Quantum criticality in the high temperature superconductors

Institut d'Etudes Scientifiques de Cargese
August 12-16, 2013

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

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Lauren Hayward

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Andrew Achkar

David Hawthorn

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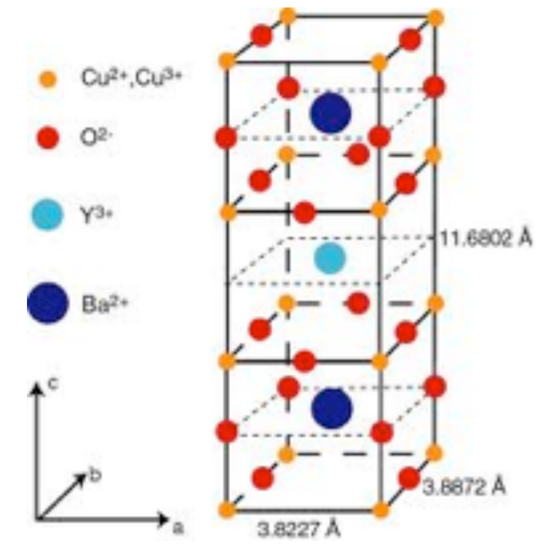
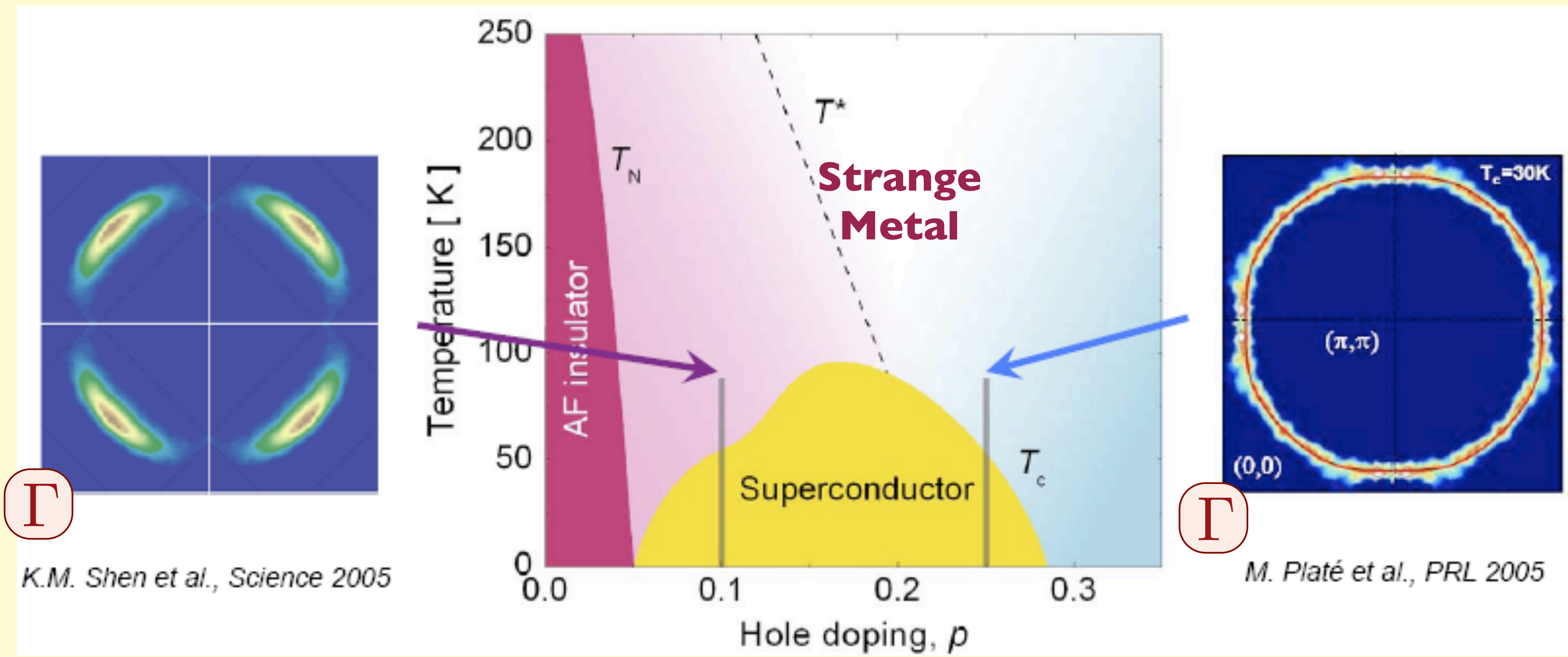




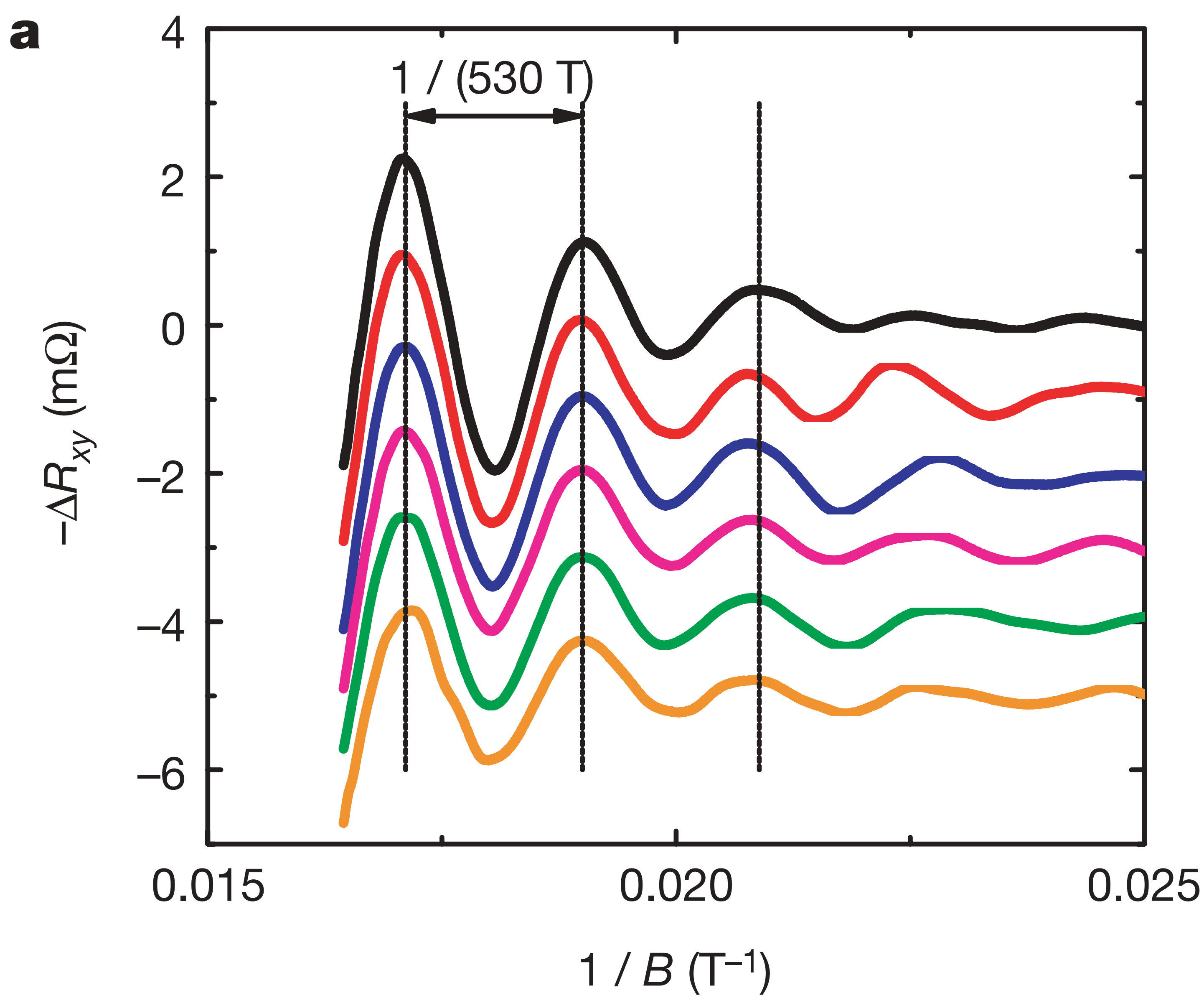
Smaller hole Fermi-pockets

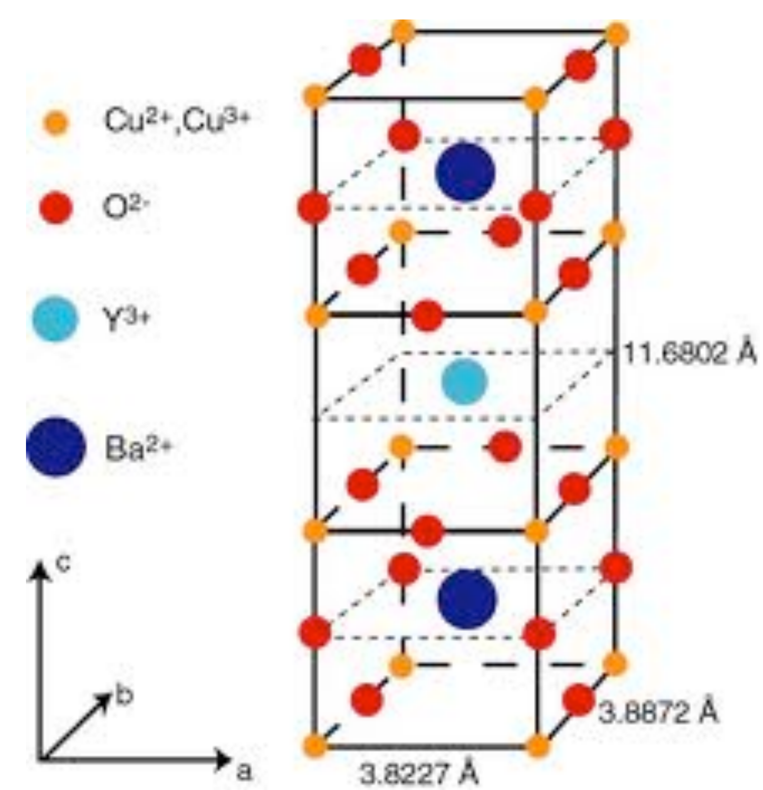
Large hole Fermi surface

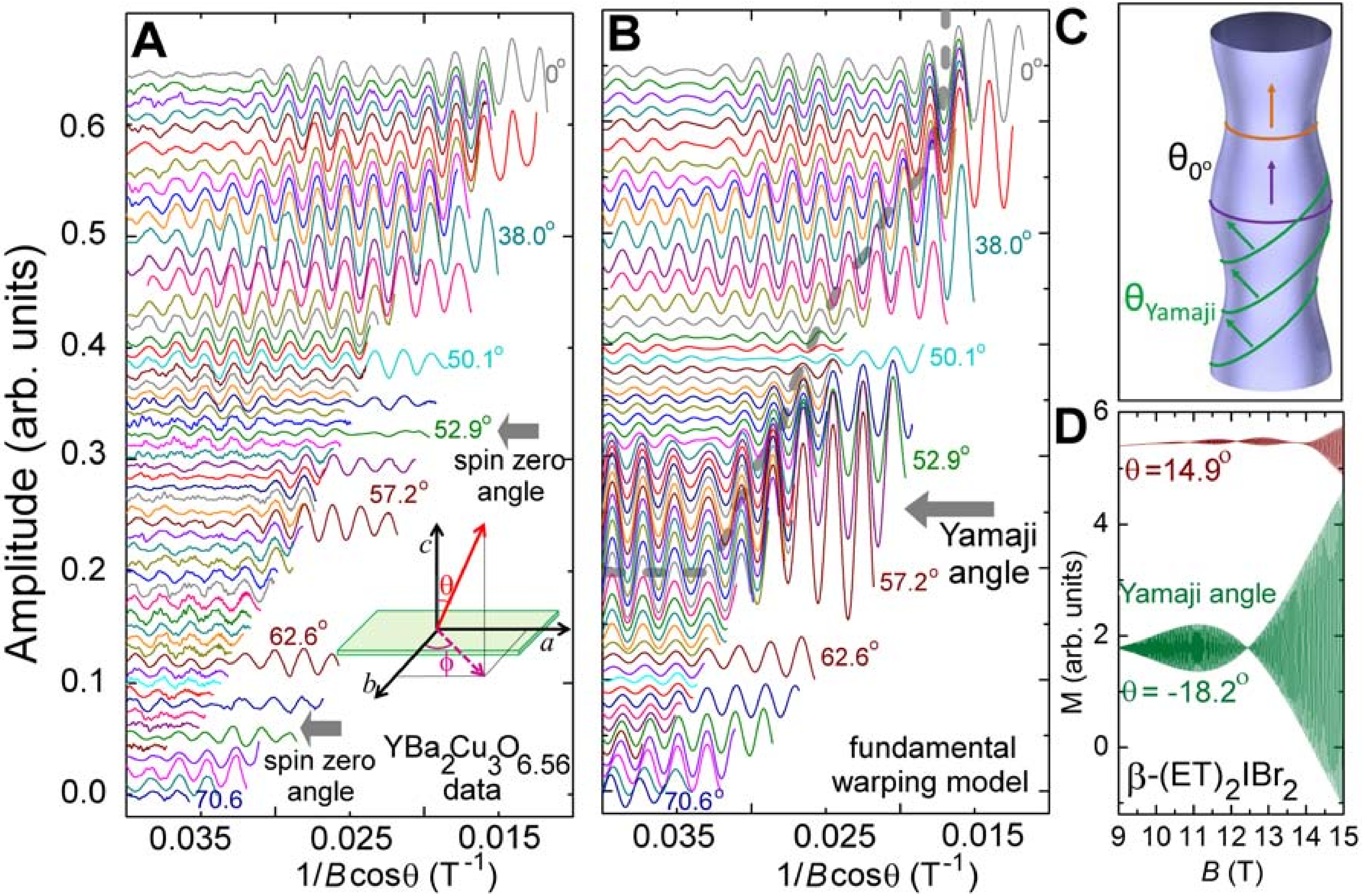
Quantum oscillations and the Fermi surface in an underdoped high-T_c superconductor

Nicolas Doiron-Leyraud[1], Cyril Proust[2], David LeBoeuf[1], Julien Levallois[2], Jean-Baptiste Bonnemaison[1], Ruixing Liang[3,4], D. A. Bonn[3,4], W. N. Hardy[3,4] & Louis Taillefer[1,4]

Nature **447**, 565 (2007)







Twofold twisted Fermi surface from staggered order in an underdoped high T_c superconductor

