Holography of compressible quantum states

New England String Meeting,
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PHYSICS

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sachdev.physics.harvard.edu

Liza Huijse

Max Metlitski

Brian Swingle

Compressible quantum matter

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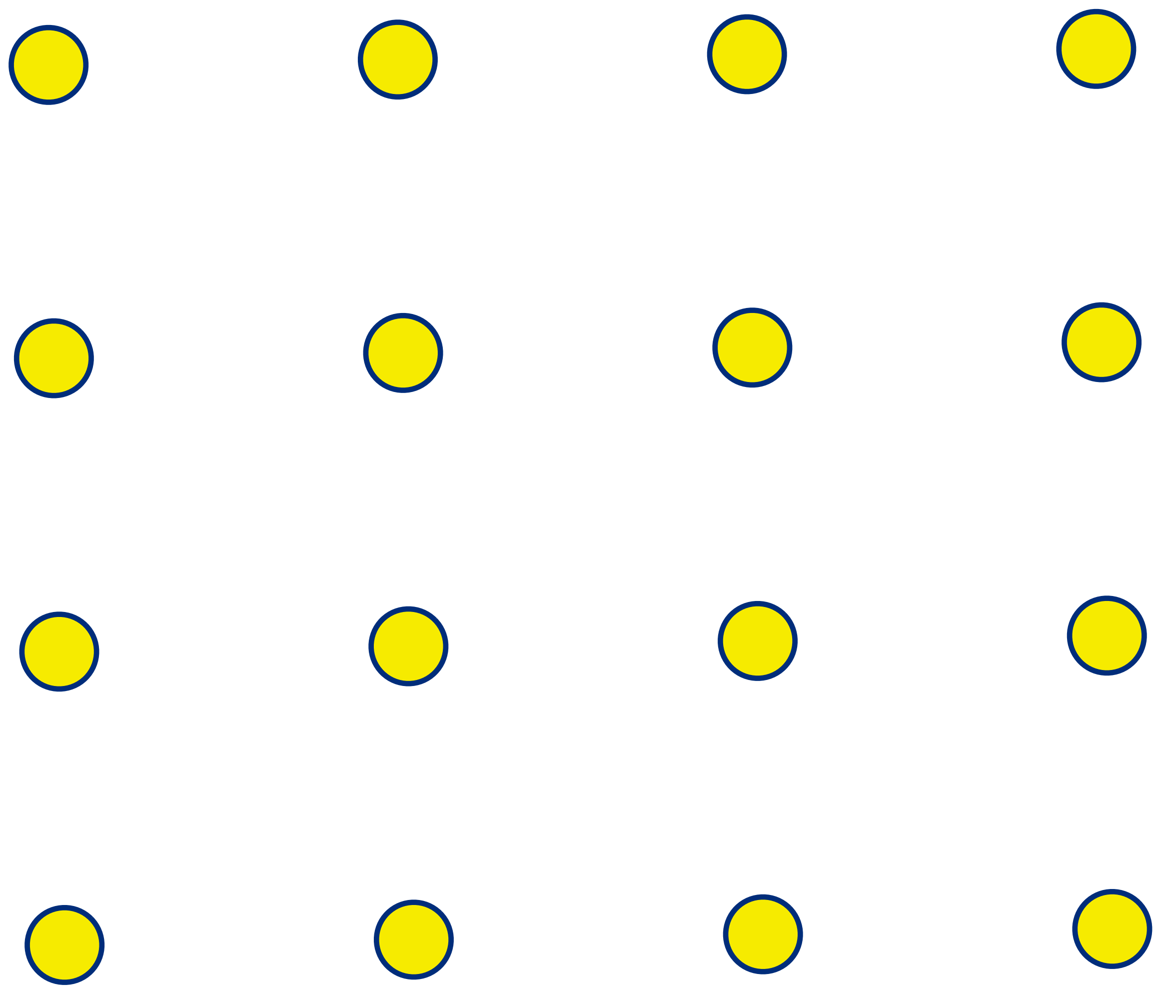
- Conformal systems are compressible in $d = 1$, but not for $d > 1$.

Compressible quantum matter

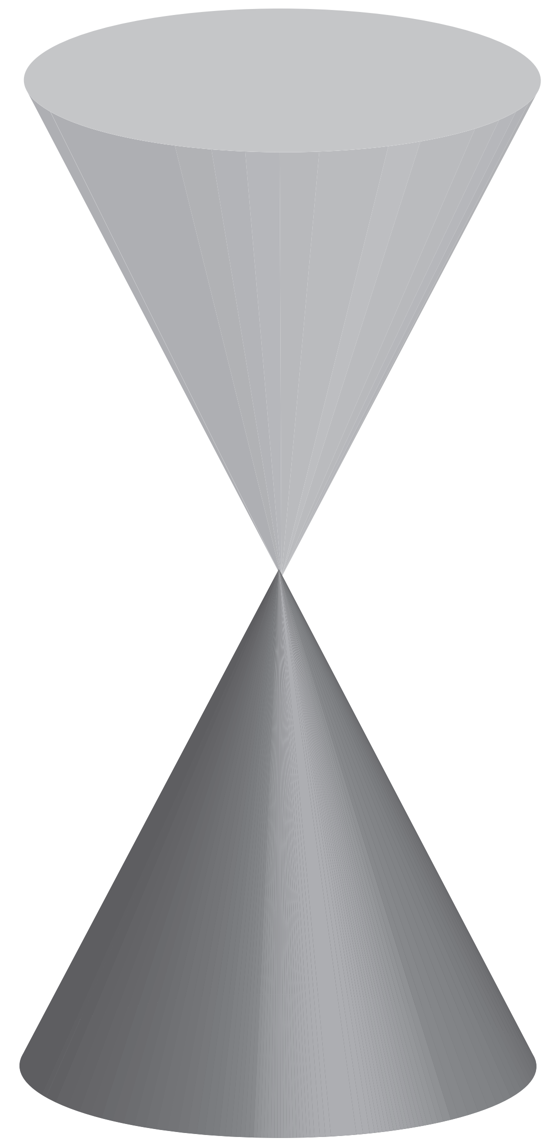
One compressible state is the **solid** (or "Wigner crystal" or "stripe").
This state breaks translational symmetry.



Compressible quantum matter

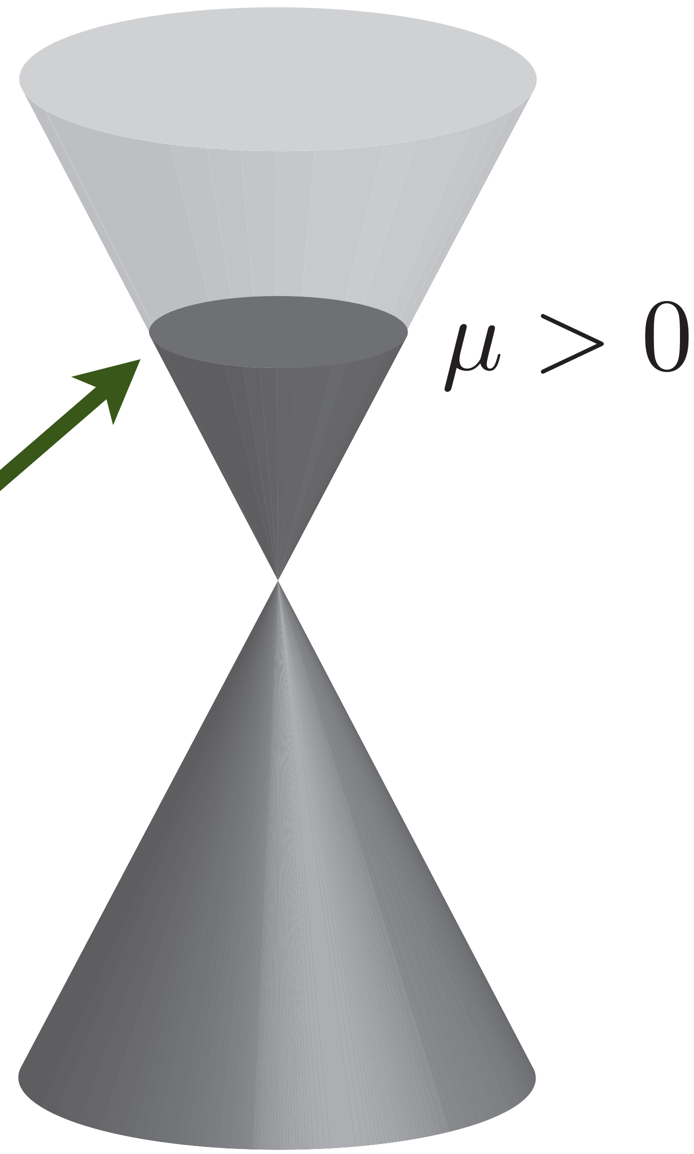
Another familiar compressible state is the **superfluid**.
This state breaks the global U(1) symmetry associated with $\mathcal{Q}$

Condensate of fermion pairs



Graphene

The only other familiar compressible phase is a **Fermi Liquid** with a **Fermi surface**



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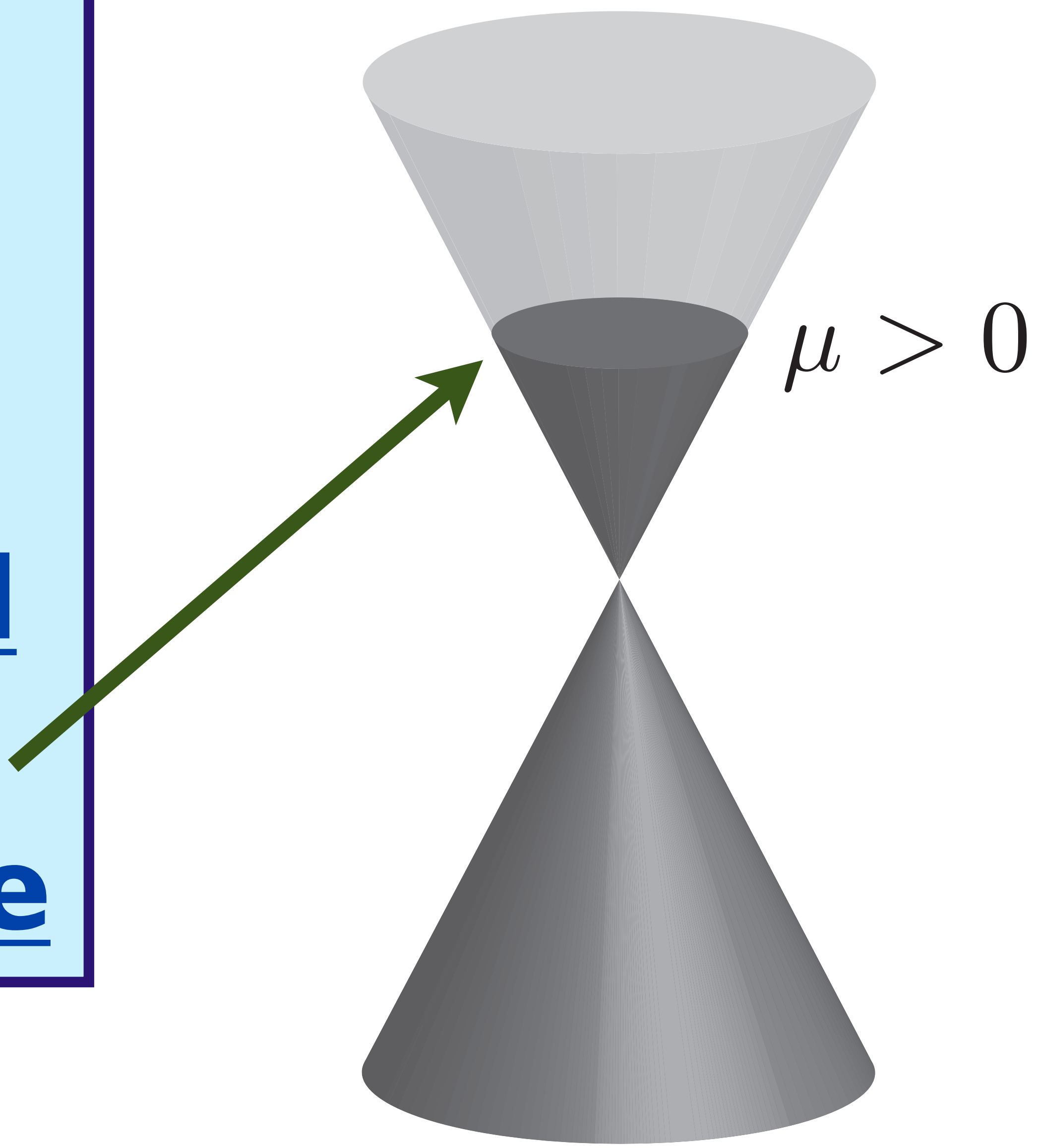
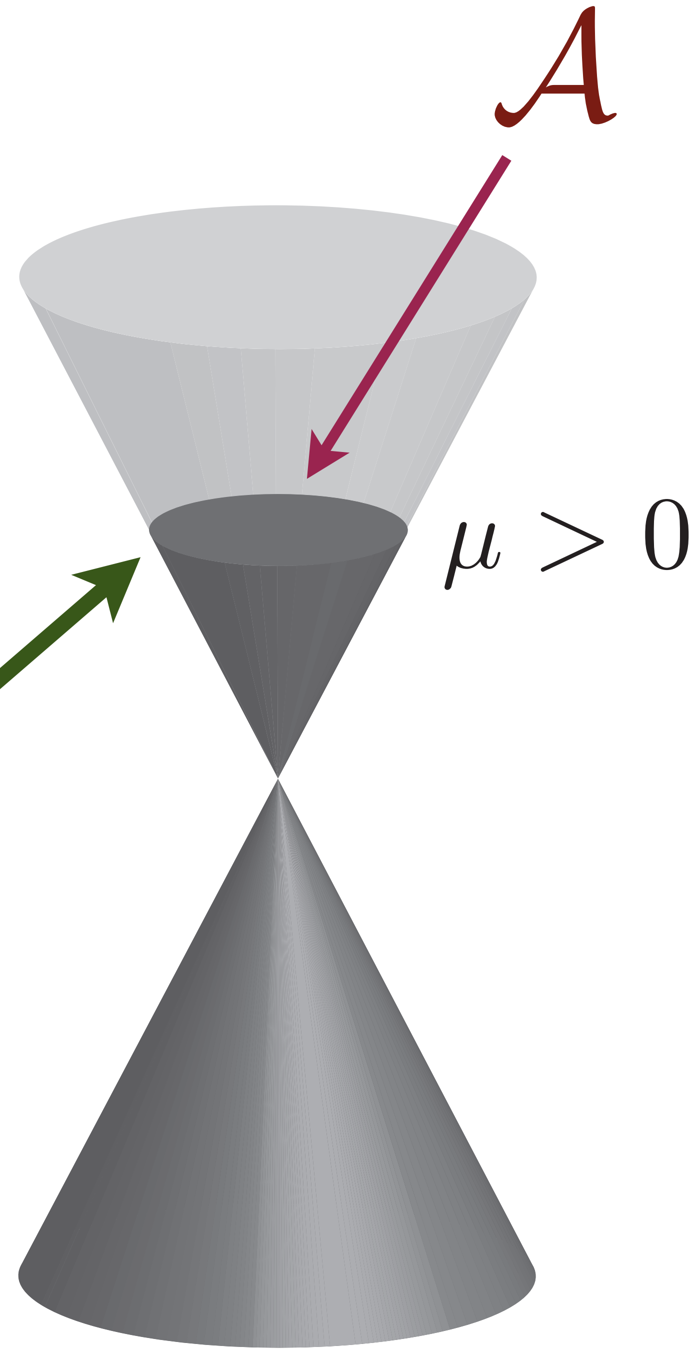


- The *only* low energy excitations are long-lived quasiparticles near the Fermi surface.

