

$$H_J = J \vec{S}_1 \cdot \vec{S}_2$$

with  $\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}$  is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow}^\dagger \end{pmatrix} \quad , \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c_{i\uparrow}^\dagger \end{pmatrix}$$

Then we can write

$$H_J = \frac{1}{8} J \left( \Psi_{1\alpha a}^\dagger \vec{\sigma}_{\alpha\beta} \Psi_{1\beta a} \right) \cdot \left( \Psi_{2\gamma b}^\dagger \vec{\sigma}_{\gamma\delta} \Psi_{2\delta b} \right)$$

where  $a, b$  are the Nambu indices. This form makes explicit the symmetry under *independent* SU(2) pseudospin transformations on each site  $\Psi_{i\alpha a} \rightarrow U_{i,ab} \Psi_{i\alpha b}$