that the lowest energy is achieved when the vector has opposite orientations on the A and B sublattices. Anticipating this, we look for a continuum limit in terms of a field φ^a where

$$J_A^a = \varphi^a \quad , \quad J_B^a = -\varphi^a \tag{13}$$

The coupling between the field φ^a and the Ψ fermions is given by

$$\begin{aligned} \sum_i J_i^a c_{i\alpha}^\dagger \sigma_{\alpha\beta}^a c_{i\beta} &= \varphi^a \left(c_{A\alpha}^\dagger \sigma_{\alpha\beta}^a c_{A\beta} - c_{B\alpha}^\dagger \sigma_{\alpha\beta}^a c_{B\beta} \right) \\ &= \varphi^a \Psi^\dagger \tau^z \sigma^a \Psi = -\varphi^a \overline{\Psi} \rho^z \sigma^a \Psi \end{aligned} \tag{14}$$

From this we motivate the low energy theory

$$\mathcal{L} = \overline{\Psi} \gamma_\mu \partial_\mu \Psi + \frac{1}{2} \left[(\partial_\mu \varphi^a)^2 + s \varphi^{a2} \right] + \frac{u}{24} \left(\varphi^{a2} \right)^2 - \lambda \varphi^a \overline{\Psi} \rho^z \sigma^a \Psi \tag{15}$$

Note that the matrix $\rho^z \sigma^a$ commutes with all the γ_μ; hence $\rho^z \sigma^a$ is a matrix in "flavor" space. This is the Gross-Neveu model, and it describes the quantum phase transition from the Dirac semi-metal to an insulating Néel state In mean-field theory, the