Strong coupling problems in condensed matter and the AdS/CFT correspondence

Reviews:

arXiv:0910.1139
arXiv:0901.4103

Talk online: sachdev.physics.harvard.edu

PHYSICS
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PHYSICS
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1. Quantum-critical transport

   Collisionless-to-hydrodynamic crossover of CFT3s

2. Exact solution from AdS/CFT

3. Quantum criticality of Fermi surfaces

   The genus expansion

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The Superfluid-Insulator transition

Boson Hubbard model

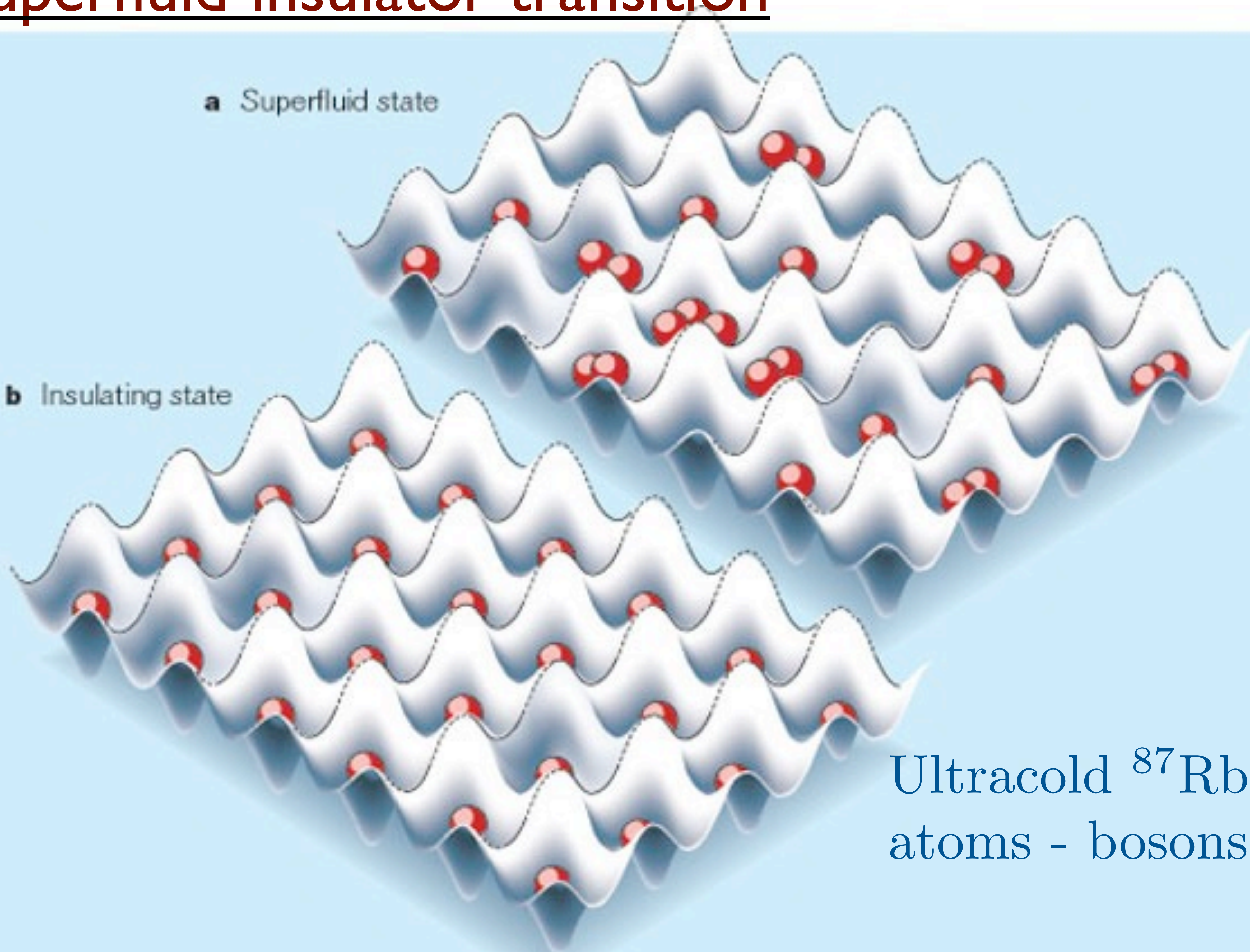
Degrees of freedom: Bosons, $b_j^\dagger$, hopping between the sites, j, of a lattice, with short-range repulsive interactions.

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2} \sum_j n_j (n_j - 1) + \cdots$$

$$n_j \equiv b_j^\dagger b_j$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).

Superfluid-insulator transition

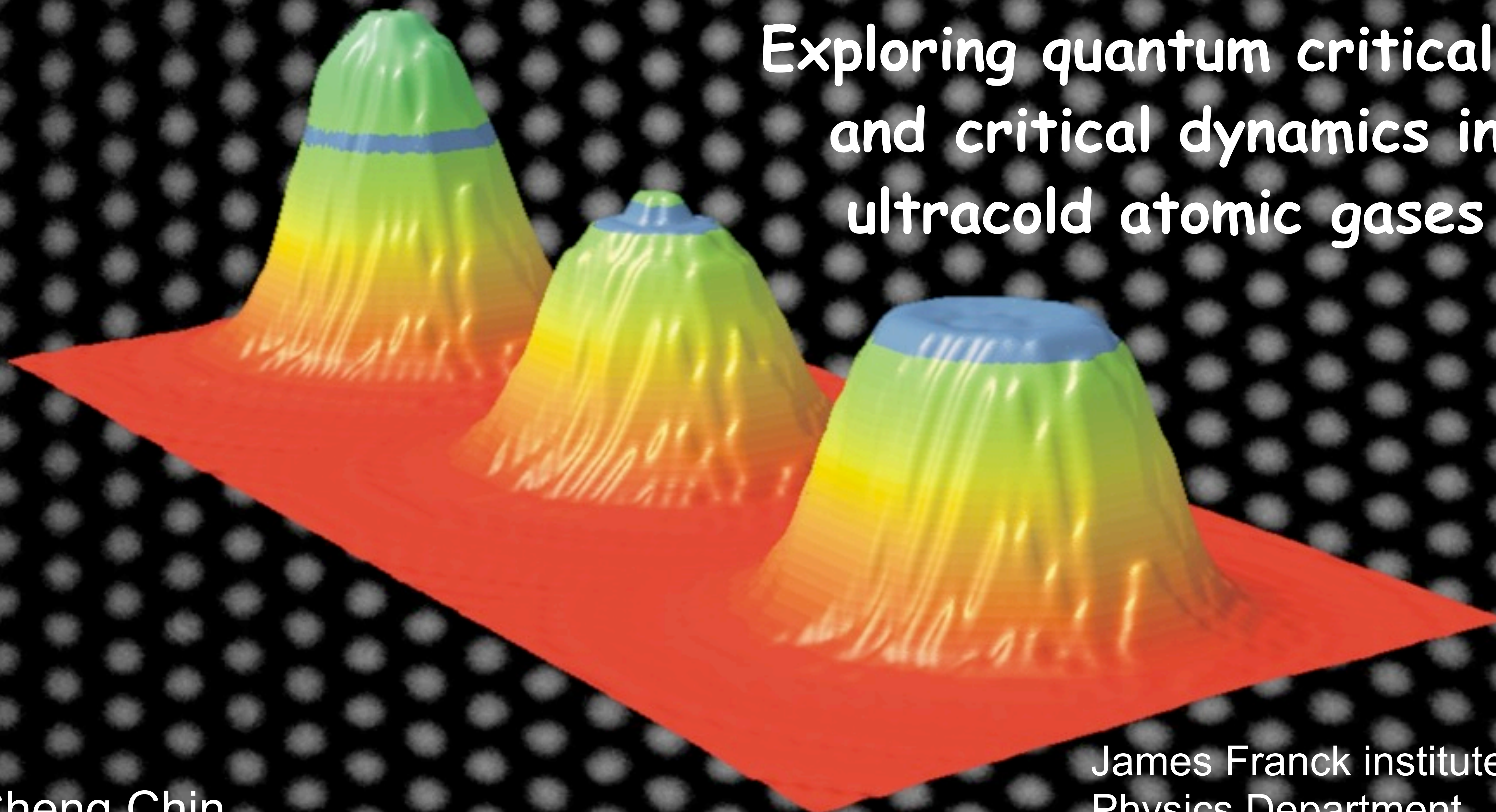


Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Having your cake and seeing it too -

Exploring quantum criticality and critical dynamics in ultracold atomic gases



Cheng Chin

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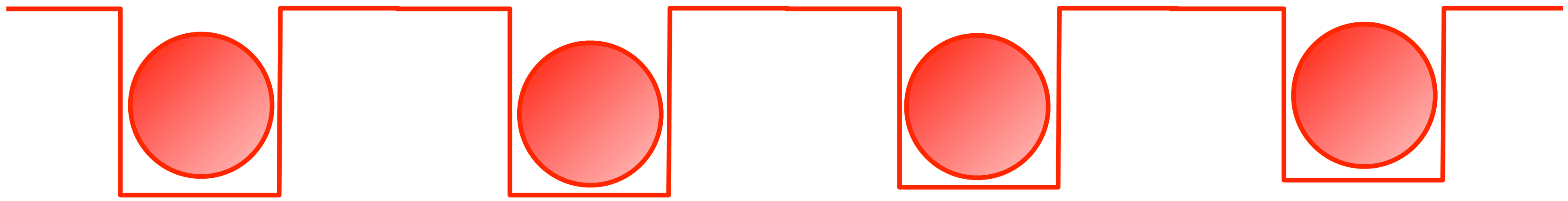
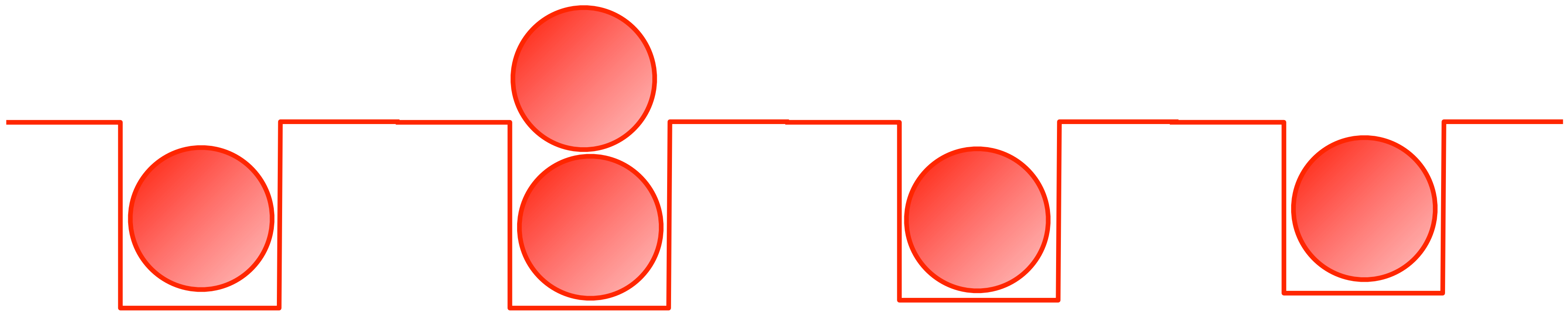


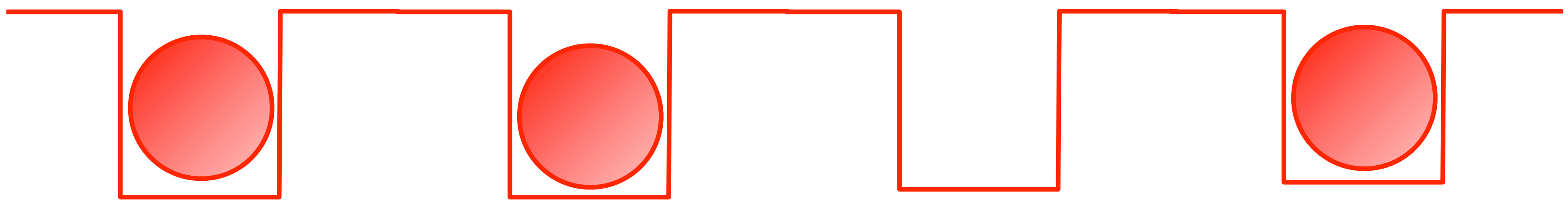
Insulator (the vacuum) at large U

Excitations:



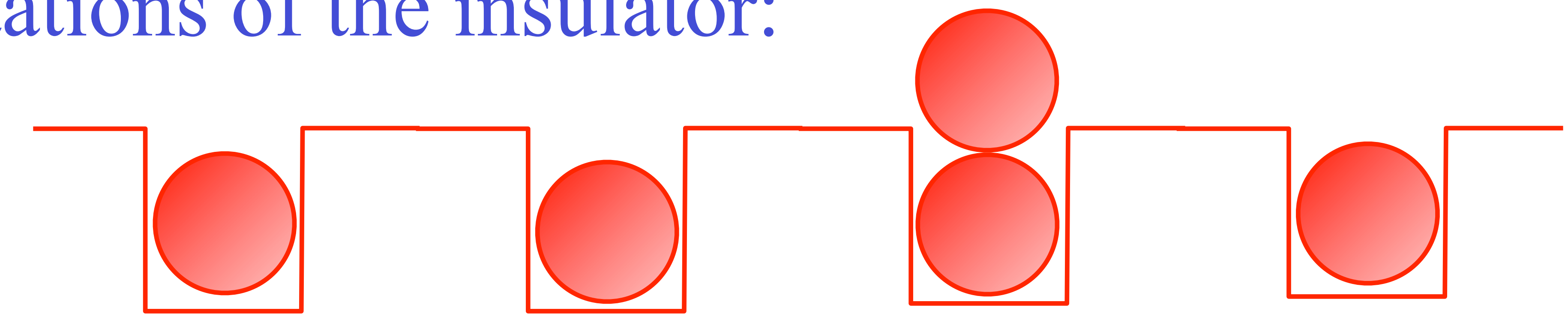
Particles ~ $\psi^\dagger$

Excitations:

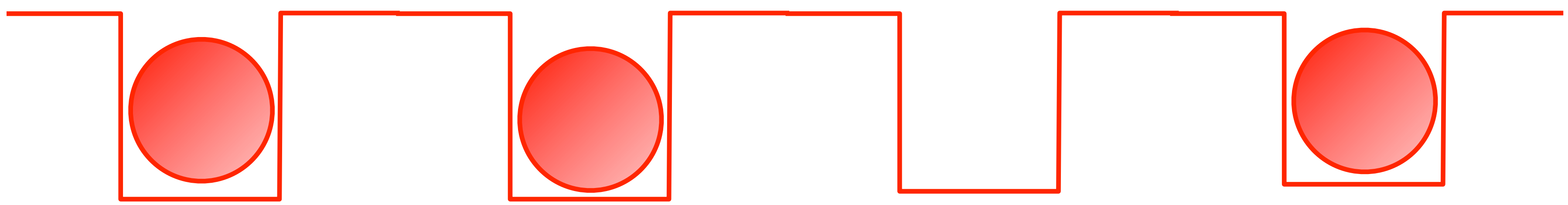


Holes ~ ψ

Excitations of the insulator:



Particles ~ $\psi^{\dagger}$



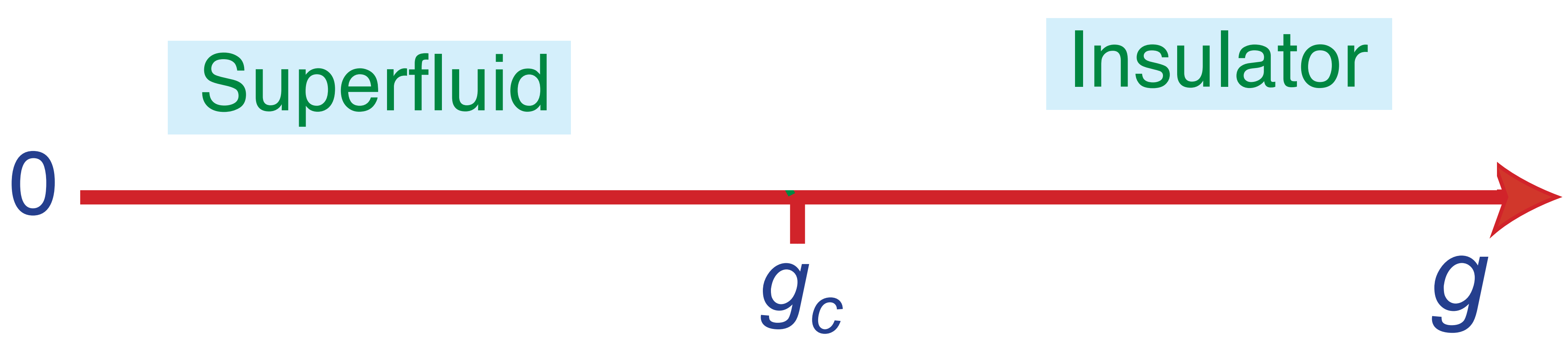
Holes ~ ψ

Density of particles = density of holes $\Rightarrow$
"relativistic" field theory for ψ:

$$\mathcal{S} = \int d^3x \left[|\partial_\mu \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

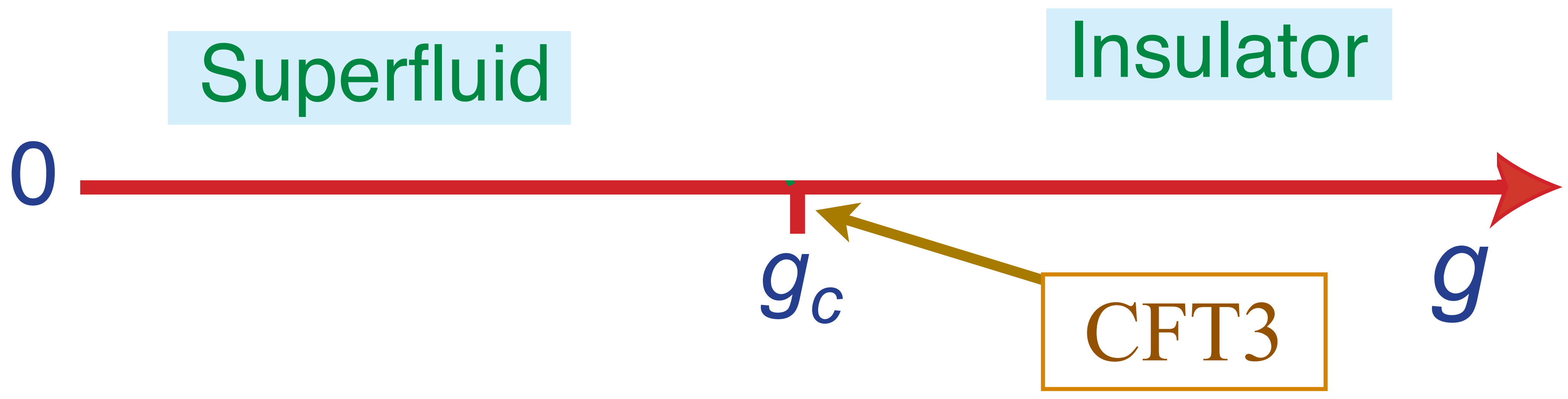
$$\text{Insulator} \Leftrightarrow \langle \psi \rangle = 0$$

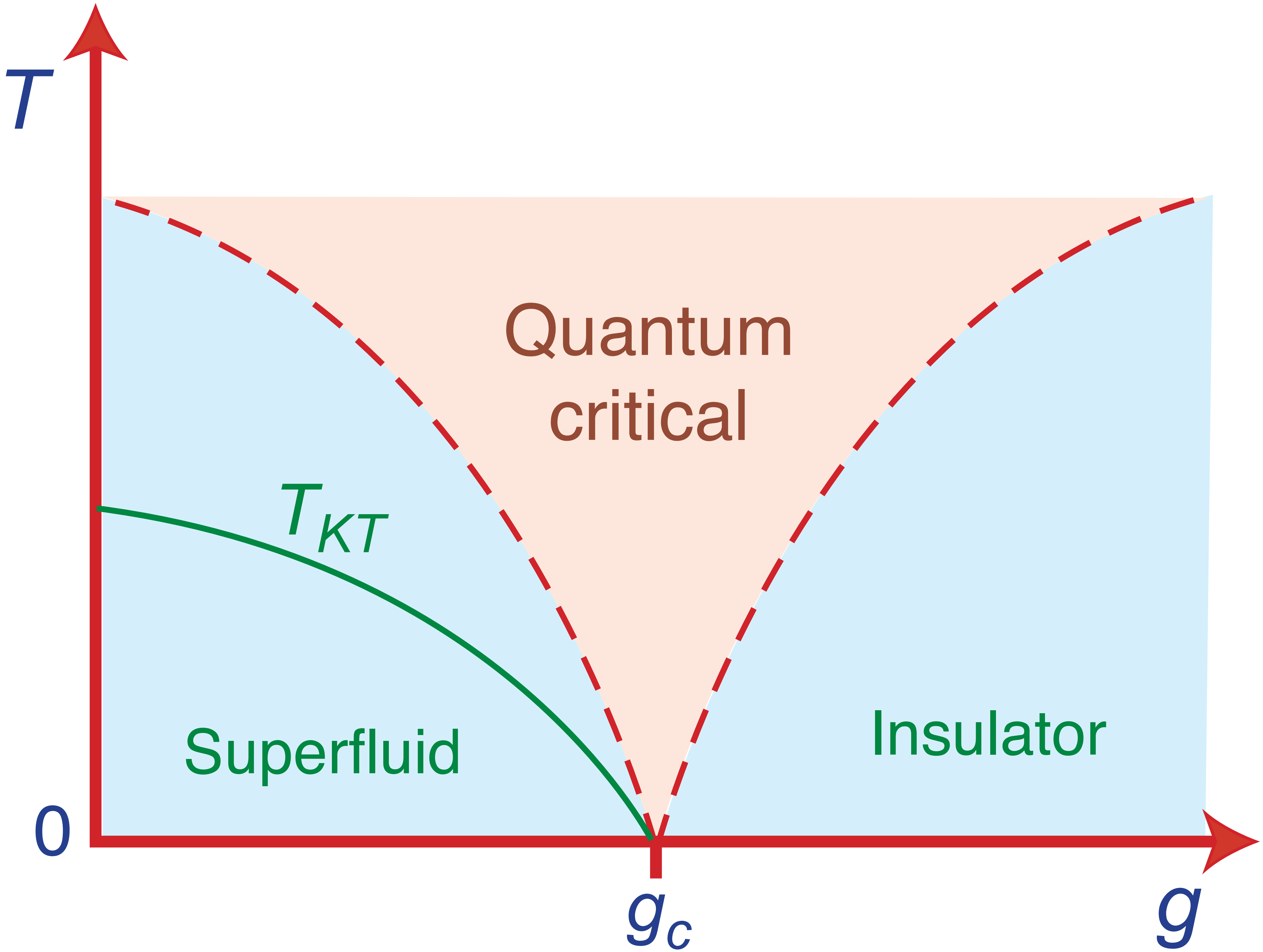
$$\text{Superfluid} \Leftrightarrow \langle \psi \rangle \neq 0$$

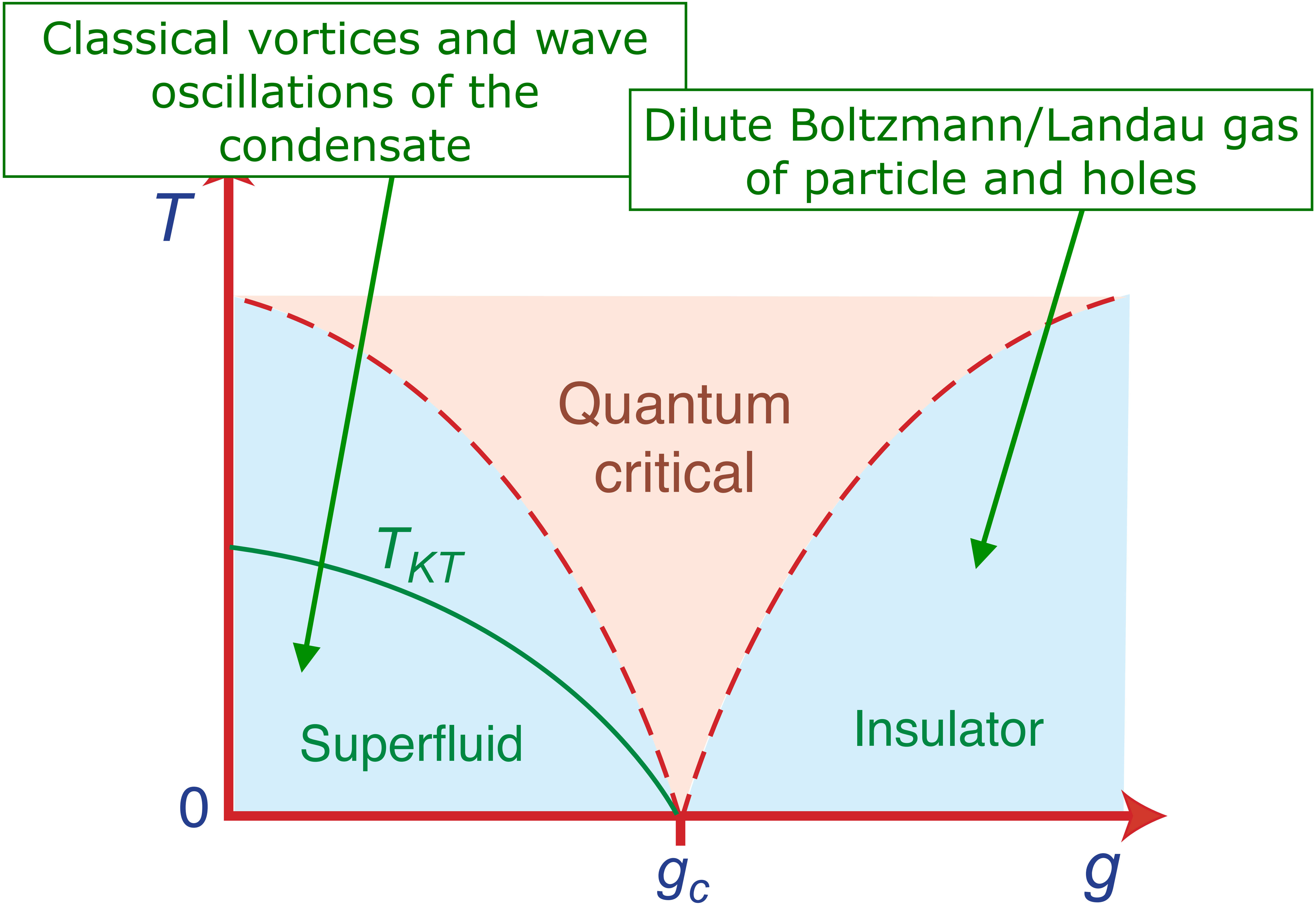

Superfluid
Insulator
0
g_c
g

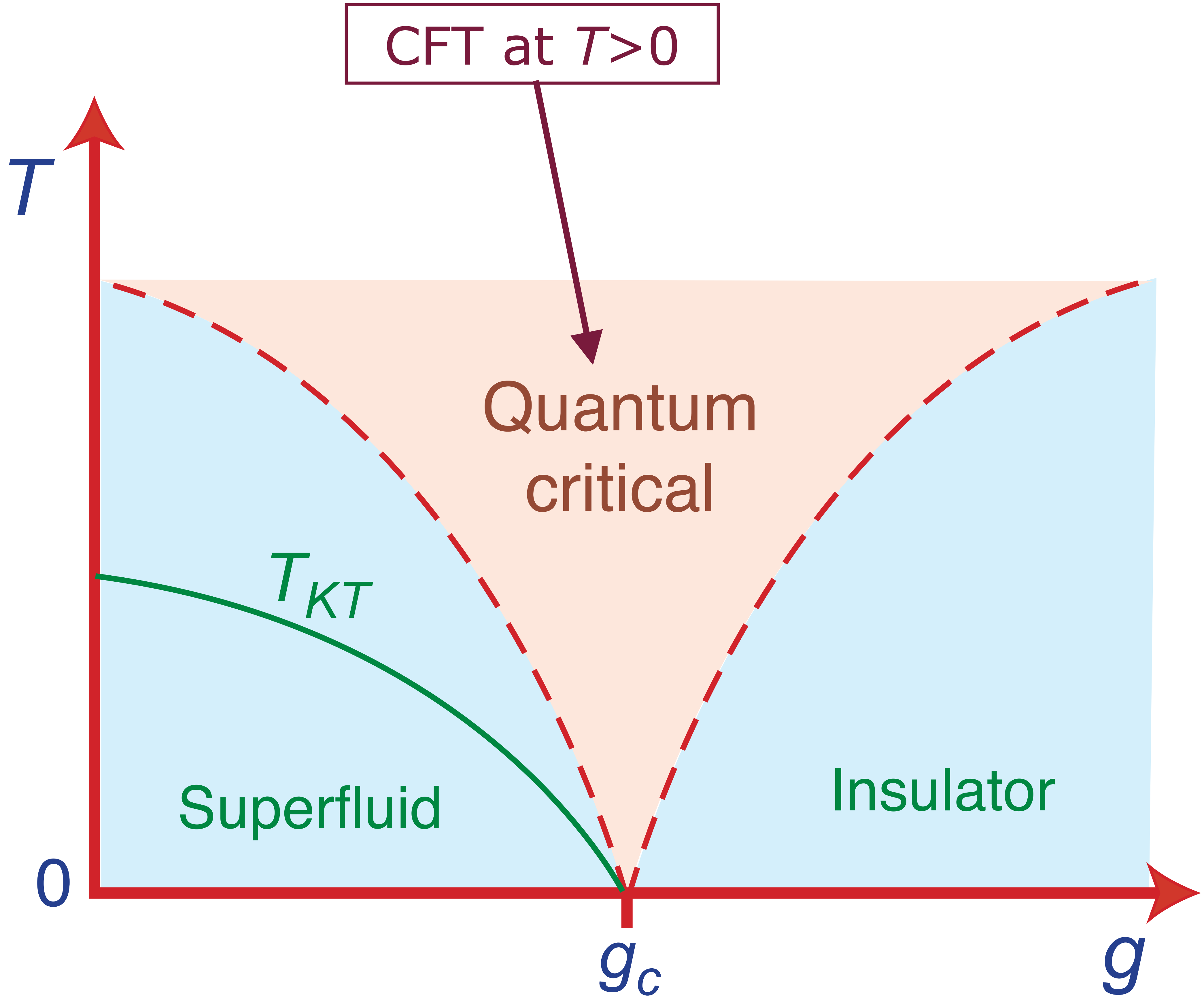
$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla} \psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\langle \psi \rangle \neq 0 \qquad\qquad \langle \psi \rangle = 0$$




