Quantum phase transitions of metals

Field Theoretic Computer Simulations
for Particle Physics and Condensed Matter
May 9, 2014
Boston University

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

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Cornell

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Cornell/ BNL

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Cornell / BNL

Stephen Edkins
Cornell / St Andrews

Michael Lawler

J. C. Seamus Davis

Eun-Ah Kim

Outline

1. Onset of antiferromagnetism in semi-metals

2. Onset of antiferromagnetism in metals

   Sign-problem-free Quantum Monte Carlo:
   evidence for d-wave superconductivity

3. A competing order

   An unconventional charge density wave

4. STM observation

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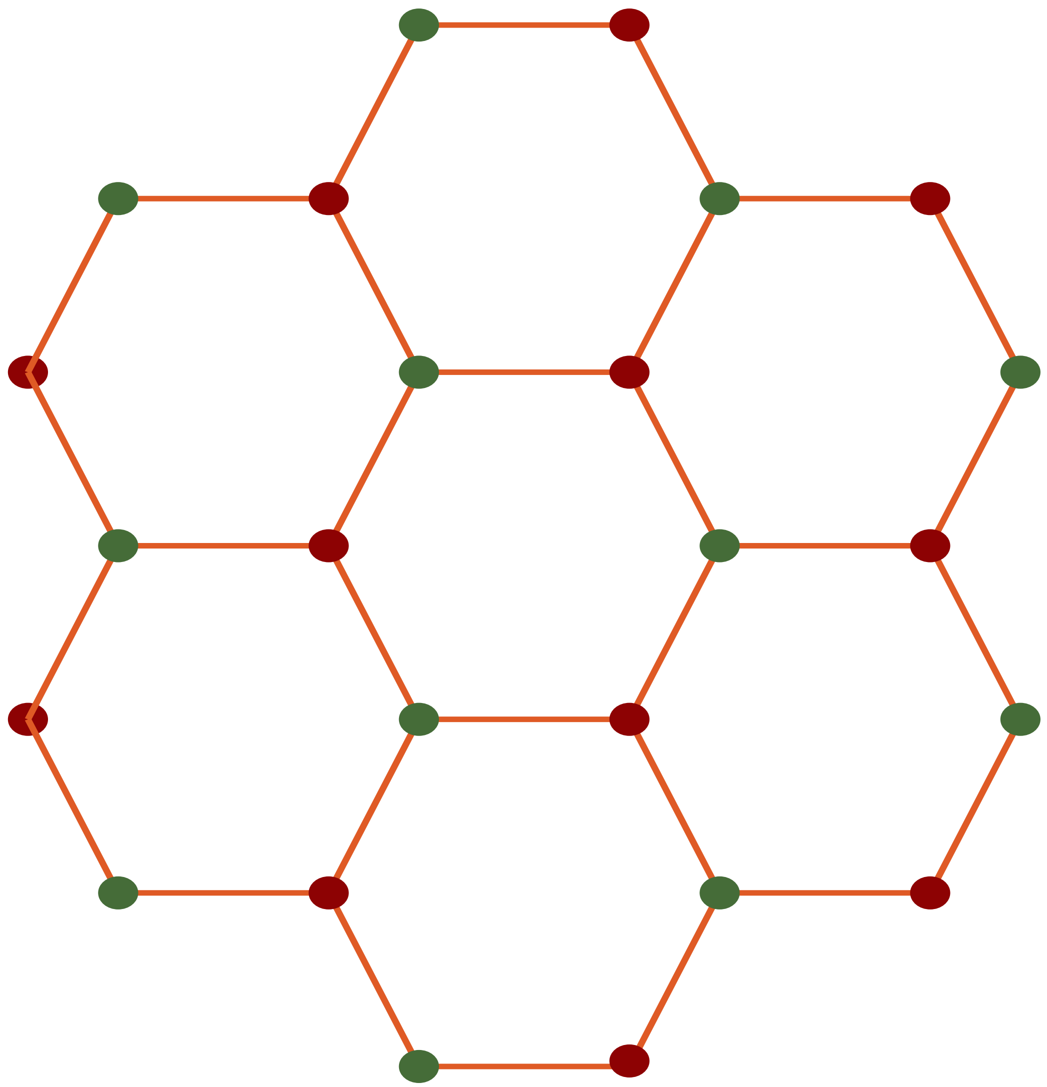
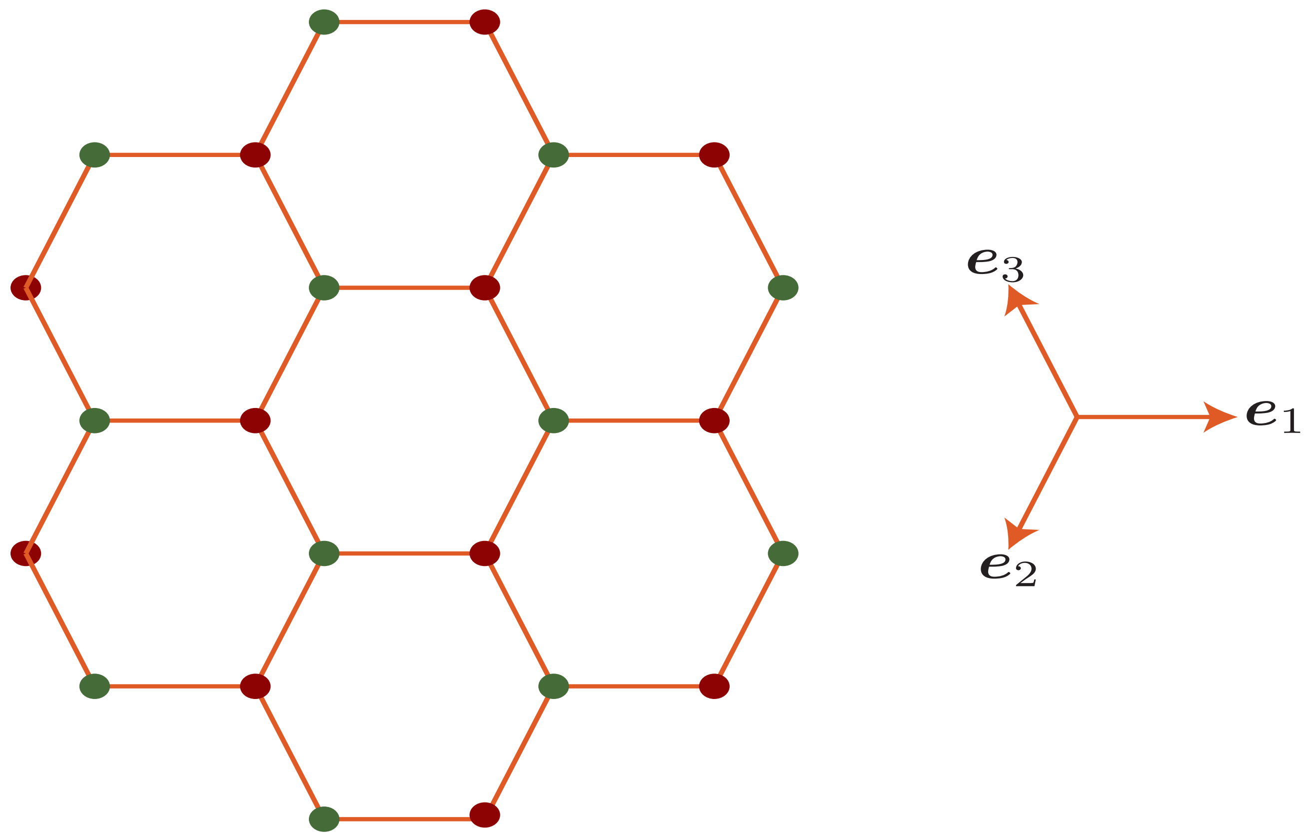
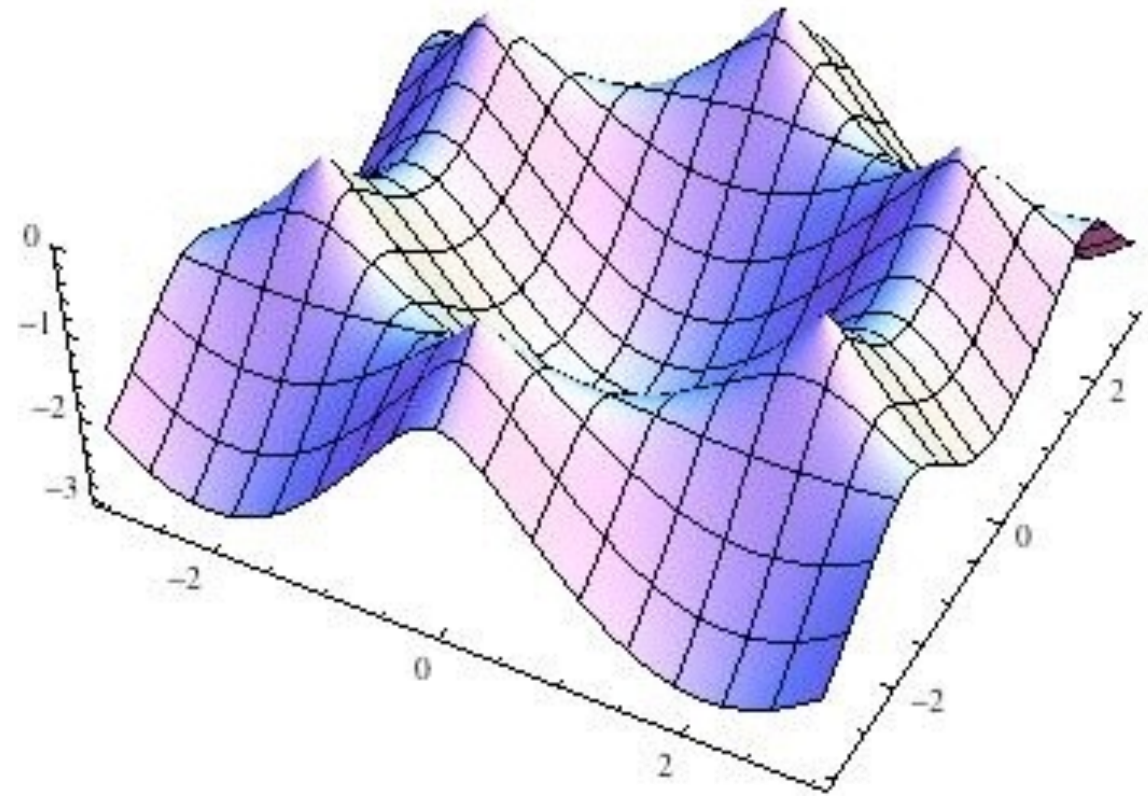
Honeycomb lattice

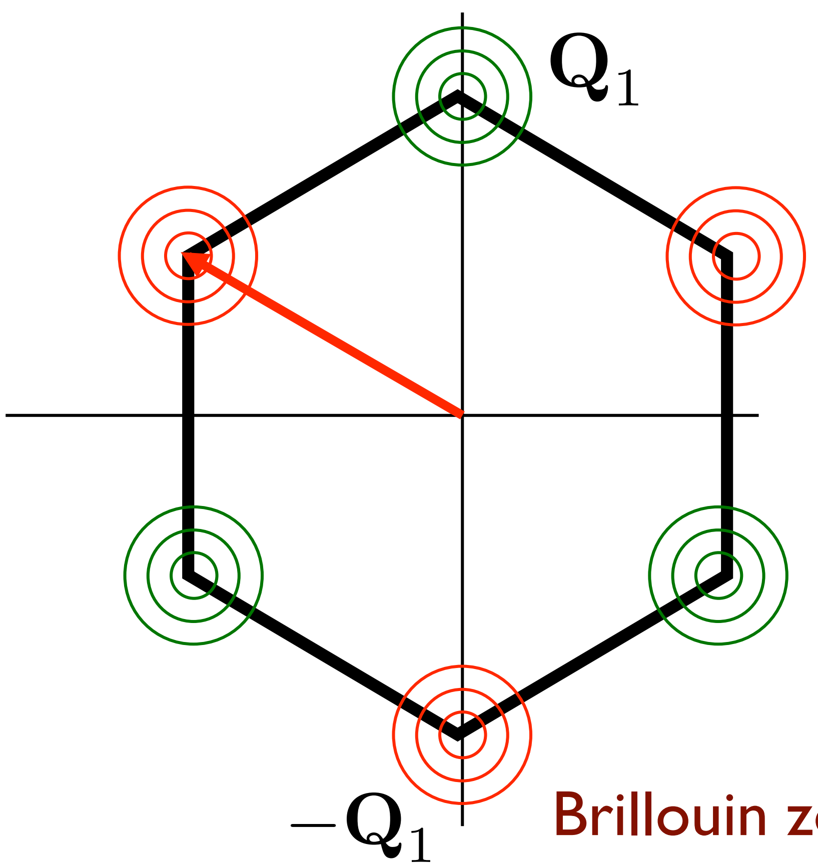
(describes graphene after adding long-range Coulomb interactions)



$$H = -t \sum_{\langle ij \rangle} c_{i\alpha}^{\dagger} c_{j\alpha} + U \sum_{i} \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right)$$







Semi-metal with
massless Dirac fermions
at small *U*/*t*

The Hubbard Model at large U

$$H = -\sum_{i,j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c^\dagger_{i\alpha} c_{i\alpha}$$

In the limit of large U, and at a density of one particle per site, this maps onto the Heisenberg antiferromagnet

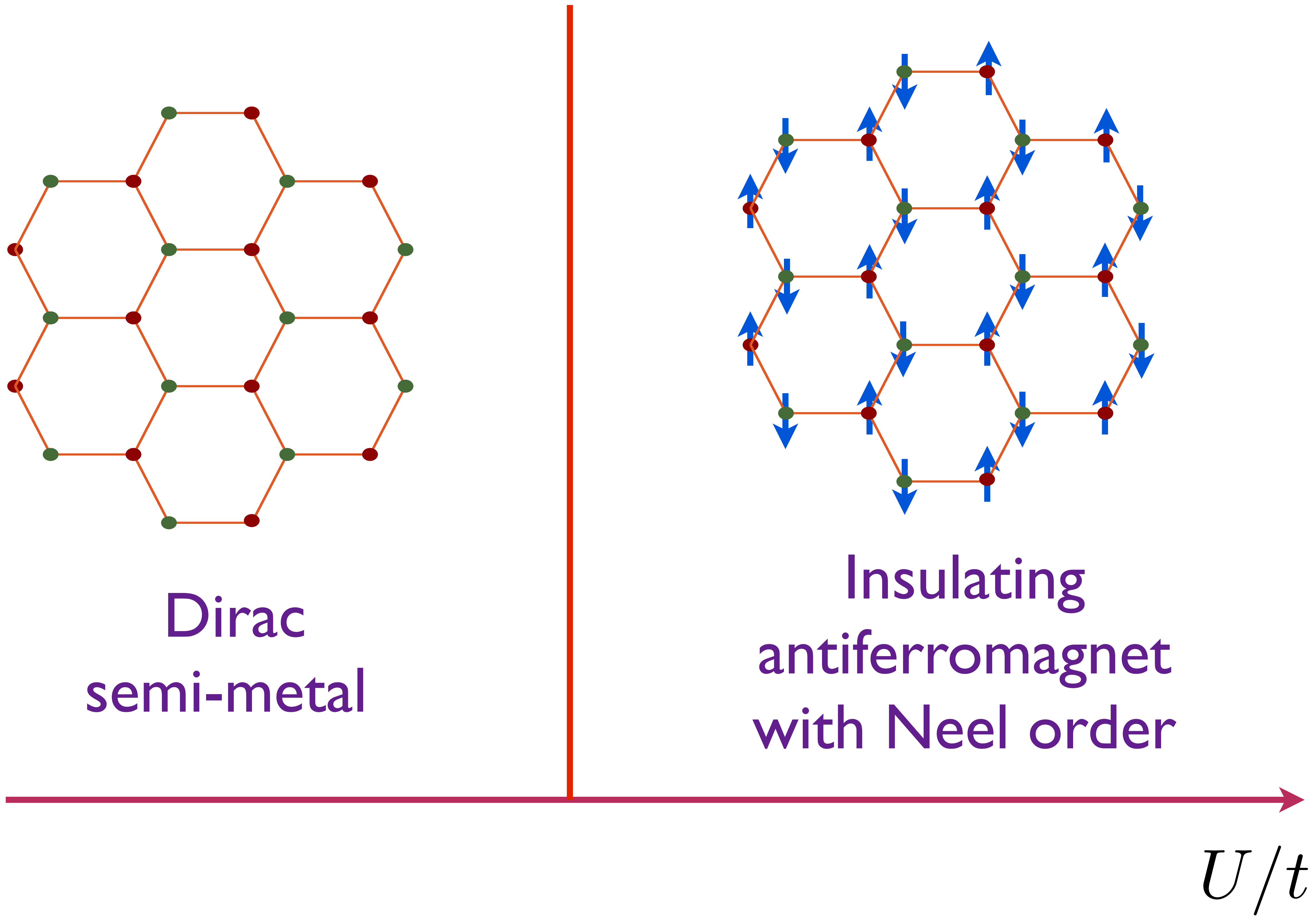
$$H_{AF} = \sum_{i<j} J_{ij} S^a_i S^a_j$$

where $a = x, y, z$,

$$S^a_i = \frac{1}{2} c^{a\dagger}_{i\alpha} \sigma^a_{\alpha\beta} c_{i\beta},$$

with σ^a the Pauli matrices and

$$J_{ij} = \frac{4t^2_{ij}}{U}$$


Dirac
semi-metal
Insulating
antiferromagnet
with Neel order
U/t

Antiferromagnetism

We use the operator equation (valid on each site i):

$$U\left(n_\uparrow - \frac{1}{2}\right)\left(n_\downarrow - \frac{1}{2}\right) = -\frac{2U}{3}S^{a2} + \frac{U}{4} \qquad (11)$$

Then we decouple the interaction via

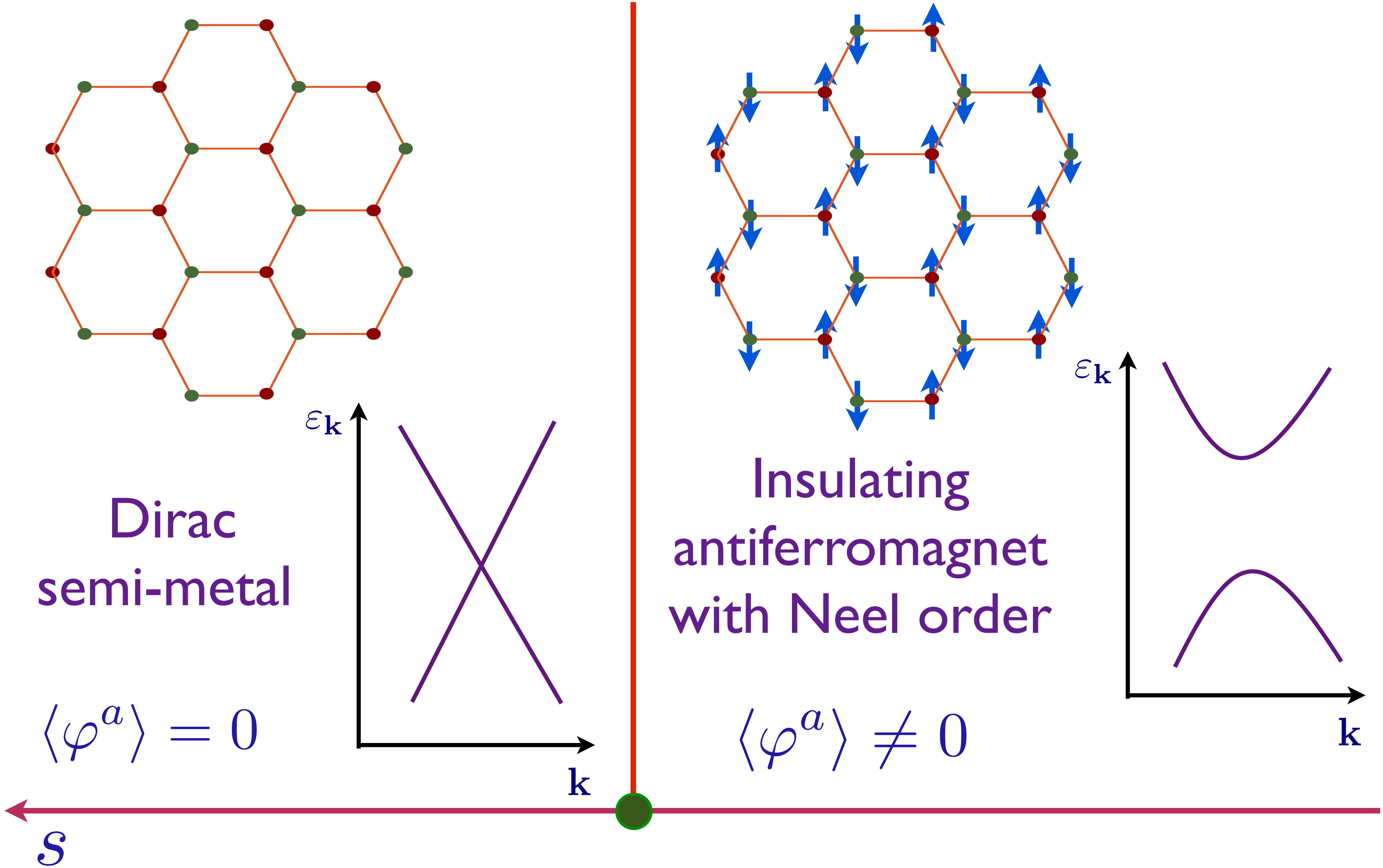
$$\exp\left(\frac{2U}{3}\sum_i \int d\tau S_i^{a2}\right) = \int \mathcal{D}J_i^a(\tau) \exp\left(-\sum_i \int d\tau \left[\frac{3}{8U}J_i^{a2} - J_i^a S_i^a\right]\right) \qquad (12)$$

We now integrate out the fermions, and look for the saddle point of the resulting effective action for J_i^a.

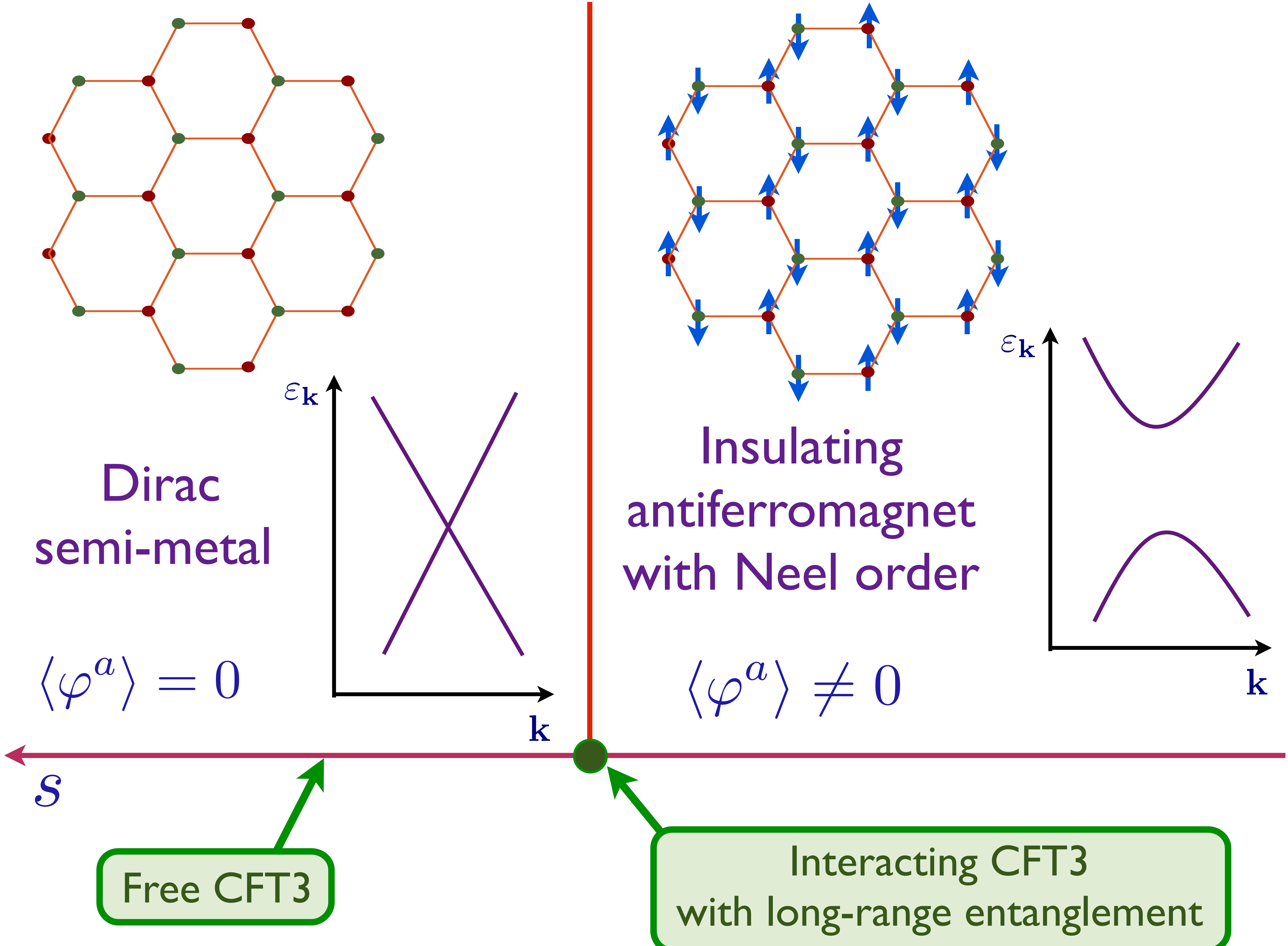
Long wavelength fluctuations about this saddle point are described by a field theory of the Néel order parameter, φ^a, coupled to the Dirac fermions in the **Gross-Neveu** model.

$$\mathcal{L} = \overline{\Psi}\gamma_\mu\partial_\mu\Psi + \frac{1}{2}\left[(\partial_\mu\varphi^a)^2 + s\varphi^{a2}\right] + \frac{u}{24}\left(\varphi^{a2}\right)^2 - \lambda\varphi^a\overline{\Psi}\rho^z\sigma^a\Psi$$

I.F. Herbut, V. Juricic, and B. Roy, *Phys. Rev. B* **79**, 085116 (2009)



$\varepsilon_{\mathbf{k}}$
Dirac
semi-metal
$\langle \varphi^a \rangle = 0$
$\mathbf{k}$
Insulating
antiferromagnet
with Neel order
$\varepsilon_{\mathbf{k}}$
$\langle \varphi^a \rangle \neq 0$
$\mathbf{k}$
s



$\varepsilon_{\mathbf{k}}$
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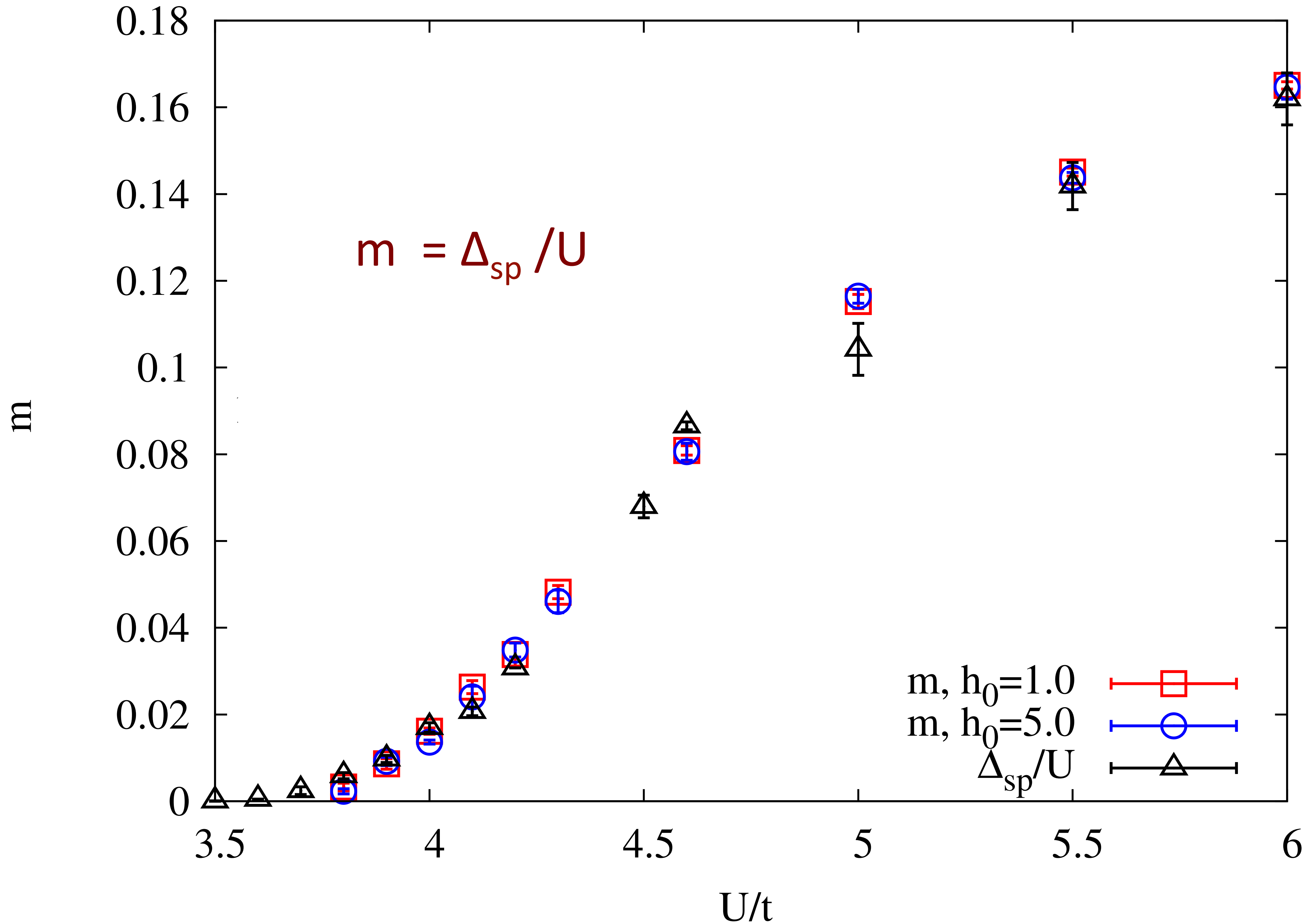
$\langle \varphi^a \rangle \neq 0$
$\varepsilon_{\mathbf{k}}$
$\mathbf{k}$
s
Free CFT3
Interacting CFT3
with long-range entanglement

Polynomial extrapolation to $L \to \infty$ of m and Δ_{sp}/U



Fakher F. Assaad and Igor F. Herbut, Phys. Rev. X **3**, 031010 (2013)

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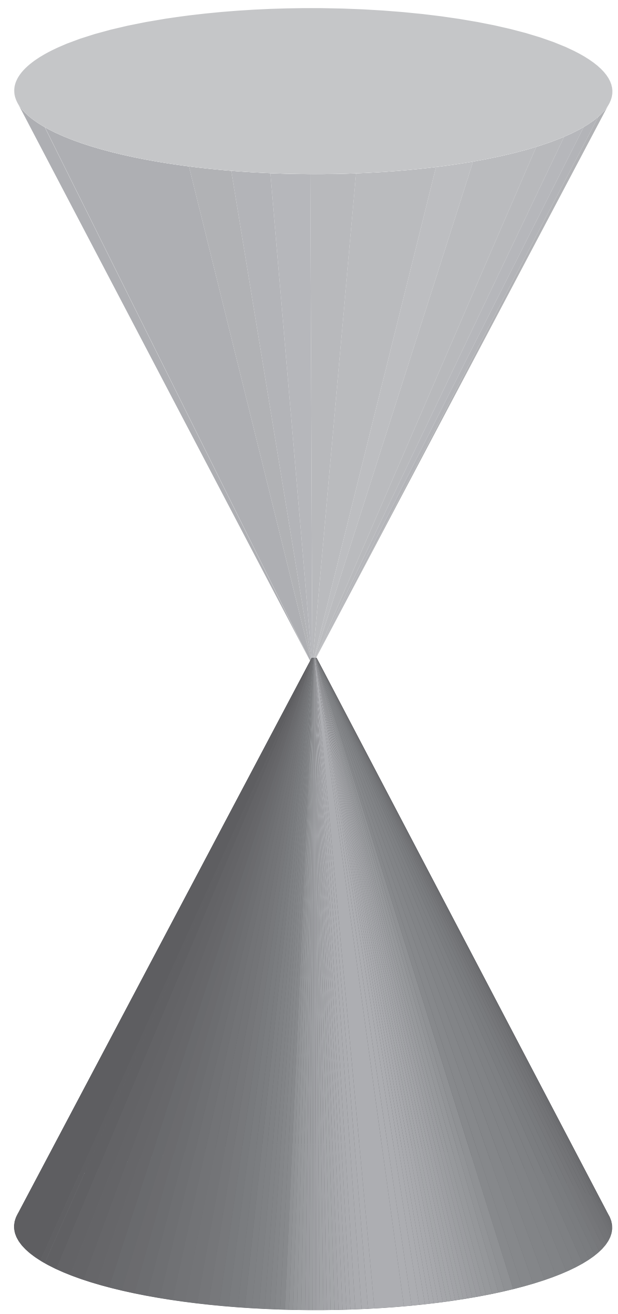
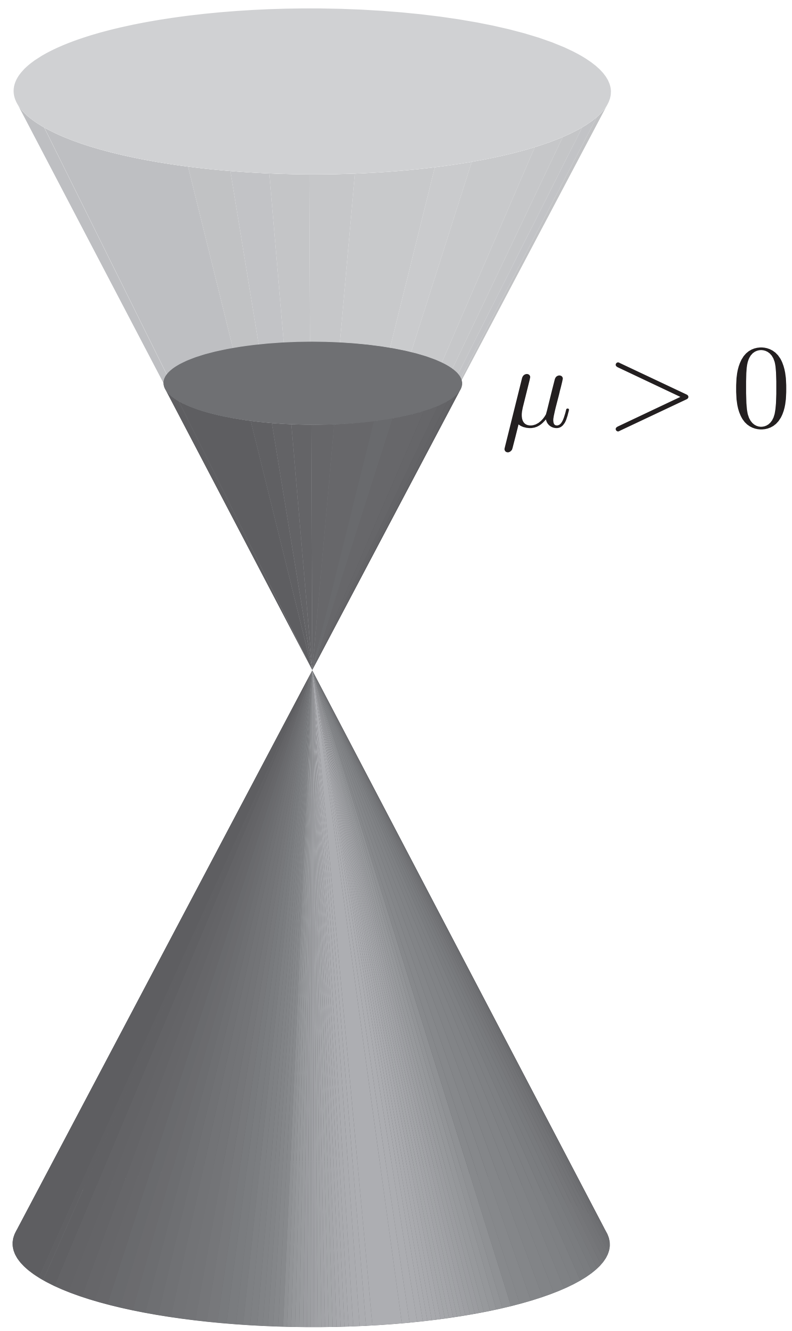
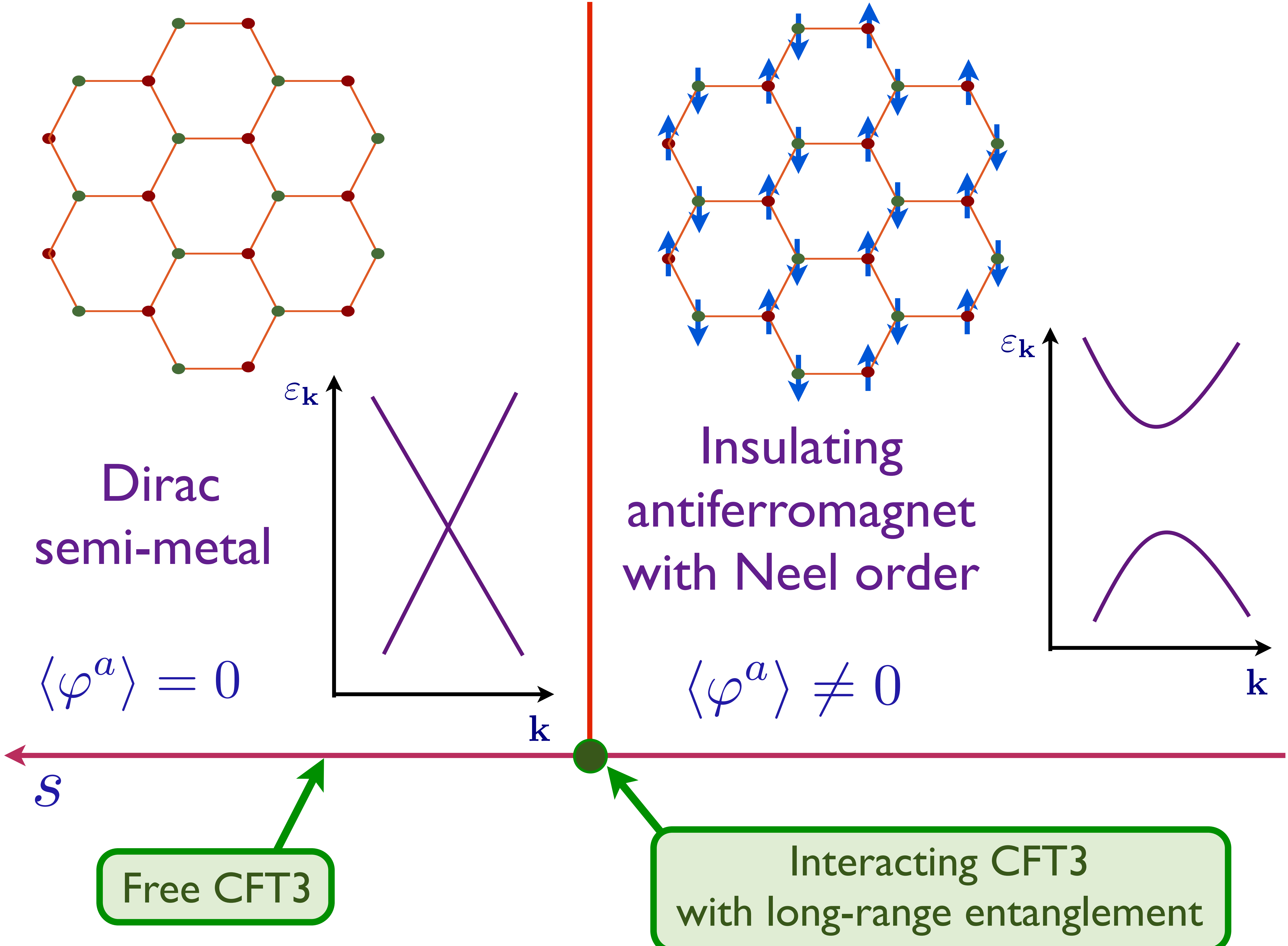