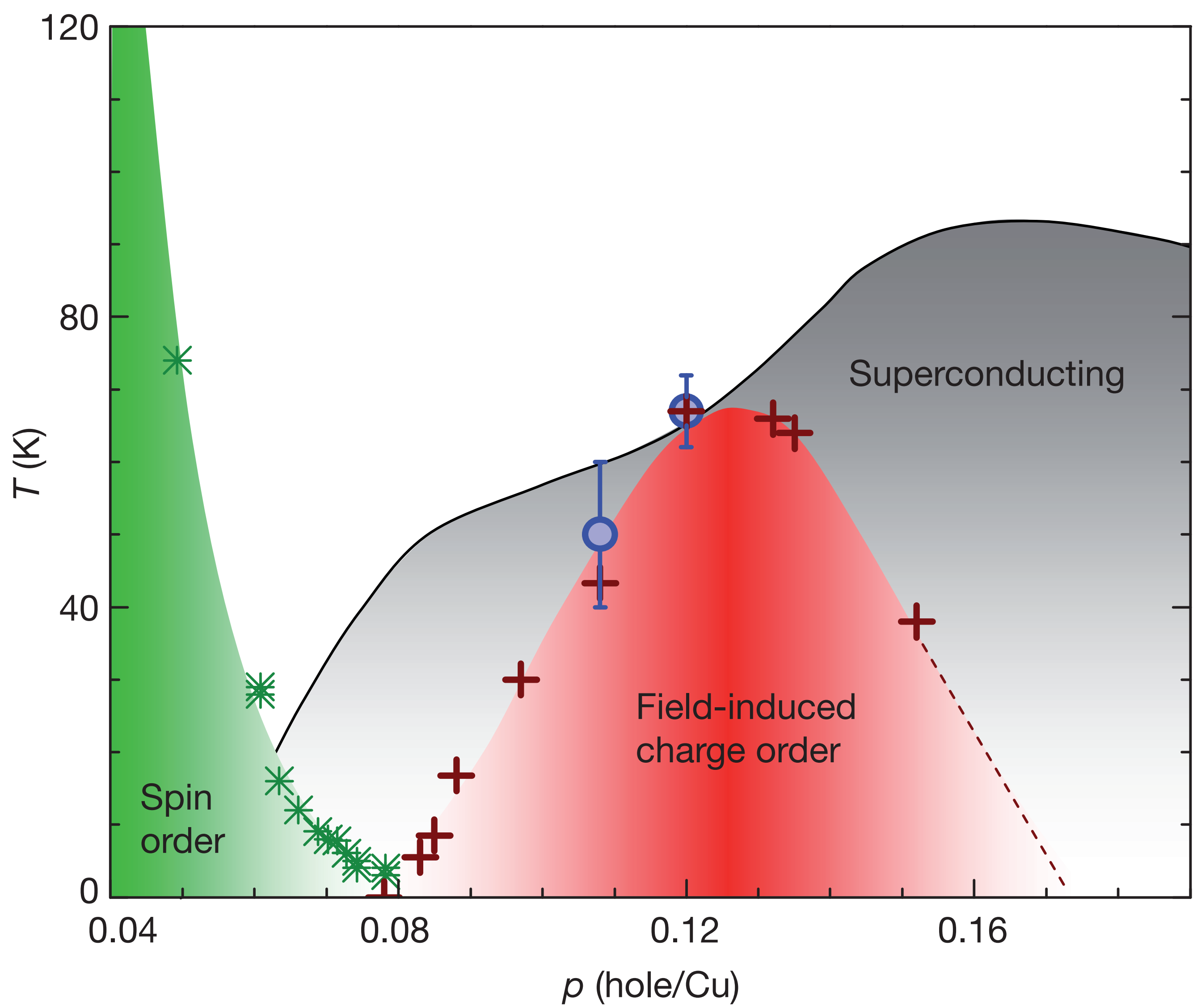
APS March meeting 2013 B2.00004

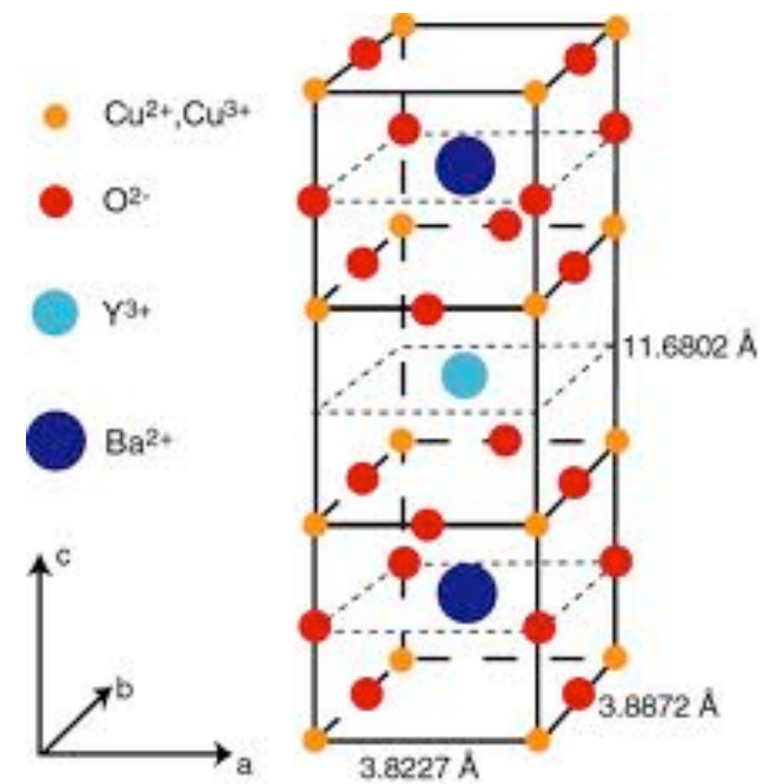
Suchitra E. Sebastian,[1*] N. Harrison,[2] F. F. Balakirev,[2] M. M. Altarawneh,[2,3] Ruixing Liang,[4,5] D. A. Bonn,[4,5] W. N. Hardy,[4,5] G. G. Lonzarich,[1]

Magnetic-field-induced charge-stripe order in the high-temperature superconductor $YBa_2Cu_3O_y$

Tao Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Mladen Horvatić[1], Claude Berthier[1], W. N. Hardy[2,3], Ruixing Liang[2,3], D. A. Bonn[2,3] & Marc-Henri Julien[1]

8 SEPTEMBER 2011 | VOL 477 | NATURE | 191





Distinct Charge Orders in the Planes and Chains of Ortho-III-Ordered $YBa_2Cu_3O_{6+\delta}$ Superconductors Identified by Resonant Elastic X-ray Scattering

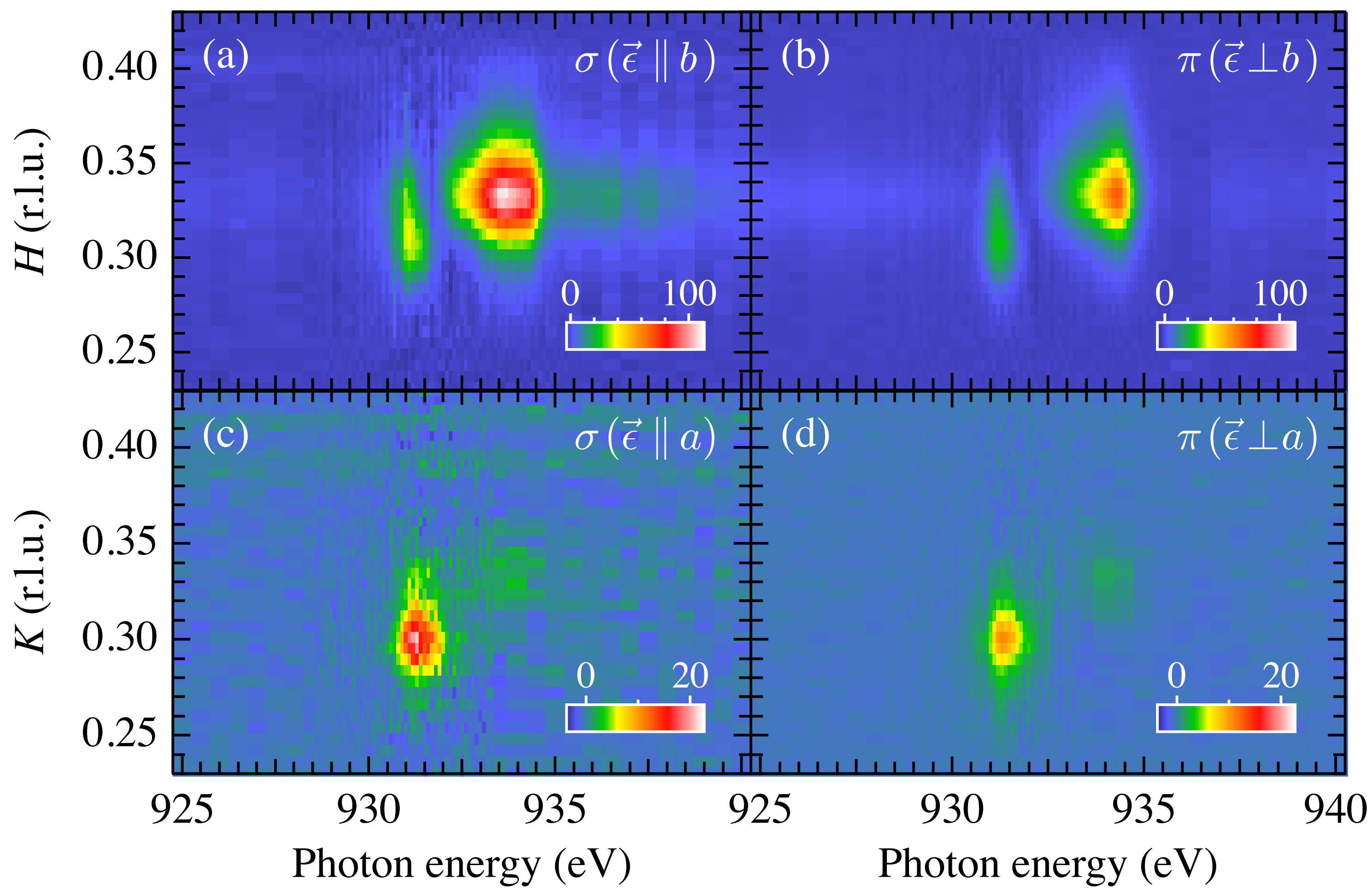
A. J. Achkar,[1] R. Sutarto,[2,3] X. Mao,[1] F. He,[3] A. Frano,[4,5] S. Blanco-Canosa,[4] M. Le Tacon,[4] G. Ghiringhelli,[6] L. Braicovich,[6] M. Minola,[6] M. Moretti Sala,[7] C. Mazzoli,[6] Ruixing Liang,[2] D. A. Bonn,[2] W. N. Hardy,[2] B. Keimer,[4] G. A. Sawatzky,[2] and D. G. Hawthorn[1,*]

PRL **109,** 167001 (2012)

Resonant X-Ray Scattering Measurements of a Spatial Modulation of the Cu 3*d* and O 2*p* Energies in Stripe-Ordered Cuprate Superconductors

A. J. Achkar,[1] F. He,[2] R. Sutarto,[3] J. Geck,[4] H. Zhang,[5] Y.-J. Kim,[5] and D. G. Hawthorn[1]

PRL **110,** 017001 (2013)



may point to a valence-bond-solid interpretation of the stripe phase.

Long-Range Incommensurate Charge Fluctuations in (Y,Nd)$Ba_2Cu_3O_{6+x}$

G. Ghiringhelli,[1]* M. Le Tacon,[2] M. Minola,[1] S. Blanco-Canosa,[2] C. Mazzoli,[1] N. B. Brookes,[3] G. M. De Luca,[4] A. Frano,[2,5] D. G. Hawthorn,[6] F. He,[7] T. Loew,[2] M. Moretti Sala,[3] D. C. Peets,[2] M. Salluzzo,[4] E. Schierle,[5] R. Sutarto,[7,8] G. A. Sawatzky,[8] E. Weschke,[5] B. Keimer,[2]* L. Braicovich[1]

SCIENCE VOL 337 17 AUGUST 2012

resonant soft x-ray scattering

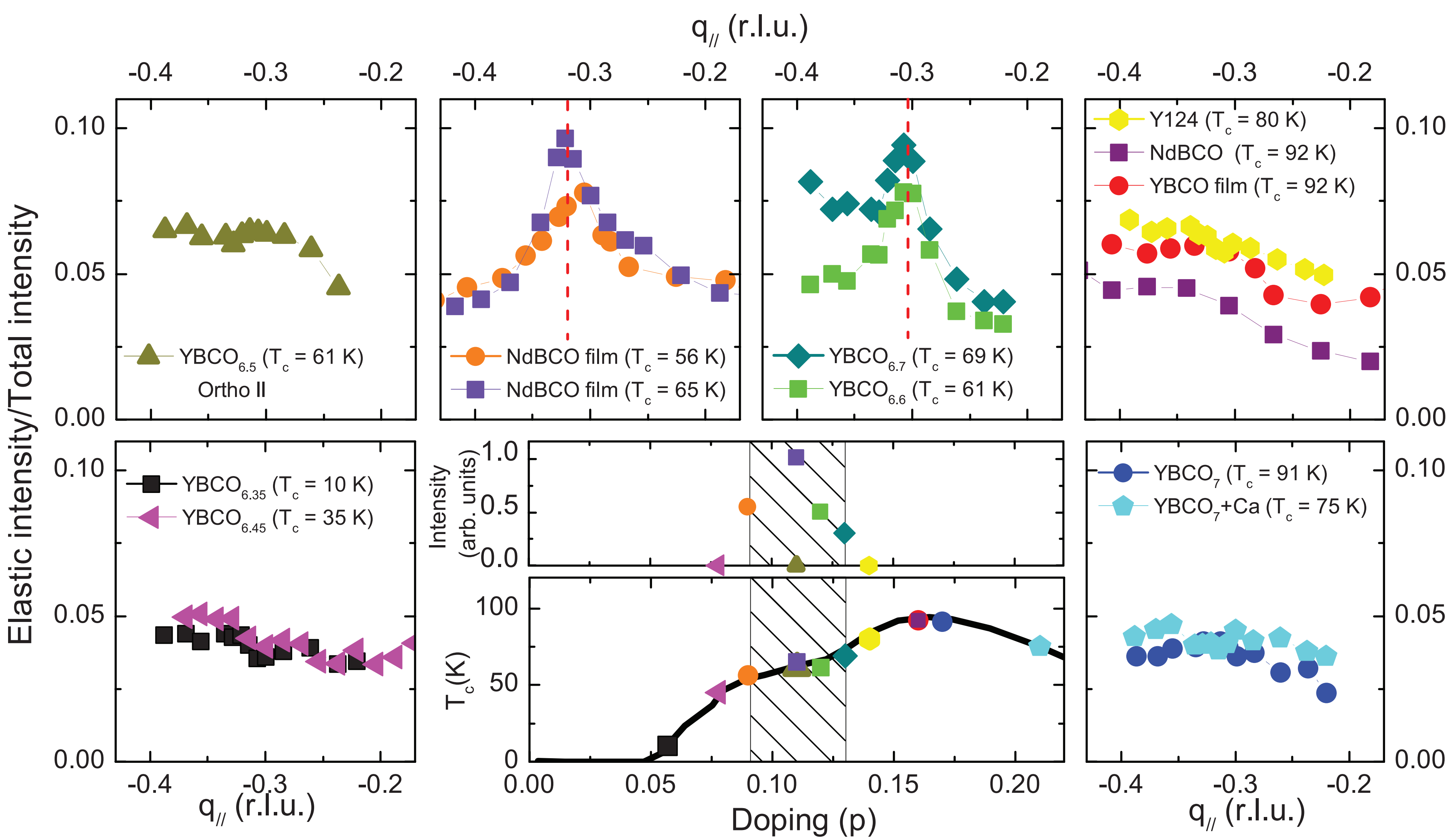
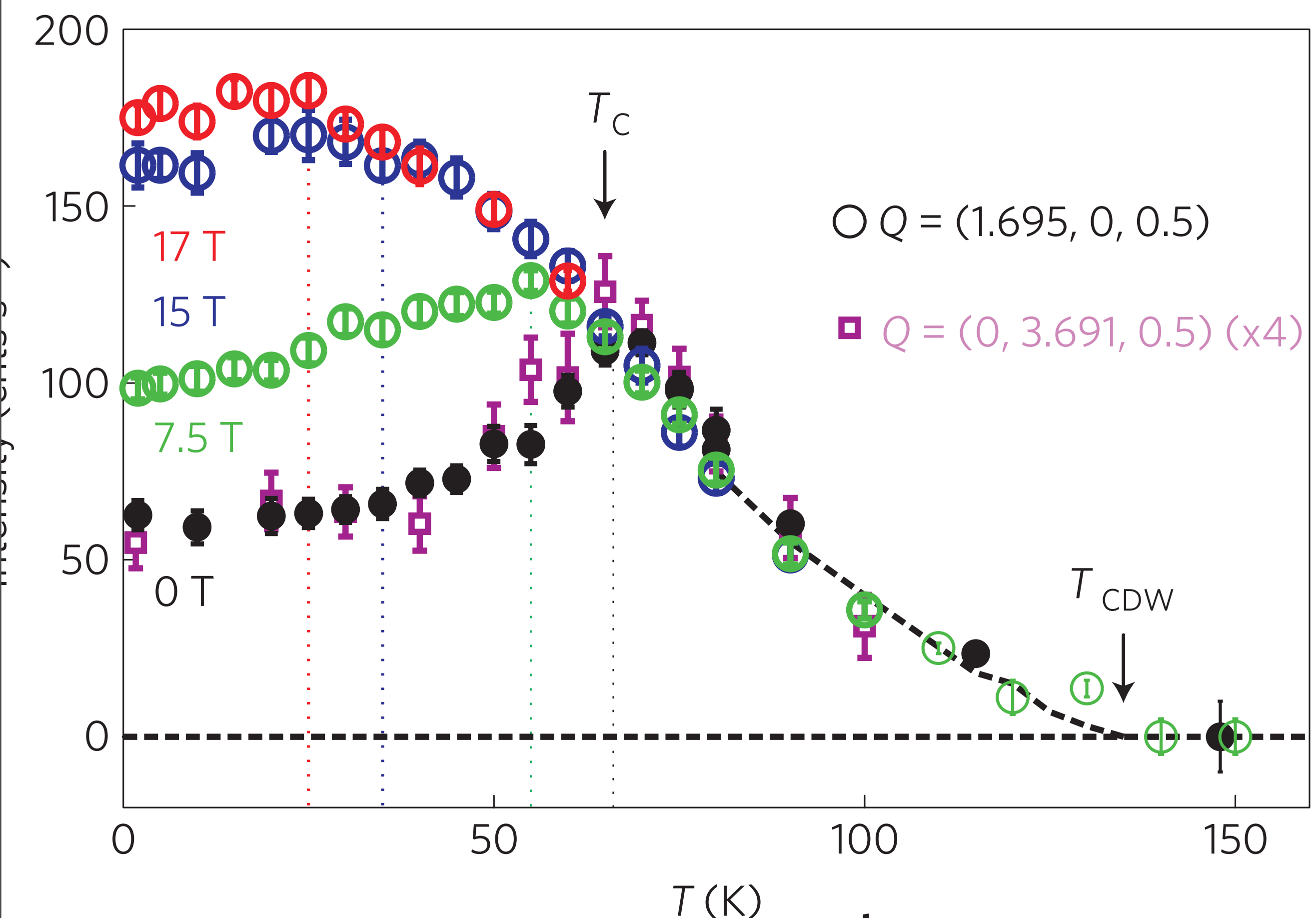


