

Fate of SYK-type strange metals at low temperatures

Non-Fermi Liquids: Recent Developments and Future Prospects
Kadanoff Center for Theoretical Physics and James Franck Institute
University of Chicago
October 28, 2023

Subir Sachdev

Talk online: sachdev.physics.harvard.edu



What is a SYK-type strange metal?

A metal with spatially random interactions
and no quasiparticle excitations,
in which the disorder self-averages

1. Infinite-range models
2. Yukawa-SYK models in $d=2$
3. Random “mass” disorder at quantum critical points in metals
 - Mapping to random transverse field Ising model (RTFIM)

1. Infinite-range models

2. Yukawa-SYK models in $d=2$

3. Random “mass” disorder at
quantum critical points in metals

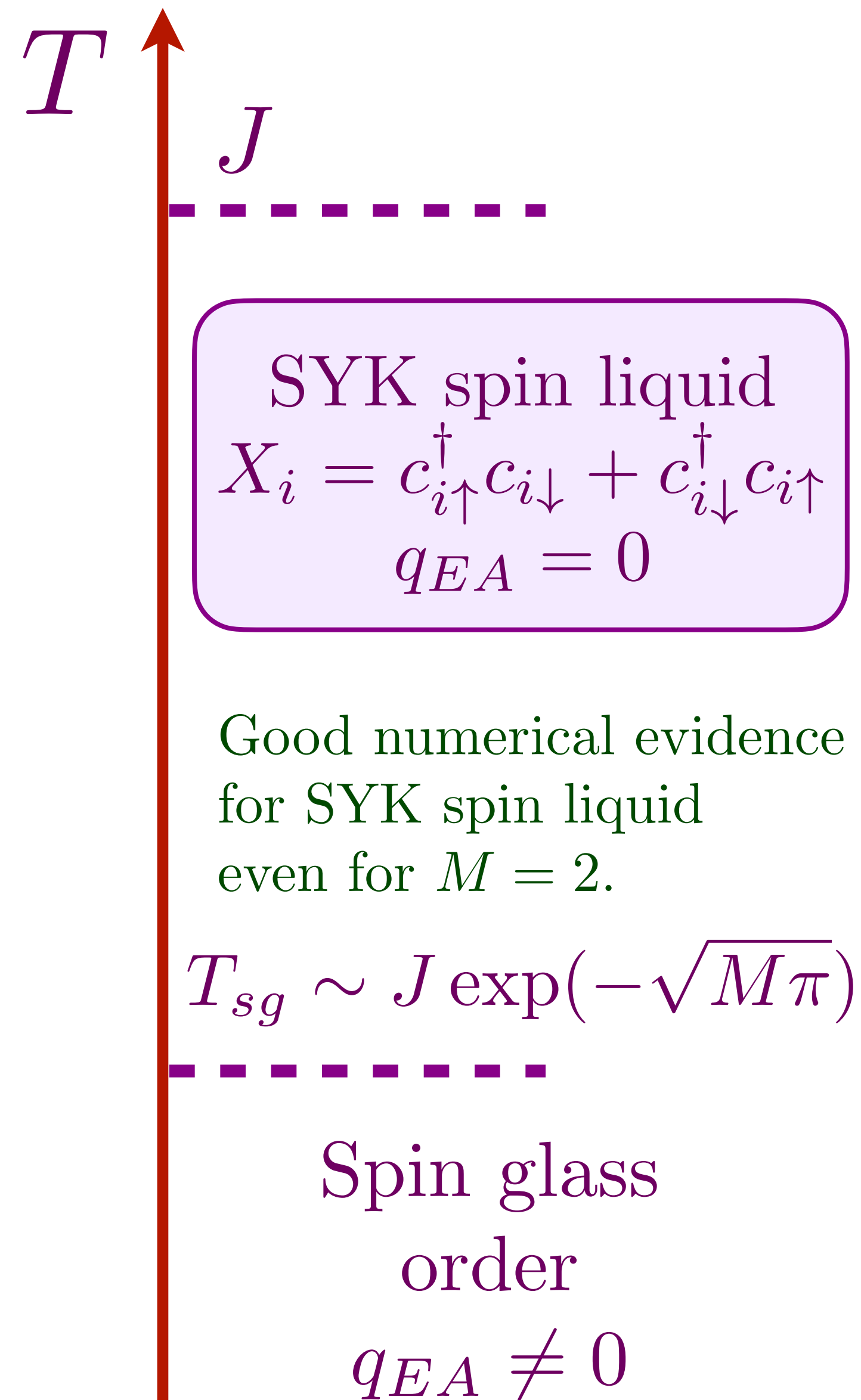
Mapping to random transverse
field Ising model (RTFIM)

Quantum Heisenberg model

$$H = \frac{1}{2\sqrt{N}} \sum_{i,j=1}^N J_{ij} (X_i X_j + Y_i Y_j + Z_i Z_j)$$

$$\overline{J_{ij}} = 0, \quad \overline{J_{ij}^2} = J^2, \quad \text{Different } J_{ij} \text{ uncorrelated.}$$

- No trivial ground state—2-fold degenerate state on each site (LSM theorem, anomalies...)
- Can be generalized to $SU(M)$ symmetry, with $M^2 - 1$ ‘Pauli’ operators.



Quantum Heisenberg model

Adding spin glass order to the $SU(M \rightarrow \infty)$ equations:

$$\Sigma(\tau) = J^2 Q_{aa}(\tau) G(\tau)$$

$$G(i\omega) = [i\omega - \Sigma(i\omega)]^{-1}$$

$$Q_{ab}(\tau) = -G(\tau)G(-\tau)\delta_{ab} + q_{ab}$$

Need only add the static spin glass order parameter q_{ab} , which is determined by the $1/M$ corrections.

T

J

SYK spin liquid
 $X_i = c_{i\uparrow}^\dagger c_{i\downarrow} + c_{i\downarrow}^\dagger c_{i\uparrow}$
 $q_{EA} = 0$

Good numerical evidence for SYK spin liquid even for $M = 2$.

