

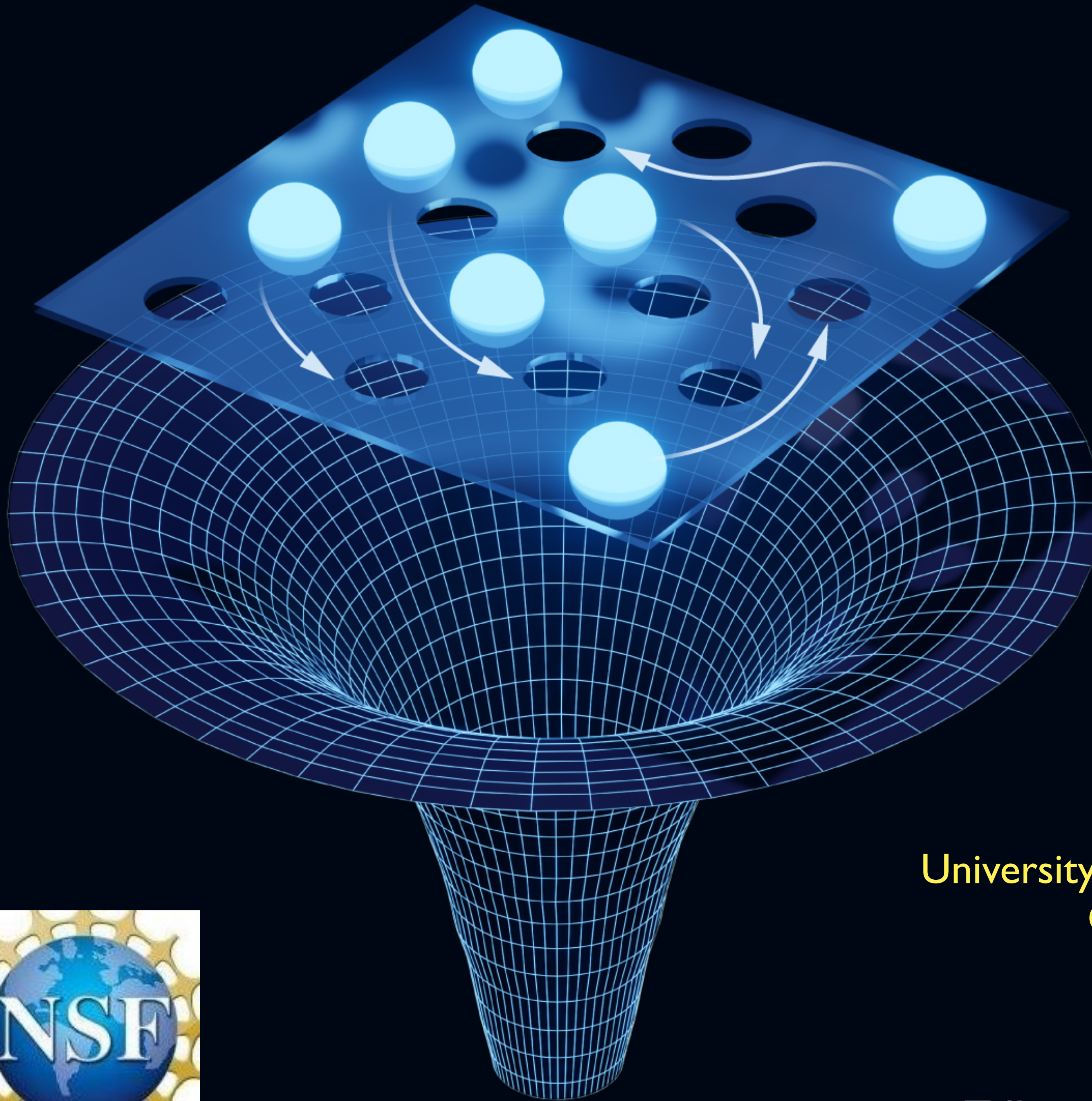
Quantum statistical mechanics of strange metals and charged black holes

D. Chowdhury, A. Georges, O. Parcollet, and S. S.,
Reviews of Modern Physics **94**, 035004 (2022)

University of Capetown, South Africa
October 14, 2022

Subir Sachdev

Talk online: sachdev.physics.harvard.edu



1. SYK model

2. Charged black holes

3. Strange metals

4. From Yukawa-SYK models to a universal theory of strange metals

SYK
model

The Sachdev-Ye-Kitaev (SYK) model

(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} - \mu \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

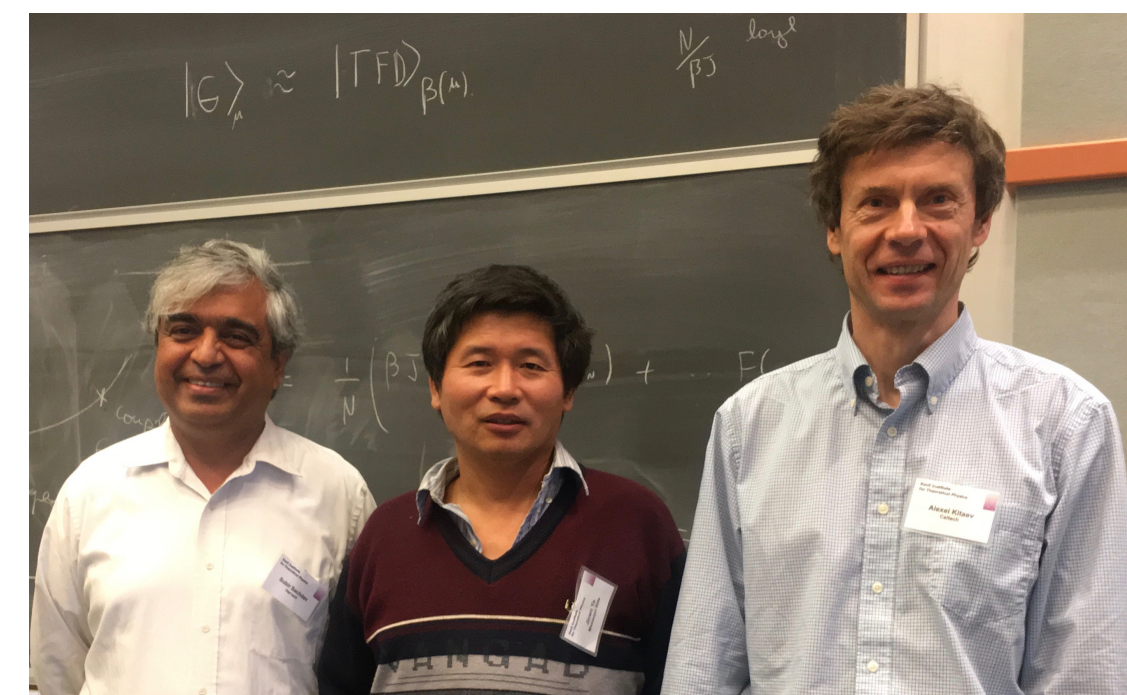
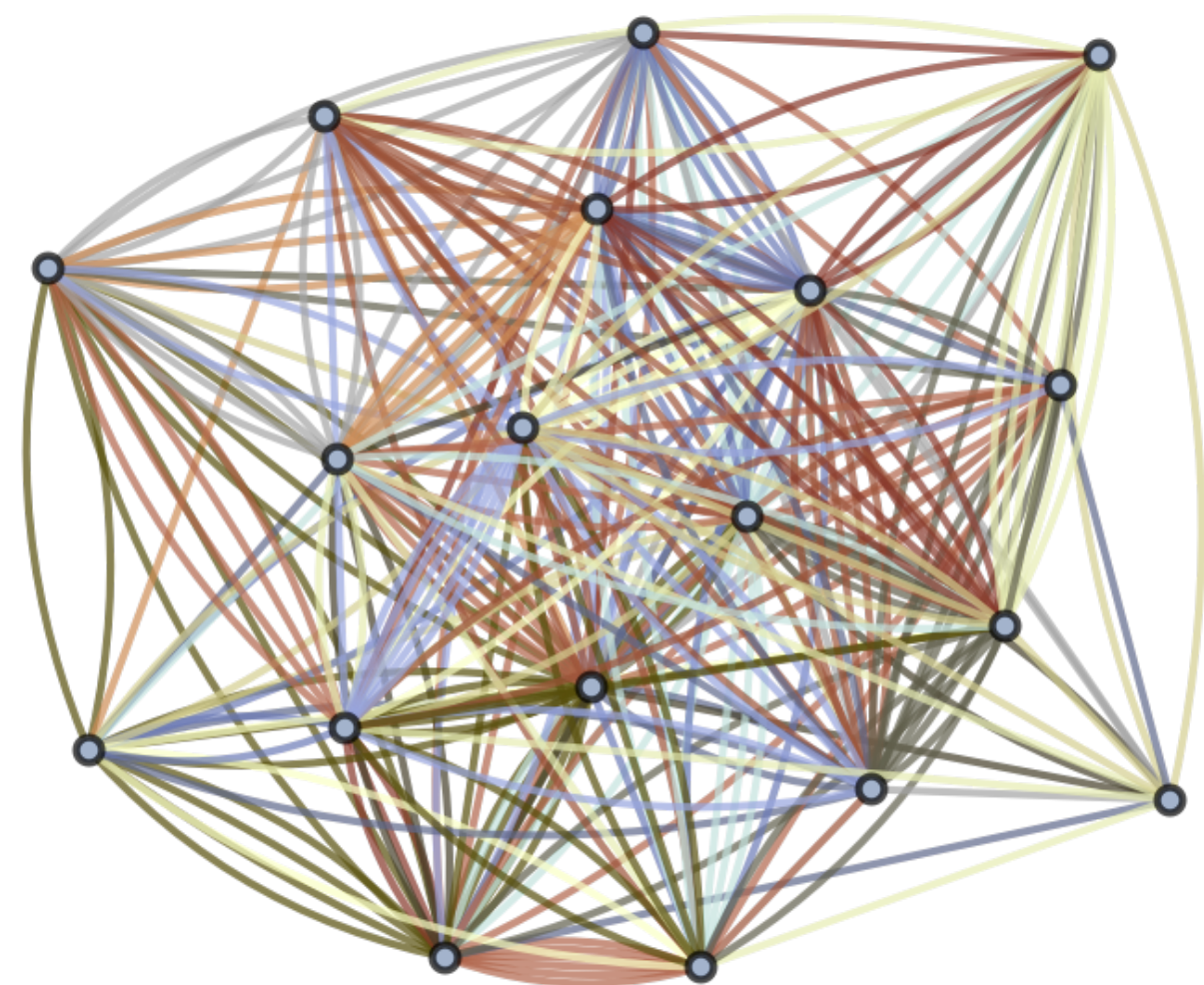
$$c_{\alpha} c_{\beta} + c_{\beta} c_{\alpha} = 0 \quad , \quad c_{\alpha} c_{\beta}^{\dagger} + c_{\beta}^{\dagger} c_{\alpha} = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}; \quad [\mathcal{H}, \mathcal{Q}] = 0; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
 $N \rightarrow \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)



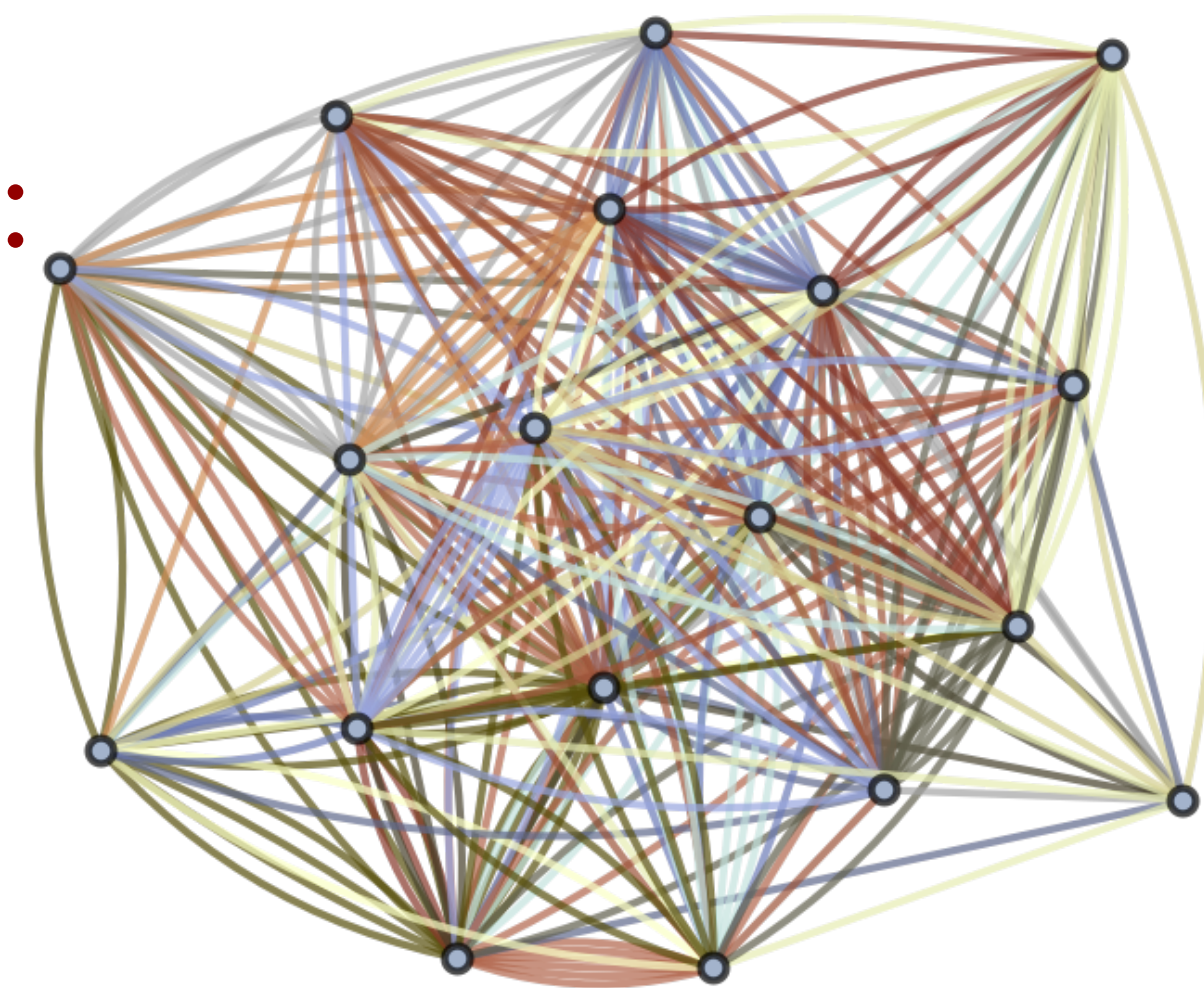
The Sachdev-Ye-Kitaev (SYK) model

The disorder-averaged partition is exactly this G - Σ theory:

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1, \tau_2) \mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NI)$$

$$I = \ln \det [\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)]$$

$$+ \int d\tau_1 d\tau_2 [\Sigma(\tau_1, \tau_2)G(\tau_2, \tau_1) + (U^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)]$$



Saddle-point equations for $G(\tau_1 - \tau_2)$ and $\Sigma(\tau_1 - \tau_2)$:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau)G(-\tau)$$

$$G(\tau = 0^-) = Q.$$

The Sachdev-Ye-Kitaev (SYK) model

$$\begin{aligned} \mathcal{Z}(Q, T) &= \text{Tr}_{\mathcal{Q}} \exp\left(-\frac{\mathcal{H}}{T}\right) = \exp(-F/T); \quad \text{Entropy } S = -\frac{\partial F}{\partial T}. \\ &\equiv \int_{E_0}^{\infty} D(E) e^{-E/T}; \quad D(E) = \sum_i \delta(E - E_i); \quad \mathcal{H} |\Psi_i\rangle = E_i |\Psi_i\rangle \end{aligned}$$

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Main result from effective theory of quantum gravity

$$D(E) \sim N^{-1} \exp(N s_0) \sinh(\sqrt{2N\gamma E}); \quad S = N(s_0 + \gamma T) - \frac{3}{2} \ln\left(\frac{N^{1/3} U}{T}\right)$$

At $\mathcal{Q} = 1/2$, we have

$$s_0 = \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4} = 0.464847699170805107492692486833 \dots$$

and $\gamma = \#/U$ is non-universal

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A. Georges, O. Parcollet, and S. Sachdev,
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JHEP 05 (2017) 118

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Yingfei Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, JHEP 02 (2020) 157

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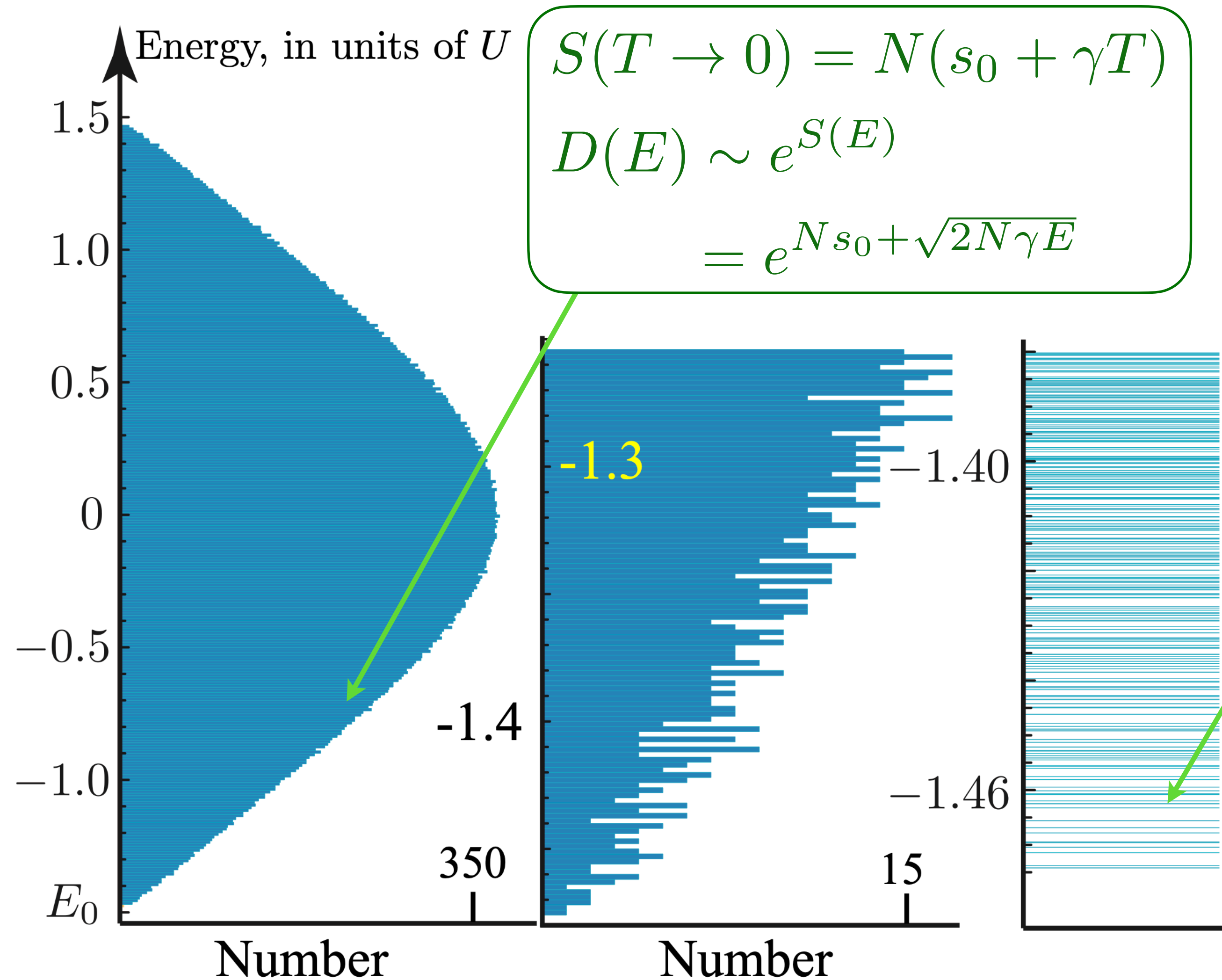
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Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

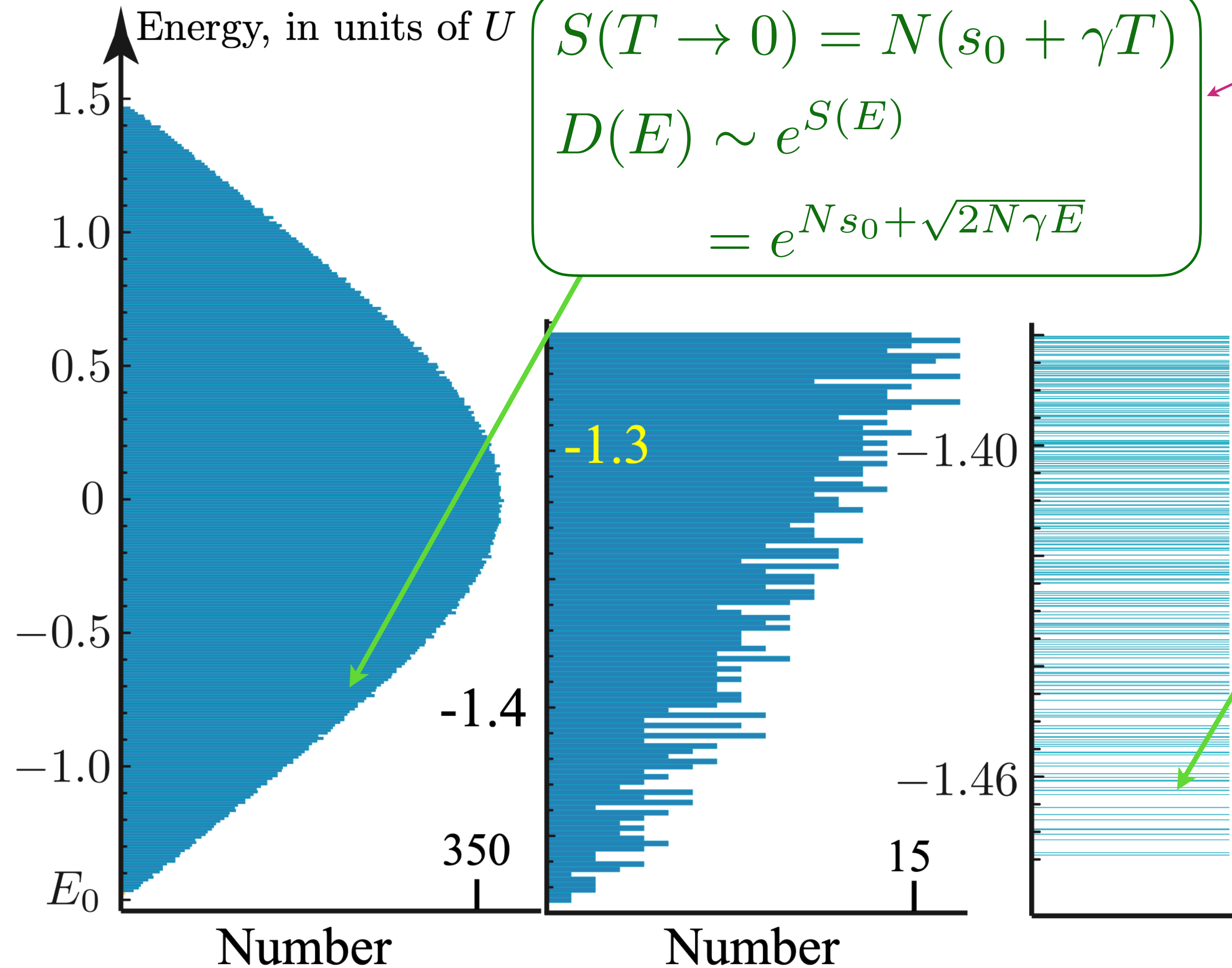


No exponentially large degeneracy, but exponentially small level spacing!
 No quasiparticle decomposition: wavefunctions change chaotically from one state to the next.