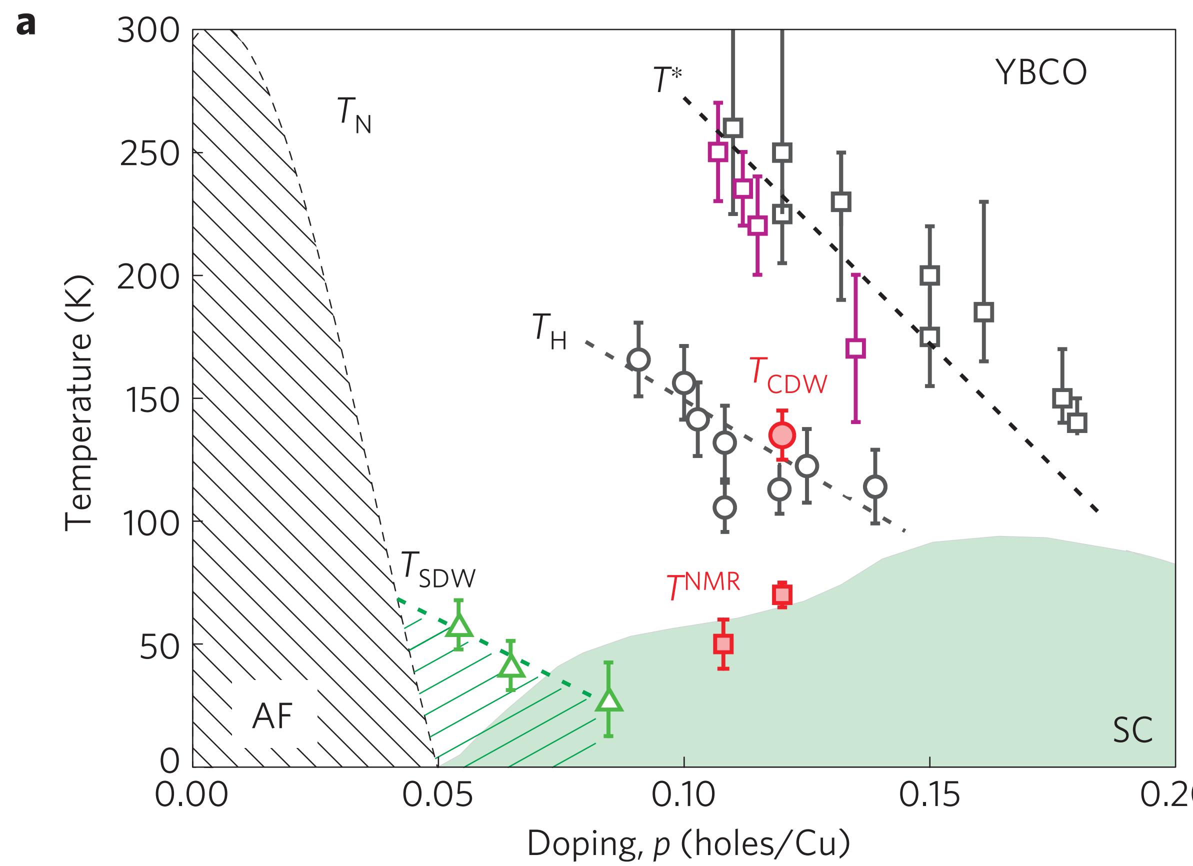
Fig. 3. Dependence of the CDW signal at 15 K on the hole doping level p. The CDW signal is present in several $YBa_2Cu_3O_{6+x}$ and $Nd_{1+y}Ba_{2-y}Cu_3O_7$ samples, but only for $0.09 \leq p \leq 0.13$. In this doping range (shaded in the central panel), the T_c-versus-p relation exhibits a plateau. The CDW peak position does not change with p outside of the experimental error, but its intensity is maximum at $p \simeq 0.11$.

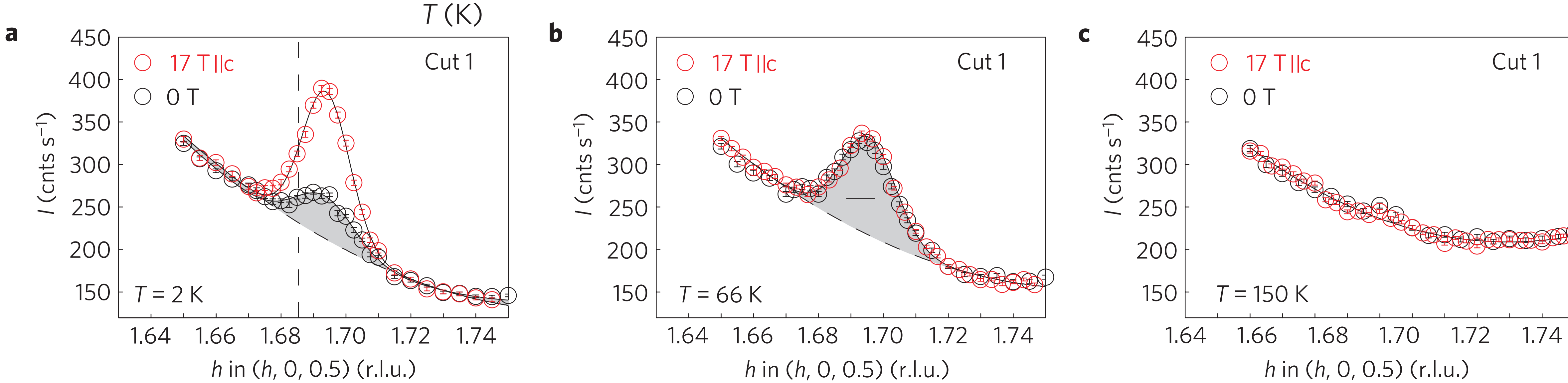
Direct observation of competition between superconductivity and charge density wave order in $YBa_2Cu_3O_{6.67}$

NATURE PHYSICS | VOL 8 | DECEMBER 2012 |

J. Chang[1,2]★, E. Blackburn[3], A. T. Holmes[3], N. B. Christensen[4], J. Larsen[4,5], J. Mesot[1,2], Ruixing Liang[6,7], D. A. Bonn[6,7], W. N. Hardy[6,7], A. Watenphul[8], M. v. Zimmermann[8], E. M. Forgan[3] and S. M. Hayden[9]



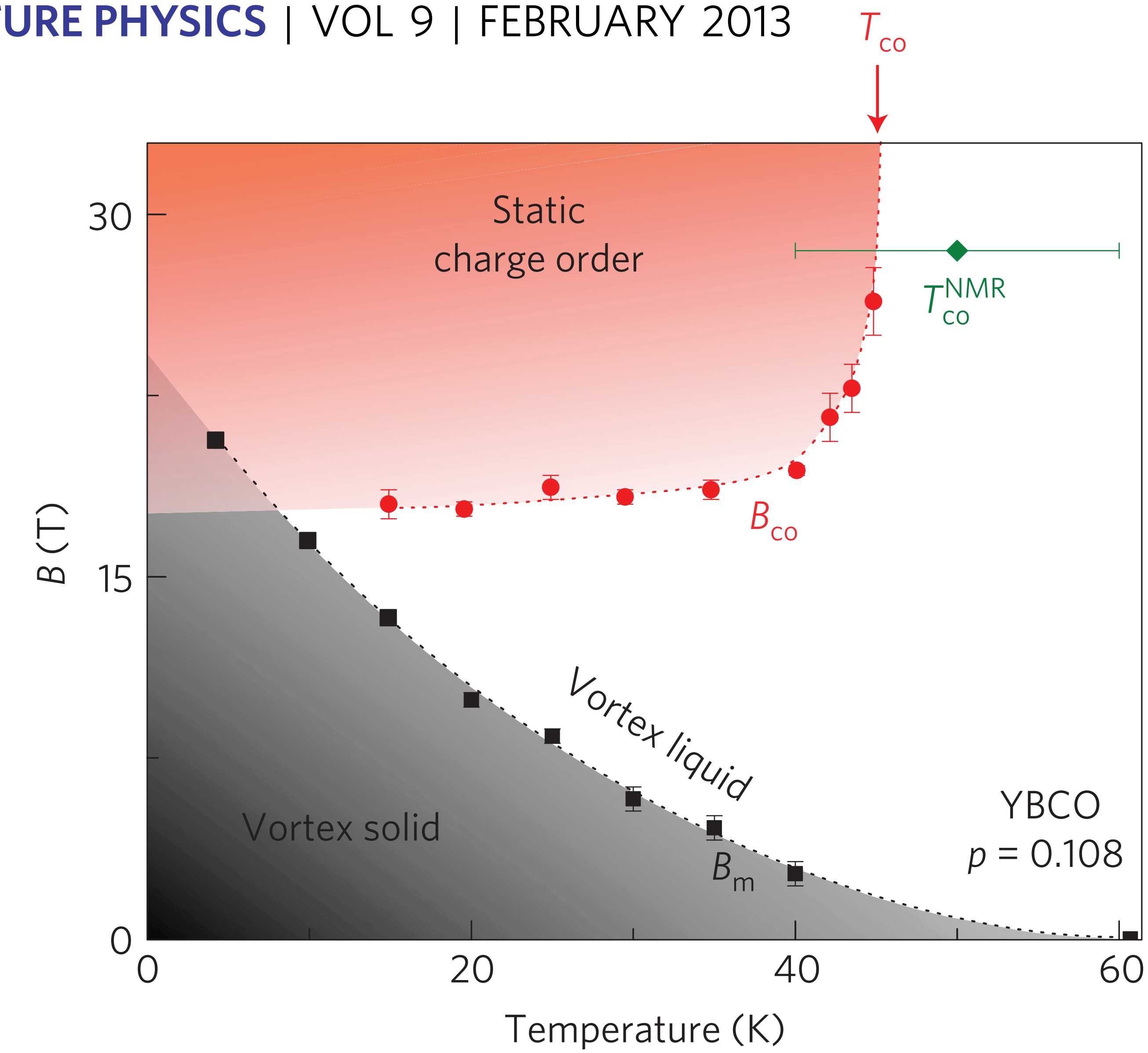




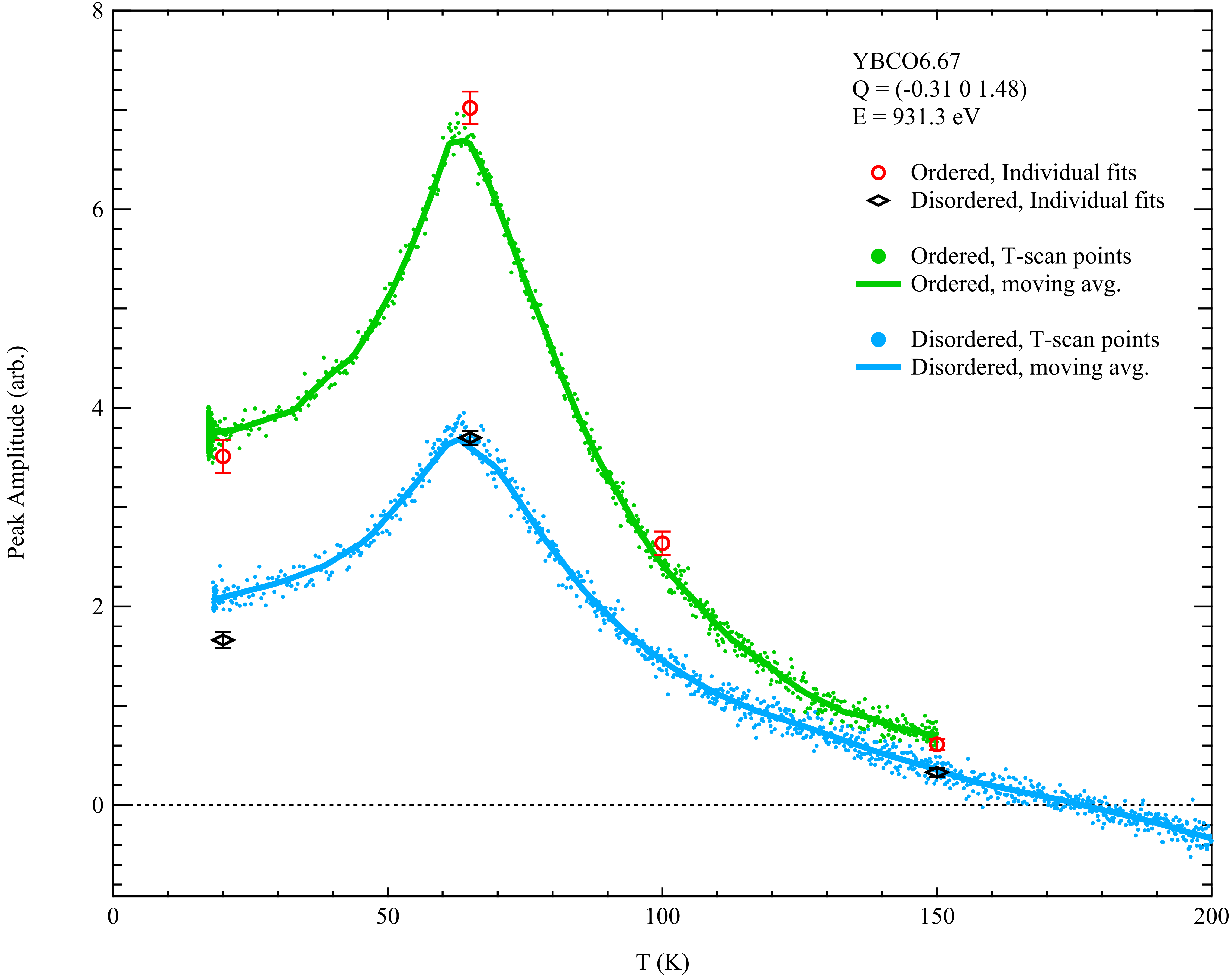
Thermodynamic phase diagram of static charge order in underdoped $YBa_2Cu_3O_y$

