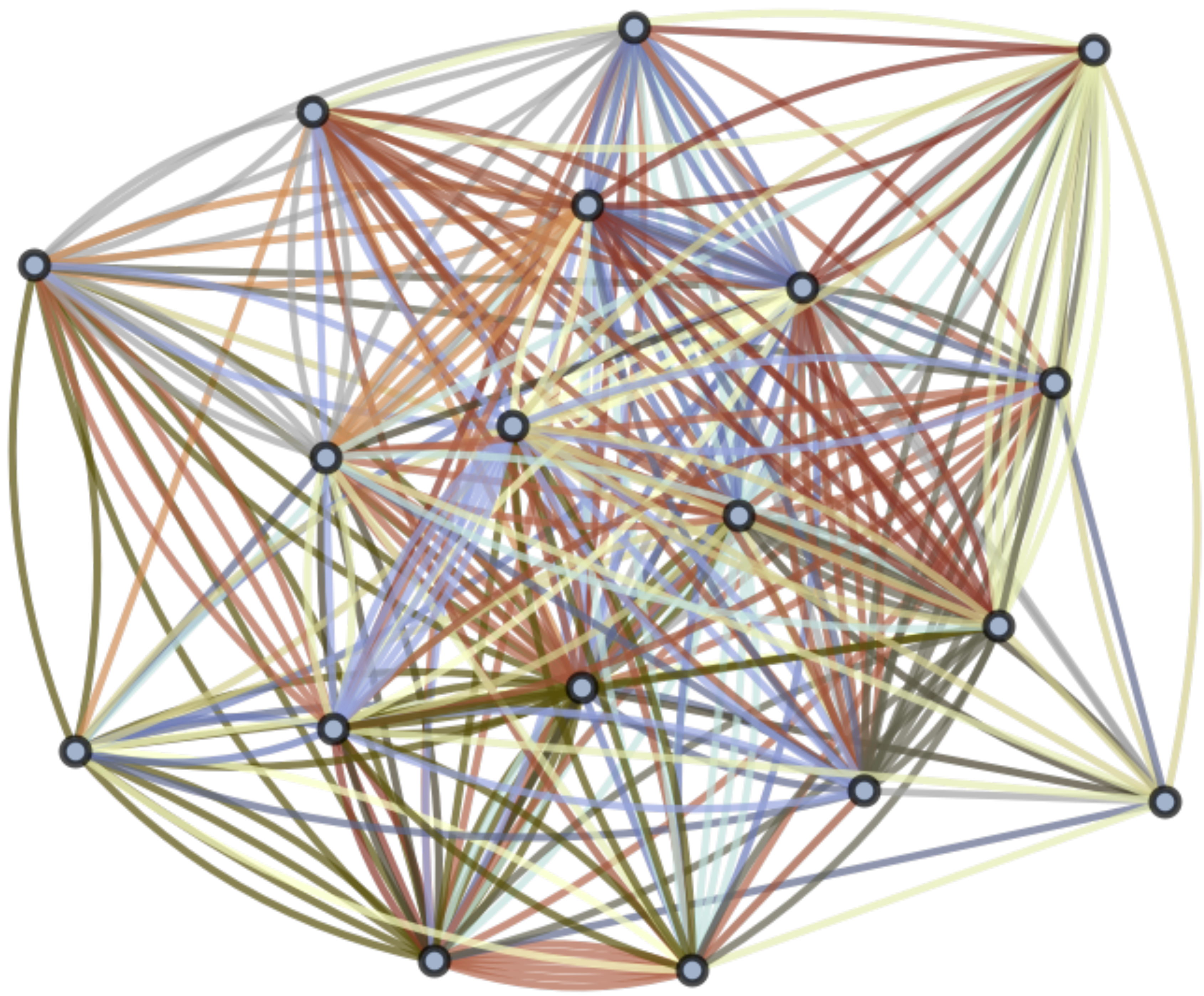
100 years of many-particle quantum mechanics: from Bose and Fermi to quantum materials and black holes



Christ University
Bengaluru

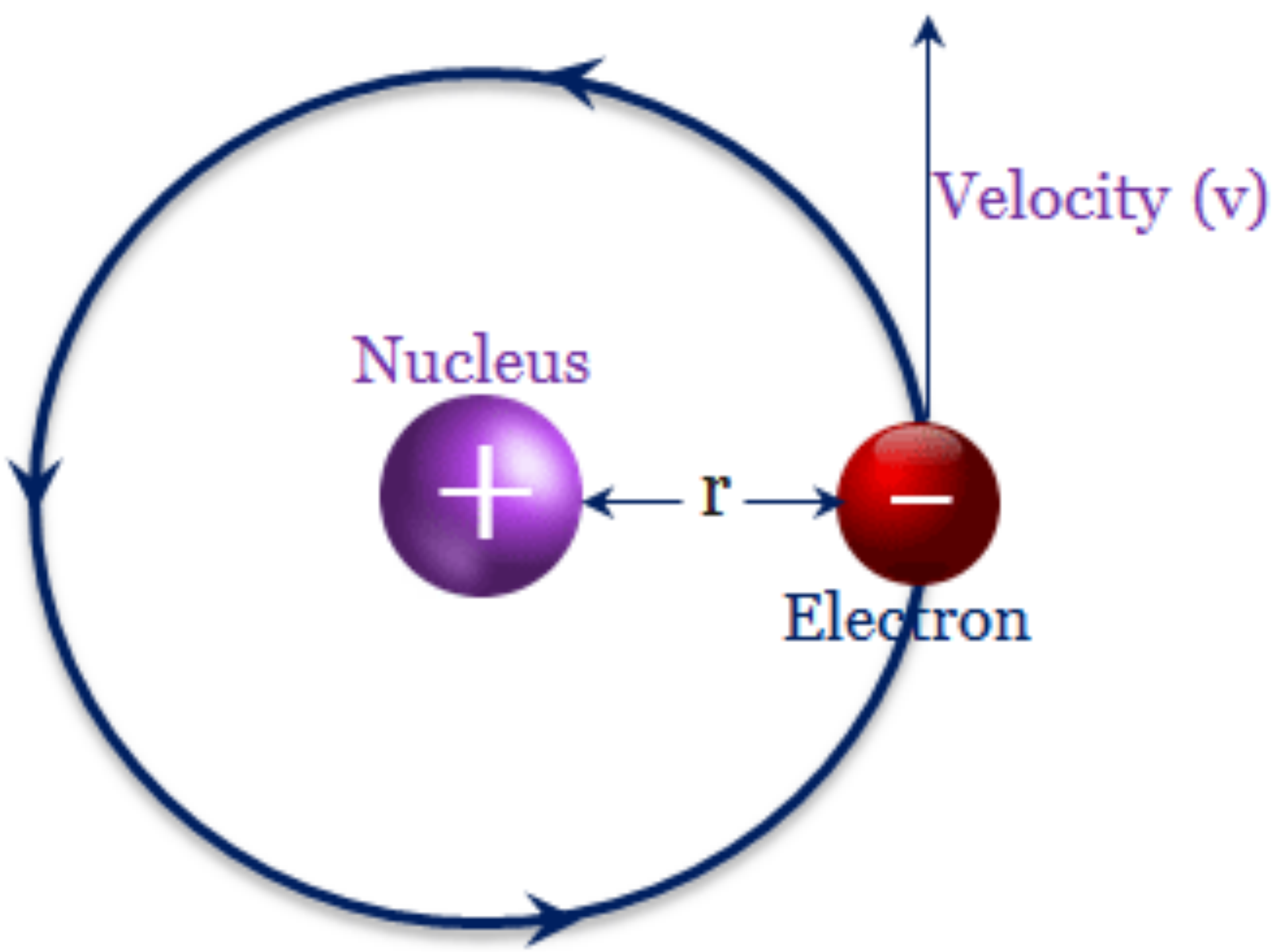
December 19, 2025

Subir Sachdev

PHYSICS
HARVARD

Quantum mechanics
to
quantum materials:
the first 100 years

Hydrogen atom

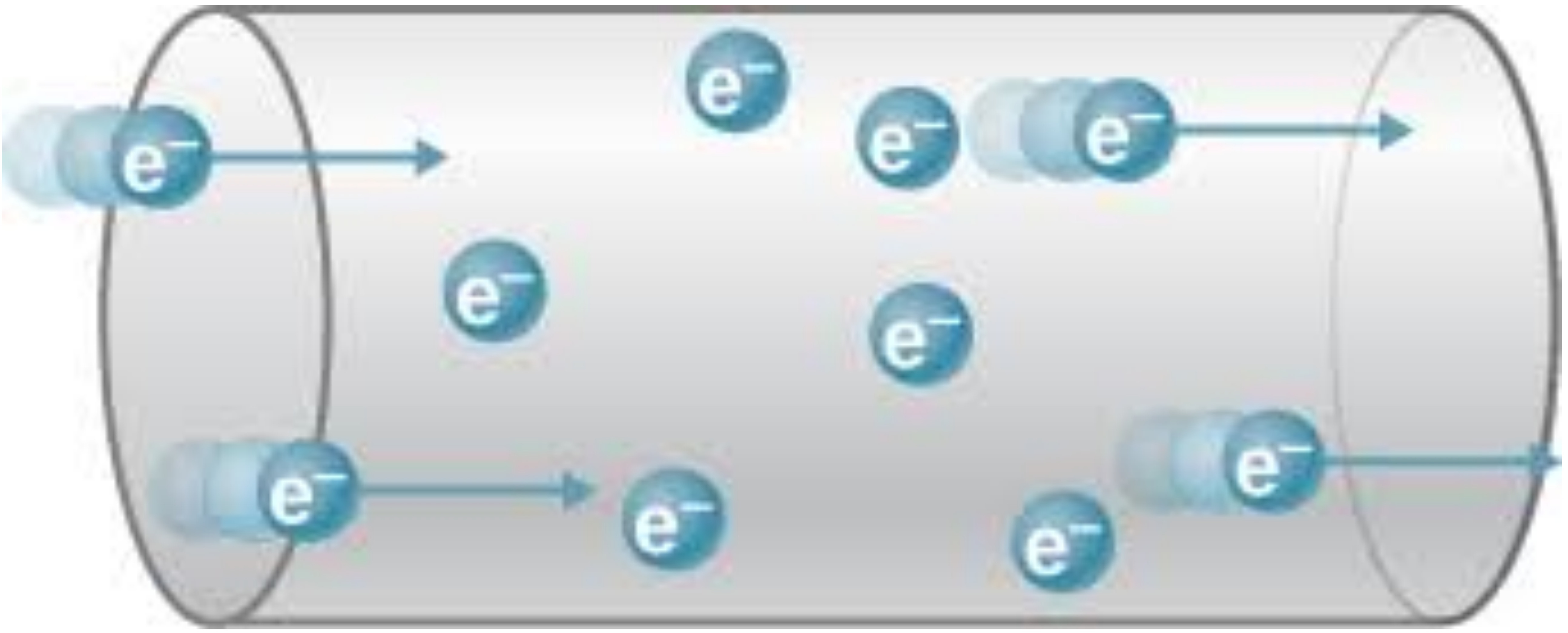


$\Rightarrow 10^{-10}$ meters $\Leftarrow$

The motion of the electron around the proton is *not* described by the same theory as the motion of the planets around the sun.

It is described by the quantum theory of Schrödinger and Heisenberg (1925).

- Sommerfeld (1927): The same equations also describe the motion of $\sim 10^{23}$ electrons in a metal. Each electron is a **fermion**, named after Fermi (1926), which obey exclusion—at most one fermion can occupy each quantum orbital.



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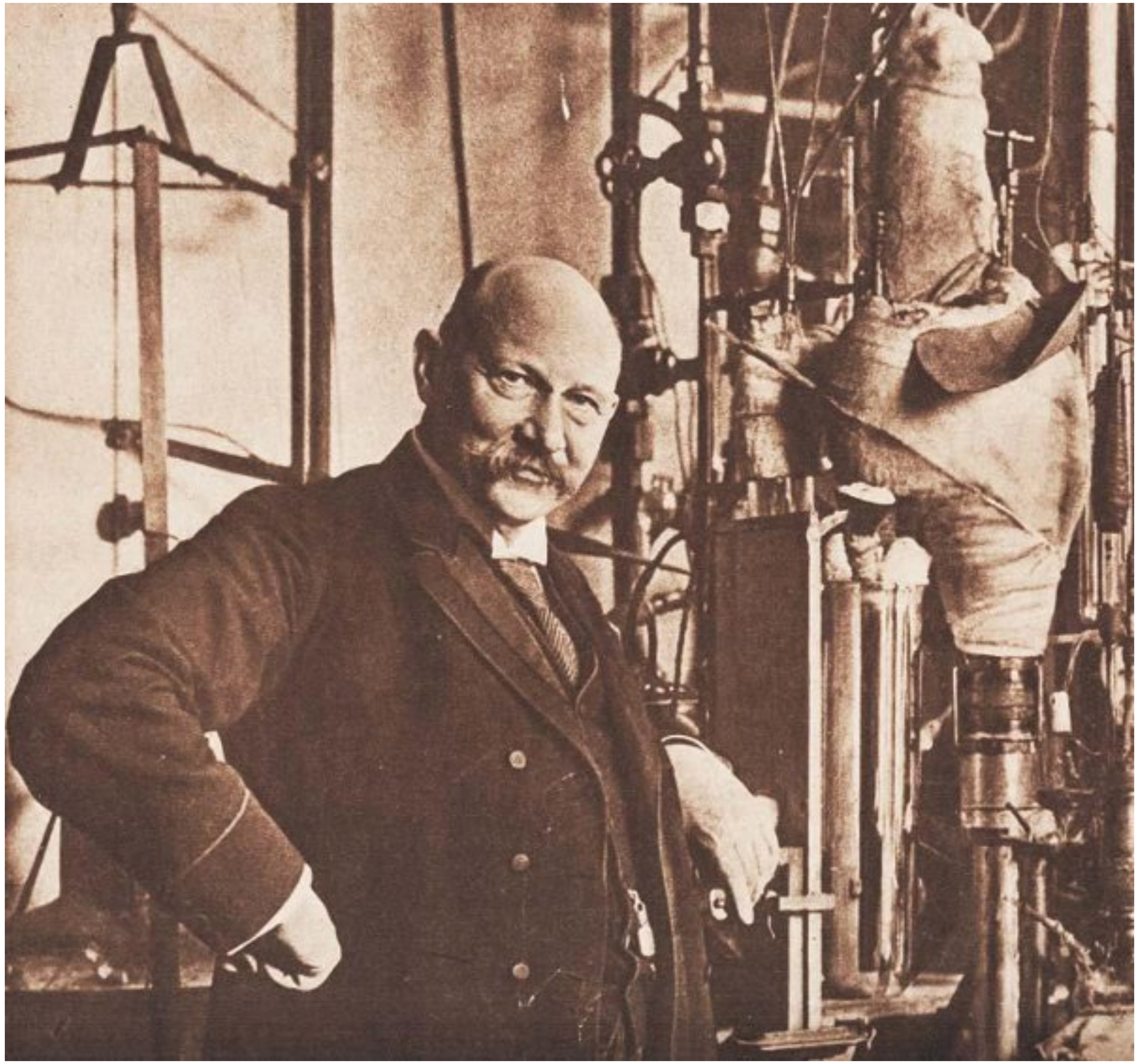
- Bardeen, Cooper, Schrieffer (1957): Pairs of electrons behave like **bosons**, and this is the explanation for superconductivity.

- Today: Many particles exhibit many *emergent phenomena*, related to **quantum entanglement**. These are crucial to understanding modern quantum materials, such as the high temperature superconductors.

- Today: Many particles exhibit many *emergent phenomena*, related to **quantum entanglement**. These are crucial to understanding modern quantum materials, such as the high temperature superconductors.

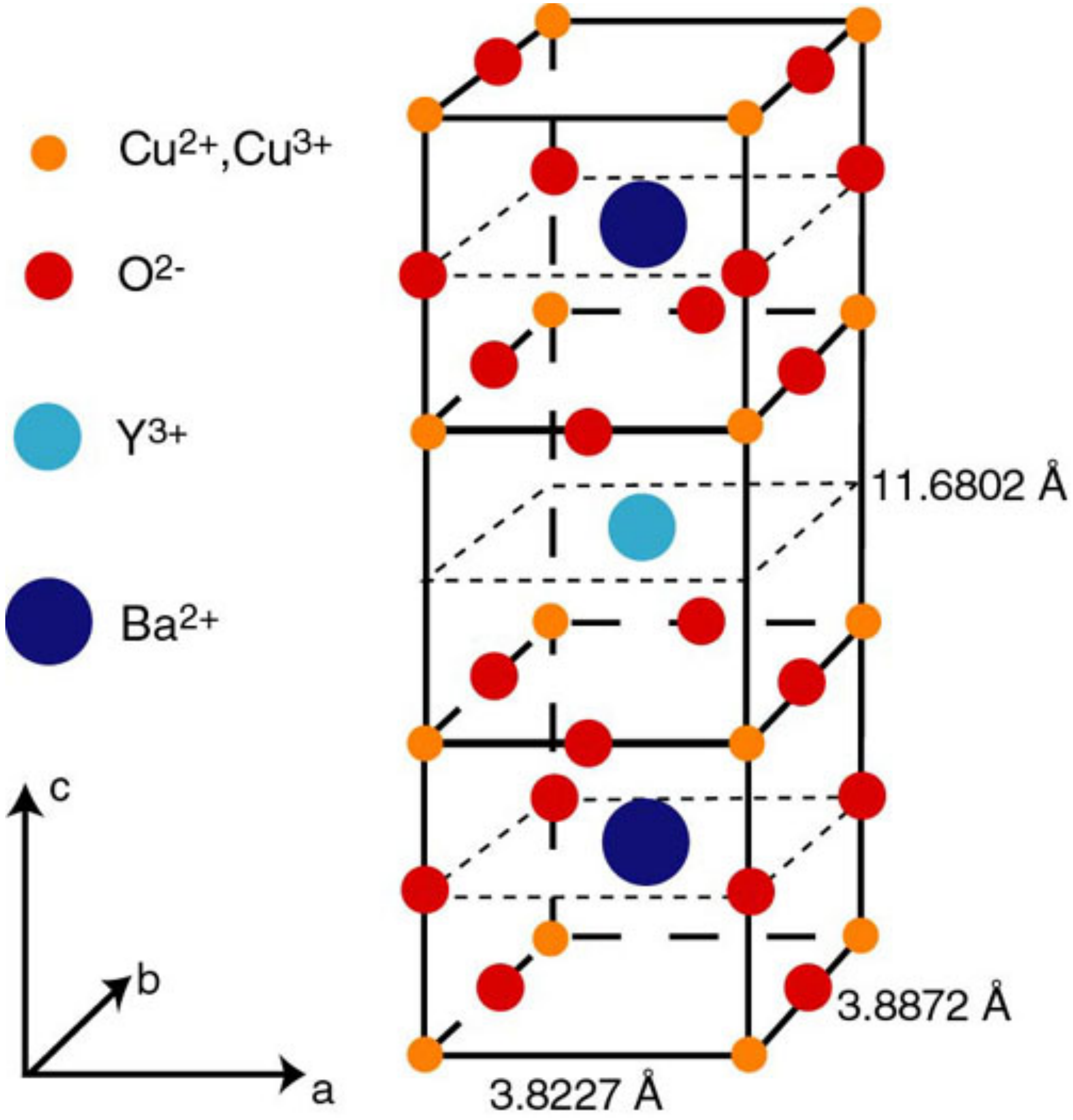
- Ideas on multi-particle entanglement in quantum materials have strongly influenced quantum computing, especially quantum error correction, and the theory of black holes (and vice versa).

Superconductors

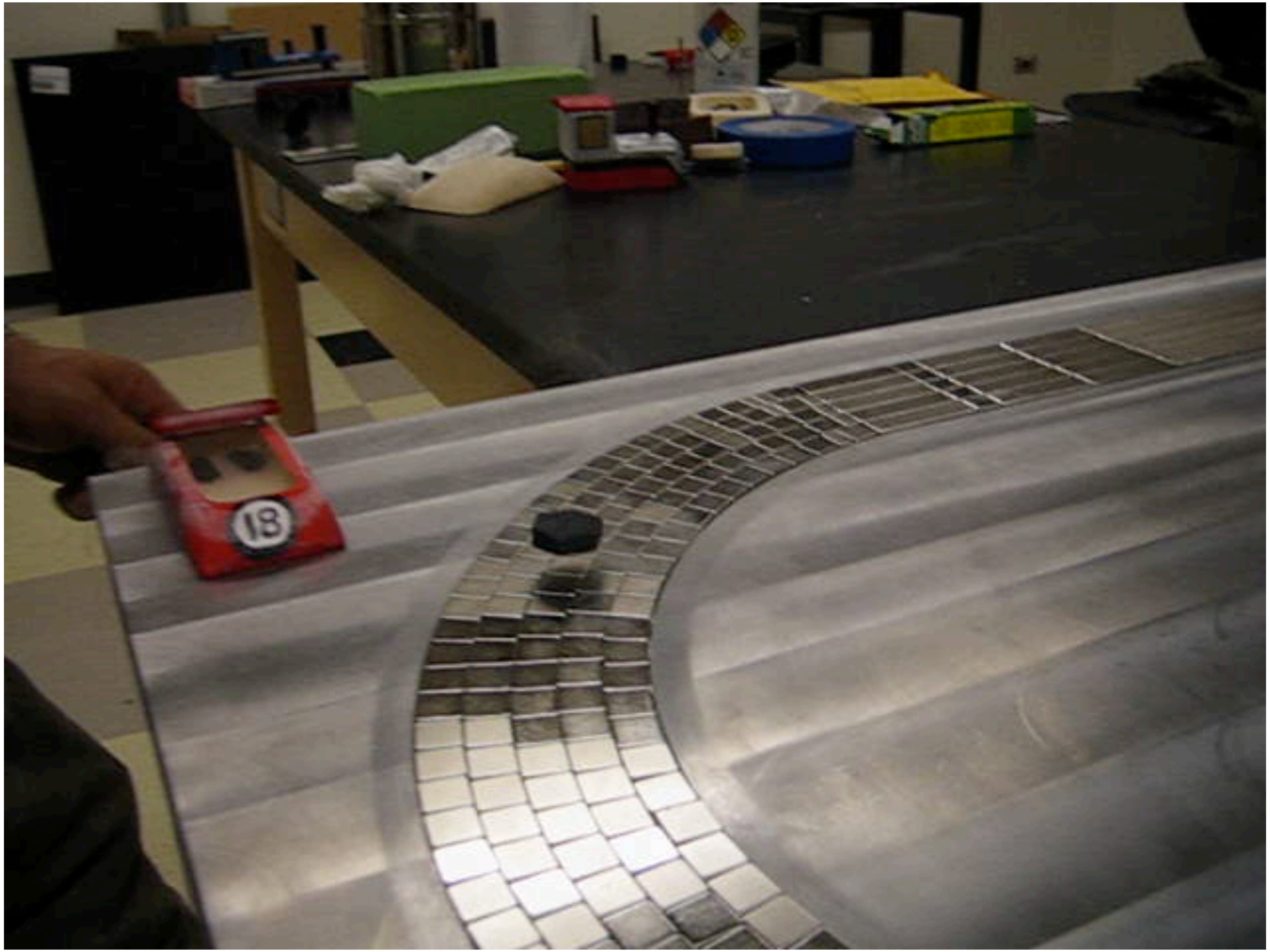


Kamerlingh Onnes 1911:
Mercury is a superconductor below -269 °C

Cuprate high temperature superconductors



$$YBa_2Cu_3O_{6+x}$$



Nd-Fe-B magnets, YBaCuO superconductor

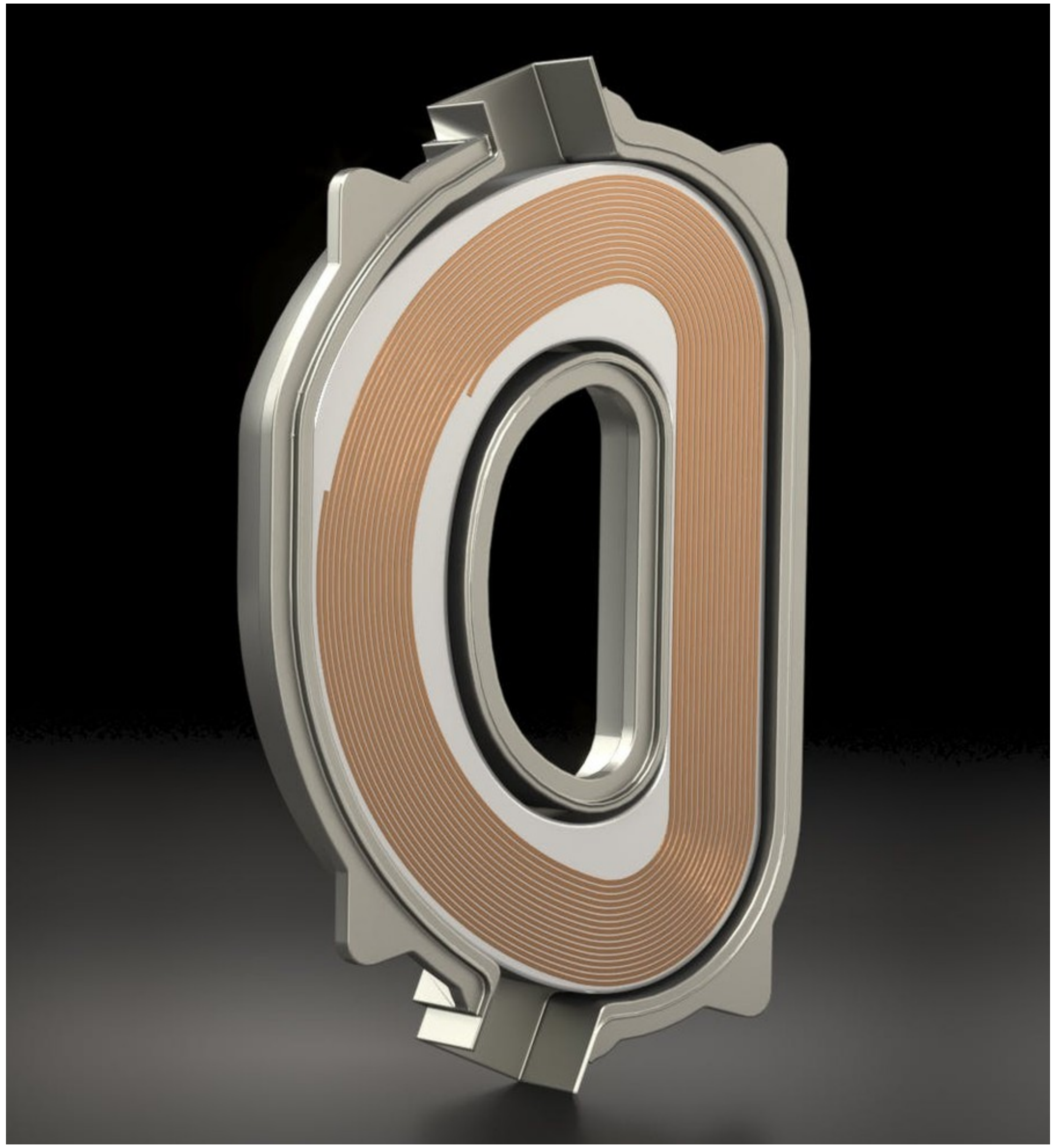
Julian Hetel and Nandini Trivedi, Ohio State University

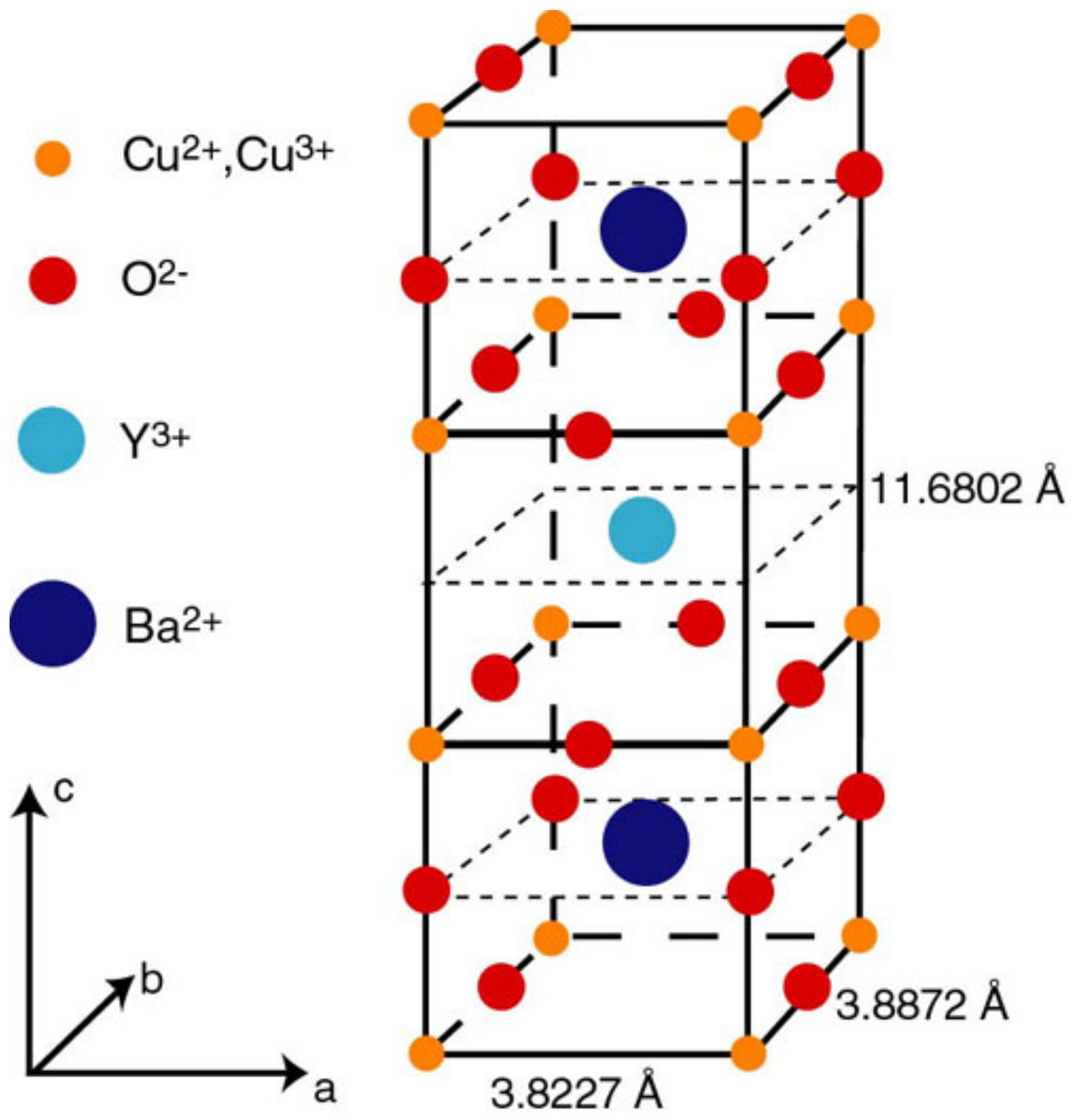
HTS Magnets: Enabling Technology

The surest path to limitless, clean, fusion energy

YBCO magnets allow for smaller, faster, and less expensive tokamaks for plasma fusion