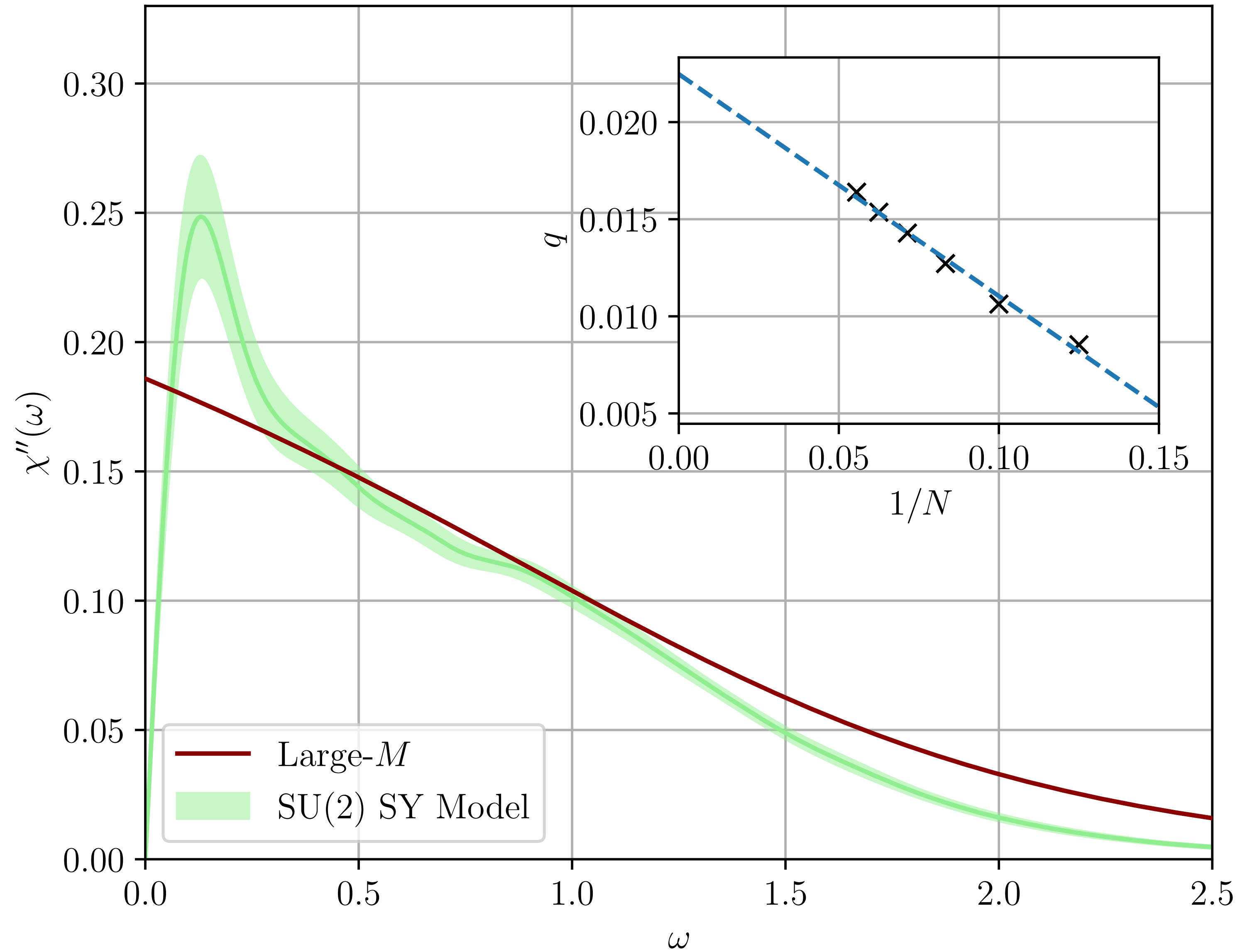
$$T_{sg} \sim J \exp(-\sqrt{M\pi})$$

Spin glass order

$$q_{EA} \neq 0$$

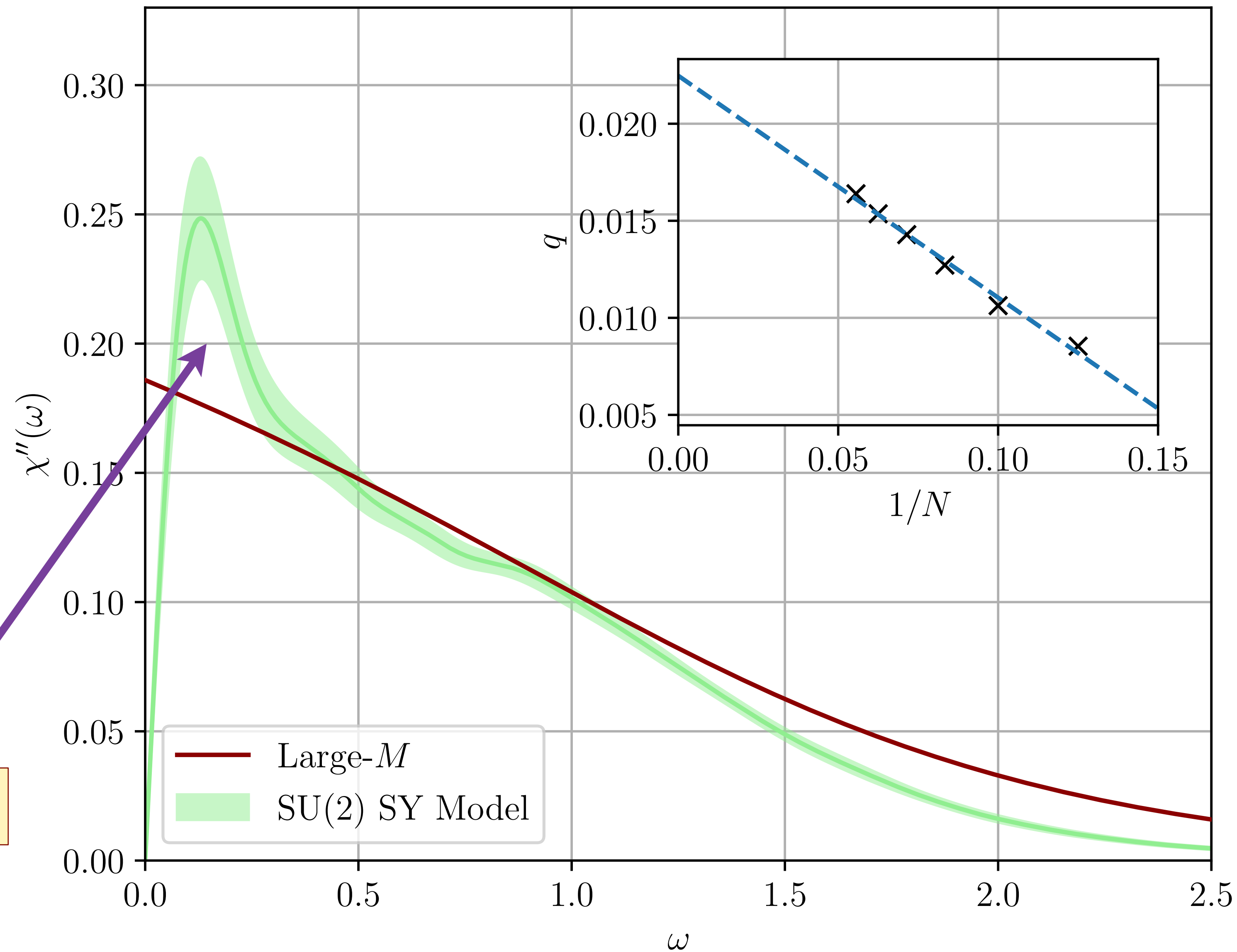


Exact diagonalization of clusters of SU(2) spins



H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

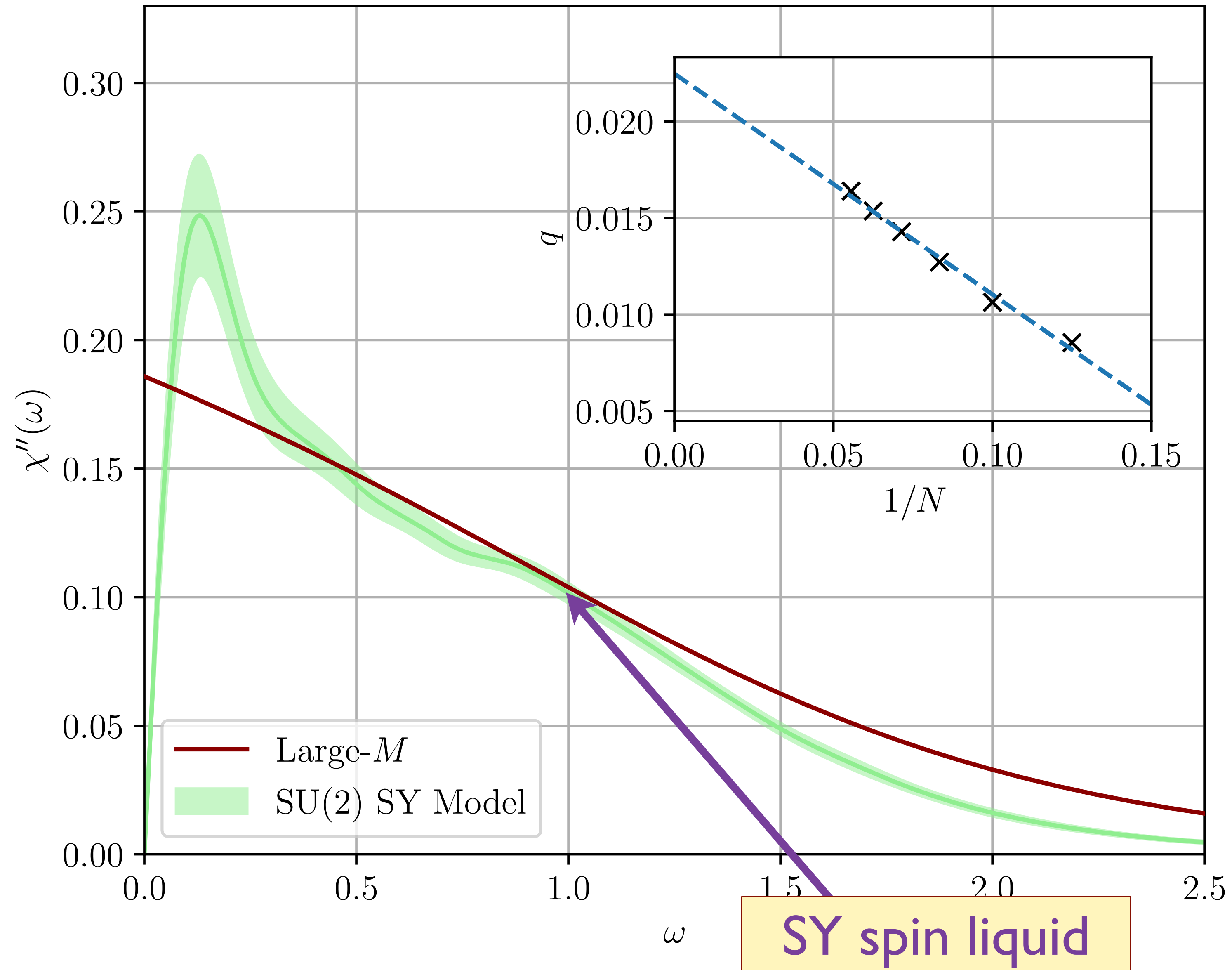
Exact diagonalization of clusters of SU(2) spins



Spin glass



Exact diagonalization of clusters of SU(2) spins



Dope the quantum Sherrington-Kirkpatrick model
with mobile electrons

$$H = \sum_{i < j} \left[-t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \text{H.c.} + J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \right]$$

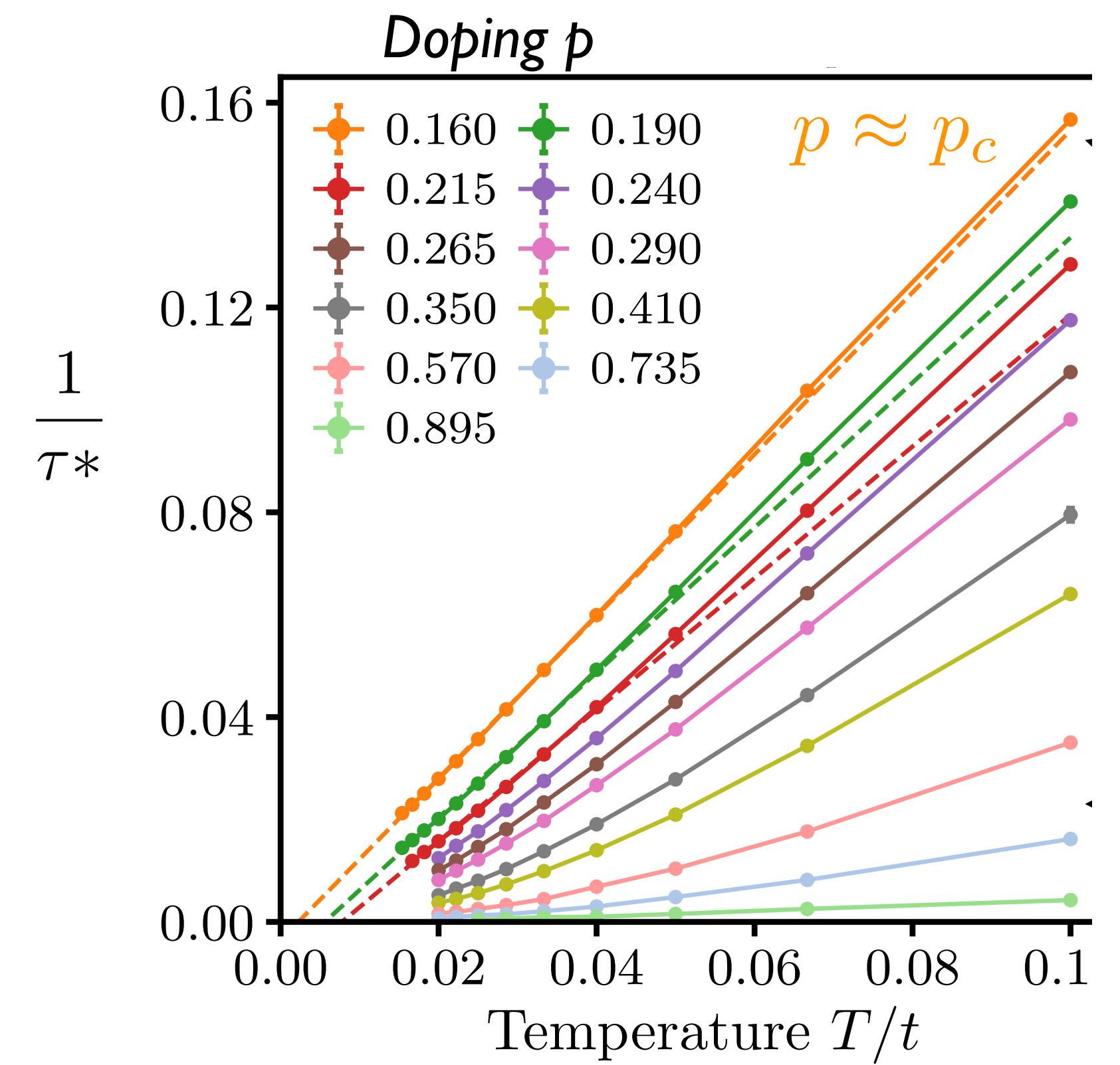
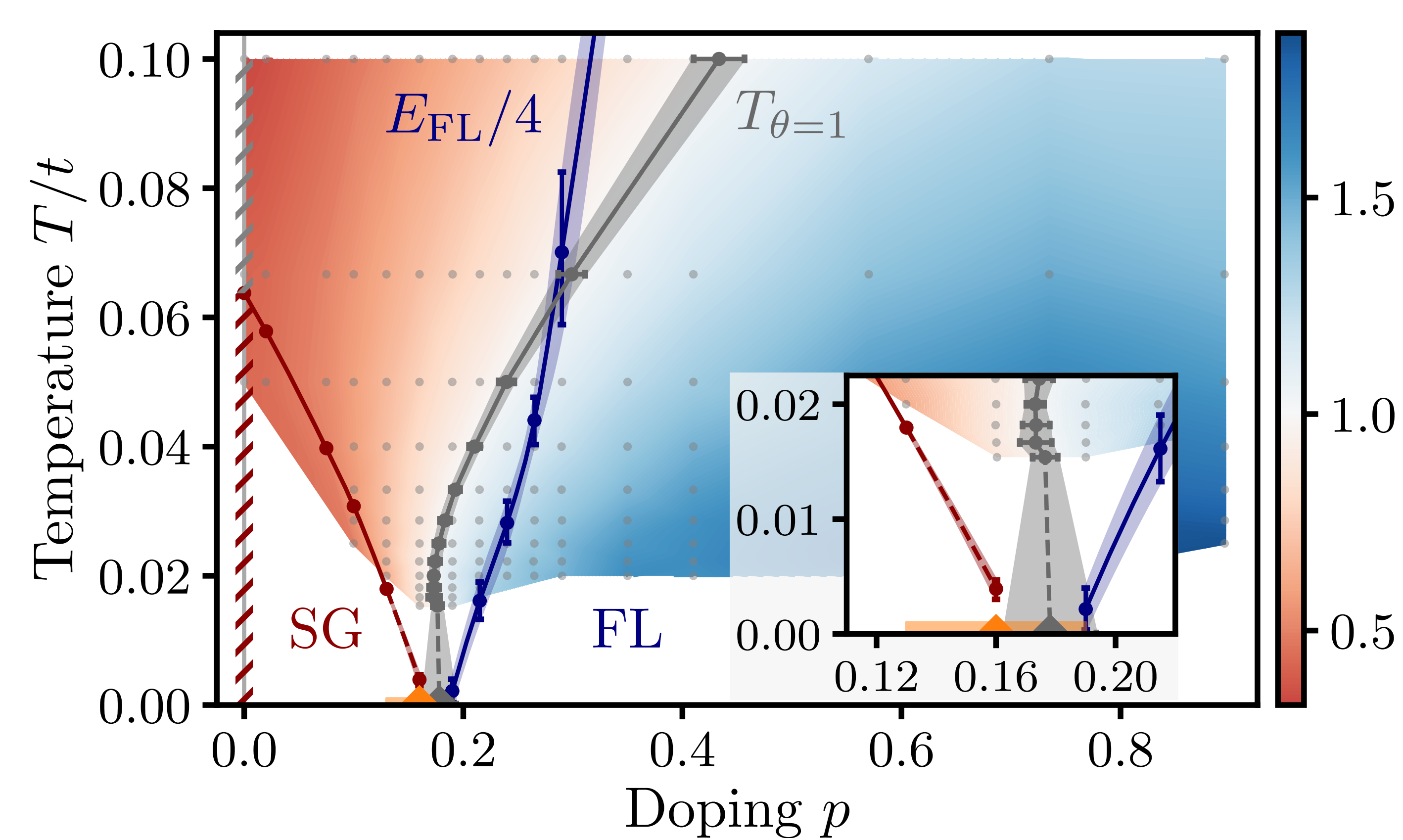
$$\mathbf{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} c_{i\beta}$$

