Detecting the fractionalized Fermi liquid in the cuprate superconductors

University of Bristol

February 9, 2026

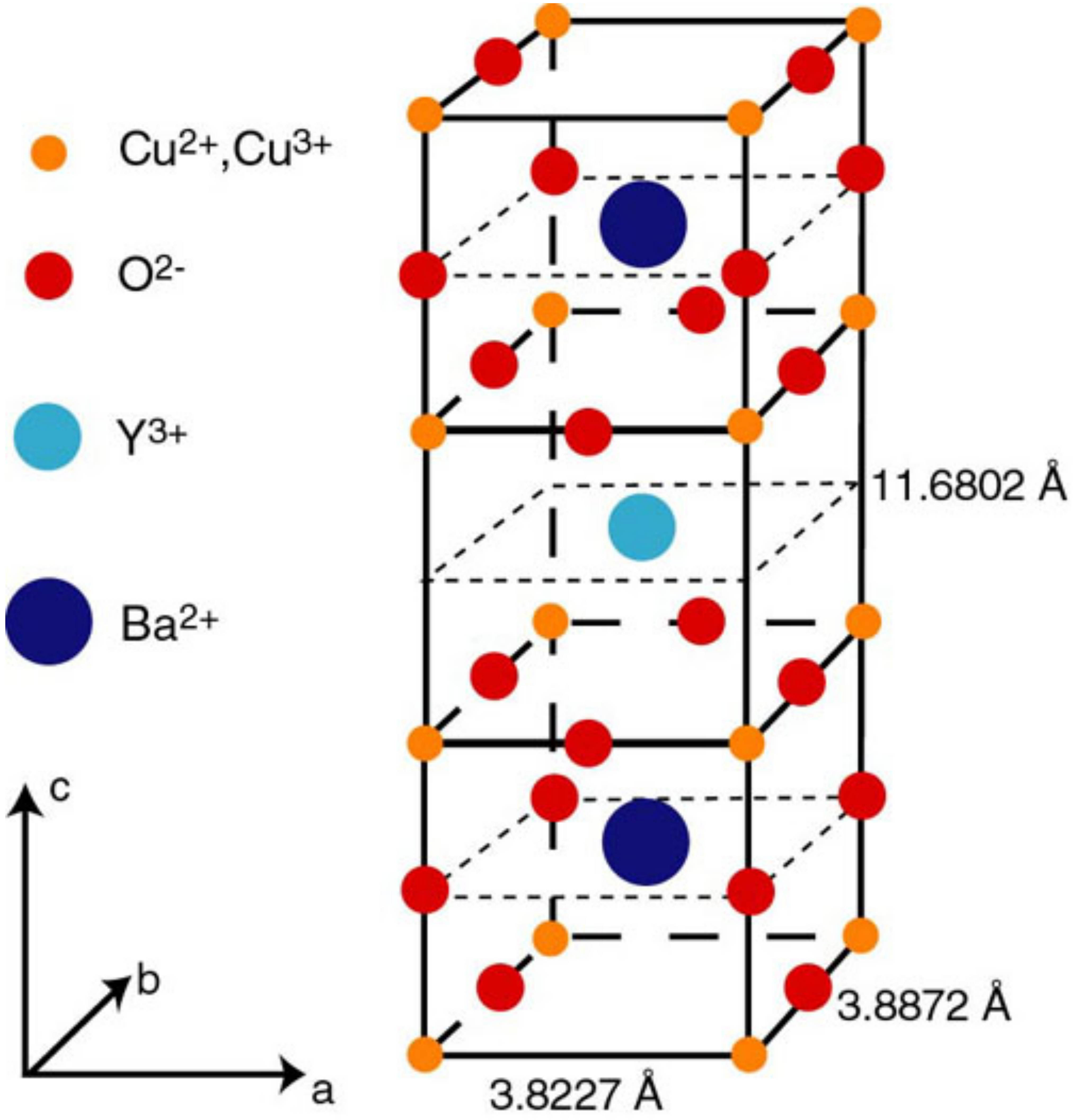
Subir Sachdev

PHYSICS

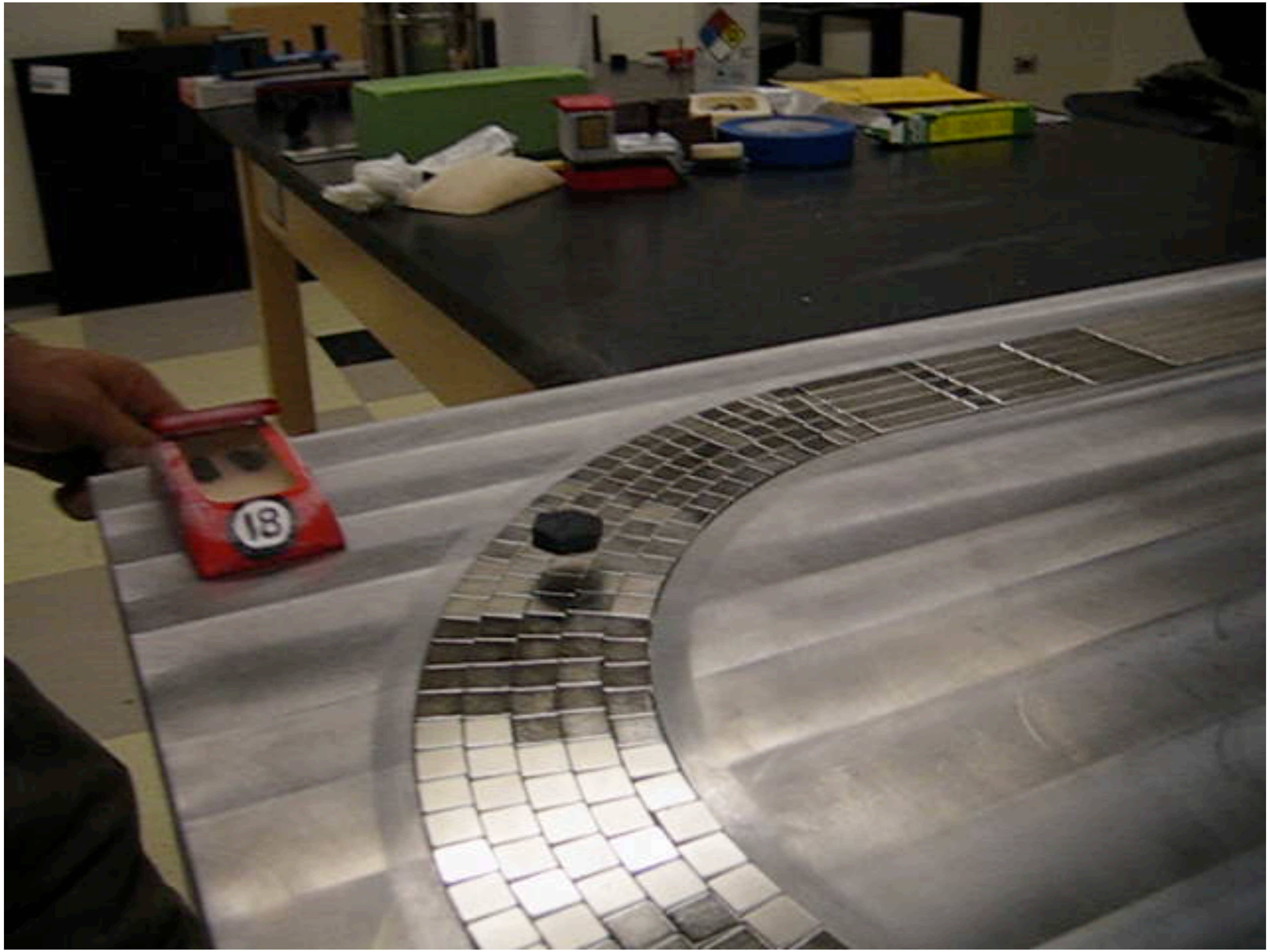
VE RI TAS

HARVARD

Cuprate
high temperature
superconductors



$YBa_2Cu_3O_{6+x}$



Nd-Fe-B magnets, YBaCuO superconductor

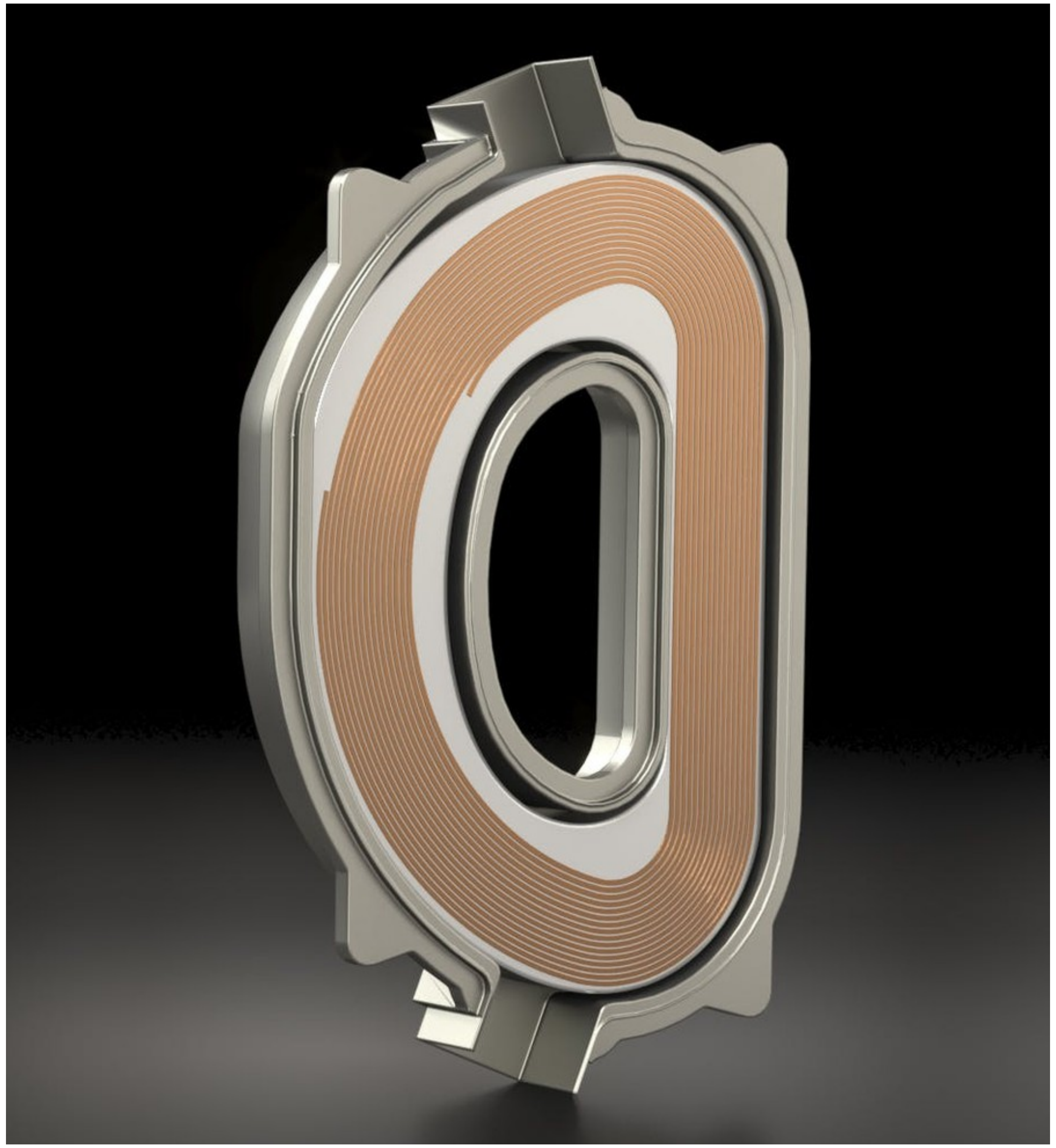
Julian Hetel and Nandini Trivedi, Ohio State University

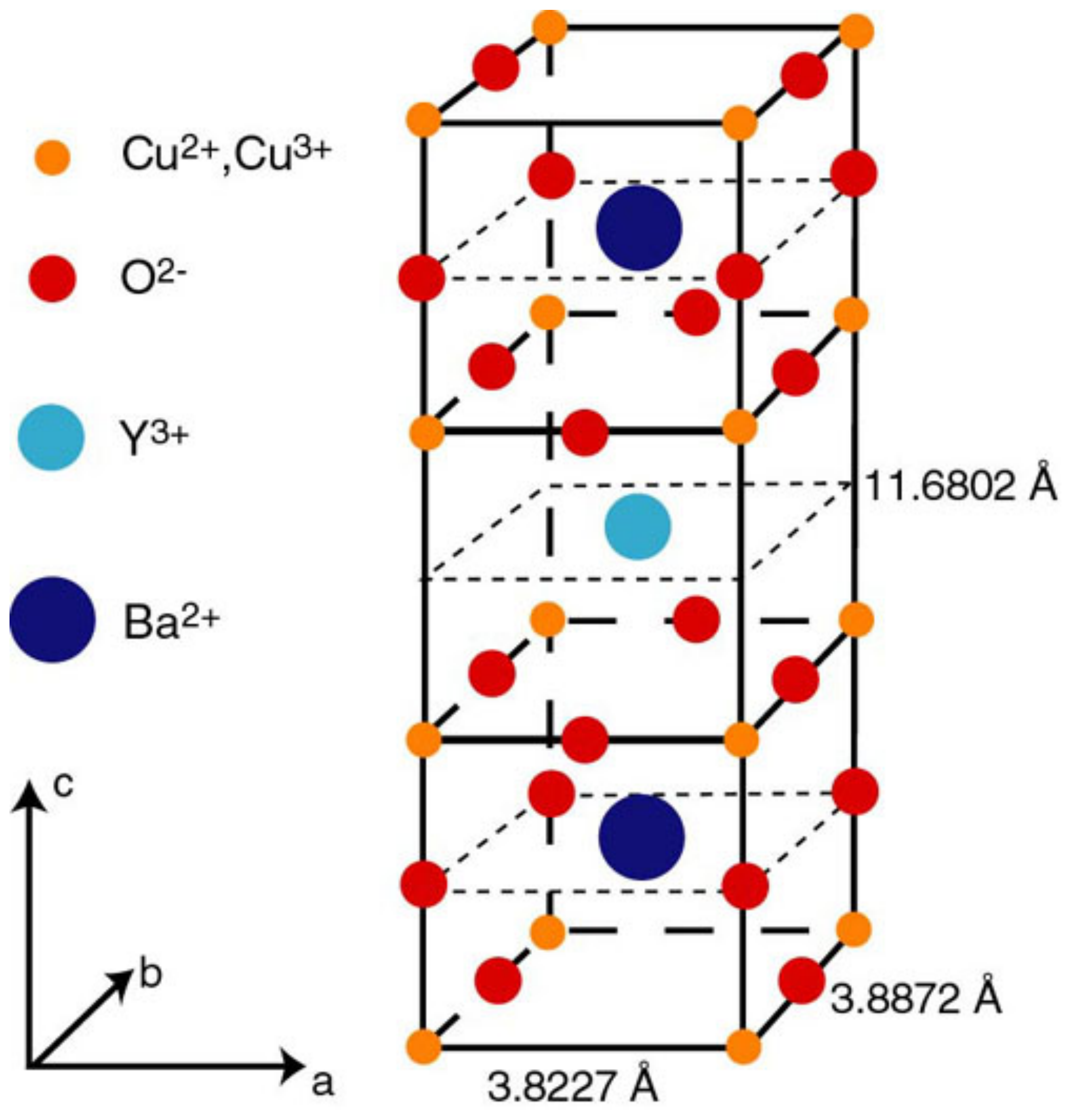
HTS Magnets: Enabling Technology

The surest path to limitless, clean, fusion energy

YBCO magnets allow for smaller, faster, and less expensive tokamaks for plasma fusion

Commonwealth Fusion Systems





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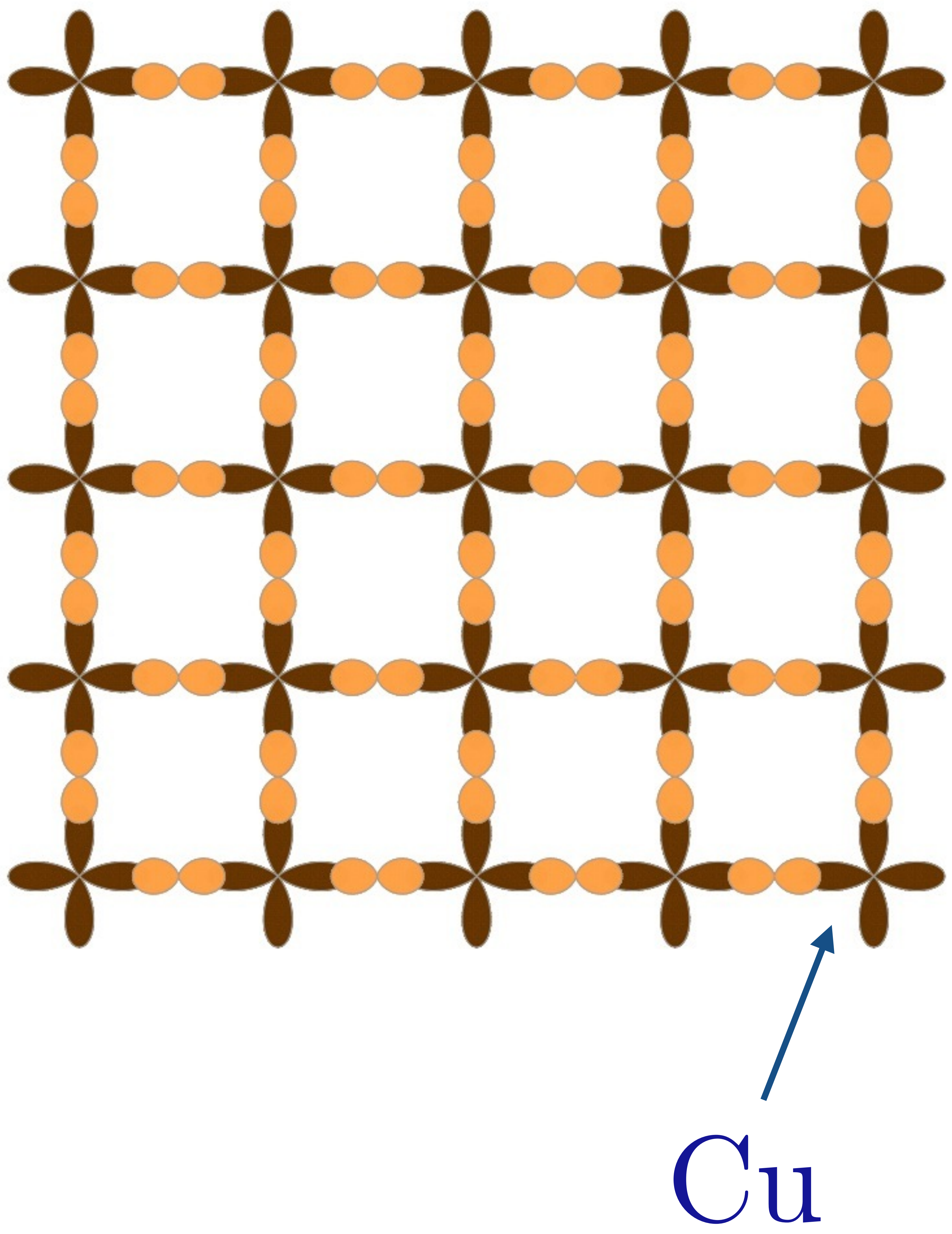

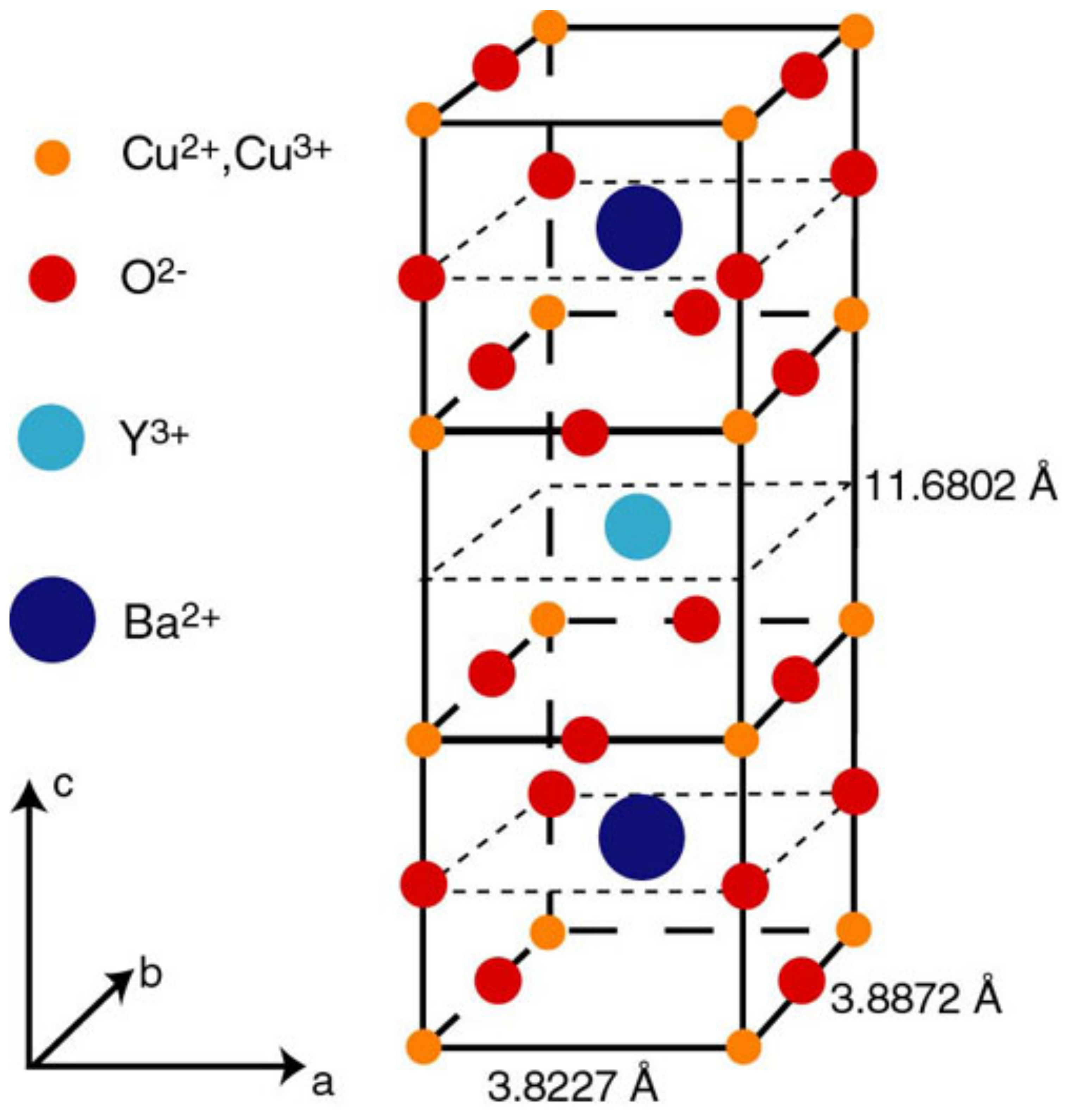

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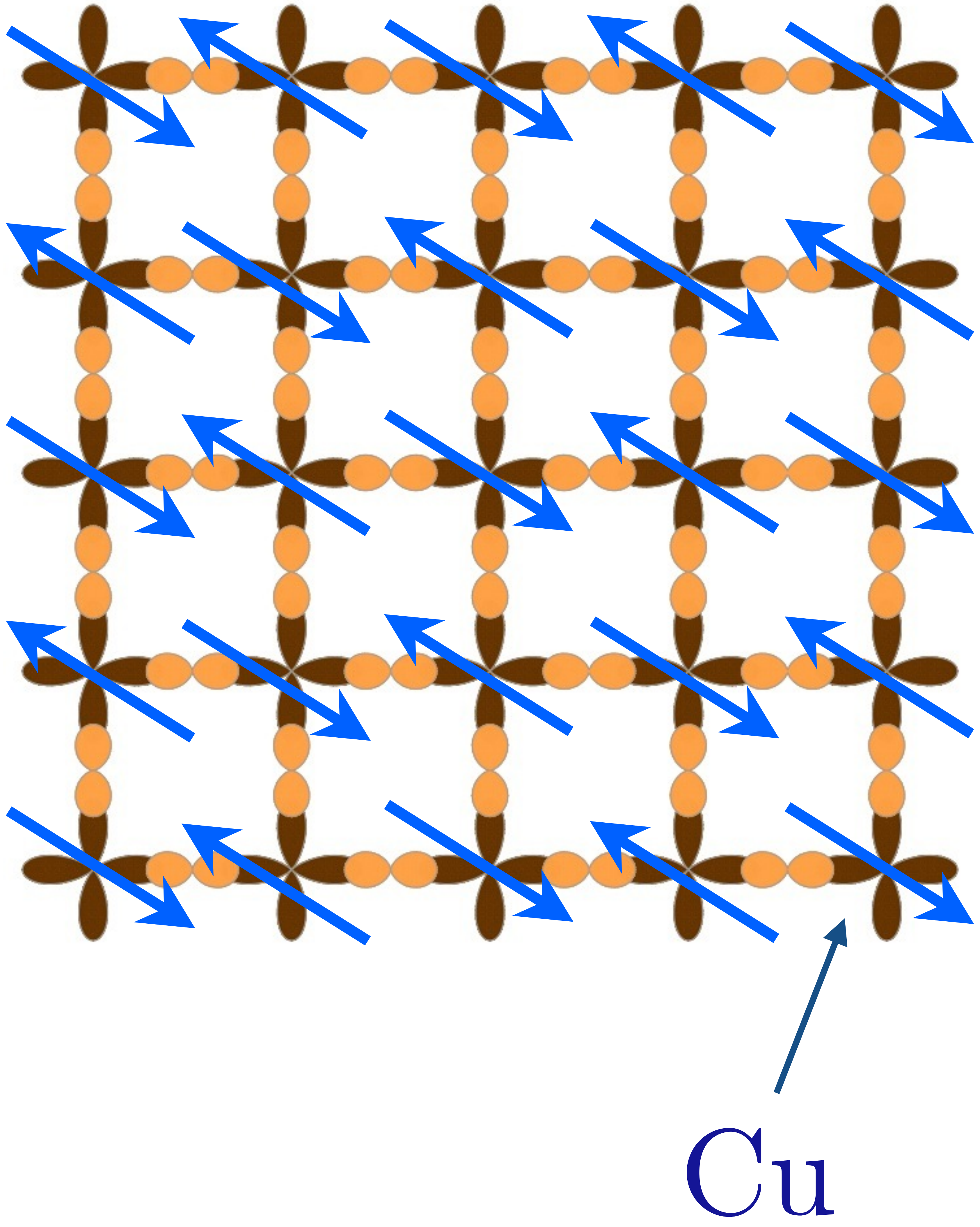


$$\mathcal{H} = J \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

$S = 1/2$ on each site

$$|\uparrow\rangle, |\uparrow\rangle$$

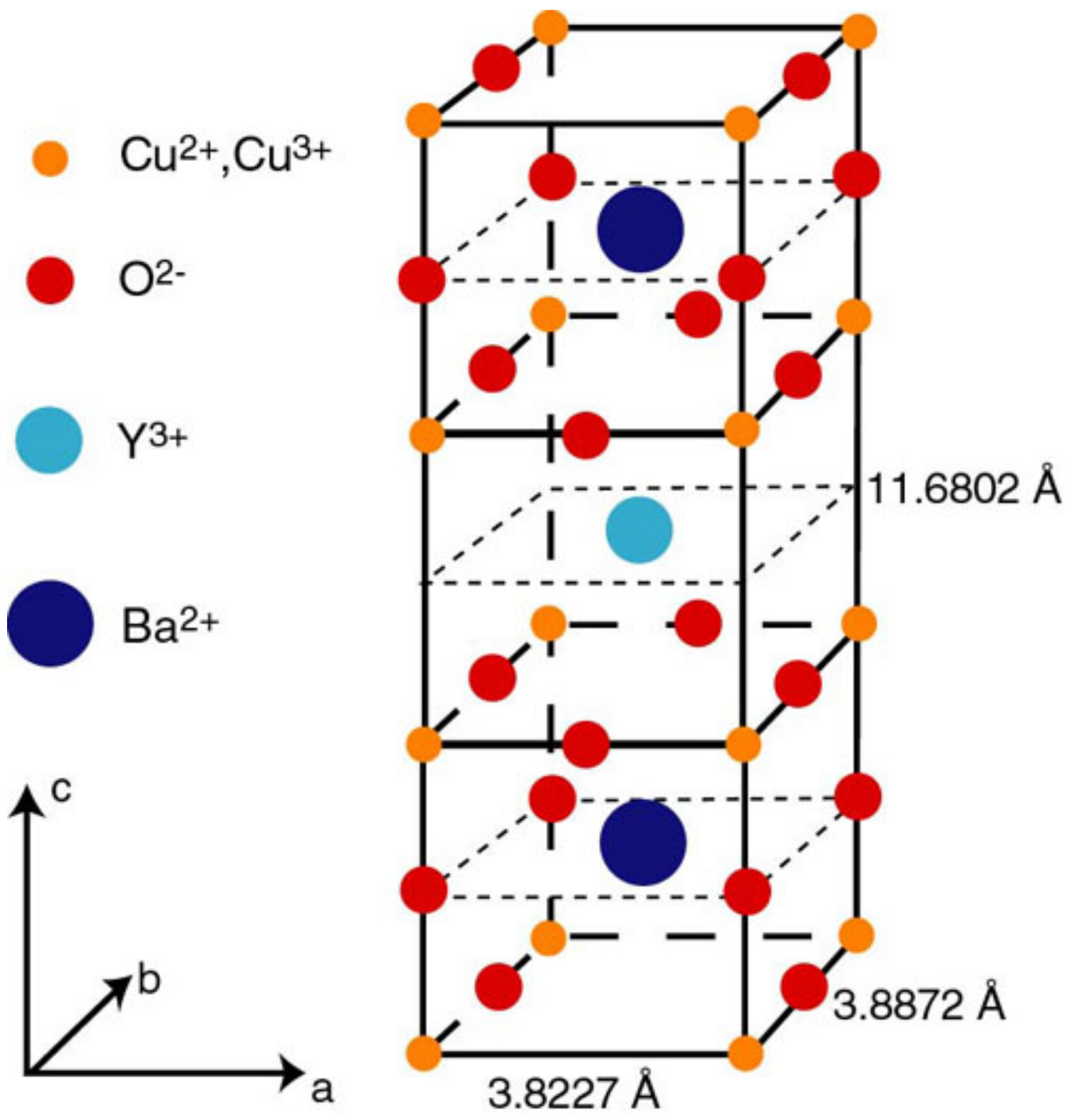
$$S_z |\uparrow\rangle = (1/2) |\uparrow\rangle$$

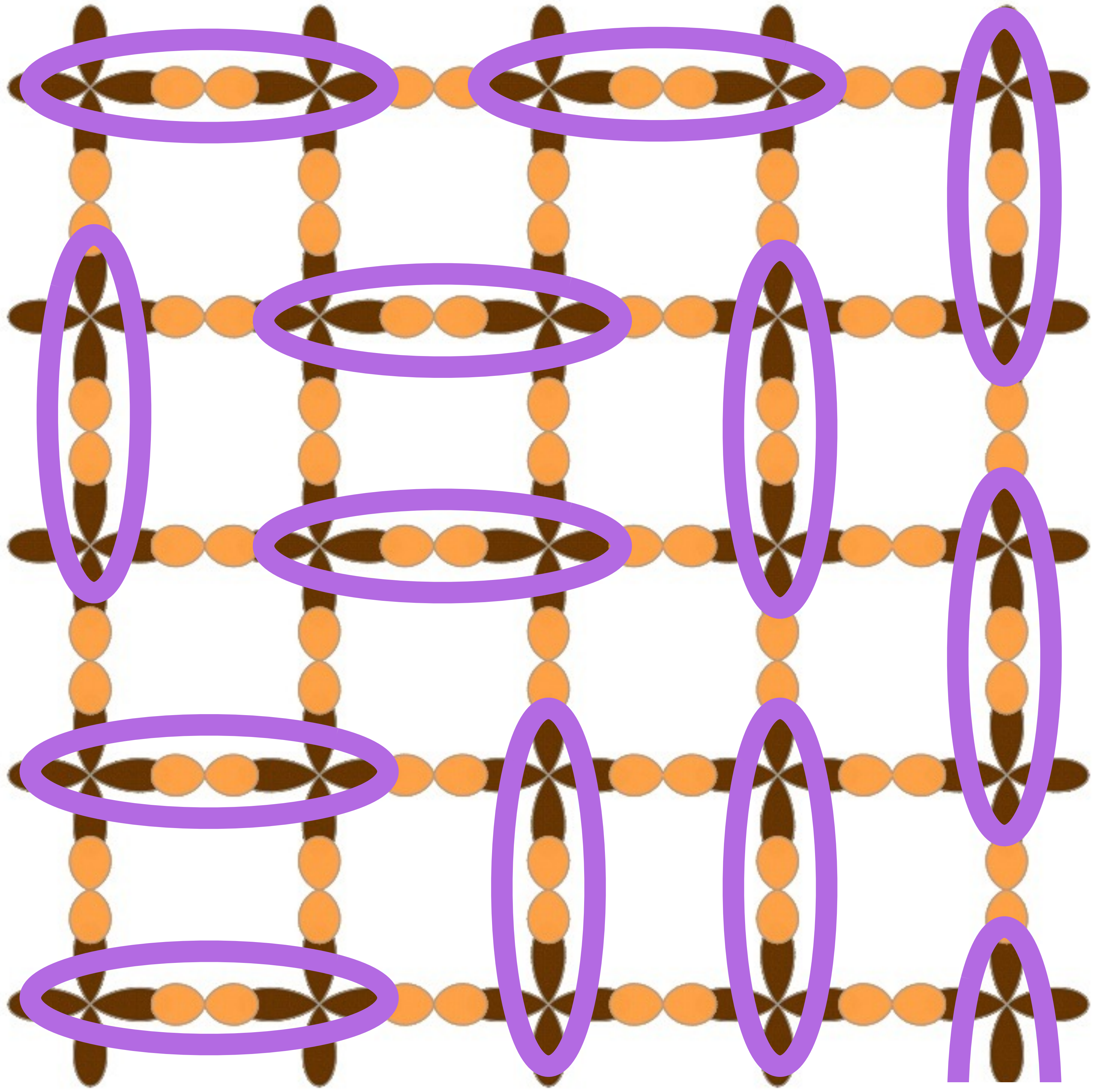
$$S_z |\downarrow\rangle = -(1/2) |\downarrow\rangle$$

$$(S_x + iS_y) |\downarrow\rangle = |\uparrow\rangle$$

$$(S_x - iS_y) |\uparrow\rangle = |\downarrow\rangle$$

Insulating antiferromagnet with one electron per site





$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

$\mathcal{D} \to$ dimer covering of lattice

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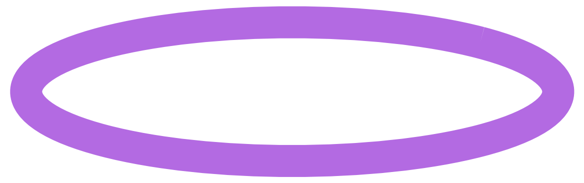
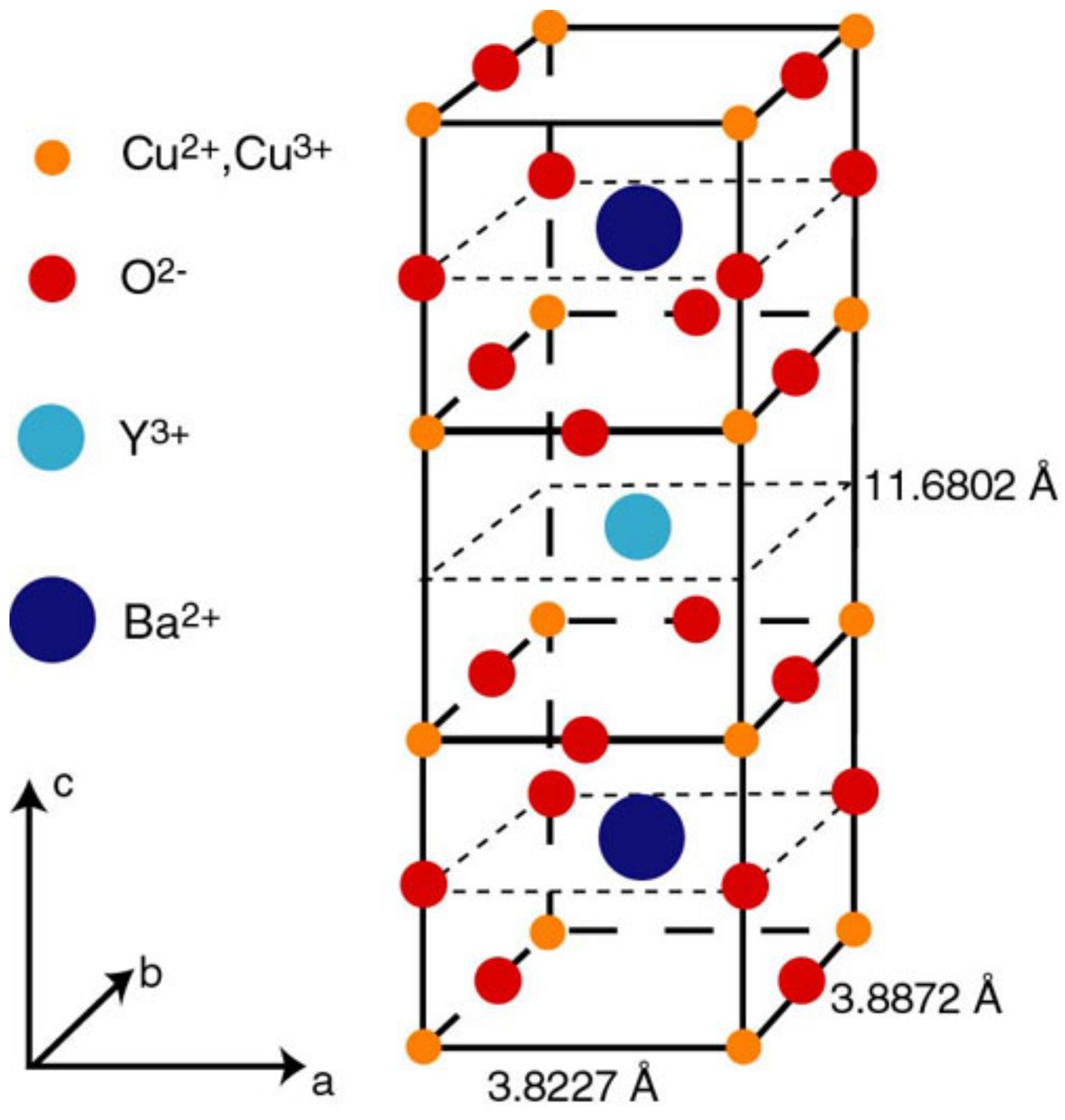
 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

P.W. Anderson (1987): The key to high temperature superconductivity is the formation of a “resonating valence bond state”.

A quantum spin liquid with many-boson (spins on Cu) **entanglement**



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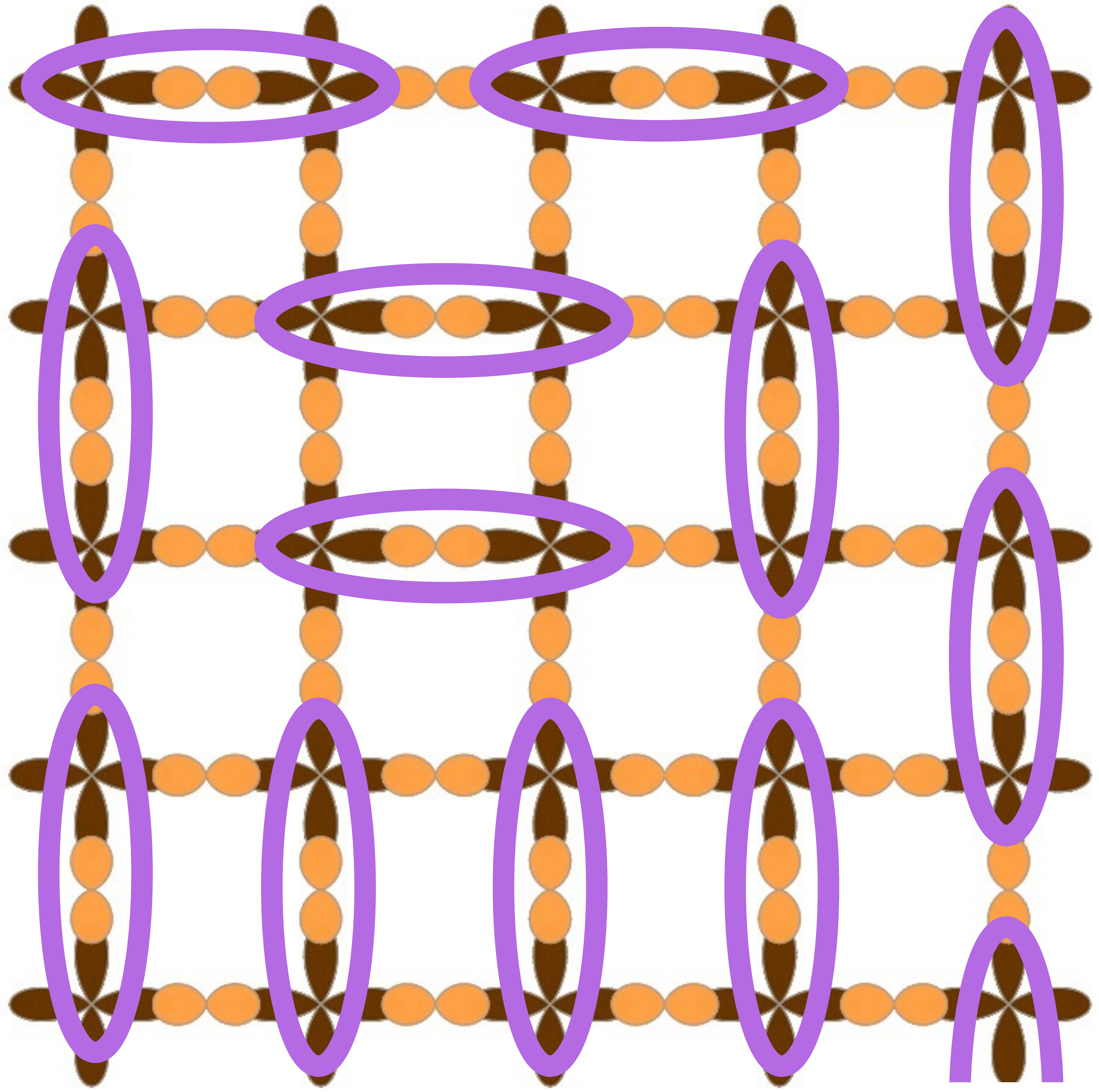

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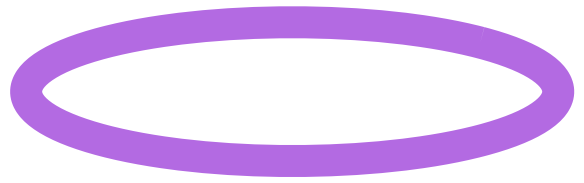

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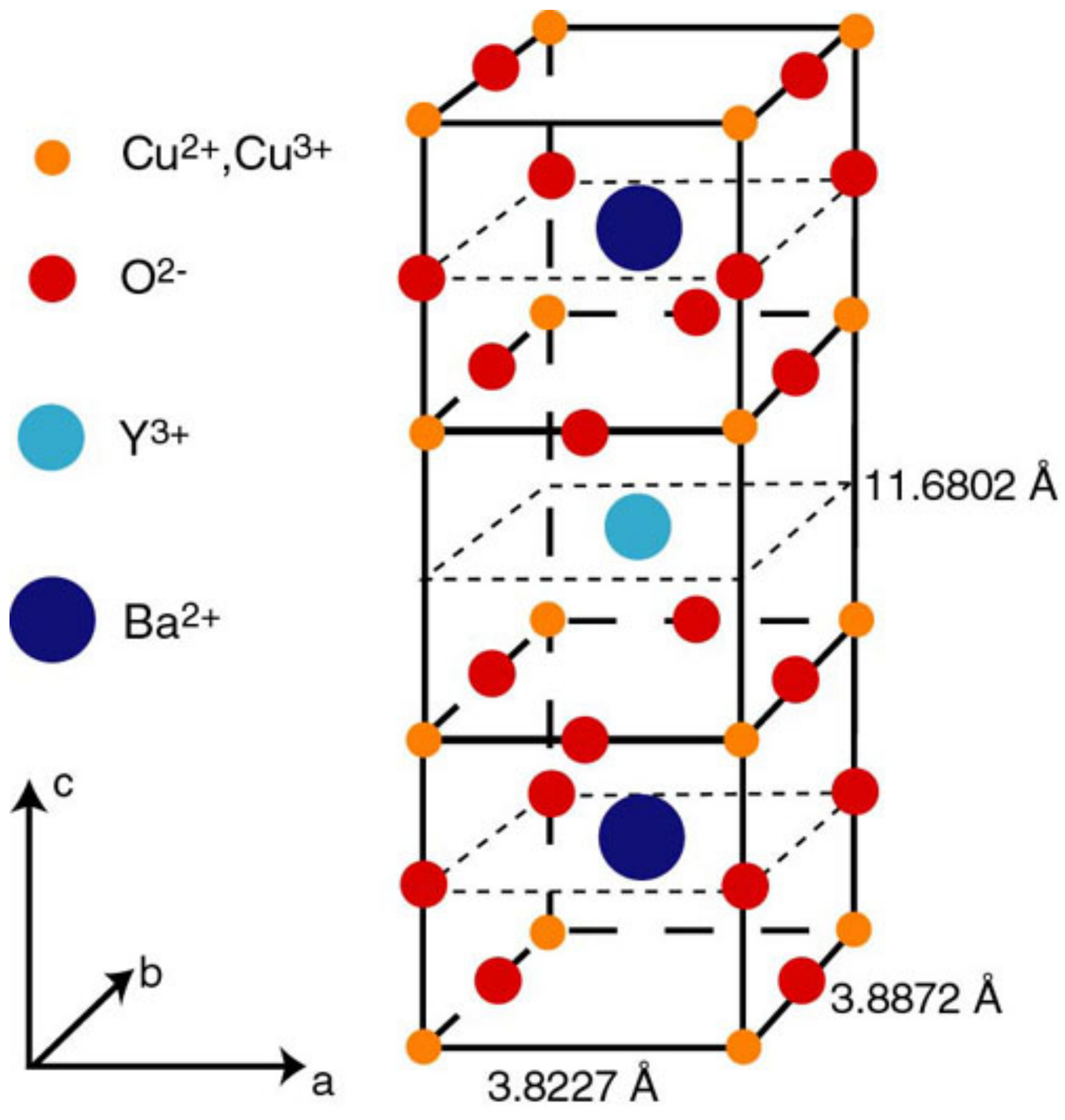

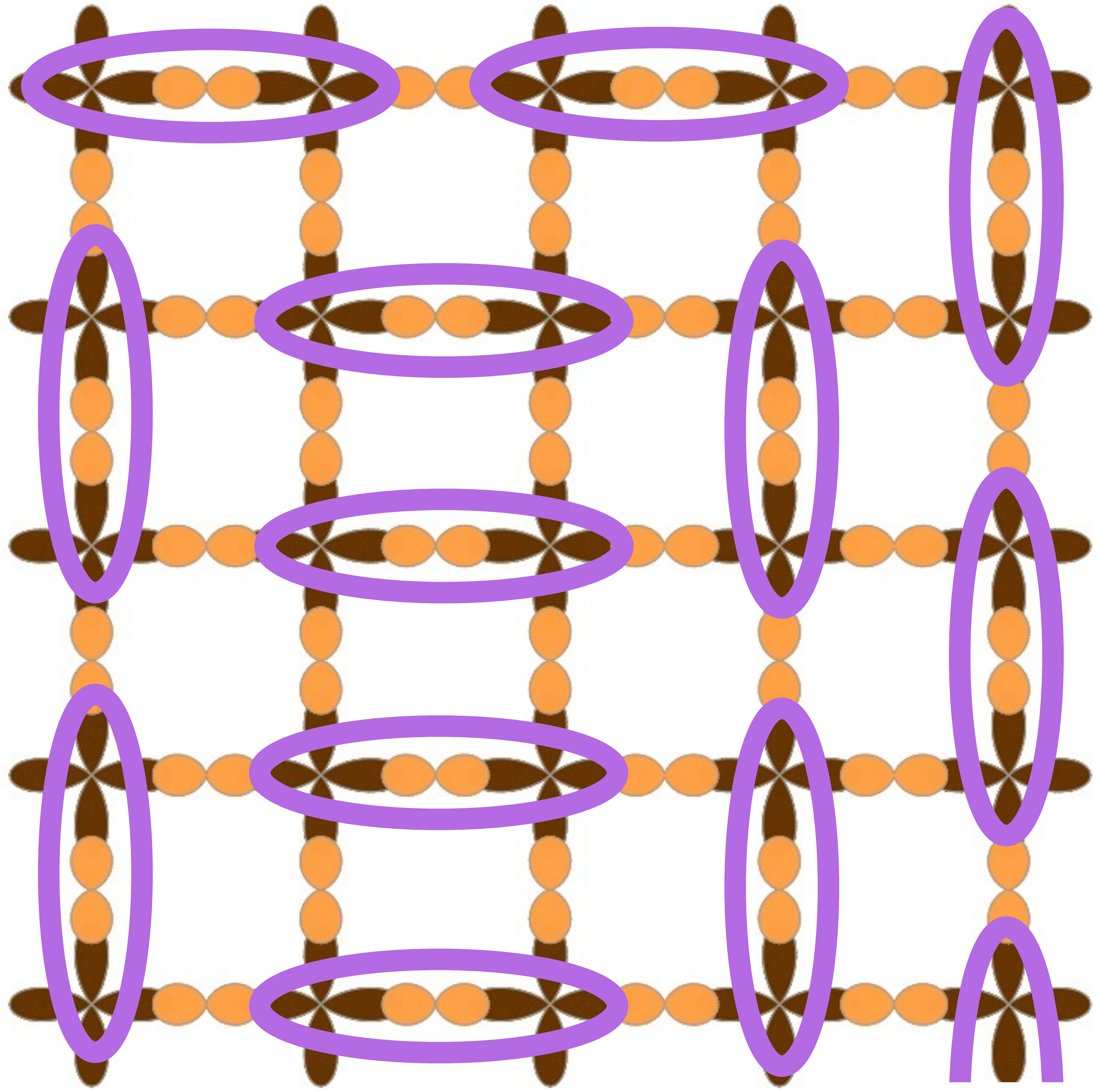

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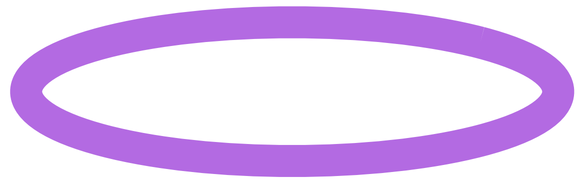 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

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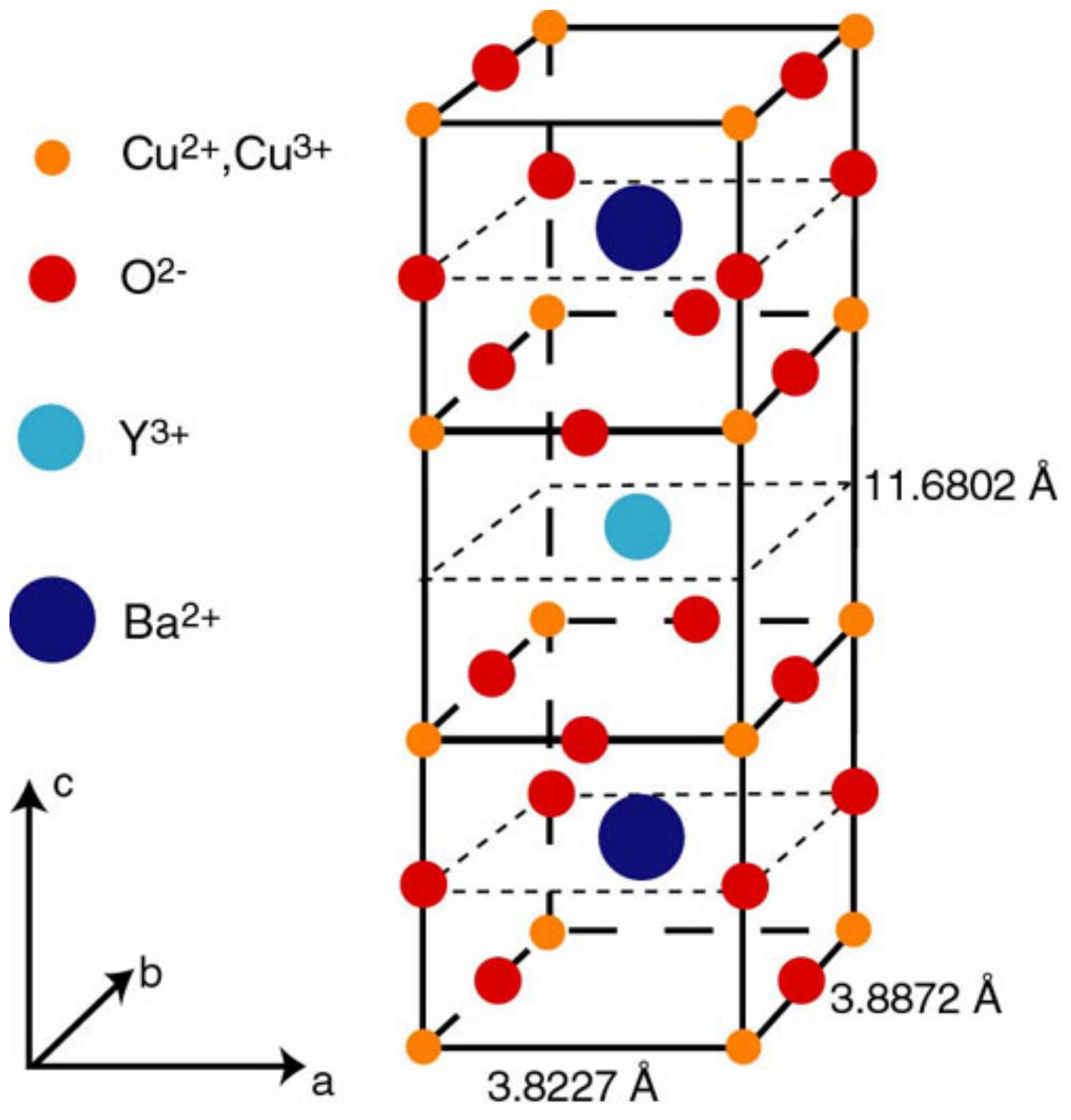

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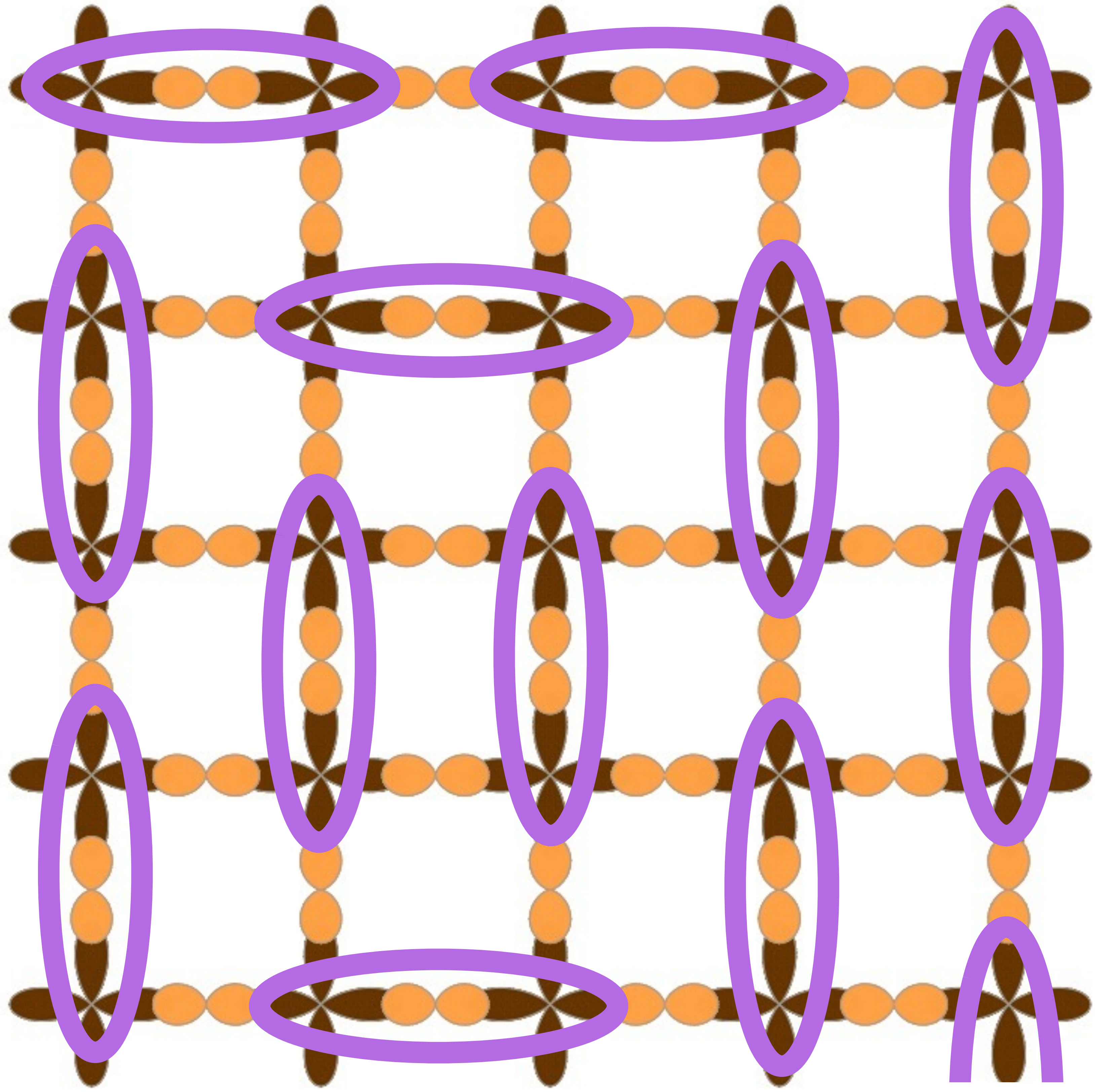

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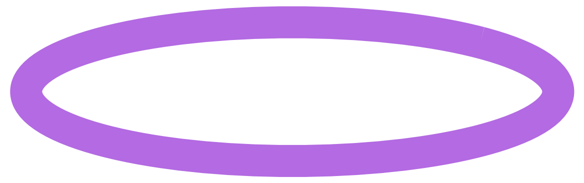

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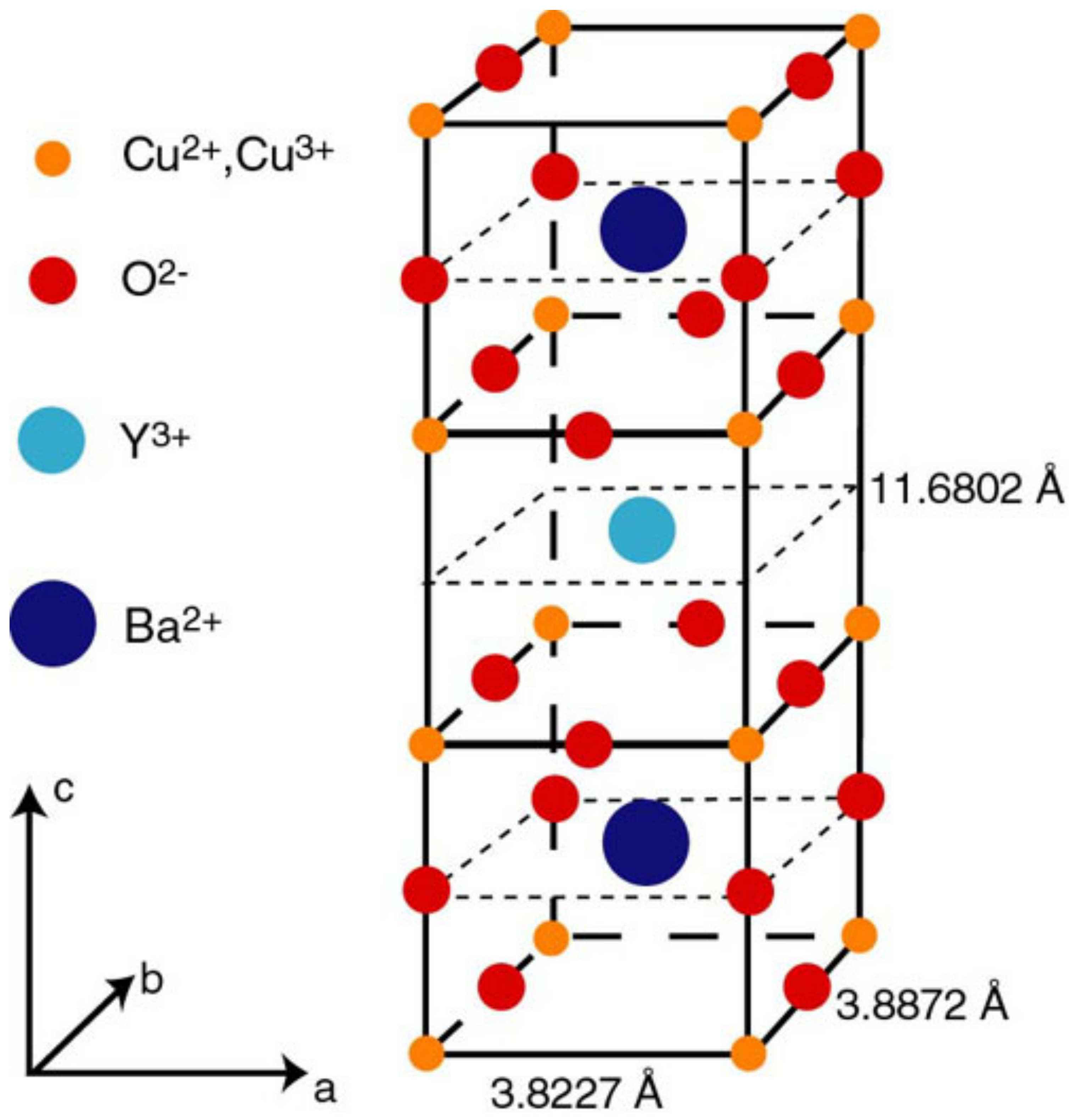

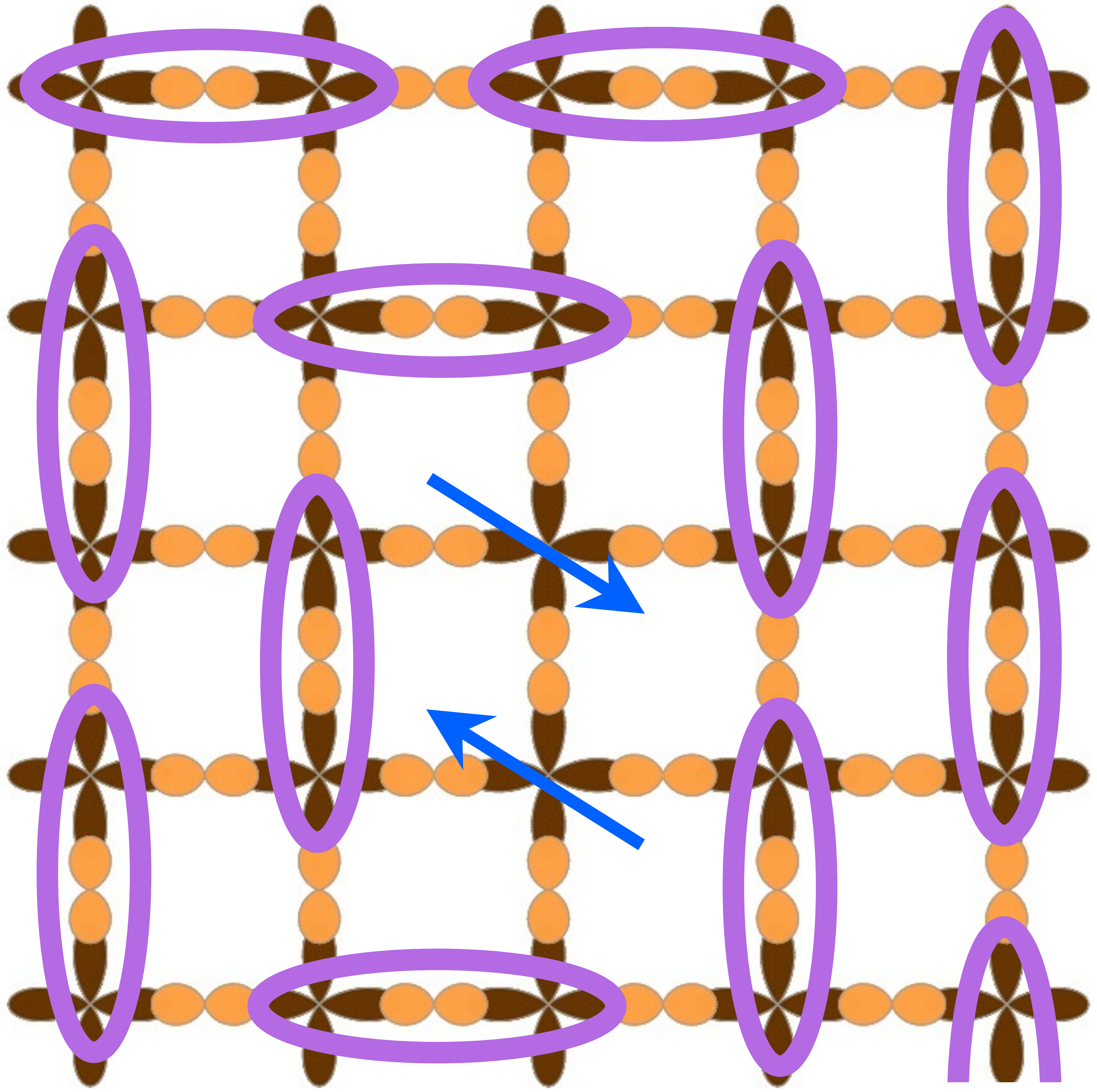

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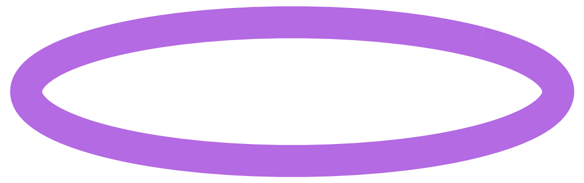
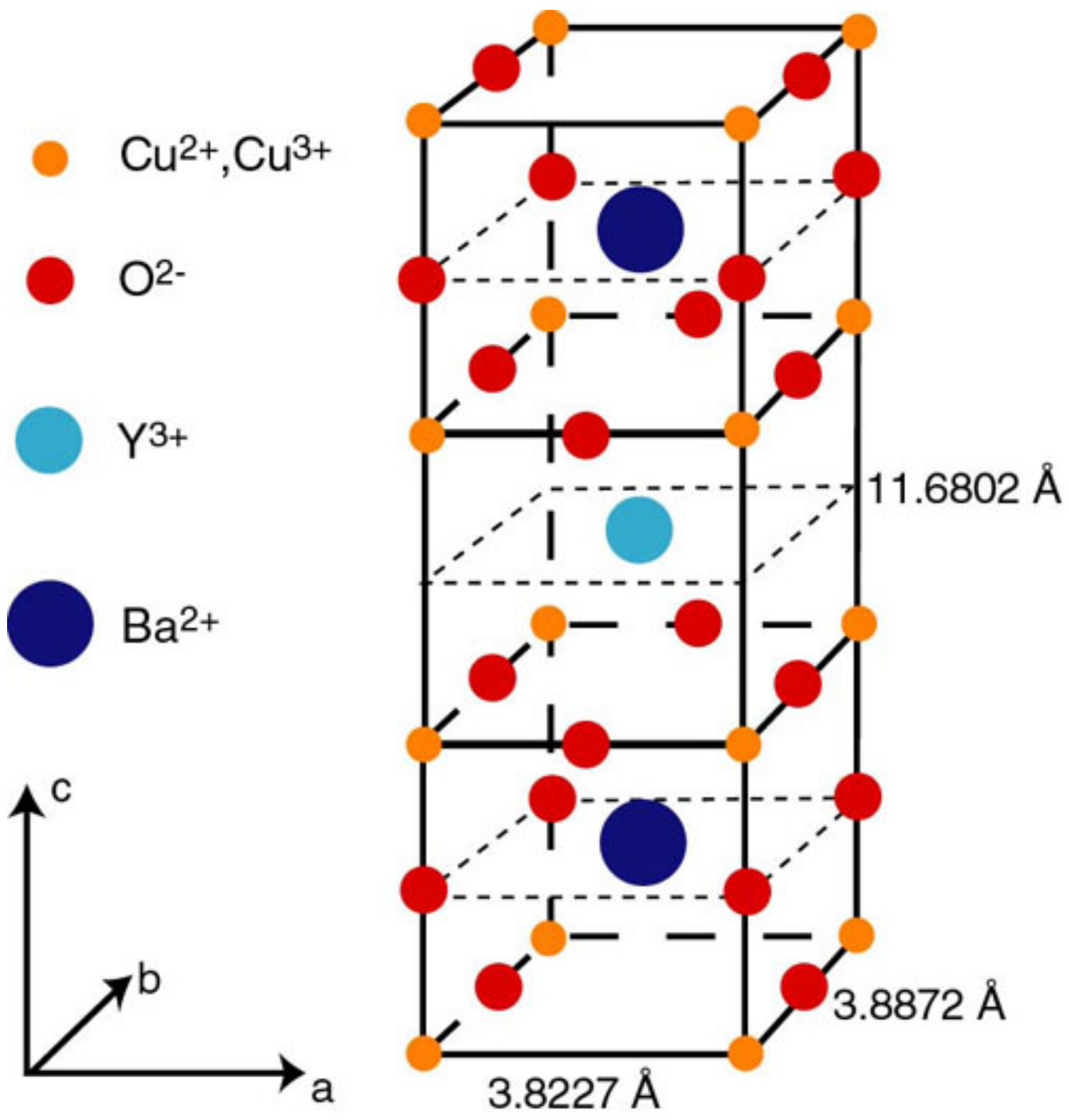
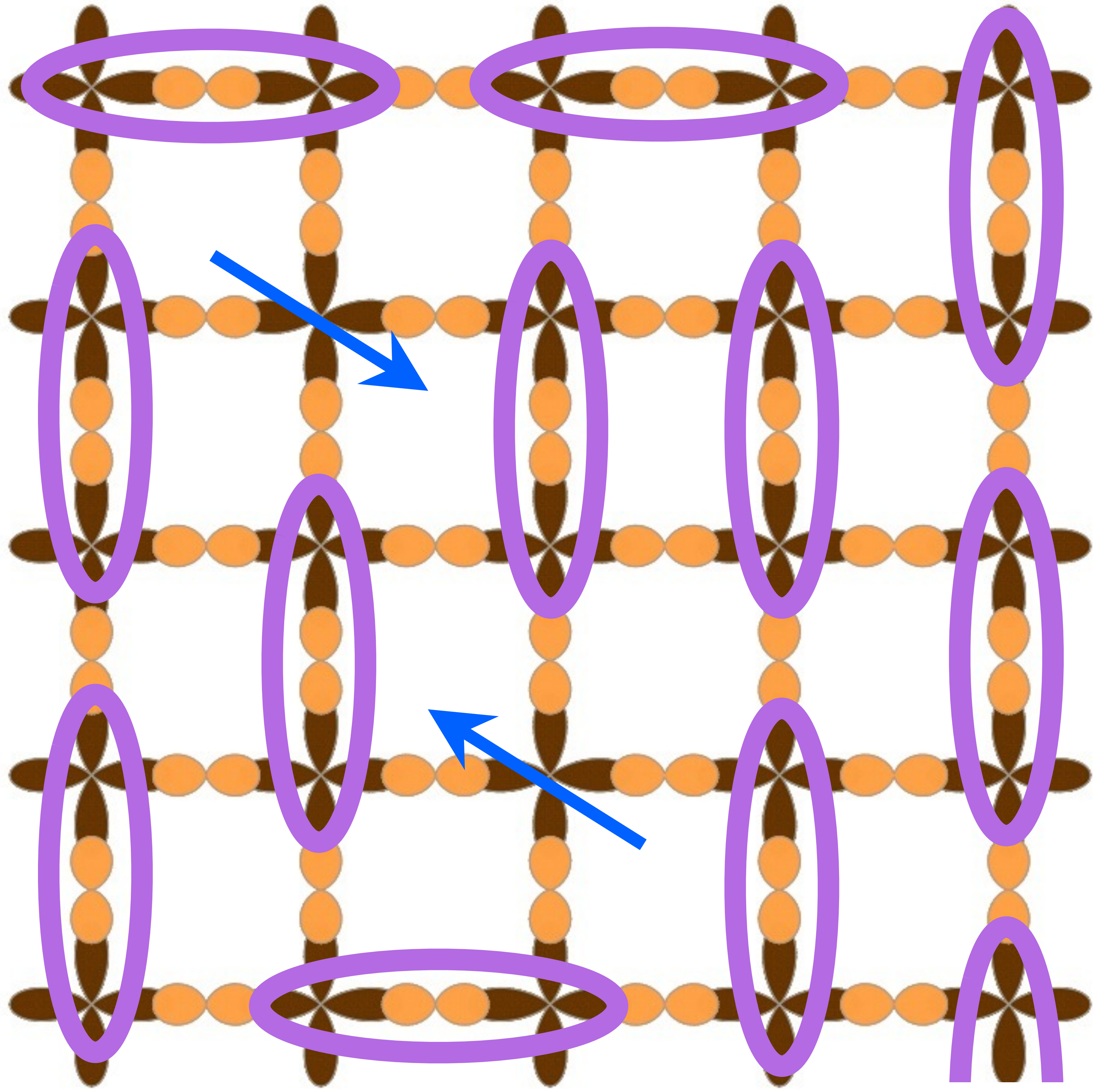
 $= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

Key feature: fractionalization. Excitations are particle-like, but cannot be created by local operators: they are classified under distinct superselection/anyon sectors.