T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g


Classical vortices and wave oscillations of the condensate
Dilute Boltzmann/Landau gas of particle and holes
T
Quantum critical
T_{KT}
Superfluid
Insulator
0
g_c
g


CFT at $T>0$
T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g

Resistivity of Bi films

Conductivity σ

$$\sigma_{\text{Superconductor}}(T \to 0) = \infty$$

$$\sigma_{\text{Insulator}}(T \to 0) = 0$$

$$\sigma_{\text{Quantum critical point}}(T \to 0) \approx \frac{4e^2}{h}$$

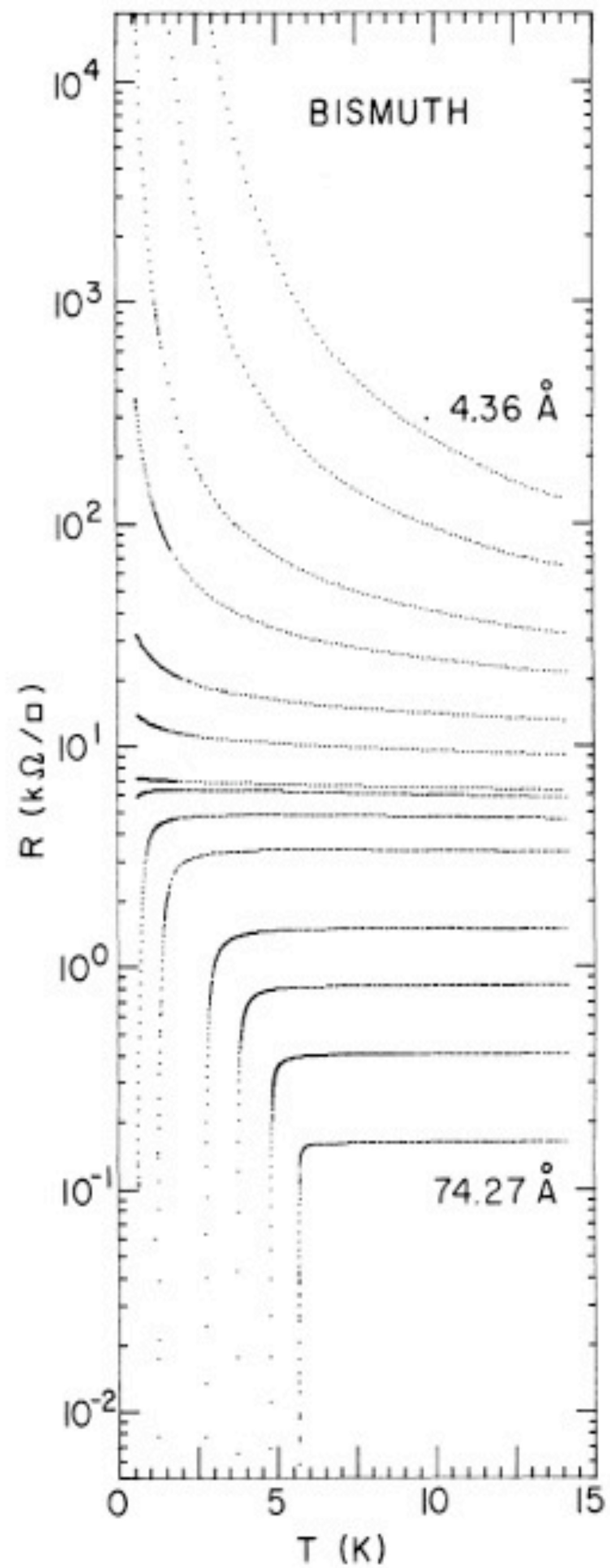


FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

Quantum critical transport

Quantum *"perfect fluid"*
with shortest possible
relaxation time, τ_R

$$\boxed{\tau_R \gtrsim \frac{\hbar}{k_B T}}$$

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Electrical conductivity

$$\sigma = \frac{e^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

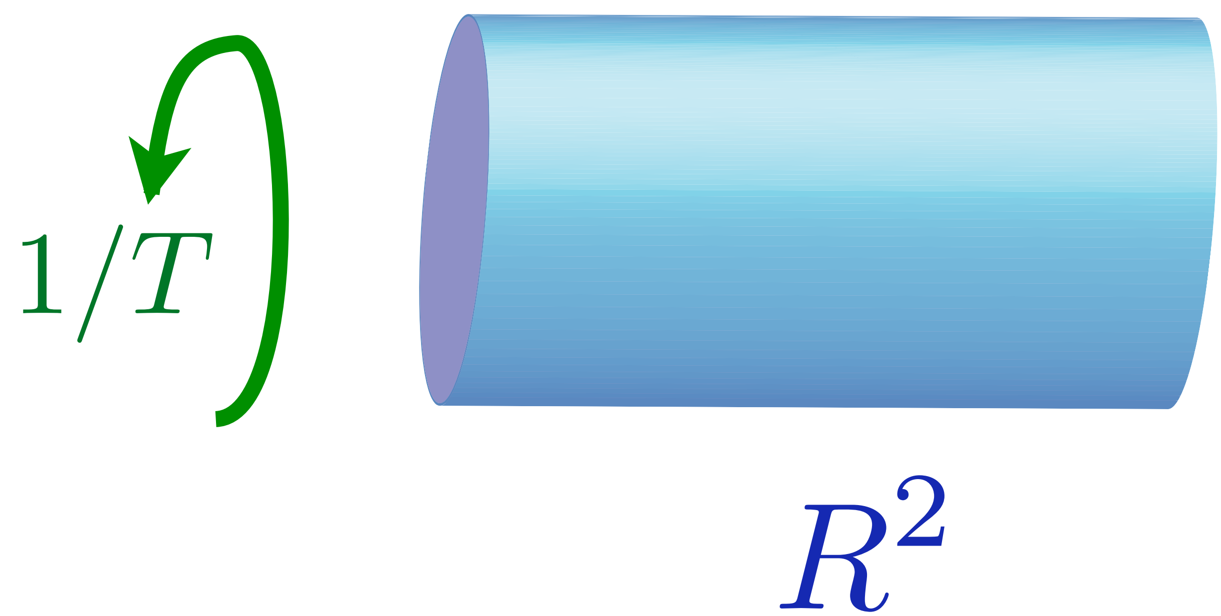
$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

Quantum critical transport

Euclidean field theory: Compute current correlations on $R^2 \times S^1$ with circumference $1/T$



Quantum critical transport

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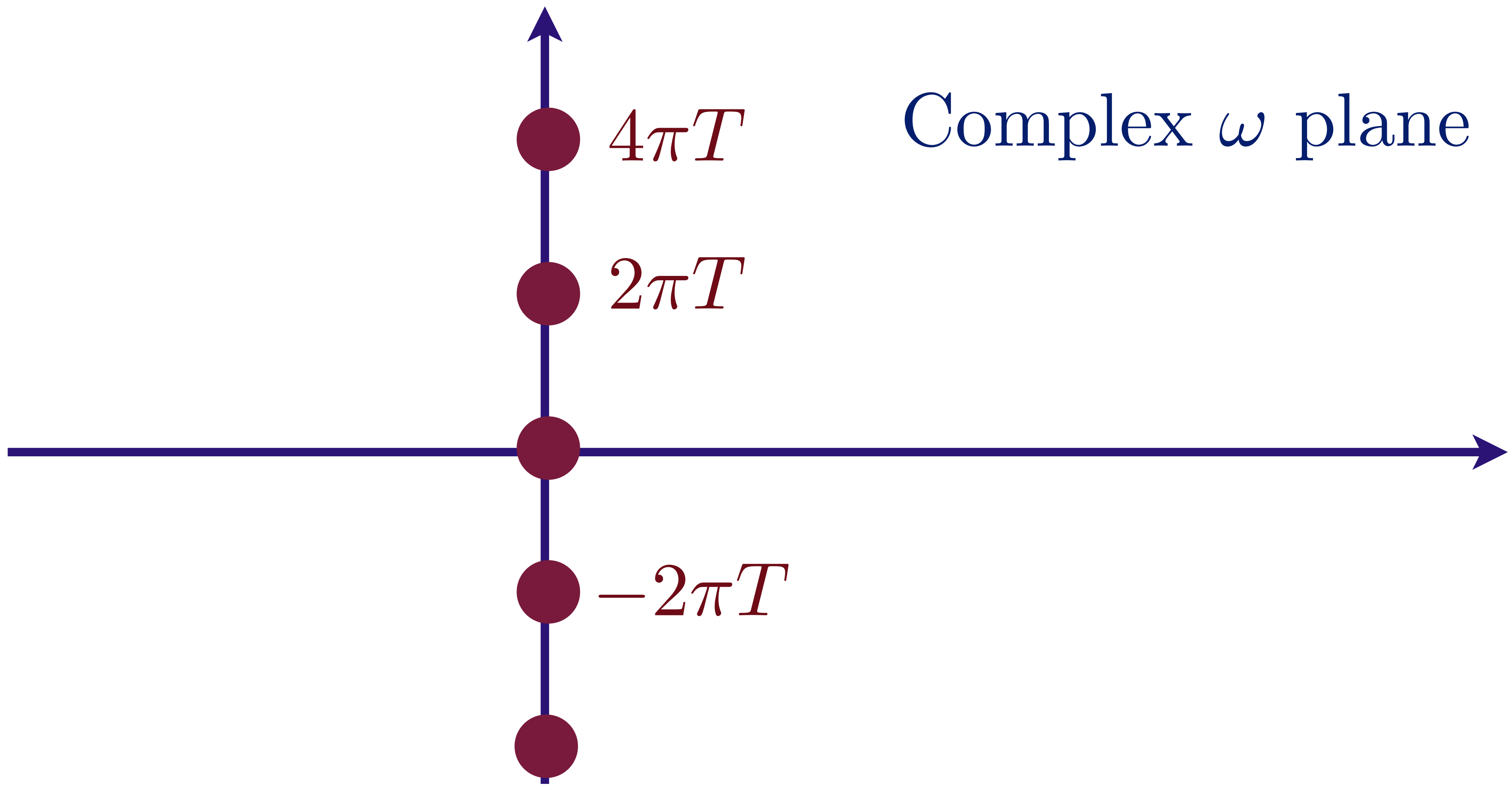
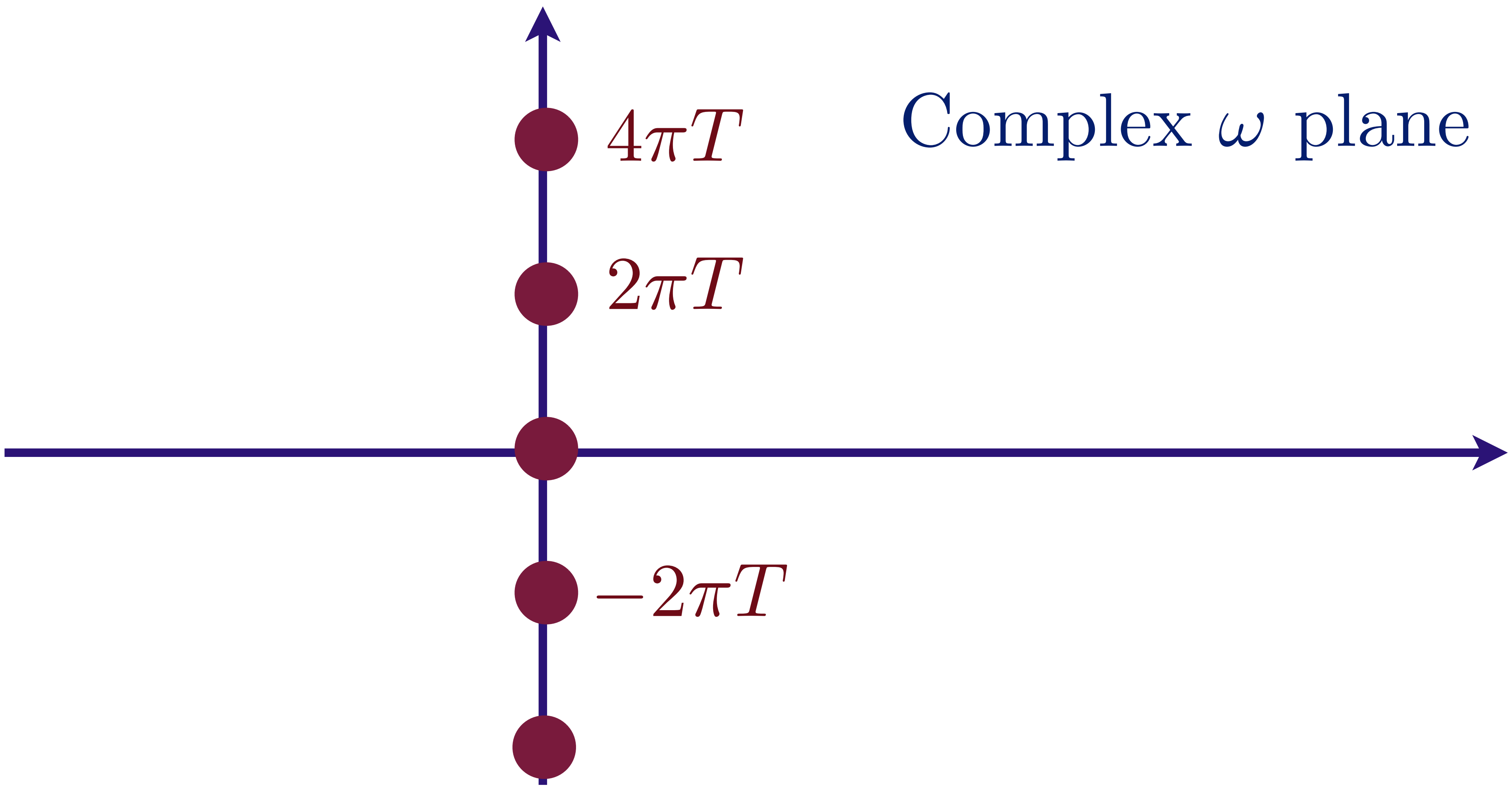


Direct $1/N$ or $\epsilon = 4 - d$ expansion for correlators at $\omega_n = 2\pi nTi$, with n integer

Quantum critical transport

Euclidean field theory: Compute current correlations on $R^2 \times S^1$ with circumference $1/T$

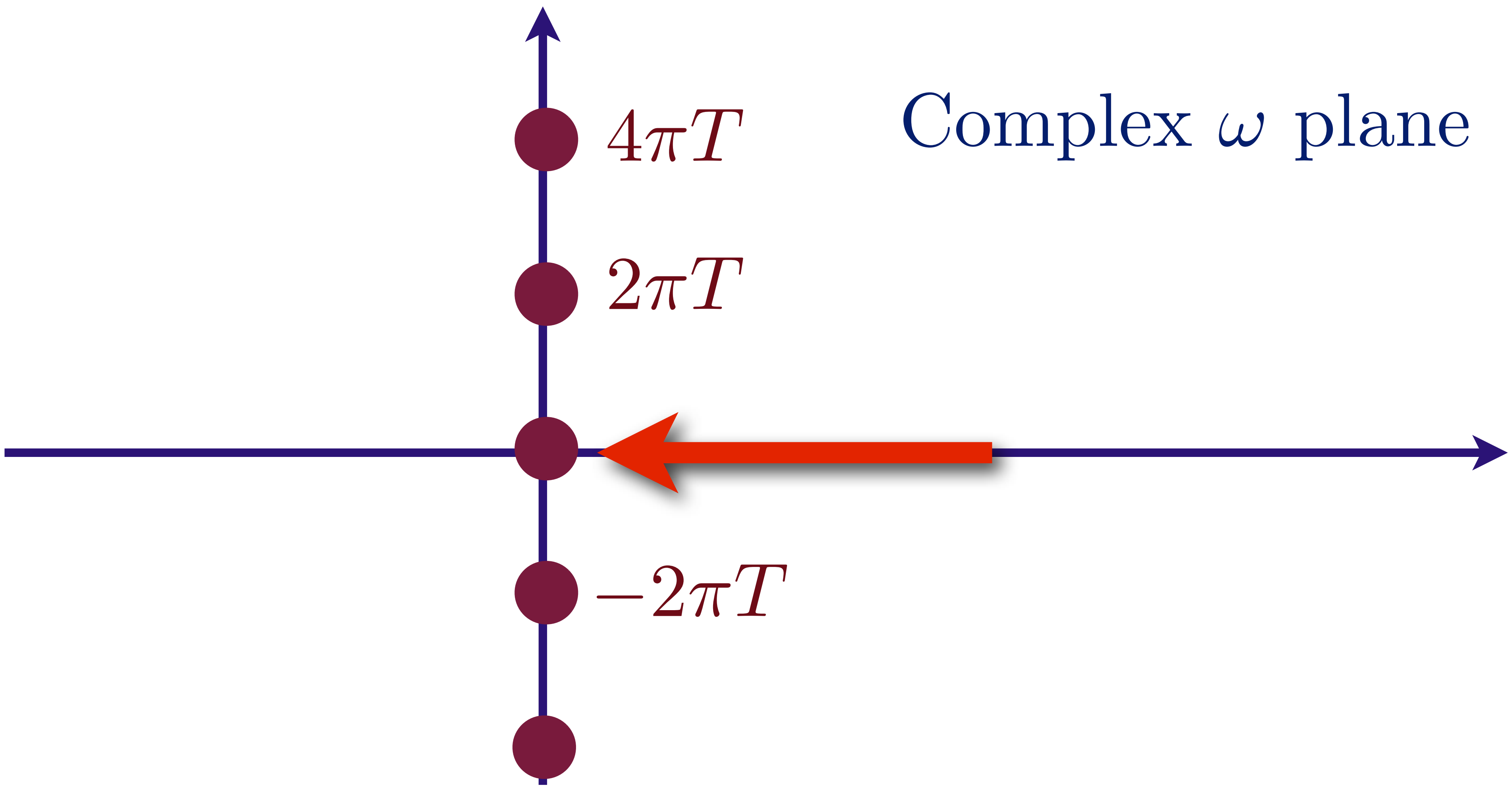


Strong coupling problem:

Correlators at $\omega \to 0$, along the real axis.

Quantum critical transport

Euclidean field theory: Compute current correlations on $R^2 \times S^1$ with circumference $1/T$



Strong coupling problem:
Correlators at $\omega \to 0$, along the real axis.

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k, \omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

For *all* CFT2s, at all $\hbar\omega / k_B T$

$$\chi(k, \omega) \quad = \quad \frac{4e^2}{h} K \frac{vk^2}{v^2 k^2 - \omega^2} \quad ; \quad \sigma(\omega) = \frac{4e^2}{h} \frac{Kv}{-i\omega}$$

where K is a universal number characterizing the CFT2 (the level number), and v is the velocity of "light".

This follows from the conformal mapping of the plane to the cylinder, which relates correlators at $T = 0$ to those at $T > 0$.

CFT correlator of $U(1)$ current J_μ in 1+1 dimensions

Charge density correlation at $T = 0$:

$$\left\langle J_R(x,\tau) J_R(0) \right\rangle \sim \frac{1}{(\tau + ix)^2}$$

$$\left\langle J_t(k,\omega) J_t(-k,-\omega) \right\rangle \sim \frac{k^2}{k^2 - \omega^2}$$

CFT correlator of $U(1)$ current J_μ in 1+1 dimensions

Charge density correlation at $T \geq 0$:

$$\left\langle J_R(x,\tau) J_R(0) \right\rangle \sim \frac{\pi^2 T^2}{\sin^2\left(\pi T(\tau + ix)\right)}$$

$$\left\langle J_t(k, i\omega_n) J_t(-k, -i\omega_n) \right\rangle \sim \frac{k^2}{k^2 + \omega_n^2}$$

Conformal mapping of plane to cylinder with circumference *1/T*

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Conformal mapping of plane to cylinder with circumference *1/T*

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k, \omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

For *all* CFT2s, at all $\hbar\omega / k_B T$

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No hydrodynamics in CFT2s.

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k,\omega)$

For *all* CFT3s, at $\hbar\omega \gg k_B T$

$$\chi(k,\omega) \quad = \quad \frac{4e^2}{h} K \frac{k^2}{\sqrt{v^2 k^2 - \omega^2}} \quad ; \quad \sigma(\omega) = \frac{4e^2}{h} K$$

where K is a universal number characterizing the CFT3, and v is the velocity of "light".

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k, \omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

However, for *all* CFT3s, at $\hbar\omega \ll k_B T$, we have the Einstein relation

$$\chi(k, \omega) = 4e^2 \chi_c \frac{Dk^2}{Dk^2 - i\omega} \quad ; \quad \sigma(\omega) = 4e^2 D \chi_c = \frac{4e^2}{h} \Theta_1 \Theta_2$$

where the **compressibility**, χ_c, and the **diffusion constant** D obey

$$\chi = \frac{k_B T}{(hv)^2} \Theta_1 \quad ; \quad D = \frac{hv^2}{k_B T} \Theta_2$$

with Θ_1 and Θ_2 universal numbers characteristic of the CFT3

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Density correlations in CFTs at $T > 0$

In CFT3s collisions are "phase" randomizing, and lead to relaxation to local thermodynamic equilibrium. So there is a crossover from collisionless behavior for $\hbar\omega \gg k_B T$, to hydrodynamic behavior for $\hbar\omega \ll k_B T$.

$$\sigma(\omega) = \begin{cases} \dfrac{4e^2}{h} K & , \quad \hbar\omega \gg k_B T \\ \dfrac{4e^2}{h} \Theta_1 \Theta_2 \equiv \sigma_Q & , \quad \hbar\omega \ll k_B T \end{cases}$$

and in general we expect $K \neq \Theta_1 \Theta_2$ (verified for Wilson-Fisher fixed point).

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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The genus expansion

Field theories in D spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u \frac{dg}{du} = \beta(g)$$

where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u.

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$$u \frac{dg}{du} = \beta(g)$$

where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u.

Key idea: $\Rightarrow$ Implement u as an extra dimension, and map to a local theory in $D+1$ dimensions.

At the RG fixed point, $\beta(g) = 0$, the D dimensional field theory is invariant under the scale transformation

$$x^\mu \to x^\mu / b \quad , \quad u \to b\, u$$

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This is an invariance of the *metric* of the theory in $D+1$ dimensions. The unique solution is

$$ds^2 = \left(\frac{u}{L}\right)^2 dx^\mu dx_\mu + L^2 \frac{du^2}{u^2}.$$

Or, using the length scale $z = L^2/u$

$$ds^2 = L^2 \frac{dx^\mu dx_\mu + dz^2}{z^2}.$$

This is the space AdS_{D+1}, and L is the AdS radius.

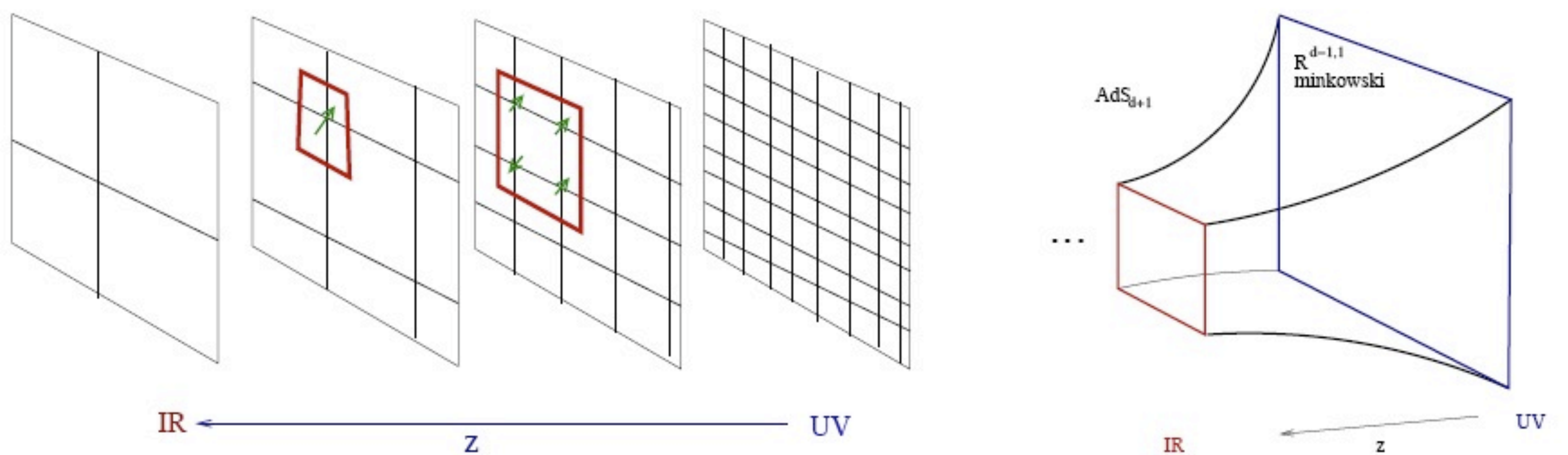

Figure 1: The extra ('radial') dimension of the bulk is the resolution scale of the field theory. The left figure indicates a series of block spin transformations labelled by a parameter z. The right figure is a cartoon of AdS space, which organizes the field theory information in the same way. In this sense, the bulk picture is a hologram: excitations with different wavelengths get put in different places in the bulk image.

Bonus: AdS_{D+1} is a solution of Einstein's equations with a negative cosmological constant, and is a symmetric space; the full group of symmetries of the metric is $\mathrm{SO}(D+1,1)$ (in Euclidean signature)

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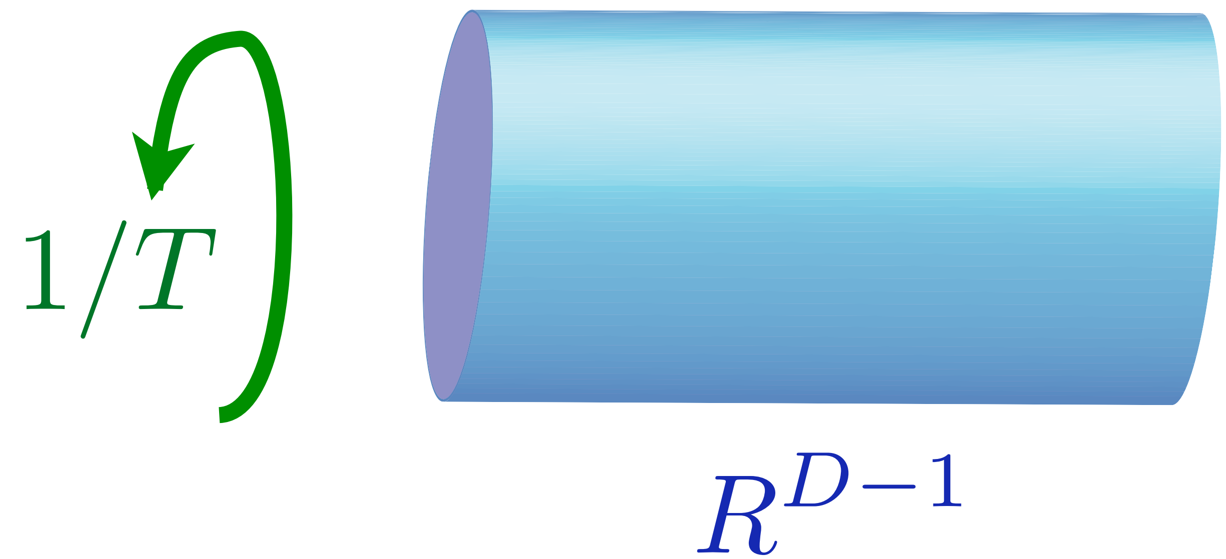
$\mathrm{SO}(D+1,1)$ is the group of conformal transformations in D dimensions, and relativistic field theories at the RG fixed point are conformally invariant.

