Vortices and impurities in the cuprate superconductors

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Talk online at
http://pantheon.yale.edu/~subir
(Search for "*Sachdev*" on Google)

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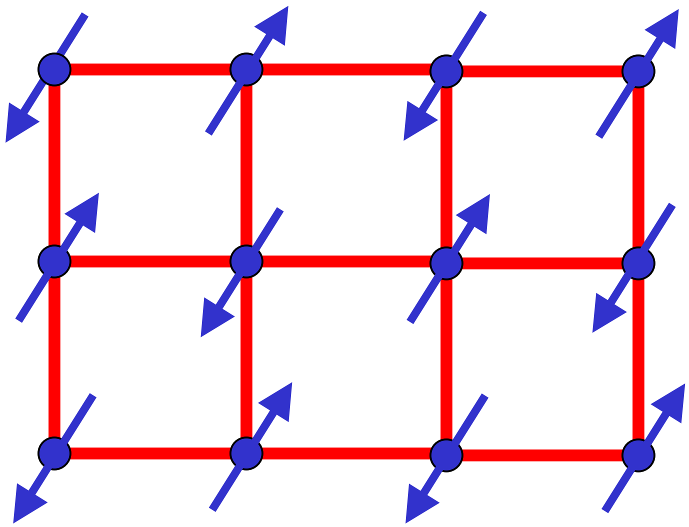
Parent compound of the high temperature superconductors: La_2CuO_4

Mott insulator: square lattice antiferromagnet



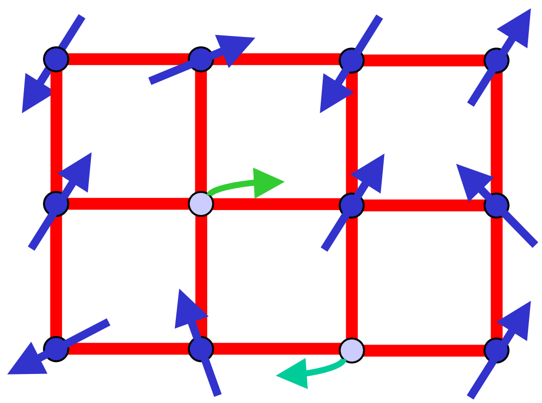
$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Ground state has long-range magnetic Néel order, or a "spin density wave (SDW)"

Néel order parameter: $\boldsymbol{n}_i \sim (-1)^{i_x + i_y} \vec{S}_i$

$$\langle \boldsymbol{n} \rangle \neq 0 \quad ; \quad \langle \vec{S}_i \rangle \neq 0$$

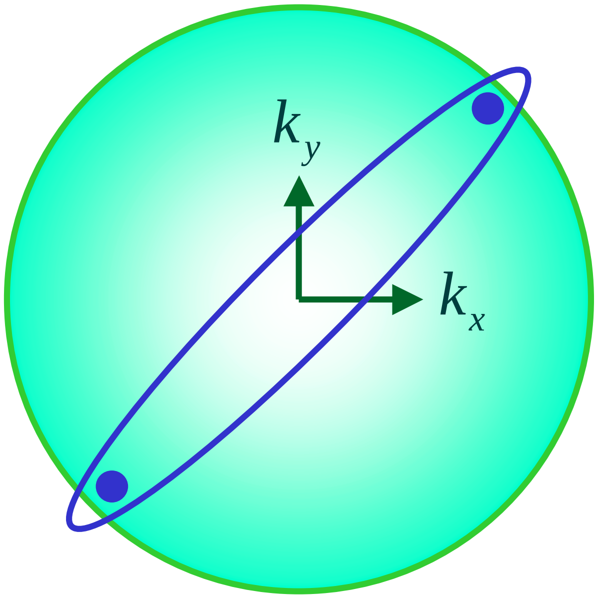
Introduce mobile carriers of density δ by substitutional doping of out-of-plane ions *e.g.* $La_{2-\delta}Sr_{\delta}CuO_4$



Exhibits superconductivity below a high critical temperature T_c

Superconductivity in a doped Mott insulator

BCS superconductor obtained by the Cooper instability of a metallic Fermi liquid



Pair wavefunction

$$\Psi = \left(k_x^2 - k_y^2\right)\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

$$\left\langle \vec{S} \right\rangle = 0$$

Observed ***low*** temperature properties of the cuprate superconductors appear to be qualitatively similar to those predicted by BCS theory.

Many experiments above T_c are not described quantitatively by BCS theory: this is probably due to strong-coupling "crossover" effects, and I will not discuss this issue further.

Superconductivity in a doped Mott insulator

$\Updownarrow$?

BCS superconductor obtained by the Cooper instability of a metallic Fermi liquid

Quantum numbers of ground state and low energy quasiparticles are the same, but characteristics of the Mott insulator are revealed near the vortices and near impurities.

S. Sachdev, *Phys. Rev*. B **45**, 389 (1992); K. Park and S. Sachdev *Phys. Rev*. B **64**, 184510 (2001).

Outline

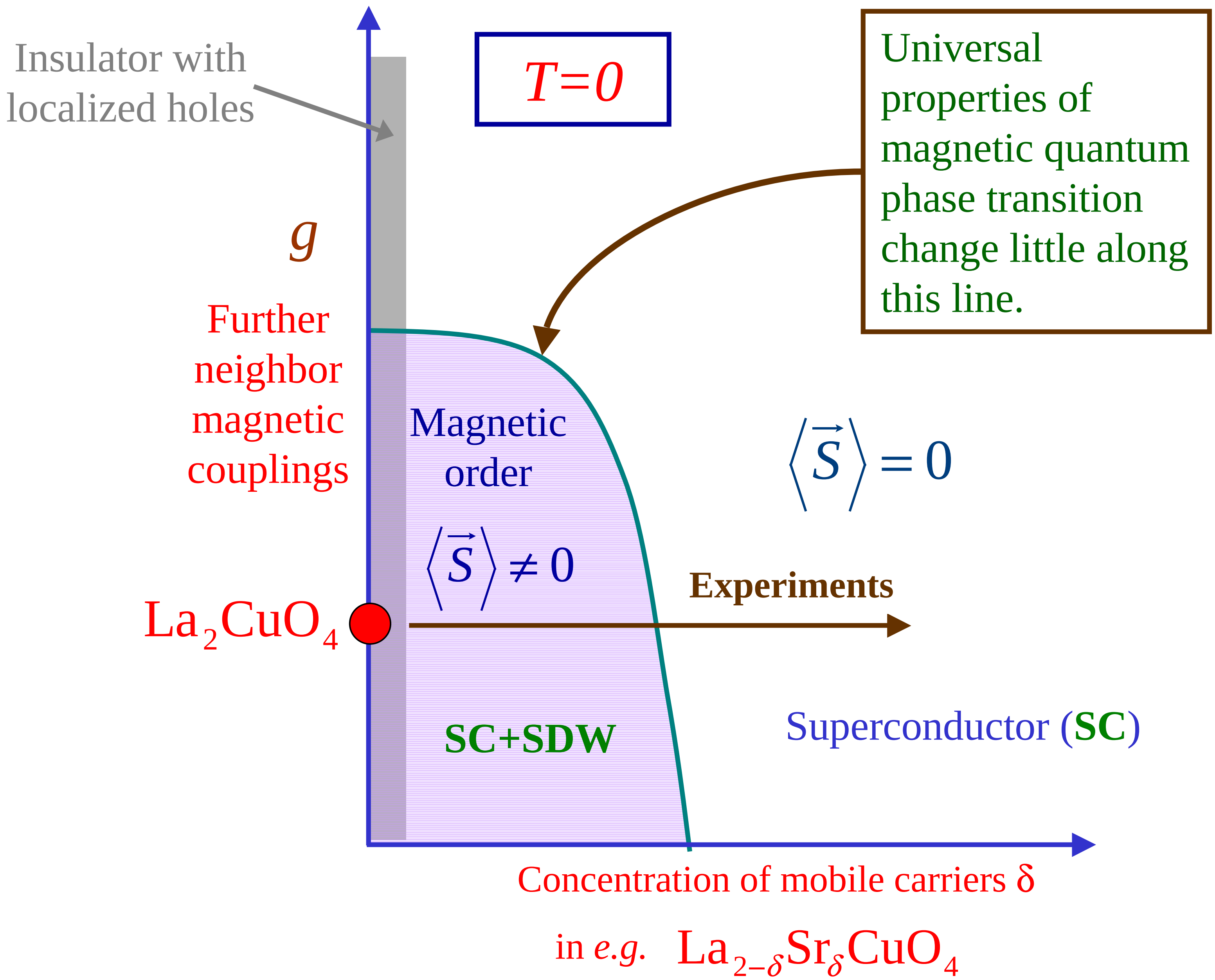
I. Mott insulators – with and without magnetic order.

II. Theory of doped paramagnetic Mott insulators.

III. Experiments on
   A. Vortices
   B. Zn/Li impurities
   C. Static charge order

IV. Competition between co-existing magnetism and superconductivity at low carrier concentrations: theory and neutron scattering experiments.

V. Conclusions



S. Sachdev and J. Ye, *Phys. Rev. Lett*. **69**, 2411 (1992).
A.V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev*. B **49**, 11919 (1994)

I. Magnetic ordering transitions in the insulator

$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$

Action for quantum spin fluctuations in spacetime

Discretize spacetime into a cubic lattice with Néel order orientation $\boldsymbol{n}_a$

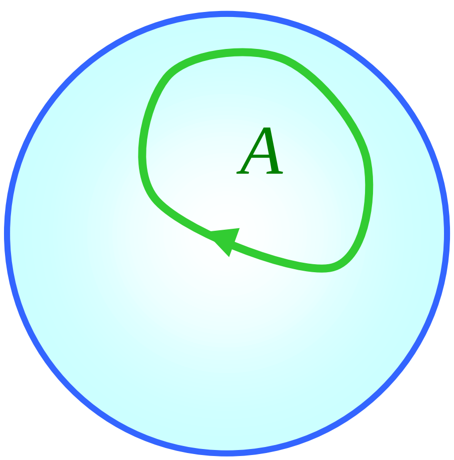
$$Z = \prod_a \int d\boldsymbol{n}_a \delta\left(\boldsymbol{n}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \boldsymbol{n}_a \cdot \boldsymbol{n}_{a+\mu}\right)$$

$a \rightarrow$ cubic lattice sites; $\mu \rightarrow x, y, \tau$;

S. Chakravarty, B.I. Halperin, and D.R. Nelson, *Phys. Rev.* B **39**, 2344 (1989).

Quantum path integral for two-dimensional quantum antiferromagnet
$\Leftrightarrow$ Partition function of a classical three-dimensional ferromagnet at a "temperature" g

Missing: Spin Berry Phases



$$e^{iSA}$$

Berry phases profoundly modify paramagnetic states with $\langle \boldsymbol{n}_a \rangle = \langle \vec{S} \rangle = 0$

Computations with Berry phases fully reproduce known states in one dimension: Bethe and Majumdar-Ghosh states for S=1/2, and Haldane states for S=1

K. Park and S. Sachdev, cond-mat/0108214

Field theory of paramagnetic ("quantum disordered") phase

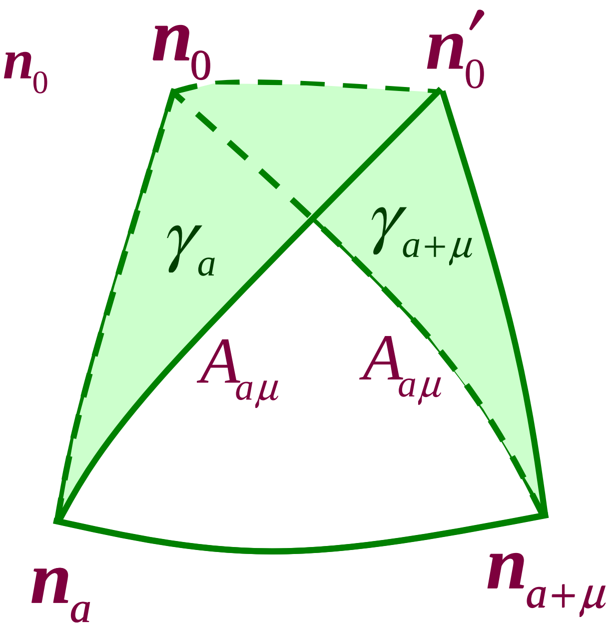
Discretize spacetime into a cubic lattice:

$$Z = \prod_a \int d\boldsymbol{n}_a \delta\left(\boldsymbol{n}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \boldsymbol{n}_a \cdot \boldsymbol{n}_{a+\mu} - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

$\eta_a \rightarrow \pm 1$ on two square sublattices ; $\boldsymbol{n}_a \sim \eta_a \vec{S}_a \rightarrow$ Neel order parameter;

$A_{a\mu} \rightarrow$ oriented area of spherical triangle

formed by $\boldsymbol{n}_a$, $\boldsymbol{n}_{a+\mu}$, and an arbitrary reference point $\boldsymbol{n}_0$



Change in choice of $\boldsymbol{n}_0$ is like a "gauge transformation"

$$A_{a\mu} \rightarrow A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

(γ_a is the oriented area of the spherical triangle formed by $\boldsymbol{n}_a$ and the two choices for $\boldsymbol{n}_0$).

The area of the triangle is uncertain modulo 4π, and the action is invariant under

$$A_{a\mu} \rightarrow A_{a\mu} + 4\pi$$

These principles strongly constrain the effective action for $A_{a\mu}$

Simplest large g effective action for the $A_{a\mu}$

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp\left(-\frac{1}{2e^2} \sum_{\square} \cos\left(\frac{1}{2} \varepsilon_{\mu\nu\lambda} \Delta_\nu A_{a\lambda} \right) - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

This is compact QED in 2+1 dimensions with Berry phases.

This theory can be reliably analyzed by a duality mapping.

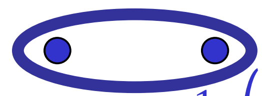
The gauge theory is *always* in a *confining* phase:

There is an energy gap and the ground state has **spontaneous bond order**.

