Emergent gauge fields and the high temperature superconductors

Nambu Memorial Symposium
University of Chicago
March 12, 2016

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

PHYSICS
VE RI TAS
HARVARD

Nambu and superconductivity

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I*

Y. NAMBU AND G. JONA-LASINIO†
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois
(Received October 27, 1960)

Quasi-Particles and Gauge Invariance in the Theory of Superconductivity*

YOICHIRO NAMBU
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois
(Received July 23, 1959)

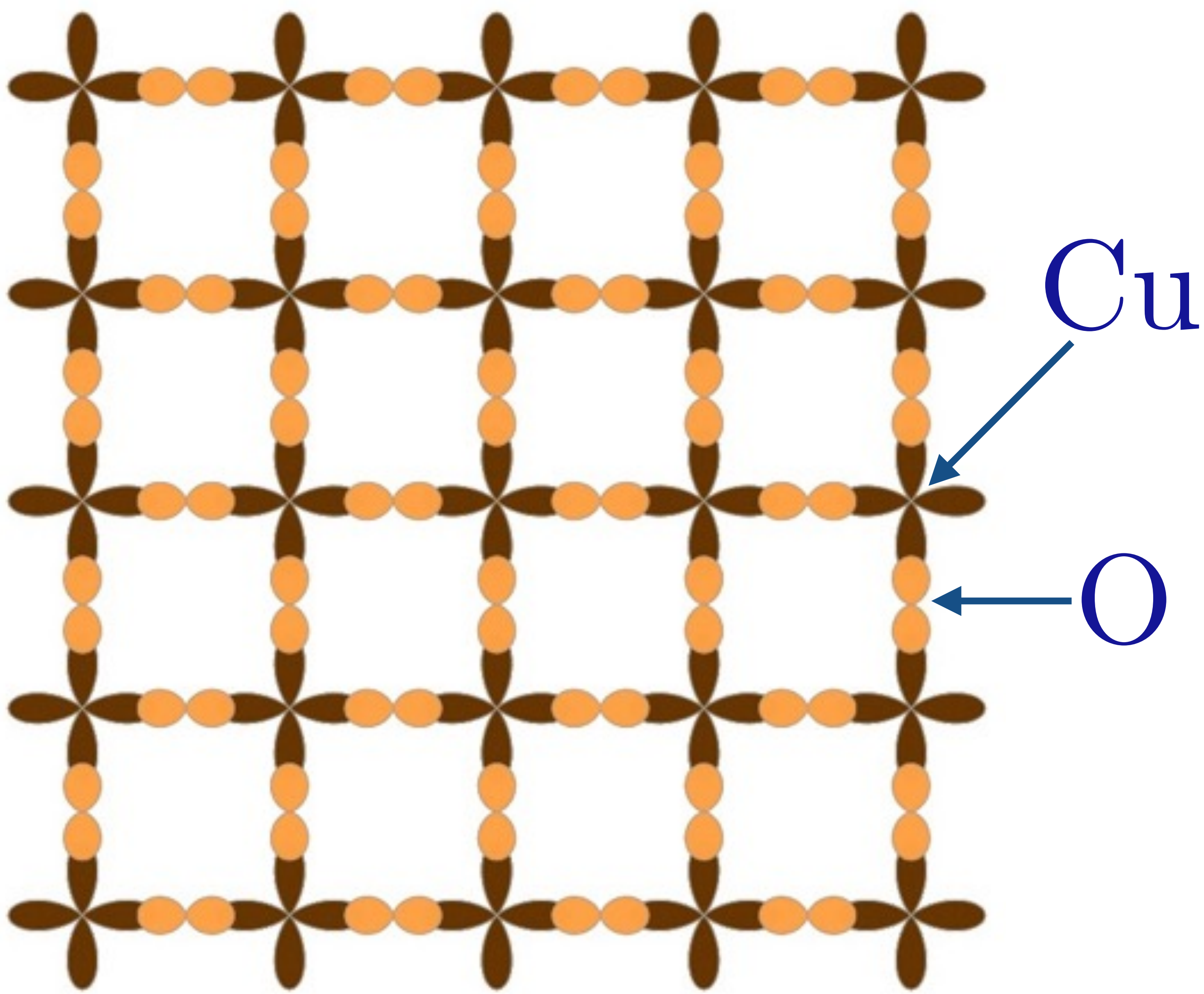
The problems of
Nambu-Goldstone bosons linked to the
spontaneous breaking of a global symmetry
and
the Higgs phase with a massive photon in
a weakly-coupled gauge theory
are closely connected

High temperature superconductors:

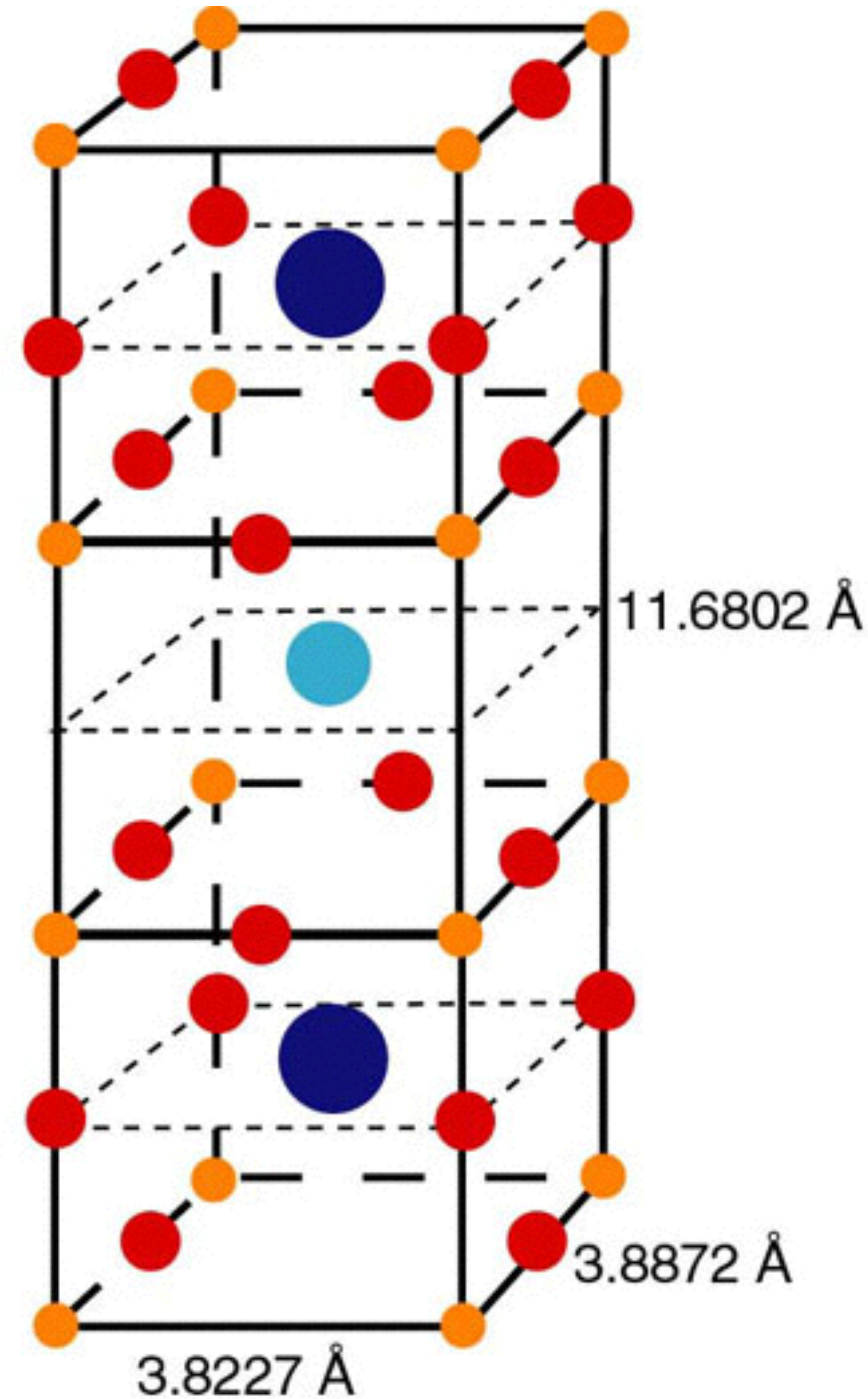
Electrons in crystals provide a novel "vacuum", and their interactions can lead to quantum ground states with long-range quantum entanglement.

The dynamics of such states is described by "emergent" gauge fields, usually at strong coupling.

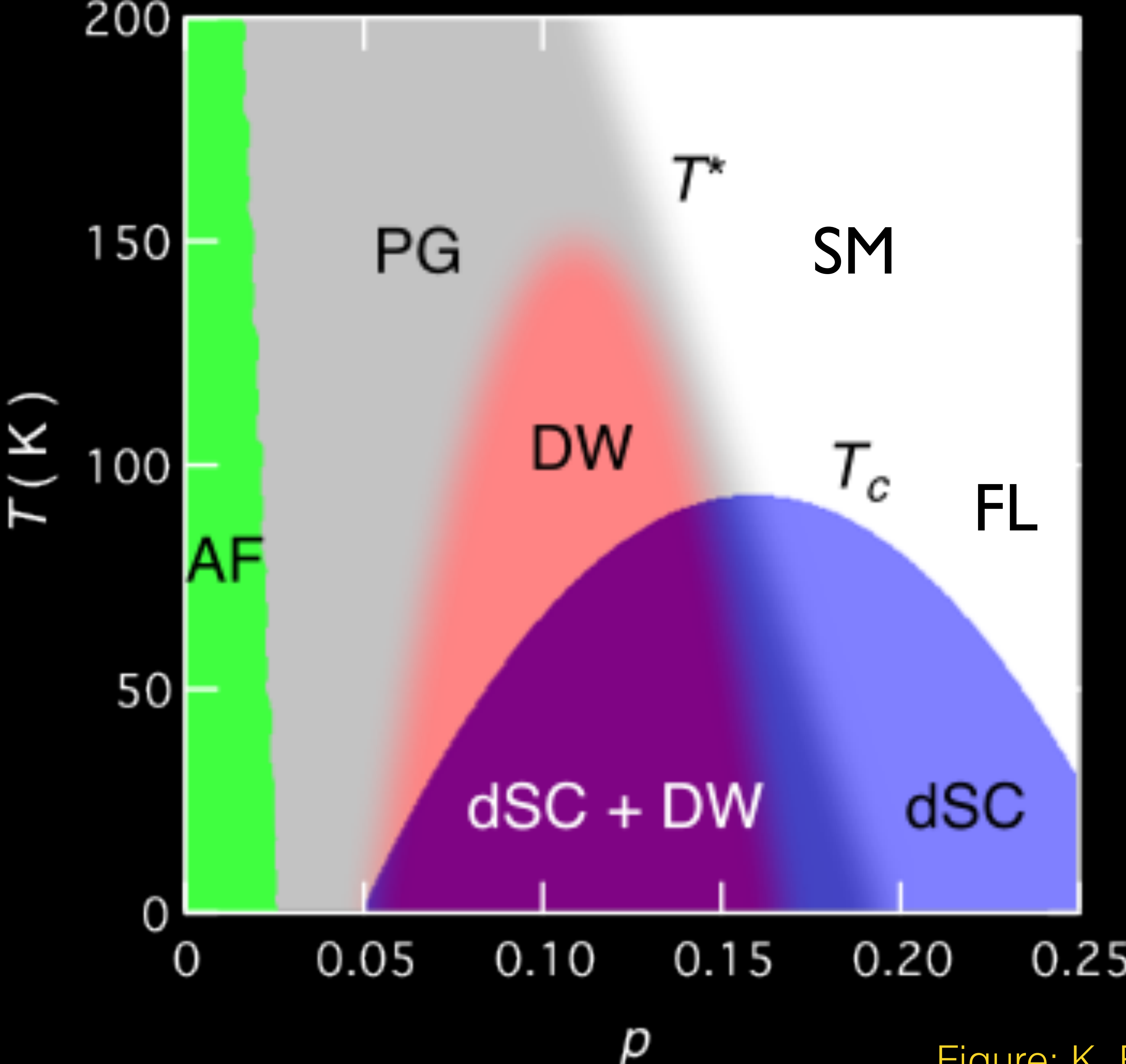
High temperature superconductors

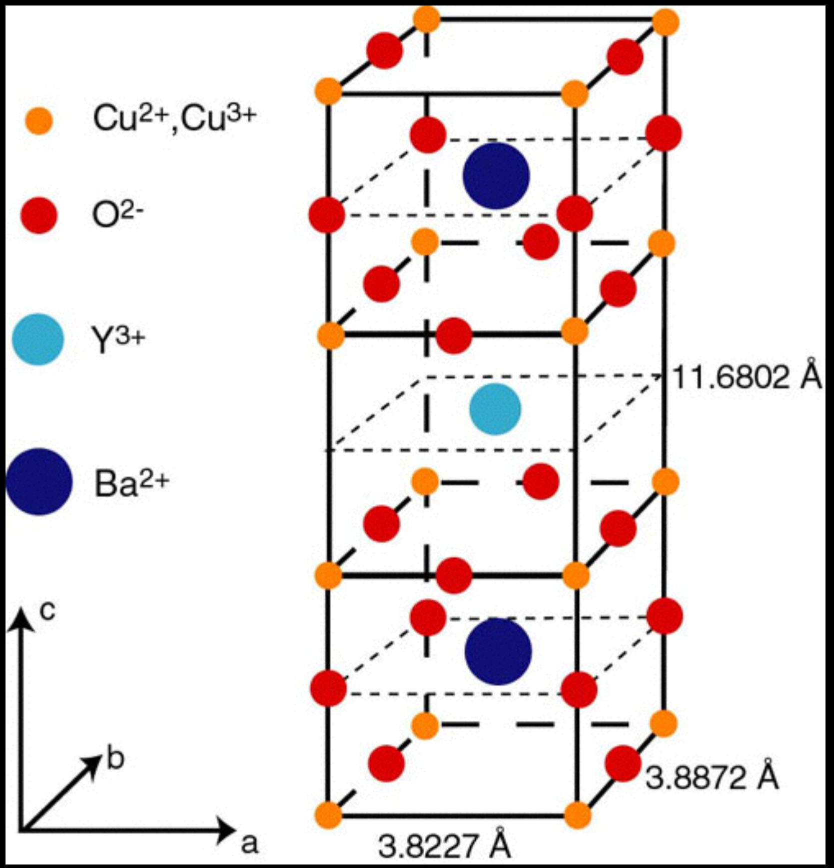


CuO_2 plane



$YBa_2Cu_3O_{6+x}$





$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

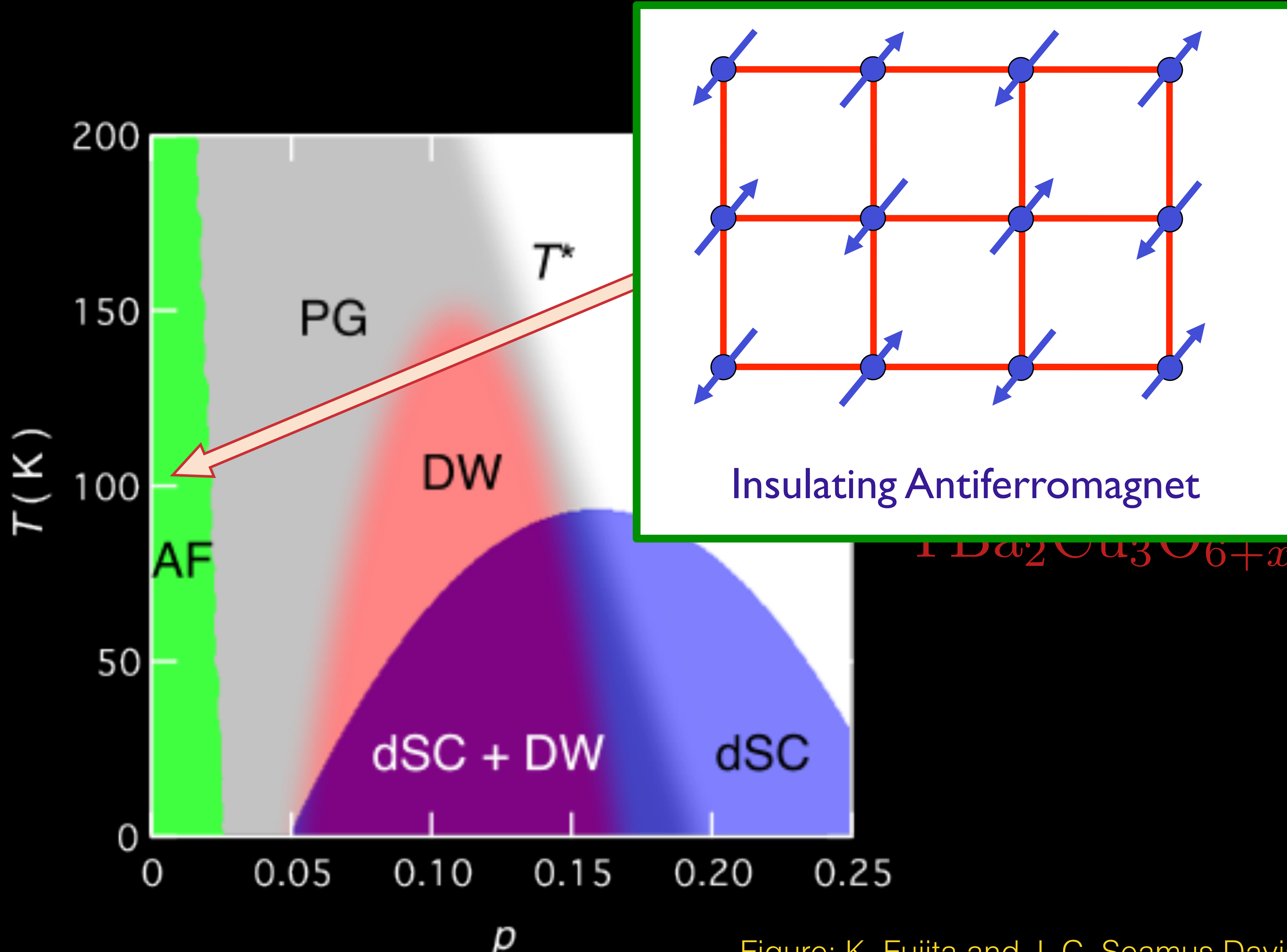
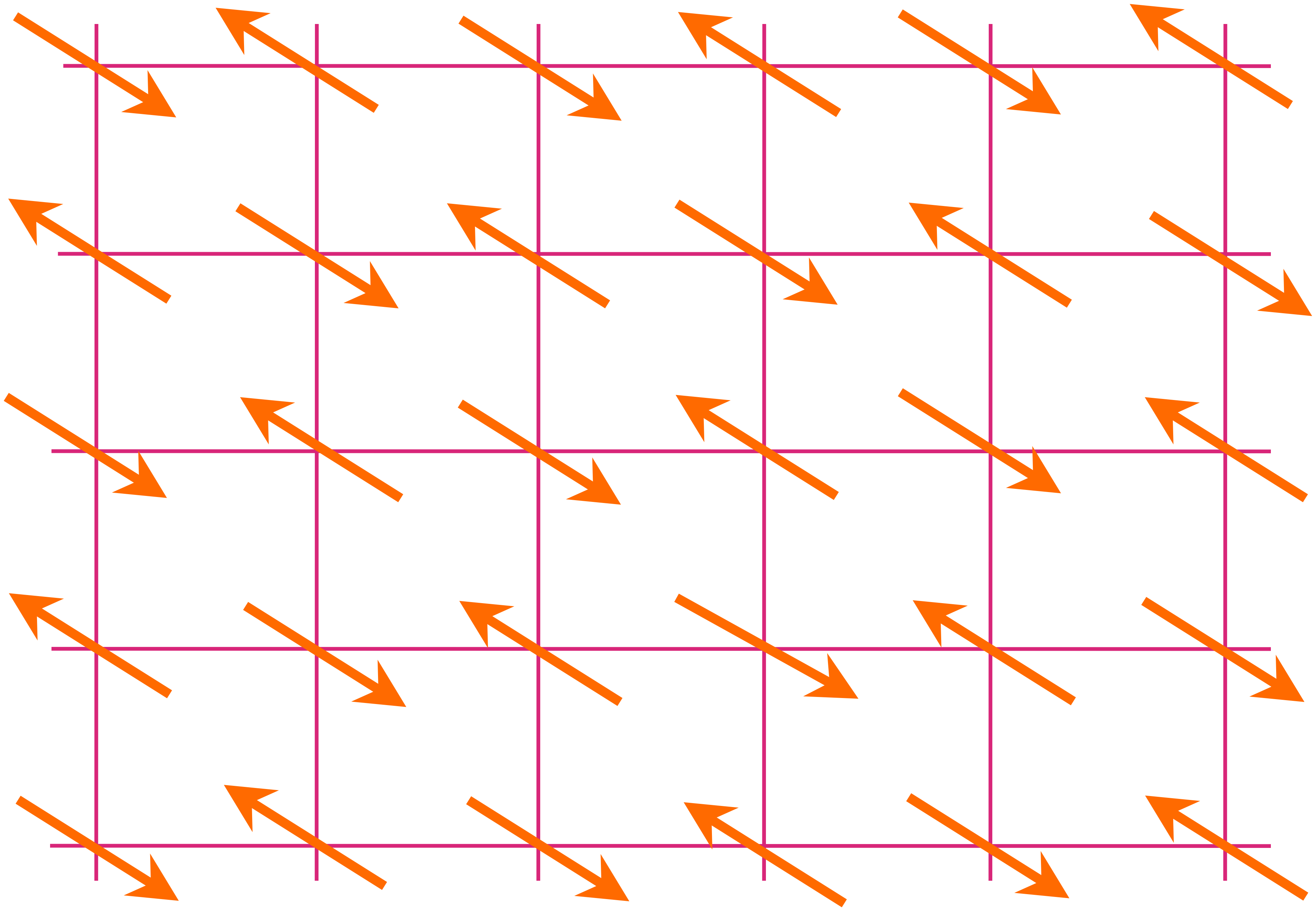
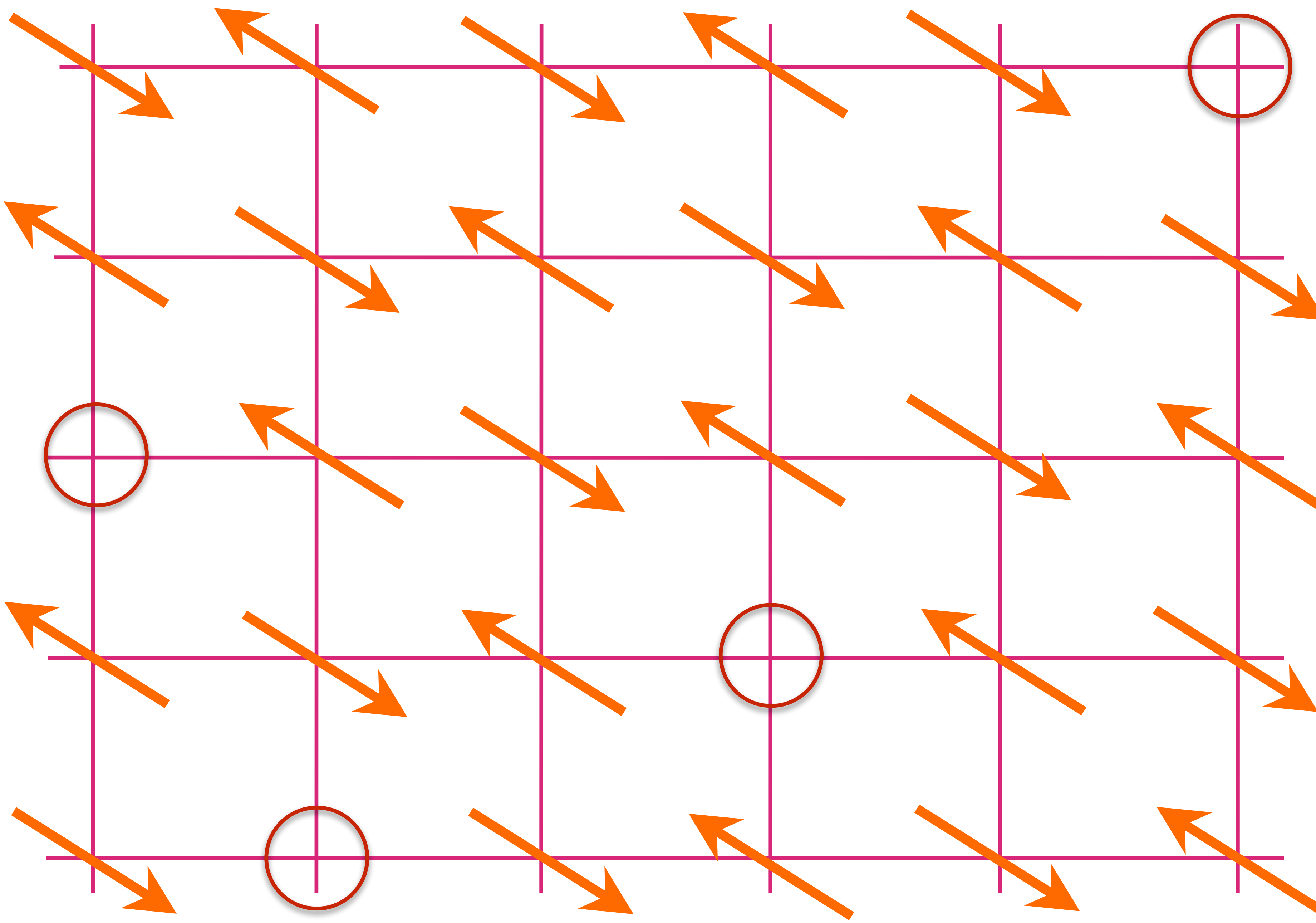


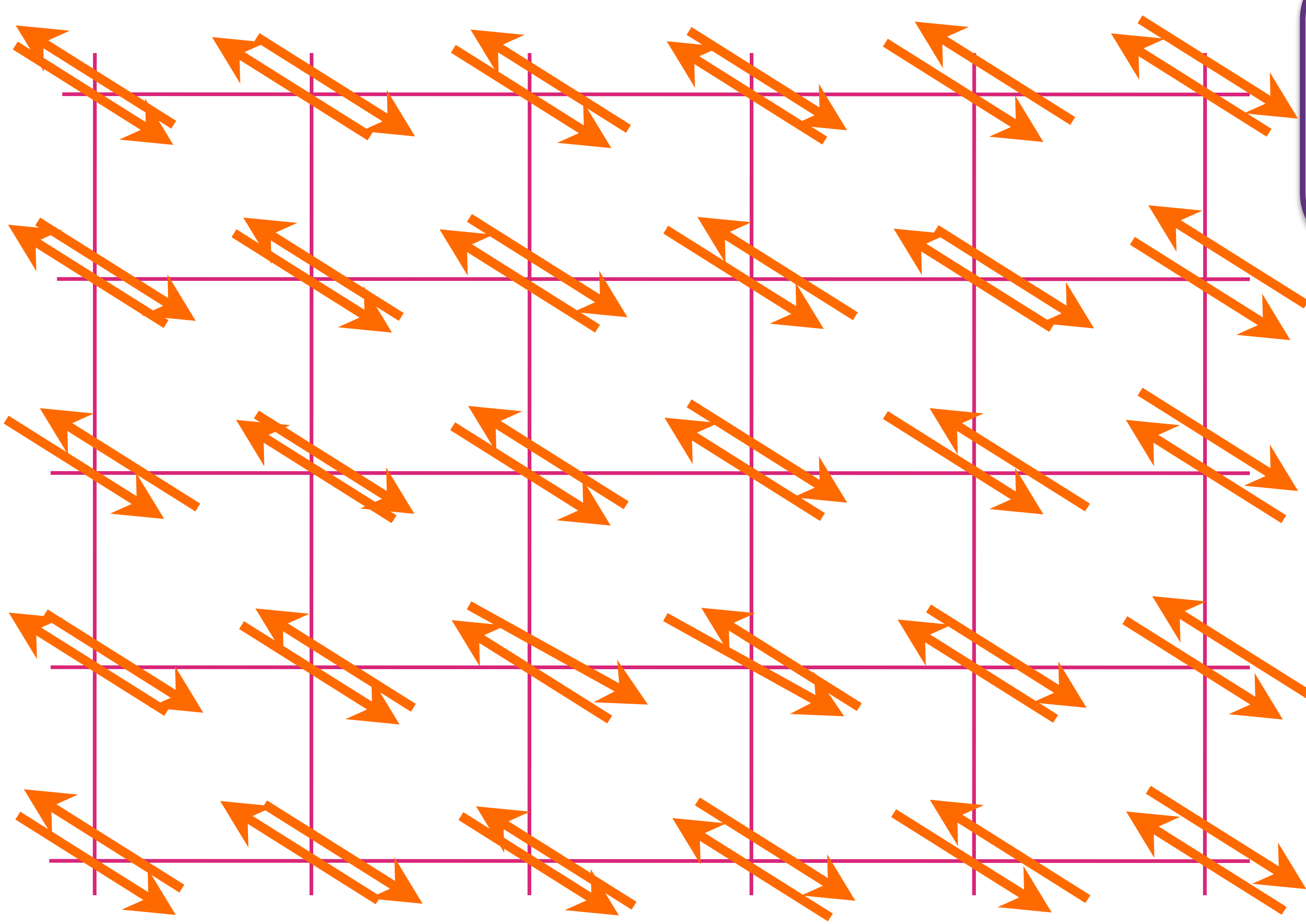
Figure: K. Fujita and J. C. Seamus Davis



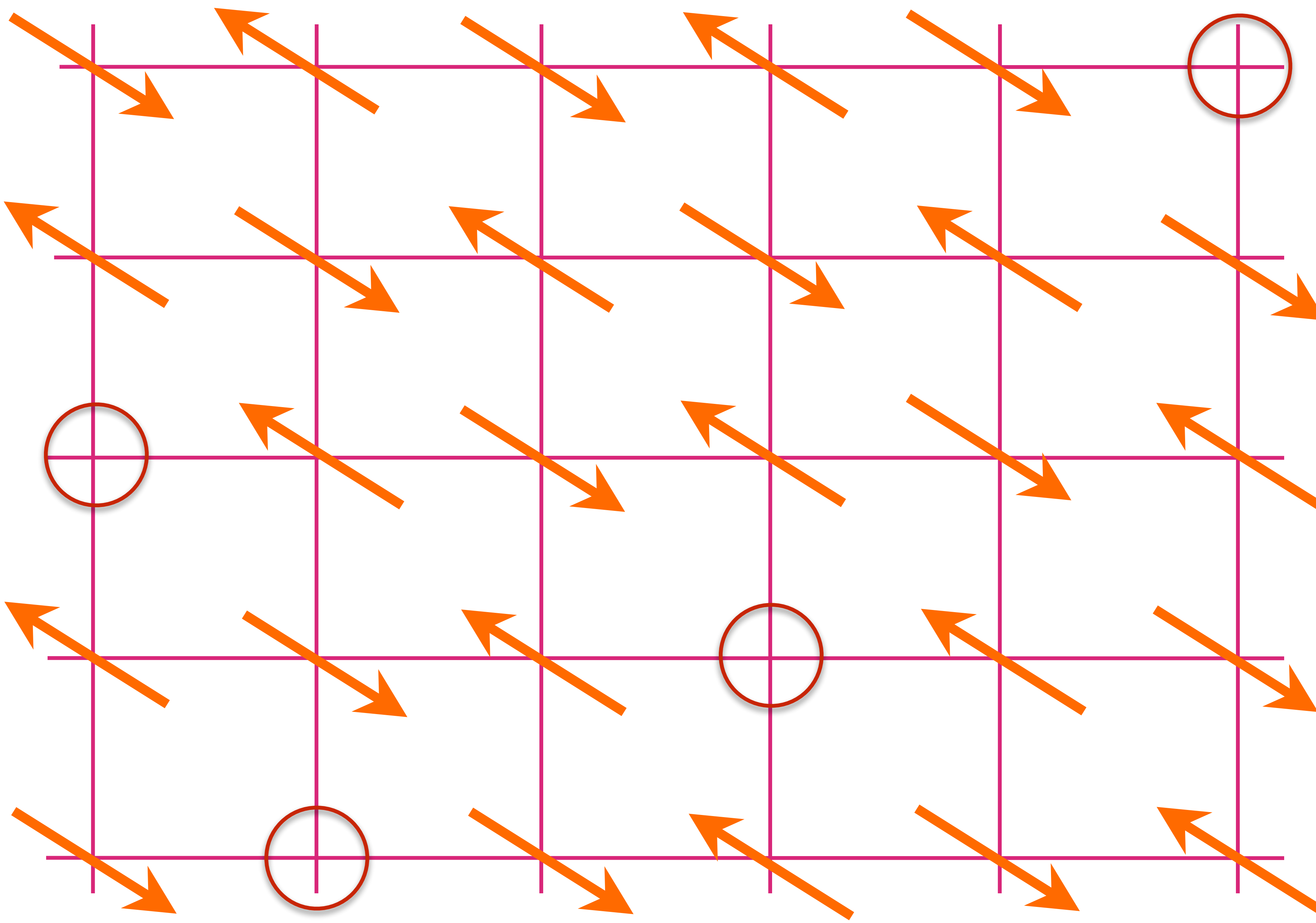
"Undoped" insulating anti-ferromagnet



Anti-
ferromagnet
with p mobile
holes
per square

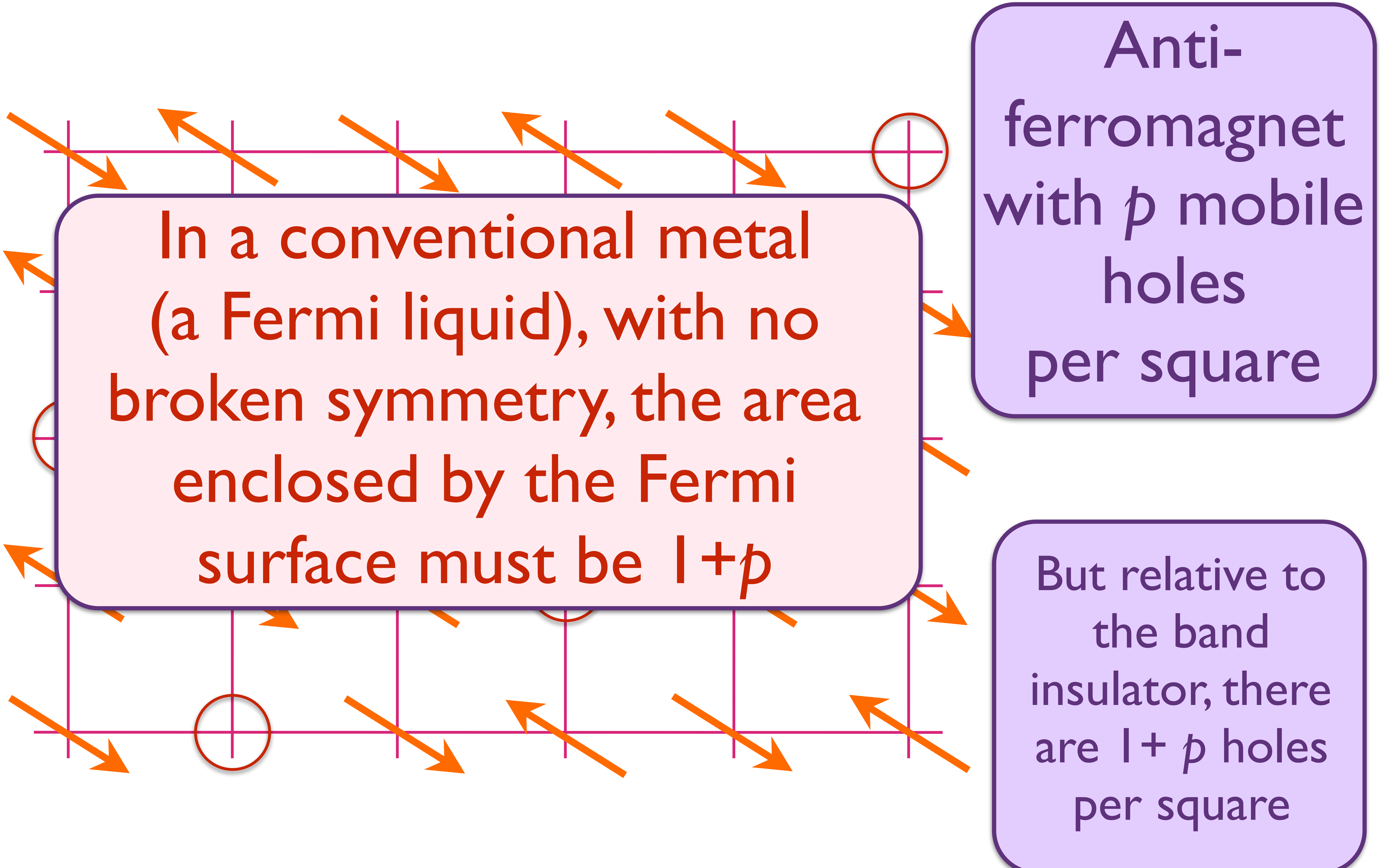


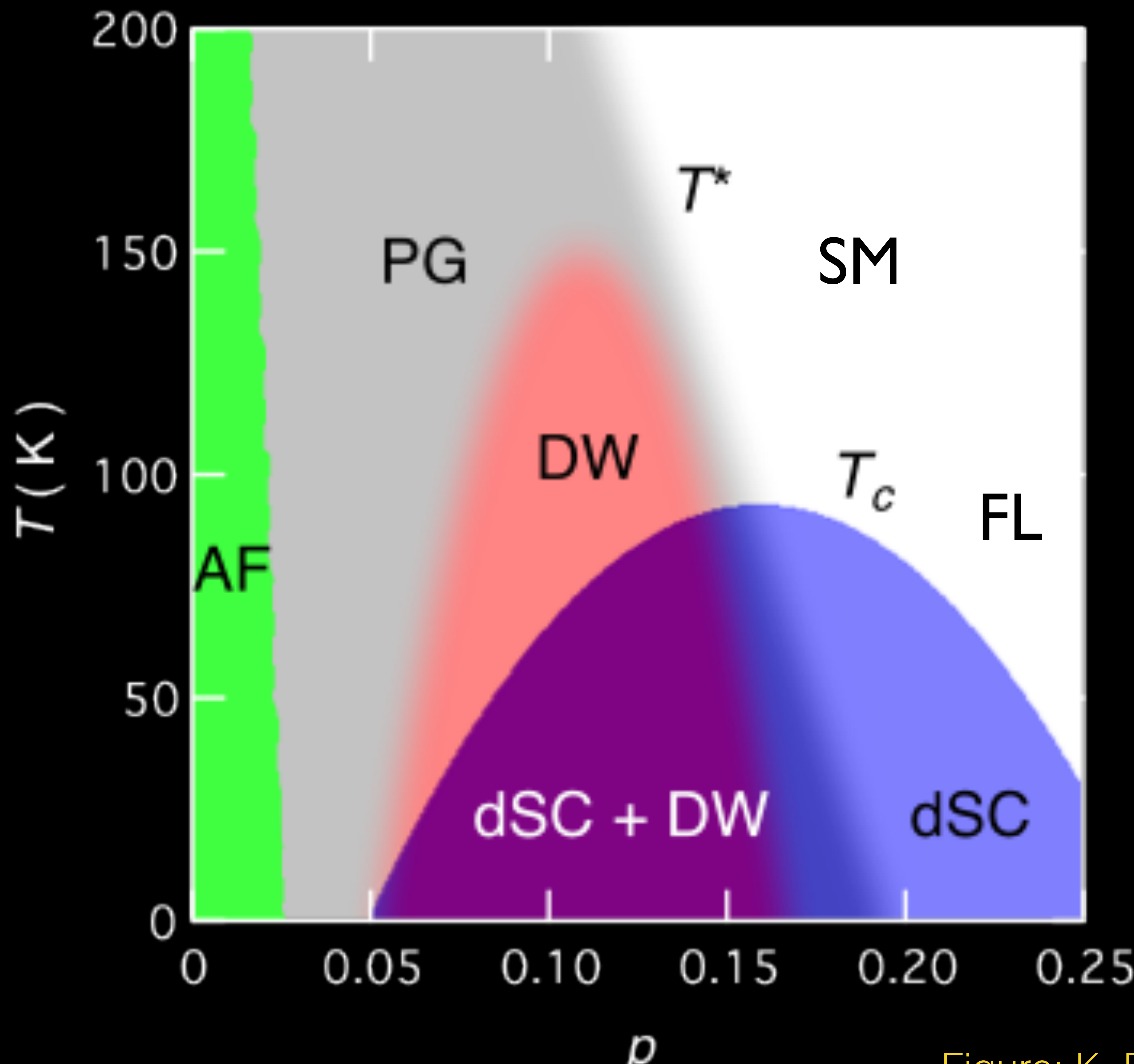
Filled
Band

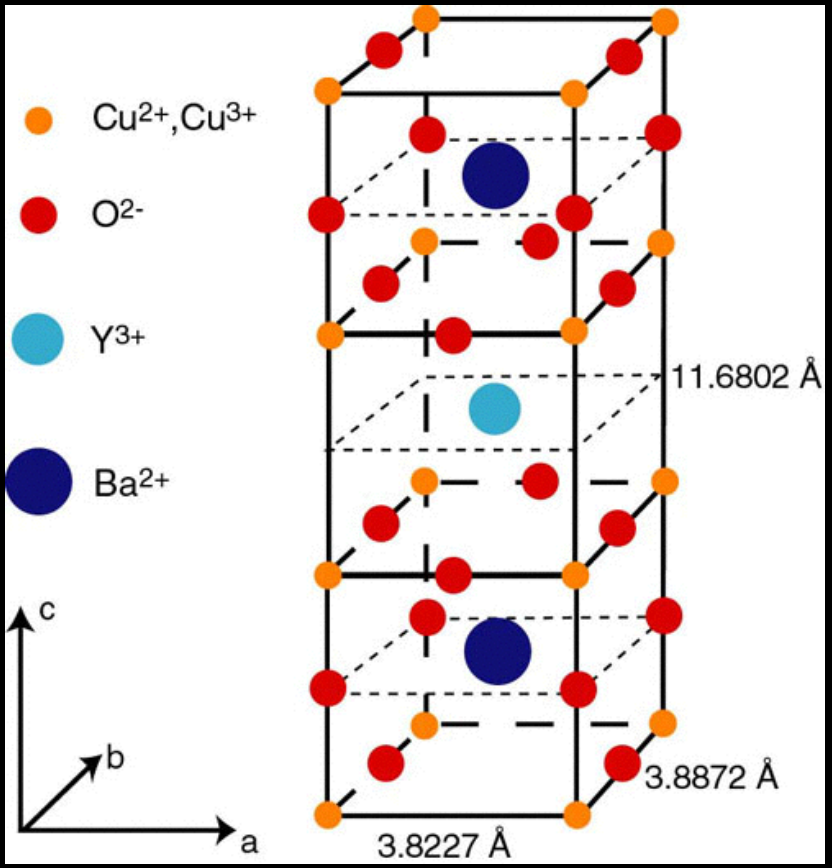


Anti-ferromagnet with p mobile holes per square

But relative to the band insulator, there are $1+p$ holes per square


Anti-
ferromagnet
with p mobile
holes
per square
In a conventional metal
(a Fermi liquid), with no
broken symmetry, the area
enclosed by the Fermi
surface must be 1+p
But relative to
the band
insulator, there
are 1+ p holes
per square

