Outline

1. Pseudogap: angular fluctuations of a multi-component order parameter

2. Instabilities of a two-dimensional metal with antiferromagnetic exchange interactions: *d*-wave superconductivity and bond order

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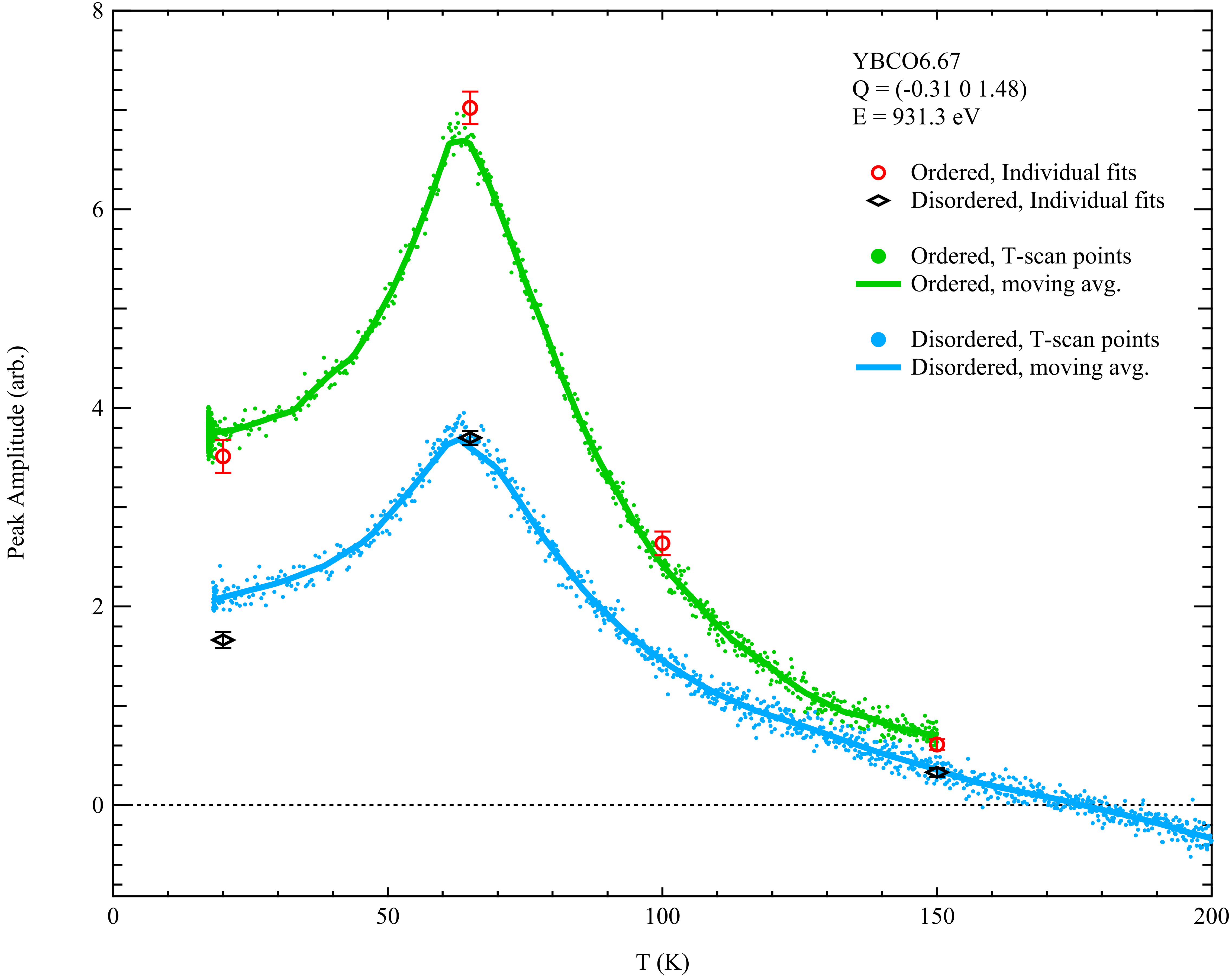
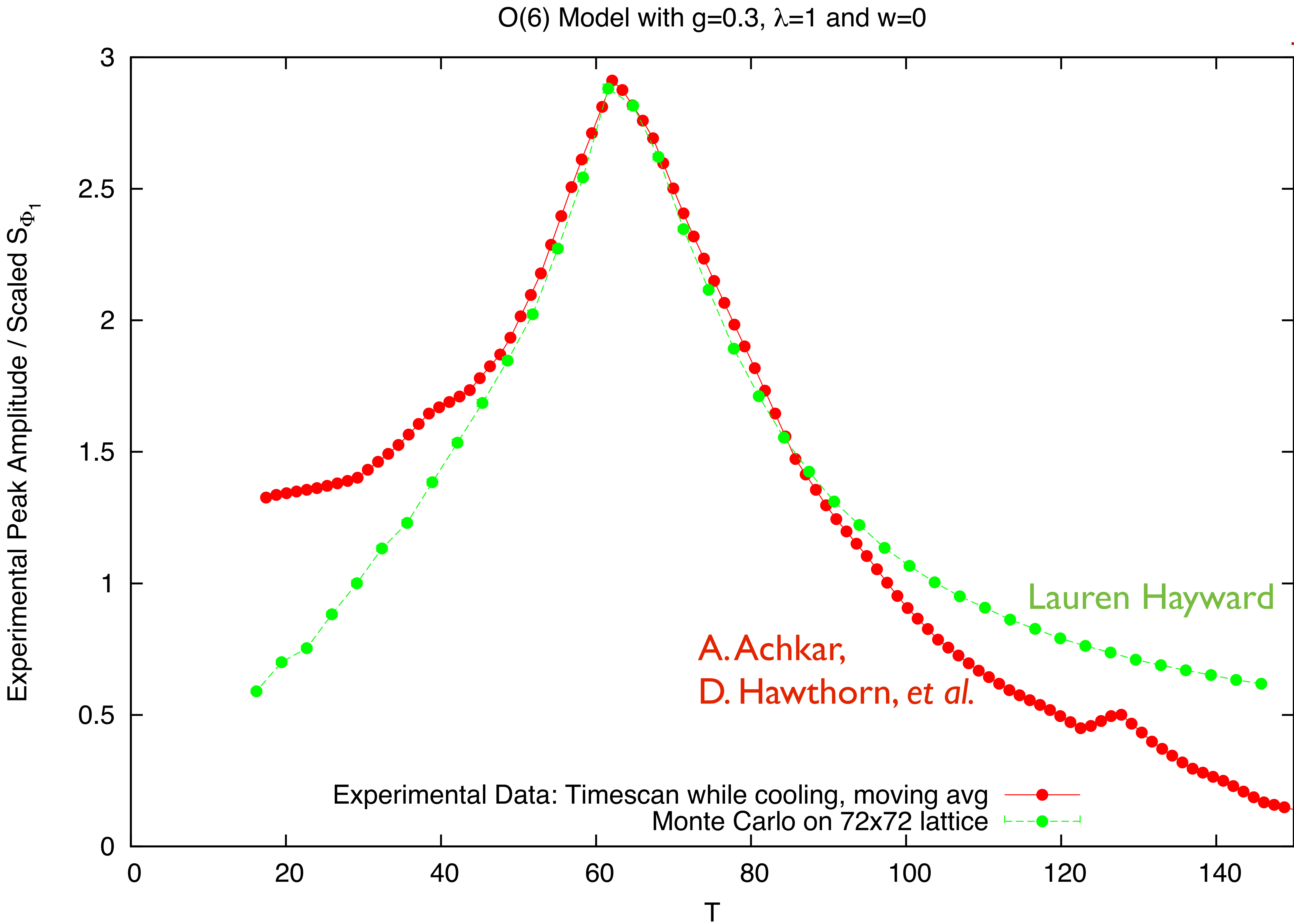


A. Achkar, D. Hawthorn, *et al.*

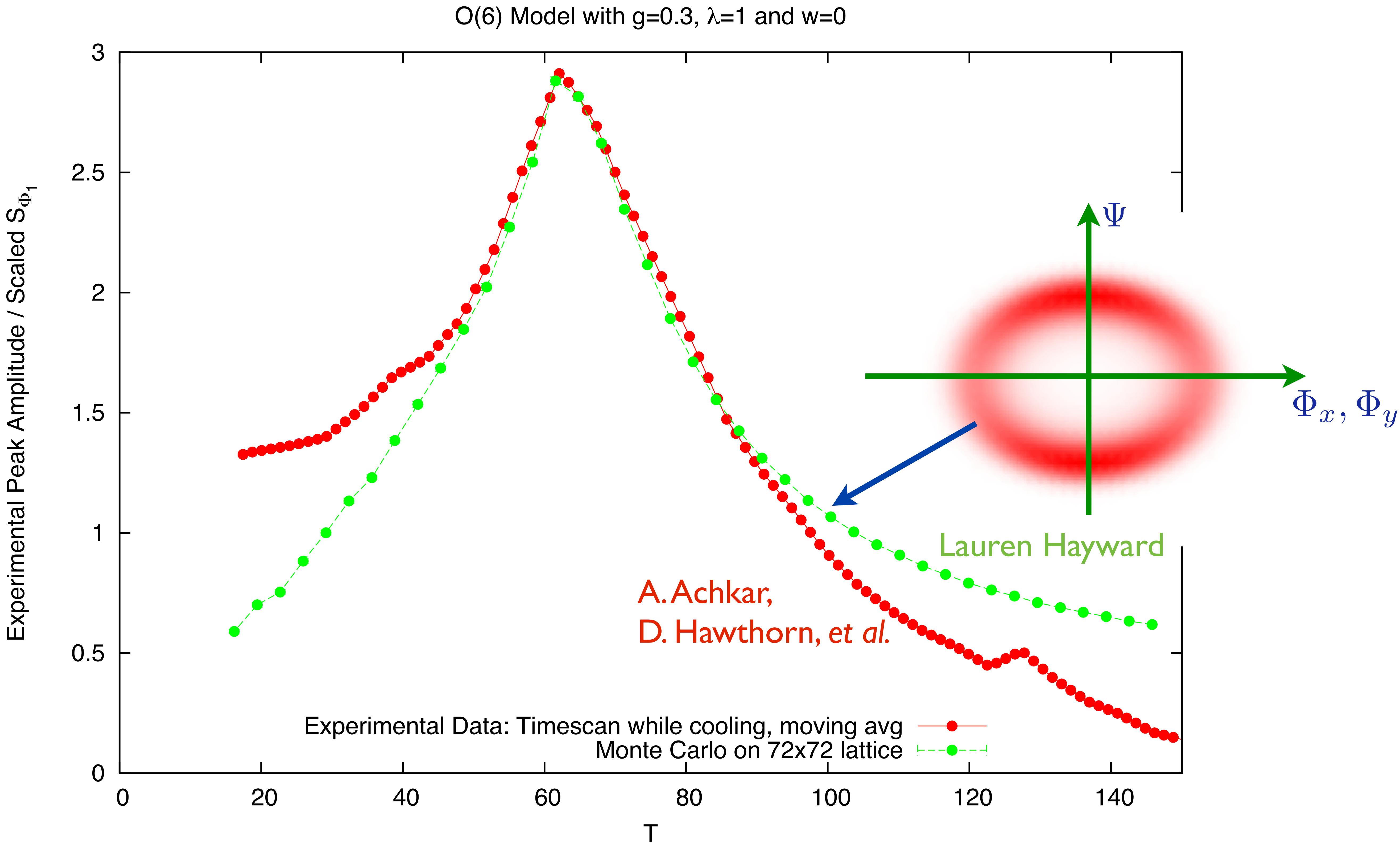
O(6) non-linear sigma model

Preliminary



Charge order structure factor S_{Φ_x}

O(6) non-linear sigma model



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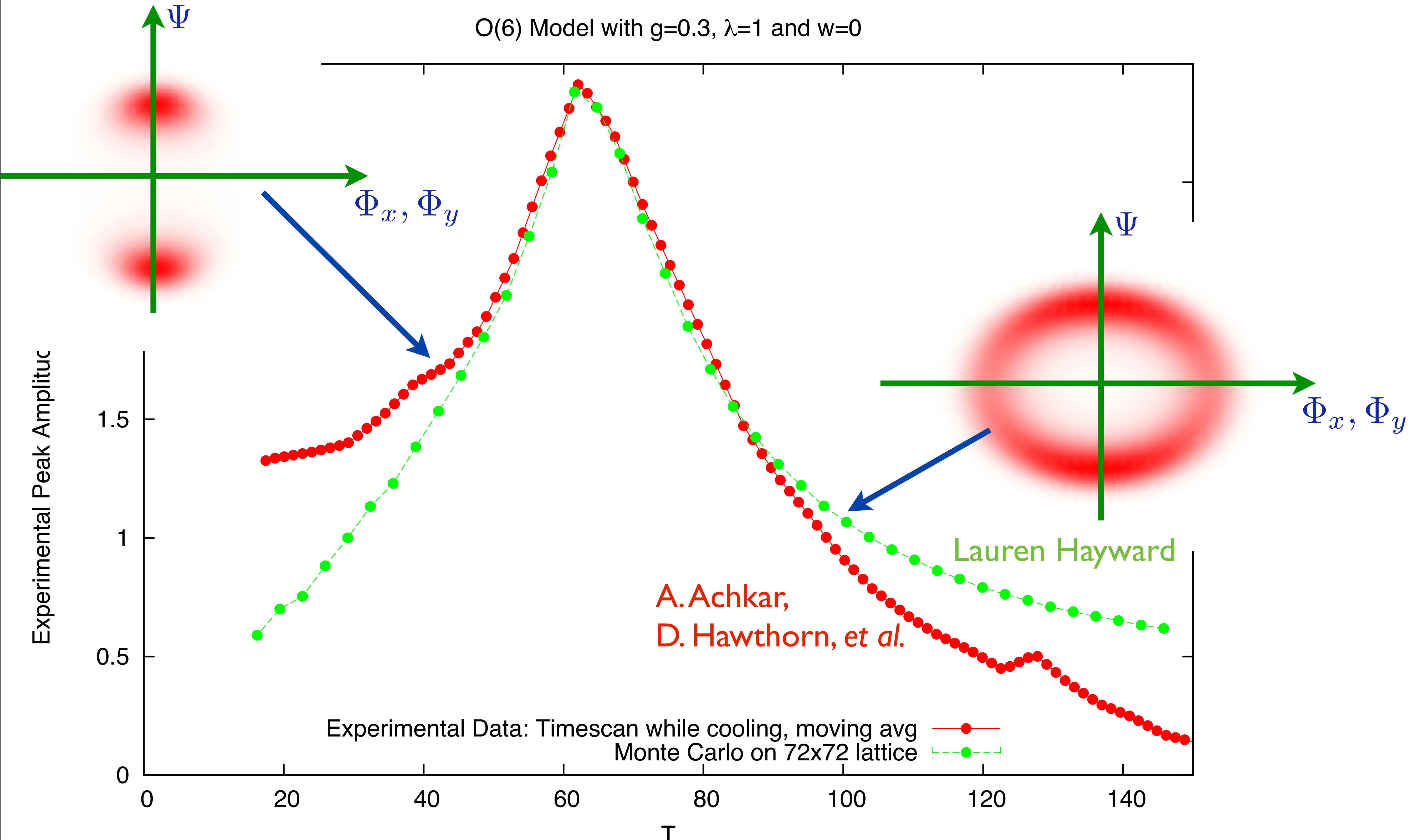

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Outline

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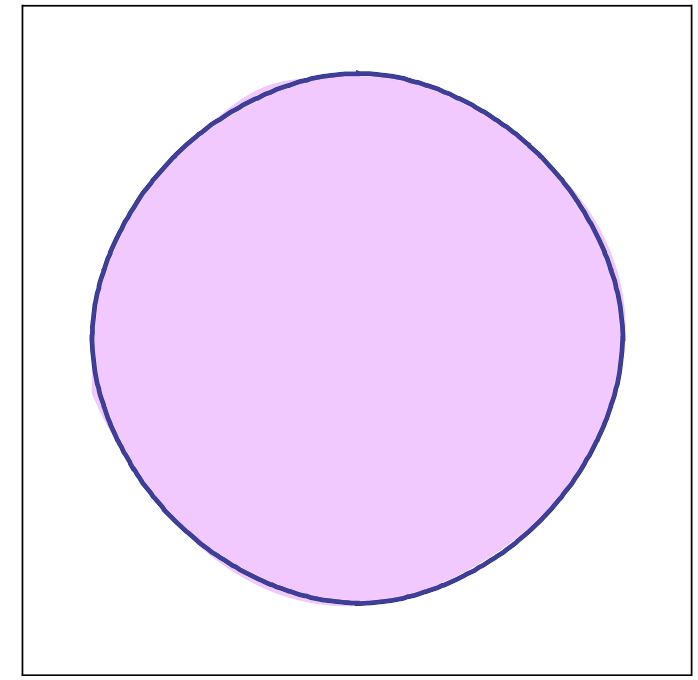
Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface



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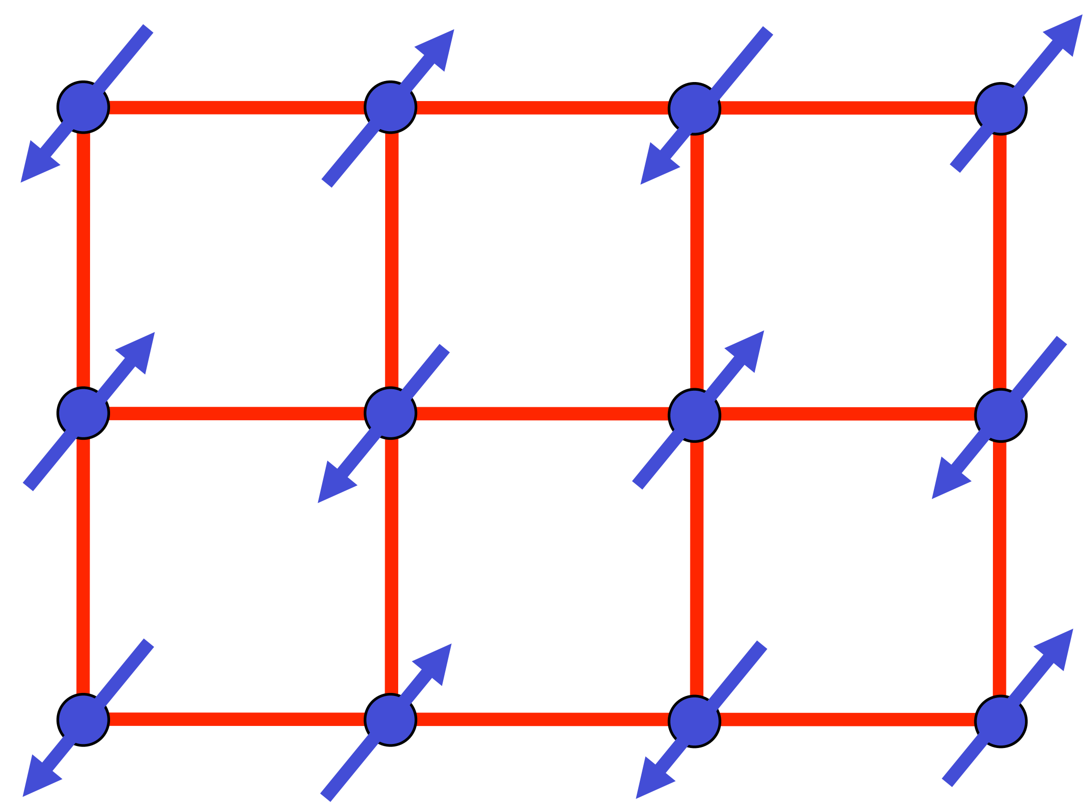
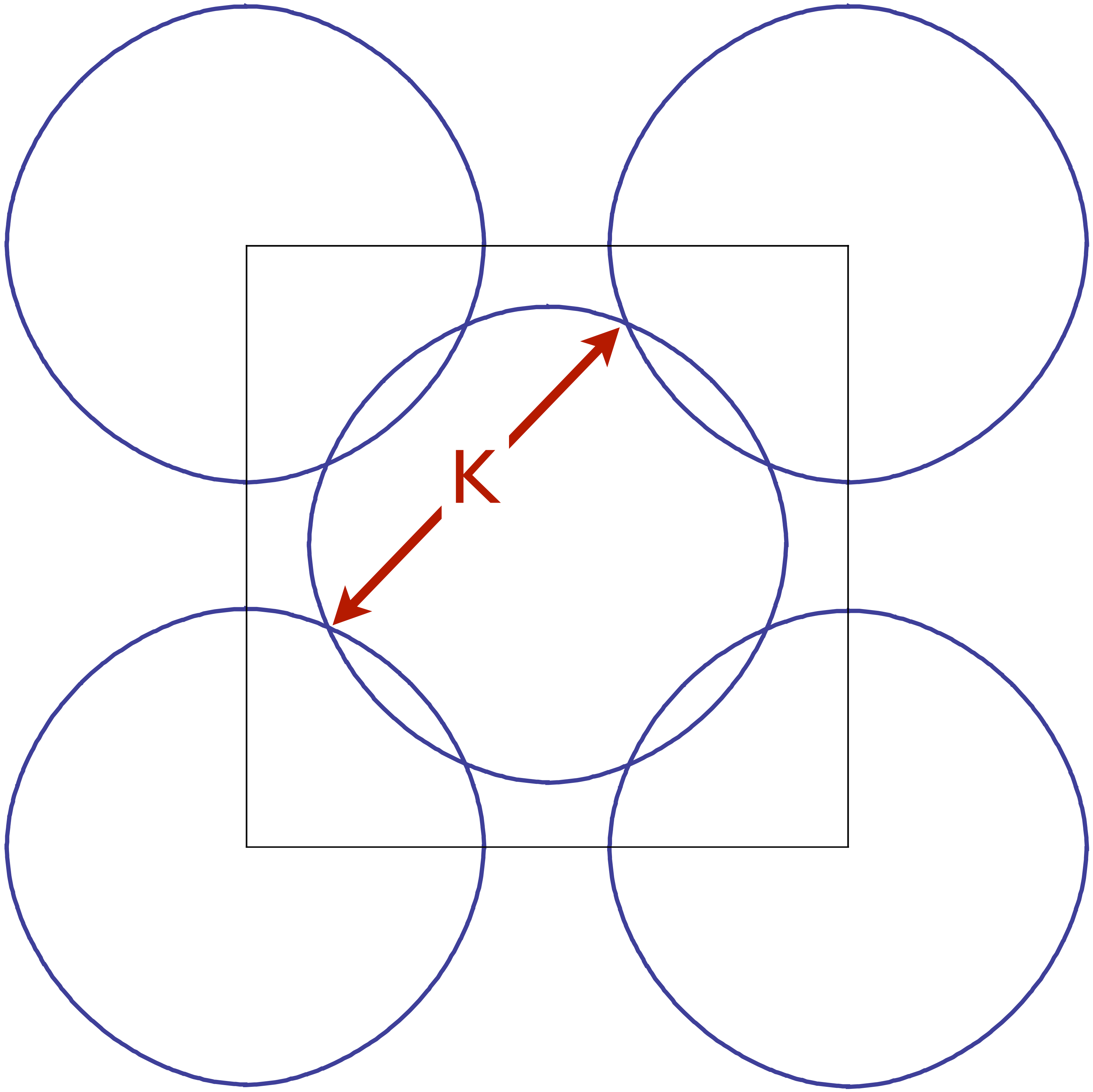


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

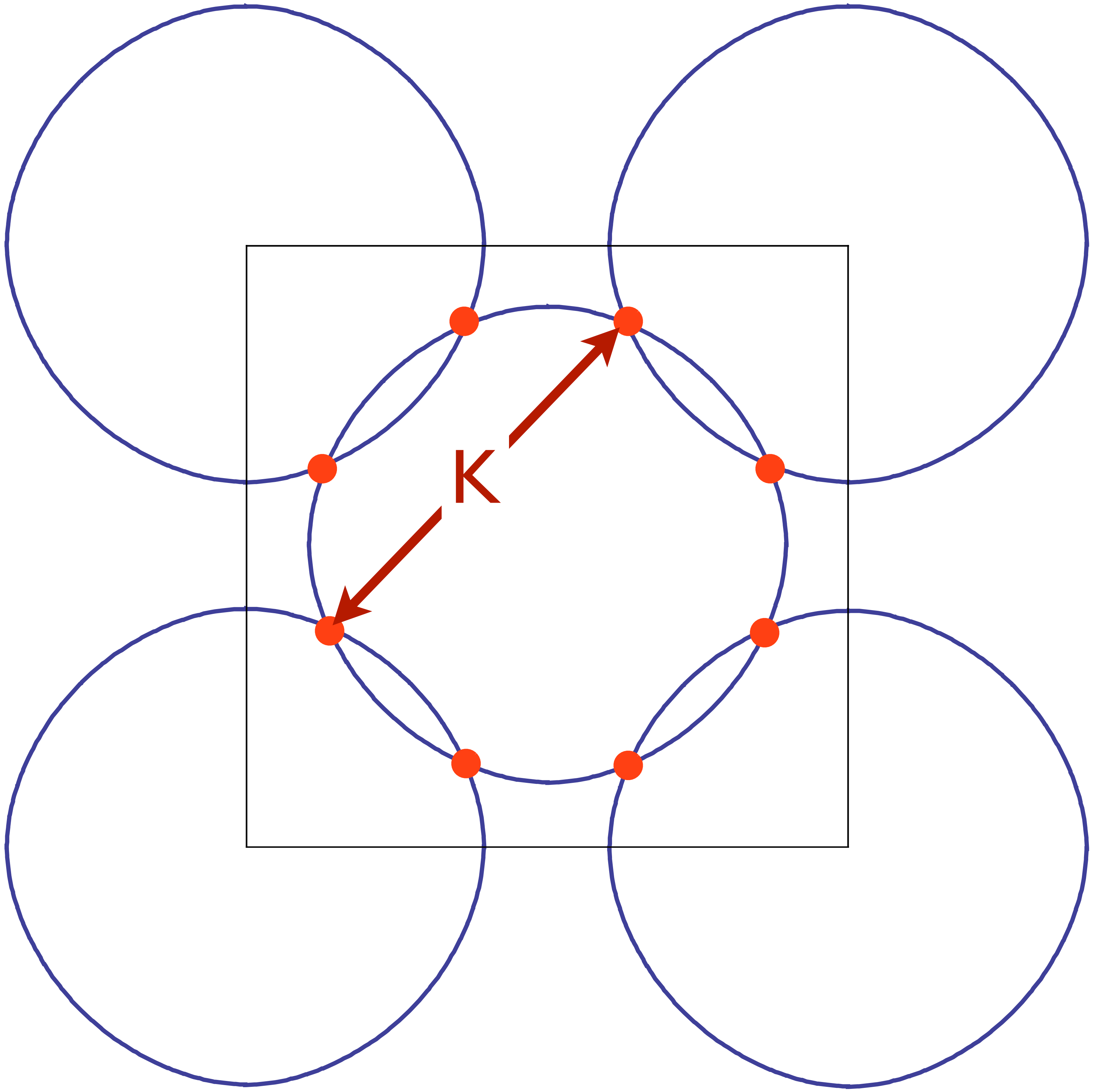
where $\mathbf{K}$ is the ordering wavevector.