At $T > 0$, the Euclidean field theory is on the cylinder $R^{D-1} \times S^1$, where the time co-ordinate is periodic under $\tau \to \tau + 1/T$.



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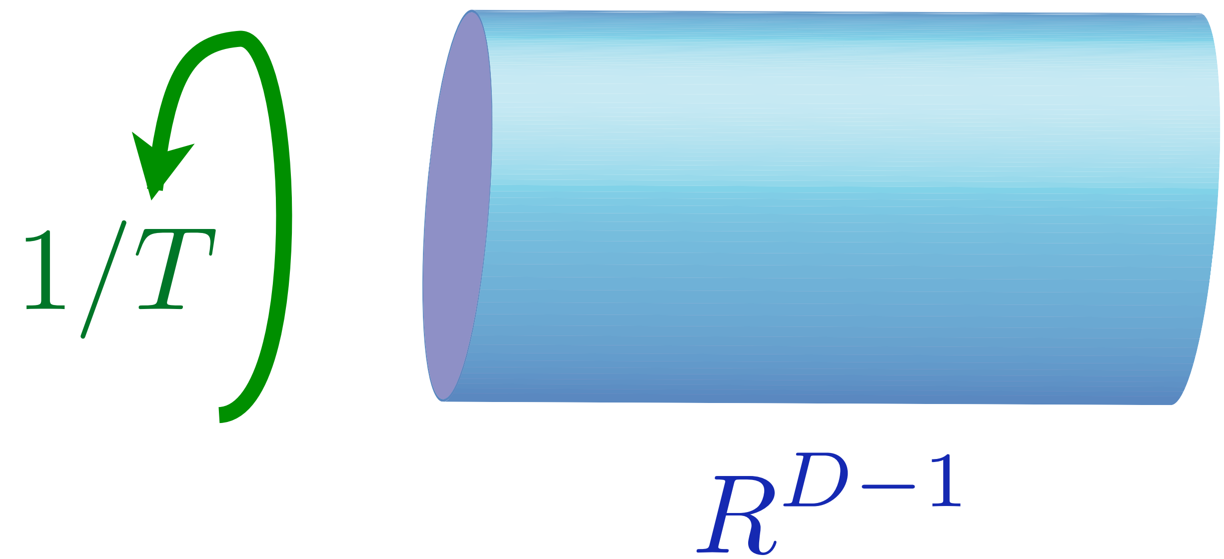


Solving Einstein's equations with a negative cosmological constant we have the solution

$$ds^2 = \frac{L^2}{z^2}\left(f(z)d\tau^2 + d\vec{x}^2 + \frac{dz^2}{f(z)}\right) \quad ; \quad f(z) = 1 - \left(\frac{z}{z_H}\right)^D$$

This is a AdS-Schwarzschild black hole with a horizon at $z = z_H$. This space is periodic in τ with period $1/T$ for

$$T = \frac{d}{4\pi z_H}$$

SU(N) SYM3 with $\mathcal{N} = 8$ supersymmetry

- Has a single dimensionful coupling constant, e_0, which flows to a strong-coupling fixed point $e_0 = e_0^*$ in the infrared.
- The CFT3 describing this fixed point resembles "critical spin liquid" theories.
- This CFT3 is the low energy limit of string theory on an M2 brane. The AdS/CFT correspondence provides a dual description using 11-dimensional supergravity on $\mathrm{AdS}_4 \times S_7$.
- The CFT3 has a global SO(8) R symmetry, and correlators of the SO(8) charge density can be computed exactly in the large N limit, even at $T > 0$.

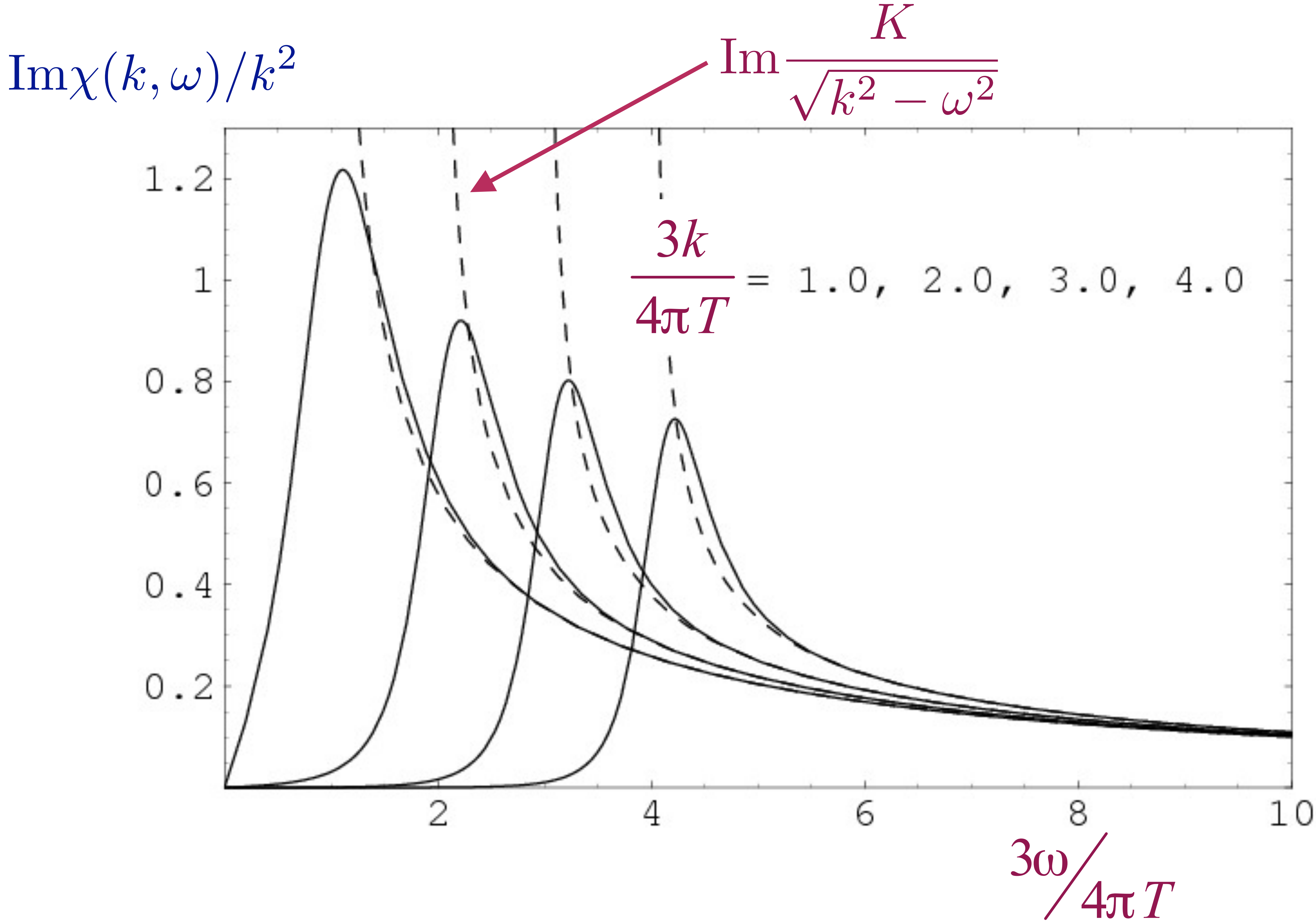
SU(N) SYM3 with $\mathcal{N} = 8$ supersymmetry

- The SO(8) charge correlators of the CFT3 are given by the usual AdS/CFT prescription applied to the following gauge theory on AdS4:

$$\mathcal{S} = -\frac{1}{4g_{4D}^2} \int d^4x \sqrt{-g} g^{MA} g^{NB} F^a_{MN} F^a_{AB}$$

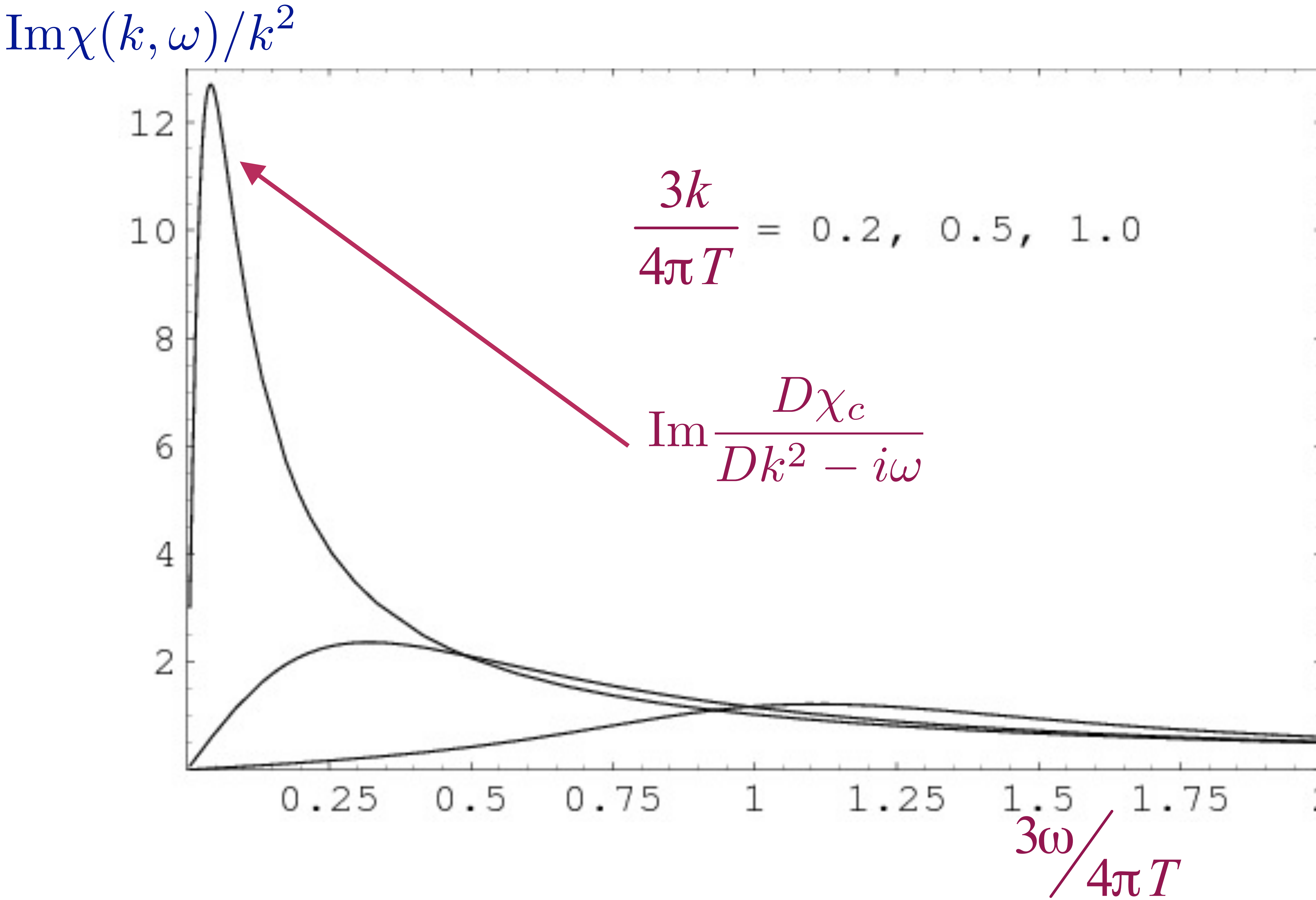
  where $a = 1 \ldots 28$ labels the generators of SO(8). Note that in large N theory, this looks like 28 copies of an Abelian gauge theory.

Collisionless to hydrodynamic crossover of SYM3



P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

Collisionless to hydrodynamic crossover of SYM3



P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

Universal constants of SYM3

$$\chi_c = \frac{k_B T}{(hv)^2}\Theta_1$$

$$D = \frac{hv^2}{k_B T}\Theta_2$$

$$\sigma(\omega) = \begin{cases} \frac{4e^2}{h}K & , \quad \hbar\omega \gg k_B T \\ \frac{4e^2}{h}\Theta_1\Theta_2 & , \quad \hbar\omega \ll k_B T \end{cases}$$

$$K = \frac{\sqrt{2}N^{3/2}}{3}$$

$$\Theta_1 = \frac{8\pi^2\sqrt{2}N^{3/2}}{9}$$

$$\Theta_2 = \frac{3}{8\pi^2}$$

C. Herzog, JHEP **0212**, 026 (2002)

P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

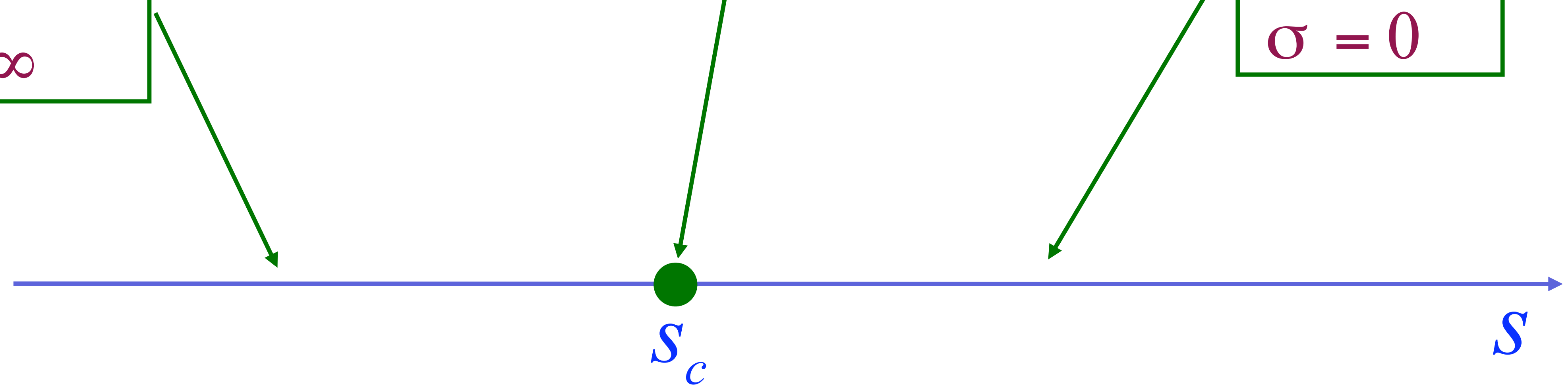
Electromagnetic self-duality

- Unexpected result, $K = \Theta_1 \Theta_2$.
- This is traced to a *four*-dimensional electromagnetic self-duality of the theory on AdS_4. In the large N limit, the SO(8) currents decouple into 28 U(1) currents with a Maxwell action for the U(1) gauge fields on AdS_4.
- This special property is not expected for generic CFT3s.

Conformal field theory:
Wilson-Fisher fixed point

Superfluid
$\langle \psi \rangle \neq 0$
$\sigma = \infty$

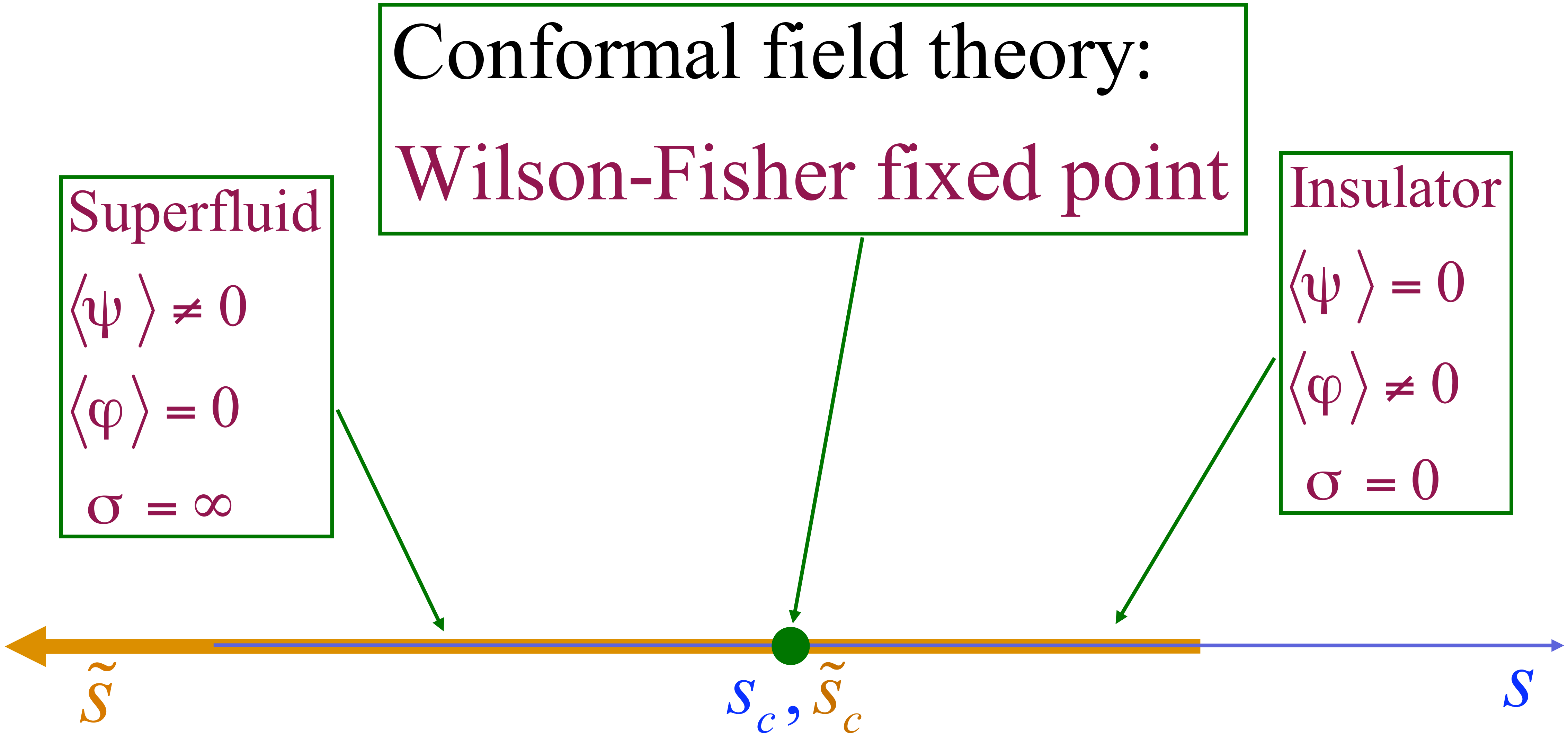
Insulator
$\langle \psi \rangle = 0$
$\sigma = 0$



Using the boson quasiparticle excitations of the insulator $\sim \psi$

$$\mathcal{S} = \int d^3x \left[|\partial_\mu \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett*. **47**, 1556 (1981)



Using the boson quasiparticle excitations of the insulator $\sim \psi$

$$\mathcal{S} = \int d^3x \left[|\partial_\mu \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

is dual to

Using the vortex quasiparticle excitations of the superfluid $\sim \varphi$

$$\mathcal{S}_{\text{dual}} = \int d^3x \left[|(\partial_\mu - iA_\mu)\varphi|^2 + \tilde{s}|\varphi|^2 + \frac{\tilde{u}}{2}|\varphi|^4 + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \right]$$

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981)

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- This special property is not expected for generic CFT3s.
- Although there is no boson-vortex *self*-duality at the Wilson-Fisher fixed point, the applicability of AdS/CFT suggests that the conductivity may be close to its self-dual value, $\sigma \approx 4e^2/h$.

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   The genus expansion

1. Quantum-critical transport

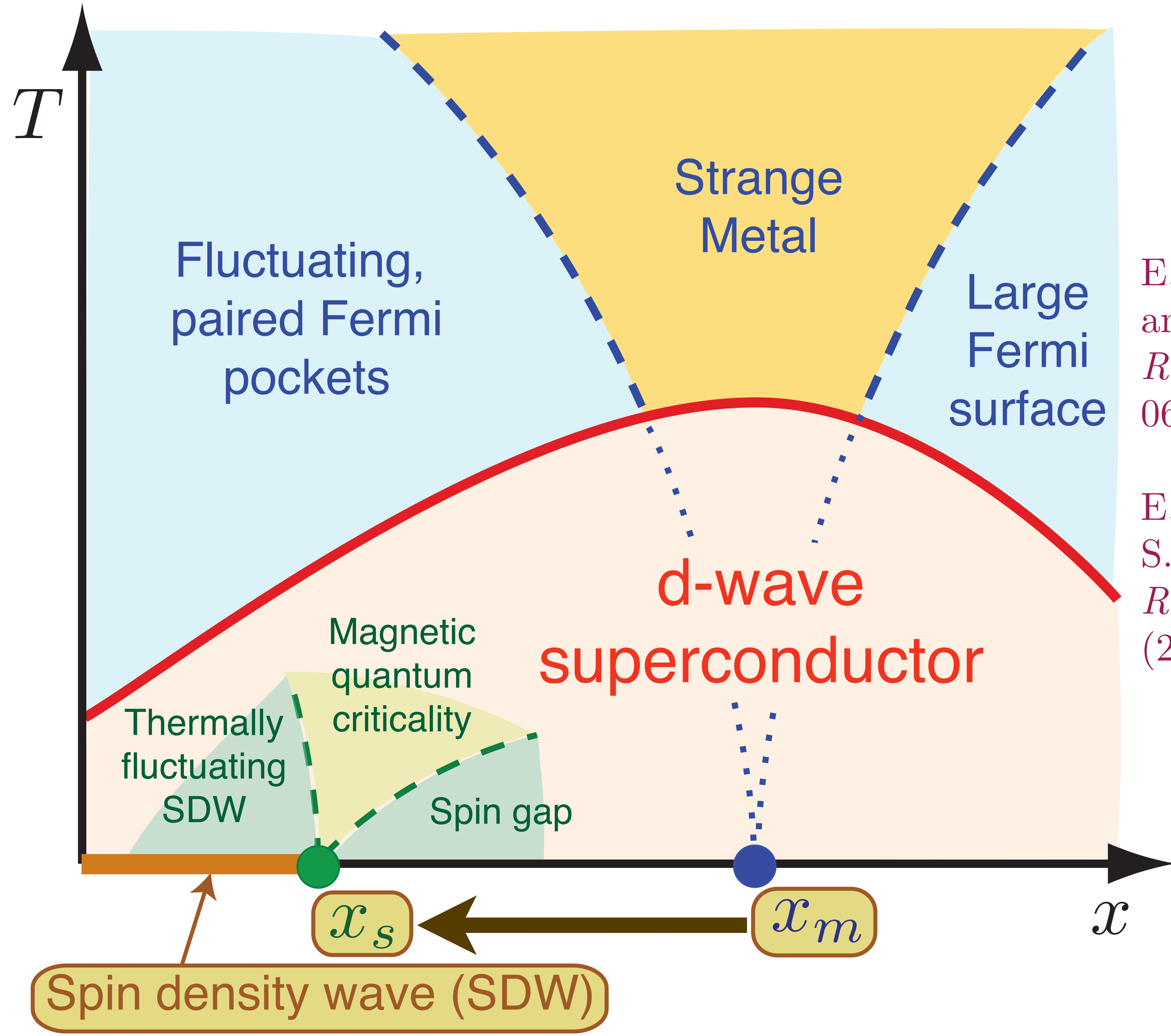
   Collisionless-to-hydrodynamic crossover of CFT3s

2. Exact solution from AdS/CFT

3. Quantum criticality of Fermi surfaces

   The genus expansion

Theory of quantum criticality in the cuprates

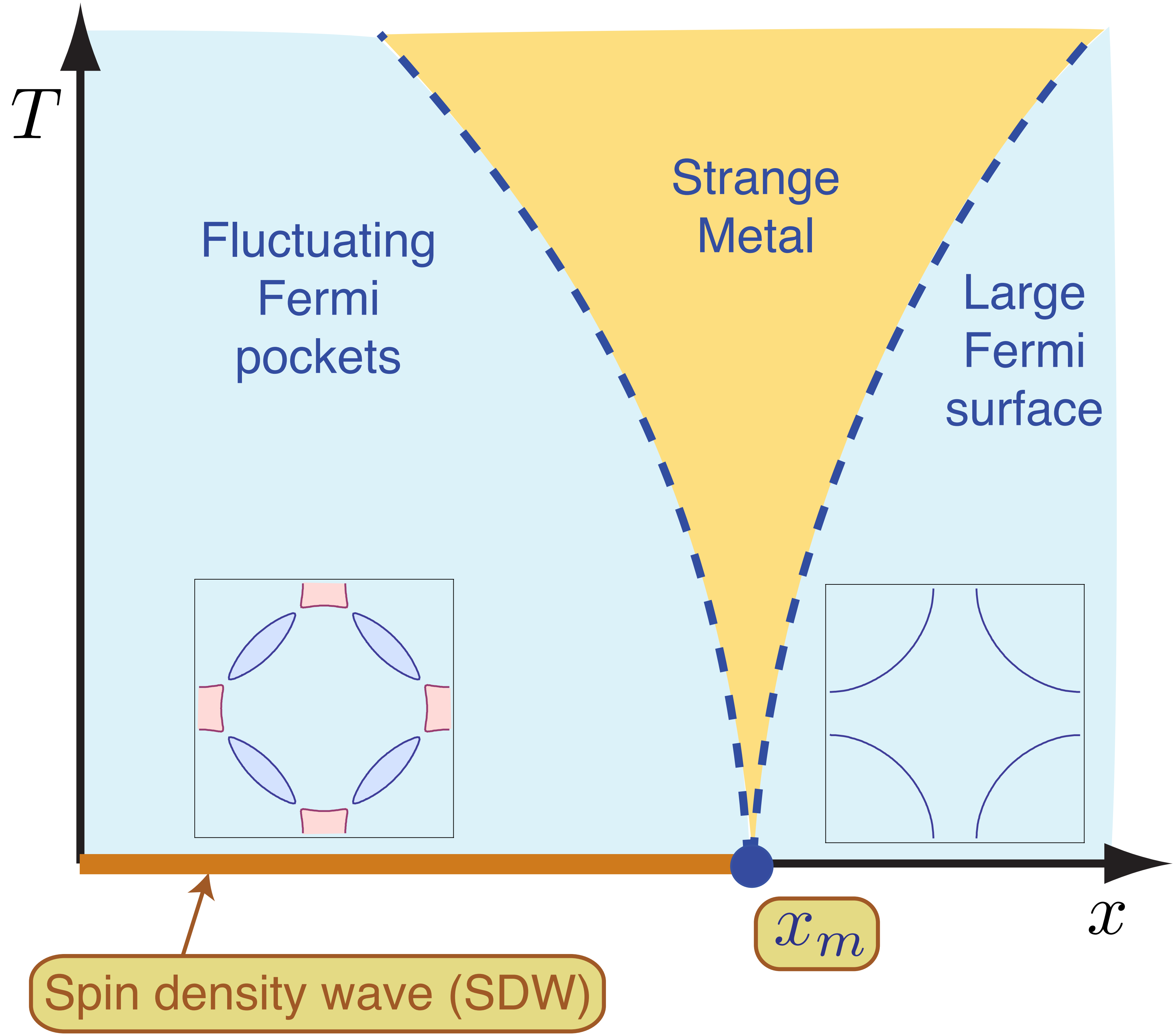


E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

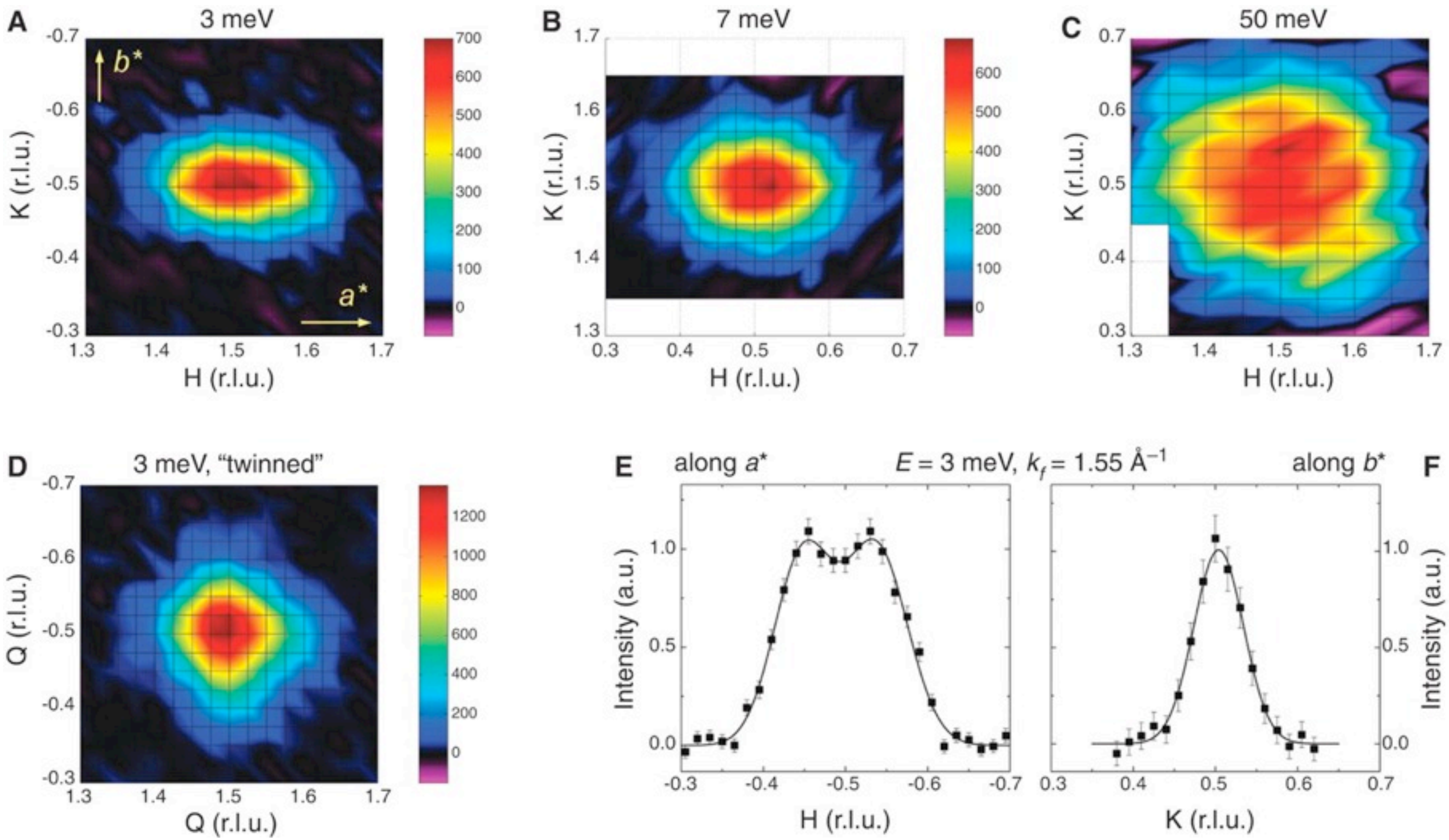
E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates

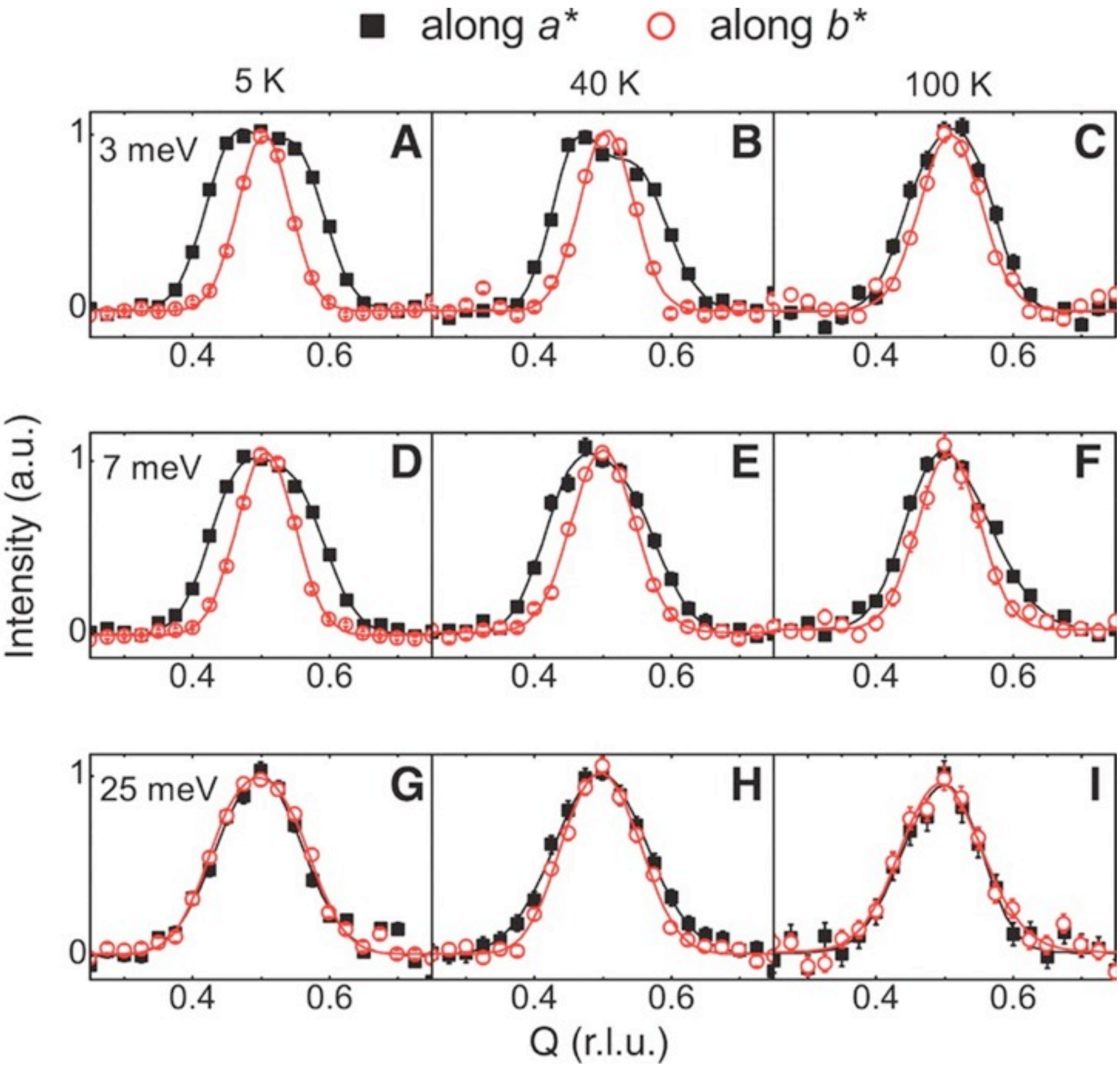


Underlying SDW ordering quantum critical point
in **metal** at $x = x_m$



Nematic order in YBCO

V. Hinkov, D. Haug, B. Fauqué, P. Bourges, Y. Sidis, A. Ivanov, C. Bernhard, C. T. Lin, and B. Keimer , *Science* **319**, 597 (2008)



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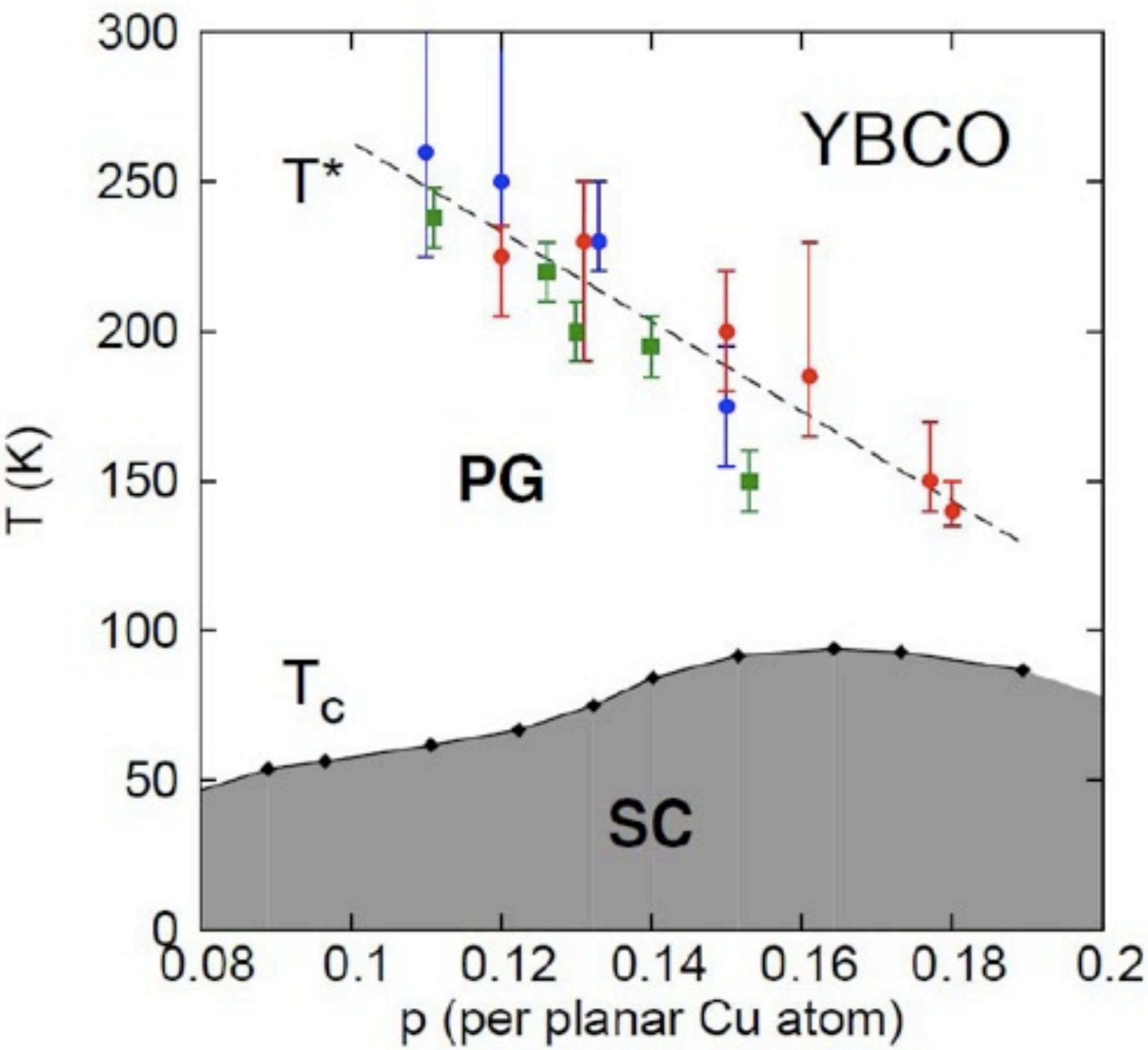
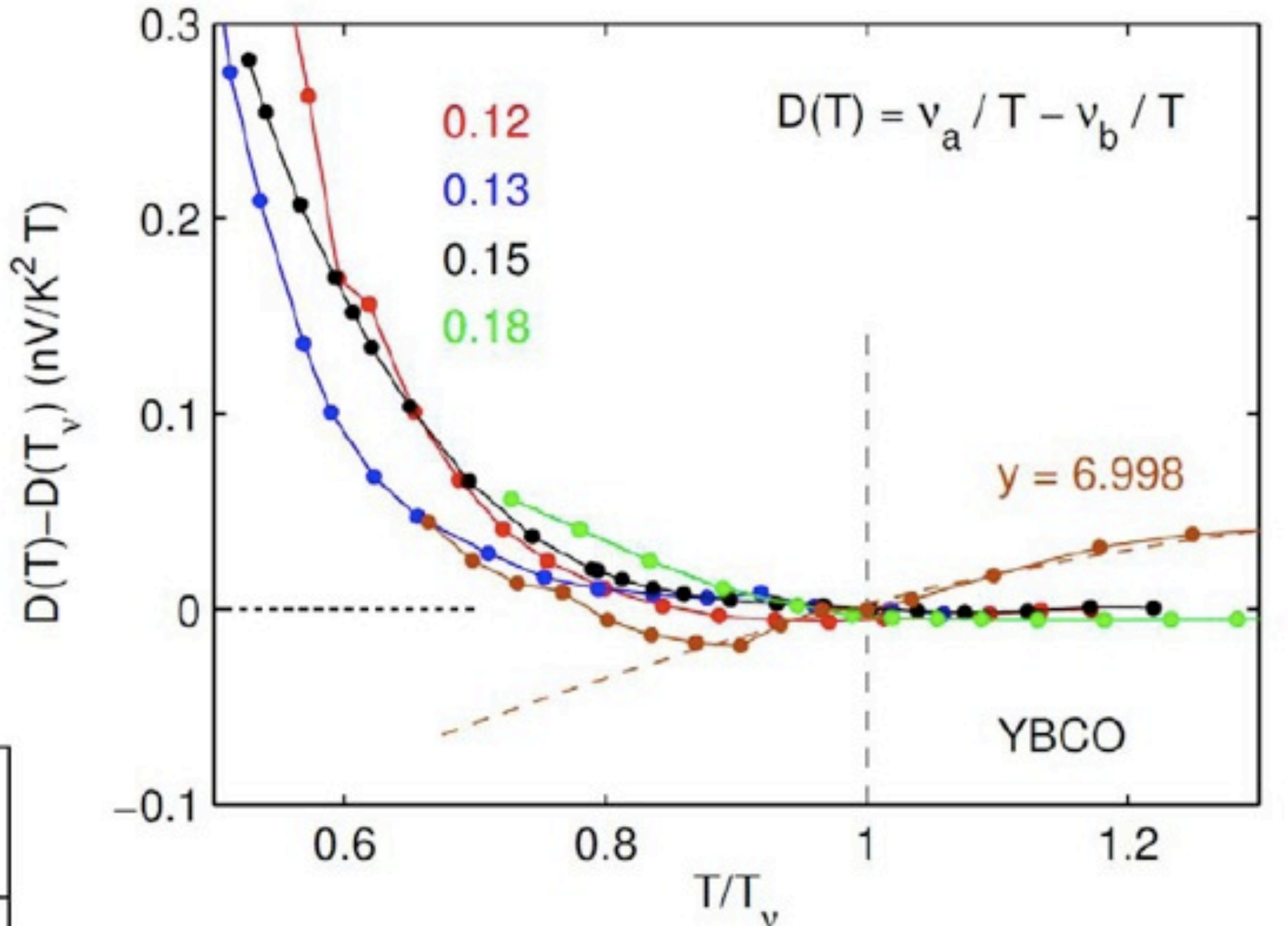
Nematic order in YBCO

Broken rotational symmetry in the pseudogap phase of a high-Tc superconductor

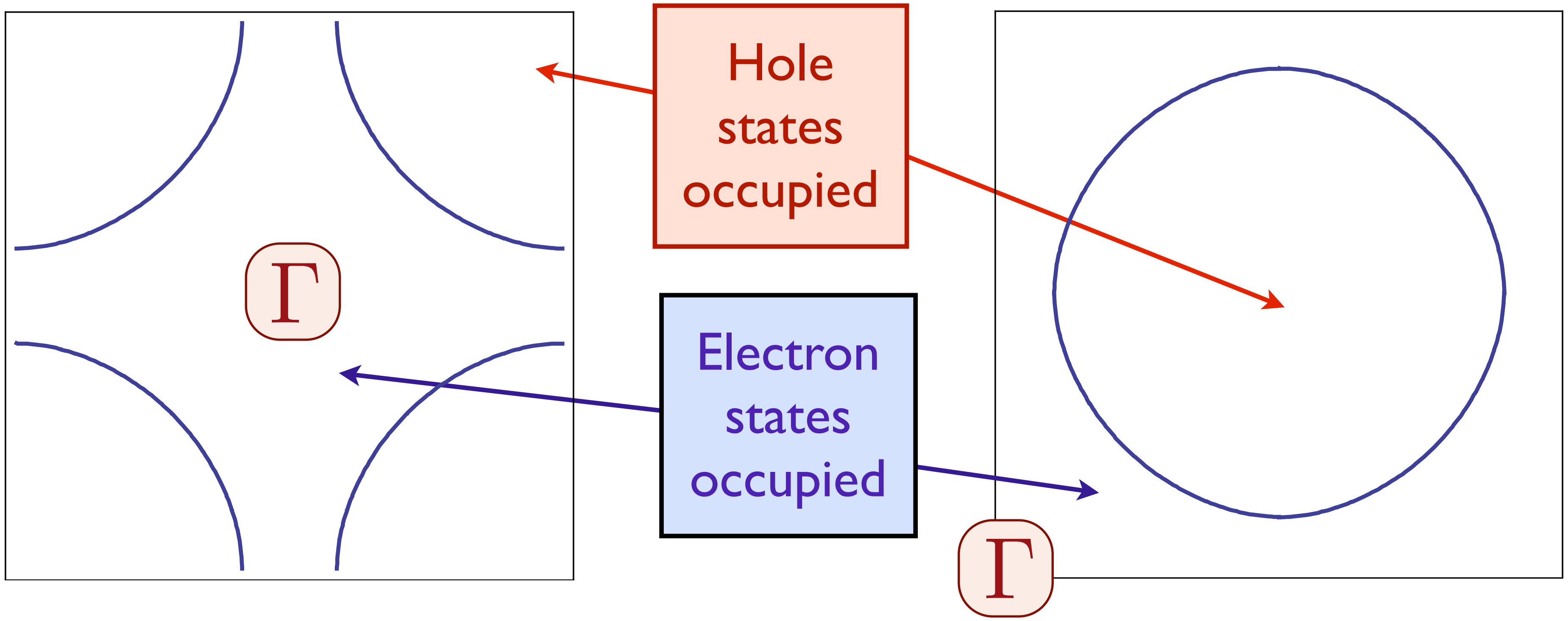
R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D.A. Bonn, W. N. Hardy, and Louis Taillefer
arXiv: 0909.4430





S. A. Kivelson, E. Fradkin, and V. J. Emery, *Nature* **393**, 550 (1998).

"Large" Fermi surfaces in cuprates



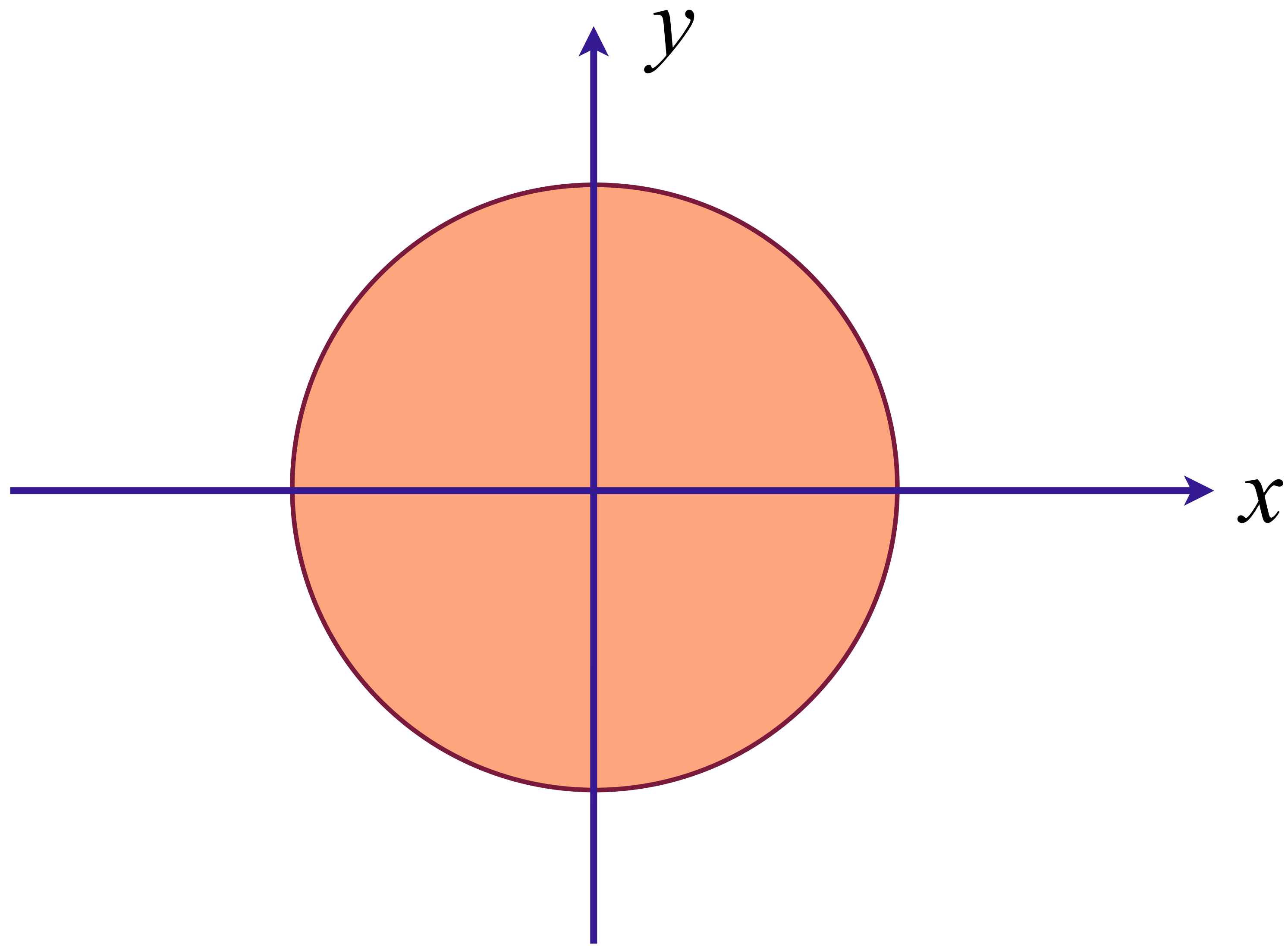
$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

The area of the occupied electron/hole states:

$$\mathcal{A}_e = \begin{cases} 2\pi^2(1-x) & \text{for hole-doping } x \\ 2\pi^2(1+p) & \text{for electron-doping } p \end{cases}$$

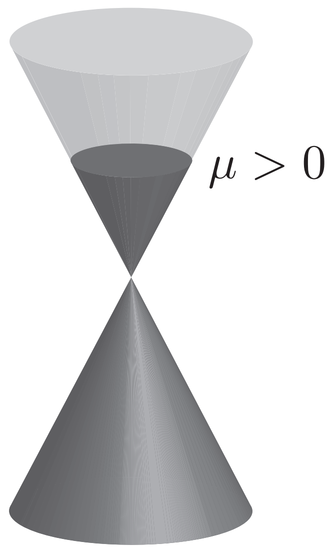
$$\mathcal{A}_h = 4\pi^2 - \mathcal{A}_e$$

Quantum criticality of Pomeranchuk instability



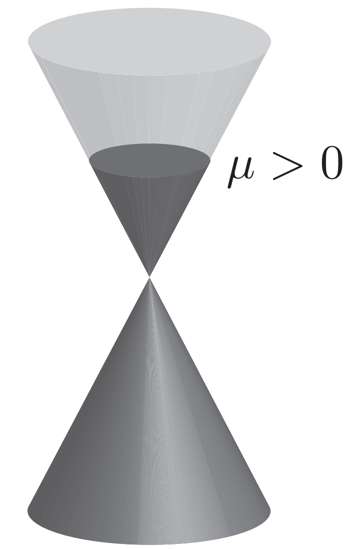
Fermi surface with full square lattice symmetry

Electron Green's function in Fermi liquid (T=0)



$$G(k,\omega) = \frac{Z}{\omega - v_F(k-k_F) - i\omega^2 \mathcal{F}\left(\frac{k-k_F}{\omega}\right)} + \ldots$$

Electron Green's function in Fermi liquid (T=0)

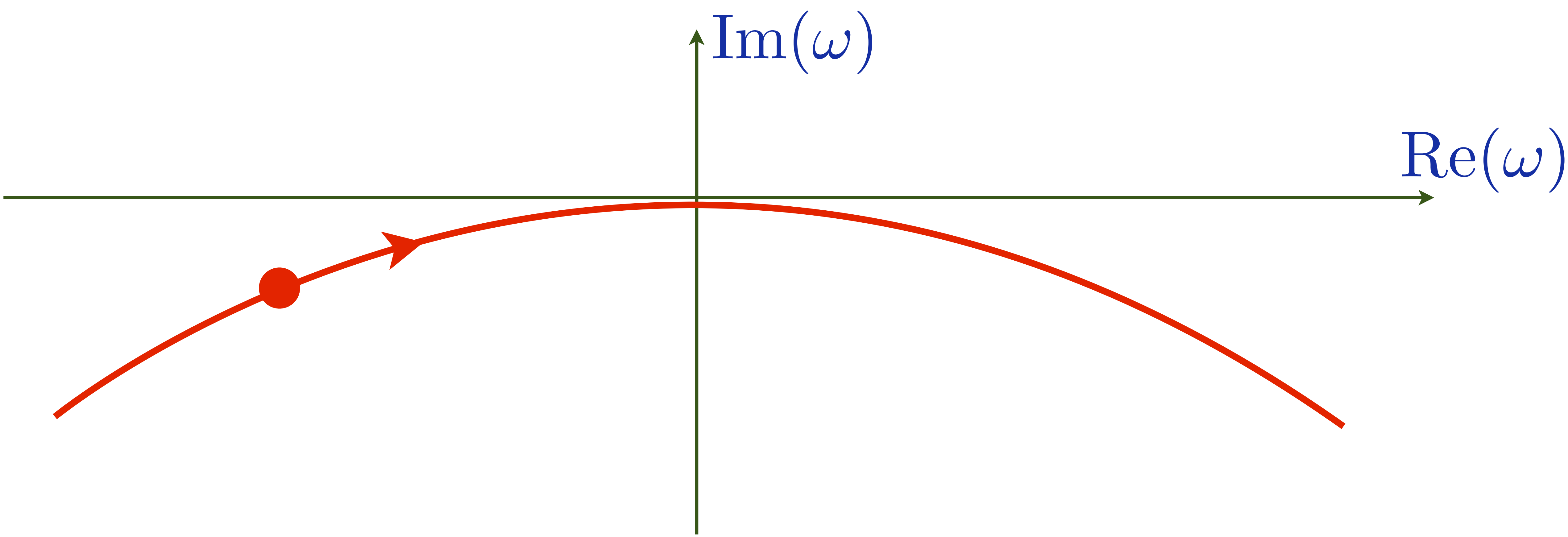


$$G(k,\omega) = \frac{Z}{\omega - v_F(k-k_F) - i\omega^2 \mathcal{F}\left(\frac{k-k_F}{\omega}\right)} + \ldots$$

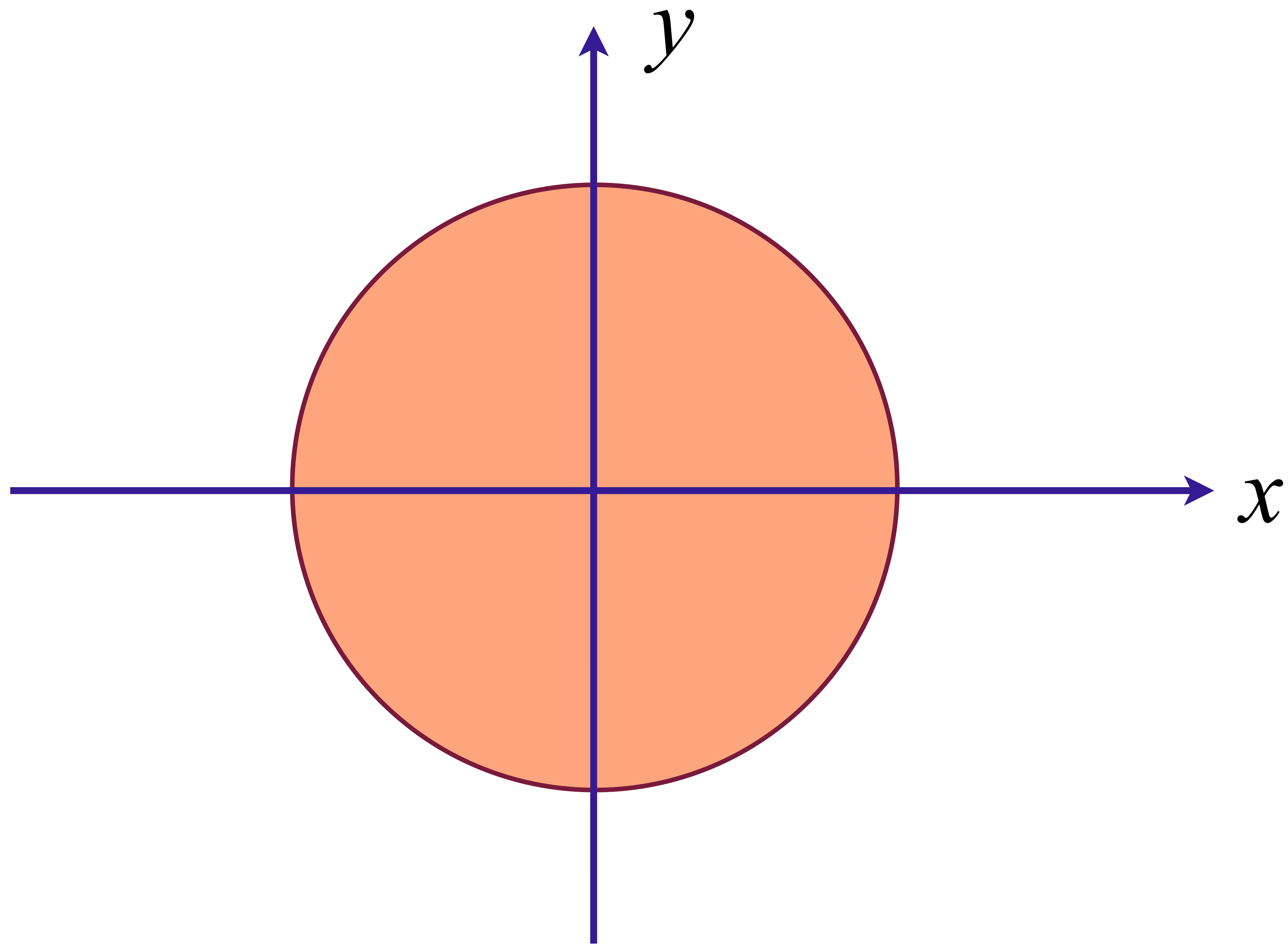
Green's function has a pole in the LHP at

$$\omega = v_F(k-k_F) - i\alpha(k-k_F)^2 + \ldots$$

Pole is at $\omega = 0$ precisely at $k = k_F$ *i.e.* on a sphere of radius k_F in momentum space. This is the *Fermi surface.*

