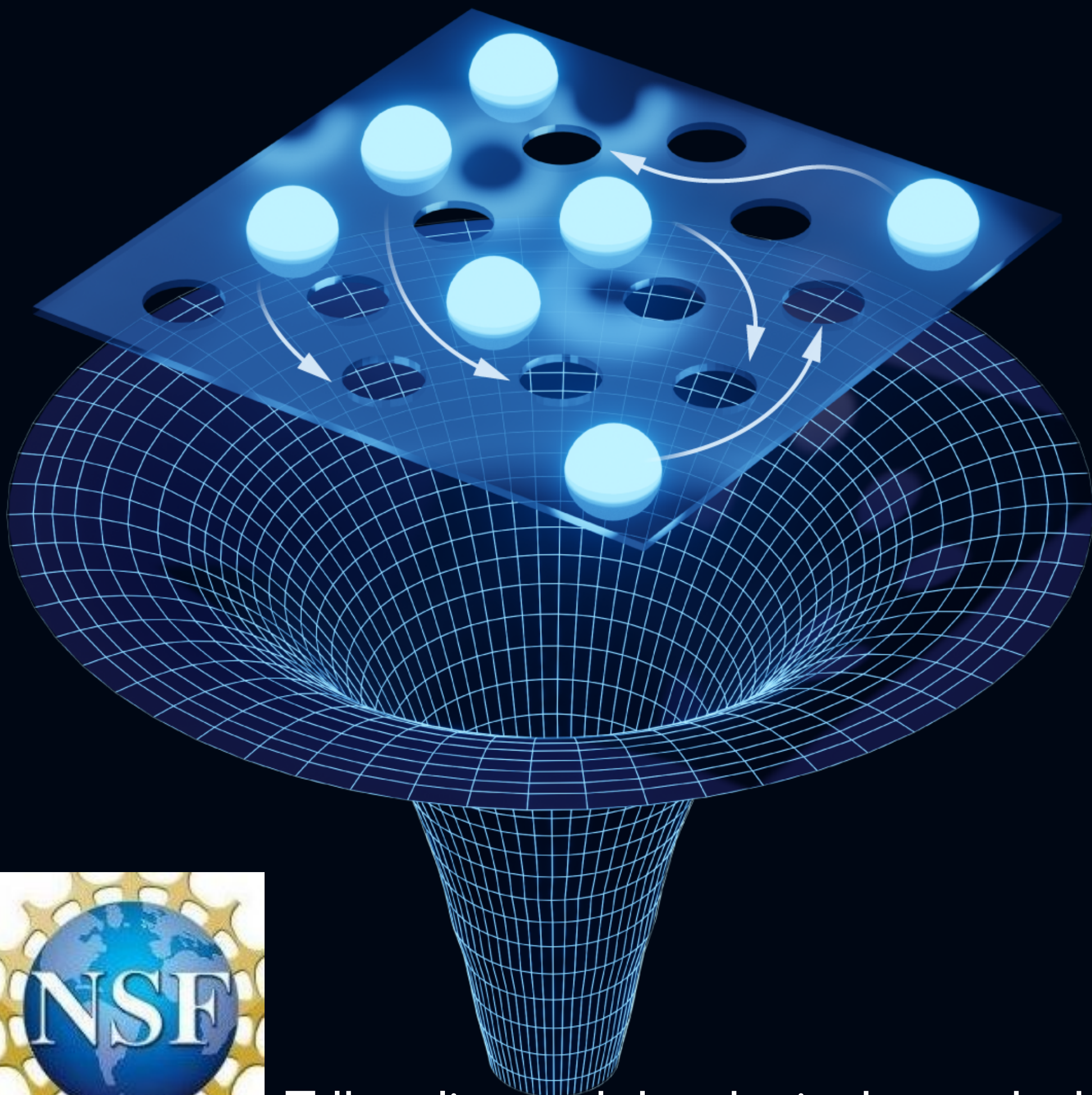


When nature entangles millions of particles: from quantum materials to black holes



Chalmers University of Technology,
Gothenberg, Sweden
November 28, 2022

Subir Sachdev



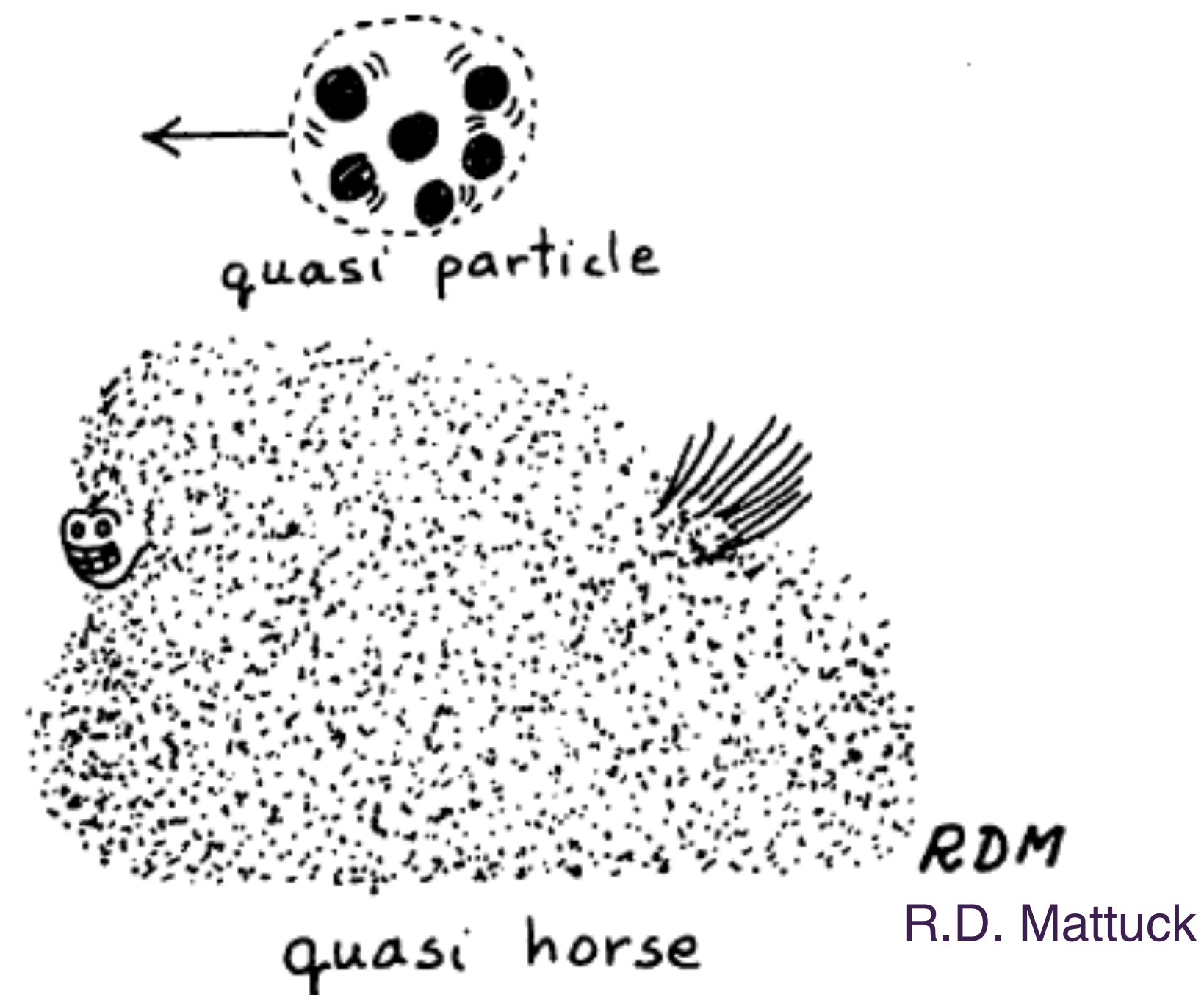
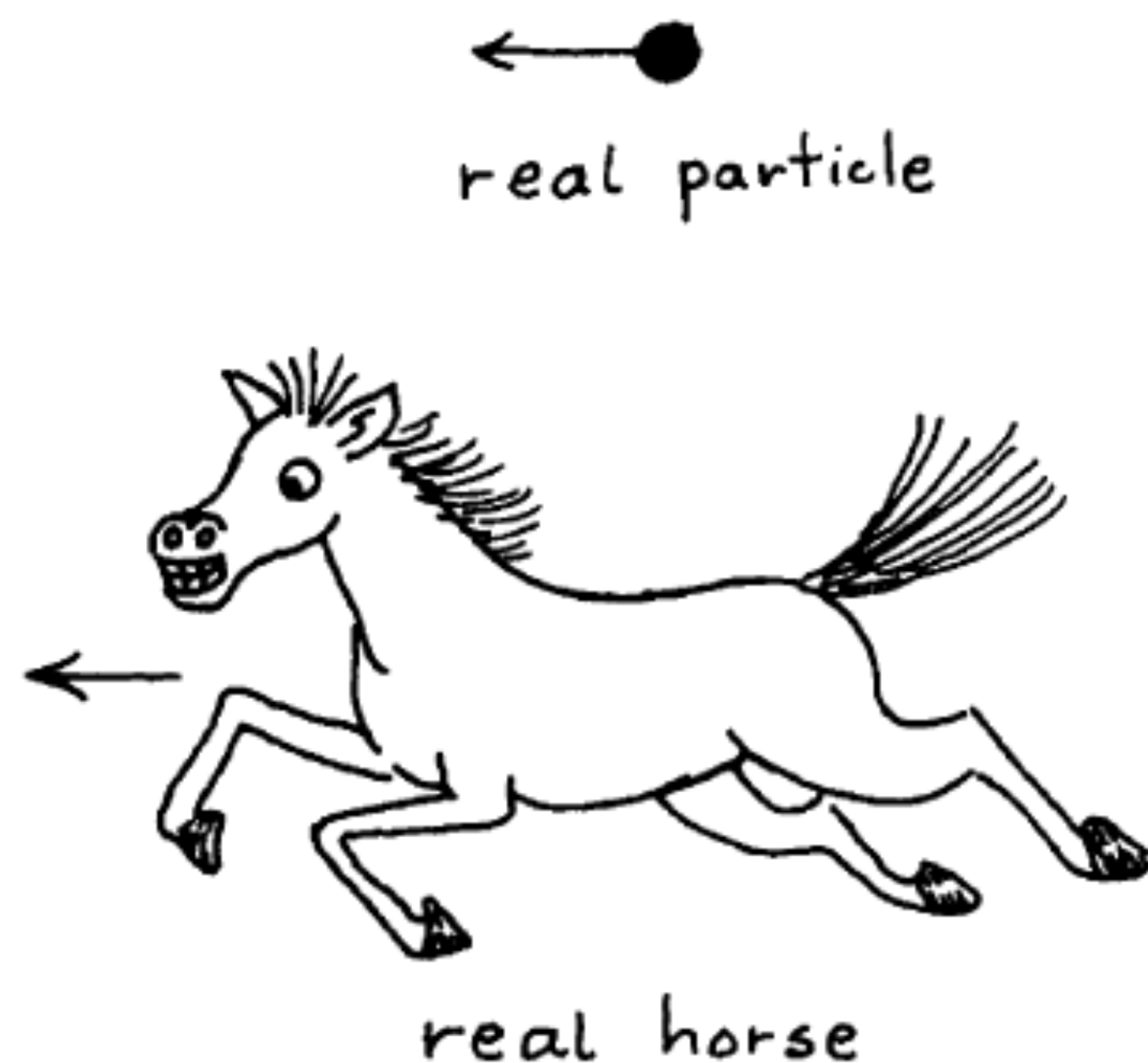
Talk online: sachdev.physics.harvard.edu



Quantum matter:
ordinary metals,
strange metals,
and superconductors

Almost all many-electron systems are described by the quasiparticle concept:

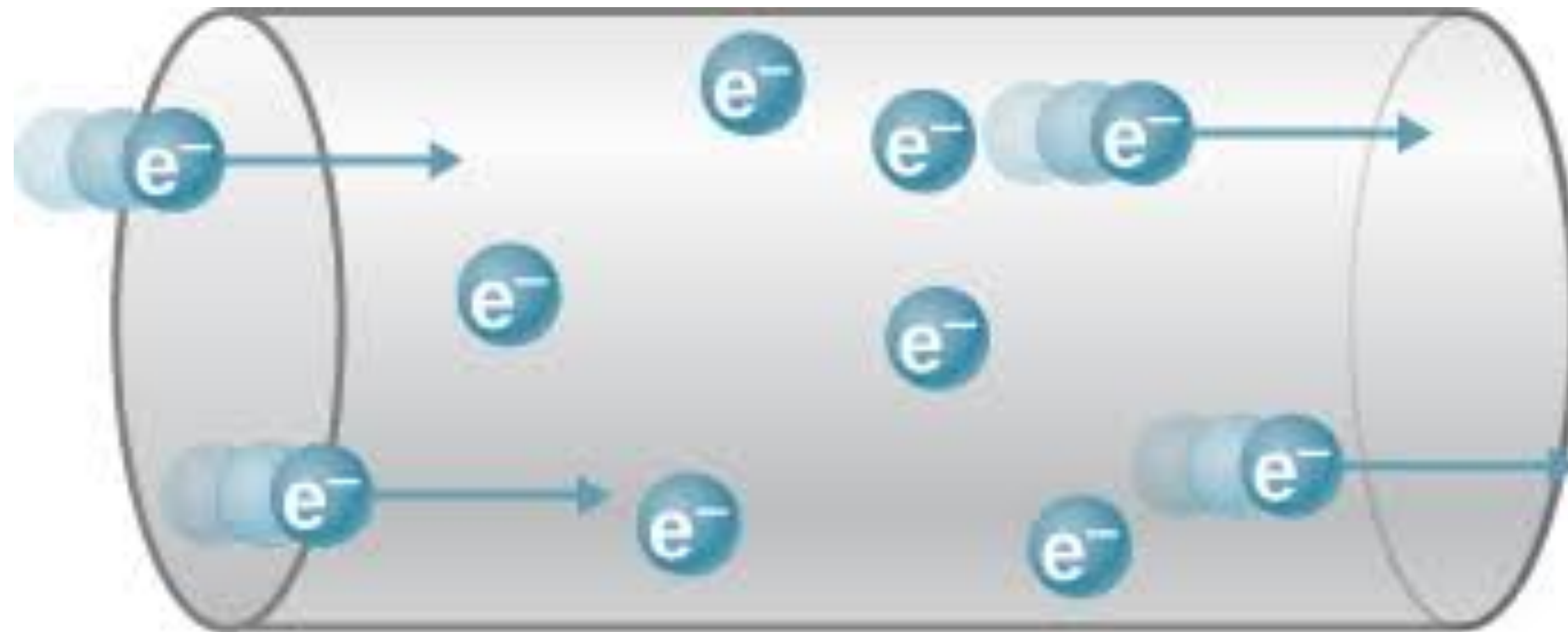
a quasiparticle is an “excited lump” in the many-electron state which responds just like an ordinary particle.



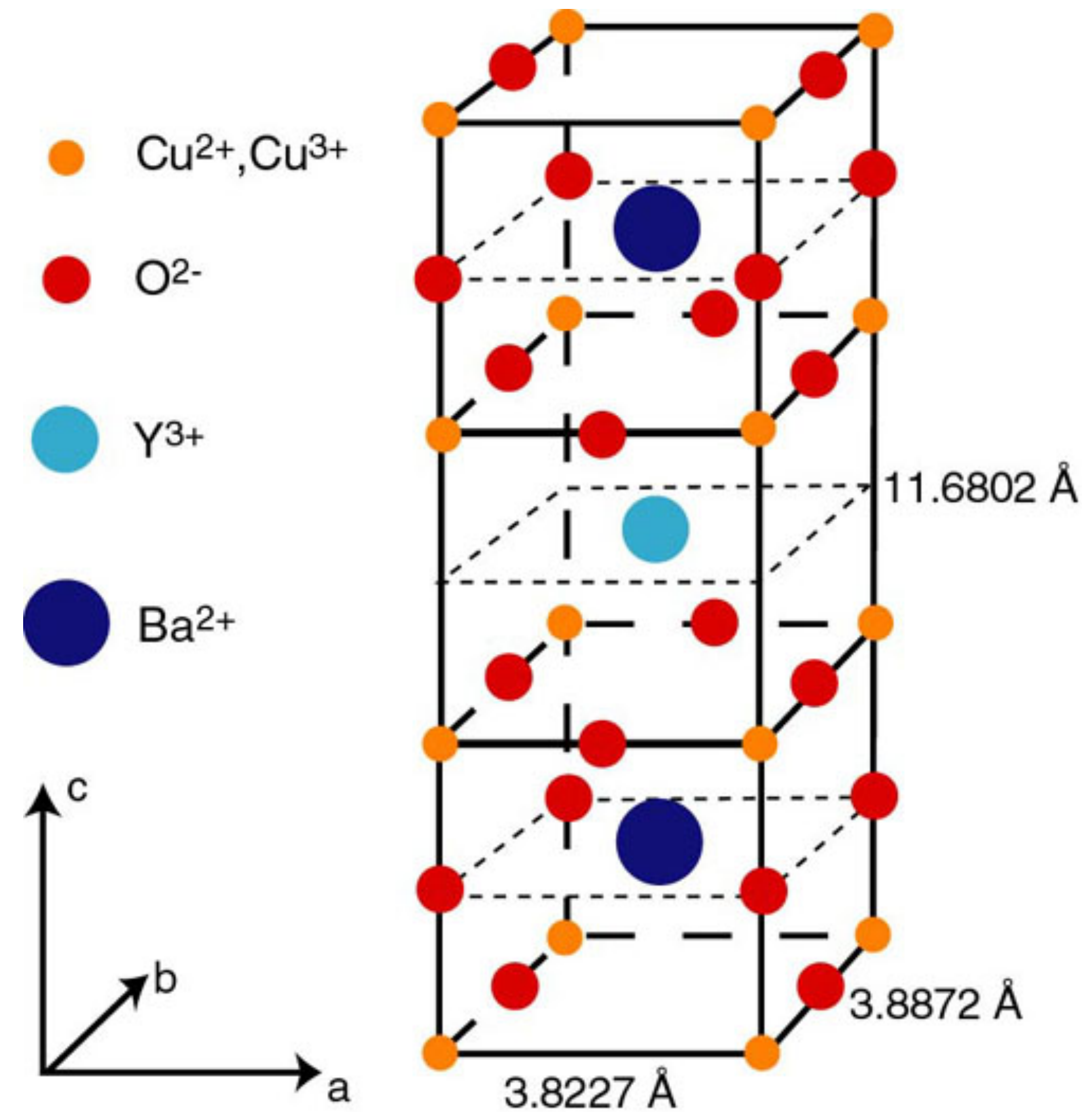
RDM

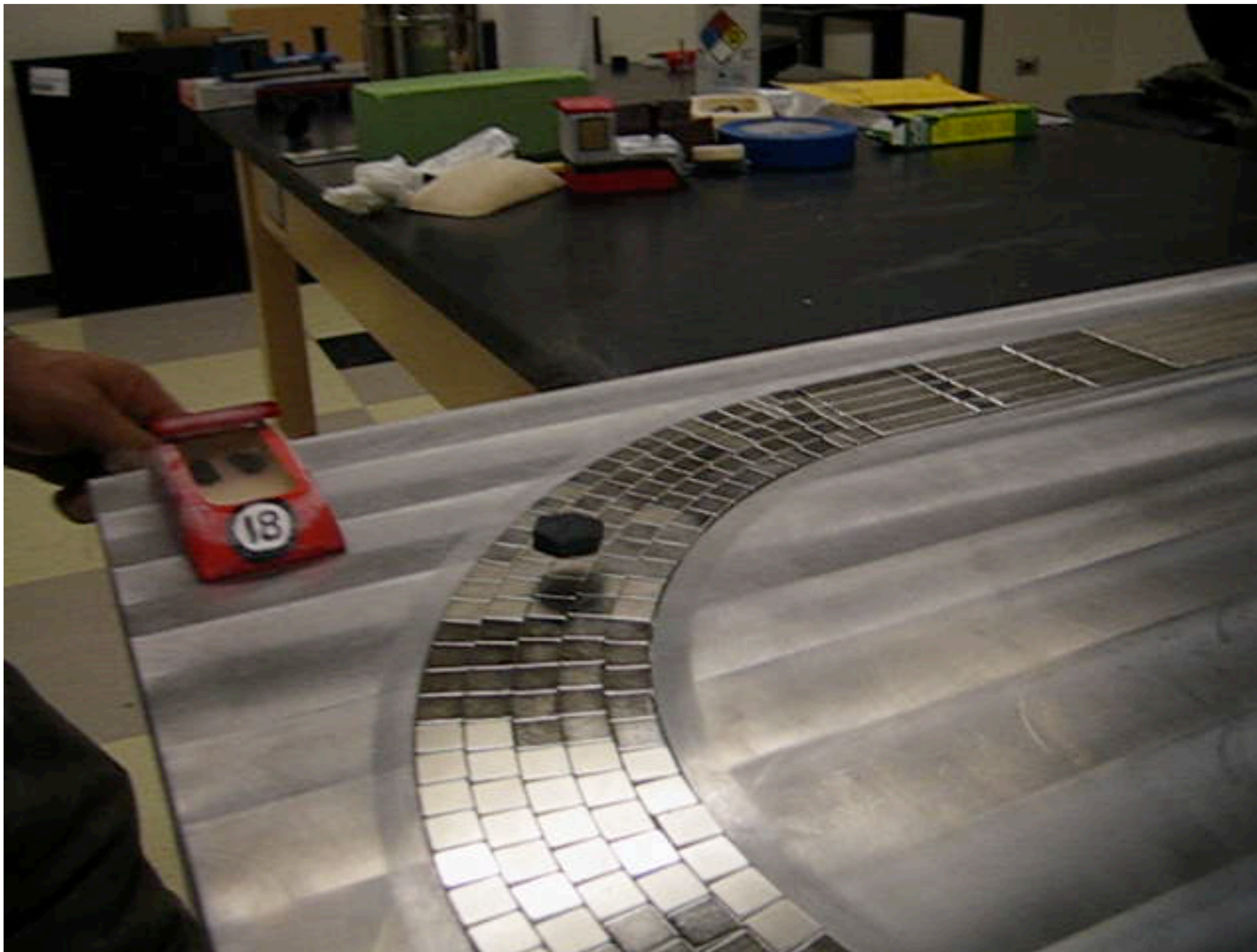
R.D. Mattuck

Current flow with quasiparticles in Copper



High temperature superconductors





Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

HTS Magnets: Enabling Technology

A new high temperature superconductor (HTS) recently reached industrial maturity: Rare Earth Barium Copper Oxide (REBCO). CFS is using HTS and has built its first-of-its-kind high-field large-bore superconducting magnet. HTS

magnets will allow for smaller, faster, and less expensive tokamaks using the science developed on Alcator C-Mod and other tokamaks.

The surest path to limitless, clean, fusion energy

● Surest

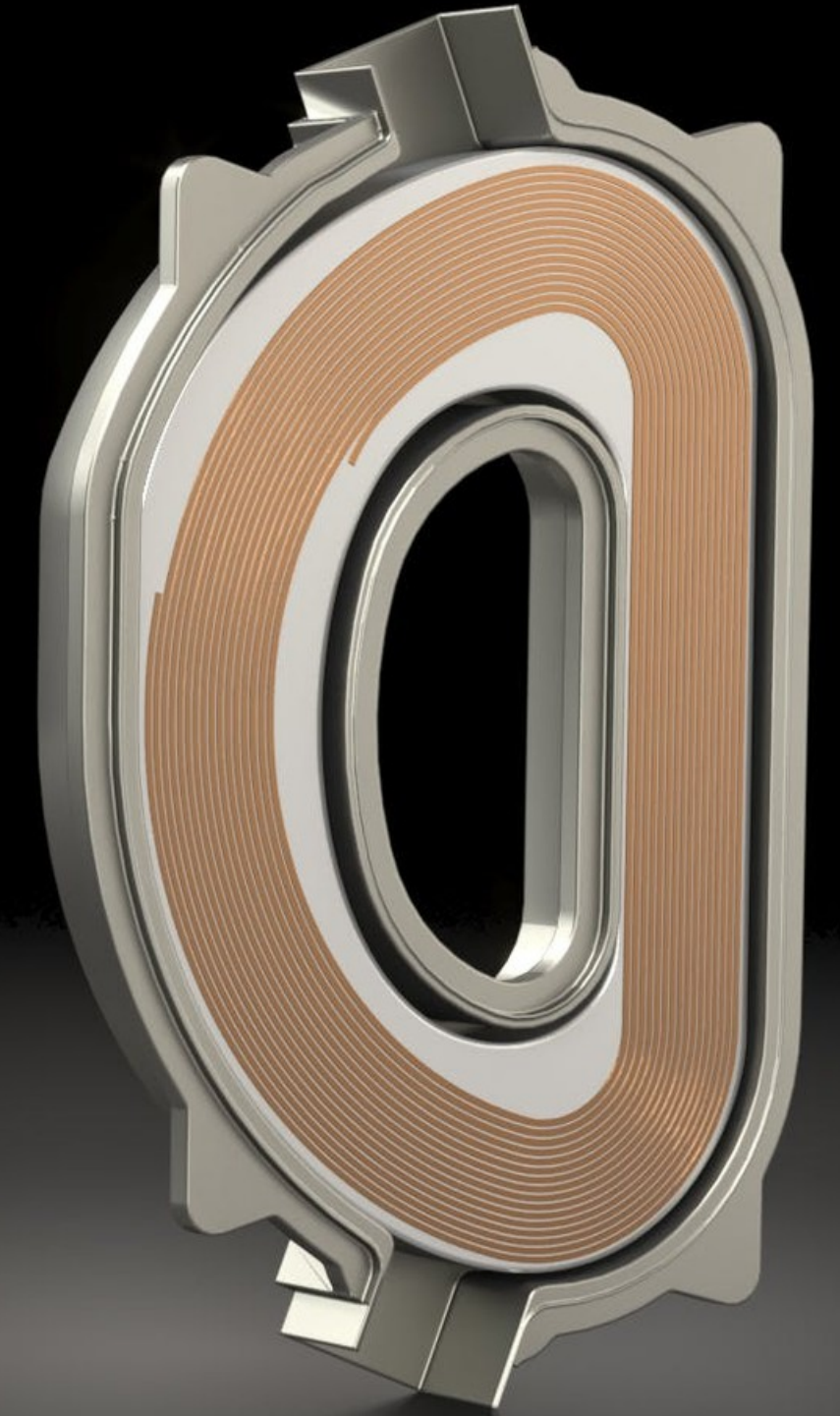
The fastest path to commercial fusion energy combining proven science with revolutionary magnet technology.

○ Limitless

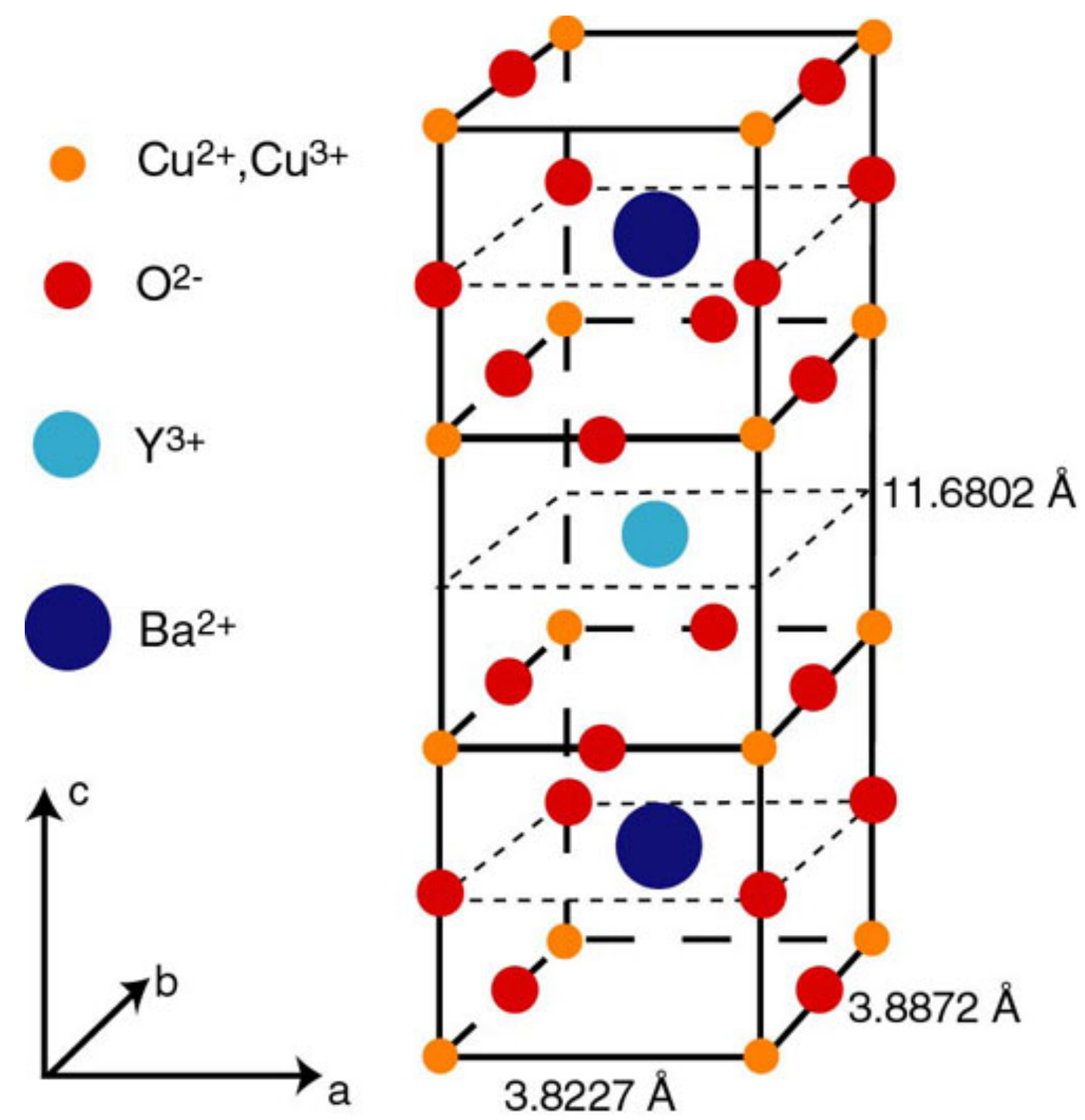
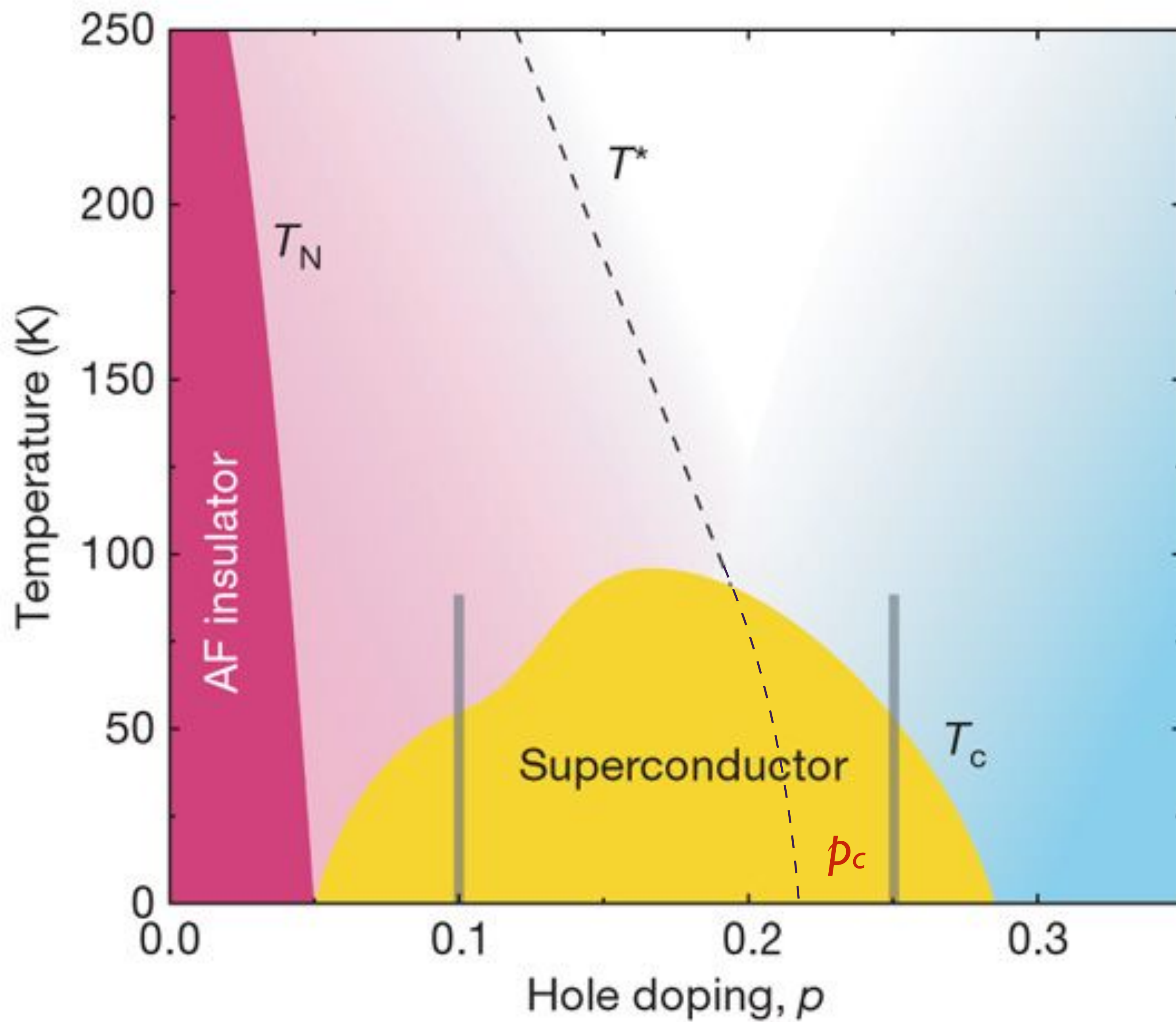
One glass of water will provide enough fusion fuel for one person's lifetime.