$$[c_{i\alpha}, c_{j\beta}^\dagger]_+ = \delta_{ij} \delta_{\alpha\beta} \quad , \quad \sum_{\alpha} c_{i\alpha}^\dagger c_{i\alpha} \leq 1$$

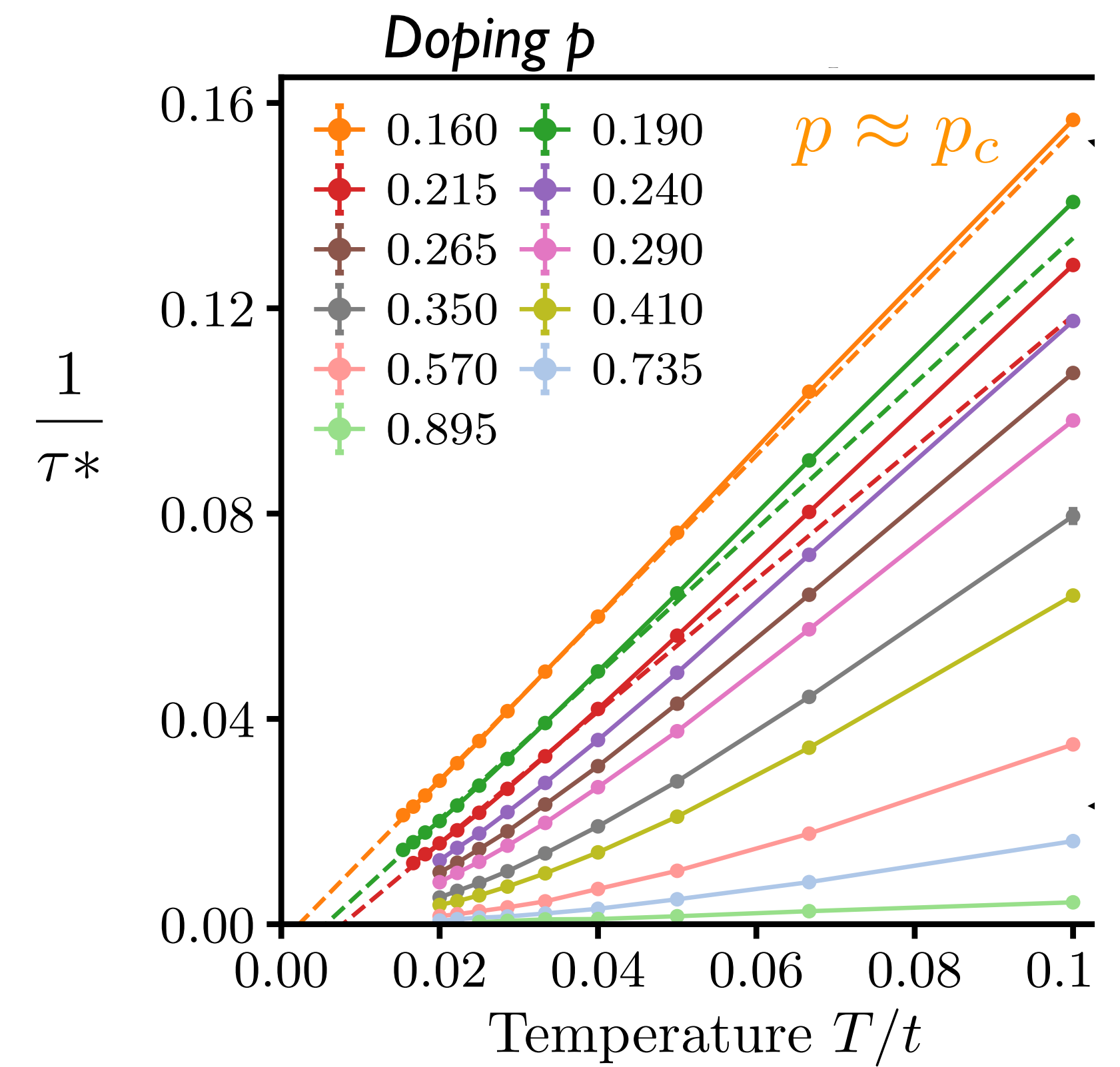
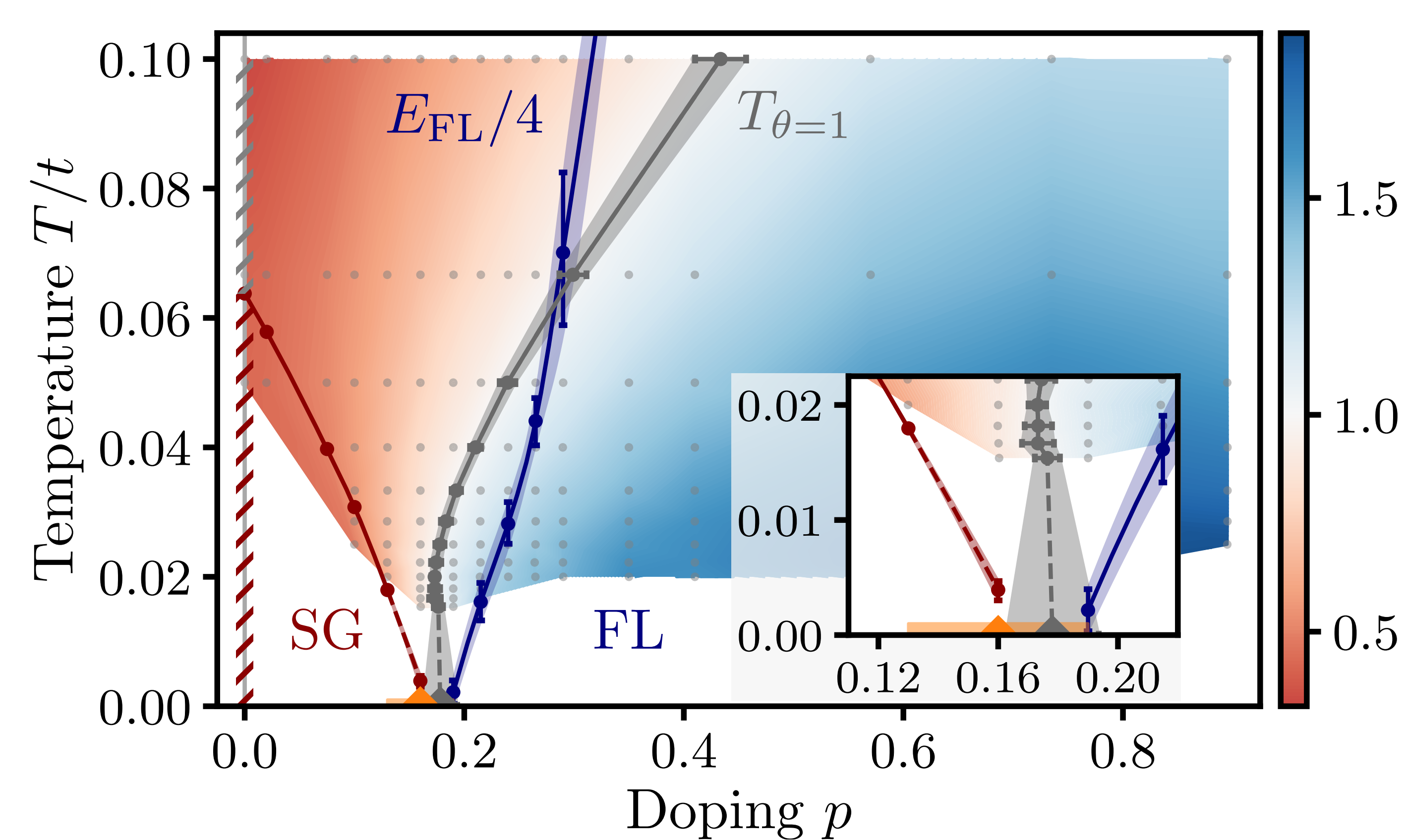
$$\frac{1}{N} \sum_{i\alpha} c_{i\alpha}^\dagger c_{i\alpha} = 1 - p$$

$$\overline{J_{ij}} = 0, \quad \overline{J_{ij}^2} = J^2, \quad \text{Different } J_{ij} \text{ uncorrelated.}$$

$$\overline{t_{ij}} = 0, \quad \overline{t_{ij}^2} = t^2, \quad \text{Different } t_{ij} \text{ uncorrelated.}$$



