Strong coupling problems in condensed matter and the AdS/CFT correspondence

Reviews:

arXiv:0910.1139
arXiv:0901.4103

Talk online: sachdev.physics.harvard.edu

PHYSICS
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Max Metlitski, Harvard

Frederik Denef, Harvard
Sean Hartnoll, Harvard
Christopher Herzog, Princeton
Pavel Kovtun, Victoria
Dam Son, Washington

PHYSICS
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1. Quantum-critical transport
   Collisionless-to-hydrodynamic crossover of CFT3s

2. Nematic quantum criticality of metals in two dimensions
   Scaling theory and planar graphs

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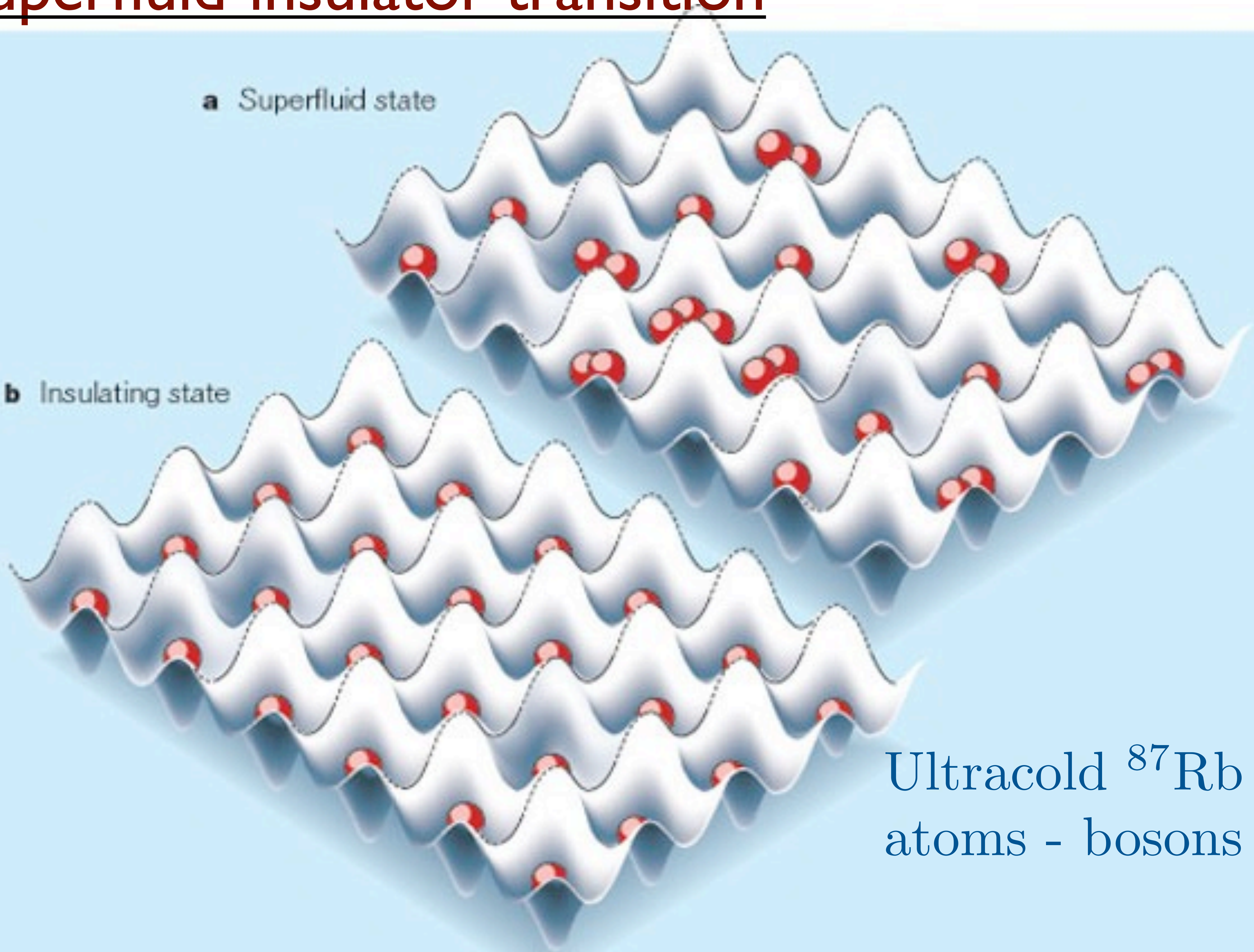
Superfluid-insulator transition

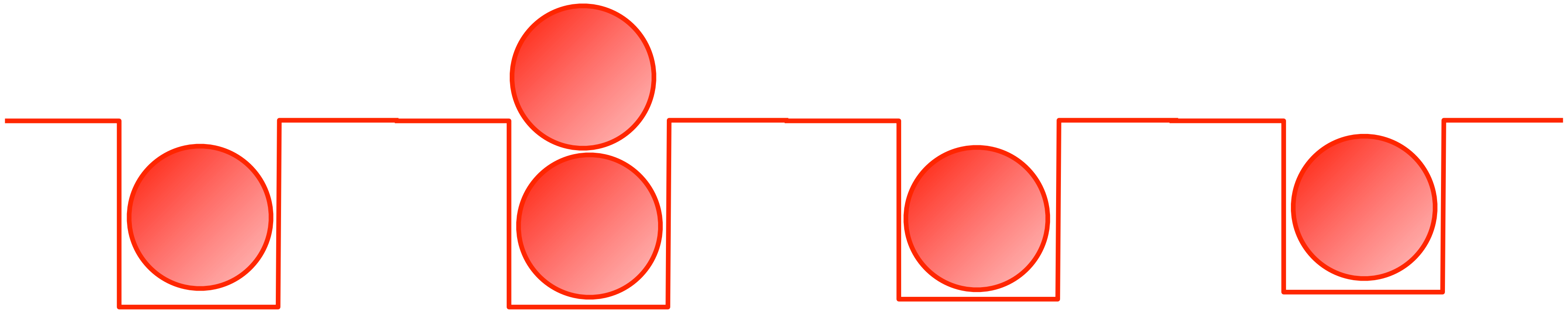


Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Insulator (the vacuum) at large U

Excitations:

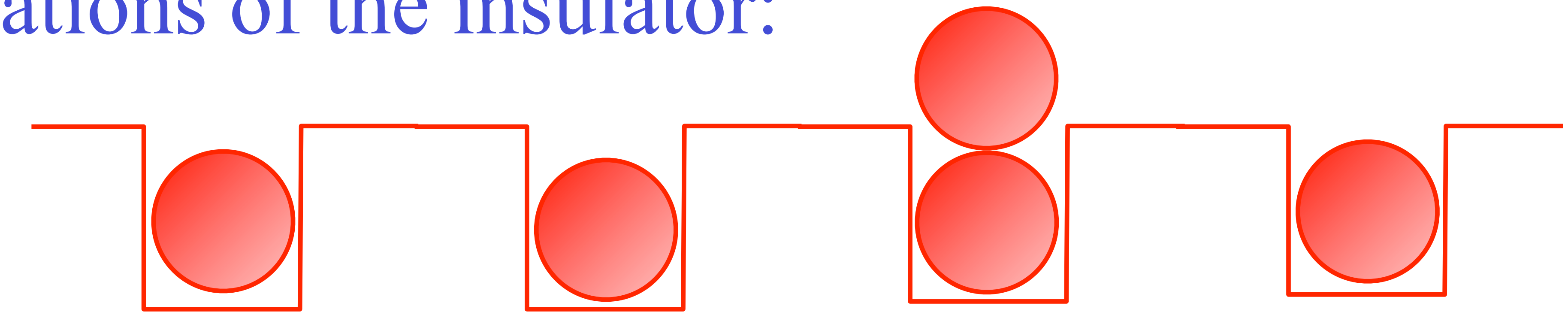


Particles $\sim \psi^\dagger$

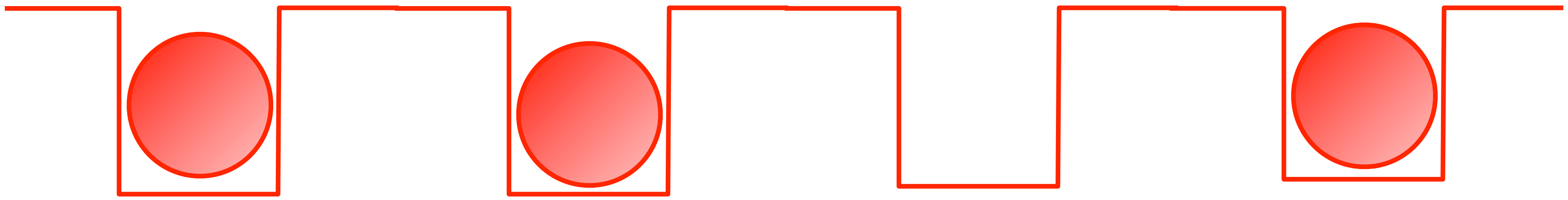
Excitations:

Holes ~ ψ

Excitations of the insulator:



Particles ~ $\psi^\dagger$



Holes ~ ψ

Density of particles = density of holes ⇒

"relativistic" field theory for ψ:

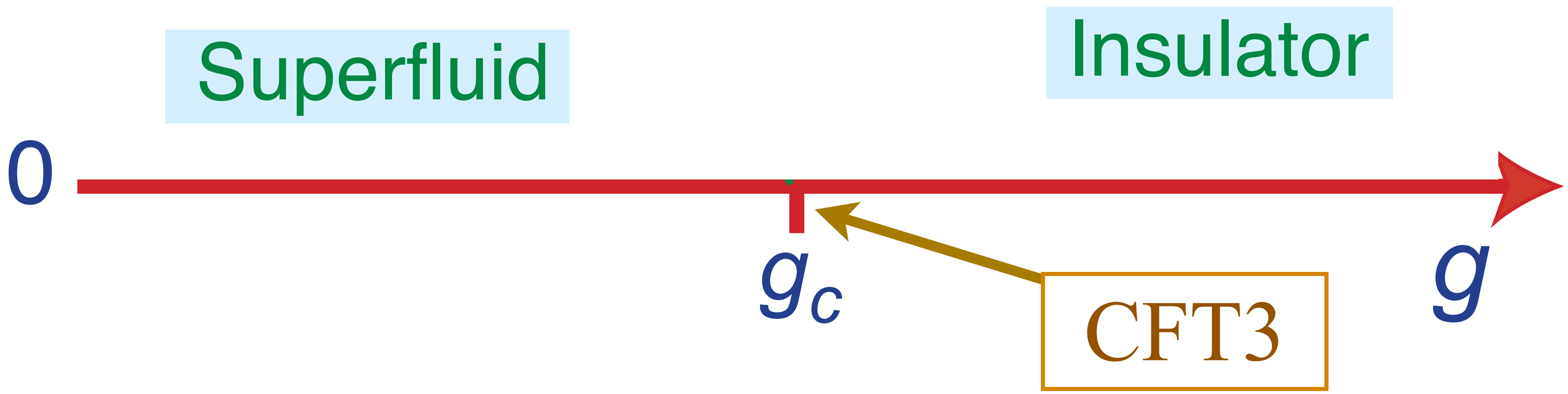
$$\mathcal{S} = \int d^2r\, d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla}\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

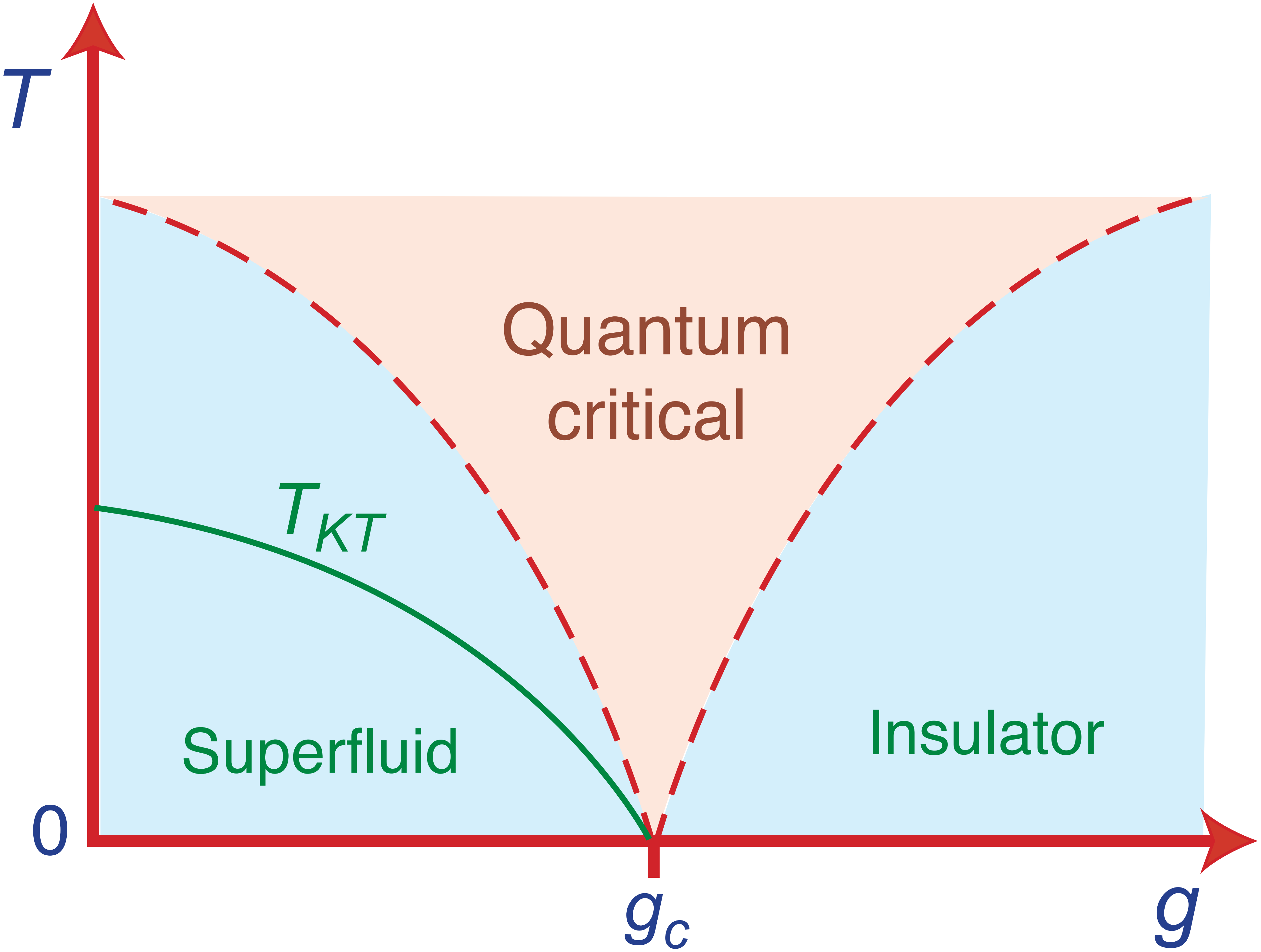
$$\text{Insulator} \quad \Leftrightarrow \quad \langle \psi \rangle = 0$$

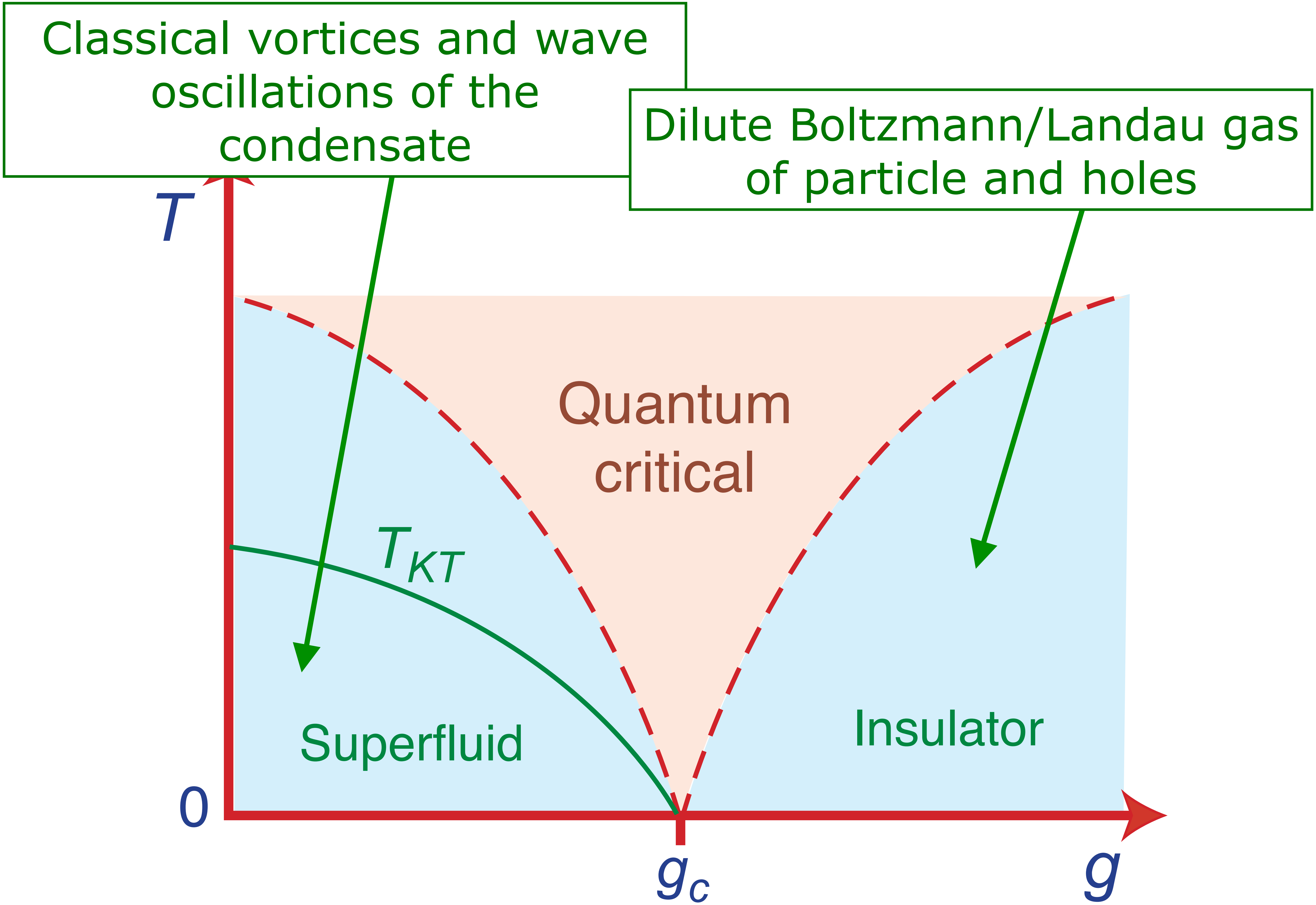
$$\text{Superfluid} \quad \Leftrightarrow \quad \langle \psi \rangle \neq 0$$

$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla}\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\langle \psi \rangle \neq 0 \qquad\qquad \langle \psi \rangle = 0$$

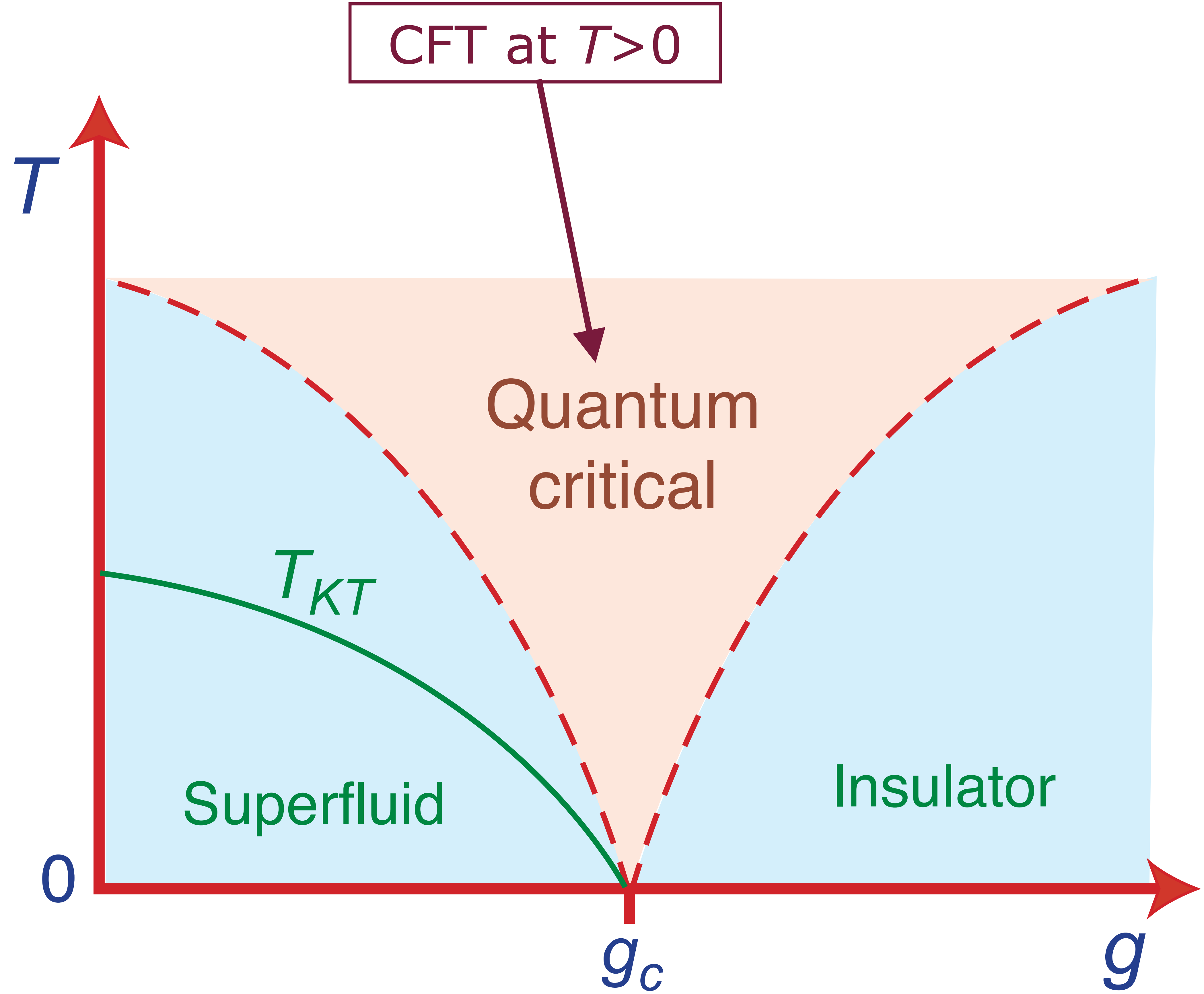



T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g



Classical vortices and wave oscillations of the condensate

Dilute Boltzmann/Landau gas of particle and holes


CFT at T>0
T
Quantum critical
T_{KT}
Superfluid
Insulator
0
g_c
g

Resistivity of Bi films

Conductivity σ

$$\sigma_{\text{Superconductor}}(T \to 0) = \infty$$

$$\sigma_{\text{Insulator}}(T \to 0) = 0$$

$$\sigma_{\text{Quantum critical point}}(T \to 0) \approx \frac{4e^2}{h}$$

D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

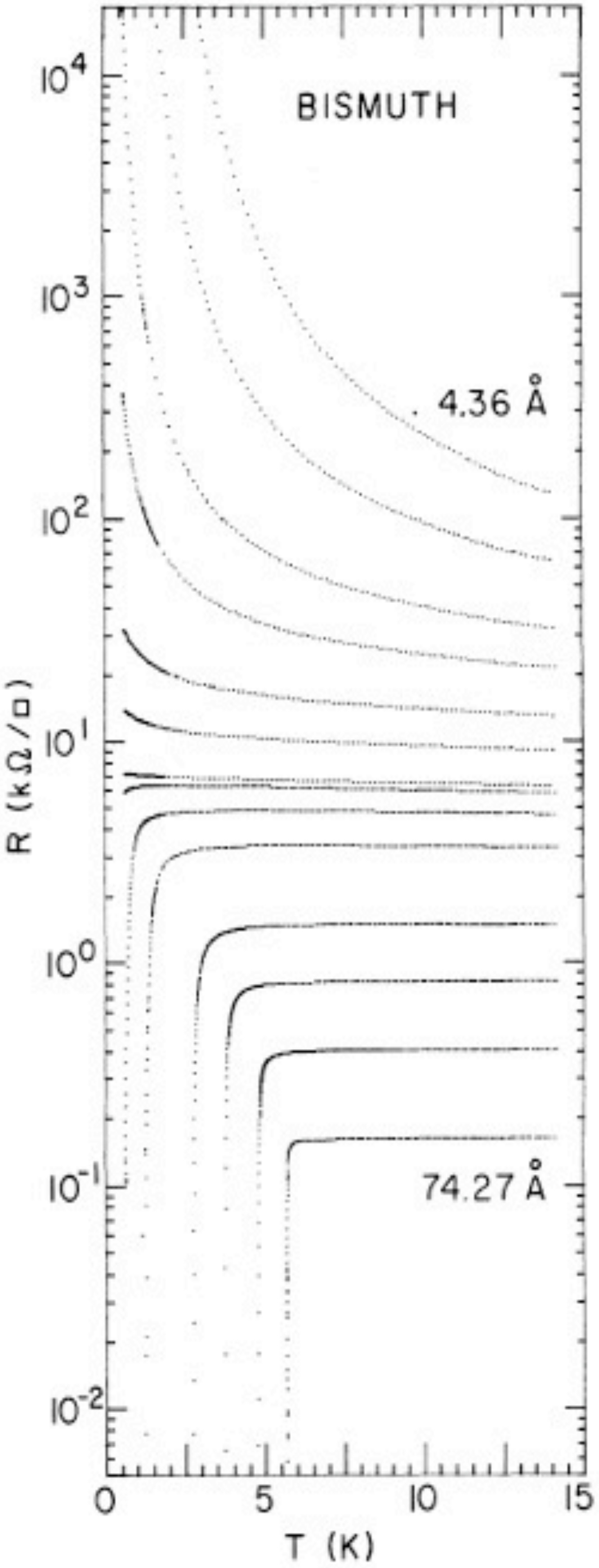


FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

Quantum critical transport

Quantum *"perfect fluid"* with shortest possible relaxation time, τ_R

$$\boxed{\tau_R \gtrsim \frac{\hbar}{k_B T}}$$

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Electrical conductivity

$$\boxed{\sigma = \frac{4e^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]}$$