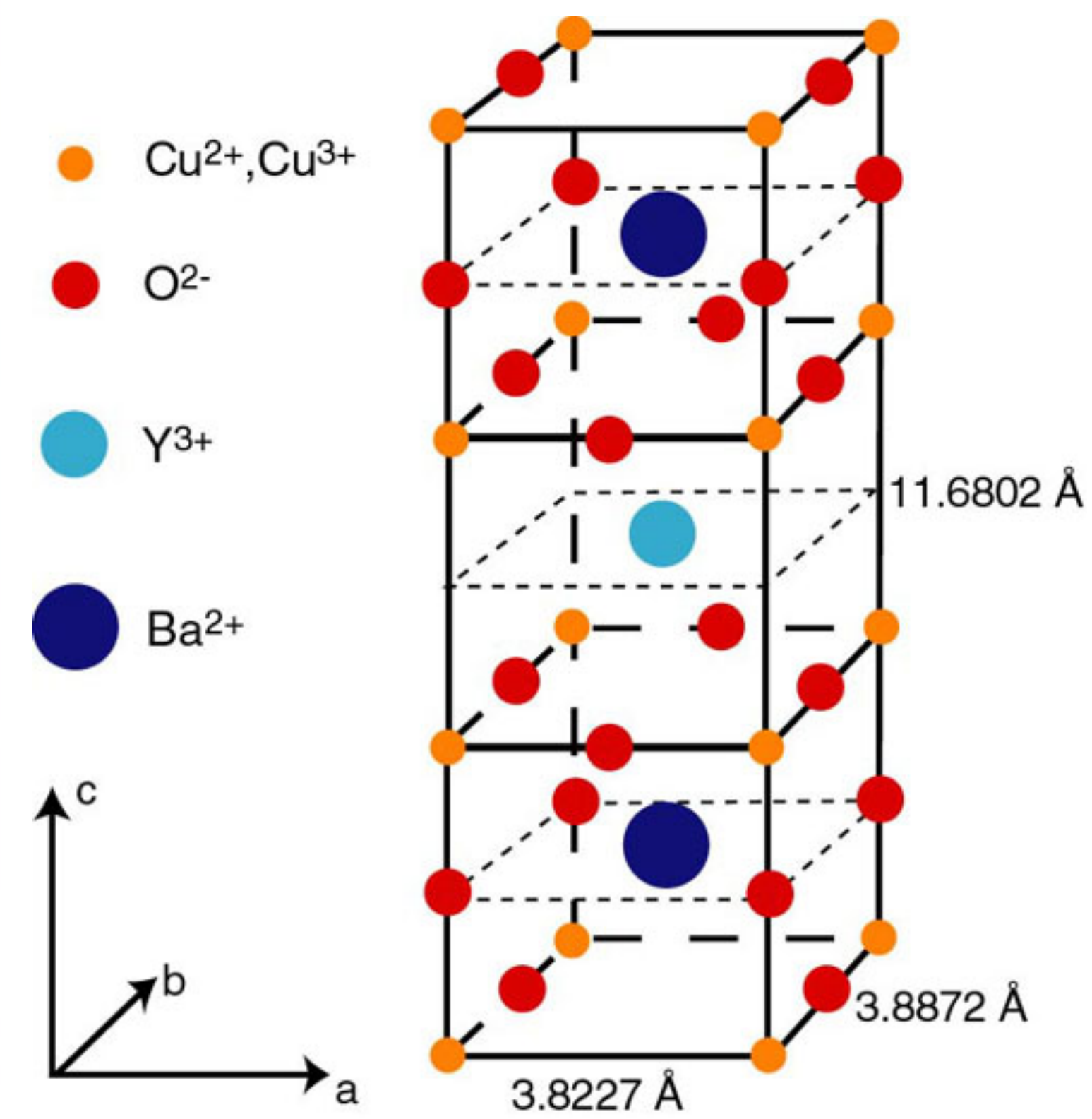
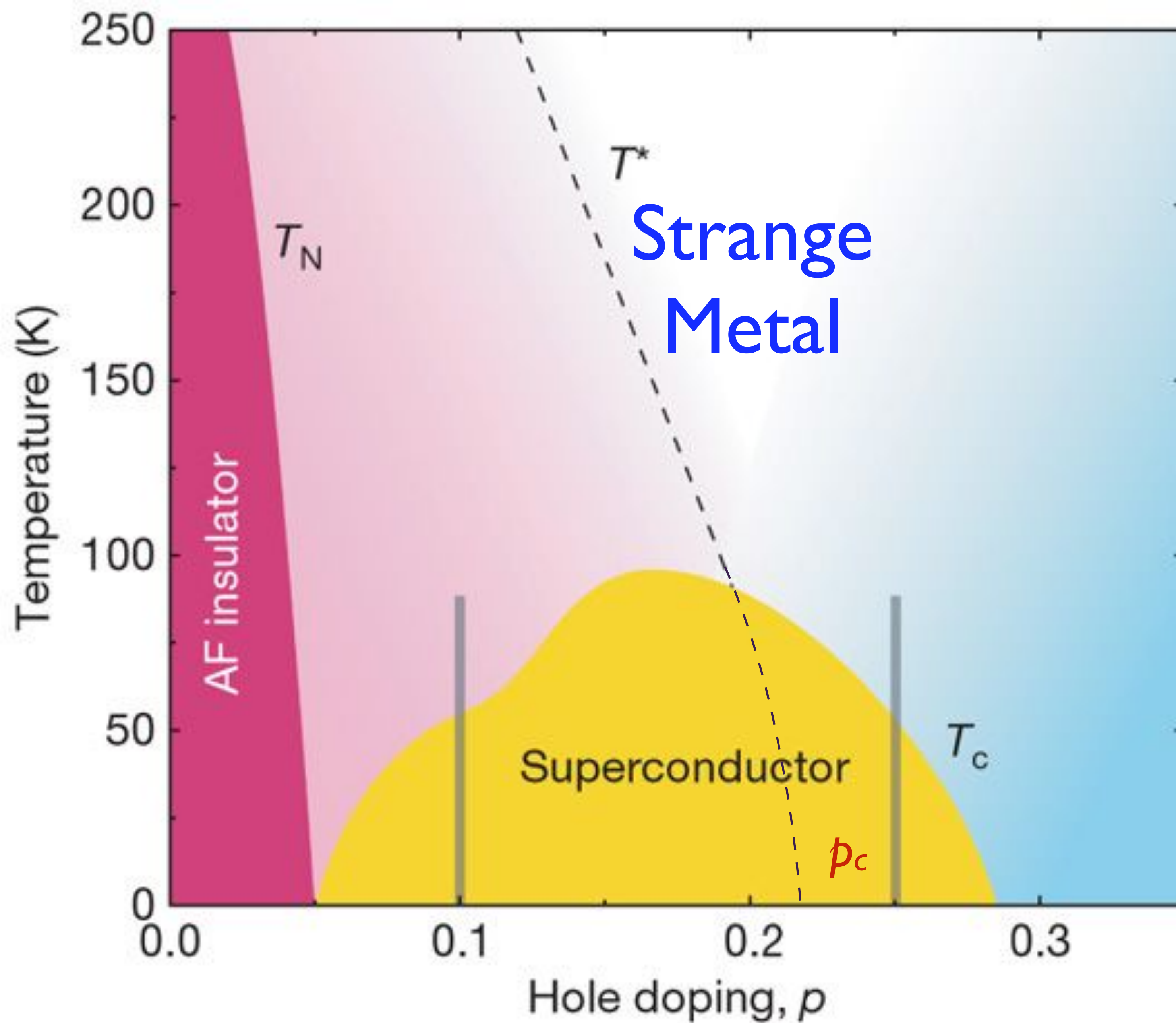
○ Clean

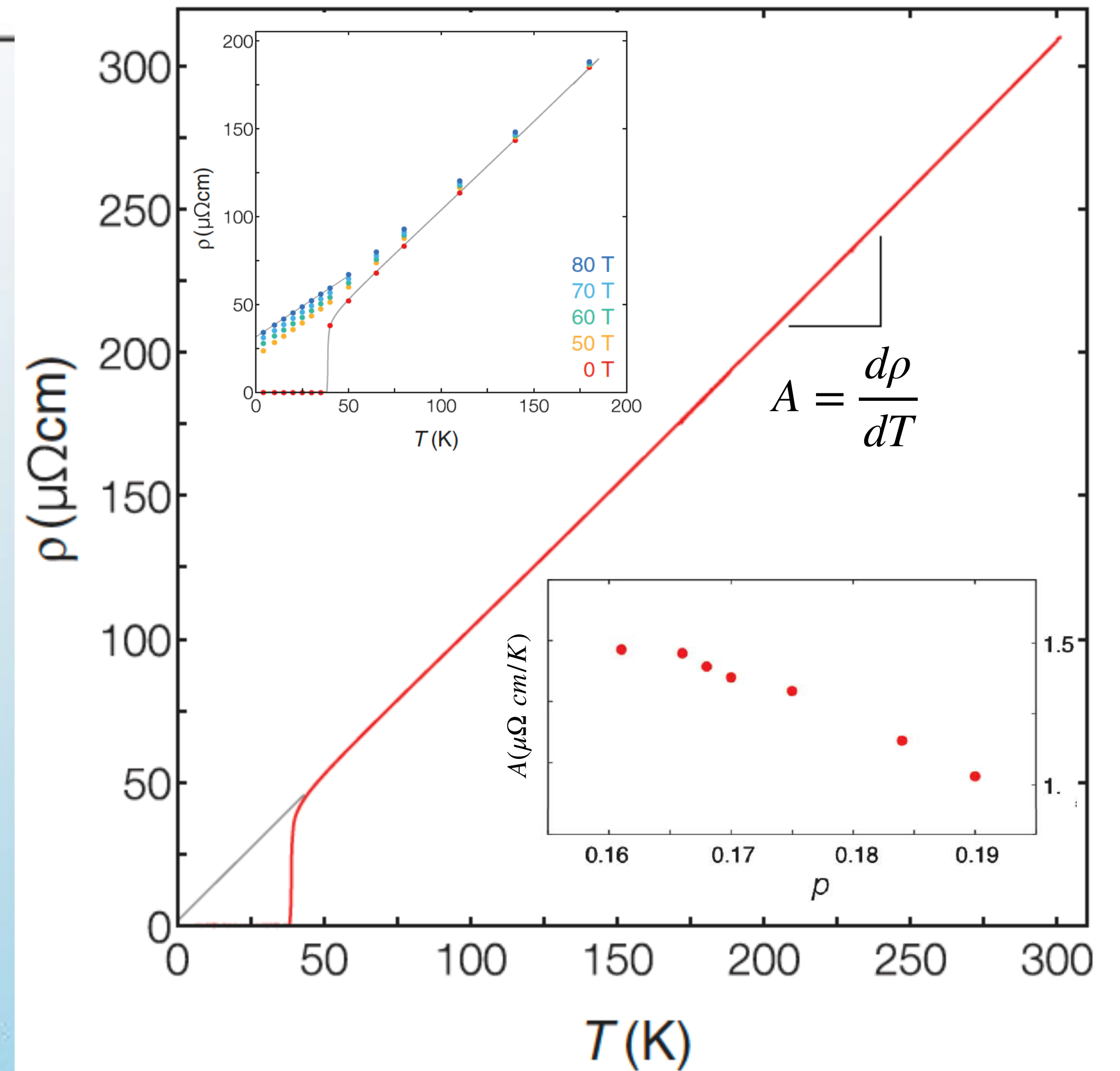
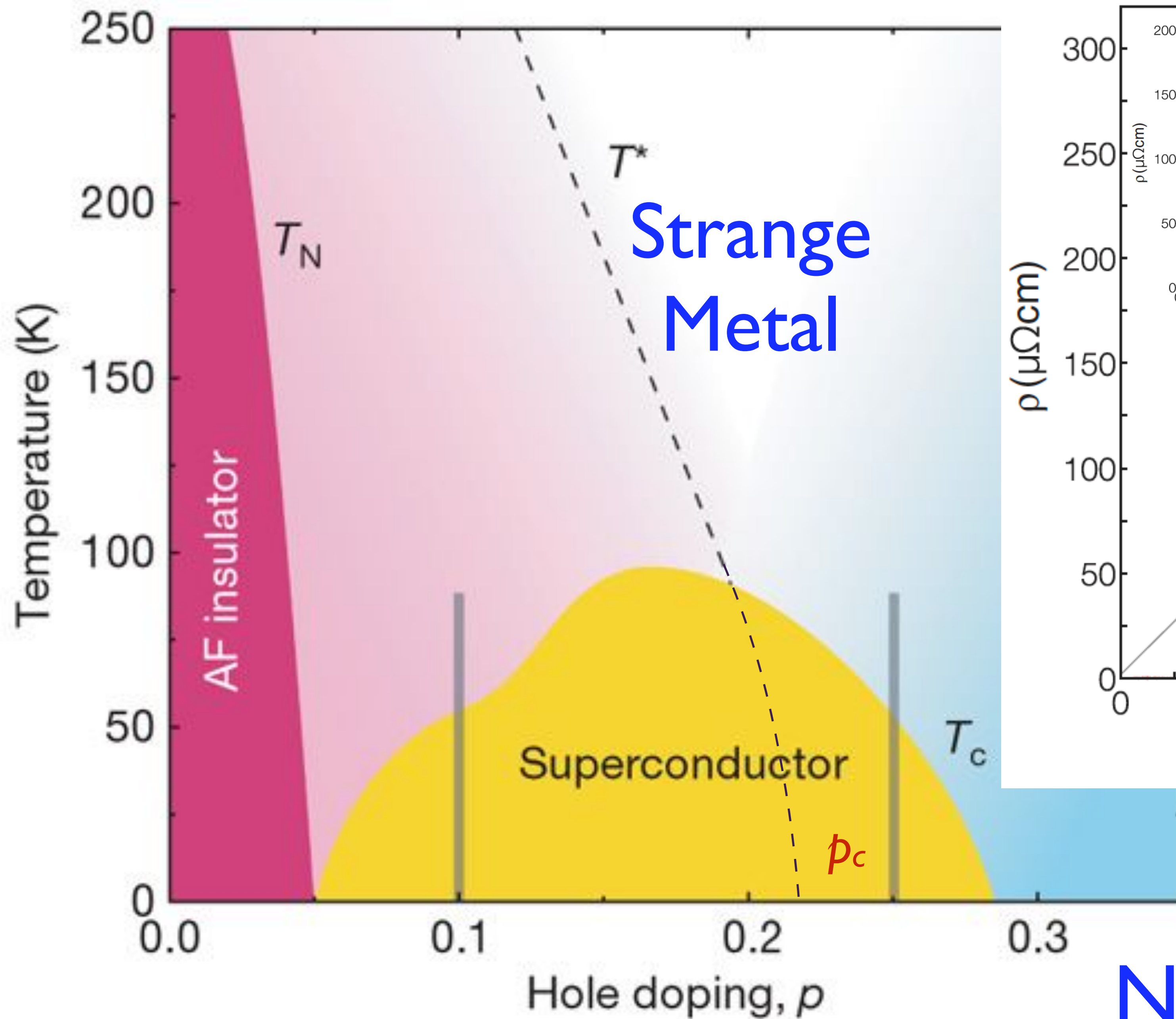
A new source of clean energy to meet our growing energy demands and combat climate change.



Commonwealth
Fusion Systems







LSCO: Giraldo-Gallo et al. 2018

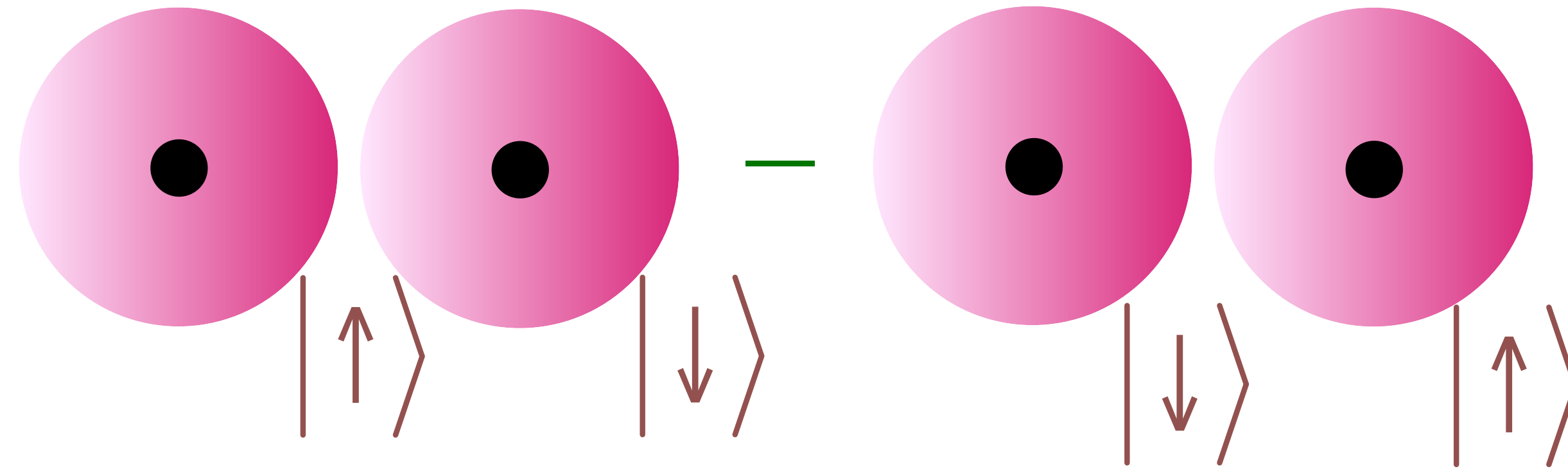
No quasiparticles ???

The Sachdev-Ye-Kitaev model
of quantum matter without
quasiparticles

The most remarkable new idea in the quantum theory is the
principle of superposition:
a physical system can be in a
superposition of two (or more) distinct states.

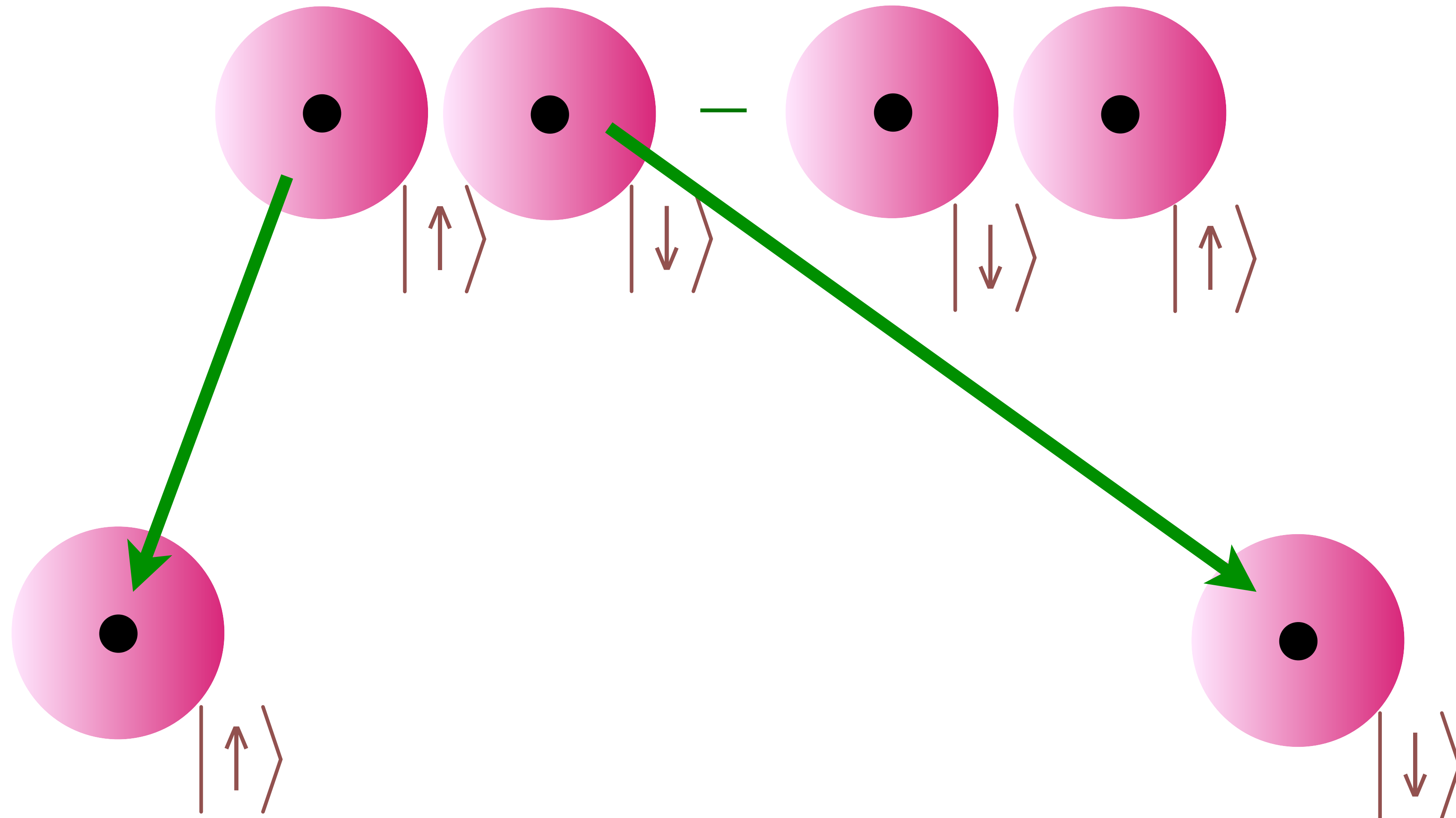
Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



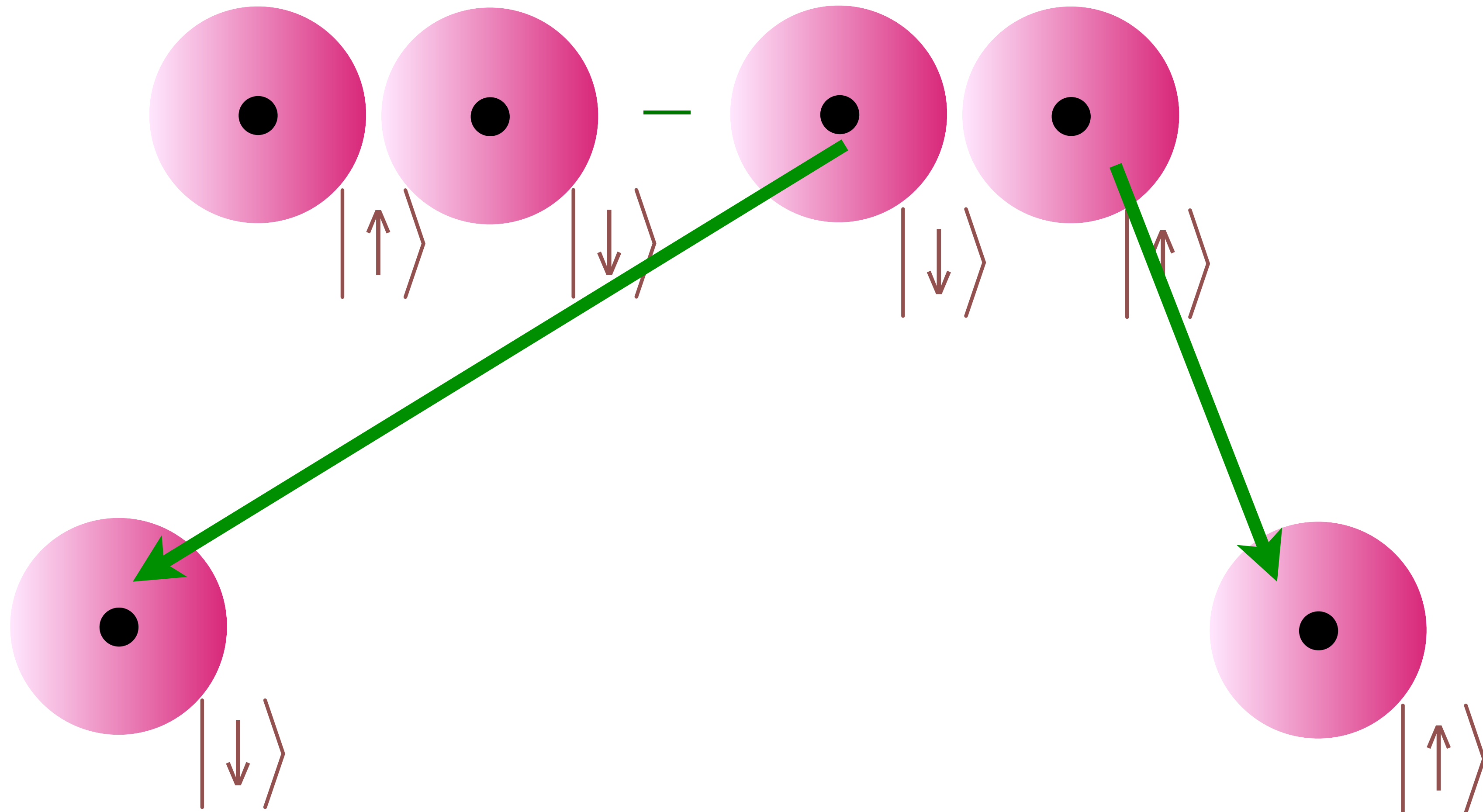
Quantum Entanglement

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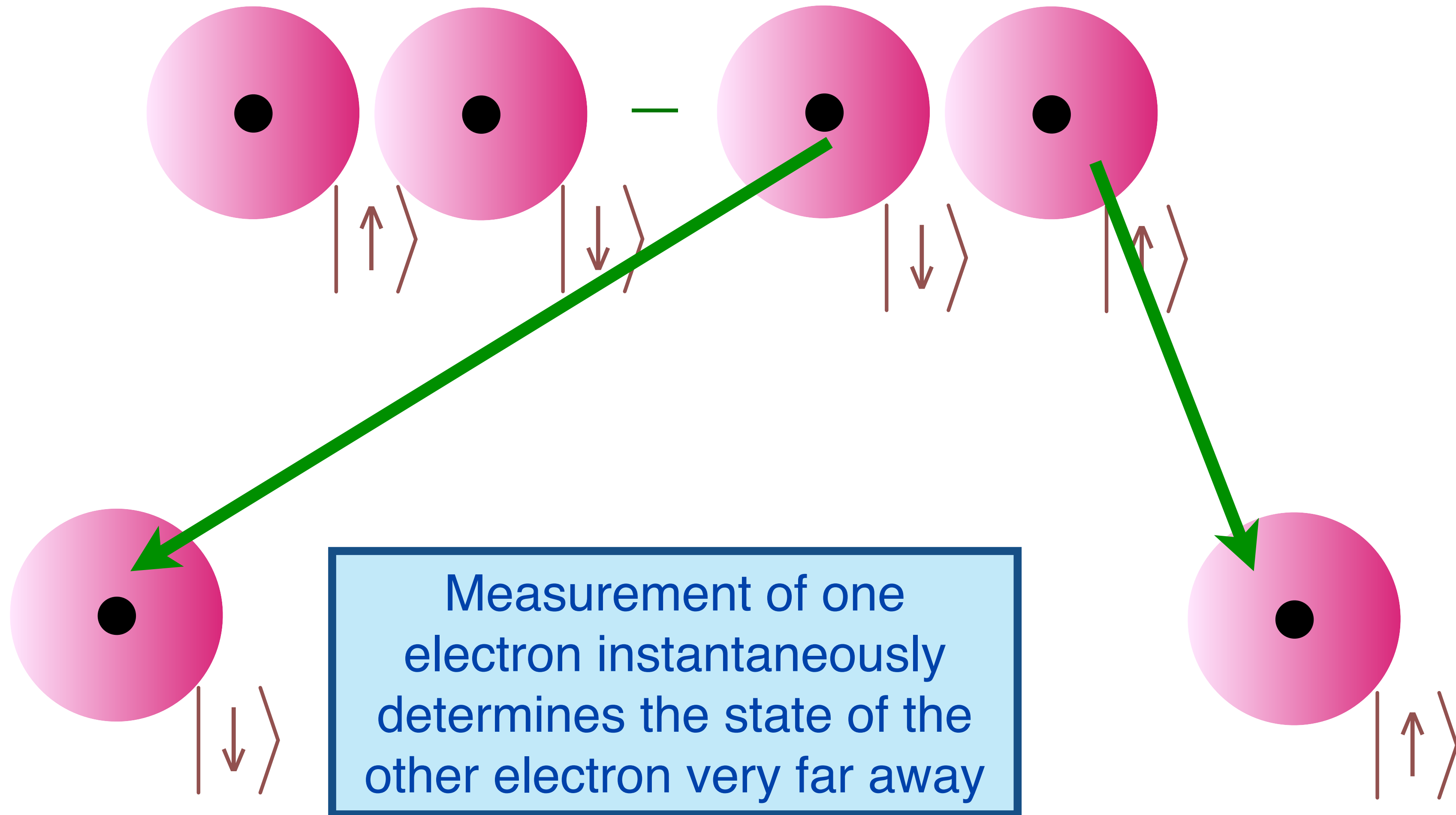
Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



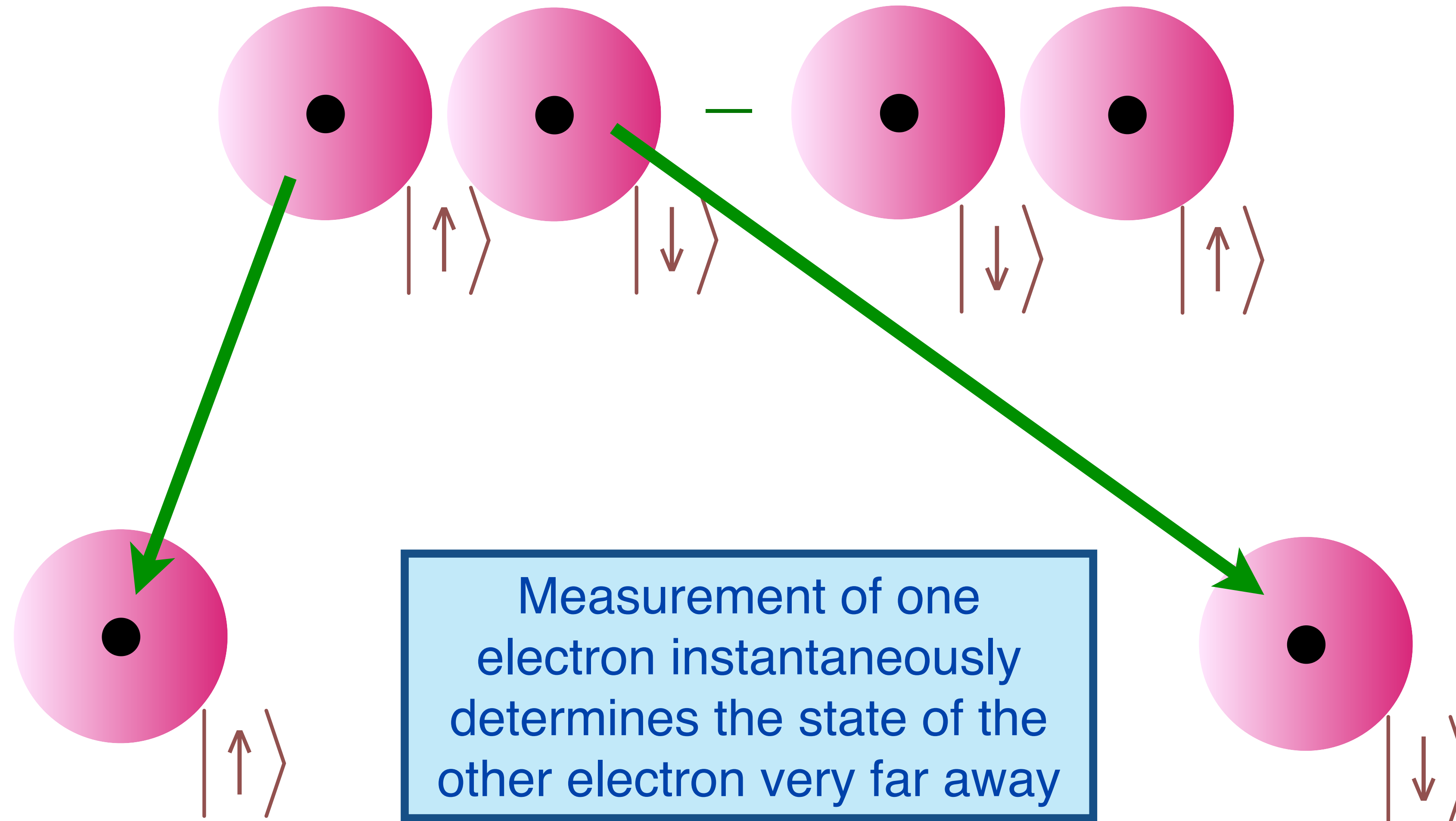
Quantum Entanglement

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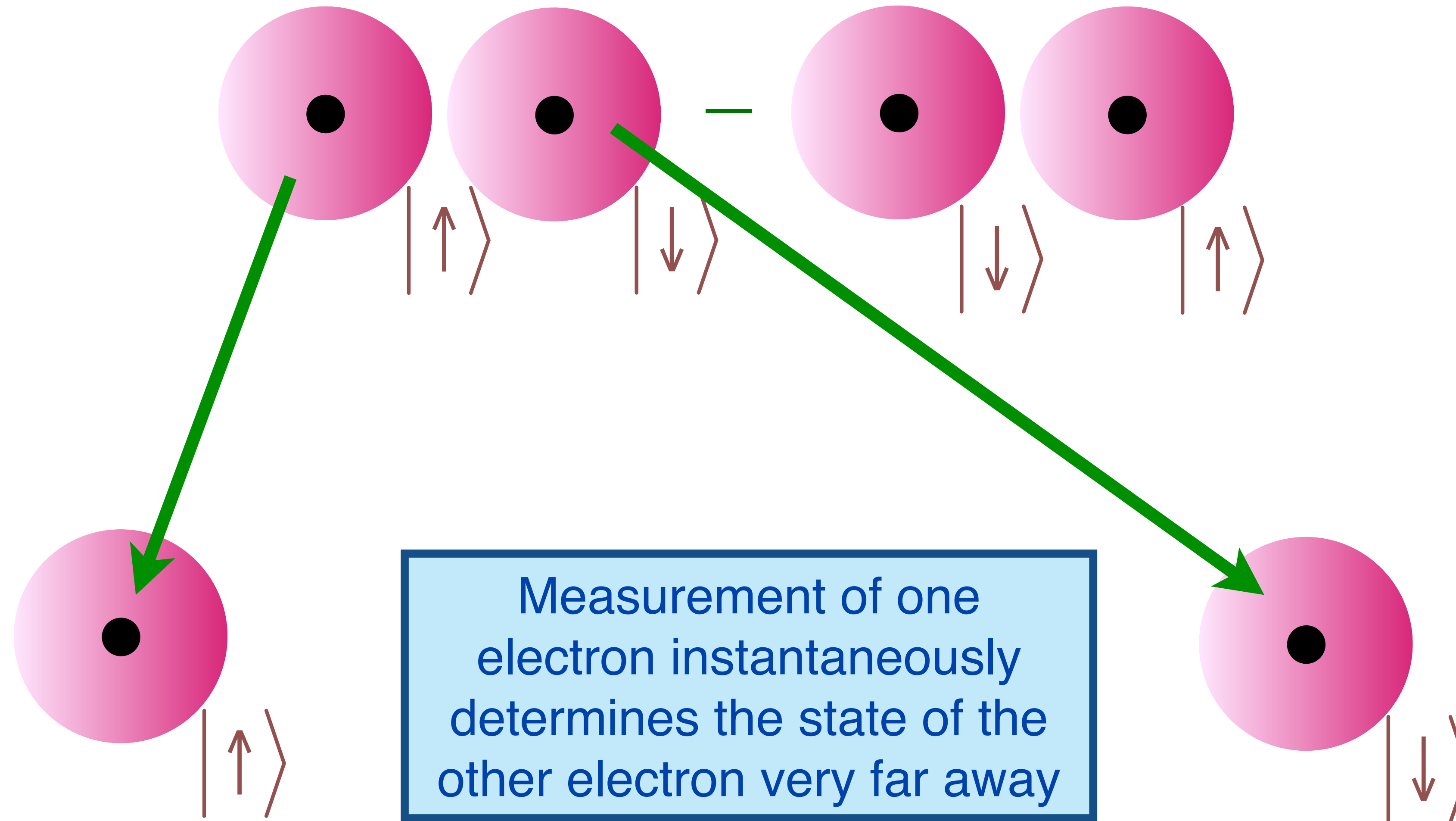
Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