Commonwealth Fusion Systems





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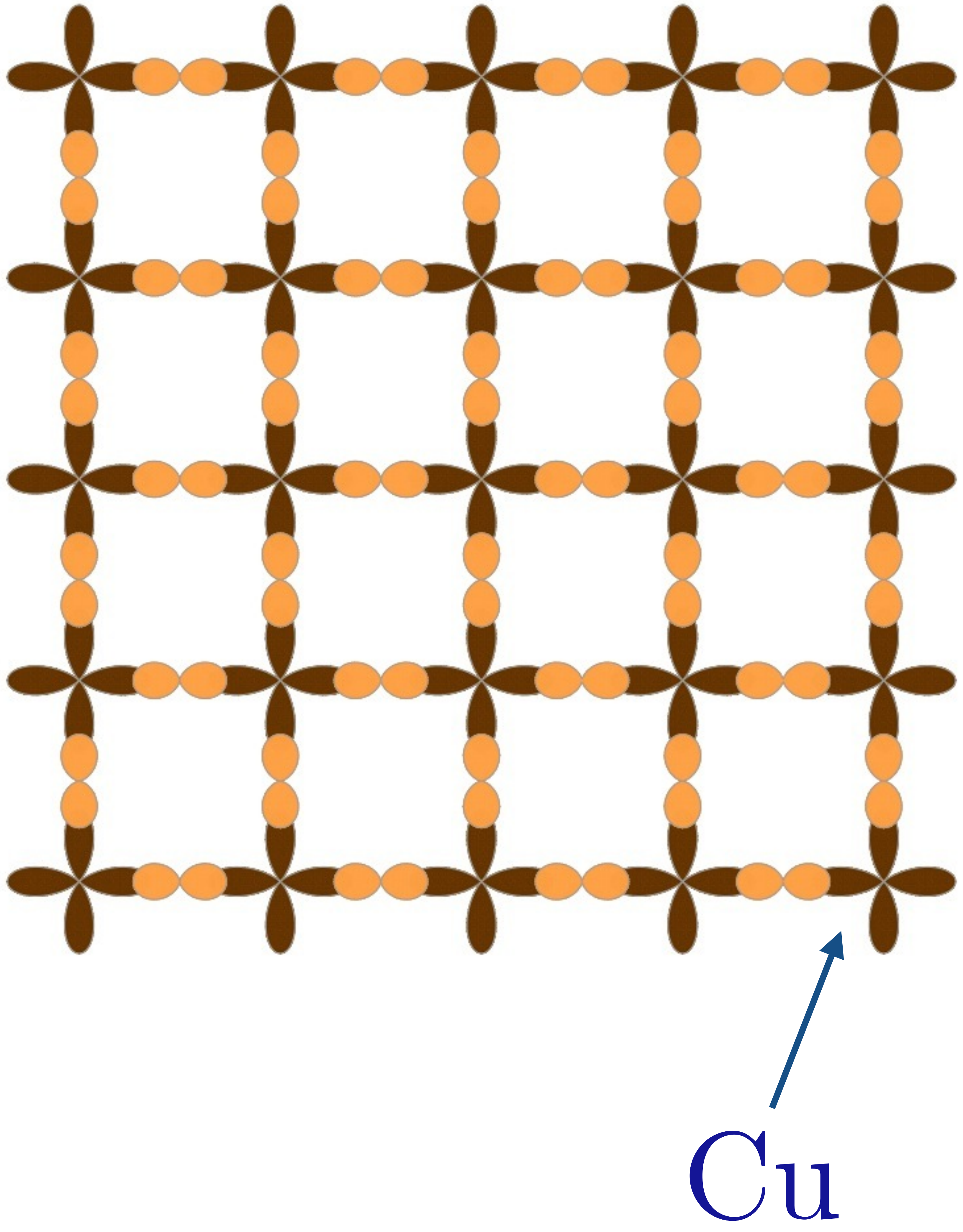
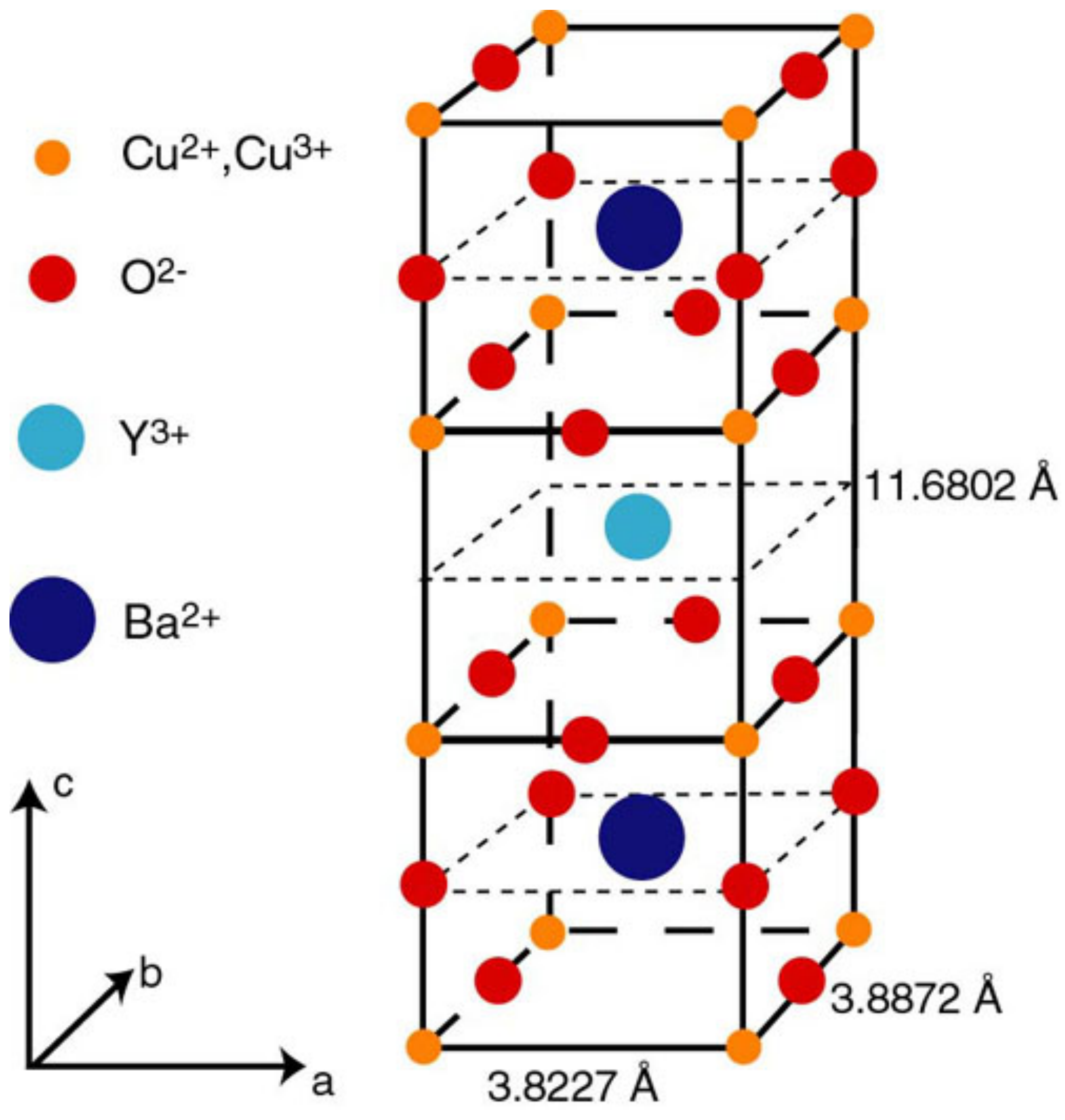
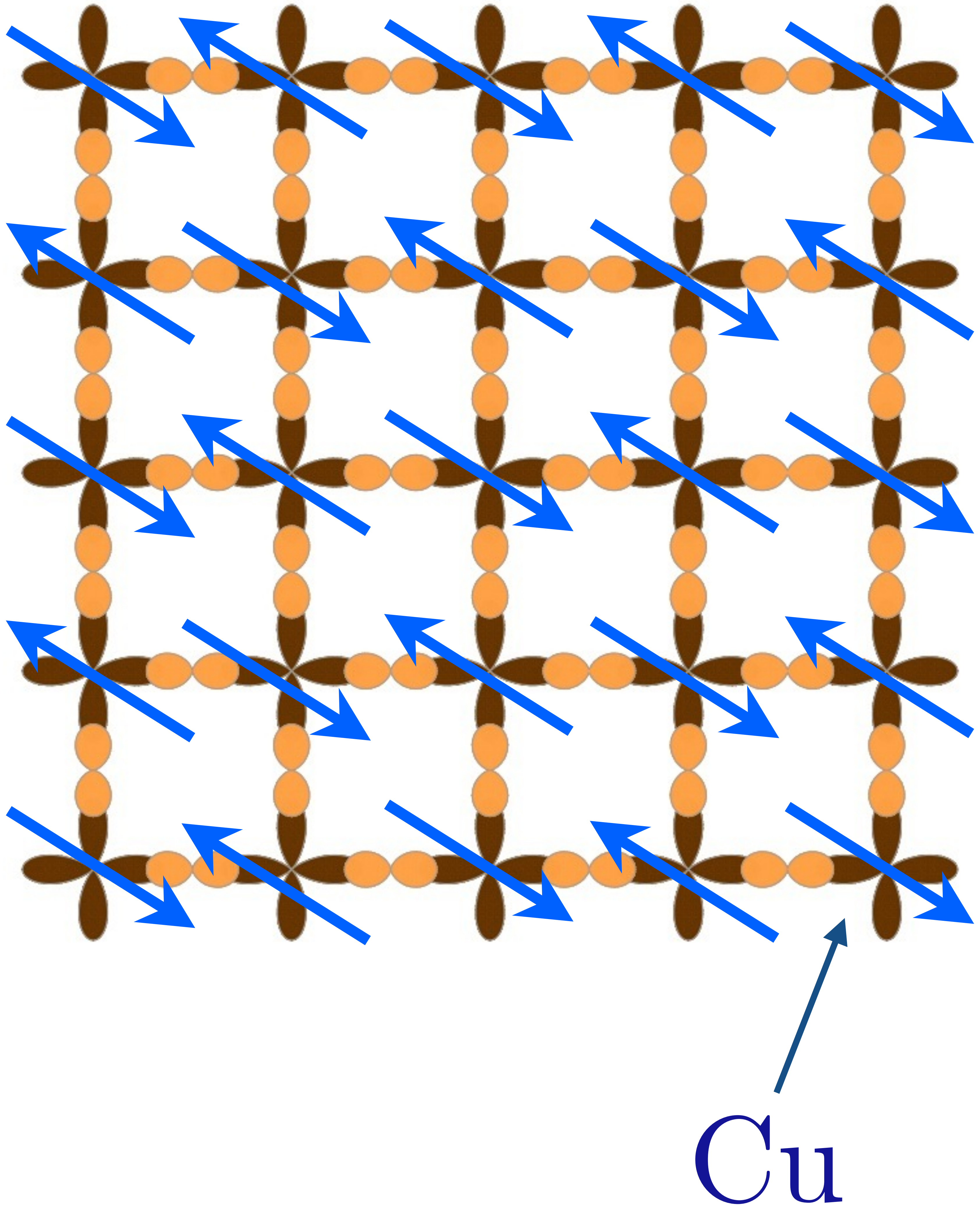




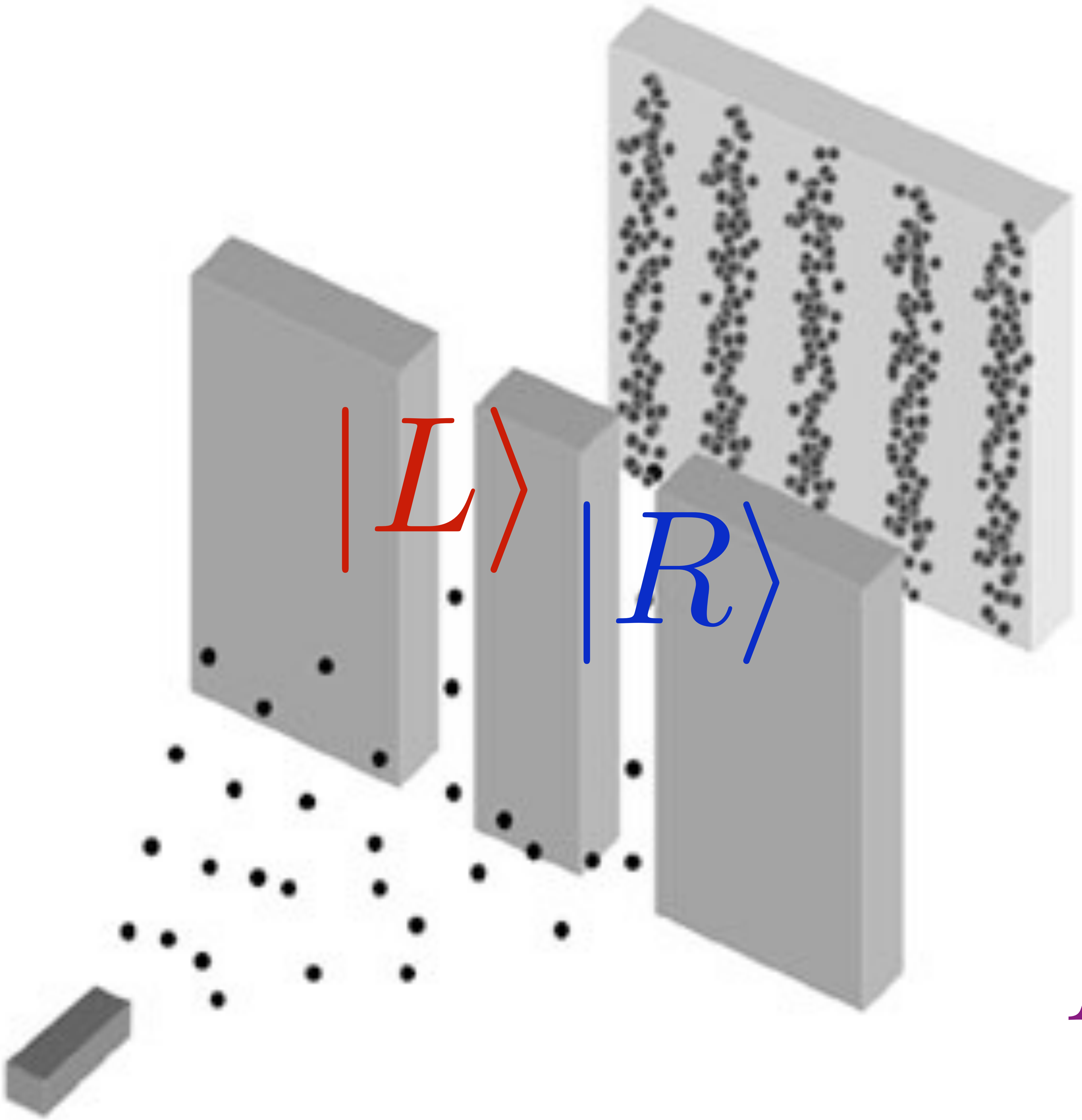
$YBa_2Cu_3O_{6+x}$



P.W. Anderson and G. Baskaran (1988): The key to high temperature superconductivity is the formation of a "resonating valence bond state" (a type of **quantum spin liquid**) which entangles the electrons on Cu

Quantum entanglement

The double slit experiment



Let $|L\rangle$ represent the state with the electron in the left slit

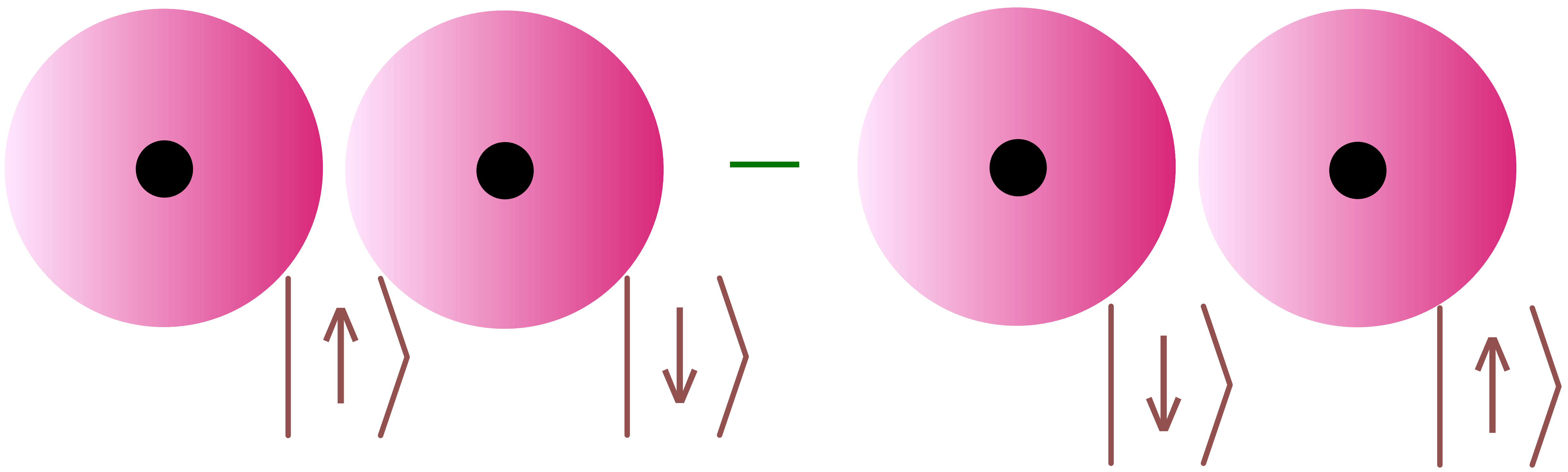
And $|R\rangle$ represents the state with the electron in the right slit

Actual state of *each* electron is

$$|L\rangle + |R\rangle$$

Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



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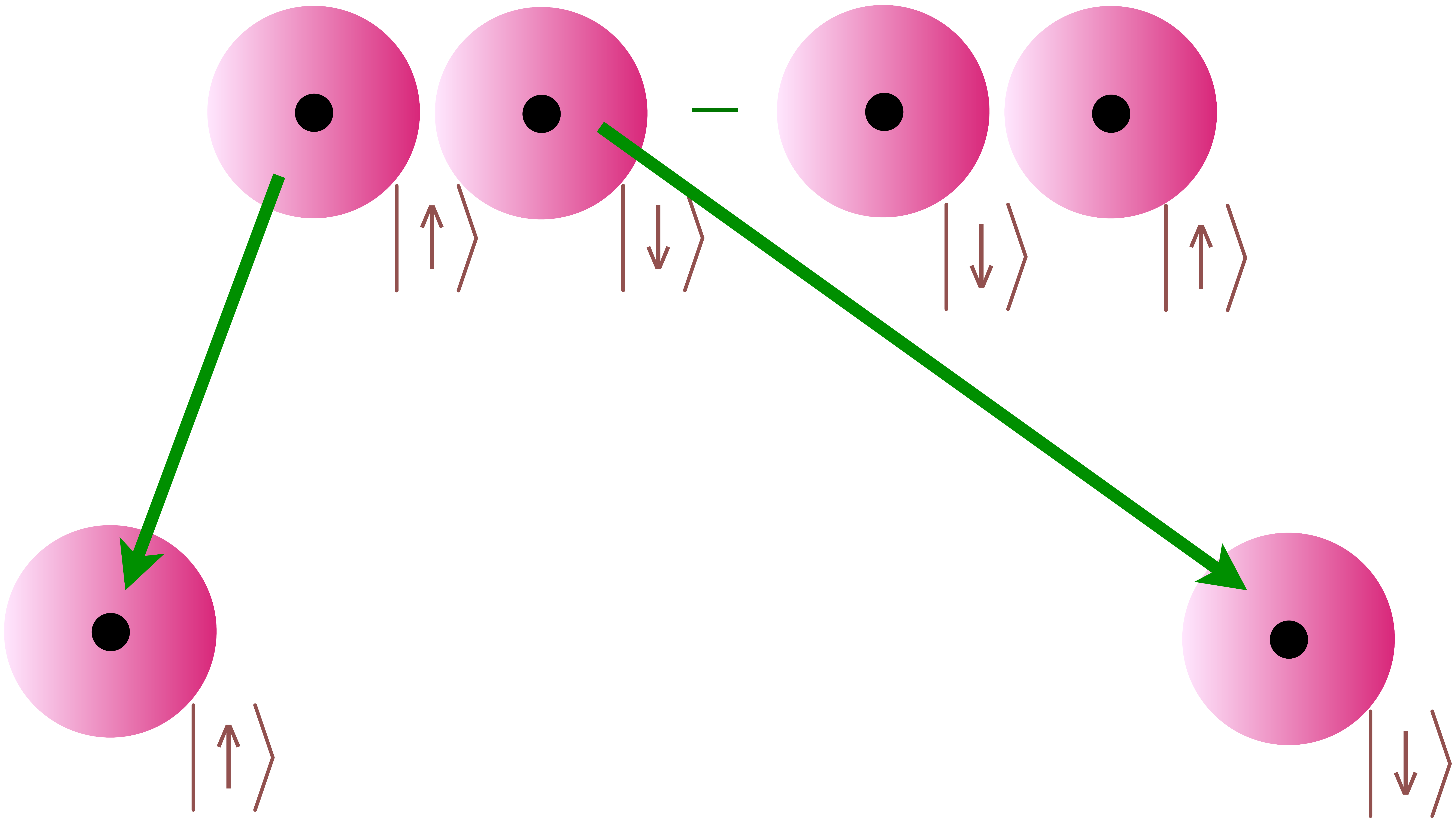

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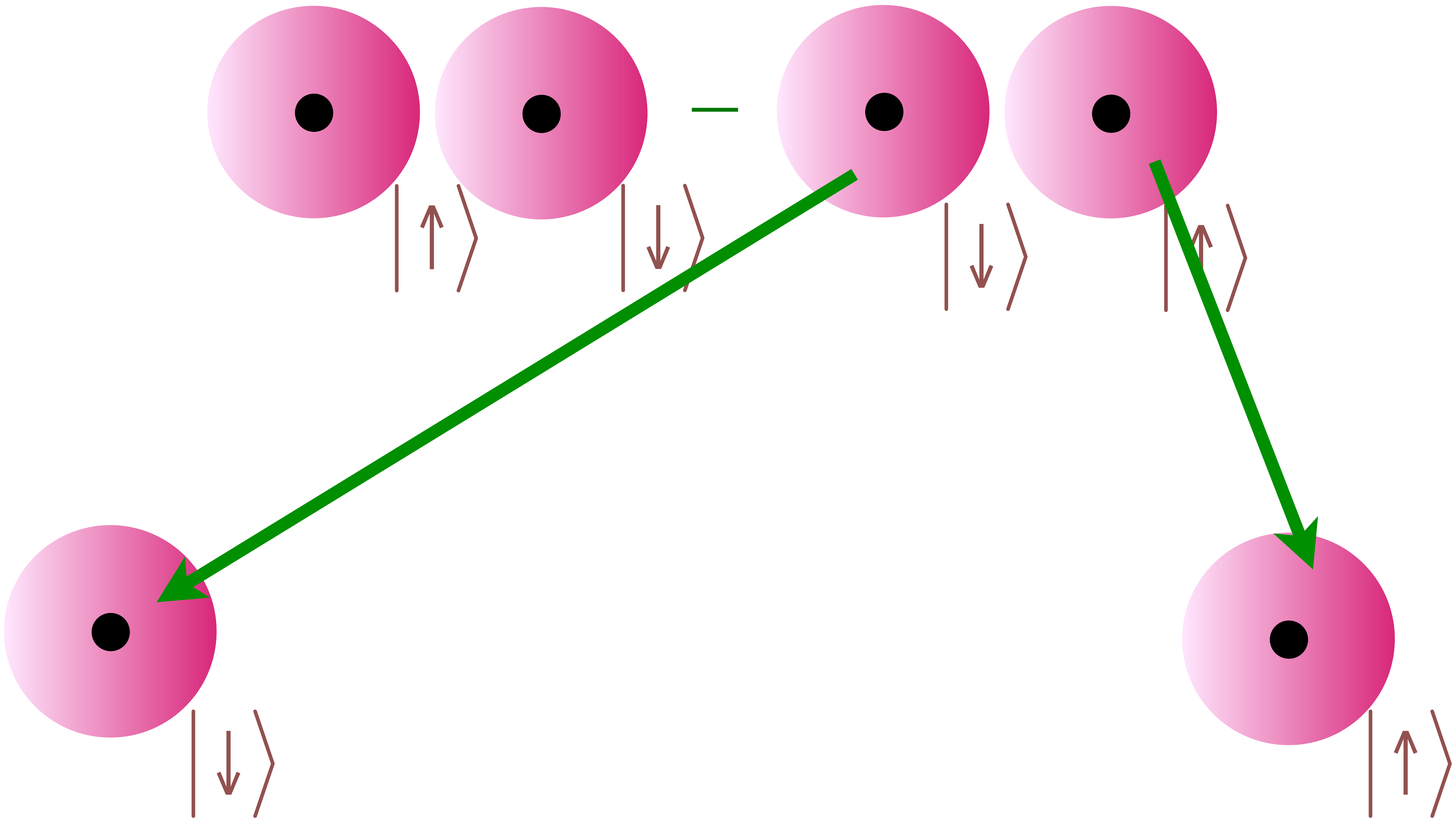

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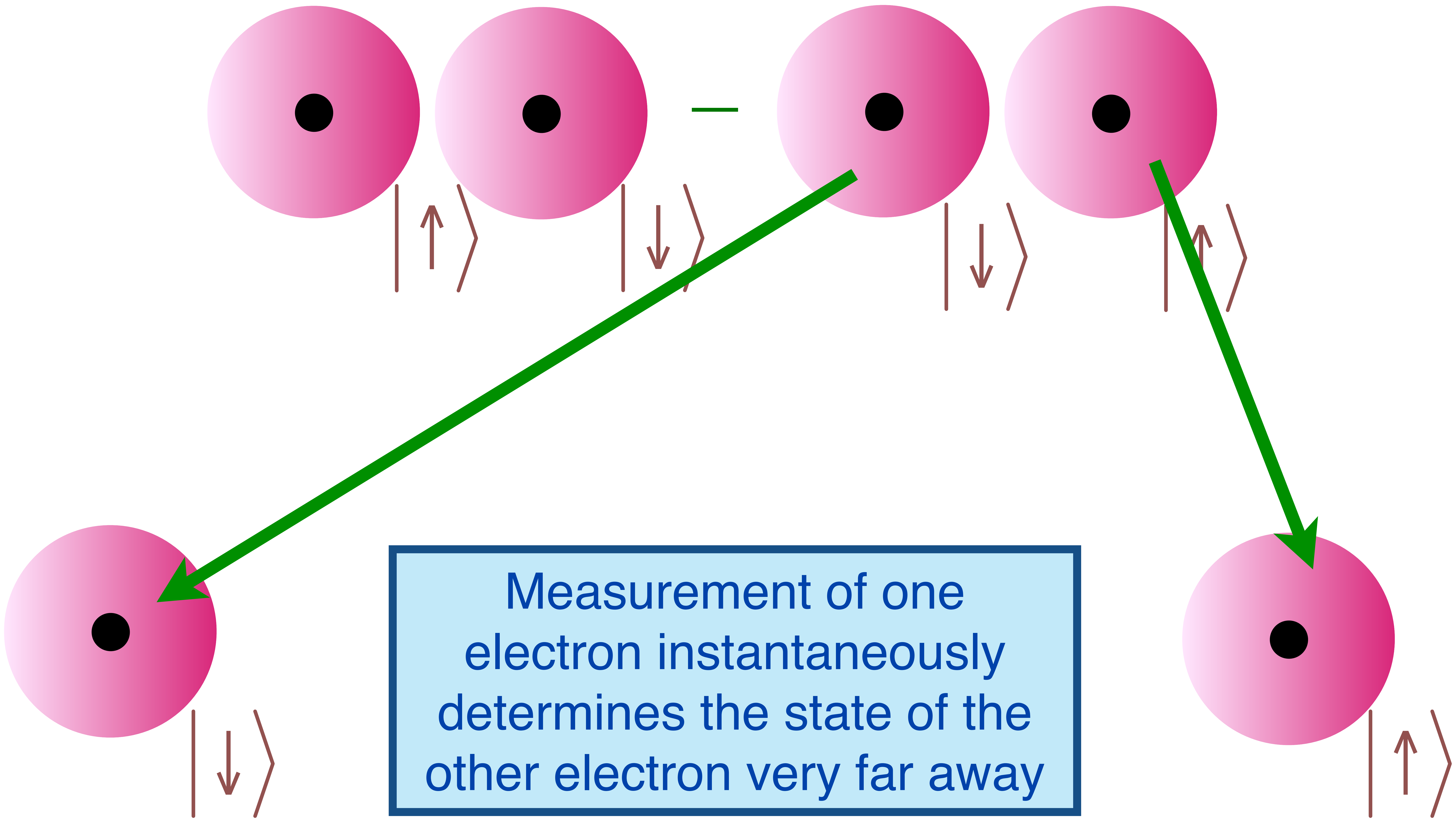

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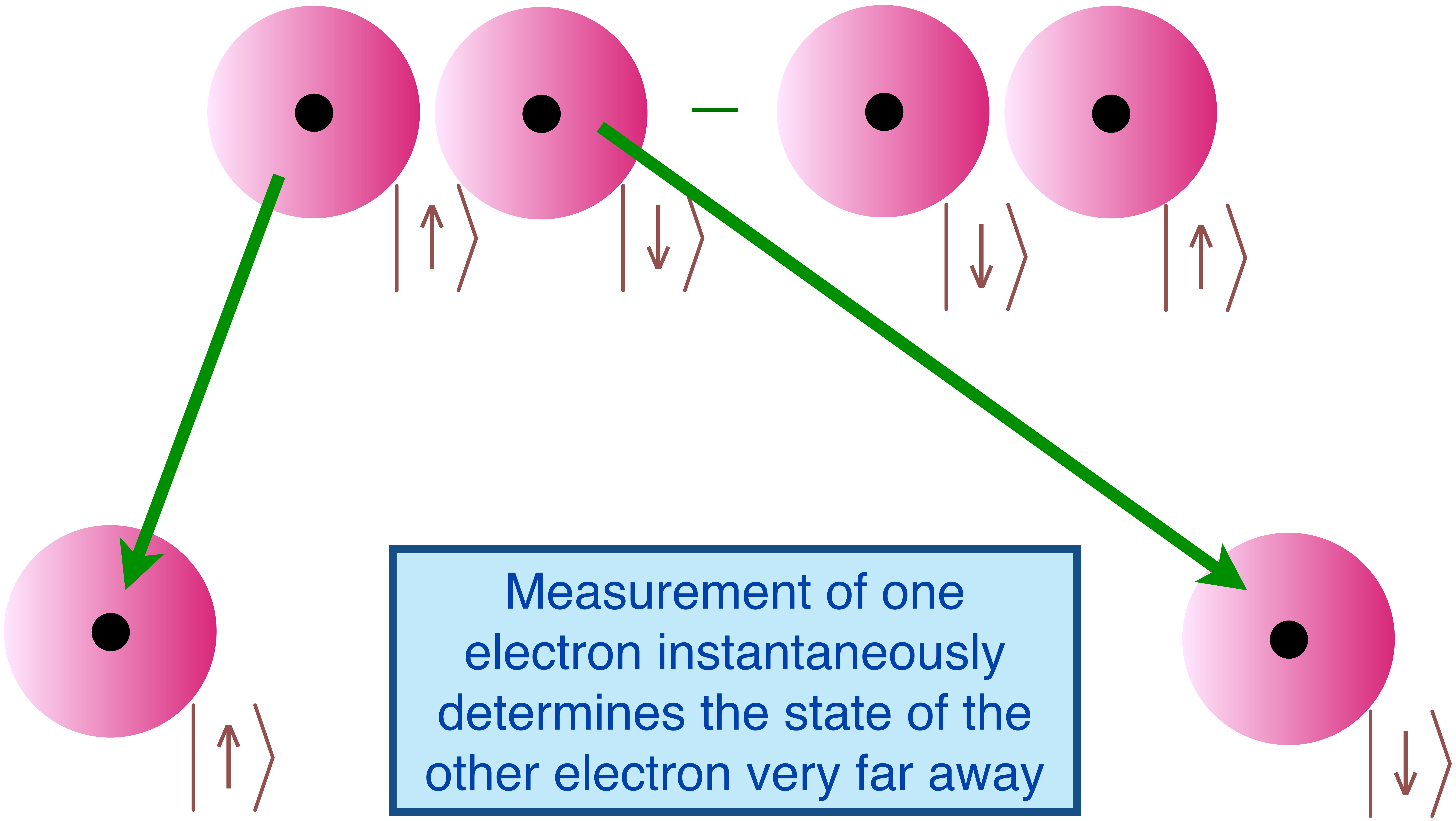

Measurement of one electron instantaneously determines the state of the other electron very far away

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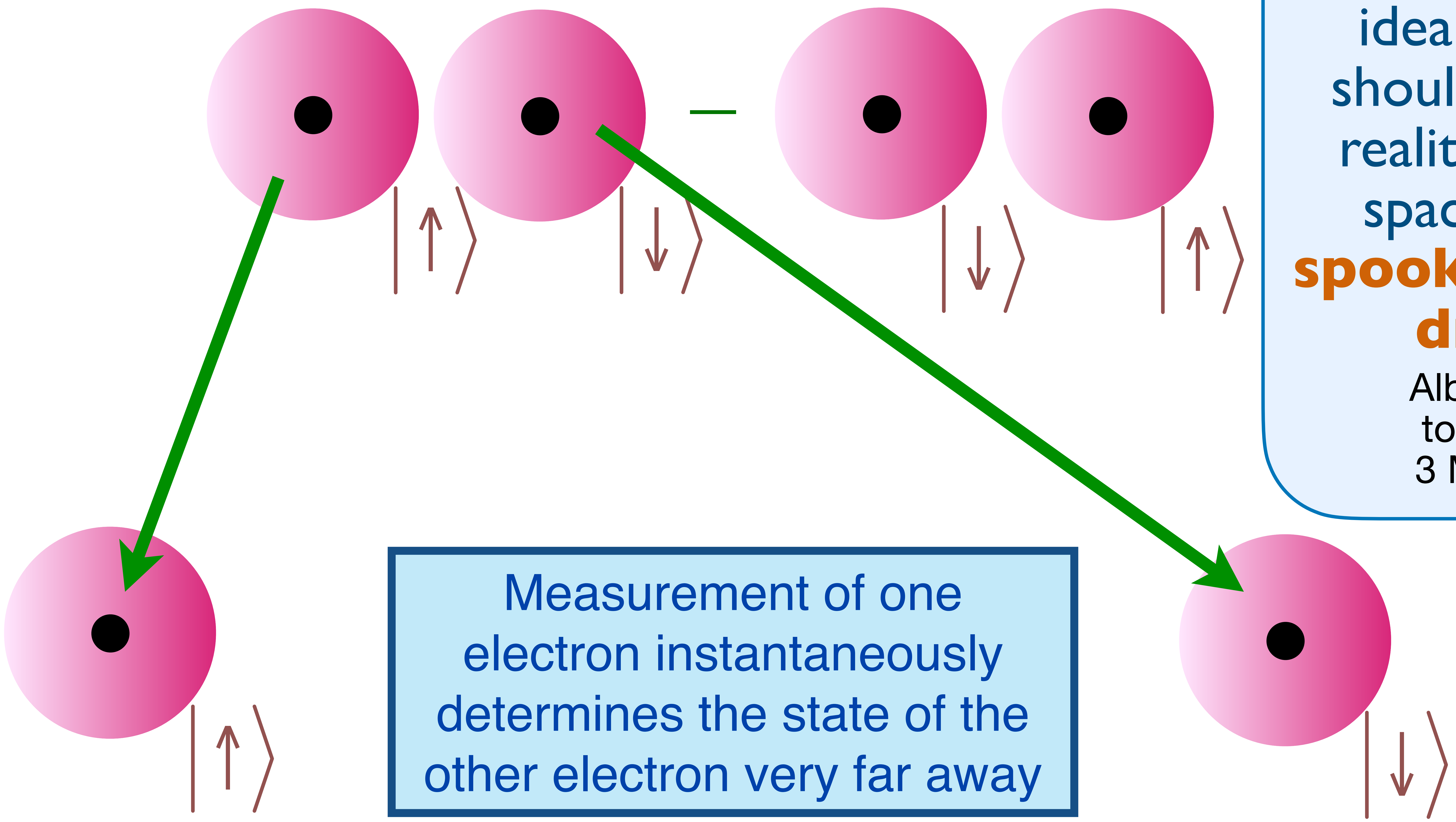


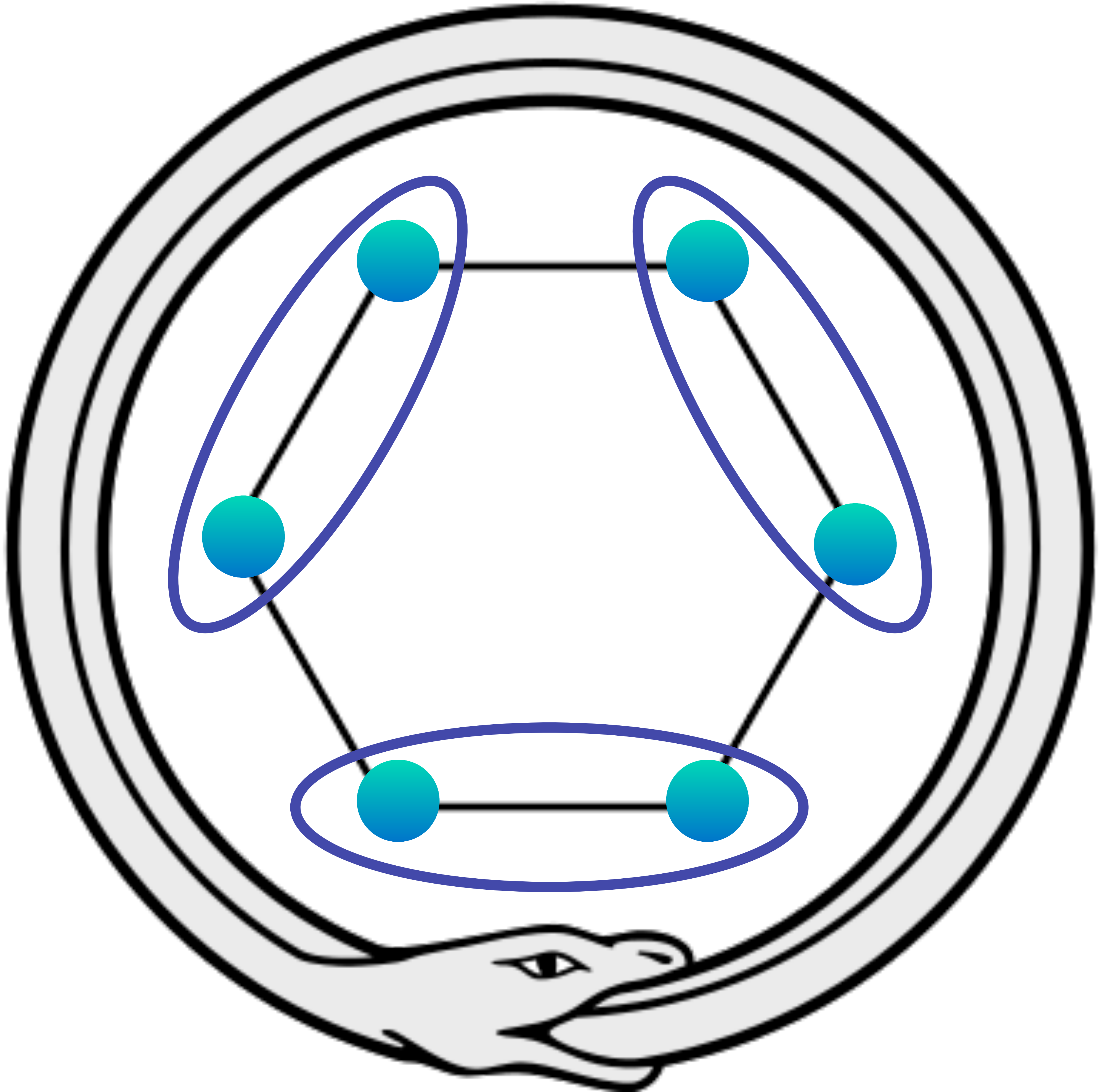
Measurement of one electron instantaneously determines the state of the other electron very far away

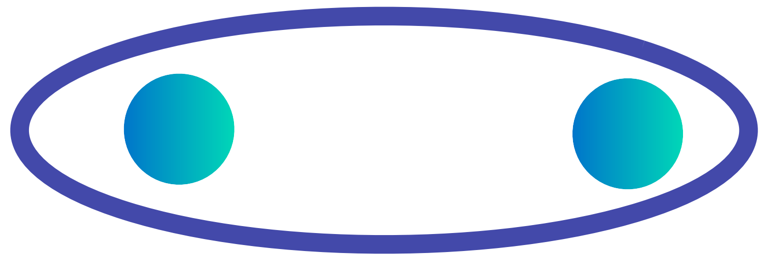
"I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from **spooky actions at distance**"

Albert Einstein
to Max Born,
3 March 1947

Kekule's spooky dream (1865)

Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*



 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Benzene

*Wikipedia

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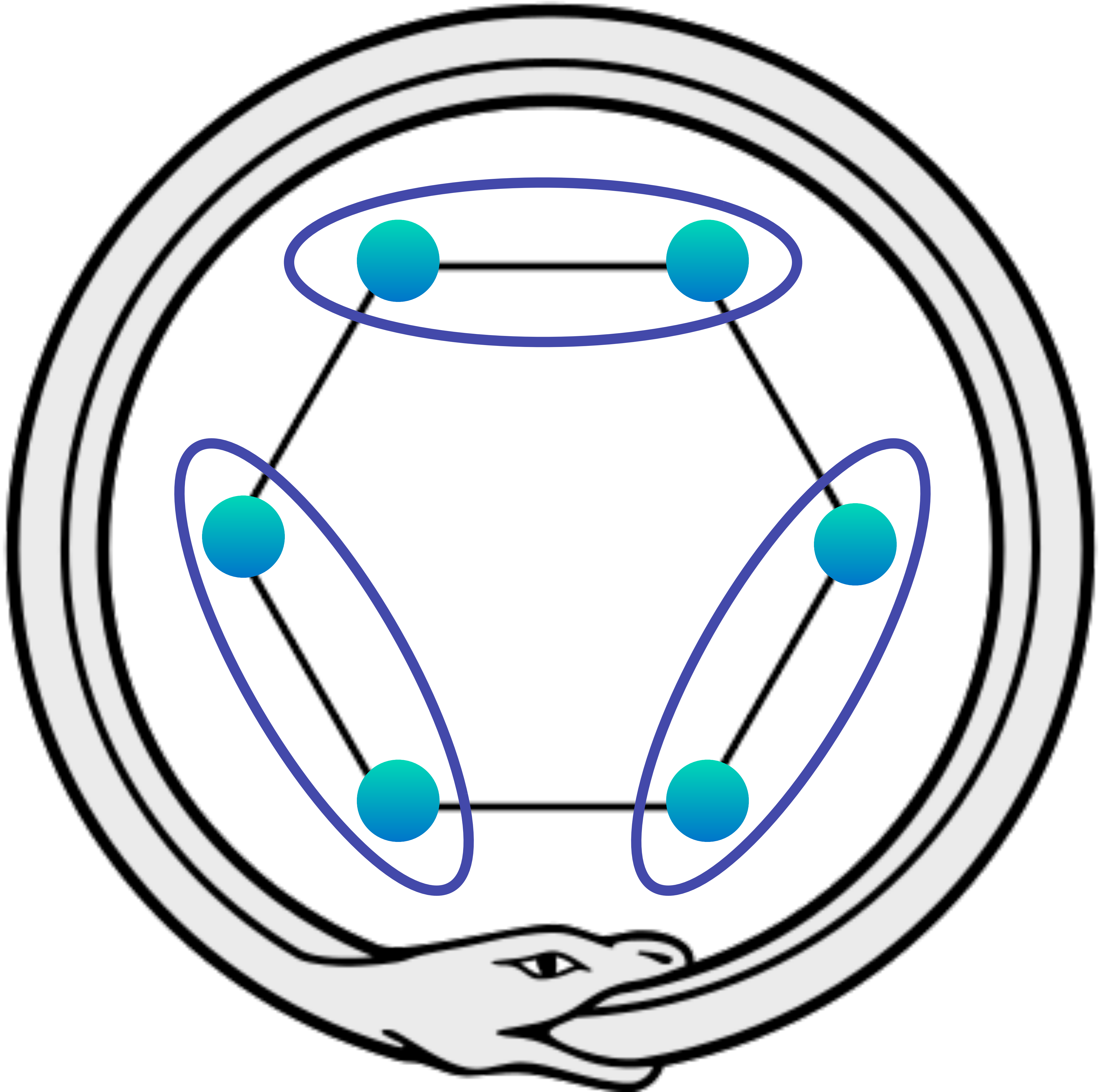

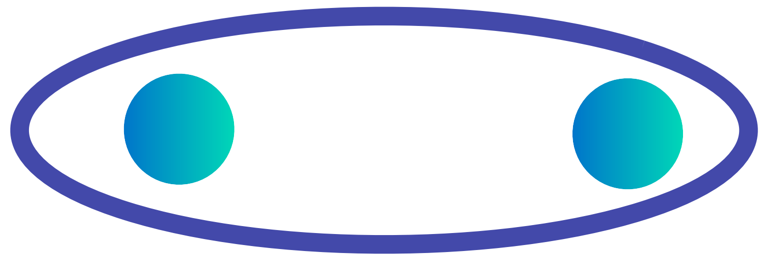

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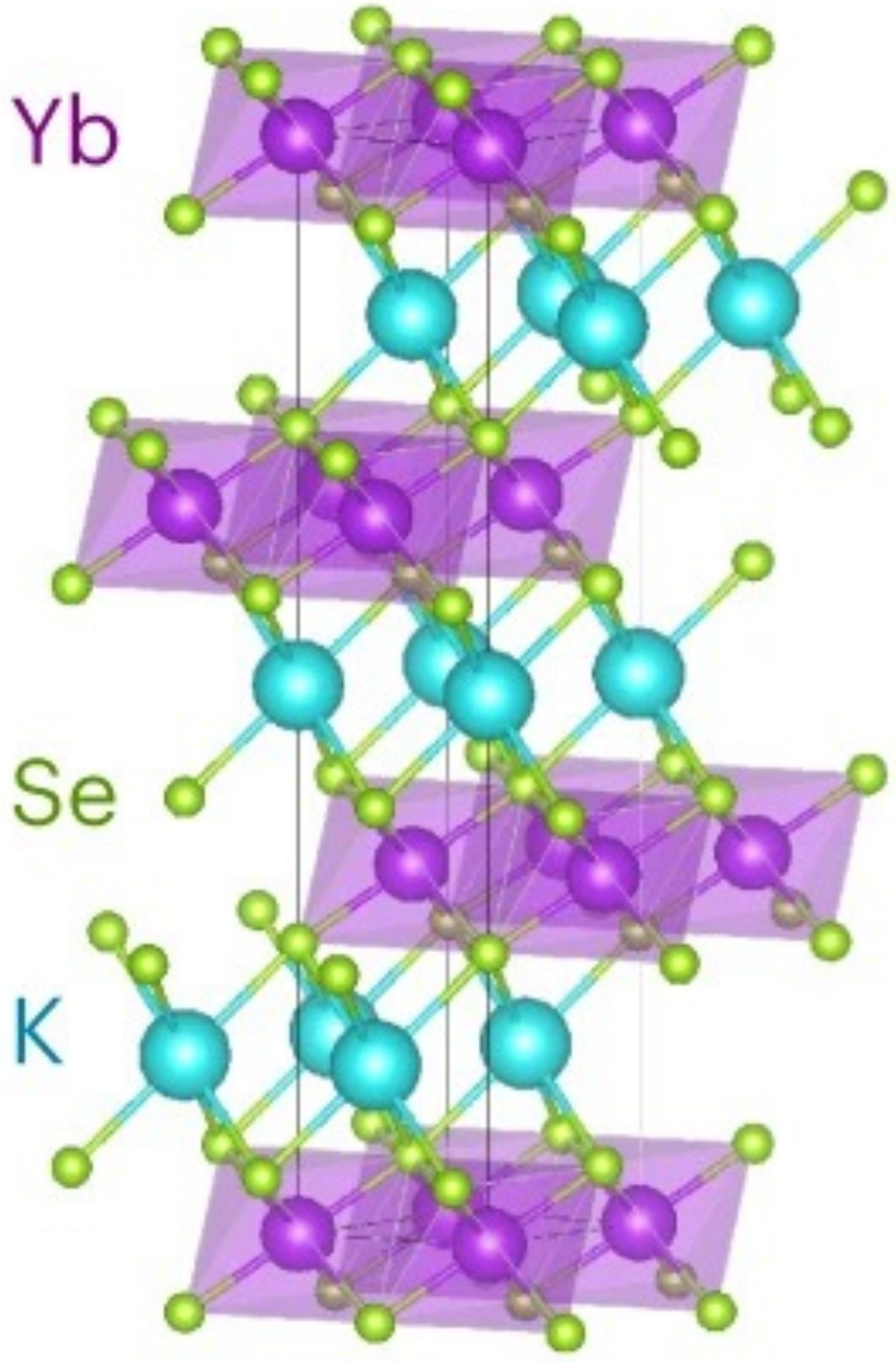
Benzene

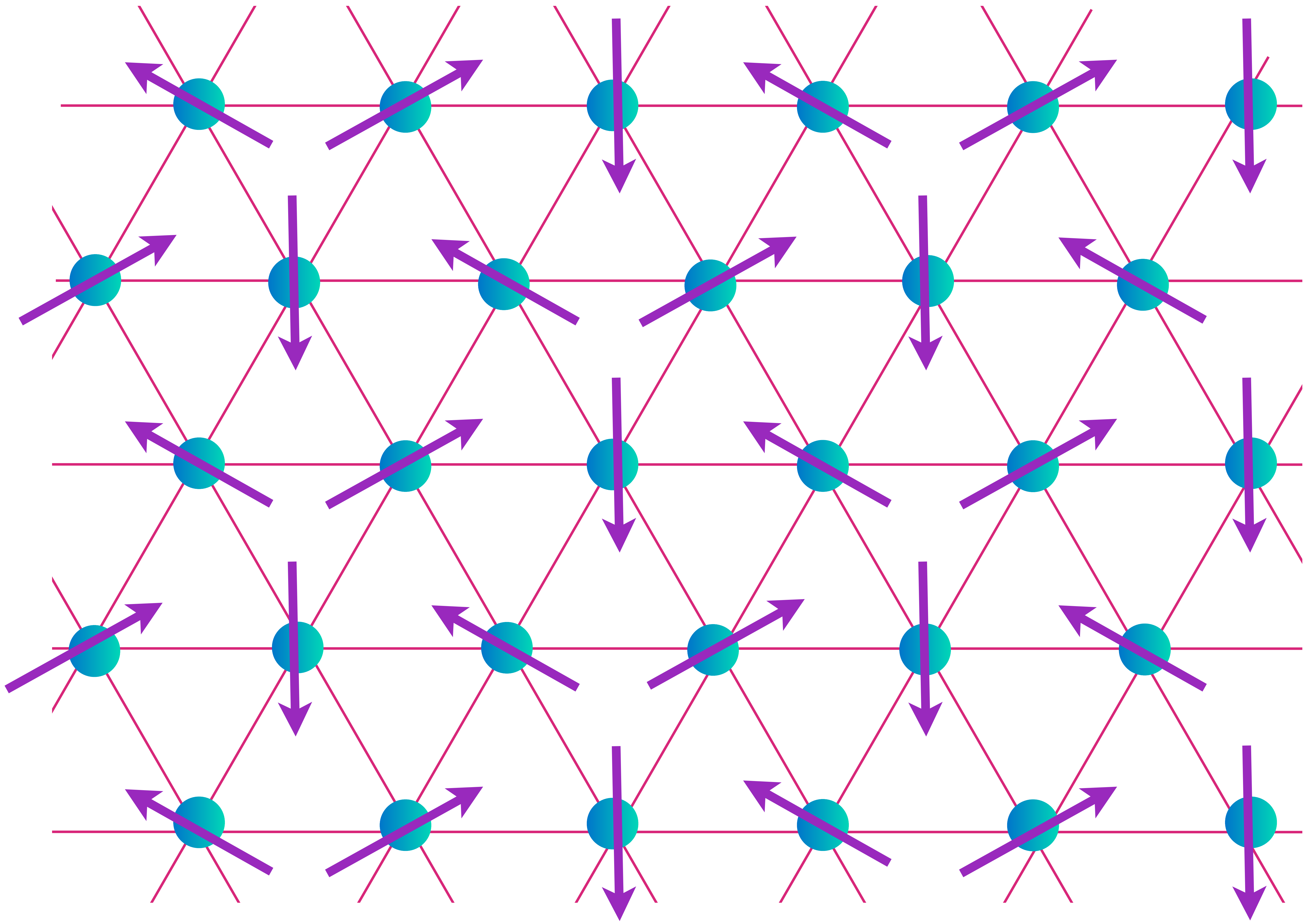
*Wikipedia

Quantum spin liquids

Triangular lattice antiferromagnet

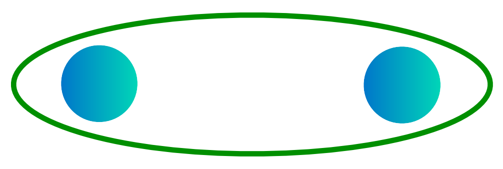
$KYbSe_2$





Nearest-neighbor model has ordered spins

Spin liquid: resonating valence bonds



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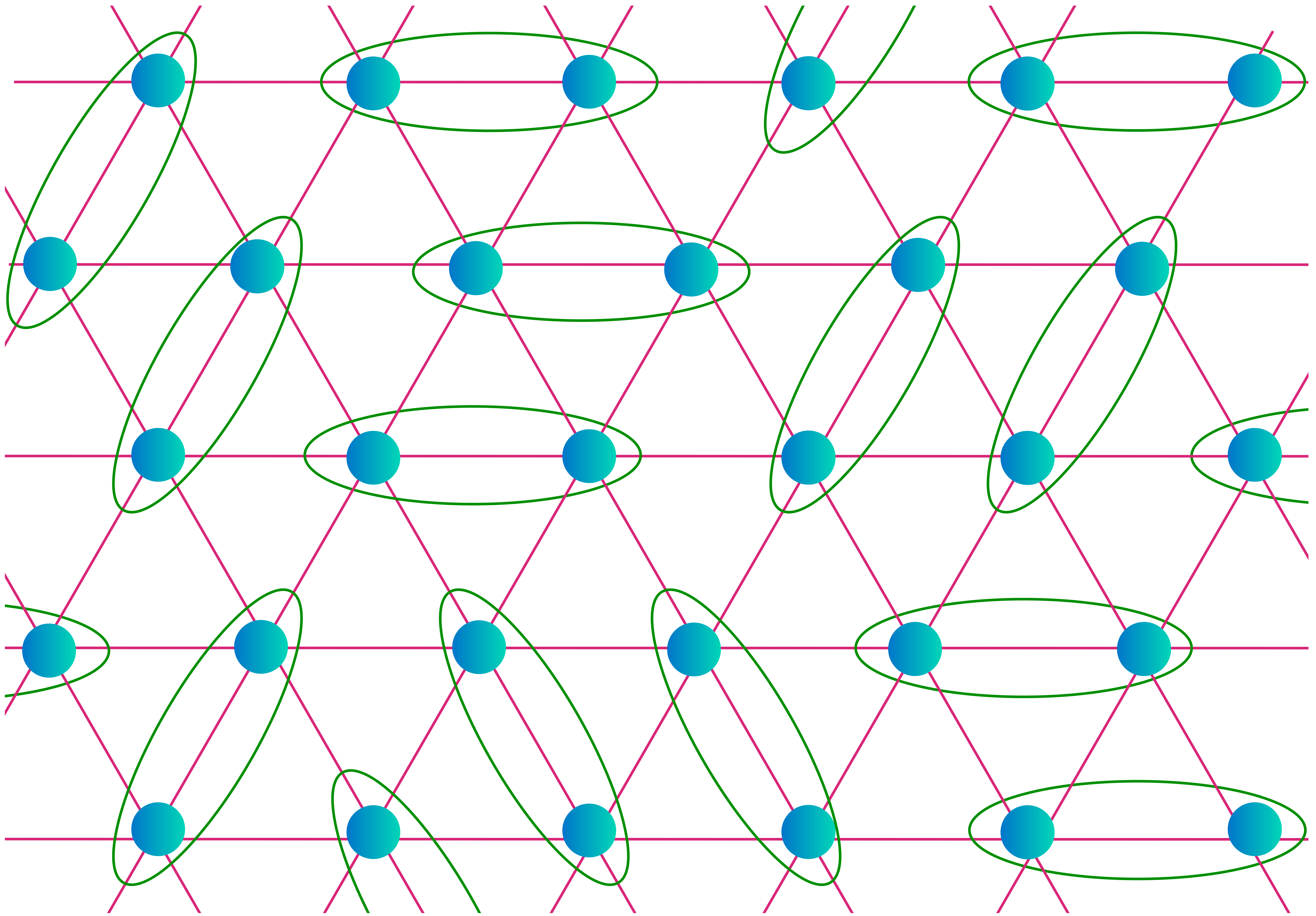
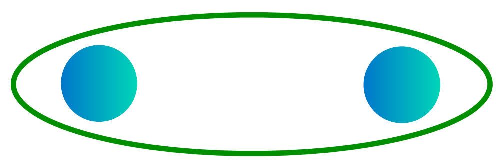


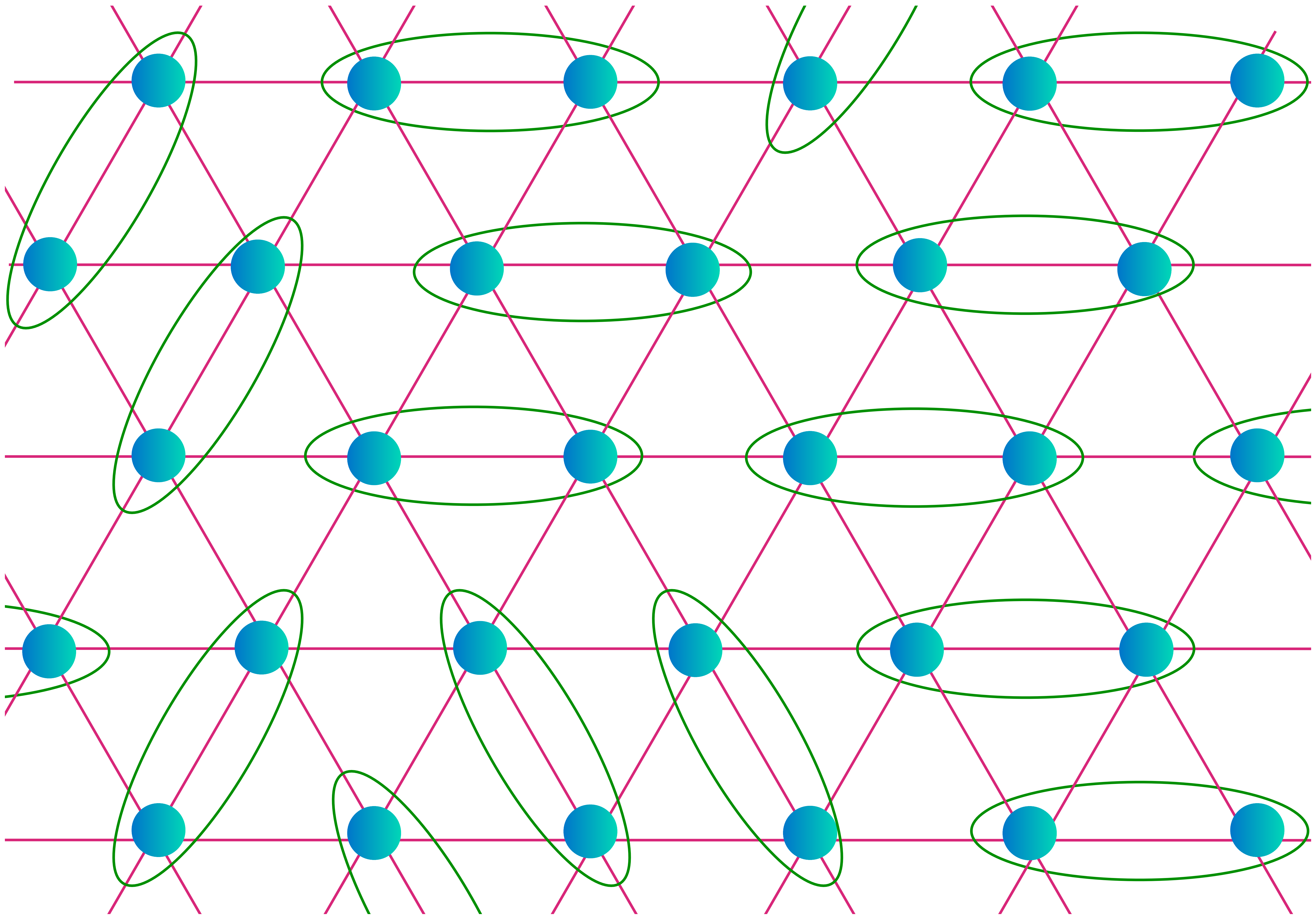
$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

$\mathcal{D} \to$ dimer covering of lattice

Patrik Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

Spin liquid: resonating valence bonds

 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



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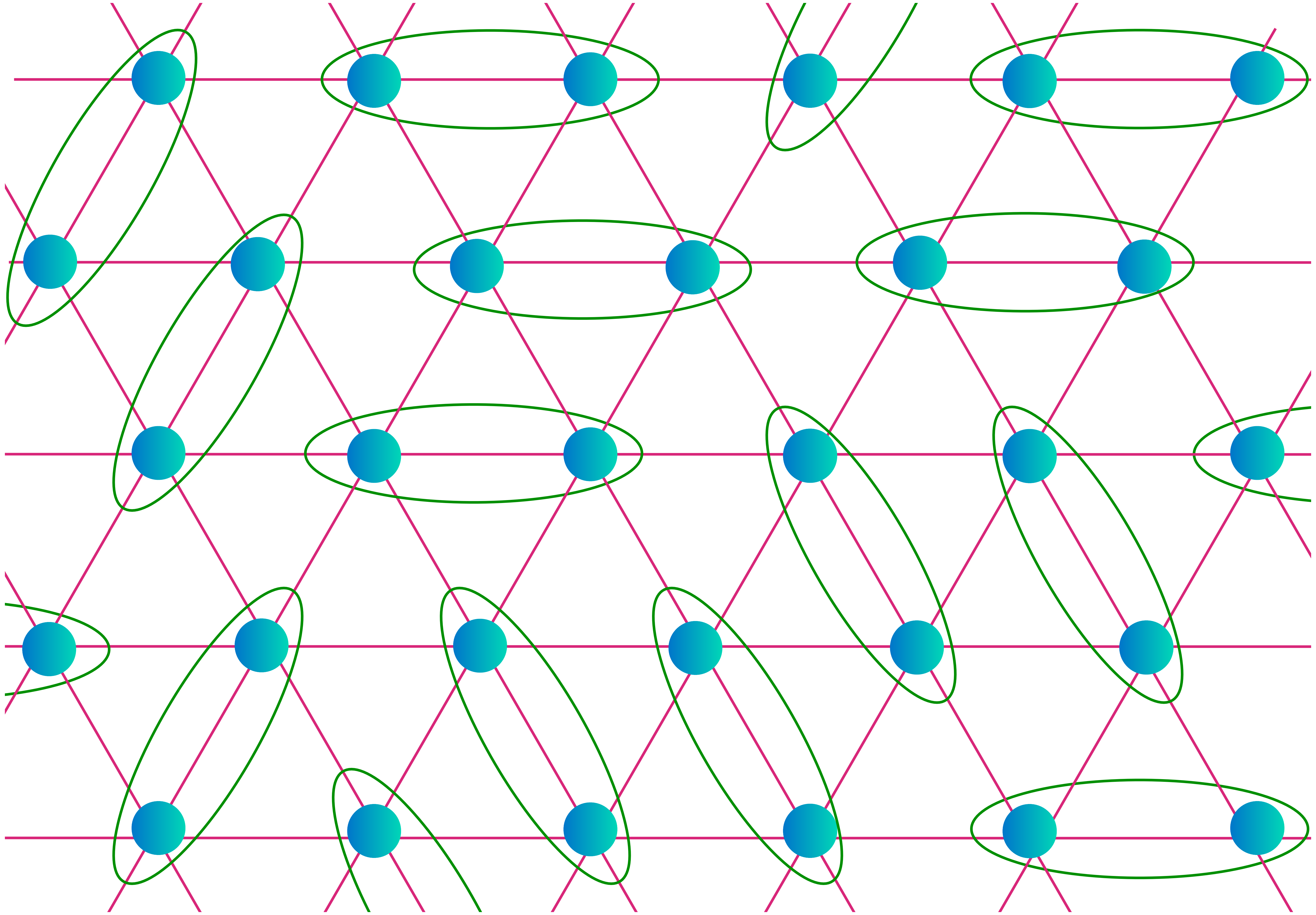

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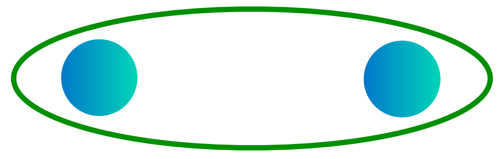
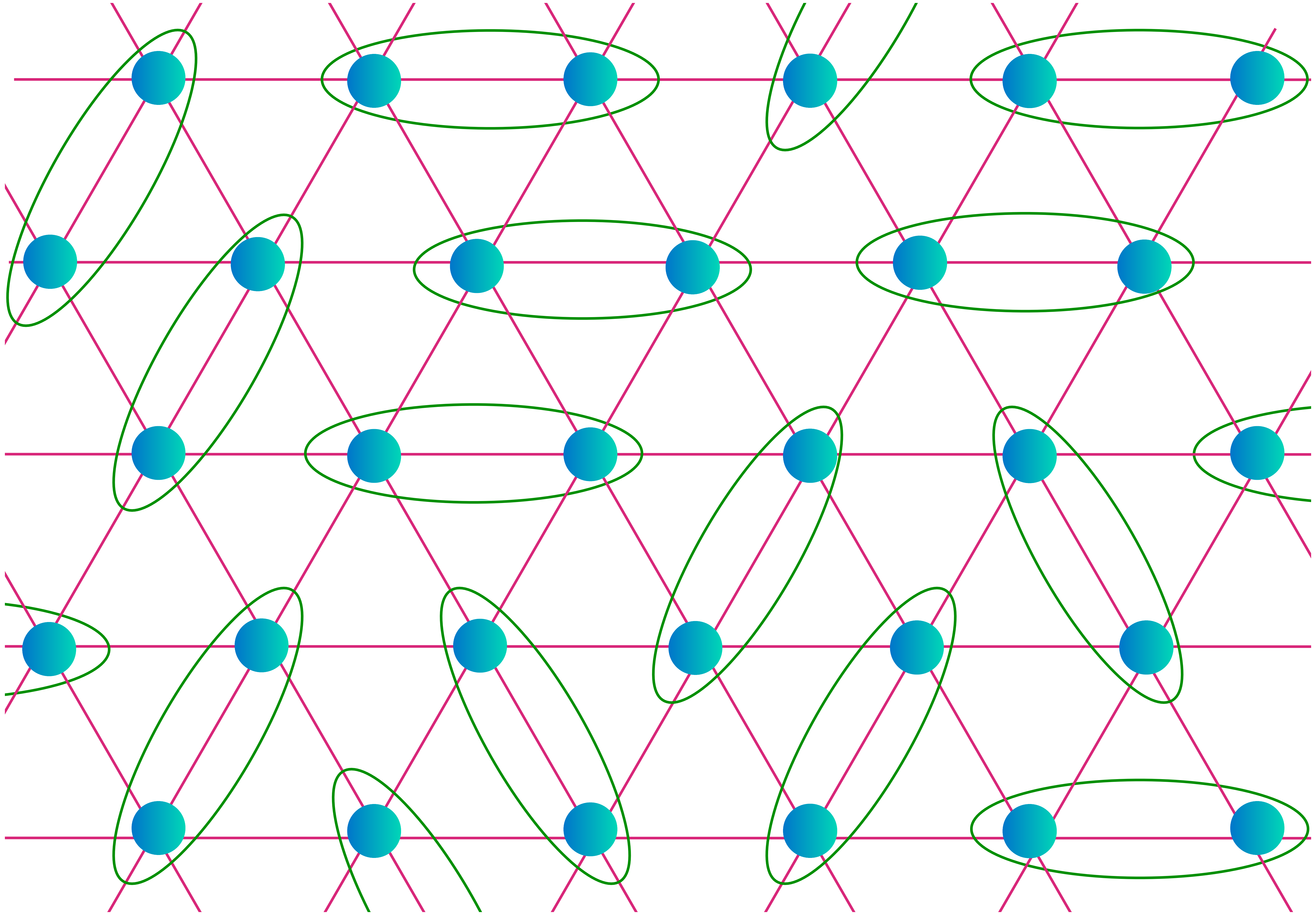
 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



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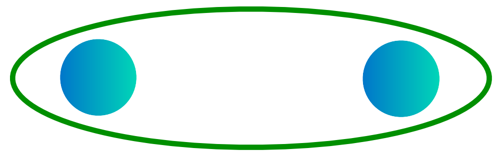

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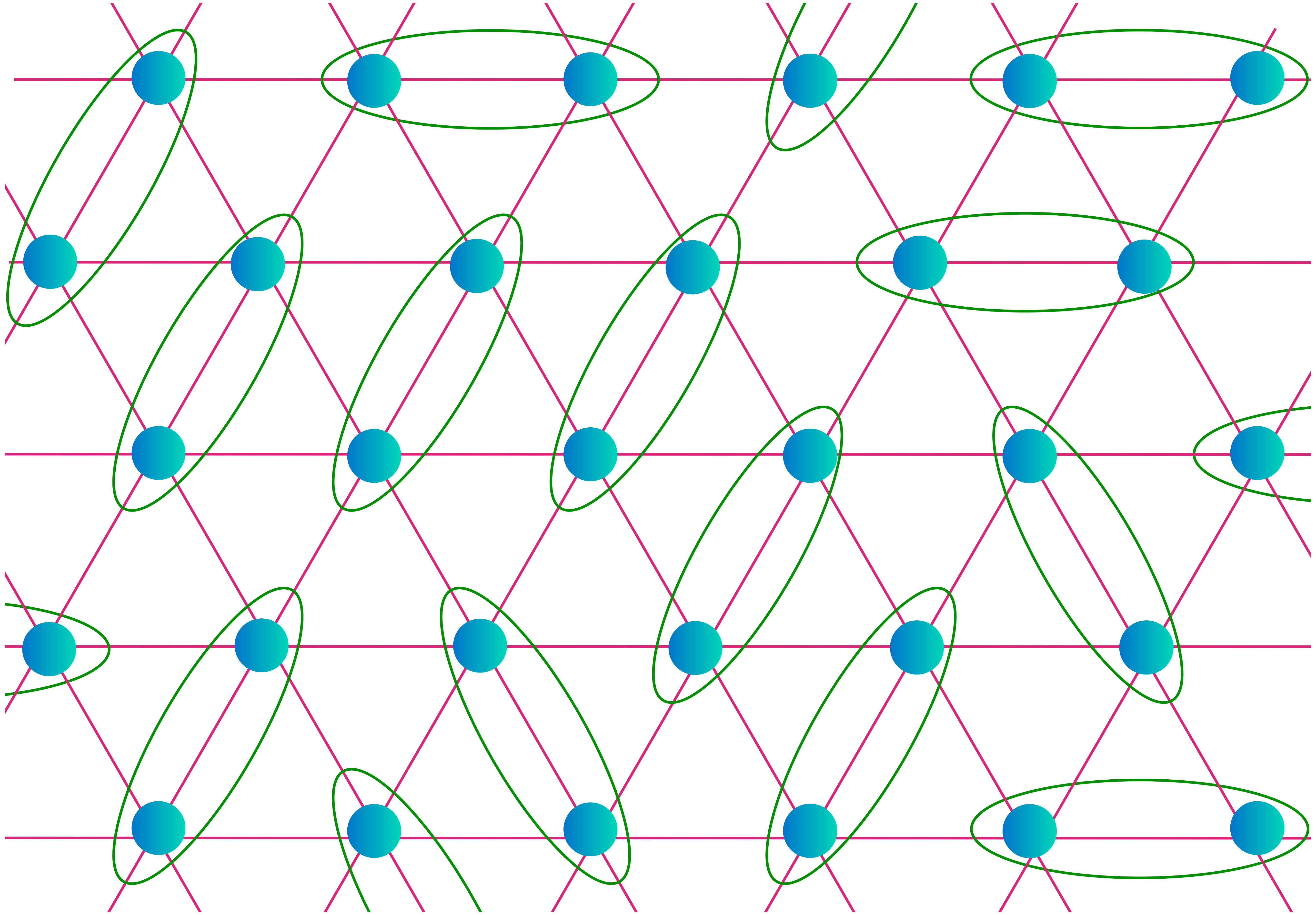

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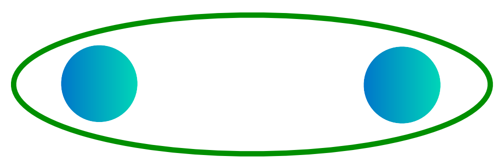

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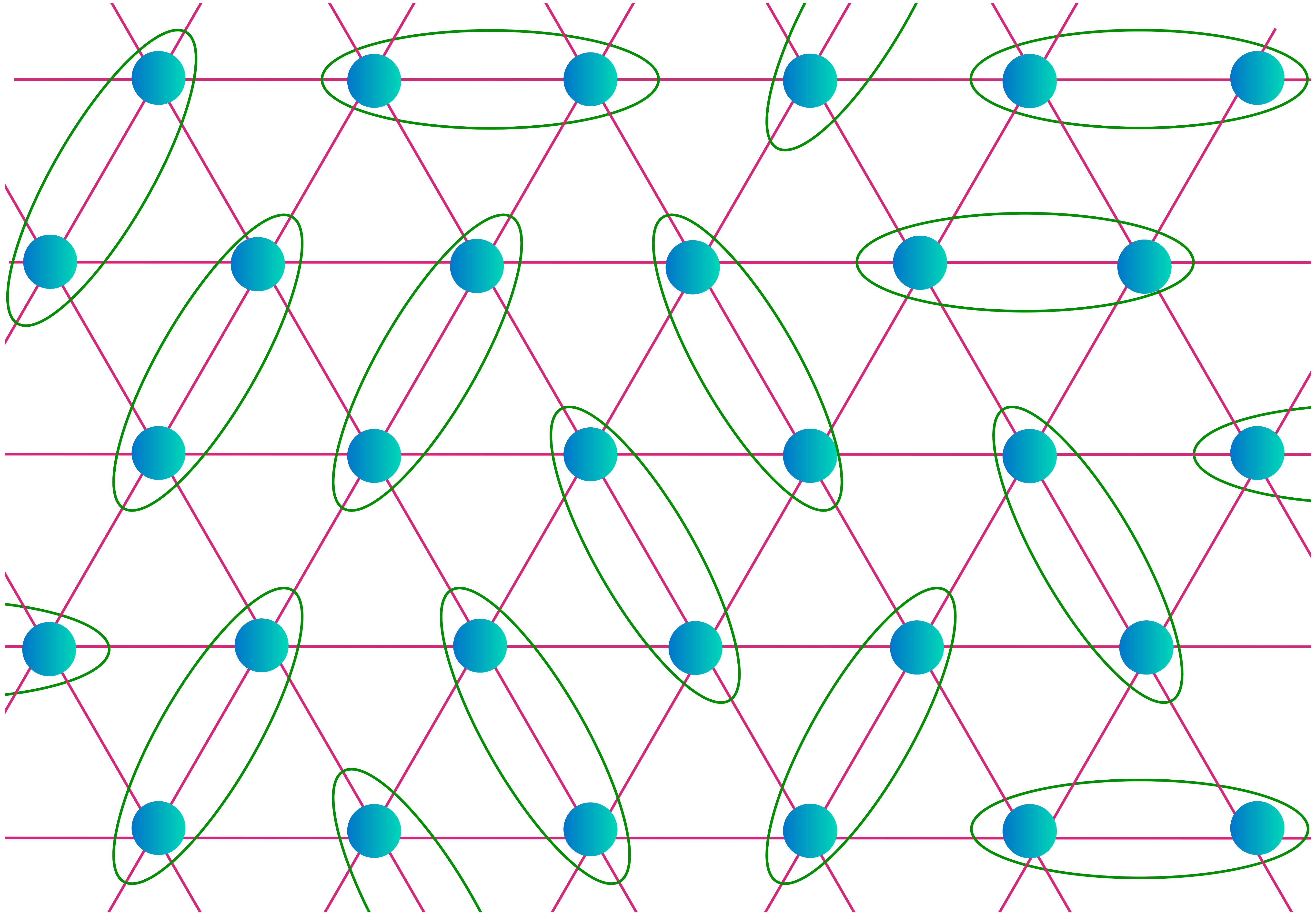

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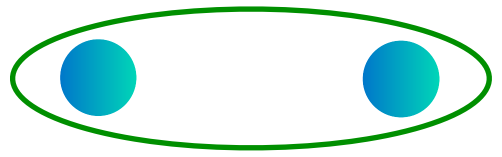
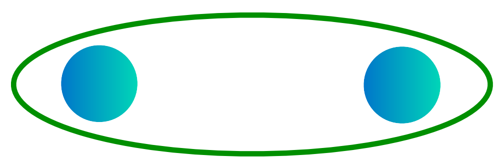


$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

$$|\Psi\rangle = C_1|\ \rangle + C_2|\ \rangle + C_3|\ \rangle + C_4|\ \rangle + C_5|\ \rangle + \ldots$$

Spin liquid: resonating valence bonds



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

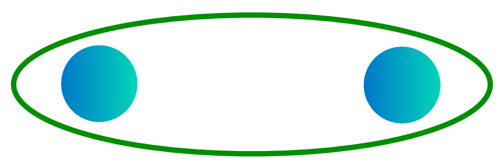
Key feature: fractionalization.