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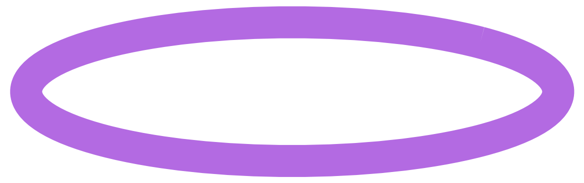 $= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

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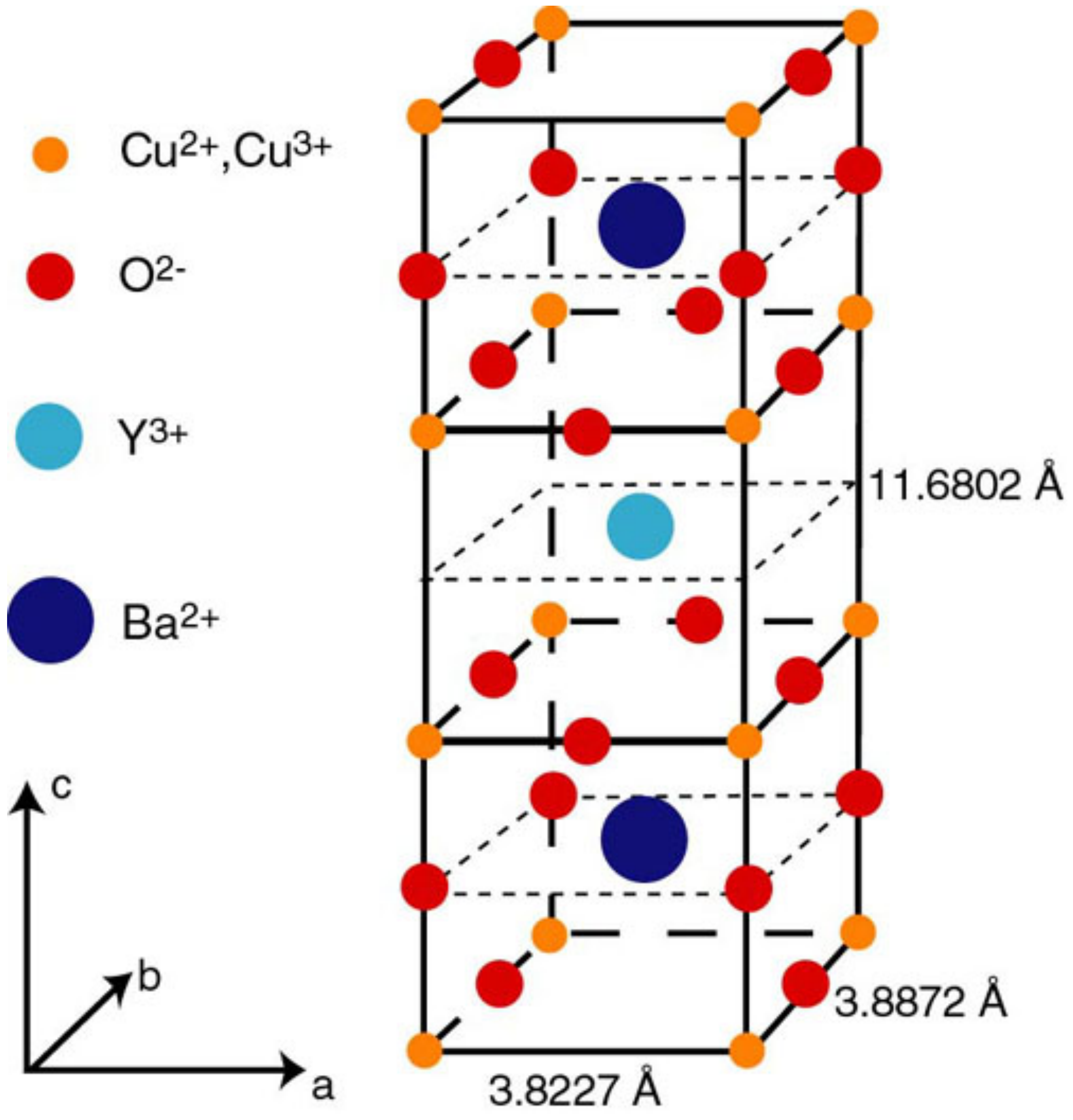

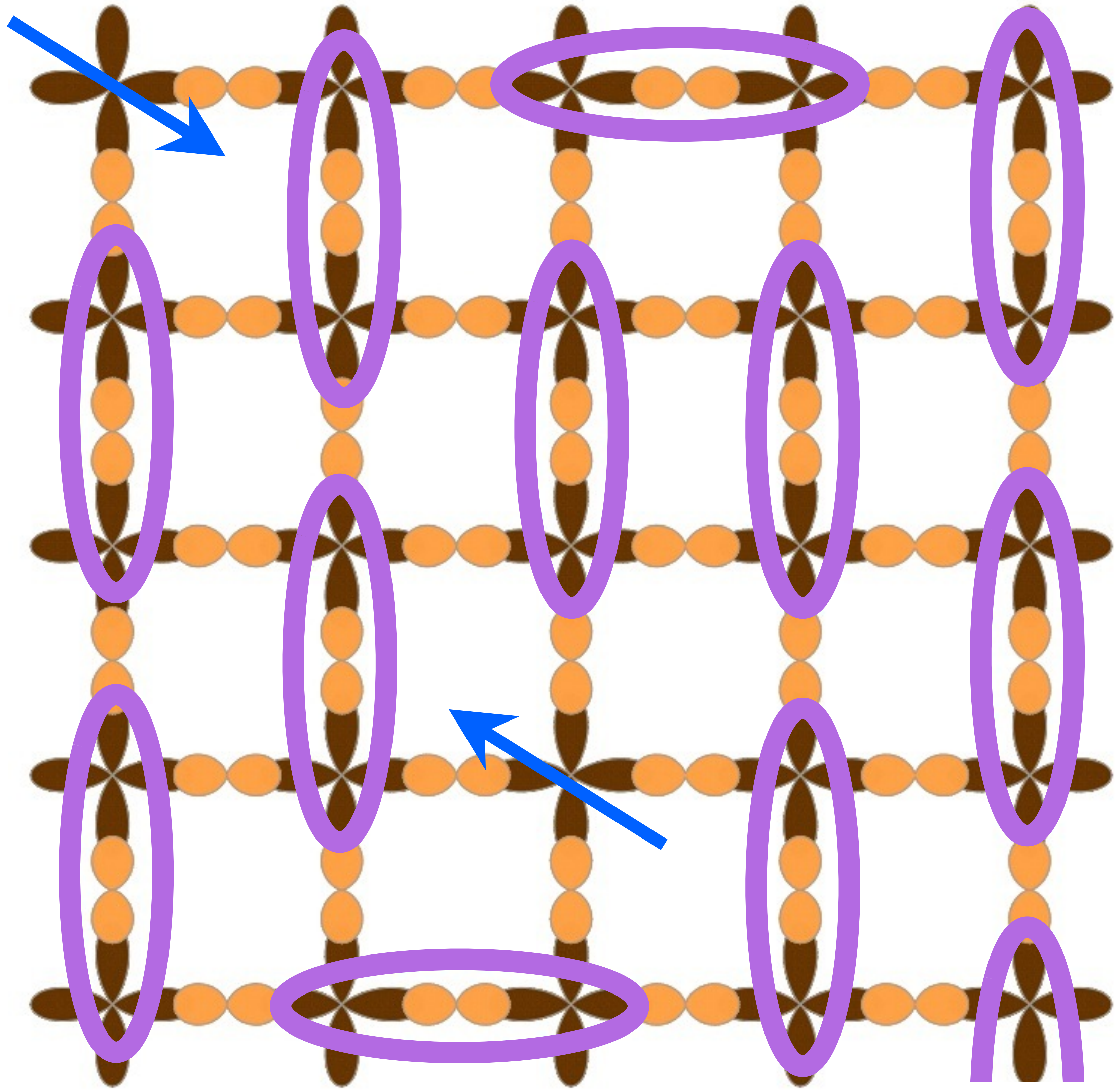

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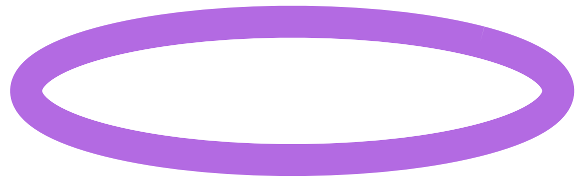 $= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

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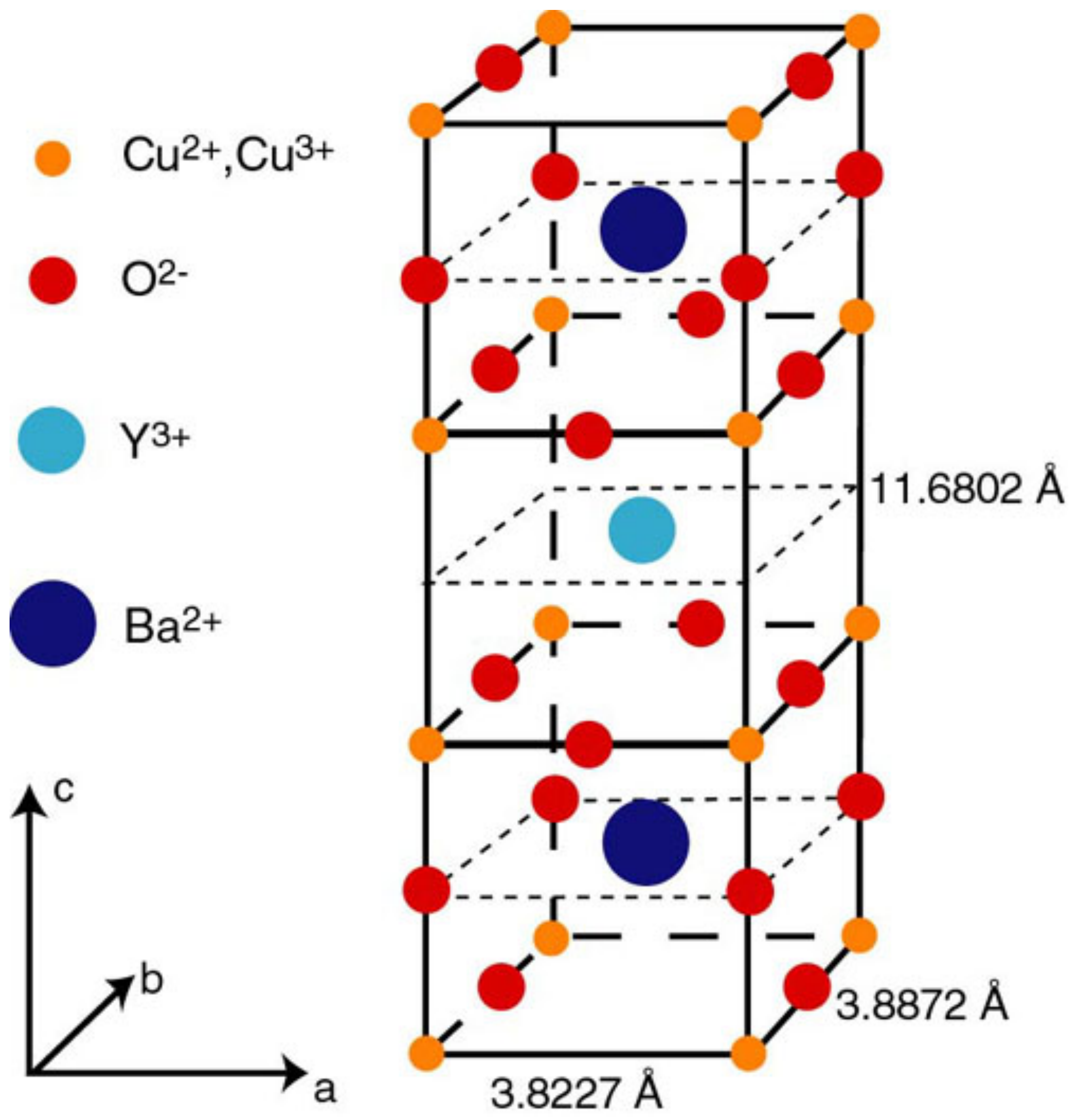

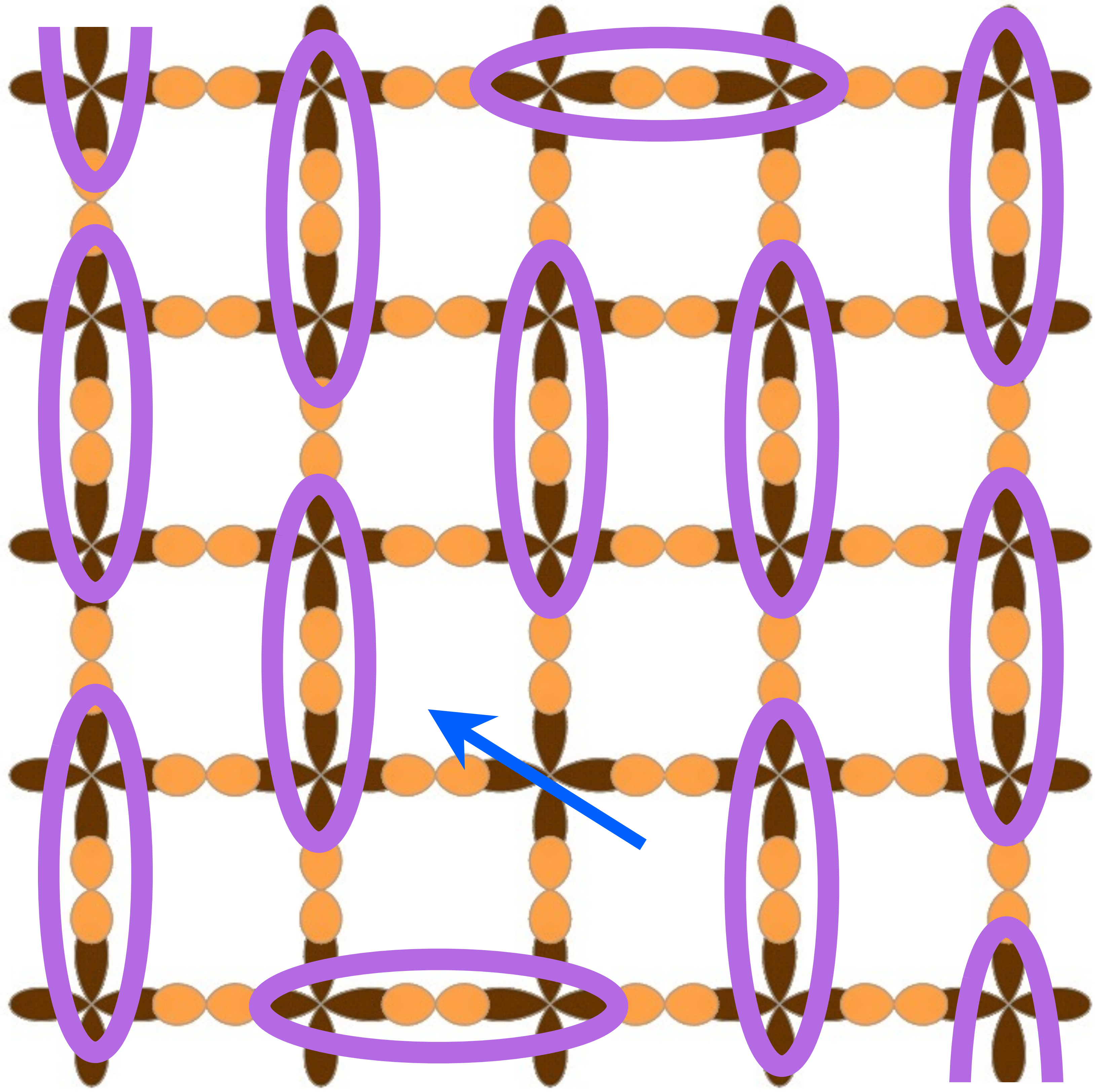

Spin S=1/2, charge neutral spinon

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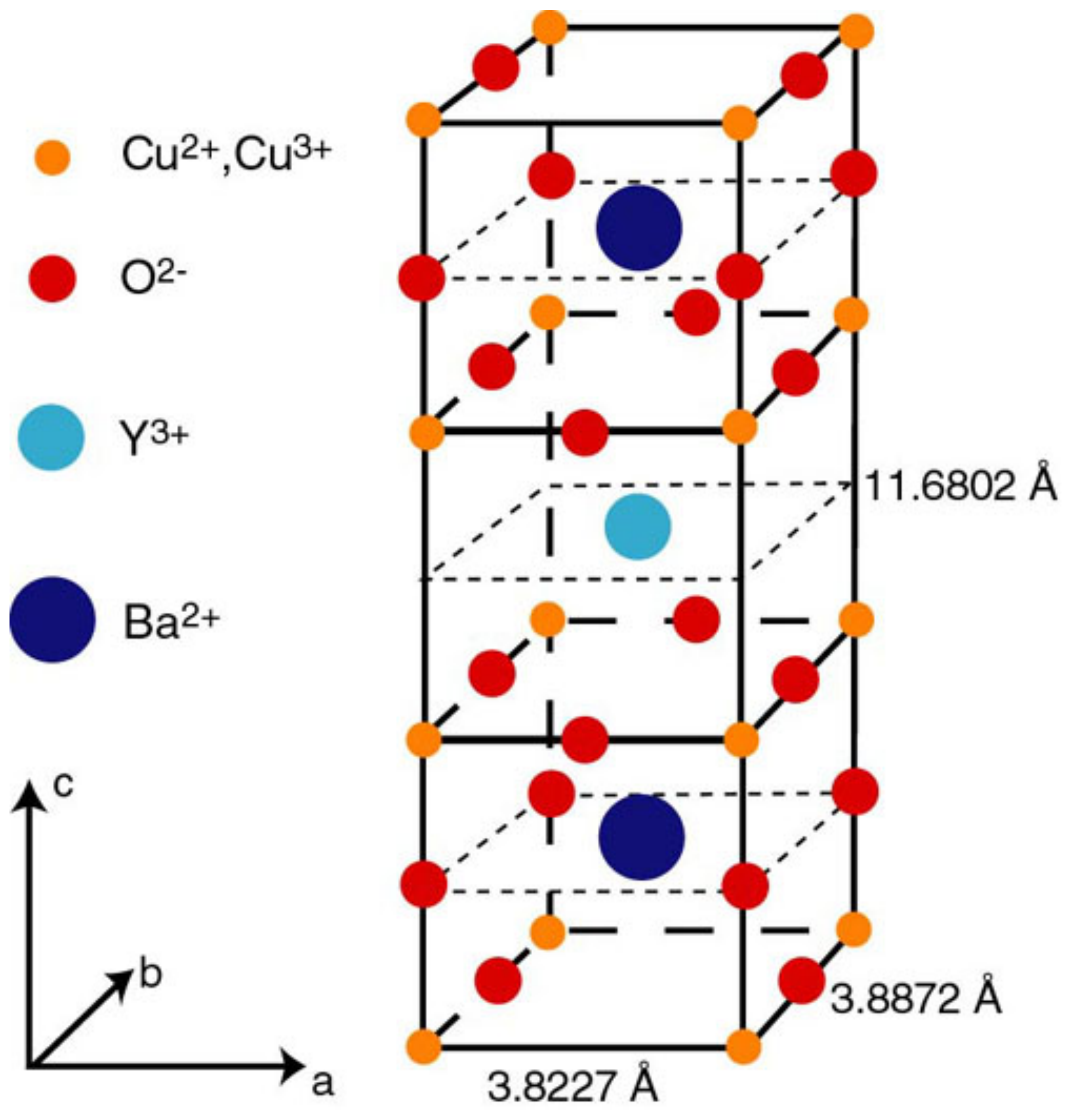

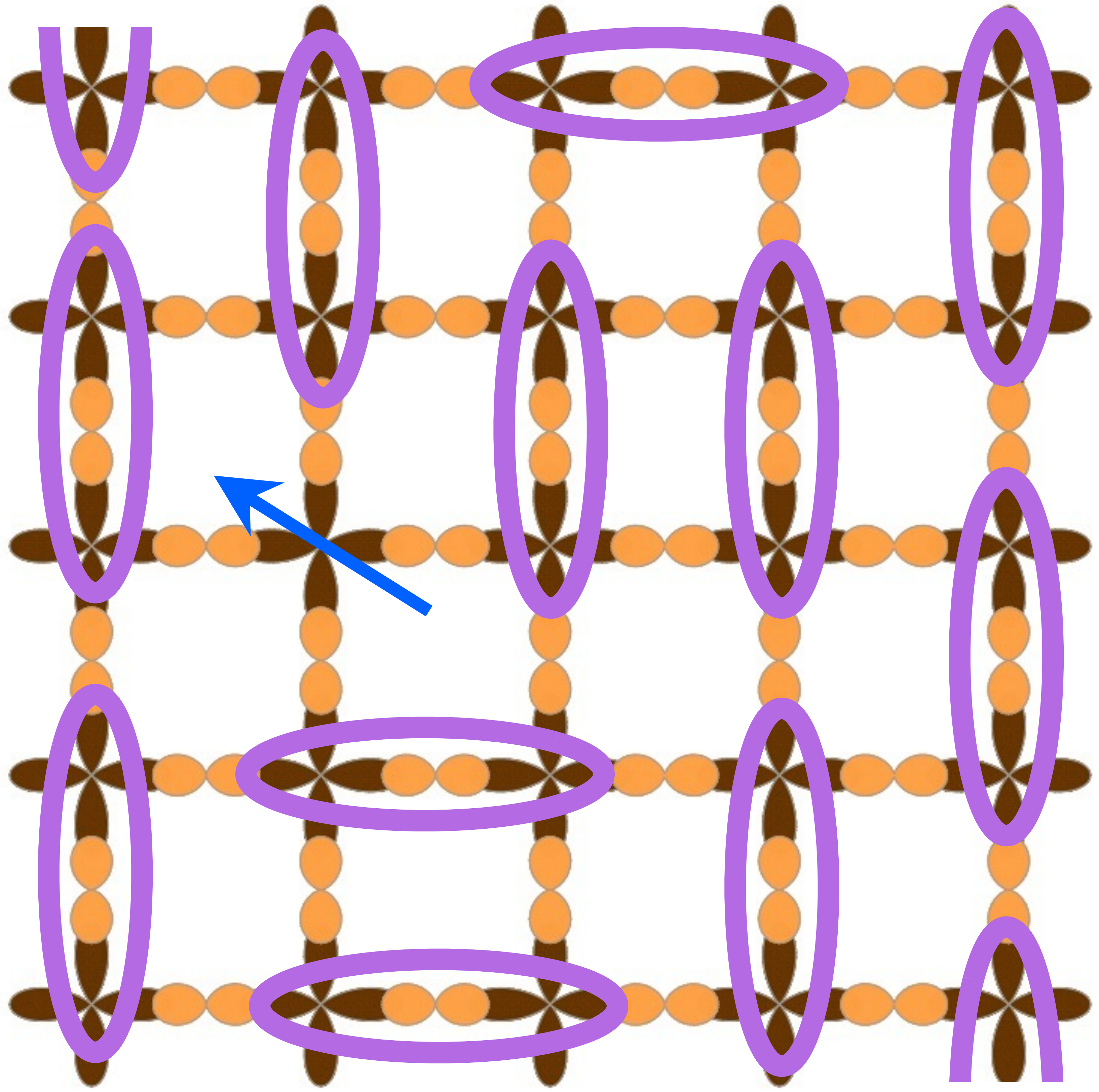

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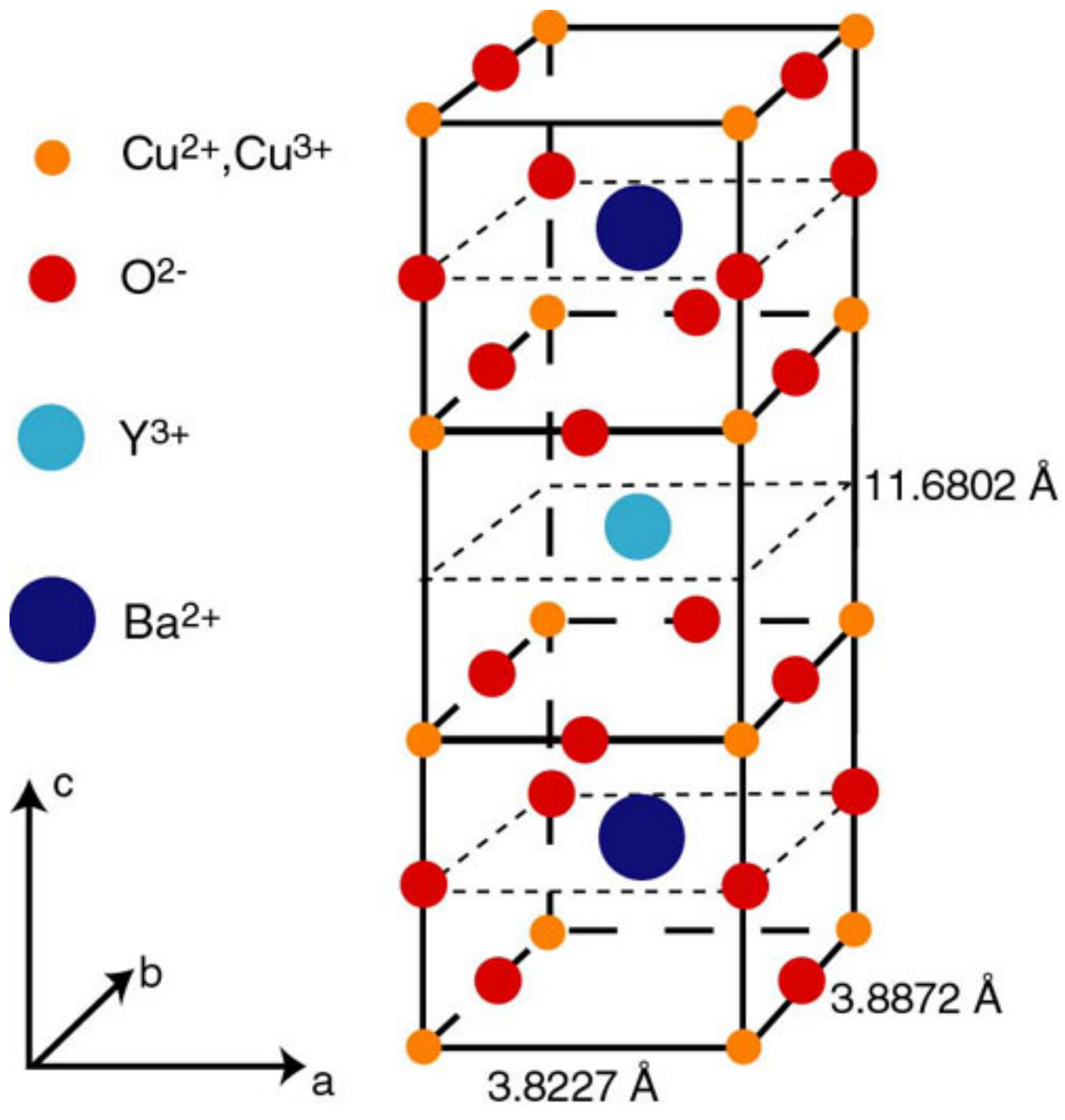

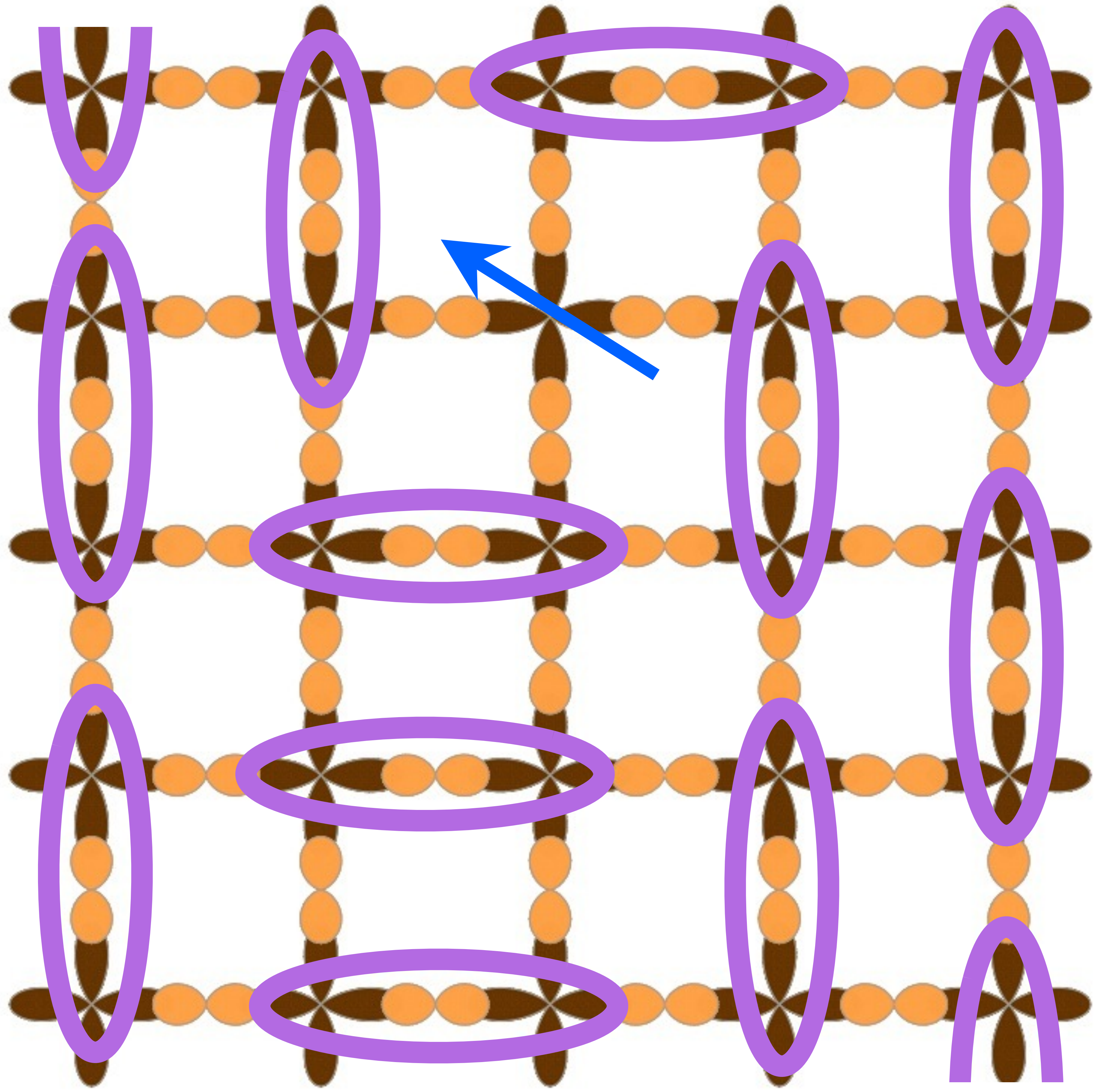

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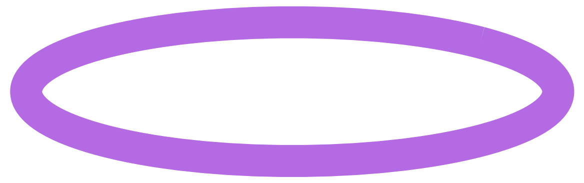 $= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

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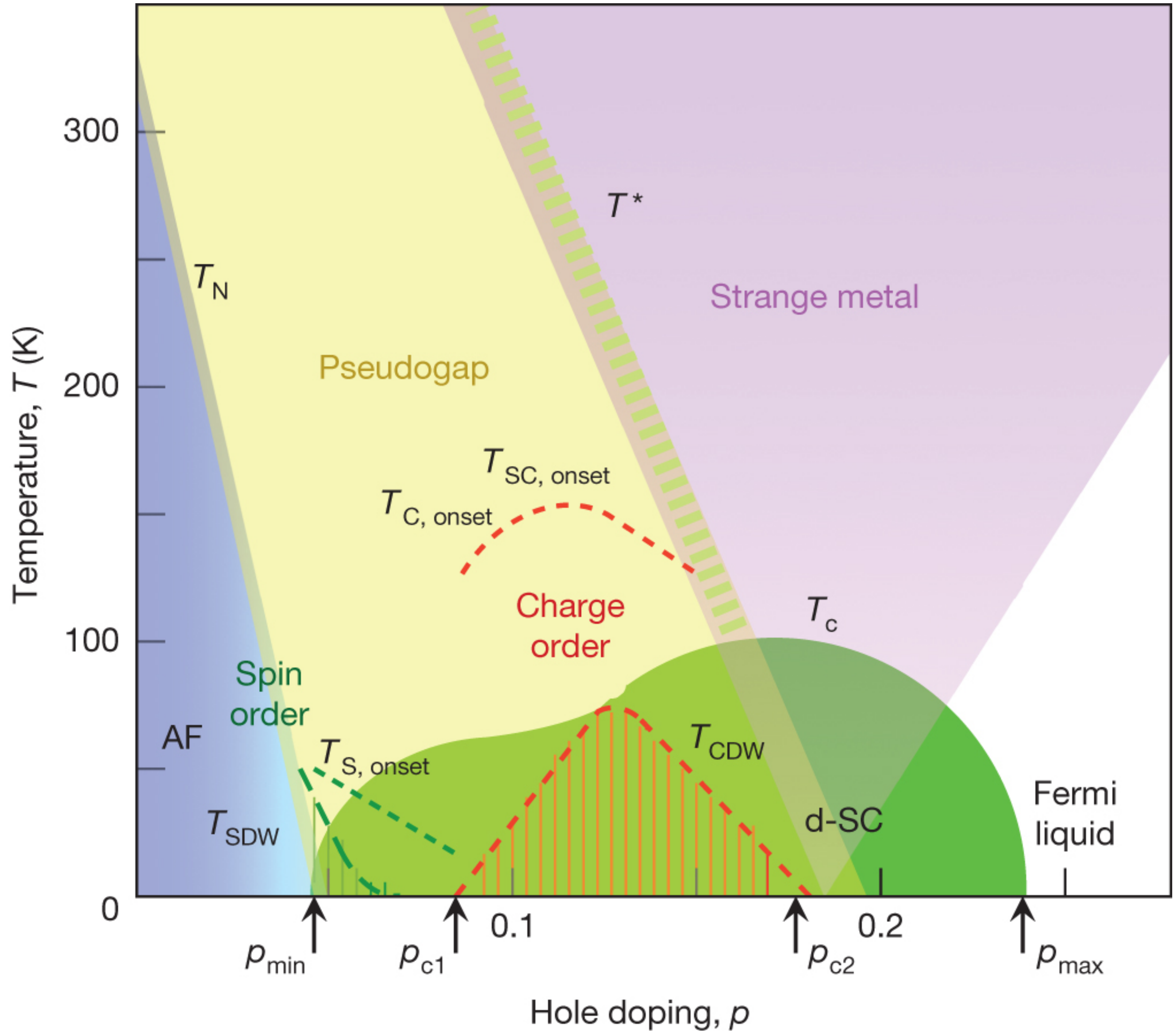
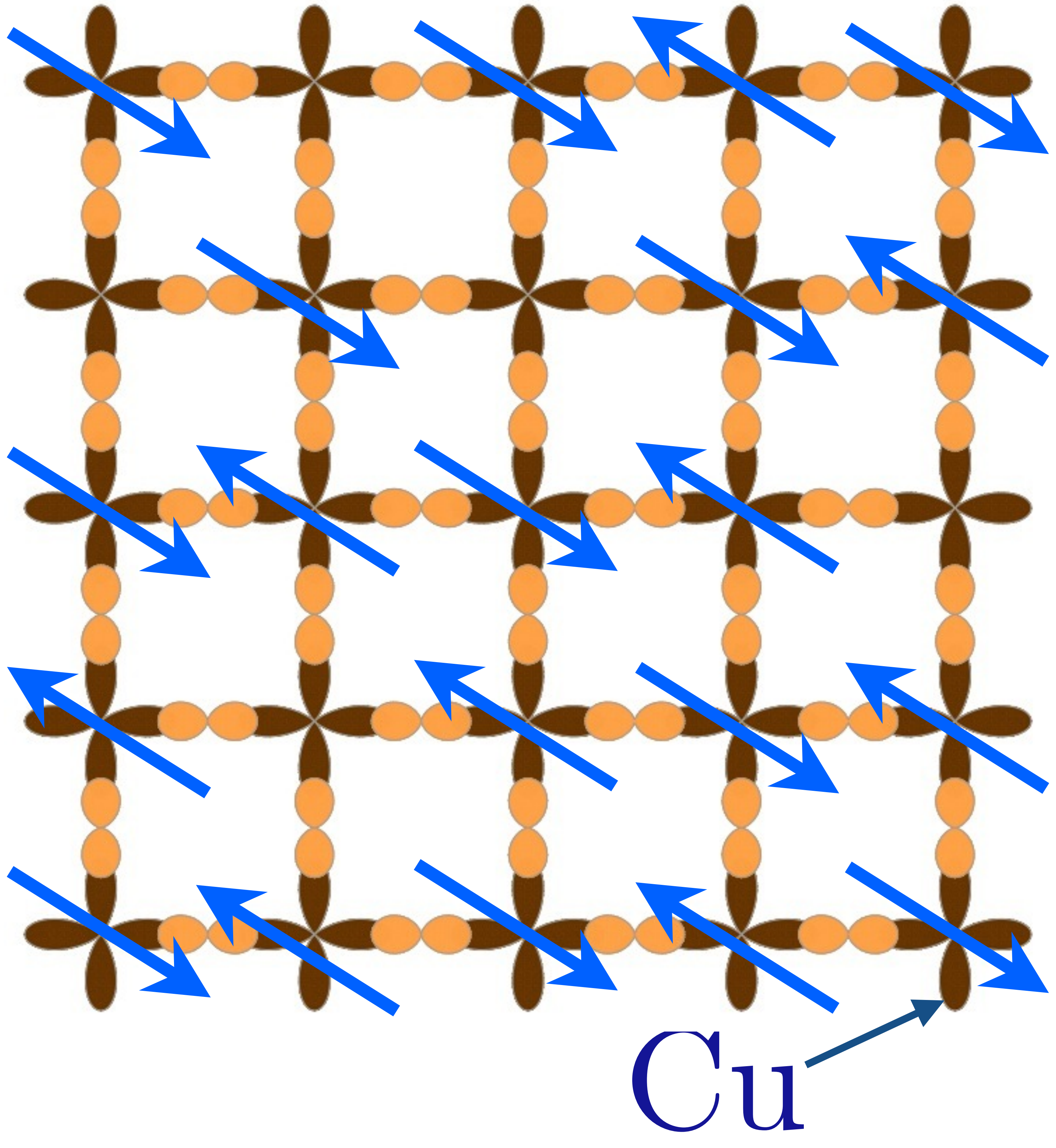
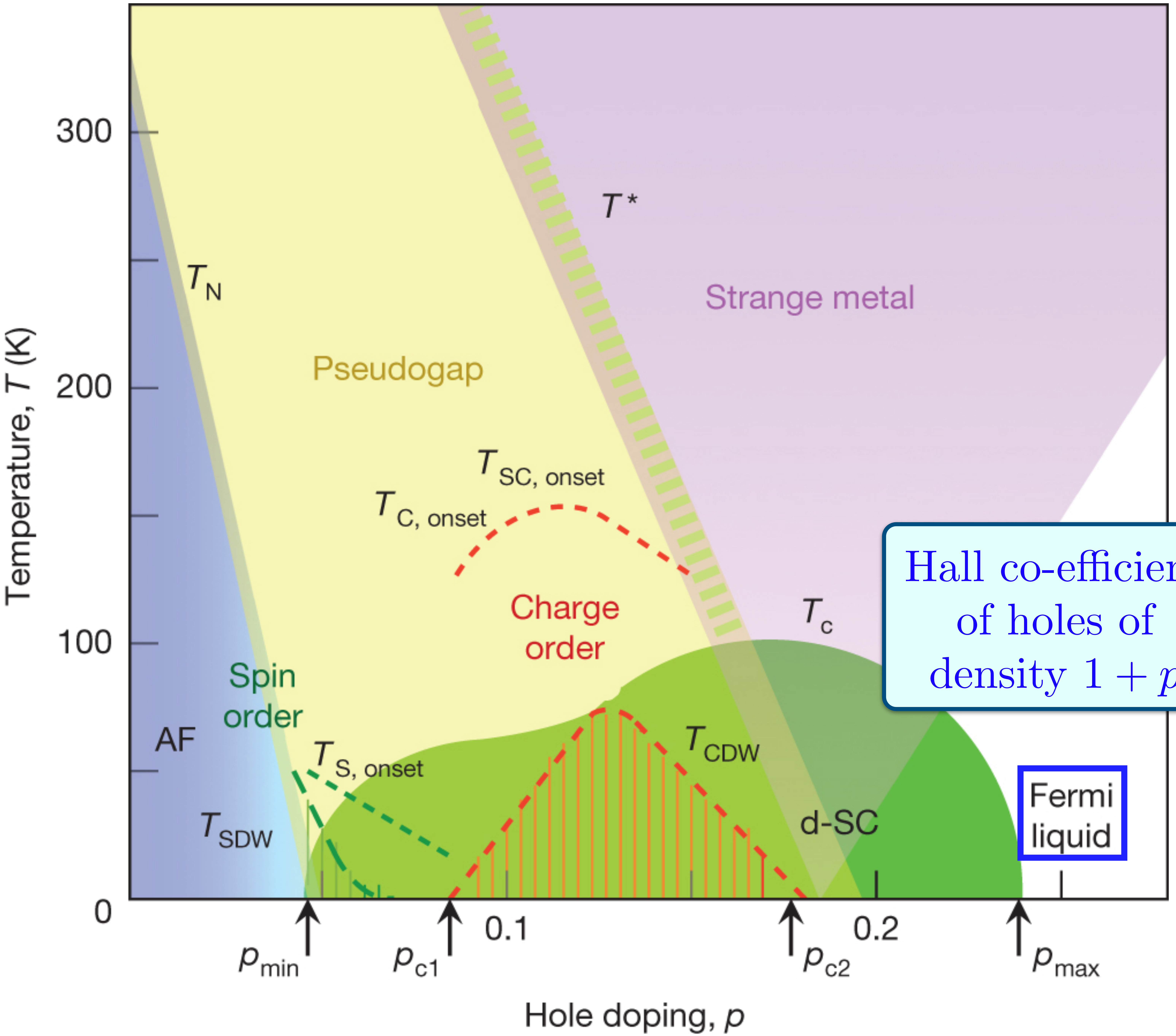
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)





d-SC obtained upon doping AF with density p holes.
Hole density relative to the filled band $\rho = 1 + p$.
Electron density relative to the empty band $\rho_e = 1 - p$.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Hall co-efficient of holes of density $1 + p$


Keimer, Kivelson, Norman, Uchida, and Zaanen, Nature 518, 179 (2015)
M. Platé et al., PRL 95, 077001 (2005)
300
200
100
0
Temperature, T (K)
T_N
T^*
Strange metal
Pseudogap
$T_{SC, onset}$
$T_{C, onset}$
Charge order
T_c
Spin order
AF
$T_{S, onset}$
T_{CDW}
T_{SDW}
d-SC
Fermi liquid
0.1
0.2
p_{min}
p_{c1}
p_{c2}
p_{max}
Hole doping, p
(π,π)
(0,0)
(d)
Hall co-efficient of holes of density $1+p$
$1-p$ electrons
$1+p$ holes

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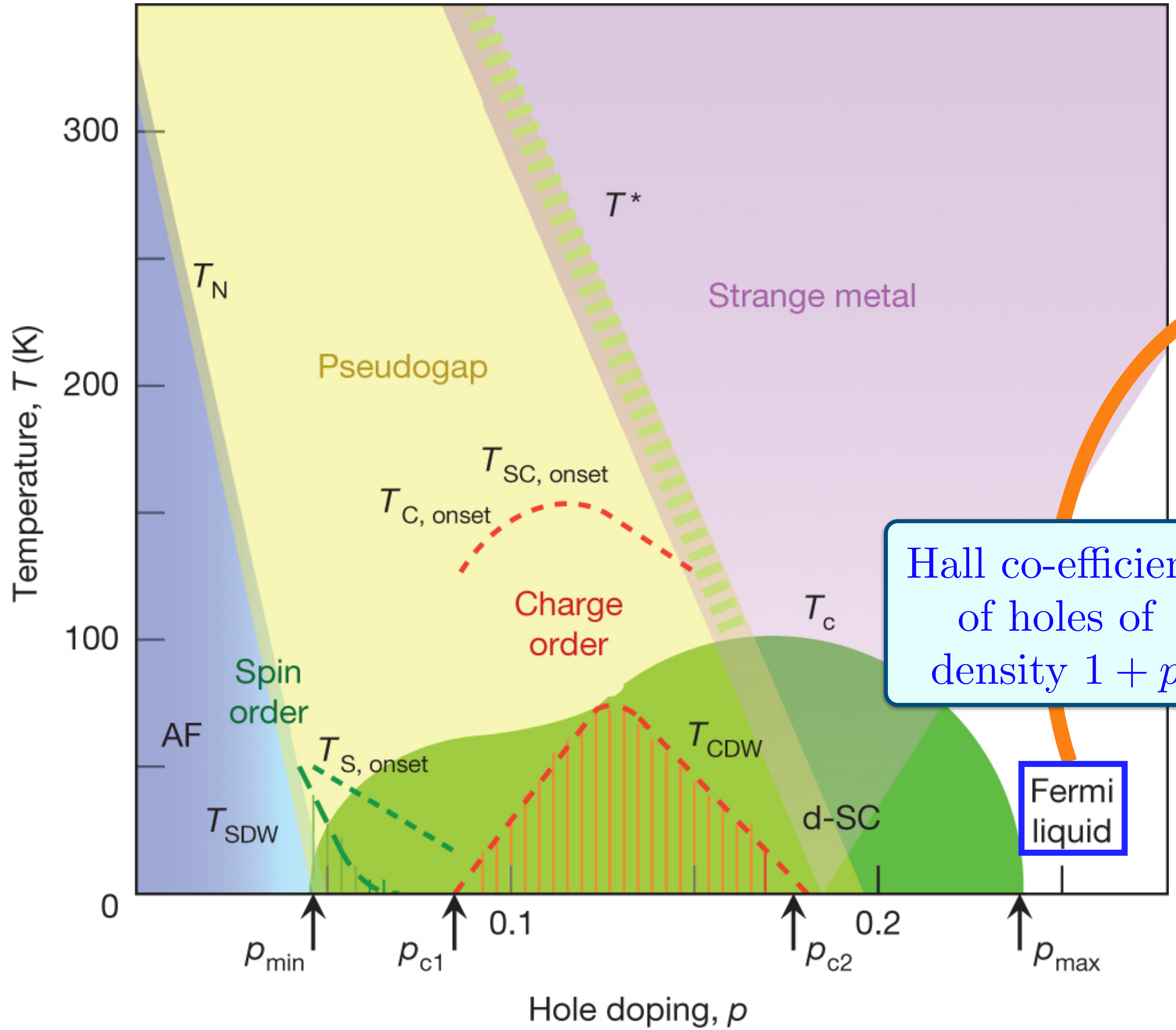

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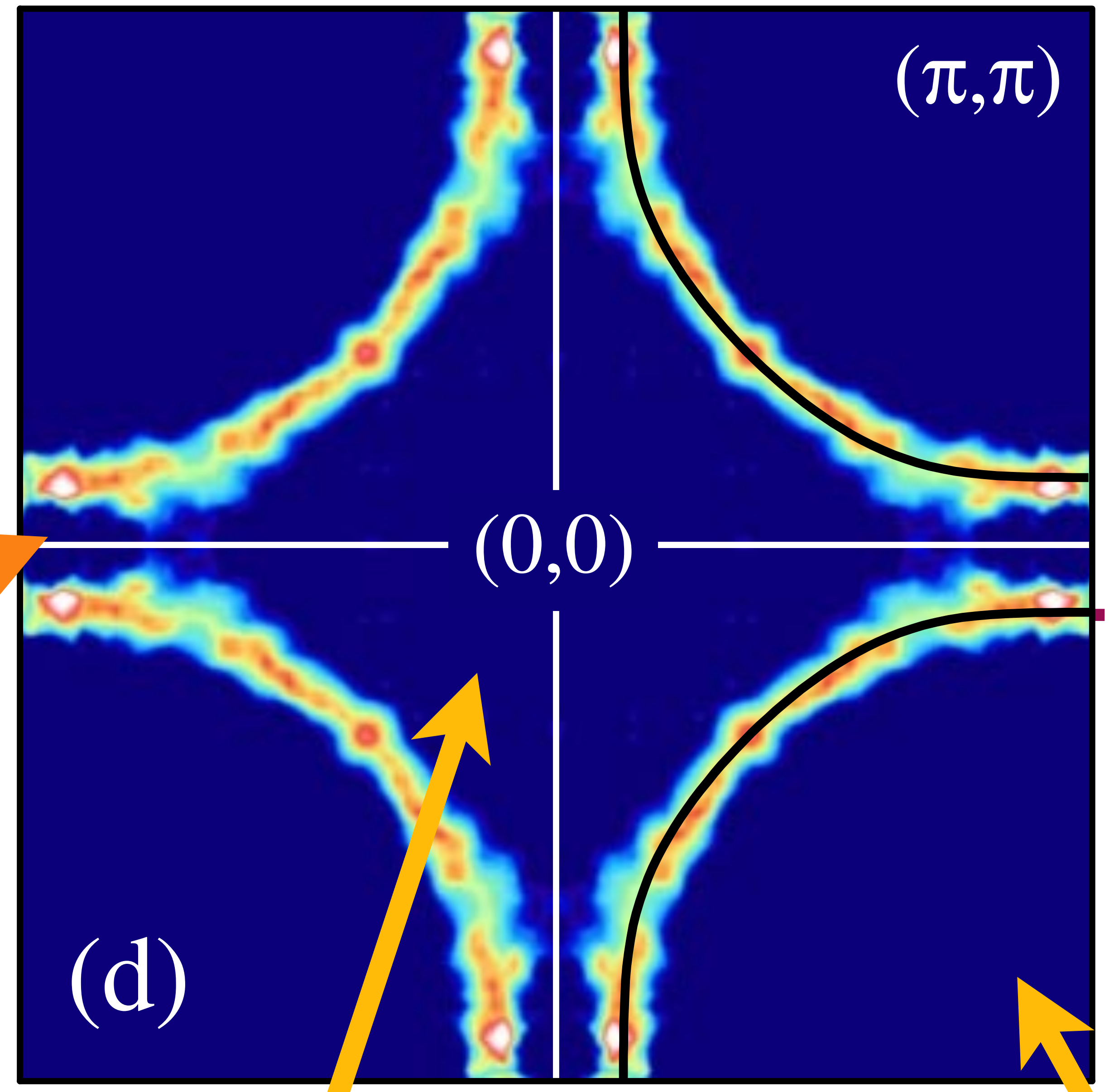

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1-p electrons

1+p holes

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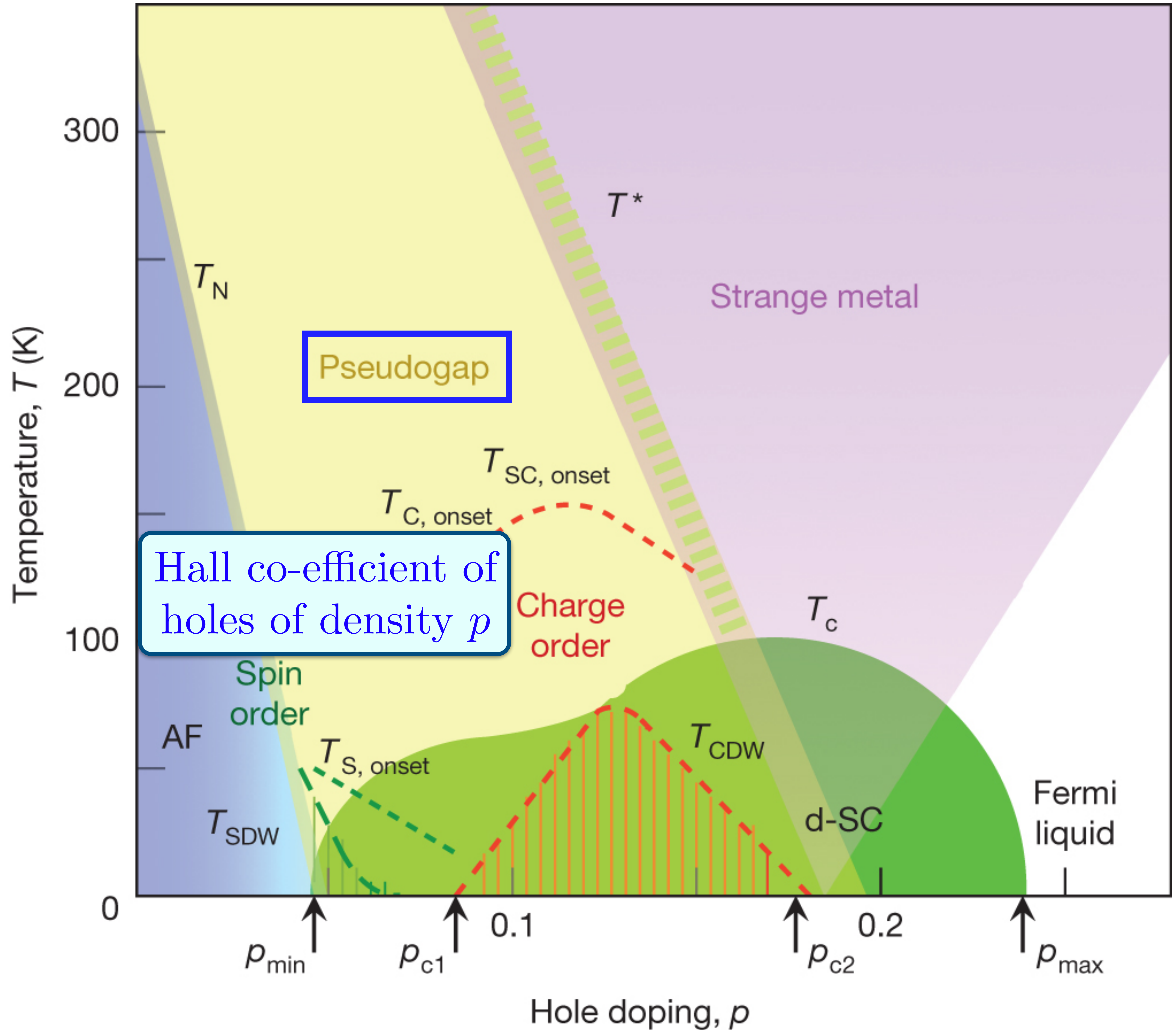
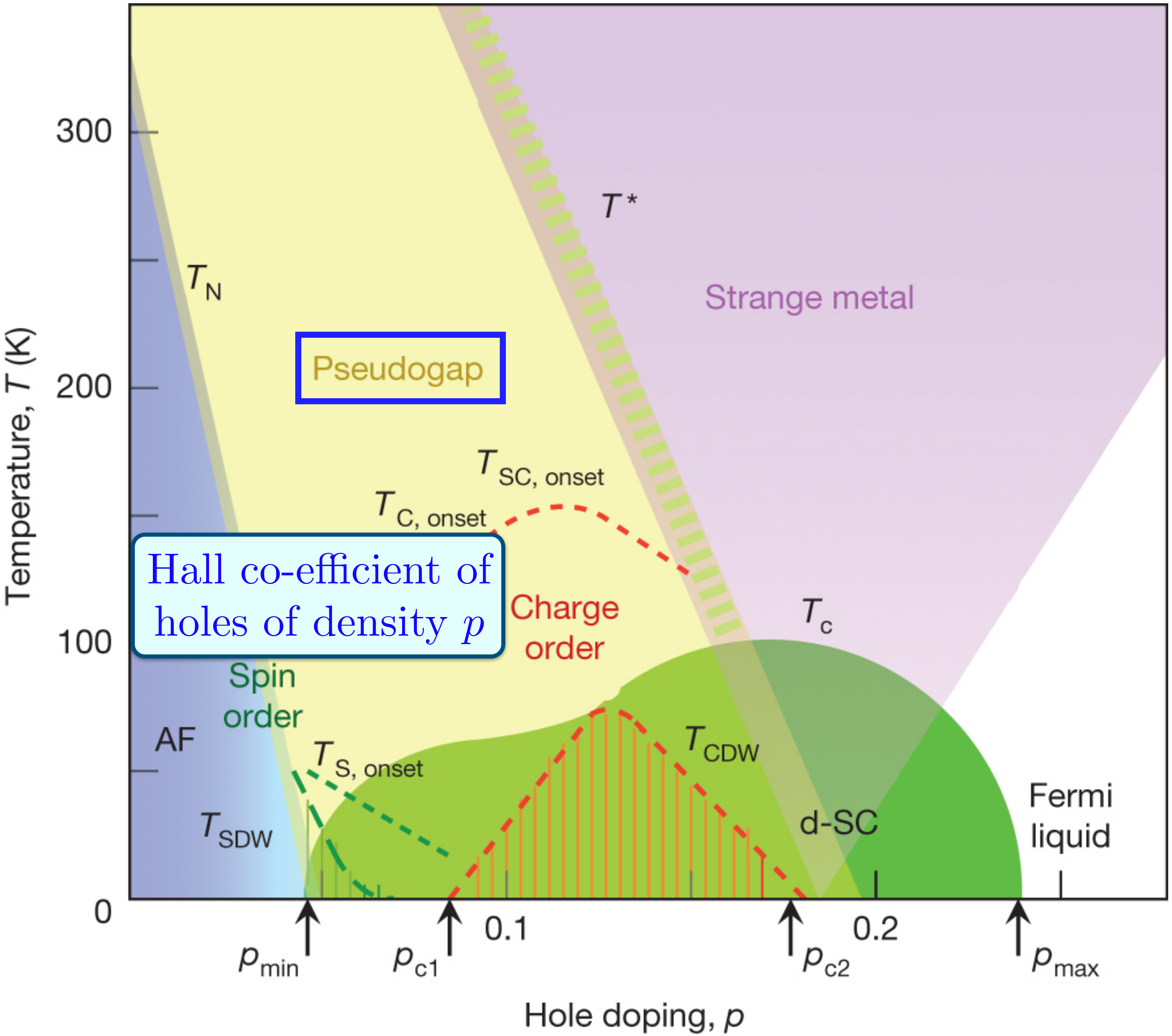


But there is no antiferromagnetic order to justify carrier density p

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



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Many theories with fluctuating AFM, d-SC and charge orders.

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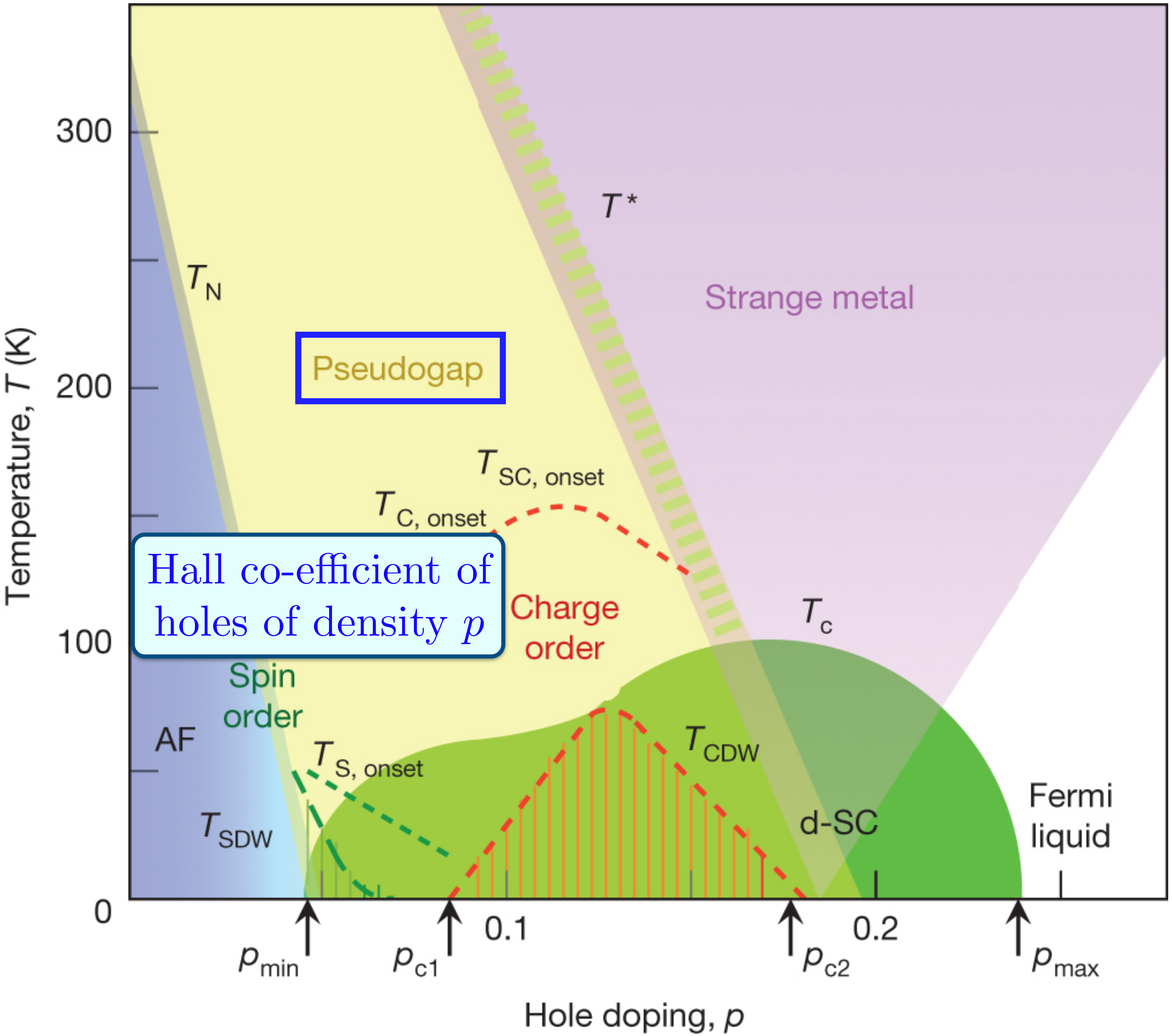

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I argue that a better starting point is a novel quantum ground state with no broken symmetry.

This approach predicted key features of recent ADMR experiments….

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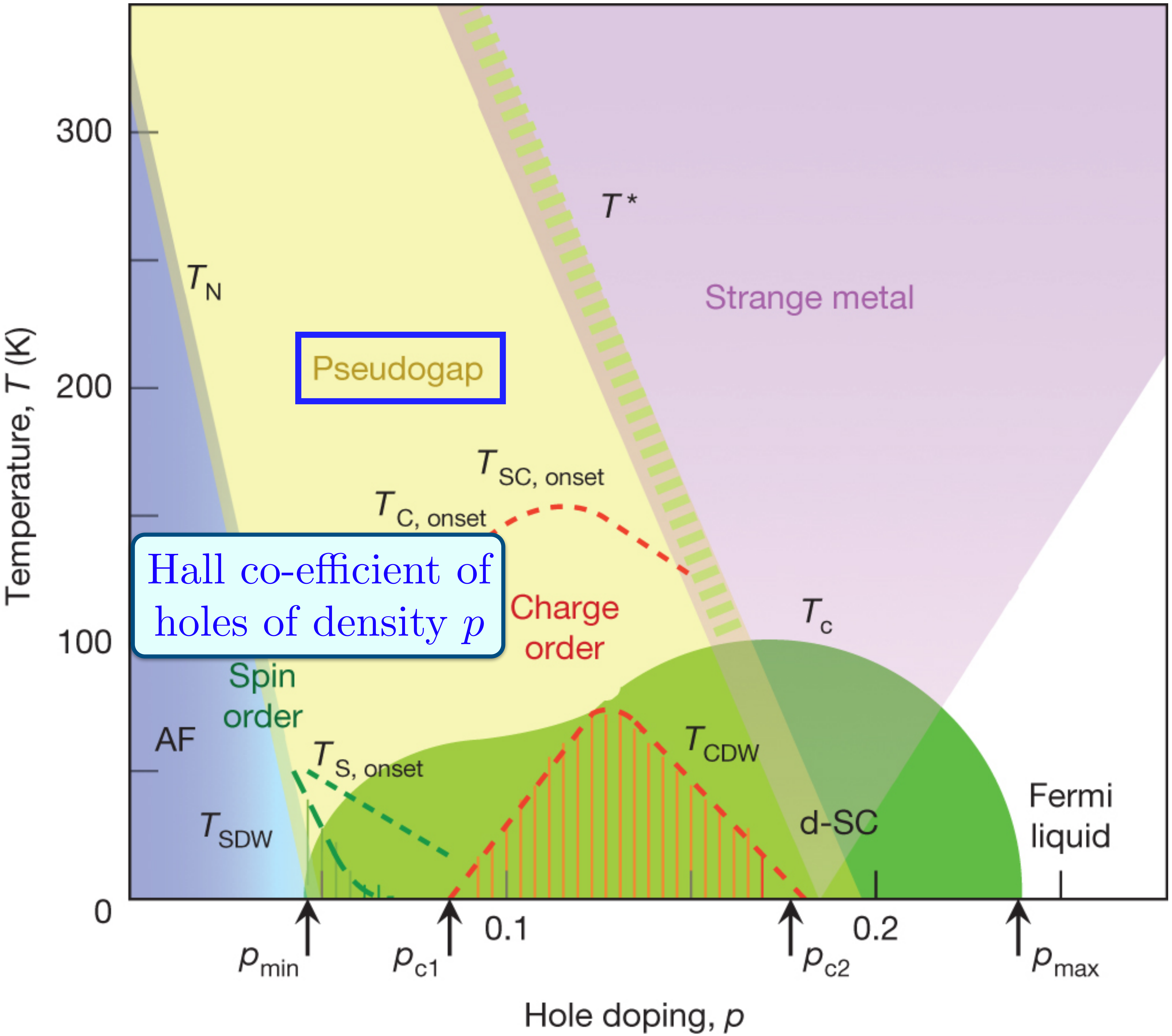

Quantum entanglement of mobile fermions

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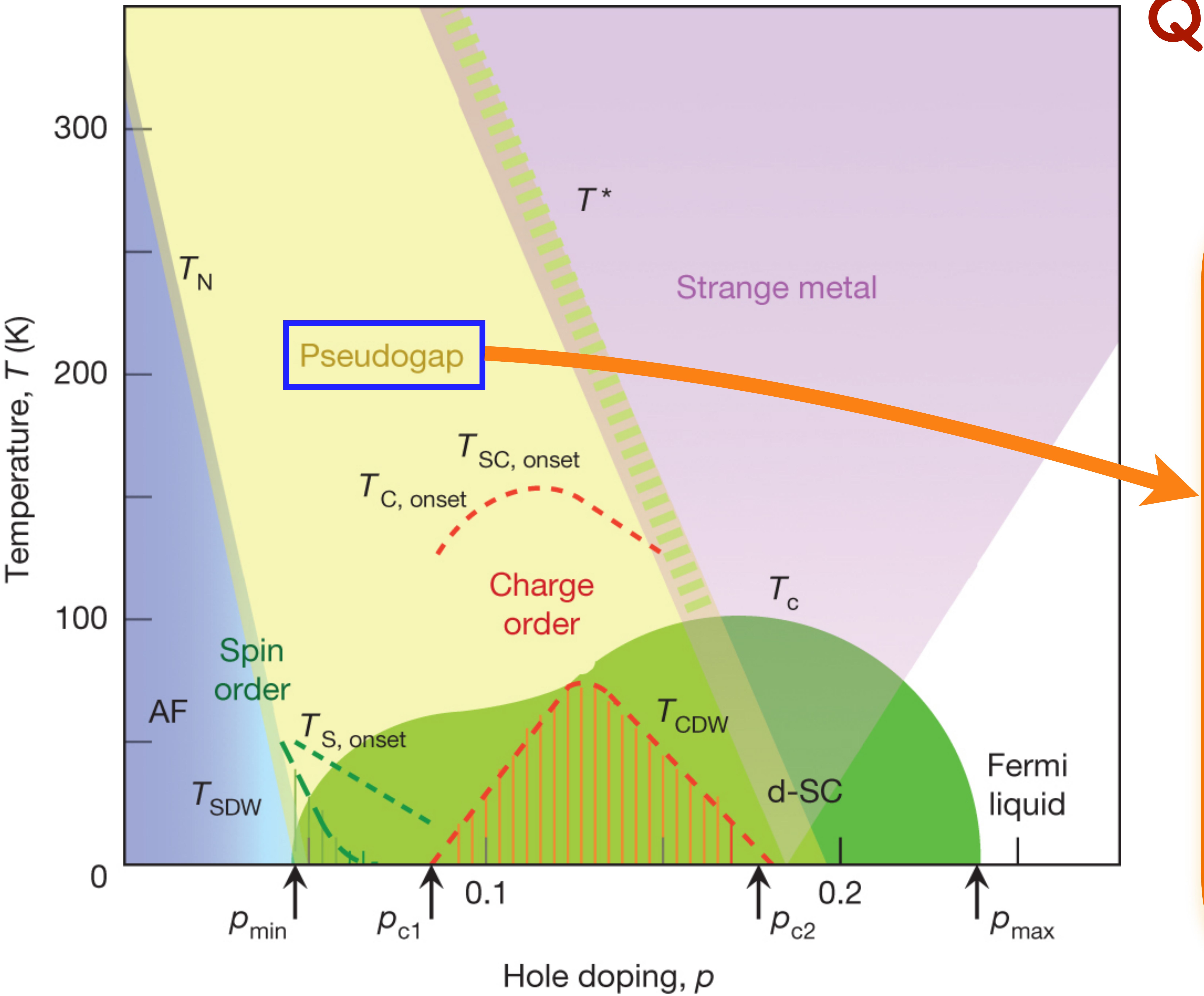
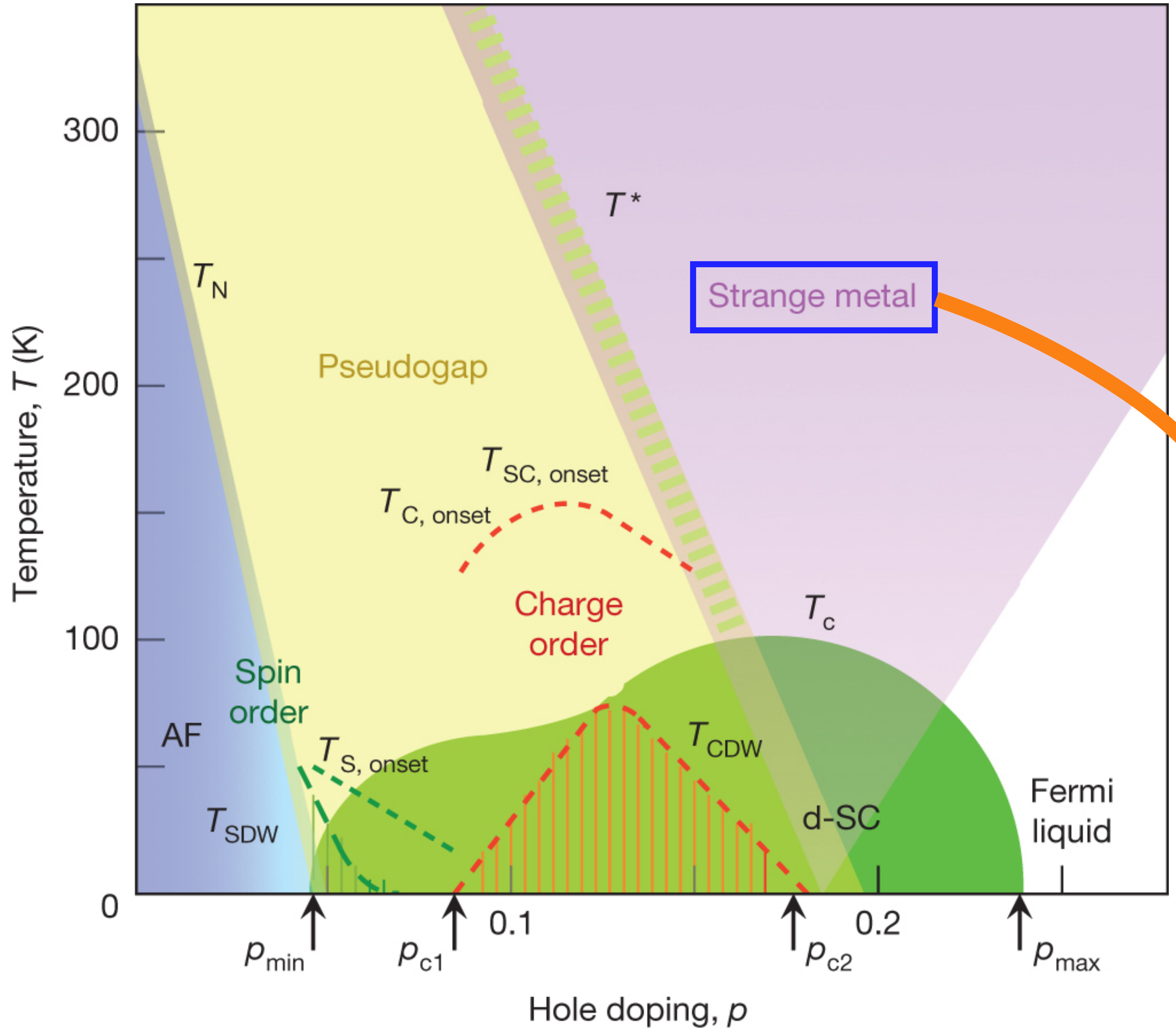


Quantum entanglement of mobile fermions

Fractionalized Fermi Liquid (FL*)

Entanglement of a (critical or gapped) quantum spin liquid coexisting with electronic Landau quasiparticles. Charge is carried by ordinary electrons, but there are also fractionalized (anyonic) spinon excitations.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

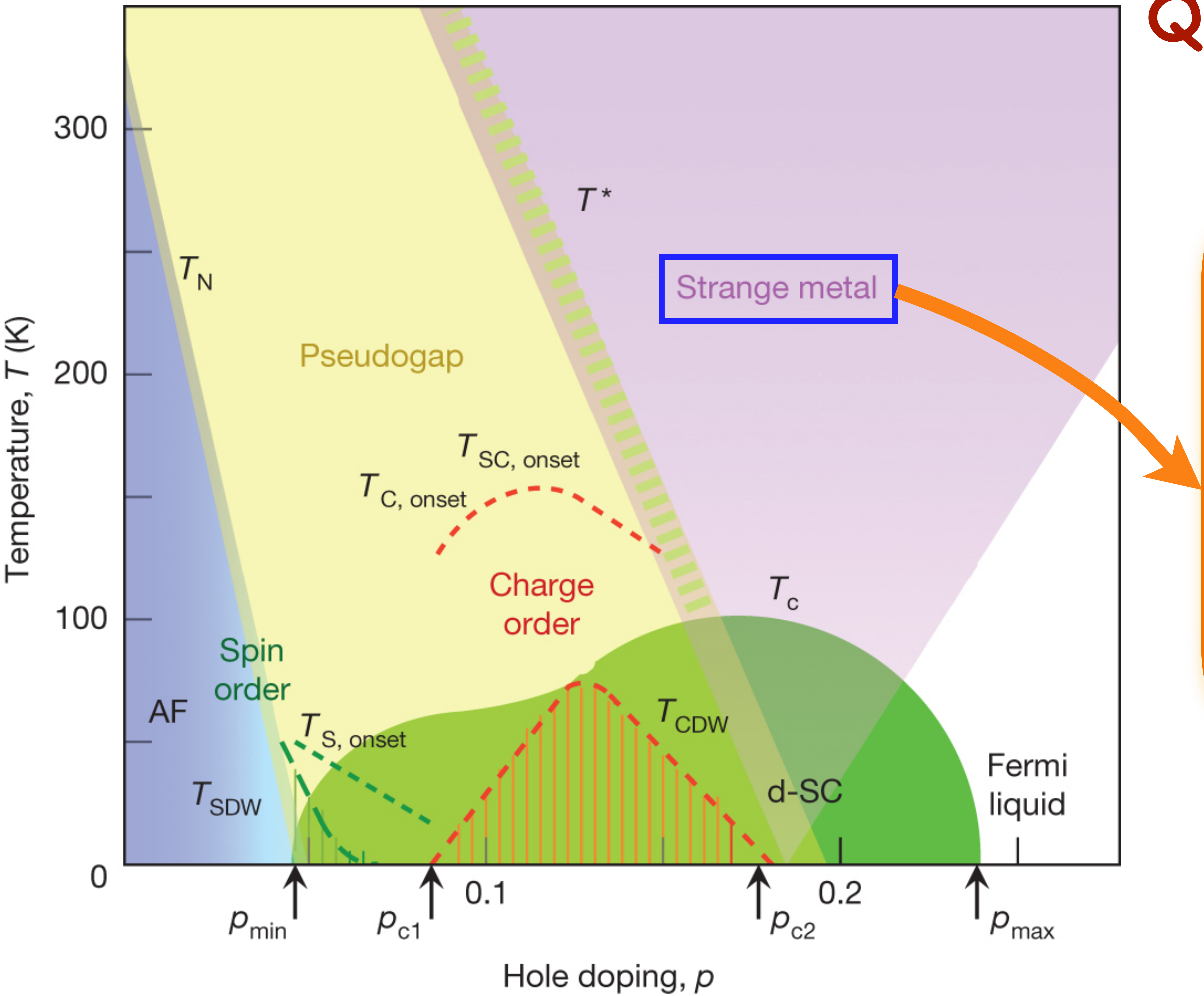


Quantum entanglement of mobile fermions

Sachdev-Ye-Kitaev (SYK) liquid

- Complex entanglement of a compressible state with no quasiparticles.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



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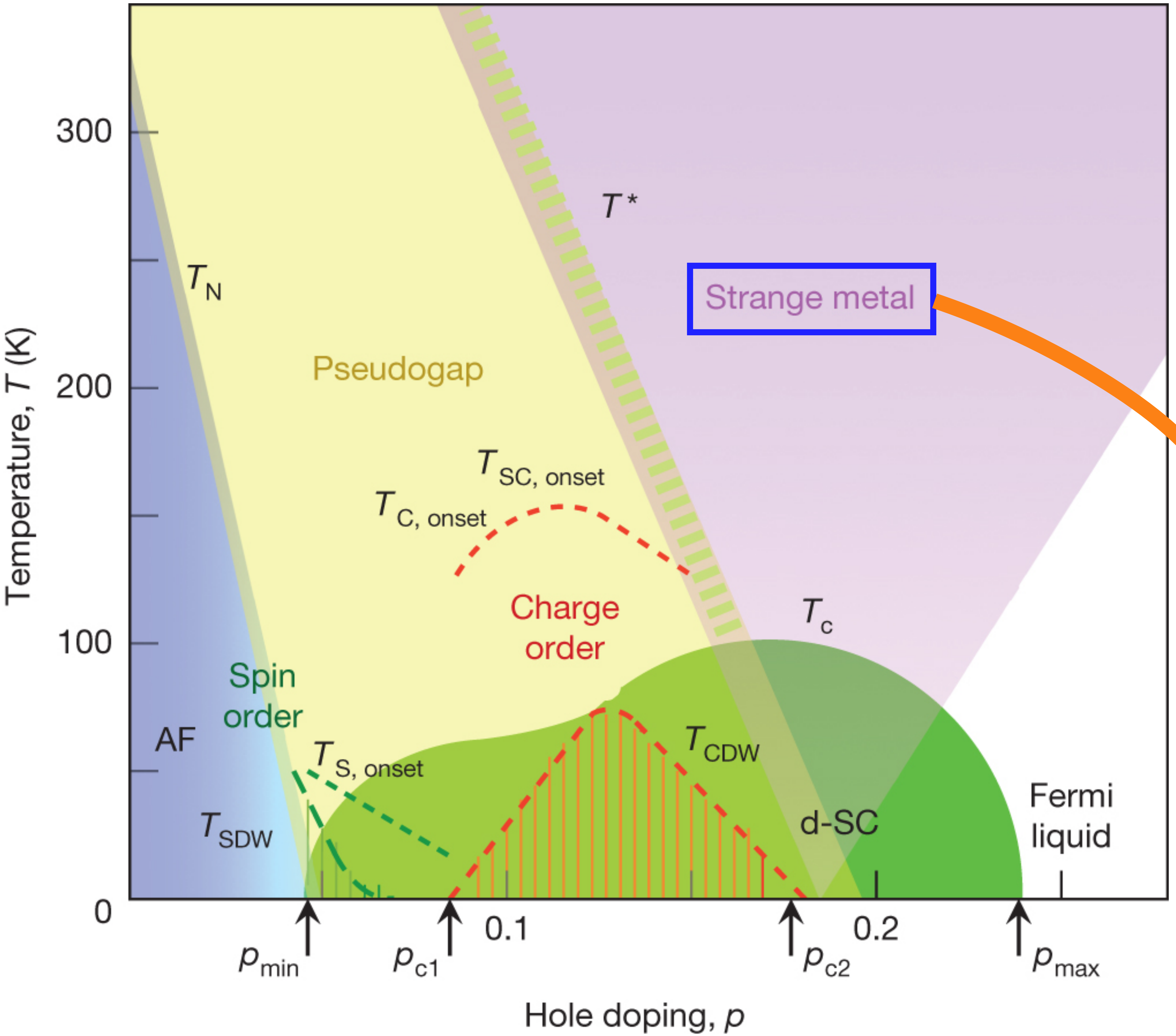


Quantum entanglement of mobile fermions

Sachdev-Ye-Kitaev (SYK) liquid

- Complex entanglement of a compressible state with no quasiparticles.
- Universal theory of strange metals
- Also applies to generic charged black holes in asymptotically flat 3+1 dimensional space.

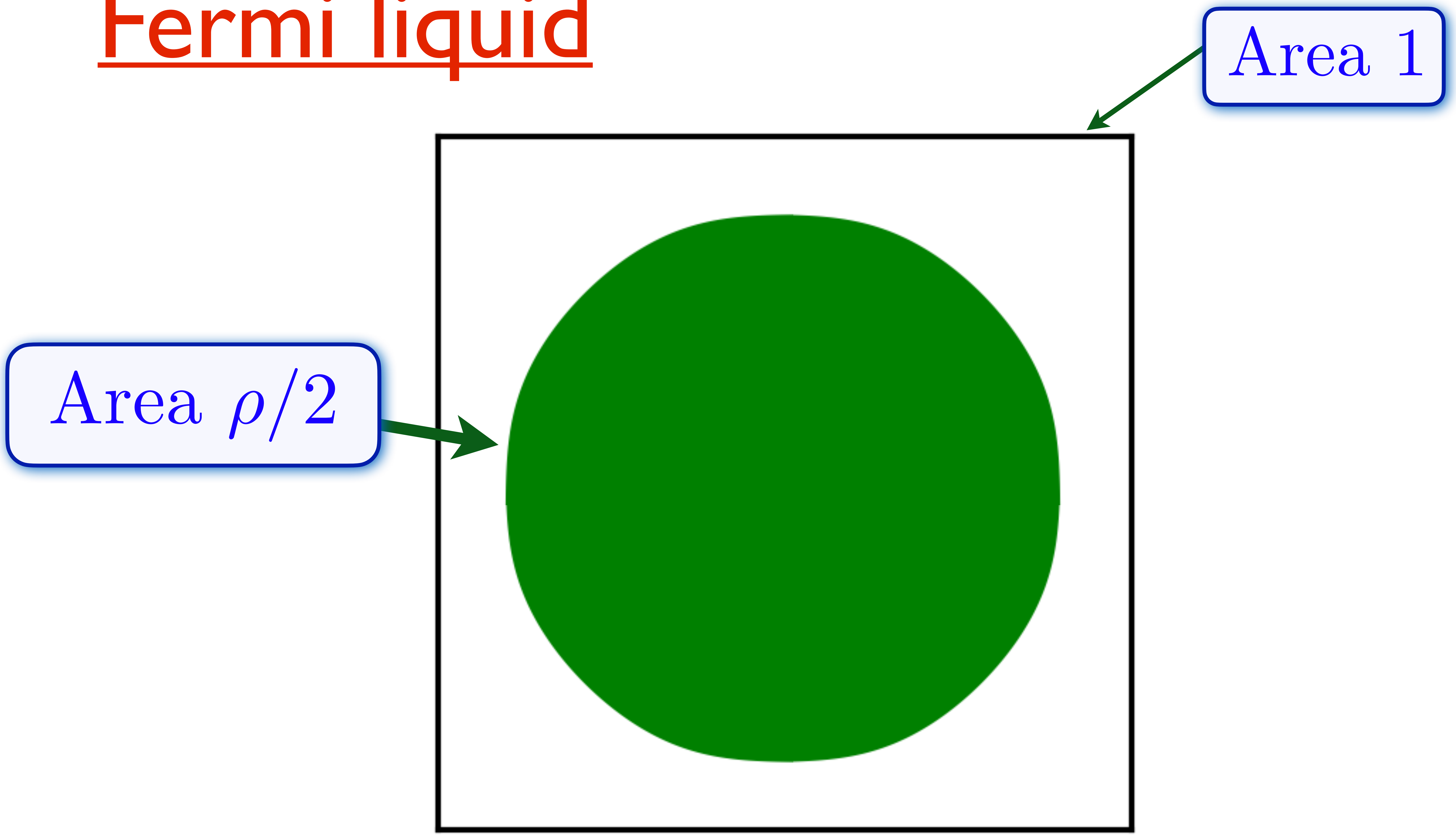
Fractionalized Fermi liquids (FL*)

Fermi liquid

Spin-1/2 holes of density

$$\rho = 1 + p$$

Positive Hall coefficient of carrier density ρ



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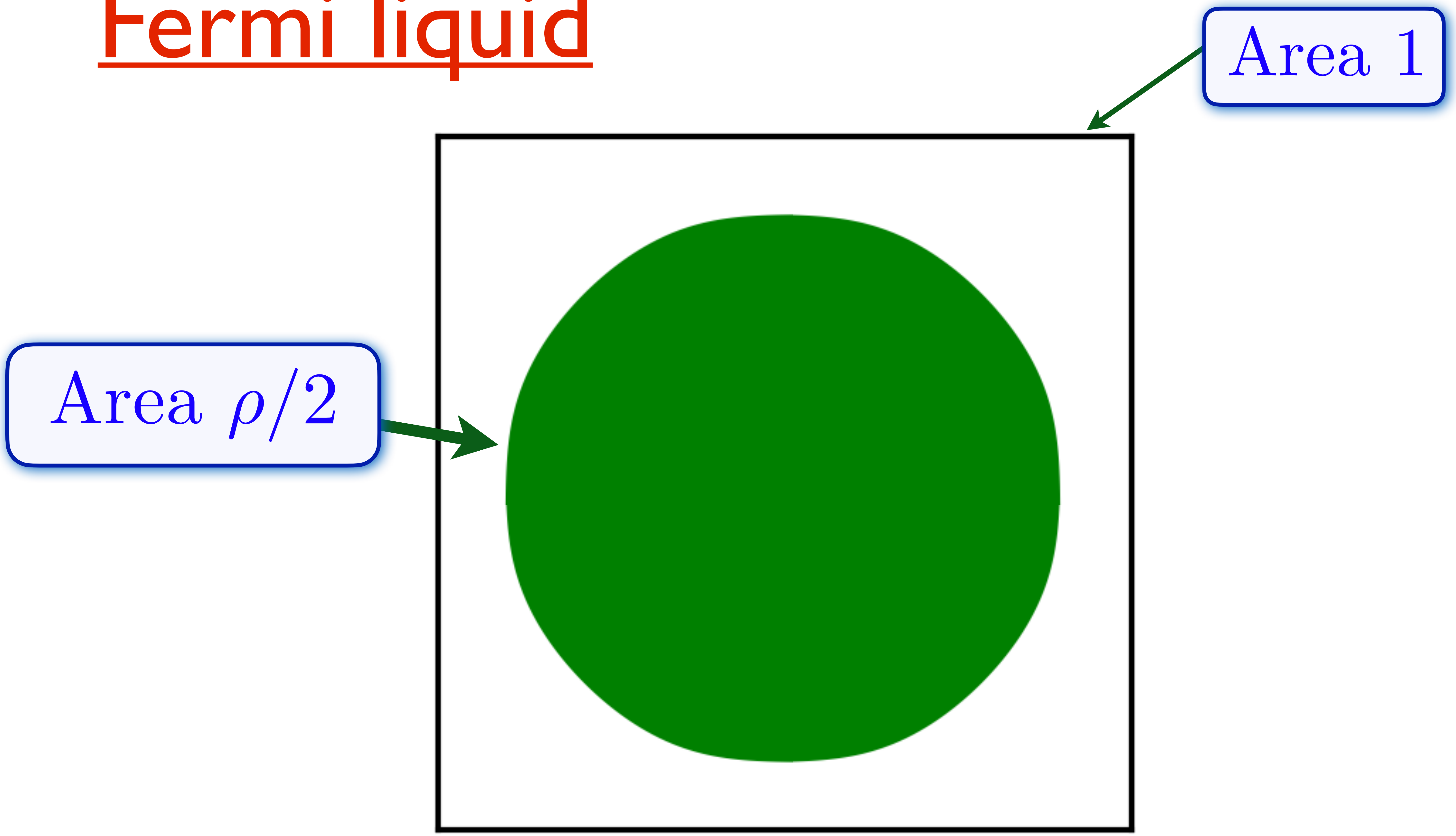

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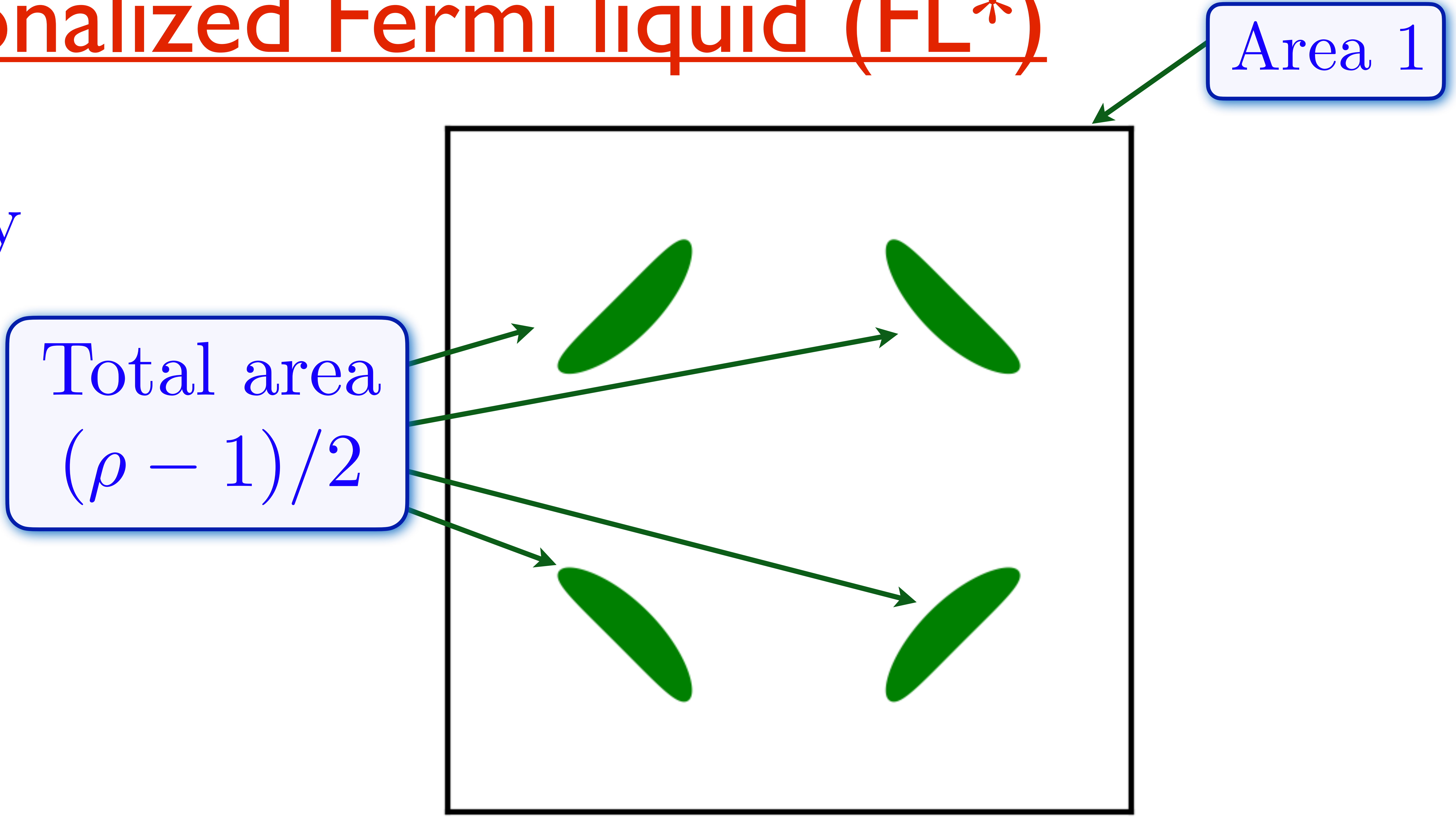
Oshikawa, 2000: Area constrained by an anomaly-argument of global U(1) and translations

Fractionalized Fermi liquid (FL*)



Spin-1/2 holes of density
$$\rho = 1 + p$$

Positive Hall coefficient
of carrier density $\rho - 1$

No broken symmetry

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

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Spin-1/2 holes of density
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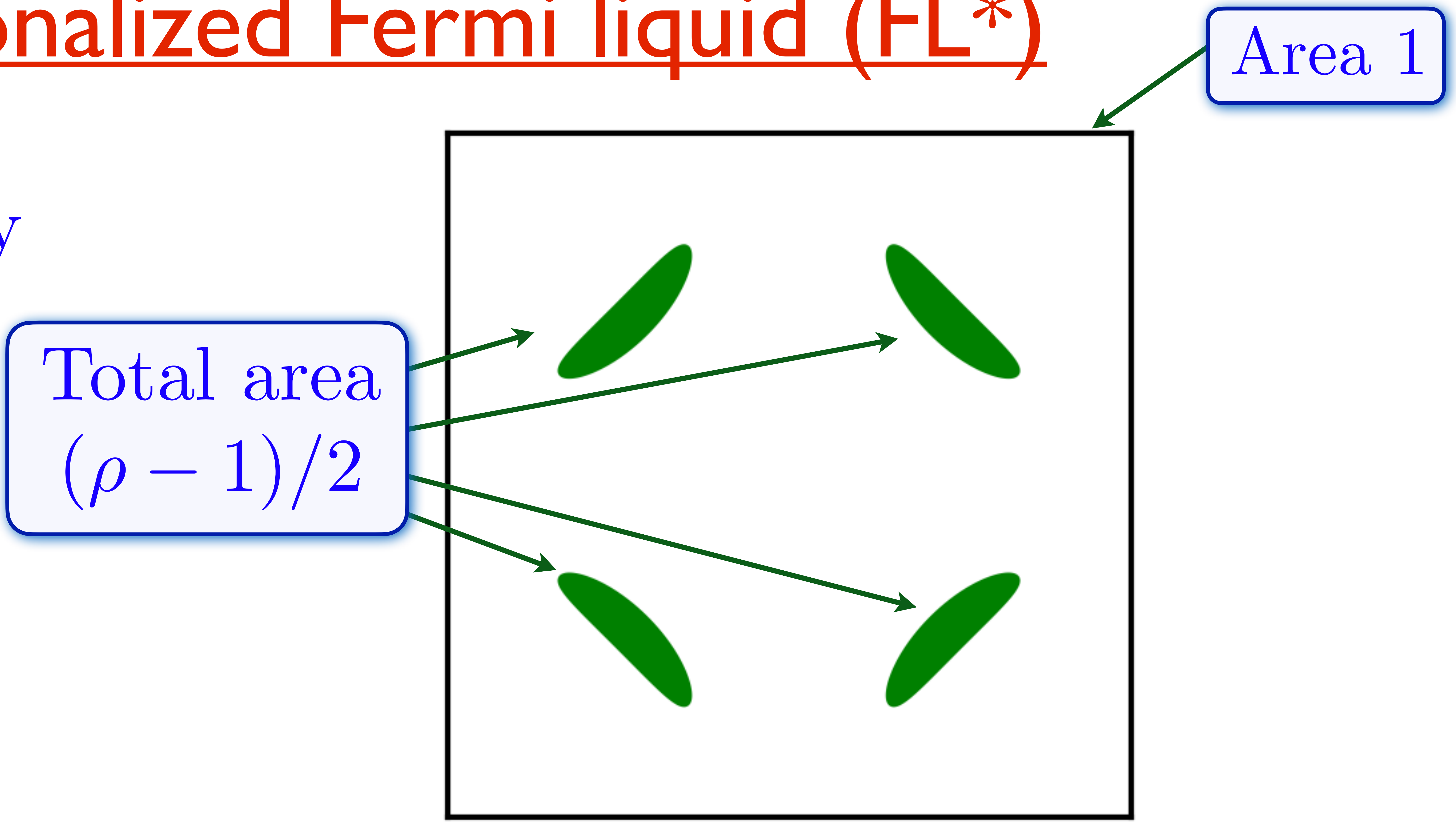
Positive Hall coefficient
of carrier density $\rho - 1$



Oshikawa anomaly-argument is satisfied by
the sum of spin liquid (1) and
Fermi surface anomalies $(\rho - 1)$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

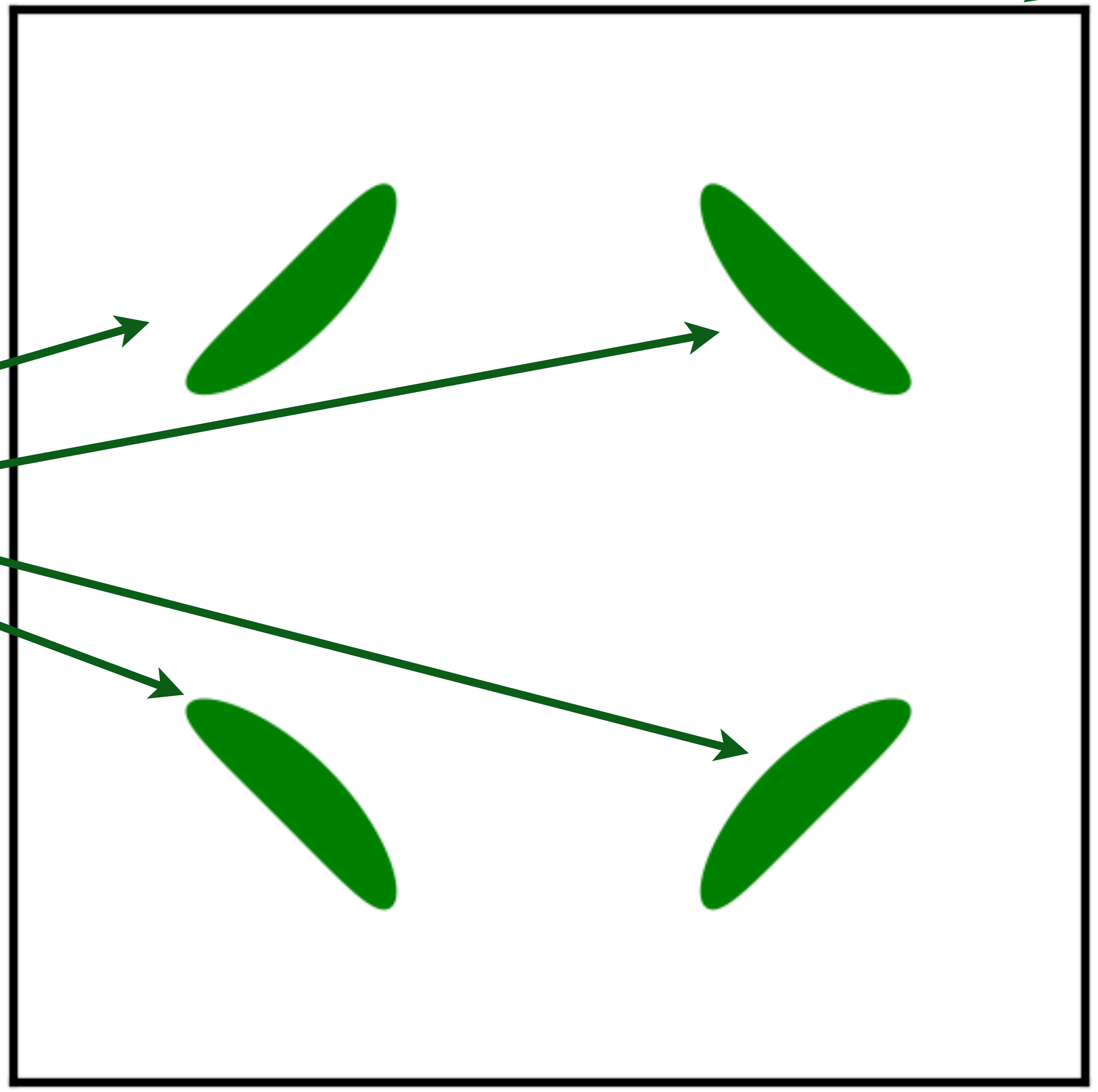
Fractionalized Fermi liquid (FL*)

Spin-1/2 holes of density

$$\rho = 1 + p$$

Positive Hall coefficient of carrier density $\rho - 1$

Total area $(\rho - 1)/2$



Area 1

The density deficit (1) in the area is **quantized** by rigid structure of the spin liquid.