Spooky action at a distance !

natürlicher
deren Notwendigkeit im Raum
mus ja zuerst von Dir klar erkannt wurde, einen Bedeutung
Wahrheitsgehalt hat. Ich kann aber deshalb nicht ernsthaft dar-
an glauben, weil die Theorie mit dem Grundsatz unvereinbar
ist, daß die Physik eine Wirklichkeit in Zeit und Raum darstel-
len soll, ohne spukhafte Fernwirkungen. Allerdings bin ich
überzeugt daß es wirklich mit der Theorie

amount of validity in the
recognise clearly as necessary given the framework of
malism. I cannot seriously believe in it because the theory cannot be rec-
onciled with the idea that physics should represent a reality in time and
space, free from spooky actions at a distance. I am, however, not yet
convinced that it can really be achieved with a continuous field
... this which so

I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at distance

Albert Einstein to Max Born, 3 March 1947

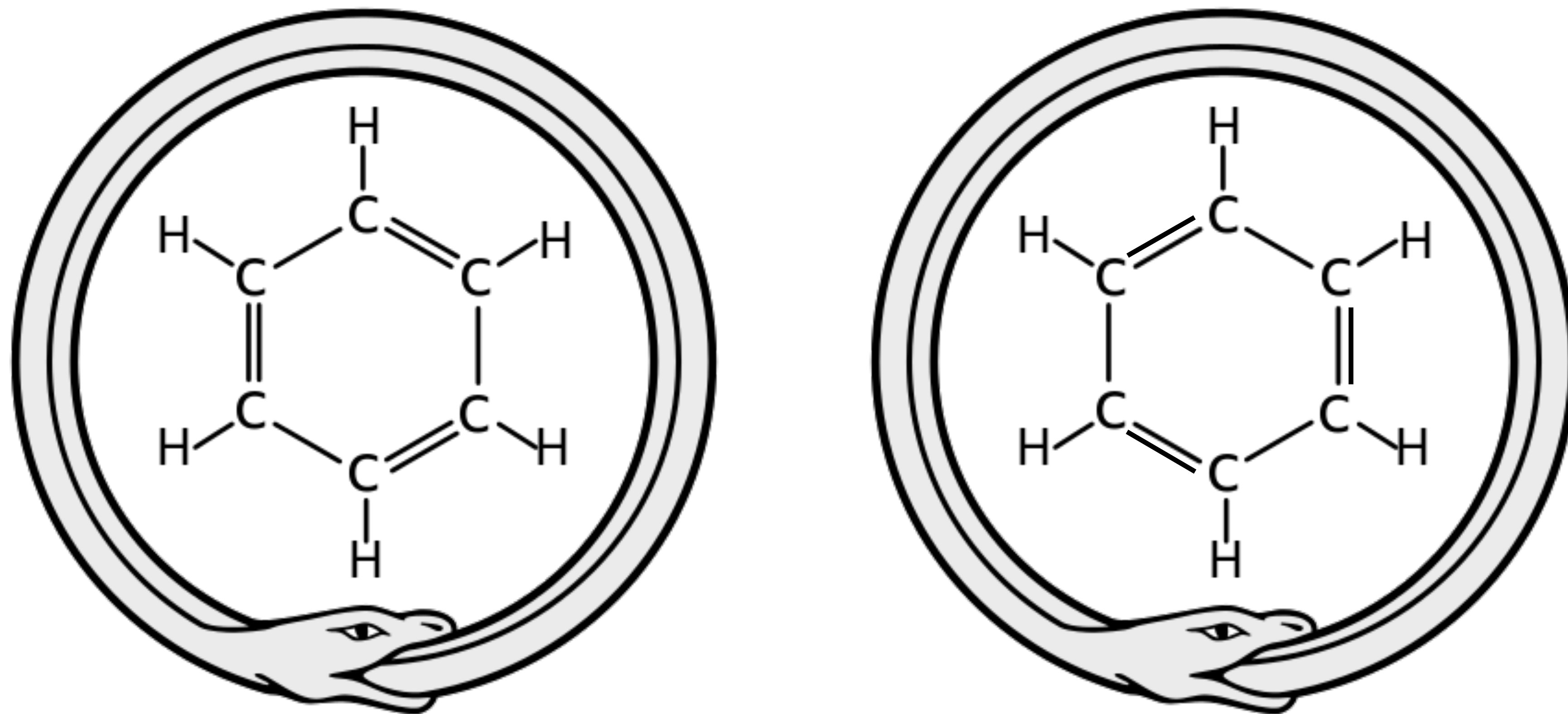
How about quantum entanglement
of 3, 4, 5, \dots ∞ particles?



August Kekule, theory of the benzene molecule, 1865

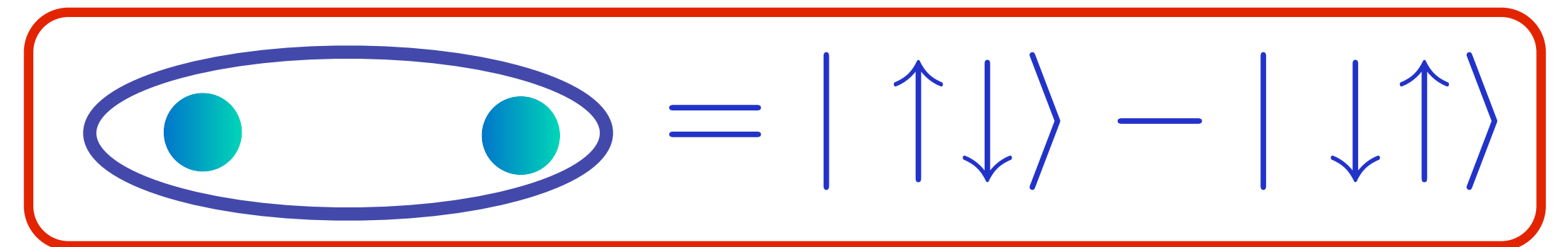
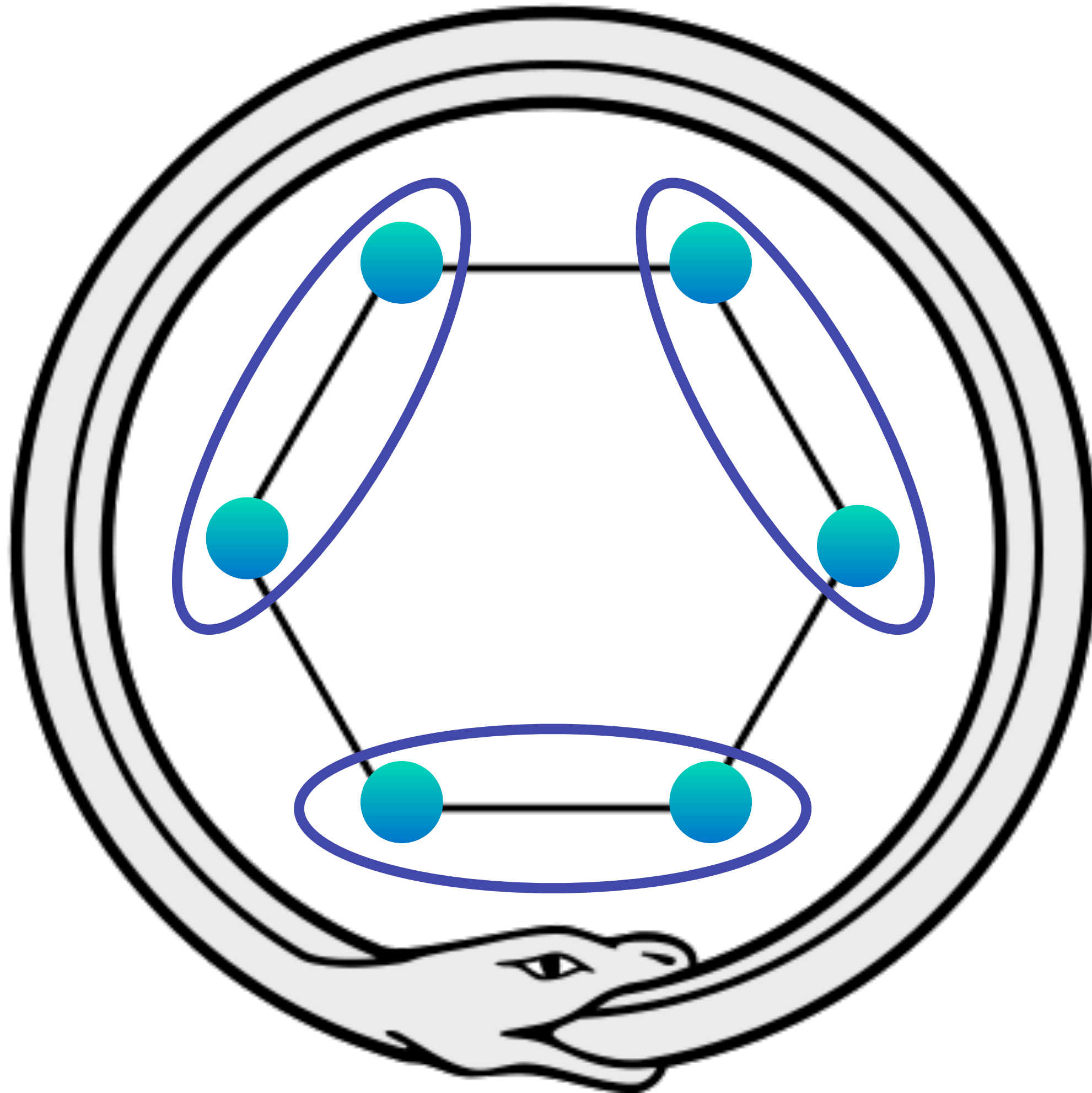
Kekulé's spooky dream

Here Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail^{*}



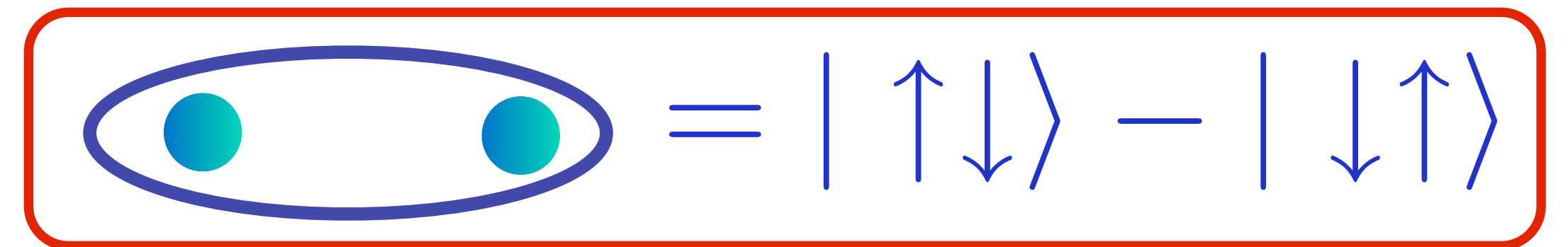
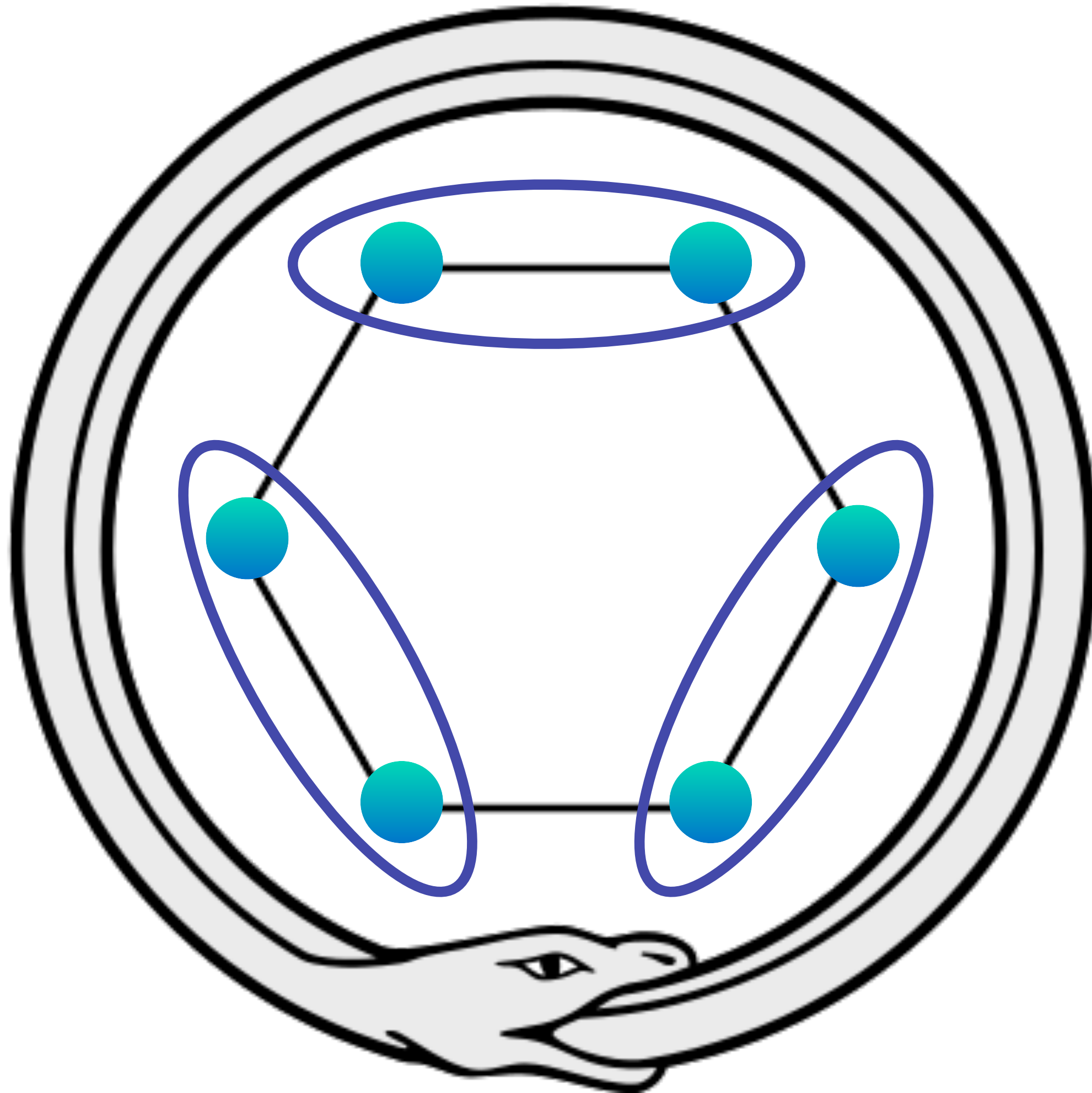
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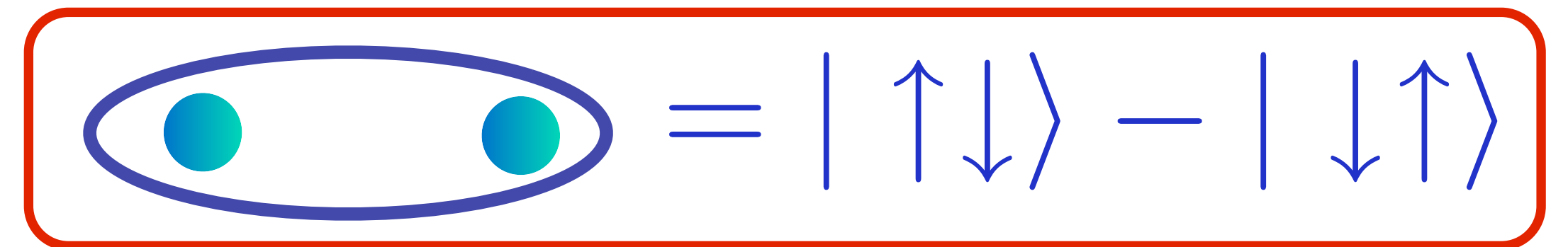
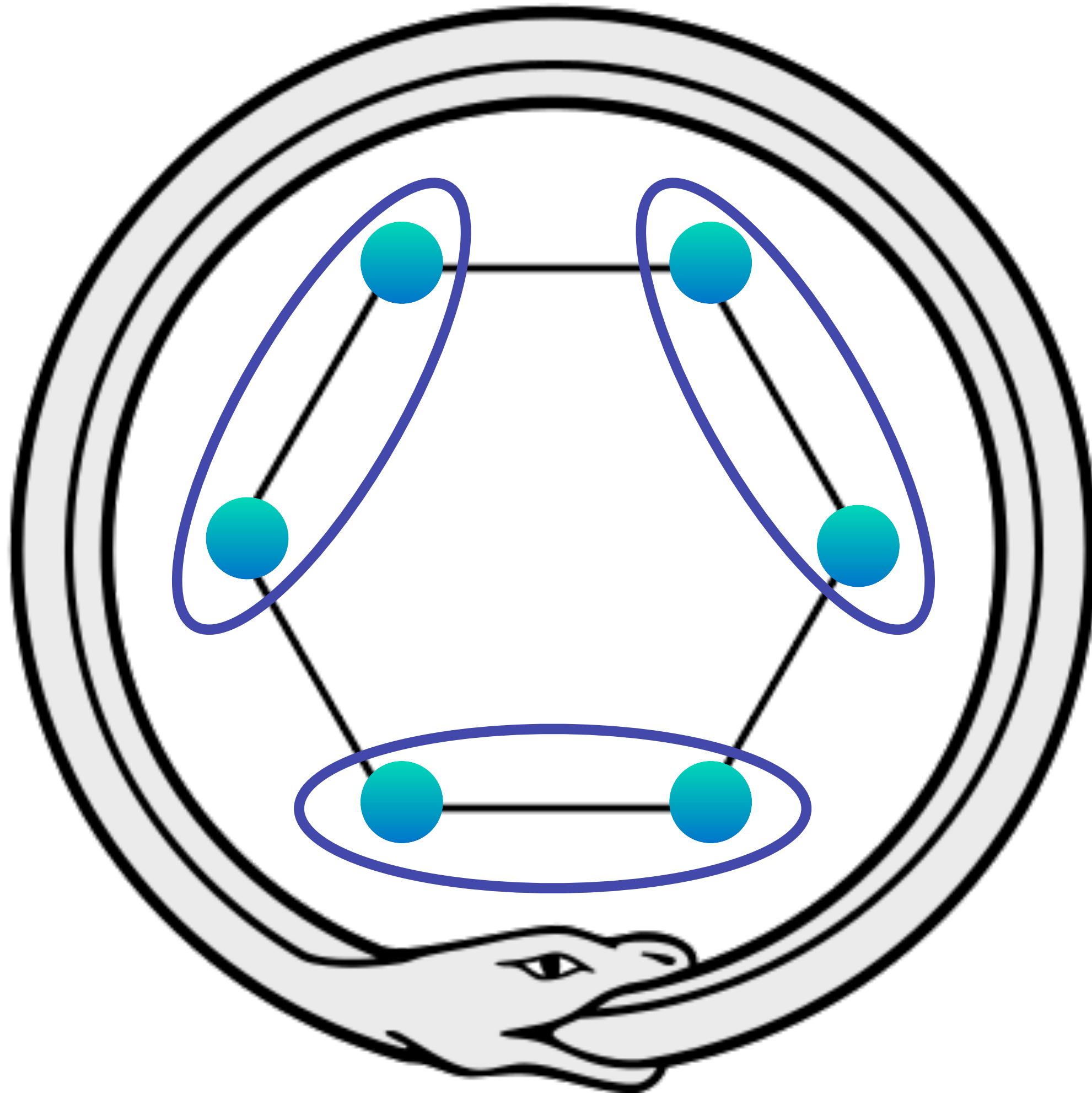
Kekulé's spooky dream

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Kekule's spooky dream

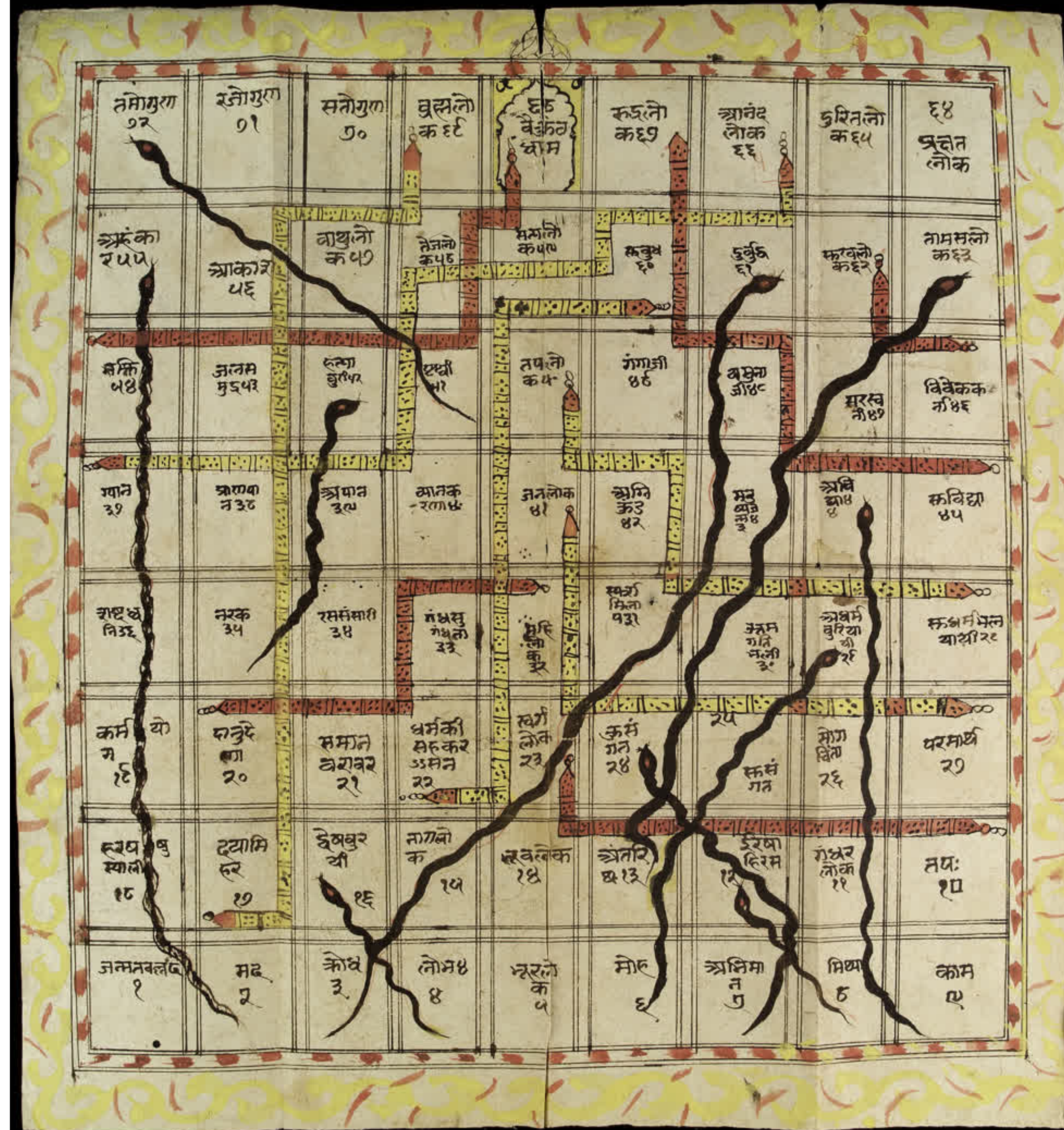
Here Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*



My
spooky
dream*

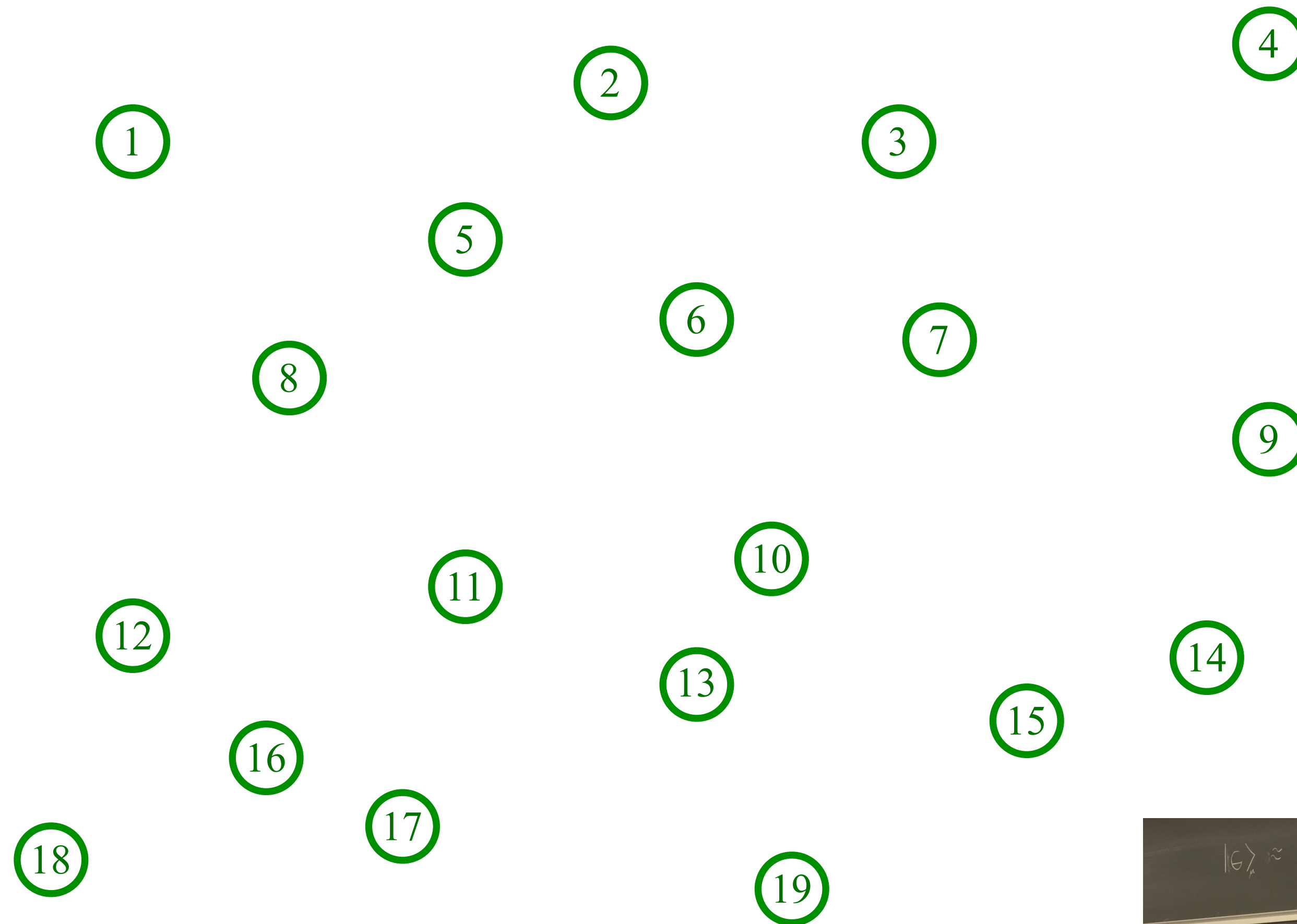
Ancient
Indian
game of
Snakes
and
Ladders

*Not true

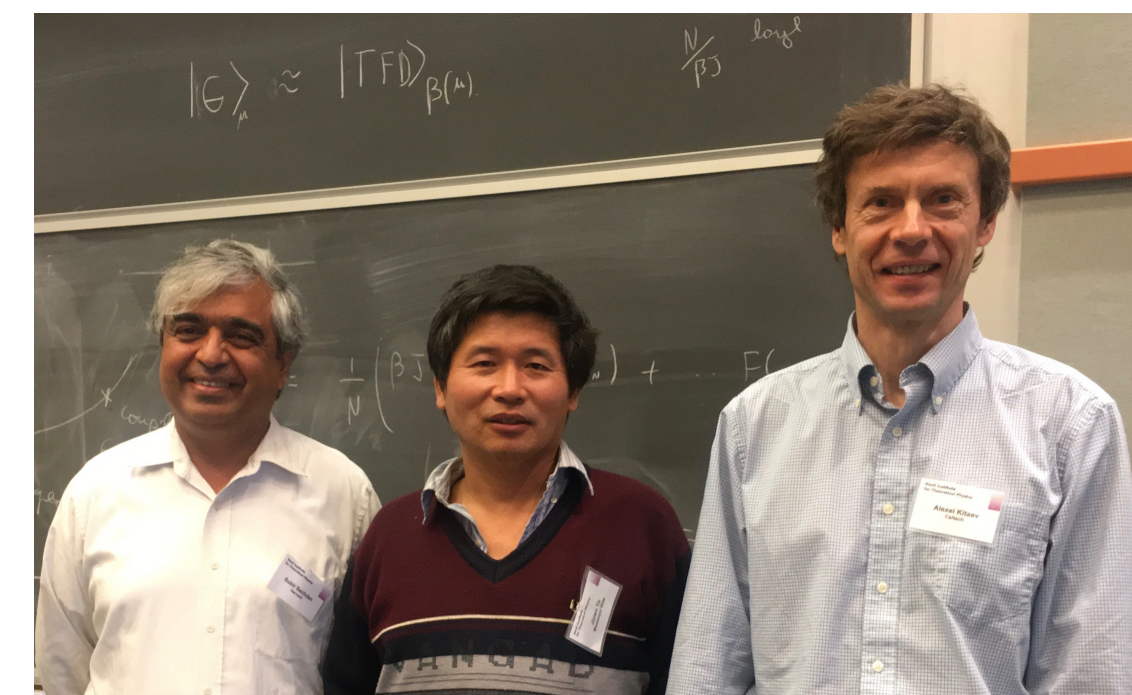


The SYK model

Sachdev, Ye (1993); Kitaev (2015)

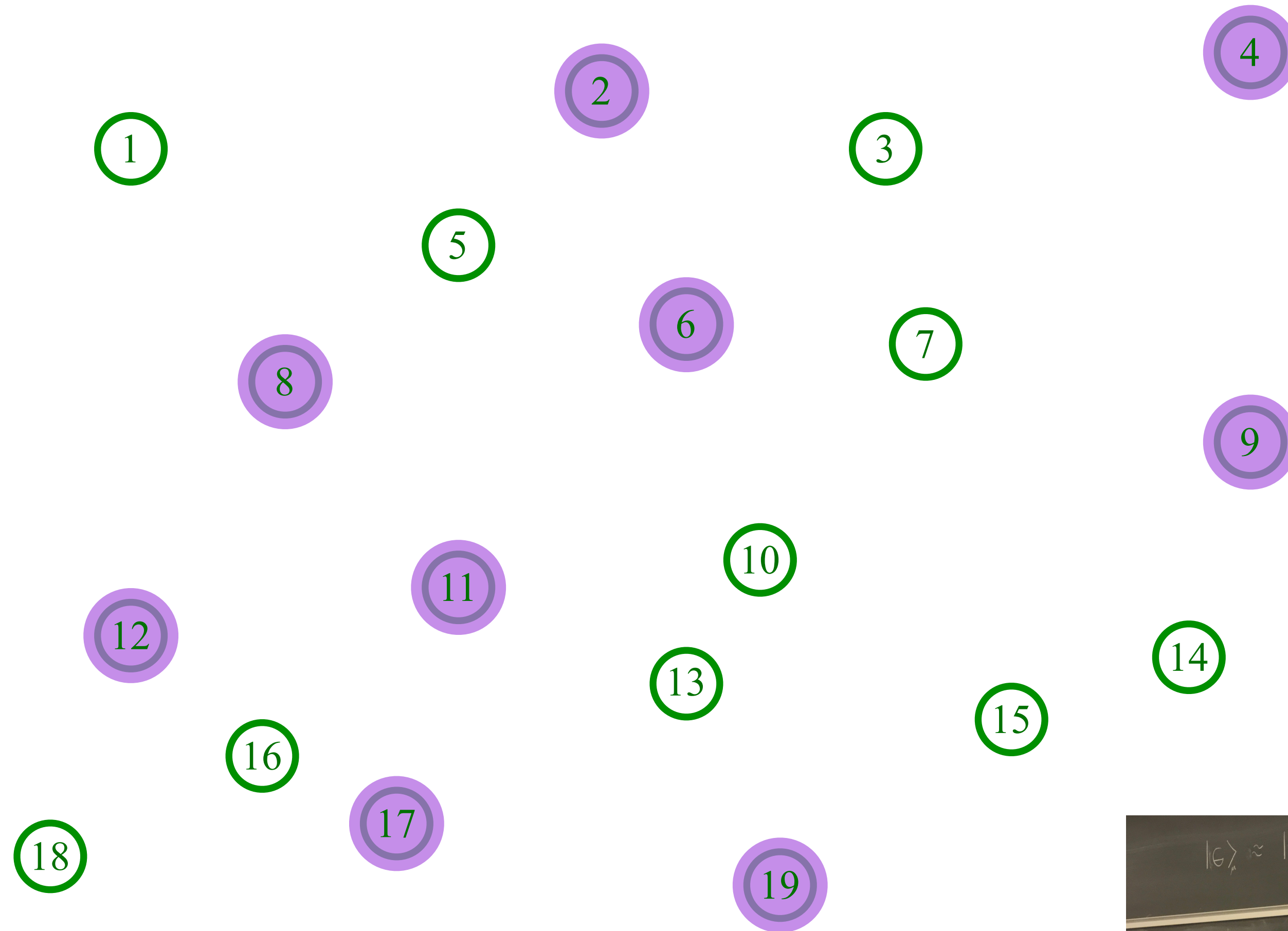


Pick a set of random positions

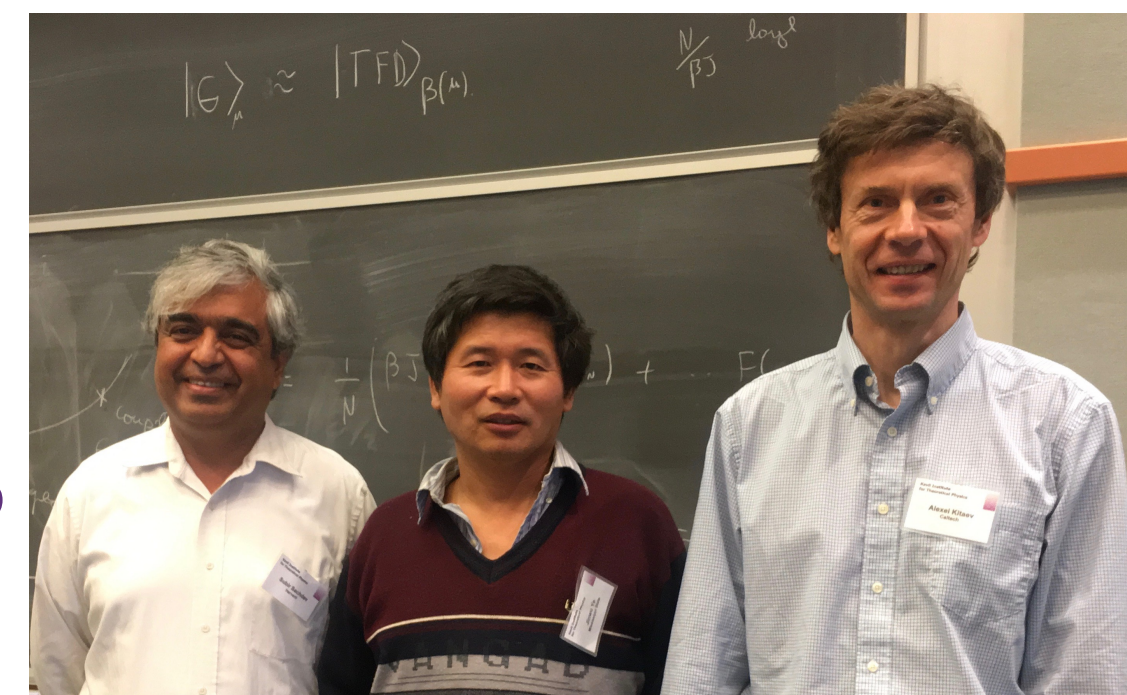


The SYK model

Sachdev, Ye (1993); Kitaev (2015)