The only other familiar compressible phase is a **Fermi Liquid** with a **Fermi surface**



- **Luttinger relation:** The total "volume (area)" $\mathcal{A}$ enclosed by the Fermi surface is equal to $\langle \mathcal{Q} \rangle$.

Exotic phases of compressible quantum matter

I. Field theory

II. Holography

Exotic phases of compressible quantum matter

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ABJM theory in D=2+1 dimensions

- $4N^2$ Weyl fermions carrying fundamental charges of $\mathrm{U}(N)\times\mathrm{U}(N)\times\mathrm{SU}(4)_R$.
- $4N^2$ complex bosons carrying fundamental charges of $\mathrm{U}(N)\times\mathrm{U}(N)\times\mathrm{SU}(4)_R$.
- $\mathcal{N} = 6$ supersymmetry

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Adding a chemical potential coupling to a SU(4) charge breaks supersymmetry and SU(4) invariance

Theory similar to ABJM

- U(1) gauge invariance and U(1) global symmetry
- Fermions, f_+ and f_- ("quarks"), carry U(1) gauge charges ± 1, and global U(1) charge 1.
- Bosons, b_+ and b_- ("squarks"), carry U(1) gauge charges ± 1, and global U(1) charge 1.
- No supersymmetry

L. Huijse and S. Sachdev, *Physical Review* D **84**, 026001 (2011).

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- Bosons, b_+ and b_- ("squarks"), carry U(1) gauge charges ± 1, and global U(1) charge 1.
- No supersymmetry
- Fermions, c ("mesinos"), gauge-invariant bound states of fermions and bosons carrying global U(1) charge 2.

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Theory similar to ABJM

$$\mathcal{L} = f_\sigma^\dagger \left[(\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m} - \mu \right] f_\sigma$$
$$+ \; b_\sigma^\dagger \left[(\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m_b} + \epsilon_1 - \mu \right] b_\sigma$$
$$+ \frac{u}{2} \left(b_\sigma^\dagger b_\sigma \right)^2 - g_1 \left(b_+^\dagger b_-^\dagger f_- f_+ + \text{H.c.} \right)$$

The index $\sigma = \pm 1$

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Theory similar to ABJM

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\begin{aligned}
\mathcal{L} &= f_\sigma^\dagger \left[(\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m} - \mu \right] f_\sigma \\
&+ b_\sigma^\dagger \left[(\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m_b} + \epsilon_1 - \mu \right] b_\sigma \\
&\quad + \frac{u}{2} \left(b_\sigma^\dagger b_\sigma \right)^2 - g_1 \left(b_+^\dagger b_-^\dagger f_- f_+ + \text{H.c.} \right) \\
&+ c^\dagger \left[\partial_\tau - \frac{\nabla^2}{2m_c} + \epsilon_2 - 2\mu \right] c \\
&\quad - g_2 \left[c^\dagger \left(f_+ b_- + f_- b_+ \right) + \text{H.c.} \right]
\end{aligned}
$$

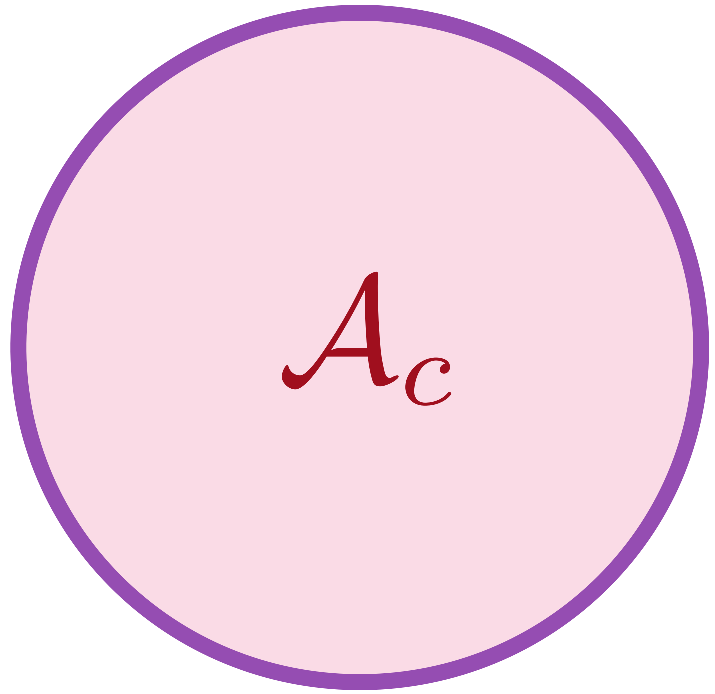
The index $\sigma = \pm 1$, and $\epsilon_{1,2}$ are tuning parameters of phase diagram

Conserved U(1) charge: $\mathcal{Q} = f_\sigma^\dagger f_\sigma + b_\sigma^\dagger b_\sigma + 2c^\dagger c$

L. Huijse and S. Sachdev, *Physical Review* D **84**, 026001 (2011).

Phases of ABJM-like theories

$\langle b_\pm \rangle = 0$



$$2\mathcal{A}_c = \langle \mathcal{Q} \rangle$$

Fermi liquid (FL)
with Fermi surface of gauge-neutral mesinos

U(1) gauge theory is in *confining* phase

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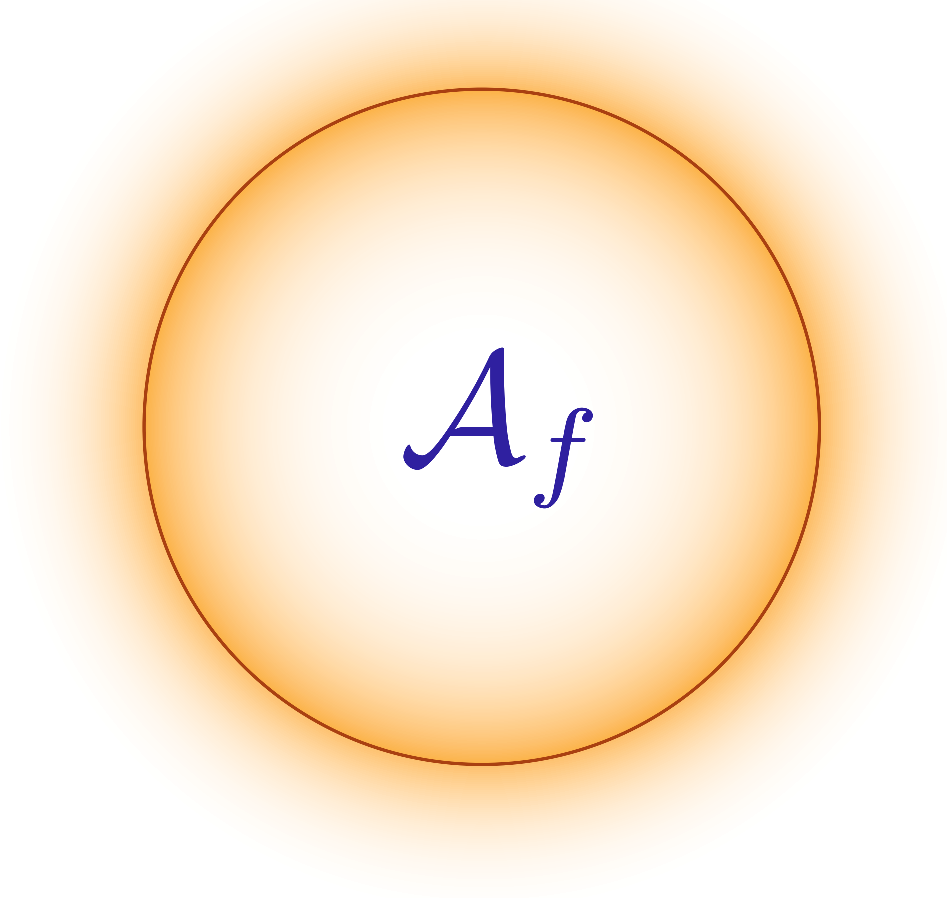

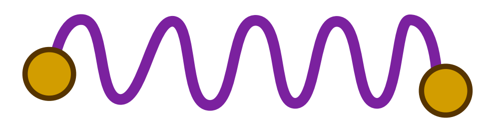

$$2\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

non-Fermi liquid (NFL)
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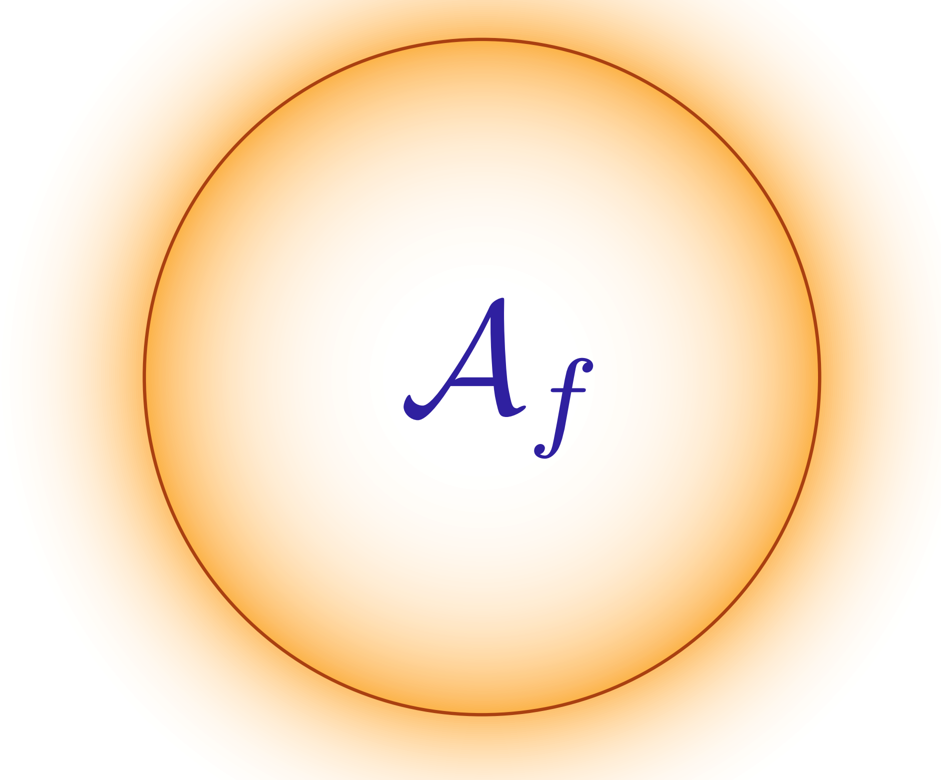

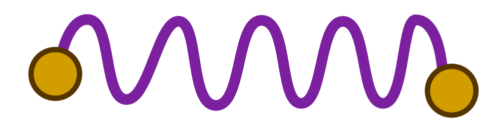

Fermi surface coupled to Abelian or non-Abelian gauge fields:

- Longitudinal gauge fluctuations are screened by the fermions.
- Transverse gauge fluctuations are unscreened, and Landau-damped. They are IR fluctuations with dynamic critical exponent $z > 1$.
- Theory is *strongly coupled in two spatial dimensions*.
- "Non-Fermi liquid" broadening of the fermion quasiparticle pole.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Phases of ABJM-like theories

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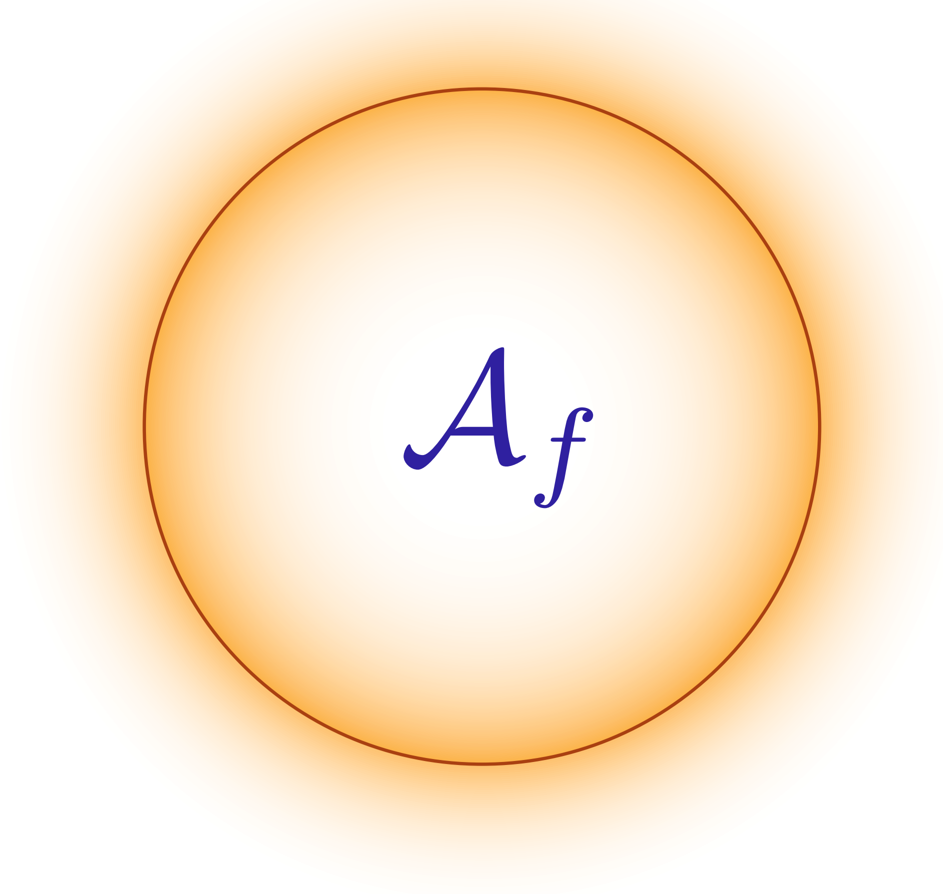