Complex SYK model

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$$S(T \rightarrow 0) = N(s_0 + \gamma T)$$

$$D(E) \sim e^{S(E)}$$

$$= e^{N s_0 + \sqrt{2N\gamma E}}$$

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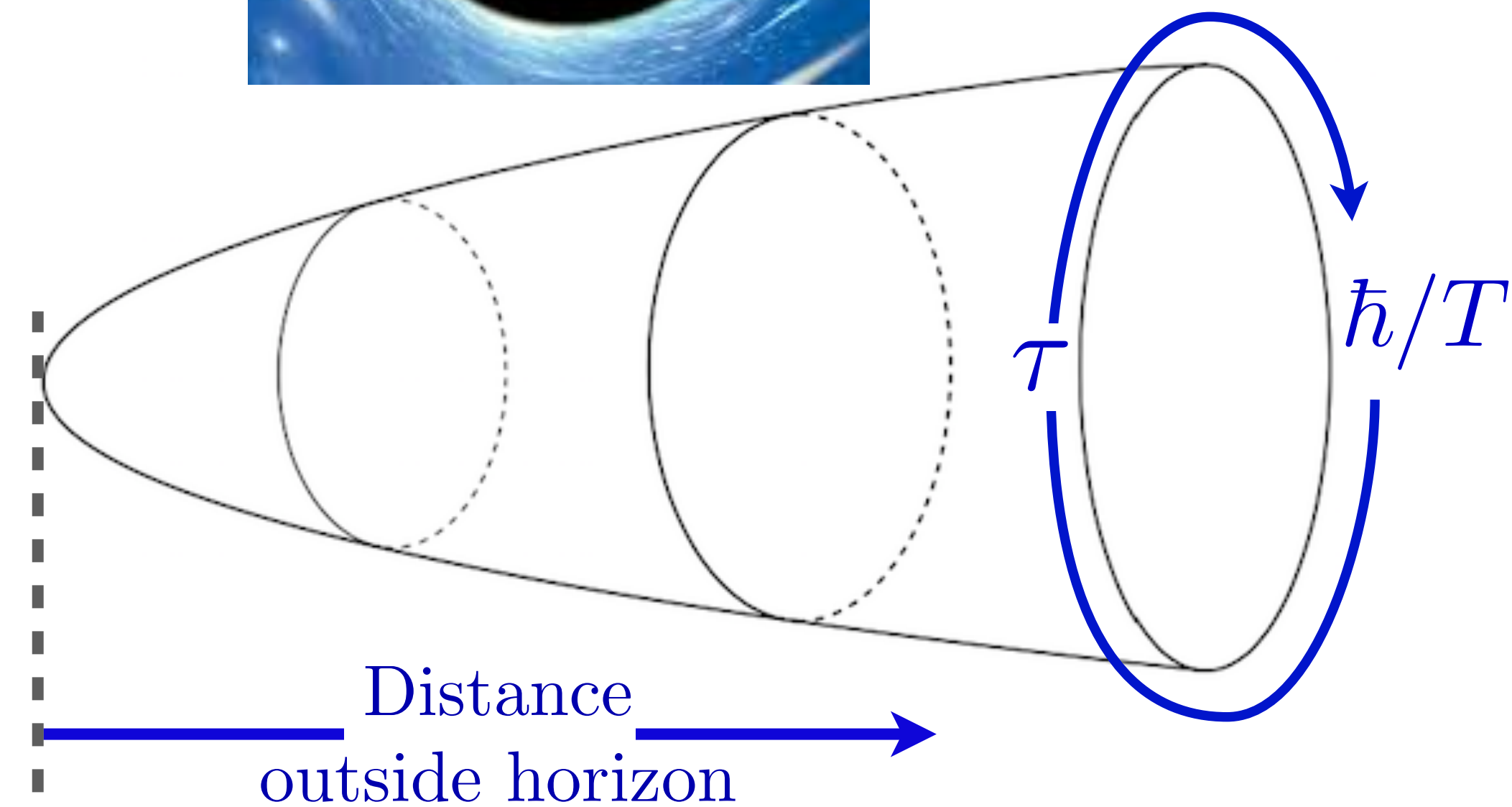
Charged black holes

[arXiv:2205.02285](https://arxiv.org/abs/2205.02285)

Thermodynamics of quantum black holes with charge Q :



$$\mathcal{Z}(Q, T) = \int \mathcal{D}g_{\mu\nu} \mathcal{D}A_{\mu} \exp \left(-\frac{1}{\hbar} I_{\text{Einstein gravity+Maxwell EM}}^{(3+1)}[g_{\mu\nu}, A_{\mu}] \right)$$



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$$= \exp(S_{BH}) \times \left(\dots????\dots \right)$$

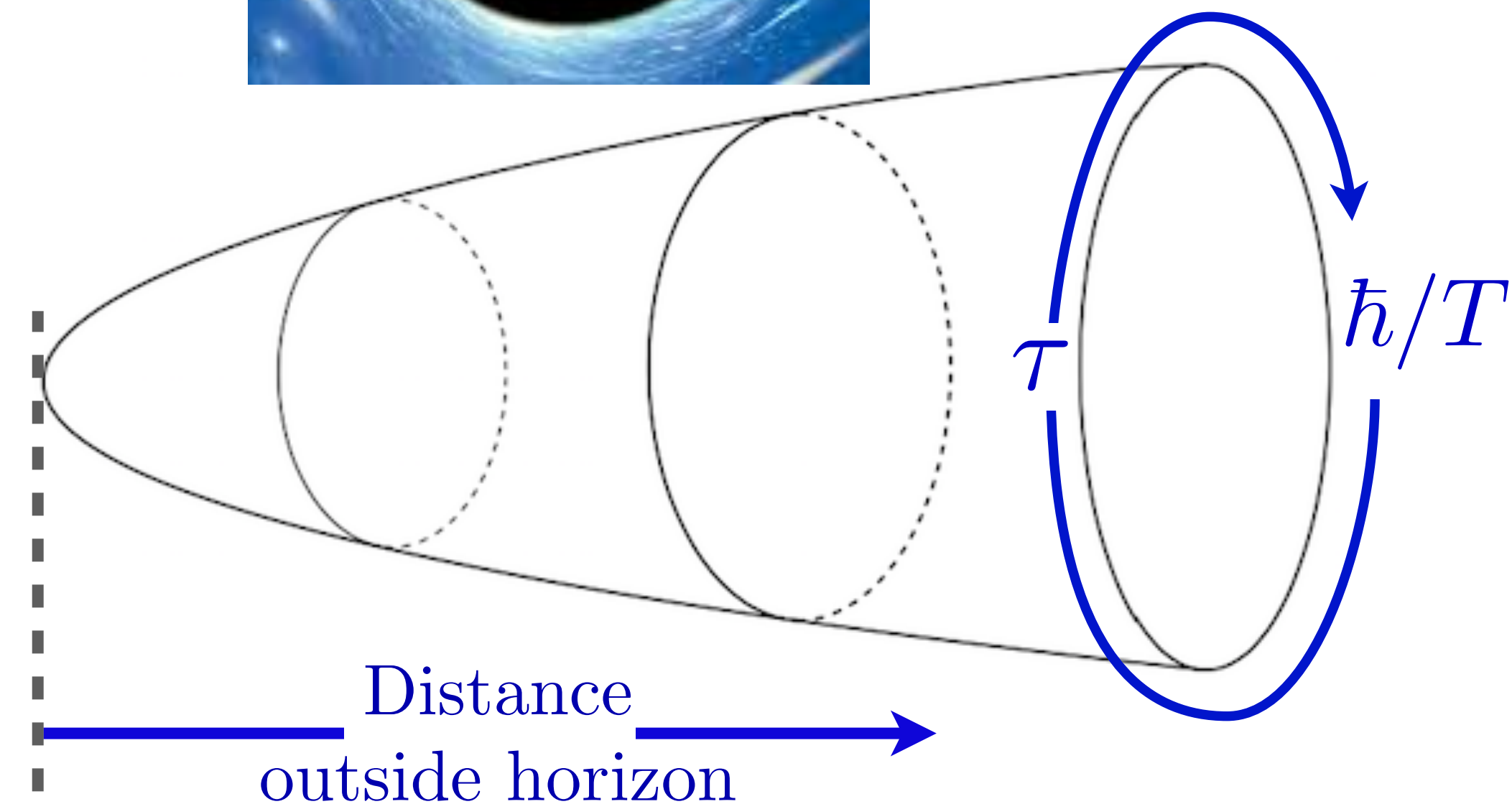
Gibbons, Hawking (1977)
Chambin, Emparan, Johnson, Myers (1999)



$$S_{BH}(T \rightarrow 0, Q) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0c^3}{4G\hbar} \left(1 + \frac{2(\pi A_0)^{1/2}T}{\hbar c} \right)$$

$A_0 = 2GQ^2/c^4$ is the area of the charged black hole horizon at $T = 0$.

Obtained from the saddle-point of the gravity path integral in the imaginary time spacetime outside the black hole.



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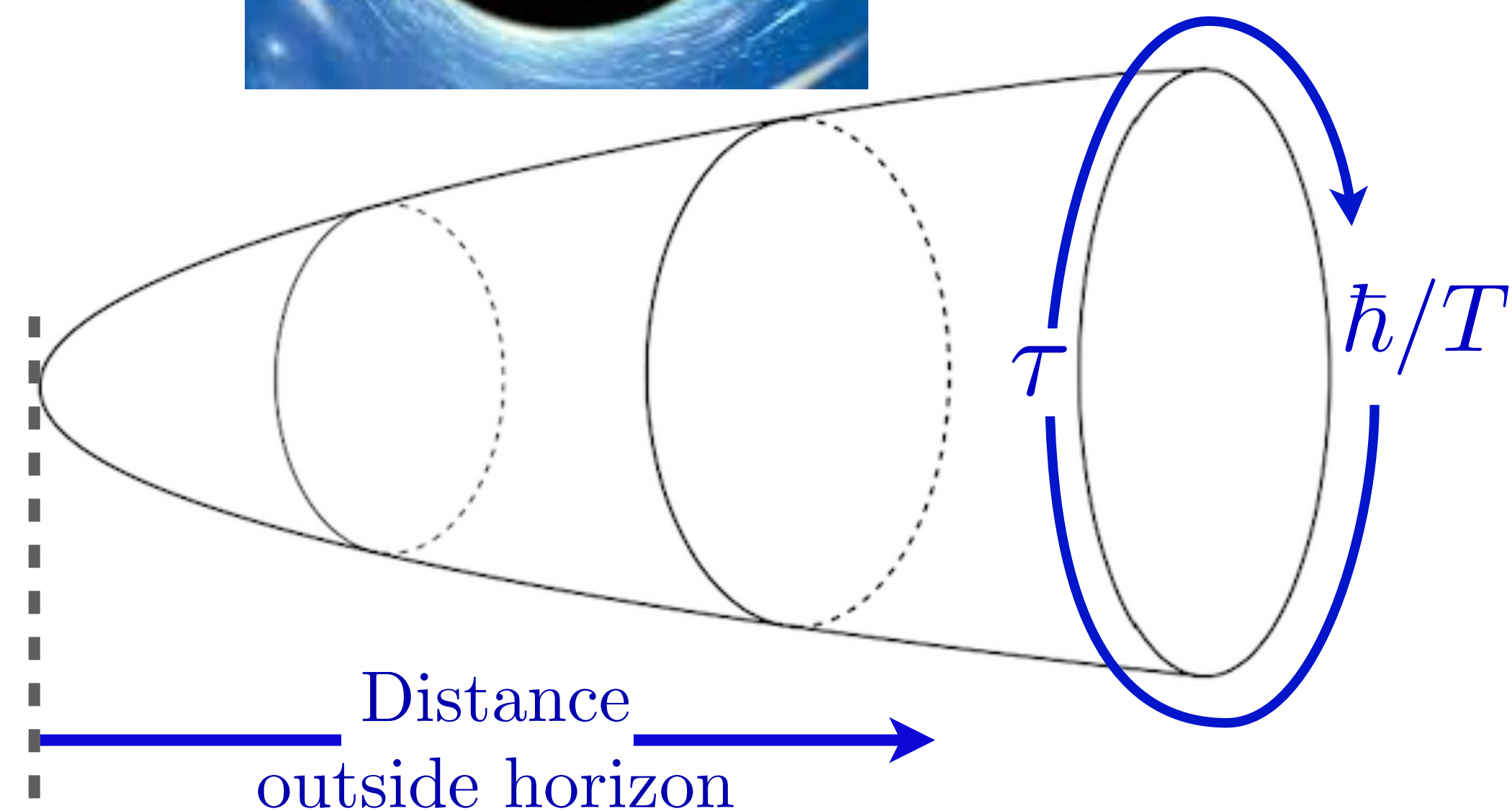


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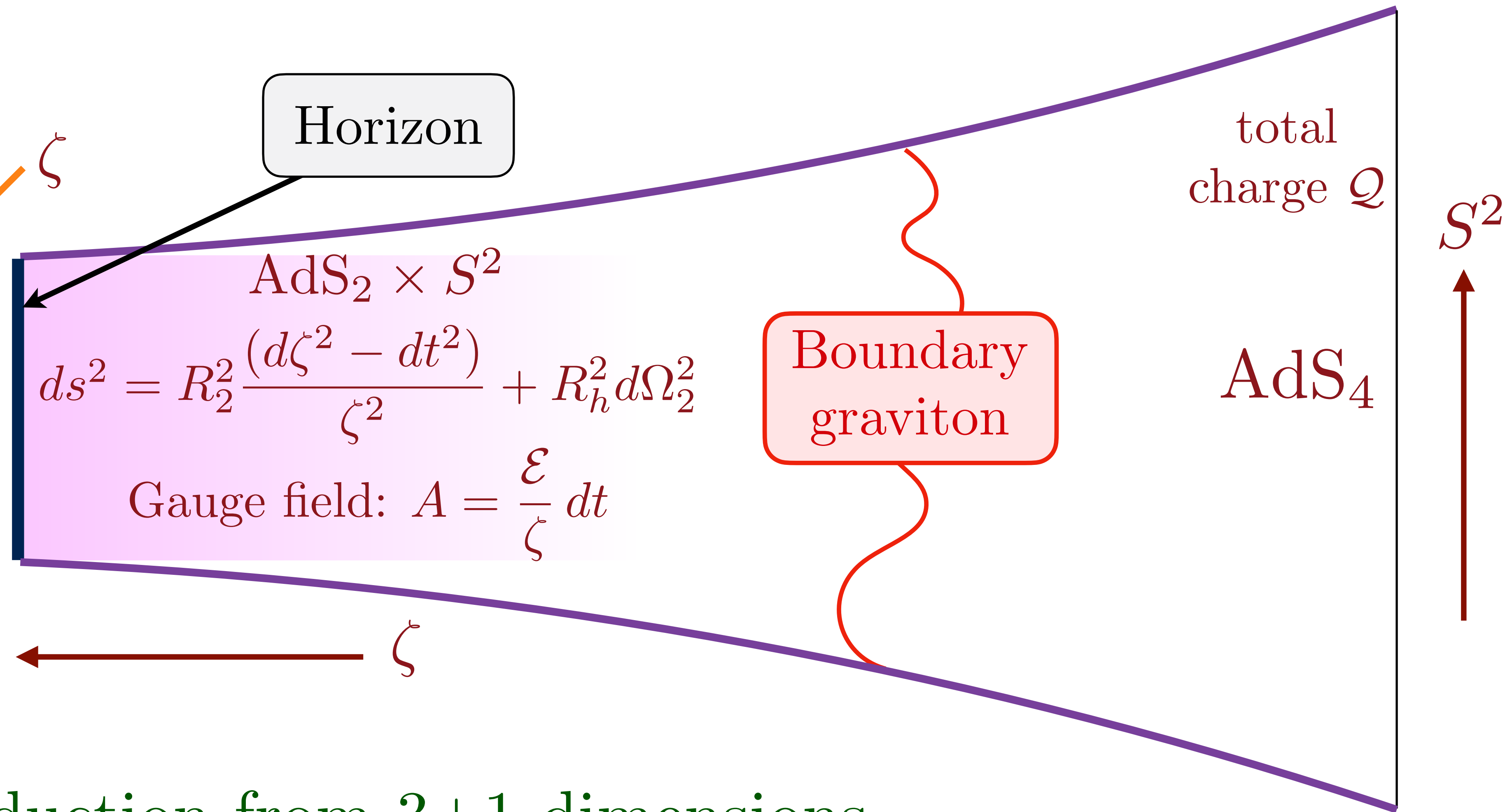
$A_0 = 2GQ^2/c^4$ is the area of the charged black hole horizon at $T = 0$.

Note the similarity to the large N entropy of the SYK model!
(along with other similarities)

Sachdev PRL 2010



Reissner-Nordstrom black hole of Einstein-Maxwell theory



Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

Thermodynamics of quantum black holes with charge Q :



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 &= \int \mathcal{D}f(\tau) \mathcal{D}\phi(\tau) \exp \left(-\frac{1}{\hbar} I_{\text{SYK}}[\text{time reparameterizations } f(\tau), \text{ phase rotations } \phi(\tau)] \right)
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$$S(T \rightarrow 0, Q) = \frac{A(T)c^3}{4G\hbar} - \frac{3}{4} \ln \left(\frac{\hbar c^5}{GT^2} \right)$$

The $\ln T$ term is the SYK/boundary-graviton correction to Bekenstein-Hawking.

There is also a
 $-\frac{559}{180} \ln \left(\frac{A_0 c^3}{\hbar G} \right)$
 term from other massless
 modes; Sen (2011)
 Iliesiu, Murthy, Turiaci (2022)

Black hole questions and answers

Can we find a quantum simulation of the inside of a black hole whose $D(E)$ matches the Bekenstein-Hawking entropy computed outside the black hole?

