

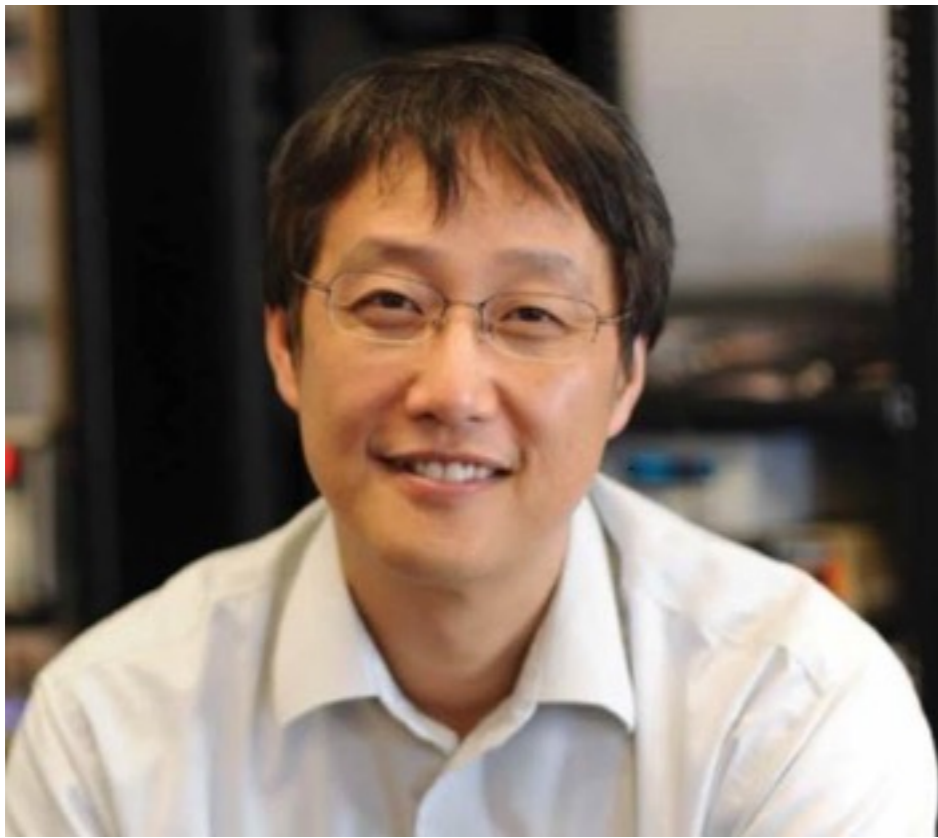
Strange metals and black holes

Boston University
September 29, 2015

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

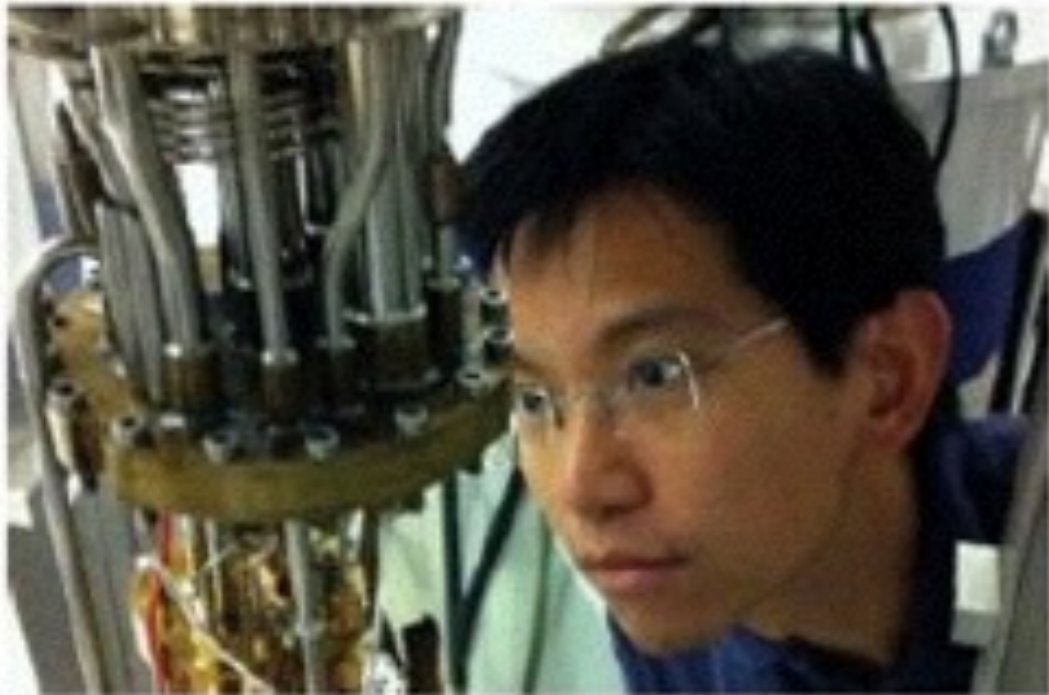




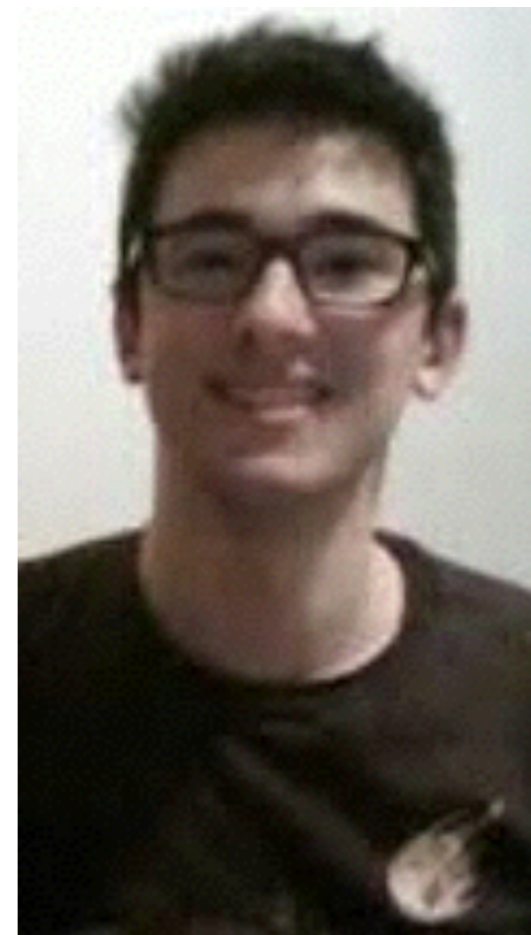
Philip Kim



Jesse Crossno



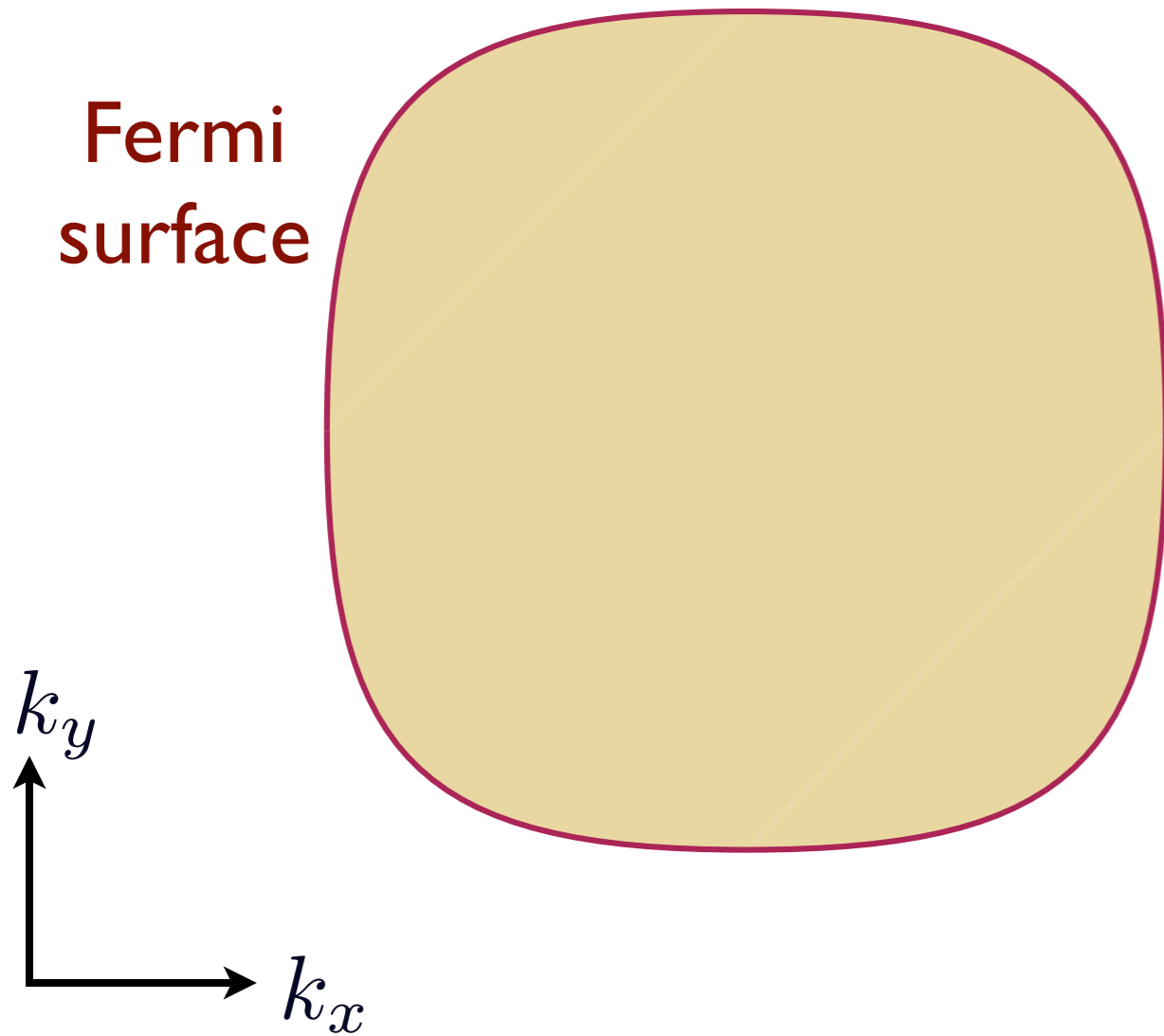
Kin Chung Fong



Andrew Lucas

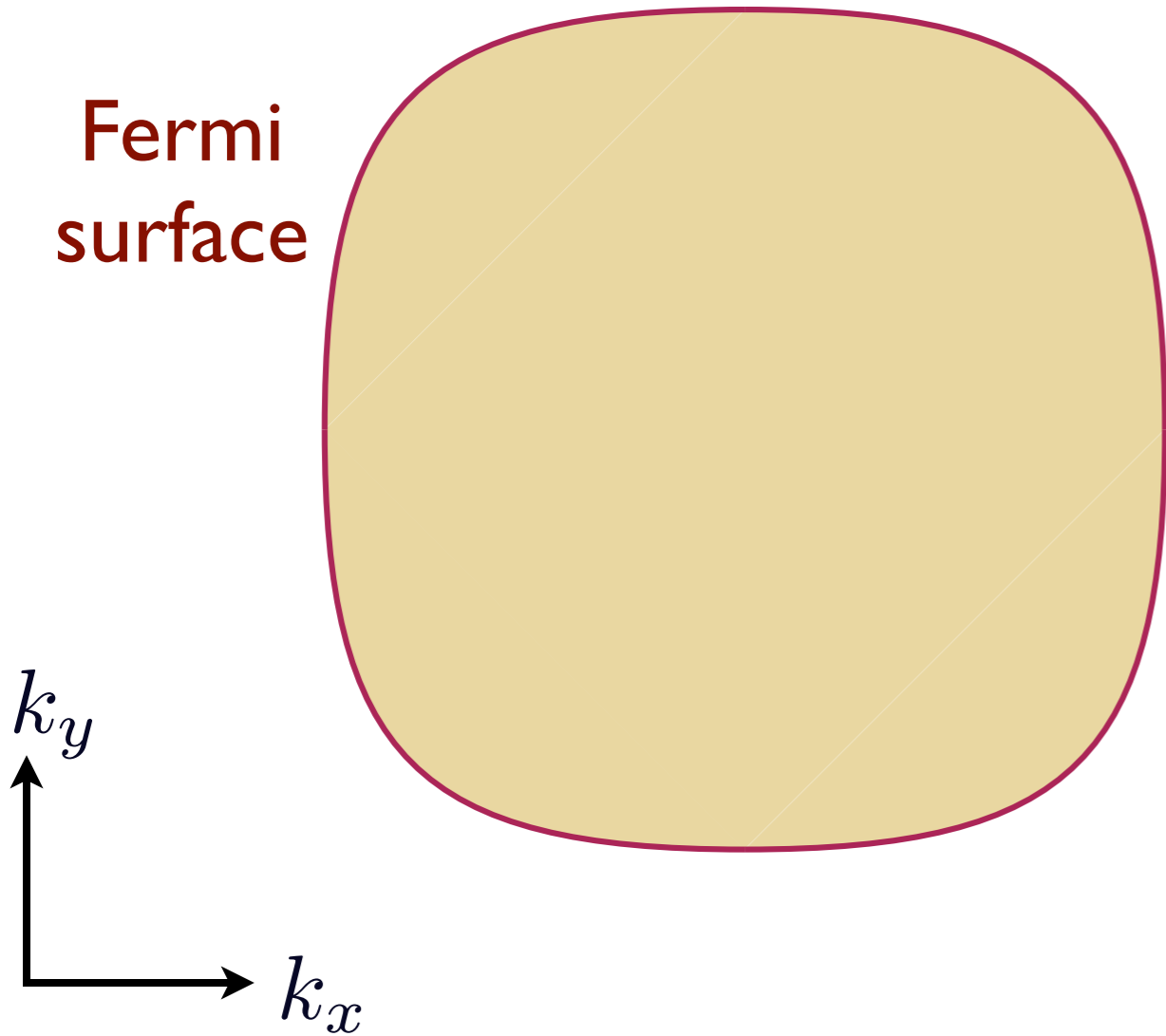
Ordinary metals: the Fermi liquid

- Fermi surface separates empty and occupied states in momentum space.



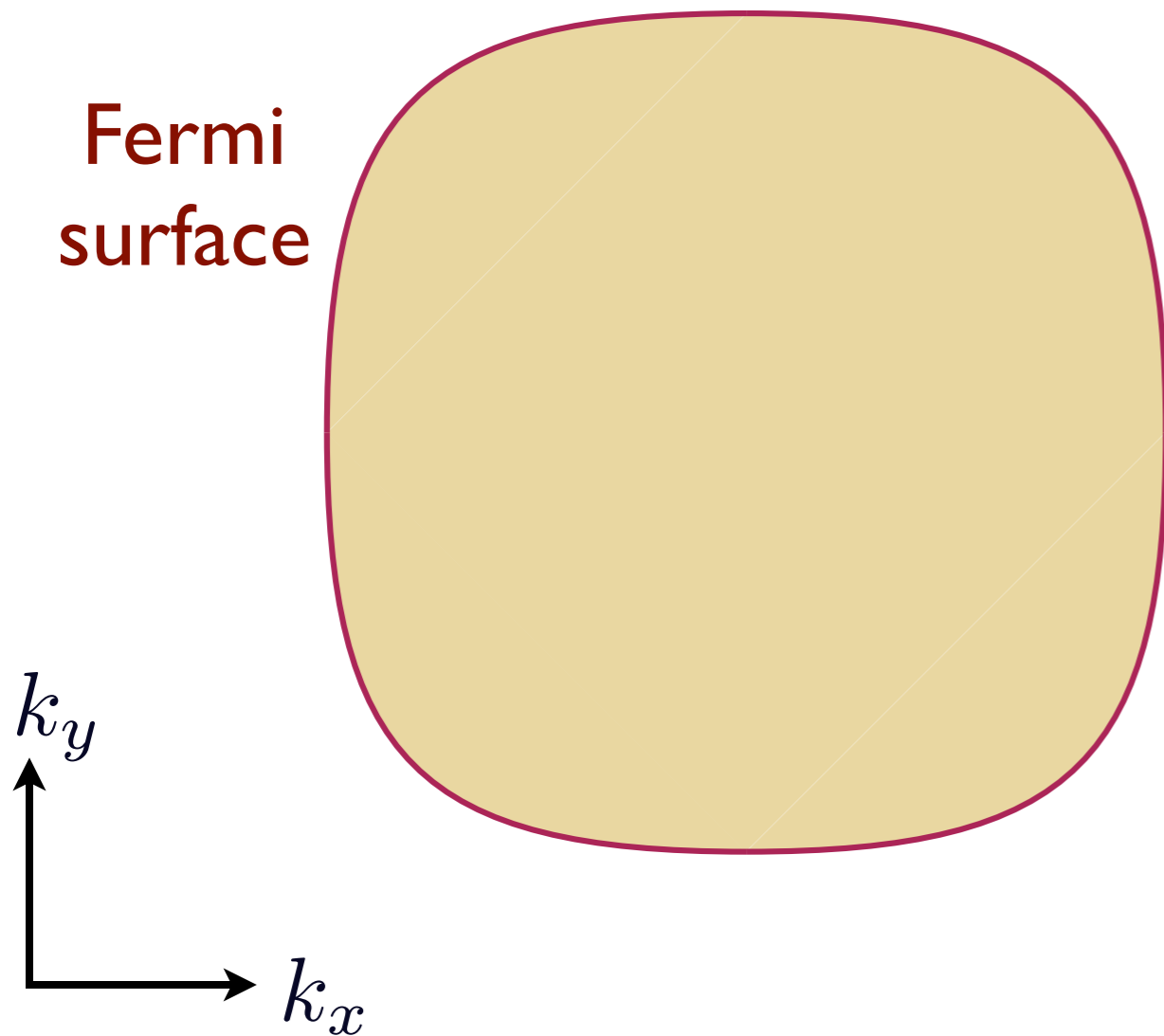
Ordinary metals: the Fermi liquid

Fermi
surface



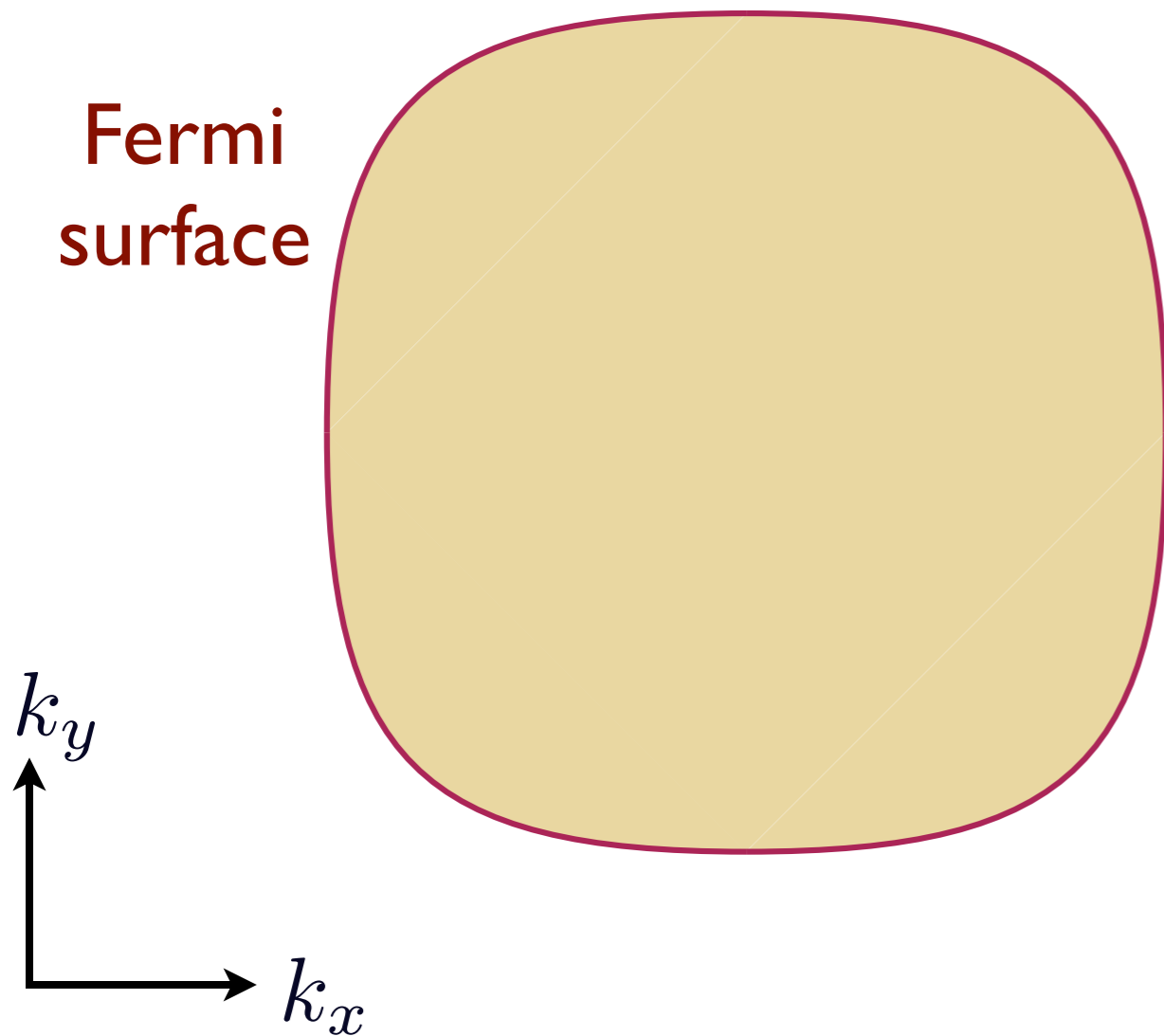
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- Area enclosed by Fermi surface = Q . Momenta of low energy excitations fixed by density of *all* electrons.

Ordinary metals: the Fermi liquid



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- Long-lived electron-like quasi-particle excitations near the Fermi surface: lifetime of quasi-particles $\sim 1/T^2$.