Fermi surface+antiferromagnetism



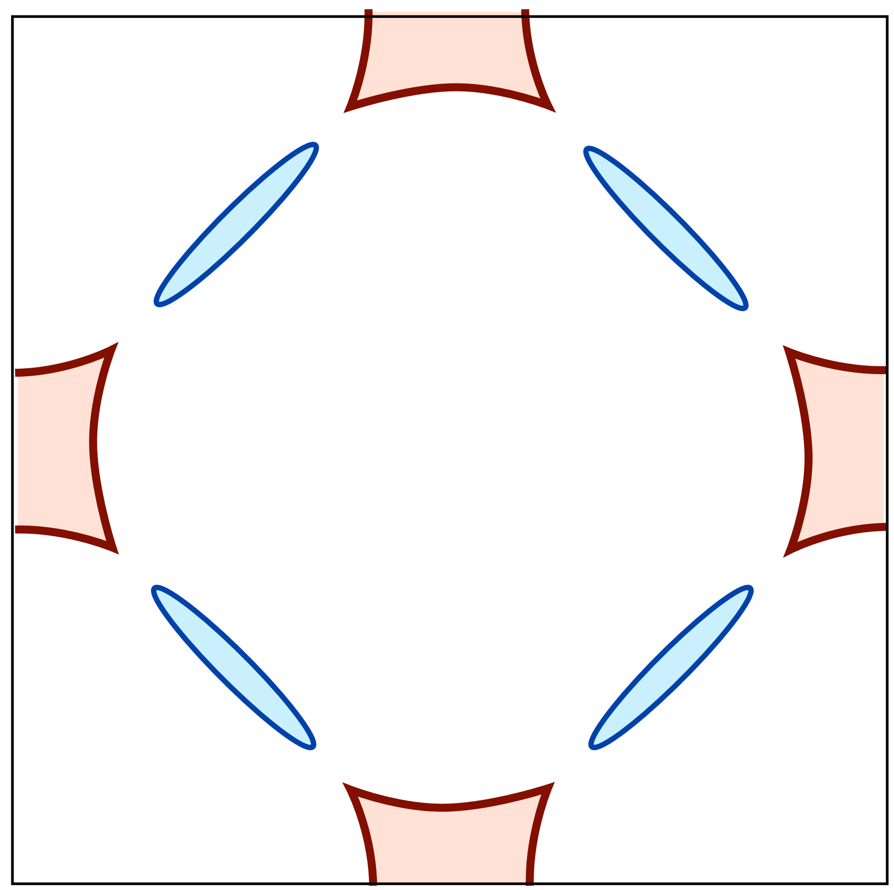
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



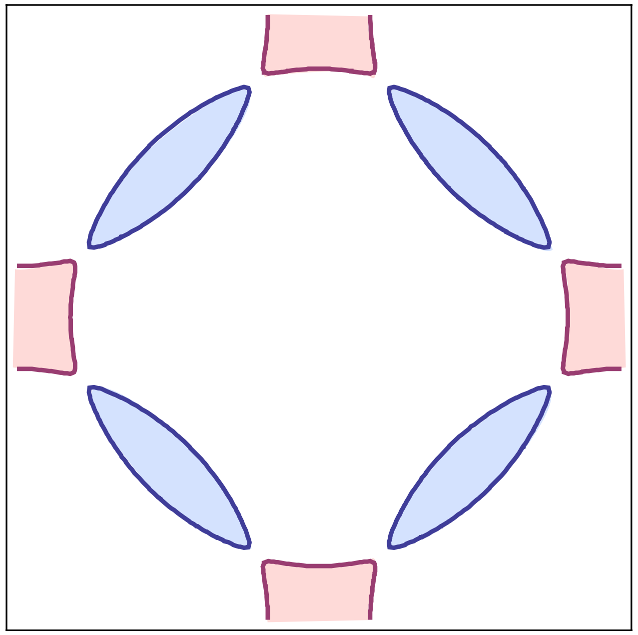
"Hot" spots

Fermi surface+antiferromagnetism



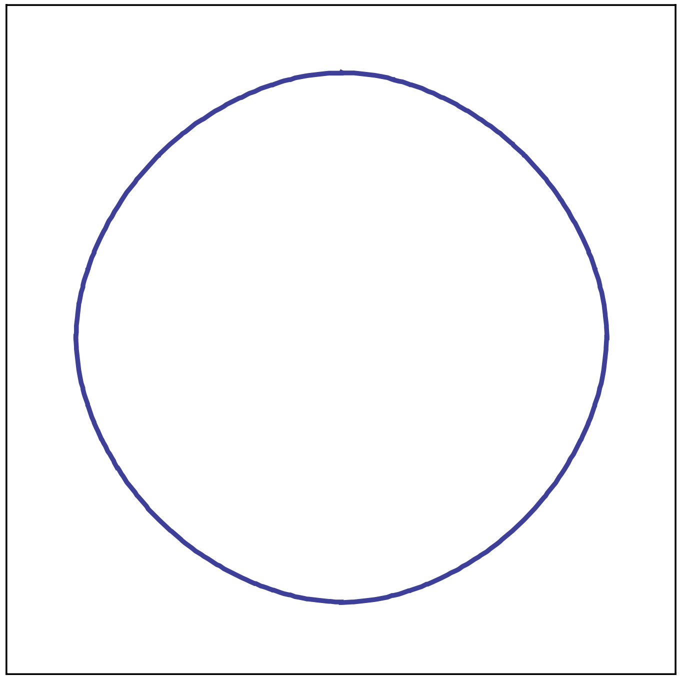
Electron and hole pockets in antiferromagnetic phase with antiferromagnetic order parameter $\langle \vec{\varphi} \rangle \neq 0$

Fermi surface+antiferromagnetism



$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets

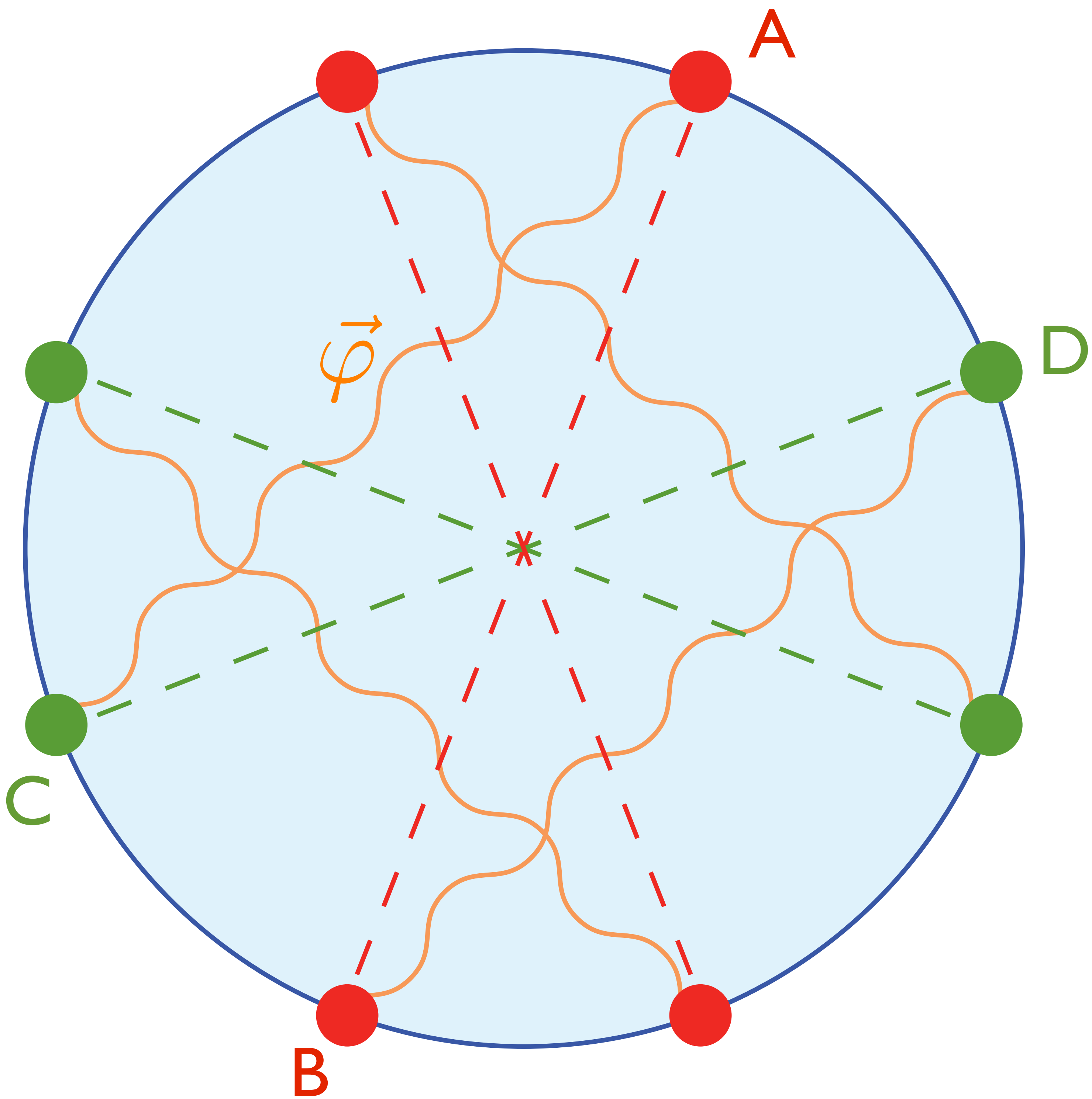


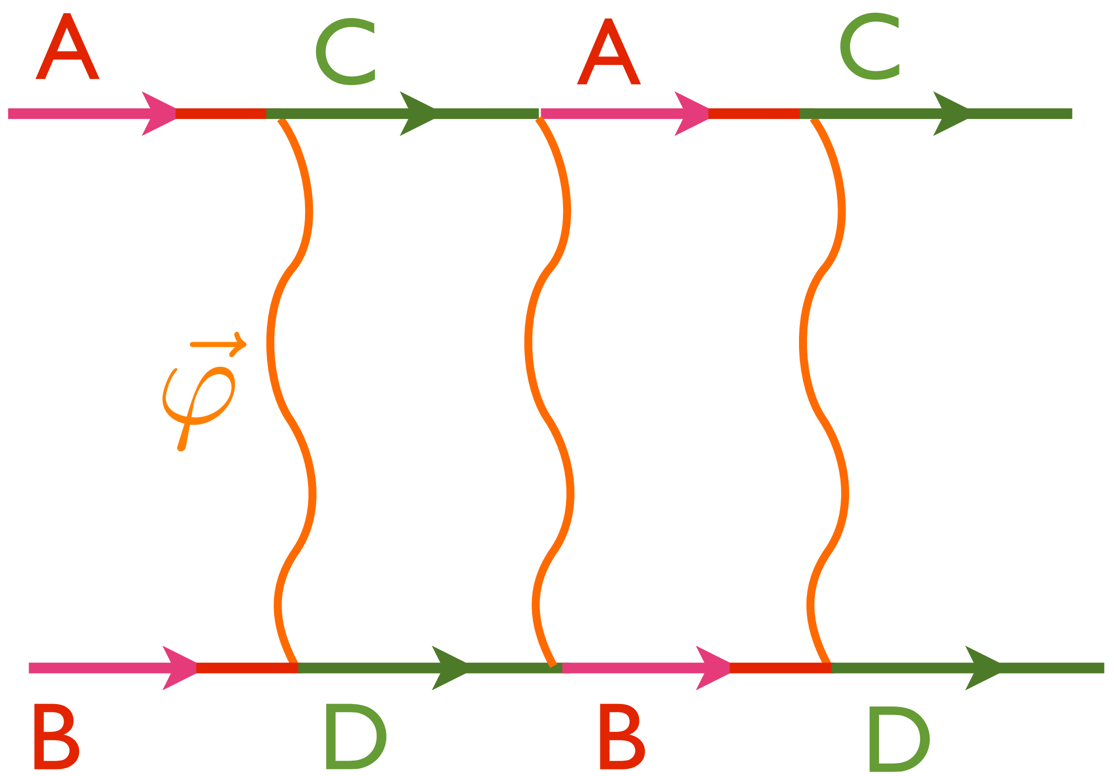
$\langle \vec{\varphi} \rangle = 0$

Metal with "large" Fermi surface

r

Pairing "glue" from antiferromagnetic fluctuations





V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
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$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y)$$

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K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
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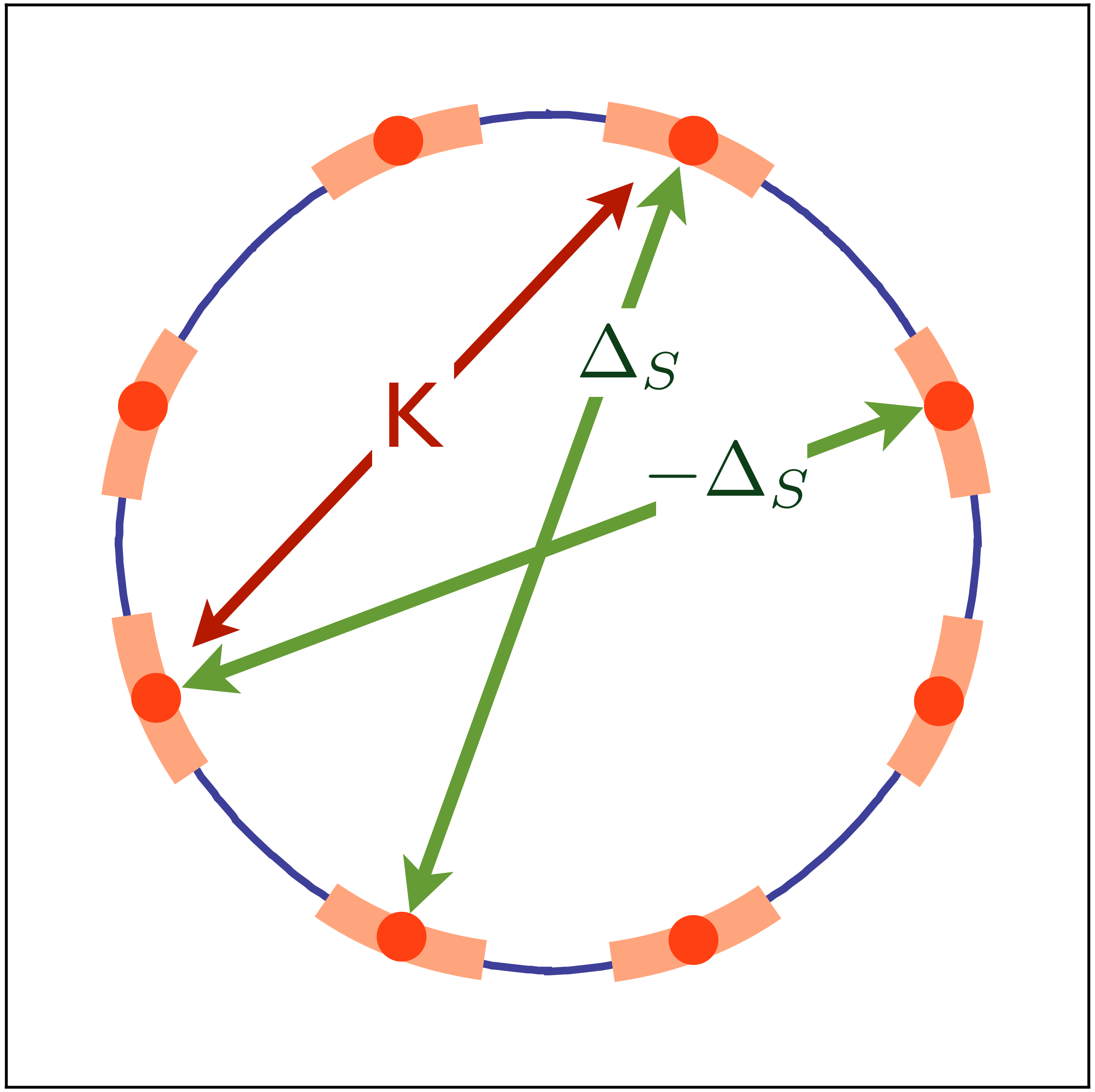
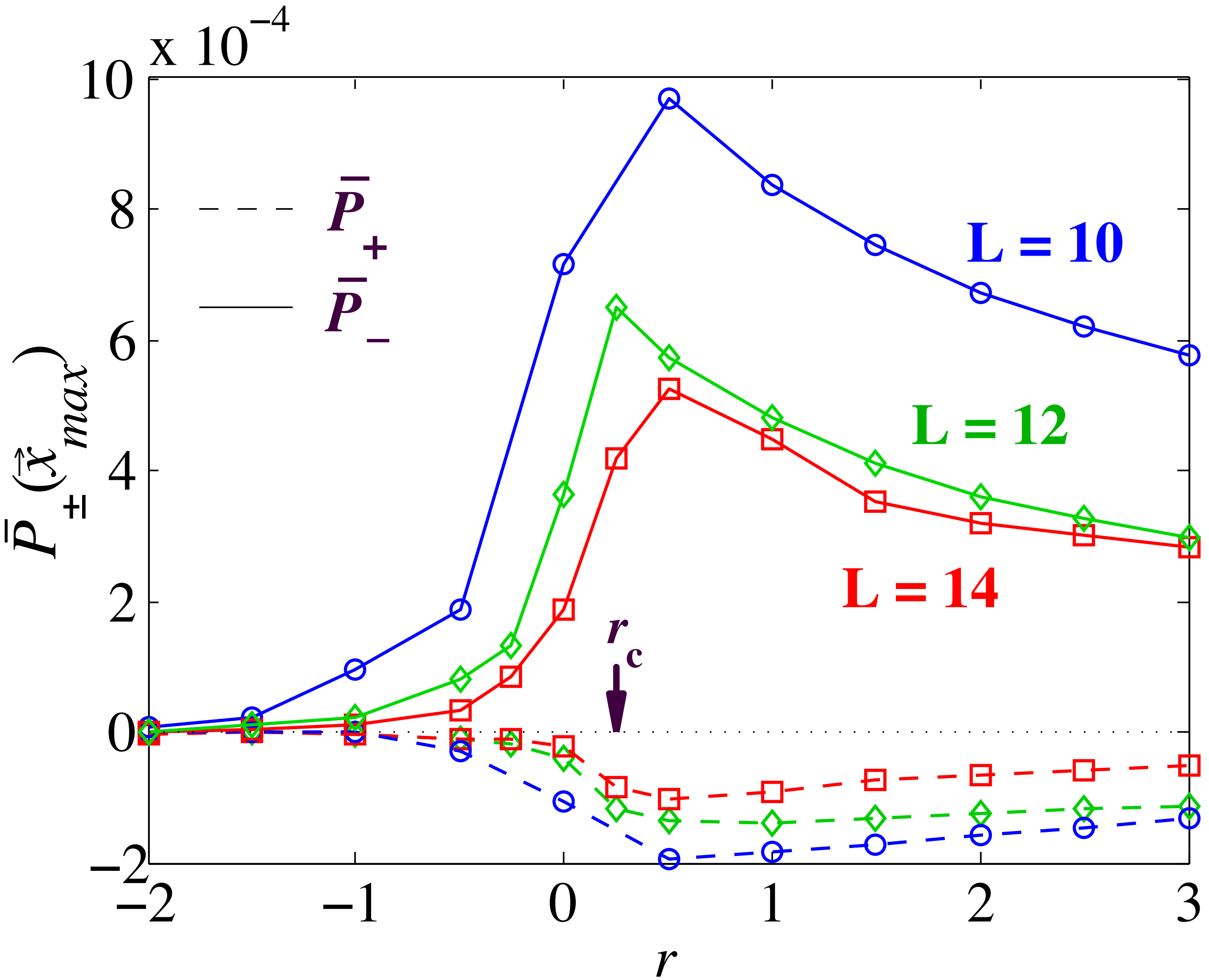


d-wave superconductor: particle-particle pairing at *and near* hot spots, with sign-changing pairing amplitude

Sign-problem-free Quantum Monte Carlo for antiferromagnetism in metals



s d P_+ P_-

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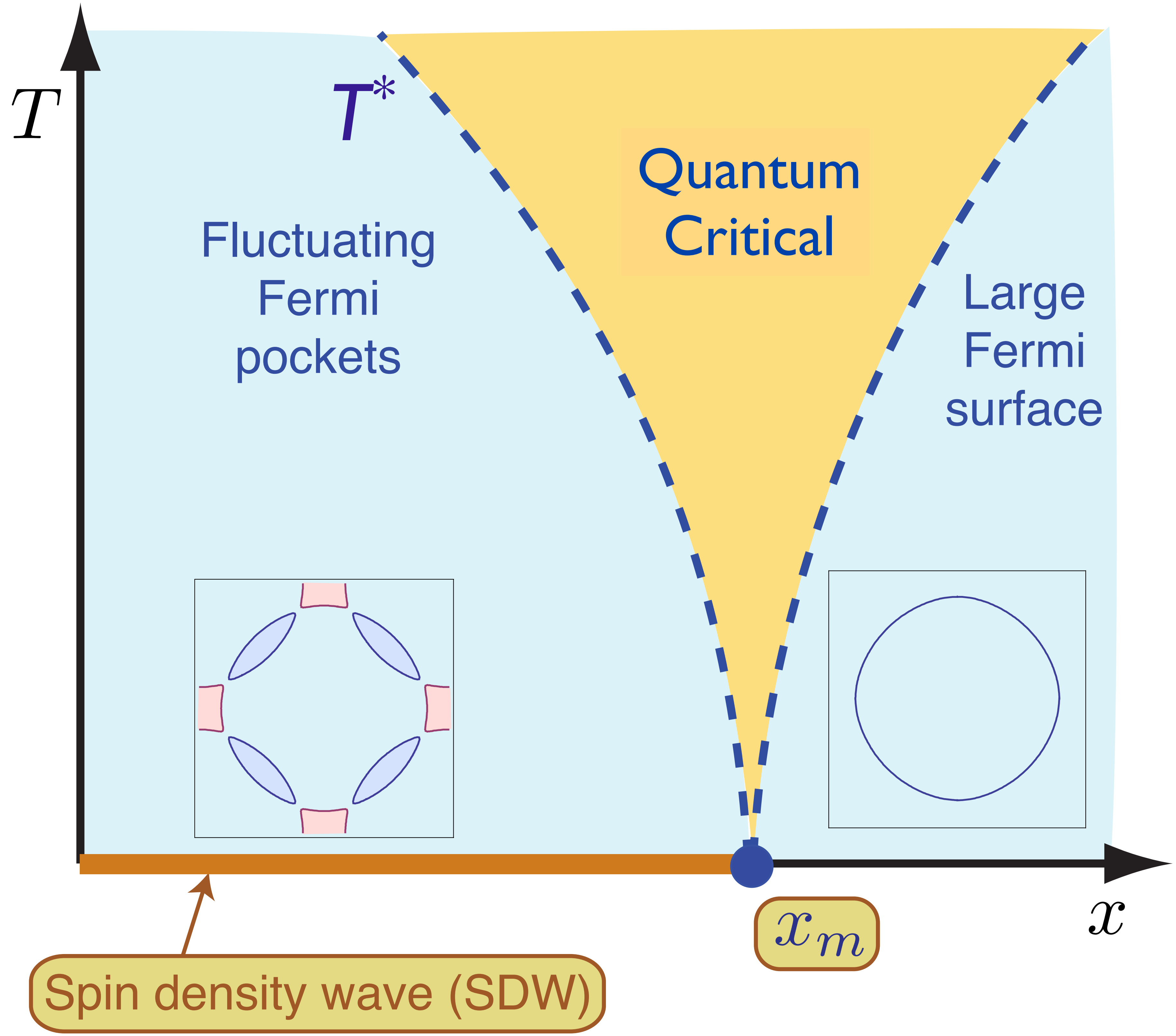
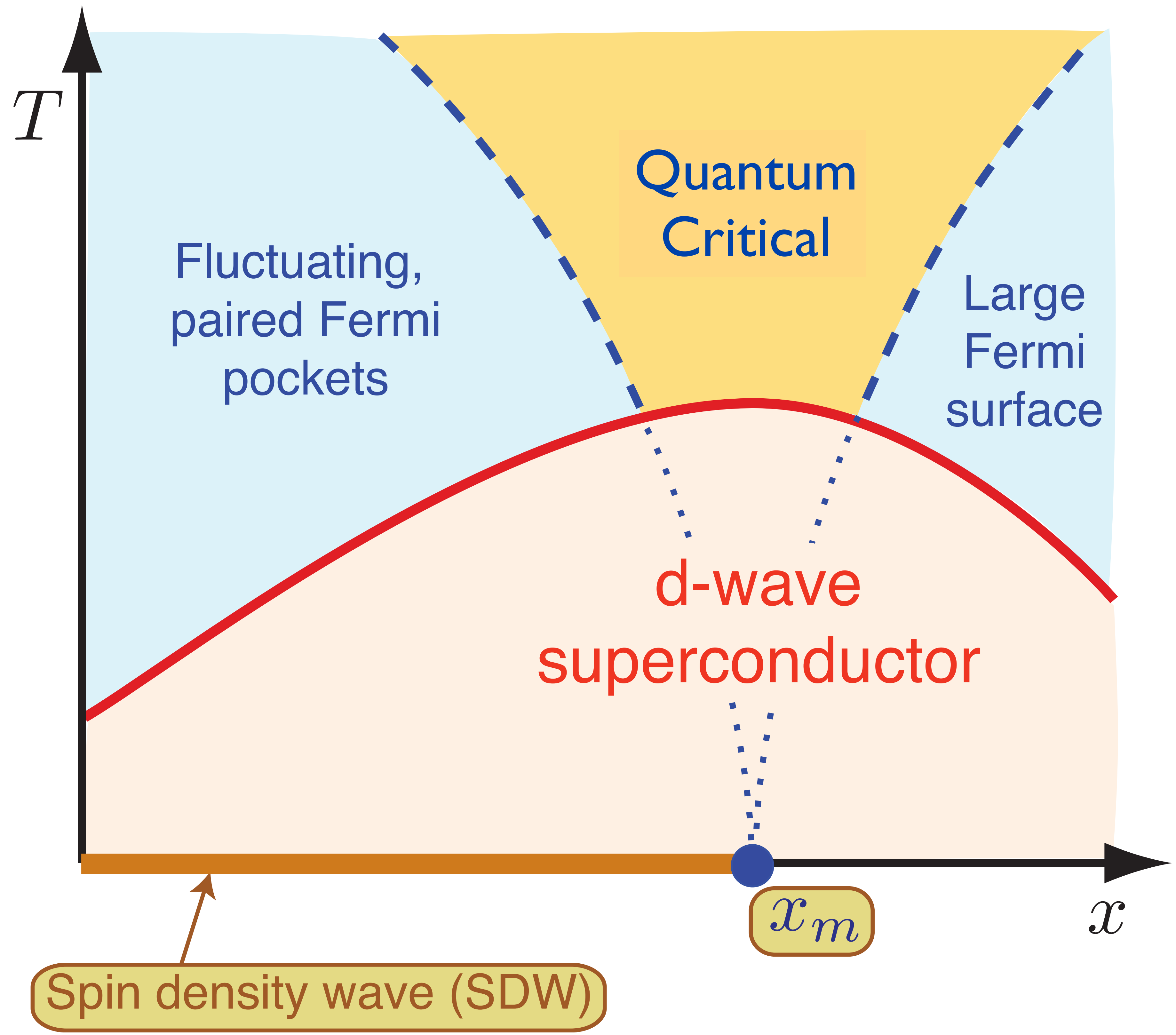
E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Fermi surface+antiferromagnetism