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).
S. Sachdev and R. Jalabert, *Mod. Phys. Lett.* B **4**, 1043 (1990).
K. Park and S. Sachdev, cond-mat/0108214

Square lattice with first(J_1) and second (J_2) neighbor exchange interactions (say)

$$H = \sum_{i<j} J_{ij} \, \vec{S}_i \cdot \vec{S}_j$$



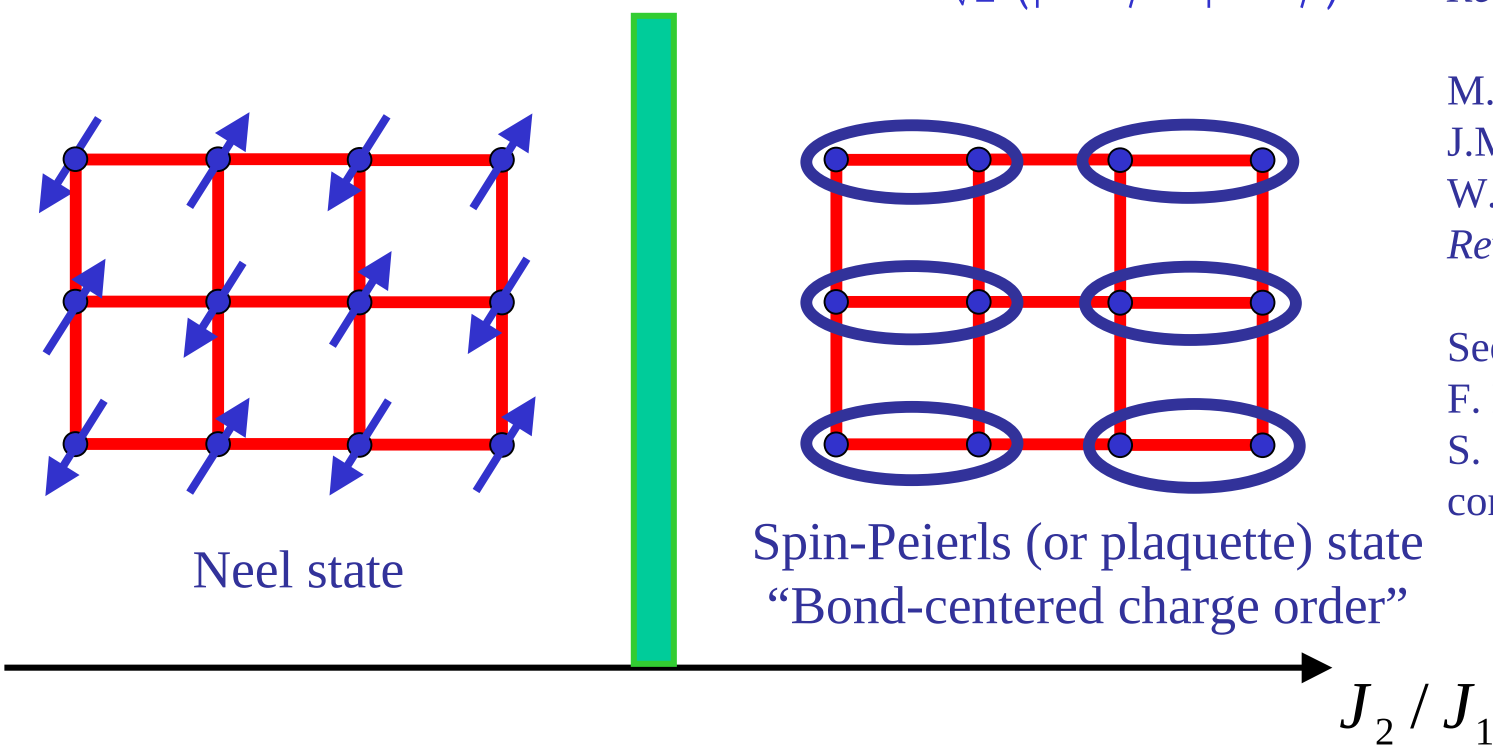
$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$$

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

O. P. Sushkov, J. Oitmaa, and Z. Weihong, *Phys. Rev.* B **63**, 104420 (2001).

M.S.L. du Croo de Jongh, J.M.J. van Leeuwen, W. van Saarloos, *Phys. Rev.* B **62**, 14844 (2000).

See however L. Capriotti, F. Becca, A. Parola, S. Sorella, cond-mat/0107204 .



Studies on the 2D pyrochlore lattice agree with related predictions of theory:

J.-B. Fouet, M. Mambrini, P. Sindzingre, C. Lhuillier, cond-mat/0108070.

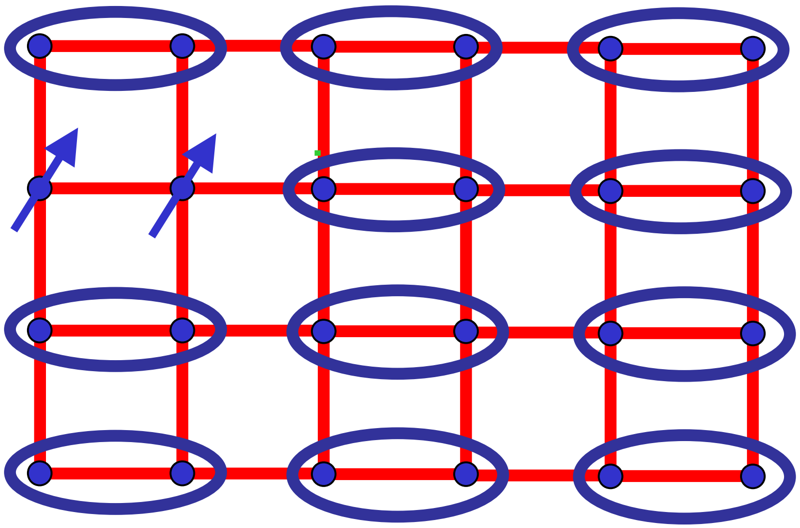
R. Moessner, Oleg Tchernyshyov, S.L. Sondhi, cond-mat/0106286.

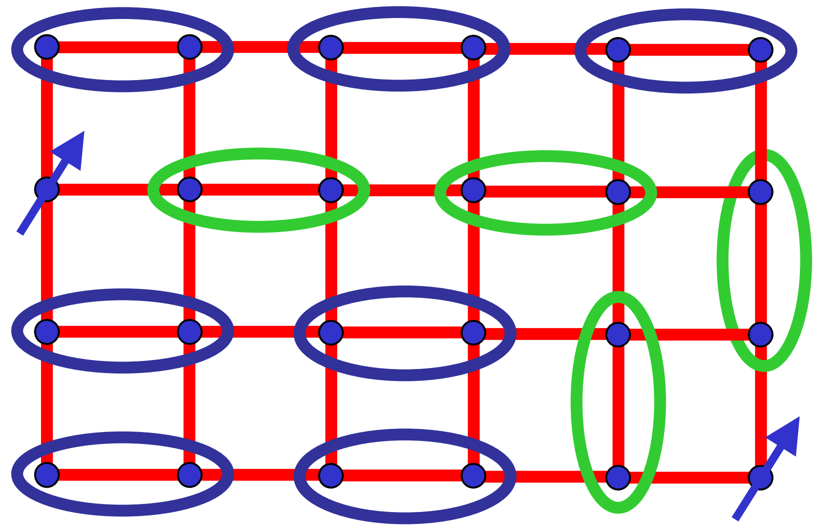
Properties of paramagnet with bond-charge-order

Stable S=1 spin exciton

$$\varepsilon_k = \Delta + \frac{c_x^2 k_x^2 + c_y^2 k_y^2}{2\Delta}$$

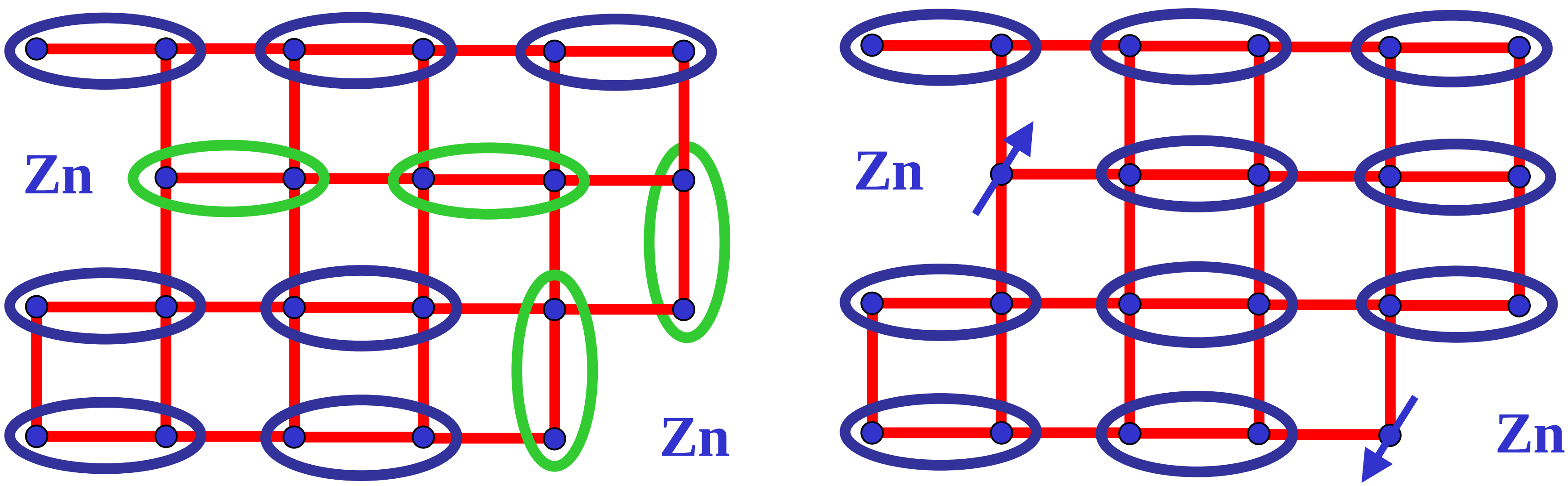
$\Delta \longrightarrow$ Spin gap





S=1/2 spinons are ***confined*** by a linear potential.

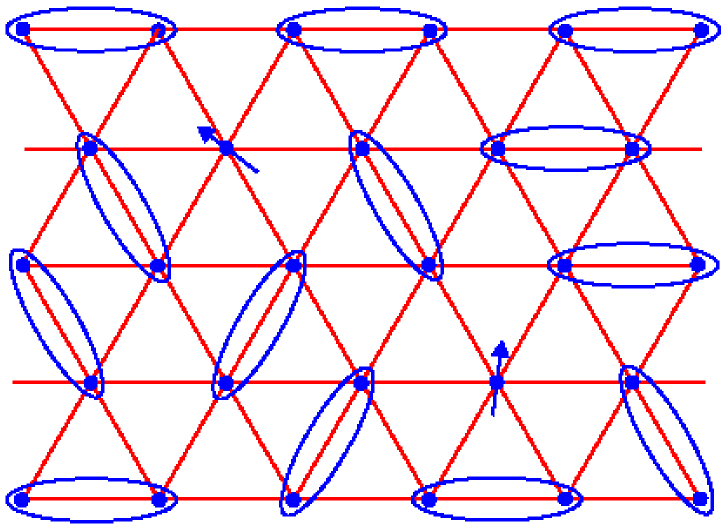
Effect of static non-magnetic impurities (Zn or Li)



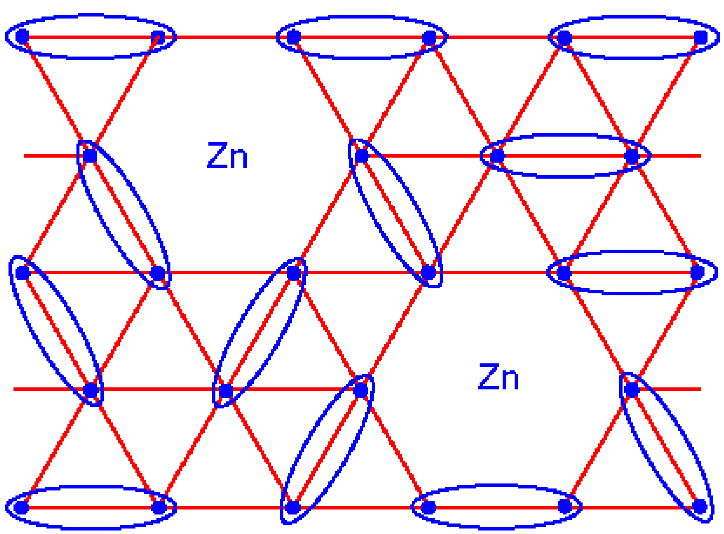
Spinon confinement implies that free S=1/2 moments form near each impurity

$$\chi_{\text{impurity}}(T \to 0) = \frac{S(S+1)}{3k_B T}$$

Impurities in paramagnets with spinon deconfinement



Spinons are deconfined



Free S=1/2 moments need not be present near the impurities

"Spin Liquid" or "RVB"