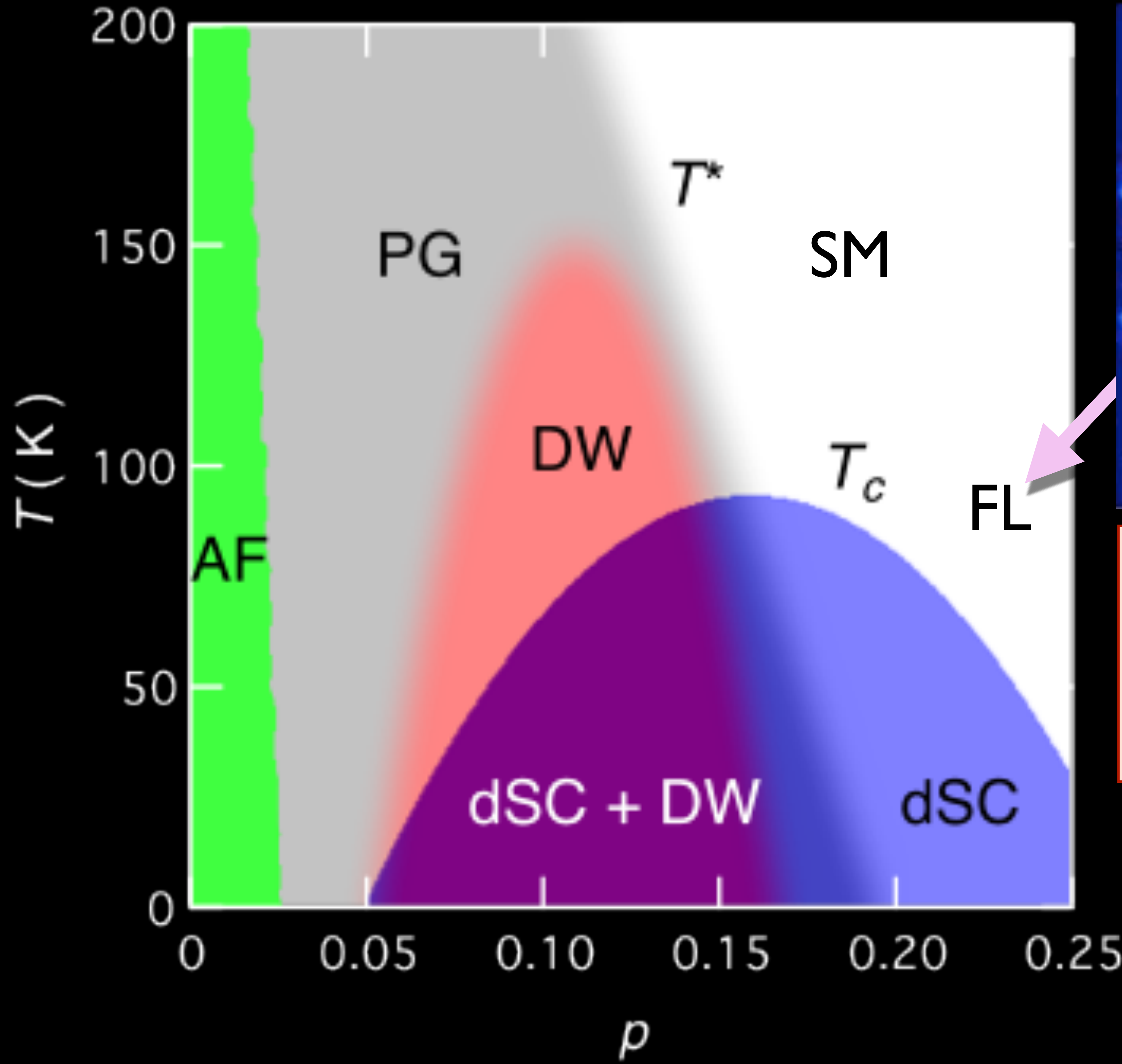


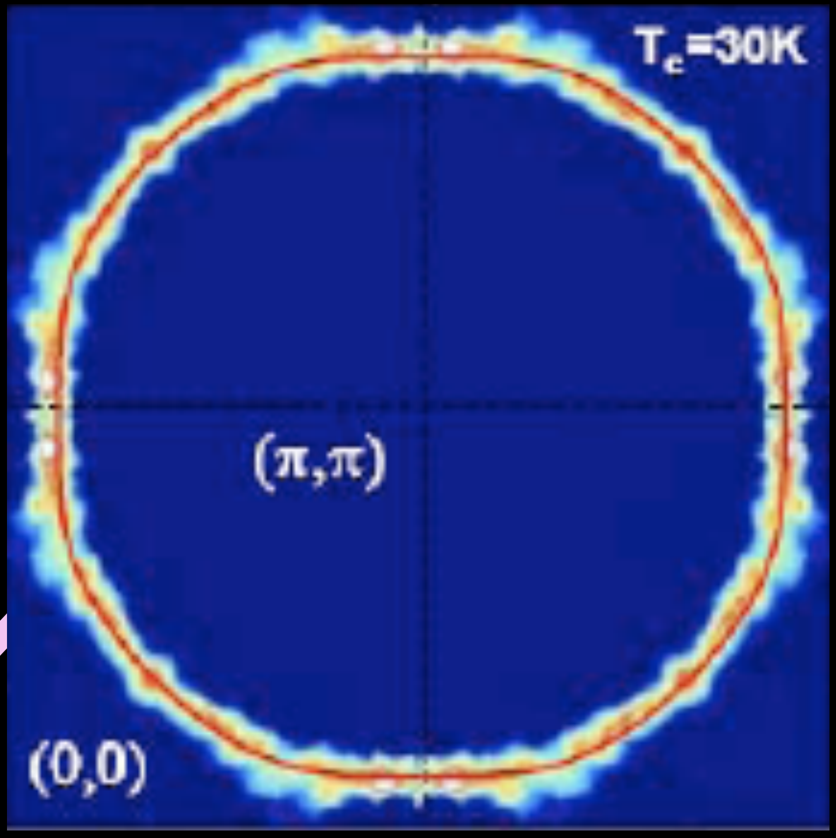


$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

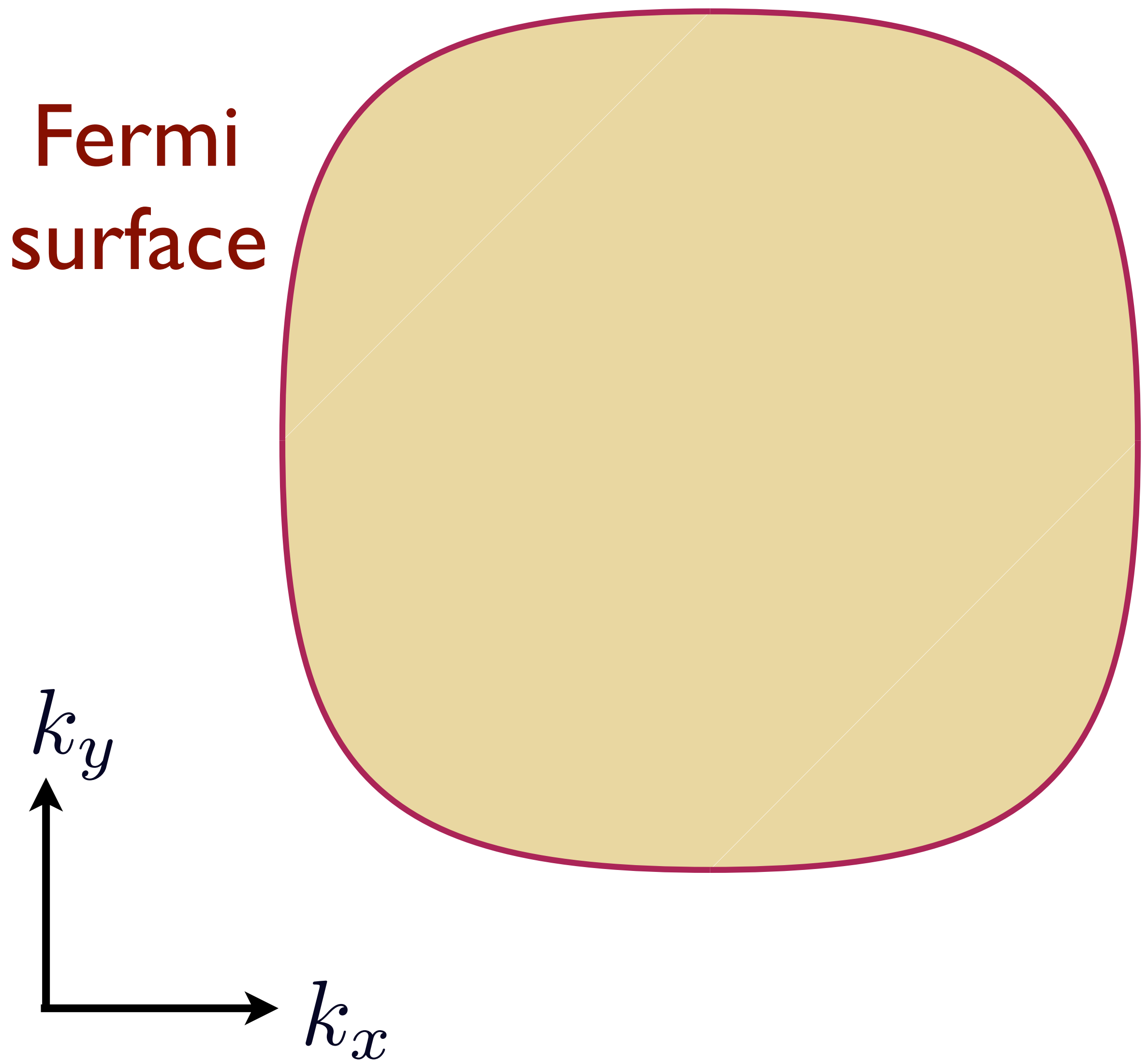
M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)



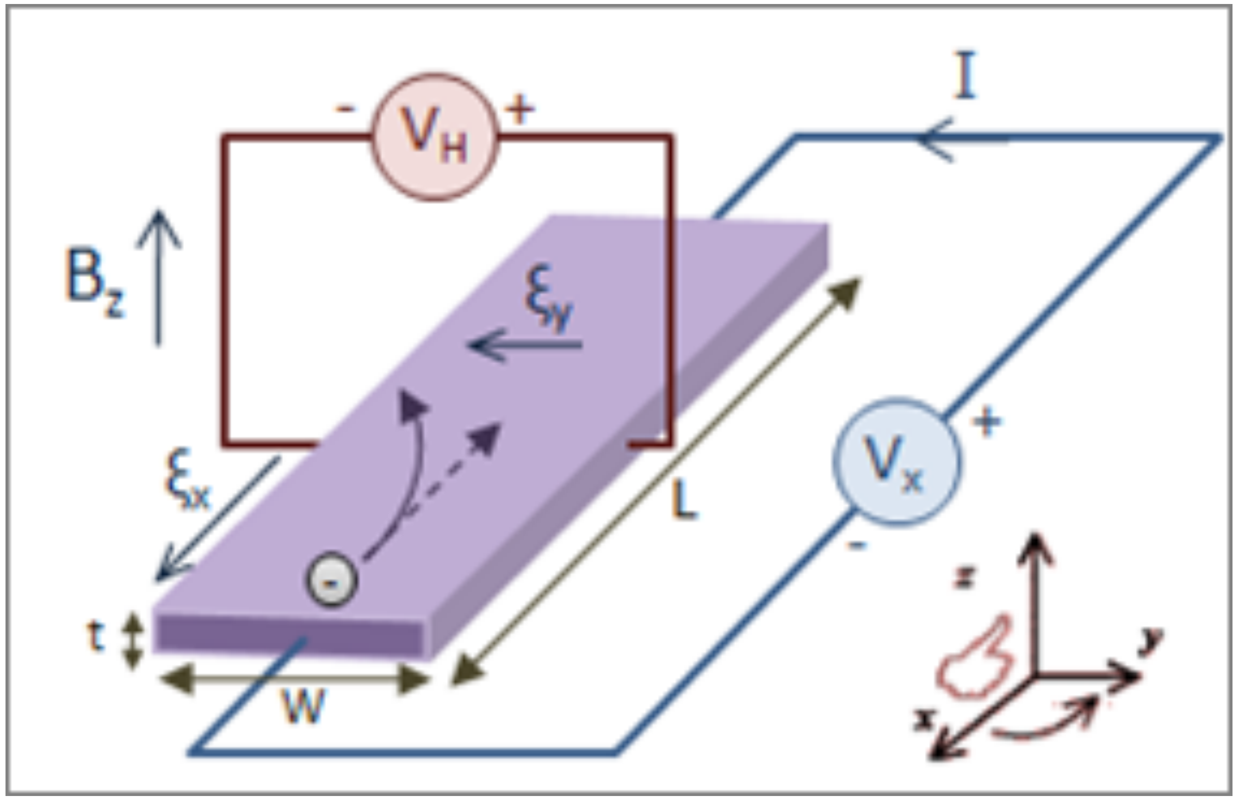


A conventional metal: the Fermi liquid

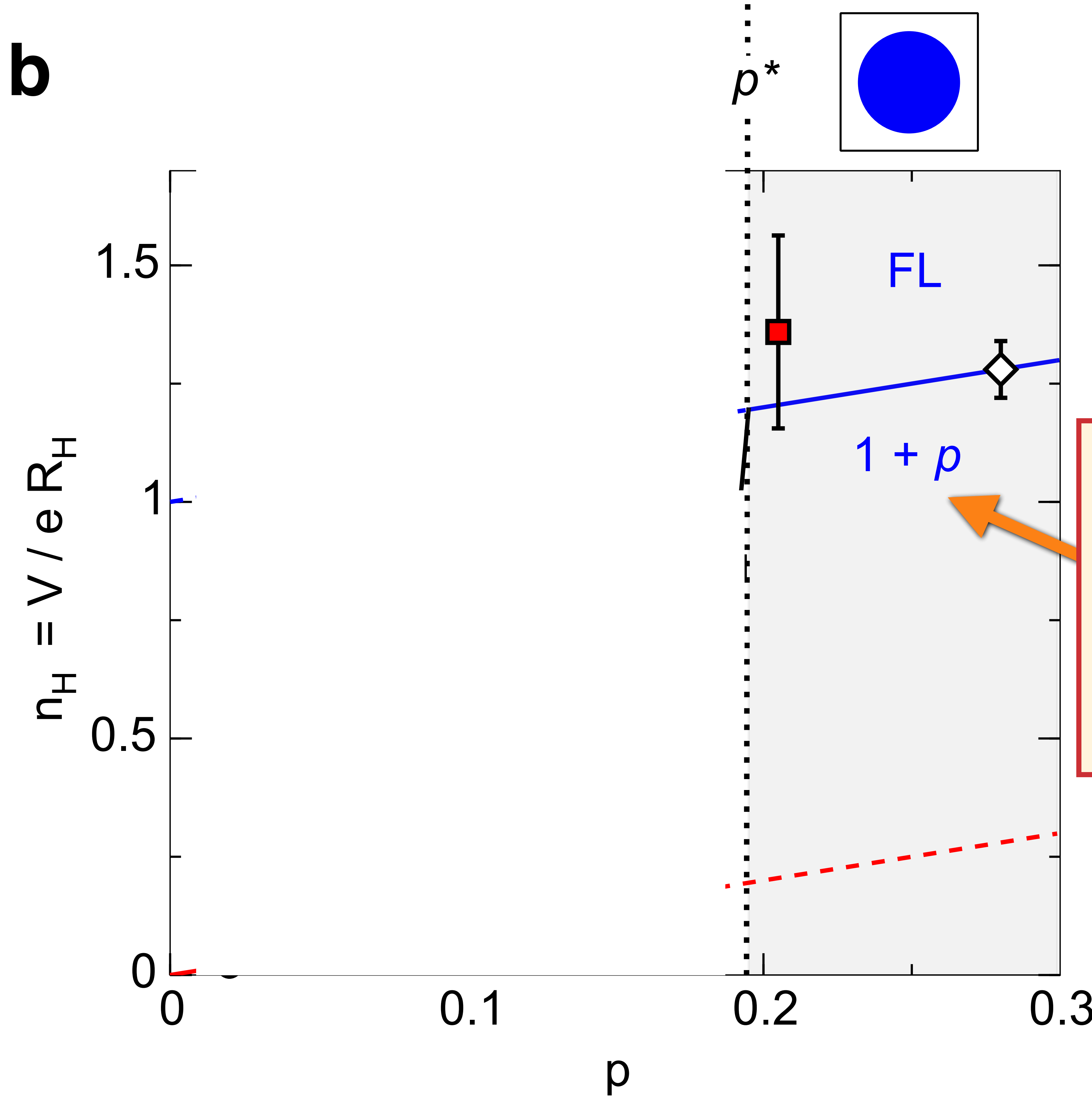
Ordinary metals: the Fermi liquid



- Fermi surface separates empty and occupied states in momentum space.
- *Luttinger Theorem*: volume (area) enclosed by Fermi surface = the electron density.
- Hall co-efficient $R_H = -1/((\text{Fermi volume}) \times e)$.



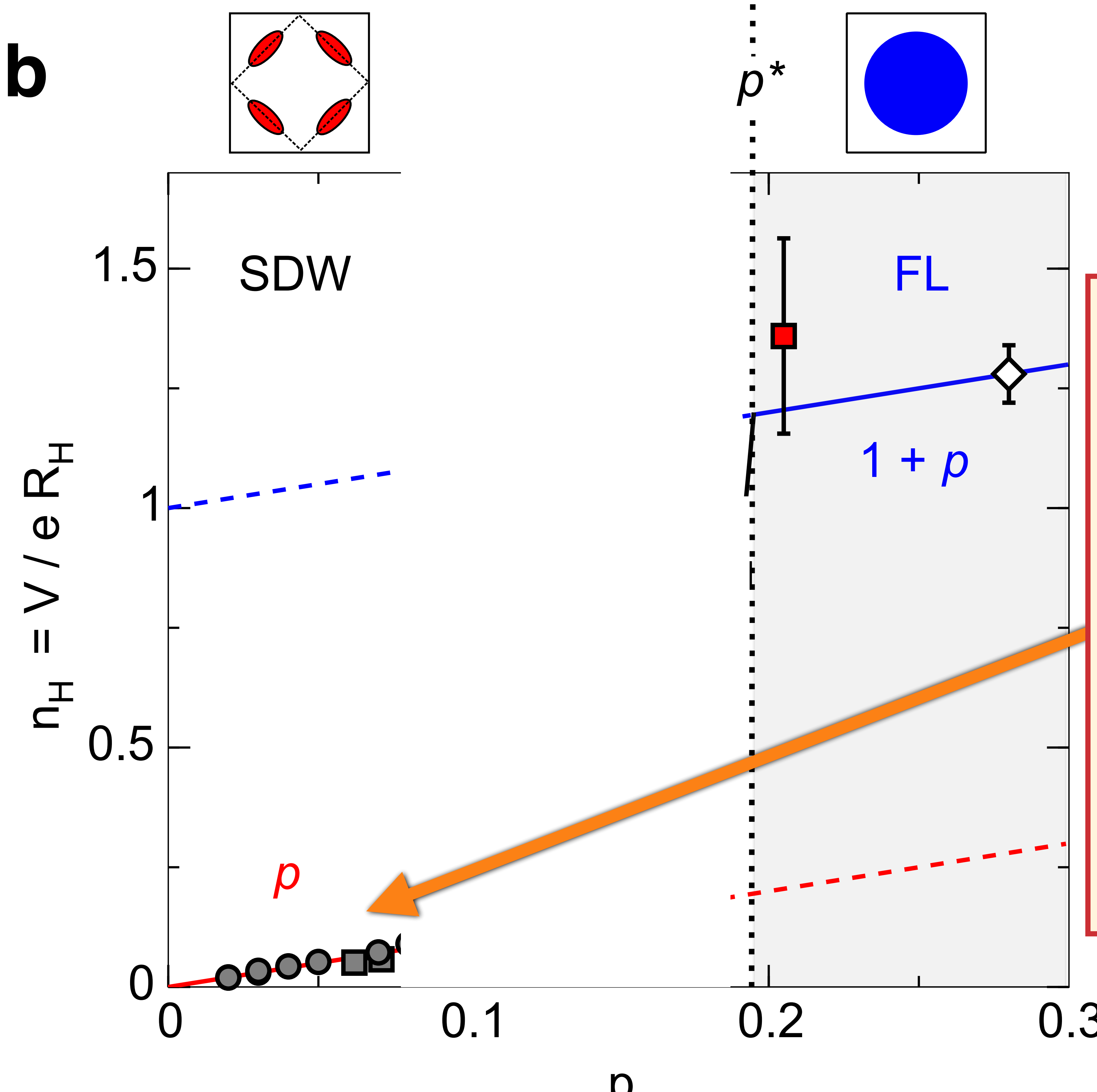
Hall effect measurements in YBCO



Fermi liquid (FL) with carrier density 1+*p*

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

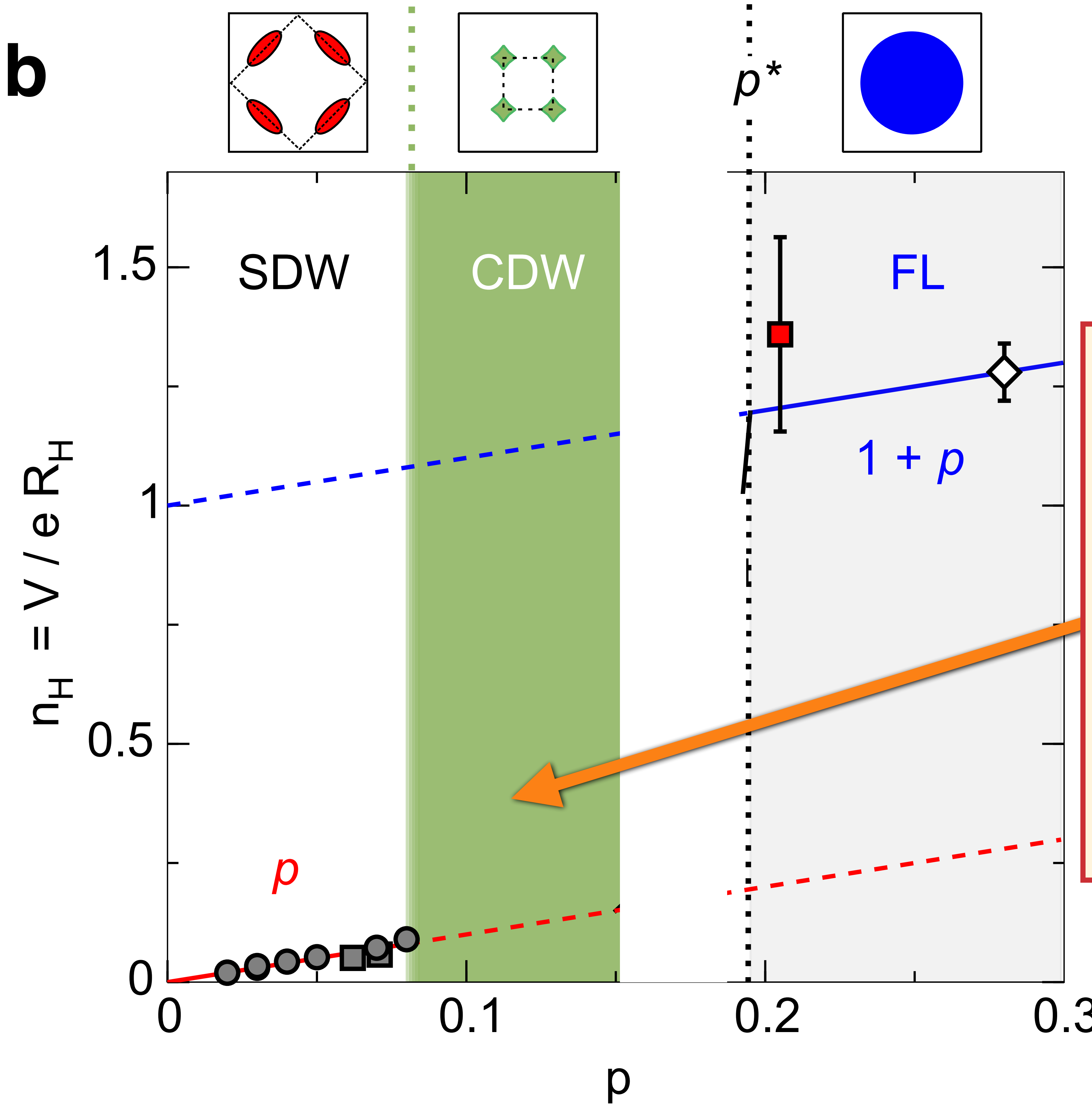
Hall effect measurements in YBCO



Spin density wave (SDW) breaks translational invariance, and the Fermi liquid then has carrier density p

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

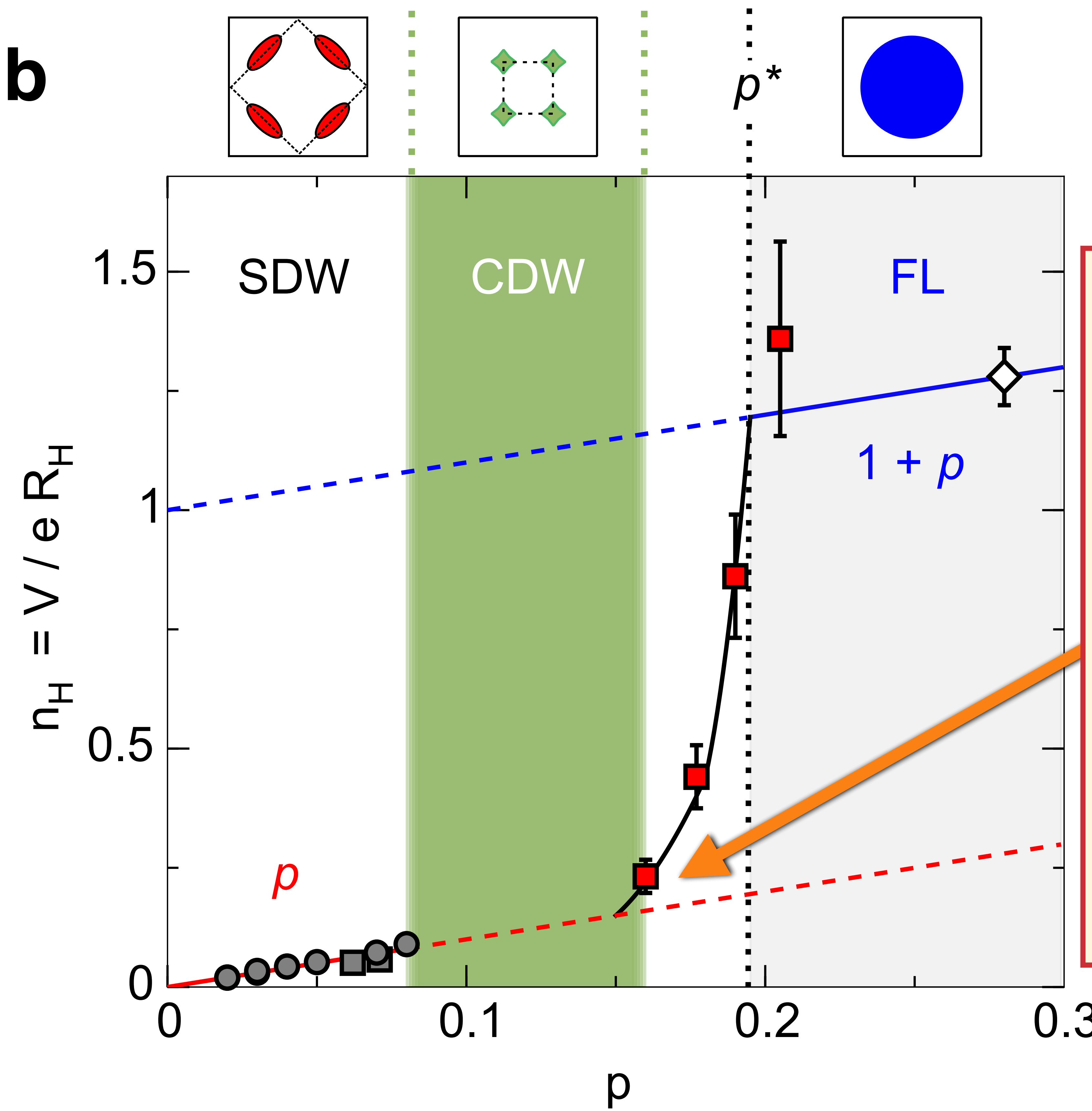
Hall effect measurements in YBCO



Charge density wave (CDW) leads to complex Fermi surface reconstruction and negative Hall resistance

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

Hall effect measurements in YBCO



Evidence for FL* metal with Fermi surface of size p and emergent gauge fields ?!

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

FL*

A metal with:

- A Fermi surface of electrons enclosing volume p, and not the Luttinger volume of $1+p$
- Emergent gauge fields and connections to topological field theories

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett* **90**, 216403 (2003)

FL*

A metal with:

- A Fermi surface of electrons enclosing volume p, and not the Luttinger volume of $1+p$
- Emergent gauge fields and connections to topological field theories

There is a general and fundamental relationship between these two characteristics.

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett* **90**, 216403 (2003)

1. The insulating spin liquid and topological field theory

2. Topology and the size of the Fermi surface

3. Transition between FL* and FL

4. Quantum matter with quasiparticles

   strange metals in superconductors,
   graphene,
   the quark-gluon plasma,
   the superfluid-insulator transition of ultra-cold atoms,
   and the dynamics of charged black holes horizons

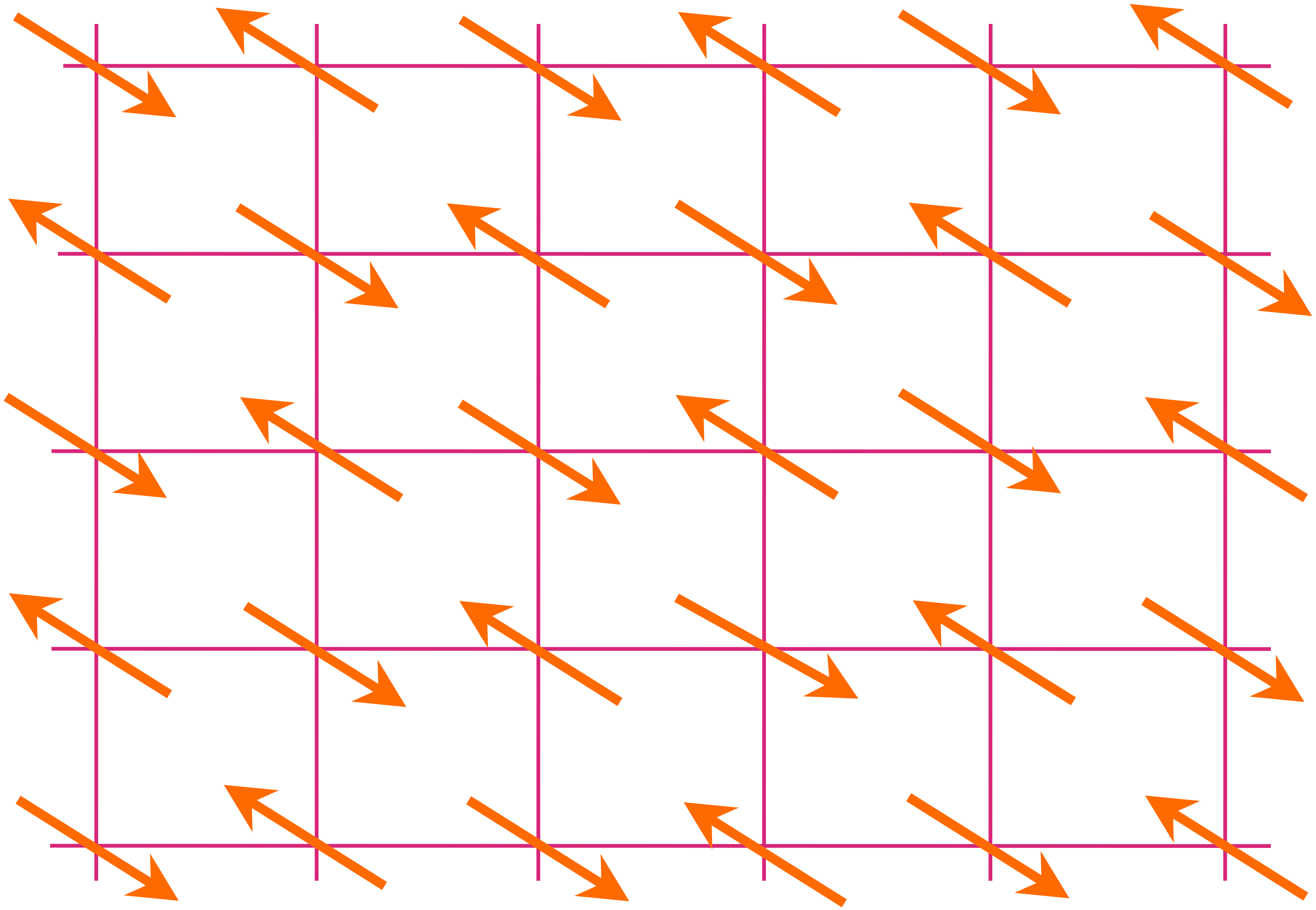
1. The insulating spin liquid and topological field theory

2. Topology and the size of the Fermi surface

3. Transition between FL* and FL

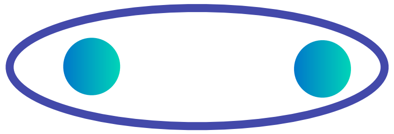
4. Quantum matter with quasiparticles

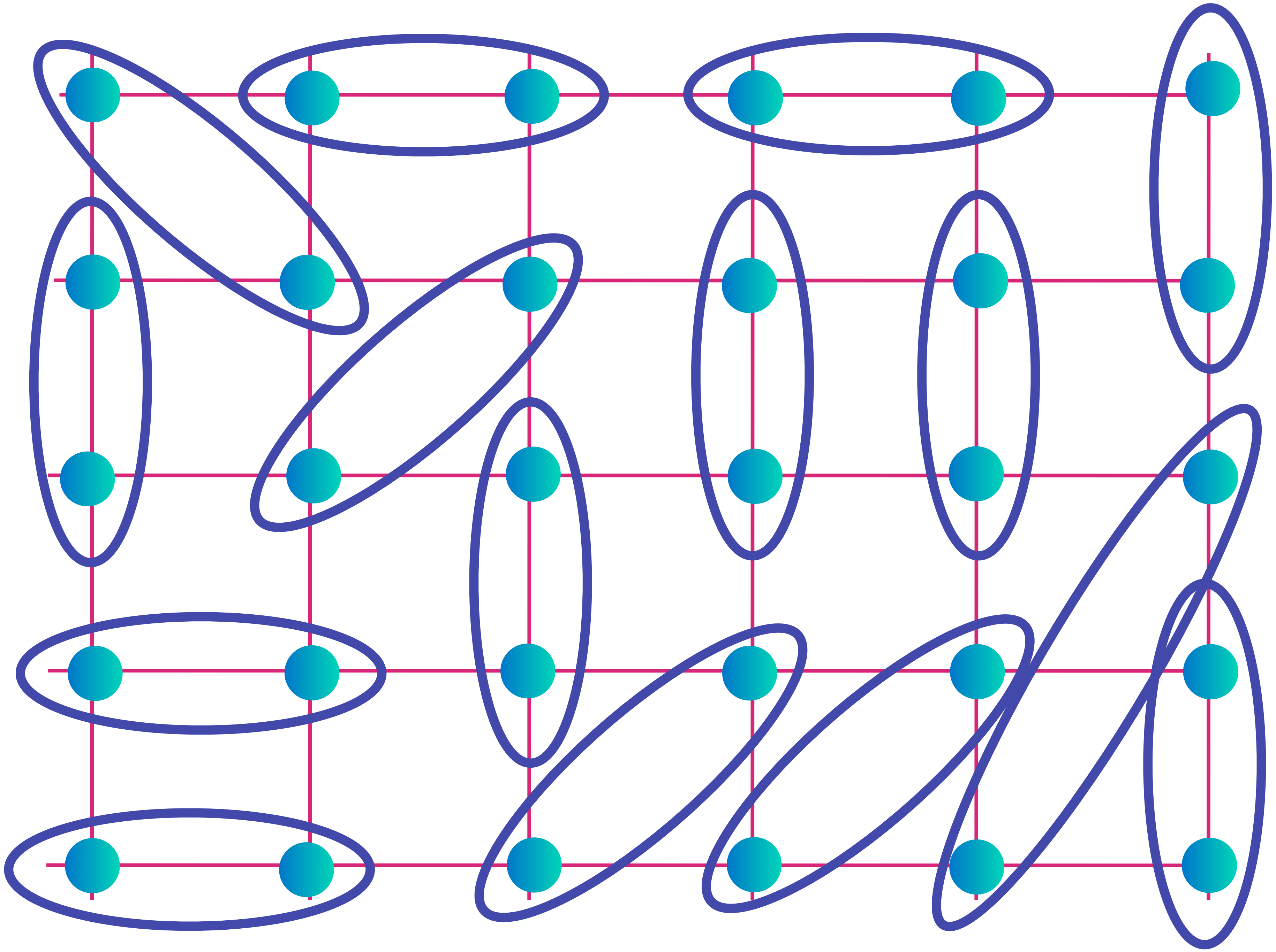
strange metals in superconductors,
graphene,
the quark-gluon plasma,
the superfluid-insulator transition of ultra-cold atoms,
and the dynamics of charged black holes horizons



“Undoped” Anti-ferromagnet

Insulating spin liquid

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

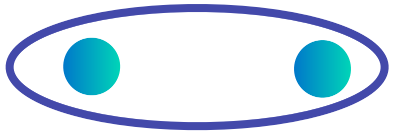


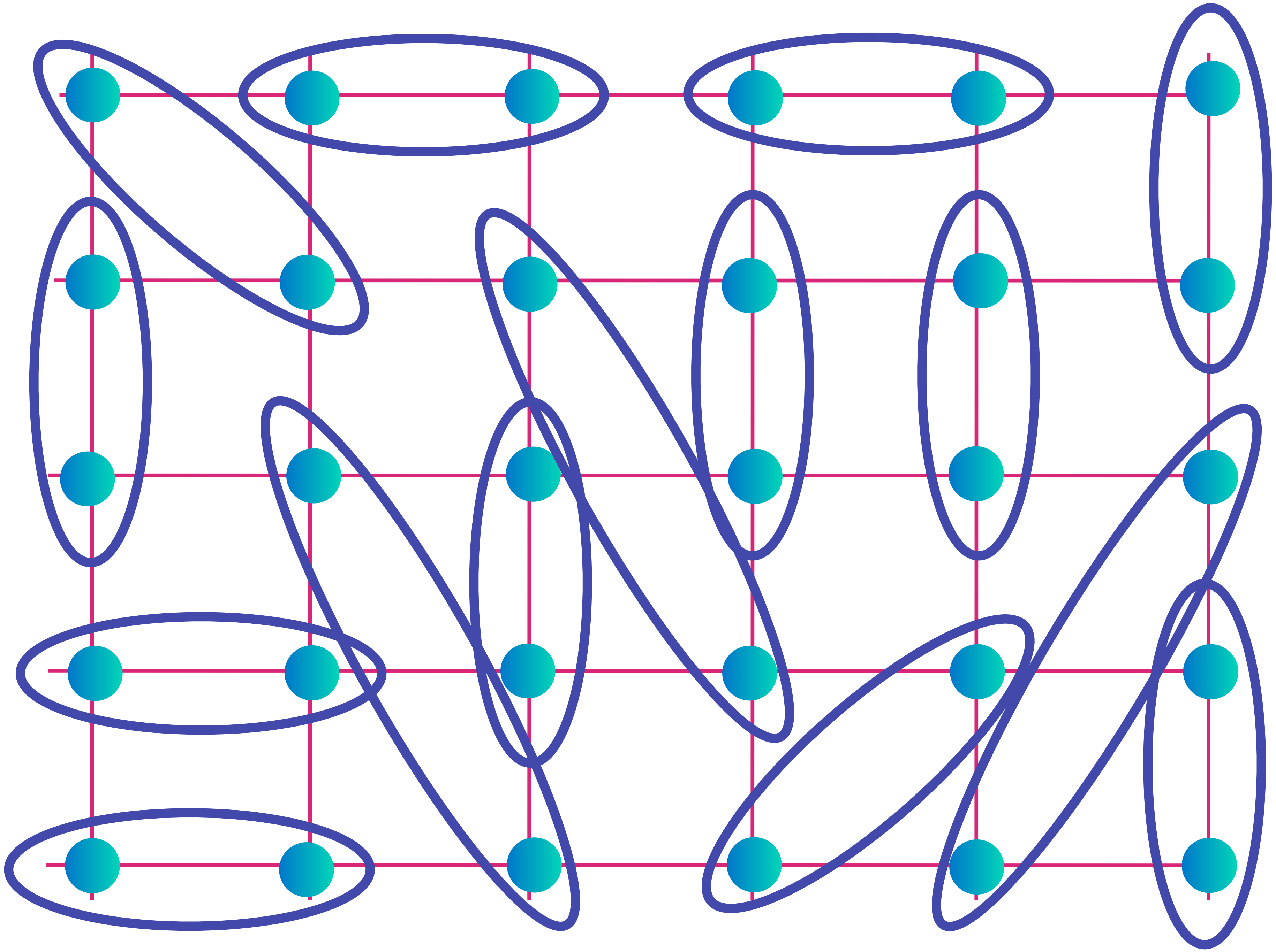
The first proposal of a quantum state with long-range entanglement

L. Pauling, Proceedings of the Royal Society London A **196**, 343 (1949)

P.W. Anderson, Materials Research Bulletin **8**, 153 (1973)

Insulating spin liquid

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

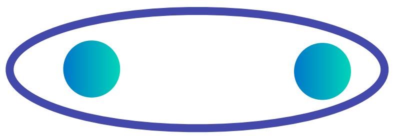


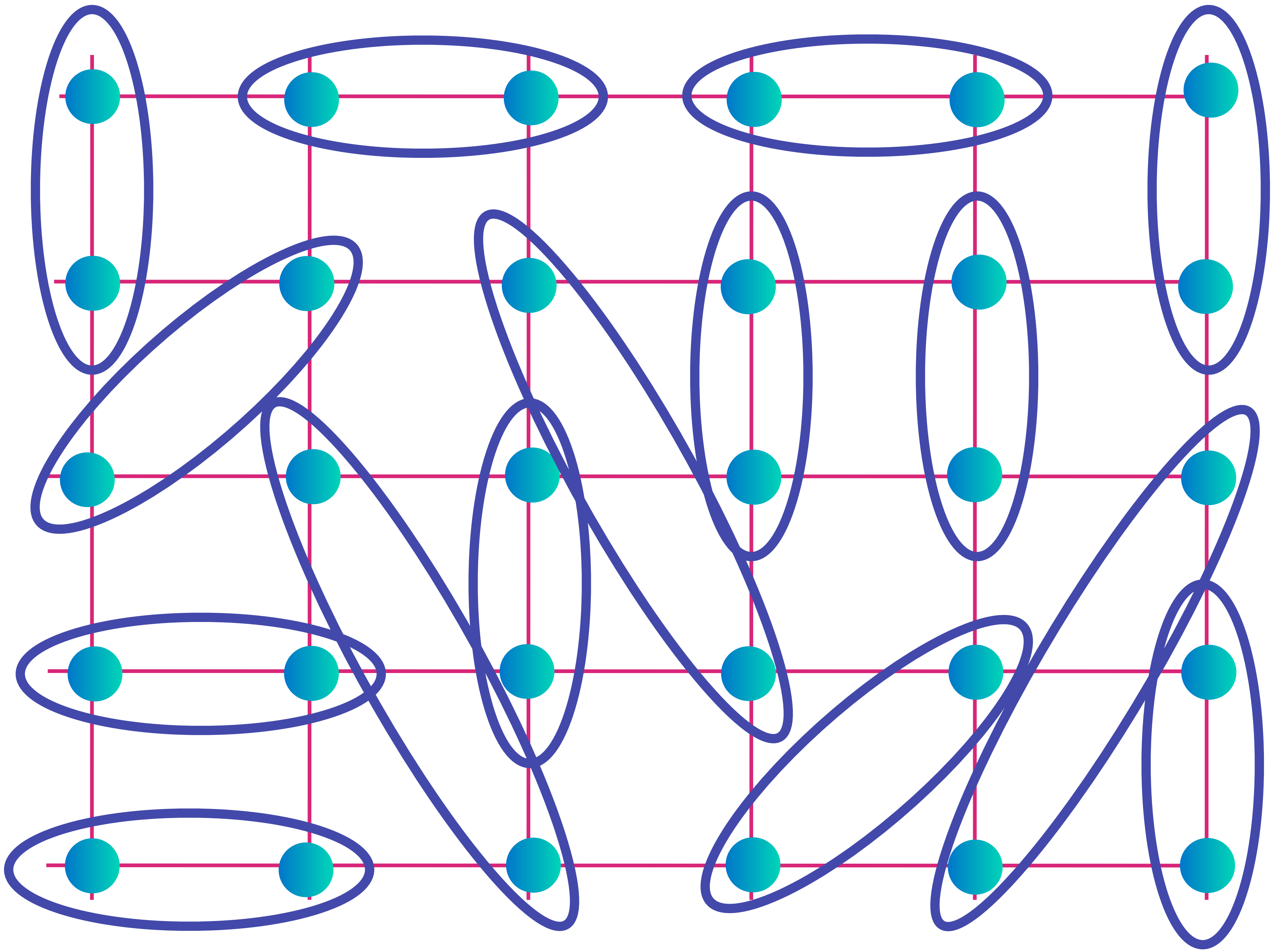
The first proposal of a quantum state with long-range entanglement

L. Pauling, Proceedings of the Royal Society London A **196**, 343 (1949)

P.W. Anderson, Materials Research Bulletin **8**, 153 (1973)

Insulating spin liquid

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

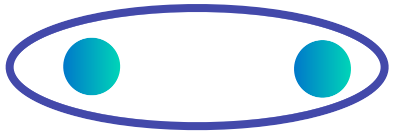


The first proposal of a quantum state with long-range entanglement

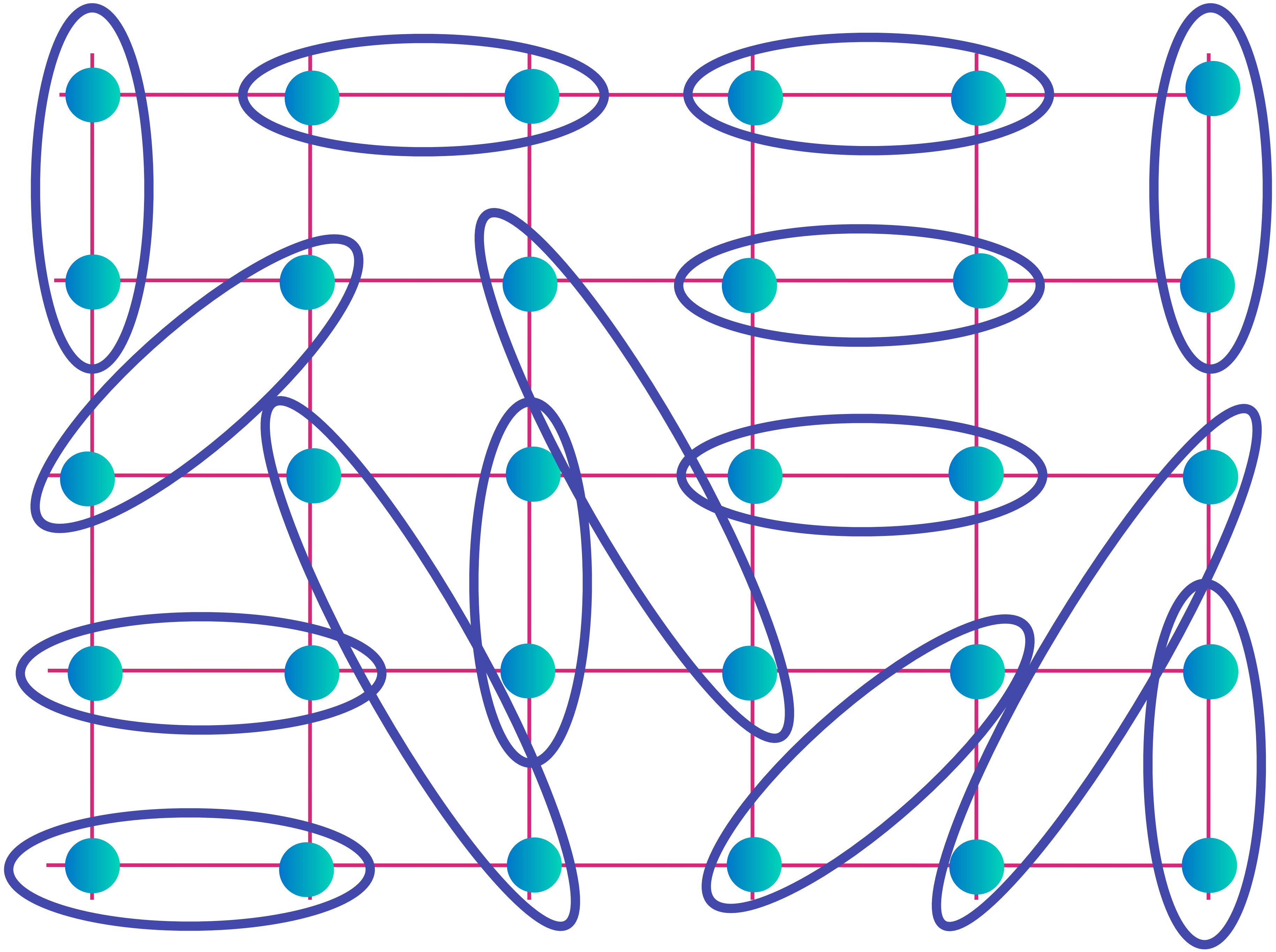
L. Pauling, Proceedings of the Royal Society London A **196**, 343 (1949)

P.W. Anderson, Materials Research Bulletin **8**, 153 (1973)

Insulating spin liquid



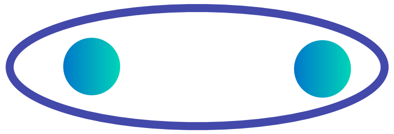
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

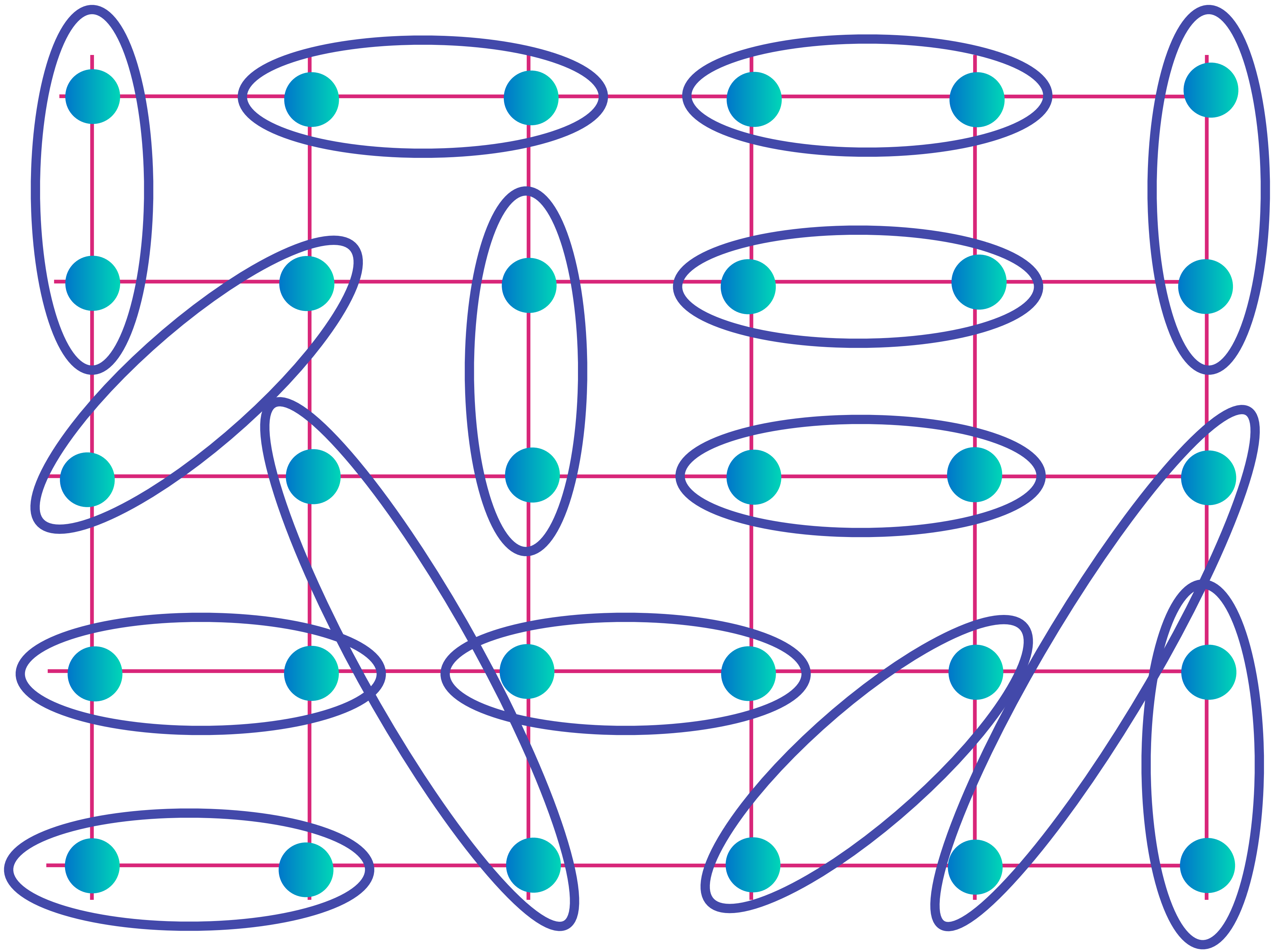


The first proposal of a quantum state with long-range entanglement

L. Pauling, Proceedings of the Royal Society London A **196**, 343 (1949)
P.W. Anderson, Materials Research Bulletin **8**, 153 (1973)