Oshikawa anomaly-argument is satisfied by the sum of spin liquid (1) and Fermi surface anomalies $(\rho - 1)$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Fractionalized Fermi liquid (FL*)

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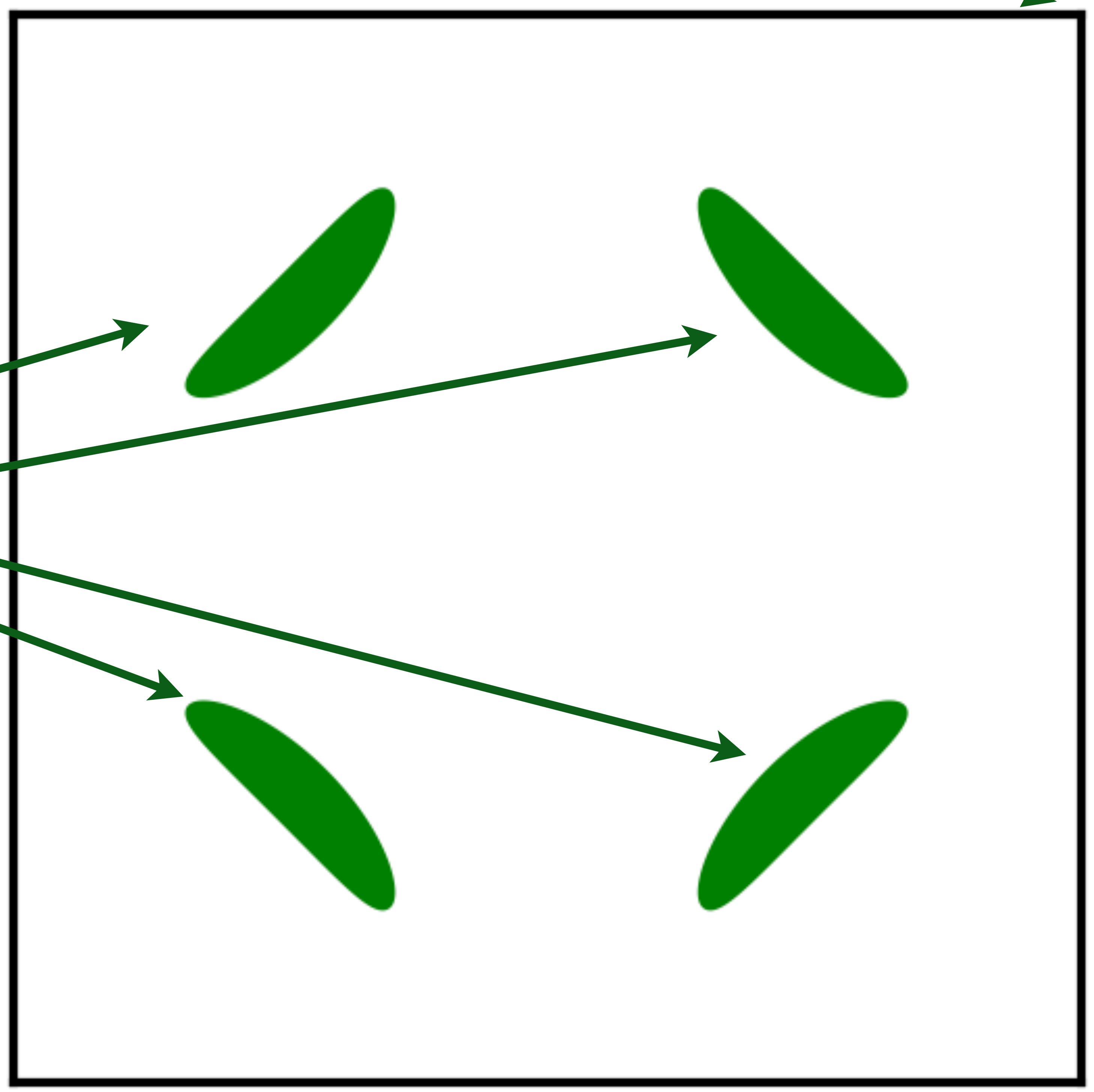

Spin-1/2 holes of density
$\rho = 1 + p$

Total area
$(\rho - 1)/2$

Positive Hall coefficient
of carrier density $\rho - 1$

Measuring non-Luttinger Fermi surface area is direct evidence for multi-fermion quantum entanglement.

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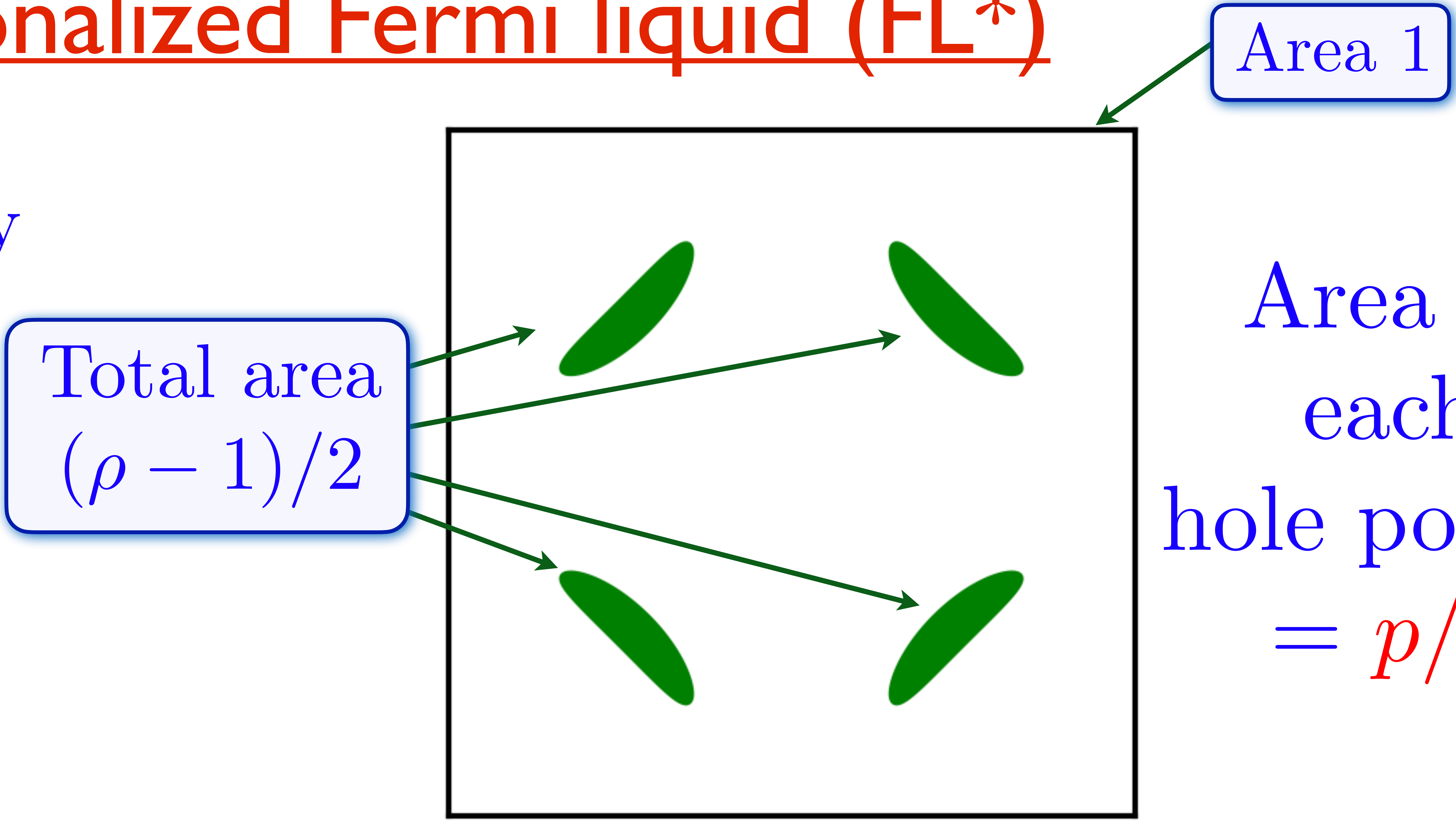
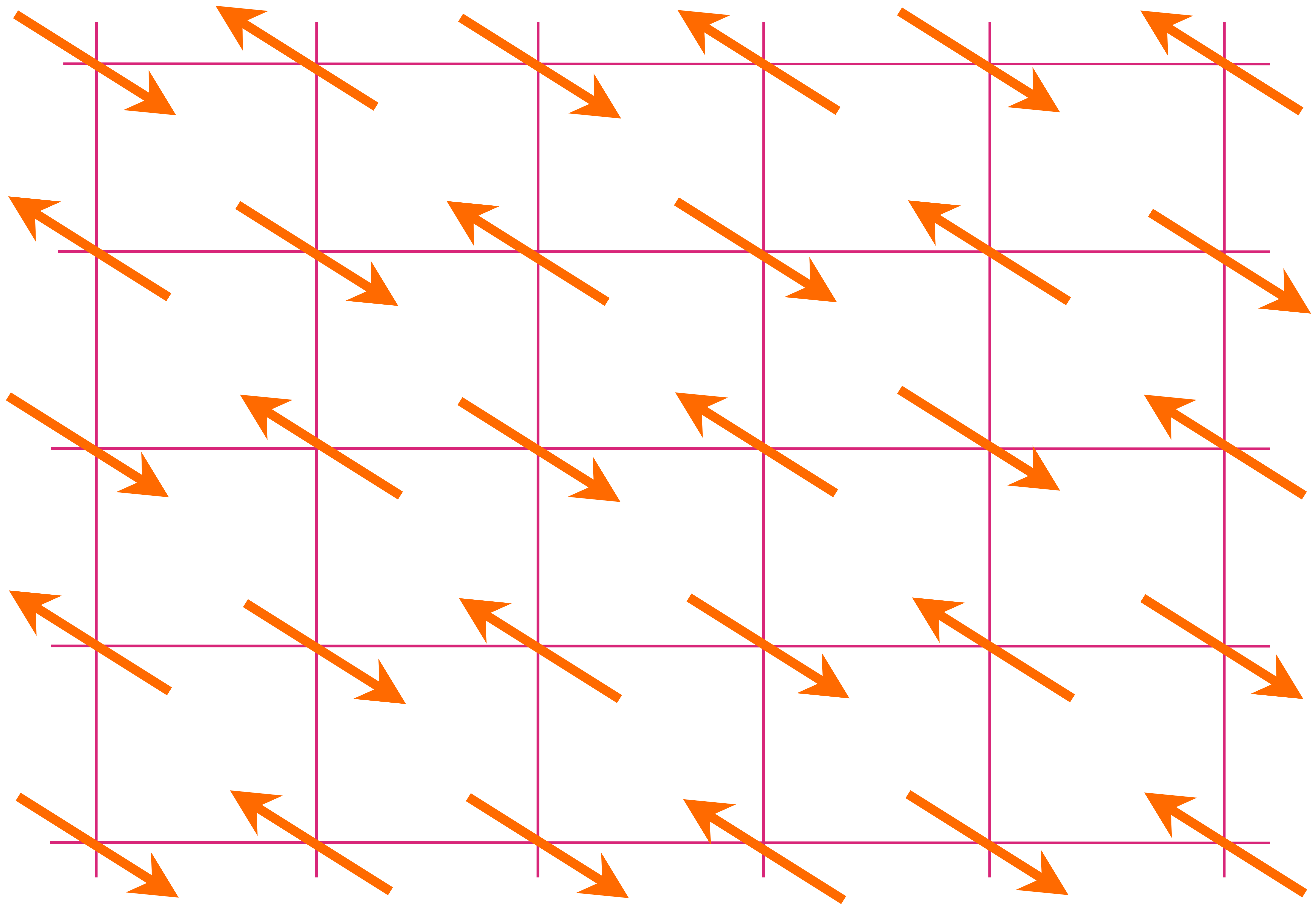


Area of each hole pocket $= p/8$

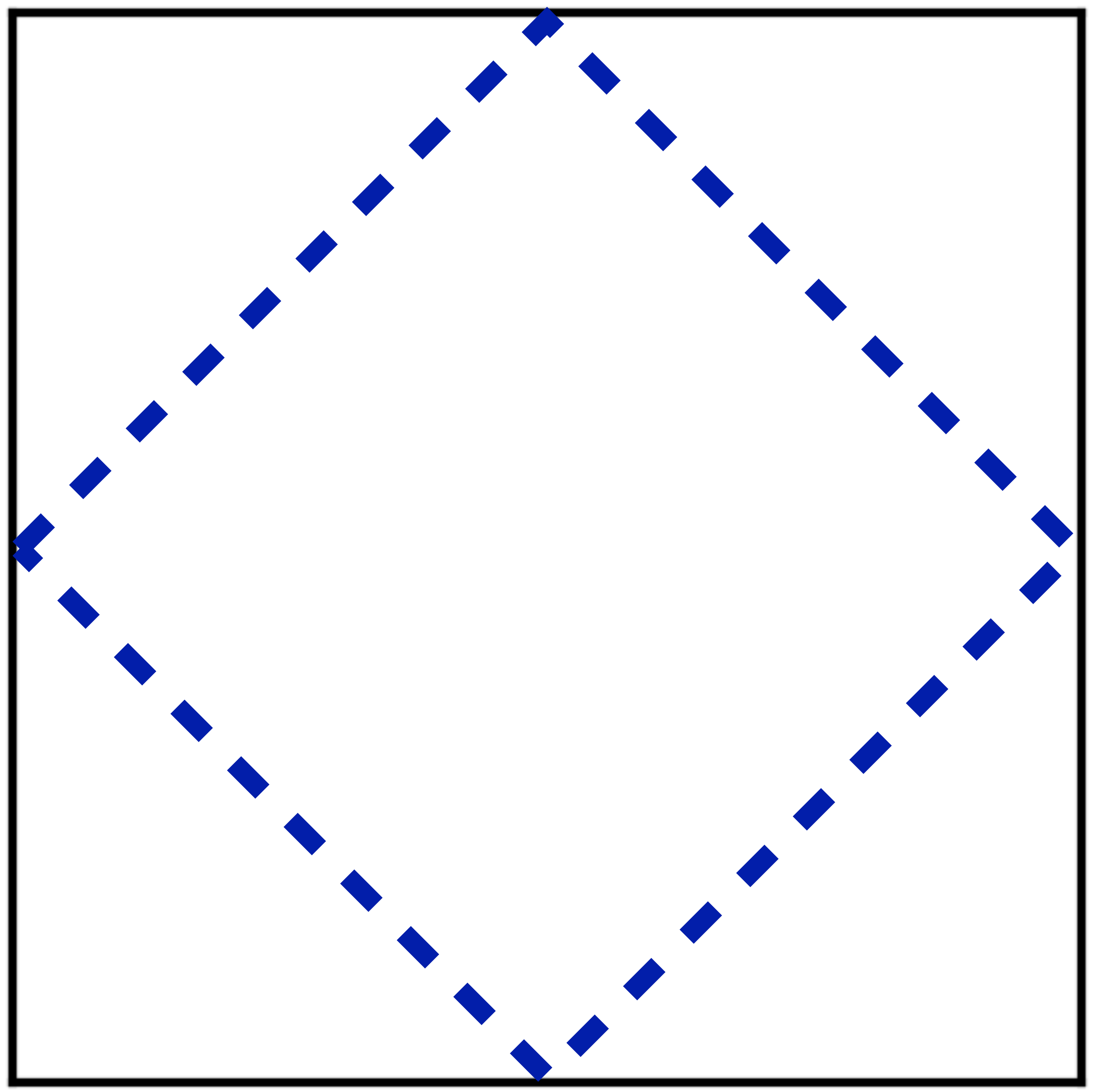
Oshikawa anomaly-argument is satisfied by the sum of spin liquid (1) and Fermi surface anomalies $(\rho - 1)$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Insulating antiferromagnet



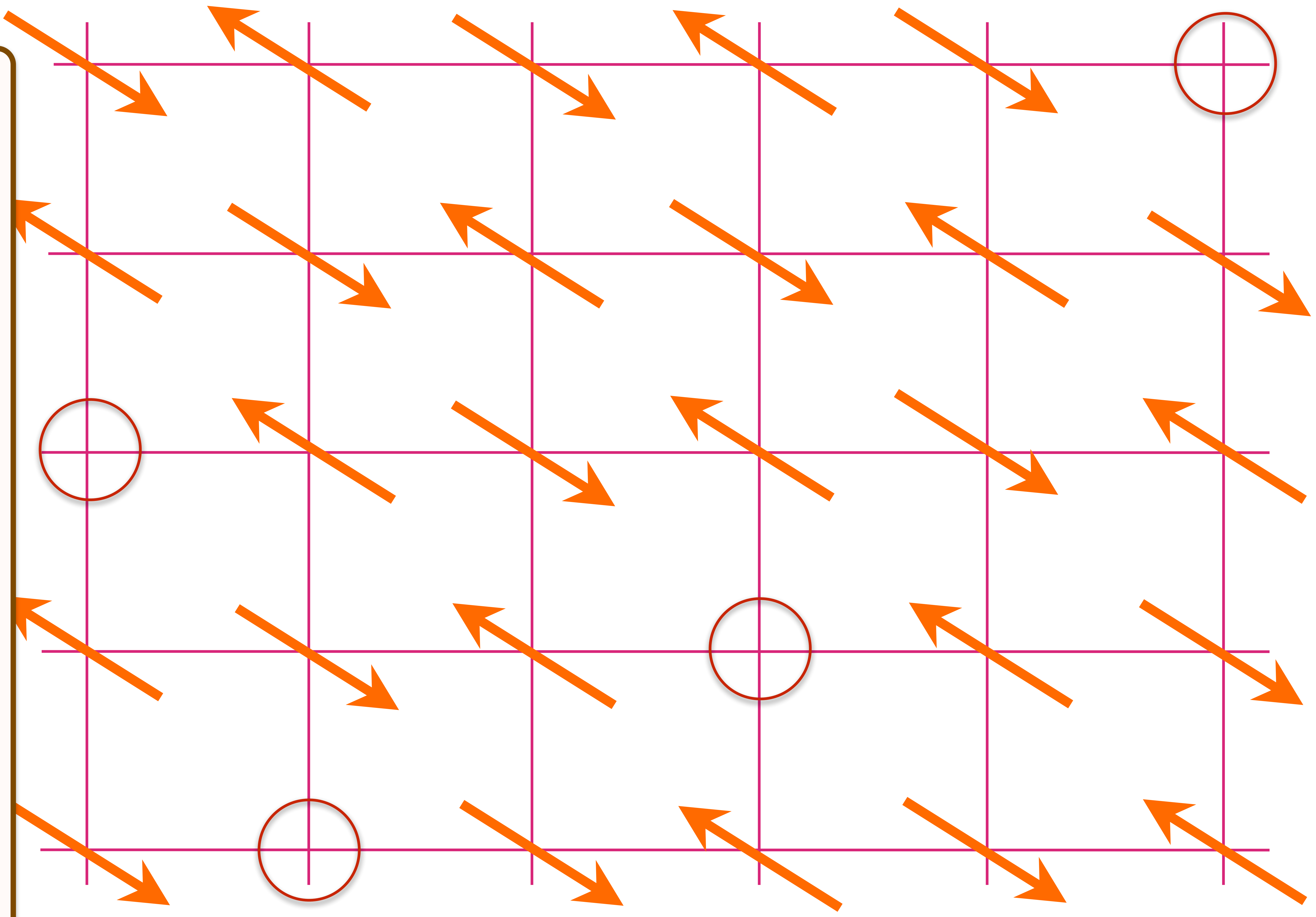
Reduced Brillouin
Zone.
Broken symmetry

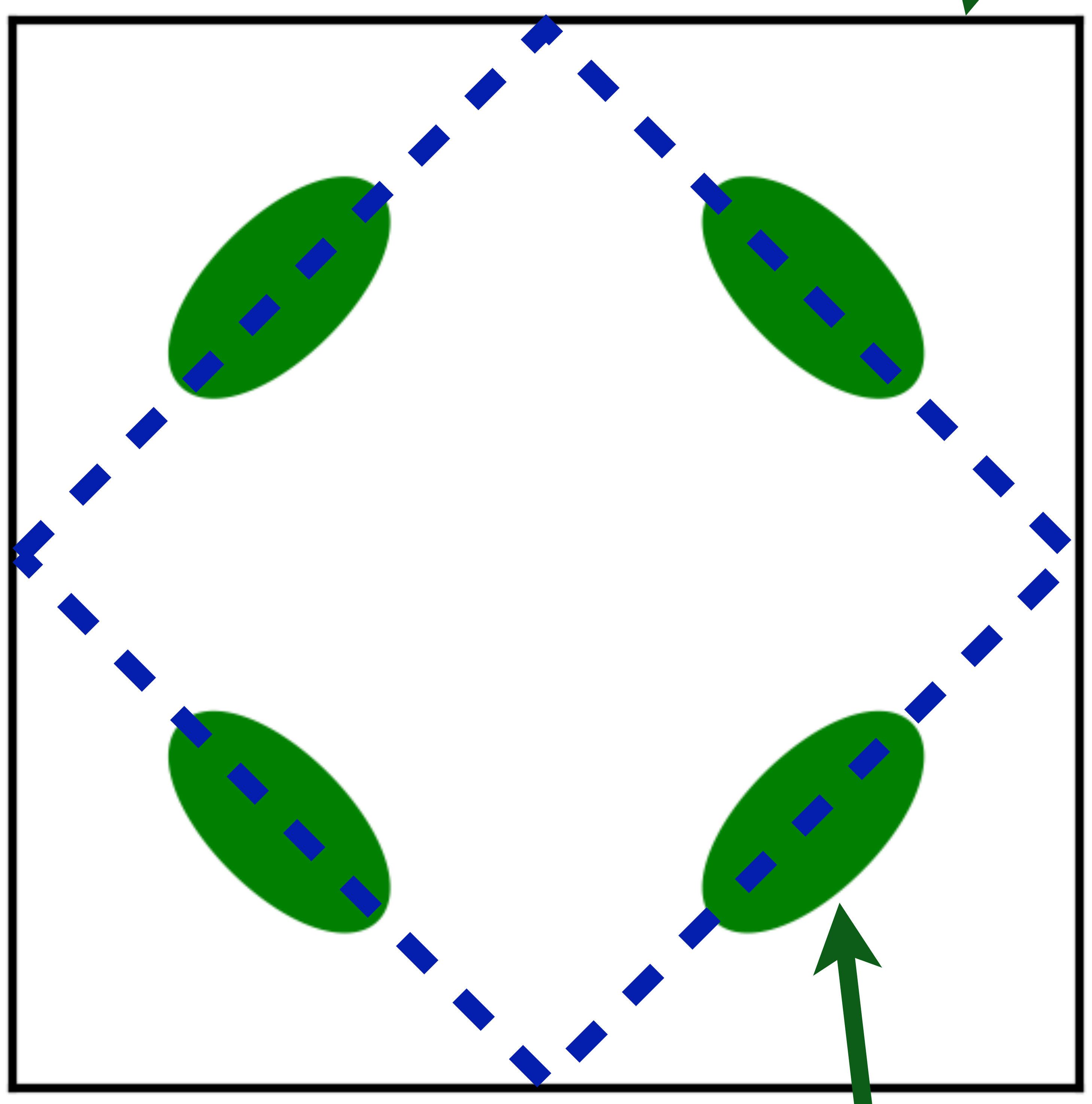


Doping an insulating antiferromagnet with holes of density p

AF metal

Fermi liquid with density p of spin 1/2, charge $+e$ holes. Coherent inter-layer transport requires inter-layer spin correlations.



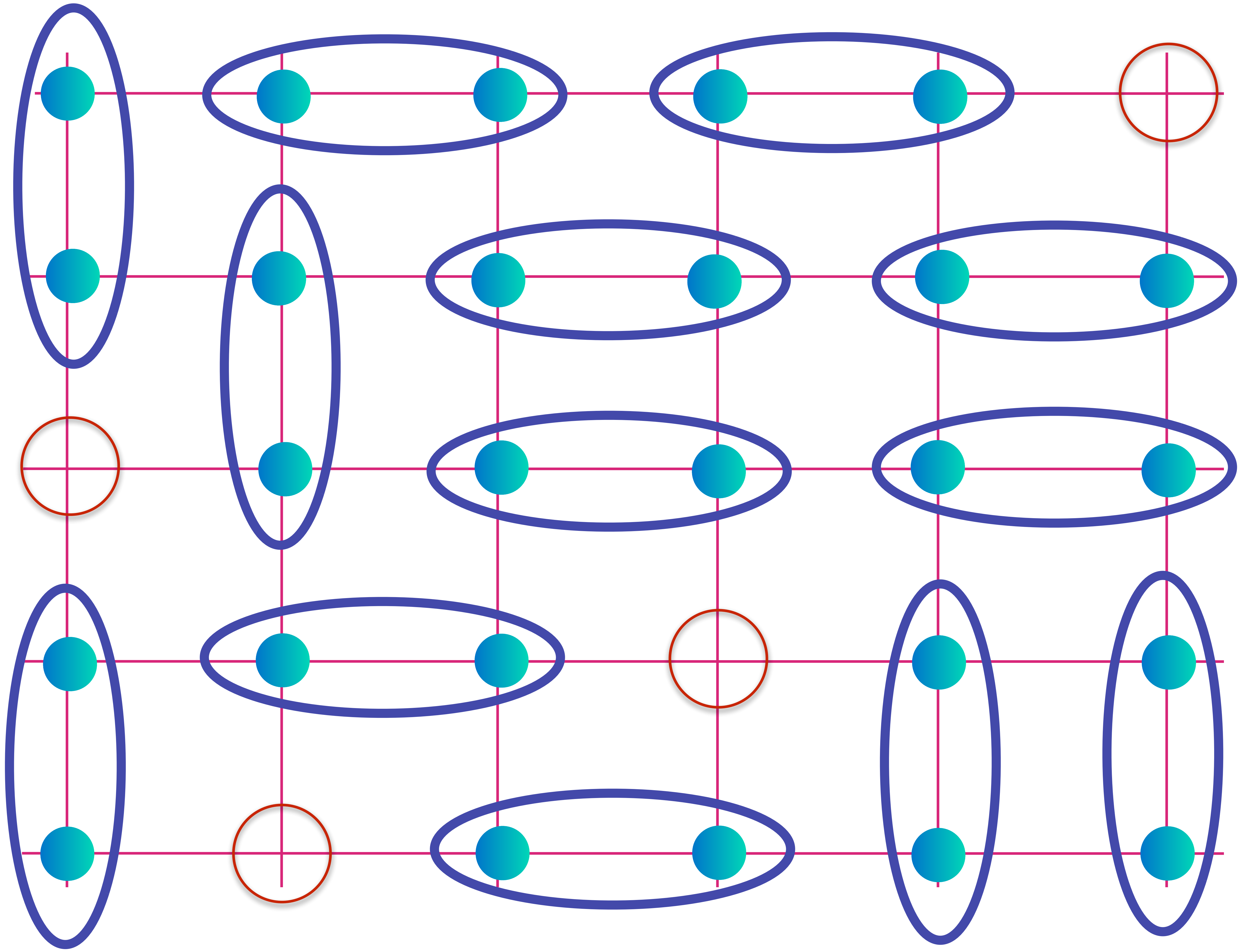


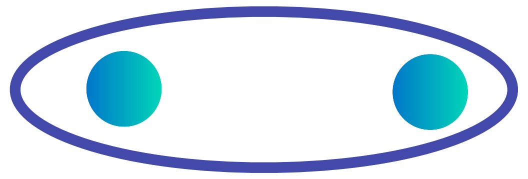
Doping an insulating antiferromagnet with holes of density p

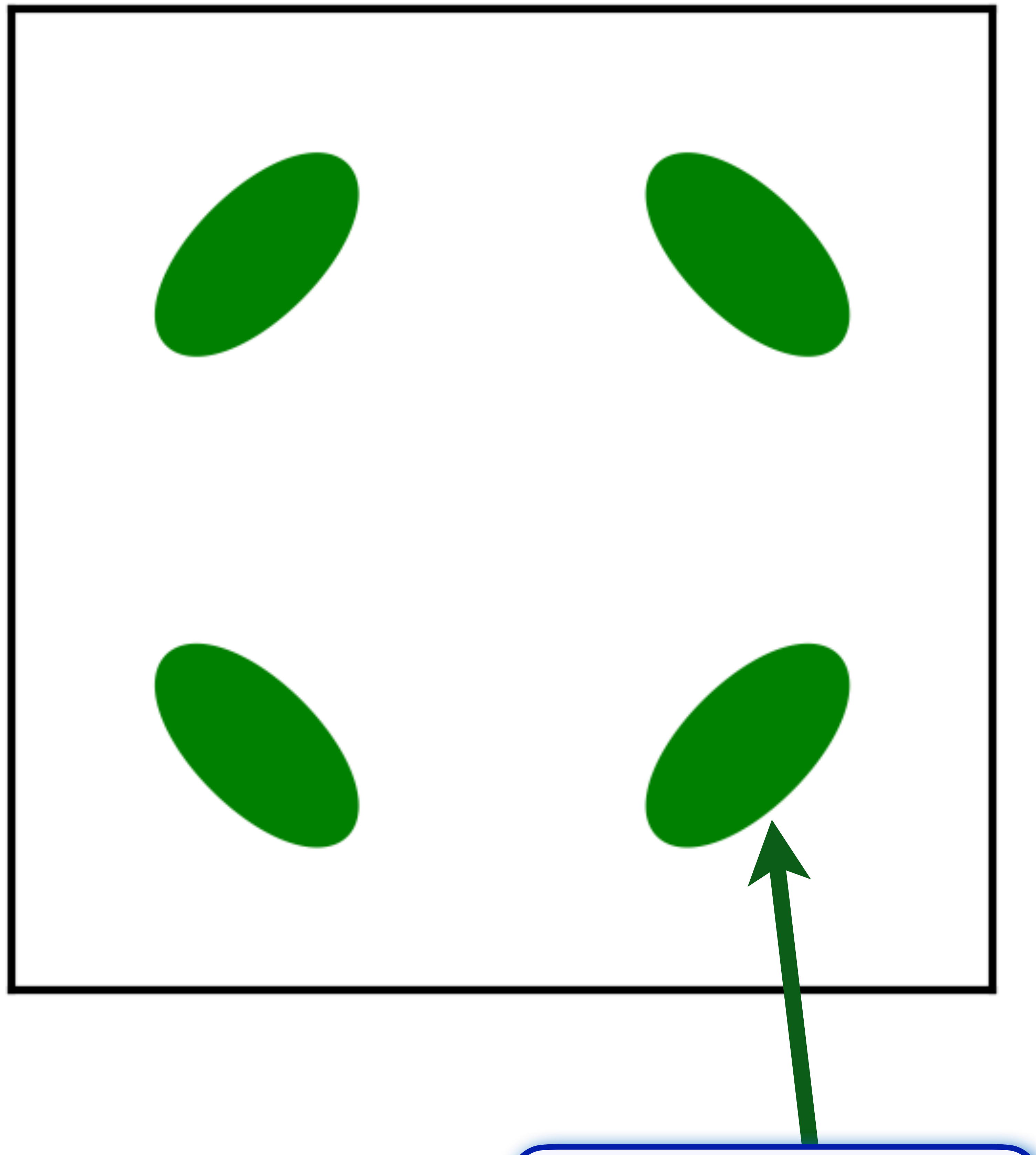
Holon metal

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Spin liquid with density p of spinless, charge $+e$ holons.
No coherent inter-layer transport.



 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



Area $p/4$

R. K. Kaul, Y. B. Kim, S. S., T. Senthil, Nature Physics **4**, 28 (2008); S.S. M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Doping an insulating antiferromagnet with holes of density p

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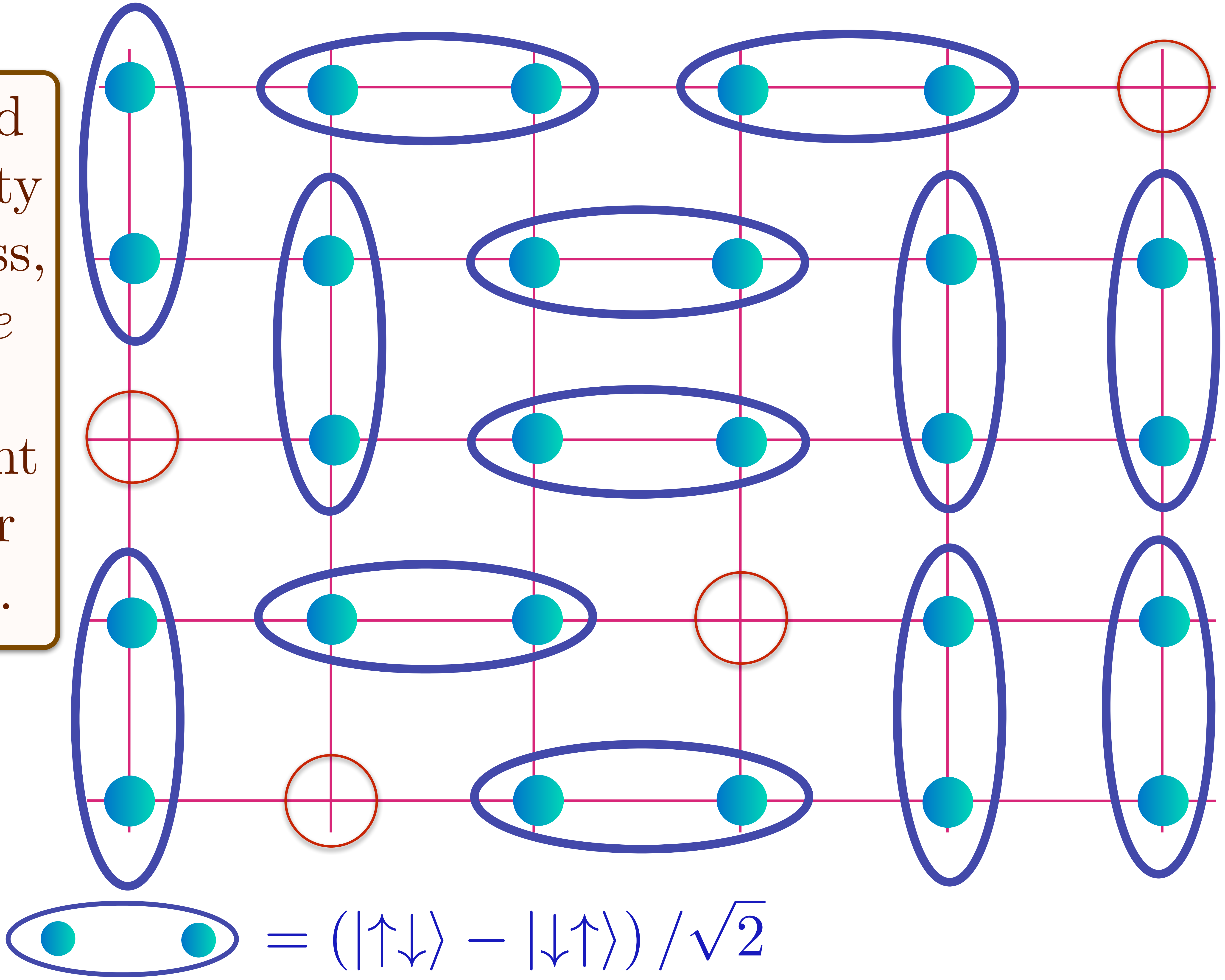

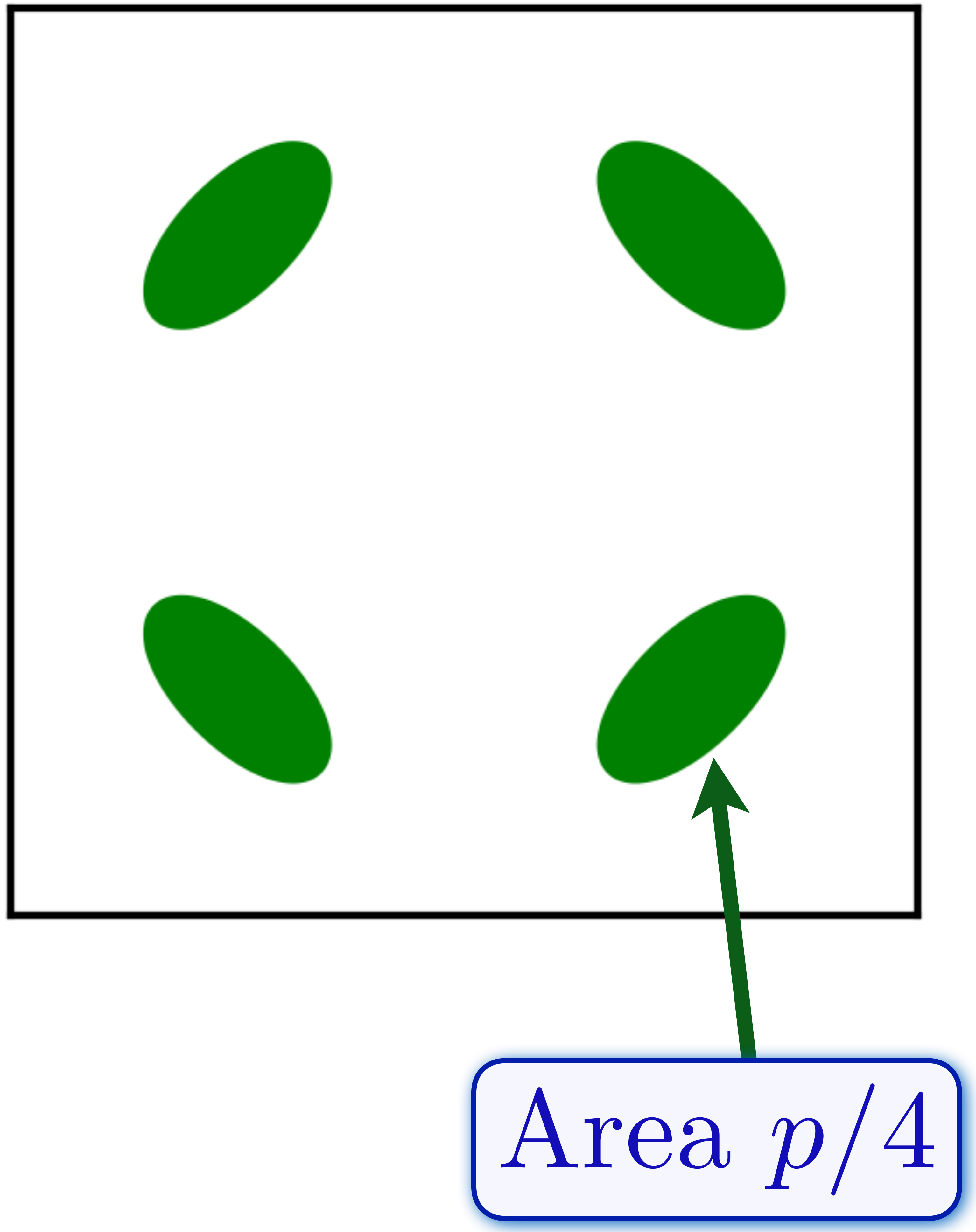

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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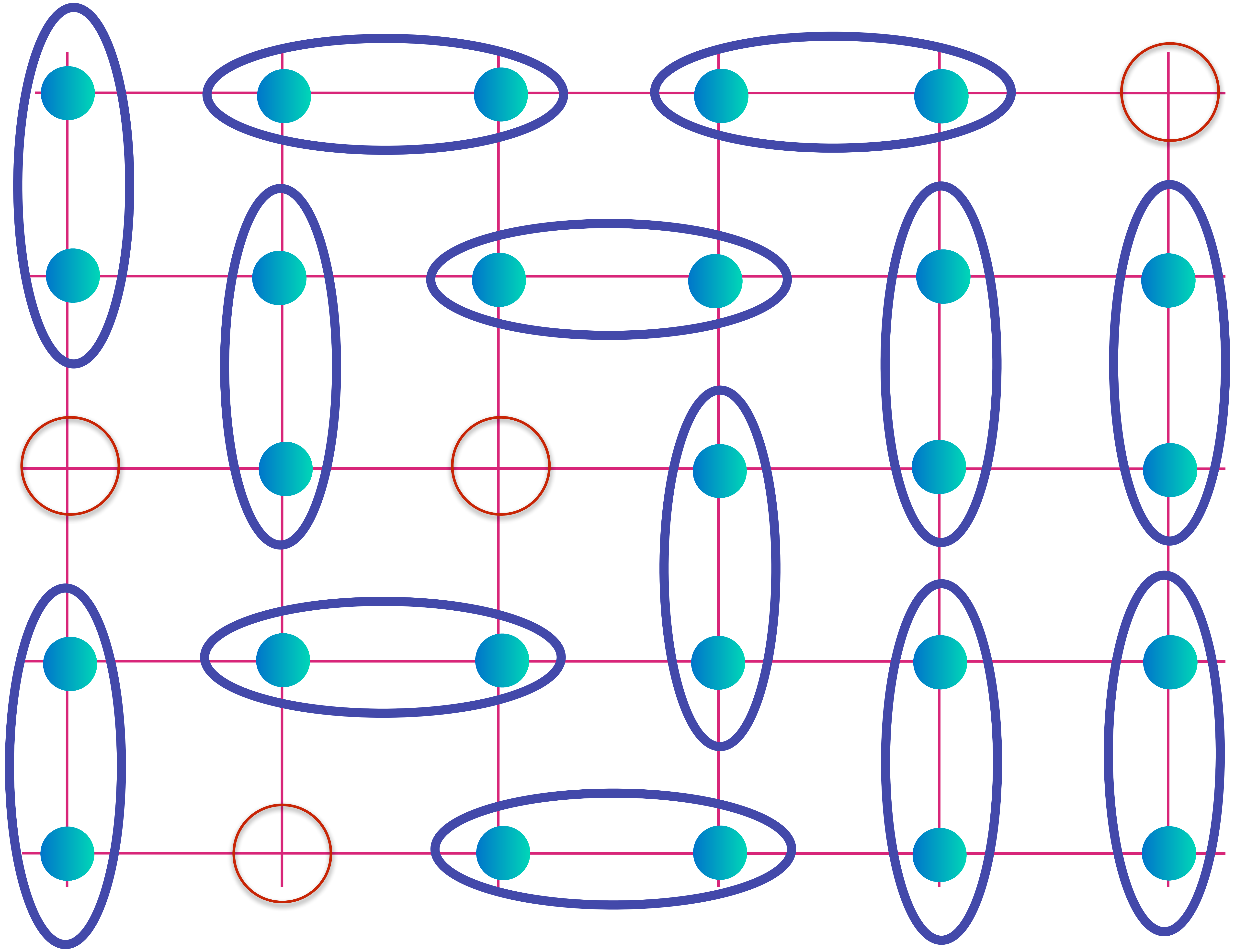
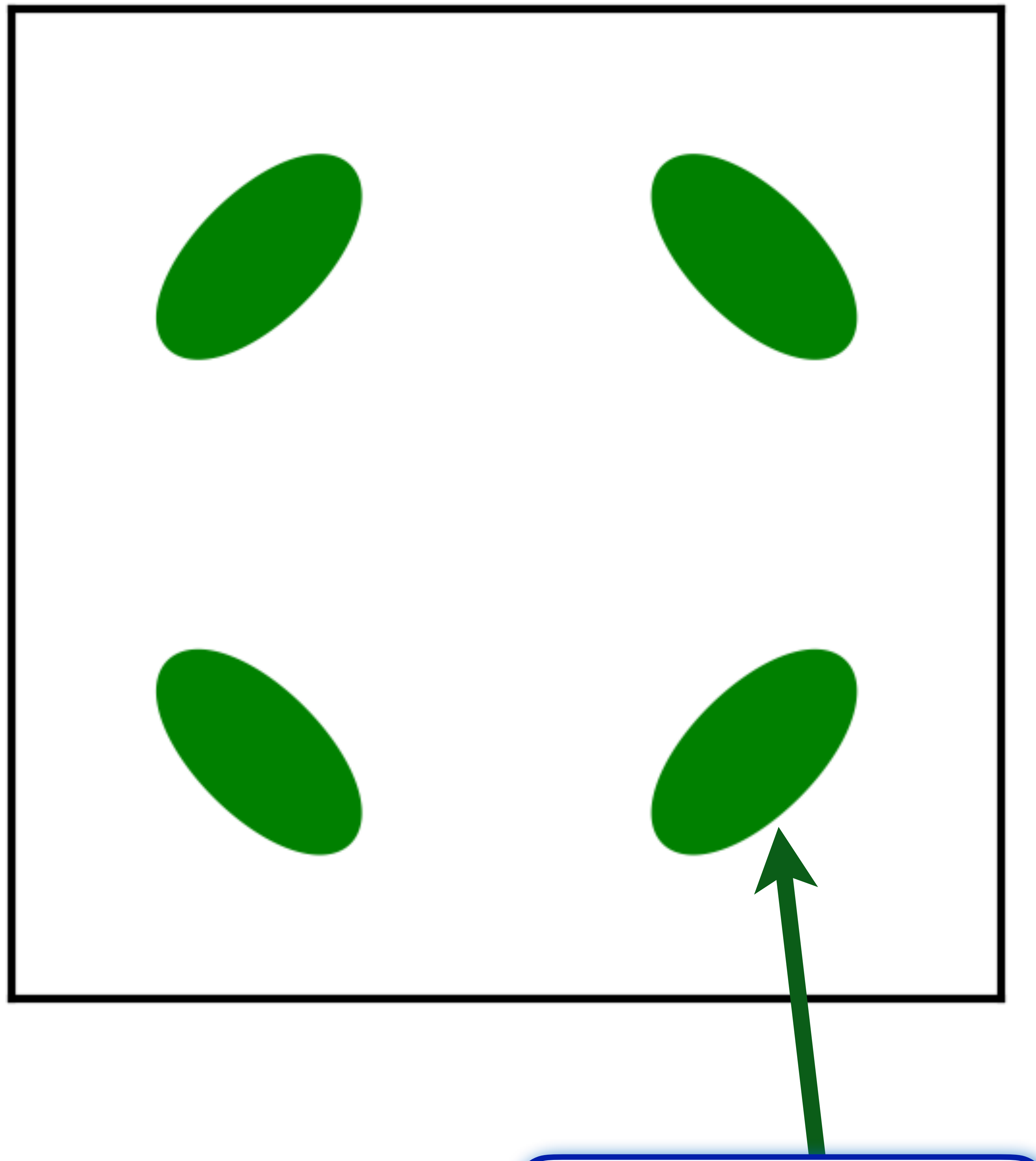


$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)



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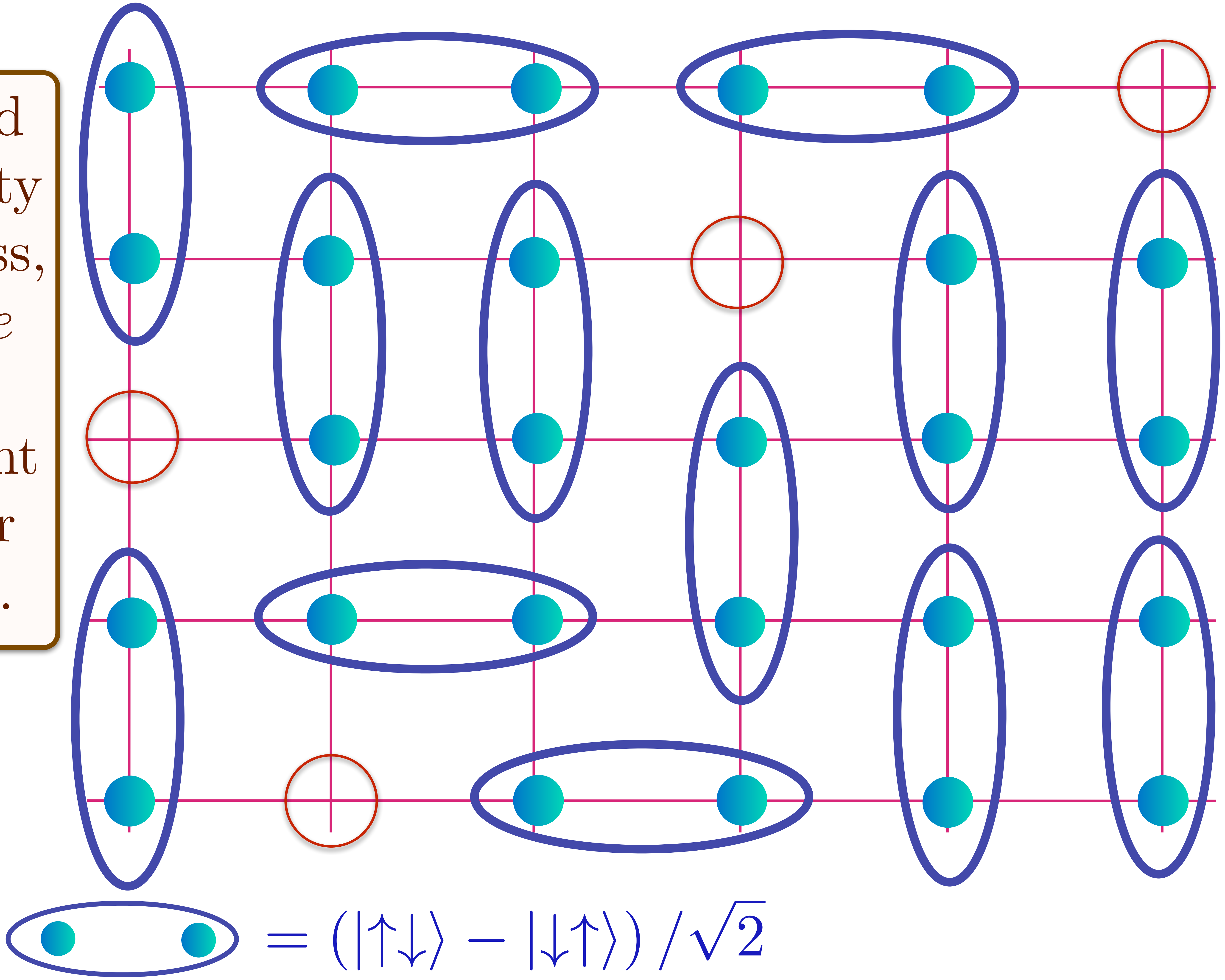

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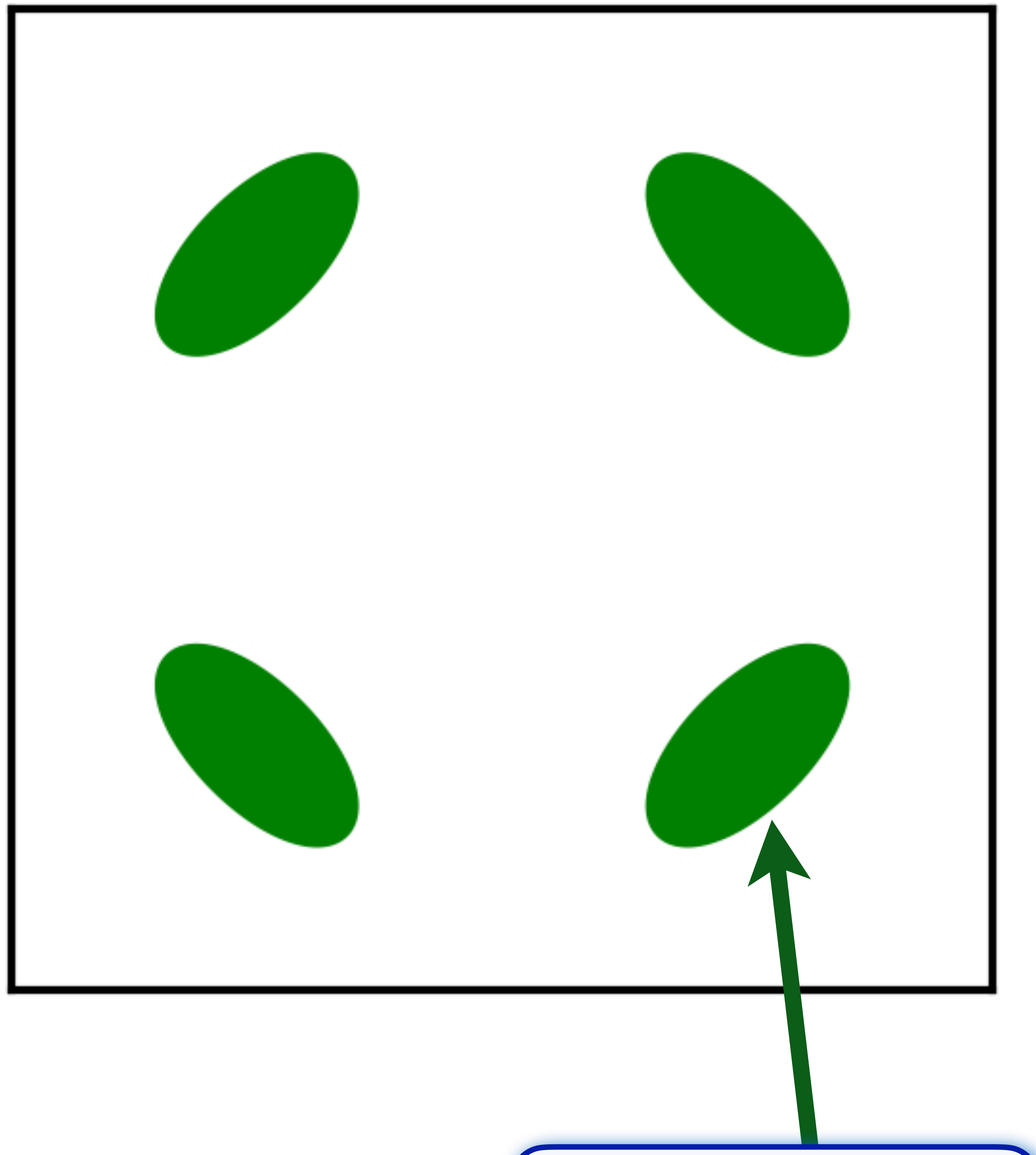

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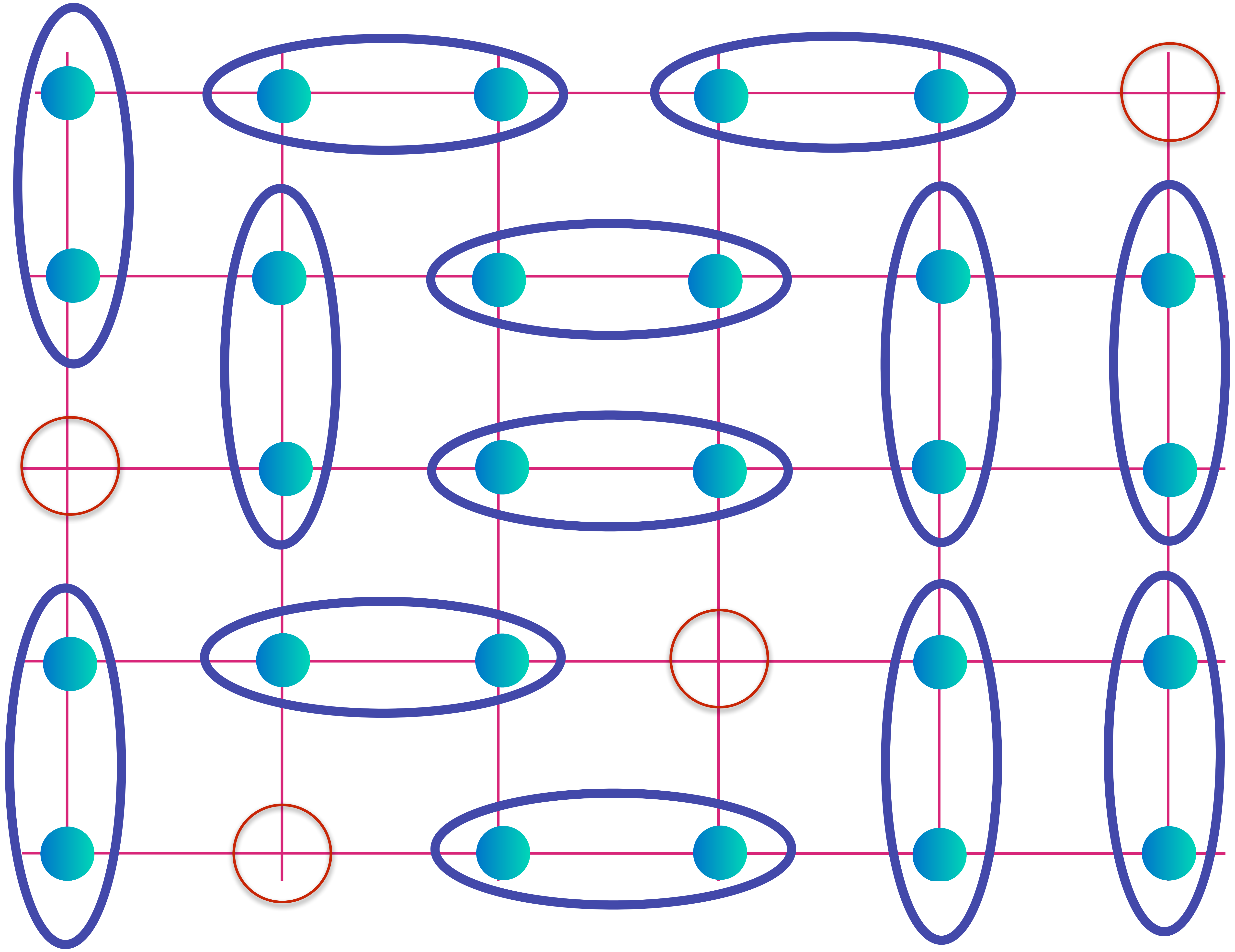

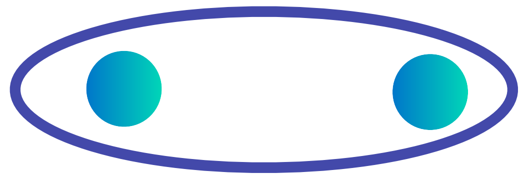

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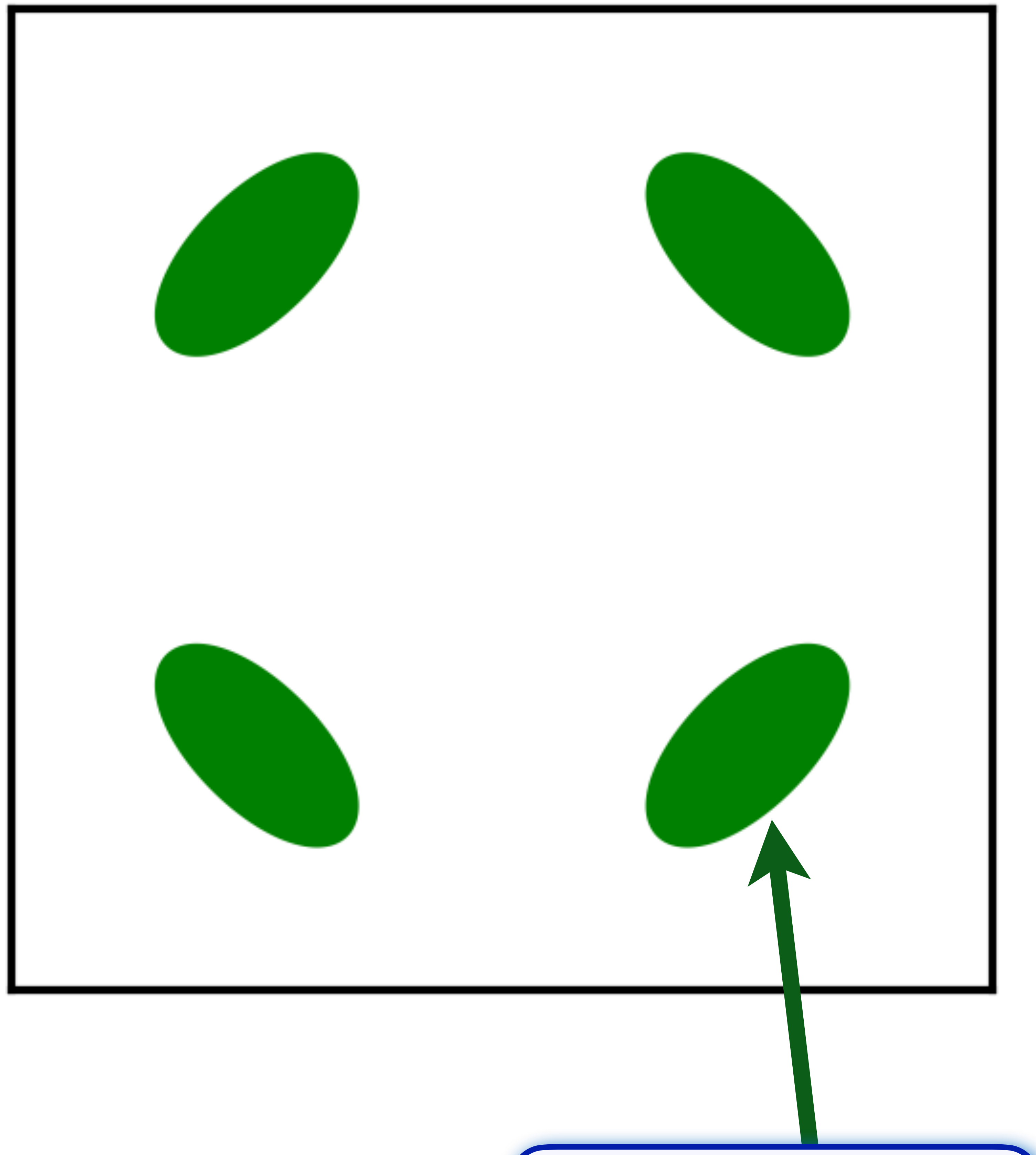

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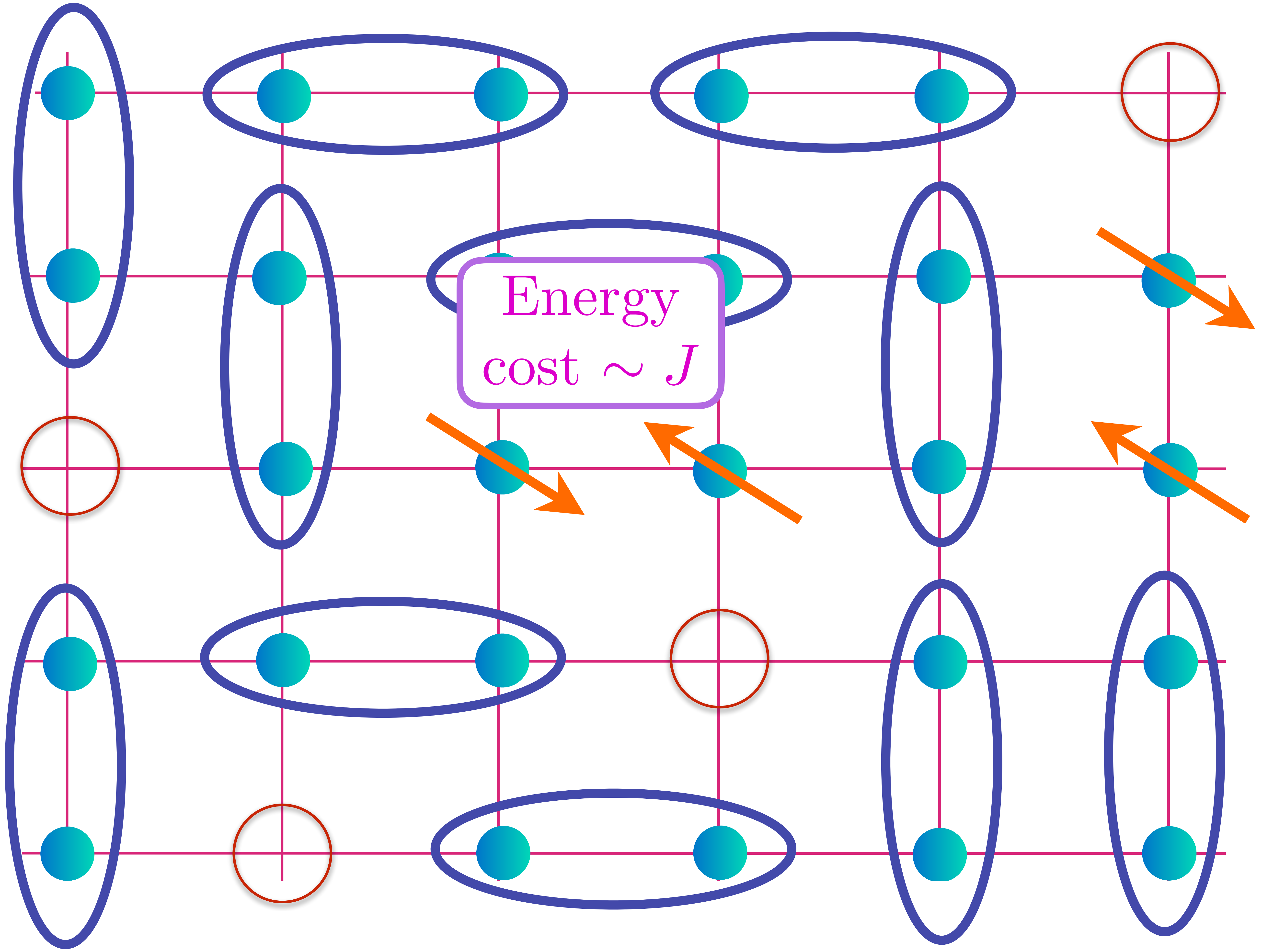
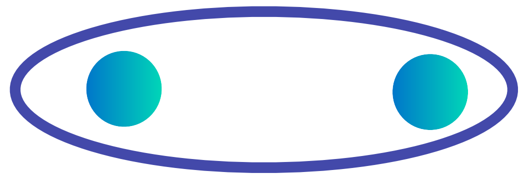
Doping an insulating antiferromagnet with holes of density p

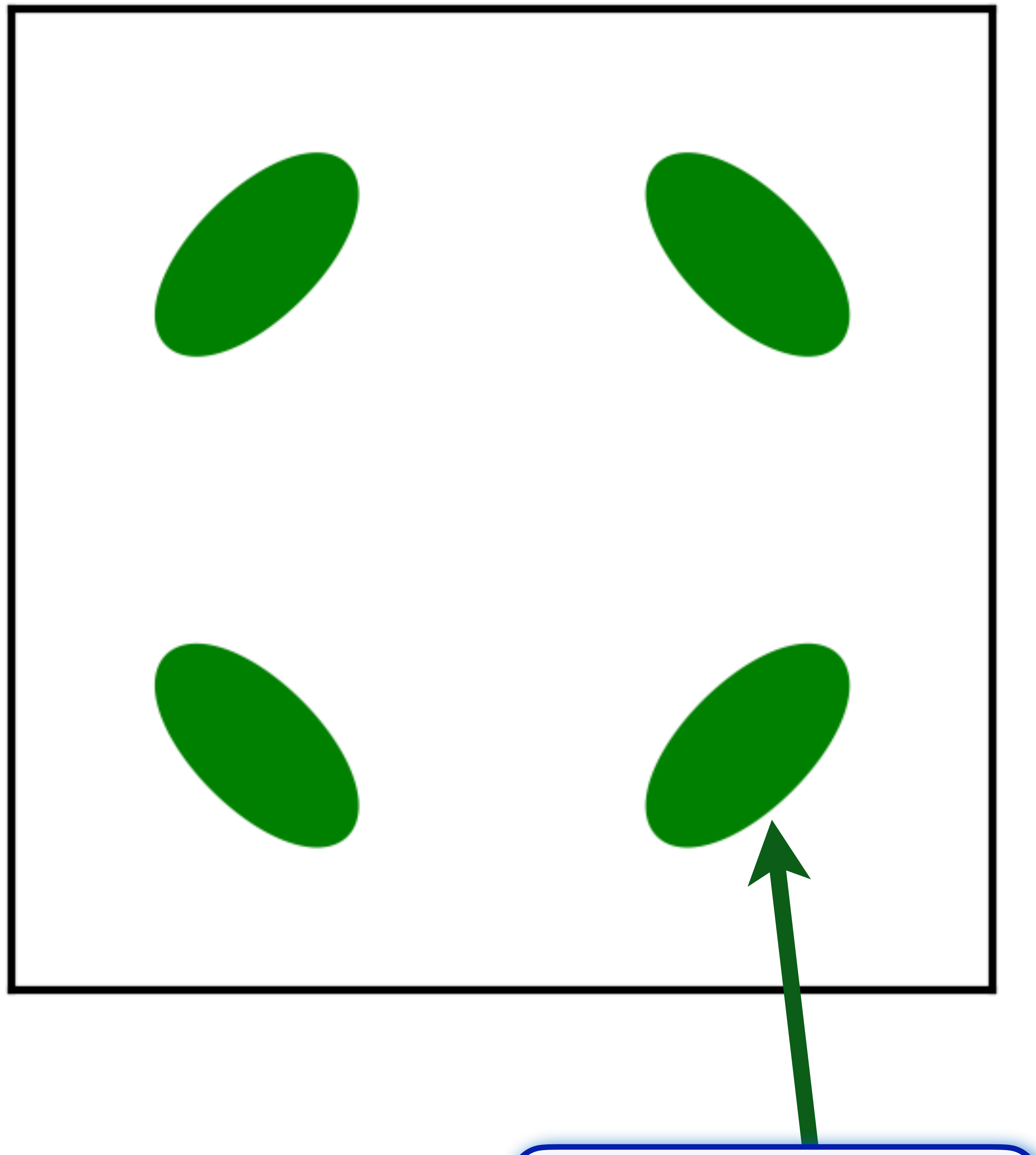
Holon metal excited states

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Charge 0, spin-1/2 spinons



 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



Area $p/4$

Doping an insulating antiferromagnet with holes of density p

Holon metal excited states

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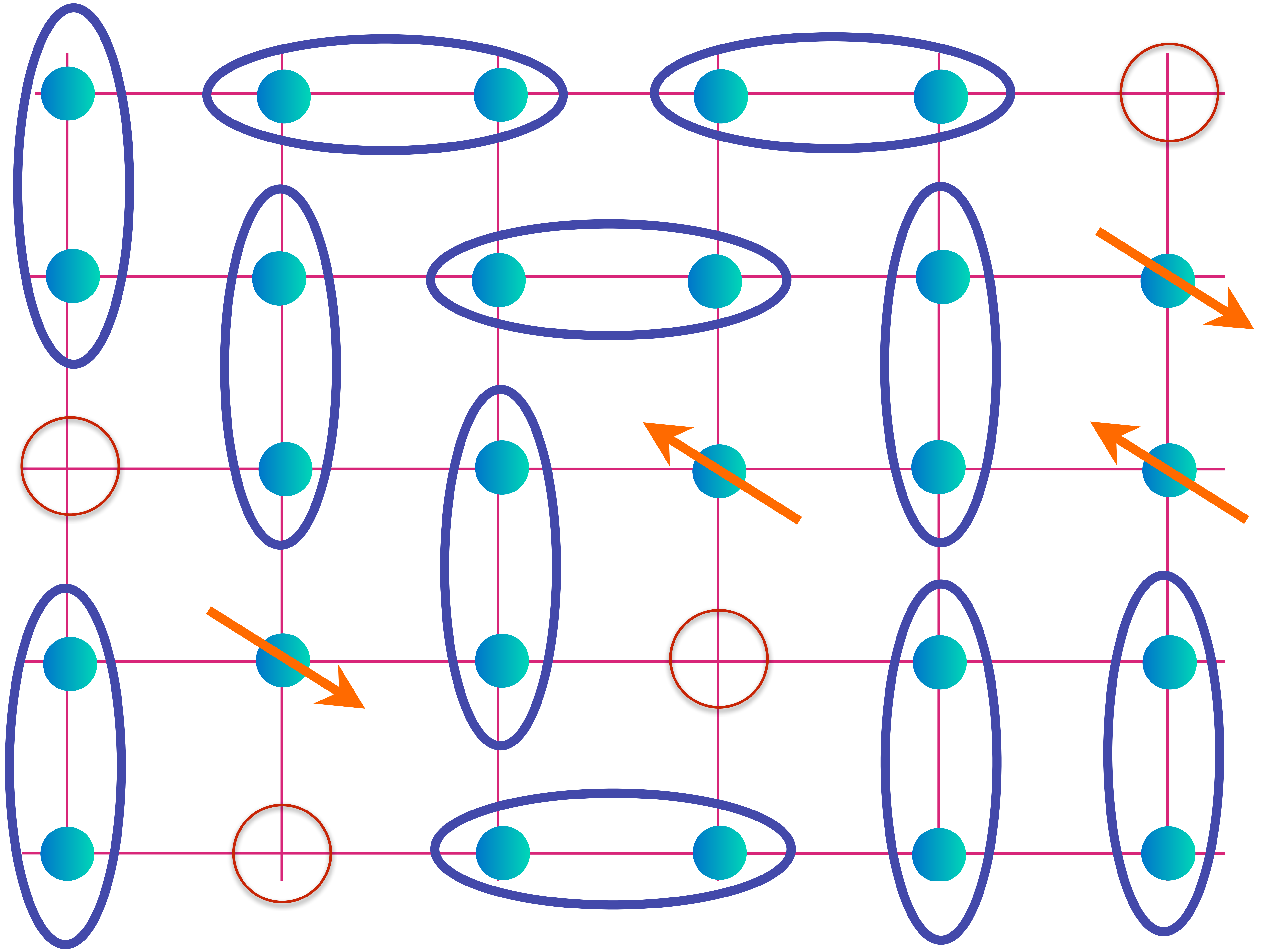