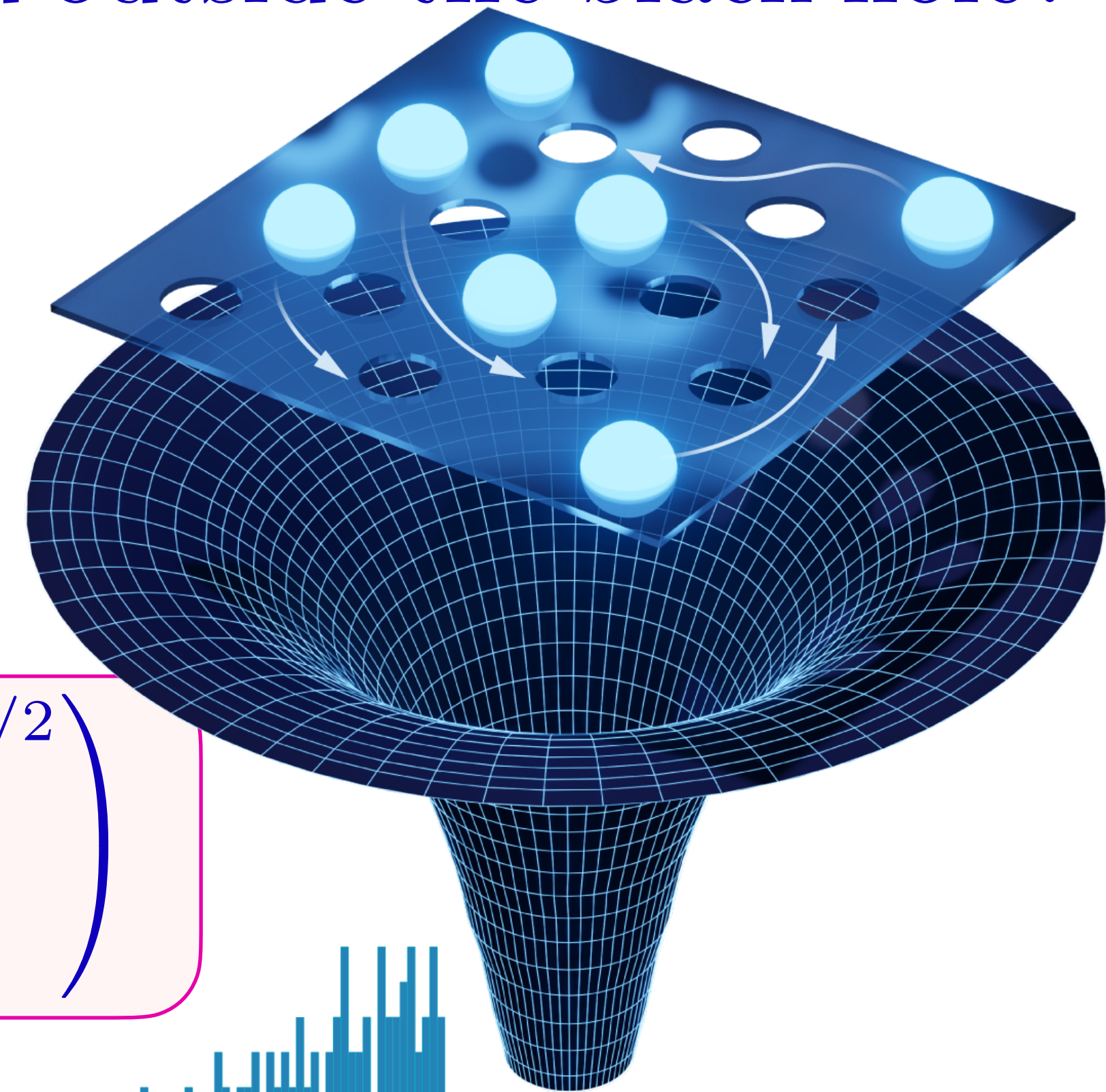
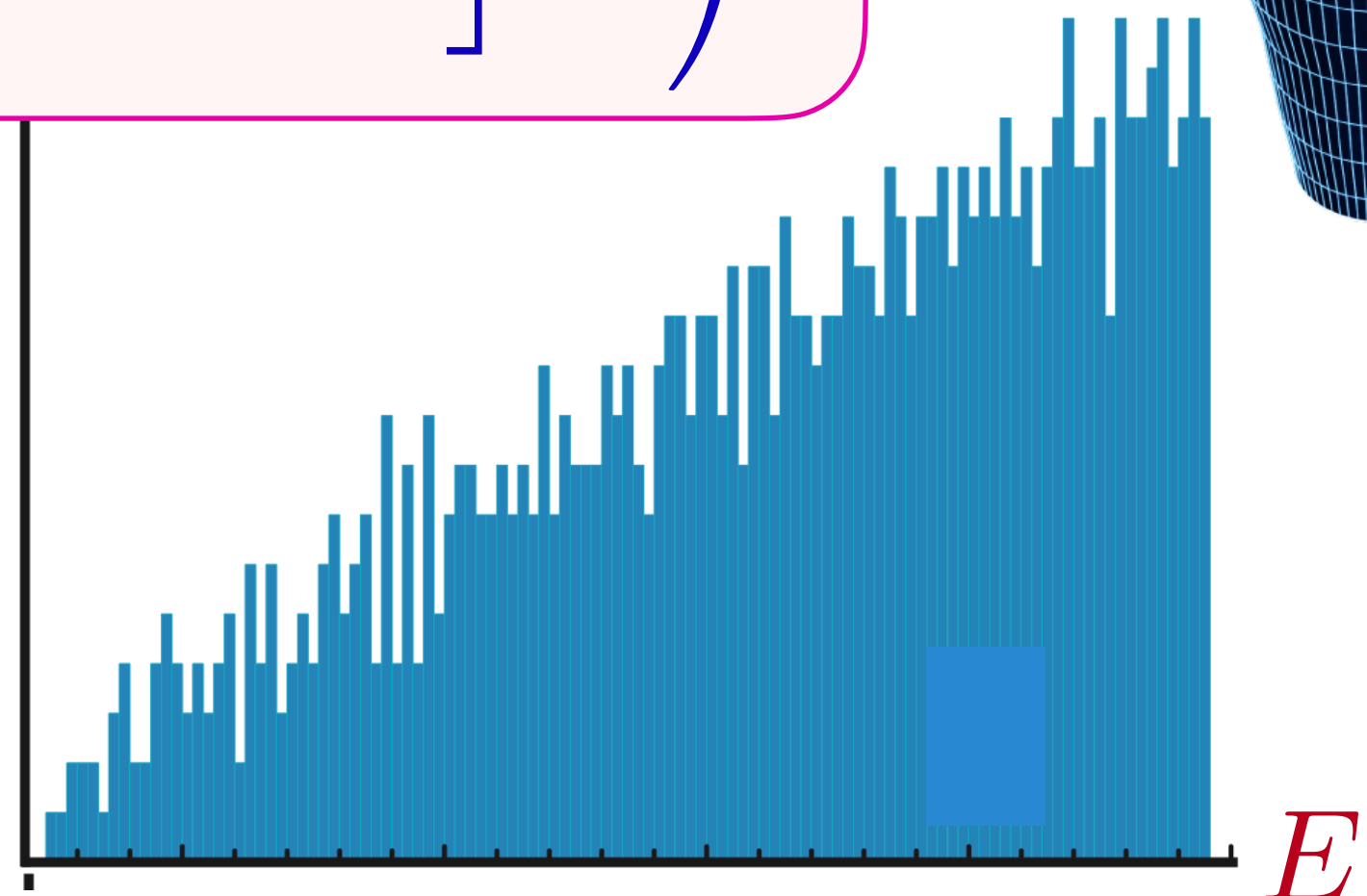
Yes, for charged black holes:

- For generic charged black holes in 3+1 dimensions, the SYK model yields, in terms of $A_0 = 2GQ^2/c^4$ the horizon area at $T = 0$:

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G} \right)^{-347/90} \exp \left(\frac{A_0 c^3}{4\hbar G} \right) \sinh \left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E \right]^{1/2} \right)$$

There is no degeneracy, but an exponentially small level spacing down to the ground state.

$D(E)$



Black hole questions and answers

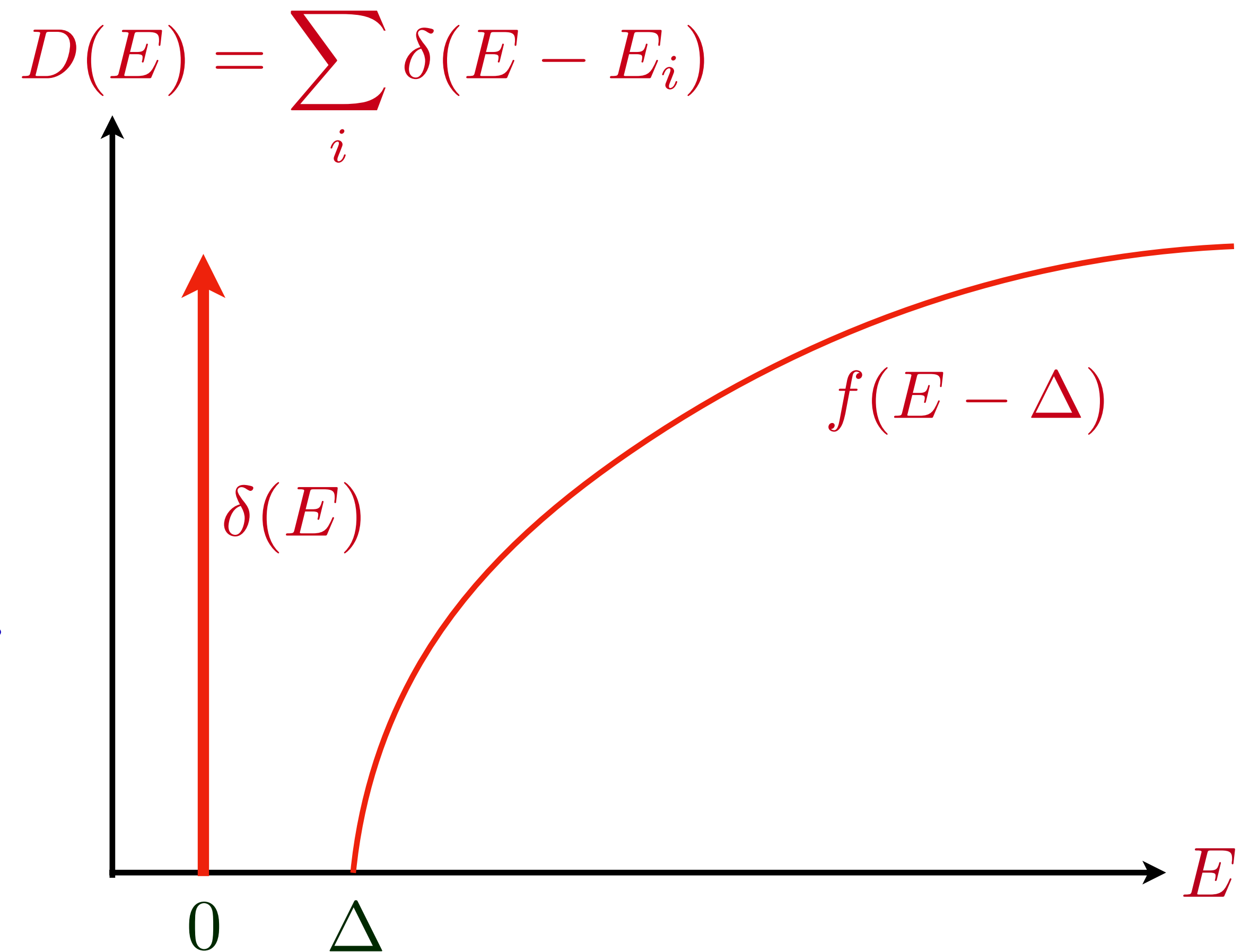
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Yes, for charged black holes:

- With sufficient low energy supersymmetry, string theory yields:

$$D(E) = \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \delta(E) + \theta(E - \Delta) f(E - \Delta) + \dots$$

There are exponentially many degenerate BPS ground states, and an energy gap Δ above the ground state.



M. Heydemann, L.V. Iliesiu, G. J. Turiaci, and W. Zhao, 2020

L.V. Iliesiu, S. Murthy, G. J. Turiaci, 2022

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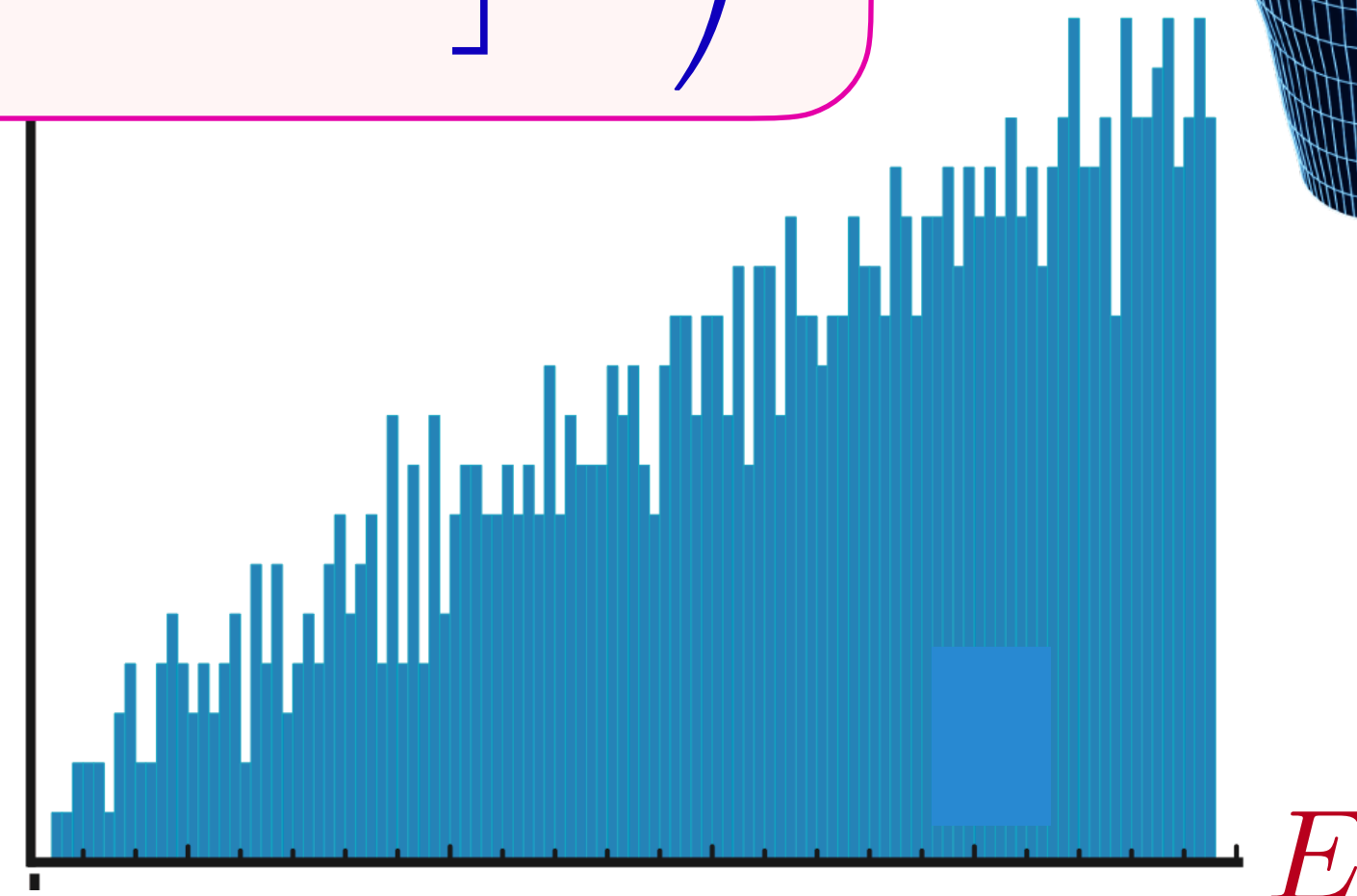
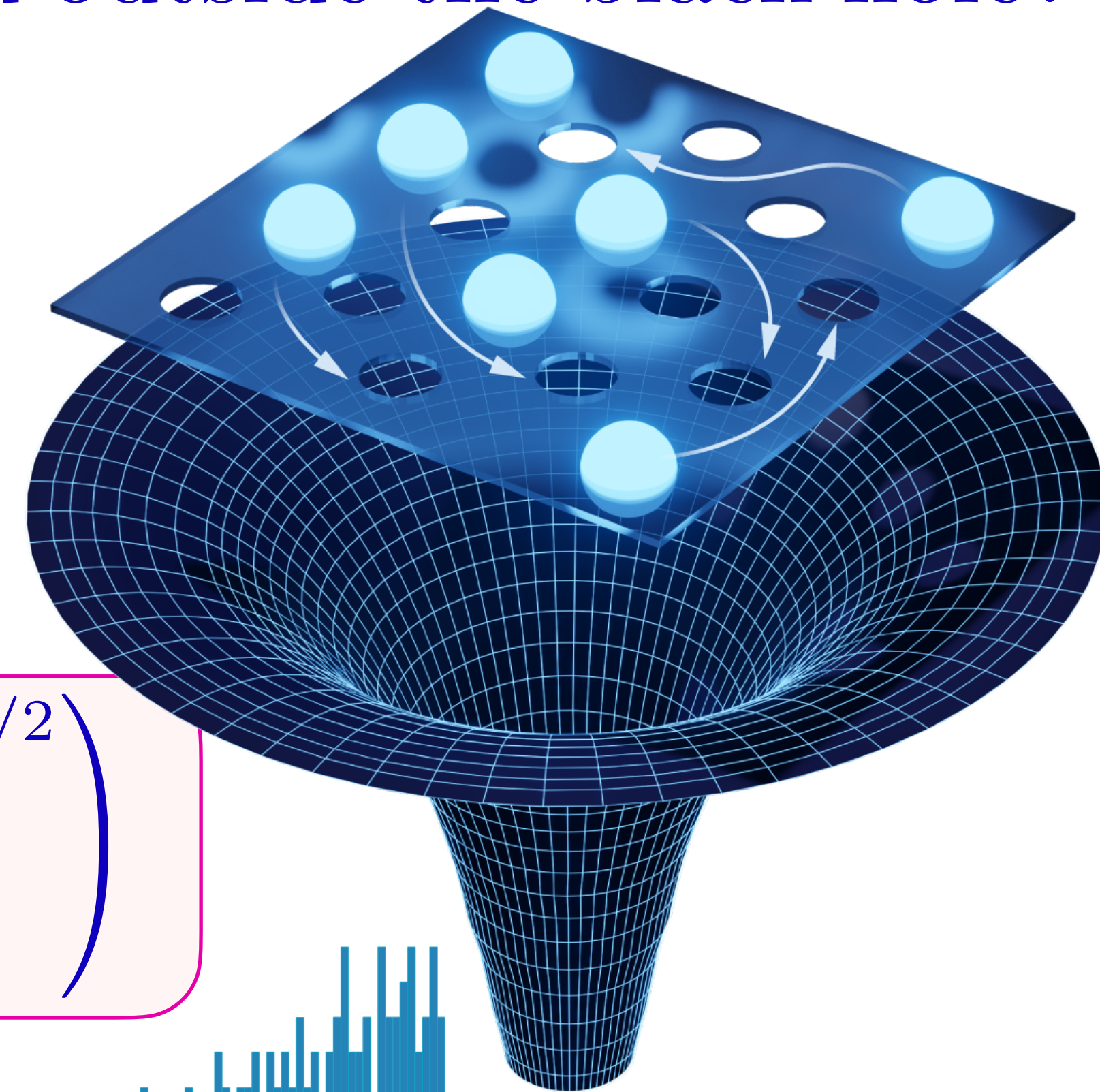
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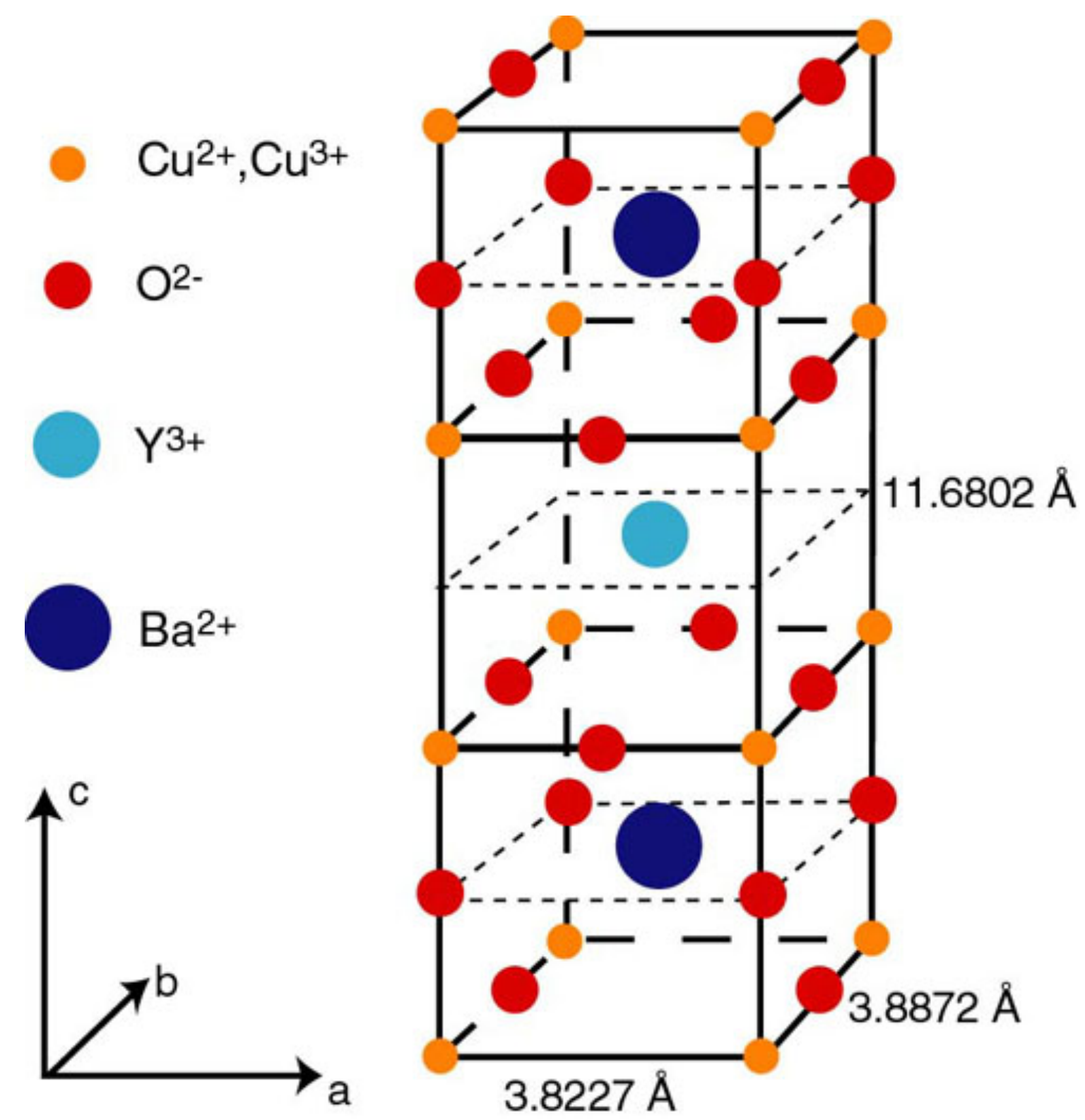
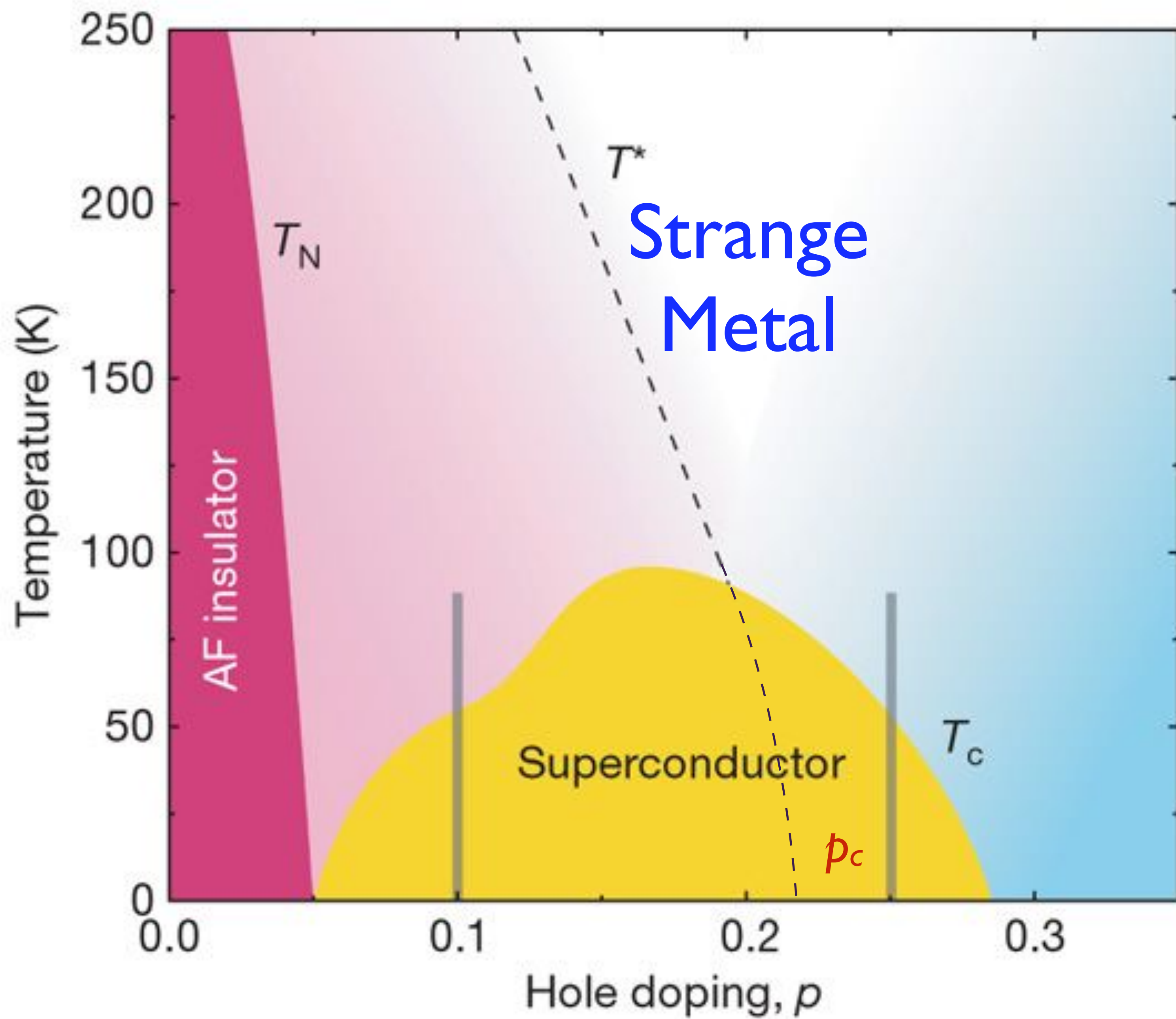
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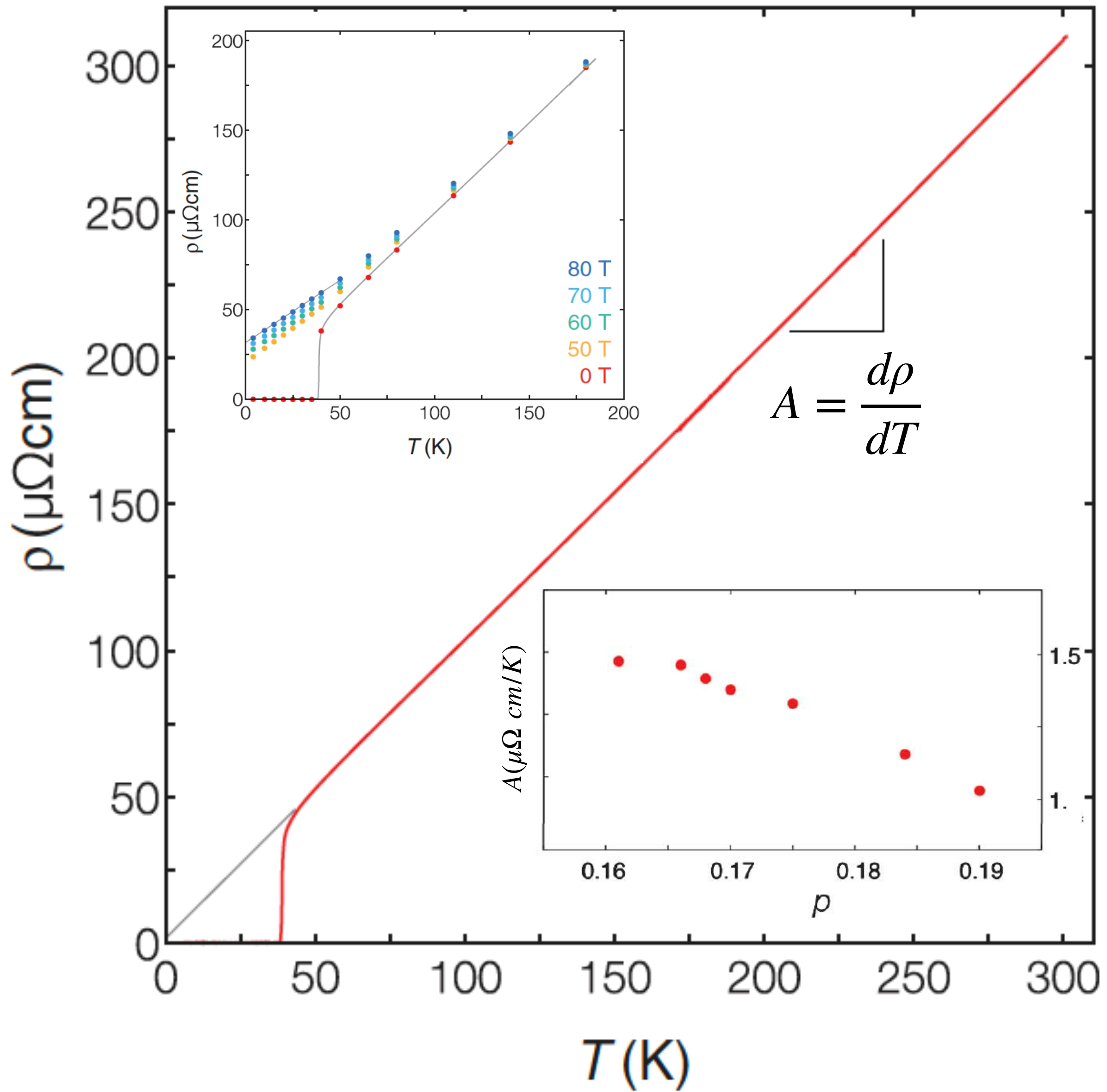
- ‘Wormhole’ contributions to this quantum simulation have led to an understanding of the Page curve of entanglement entropy of evaporating black holes.

