

Exercise: Observe that \mathcal{L}_0 is invariant under the scaling transformation $x' = xe^{-\ell}$ and $\tau' = \tau e^{-\ell}$. Write the Hubbard interaction U in terms of the Dirac fermions, and show that it has the tree-level scaling transformation $U' = Ue^{-\ell}$. So argue that all short-range interactions are *irrelevant* in the Dirac semi-metal phase.

Antiferromagnetism

We use the operator equation (valid on each site i):

$$U \left(n_{\uparrow} - \frac{1}{2} \right) \left(n_{\downarrow} - \frac{1}{2} \right) = -\frac{2U}{3} S_i^{a2} + \frac{U}{4} \quad (11)$$

Then we decouple the interaction via

$$\exp \left(\frac{2U}{3} \sum_i \int d\tau S_i^{a2} \right) = \int \mathcal{D}J_i^a(\tau) \exp \left(- \sum_i \int d\tau \left[\frac{3}{8U} J_i^{a2} - J_i^a S_i^a \right] \right) \quad (12)$$

We now integrate out the fermions, and look for the saddle point of the resulting effective action for J_i^a . At the saddle-point we find