Excitations are particle-like, but cannot be created by local operators.

The excitations are classified under distinct **anyon** sectors.

Spin liquid: resonating valence bonds

Anyon: a "spinon"



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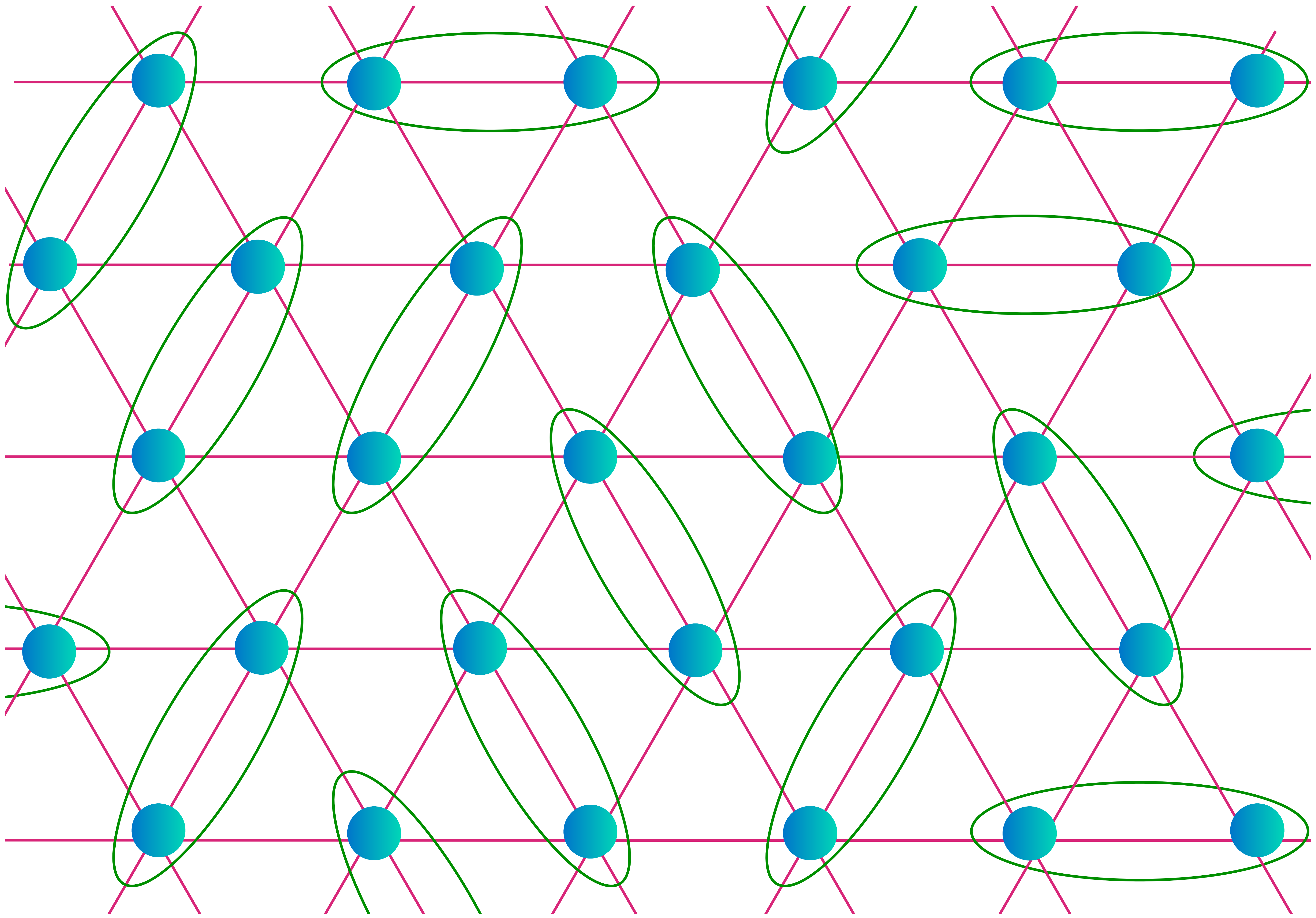

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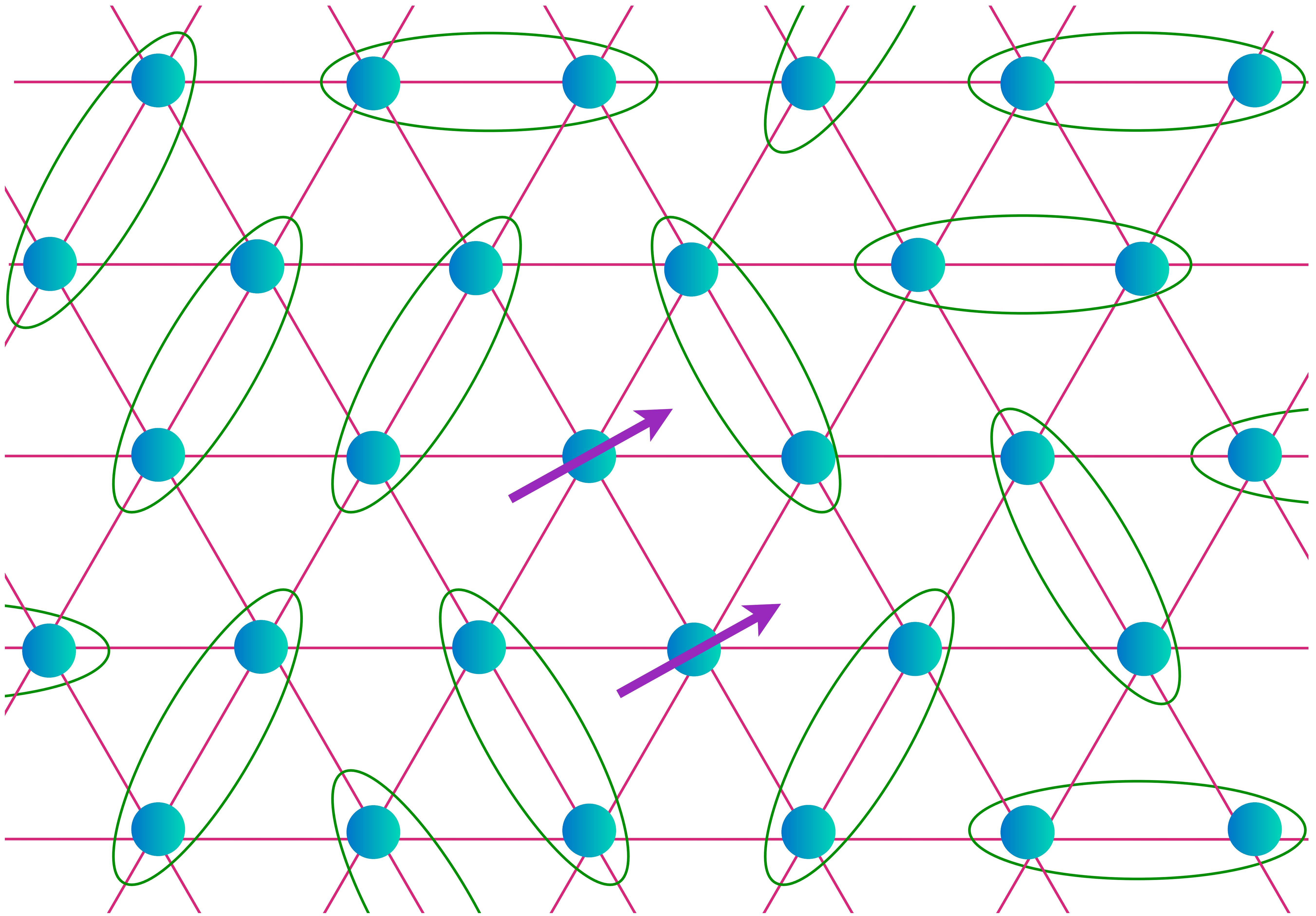

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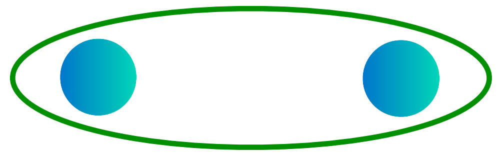

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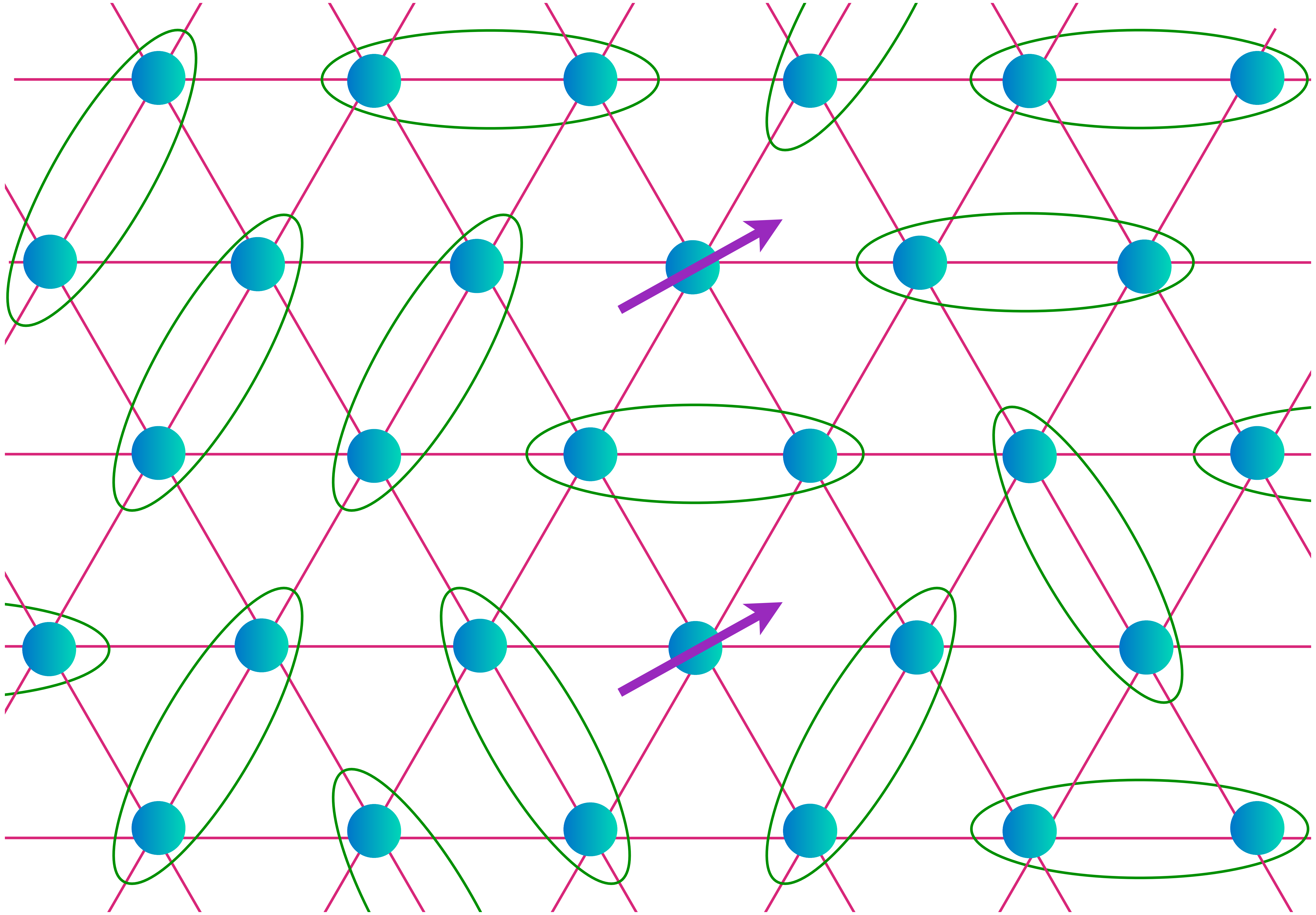

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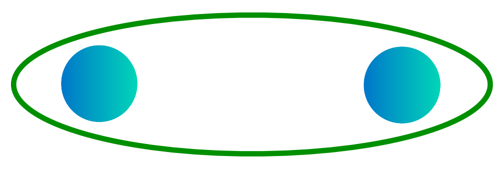

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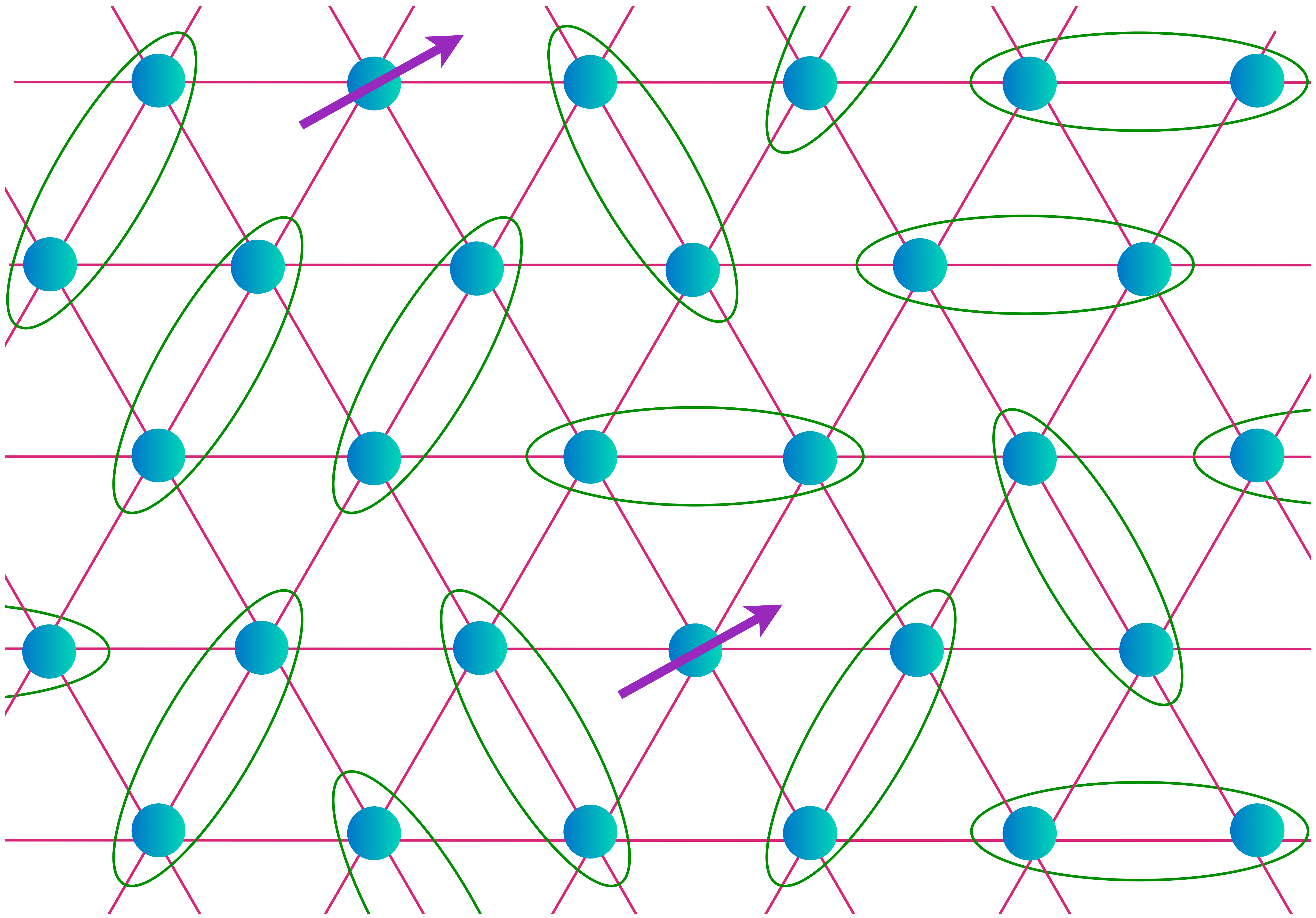

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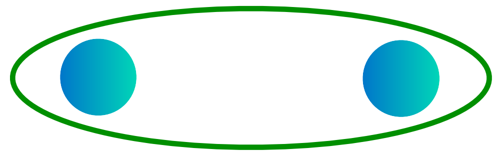
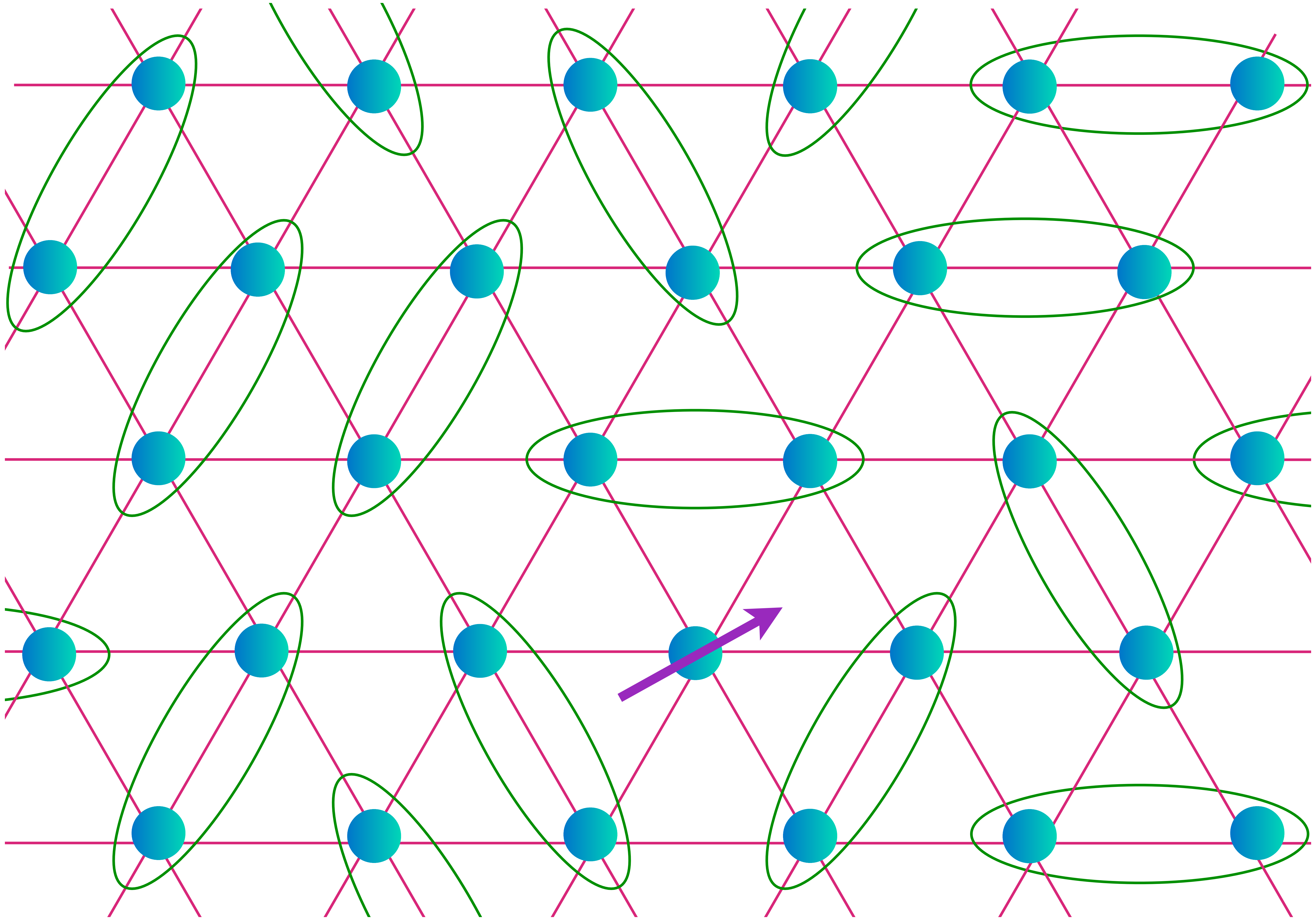
 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



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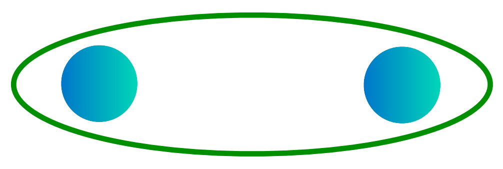

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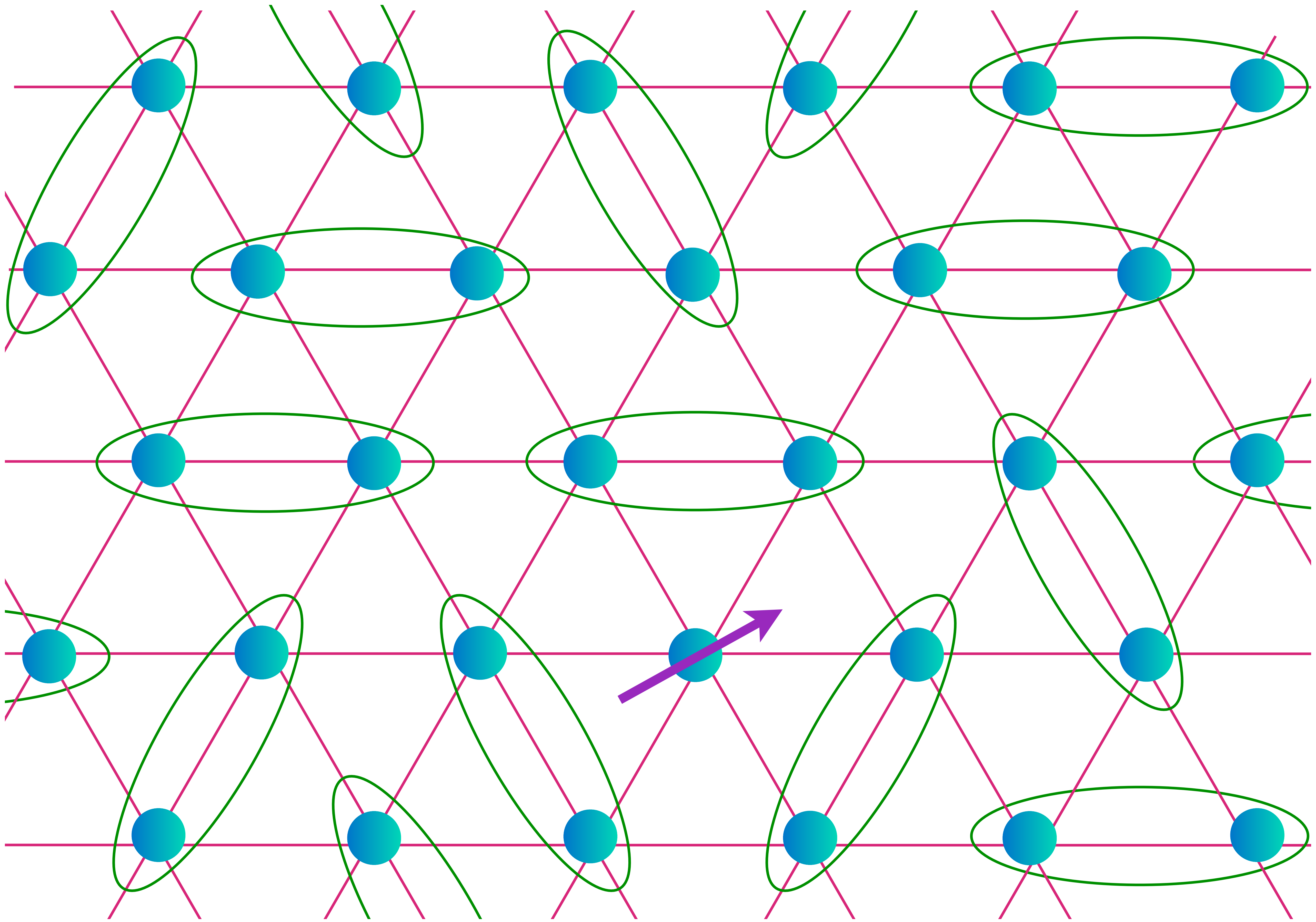

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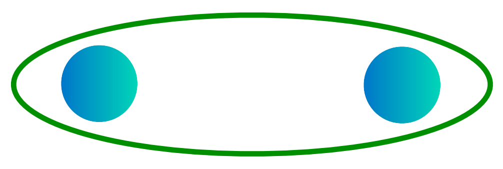

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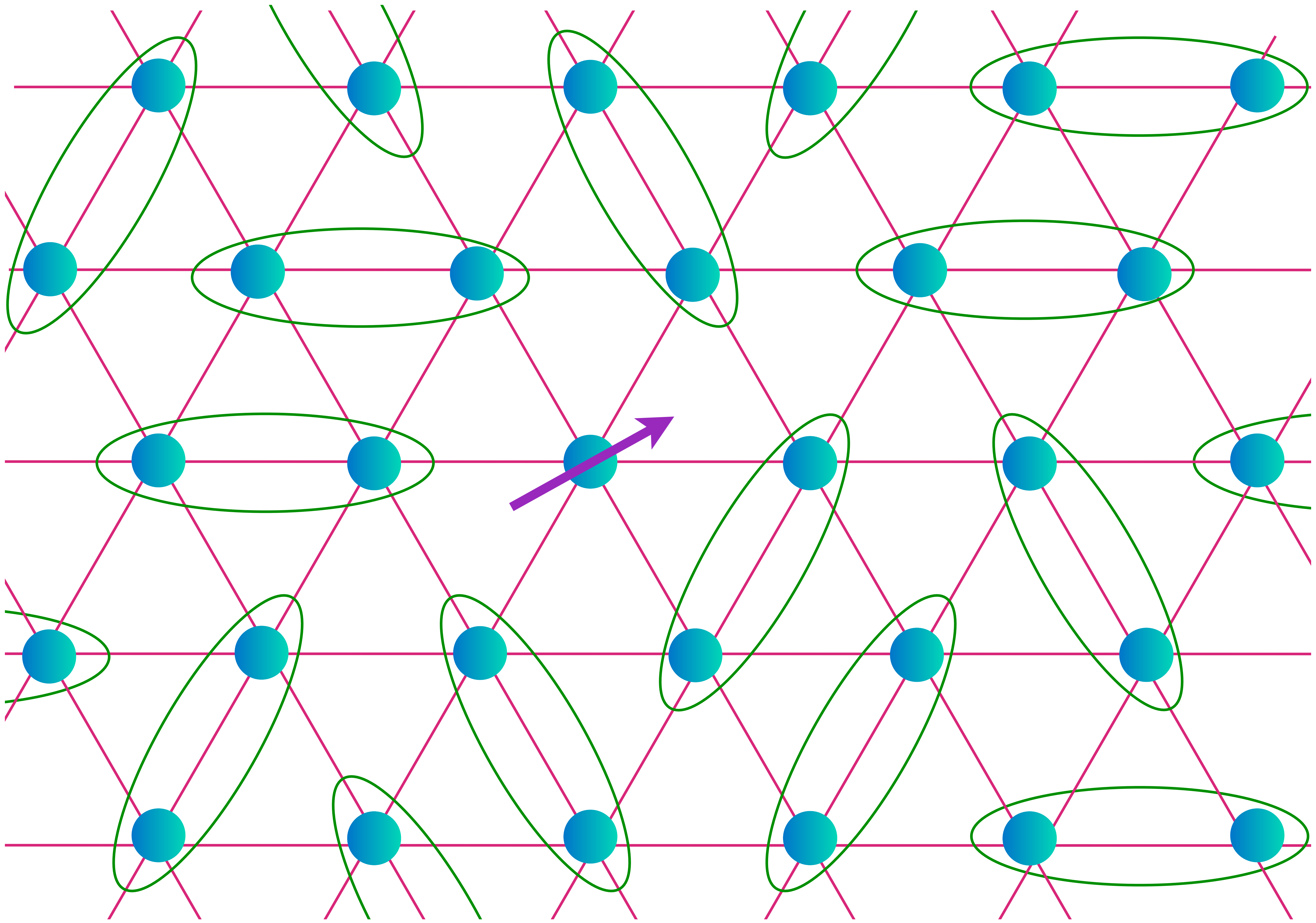

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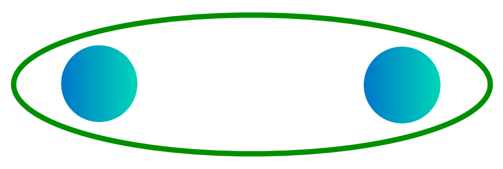

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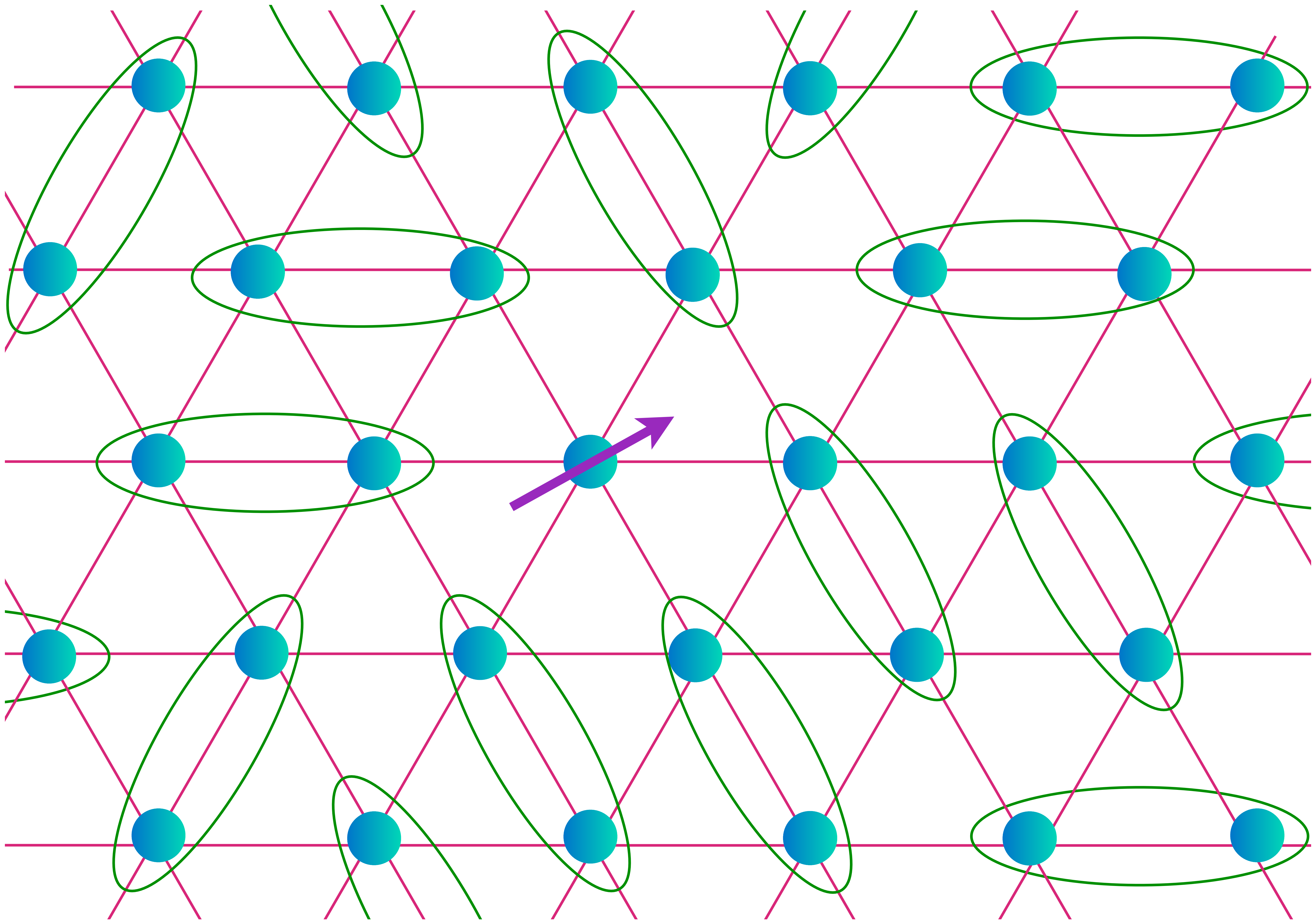

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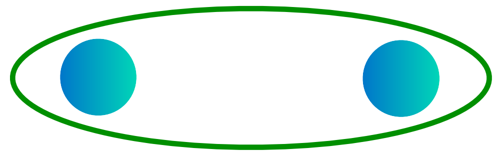
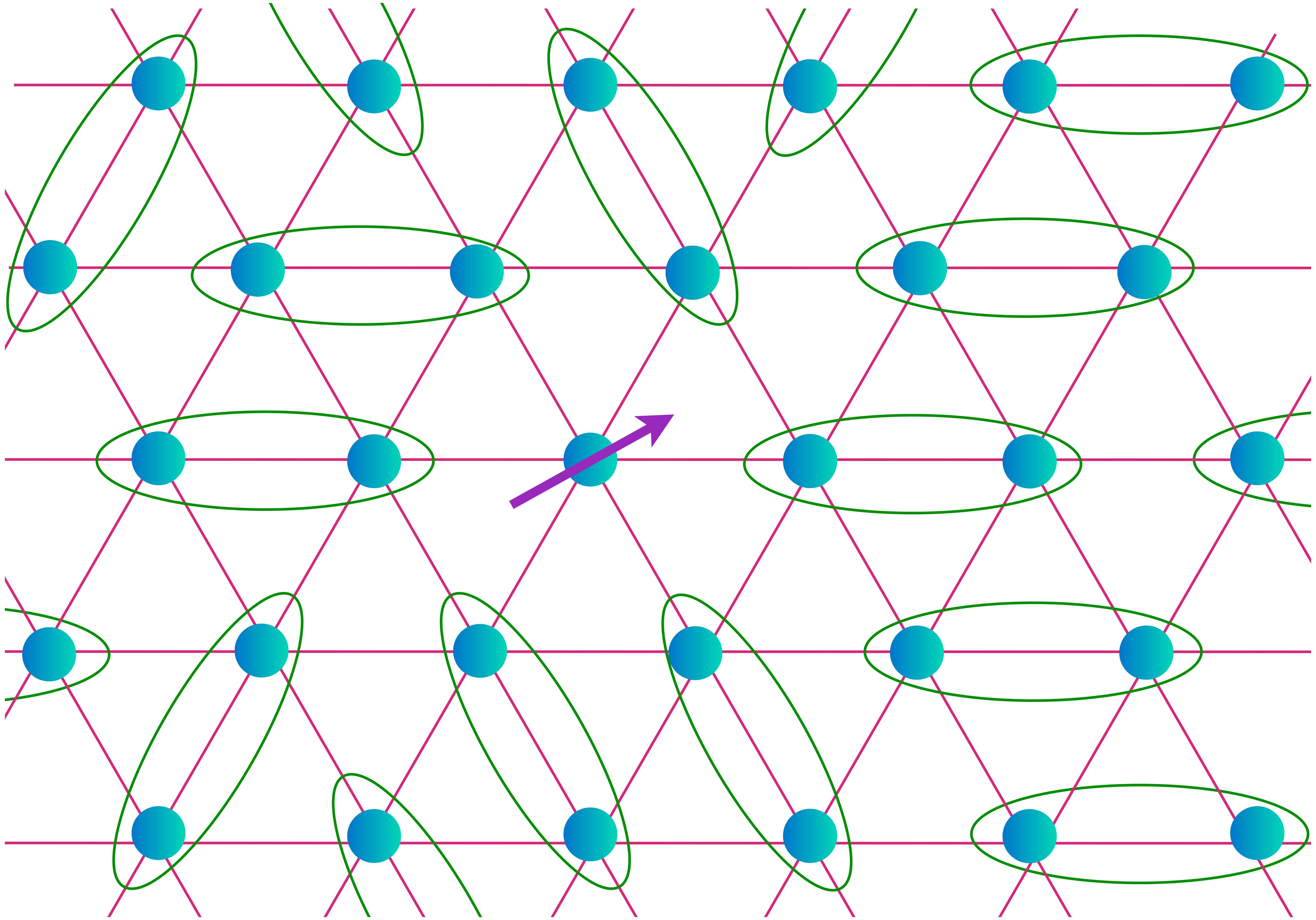
 $= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$



- The theory of spin liquids with well-defined anyons has applications to insulators (such as $KYbSe_2$), and to quantum devices built by Google and IBM.