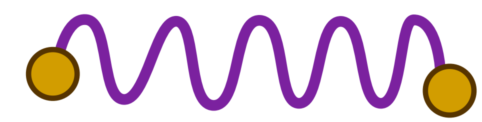

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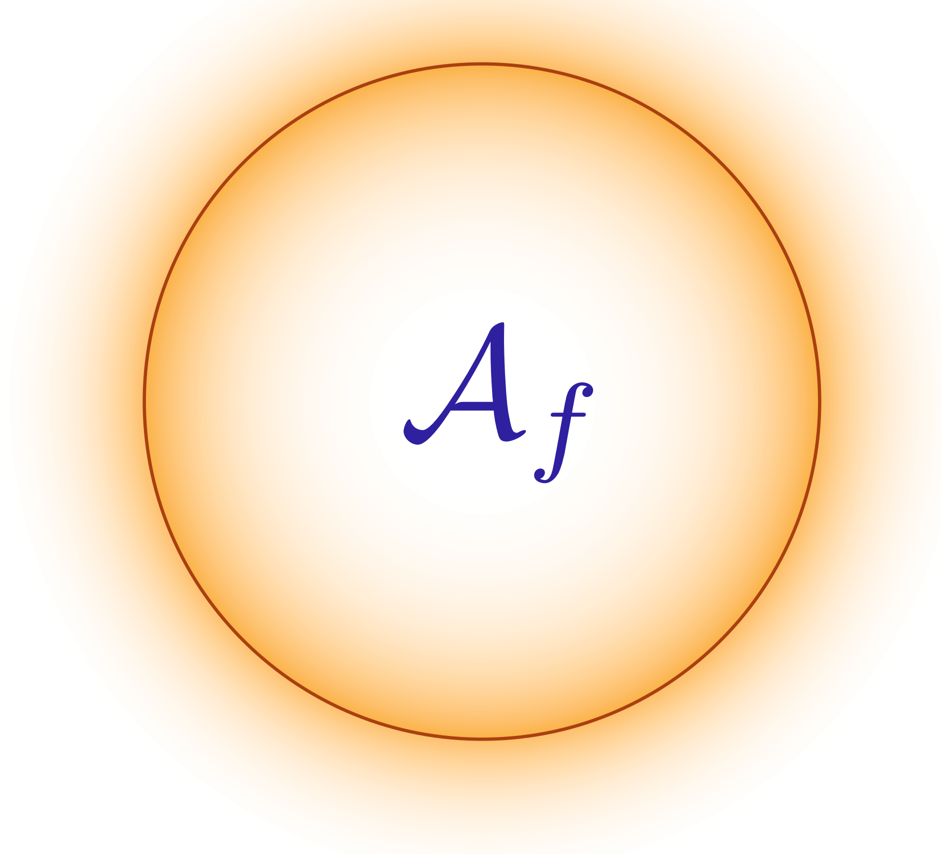

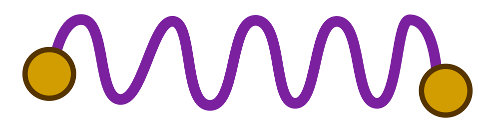

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"Hidden" Fermi surface

Phases of ABJM-like theories

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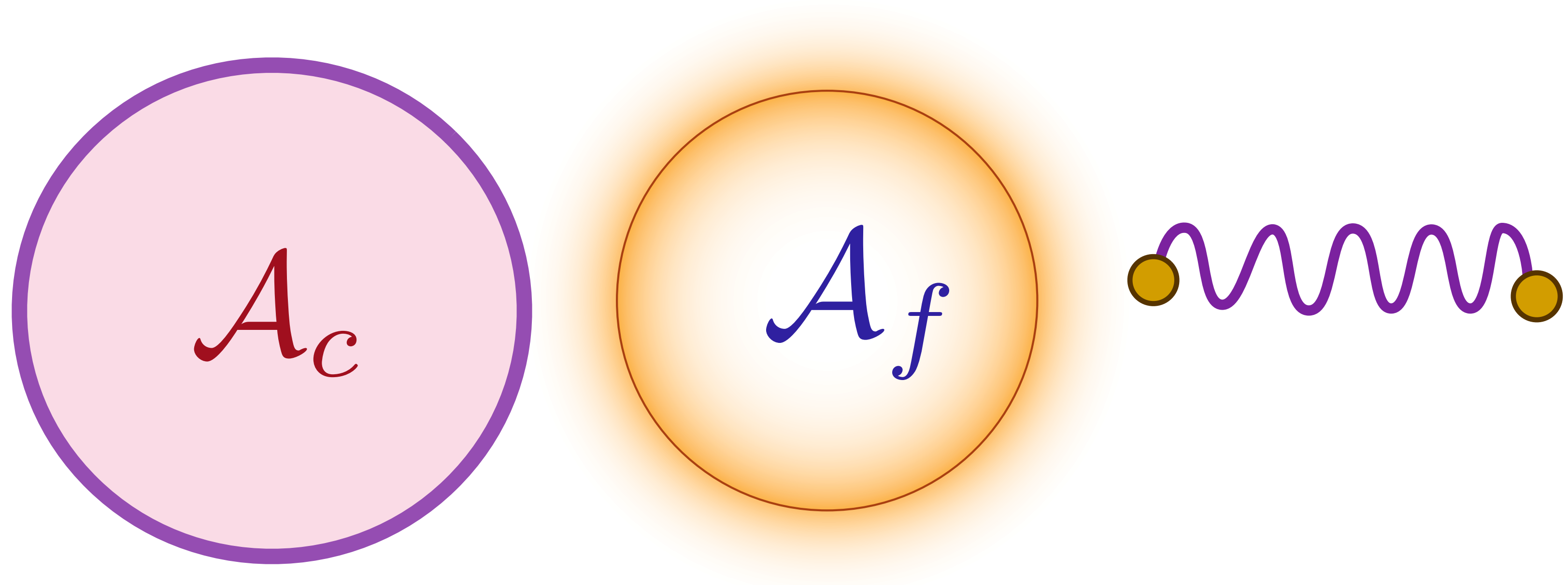

$$2\mathcal{A}_c + 2\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

Fractionalized Fermi liquid (FL*)
with Fermi surfaces of *both* quarks and mesinos

U(1) gauge theory is in *deconfined* phase

Phases of ABJM-like theories

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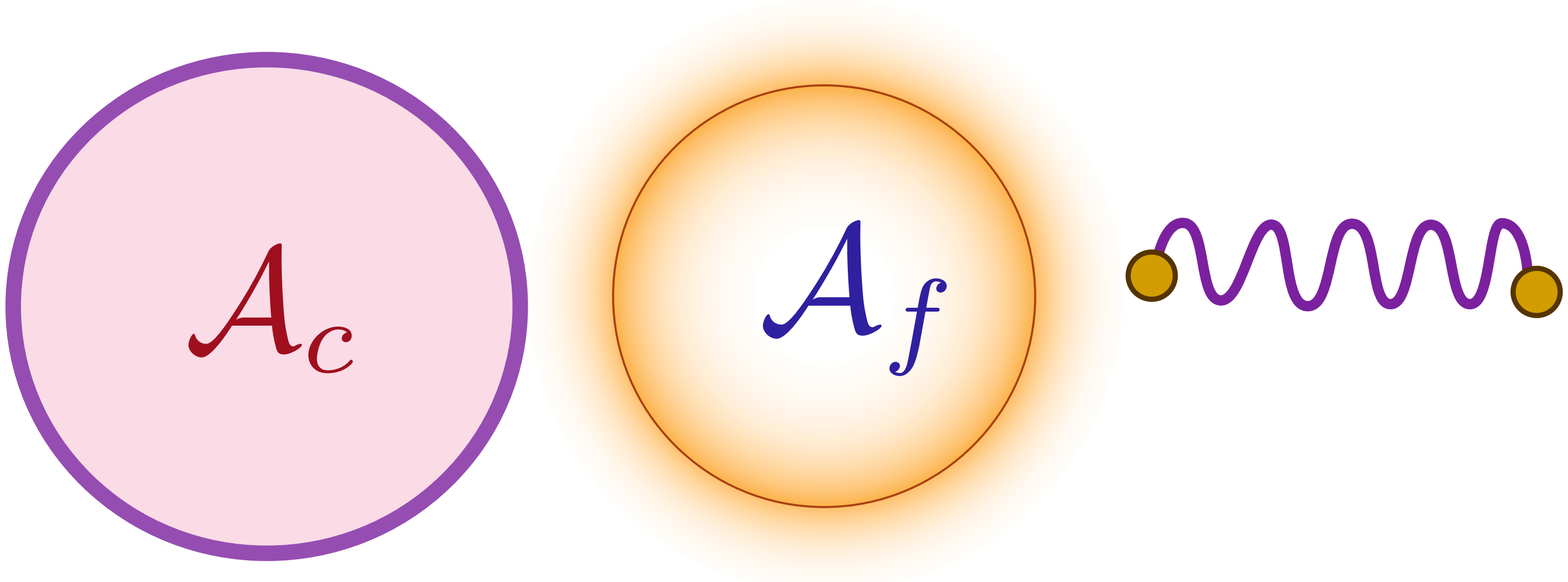

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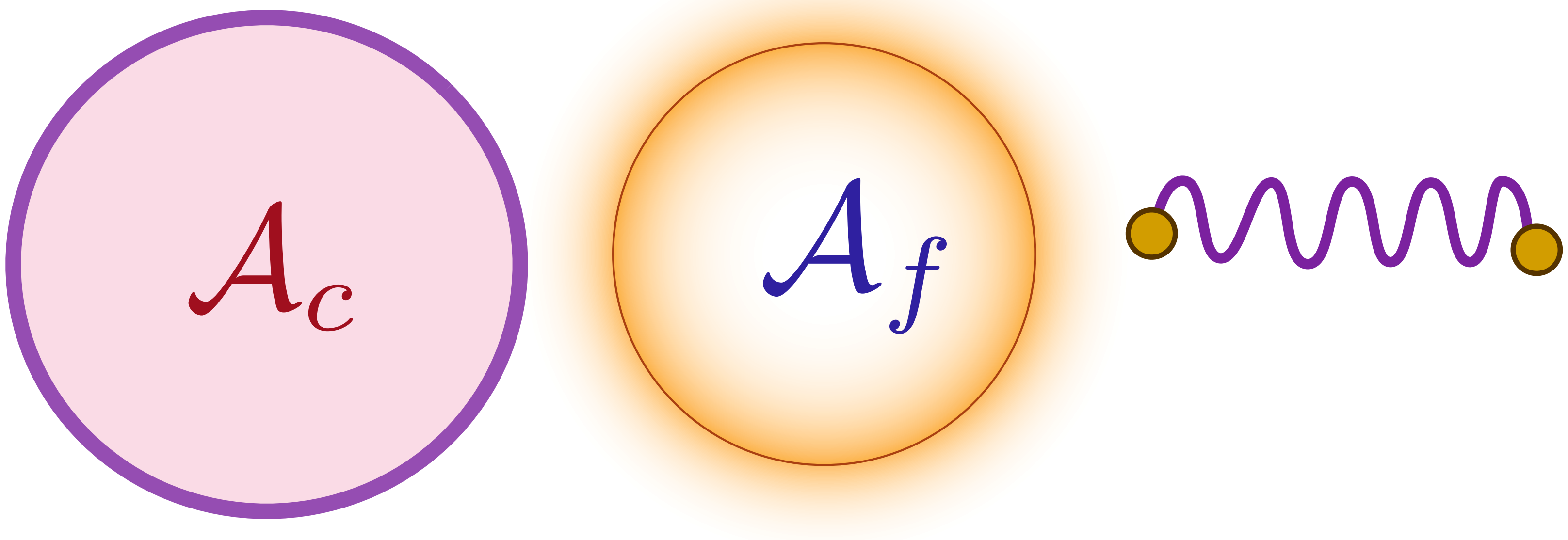

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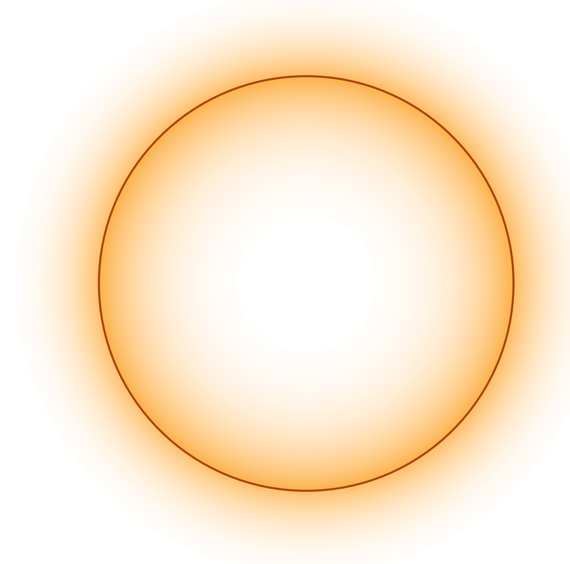
"Hidden" and visible Fermi surfaces co-exist

Key question:

How do we detect the
"hidden Fermi surfaces"
of fermions with gauge charges
in the non-Fermi liquid phases ?