Dirac
semi-metal



Electron
Fermi surface



$\varepsilon_{\mathbf{k}}$
$\mathbf{k}$
Dirac
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$\langle \varphi^a \rangle = 0$
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Free CFT3
Interacting CFT3
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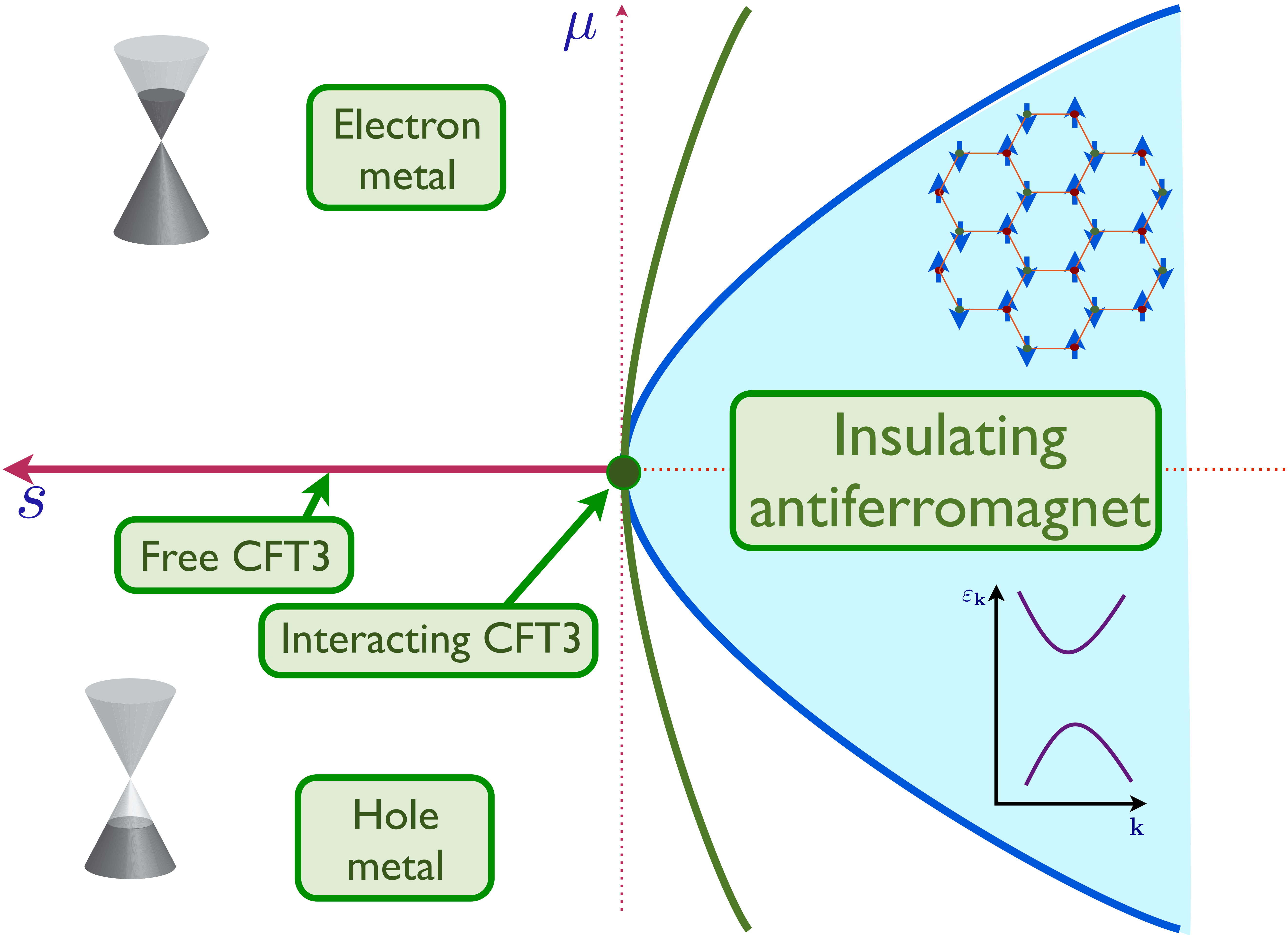
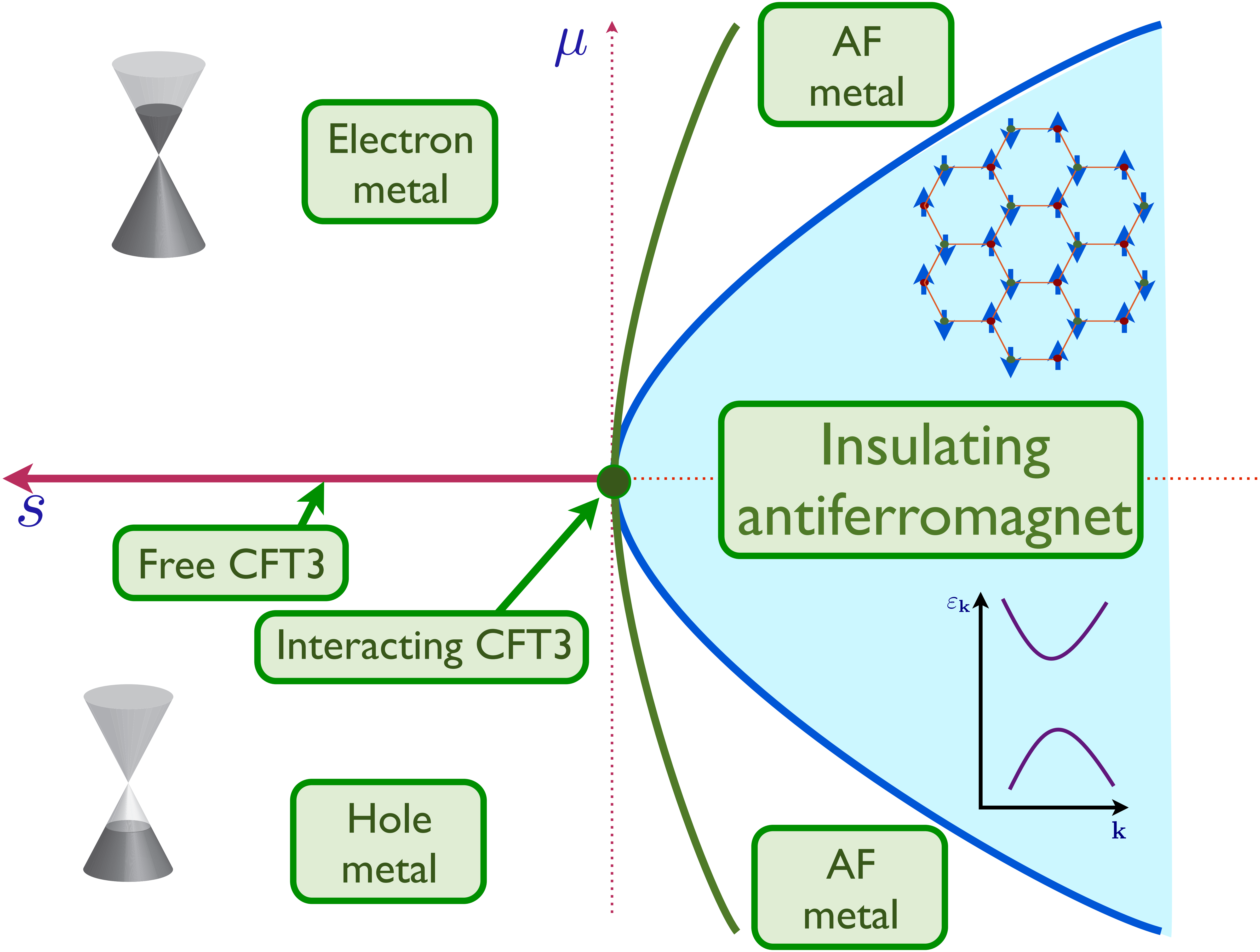


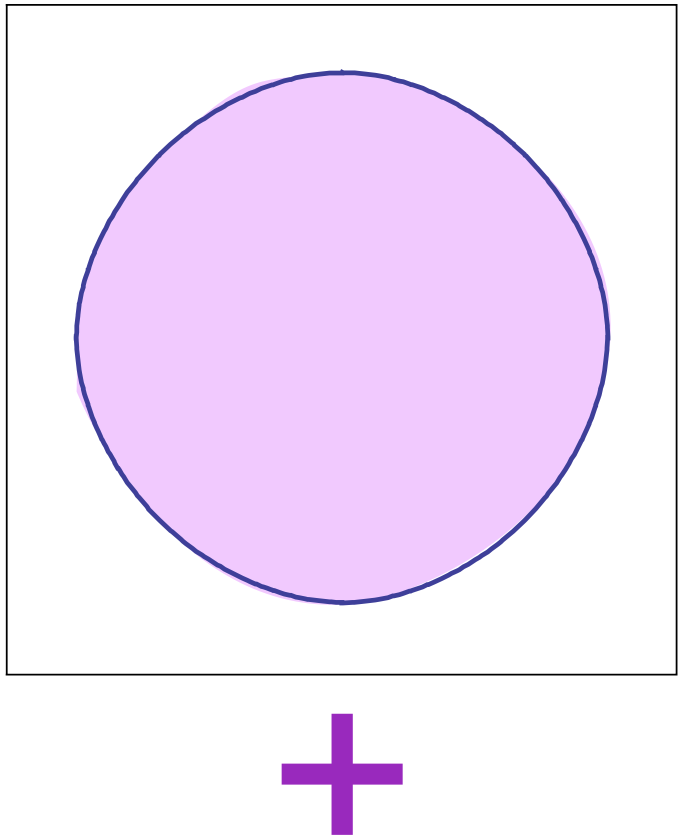
μ
Electron metal
Insulating antiferromagnet
s
Free CFT3
Interacting CFT3
Hole metal
$\varepsilon_{\mathbf{k}}$
$\mathbf{k}$

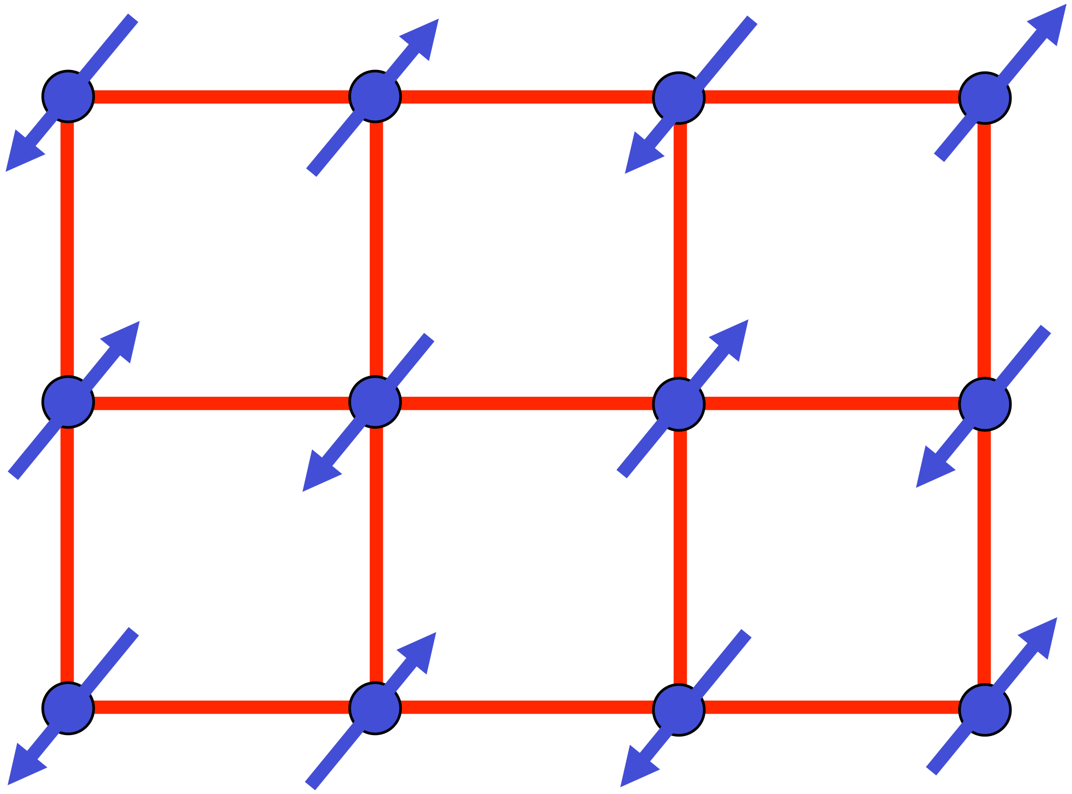


μ
AF metal
Electron metal
Insulating antiferromagnet
s
Free CFT3
Interacting CFT3
$\varepsilon_{\mathbf{k}}$
$\mathbf{k}$
Hole metal
AF metal

Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface



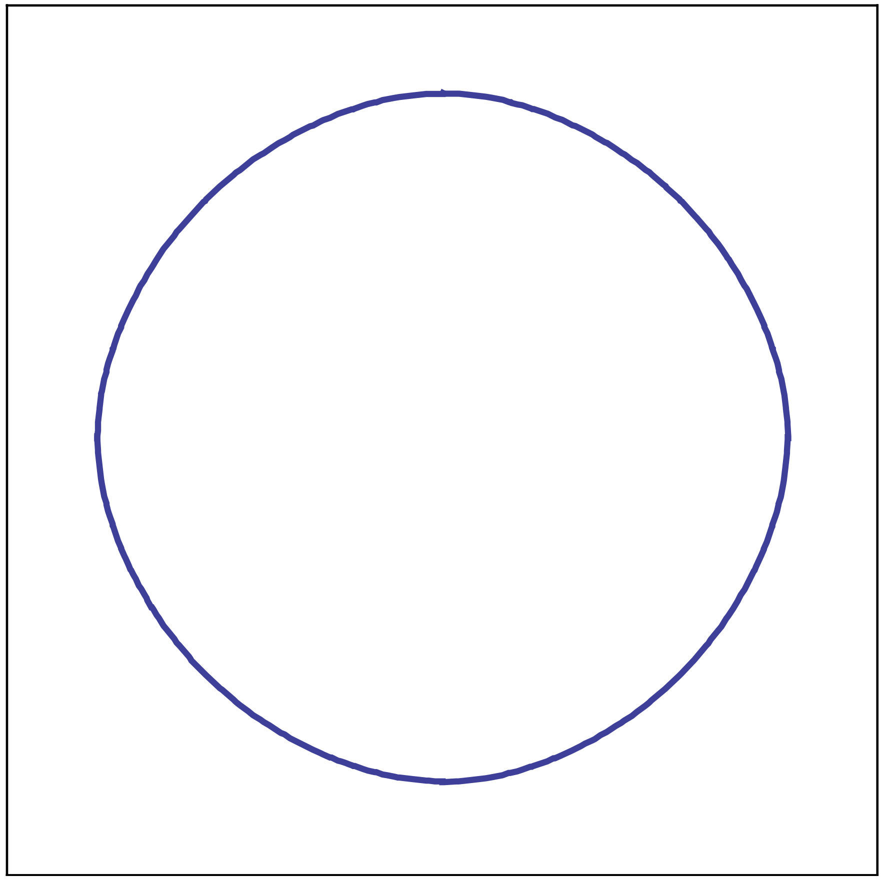


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

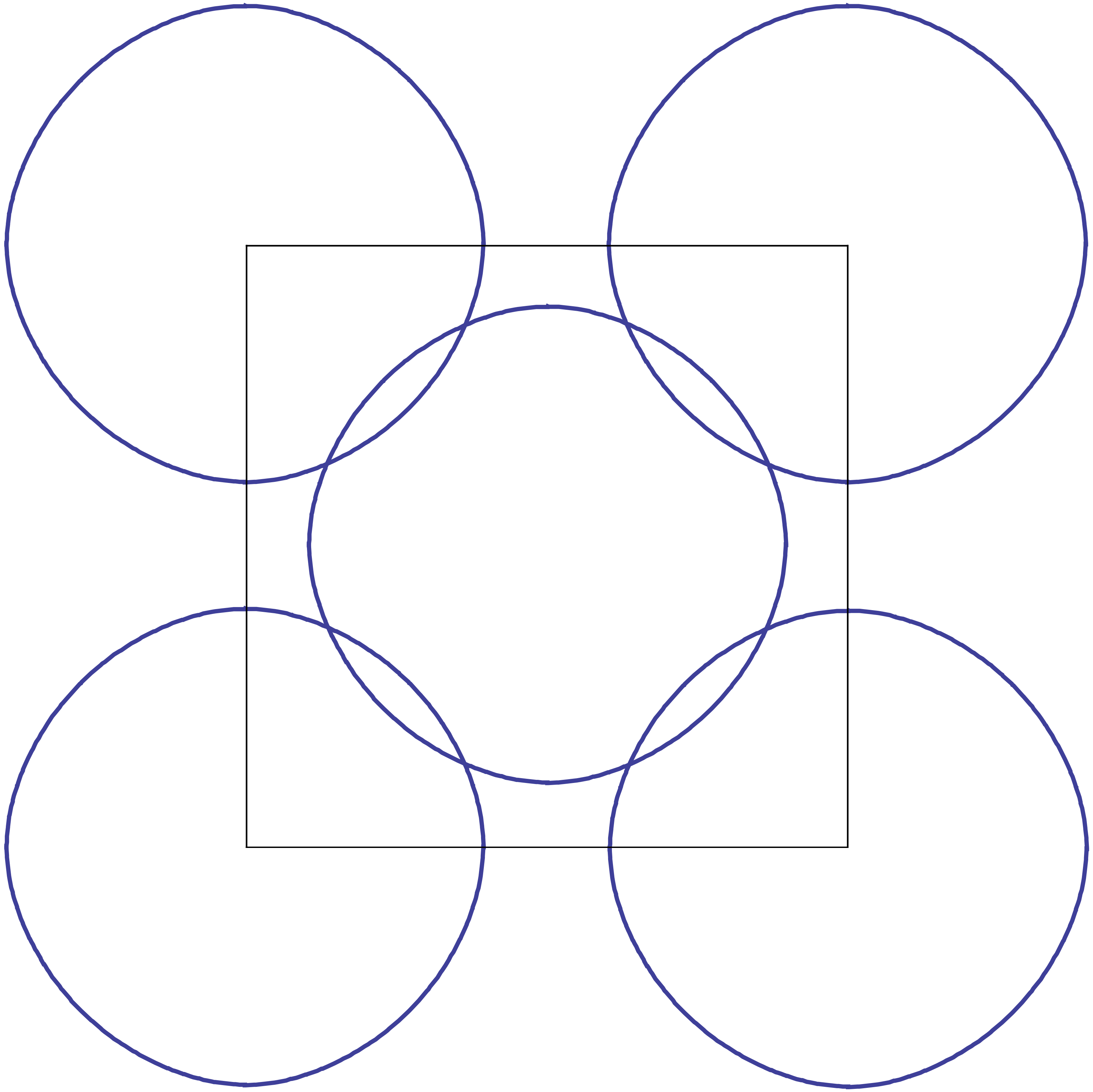
where $\mathbf{K} = (\pi, \pi)$ is the ordering wavevector.

Fermi surface+antiferromagnetism



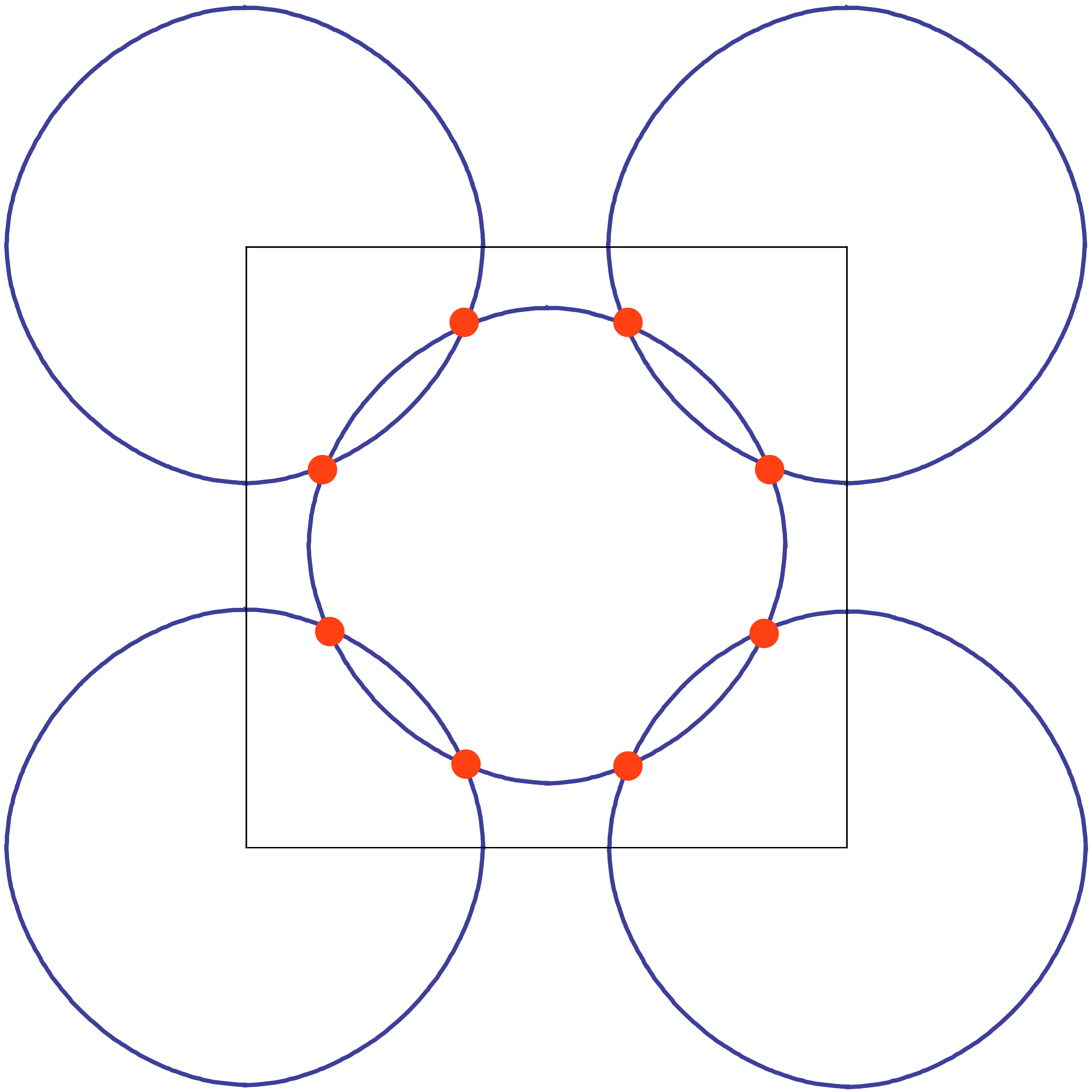
Metal with "large" Fermi surface

Fermi surface+antiferromagnetism



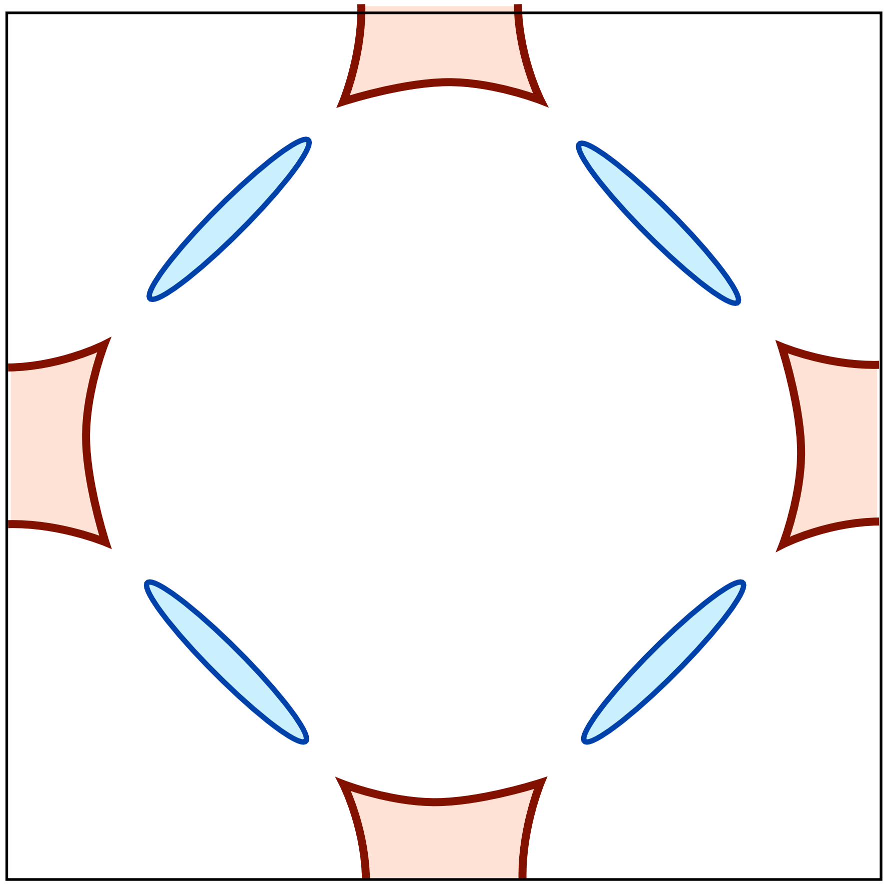
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



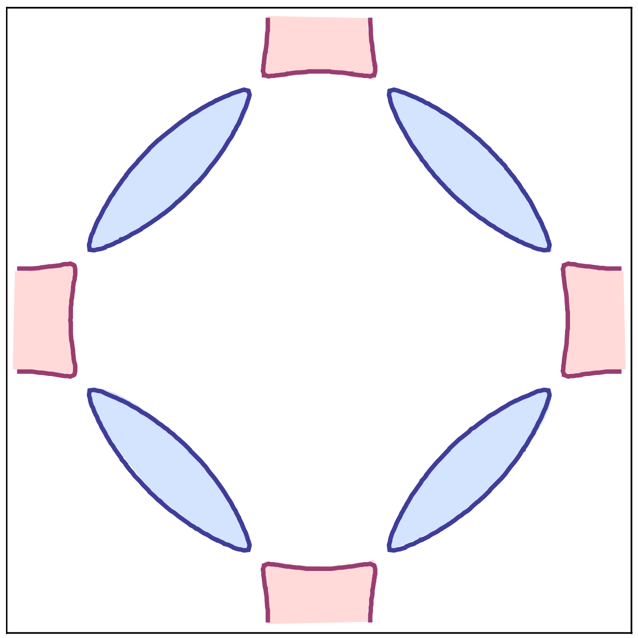
“Hot” spots

Fermi surface+antiferromagnetism



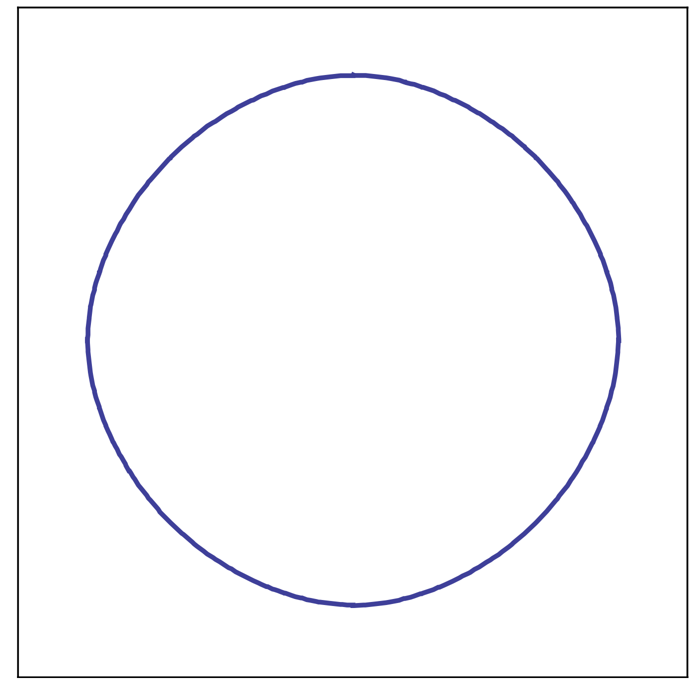
Electron and hole pockets in
antiferromagnetic phase
with antiferromagnetic order parameter $\langle \vec{\varphi} \rangle \neq 0$

Quantum phase transition with onset of antiferromagnetism in a metal



$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets

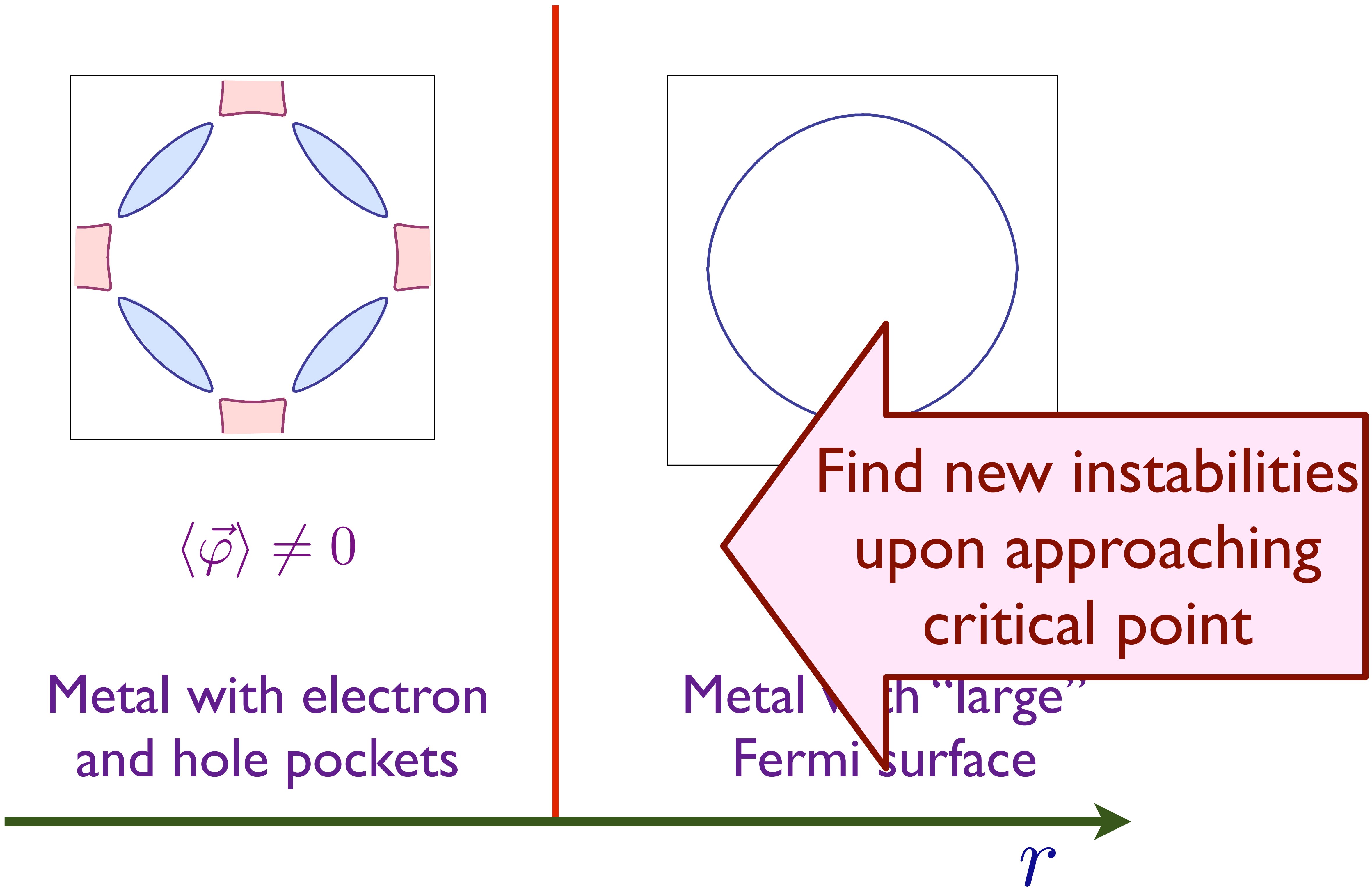


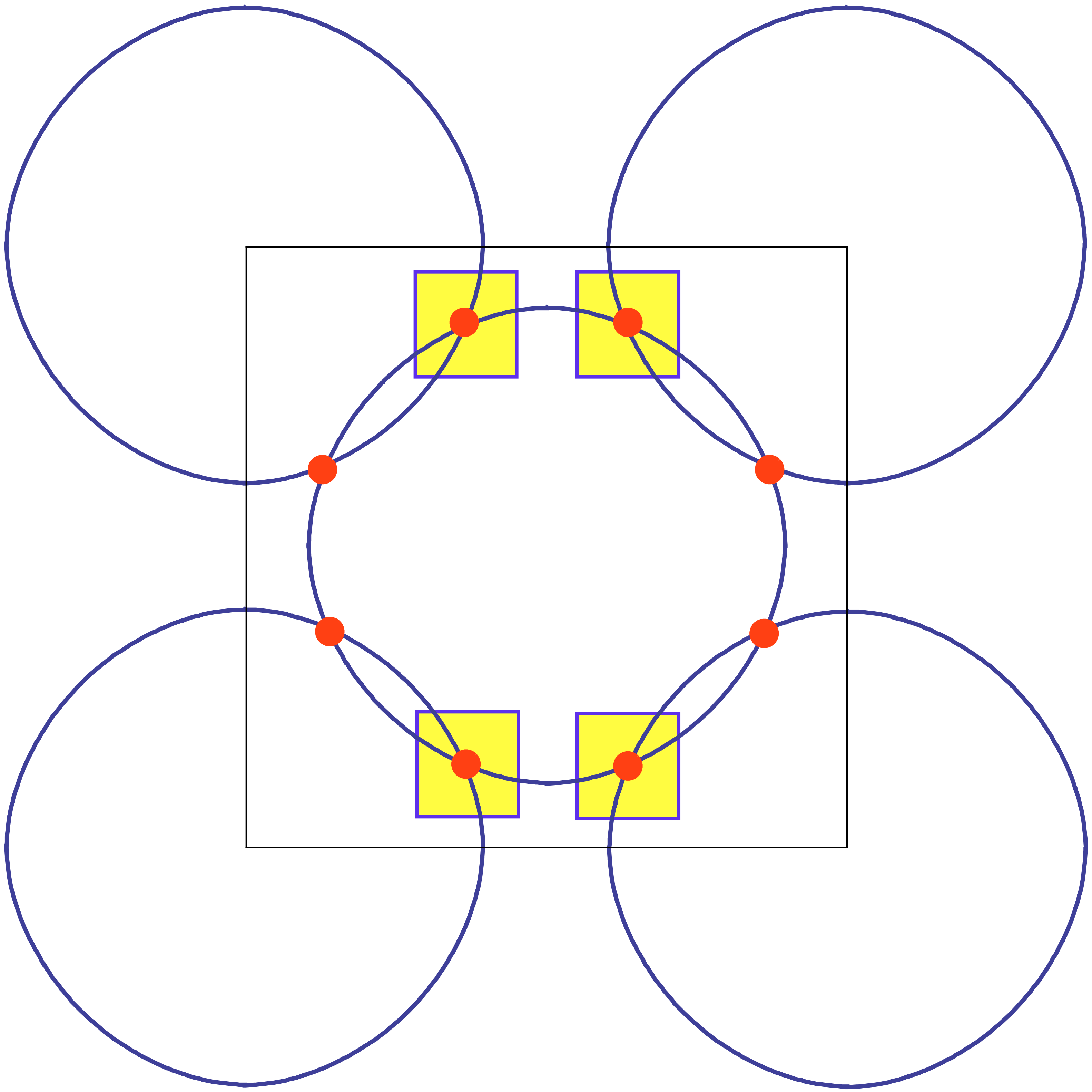
$\langle \vec{\varphi} \rangle = 0$

Metal with "large" Fermi surface

r

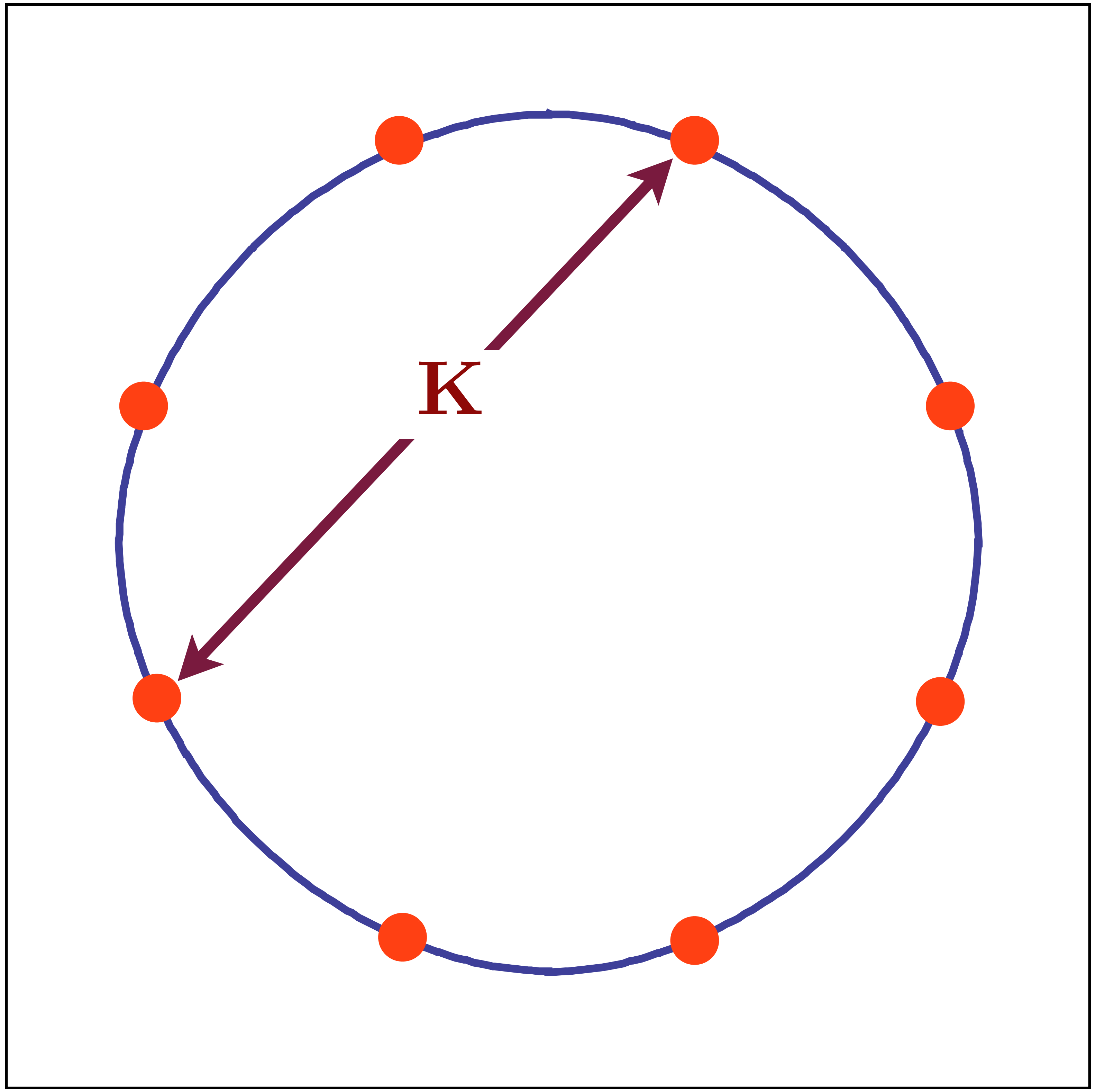
Quantum phase transition with onset of antiferromagnetism in a metal