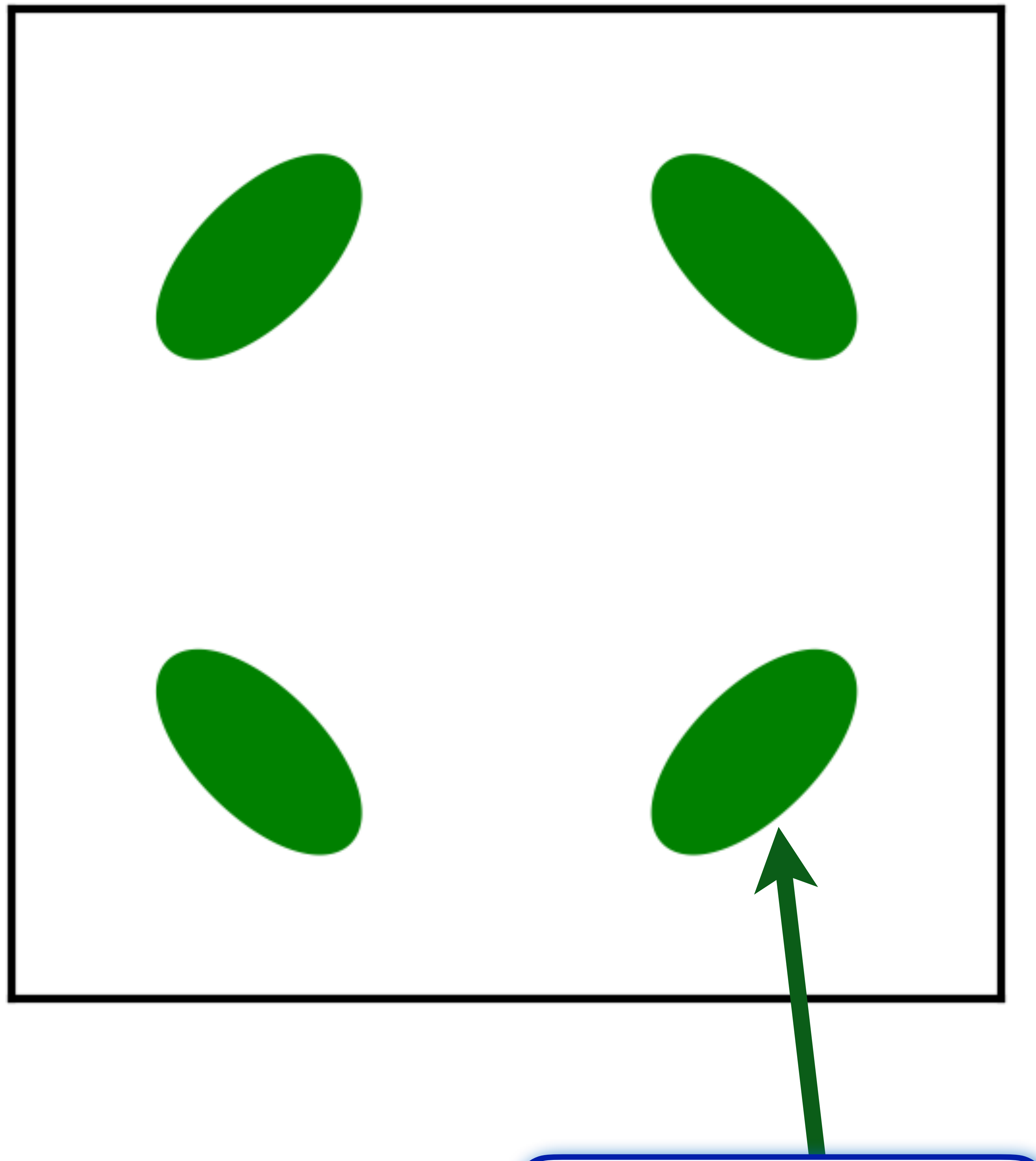

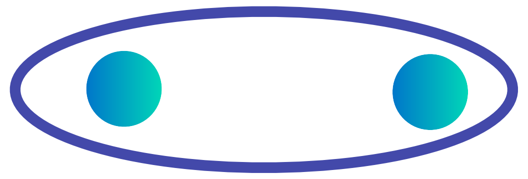 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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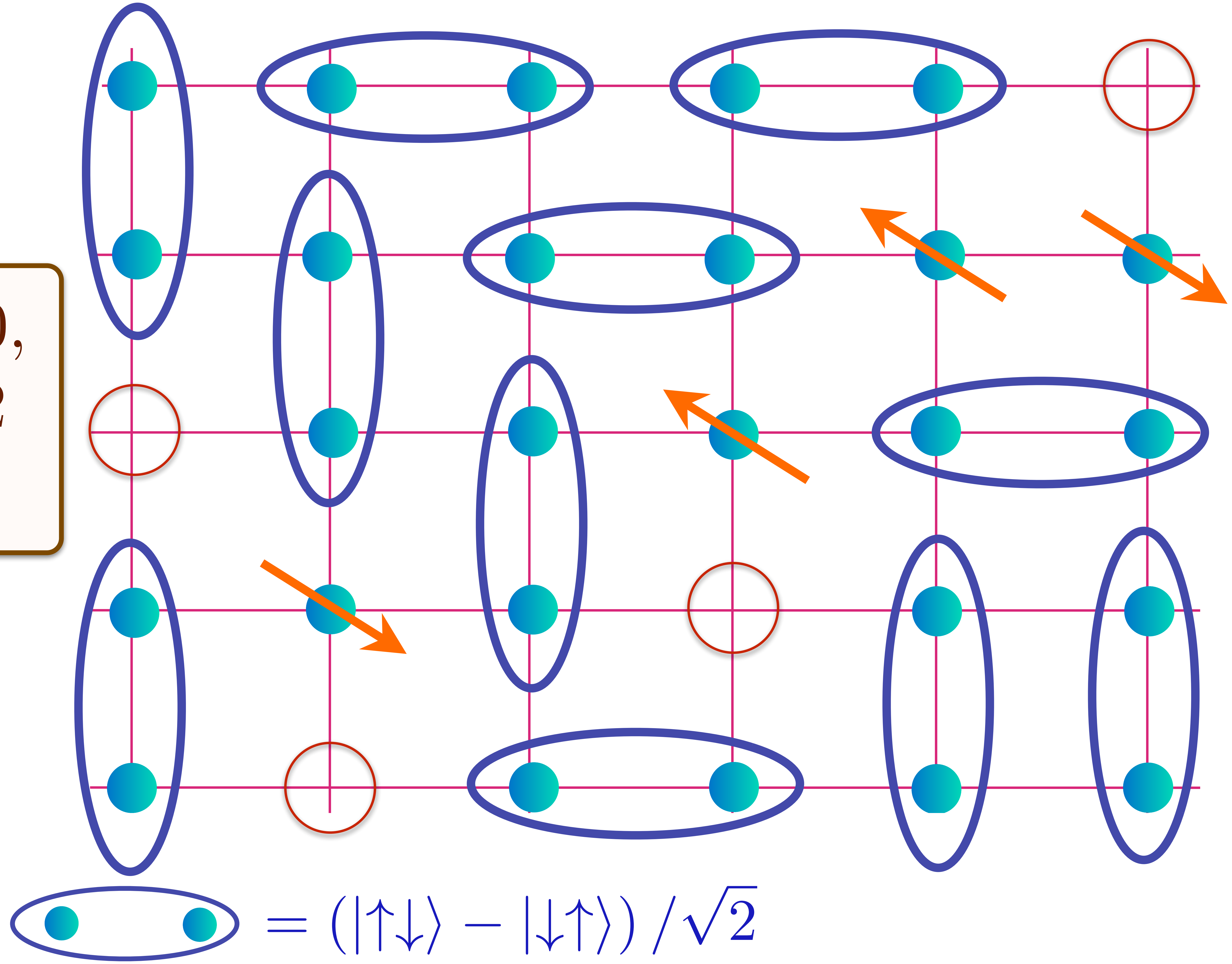

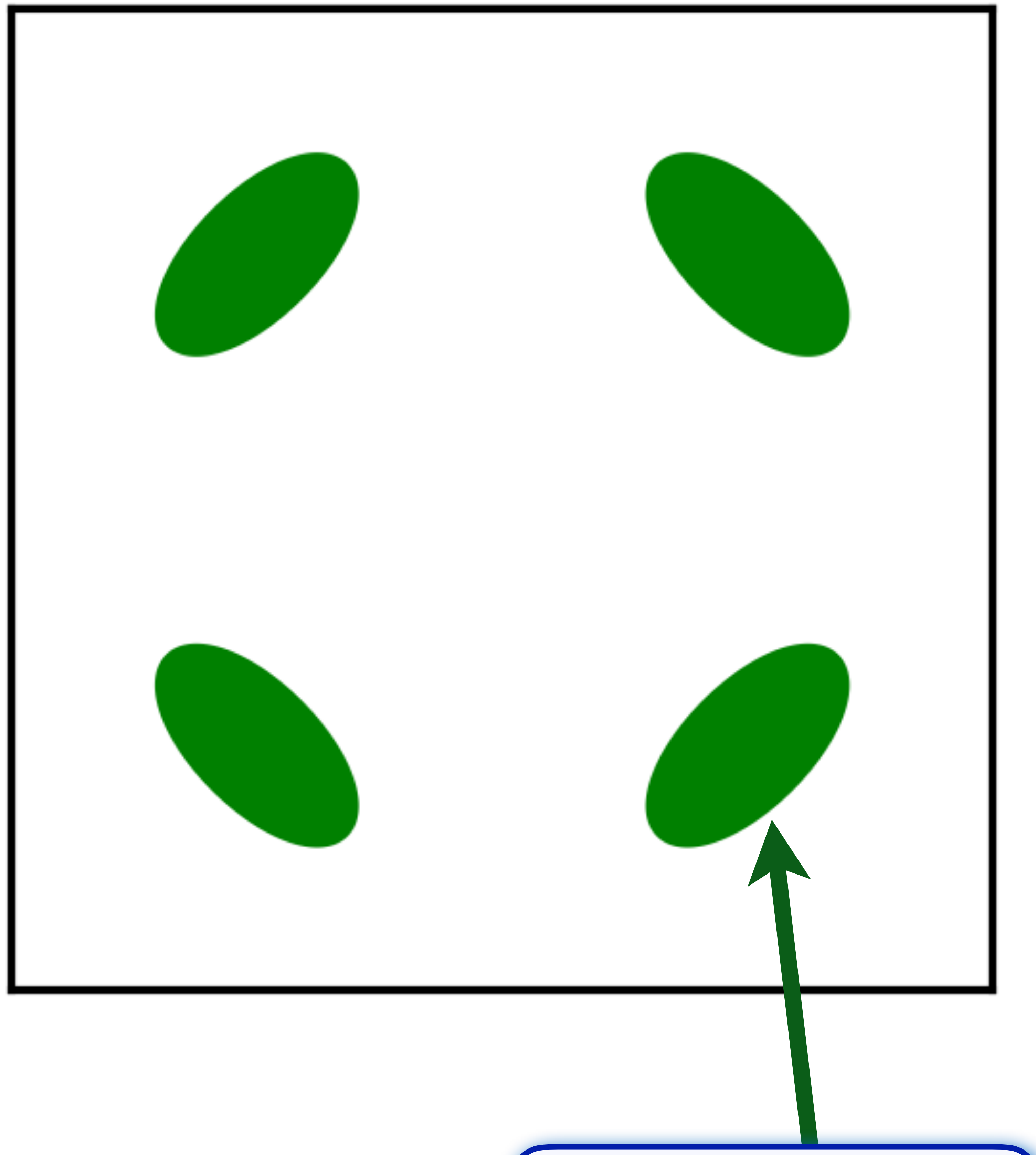

Area $p/4$

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Doping an insulating antiferromagnet with holes of density p

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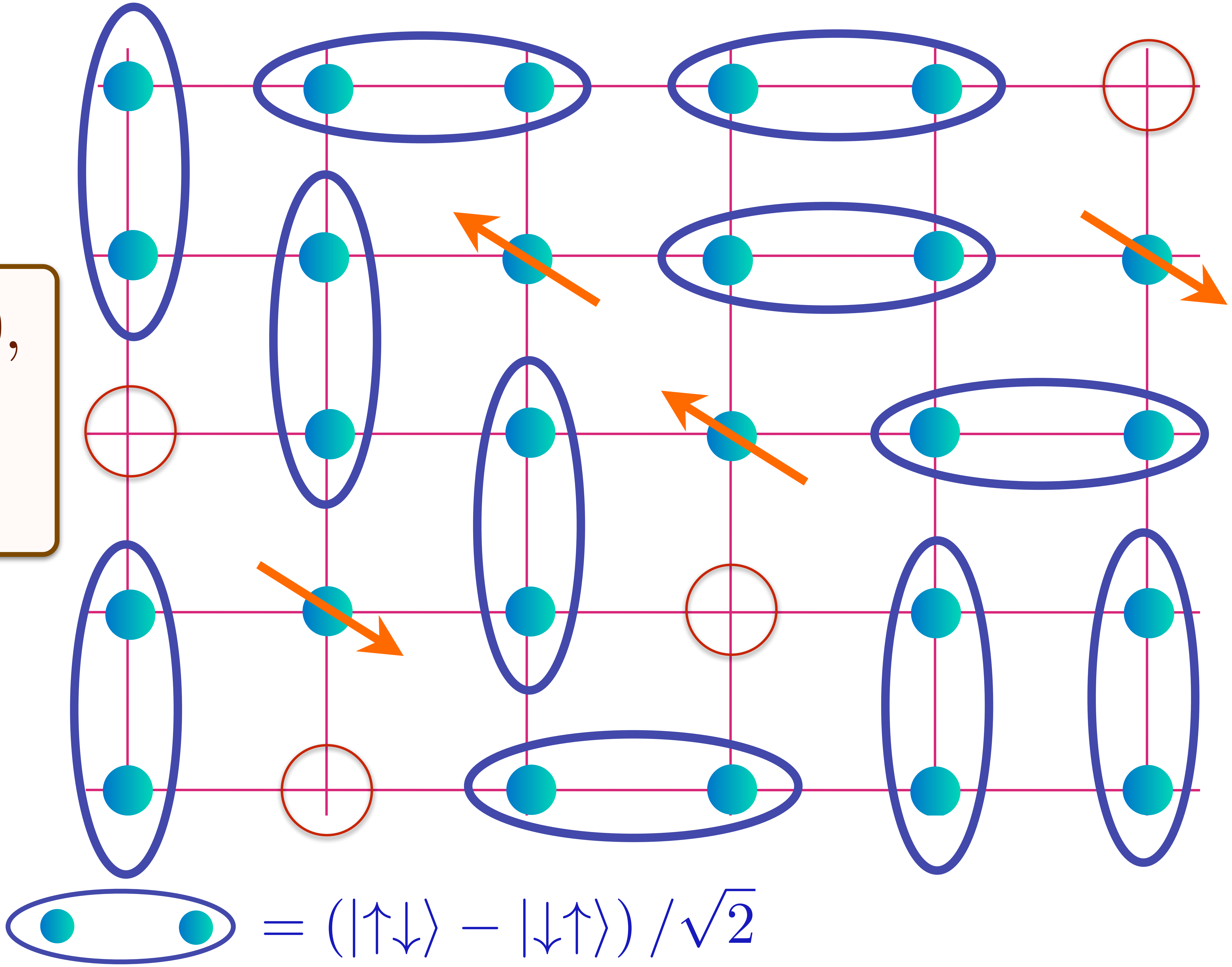

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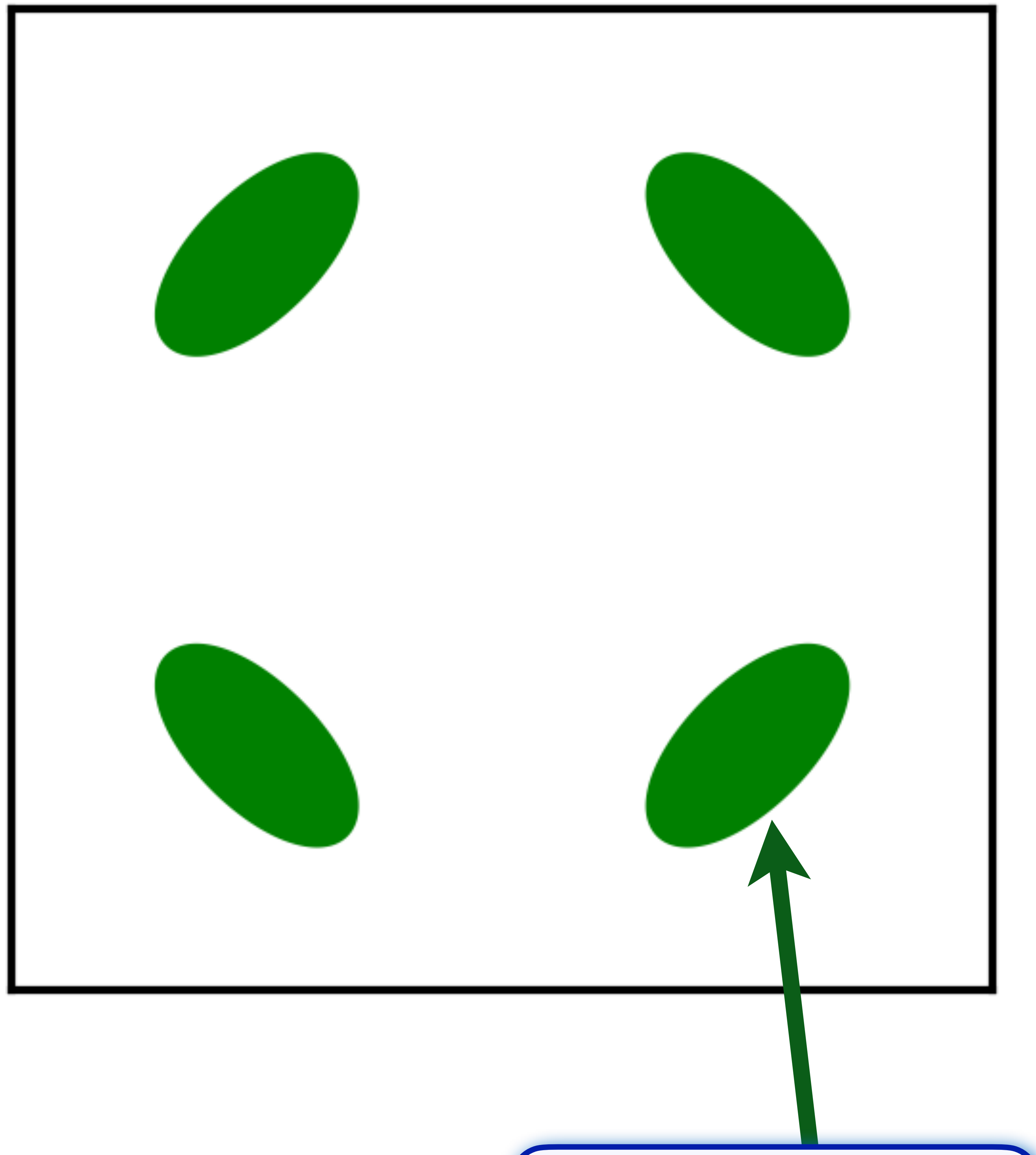

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Doping an insulating antiferromagnet with holes of density p

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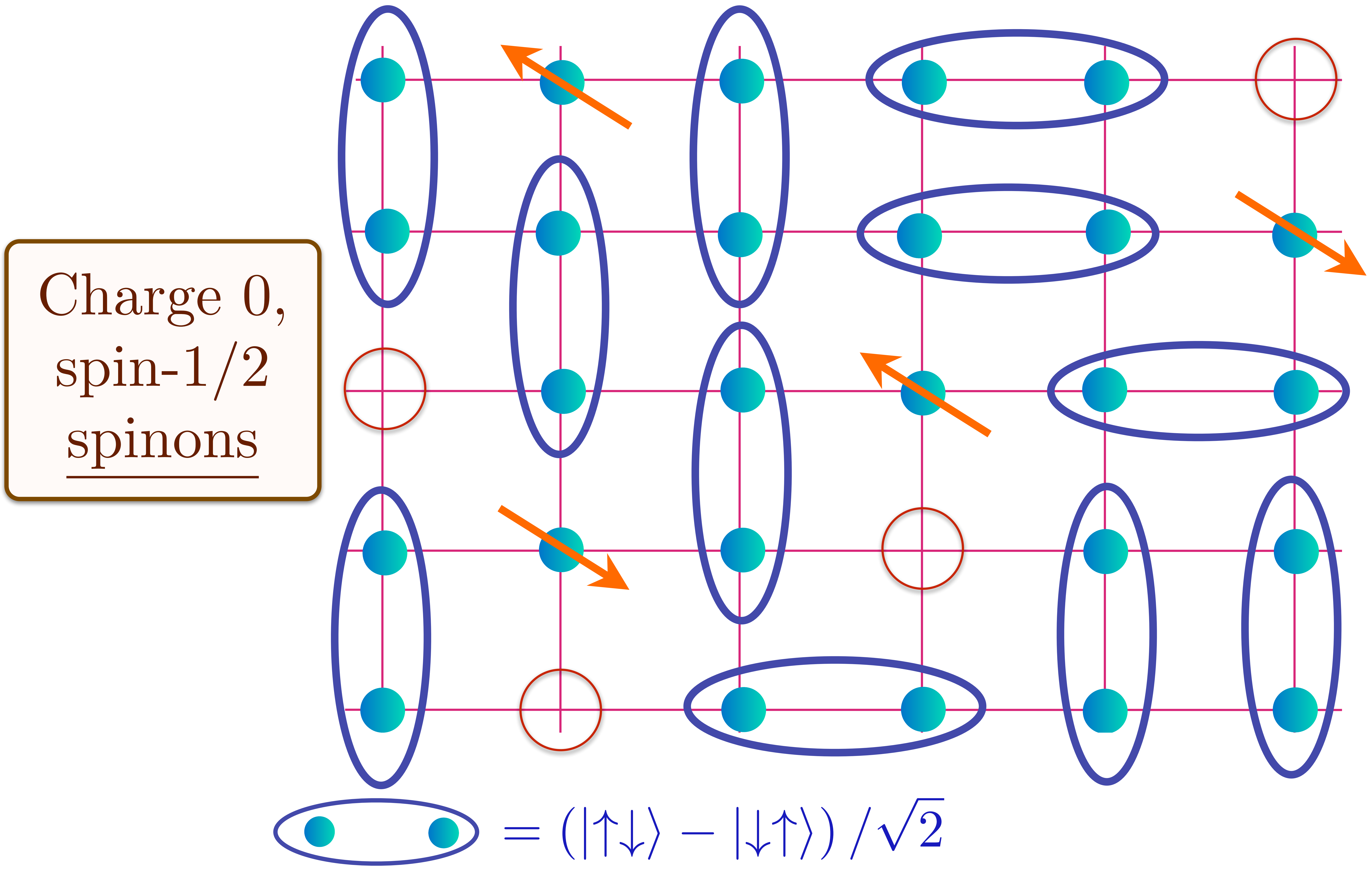

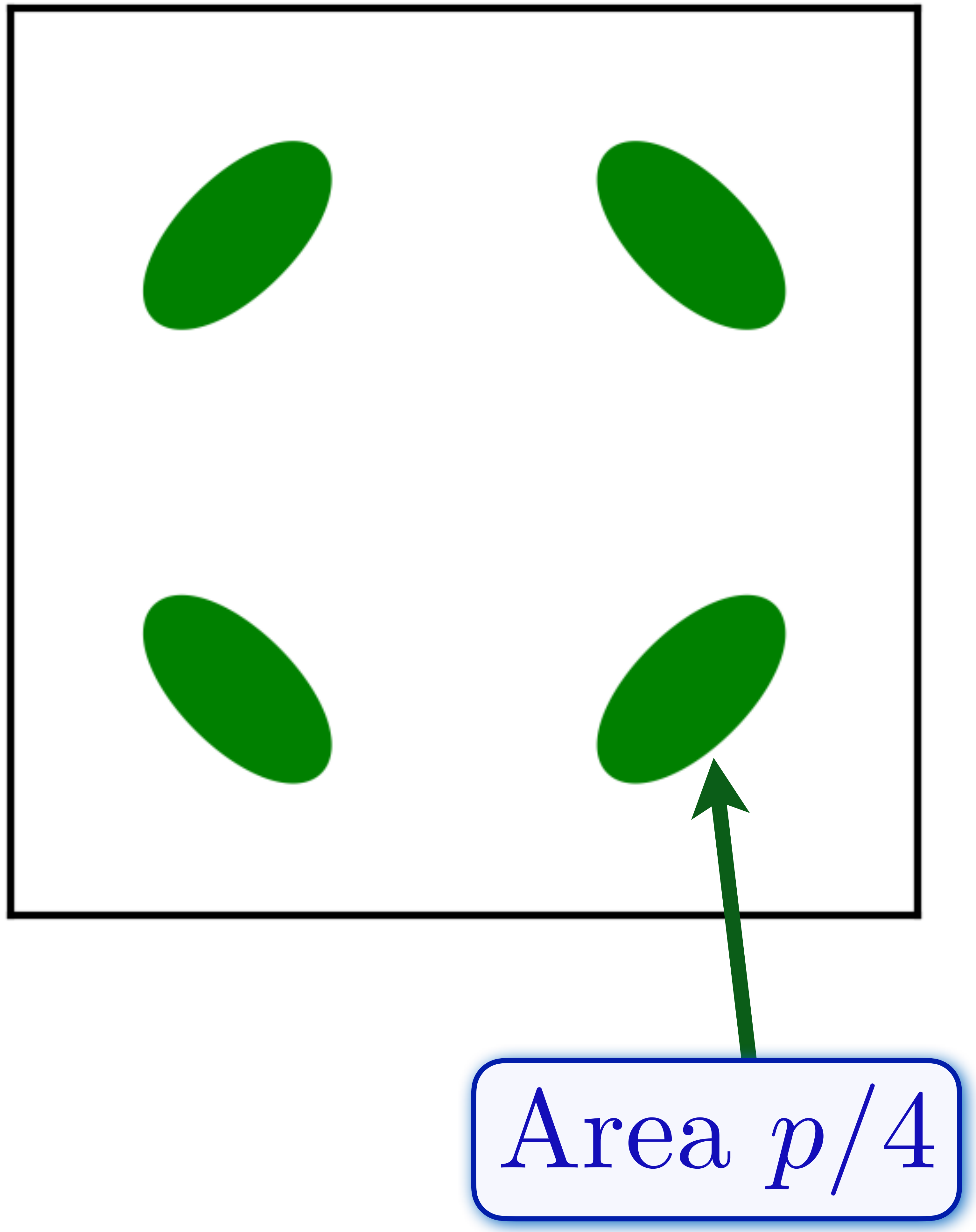

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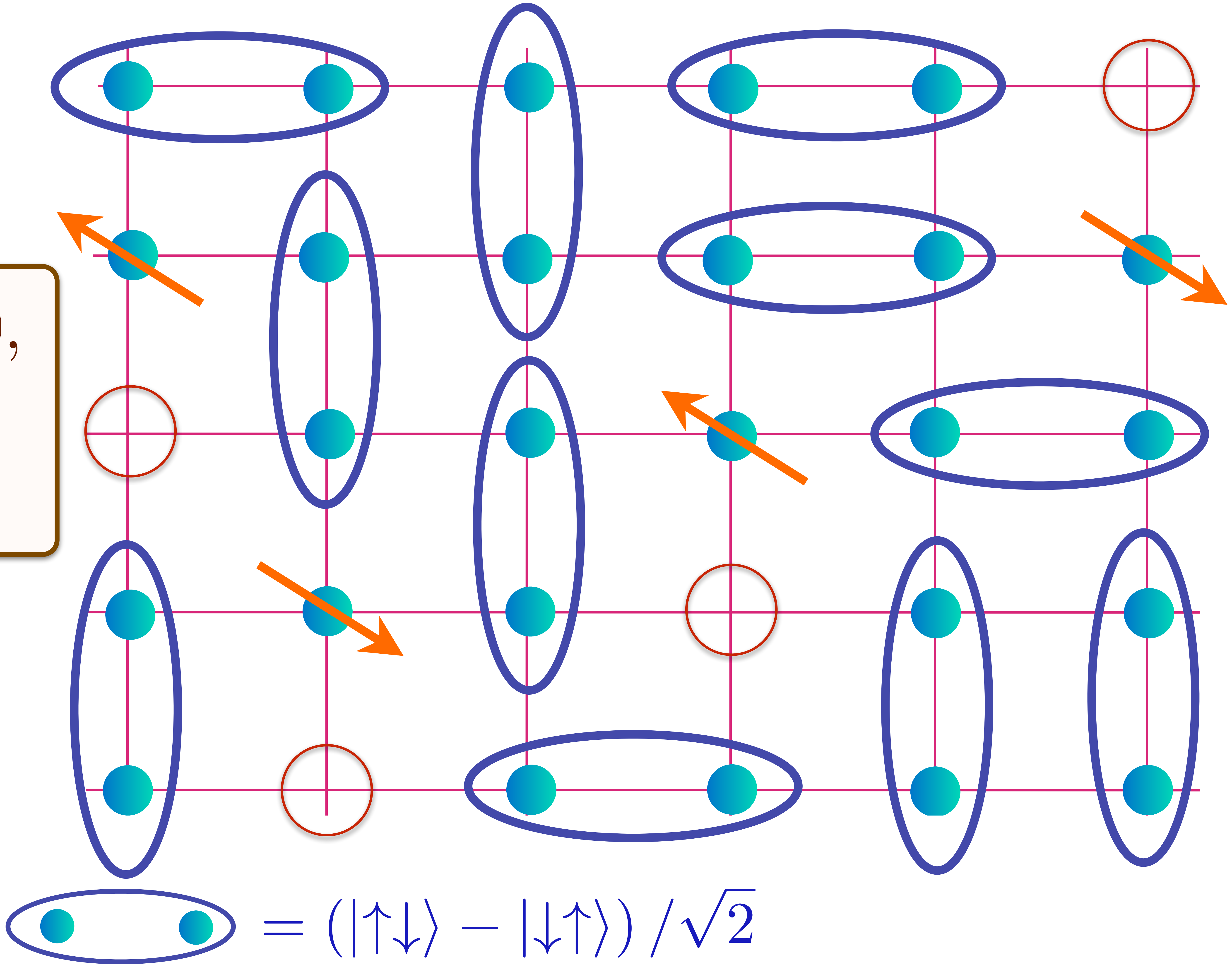

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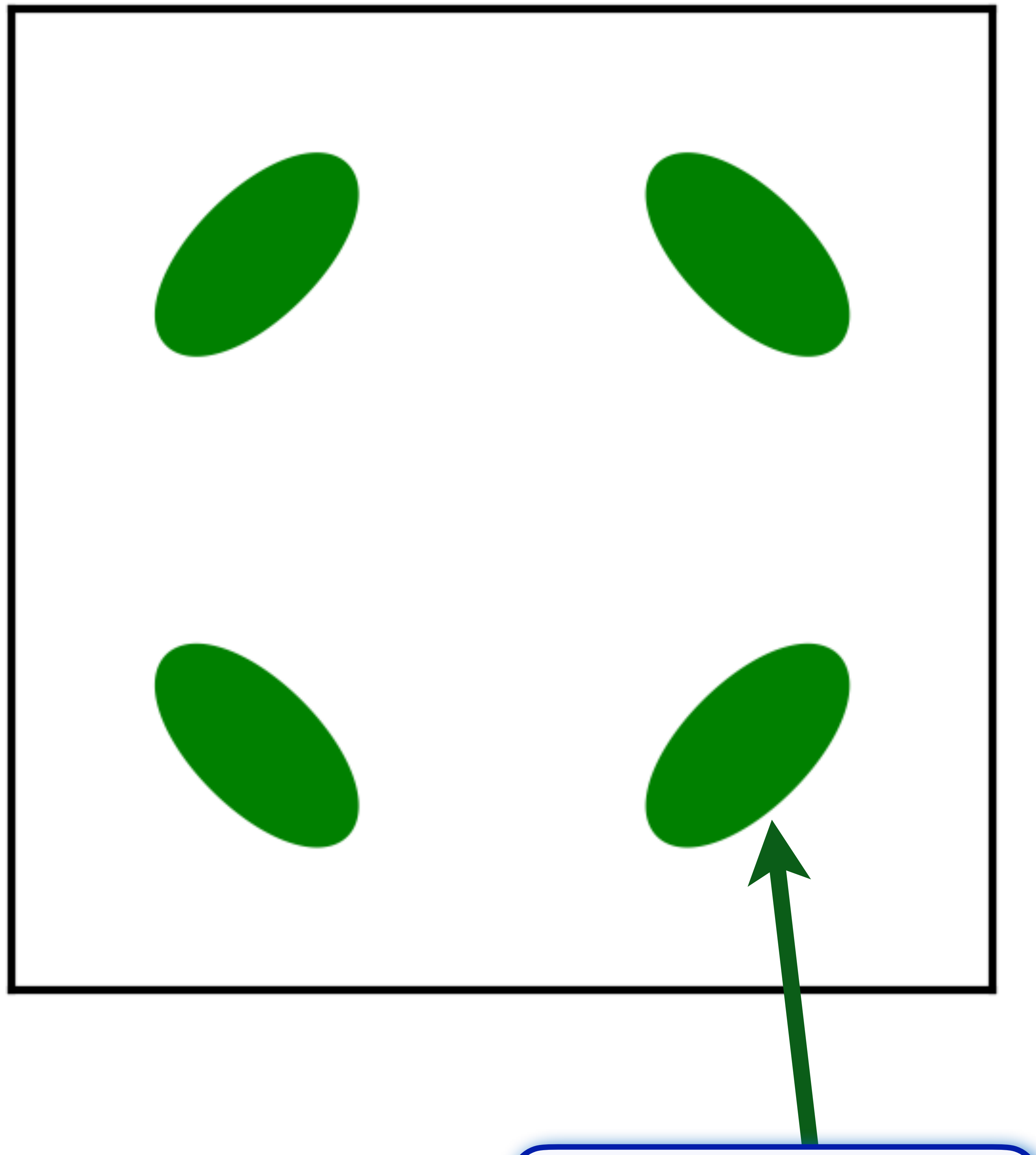

Area $p/4$

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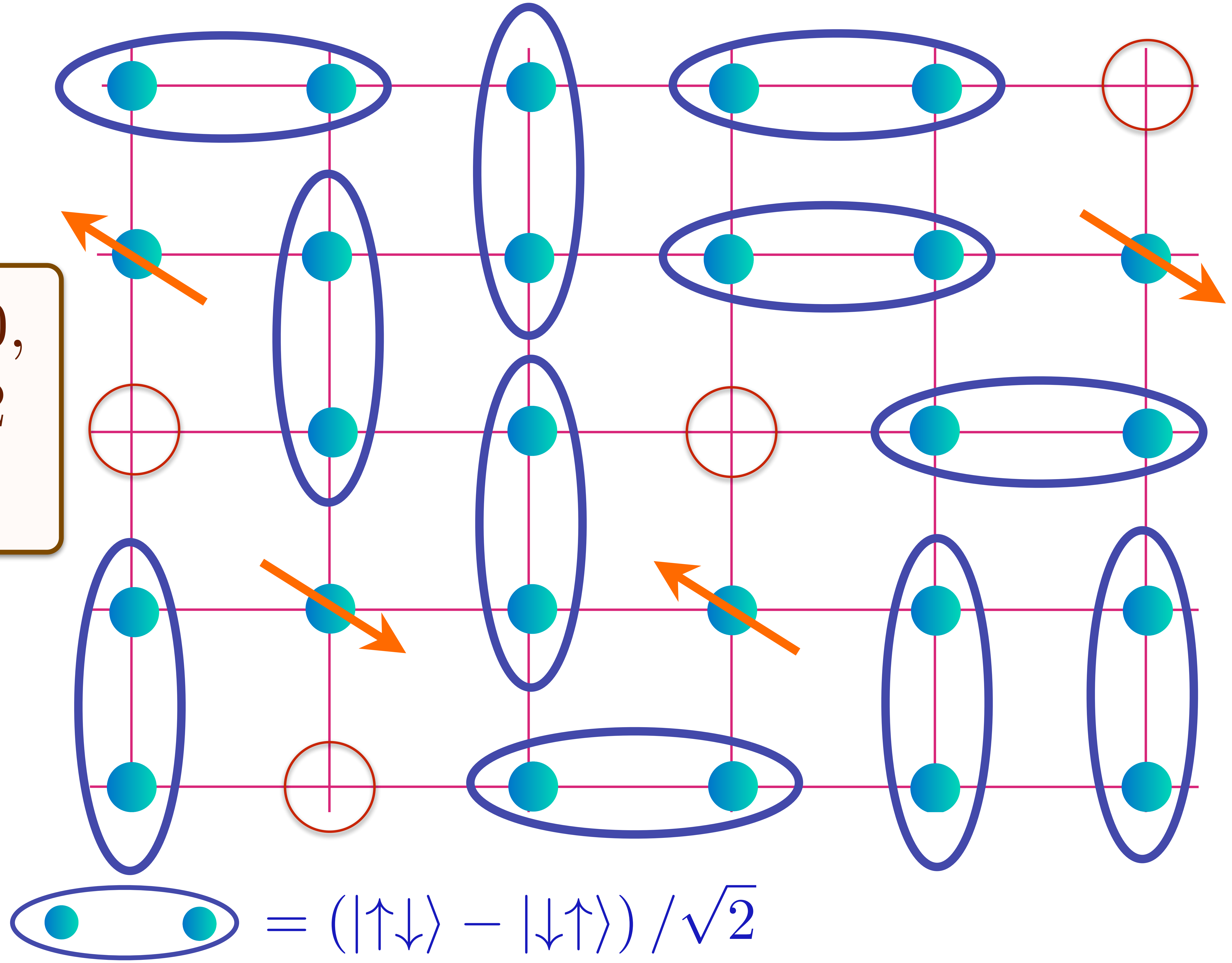

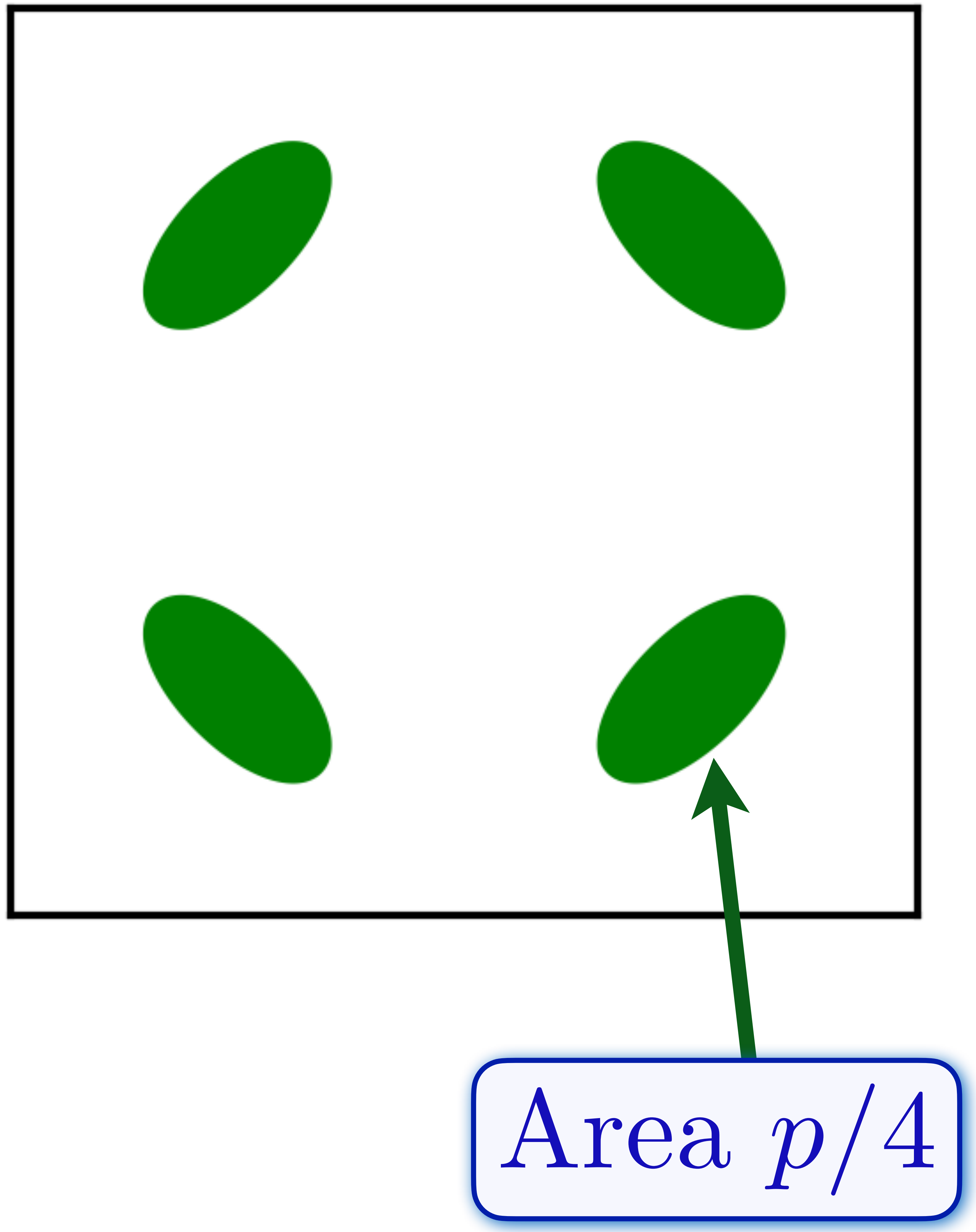

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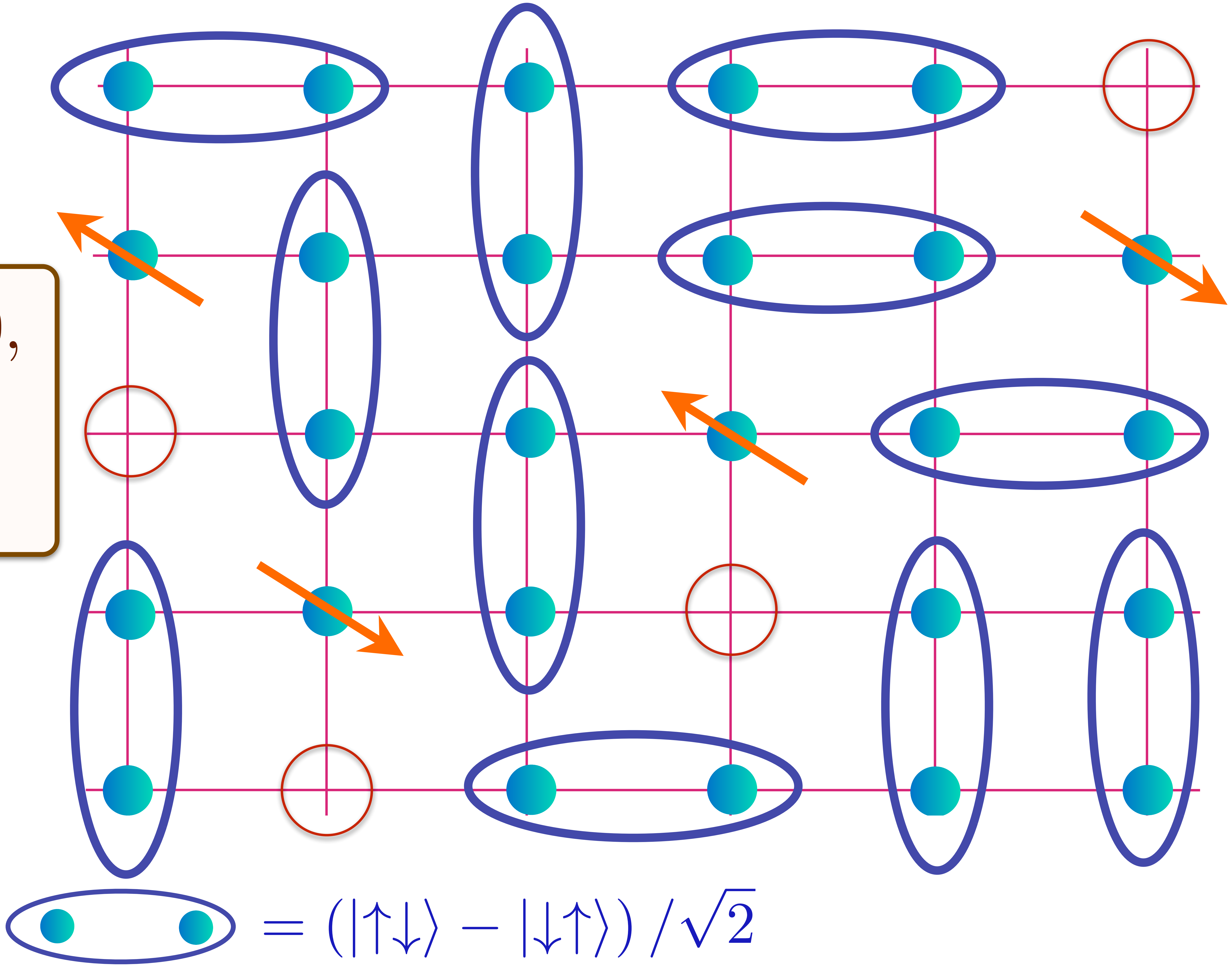
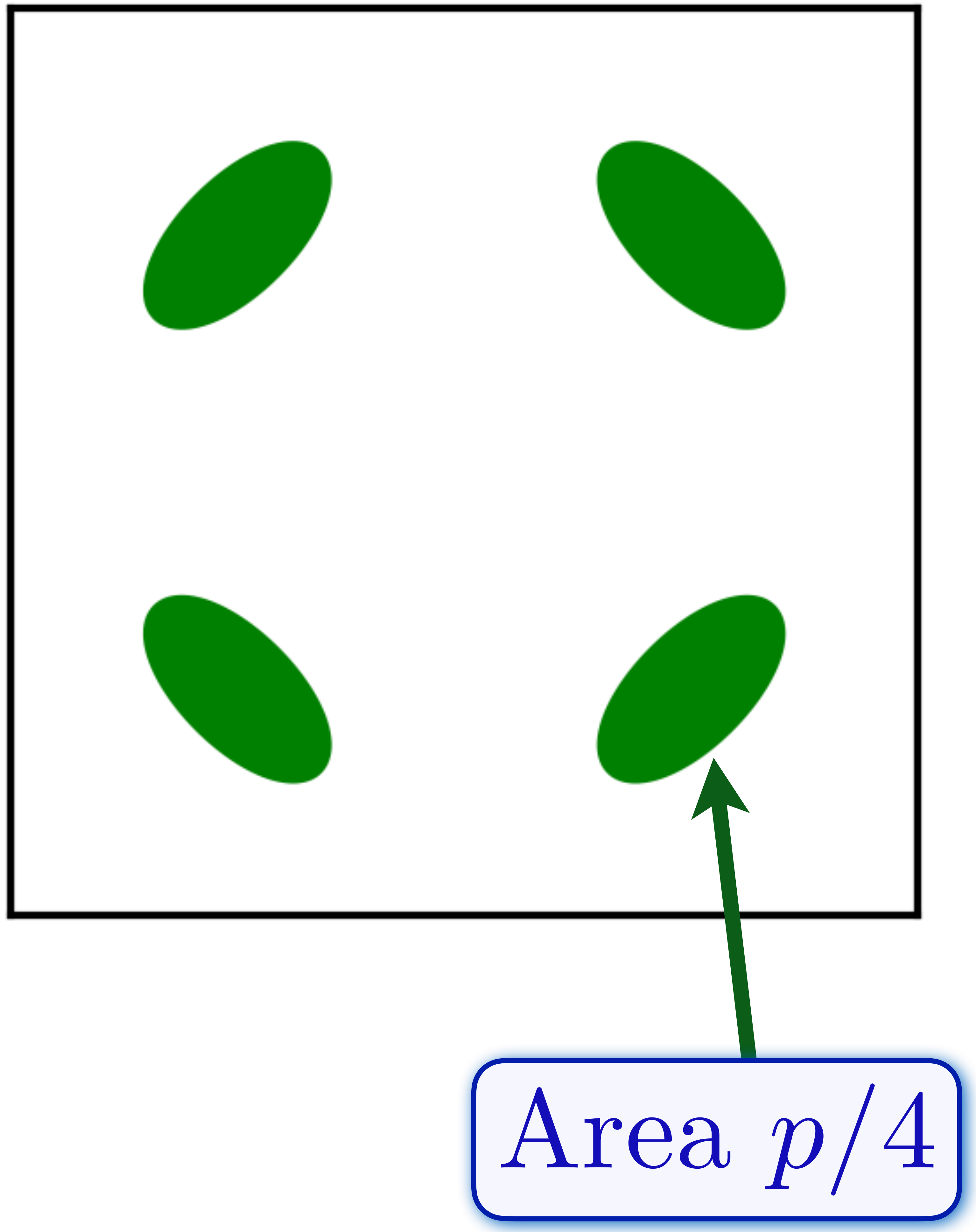
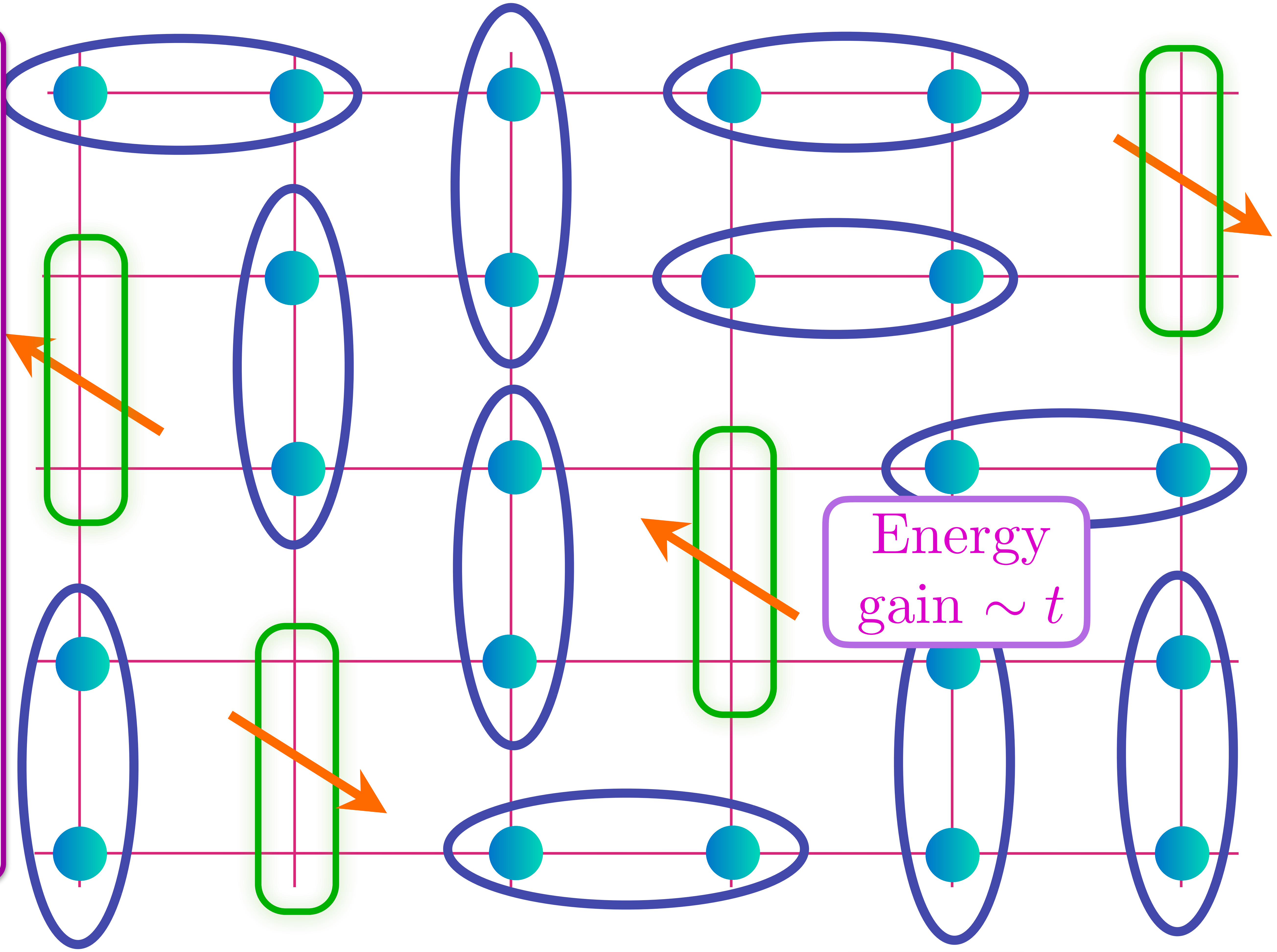




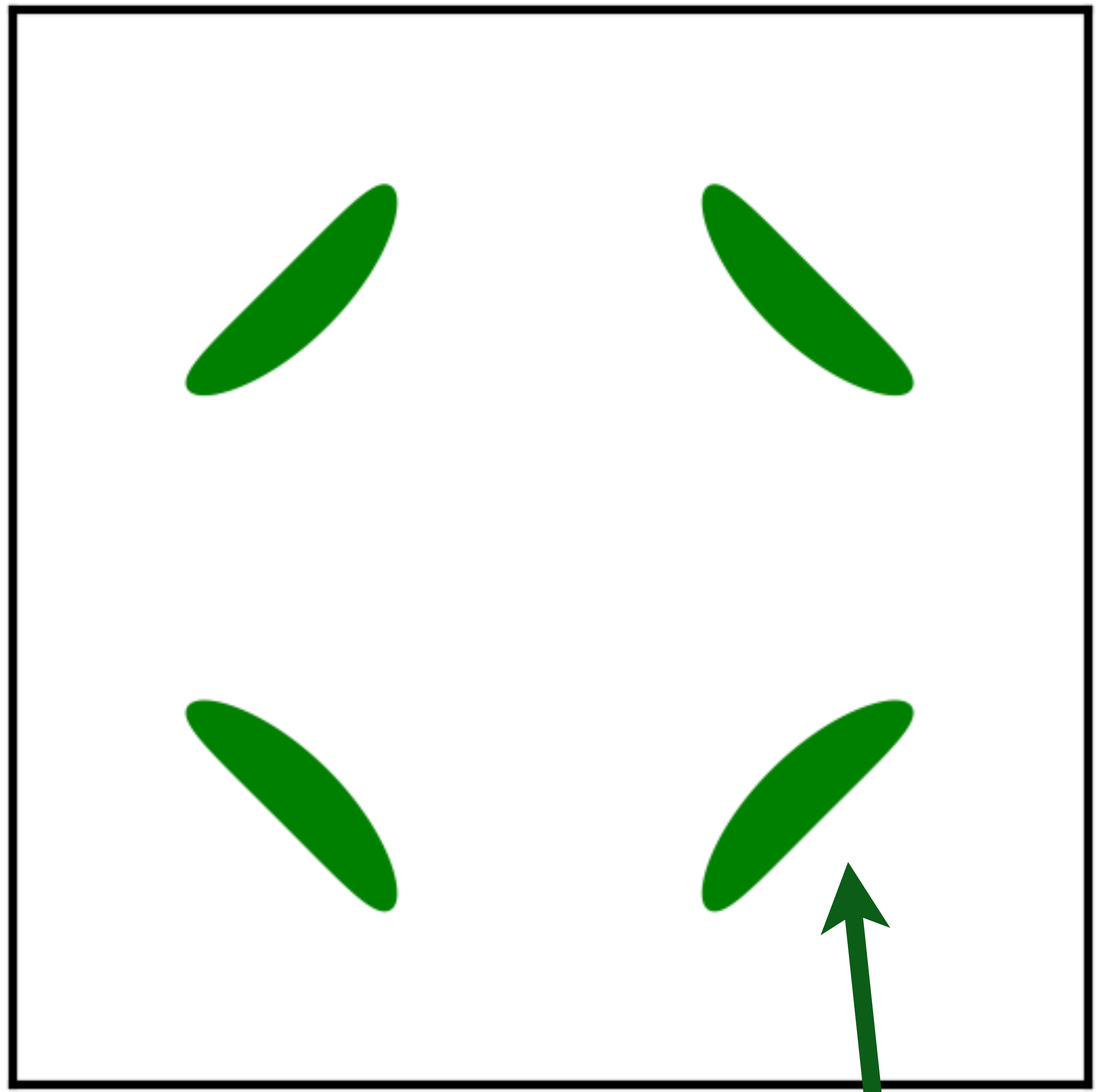
Doping an insulating antiferromagnet with holes of density p

FL*

Metal with density p of spin-1/2, charge $+e$ 'holes' (or 'magnetic polarons') with coherent inter-layer transport.



Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)



Area $p/8$

(blue ellipse) $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$ (green box) $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

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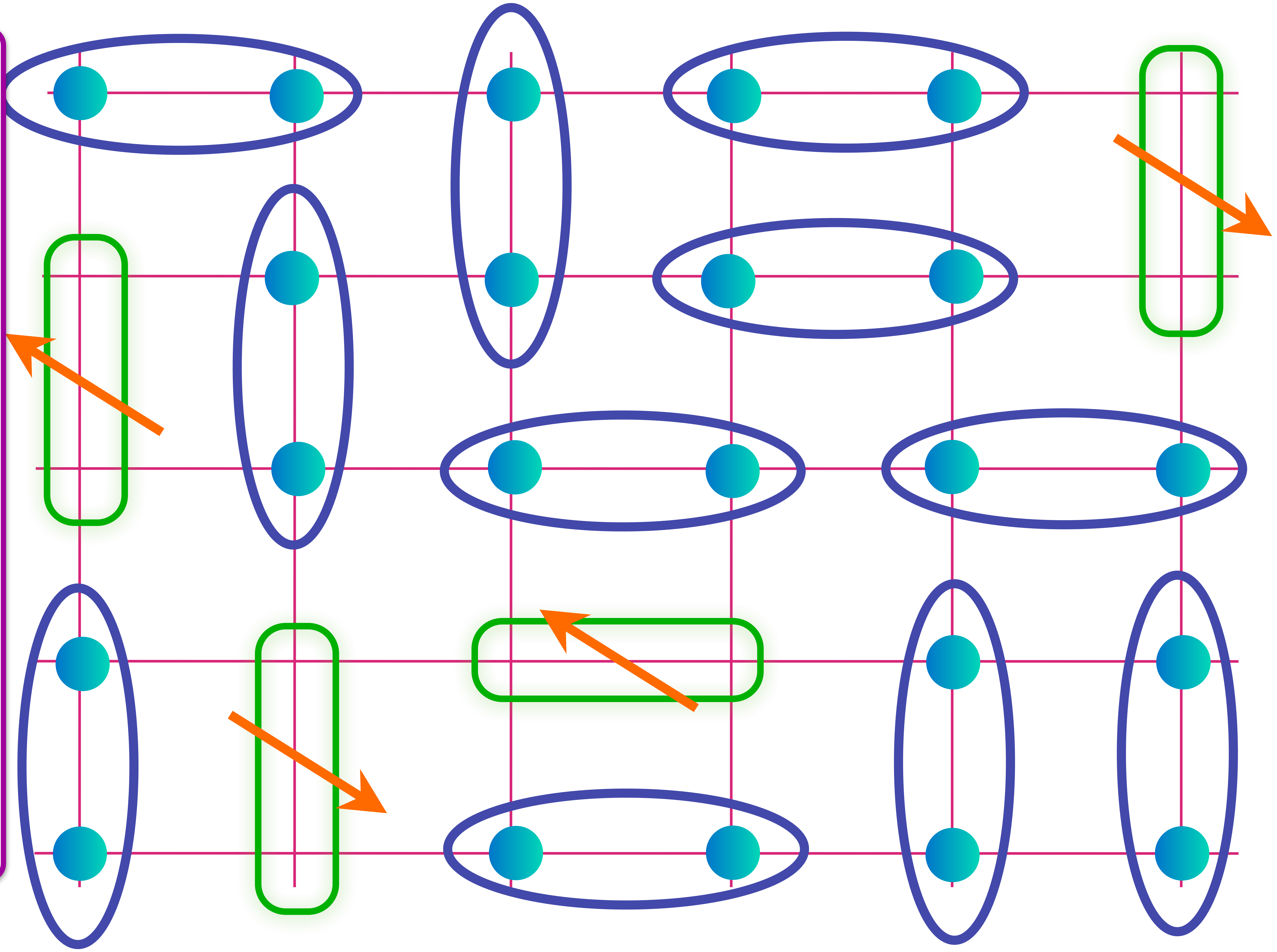
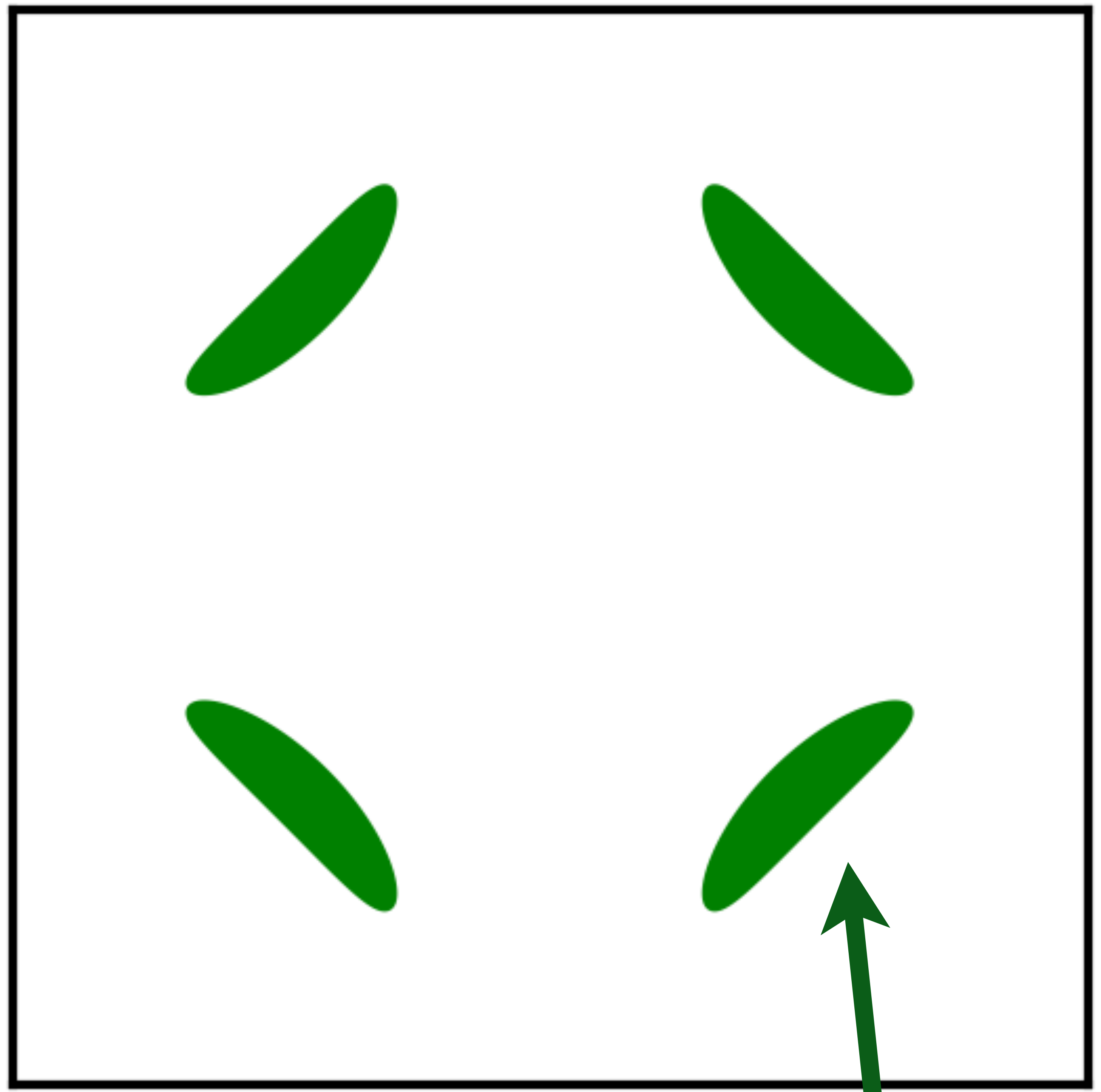


Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)



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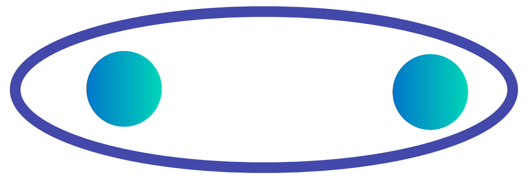
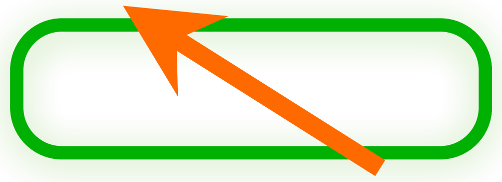
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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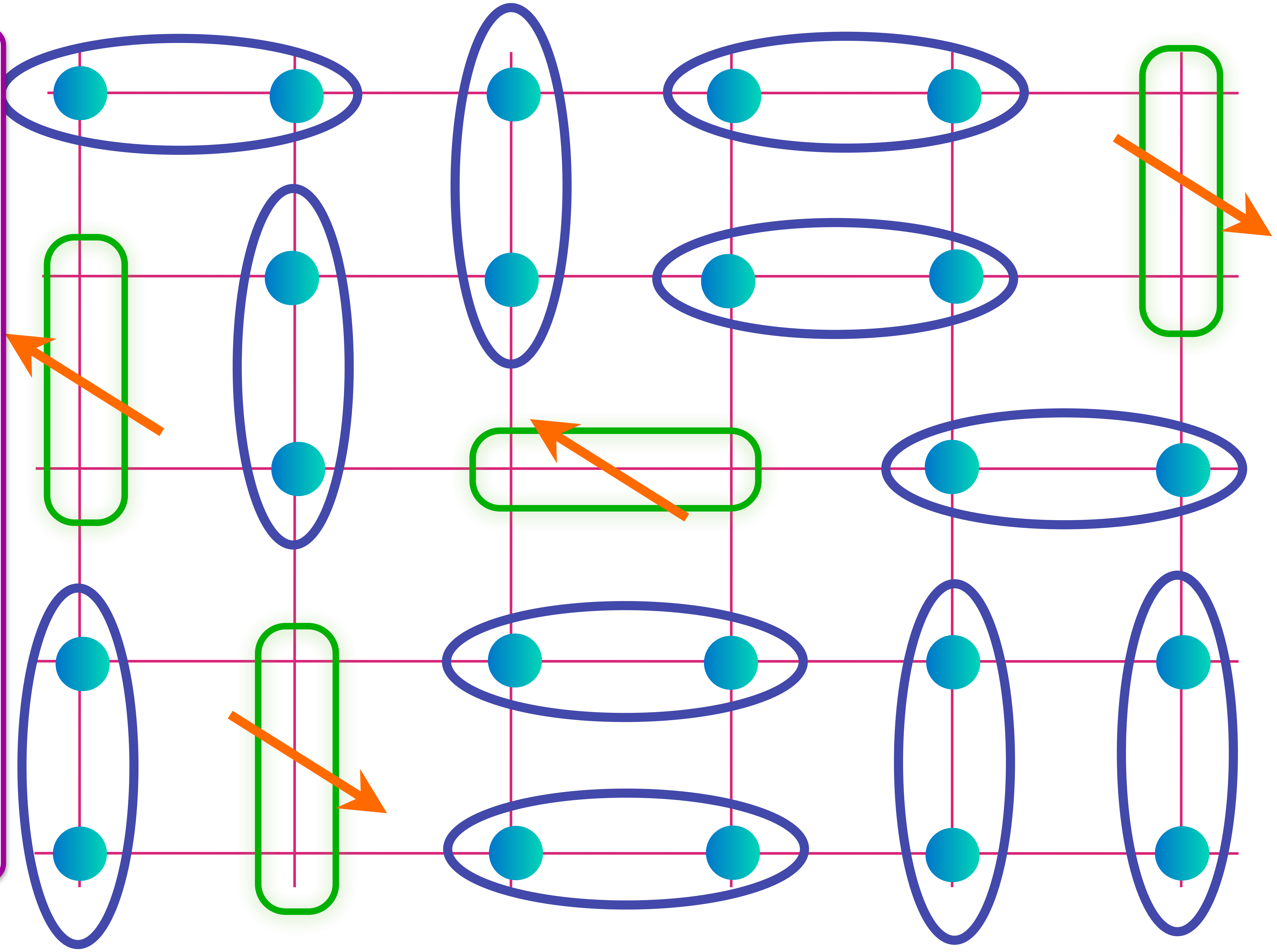

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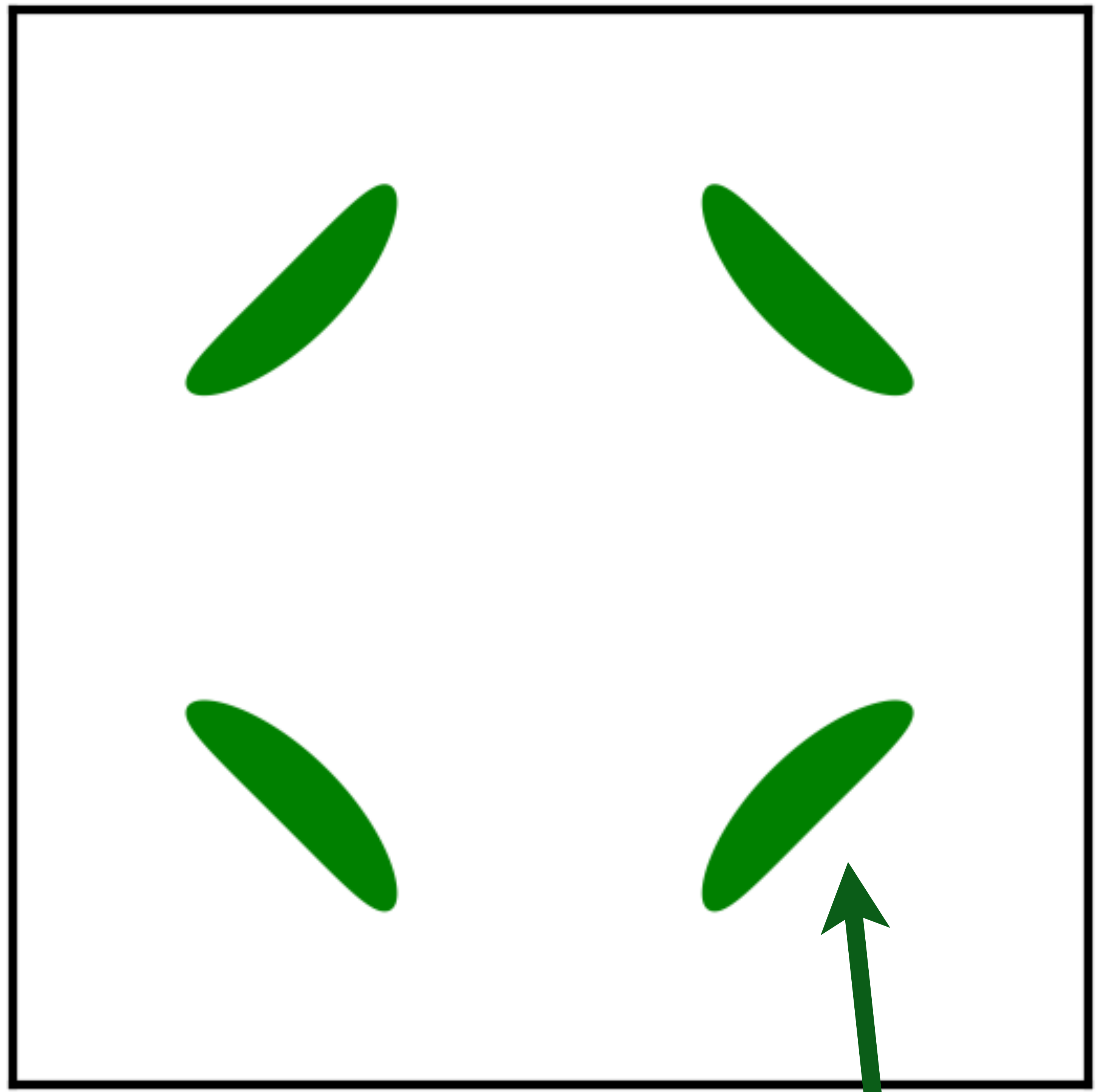

Area $p/8$

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
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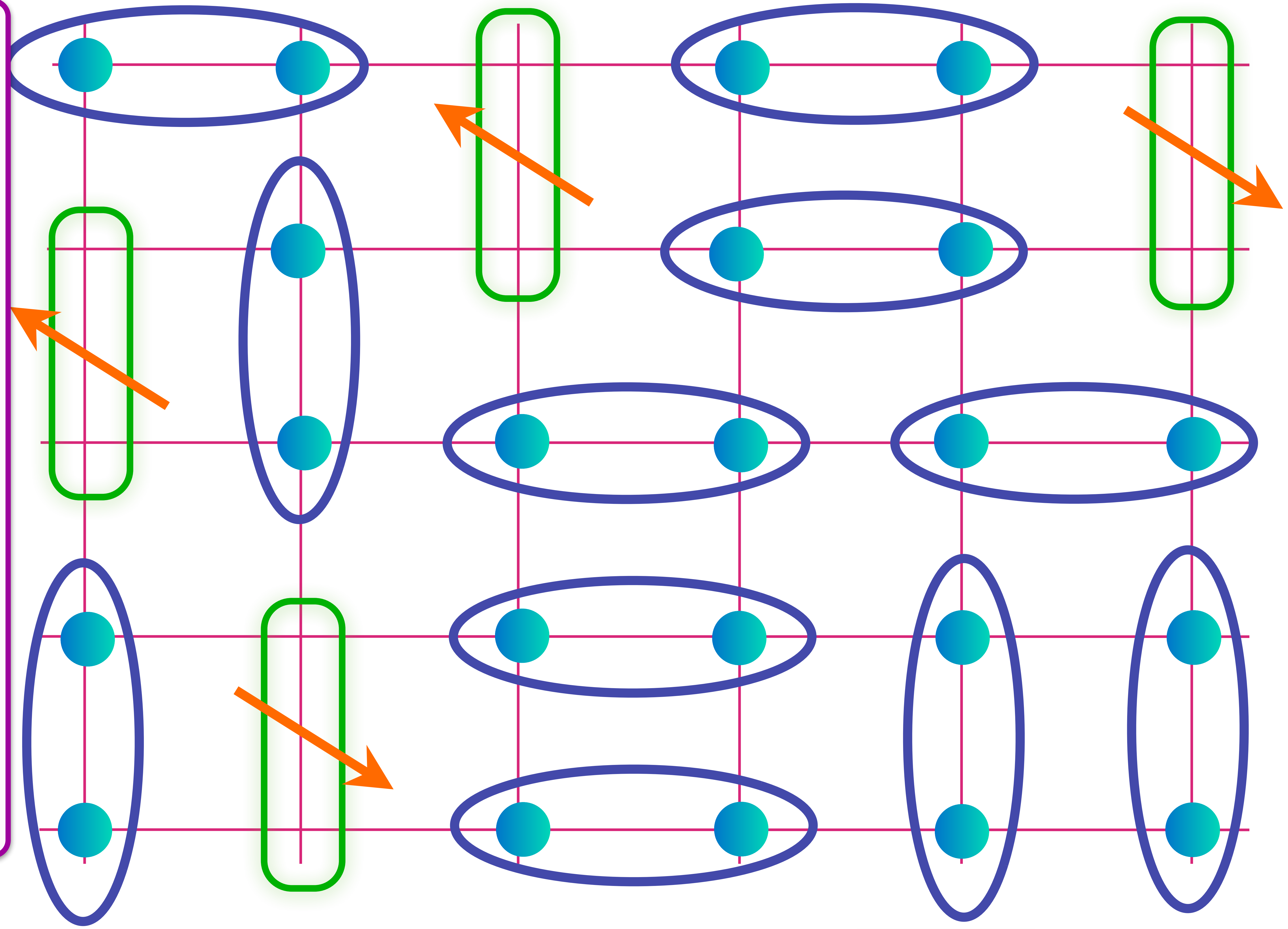

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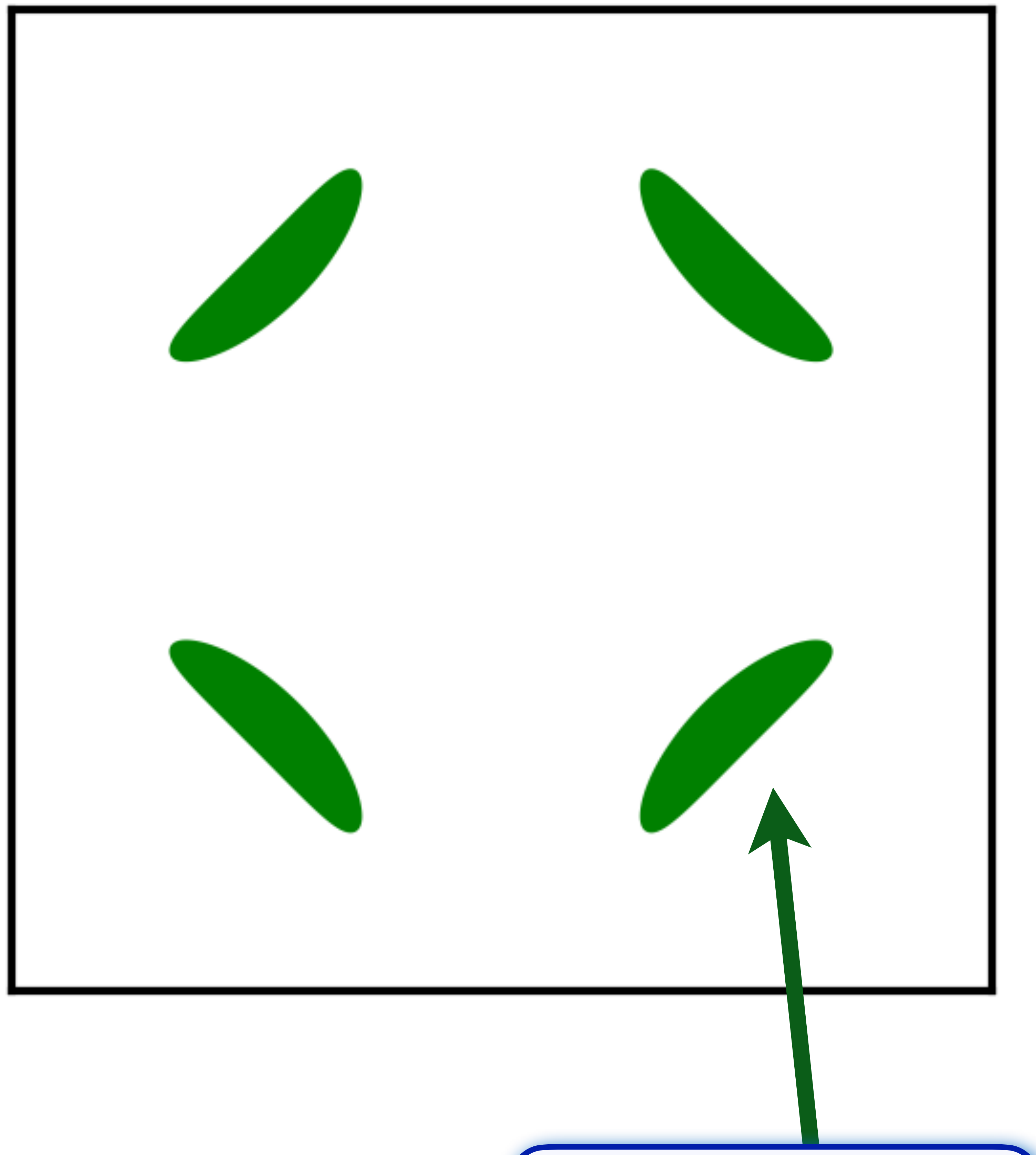
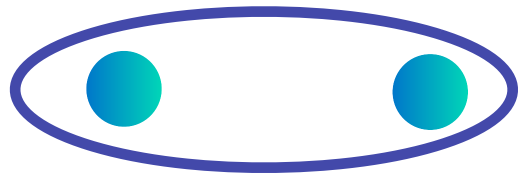
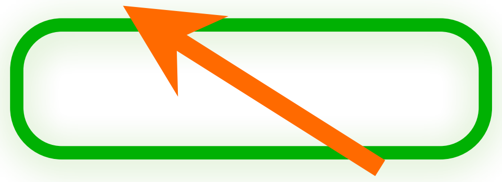


Area $p/8$

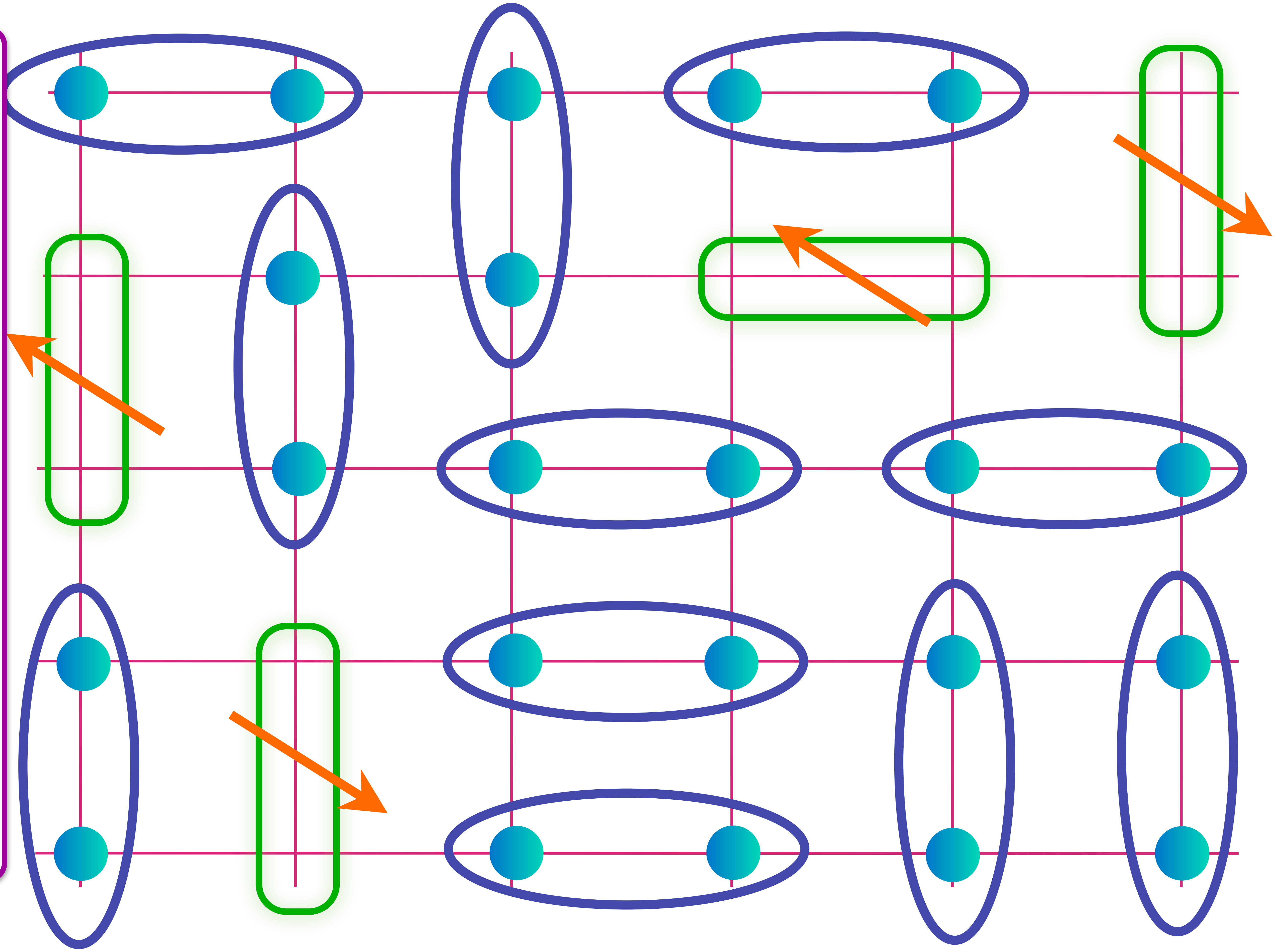
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

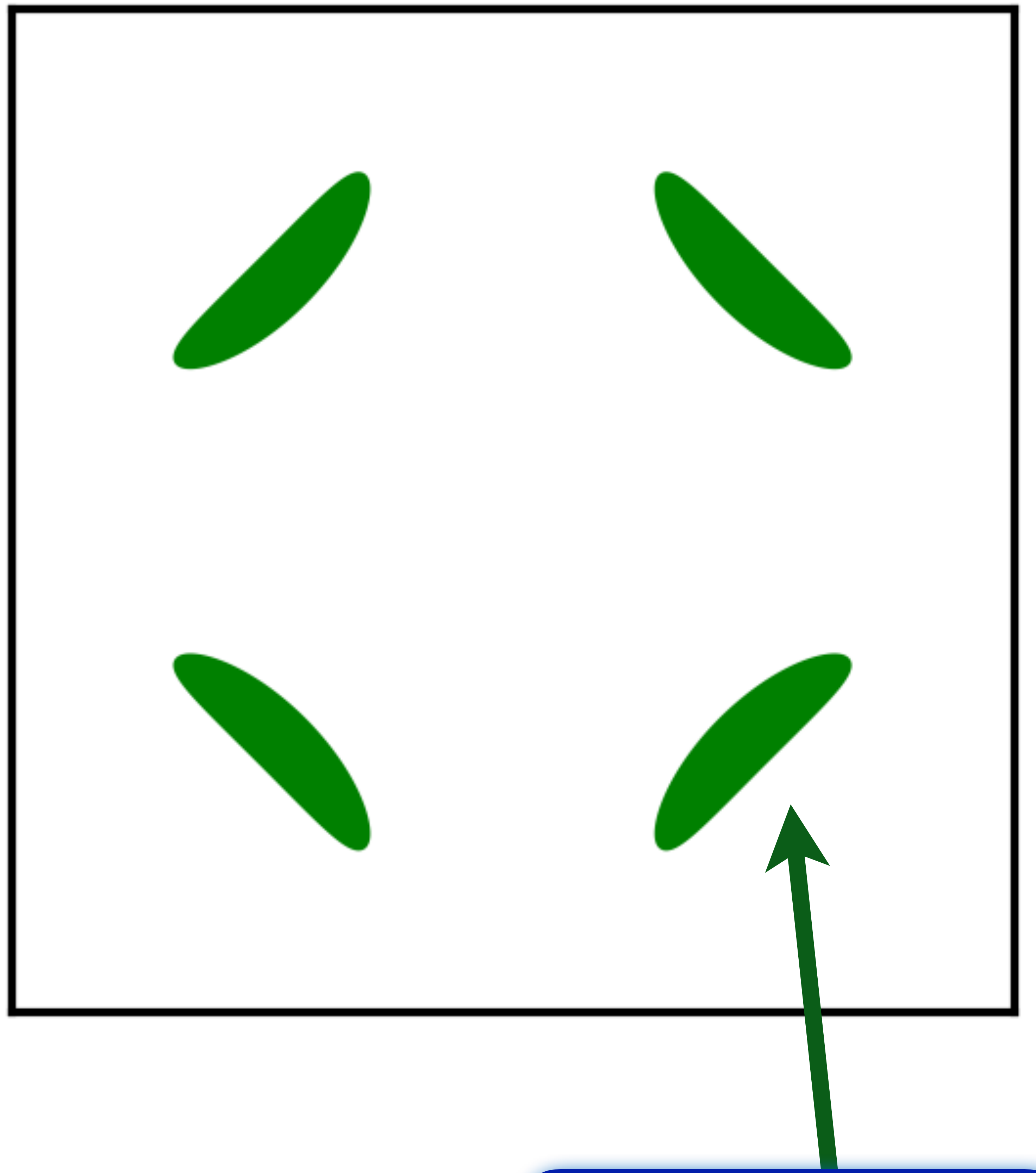
Doping an insulating antiferromagnet with holes of density p

FL*

Metal with density p of spin-1/2, charge $+e$ 'holes' (or 'magnetic polarons') with coherent inter-layer transport.



Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)



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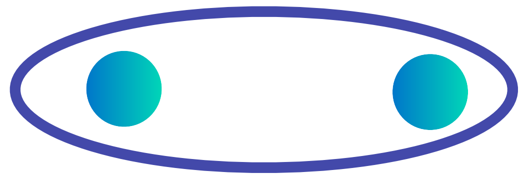
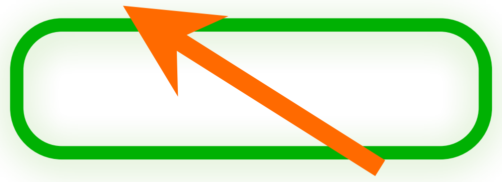
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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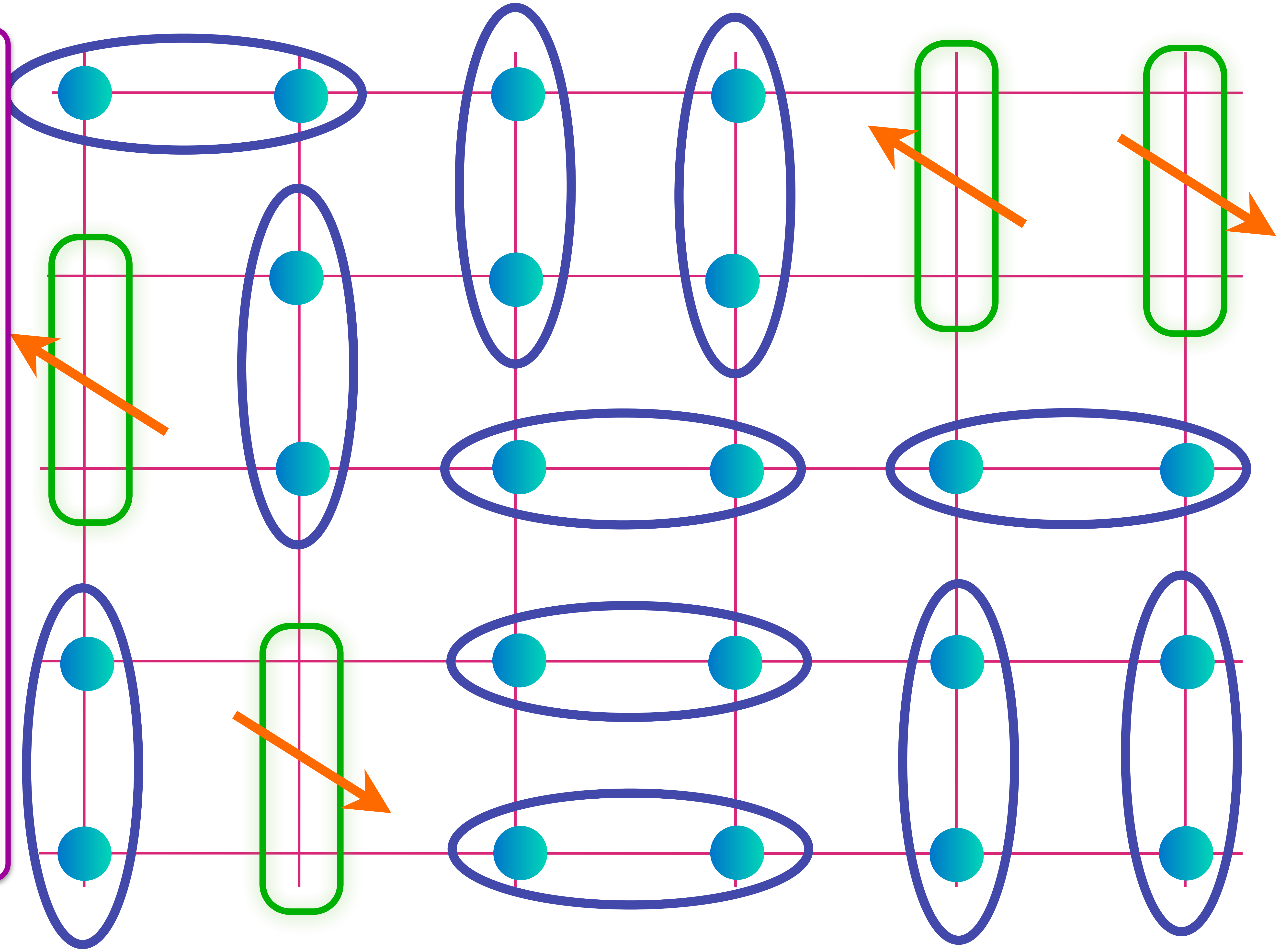

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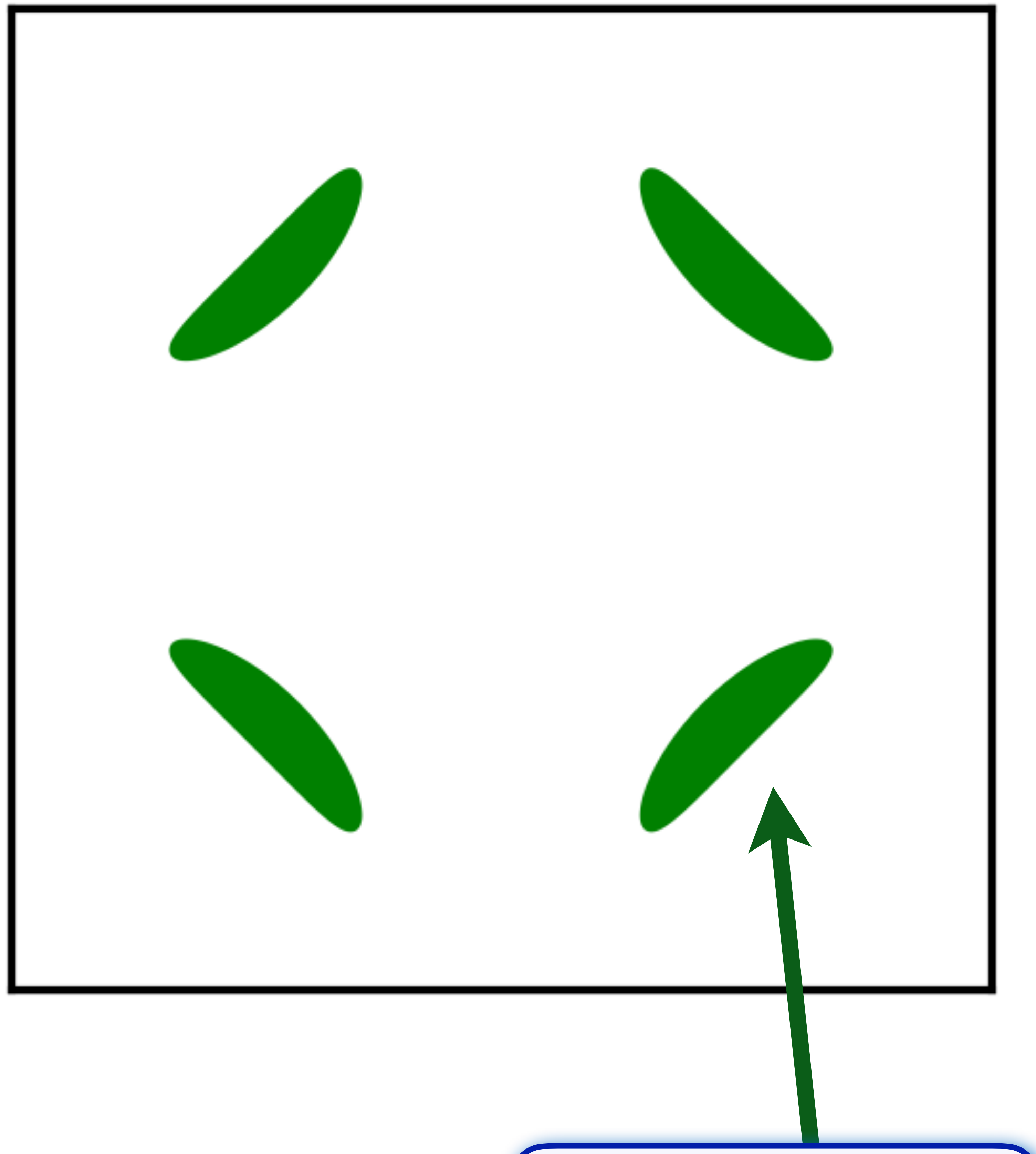
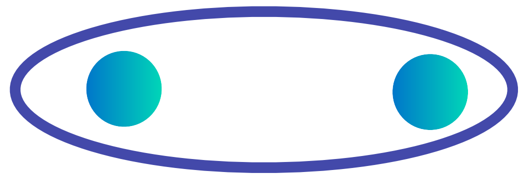
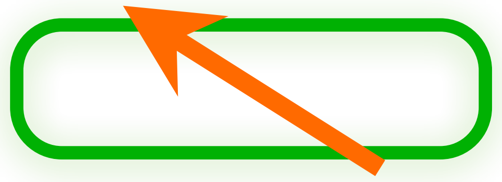


Area $p/8$

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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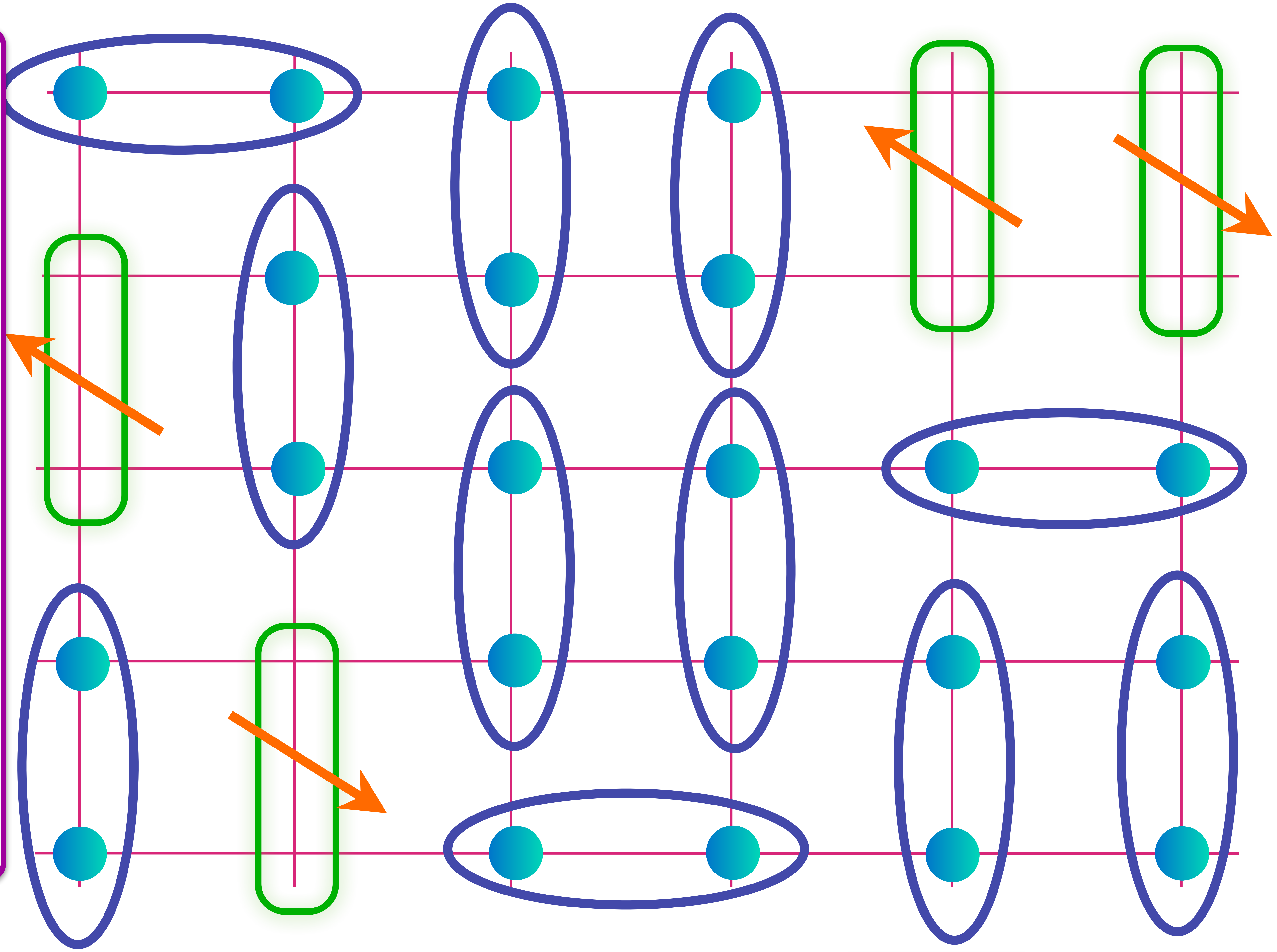

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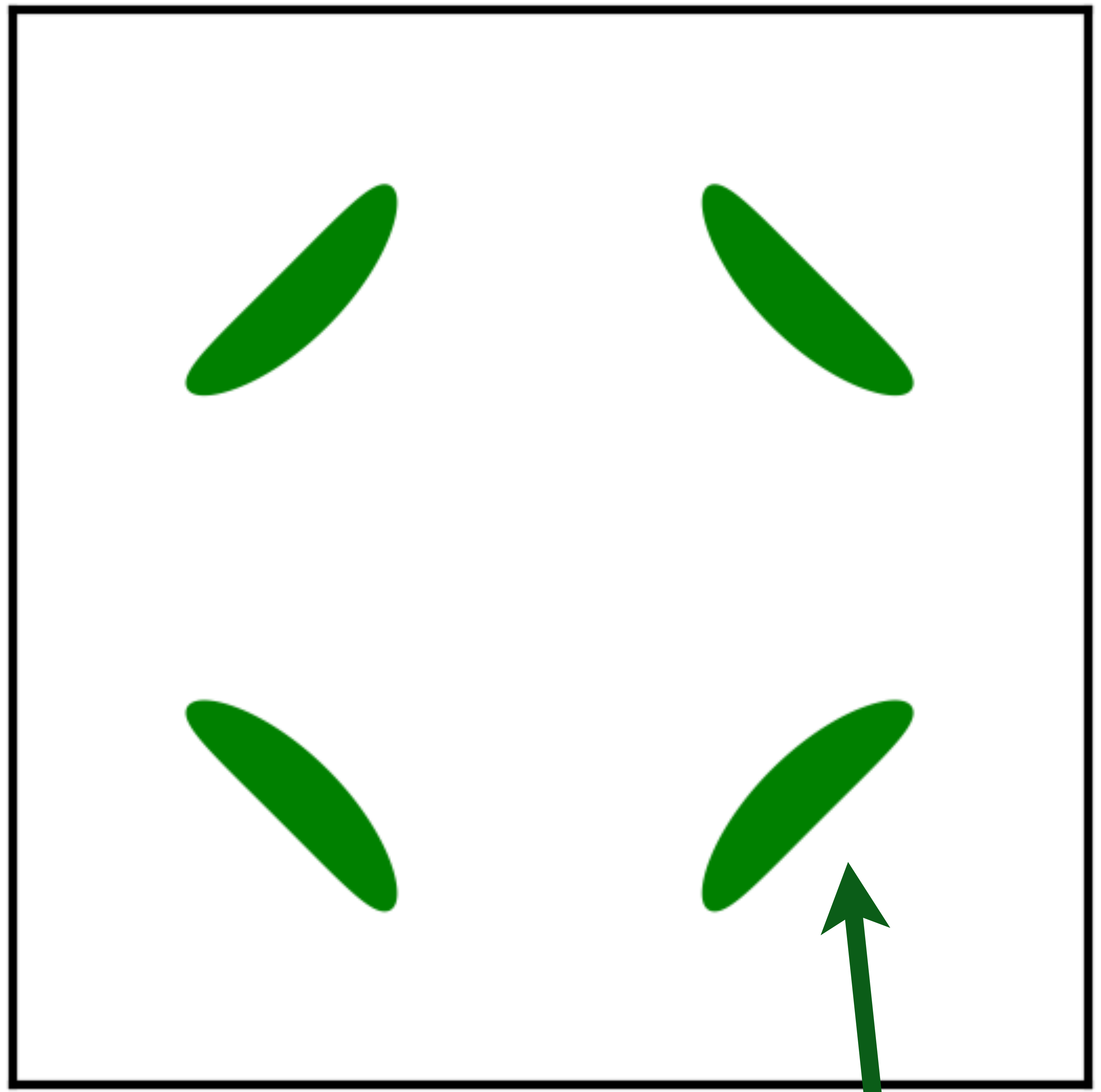

Area $p/8$

$$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2} \qquad = (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$$

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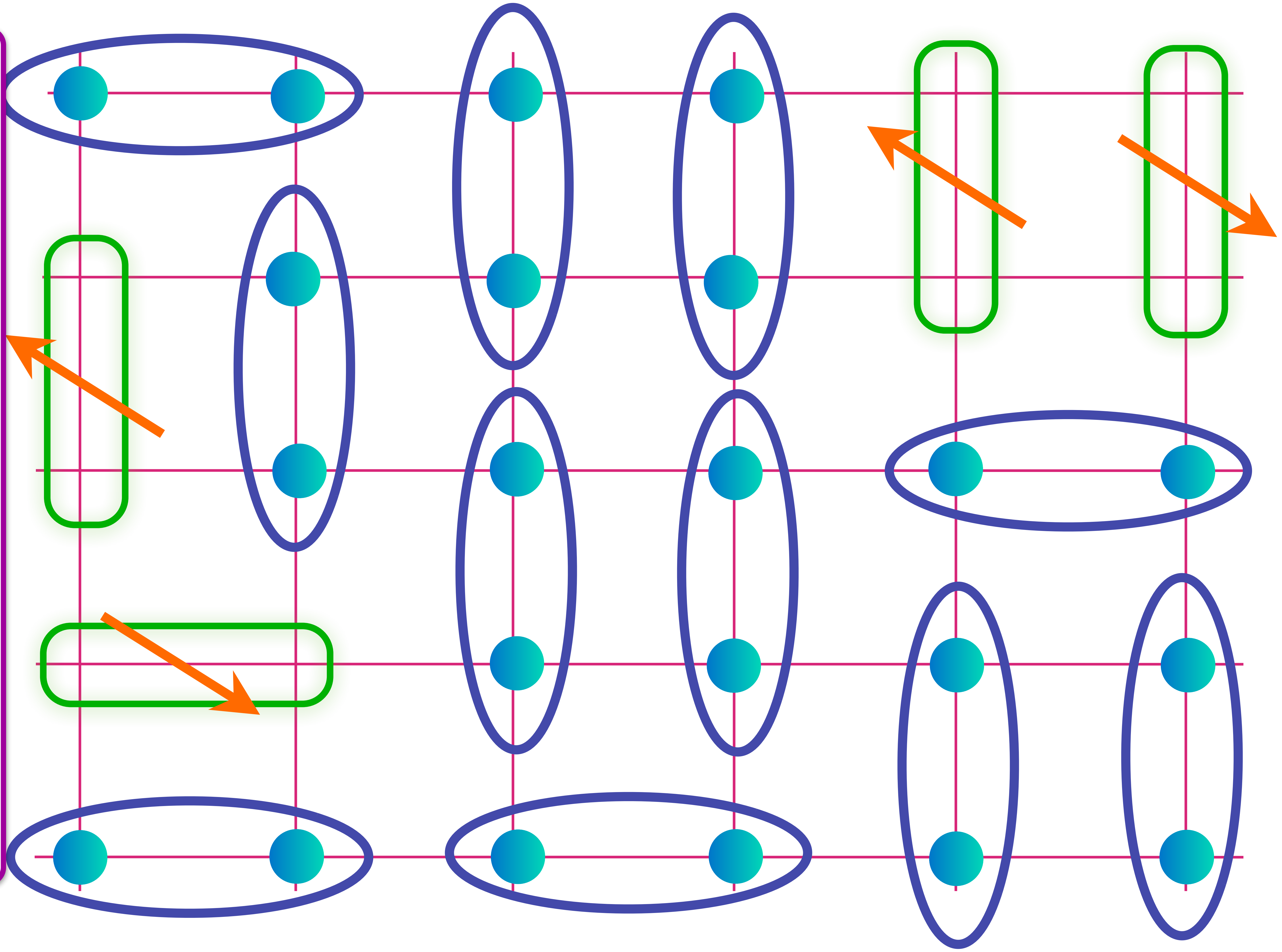

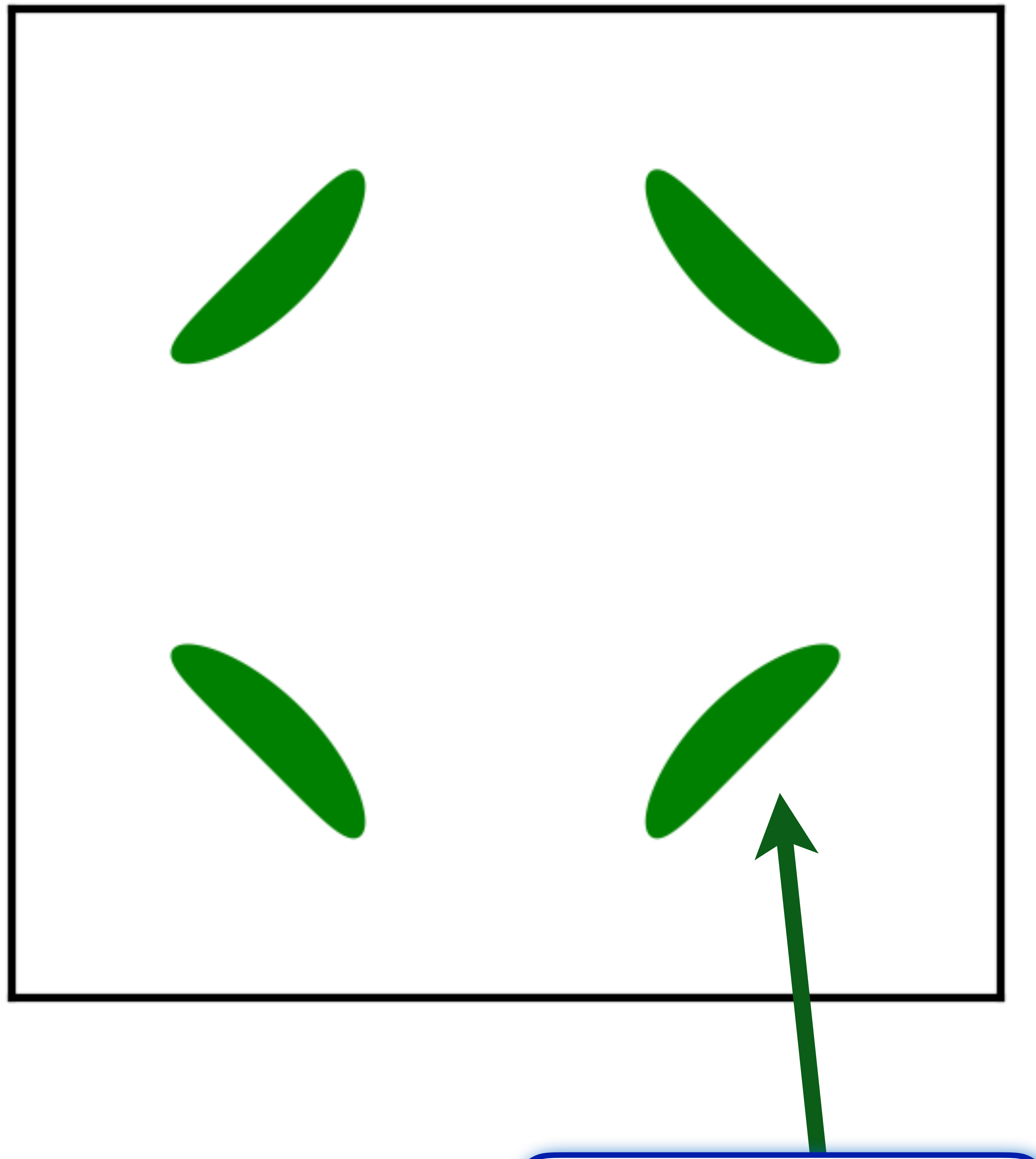

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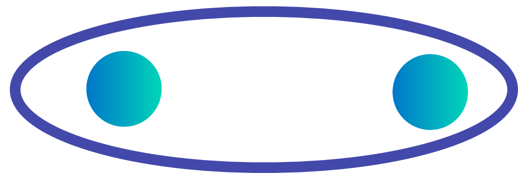
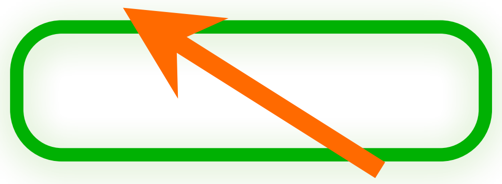
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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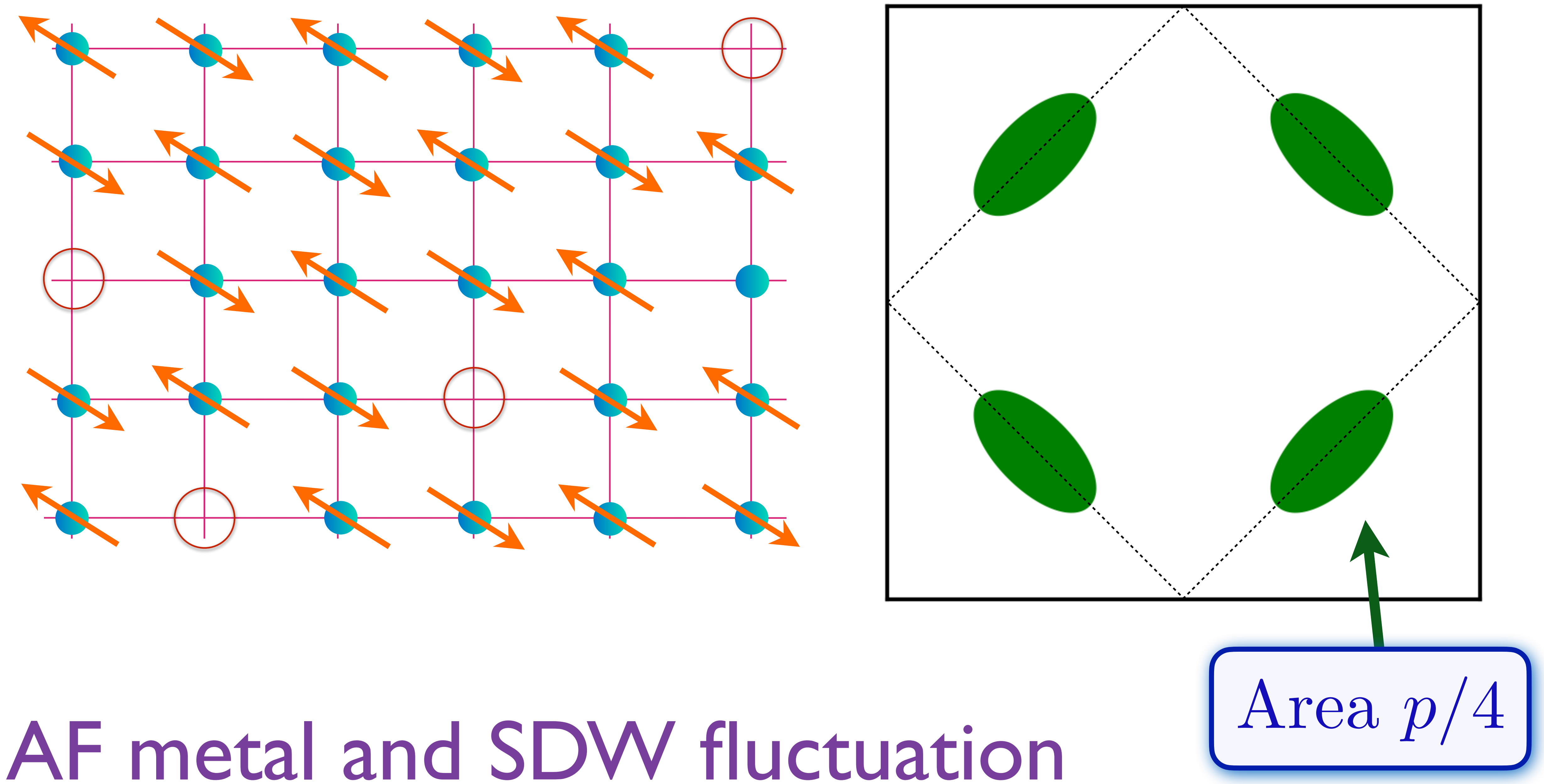
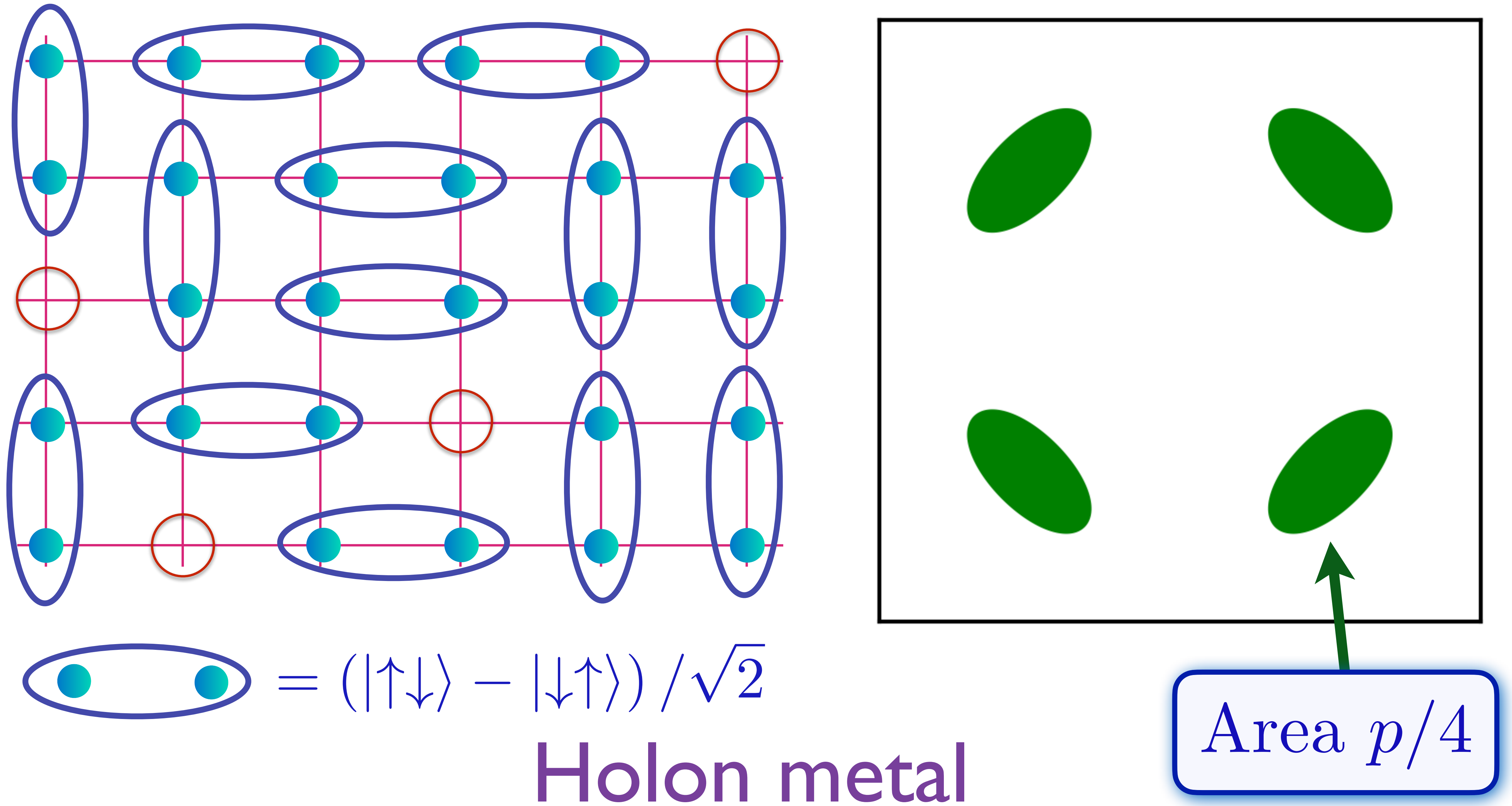
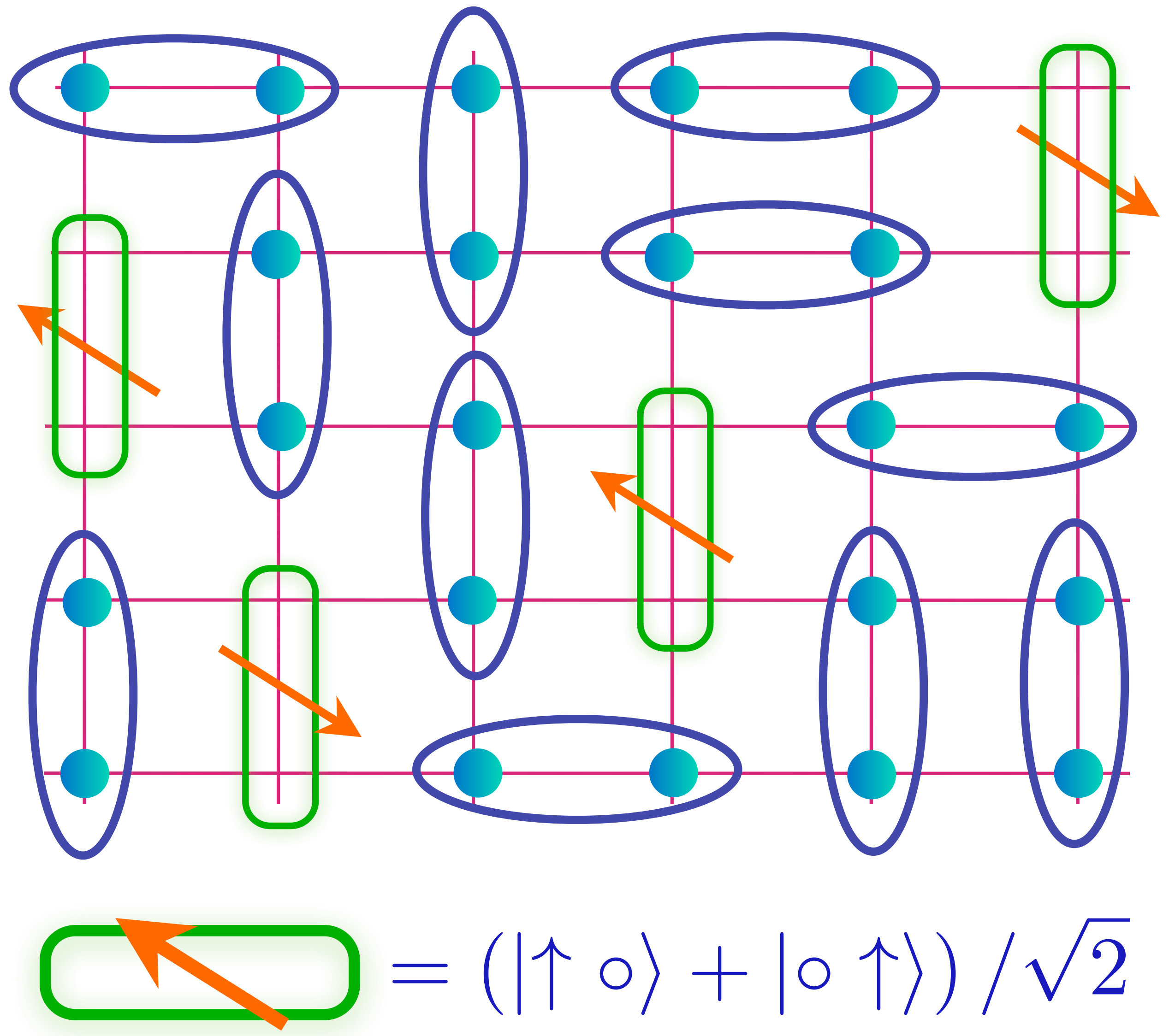


AF metal and SDW fluctuation

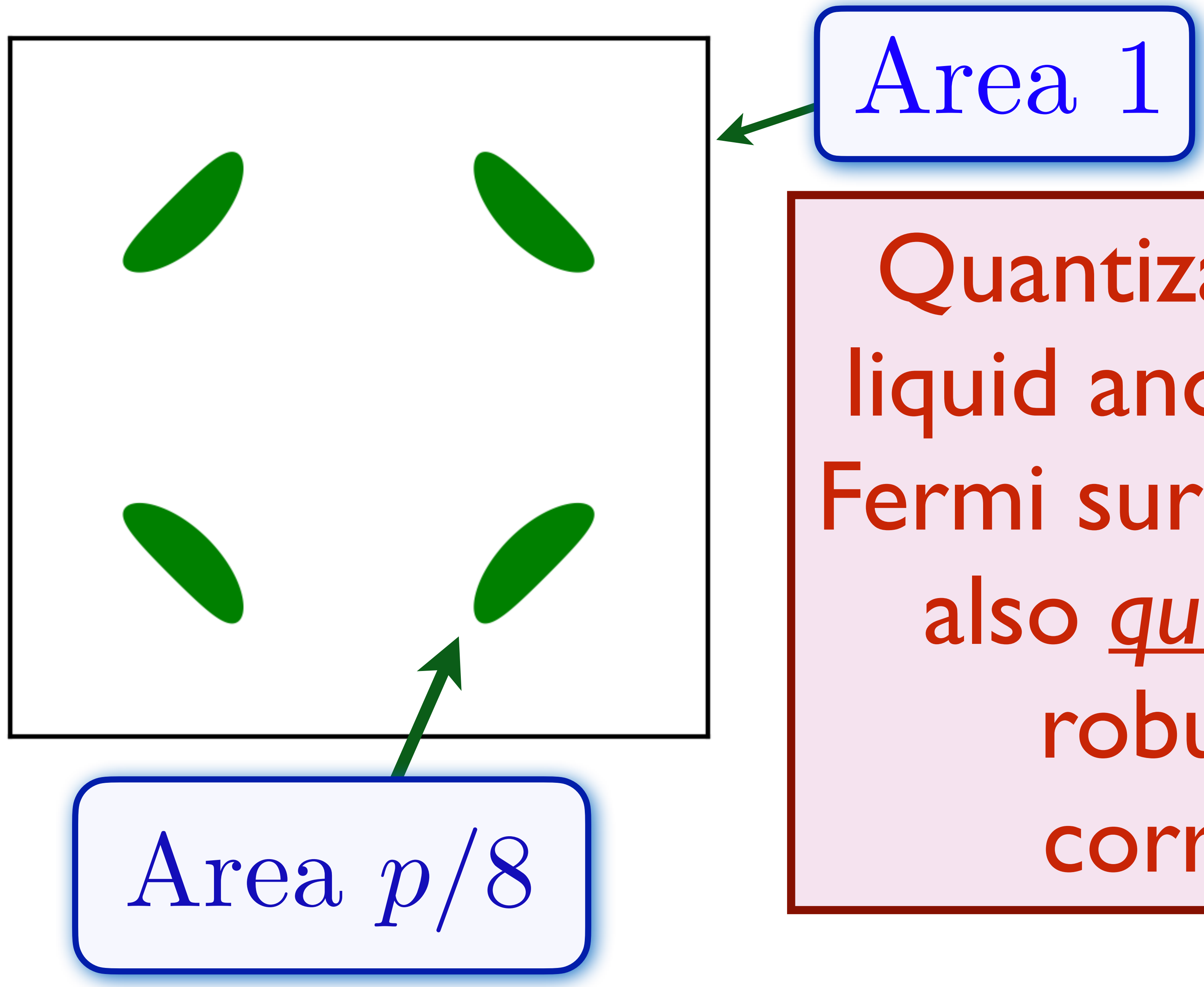


Holon metal

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003);
R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, S. S., PRB **105**, 075146 (2022)



FL*

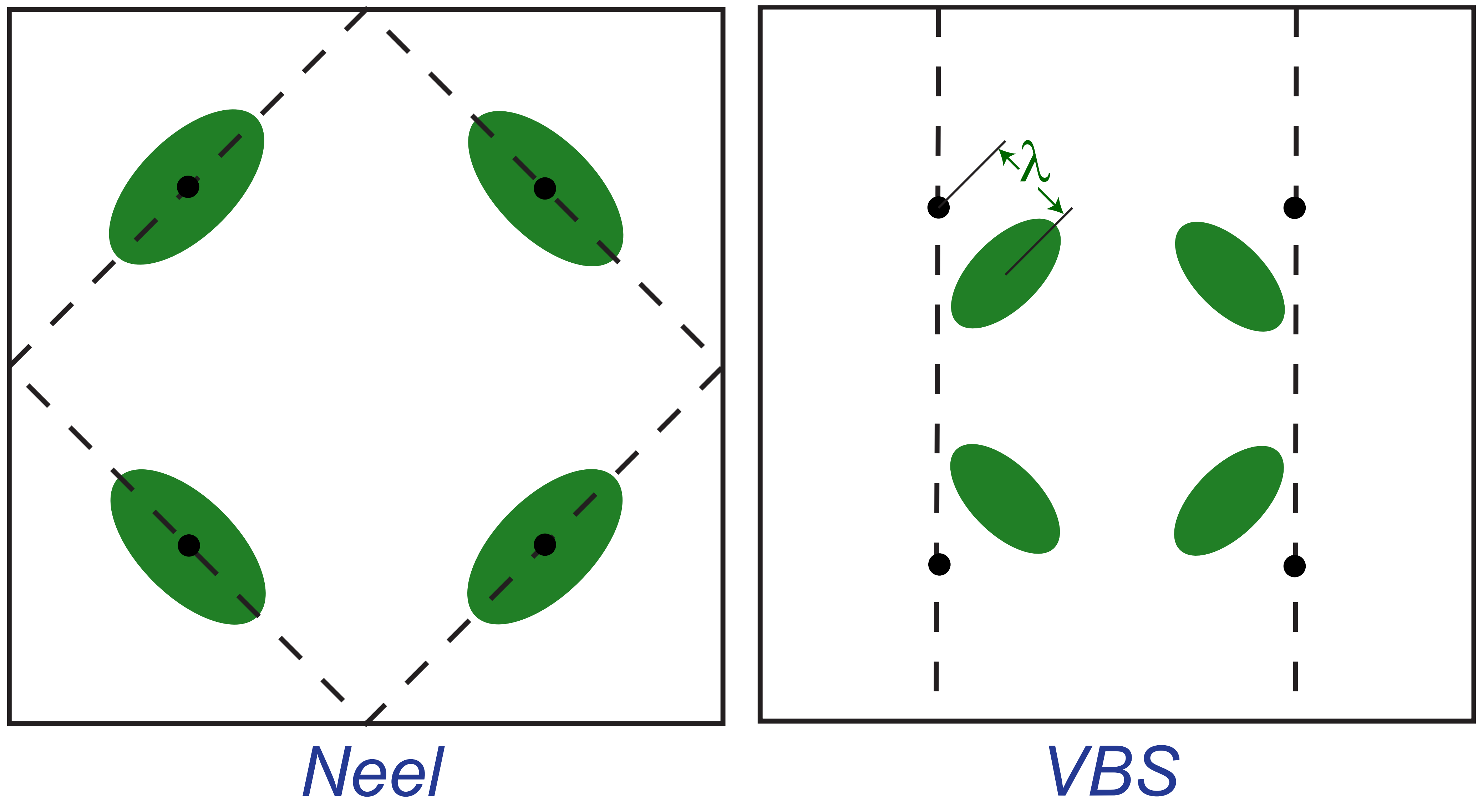


Quantization of spin liquid anomaly implies Fermi surface areas are also *quantized* and robust to all corrections.

PHYSICAL REVIEW B **75**, 235122 (2007)

Hole dynamics in an antiferromagnet across a deconfined quantum critical point

Ribhu K. Kaul,[1] Alexei Kolezhuk,[1,2] Michael Levin,[1] Subir Sachdev,[1] and T. Senthil[3,4]



The dashed line in the Néel phase indicates the boundary of the magnetic Brillouin zone. Only the Fermi surfaces within this zone contribute to the Luttinger counting, and so the area of each ellipse is $\mathcal{A}_F=(2\pi)^2\delta/4$. In the VBS phase, all four pockets are inequivalent, and so the area of each ellipse is $\mathcal{A}_F=(2\pi)^2\delta/8$.

Factor of 2 between SDW fluctuation and FL*

Observation of the Yamaji effect in the cuprate pseudogap

See also:

Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, *Nature Physics* **18**, 558 (2022)

Angle-dependent magnetoresistance (ADMR) of $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$

Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan[1], Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

nature physics
21, 1753 (2025)
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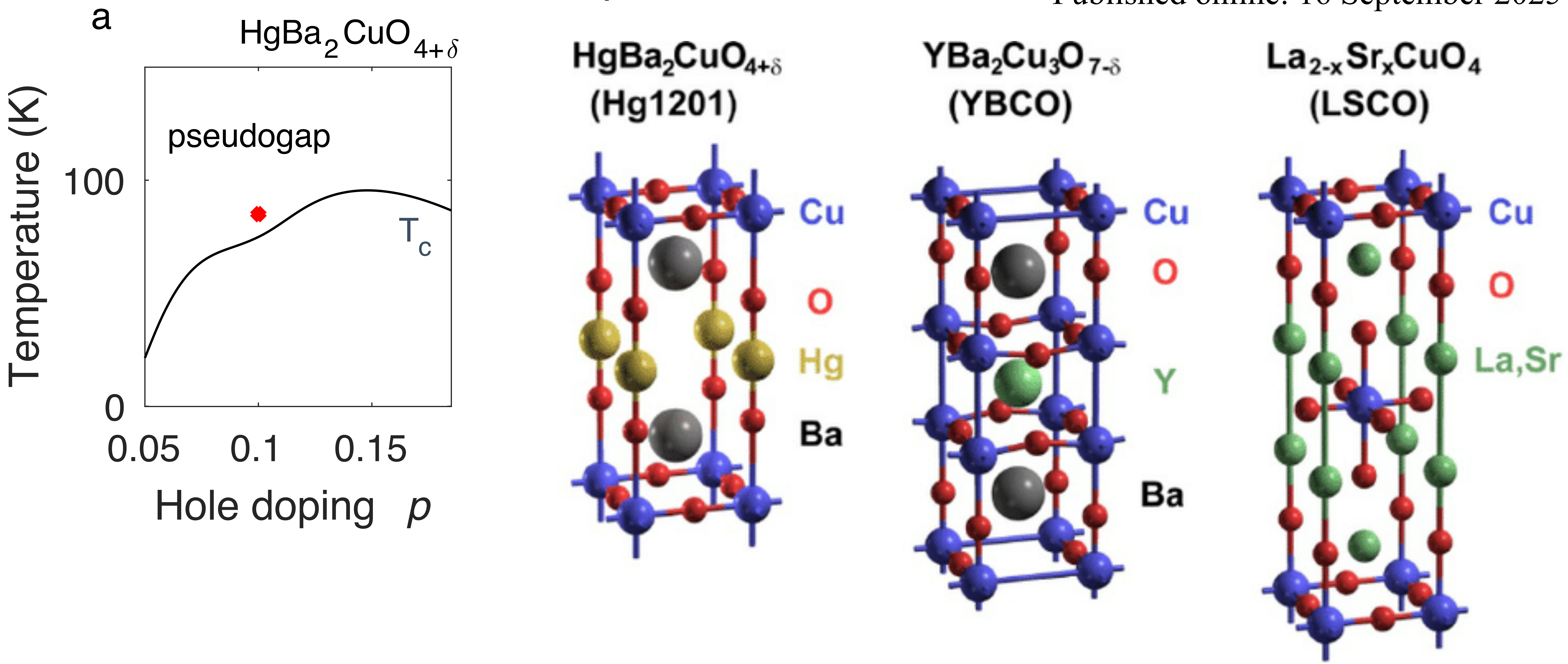

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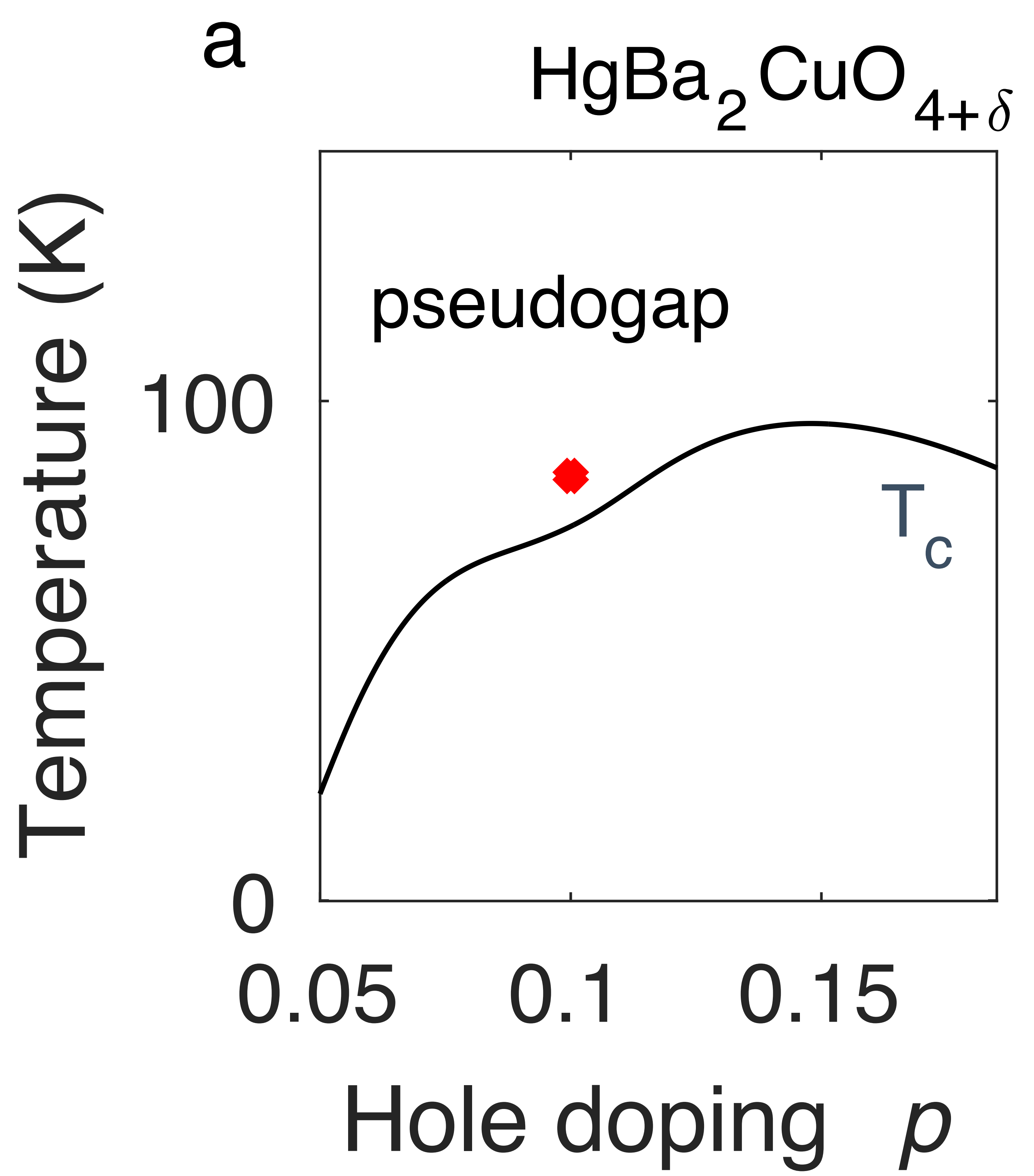

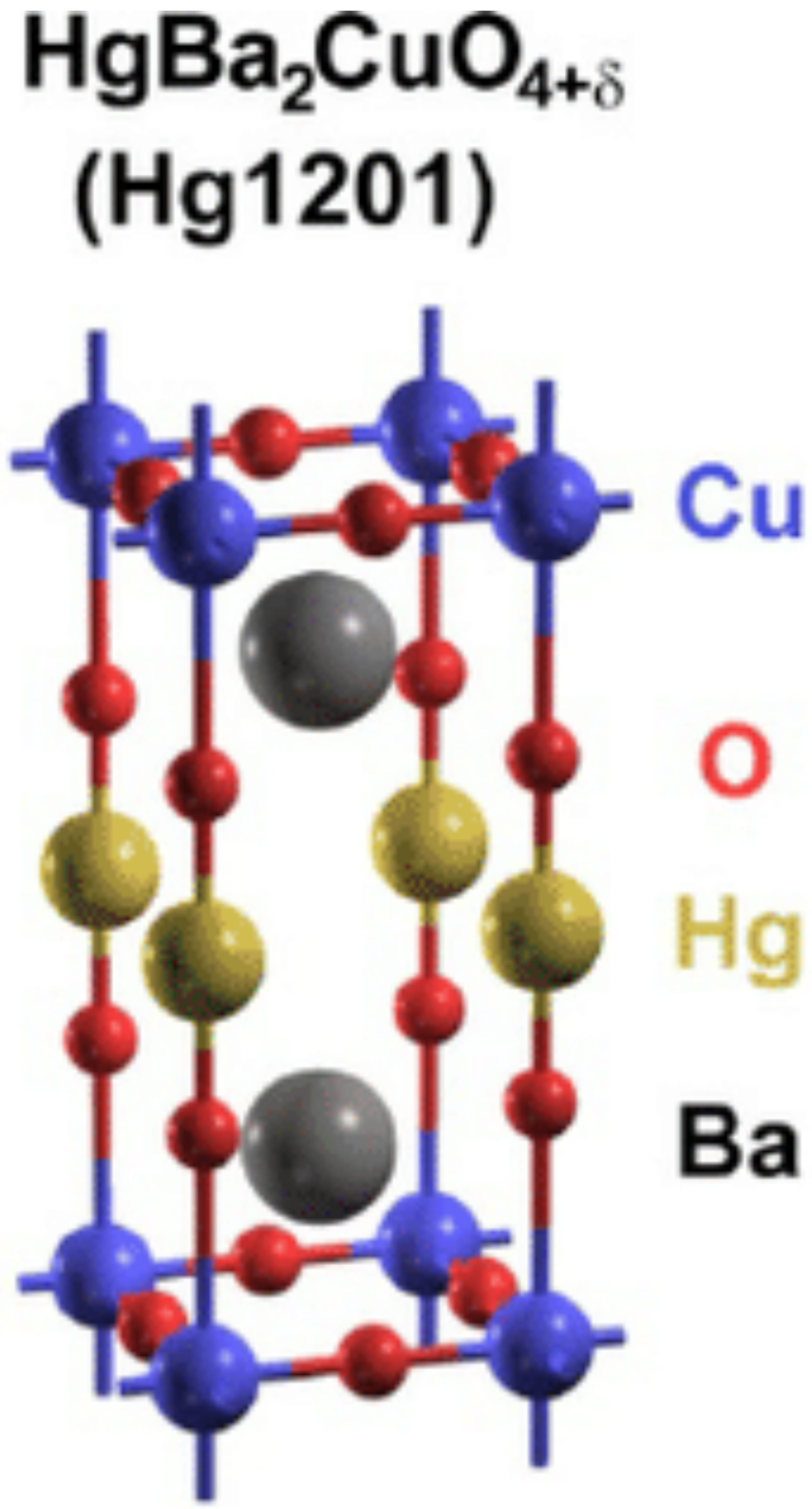

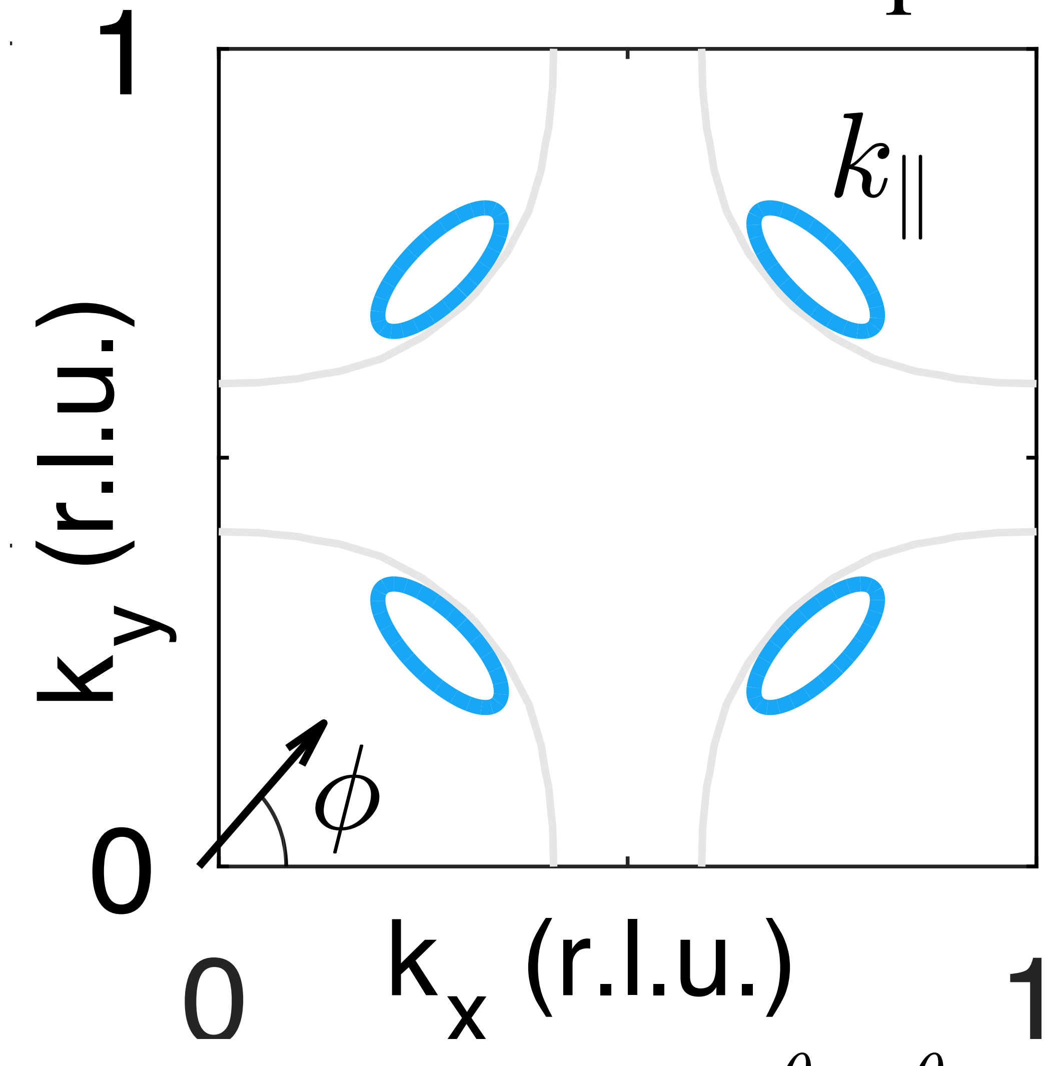

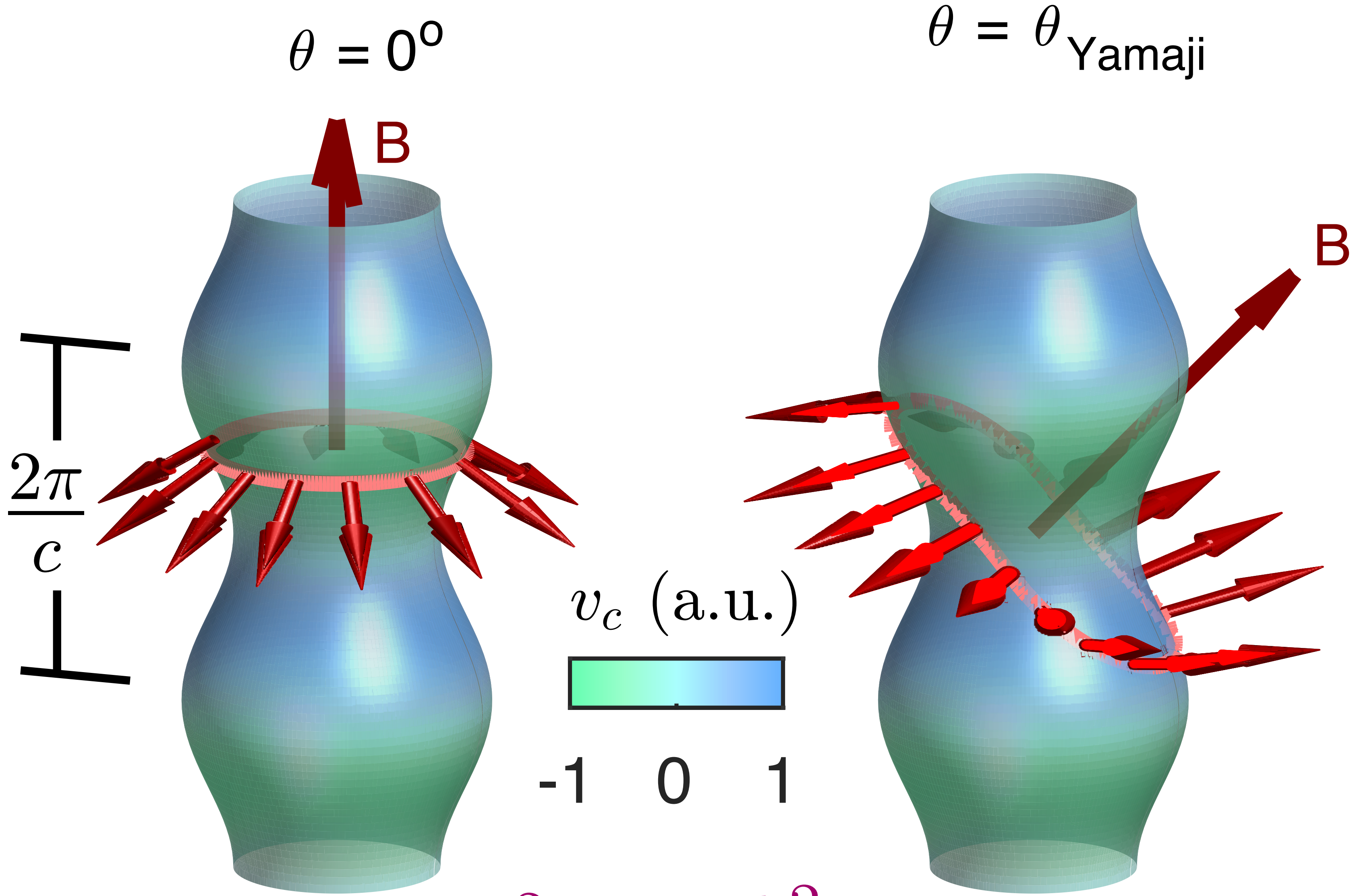

At the Yamaji angle, the orbits in the plane orthogonal to $\boldsymbol{B}$ have an area which is independent of momentum in the c direction, to first order in the hopping along the c direction.

$$\epsilon(\mathbf{k}) = \frac{k_x^2}{2m_1} + \frac{k_y^2}{2m_2} - 2t_\perp \cos(k_z c)$$

K.Yamaji JPSJ **58**, 1520 (1989)

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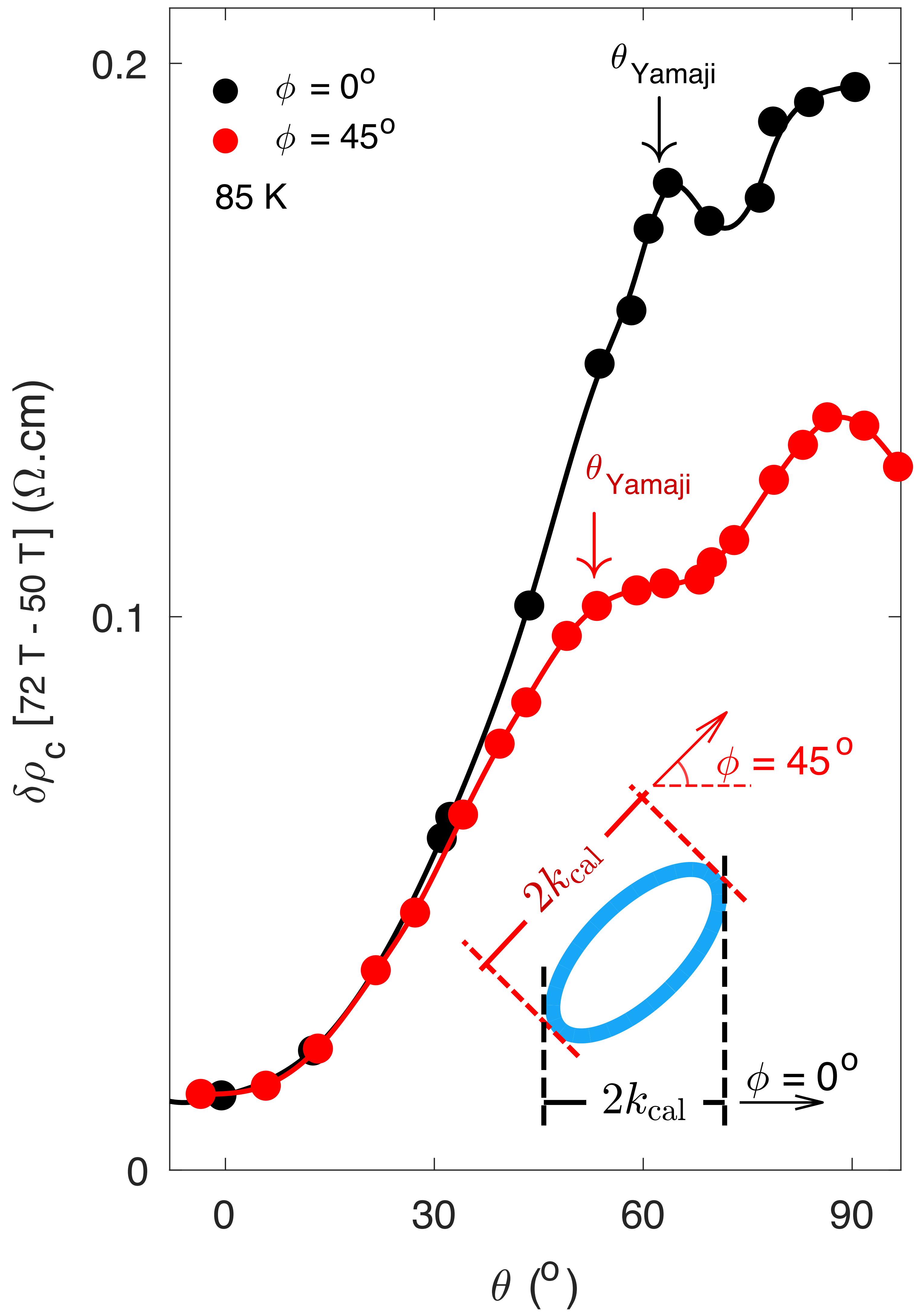

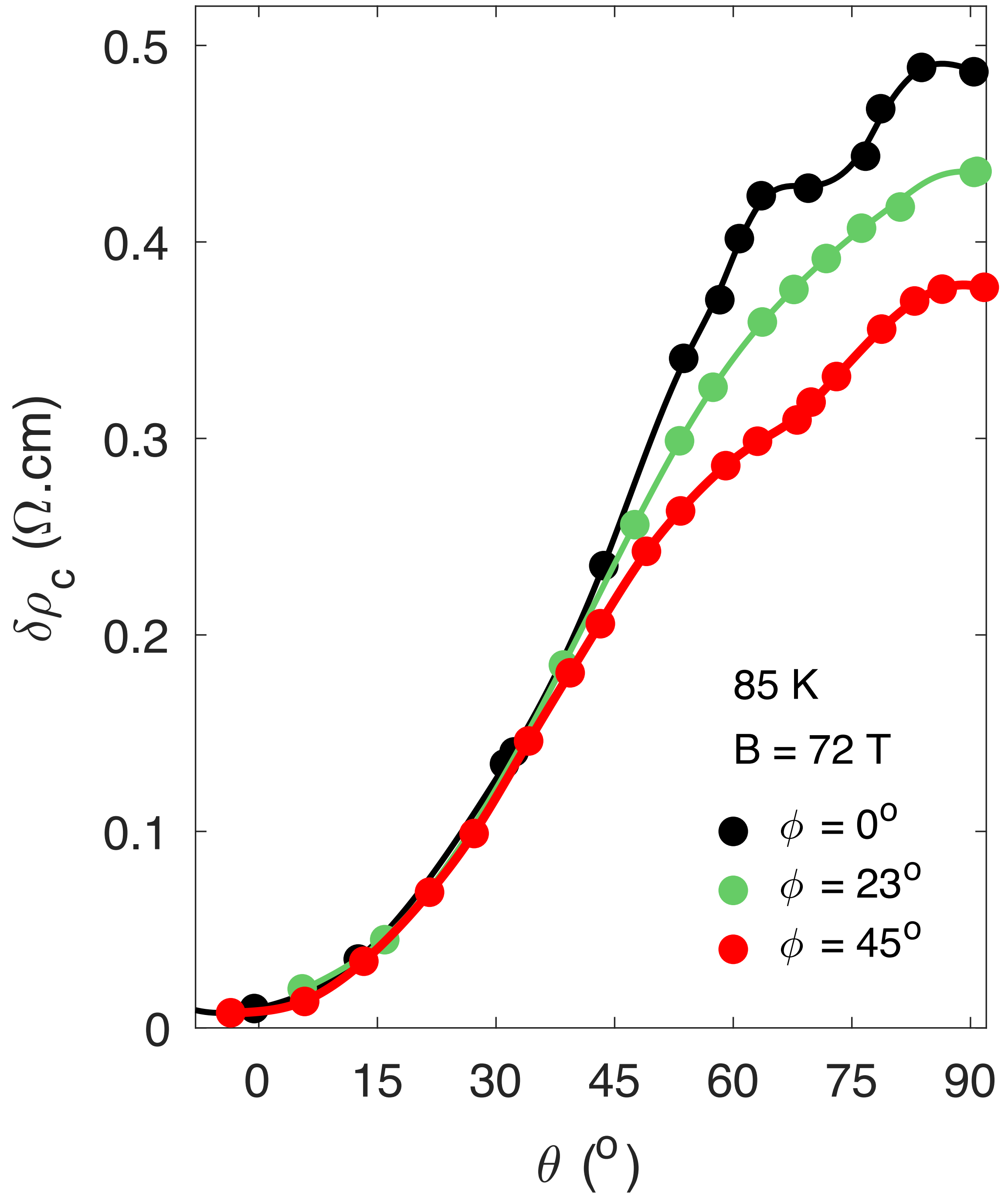

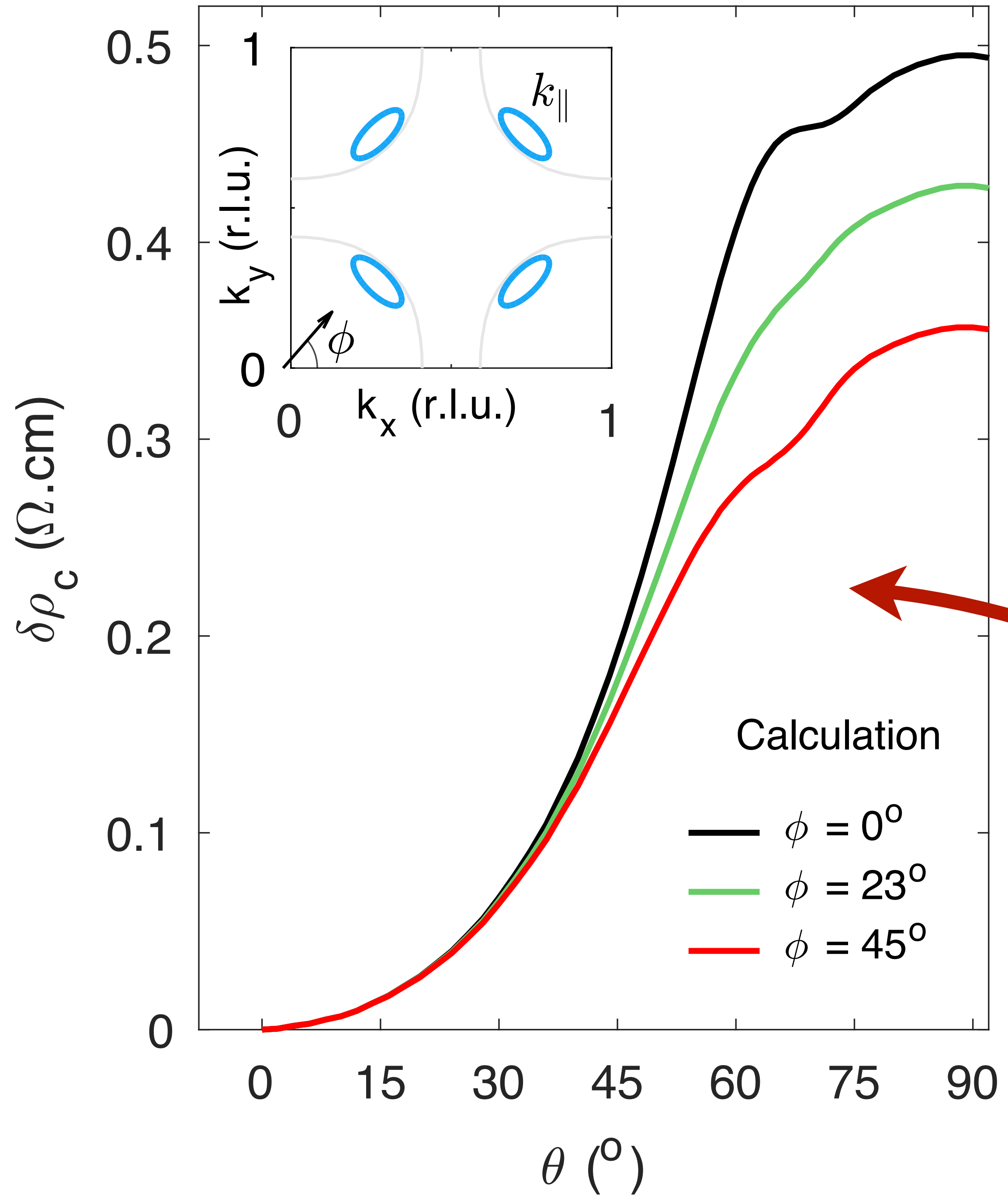

Doping
$p = 0.1$

The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase.

$$\frac{\partial f}{\partial t} + e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \mathbf{v} \left(-\frac{\partial f}{\partial \epsilon} \right) = -\frac{f - f_0}{\tau}$$

$$\mathbf{v} = \nabla_{\mathbf{k}} \epsilon(\mathbf{k}) \; ; \; f_0(\epsilon) = \frac{1}{e^{(\epsilon - \mu)/T} + 1}$$

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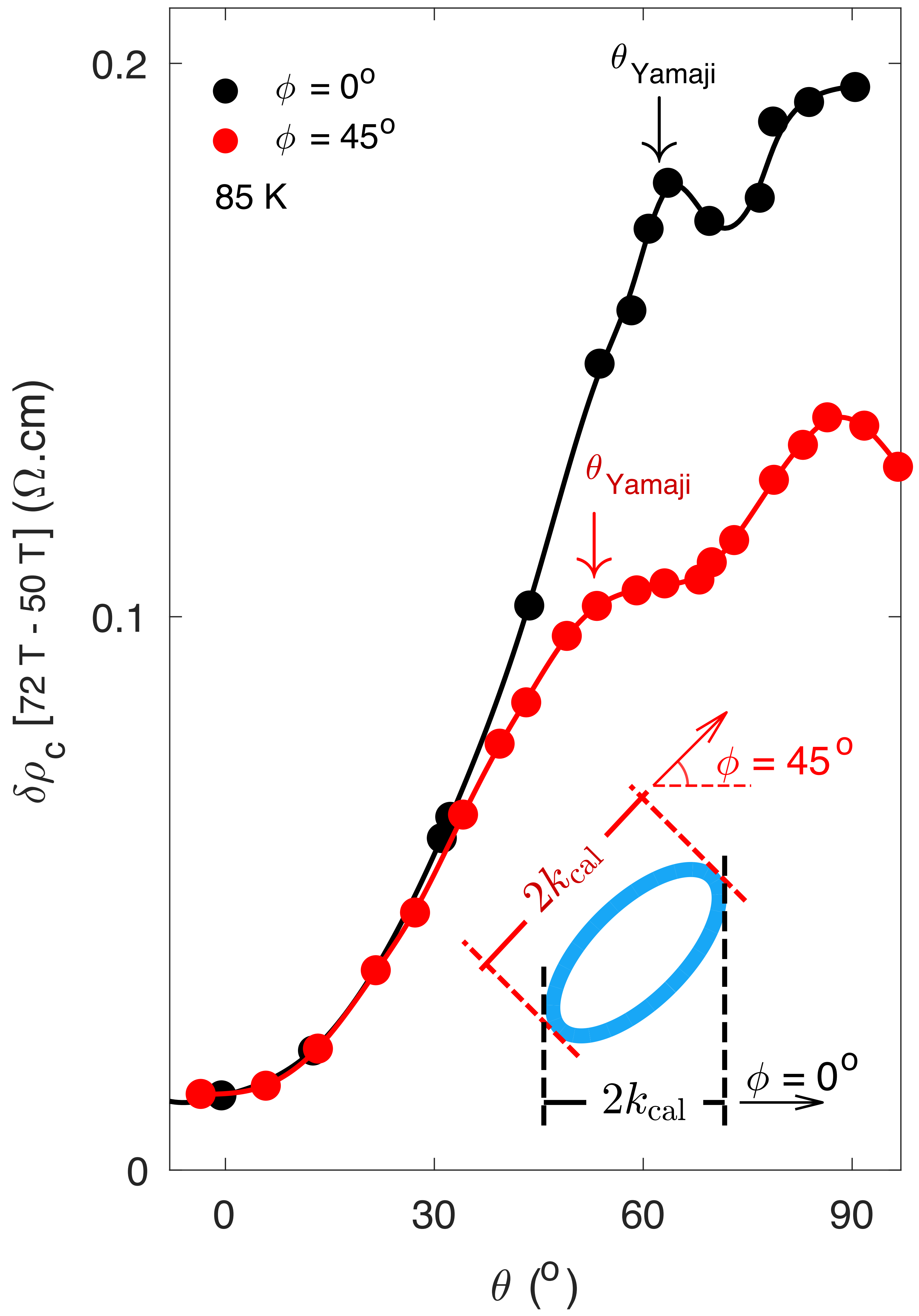

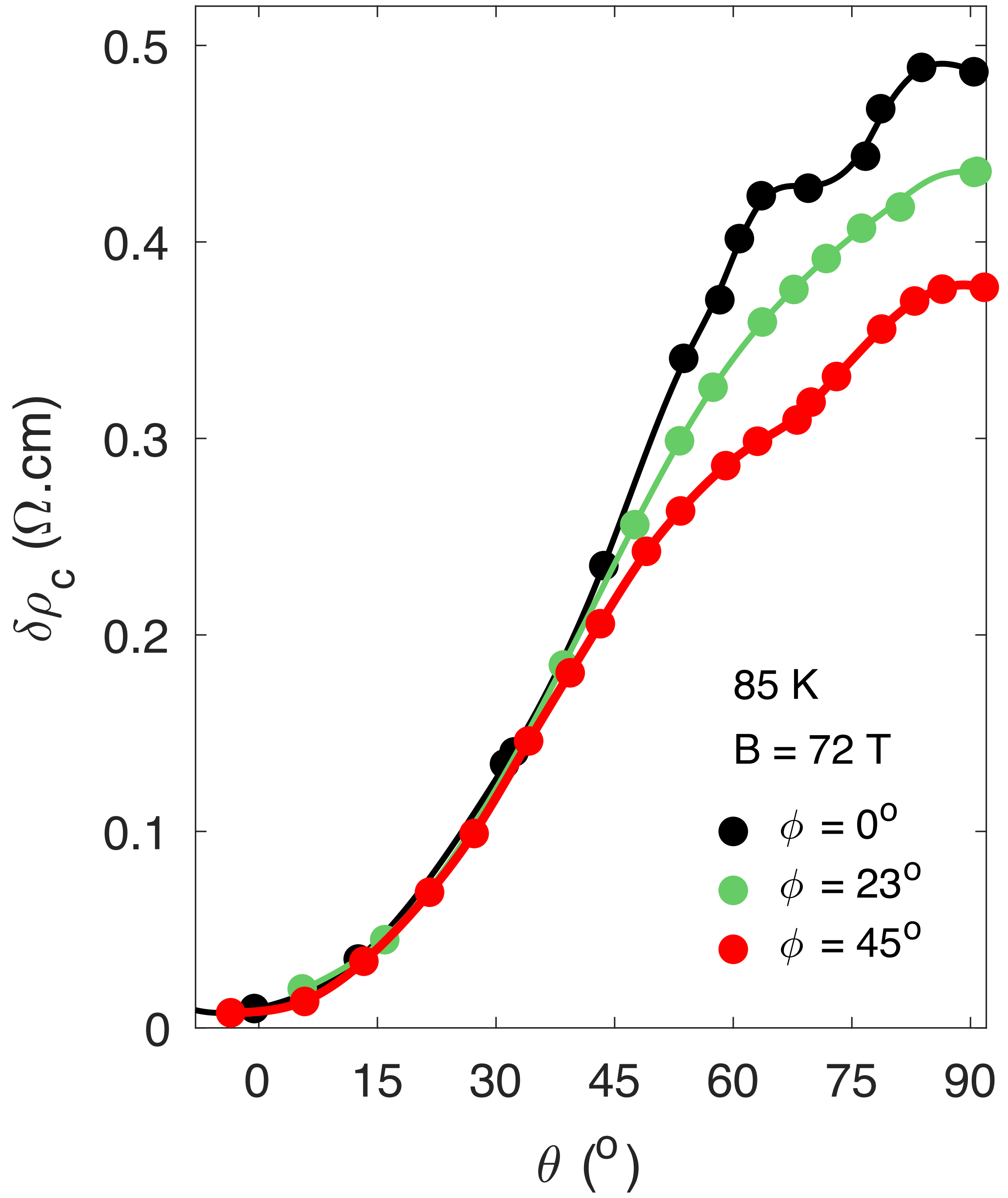

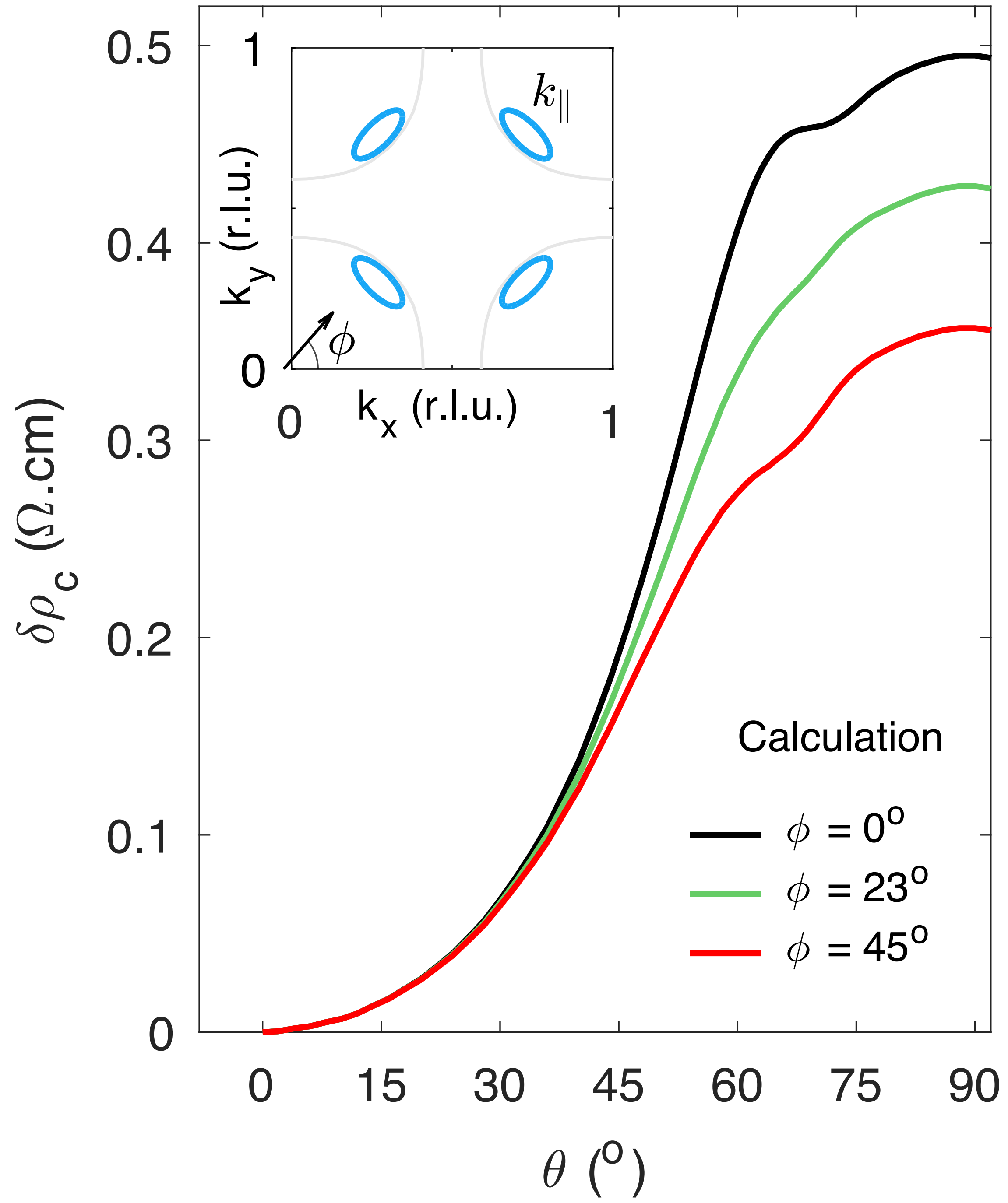

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Excellent evidence for hole pockets with coherent interlayer-transport.