Saad, Shenker, Stanford (2019)

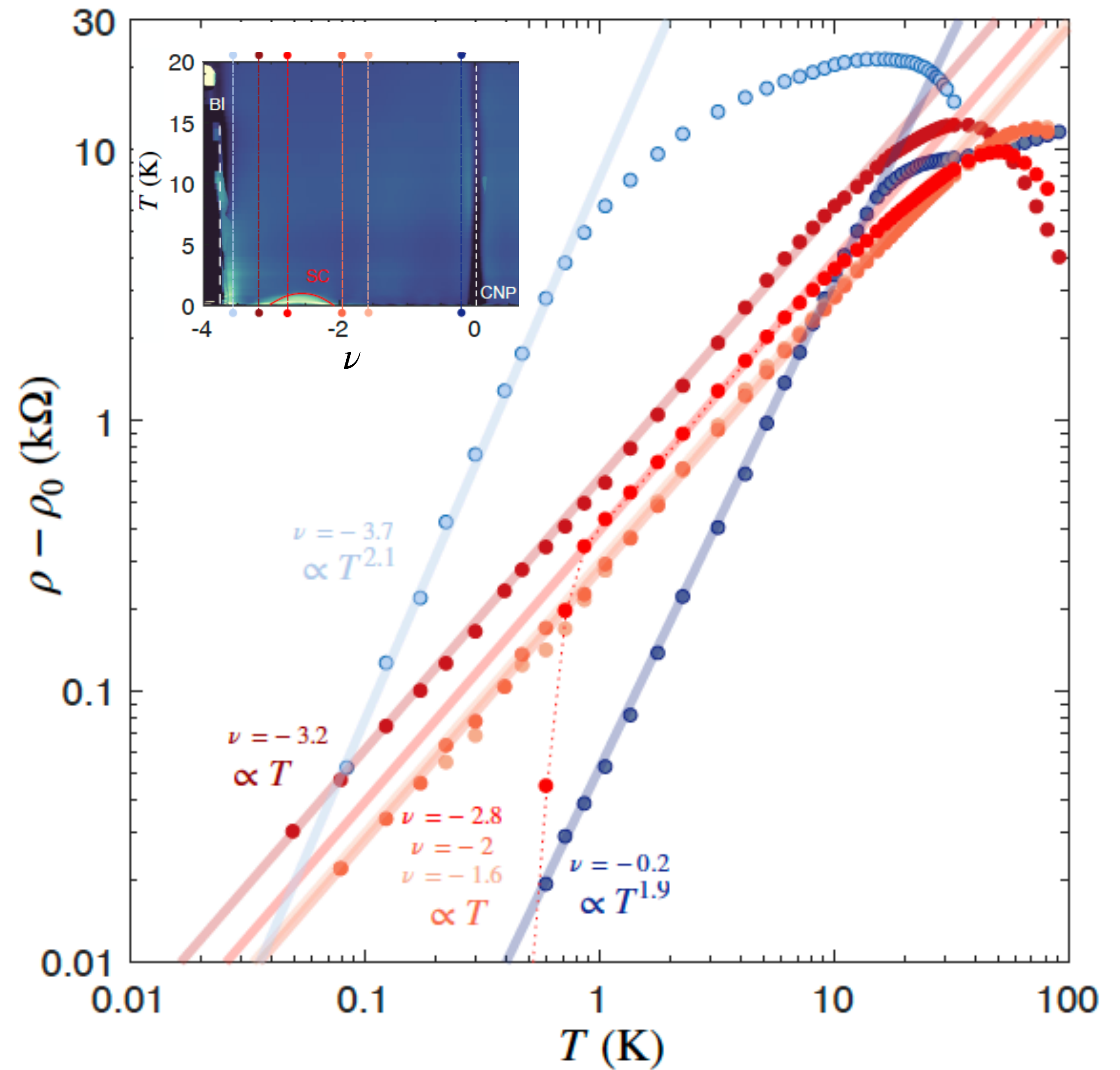


**Strange
metals**





LSCO: Giraldo-Gallo et al. 2018



MATBG: Jaoui et al. 2021

Properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \dots$ as $T \rightarrow 0$
and $\rho(T) < h/e^2$ (in $d = 2$).
Metals with $\rho(T) > h/e^2$ are bad metals.

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S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

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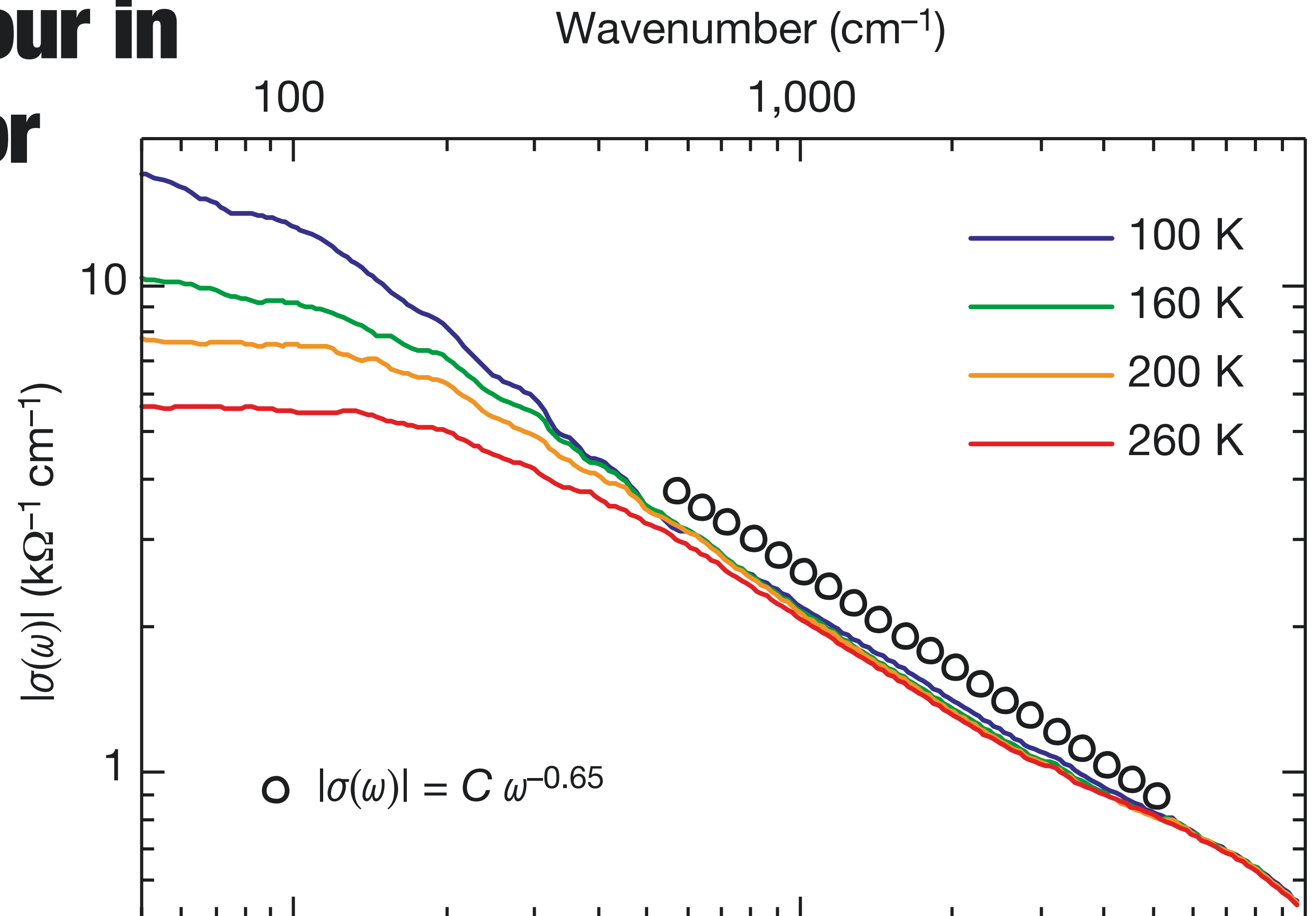
- Optical conductivity

Nature **425**, 271 (2003)

D. van der Marel^{1*}, H. J. A. Molegraaf^{1*}, J. Zaanen², Z. Nussinov^{2*},
F. Carbone^{1*}, A. Damascelli^{3*}, H. Eisaki^{3*}, M. Greven³, P. H. Kes² & M. Li²

Quantum critical behaviour in a high- T_c superconductor

But no $\hbar\omega/(k_B T)$ scaling.



- Optical conductivity

B. Michon,^{1,2,3} C. Berthod,³ C. W. Rischau,³ A. Ataei,⁴ L. Chen,⁴
S. Komiya,⁵ S. Ono,⁵ L. Taillefer,^{4,6} D. van der Marel,³ and A. Georges^{7,8,3}

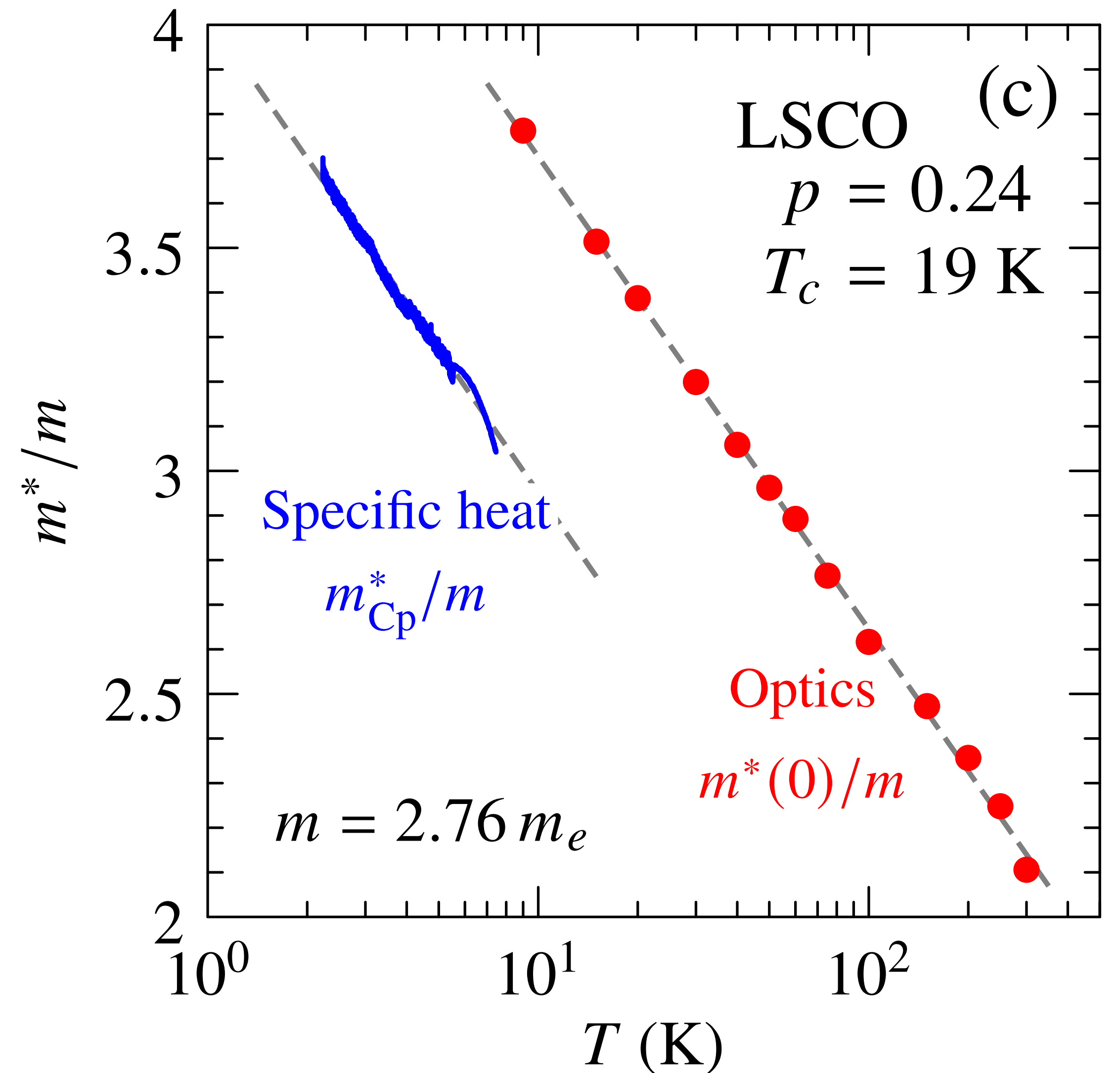
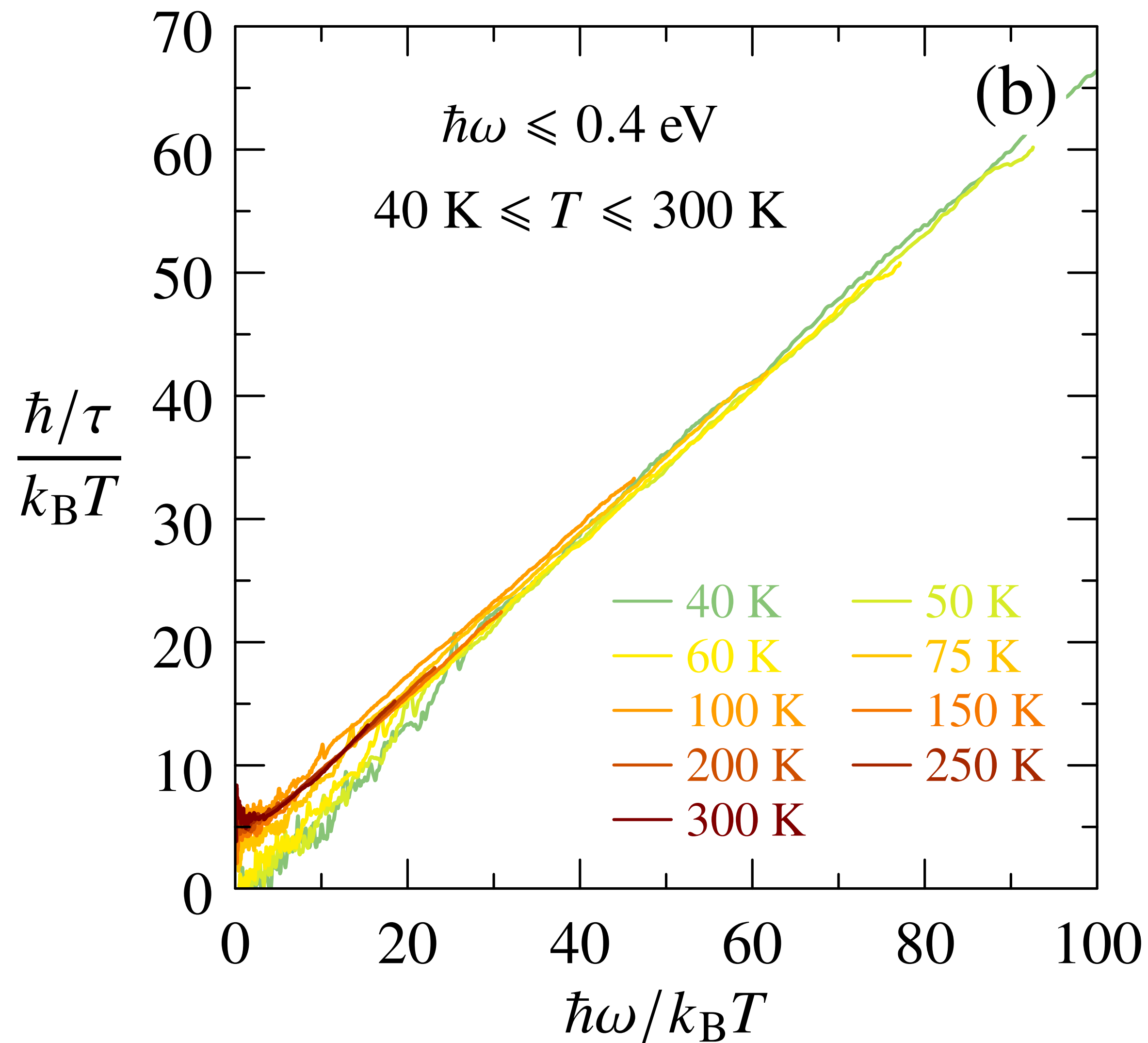
Planckian Behavior of Cuprate Superconductors: Reconciling the Scaling of Optical Conductivity with Resistivity and Specific Heat

$$\sigma(\omega) = \frac{K}{\frac{1}{\tau_{\text{trans}}(\omega)} - i\omega \frac{m_{\text{trans}}^*(\omega)}{m}} ; \quad \frac{1}{\tau_{\text{trans}}(\omega)} \sim |\omega| \Phi_{\sigma} \left(\frac{\hbar\omega}{k_B T} \right)$$

$$\text{Causality: } \frac{m_{\text{trans}}^*(\omega)}{m} \sim \ln \left(\frac{\Lambda}{\text{Max}(\hbar\omega, k_B T)} \right)$$