These are not directly visible in the
gauge-invariant fermion correlations
computable via holography

One promising answer:

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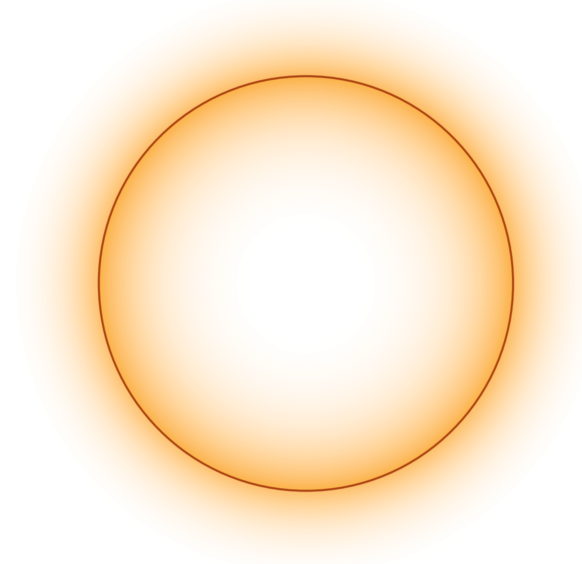
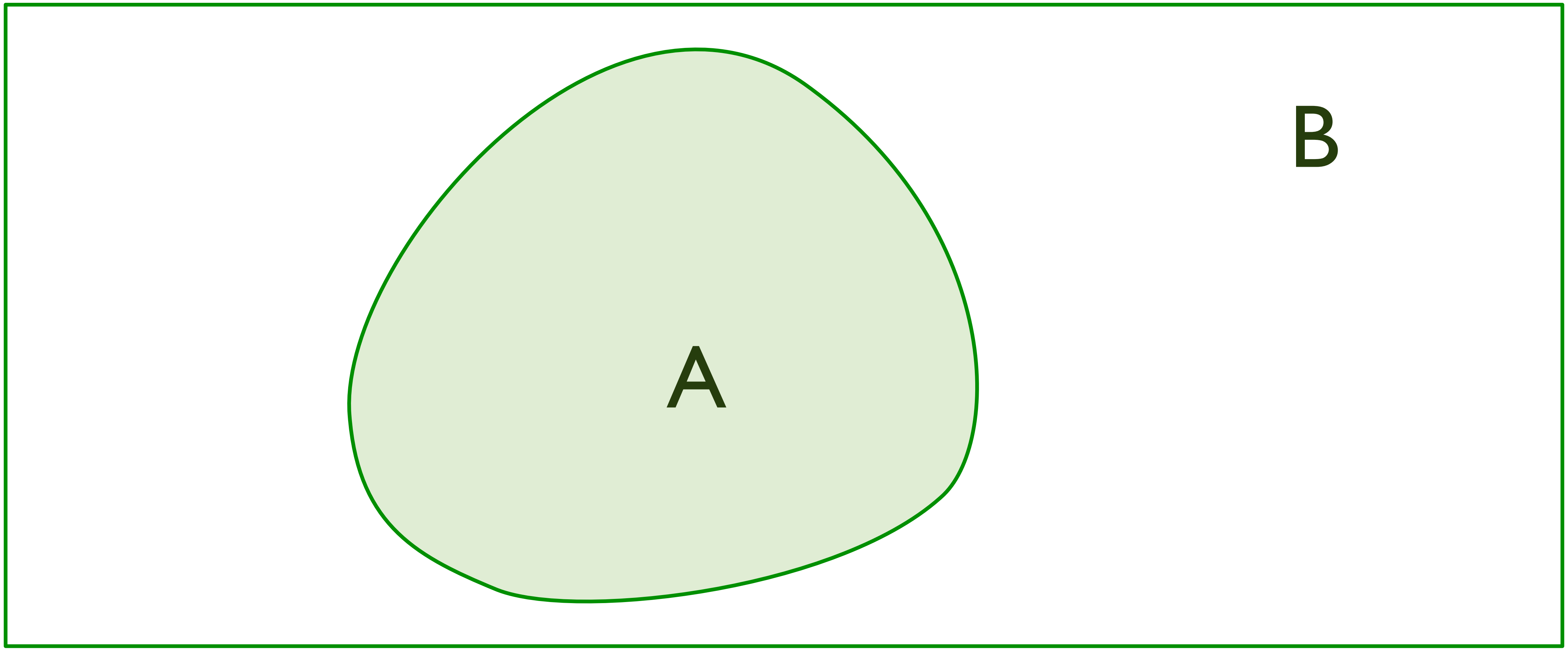


Compute
entanglement entropy

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023
L. Huijse, B. Swingle, and S. Sachdev to appear.

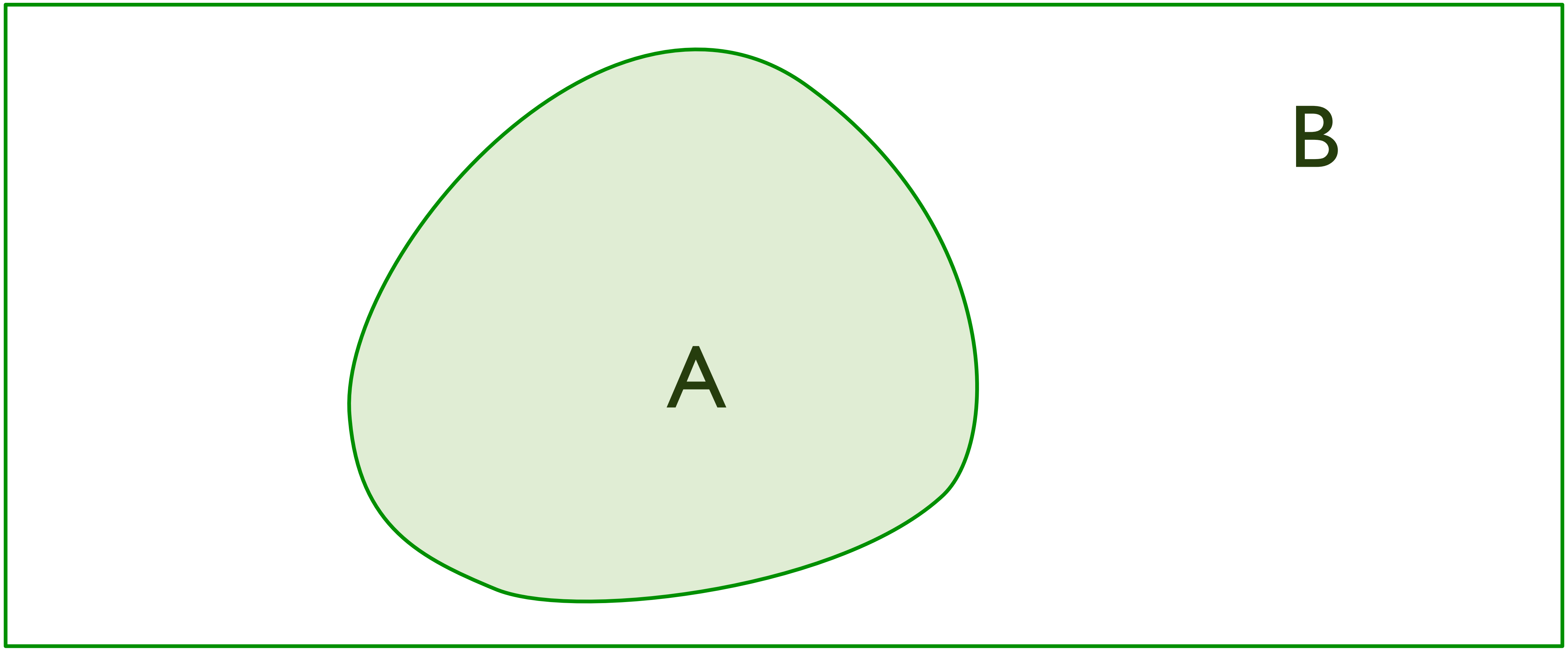
Entanglement entropy of Fermi surfaces



$$\rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_{EE} = -\text{Tr}\left(\rho_A \ln \rho_A\right)$$

Entanglement entropy of Fermi surfaces



Logarithmic violation of "area law": $S_{EE} = \frac{1}{12}(k_F P) \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

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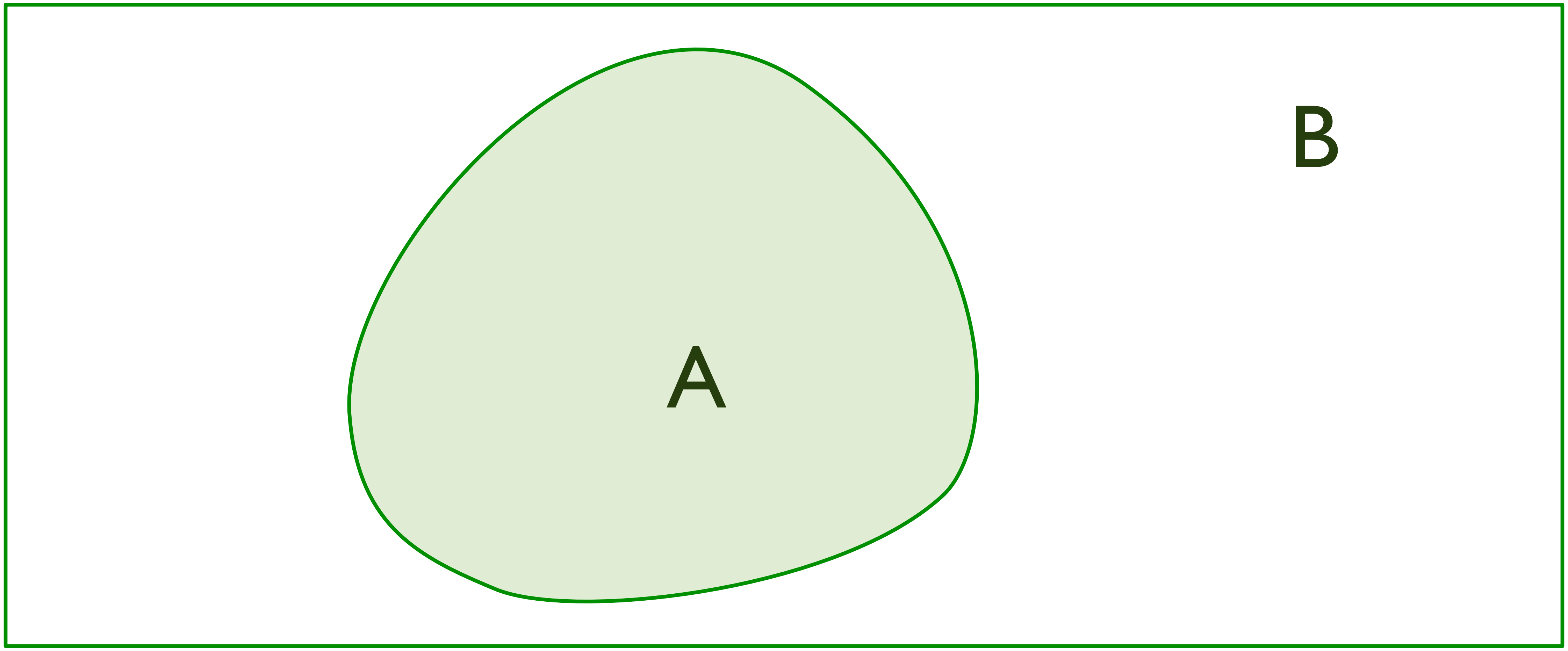

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Non-Fermi liquids have, at most, the "1/12" prefactor modified.

Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

Exotic phases of compressible quantum matter

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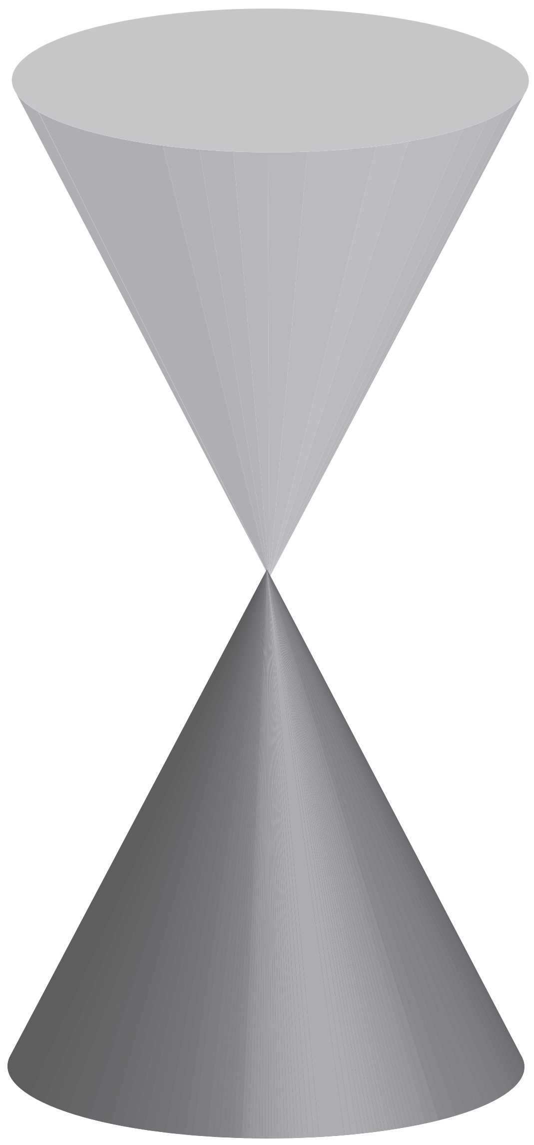
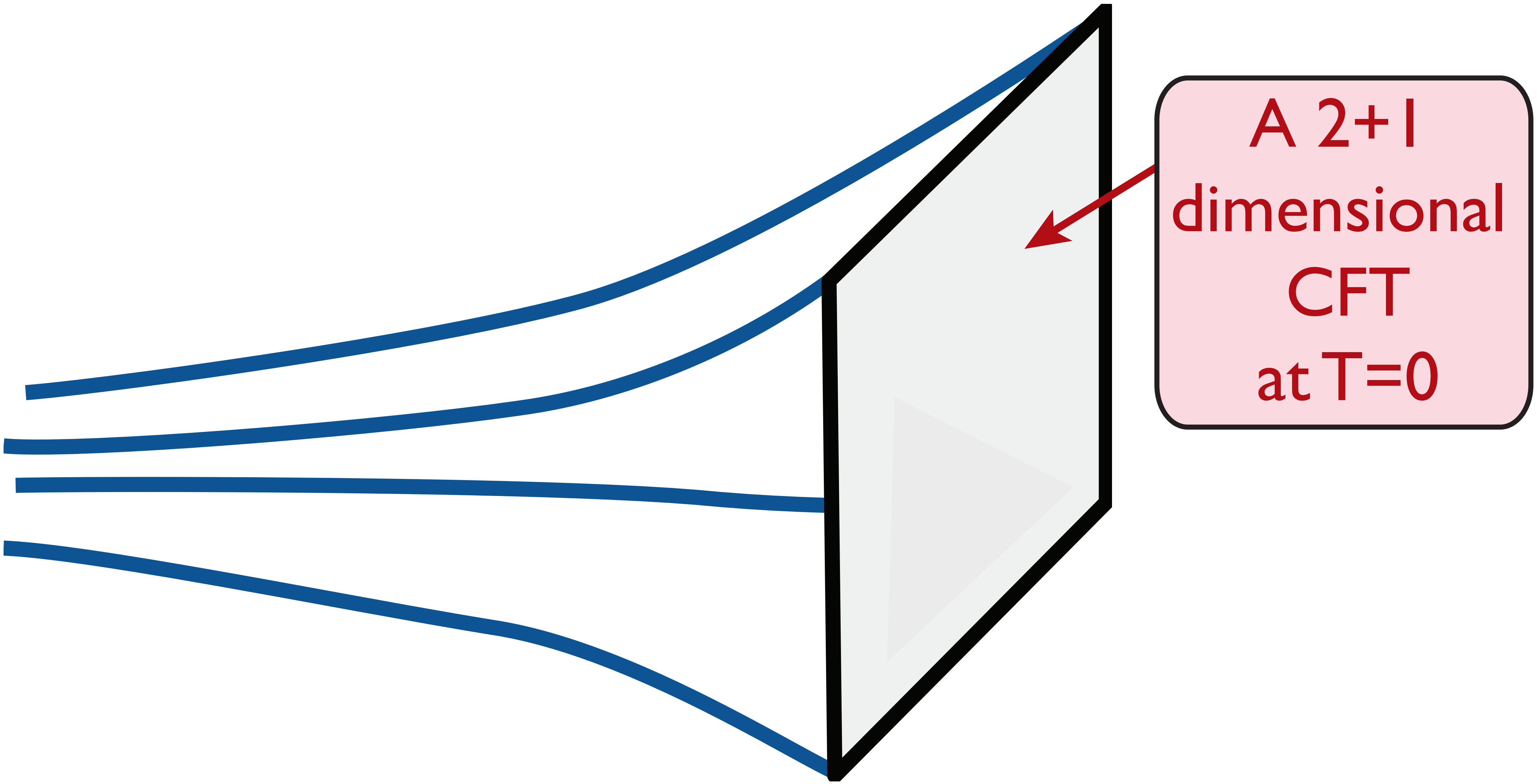
Begin with a CFT