David LeBoeuf[1]★, S. Krämer[2], W. N. Hardy[3,4], Ruixing Liang[3,4], D. A. Bonn[3,4] and Cyril Proust[1,4]★

NATURE PHYSICS | VOL 9 | FEBRUARY 2013



The comparison of different acoustic modes indicates that the charge modulation is biaxial, which differs from a uniaxial stripe charge order.



A. Achkar, D. Hawthorn, *et al.*

Outline

1. Pseudogap: angular fluctuations of a multi-component order parameter

2. Instabilities of a two-dimensional metal with antiferromagnetic exchange interactions: *d*-wave superconductivity and bond order

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PHYSICAL REVIEW B **69**, 144504 (2004)

Competing orders in thermally fluctuating superconductors in two dimensions

Subir Sachdev
Department of Physics, Yale University, P.O. Box 208120, New Haven, Connecticut 06520-8120, USA

Eugene Demler
Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA
(Received 6 August 2003; revised manuscript received 24 November 2003; published 6 April 2004)

We extend recent low-temperature analyses of competing orders in the cuprate superconductors to the pseudogap regime where all orders are fluctuating. A universal continuum limit of a classical Ginzburg-Landau functional is used to characterize fluctuations of the superconducting order: this describes the crossover from Gaussian fluctuations at high temperatures to the vortex-binding physics near the onset of global phase coherence. These fluctuations induce affiliated corrections in the correlations of other orders, and in particular, in the different realizations of charge order. Implications for scanning tunneling spectroscopy and neutron-scattering experiments are noted: there may be a regime of temperatures near the onset of superconductivity where the charge order is enhanced with increasing temperatures.

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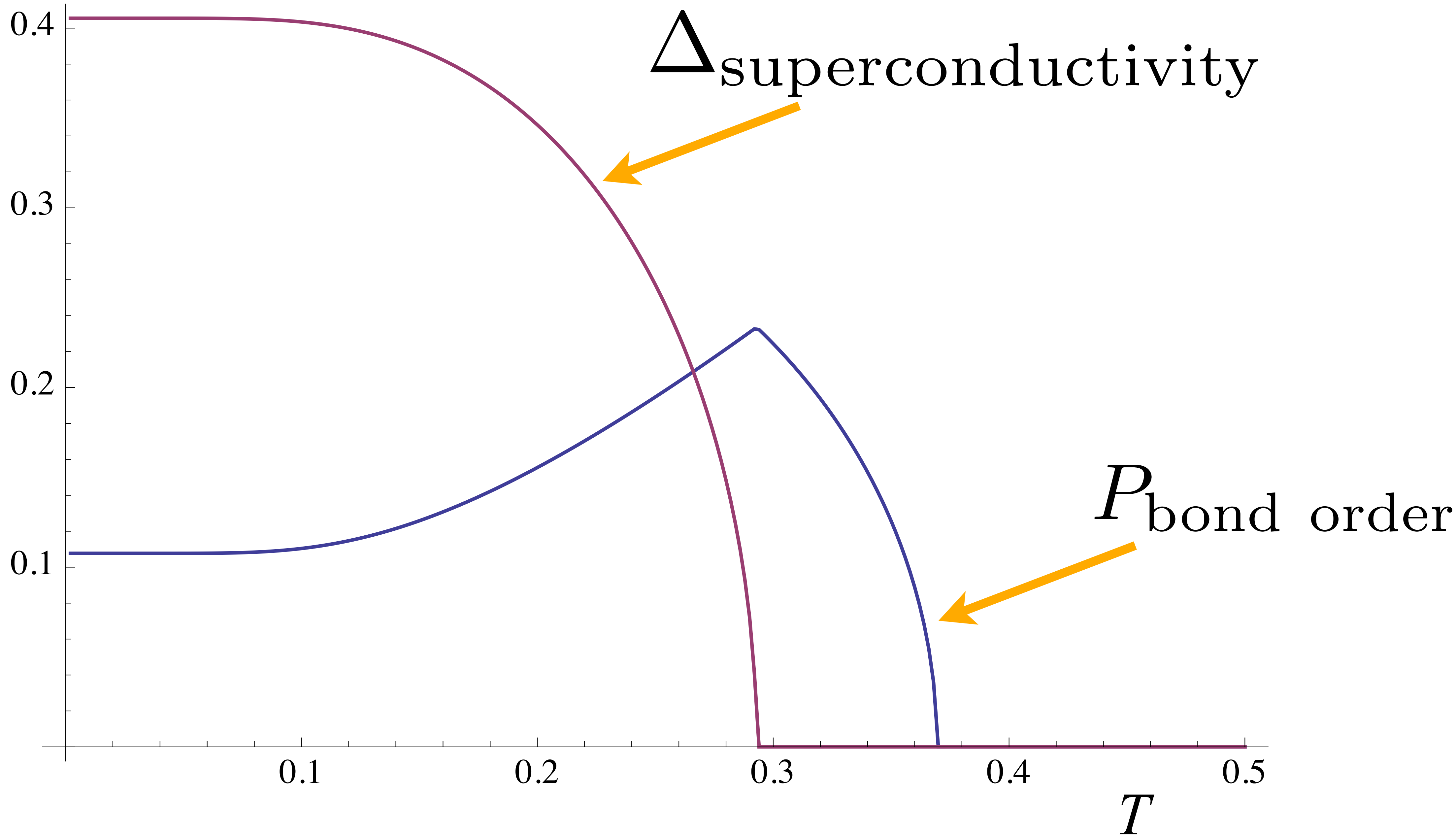
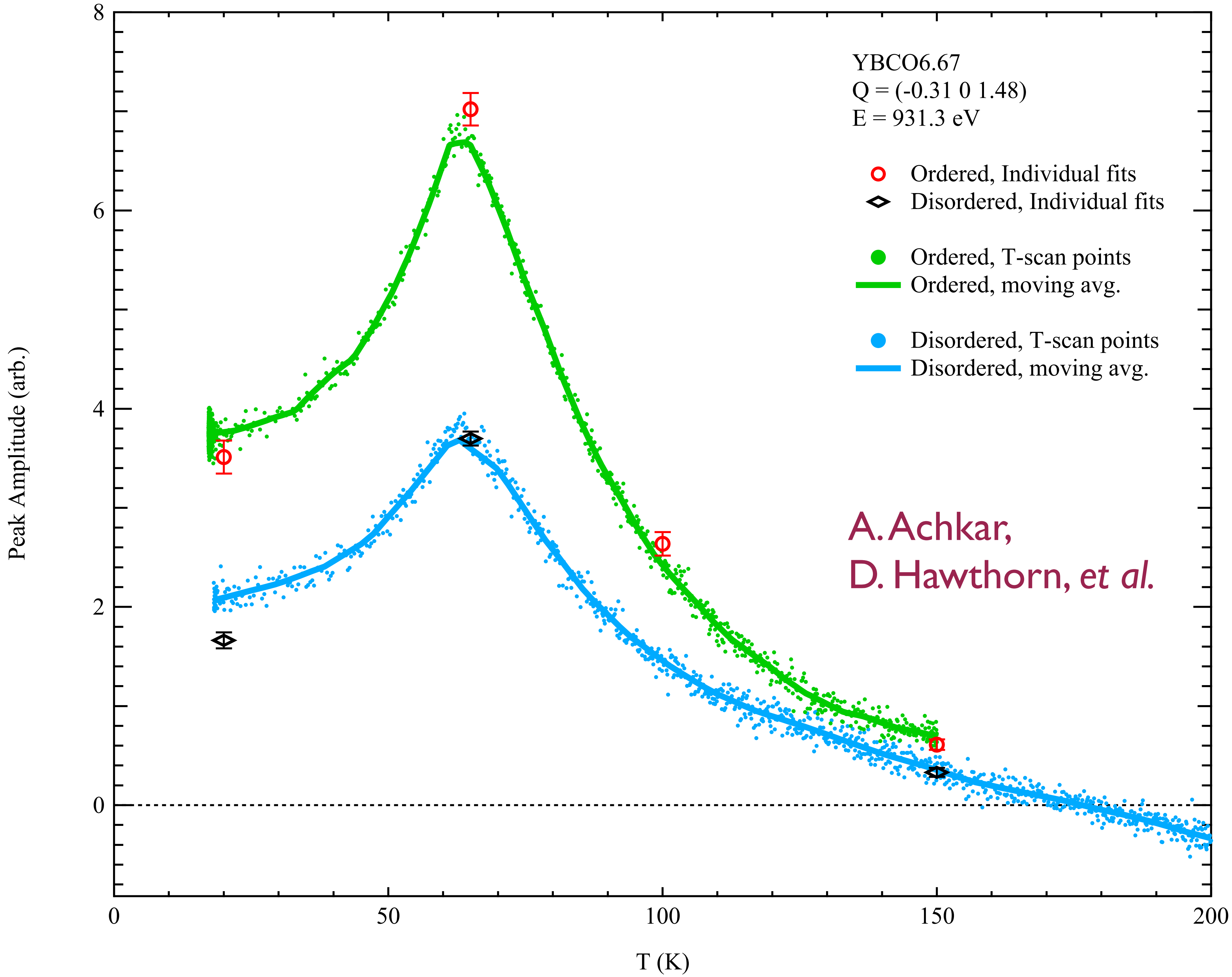
Thursday, August 29, 13

Mean-field theory of charge/bond order and *d*-wave superconductivity

Both orders are induced by a "glue" provided by the antiferromagnetic exchange interaction (details in part 2)

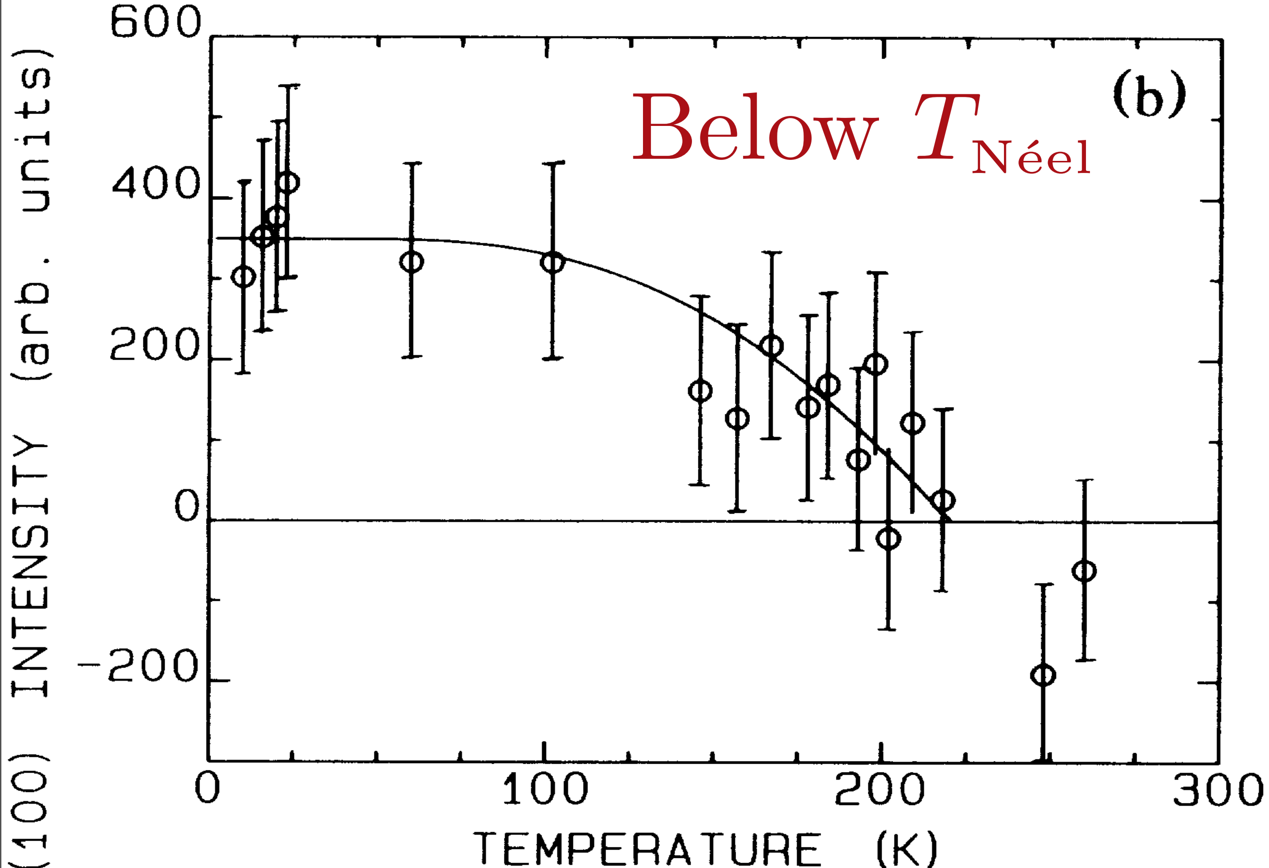


S. Sachdev et al., to appear



<u>Needed:</u> a theory to explain the maximum in the charge order at the superconducting T_c, <u>*and*</u> its gradual onset over a very wide range of T: *unlike* precursor critical fluctuations above a finite temperature ordering transition.