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3. Optical conductivity

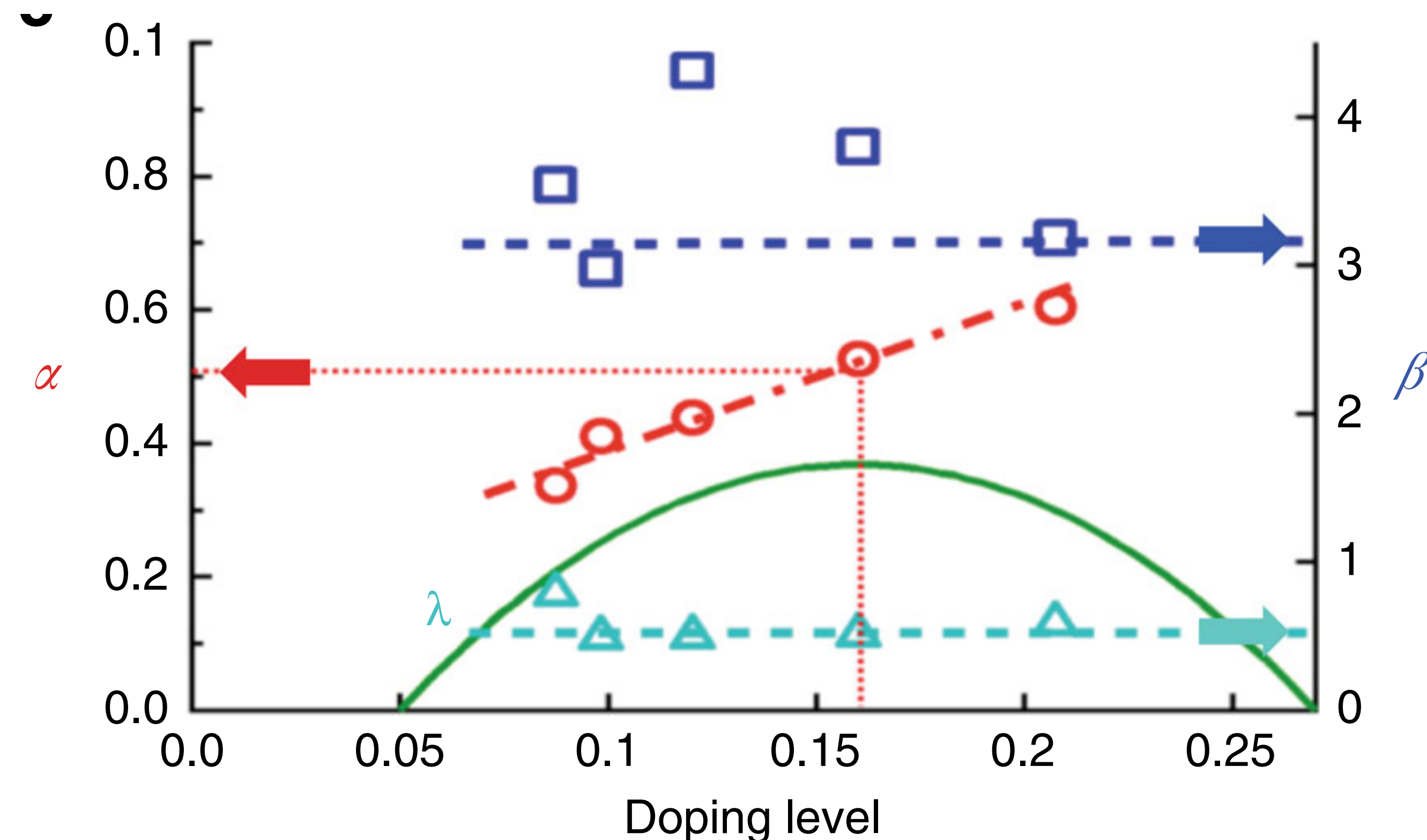
$$\sigma(\omega) = \frac{K}{\frac{1}{\tau_{\text{trans}}(\omega)} - i\omega \frac{m_{\text{trans}}^*(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\text{trans}}(\omega)} \sim |\omega| \Phi_{\sigma} \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon.....A. Georges, arXiv:2205.04030

● Photoemission

A unified form of low-energy nodal electronic interactions in hole-doped cuprate superconductors

T.J. Reber^{1,5*}, X. Zhou^{1*}, N.C. Plumb^{1,6}, S. Parham¹, J.A. Waugh¹, Y. Cao¹, Z. Sun^{1,7}, H. Li¹, Q. Wang¹, J.S. Wen², Z.J. Xu², G. Gu², Y. Yoshida³, H. Eisaki³, G.B. Arnold¹ & D.S. Dessau^{1,4*}



$$\Sigma''_{\text{PLL}}(\omega) = \Gamma_0 + \lambda \frac{[(\hbar\omega)^2 + (\beta k_B T)^2]^\alpha}{(\hbar\omega_N)^{2\alpha-1}}$$

Properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \dots$ as $T \rightarrow 0$
and $\rho(T) < h/e^2$ (in $d = 2$).
Metals with $\rho(T) > h/e^2$ are bad metals.

2. Specific heat $\sim T \ln(1/T)$ as $T \rightarrow 0$.

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

3. Optical conductivity

$$\sigma(\omega) = \frac{K}{\frac{1}{\tau_{\text{trans}}(\omega)} - i\omega \frac{m_{\text{trans}}^*(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\text{trans}}(\omega)} \sim |\omega| \Phi_{\sigma} \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon.....A. Georges, arXiv:2205.04030

4. Photoemission: nearly “marginal Fermi liquid” electron spectral density:

$$\text{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_{\Sigma} \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2$$

T.J. Reber....D. Dessau,
Nature Communications **10**, 5737 (2019)

From
Yukawa-SYK models
to a universal theory of
strange metals



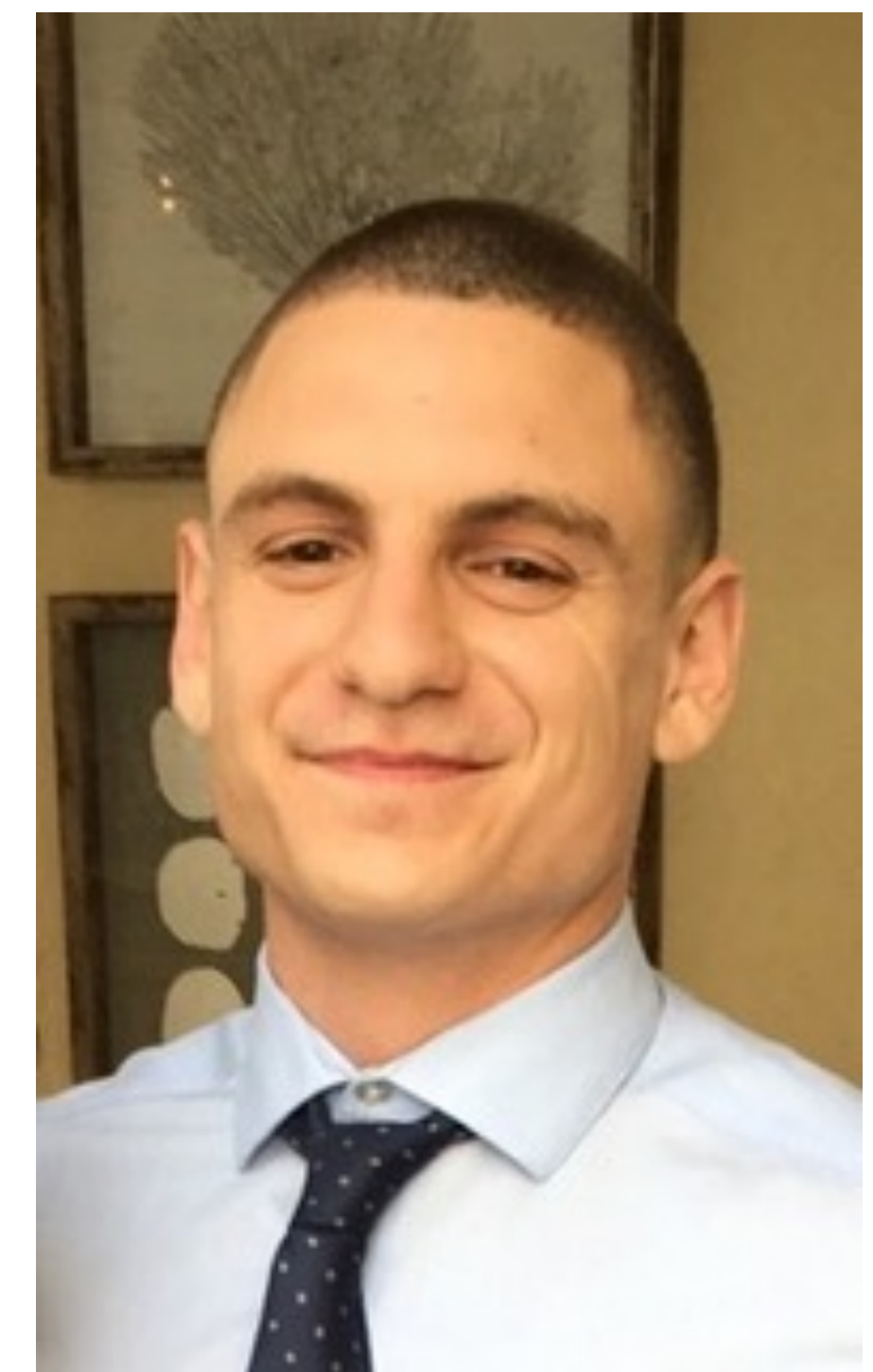
Aavishkar Patel

Flatiron Institute, NYC



Haoyu Guo

Harvard



Ilya Esterlis

Harvard → Wisconsin

arXiv: 2103.08615, 2203.04990, 2207.08841

E. E. Aldape, T. Cookmeyer, Aavishkar A. Patel, and Ehud Altman, arXiv:2012.00763

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2} (\pi_\ell^2 + \omega_0^2 \phi_\ell^2) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell,$$

$g_{ij\ell}$ independent random numbers with zero mean. Large N limit leads to Migdal-Eliashberg equations $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)

J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)

A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)

E. Marcus and S. Vandoren, JHEP 01, 166 (2018)

Yuxuan Wang, PRL **124**, 017002 (2020)

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)

Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)

E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763

Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)

W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)

I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi + \sum_\ell \frac{1}{2} (\pi_\ell^2 + \omega_0^2 \phi_\ell^2) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean.

The disorder-averaged partition function is a G - Σ - D - Π theory:

$$\begin{aligned} \mathcal{Z} &= \int \mathcal{D}G \mathcal{D}\Sigma \mathcal{D}D \mathcal{D}\Pi \exp(-N S_{\text{all}}) \\ S_{\text{all}} &= -\ln \det(\partial_\tau - \mu + \Sigma) + \frac{1}{2} \ln \det(-\partial_\tau^2 + \omega_0^2 - \Pi) \\ &+ \int d\tau \int d\tau' \left[-\Sigma(\tau'; \tau) G(\tau, \tau') + \frac{1}{2} \Pi(\tau' \tau) D(\tau, \tau') \right. \\ &\quad \left. + \frac{g^2}{2} G(\tau, \tau') G(\tau', \tau) D(\tau, \tau') \right]. \end{aligned}$$

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi + \sum_\ell \frac{1}{2} (\pi_\ell^2 + \omega_0^2 \phi_\ell^2) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large N saddle point equations are

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)}$$
$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$

Make the low frequency ansatz

$$G(i\omega) \sim -i \operatorname{sgn}(\omega) |\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

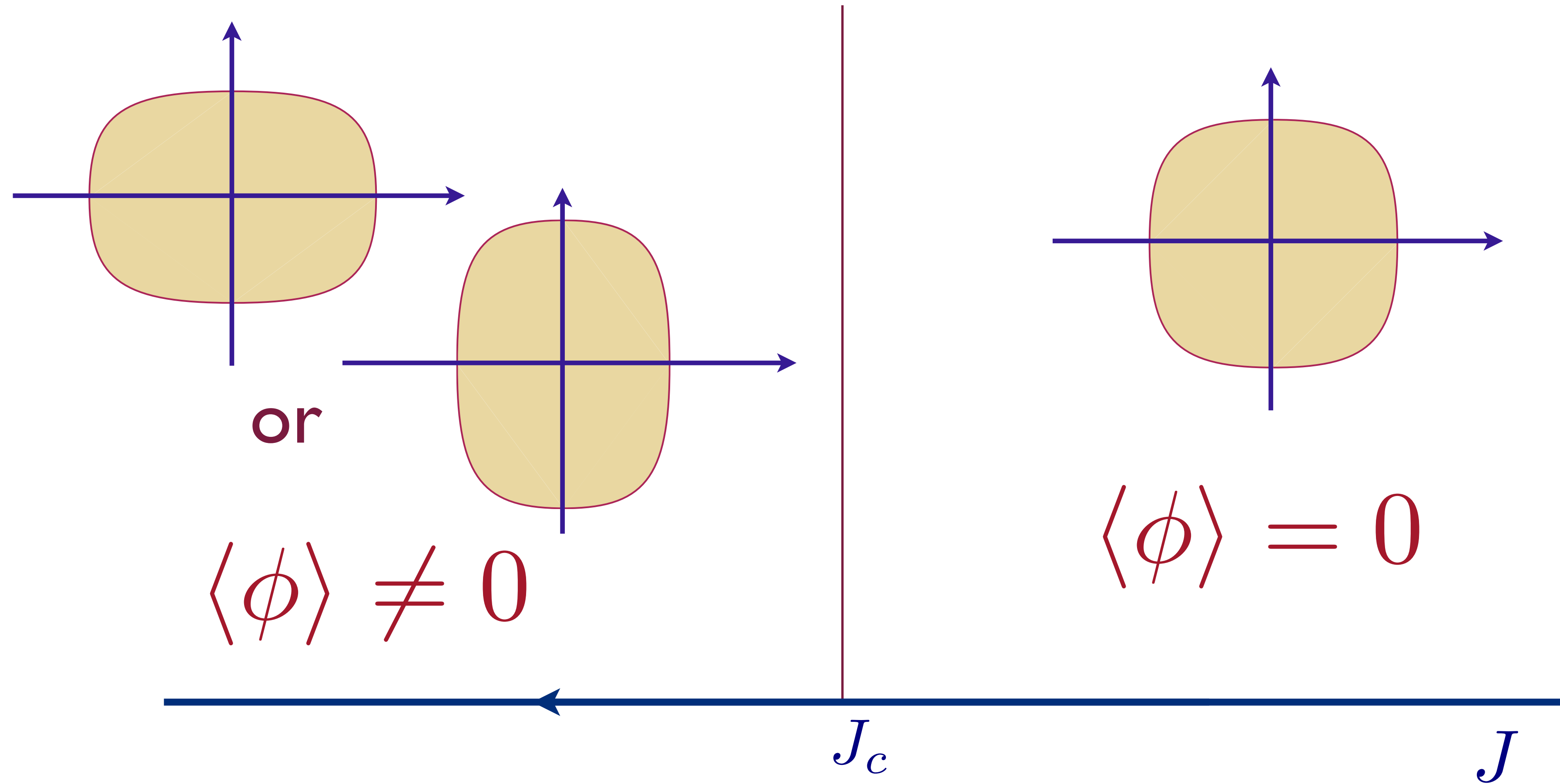
A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037 \dots$$

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)

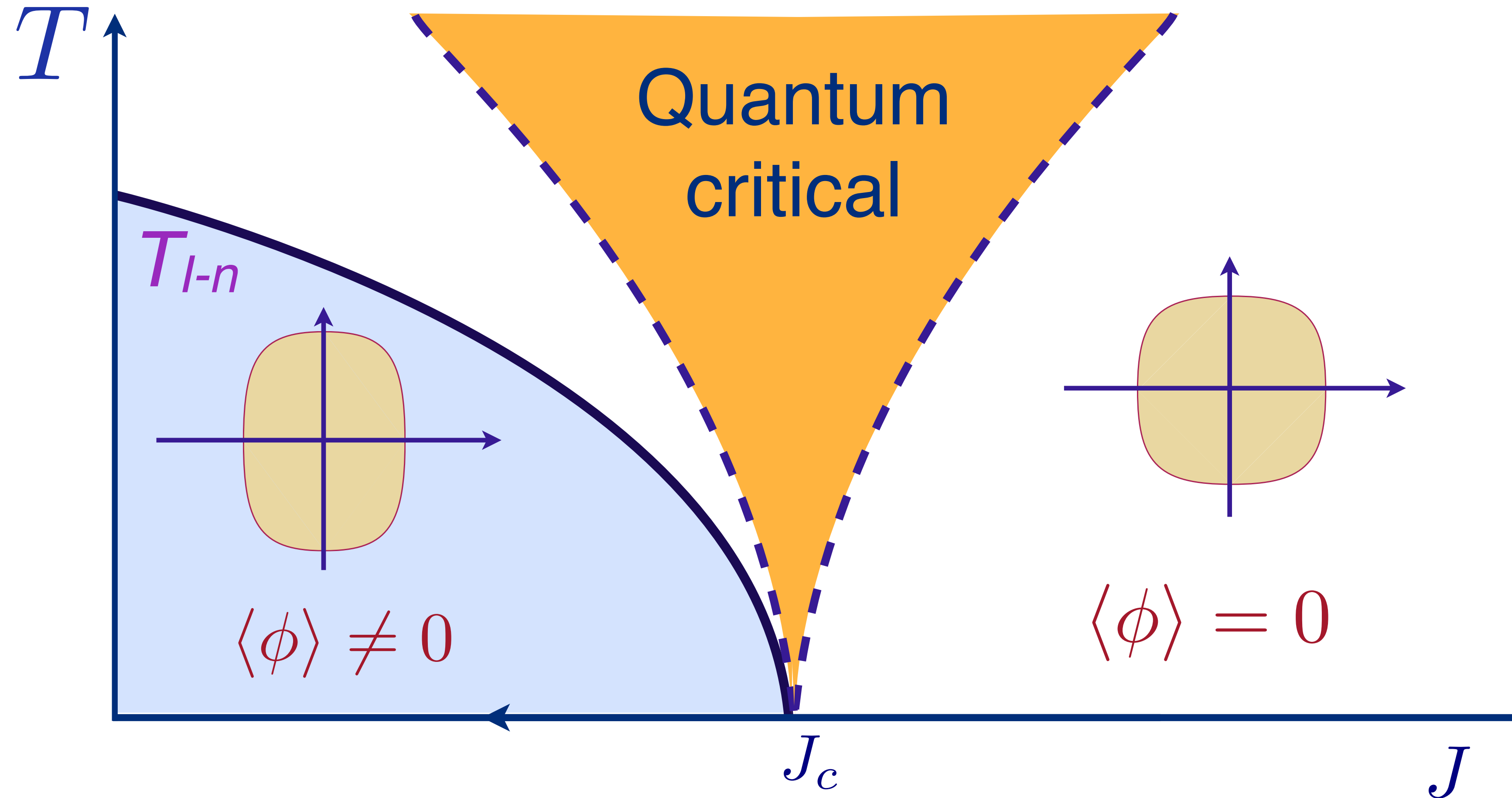
See also Yuxuan Wang, PRL **124**, 017002 (2020)

Quantum criticality of Ising-nematic ordering in a metal



Pomeranchuk instability as a function of coupling J

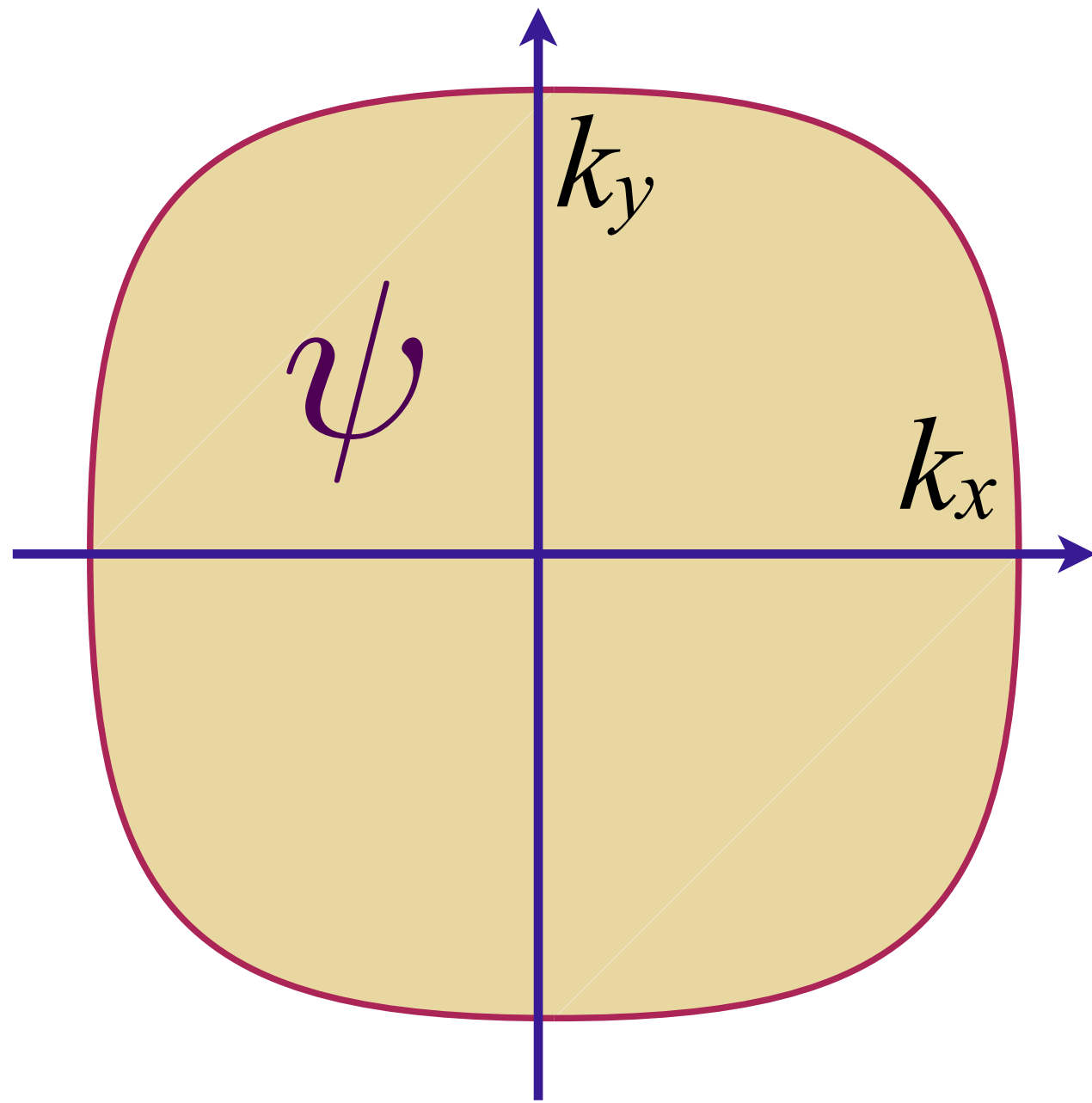
Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and J