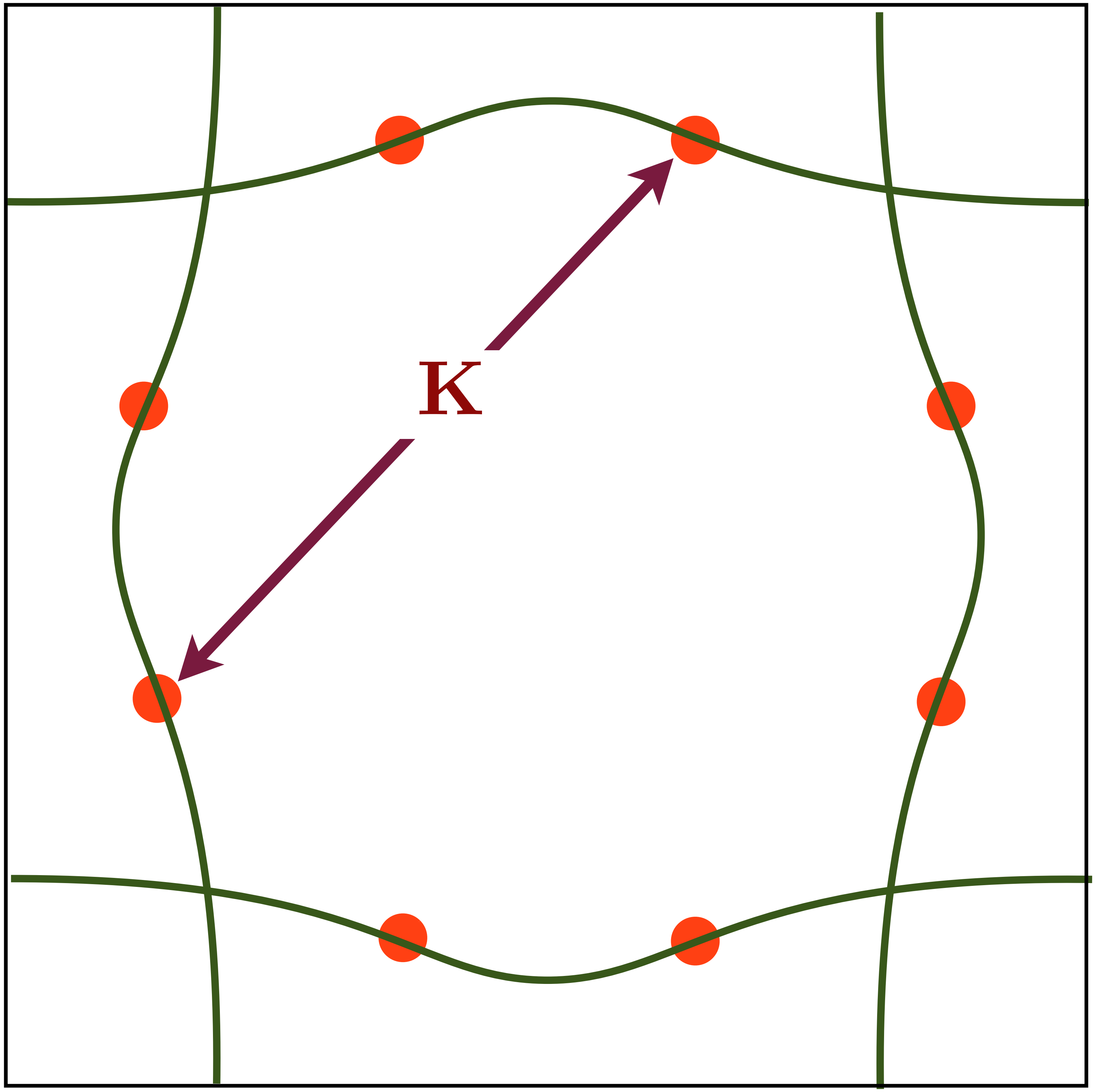
Low energy theory for critical point near hot spots

QMC for the onset of antiferromagnetism



Hot spots in a single band model

QMC for the onset of antiferromagnetism

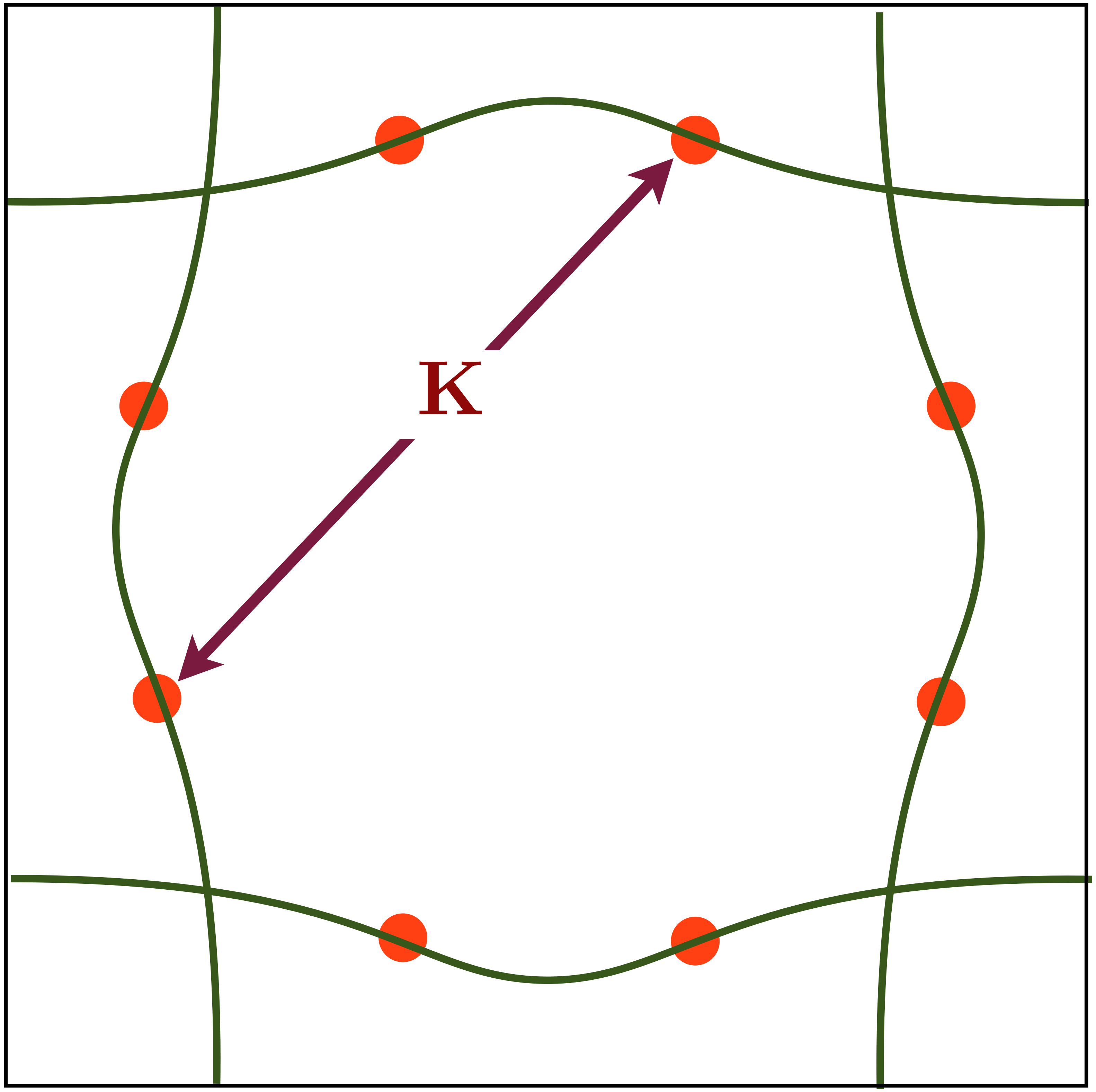


Hot spots in a two band model

E. Berg,
M. Metlitski, and
S. Sachdev,
Science **338**, 1606
(2012).

QMC for the onset of antiferromagnetism

Faithful realization of the *generic* universal low energy theory for the onset of antiferro-magnetism.



Hot spots in a two band model

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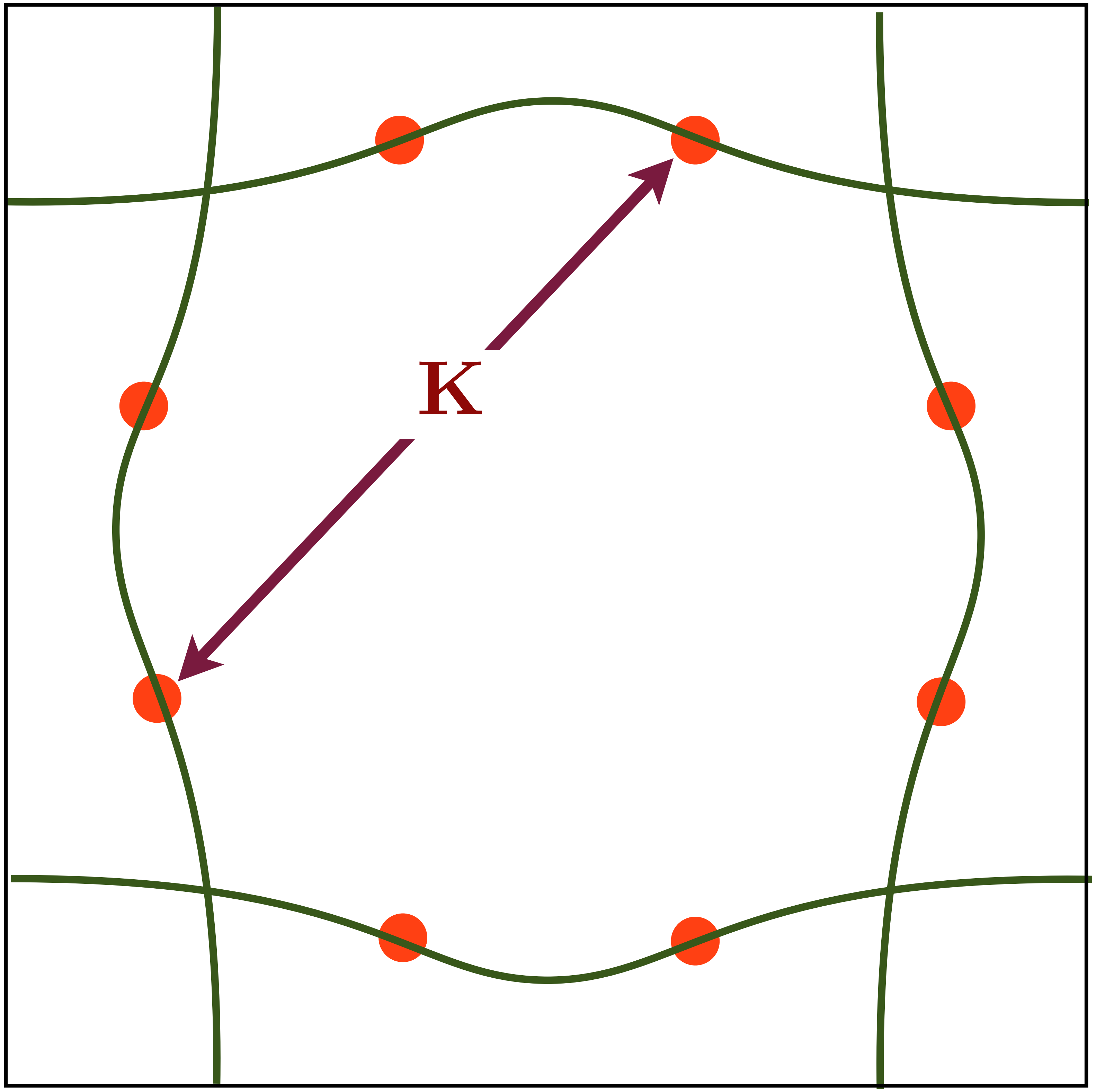
QMC for the onset of antiferromagnetism

Sign problem is absent as long as *K* connects hotspots in distinct bands



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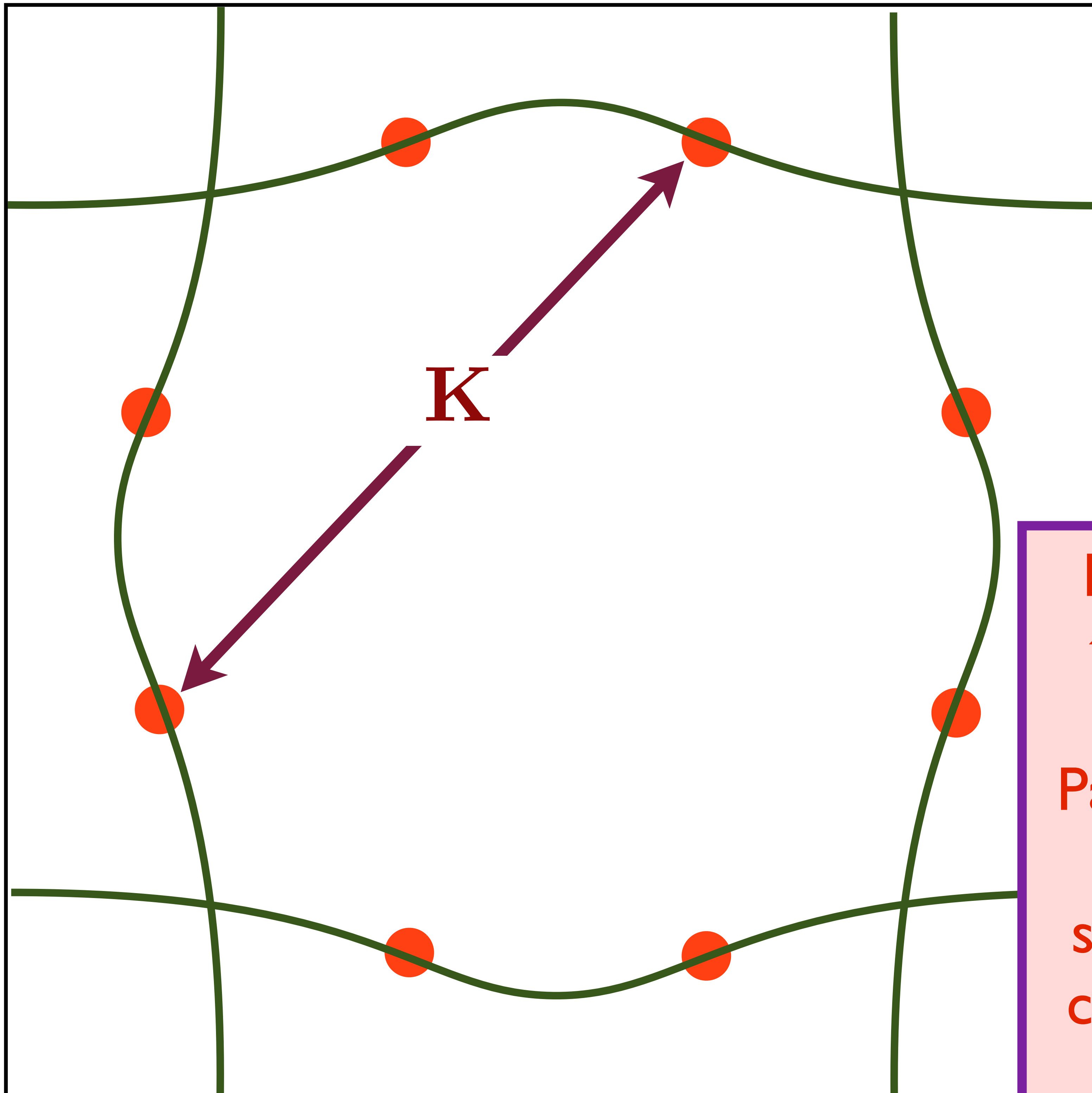


Hot spots in a two band model

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Requires only time-reversal symmetry. Particle-hole or point-group symmetries or commensurate densities ***not*** required !

QMC for the onset of antiferromagnetism

Electrons with dispersion $\varepsilon_{\mathbf{k}}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$+ \int d\tau d^2 x \left[\frac{1}{2} \left(\boldsymbol{\nabla}_x \vec{\varphi} \right)^2 + \frac{r}{2} \vec{\varphi}^2 + \ldots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

QMC for the onset of antiferromagnetism

Electrons with dispersions $\varepsilon_{\mathbf{k}}^{(x)}$ and $\varepsilon_{\mathbf{k}}^{(y)}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_{\alpha}^{(x)}\mathcal{D}c_{\alpha}^{(y)}\mathcal{D}\vec{\varphi}\exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(x)\dagger}\left(\frac{\partial}{\partial\tau} - \varepsilon_{\mathbf{k}}^{(x)}\right) c_{\mathbf{k}\alpha}^{(x)}$$

$$+ \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(y)\dagger}\left(\frac{\partial}{\partial\tau} - \varepsilon_{\mathbf{k}}^{(y)}\right) c_{\mathbf{k}\alpha}^{(y)}$$

$$+ \int d\tau d^2x \left[\frac{1}{2}\left(\boldsymbol{\nabla}_x\vec{\varphi}\right)^2 + \frac{r}{2}\vec{\varphi}^2 + \ldots\right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c_{i\alpha}^{(x)\dagger}\vec{\sigma}_{\alpha\beta}c_{i\beta}^{(y)} + \text{H.c.}$$

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

No sign problem !

QMC for the onset of antiferromagnetism

Electrons with dispersions $\varepsilon_{\mathbf{k}}^{(x)}$ and $\varepsilon_{\mathbf{k}}^{(y)}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha^{(x)} \mathcal{D}c_\alpha^{(y)} \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(x)\dagger} \left(\frac{\partial}{\partial\tau} - \varepsilon_{\mathbf{k}}^{(x)}\right) c_{\mathbf{k}\alpha}^{(x)}$$

$$+ \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(y)\dagger} \left(\frac{\partial}{\partial\tau} - \varepsilon_{\mathbf{k}}^{(y)}\right) c_{\mathbf{k}\alpha}^{(y)}$$

$$+ \int d\tau d^2x \left[\frac{1}{2}\left(\boldsymbol{\nabla}_x \vec{\varphi}\right)^2 + \frac{r}{2}\vec{\varphi}^2 + \ldots\right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c_{i\alpha}^{(x)\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}^{(y)} + \text{H.c.}$$

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Applies without changes to the microscopic band structure in the iron-based superconductors

QMC for the onset of antiferromagnetism

Electrons with dispersions $\varepsilon_{\mathbf{k}}^{(x)}$ and $\varepsilon_{\mathbf{k}}^{(y)}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha^{(x)} \mathcal{D}c_\alpha^{(y)} \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(x)\dagger} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}}^{(x)} \right) c_{\mathbf{k}\alpha}^{(x)}$$

$$+ \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(y)\dagger} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}}^{(y)} \right) c_{\mathbf{k}\alpha}^{(y)}$$

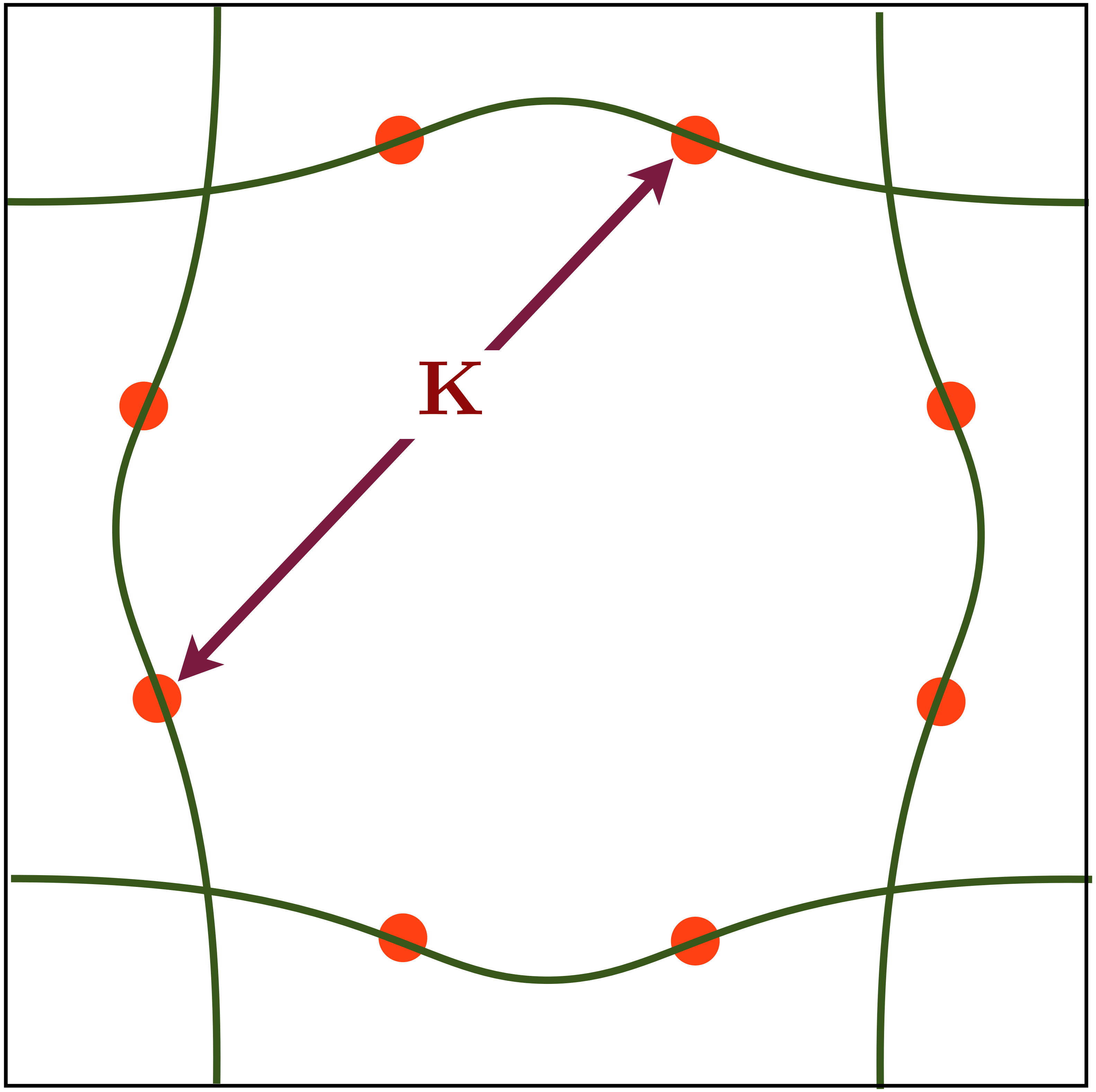
$$+ \int d\tau d^2 x \left[\frac{1}{2} \left(\boldsymbol{\nabla}_x \vec{\varphi} \right)^2 + \frac{r}{2} \vec{\varphi}^2 + \ldots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c_{i\alpha}^{(x)\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}^{(y)} + \text{H.c.}$$

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Can integrate out $\vec{\varphi}$ to obtain an extended Hubbard model. The interactions in this model only couple electrons in separate bands.

QMC for the onset of antiferromagnetism



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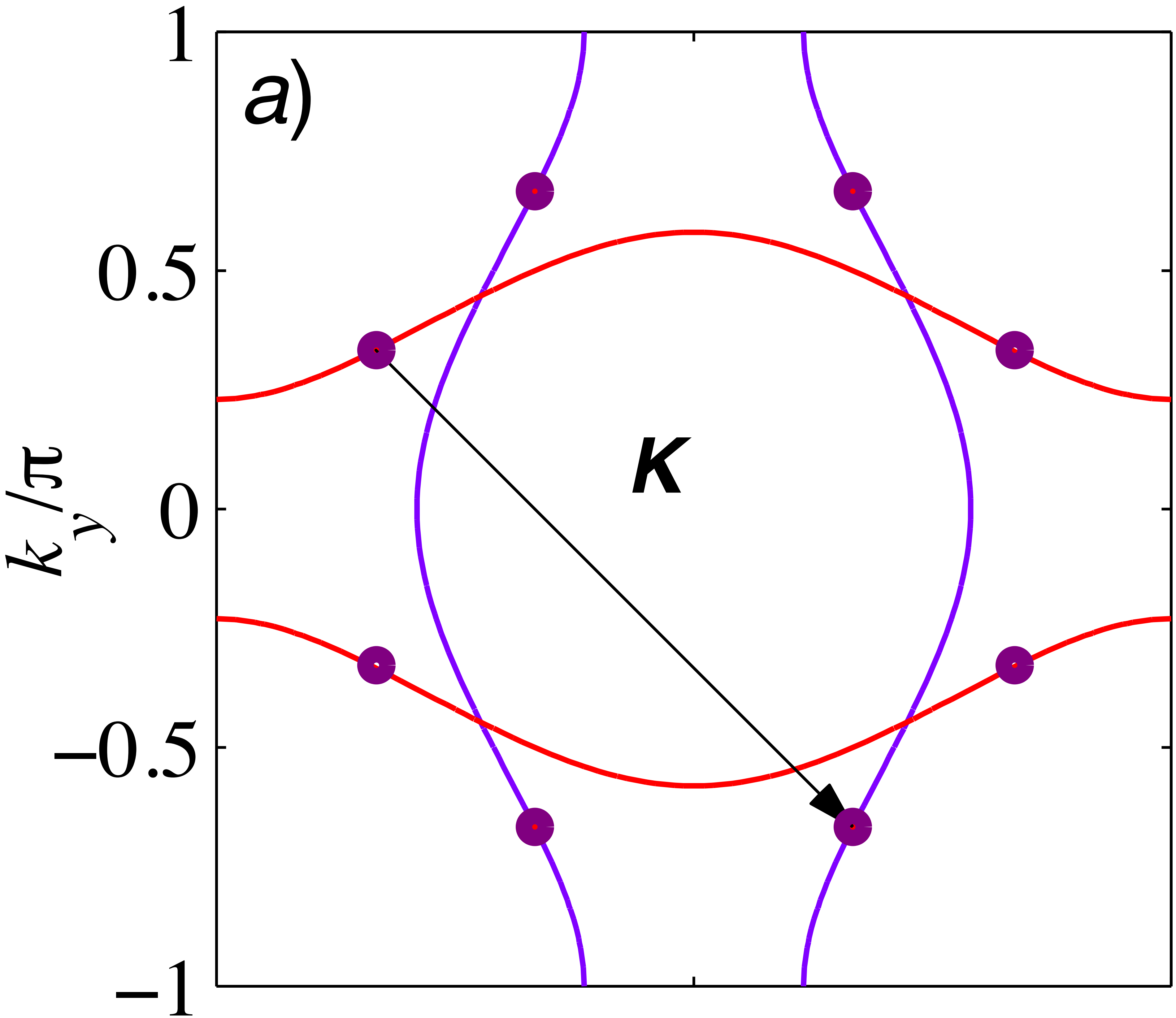
Hot spots in a two band model

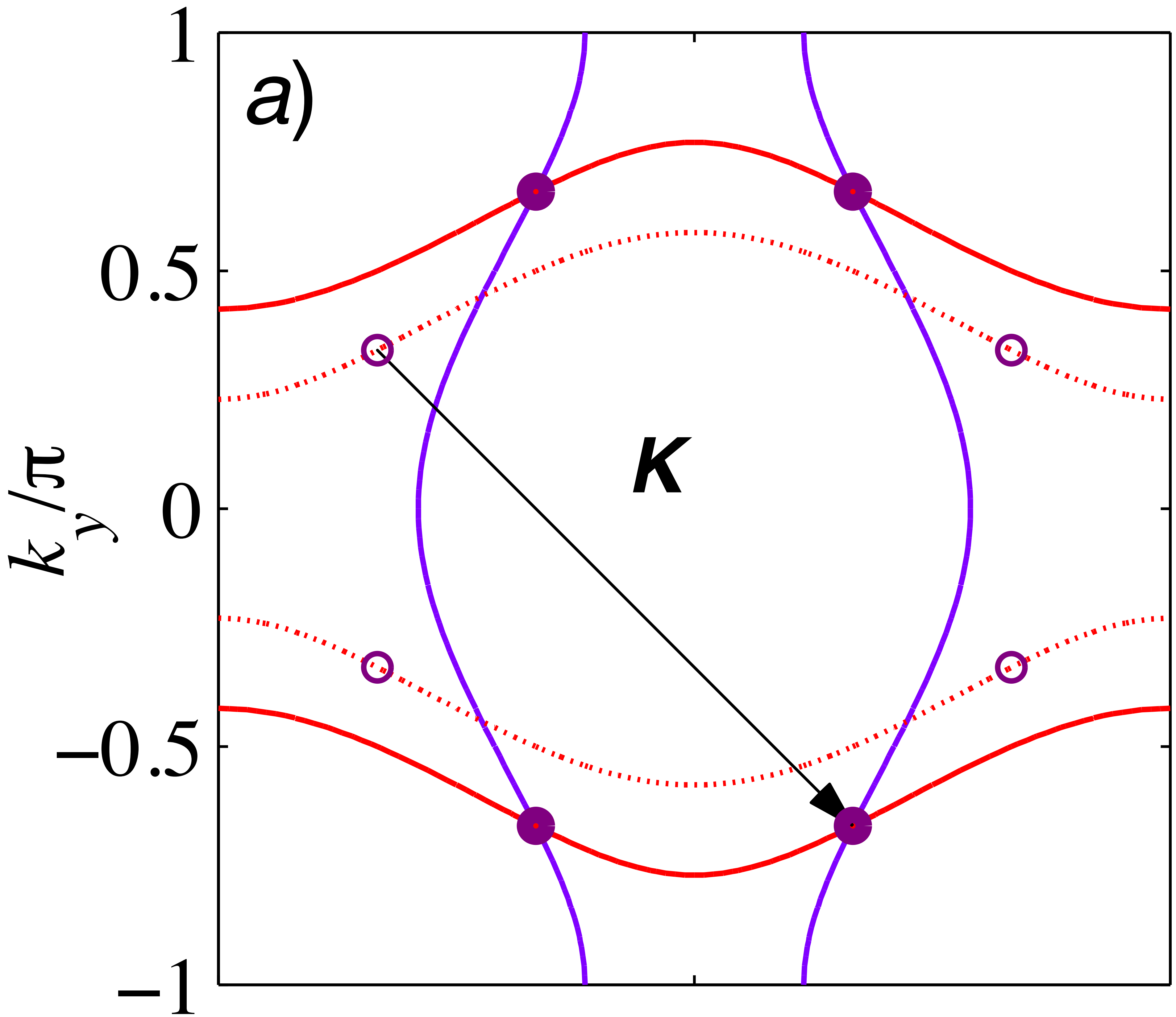
QMC for the onset of antiferromagnetism



Center Brillouin zone at (π,π,)

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

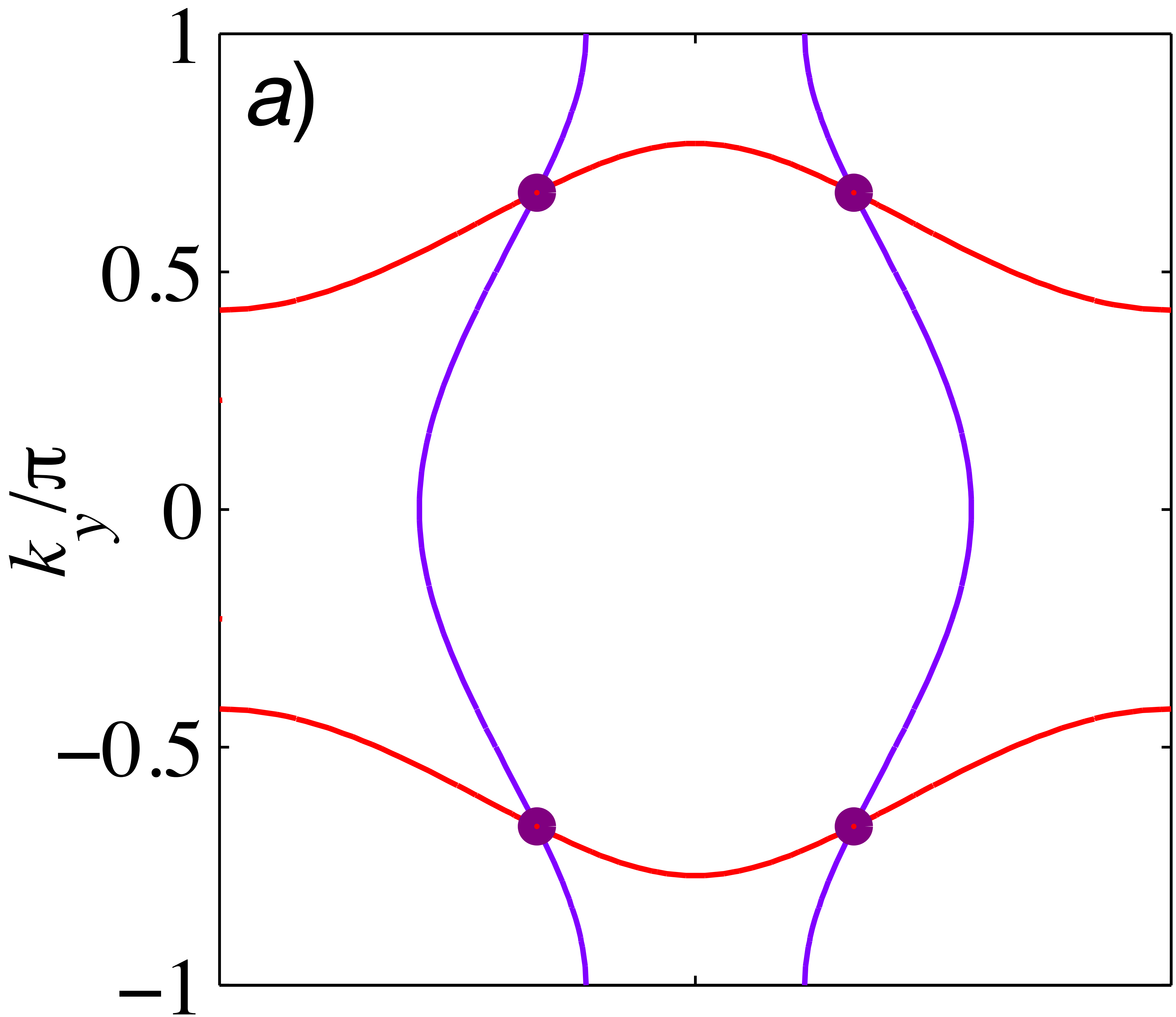
QMC for the onset of antiferromagnetism



E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Move one of the Fermi surface by (π,π,)

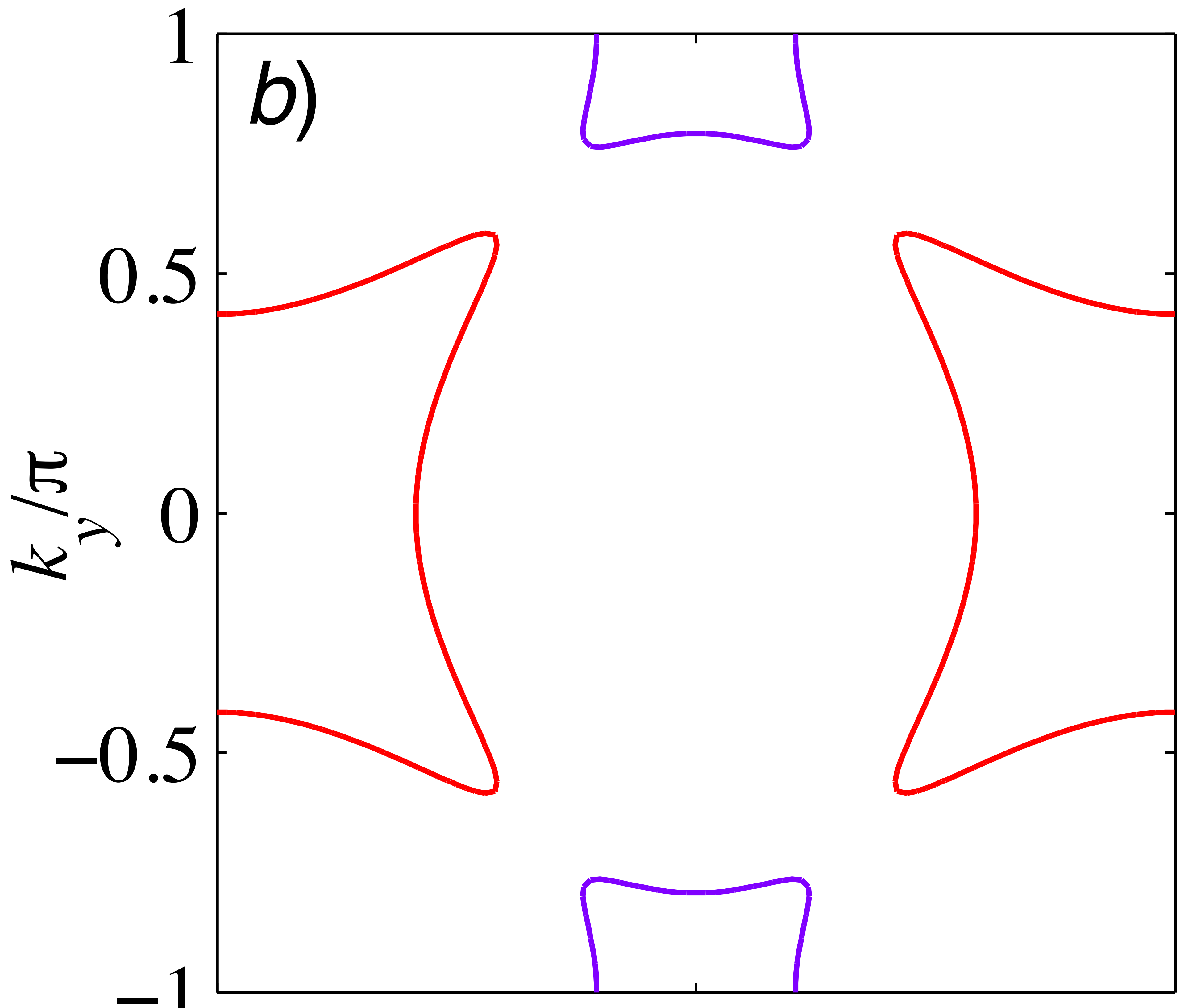
QMC for the onset of antiferromagnetism



E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Now hot spots are at Fermi surface intersections