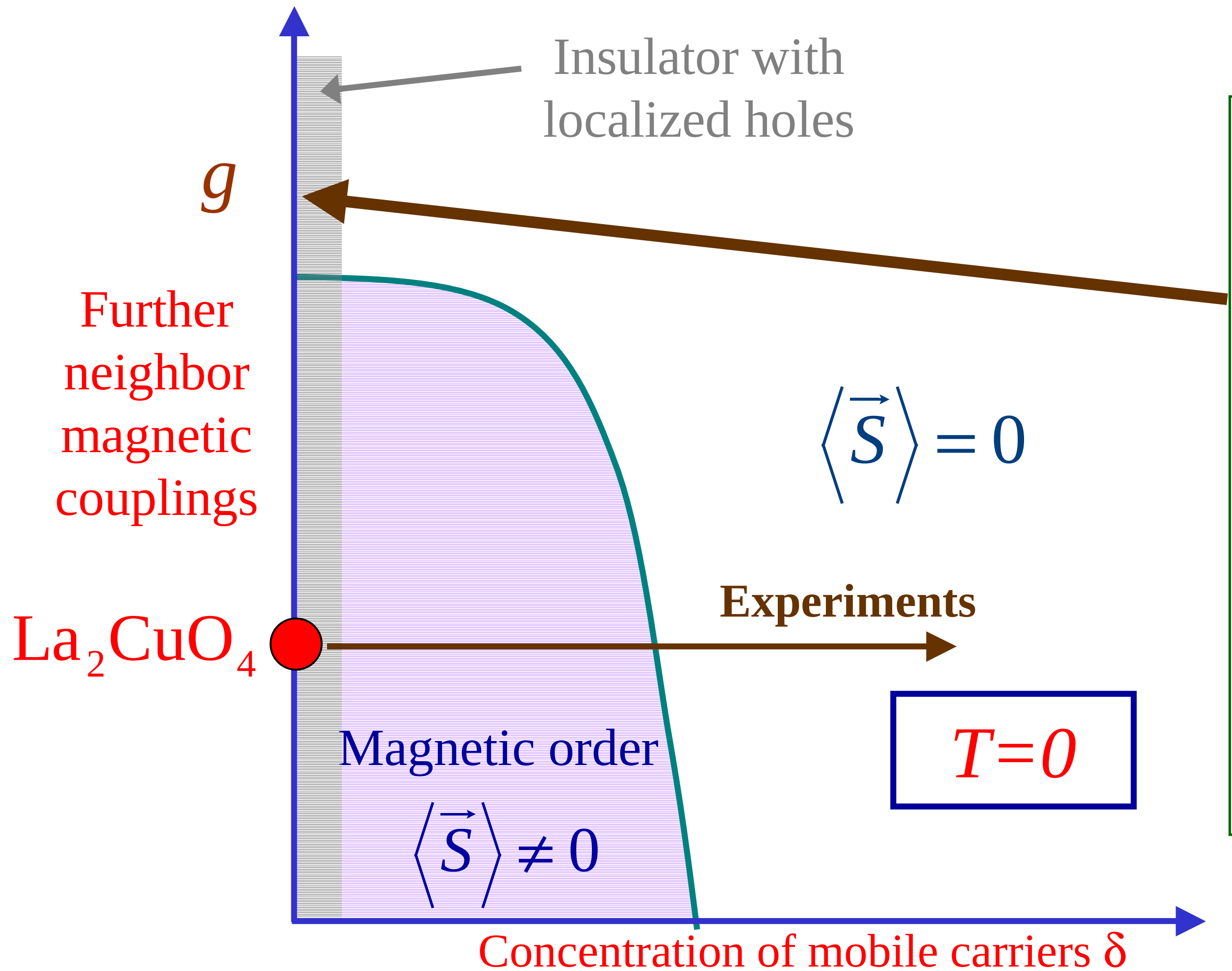
P. Fazekas and P.W. Anderson, *Phil Mag* **30**, 23 (1974).

S. Sachdev, *Phys. Rev*. B **45**, 12377 (1992).

G. Misguich and C. Lhuillier, cond-mat/0002170.

R. Moessner and S.L. Sondhi, *Phys. Rev. Lett.* **86**, 1881 (2001).

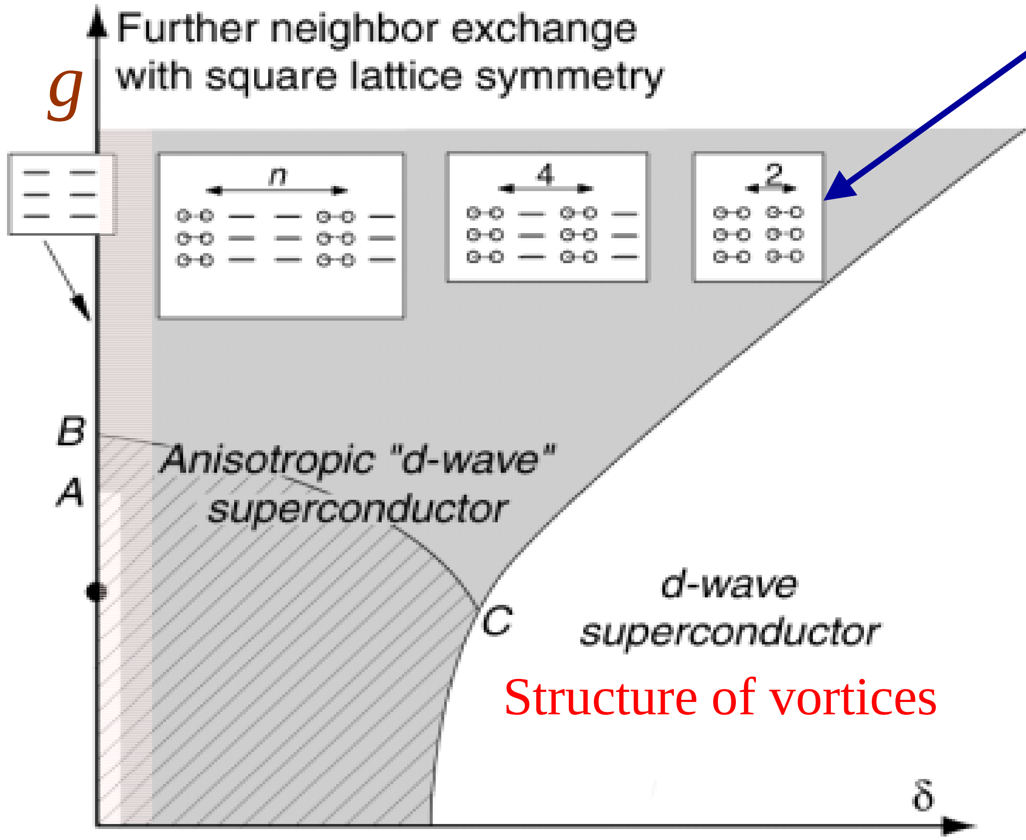
Framework for spin/charge order in cuprate superconductors



Confined, paramagnetic Mott insulator has

1. Stable *S*=1 spin exciton.
2. Broken translational symmetry:- bond-centered charge order.
3. *S*=1/2 moments near non-magnetic impurities

II. Doping the Mott insulator

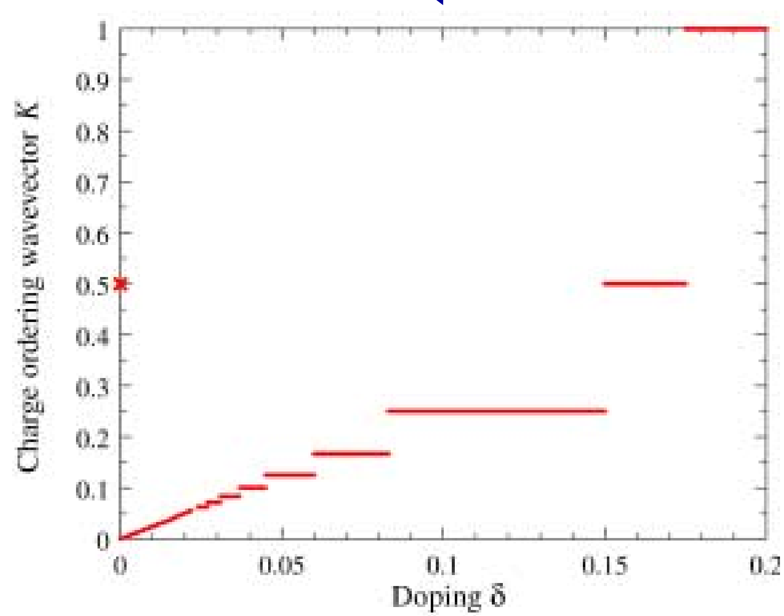


Structure of vortices

Hatched region --- spin order
Shaded region ---- charge order

"Large *N*" theory in region with preserved spin rotation symmetry

S. Sachdev and N. Read, *Int. J. Mod. Phys*. B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett*. **83**, 3916 (1999).
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev*. B **62**, 6721 (2000).



See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. Kivelson, E. Fradkin and V. Emery, *Nature* **393**, 550 (1998),
S. White and D. Scalapino, *Phys. Rev. Lett*. **80**, 1272 (1998).

III.A Charge order nucleated by vortices

Memory of the Mott insulator should survive in and around vortices in superconducting order: superconductivity is suppressed in the vortex core, but the electrons should still strive to retain the exchange correlation energy of the Mott insulator. The vortex core is *not* a "normal Fermi liquid" as in BCS theory. This is the primary failure of BCS theory in the cuprate superconductors.

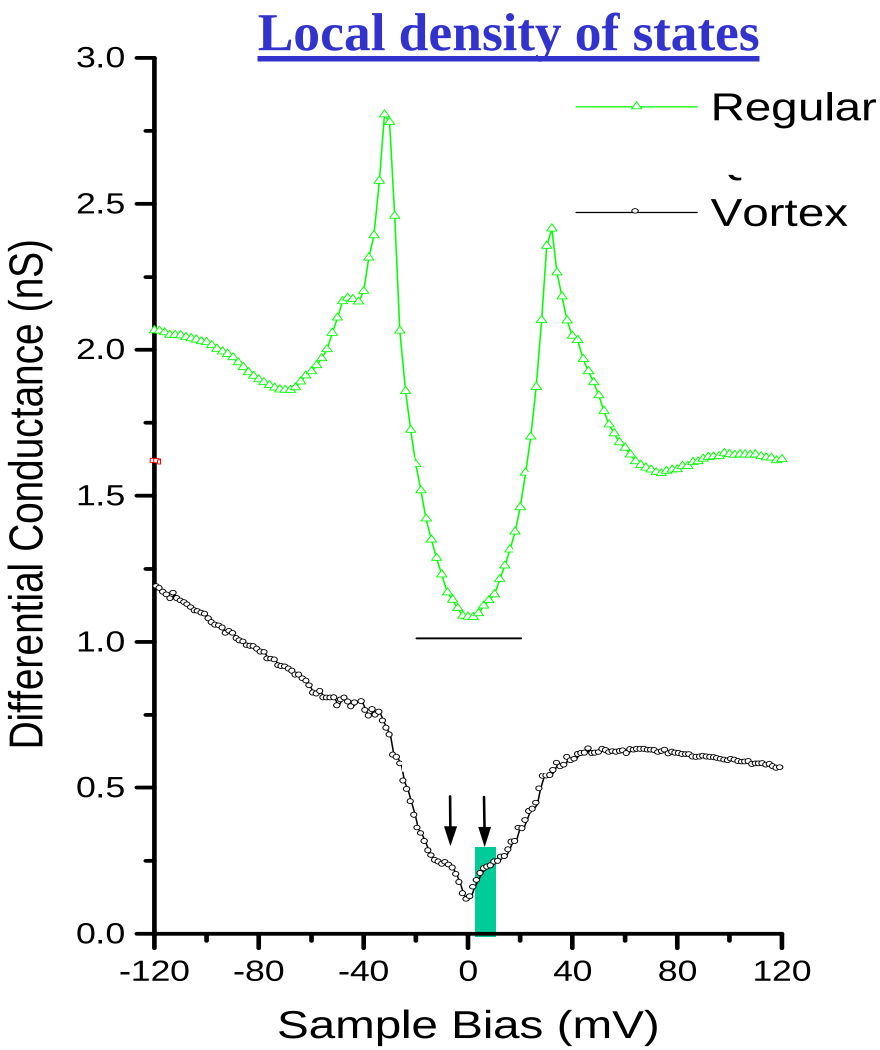
S. Sachdev, *Phys. Rev.* B **45**, 389 (1992);
N. Nagaosa and P.A. Lee, *Phys. Rev.* B **45**, 966 (1992);
D.P. Arovas, A. J. Berlinsky, C. Kallin, and S.-C. Zhang *Phys. Rev. Lett.* **79**, 2871 (1997).
J. H. Han and D. H. Lee, *Phys. Rev. Lett.* **85**, 1100 (2000).
M. Franz and Z. Tesanovic, *Phys. Rev.* B **63**, 064516 (2001);
J. I. Kishine, P.A. Lee, and X.-G. Wen, *Phys. Rev. Lett.* **86**, 5365 (2001).

Local magnetic order in the vortex core is "quantum-disordered": so there is a spin gap and charge order should appear, as in the doped paramagnetic Mott insulator.

K. Park and S. Sachdev, *Phys. Rev.* B **64**, 184510 (2001).

III.A STM around vortices induced by a magnetic field in the superconducting state

J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).

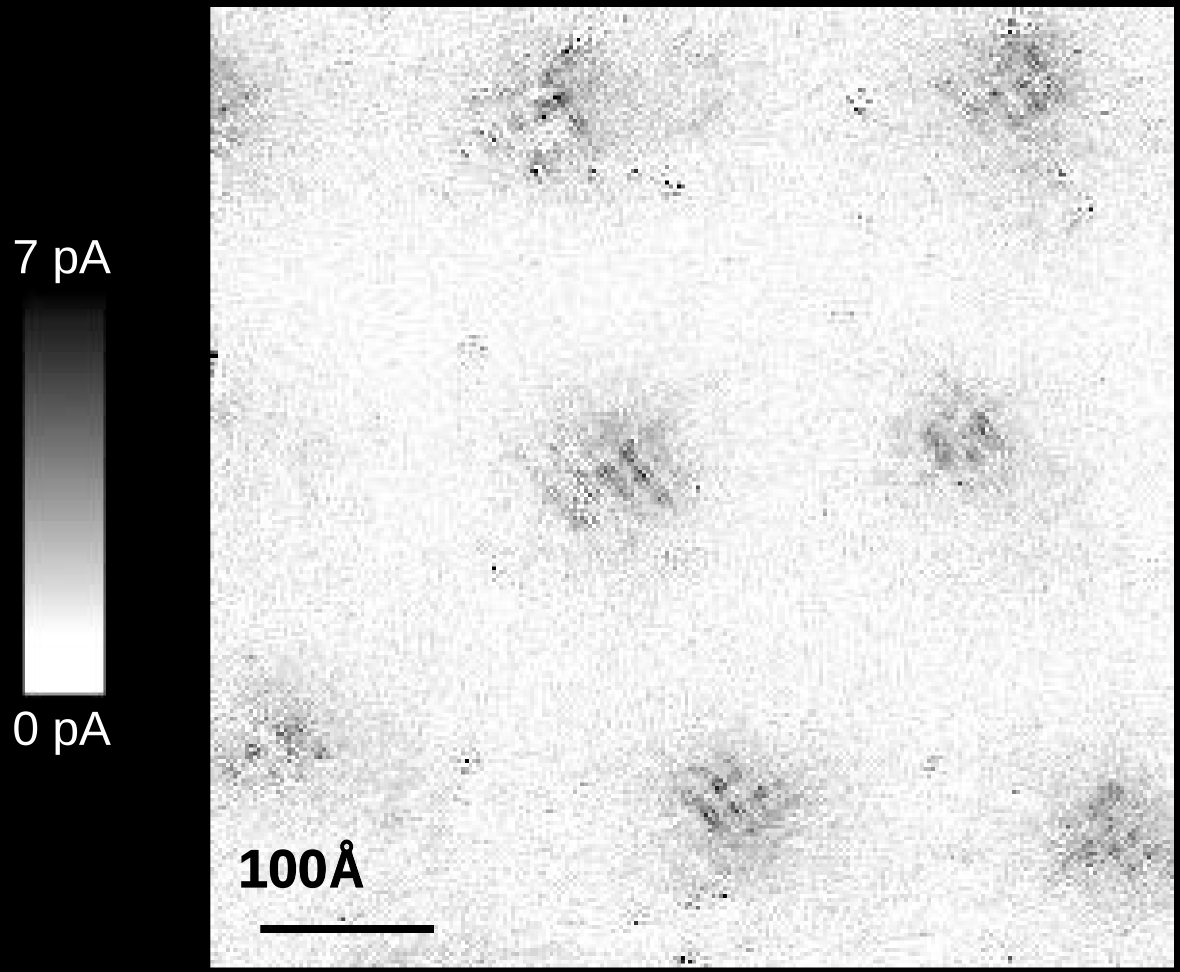


S.H. Pan *et al*. *Phys. Rev. Lett*. **85**, 1536 (2000).