Key idea: analogy with the onset of antiferromagnetism in the *insulator* La_2CuO_4



D. Vaknin *et al.*,
Phys. Rev. Lett. **58**, 2802 (1987).

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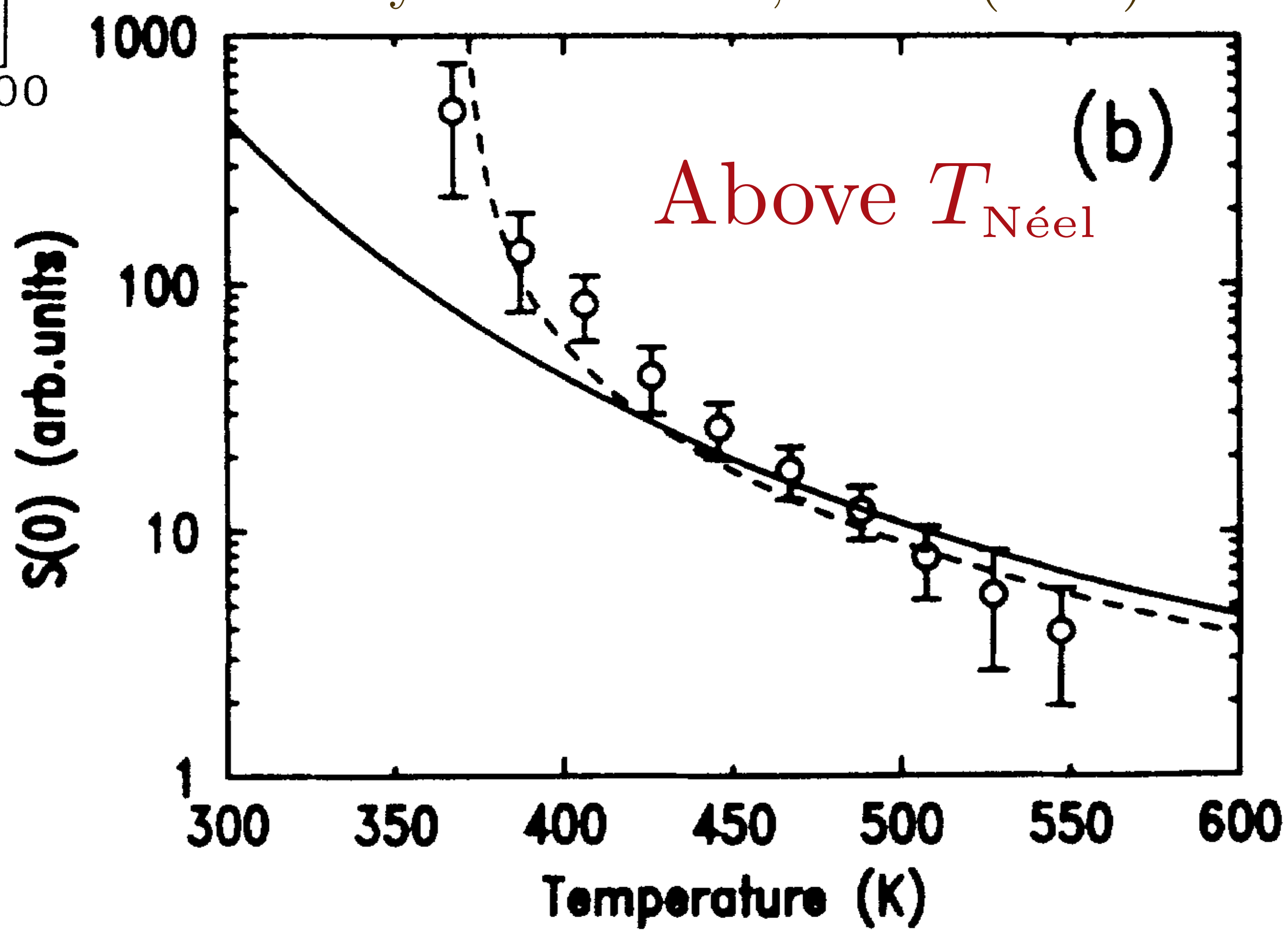

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Gradual onset of intensity over a wide range of T is a consequence of *angular thermal* fluctuations of an order parameter with 3 or more components in 2 spatial dimensions

Polyakov, 1975

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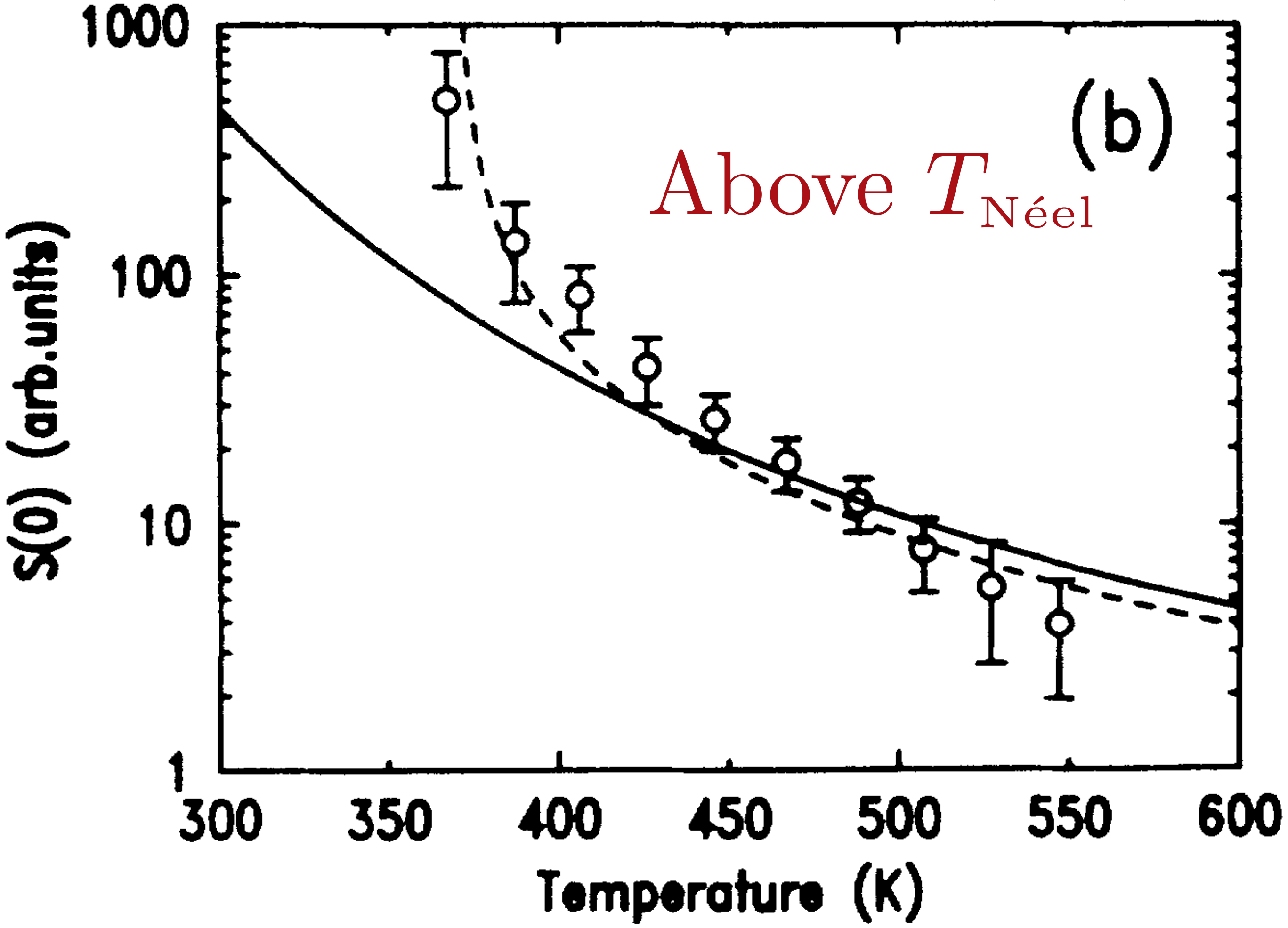

O(3) non-linear sigma model

$$Z = \int \mathcal{D}\vec{n}(x)\delta\left(\vec{n}^2(x) - 1\right)\exp\left(-\frac{\rho_s}{2T}\int d^2x\left(\nabla_x\vec{n}\right)^2\right)$$

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Generalize $\vec{n}$ to a N-component vector n_α, $\alpha = 1 \ldots N$, and take the $N \to \infty$ limit while taking $\rho_s \propto N$. This is implemented by a Lagrange multiplier λ

$$Z = \int \mathcal{D}\lambda(x)\mathcal{D}n_\alpha(x)\exp\left(-\frac{\rho_s}{2T}\int d^2x\Big[\left(\nabla_x n_\alpha\right)^2 + i\lambda(n_\alpha^2 - 1)\Big]\right)$$

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We can now perform the Gaussian integral over n_α

$$Z = \int \mathcal{D}\lambda(x)\exp\left(-\frac{N}{2}\mathrm{Tr}\ln\left(-\nabla_x^2 + i\lambda\right) + \frac{\rho_s}{2T}\int d^2x\, i\lambda\right)$$

Because $\rho_s \propto N$, in the $N \to \infty$ limit the partition function is dominated by the saddle point.

O(3) non-linear sigma model

At the saddle point, we set $i\lambda(x) = \xi^{-1}$, and then the "structure factor" $S(k)$ of the order parameter is

$$S(k) = \int d^2x \, \langle n_\alpha(x) n_\alpha(0) \rangle \, e^{ikx} = \frac{NT}{\rho_s} \frac{1}{(k^2 + \xi^{-2})}$$

This identifies ξ as the correlation length.

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This identifies ξ as the correlation length. The value of ξ is determined by the saddle-point equation, which simply enforces the constraint $n_\alpha^2(x) = 1$. So we have

$$\frac{NT}{\rho_s} \int \frac{d^2k}{4\pi^2} \frac{1}{k^2 + \xi^{-2}} = 1$$

Performing the k integral with a momentum cutoff Λ we obtain

$$\frac{NT}{4\pi\rho_s} \ln\left(1 + \Lambda^2 \xi^2\right) = 1 \quad \Rightarrow \quad \xi = \Lambda^{-1} \exp\left(\frac{2\pi\rho_s}{NT}\right)$$

So ξ is *finite* at all non-zero T (no LRO), and diverges exponentially as $T \to 0$ (consistent with Mermin-Wagner theorem).

O(3) non-linear sigma model

The *exact* result (for the exponential) at finite N is

$$\xi = \Lambda^{-1} \exp\left(\frac{2\pi\rho_s}{(N-2)T}\right)$$

O(3) non-linear sigma model

The *exact* result (for the exponential) at finite N is

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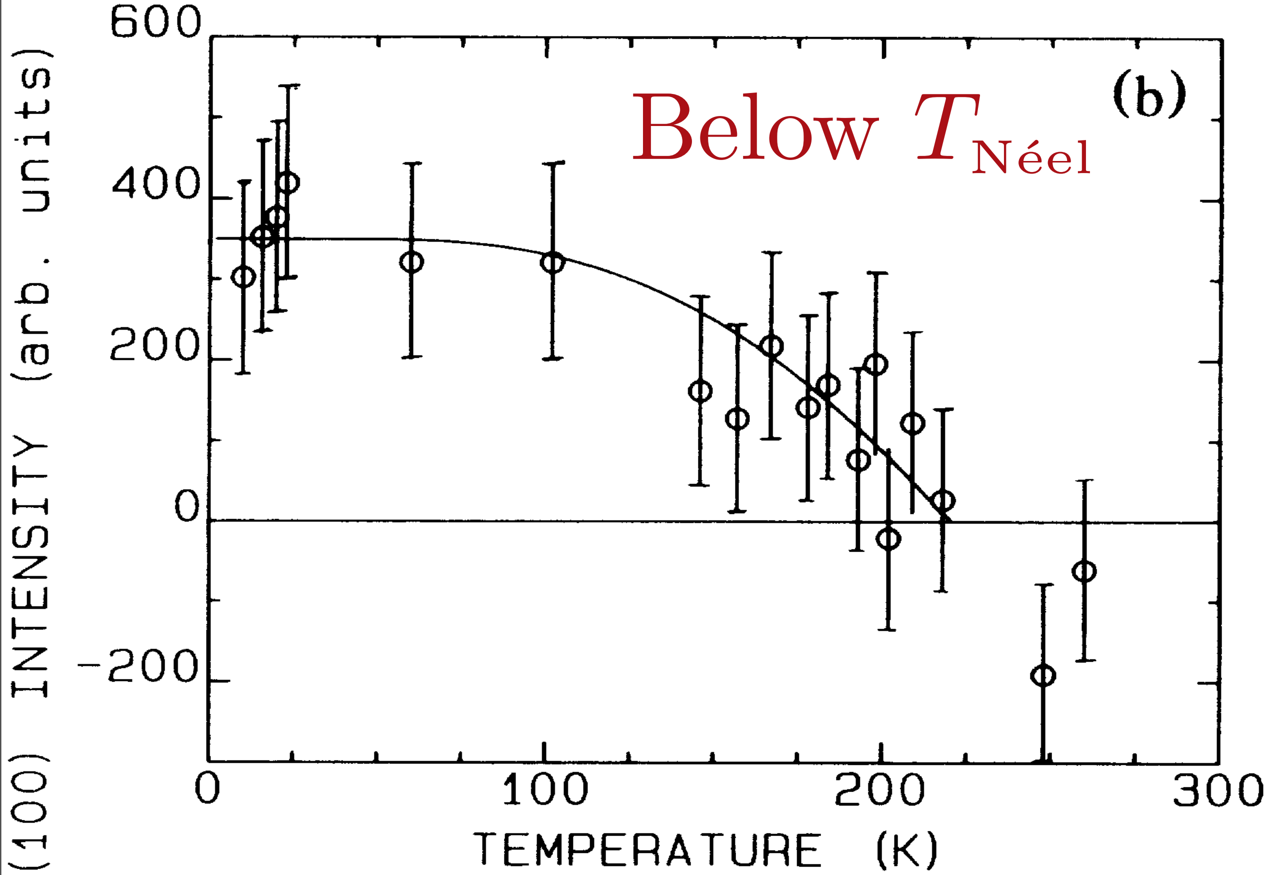
Neutron scattering measures the structure factor, and the peak value is $S(0)$

$$S(0) = \frac{NT}{\rho_s}\xi^2 = \frac{NT}{\Lambda^2\rho_s} \exp\left(\frac{4\pi\rho_s}{(N-2)T}\right)$$

So there is no Bragg peak at the ordering wavevector for any two-dimensional antiferromagnet.

La_2CuO_4 has a non-zero ordering temperature $T_N =$ 325K, and this arises solely from the *inter-layer* coupling.