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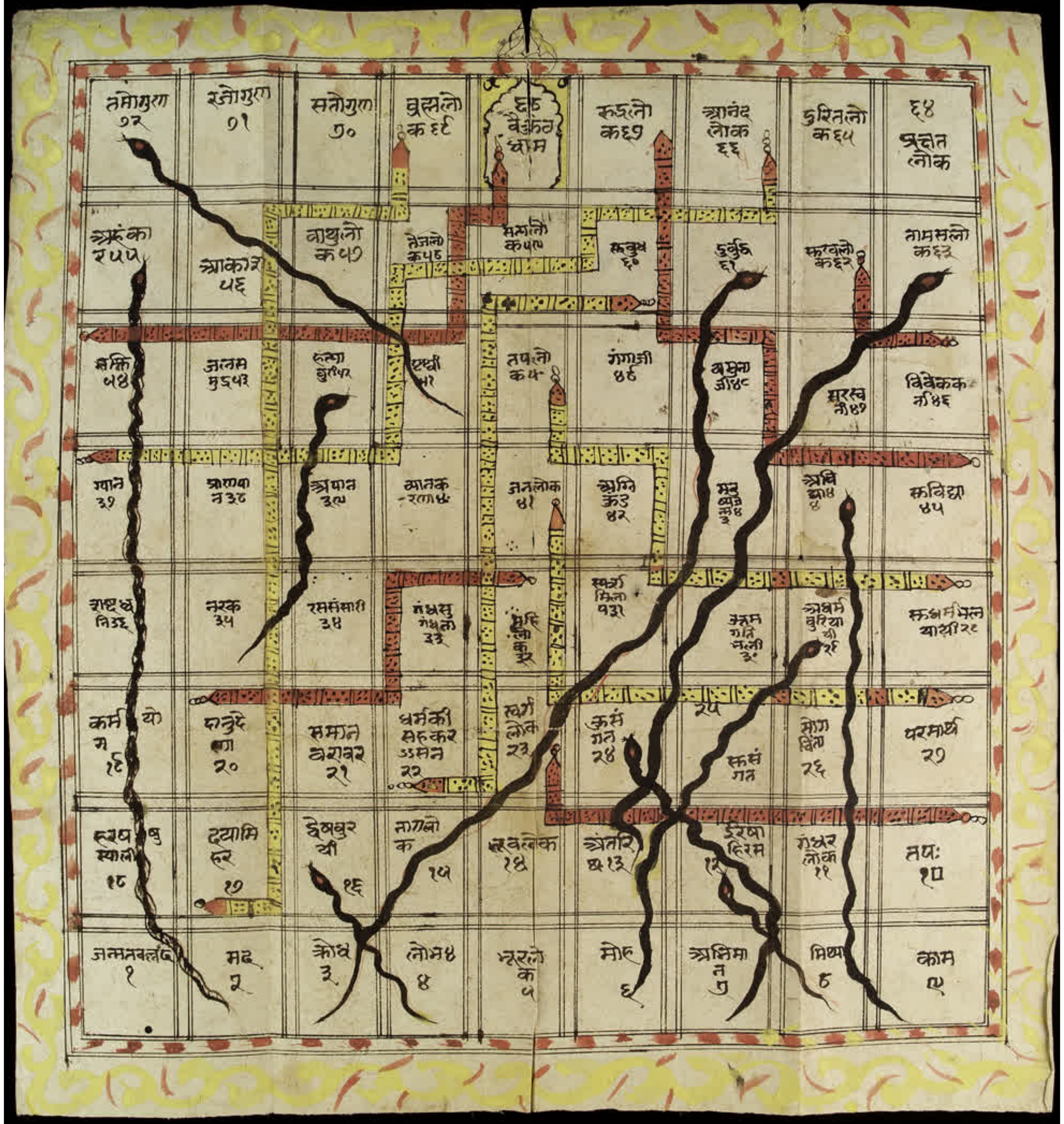
- However, it does *not* describe high temperature superconductivity in the cuprates, which requires a *different* type of multi-particle quantum entanglement.

The Sachdev-Ye-Kitaev model of entanglement of mobile fermions



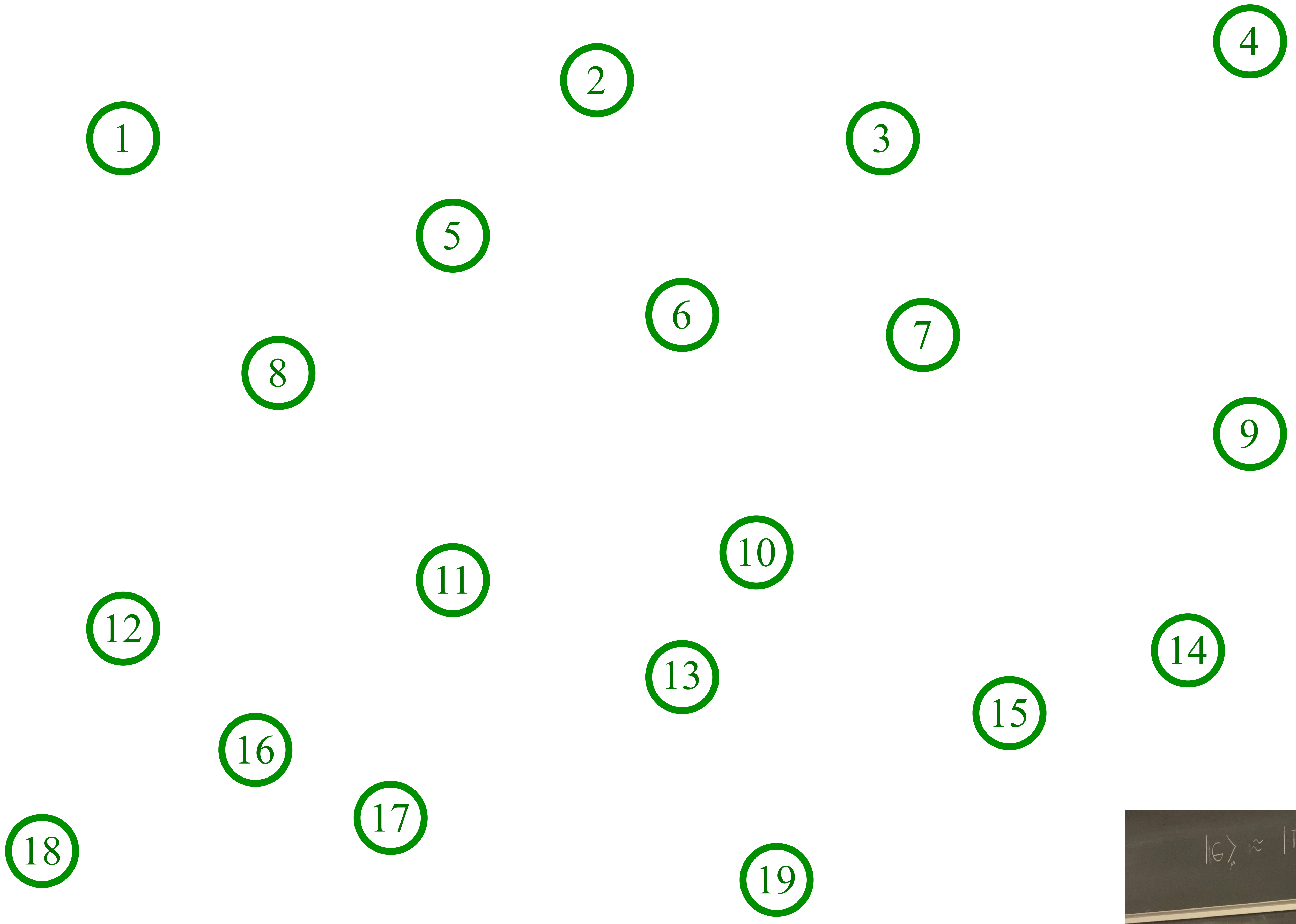
My spooky dream*

Ancient Indian game of Snakes and Ladders

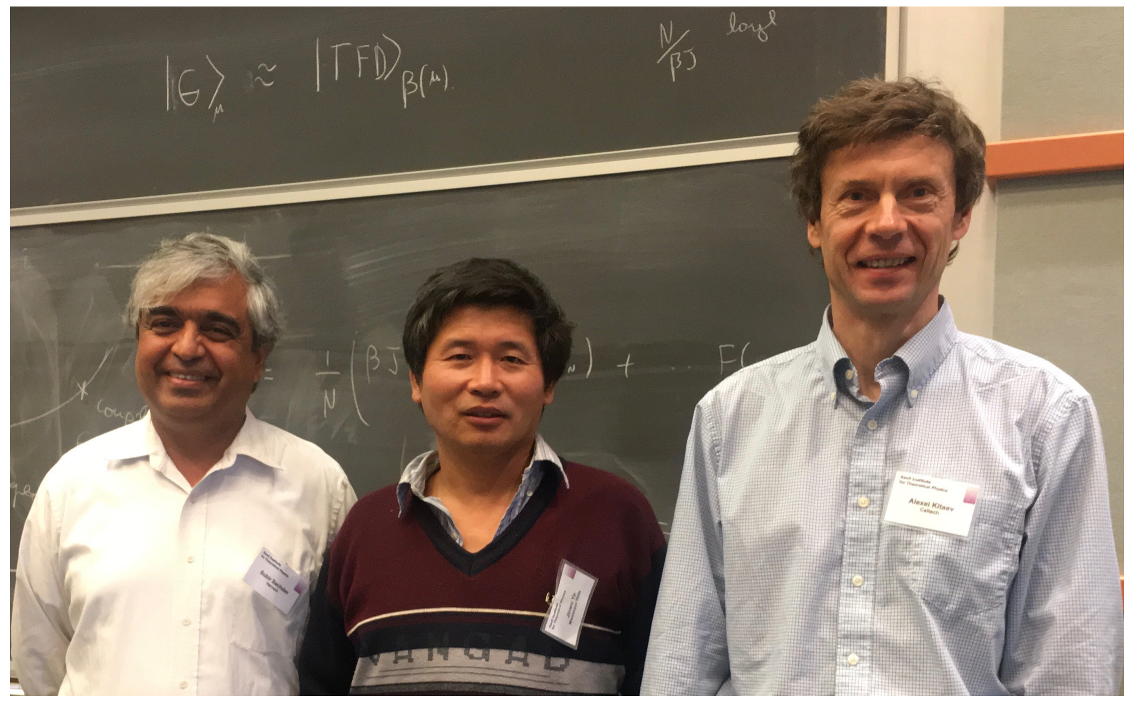
*Not true

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)



Pick a set of random positions



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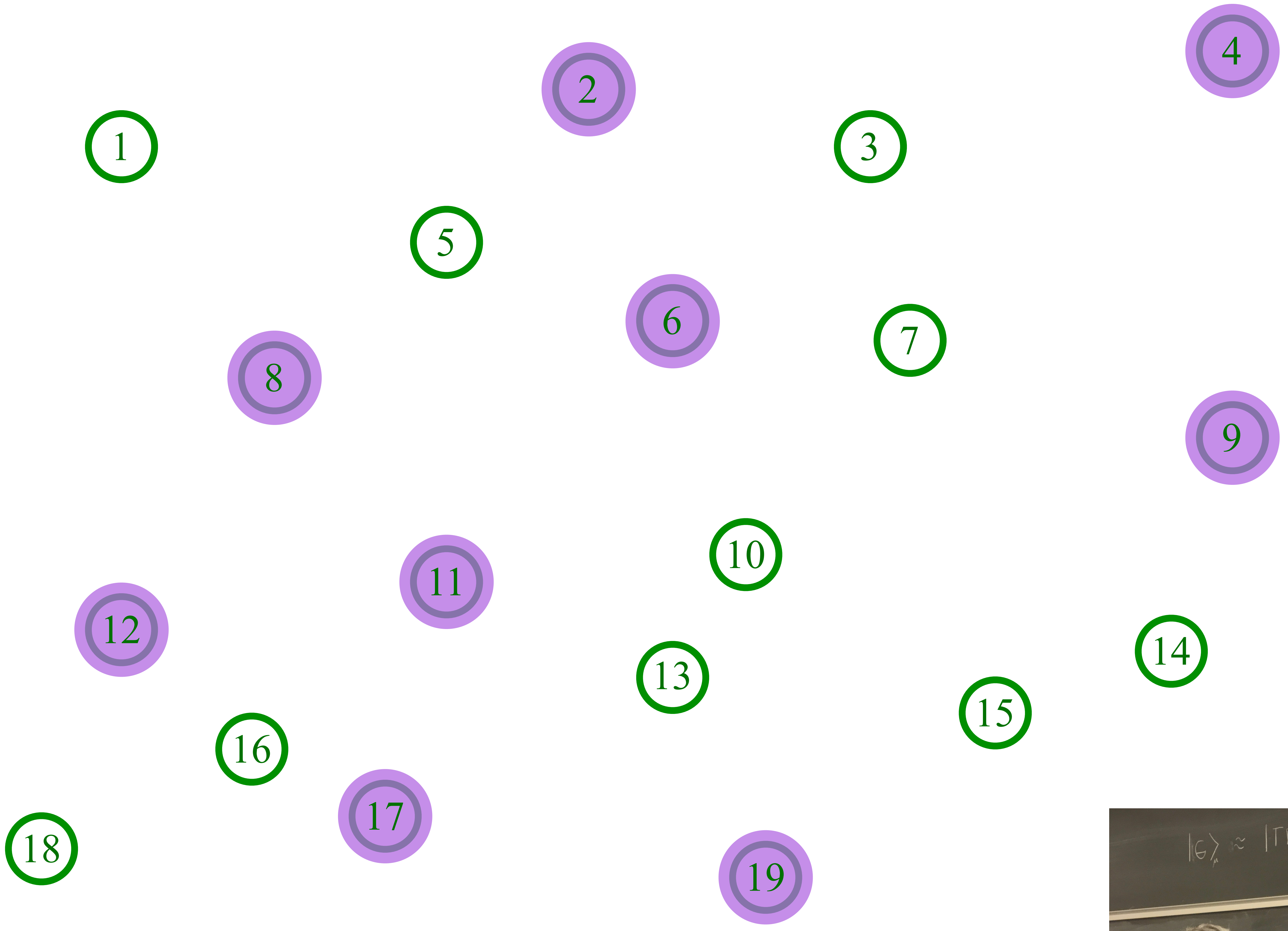
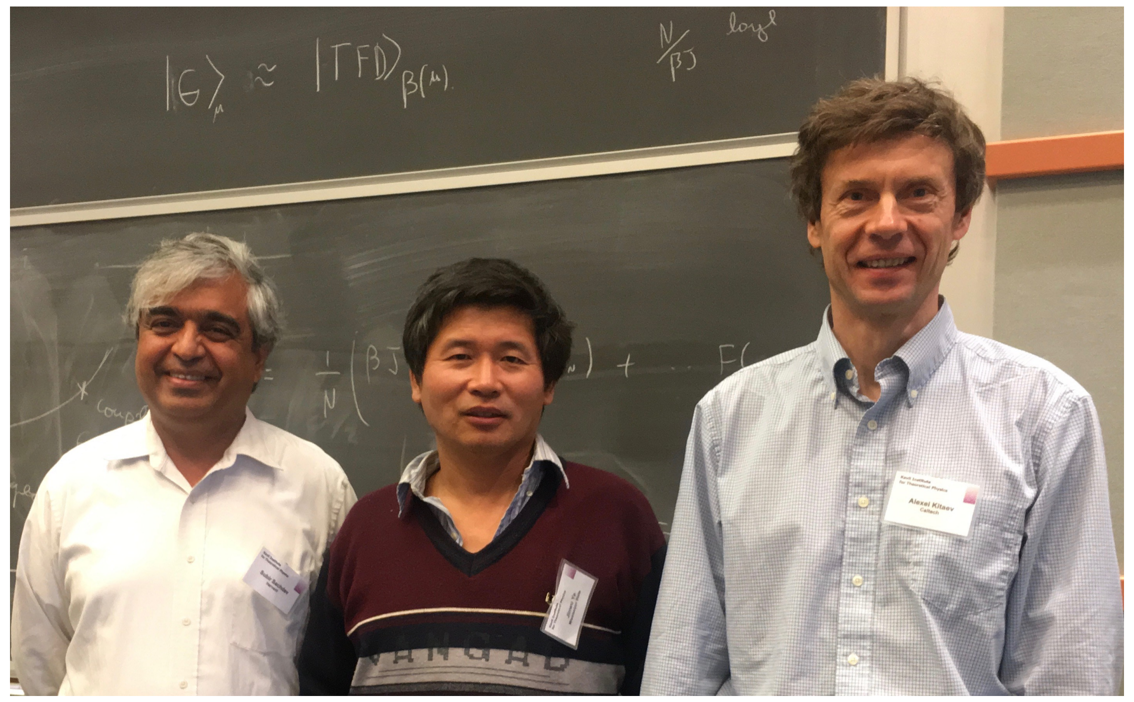


Place electrons randomly on some sites



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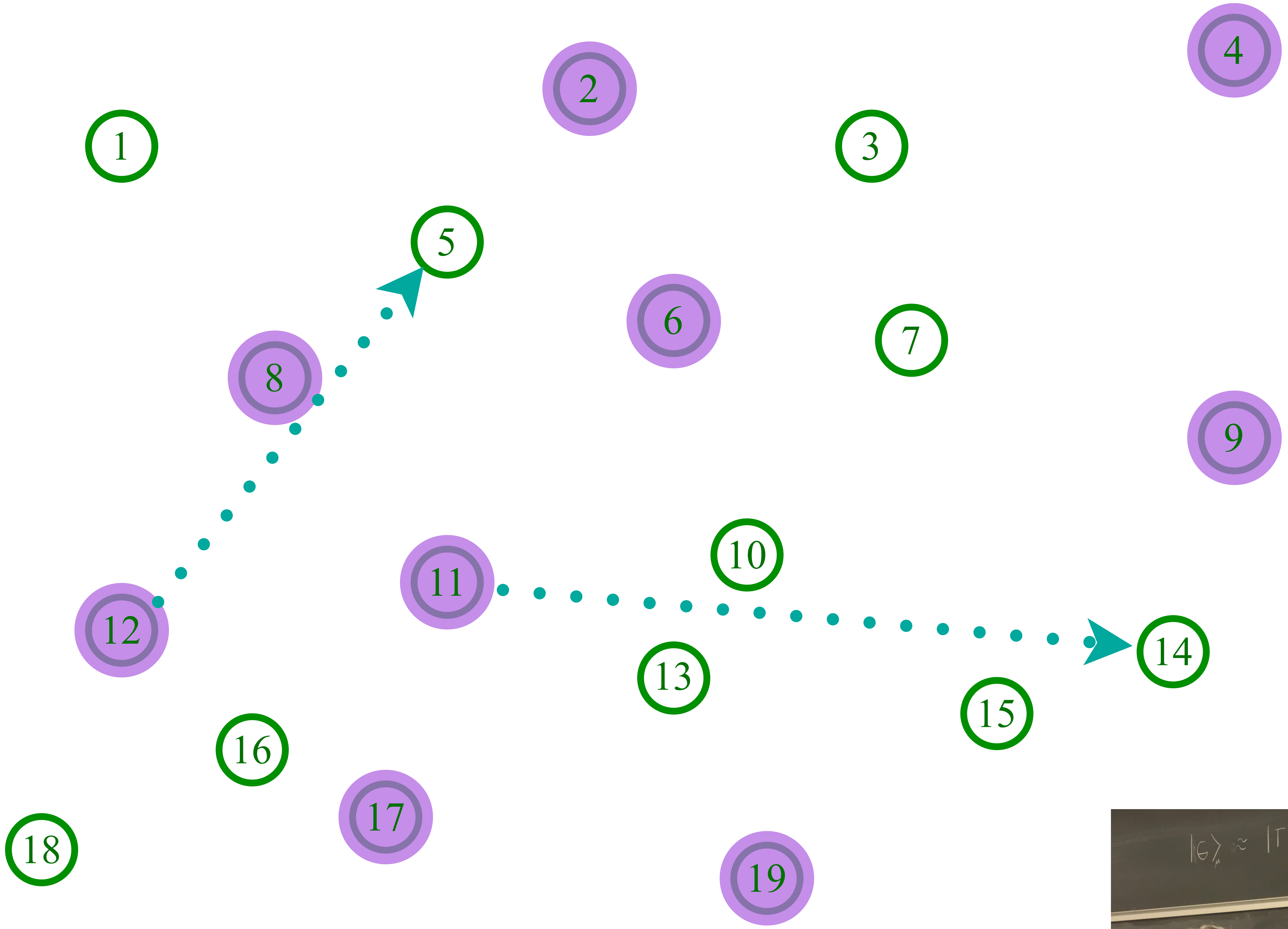
Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



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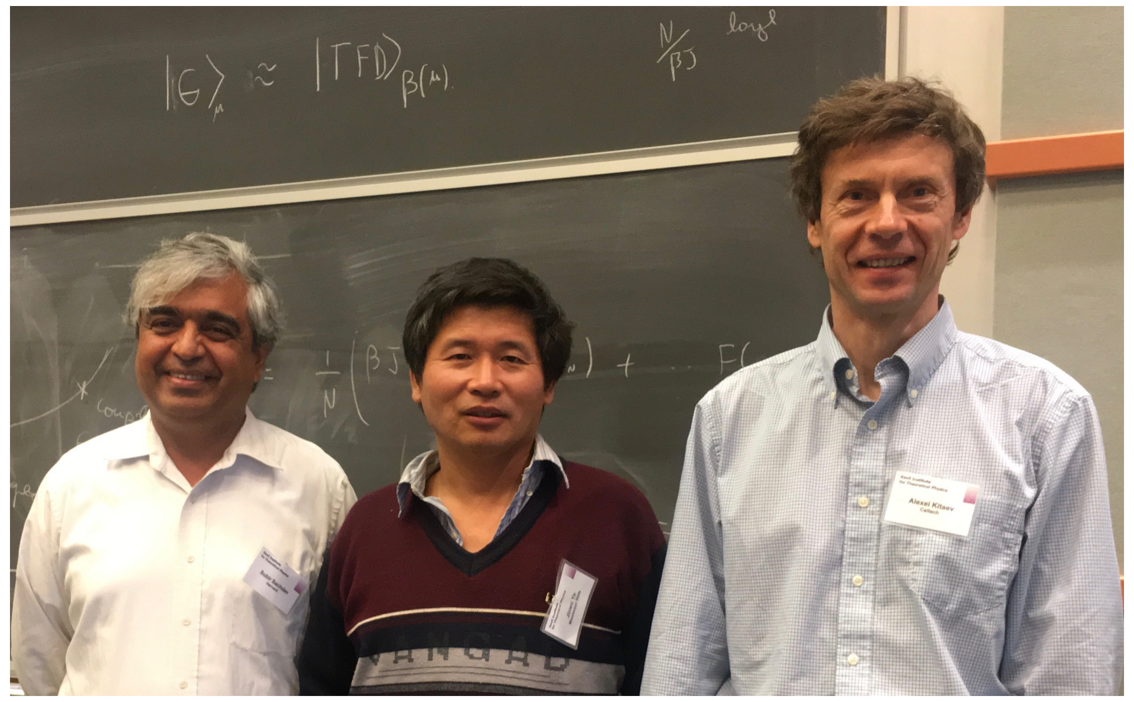

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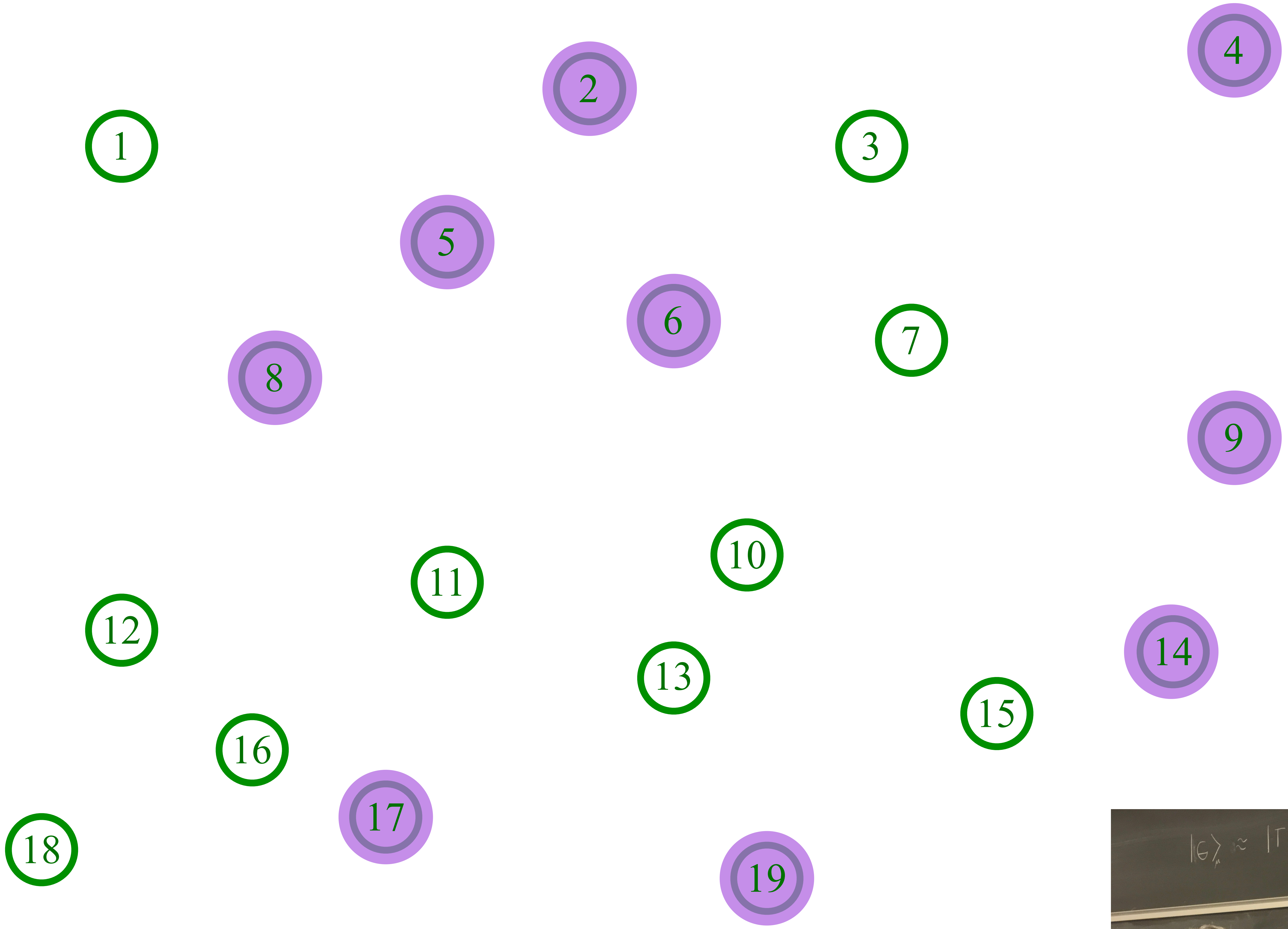
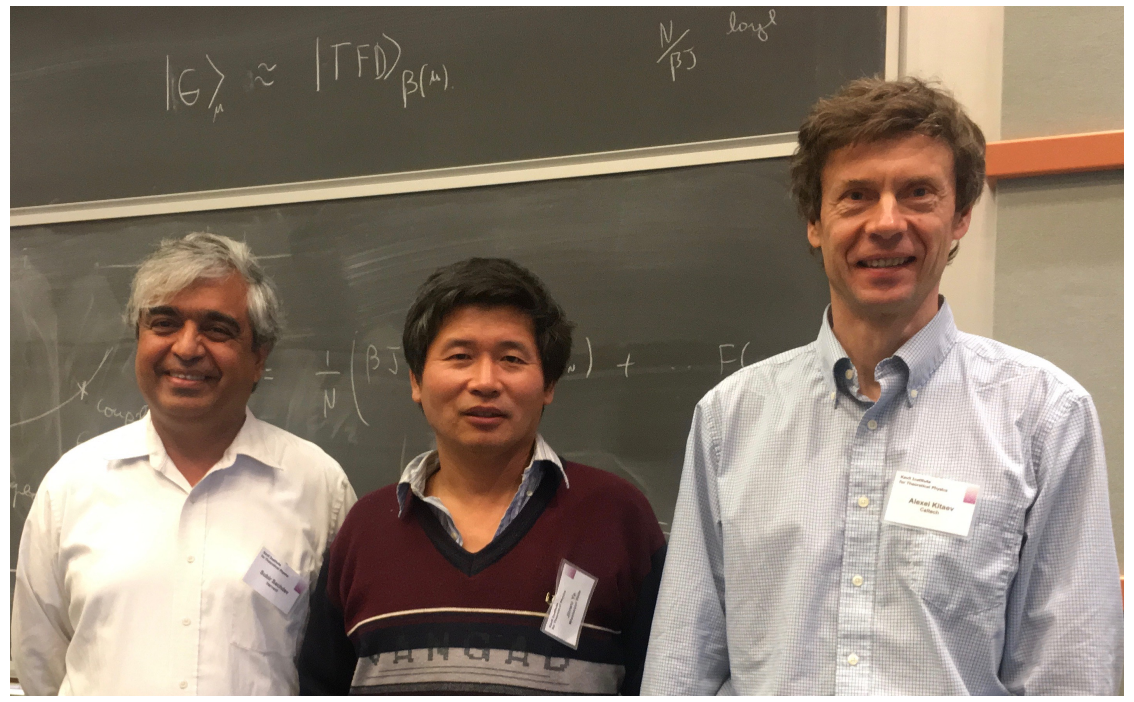


Entangle electrons pairwise randomly



The Sachdev-Ye-Kitaev (SYK) model

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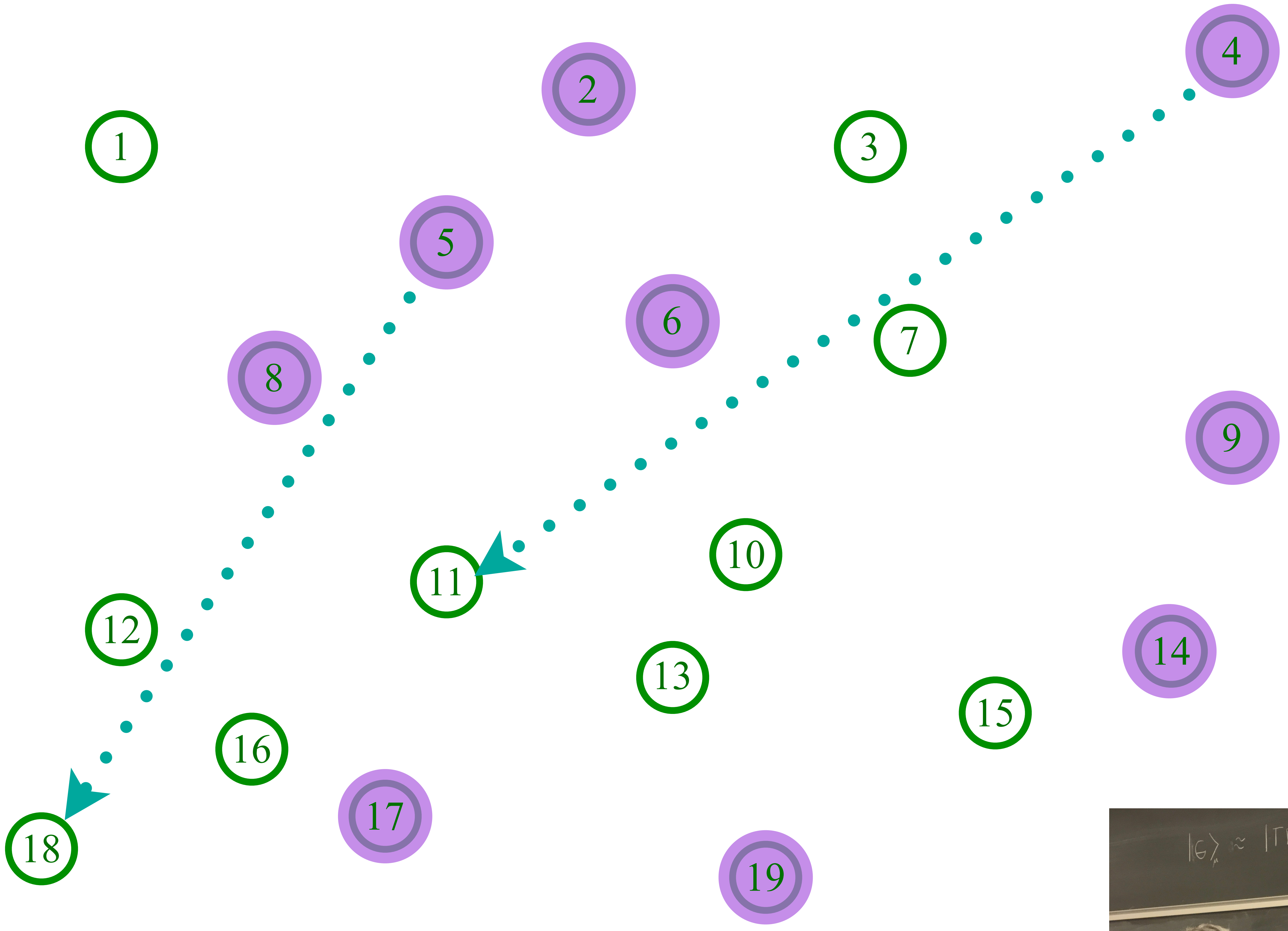
$$U_{4,5;11,18}$$



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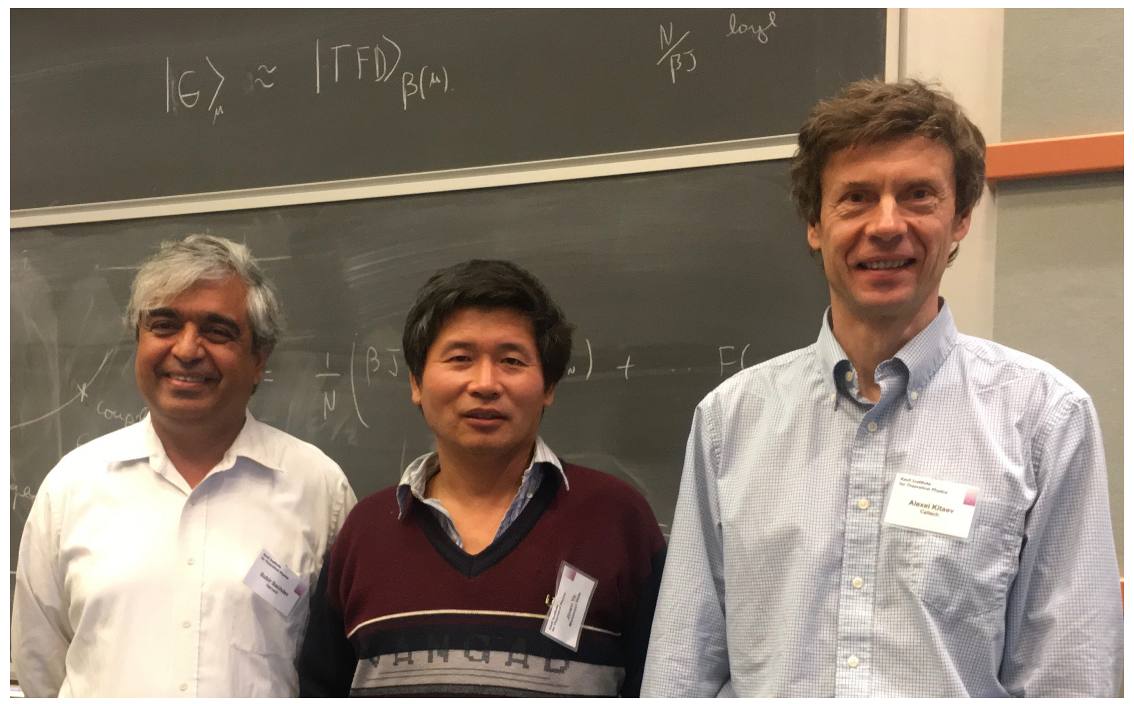