Insulating spin liquid

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

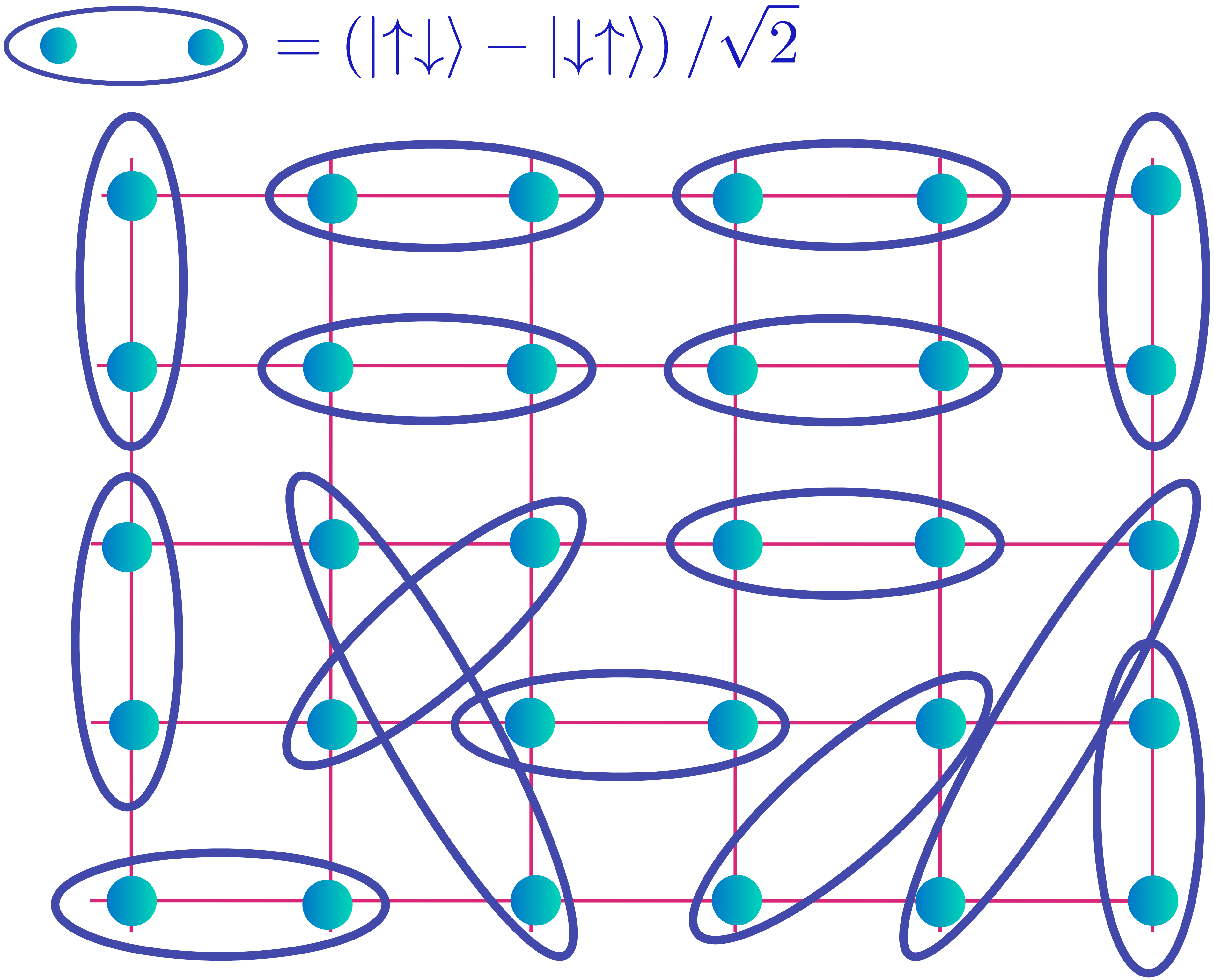


The first proposal of a quantum state with long-range entanglement

L. Pauling, Proceedings of the Royal Society London A **196**, 343 (1949)

P.W. Anderson, Materials Research Bulletin **8**, 153 (1973)

Insulating spin liquid



The first proposal of a quantum state with long-range entanglement

L. Pauling, Proceedings of the Royal Society London A **196**, 343 (1949)
P.W. Anderson, Materials Research Bulletin **8**, 153 (1973)

Insulating spin liquid

Modern description:

The $\mathbb{Z}_2$ spin liquid: Described by the simplest, non-trivial, topological field theory with time-reversal symmetry:

$$\mathcal{L} = \frac{1}{4\pi} K_{IJ} \int d^3x \, a^I \wedge da^J$$

where a^I, $I = 1, 2$ are U(1) gauge connections, and the K matrix is

$$K = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$$

N. Read and S. Sachdev, Phys. Rev. Lett. 66, 1773 (1991)
X.-G. Wen, Phys. Rev. B 44, 2664 (1991)
M. Freedman, C. Nayak, K. Shtengel, K. Walker, and Z. Wang, Annals of Physics **310**, 428 (2004)

Insulating spin liquid

Modern description:

The $\mathbb{Z}_2$ spin liquid: Described by the simplest, non-trivial, topological field theory with time-reversal symmetry:

$$\mathcal{L} = \frac{1}{4\pi} K_{IJ} \int d^3x \, a^I \wedge da^J$$

where a^I, $I = 1, 2$ are U(1) gauge connections, and the K matrix is

$$K = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$$

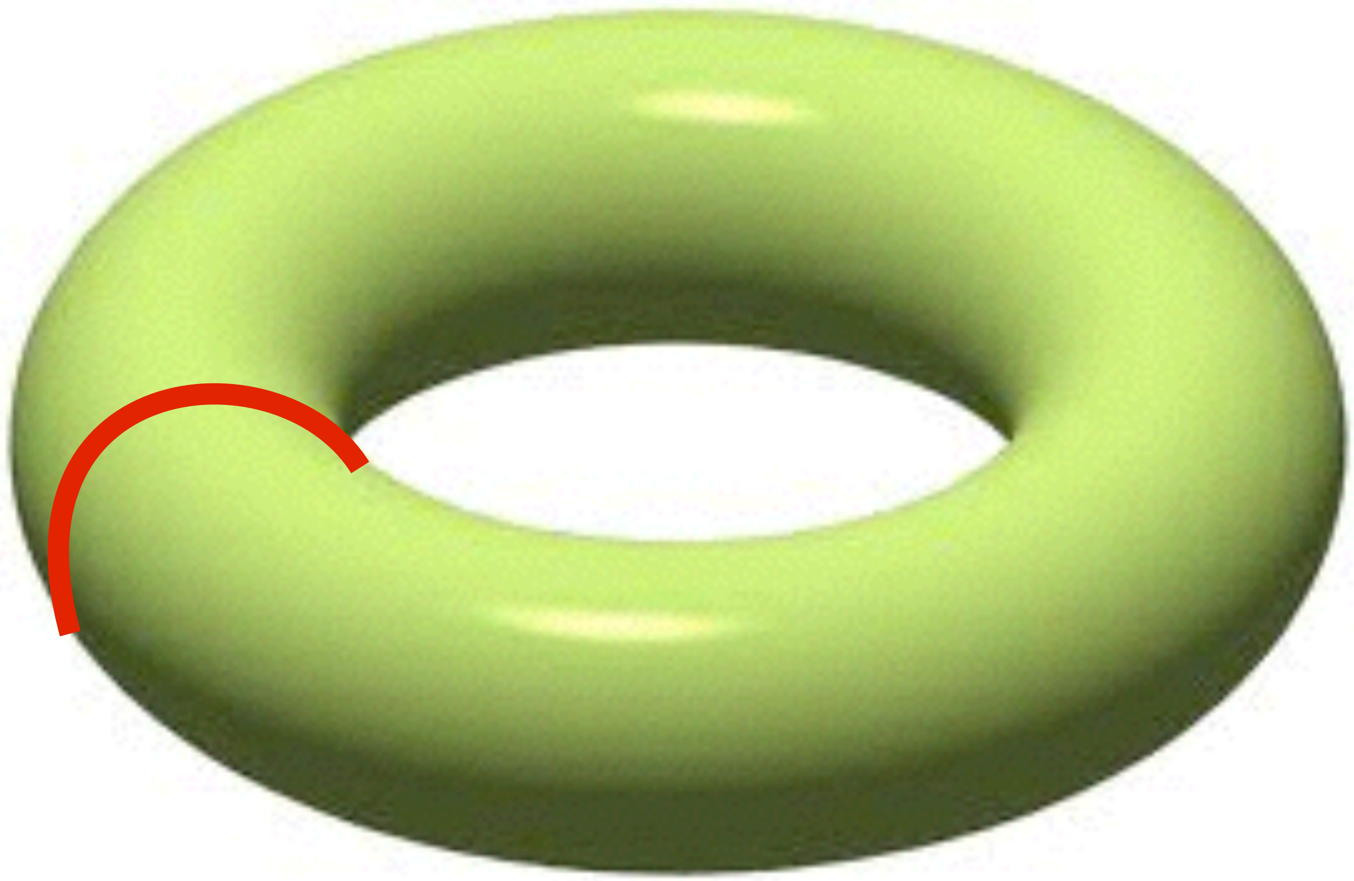
See also E. Fradkin and S. H. Shenker, "Phase diagrams of lattice gauge theories with Higgs fields," Phys. Rev. D **19**, 3682 (1979);
J. M. Maldacena, G. W. Moore and N. Seiberg, "D-brane charges in five-brane backgrounds," JHEP 0110, 005 (2001).

N. Read and S. Sachdev, Phys. Rev. Lett. 66, 1773 (1991)
X.-G. Wen, Phys. Rev. B 44, 2664 (1991)
M. Freedman, C. Nayak, K. Shtengel, K. Walker, and Z. Wang, Annals of Physics **310**, 428 (2004)

Ground state degeneracy

Place insulator on a torus:

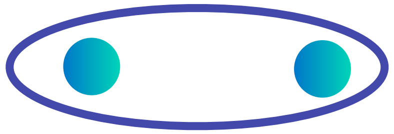
Ground state degeneracy



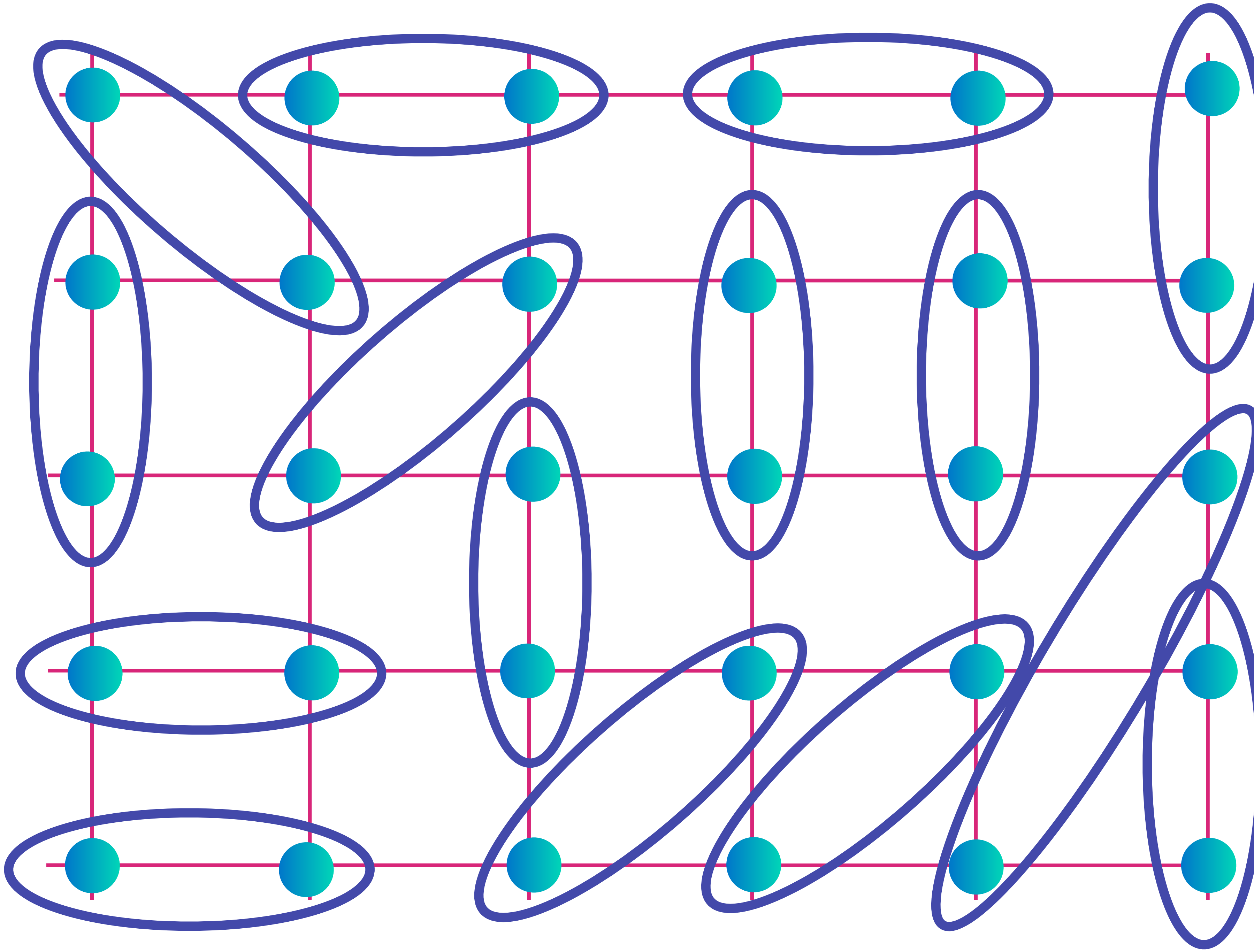
Place insulator on a torus:

Ground state becomes degenerate due to gauge fluxes enclosed by cycles of the torus

Ground state degeneracy



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

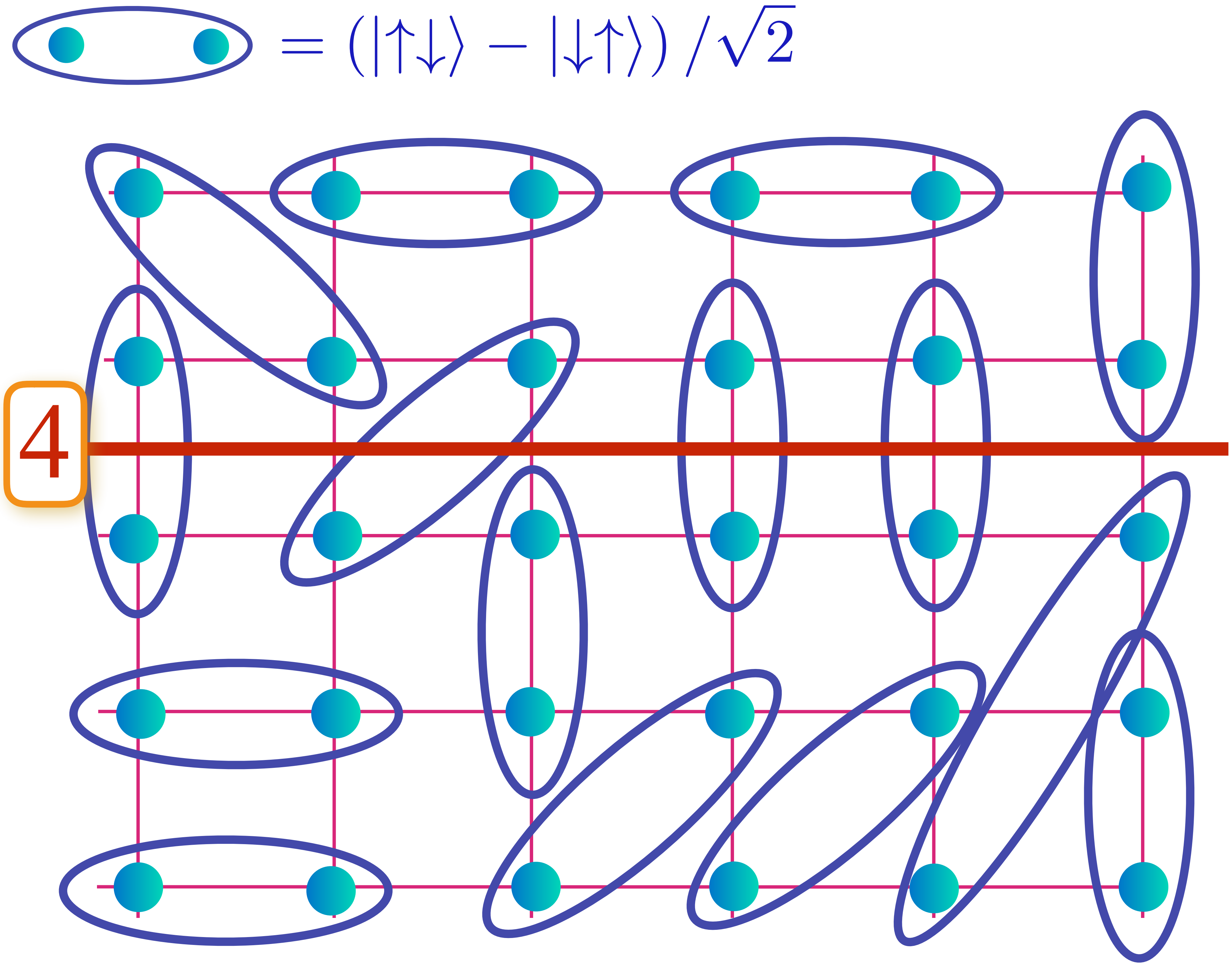


Place insulator on a torus:

obtain "topological" states nearly degenerate with the ground state: number of dimers crossing red line is conserved modulo 2

D.J. Thouless, PRB **36**, 7187 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. **6**, 353 (1988)

Ground state degeneracy

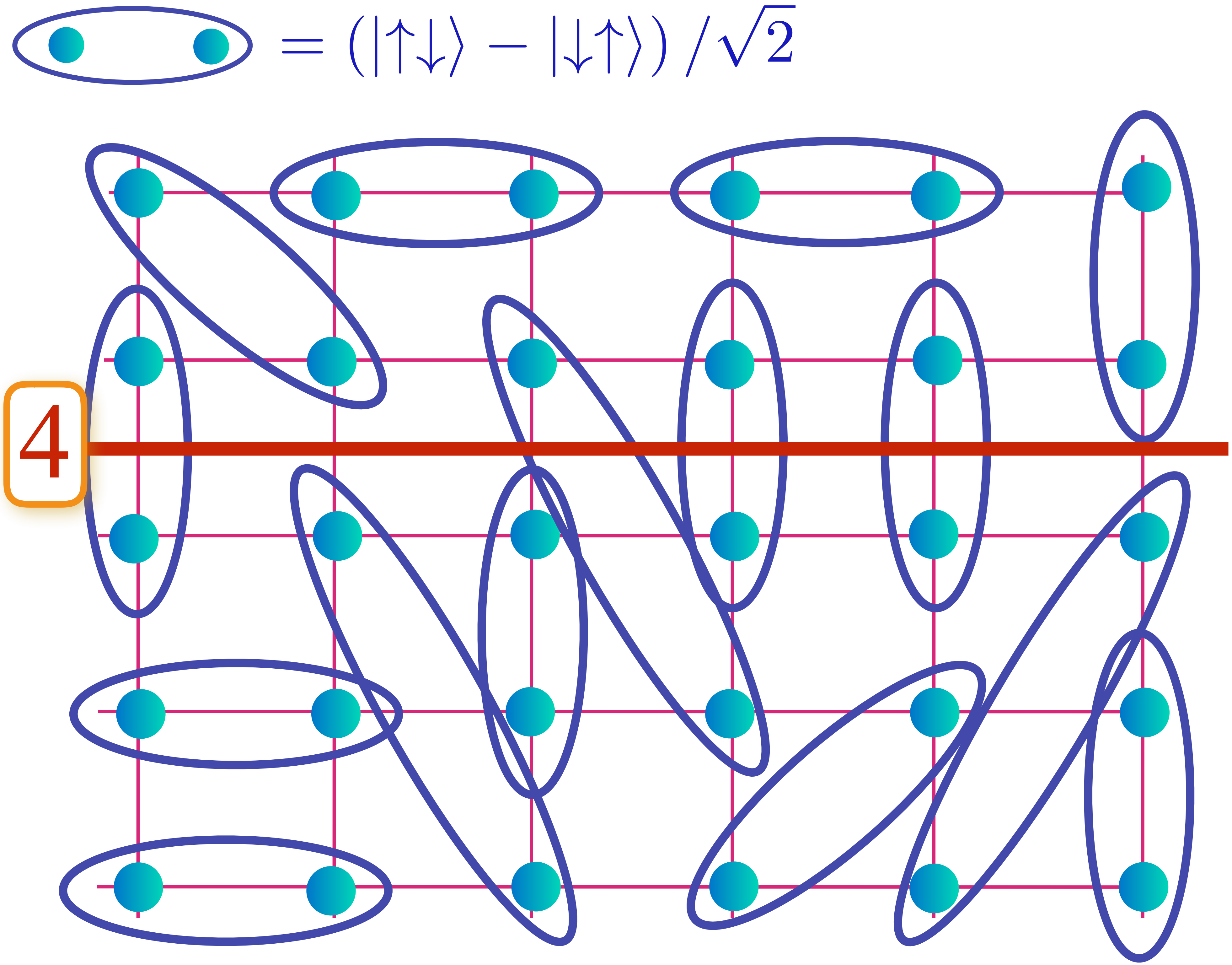


Place insulator on a torus:

obtain "topological" states nearly degenerate with the ground state: number of dimers crossing red line is conserved modulo 2

D.J. Thouless, PRB **36**, 7187 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. **6**, 353 (1988)

Ground state degeneracy

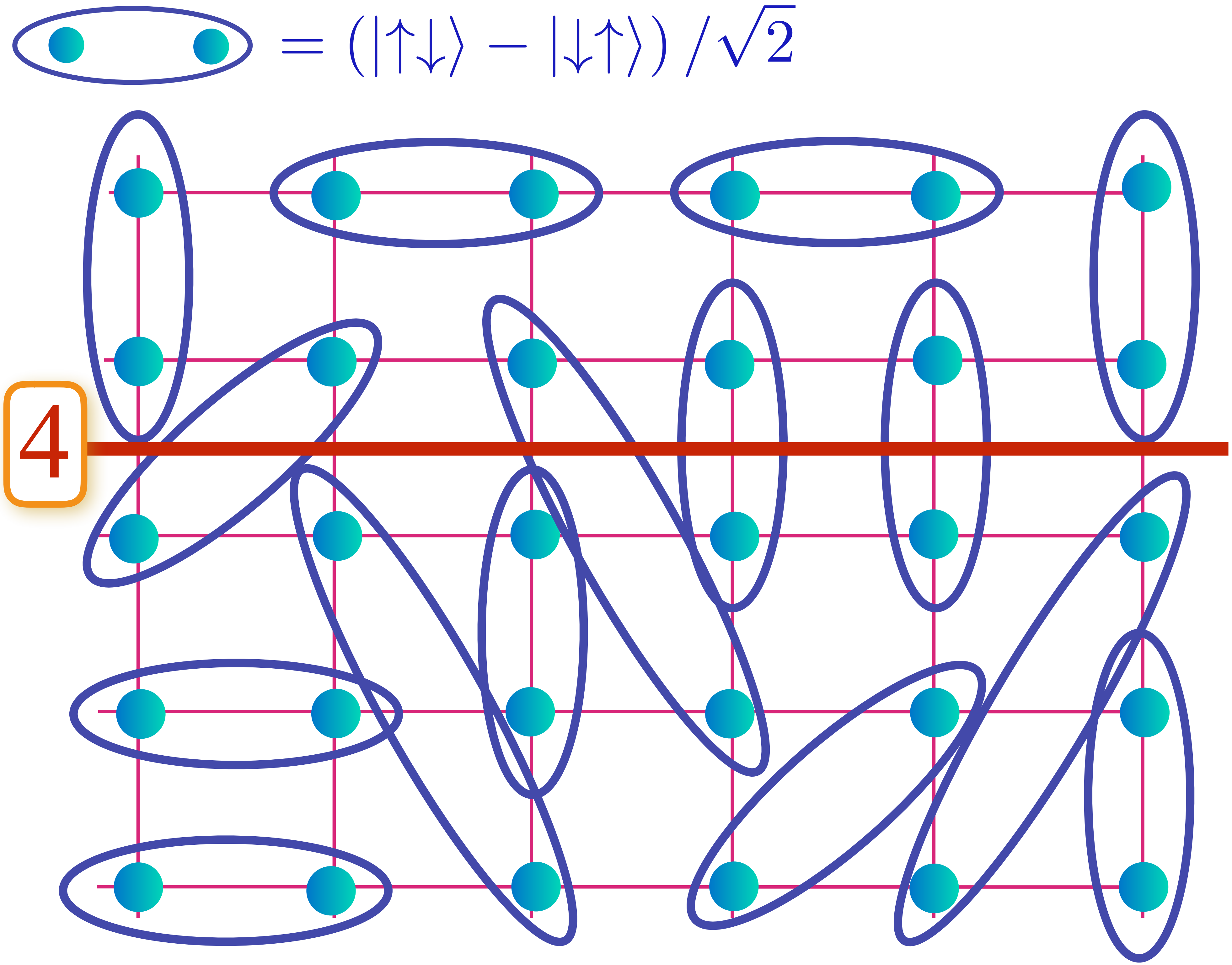


Place insulator on a torus:

obtain "topological" states nearly degenerate with the ground state: number of dimers crossing red line is conserved modulo 2

D.J. Thouless, PRB **36**, 7187 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. **6**, 353 (1988)

Ground state degeneracy

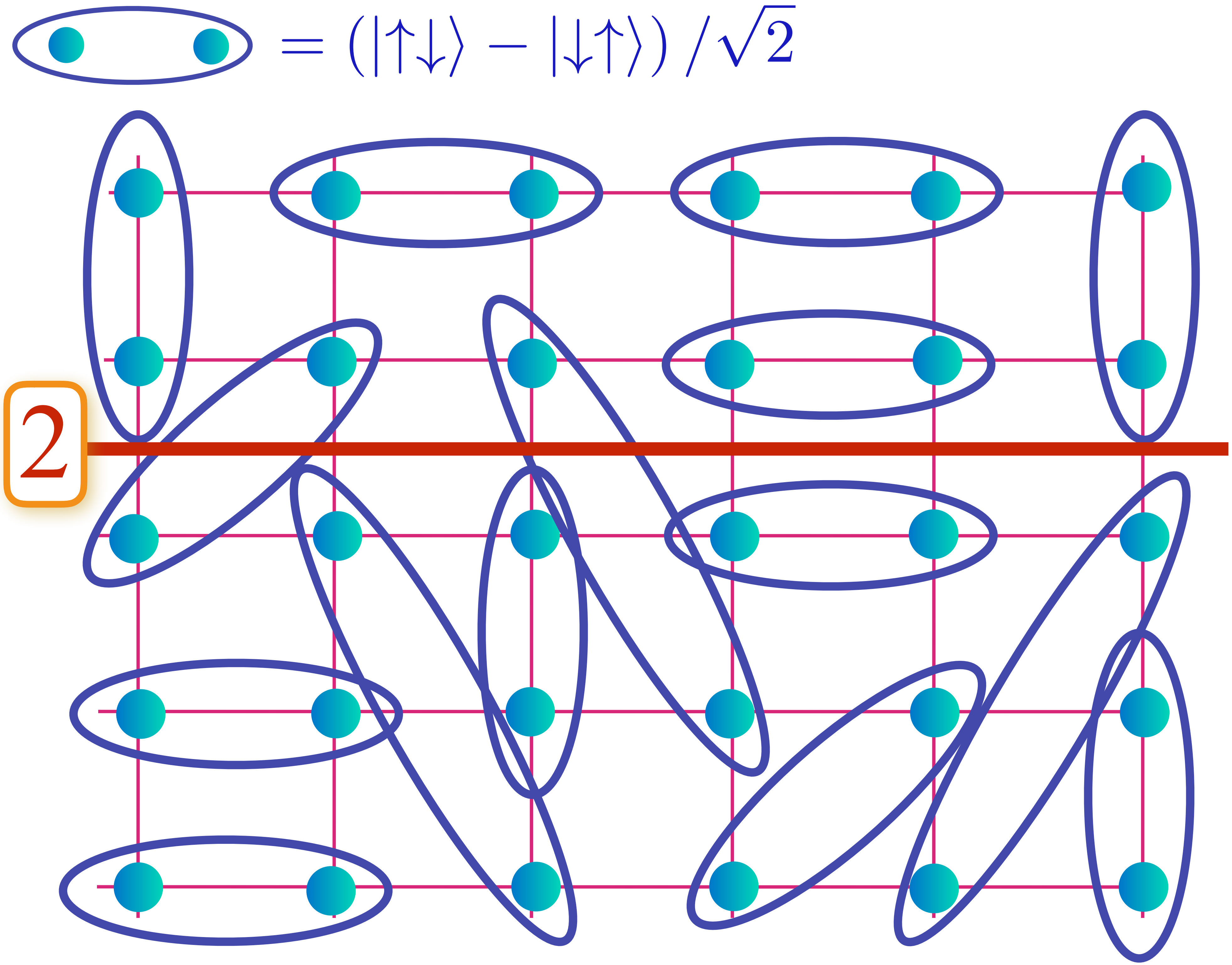


Place insulator on a torus:

obtain "topological" states nearly degenerate with the ground state: number of dimers crossing red line is conserved modulo 2

D.J. Thouless, PRB **36**, 7187 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. **6**, 353 (1988)

Ground state degeneracy

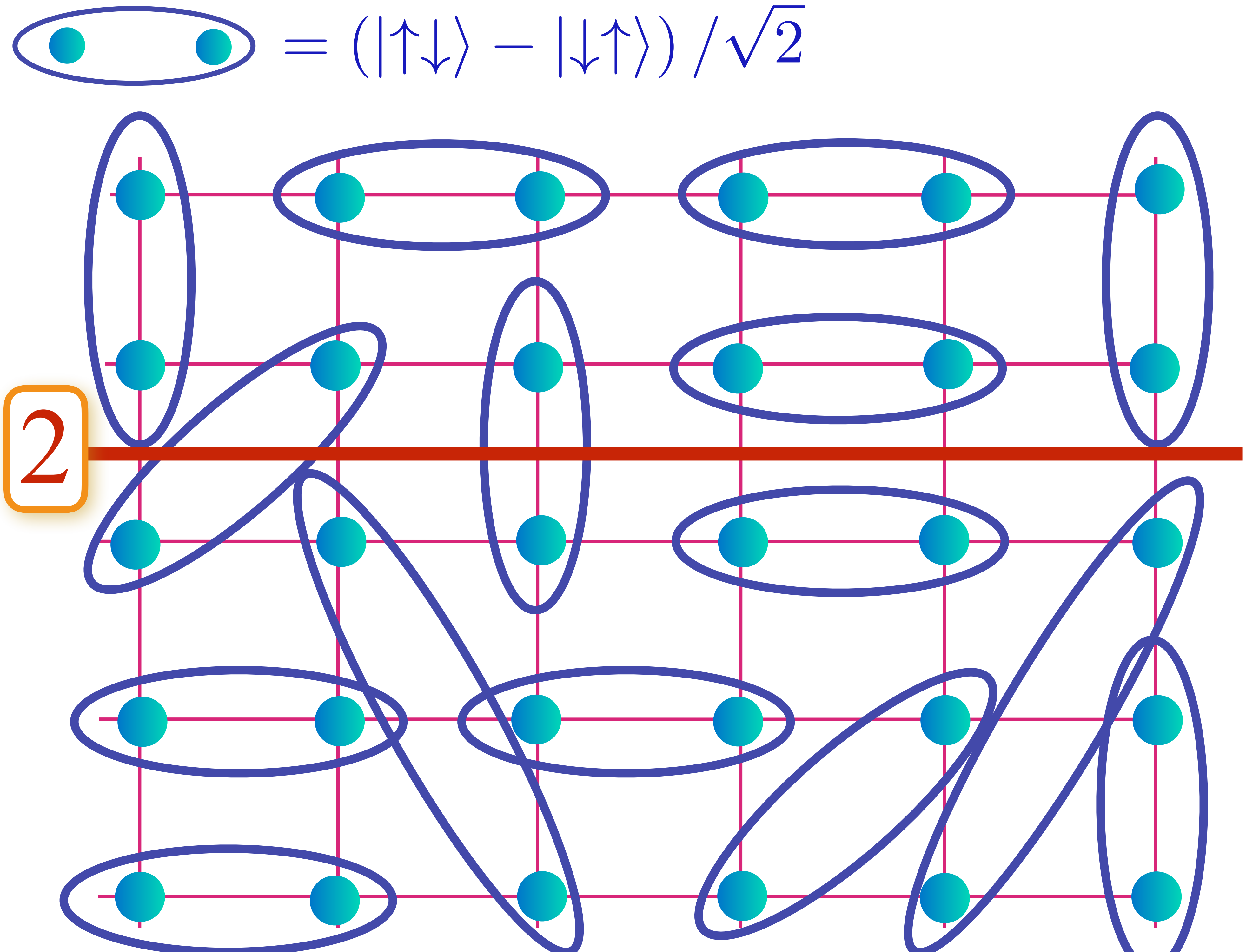


Place insulator on a torus:

obtain "topological" states nearly degenerate with the ground state: number of dimers crossing red line is conserved modulo 2

D.J. Thouless, PRB **36**, 7187 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. **6**, 353 (1988)

Ground state degeneracy

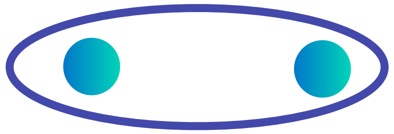


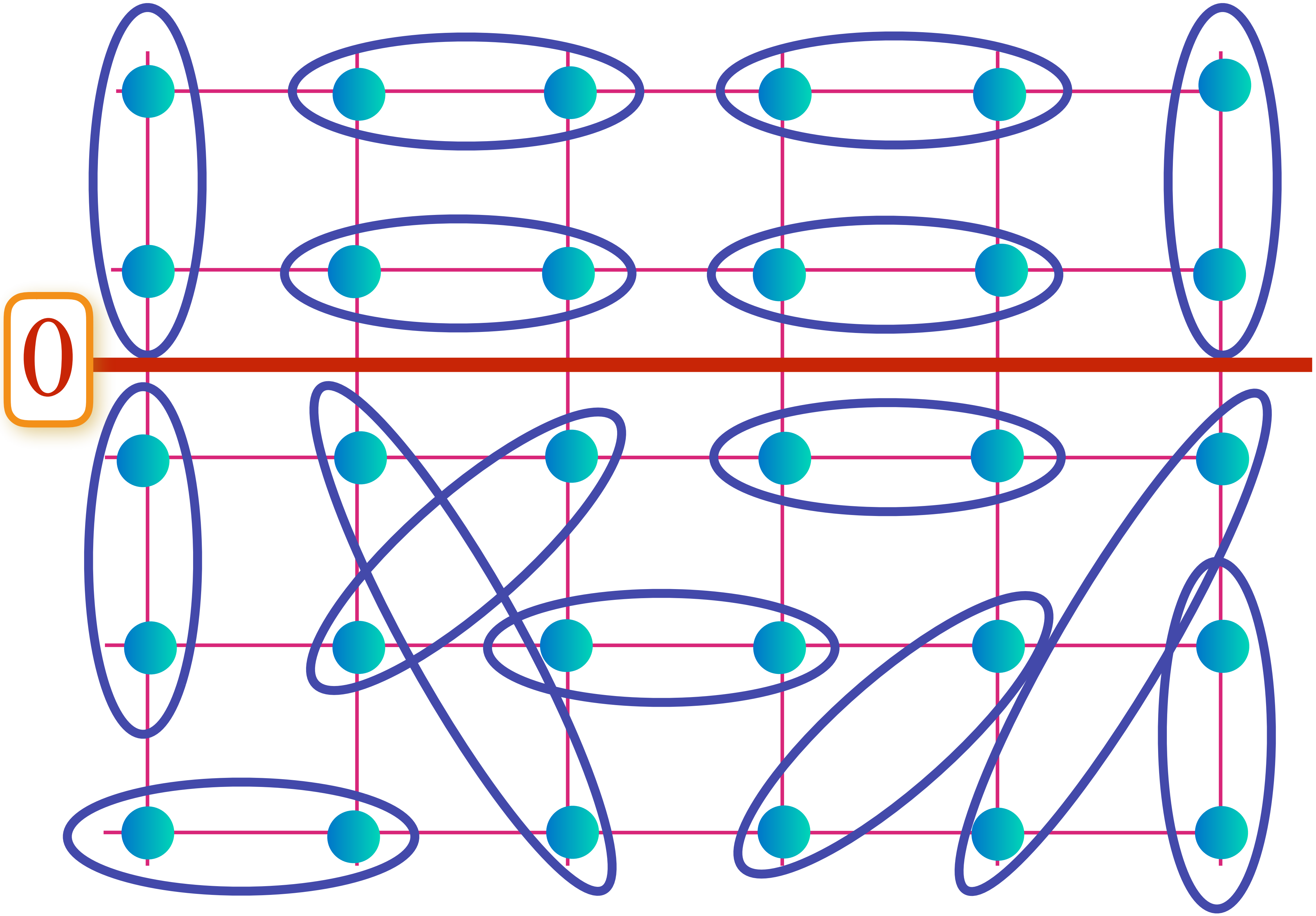
Place insulator on a torus:

obtain "topological" states nearly degenerate with the ground state: number of dimers crossing red line is conserved modulo 2

D.J. Thouless, PRB **36**, 7187 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. **6**, 353 (1988)

Ground state degeneracy

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

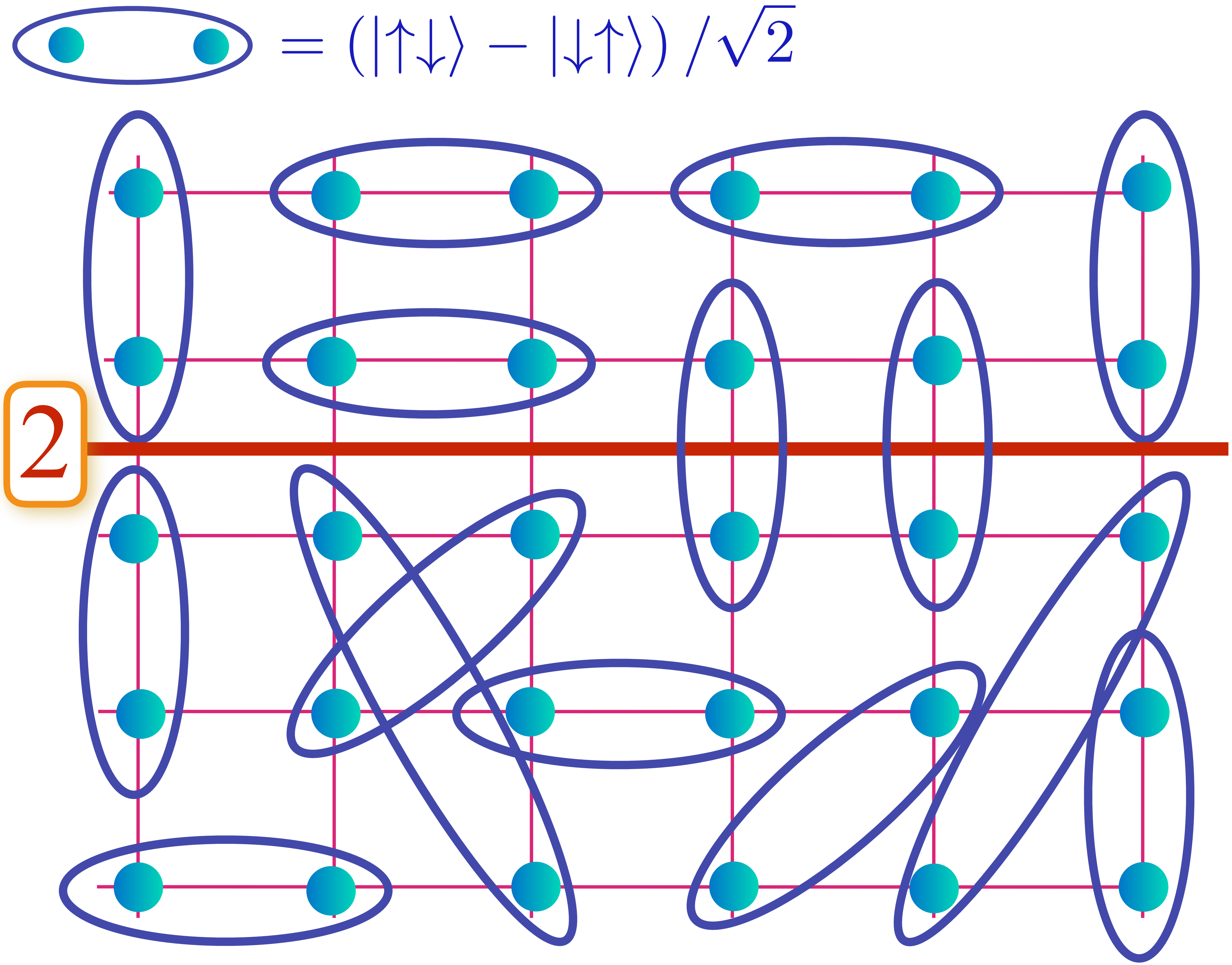


Place insulator on a torus:

obtain "topological" states nearly degenerate with the ground state: number of dimers crossing red line is conserved modulo 2

D.J. Thouless, PRB **36**, 7187 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. **6**, 353 (1988)

Ground state degeneracy



Place insulator on a torus:

obtain "topological" states nearly degenerate with the ground state: number of dimers crossing red line is conserved modulo 2

D.J. Thouless, PRB **36**, 7187 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. **6**, 353 (1988)

1. The insulating spin liquid and topological field theory

2. Topology and the size of the Fermi surface

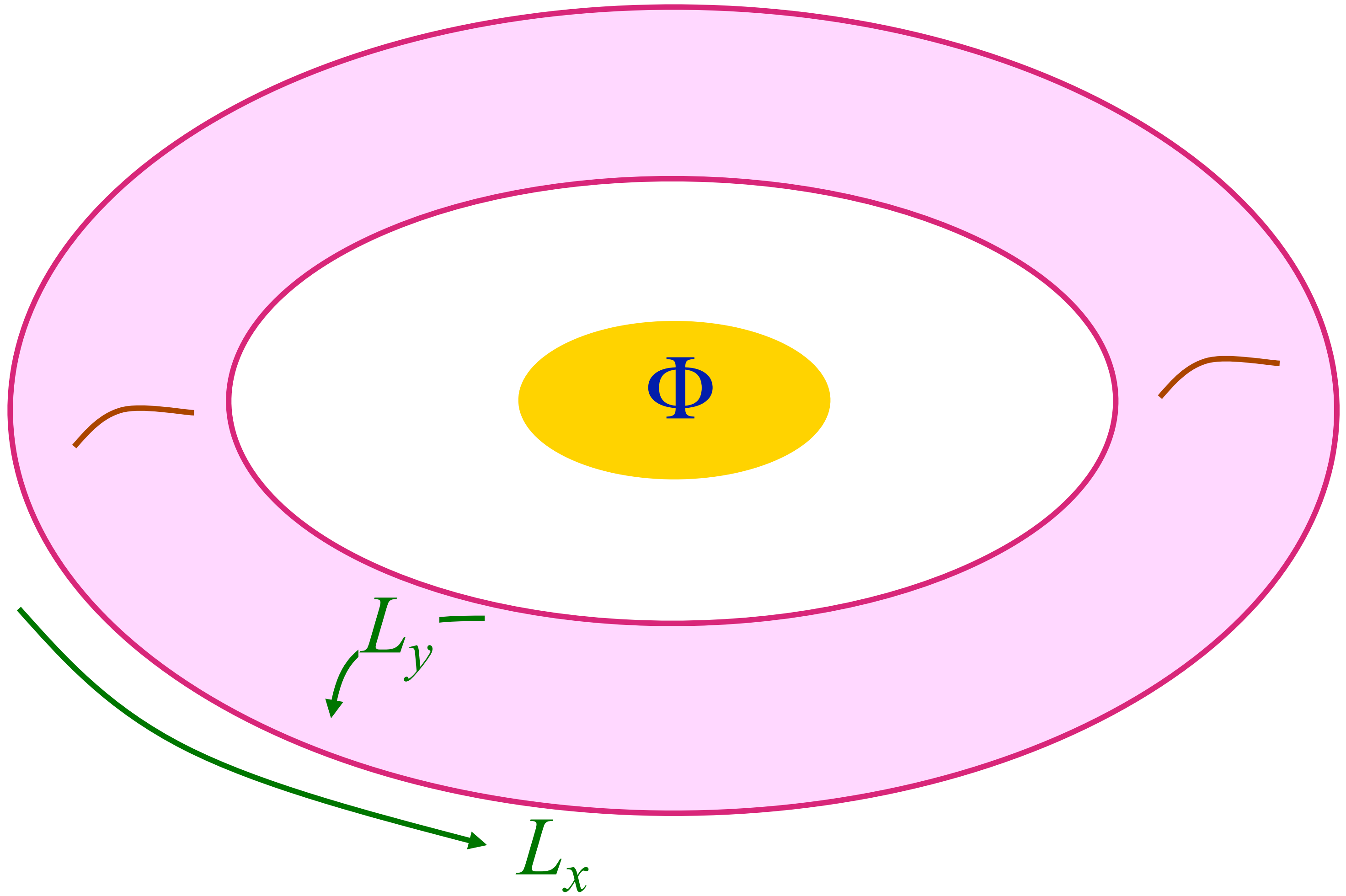
3. Transition between FL* and FL

4. Quantum matter with quasiparticles

strange metals in superconductors,
graphene,
the quark-gluon plasma,
the superfluid-insulator transition of ultra-cold atoms,
and the dynamics of charged black holes horizons

Topology and the Fermi surface size

M. Oshikawa, PRL **84**, 3370 (2000)
A. Paramekanti and A. Vishwanath,
PRB **70**, 245118 (2004)



We take N particles, each with charge Q, on a $L_x \times L_y$ lattice on a torus.
We pierce flux $\Phi = hc/Q$ through a hole of the torus.
An exact computation shows that the change in crystal momentum of the many-body state due to flux piercing is

$$P_{xf} - P_{xi} = \frac{2\pi N}{L_x} (\text{mod } 2\pi) = 2\pi \nu L_y (\text{mod } 2\pi)$$

where $\nu = N/(L_x L_y)$ is the density.