Key idea: analogy with the onset of antiferromagnetism in the *insulator* La_2CuO_4



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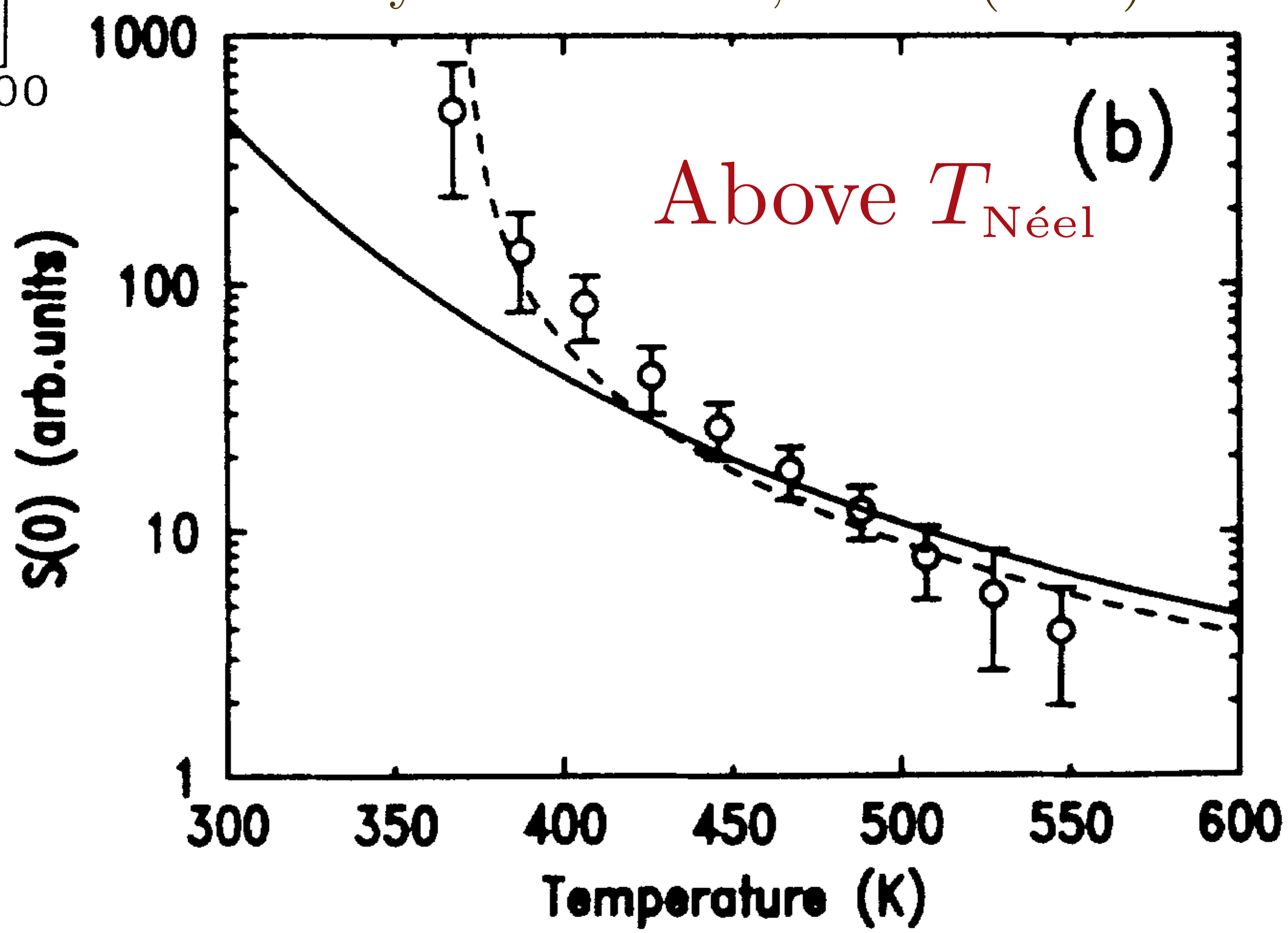

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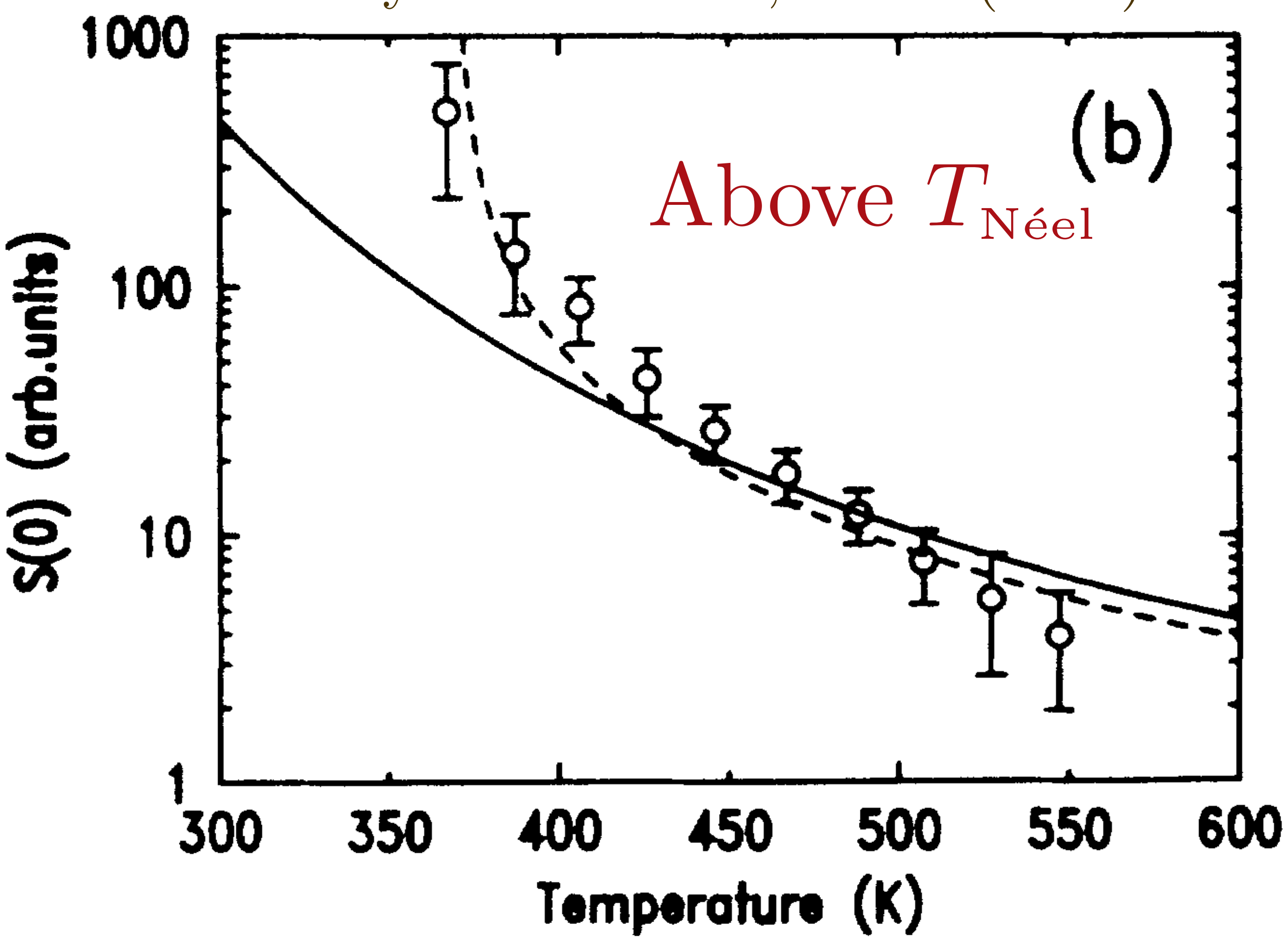

Multi-component order parameter

Superconducting order $\Psi(\mathbf{r})$:

$$\left\langle c_{i\alpha}^{\dagger} c_{j\beta}^{\dagger} \right\rangle = \varepsilon_{\alpha\beta} \left[\sum_{\mathbf{k}} \Delta_0(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] \Psi\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right)$$

Charge/bond order $\Phi_{x,y}(\mathbf{r})$ at wavevectors $\mathbf{Q}_{x,y}$:

$$\begin{aligned} \left\langle c_{i\alpha}^{\dagger} c_{j\beta} \right\rangle = & \delta_{\alpha\beta} \left[\sum_{\mathbf{k}} P_x(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}_x\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_x\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right) \\ & + \delta_{\alpha\beta} \left[\sum_{\mathbf{k}} P_y(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}_y\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_y\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right) \end{aligned}$$

Multi-component order parameter

Symmetries:
Charge conservation (O(2)), x translations (O(2)), y translations (O(2)), $x \leftrightarrow y$ ($\mathbb{Z}_2$), inversion, time-reversal.
Landau-Ginzburg free energy:

$$
\begin{aligned}
F = \int d^2 r \Big[& |\nabla \Psi|^2 + s_1 |\Psi|^2 + u_1 |\Psi|^4 \\
& + |\nabla \Phi_x|^2 + |\nabla \Phi_y|^2 + s_2 \left(|\Phi_x|^2 + |\Phi_y|^2 \right) \\
& + u_2 \left(|\Phi_x|^2 + |\Phi_y|^2 \right)^2 + w \left(|\Phi_x|^4 + |\Phi_y|^4 \right) \\
& + v |\Psi|^2 \left(|\Phi_x|^2 + |\Phi_y|^2 \right) \Big]
\end{aligned}
$$

Competing orders: v is positive (and large).

PHYSICAL REVIEW B **69**, 144504 (2004)

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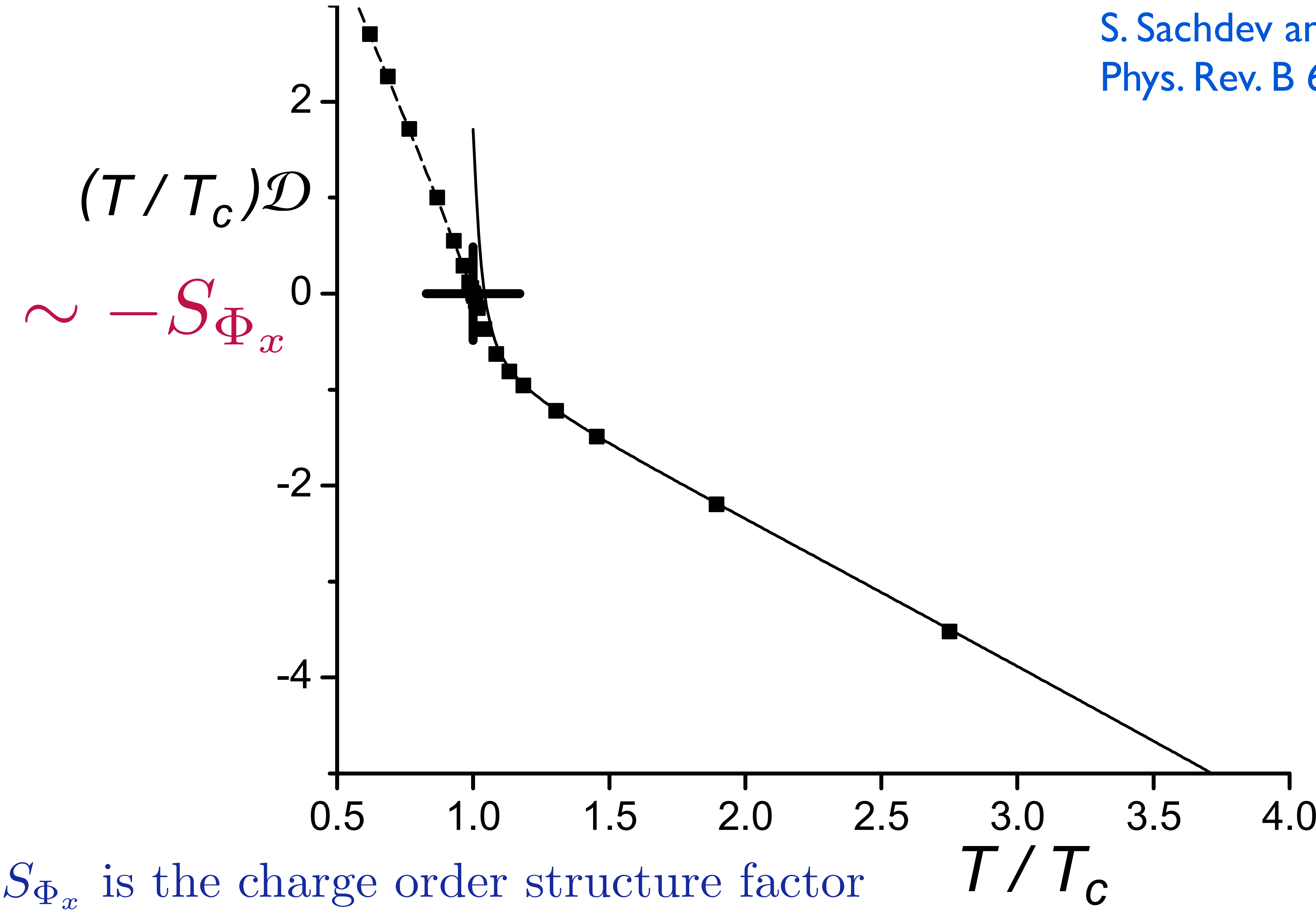
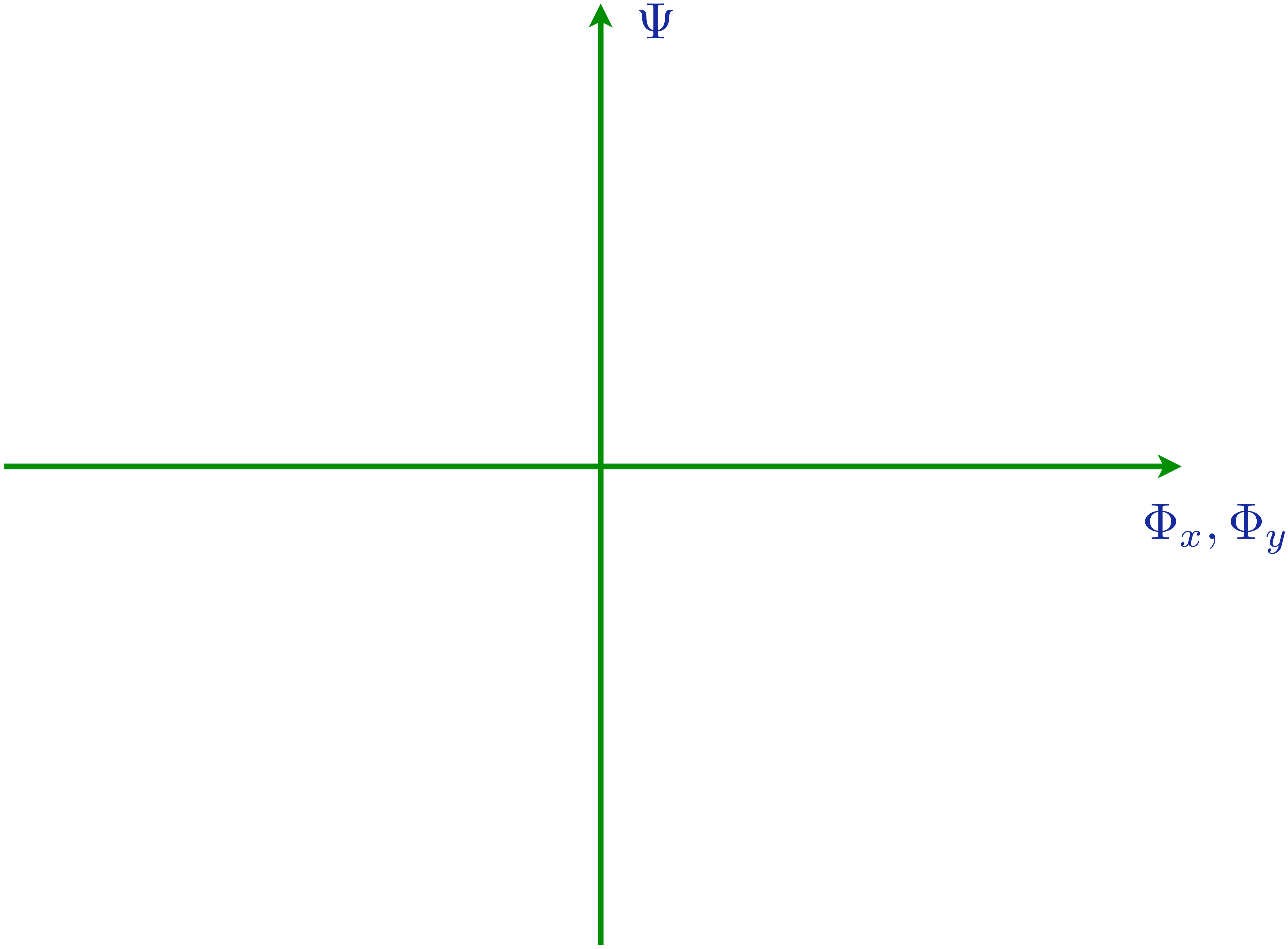