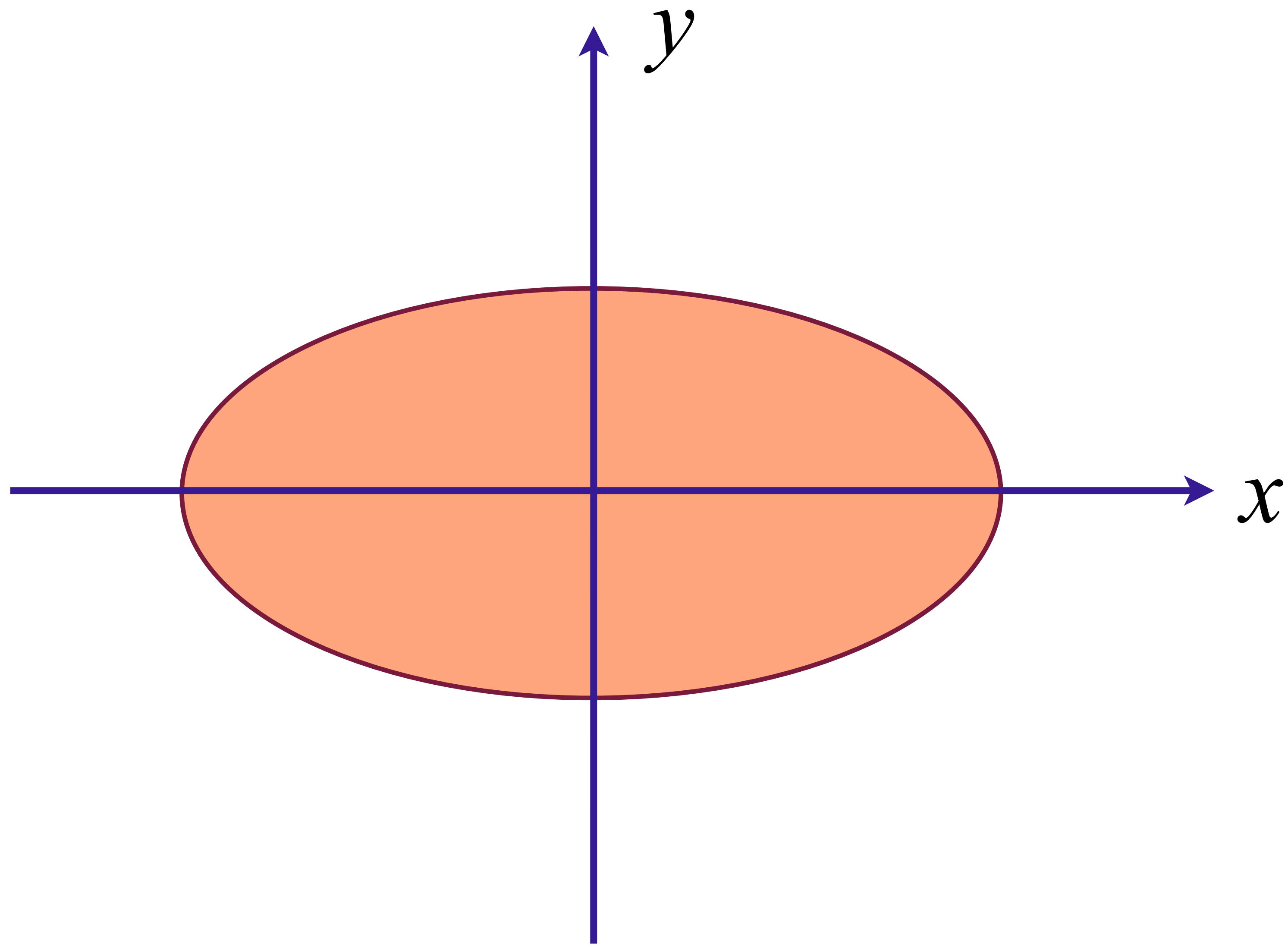


Quantum criticality of Pomeranchuk instability



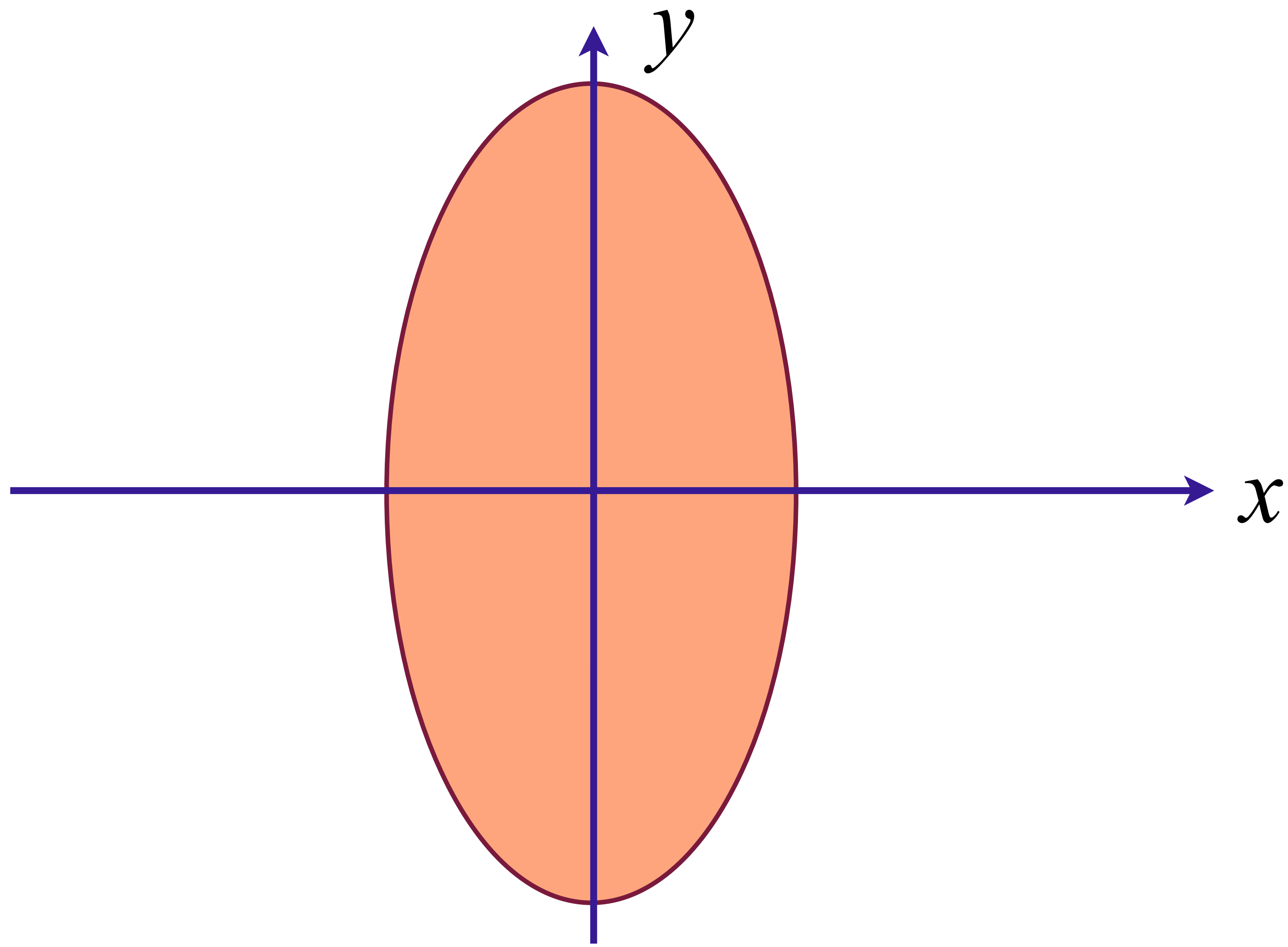
Fermi surface with full square lattice symmetry

Quantum criticality of Pomeranchuk instability



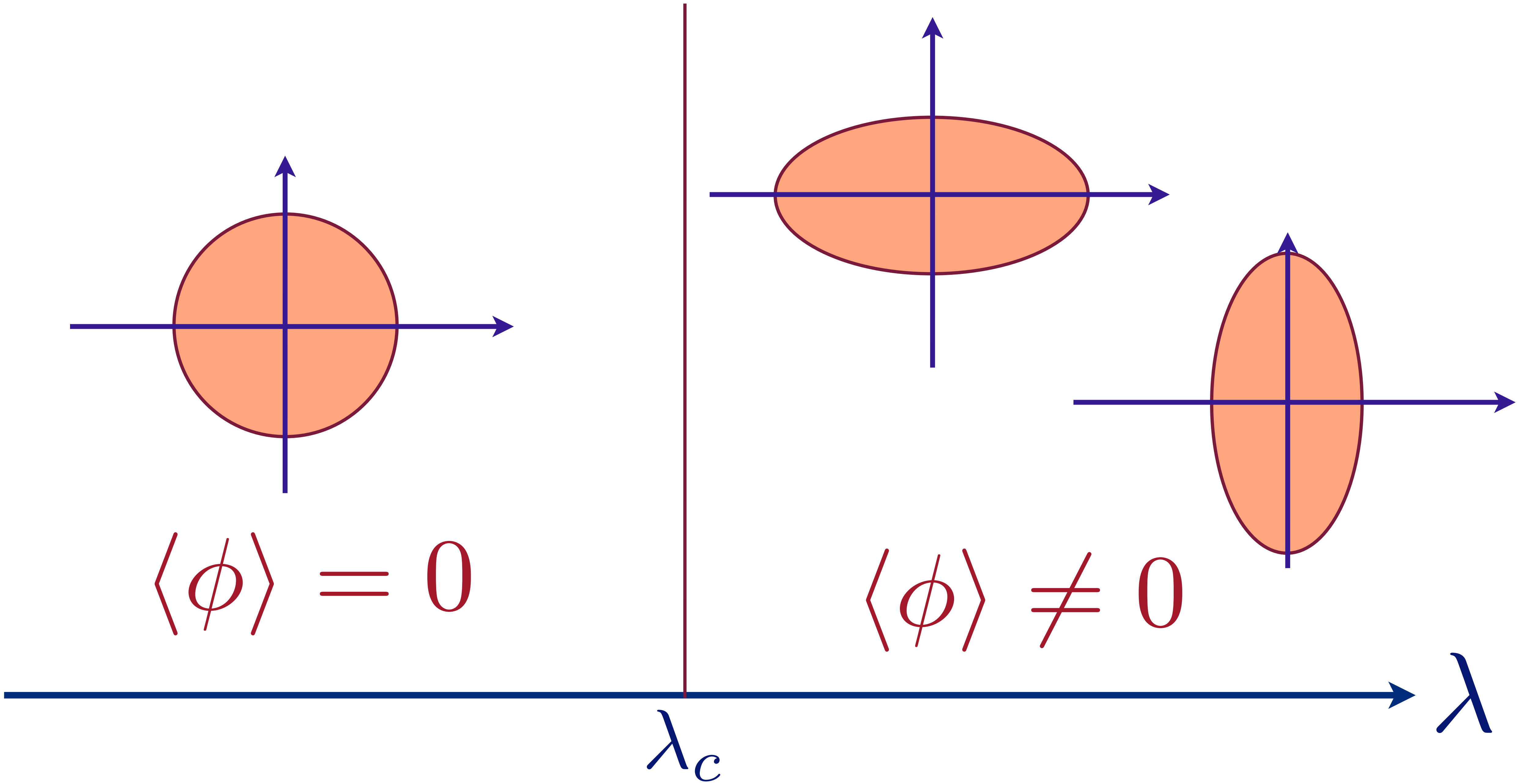
Spontaneous elongation along x direction:
Ising order parameter $\phi > 0$.

Quantum criticality of Pomeranchuk instability



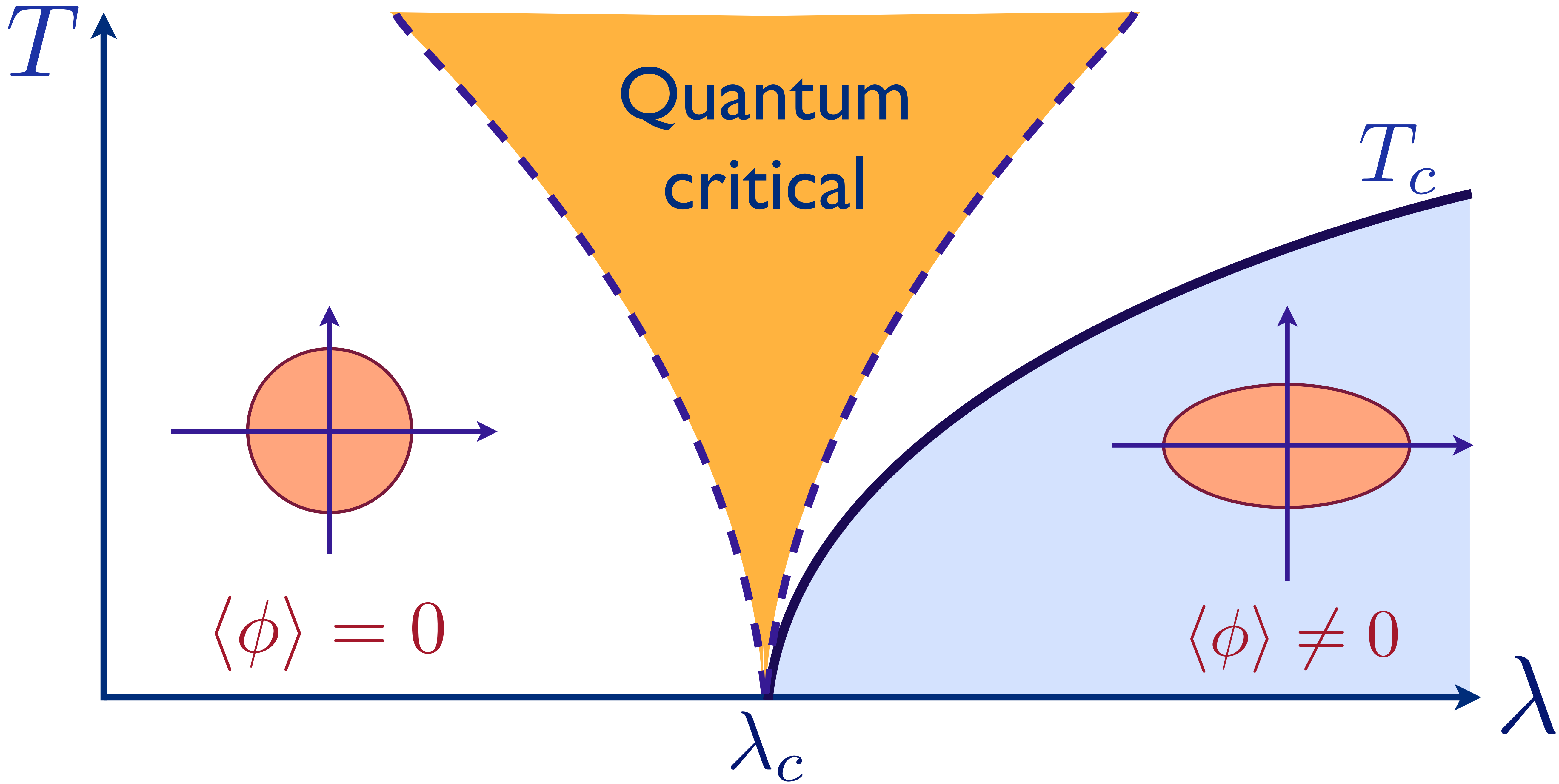
Spontaneous elongation along y direction:
Ising order parameter $\phi < 0$.

Quantum criticality of Pomeranchuk instability



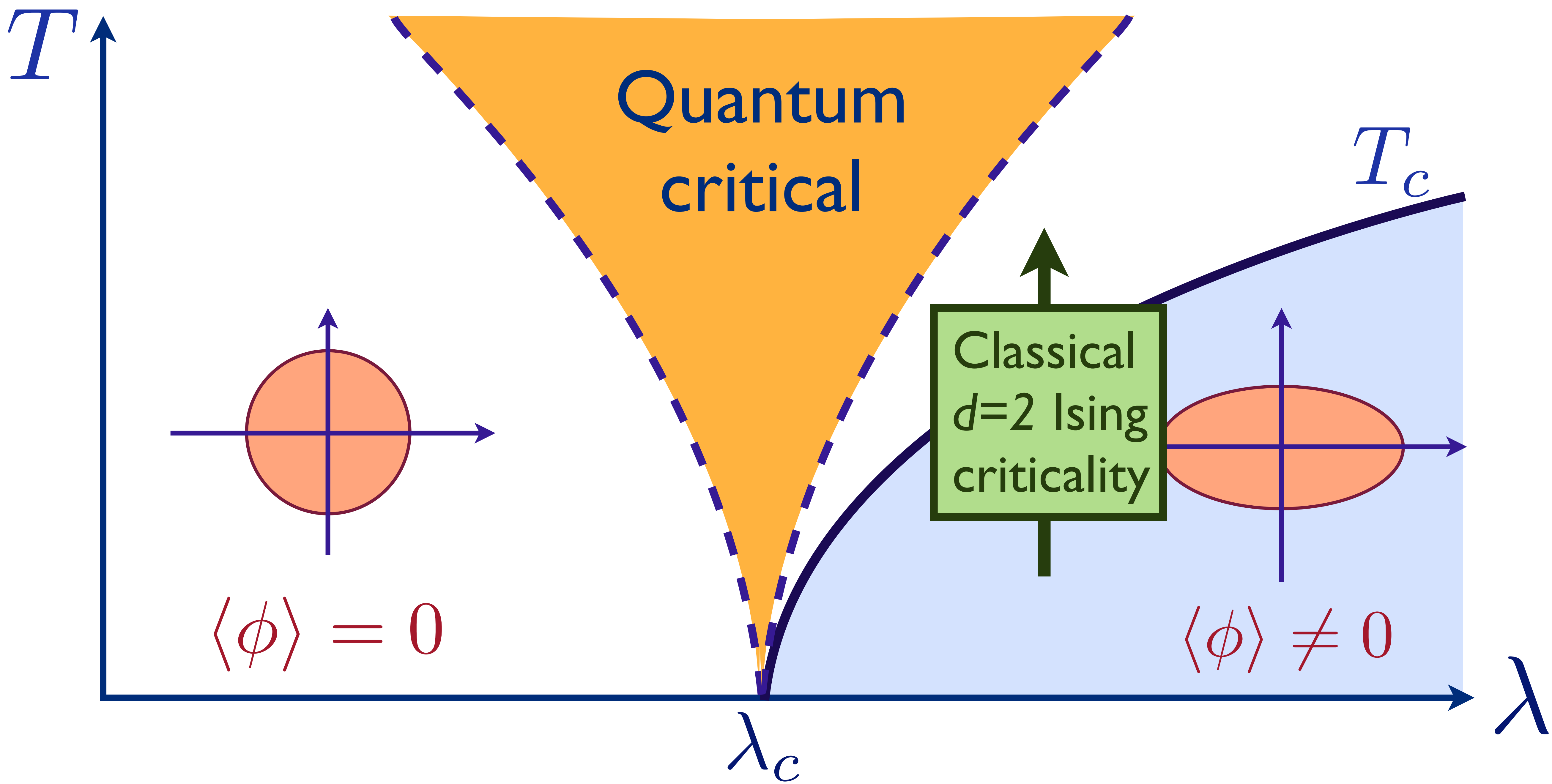
Pomeranchuk instability as a function of coupling λ

Quantum criticality of Pomeranchuk instability



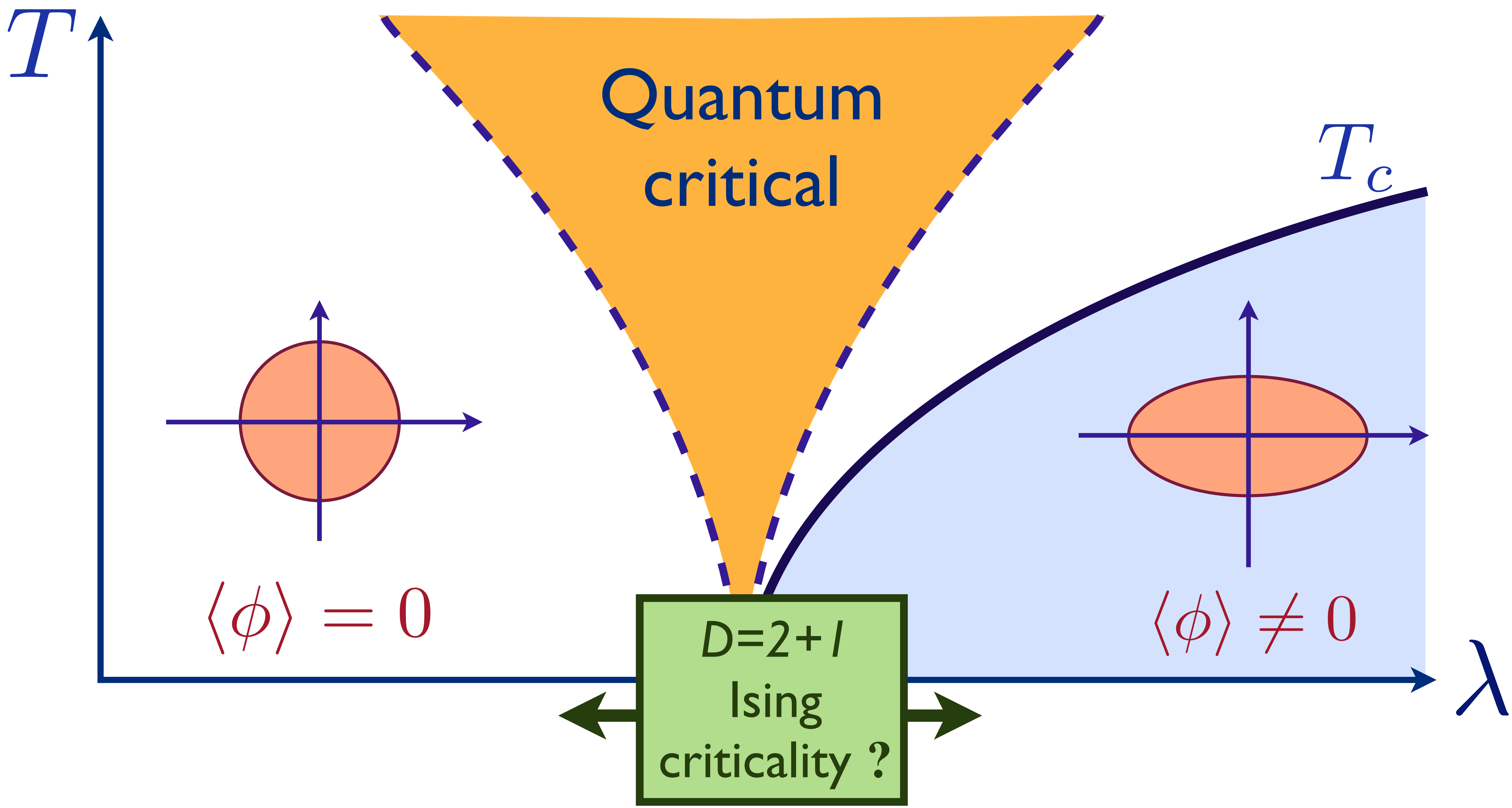
Phase diagram as a function of T and λ

Quantum criticality of Pomeranchuk instability



Phase diagram as a function of T and λ

Quantum criticality of Pomeranchuk instability



Phase diagram as a function of T and λ

Quantum criticality of Pomeranchuk instability

Effective action for Ising order parameter

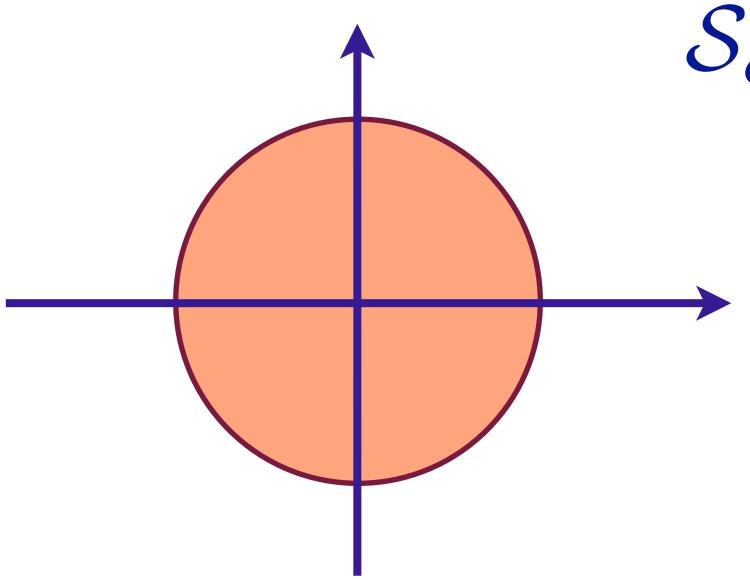
$$\mathcal{S}_\phi = \int d^2 r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

Quantum criticality of Pomeranchuk instability

Effective action for Ising order parameter

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

Effective action for electrons:



$$\mathcal{S}_c = \int d\tau \sum_{\alpha=1}^{N_f} \left[\sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \right]$$

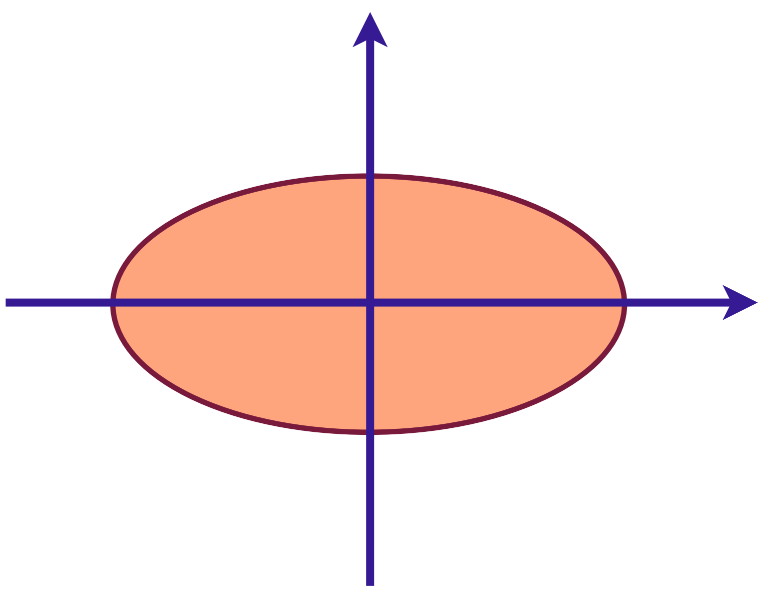
$$\equiv \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