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

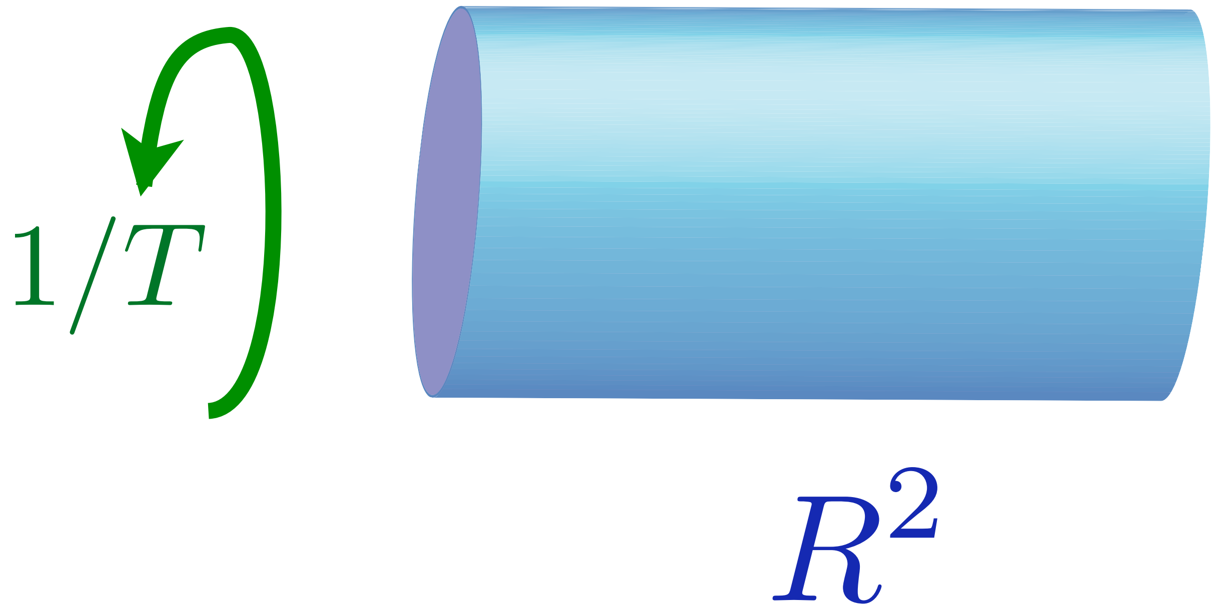
$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

Quantum critical transport

Euclidean field theory:

Compute current correlations on $R^2 \times S^1$ with circumference $1/T$



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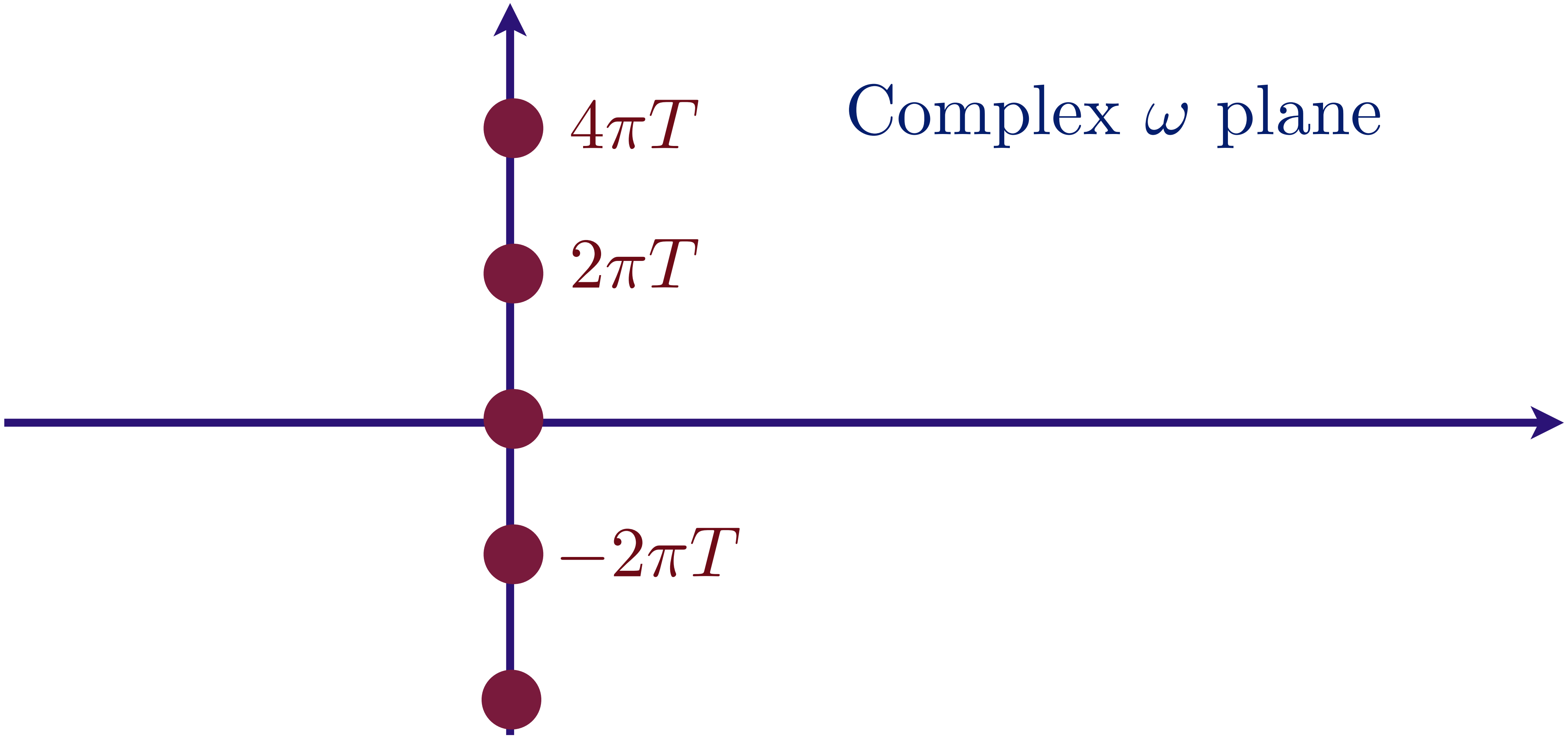
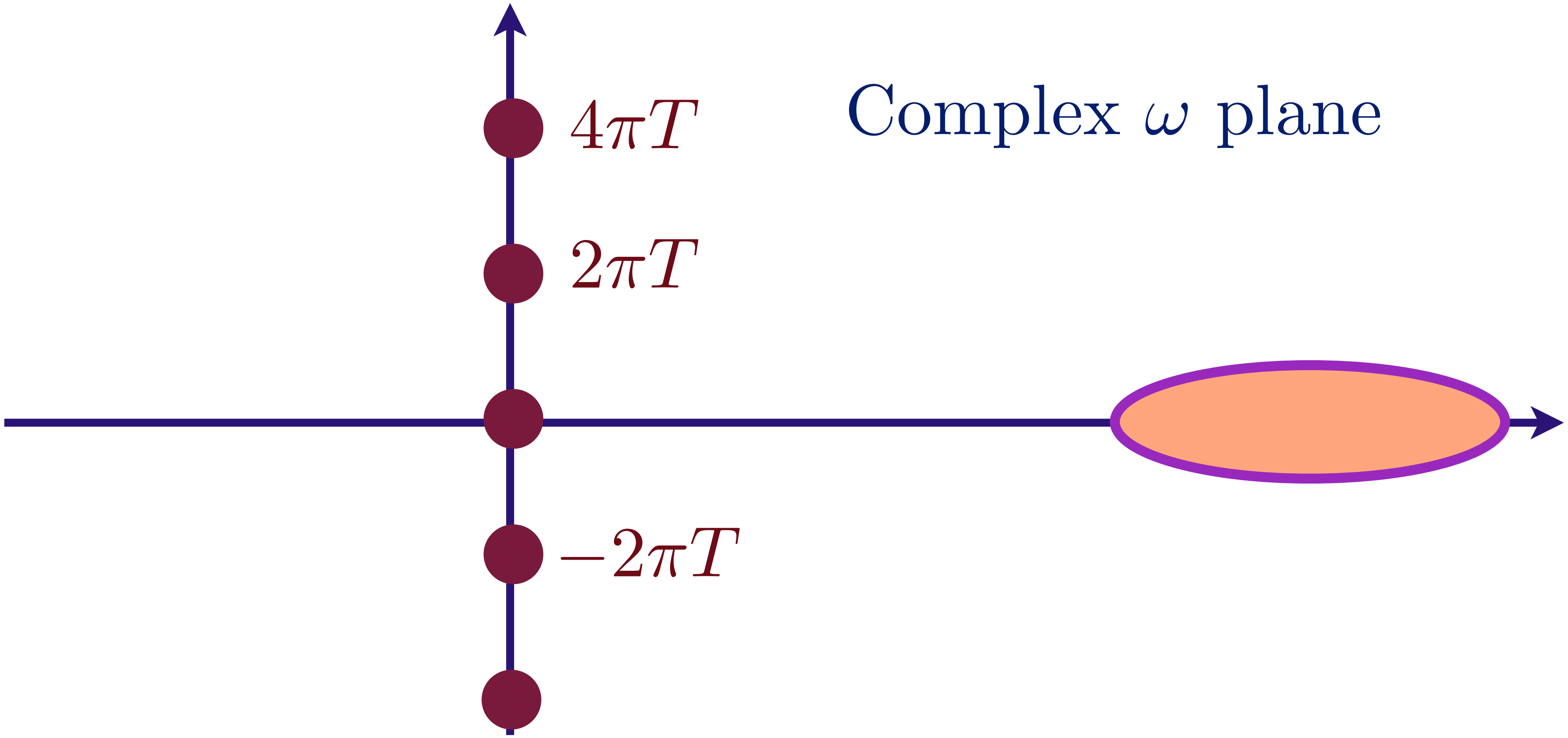


Direct $1/N$ or ϵ expansions for correlators at the Euclidean frequencies $\omega_n = 2\pi n T i$ (n integer) or in the conformal "collisionless" regime, $\hbar\omega \gg k_B T$.

Quantum critical transport

Euclidean field theory: Compute current correlations on $R^2 \times S^1$ with circumference $1/T$



Complex ω plane

Direct $1/N$ or ϵ expansions for correlators at the Euclidean frequencies $\omega_n = 2\pi n T i$ (n integer)
or in the conformal "collisionless" regime, $\hbar\omega \gg k_B T$.

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2}\chi(k,\omega)$

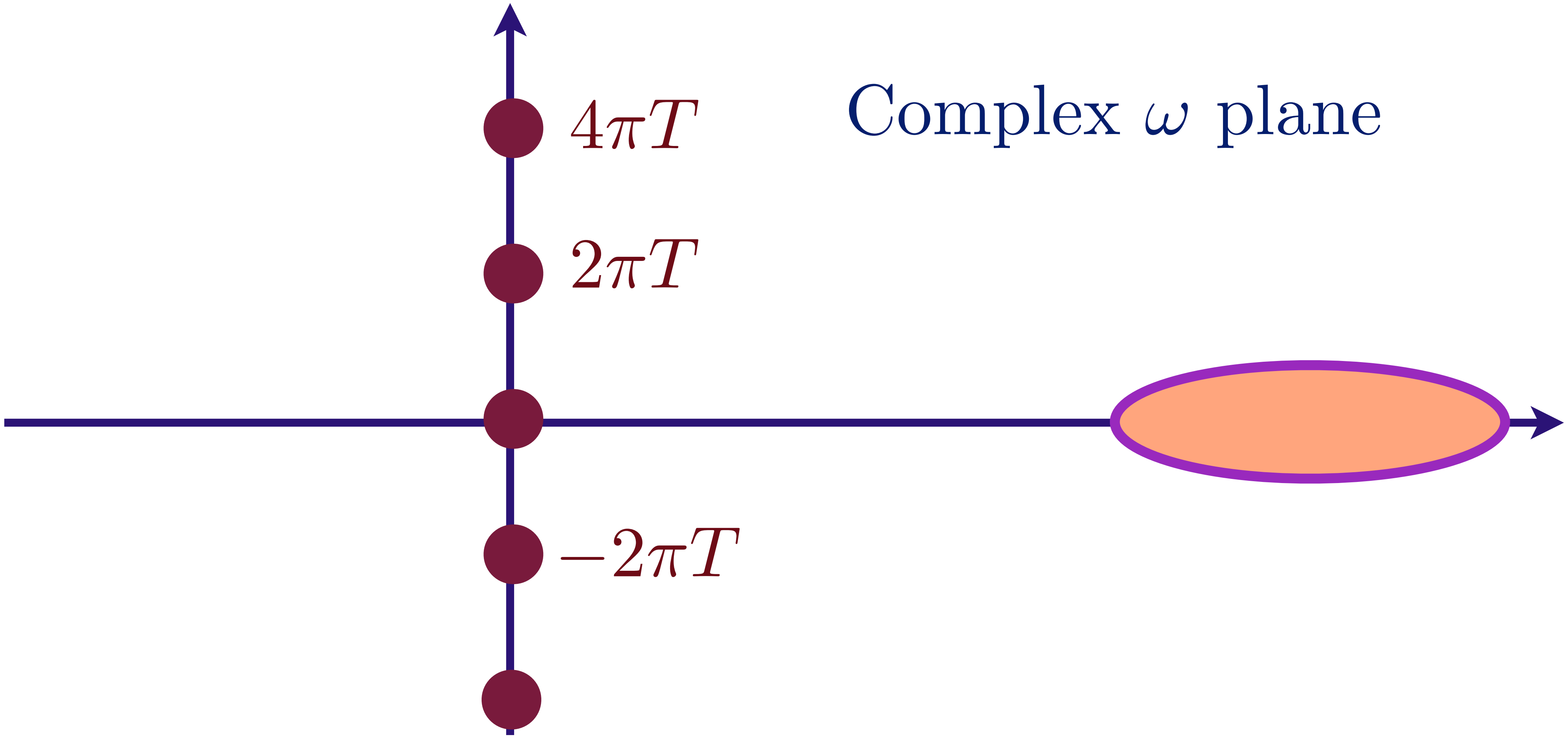
For *all* CFT3s, at $\underline{\hbar\omega \gg k_B T}$

$$\chi(k,\omega) \quad = \quad \frac{4e^2}{h} K \frac{k^2}{\sqrt{v^2 k^2 - \omega^2}} \quad ; \quad \sigma(\omega) = \frac{4e^2}{h} K$$

where K is a universal number characterizing the CFT3, and v is the velocity of “light”.

Quantum critical transport

Euclidean field theory: Compute current correlations on $R^2 \times S^1$ with circumference $1/T$



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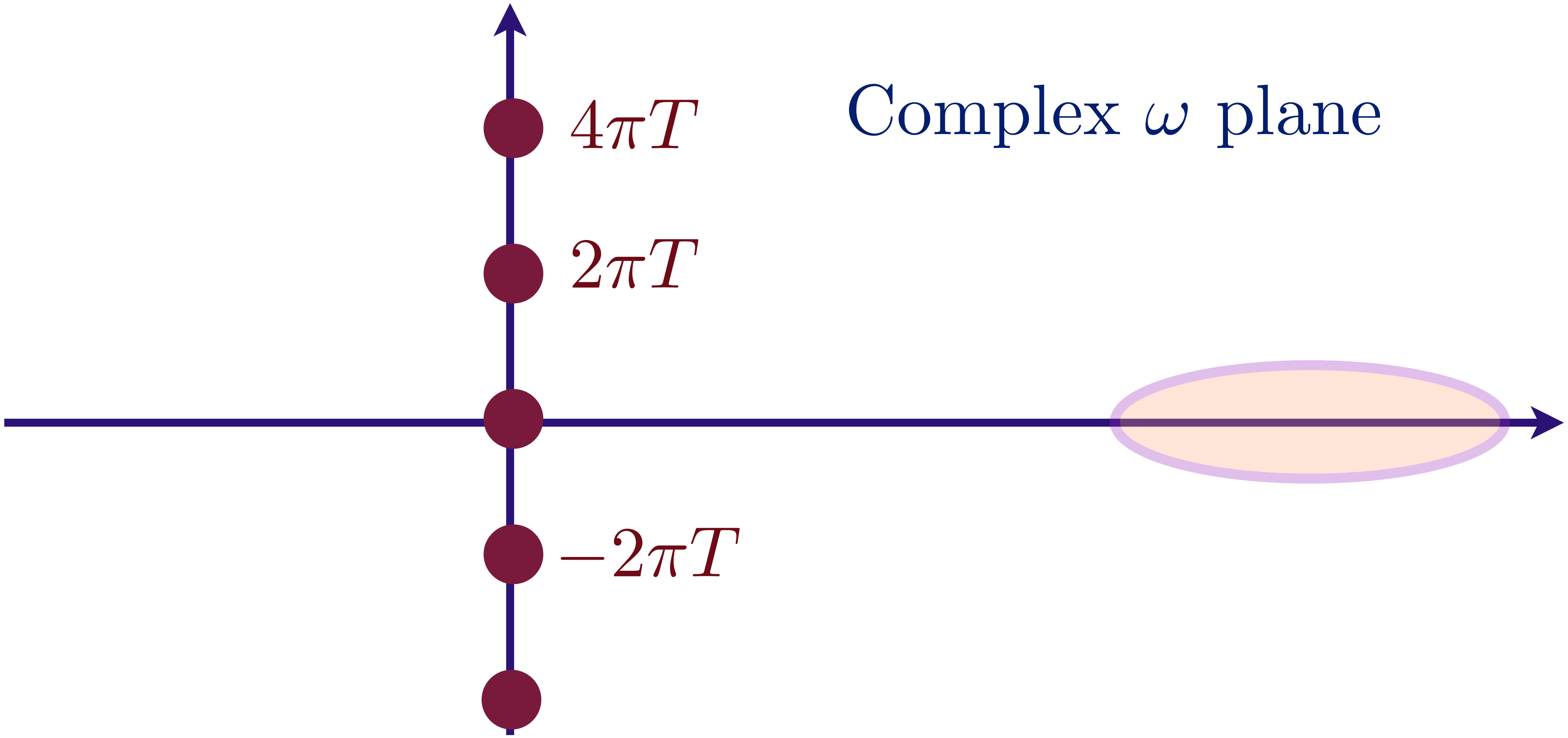

Strong coupling problem:
Correlators at $\hbar\omega \ll k_B T$, along the real axis, in the collision-dominated hydrodynamic regime.

Quantum critical transport

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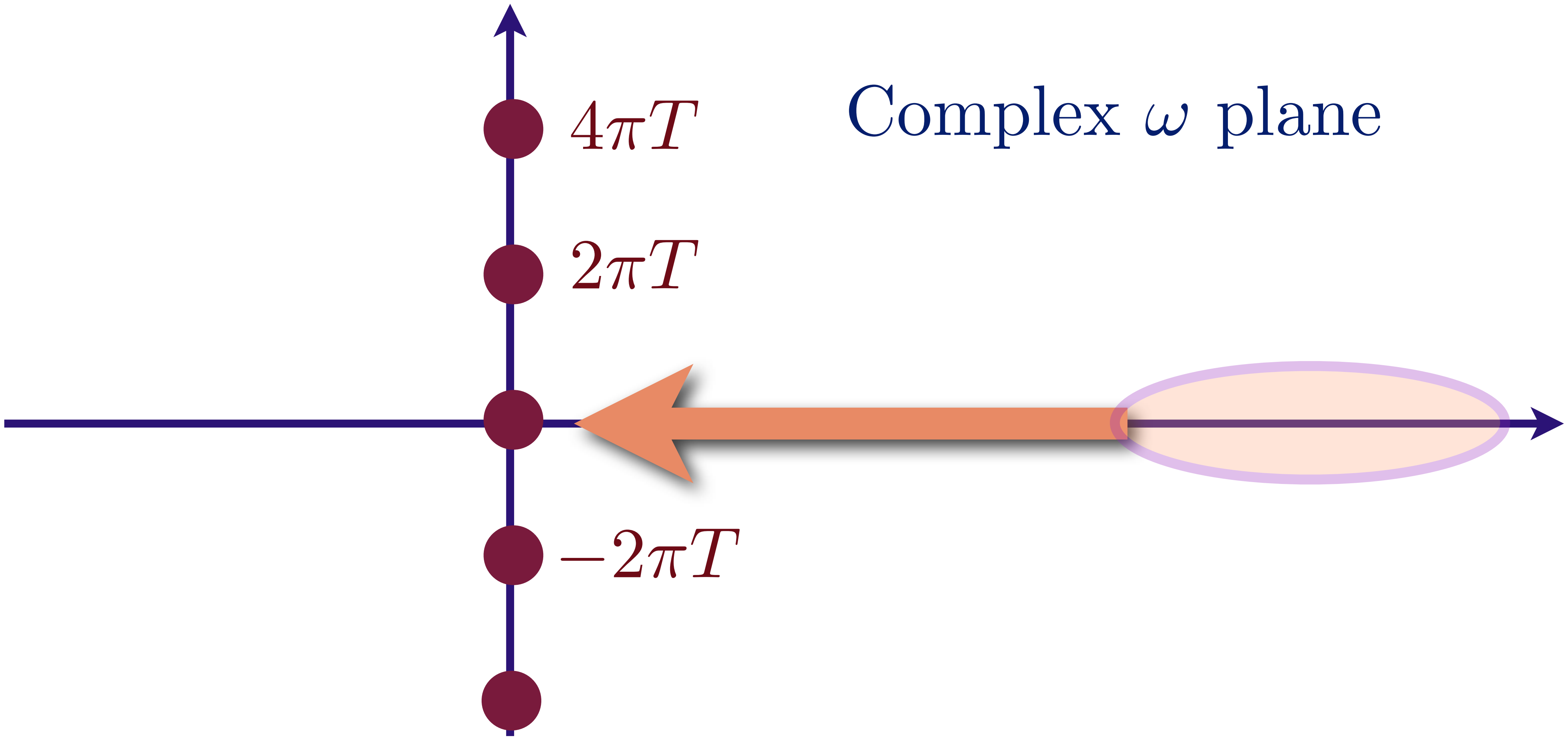

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Density correlations in CFTs at $T>0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k\to 0} \frac{-i\omega}{k^2}\chi(k,\omega)$

For *all* CFT3s, at $\underline{\hbar\omega \ll k_B T}$, we have the Einstein relation

$$\chi(k,\omega) \quad = \quad 4e^2\chi_c \frac{Dk^2}{Dk^2 - i\omega} \quad ; \quad \sigma(\omega) = 4e^2 D\chi_c = \frac{4e^2}{h}\Theta_1\Theta_2$$

where the **compressibility**, χ_c, and the **diffusion constant** D obey

$$\chi = \frac{k_B T}{(hv)^2}\Theta_1 \quad ; \quad D = \frac{hv^2}{k_B T}\Theta_2$$

with Θ_1 and Θ_2 universal numbers characteristic of the CFT3

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Density correlations in CFTs at $T>0$

In CFT3s collisions are "phase" randomizing, and lead to relaxation to local thermodynamic equilibrium. So there is a crossover from collisionless behavior for $\hbar\omega \gg k_B T$, to hydrodynamic behavior for $\hbar\omega \ll k_B T$.

$$\sigma(\omega) = \begin{cases} \dfrac{4e^2}{h} K & , \quad \hbar\omega \gg k_B T \\[2em] \dfrac{4e^2}{h}\Theta_1\Theta_2 \equiv \sigma_Q & , \quad \hbar\omega \ll k_B T \end{cases}$$

and in general we expect $K \neq \Theta_1\Theta_2$ (verified for Wilson-Fisher fixed point).