Topology and the Fermi surface size

Proof of

$$P_{xf} - P_{xi} = \frac{2\pi N}{L_x} (\text{mod } 2\pi) = 2\pi \nu L_y (\text{mod } 2\pi).$$

The initial and final Hamiltonians are related by a gauge transformation

$$\mathcal{U}_G H_f \mathcal{U}_G^{-1} = H_i \quad , \quad \mathcal{U}_G = \exp\left(i \frac{2\pi}{L_x} \sum_i x_i \hat{n}_i \right).$$

while the wavefunction evolves from $|\Psi_i\rangle$ to $\mathcal{U}_T |\Psi_i\rangle$, where $\mathcal{U}_T$ is the time evolution operator. We want to work in a fixed gauge in which the initial and final Hamiltonians are the same: in this gauge, the final state is $|\Psi_f\rangle = \mathcal{U}_G \mathcal{U}_T |\Psi_i\rangle$. Let $\hat{T}_x$ be the lattice translation operator. Then we can establish the above result using the definitions

$$\hat{T}_x |\Psi_i\rangle = e^{-iP_{xi}} |\Psi_i\rangle \quad , \quad \hat{T}_x |\Psi_f\rangle = e^{-iP_{xf}} |\Psi_f\rangle ,$$

and the easily established properties

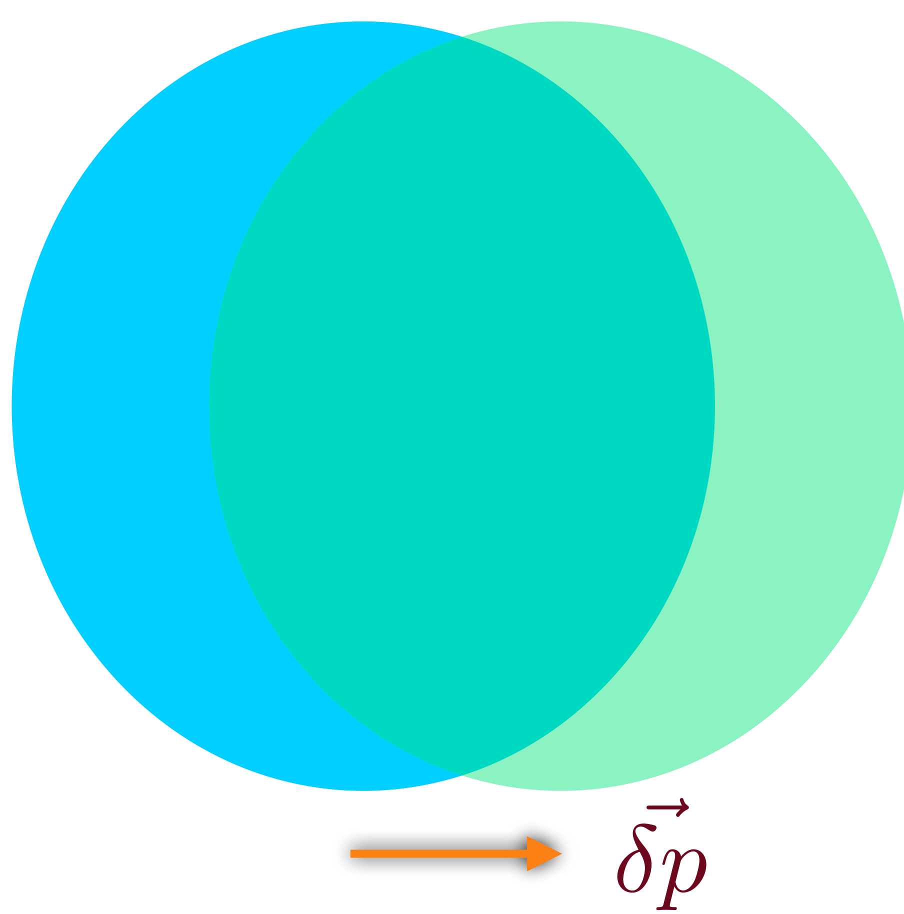
$$\hat{T}_x \mathcal{U}_T = \mathcal{U}_T \hat{T}_x \quad , \quad \hat{T}_x \mathcal{U}_G = \exp\left(-i2\pi \frac{N}{L_x} \right) \mathcal{U}_G \hat{T}_x$$

Topology and the Fermi surface size

$$\Delta P_x = 2\pi\nu L_y (\text{mod } 2\pi) \quad , \quad \Delta P_y = 2\pi\nu L_x (\text{mod } 2\pi)$$

Now we compute the momentum balance *assuming* that the only low energy excitations are quasiparticles near the Fermi surface, and these react like free particles to a sufficiently slow flux insertion. So each quasiparticle picks up a momentum $\vec{\delta p} \equiv (2\pi/L_x, 0)$, and then we can write (with δn_p the quasiparticle density excited by the flux insertion)

$$\Delta P_x = \sum_p \delta n_p p_x .$$



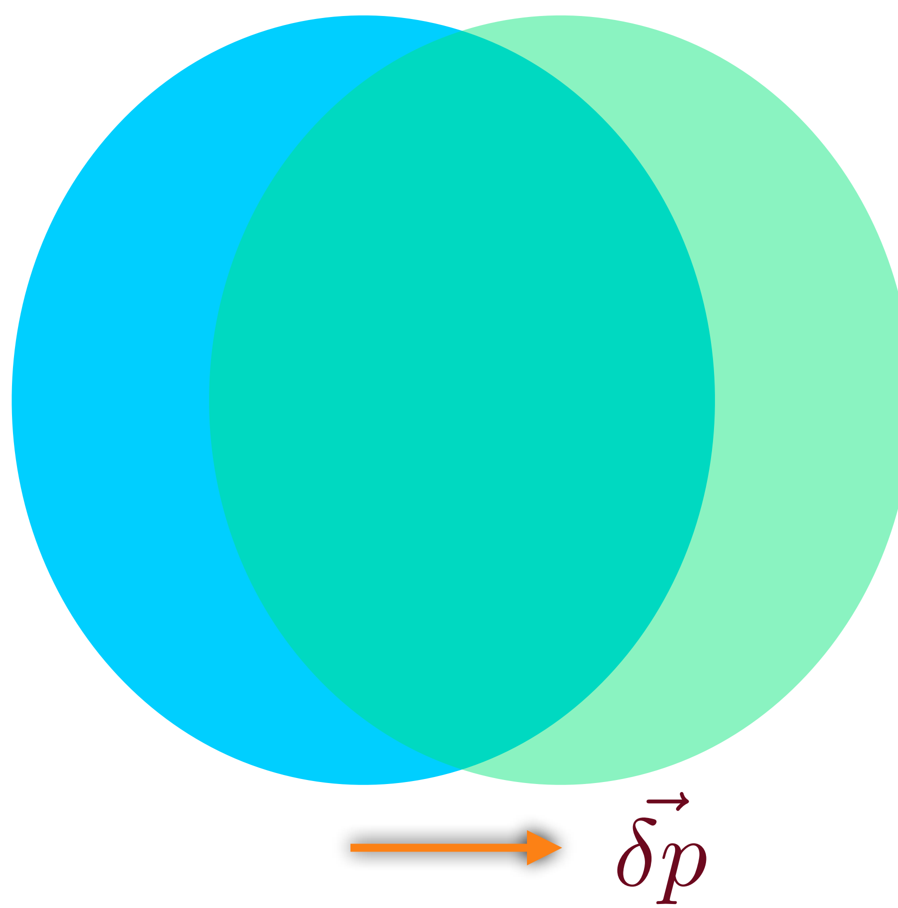
Topology and the Fermi surface size

$$\Delta P_x = 2\pi\nu L_y (\text{mod } 2\pi) \quad , \quad \Delta P_y = 2\pi\nu L_x (\text{mod } 2\pi)$$

Now $\delta n_p = \pm 1$ on a shell of thickness $\vec{\delta p} \cdot d\vec{S}_p$ on the Fermi surface (where $\vec{S}_p$ is an area element on the Fermi surface). So we can write the above as a surface integral

$$\begin{aligned}\Delta P_x &= \oint_{\text{FS}} p_x \left(\frac{L_x L_y}{4\pi^2}\right) \vec{\delta p} \cdot d\vec{S}_p \\ &= (\vec{\delta p} \cdot \hat{x}) \int_{\text{FV}} \left(\frac{L_x L_y}{4\pi^2}\right) dV\end{aligned}$$

by the divergence theorem. So



Topology and the Fermi surface size

$$\Delta P_x = 2\pi\nu L_y (\text{mod } 2\pi) \quad , \quad \Delta P_y = 2\pi\nu L_x (\text{mod } 2\pi)$$

$$\Delta P_x = \left(\frac{2\pi}{L_x}\right) \frac{L_x L_y}{4\pi^2} V_{\text{FS}} \quad , \quad \Delta P_y = \left(\frac{2\pi}{L_y}\right) \frac{L_x L_y}{4\pi^2} V_{\text{FS}}$$

where V_{FS} is the volume of the Fermi surface. So, although the quasiparticles are only defined near the Fermi surface, by using Gauss's Law on the momentum acquired by quasiparticles near the Fermi surface, we have converted the answer to an integral over the volume enclosed by the Fermi surface.
Now we equate these values to those obtained above, and obtain

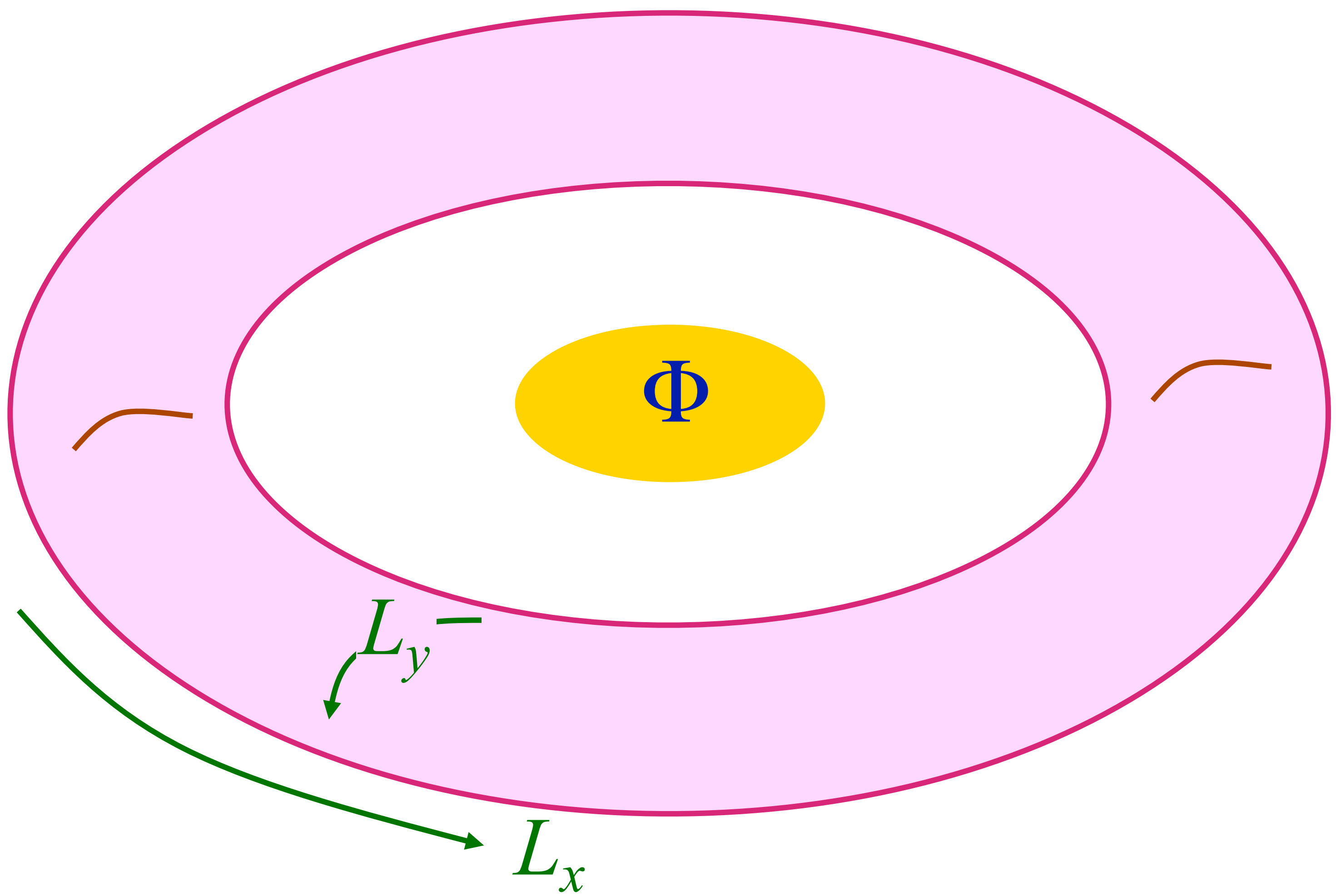
$$N - L_x L_y \frac{V_{\text{FS}}}{4\pi^2} = L_x m_x \quad , \quad N - L_x L_y \frac{V_{\text{FS}}}{4\pi^2} = L_y m_y$$

for some integers m_x, m_y. Now choose L_x, L_y mutually prime integers; then $m_x L_x = m_y L_y$ implies that $m_x L_x = m_y L_y = p L_x L_y$ for some integer p. Then we obtain

$$\nu = \frac{N}{L_x L_y} = \frac{V_{\text{FS}}}{4\pi^2} + p.$$

This is Luttinger's theorem.

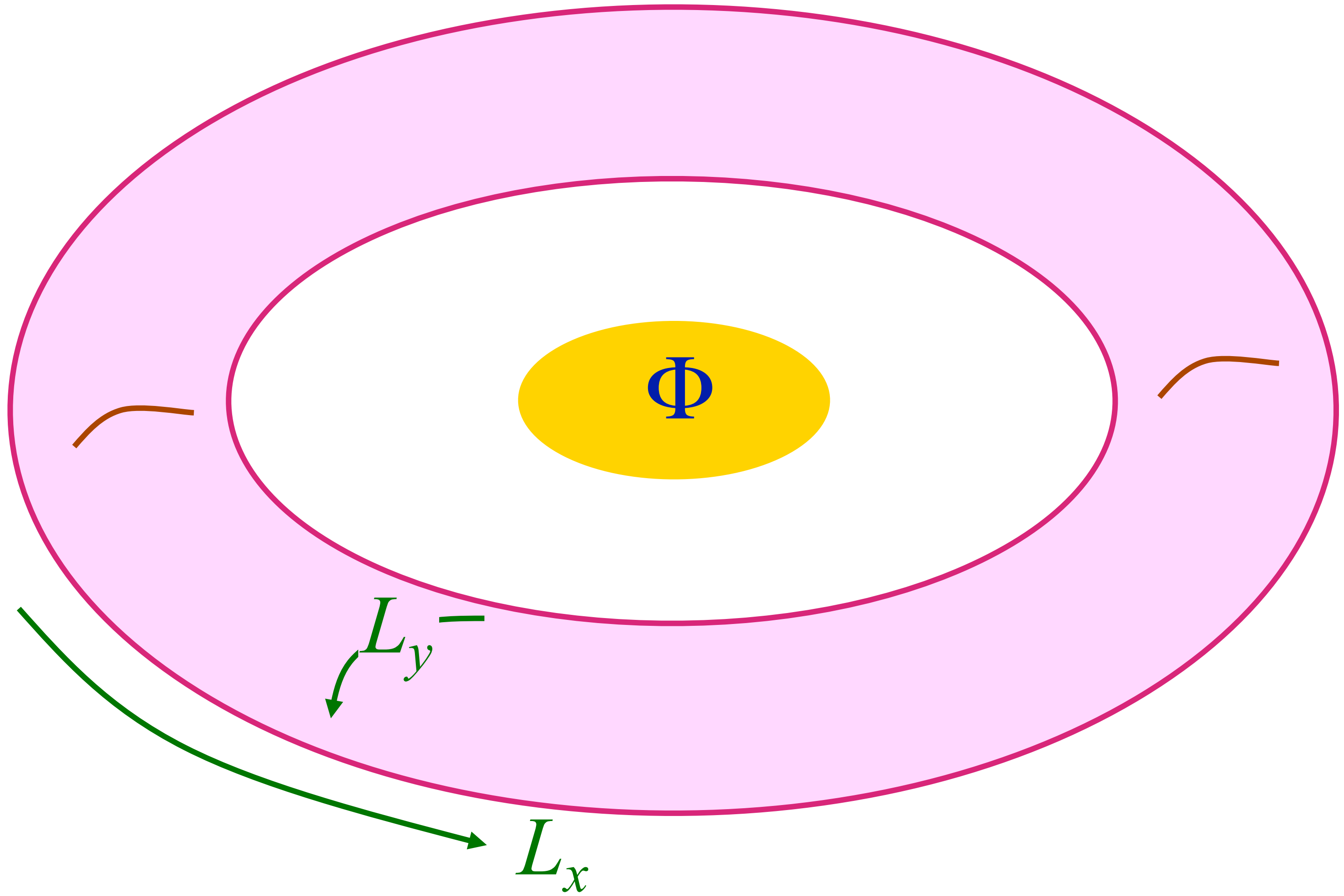
Topology and the Fermi surface size in FL*



To obtain a different Fermi surface size, we need low energy excitations on a torus which are *not* composites of quasiparticles around the Fermi surface. The degenerate ground states of a $\mathbb{Z}_2$ spin liquid can provide the needed excitation, and lead to a $\mathbb{Z}_2$-FL* state with a Fermi surface size of p, rather than the Luttinger size of $1 + p$.

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

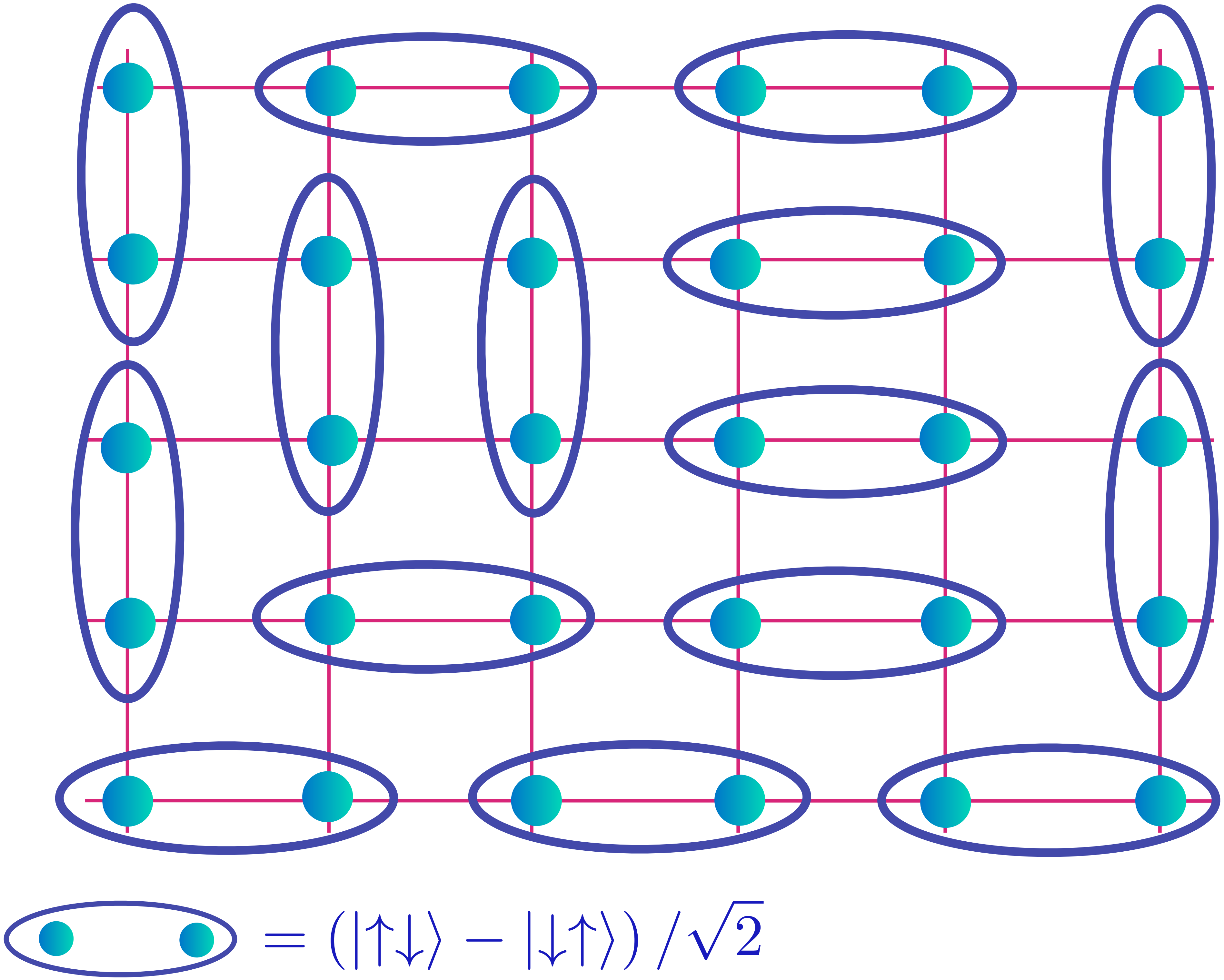
Topology and the Fermi surface size in FL*



The exact momentum transfers $\Delta P_x = 2\pi(1+p)L_y(\text{mod}2\pi)$ and $\Delta P_y = 2\pi(1+p)L_x(\text{mod}2\pi)$ due to flux piercing arise from

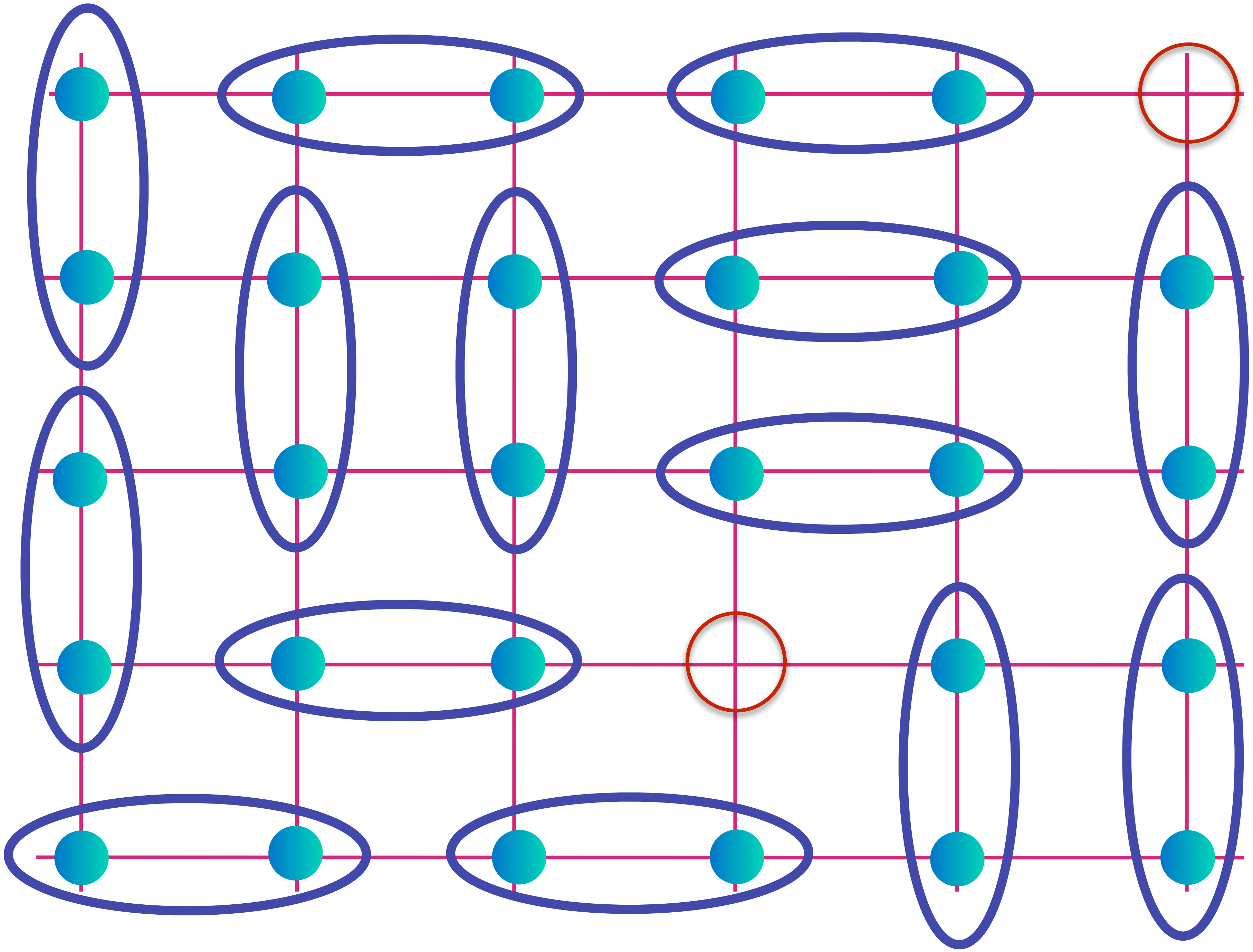
- A contribution $2\pi p L_{x,y}$ from the small Fermi surface of quasiparticles of size p.
- The remainder is made up by the topological sector: flux insertion creates a "vison" in the hole of the torus.

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

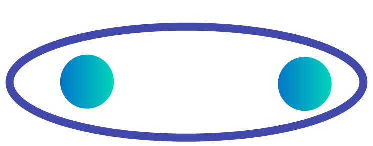


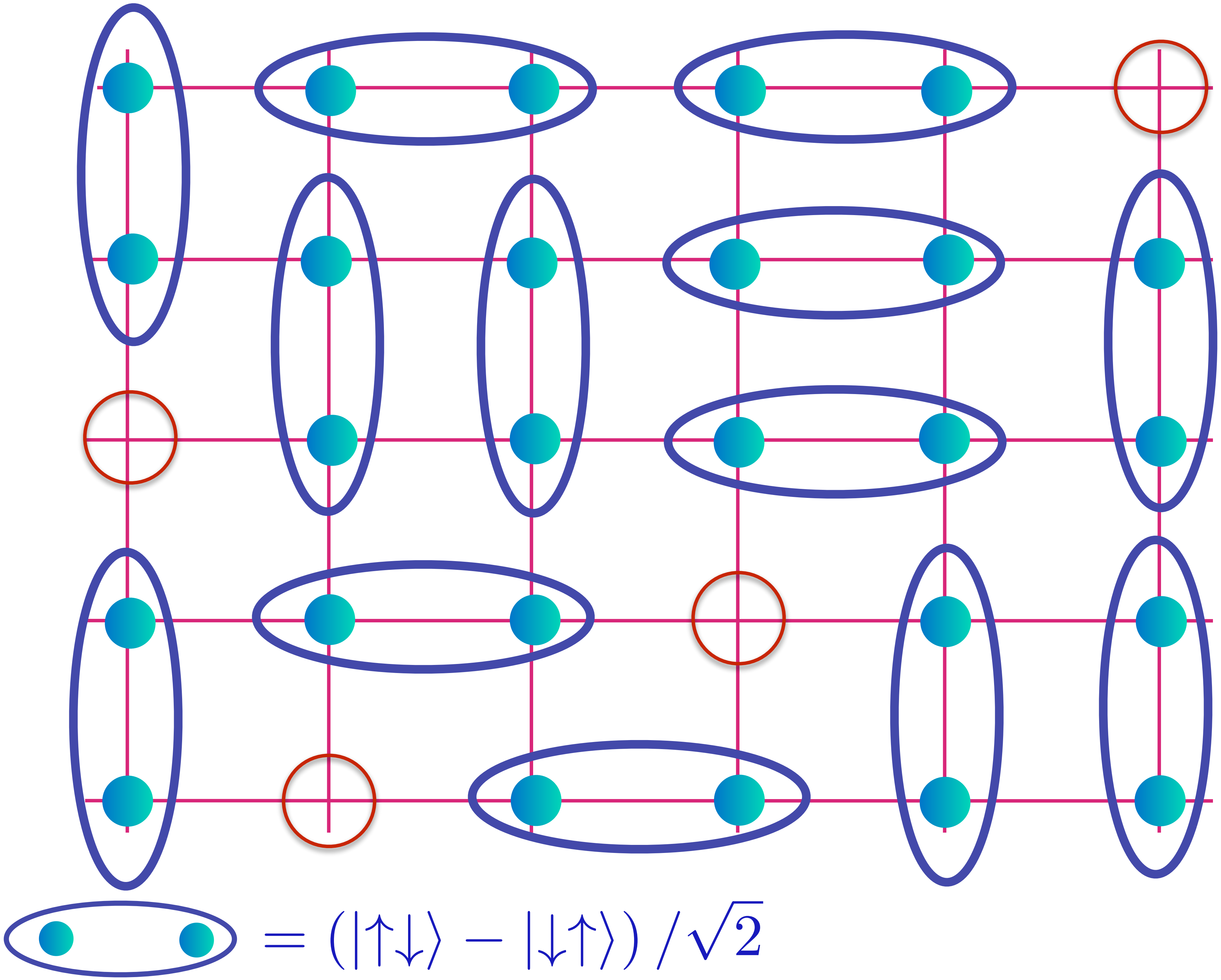
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Start with a spin liquid and then remove electrons



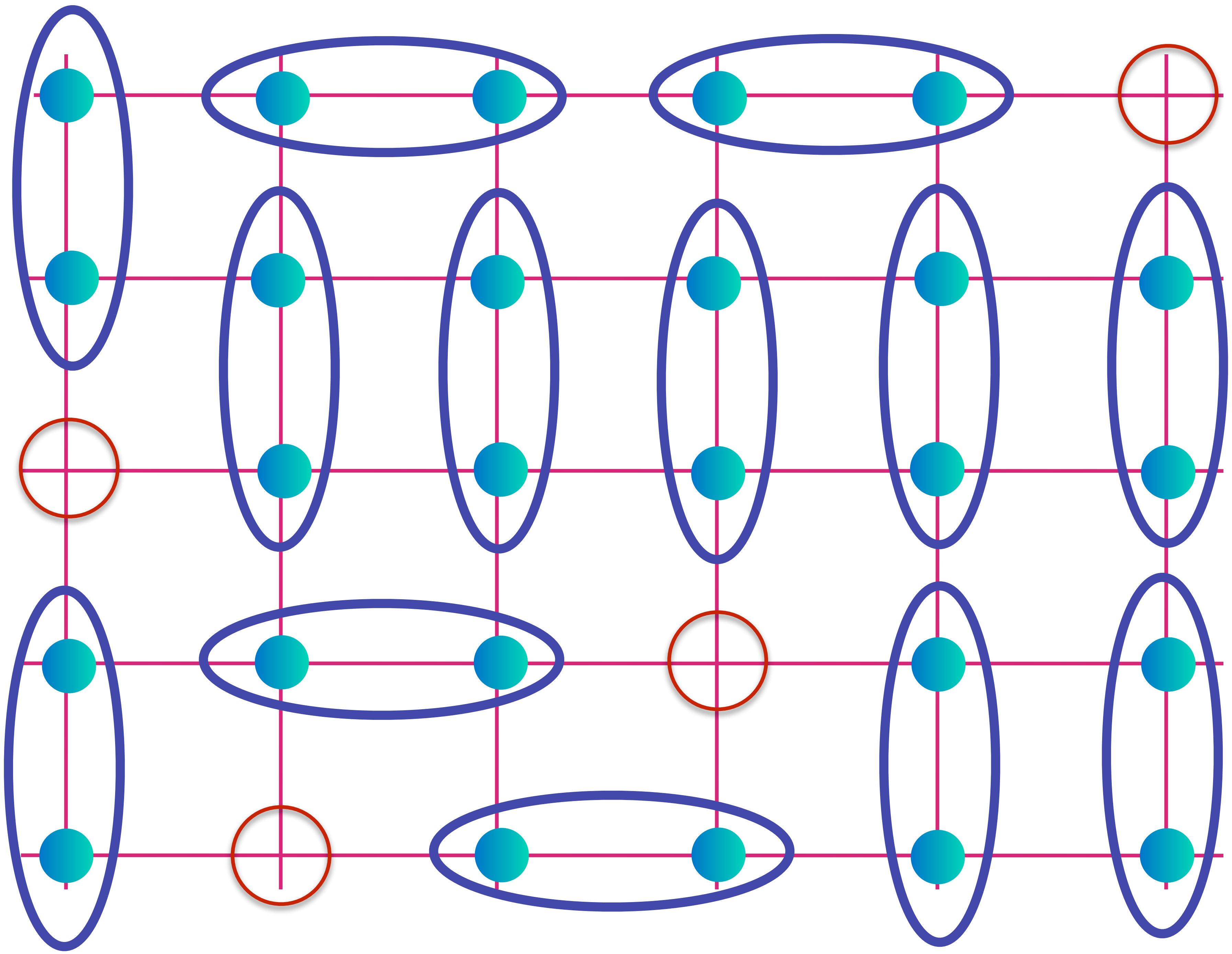
Start with a spin liquid and then remove electrons

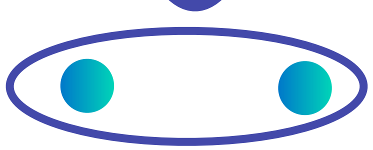
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



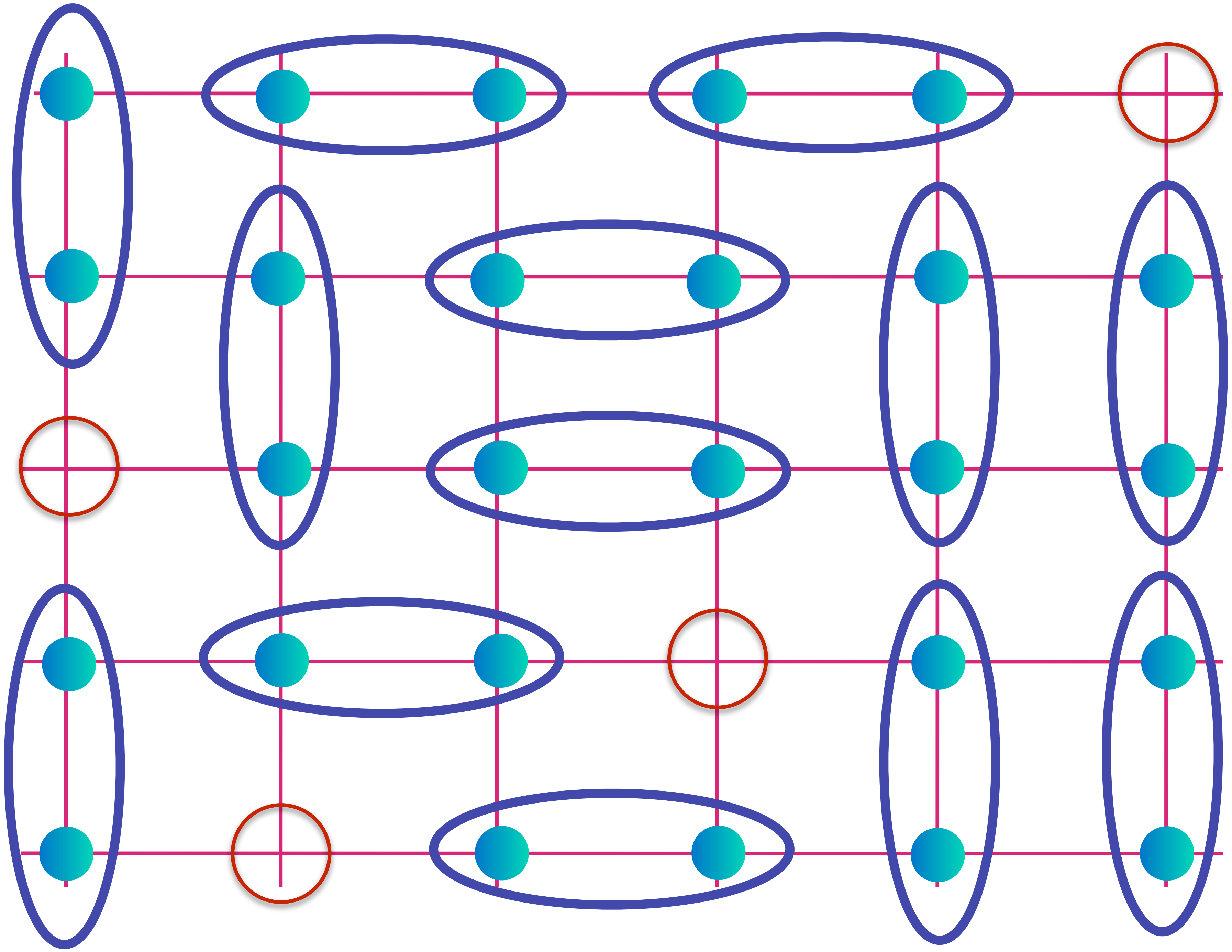
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Start with a spin liquid and then remove electrons

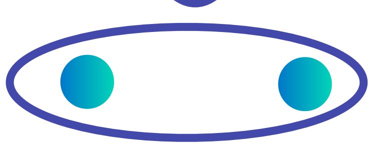


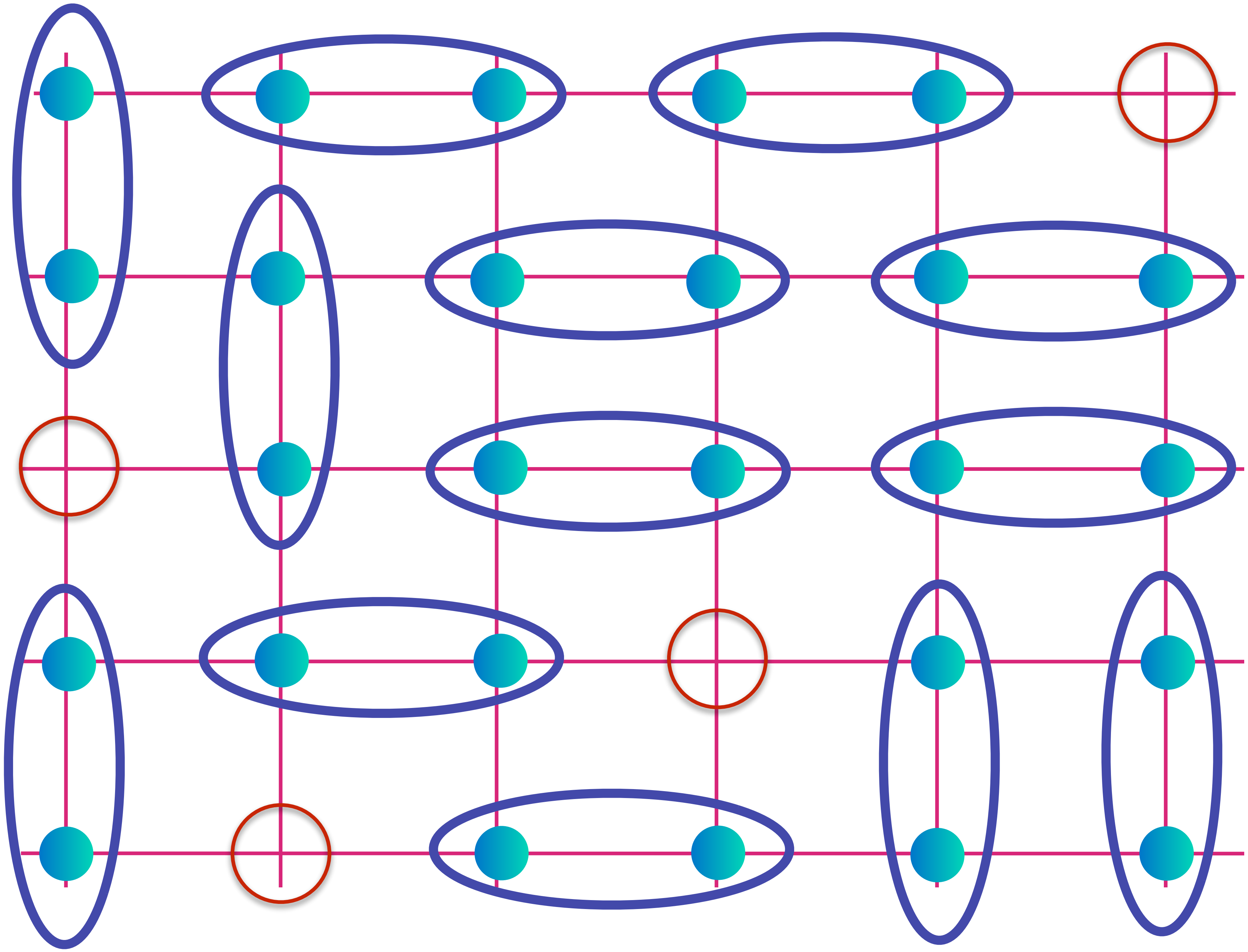
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) / \sqrt{2}$

Start with a spin liquid and then remove electrons



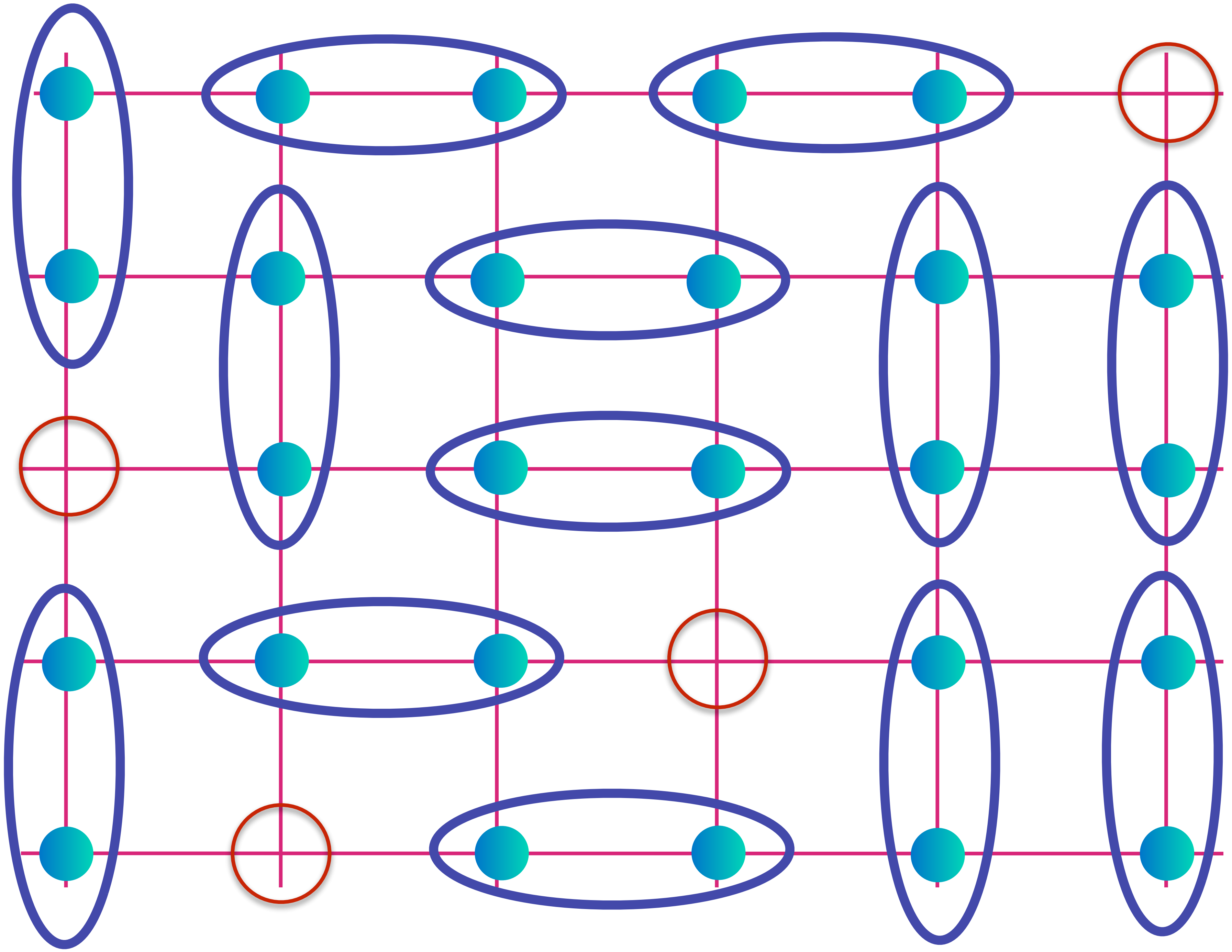
Start with a spin liquid and then remove electrons

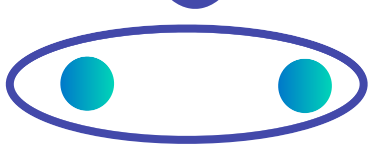
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



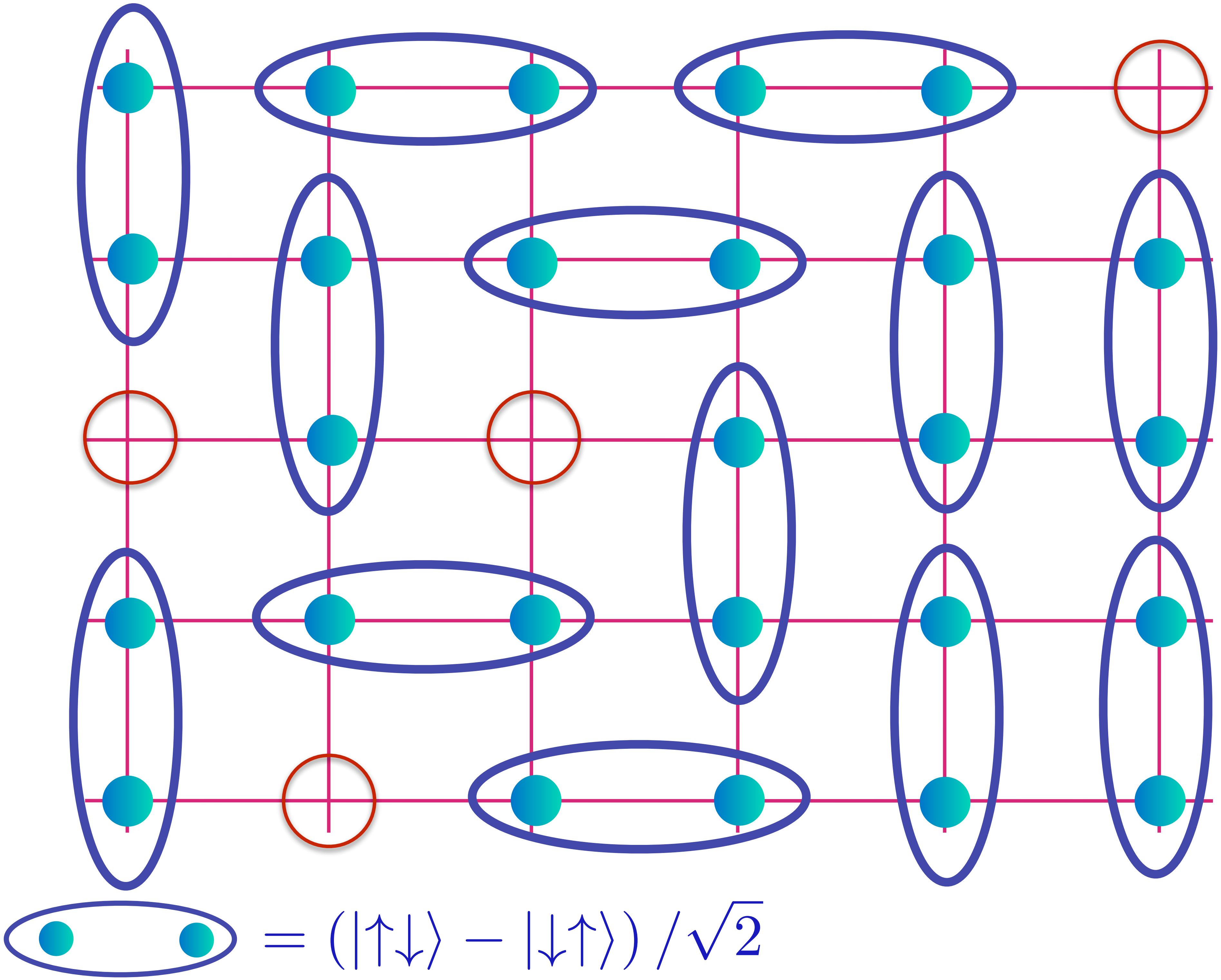
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Start with a spin liquid and then remove electrons