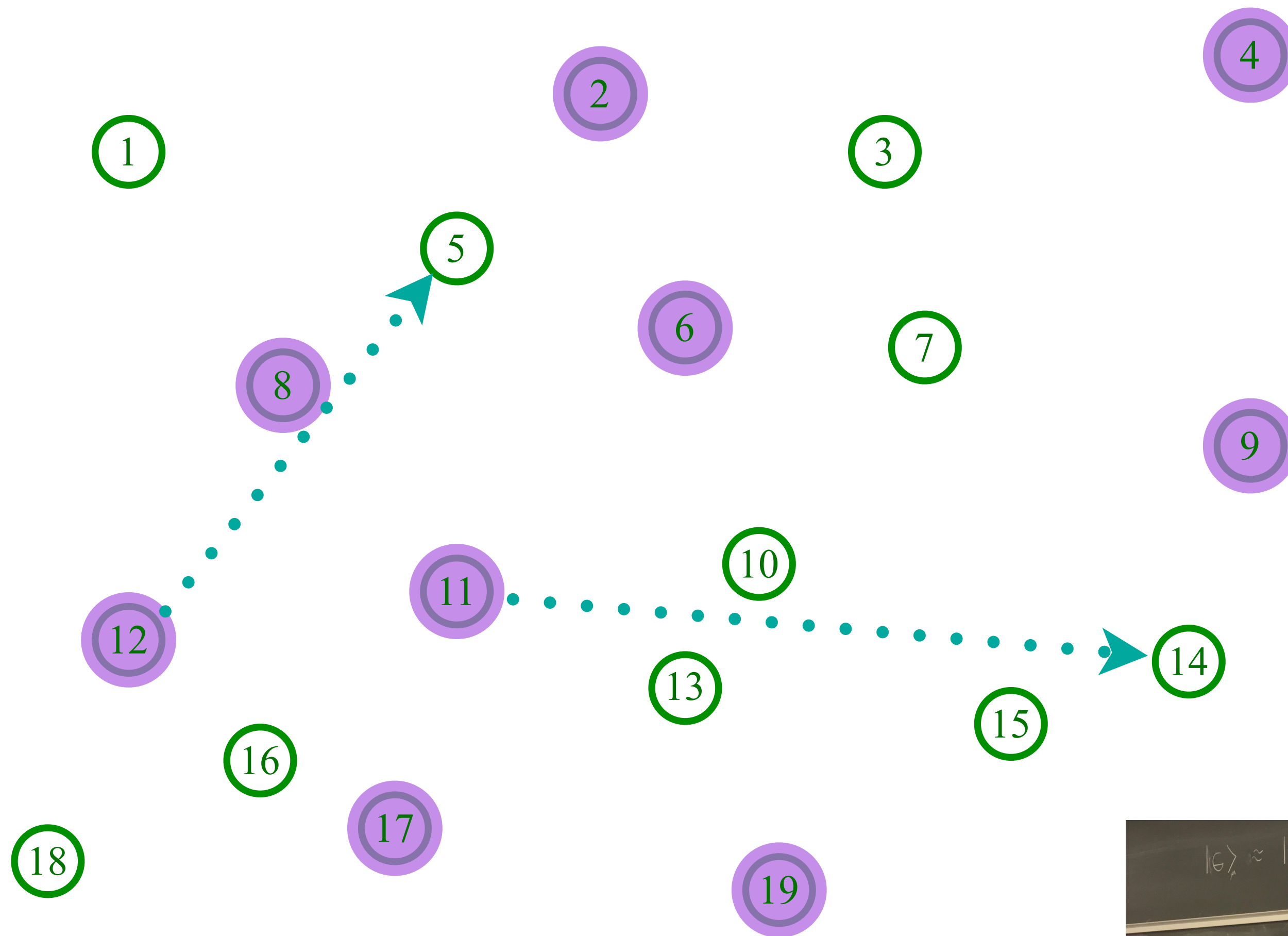
Place electrons randomly on some sites



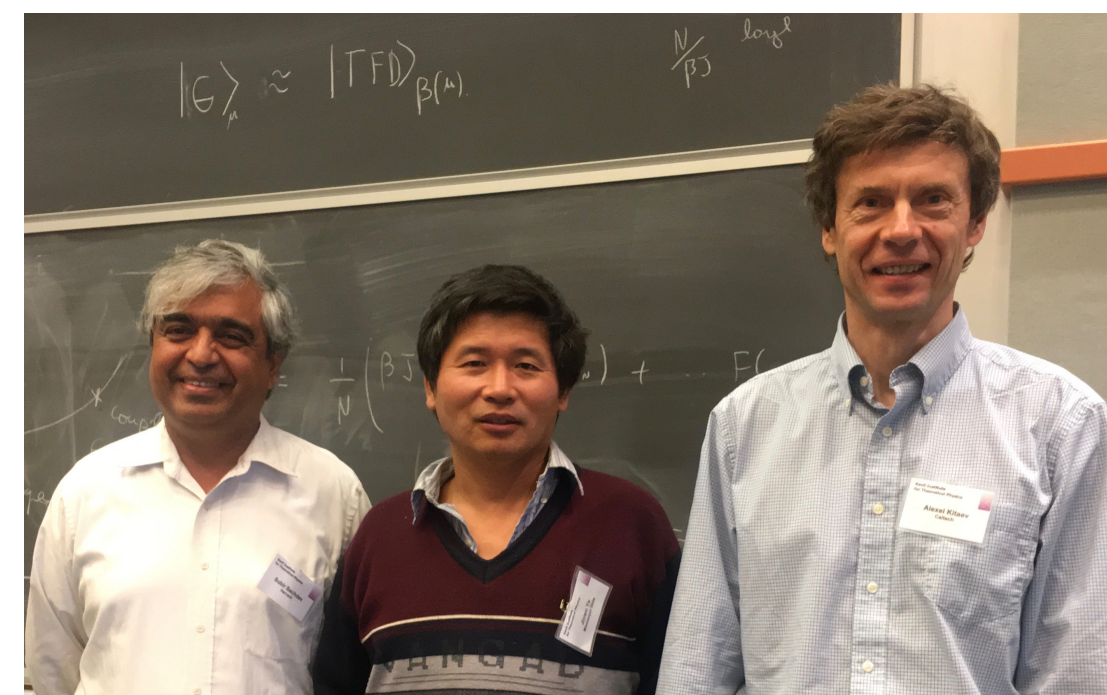
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



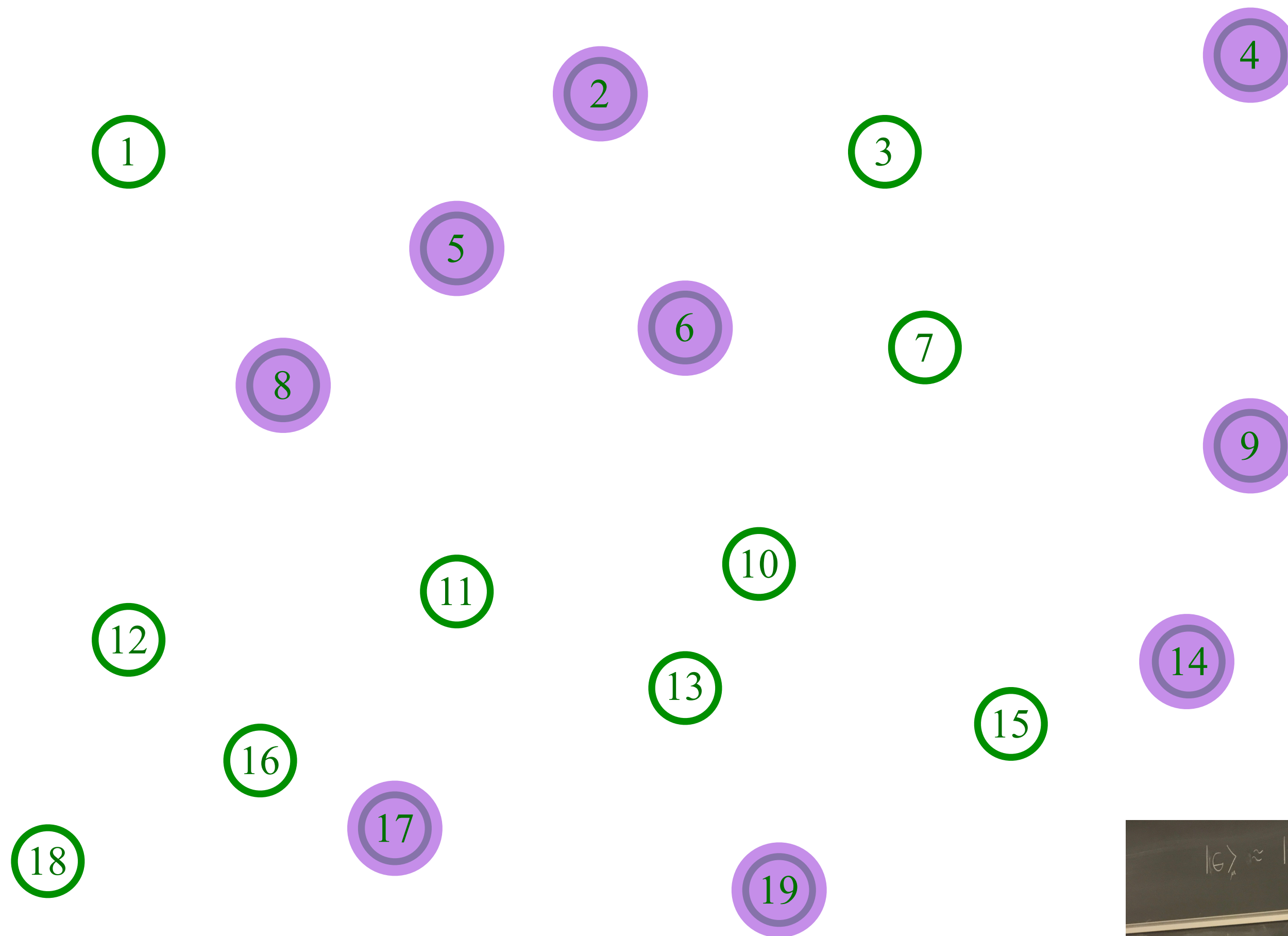
Place electrons randomly on some sites



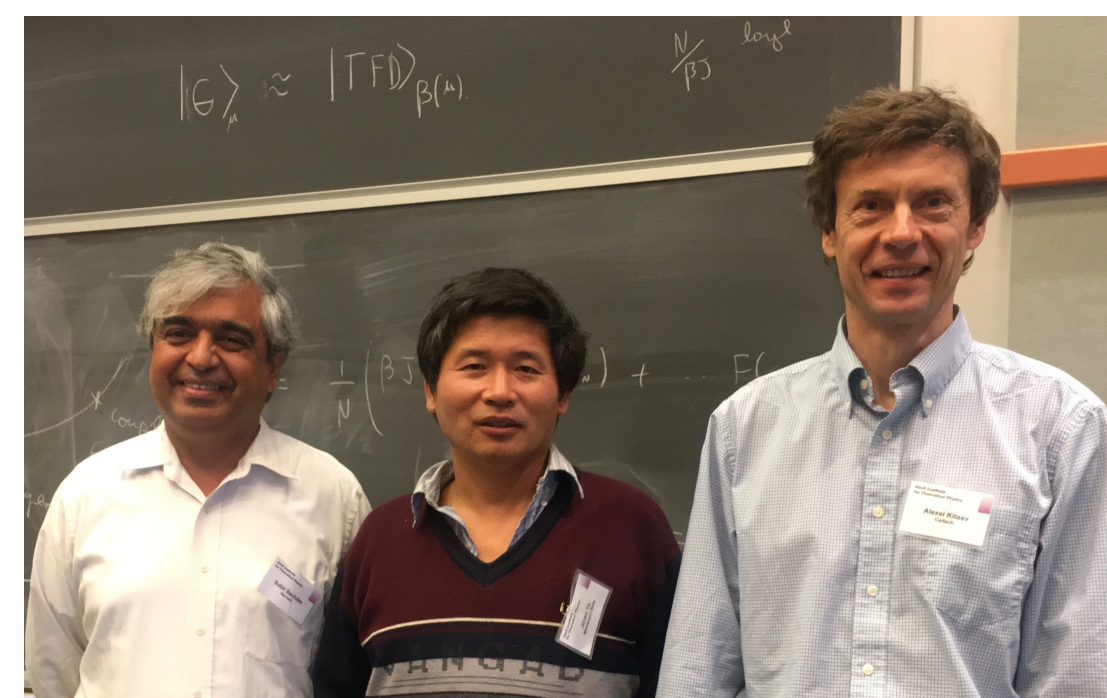
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

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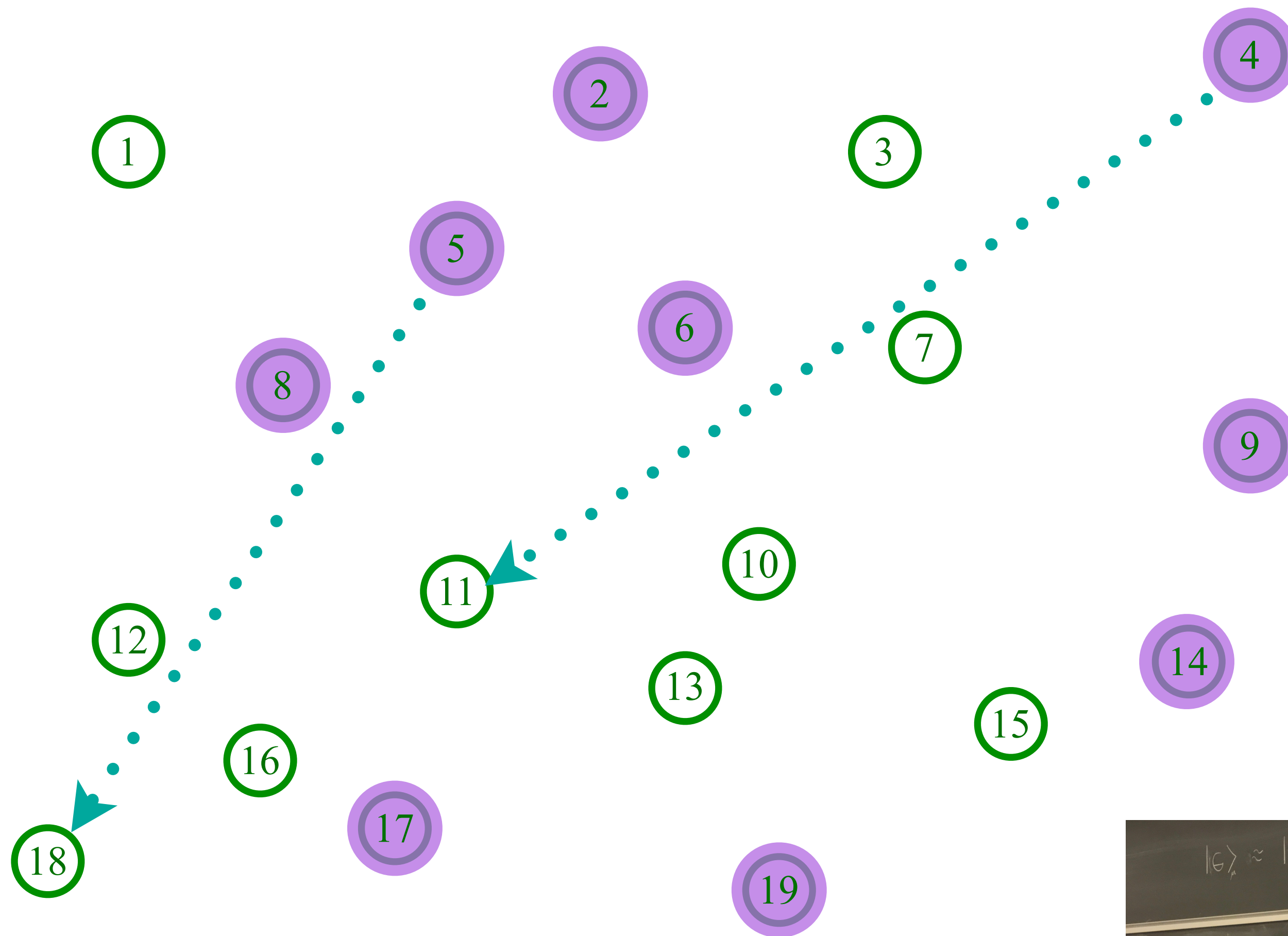
Entangle electrons pairwise randomly



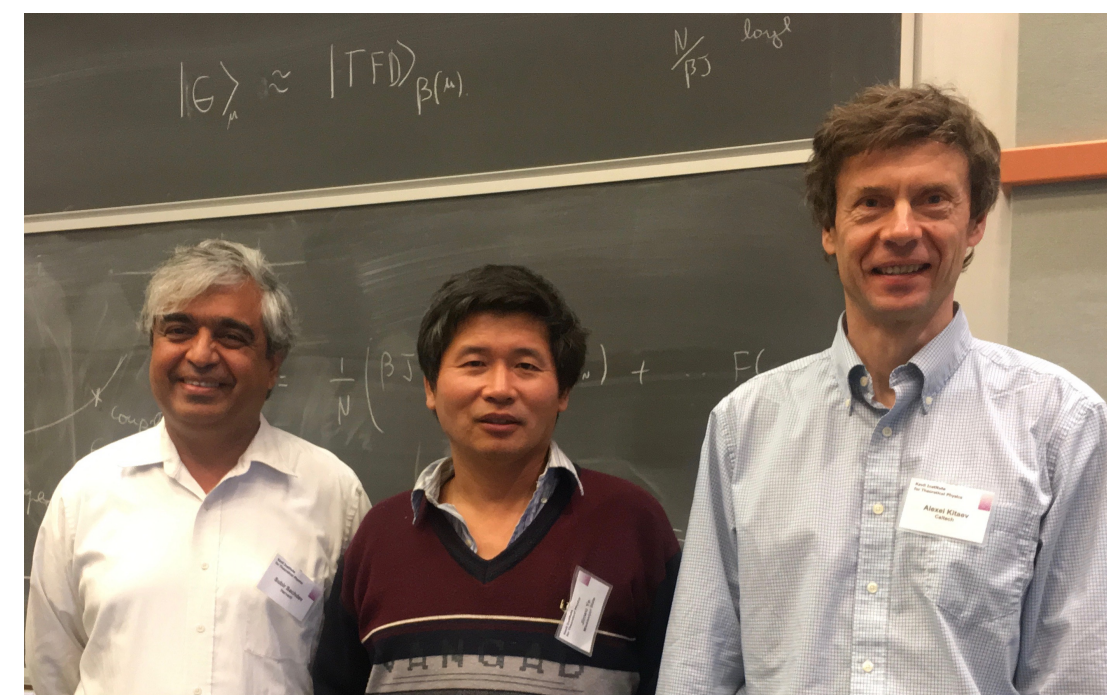
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



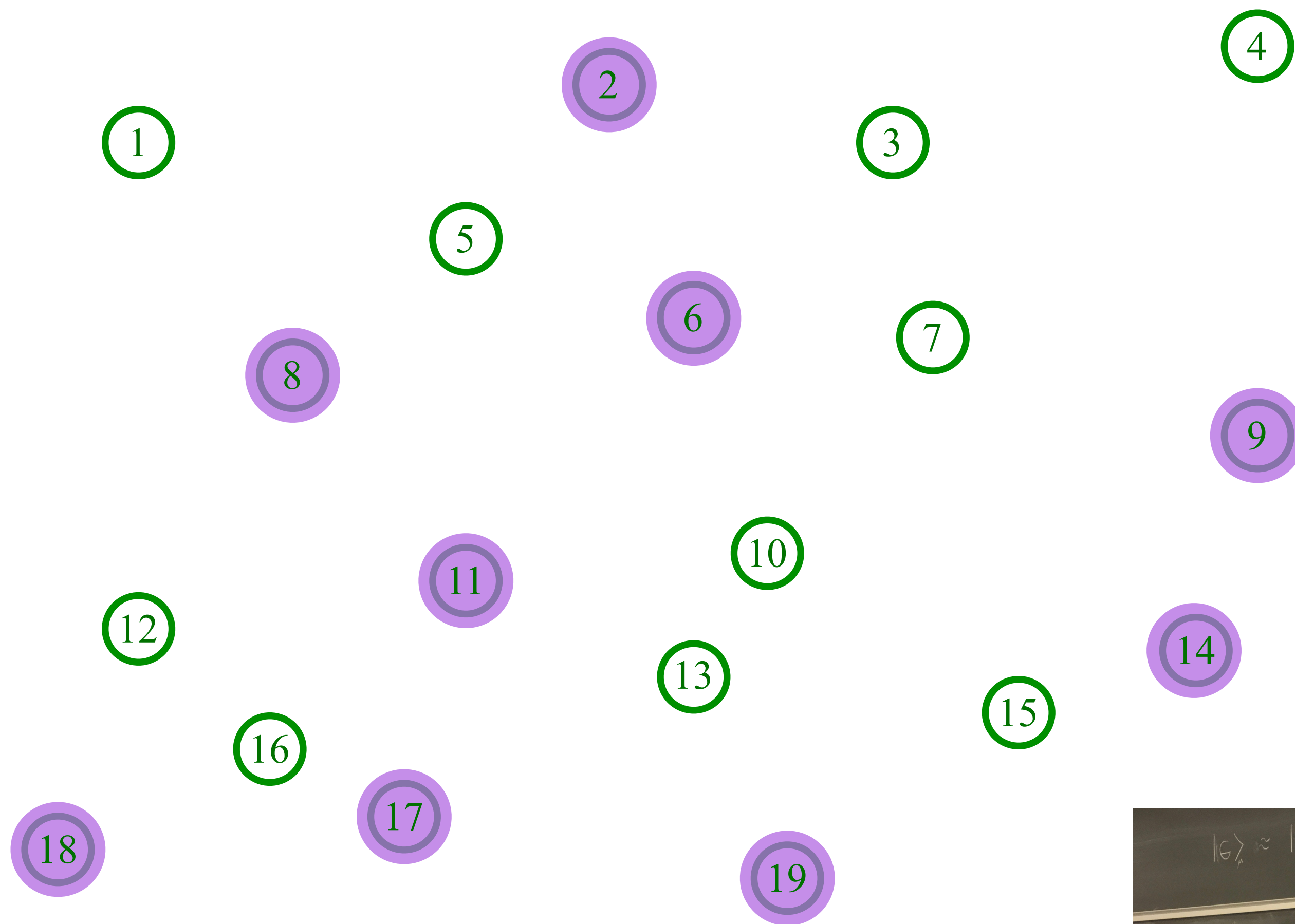
Entangle electrons pairwise randomly



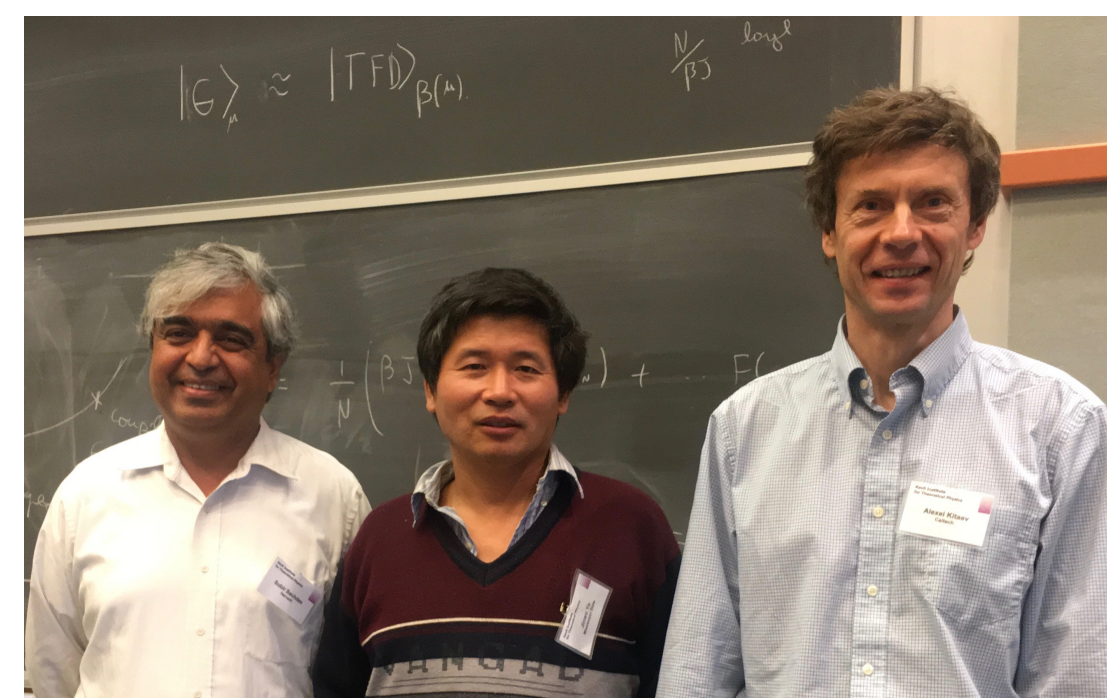
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Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



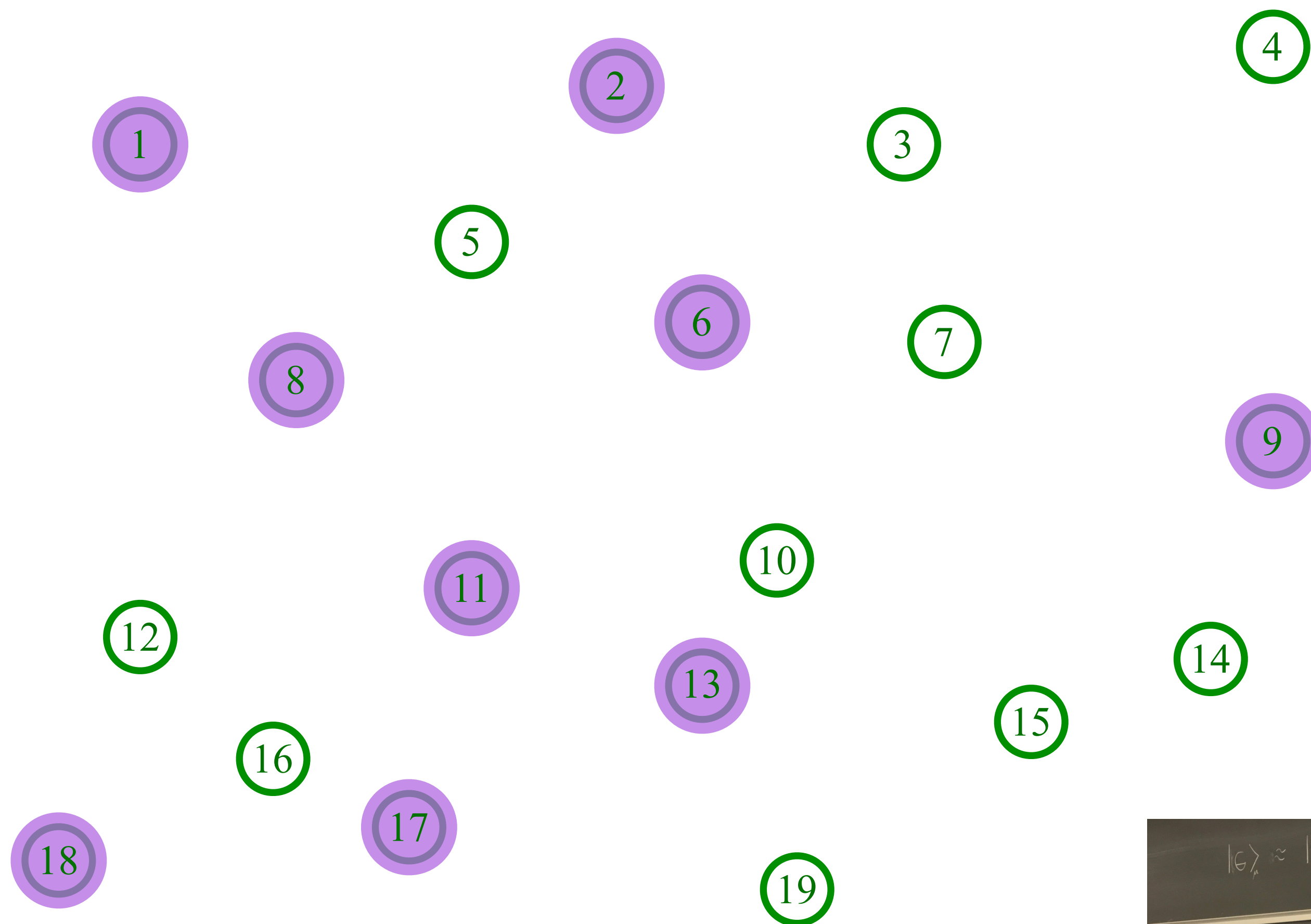
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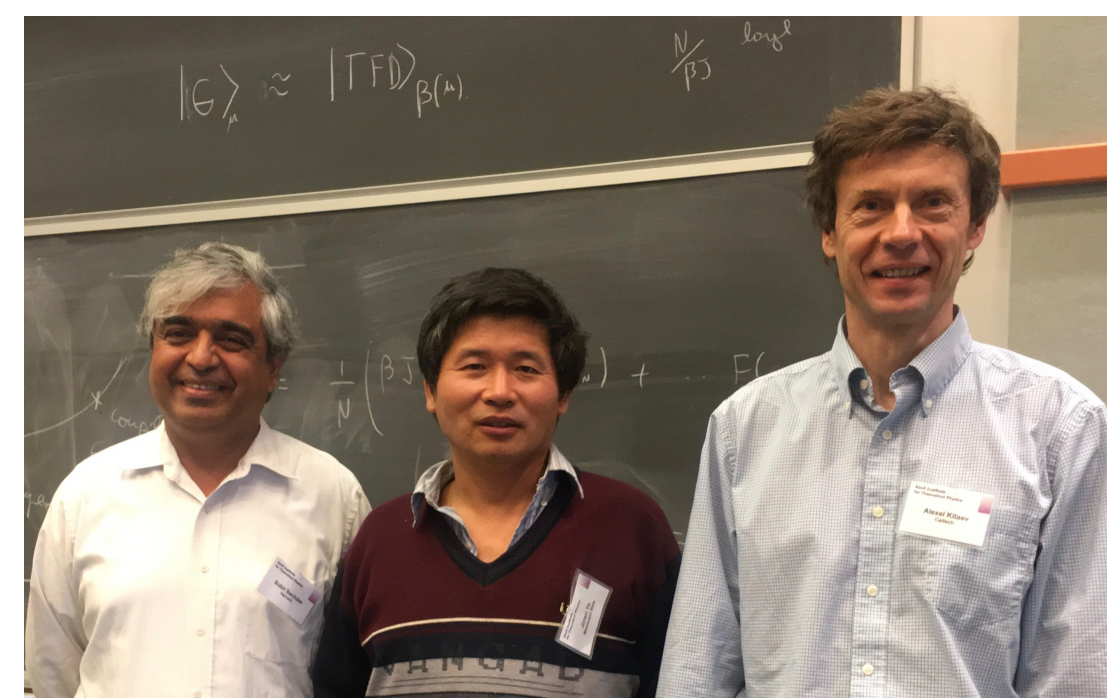
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