Quantum criticality of Pomeranchuk instability

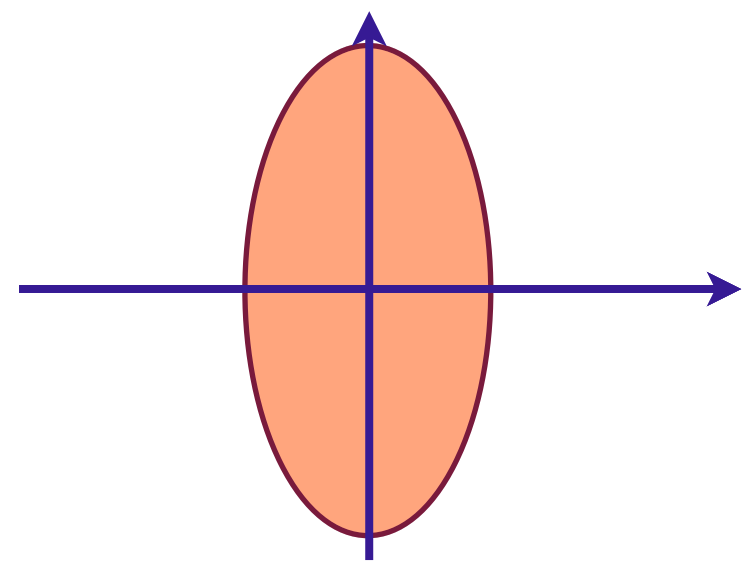
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \, \phi \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} (\cos k_x - \cos k_y) c_{\mathbf{k}\alpha}^{\dagger} c_{\mathbf{k}\alpha}$$

for spatially independent ϕ



$$\langle \phi \rangle > 0$$



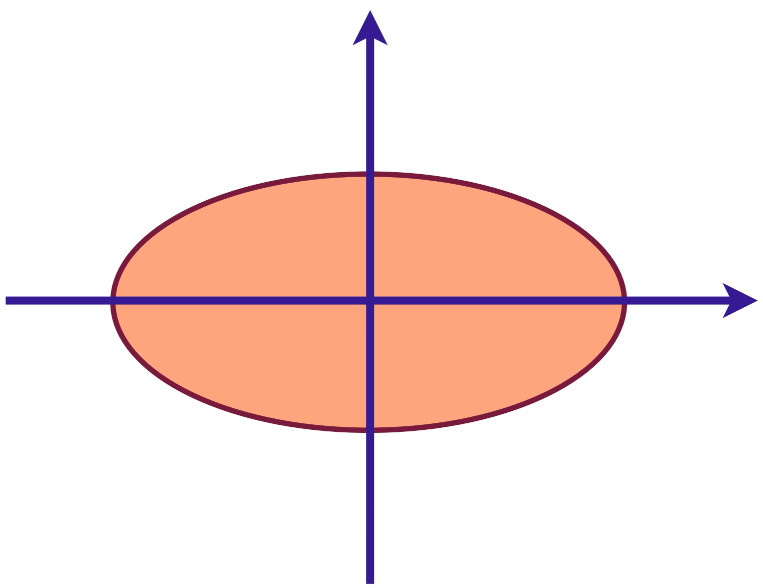
$$\langle \phi \rangle < 0$$

Quantum criticality of Pomeranchuk instability

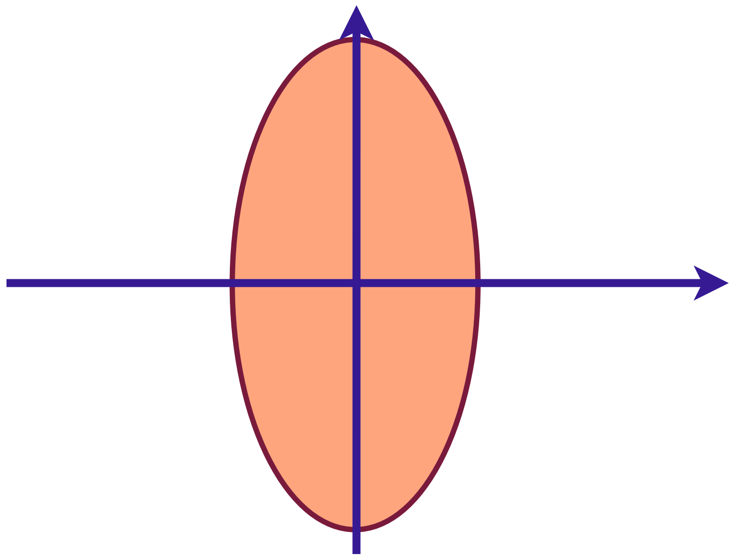
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y\right) c^{\dagger}_{\mathbf{k}+\mathbf{q}/2,\alpha} c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

for spatially dependent ϕ



$\langle \phi \rangle > 0$



$\langle \phi \rangle < 0$

Quantum criticality of Pomeranchuk instability

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y \right) c_{\mathbf{k}+\mathbf{q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

Quantum critical field theory

$$\mathcal{Z} = \int \mathcal{D}\phi \mathcal{D}c_{i\alpha} \exp\left(-\mathcal{S}_\phi - \mathcal{S}_c - \mathcal{S}_{\phi c} \right)$$

Quantum criticality of Pomeranchuk instability

Hertz theory

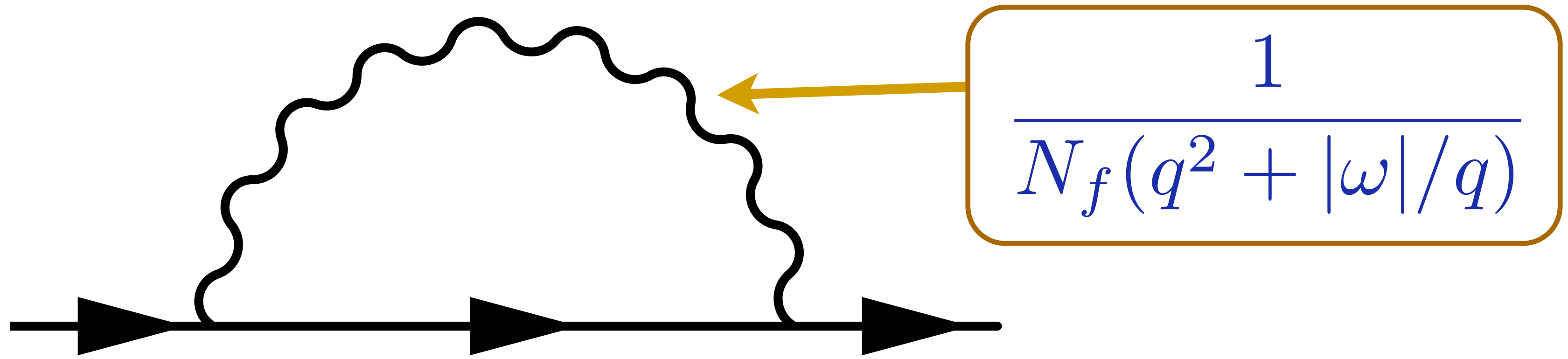
Integrate out c_α fermions and obtain non-local corrections to ϕ action

$$\delta \mathcal{S}_\phi \sim N_f \gamma^2 \int \frac{d^2 q}{4\pi^2} \int \frac{d\omega}{2\pi} |\phi(\mathbf{q},\omega)|^2 \left[\frac{|\omega|}{q} + q^2 \right] + \ldots$$

This leads to a critical point with dynamic critical exponent $z = 3$ and quantum criticality controlled by the Gaussian fixed point.

Quantum criticality of Pomeranchuk instability

Hertz theory



Self energy of c_α fermions to order $1/N_f$

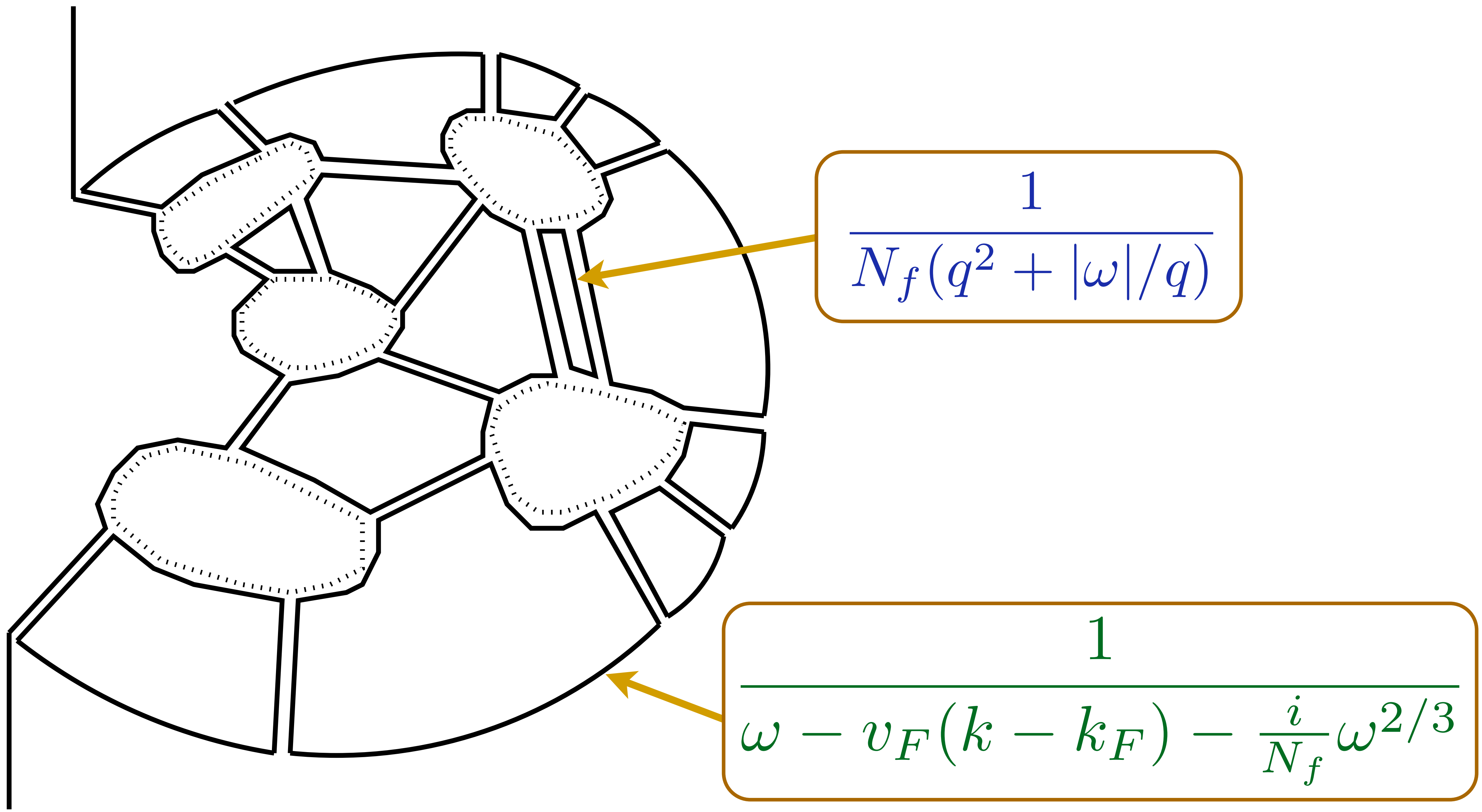
$$\Sigma_c(k,\omega) \sim \frac{i}{N_f}\omega^{2/3}$$

This leads to the Green's function

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - \frac{i}{N_f}\omega^{2/3}}$$

Note that the order $1/N_f$ term is more singular in the infrared than the bare term; this leads to problems in the bare $1/N_f$ expansion in terms that are dominated by low frequency fermions.

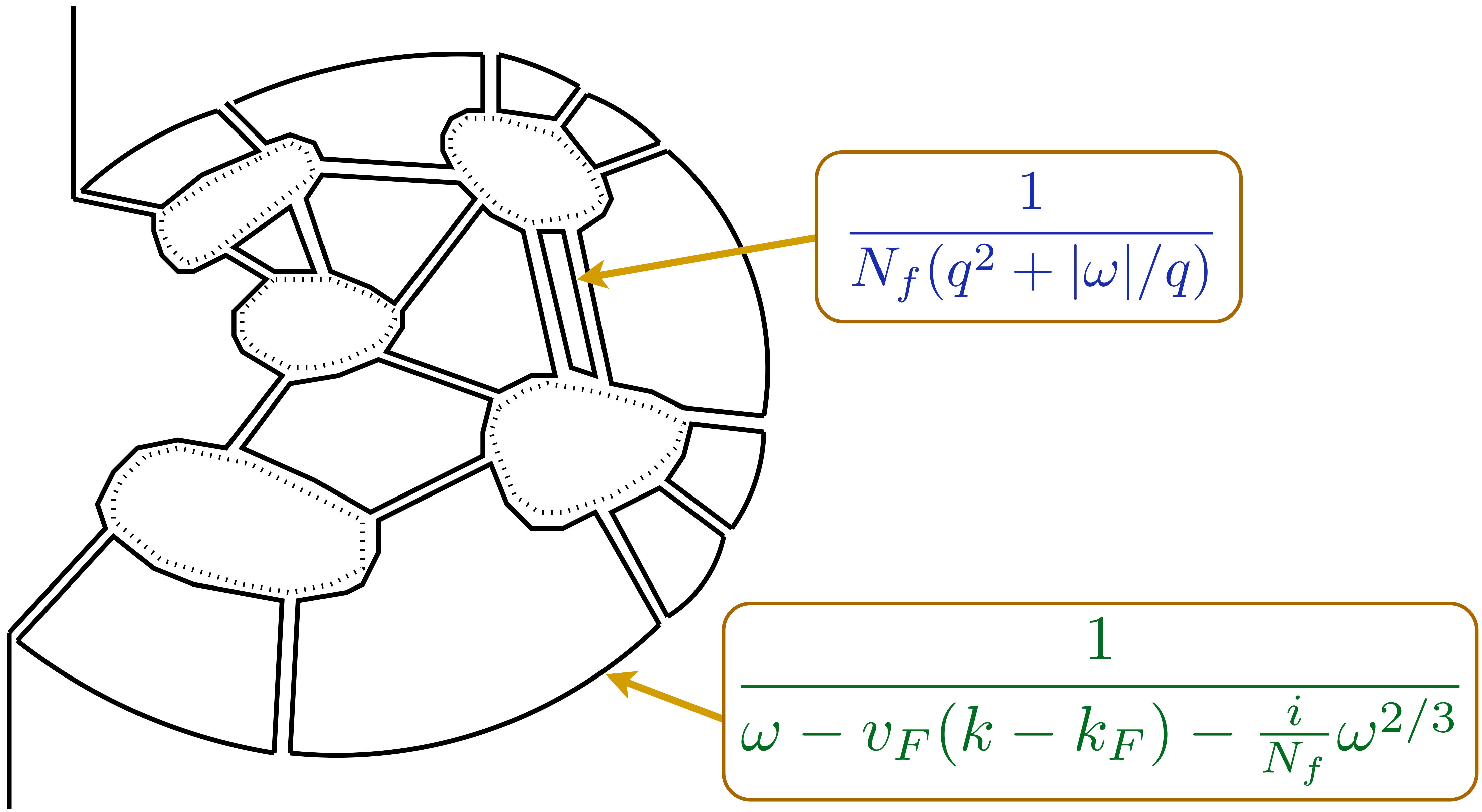
Quantum criticality of Pomeranchuk instability



The infrared singularities of fermion particle-hole pairs are most severe on planar graphs: these all contribute at leading order in $1/N_f$.

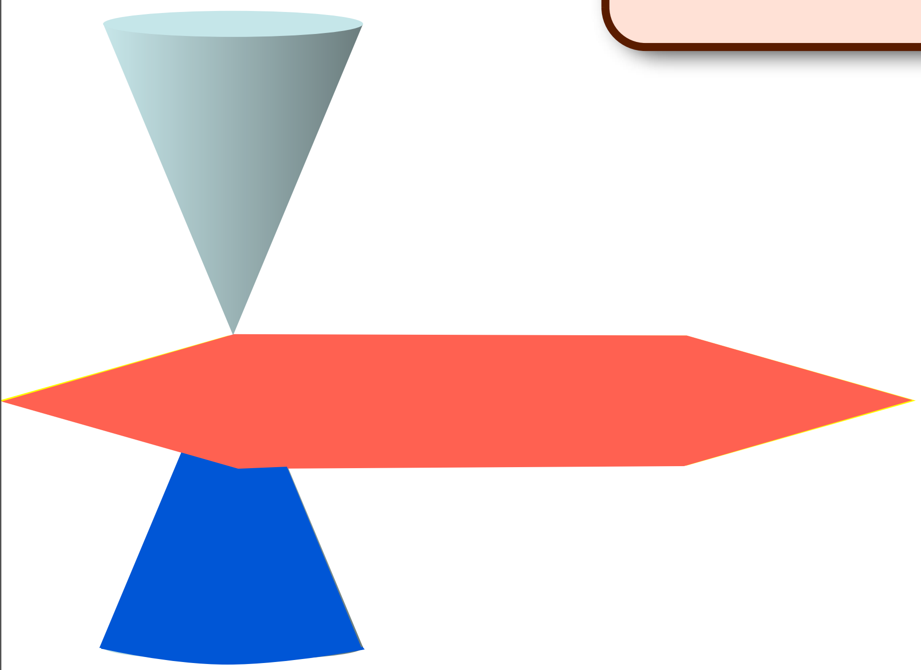
Sung-Sik Lee, *Physical Review* B **80**, 165102 (2009)

Quantum criticality of Pomeranchuk instability



A string theory for the Fermi surface ?

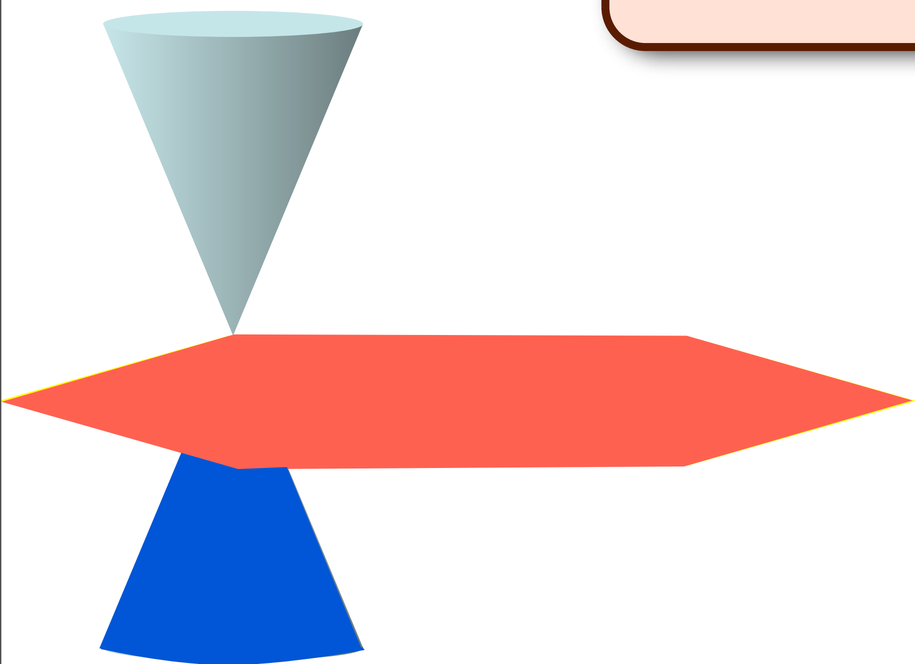
Conformal field theory
in 2+1 dimensions at $T = 0$



$\Updownarrow$

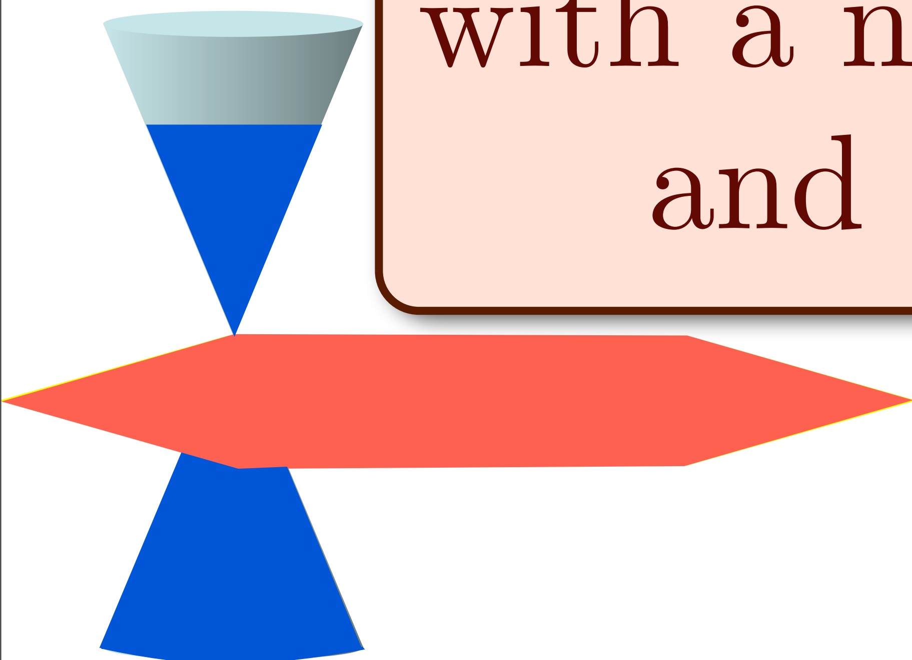
Einstein gravity
on AdS_4

Conformal field theory
in 2+1 dimensions at $T > 0$

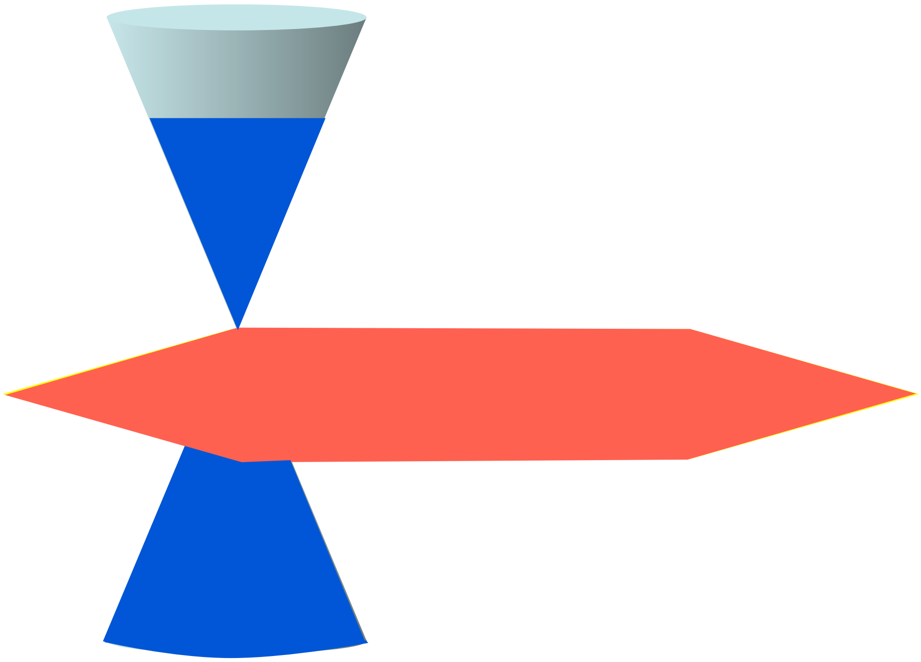


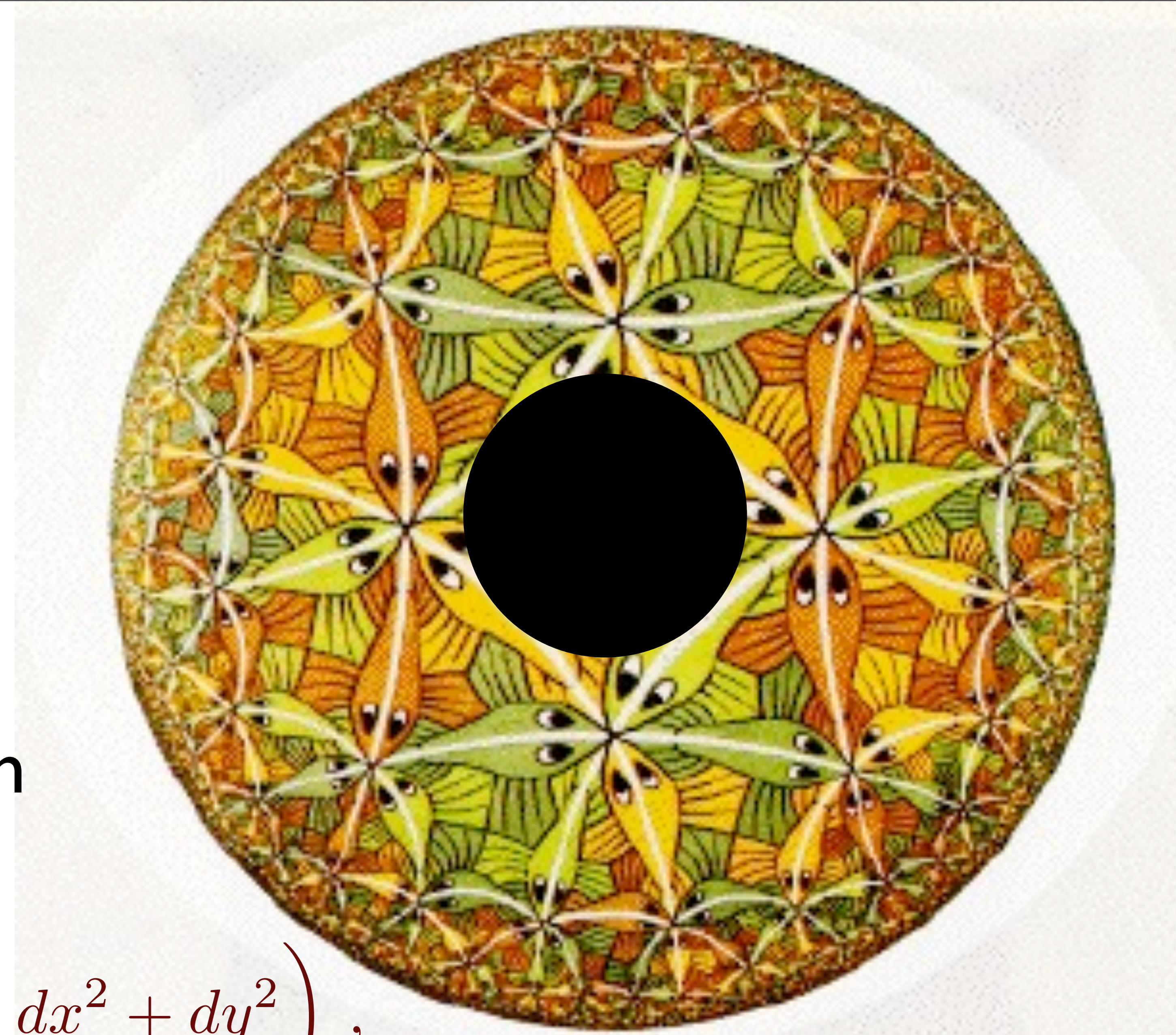
Einstein gravity on AdS_4
with a Schwarzschild
black hole

Conformal field theory
in 2+1 dimensions at $T > 0$,
with a non-zero chemical potential, μ
and applied magnetic field, B



Einstein gravity on AdS_4
with a Reissner-Nordstrom
black hole carrying electric
and magnetic charges