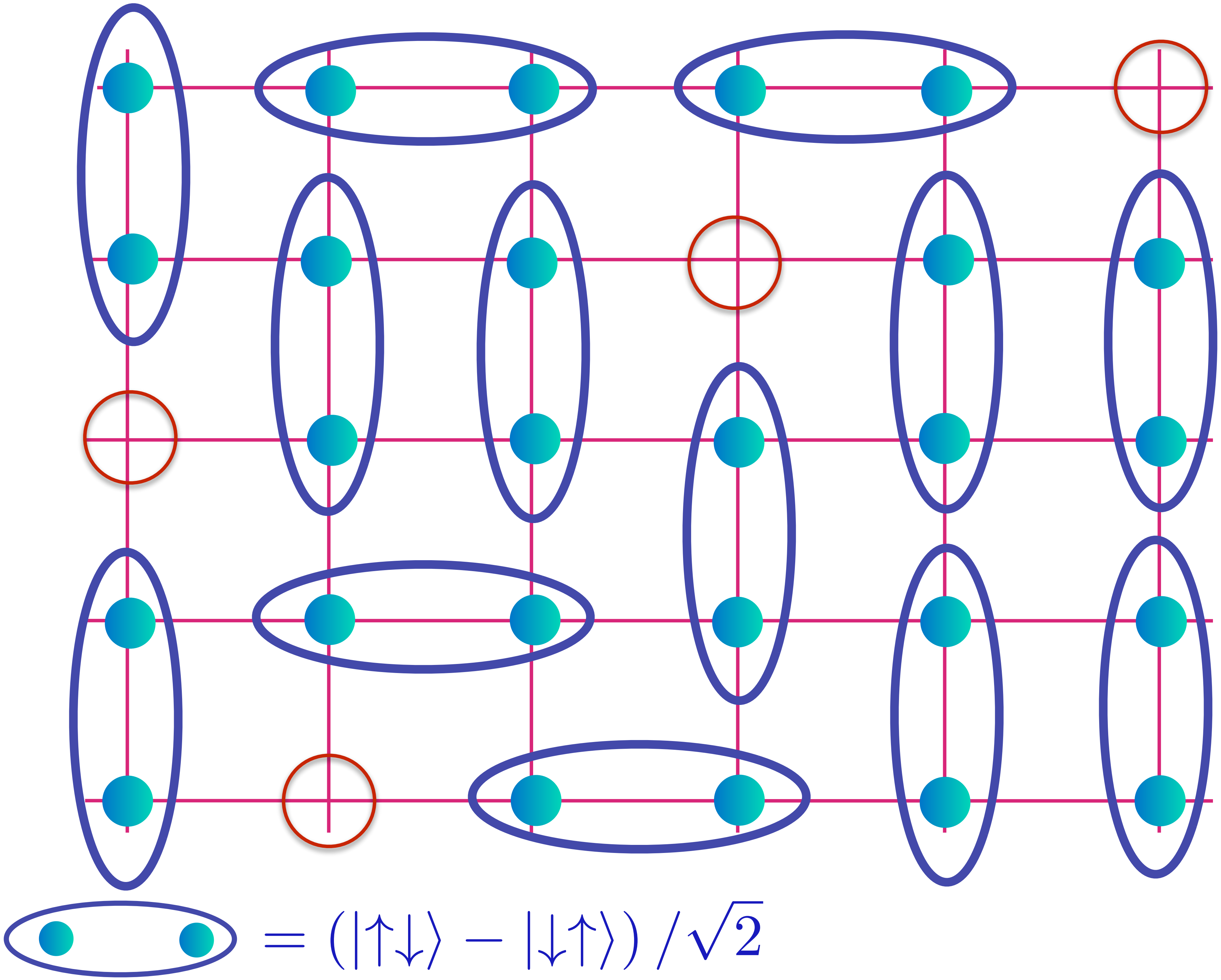
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

Start with a spin liquid and then remove electrons



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

A mobile charge +e, but carrying no spin

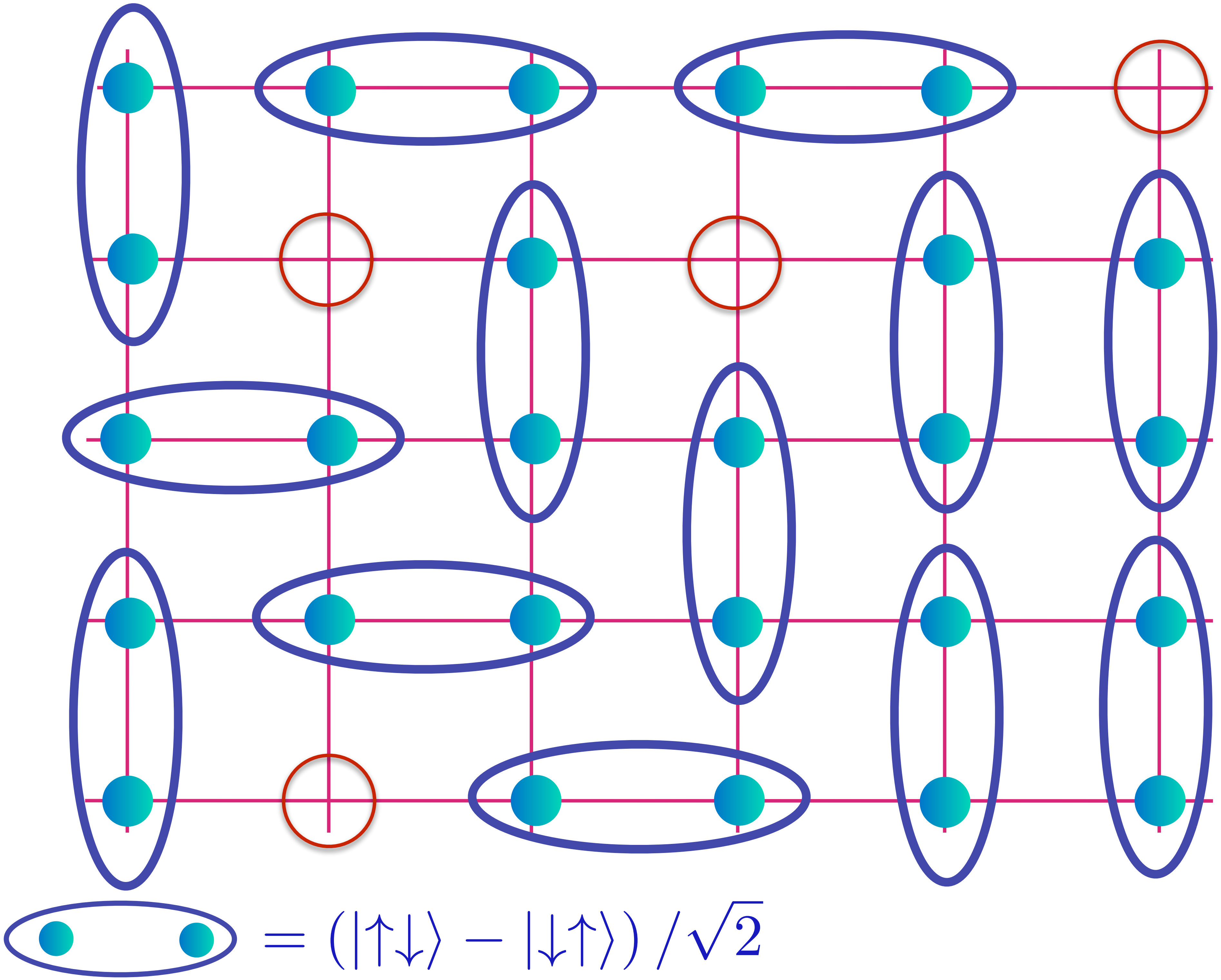


$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

A mobile charge +e, but carrying no spin

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

N. Read and B. Chakraborty, PRB **40**, 7133 (1989)

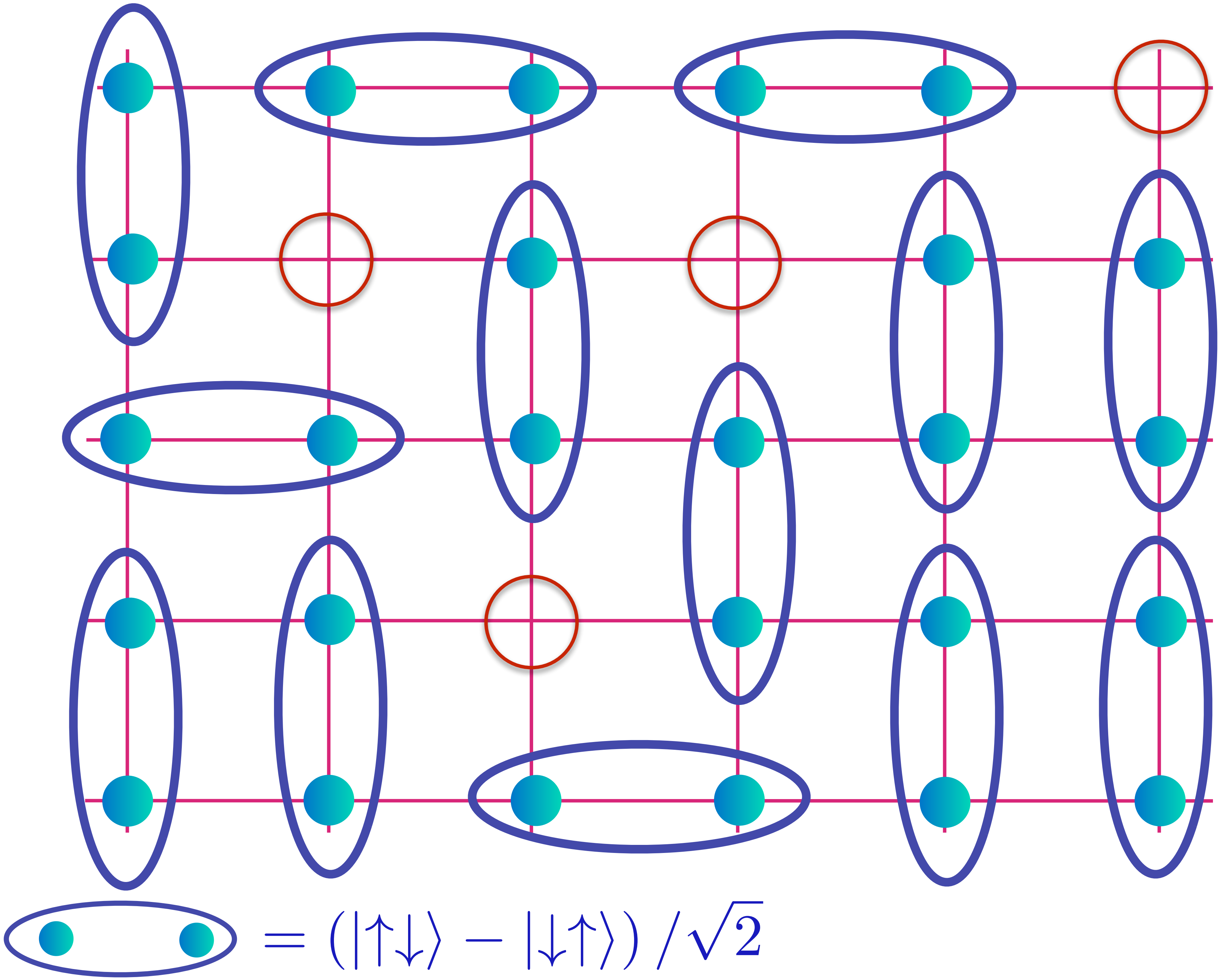


$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Spin liquid with density p of spinless, charge +e "holons". These can form a Fermi surface of size p, but not of electrons

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

N. Read and B. Chakraborty, PRB **40**, 7133 (1989)

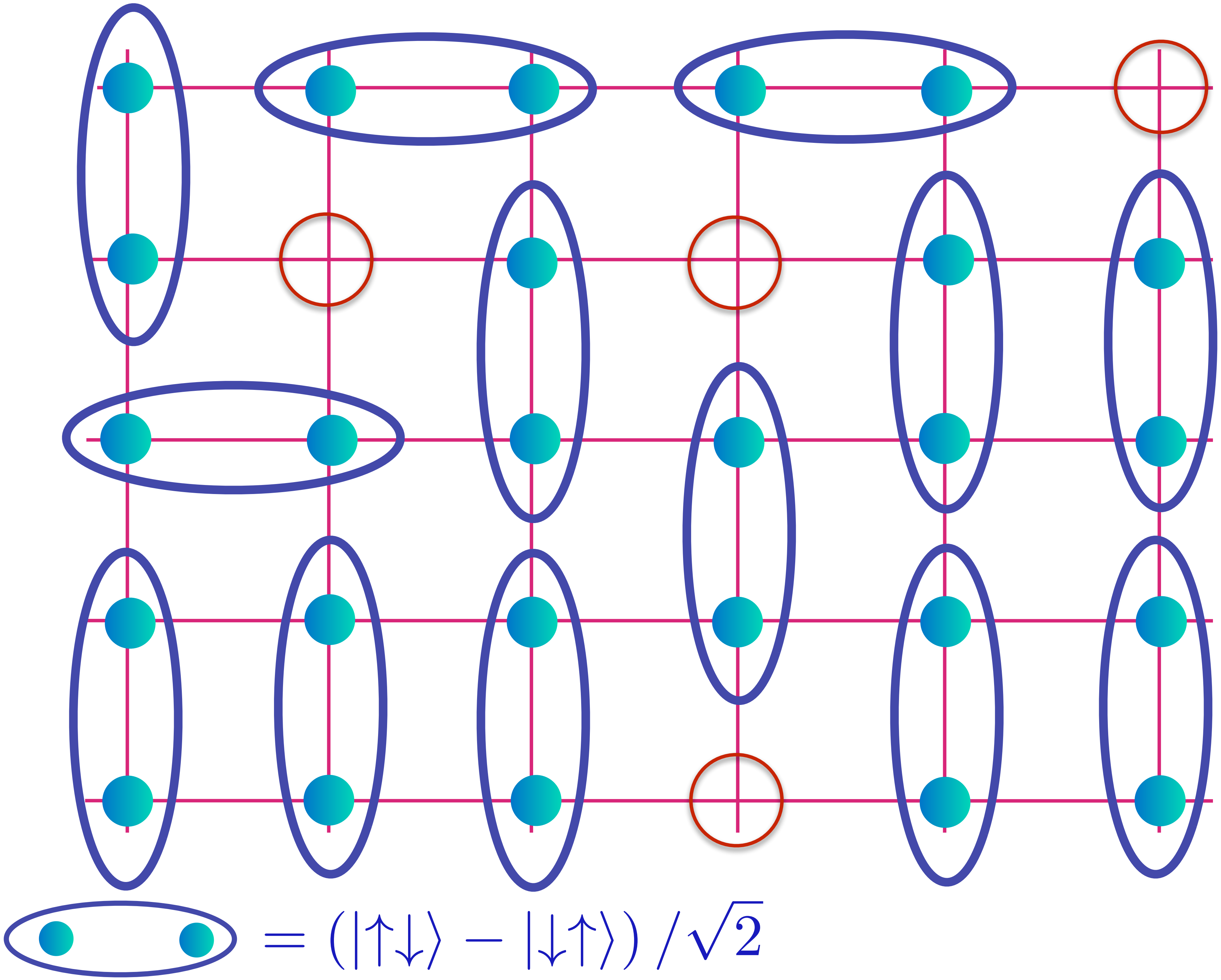


$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

Spin liquid with density *p* of spinless, charge +e "holons". These can form a Fermi surface of size p, but not of electrons

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

N. Read and B. Chakraborty, PRB **40**, 7133 (1989)

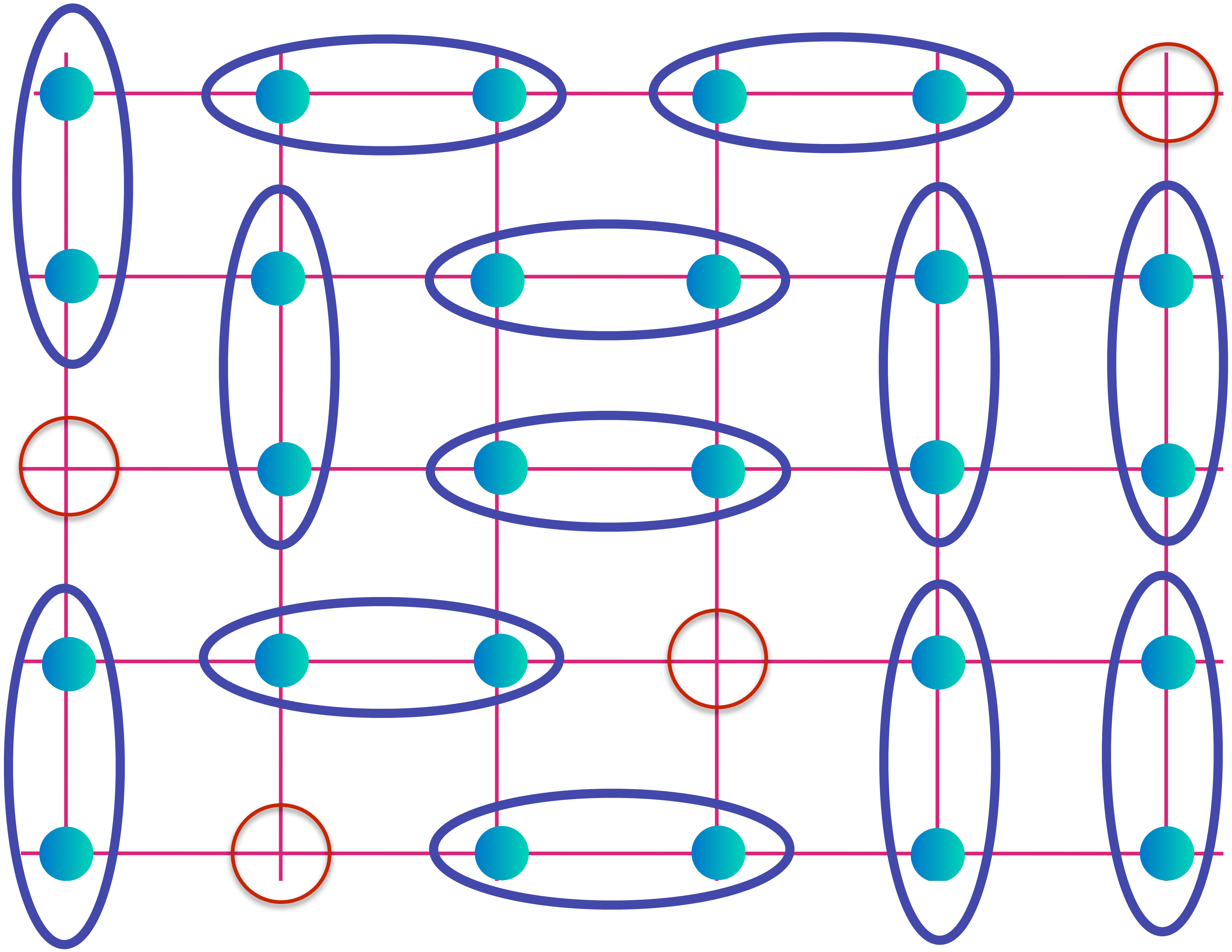


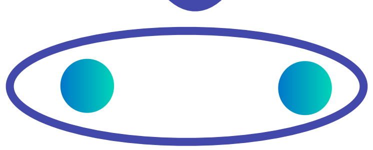
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

Spin liquid with density p of spinless, charge +e "holons". These can form a Fermi surface of size p, but not of electrons

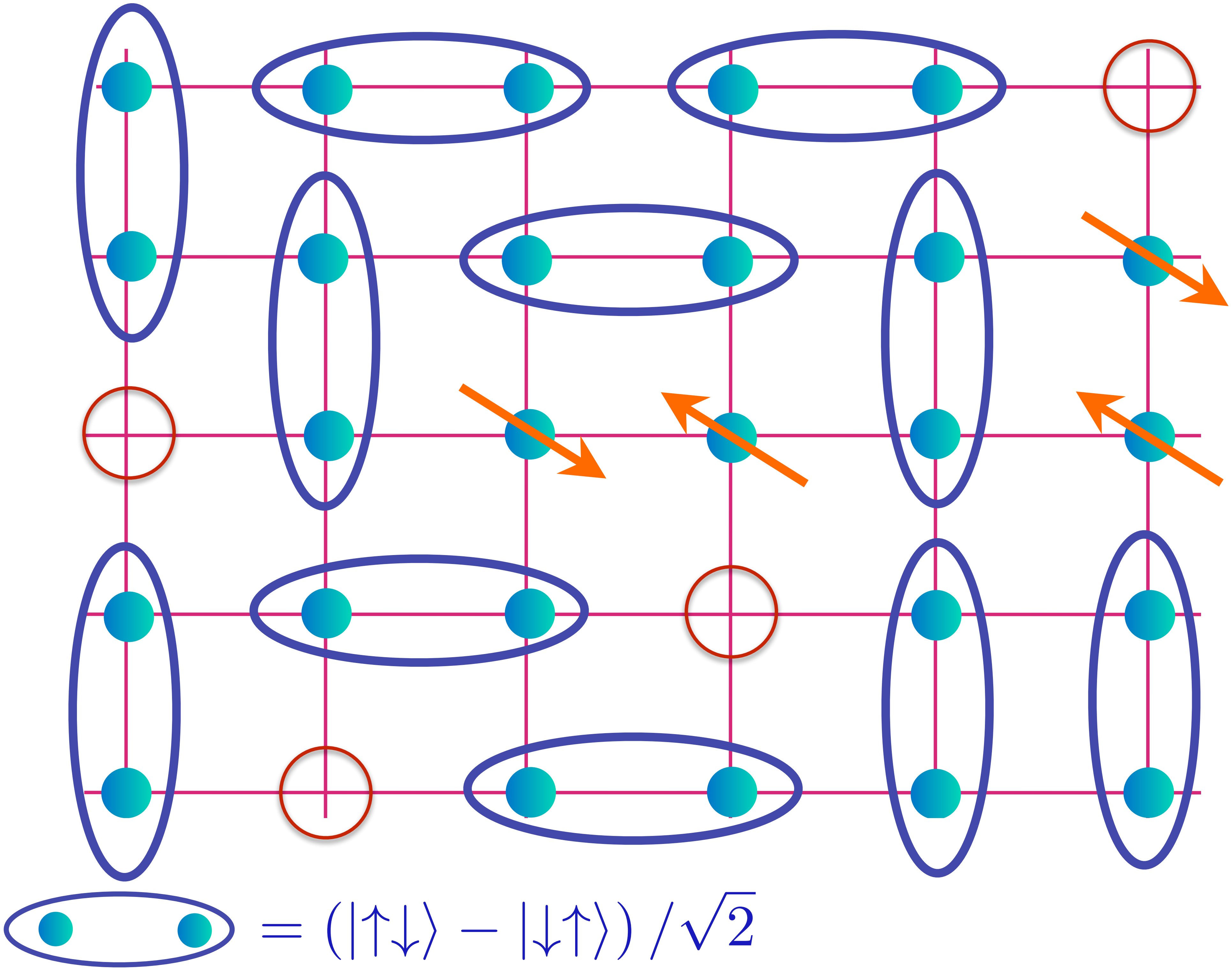
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

N. Read and B. Chakraborty, PRB **40**, 7133 (1989)

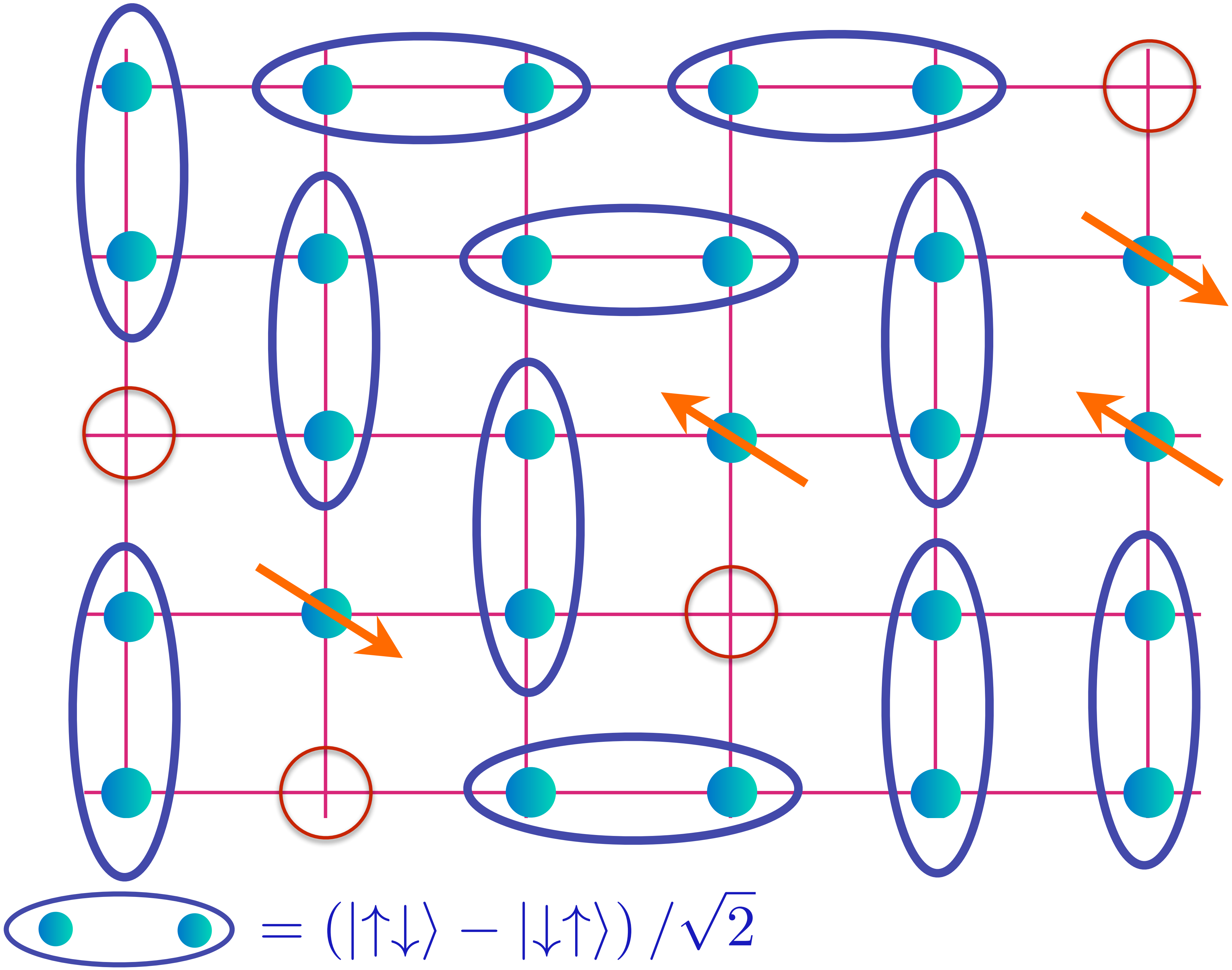


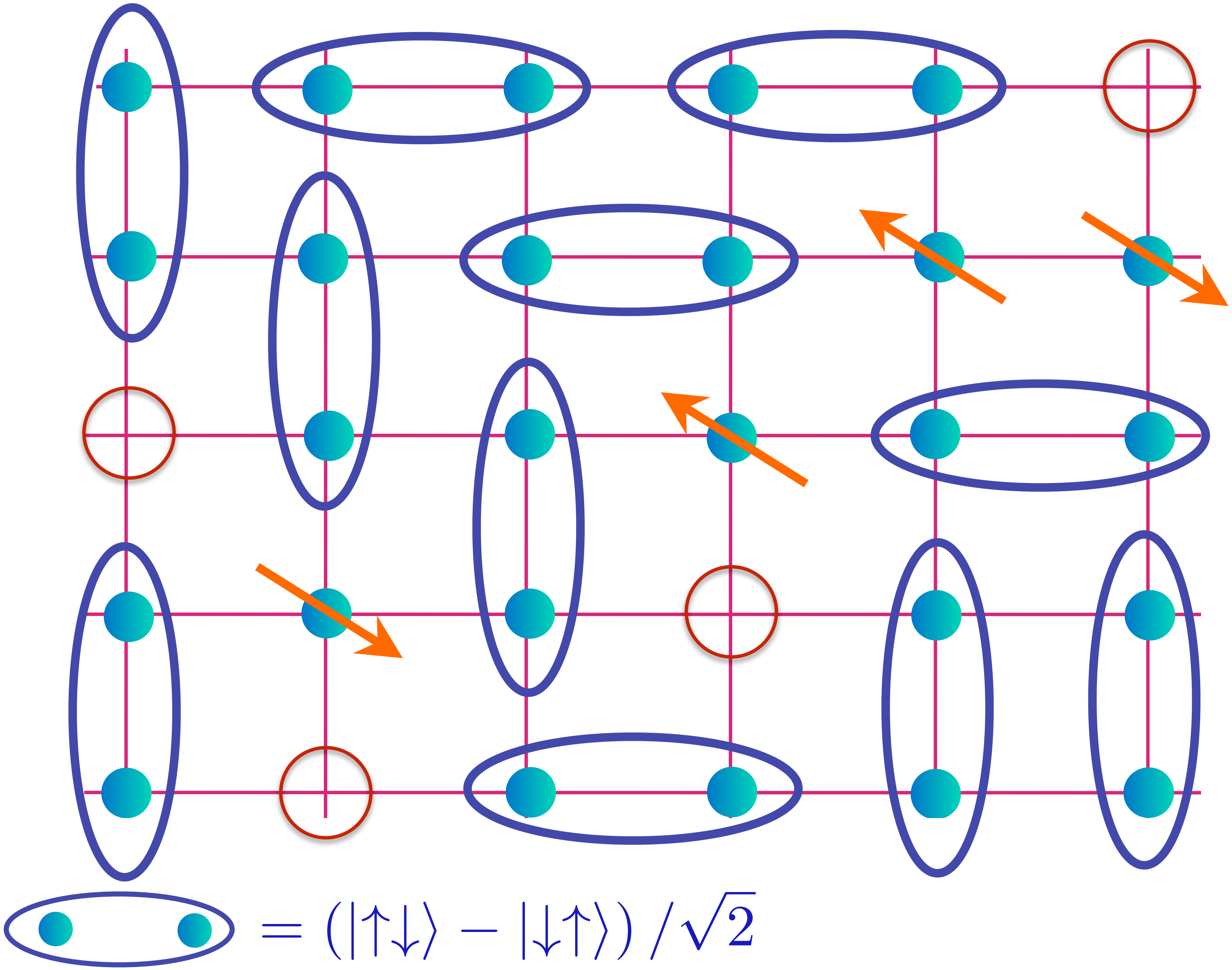
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

Spin liquid with density p of spinless, charge +e "holons". These can form a Fermi surface of size p, but not of electrons

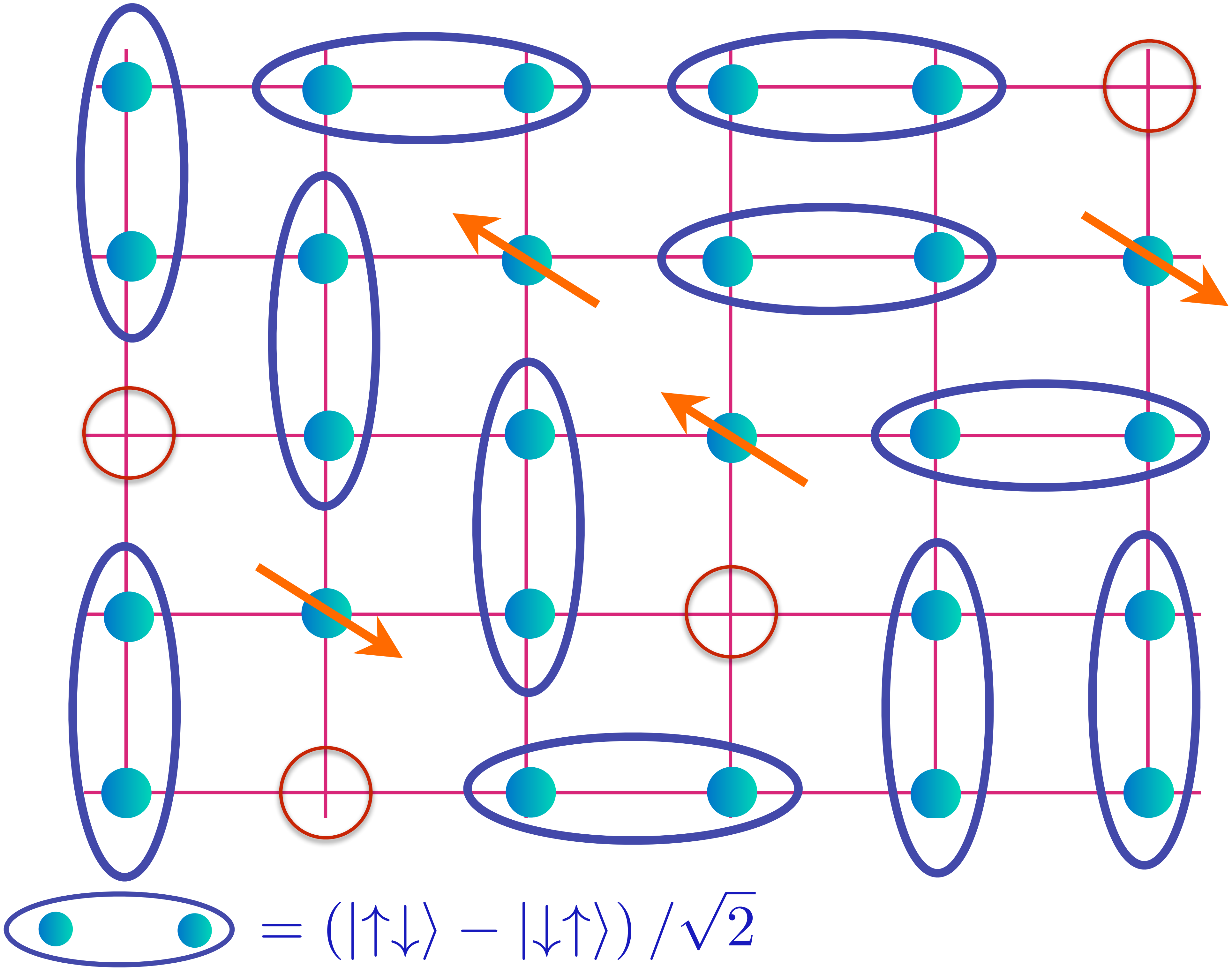


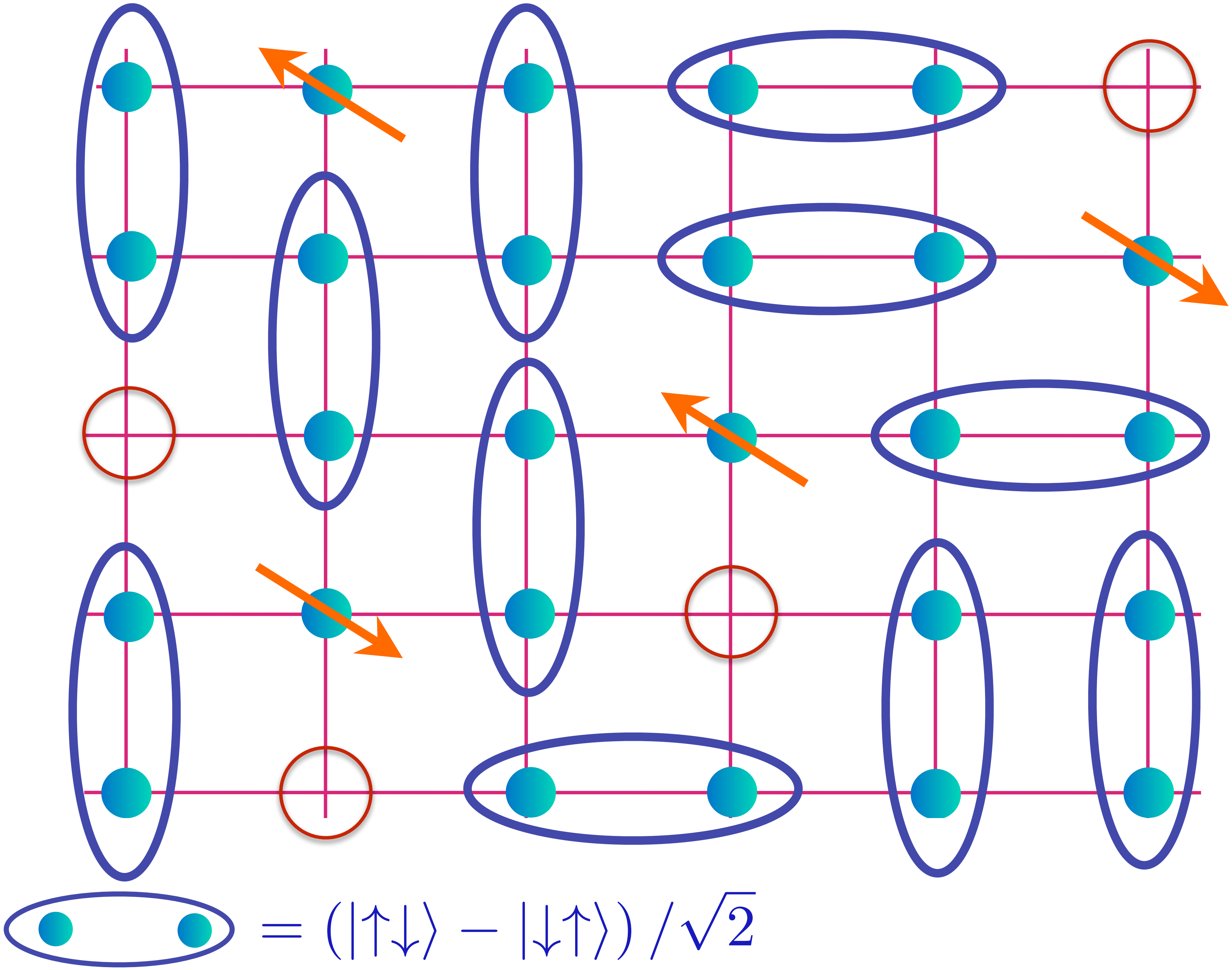
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

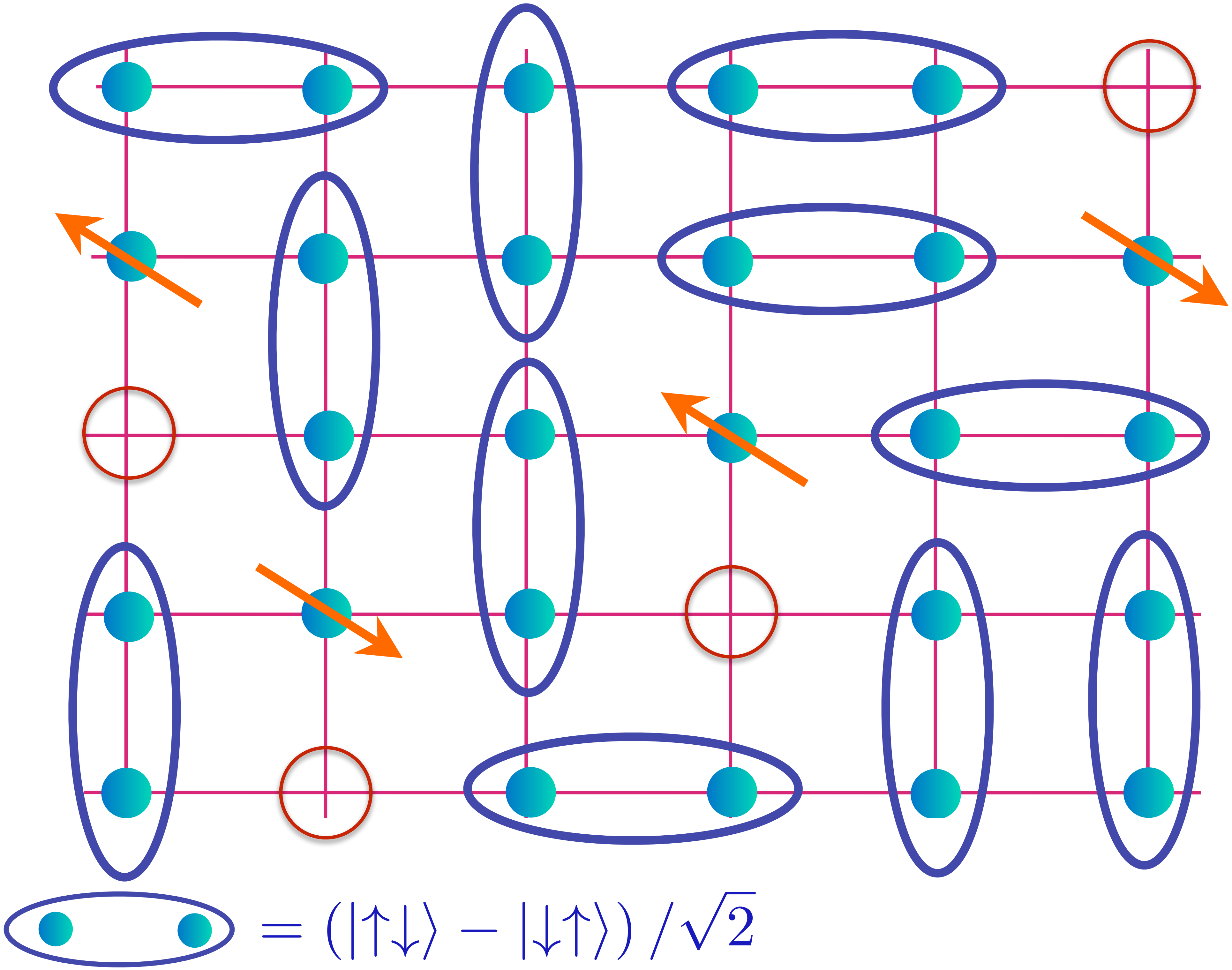

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

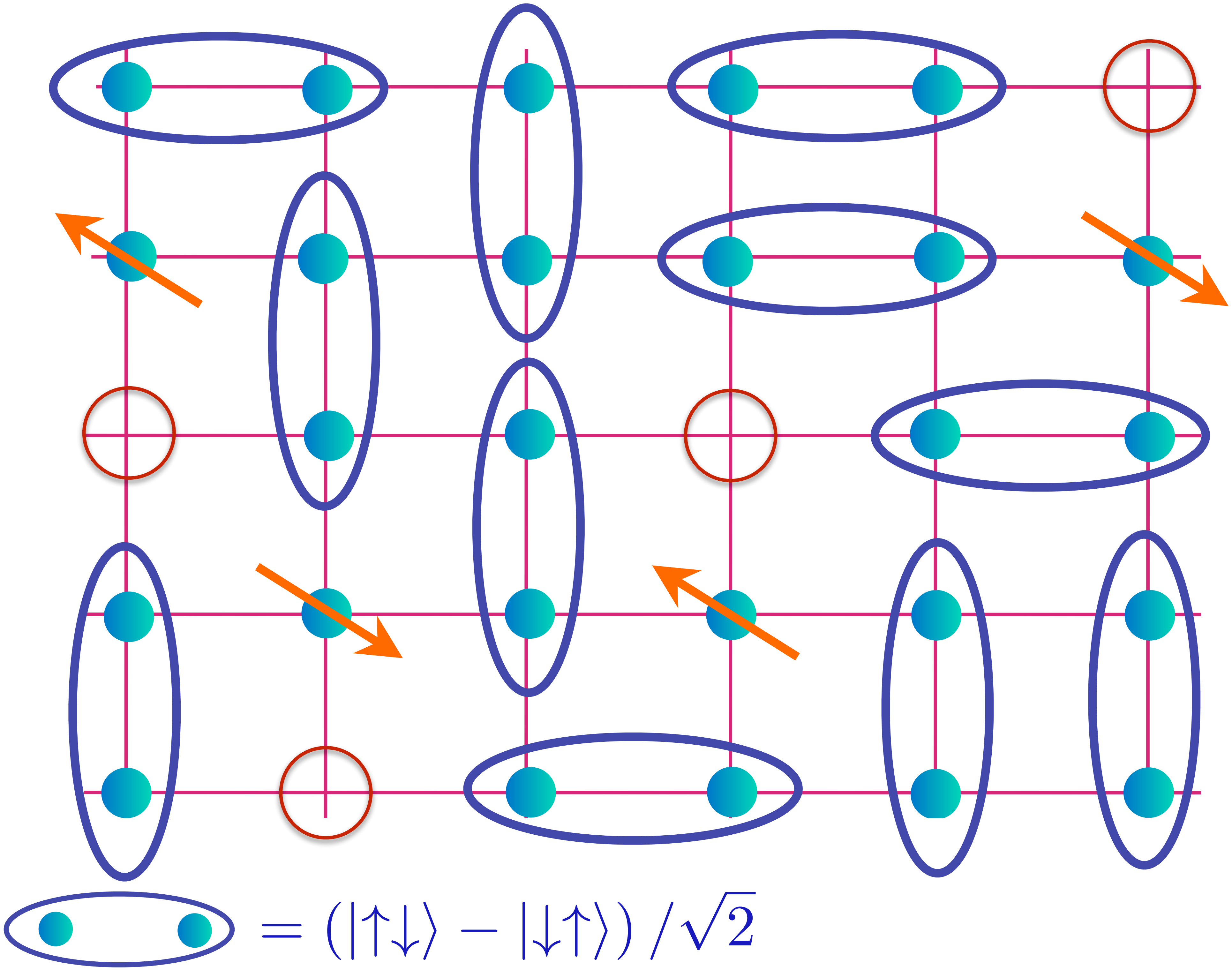


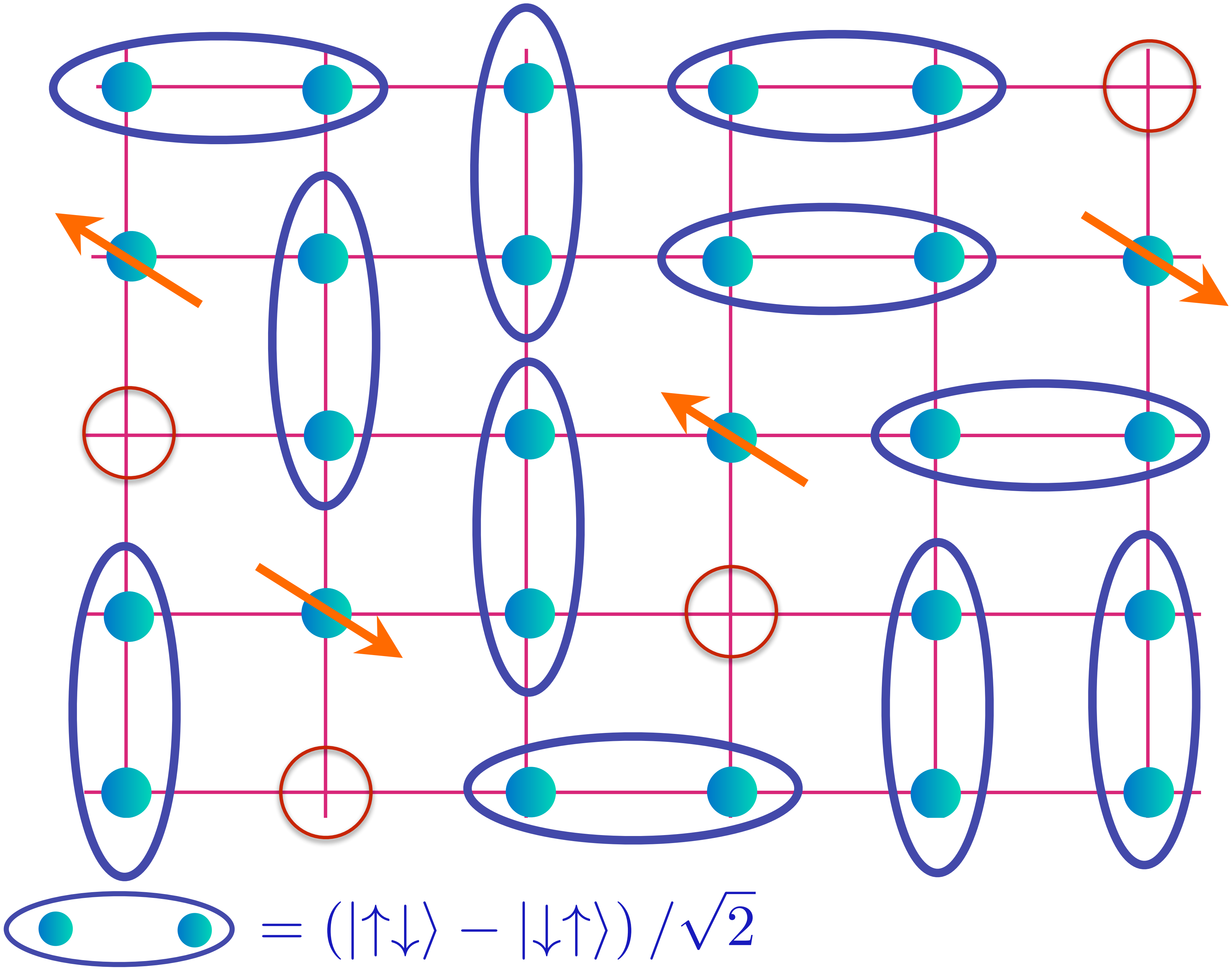
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$