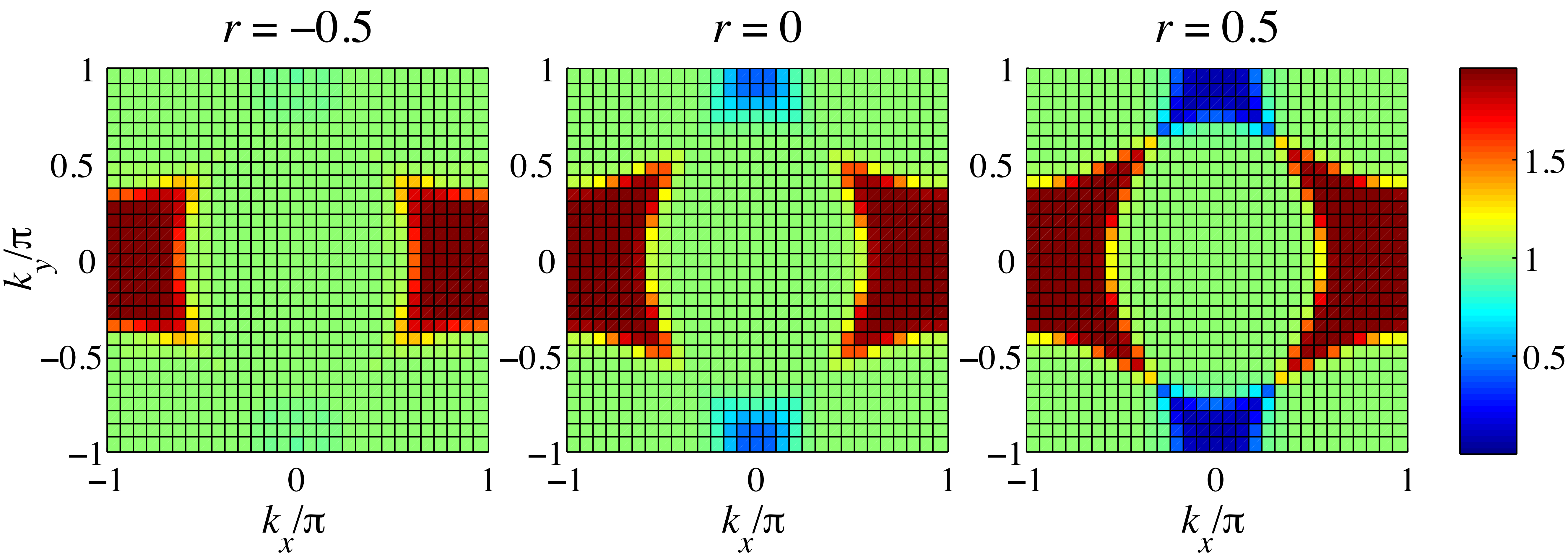
QMC for the onset of antiferromagnetism



E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Expected Fermi surfaces in the AFM ordered phase

QMC for the onset of antiferromagnetism



Electron occupation number $n_{\mathbf{k}}$
as a function of the tuning parameter r

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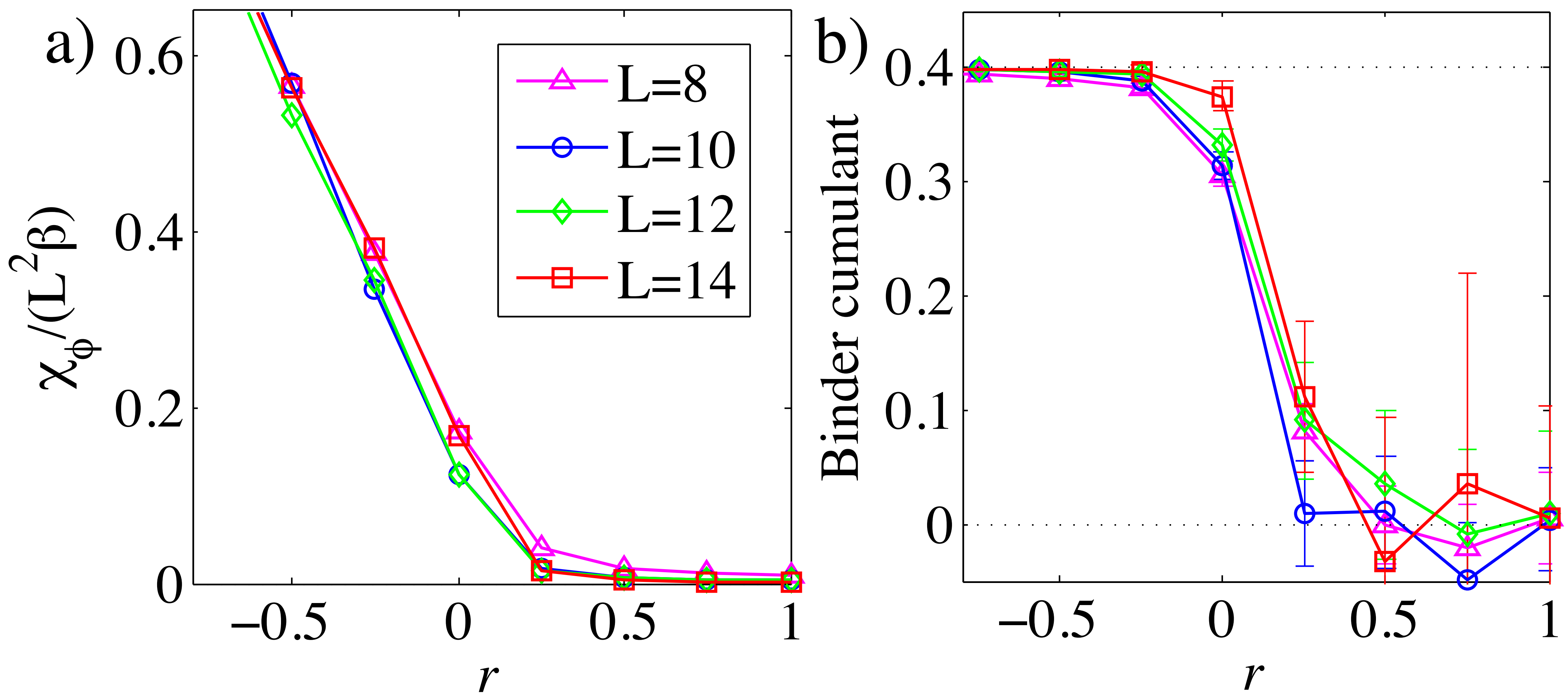
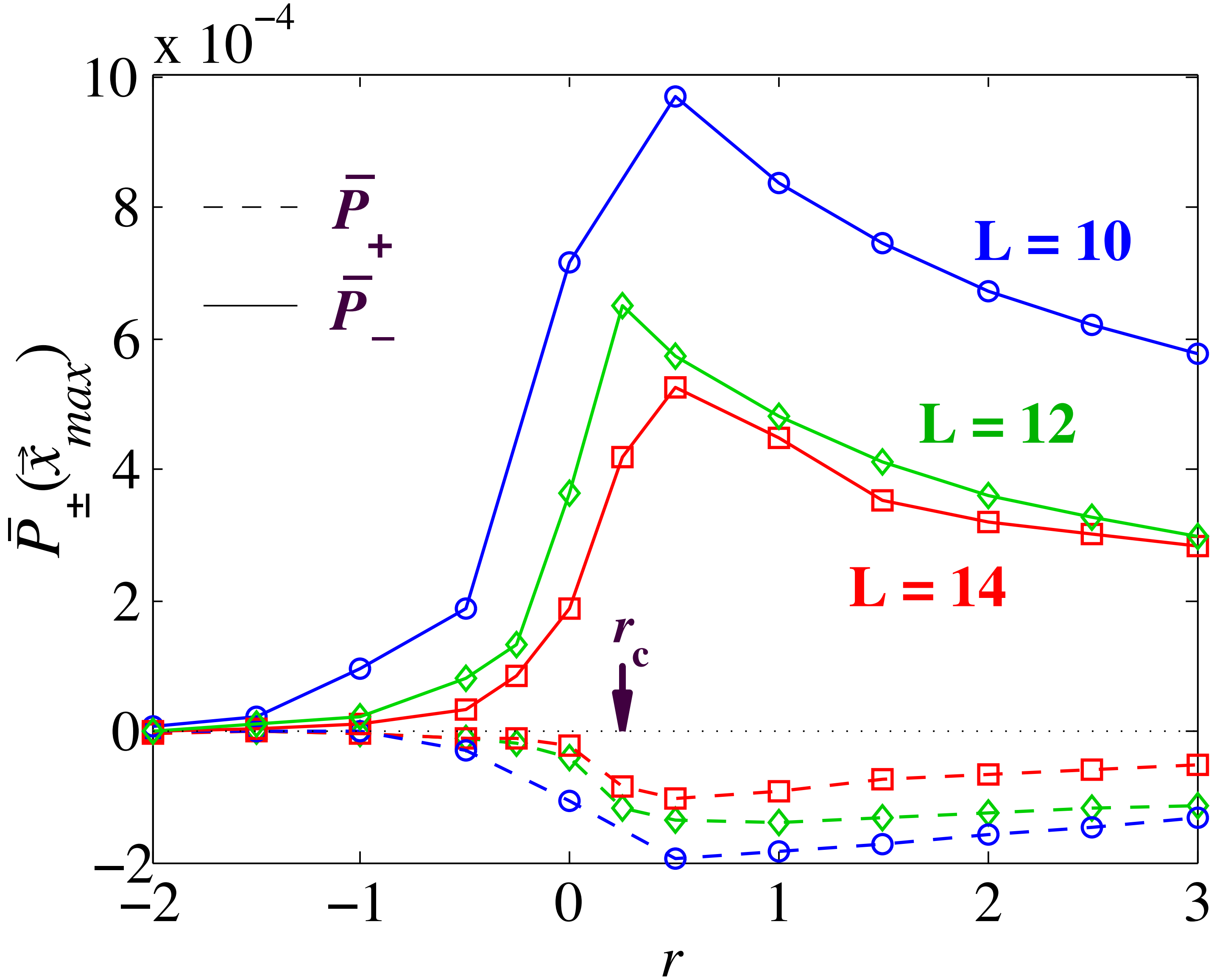
QMC for the onset of antiferromagnetism



AF susceptibility, χ_φ, and Binder cumulant
as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

QMC for the onset of antiferromagnetism



s/d pairing amplitudes P_+/P_-

as a function of the tuning parameter r

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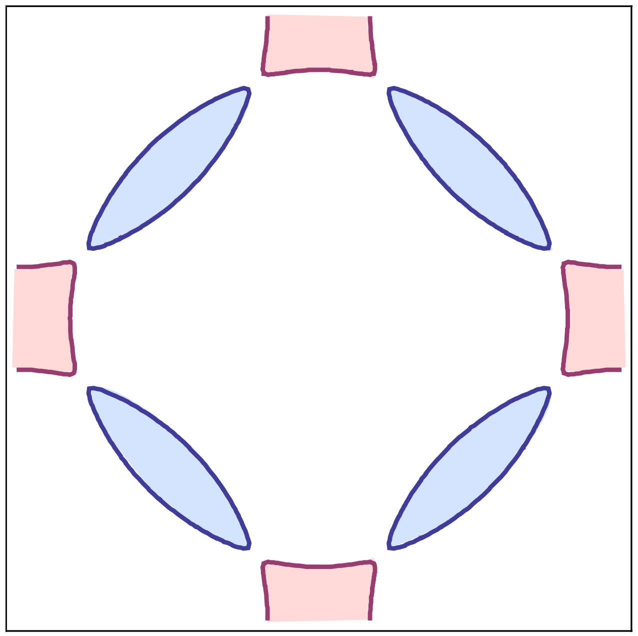
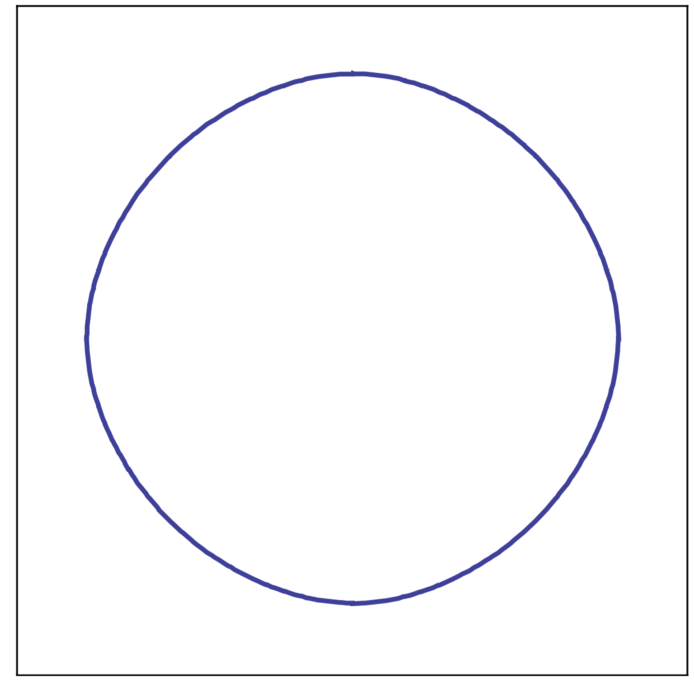
Quantum phase transition with onset of antiferromagnetism in a metal





$\langle \vec{\varphi} \rangle \neq 0$

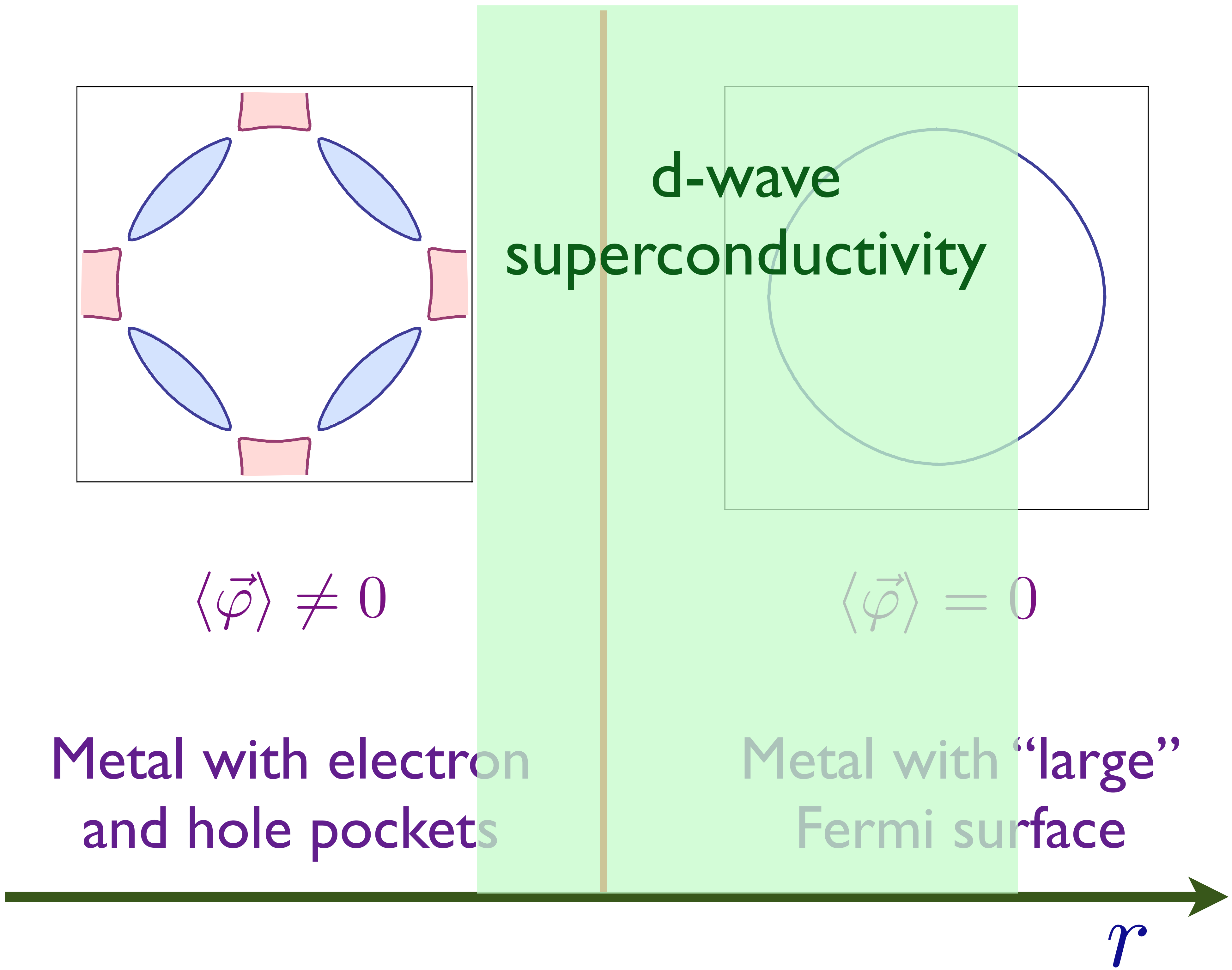
$\langle \vec{\varphi} \rangle = 0$

Metal with electron and hole pockets

Metal with "large" Fermi surface

r

Quantum phase transition with onset of antiferromagnetism in a metal



Outline

1. Onset of antiferromagnetism in semi-metals

2. Onset of antiferromagnetism in metals

   Sign-problem-free Quantum Monte Carlo: evidence for d-wave superconductivity

3. A competing order

   An unconventional charge density wave

4. STM observation

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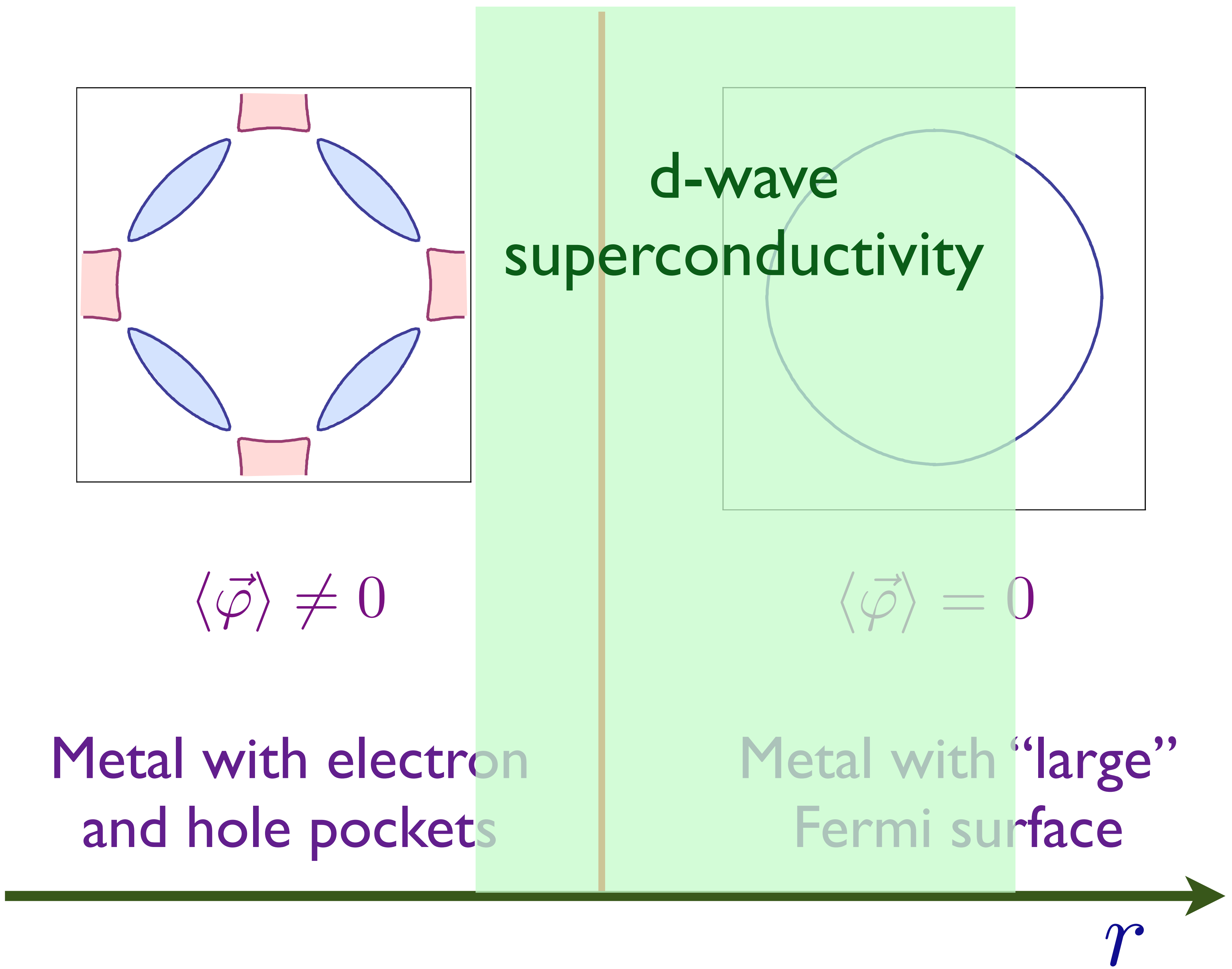
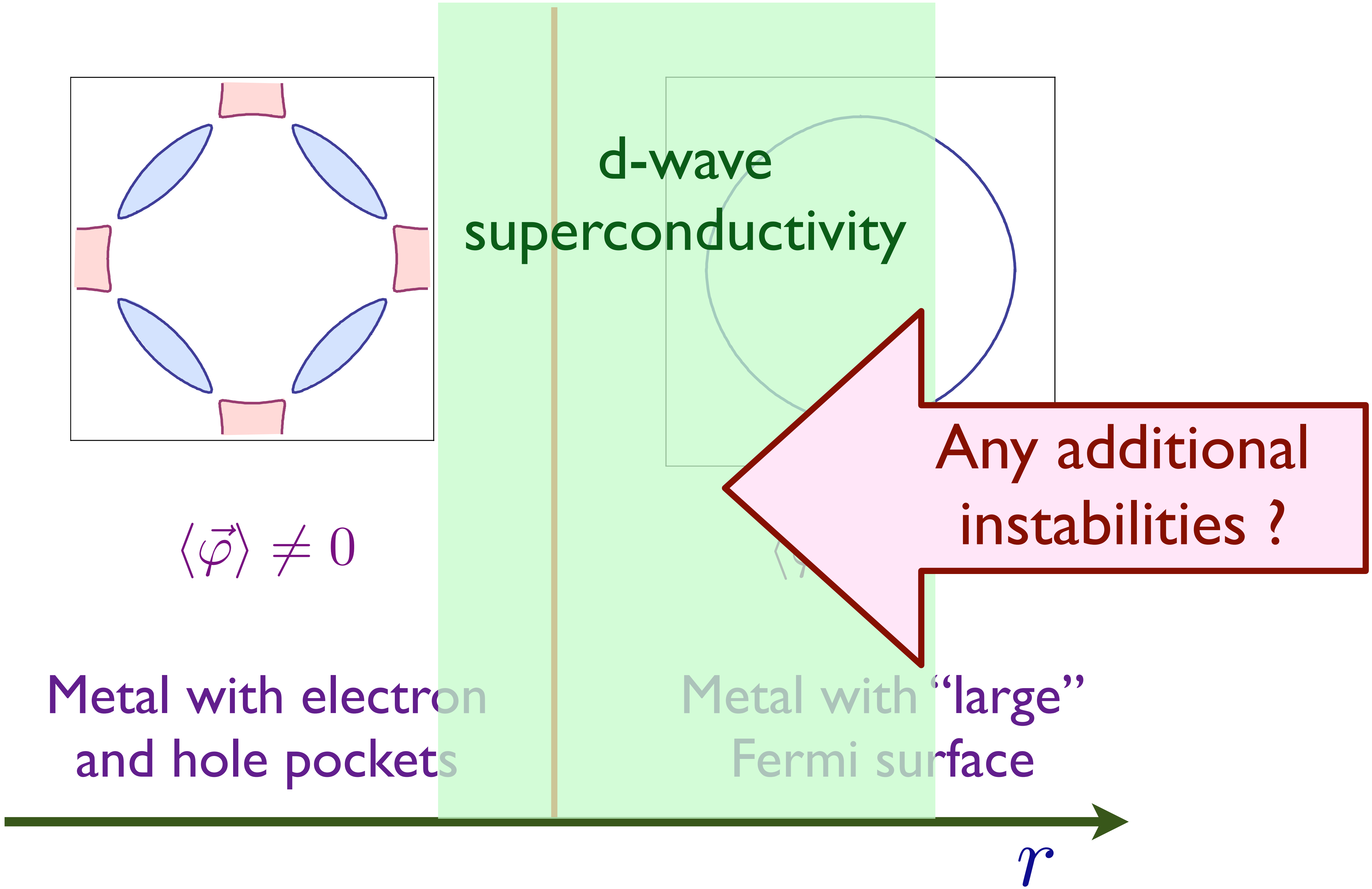
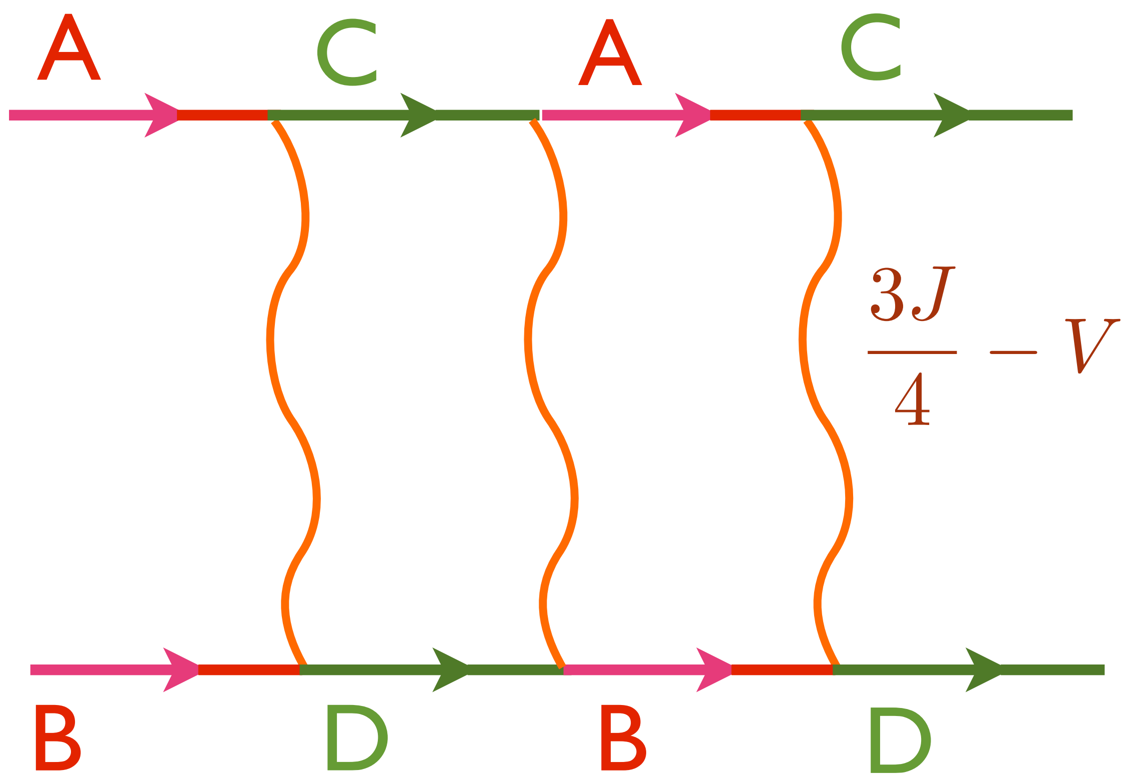


Quantum phase transition with onset of antiferromagnetism in a metal



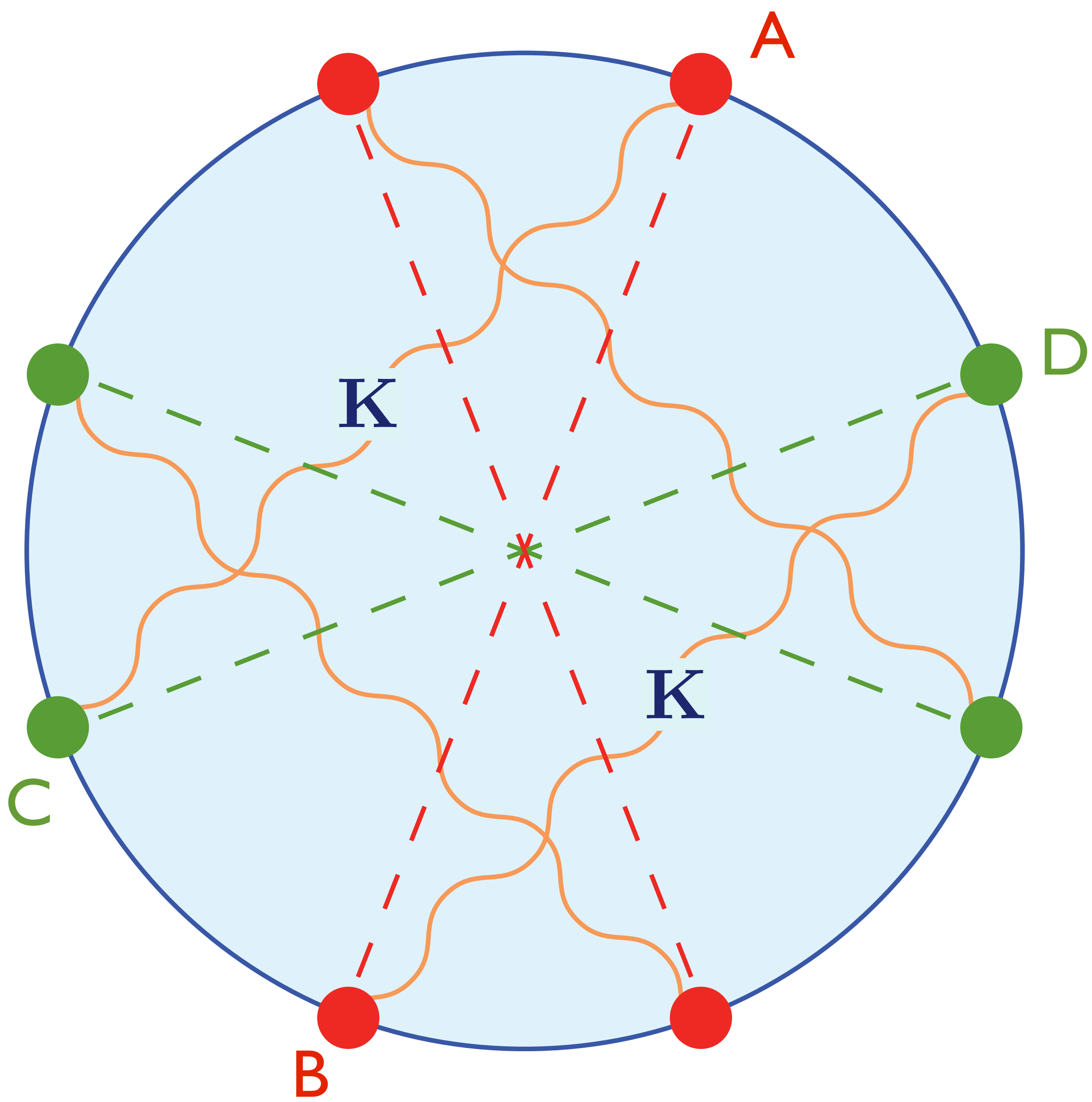
Pairing "glue" from antiferromagnetic fluctuations

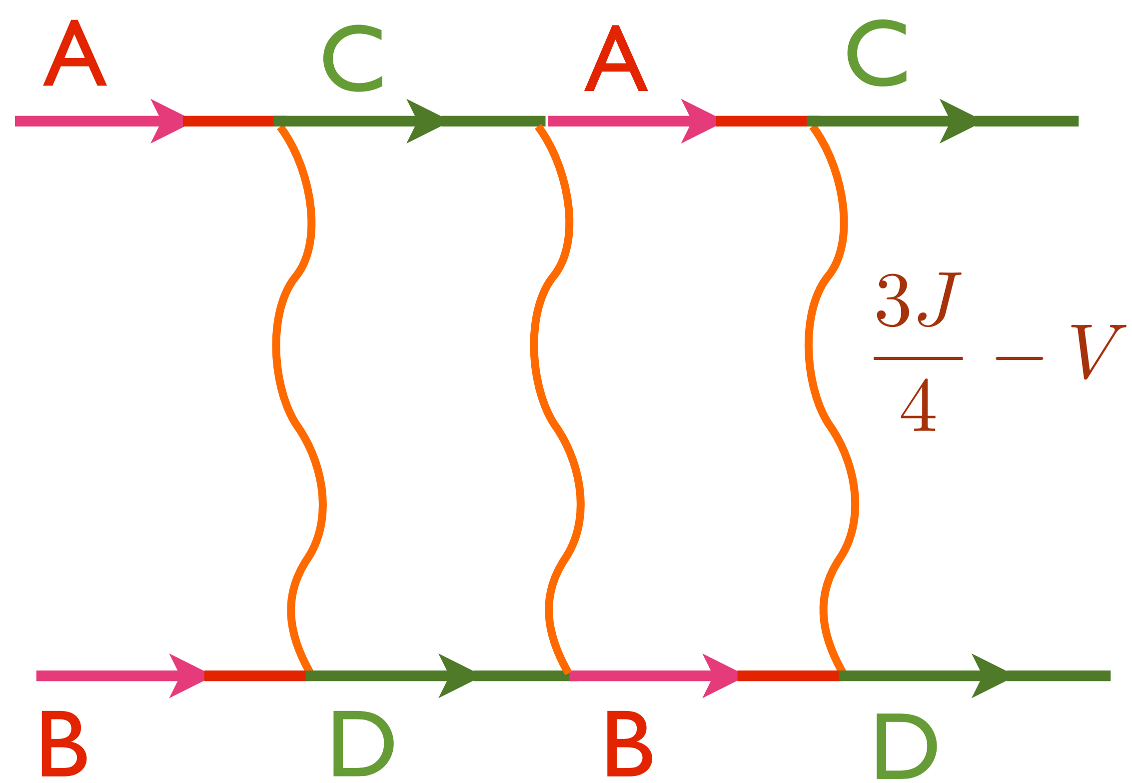
$$H = -\sum_{i,j} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + V \sum_{\langle ij \rangle} n_i n_j + \ldots$$



V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)

Pairing "glue" from antiferromagnetic fluctuations





Pairing amplitude of AB =
- Pairing amplitude of CD

V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
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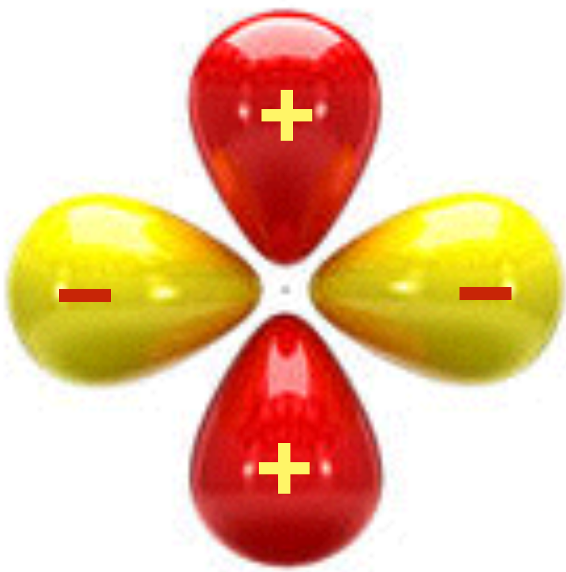
Superconductivity:
Bose condensation of Cooper pairs of electrons

$$\varepsilon^{\alpha\beta}\left\langle c_\alpha^\dagger(\mathbf{r}_1)c_\beta^\dagger(\mathbf{r}_2)\right\rangle = \Big[P(\mathbf{r}_1-\mathbf{r}_2)\Big] \times \Psi_{SC}\left(\frac{\mathbf{r}_1+\mathbf{r}_2}{2}\right)$$



Internal Cooper-pair wavefunction has *d*-wave form in cuprates: ***Unconventional*** superconductivity

$$\alpha,\beta=\uparrow,\downarrow \quad ; \quad \varepsilon^{\uparrow\downarrow}=-\varepsilon^{\downarrow\uparrow}=1\,;\,\varepsilon^{\uparrow\uparrow}=\varepsilon^{\downarrow\downarrow}=0$$

Pseudospin symmetry of the exchange interaction

$$H_J = J\,\vec{S}_1 \cdot \vec{S}_2$$

with $\vec{S}_i = \frac{1}{2} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c^\dagger_{i\downarrow} \end{pmatrix} \quad , \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c^\dagger_{i\uparrow} \end{pmatrix}$$