Fermi surface

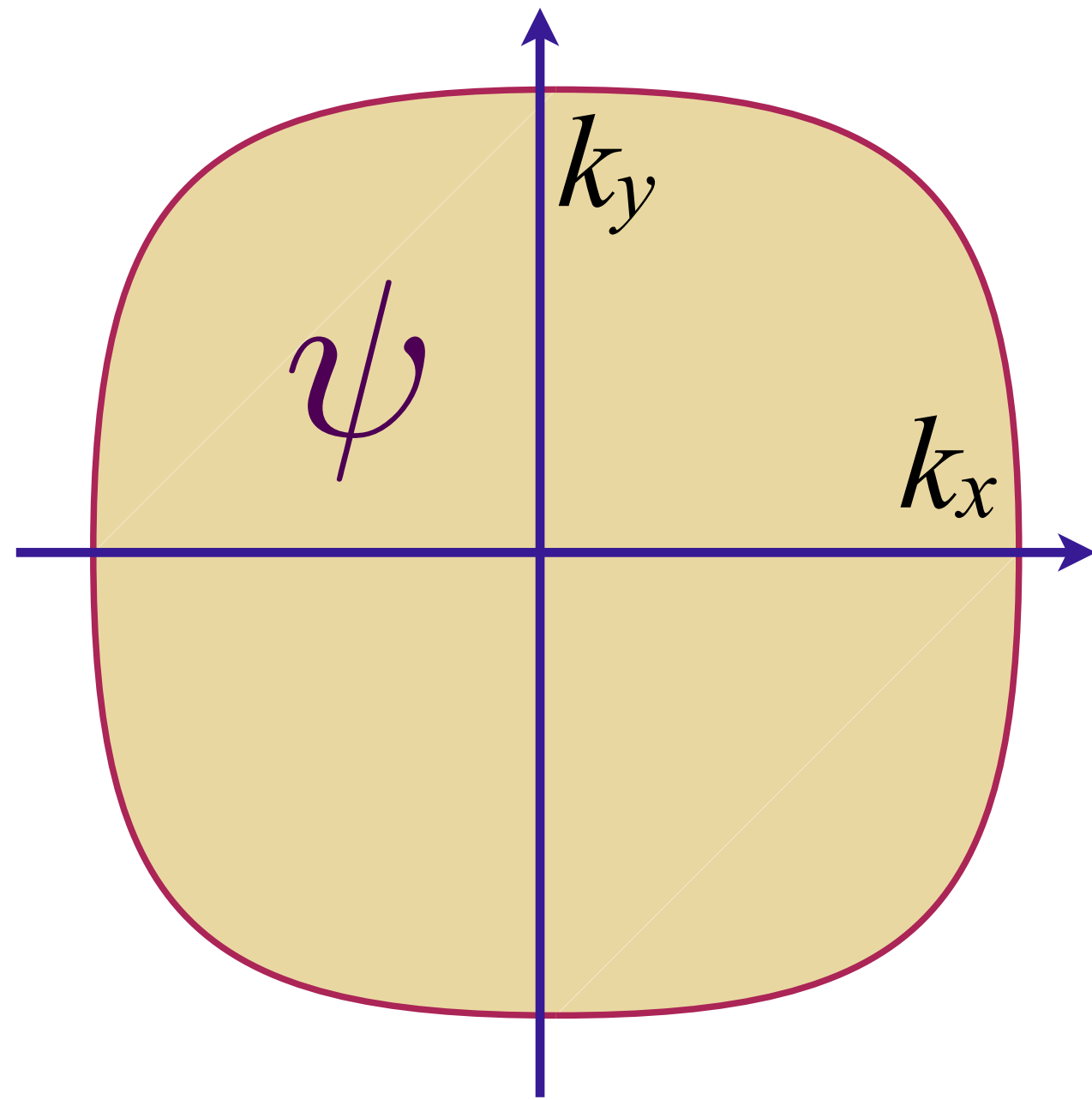
$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



$$-J \psi^\dagger(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \psi(\mathbf{r})$$

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



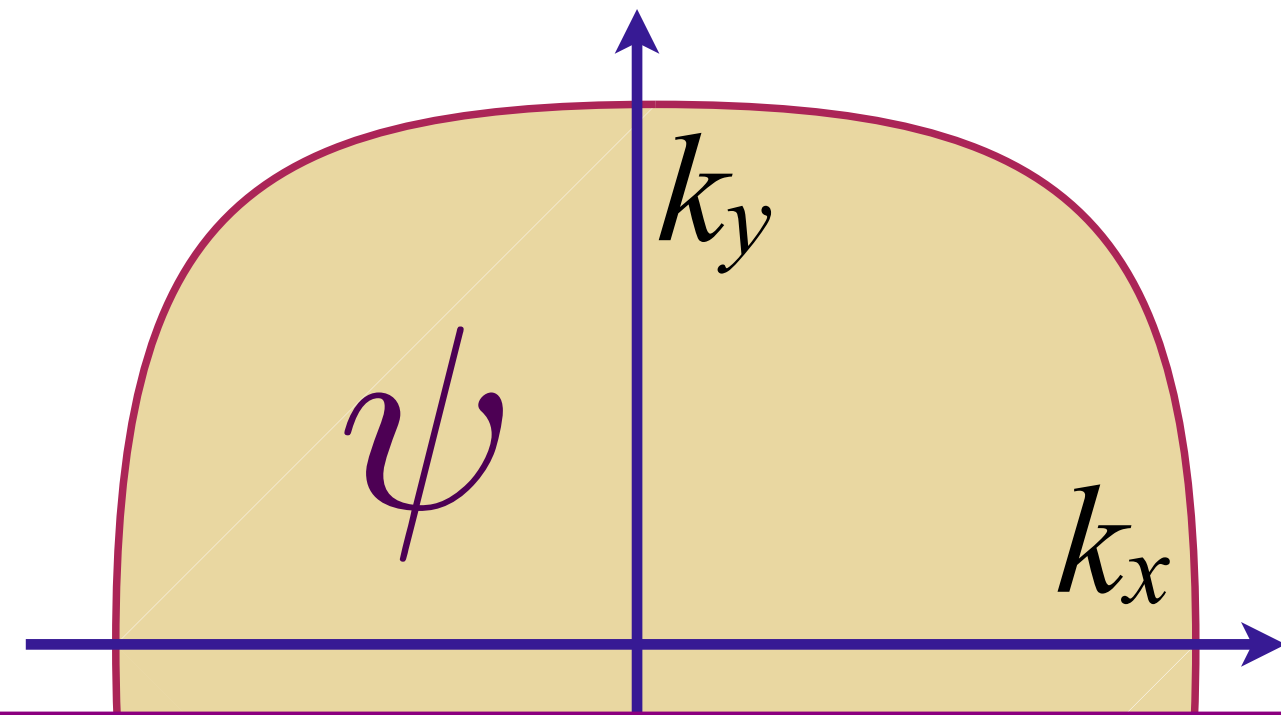
a critical boson ϕ
e.g. Ising-nematic order

$$\frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r})$$

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$\frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r})$$

Solve in a large N limit with Yukawa coupling

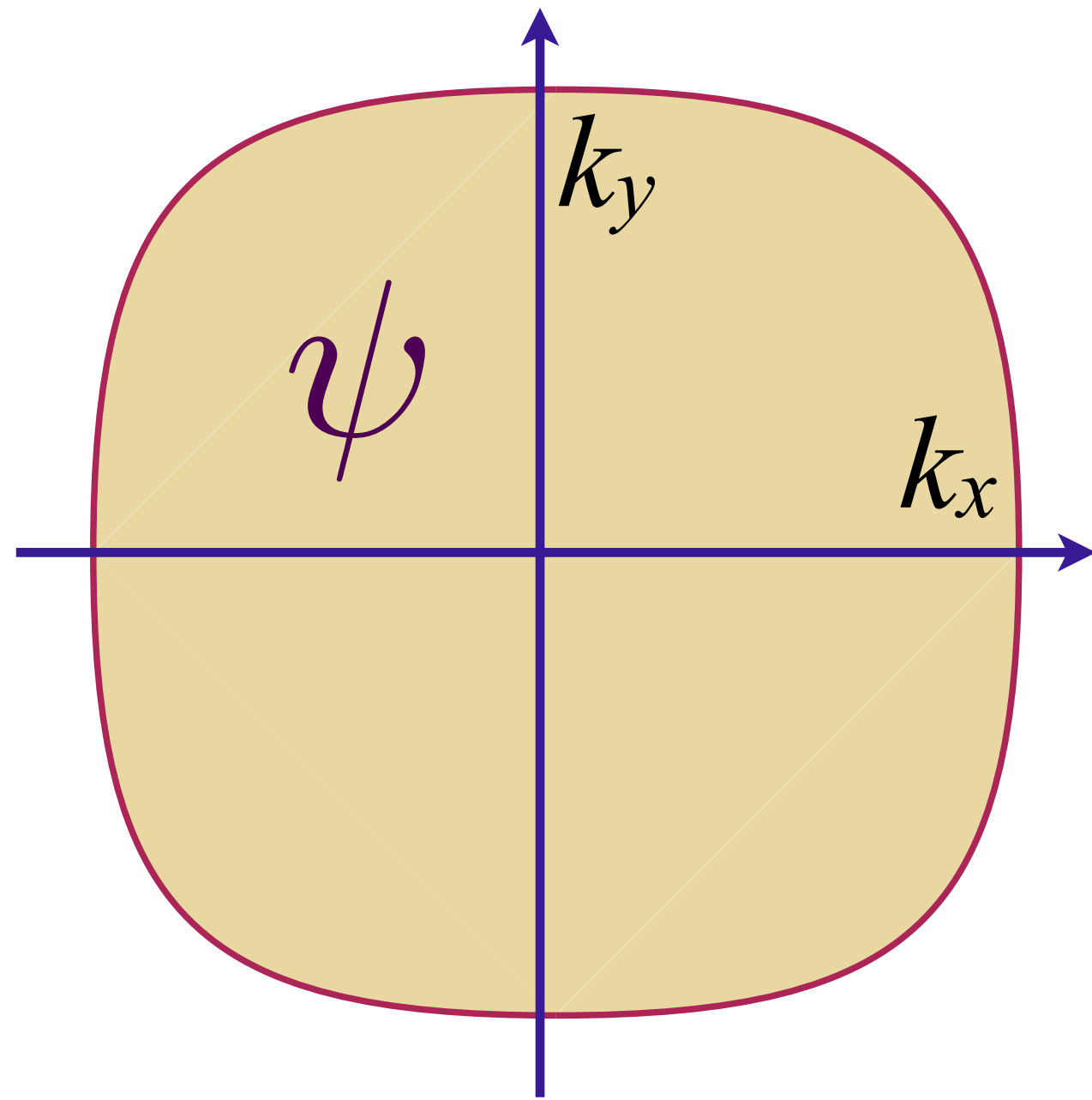
$$\frac{g_{ijl}}{N} \int d^2r d\tau \psi_i^\dagger(\mathbf{r}, \tau) \psi_j(\mathbf{r}, \tau) \phi_l(\mathbf{r}, \tau) \quad , \quad \overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

to obtain Eliashberg solution for electron (G) and boson (D) Green's functions at small ω :

$$\Sigma(\hat{\mathbf{k}}, i\omega) \sim -i \text{sgn}(\omega) |\omega|^{2/3} \quad , \quad G(\mathbf{k}, i\omega) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) - \Sigma(\hat{\mathbf{k}}, i\omega)} \quad , \quad D(\mathbf{q}, i\Omega) = \frac{1}{\Omega^2 + q^2 + \gamma|\Omega|/q}$$

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



Transport—a perfect metal!

Conservation of momentum and fermion-boson drag imply:

$$\text{Re} [\sigma(\omega)] = D\delta(\omega) + \dots$$

a critical boson ϕ
e.g. Ising-nematic order

$$\frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r})$$

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)

D. L. Maslov, V. I. Yudson, and A. V. Chubukov PRL **106**, 106403 (2011)

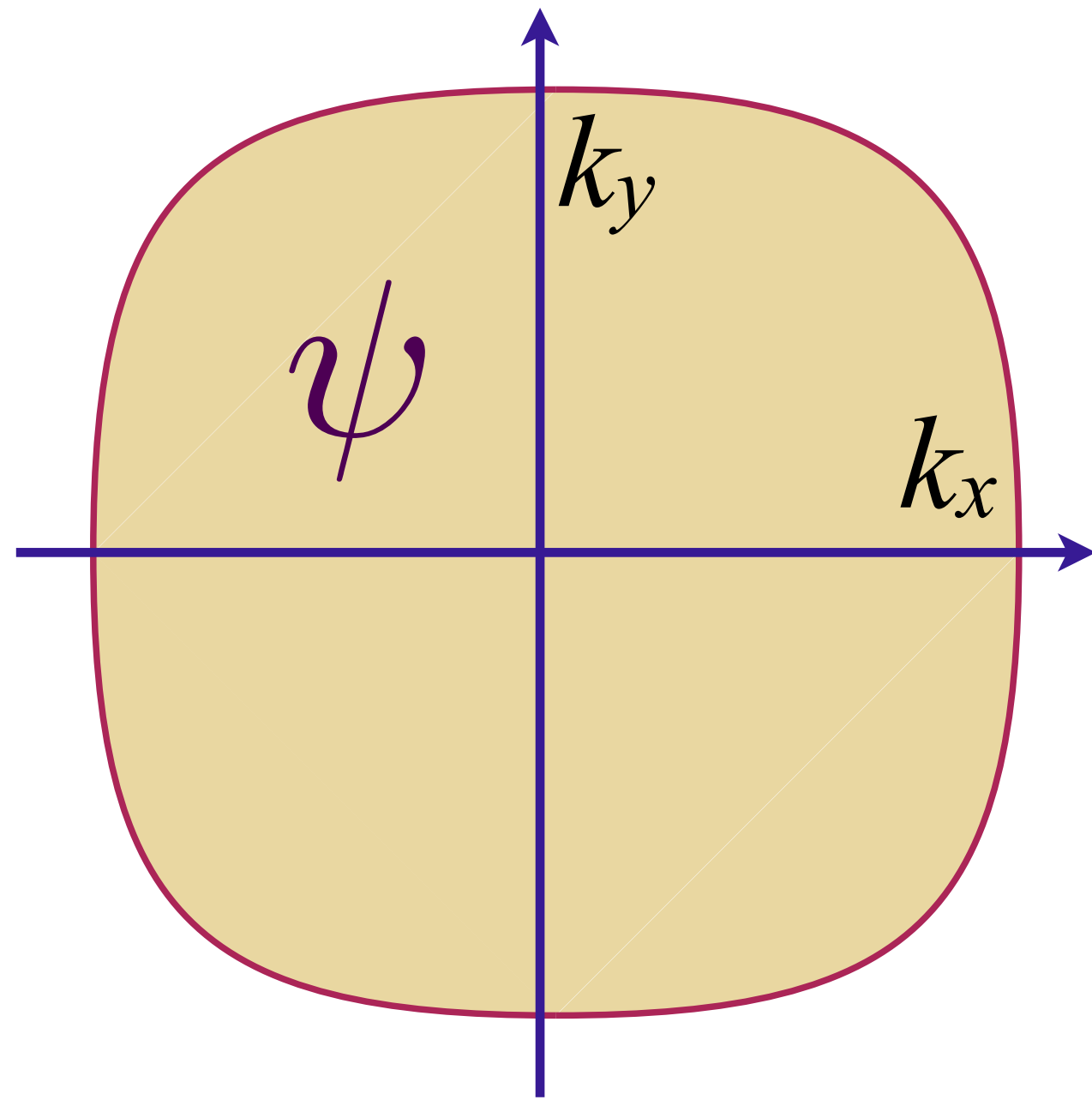
S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)

A. Eberlein, I. Mandal, and S.S. PRB **94**, 045133 (2016)

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$\frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r})$$

Transport—a perfect metal!

Conservation of momentum and fermion-boson drag imply:

$$\sigma(\omega) \sim \frac{1}{-i\omega} + |\omega|^0 + \dots \quad (\omega^{-2/3} \text{ term has vanishing co-efficient})$$

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)

D. L. Maslov, V. I. Yudson, and A. V. Chubukov PRL **106**, 106403 (2011)

S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)

A. Eberlein, I. Mandal, and S.S. PRB **94**, 045133 (2016)

Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S. PRB **106**, 115151 (2022)

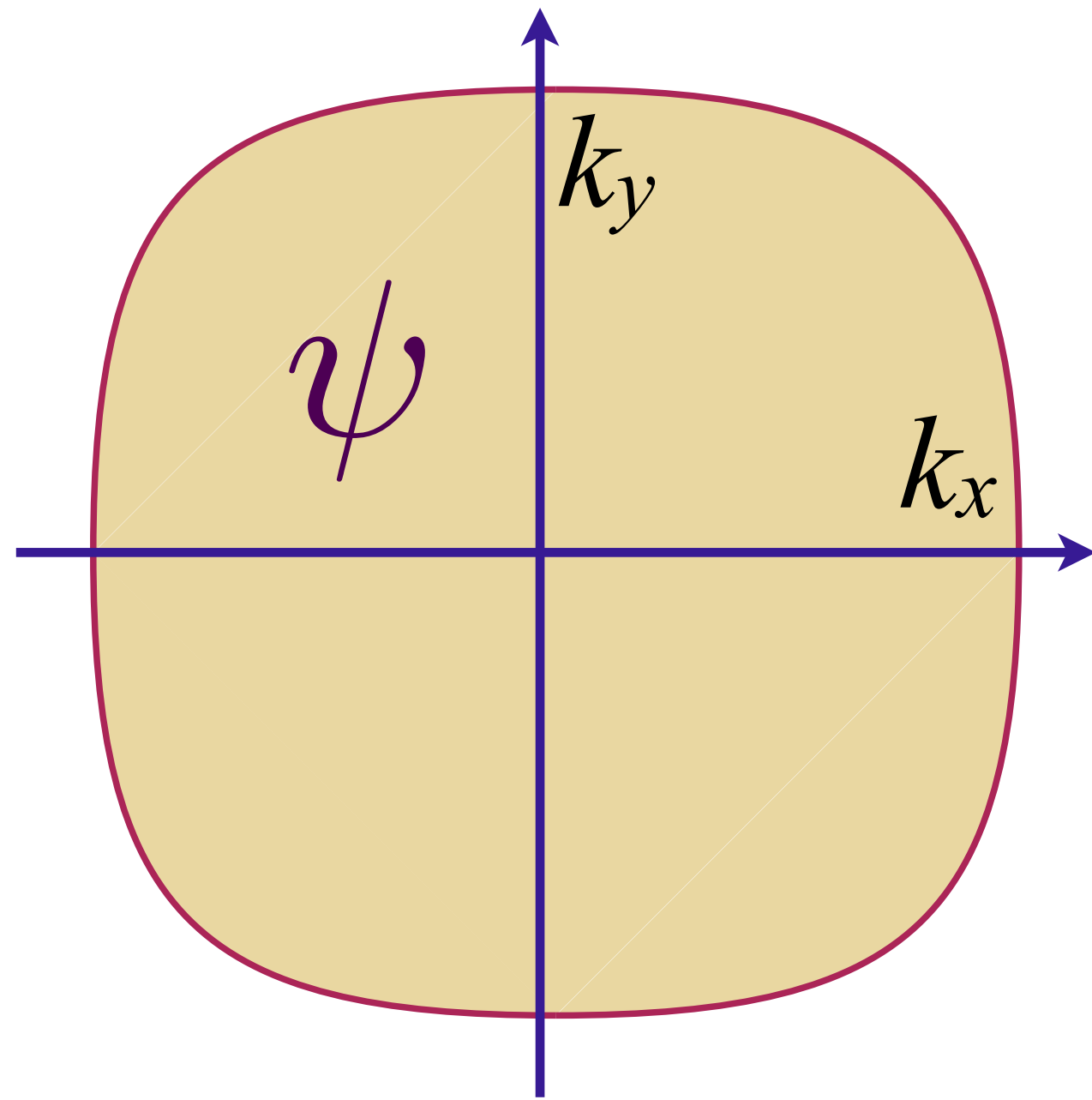
Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



a critical boson ϕ
e.g. Ising-nematic order

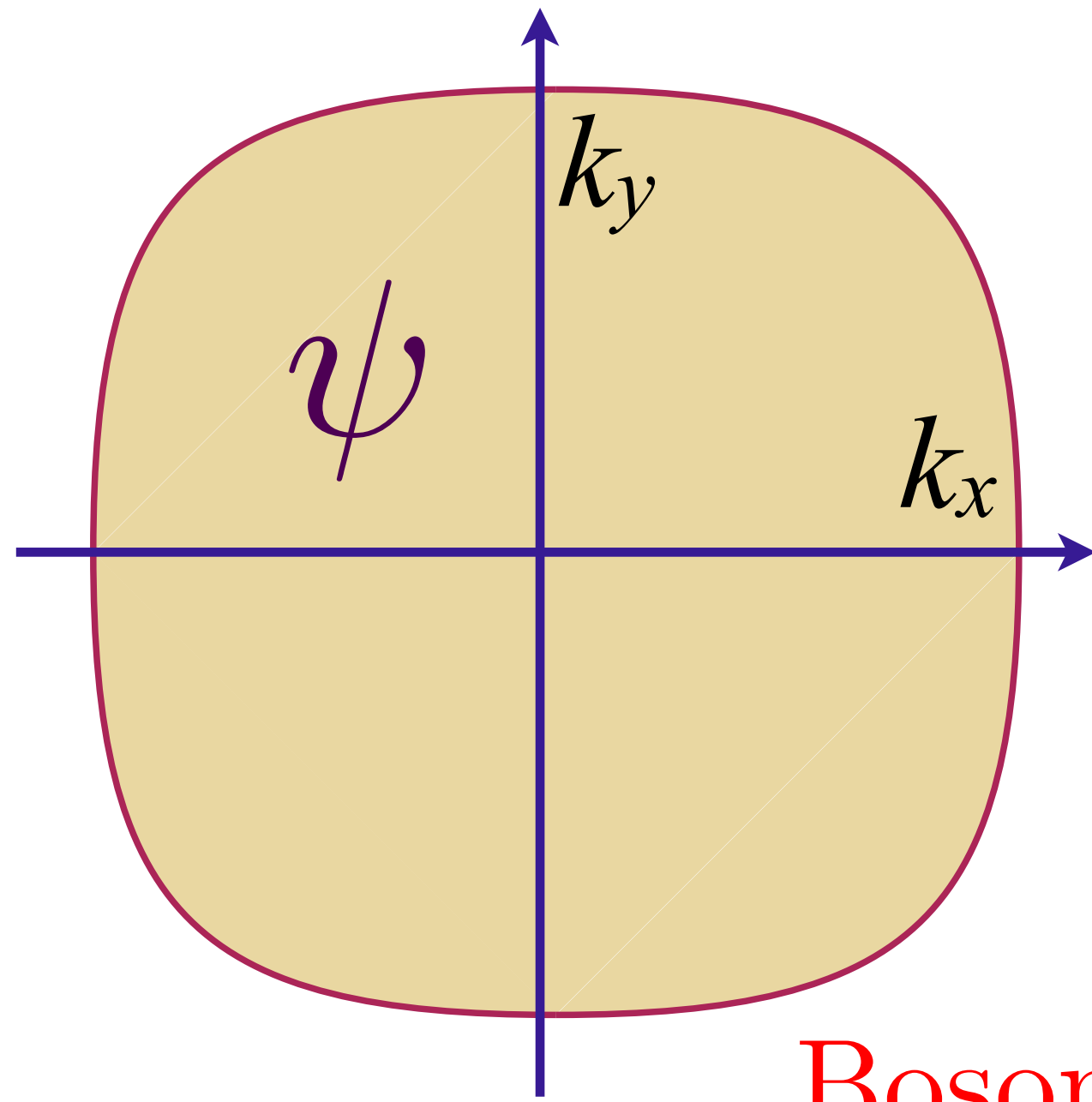
$$\begin{aligned} & \frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r}) \\ & + v(\mathbf{r})\psi^\dagger(\mathbf{r})\psi(\mathbf{r}) \end{aligned}$$

Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')}$ = $v^2\delta(\mathbf{r} - \mathbf{r}')$

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$\frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r}) + v(\mathbf{r})\psi^\dagger(\mathbf{r})\psi(\mathbf{r})$$

Boson Green's function: $D(q, i\Omega) \sim 1/(q^2 + \gamma|\Omega|)$

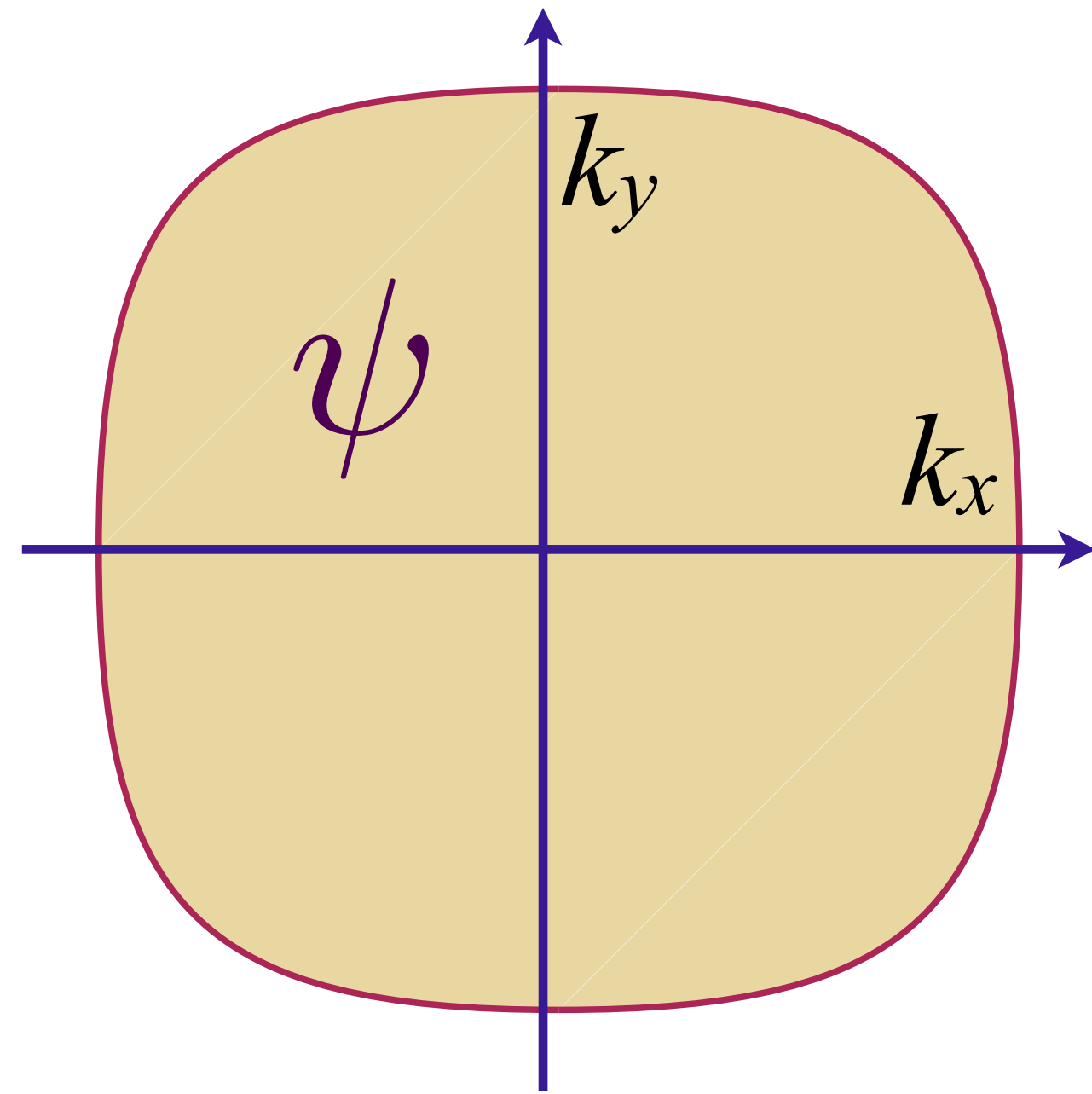
Fermion self energy: $\Sigma(i\omega) \sim -iv^2 \text{sgn}(\omega) - i\frac{g^2}{v^2}\omega \ln(1/|\omega|)$; $\frac{1}{\tau(\omega)} \sim \frac{g^2}{v^2}|\omega|$

Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$\frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r}) + v(\mathbf{r})\psi^\dagger(\mathbf{r})\psi(\mathbf{r})$$

But resistivity and optical conductivity are like a Fermi liquid,
with residual resistivity $\sim v^2$.

Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S. PRB **106**, 115151 (2022)

T. C. Wu, Y. Liao, and M. S. Foster, PRB **106**, 155108 (2022)

Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:

Potential disorder ν

A marginal Fermi liquid but NOT a strange metal

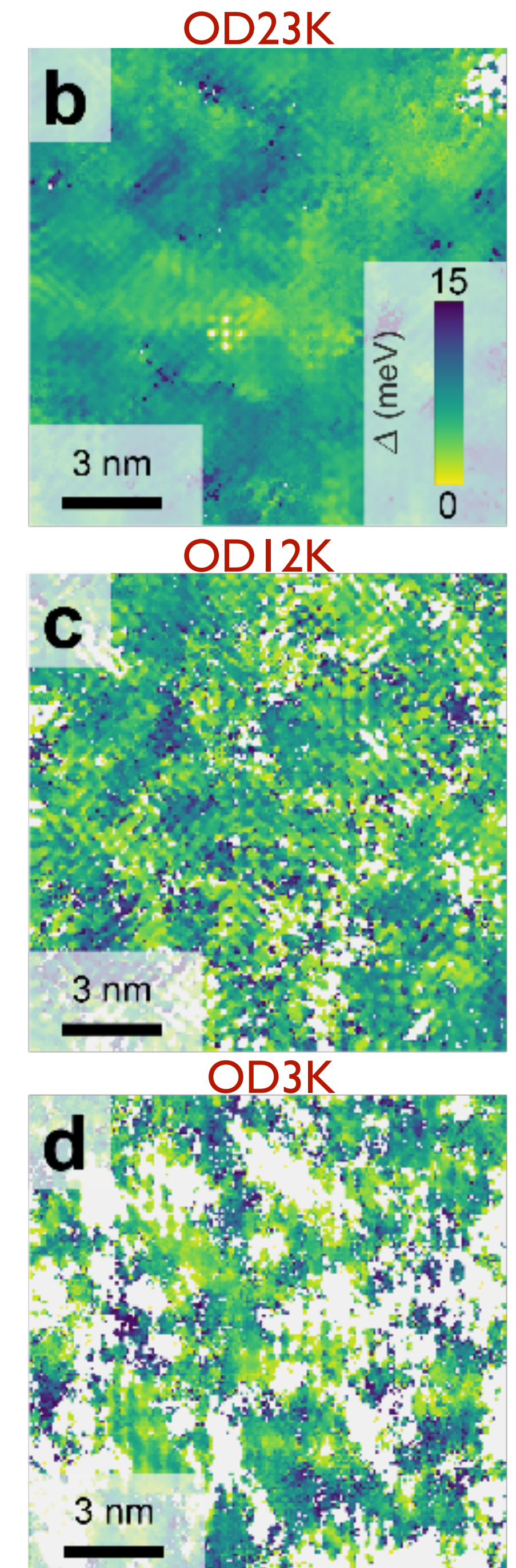
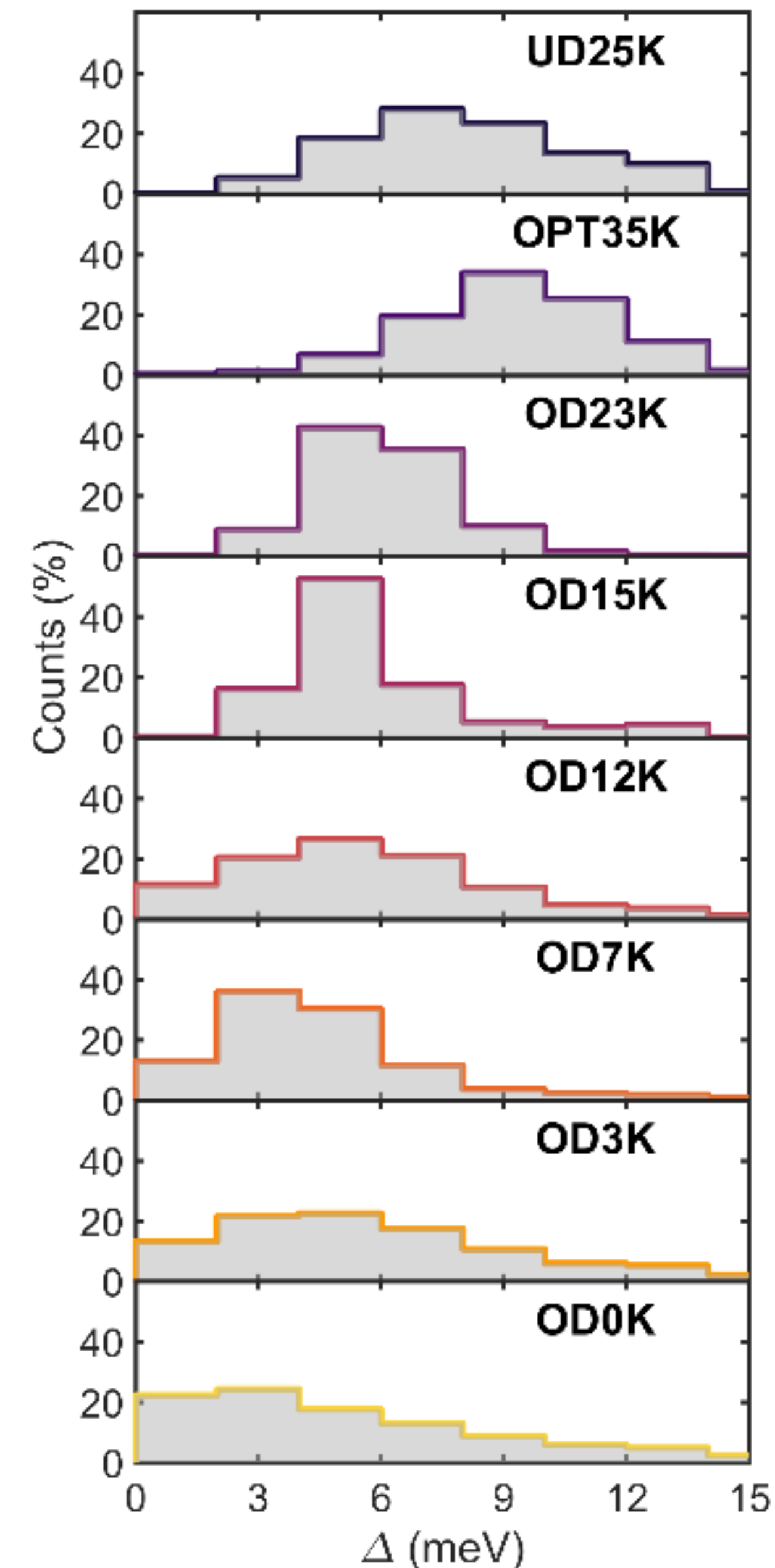
Spatially random interactions!

Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped $(\text{Pb,Bi})_2\text{Sr}_2\text{CuO}_{6+\delta}$

Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge, Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos, Amber Vervloet, Steef Smit, Erik van Heumen, Mark S. Golden, Yinkai Huang, Takeshi Kondo, Yi Yin, Jennifer E. Hoffman, Miguel Antonio Sulangi, Jan Zaanen, Milan P. Allan

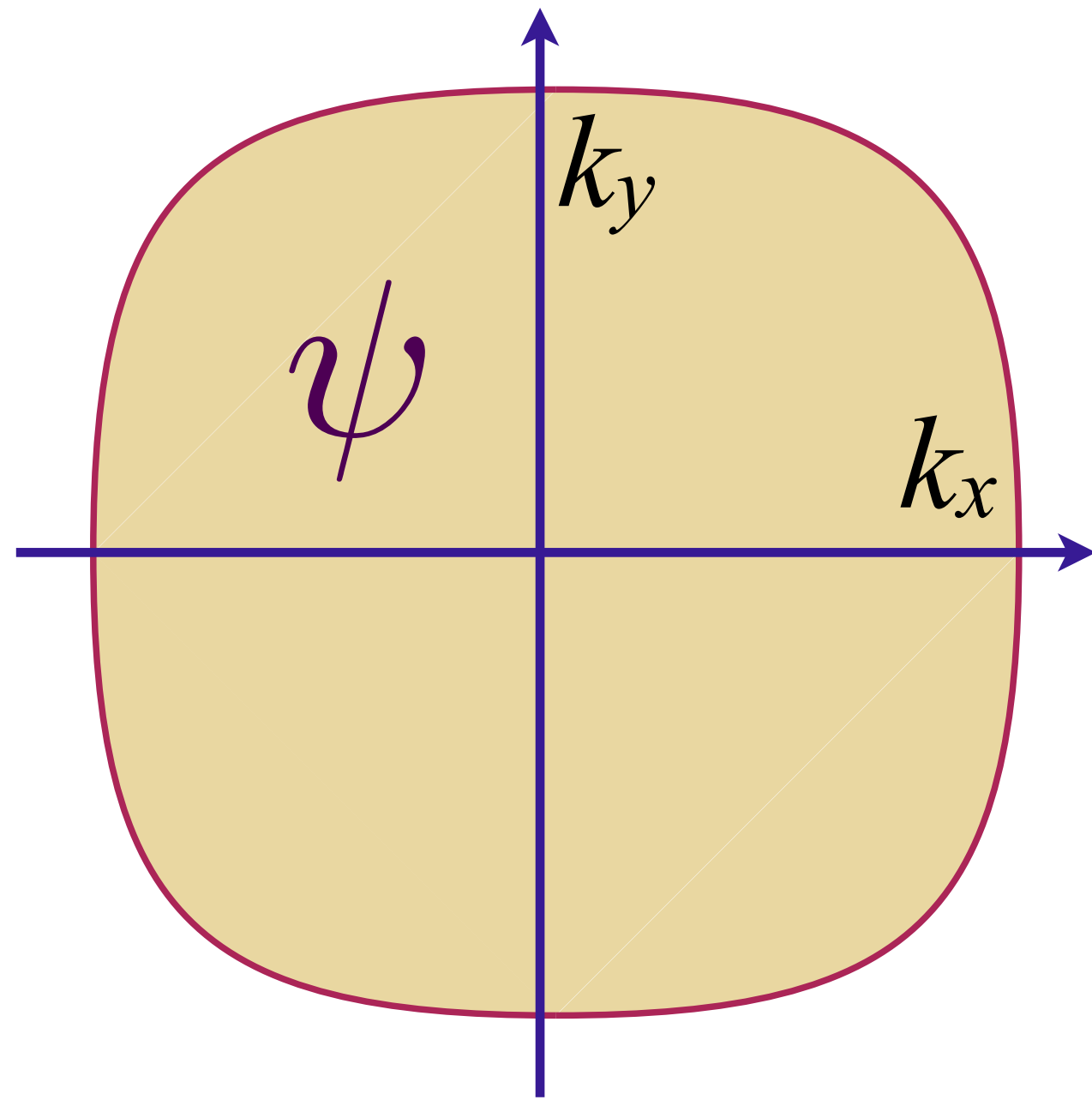
Our scanning tunneling spectroscopy measurements in the overdoped regime of the $(\text{Pb,Bi})_2\text{Sr}_2\text{CuO}_{6+\delta}$ high-temperature superconductor show the emergence of puddled superconductivity, featuring nanoscale superconducting islands in a metallic matrix

arXiv:2205.09740



Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



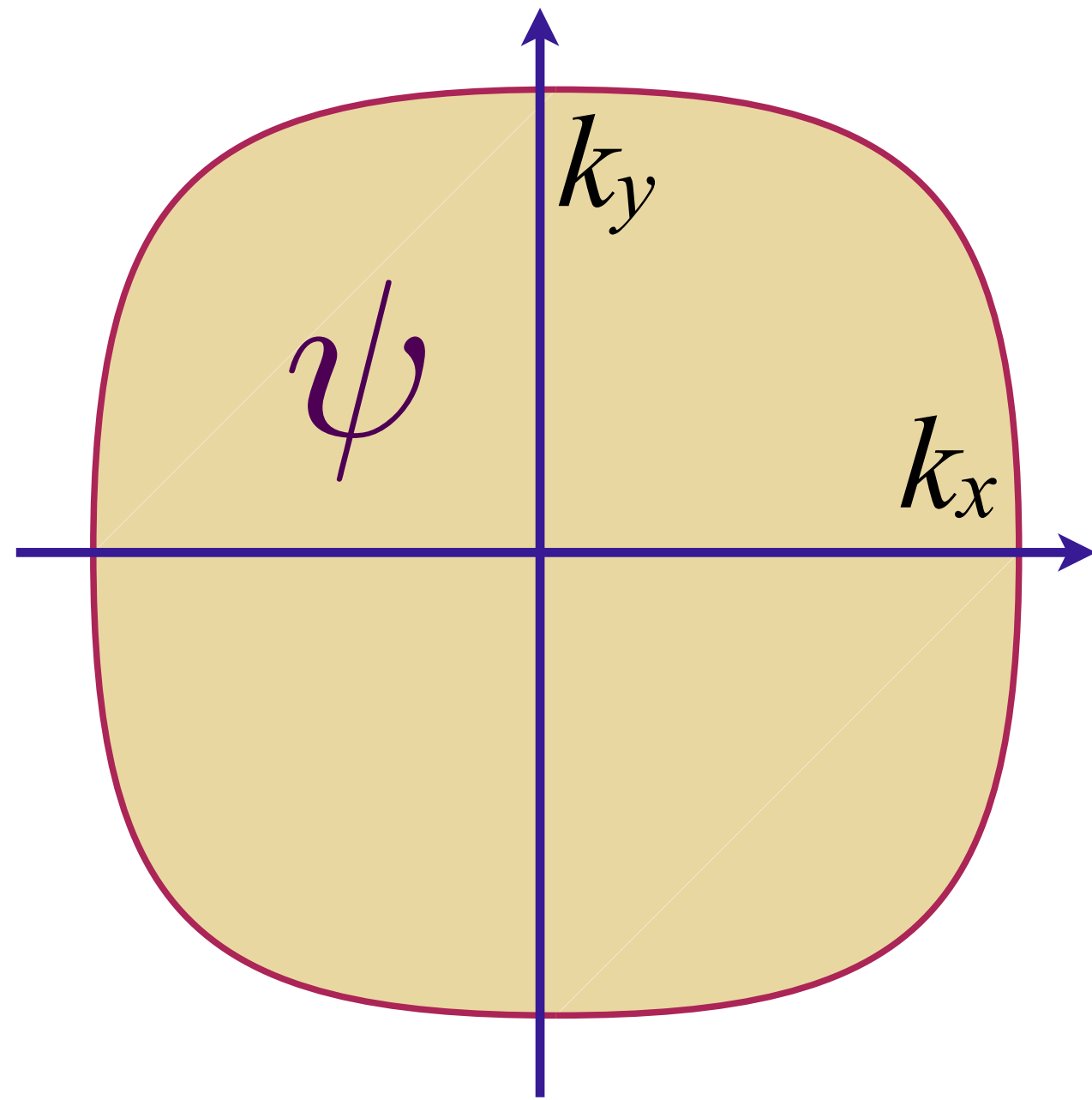
a critical boson ϕ
e.g. Ising-nematic order

$$\frac{[\phi(\mathbf{r})]^2}{J + J'(\mathbf{r})} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r}) + v(\mathbf{r})\psi^\dagger(\mathbf{r})\psi(\mathbf{r})$$

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$[\phi(\mathbf{r})]^2 + [g + g'(\mathbf{r})] \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r}) + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

ϕ^2 “mass” disorder $J'(\mathbf{r})$ is strongly relevant;
 rescale ϕ to move disorder to the Yukawa coupling;

Spatially random Yukawa coupling $g'(\mathbf{r})$ with $\overline{g'(\mathbf{r})} = 0$, $\overline{g'(\mathbf{r})g'(\mathbf{r}')} = g'^2 \delta(\mathbf{r} - \mathbf{r}')$

Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')} = v^2 \delta(\mathbf{r} - \mathbf{r}')$

Fermi surface coupled to a critical boson with disorder

All results are obtained from the large N saddle-point and response functions of this G - Σ - D - Π theory:

$$\mathcal{Z} = \int \mathcal{D}G \mathcal{D}\Sigma \mathcal{D}D \mathcal{D}\Pi \exp(-N S_{\text{all}})$$

$$S_{\text{all}} = -\ln \det(\partial_\tau + \varepsilon(\mathbf{k}) - \mu + \Sigma) + \frac{1}{2} \ln \det(-\partial_\tau^2 + \mathbf{q}^2 + m_b^2 - \Pi)$$

$$+ \int d\tau d^2r \int d\tau' d^2r' \left[-\Sigma(\tau', \mathbf{r}'; \tau, \mathbf{r}) G(\tau, \mathbf{r}; \tau', \mathbf{r}') + \frac{1}{2} \Pi(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') \right.$$

$$+ \frac{g^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') + \frac{v^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) \delta(\mathbf{r} - \mathbf{r}')$$

$$\left. + \frac{g'^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') \delta(\mathbf{r} - \mathbf{r}') \right].$$