Ordinary metals: the Fermi liquid

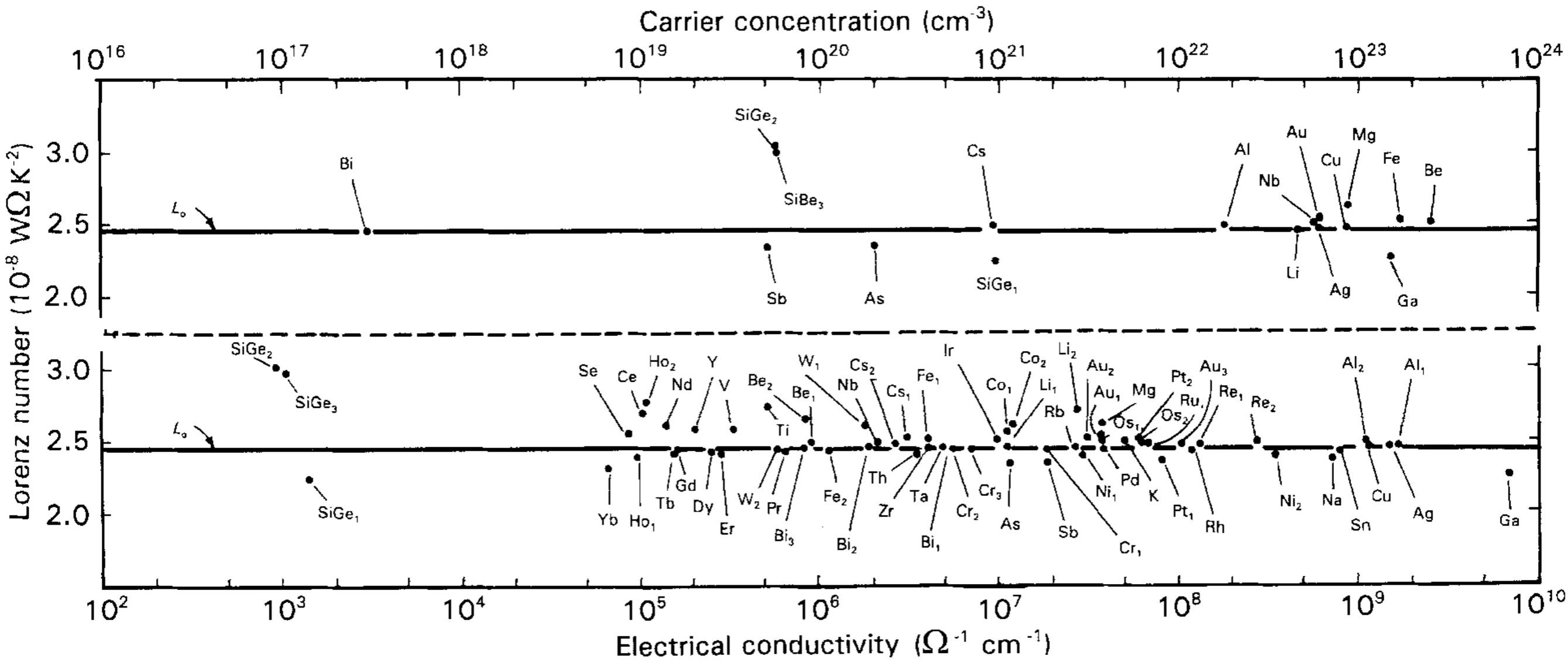


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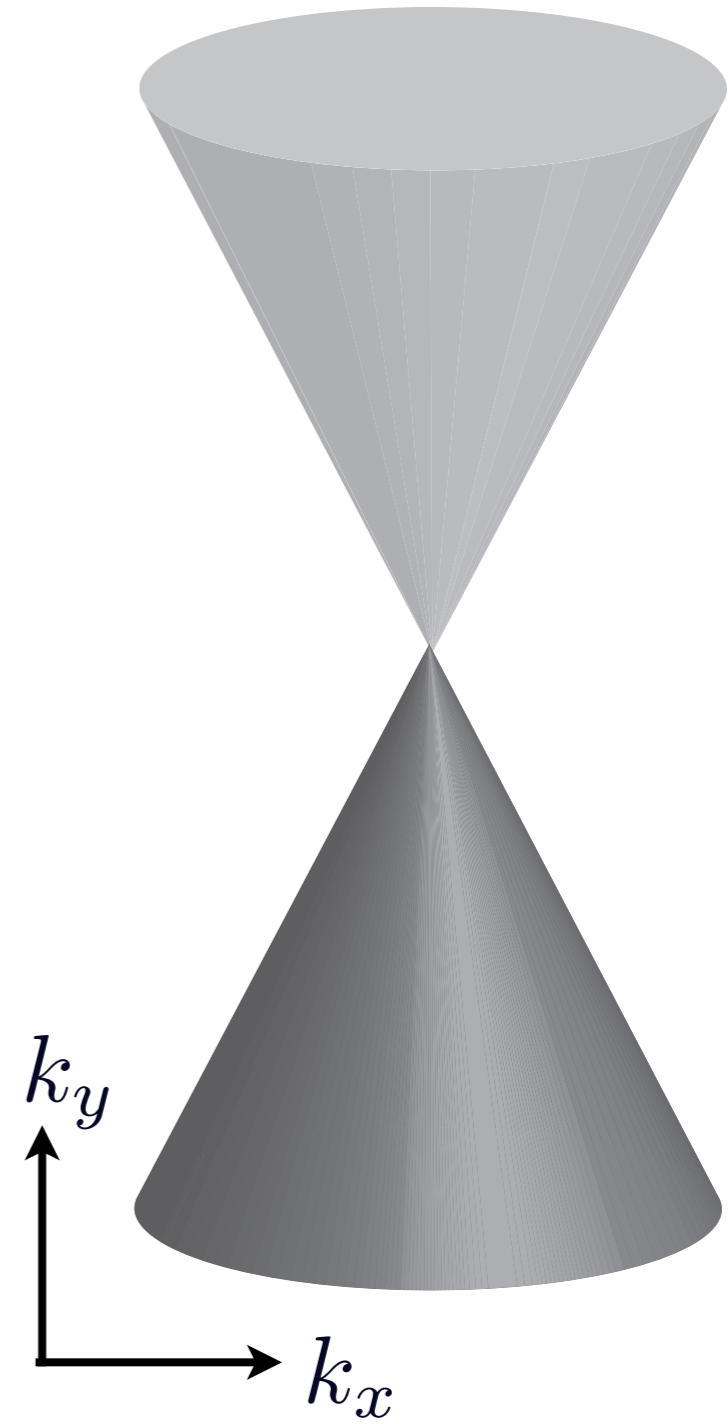
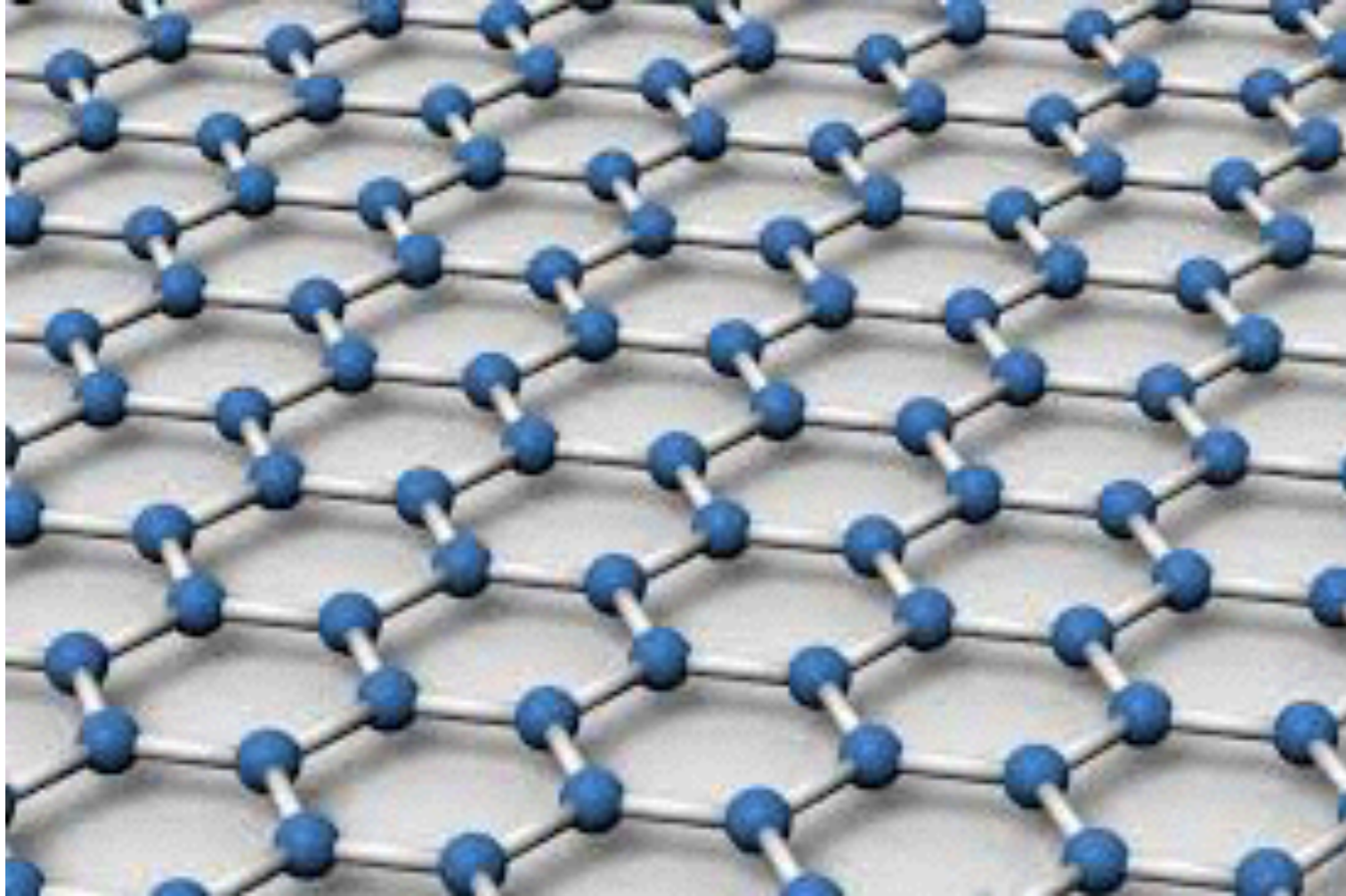
- $$\frac{(\text{Thermal conductivity})}{T (\text{Electrical conductivity})} = \frac{\pi^2 k_B^2}{3e^2}$$

► Wiedemann-Franz law in a Fermi liquid:

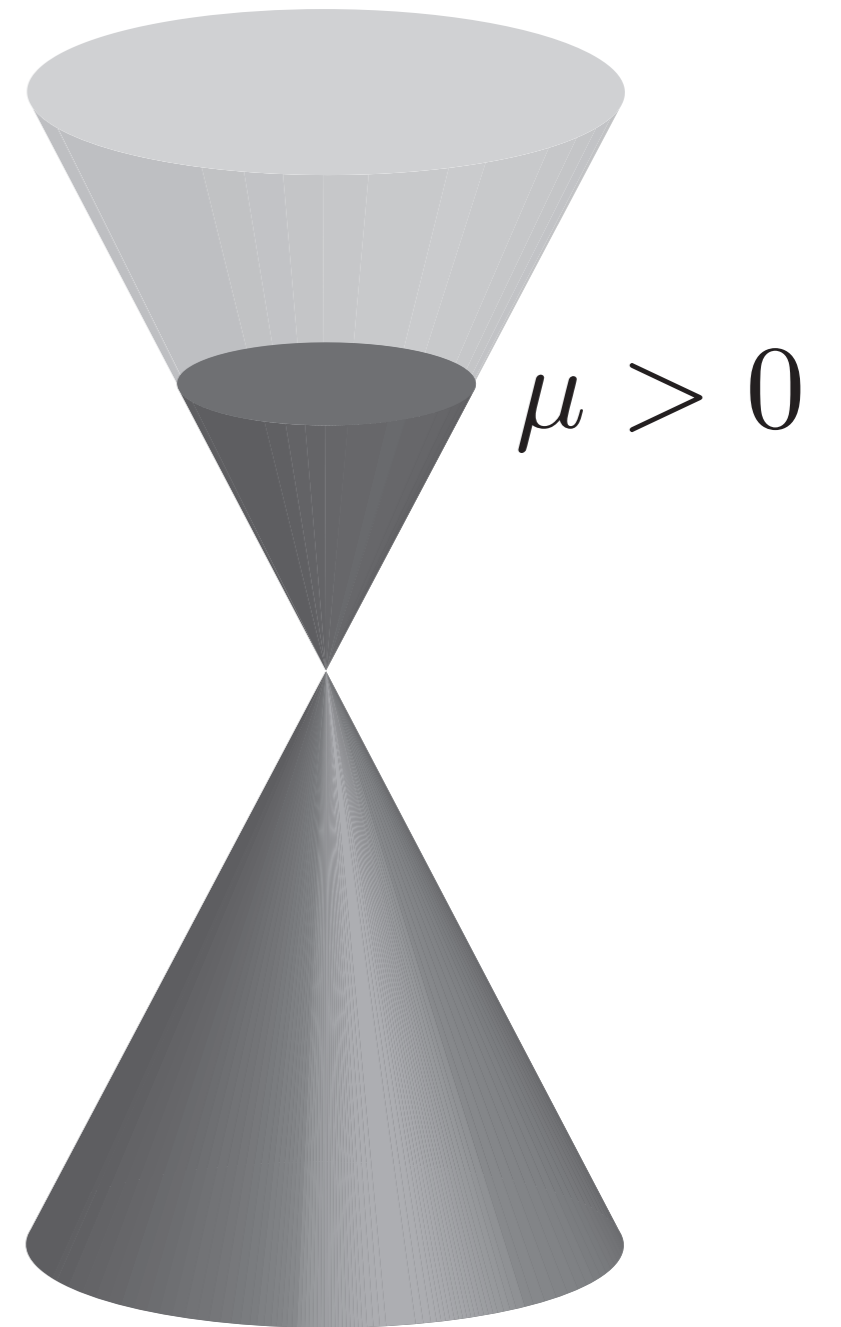
$$\frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8} \frac{W \cdot \Omega}{K^2}.$$



Graphene

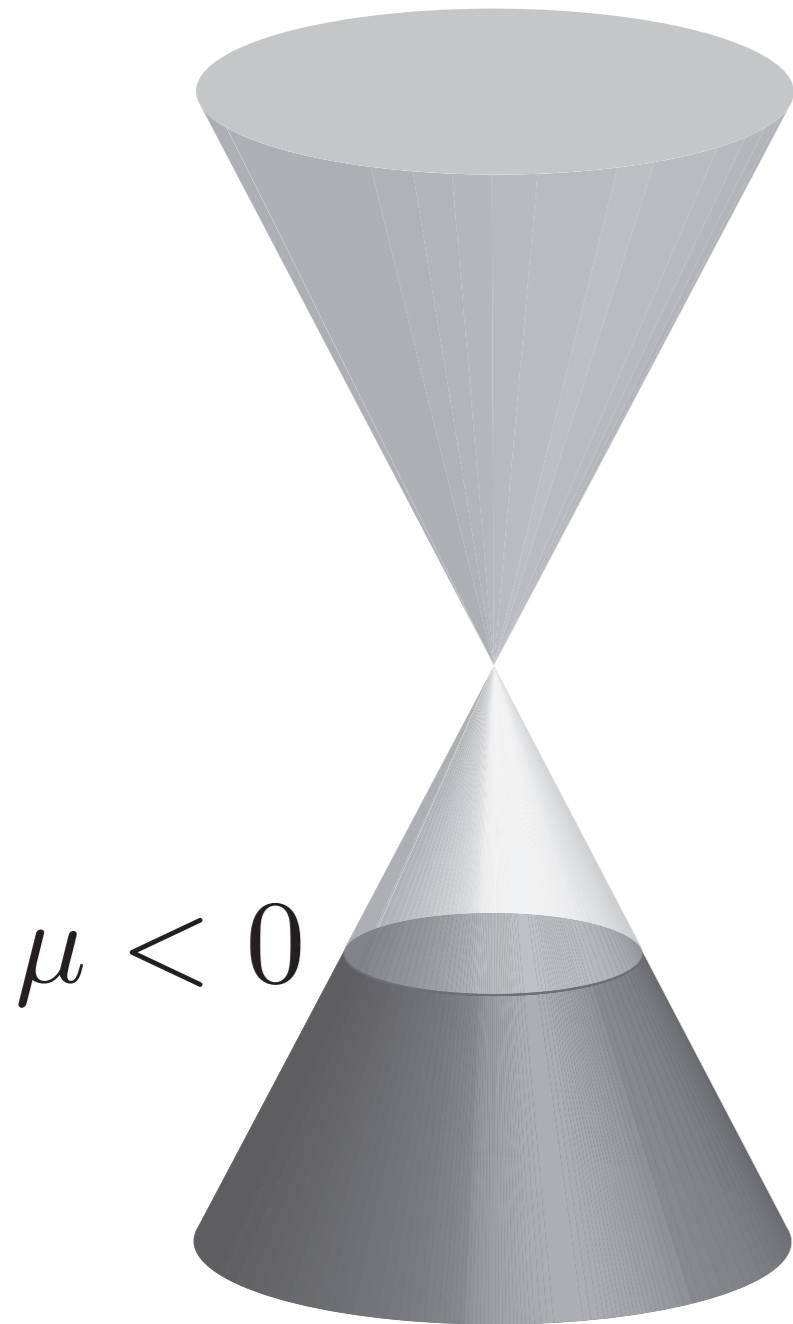


Graphene

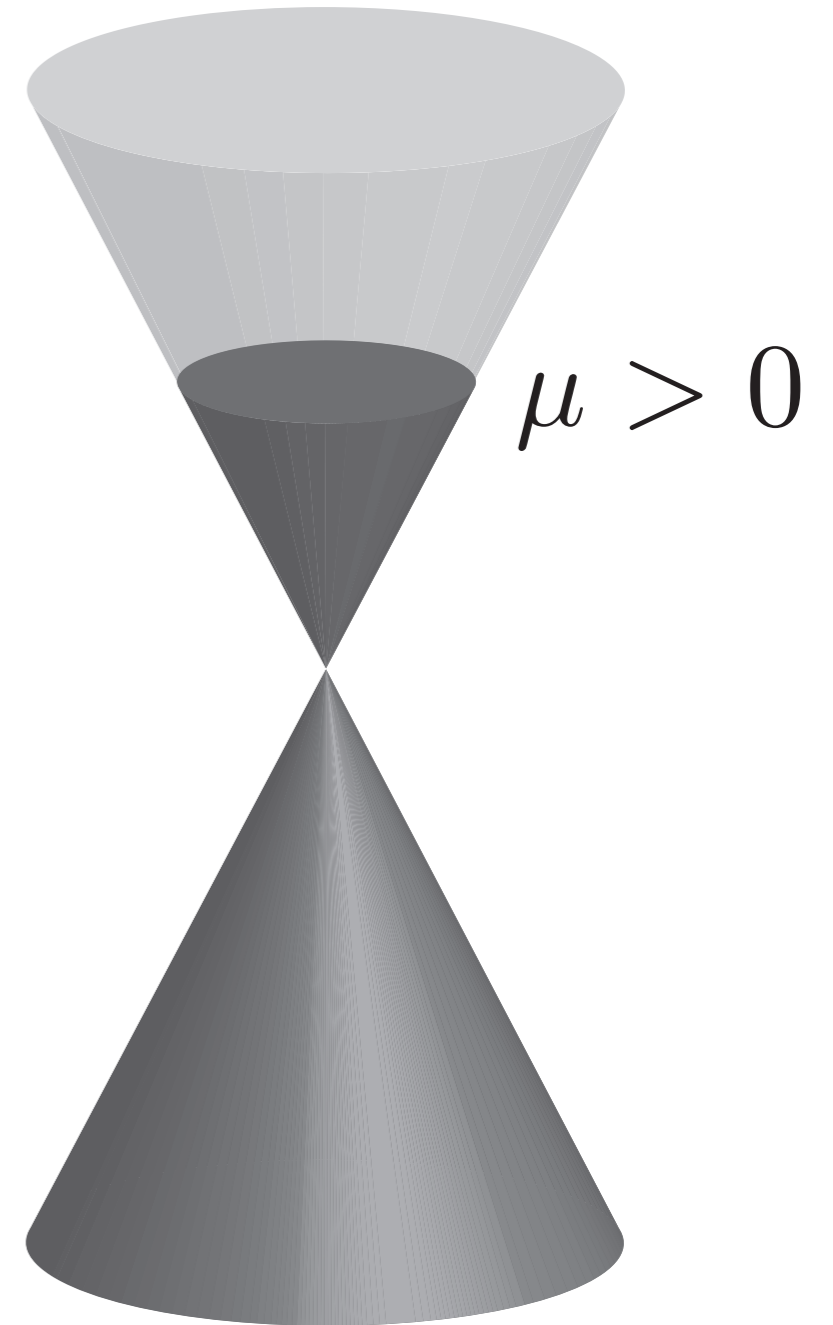


**Electron
Fermi surface**

Graphene

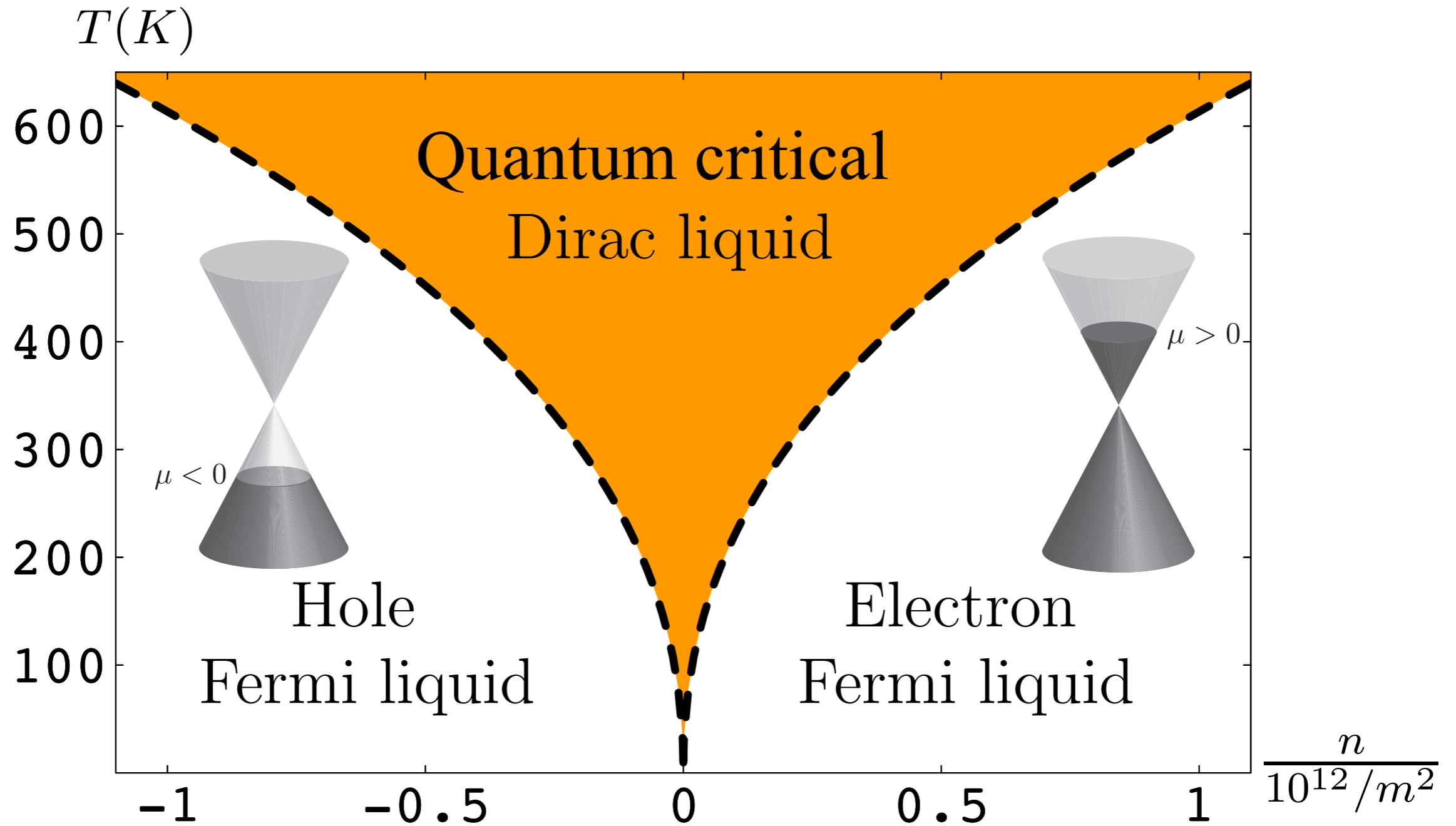


**Hole
Fermi surface**



**Electron
Fermi surface**

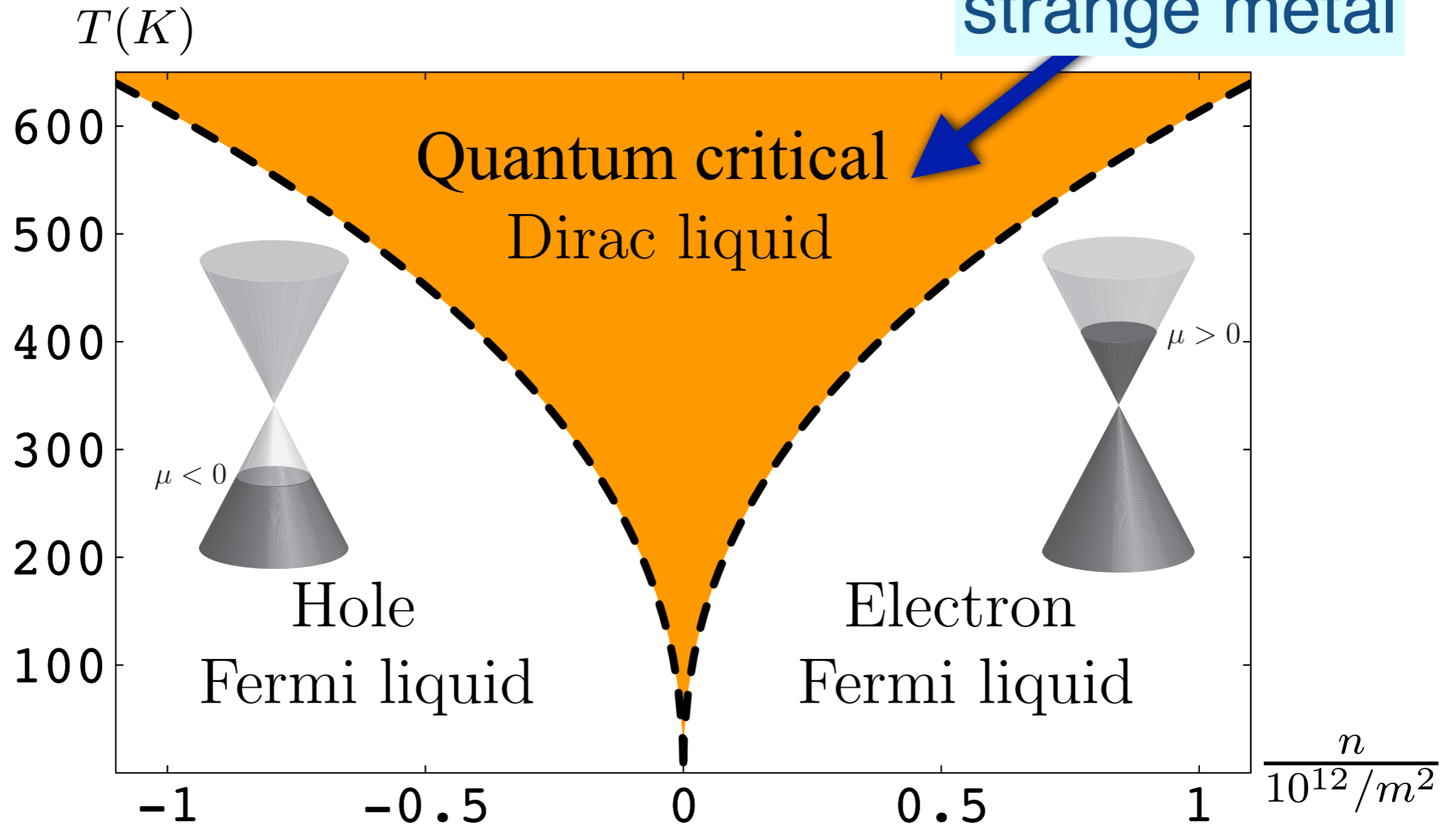
Graphene



D. E. Sheehy and J. Schmalian, PRL **99**, 226803 (2007)
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Graphene

Predicted
strange metal



M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)

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Key properties of a strange metal

- No quasiparticle excitations

Key properties of a strange metal

- No quasiparticle excitations
- Shortest possible “collision time”, or more precisely, fastest possible local equilibration time $\sim \frac{\hbar}{k_B T}$

Prediction for transport in the graphene strange metal

Recall that in a Fermi liquid, the Lorenz ratio $L = \kappa / (T\sigma)$, where κ is the thermal conductivity, and σ is the conductivity, is given by

$$L = \frac{\pi^2 k_B^2}{3e^2}$$

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

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In contrast, for a strange metal with a “relativistic” Hamiltonian, L diverges as the charge density $Q \rightarrow 0$, and then the impurity momentum relaxation time $\tau_{\text{imp}} \rightarrow \infty$:

$$L = \frac{\mathcal{H}\tau_{\text{imp}}}{T^2\sigma_Q} \frac{1}{(1 + Q^2\tau_{\text{imp}}/(\mathcal{H}\sigma_Q))^2},$$

where \mathcal{H} is the enthalpy density. This divergence happens because at $Q = 0$, κ diverges as $\tau_{\text{imp}} \rightarrow \infty$, while $\sigma = \sigma_Q$, an intrinsic, finite, “quantum critical” conductivity.