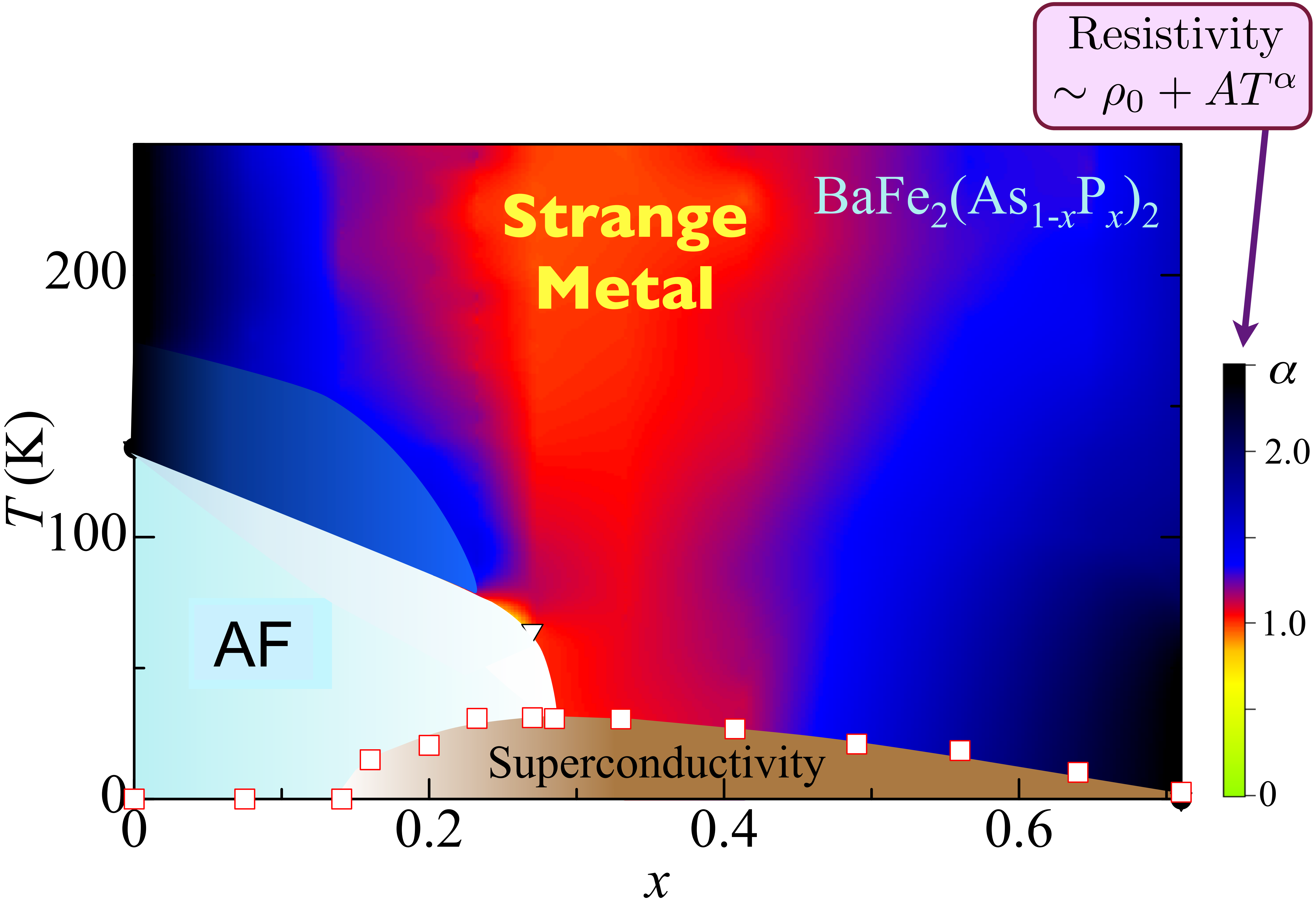


Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Fermi surface+antiferromagnetism



QCP for the onset of SDW order is actually within a superconductor



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

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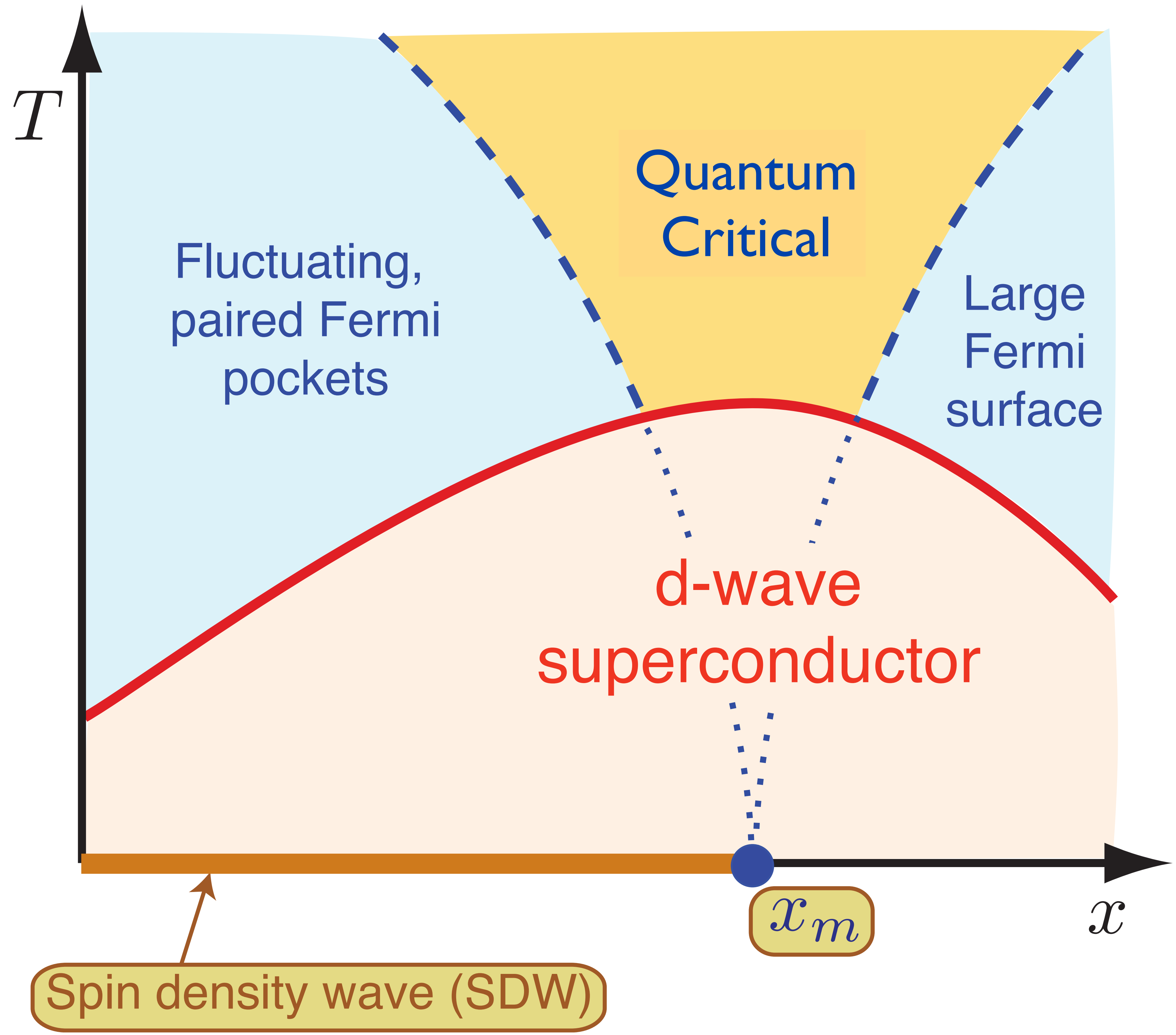

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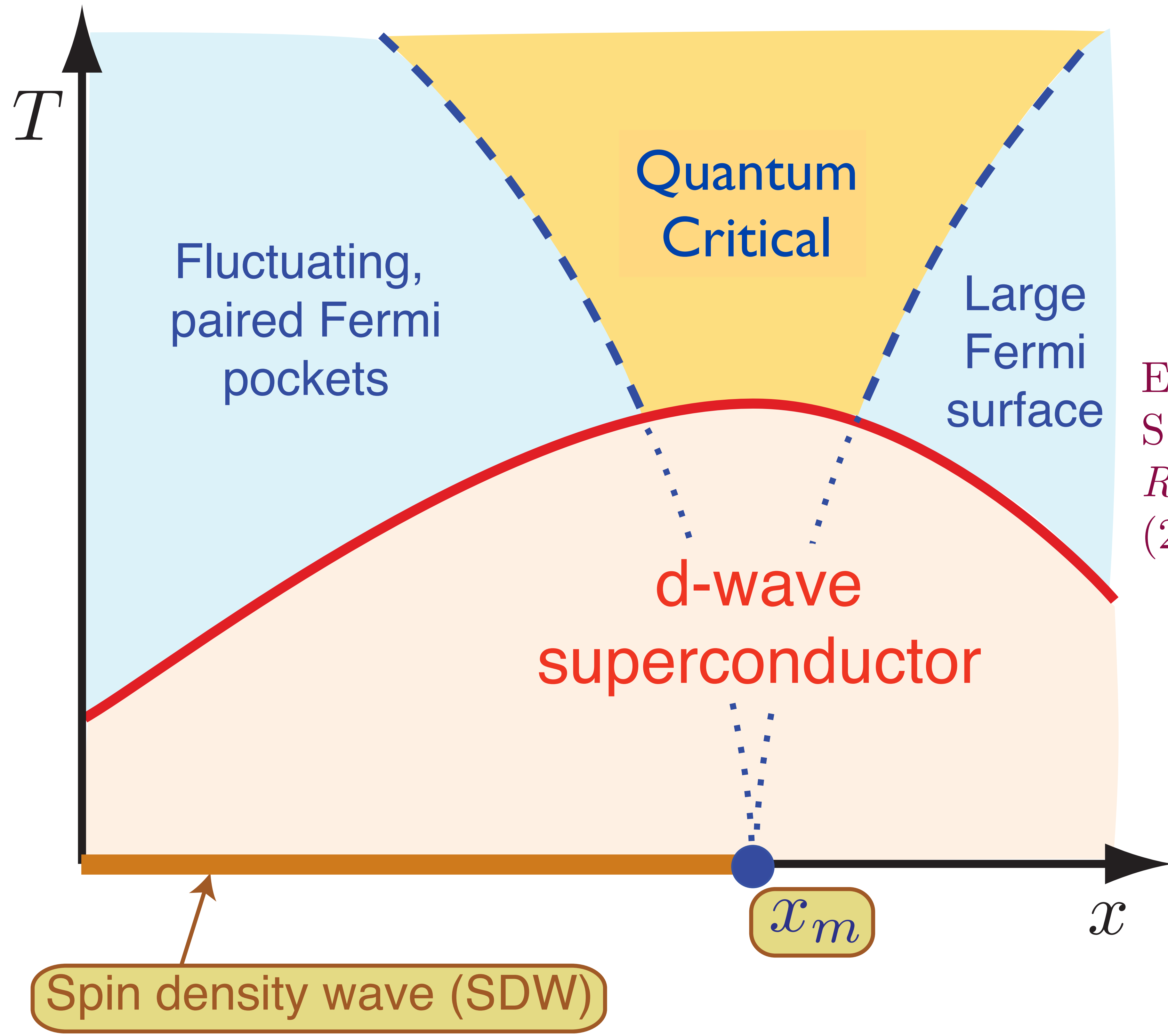
Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

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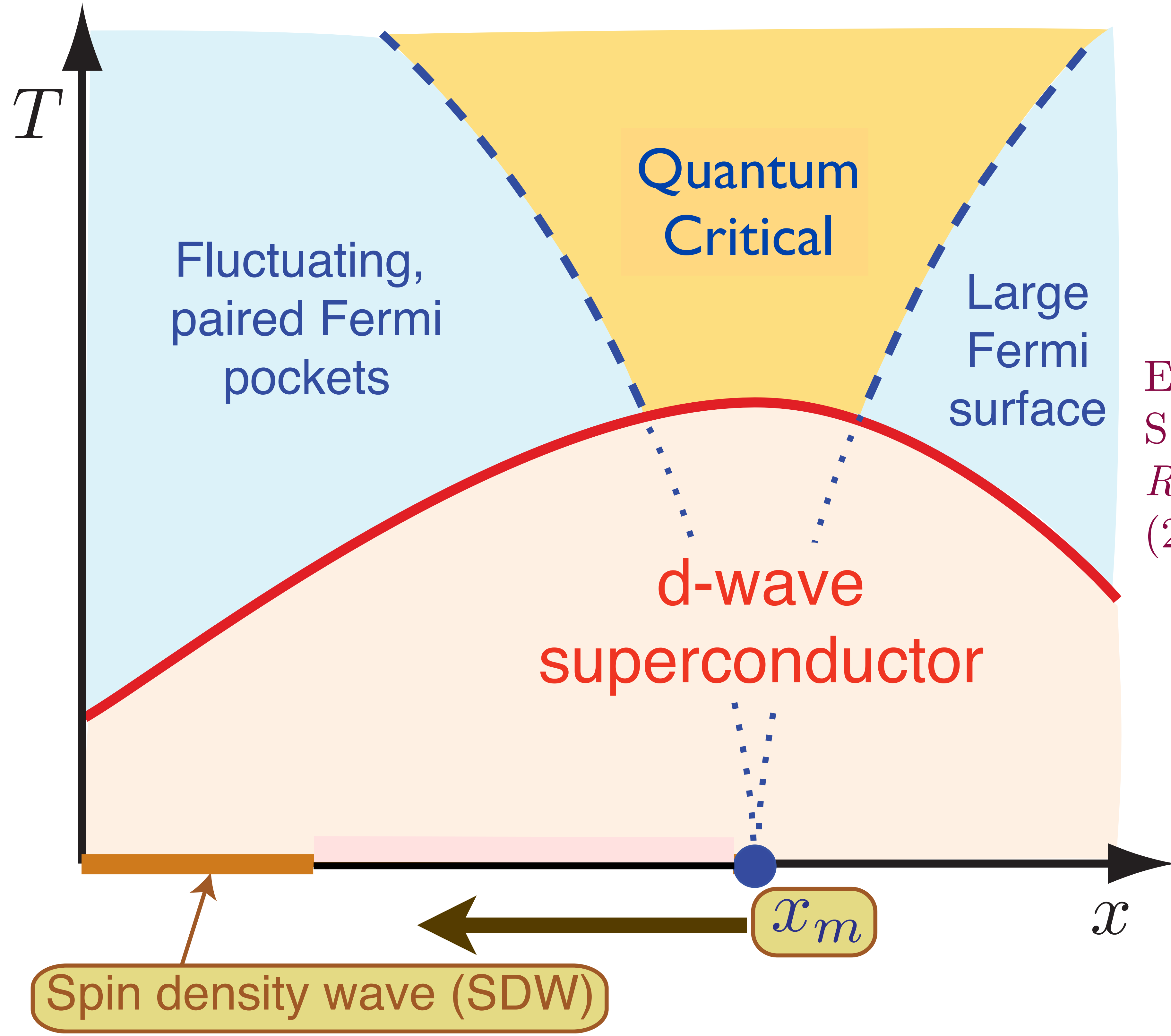

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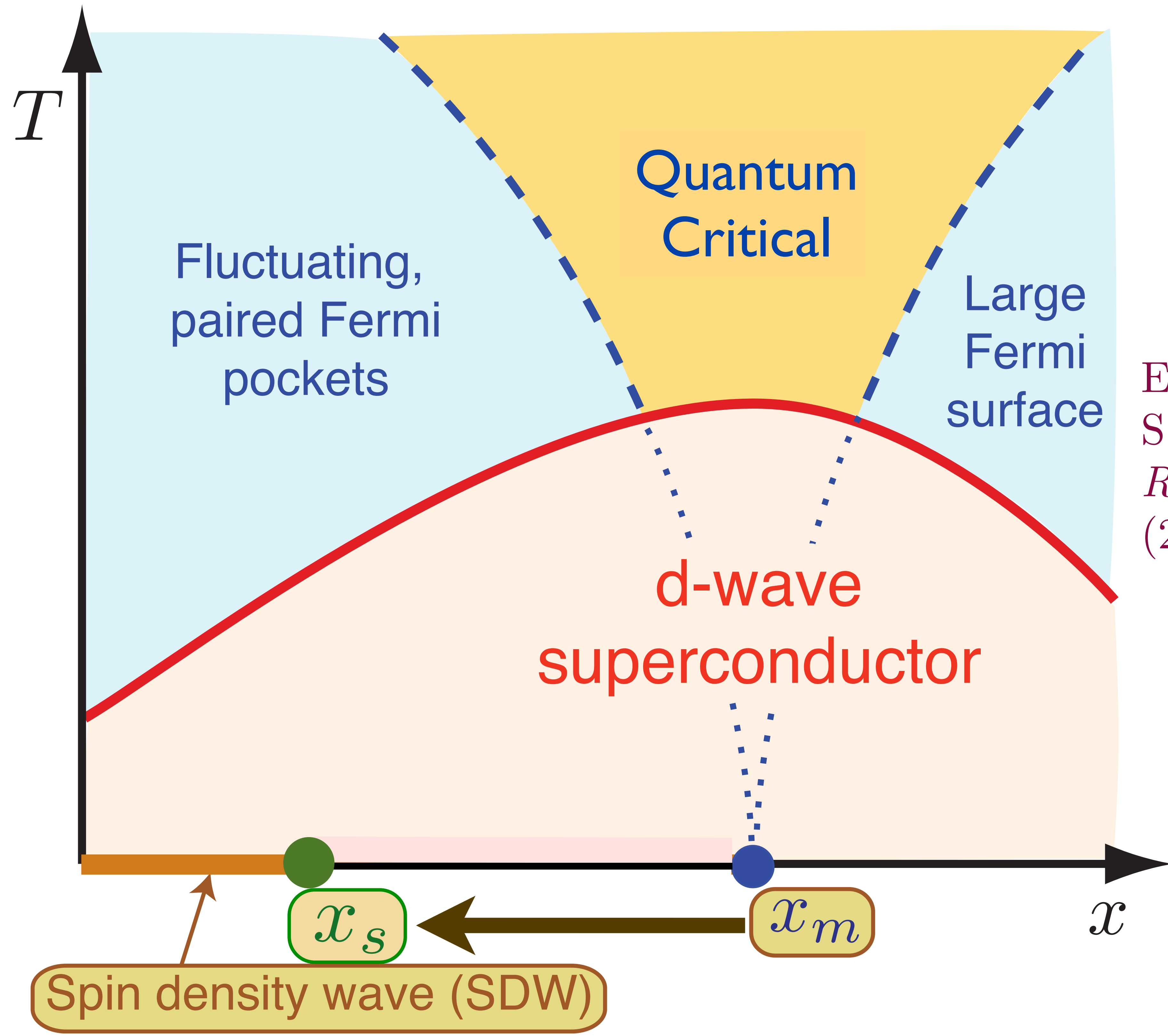

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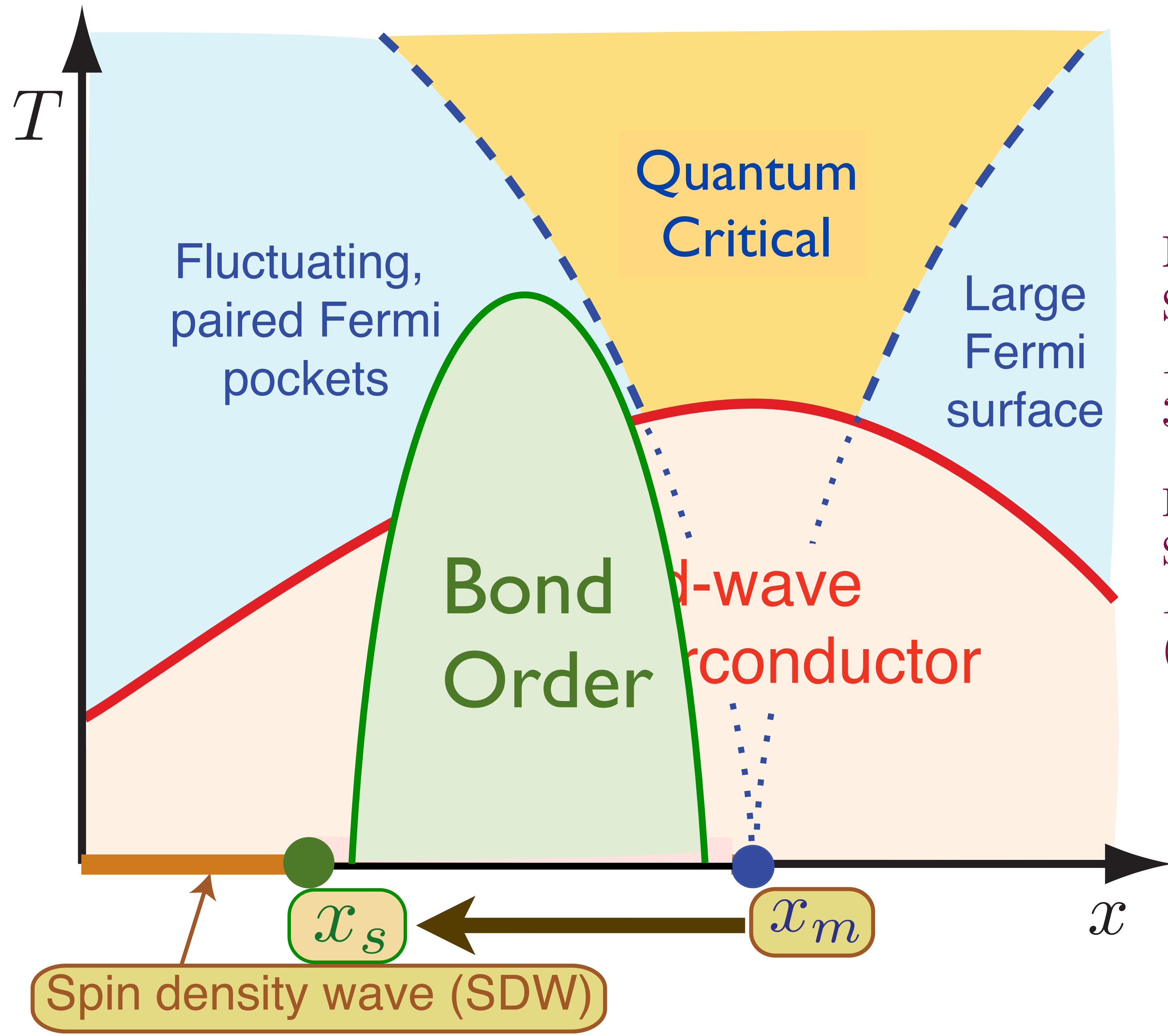


M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999)

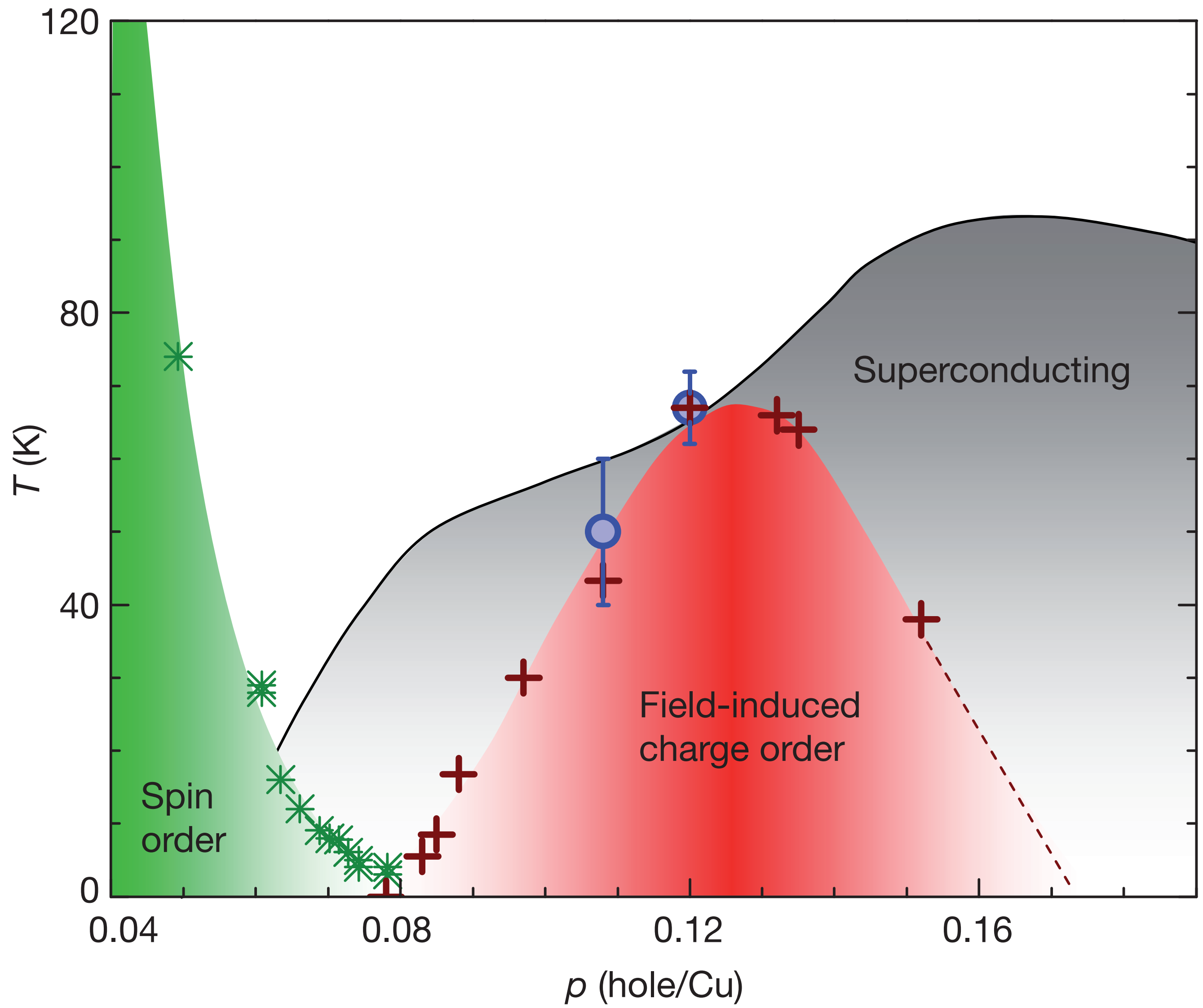
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

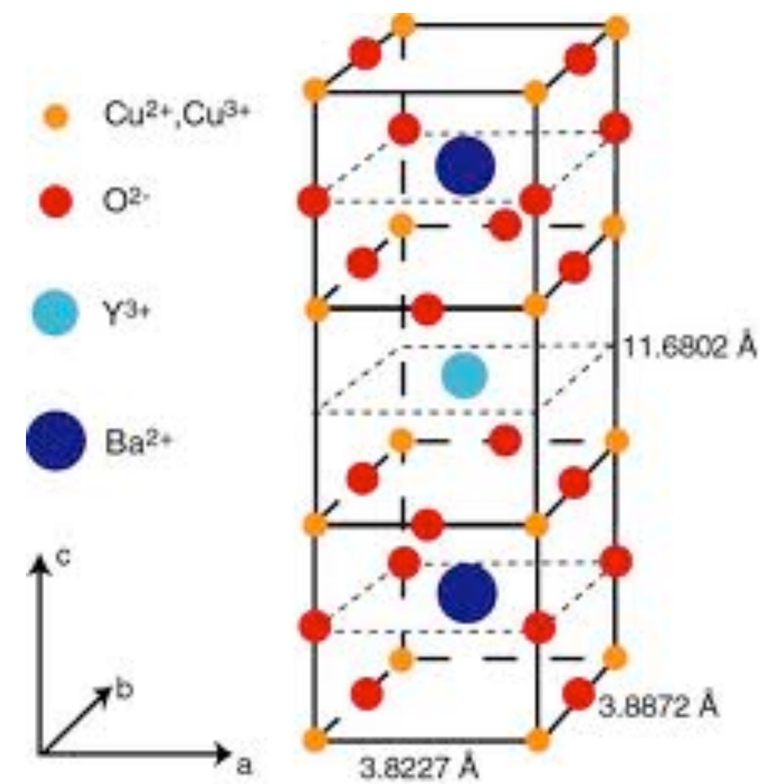
The metal has an instability to *both* d-wave superconductivity and a d-wave charge density wave (bond order).