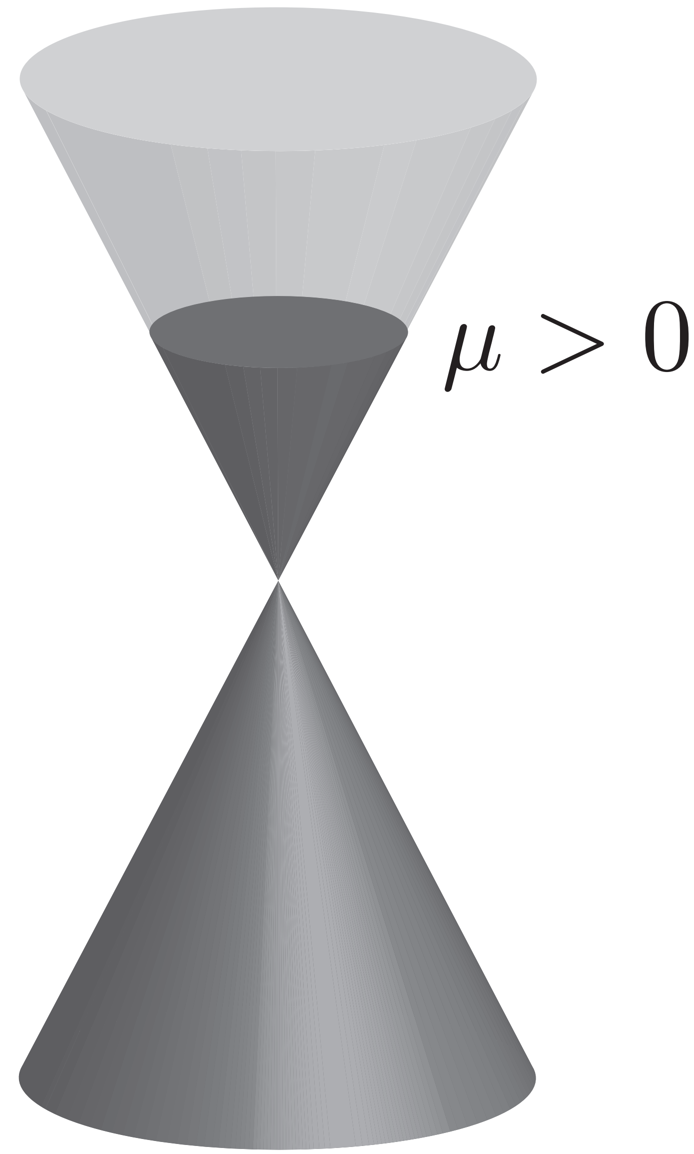
Dirac fermions + gauge field + ...

Holographic representation: AdS$_4$

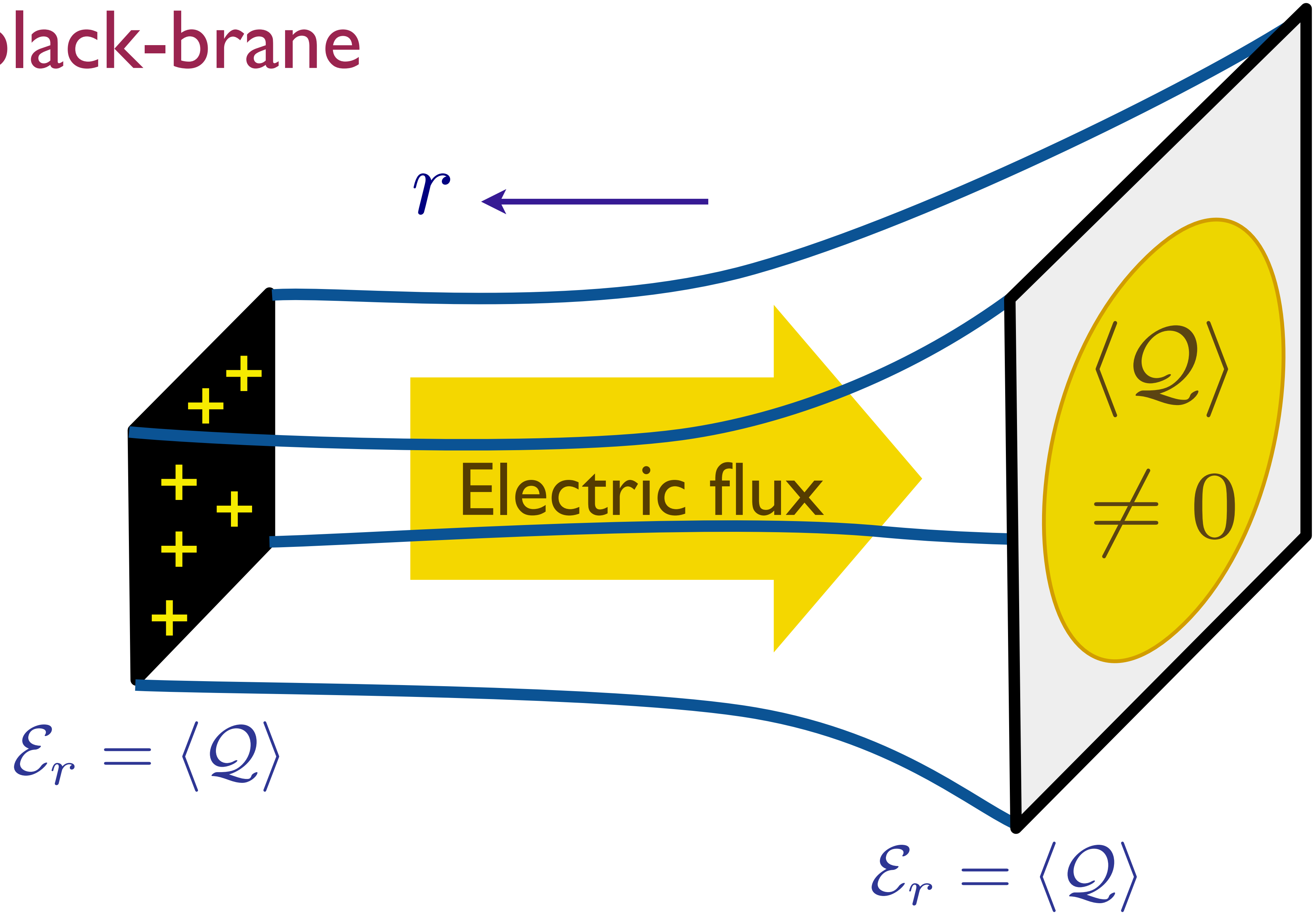


$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

Apply a chemical potential to the "deconfined" CFT



The Maxwell-Einstein theory of the applied chemical potential yields a AdS4-Reissner-Nordtröm black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4e^2} F_{ab} F^{ab} \right]$$

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Physical Review B **76**, 144502 (2007)

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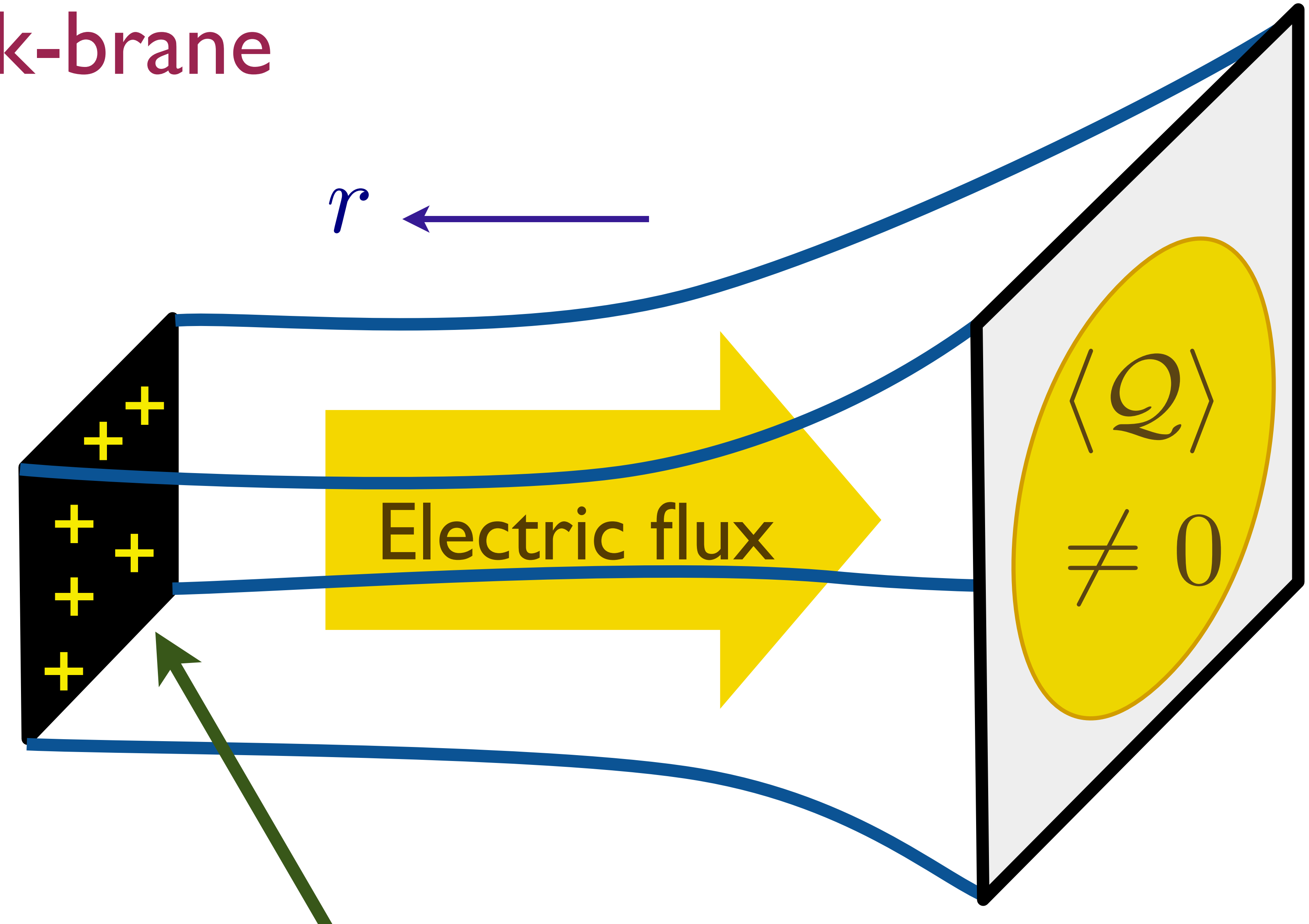


At $T = 0$, we obtain an extremal black-brane, with a near-horizon (IR) metric of $\mathrm{AdS}_2 \times R^2$

$$ds^2 = \frac{L^2}{6}\left(\frac{-dt^2 + dr^2}{r^2}\right) + dx^2 + dy^2$$

Artifacts of AdS_2 X R^2

- Non-zero entropy density at $T = 0$

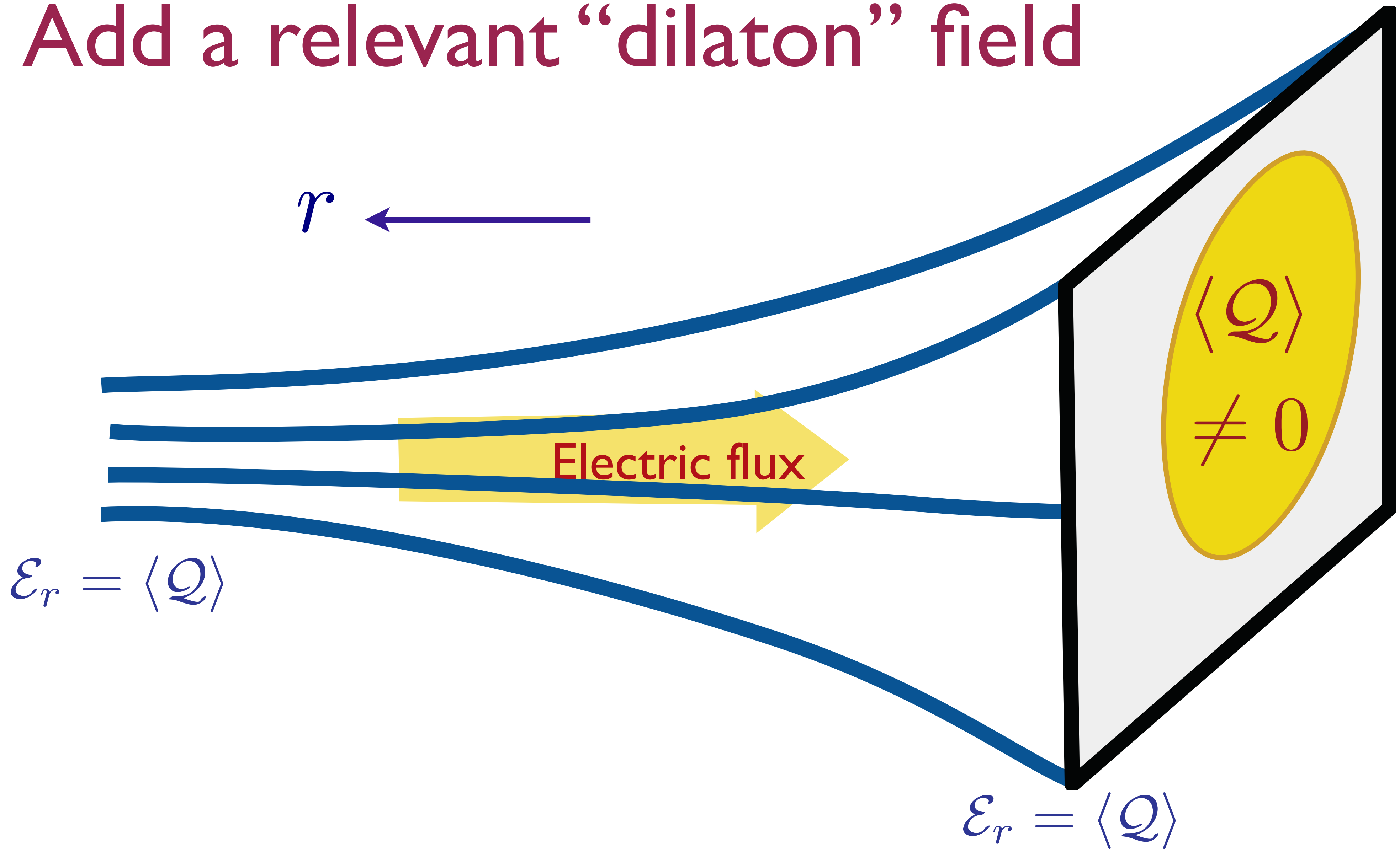
- Green's function of a probe fermion (a *mesino*) can have a Fermi surface, but self energies are momentum independent, and the singular behavior is the same on and off the Fermi surface

- Deficit of order $\sim N^2$ in the volume enclosed by the mesino Fermi surfaces: presumably associated with "hidden Fermi surfaces" of gauge-charged particles (the *quarks*).