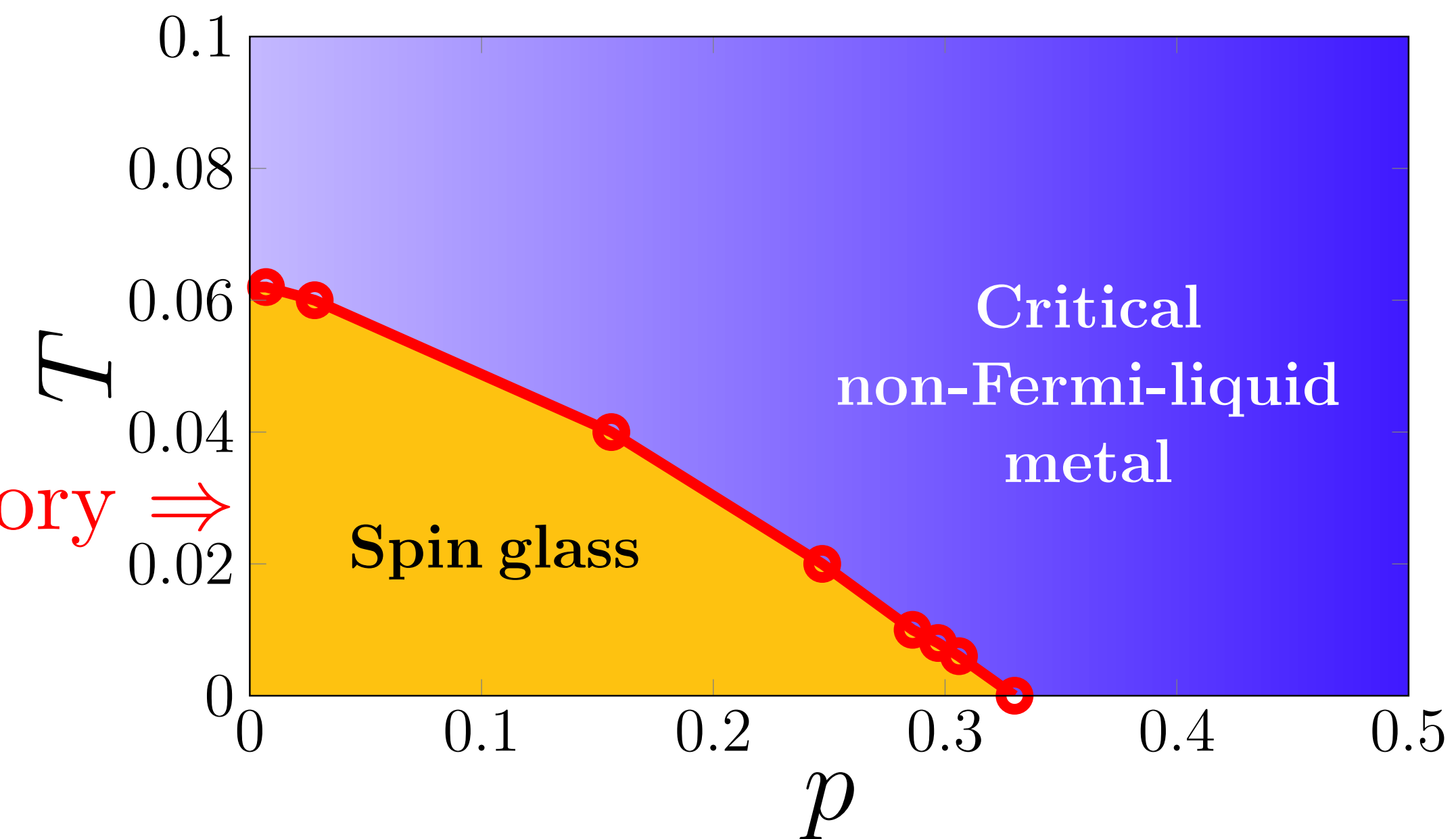
P. T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet, PRB **105**, L180404 (2022)
H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)



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Large M theory \Rightarrow



M. Christos, D. G. Joshi, S. Sachdev, and M. Tikhanovskaya, PNAS **119**, e2206921119 (2022)

Linear- T resistivity from Schwarzian mode at all p in overdoped metal

Haoyu Guo, Yingfei Gu, and S. Sachdev, Annals of Physics **418**, 168202 (2020)

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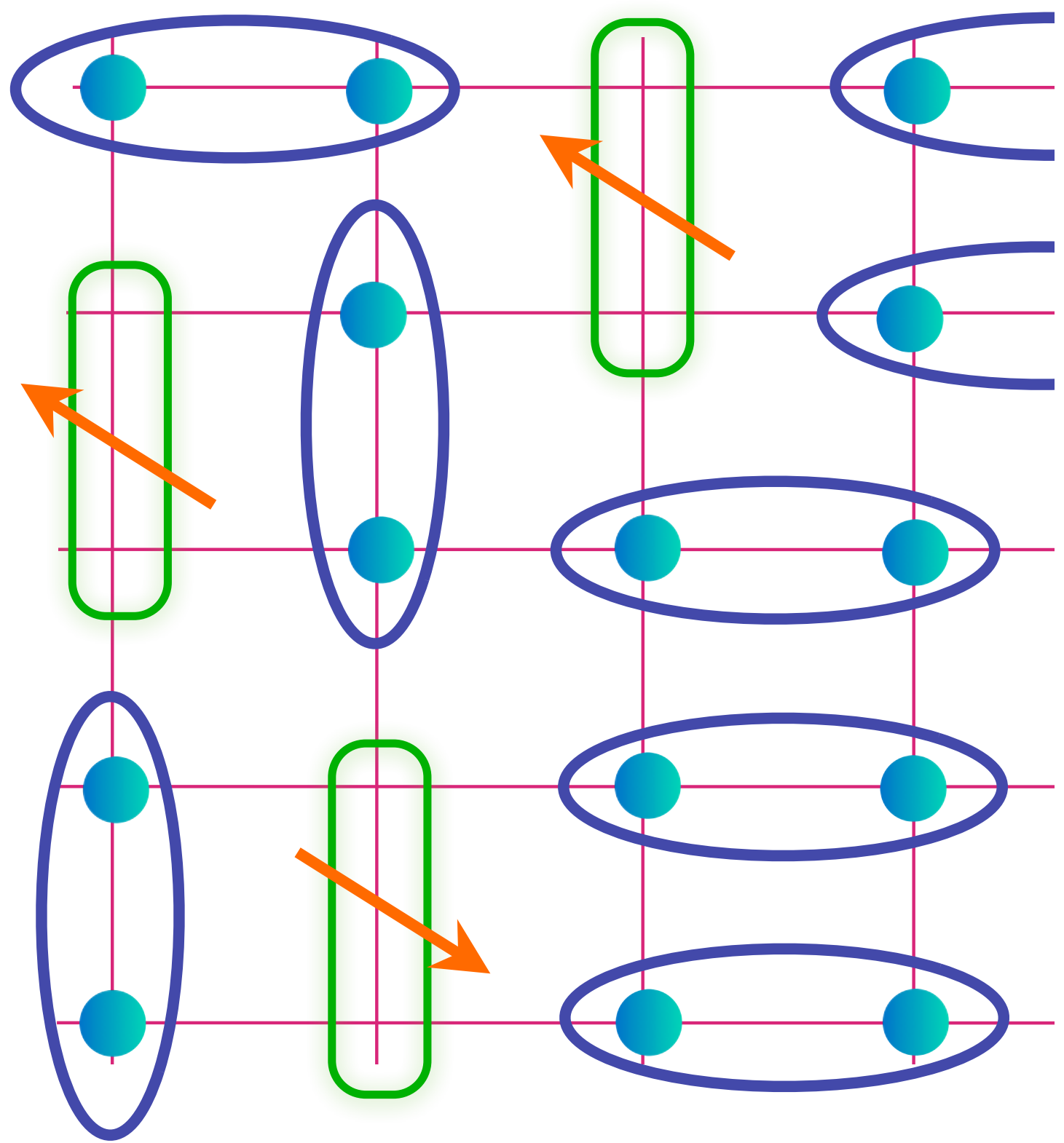
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3. Random “mass” disorder at

quantum critical points in metals:

Mapping to random transverse
field Ising model (RTFIM)

Pseudogap metal to Fermi liquid in single band model



Higgs boson with Φ the fundamental gauge charge of an emergent SU(2) gauge field.

$$\text{blue oval} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$

$$\text{green rectangle} = (|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2}$$

Small Fermi surface of size p

+
spin liquid.