Magnetic-field-induced charge-stripe order in the high-temperature superconductor $YBa_2Cu_3O_y$

Tao Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Mladen Horvatić[1], Claude Berthier[1], W. N. Hardy[2,3], Ruixing Liang[2,3], D. A. Bonn[2,3] & Marc-Henri Julien[1]

8 SEPTEMBER 2011 | VOL 477 | NATURE | 191





- There is an approximate pseudospin symmetry in metals with antiferromagnetic spin correlations.

- The pseudospin partner of *d*-wave superconductivity is an incommensurate *d*-wave bond order

- These orders form a pseudospin doublet, which is responsible for the "pseudogap" phase.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **85**, 075127 (2010)
T. Holder and W. Metzner, Phys. Rev. B **85**, 165130 (2012)
C. Husemann and W. Metzner, Phys. Rev. B **86**, 085113 (2012)
M. Bejas, A. Greco, and H. Yamase, Phys. Rev. B **86**, 224509 (2012)
K. B. Efetov, H. Meier, and C. Pépin, Nature Physics **9**, 442 (2013).
S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013)

Pseudospin symmetry of the exchange interaction

$$H_J = \sum_{i<j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

with $\vec{S}_i = \frac{1}{2} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c^\dagger_{i\downarrow} \end{pmatrix} \quad , \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c^\dagger_{i\uparrow} \end{pmatrix}$$

Then we can write

$$H_J = \frac{1}{8} \sum_{i<j} J_{ij} \left(\Psi^\dagger_{i\alpha a} \vec{\sigma}_{\alpha\beta} \Psi_{i\beta a} \right) \cdot \left(\Psi^\dagger_{j\gamma b} \vec{\sigma}_{\gamma\delta} \Psi_{j\delta b} \right)$$

where a, b are the Nambu indices. This form makes explicit the symmetry under *independent* SU(2) pseudospin transformations on each site

$$\Psi_{i\alpha a} \to U_{i,ab} \Psi_{i\alpha b}$$

This pseudospin (gauge) symmetry is important in classifying spin liquid ground states of H_J.

I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988)
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Pseudospin symmetry of the exchange interaction

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which is invariant under independent SU(2) pseudospin transformations on each site

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We will find important consequences of the pseudospin symmetry in ordinary metals with antiferromagnetic correlations.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Pseudospin symmetry of the exchange interaction

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The familiar BCS factorization of the exchange interaction leads to

$$H_{BCS} = -\sum_{i,j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + \sum_{i<j} \frac{3J_{ij}}{4} \left[\Delta_{ij} \varepsilon_{\alpha\beta} c_{j\alpha} c_{i\beta} + \Delta^*_{ij} \varepsilon_{\alpha\beta} c^\dagger_{i\alpha} c^\dagger_{j\beta} + 2|\Delta_{ij}|^2 \right]$$

Pseudospin symmetry of the exchange interaction

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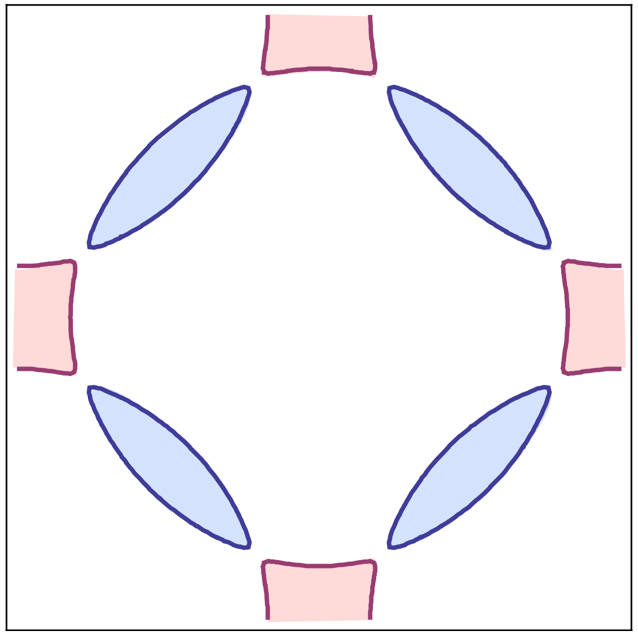
However, we can also factorize in the particle-hole channel, by introducing the *bond* order parameter P_{ij}, and then the complete Hartree-Fock-BCS factorization of the exchange interaction is

$$\begin{aligned} H_{HF-BCS} &= -\sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \\ &+ \sum_{i<j} \frac{3J_{ij}}{4} \left[\Delta_{ij} \varepsilon_{\alpha\beta} c_{j\alpha} c_{i\beta} + \Delta_{ij}^* \varepsilon_{\alpha\beta} c_{i\alpha}^\dagger c_{j\beta}^\dagger + 2|\Delta_{ij}|^2 \right] \\ &+ \sum_{i<j} \frac{3J_{ij}}{4} \left[P_{ij} c_{j\alpha}^\dagger c_{i\alpha} + P_{ij}^* c_{i\alpha}^\dagger c_{j\alpha} + 2|P_{ij}|^2 \right] \end{aligned}$$

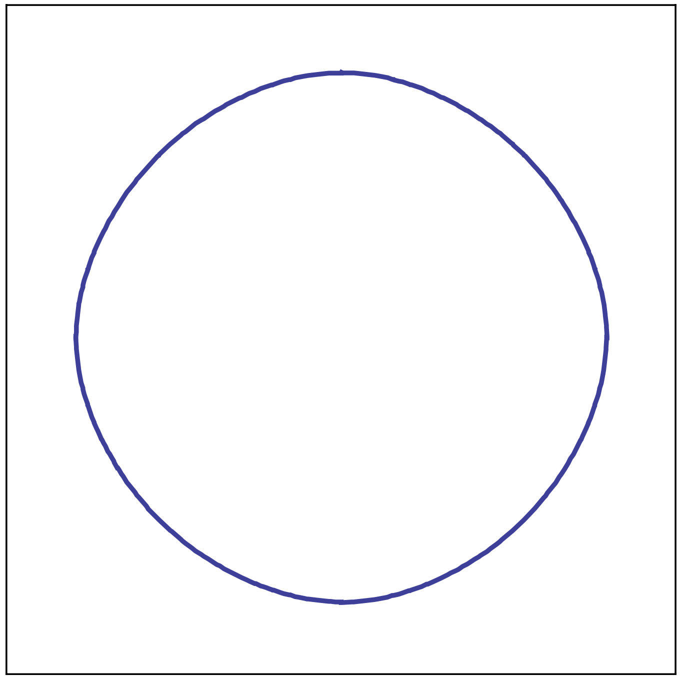
Note that Δ_{ij} and P_{ij} appear with the *same* couplings: this is a manifestation of the SU(2) pseudo-spin symmetry of the exchange interactions. *If* the fermion hopping t_{ij} was pseudo-spin invariant, the energy would depend only on the combination $|P_{ij}|^2 + |\Delta_{ij}|^2$.

Fermi surface+antiferromagnetism



$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets

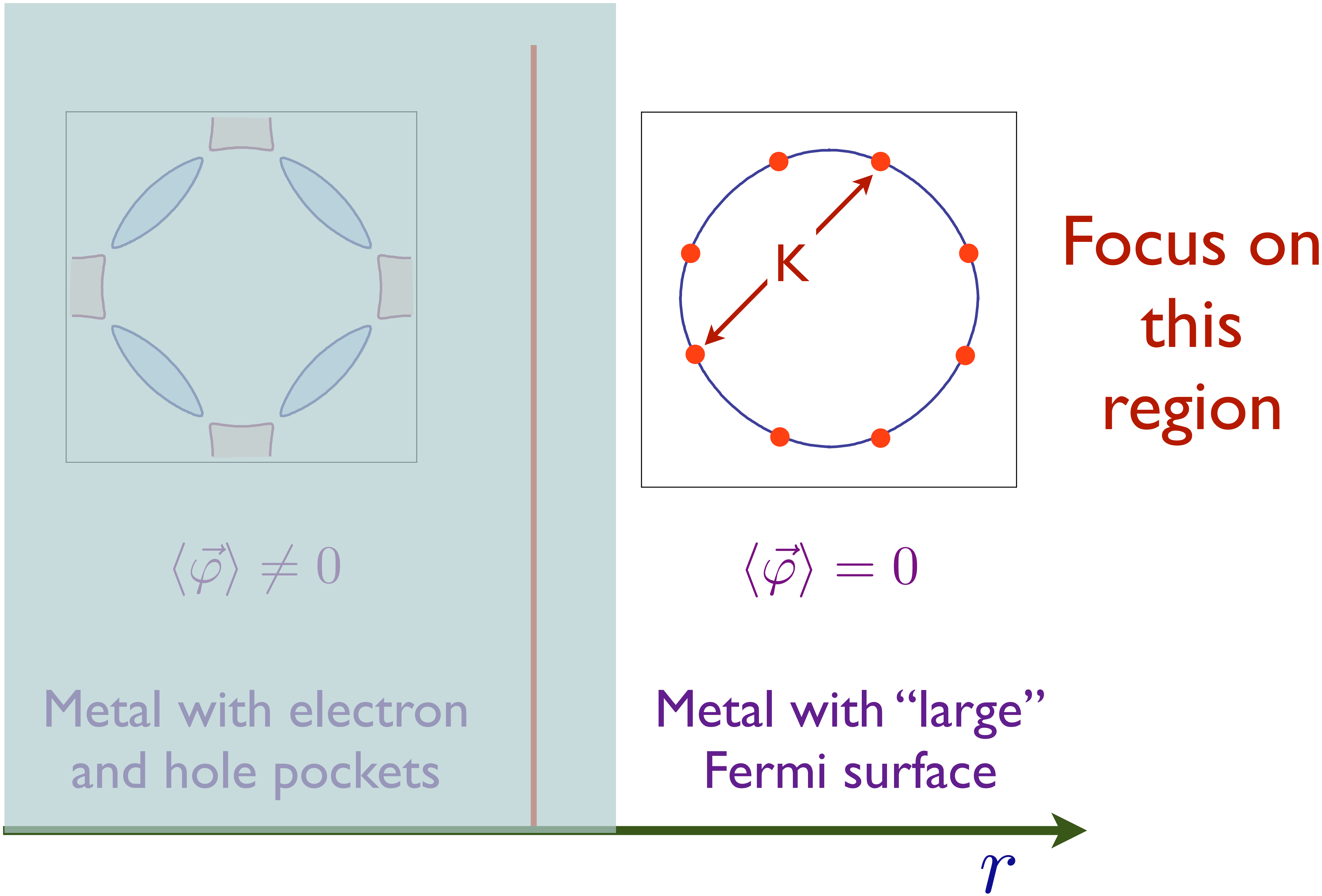


$\langle \vec{\varphi} \rangle = 0$

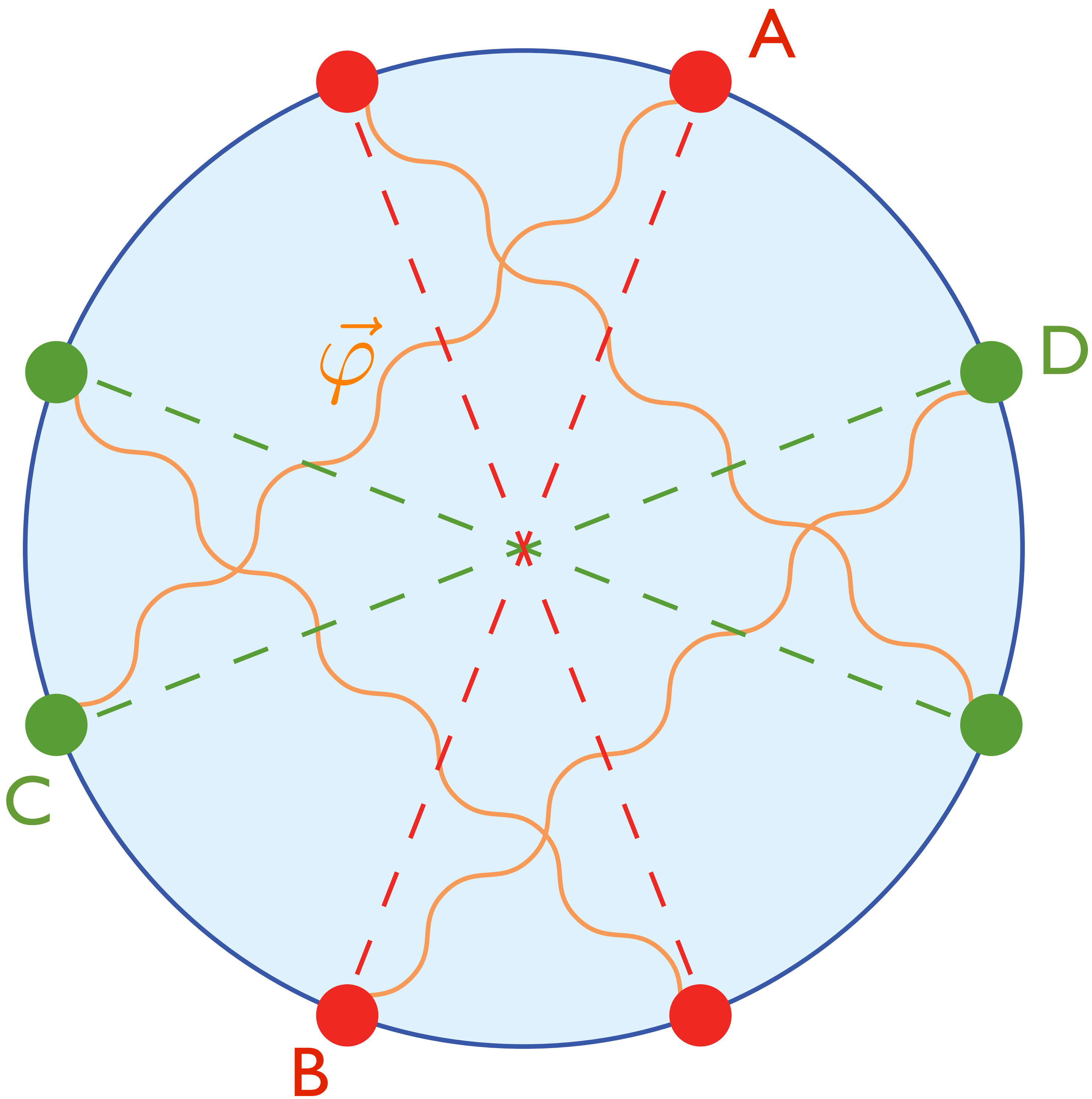
Metal with "large" Fermi surface

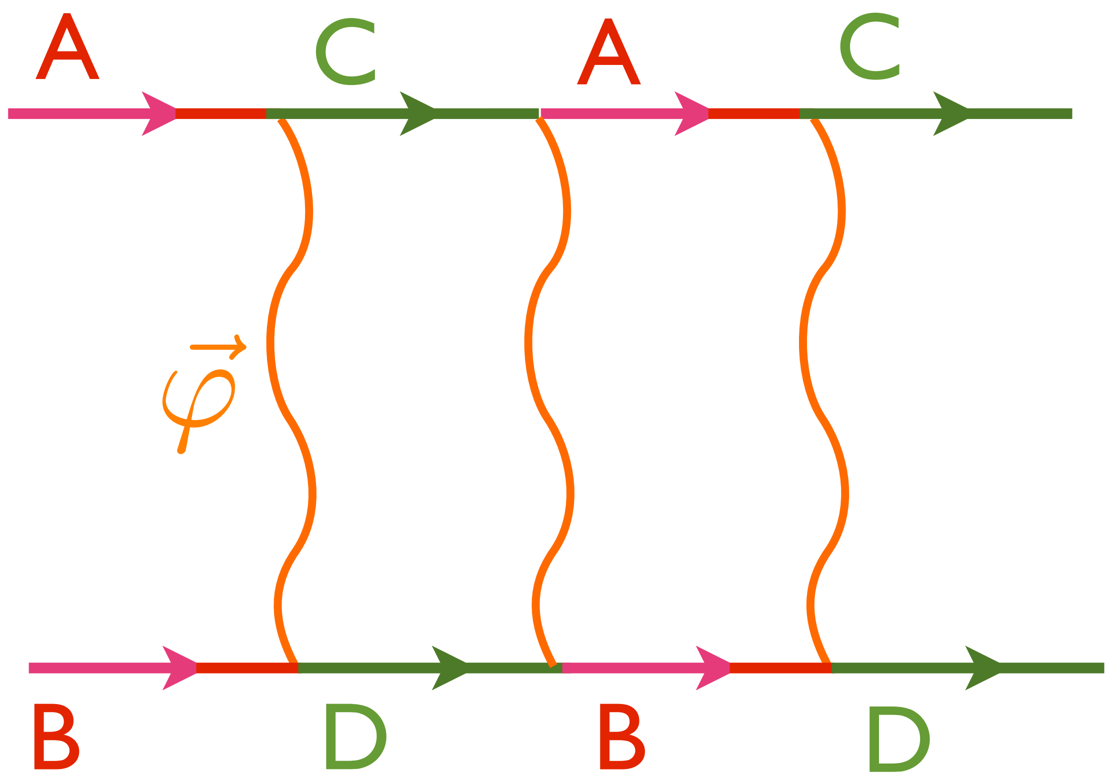
r

Fermi surface+antiferromagnetism



Pairing "glue" from antiferromagnetic fluctuations





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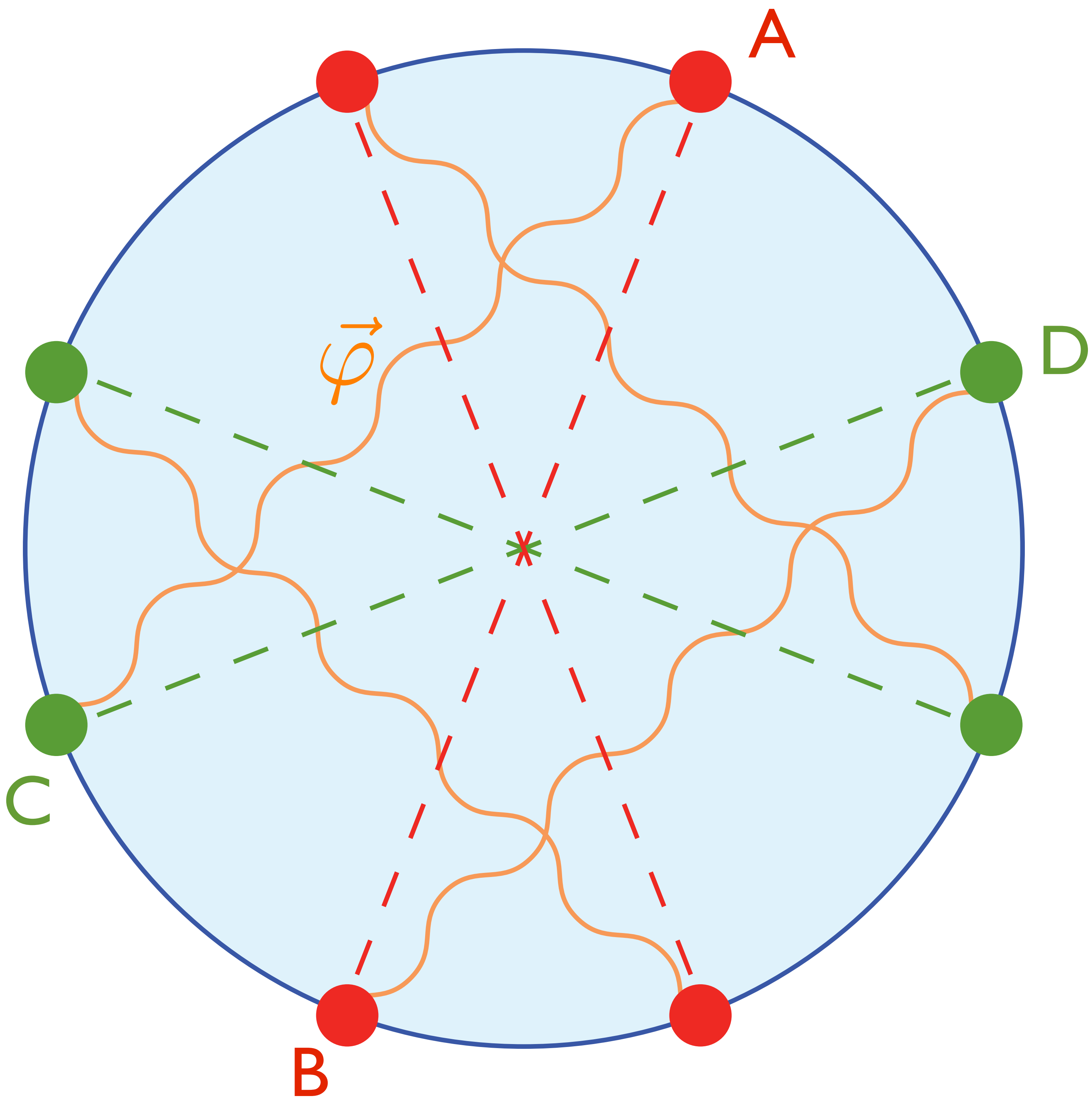
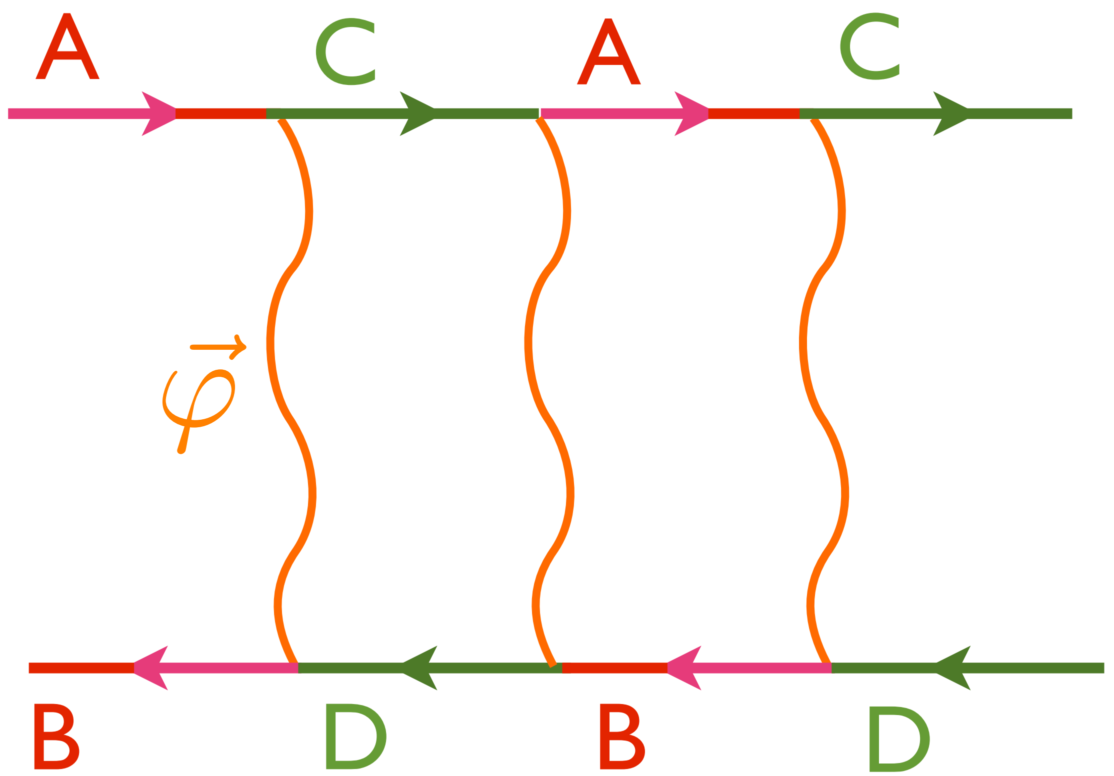
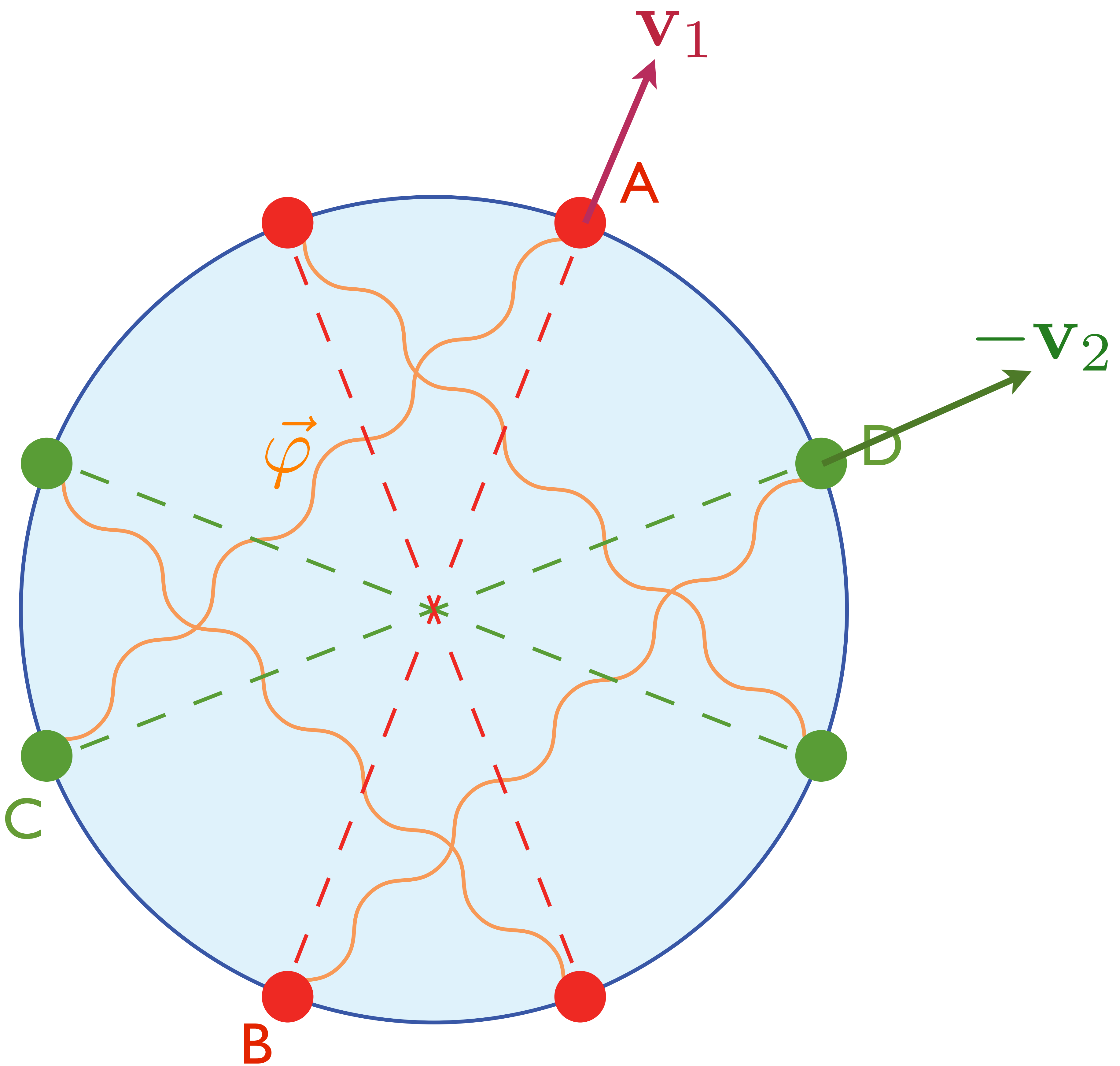
Same "glue" leads to particle-hole pairing