1Å spatial resolution image of integrated LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$

(1meV to 12 meV) at B=5 Tesla.

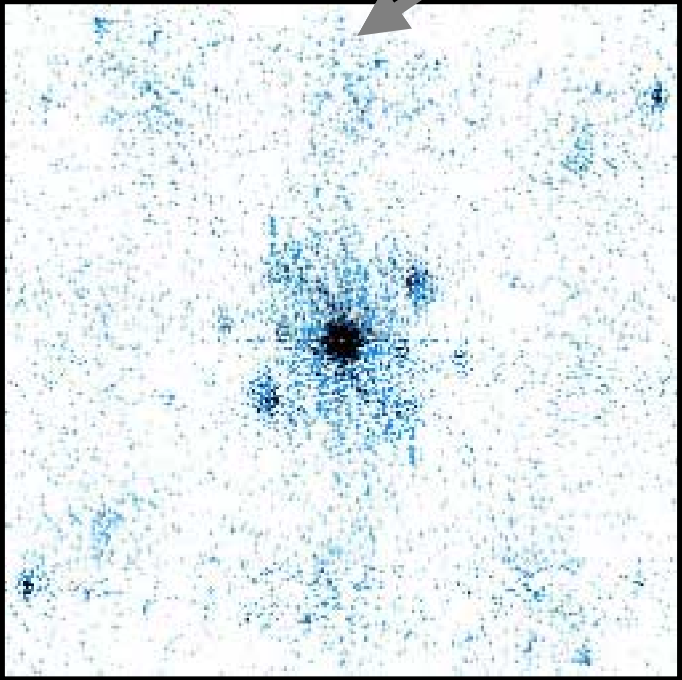
Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV

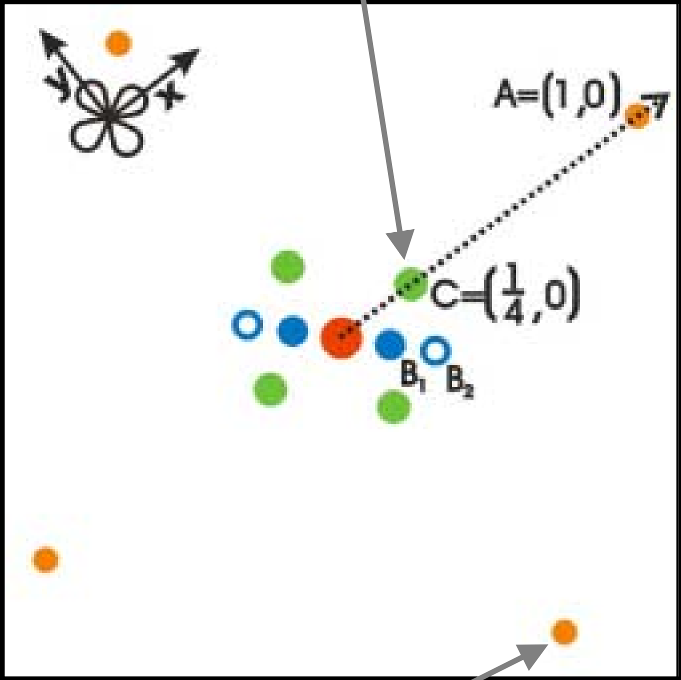


J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Fourier Transform of Vortex-Induced LDOS map

K-space locations of vortex induced LDOS



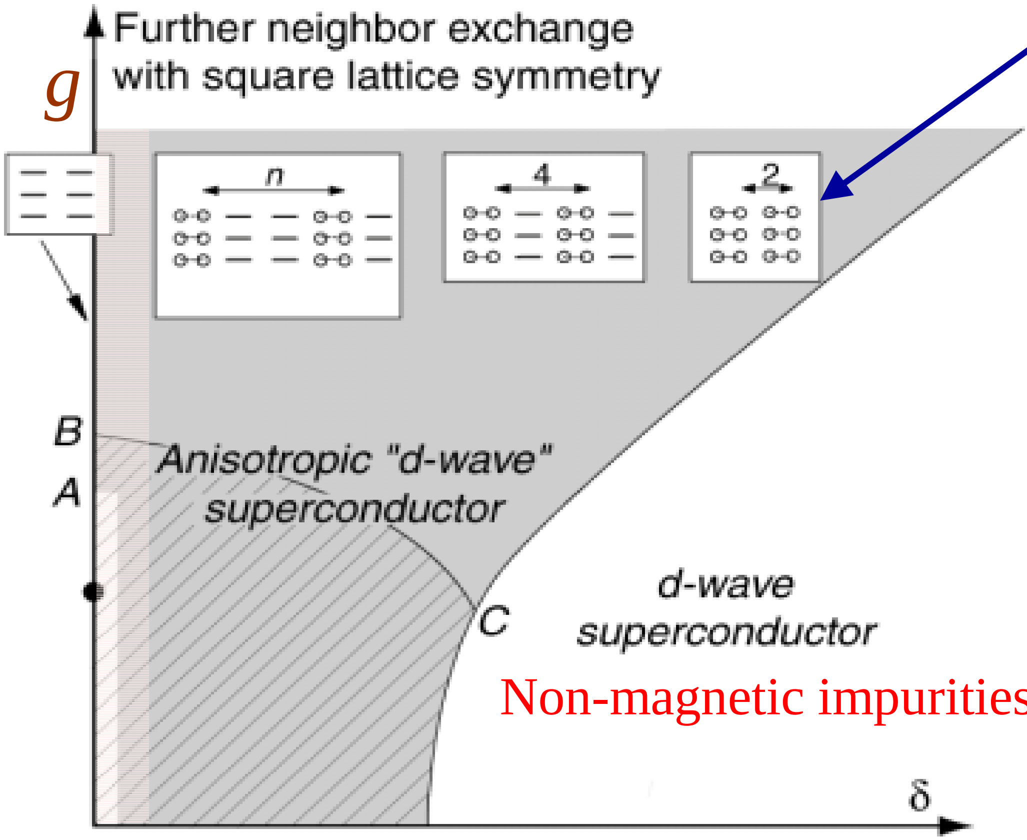


K-space locations of Bi and Cu atoms

Distances in k –space have units of $2\pi/a_0$
a_0=3.83 Å is Cu-Cu distance

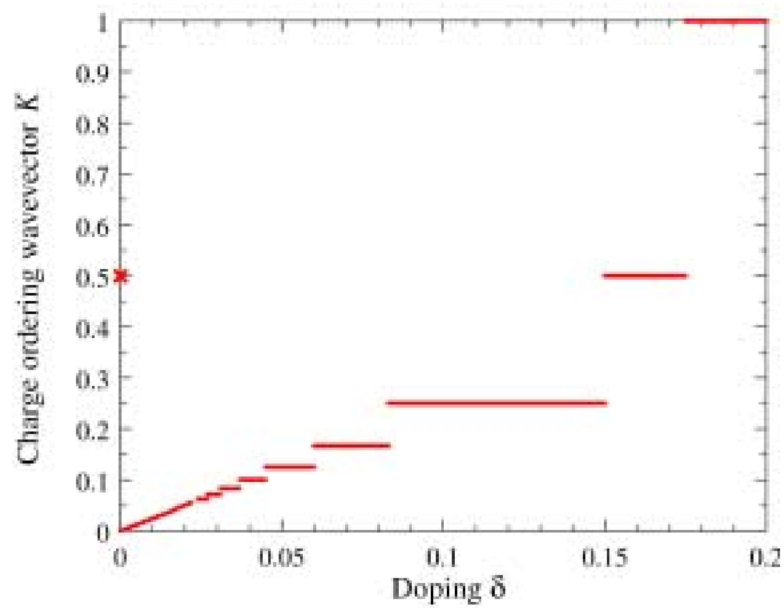
J. Hoffman *et al. Science*, **295**, 466 (2002).

III.B Non-magnetic impurities



“Large *N*” theory in region with preserved spin rotation symmetry

S. Sachdev and N. Read, *Int. J. Mod. Phys*. B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett*. **83**, 3916 (1999).
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev*. B **62**, 6721 (2000).

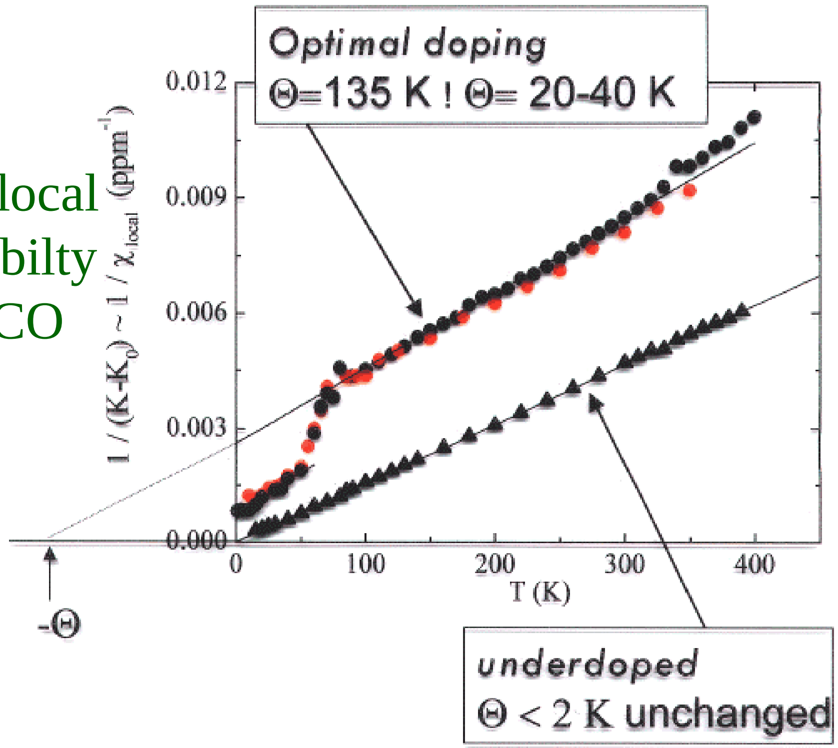


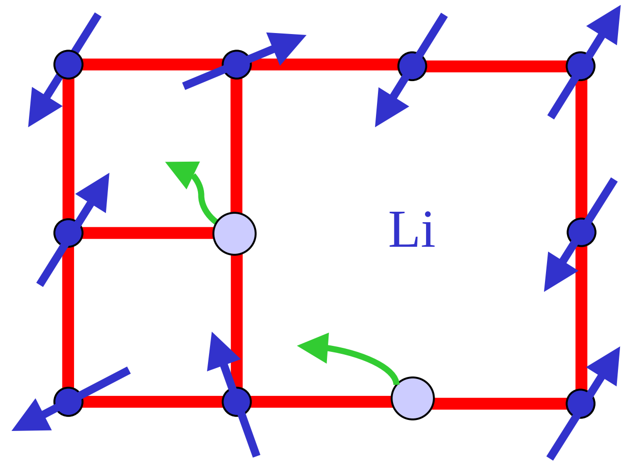
Hatched region --- spin order
Shaded region ---- charge order

See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. Kivelson, E. Fradkin and V. Emery, *Nature* **393**, 550 (1998),
S. White and D. Scalapino, *Phys. Rev. Lett*. **80**, 1272 (1998).

III.B Spatially resolved NMR of Zn/Li impurities in the superconducting state

Inverse local susceptibilty in YBCO





^{7}Li NMR below T_c

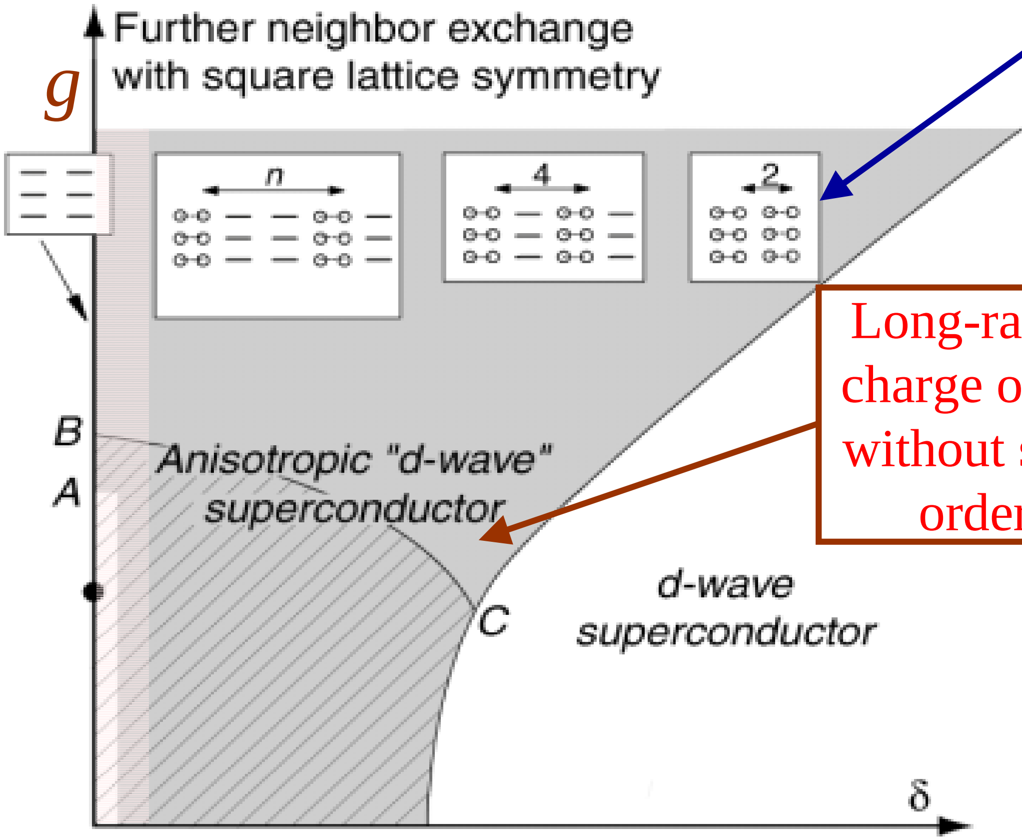
J. Bobroff, H. Alloul, W.A. MacFarlane, P. Mendels, N. Blanchard, G. Collin, and J.-F. Marucco, *Phys. Rev. Lett*. **86**, 4116 (2001).