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

The AdS/CFT correspondence

Field theories in D spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u\frac{dg}{du} = \beta(g)$$

where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u.

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Key idea: $\Rightarrow$ Implement u as an extra dimension, and map to a local theory in $D+1$ dimensions.

At the RG fixed point, $\beta(g) = 0$, the D dimensional field theory is invariant under the scale transformation

$$x^\mu \longrightarrow x^\mu / b \quad , \quad u \longrightarrow b\, u$$

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This is an invariance of the *metric* of the theory in $D+1$ dimensions. The unique solution is

$$ds^2 = \left(\frac{u}{L}\right)^2 dx^\mu dx_\mu + L^2 \frac{du^2}{u^2}.$$

Or, using the length scale $z = L^2/u$

$$ds^2 = L^2 \frac{dx^\mu dx_\mu + dz^2}{z^2}.$$

This is the space AdS_{D+1}, and L is the AdS radius.

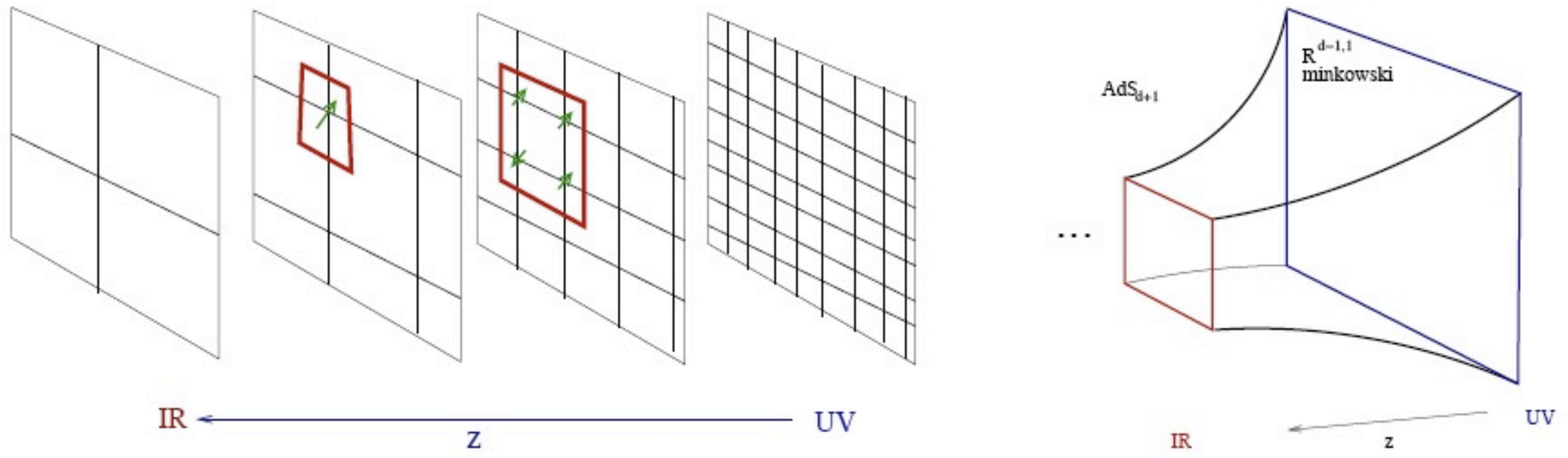

Figure 1: The extra ('radial') dimension of the bulk is the resolution scale of the field theory. The left figure indicates a series of block spin transformations labelled by a parameter z. The right figure is a cartoon of AdS space, which organizes the field theory information in the same way. In this sense, the bulk picture is a hologram: excitations with different wavelengths get put in different places in the bulk image.

J. McGreevy, arXiv0909.0518

Bonus: AdS_{D+1} is a solution of Einstein's equations with a negative cosmological constant, and is a symmetric space; the full group of symmetries of the metric is $\mathrm{SO}(D+1,1)$ (in Euclidean signature)

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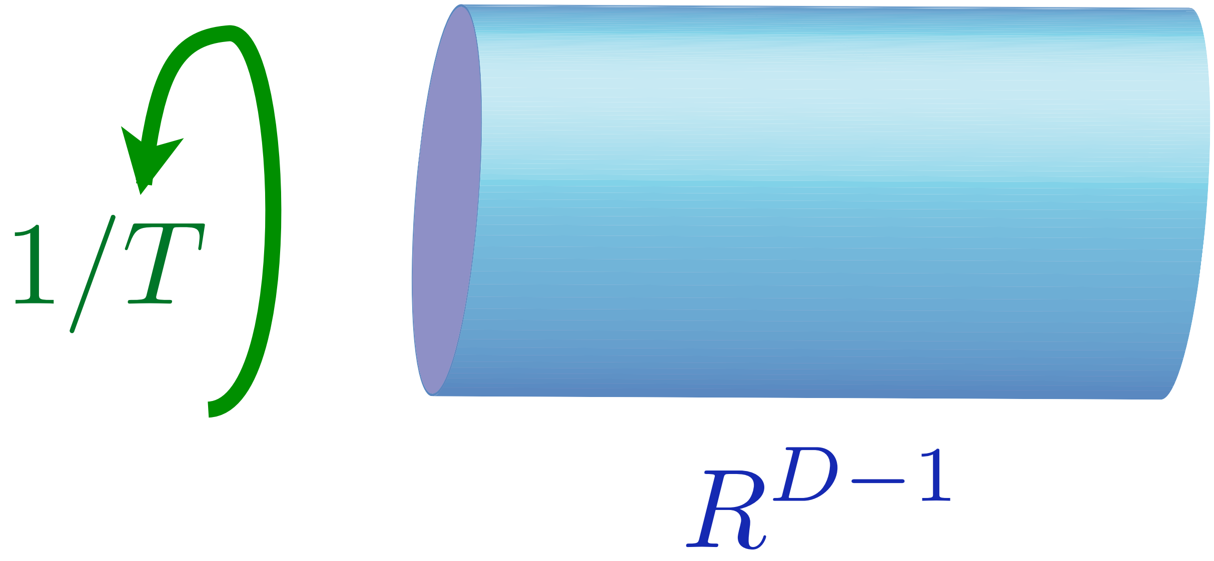
$\mathrm{SO}(D+1,1)$ is the group of conformal transformations in D dimensions, and relativistic field theories at the RG fixed point are conformally invariant.

At $T > 0$, the Euclidean field theory is on the cylinder $R^{D-1} \times S^1$, where the time co-ordinate is periodic under $\tau \to \tau + 1/T$.



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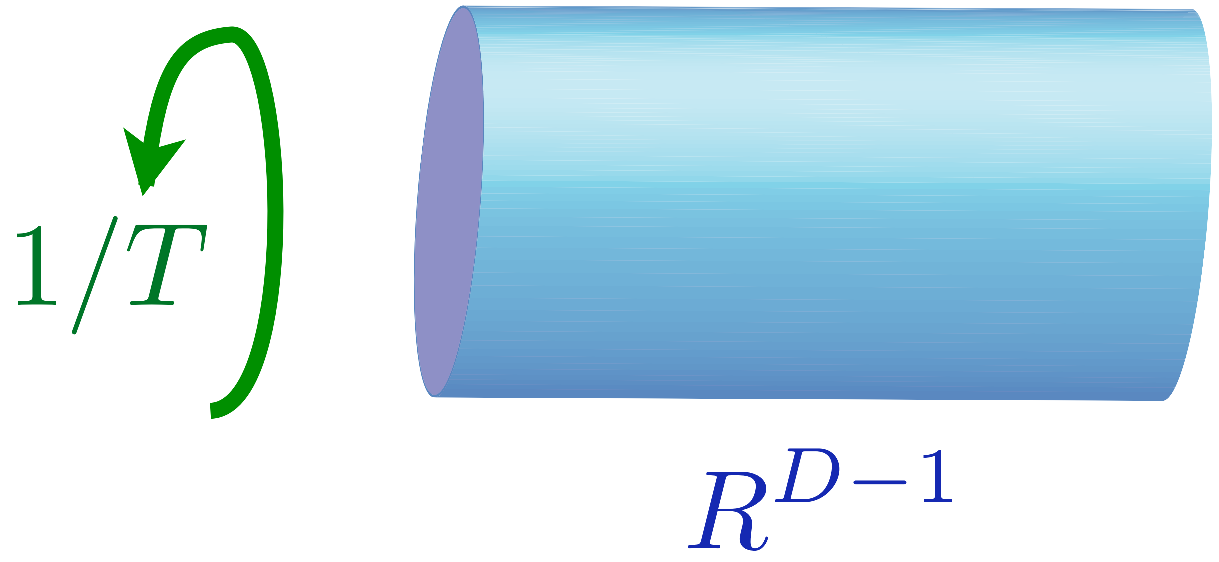


Solving Einstein's equations with a negative cosmological constant we have the solution

$$ds^2 = \frac{L^2}{z^2}\left(f(z)d\tau^2 + d\vec{x}^2 + \frac{dz^2}{f(z)}\right) \quad ; \quad f(z) = 1 - \left(\frac{z}{z_H}\right)^D$$

This is a AdS-Schwarzschild black hole with a horizon at $z = z_H$. This space is periodic in τ with period $1/T$ for

$$T = \frac{d}{4\pi z_H}$$

SU(N) SYM3 with $\mathcal{N} = 8$ supersymmetry

- Has a single dimensionful coupling constant, e_0, which flows to a strong-coupling fixed point $e_0 = e_0^*$ in the infrared.
- The CFT3 describing this fixed point resembles "critical spin liquid" theories.
- This CFT3 is the low energy limit of string theory on an M2 brane. The AdS/CFT correspondence provides a dual description using 11-dimensional supergravity on $\text{AdS}_4 \times S_7$.
- The CFT3 has a global SO(8) R symmetry, and correlators of the SO(8) charge density can be computed exactly in the large N limit, even at $T > 0$.

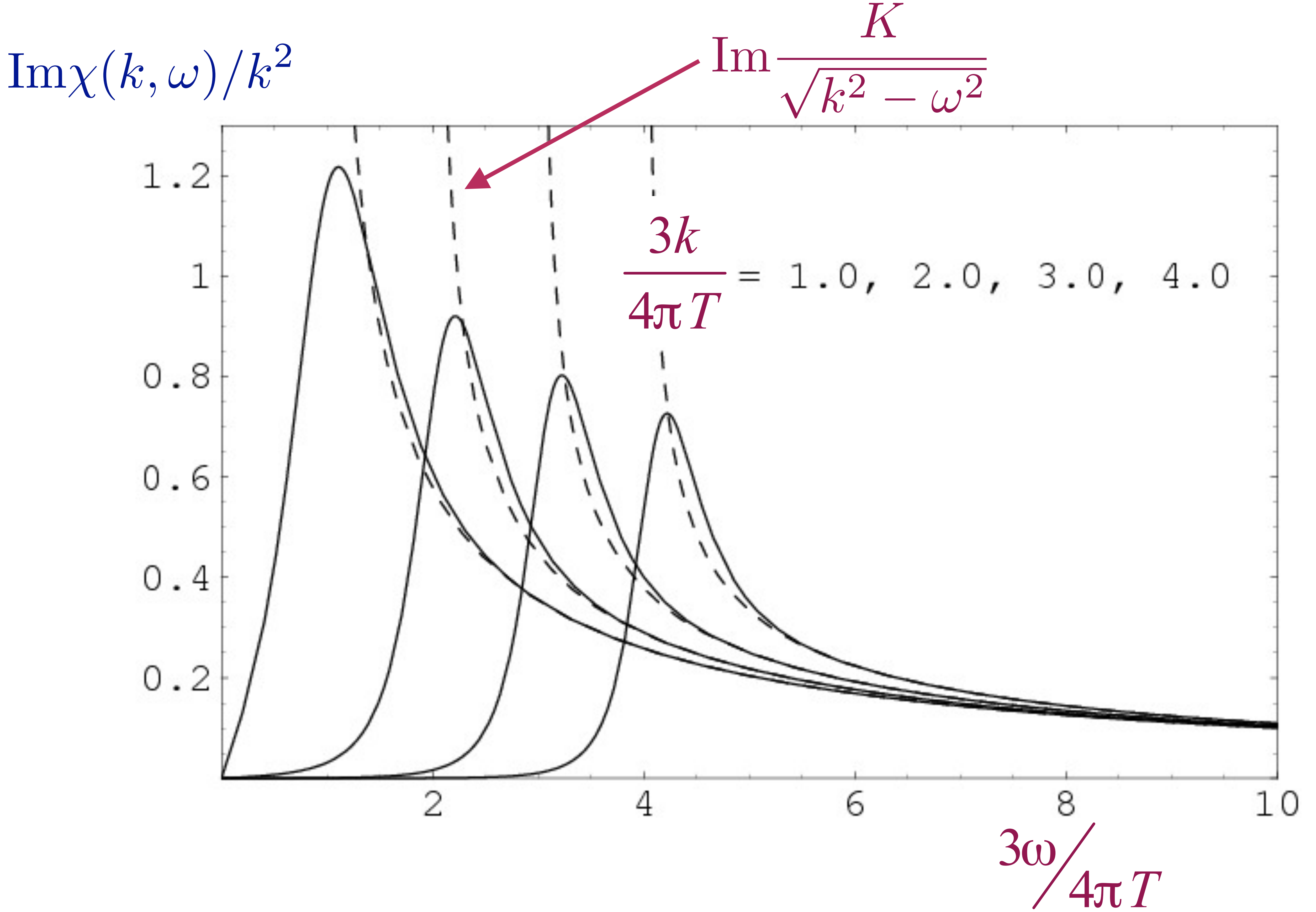
SU(N) SYM3 with $\mathcal{N}=8$ supersymmetry

- The SO(8) charge correlators of the CFT3 are given by the usual AdS/CFT prescription applied to the following gauge theory on AdS4:

  $$\mathcal{S} = -\frac{1}{4g_{4D}^2}\int d^4x\sqrt{-g}g^{MA}g^{NB}F^a_{MN}F^a_{AB}$$

  where $a = 1 \ldots 28$ labels the generators of SO(8). Note that in large N theory, this looks like 28 copies of an Abelian gauge theory.

Collisionless to hydrodynamic crossover of SYM3



P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

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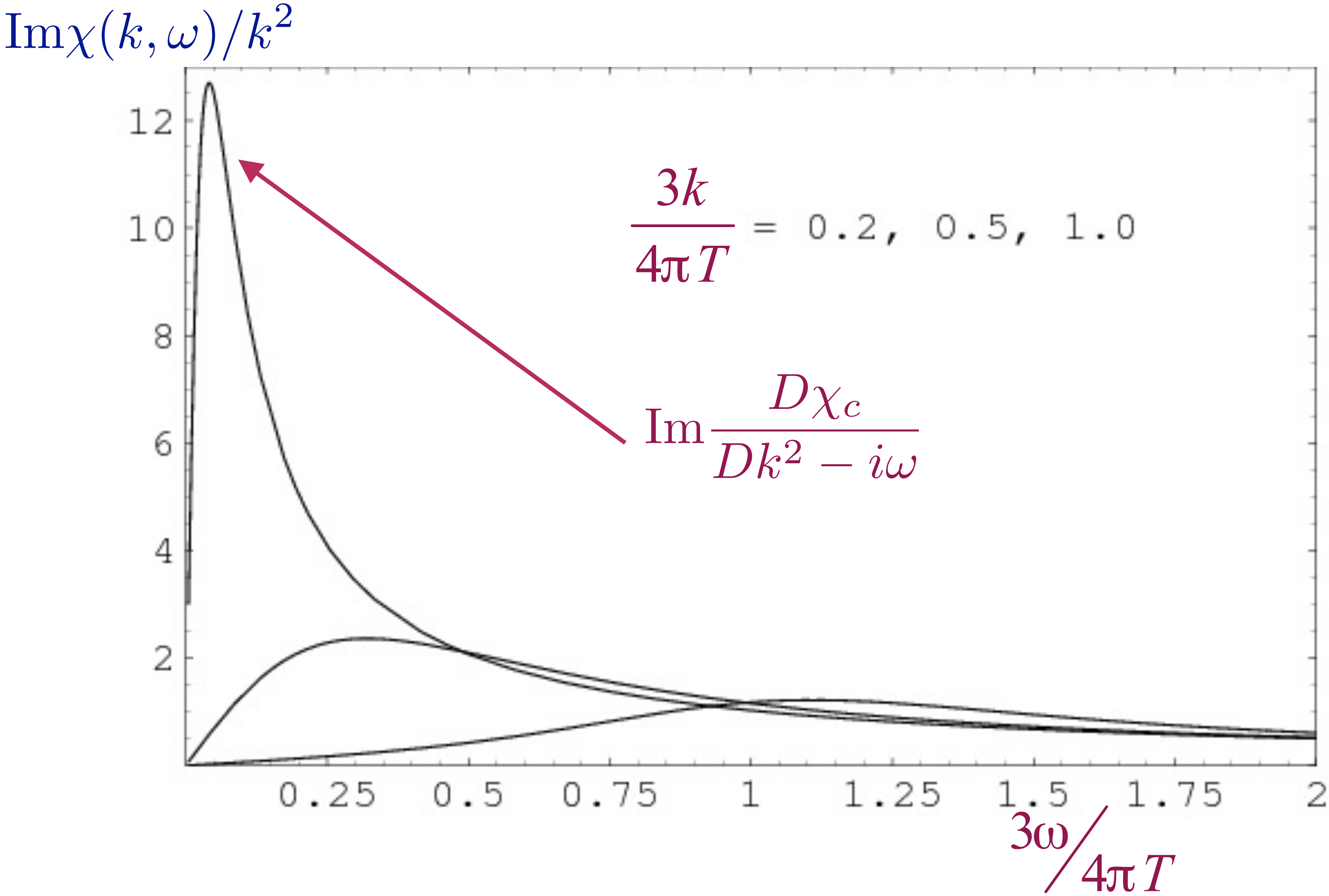

P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

Universal constants of SYM3

$$\chi_c = \frac{k_B T}{(hv)^2}\Theta_1$$

$$D = \frac{hv^2}{k_B T}\Theta_2$$

$$\sigma(\omega) = \begin{cases} \frac{4e^2}{h}K & , \quad \hbar\omega \gg k_B T \\ \frac{4e^2}{h}\Theta_1\Theta_2 & , \quad \hbar\omega \ll k_B T \end{cases}$$

$$\boxed{\begin{aligned} K &= \frac{\sqrt{2}N^{3/2}}{3} \\ \Theta_1 &= \frac{8\pi^2\sqrt{2}N^{3/2}}{9} \\ \Theta_2 &= \frac{3}{8\pi^2} \end{aligned}}$$

C. Herzog, JHEP **0212**, 026 (2002)

P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

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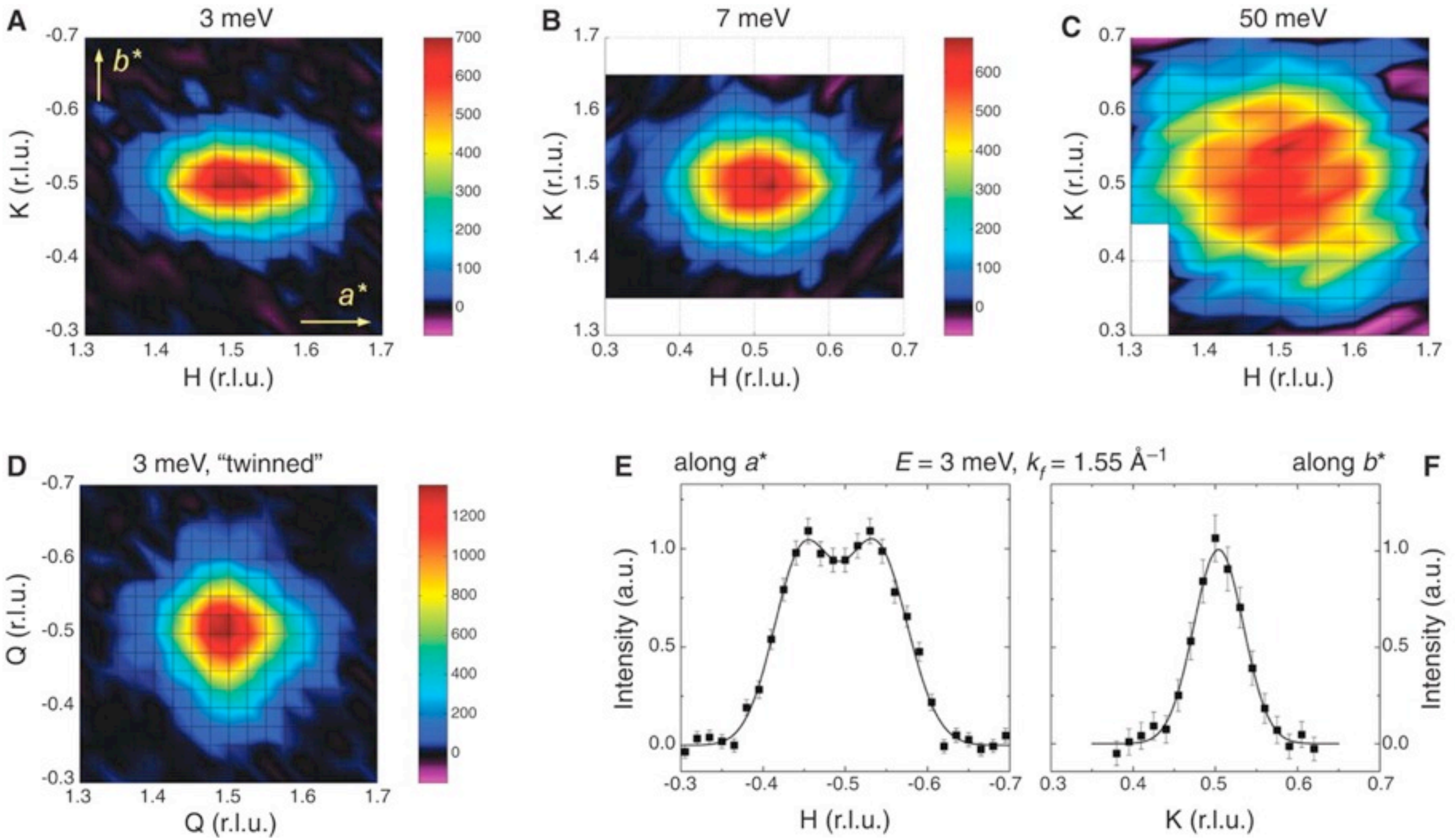
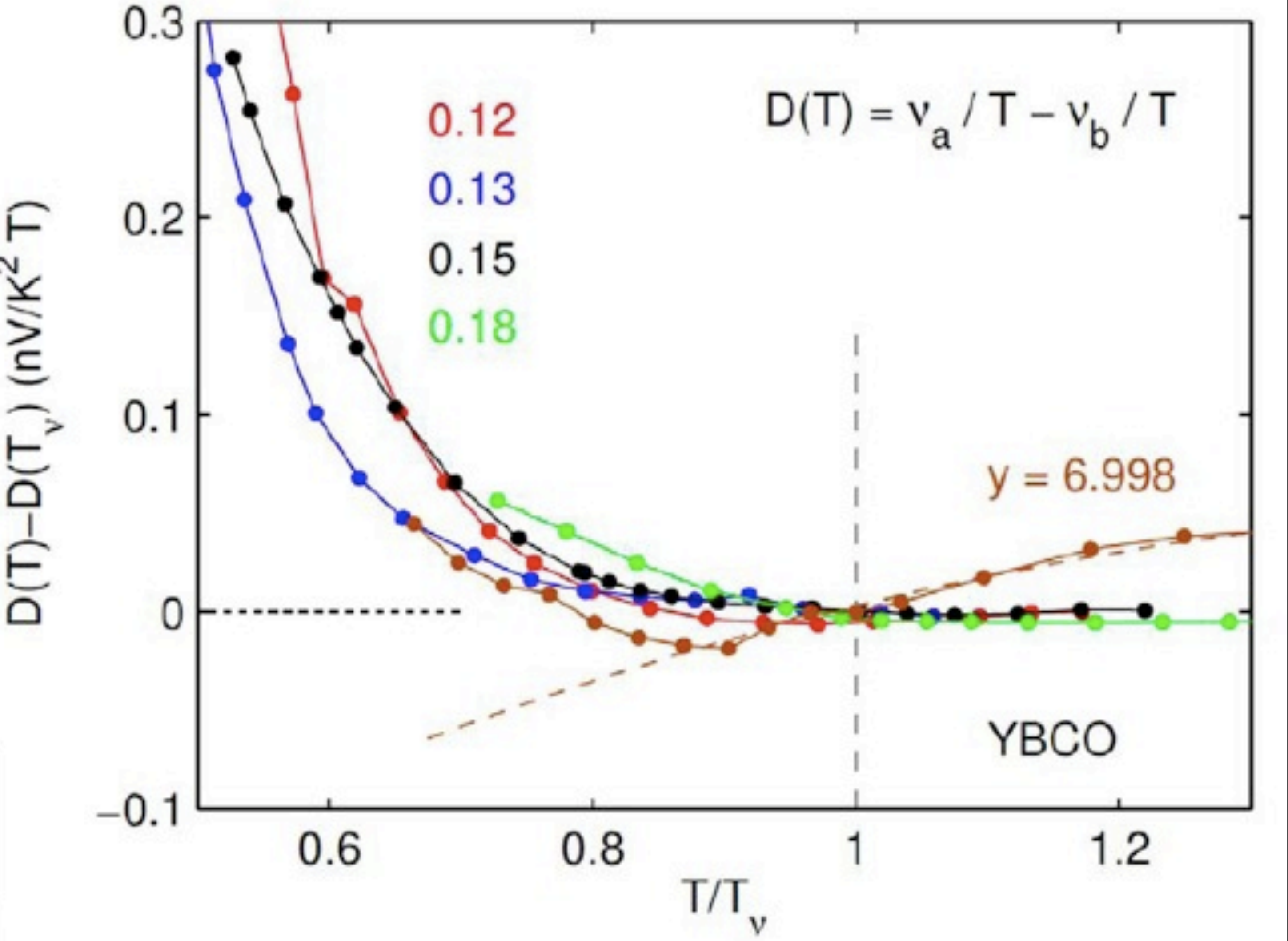