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).

Holographic theory of a non-Fermi liquid (NFL)

Add a relevant "dilaton" field



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2} \right) - \frac{Z(\Phi)}{4e^2} F_{ab}F^{ab} \right]$$

with $Z(\Phi) = Z_0 e^{\alpha\Phi}$, $V(\Phi) = -V_0 e^{\delta\Phi}$, as $\Phi \to \infty$.

C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, arXiv:1105.1162 [hep-th].

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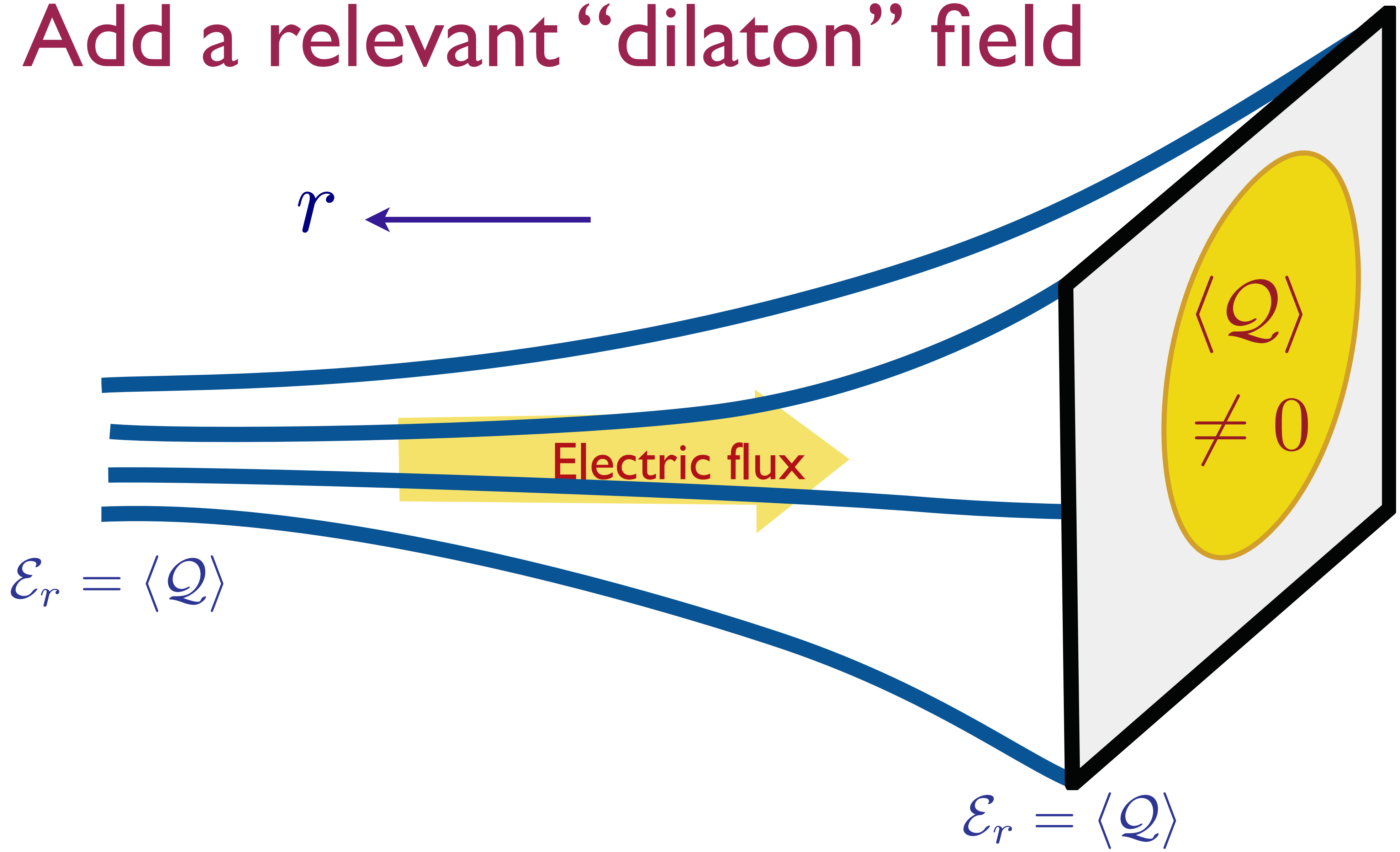

Leads to metric $ds^2 = L^2 \left(-f(r)dt^2 + g(r)dr^2 + \frac{dx^2 + dy^2}{r^2} \right)$

with $f(r) \sim r^{-\gamma}$, $g(r) \sim r^{\beta}$ as $r \to \infty$.

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Holographic theory of a non-Fermi liquid (NFL)

With the choice of the exponents, α, δ, a large zoo of NFL phases appear possible. But the fate of the Luttinger count of Fermi surfaces seems unclear.

Holographic theory of a non-Fermi liquid (NFL)

Key idea:

Restrict attention to those models in which there is logarithmic violation of area law in the entanglement entropy. This restricts $\delta = -\alpha/3$ and $g(r) \sim$ constant, as $r \to \infty$.

$$ds^2 = L^2 \left(-\frac{dt^2}{r^\gamma} + dr^2 + \frac{dx^2 + dy^2}{r^2} \right)$$

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023

Evidence for "hidden" Fermi surface:

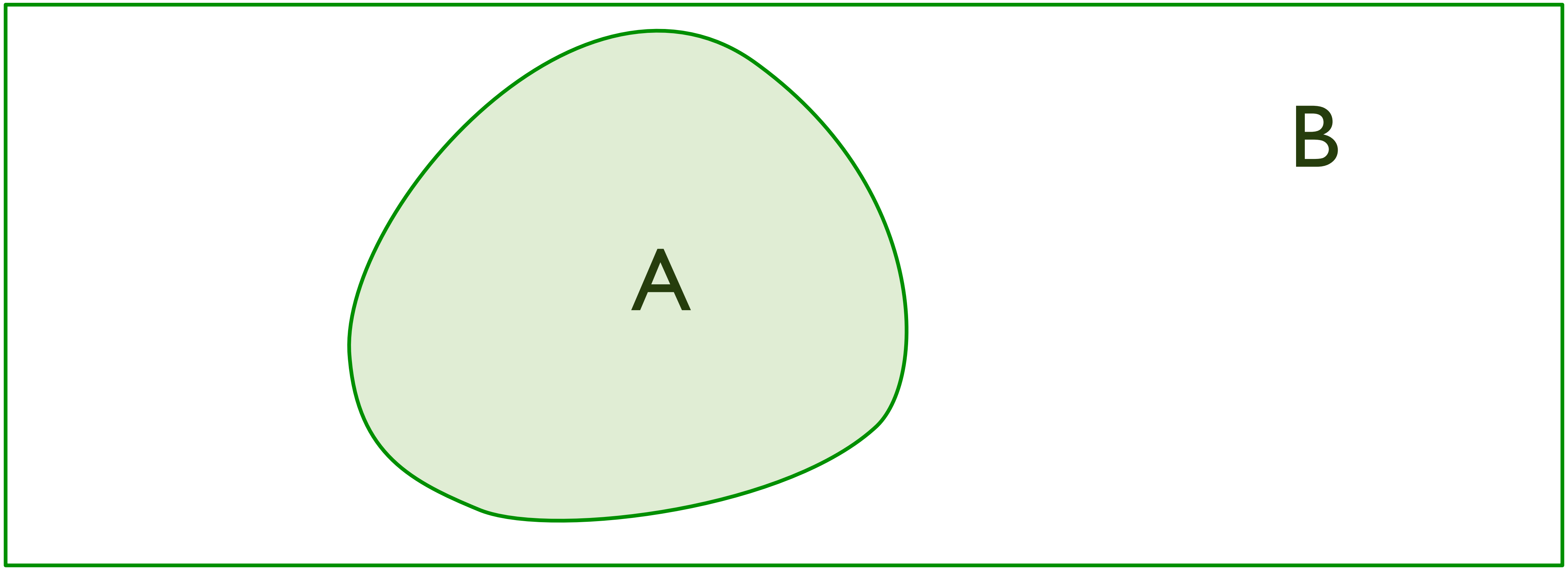
$$S_{EE} = F(\alpha)(P\sqrt{Q})\ln(P\sqrt{Q})$$

where P is the perimeter of the entangling region, and $F(\alpha)$ is a known function of α only.

- The dependence on Q and the shape of the entangling region is just as expected for a Fermi surface.

L. Huijse, B. Swingle, and S. Sachdev to appear.

Entanglement entropy of Fermi surfaces



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Holographic theory of a non-Fermi liquid (NFL) and a fractionalized Fermi liquid (FL*)

$$S_{EE} = F(\alpha)(P\sqrt{\mathcal{Q}})\ln(P\sqrt{\mathcal{Q}})$$

where P is the perimeter of the entangling region, and $F(\alpha)$ is a known function of α only.

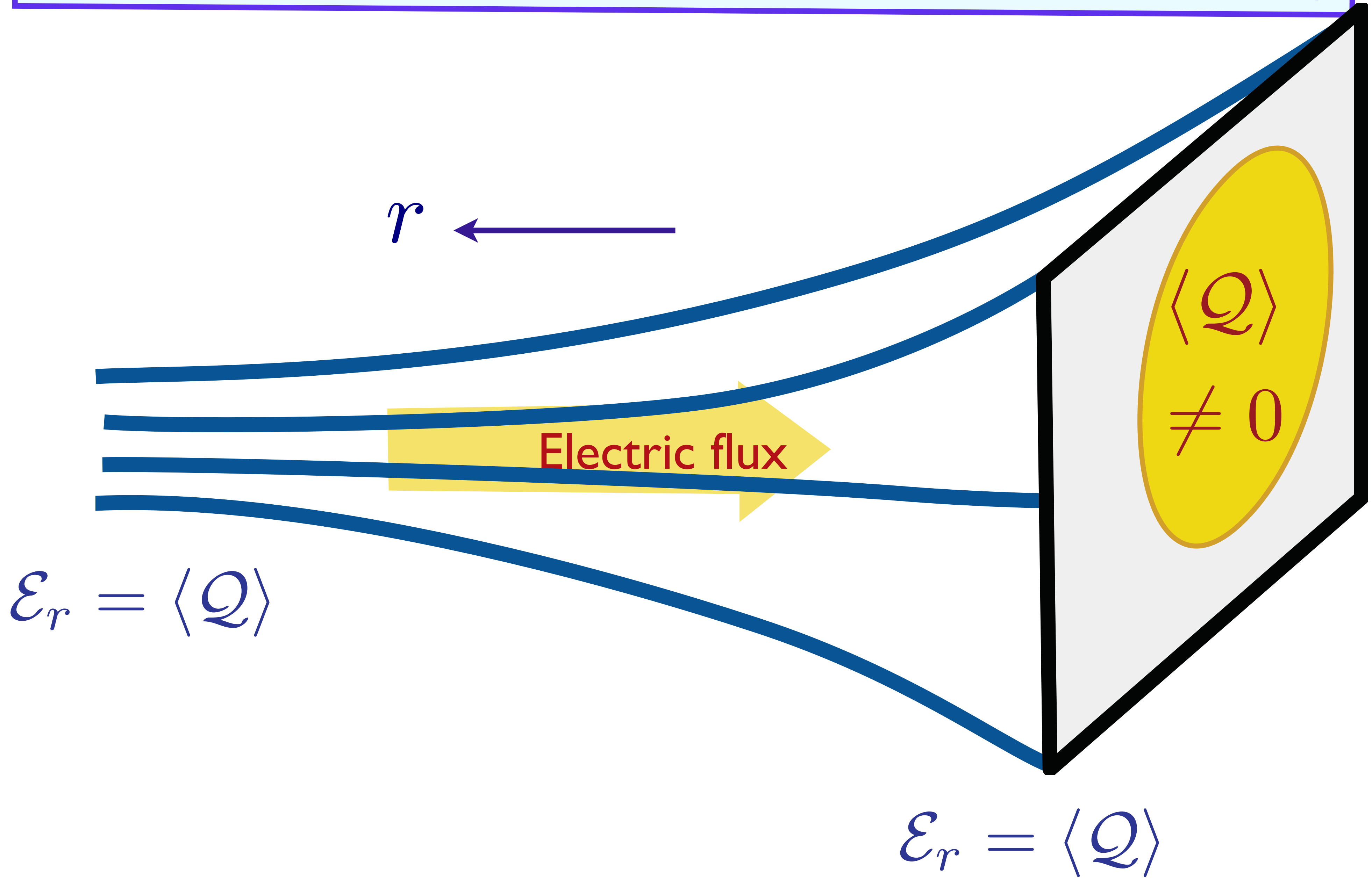
- Adding probe fermions leads to a (visible) Fermi surface of "mesinos" as in a FL* phase, and a holographic entanglement entropy in which

$$\mathcal{Q} \to \mathcal{Q} - \mathcal{Q}_{\text{mesinos}}.$$

So only the "hidden" Fermi surfaces of "quarks" are measured by the holographic minimal area formula.