Measured $\chi_{\text{impurity}}(T \to 0) = \dfrac{S(S+1)}{3k_B T}$ with $S = 1/2$ in underdoped sample.

This behavior does not emerge out of BCS theory.

A.M Finkelstein, V.E. Kataev, E.F. Kukovitskii, G.B. Teitel'baum, *Physica* C **168**, 370 (1990).

III.C Observation of static charge order.

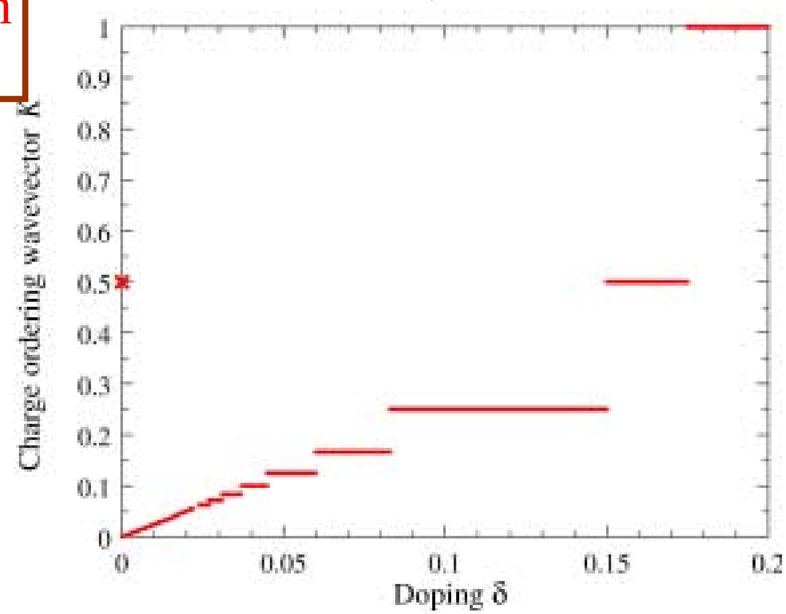


Long-range charge order without spin order

Hatched region --- spin order
Shaded region ---- charge order

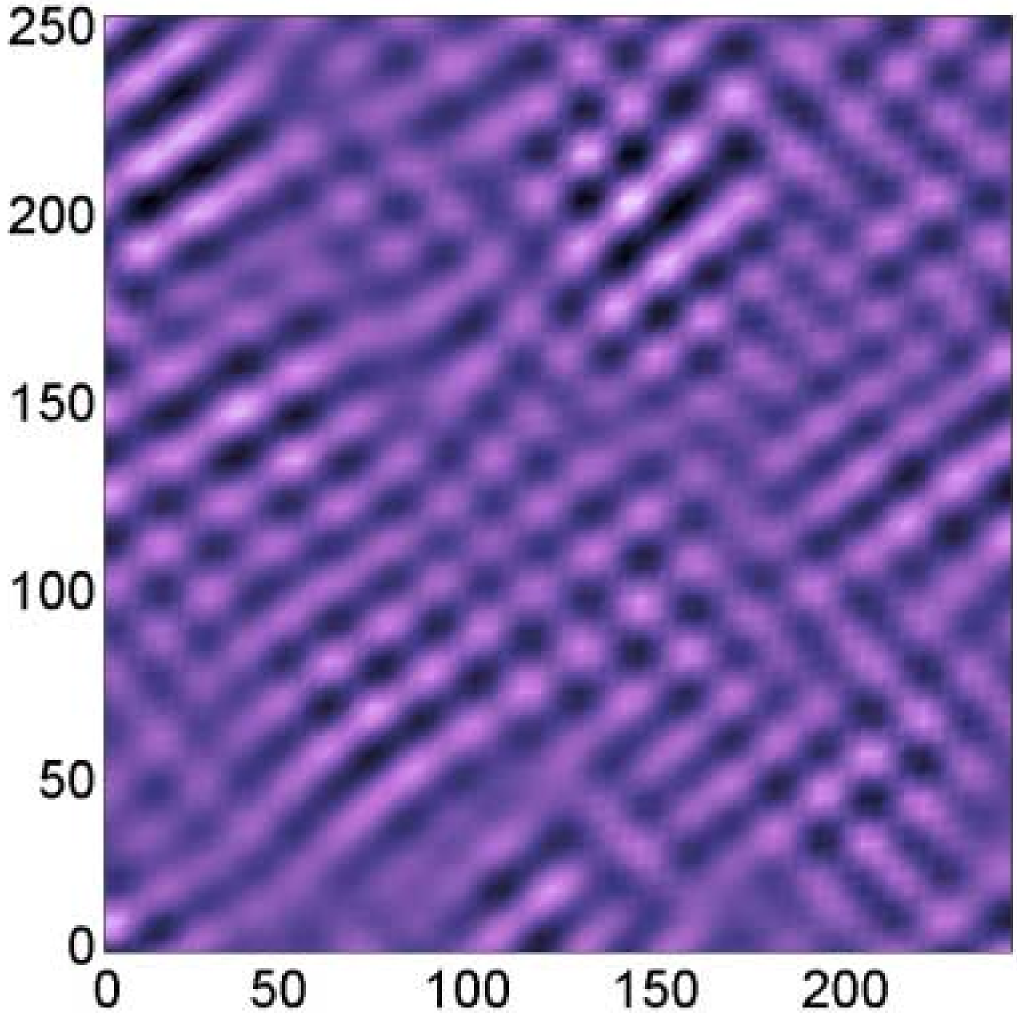
"Large *N*" theory in region with preserved spin rotation symmetry

S. Sachdev and N. Read, *Int. J. Mod. Phys*. B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett*. **83**, 3916 (1999).
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev*. B **62**, 6721 (2000).



See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. Kivelson, E. Fradkin and V. Emery, *Nature* **393**, 550 (1998),
S. White and D. Scalapino, *Phys. Rev. Lett*. **80**, 1272 (1998).

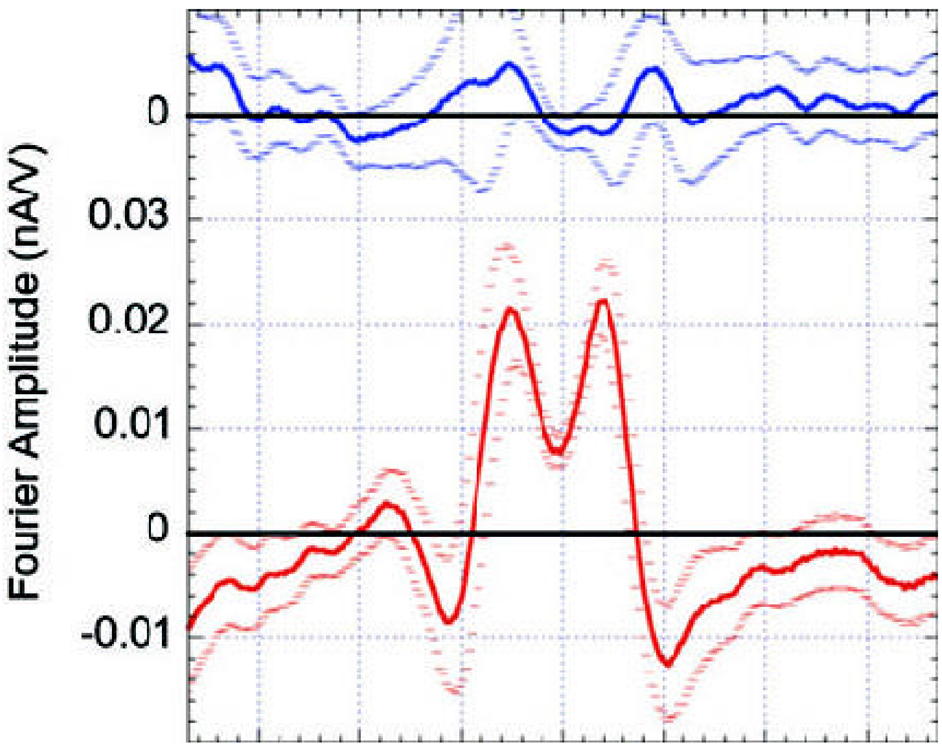
STM image of pinned charge order in $Bi_2Sr_2CaCu_2O_{8+\delta}$ in zero magnetic field



Charge order period = 4 lattice spacings

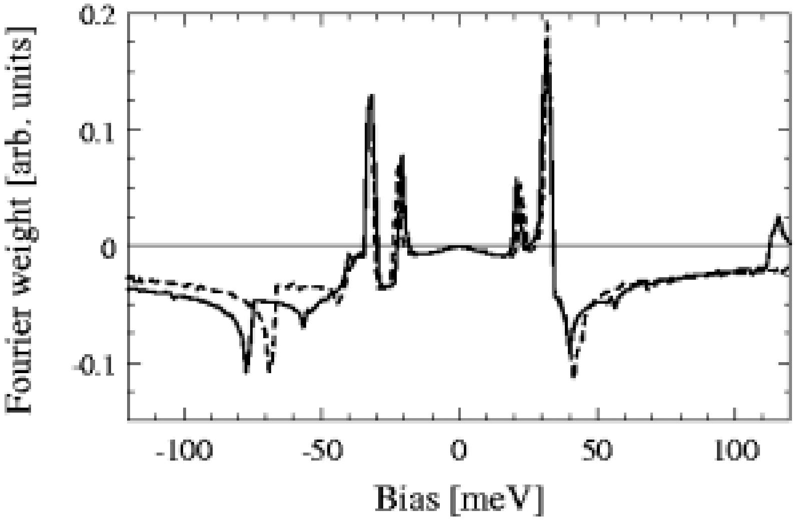
C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546

Spectral properties of the STM signal are sensitive to the microstructure of the charge order



Measured energy dependence of the Fourier component of the density of states which modulates with a period of 4 lattice spacings

C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546



Theoretical modeling shows that this spectrum is best obtained by a modulation of bond variables, such as the exchange, kinetic or pairing energies.

M. Vojta, cond-mat/0204284

D. Podolsky, E. Demler, K. Damle, and B.I. Halperin, cond-mat/0204011

Observation of static charge order in $YBa_2Cu_3O_{6.35}$
(spin correlations are dynamic)

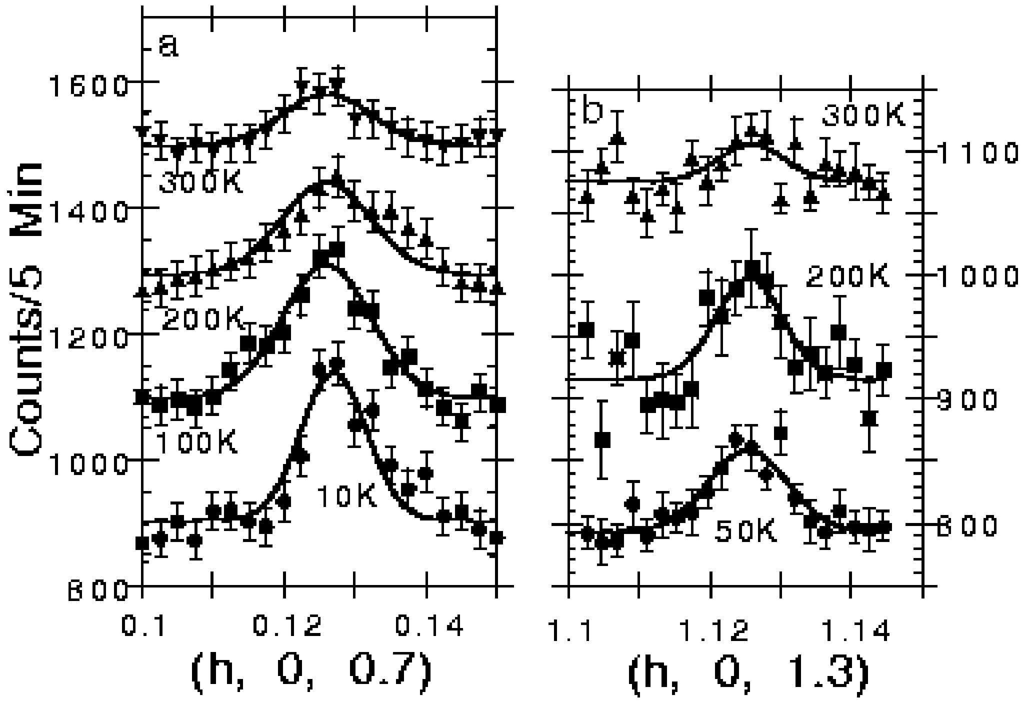
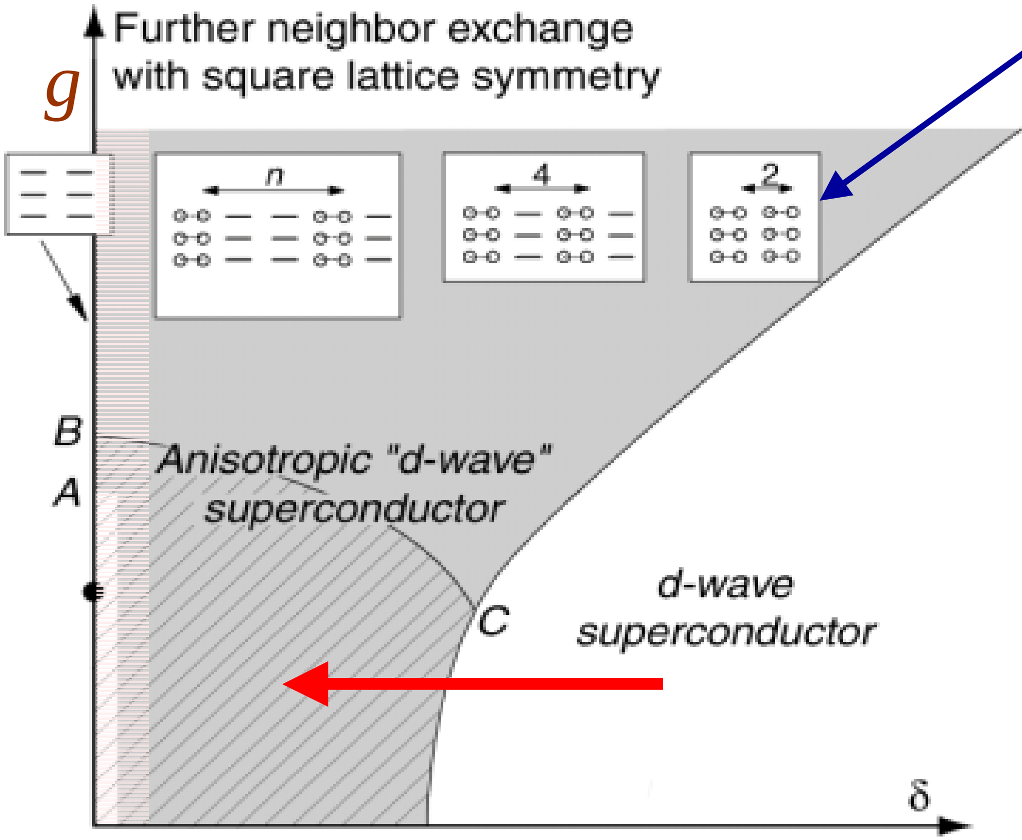