M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



$\mathbf{v}_1$
A
$-\mathbf{v}_2$
D
$\vec{\varphi}$
C
B

Hamiltonian at and *near* hot spots

$$H = \sum_{\mathbf{k}} \Bigg[\left[\mathbf{v}_1 \cdot \mathbf{k} \qquad \right] \Psi^\dagger_{A\alpha}(\mathbf{k}) \Psi_{A\alpha}(\mathbf{k})$$

$$+ \left[\mathbf{v}_2 \cdot \mathbf{k} \qquad \right] \Psi^\dagger_{C\alpha}(\mathbf{k}) \Psi_{C\alpha}(\mathbf{k})$$

$$+ \left[-\mathbf{v}_1 \cdot \mathbf{k} \qquad \right] \Psi^\dagger_{B\alpha}(\mathbf{k}) \Psi_{B\alpha}(\mathbf{k})$$

$$+ \left[-\mathbf{v}_2 \cdot \mathbf{k} \qquad \right] \Psi^\dagger_{D\alpha}(\mathbf{k}) \Psi_{D\alpha}(\mathbf{k})$$

$$+ \int d^2x \Bigg[-J \left(\Psi^\dagger_{A\alpha} \vec{\sigma}_{\alpha\beta} \Psi_{C\beta} + \Psi^\dagger_{C\alpha} \vec{\sigma}_{\alpha\beta} \Psi_{A\beta} \right)$$

$$\cdot \left(\Psi^\dagger_{B\gamma} \vec{\sigma}_{\gamma\delta} \Psi_{D\delta} + \Psi^\dagger_{D\gamma} \vec{\sigma}_{\gamma\delta} \Psi_{B\delta} \right)$$

$$\Bigg]$$

Hamiltonian at and *near* hot spots

$$H = \sum_{\mathbf{k}} \Big[\left[\mathbf{v}_1 \cdot \mathbf{k} \right] \Psi^\dagger_{A\alpha}(\mathbf{k}) \Psi_{A\alpha}(\mathbf{k})$$

$$+ \left[\mathbf{v}_2 \cdot \mathbf{k} \right] \Psi^\dagger_{C\alpha}(\mathbf{k}) \Psi_{C\alpha}(\mathbf{k})$$

$$+ \left[-\mathbf{v}_1 \cdot \mathbf{k} \right] \Psi^\dagger_{B\alpha}(\mathbf{k}) \Psi_{B\alpha}(\mathbf{k})$$

$$+ \left[-\mathbf{v}_2 \cdot \mathbf{k} \right] \Psi^\dagger_{D\alpha}(\mathbf{k}) \Psi_{D\alpha}(\mathbf{k})$$

$$+ \int d^2x \Big[-J \left(\Psi^\dagger_{A\alpha} \vec{\sigma}_{\alpha\beta} \Psi_{C\beta} + \Psi^\dagger_{C\alpha} \vec{\sigma}_{\alpha\beta} \Psi_{A\beta} \right)$$

$$\cdot \left(\Psi^\dagger_{B\gamma} \vec{\sigma}_{\gamma\delta} \Psi_{D\delta} + \Psi^\dagger_{D\gamma} \vec{\sigma}_{\gamma\delta} \Psi_{B\delta} \right) \Big]$$

This Hamiltonian has an exact SU(2)xSU(2) pseudospin symmetry !

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Hamiltonian at and *near* hot spots

$$H = \sum_{\mathbf{k}} \Big[[\mathbf{v}_1 \cdot \mathbf{k} \qquad] \Psi^\dagger_{A\alpha}(\mathbf{k}) \Psi_{A\alpha}(\mathbf{k})$$

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$$+ [-\mathbf{v}_2 \cdot \mathbf{k} \qquad] \Psi^\dagger_{D\alpha}(\mathbf{k}) \Psi_{D\alpha}(\mathbf{k})$$

$$+ \int d^2x \Big[-J \left(\Psi^\dagger_{A\alpha} \vec{\sigma}_{\alpha\beta} \Psi_{C\beta} + \Psi^\dagger_{C\alpha} \vec{\sigma}_{\alpha\beta} \Psi_{A\beta} \right)$$

$$\cdot \left(\Psi^\dagger_{B\gamma} \vec{\sigma}_{\gamma\delta} \Psi_{D\delta} + \Psi^\dagger_{D\gamma} \vec{\sigma}_{\gamma\delta} \Psi_{B\delta} \right) \Big]$$

This Hamiltonian has an exact SU(2)xSU(2) pseudospin symmetry !

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Perform standard Hartree-Fock-BCS factorizations into

$$\Delta_S = \langle \varepsilon_{\alpha\beta} \Psi_{A\alpha} \Psi_{B\beta} \rangle = - \langle \varepsilon_{\alpha\beta} \Psi_{C\alpha} \Psi_{D\beta} \rangle$$

$$P_Q = \left\langle \Psi^\dagger_{A\alpha} \Psi_{B\alpha} \right\rangle = - \left\langle \Psi^\dagger_{C\alpha} \Psi_{D\alpha} \right\rangle$$

With pseudospin symmetry, energy depends only on $|\Delta_S|^2 + |P_Q|^2$.

Hamiltonian at and *near* hot spots

$$H = \sum_{\mathbf{k}} \Big[\left[\mathbf{v}_1 \cdot \mathbf{k} + \alpha(\mathbf{v}_1 \times \mathbf{k})^2\right] \Psi^\dagger_{A\alpha}(\mathbf{k})\Psi_{A\alpha}(\mathbf{k})$$

$$+ \left[\mathbf{v}_2 \cdot \mathbf{k} + \alpha(\mathbf{v}_2 \times \mathbf{k})^2\right] \Psi^\dagger_{C\alpha}(\mathbf{k})\Psi_{C\alpha}(\mathbf{k})$$

$$+ \left[-\mathbf{v}_1 \cdot \mathbf{k} + \alpha(\mathbf{v}_1 \times \mathbf{k})^2\right] \Psi^\dagger_{B\alpha}(\mathbf{k})\Psi_{B\alpha}(\mathbf{k})$$

$$+ \left[-\mathbf{v}_2 \cdot \mathbf{k} + \alpha(\mathbf{v}_2 \times \mathbf{k})^2\right] \Psi^\dagger_{D\alpha}(\mathbf{k})\Psi_{D\alpha}(\mathbf{k})$$

$$+ \int d^2x \Big[-J \left(\Psi^\dagger_{A\alpha} \vec{\sigma}_{\alpha\beta} \Psi_{C\beta} + \Psi^\dagger_{C\alpha} \vec{\sigma}_{\alpha\beta} \Psi_{A\beta} \right)$$

$$\cdot \left(\Psi^\dagger_{B\gamma} \vec{\sigma}_{\gamma\delta} \Psi_{D\delta} + \Psi^\dagger_{D\gamma} \vec{\sigma}_{\gamma\delta} \Psi_{B\delta} \right) \Big]$$

Fermi surface curvature; breaks pseudospin symmetry

Perform standard Hartree-Fock-BCS factorizations into

$$\Delta_S = \langle \varepsilon_{\alpha\beta} \Psi_{A\alpha} \Psi_{B\beta} \rangle = - \langle \varepsilon_{\alpha\beta} \Psi_{C\alpha} \Psi_{D\beta} \rangle$$

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With pseudospin symmetry, energy depends only on $|\Delta_S|^2 + |P_Q|^2$.

Hamiltonian at and *near* hot spots

$$H = \sum_{\mathbf{k}} \Big[\left[\mathbf{v}_1 \cdot \mathbf{k} + \alpha(\mathbf{v}_1 \times \mathbf{k})^2\right] \Psi^\dagger_{A\alpha}(\mathbf{k})\Psi_{A\alpha}(\mathbf{k})$$

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$$\cdot \left(\Psi^\dagger_{B\gamma} \vec{\sigma}_{\gamma\delta} \Psi_{D\delta} + \Psi^\dagger_{D\gamma} \vec{\sigma}_{\gamma\delta} \Psi_{B\delta} \right)$$

$$-V \left(\Psi^\dagger_{A\alpha} \Psi_{C\alpha} + \Psi^\dagger_{C\alpha} \Psi_{A\alpha} \right) \left(\Psi^\dagger_{B\beta} \Psi_{D\beta} + \Psi^\dagger_{D\beta} \Psi_{B\beta} \right) \Big]$$

Fermi surface curvature; breaks pseudospin symmetry

Coulomb repulsion; breaks pseudospin symmetry

Perform standard Hartree-Fock-BCS factorizations into

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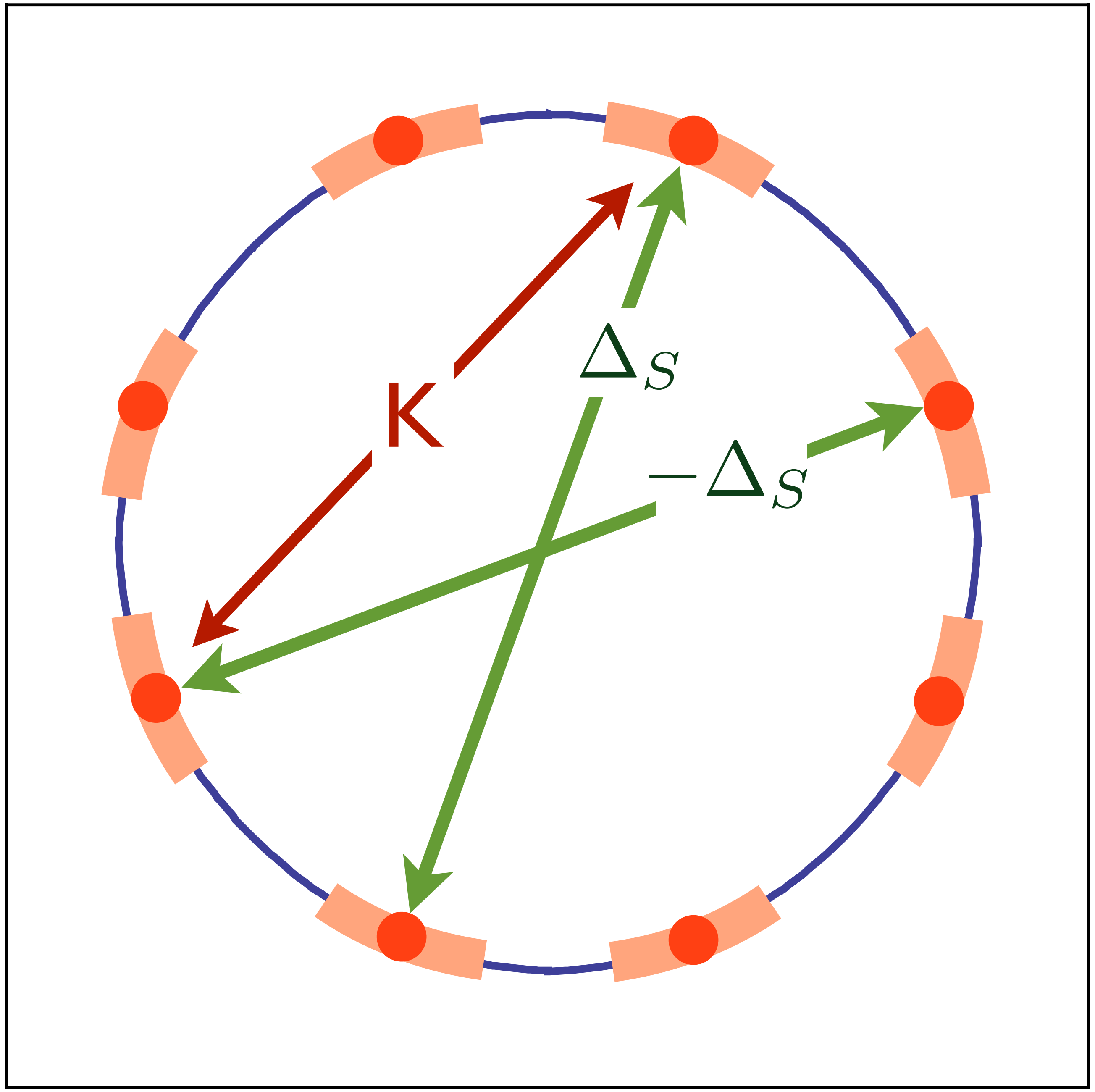
$$P_Q = \left\langle \Psi^\dagger_{A\alpha} \Psi_{B\alpha} \right\rangle = - \left\langle \Psi^\dagger_{C\alpha} \Psi_{D\alpha} \right\rangle$$

With pseudospin symmetry, energy depends only on $|\Delta_S|^2 + |P_Q|^2$.

$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y)$$

V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

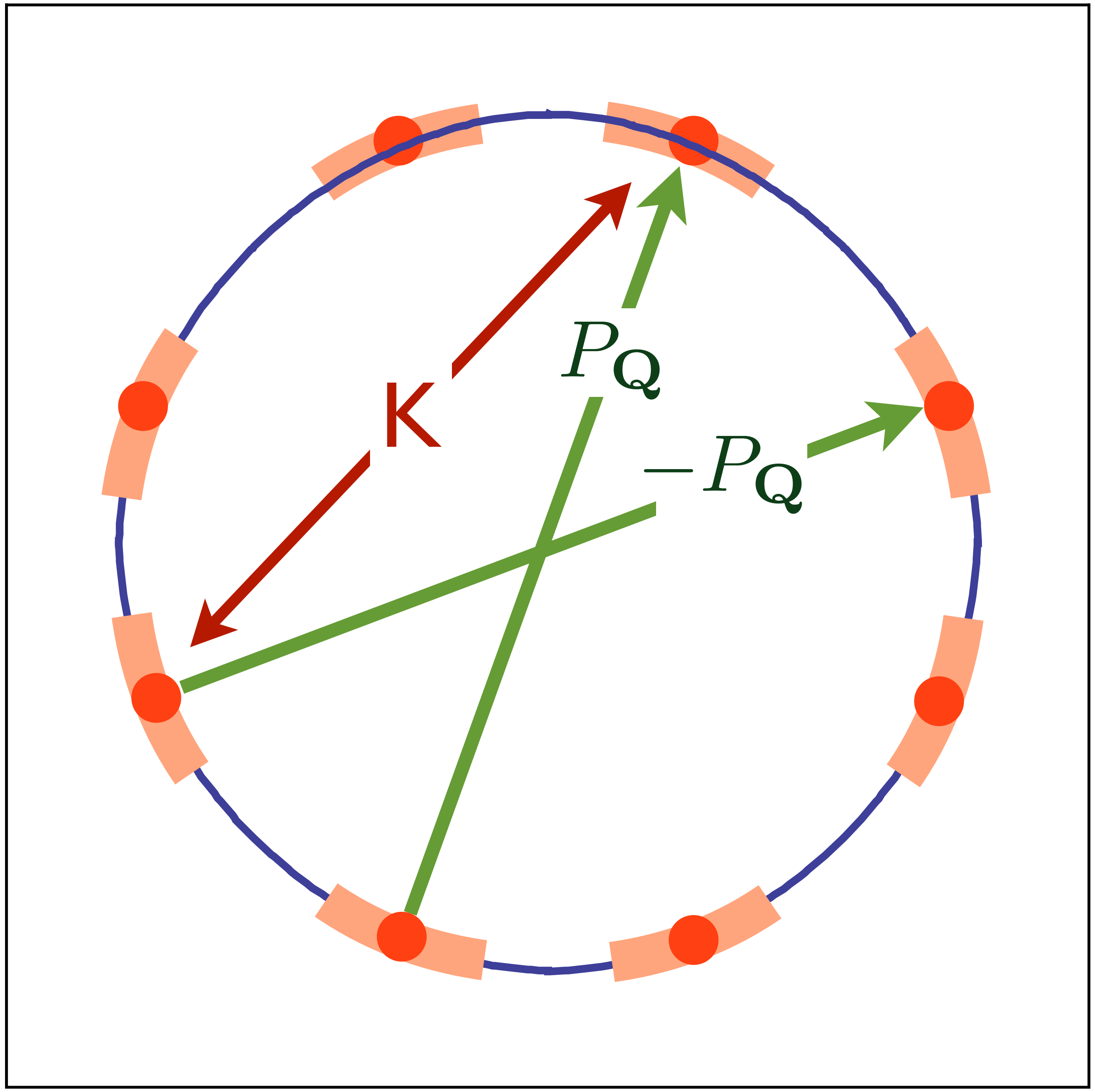


d-wave superconductor: particle-particle pairing at *and near* hot spots, with sign-changing pairing amplitude

$$\left\langle c^{\dagger}_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = P_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

After pseudospin rotation on *half* the hot-spots

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

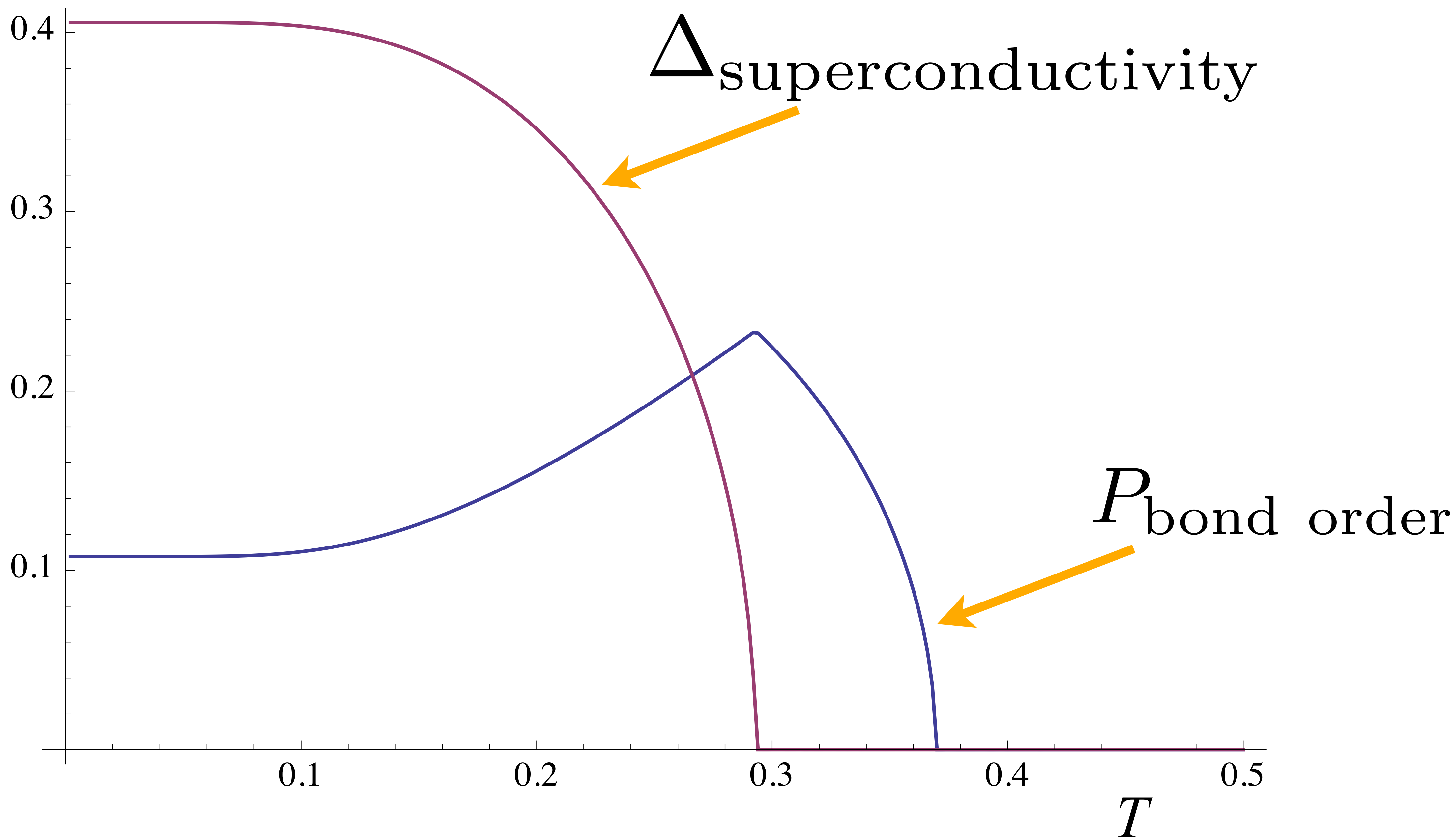


$\mathbf{Q}$ is '$2k_F$' wavevector

Incommensurate d-wave bond order: particle-hole pairing at *and near* hot spots, with sign-changing pairing amplitude

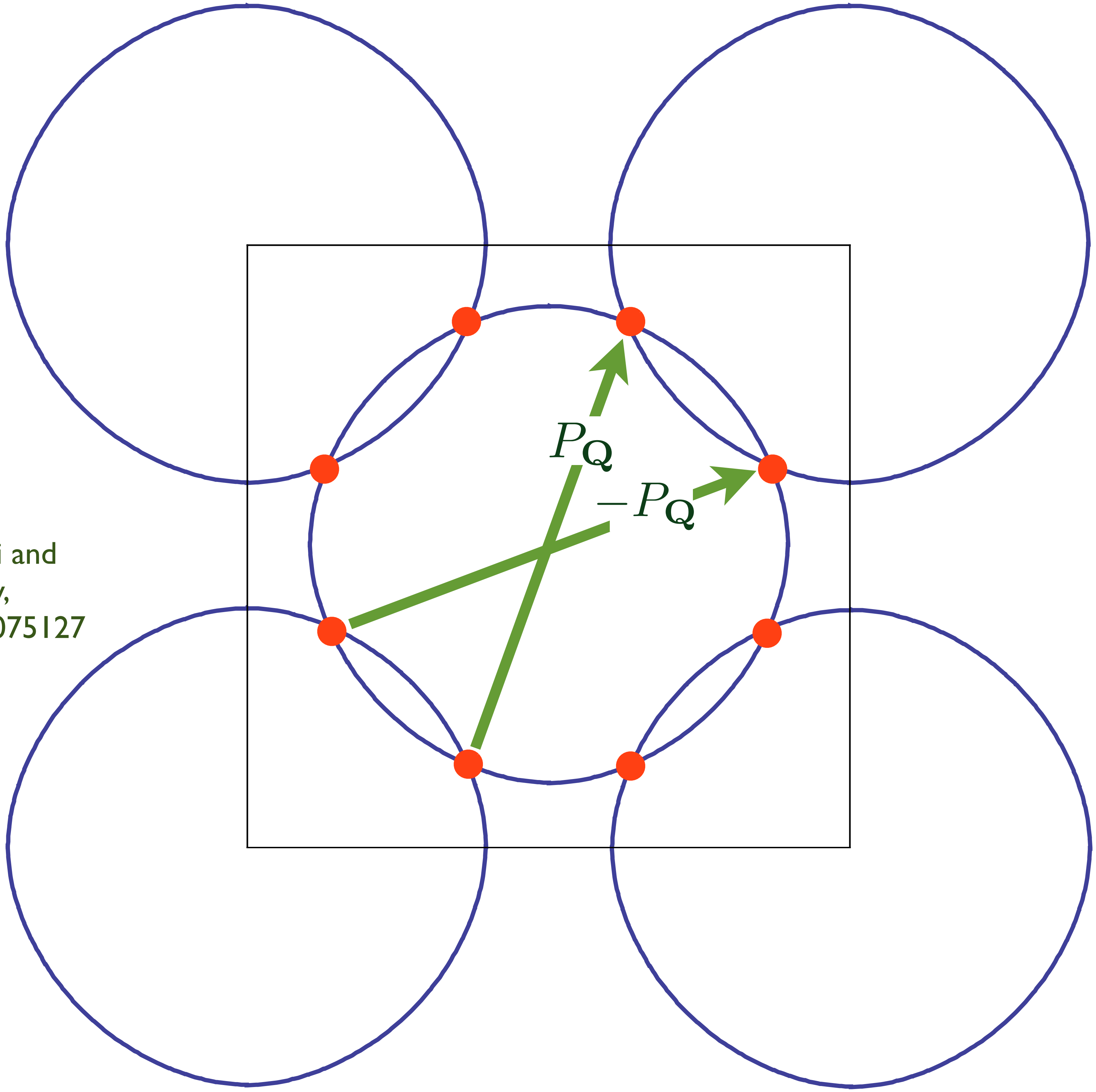
Mean-field theory of charge/bond order and *d*-wave superconductivity