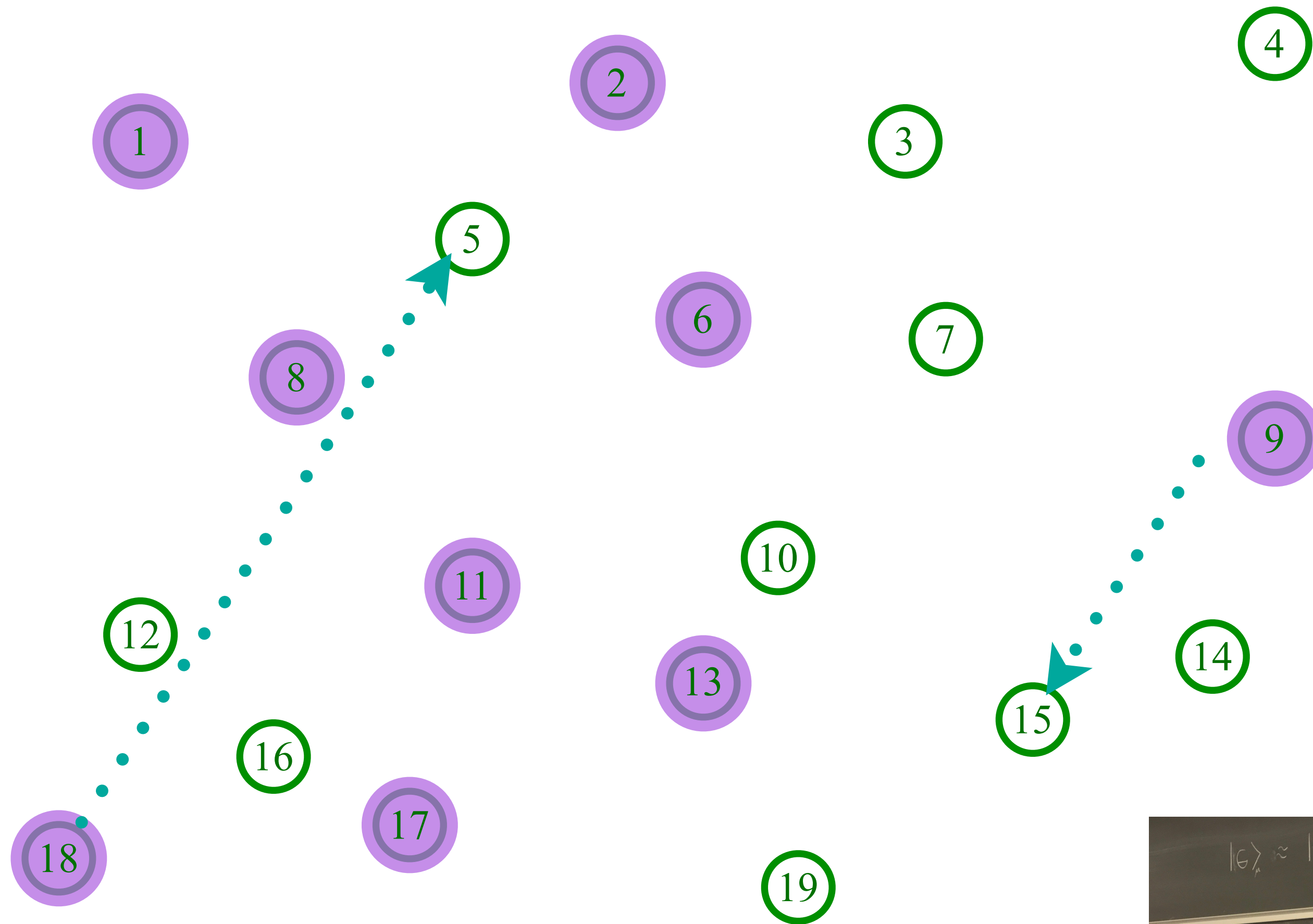
Entangle electrons pairwise randomly



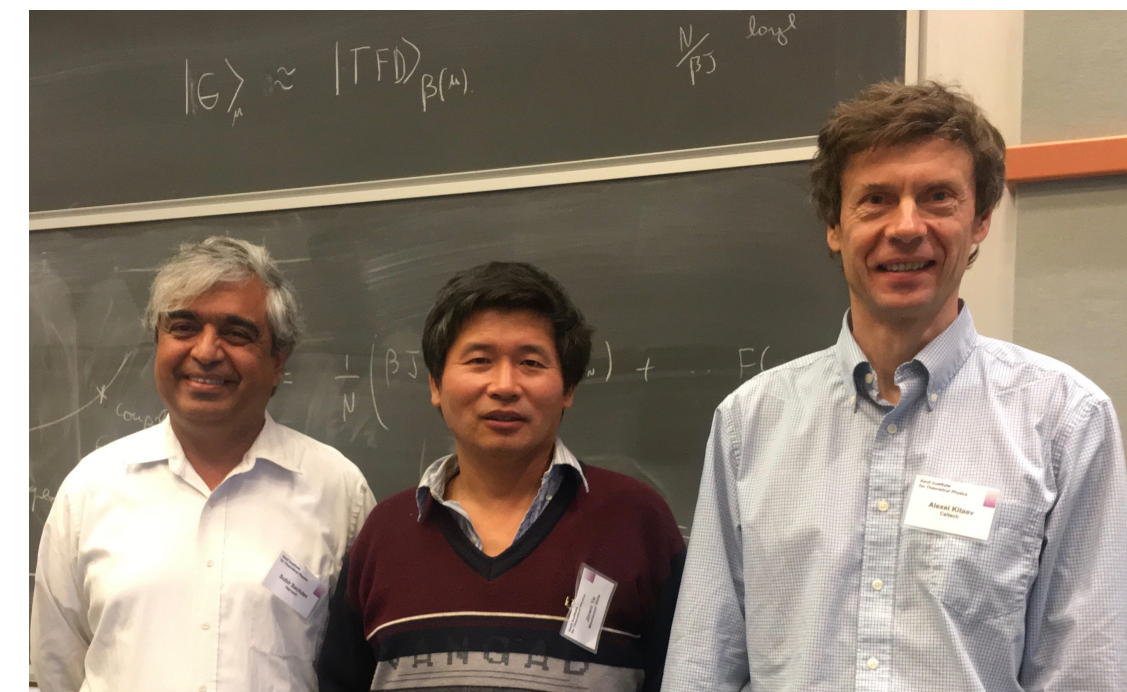
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



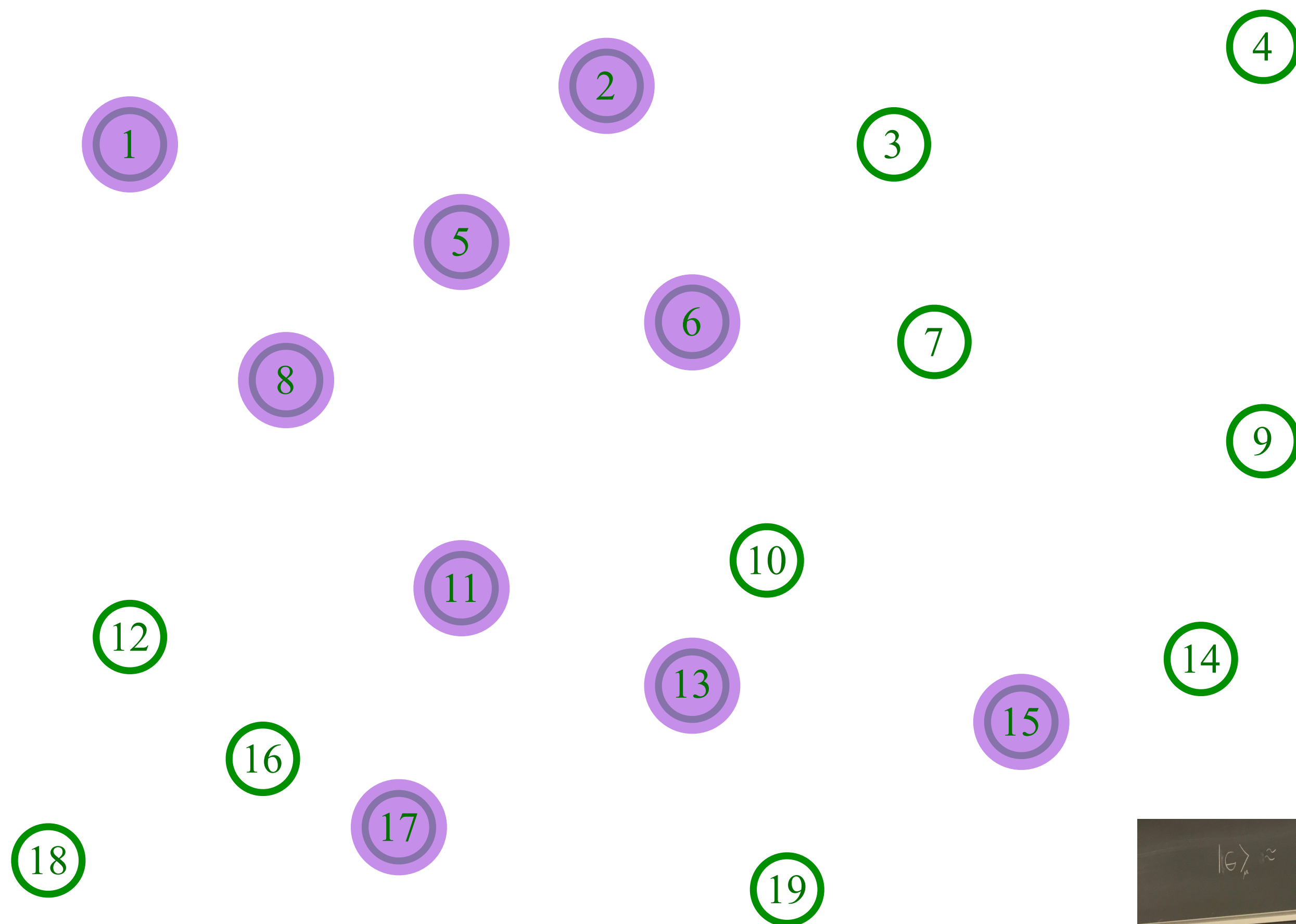
Entangle electrons pairwise randomly



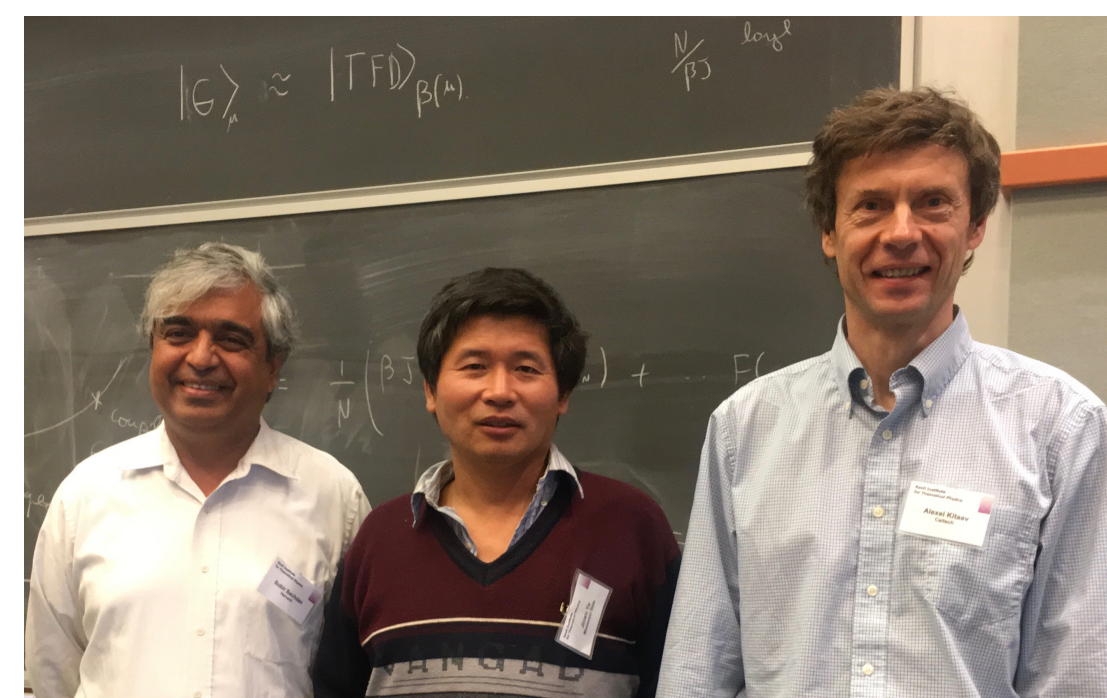
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



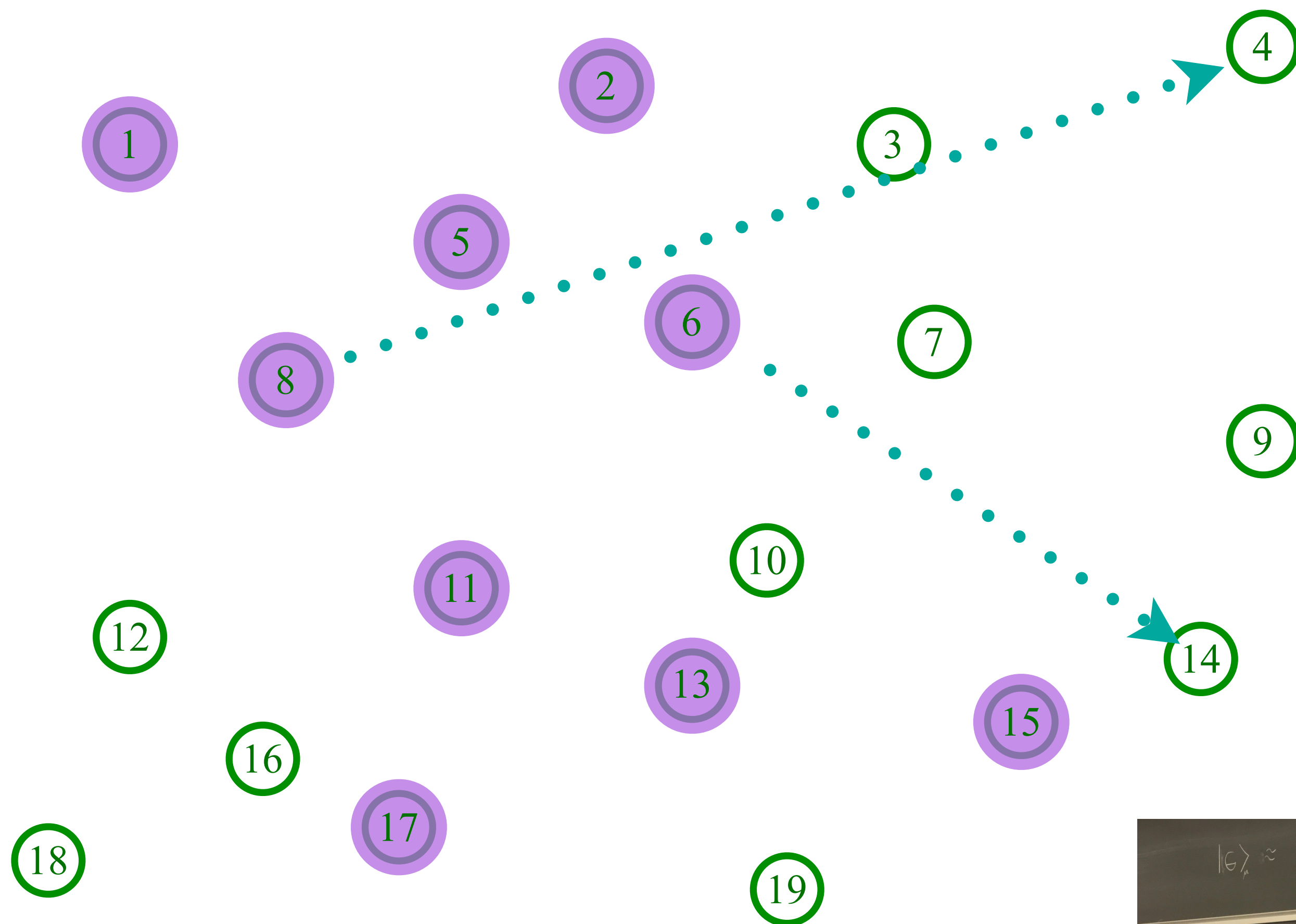
Entangle electrons pairwise randomly



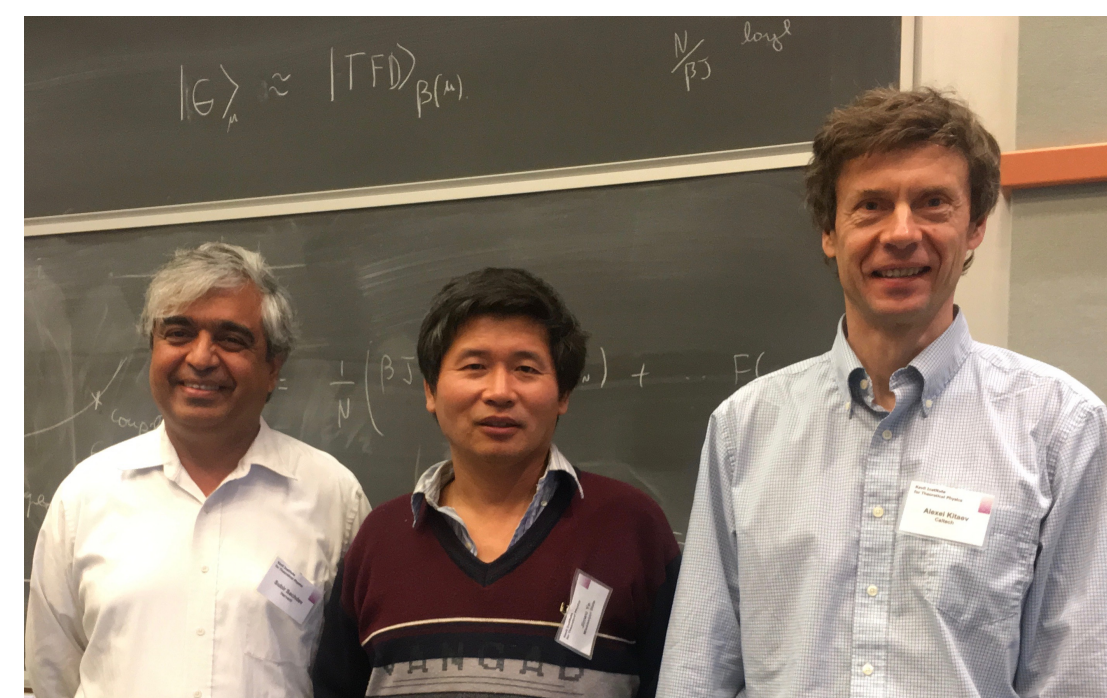
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



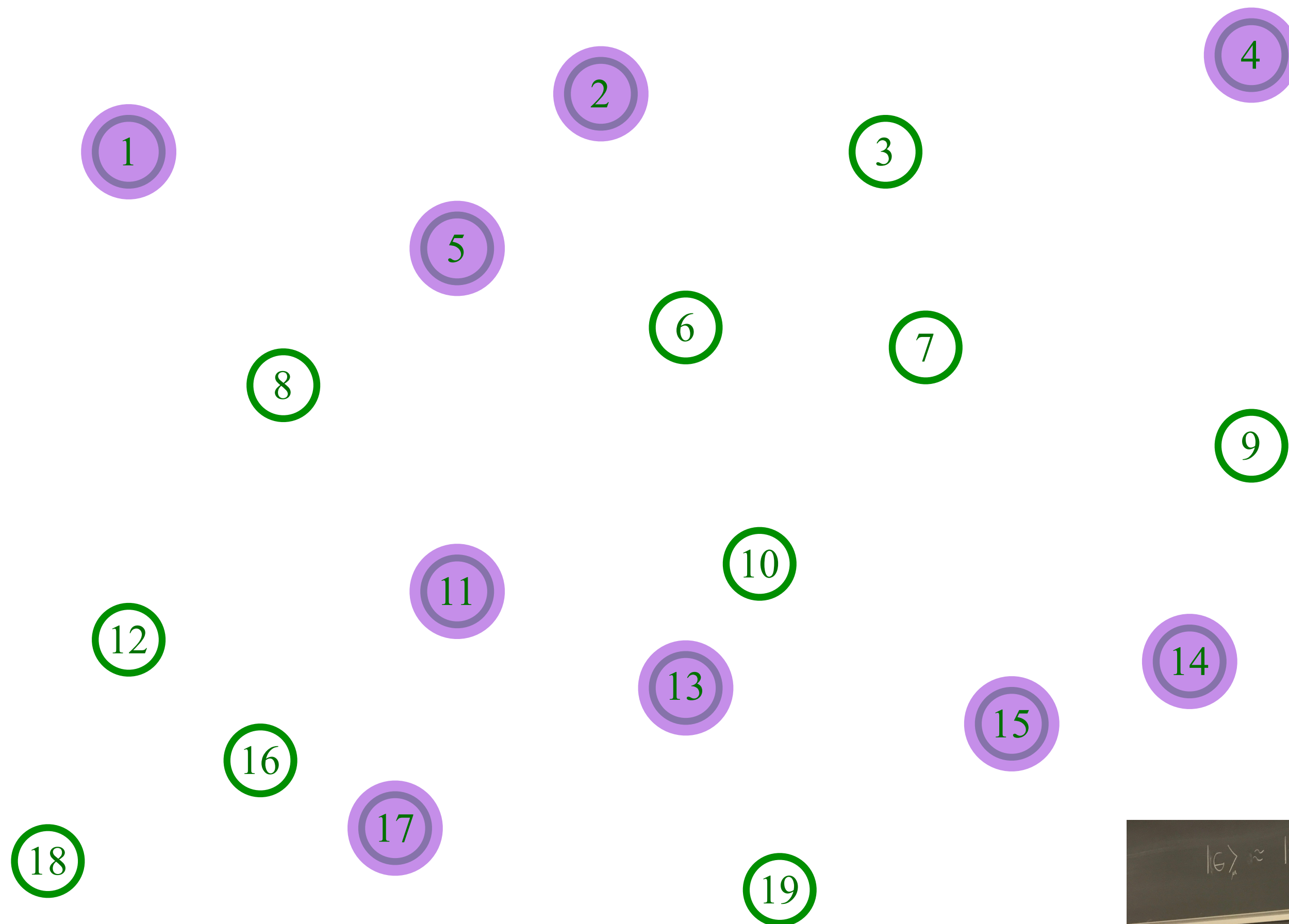
Entangle electrons pairwise randomly



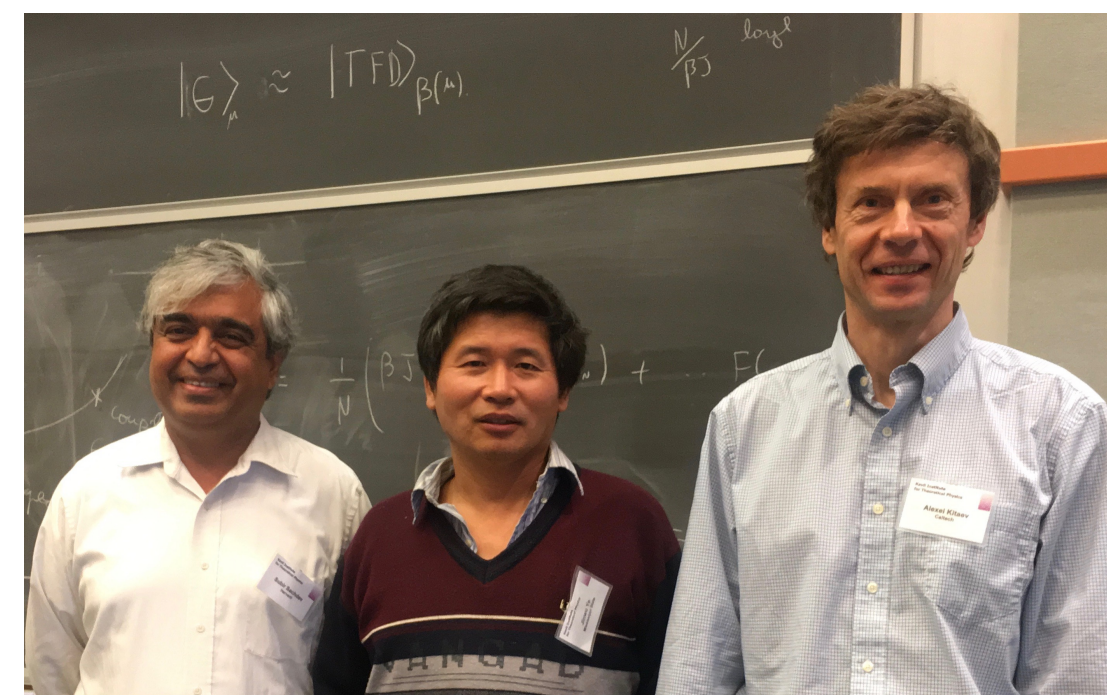
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



Entangle electrons pairwise randomly



The SYK model

(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} - \mu \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

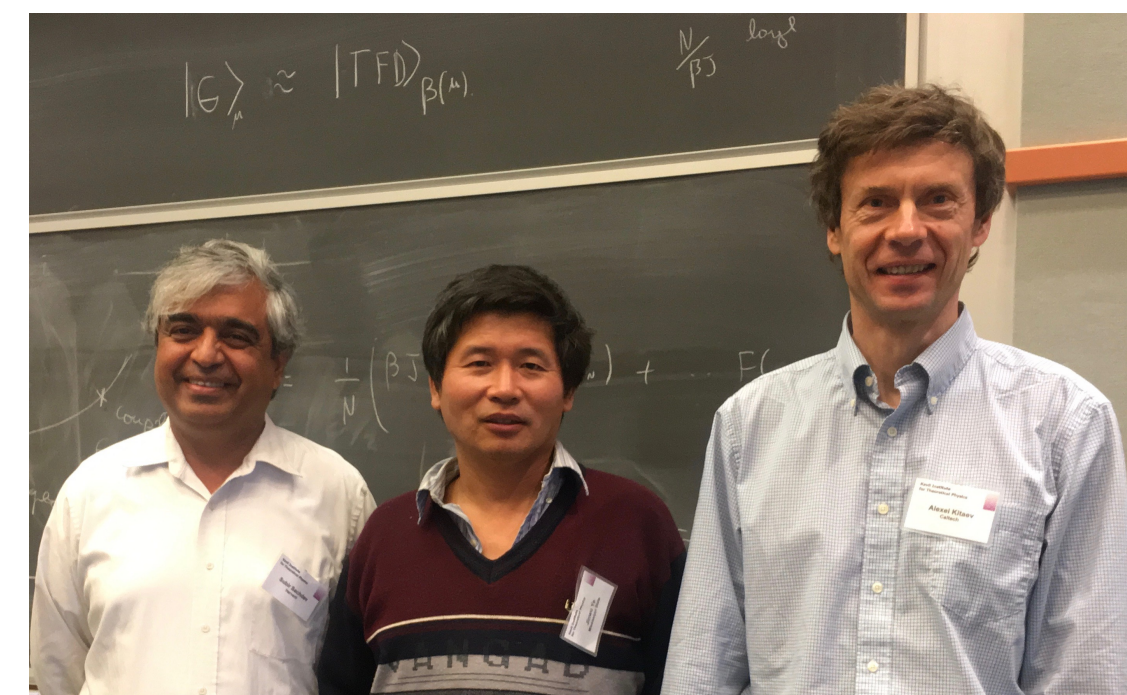
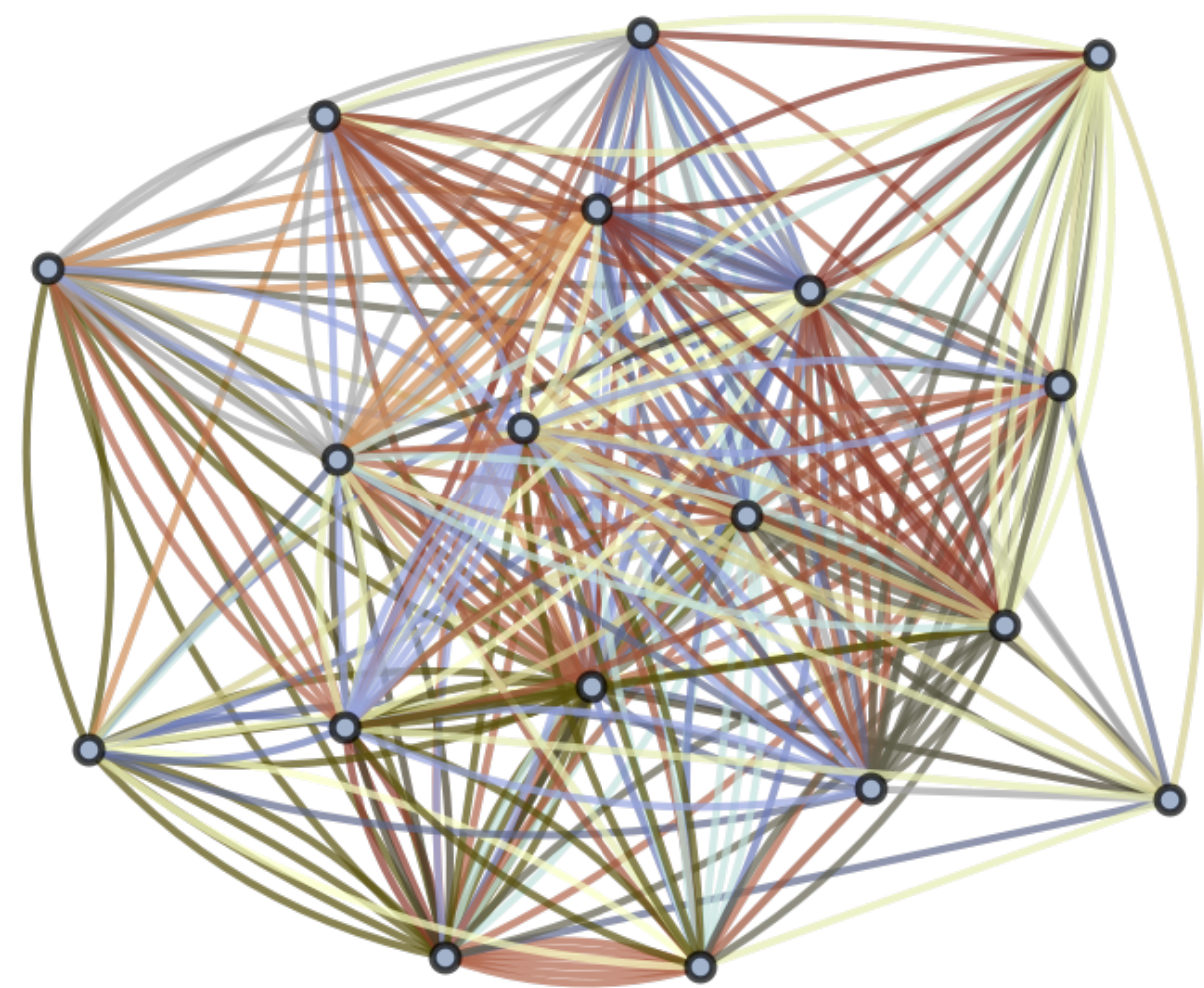
$$c_{\alpha} c_{\beta} + c_{\beta} c_{\alpha} = 0 \quad , \quad c_{\alpha} c_{\beta}^{\dagger} + c_{\beta}^{\dagger} c_{\alpha} = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}; \quad [\mathcal{H}, \mathcal{Q}] = 0; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
 $N \rightarrow \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)



The Sachdev-Ye-Kitaev (SYK) model

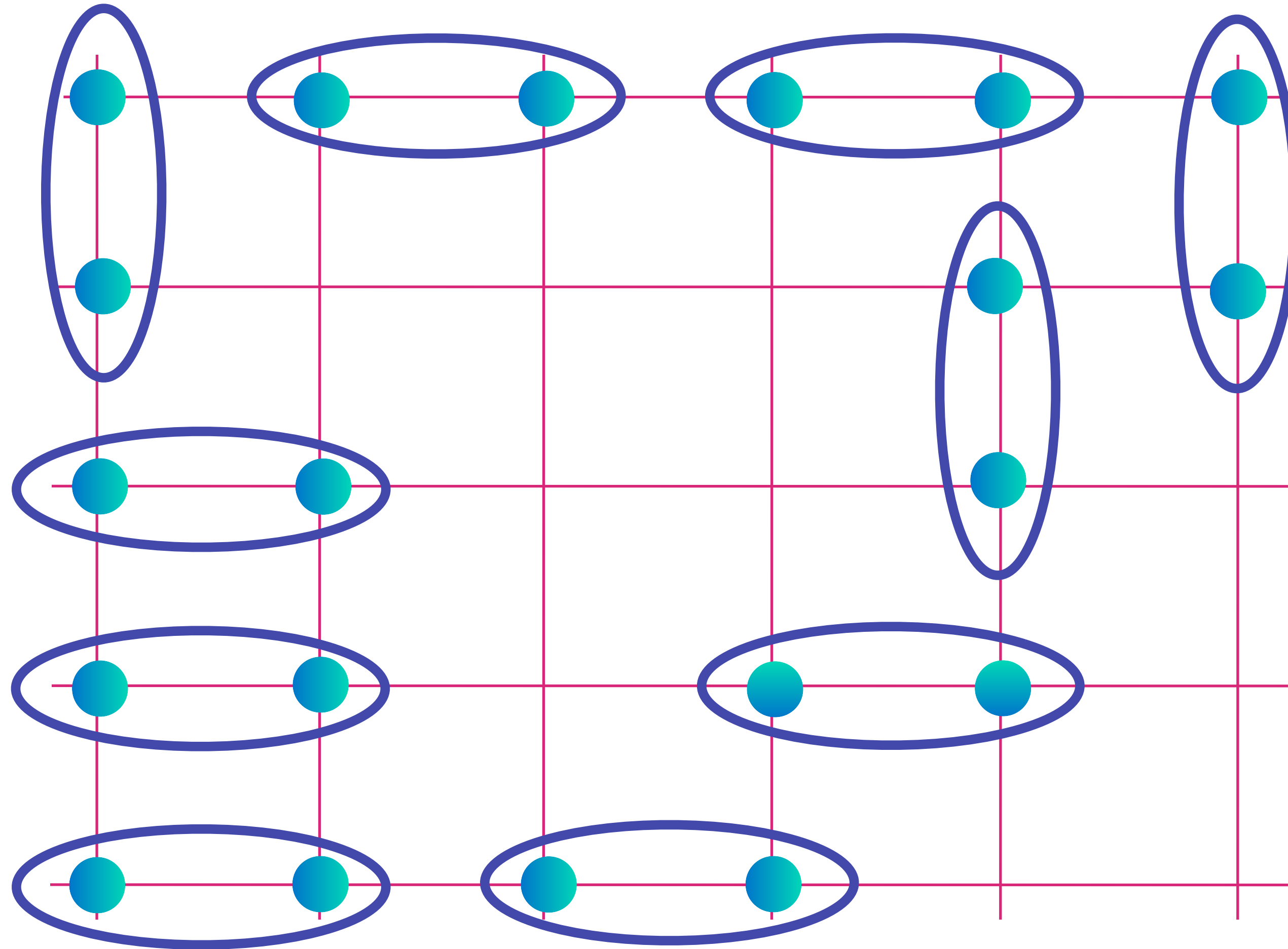
Sachdev, Ye (1993); Kitaev (2015)

A solvable model of multi-particle
quantum entanglement.