FIG. 1. Measurements of the charge order for YBCO6.35. (a) Measurements obtained at a small momentum transfer so the results are not affected by impurity powder lines. Powder lines were also avoided around the (1.125, 0, 1.3) r.l.u. position shown in (b). The lines are Gaussian fits to the data. In (a) 200 and (b) 100 additional counts were added onto successive scans so the data could be presented on the same plot. The scattering broadens at higher temperatures.

Charge order period
= 8 lattice spacings

H. A. Mook, Pengcheng Dai, and F. Dogan
Phys. Rev. Lett. **88**, 097004 (2002).

IV. SC+SDW to SC transition: influence of an applied magnetic field



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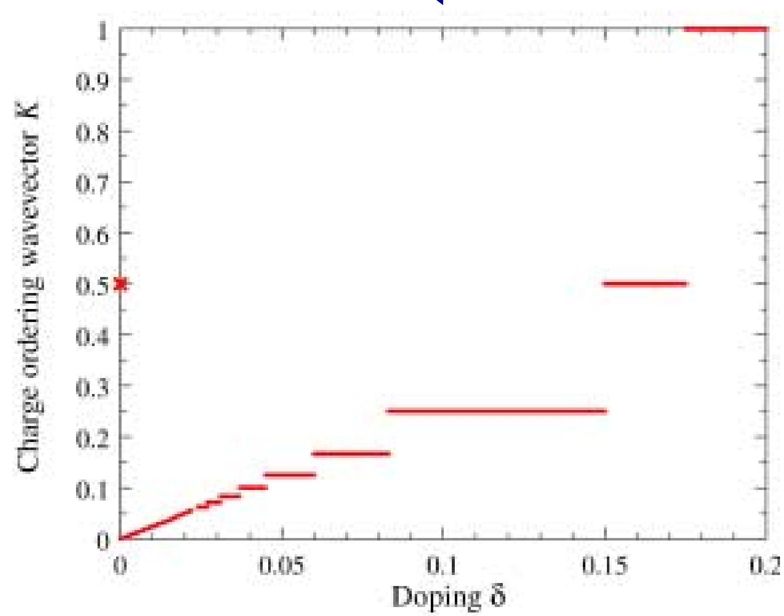
M. Vojta and S. Sachdev, *Phys. Rev. Lett*. **83**, 3916 (1999).
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev*. B **62**, 6721 (2000).



Hatched region --- spin order
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S. Kivelson, E. Fradkin and V. Emery, *Nature* **393**, 550 (1998),
S. White and D. Scalapino, *Phys. Rev. Lett*. **80**, 1272 (1998).

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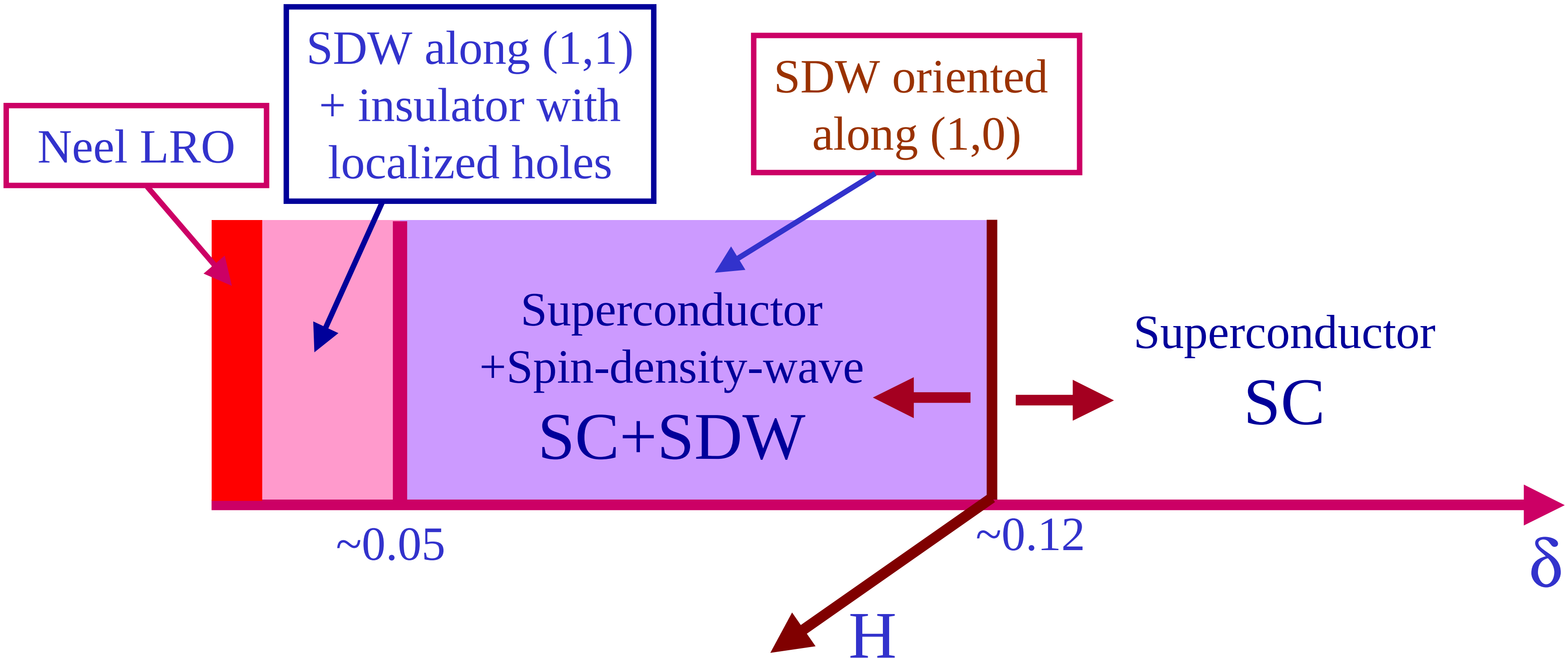


Theory for a system with strong interactions: describe SC and SC+SDW phases by expanding in the deviation from the quantum critical point between them.

B. Keimer *et al*. Phys. Rev. B **46**, 14034 (1992).

S. Wakimoto, G. Shirane *et al*., Phys. Rev. B **60**, R769 (1999).

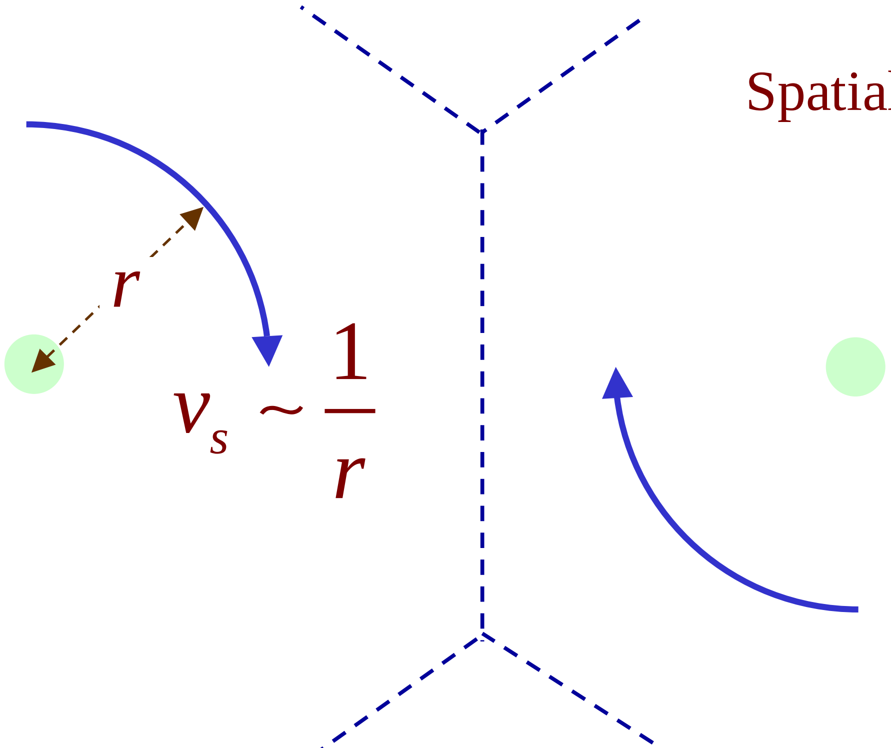
G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, Science **278**, 1432 (1997).

Y. S. Lee, R. J. Birgeneau, M. A. Kastner *et al*., Phys. Rev. B **60**, 3643 (1999).

J. E. Sonier *et al*., cond-mat/0108479.

C. Panagopoulos, B. D. Rainford, J. L. Tallon, T. Xiang, J. R. Cooper, and C. A. Scott, preprint.

Dominant effect: **uniform** softening of spin excitations by superflow kinetic energy



$$v_s \sim \frac{1}{r}$$

Spatially averaged superflow kinetic energy

$$\sim \left\langle v_s^2 \right\rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

Coupling determining spin excitation energy, *s*,

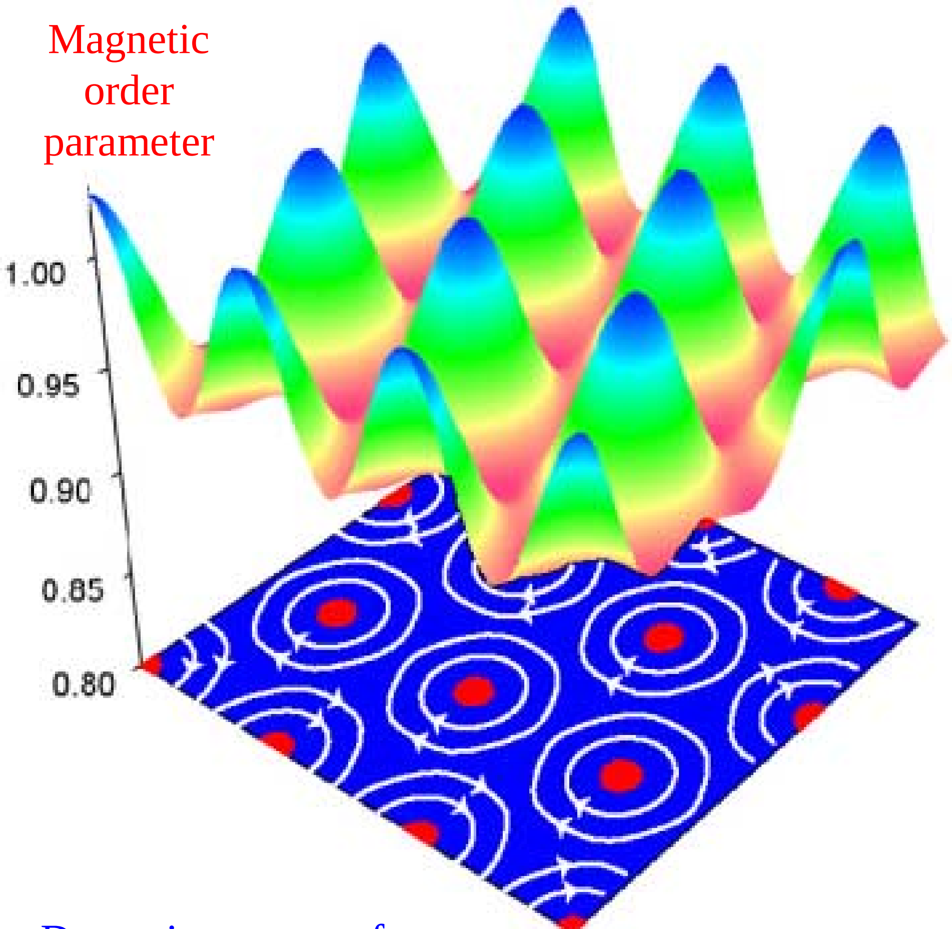
replaced by $s_{eff}\left(H\right) = s - C\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$

E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

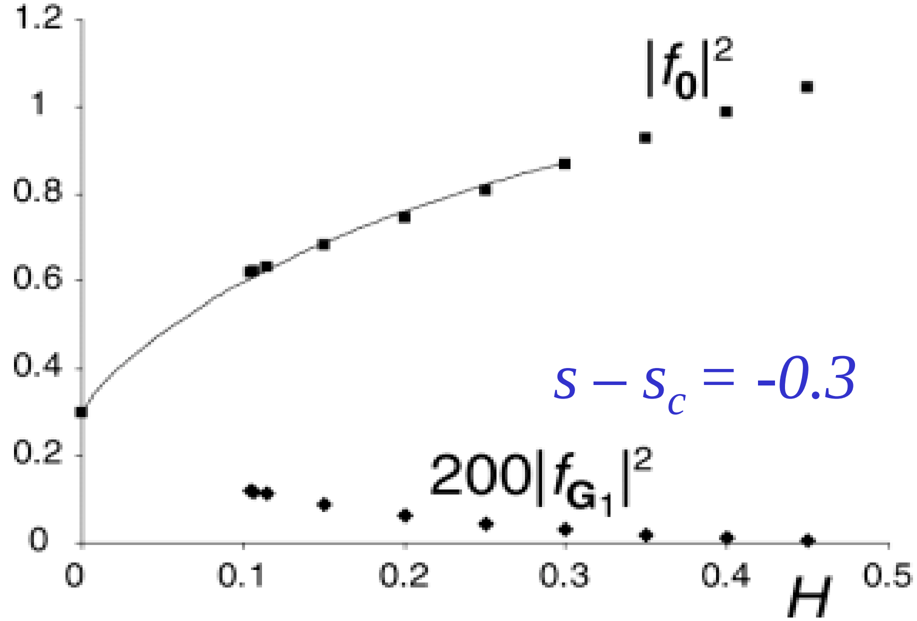
Structure of *long-range* SDW order in SC+SDW phase

E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Magnetic order parameter



$$\delta |f_0|^2 \propto H \ln(1/H)$$



$s - s_c = -0.3$

Dynamic structure factor

$$S(\mathbf{k},\omega) = (2\pi)^3 \delta(\omega) \sum_{\mathbf{G}} |f_{\mathbf{G}}|^2 \delta(\mathbf{k}-\mathbf{G}) + \cdots$$

$\mathbf{G} \rightarrow$ reciprocal lattice vectors of vortex lattice.