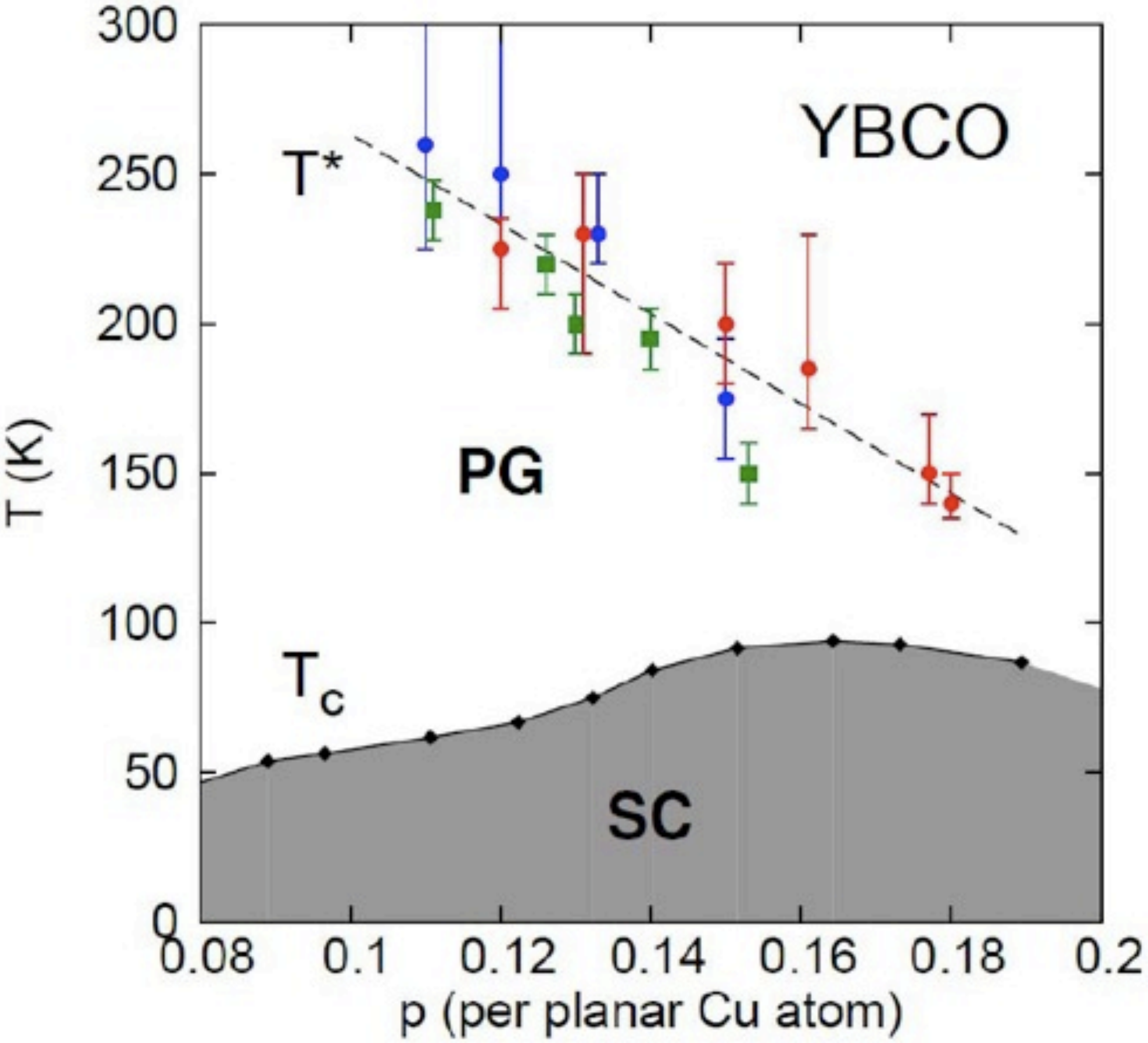
Nematic order in YBCO

V. Hinkov, D. Haug, B. Fauqué, P. Bourges, Y. Sidis, A. Ivanov,
C. Bernhard, C. T. Lin, and B. Keimer , *Science* **319**, 597 (2008)

Broken rotational symmetry in the pseudogap phase of a high-Tc superconductor

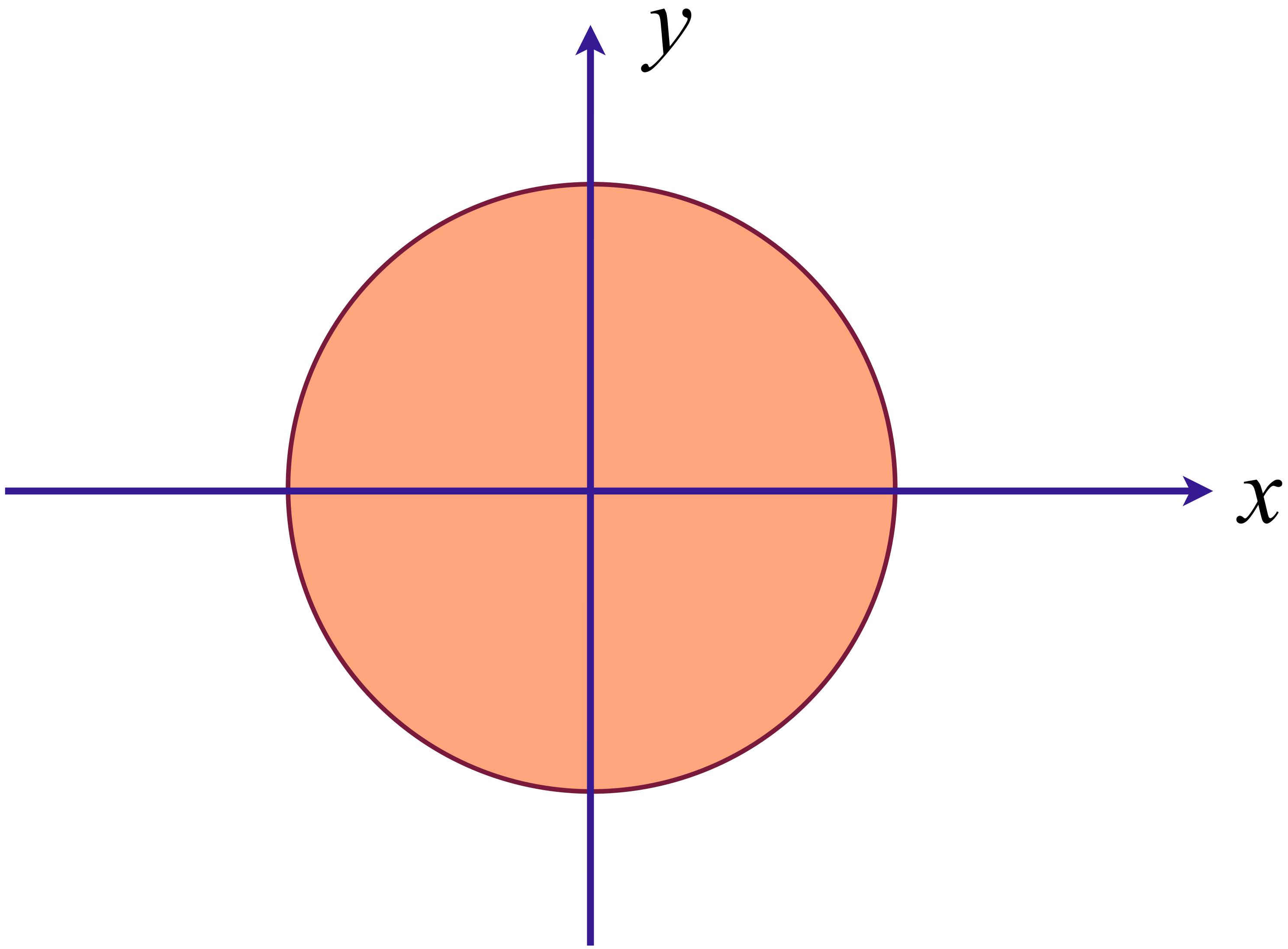
R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D. A. Bonn, W. N. Hardy, and Louis Taillefer
arXiv: 0909.4430





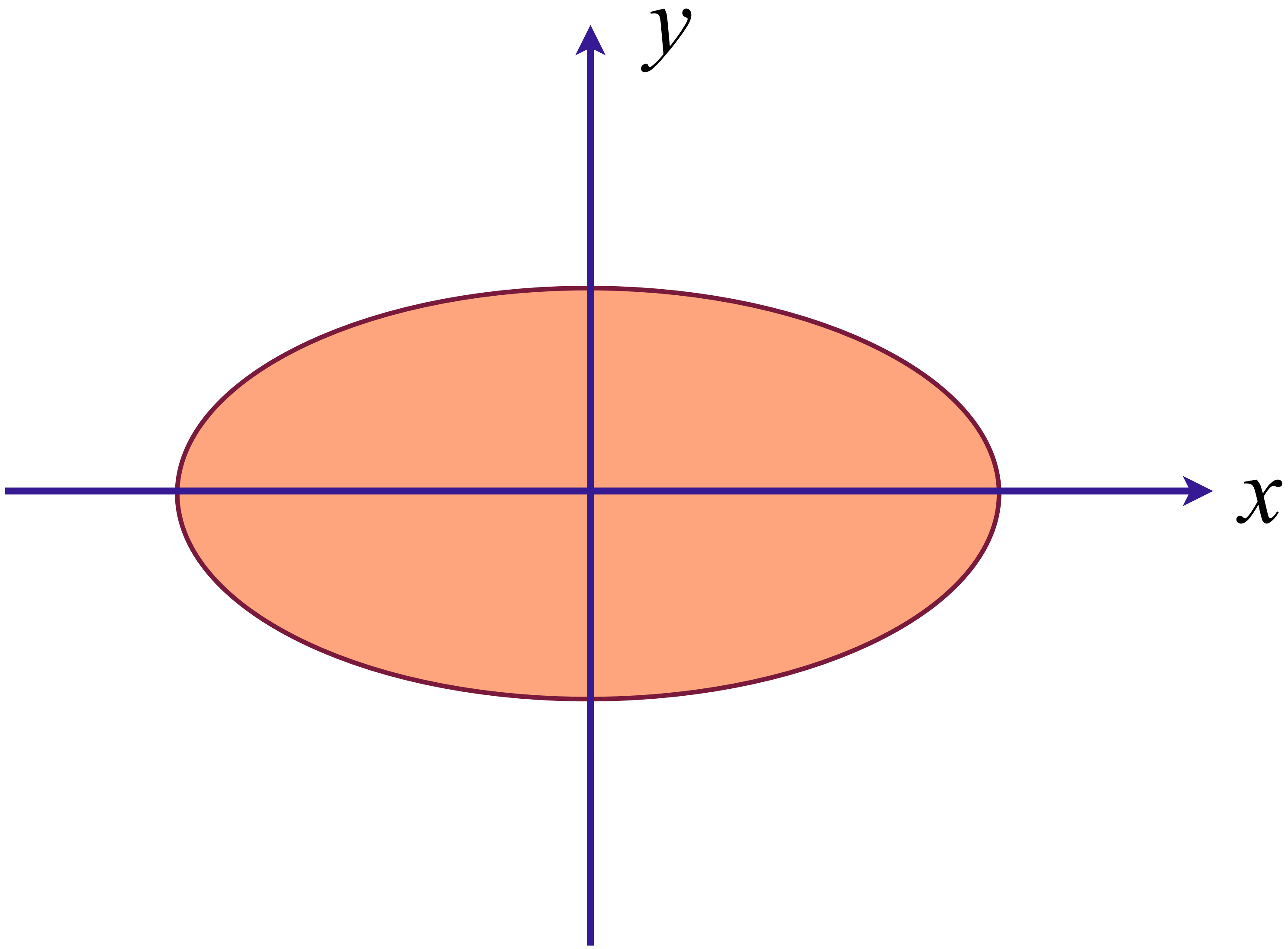
S. A. Kivelson, E. Fradkin, and V. J. Emery, *Nature* **393**, 550 (1998).

Quantum criticality of Pomeranchuk instability



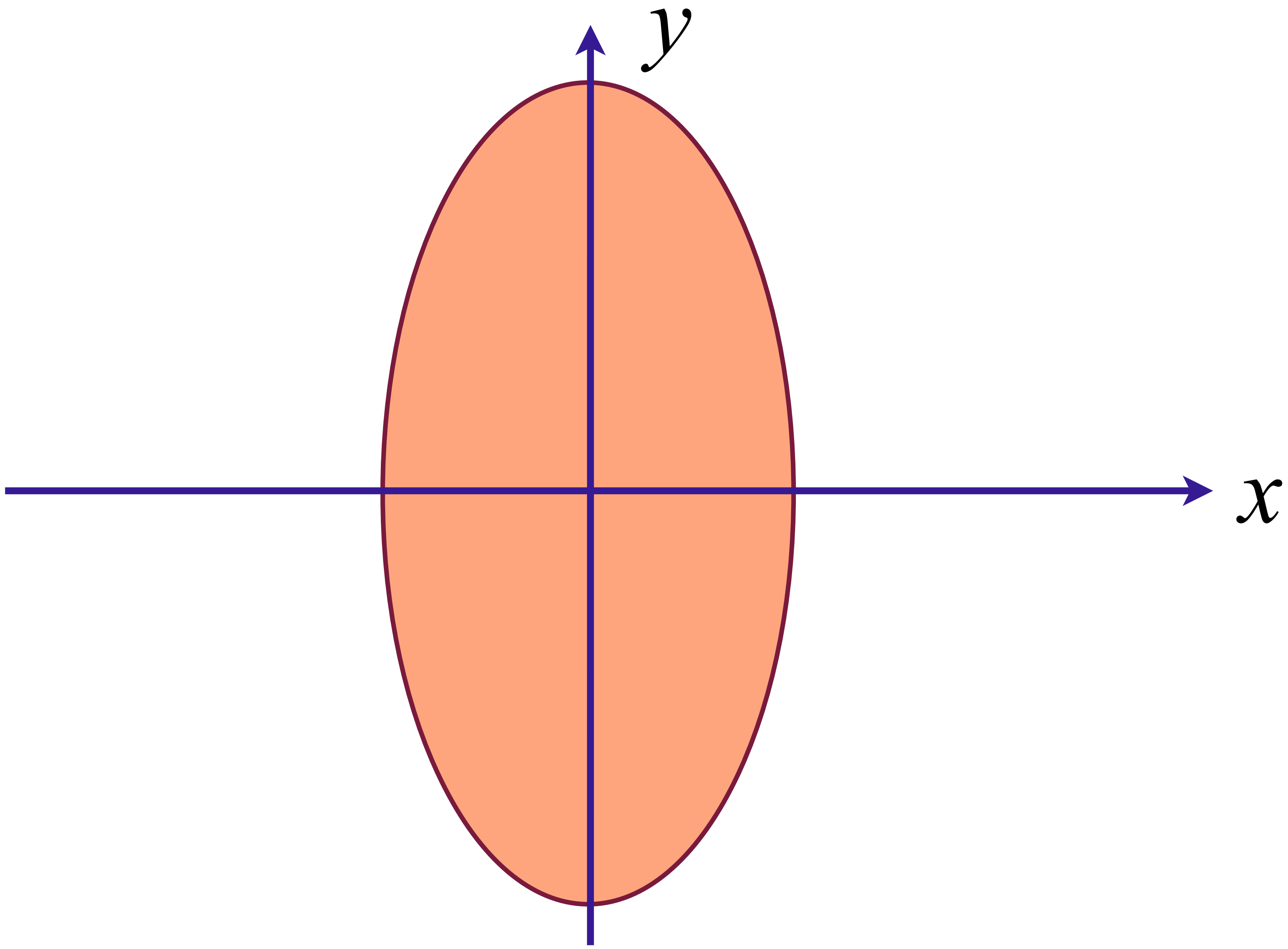
Fermi surface with full square lattice symmetry

Quantum criticality of Pomeranchuk instability



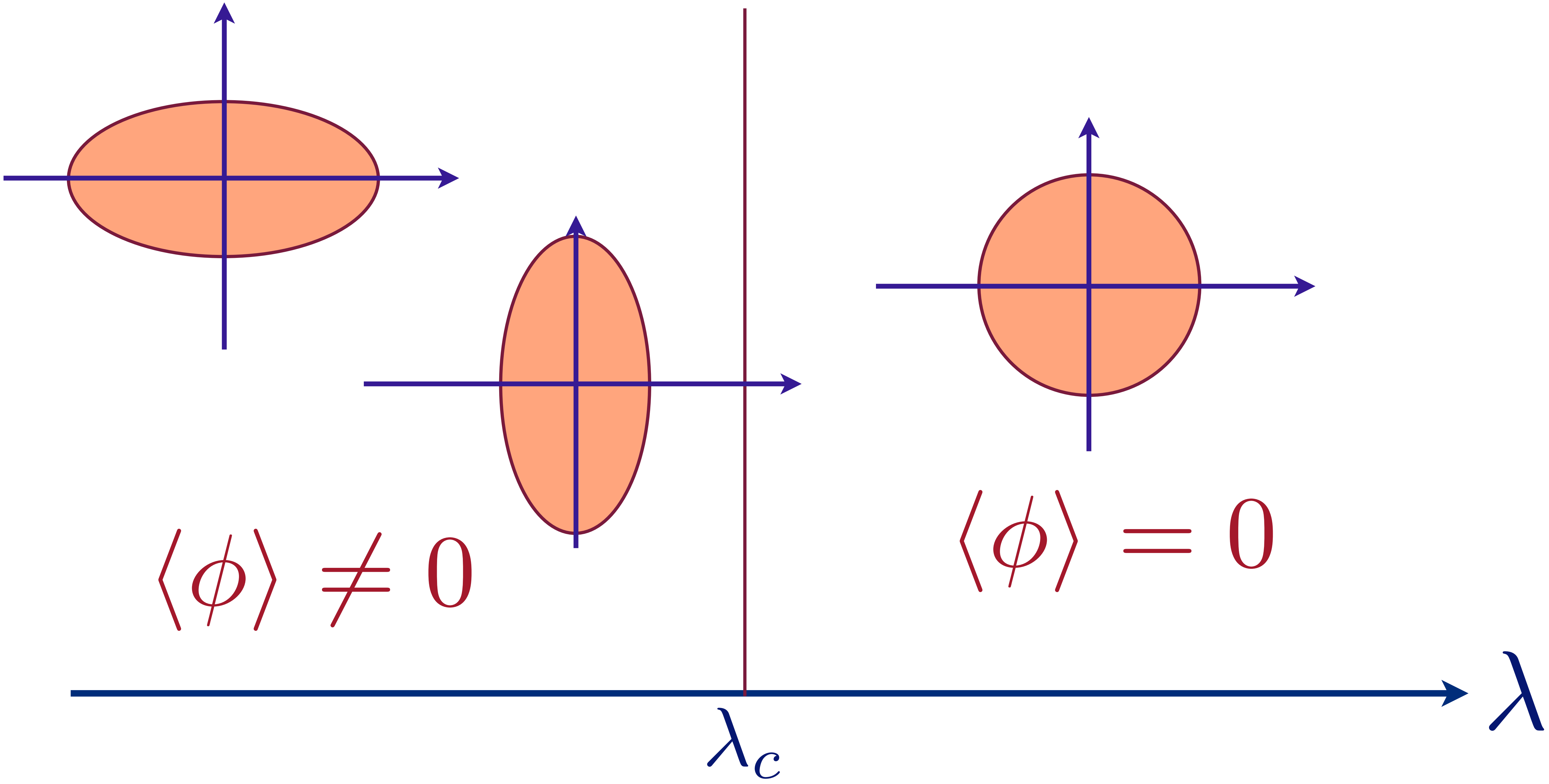
Spontaneous elongation along x direction:
Ising order parameter $\phi > 0$.

Quantum criticality of Pomeranchuk instability



Spontaneous elongation along y direction:
Ising order parameter $\phi < 0$.