Both orders are induced by a "glue" provided by the antiferromagnetic exchange interaction



S. Sachdev et al., to appear

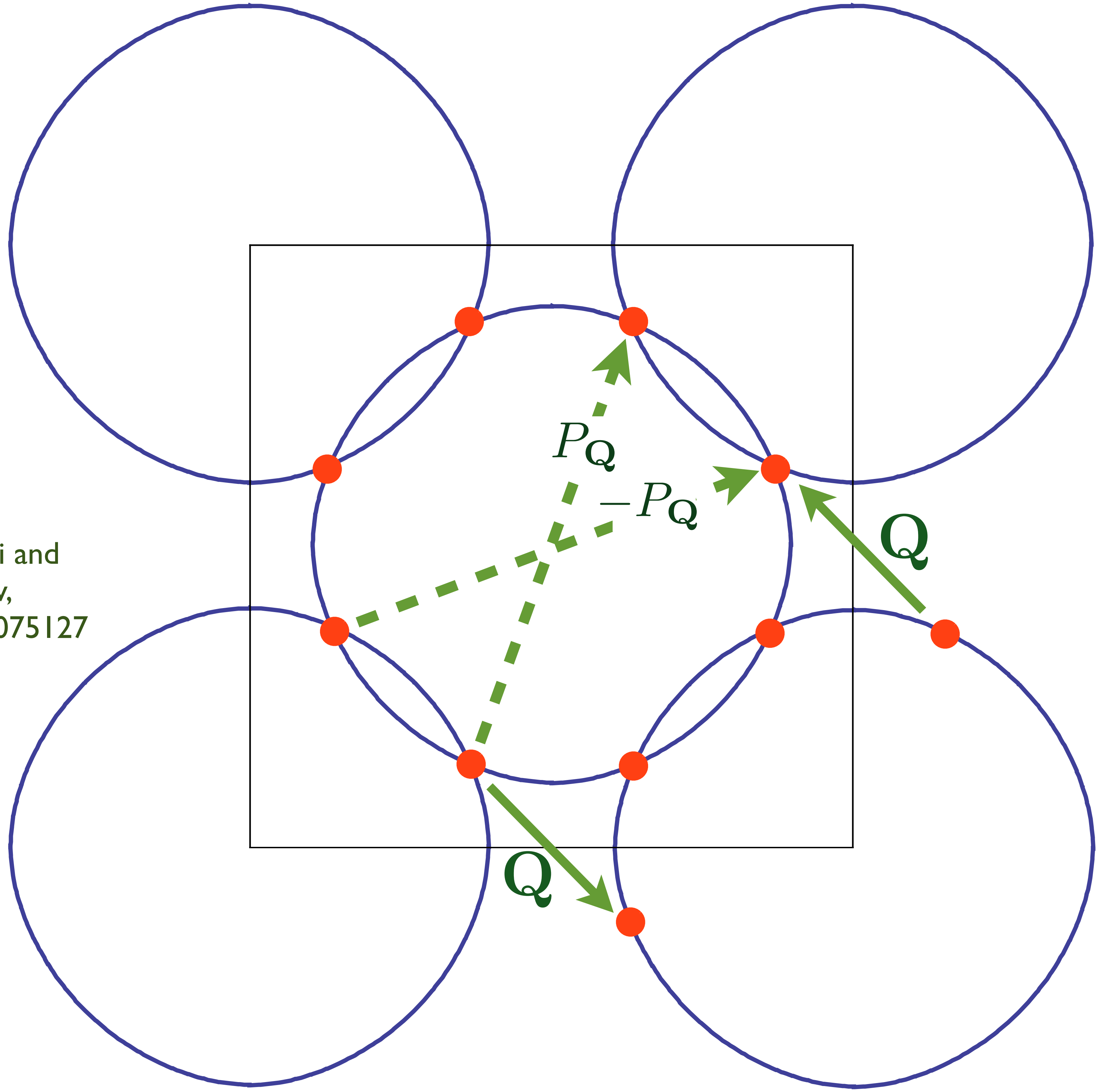
Incommensurate *d*-wave bond order



M.A. Metlitski and
S. Sachdev,
Phys. Rev. B **85**, 075127
(2010)

$$\left\langle c^{\dagger}_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

Incommensurate *d*-wave bond order



M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

Particle-hole instabilities of the full Fermi surface

$$H_{tJ} = -\sum_{i,j} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + \sum_{i<j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Optimize the free energy w.r.t. a mean field Hamiltonian which allows for spin-singlet charge order ($P_{\mathbf{Q}}(\mathbf{k})$):

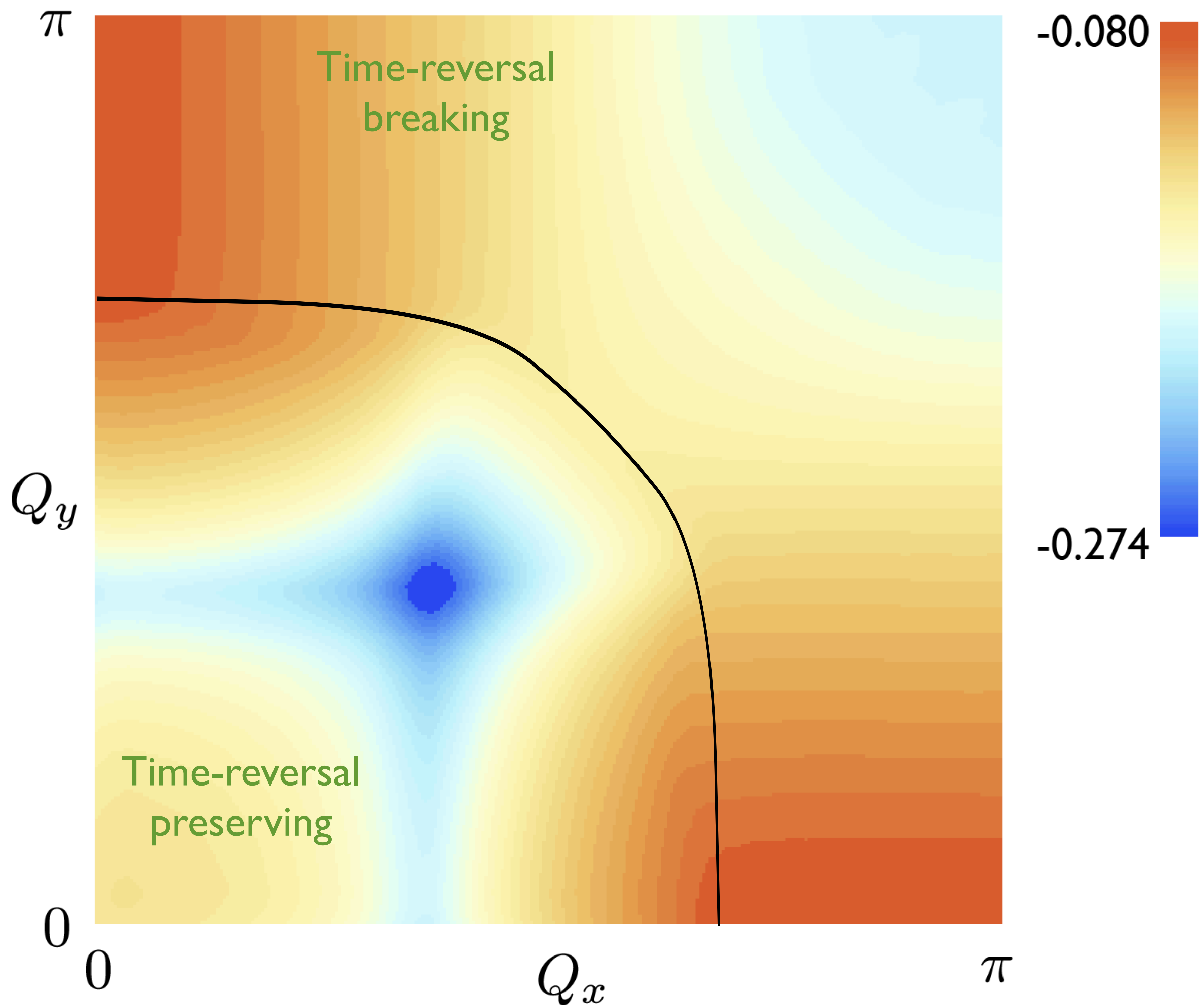
$$H_{MF} = -\sum_{i,j} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + \sum_{\mathbf{k},\mathbf{Q}} P_{\mathbf{Q}}(\mathbf{k}) c_{\mathbf{k}+\mathbf{Q}/2,\alpha}^{\dagger} c_{\mathbf{k}-\mathbf{Q}/2,\alpha}$$

Expanding the free energy in powers of the order parameters we obtain

$$F = F_0 + \sum_{\mathbf{k},\mathbf{Q}} P_{\mathbf{Q}}^{*}(\mathbf{k}) \mathcal{M}_{\mathbf{Q}}(\mathbf{k},\mathbf{k}') P_{\mathbf{Q}}(\mathbf{k}')$$

We compute the eigenvalues, $1+\lambda_{\mathbf{Q}}$, and eigenfunctions, $P_{\mathbf{Q}}(\mathbf{k})$ of the kernel $\mathcal{M}_{\mathbf{Q}}(\mathbf{k},\mathbf{k}')$

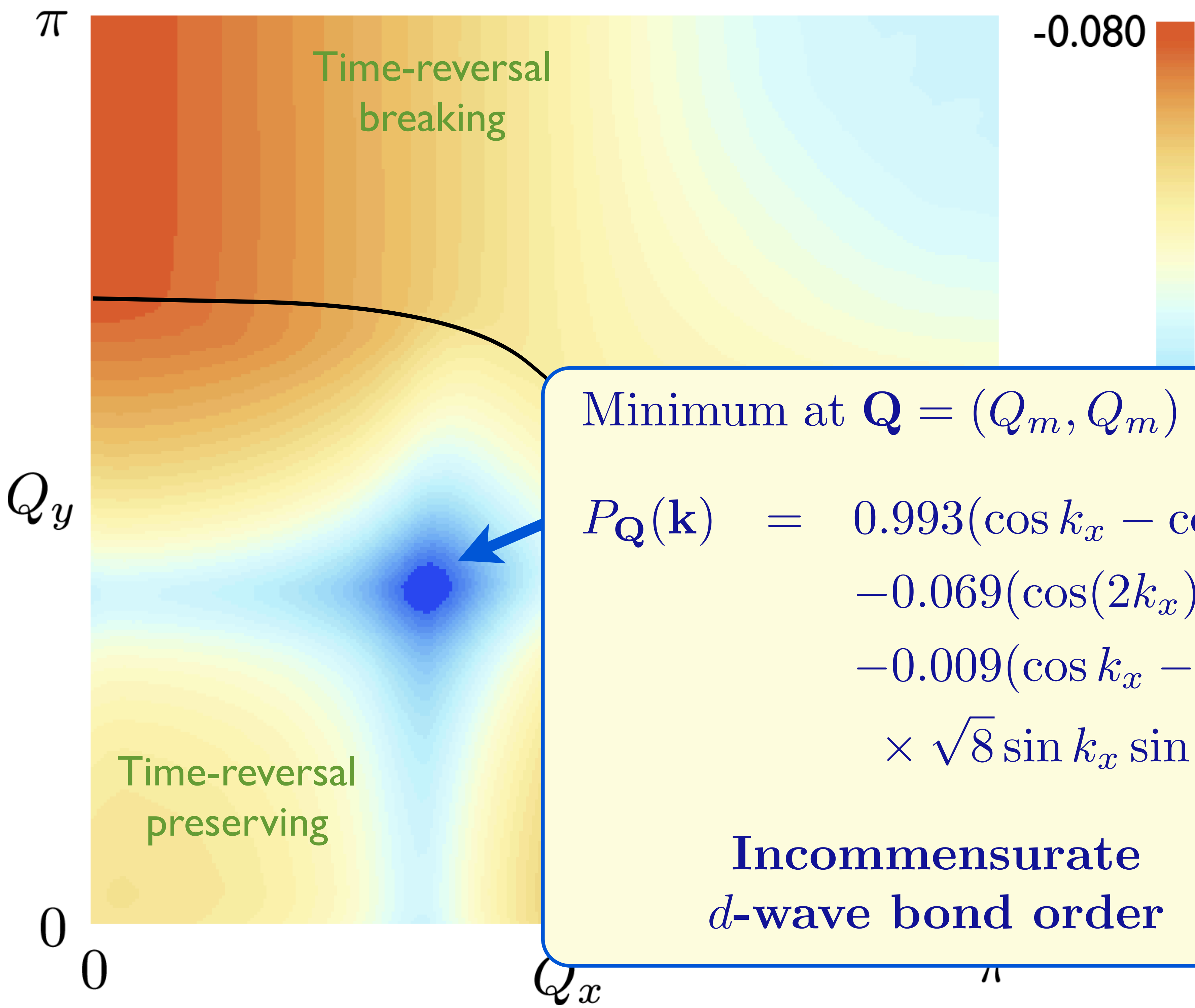
S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013)



Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$ of particle-hole propagator. The corresponding eigenvector is $P_{\mathbf{Q}}(\mathbf{k})$ and this leads to the order

$$\left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle = \left[\sum_{\mathbf{k}} P_{\mathbf{Q}}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$$

S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013)



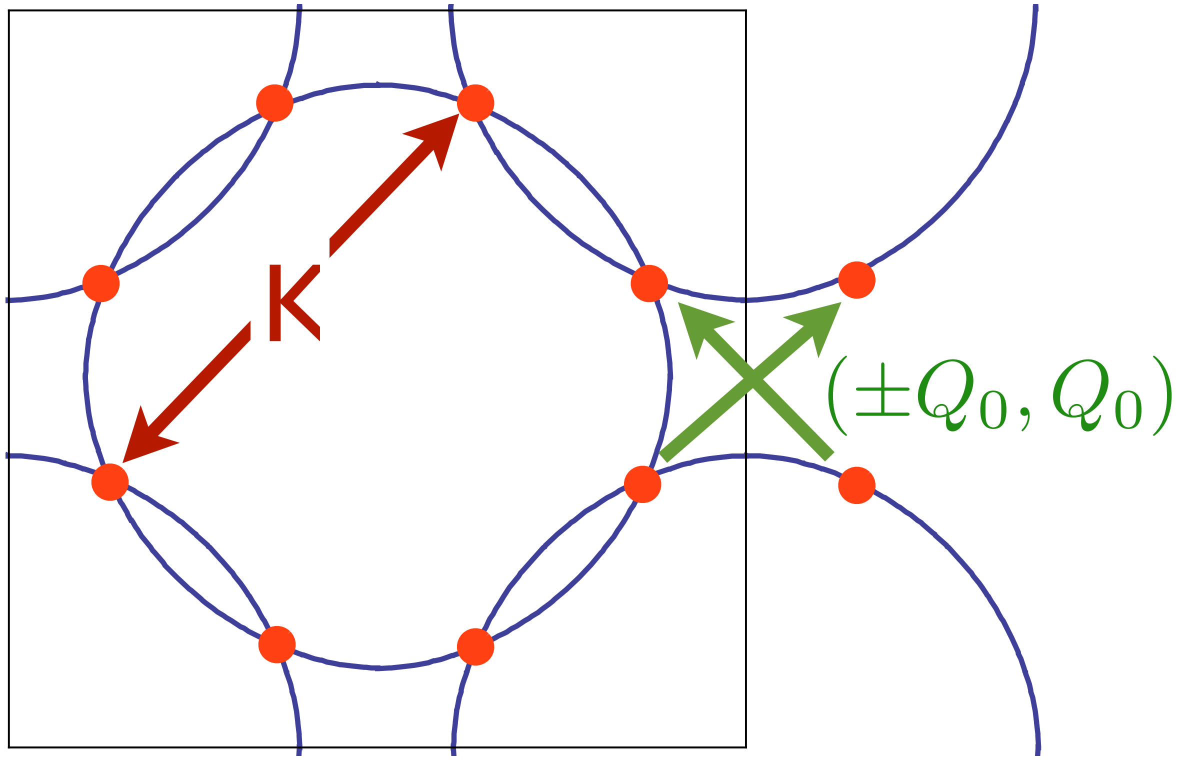
Minimum at $\mathbf{Q} = (Q_m, Q_m)$ with

$$
\begin{aligned}
P_{\mathbf{Q}}(\mathbf{k}) \quad = \quad & 0.993(\cos k_x - \cos k_y) \\
& -0.069(\cos(2k_x) - \cos(2k_y)) \\
& -0.009(\cos k_x - \cos k_y) \\
& \times \sqrt{8} \sin k_x \sin k_y
\end{aligned}
$$

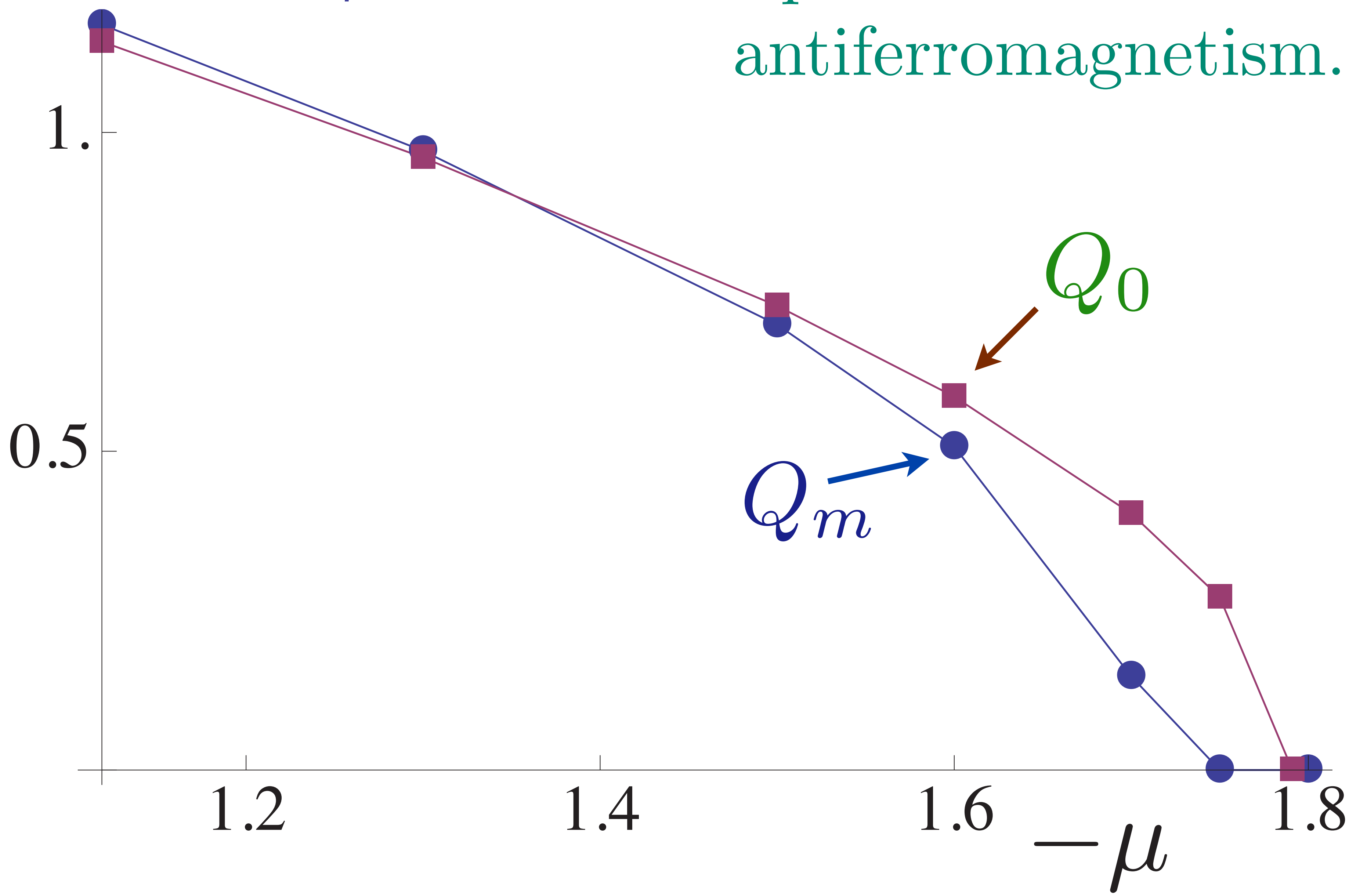
Incommensurate
***d*-wave bond order**

Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$ of particle-hole propagator.
The corresponding eigenvector is $P_{\mathbf{Q}}(\mathbf{k})$ and this leads to the order

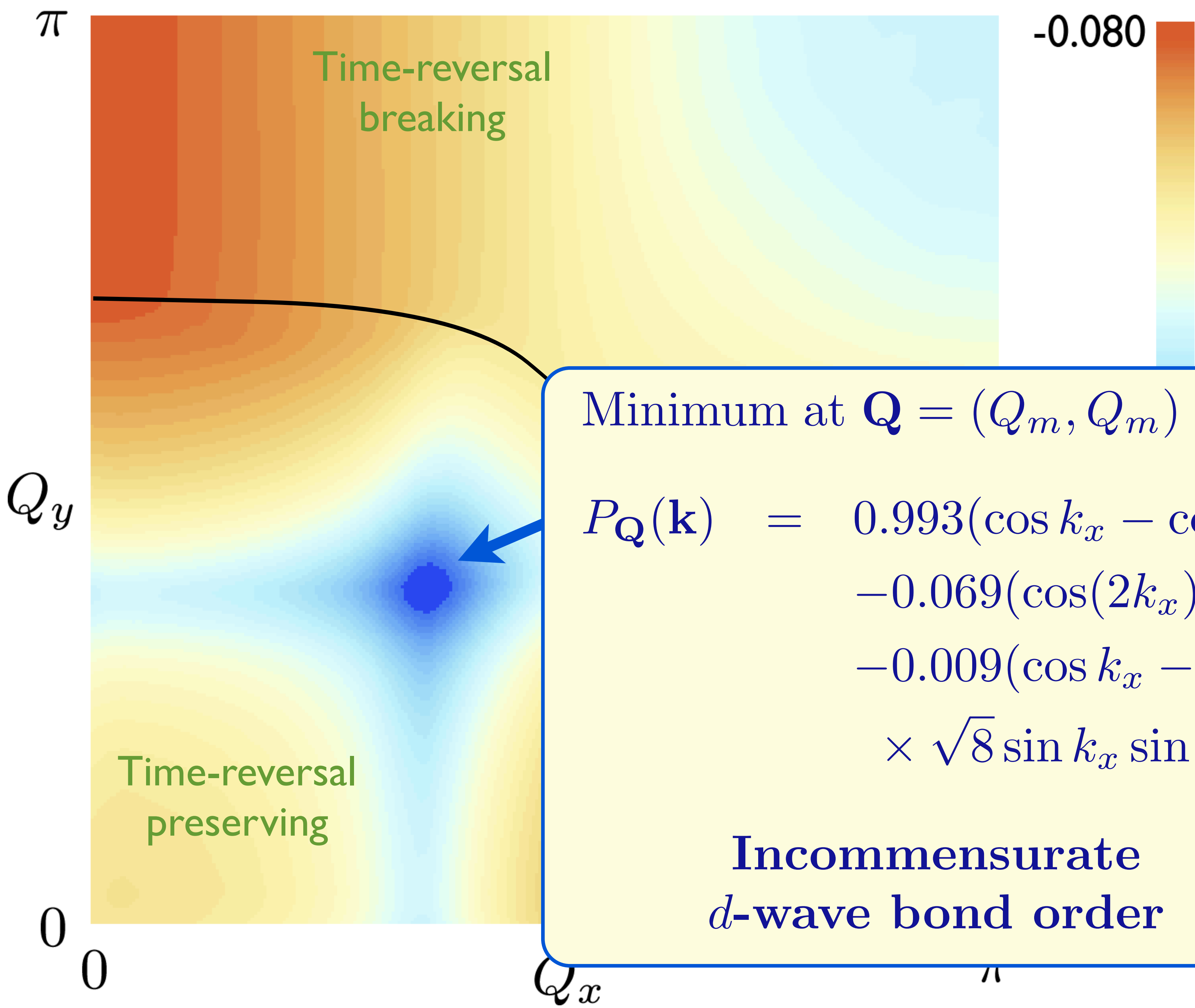
$$\left\langle c_{i\alpha}^\dagger c_{j\alpha} \right\rangle = \left[\sum_{\mathbf{k}} P_{\mathbf{Q}}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$$



Remarkable agreement between the value of Q_m from Hartree-Fock in a metal with short-range *incommensurate* spin correlations, and the value of Q_0 from hot spots of *commensurate* antiferromagnetism.