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene

Jesse Crossno,^{1,2} Jing K. Shi,¹ Ke Wang,¹ Xiaomeng Liu,¹ Achim Harzheim,¹ Andrew Lucas,¹ Subir Sachdev,^{1,3}
Philip Kim,^{1,2,*} Takashi Taniguchi,⁴ Kenji Watanabe,⁴ Thomas A. Ohki,⁵ and Kin Chung Fong^{5,†}

¹*Department of Physics, Harvard University, Cambridge, MA 02138, USA*

²*John A. Paulson School of Engineering and Applied Sciences,
Harvard University, Cambridge, MA 02138, USA*

³*Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada*

⁴*National Institute for Materials Science, Namiki 1-1, Tsukuba, Ibaraki 305-0044, Japan*

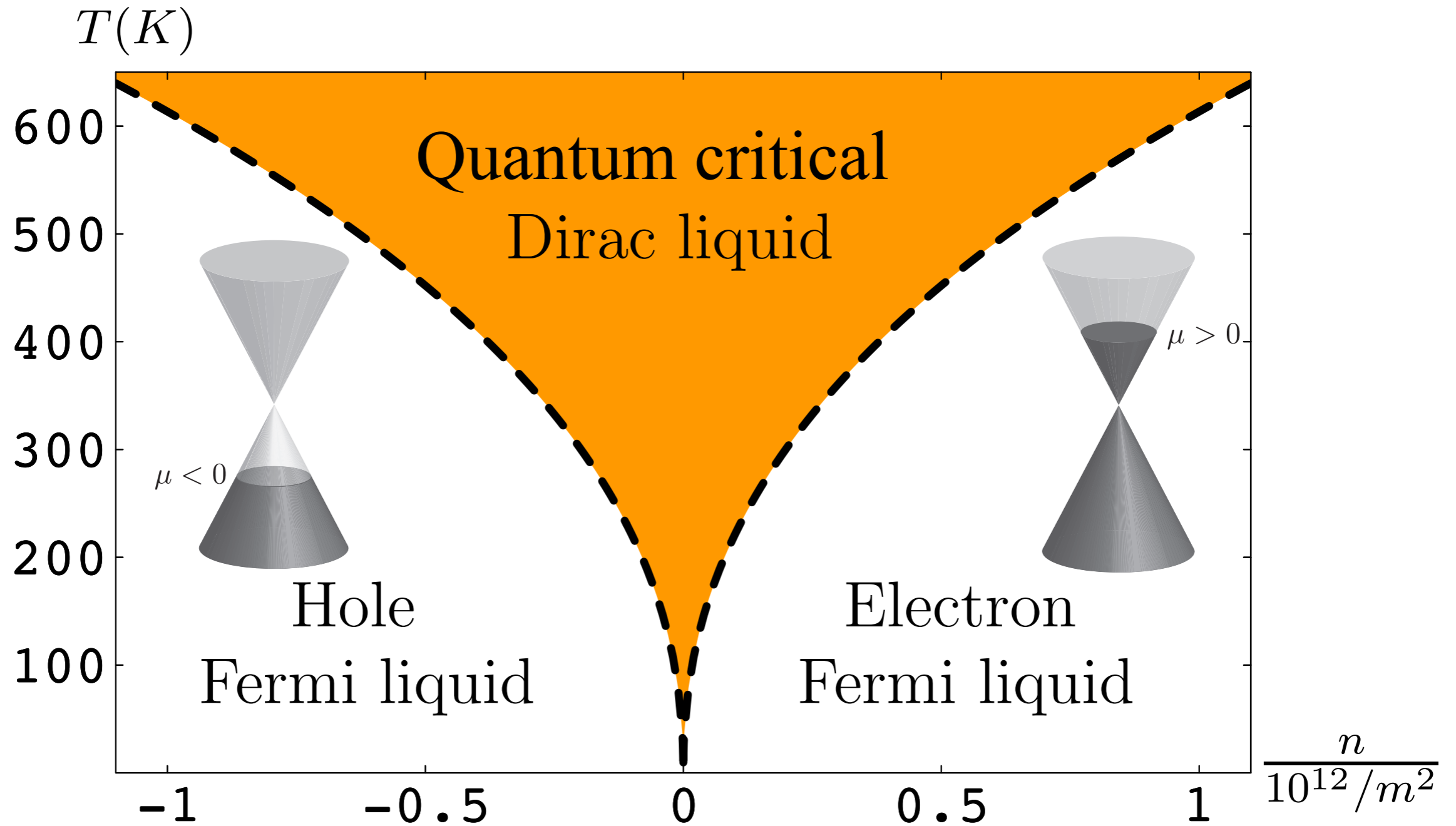
⁵*Raytheon BBN Technologies, Quantum Information Processing Group, Cambridge, Massachusetts 02138, USA*

(Dated: September 28, 2015)

Interactions between particles in quantum many-body systems can lead to collective behavior described by hydrodynamics. One such system is the electron-hole plasma in graphene near the charge neutrality point which can form a strongly coupled Dirac fluid. This charge neutral plasma of quasi-relativistic fermions is expected to exhibit a substantial enhancement of the thermal conductivity, due to decoupling of charge and heat currents within hydrodynamics. Employing high sensitivity Johnson noise thermometry, we report the breakdown of the Wiedemann-Franz law in graphene, with a thermal conductivity an order of magnitude larger than the value predicted by Fermi liquid theory. This result is a signature of the Dirac fluid, and constitutes direct evidence of collective motion in a quantum electronic fluid.

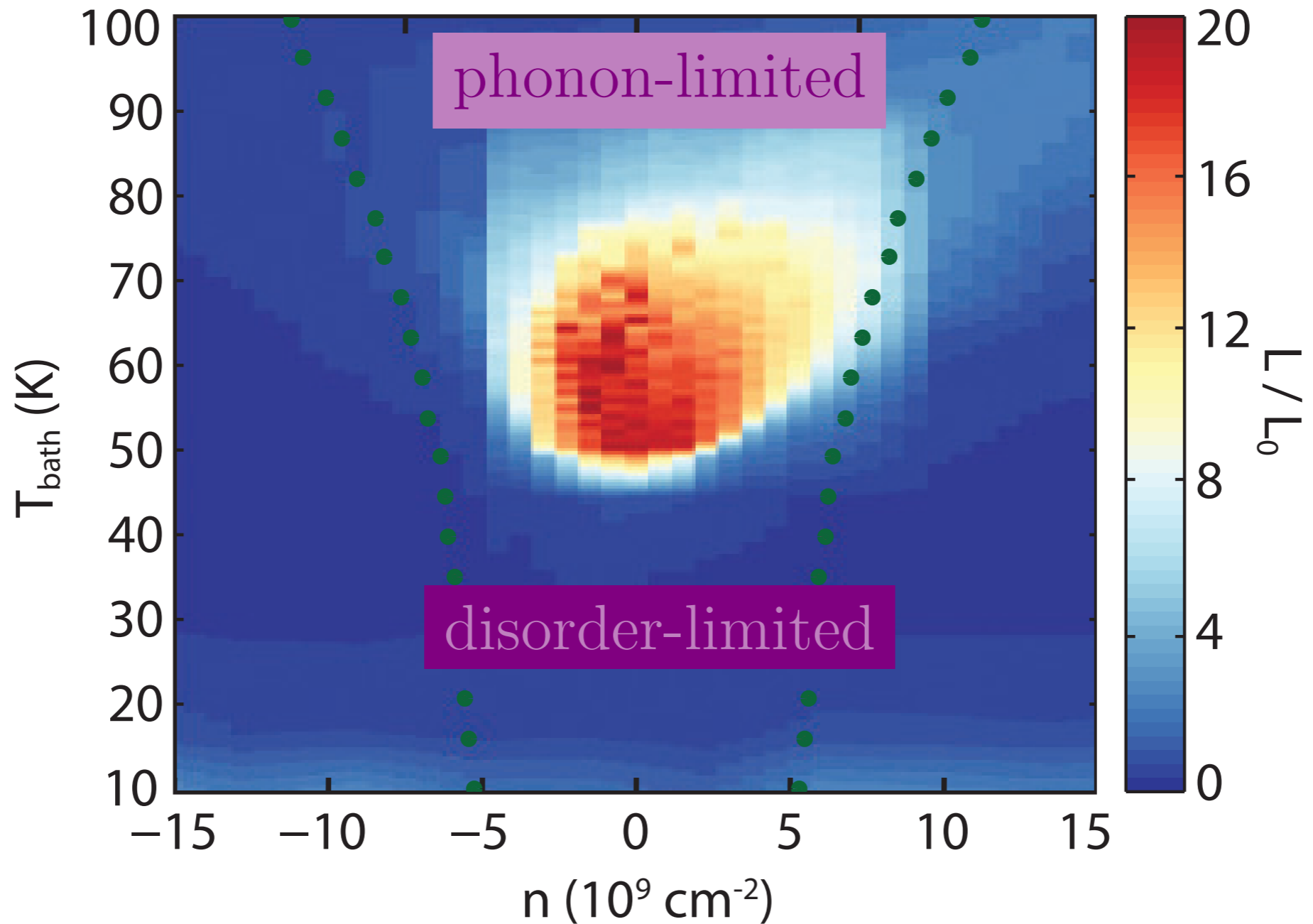
arXiv:1509.04713

Graphene

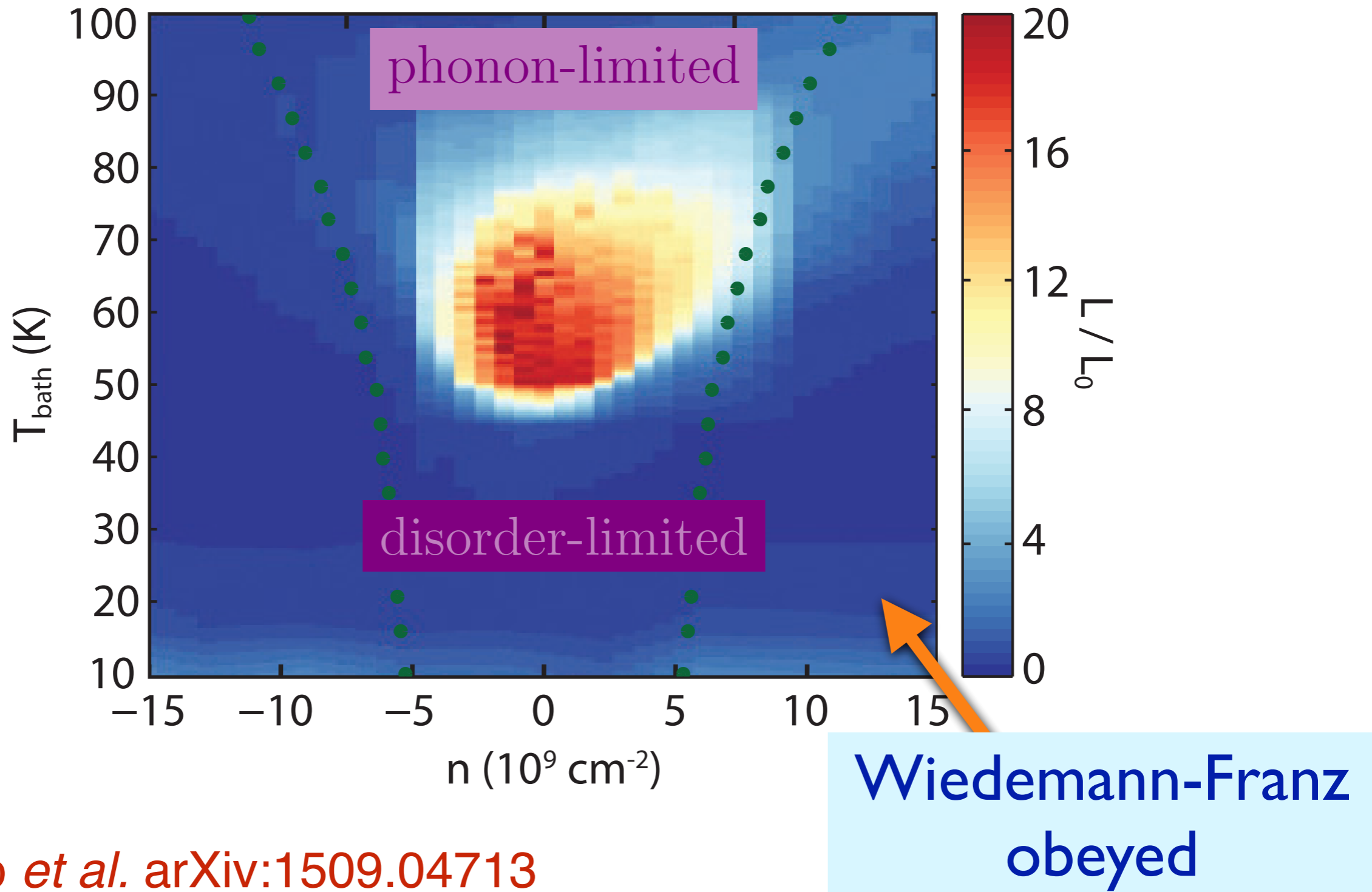


D. E. Sheehy and J. Schmalian, PRL **99**, 226803 (2007)
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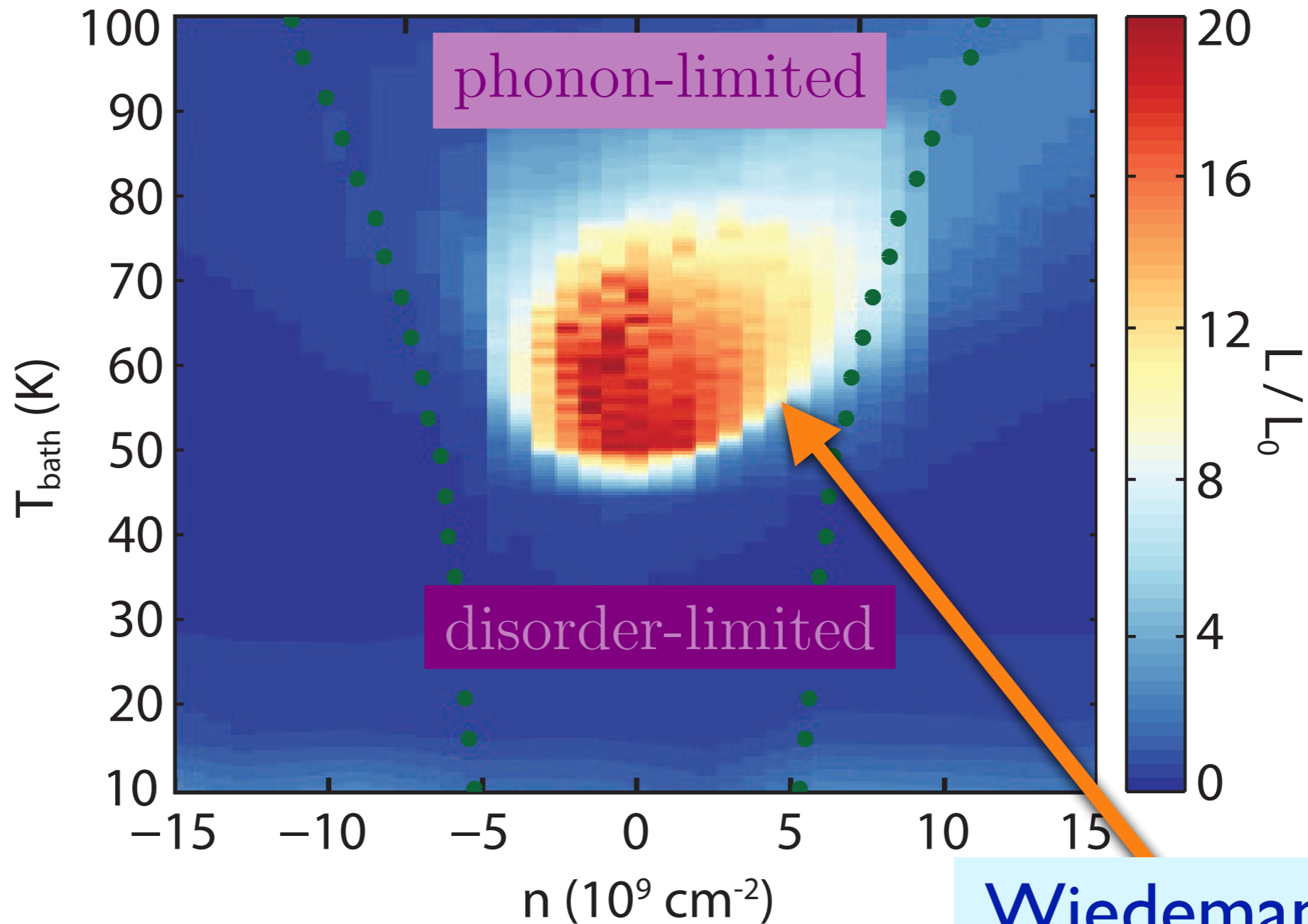
Strange metal in graphene



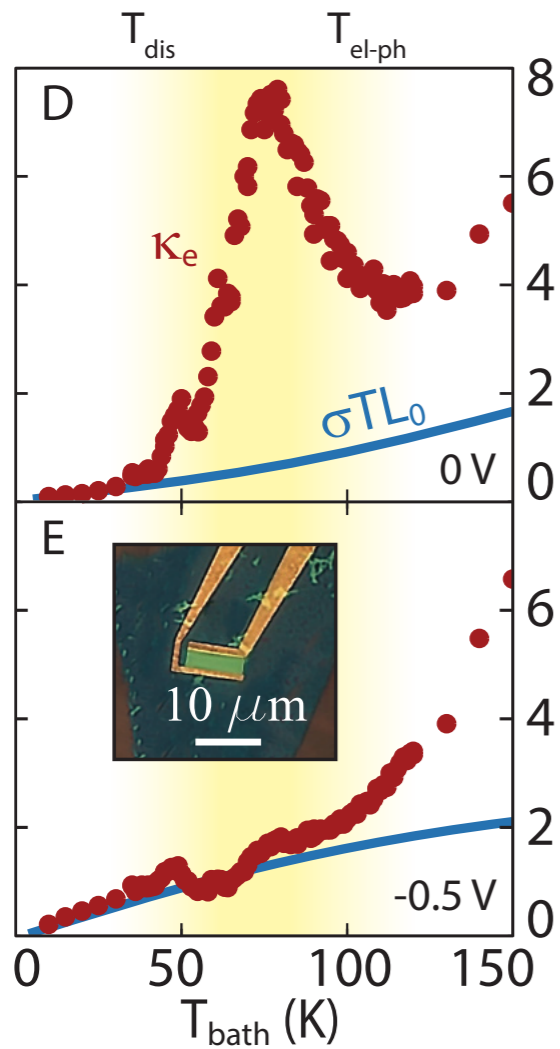
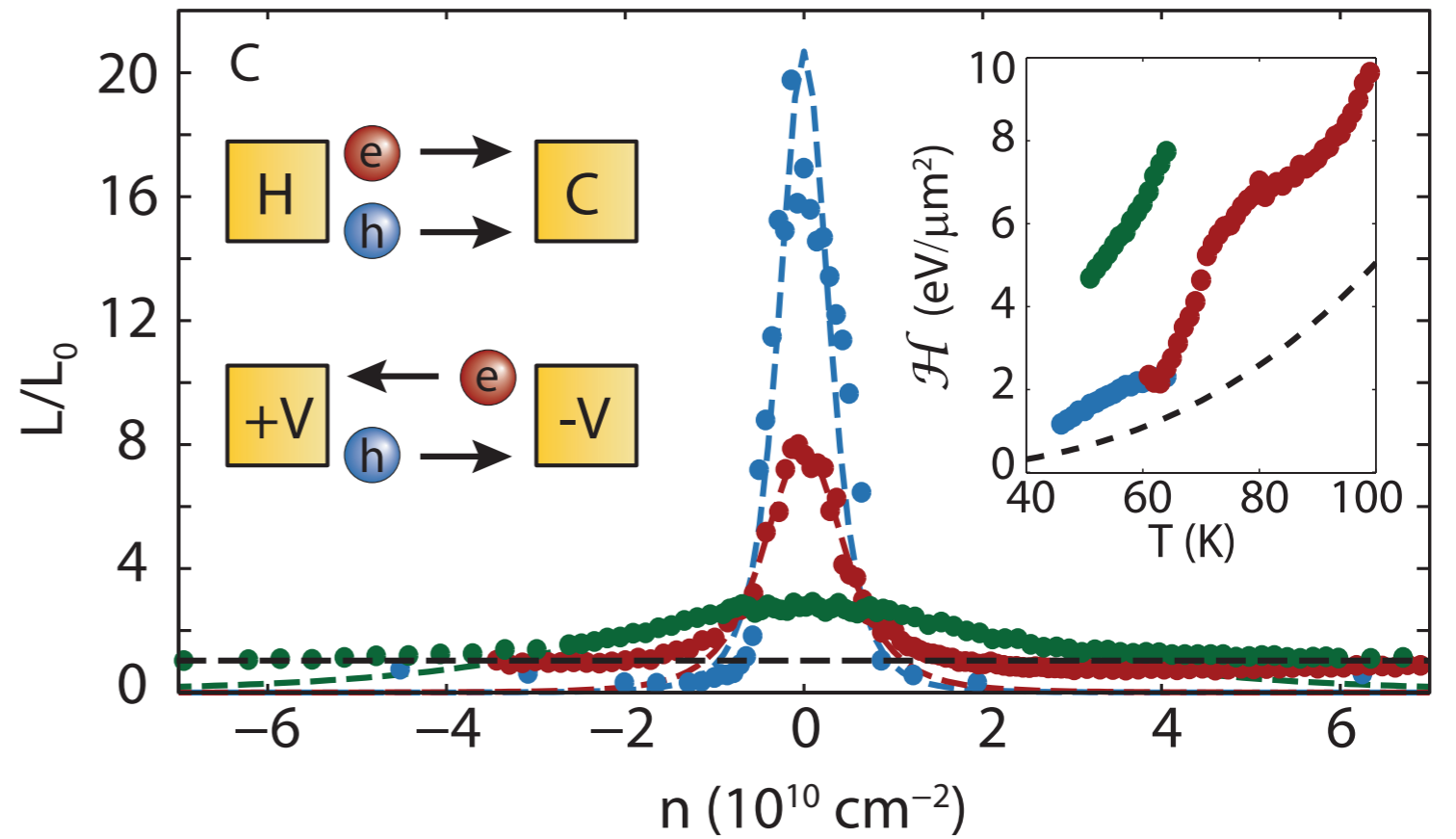
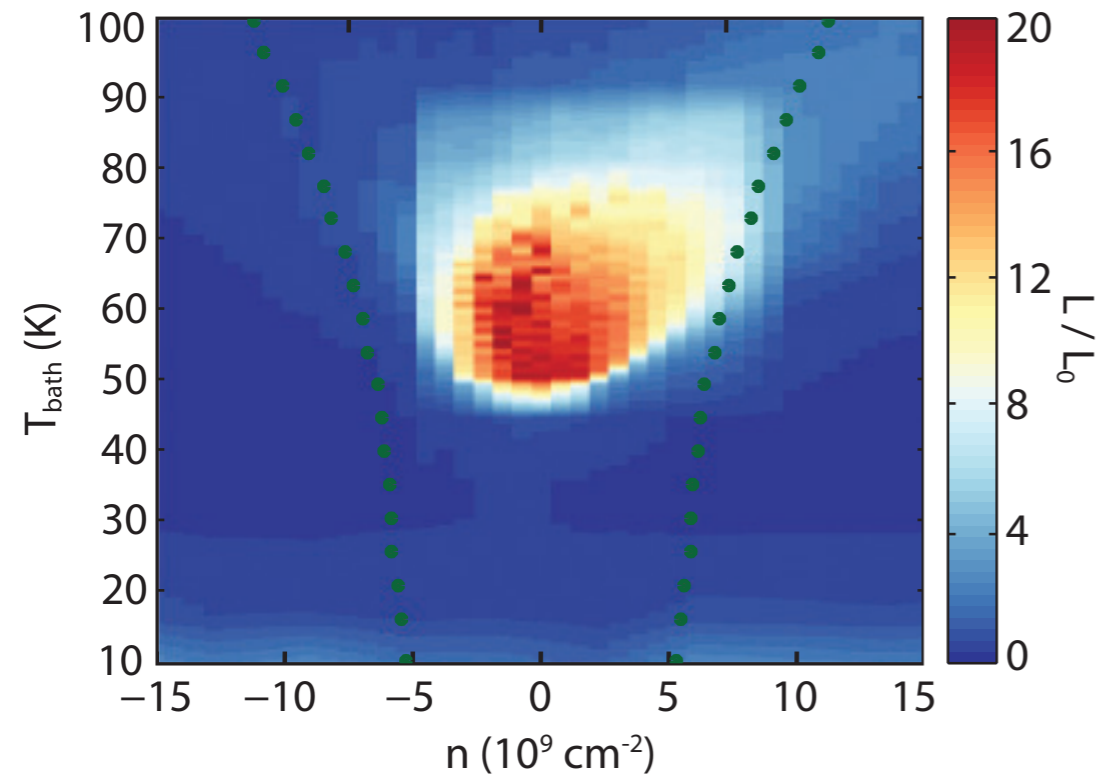
Strange metal in graphene