Then we can write

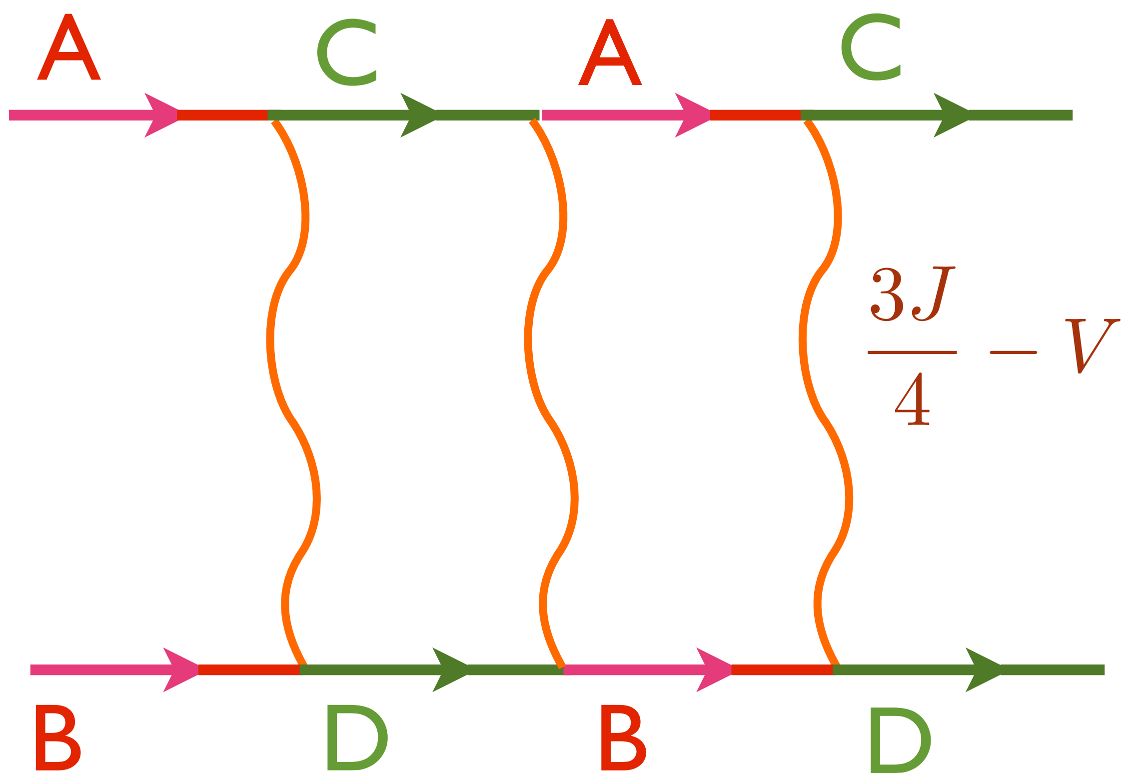
$$H_J = \frac{1}{8} J \left(\Psi^\dagger_{1\alpha a} \vec{\sigma}_{\alpha\beta} \Psi_{1\beta a} \right) \cdot \left(\Psi^\dagger_{2\gamma b} \vec{\sigma}_{\gamma\delta} \Psi_{2\delta b} \right)$$

where a, b are the Nambu indices. This form makes explicit the symmetry under *independent* SU(2) pseudospin transformations on each site $\Psi_{i\alpha a} \rightarrow U_{i,ab} \Psi_{i\alpha b}$

I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988)
E. Dagotto, E. Fradkin, and A. Moreo, Phys. Rev. B **38**, 2926 (1988)
P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006)

Pairing "glue" from antiferromagnetic fluctuations

$$H = -\sum_{i,j} t_{ij} c^{\dagger}_{i\alpha} c_{j\alpha} + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + V \sum_{\langle ij \rangle} n_i n_j + \ldots$$



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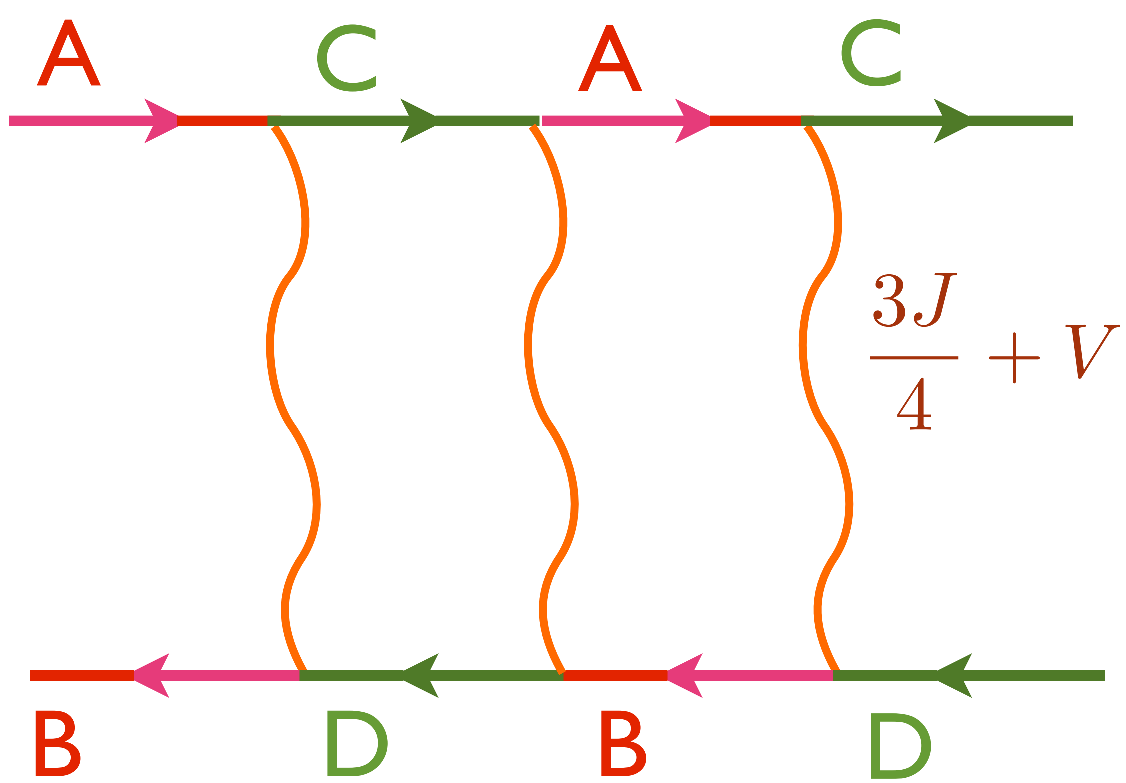
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)

Same "glue" leads to *d*-wave particle-hole pairing !

$$H = -\sum_{i,j} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + V \sum_{\langle ij \rangle} n_i n_j + \ldots$$



M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)
S. Sachdev and R. LaPlaca *Phys. Rev. Lett.* **111**, 027202 (2013)
J. C. Davis and Dung-Hai Lee, Proc. Natl. Acad. Sci. **110**, 17623 (2013)
J. D. Sau and S. Sachdev, *Phys. Rev. B* **89**, 075129 (2014)

Charge density wave (CDW) order

$$\langle c_\alpha^\dagger(\mathbf{r}) c_\alpha(\mathbf{r}) \rangle = \Psi_{CDW}(\mathbf{r})\, e^{i\mathbf{Q}\cdot\mathbf{r}} + \text{c.c.}$$

Unconventional charge density wave (CDW) : Bose condensation of particle-hole pairs

$$\left\langle c_\alpha^\dagger(\mathbf{r}_1) c_\alpha(\mathbf{r}_2) \right\rangle$$

$$= \Big[\mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2)\Big] \times \Psi_{CDW}\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}\right) e^{i\mathbf{Q}\cdot(\mathbf{r}_1+\mathbf{r}_2)/2} + \text{c.c.}$$

Internal particle-hole pair wavefunction

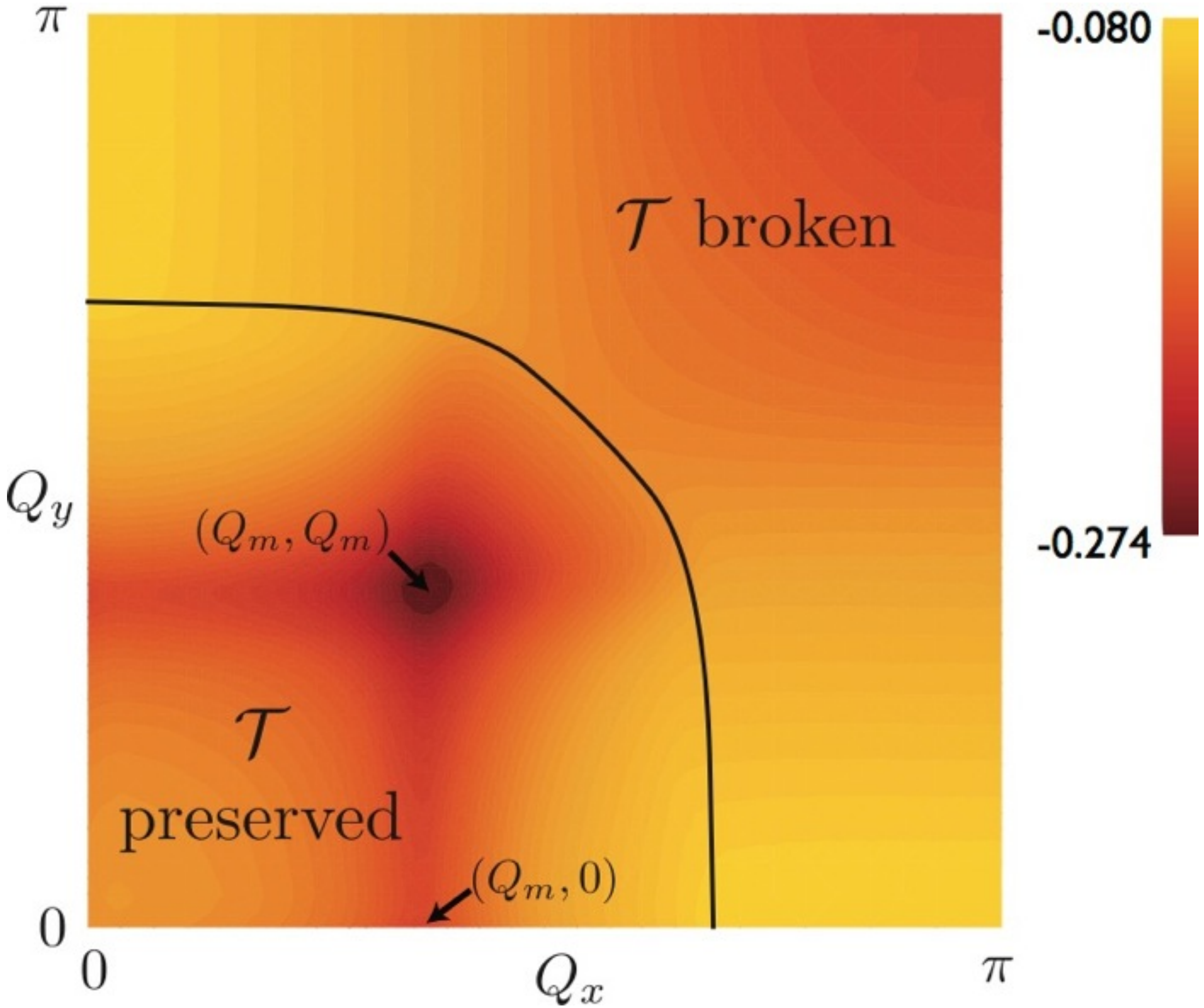
$$\mathcal{P}(\mathbf{r}) = \int \frac{d^2k}{4\pi^2} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}}$$

Time-reversal symmetry requires $\mathcal{P}(\mathbf{k}) = \mathcal{P}(-\mathbf{k})$.

We expand (using reflection symmetry for $\mathbf{Q}$ along axes or diagonals)

$$\mathcal{P}(\mathbf{k}) = \mathcal{P}_s + \mathcal{P}_{s'}(\cos k_x + \cos k_y) + \mathcal{P}_d(\cos k_x - \cos k_y)$$

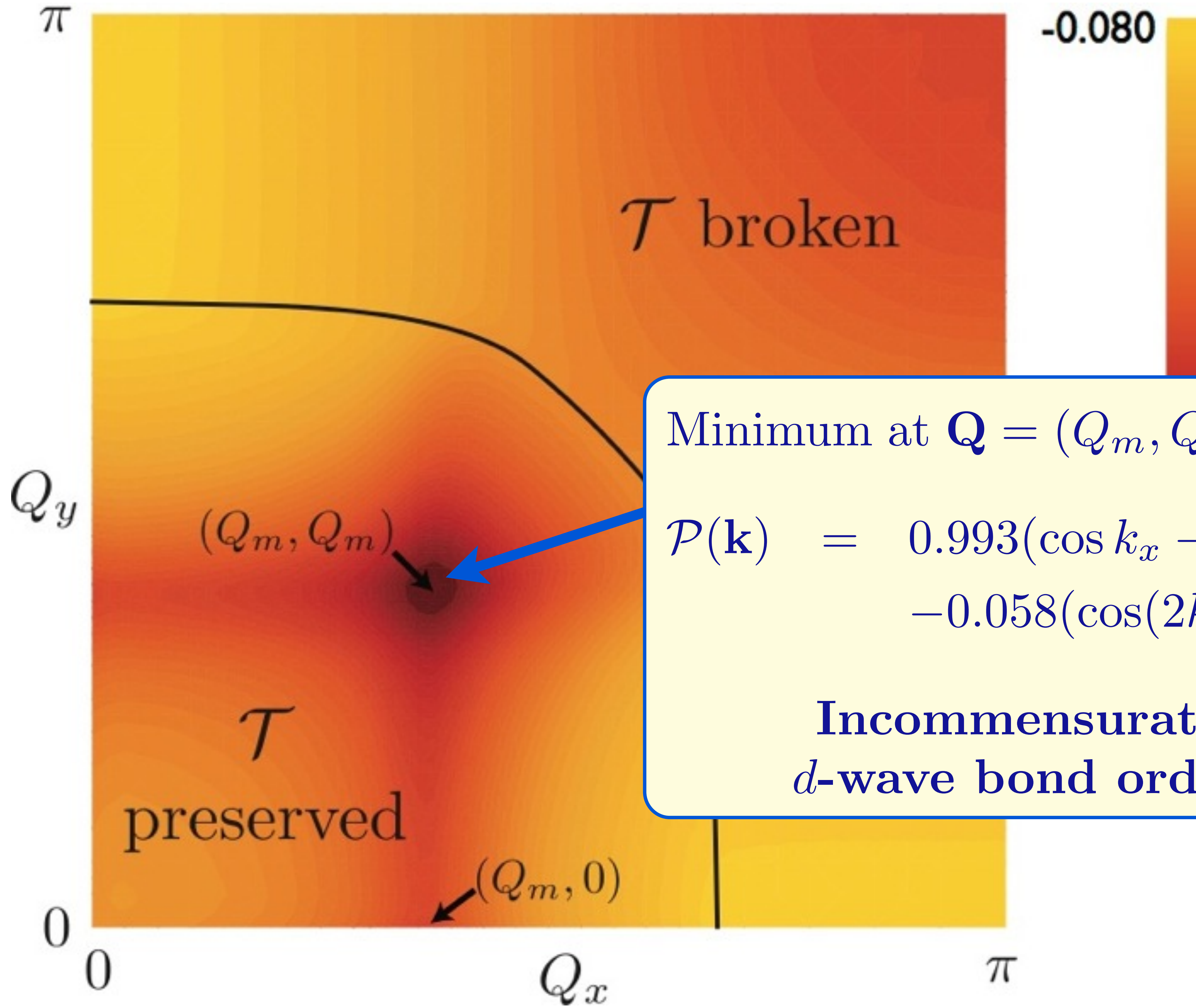
S. Sachdev and R. La Placa, Phys. Rev. Lett. 111, 027202 (2013); A. Allais, J. Bauer, and S. Sachdev, arXiv:1402.6311



Eigenvalues, $\lambda(\mathbf{Q})$, of the spin-singlet, particle-hole propagator. The corresponding eigenvector is $\mathcal{P}(\mathbf{k})$ and this leads to the order

$$\left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$$

S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013); A. Allais, J. Bauer, and S. Sachdev, arXiv:1402.6311



Minimum at $\mathbf{Q} = (Q_m, Q_m)$ with

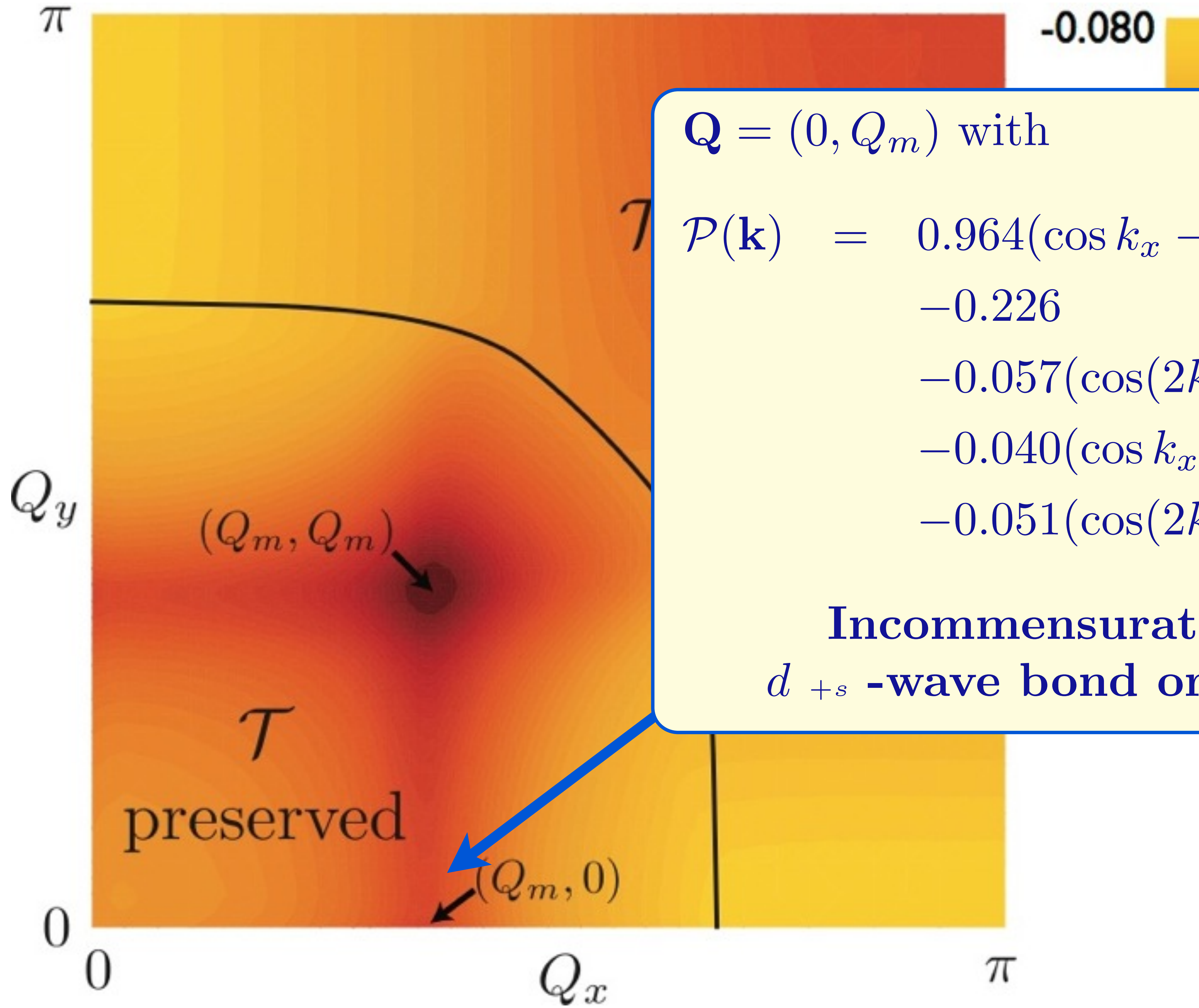
$$\begin{aligned}\mathcal{P}(\mathbf{k}) &= 0.993(\cos k_x - \cos k_y) \\ &\quad -0.058(\cos(2k_x) - \cos(2k_y))\end{aligned}$$

Incommensurate
***d*-wave bond order**

Eigenvalues, $\lambda(\mathbf{Q})$, of the spin-singlet, particle-hole propagator. The corresponding eigenvector is $\mathcal{P}(\mathbf{k})$ and this leads to the order

$$\left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$$

S. Sachdev and R. La Placa, Phys. Rev. Lett. 111, 027202 (2013); A. Allais, J. Bauer, and S. Sachdev, arXiv:1402.6311



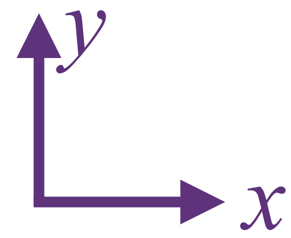
$\mathbf{Q} = (0, Q_m)$ with

$$
\begin{aligned}
\mathcal{P}(\mathbf{k}) \quad = \quad & 0.964(\cos k_x - \cos k_y) \\
& -0.226 \\
& -0.057(\cos(2k_x) - \cos(2k_y)) \\
& -0.040(\cos k_x + \cos k_y) \\
& -0.051(\cos(2k_x) + \cos(2k_y))
\end{aligned}
$$

Incommensurate
d_{+s} -wave bond order

Eigenvalues, $\lambda(\mathbf{Q})$, of the spin-singlet, particle-hole propagator. The corresponding eigenvector is $\mathcal{P}(\mathbf{k})$ and this leads to the order $\left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$

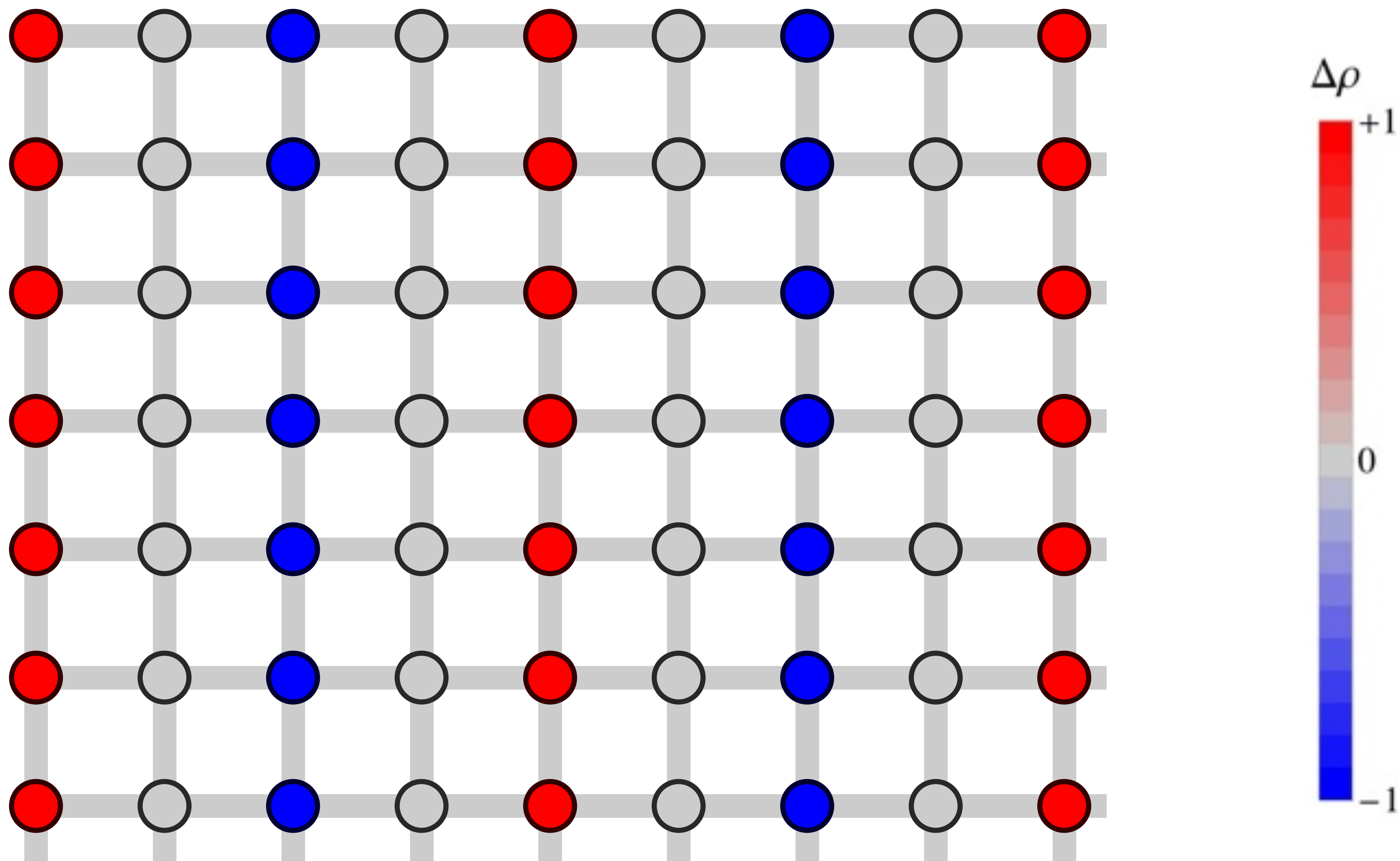
(Un)conventional CDW order: s-wave



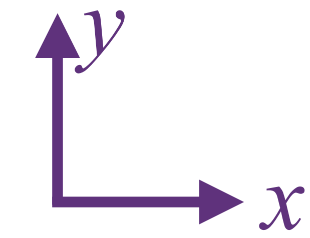
Plot of $P_{ij} = \left\langle c^\dagger_{i\alpha} c_{j\alpha} \right\rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = 1 \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$



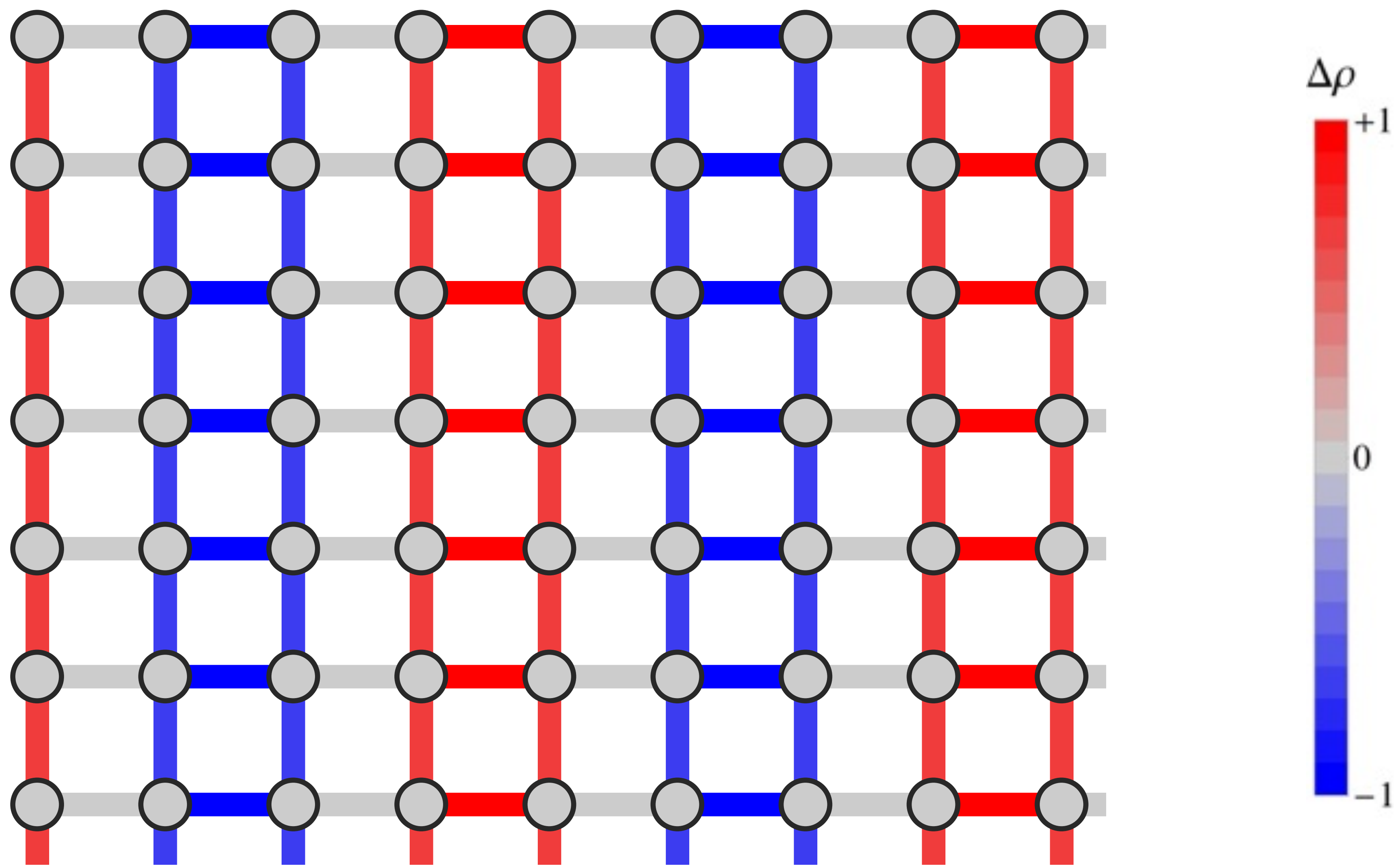
Unconventional CDW order: s'-wave



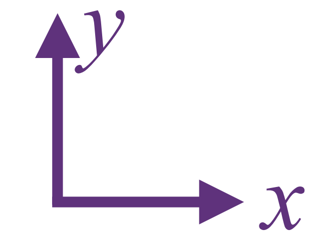
Plot of $P_{ij} = \left\langle c_{i\alpha}^\dagger c_{j\alpha} \right\rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} \left[\cos(k_x) + \cos(k_y)\right] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$



Unconventional CDW order: d-wave



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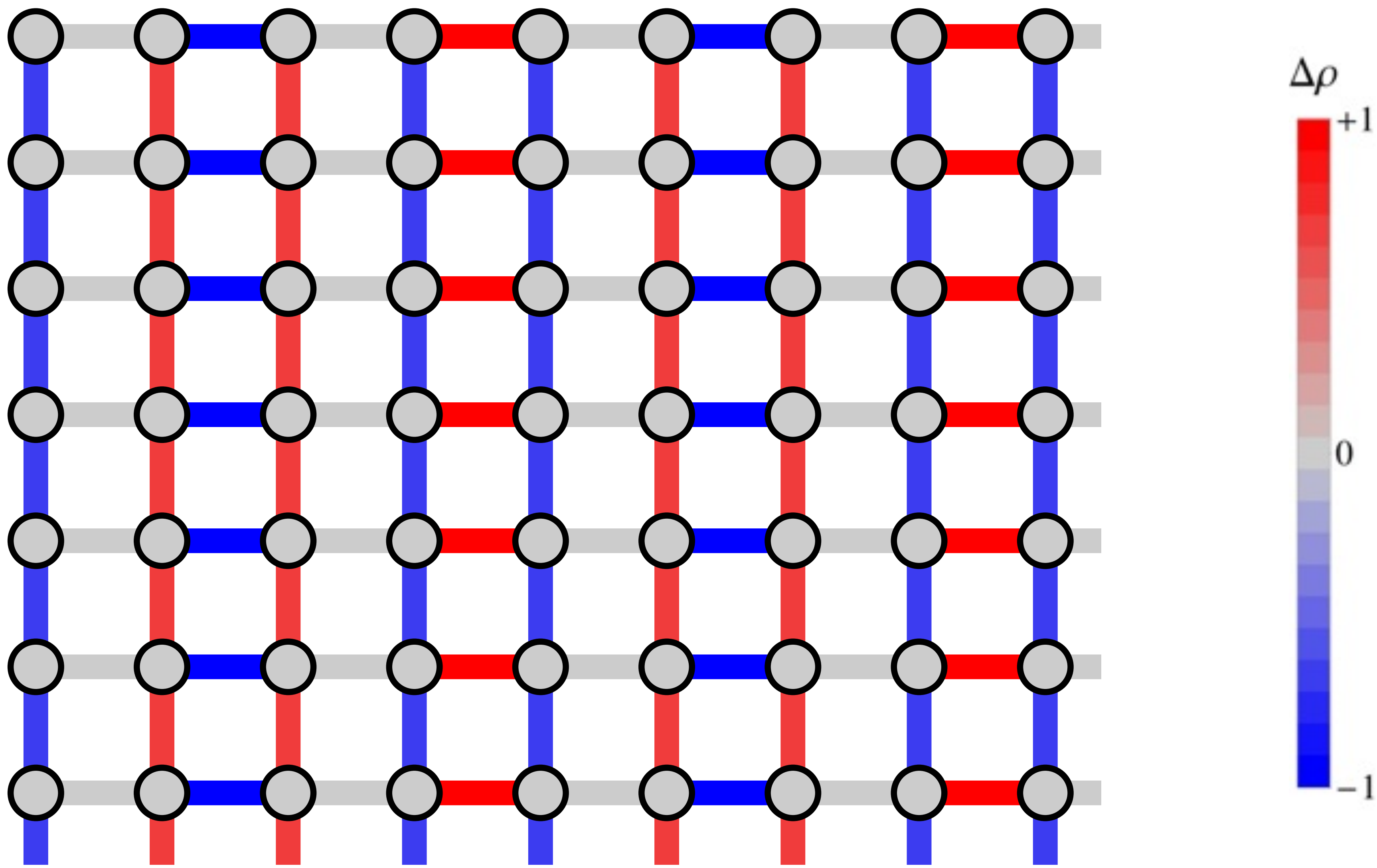
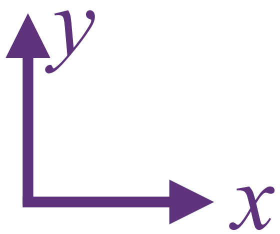
$$\mathcal{P}(\mathbf{k}) = e^{i\phi}\left[\cos(k_x) - \cos(k_y)\right] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$



This specific d-wave bond order (with $\mathbf{Q}$ along the axes) was first predicted in S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

Unconventional CDW order: d-wave



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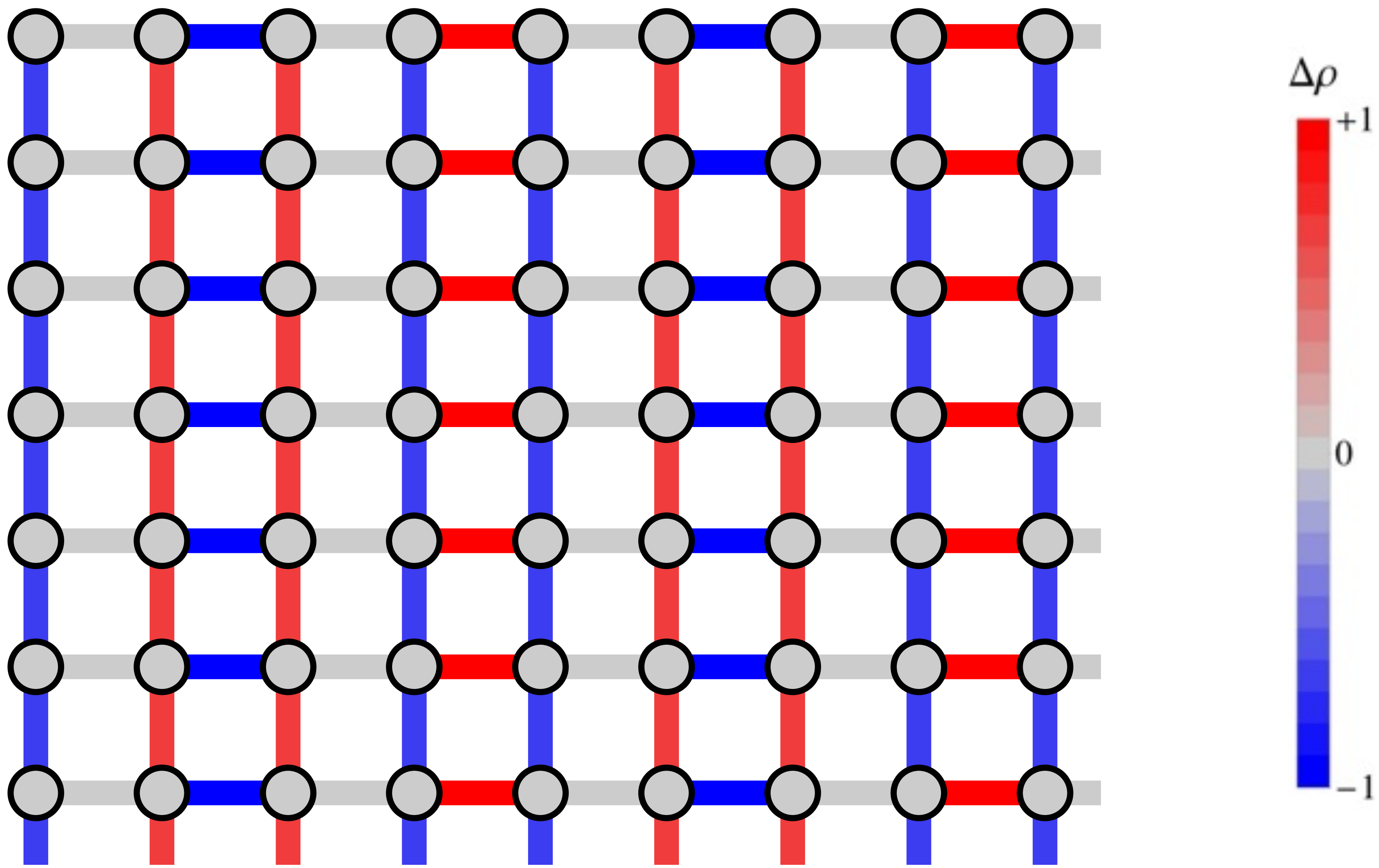
$$\mathcal{P}(\mathbf{k}) = e^{i\phi} \left[\cos(k_x) - \cos(k_y)\right] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



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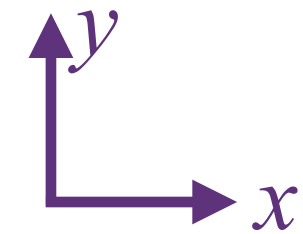
Unconventional CDW order: $d + s'$-wave



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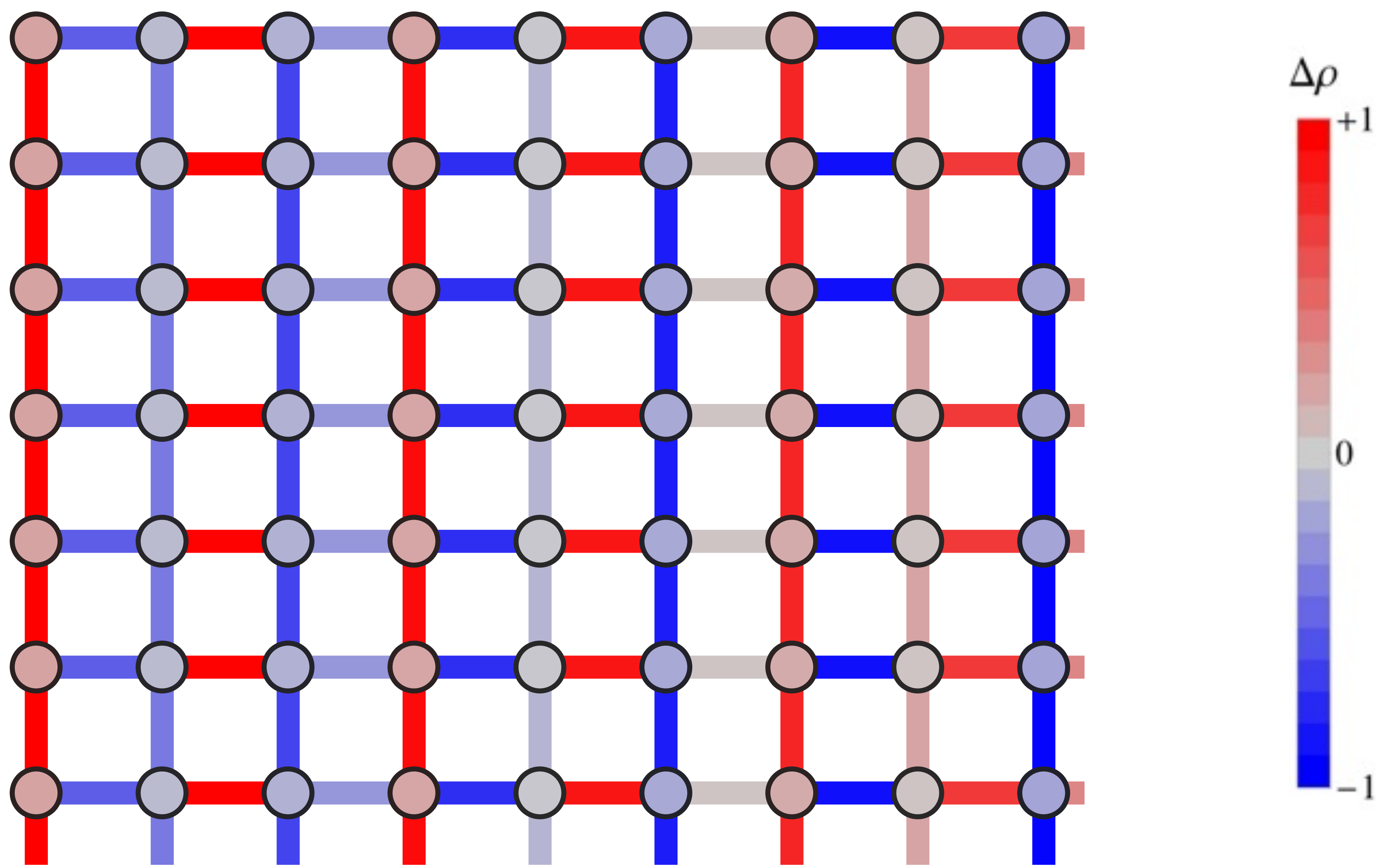
$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi}\left[0.2 + \cos(k_x) - \cos(k_y)\right] \quad \text{and} \quad \mathbf{Q} = 2\pi(0.317, 0)$$

Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



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See also
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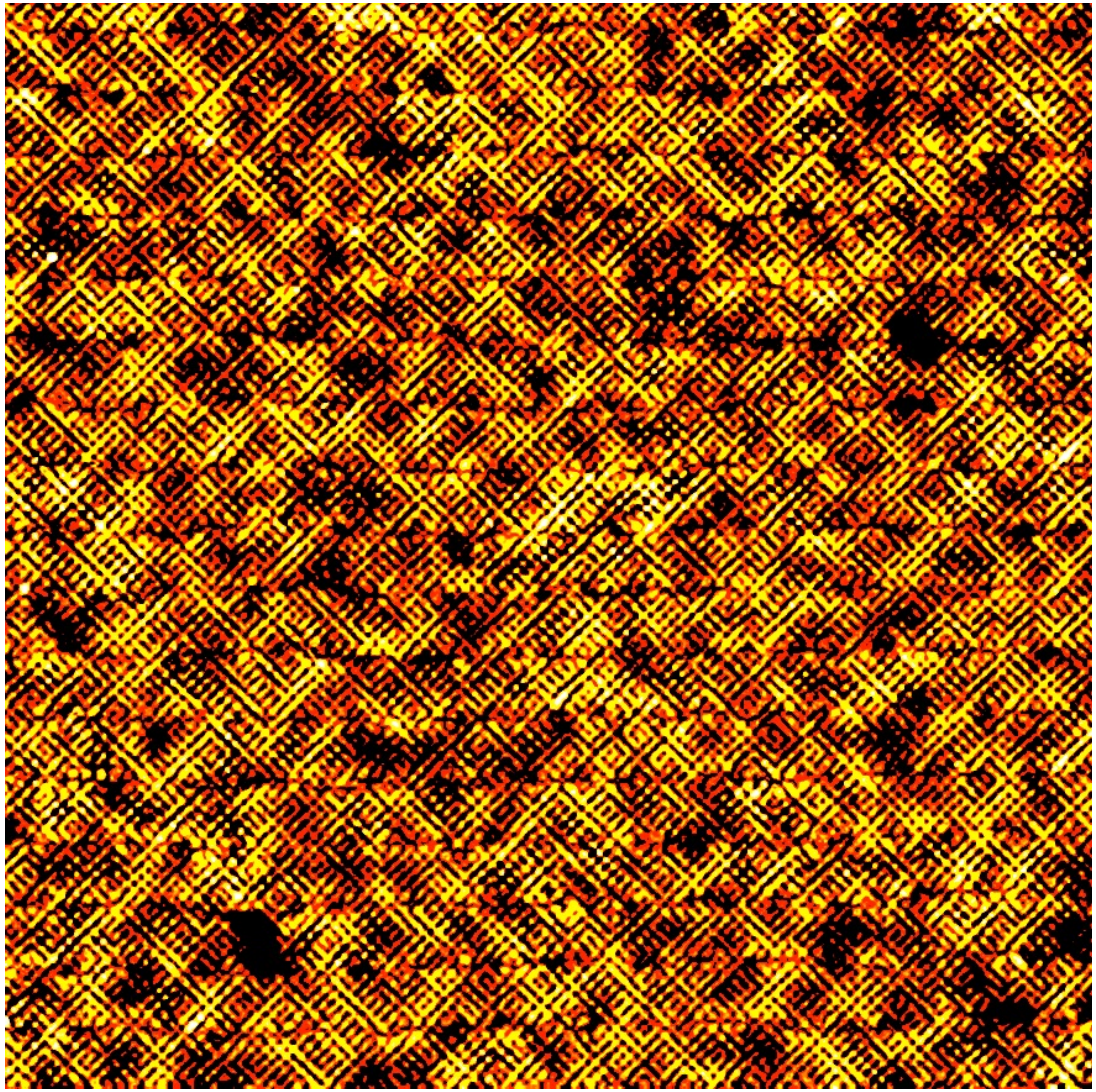
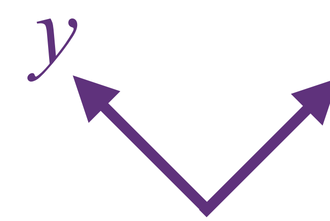
M. Vershinin, S. Misra, S. Ono, Y. Abe, Yoichi Ando, and A. Yazdani, Science **303**, 1995 (2004).

W. D. Wise, M. C. Boyer, K. Chatterjee, T. Kondo, T. Takeuchi, H. Ikuta, Y. Wang, and E. W. Hudson, Nature Phys. **4**, 696 (2008).





"R-map" of BSCCO in zero magnetic field, similar to those published in Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007).

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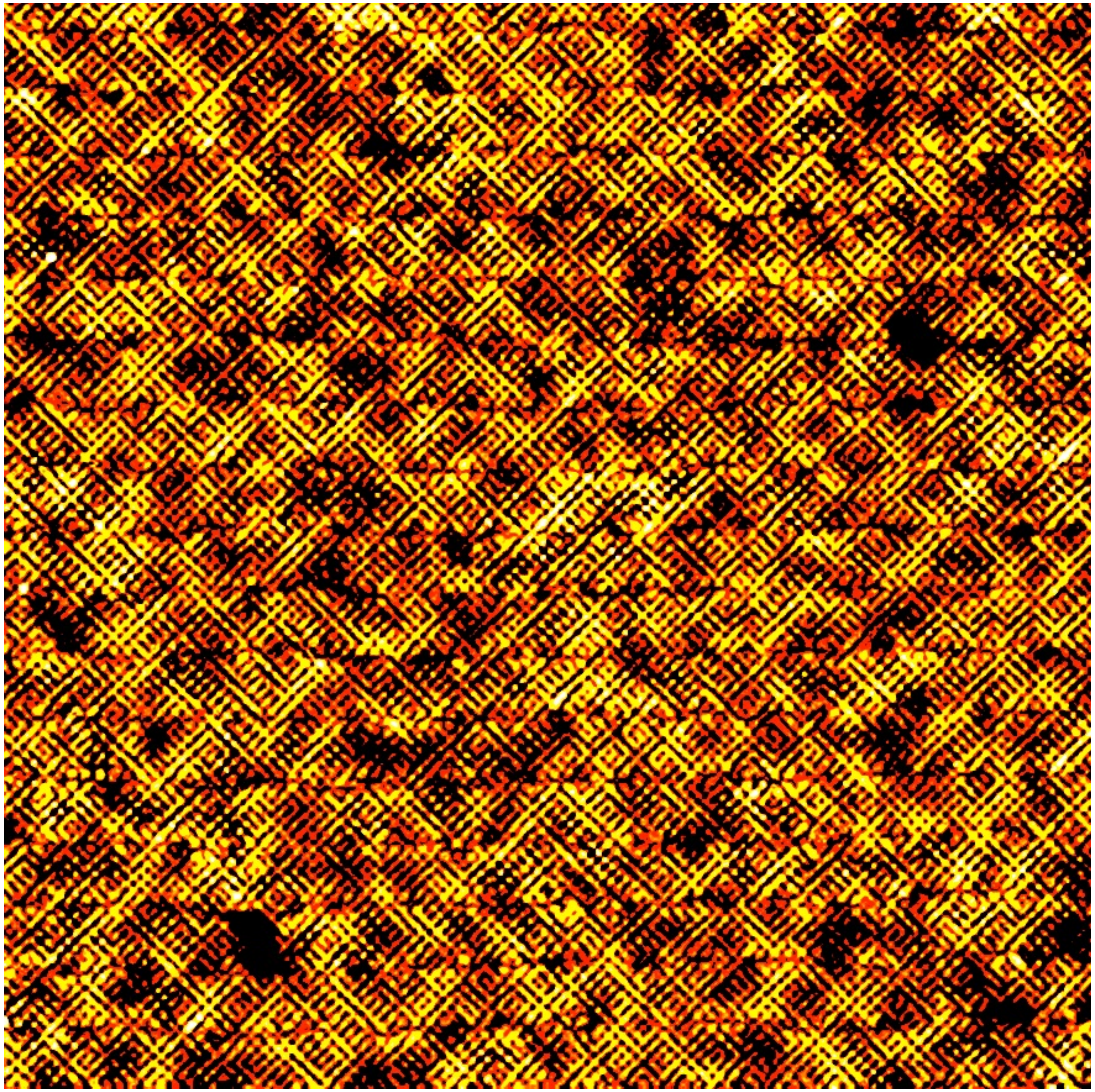

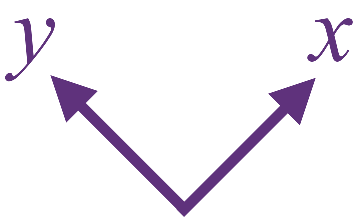

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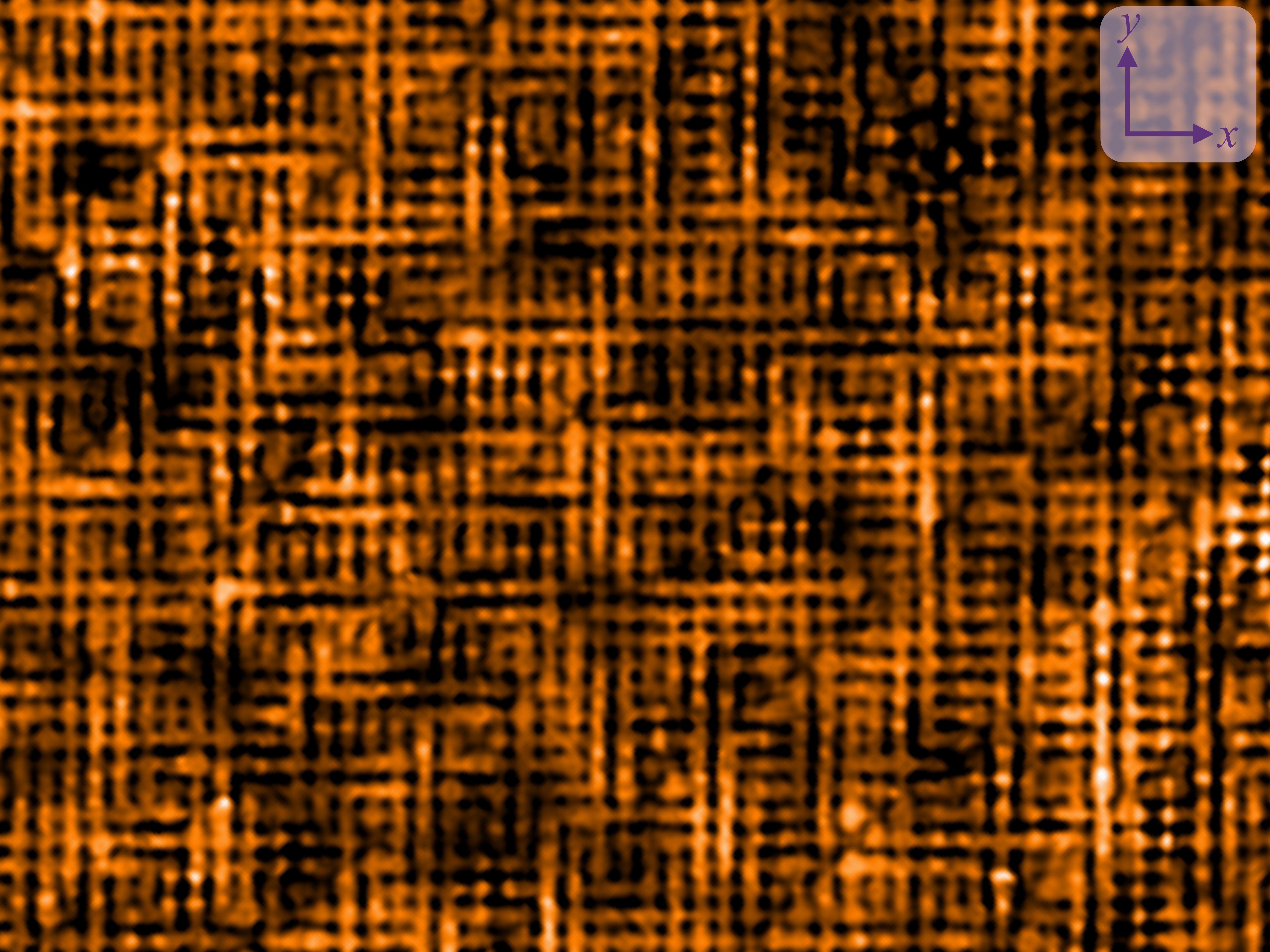
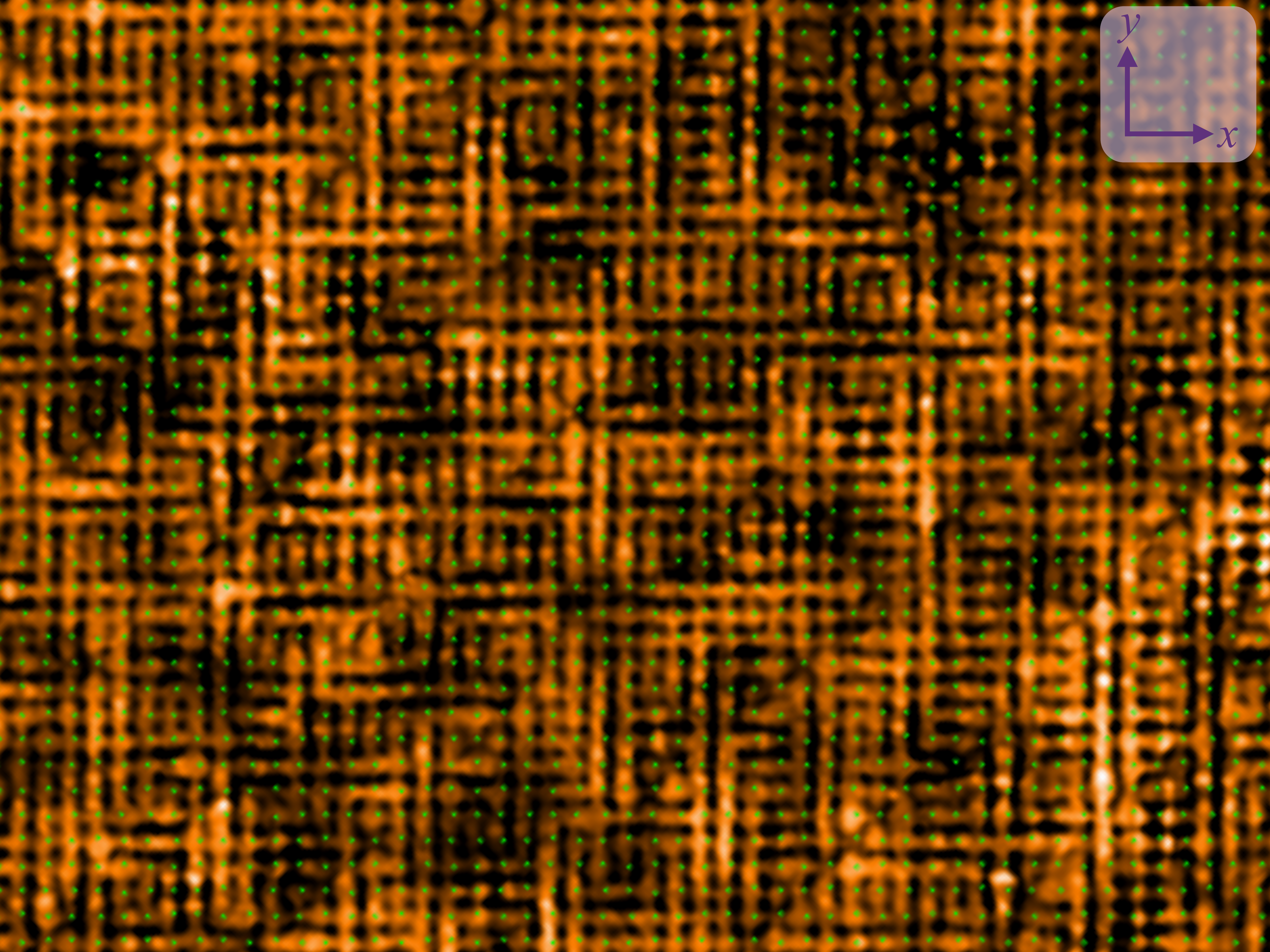
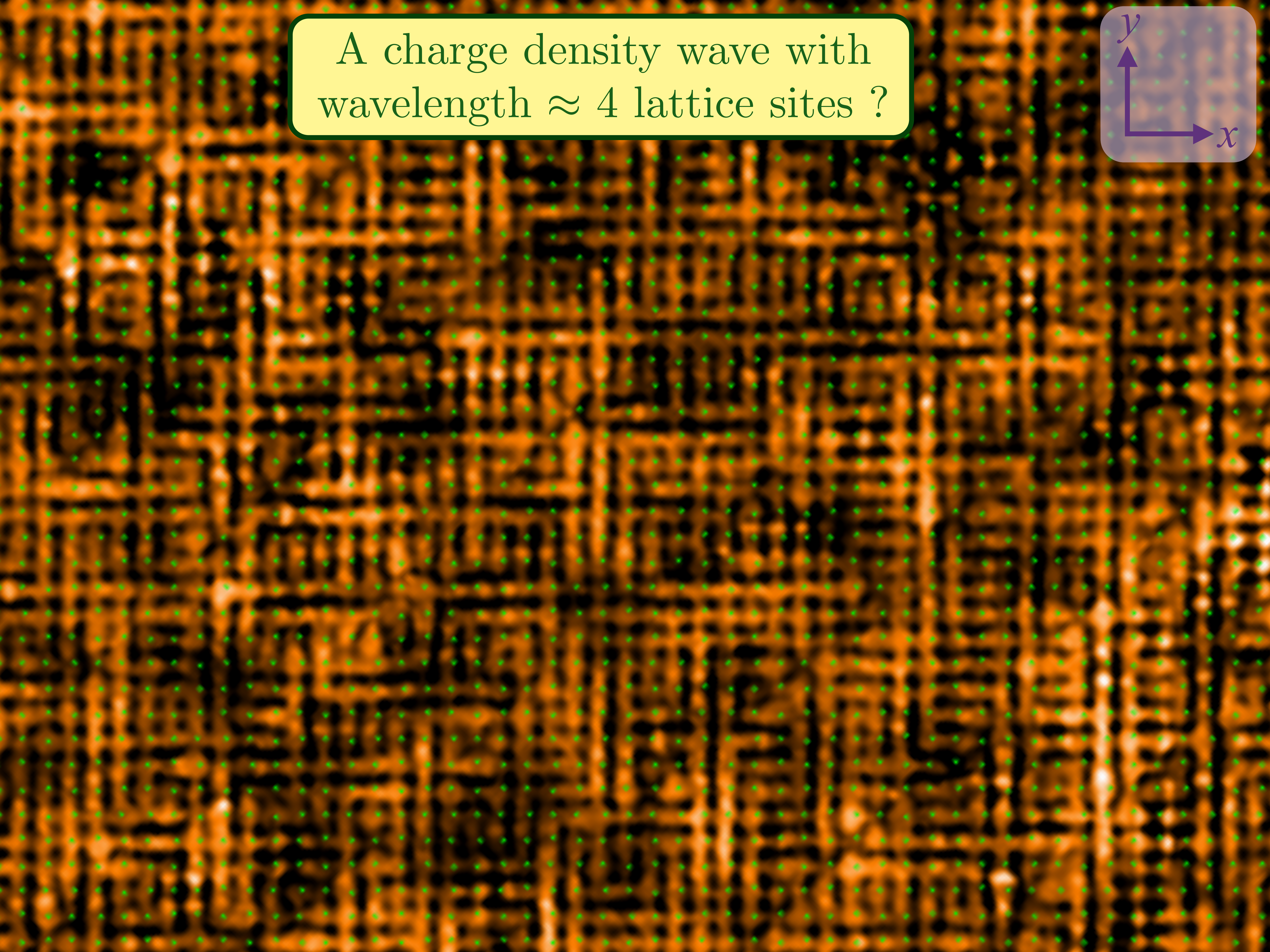
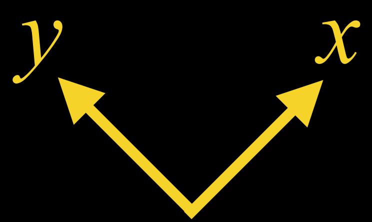

y
x


y
x

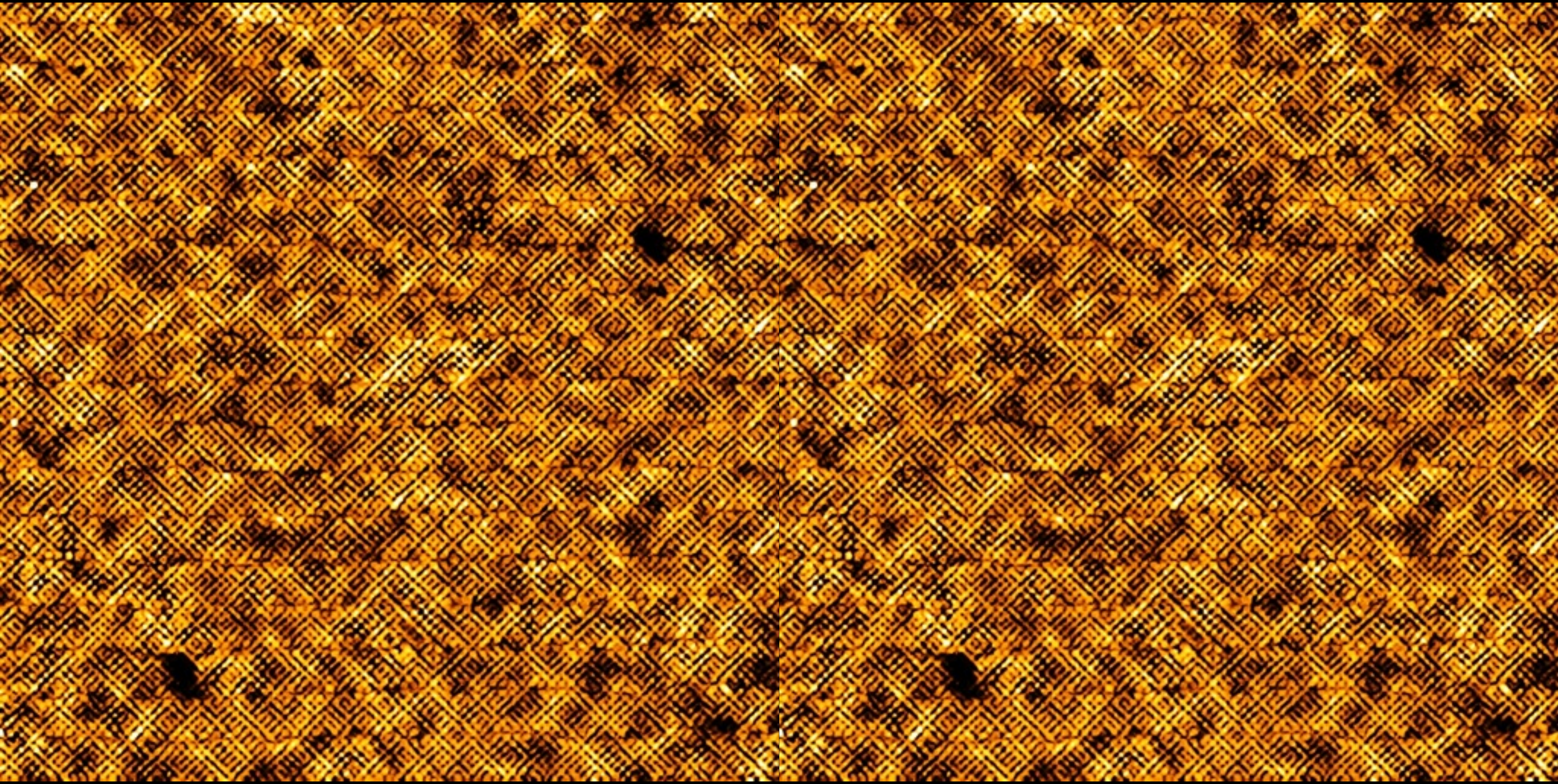

A charge density wave with
wavelength ≈ 4 lattice sites ?
y
x

UD45K
BSCCO

R(r,150mV)

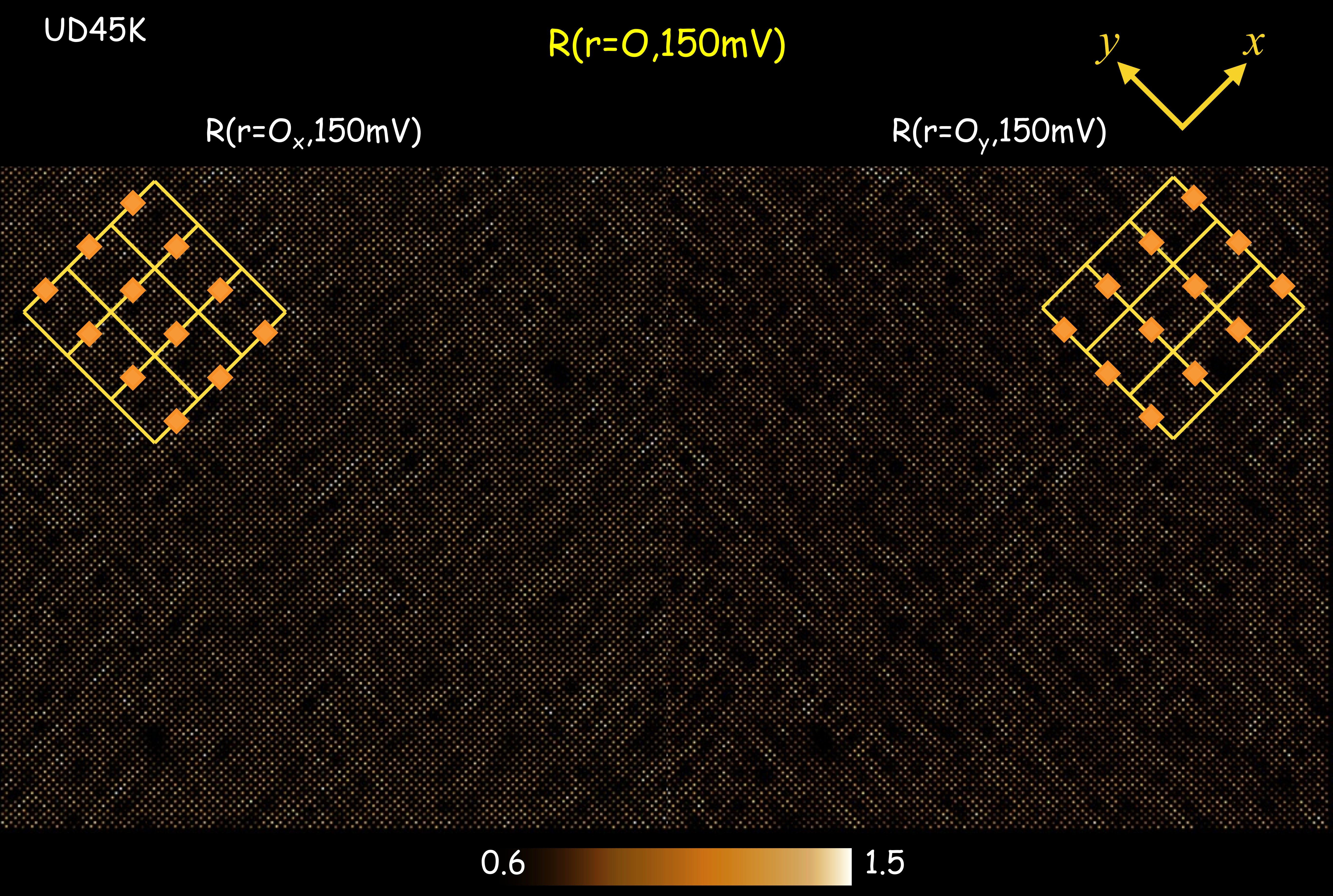


R(r,150mV) R(r,150mV)



Note that these are identical images.

K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, arXiv:1404.0362



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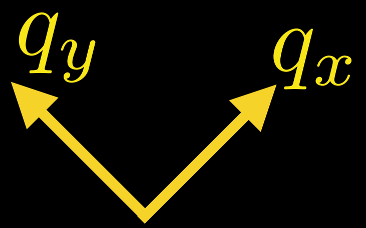
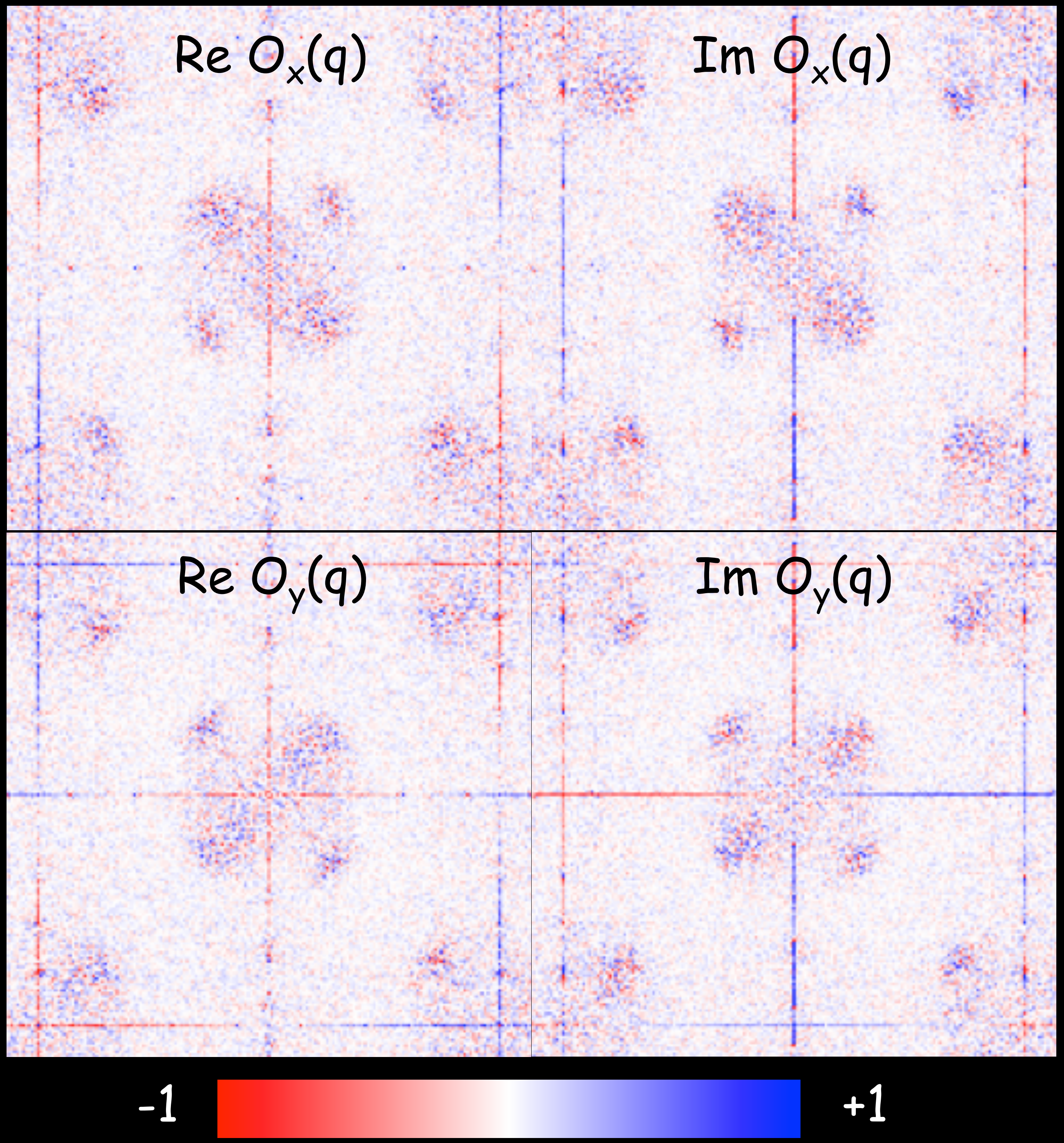
UD45K

Broad (0,Q) and (Q,0) DW Features





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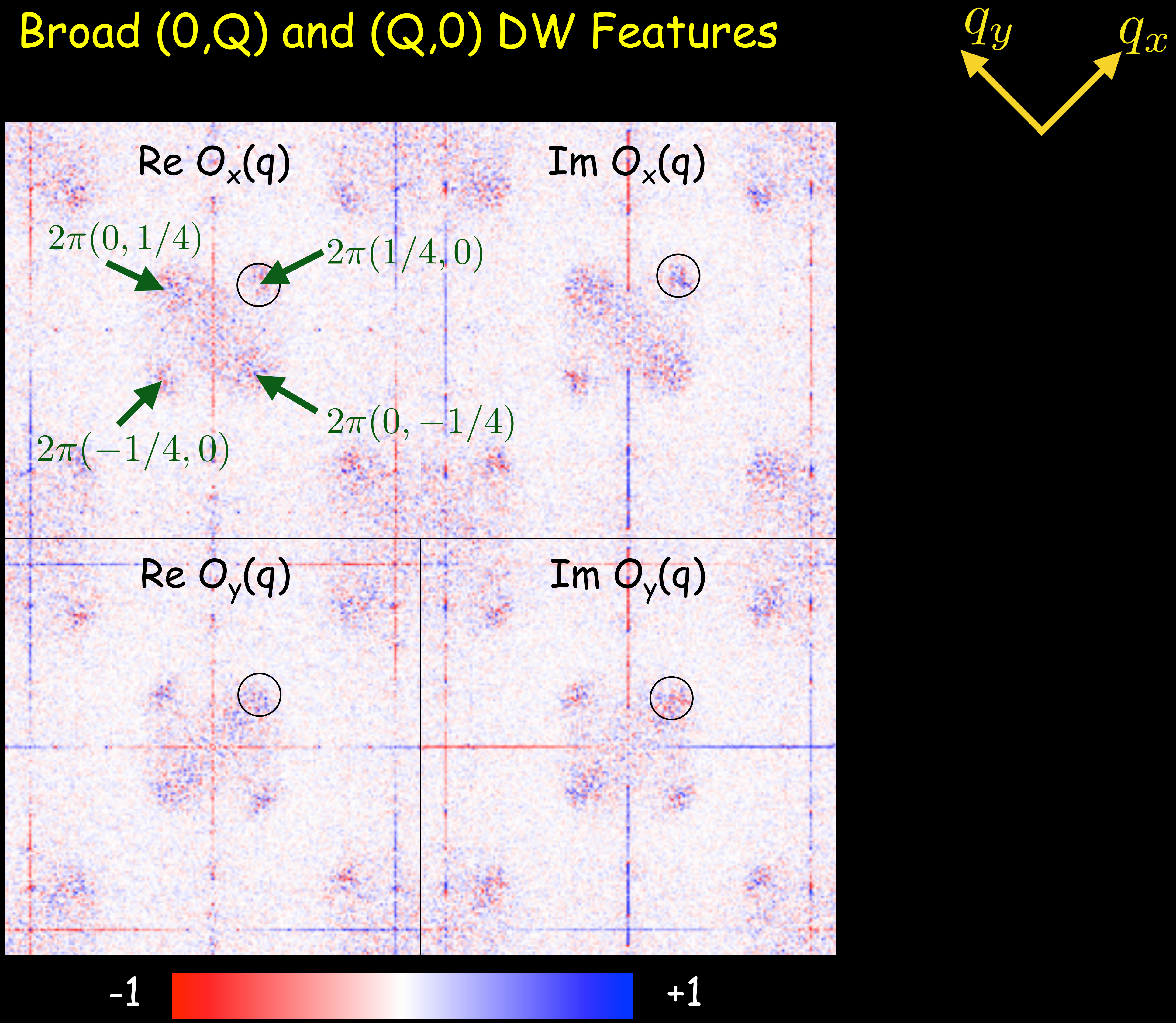

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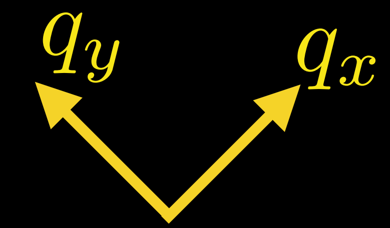
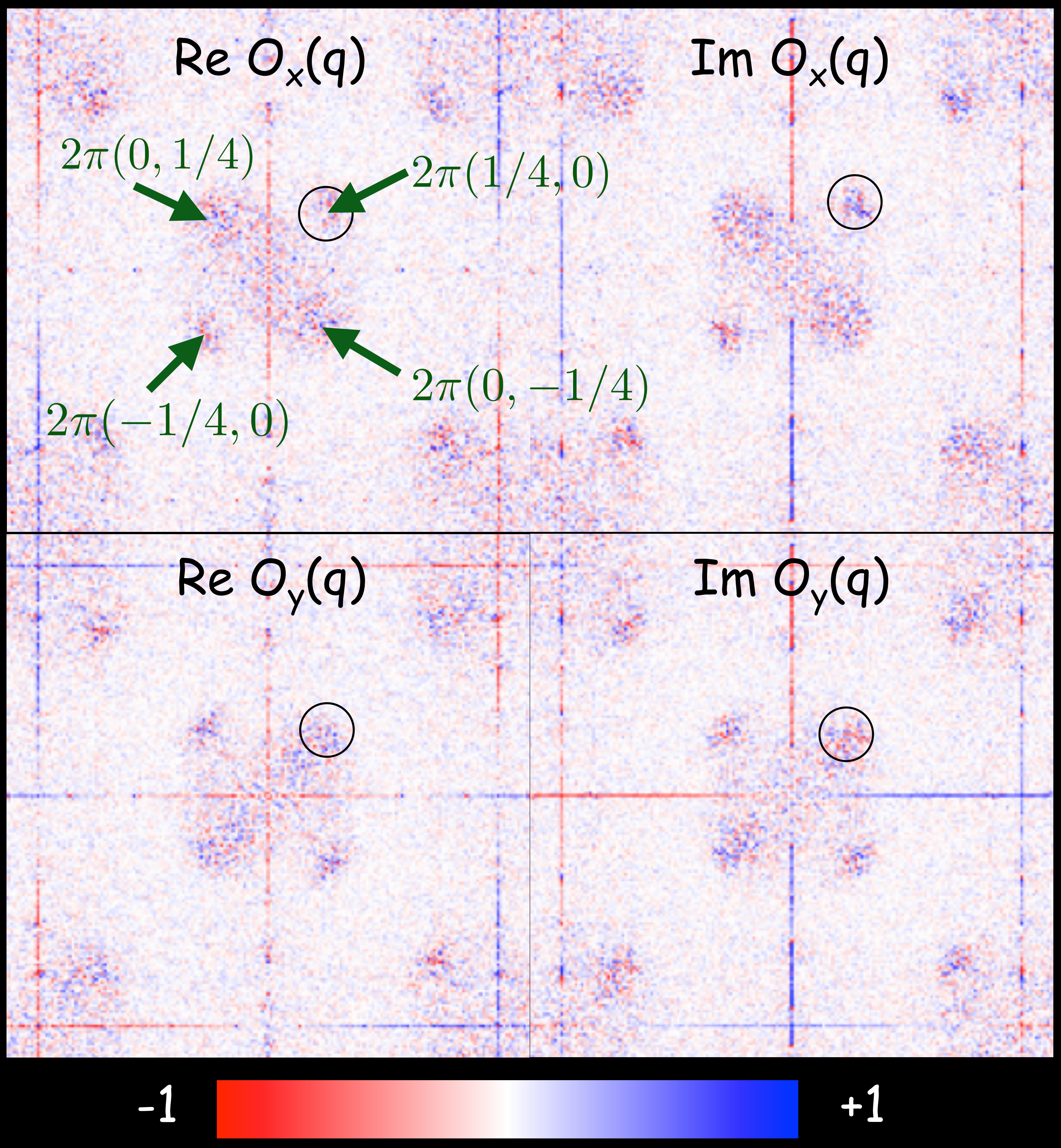


For each pixel in the circles, we obtain 2 complex numbers, $O_x(q)$ and $O_y(q)$.