S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013)



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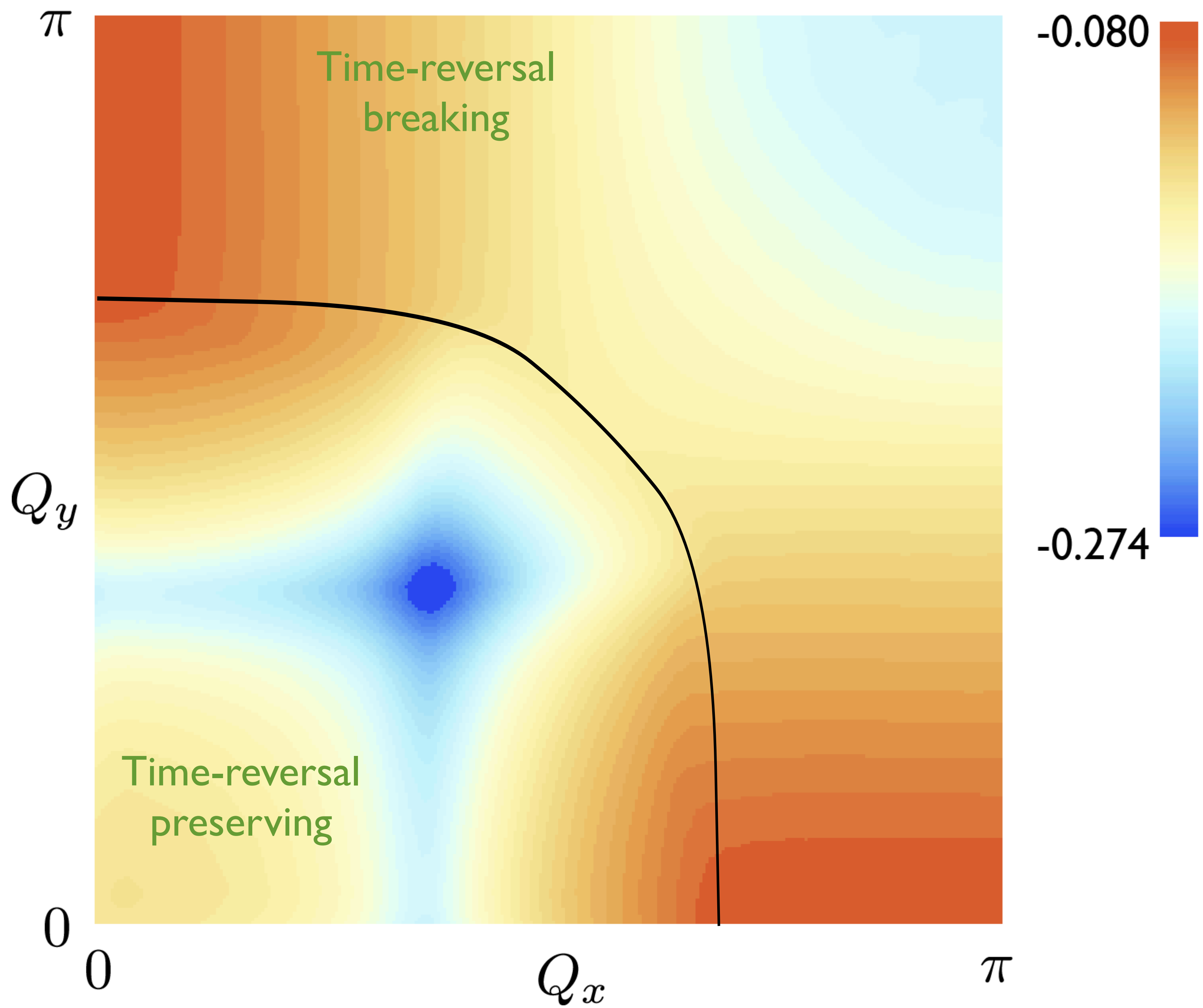
Incommensurate
***d*-wave bond order**

Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$ of particle-hole propagator. The corresponding eigenvector is $P_{\mathbf{Q}}(\mathbf{k})$ and this leads to the order

$$\left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle = \left[\sum_{\mathbf{k}} P_{\mathbf{Q}}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$$

S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013)



Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$ of particle-hole propagator. The corresponding eigenvector is $P_{\mathbf{Q}}(\mathbf{k})$ and this leads to the order

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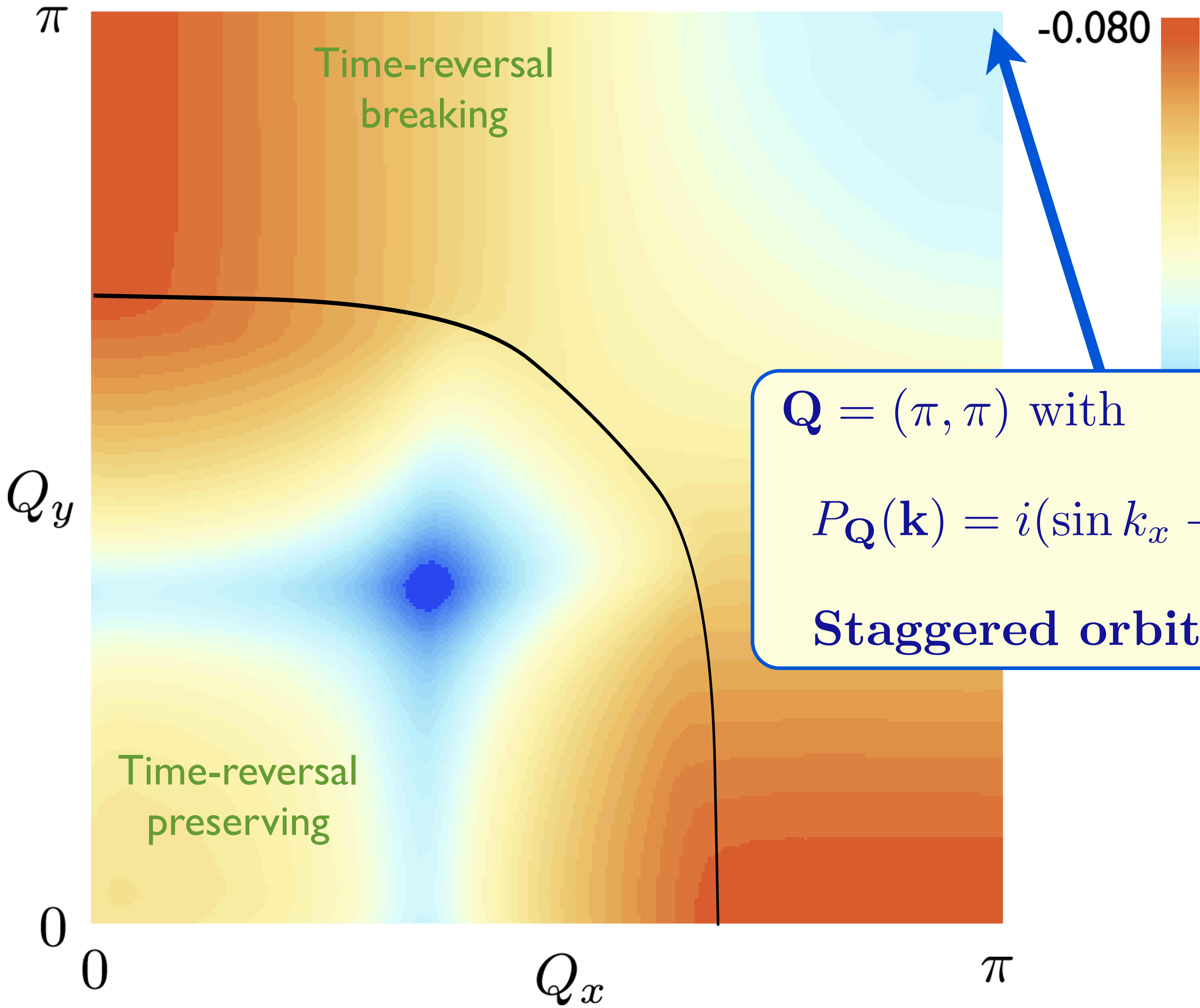
S. Sachdev and R. La Placa, Phys. Rev. Lett. 111, 027202 (2013)



$\mathbf{Q} = (\pi, \pi)$ with

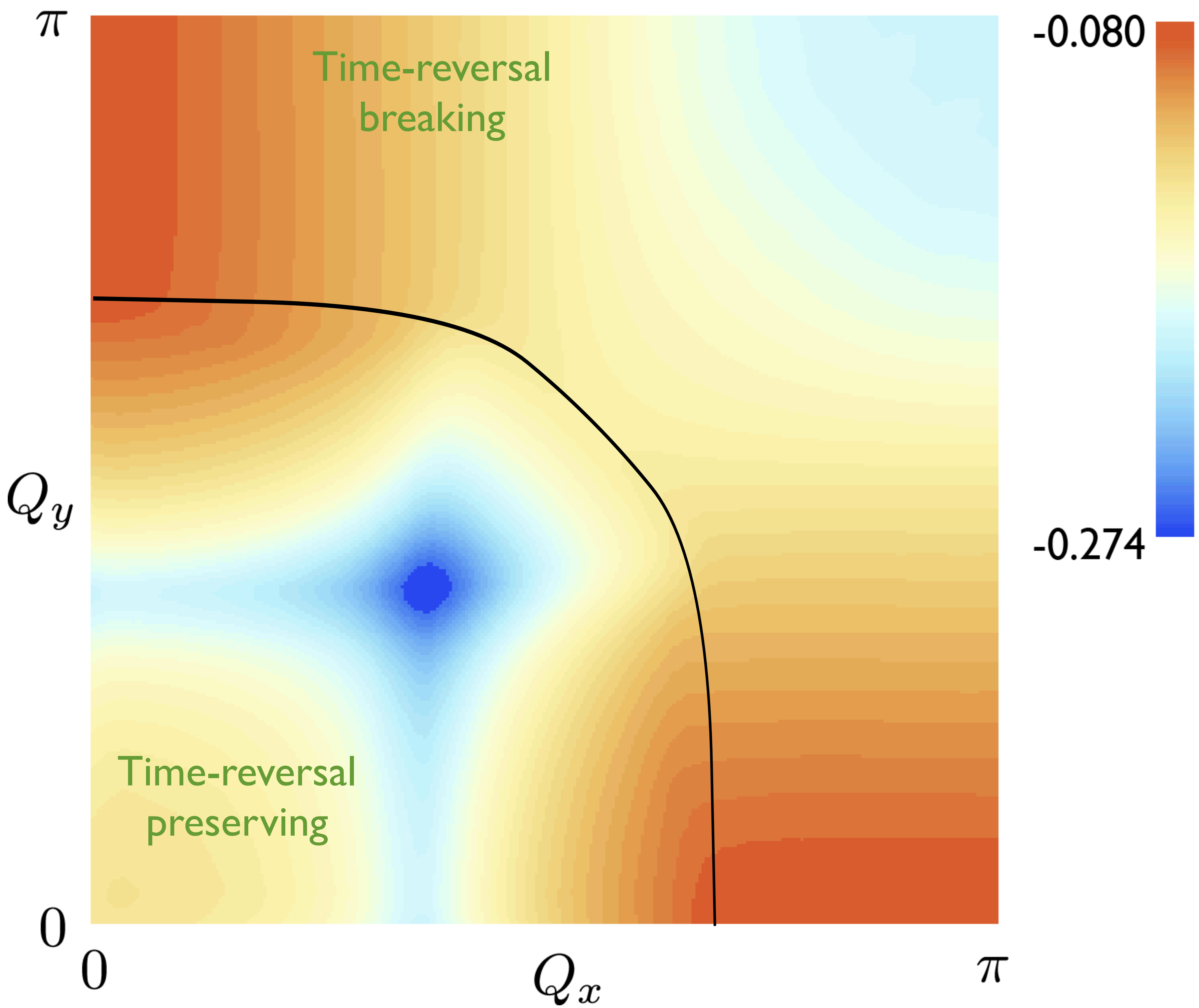
$P_{\mathbf{Q}}(\mathbf{k}) = i(\sin k_x - \sin k_y)$

Staggered orbital currents

Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$ of particle-hole propagator. The corresponding eigenvector is $P_{\mathbf{Q}}(\mathbf{k})$ and this leads to the order

$$\left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle = \left[\sum_{\mathbf{k}} P_{\mathbf{Q}}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$$

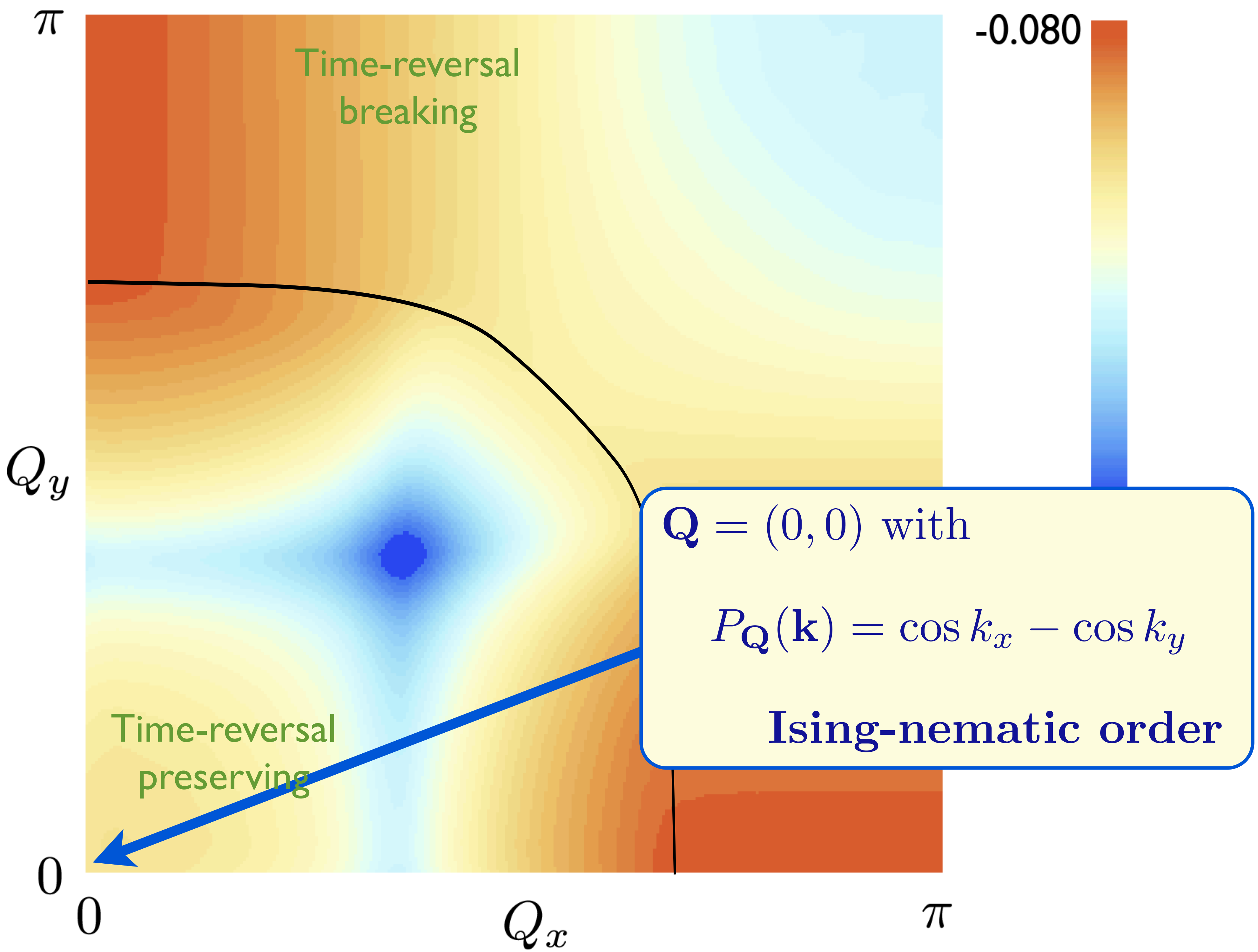
S. Sachdev and R. La Placa, Phys. Rev. Lett. 111, 027202 (2013)



Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$ of particle-hole propagator. The corresponding eigenvector is $P_{\mathbf{Q}}(\mathbf{k})$ and this leads to the order

$$\left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle = \left[\sum_{\mathbf{k}} P_{\mathbf{Q}}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}\right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$$

S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013)



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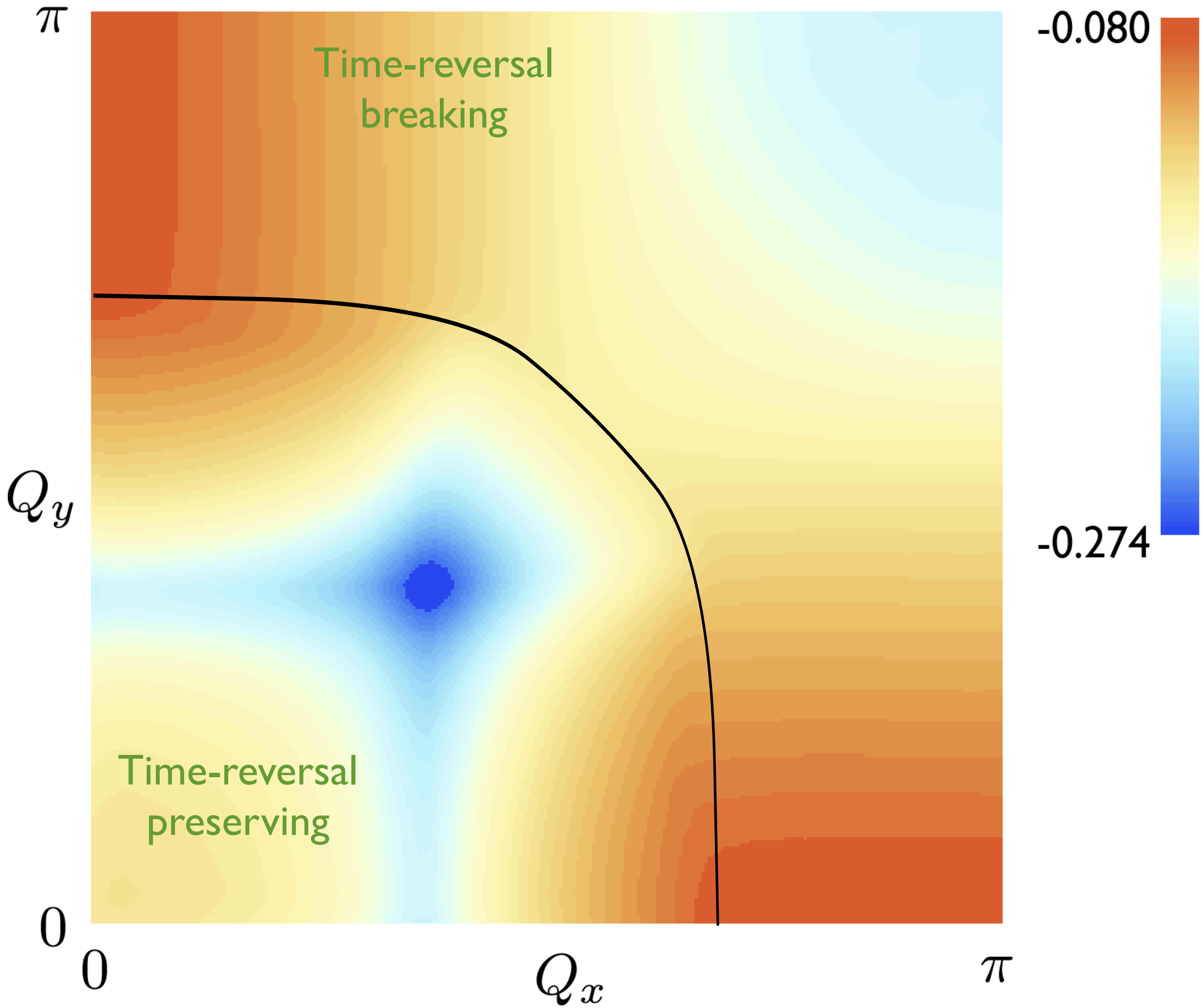
S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013)



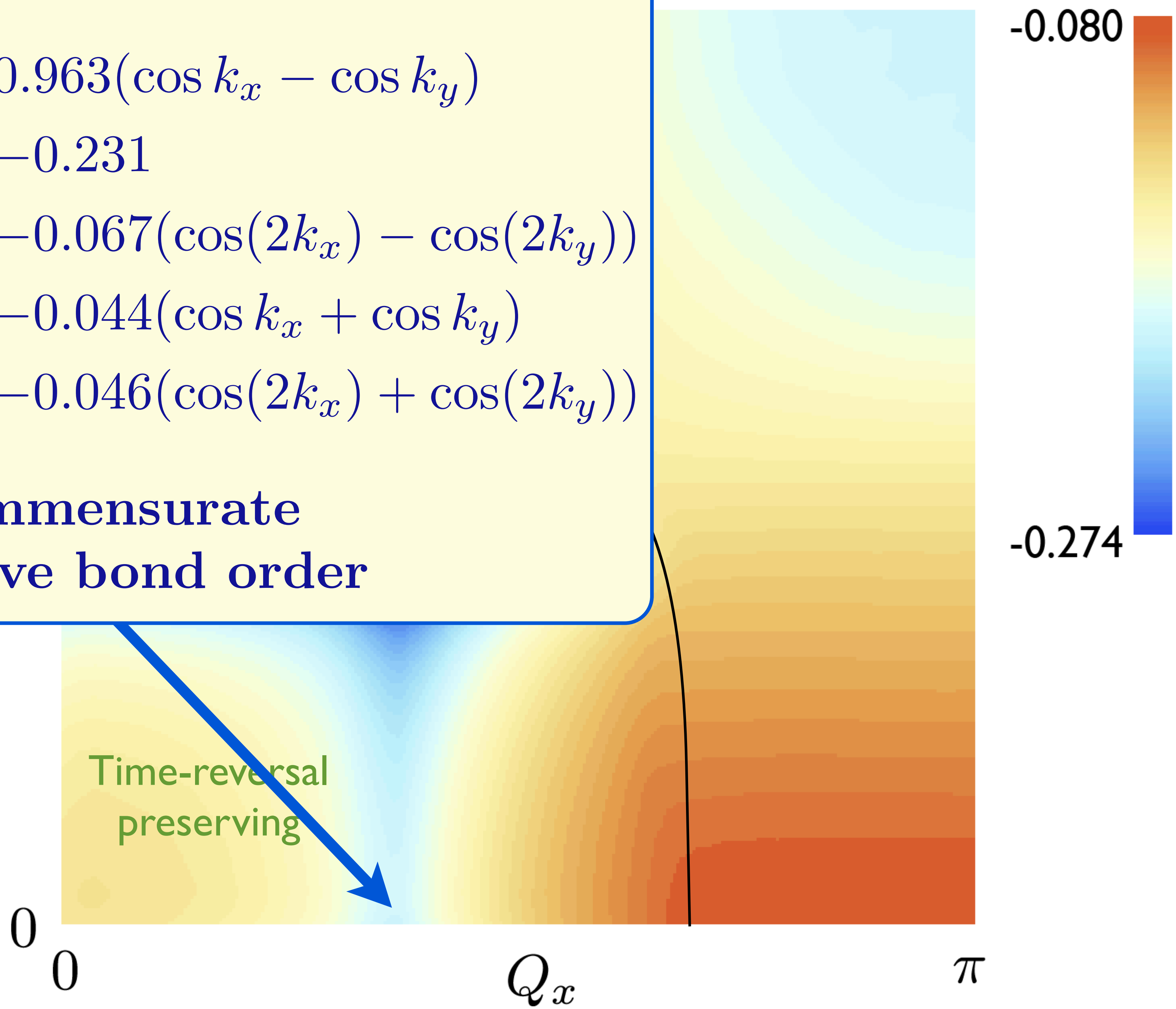
Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$ of particle-hole propagator. The corresponding eigenvector is $P_{\mathbf{Q}}(\mathbf{k})$ and this leads to the order

$$\left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle = \left[\sum_{\mathbf{k}} P_{\mathbf{Q}}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$$

$\mathbf{Q} = (Q_m, 0)$ with

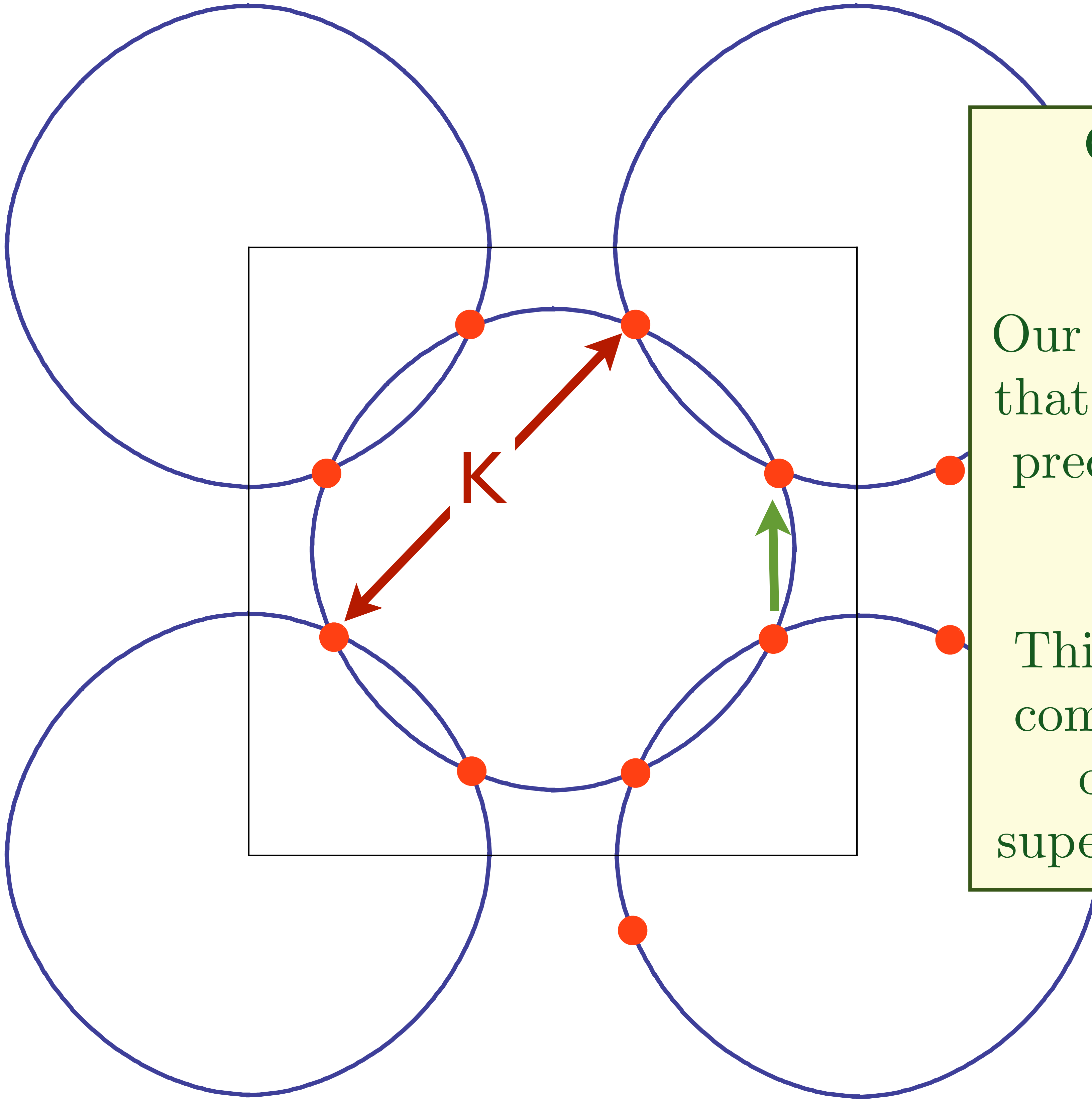
$$\begin{aligned} P_{\mathbf{Q}}(\mathbf{k}) \;=\; & 0.963(\cos k_x - \cos k_y) \\ & -0.231 \\ & -0.067(\cos(2k_x) - \cos(2k_y)) \\ & -0.044(\cos k_x + \cos k_y) \\ & -0.046(\cos(2k_x) + \cos(2k_y)) \end{aligned}$$

Incommensurate
d_{+s} **-wave bond order**



Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$ of particle-hole propagator.
The corresponding eigenvector is $P_{\mathbf{Q}}(\mathbf{k})$ and this leads to the order
$\left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle = \left[\sum_{\mathbf{k}} P_{\mathbf{Q}}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}\right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$

Incommensurate *d*-wave bond order



Observed low T ordering.