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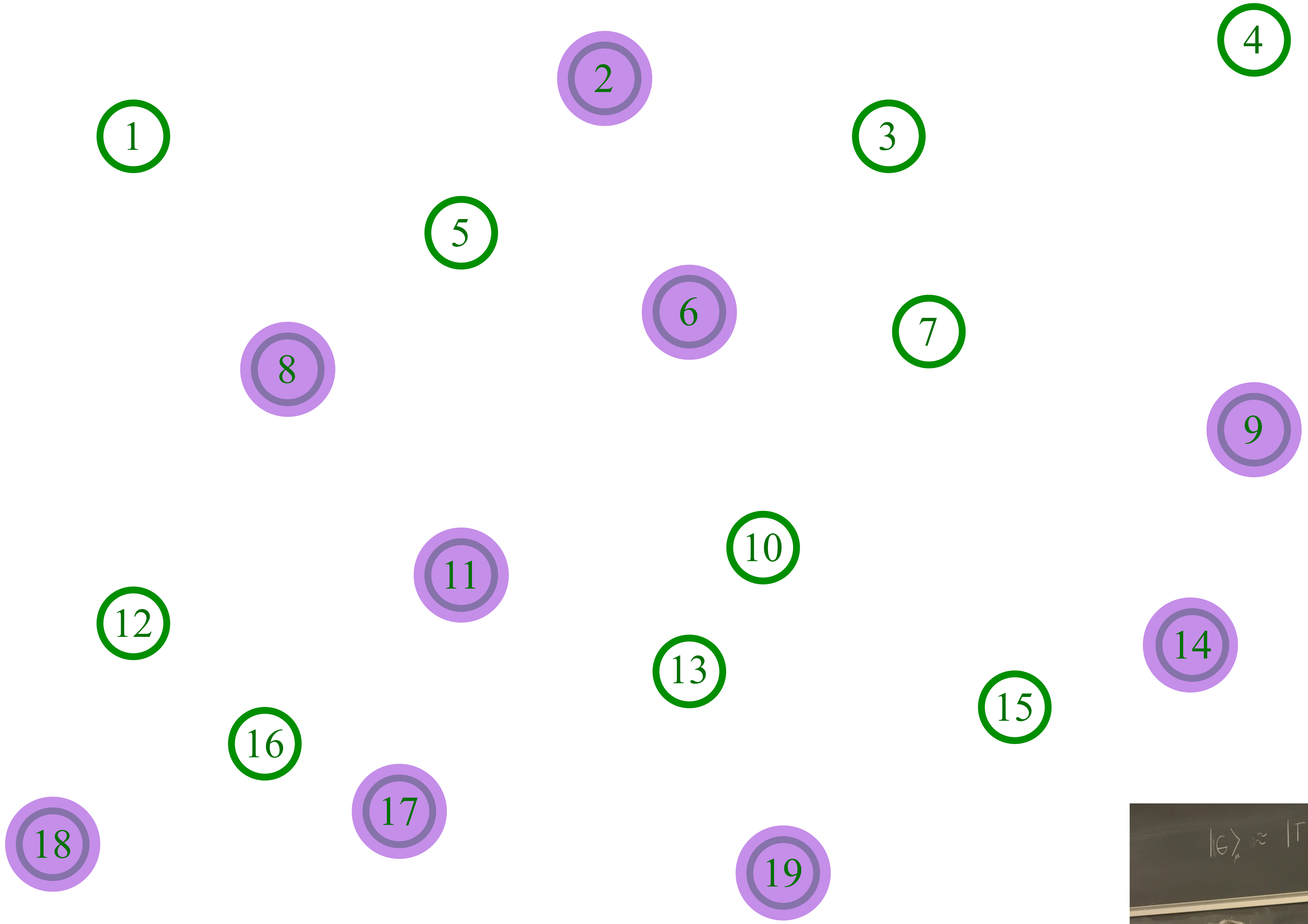

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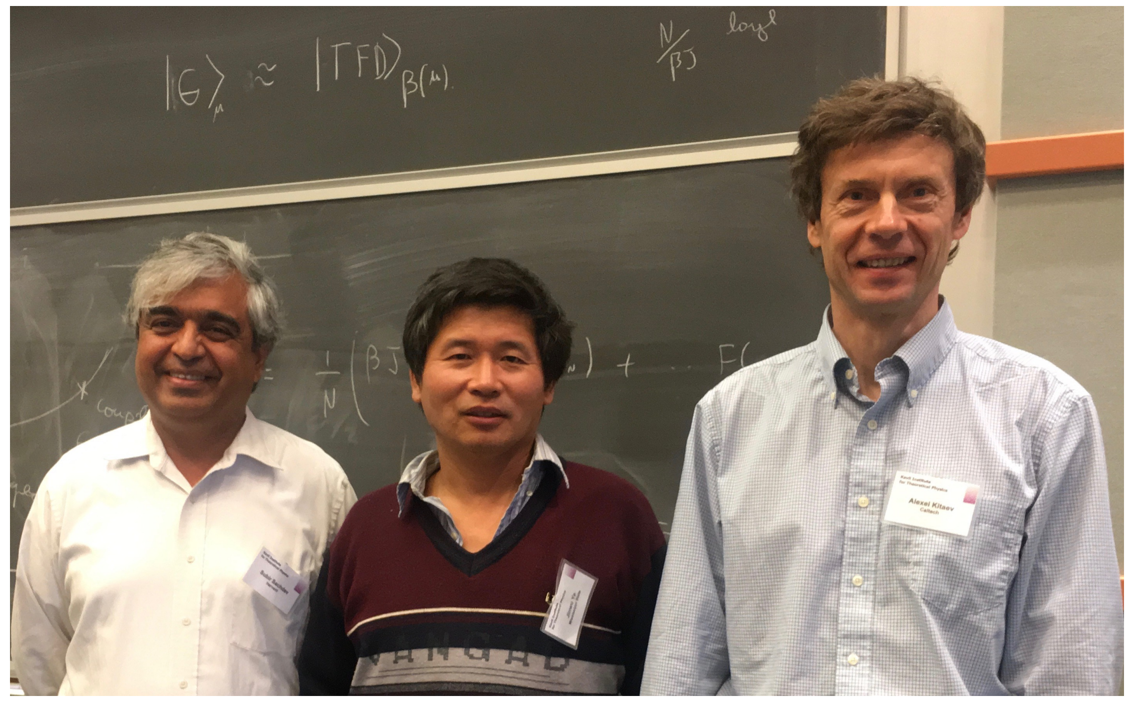

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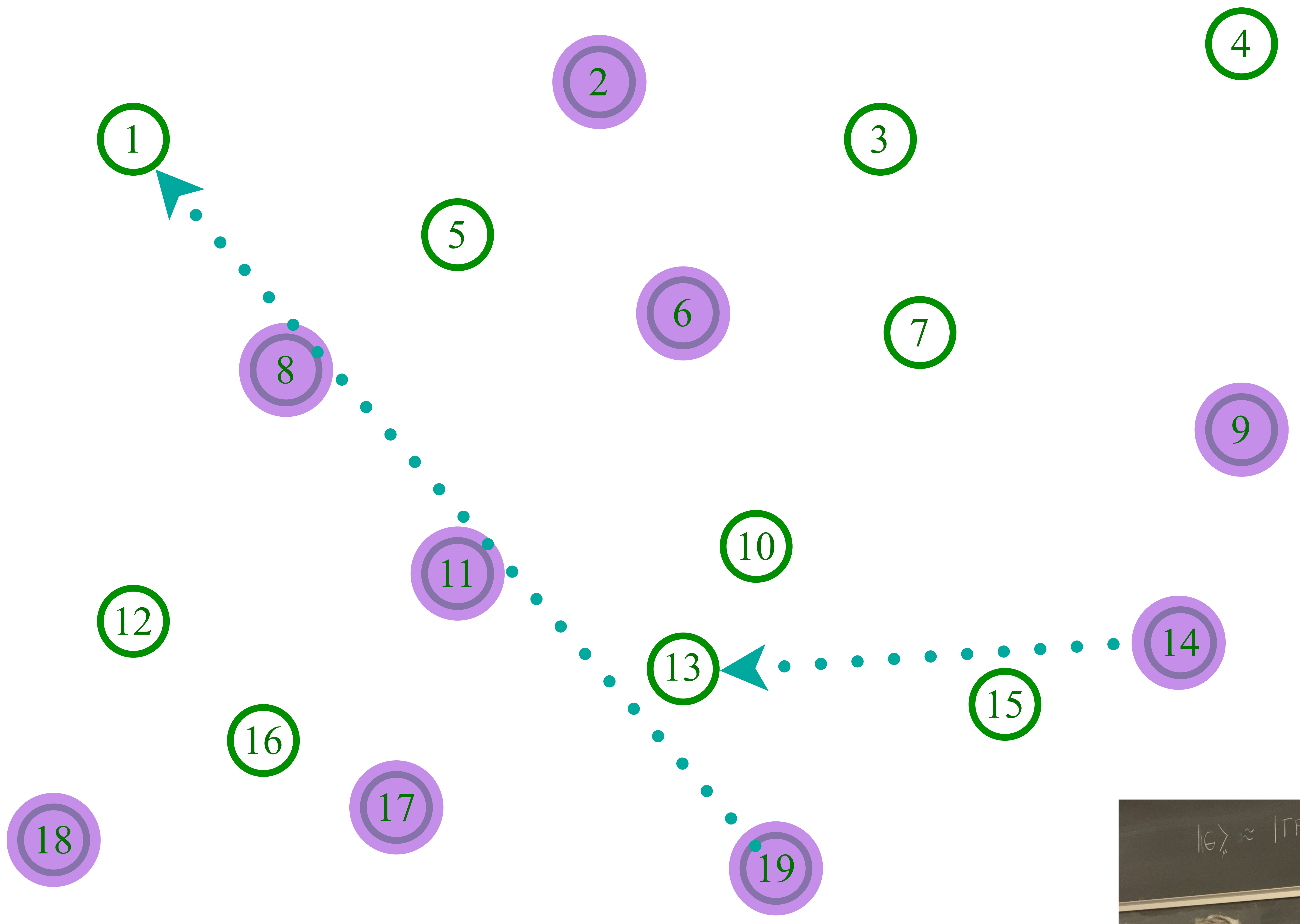

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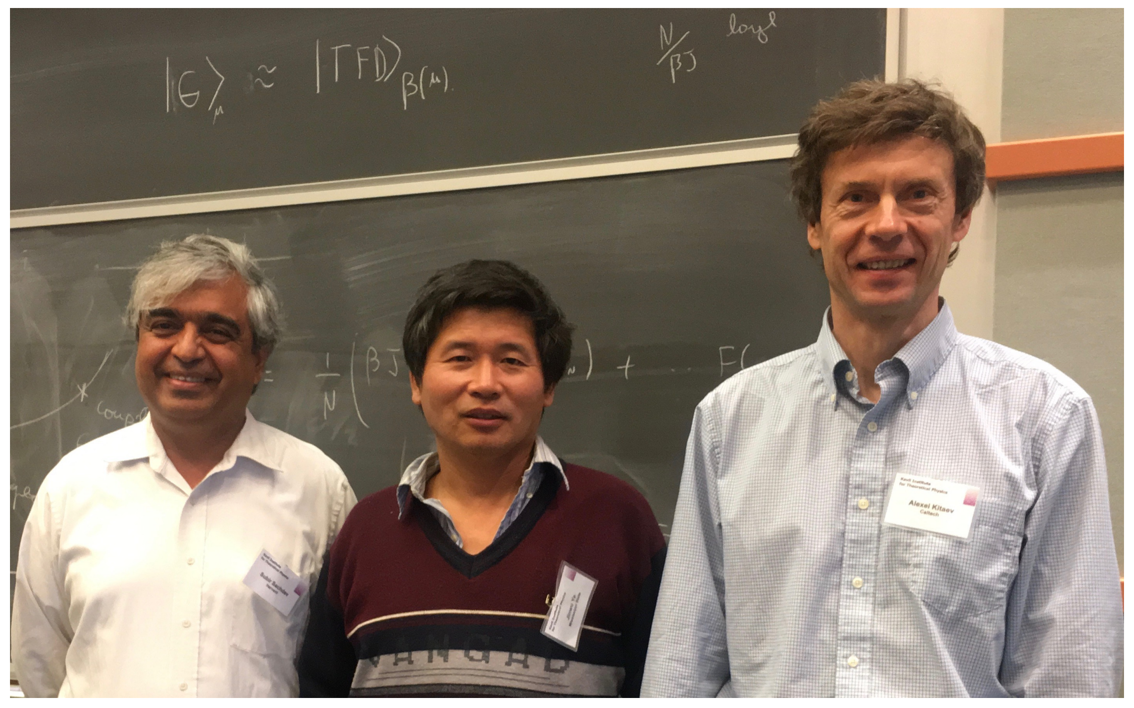

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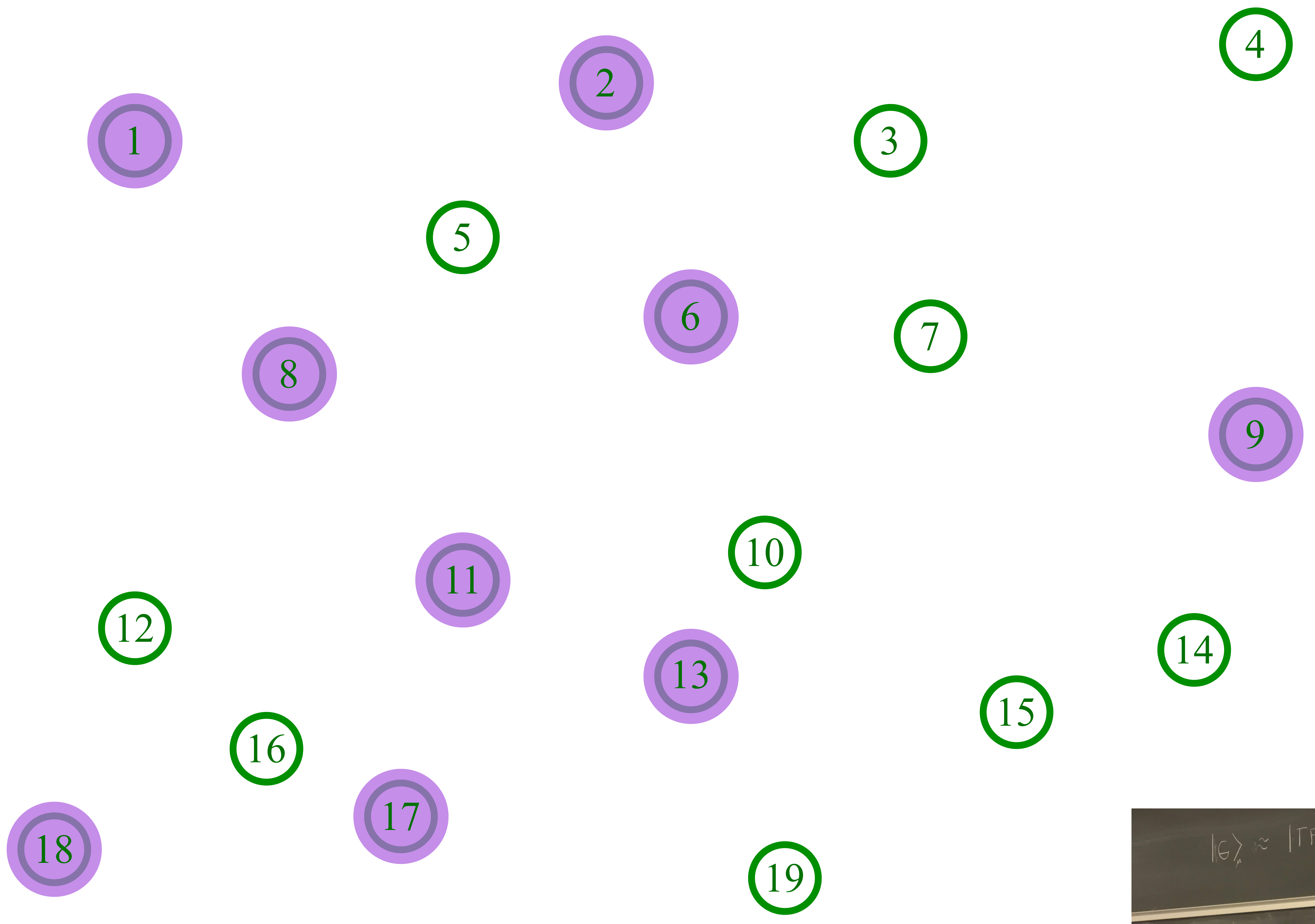

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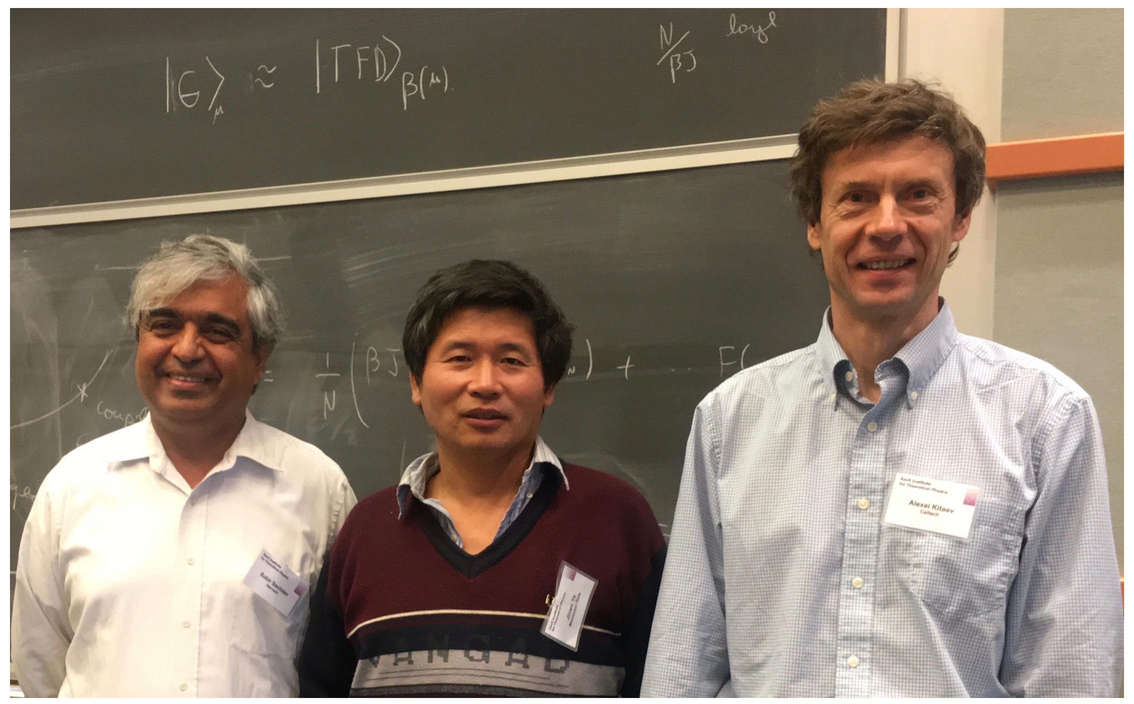
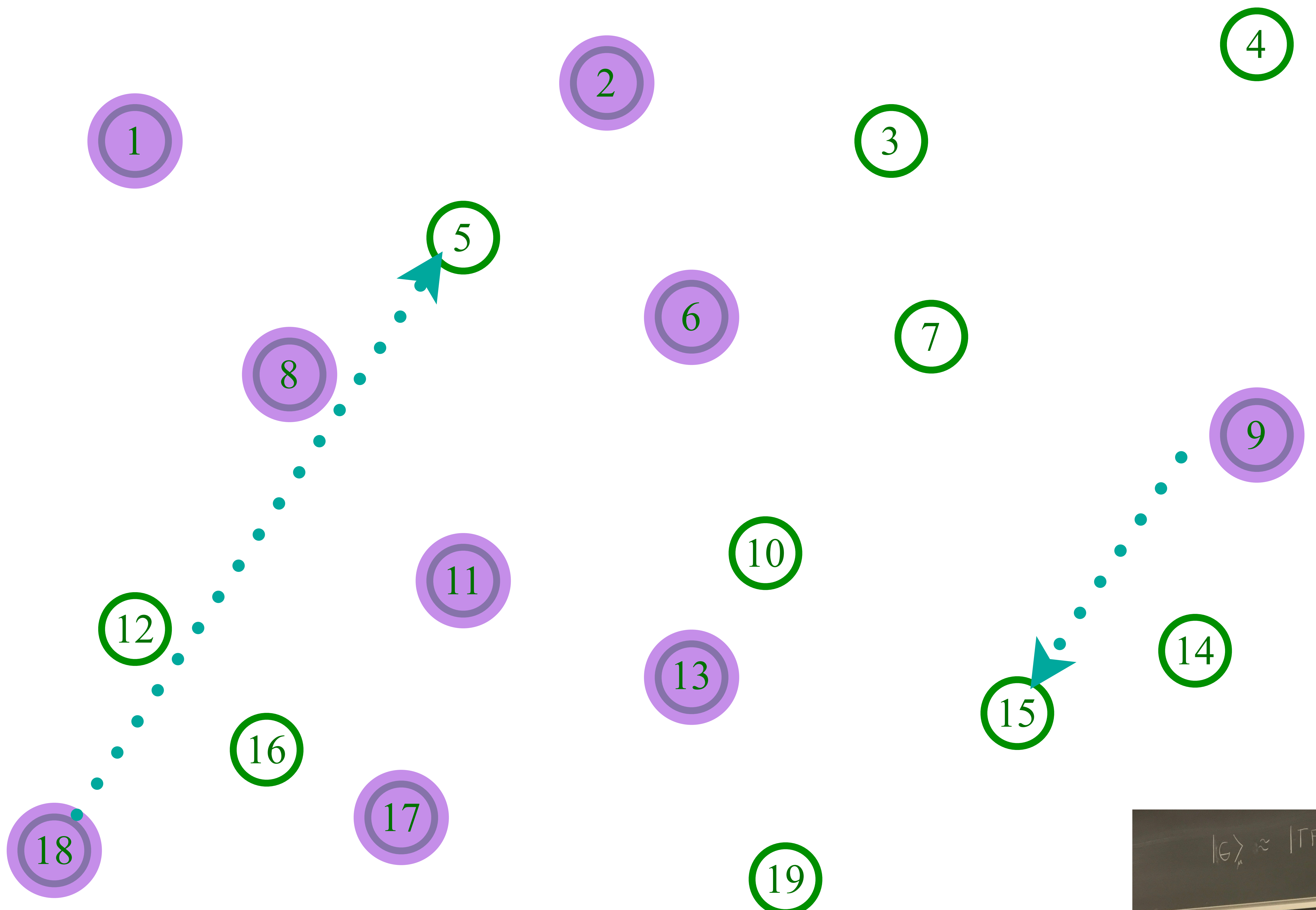


The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



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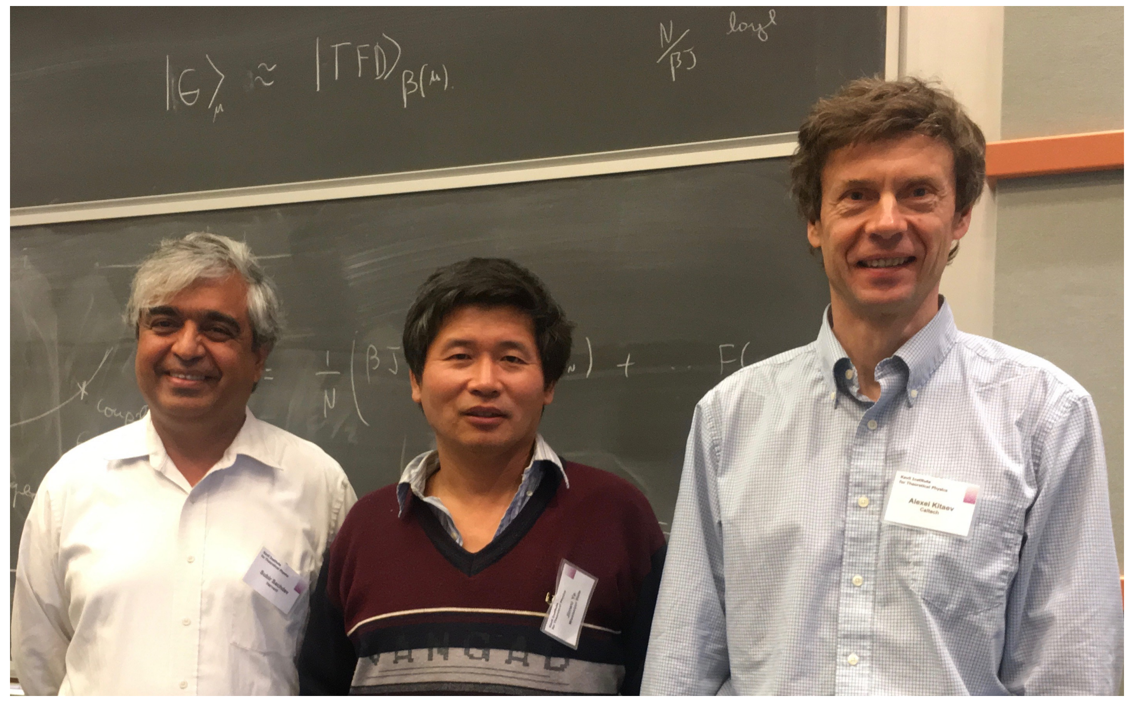

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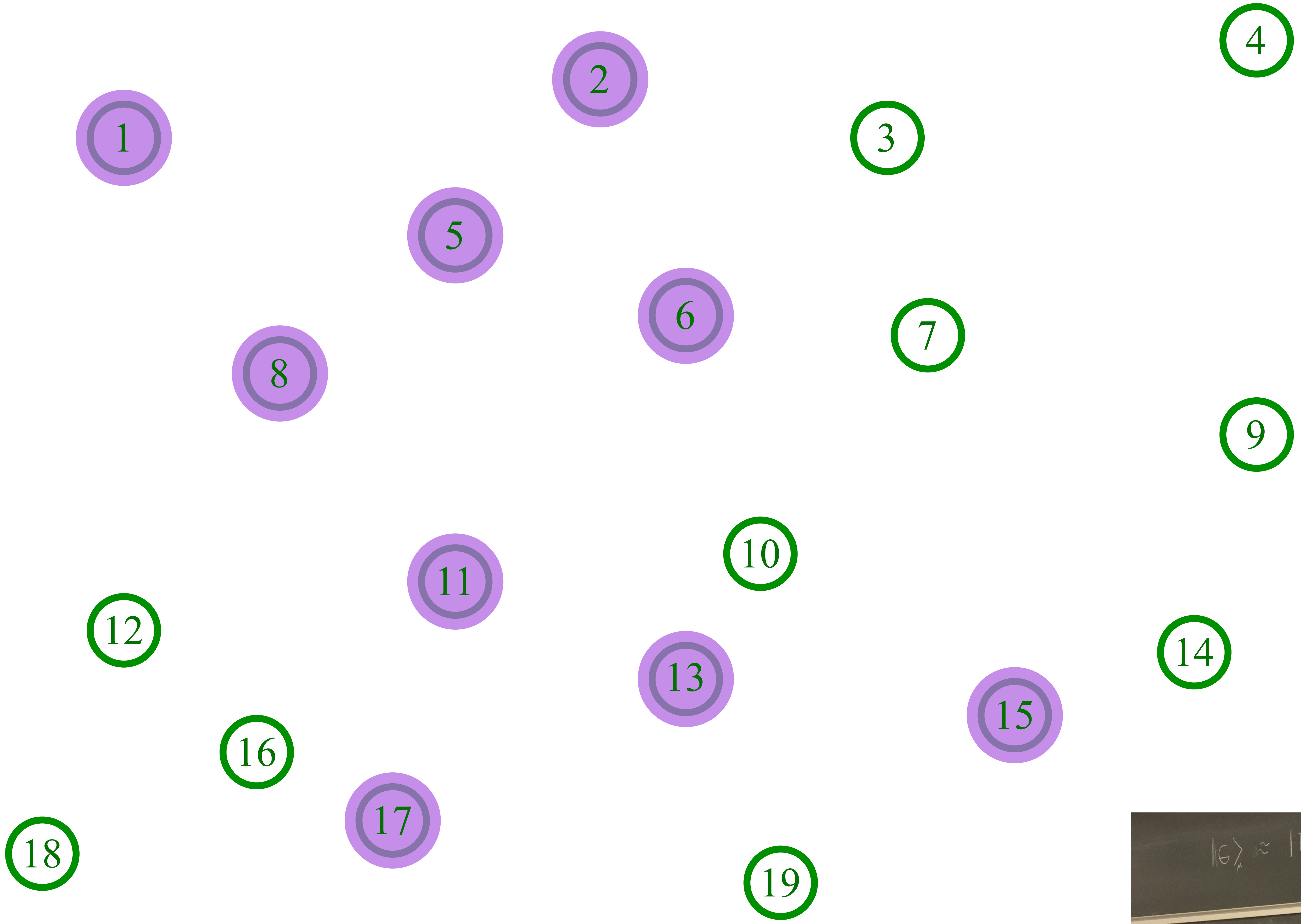

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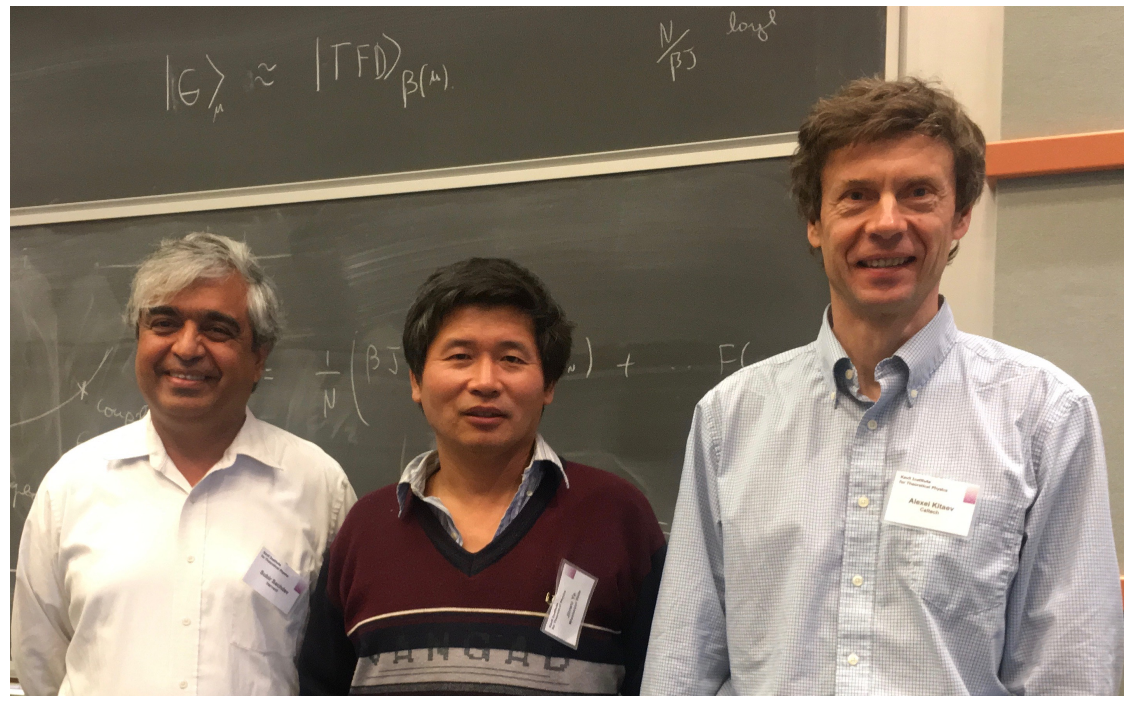
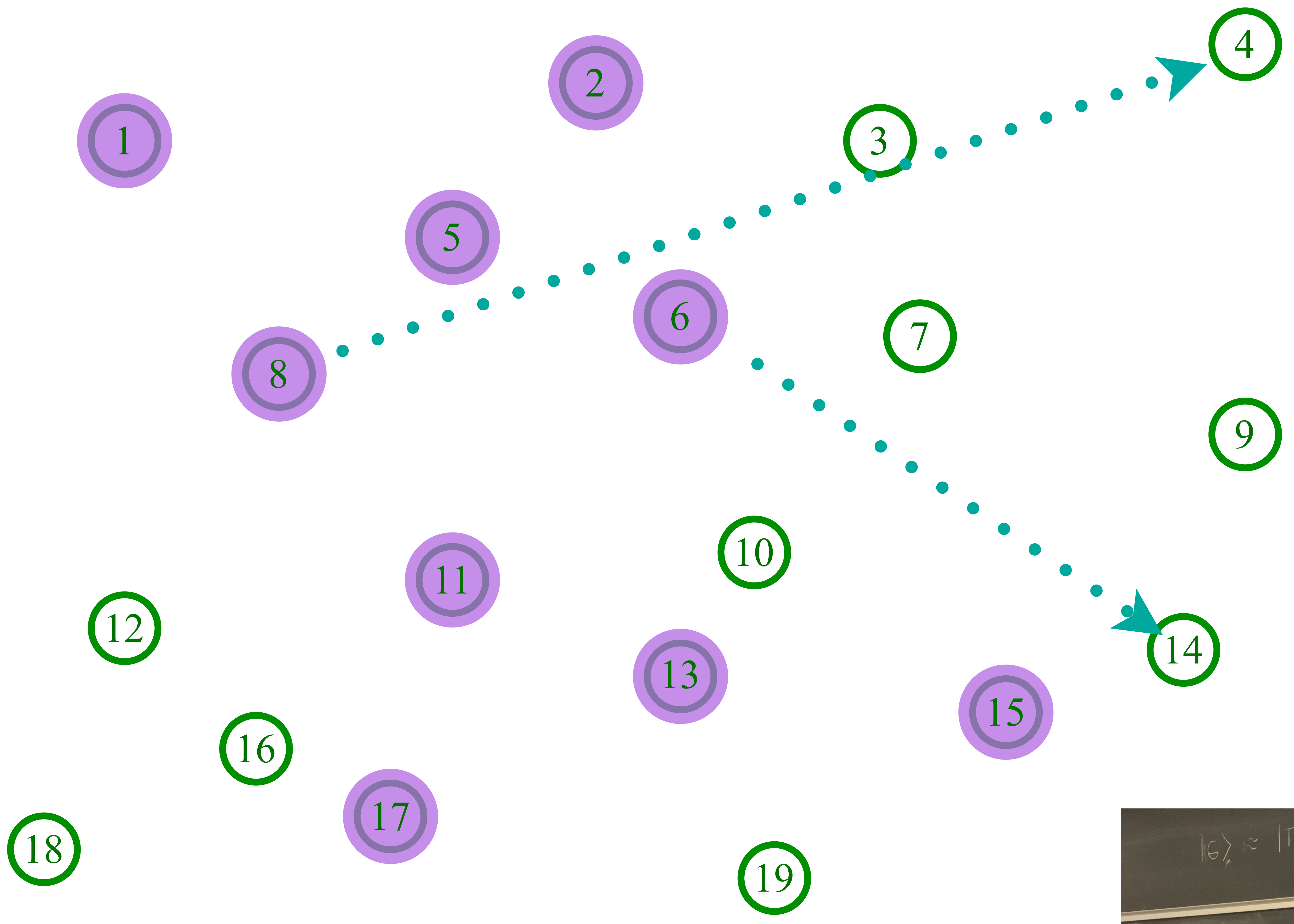