Strange metal in graphene



**Wiedemann-Franz
violated !**



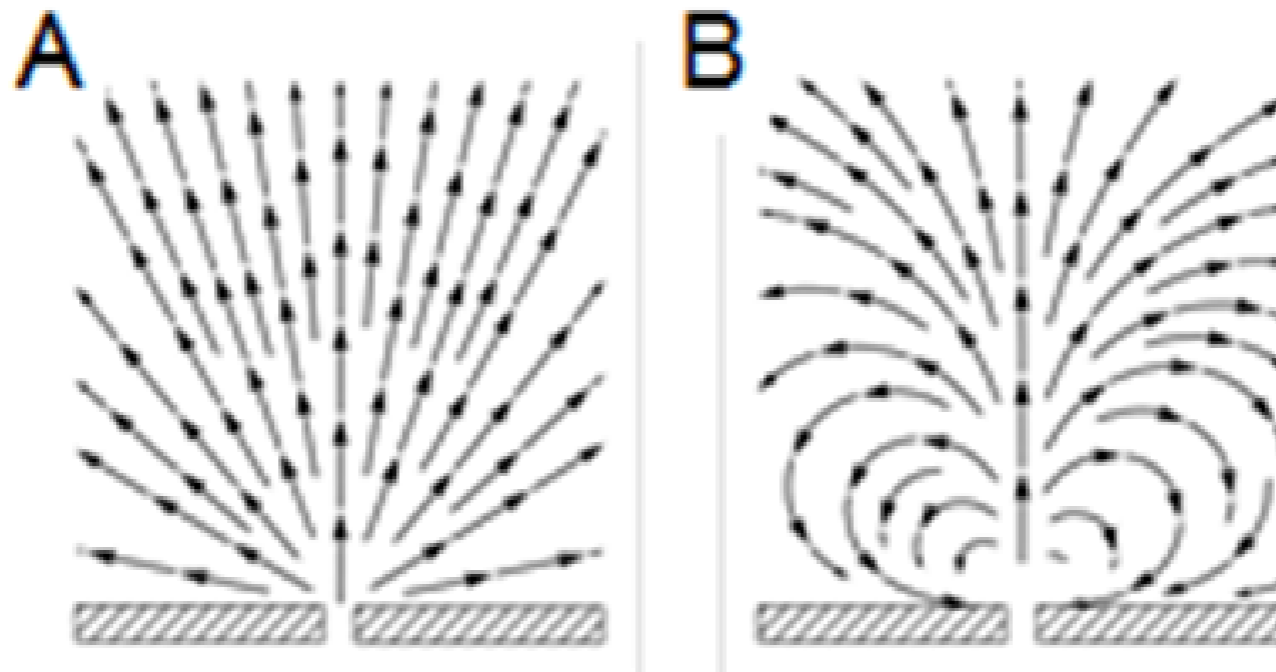
$$\begin{aligned} \text{Lorentz ratio } L &= \frac{\kappa}{(T\sigma)} \\ &= \frac{\mathcal{H}\tau_{\text{imp}}}{T^2\sigma_Q} \frac{1}{(1 + Q^2\tau_{\text{imp}}/(\mathcal{H}\sigma_Q))^2} \end{aligned}$$

Strange metal in graphene

Negative local resistance due to viscous electron backflow in graphene

D. A. Bandurin¹, I. Torre^{2,3}, R. Krishna Kumar^{1,4}, M. Ben Shalom^{1,5}, A. Tomadin⁶, A. Principi⁷, G. H. Auton⁵, E. Khestanova^{1,5}, K. S. Novoselov⁵, I. V. Grigorieva¹, L. A. Ponomarenko^{1,4}, A. K. Geim¹, M. Polini^{3,6}

Graphene hosts a unique electron system that due to weak electron-phonon scattering allows micrometer-scale ballistic transport even at room temperature whereas the local equilibrium is provided by frequent electron-electron collisions. Under these conditions, electrons can behave as a viscous liquid and exhibit hydrodynamic phenomena similar to classical liquids. Here we report unambiguous evidence for this long-sought transport regime. In particular, doped graphene exhibits an anomalous (negative) voltage drop near current injection contacts, which is attributed to the formation of submicrometer-size whirlpools in the electron flow. The viscosity of graphene's electron liquid is found to be an order of magnitude larger than that of honey, in quantitative agreement with many-body theory. Our work shows a possibility to study electron hydrodynamics using high quality graphene.



Quantum matter without quasiparticles

1. A solvable model of an ordinary metal
2. A solvable model of a strange metal
3. Holography and charged black holes
4. Transport in strange metals

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Infinite-range model of a Fermi liquid

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^N t_{ij} c_i^\dagger c_j + \dots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = Q$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

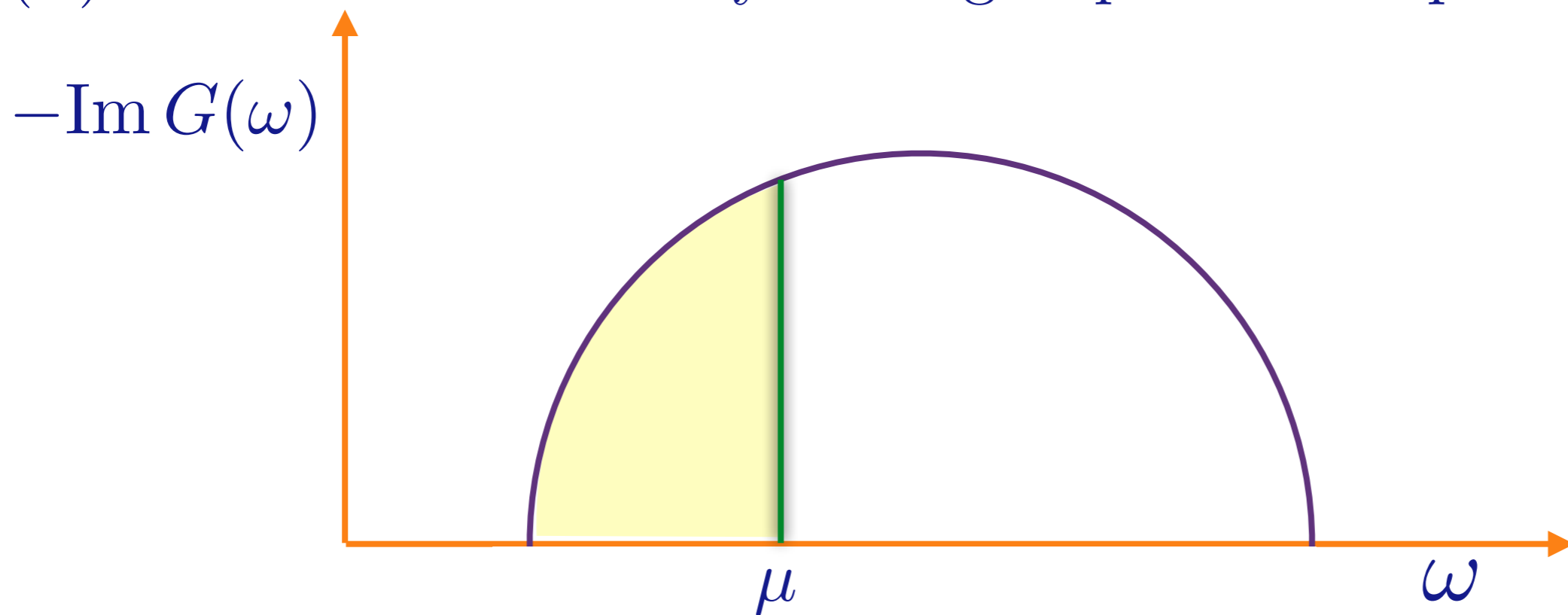
Fermions occupying the eigenstates of a
 $N \times N$ random matrix

Infinite-range model of a Fermi liquid

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$
$$G(\tau = 0^-) = Q.$$

$G(\omega)$ can be determined by solving a quadratic equation.



Fermions occupying eigenstates with a “semi-circular” density of states

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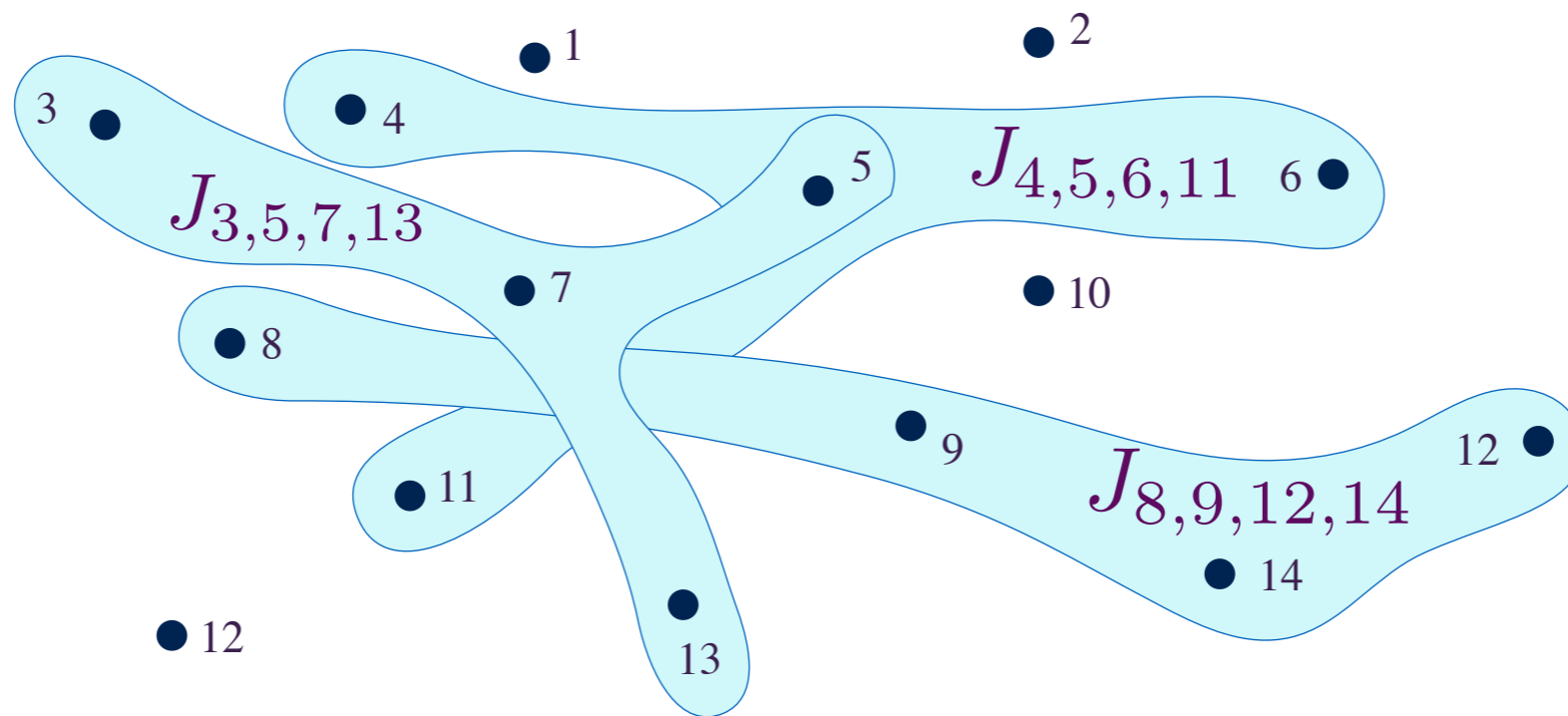
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Infinite-range model of a strange metal

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,l=1}^N J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_l - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$Q = \frac{1}{N} \sum_i c_i^\dagger c_i$$



$J_{ij;kl}$ are independent random variables with $\overline{J_{ij;kl}} = 0$ and $\overline{|J_{ij;kl}|^2} = J^2$
 $N \rightarrow \infty$ yields same critical strange metal; simpler to study numerically

Infinite-range strange metals

Local fermion density of states

$$\rho(\omega) = -\text{Im } G(\omega) \sim \begin{cases} \omega^{-1/2}, & \omega > 0 \\ e^{-2\pi\mathcal{E}} |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

\mathcal{E} encodes the particle-hole asymmetry

While \mathcal{E} determines the *low* energy spectrum, it is determined by the *total* fermion density Q :

$$Q = \frac{1}{4}(3 - \tanh(2\pi\mathcal{E})) - \frac{1}{\pi} \tan^{-1}(e^{2\pi\mathcal{E}}).$$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

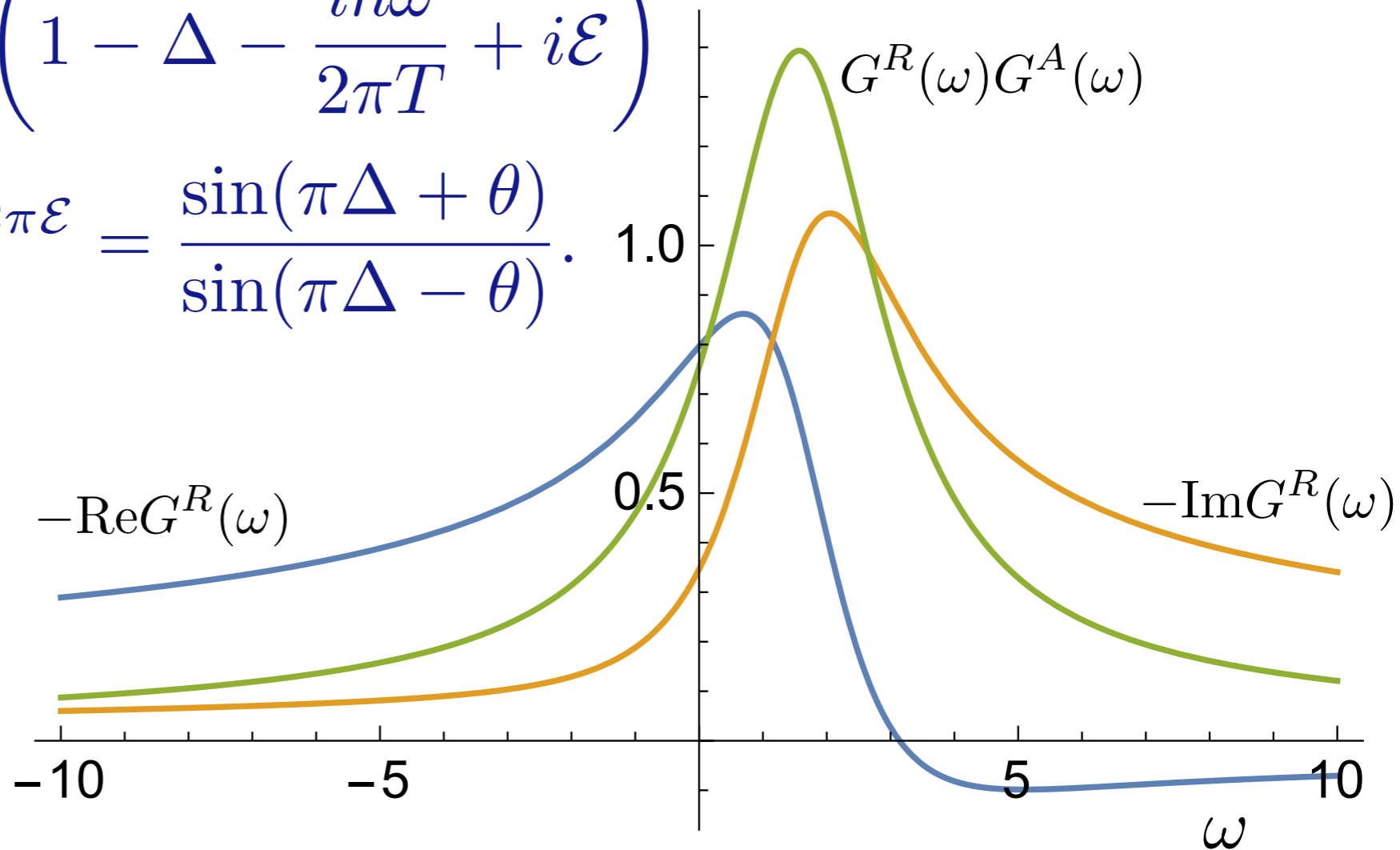
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

Infinite-range strange metals

At non-zero temperature, T , the Green's function also fully determined by \mathcal{E} .

$$G^R(\omega) = \frac{-iC e^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \frac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \frac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}$$

where $\Delta = 1/4$ and $e^{2\pi\mathcal{E}} = \frac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$.



S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

A. Georges and O. Parcollet PRB **59**, 5341 (1999)

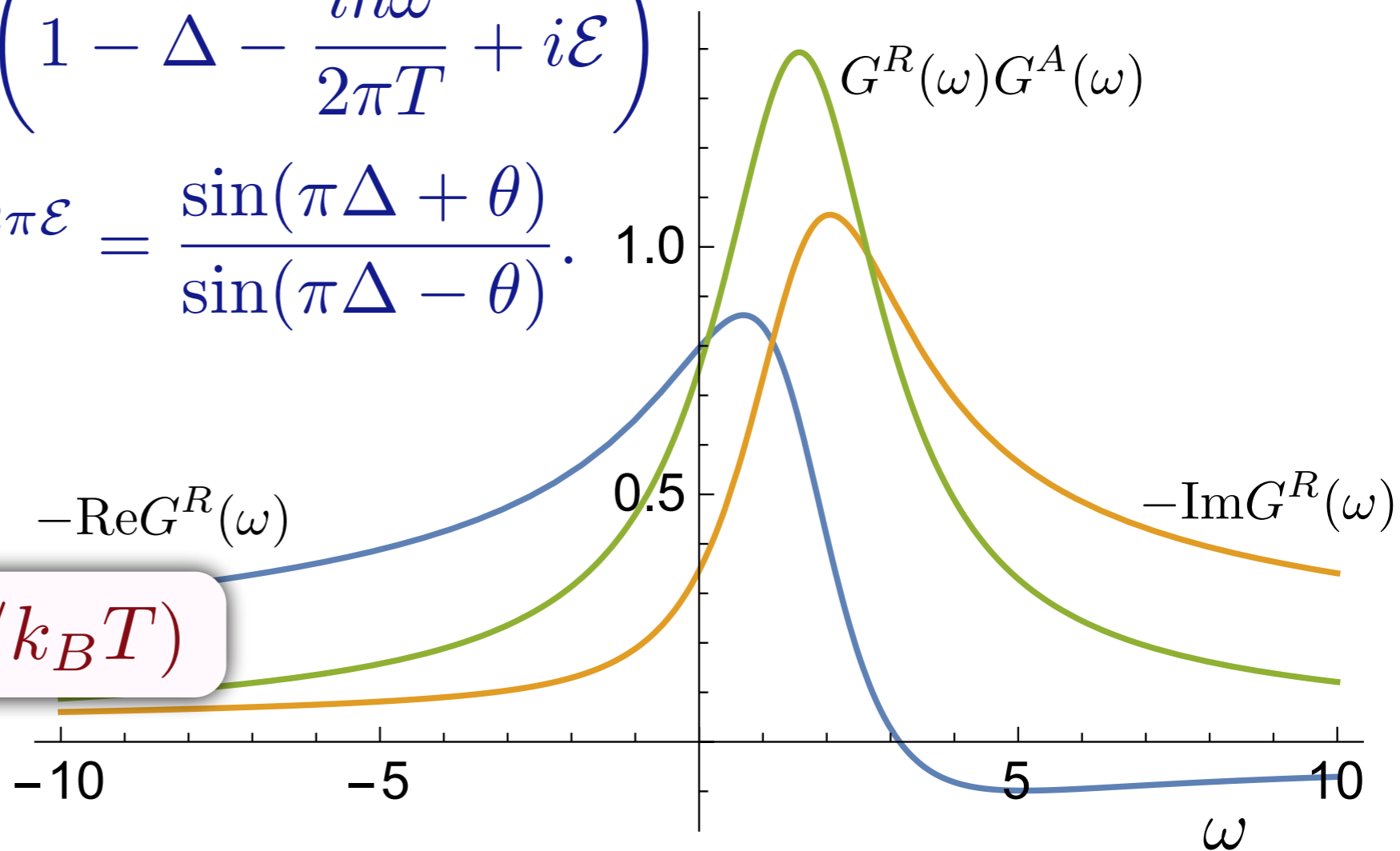
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where $\Delta = 1/4$ and $e^{2\pi\mathcal{E}} = \frac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$.



Note $G(\omega) \equiv f(\hbar\omega/k_B T)$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

A. Georges and O. Parcollet PRB **59**, 5341 (1999)

A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

Infinite-range strange metals

The entropy per site, \mathcal{S} , has a non-zero limit as $T \rightarrow 0$, and can be viewed as each site acquiring the universal boundary entropy of the multichannel Kondo problem.