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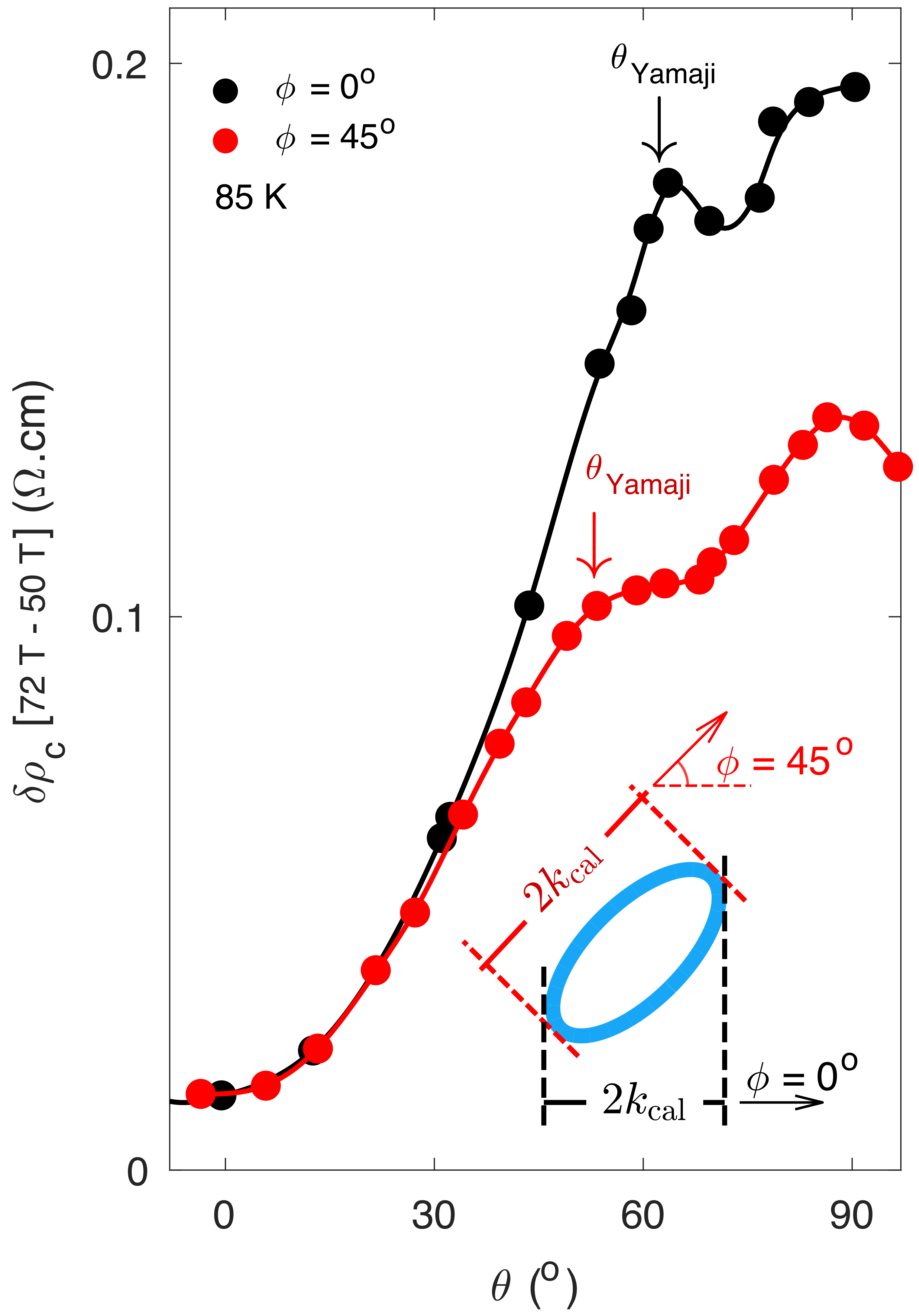

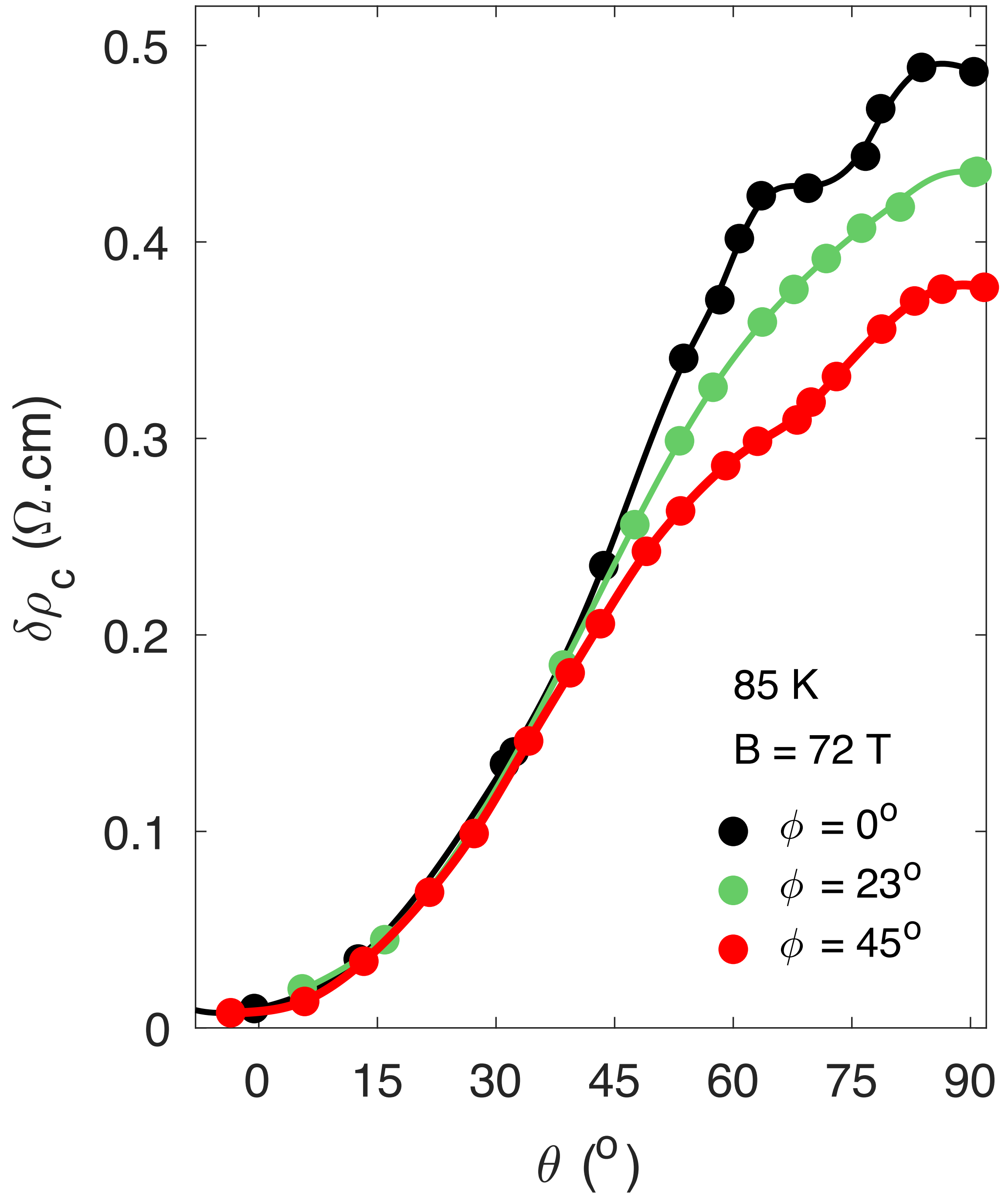

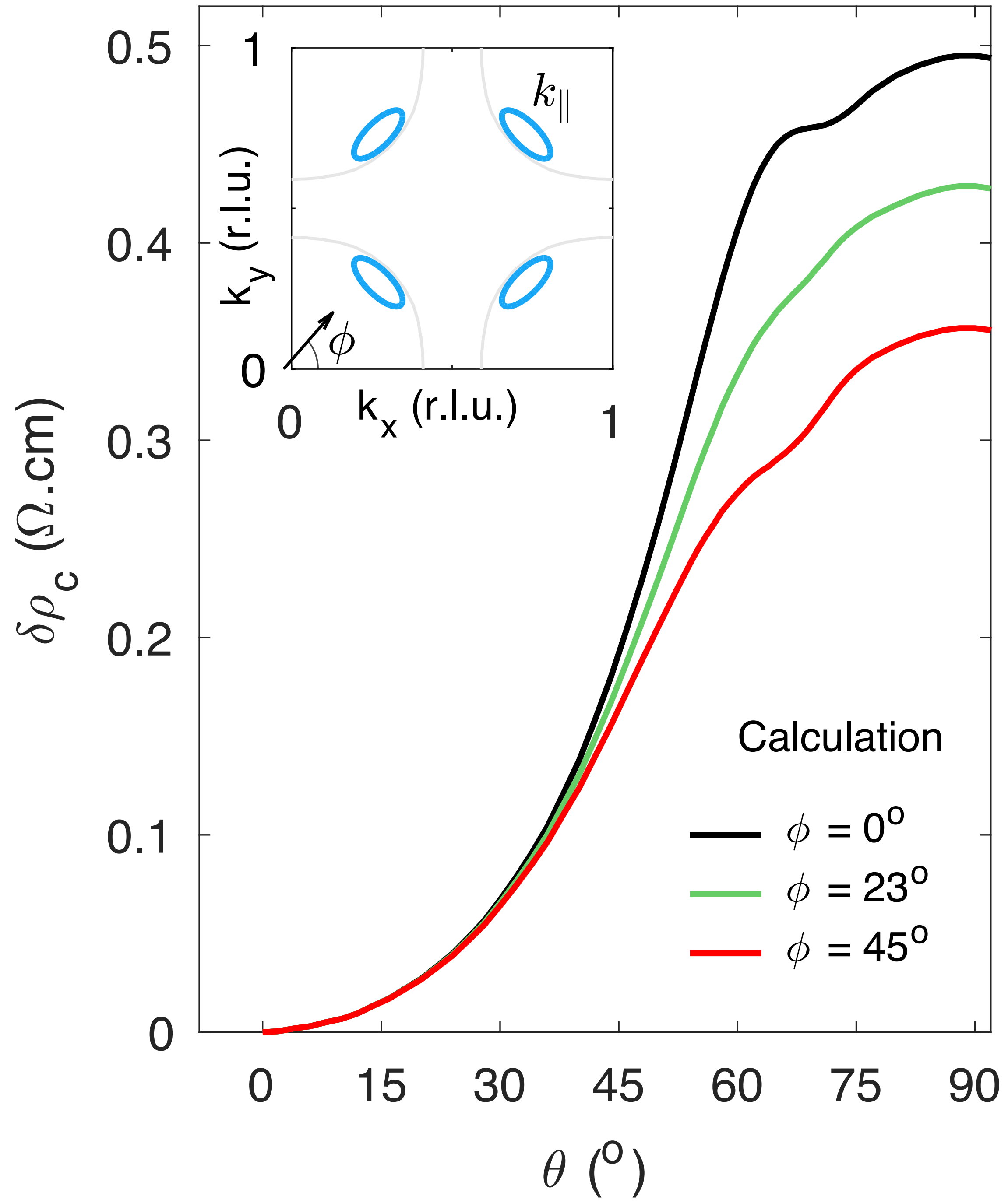

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Rules out holon metal

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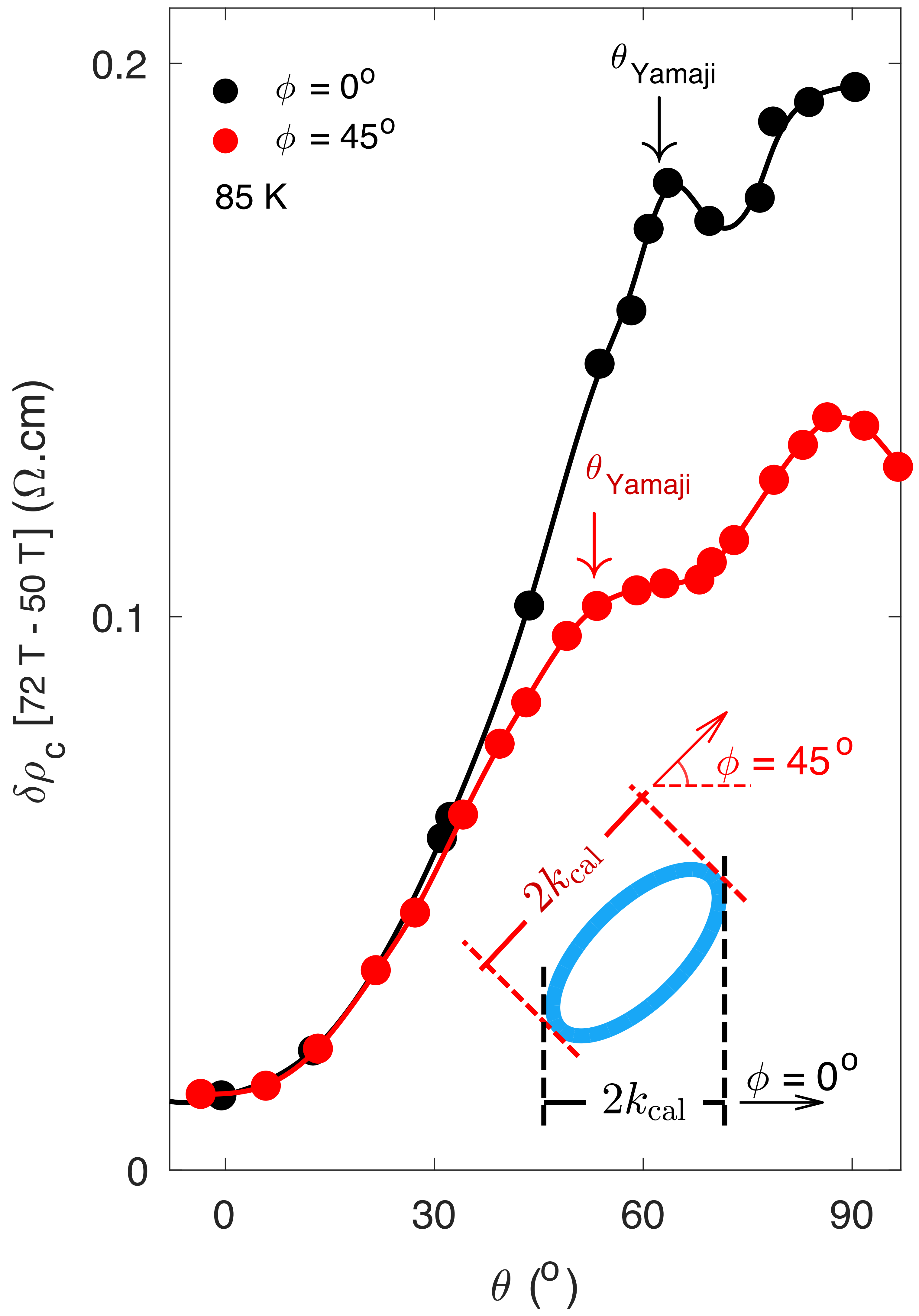

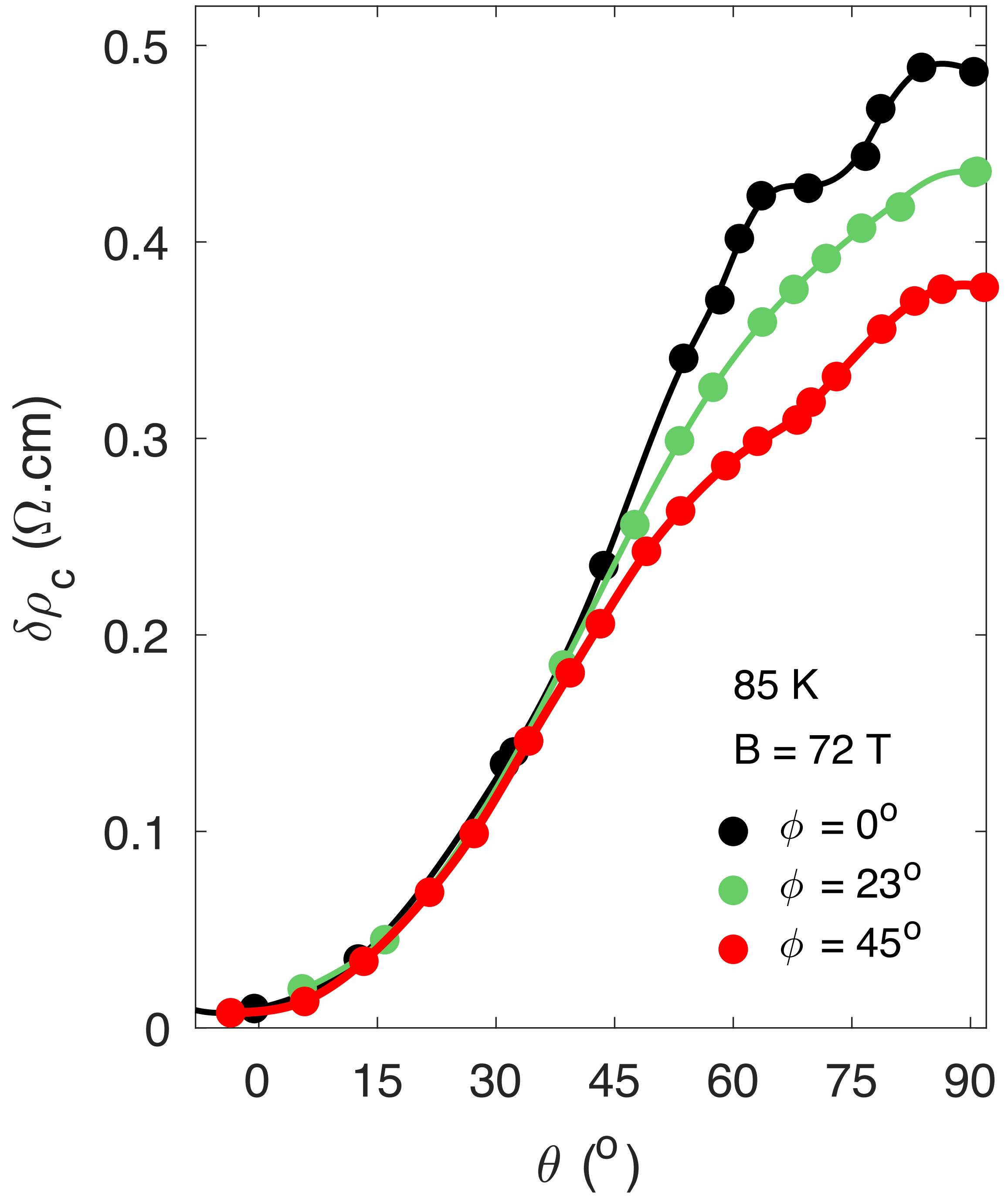

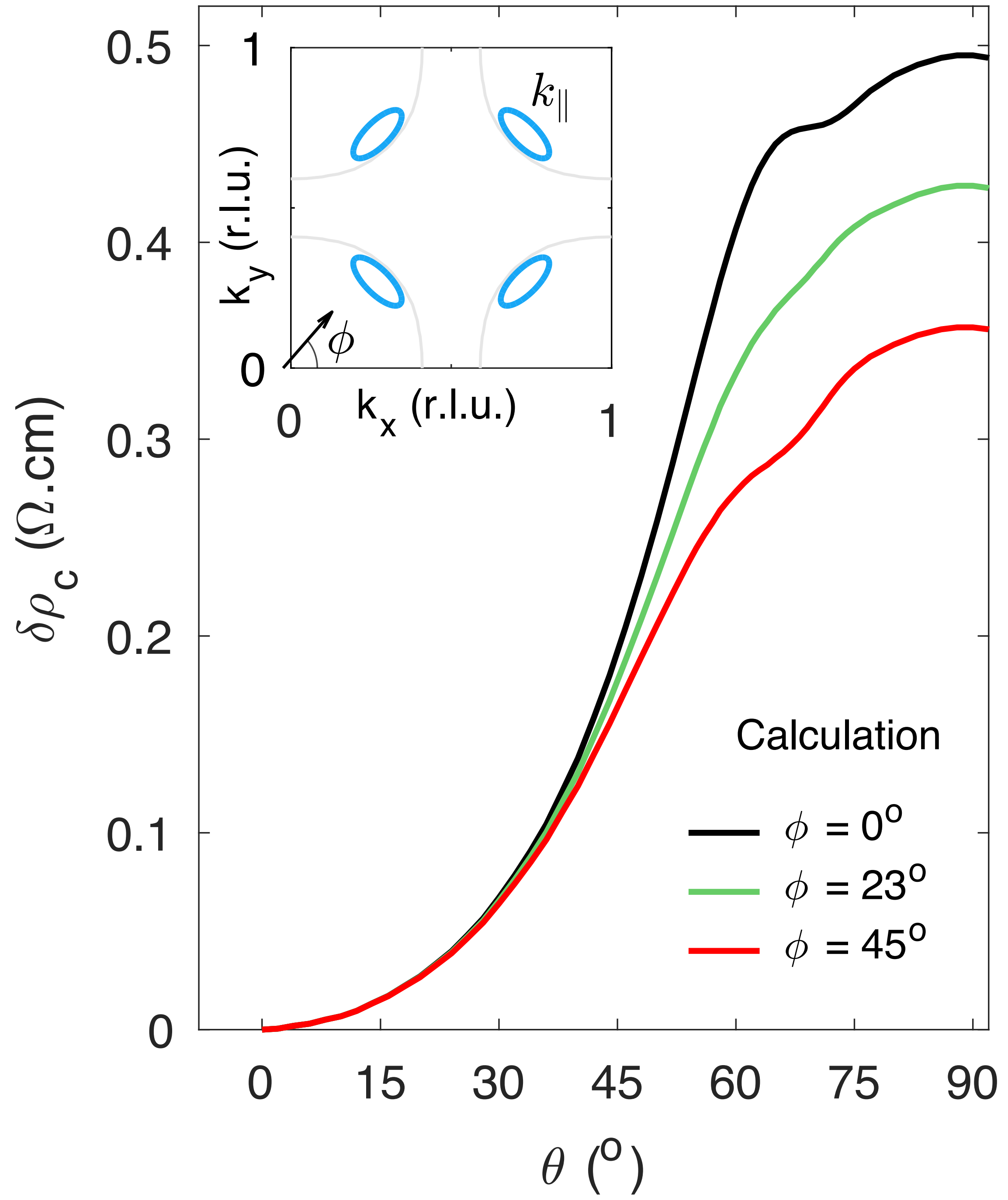

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Rules out holon metal
and possibly SDW metal

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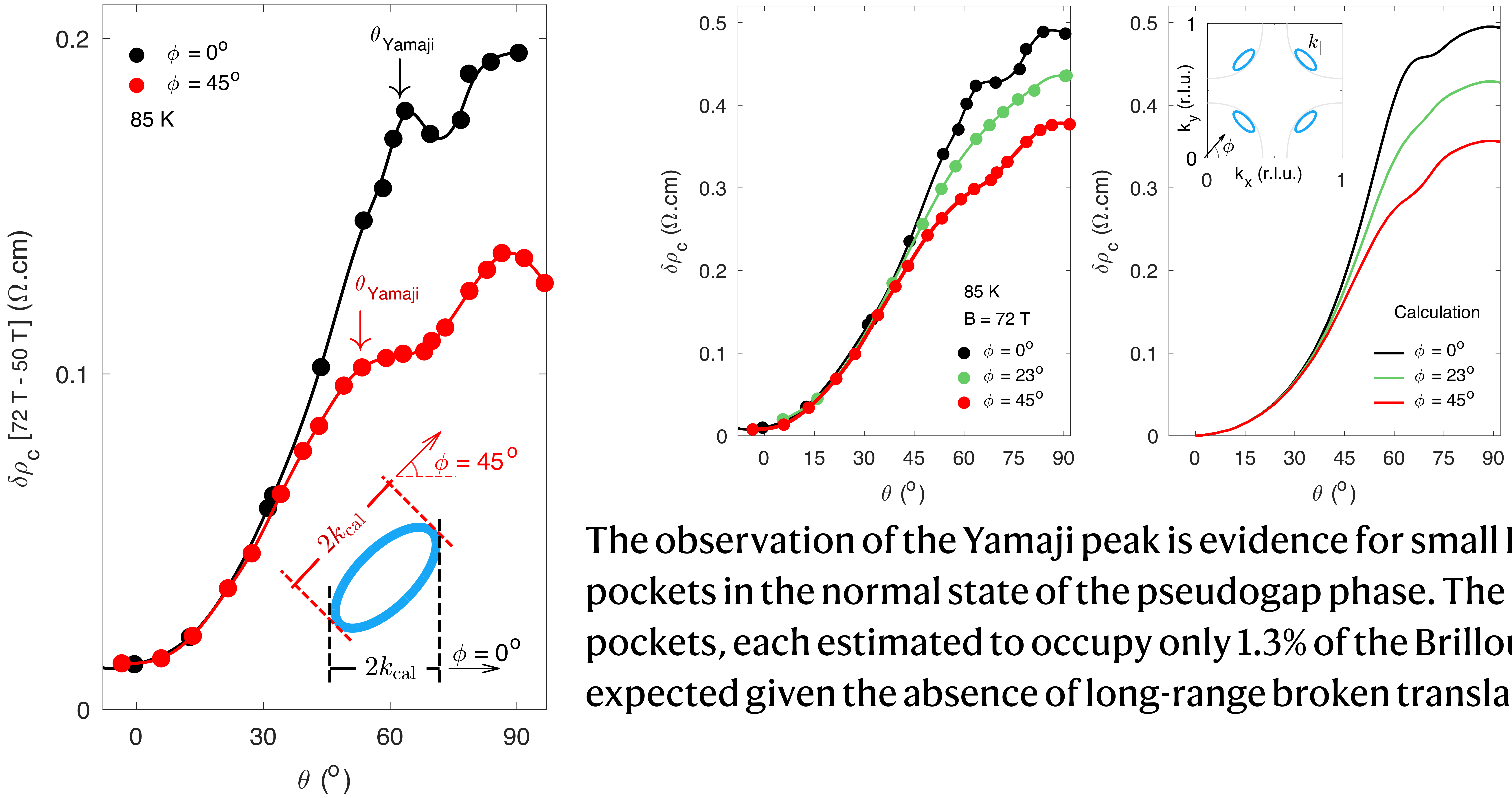

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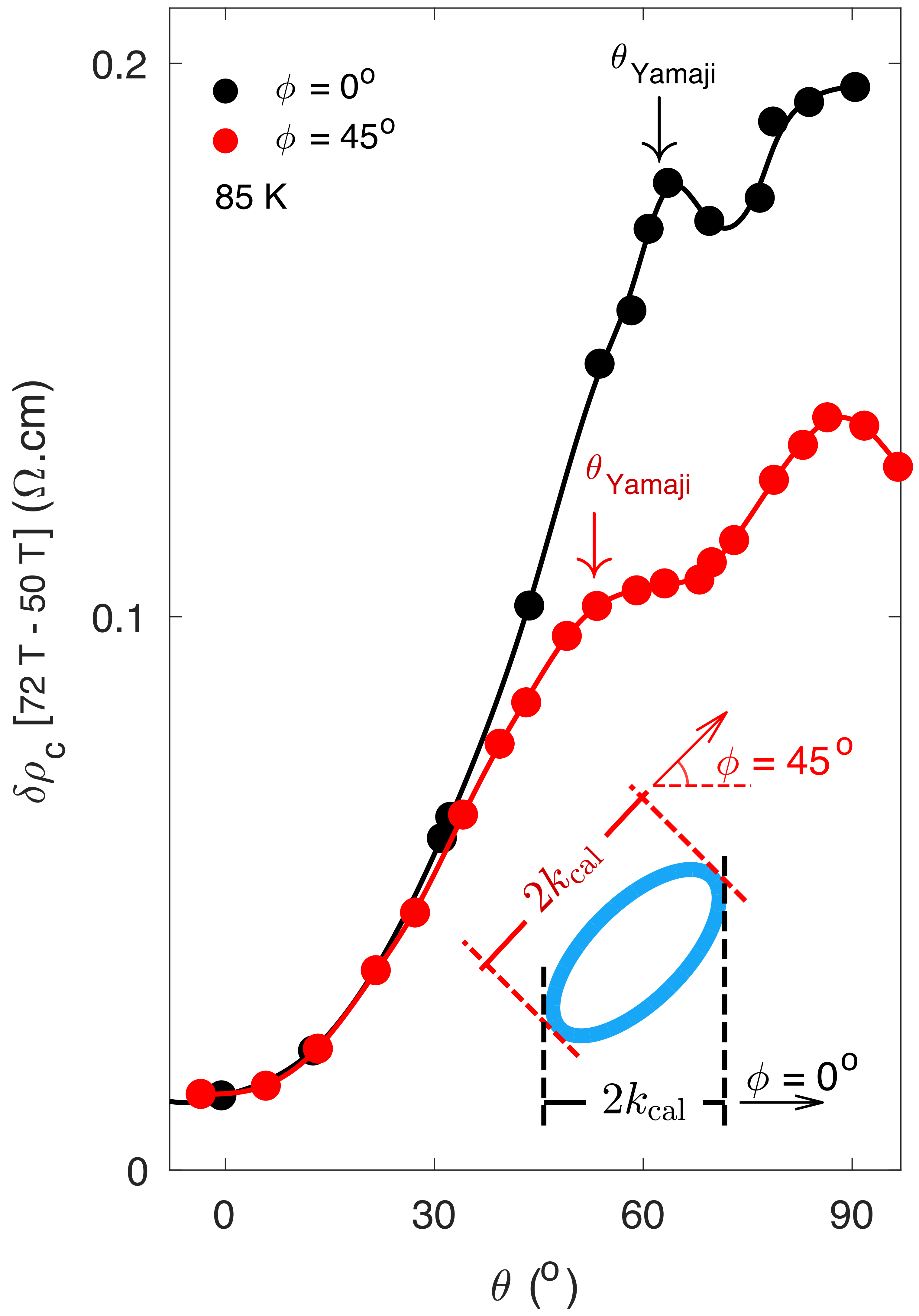

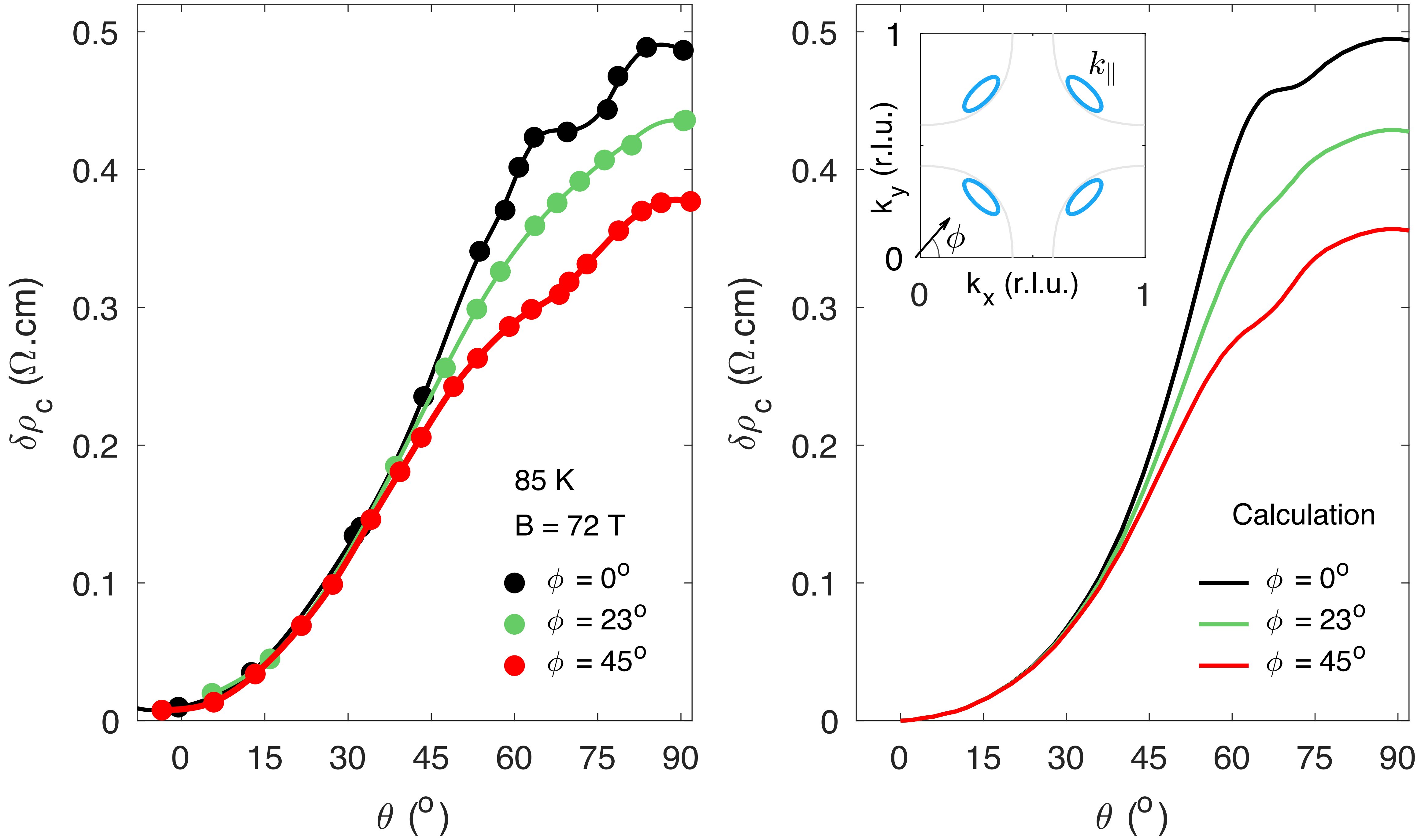

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(was expected by us!)

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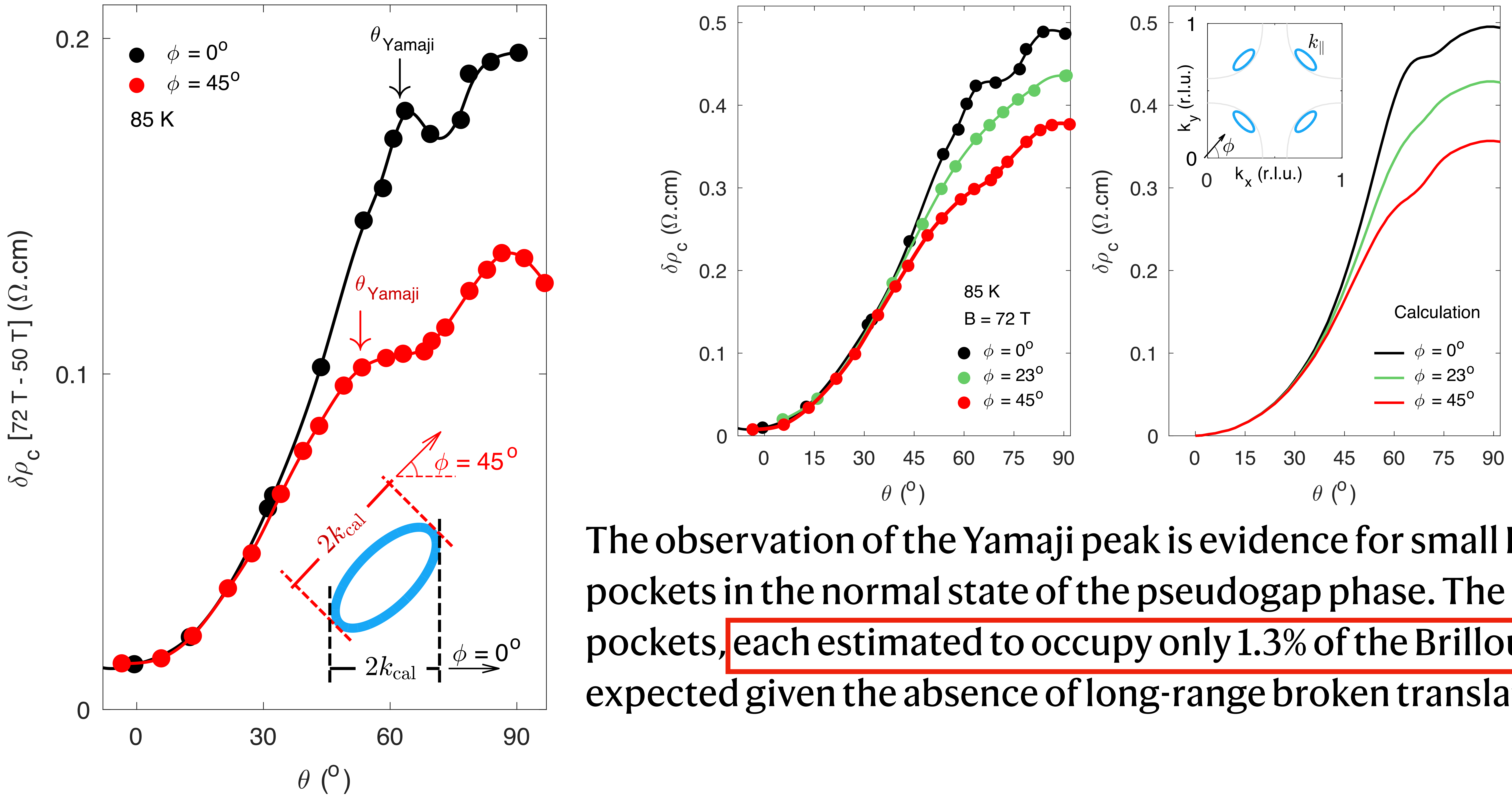

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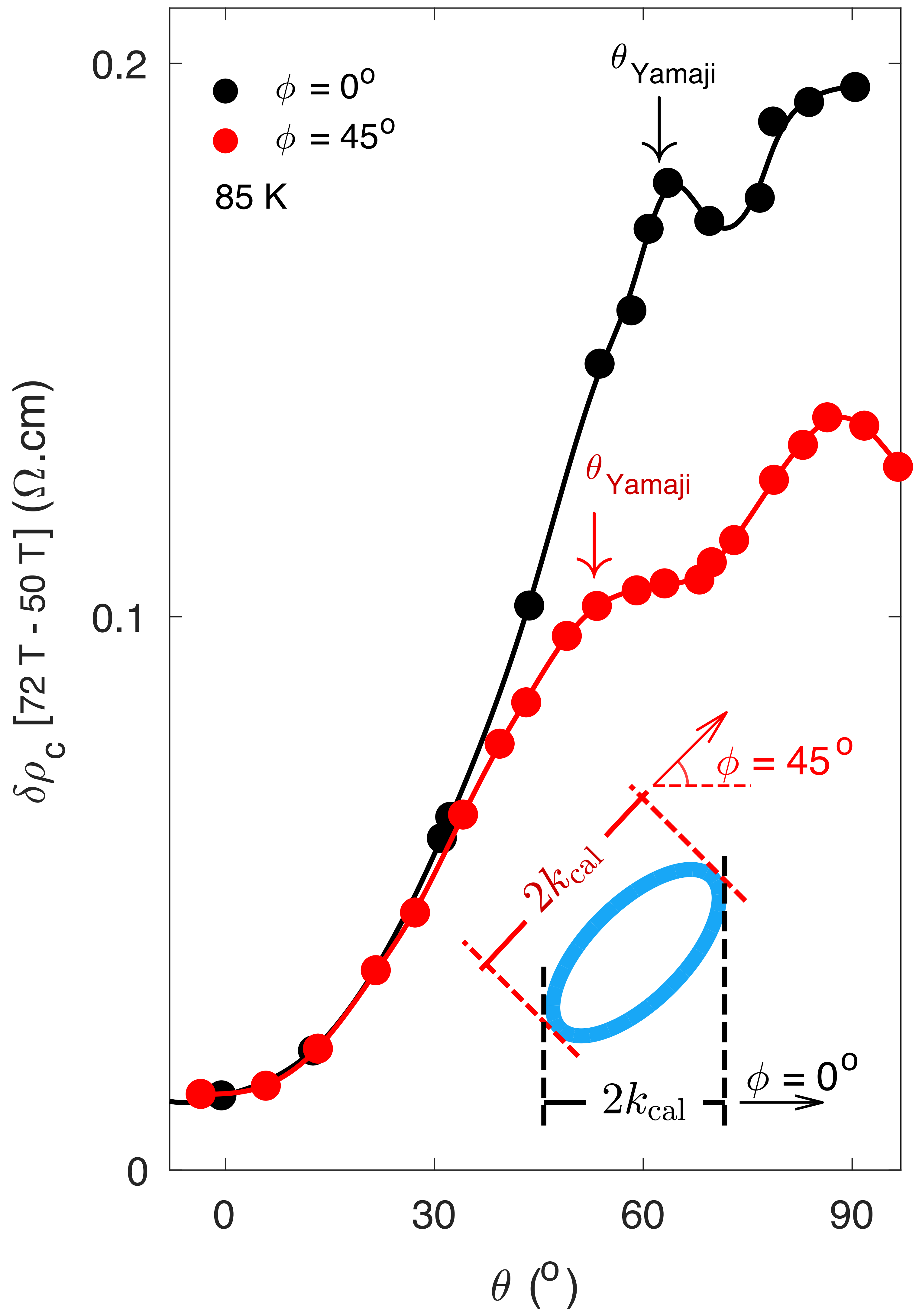

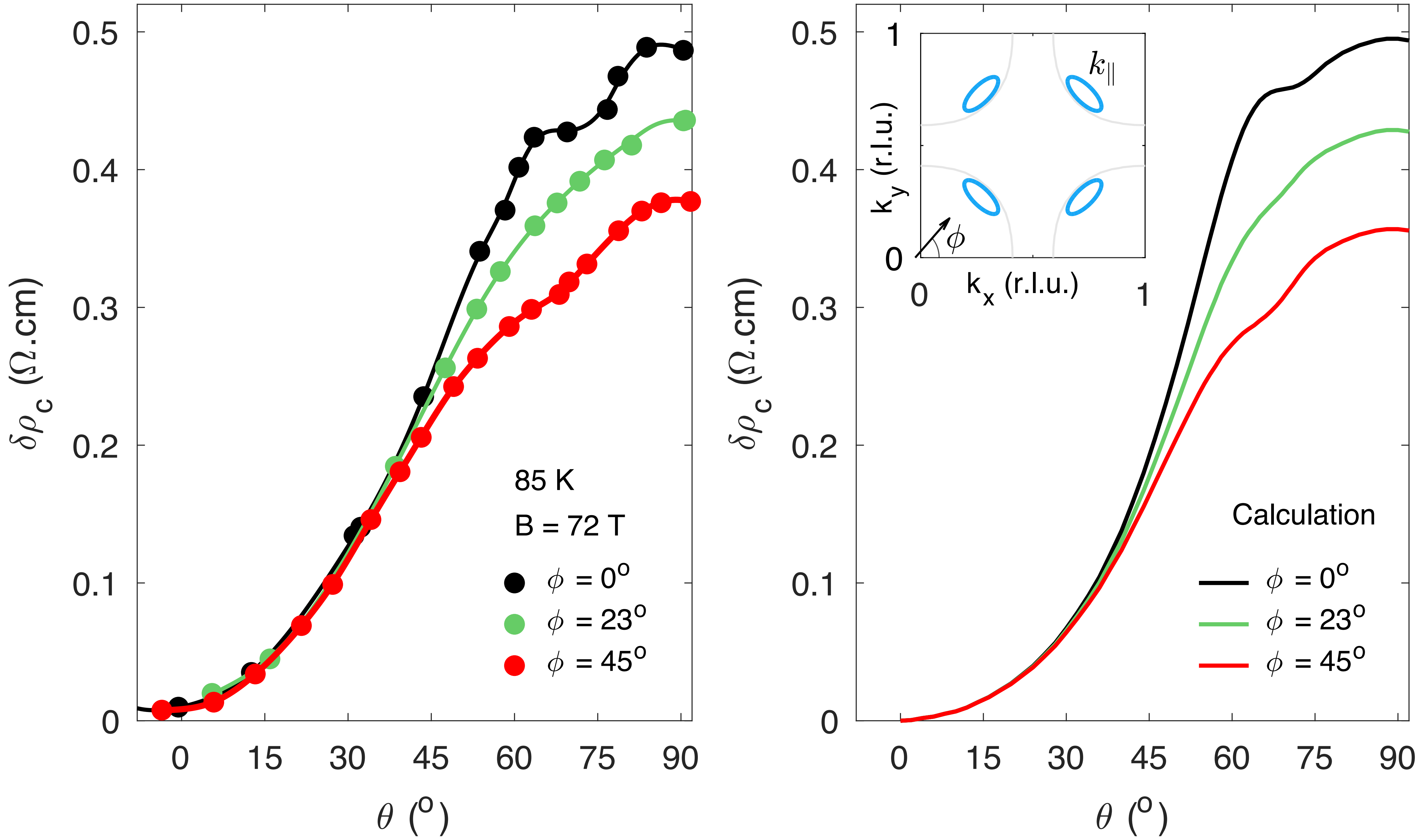

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Predicted FL* pocket fraction $= p/8 = 1.25\%$!

Fluctuating AF metal fraction $= p/4 = 2.5\%$.

(p/8 also in Yang-Rice-Zjang ansatz, Peter Johnson photoemission, and Jenny Hoffman and Seamus Davis STMs; Stanescu-Kotliar)

Jing-Yu Zhao, S. Chatterjee, S. S., Ya-Hui Zhang, arXiv:2510.13943

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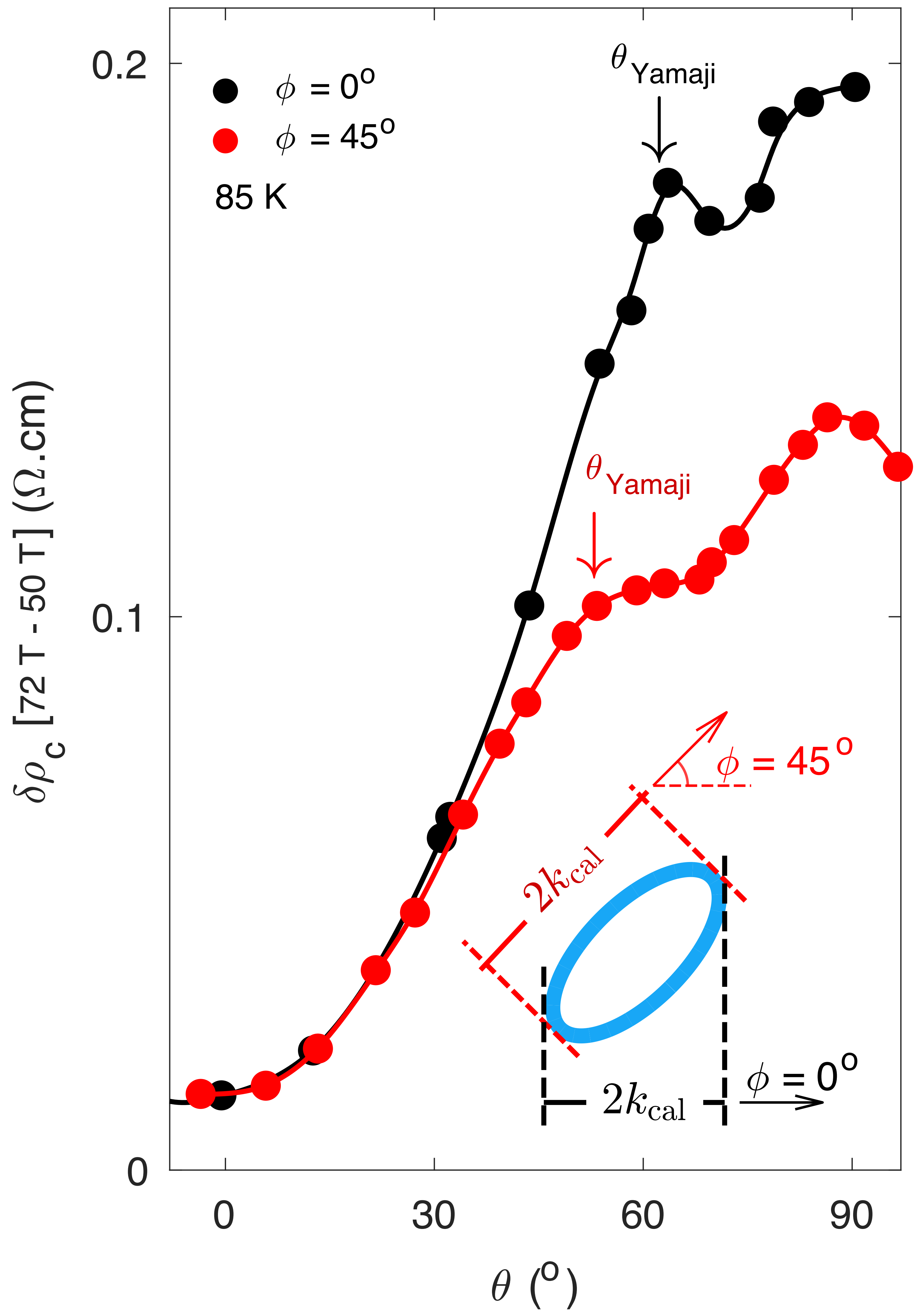

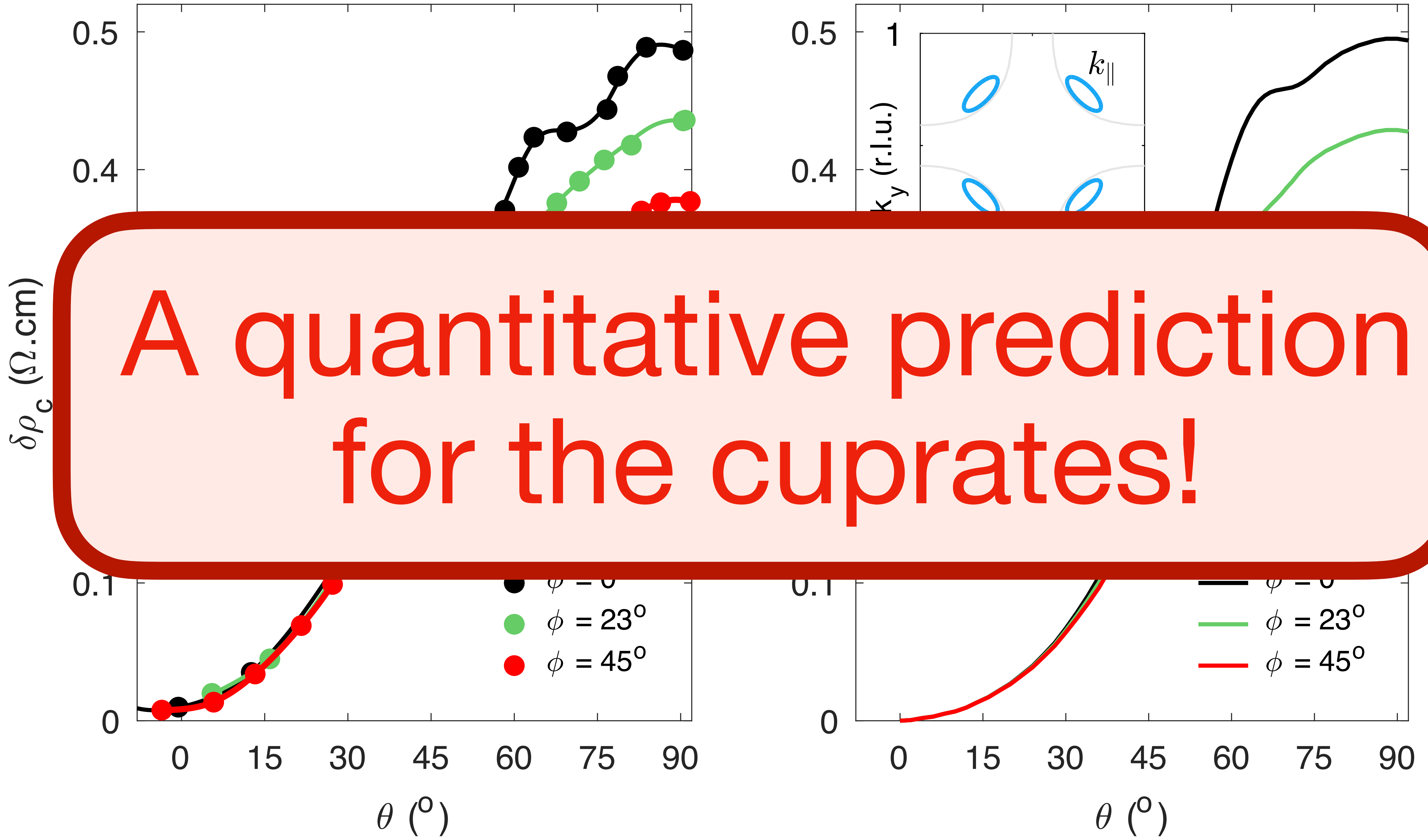

A quantitative prediction for the cuprates!

Doping
$p = 0.1$

The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase. The small size of the pockets, each estimated to occupy only 1.3% of the Brillouin zone area, is not expected given the absence of long-range broken translational symmetry.

Predicted FL* pocket fraction $= p/8 = 1.25\%$!

Fluctuating AF metal fraction $= p/4 = 2.5\%$.

(p/8 also in Yang-Rice-Zhang ansatz, Peter Johnson photoemission, and Jenny Hoffman and Seamus Davis STMs; Stanescu-Kotliar)

Jing-Yu Zhao, S. Chatterjee, S. S., Ya-Hui Zhang, arXiv:2510.13943

Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan[1], Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

nature physics
21, 1753 (2025)
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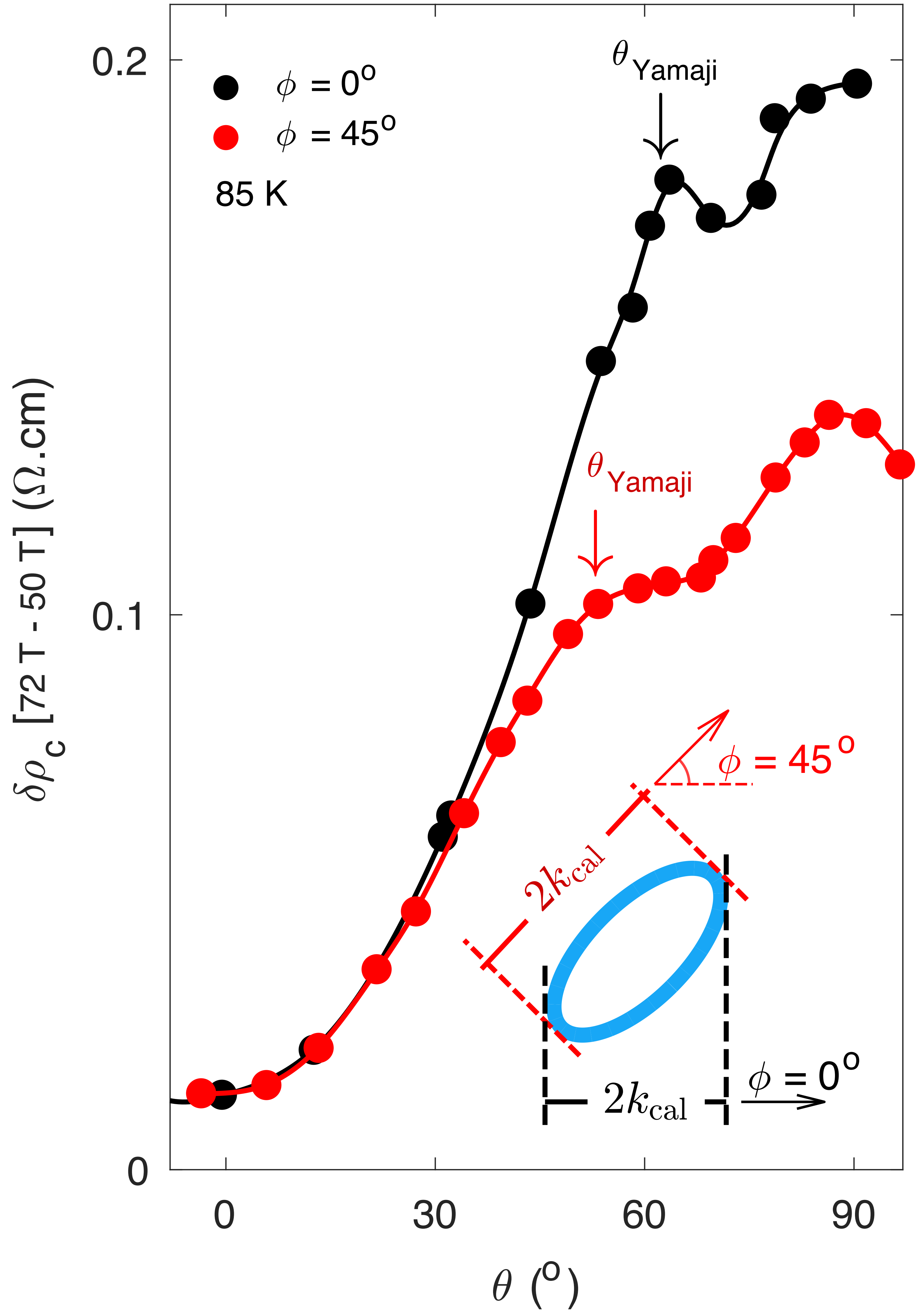

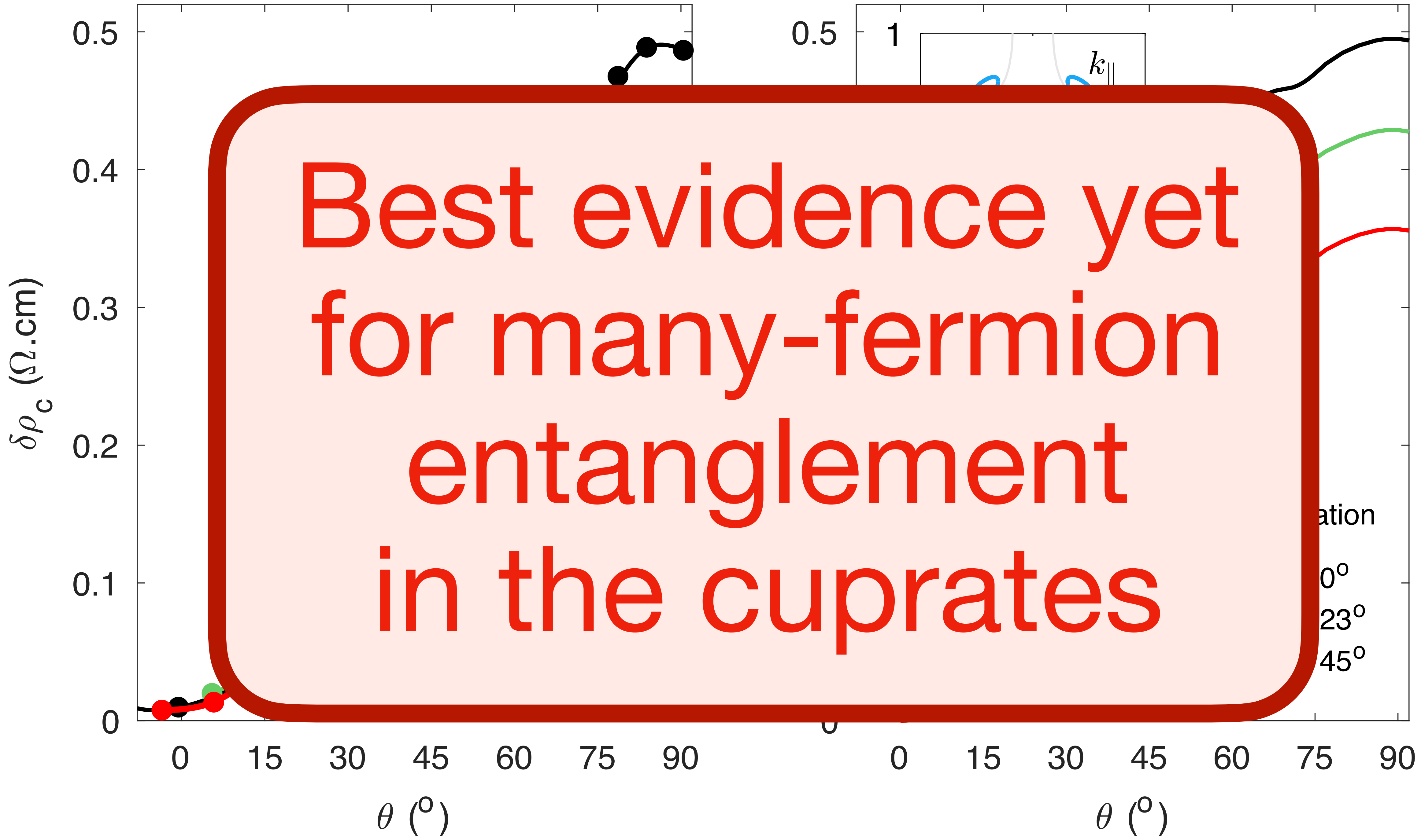

Best evidence yet for many-fermion entanglement in the cuprates

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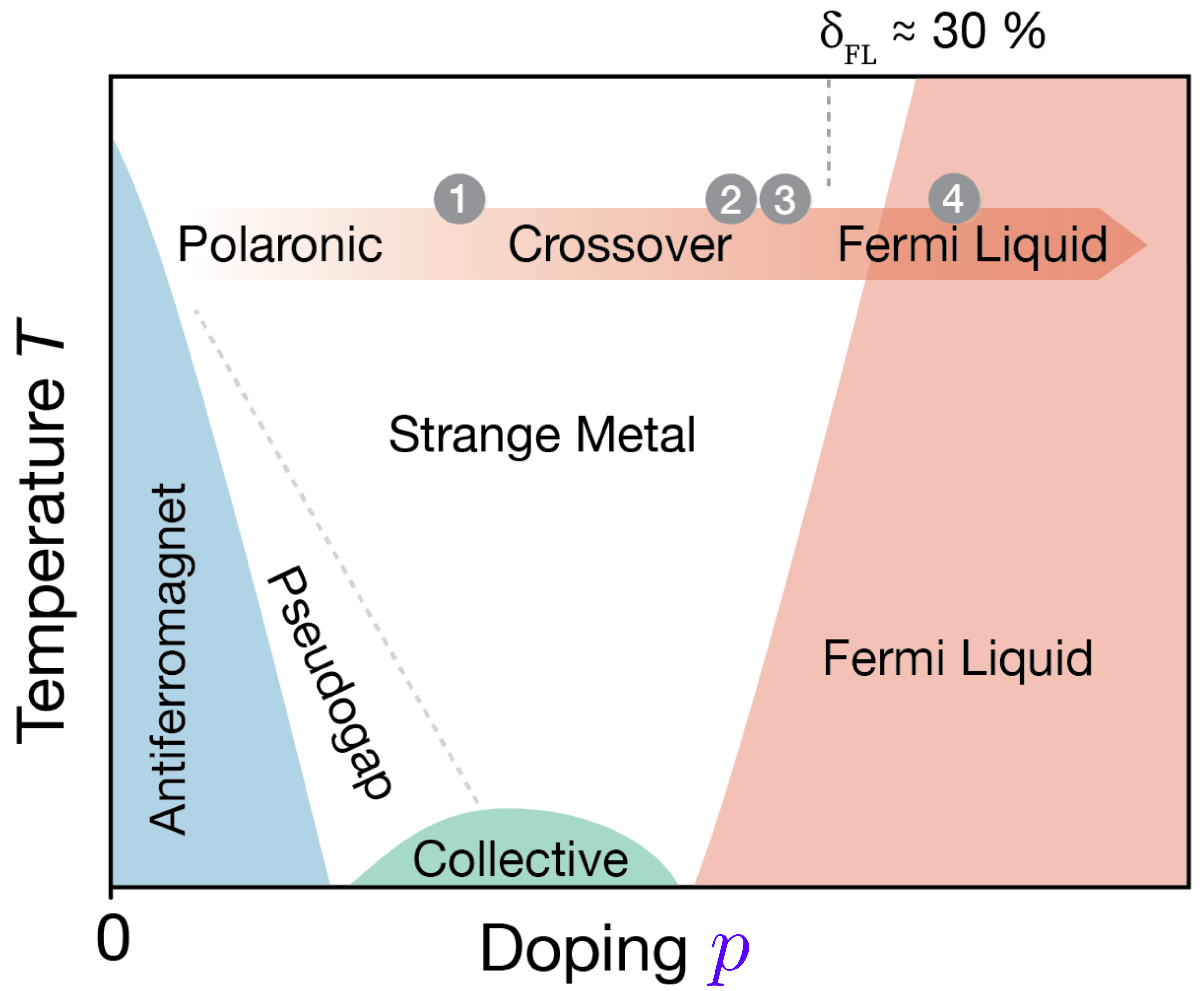
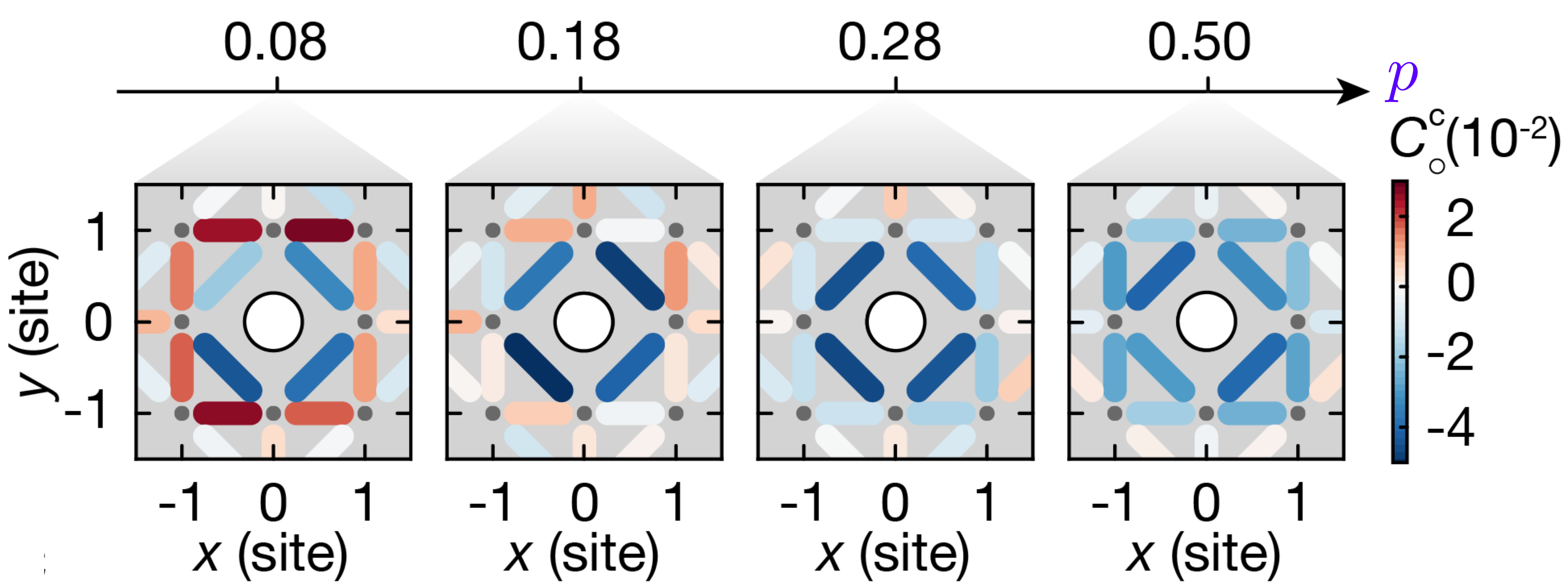
Jing-Yu Zhao, S. Chatterjee, S. S., Ya-Hui Zhang, arXiv:2510.13943

Wavefunction for FL*
and
observations on
ultracold atoms





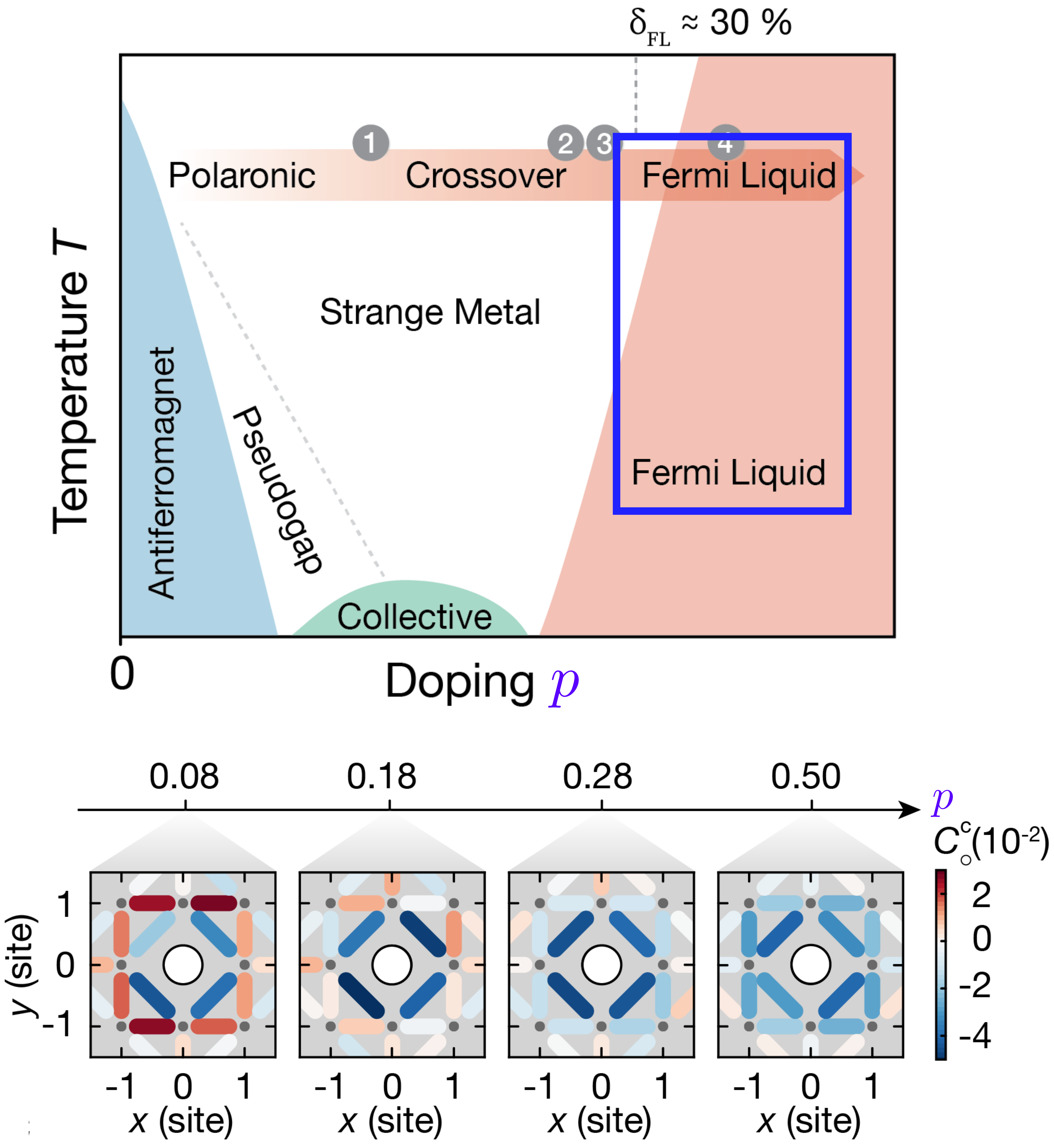
Microscopic evolution of doped Mott insulators from polaronic metal to Fermi liquid

Joannis Koepsell, Dominik Bourgund, Pimonpan Sompet, Sarah Hirthe, Annabelle Bohrdt, Yao Wang, Fabian Grusdt, Eugene Demler, Guillaume Salomon, Christian Gross, Immanuel Bloch

Science **374** (2021) 82

Chalopin….Bloch, PNAS **123**, e2525539123 (2026)

Max Planck Institute of Quantum Optics, Garching



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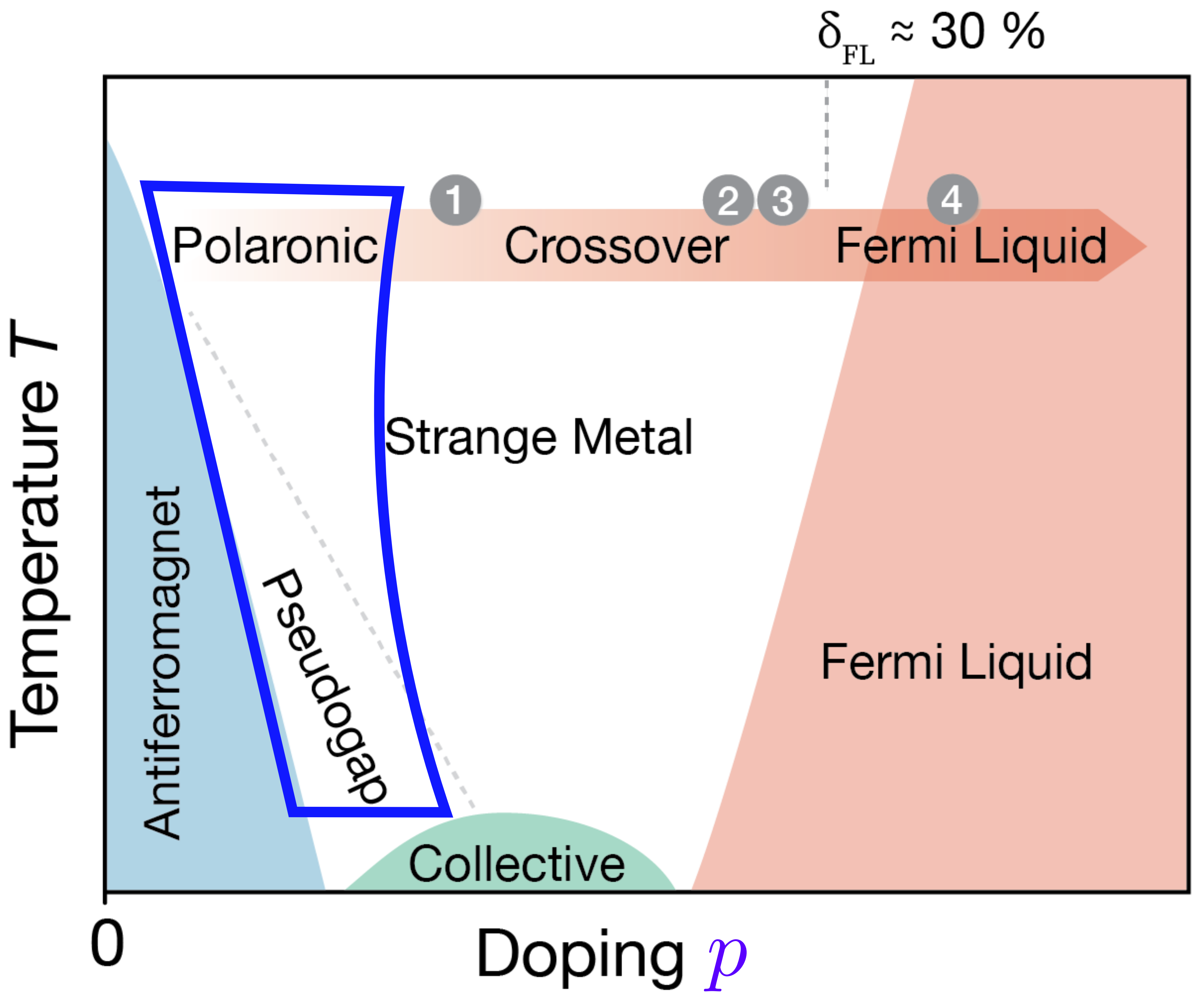

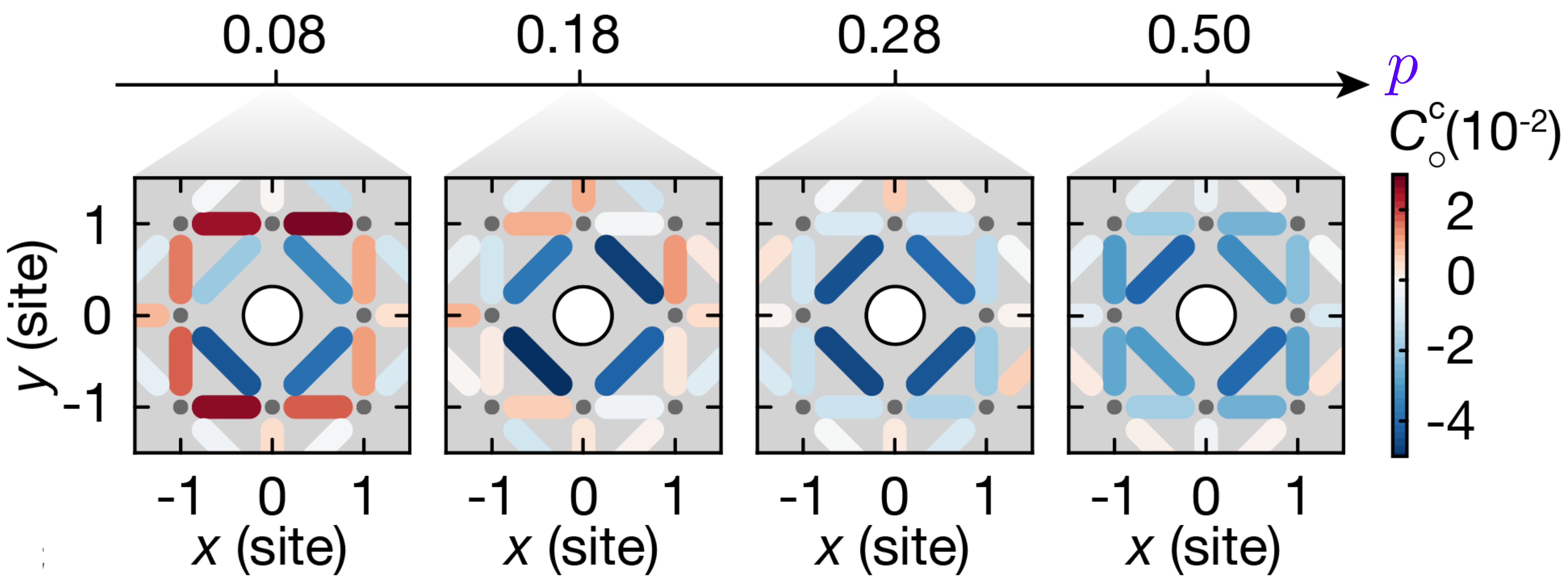

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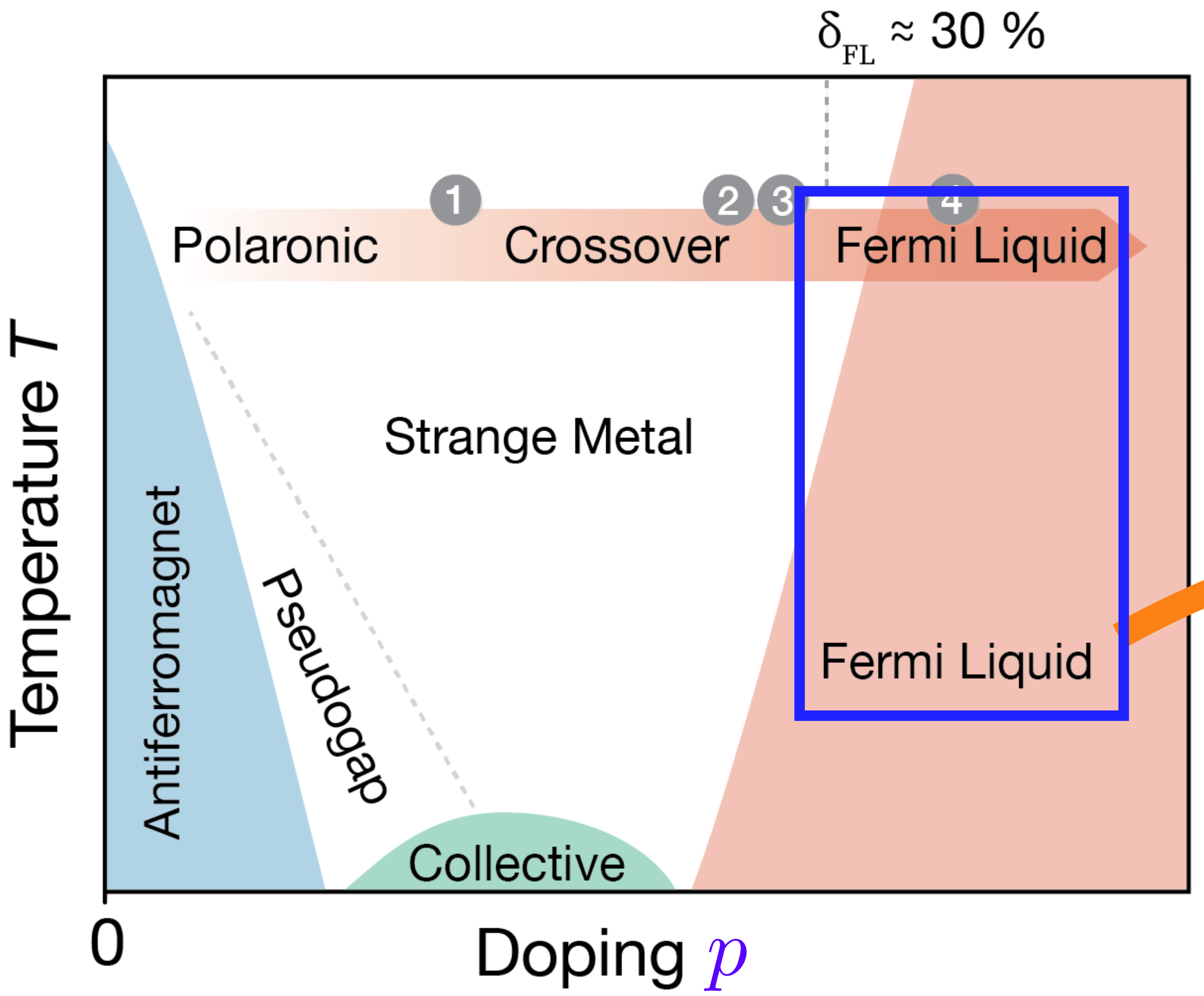
Max Planck Institute of Quantum Optics, Garching

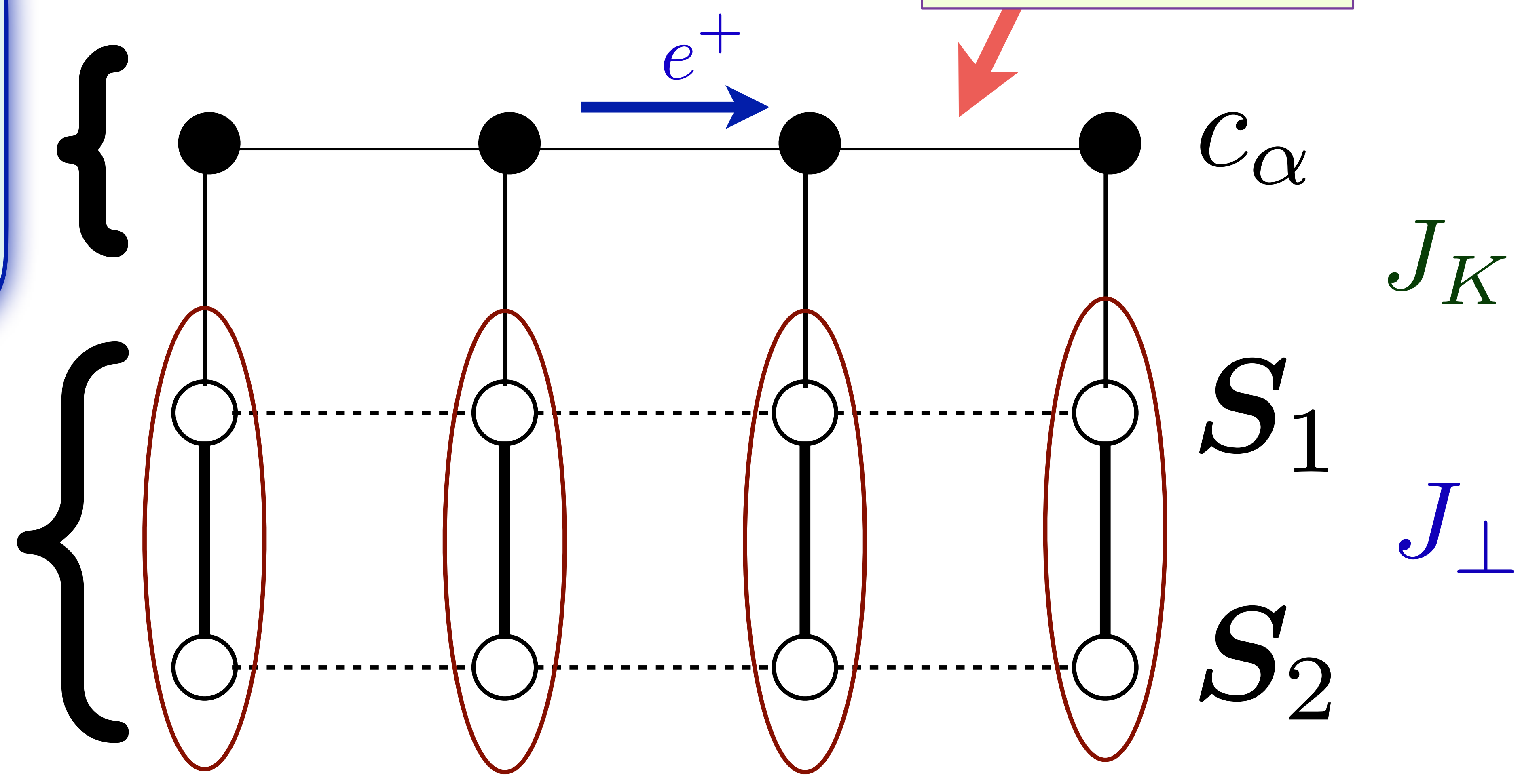
Ancilla Layer Model

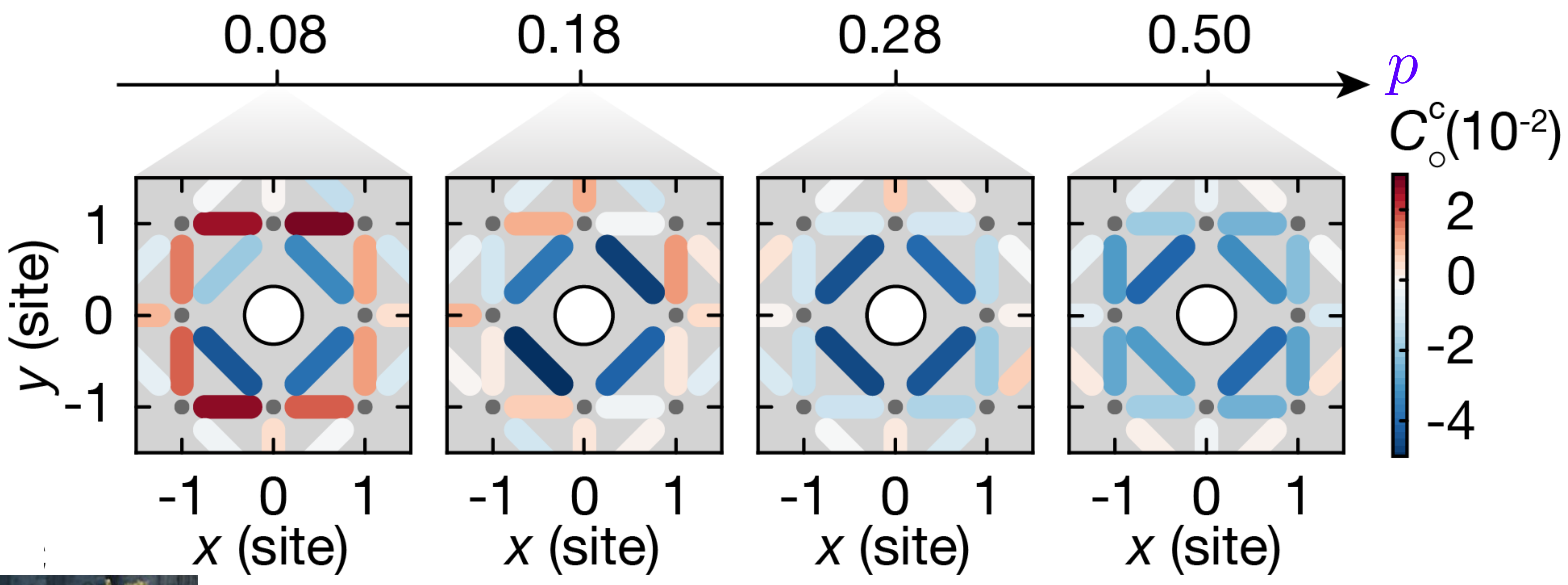


Fermi liquid. Fermi surface encloses area $(1+p)/2$.