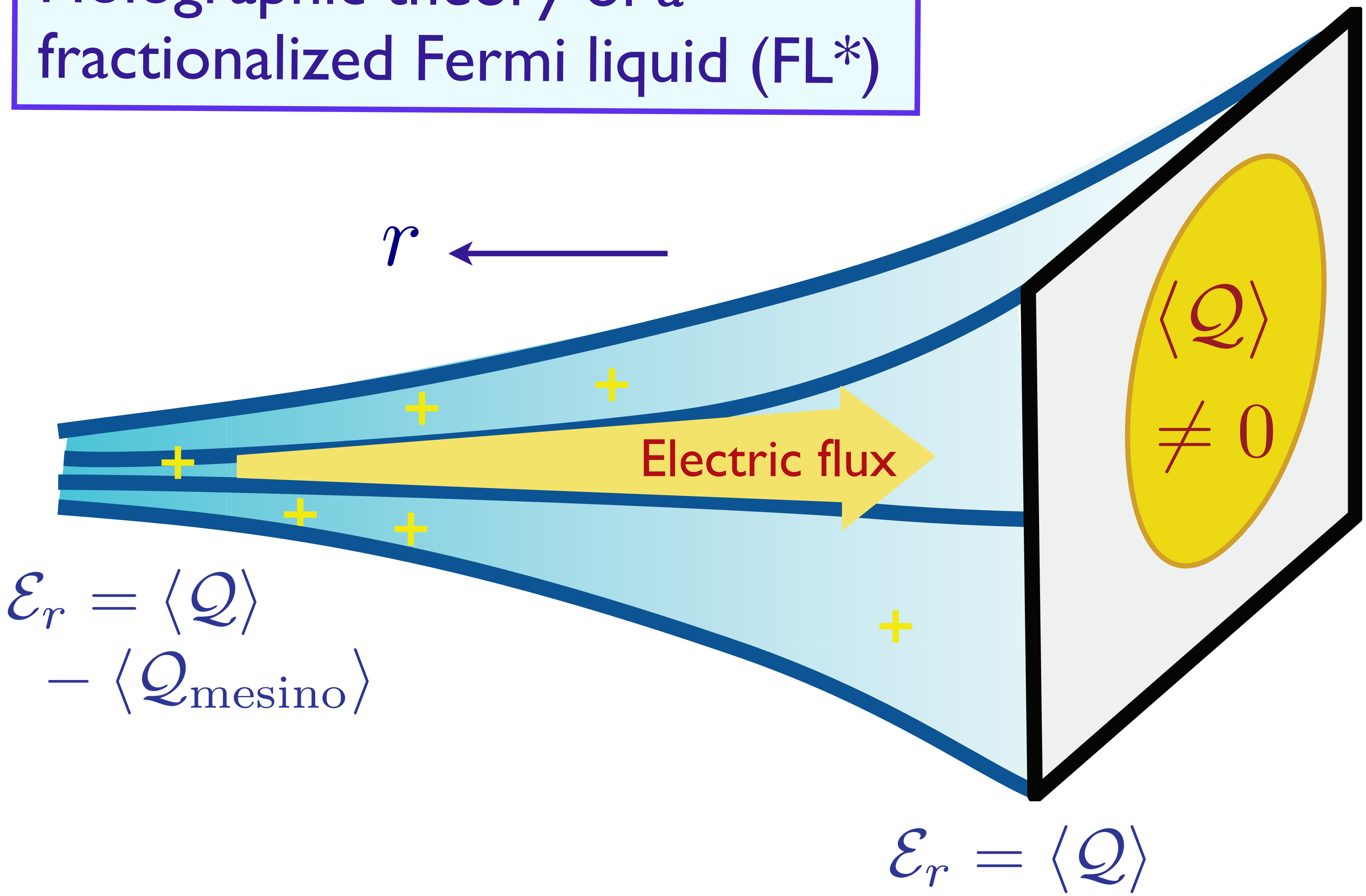
L. Huijse, B. Swingle, and S. Sachdev to appear.

Holographic theory of a non-Fermi liquid (NFL)



In a deconfined NFL phase, the metric extends to infinity (representing critical IR modes), and ***all*** of the electric flux "leaks out".

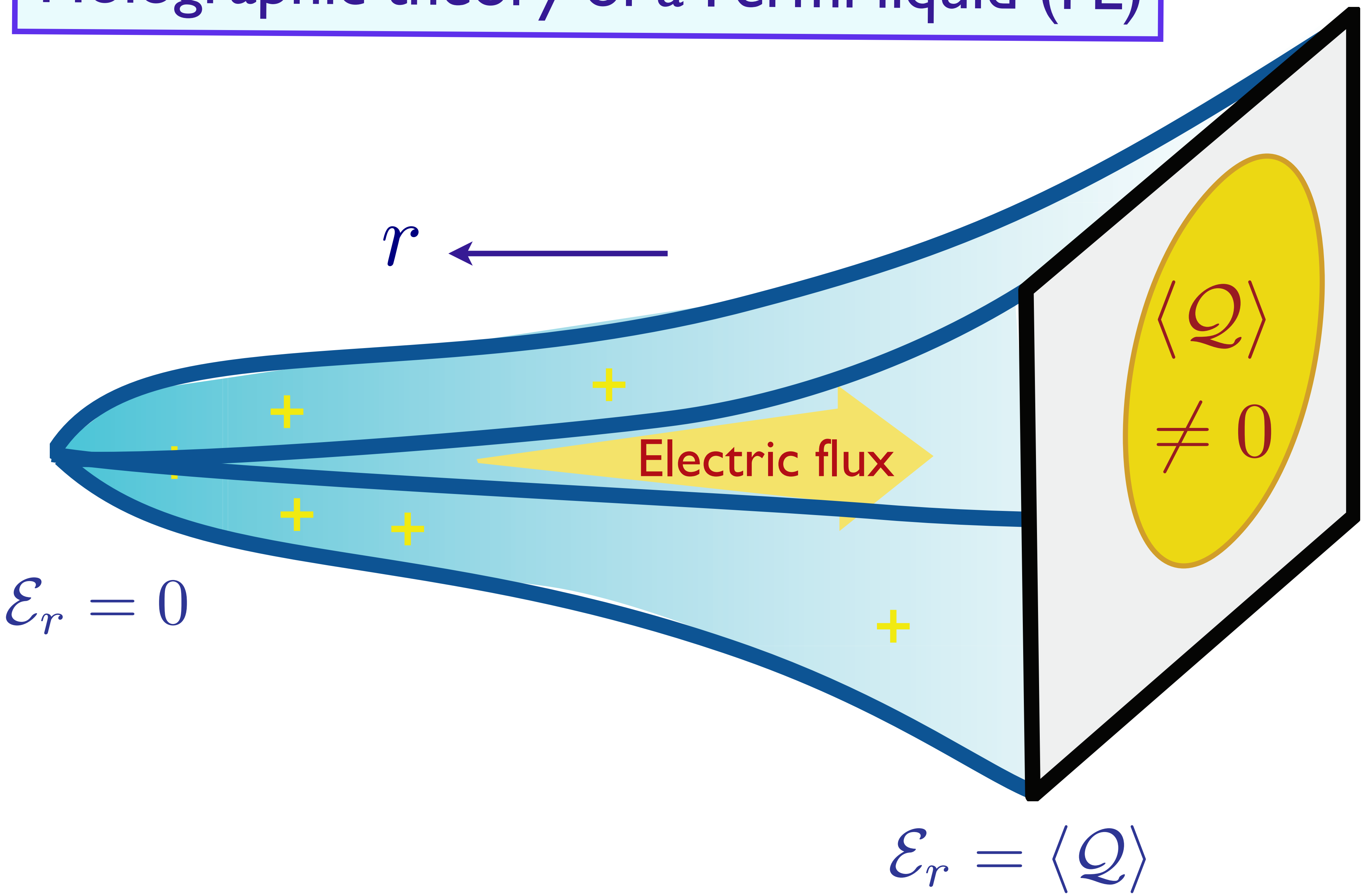
Holographic theory of a fractionalized Fermi liquid (FL*)



In a deconfined FL* phase, the metric extends to infinity,
there is a mesino charge density in the bulk,
and ***only part*** of the electric flux "leaks out".

Holographic theory of a Fermi liquid (FL)

S. Sachdev
arXiv:1107.5321



In a confining FL phase, the metric terminates,
all of the mesino density is in the bulk spacetime,
and ***<u>none</u>*** of the electric flux "leaks out".

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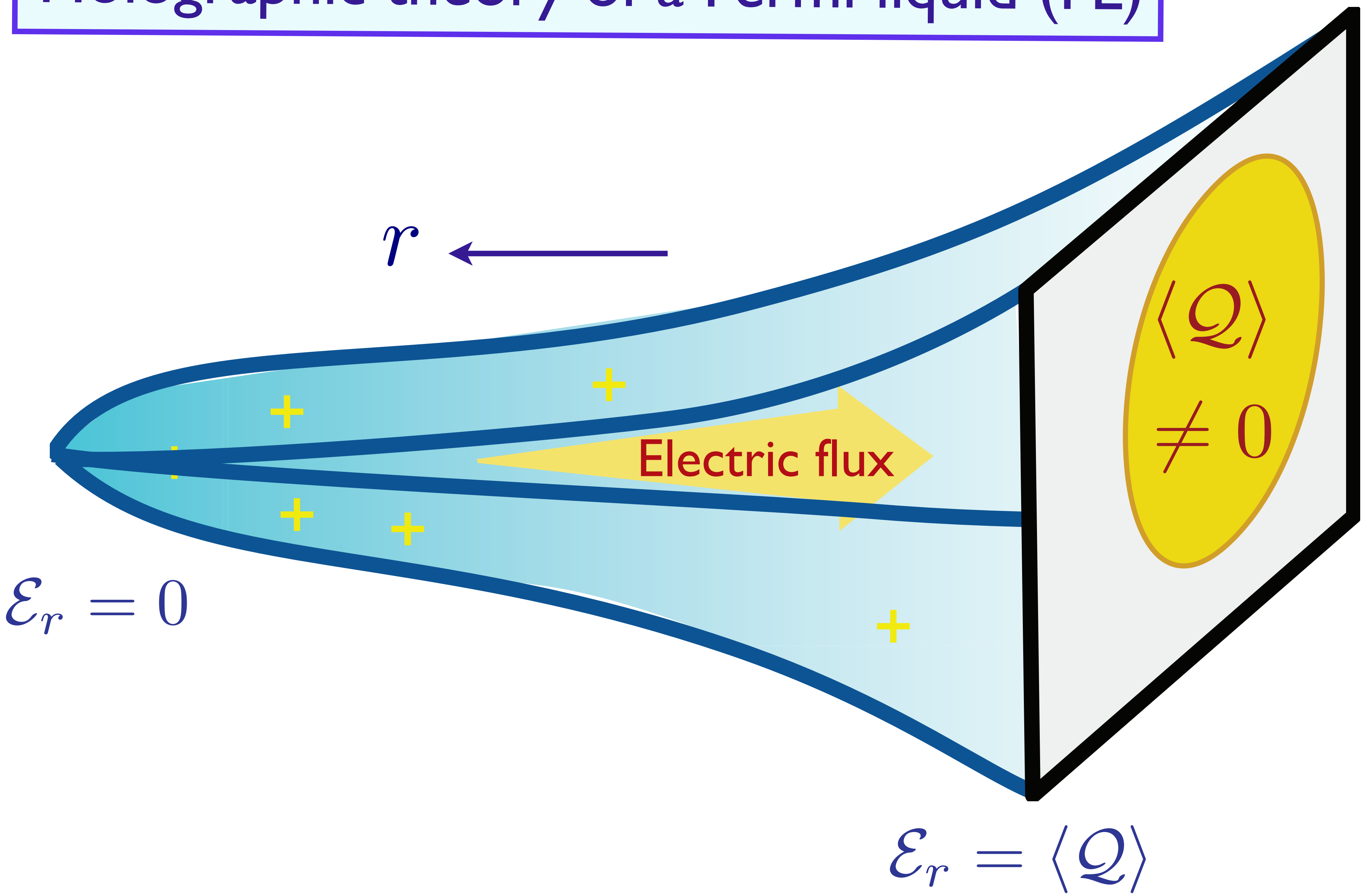

Gauss Law in the bulk
$\Leftrightarrow$ Luttinger theorem on the boundary

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Consider QED_4, with *full* quantum fluctuations,

$$S = \int d^4x \sqrt{g} \left[\frac{1}{4e^2} F_{ab} F^{ab} + i \left(\overline{\psi} \Gamma^M D_M \psi + m \overline{\psi} \psi \right) \right].$$

in a metric which is AdS_4 in the UV, and confining in the IR. A simple model

$$ds^2 = \frac{1}{r^2} (dr^2 - dt^2 + dx^2 + dy^2) \quad , \quad r < r_m$$

with r_m determined by the confining scale.

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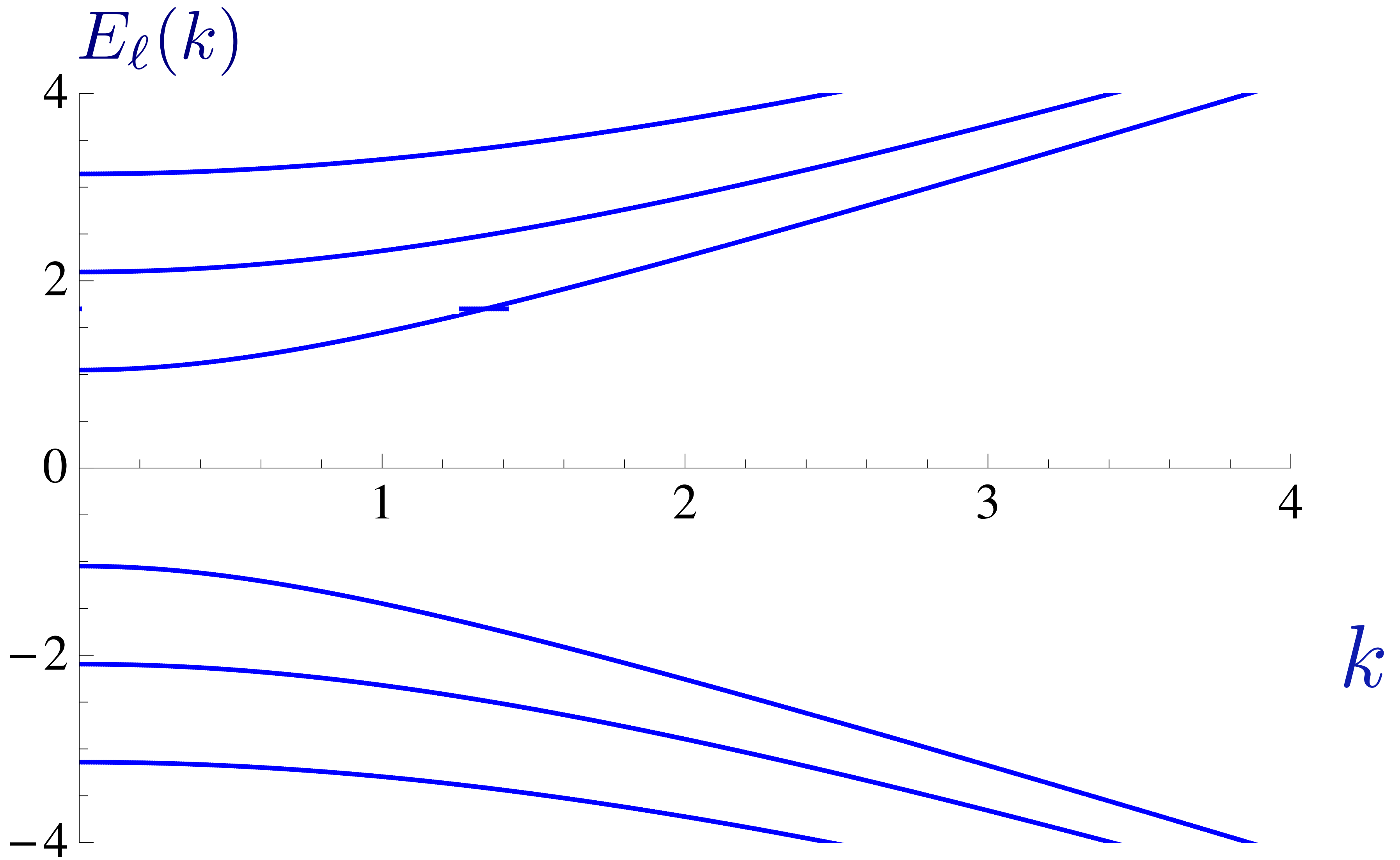