$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$


$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$


$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

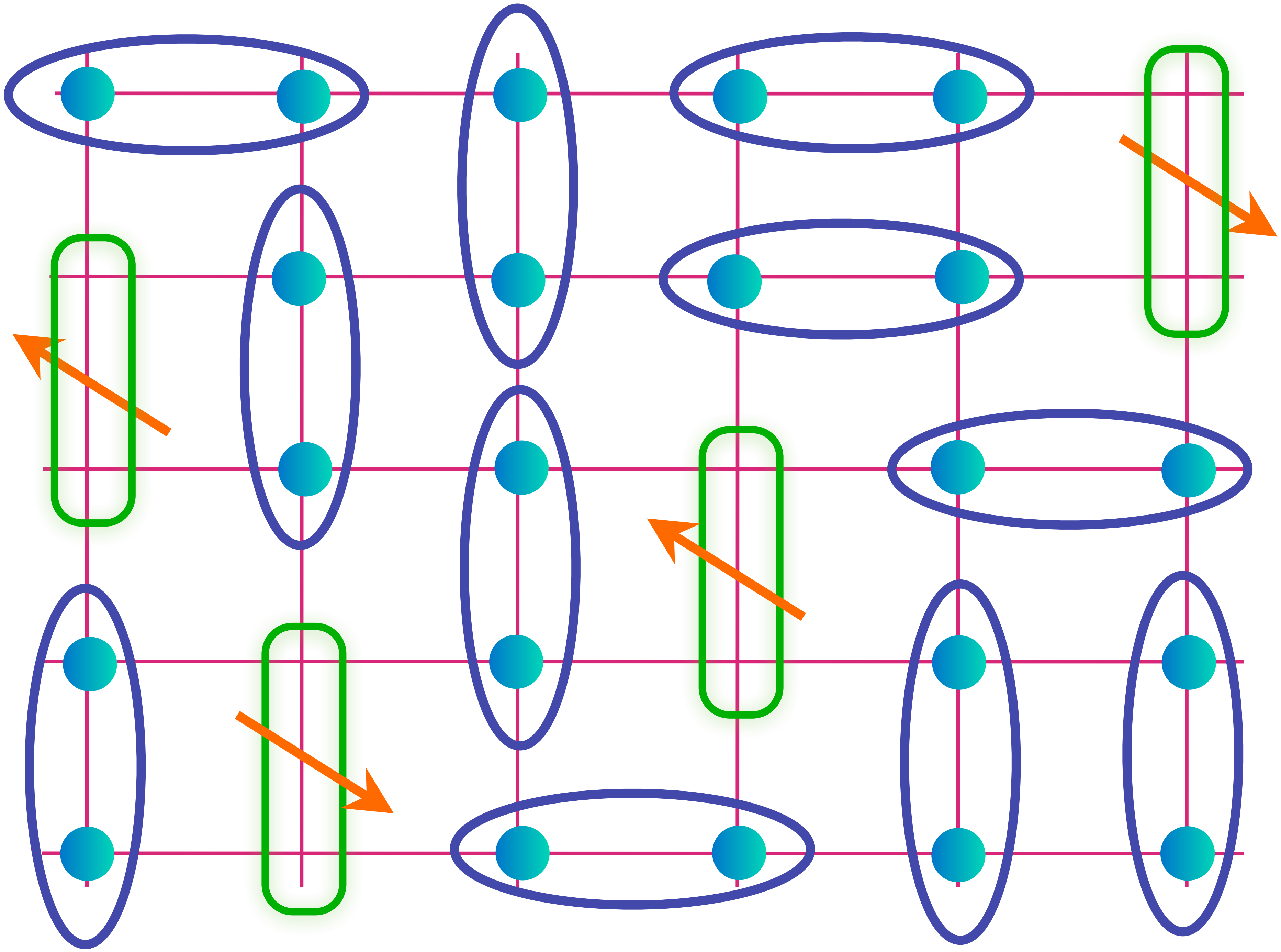

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



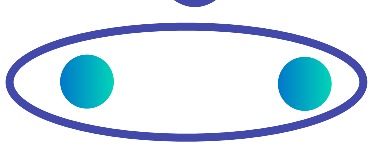
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

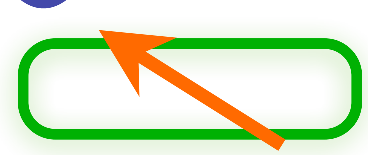
FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p of electrons

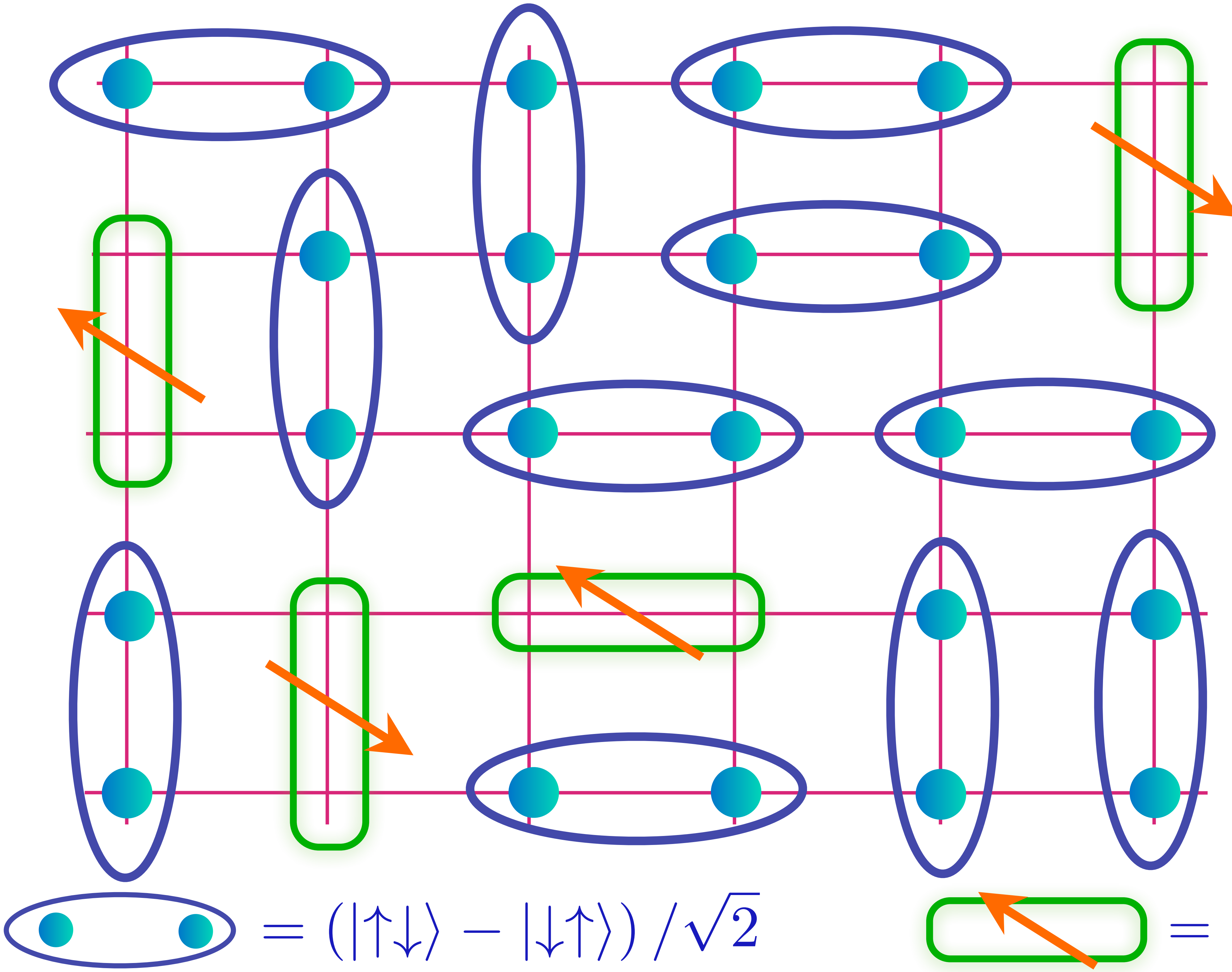
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p of electrons

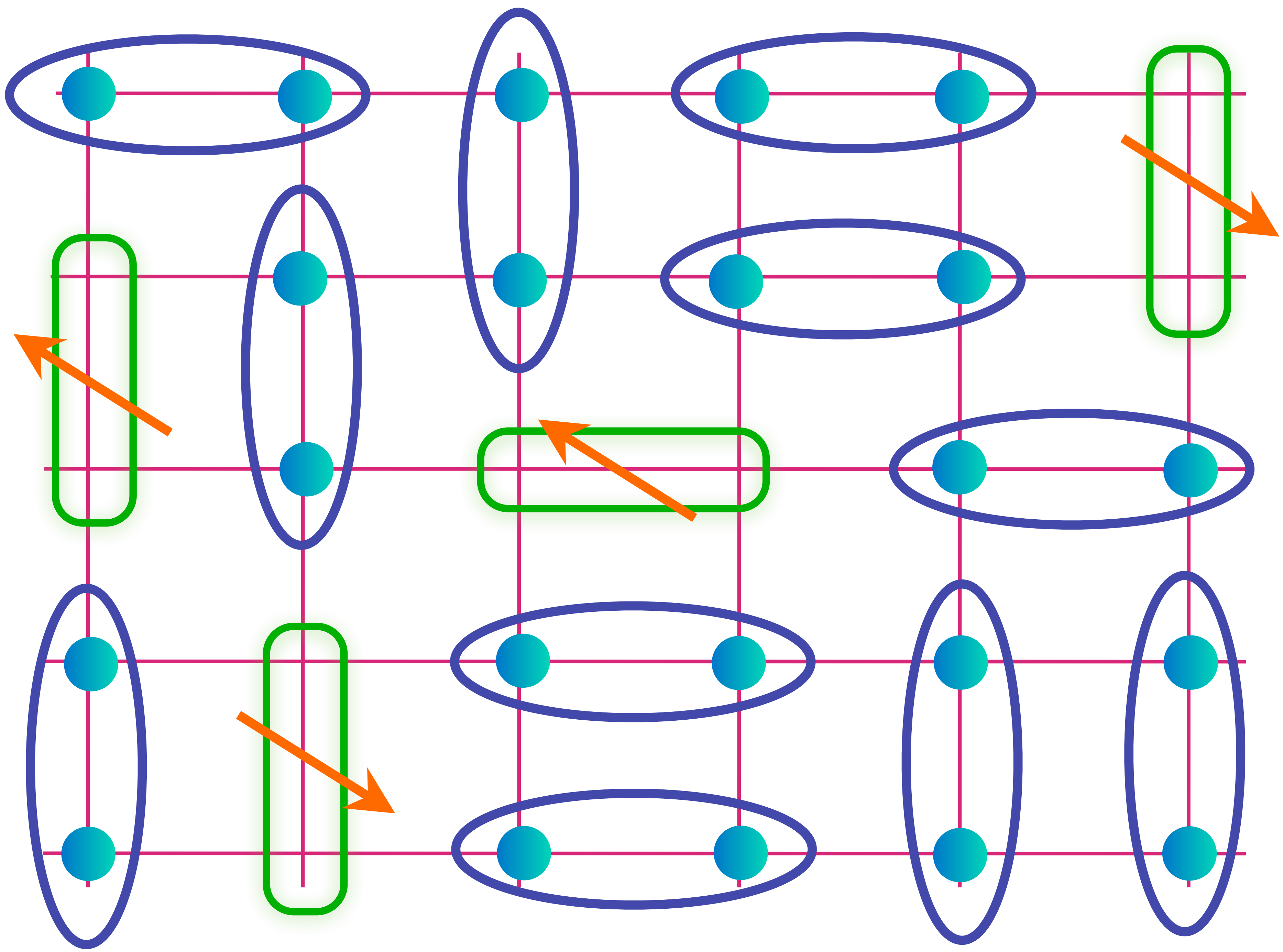
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

$$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p of electrons

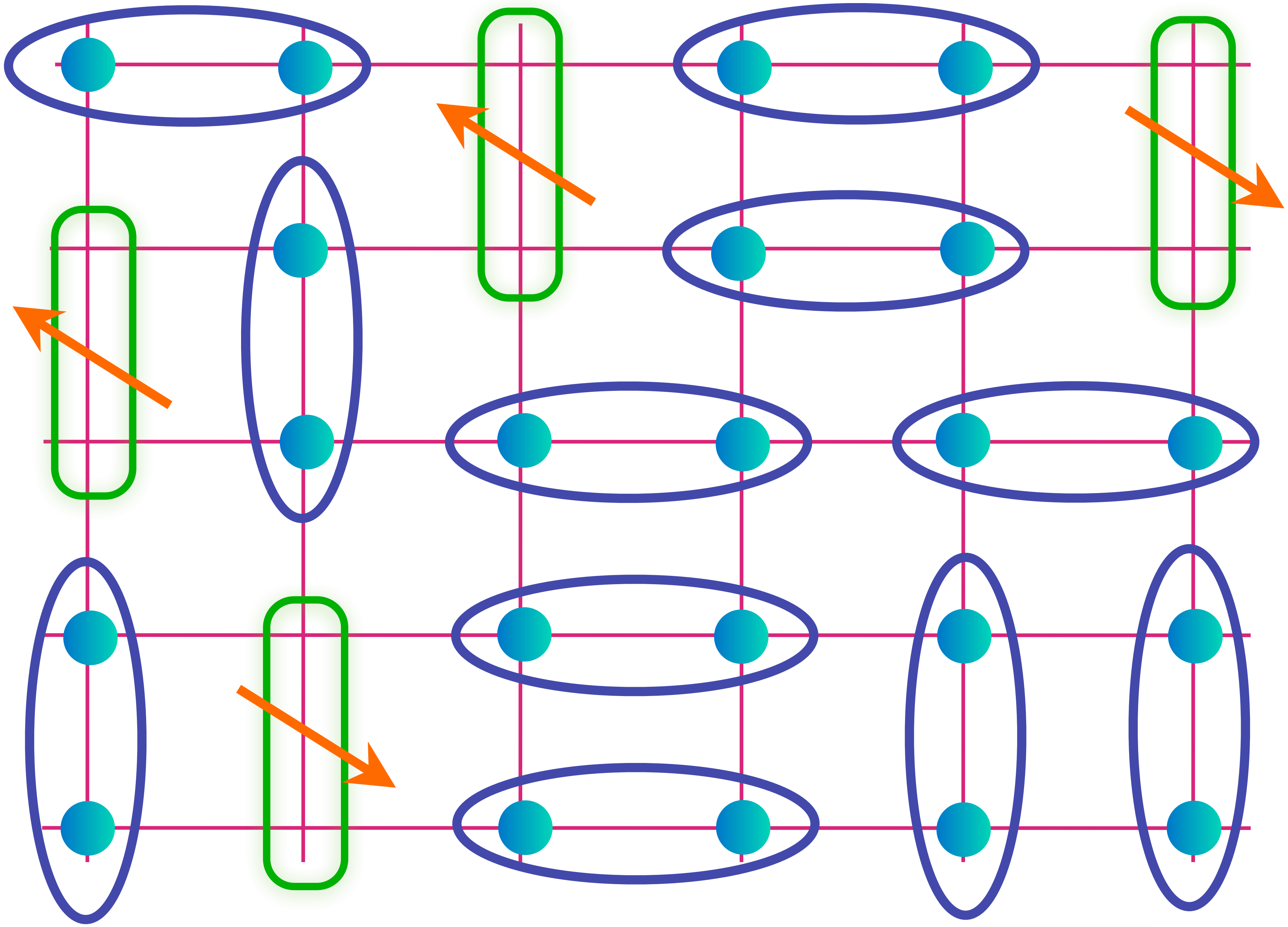
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

$$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p of electrons

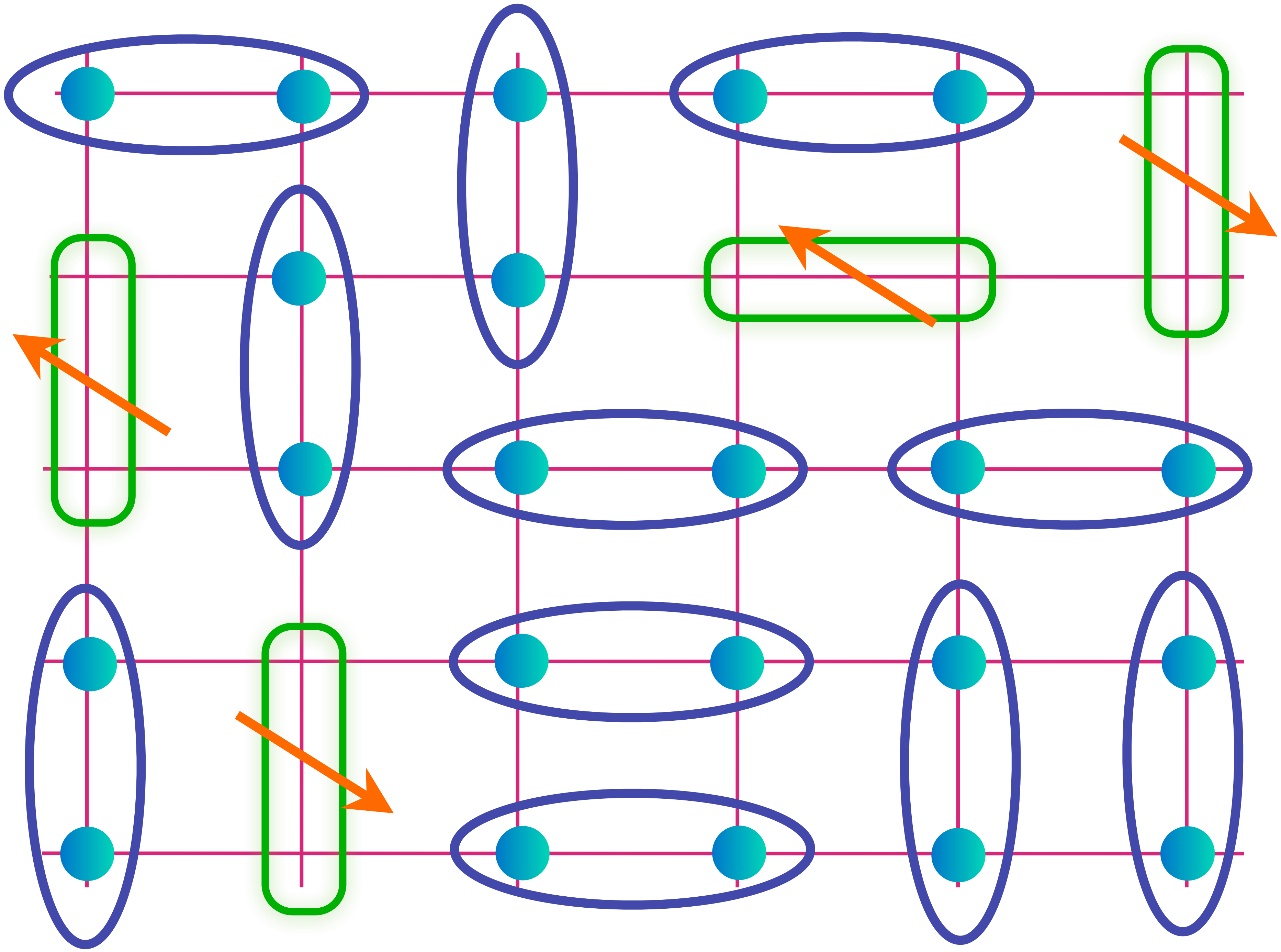
$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

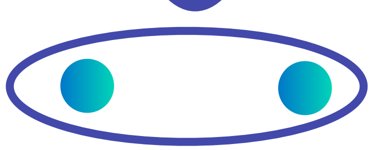
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

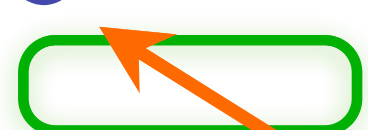
FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile $S=1/2$, charge $+e$ fermionic dimers: form a Fermi surface of size p of electrons

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

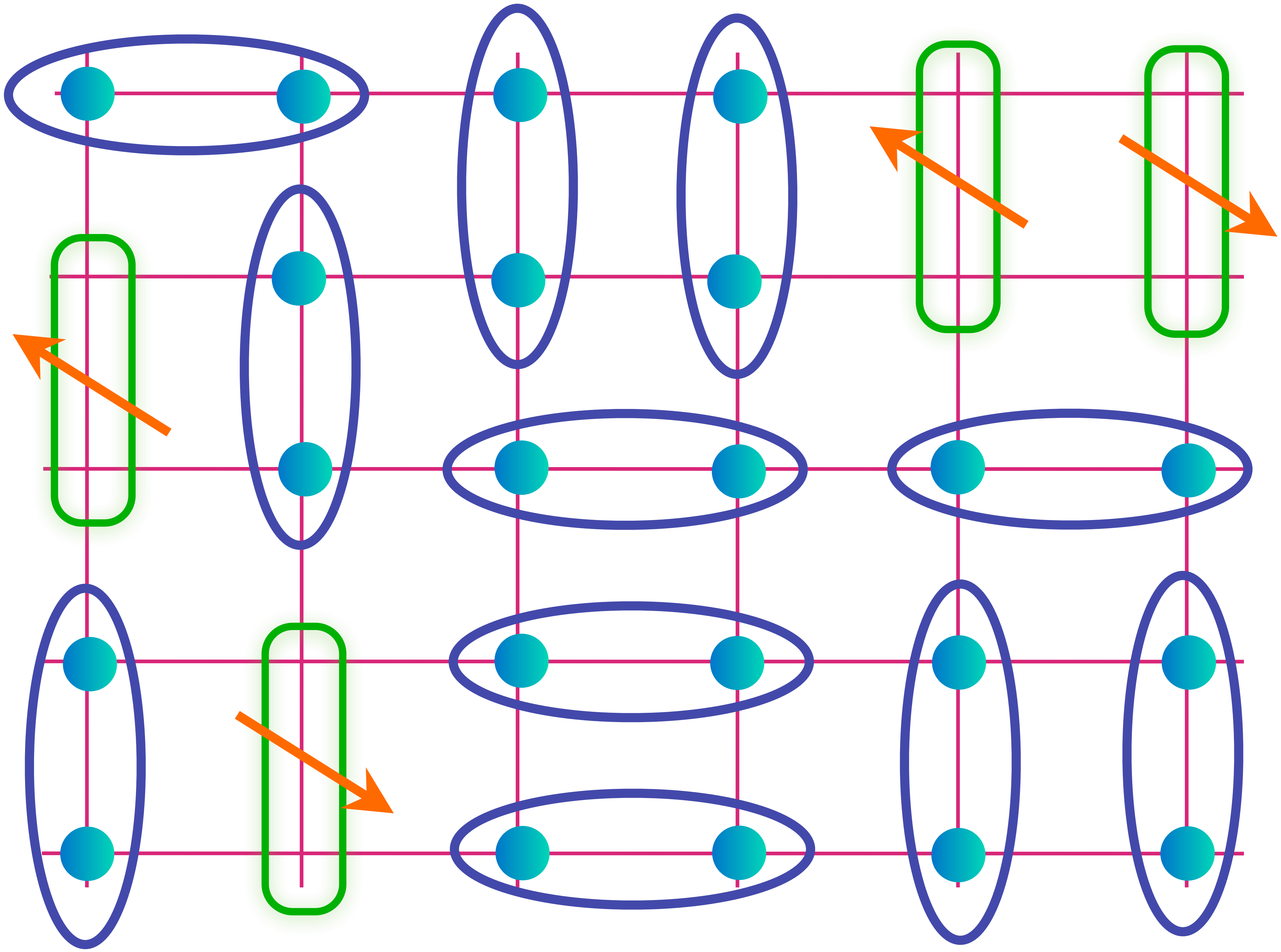
 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

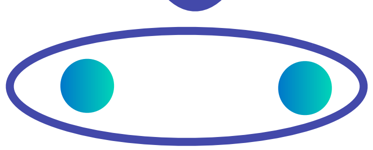
FL*

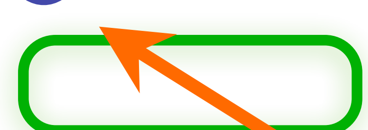
S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p of electrons

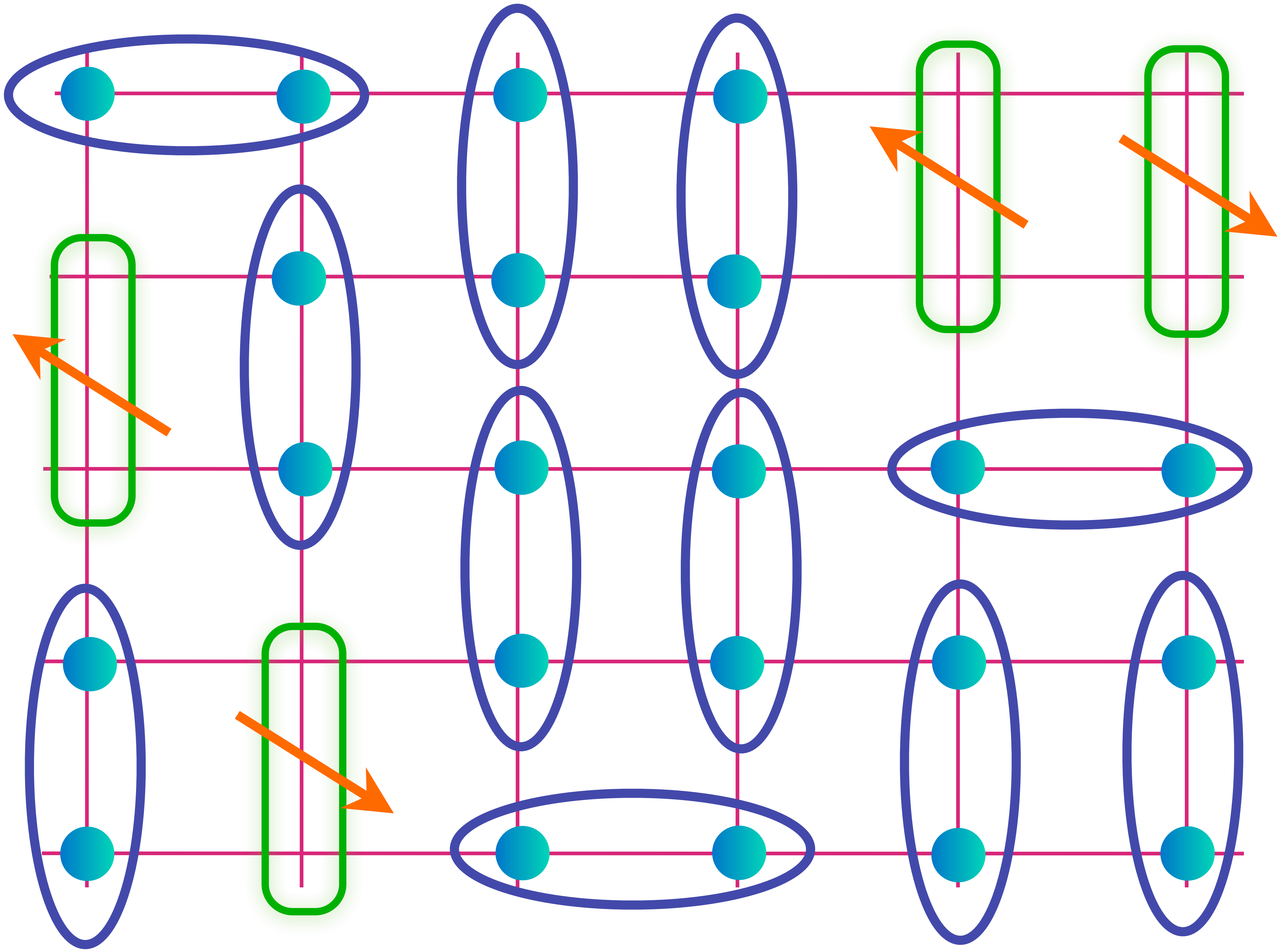
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p of electrons

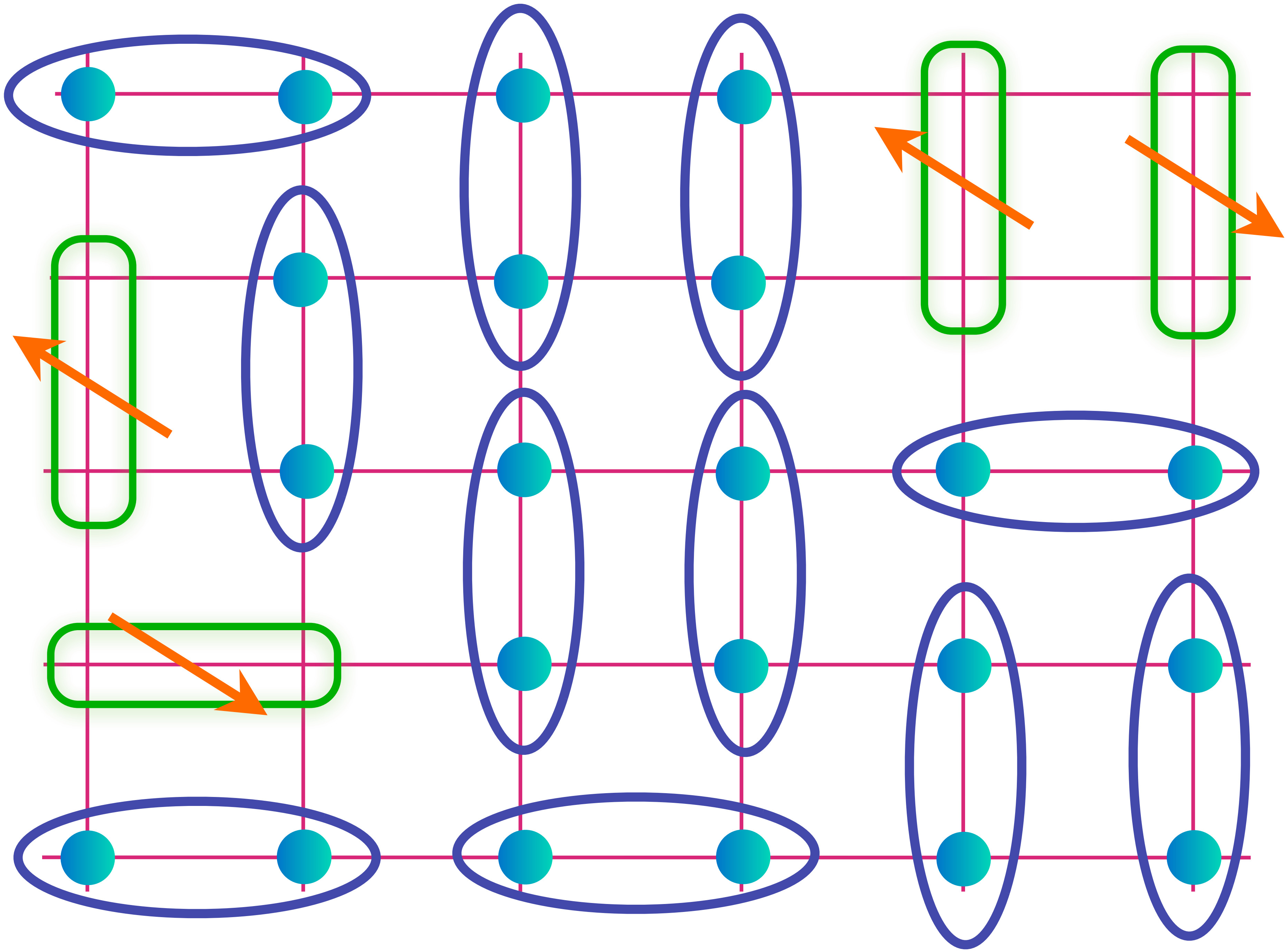
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

$$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$$

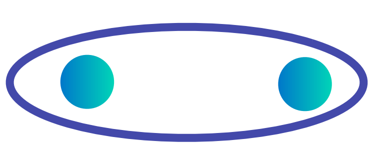
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*

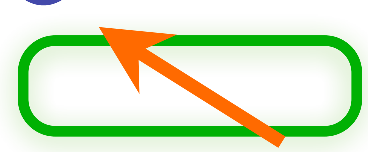
S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size *p* of electrons



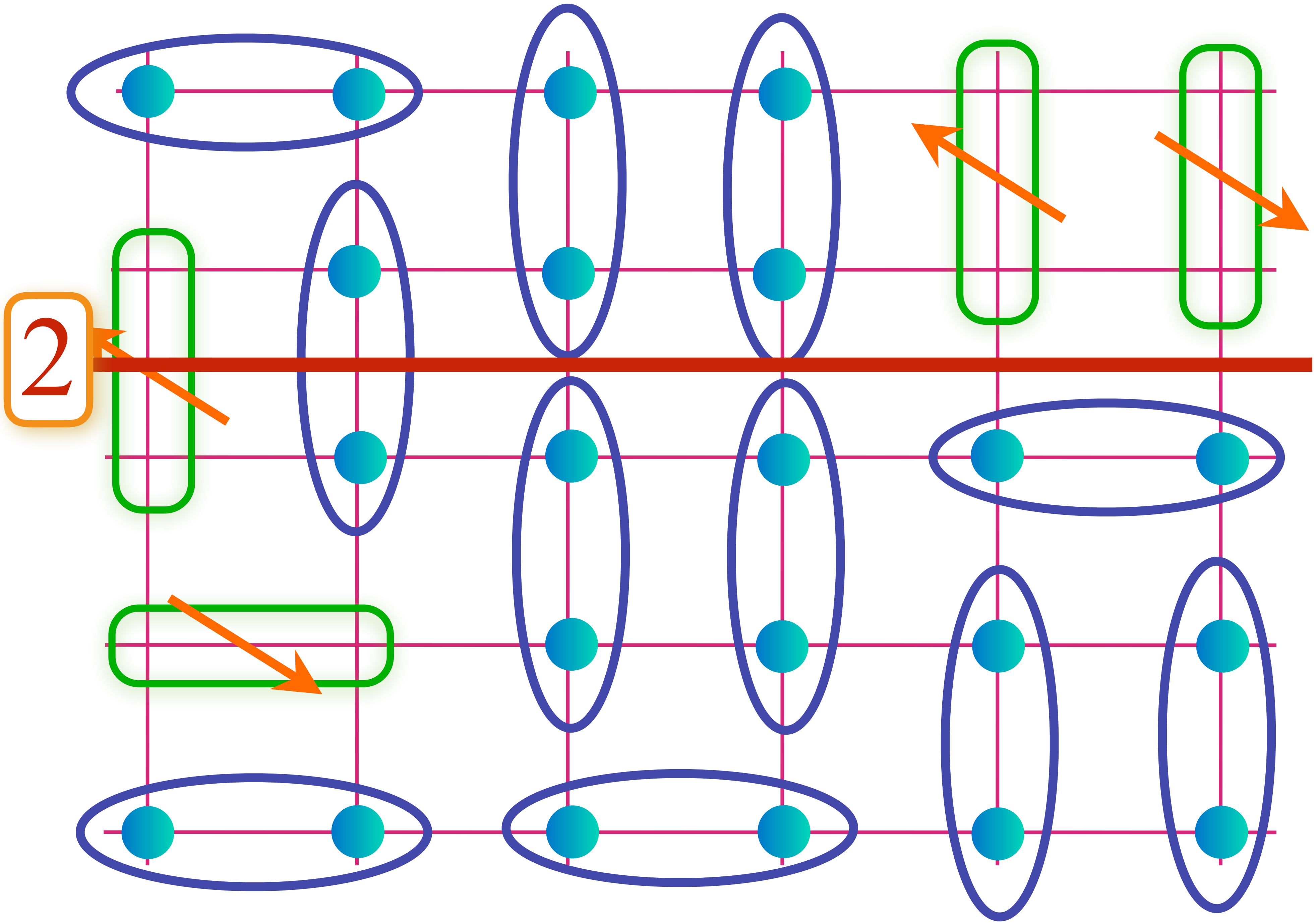
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$



$$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*



Place FL* on a torus:

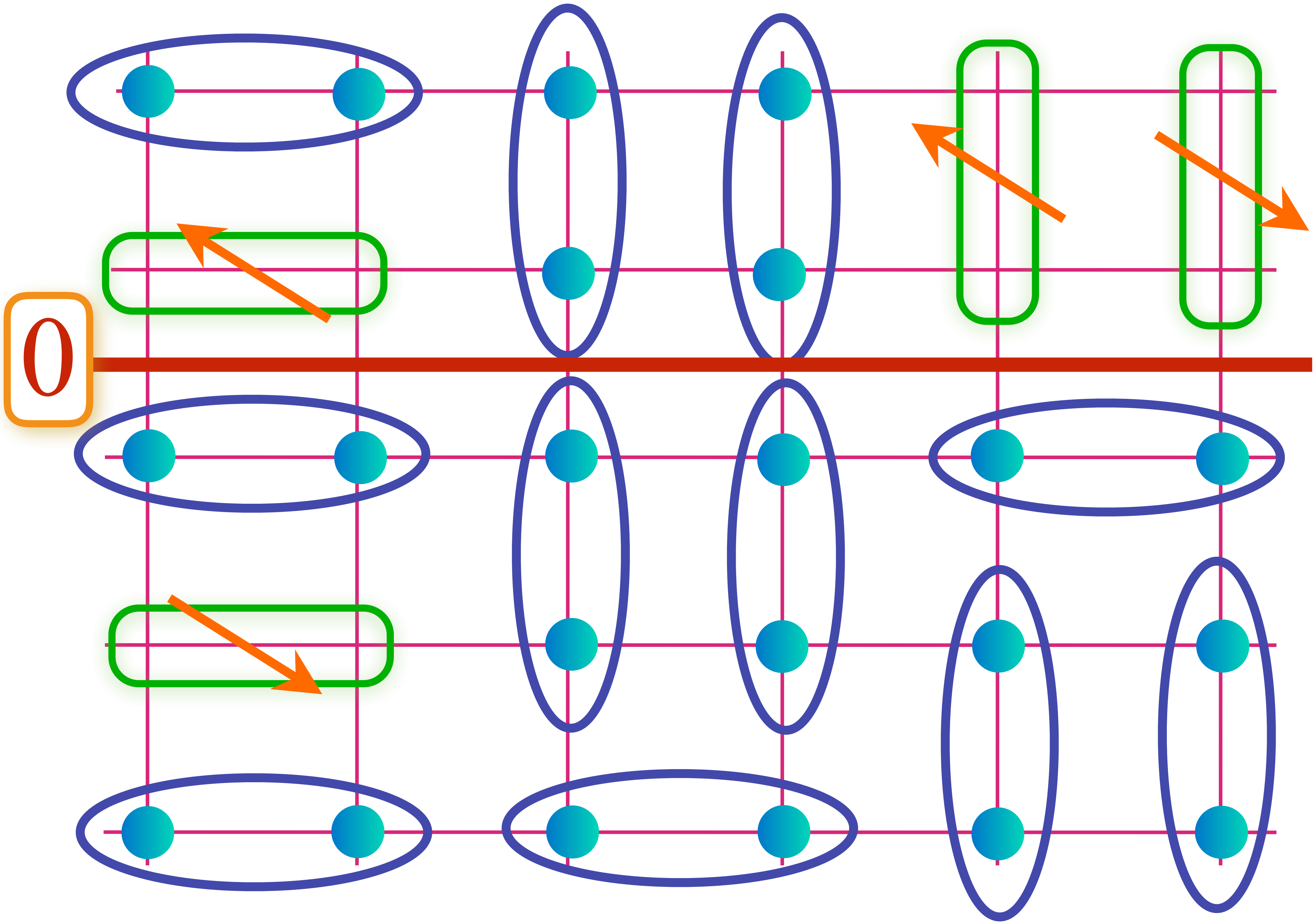
obtain "topological" states nearly degenerate with quasiparticle states: number of dimers crossing red line is conserved modulo 2

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

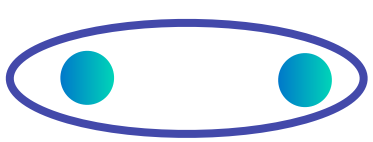
T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

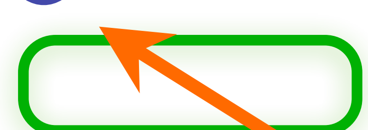
FL*



Place FL* on a torus:

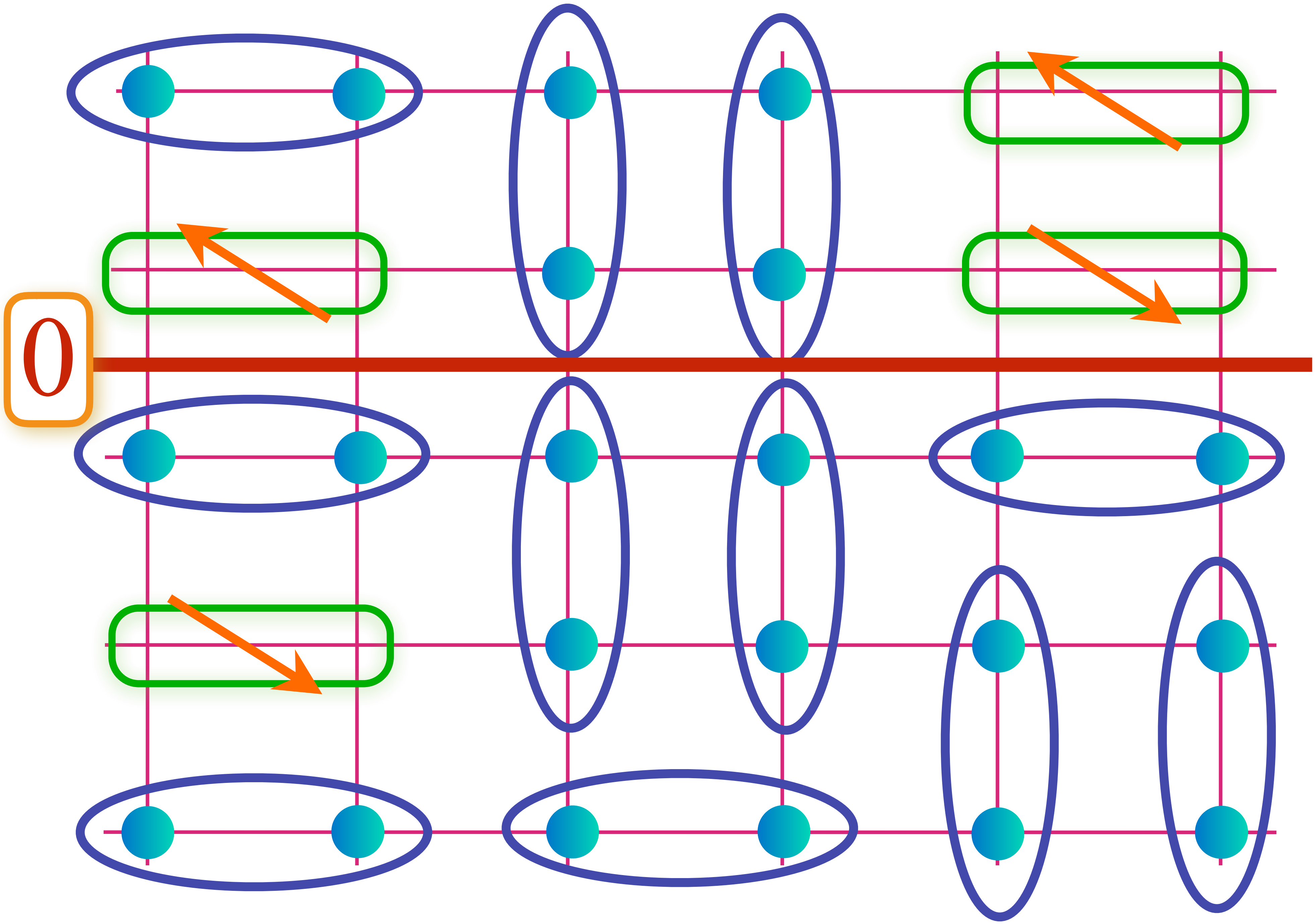
obtain "topological" states nearly degenerate with quasiparticle states: number of dimers crossing red line is conserved modulo 2

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

FL*



Place FL* on a torus:

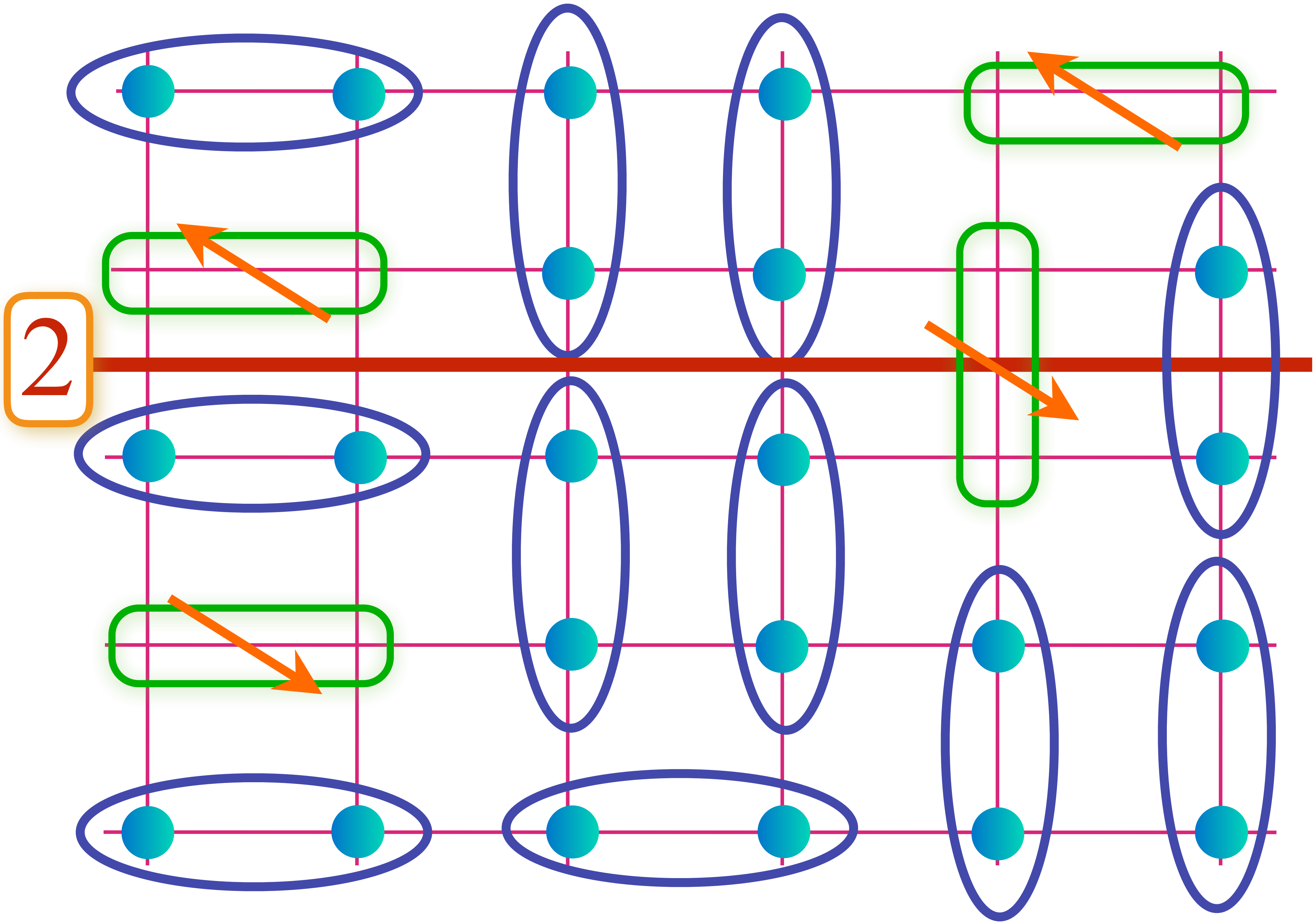
obtain “topological” states nearly degenerate with quasiparticle states: number of dimers crossing red line is conserved modulo 2