The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



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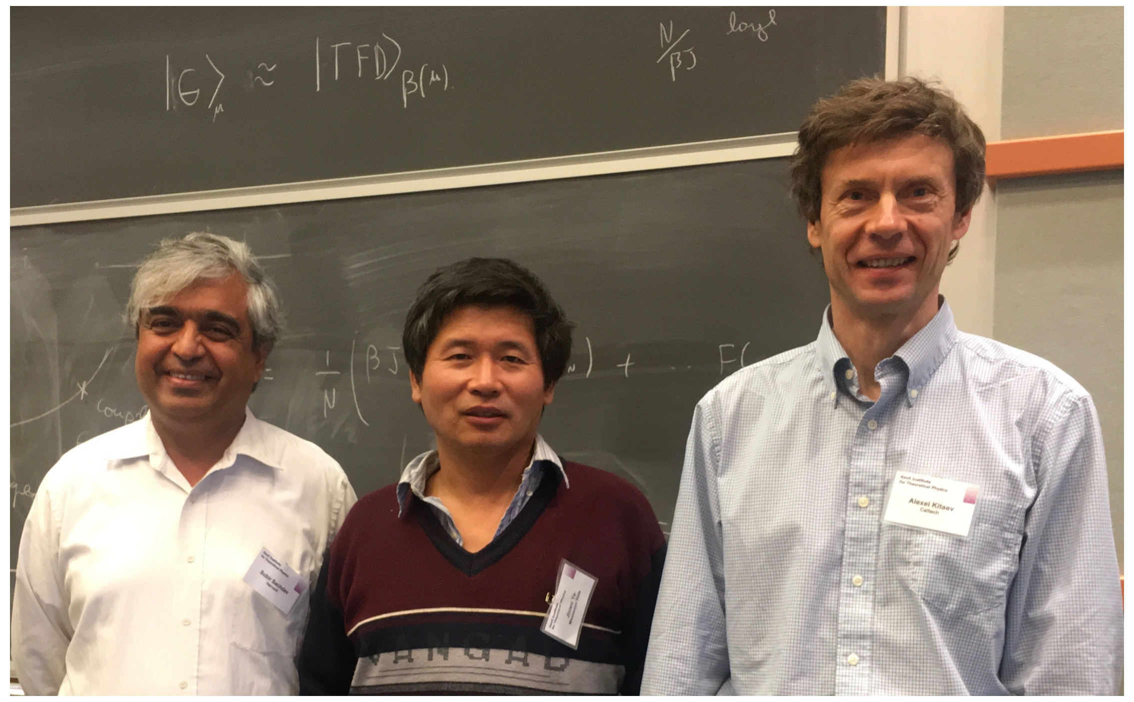

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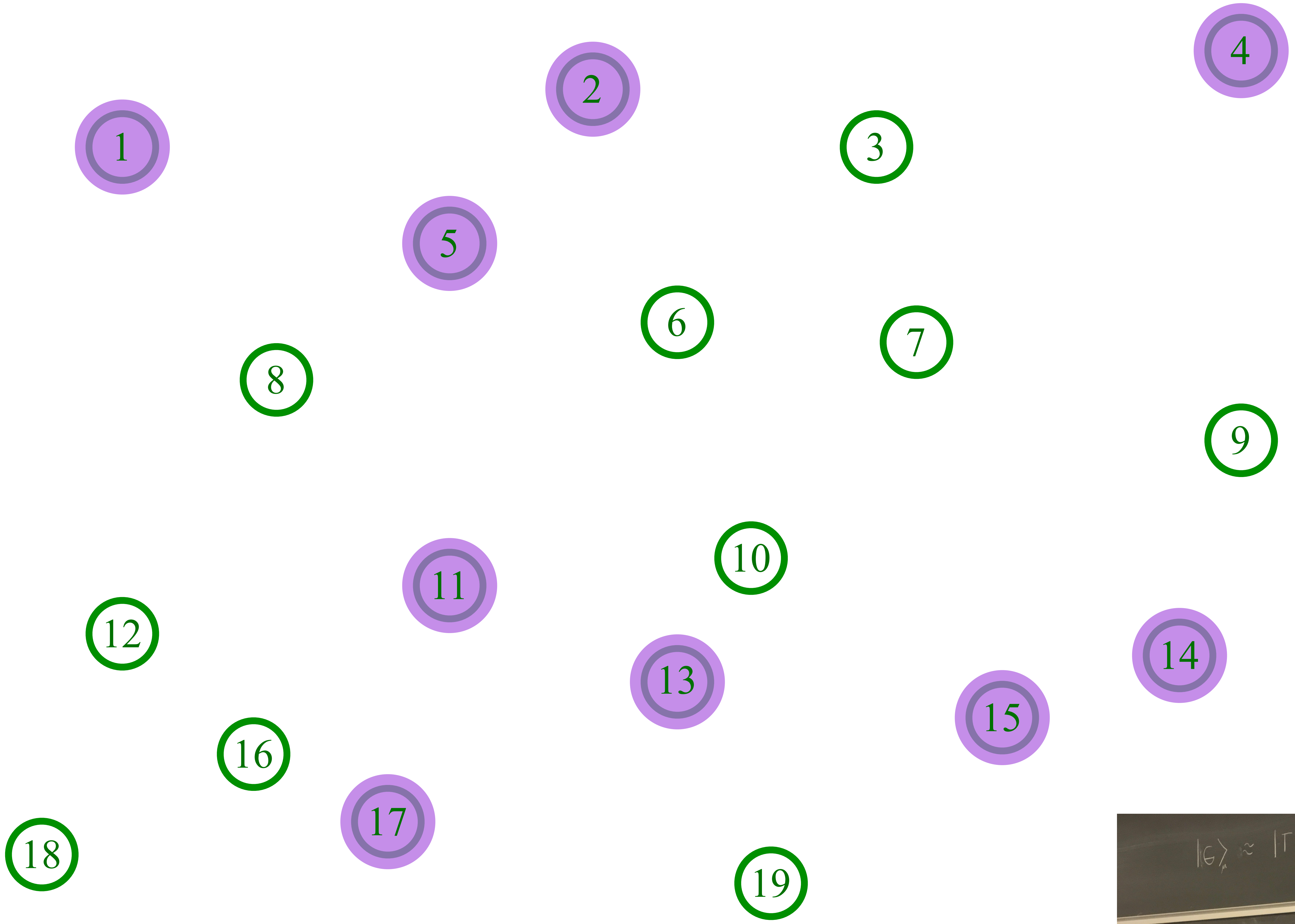

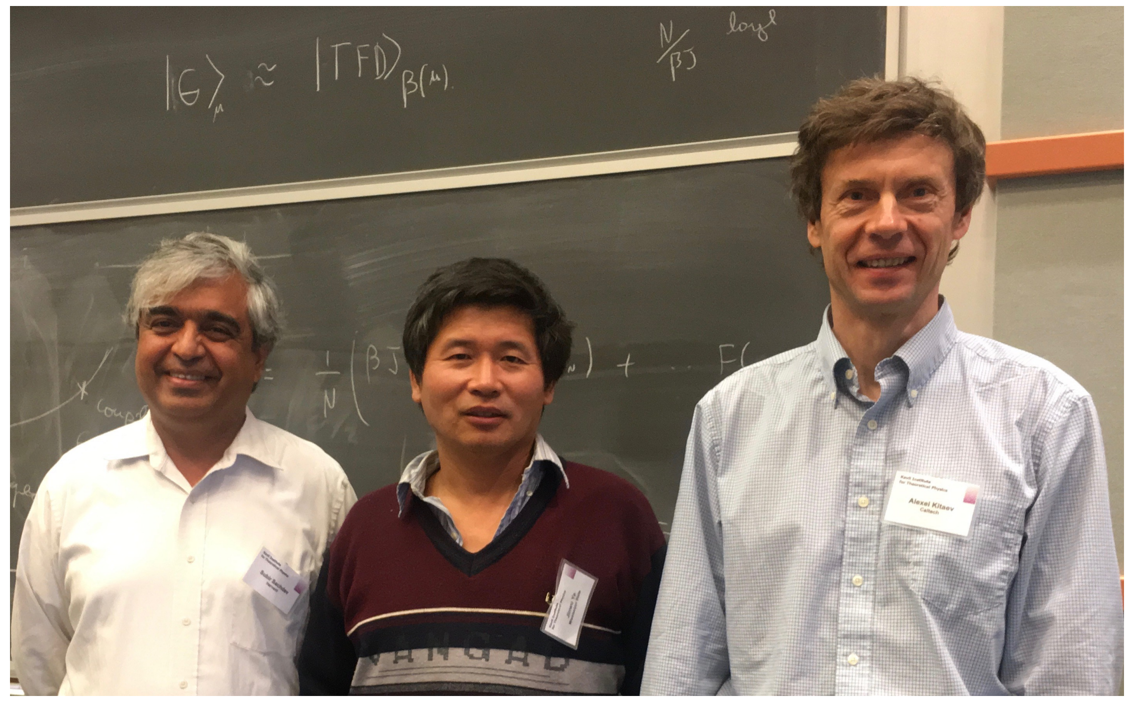

Entangle electrons pairwise randomly

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

A solvable model of multi-particle quantum entanglement.

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

Yields a quantum state whose excitations are not particle-like i.e. no bosons, fermions, anyons….

The Sachdev-Ye-Kitaev (SYK) model

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Current is carried by an "entangled quantum soup"

The Sachdev-Ye-Kitaev (SYK) model

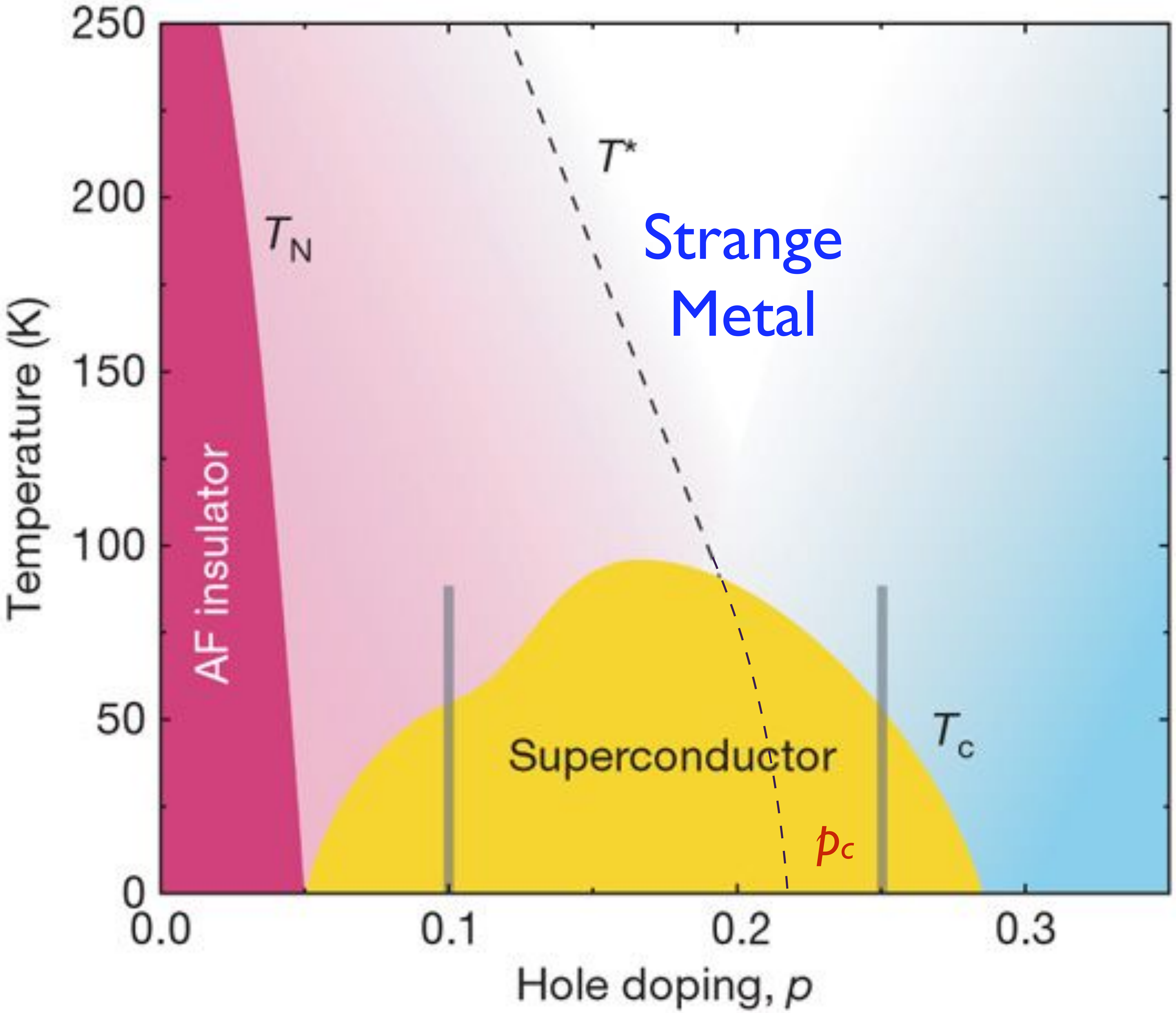
Sachdev, Ye (1993); Kitaev (2015)

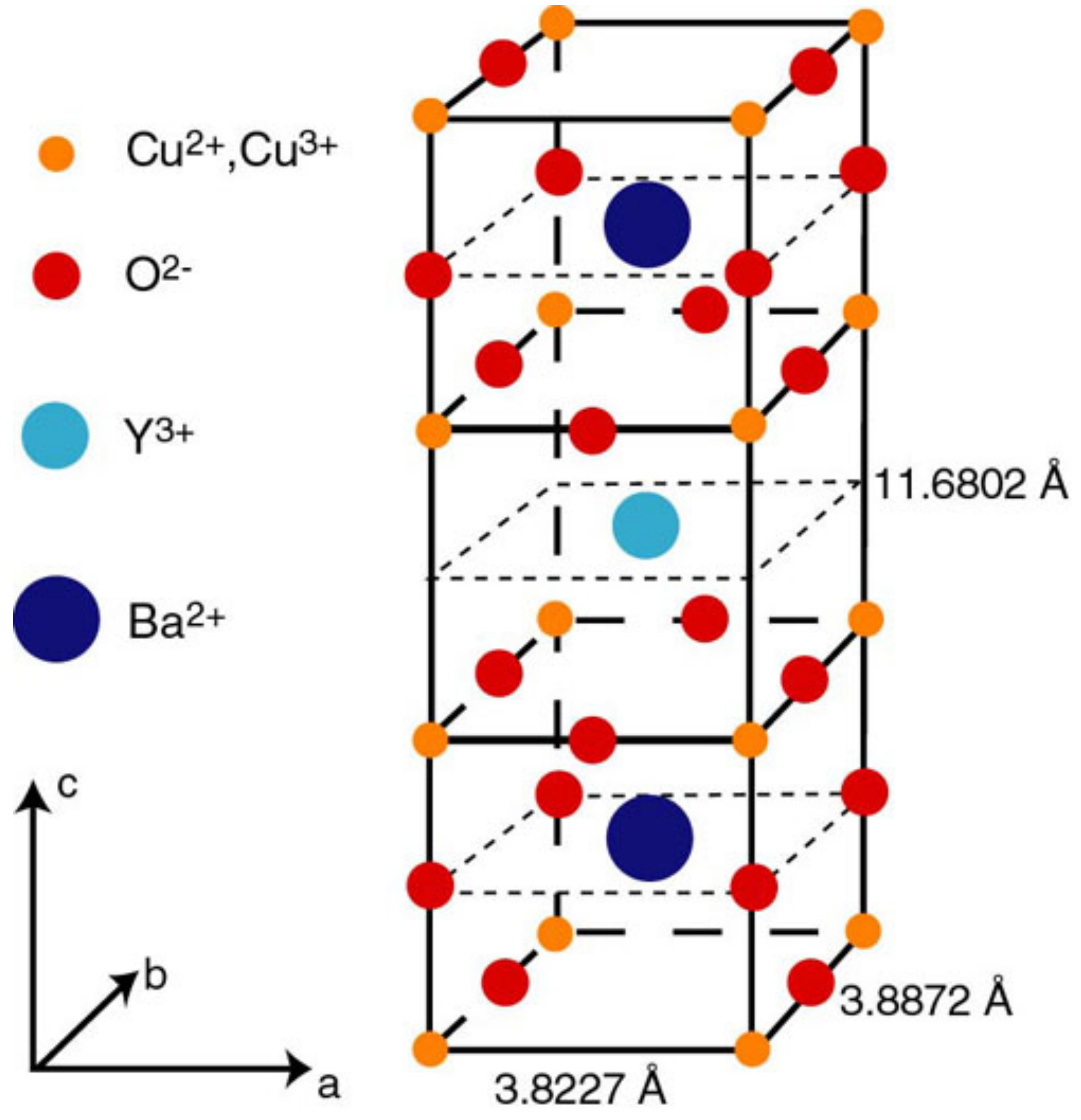
Yields a quantum state whose excitations are not particle-like i.e. no bosons, fermions, anyons….

A key consequence of the absence of the particle-like excitations is *Universal Planckian Dissipation*.
The relaxation time, τ, when perturbed at a frequency ω is given by

$$\tau = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

where $\hbar$ is Planck's constant, T is temperature, and the function F is independent of the strength of interaction between the particles.

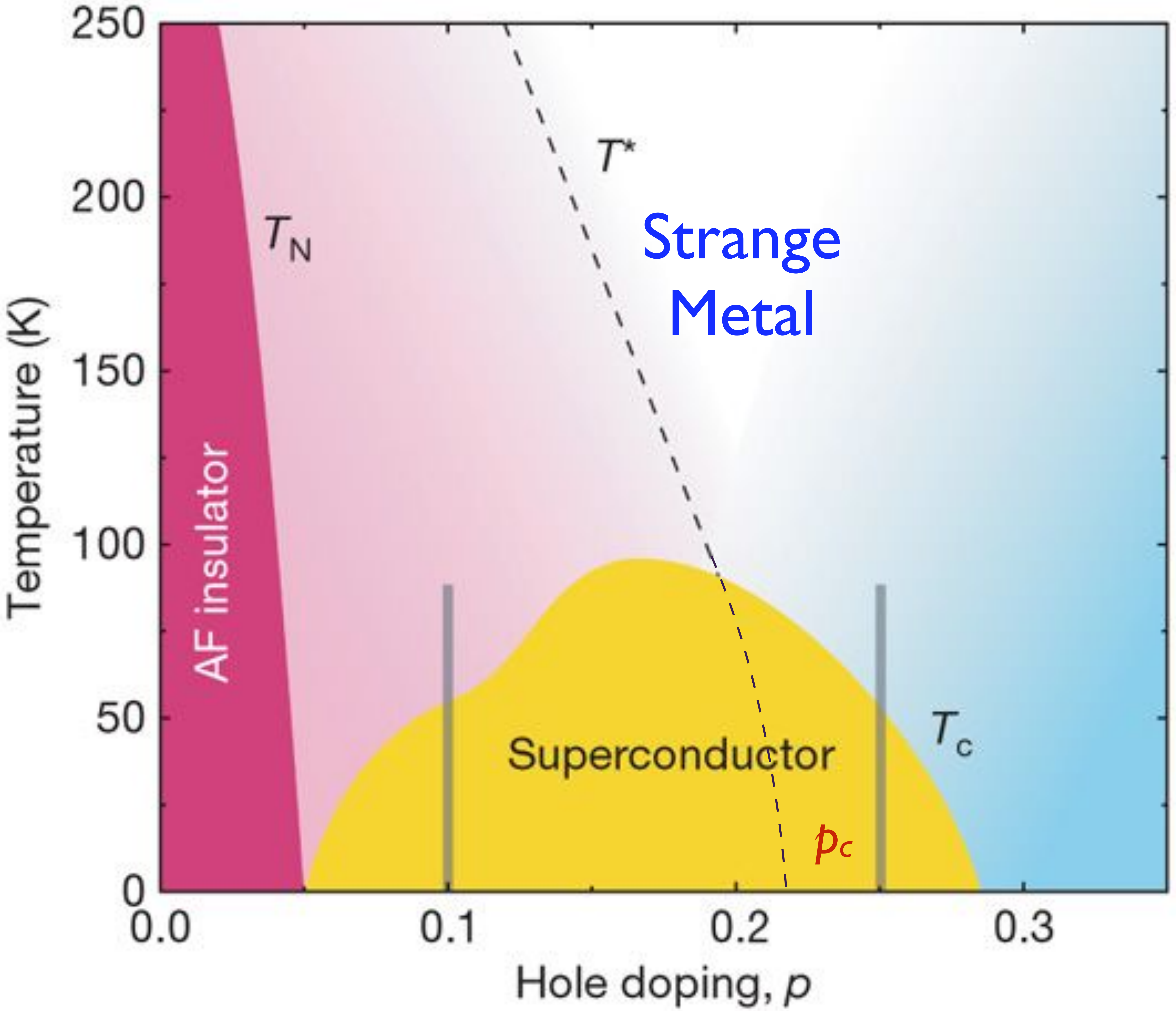




$YBa_2Cu_3O_{6+x}$

The "strange metal" has no particle-like/anyon excitations and is described by a SYK-type theory

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)

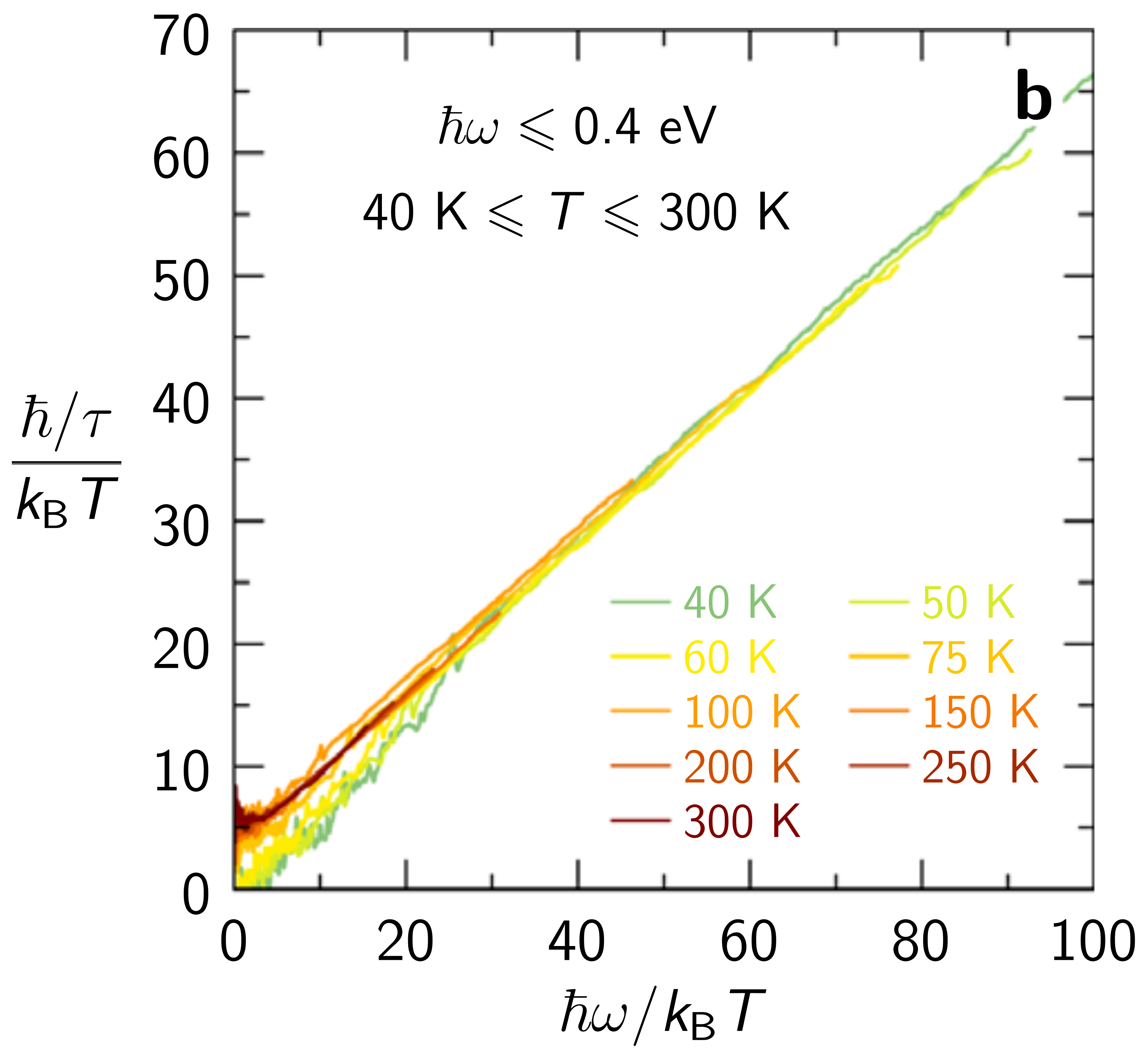


Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)

Planckian dynamics !

Electron scattering time τ from optical conductivity

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

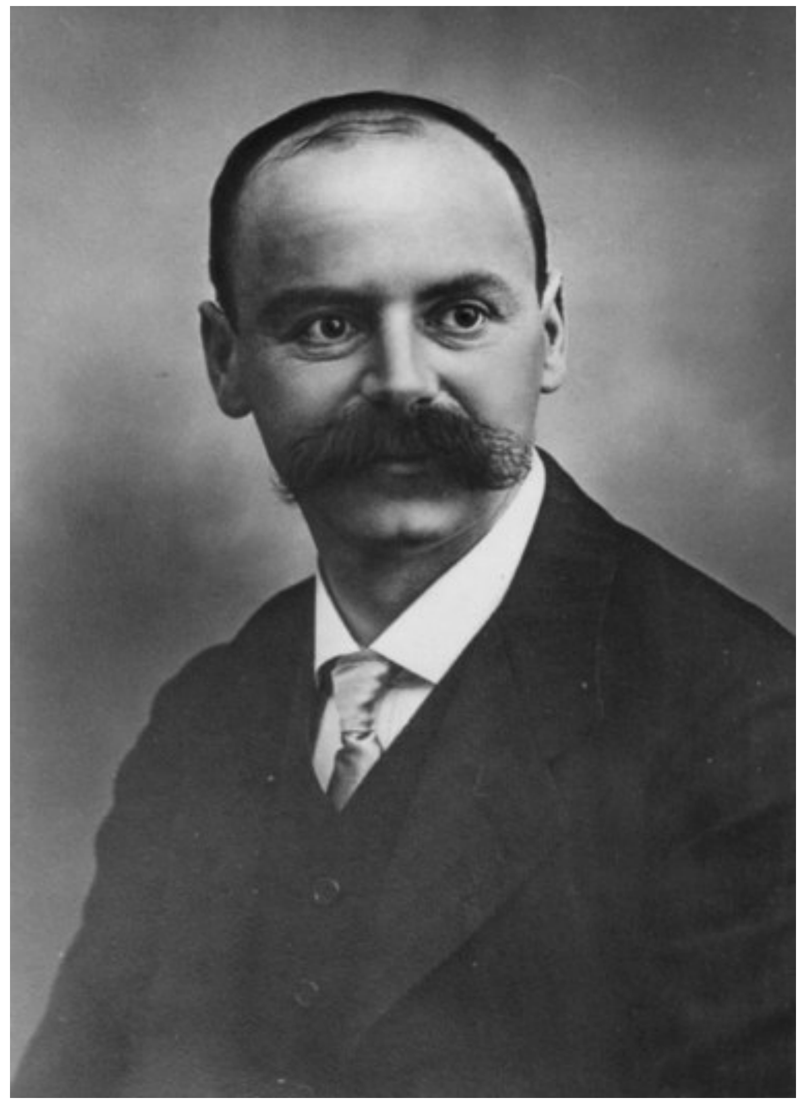


B. Michon, C. Berthod, C.W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges, Nature Comm. **14**, 3033 (2023)

Quantum entanglement, the SYK model, and black holes

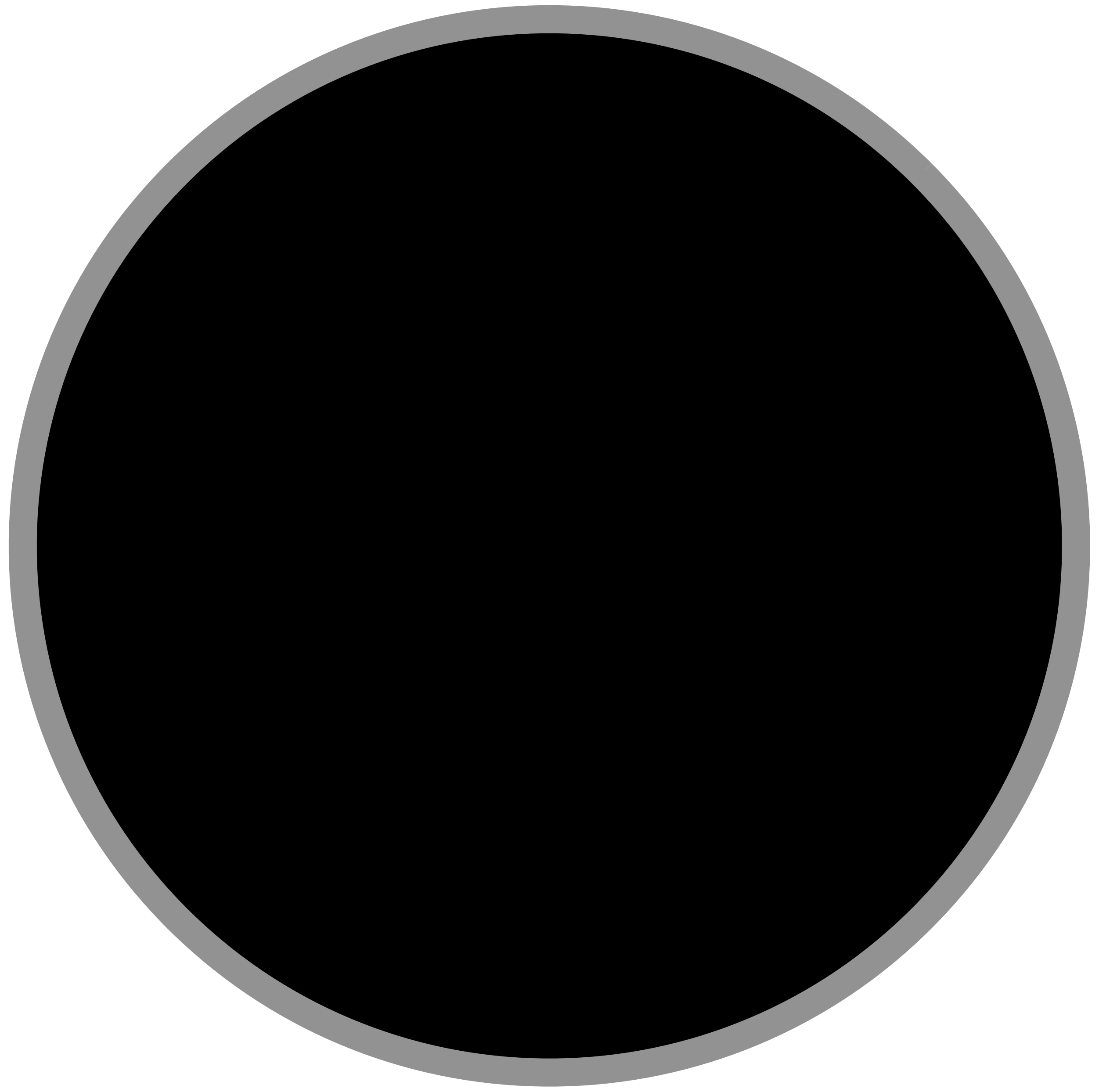
Black Holes

Objects so dense that light is gravitationally bound to them.



Karl Schwarzschild (1916)

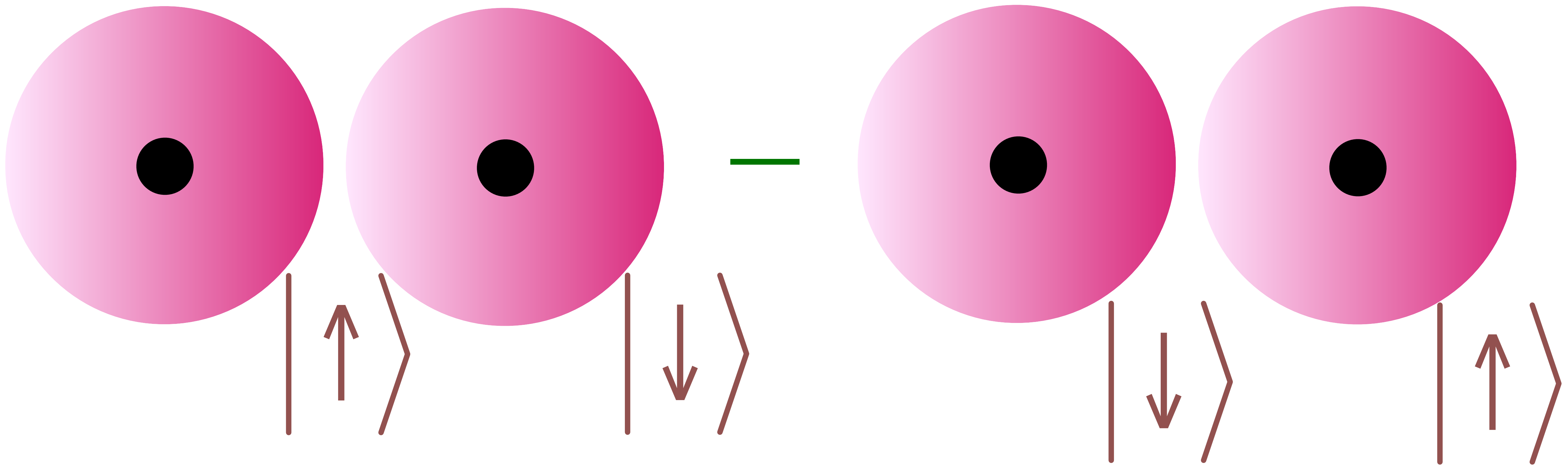
Horizon radius $R = \frac{2GM}{c^2}$



G Newton's constant, c velocity of light, M mass of black hole

For $M =$ earth's mass, $R \approx 9\,mm$!