Quantum criticality of Pomeranchuk instability



$\langle \phi \rangle \neq 0$

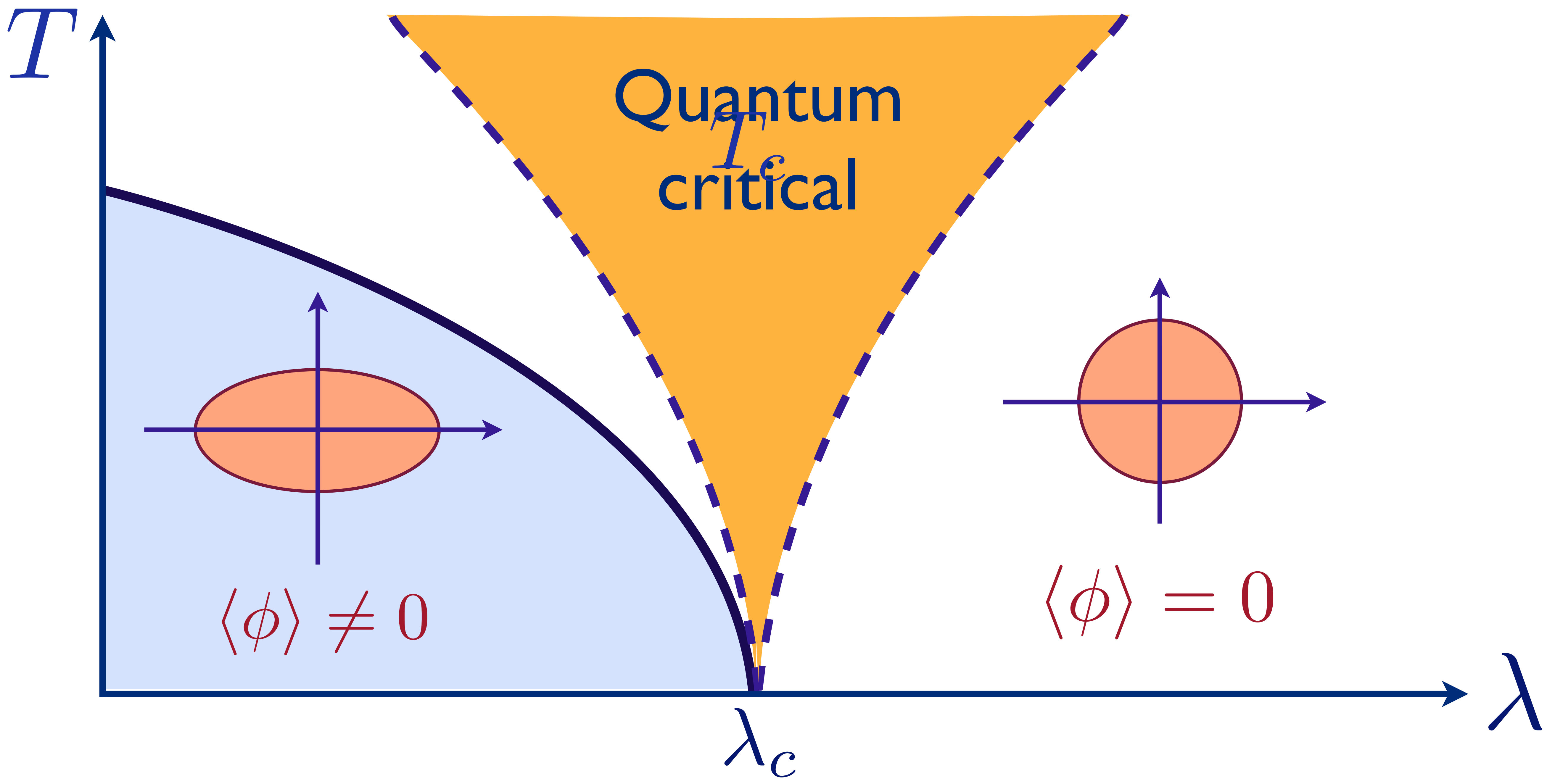
$\langle \phi \rangle = 0$

λ

λ_c

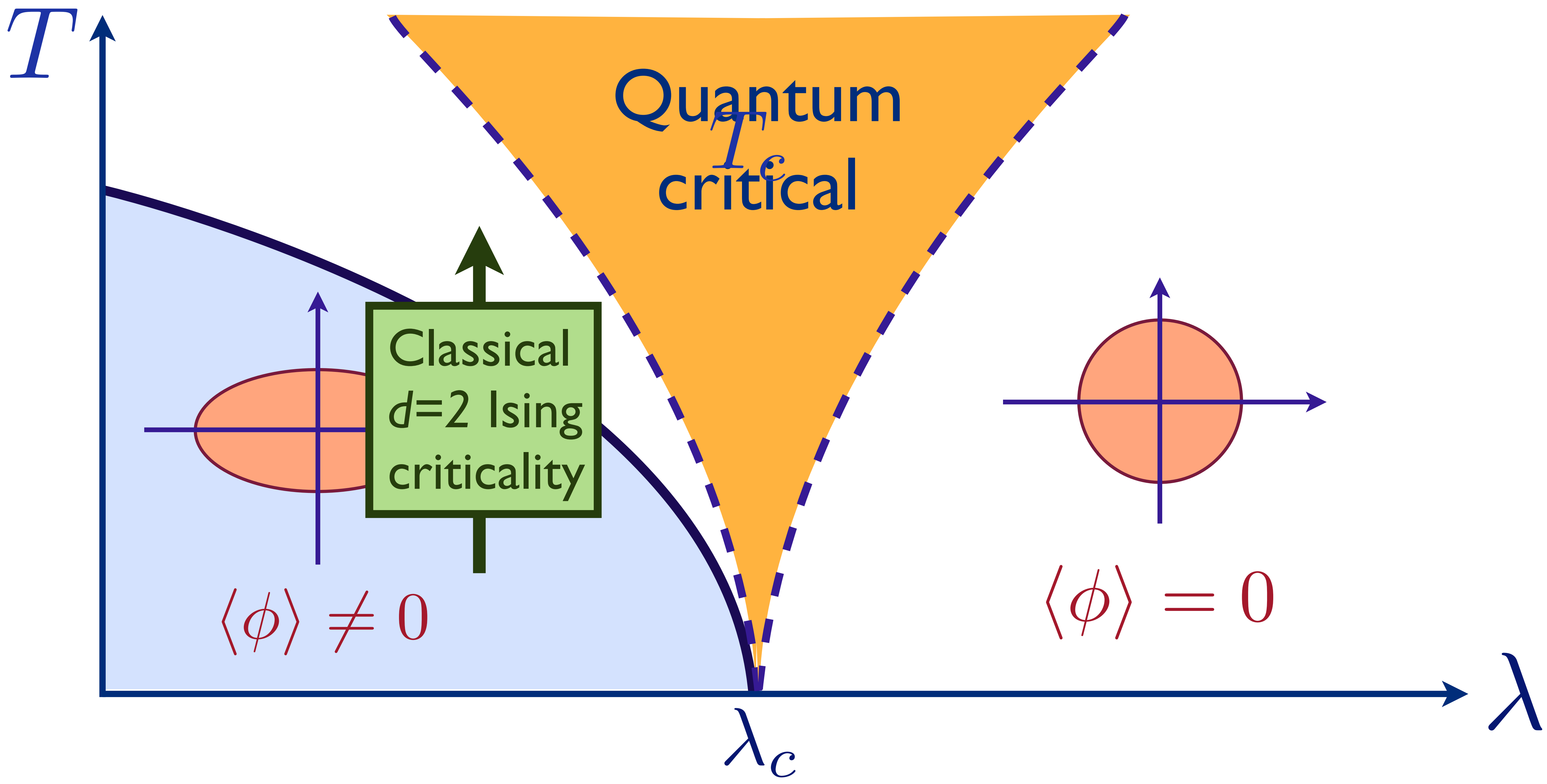
Pomeranchuk instability as a function of coupling λ

Quantum criticality of Pomeranchuk instability



Phase diagram as a function of T and λ

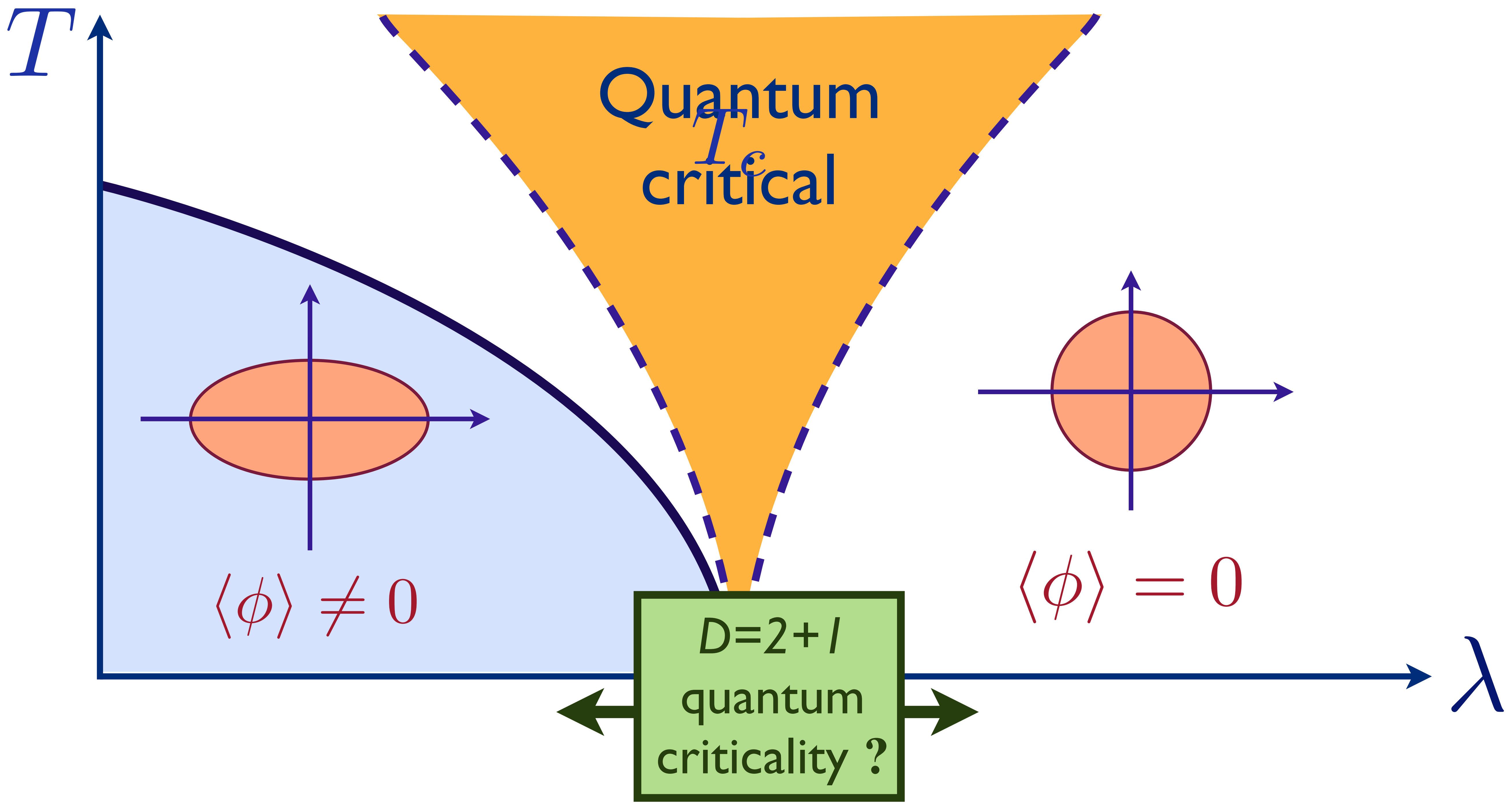
Quantum criticality of Pomeranchuk instability



Classical
d=2 Ising
criticality

Phase diagram as a function of T and λ

Quantum criticality of Pomeranchuk instability



Phase diagram as a function of T and λ

Quantum criticality of Pomeranchuk instability

Effective action for Ising order parameter

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4 \right]$$

Quantum criticality of Pomeranchuk instability

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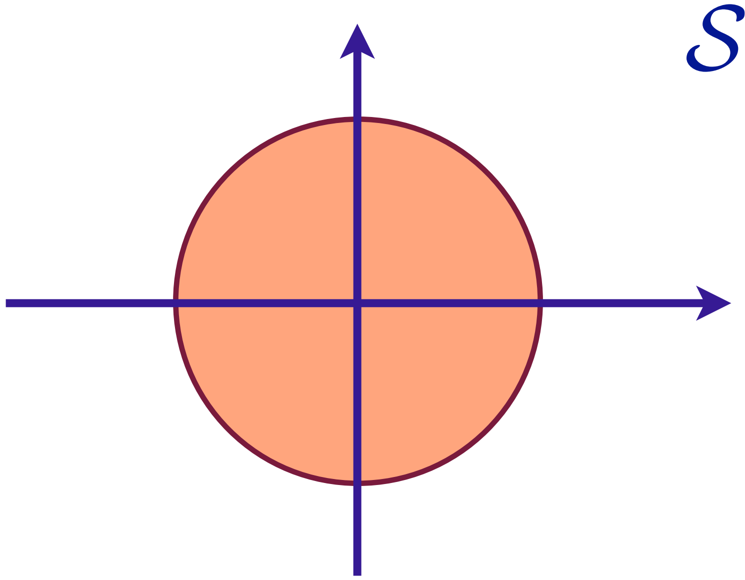
Effective action for electrons:



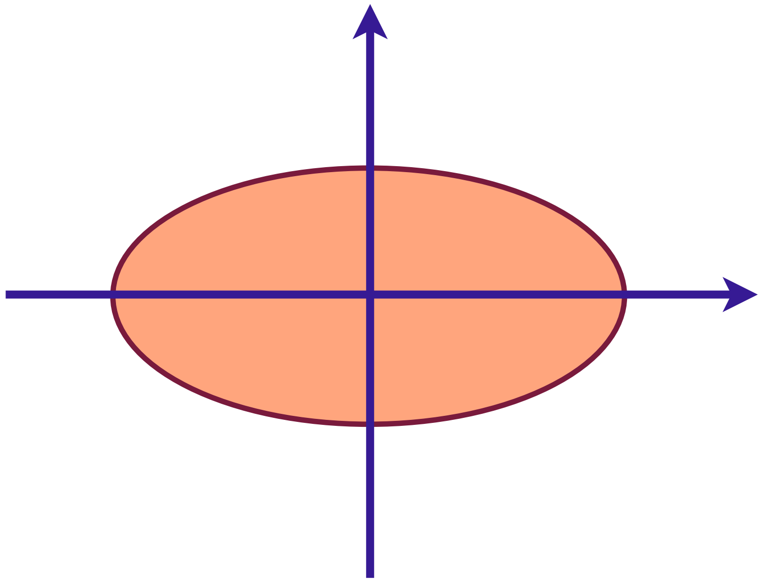
$$\begin{aligned} \mathcal{S}_c &= \int d\tau \sum_{\alpha=1}^{N_f} \left[\sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \right] \\ &\equiv \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha} \end{aligned}$$

Quantum criticality of Pomeranchuk instability

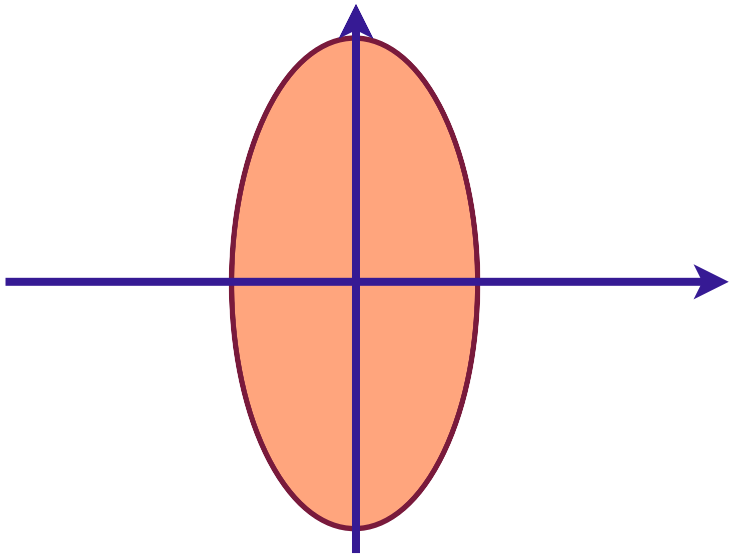
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \, \phi \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} (\cos k_x - \cos k_y) c_{\mathbf{k}\alpha}^{\dagger} c_{\mathbf{k}\alpha}$$

for spatially independent ϕ



$$\langle \phi \rangle > 0$$



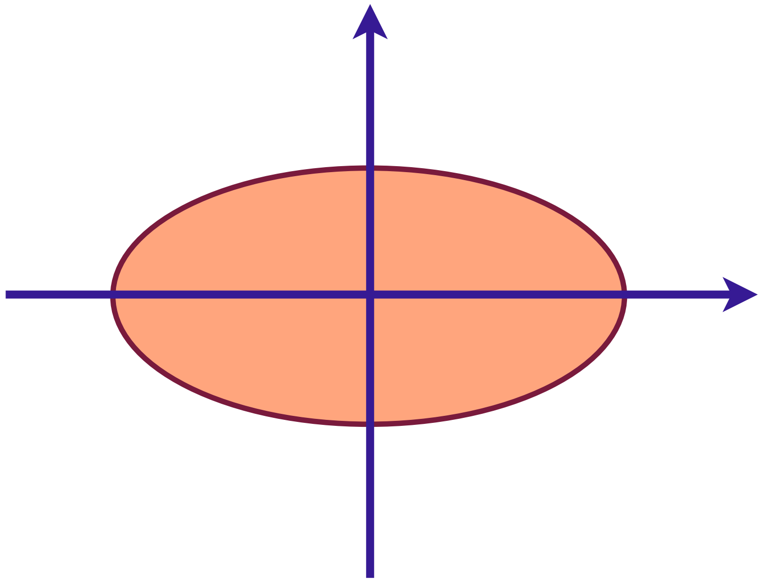
$$\langle \phi \rangle < 0$$

Quantum criticality of Pomeranchuk instability

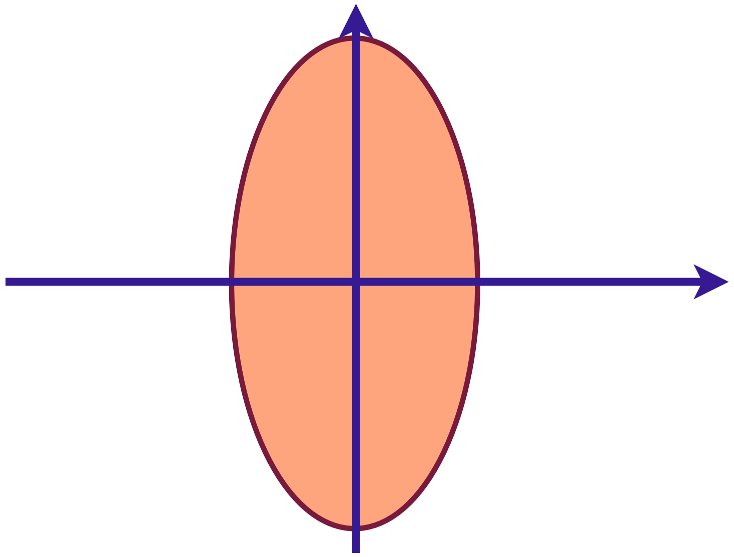
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y\right) c^\dagger_{\mathbf{k}+\mathbf{q}/2,\alpha} c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

for spatially dependent ϕ



$$\langle \phi \rangle > 0$$



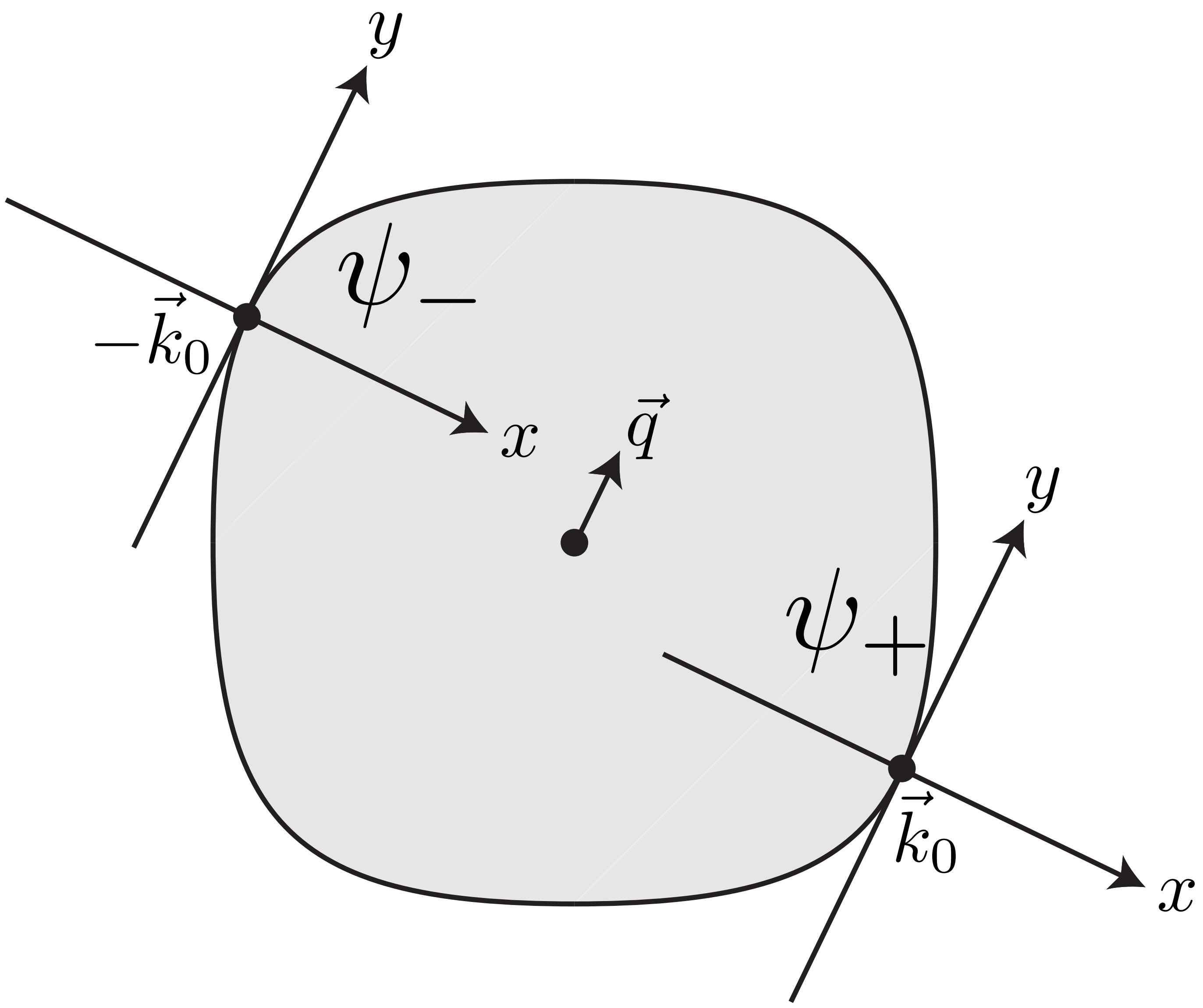
$$\langle \phi \rangle < 0$$

Quantum criticality of Pomeranchuk instability

$$\mathcal{S}_\phi = \int d^2 r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4 \right]$$

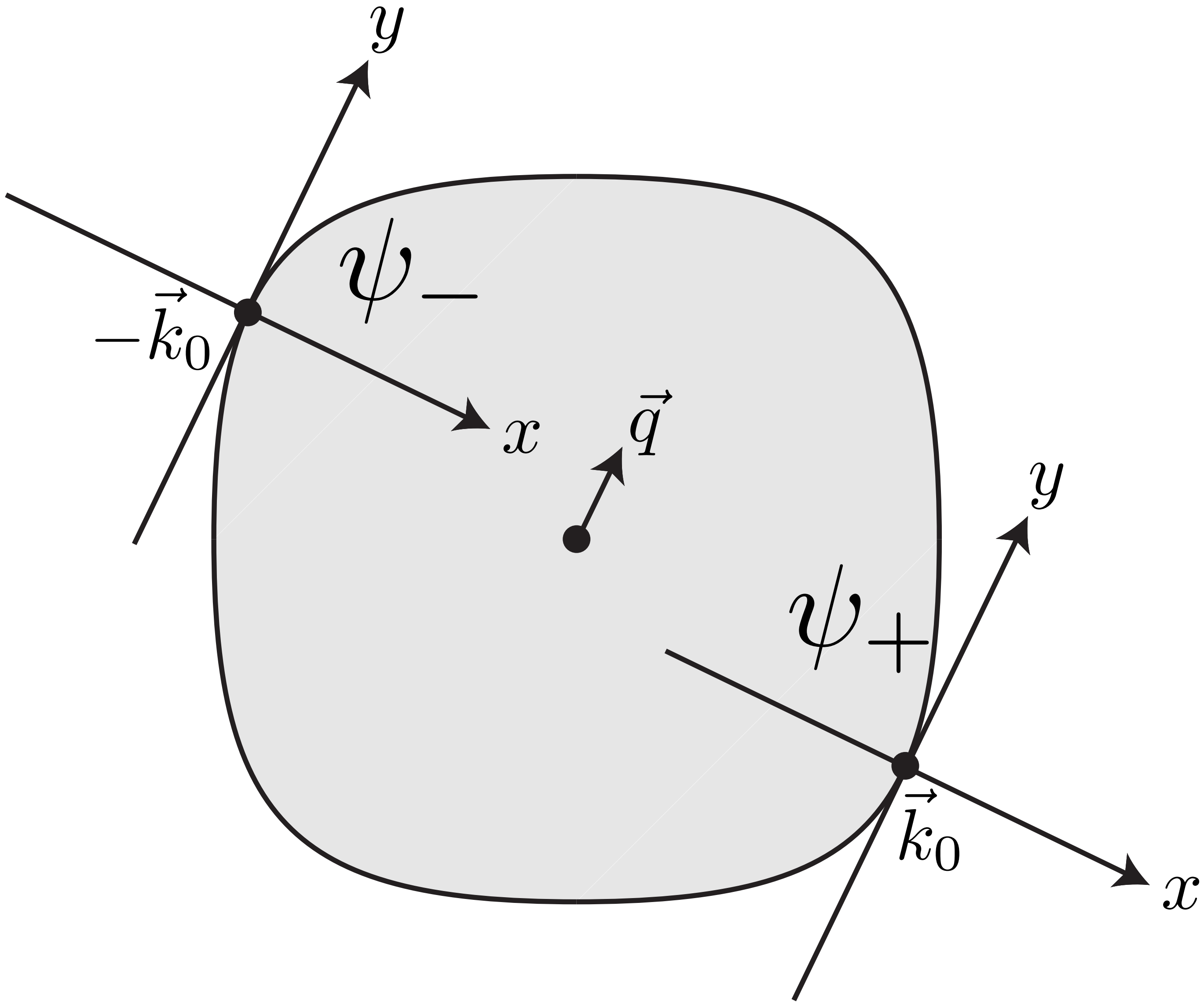
$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y \right) c_{\mathbf{k}+\mathbf{q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