S_{Φ_x} is the charge order structure factor

FIG. 2: Plots of the universal function $(T/T_c)\mathcal{D}(g, T/T_c)$ as a function of T/T_c for $\rho_s(0)/(k_B T_c) = 6.8$. From (2.3) we see that $\langle|\Psi|^2\rangle_T = (T/T_c)\langle|\Psi|^2\rangle_{T_c} + (m^* k_B T_c/\hbar^2)(T/T_c)\mathcal{D}(g, T/T_c)$; so $\langle|\Psi|^2\rangle_T$ is determined from the above plot up to an additive, non-singular, linear dependence on T determined by $\langle|\Psi|^2\rangle_{T_c}$. This linear T dependence can compensate for the the linear T dependence in the plot above so that $\langle|\Psi|^2\rangle_T$ saturates at high T. Also, as noted in the text, the present theory breaks down at large enough T, and its main utility is in capturing the singular increase in $\langle|\Psi|^2\rangle_T$ as T crosses T_c. The solid line is the small $\mathcal{G}$ approximation obtained by solving (3.5), (3.7), (3.10),

Multi-component order parameter



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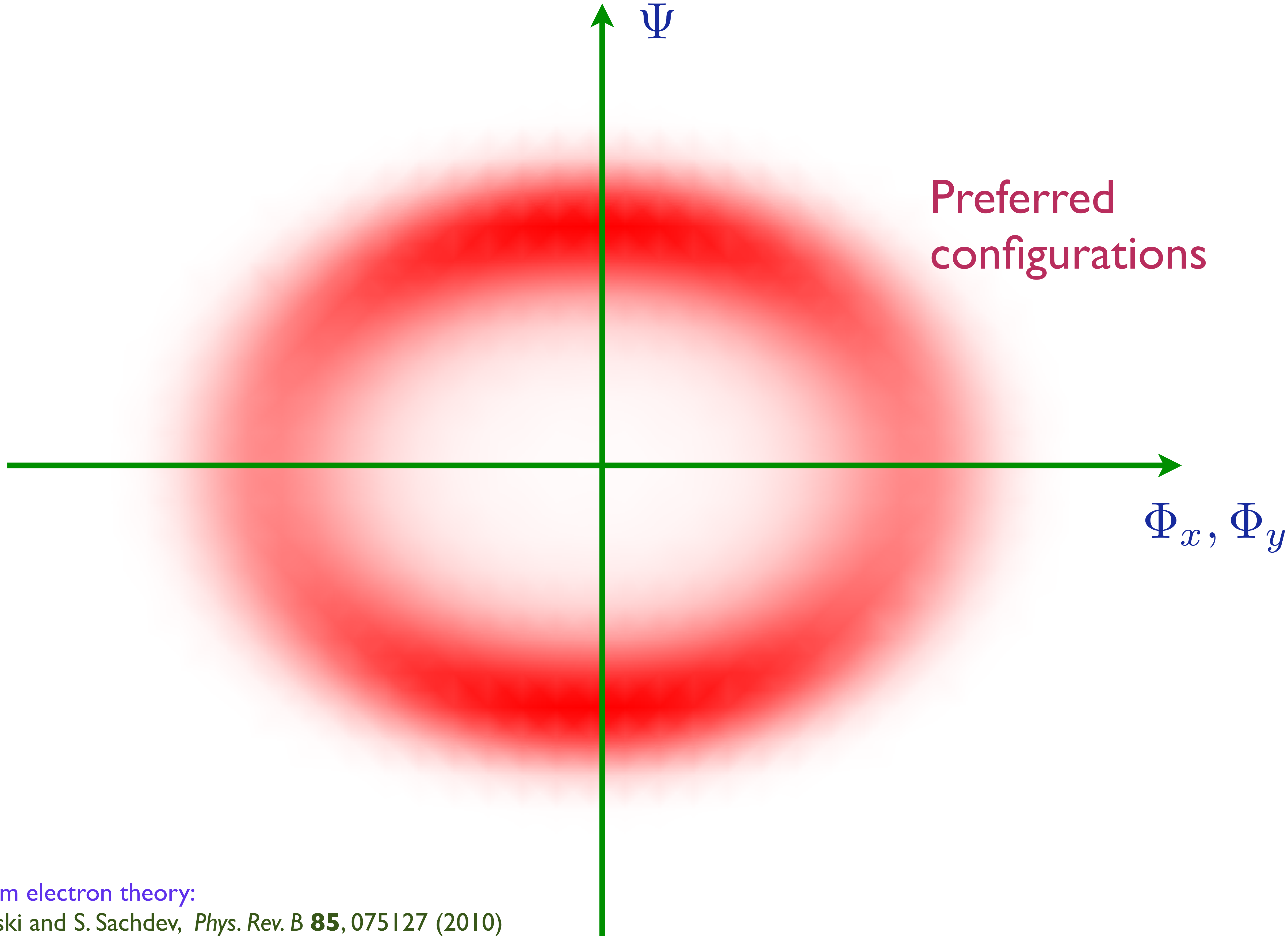

Support from electron theory:

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

K. B. Efetov, H. Meier, and C. Pepin, *Nature Physics* **9**, 442 (2013)

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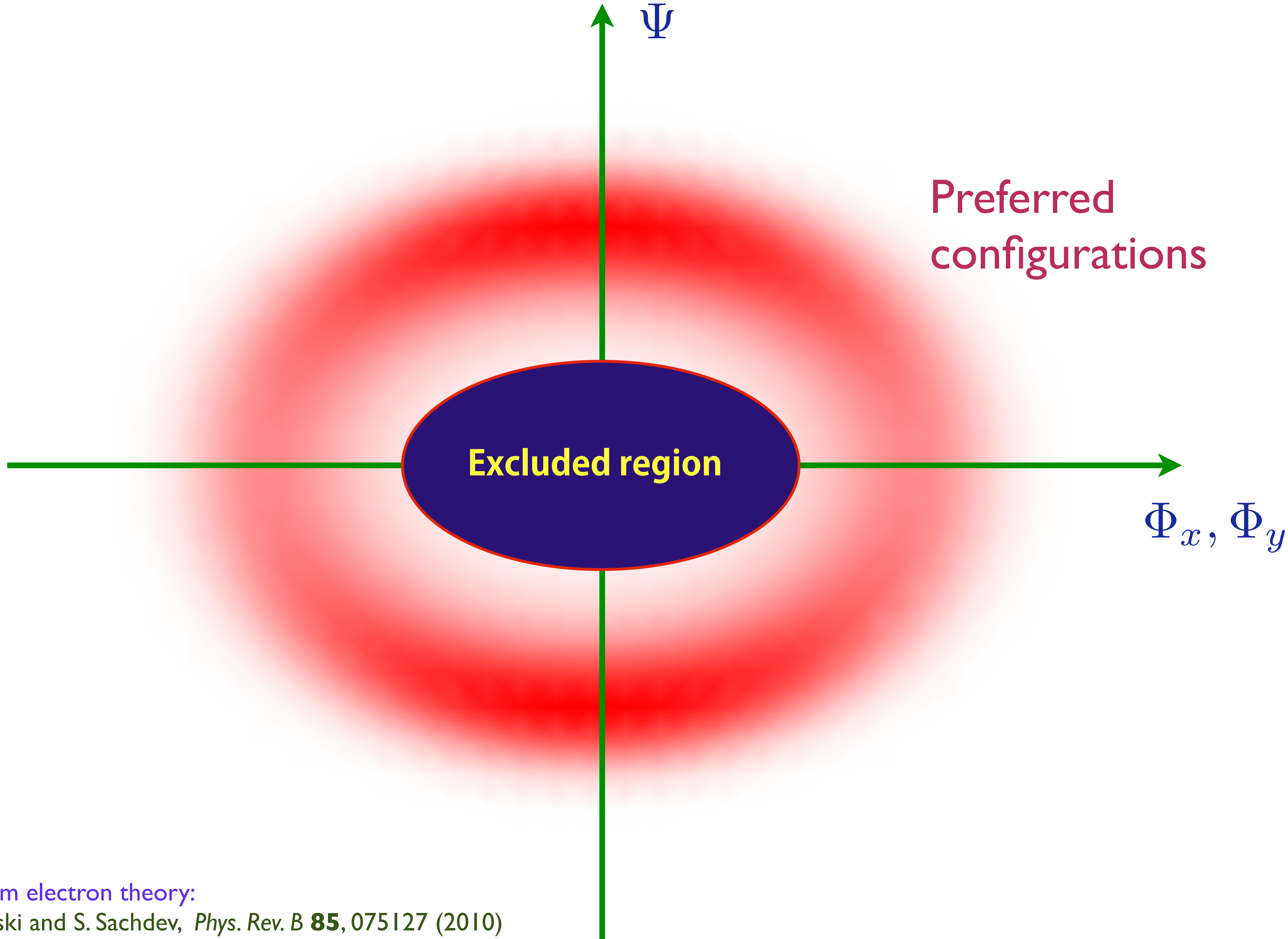

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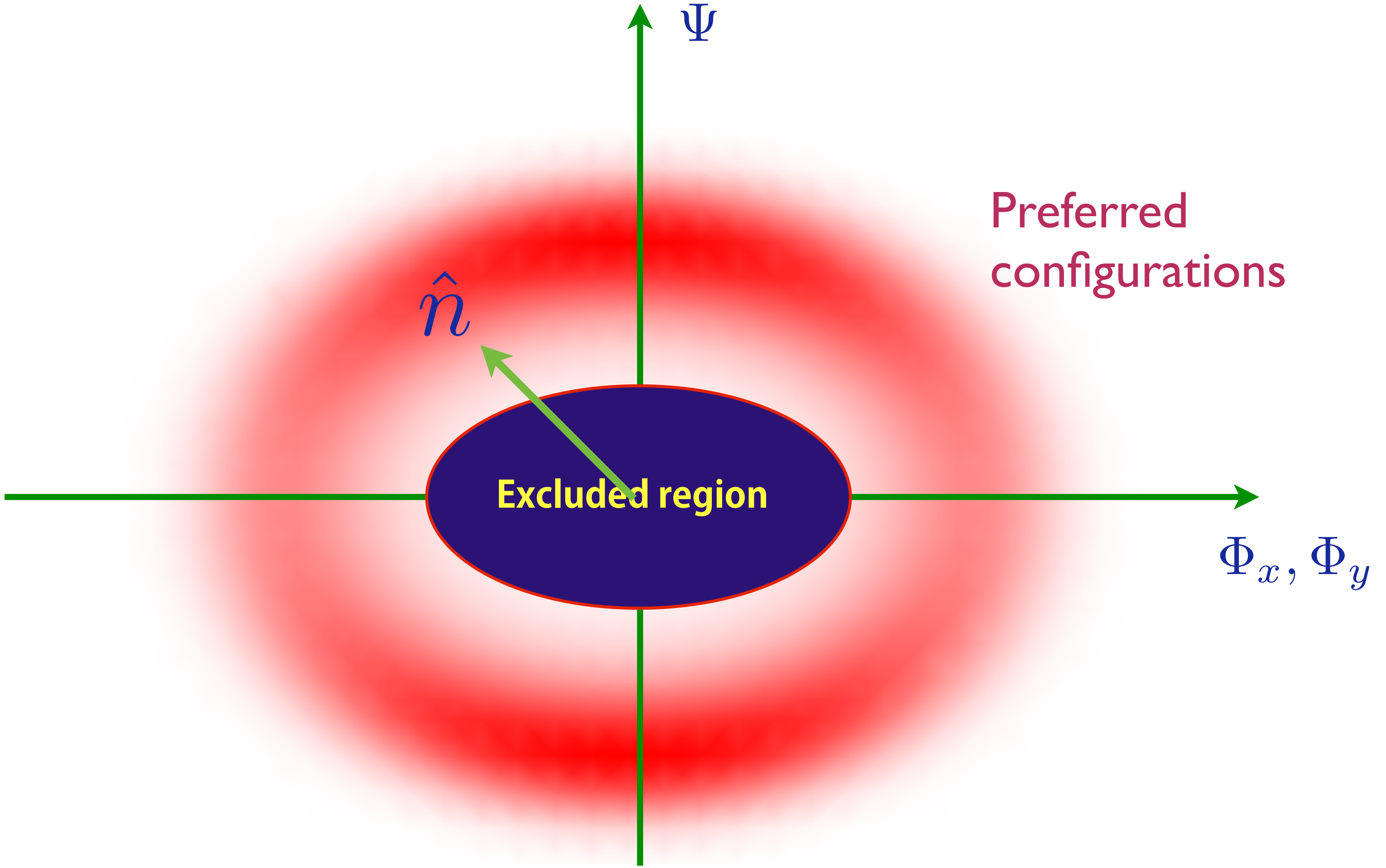
K. B. Efetov, H. Meier, and C. Pepin, *Nature Physics* **9**, 442 (2013)

Multi-component order parameter



Label order parameter by a
6-component unit vector n_α with $\sum_\alpha n_\alpha^2 = 1$