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

FL*



Place FL* on a torus:

obtain "topological" states nearly degenerate with quasiparticle states: number of dimers crossing red line is conserved modulo 2

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

$$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$$

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

1. The insulating spin liquid and topological field theory

2. Topology and the size of the Fermi surface

3. Transition between FL* and FL

4. Quantum matter with quasiparticles

strange metals in superconductors,
graphene,
the quark-gluon plasma,
the superfluid-insulator transition of ultra-cold atoms,
and the dynamics of charged black holes horizons

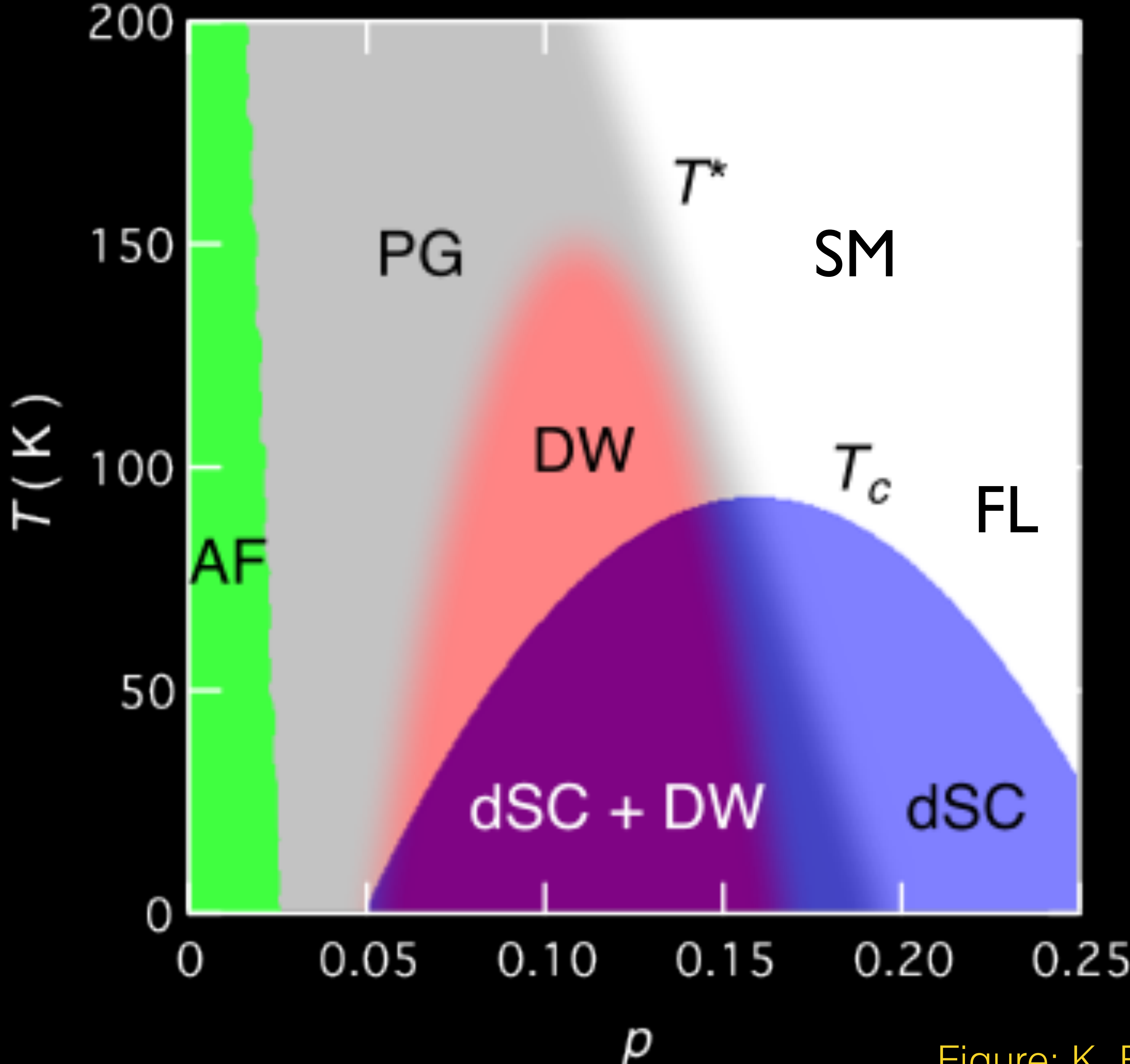


Figure: K. Fujita and J. C. Seamus Davis

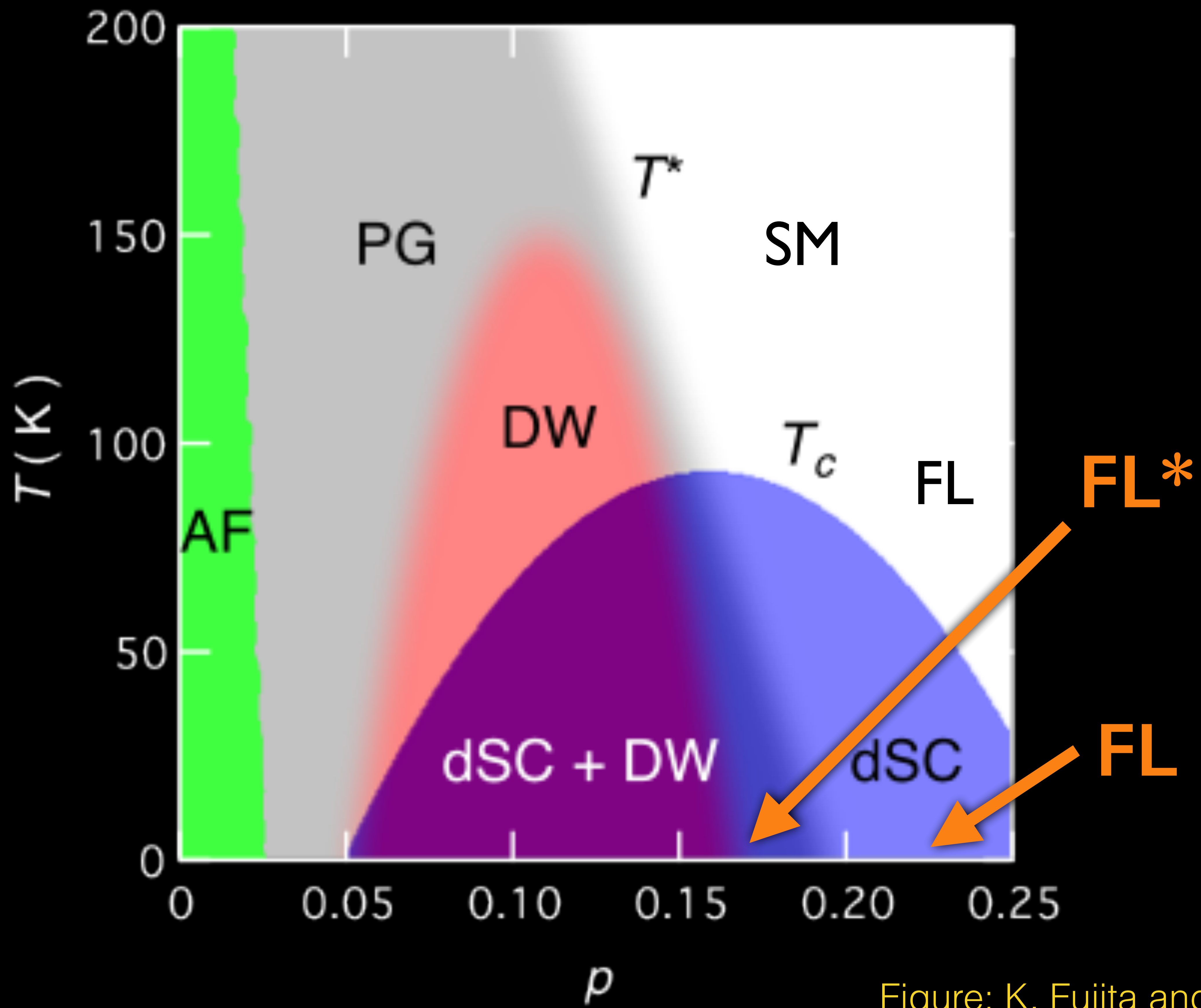


Figure: K. Fujita and J. C. Seamus Davis

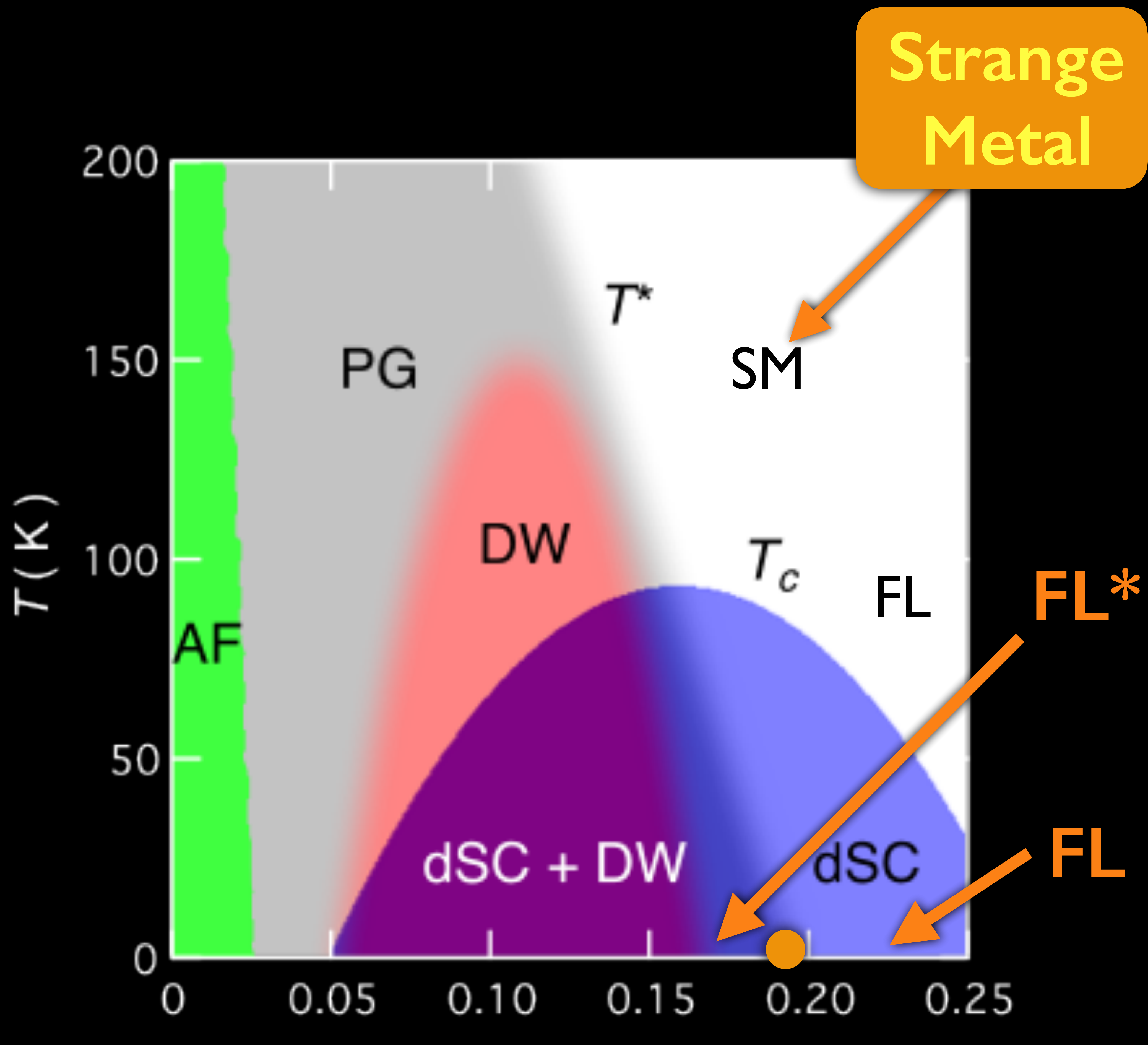


Figure: K. Fujita and J. C. Seamus Davis

SU(2) gauge theory for transition between $\mathbb{Z}_2$-FL* and FL

- A SU(2) gauge boson.
- Fermion ψ, transforming as a gauge SU(2) fundamental, with dispersion $\varepsilon_{\mathbf{k}}$ from the band structure, at a non-zero chemical potential: has a "large" Fermi surface, and carries electromagnetic charge
- A complex Higgs field, H, transforming as a gauge SU(2) adjoint, carrying non-zero lattice momentum.
- A SU(2) fundamental scalar z_α, carrying electron-spin and electromagnetically neutral.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)
D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015)

SU(2) gauge theory for transition between $\mathbb{Z}_2$-FL* and FL

- The 'Higgs' phase with $\langle H \rangle$ has a deconfined $\mathbb{Z}_2$ gauge field, and realizes a $\mathbb{Z}_2$-FL* with Ising-nematic order.
- The 'confining' phase is a FL or a superconductor, and no Ising-nematic order.
- When the Higgs potential is critical, we obtain a non-Fermi liquid of a ψ Fermi surface coupled to Landau-damped gauge bosons, and critical Landau-damped Higgs field. This is a candidate for describing the strange metal.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)
D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015)

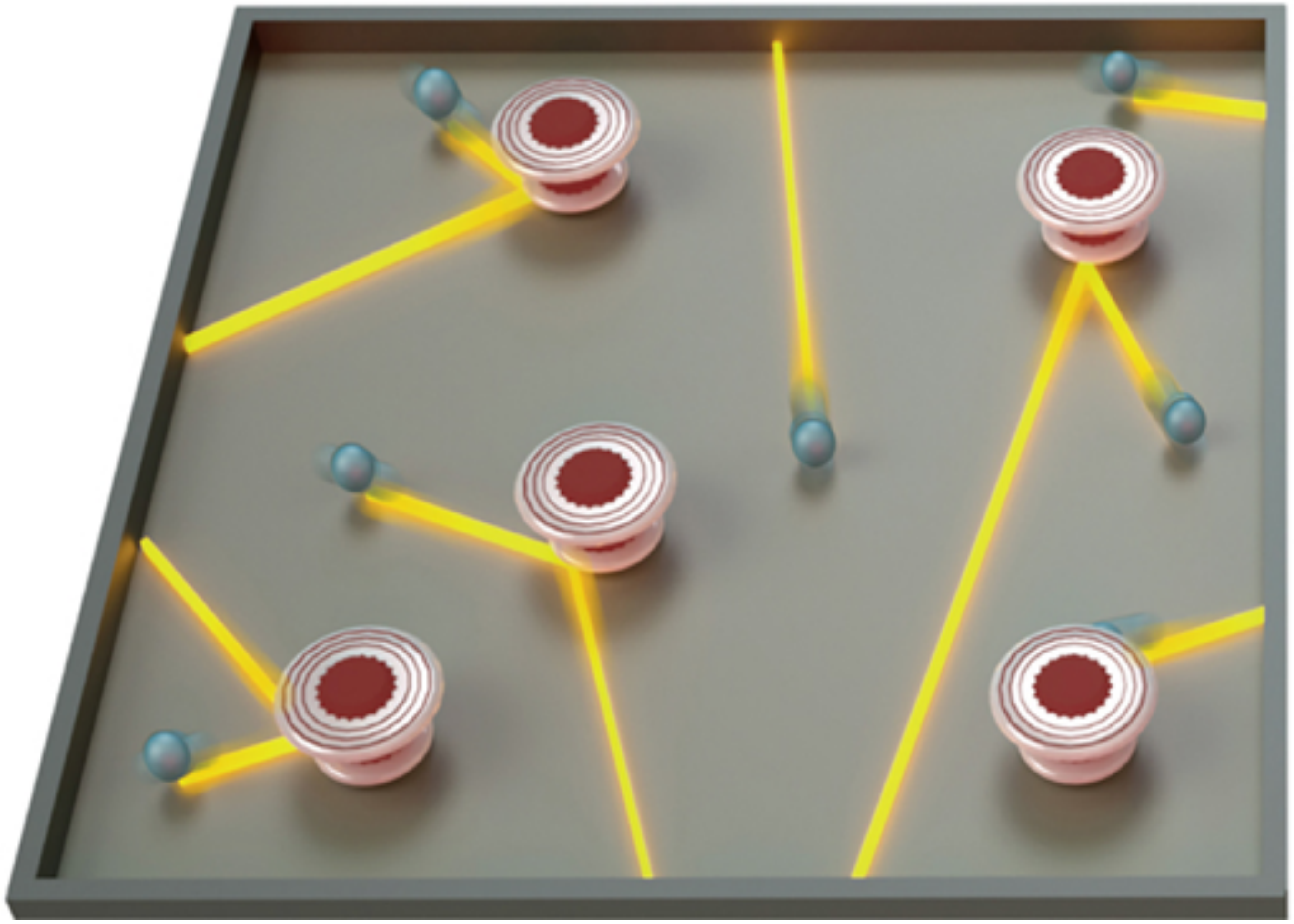
1. The insulating spin liquid and topological field theory

2. Topology and the size of the Fermi surface

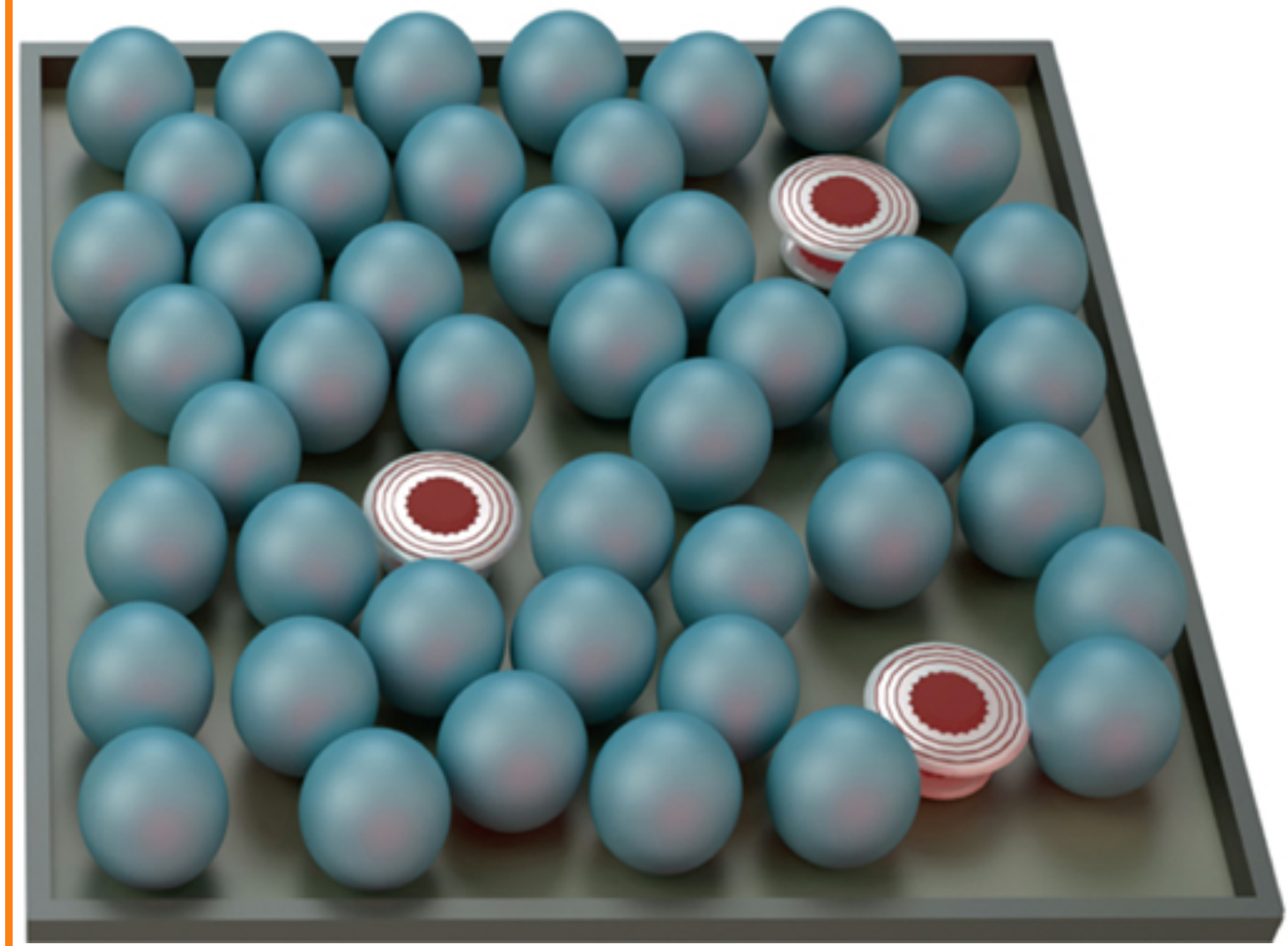
3. Transition between FL* and FL

4. Quantum matter with quasiparticles

   strange metals in superconductors,
   graphene,
   the quark-gluon plasma,
   the superfluid-insulator transition of ultra-cold atoms,
   and the dynamics of charged black holes horizons

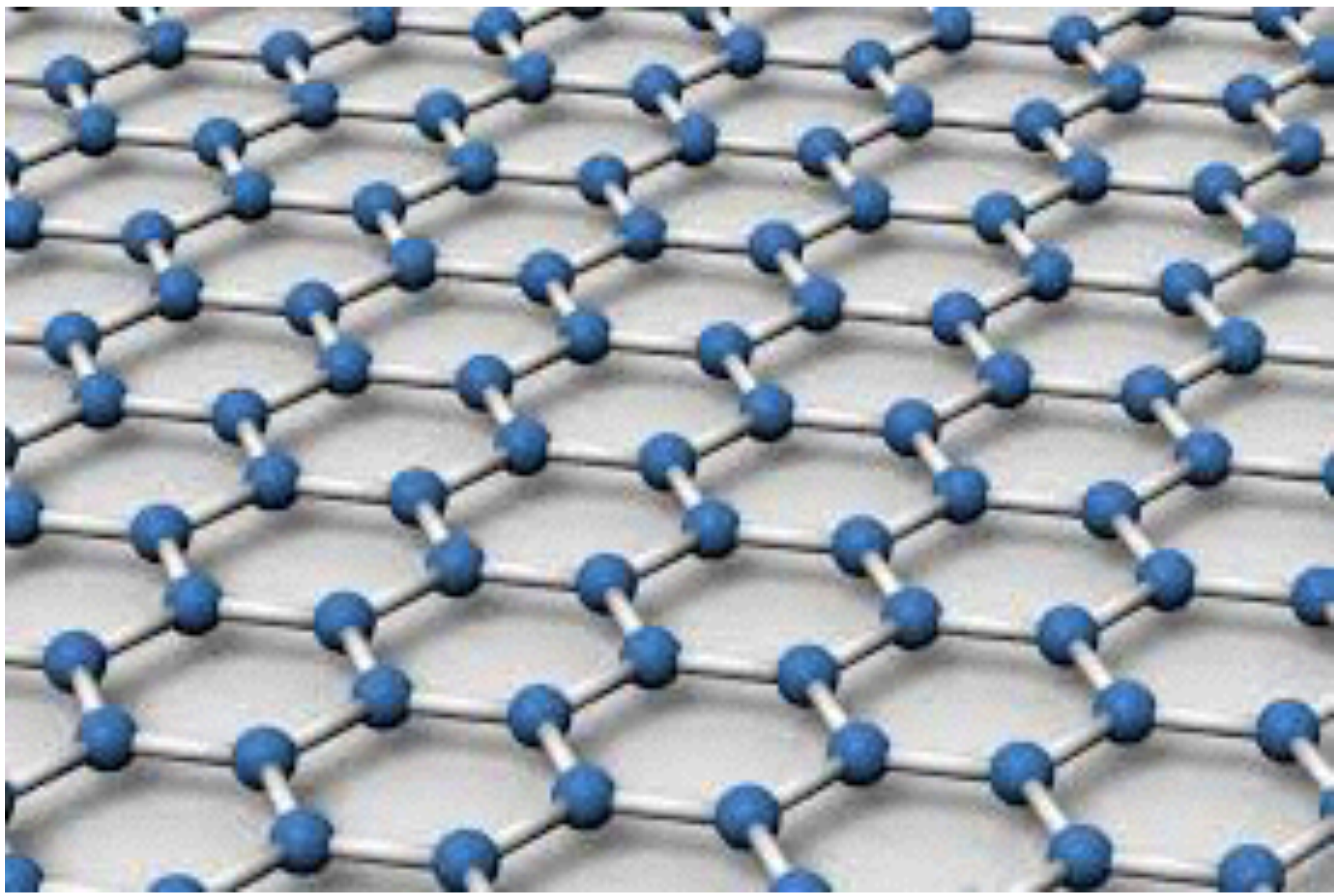


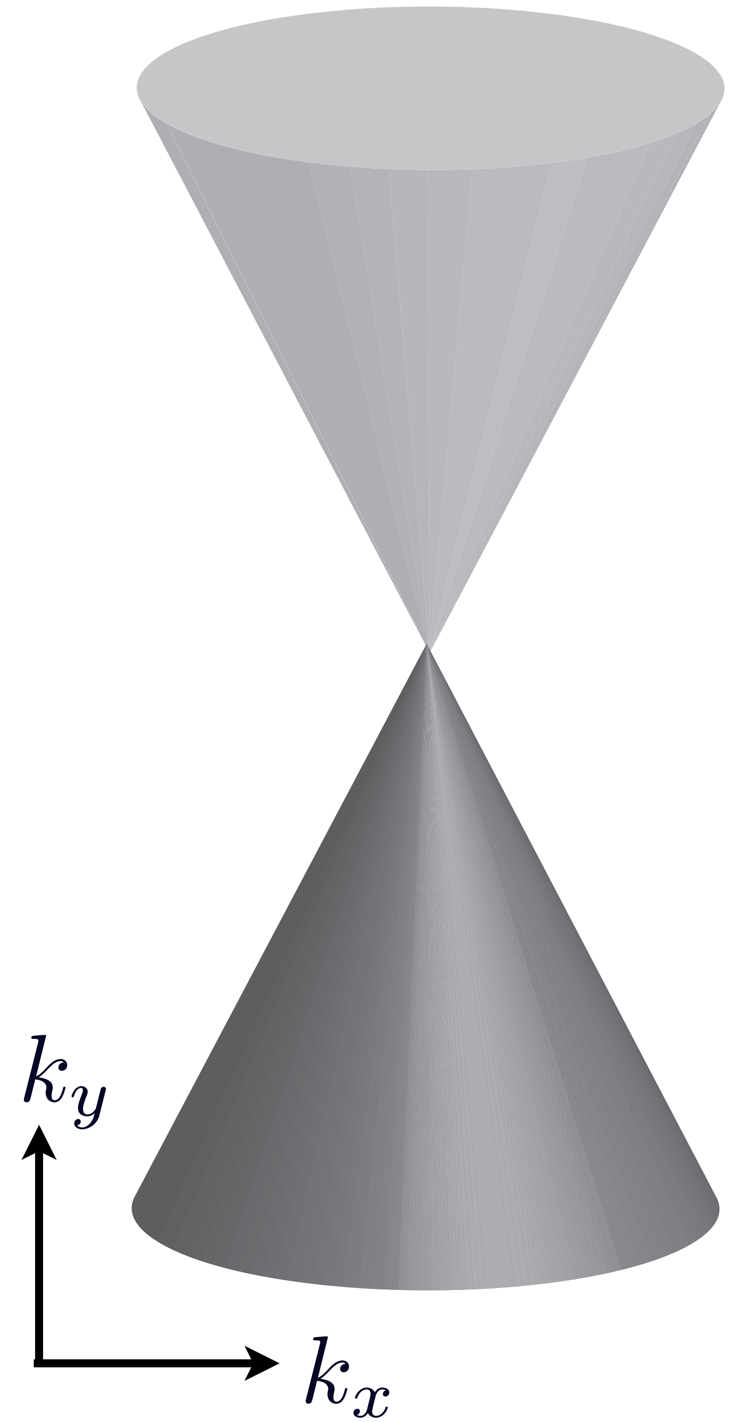
Fermi liquids: quasiparticles moving ballistically between impurity (red circles) scattering events



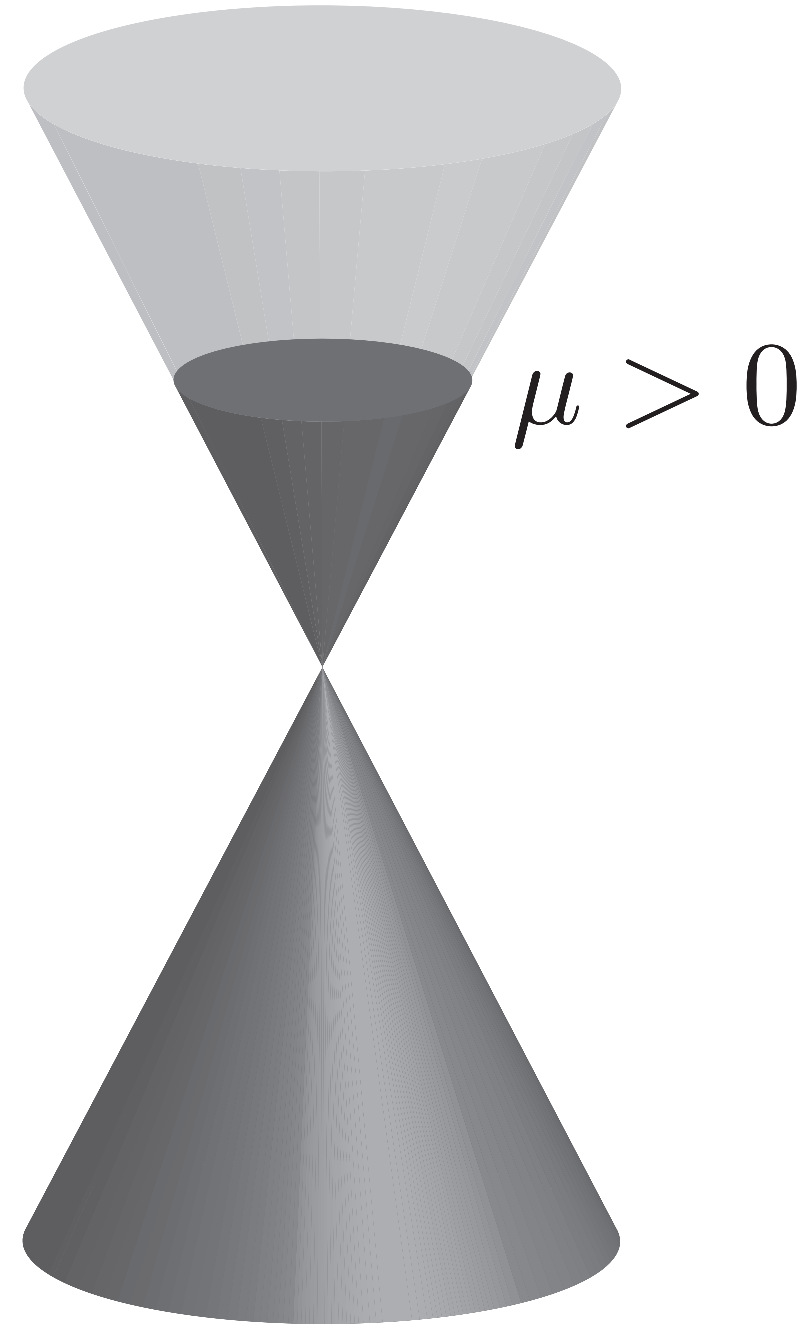
Strange metals: electrons scatter frequently off each other, so there is no regime of ballistic quasiparticle motion. The electron "liquid" then "flows" around impurities

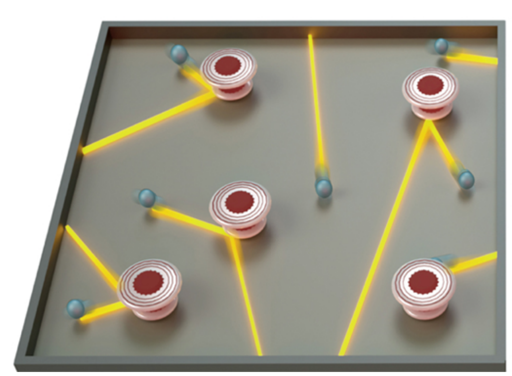
Graphene





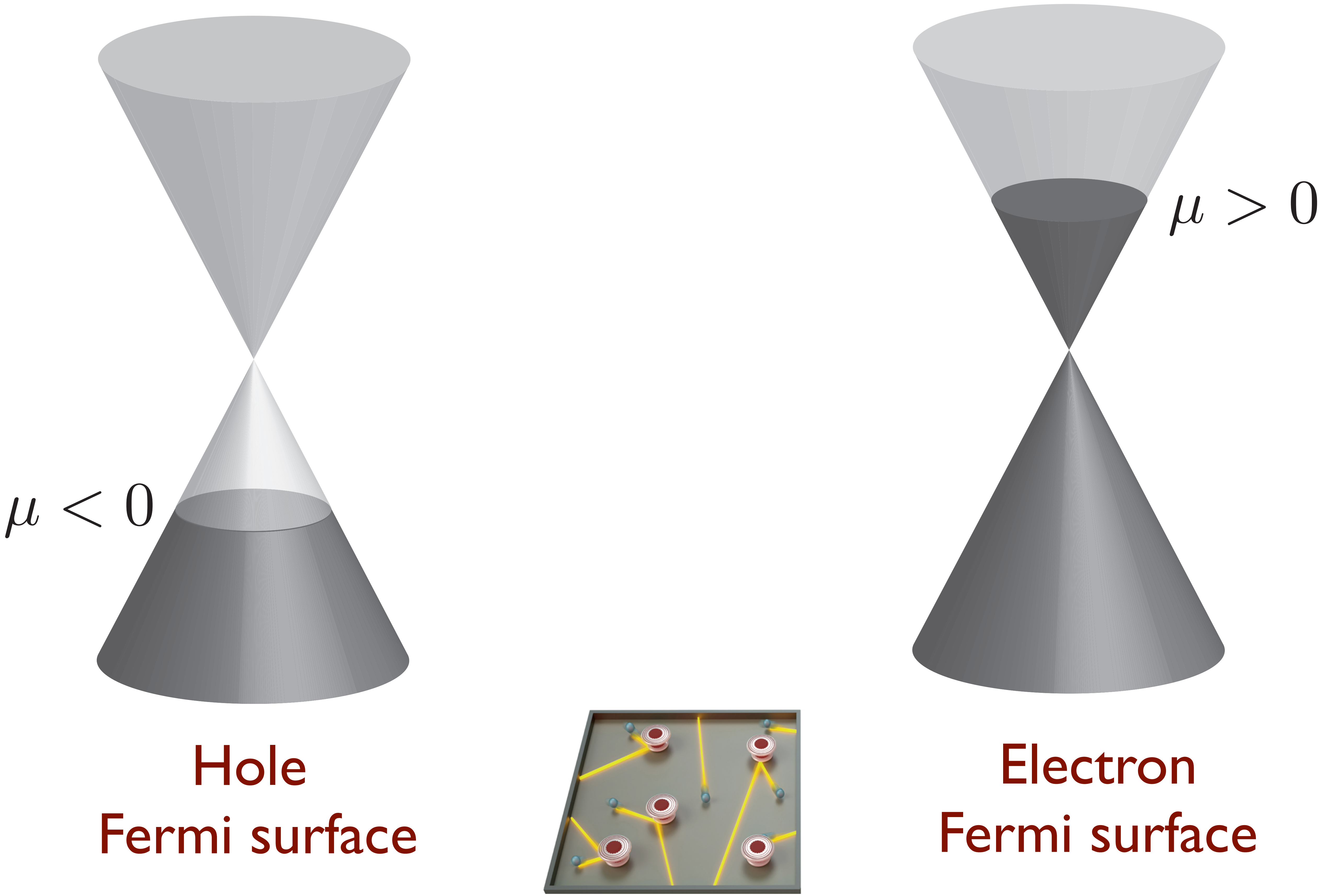
Graphene



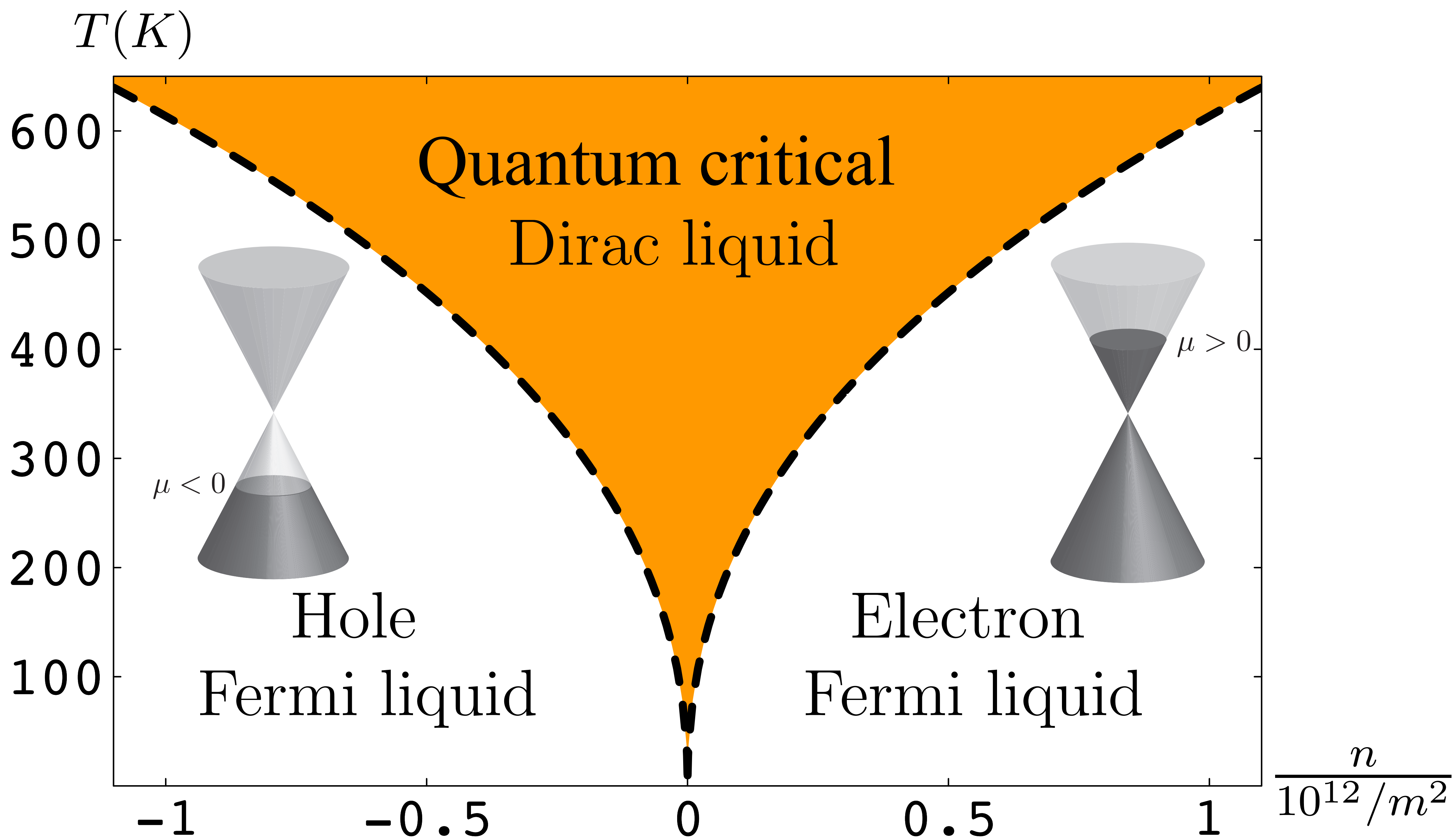


Electron
Fermi surface

Graphene

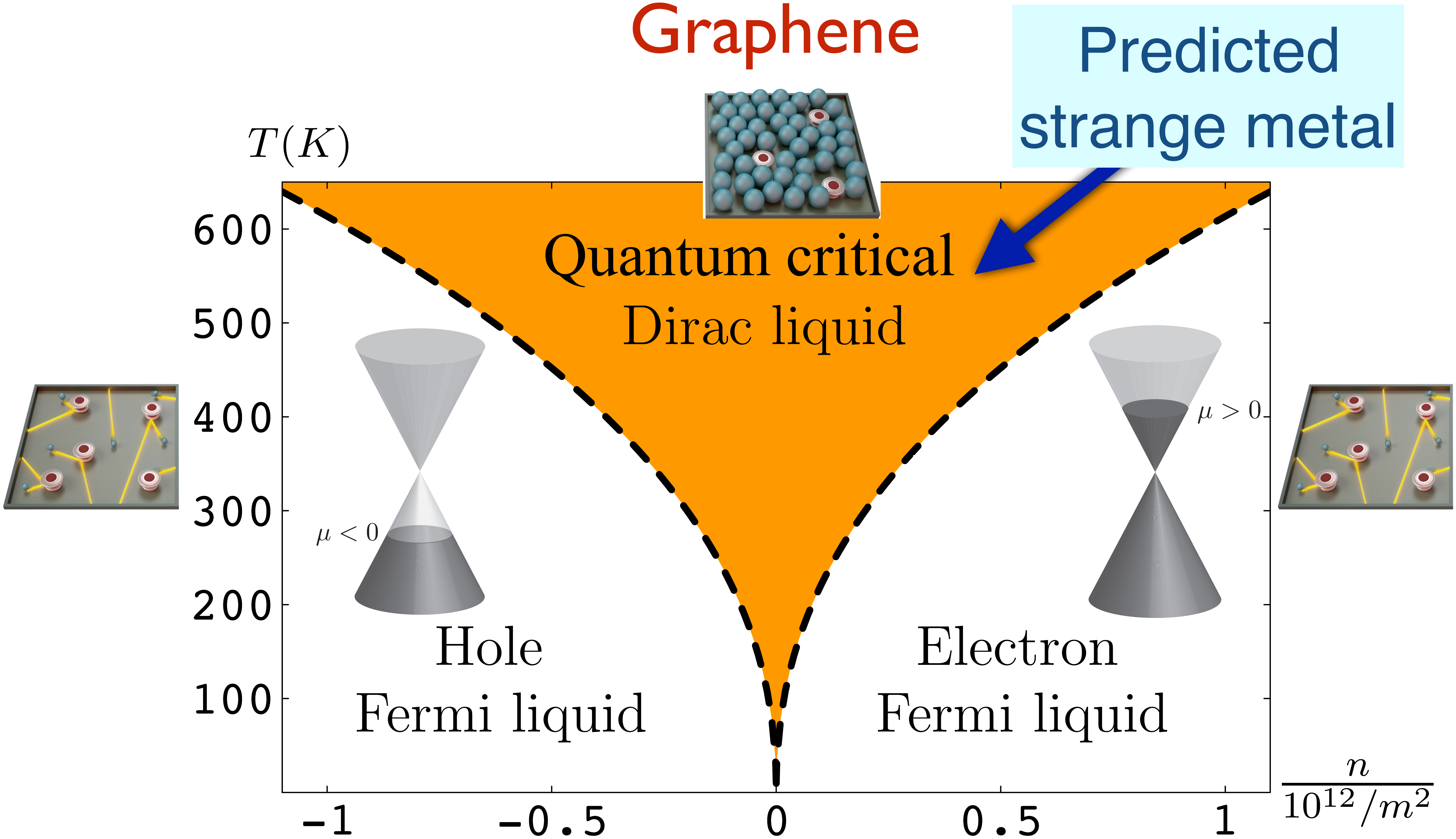


Graphene



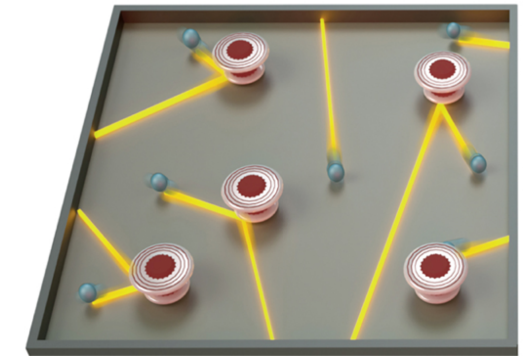
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Graphene



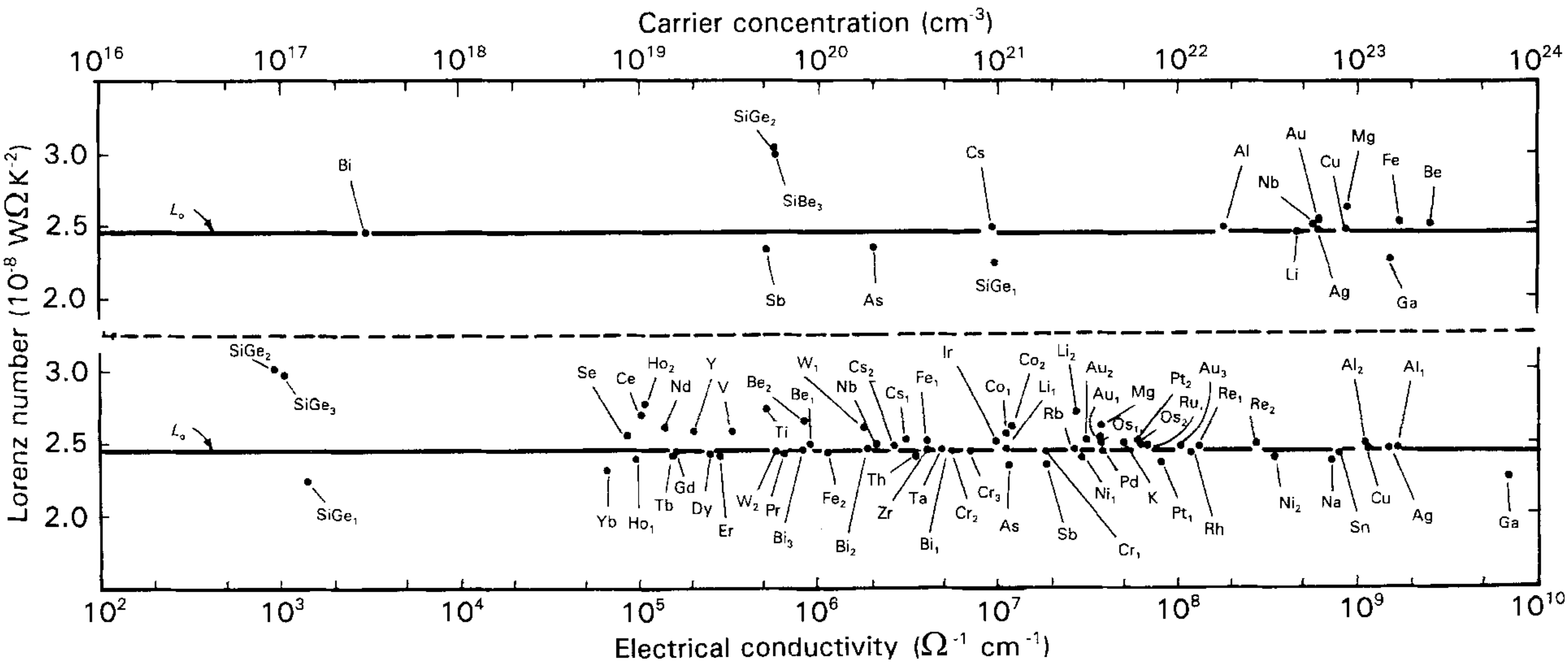
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Thermal and electrical conductivity with quasiparticles