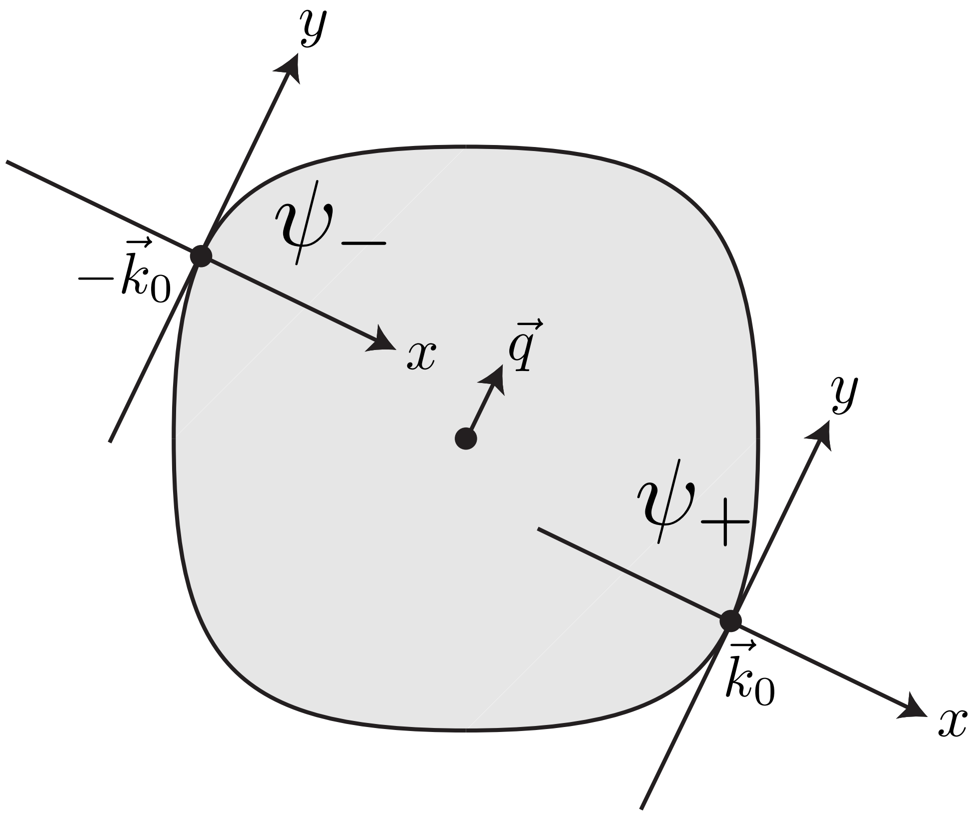


A ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

Expand fermion kinetic energy at wavevectors about $\vec{k}_0$



$$
\begin{aligned}
\mathcal{L} \quad = \quad & \psi_+^\dagger \left(\zeta \partial_\tau - i \partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\zeta \partial_\tau + i \partial_x - \partial_y^2\right) \psi_- \\
& - \lambda \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g} \left(\partial_y \phi\right)^2 + \frac{r}{2} \phi^2
\end{aligned}
$$

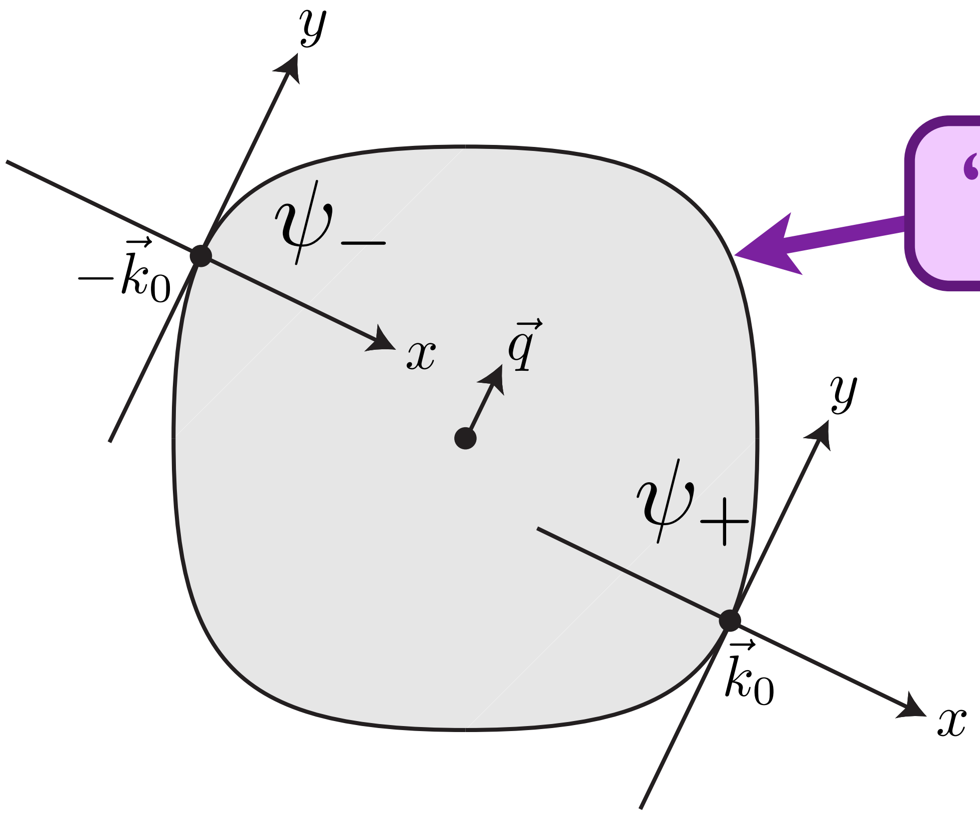


Emergent "Galilean invariance" at low energy ($s = \pm$):

$$\phi(x,y) \to \phi(x, y+\theta x), \quad \psi_s(x,y) \to e^{-is(\frac{\theta}{2}y + \frac{\theta^2}{4}x)}\psi_s(x, y+\theta x)$$

which implies for the fermion Green's function

$$G(q_x, q_y) = G(sq_x + q_y^2).$$



“Hot” Fermi surfaces

Emergent “Galilean invariance” at low energy ($s = \pm$):

$$\phi(x, y) \rightarrow \phi(x, y + \theta x), \quad \psi_s(x, y) \rightarrow e^{-is(\frac{\theta}{2}y + \frac{\theta^2}{4}x)}\psi_s(x, y + \theta x)$$

which implies for the fermion Green’s function

$$G(q_x, q_y) = G(sq_x + q_y^2).$$

Every point on the Fermi surface $sq_x + q_y^2 = 0$ has the same singularity: “Hot Fermi surface”.

$$\mathcal{L} = \psi_+^\dagger \left(\zeta\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\zeta\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - \lambda\phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g}\left(\partial_y\phi\right)^2 + \frac{r}{2}\phi^2$$

After tuning the single parameter $r \sim \lambda - \lambda_c$, and sending $\zeta \to 0$, $\mathcal{L}$ describes a critical theory with no coupling constants. There is a separate copy of this critical theory for each direction $\hat{q}$. This theory has 2 independent exponents z and η, and the correlation length and susceptibility exponents are given by

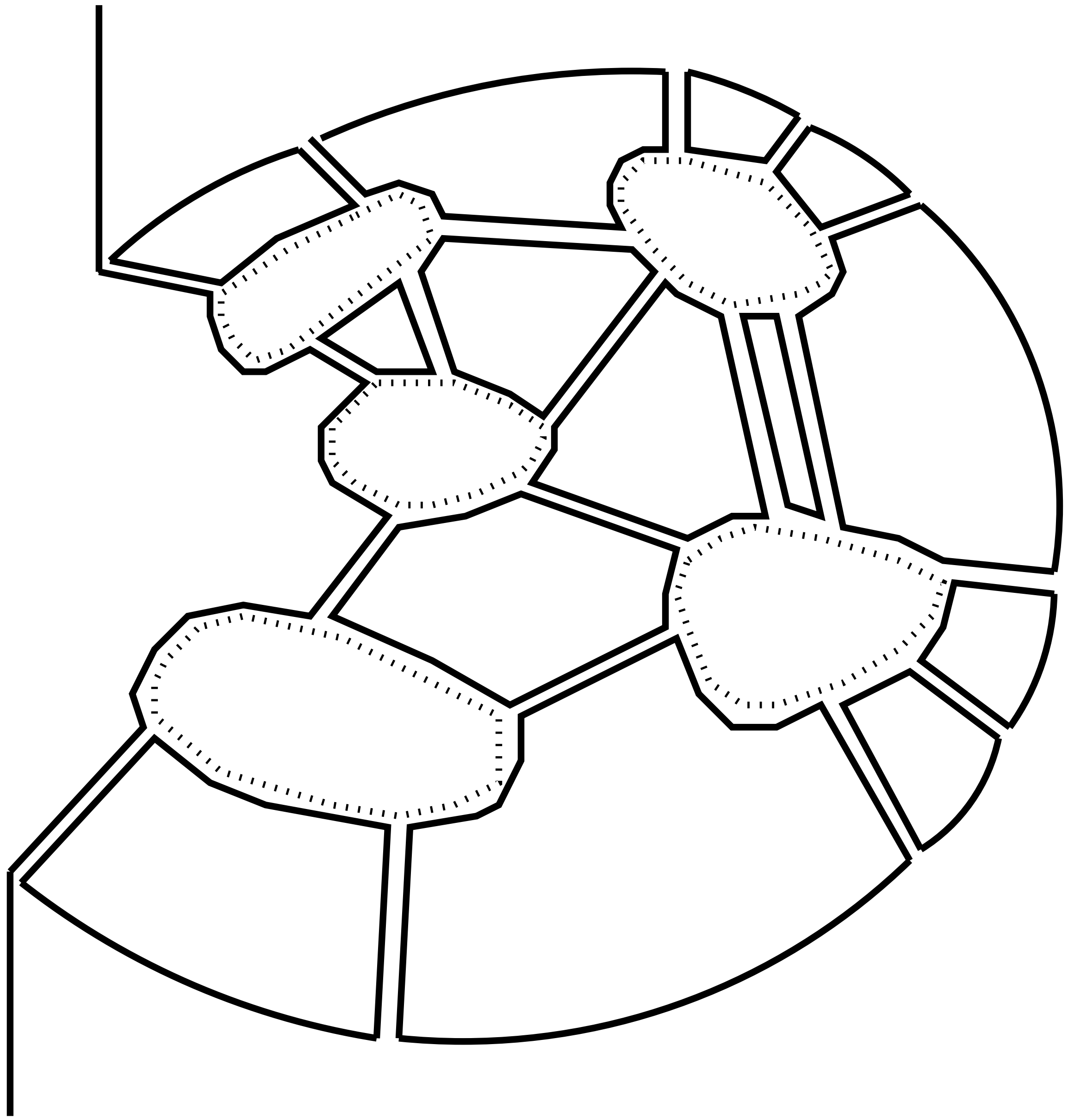
$$\nu = \frac{1}{z-1} \quad ; \quad \gamma = 1$$

The fermion and order parameter Green's functions obey the scaling forms

$$G(\vec{q},\omega) = \xi^{2-\eta}\Phi_\psi\Big((q_x+q_y^2)\xi^2, \omega\xi^z\Big) \quad ; \quad D(\vec{q},\omega) = \xi^{z-1}\Phi_\phi\Big(q_y\xi, \omega\xi^z\Big)$$

We have computed the exponents to three loops, and find $z = 3$ and $\eta = -0.0868$ at this order.

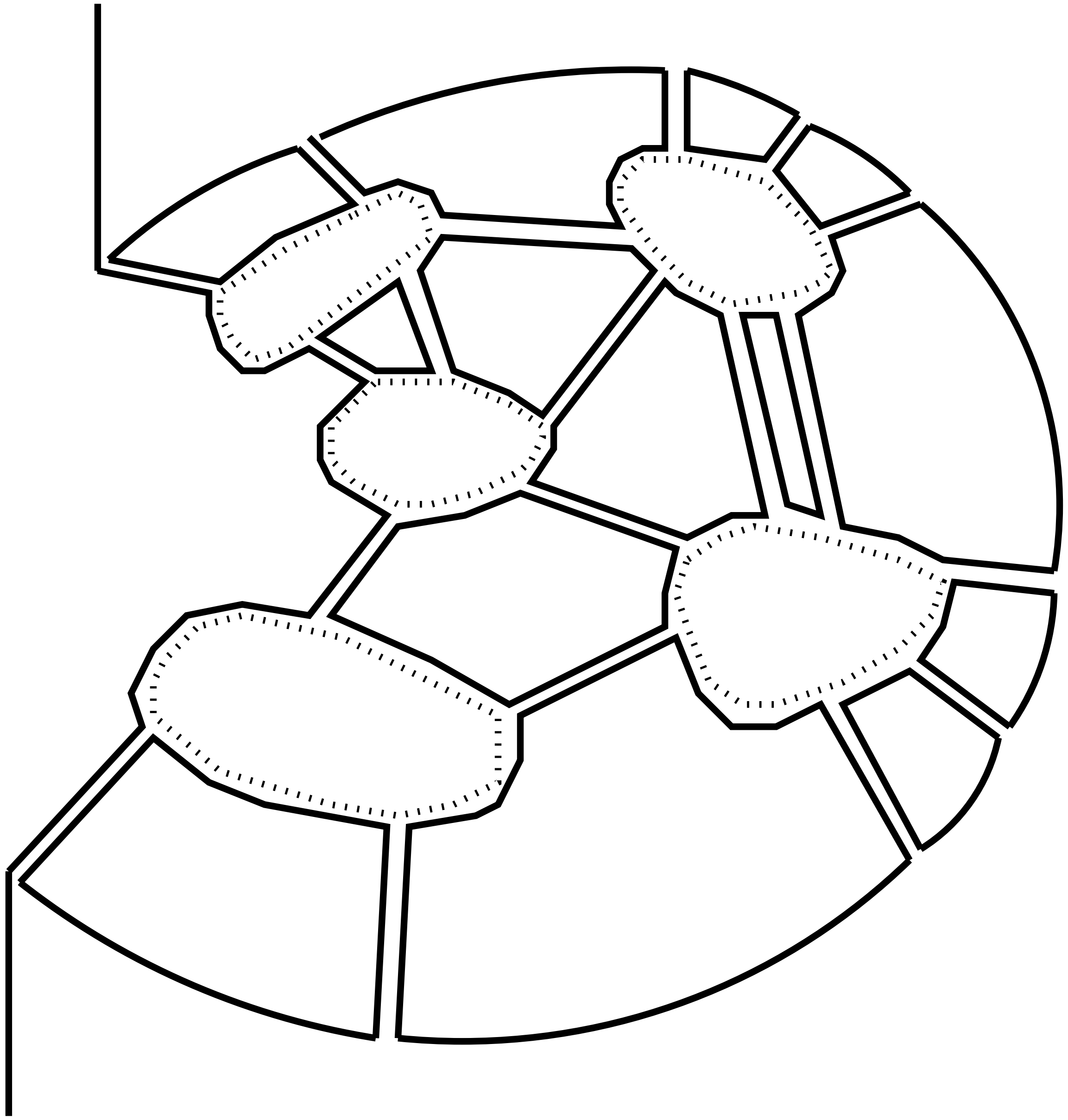
Computations in the $1/N$ expansion



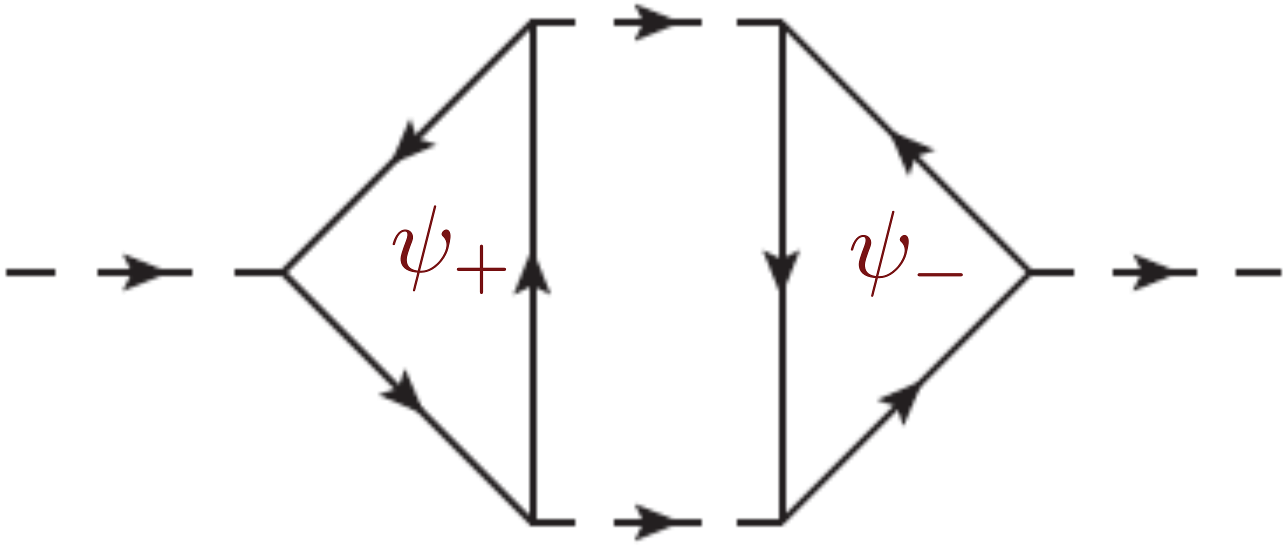
All planar graphs of ψ_+ alone are as important as the leading term

Sung-Sik Lee, *Physical Review* B **80**, 165102 (2009)

Computations in the 1/N expansion



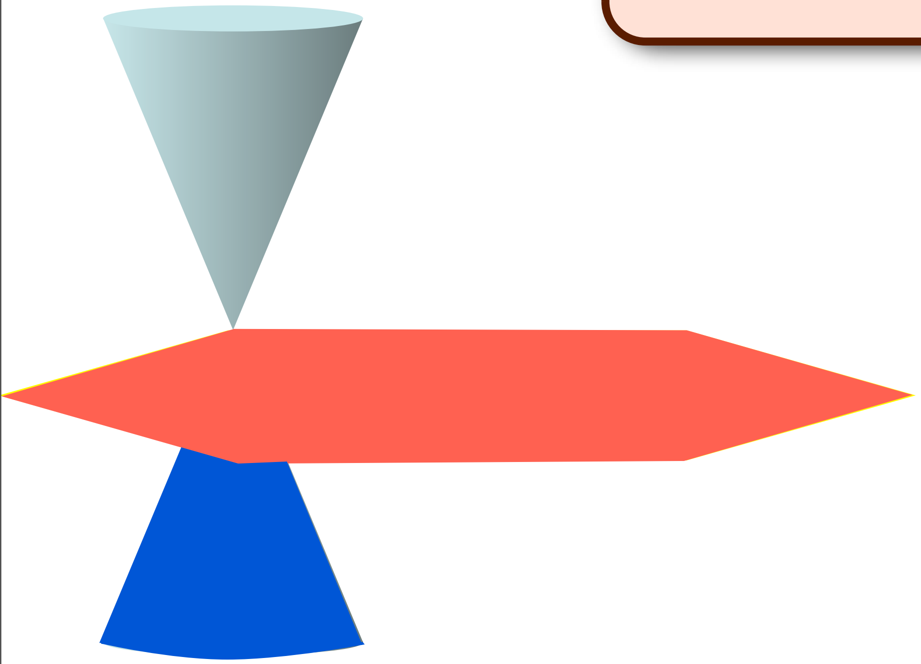
All planar graphs of ψ_+ alone are as important as the leading term



Graph mixing ψ_+ and ψ_- is $\mathcal{O}\left(N^{3/2}\right)$ (instead of $\mathcal{O}\left(N\right)$), violating genus expansion

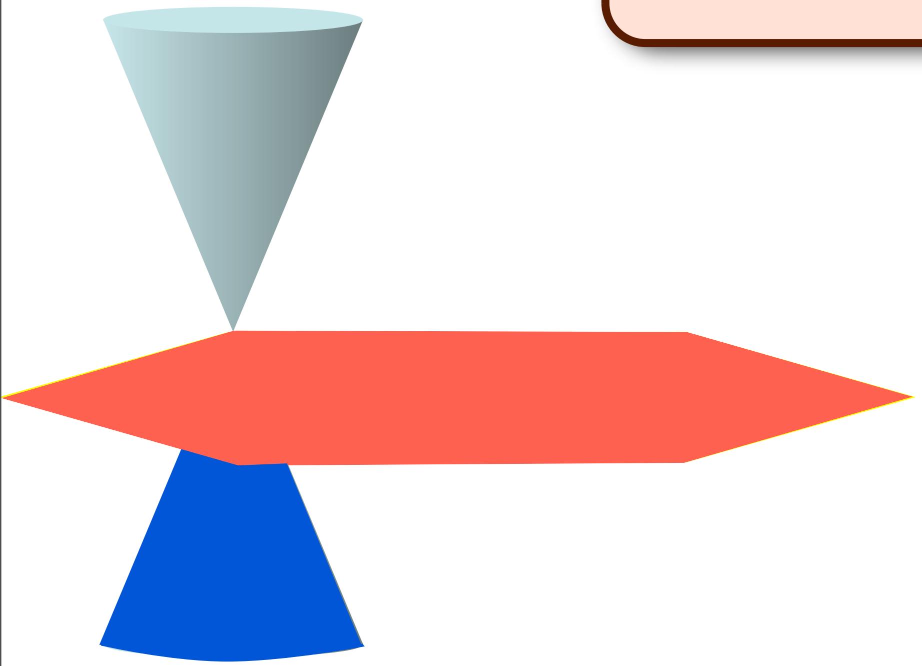
Sung-Sik Lee, *Physical Review* B **80**, 165102 (2009)

Conformal field theory
in 2+1 dimensions at $T = 0$



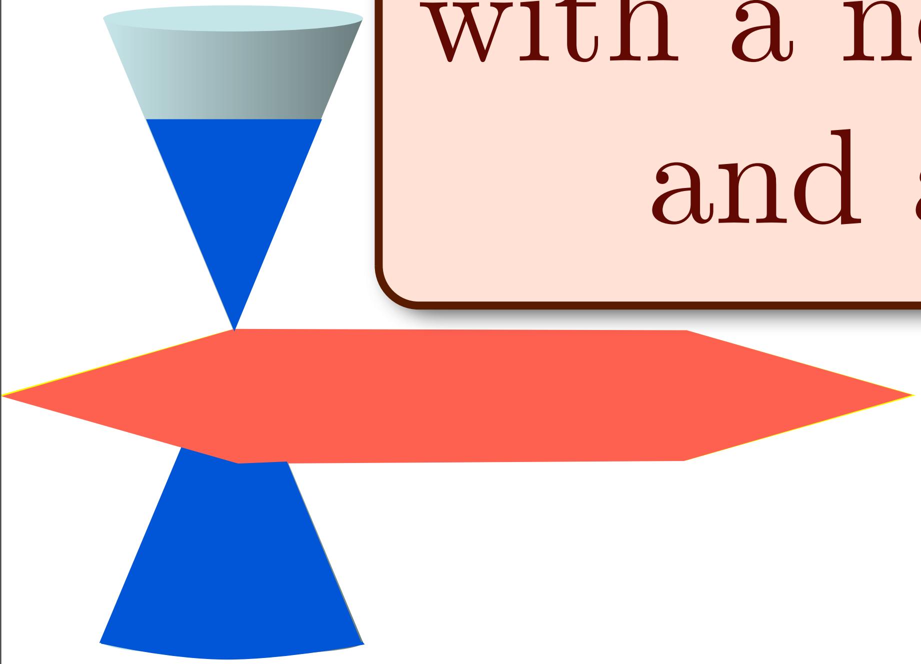
Einstein gravity
on AdS_4

Conformal field theory
in 2+1 dimensions at $T > 0$



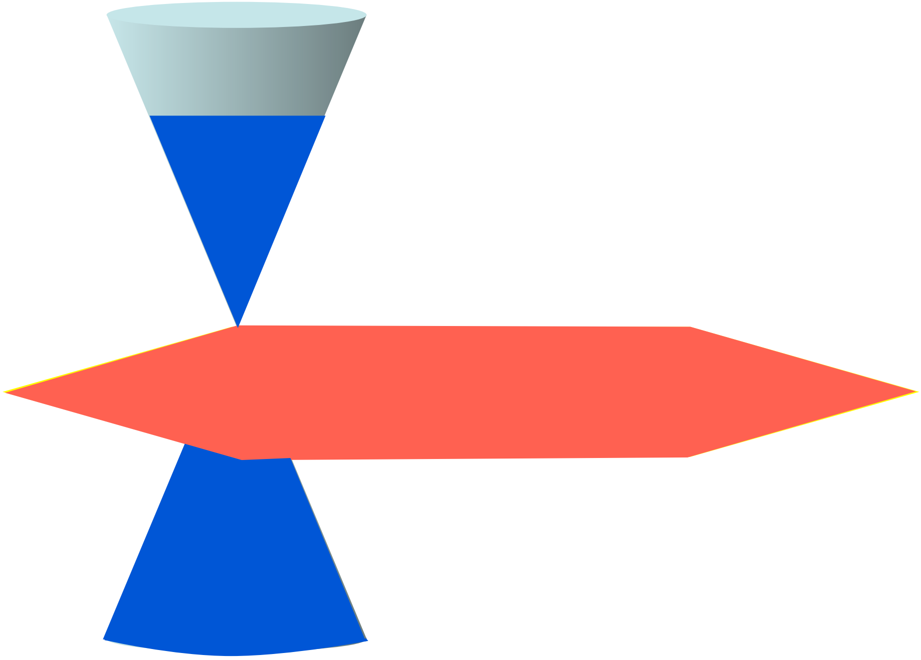
$\updownarrow$

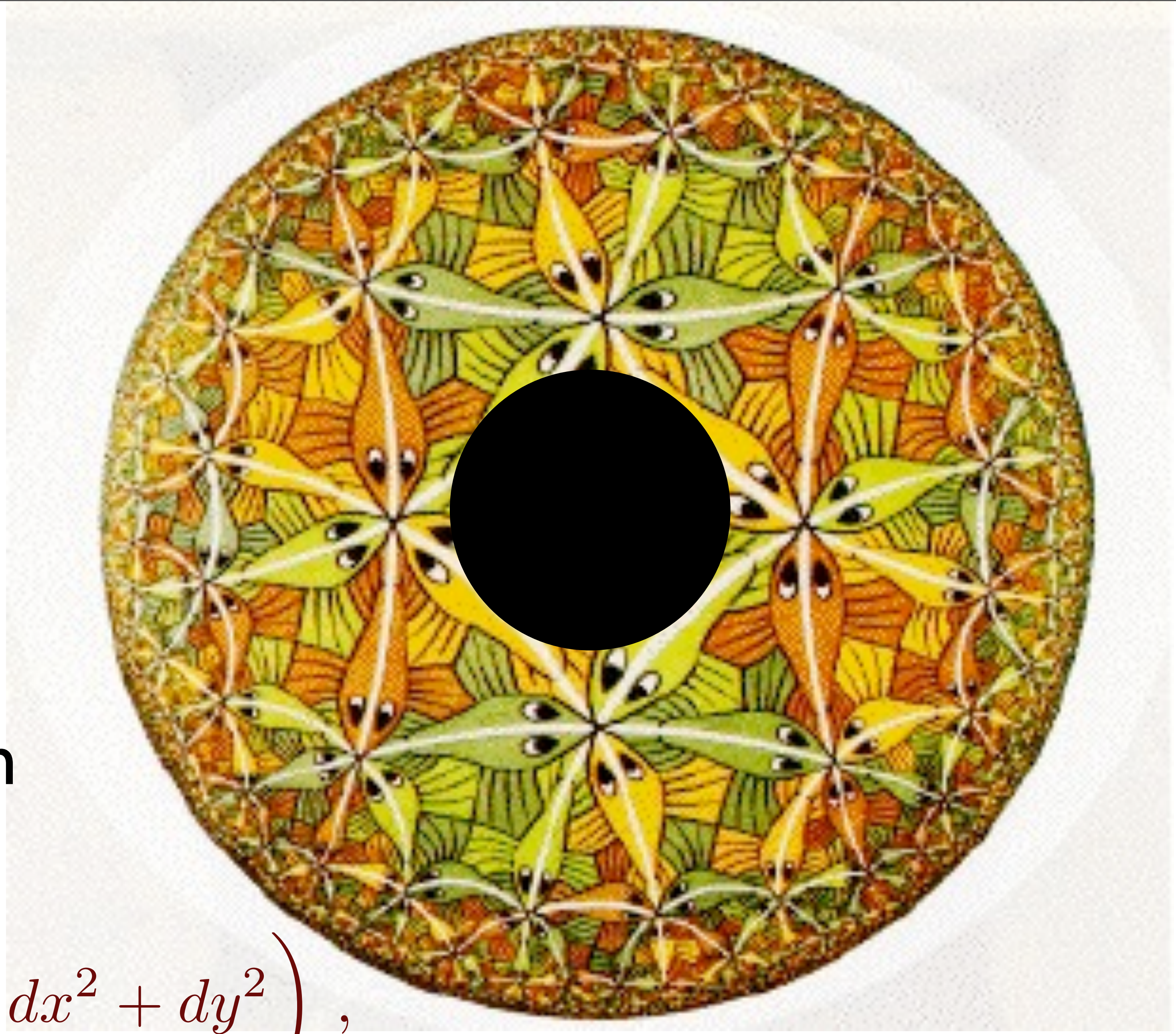
Einstein gravity on AdS_4
with a Schwarzschild
black hole