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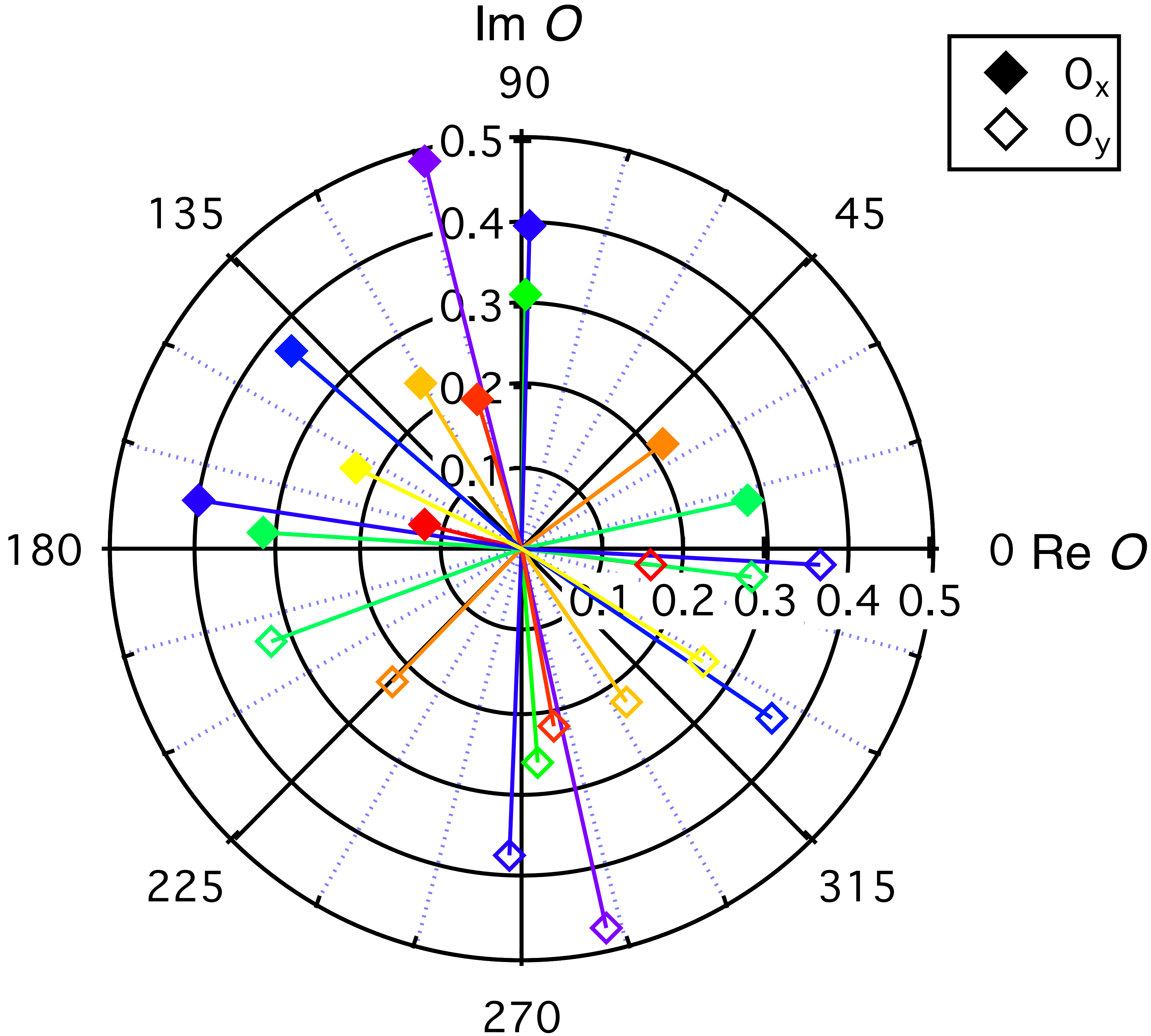

Phase-sensitive measurement of the *d* symmetry of charge density wave order

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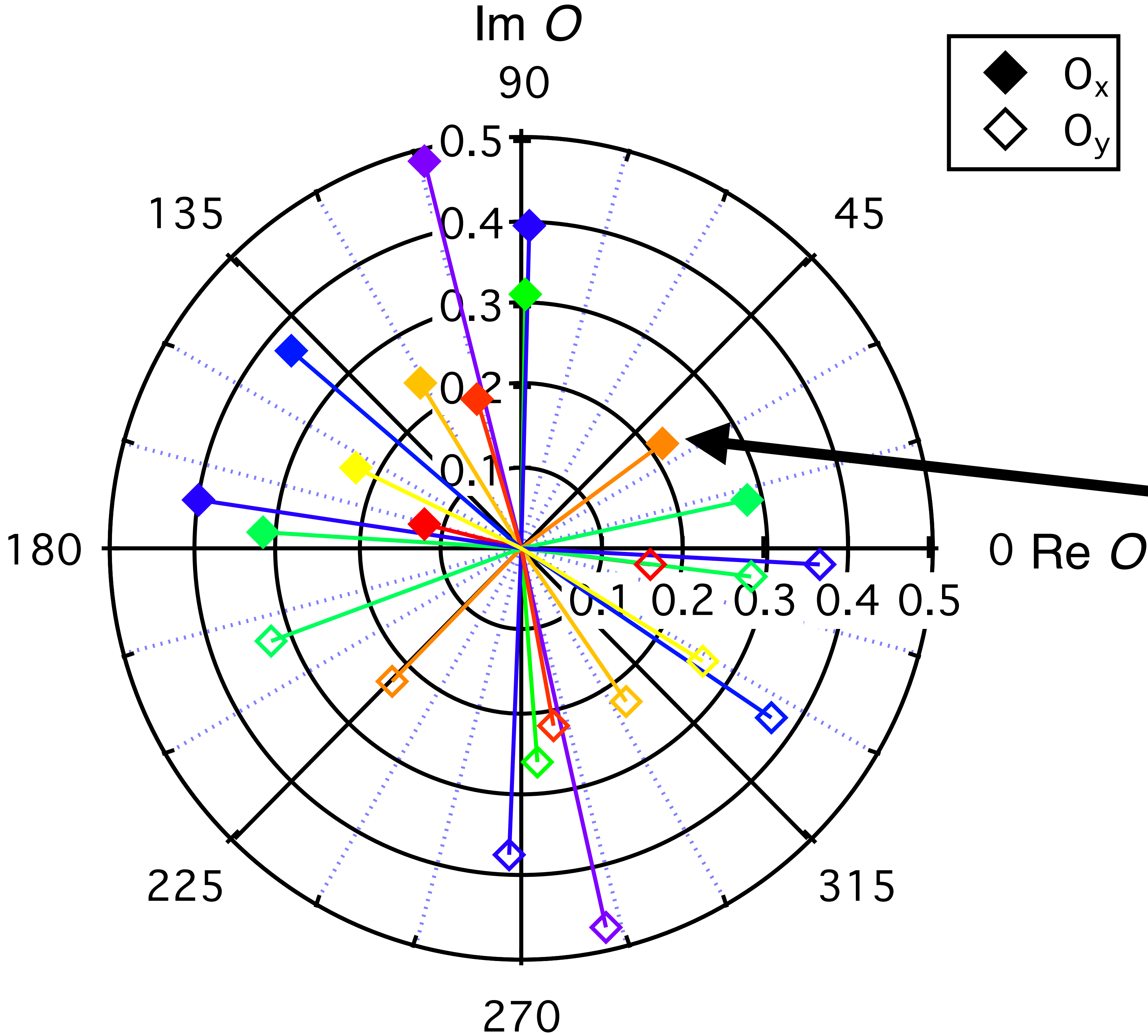

Phase-sensitive measurement of the *d* symmetry of charge density wave order

Complex value of O_x at a pixel

K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, arXiv:1404.0362

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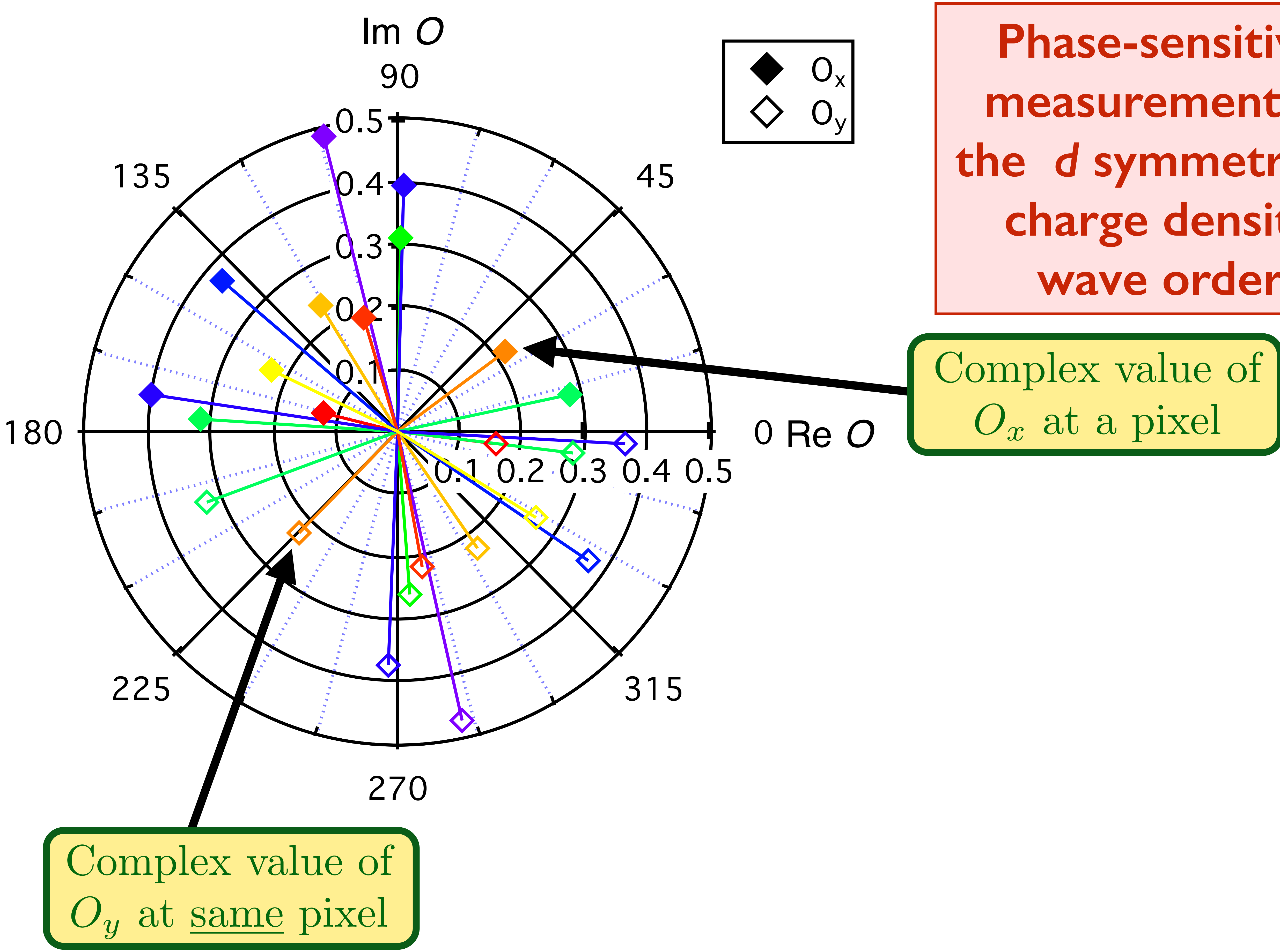

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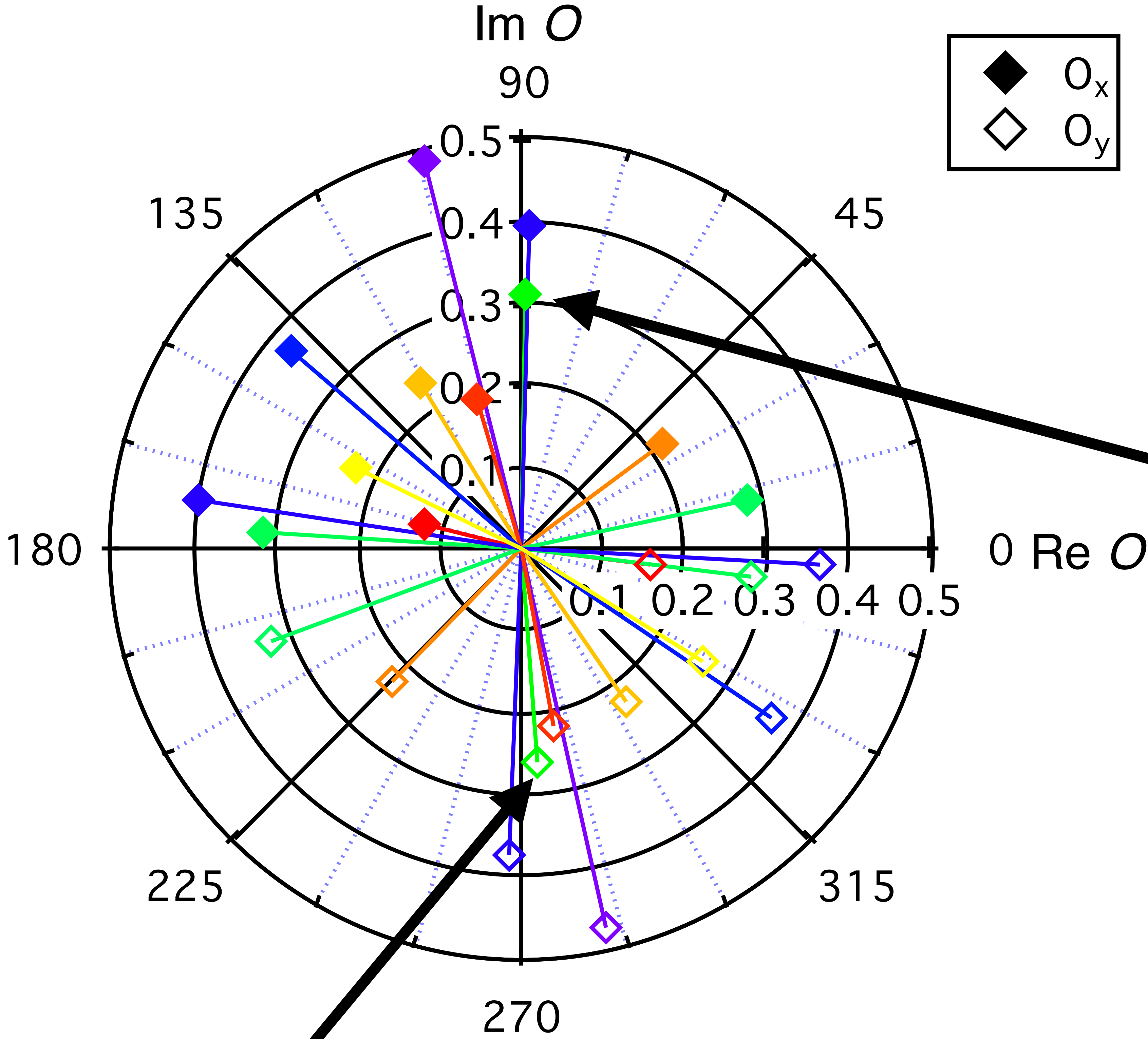

Complex value of O_x at a pixel

Complex value of O_y at <u>same</u> pixel

K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, arXiv:1404.0362

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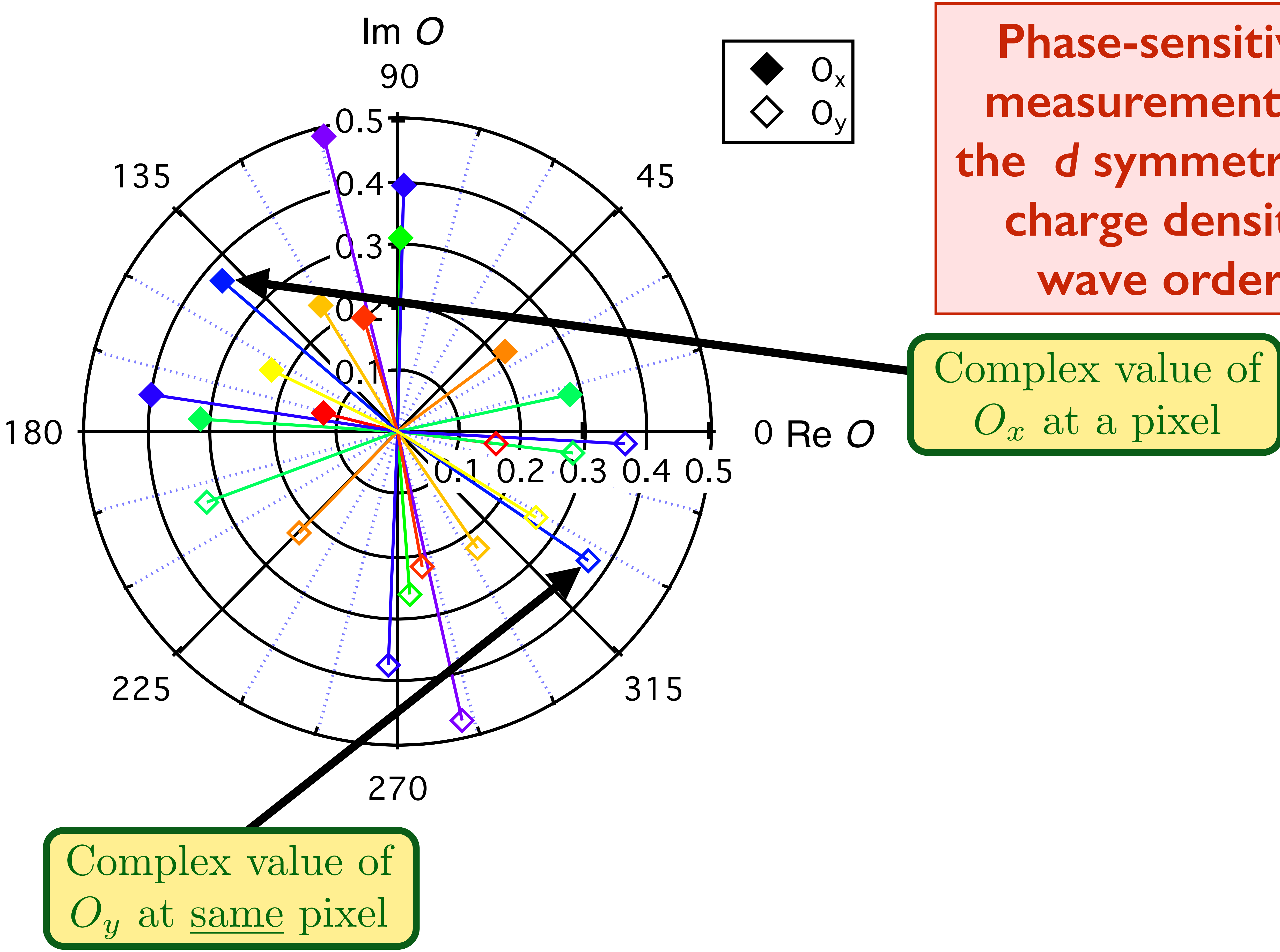

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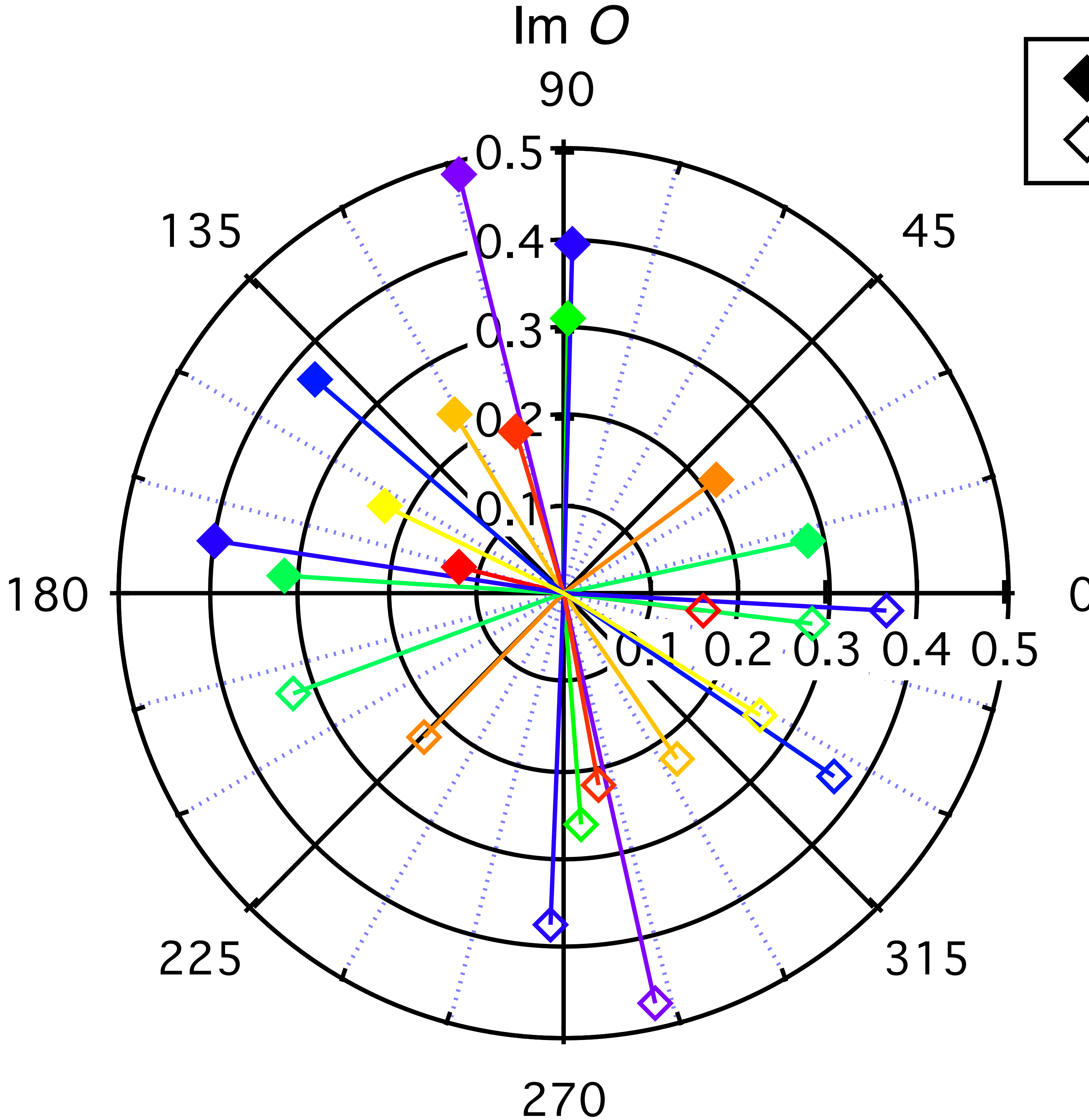

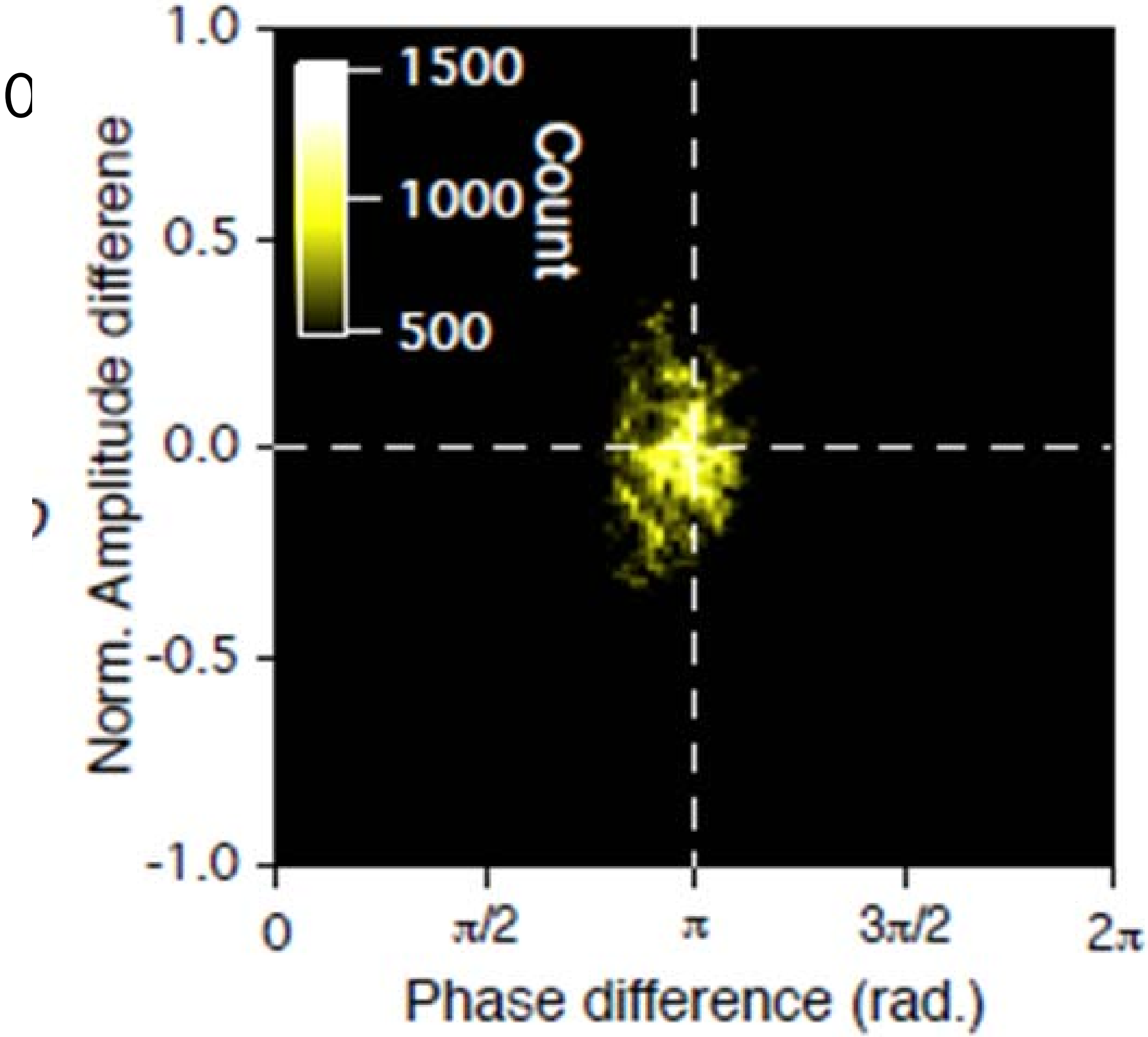

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See also
C. Howald, H. Eisaki, N. Kaneko, M. Greven, and A. Kapitulnik, Phys. Rev. B **67**, 014533 (2003);

M. Vershinin, S. Misra, S. Ono, Y. Abe, Yoichi Ando, and A. Yazdani, Science **303**, 1995 (2004).

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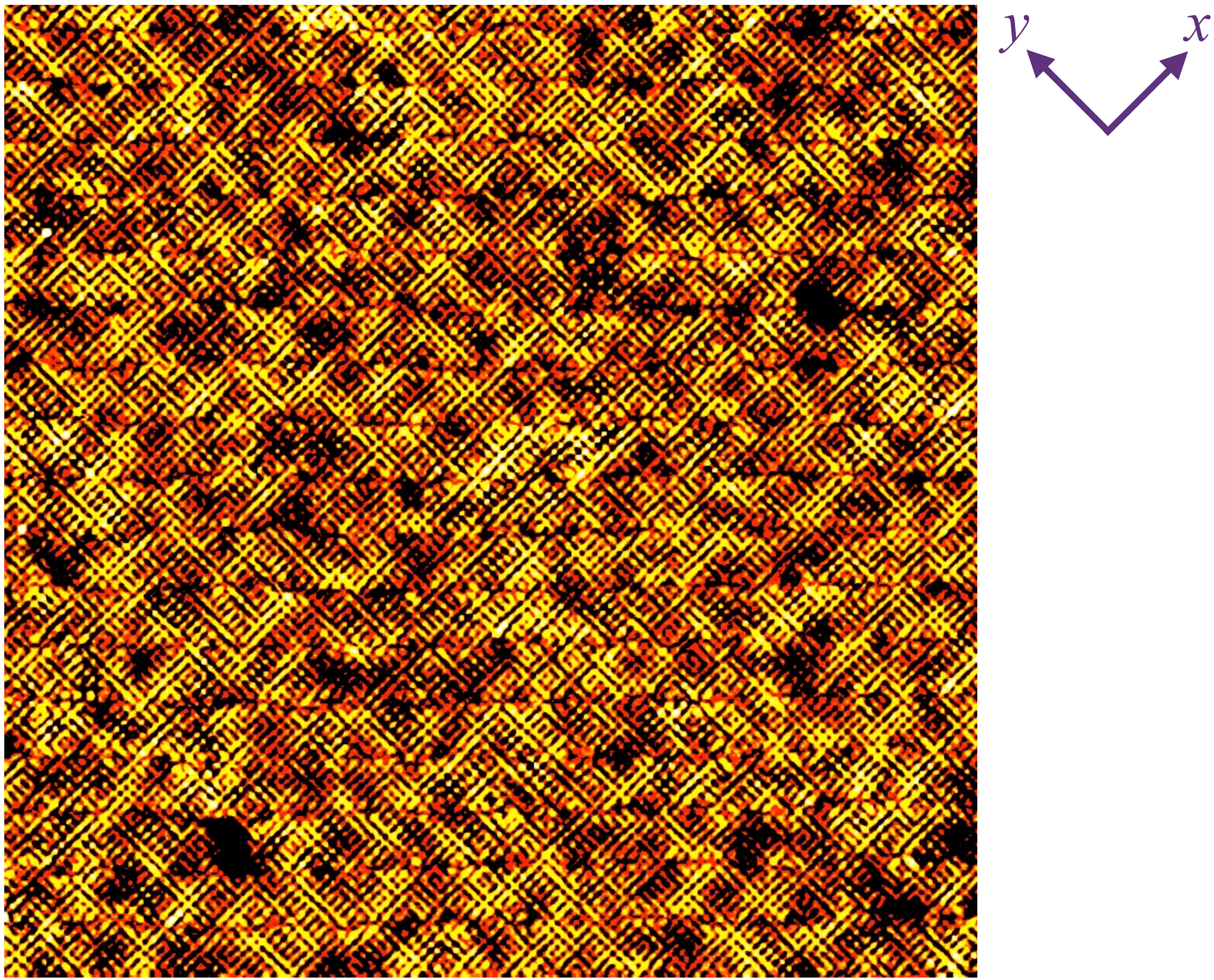

“R-map” of BSCCO in zero magnetic field, similar to those published in Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007).

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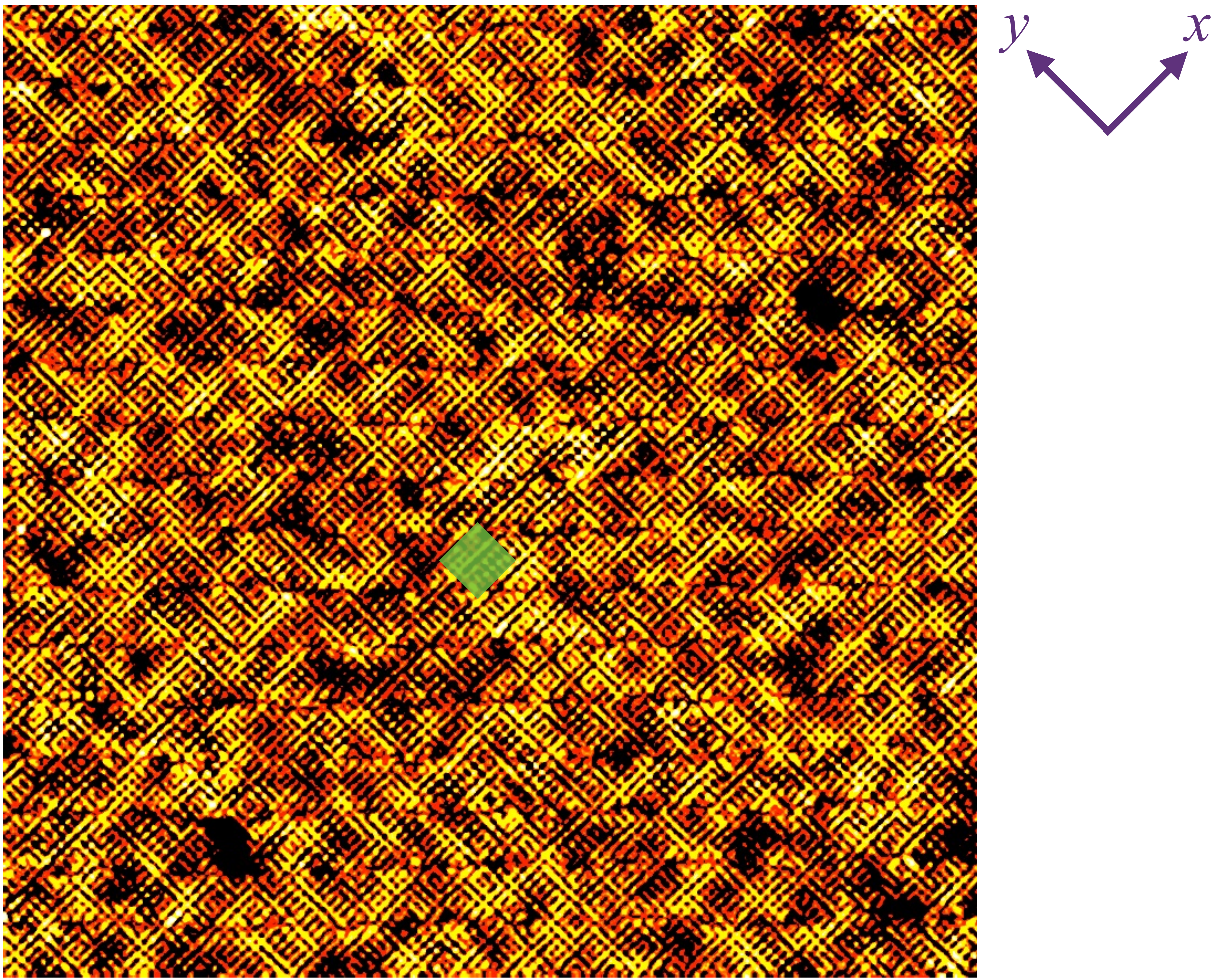