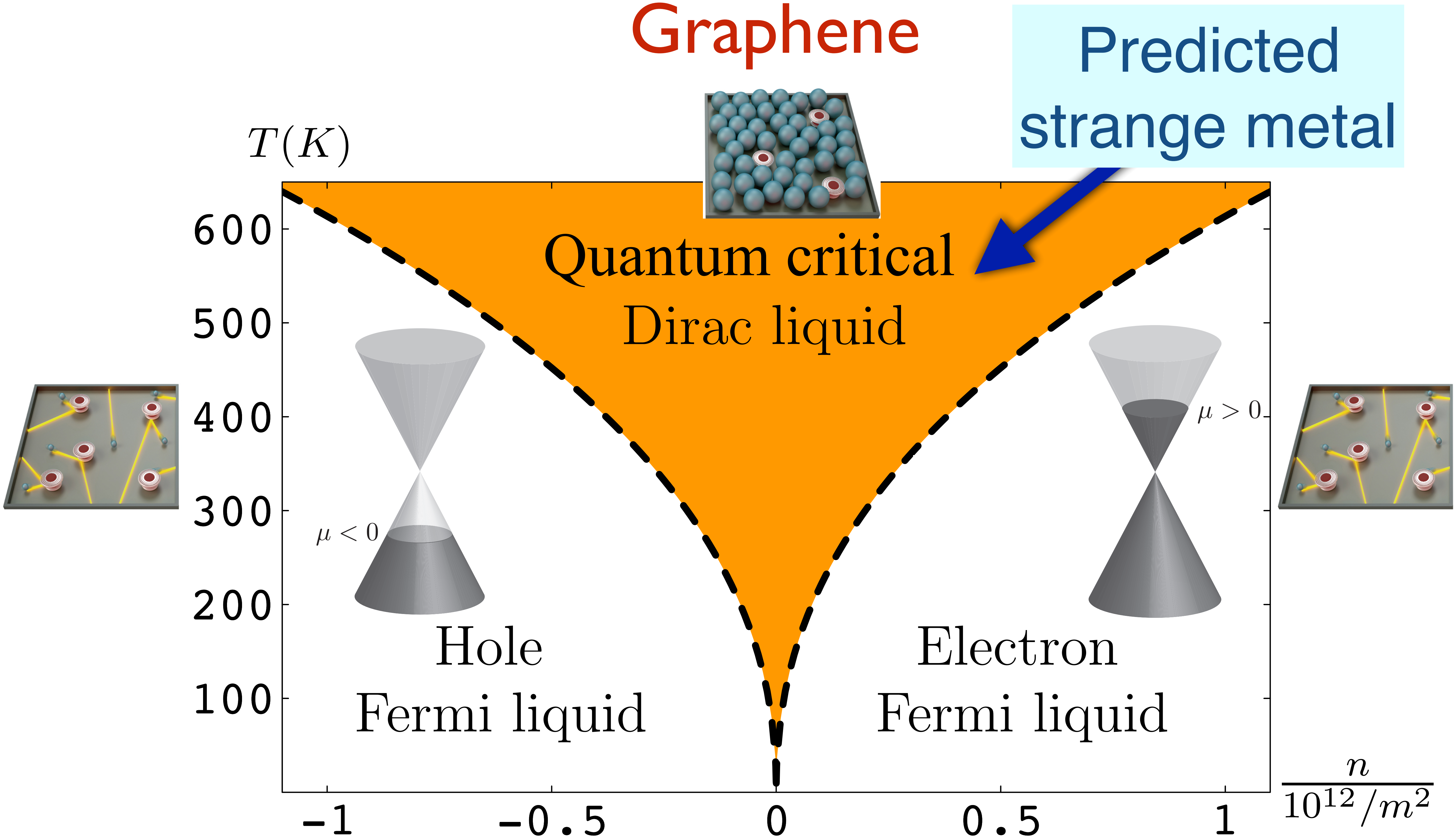
- Wiedemann-Franz law in a Fermi liquid:

$$L_0 = \frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8} \, \frac{\mathrm{W} \cdot \Omega}{\mathrm{K}^2}.$$



G. S. Kumar, G. Prasad, and R.O. Pohl, J. Mat. Sci. **28**, 4261 (1993)

Graphene



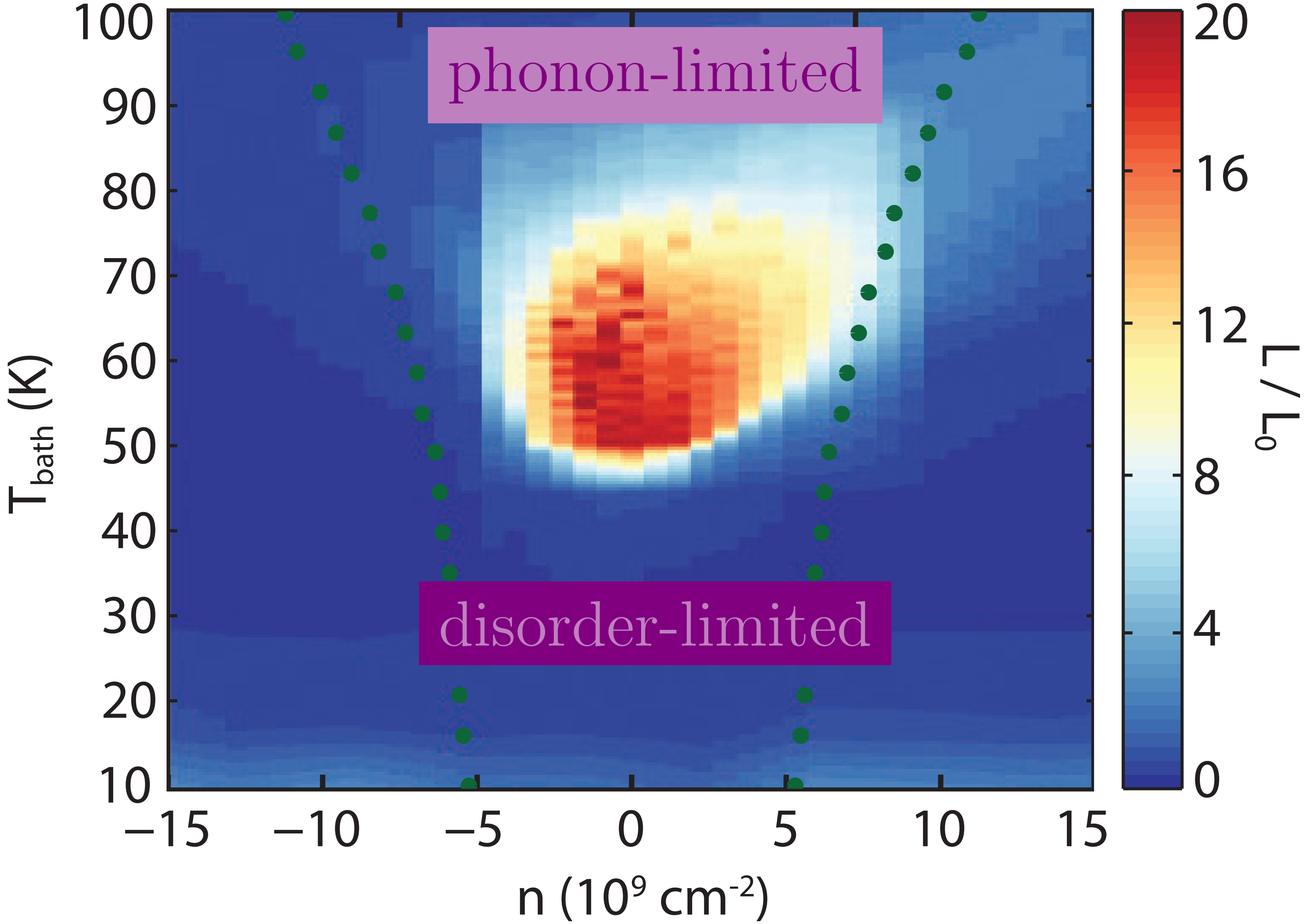
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

J. Crossno, Jing K. Shi, Ke Wang, Xiaomeng Liu, A. Harzheim, A. Lucas, S. Sachdev, Philip Kim, Takashi Taniguchi, Kenji Watanabe, T. A. Ohki, and Kin Chung Fong, Science **351**, 1058 (2016)

Strange metal in graphene

$$L = \frac{\kappa}{T\sigma}$$

$$L_0 = \frac{\pi^2 k_B^2}{3e^2}$$

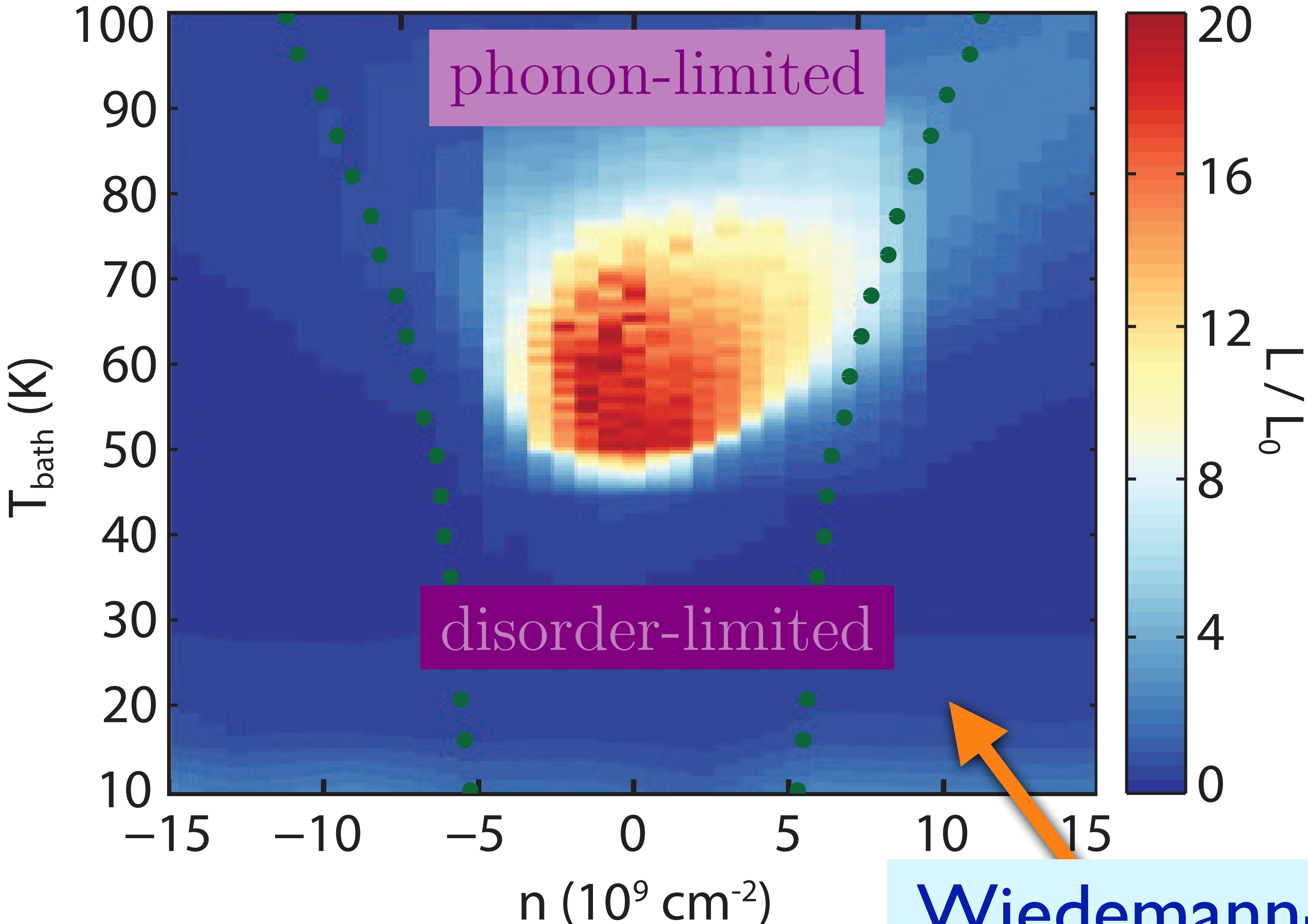


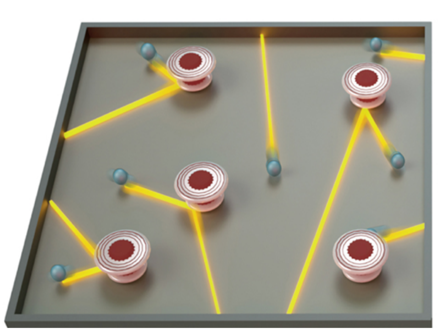
J. Crossno, Jing K. Shi, Ke Wang, Xiaomeng Liu, A. Harzheim, A. Lucas, S. Sachdev, Philip Kim, Takashi Taniguchi, Kenji Watanabe, T. A. Ohki, and Kin Chung Fong, Science **351**, 1058 (2016)

Strange metal in graphene

$$L = \frac{\kappa}{T\sigma}$$

$$L_0 = \frac{\pi^2 k_B^2}{3e^2}$$





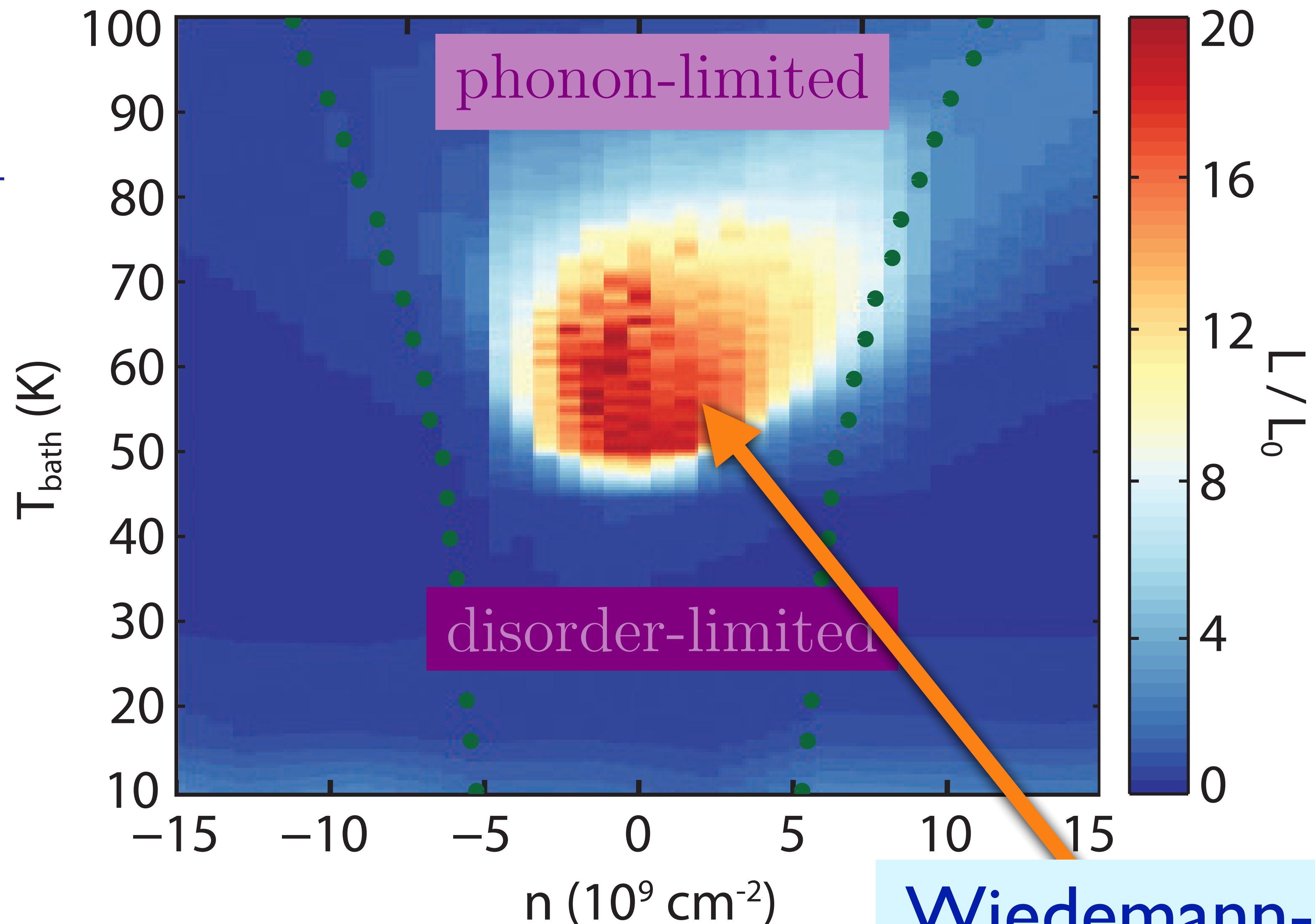
Wiedemann-Franz obeyed

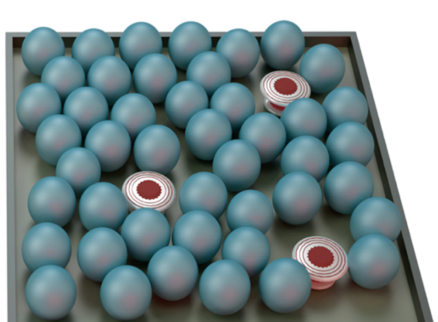
J. Crossno, Jing K. Shi, Ke Wang, Xiaomeng Liu, A. Harzheim, A. Lucas, S. Sachdev, Philip Kim, Takashi Taniguchi, Kenji Watanabe, T. A. Ohki, and Kin Chung Fong, Science **351**, 1058 (2016)

Strange metal in graphene

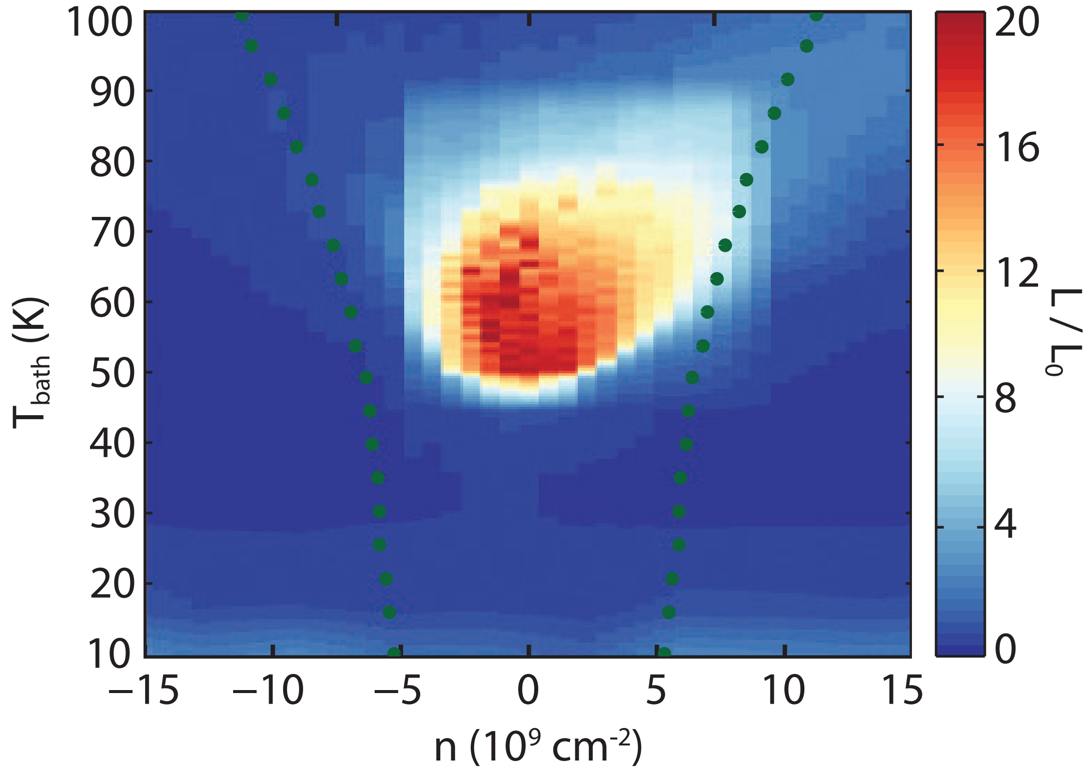
$$L = \frac{\kappa}{T\sigma}$$

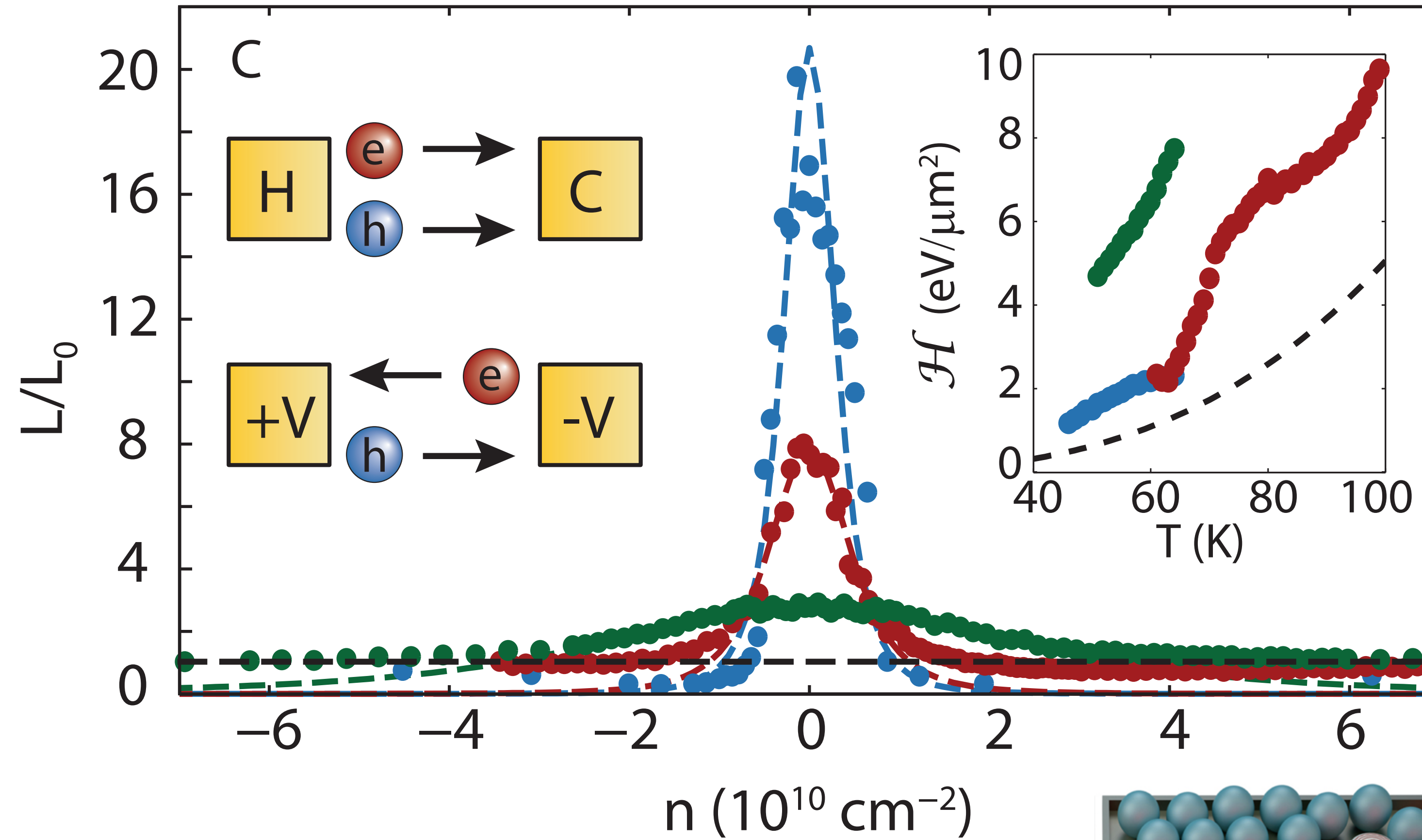
$$L_0 = \frac{\pi^2 k_B^2}{3e^2}$$

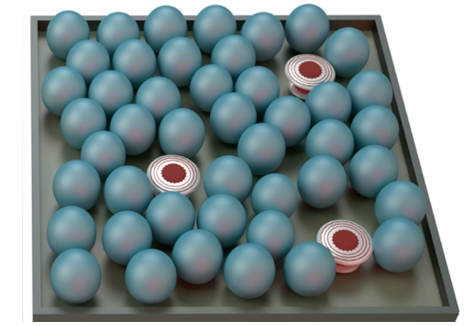


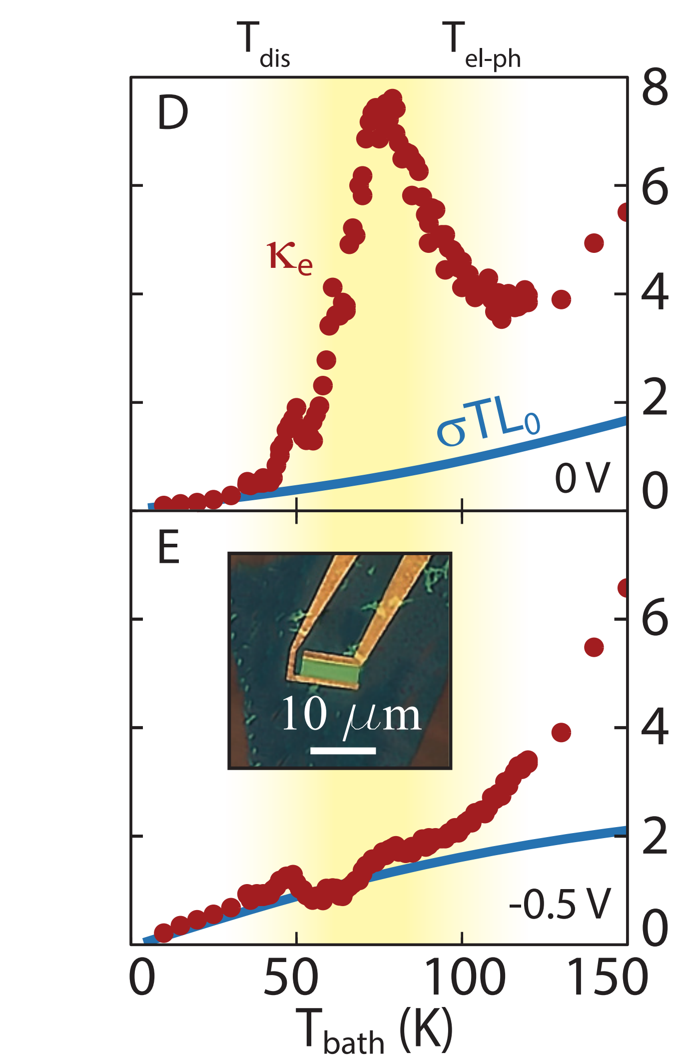


Wiedemann-Franz violated !









$$\text{Lorentz ratio } L = \kappa/(T\sigma)$$

$$= \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T^2 \sigma_Q} \frac{1}{\left(1 + e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}/(\mathcal{H}\sigma_Q)\right)^2}$$

$\mathcal{Q} \rightarrow$ electron density; $\mathcal{H} \rightarrow$ enthalpy density

$\sigma_Q \rightarrow$ quantum critical conductivity

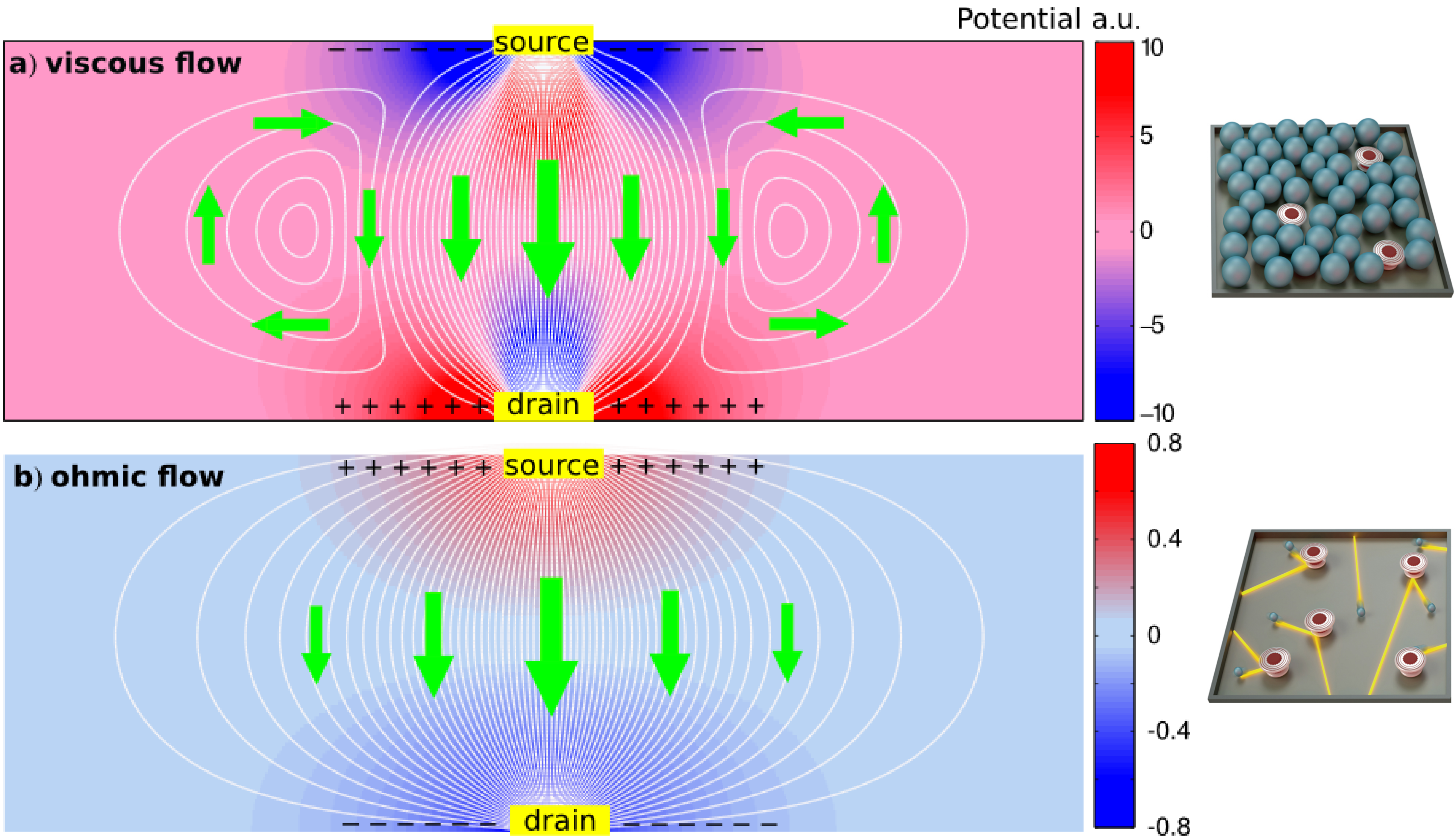
$\tau_{\rm imp} \rightarrow$ momentum relaxation time from impurities

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

J. Crossno et al., Science **351**, 1058 (2016)

Strange metal in graphene

Negative local resistance due to viscous electron backflow in graphene



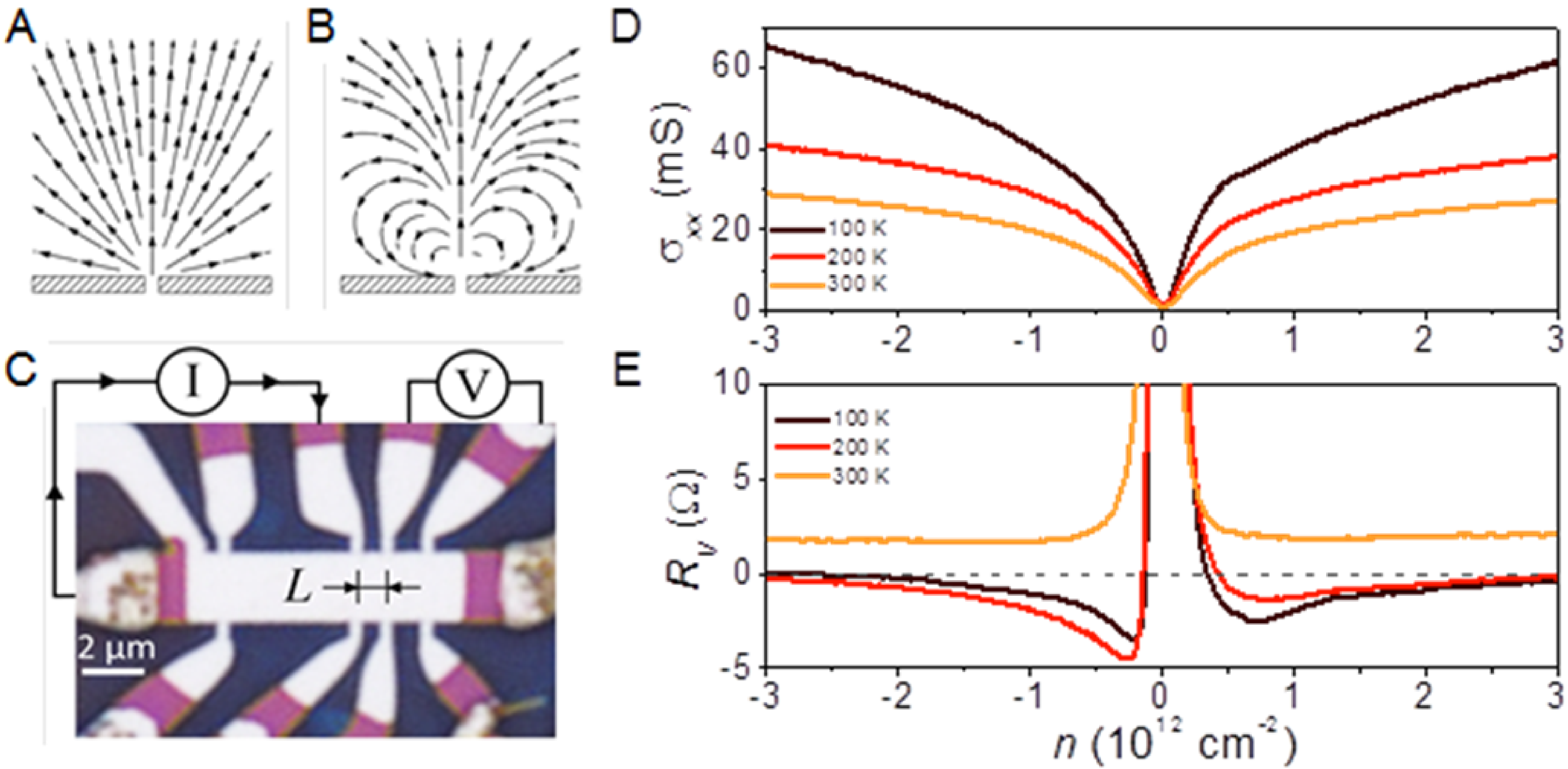
L. Levitov and G. Falkovich, arXiv:1508.00836, *Nature Physics online*

Science **351**, 1055 (2016)

Strange metal in graphene

Negative local resistance due to viscous electron backflow in graphene

D. A. Bandurin[1], I. Torre[2,3], R. Krishna Kumar[1,4], M. Ben Shalom[1,5], A. Tomadin[6], A. Principi[7], G. H. Auton[5], E. Khestanova[1,5], K. S. Novoselov[5], I. V. Grigorieva[1], L. A. Ponomarenko[1,4], A. K. Geim[1], M. Polini[3,6]



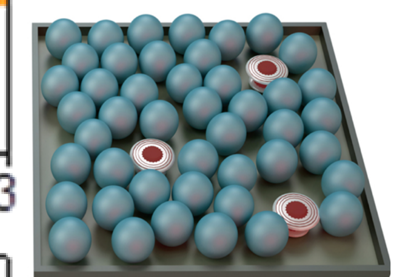
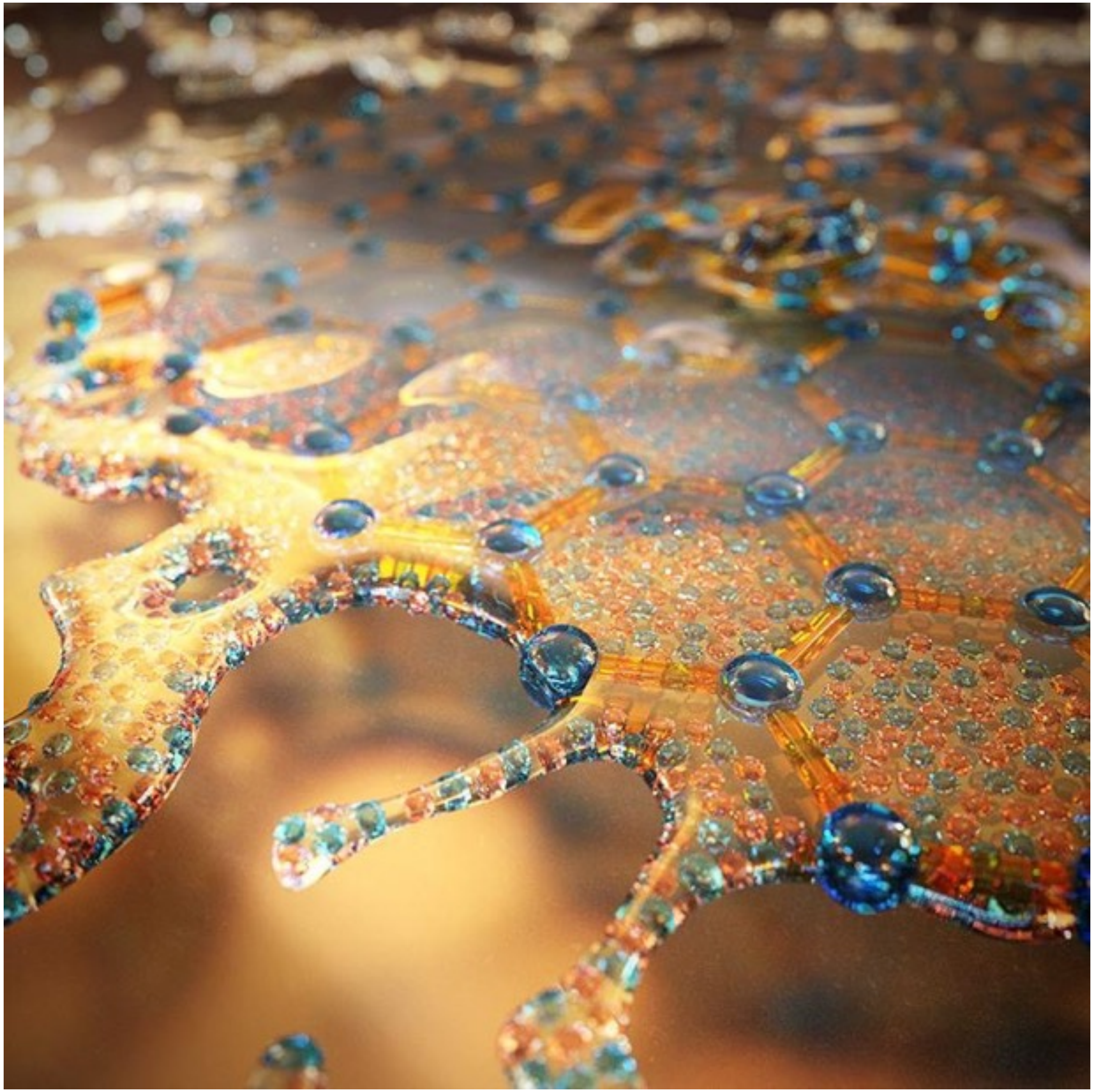
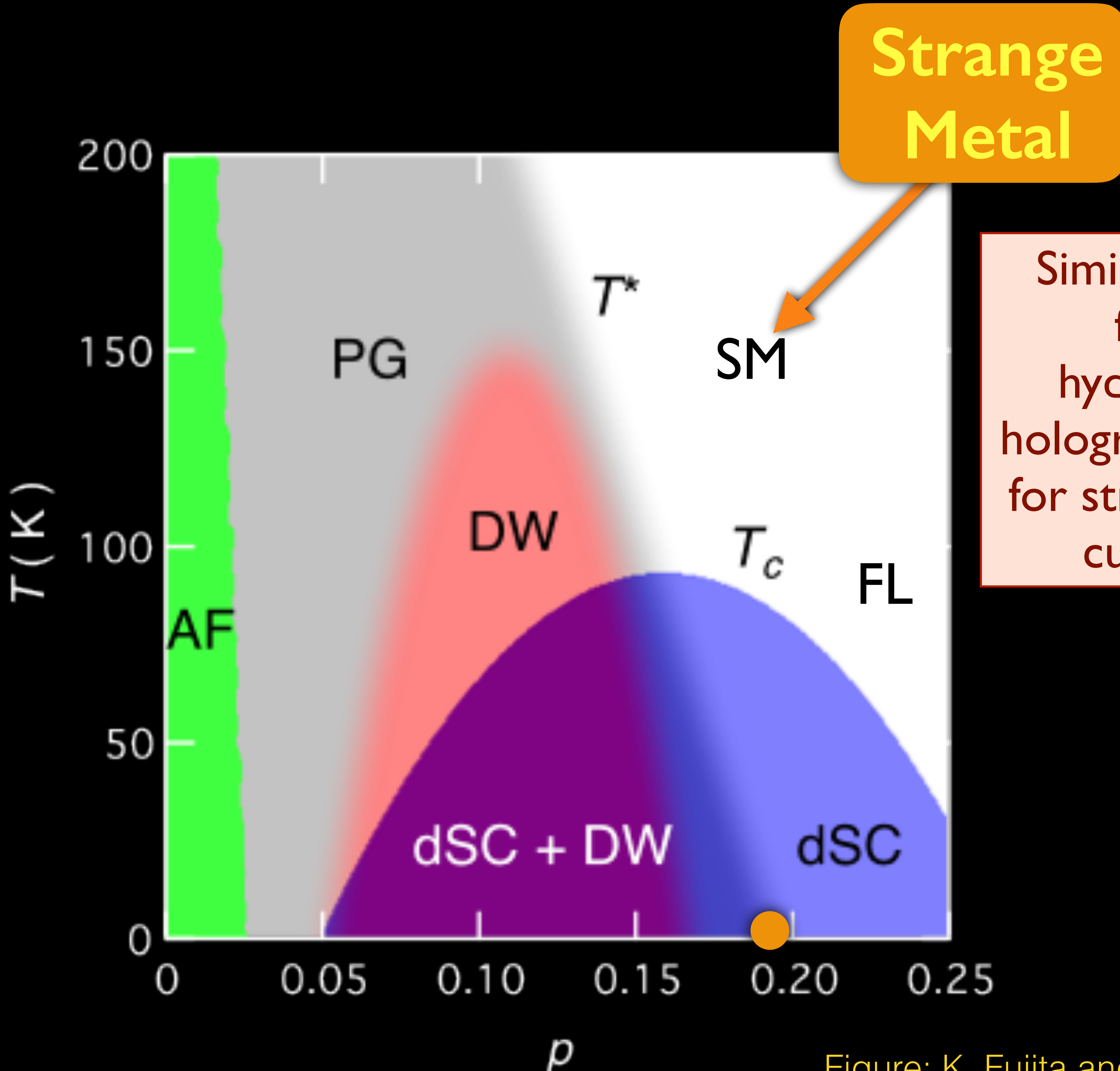


Figure 1. Viscous backflow in doped graphene. (**a**,**b**) Steady-state distribution of current injected through a narrow slit for a classical conducting medium with zero ν (a) and a viscous Fermi liquid (b). (**c**) Optical micrograph of one of our SLG devices. The schematic explains the measurement geometry for vicinity resistance. (**d**,**e**) Longitudinal conductivity σ_{xx} and R_{V} for this device as a function of n induced by applying gate voltage. $I = 0.3$ μA; $L = 1$ μm. For more detail, see Supplementary Information.

Graphene: "a metal that behaves like water"





Similar memory-function/ hydrodynamic/ holographic analyses for strange metal in cuprates…..

Figure: K. Fujita and J. C. Seamus Davis

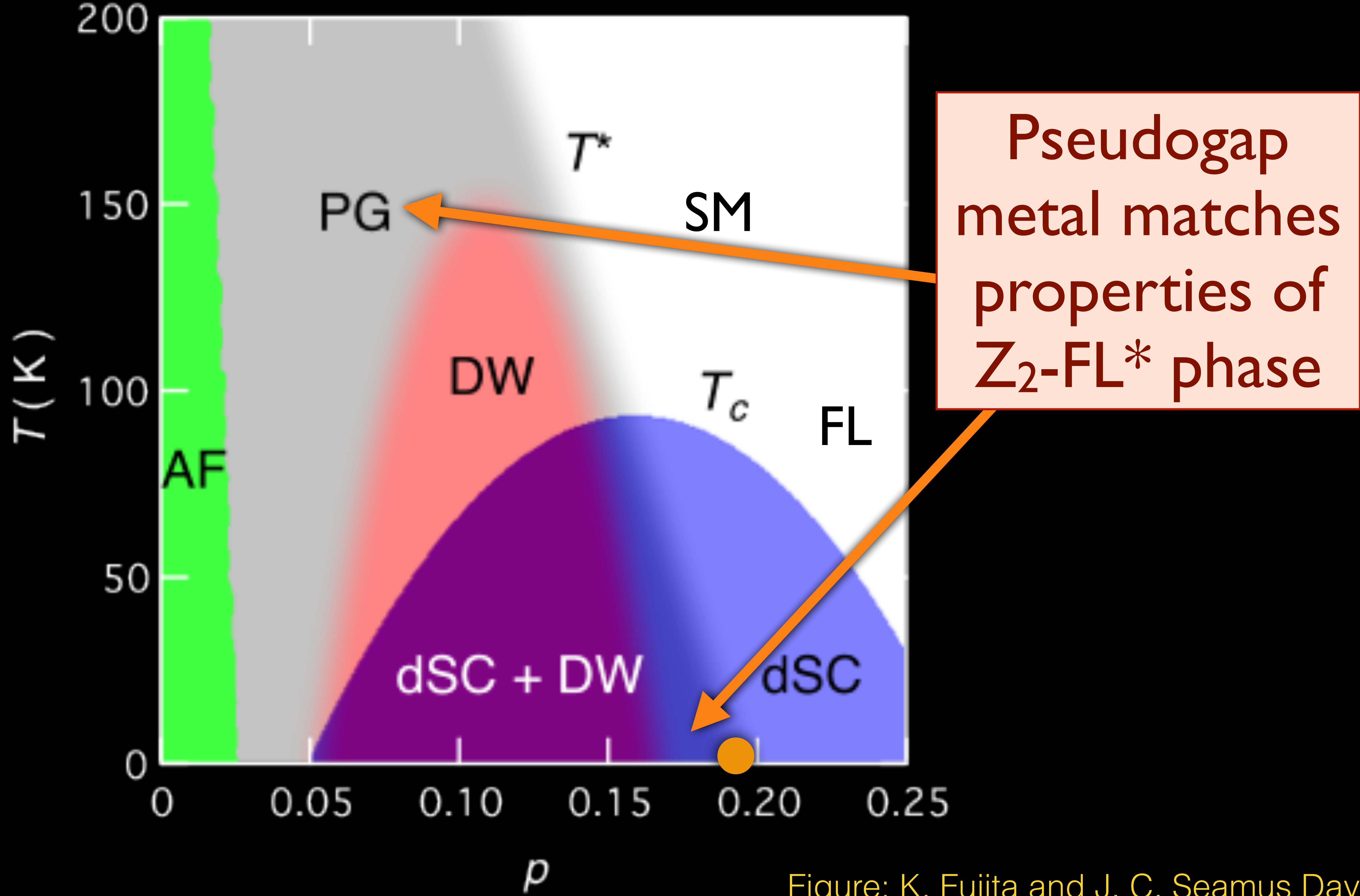


Figure: K. Fujita and J. C. Seamus Davis

FL*

We have described a metal with:

- A Fermi surface of electrons enclosing volume p, and not the Luttinger volume of $1+p$
- Additional low energy quantum states on a torus not associated with quasiparticle excitations *i.e.* emergent gauge fields

FL*

We have described a metal with:

- A Fermi surface of electrons enclosing volume p, and not the Luttinger volume of $1+p$
- Additional low energy quantum states on a torus not associated with quasiparticle excitations *i*.e. emergent gauge fields

There is a general and fundamental relationship between these two characteristics. Promising indications that such a metal describes the pseudogap of the cuprate supercondutors