N. Andrei and C. Destri, PRL **52**, 364 (1984).

A. M. Tsvelick, J. Phys. C **18**, 159 (1985).

I. Affleck and A. W. W. Ludwig, PRL **67**, 161 (1991).

S. Sachdev, C. Buragohain, and M. Vojta, Science **286**, 2479 (1999).

This entropy obeys

$$\left(\frac{\partial \mathcal{S}}{\partial \mathcal{Q}}\right)_T = - \left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi \mathcal{E}$$

Note that \mathcal{S} and \mathcal{E} involve low-lying states, while \mathcal{Q} depends upon *all* states, and details of the UV structure

O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta Phys. Rev. B **58**, 3794 (1998)

A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

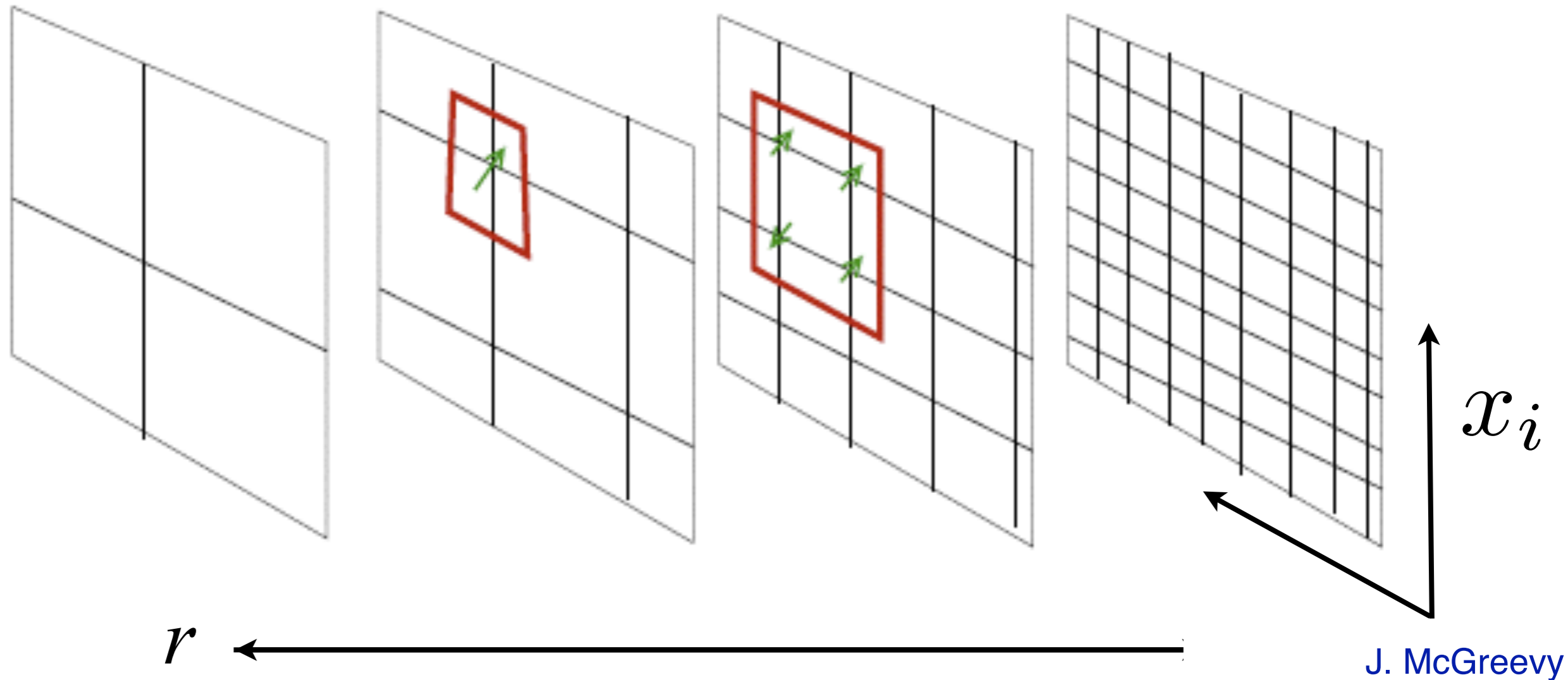
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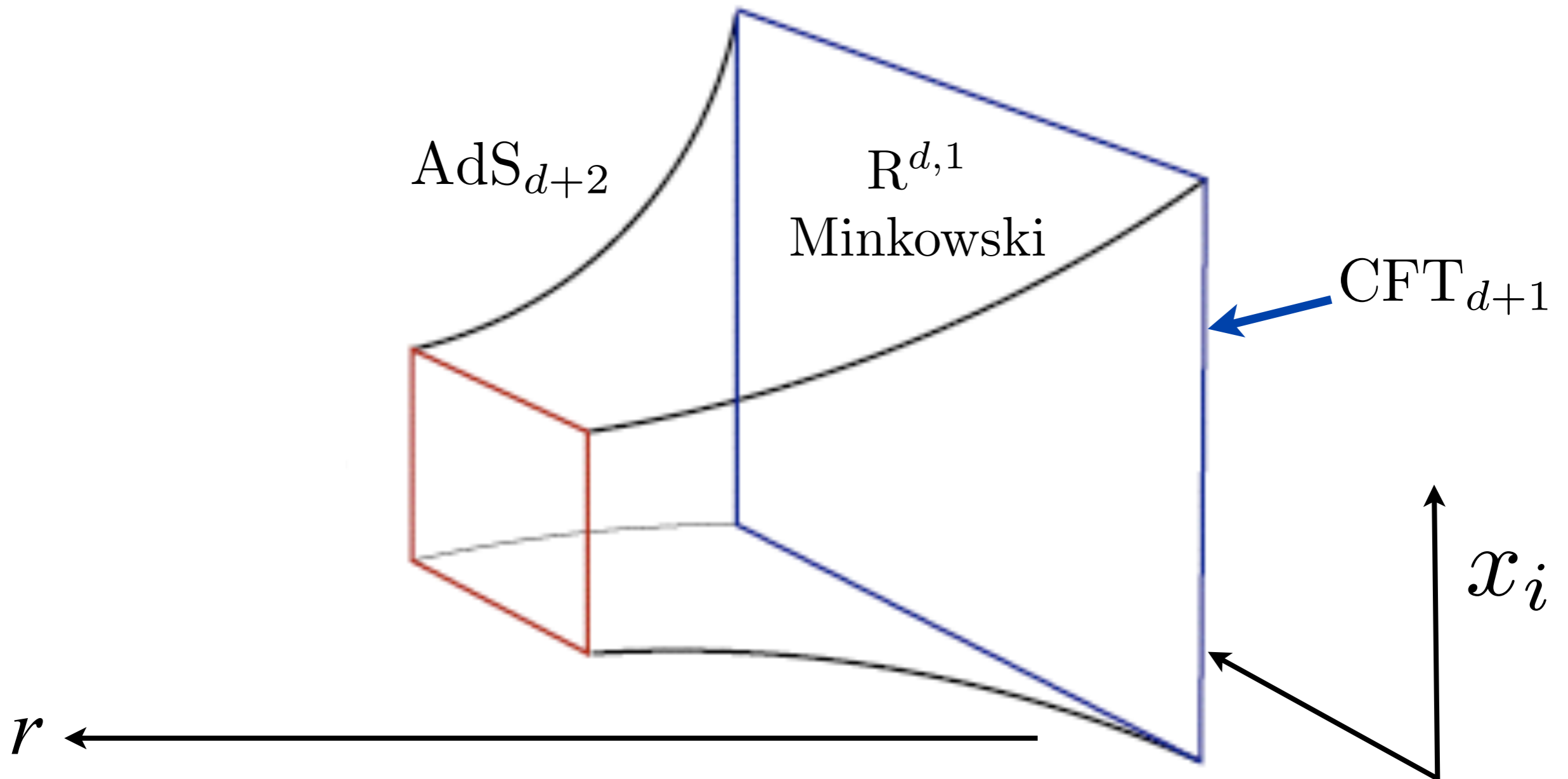
Holography



Key idea: \Rightarrow Implement r as an extra dimension, and map to a local theory in $d+2$ spacetime dimensions.

AdS/CFT correspondence at zero temperature

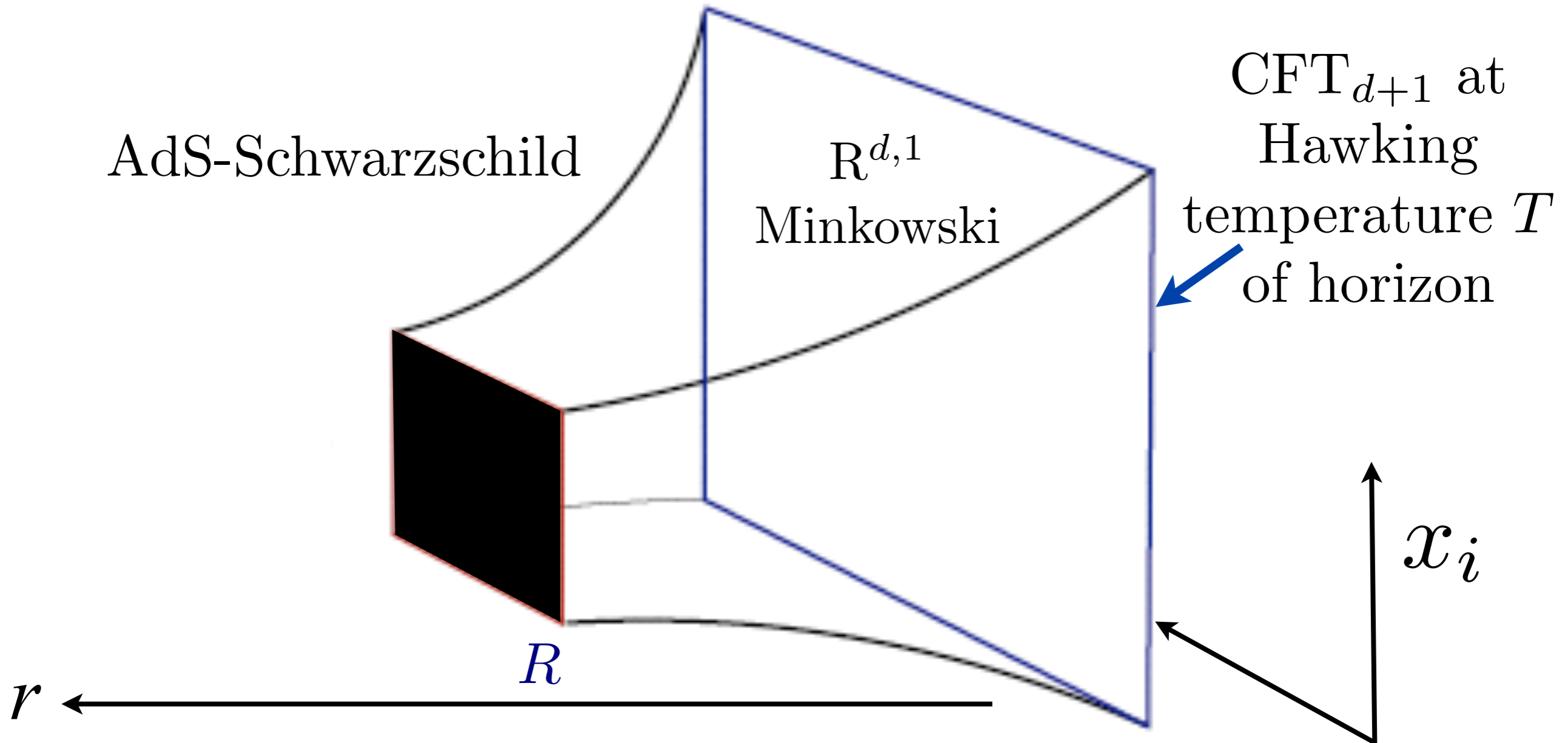
Einstein gravity $\mathcal{S}_E = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(\mathcal{R} + \frac{d(d+1)}{L^2} \right) \right]$



$$ds^2 = \left(\frac{L}{r} \right)^2 [dr^2 - dt^2 + d\vec{x}^2]$$

AdS/CFT correspondence at non-zero temperature

Einstein gravity $\mathcal{S}_E = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(\mathcal{R} + \frac{d(d+1)}{L^2} \right) \right]$

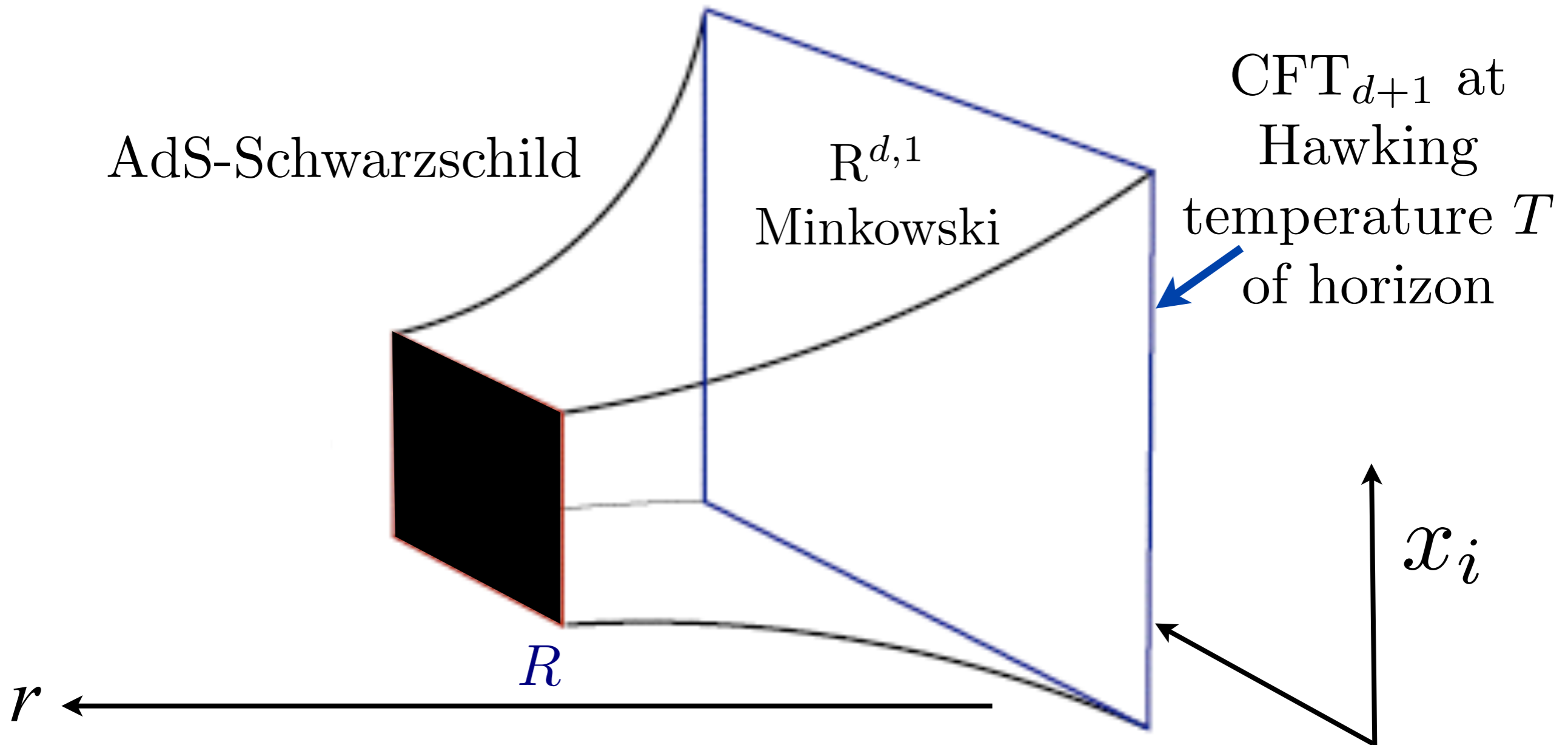


$$ds^2 = \left(\frac{L}{r} \right)^2 \left[\frac{dr^2}{f(r)} - f(r) dt^2 + d\vec{x}^2 \right]$$

with $f(r) = 1 - (r/R)^{d+1}$ and $T = (d+1)/(4\pi R)$.

AdS/CFT correspondence at non-zero temperature

Einstein gravity $\mathcal{S}_E = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(\mathcal{R} + \frac{d(d+1)}{L^2} \right) \right]$

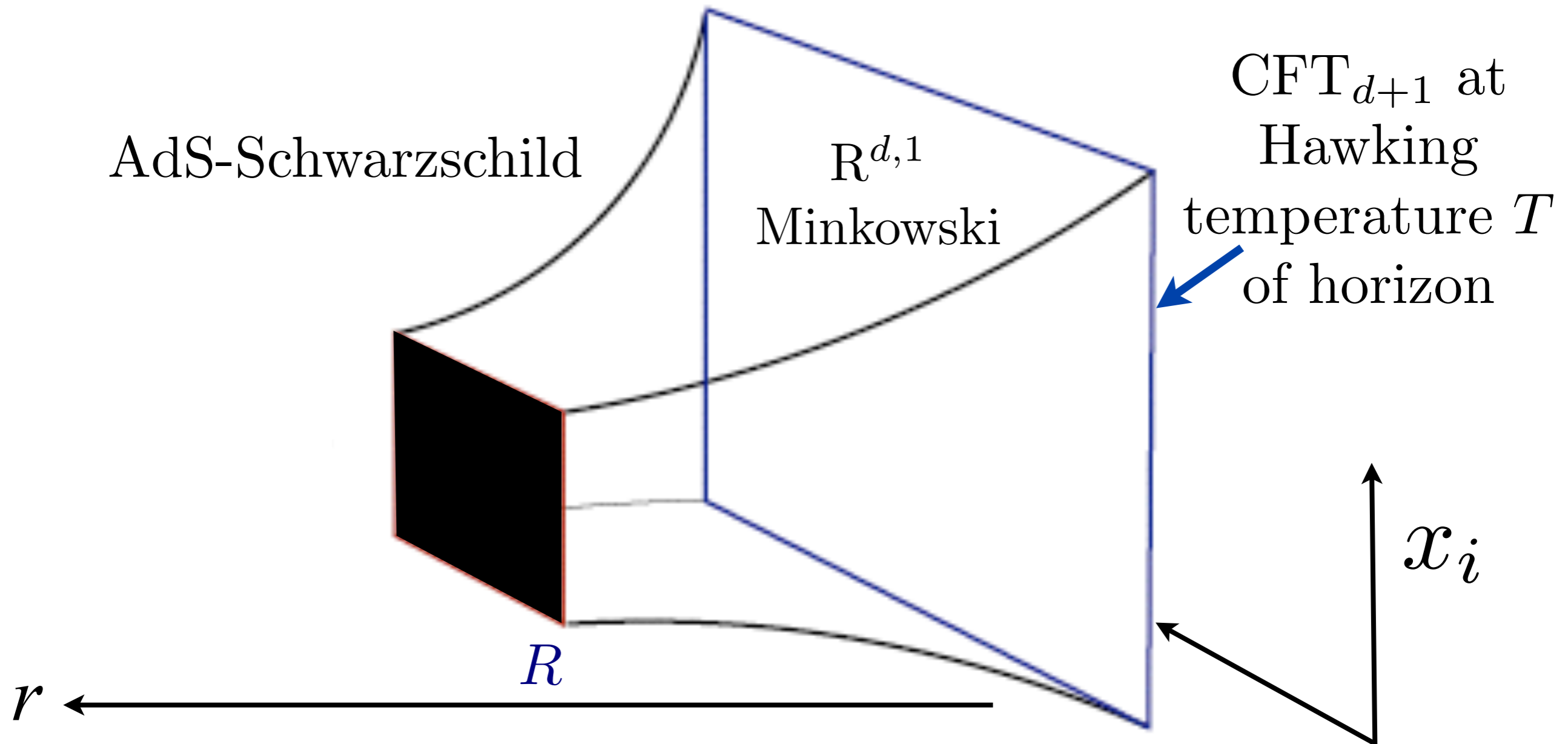


Entropy density of CFT_{d+1} , $\mathcal{S} \sim T^d$

Bekenstein-Hawking entropy density, $\mathcal{S}_{\text{BH}} \sim T^d$

AdS/CFT correspondence at non-zero temperature

Einstein gravity $\mathcal{S}_E = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(\mathcal{R} + \frac{d(d+1)}{L^2} \right) \right]$

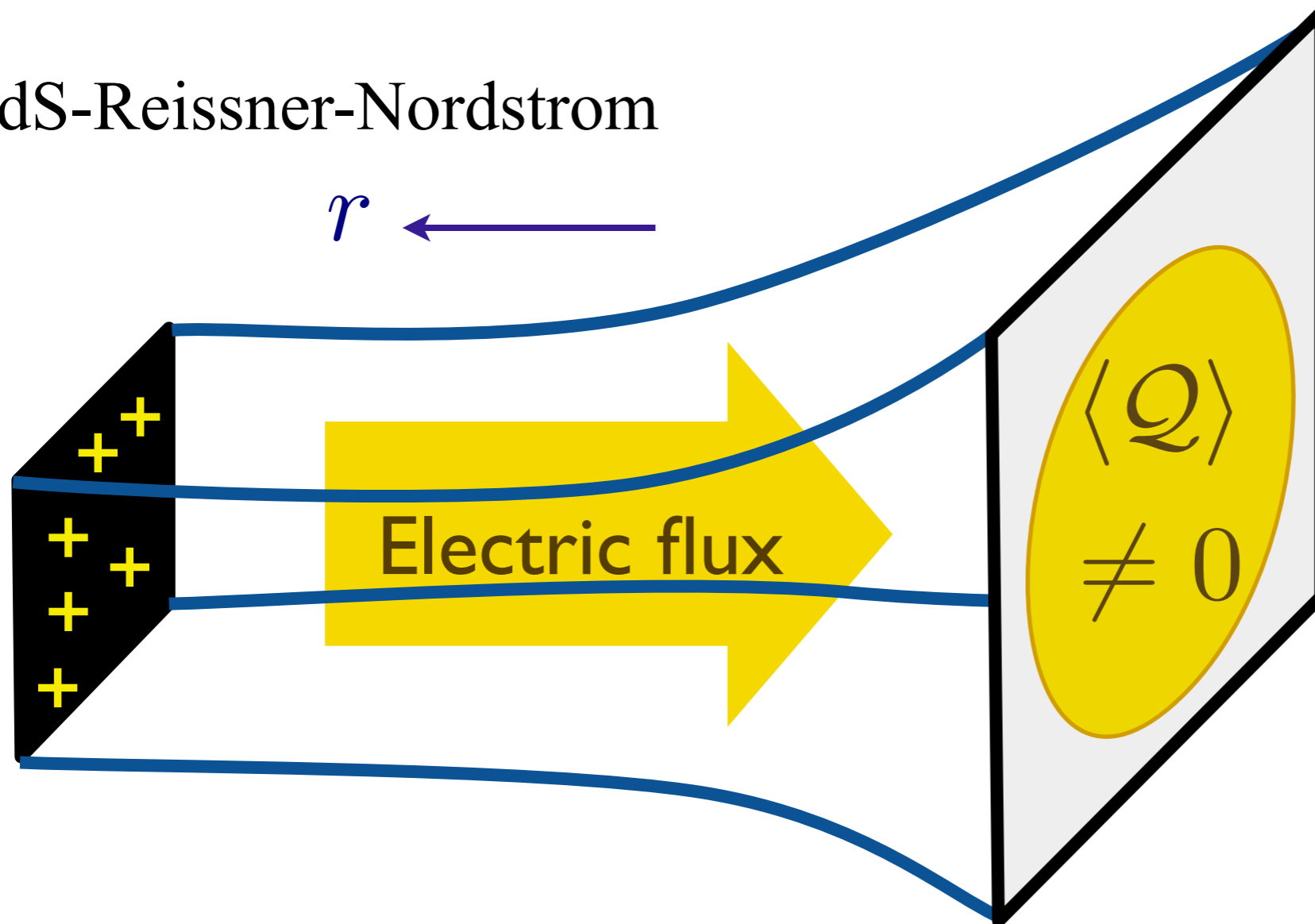


For $\text{SU}(N)$ SYM in $d = 3$, $\mathcal{S}_{\text{BH}} = (\pi^2/2)N^2T^3$. But there is (still) no confirmation of this from a field-theory computation on SYM.

Charged black branes

Einstein-Maxwell theory $\mathcal{S}_{EM} = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(\mathcal{R} + \frac{d(d+1)}{L^2} - \frac{R^2}{g_F^2} F^2 \right) \right]$

AdS-Reissner-Nordstrom

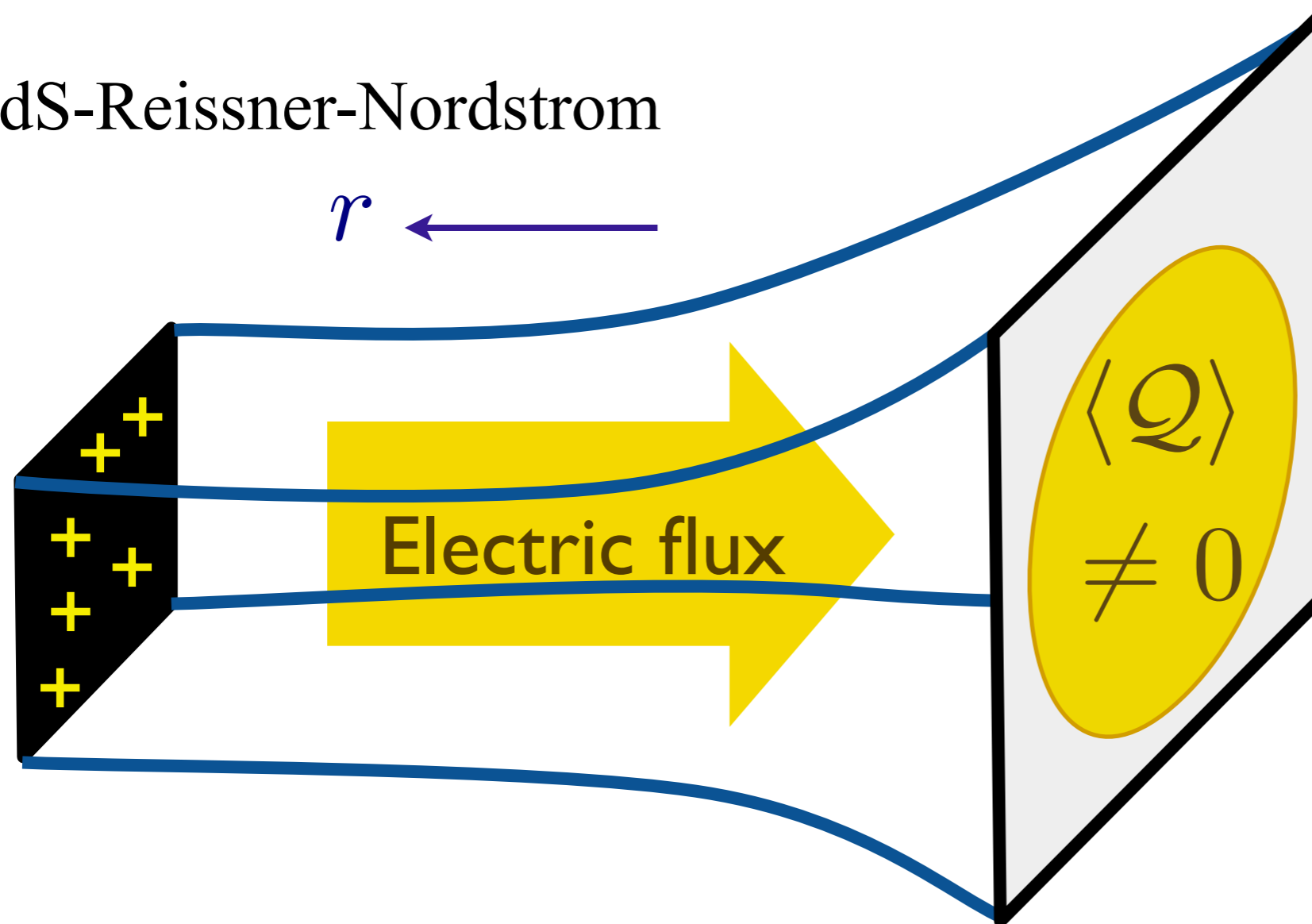


Quantum matter on the boundary with a variable charge density \mathcal{Q} of a global U(1) symmetry.