O(6) non-linear sigma model

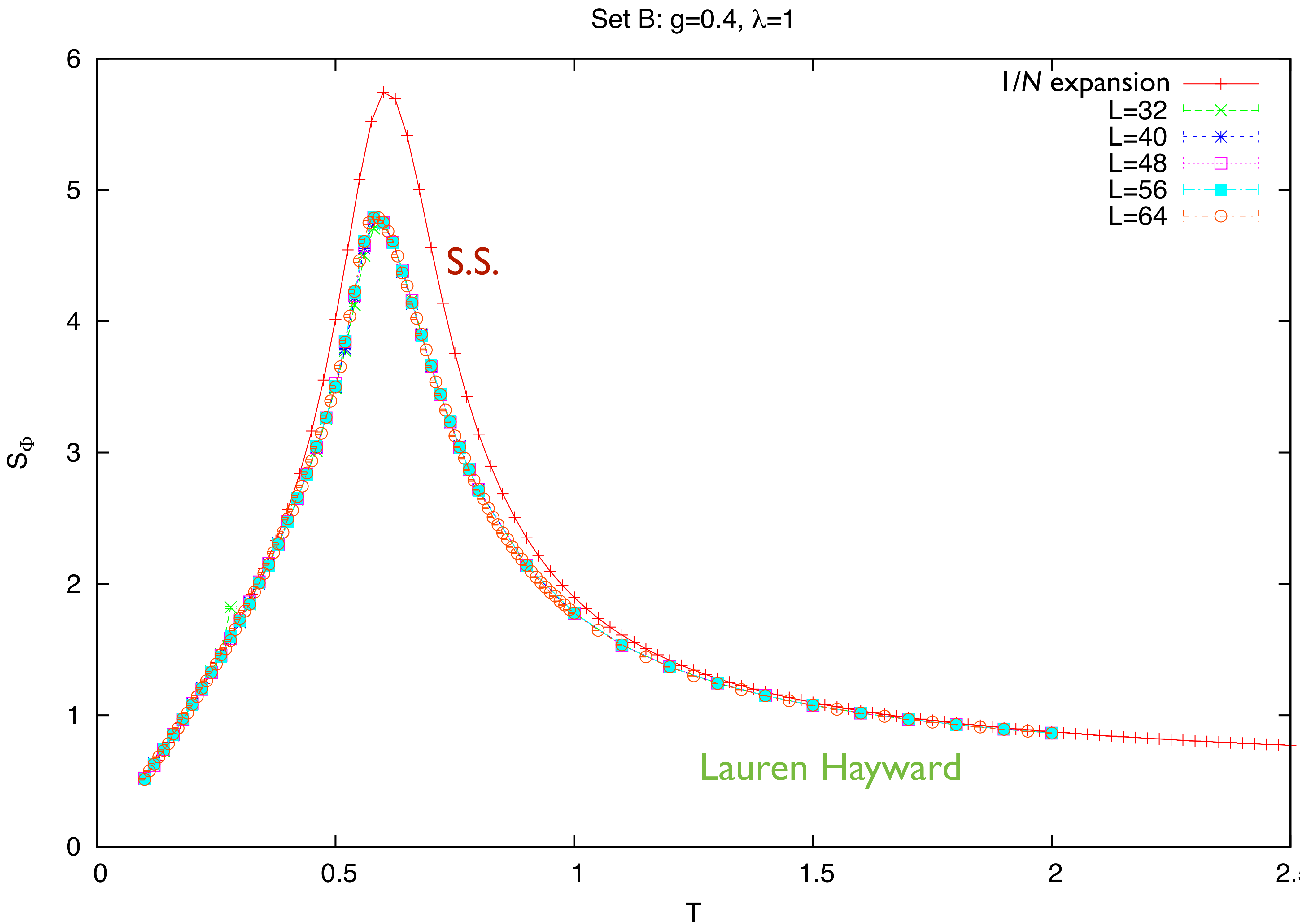
$$Z = \prod_i \left[\int dn_{i\alpha} \delta \left(\sum_{\alpha=1}^{6} n_{i\alpha}^2 - 1 \right) \right] \exp\Bigg(- \frac{1}{2T} \sum_{\langle ij \rangle} \Big[\sum_{\alpha=1}^{2} (n_{i\alpha} - n_{j\alpha})^2 + \lambda \sum_{\alpha=3}^{6} (n_{i\alpha} - n_{j\alpha})^2 \Big] - \frac{g}{2T} \sum_i \sum_{\alpha=3}^{6} n_{i\alpha}^2 - \frac{w}{2T} \sum_i \left[\left(n_{i3}^2 + n_{i4}^2 \right)^2 + \left(n_{i5}^2 + n_{i6}^2 \right)^2 \right] \Bigg)$$

where $\Psi \propto n_1 + i n_2$, $\Phi_x \propto n_3 + i n_4$, $\Phi_y \propto n_5 + i n_6$.

Describes $\mathrm{O}(6) \Rightarrow \mathrm{O}(2) \times \mathrm{O}(2) \times \mathrm{O}(2) \rtimes \mathbb{Z}_2$.

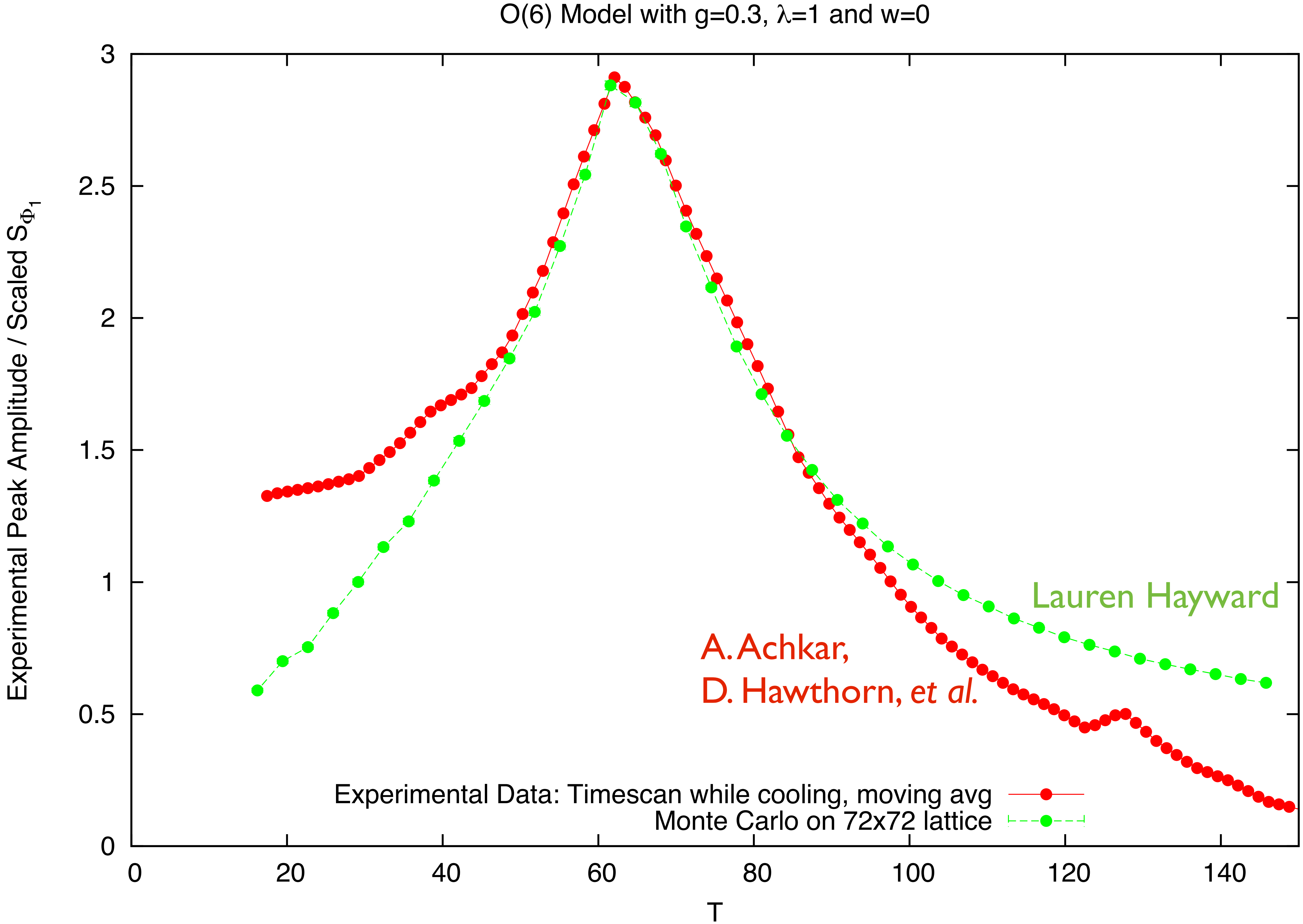
Solve by cluster Monte Carlo and $1/N$ expansion.

O(6) non-linear sigma model



Charge order structure factor S_{Φ_x}

O(6) non-linear sigma model



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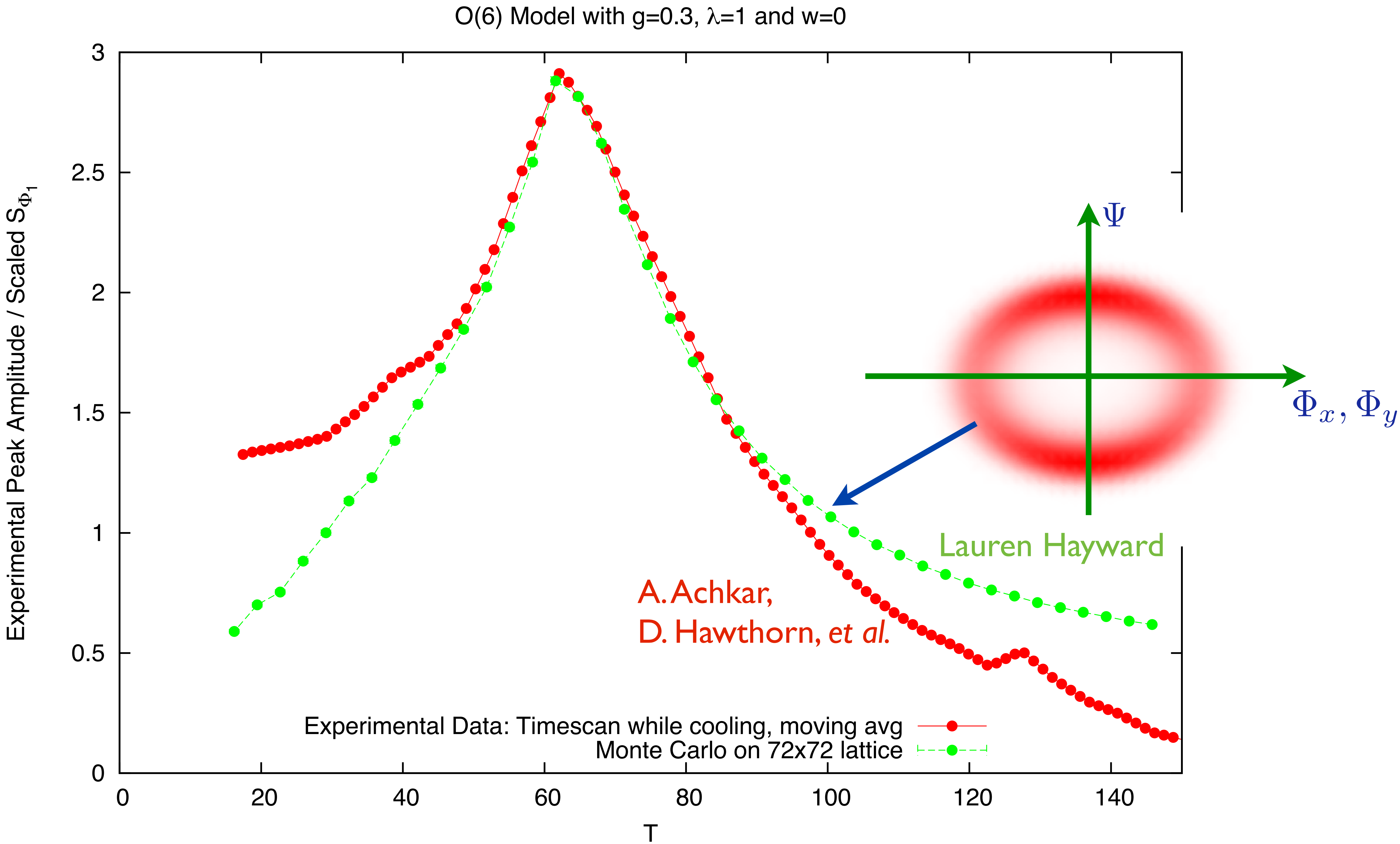
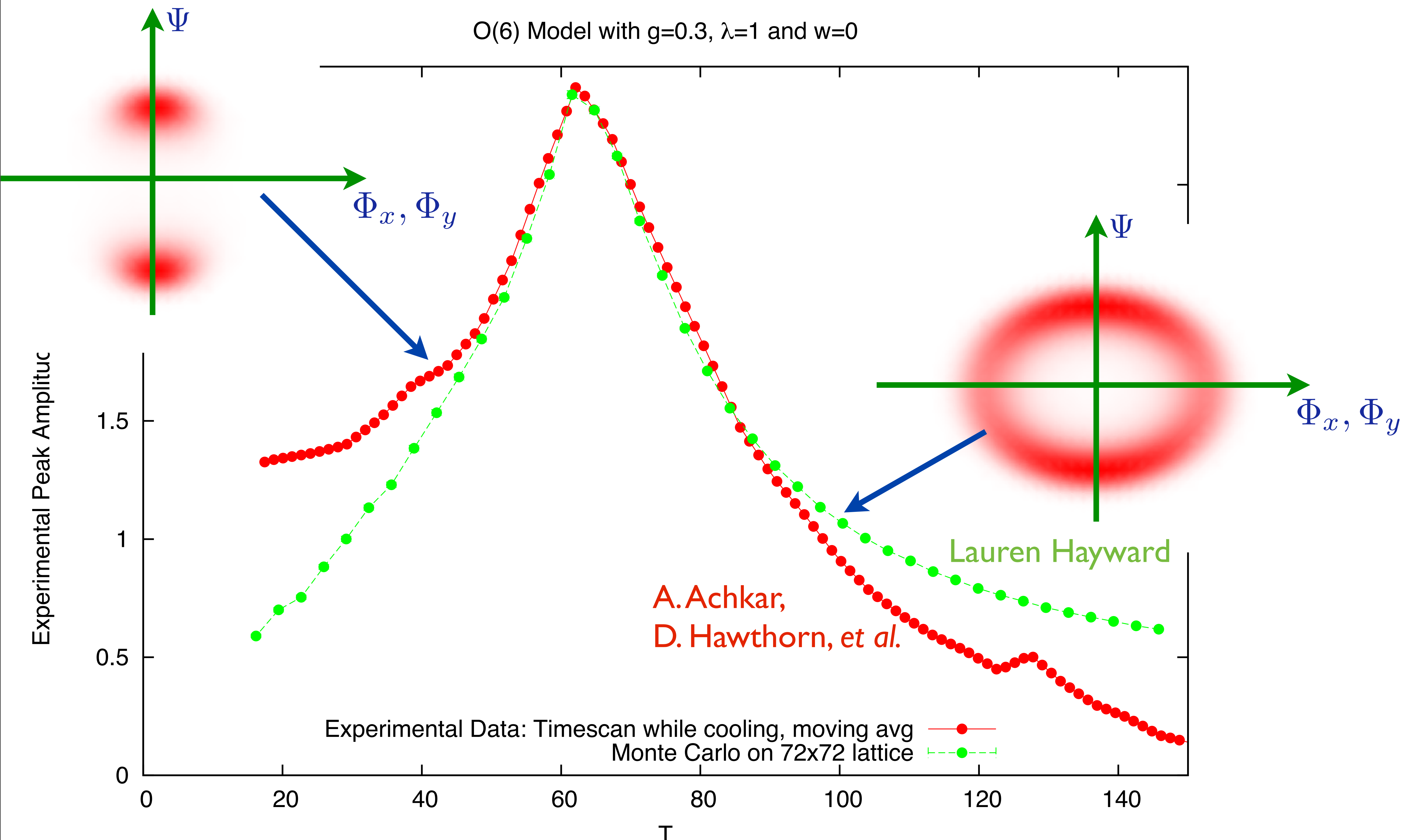
O(6) non-linear sigma model



Charge order structure factor S_{Φ_x}

O(6) non-linear sigma model



Charge order structure factor S_{Φ_x}

O(6) non-linear sigma model

In progress: possibility of Ising transition in *nematic* order parameter $m = |\Phi_x|^2 - |\Phi_y|^2$.

This is expected to be present for $w < 0$ at sufficiently small T, and is found in the $N = \infty$ theory.

- Pseudogap:
Angular fluctuations of a multi-component order parameter