Yields a metal in which current is carried
not by individual electrons,
but by an entangled “quantum soup”

From the SYK model
to a theory of strange
metals and their
superconductivity

The dance of electrons on Cu atoms in YBCO

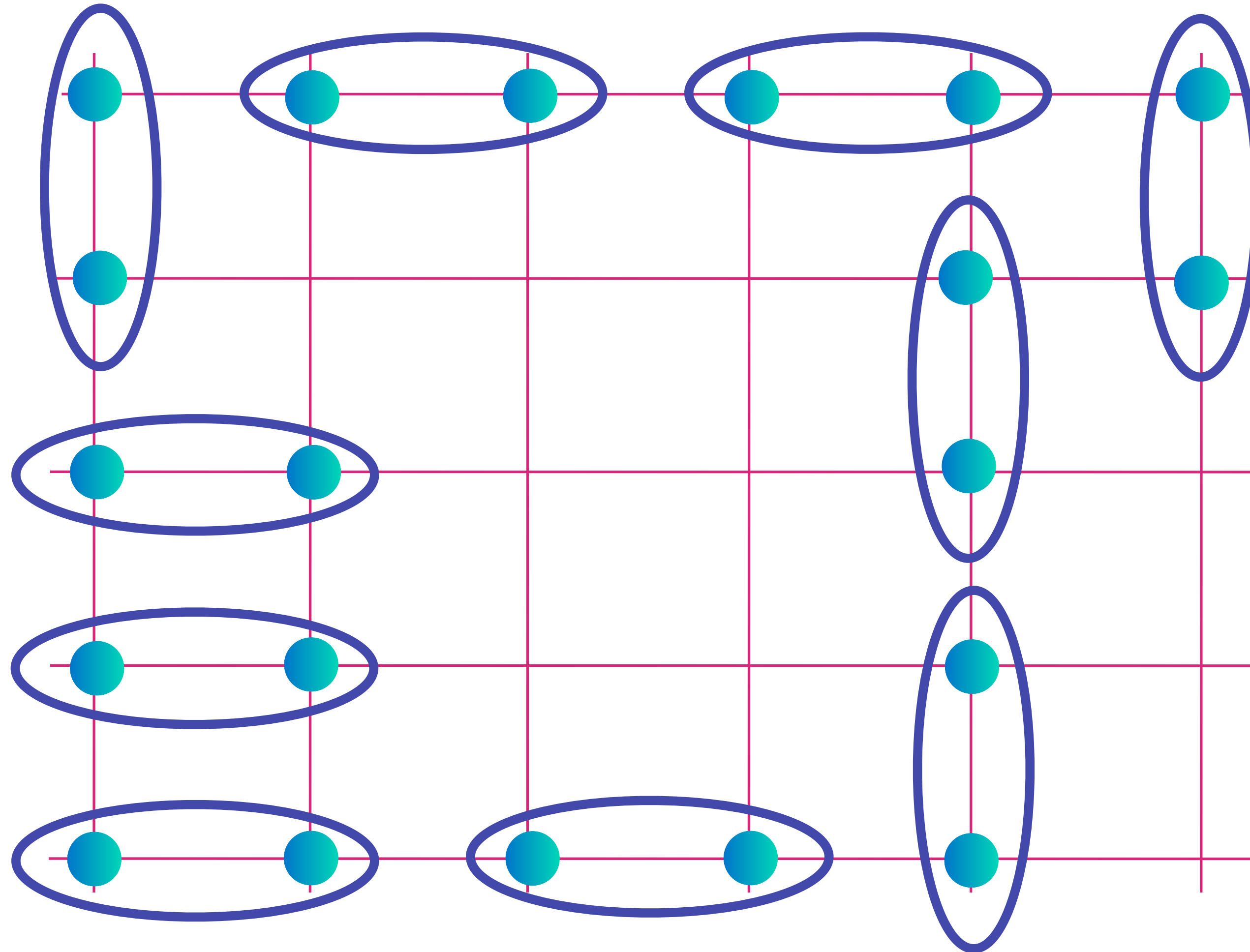


Superconductivity

Electrons form entangled pairs, and the motion of the pairs leads to superconductivity

$$\text{[Diagram of a pair]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

The dance of electrons on Cu atoms in YBCO

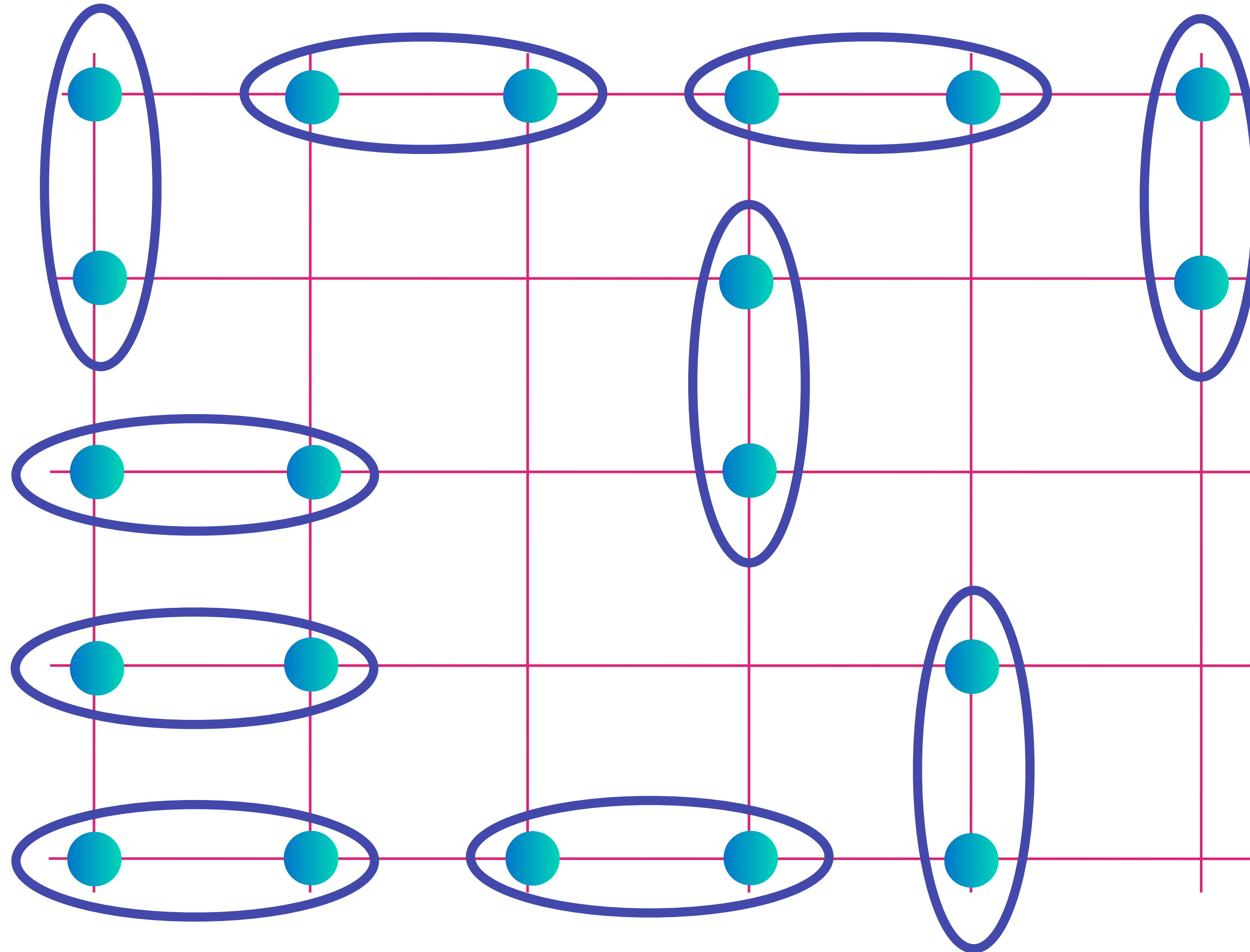


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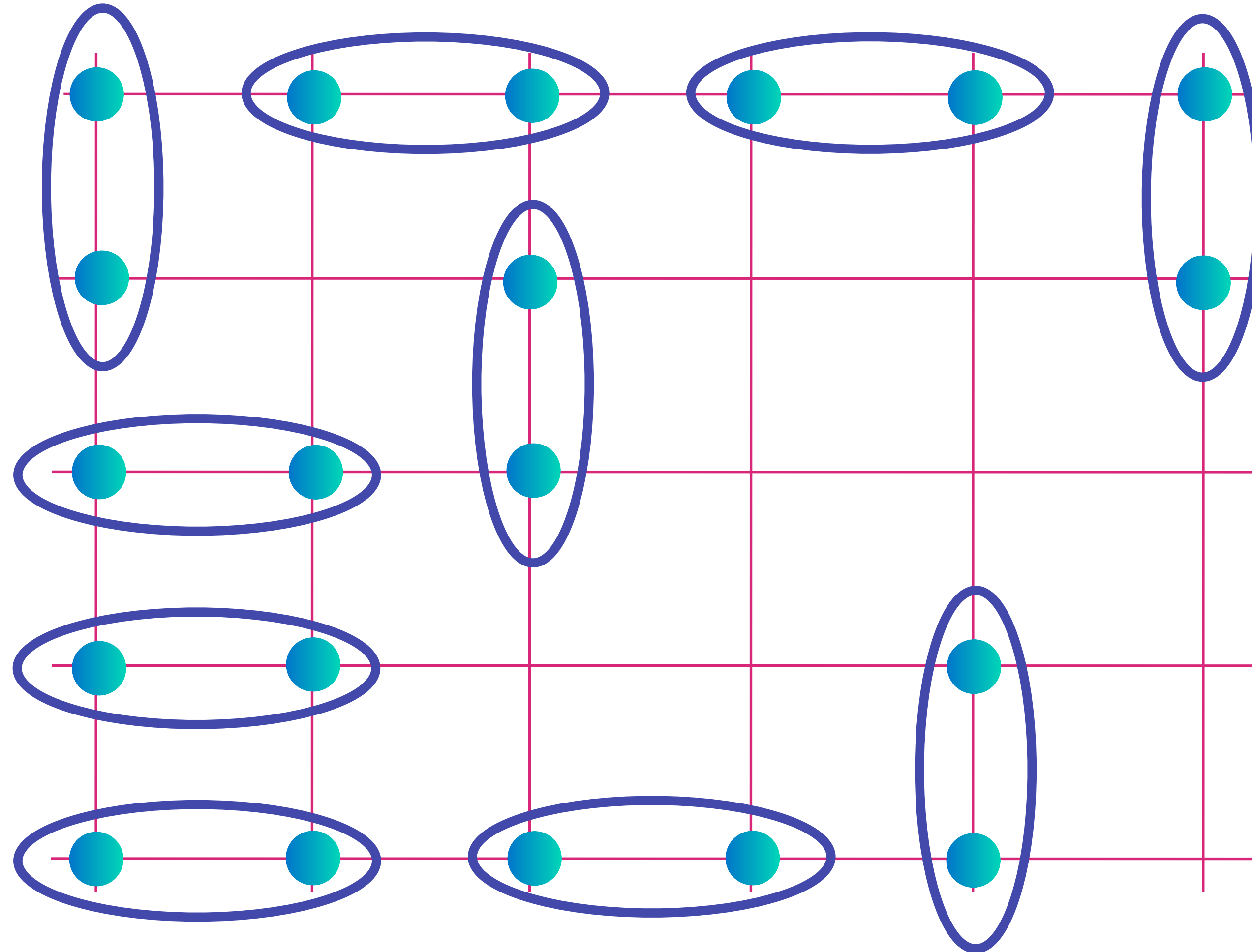


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The dance of electrons on Cu atoms in YBCO



Superconductivity

Electrons form entangled pairs, and the motion of the pairs leads to superconductivity

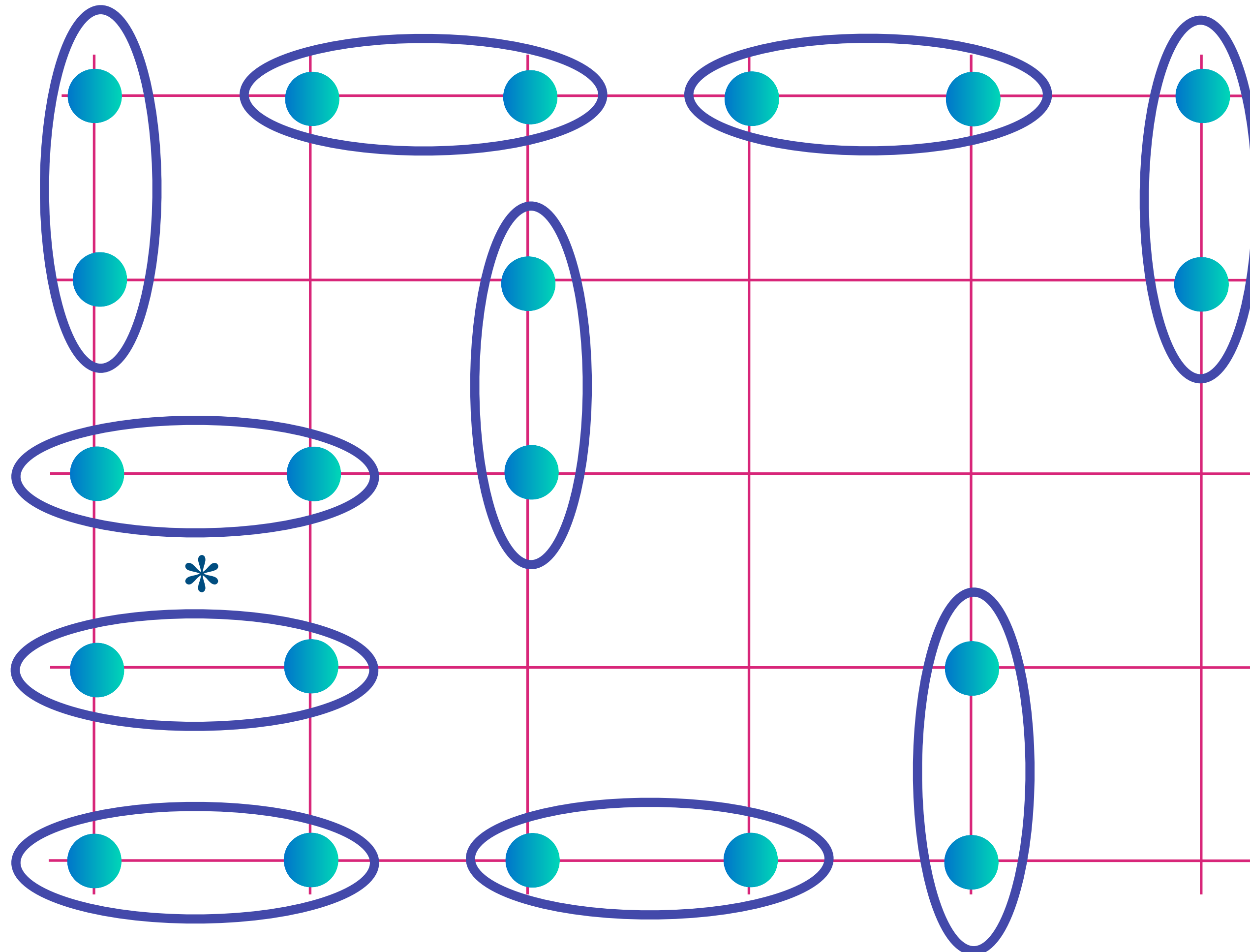
$$\text{[Diagram of a pair]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

The dance of electrons on Cu atoms in YBCO

Strange Metal

Electrons entangle “en masse” by exchanging partners.
A theory extending the SYK model shows that this leads to the “strange metal”.

2



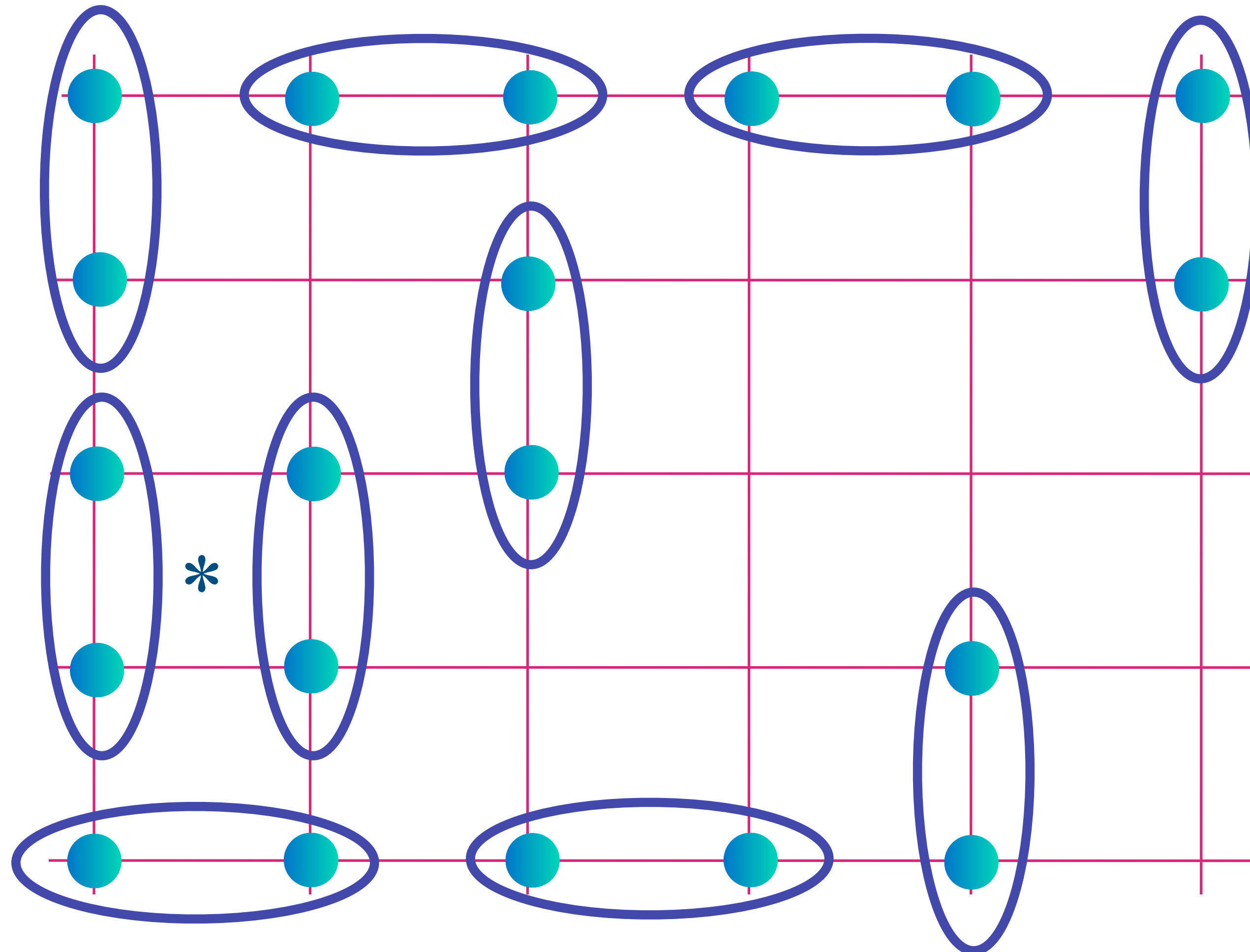
$$\text{[Diagram of two dots in an oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

The dance of electrons on Cu atoms in YBCO

Strange Metal

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4



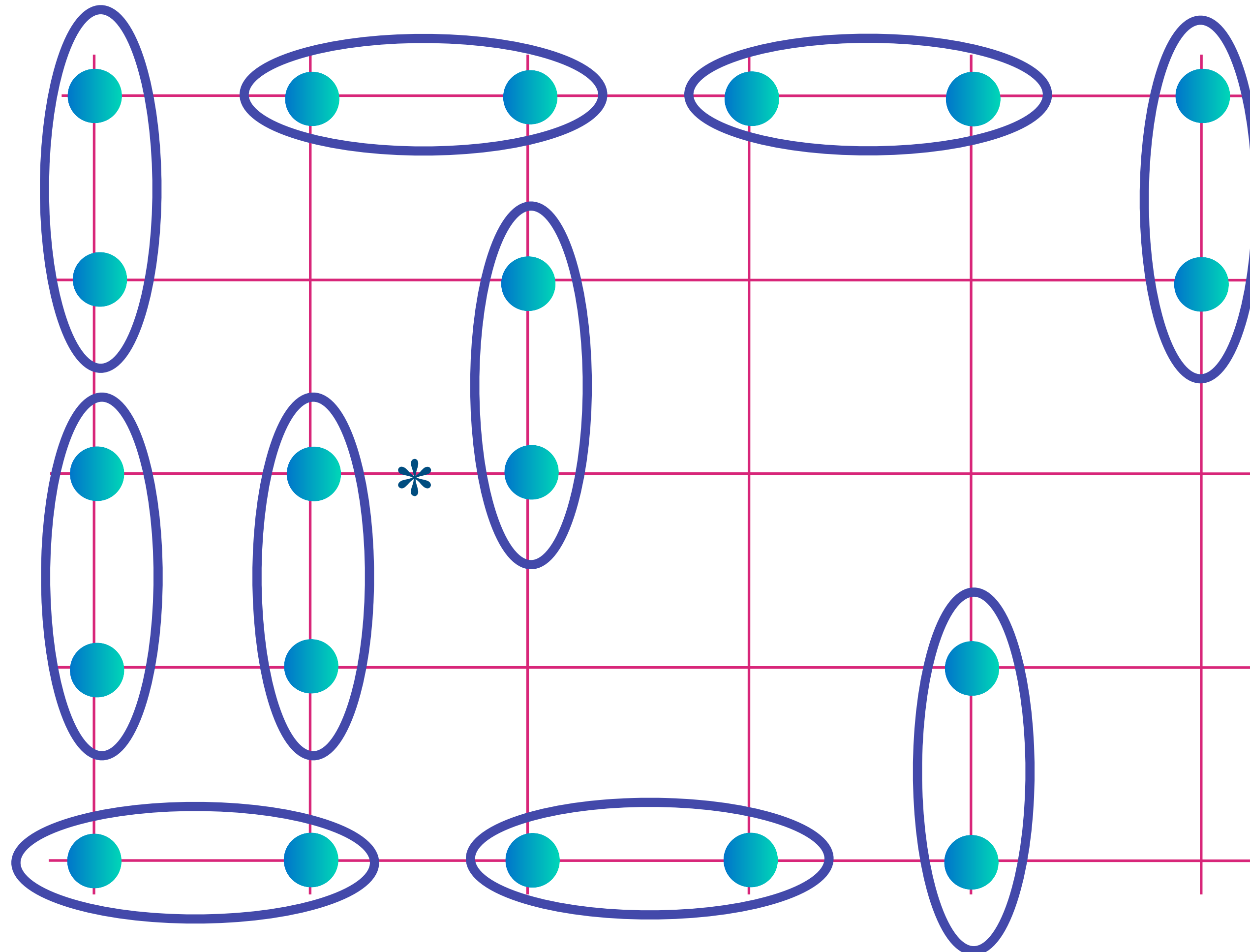
$$\text{[Diagram of two electrons in an oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

The dance of electrons on Cu atoms in YBCO

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4



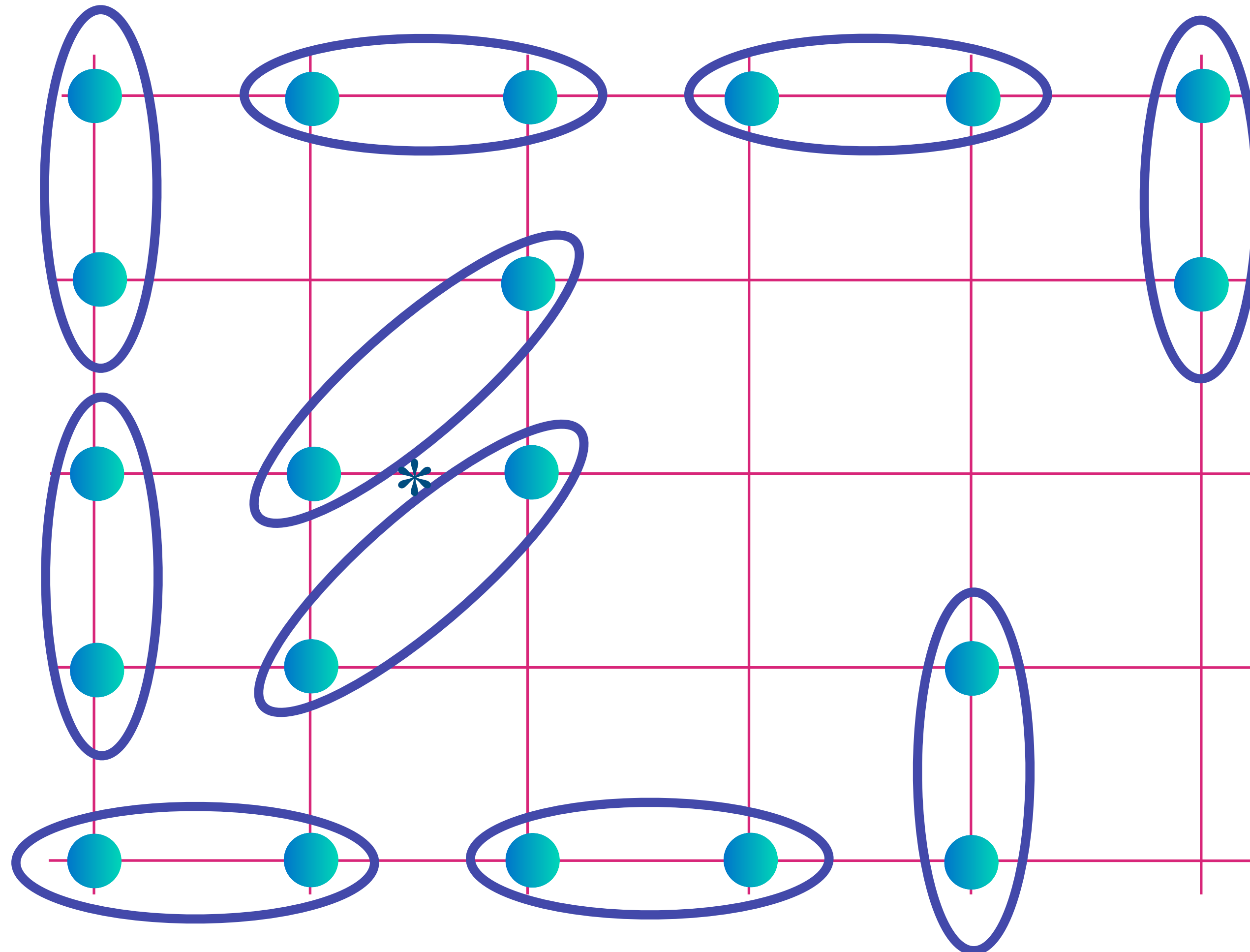
$$\text{Diagram of two electrons in a pair} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

The dance of electrons on Cu atoms in YBCO

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6



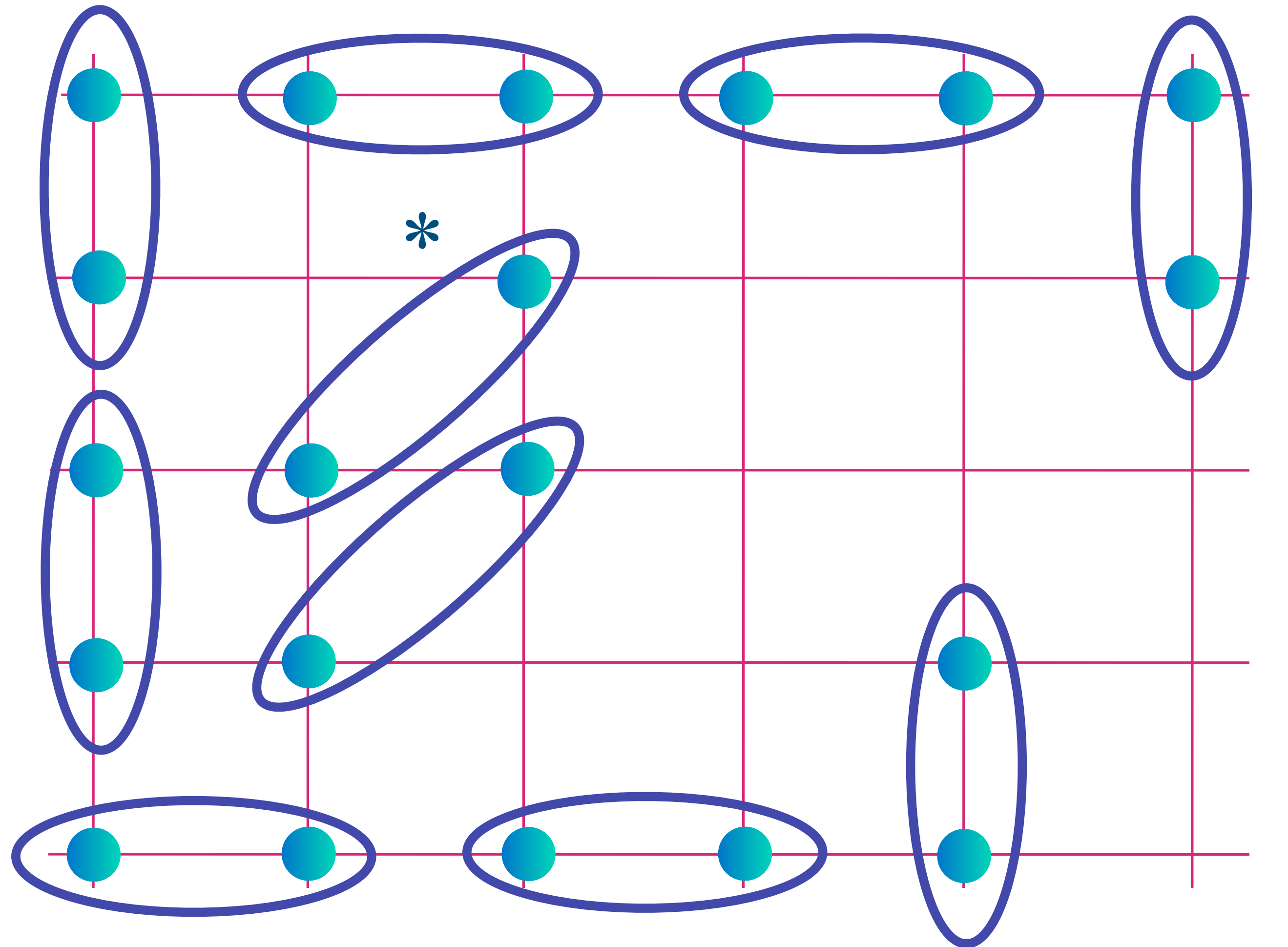
$$\text{[Diagram of two dots in a blue oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

The dance of electrons on Cu atoms in YBCO

Strange Metal

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A theory extending the SYK model shows that this leads to the “strange metal”.

6



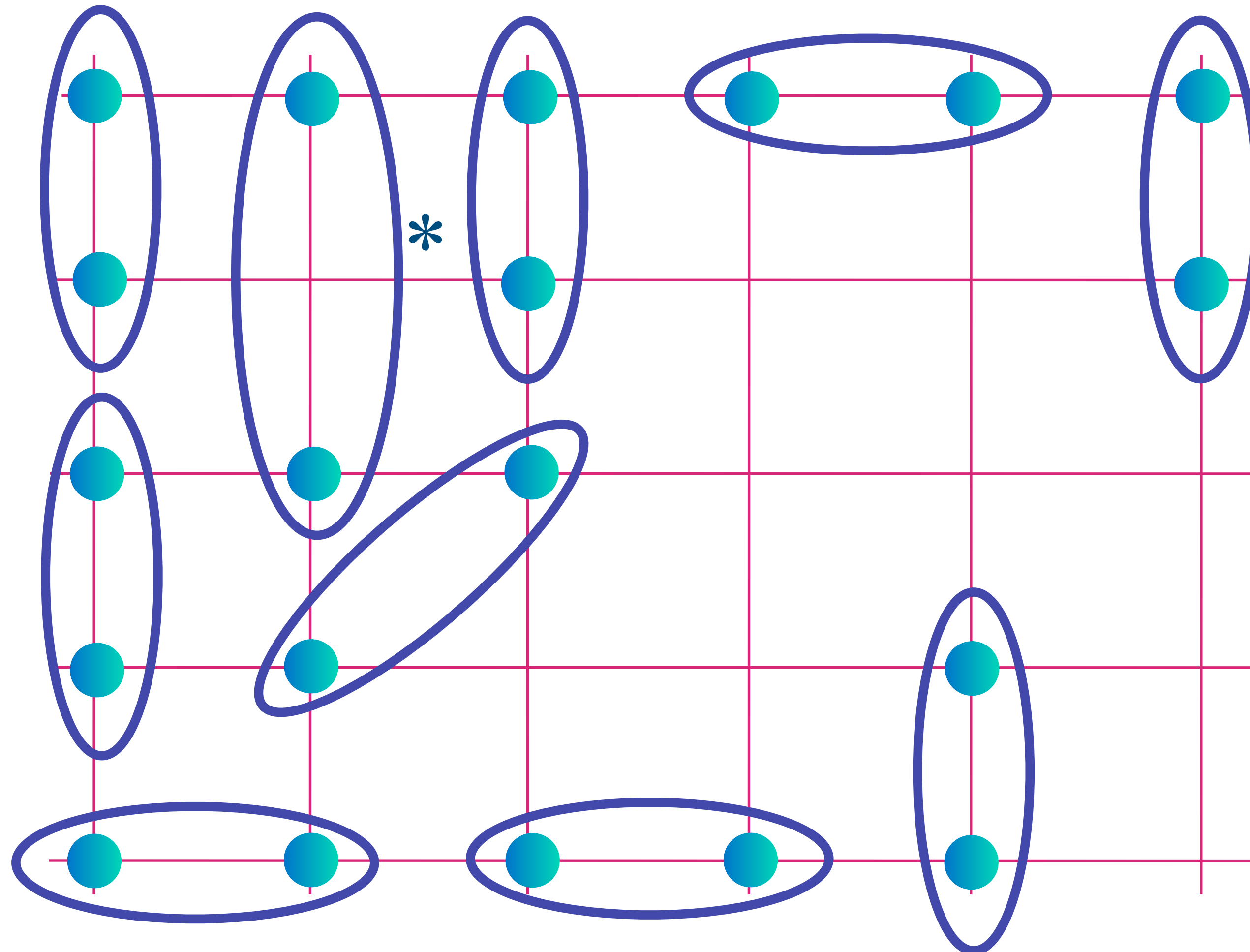
$$\text{Diagram of two electrons in a pair} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

The dance of electrons on Cu atoms in YBCO

Strange Metal

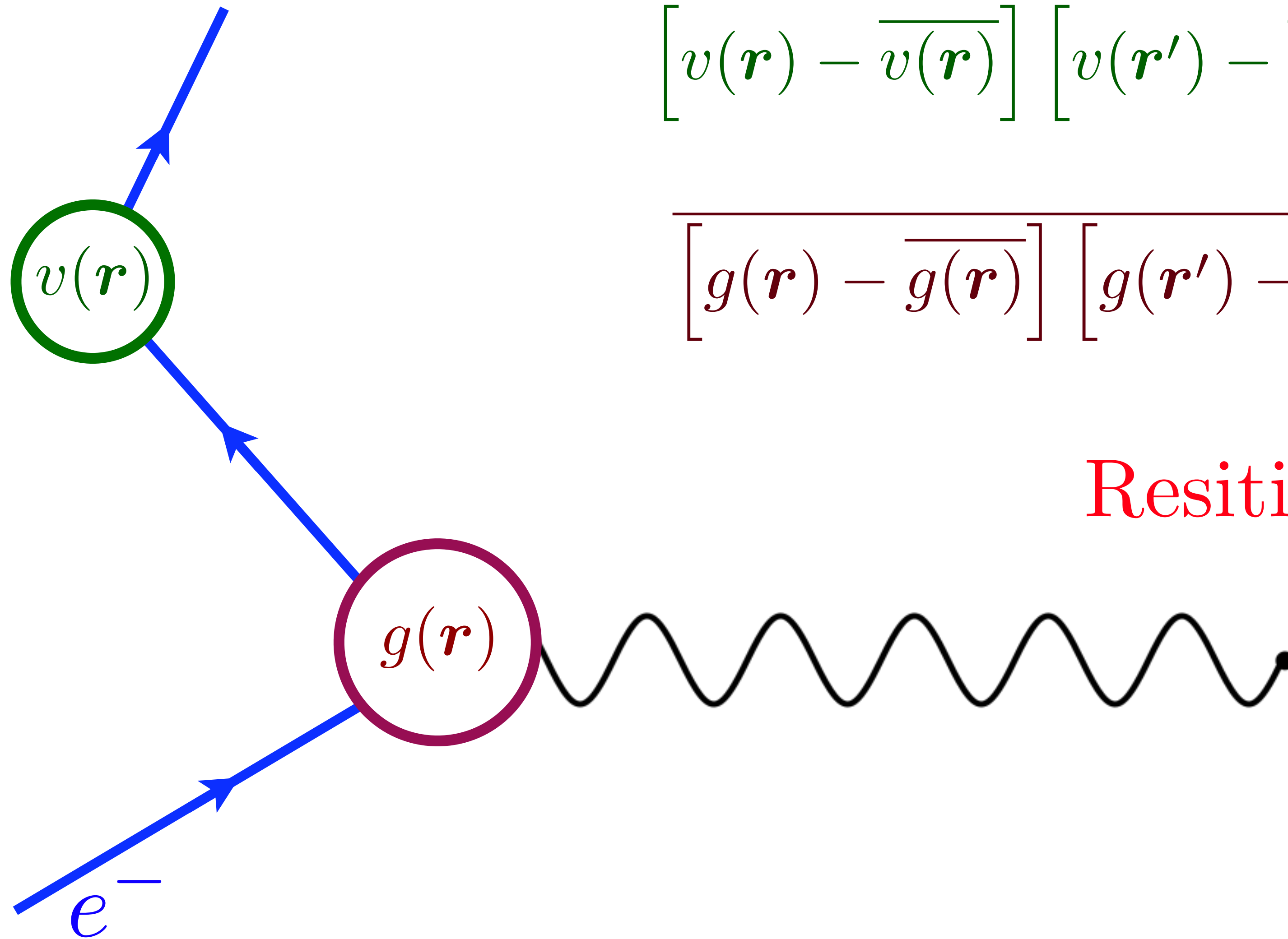
Electrons entangle “en masse” by exchanging partners.
A theory extending the SYK model shows that this leads to the “strange metal”.