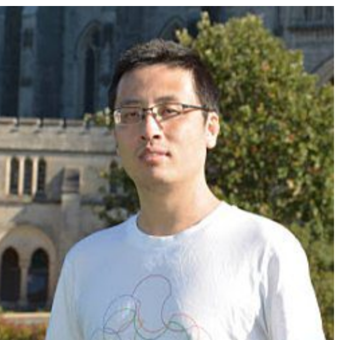
Rung singlets of ancilla spins $\boldsymbol{S}_1$, $\boldsymbol{S}_2$.

Holes of density 1+p





$$\text{Fermi liquid} = \begin{array}{c} \text{FL of } c_\alpha \\ \oplus \\ \textit{Trivial, gapped} \text{ state of } \boldsymbol{S}_1 \text{ and } \boldsymbol{S}_2 \end{array}$$

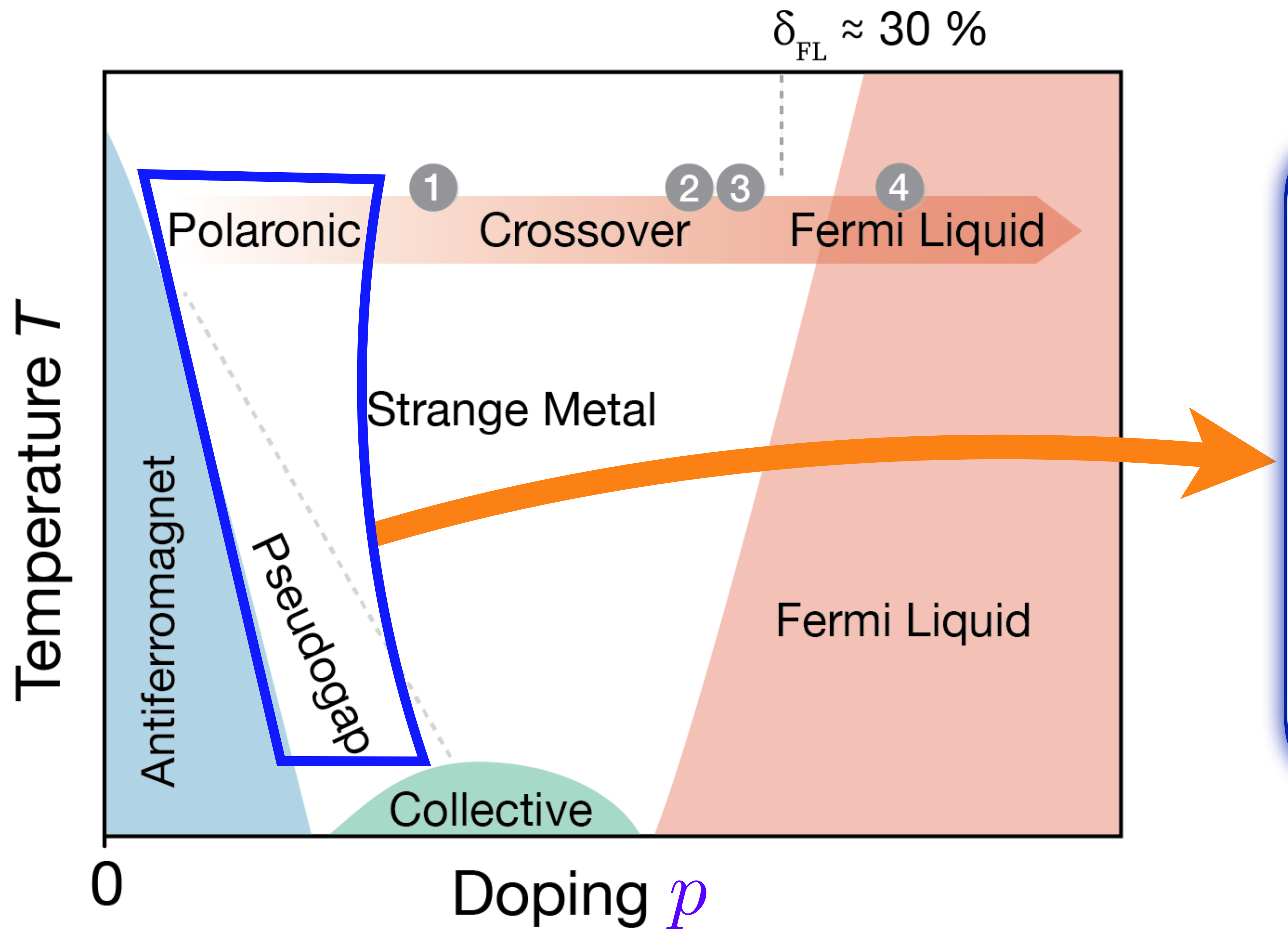


Ya-Hui Zhang

Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, S. Sachdev, PRB **105,** 075146 (2022)

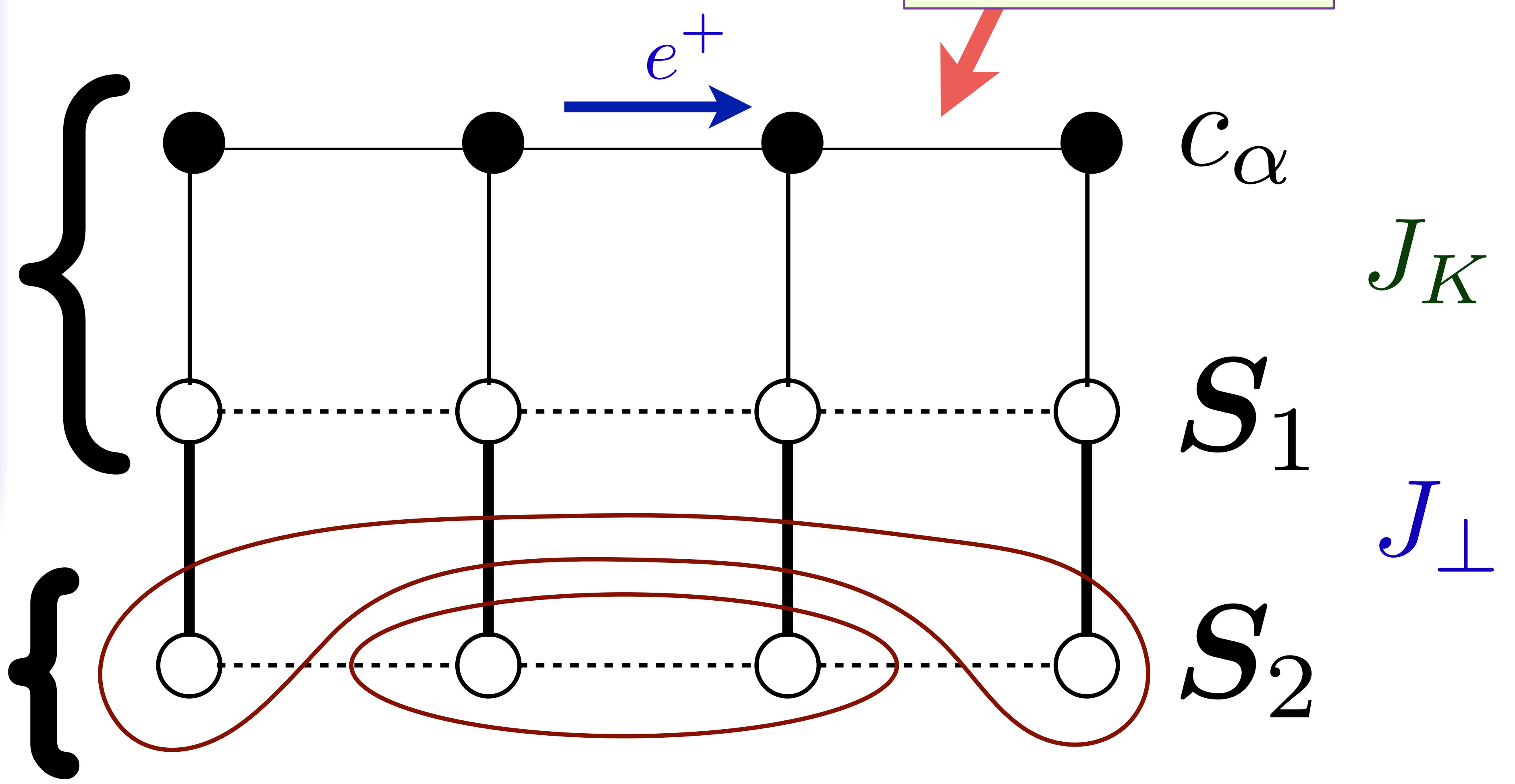
Ancilla Layer Model



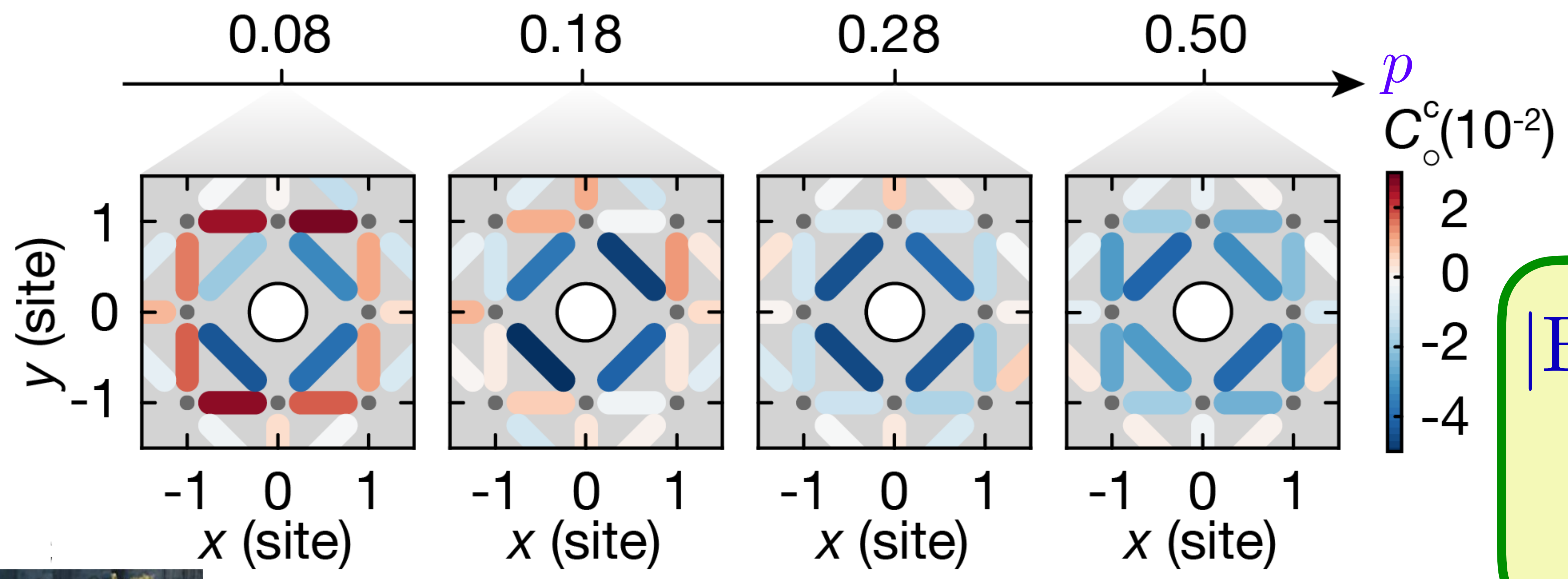
Kondo lattice heavy Fermi liquid.
Hybridization
$\Phi \sim f^{\dagger}_{1\alpha} c_{\alpha}$.
Fermi surfaces enclose area
$(1+p+1)/2$
$= p/2 \pmod 1$.

Your favorite spin liquid

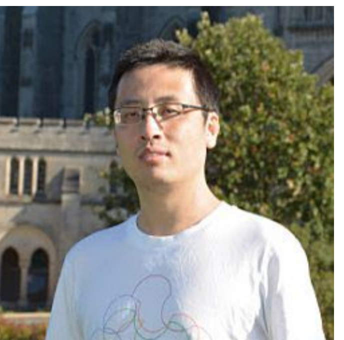
Holes of density 1+p



$$\boldsymbol{S}_{1i} = \frac{1}{2} f^{\dagger}_{1i\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{1i\beta}, \quad \boldsymbol{S}_{2i} = \frac{1}{2} f^{\dagger}_{i\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta}$$



$$|\mathrm{FL}*\rangle_{\mathrm{Hubbard}} = [\text{Projection onto rung singlets of } \boldsymbol{S}_1, \boldsymbol{S}_2] \bowtie |\text{Slater determinant of } (c, f_1)\rangle \otimes |\text{Slater determinant of } f\rangle$$

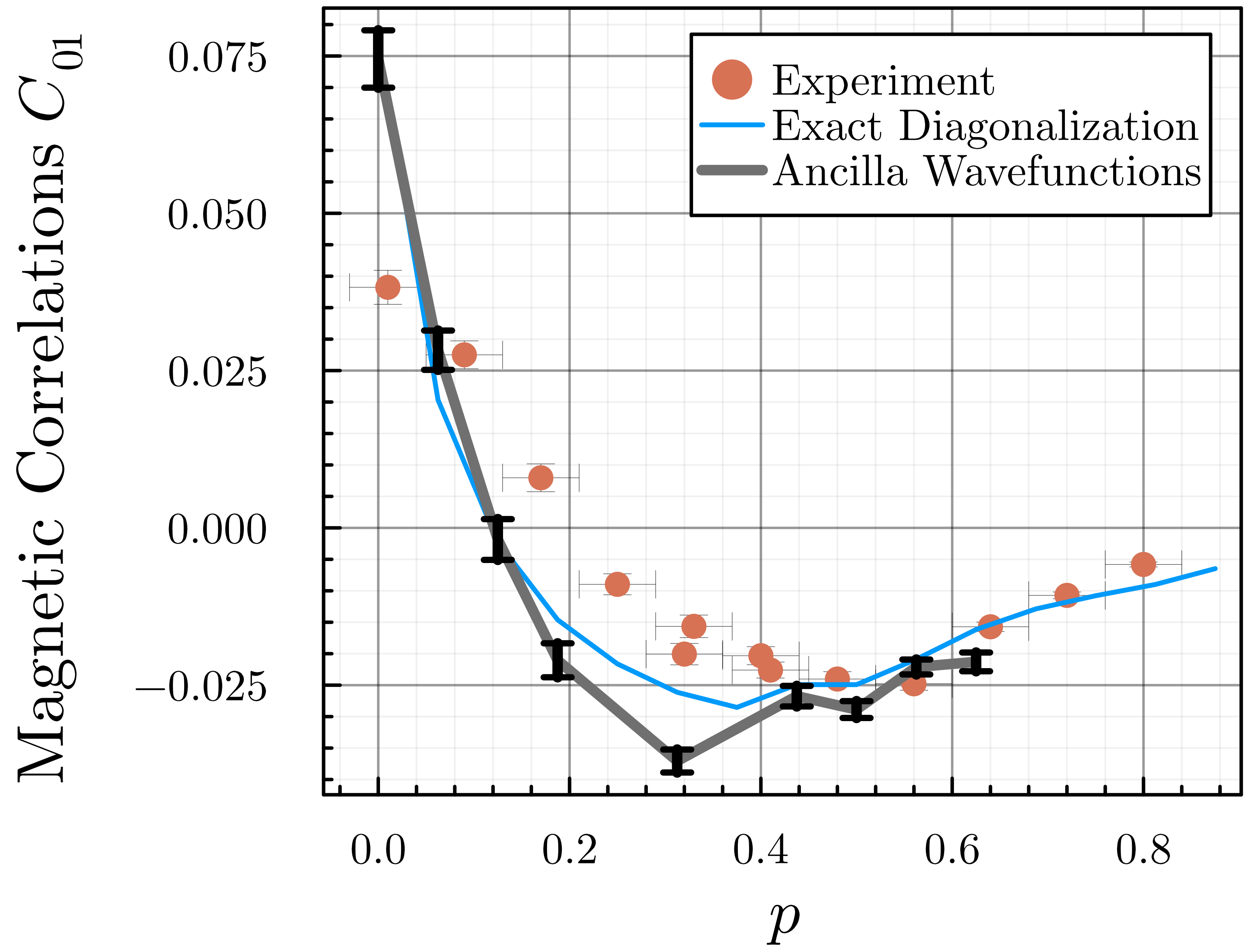


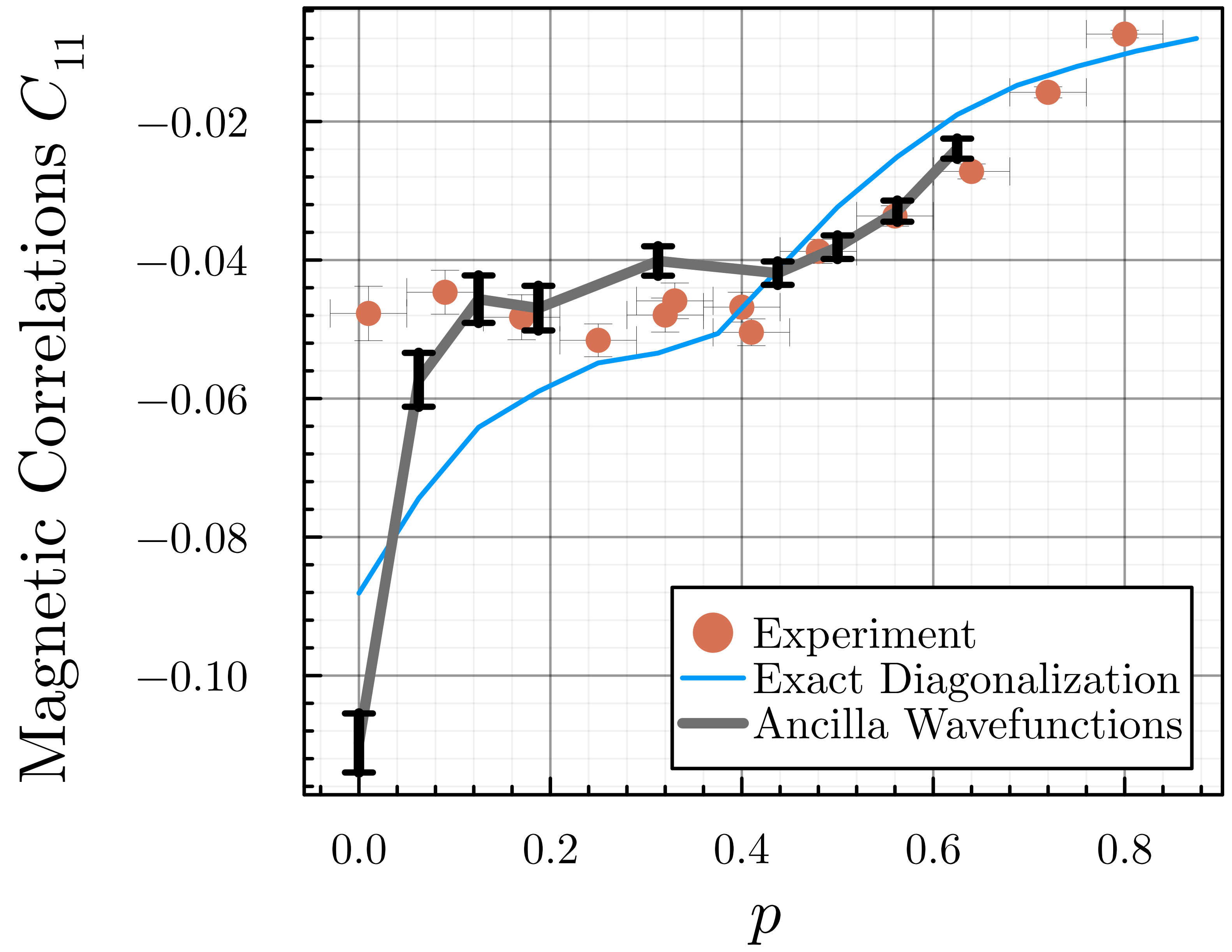
Ya-Hui Zhang

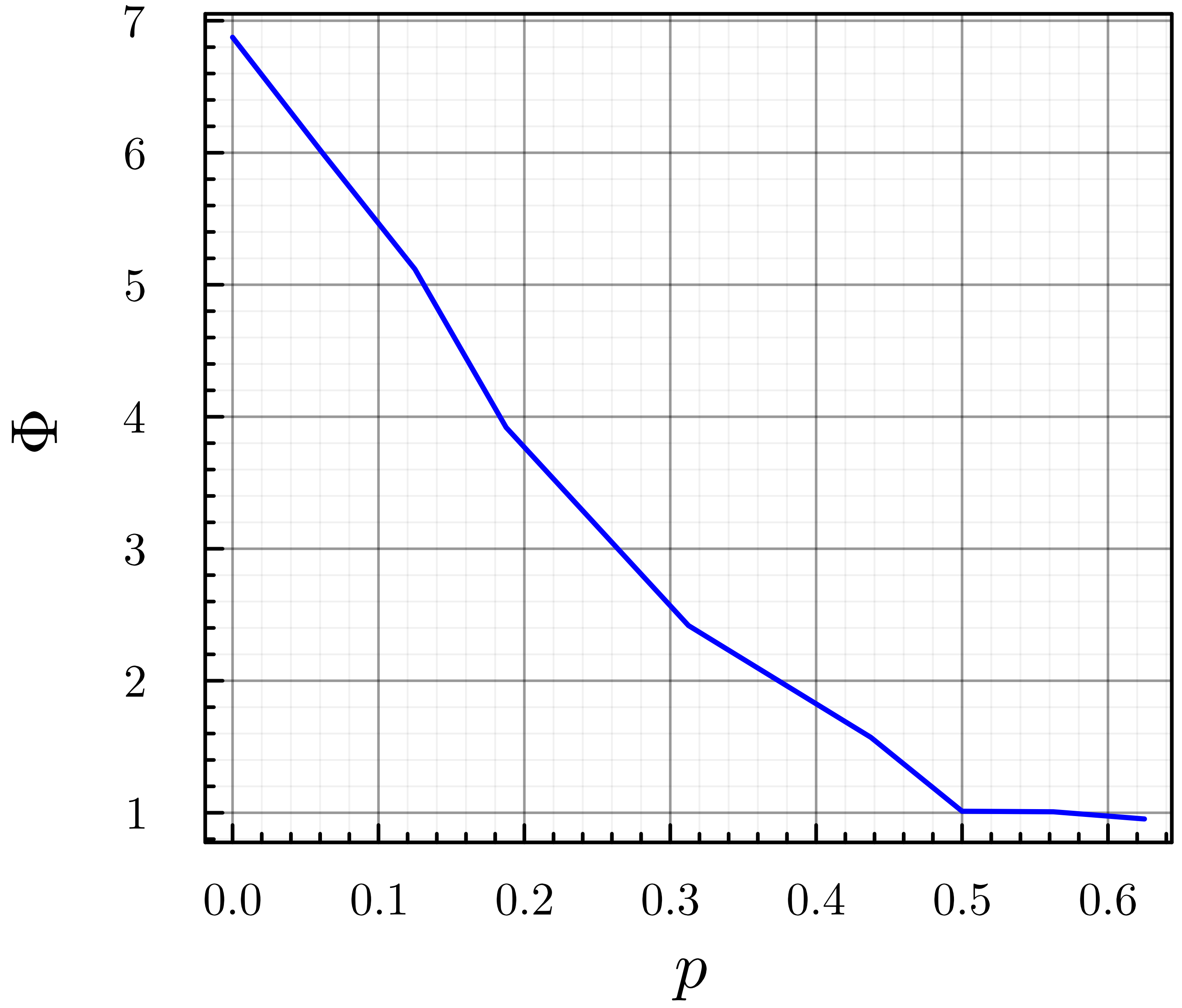
Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, S. Sachdev, PRB **105,** 075146 (2022)

Ancilla wavefunction for FL* of Hubbard model










Hybridization (Higgs boson) $\Phi \sim f_{1\alpha}^{\dagger} c_{\alpha}$.

FL*: $\Phi \neq 0$; FL: $\Phi = 0$

Φ: variational parameter with Hubbard Hamiltonian

$$|\mathrm{FL}*\rangle_{\mathrm{Hubbard}} = [\text{Projection onto rung singlets of } \boldsymbol{S}_1, \boldsymbol{S}_2] \bowtie |\text{Slater determinant of } (c, f_1)\rangle \otimes |\text{Slater determinant of } f\rangle$$

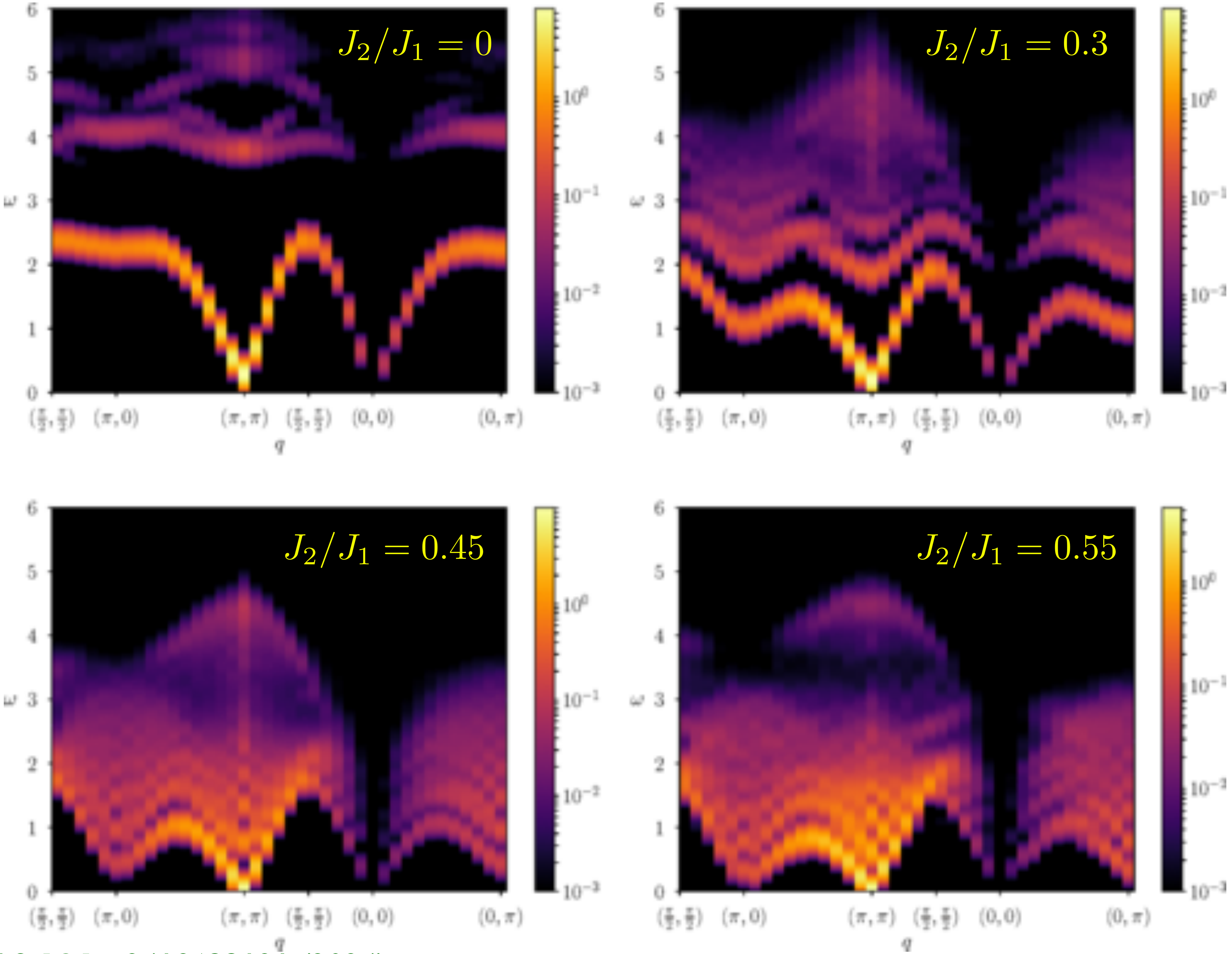
L. Shackleton and Shiwei Zhang, arXiv:2408.02190

Tobias Müller, Yasir Iqbal, S.S., Ronny Thomale, PNAS **122**, e2504261122 (2025)

Detecting the spinons of FL* ?

Numerical study of a square lattice antiferromagnet with first (J_1) and second (J_2) neighbor exchange.

Francesco Ferrari and Federico Becca, PRB **98**, 100405(R) (2018)



- Spin waves only present at low energies in the presence of AF order
- Spinon continuum near the quantum transition involving loss of AF order.

Pietro Bonetti, Maine Christos and S.S. (BCS), PNAS **121**, e2418633121 (2024)

Anisotropic damping and wave vector dependent susceptibility of the spin fluctuations in $La_{2-x}Sr_xCuO_4$ studied by resonant inelastic x-ray scattering

H. C. Robarts, M. Barthélemy, K. Kummer, M. García-Fernández, J. Li, A. Nag, A. C. Walters, K. J. Zhou, and S. M. Hayden

PHYSICAL REVIEW B **100**, 214510 (2019)

- Spin waves only present at low energies in the presence of AF order
- Most natural interpretation is a spinon continuum, similar to that found in the frustrated square lattice antiferromagnet

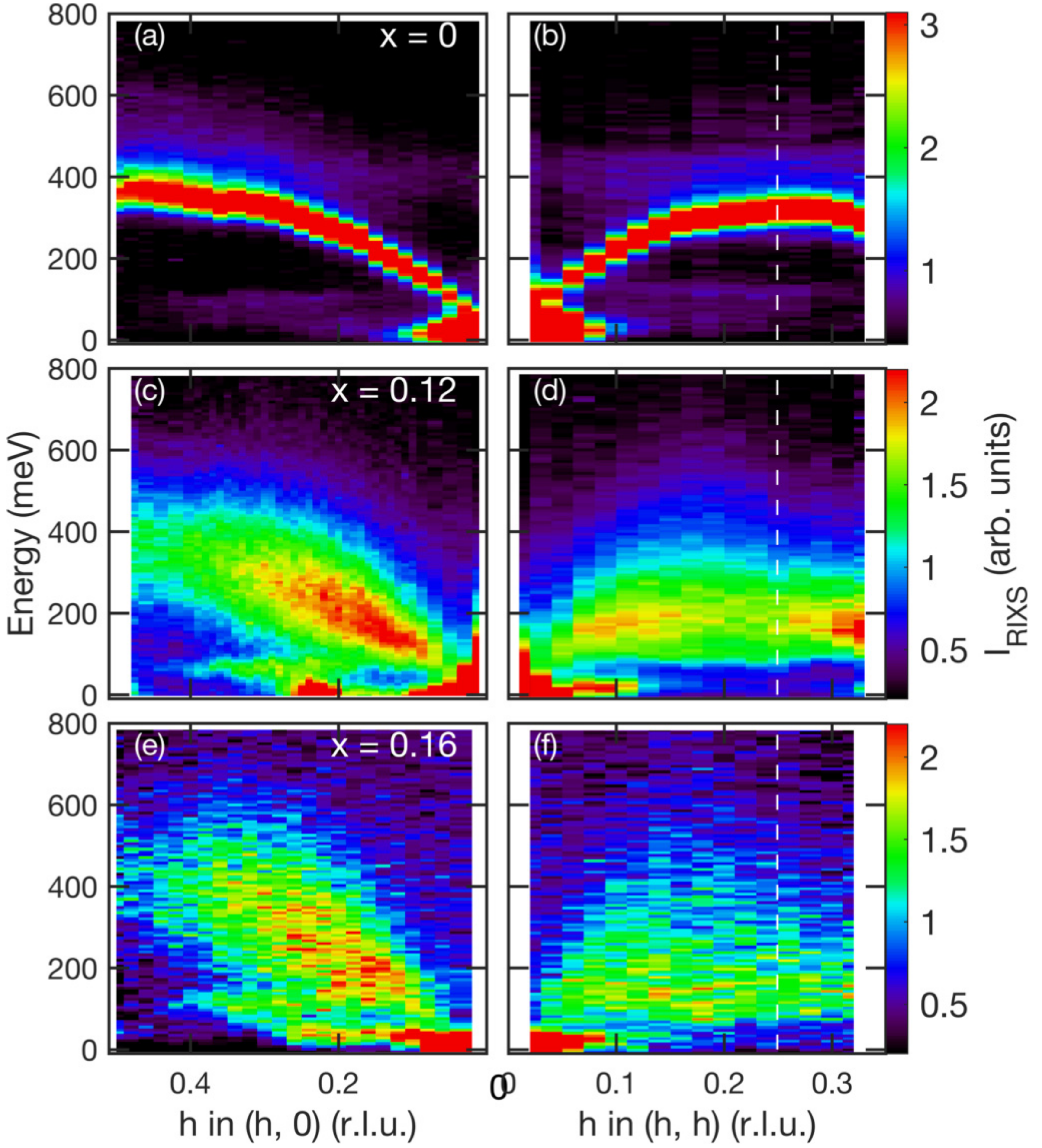
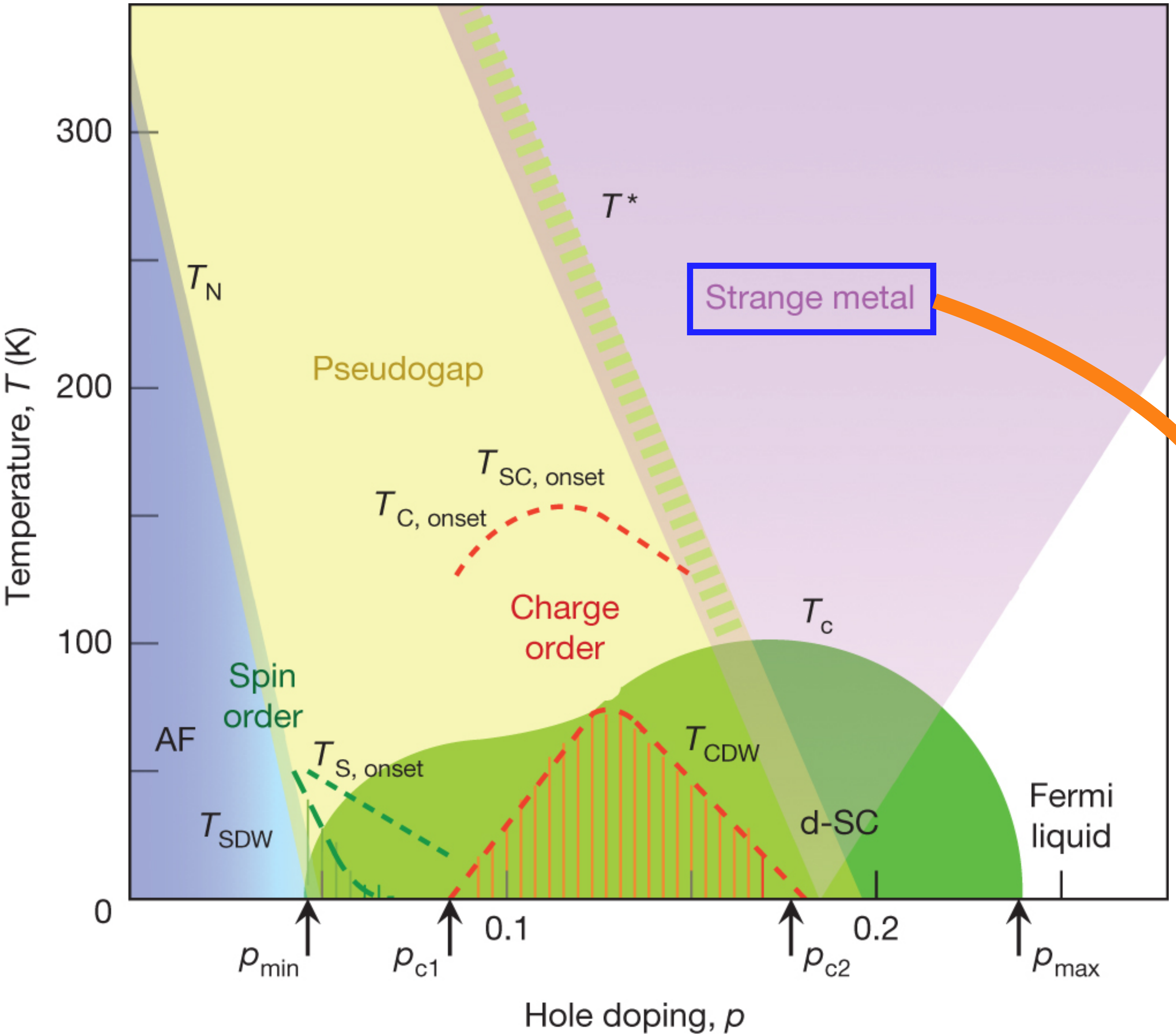


FIG. 2. I_{RIXS} intensity maps as a function of **Q** in LSCO $x = 0$ ($T \approx 20$ K), 0.12, and 0.16 ($T \approx 30$ K).

Pietro Bonetti, Maine Christos and S.S. (BCS), PNAS **121**, e2418633121 (2024)

Sachdev-Ye-Kitaev liquids

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



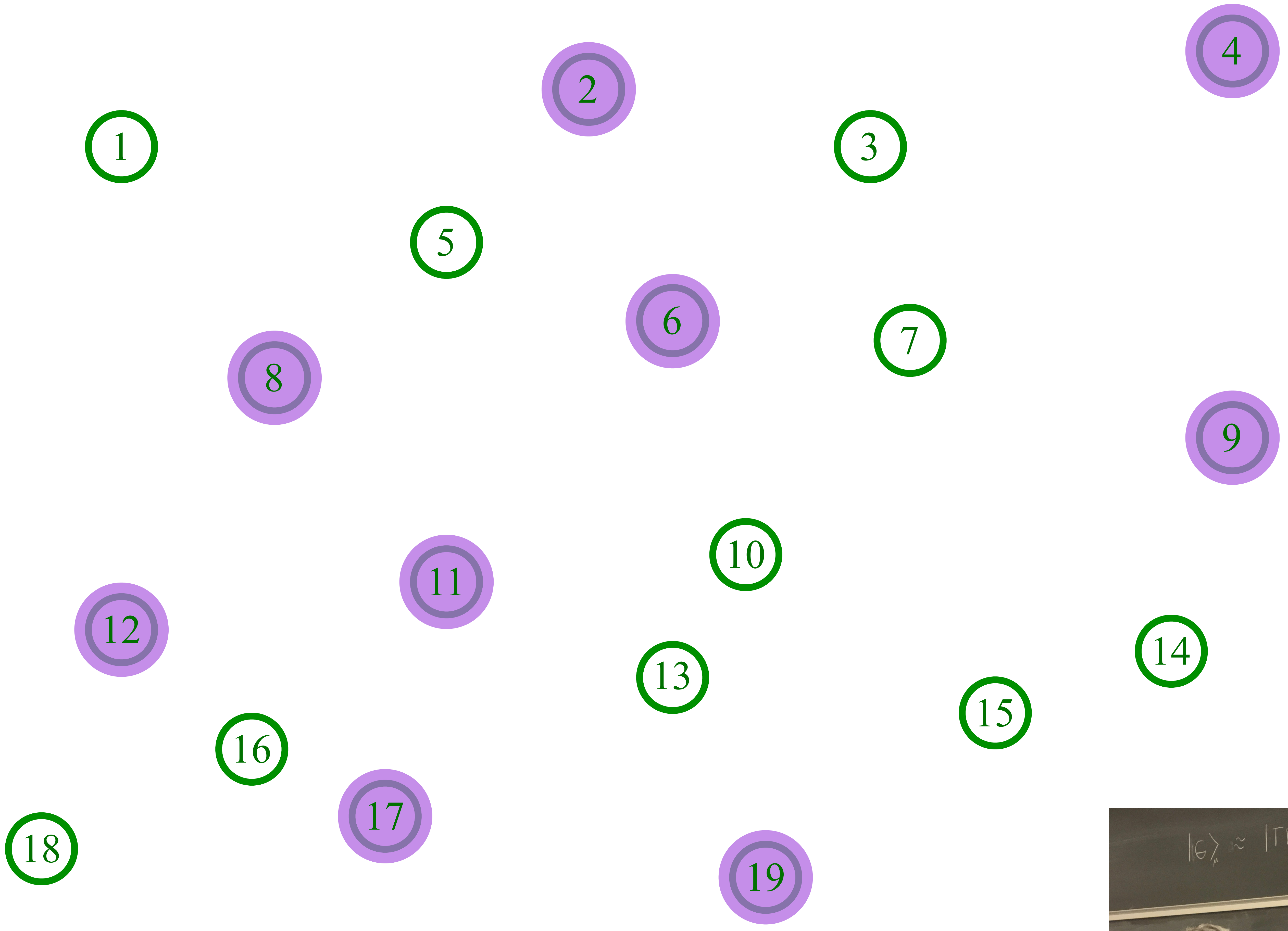
Quantum entanglement of mobile fermions

Sachdev-Ye-Kitaev (SYK) liquid

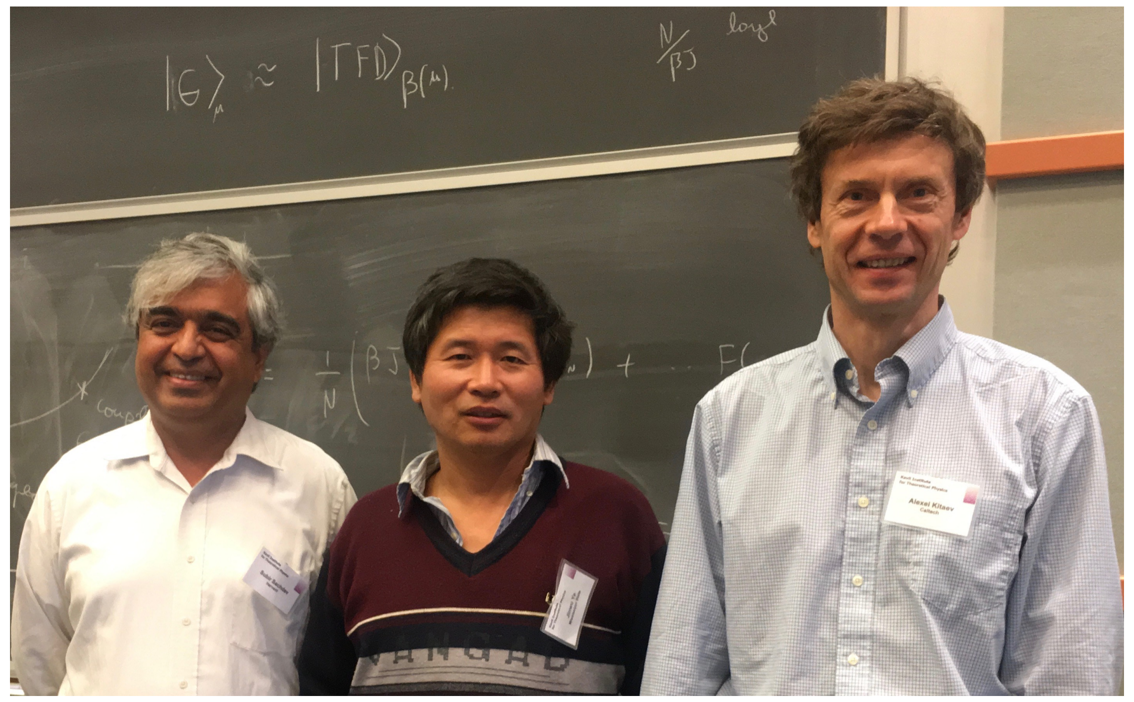
- Complex entanglement of a compressible state with no quasiparticles.
- Universal theory of strange metals
- Also applies to generic charged black holes in asymptotically flat 3+1 dimensional space.

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)



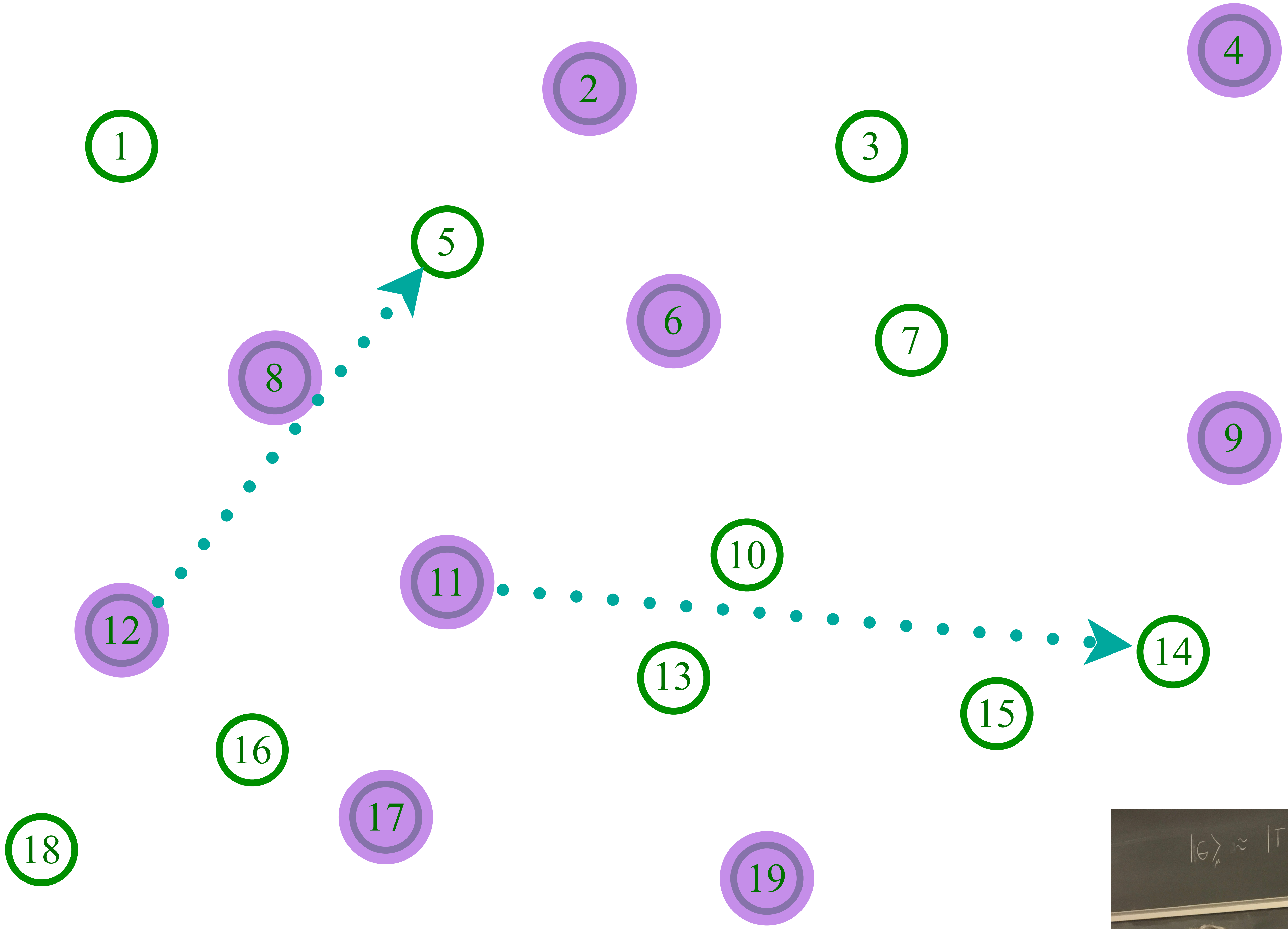
Place electrons randomly on some sites



The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



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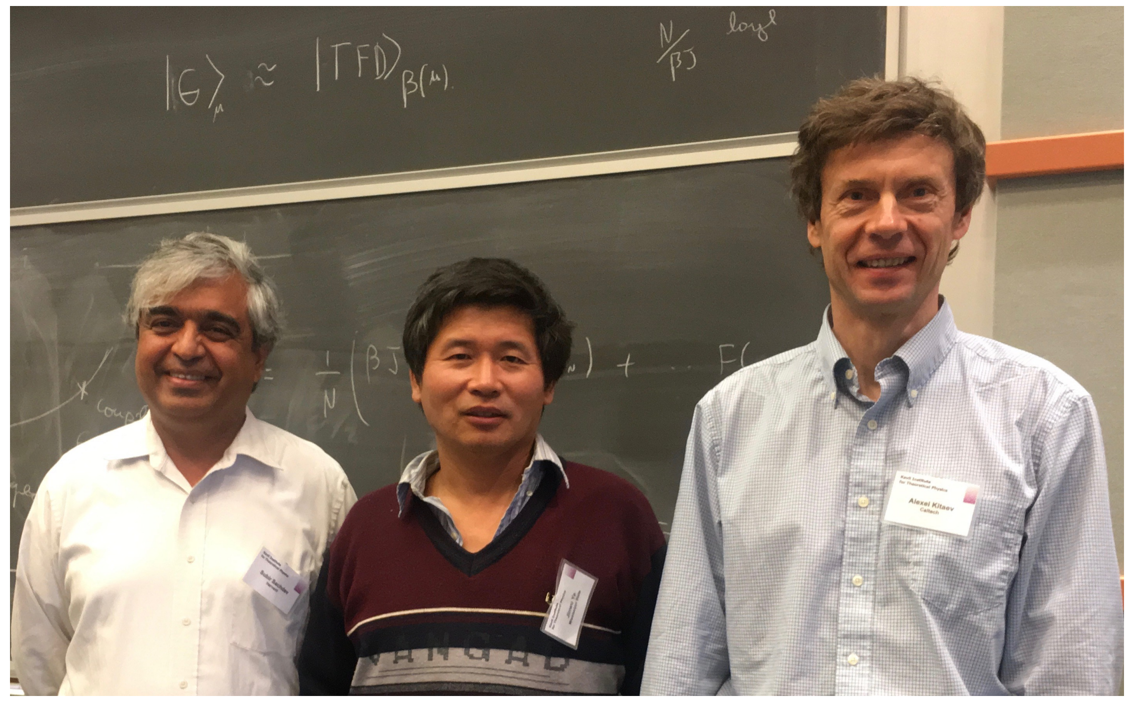

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Sachdev, Ye (1993); Kitaev (2015)

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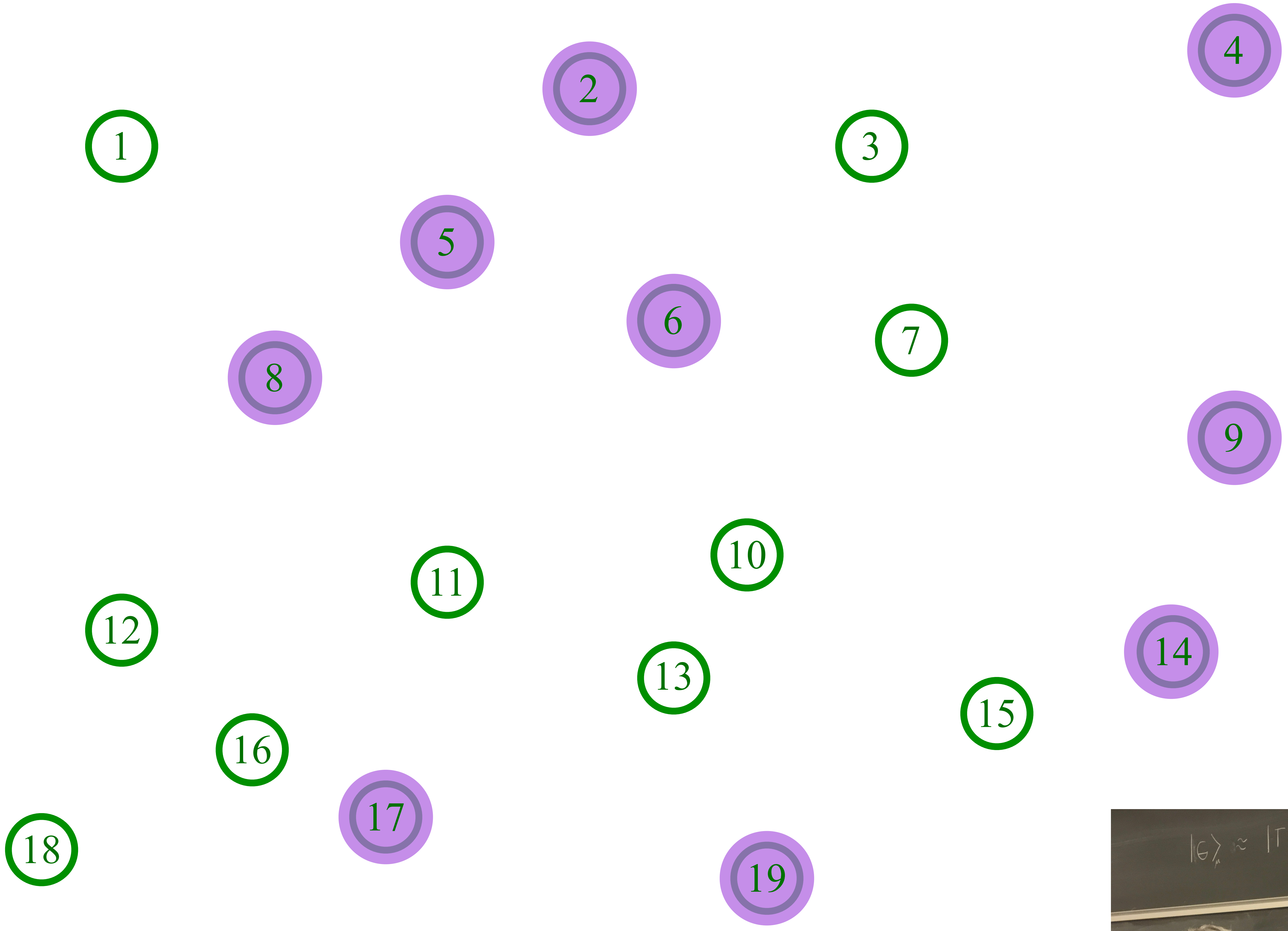
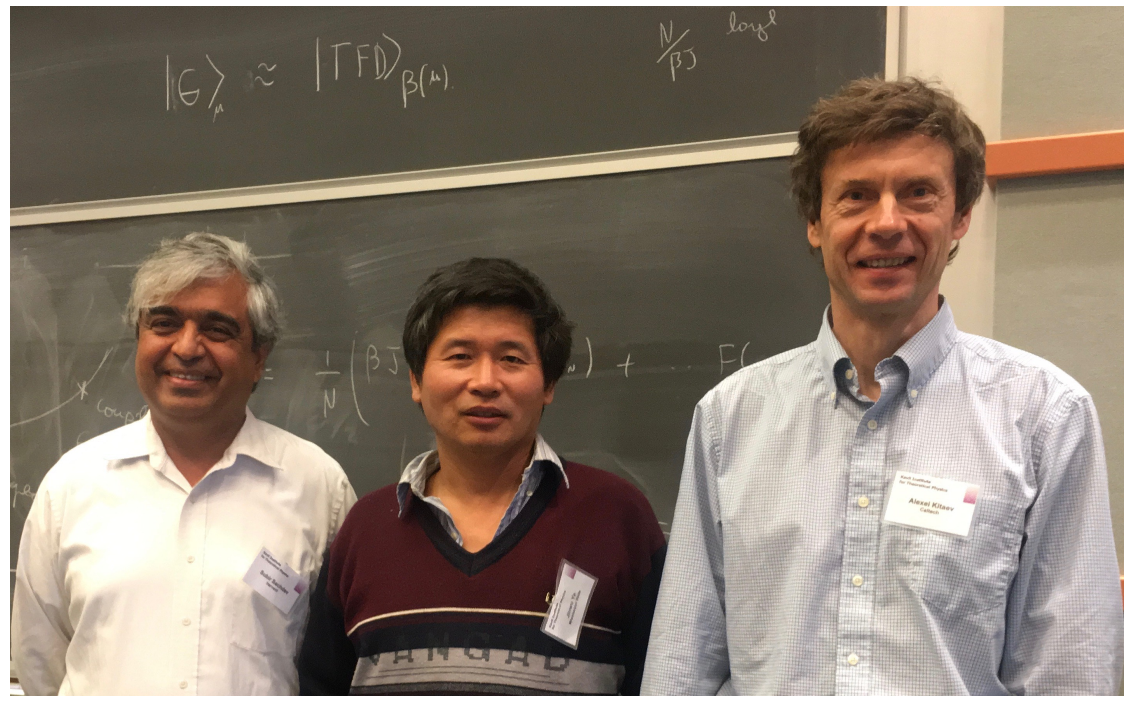


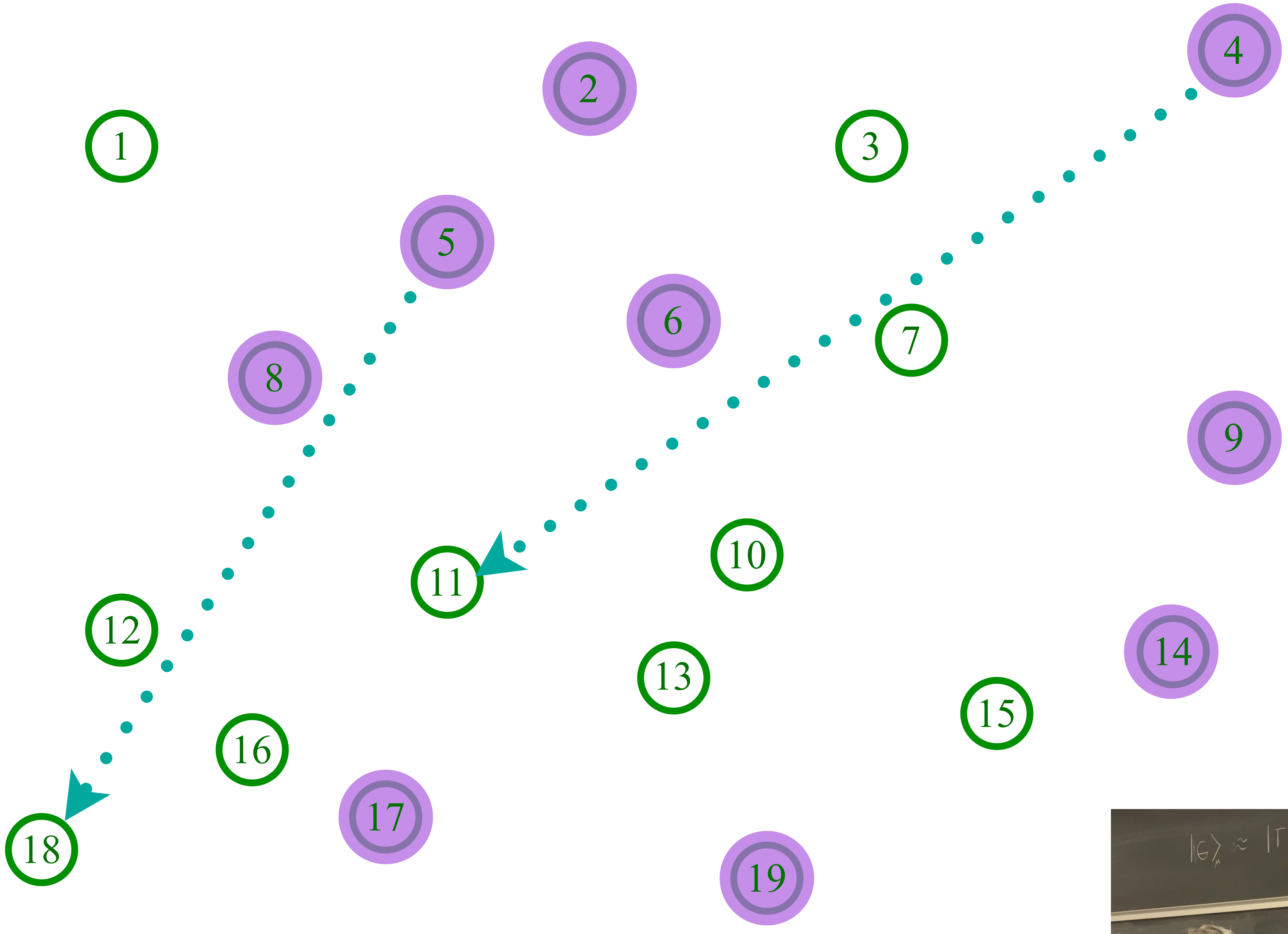
Entangle electrons pairwise randomly



The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



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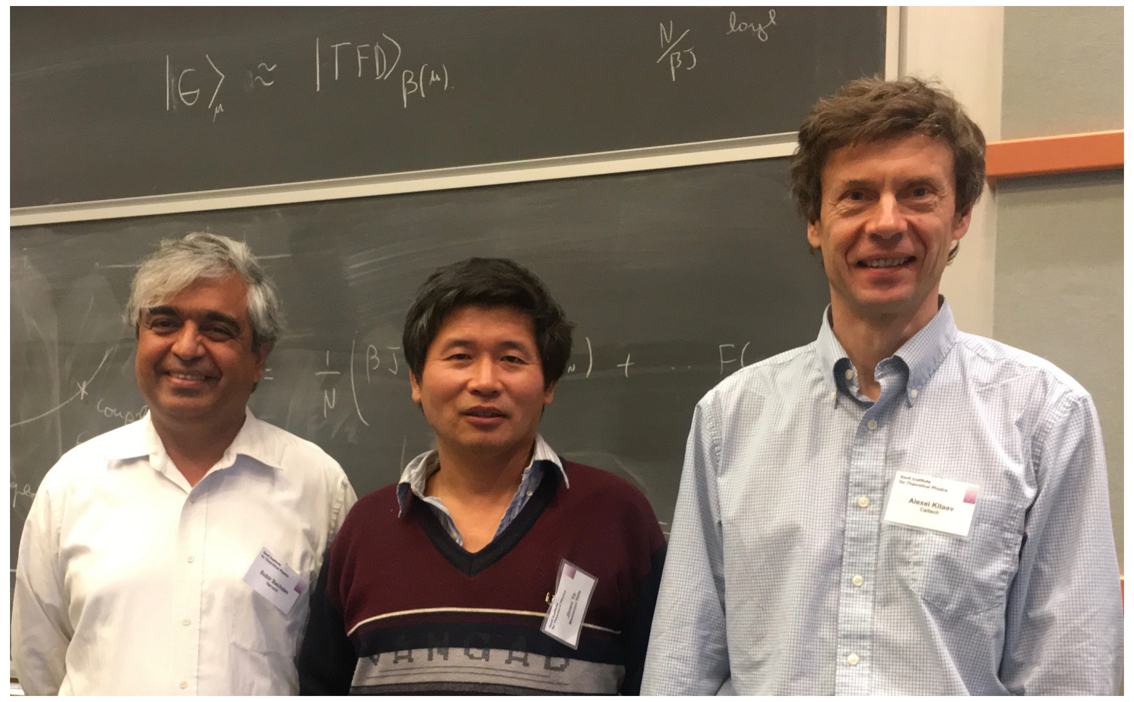

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

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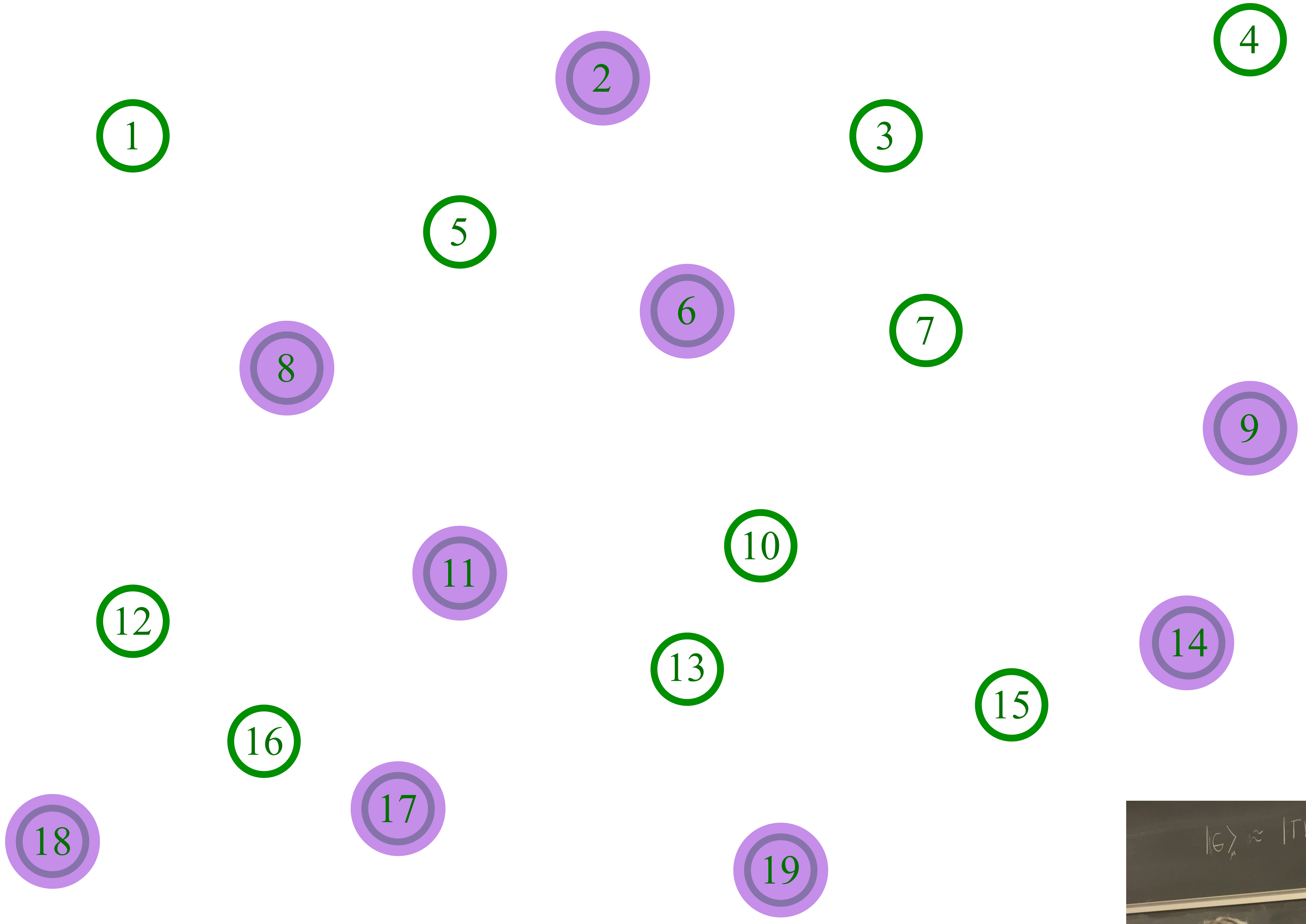

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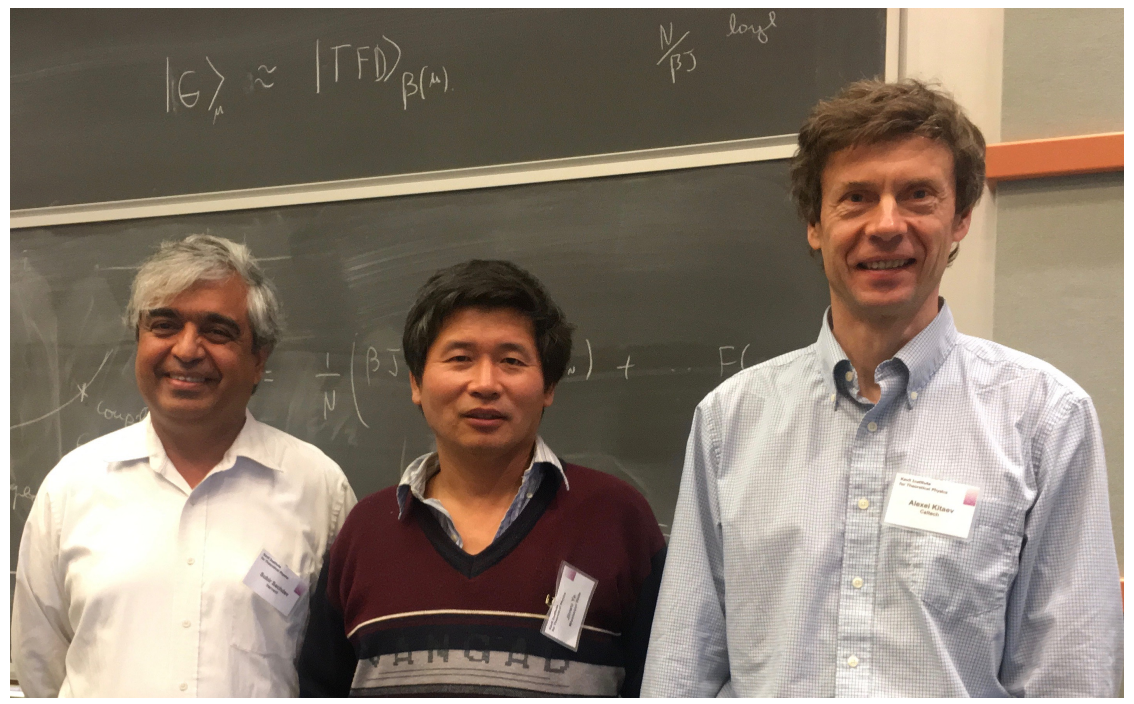
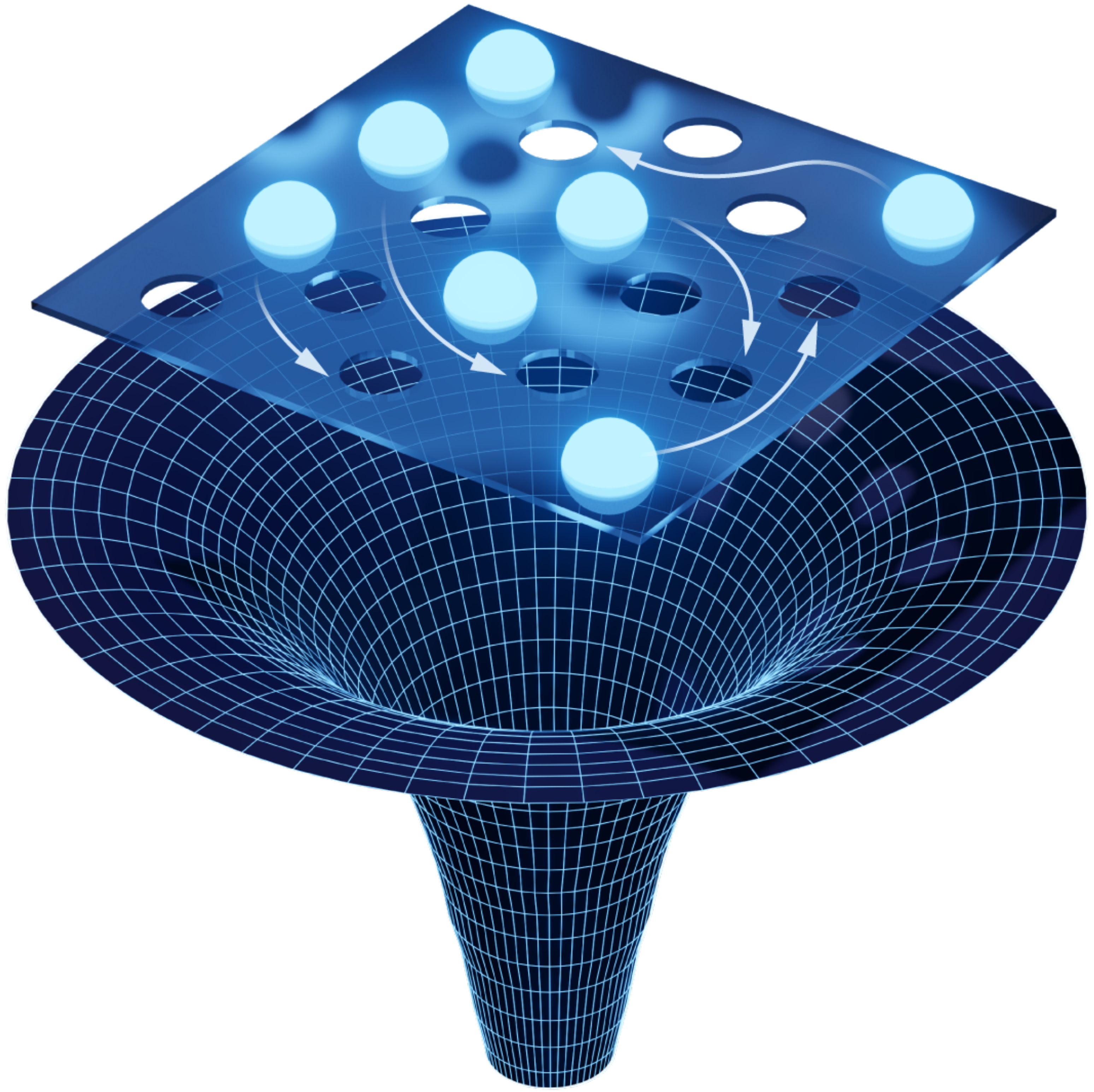


The Sachdev-Ye-Kitaev (SYK) models

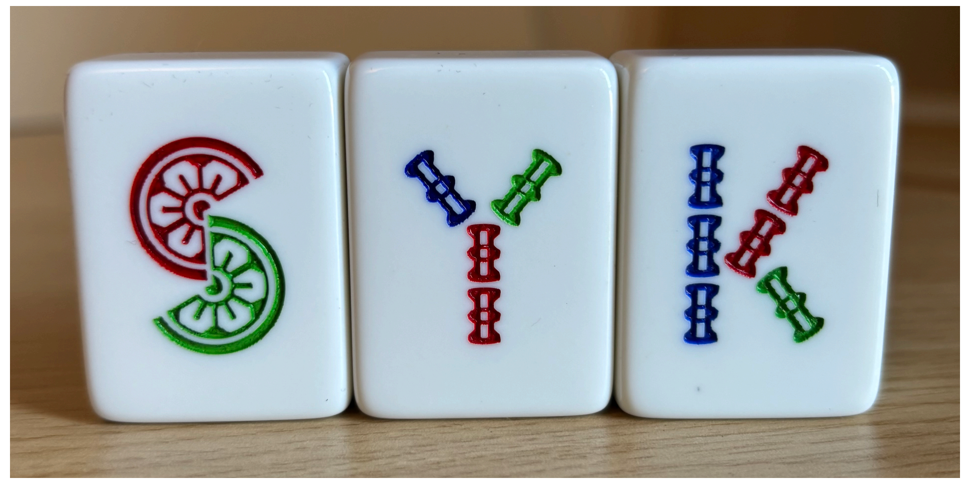
Sachdev, Ye (1993); Kitaev (2015)

Solvable models of multi-particle quantum entanglement with mobile fermions.



Yields a metal whose excitations are not particle-like i.e. no bosons, fermions, anyons….