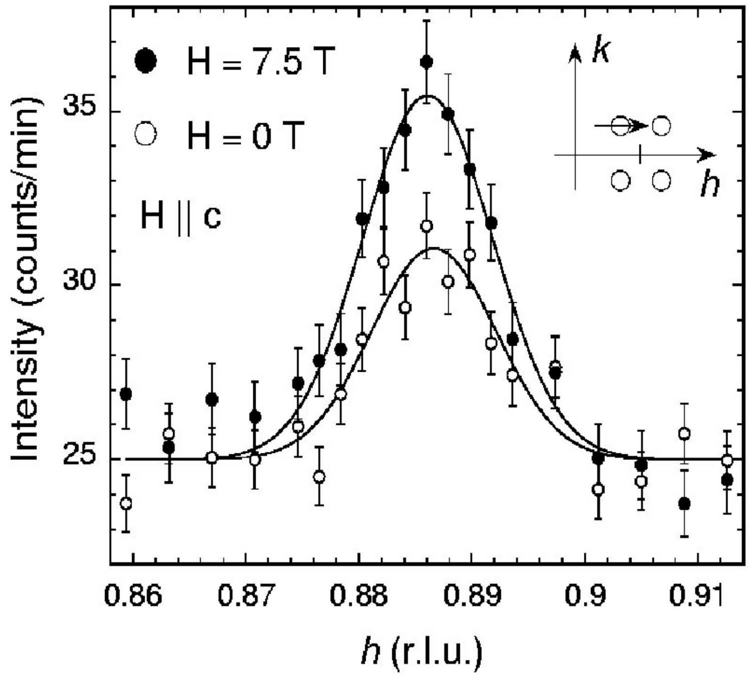
$\mathbf{k}$ measures deviation from SDW ordering wavevector $\mathbf{K}$

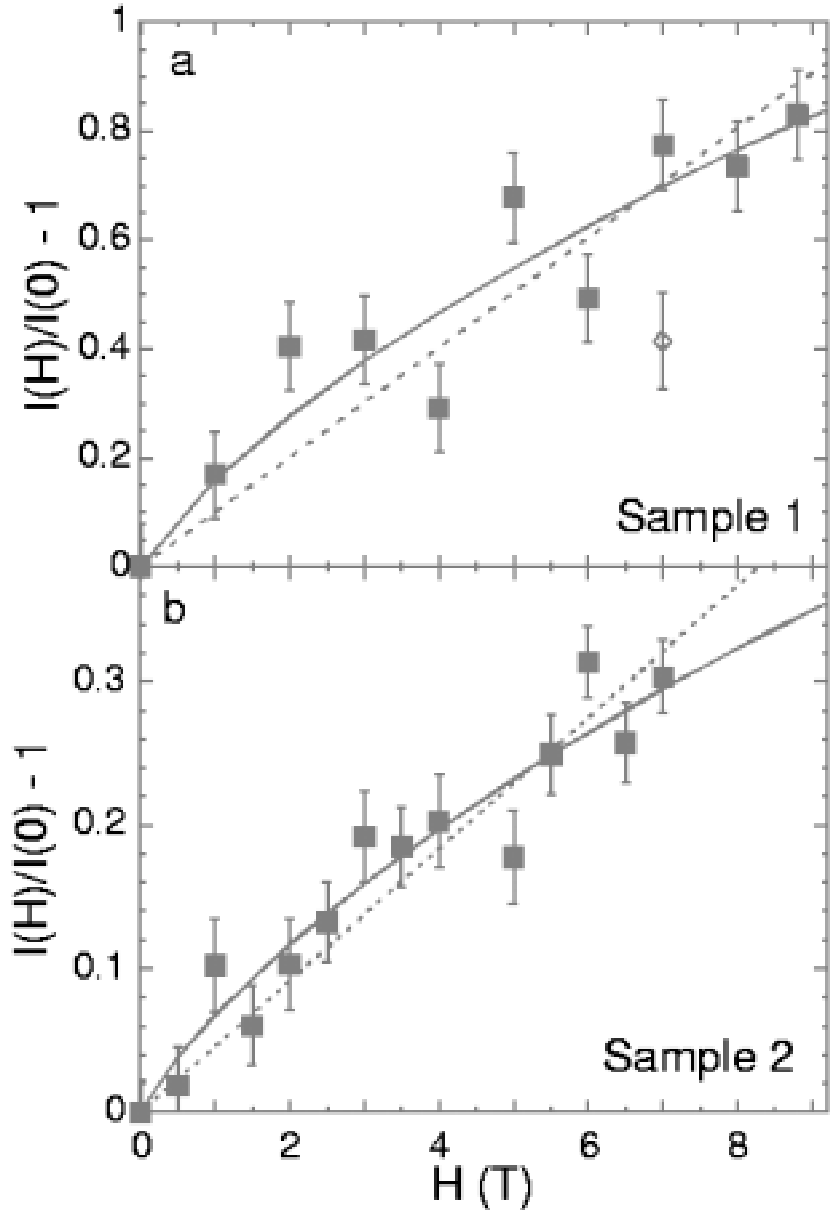
D. P. Arovas, A. J. Berlinsky, C. Kallin, and S.-C. Zhang, *Phys. Rev. Lett.* **79**, 2871 (1997) discussed static magnetism within the vortex cores in the SC phase.Their model implies a ~*H* dependence of the intensity

Neutron scattering measurements of static spin correlations of the superconductor+spin-density-wave (SC+SDW) in a magnetic field

Elastic neutron scattering off La_2CuO_{4+y}

B. Khaykovich, Y. S. Lee, S. Wakimoto,
K. J. Thomas, M. A. Kastner,
and R.J. Birgeneau, cond-mat/0112505.



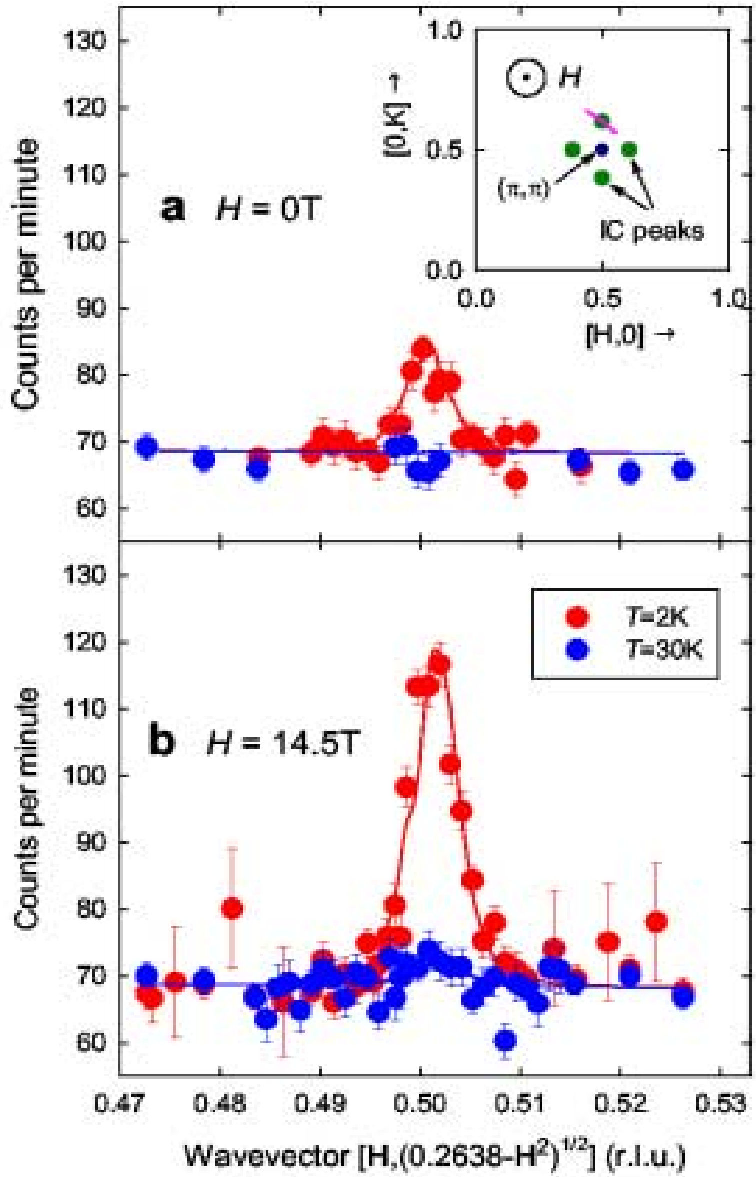


Solid line --- fit to : $\dfrac{I(H)}{I(0)} = 1 + a\dfrac{H}{H_{c2}}\ln\left(\dfrac{3.0H_{c2}}{H}\right)$

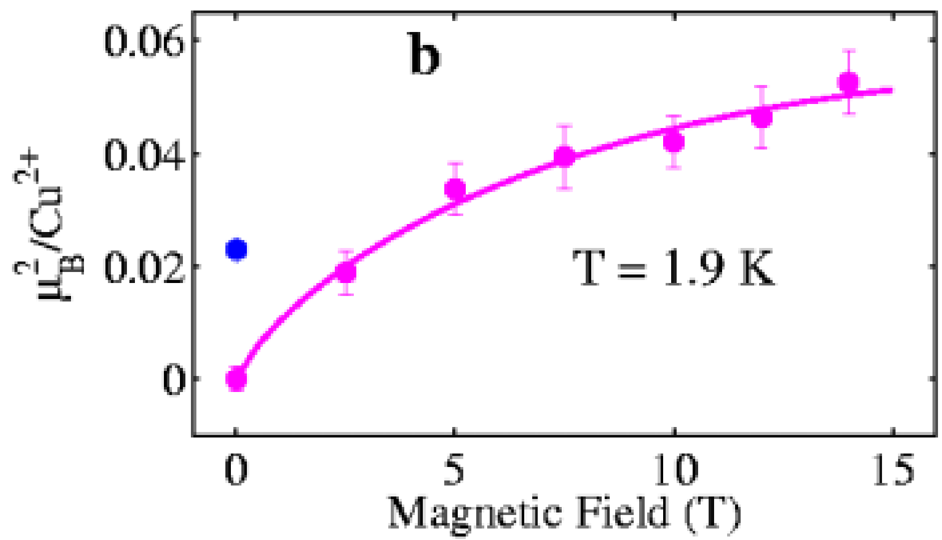
a is the only fitting parameter

Best fit value - a = 2.4 with H_{c2} = 60 T

Neutron scattering of $La_{2-x}Sr_xCuO_4$ at x=0.1



B. Lake, H. M. Rønnow, N. B. Christensen, G. Aeppli, K. Lefmann, D. F. McMorrow, P. Vorderwisch, P. Smeibidl, N. Mangkorntong, T. Sasagawa, M. Nohara, H. Takagi, T. E. Mason, *Nature*, **415**, 299 (2002).