Charged black branes

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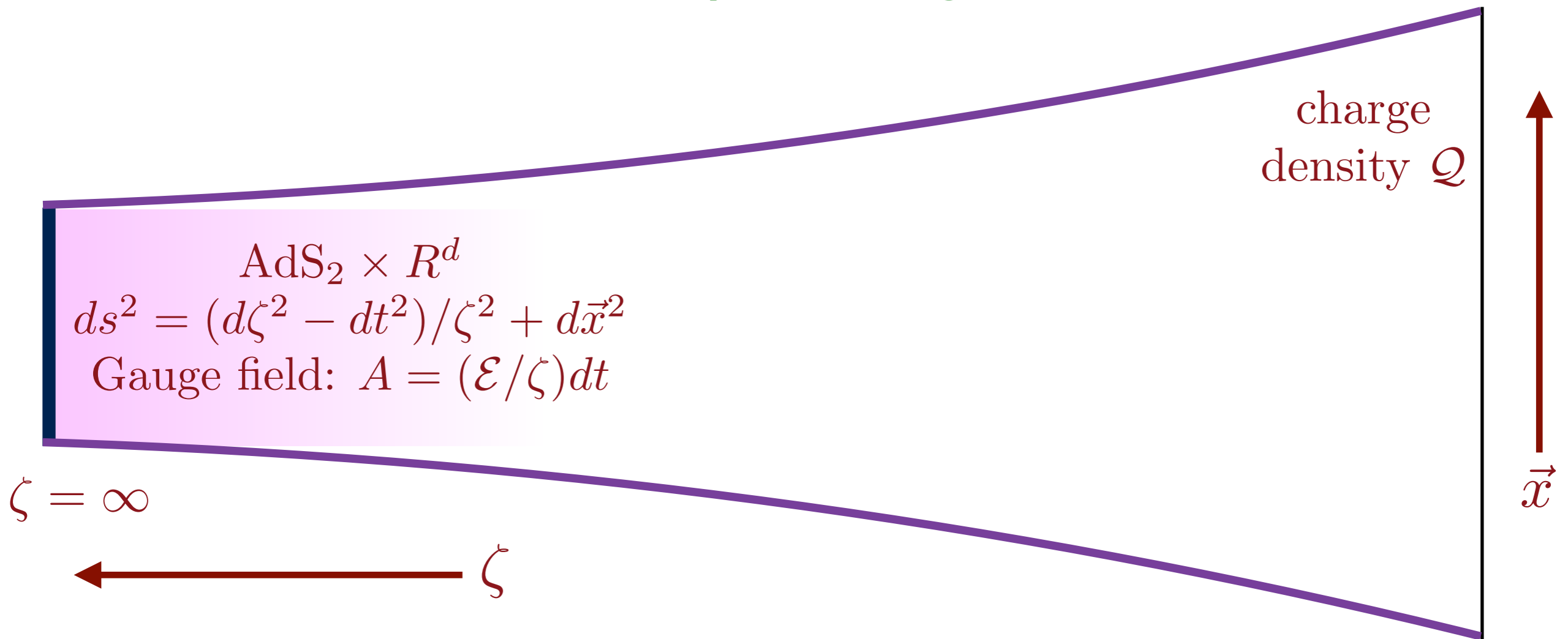
AdS-Reissner-Nordstrom



Quantum matter on the boundary with a variable charge density \mathcal{Q} of a global U(1) symmetry.

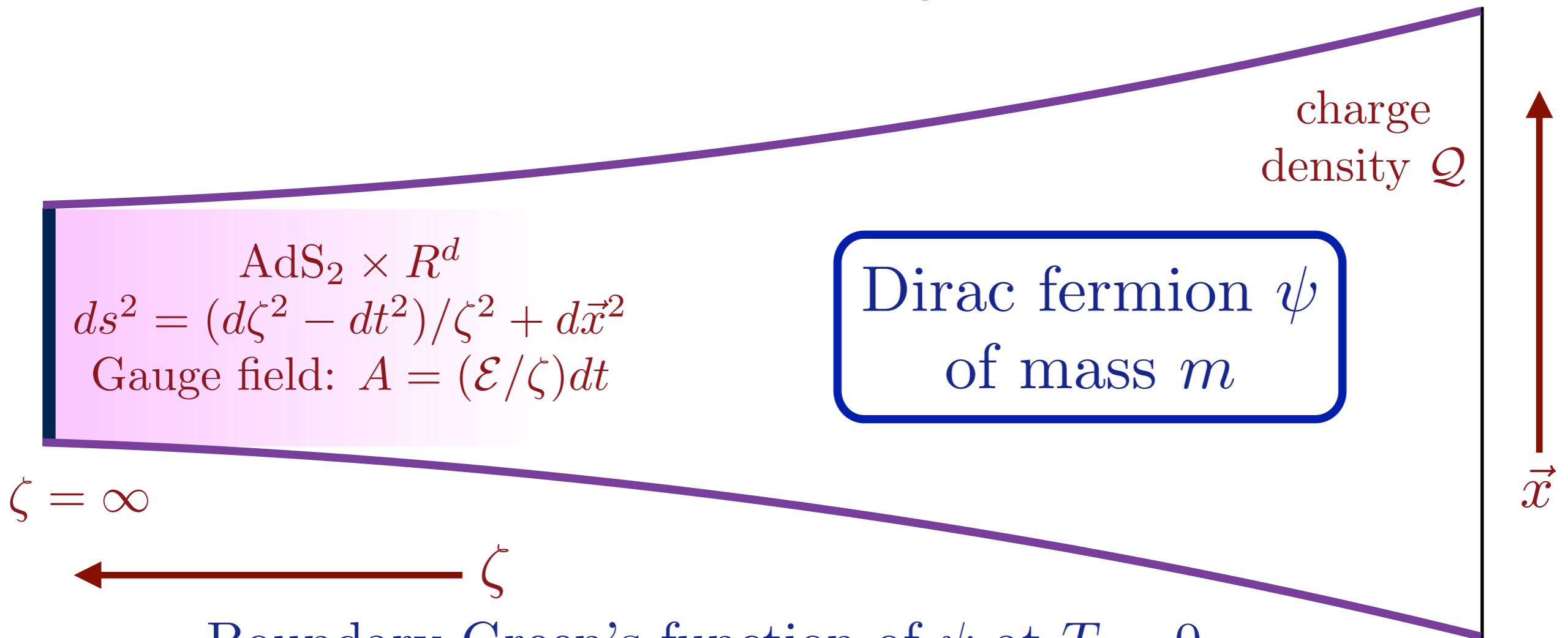
Realizes a strange metal: a state with an unbroken global U(1) symmetry with a continuously variable charge density, \mathcal{Q} , at $T = 0$ which does not have any quasiparticle excitations.

General Relativity of charged black branes



- Near-horizon metric is AdS_2 , with near-horizon electric field \mathcal{E} .

Quantum fields on charged black branes



Boundary Green's function of ψ at $T = 0$

$$\text{Im}G(\omega) \sim \begin{cases} \omega^{-(1-2\Delta)}, & \omega > 0 \\ e^{-2\pi\mathcal{E}} |\omega|^{-(1-2\Delta)}, & \omega < 0. \end{cases}$$

where the fermion scaling dimension Δ is a function of m

\mathcal{E} encodes the particle-hole asymmetry

Quantum fields on charged black branes

Conformal mapping to $T > 0$

$$\zeta = \zeta_0$$

charge density Q

$$ds^2 = [d\zeta^2 / (1 - \zeta^2 / \zeta_0^2) - (1 - \zeta^2 / \zeta_0^2) dt^2] / \zeta^2 + d\vec{x}^2$$

$$\text{Gauge field: } A = \mathcal{E}(1/\zeta - 1/\zeta_0) dt \text{ with } \zeta_0 = 1/(2\pi T)$$

Dirac fermion ψ
of mass m

$$\zeta = \infty$$



Boundary Green's function of ψ at $T > 0$

is fully determined by \mathcal{E}

$$G^R(\omega) = \frac{-iC e^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \frac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \frac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}$$

where
$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}.$$

General Relativity of charged black branes

Conformal mapping to $T > 0$

$$\zeta = \zeta_0$$

charge
density Q

$$ds^2 = [d\zeta^2 / (1 - \zeta^2 / \zeta_0^2) - (1 - \zeta^2 / \zeta_0^2) dt^2] / \zeta^2 + d\vec{x}^2$$

$$\text{Gauge field: } A = \mathcal{E}(1/\zeta - 1/\zeta_0) dt \text{ with } \zeta_0 = 1/(2\pi T)$$

$$\zeta = \infty$$

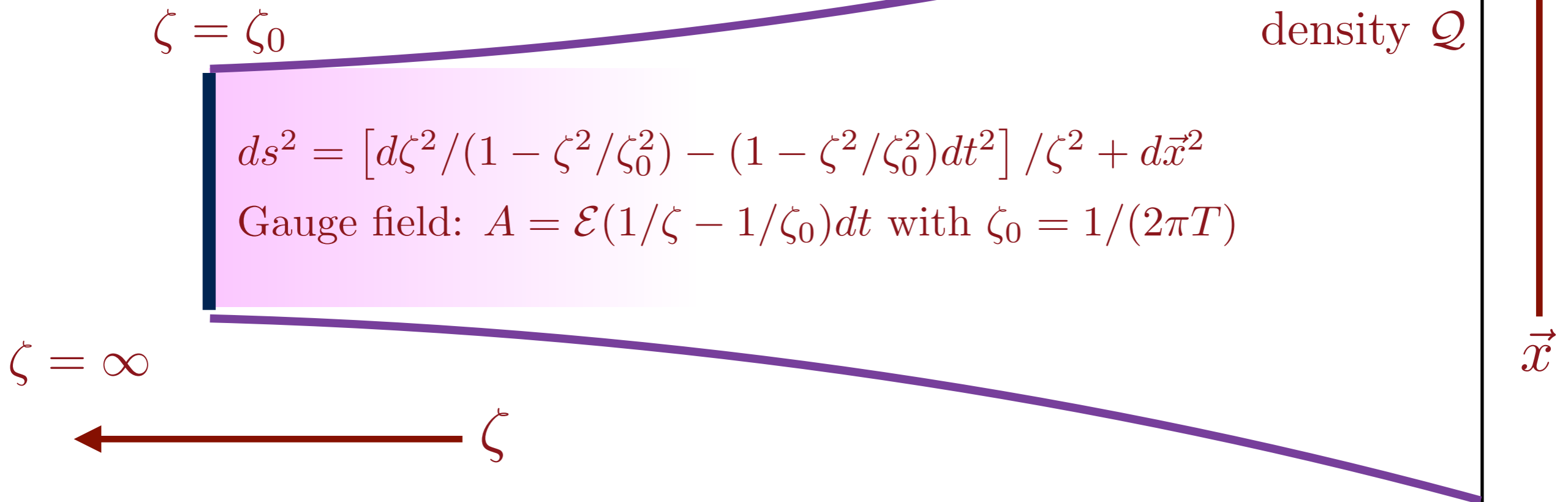
$$\zeta$$

$$\vec{x}$$

- As $T \rightarrow 0$, there is a non-zero Bekenstein-Hawking entropy, \mathcal{S}_{BH} .

General Relativity of charged black branes

Conformal mapping to $T > 0$



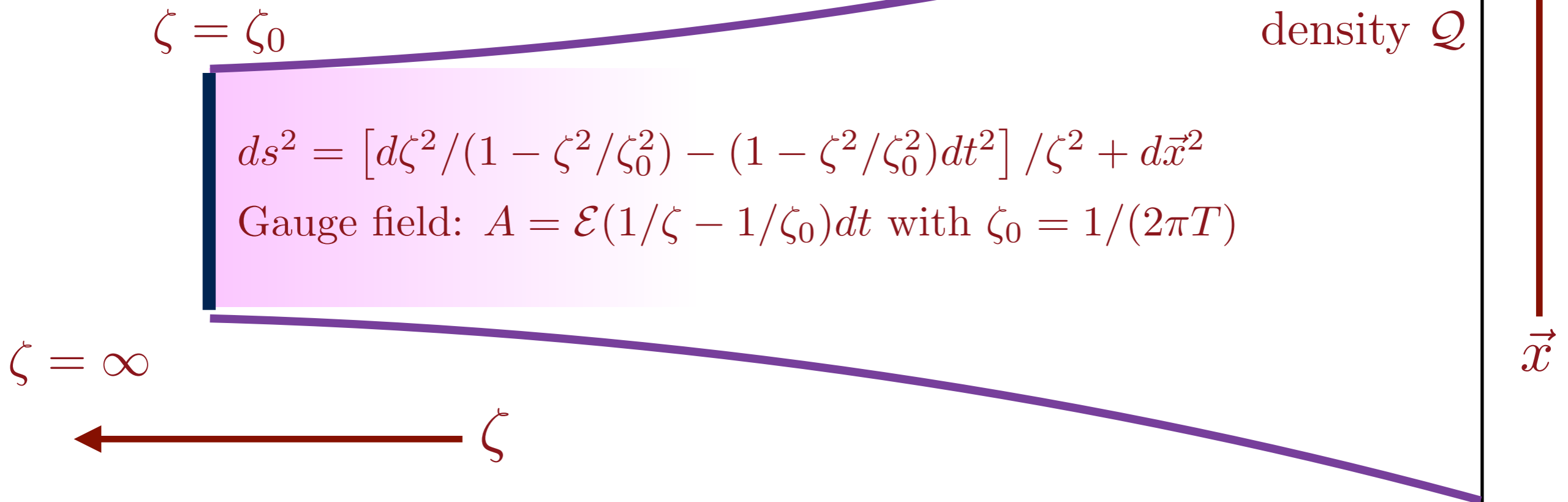
- As $T \rightarrow 0$, there is a non-zero Bekenstein-Hawking entropy, \mathcal{S}_{BH} .
- Using Gauss's Law, it can be shown that $\mu(T) = -2\pi\mathcal{E}T + \text{constant}$ as $T \rightarrow 0$.
- Using a thermodynamic Maxwell relation (also obeyed by gravity),

A. Sen
hep-th/0506177
S. Sachdev
1506.05111

$$\left(\frac{\partial \mathcal{S}_{BH}}{\partial Q} \right)_T = - \left(\frac{\partial \mu}{\partial T} \right)_Q = 2\pi\mathcal{E}$$

General Relativity of charged black branes

Conformal mapping to $T > 0$



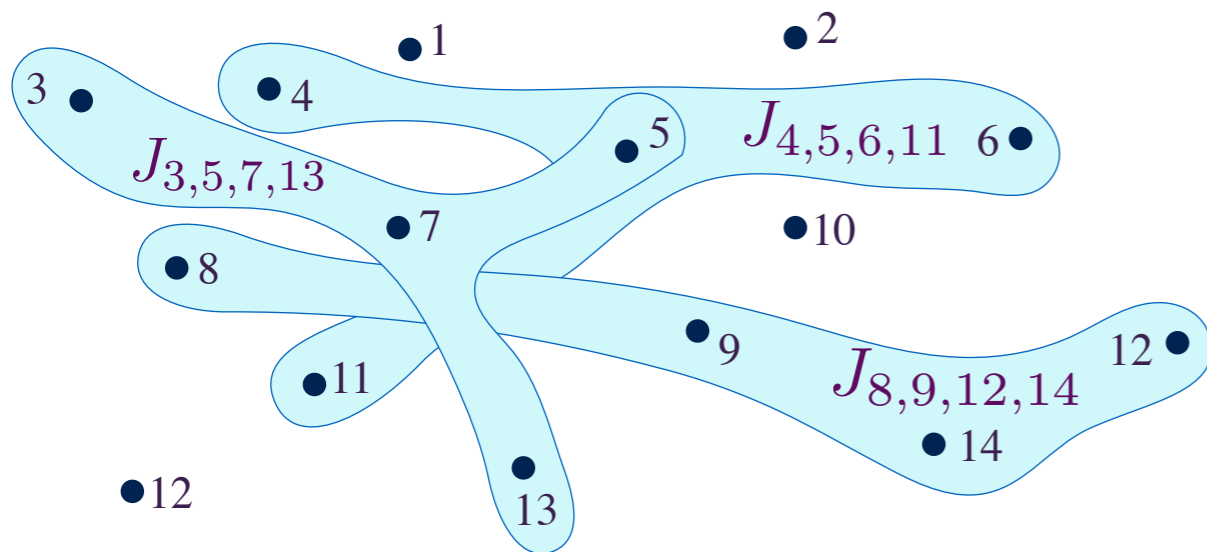
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$$\left(\frac{\partial \mathcal{S}_{BH}}{\partial Q} \right)_T = - \left(\frac{\partial \mu}{\partial T} \right)_Q = 2\pi\mathcal{E}$$

Also obeyed by
 Wald entropy
 in higher-derivative
 gravity.

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^N J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell$$



$$Q = \frac{1}{N} \sum_i \langle c_i^\dagger c_i \rangle.$$

Local fermion density of states

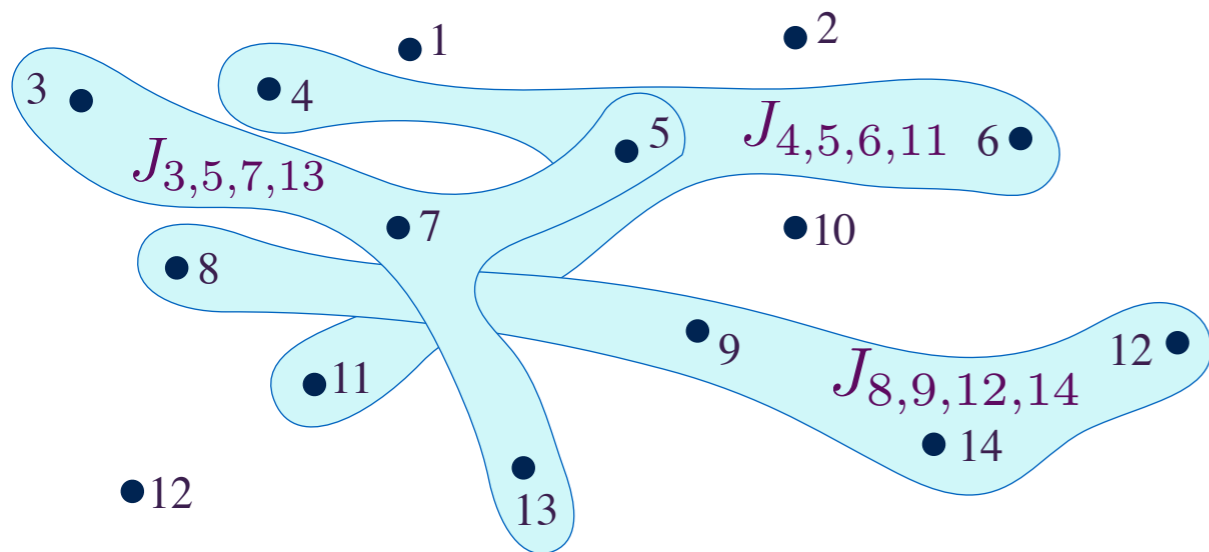
$$\rho(\omega) \sim \begin{cases} \omega^{-1/2}, & \omega > 0 \\ e^{-2\pi\mathcal{E}} |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

Known 'equation of state'
determines \mathcal{E} as a function of Q

Microscopic zero temperature
entropy density, \mathcal{S} , obeys

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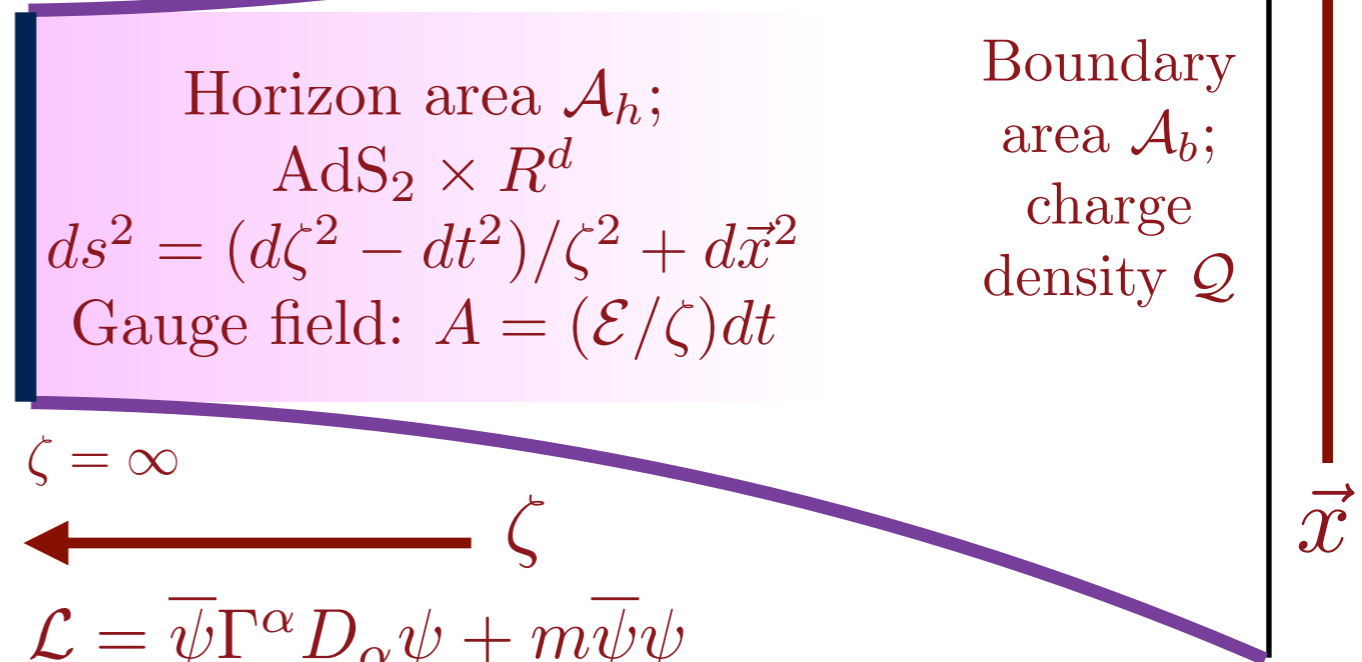
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Einstein-Maxwell theory
+ cosmological constant



Horizon area \mathcal{A}_h ;
 $\text{AdS}_2 \times R^d$
 $ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$
Gauge field: $A = (\mathcal{E}/\zeta)dt$

Boundary area \mathcal{A}_b ;
charge density \mathcal{Q}

$\zeta = \infty$

ζ

$$\mathcal{L} = \bar{\psi} \Gamma^\alpha D_\alpha \psi + m \bar{\psi} \psi$$

Local fermion density of states

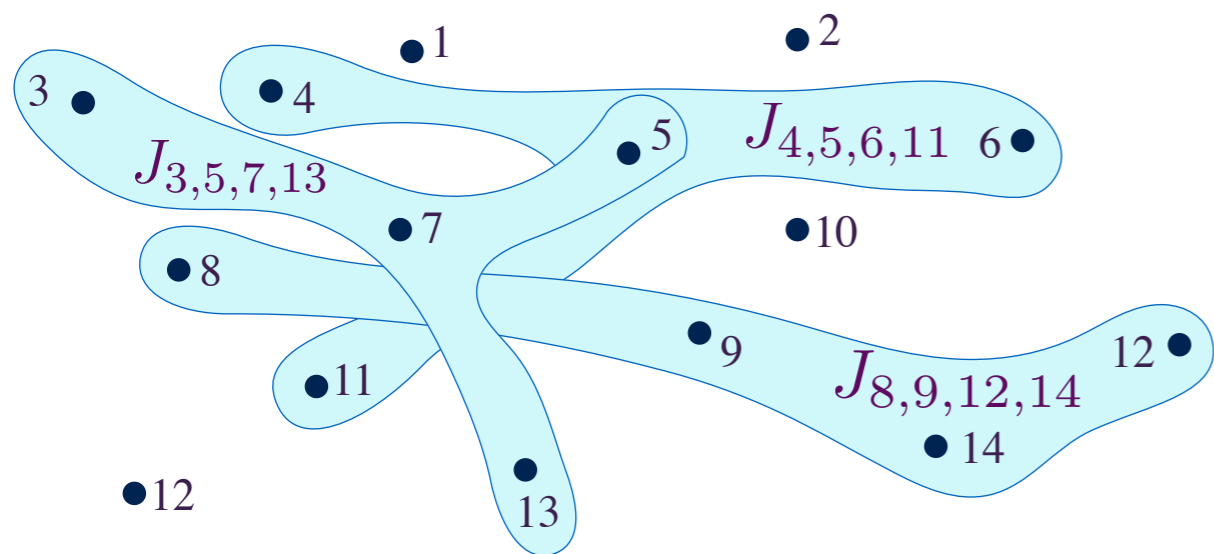
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‘Equation of state’ relating \mathcal{E} and Q depends upon the geometry of spacetime far from the AdS_2