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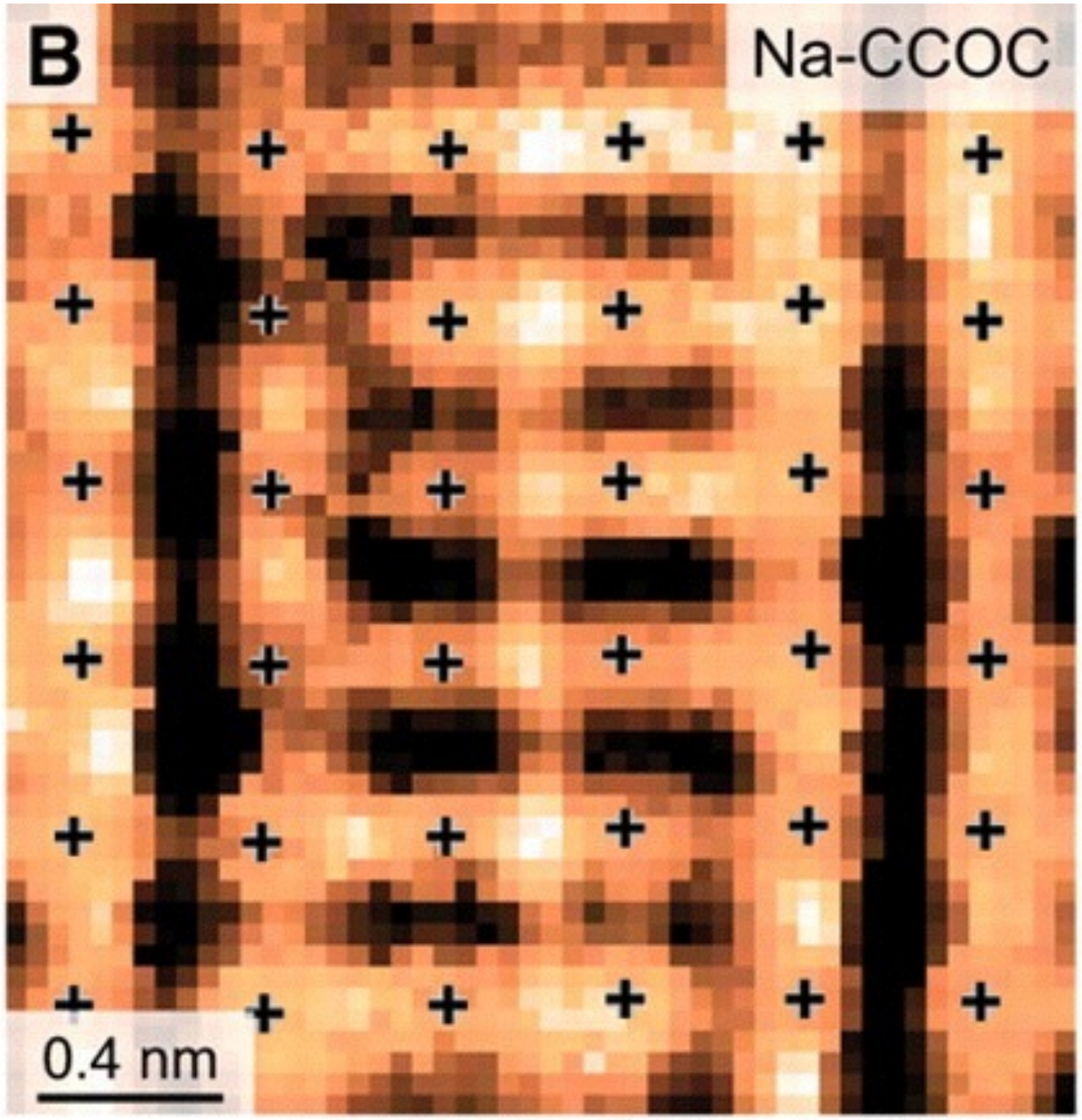
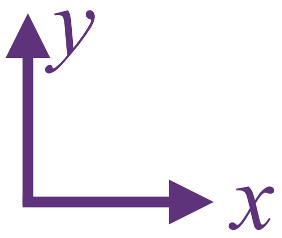
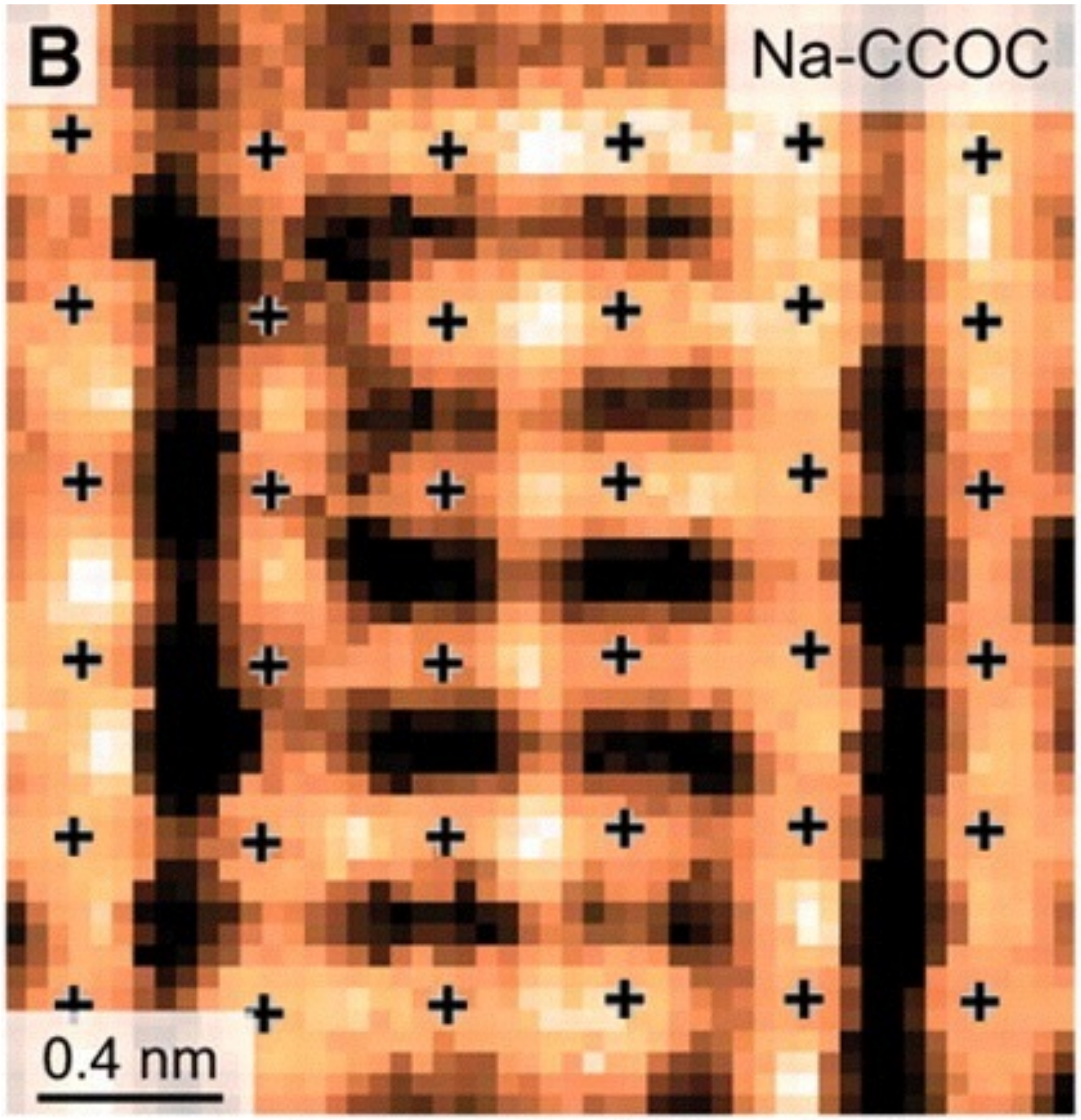


Y. Kohsaka *et al.*, Science **315**, 1380 (2007)





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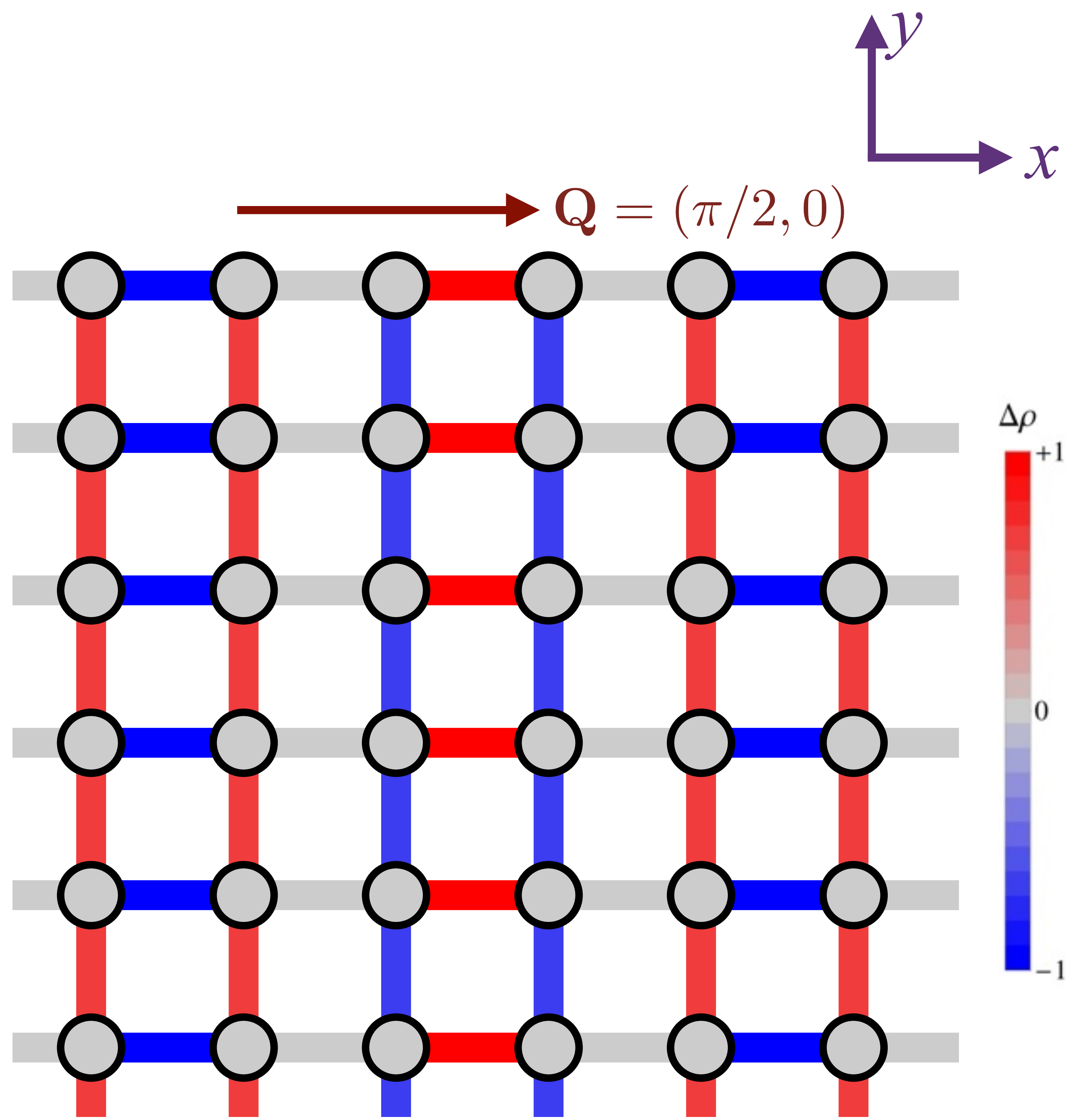

d-wave bond order

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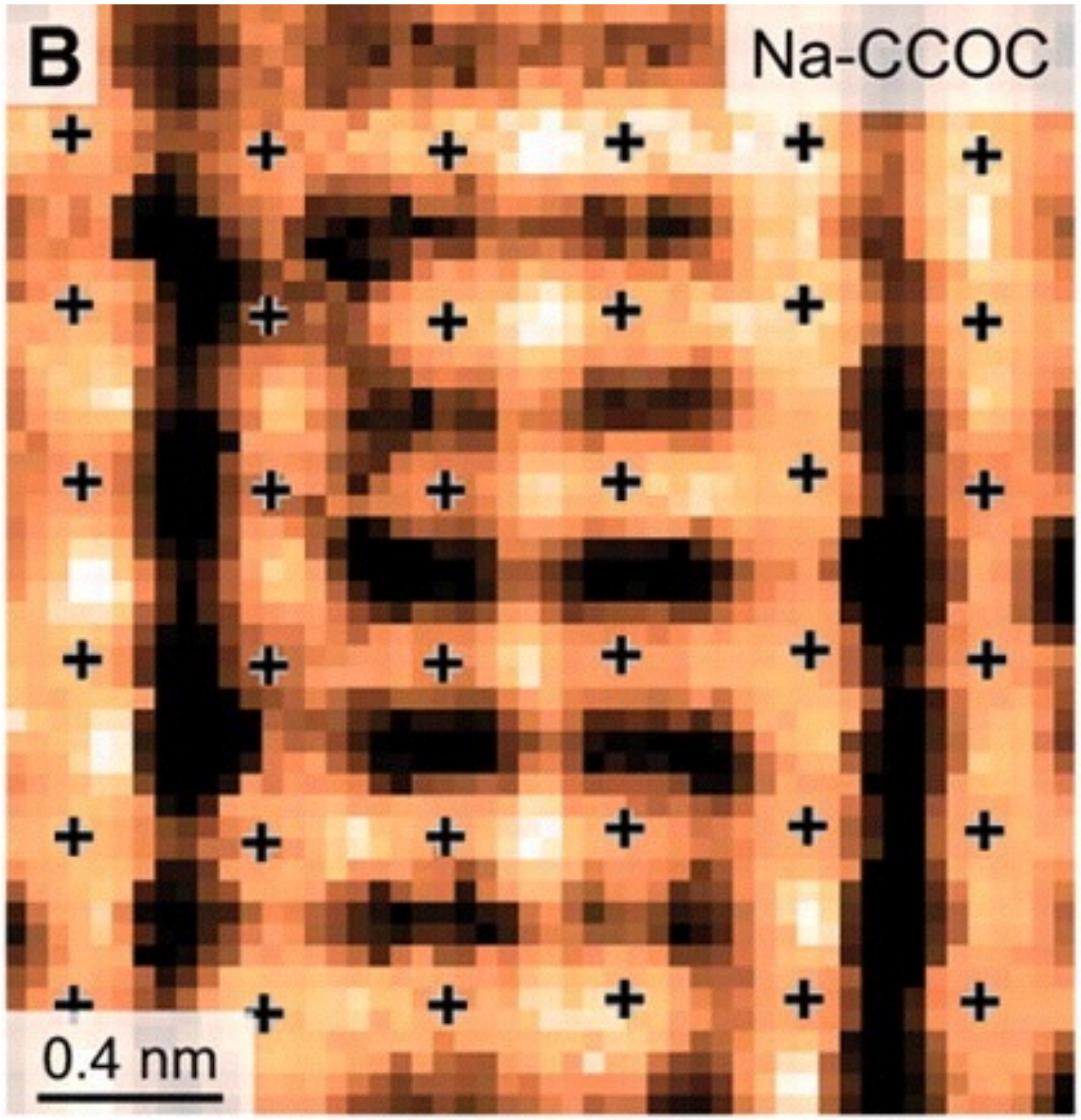
S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).



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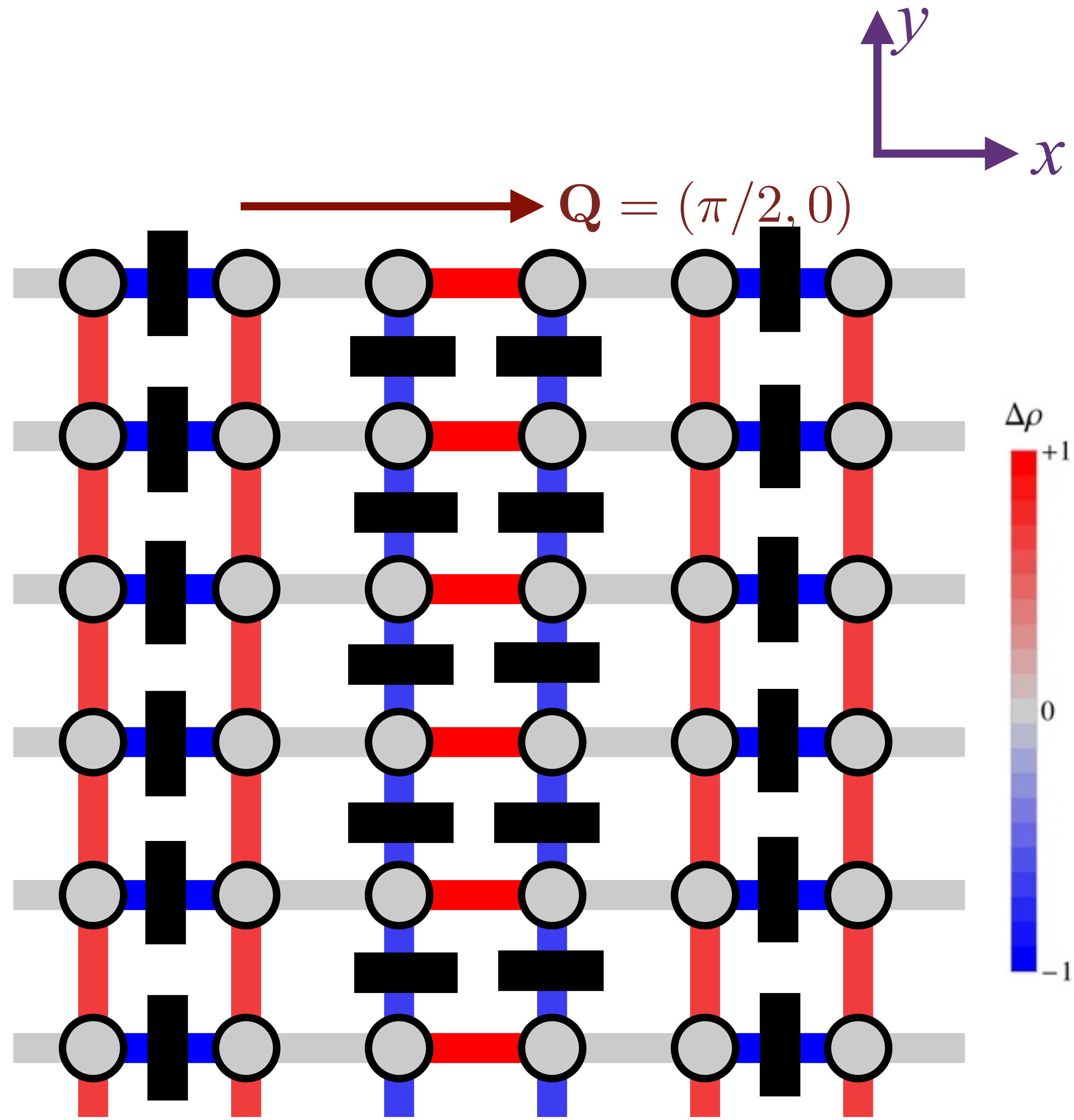

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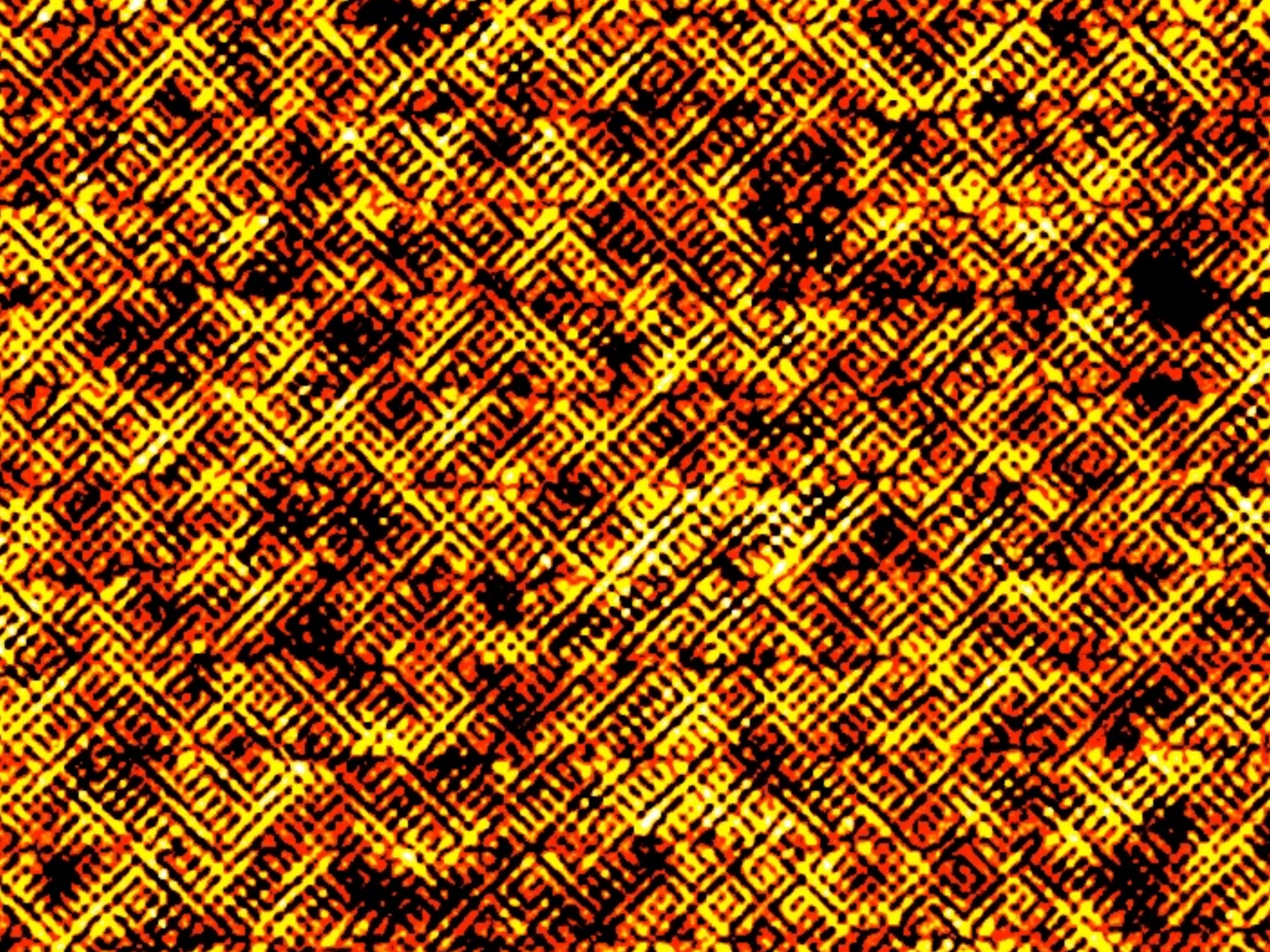
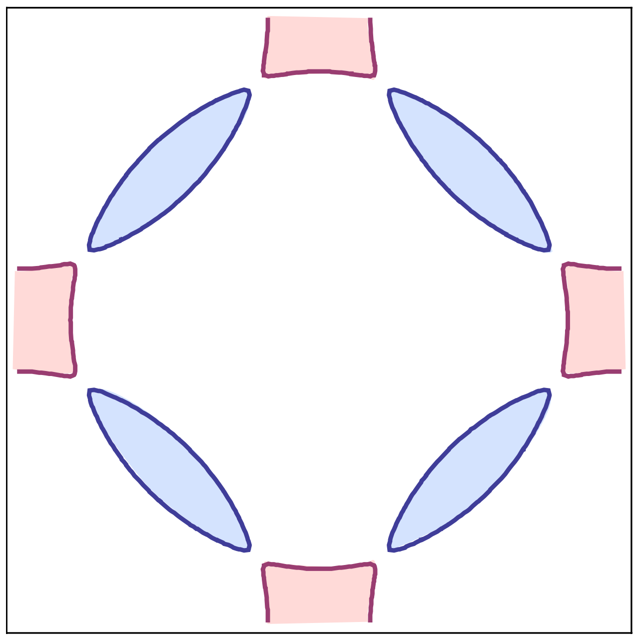
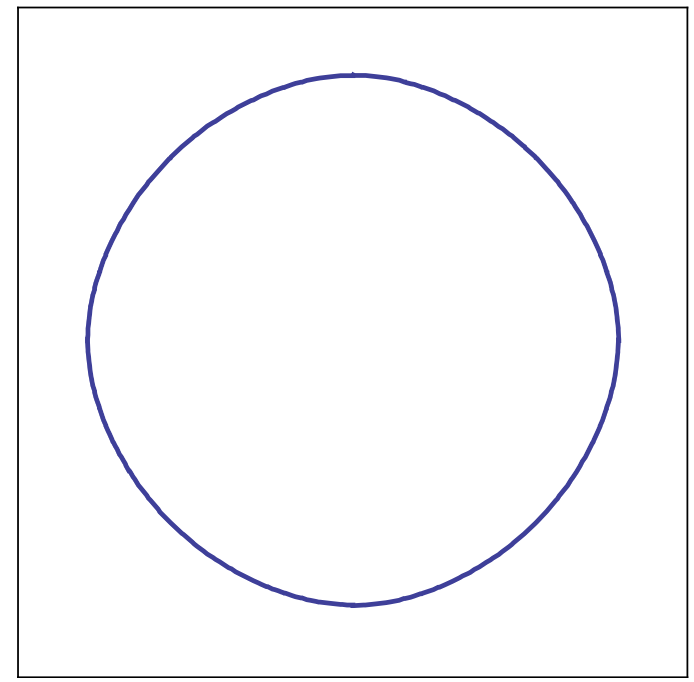


Quantum phase transition with onset of antiferromagnetism in a metal





$\langle \vec{\varphi} \rangle \neq 0$

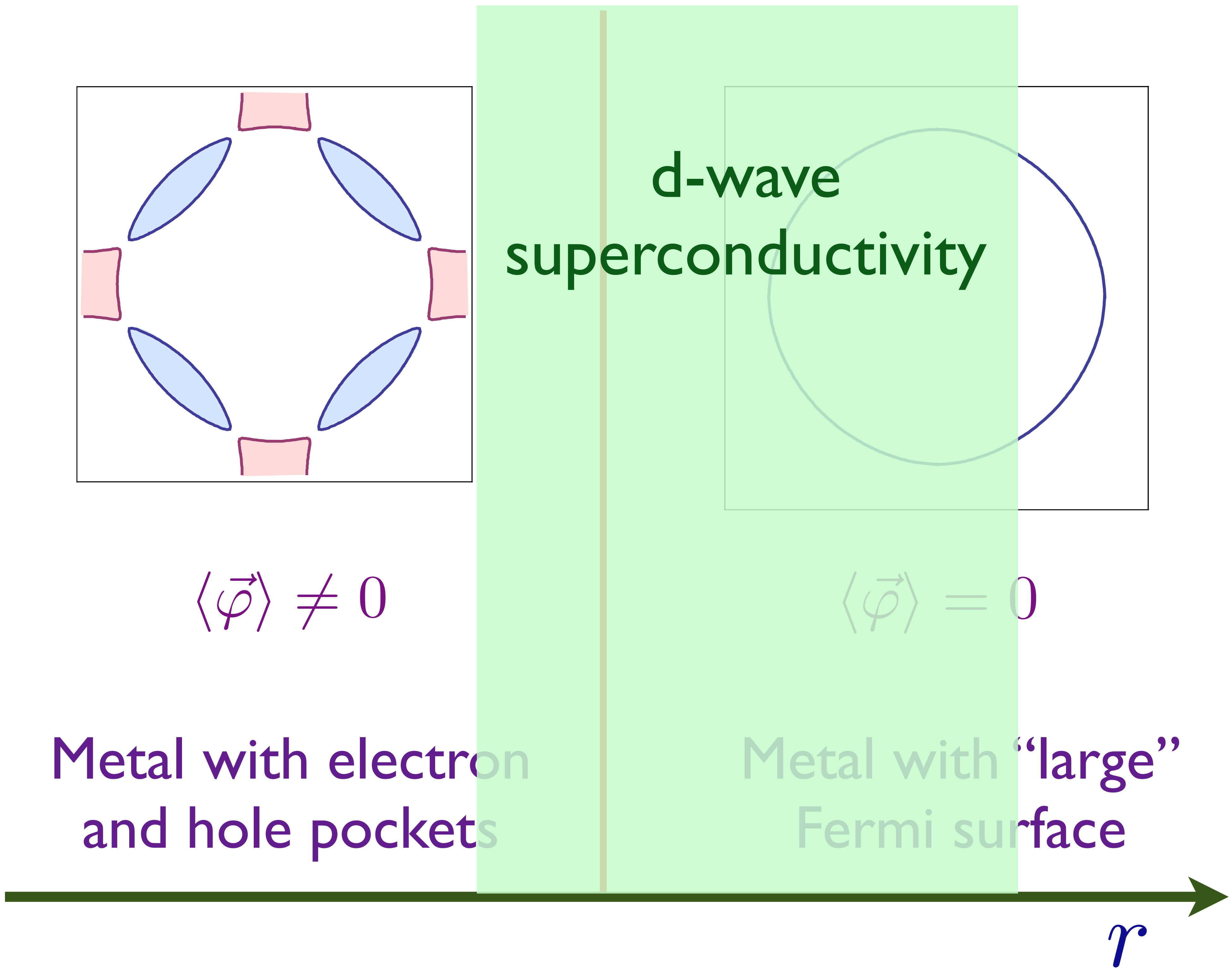
$\langle \vec{\varphi} \rangle = 0$

Metal with electron and hole pockets

Metal with "large" Fermi surface

r

Quantum phase transition with onset of antiferromagnetism in a metal



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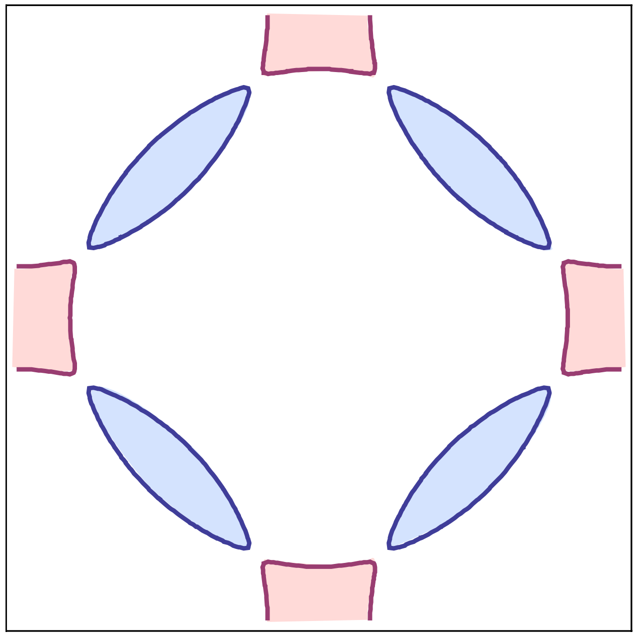

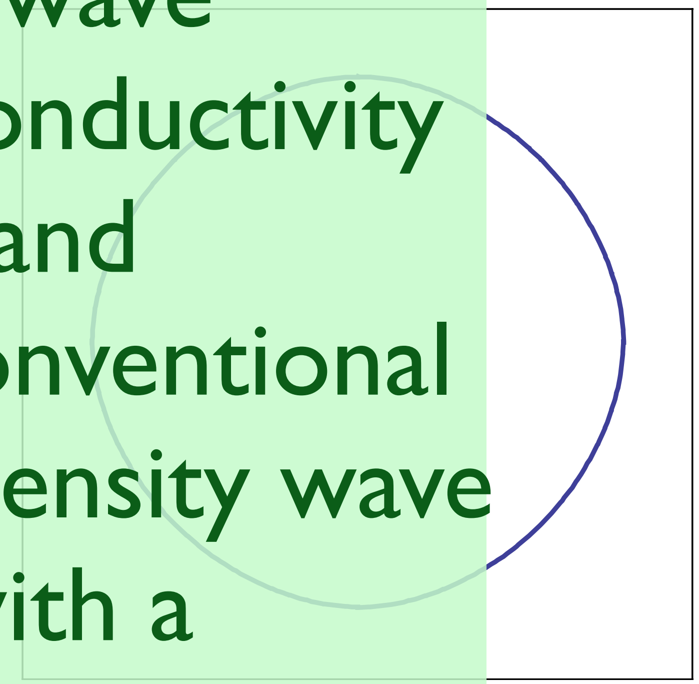

d-wave
superconductivity
and
an unconventional
charge density wave
with a
d-wave form factor

$\langle \vec{\varphi} \rangle \neq 0$

$\langle \vec{\varphi} \rangle = 0$

Metal with electron and hole pockets

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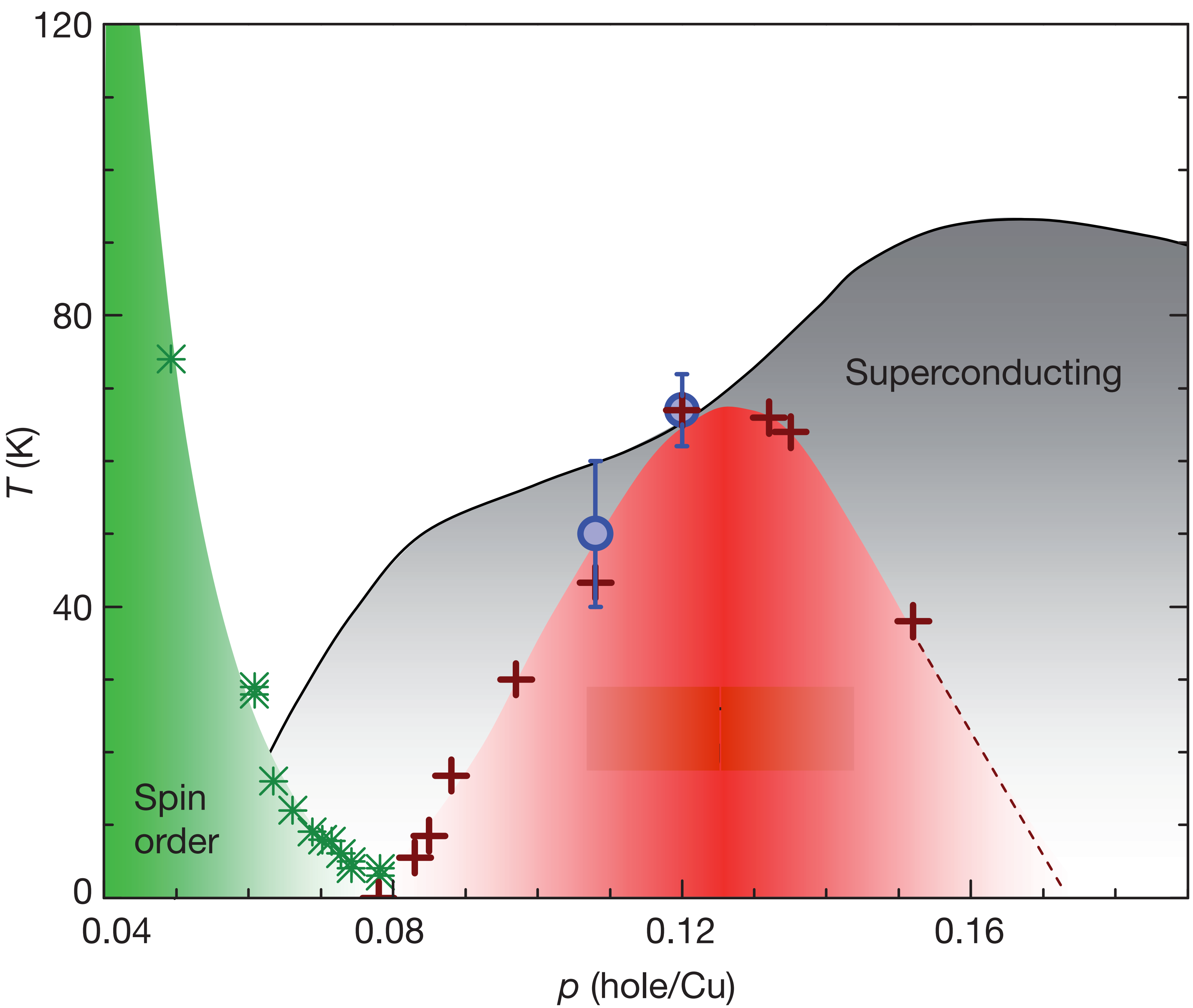

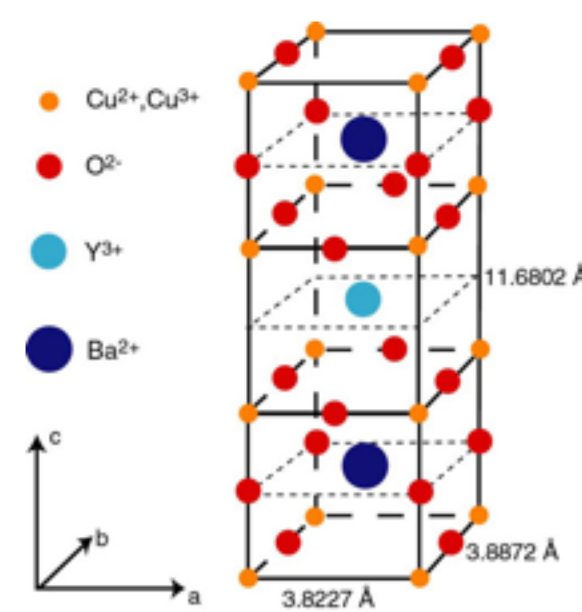

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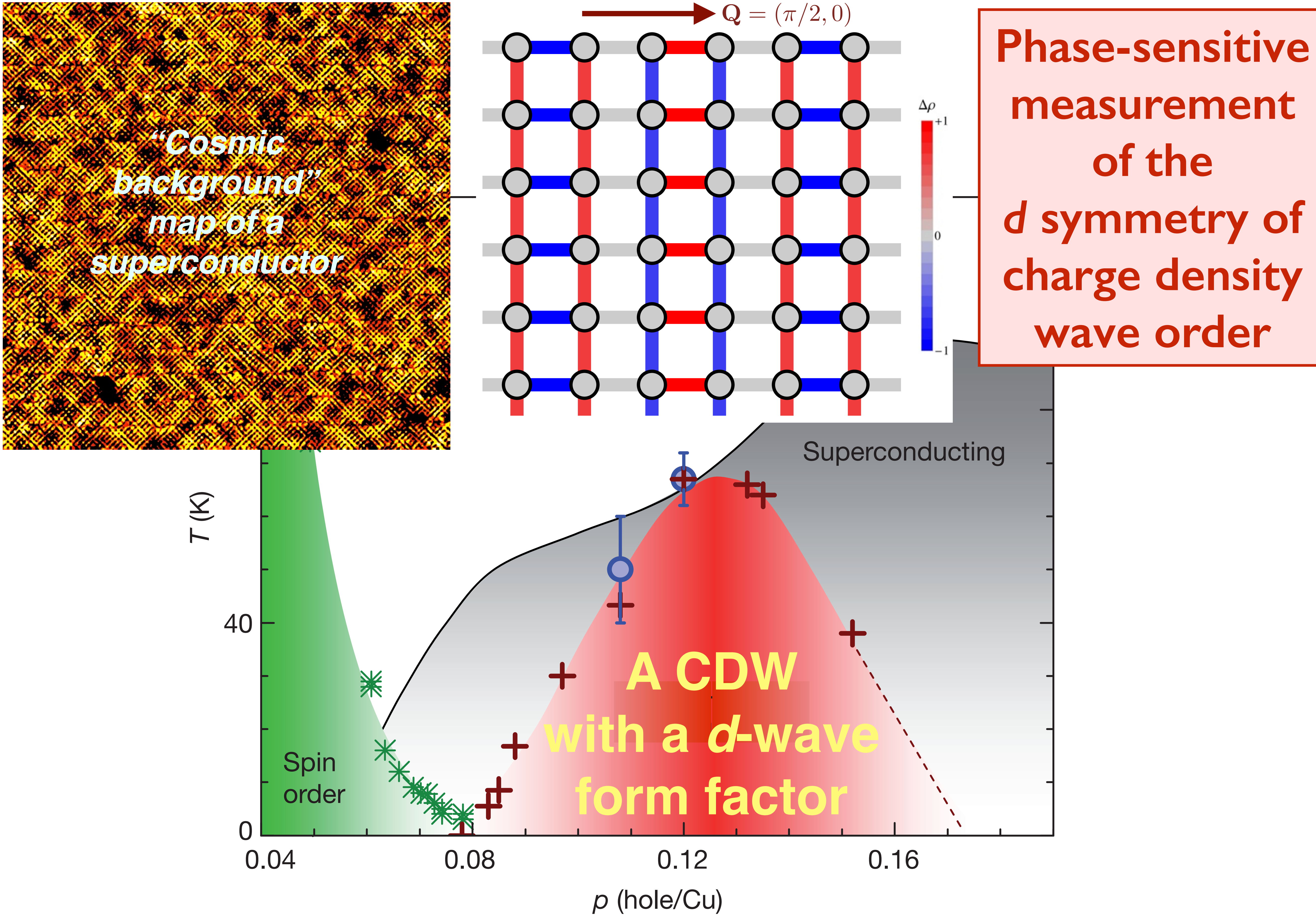

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