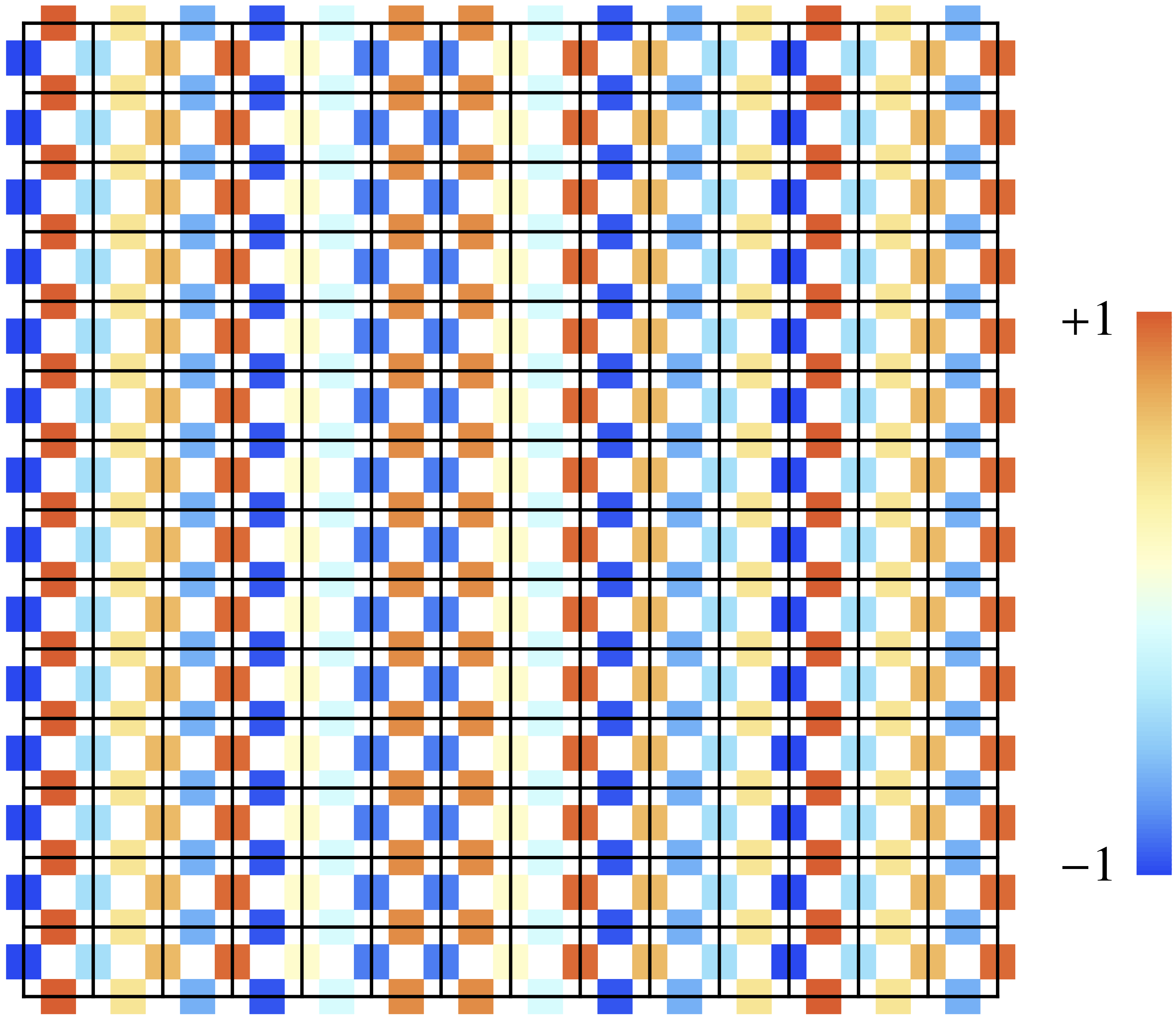
Our computations show that the charge order is predominantly d-wave also at this $\mathbf{Q}$.

This $\mathbf{Q}$ is preferred in computations of bond order within the superconducting phase.

S. Sachdev and R. La Placa, Physical Review Letters in press; arXiv:1303.2114
M. Vojta and S. Sachdev, Physical Review Letters **83**, 3916 (1999)
M. Vojta and O. Rosch, Physical Review B 77, 094504 (2008)

Incommensurate *d*-wave bond order

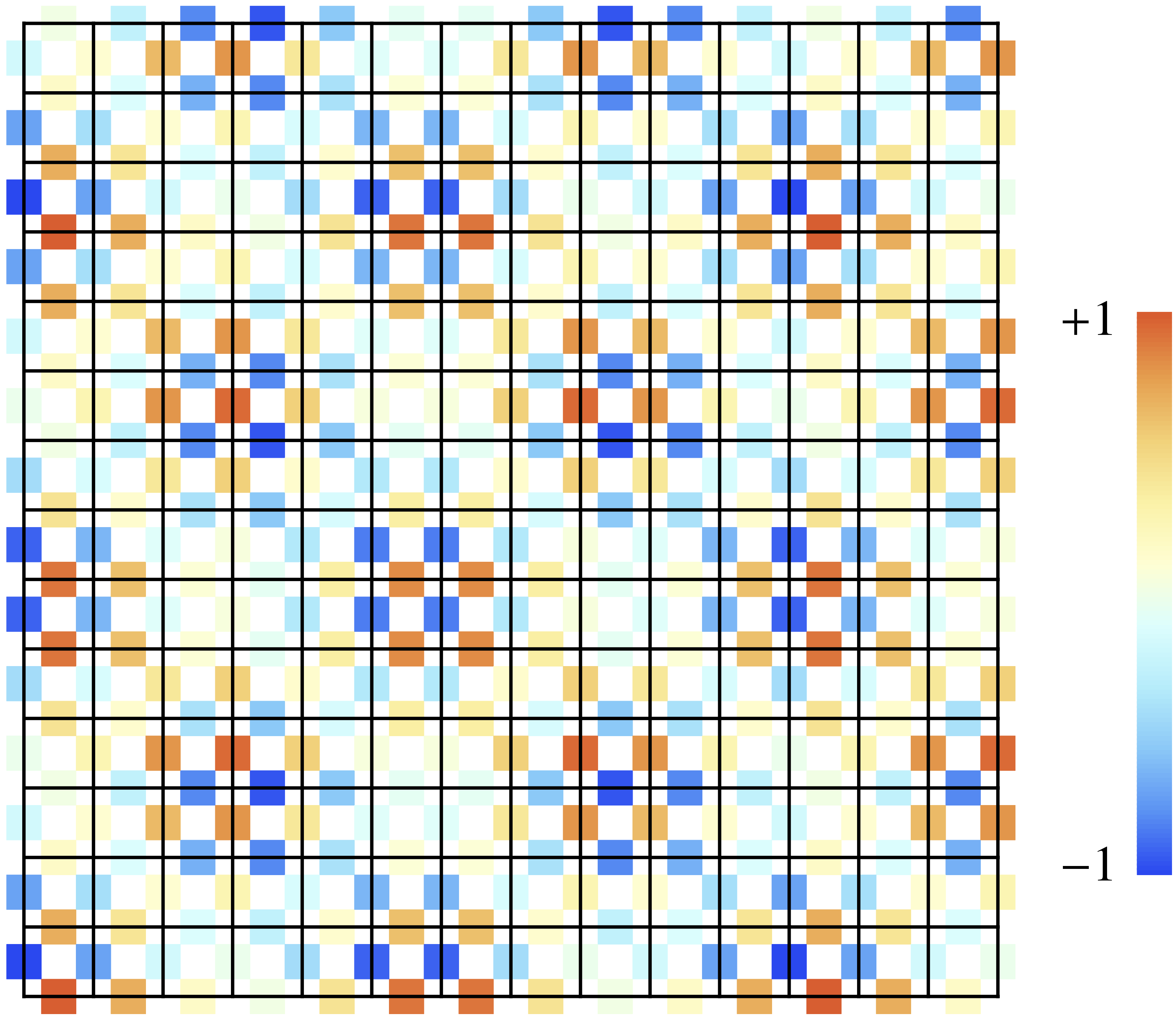
Plot of $P_{ij} = \left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle$ for i, j nearest neighbors.



Unidirectional order along $\mathbf{Q} = (4\pi/11, 0)$

Incommensurate *d*-wave bond order

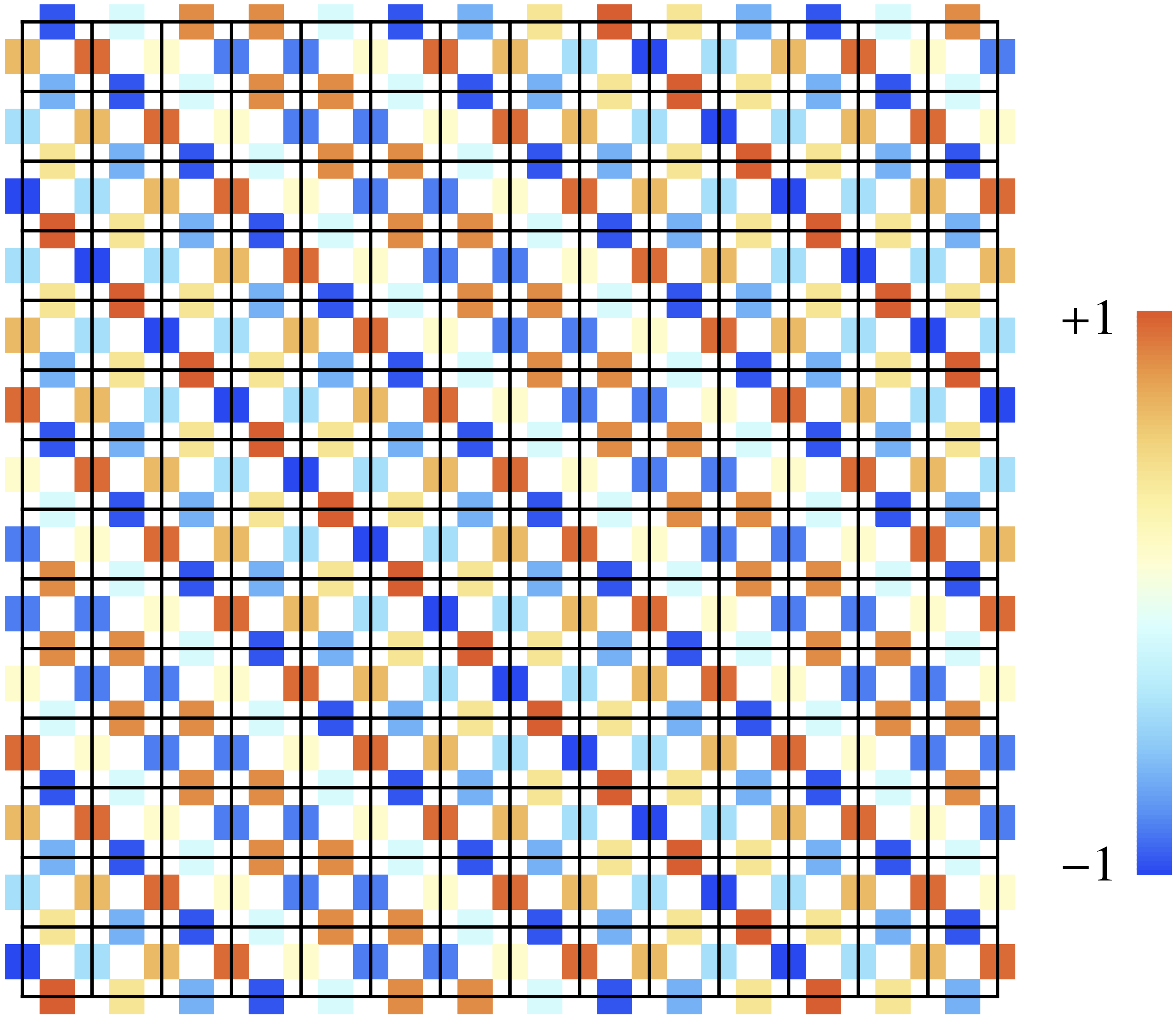
Plot of $P_{ij} = \left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle$ for i, j nearest neighbors.



Bi-directional order along $\mathbf{Q} = (4\pi/11, 0)$ and $(0, 4\pi/11)$.

Incommensurate *d*-wave bond order

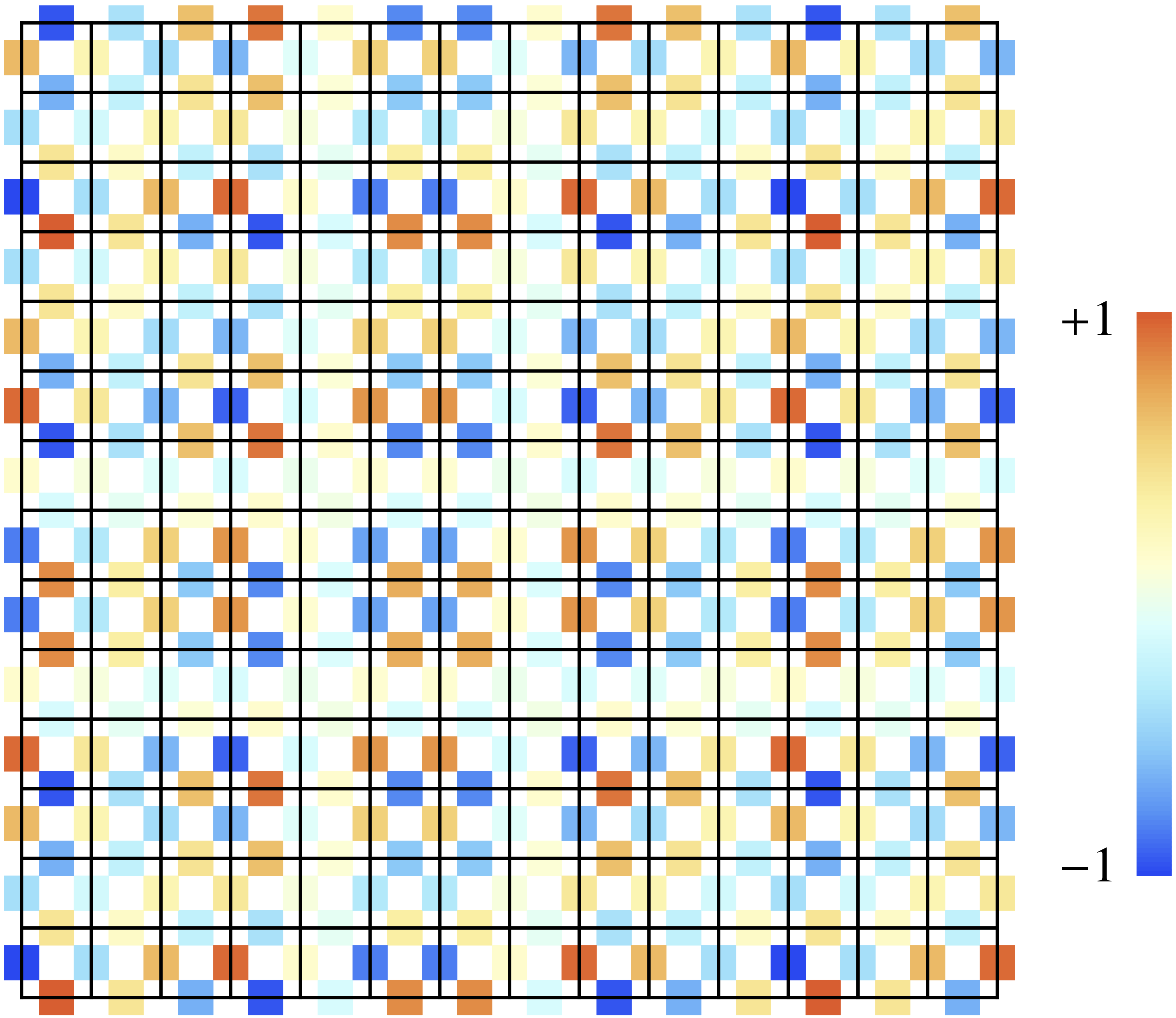
Plot of $P_{ij} = \left\langle c^{\dagger}_{i\alpha} c_{j\alpha} \right\rangle$ for i, j nearest neighbors.



Unidirectional order along $\mathbf{Q} = (4\pi/11, 4\pi/11)$

Incommensurate *d*-wave bond order

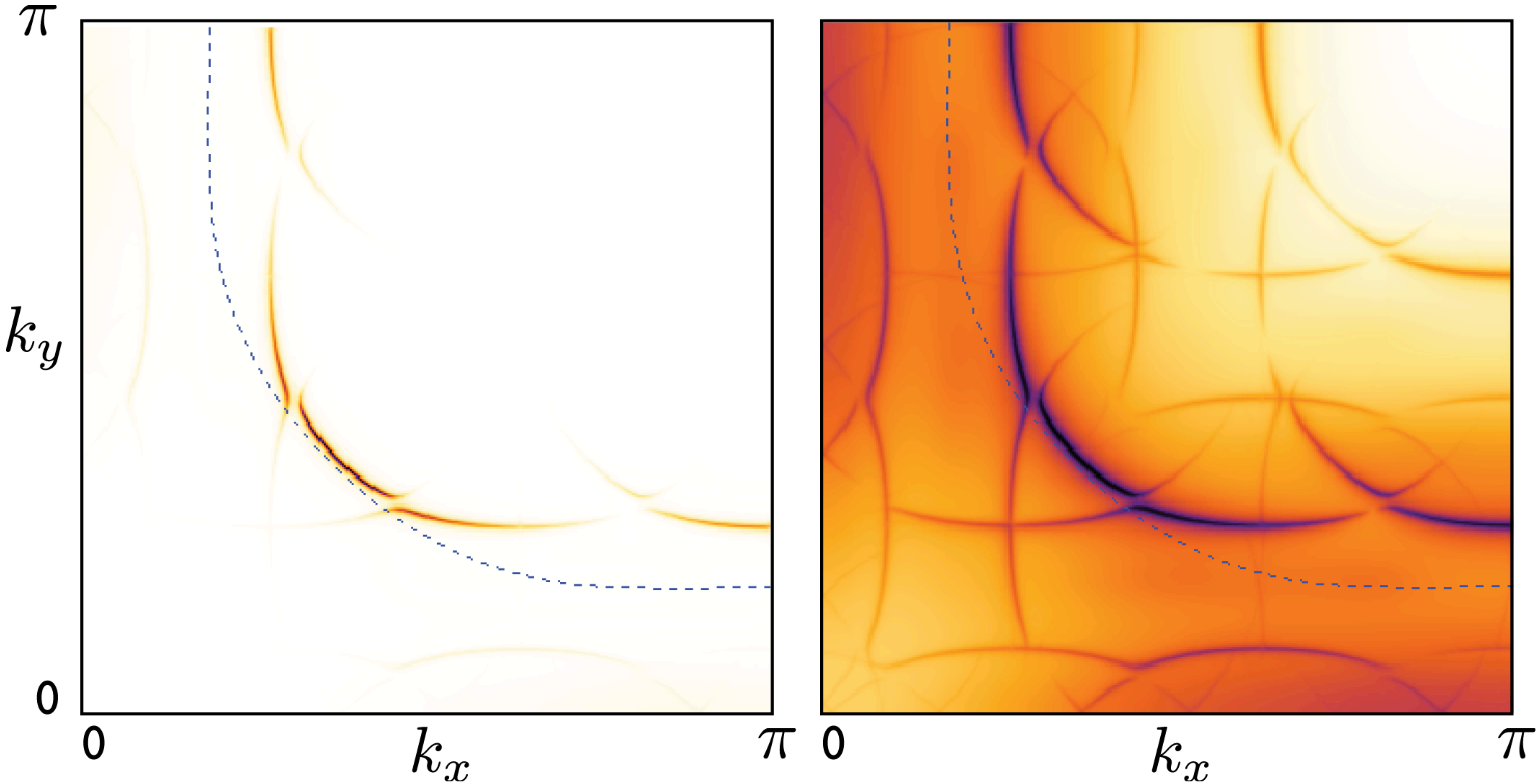
Plot of $P_{ij} = \left\langle c^\dagger_{i\alpha} c_{j\alpha} \right\rangle$ for i, j nearest neighbors.



Bi-directional order along $\mathbf{Q} = (4\pi/11, 4\pi/11)$ and $(4\pi/11, -4\pi/11)$

Incommensurate *d*-wave bond order

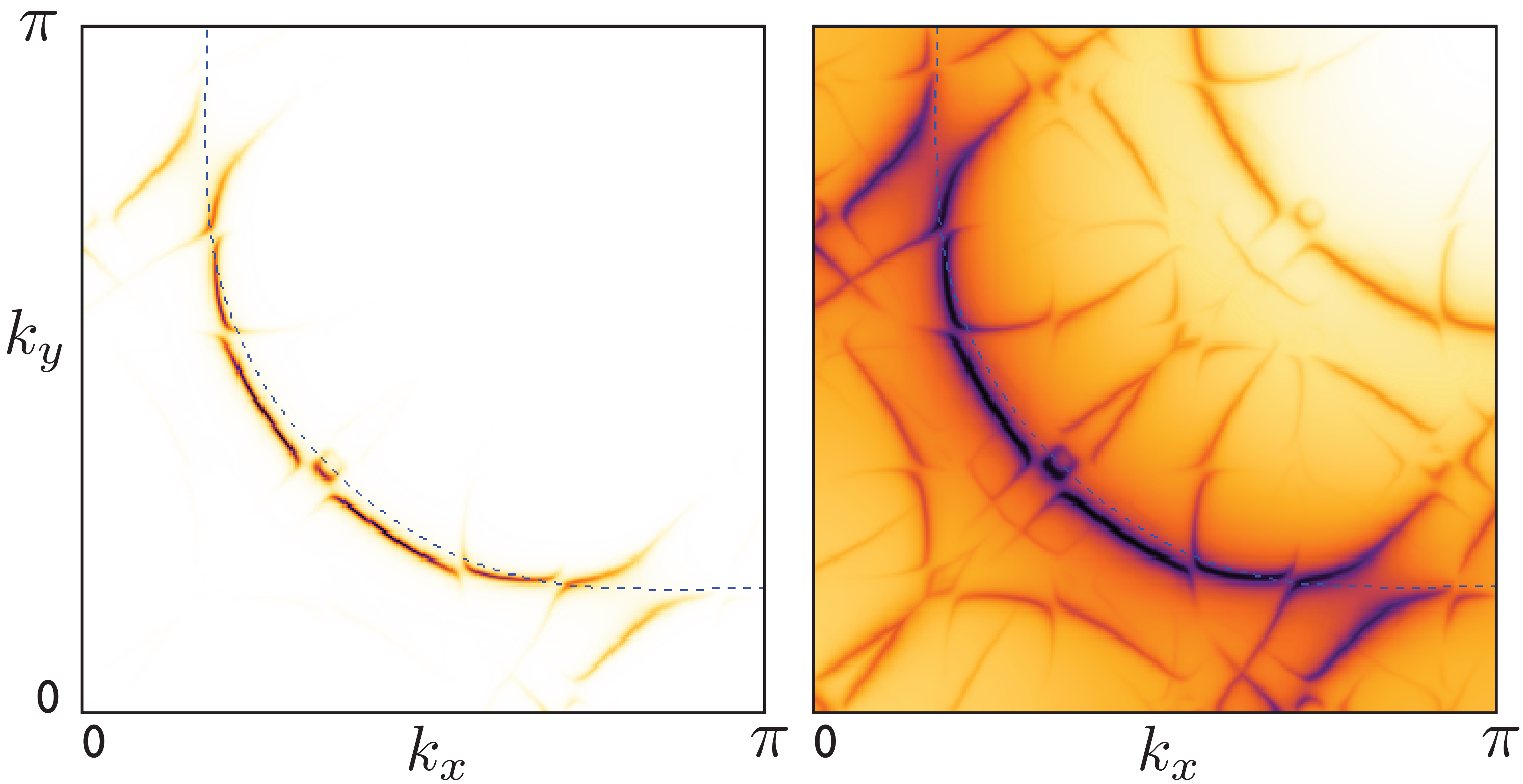
Photoemission spectrum $\text{Im}G(\mathbf{k}, \omega = i0^+)$;
right panel shows $\log \text{Im}G(\mathbf{k}, \omega = i0^+)$



Bi-directional order along $\mathbf{Q} = (4\pi/11, 0)$ and $(0, 4\pi/11)$.

Incommensurate *d*-wave bond order

Photoemission spectrum $\mathrm{Im} G(\mathbf{k}, \omega = i0^+)$;
right panel shows $\log \mathrm{Im} G(\mathbf{k}, \omega = i0^+)$



Bi-directional order along $\mathbf{Q} = (4\pi/11, 4\pi/11)$ and $(4\pi/11, -4\pi/11)$

Summary

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- Antiferromagnetic quantum criticality leads to d-wave superconductivity (supported by sign-problem-free Monte Carlo simulations)

- Metals with antiferromagnetic spin correlations have nearly degenerate instabilities: to *d*-wave superconductivity, and to a charge density wave with a *d*-wave form factor. This is a promising explanation of the pseudogap regime.