Massive Dirac fermions at zero chemical potential

Dispersion $E_\ell(k) = \sqrt{k^2 + M_\ell^2}$

Masses $M_\ell \sim 1/r_m$

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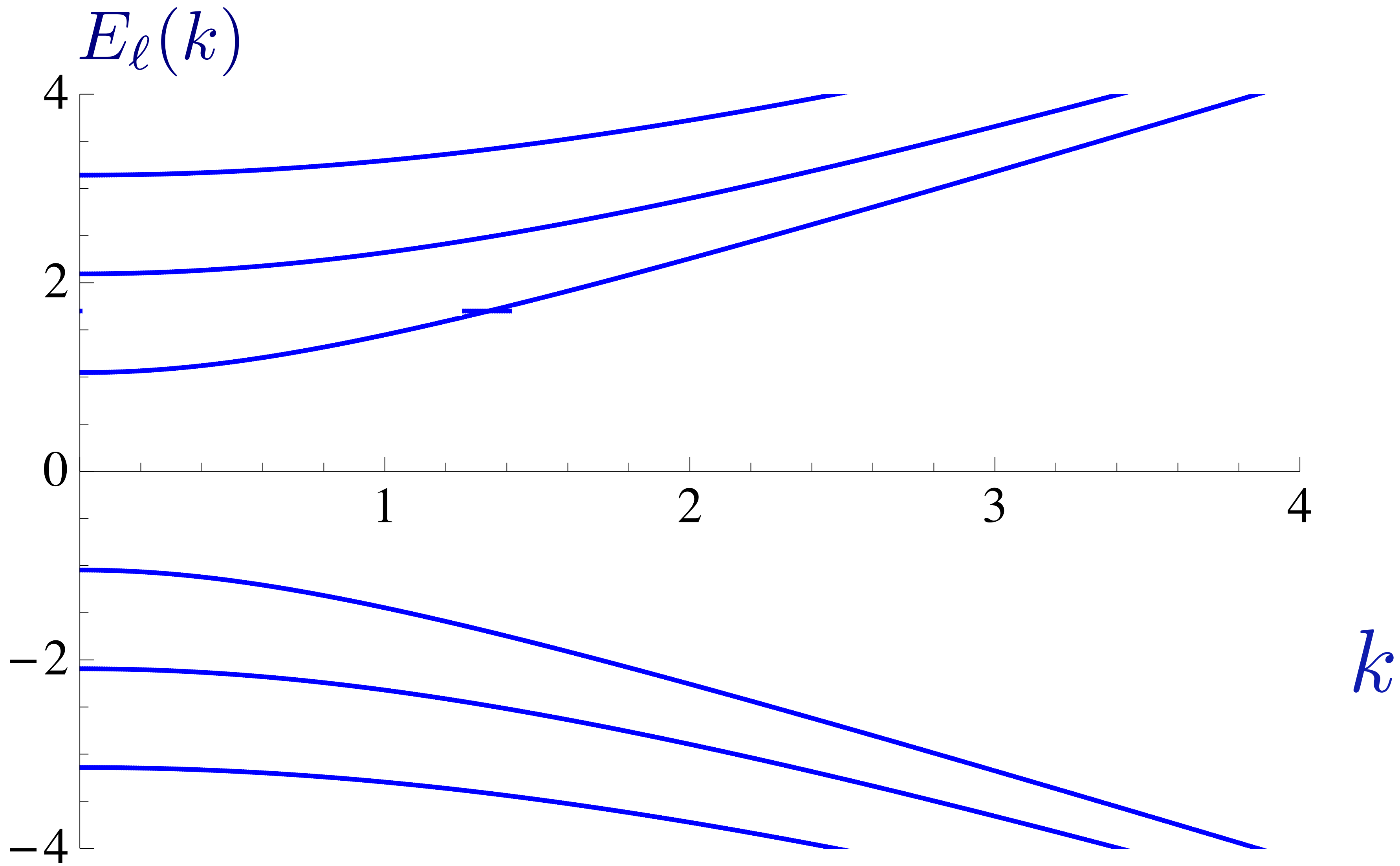

Massive Dirac fermions at zero chemical potential

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Almost all previous holographic theories have considered the situation where the spacing between the $E_\ell(k)$ vanishes, and an infinite number of $E_\ell(k)$ are relevant.

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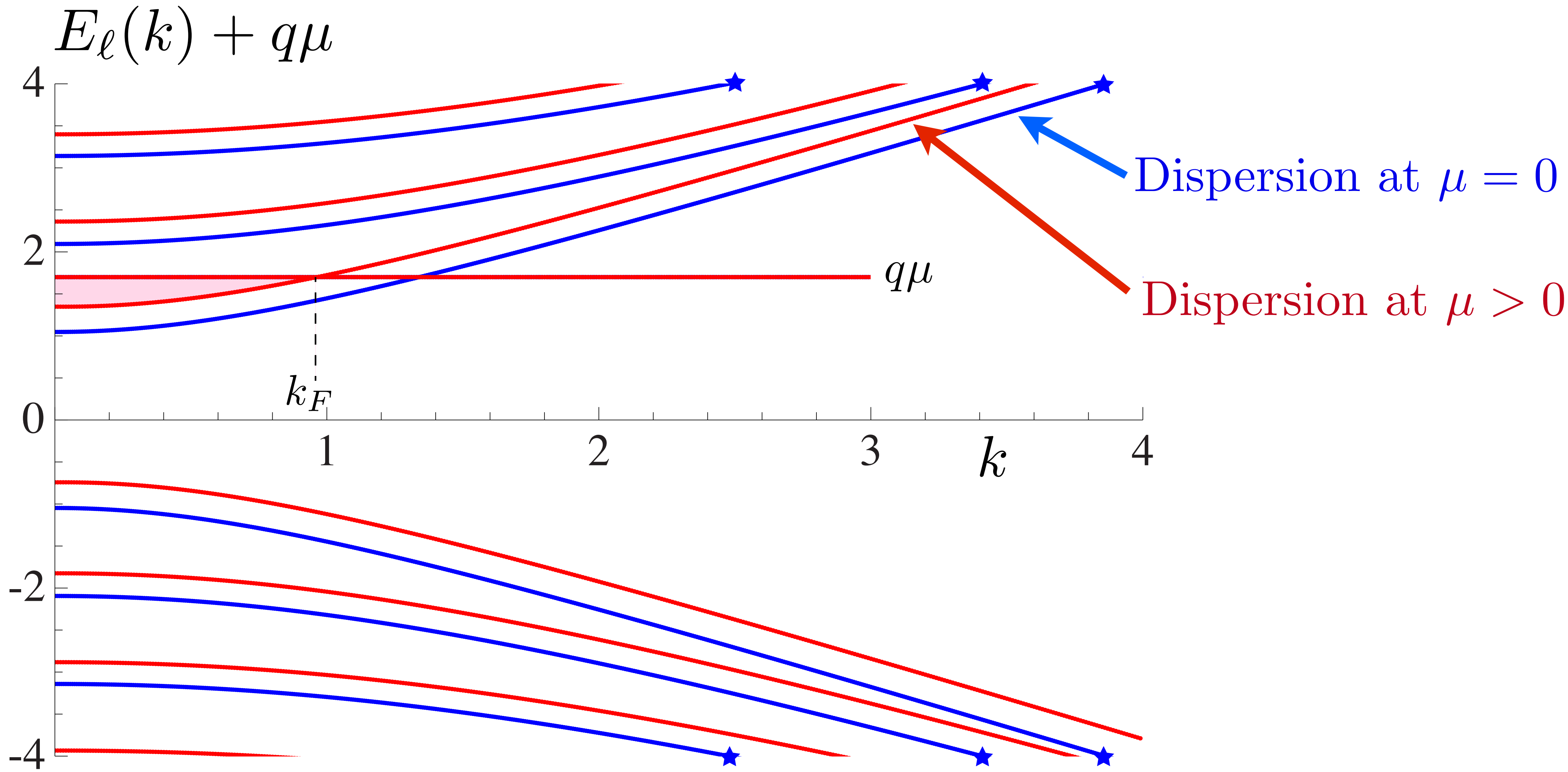

The spectrum at non-zero chemical potential is determined by self-consistently solving the Dirac equation and Gauss's law:

$$\left(\vec{\Gamma} \cdot \vec{D} + m\right) \Psi_\ell = E_\ell \Psi_\ell \ ; \ \nabla_r \mathcal{E}_r = \sum_\ell \int \frac{d^2k}{4\pi^2} \Psi_\ell^\dagger(k,z) \Psi_\ell(k,z) f(E_\ell(k))$$

where $\mathcal{E}$ is the electric field, and $f(E)$ is the Fermi function

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- Integrating Gauss's Law, we obtain

$$\mathcal{E}_r(\text{boundary}) - \mathcal{E}_r(\text{IR}) = \mathcal{A}$$

But $\mathcal{E}_r(\text{boundary}) = \langle \mathcal{Q} \rangle$, by the rules of AdS/CFT. So we obtain the usual Luttinger theorem of a Landau Fermi liquid,

$$\mathcal{A} = \langle \mathcal{Q} \rangle$$

provided $\mathcal{E}_r(\text{IR}) = 0$.

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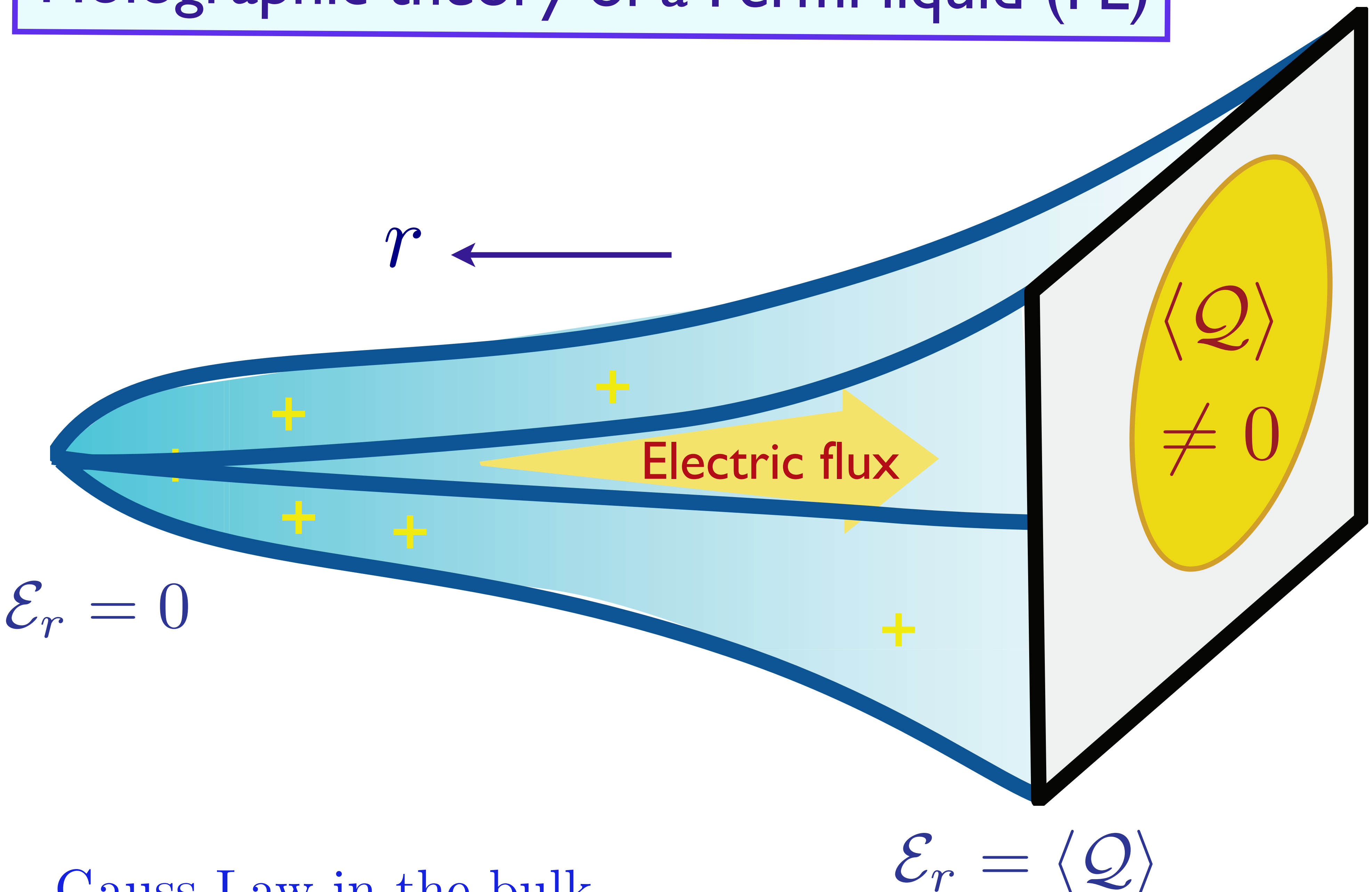

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Conclusions

Compressible quantum matter

- Evidence for *hidden Fermi surfaces* in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a *non-Fermi liquid* (NFL) state of gauge theories at non-zero density.

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Conclusions

Compressible quantum matter

- Evidence for *hidden Fermi surfaces* in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a *non-Fermi liquid* (NFL) state of gauge theories at non-zero density.

- Fermi liquid (FL) state described by a confining holographic geometry

- Hidden Fermi surfaces can co-exist with Fermi surfaces of mesinos, leading to a *fractionalized Fermi liquid* (FL*)