FL*

$$\langle \Phi \rangle \neq 0$$

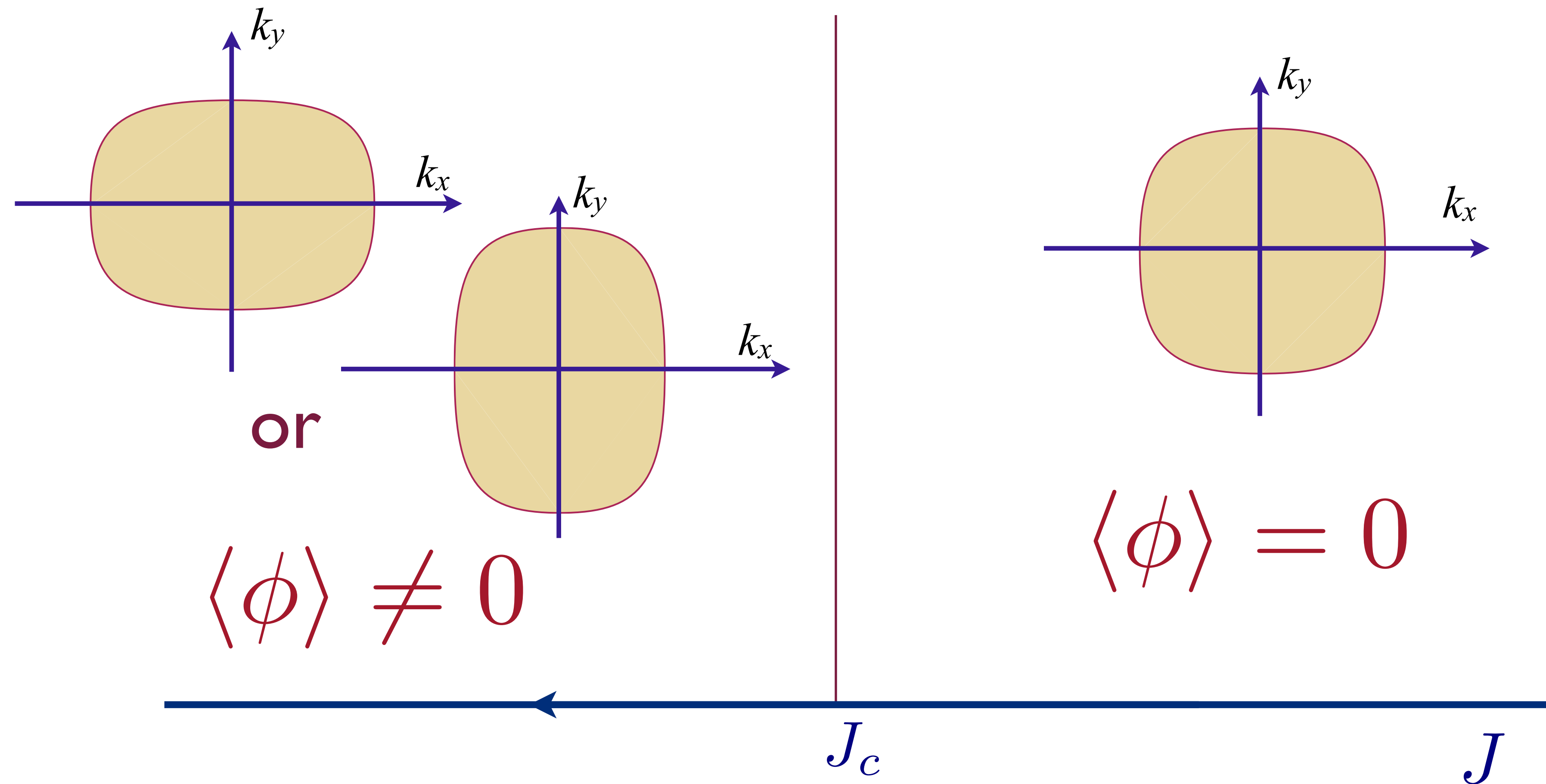
Large Fermi surface of size $1 + p$

FL

$$\langle \Phi \rangle = 0$$

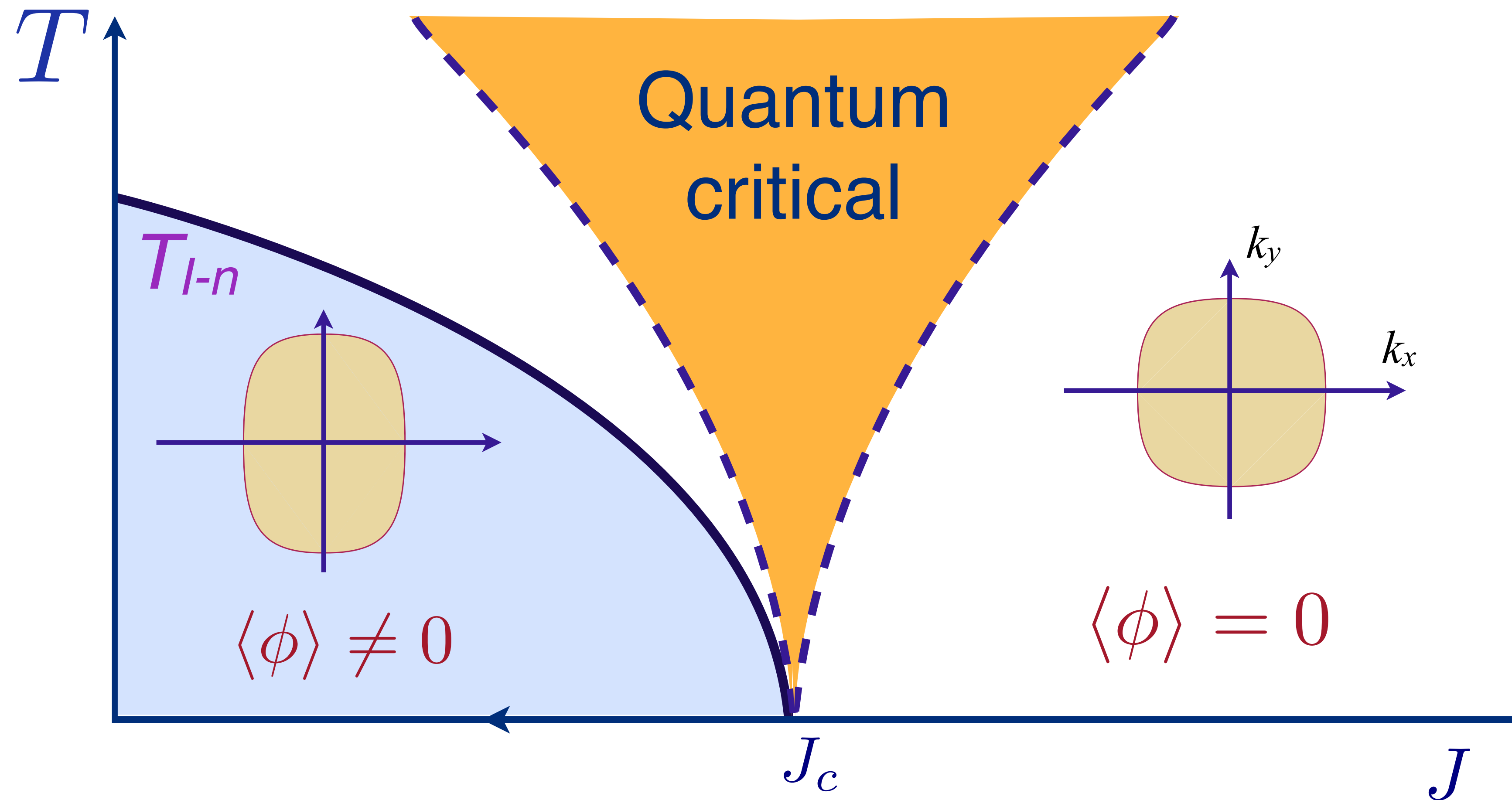
doping p

Quantum criticality of Ising-nematic ordering in a metal



Pommeranchuk instability as a function of coupling J

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and J



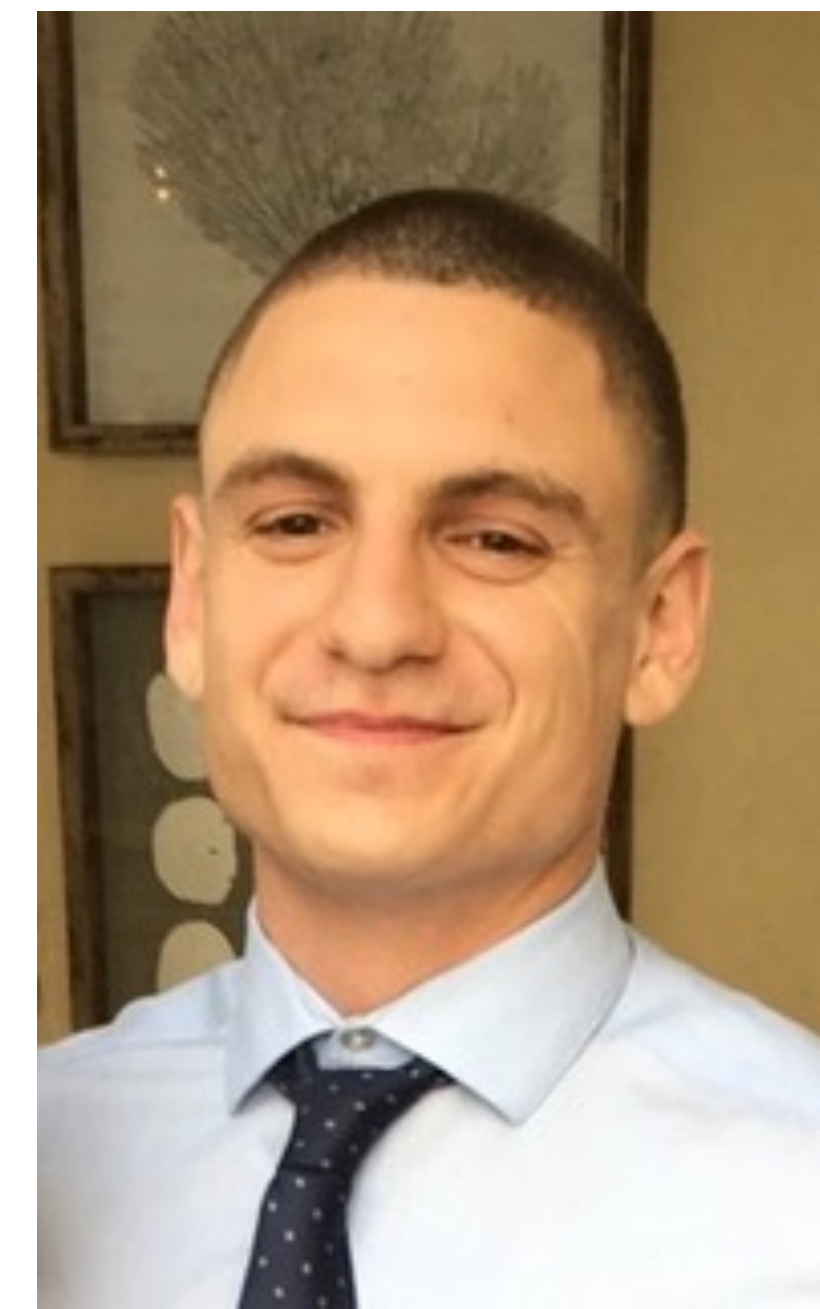
Aavishkar Patel

Flatiron Institute, NYC



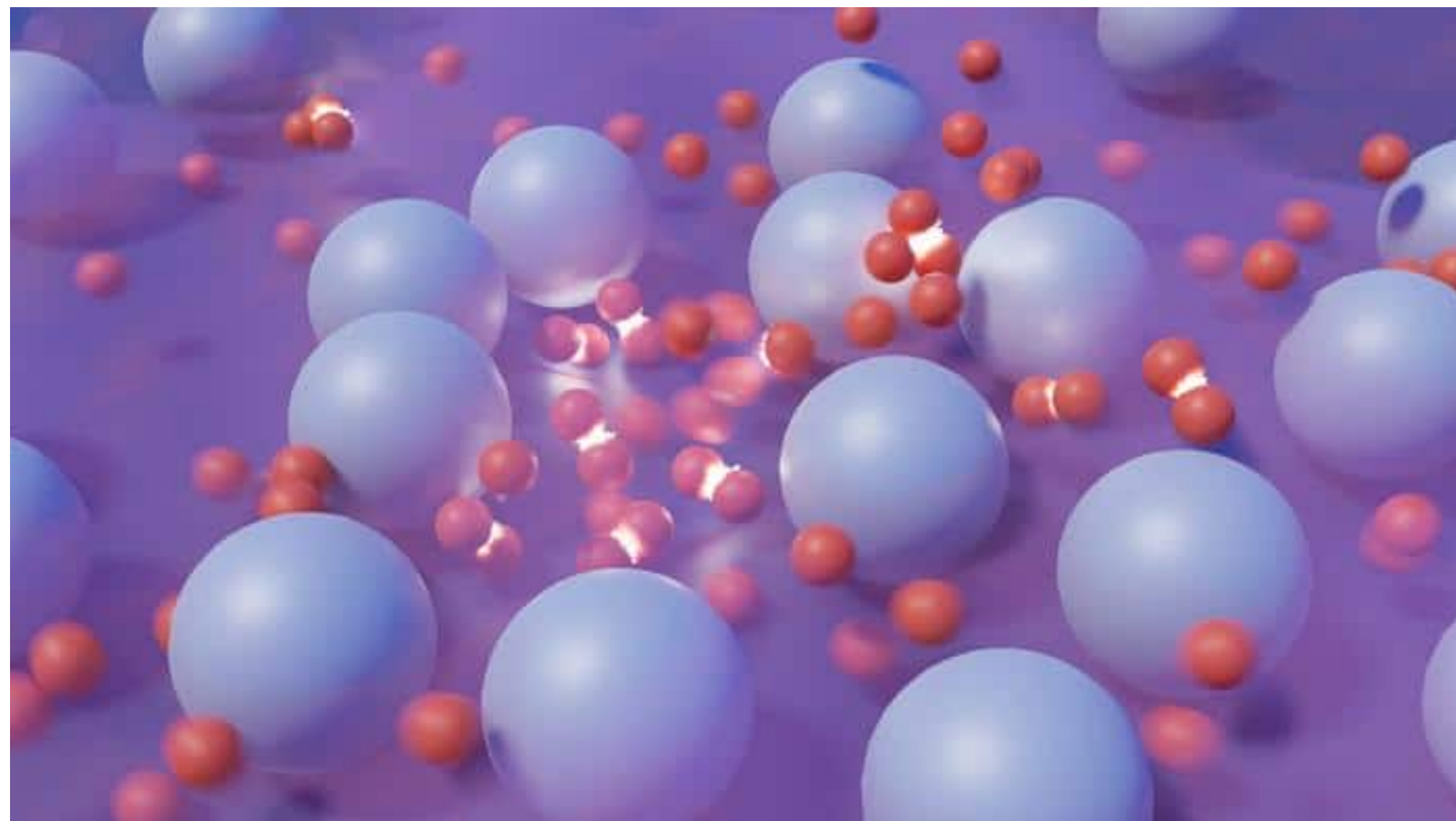
Haoyu Guo

Cornell



Ilya Esterlis

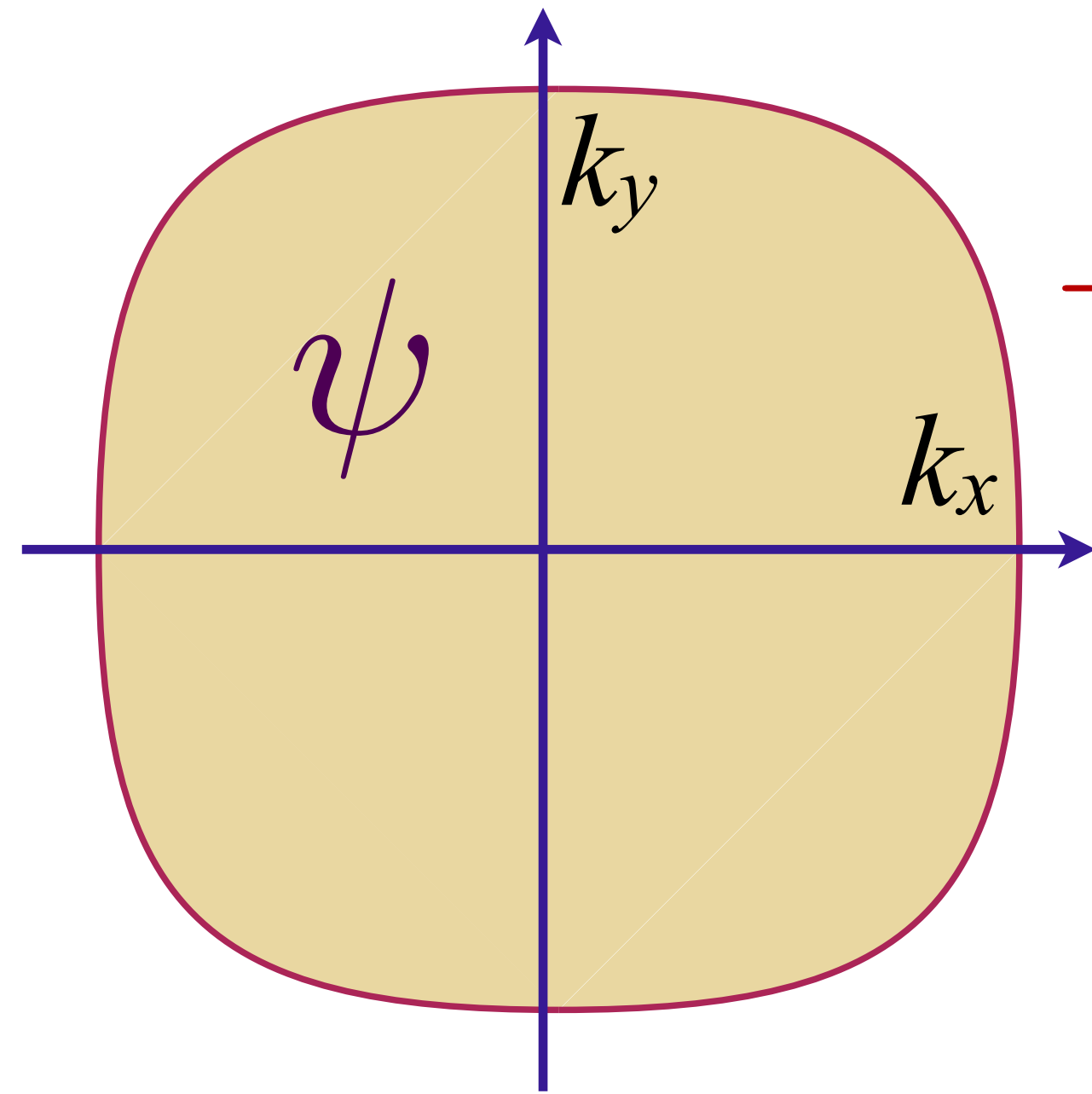
Wisconsin



Universal theory of strange metals from
spatially random interactions,
Aavishkar A. Patel, Haoyu Guo,
Ilya Esterlis, and S. Sachdev,
Science **381**, 790 (2023)

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
 spin-density wave order,

Higgs boson for Fermi-volume changing transition

$$+ [s + \delta s(\mathbf{r})] [\phi(\mathbf{r})]^2 + [g + g'(\mathbf{r})] \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r})$$

$$+ v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

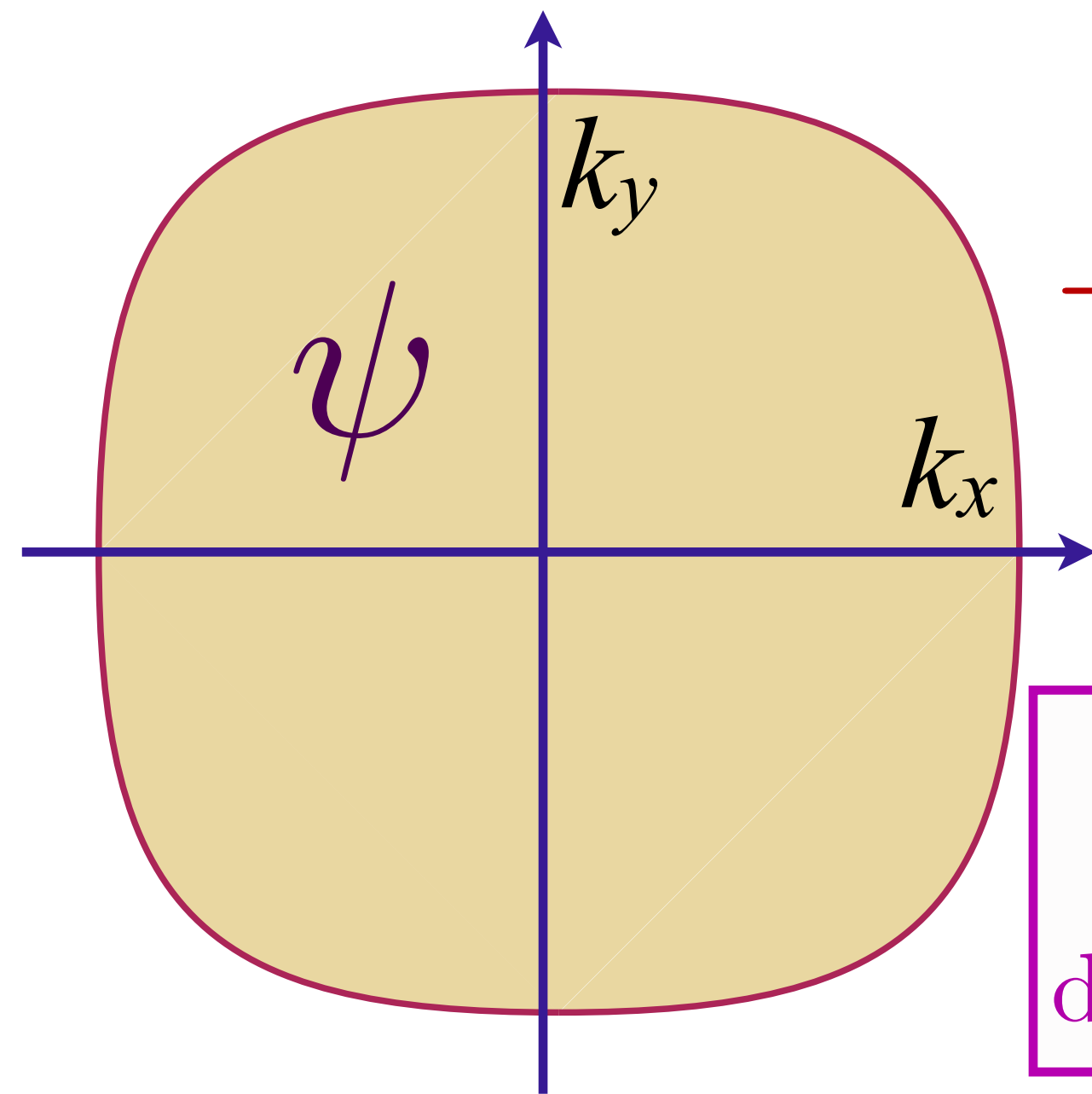
Spatially random Yukawa coupling $g'(\mathbf{r})$ with $\overline{g'(\mathbf{r})} = 0$, $\overline{g'(\mathbf{r})g'(\mathbf{r}')} = g'^2 \delta(\mathbf{r} - \mathbf{r}')$