Black hole thermodynamics (classical general relativity) yields

$$\frac{\partial \mathcal{S}_{\text{BH}}}{\partial Q} = 2\pi\mathcal{E}$$

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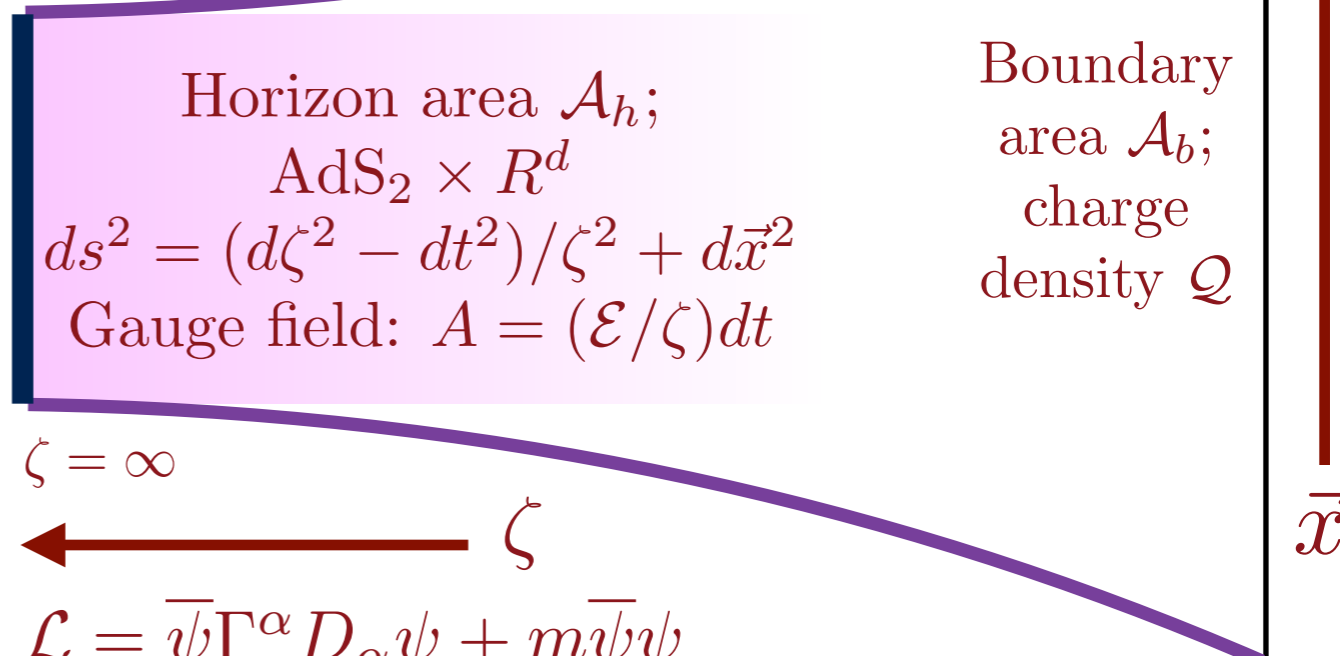
Known 'equation of state' determines \mathcal{E} as a function of Q

Microscopic zero temperature entropy density, \mathcal{S} , obeys

$$\frac{\partial \mathcal{S}}{\partial Q} = 2\pi\mathcal{E}$$

Evidence for AdS₂ gravity dual of H

Einstein-Maxwell theory + cosmological constant



$$\zeta = \infty$$

$$\leftarrow \zeta$$

$$\mathcal{L} = \bar{\psi} \Gamma^\alpha D_\alpha \psi + m \bar{\psi} \psi$$

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Quantum matter without quasiparticles

1. A solvable model of an ordinary metal
2. A solvable model of a strange metal
3. Holography and charged black holes
4. Transport in strange metals

Quantum matter without quasiparticles

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Transport in Strange Metals

universal constraints on transport

hydrodynamics

[Forster '70s]

[Hartnoll, others]

[Lucas, Sachdev PRB]

few conserved quantities

memory matrix

appropriate microscopics
for cuprates

perturbative
limit

[Lucas JHEP]

holography

matrix large N theory;
non-perturbative computations

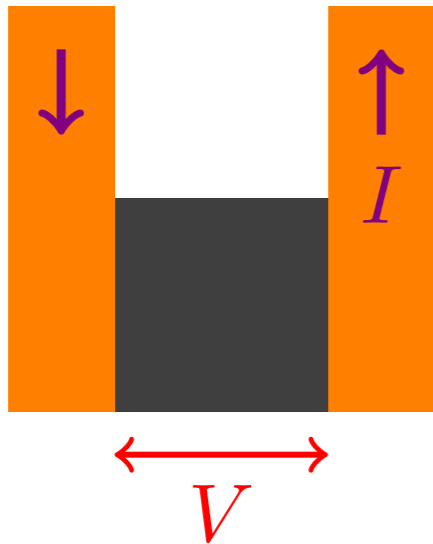
long time dynamics;
“renormalized IR fluid”
emerges

[Lucas 1506]

[Donos, Gauntlett 1506]

figure from [Lucas, Sachdev, *Physical Review* **B91** 195122 (2015)]

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)



$$V = IR \quad R \sim \frac{1}{\sigma}$$

- ▶ more generally, measure thermoelectric transport:

$$\begin{pmatrix} \delta J_i \\ \delta Q_i \end{pmatrix} = \begin{pmatrix} \sigma_{ij} & \alpha_{ij} \\ T\bar{\alpha}_{ij} & \bar{\kappa}_{ij} \end{pmatrix} \begin{pmatrix} \delta E_j \\ -\partial_j \delta T \equiv T\delta\zeta_j \end{pmatrix}.$$

- ▶ σ = easy experiment; related to QFT correlators:

$$\sigma_{ij}(\omega) = \frac{i}{\omega} \langle J_i(-\omega) J_j(\omega) \rangle, \quad \text{etc.}$$

Thermoelectric transport coefficients

Transport has two components: a “momentum drag” term, and a “quantum critical” term.

$$\sigma = \frac{Q^2}{\mathcal{M}} \pi \delta(\omega) + \sigma_Q(\omega)$$

$$\alpha = \frac{SQ}{\mathcal{M}} \pi \delta(\omega) + \alpha_Q(\omega)$$

$$\bar{\kappa} = \frac{TS^2}{\mathcal{M}} \pi \delta(\omega) + \bar{\kappa}_Q(\omega)$$

with entropy density \mathcal{S} , $Q \equiv \chi_{J_x, P_x}$, and $\mathcal{M} \equiv \chi_{P_x, P_x}$.

**Obtained in hydrodynamics, holography, and
by memory functions**

S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

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with entropy density \mathcal{S} , $Q \equiv \chi_{J_x, P_x}$, and $\mathcal{M} \equiv \chi_{P_x, P_x}$.

In theories which are relativistic at high energies (including graphene), $T\alpha_Q(\omega) = -\mu\sigma_Q(\omega)$, $T\bar{\kappa}_Q(\omega) = \mu^2\sigma_Q(\omega)$, $\mathcal{M} = T\mathcal{S} + \mu Q = \mathcal{H}$ the enthalpy density, and $Q = n$ the electron density

S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

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Transport has two components: a “momentum drag” term, and a “quantum critical” term.

$$\begin{aligned}\sigma &= \frac{Q^2}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau)} + \sigma_Q(\omega) \\ \alpha &= \frac{SQ}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau)} + \alpha_Q(\omega) \\ \bar{\kappa} &= \frac{TS^2}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau)} + \bar{\kappa}_Q(\omega)\end{aligned}$$

Momentum relaxation by an external source h coupling to the operator \mathcal{O}

$$\begin{aligned}H &= H_0 - \int d^d x h(x) \mathcal{O}(x). \\ \frac{\mathcal{M}}{\tau} &= \lim_{\omega \rightarrow 0} \int d^d q |h(q)|^2 q_x^2 \frac{\text{Im} (G_{\mathcal{O}\mathcal{O}}^R(q, \omega))_{H_0}}{\omega} + \text{higher orders in } h\end{aligned}$$

S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

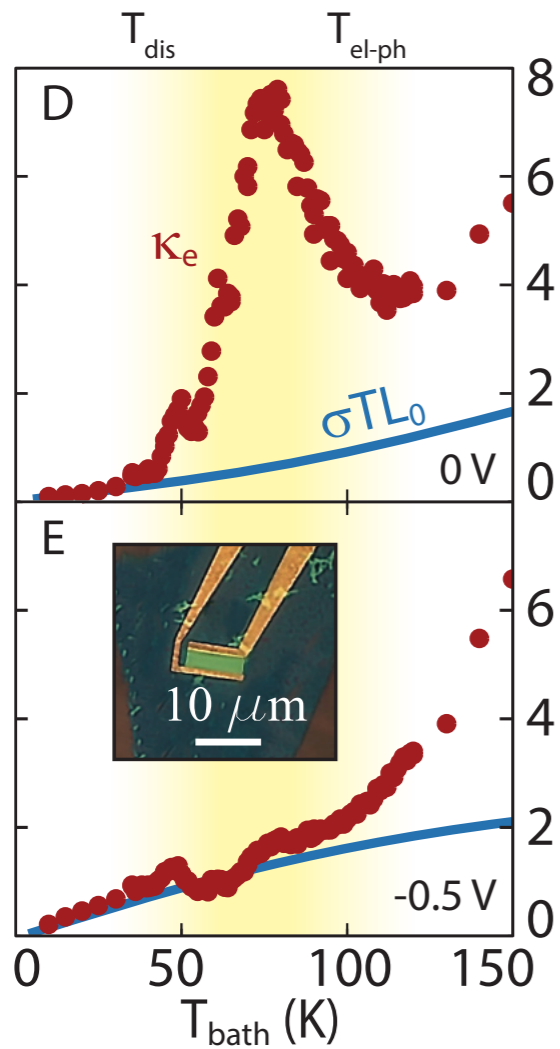
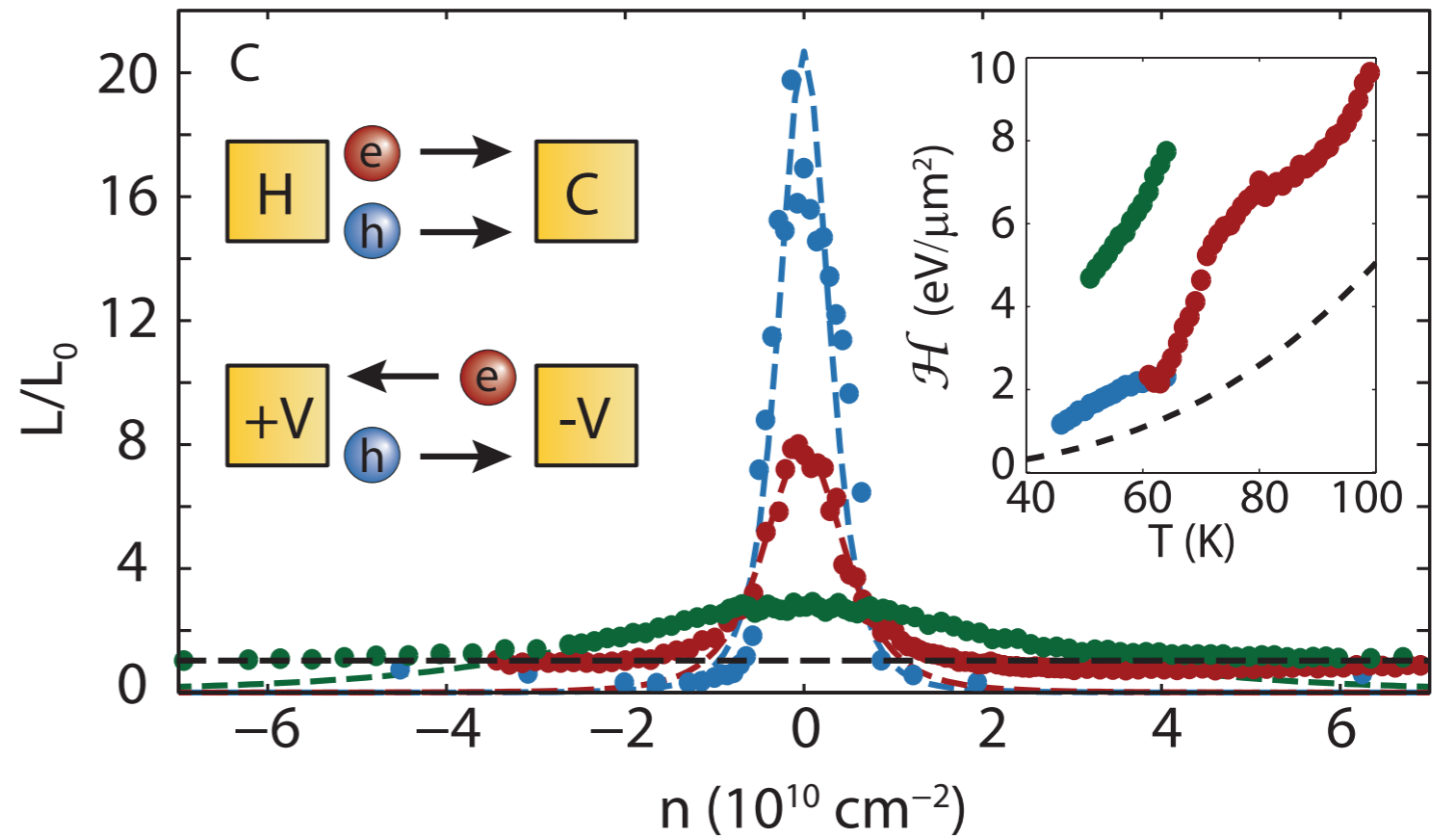
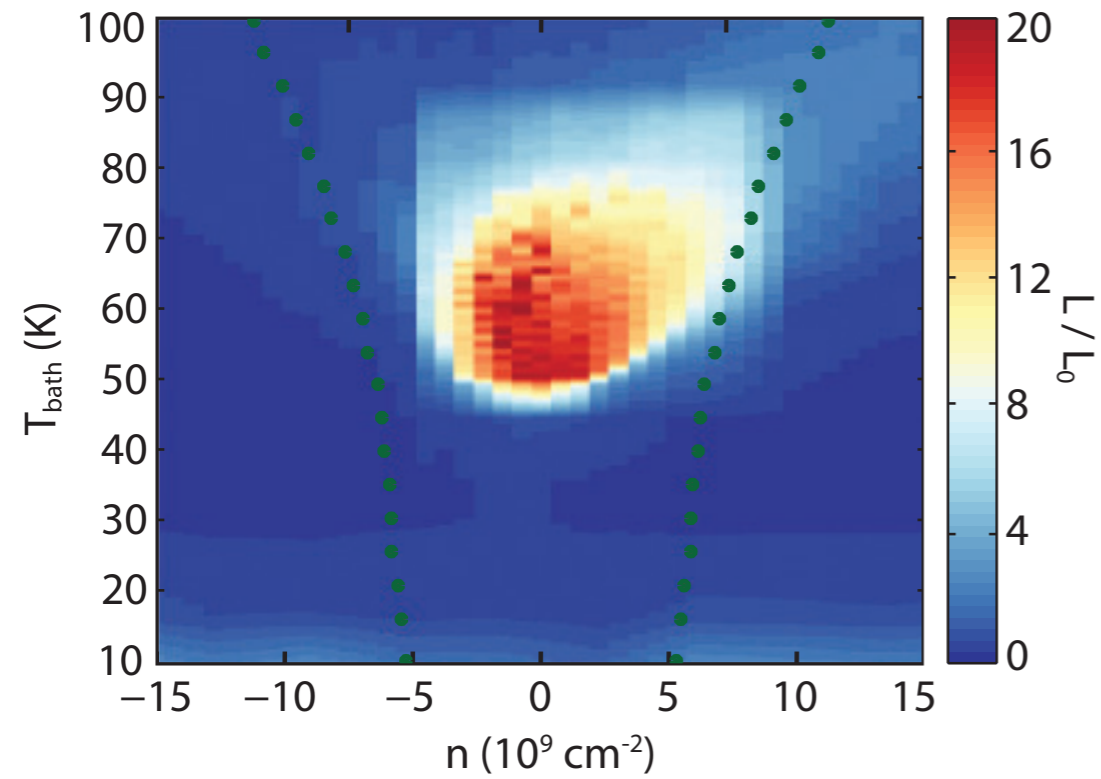
Thermoelectric transport coefficients

Transport has two components: a “momentum drag” term, and a “quantum critical” term.

$$\sigma_{xx} = \frac{(\tau^{-1} - i\omega)\mathcal{M}\sigma_Q + Q^2 + B^2\sigma_Q^2}{Q^2B^2 + ((\tau^{-1} - i\omega)\mathcal{M} + B^2\sigma_Q)^2} \mathcal{M} \left(\frac{1}{\tau} - i\omega \right),$$
$$\sigma_{xy} = \frac{2(\tau^{-1} - i\omega)\mathcal{M}\sigma_Q + Q^2 + B^2\sigma_Q^2}{Q^2B^2 + ((\tau^{-1} - i\omega)\mathcal{M} + B^2\sigma_Q)^2} BQ.$$

**Electrical and thermal magnetotransport
in a magnetic field B with no additional parameters**

(assuming σ_Q is field-independent)

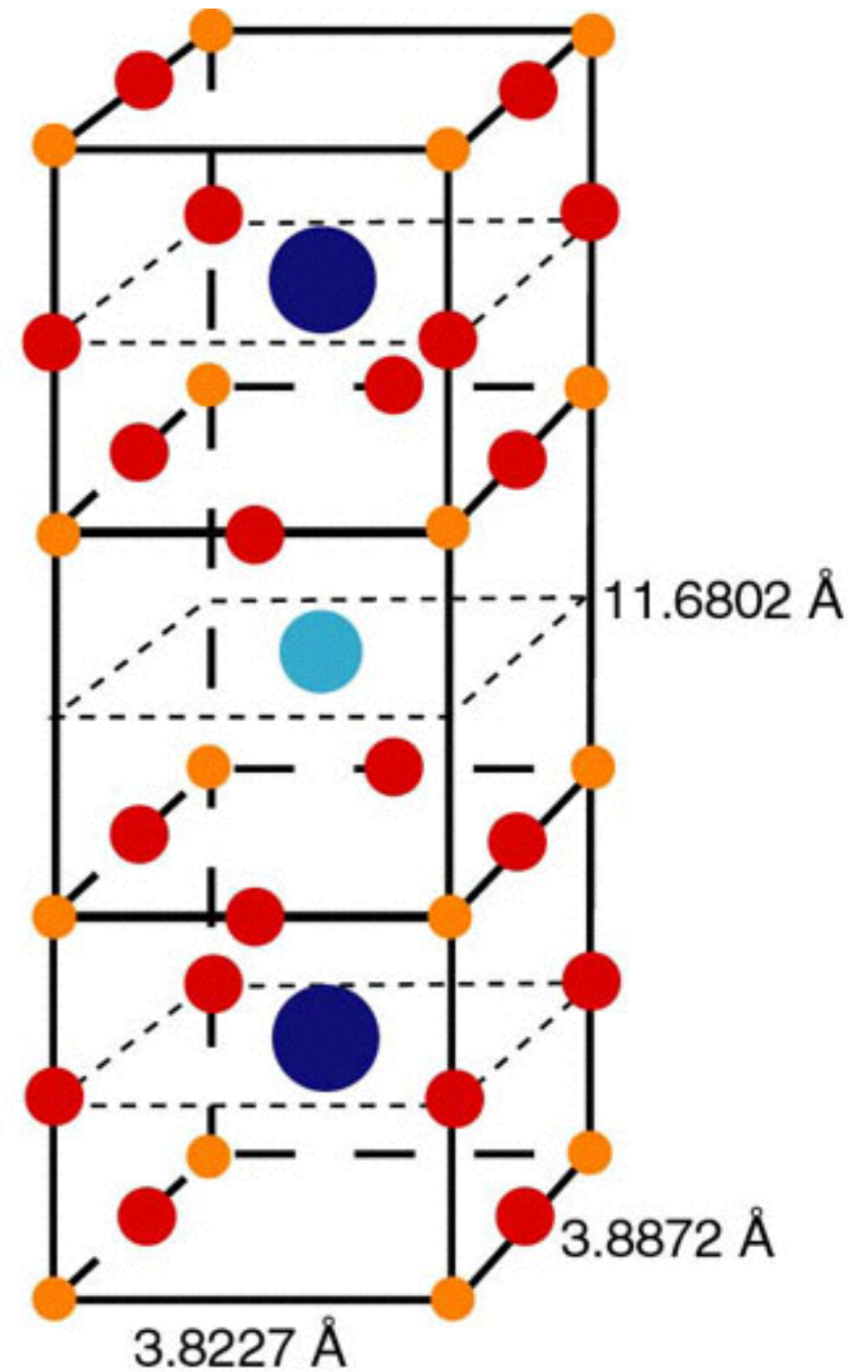
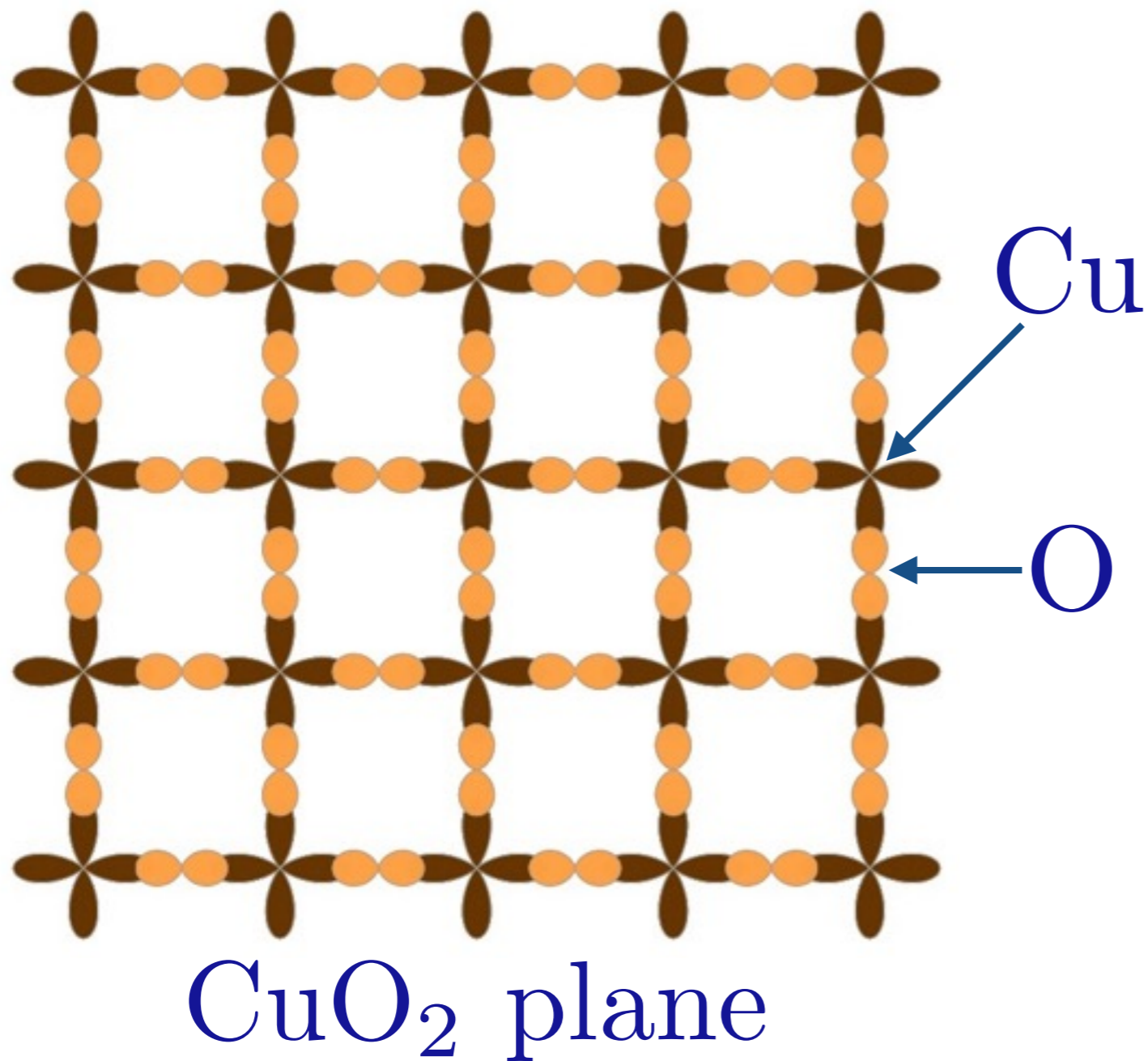