Fermi surface coupled to a critical boson with disorder

All results are obtained from the large N saddle-point and response functions of this G - Σ - D - Π theory:

$$\mathcal{Z} = \int \mathcal{D}G \mathcal{D}\Sigma \mathcal{D}D \mathcal{D}\Pi \exp(-N S_{\text{all}})$$

Saddle-point equations

$$\Sigma(\tau, \mathbf{r}) = g^2 D(\tau, \mathbf{r}) G(\tau, \mathbf{r}) + v^2 G(\tau, \mathbf{r}) \delta^2(\mathbf{r}) + g'^2 G(\tau, \mathbf{r}) D(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

$$\Pi(\tau, \mathbf{r}) = -g^2 G(-\tau, -\mathbf{r}) G(\tau, \mathbf{r}) - g'^2 G(-\tau, \mathbf{r}) G(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

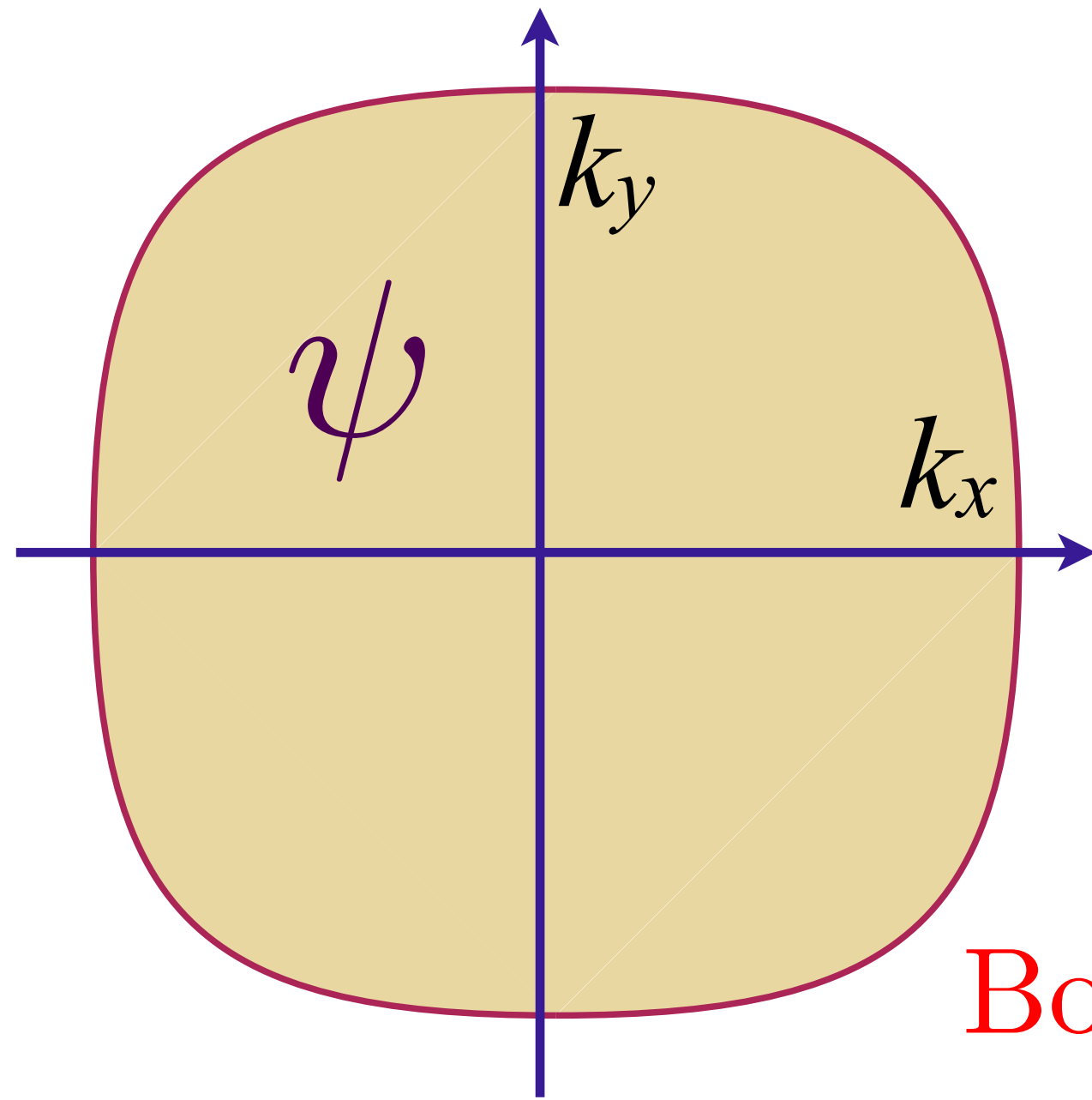
$$G(i\omega, \mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega, \mathbf{k})},$$

$$D(i\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega, \mathbf{q})}.$$

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$[\phi(\mathbf{r})]^2 + [g + g'(\mathbf{r})] \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r}) + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

Boson Green's function: $D(q, i\Omega) \sim 1/(q^2 + \gamma|\Omega|)$

Fermion self energy:

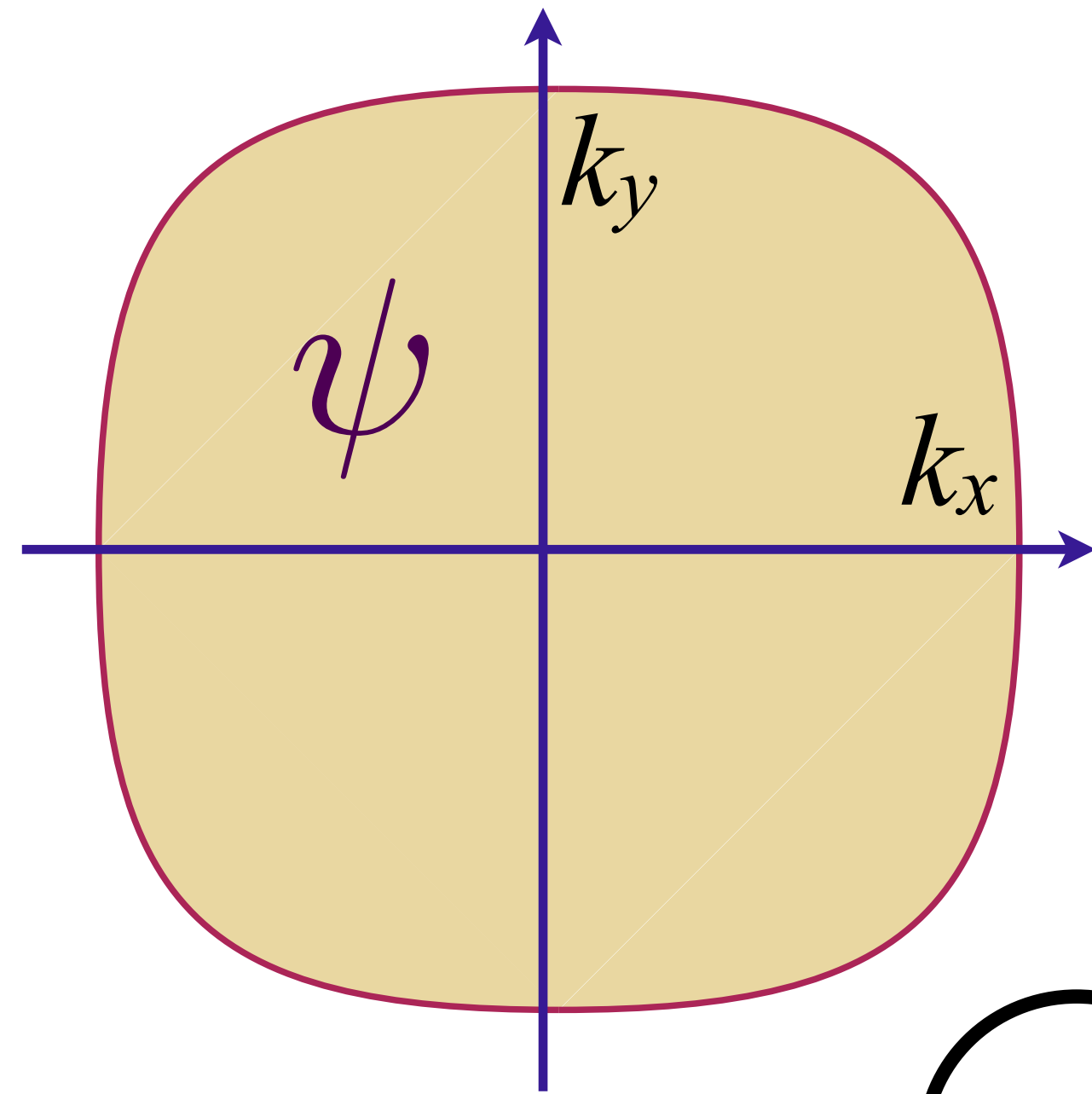
$$\Sigma(i\omega) \sim -iv^2 \text{sgn}(\omega) - i \left(\frac{g^2}{v^2} + g'^2 \right) \omega \ln(1/|\omega|); \quad \frac{1}{\tau(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2 \right) |\omega|$$

Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat

Fermi surface coupled to a critical boson with disorder

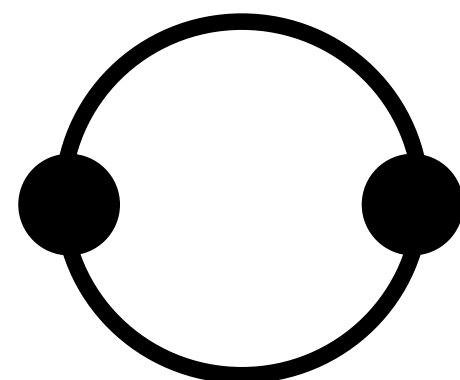
$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



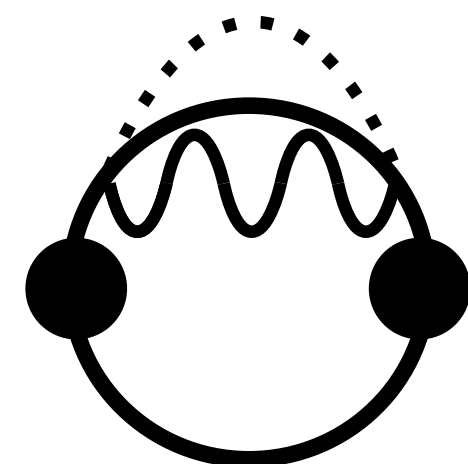
$$[\phi(\mathbf{r})]^2 + [g + g'(\mathbf{r})] \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r}) + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

Conductivity:



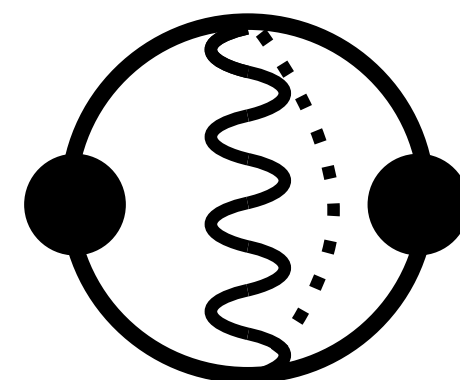
(a)

$$\sigma_v$$



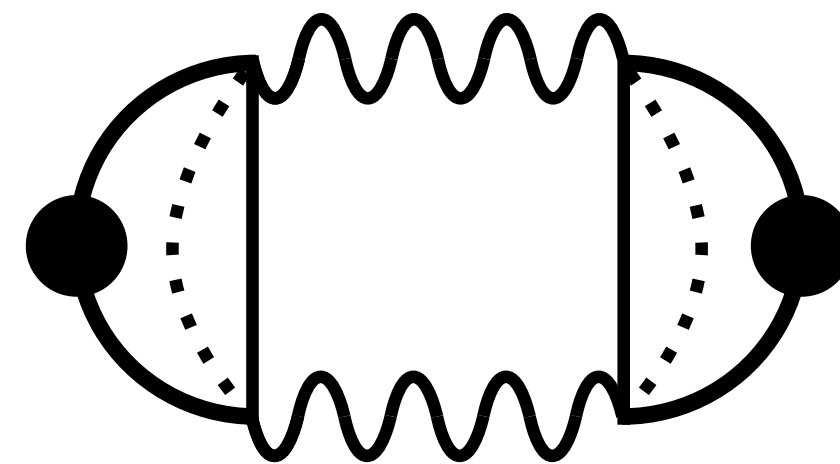
(b)

$$\frac{\sigma_{\Sigma, g}}{2}, \frac{\sigma_{\Sigma, g'}}{2}$$

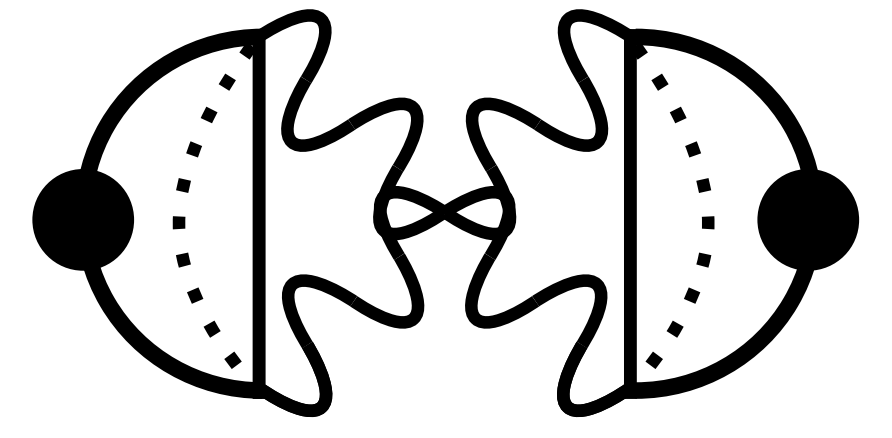


(c)

$$\sigma_{V, g}$$



(d)



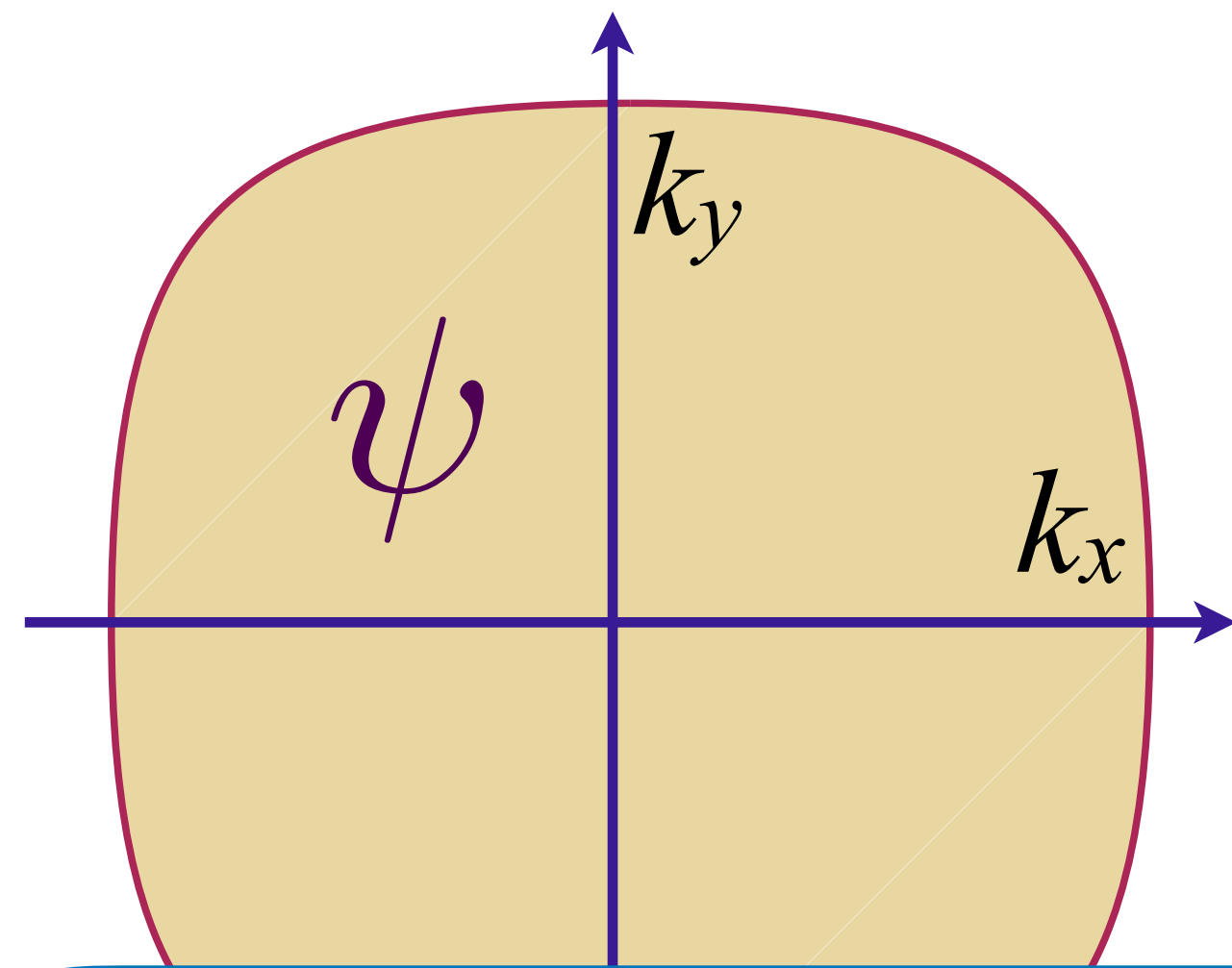
(e)

+ all ladders and bubbles.....

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$[\phi(\mathbf{r})]^2 + [g + g'(\mathbf{r})] \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r}) + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

Conductivity: $\sigma(\omega) \sim [1/\tau_{\text{trans}}(\omega) - i\omega m_{\text{trans}}^*(\omega)/m]^{-1}$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2 |\omega| \quad ; \quad \frac{m_{\text{trans}}^*(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2 ; Linear-in- T resistivity determined by g'^2 ;
 Transport insensitive to g ; Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat.

Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:

Potential disorder v

A marginal Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:

Interaction disorder g'

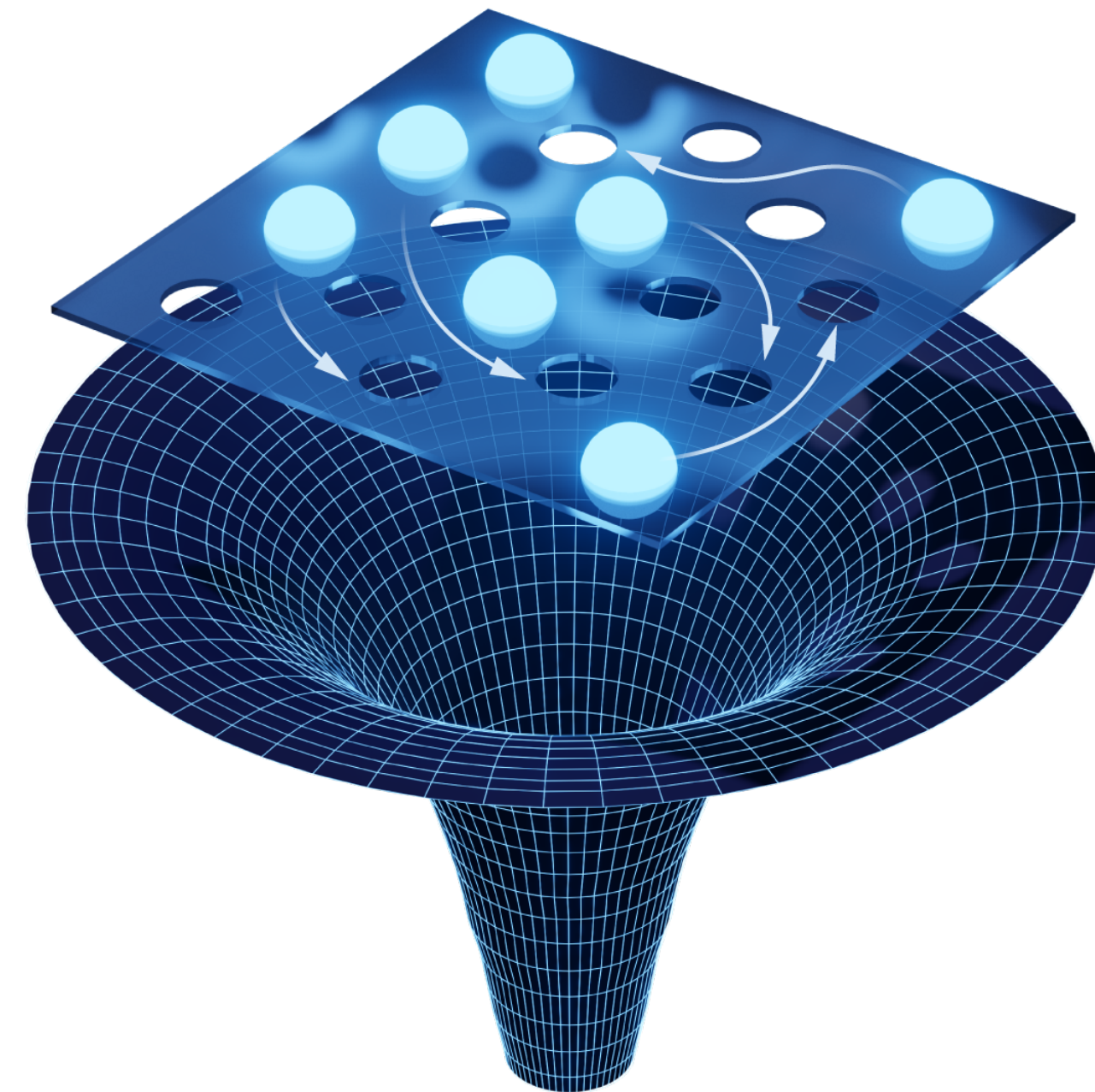
A marginal Fermi liquid AND a strange metal

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- Toy SYK model captures the correct universal low energy quantum theory of charged black holes, and provides a Hamiltonian realization of black hole microstates.
- Linear- T resistivity, $T \ln(1/T)$ specific heat, $\sim 1/\omega$ optical conductivity, and marginal Fermi liquid electron spectrum *all* arise from a SYK-like model with spatially random interactions in a two-dimensional quantum-critical metal.