Spatially random mass $\delta s(\mathbf{r})$ with $\overline{\delta s(\mathbf{r})} = 0$, $\overline{\delta s(\mathbf{r})\delta s(\mathbf{r}')} = \delta s^2 \delta(\mathbf{r} - \mathbf{r}')$

Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')} = v^2 \delta(\mathbf{r} - \mathbf{r}')$

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$$+ v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

Key assumption:

can 'gauge away' $\delta s(\mathbf{r})$ disorder by rescaling $\phi(\mathbf{r})$, and the remaining disorder can be treated in a SYK-like, self-averaging, self-consistent theory.

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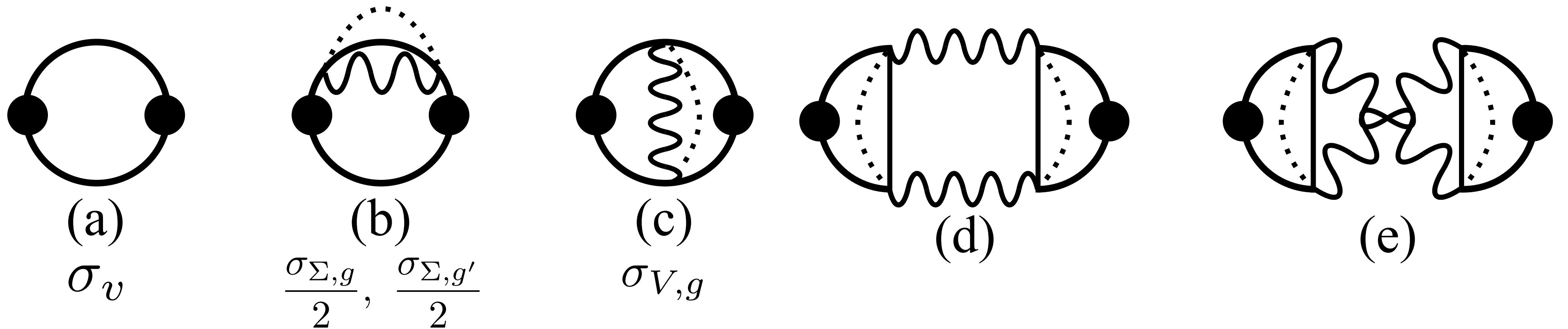
SYK-type self-consistent equations

$$\Sigma(\tau, \mathbf{r}) = g^2 D(\tau, \mathbf{r}) G(\tau, \mathbf{r}) + v^2 G(\tau, \mathbf{r}) \delta^2(\mathbf{r}) + g'^2 G(\tau, \mathbf{r}) D(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

$$\Pi(\tau, \mathbf{r}) = -g^2 G(-\tau, -\mathbf{r}) G(\tau, \mathbf{r}) - g'^2 G(-\tau, \mathbf{r}) G(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

$$G(i\omega, \mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega, \mathbf{k})},$$

$$D(i\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega, \mathbf{q})}.$$



+ all ladders and bubbles.....