Conformal field theory
in 2+1 dimensions at $T > 0$,
with a non-zero chemical potential, μ
and applied magnetic field, B

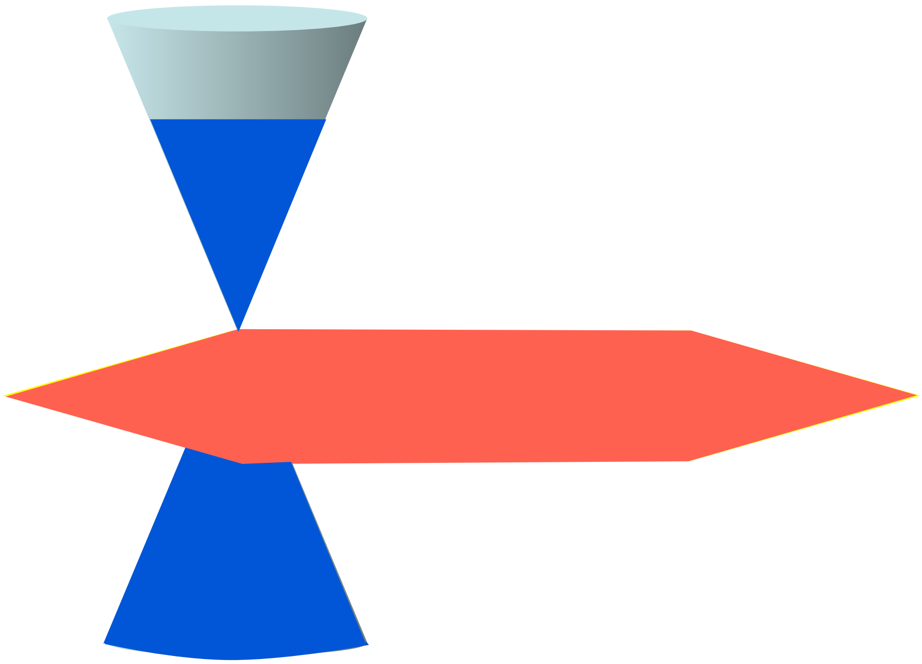
Einstein gravity on AdS_4
with a Reissner-Nordstrom
black hole carrying electric
and magnetic charges

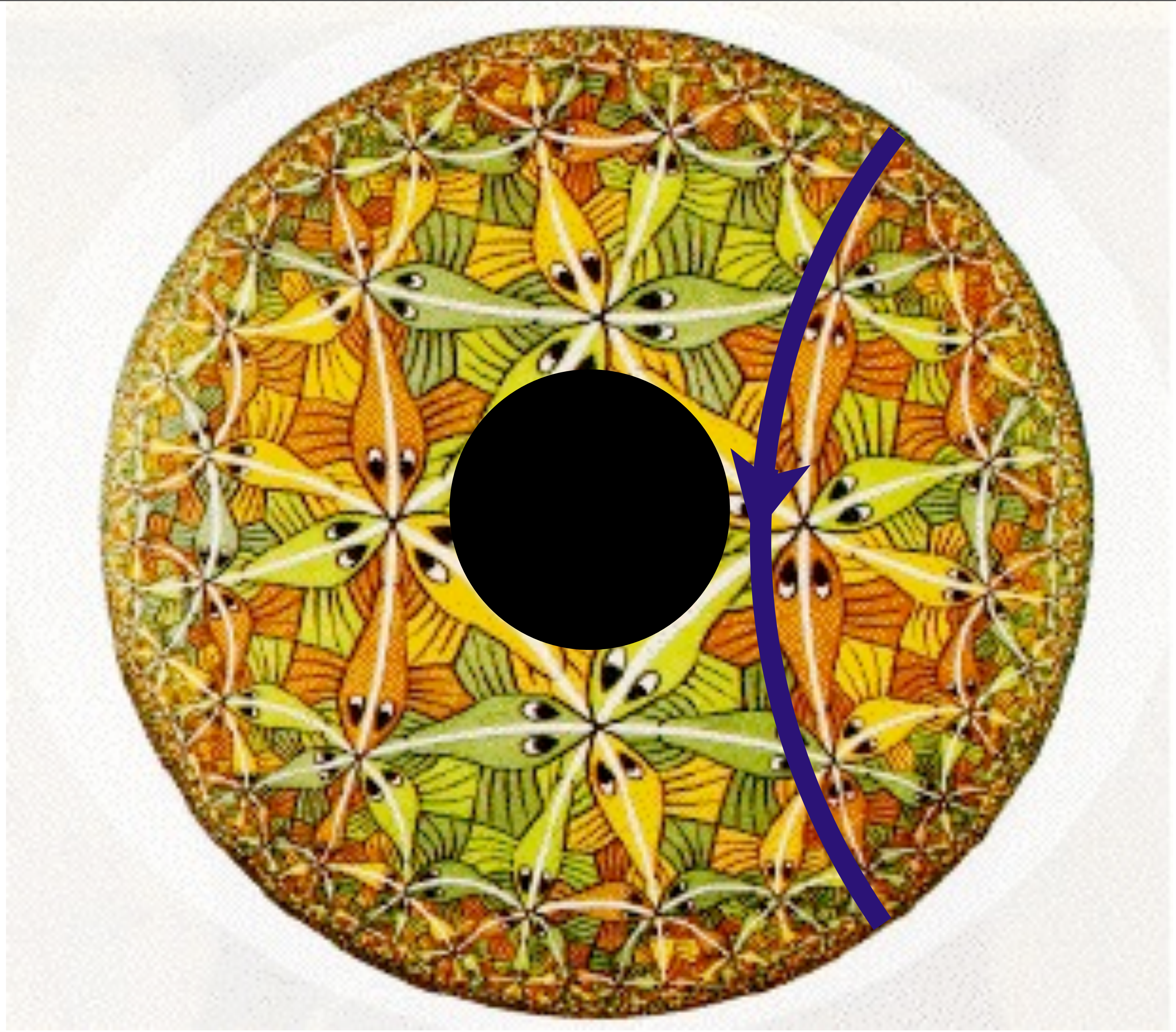




AdS$_4$-Reissner-Nordstrom black hole

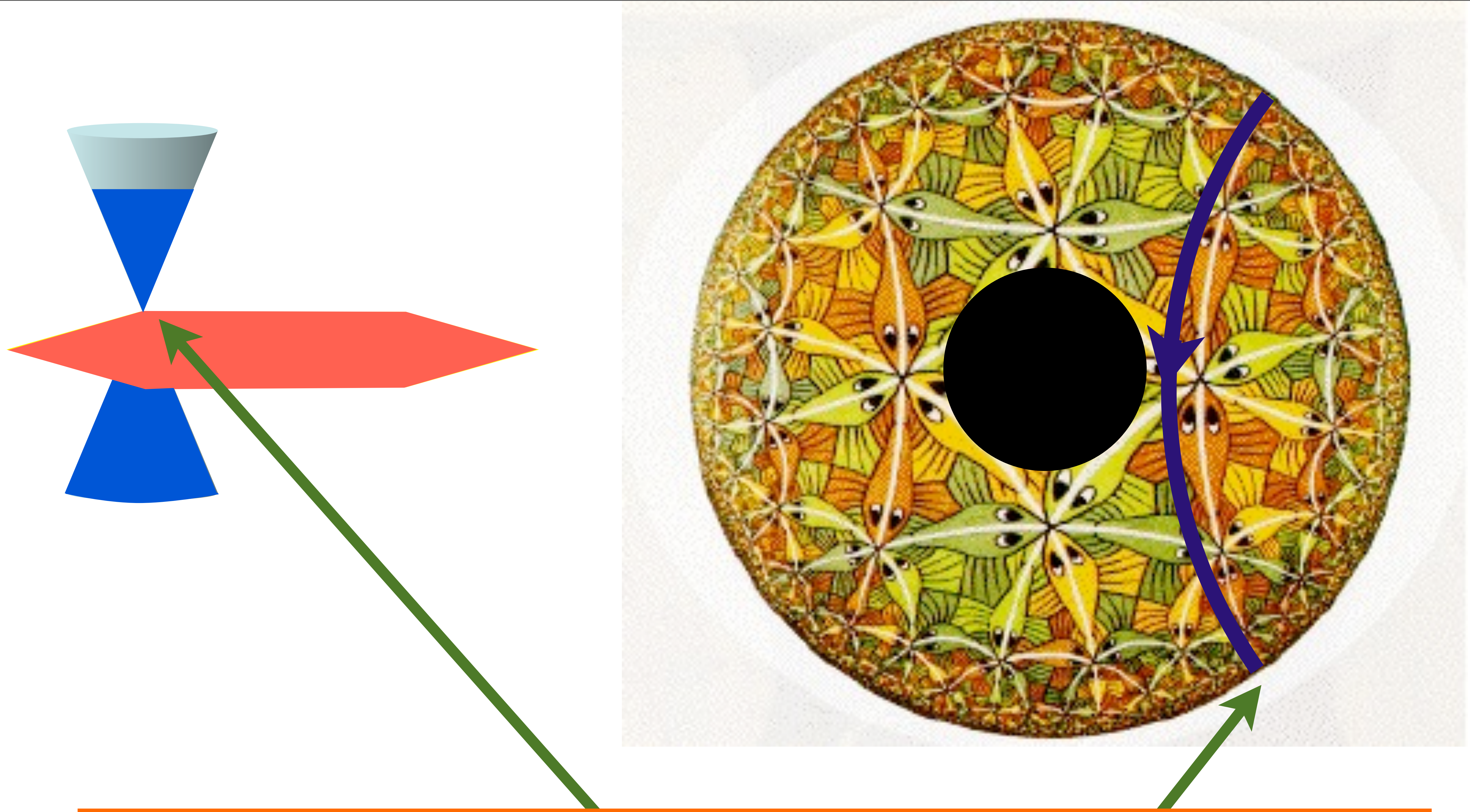
$$\begin{aligned}
ds^2 &= \frac{L^2}{r^2}\left(f(r)d\tau^2 + \frac{dr^2}{f(r)} + dx^2 + dy^2\right),\\
f(r) &= 1 - \left(1 + \frac{(r_+^2\mu^2 + r_+^4 B^2)}{\gamma^2}\right)\left(\frac{r}{r_+}\right)^3 + \frac{(r_+^2\mu^2 + r_+^4 B^2)}{\gamma^2}\left(\frac{r}{r_+}\right)^4,\\
A &= i\mu\left[1 - \frac{r}{r_+}\right]d\tau + Bx\,dy\,.\\
T &= \frac{1}{4\pi r_+}\left(3 - \frac{r_+^2\mu^2}{\gamma^2} - \frac{r_+^4 B^2}{\gamma^2}\right).
\end{aligned}$$





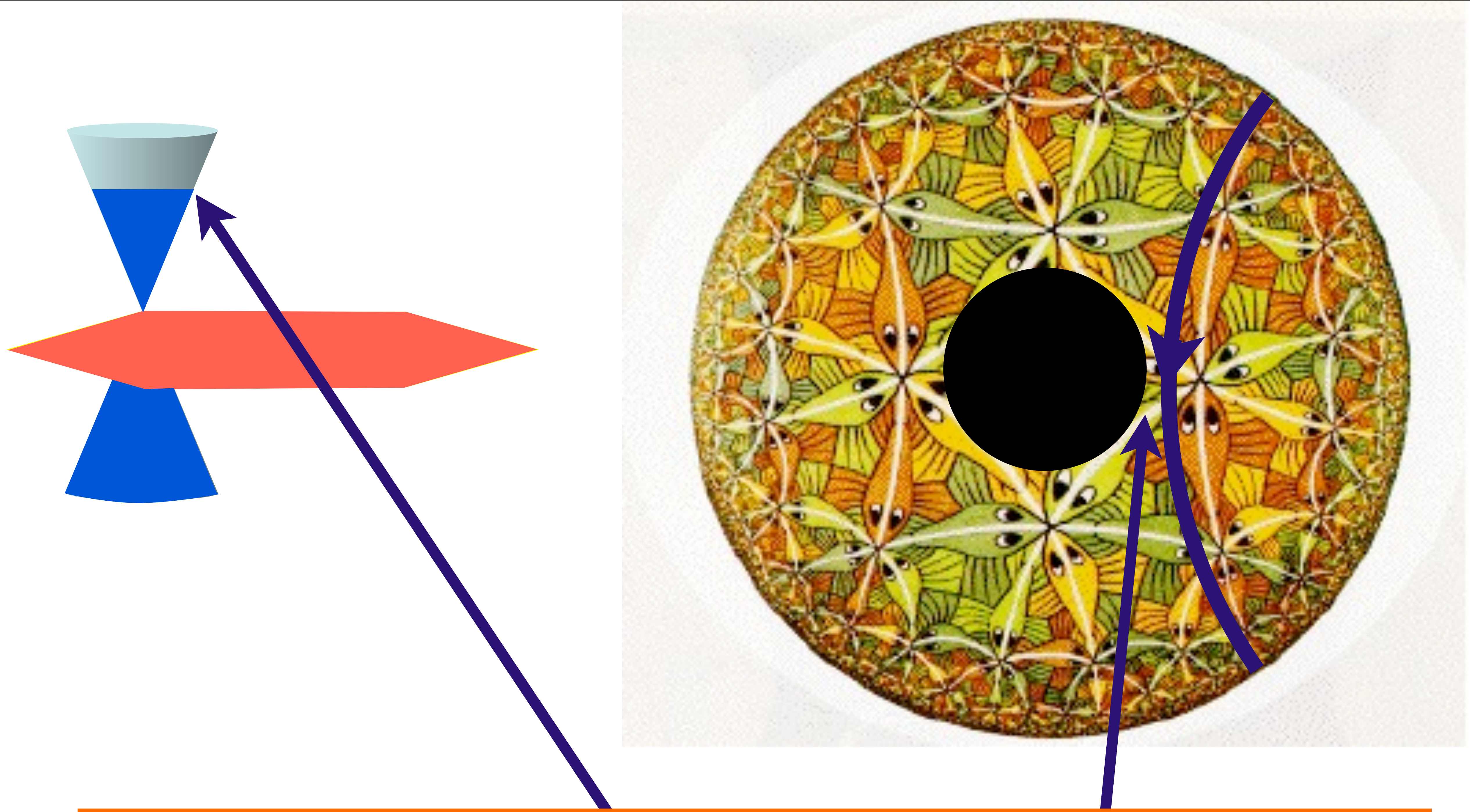
Examine free energy and Green’s function of a probe particle

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788



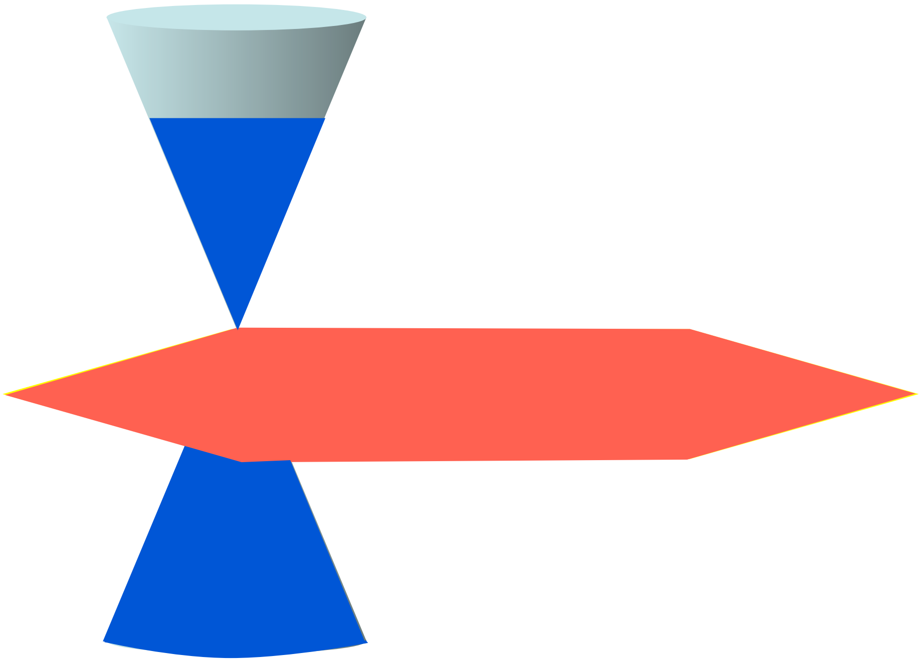
Short time behavior depends upon conformal AdS_4 geometry near boundary

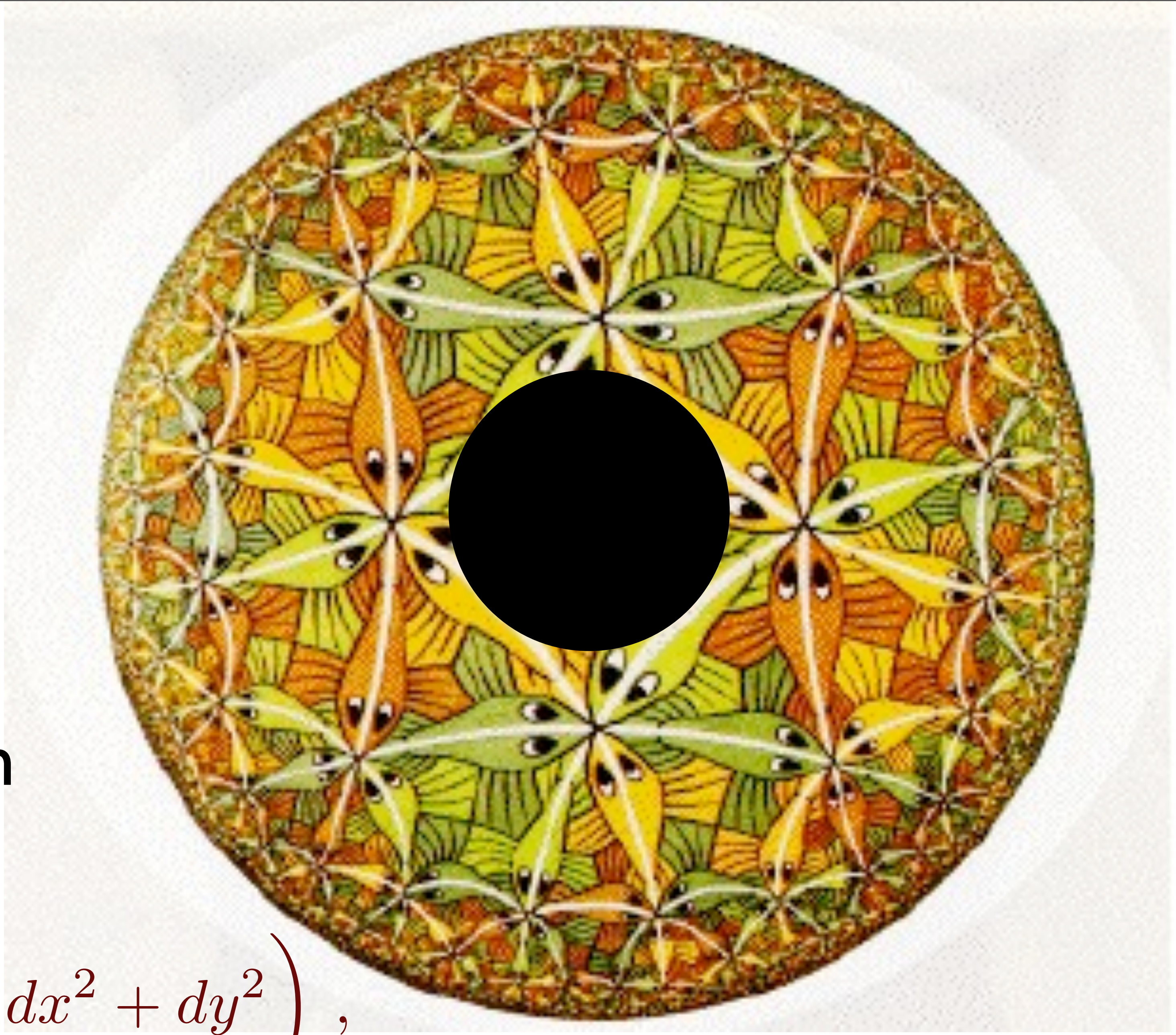
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788



Long time behavior depends upon near-horizon geometry of black hole

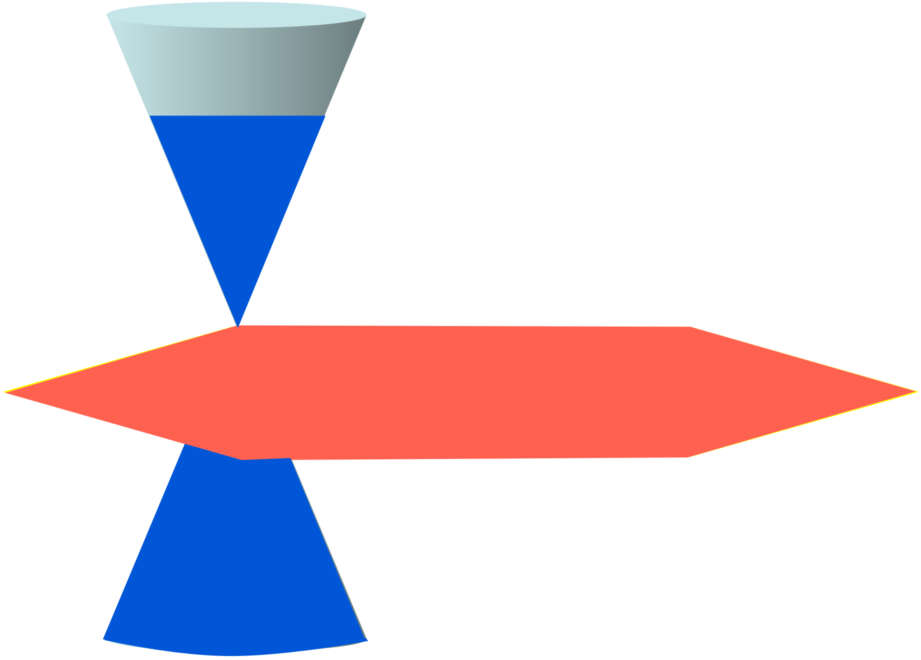
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788