8



$$\text{Diagram of two atoms in a blue oval} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

The dance of electrons on Cu atoms in YBCO



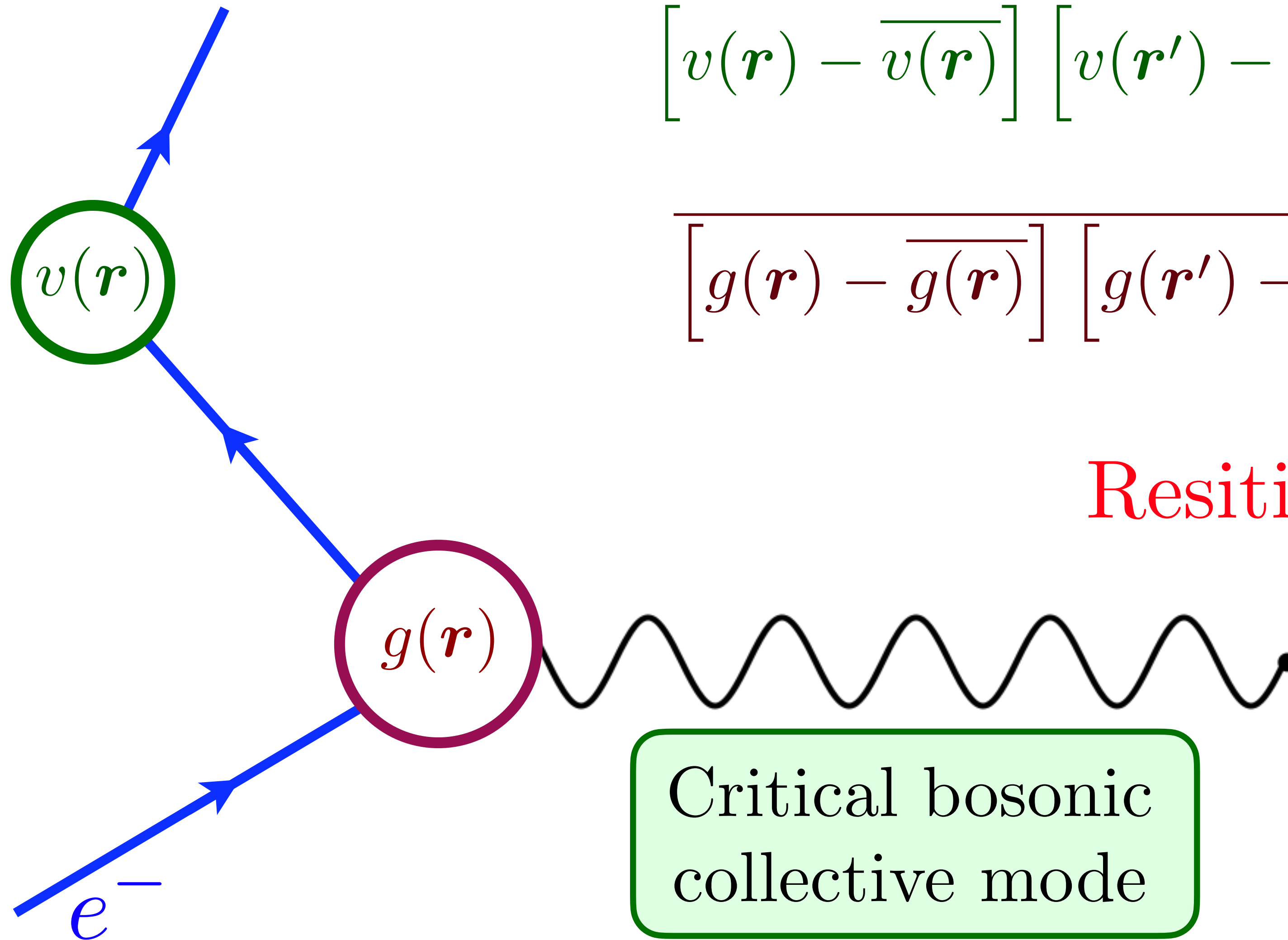
$$\overline{[v(\mathbf{r}) - \overline{v(\mathbf{r})}] [v(\mathbf{r}') - \overline{v(\mathbf{r}')}] = v^2 \delta(\mathbf{r} - \mathbf{r}')$$

$$\overline{[g(\mathbf{r}) - \overline{g(\mathbf{r})}] [g(\mathbf{r}') - \overline{g(\mathbf{r}')}] = g^2 \delta(\mathbf{r} - \mathbf{r}')$$

$$\text{Resistivity } \rho(T) \sim v^2 + g^2 T$$



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Critical bosonic
collective mode

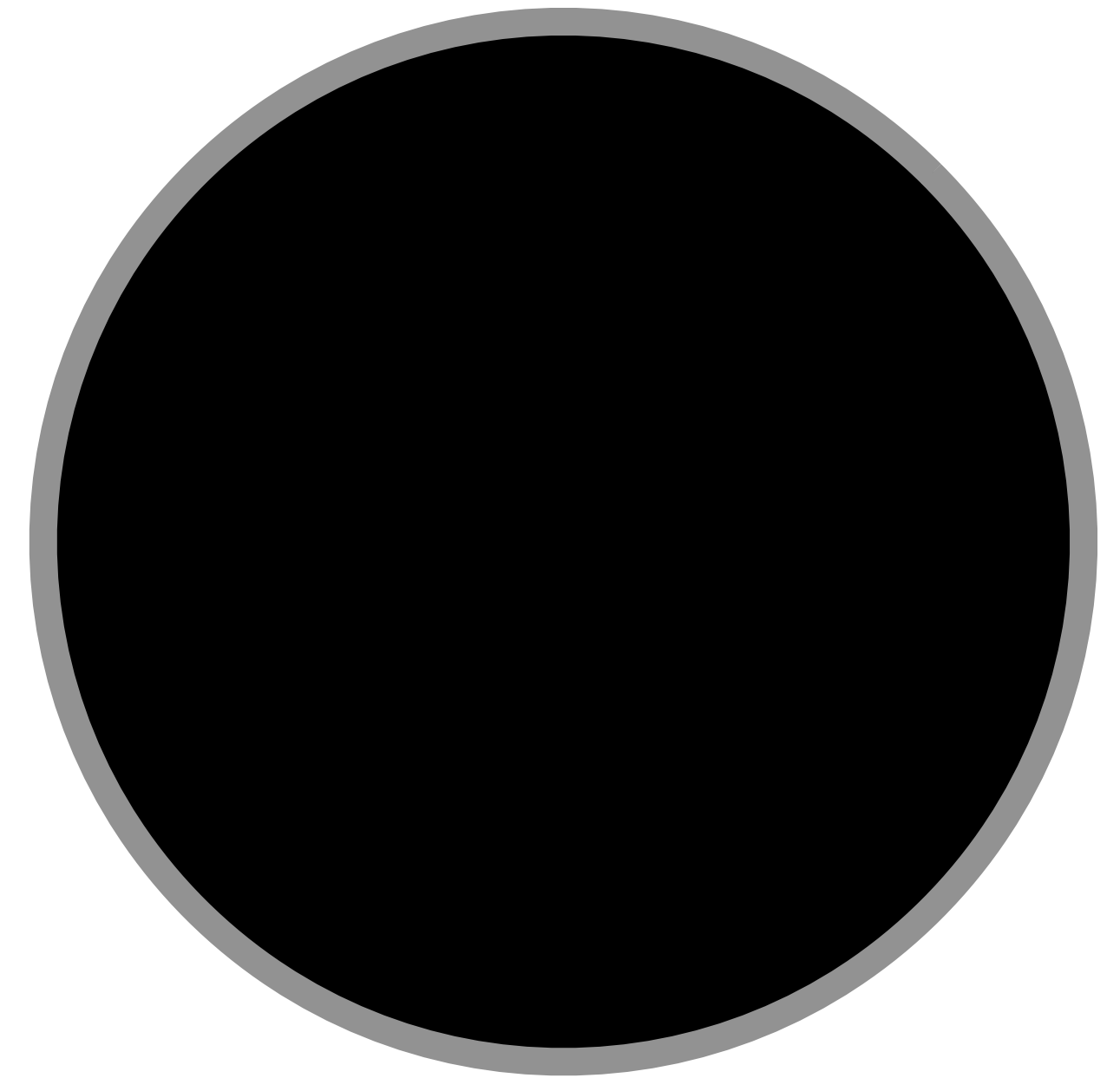


**Black
holes**

Black Holes

Objects so dense that light is gravitationally bound to them.

Horizon radius $R = \frac{2GM}{c^2}$



G Newton's constant, c velocity of light, M mass of black hole
For $M = \text{earth's mass}$, $R \approx 9 \text{ mm!}$

The supermassive black hole lurking at the heart of the Milky Way – Sagittarius A* contains about 4.3 million solar masses, and, as it turns out, nearly all of the mass at the very center of the galaxy.

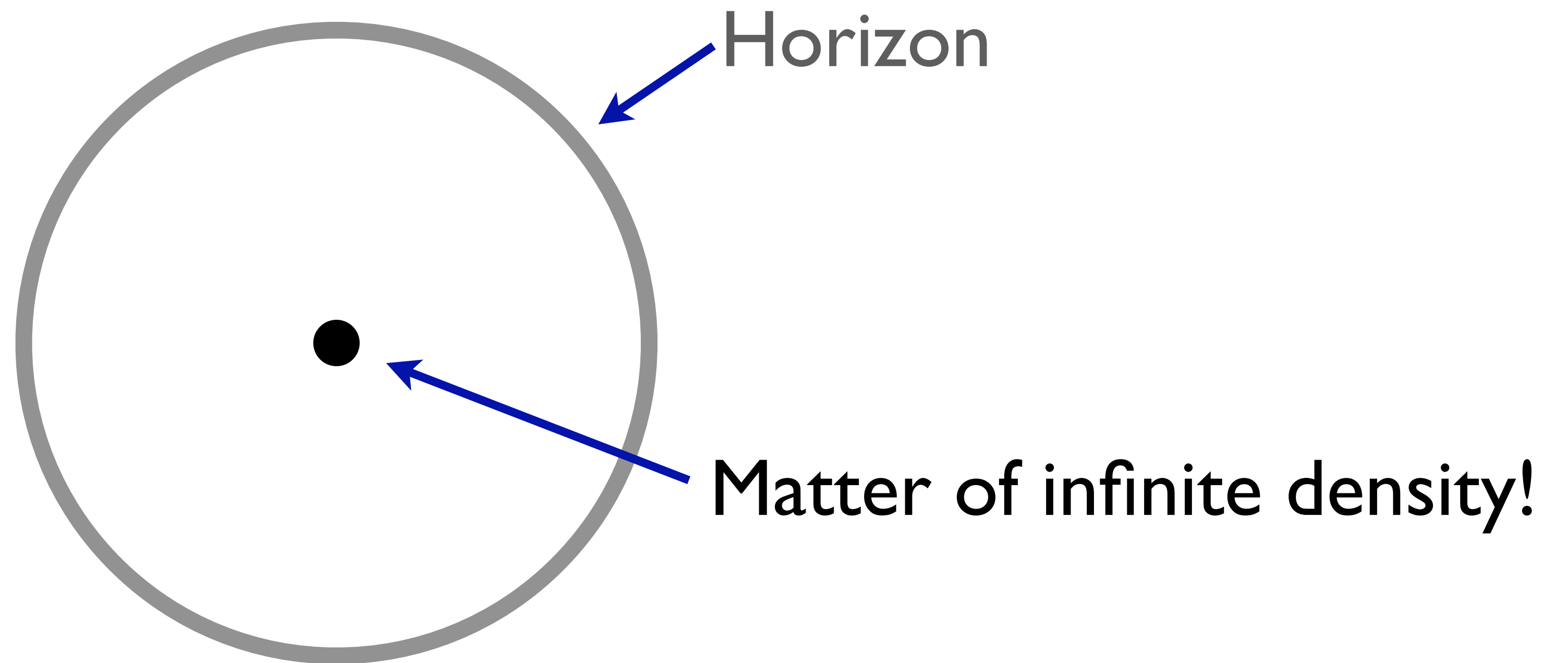
$$R = 1.3 \times 10^{11} \text{ m}$$

\approx earth's orbit

An artist's impression of Sagittarius A*, the supermassive black hole at the heart of the Milky Way. Image: International Gemini Observatory/NOIRLab/NSF/AURA/J. da Silva/(Spaceengine); M. Zamani (NSF's NOIRLab)

What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



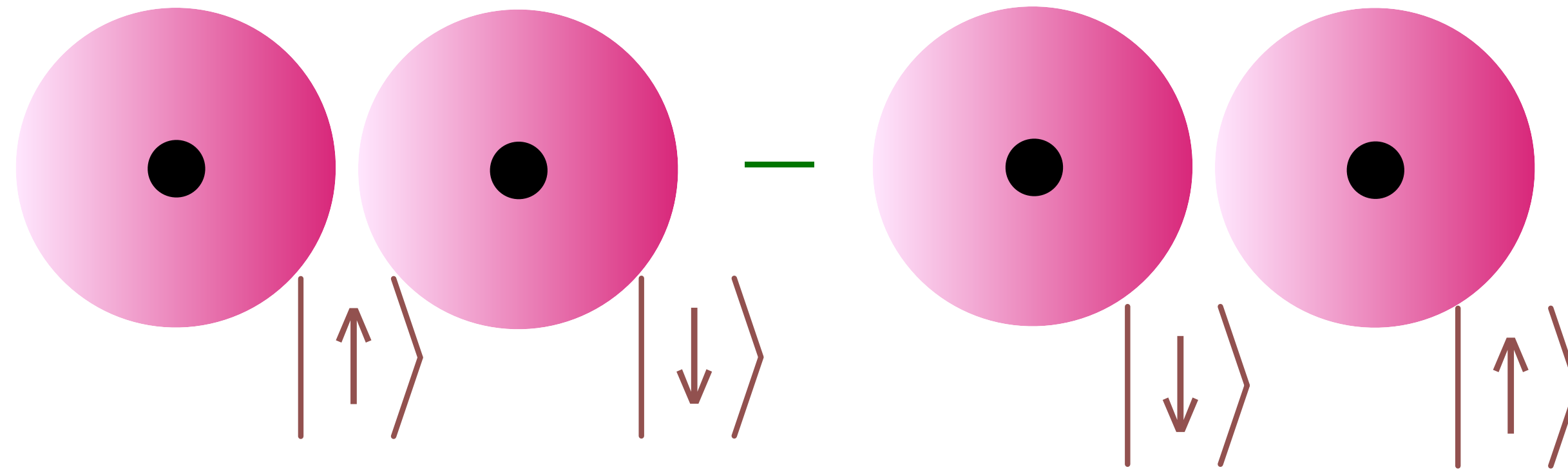
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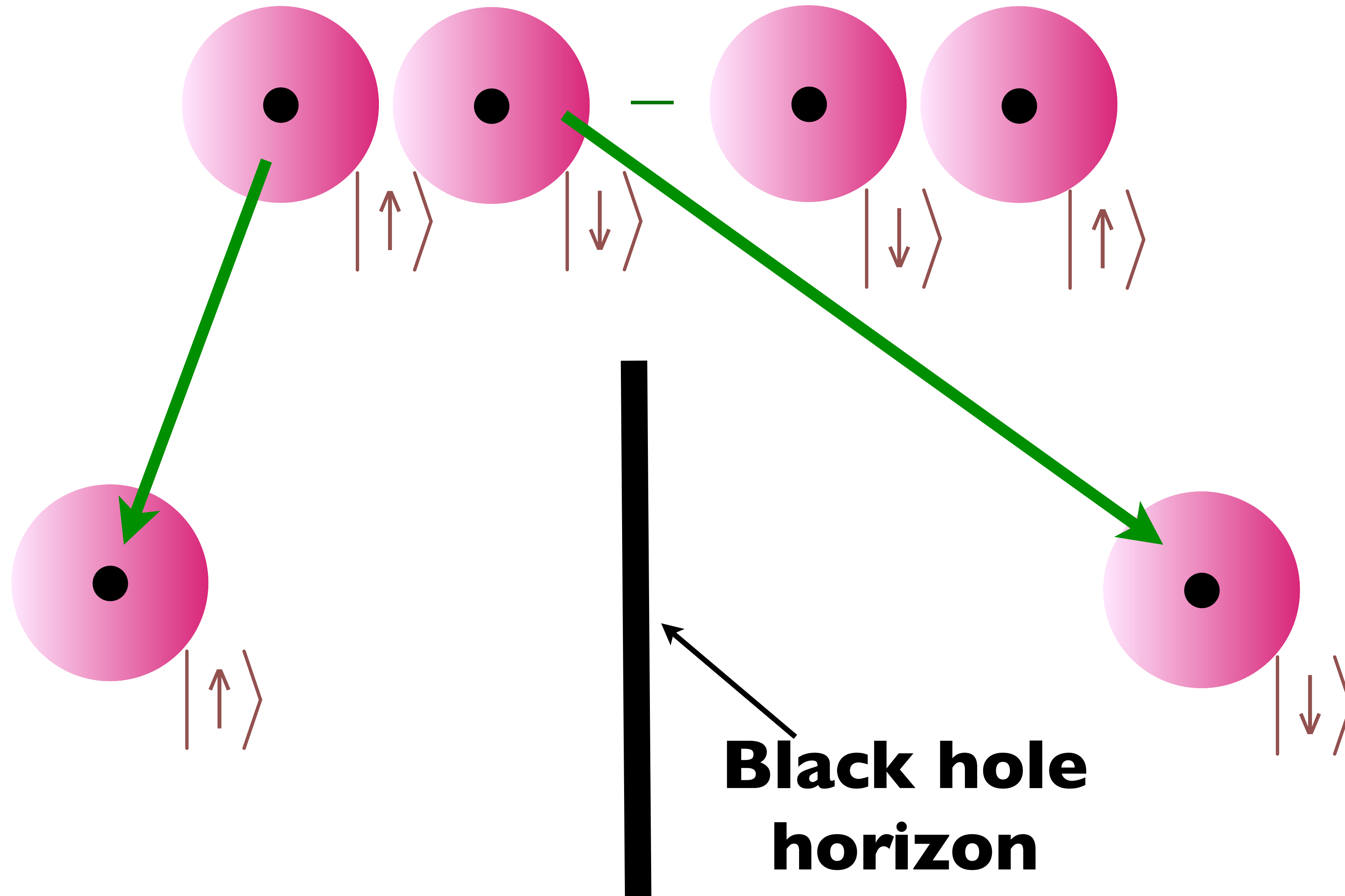
This singularity convinced many early on that black holes were unphysical solutions of Einstein's equations, and did not exist in our universe.

In any case, it was clear that quantum theory should be applied to the collapsed matter, but no one knew how to.

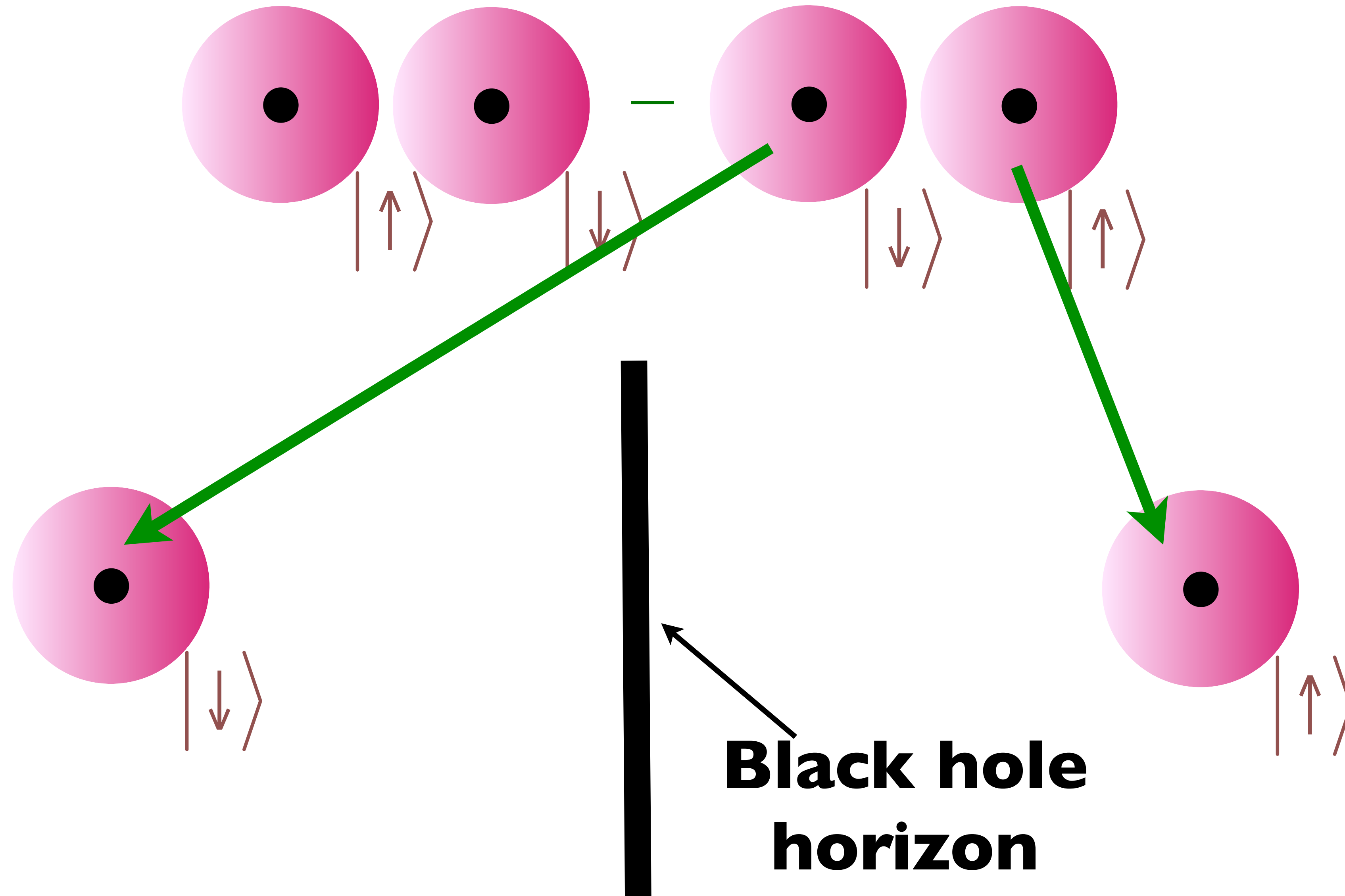
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

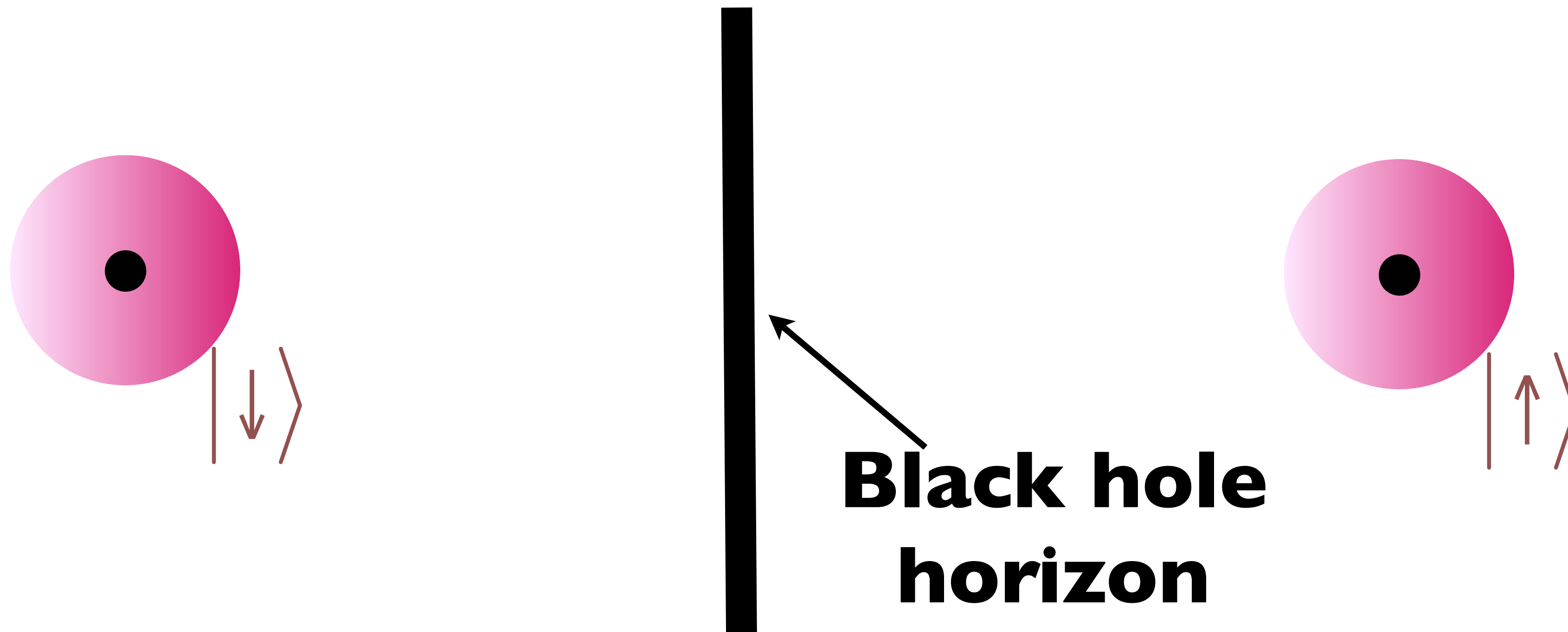


Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

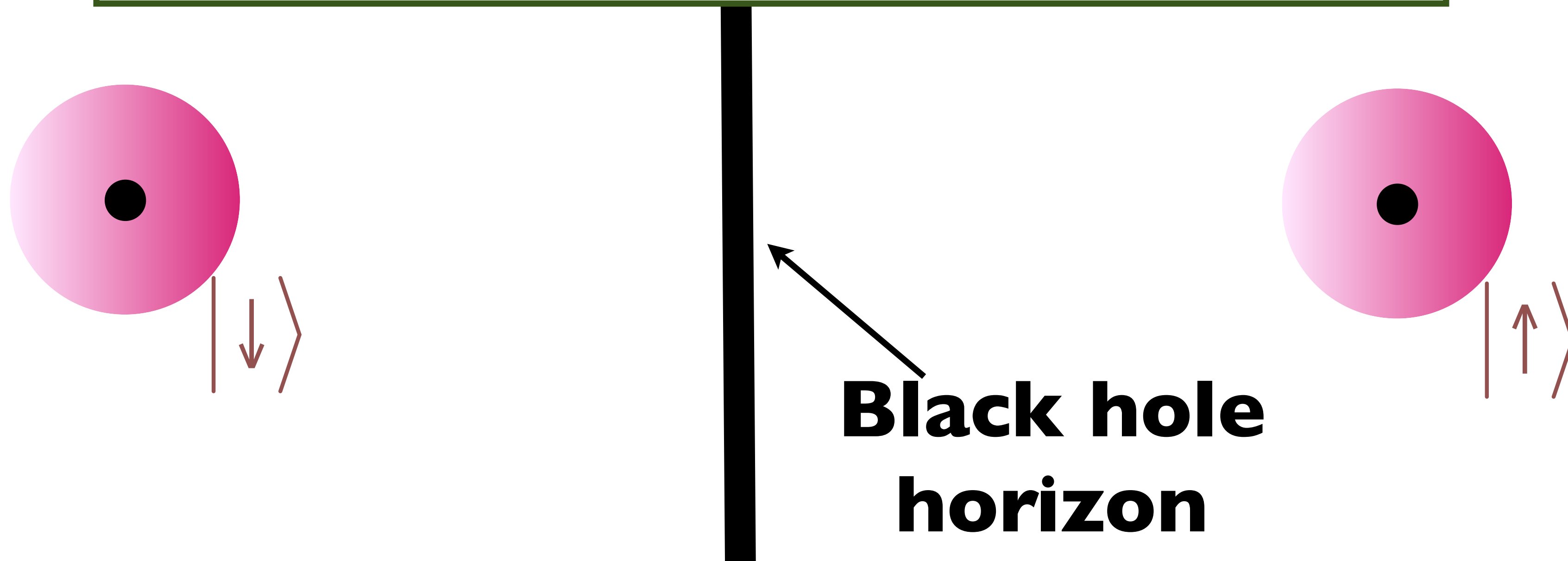
There is quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

Hawking (1975) used other arguments to show that black hole horizons have a temperature and an entropy

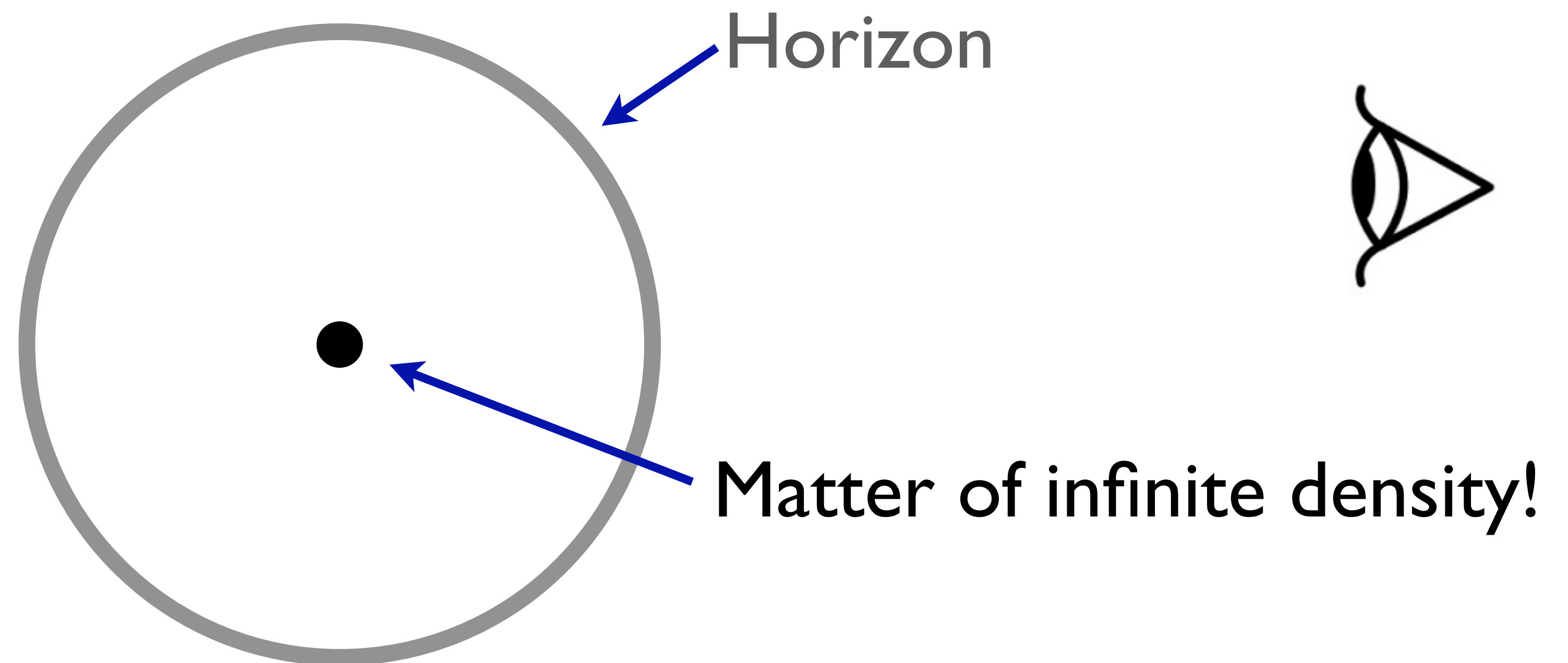
(The entanglement reasoning: to an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.)



Quantum Black Holes

Hawking obtained the black hole entropy by semiclassical computations for an observer outside the black hole horizon.

This allowed Hawking to avoid the contradictions associated with the singularity at the center of the black hole.