Conductivity:

Fermi surface + critical boson with potential and interaction disorder

$$\text{Conductivity: } \sigma(\omega) \sim \frac{1}{\frac{1}{\tau_{\text{trans}}(\omega)} - i\omega \frac{m_{\text{trans}}^*(\omega)}{m}}$$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2 |\omega| \quad ; \quad \frac{m_{\text{trans}}^*(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

$$\text{Electron Green's function: } G(\omega) \sim \frac{1}{\omega \frac{m^*(\omega)}{m} - \varepsilon(\mathbf{k}) + i \left(\frac{1}{\tau_e} + \frac{1}{\tau_{\text{in}}(\omega)} \right) \text{sgn}(\omega)}$$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\text{in}}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2 \right) |\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi} \left(\frac{g^2}{v^2} + g'^2 \right) \ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2 ; Linear-in- T resistivity determined by g'^2 ; Transport insensitive to g ; Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat.

1. Resistivity $\rho(T) \sim v^2 + g'^2 T$.

2. Optical conductivity: $\sigma(\omega) = \frac{K}{\frac{1}{\tau_{\text{trans}}(\omega)} - i\omega \frac{m_{\text{trans}}^*(\omega)}{m}}$; $\frac{1}{\tau_{\text{trans}}(\omega)} \sim |\omega| \Phi_{\sigma} \left(\frac{\hbar\omega}{k_B T} \right)$

B. Michon.....A. Georges, Nat. Commun. **14, 3033 (2023)**

3. Photoemission: nearly marginal Fermi liquid electron spectral density.

T.J. Reber....D. Dessau, Nature Communications **10, 5737 (2019)**

4. Specific heat $\sim T \ln(1/T)$ as $T \rightarrow 0$.

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10. 'Rare region' or 'Griffiths' effects, and a theory of the extended 'foot' of strange metal behavior that is often observed at the lowest temperatures (A.A. Patel, P. Lunts and S.S., in progress).

1. Infinite-range models
2. Yukawa-SYK models in $d=2$
3. Random “mass” disorder at quantum critical points in metals:
Mapping to random transverse field Ising model (RTFIM)

with Aavishkar Patel and Peter Lunts

- We expect the ‘gauging away’ of $\delta s(\boldsymbol{r})$, and the self-averaging, to be valid when the bosonic eigenmodes are extended.

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- At low T , rare-region fluctuations of $\delta s(\mathbf{r})$ could become important, leading to localized low energy modes (analogous to ‘two level systems’ of glasses).
- We can account for these localized bosonic modes by integrating out the fermions (which remain extended), and considering the Landau-damped Hertz theory for the boson alone, in the presence of a random mass.

$$\mathcal{S}_b = \int d\tau \left(- \sum_{\langle ij \rangle} J_{ij} \phi_{ia} \phi_{ja} + \sum_j \left[\frac{s_j}{2} \phi_{ja}^2 + \frac{u}{4} (\phi_{ja}^2)^2 \right] \right) + \frac{T\gamma}{2} \sum_{\omega_n} \sum_j |\omega_n| |\phi_{ja}(\omega_n)|^2$$

where $a = 1 \dots N$ is a flavor index for an order parameter with $O(N)$ symmetry.

Effects of Dissipation on a Quantum Critical Point with Disorder

José A. Hoyos, Chetan Kotabage, and Thomas Vojta

Department of Physics, University of Missouri-Rolla, Rolla, Missouri 65409, USA

(Received 19 May 2007; published 4 December 2007)

We study the effects of dissipation on a disordered quantum phase transition with $O(N)$ order-parameter symmetry by applying a strong-disorder renormalization group to the Landau-Ginzburg-Wilson field theory of the problem. We find that Ohmic dissipation results in a nonperturbative infinite-randomness critical point

Strong disorder RG identical to that for the RTFIM (D.S. Fisher)

$$\tilde{J}_{ij} = J_{ij} + \frac{J_{i2}J_{2j}}{s_2}$$

$$\tilde{s}_2 = 2 \frac{s_2 s_3}{J_{23}}$$

$$H_{\text{RTFIM}} = - \sum_{\langle ij \rangle} J_{ij} Z_i Z_j - \sum_j s_j X_j$$

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- Each rare region is described by a one-dimensional classical $O(N)$ model with a long-range $1/\tau^2$ interaction.
- For $N \geq 2$, the classical model has an exponentially long correlation time at weak coupling (low ‘temperature’) - Dyson, 1969
- This is similar to the classical Ising chain with short-range interactions.

Infinite Randomness Fixed Point of the Superconductor-Metal Quantum Phase Transition

Adrian Del Maestro, Bernd Rosenow, Markus Müller, and Subir Sachdev

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 29 February 2008; revised manuscript received 20 April 2008; published 17 July 2008)

We examine the influence of quenched disorder on the superconductor-metal transition, as described by a theory of overdamped Cooper pairs which repel each other. The self-consistent pairing eigenmodes of a quasi-one-dimensional wire are determined numerically. Our results support the recent proposal by Hoyos *et al.* [Phys. Rev. Lett. **99**, 230601 (2007)] that the transition is characterized by the same strong-disorder fixed point describing the onset of ferromagnetism in the random quantum Ising chain in a transverse field.

Crossover from extended to localized regime can be described by numerical solution of self-consistent large N equations in $d = 1$

$$\mathcal{S}_{b0} = \int d\tau \left(\frac{K}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \frac{\bar{s}_j}{2} \phi_{ja}^2 \right) + \frac{T\gamma}{2} \sum_{\omega_n} \sum_j |\omega_n| |\phi_{ja}(\omega_n)|^2,$$

where the renormalized masses have to be obtained by solving

$$\bar{s}_j = s_j + u \langle \phi_{ja}^2 \rangle_{\mathcal{S}_{b0}}.$$

These equations describe the exponentially long correlation time of rare regions.

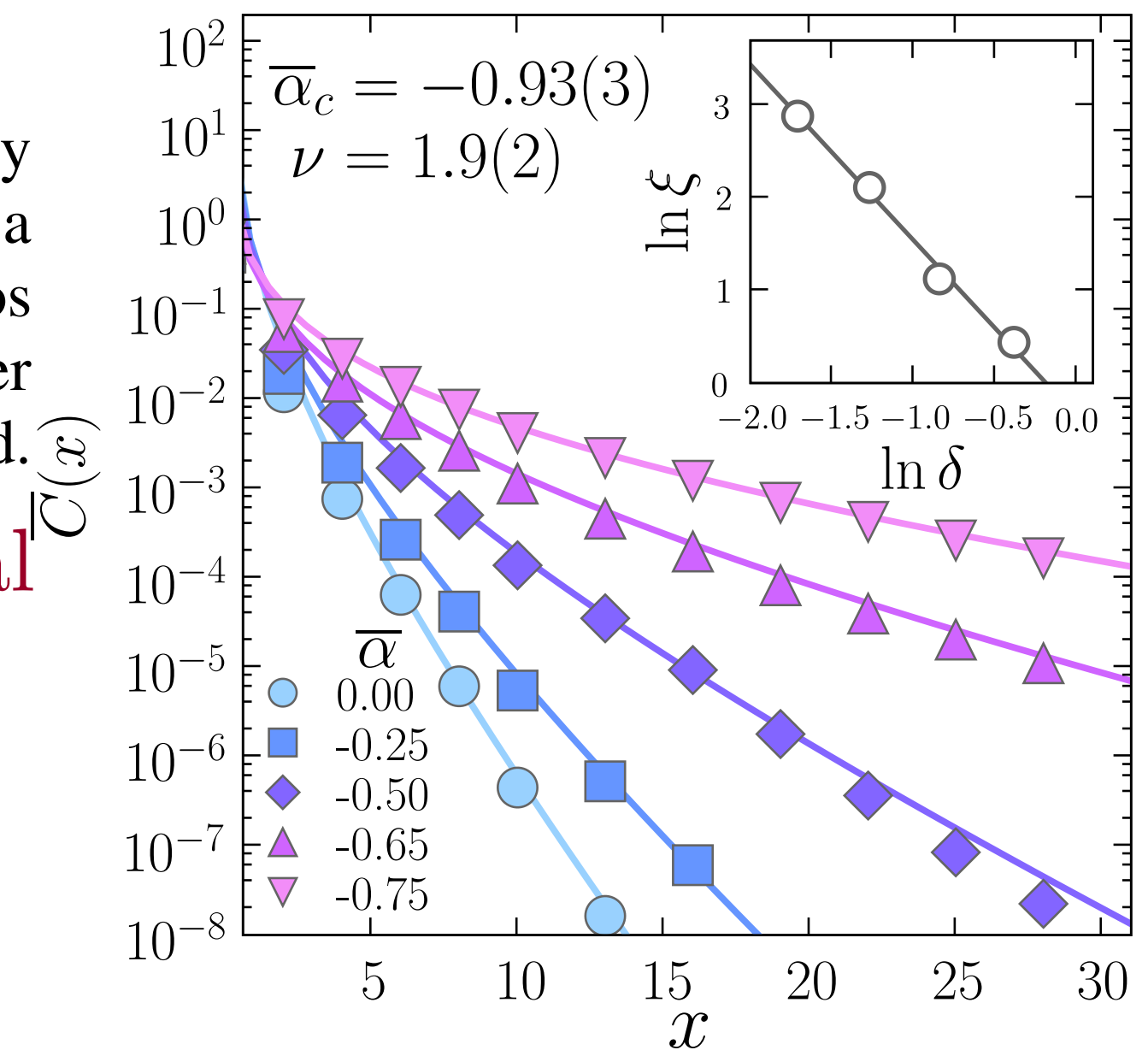


FIG. 1 (color online). The equal-time disorder averaged correlation functions for $L = 64$ and five values of the mean of α_j , $\bar{\alpha}$. The solid lines are fits to Eq. (4) via ξ and an overall scale parameter. The inset displays a fit to the power law form of the finite size scaled correlation length, providing an estimate for the location of the critical point $\bar{\alpha}_c = -0.93(3)$ and the correlation length exponent $\nu = 1.9(2)$.