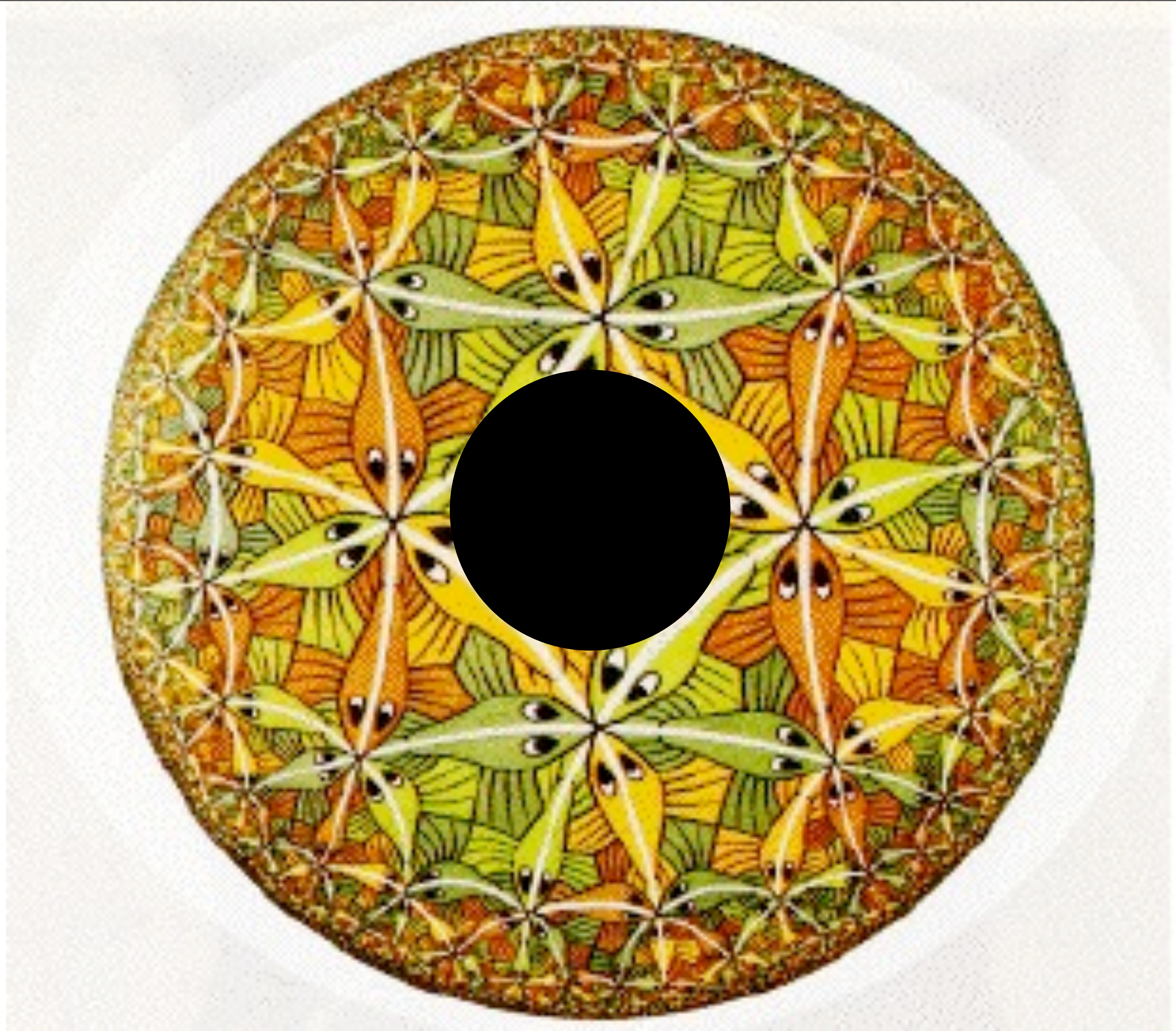


AdS$_4$-Reissner-Nordstrom black hole

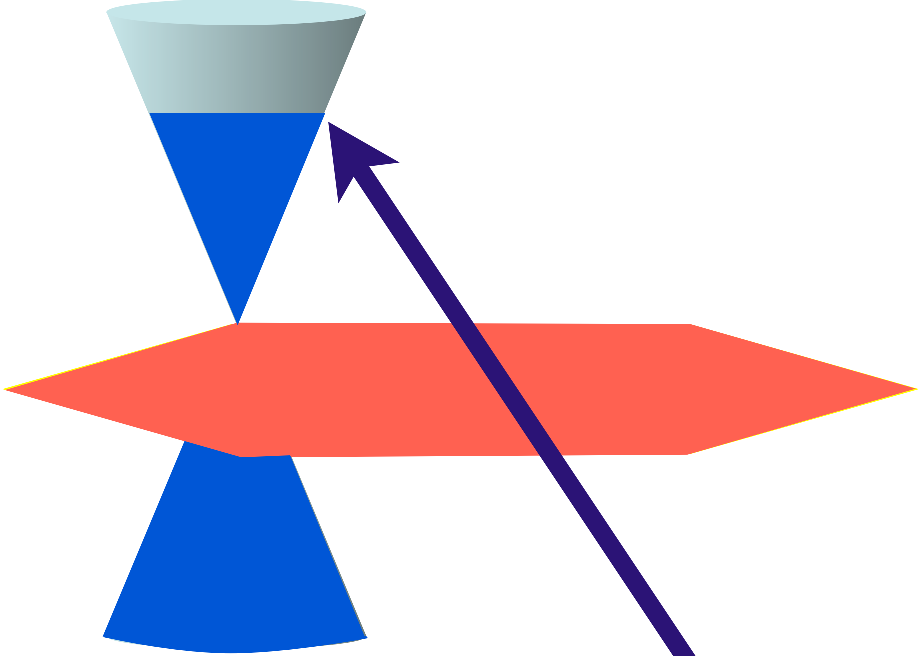
$$\begin{aligned}
ds^2 &= \frac{L^2}{r^2}\left(f(r)d\tau^2 + \frac{dr^2}{f(r)} + dx^2 + dy^2\right), \\
f(r) &= 1 - \left(1 + \frac{(r_+^2\mu^2 + r_+^4 B^2)}{\gamma^2}\right)\left(\frac{r}{r_+}\right)^3 + \frac{(r_+^2\mu^2 + r_+^4 B^2)}{\gamma^2}\left(\frac{r}{r_+}\right)^4, \\
A &= i\mu\left[1 - \frac{r}{r_+}\right]d\tau + Bx\,dy\,. \\
T &= \frac{1}{4\pi r_+}\left(3 - \frac{r_+^2\mu^2}{\gamma^2} - \frac{r_+^4 B^2}{\gamma^2}\right).
\end{aligned}$$

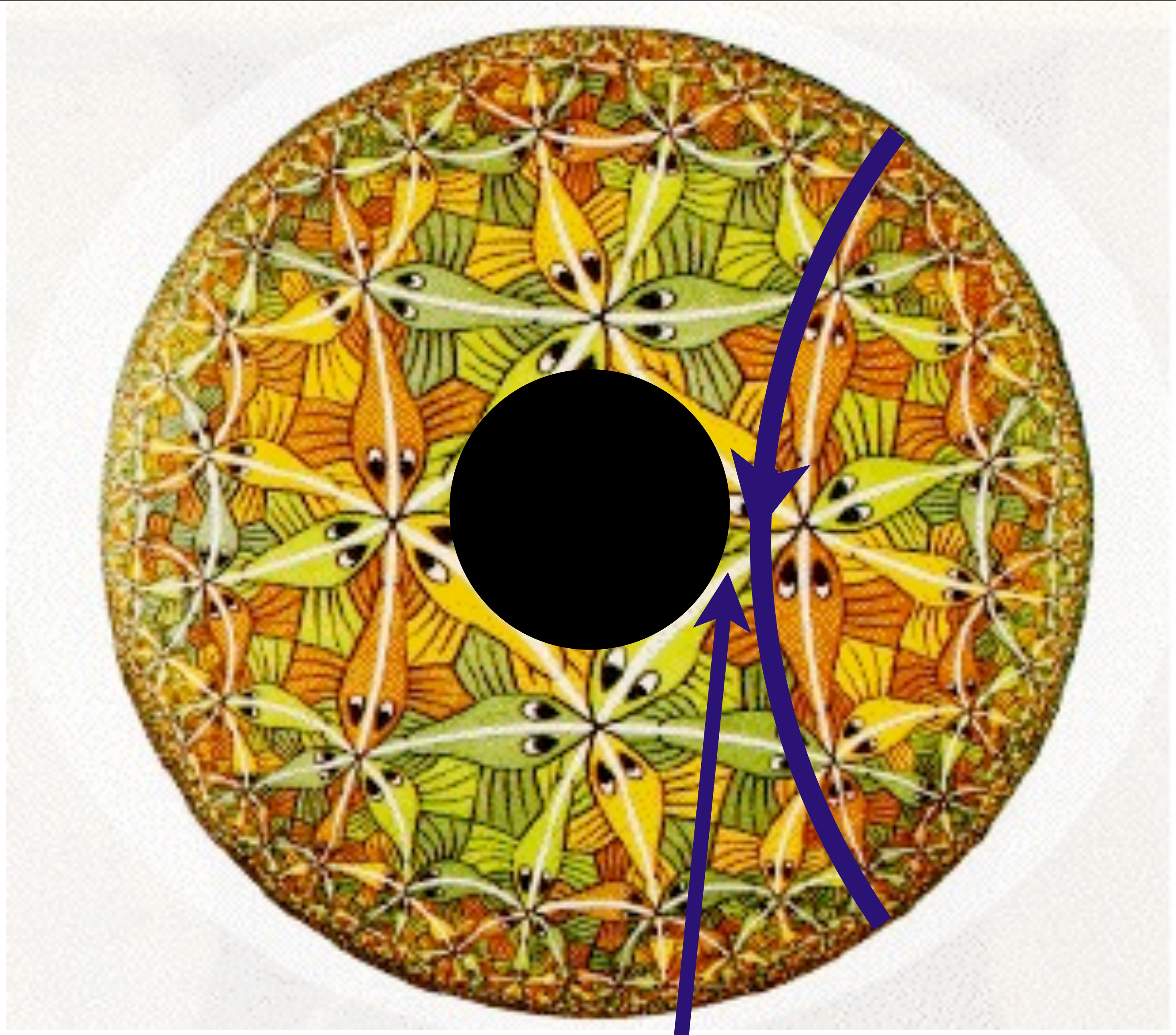


AdS$_2$ x R^2 near-horizon geometry



$$
\begin{aligned}
r - r_+ &\sim \frac{1}{\zeta} \\
ds^2 &= \frac{R^2}{\zeta^2}\left(-d\tau^2 + d\zeta^2\right) + \frac{r_+^2}{R^2}\left(dx^2 + dy^2\right)
\end{aligned}
$$

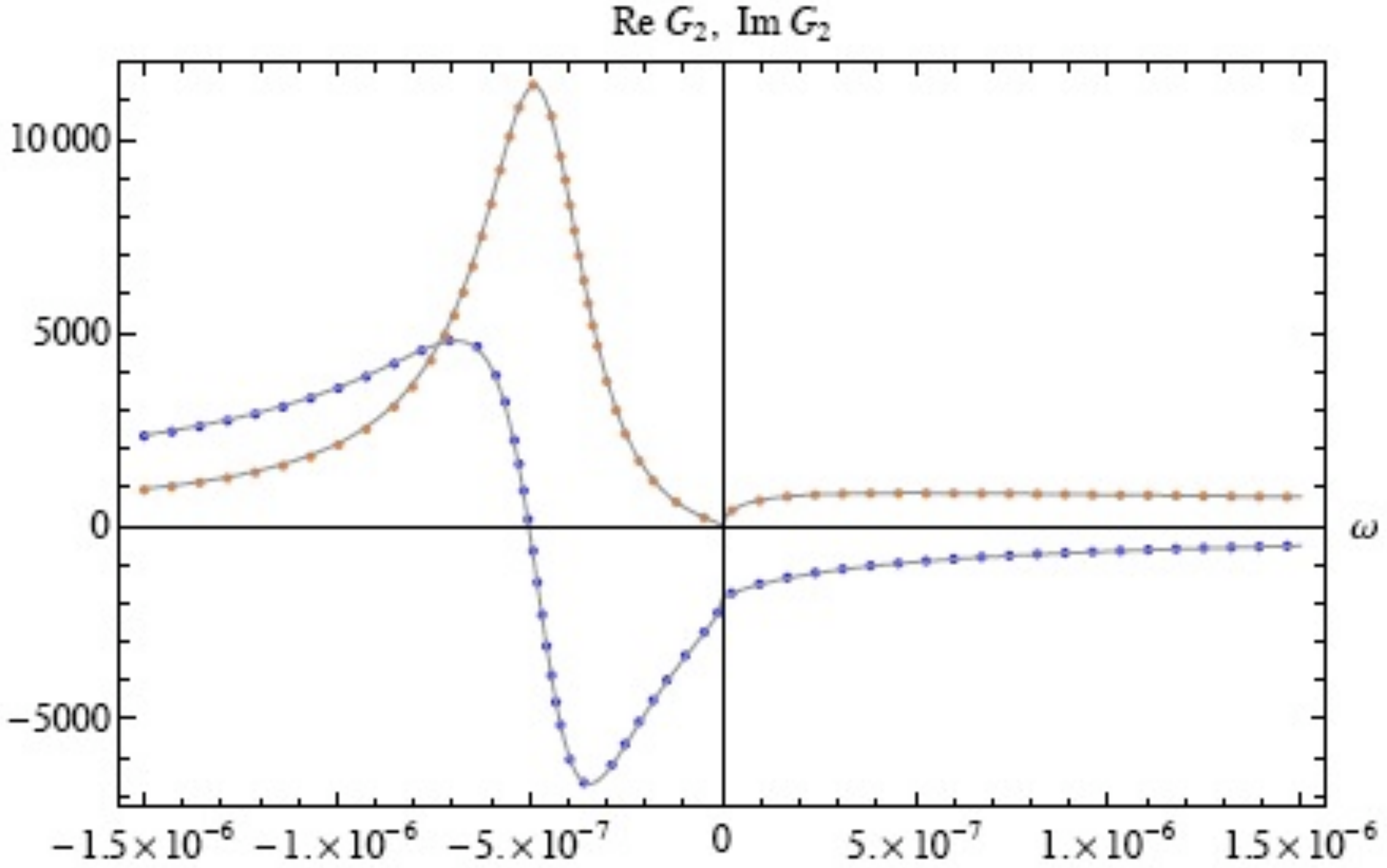




Infrared physics of Fermi surface is linked to the near horizon AdS_2 geometry of Reissner-Nordstrom black hole

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

Green’s function of a fermion

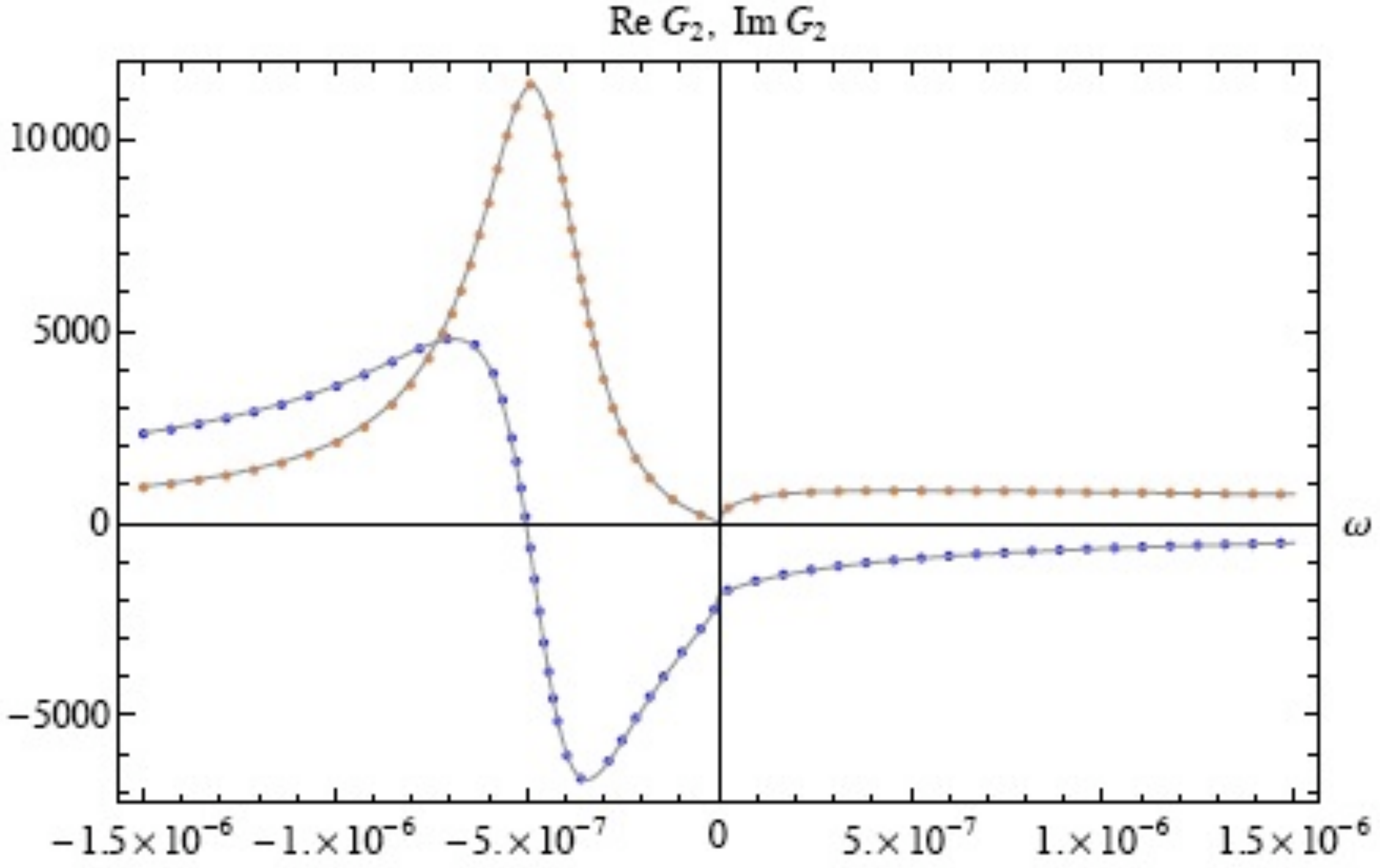


T. Faulkner, H. Liu,
J. McGreevy, and
D. Vegh,
arXiv:0907.2694

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

M. Cubrovic, J Zaanen, and K. Schalm, arXiv:0904.1993

Green’s function of a fermion



T. Faulkner, H. Liu,
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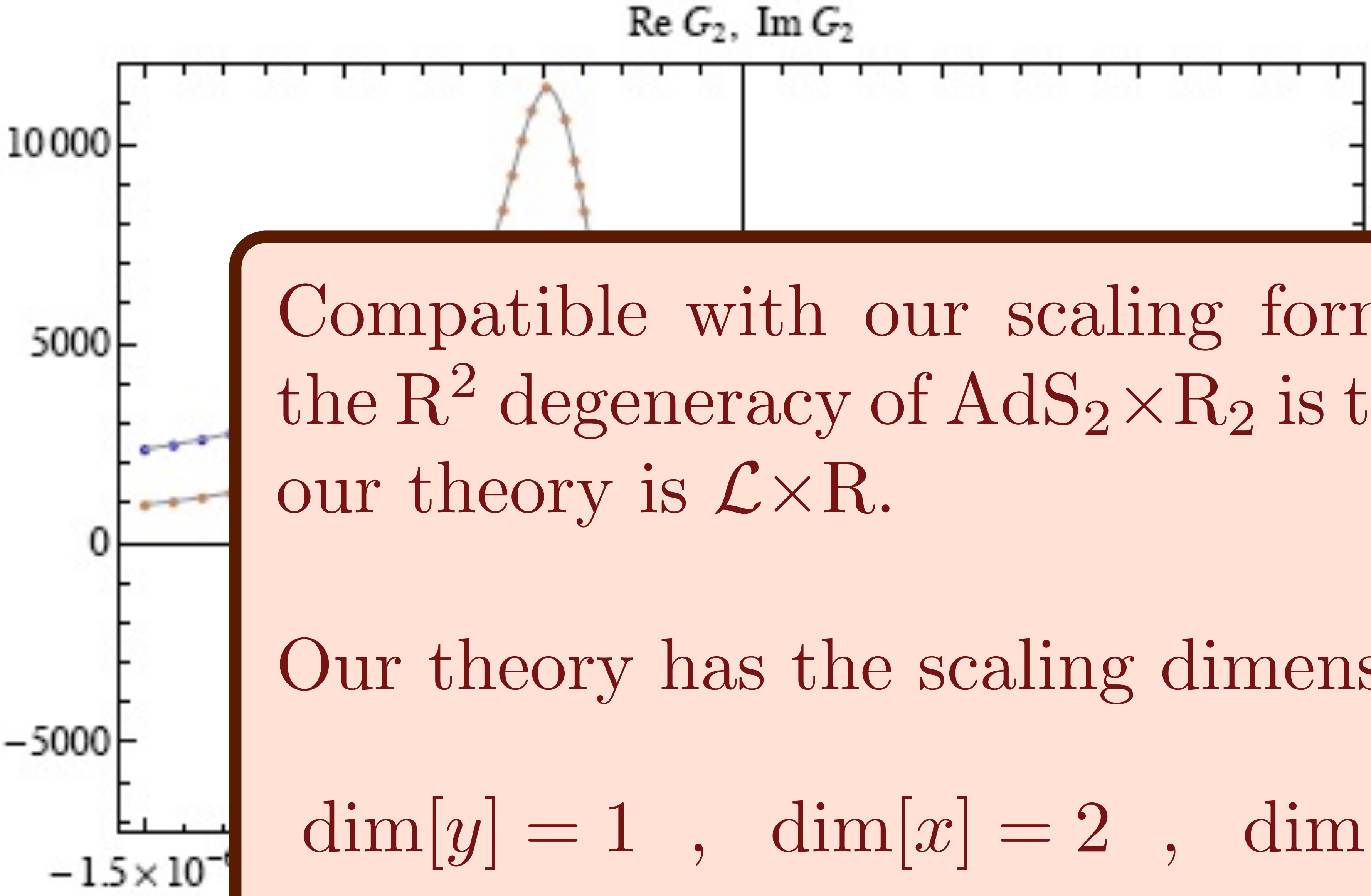

Compatible with our scaling forms, but the R^2 degeneracy of $\mathrm{AdS}_2\times\mathrm{R}_2$ is too large: our theory is $\mathcal{L}\times\mathrm{R}$.

Our theory has the scaling dimensions

$$\dim[y] = 1 \quad , \quad \dim[x] = 2 \quad , \quad \dim[\tau] = z$$

lkner, H. Liu,
Greevy, and
D. Vegh,
:0907.2694

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

M. Cubrovic, J Zaanen, and K. Schalm, arXiv:0904.1993

Free energy from gravity theory

The free energy is expressed as a sum over the "quasinormal frequencies", z_ℓ, of the black hole. Here ℓ represents any set of quantum numbers:

$$\begin{aligned}\mathcal{F}_{\text{boson}} &= -T\sum_\ell \ln\left(\frac{|z_\ell|}{2\pi T}\left|\Gamma\left(\frac{iz_\ell}{2\pi T}\right)\right|^2\right)\\ \mathcal{F}_{\text{fermion}} &= T\sum_\ell \ln\left(\left|\Gamma\left(\frac{iz_\ell}{2\pi T}+\frac{1}{2}\right)\right|^2\right)\end{aligned}$$

Application of this formula shows that the fermions exhibit the dHvA quantum oscillations with expected period (2π/(Fermi surface ares)) in $1/B$, but with an amplitude corrected from the Fermi liquid formula of Lifshitz-Kosevich.

F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788

Conclusions

General theory of finite temperature dynamics and transport near quantum critical points, with applications to antiferromagnets, graphene, and superconductors

Conclusions

The AdS/CFT offers promise in providing a new understanding of strongly interacting quantum matter at non-zero density