Statistical interpretation of entropy (1870)

$$S = k_B \log W$$

Density of quantum states $D(E) = \exp(S(E)/k_B)$



Ludwig Boltzmann

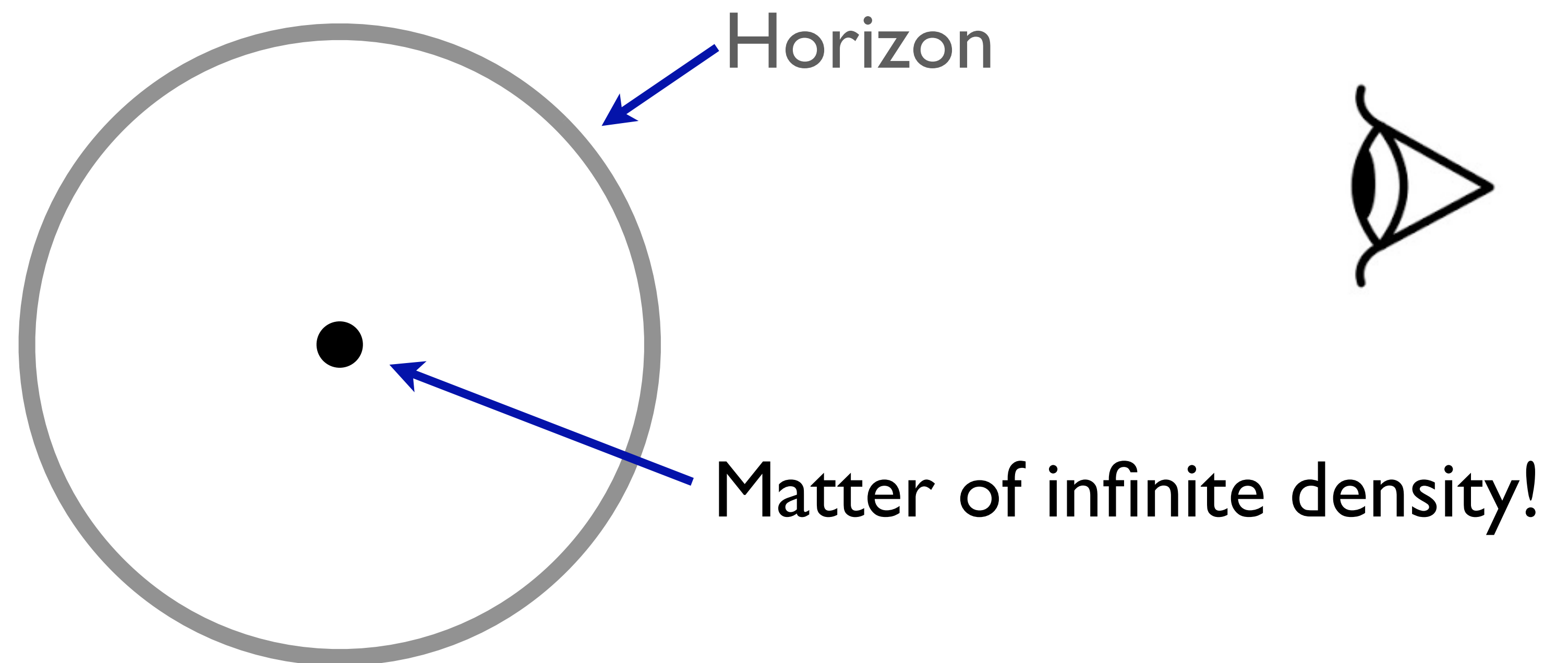
20 February 1844 - September 5, 1906

Vienna, Austria

Quantum Black Holes

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Quantum Black Holes

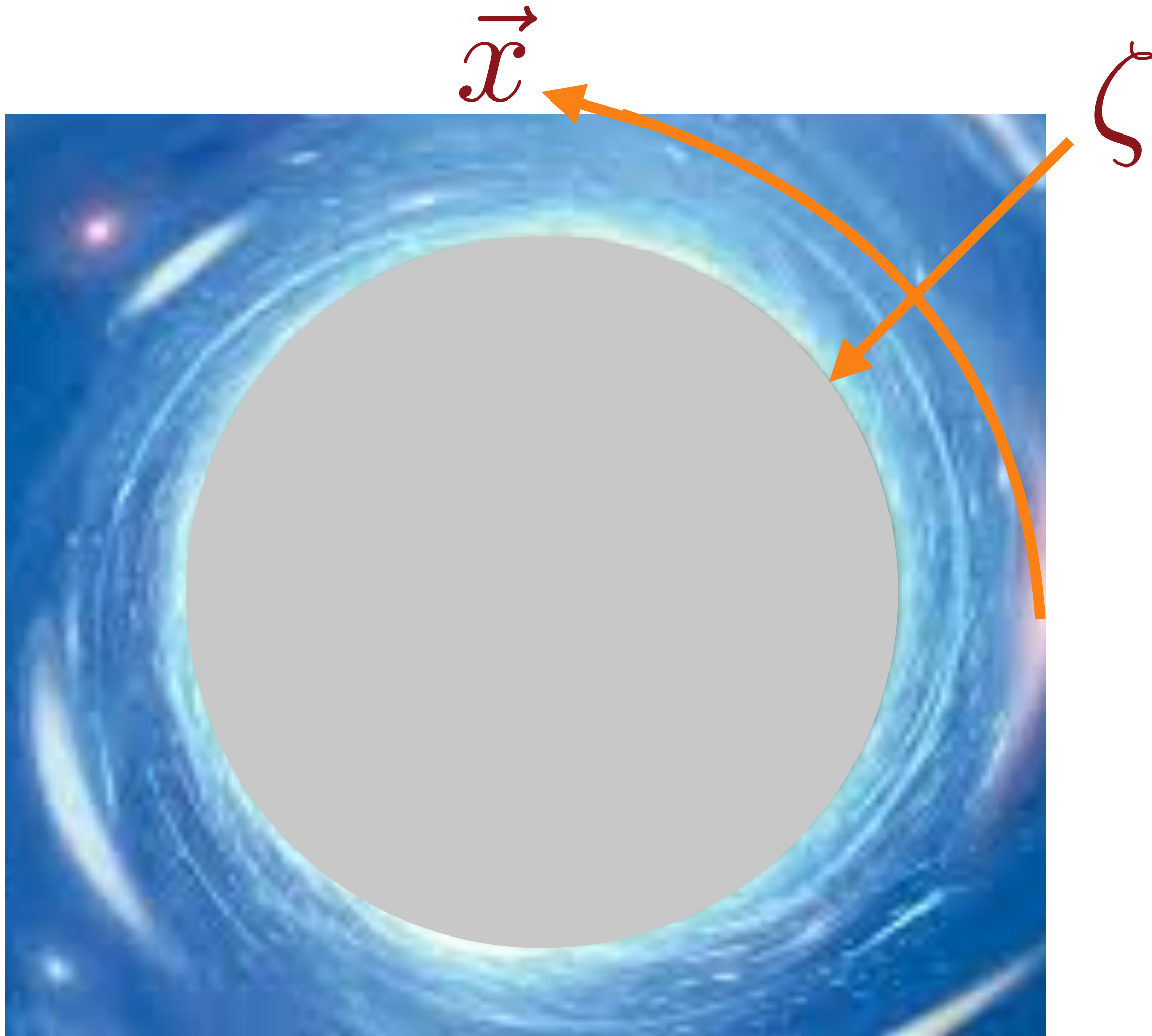
Hawking obtained the black hole entropy by semiclassical computations for an observer outside the black hole horizon.

Can we find a quantum theory for the collapsed matter at the center of the black hole, whose density of quantum states matches the Bekenstein-Hawking entropy, in accordance with Boltzmann's principles of statistical mechanics ?

From the SYK model
to a quantum theory of
charged black holes

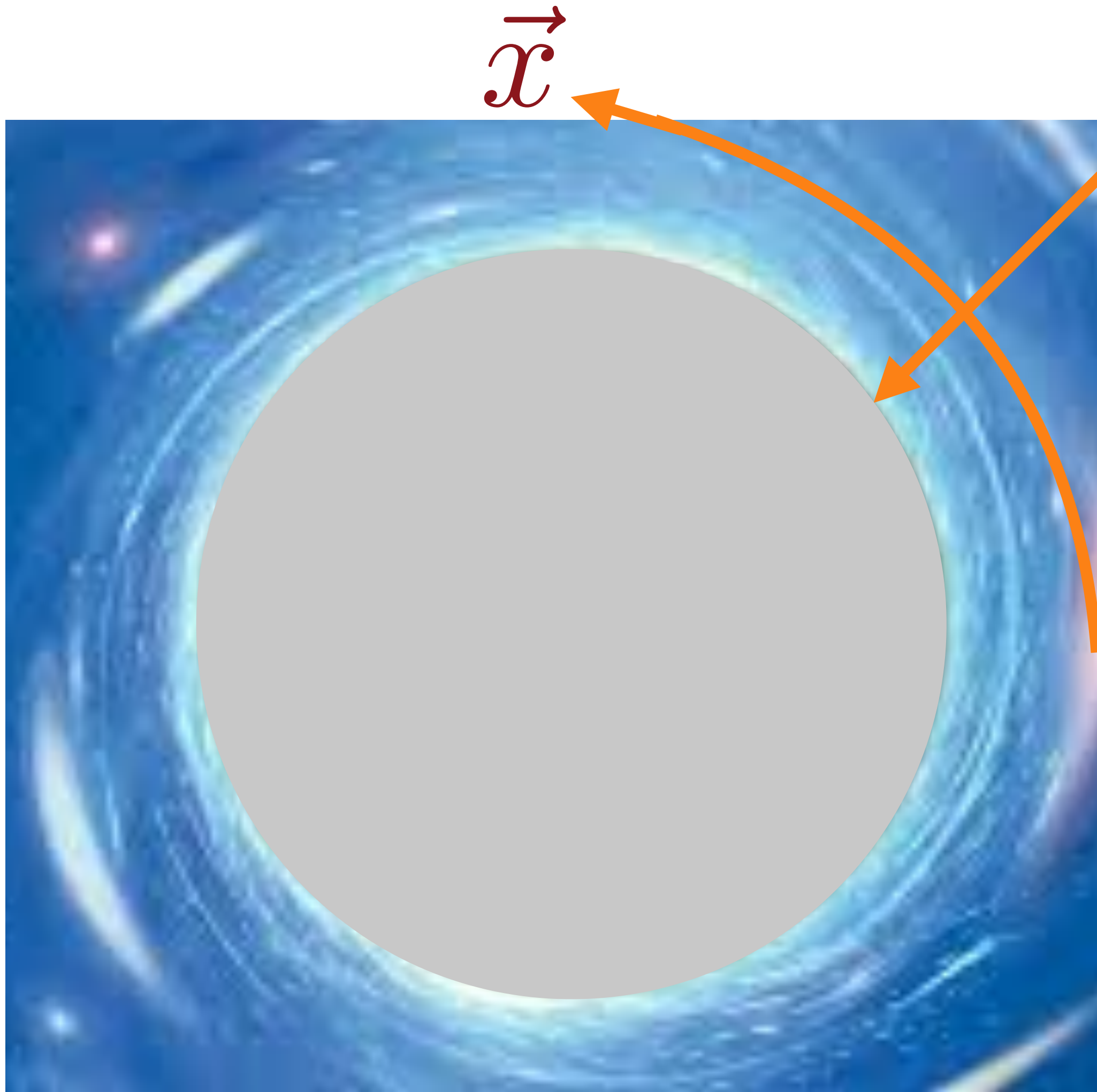


Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge





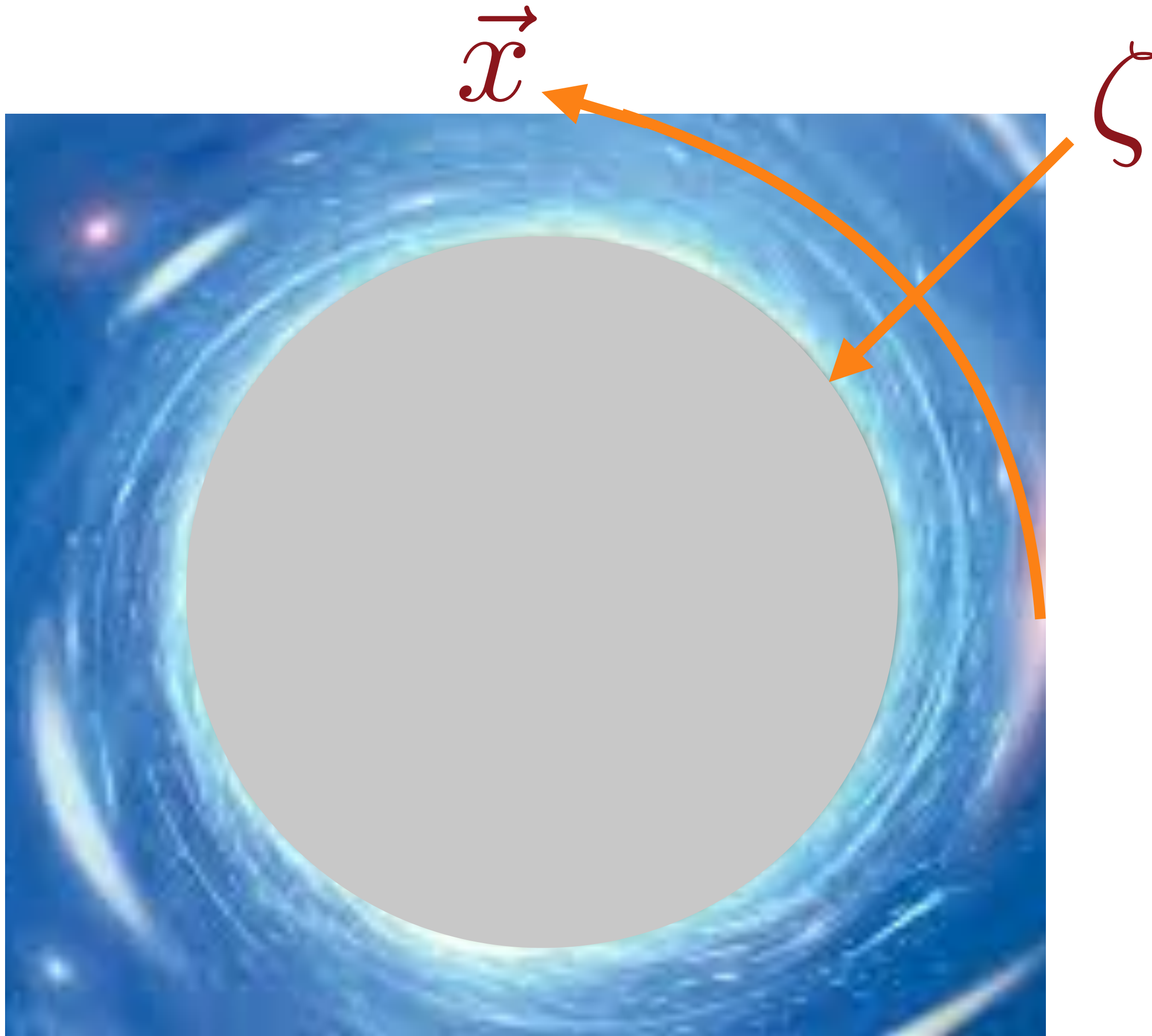
Maxwell's electromagnetism
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Zooming into the near-
horizon region of a charged
black hole at low
temperature, yields a
quantum theory in one
space (ζ) and one time
dimension



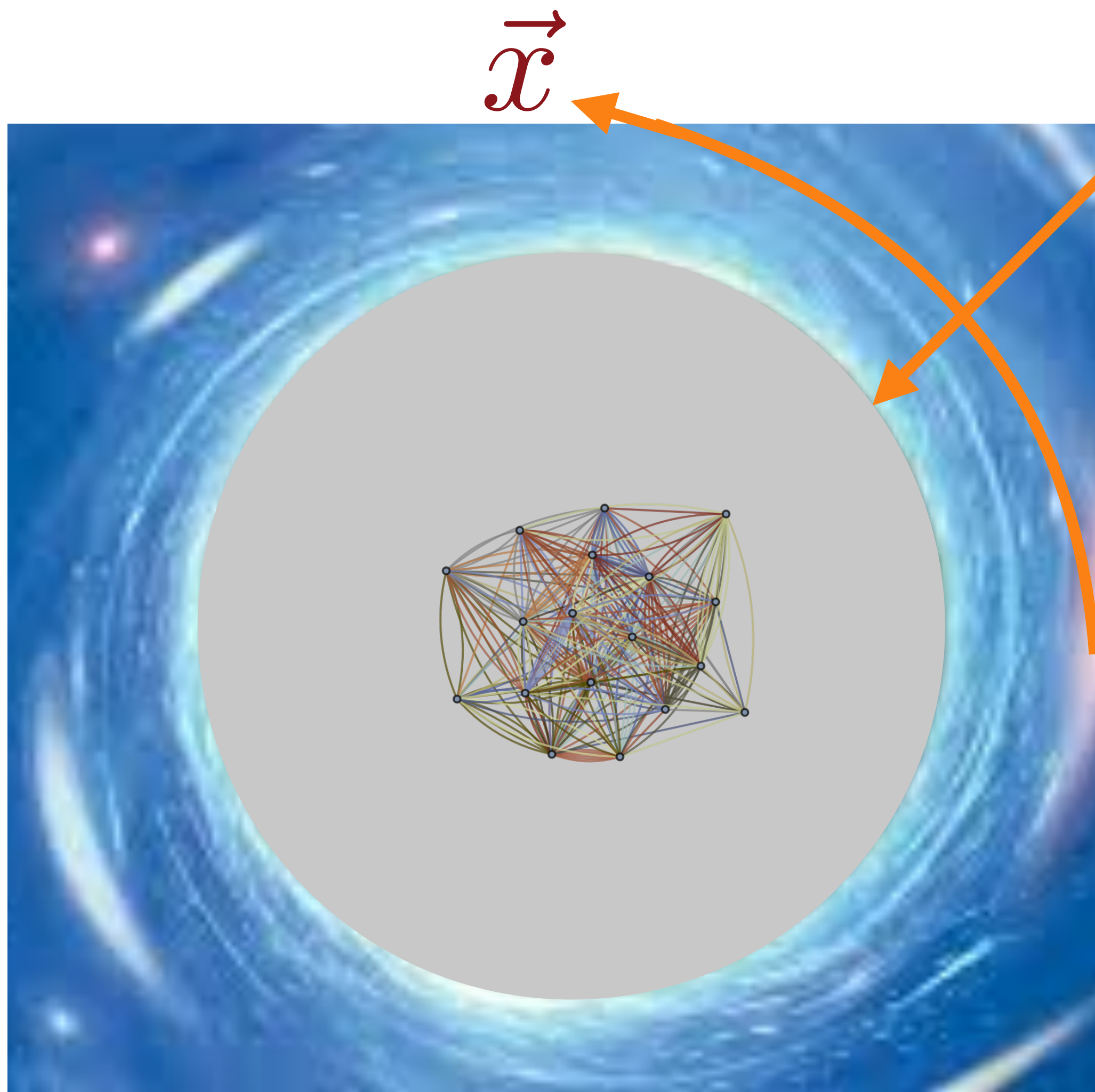
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The quantum versions of
Maxwell's and Einstein's
equations in this
two-dimensional spacetime are
also the equations describing
electron entanglement in the
SYK model!



Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge

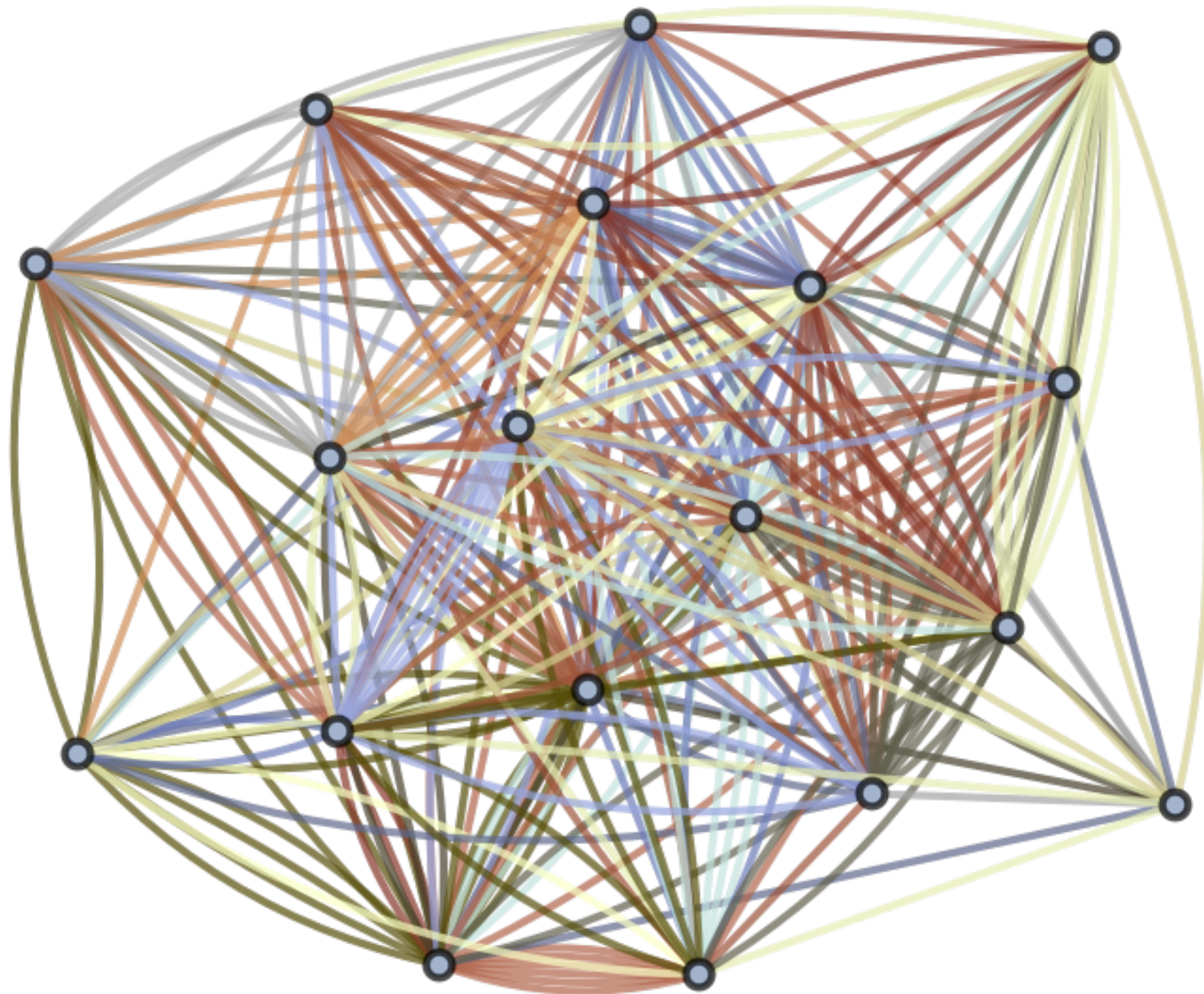


The SYK provides the
needed realization of the
black hole interior, and its
density of quantum states
matches gravitational
entropy computations for
charged black holes !

Recap

The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles

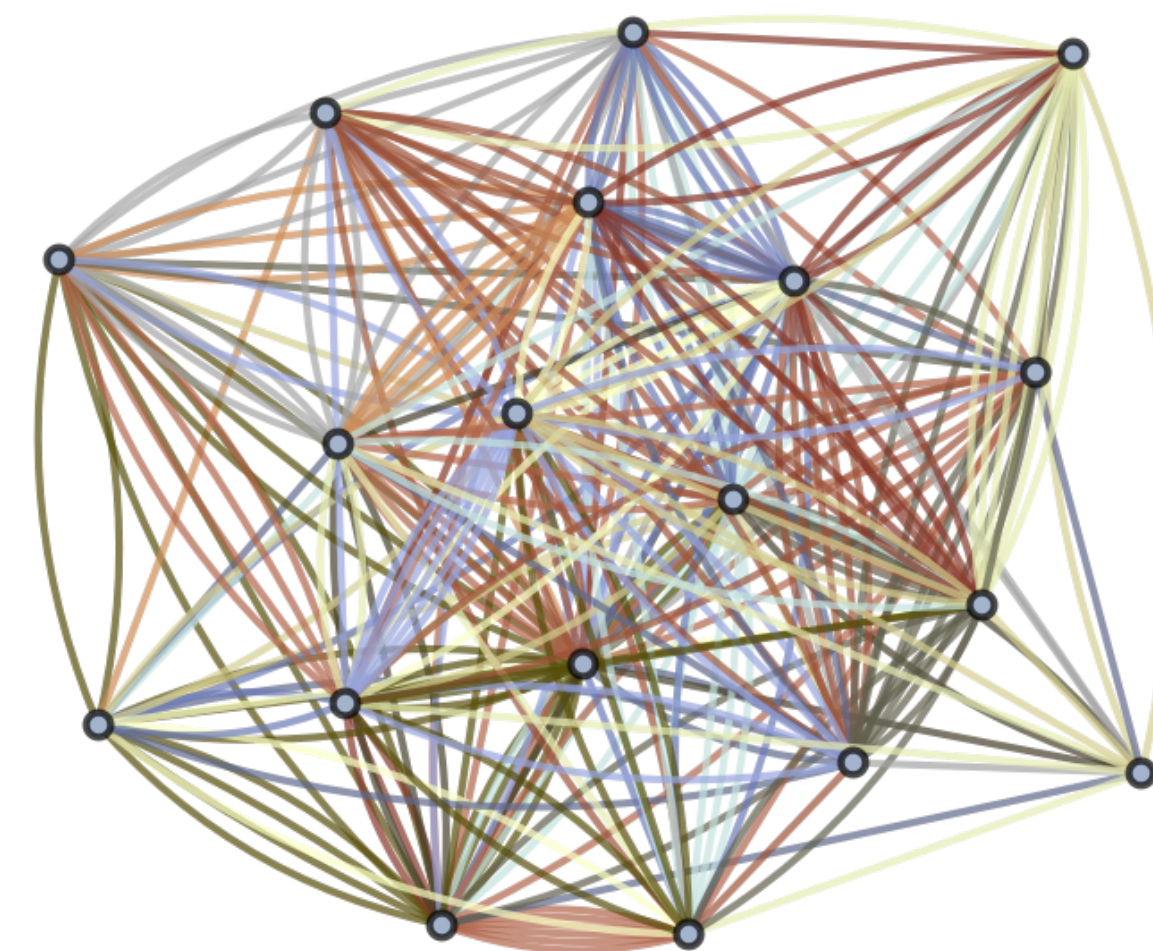
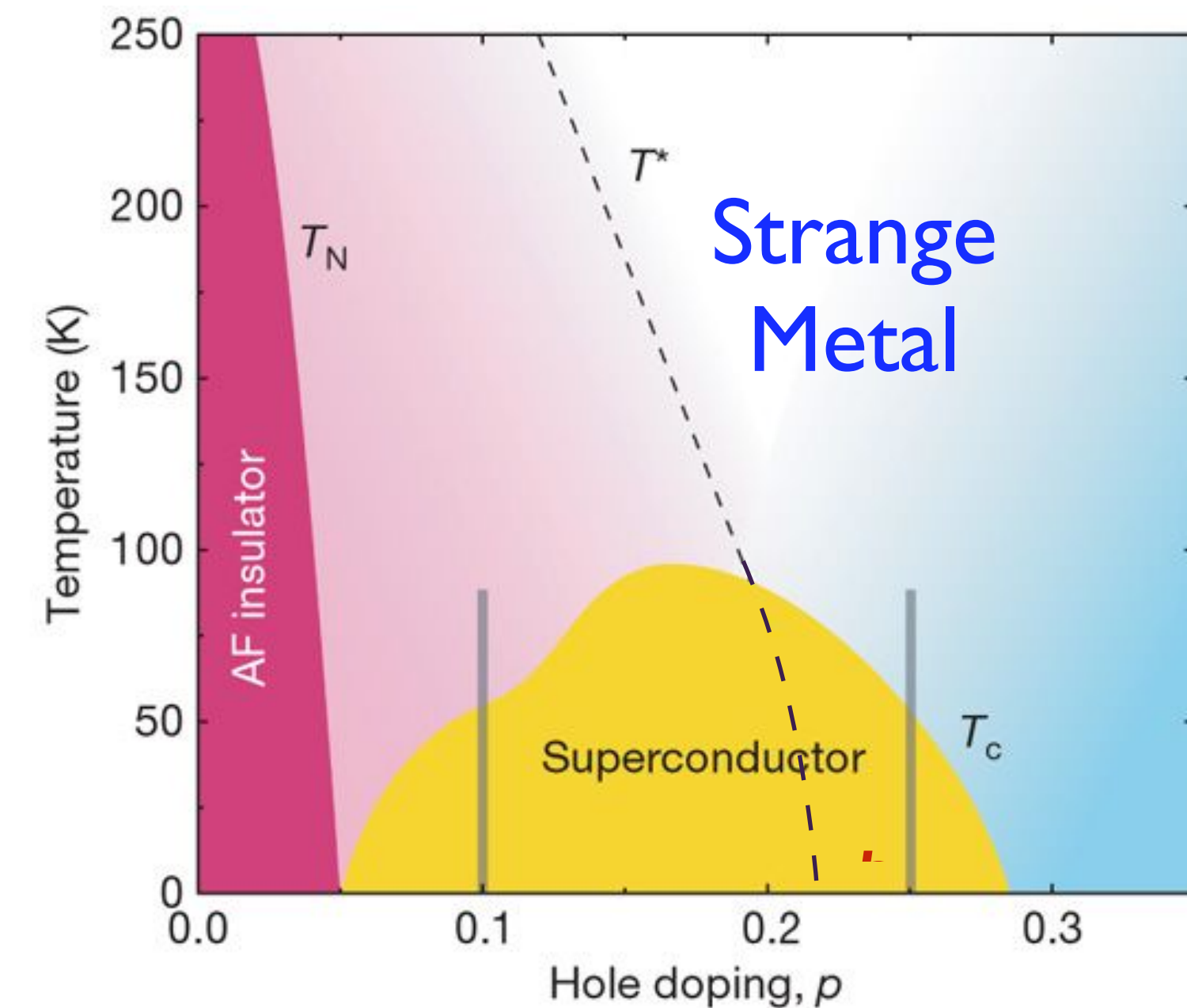


The Sachdev-Ye-Kitaev (SYK) model

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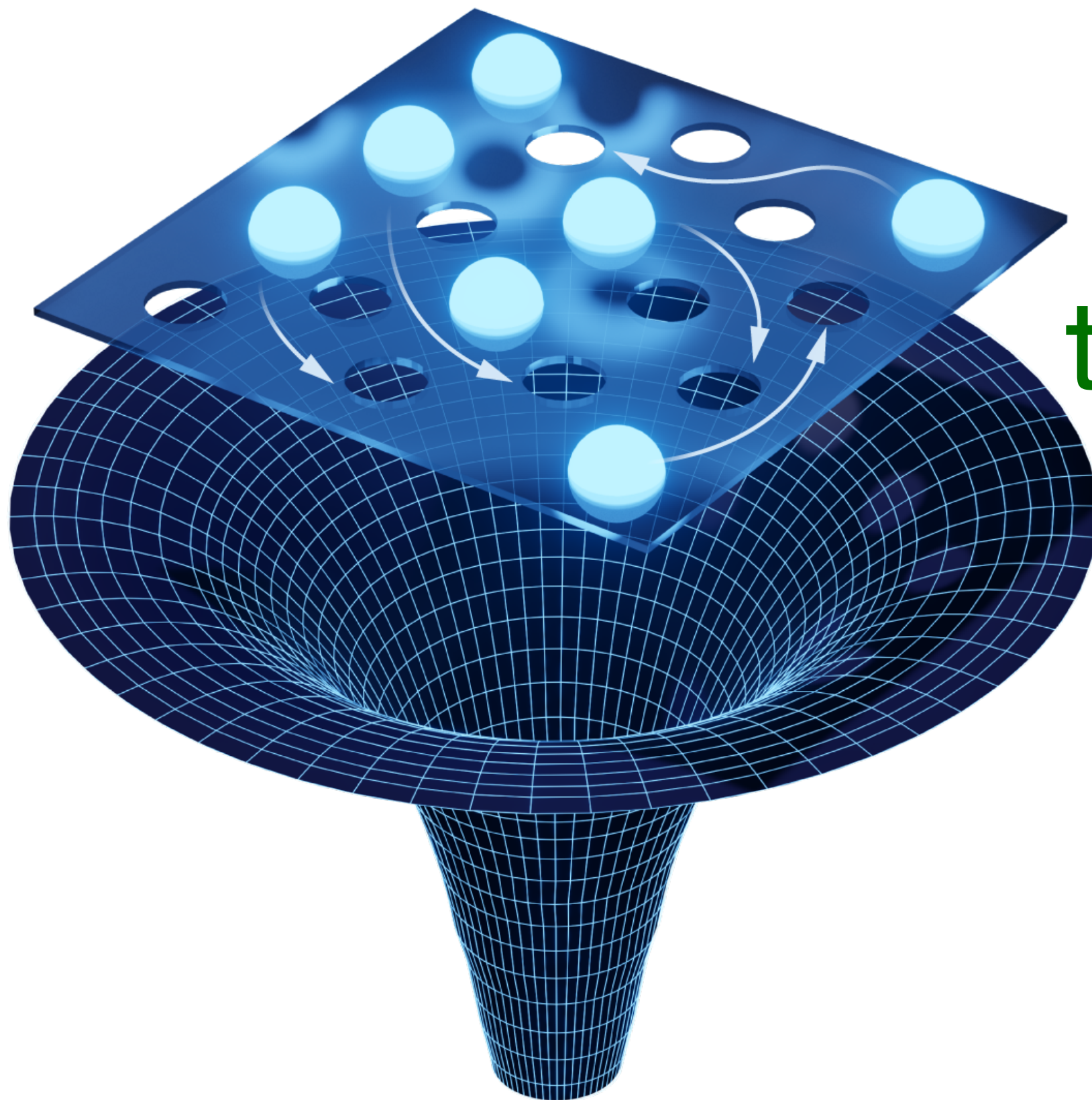
In one set of variables, it helps describe the *strange* electrical properties of YBCO

Sachdev, Ye (1993)



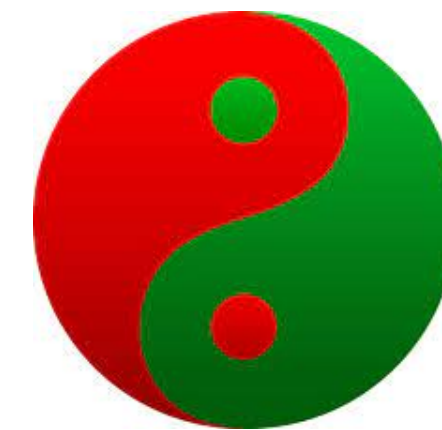
The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles



In one set of variables, it helps describe the ***strange*** electrical properties of YBCO

Sachdev, Ye (1993)



In a ***dual*** set of variables it describes ***charged black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)