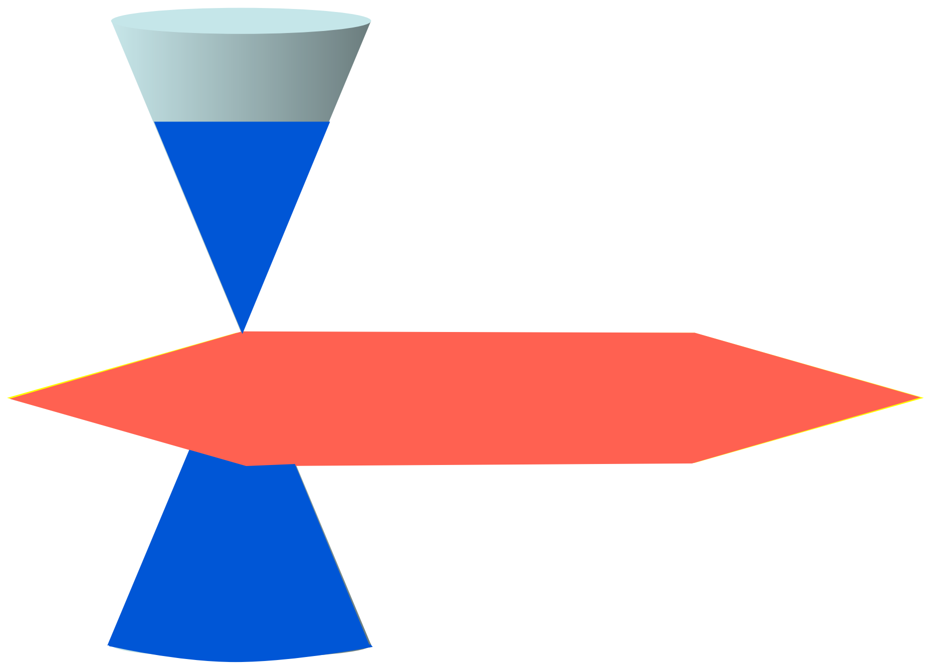
AdS_4-Reissner-Nordstrom black hole

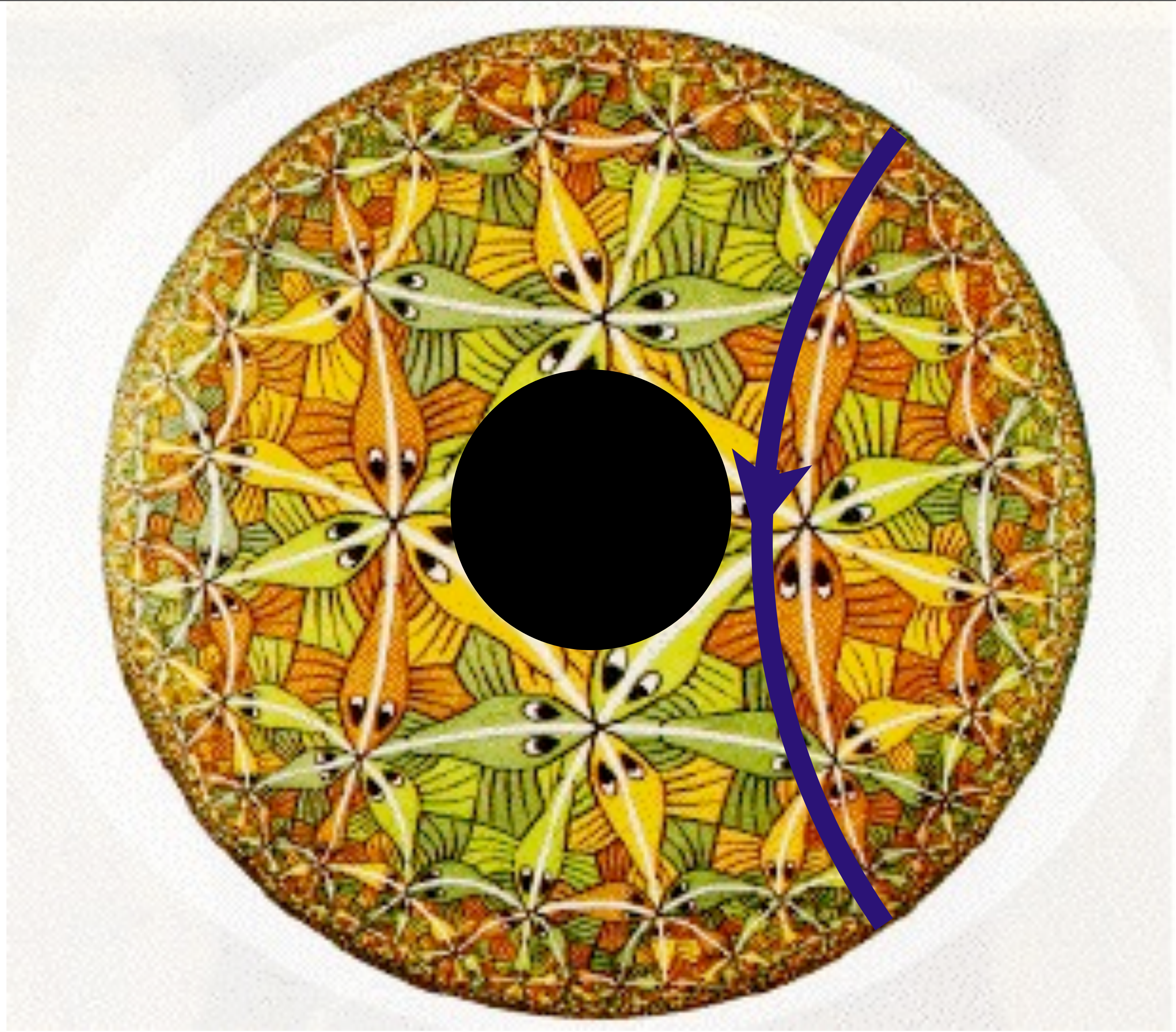
$$ds^2 = \frac{L^2}{r^2}\left(f(r)d\tau^2 + \frac{dr^2}{f(r)} + dx^2 + dy^2\right),$$

$$f(r) = 1 - \left(1 + \frac{(r_+^2\mu^2 + r_+^4 B^2)}{\gamma^2}\right)\left(\frac{r}{r_+}\right)^3 + \frac{(r_+^2\mu^2 + r_+^4 B^2)}{\gamma^2}\left(\frac{r}{r_+}\right)^4,$$

$$A = i\mu\left[1 - \frac{r}{r_+}\right]d\tau + Bx\,dy\,.$$

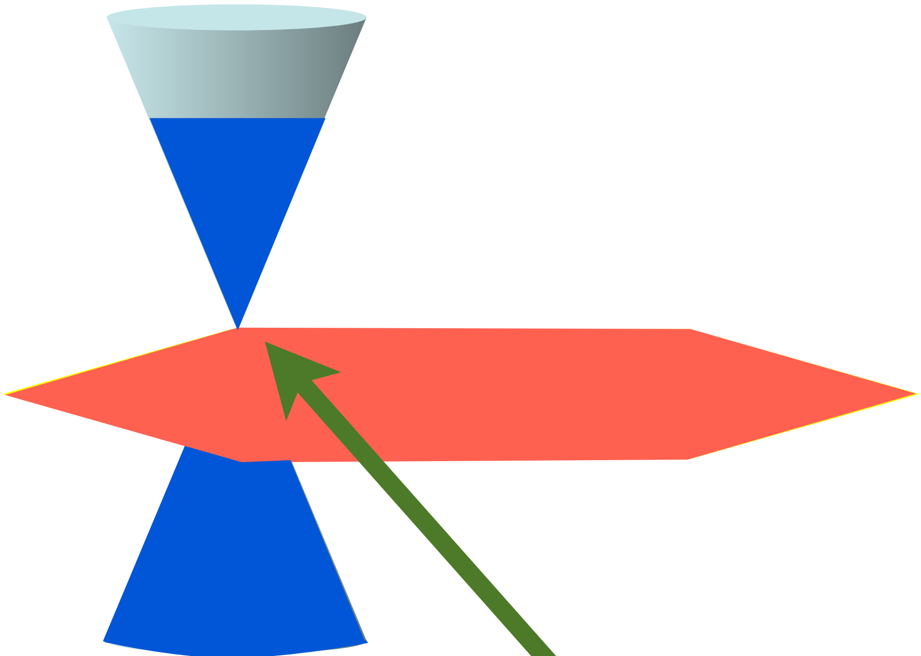
$$T = \frac{1}{4\pi r_+}\left(3 - \frac{r_+^2\mu^2}{\gamma^2} - \frac{r_+^4 B^2}{\gamma^2}\right).$$

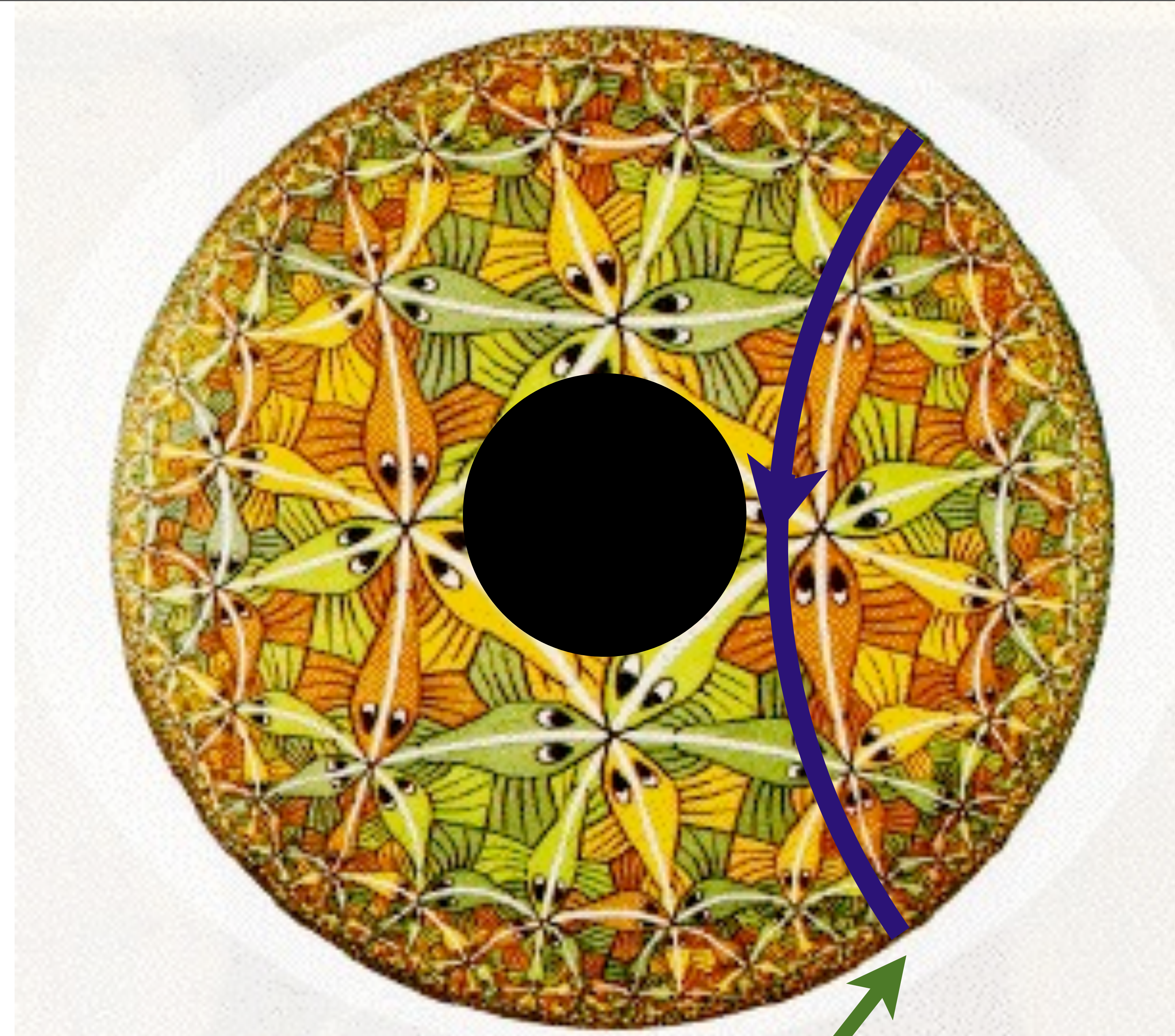




Examine free energy and Green's function of a probe particle

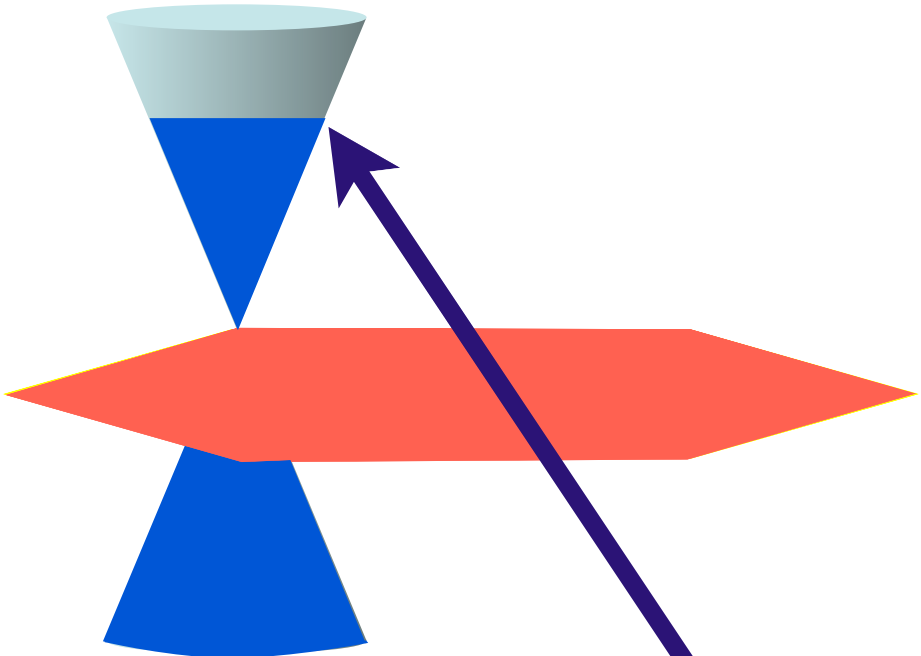
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788

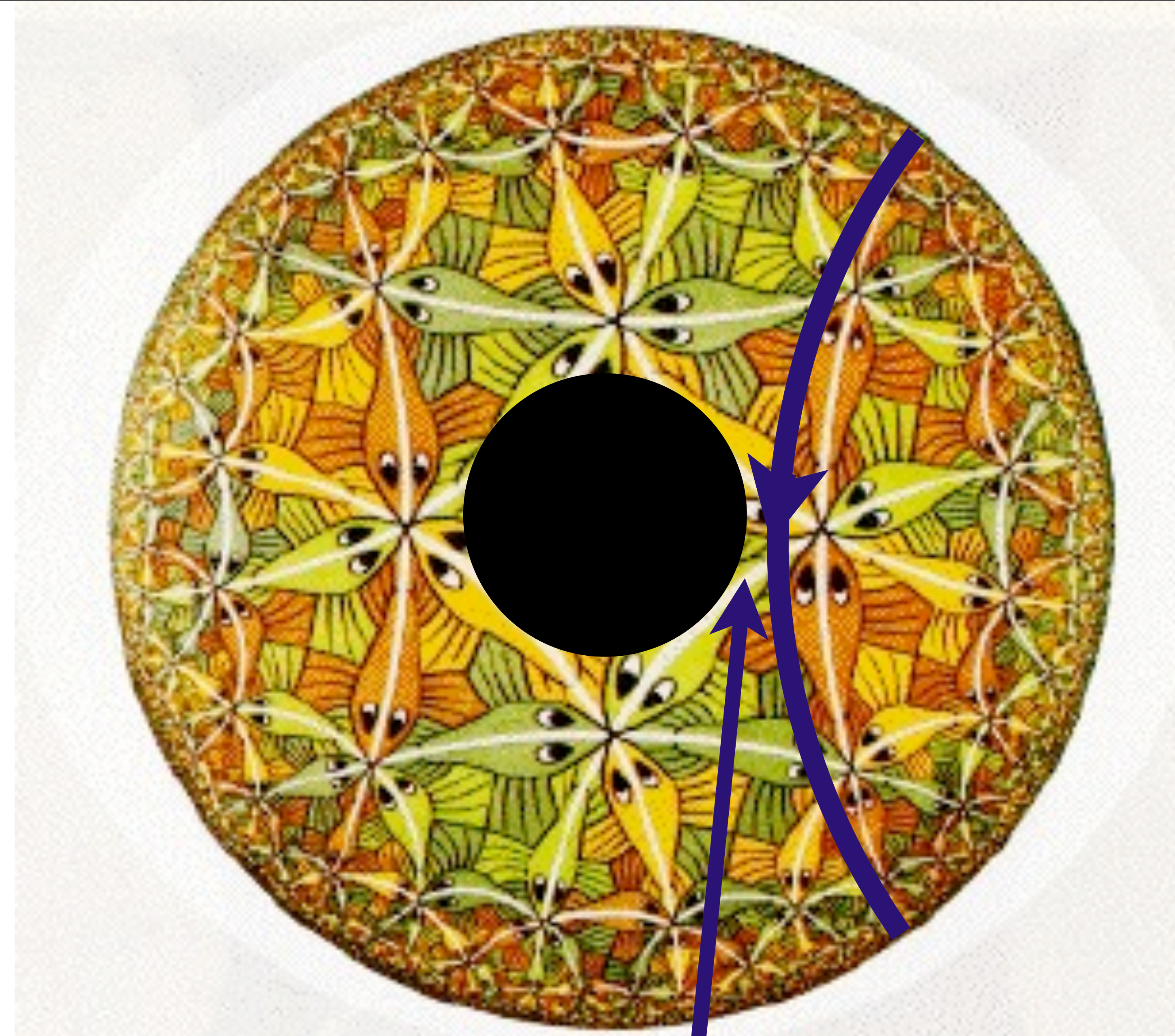




Short time behavior depends upon conformal AdS_4 geometry near boundary

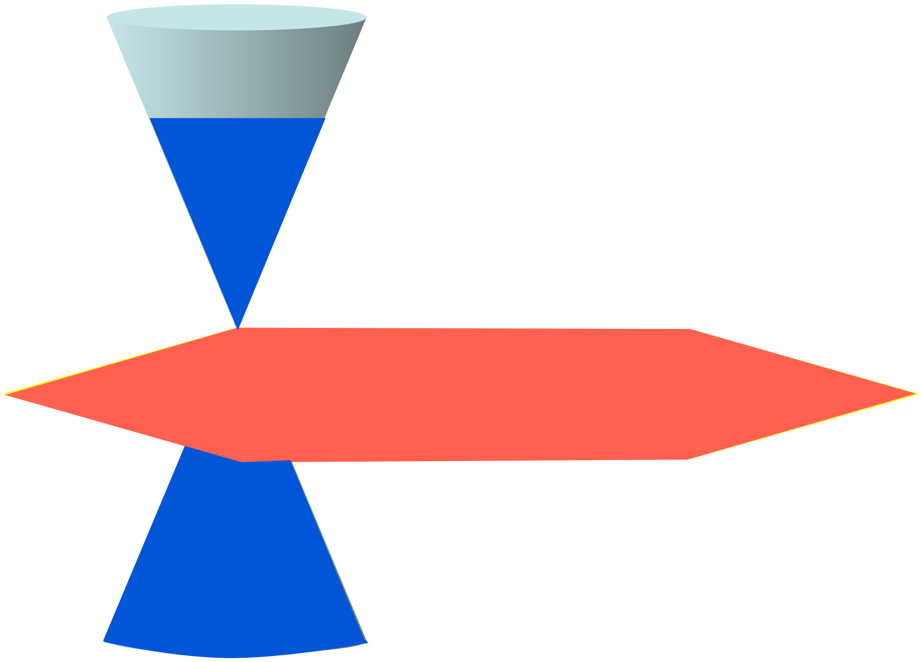
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788

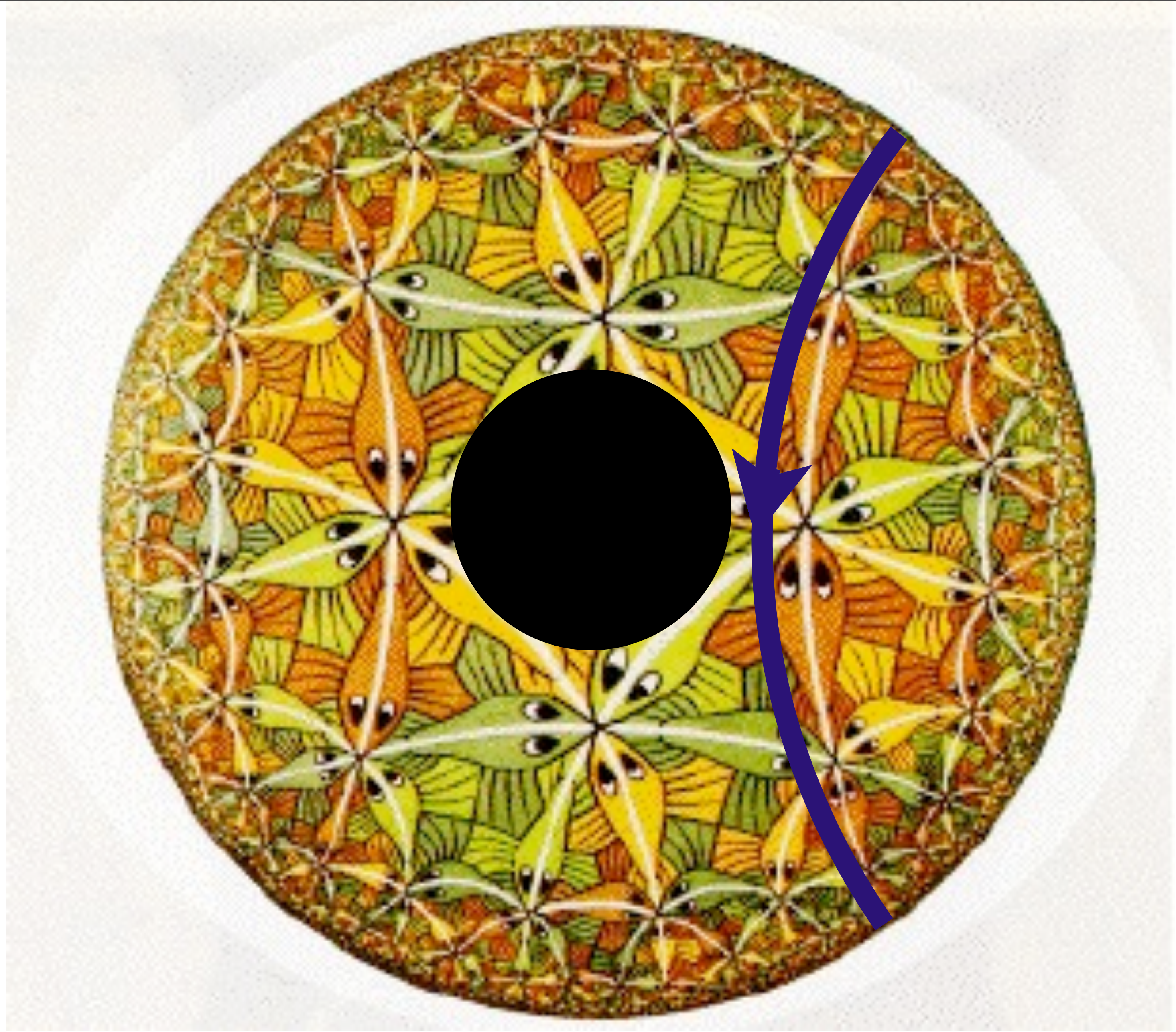




Long time behavior depends upon near-horizon geometry of black hole

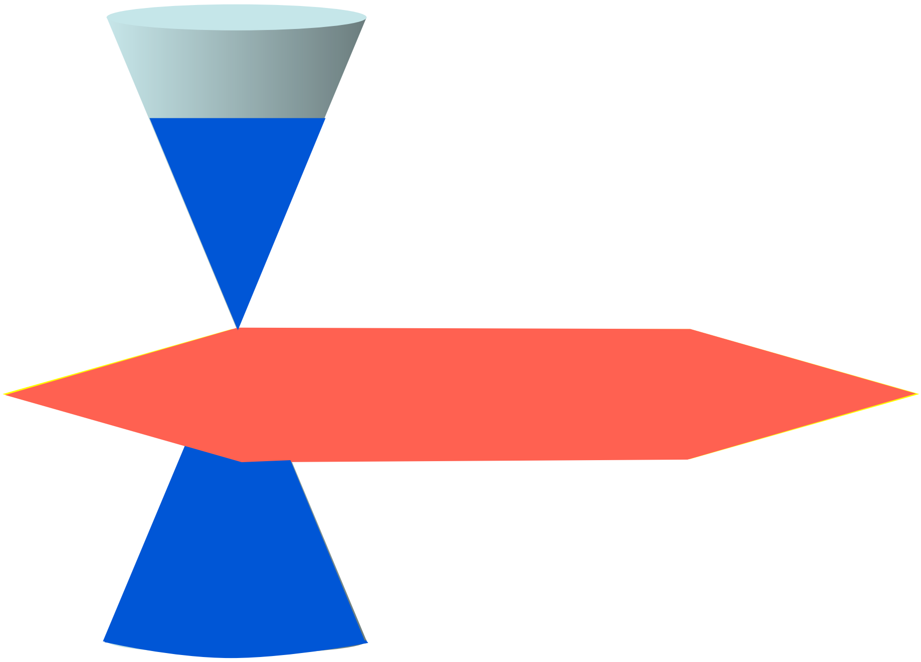
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788

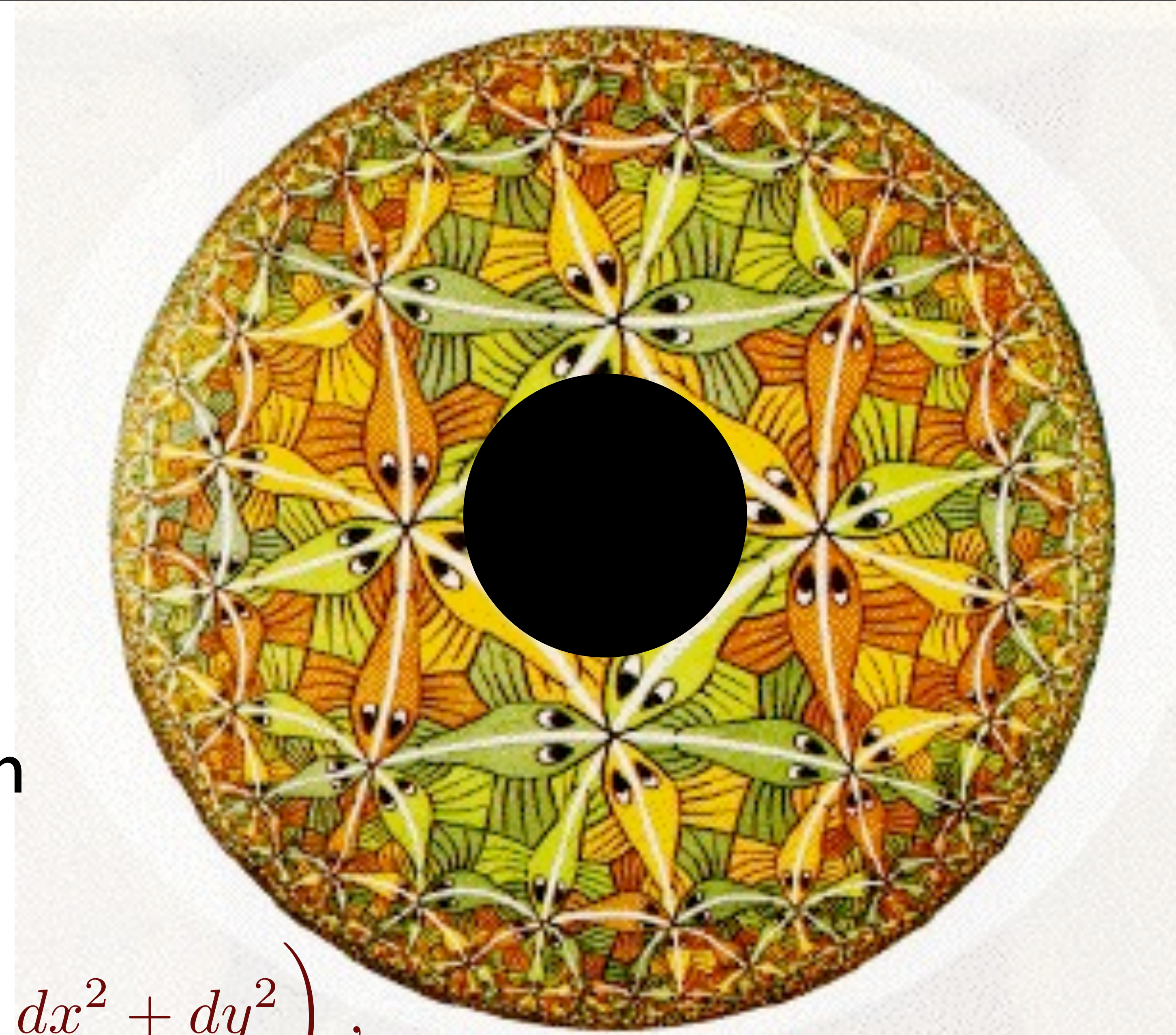




Radial direction of gravity theory is measure of energy scale in CFT

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788





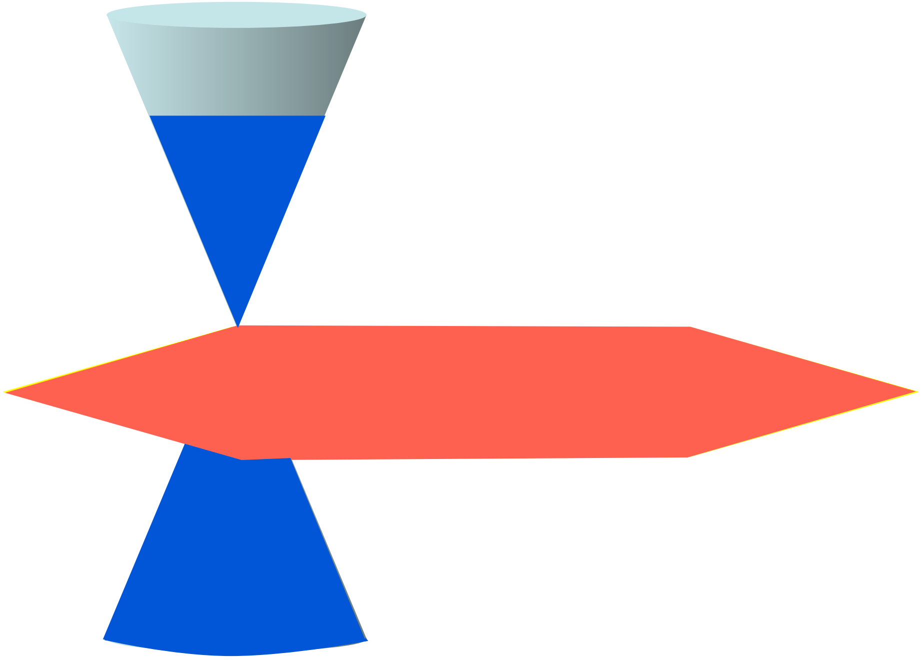
AdS_4-Reissner-Nordstrom black hole

$$ds^2 = \frac{L^2}{r^2}\left(f(r)d\tau^2 + \frac{dr^2}{f(r)} + dx^2 + dy^2\right),$$