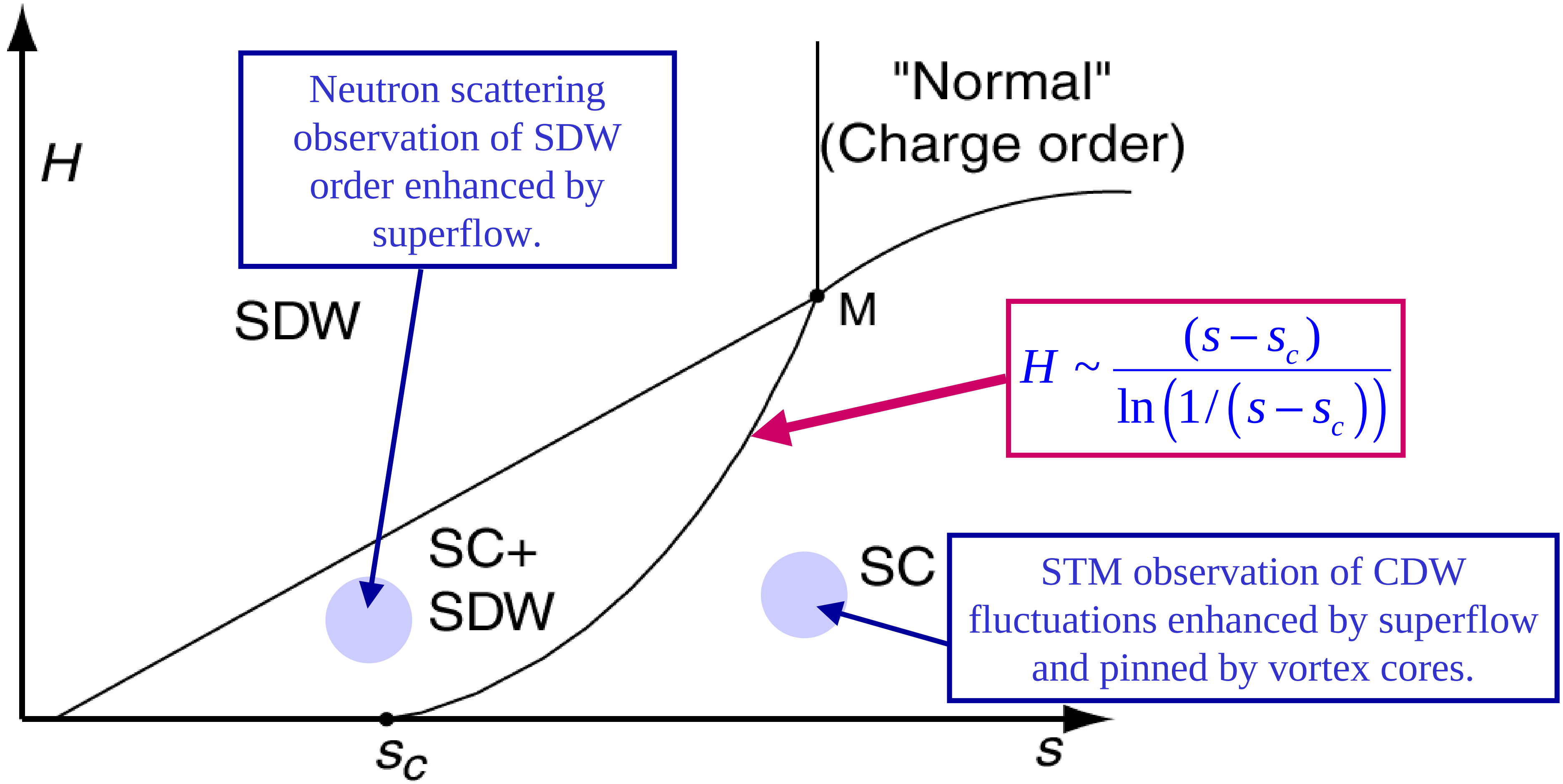
Solid line - fit to : $I(H) = a\frac{H}{H_{c2}}\ln\left(\frac{H_{c2}}{H}\right)$

Effect of magnetic field on SDW+SC to SC transition

(extreme Type II superconductivity)

$T=0$

Main results



E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

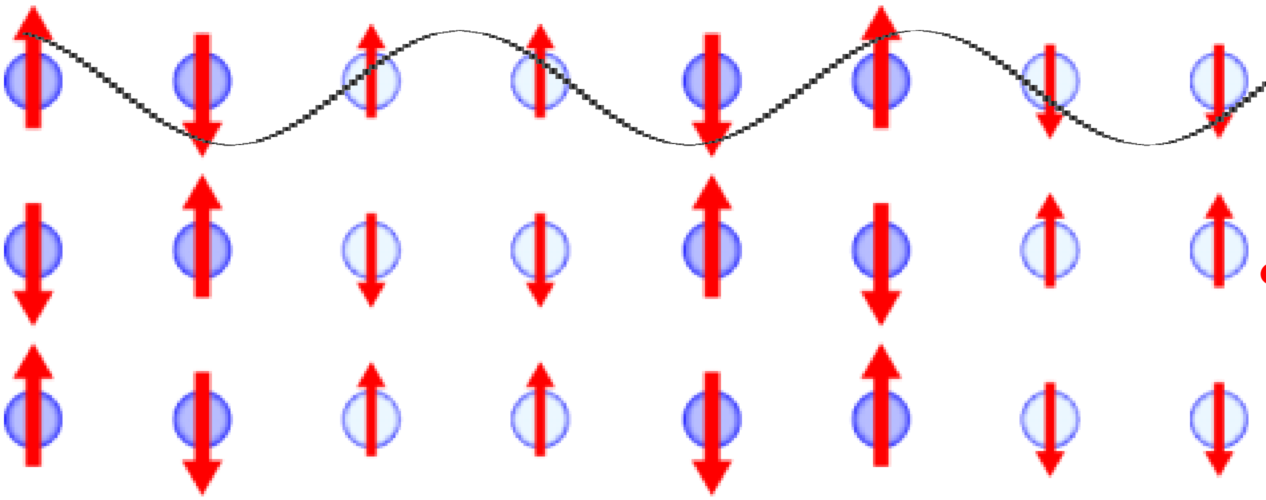
Quantitative connection between the two experiments ?

Theory of SC+SDW to SC quantum transition

Spin density wave order parameter for general ordering wavevector

$$S_\alpha(\boldsymbol{r}) = \Phi_\alpha(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

$\Phi_\alpha(\boldsymbol{r})$ is a complex field $\left(\text{except for } \boldsymbol{K}=(\pi,\pi) \text{ when } e^{i\boldsymbol{K}\cdot\boldsymbol{r}} = (-1)^{r_x+r_y}\right)$



Wavevector $\boldsymbol{K}=(3\pi/4,\pi)$

Exciton wavefunction $\Phi_\alpha(\boldsymbol{r})$ describes *envelope* of this order. Phase of $\Phi_\alpha(\boldsymbol{r})$ represents sliding degree of freedom

Associated "charge" density wave order

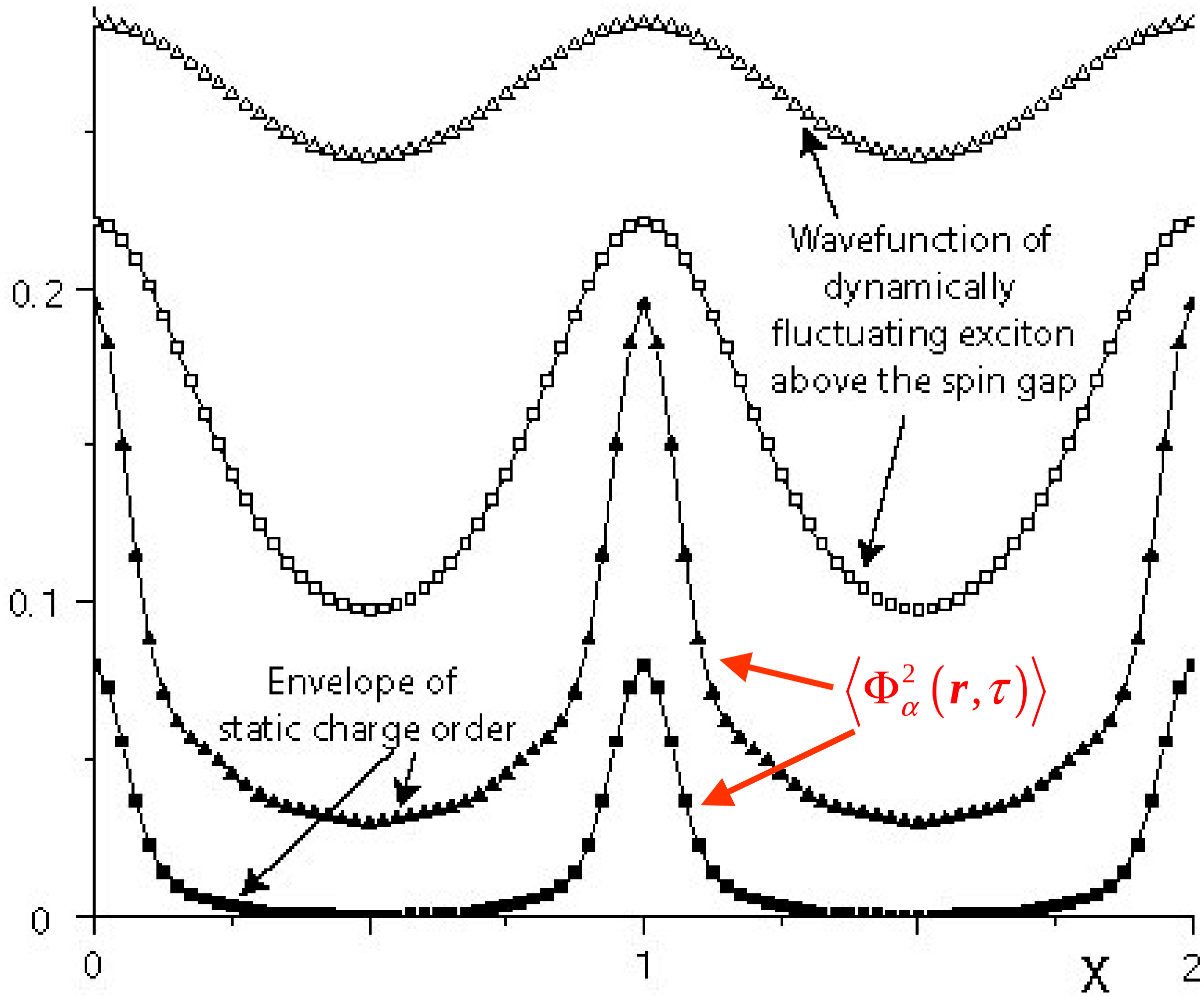
$$\delta\rho(\boldsymbol{r}) \propto S_\alpha^2(\boldsymbol{r}) = \sum_\alpha \Phi_\alpha^2(\boldsymbol{r}) e^{i2\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

J. Zaanen and O. Gunnarsson, *Phys. Rev*. B **40**, 7391 (1989).
H. Schulz, *J. de Physique* **50**, 2833 (1989).
O. Zachar, S. A. Kivelson, and V. J. Emery, *Phys. Rev*. B **57**, 1422 (1998).

Pinning of CDW order by vortex cores in SC phase

Y. Zhang, E. Demler, and S. Sachdev, cond-mat/0112343.

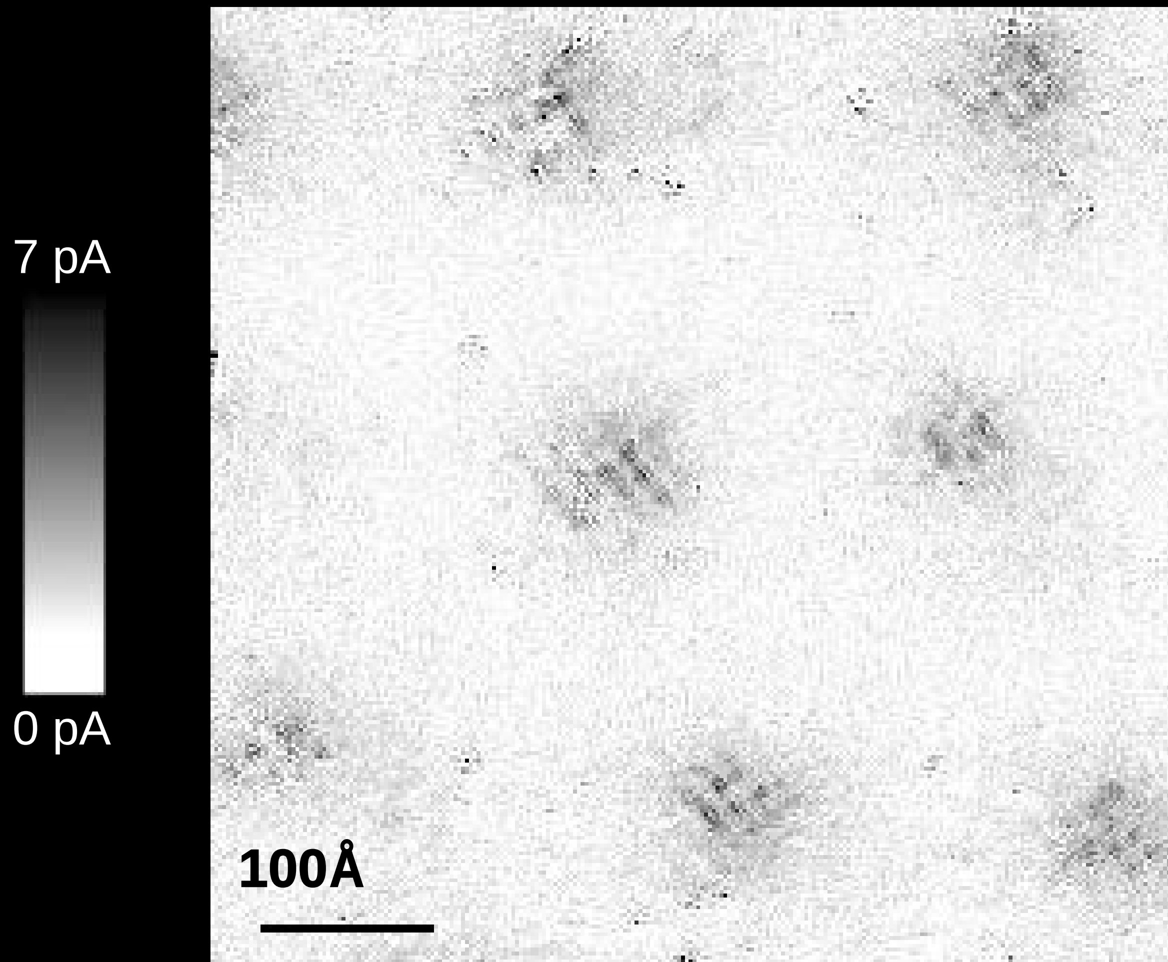
$$\left\langle \Phi_\alpha^2(\boldsymbol{r},\tau)\right\rangle \propto \zeta \int d\tau_1 \left\langle \Phi_\alpha(\boldsymbol{r},\tau)\,\Phi_\alpha^*(\boldsymbol{r}_v,\tau_1)\right\rangle^2$$



□ → low magnetic field

△ → high magnetic field near the boundary to the SC+SDW phase

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV



J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Conclusions

I. Cuprate superconductivity is associated with doping Mott insulators with charge carriers

II. The correct paramagnetic Mott insulator has charge-order and confinement of spinons

III. Mott insulator reveals itself vortices and near impurities. Predicted effects seen recently in STM and NMR experiments.

IV. Semi-quantitative predictions for neutron scattering measurements of spin-density-wave order in superconductors; theory also establishes connection to STM experiments.

V. Future experiments should search for SC+SDW to SC quantum transition driven by a magnetic field.

VI. Major open question: how does understanding of low temperature order parameters help explain anomalous behavior at high temperatures ?