Quantum Entanglement across a black hole horizon



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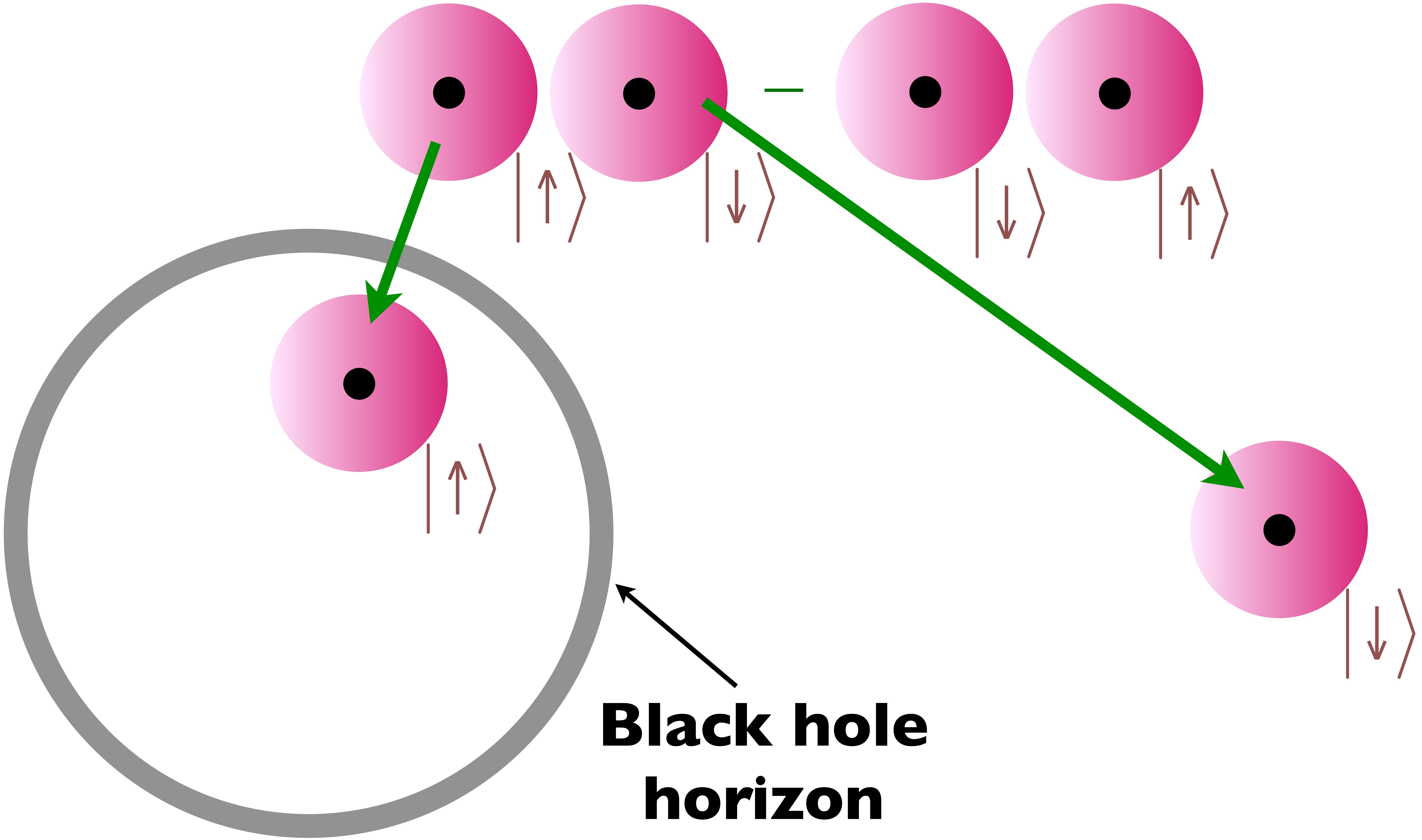

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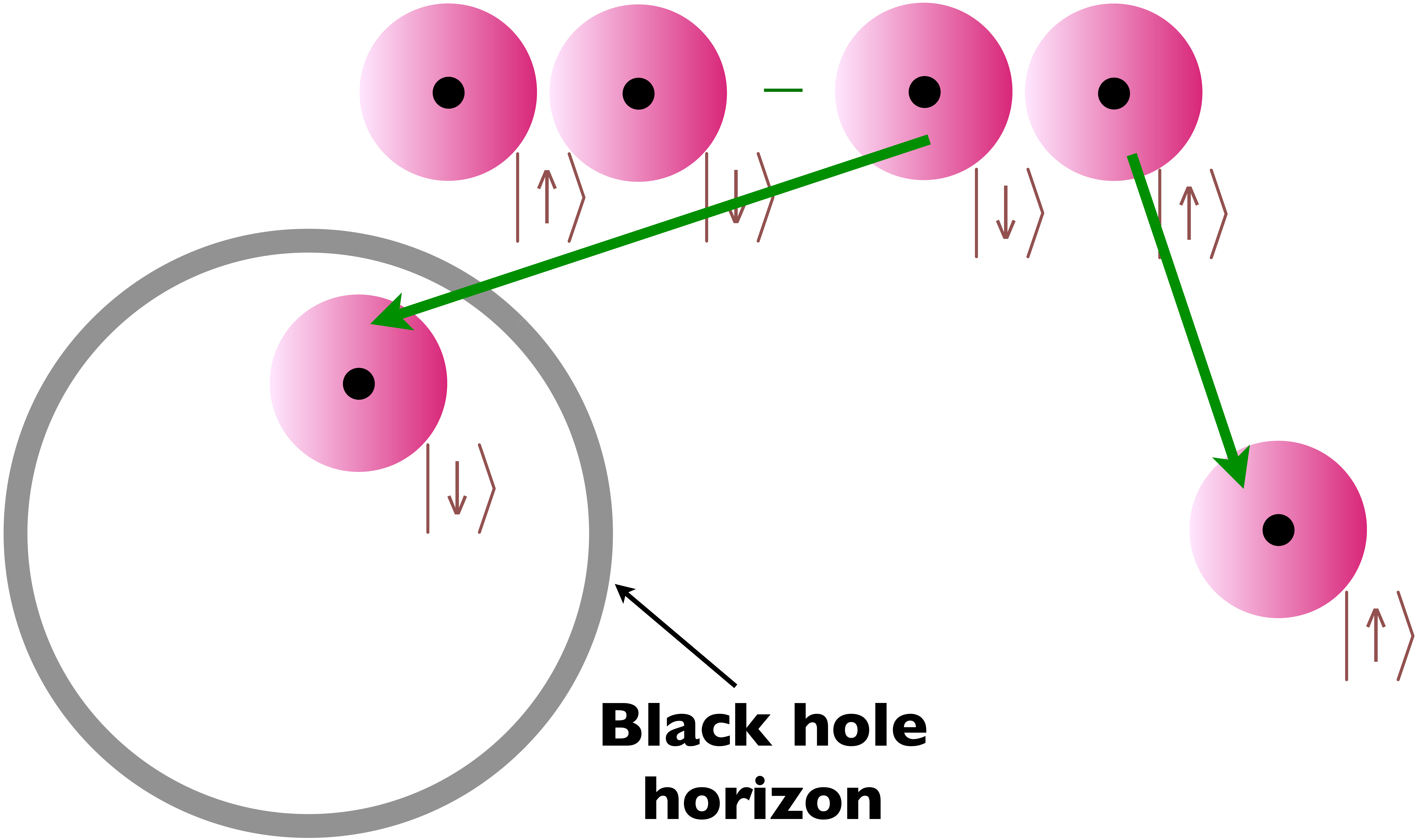
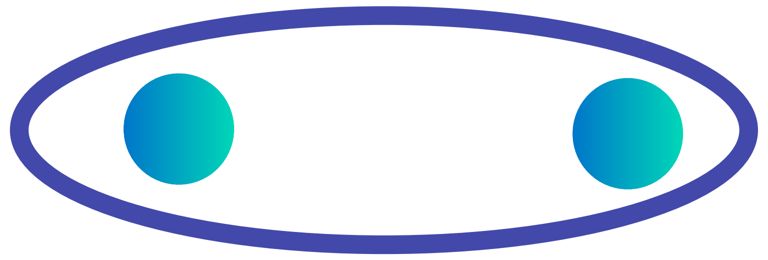
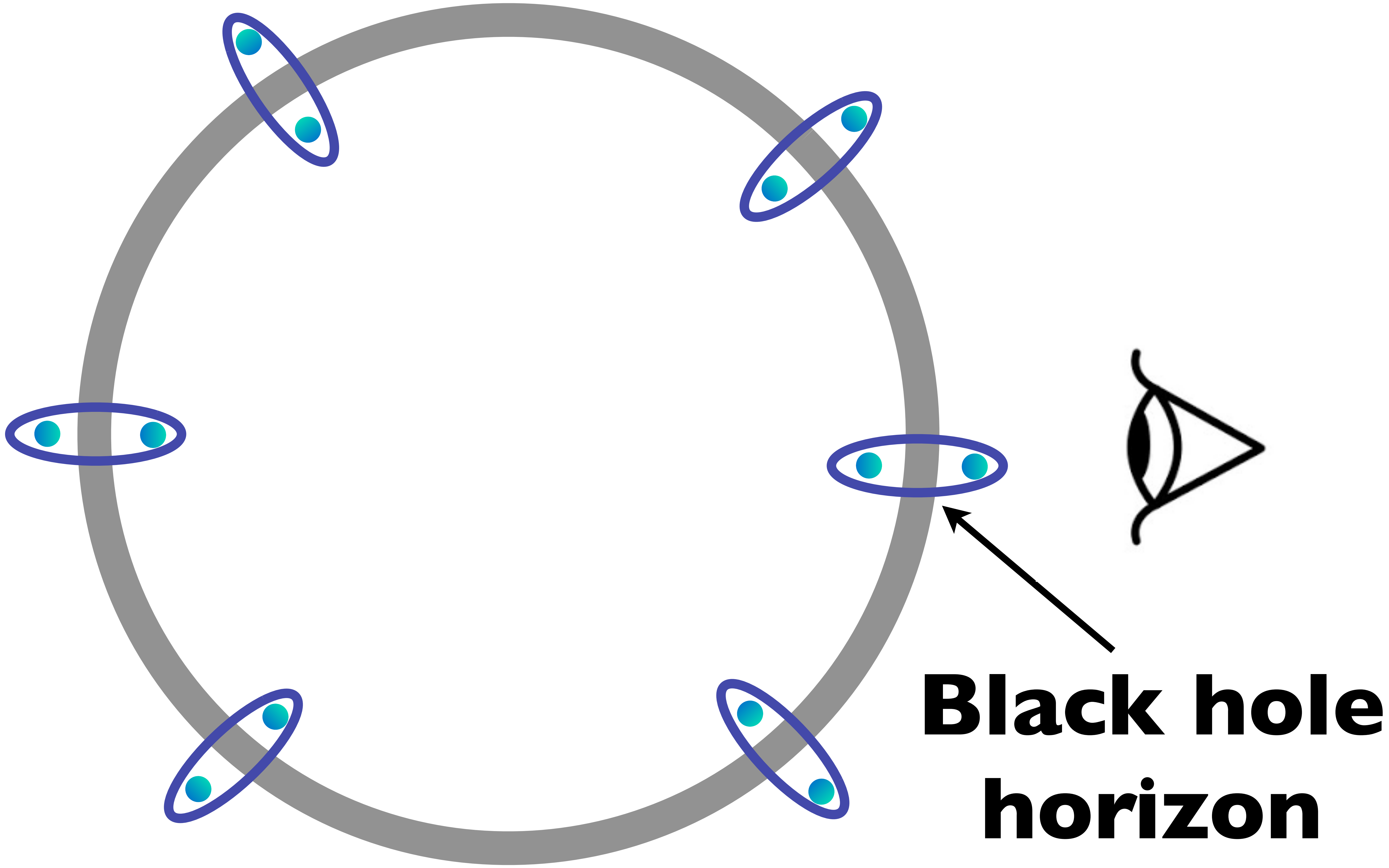


Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface



$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



By computations *outside* the black hole, Hawking obtained the black hole entropy

$$S = \frac{Ac^3}{4G\hbar}$$

where A is area of the black hole horizon.

All other systems have entropy proportional to their volume.

Quantum Entanglement across a black hole horizon

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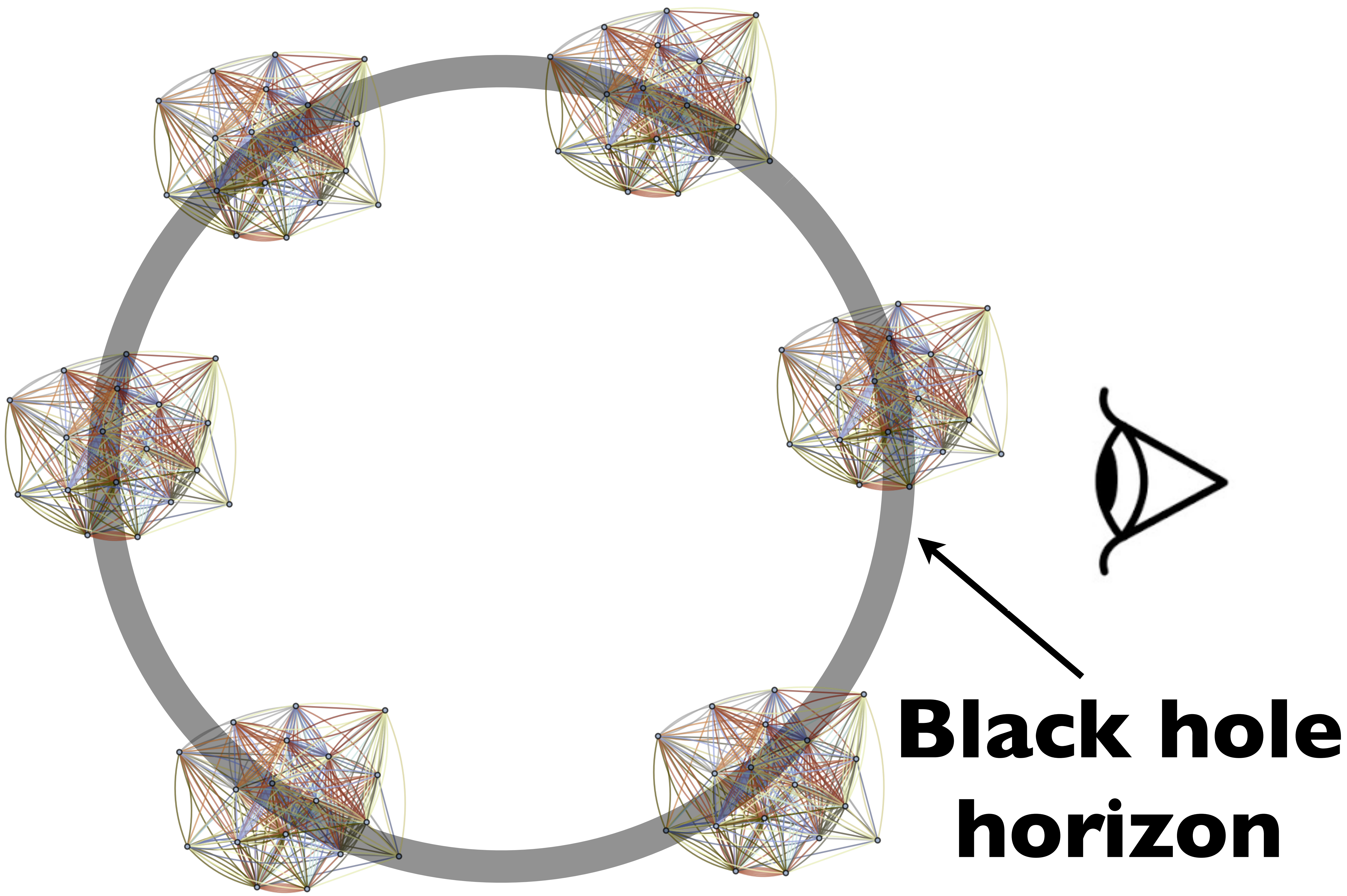

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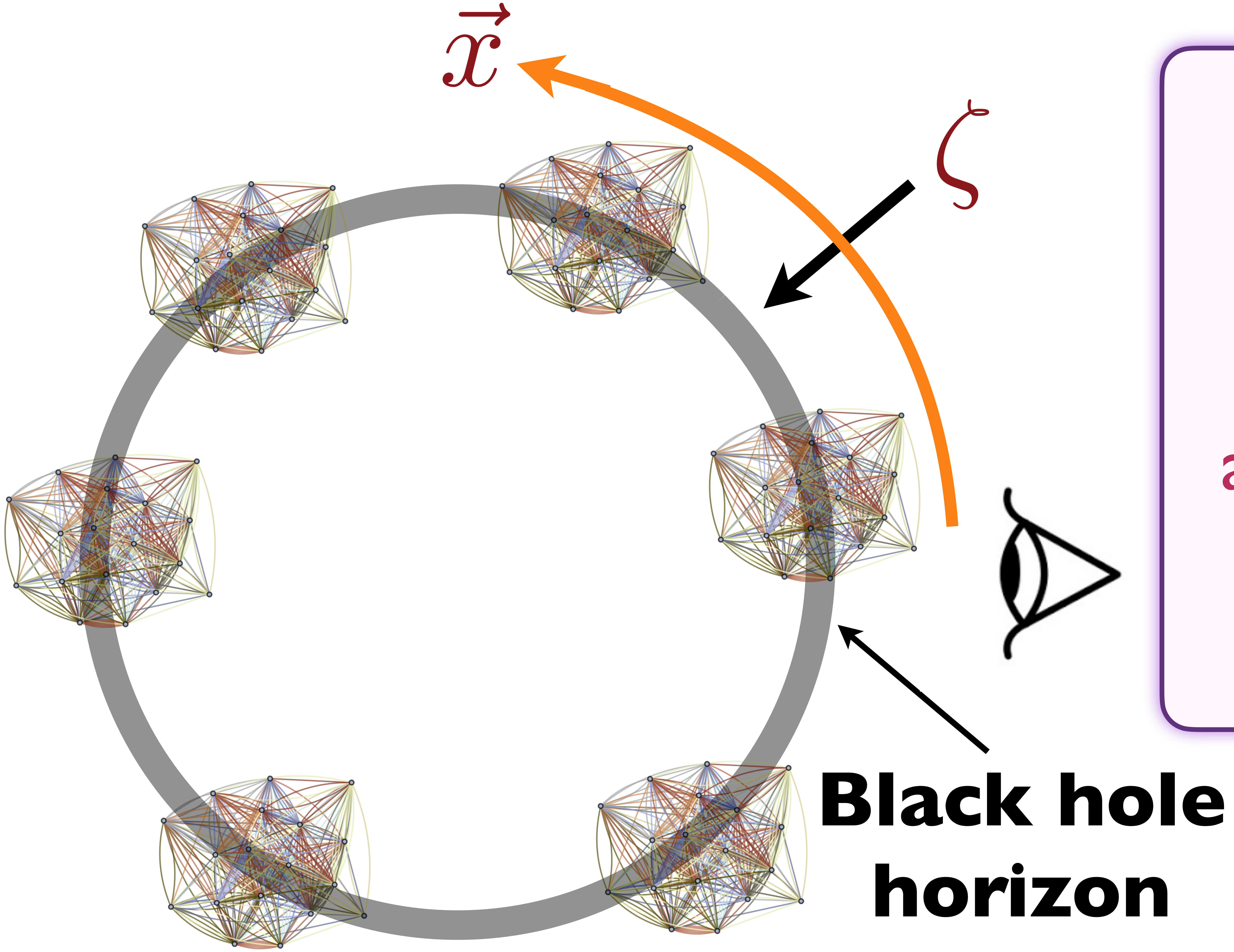
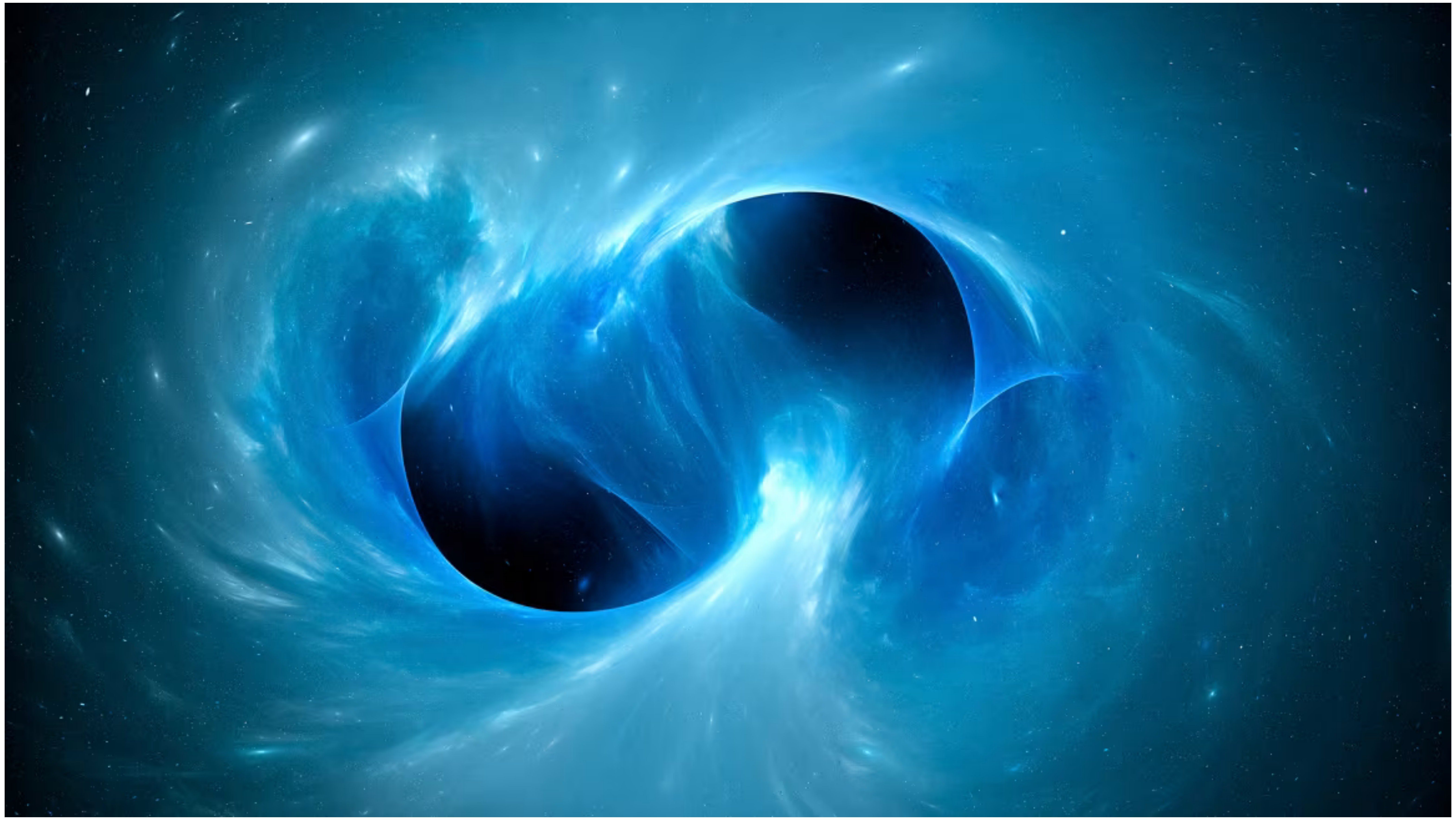
Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge



The quantum versions of Maxwell's and Einstein's equations in ζ space and time are also the equations describing electron entanglement in the SYK model!

Quantum Entanglement across a black hole horizon



Sakkmesterke/Science Photo Library RF/Getty Images

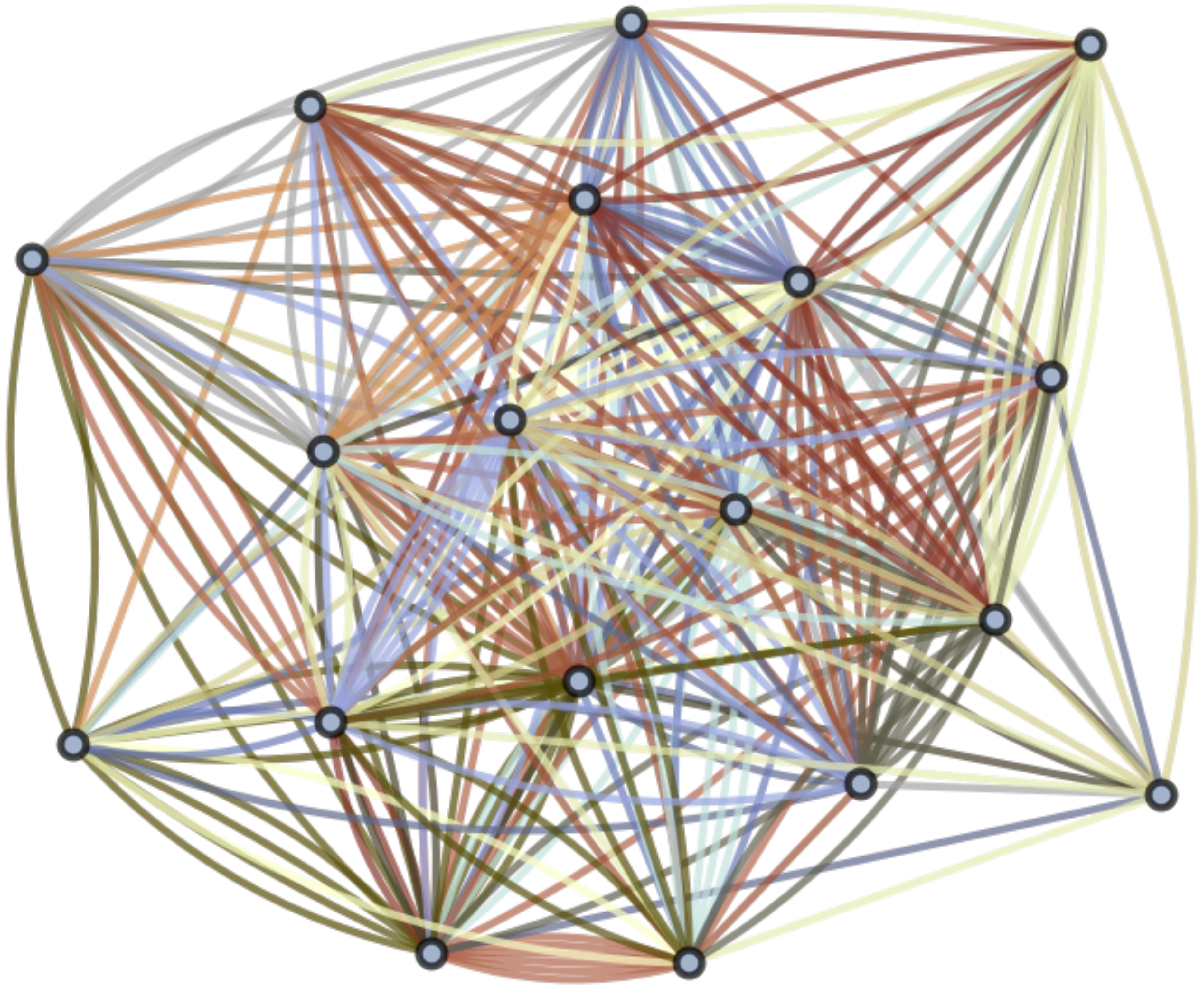
$$\tau_{\text{ring-down}} \sim \frac{\hbar}{k_B T}$$

Planckian dynamics!

T is the Hawking temperature of the black hole

The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles



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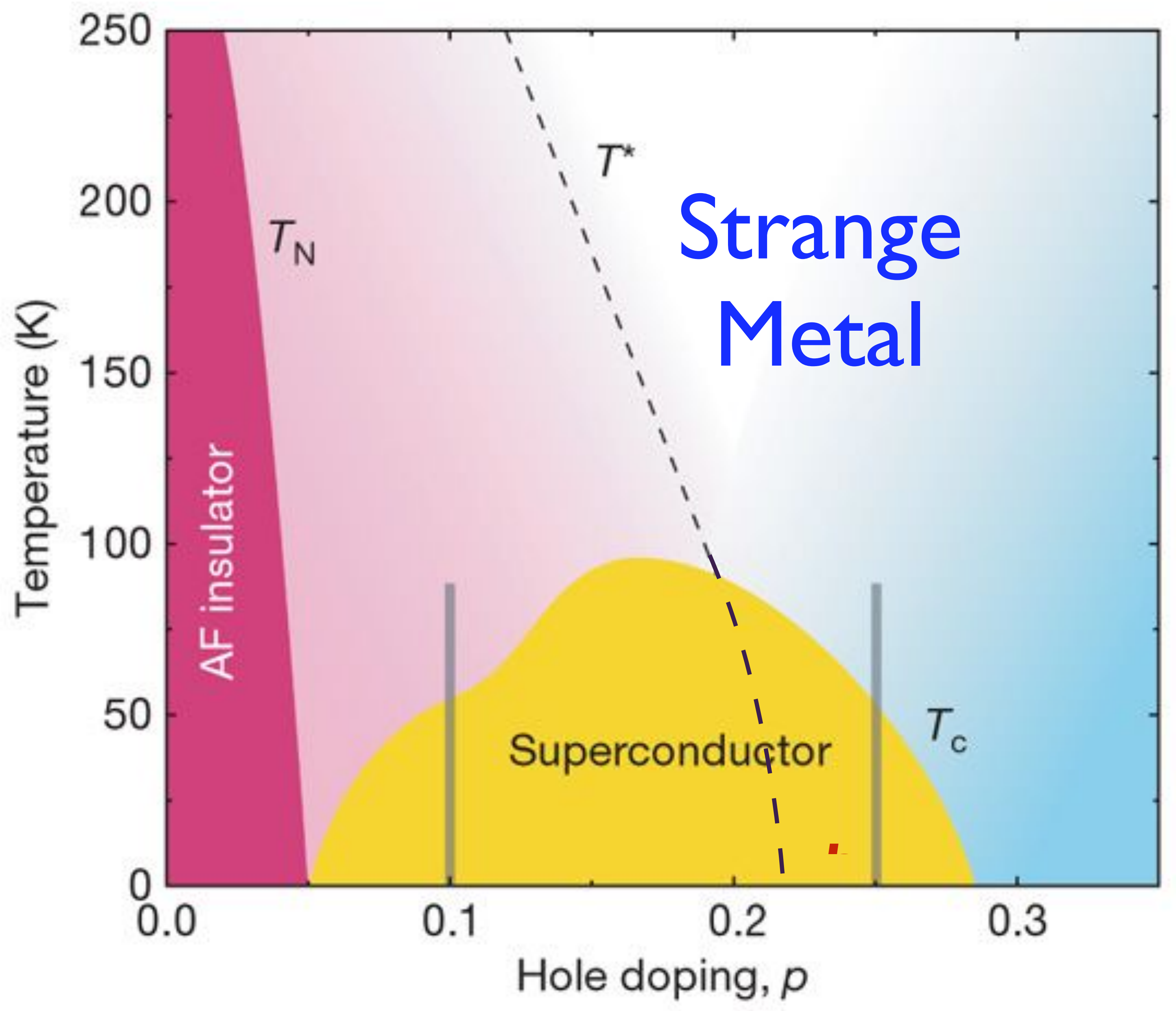
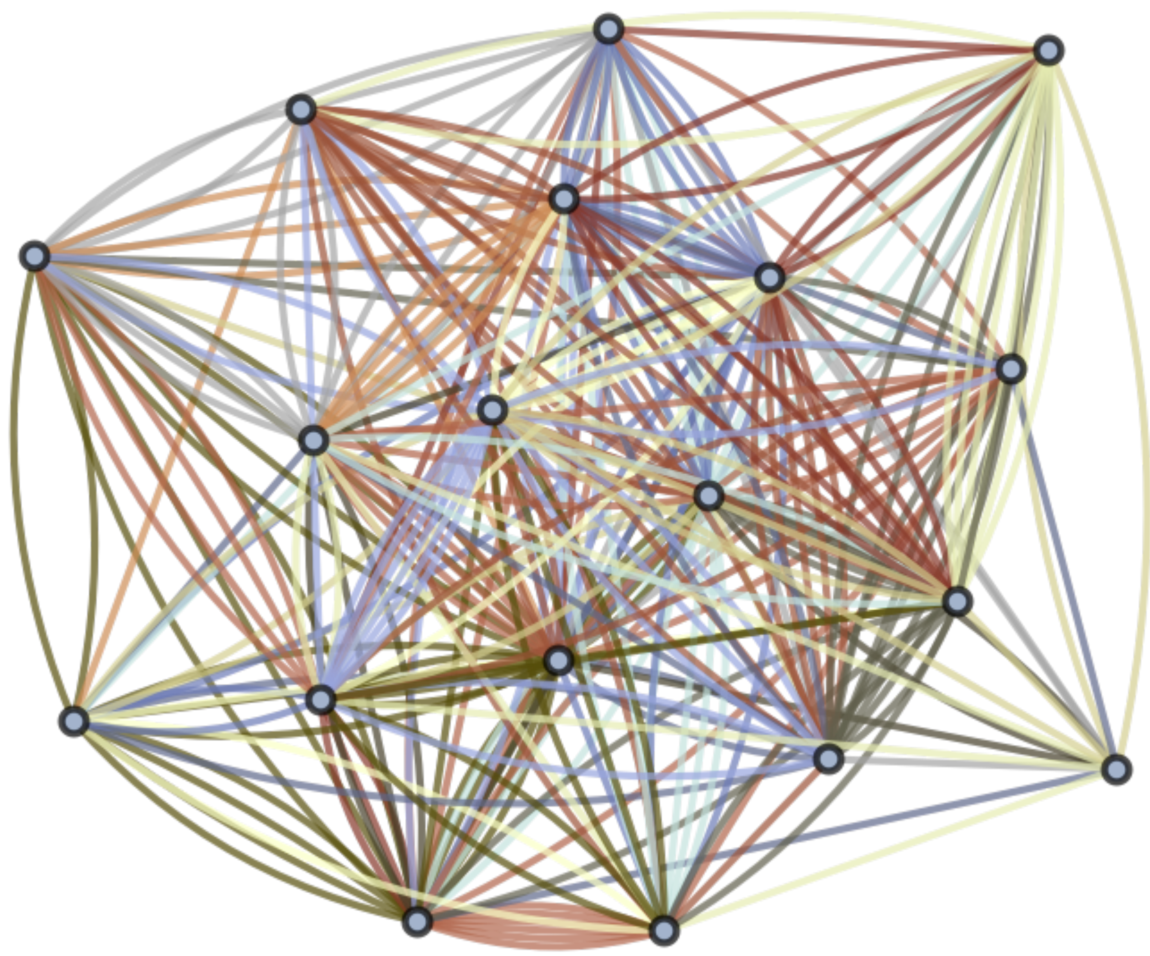


In one set of variables, it helps describe the ***strange*** electrical properties of YBCO

Sachdev, Ye (1993)



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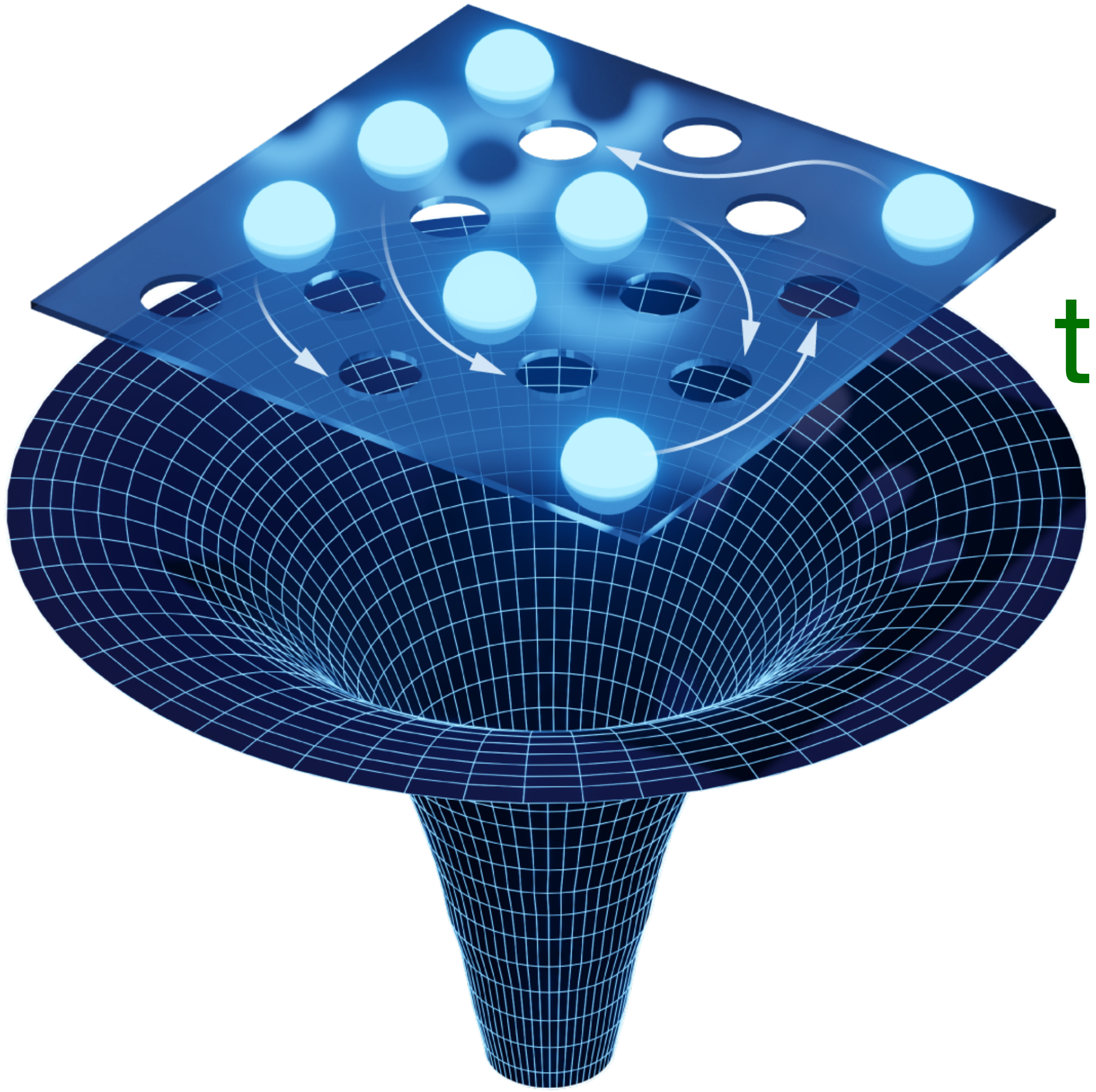

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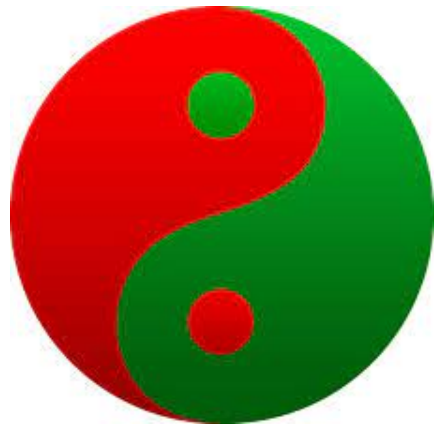

In a *dual* set of variables it describes the interior of ***charged black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)