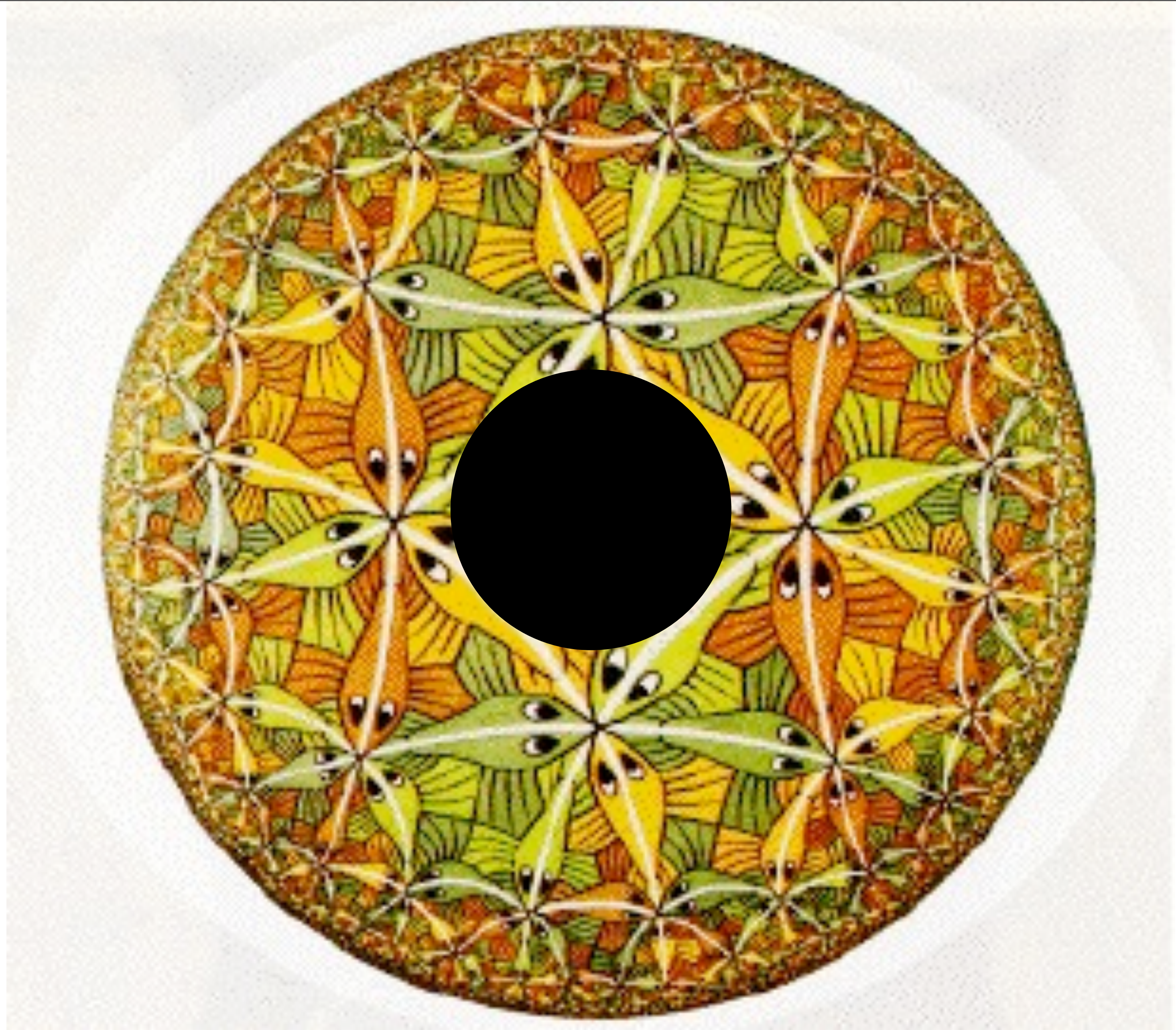
$$f(r) = 1 - \left(1 + \frac{(r_+^2\mu^2 + r_+^4 B^2)}{\gamma^2}\right)\left(\frac{r}{r_+}\right)^3 + \frac{(r_+^2\mu^2 + r_+^4 B^2)}{\gamma^2}\left(\frac{r}{r_+}\right)^4,$$

$$A = i\mu\left[1 - \frac{r}{r_+}\right]d\tau + Bx\,dy\,.$$

$$T = \frac{1}{4\pi r_+}\left(3 - \frac{r_+^2\mu^2}{\gamma^2} - \frac{r_+^4 B^2}{\gamma^2}\right).$$

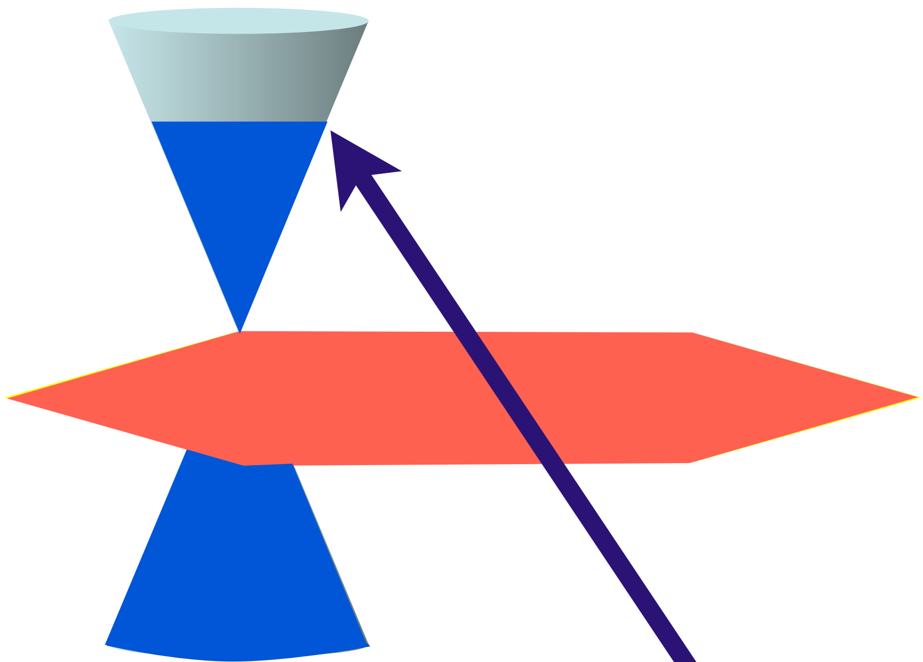


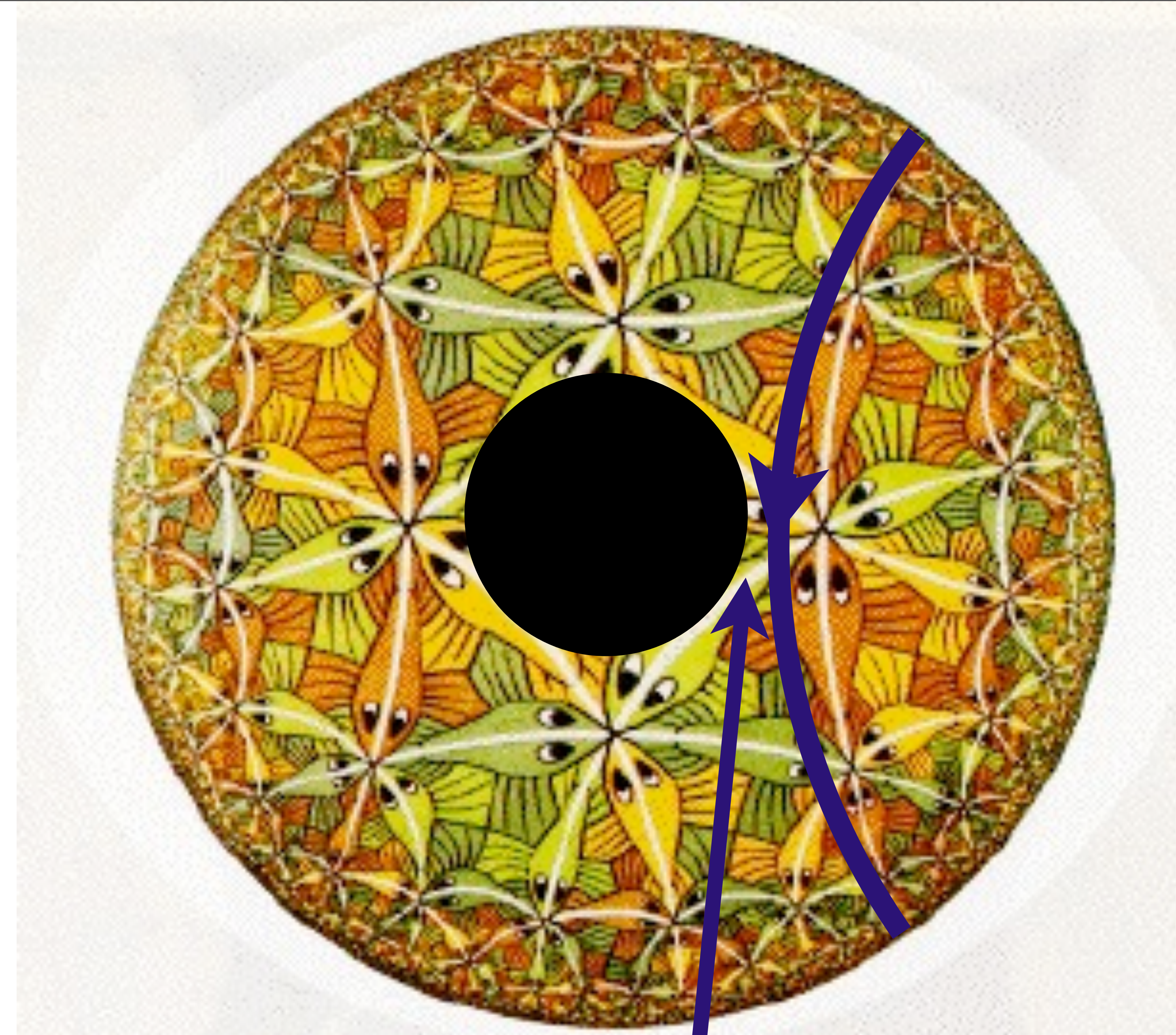
AdS$_2$ x R^2 near-horizon geometry



$$r - r_+ \sim \frac{1}{\zeta}$$

$$ds^2 = \frac{R^2}{\zeta^2}\left(-d\tau^2 + d\zeta^2\right) + \frac{r_+^2}{R^2}\left(dx^2 + dy^2\right)$$

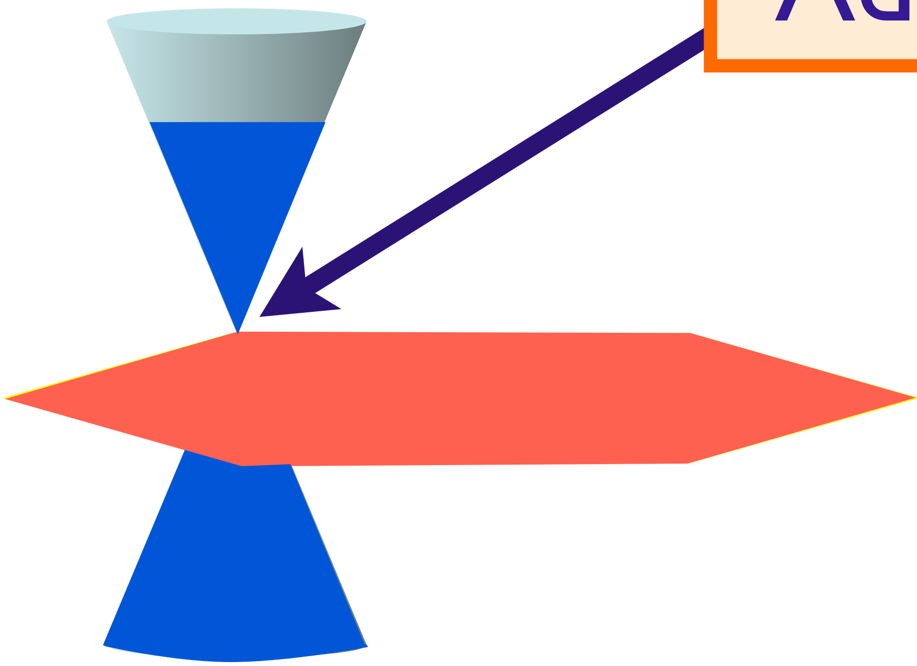


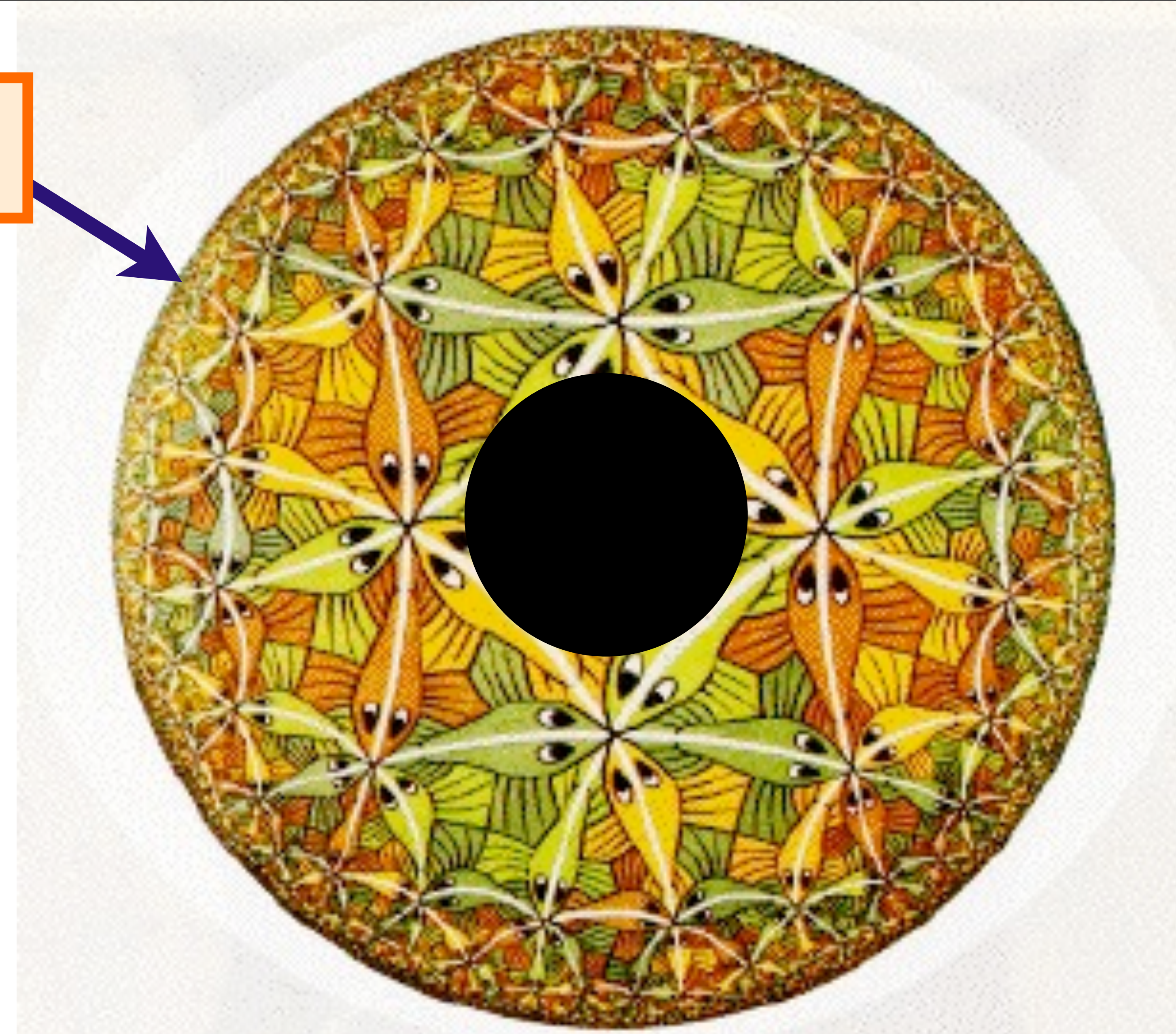


Infrared physics of Fermi surface is linked to the near horizon AdS_2 geometry of Reissner-Nordstrom black hole

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

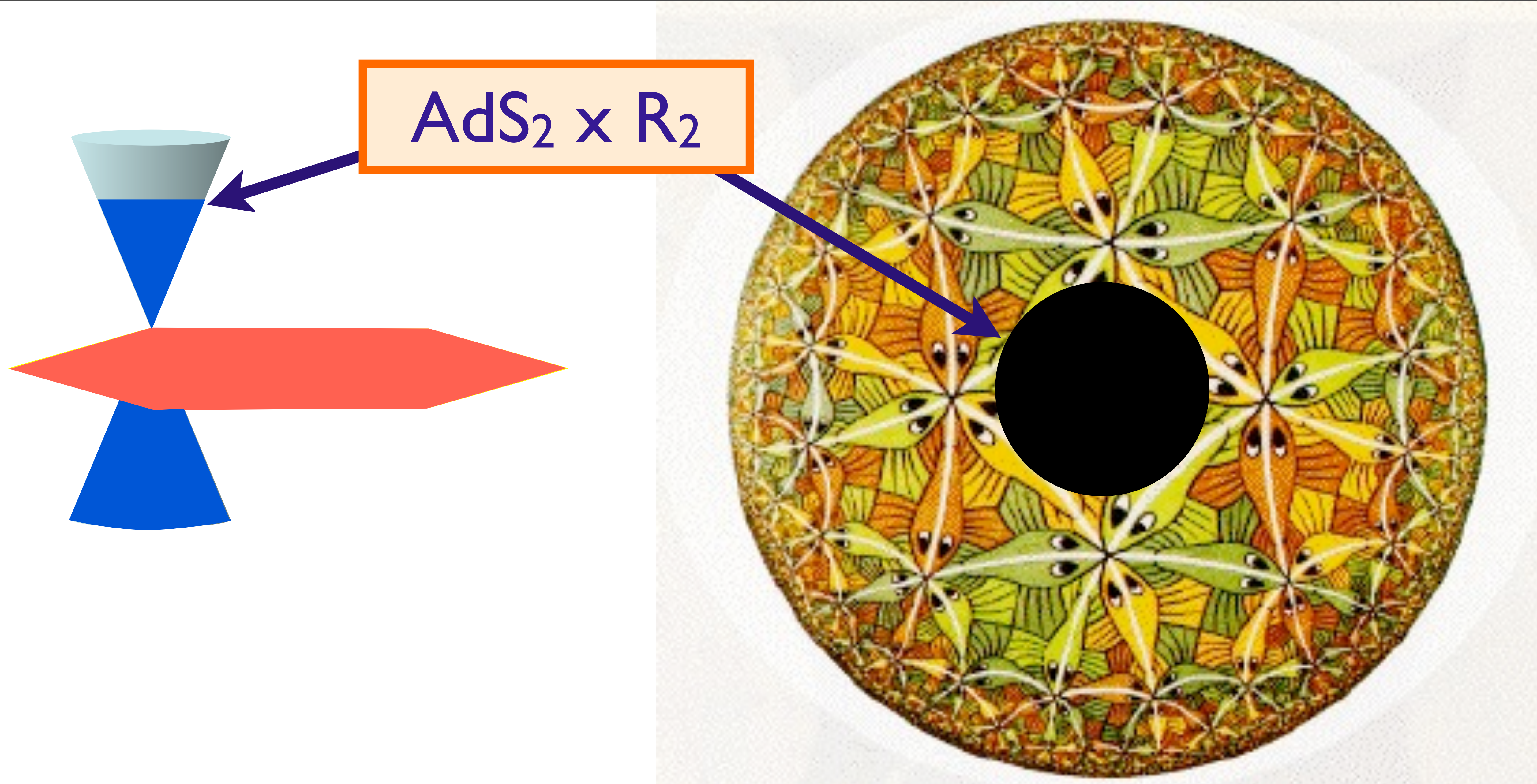
AdS_4





Geometric interpretation of RG flow

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694



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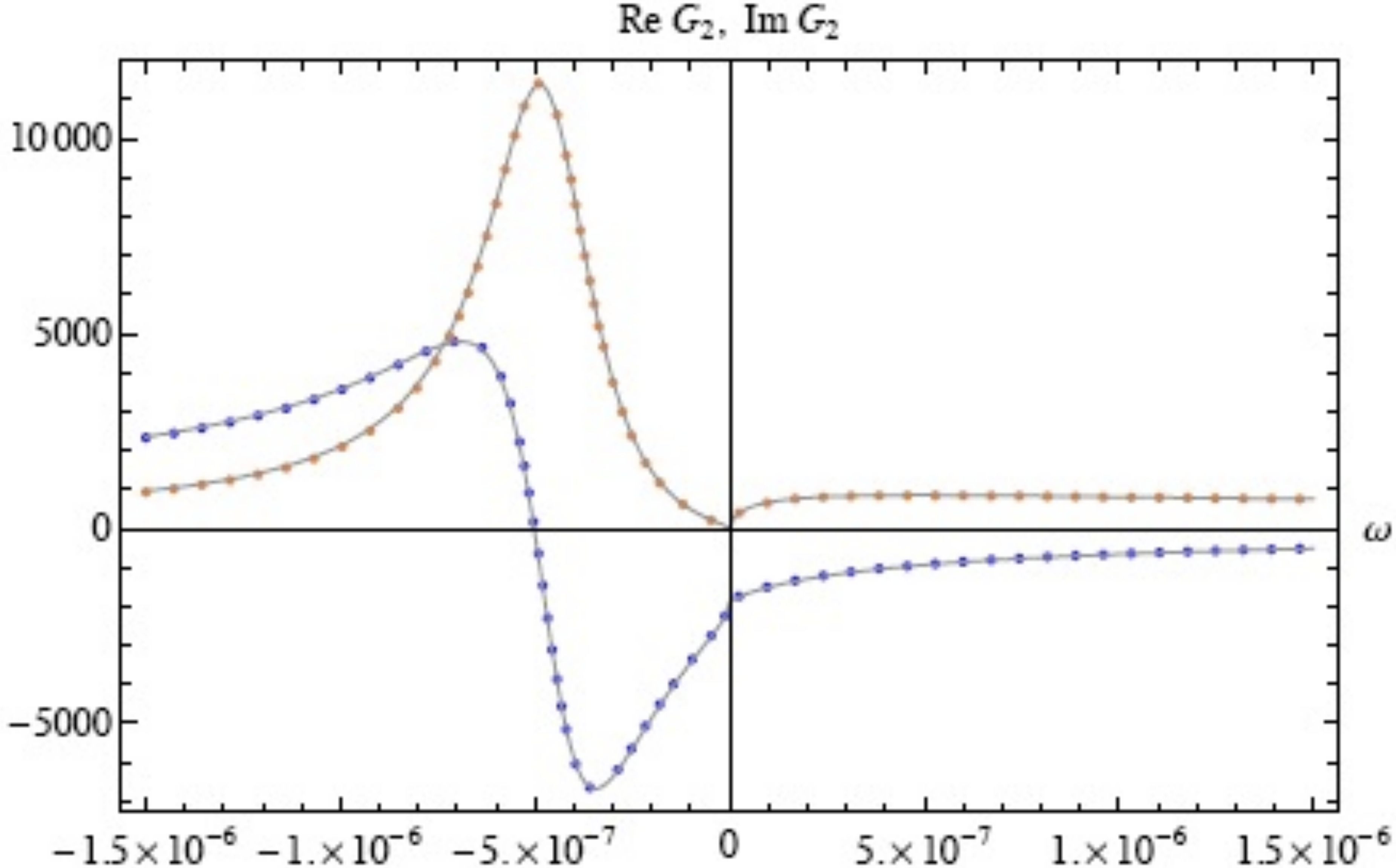
Green's function of a fermion

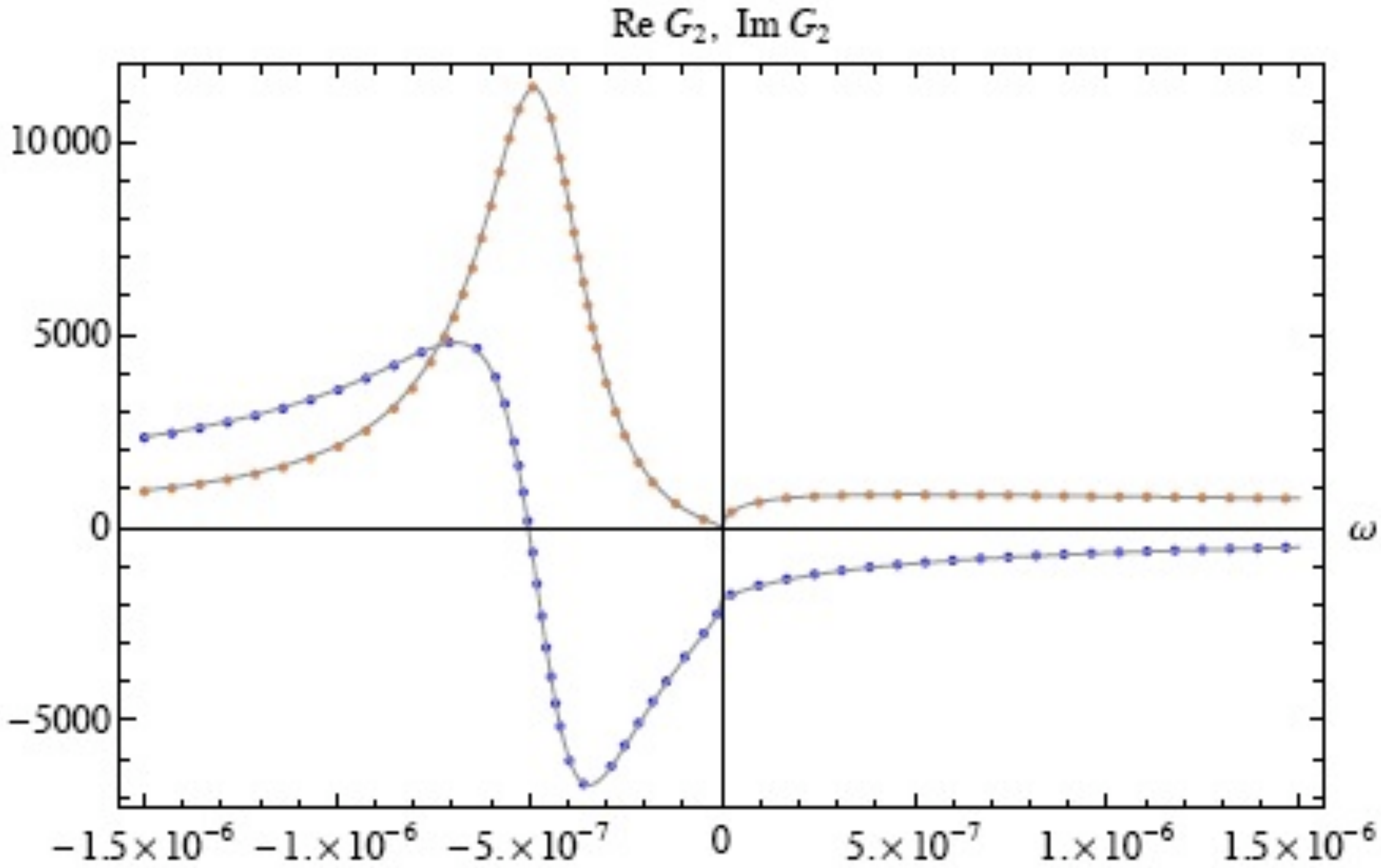


T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

See also M. Cubrovic, J Zaanen, and K. Schalm, arXiv:0904.1993

Green's function of a fermion



T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

Similar to non-Fermi liquid theories of Fermi surfaces coupled to gauge fields, and at quantum critical points

Conclusions

General theory of finite temperature dynamics and transport near quantum critical points, with applications to antiferromagnets, graphene, and superconductors

Conclusions

The AdS/CFT offers promise in providing a new understanding of strongly interacting quantum matter at non-zero density