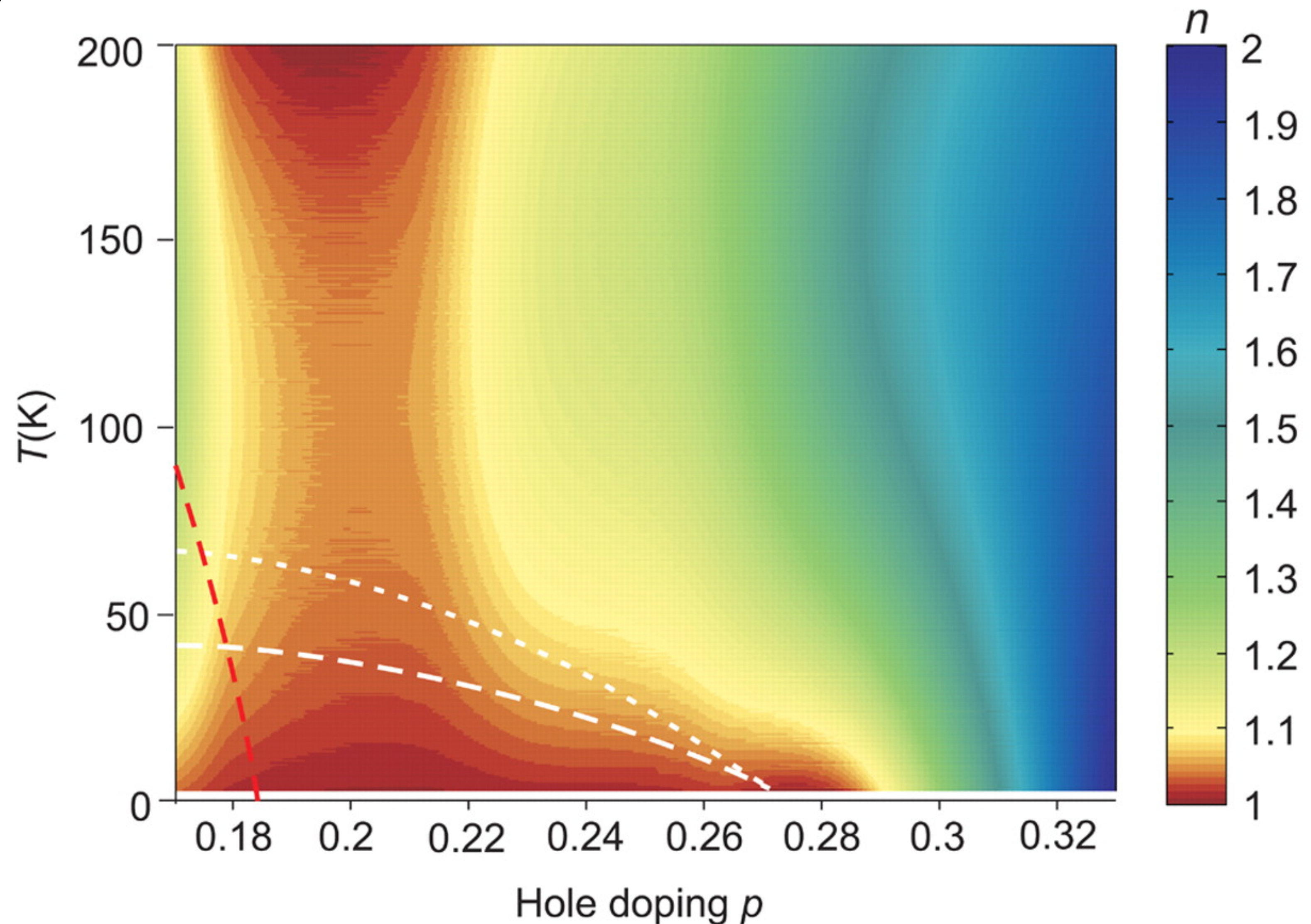
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Proposal: 'Foot' due
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See talks tomorrow
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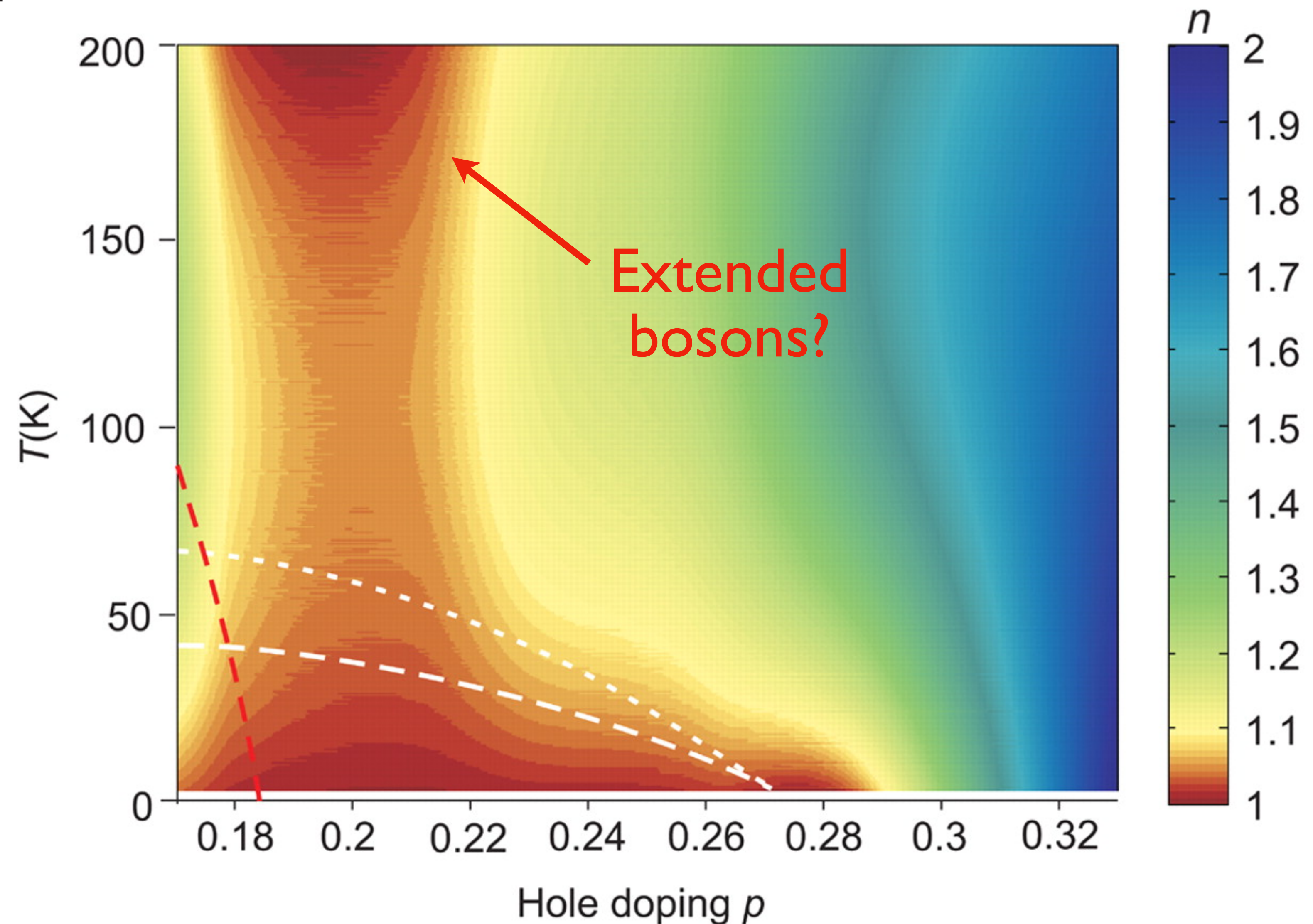
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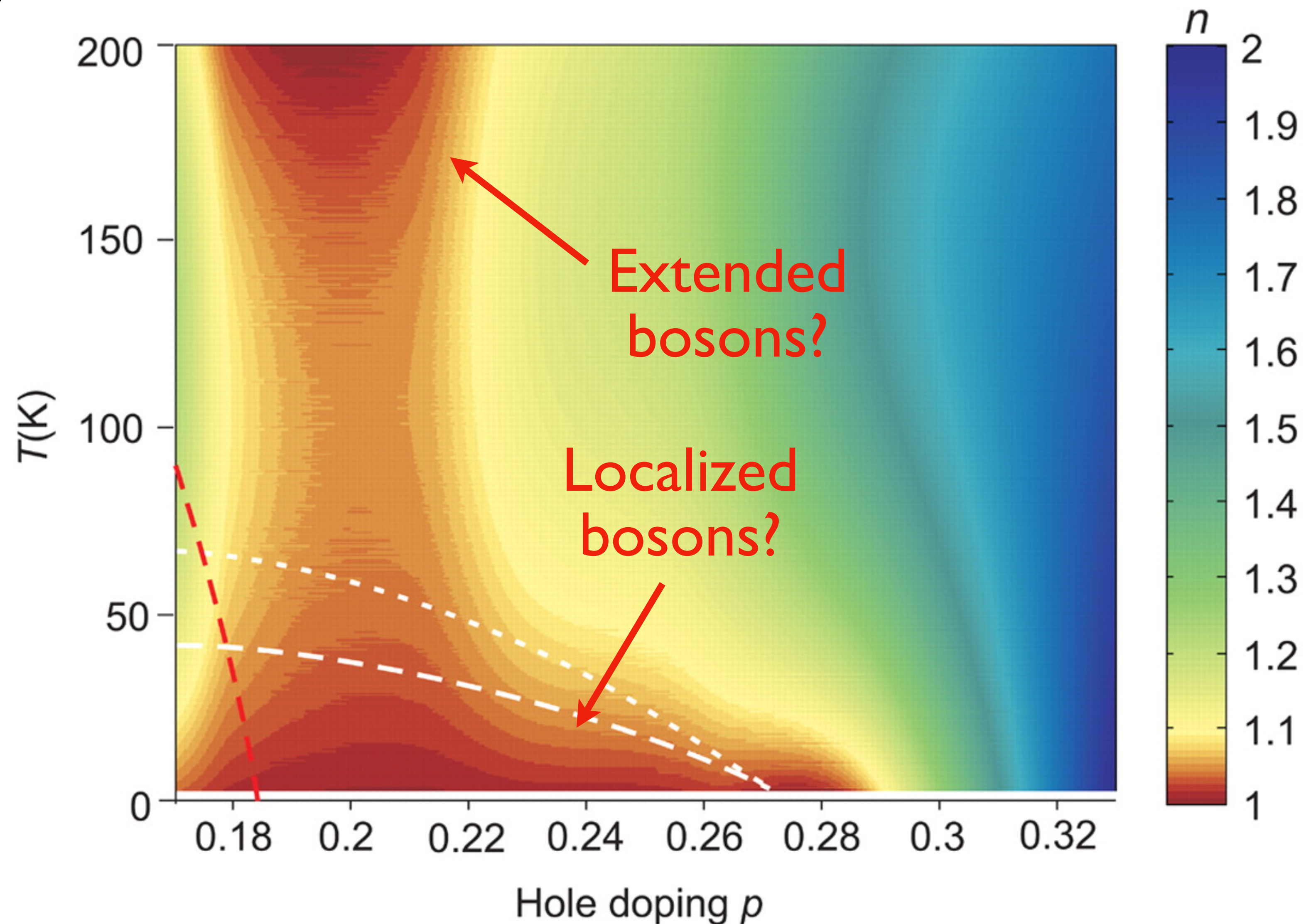
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3. Random “mass” disorder at quantum critical points in metals
 - Mapping to random transverse field Ising model (RTFIM)