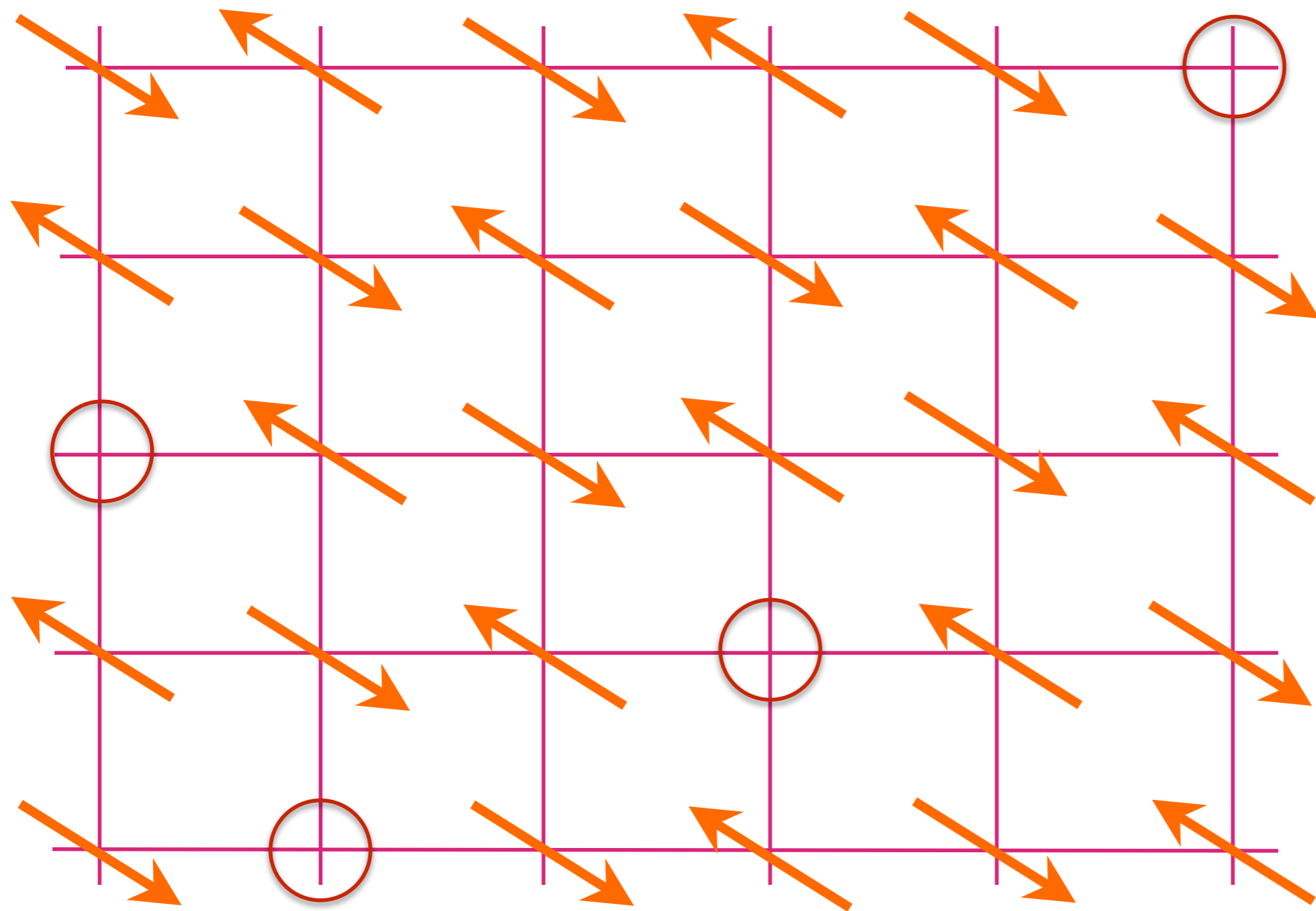
$$\begin{aligned} \text{Lorentz ratio } L &= \kappa / (T\sigma) \\ &= \frac{\mathcal{H}\tau_{\text{imp}}}{T^2\sigma_Q} \frac{1}{(1 + Q^2\tau_{\text{imp}}/(\mathcal{H}\sigma_Q))^2} \end{aligned}$$

Quantum matter without quasiparticles

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High temperature superconductors





Anti-ferromagnet
with p holes
per square

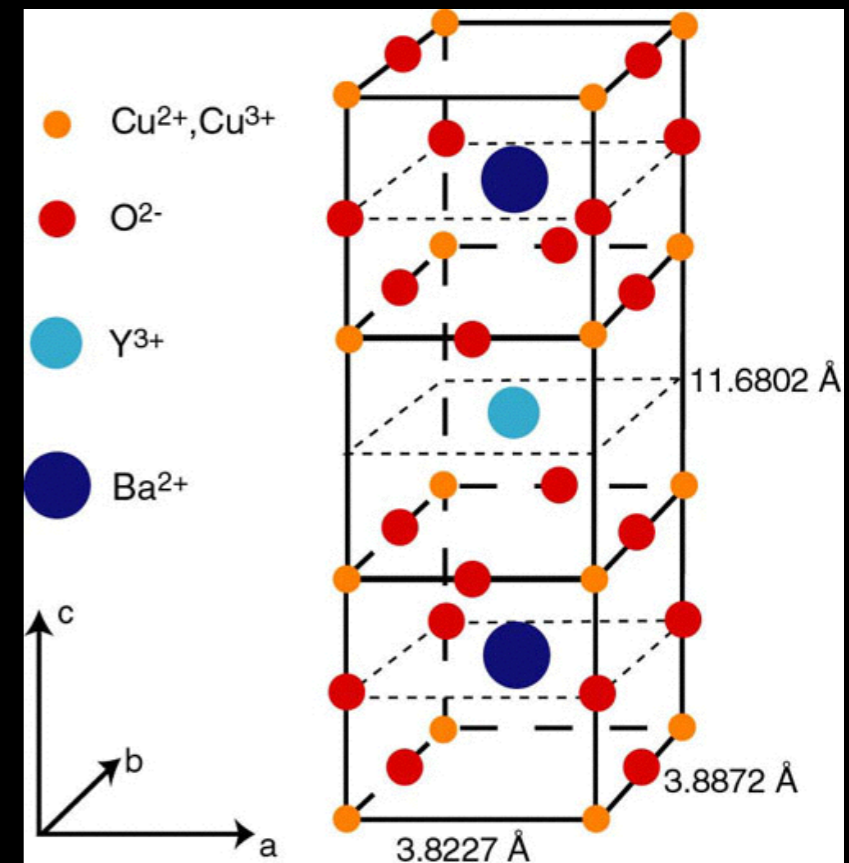
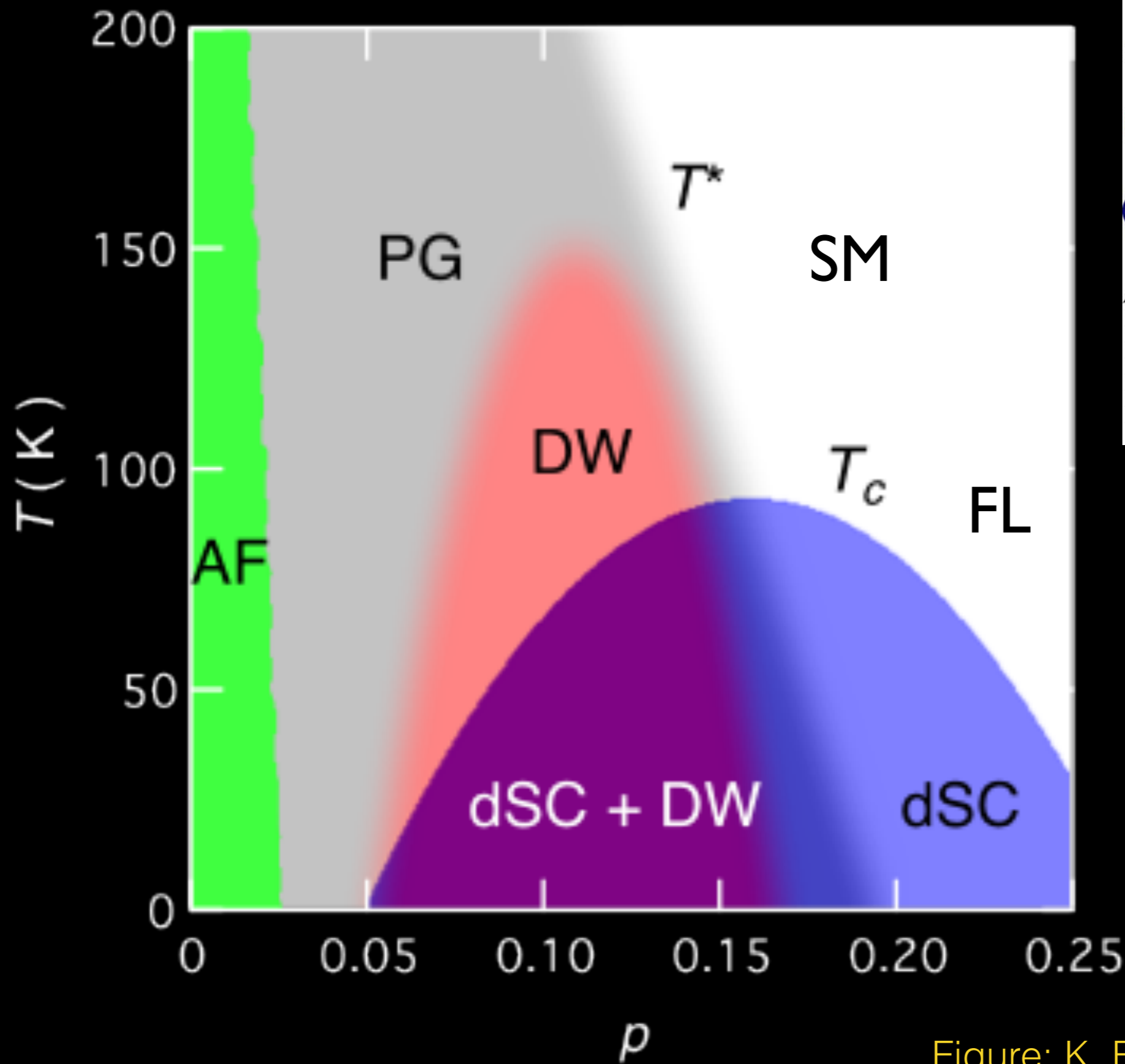
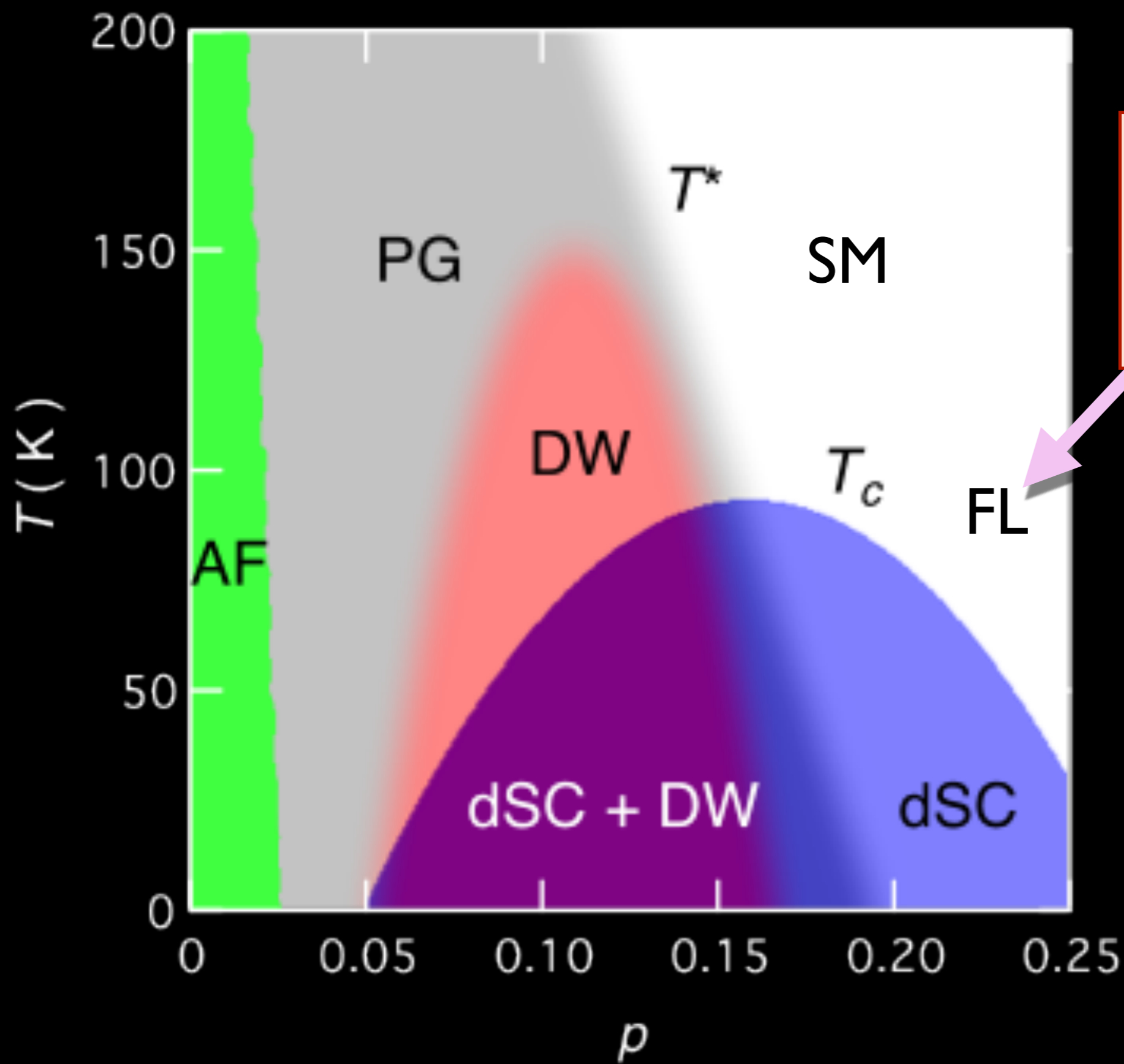
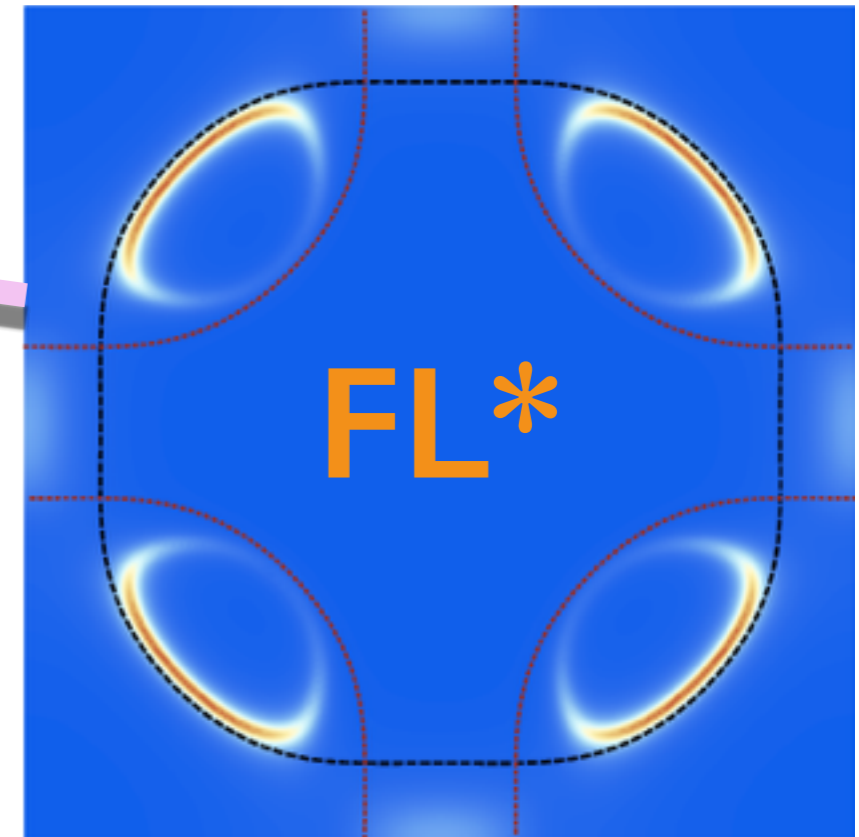
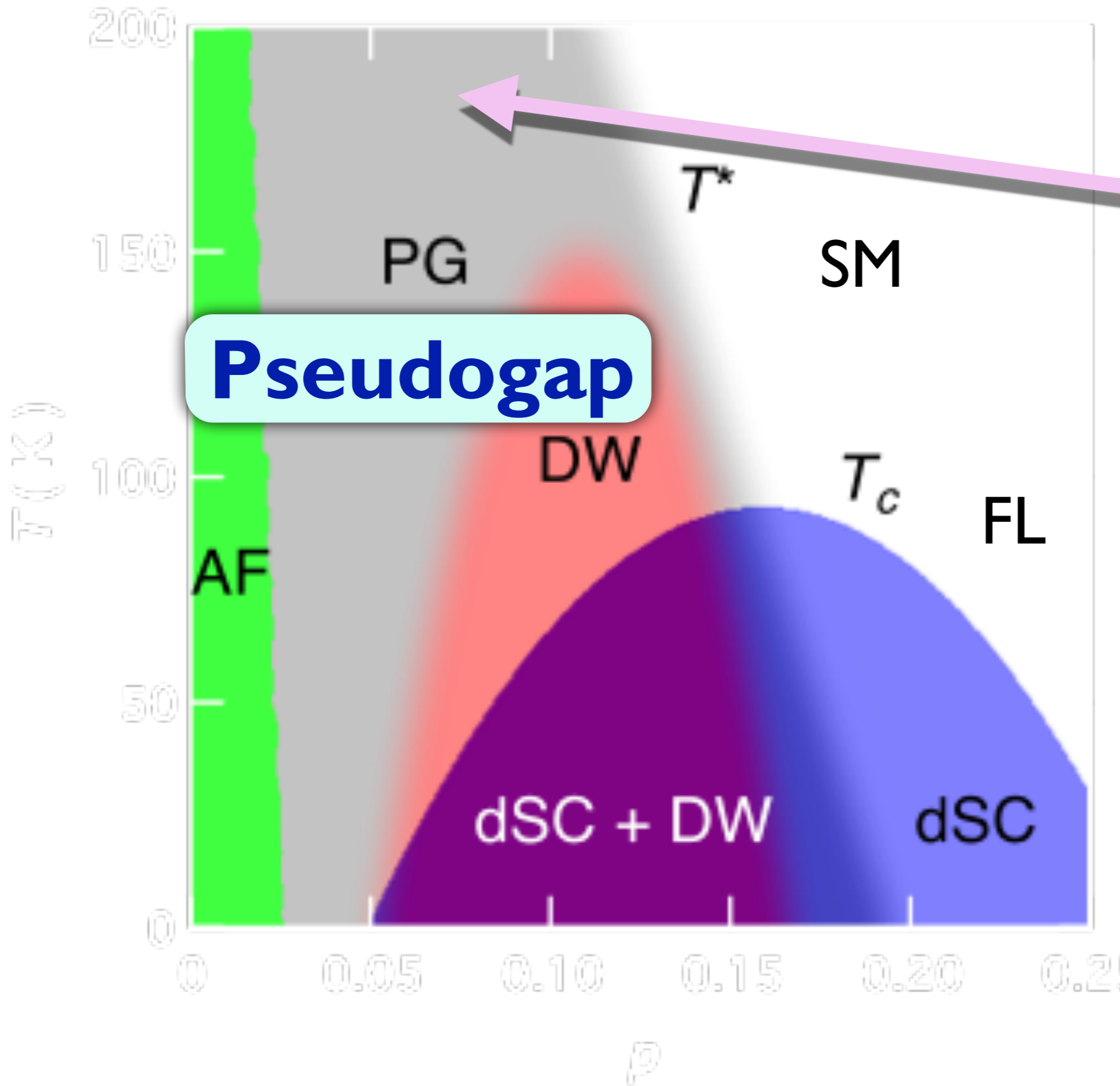


Figure: K. Fujita and J. C. Seamus Davis

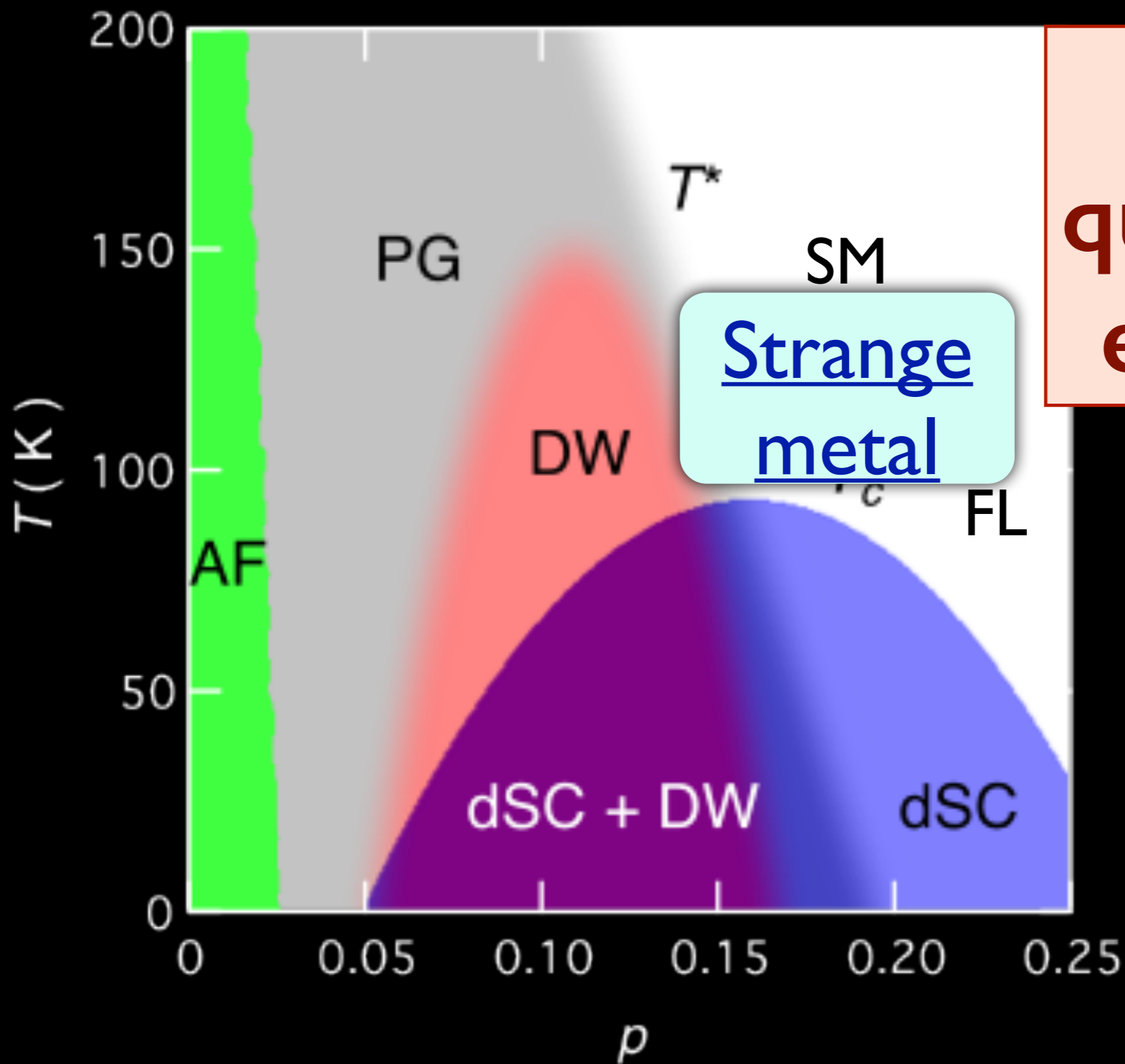


Fermi liquid:
a conventional
metal

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)



A new metal — a fractionalized Fermi liquid (FL*) — with electron-like quasiparticles on a Fermi surface of size p



**No
quasiparticle
excitations**