Current is carried by an "entangled quantum soup"



Yukawa-Sachdev-Ye-Kitaev model

$$\mathcal{H} = -\mu \sum_i c_i^\dagger c_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \Phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell} c_i^\dagger c_j \Phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, PRB **105**, 235111 (2022)
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Yukawa-Sachdev-Ye-Kitaev model

$$\mathcal{H} = -\mu \sum_i c_i^\dagger c_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \Phi_\ell^2\right) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} c_i^\dagger c_j \Phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large N equations for the Green's functions and self energies of the fermions (G, Σ) and bosons (D, Π) are

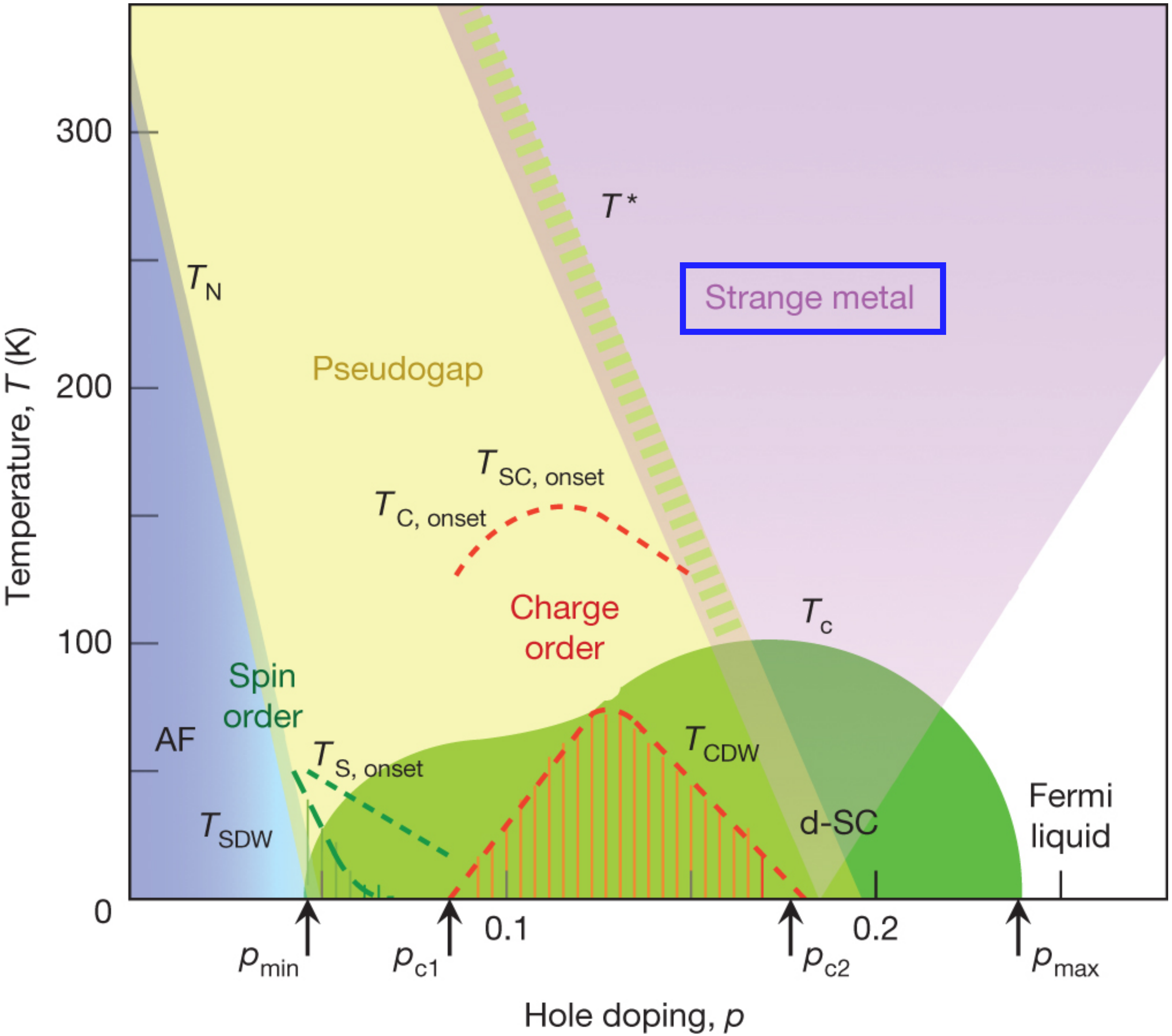
$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)}$$
$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$

At $T > 0$, solutions are fully characterized by a universal frequency-dependent relaxation time,

$$\frac{\hbar}{\tau(\omega)} = k_B T \, \Phi_\tau\left(\frac{\hbar\omega}{k_B T}\right)$$

where Φ_τ is a known universal function.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Non-Boltzmann

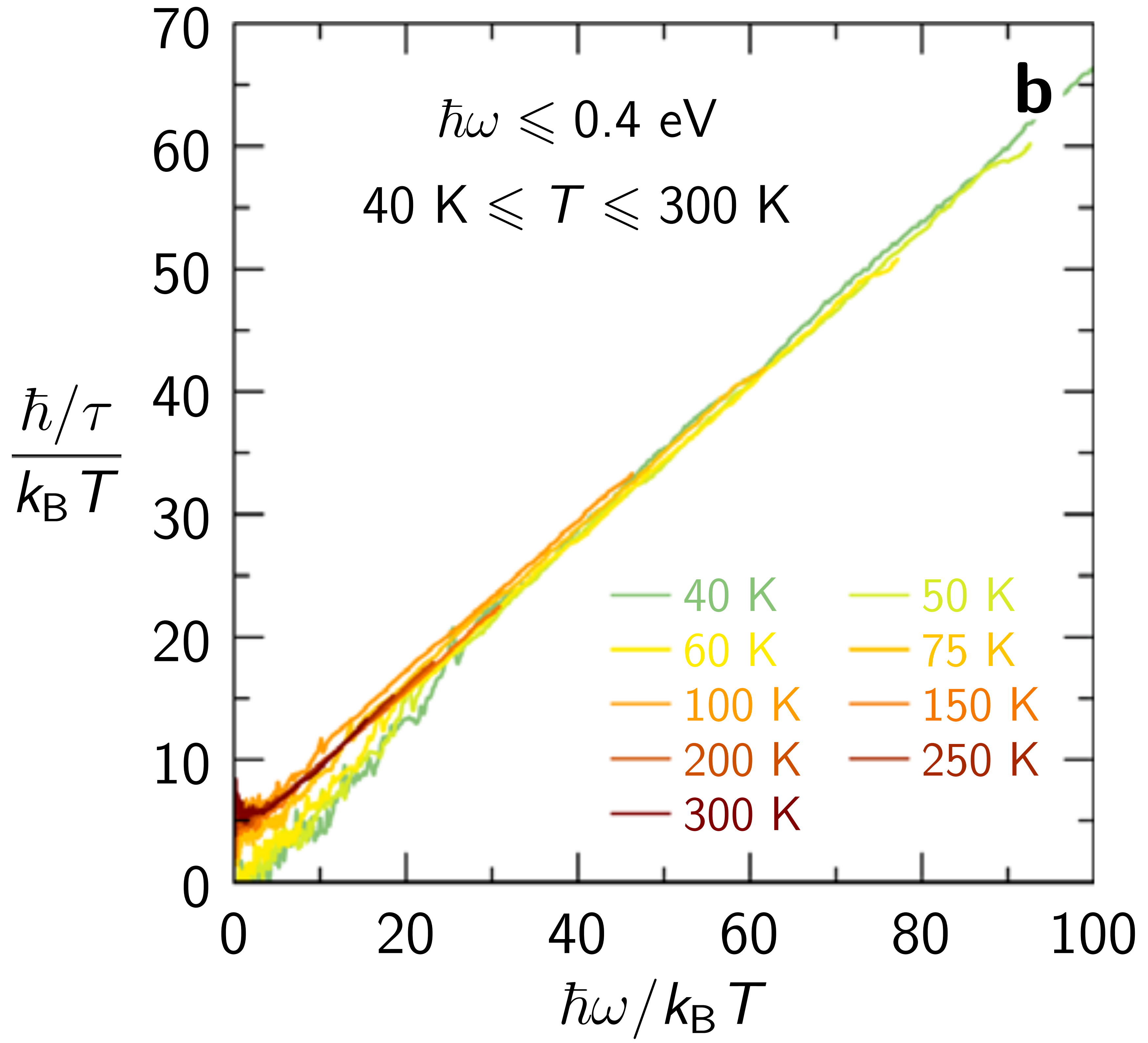
Planckian dynamics

of large Fermi surface

Electron scattering time τ

from optical conductivity

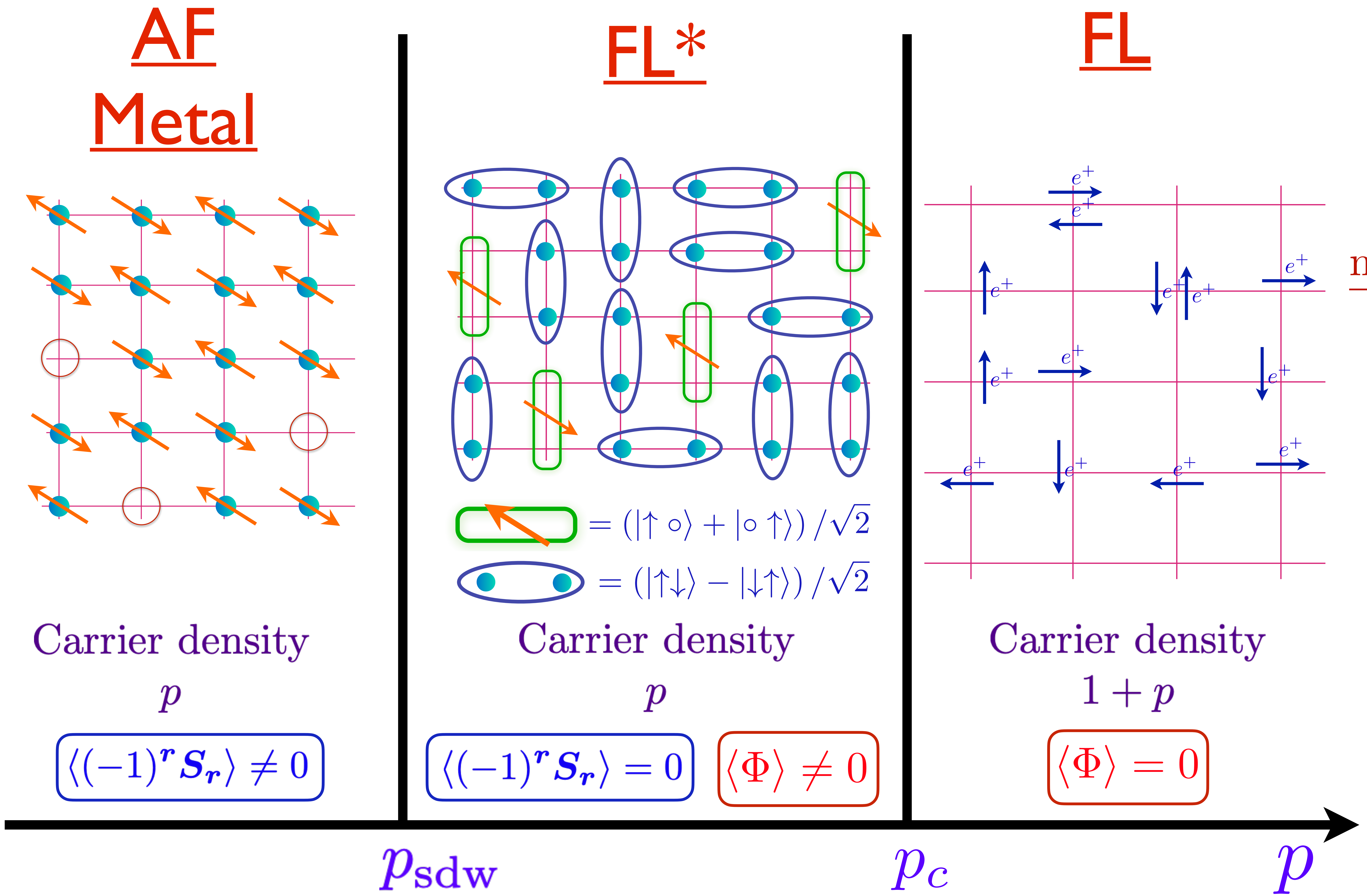
$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$



B. Michon, C. Berthod, C.W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges, Nature Comm. **14**, 3033 (2023)

From FL* to FL
via
the strange metal
using the 2D-YSYK model

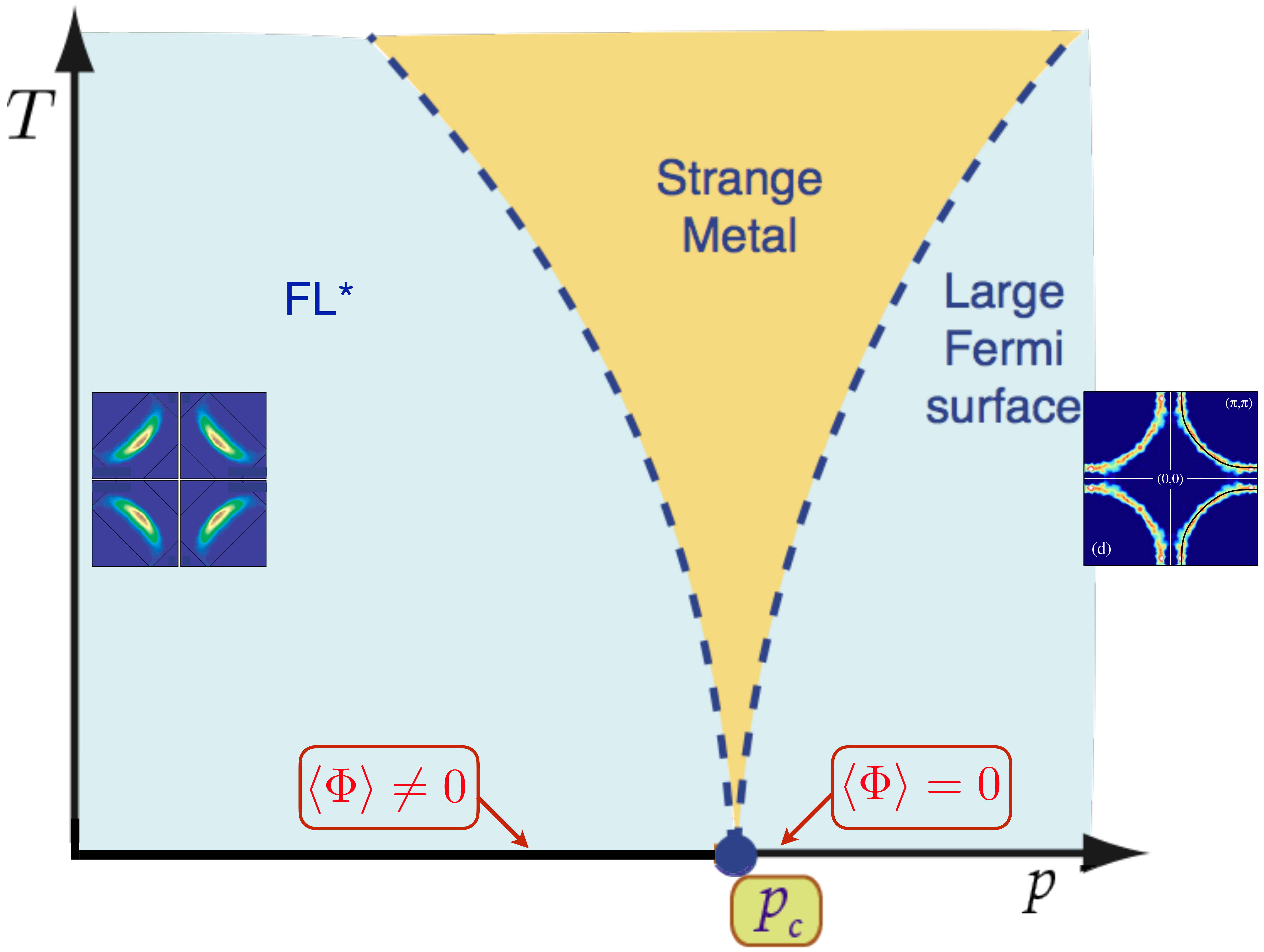
S. Sachdev, M. A. Metlitski and M. Punk, Journal of Physics Condensed Matter **24**, 294205 (2012)



Quantum phase transition between two metals (FL* and FL) at $p = p_c$, with no symmetry breaking.

Described by the condensation of a *Higgs* field Φ.

S. Sachdev, M.A. Metlitski and M. Punk, Journal of Physics Condensed Matter **24**, 294205 (2012)
Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

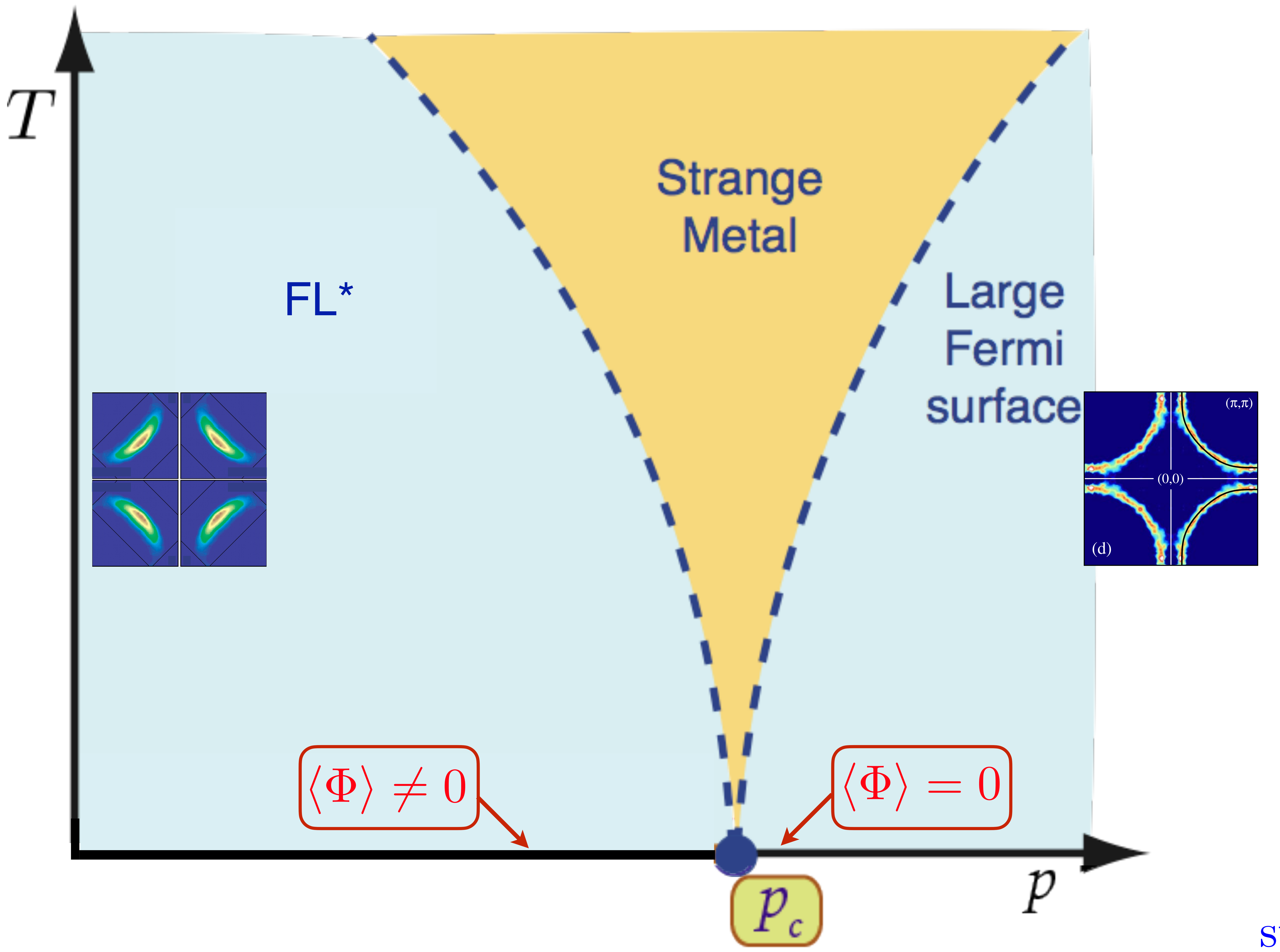


Quantum phase transition between two metals (FL* and FL) at $p = p_c$, with <u>no symmetry breaking</u>.

Described by the condensation of a <u>*Higgs*</u> field Φ.

Strange metal is obtained from the $T > 0$ quantum criticality of the FL-FL* transition, *provided* there is momentum relaxation.

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)



Quantum phase transition between two metals (FL* and FL) at $p = p_c$, with no symmetry breaking.

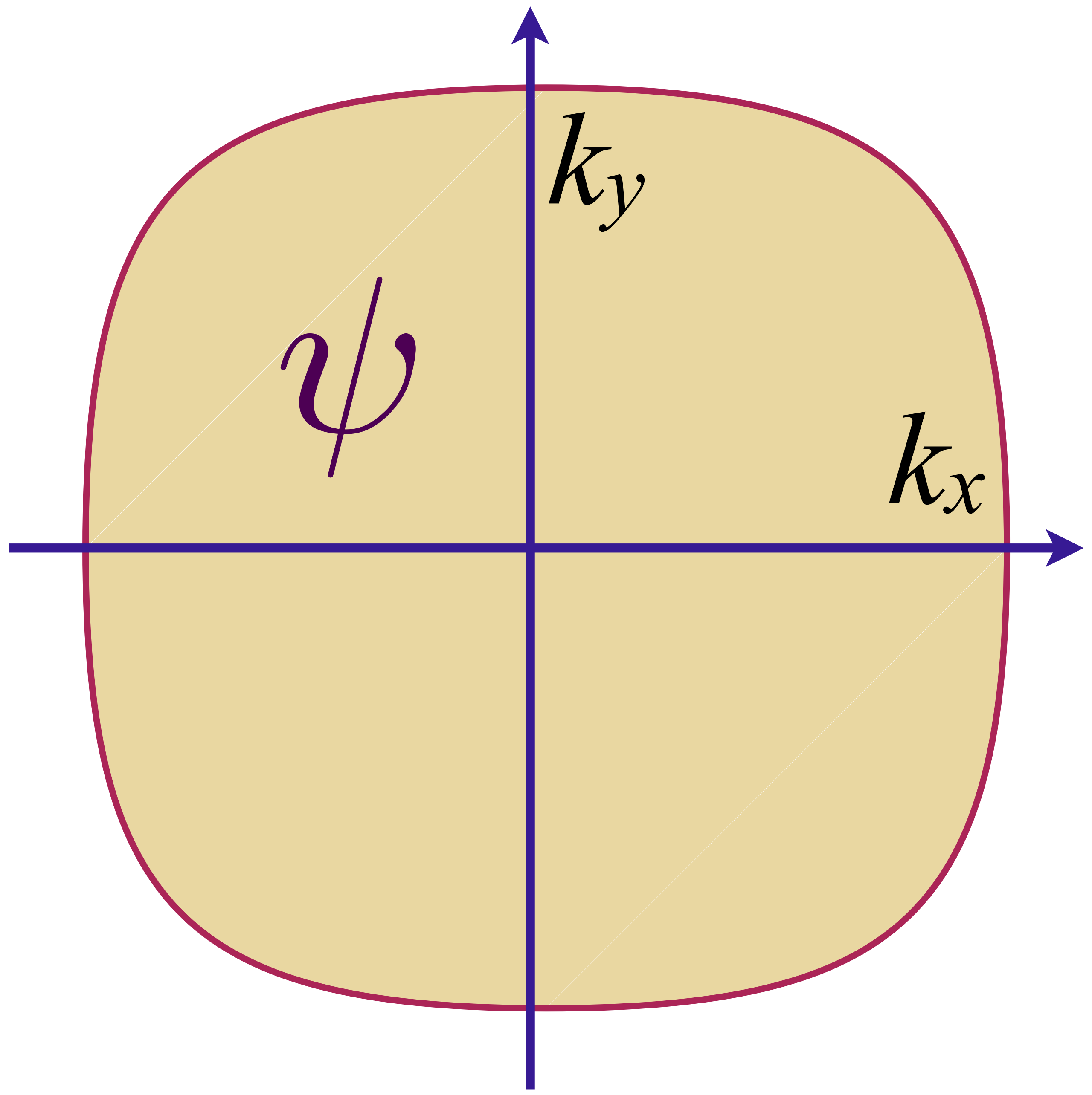
Described by the condensation of a *Higgs* field Φ.

Strange metal is obtained from the $T > 0$ quantum criticality of the FL-FL* transition, *provided* there is momentum relaxation.
At low T this requires spatial disorder.
Most relevant is Harris disorder: spatial variation in the value of p_c.

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

2D-YSYK model: Fermi surface + Higgs boson with interaction disorder

$$\mathcal{L} = c^\dagger_{\boldsymbol{k}\alpha}\left(\frac{\partial}{\partial\tau} + \varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\alpha} + f^\dagger_{1\boldsymbol{k}\alpha}\left(\frac{\partial}{\partial\tau} + \widetilde{\varepsilon}(\boldsymbol{k})\right)f_{1\boldsymbol{k}\alpha}$$



$$+ [\nabla\Phi(\boldsymbol{r})]^2 + s\,[\Phi(\boldsymbol{r})]^2 + u\,[\Phi(\boldsymbol{r})]^4$$

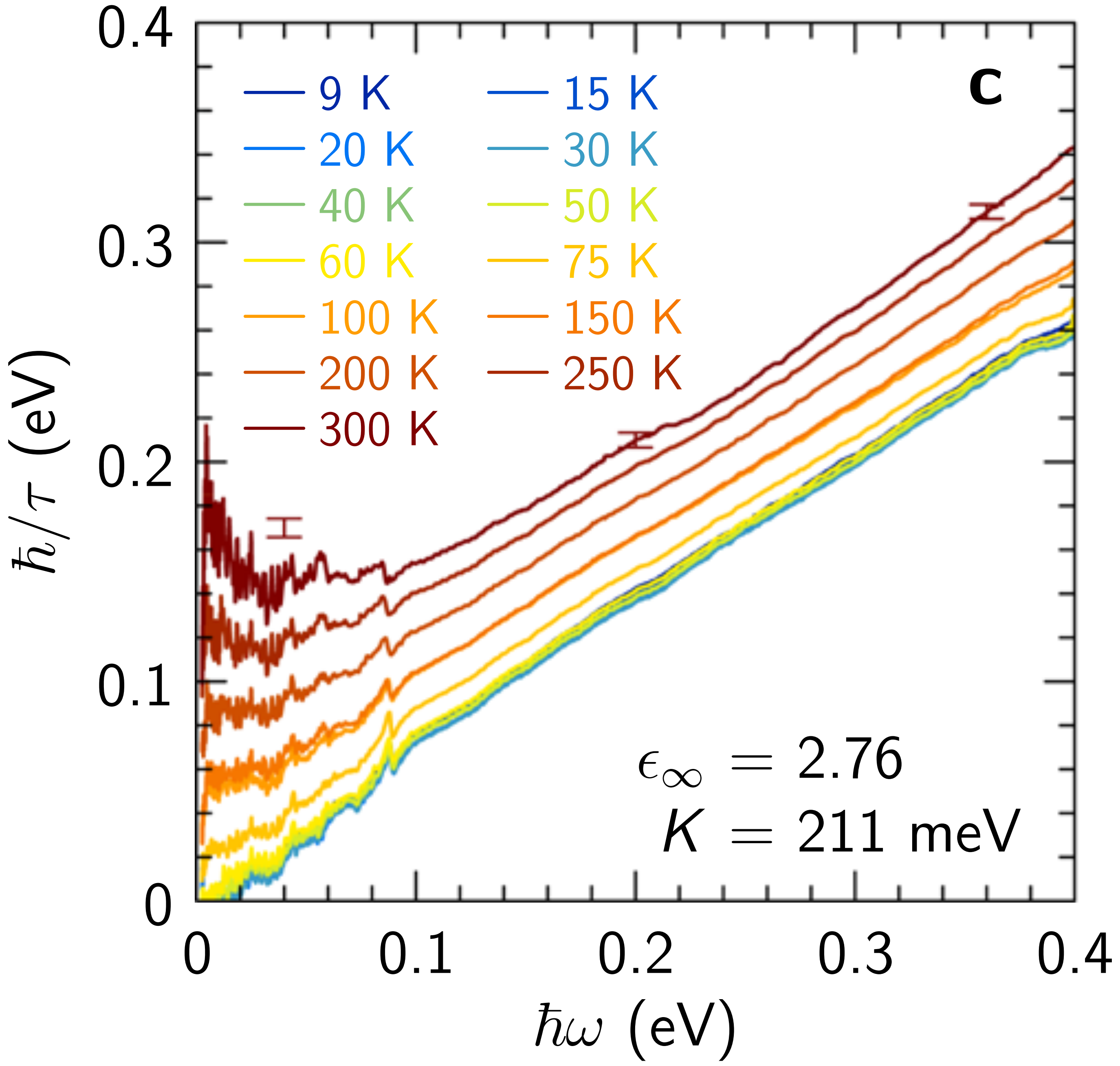
$$+ [g + g'(\boldsymbol{r})]\, c^\dagger_\alpha(\boldsymbol{r}) f_{1\alpha}(\boldsymbol{r})\,\Phi(\boldsymbol{r}) + \text{H.c.}$$

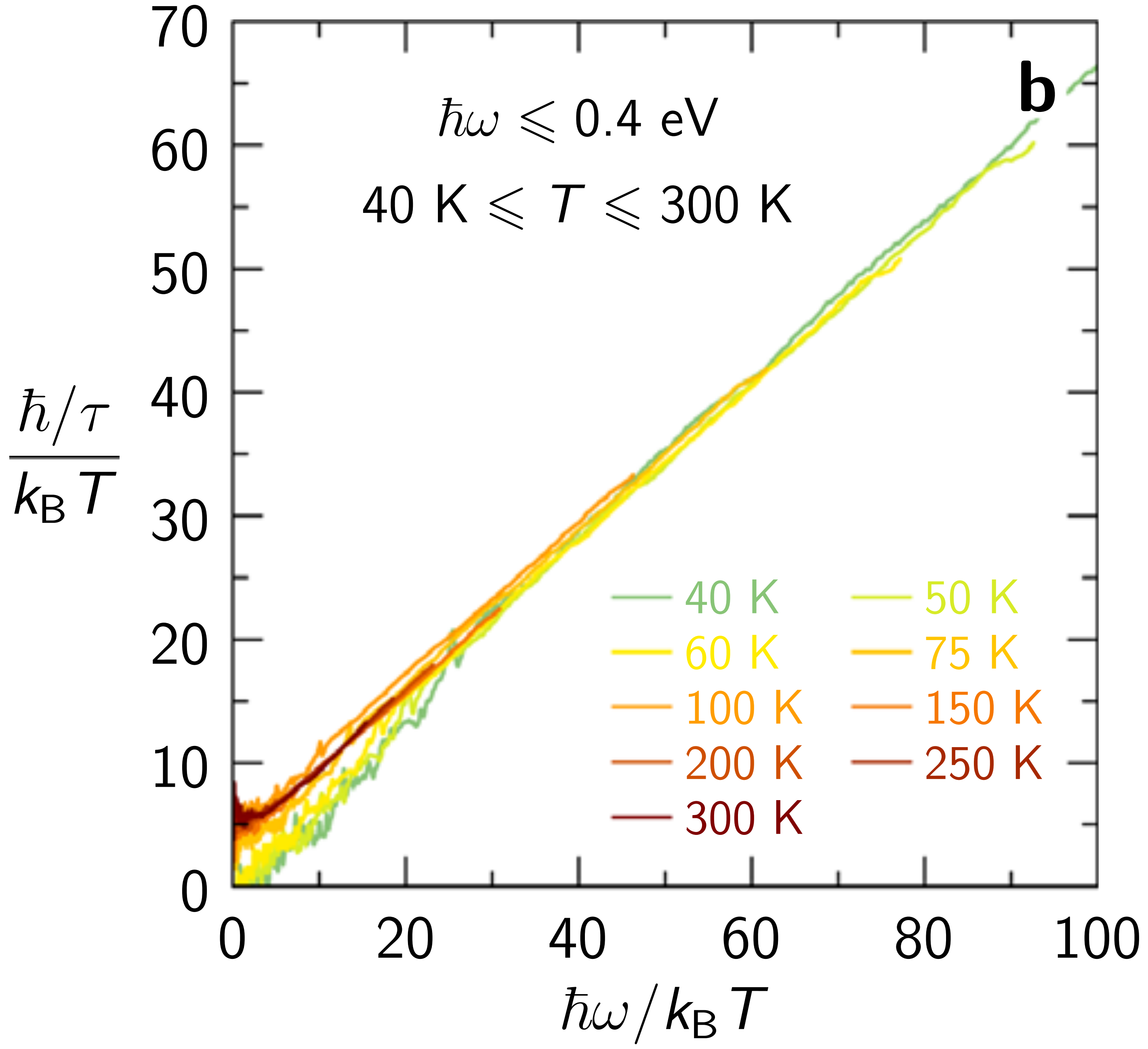
$$+ v(\boldsymbol{r}) c^\dagger_\alpha(\boldsymbol{r}) c_\alpha(\boldsymbol{r})$$

Φ^2 "mass" disorder $s \to s + \delta s(\boldsymbol{r})$ is strongly relevant; rescale Φ to move disorder to the Yukawa coupling.

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

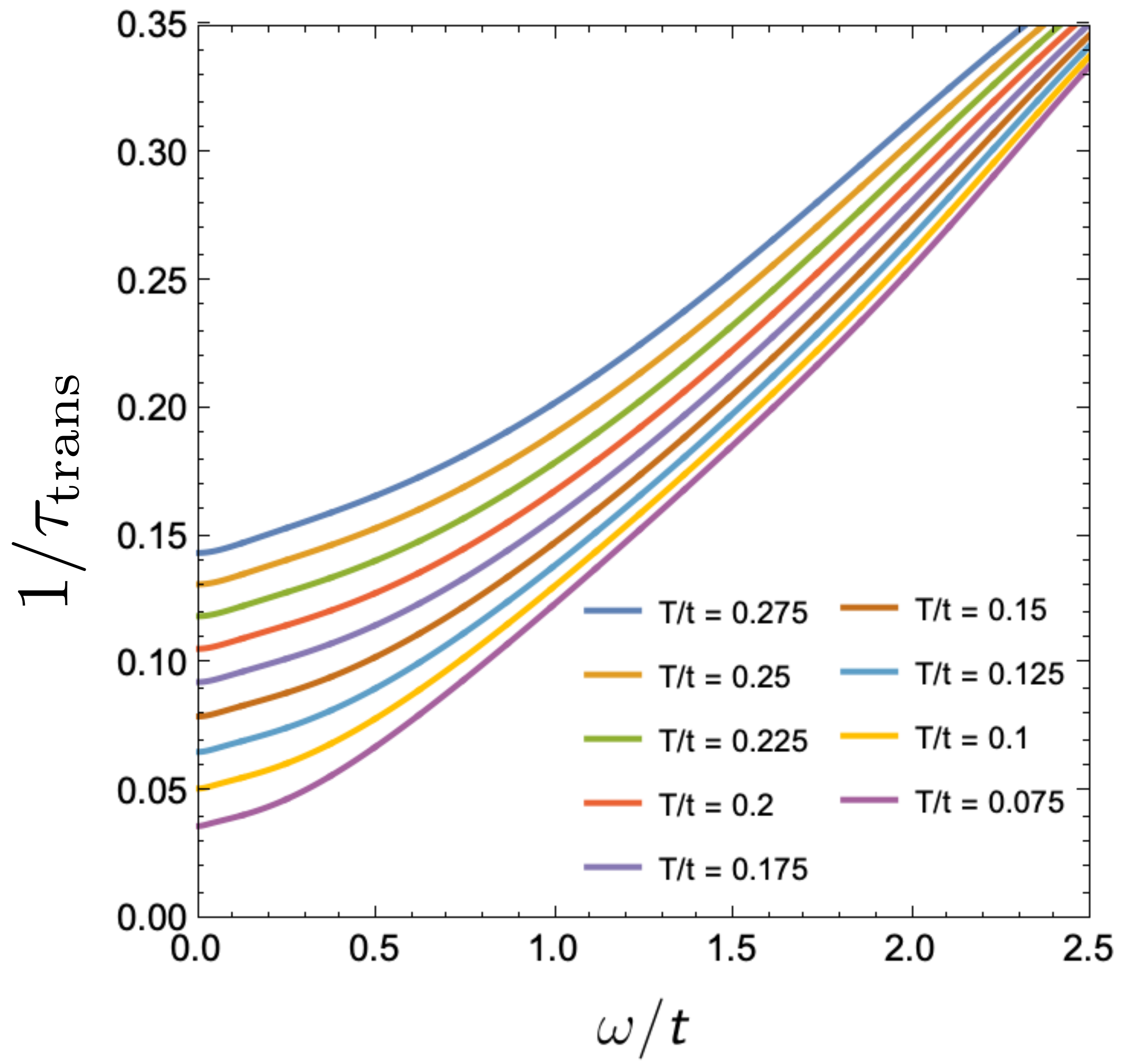
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

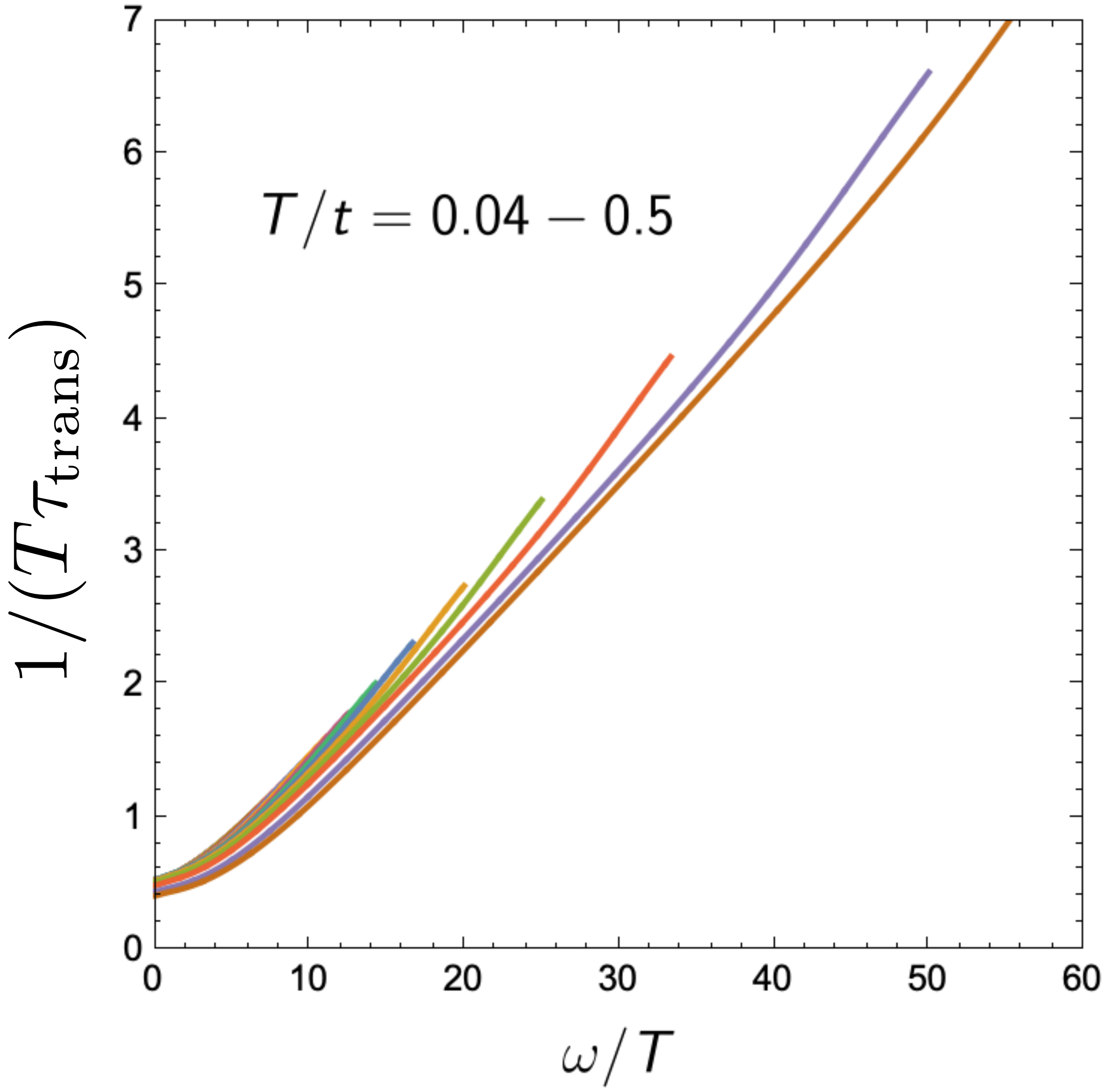




$$\sigma(\omega) = i \frac{e^2 K/(\hbar d_c)}{\hbar\omega \dfrac{m^*(\omega)}{m} + i \dfrac{\hbar}{\tau(\omega)}}$$

From optical conductivity data of Michon et al. (2023)



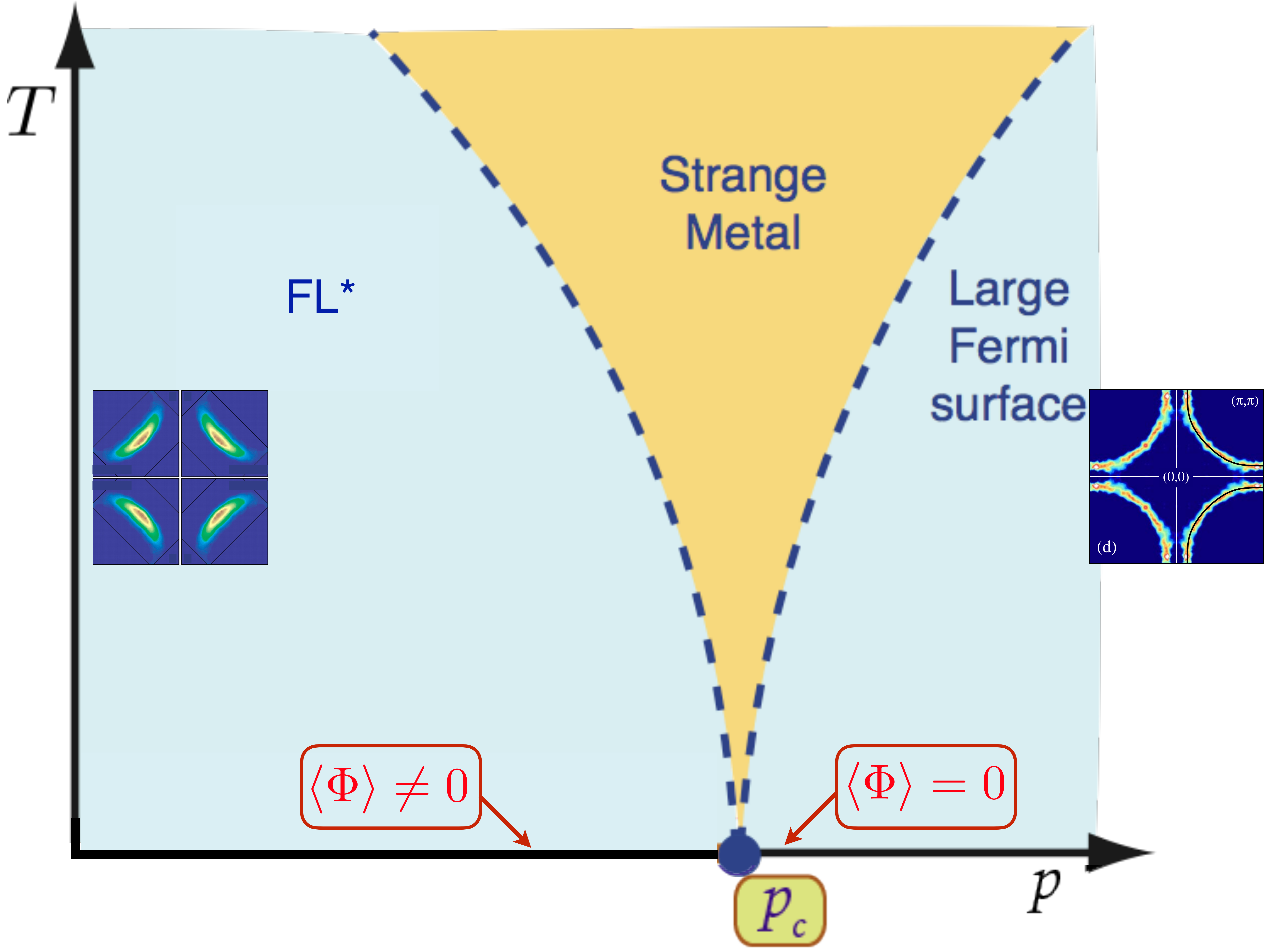


$$\frac{\hbar}{\tau(\omega)} = k_B T \, \Phi_\tau \left(\frac{\hbar\omega}{k_B T} \right)$$

2d-YSYK theory

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

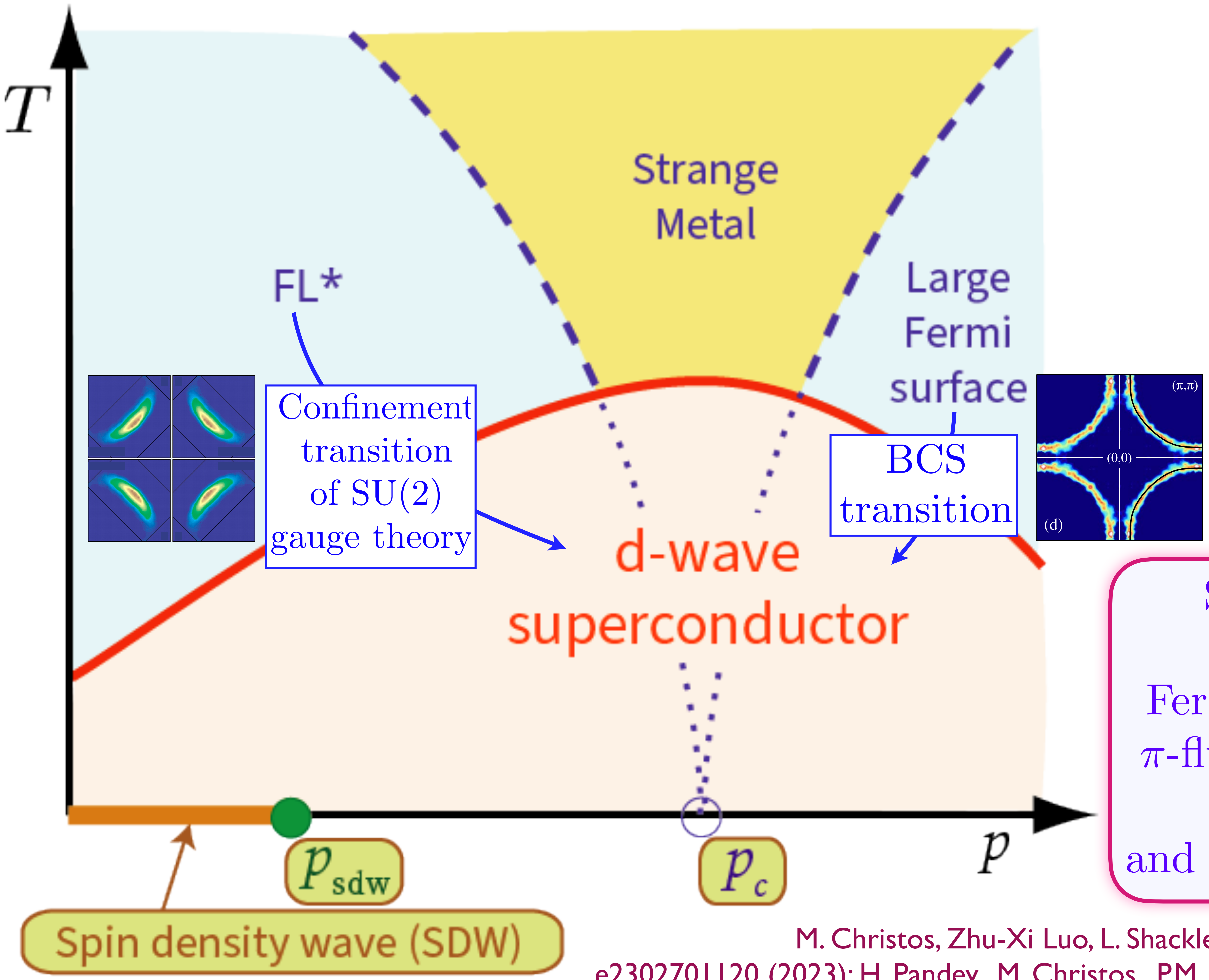


Quantum phase transition between two metals (FL* and FL) at $p = p_c$, with no symmetry breaking.

Described by the condensation of a *Higgs* field Φ.

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

S. S., Physica Status Solidi B Basic Research **247**, 537 (2010)



Both metals lead to the same d-wave superconductor at lower temperatures, and so there is no transition at $p = p_c$ within the superconducting state.

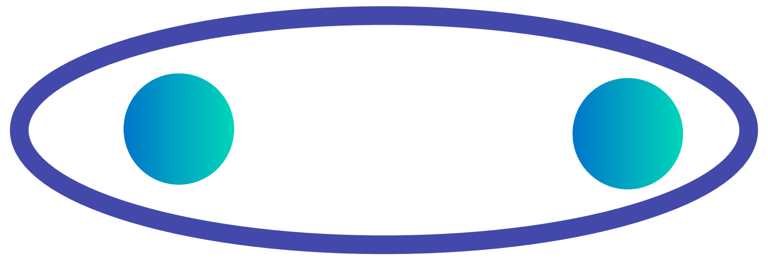
SU(2) gauge theory similar to Weinberg-Salam theory:
Fermionic spinons (*cf.* neutrinos) of π-flux state (Affleck-Marston, 1988),
electrons,
and SU(2) fundamental Higgs field B.

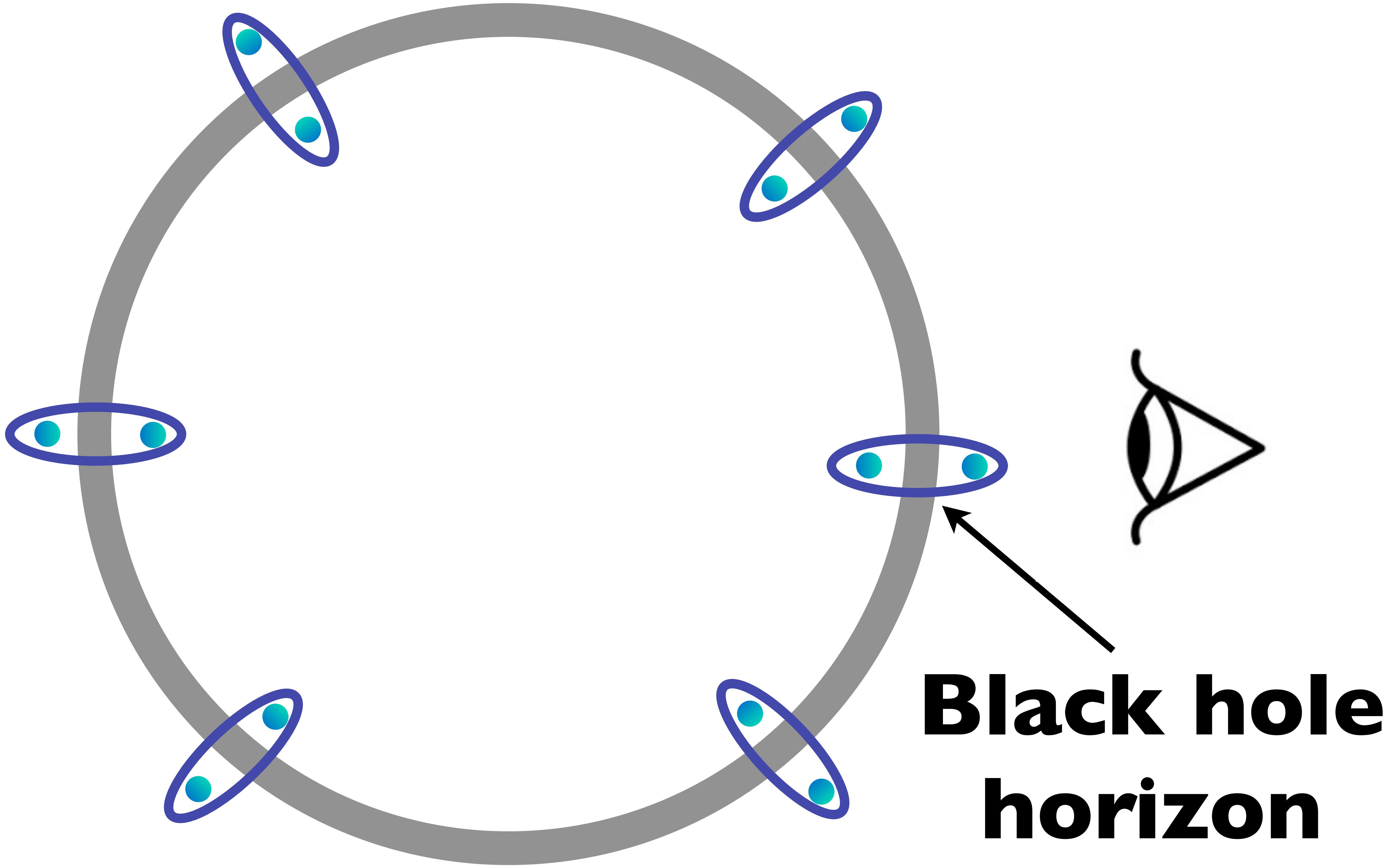
M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120**, e2302701120 (2023); H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

Quantum entanglement, the SYK model, and black holes

Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



By computations *outside* the black hole, Hawking obtained the black hole entropy

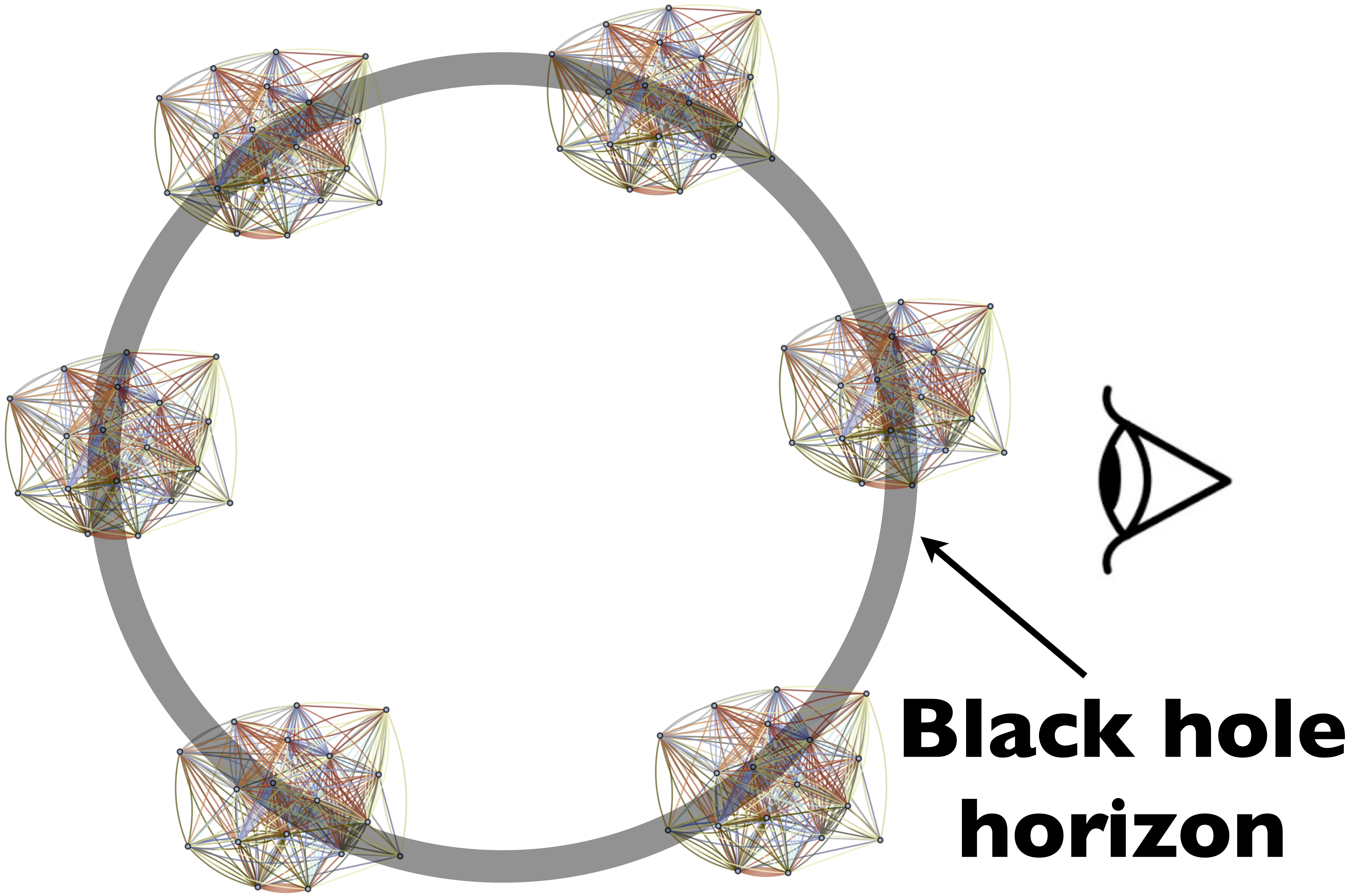
$$S = \frac{Ac^3}{4G\hbar}$$

where A is area of the black hole horizon.

All other systems have entropy proportional to their volume.

Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface



By computations *outside* the black hole, Hawking obtained the black hole entropy

$$S = \frac{Ac^3}{4G\hbar}$$

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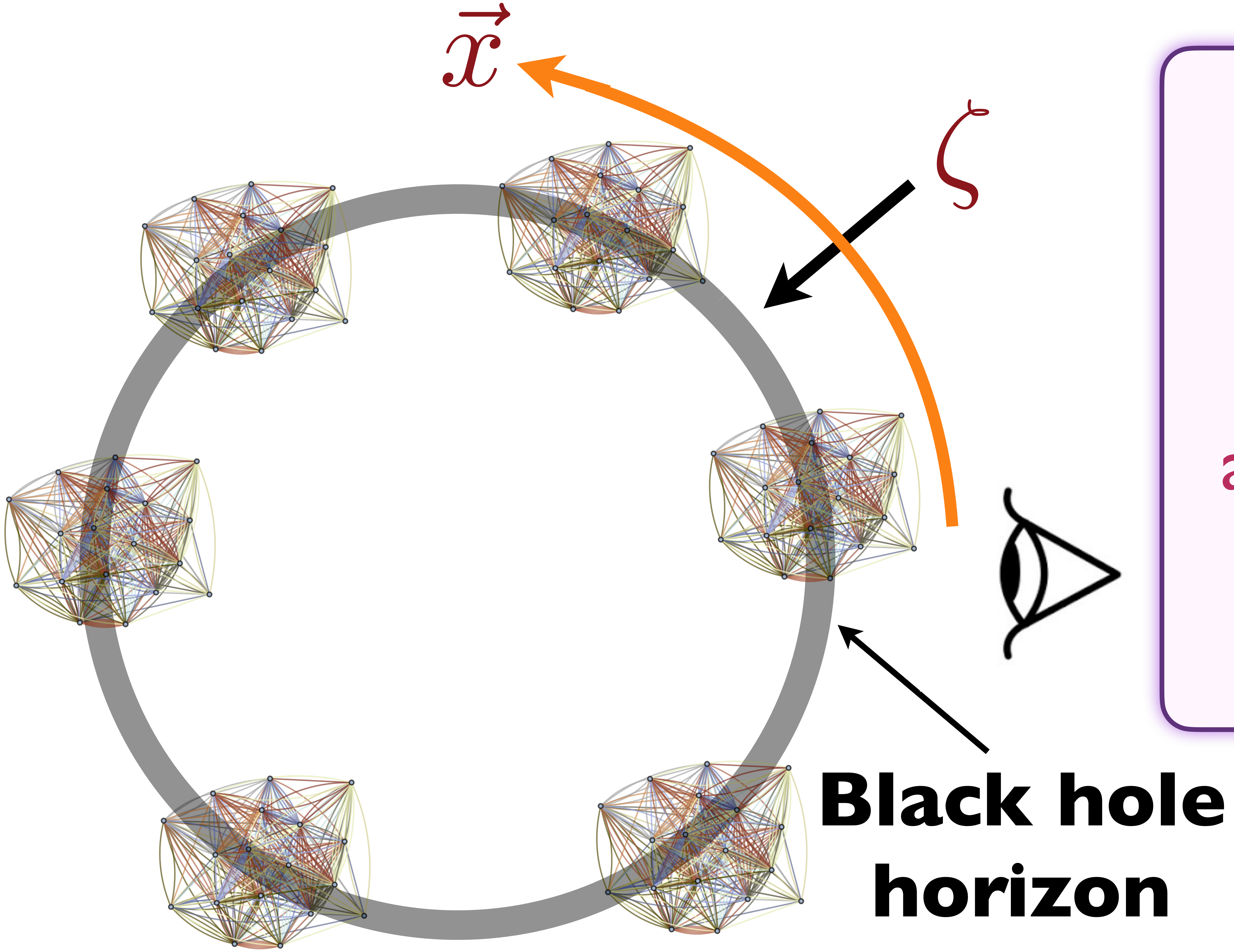
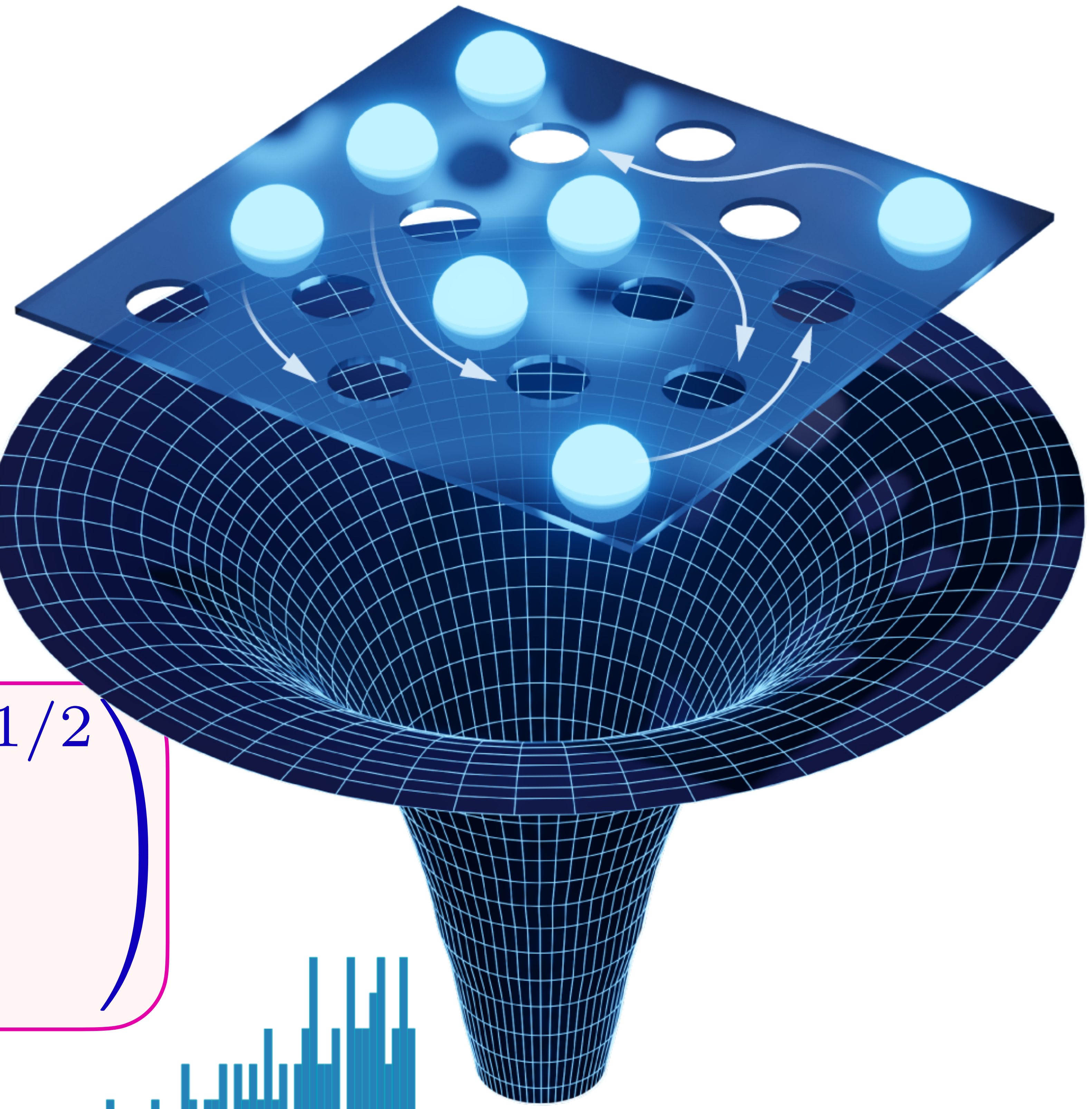
All other systems have entropy proportional to their volume.

Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge



The quantum versions of Maxwell's and Einstein's equations in ζ space and time are also the equations describing electron entanglement in the SYK model!

D(E) of charged black holes from the SYK model



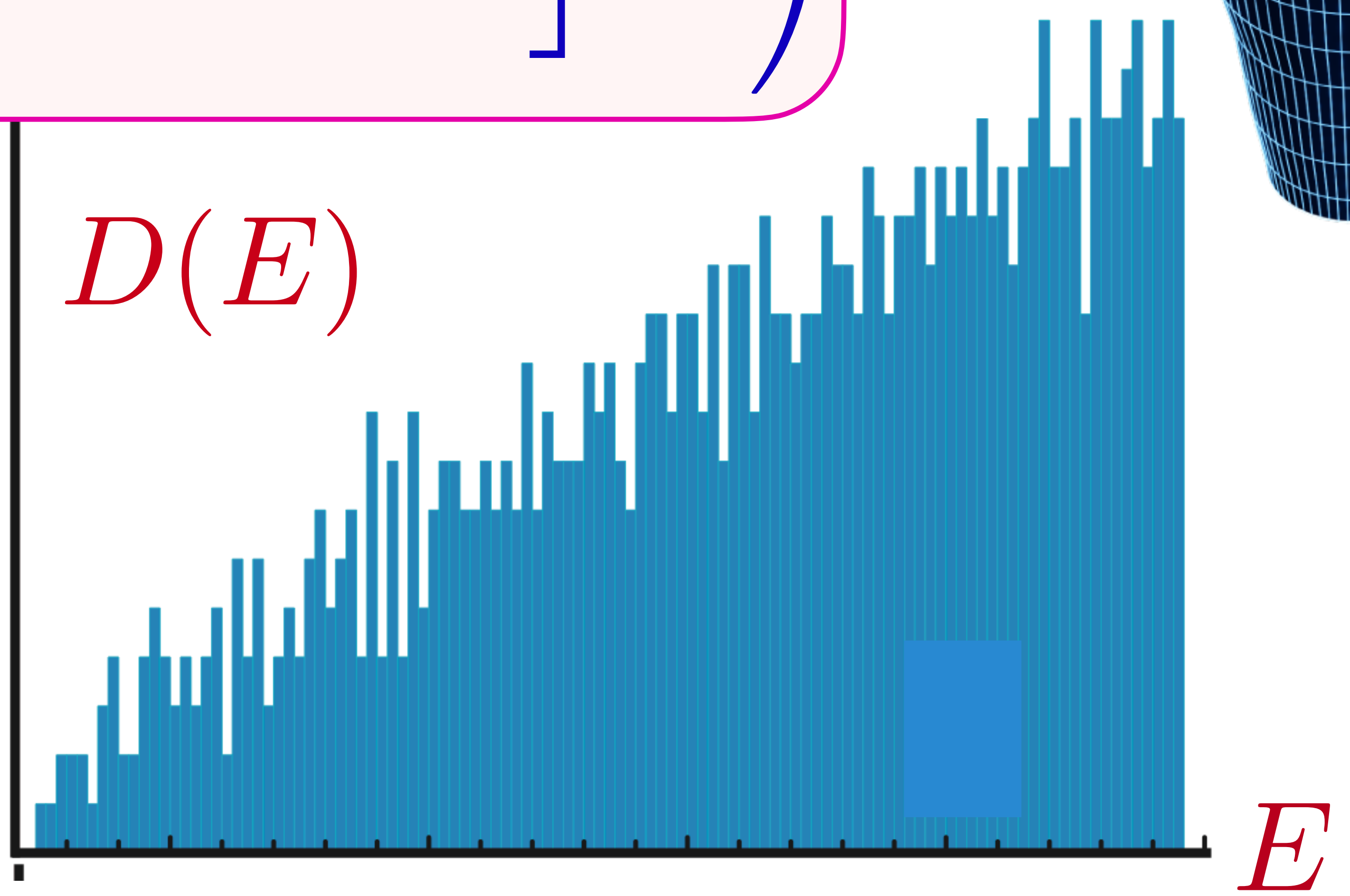
- For generic charged black holes in 3+1 dimensions with horizon area A_0 at $T = 0$ and fixed charge $\mathcal{Q}$ ($A_0 = 2G\mathcal{Q}^2/c^4$), the density of quantum states at small energy E is

Bekenstein-Hawking

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} {A_0}^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

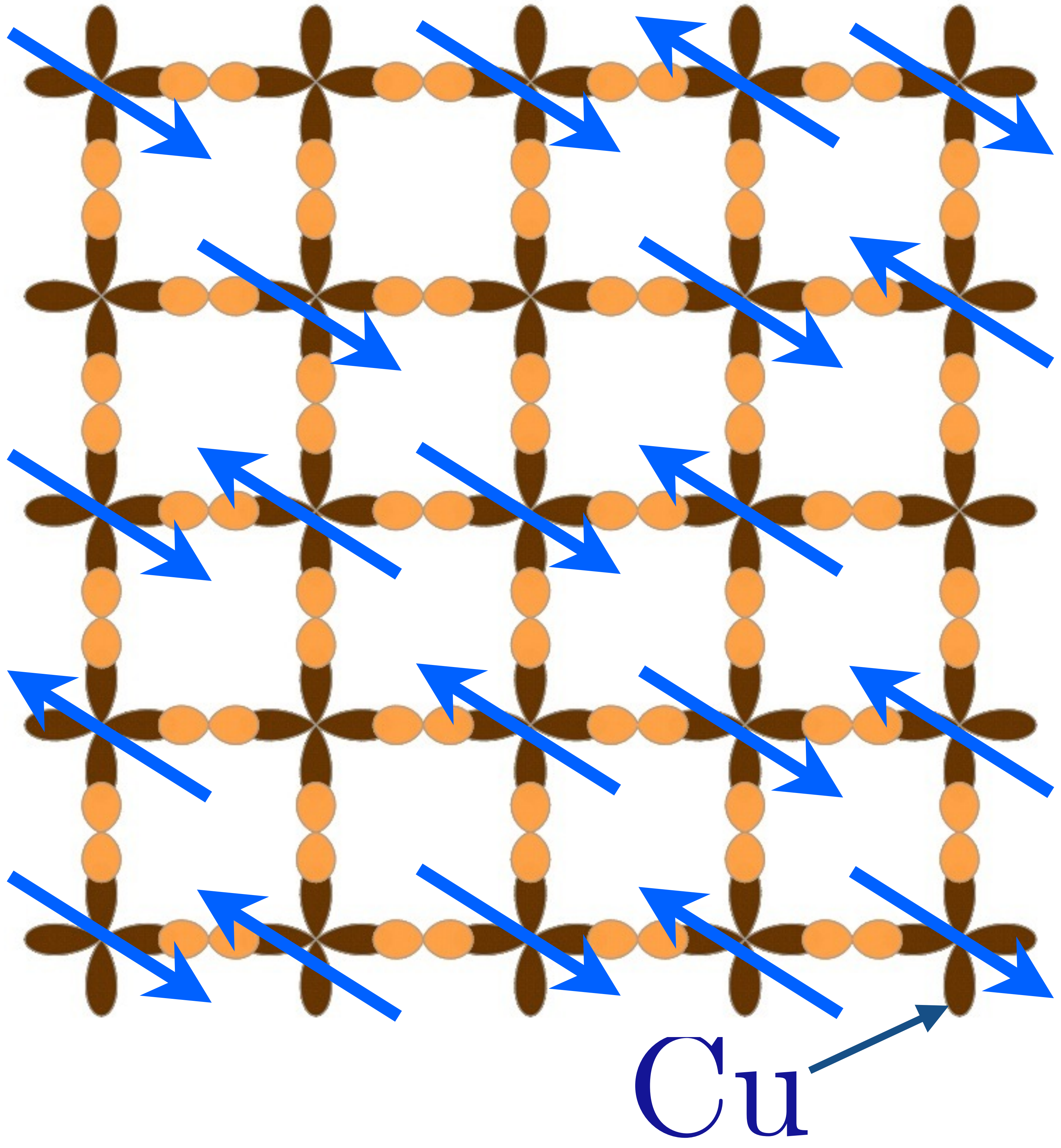
Iliesiu, Murthy, Turiaci (2022)

Developments from the SYK model



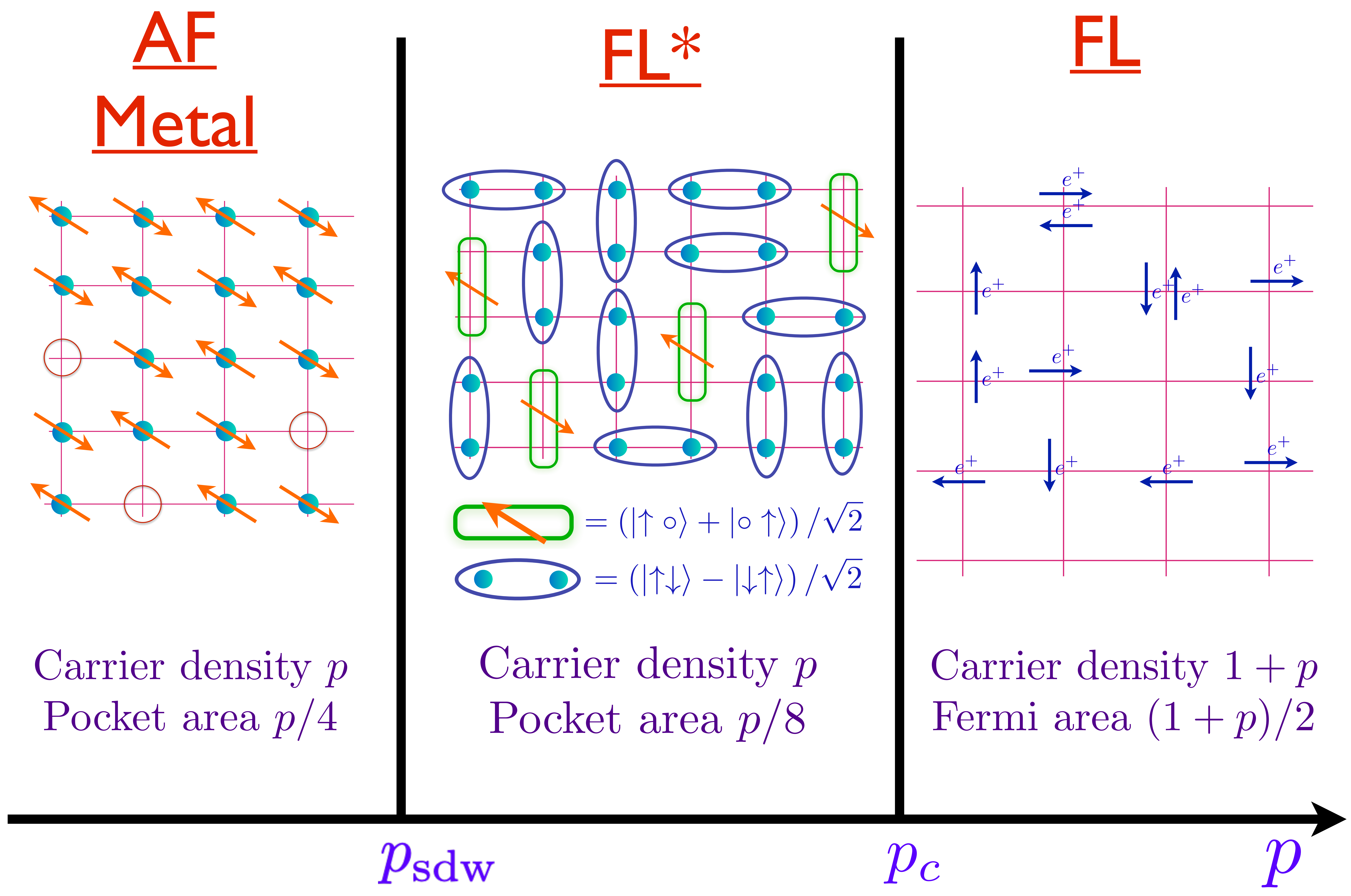
Similar remarks apply to rotating neutral black holes.

D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)



d-SC obtained upon doping AF with density p holes. Hole density relative to the filled band $\rho = 1 + p$. Electron density relative to the empty band $\rho_e = 1 - p$.

Cuprate recap

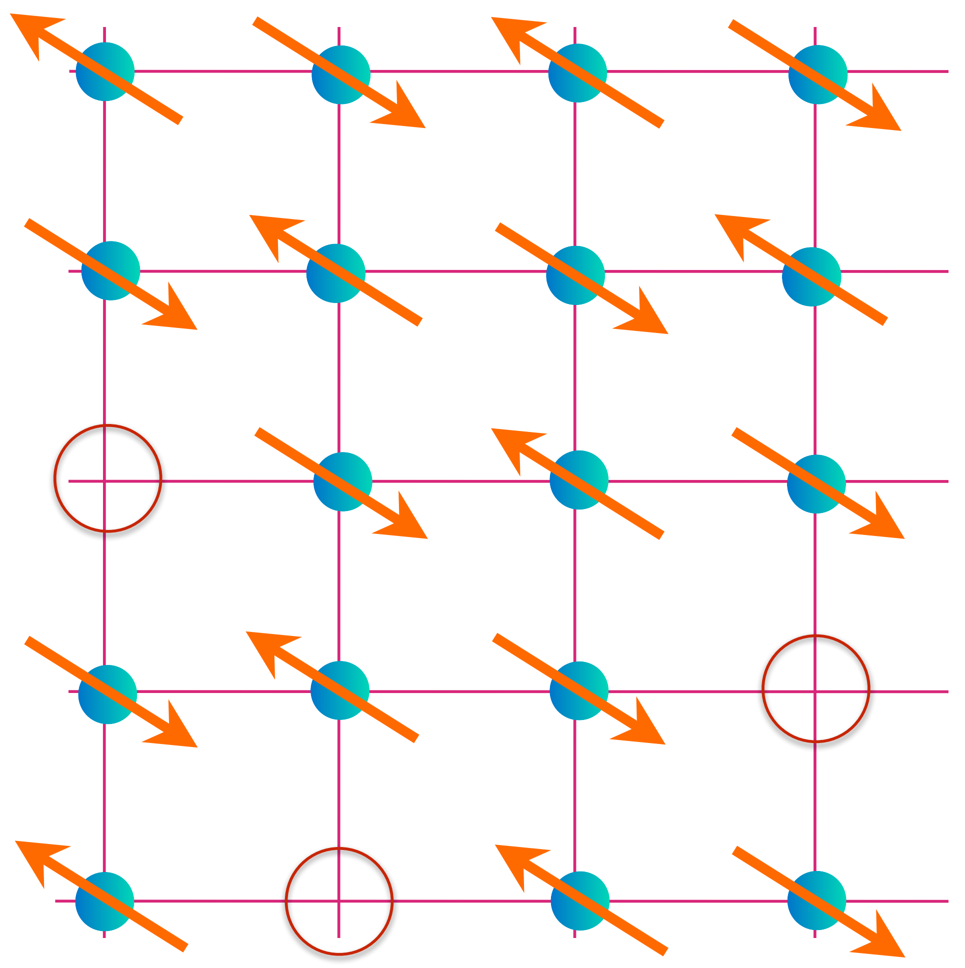


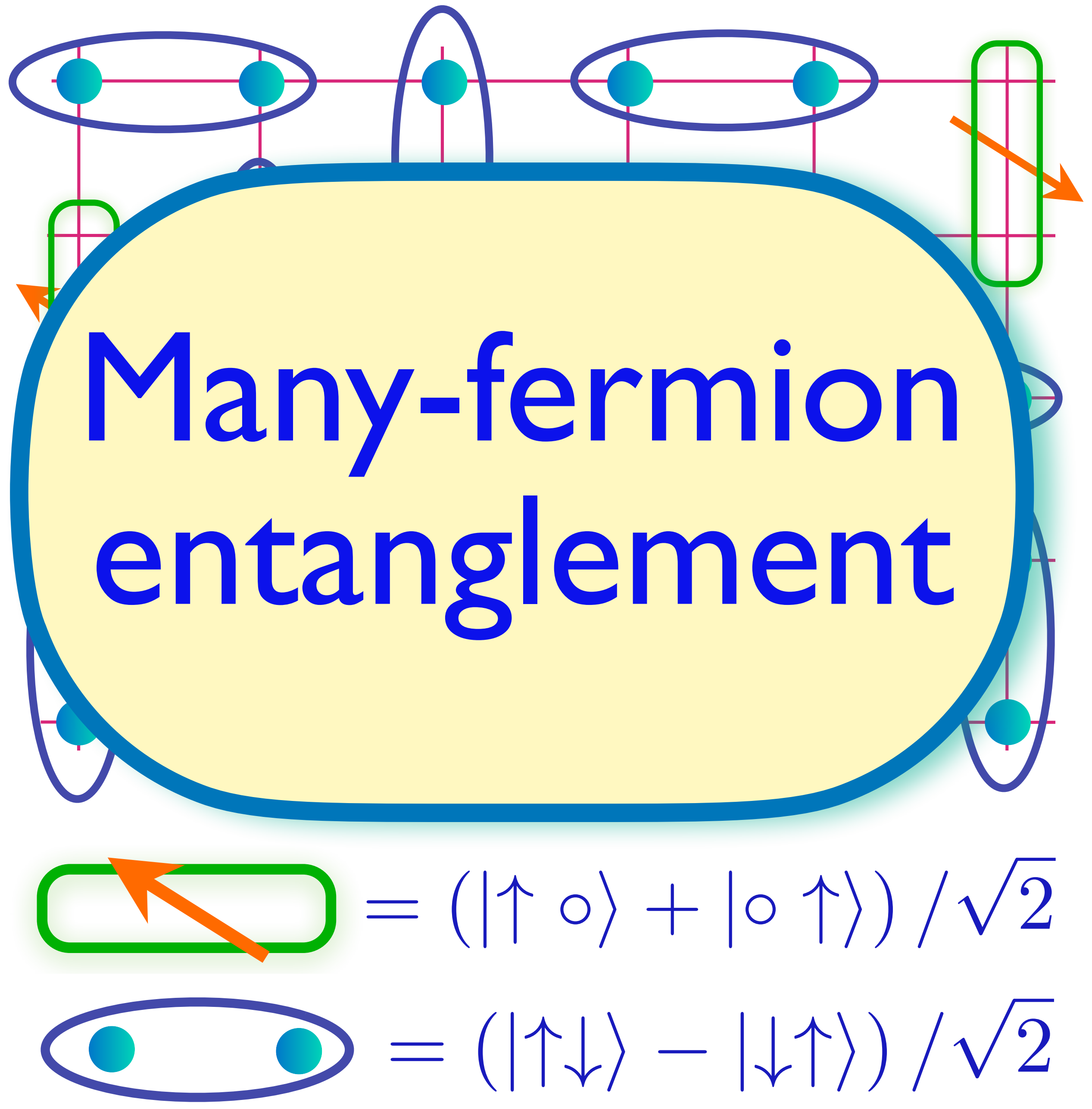
S. Sachdev, M.A. Metlitski and M. Punk, Journal of Physics Condensed Matter **24**, 294205 (2012)

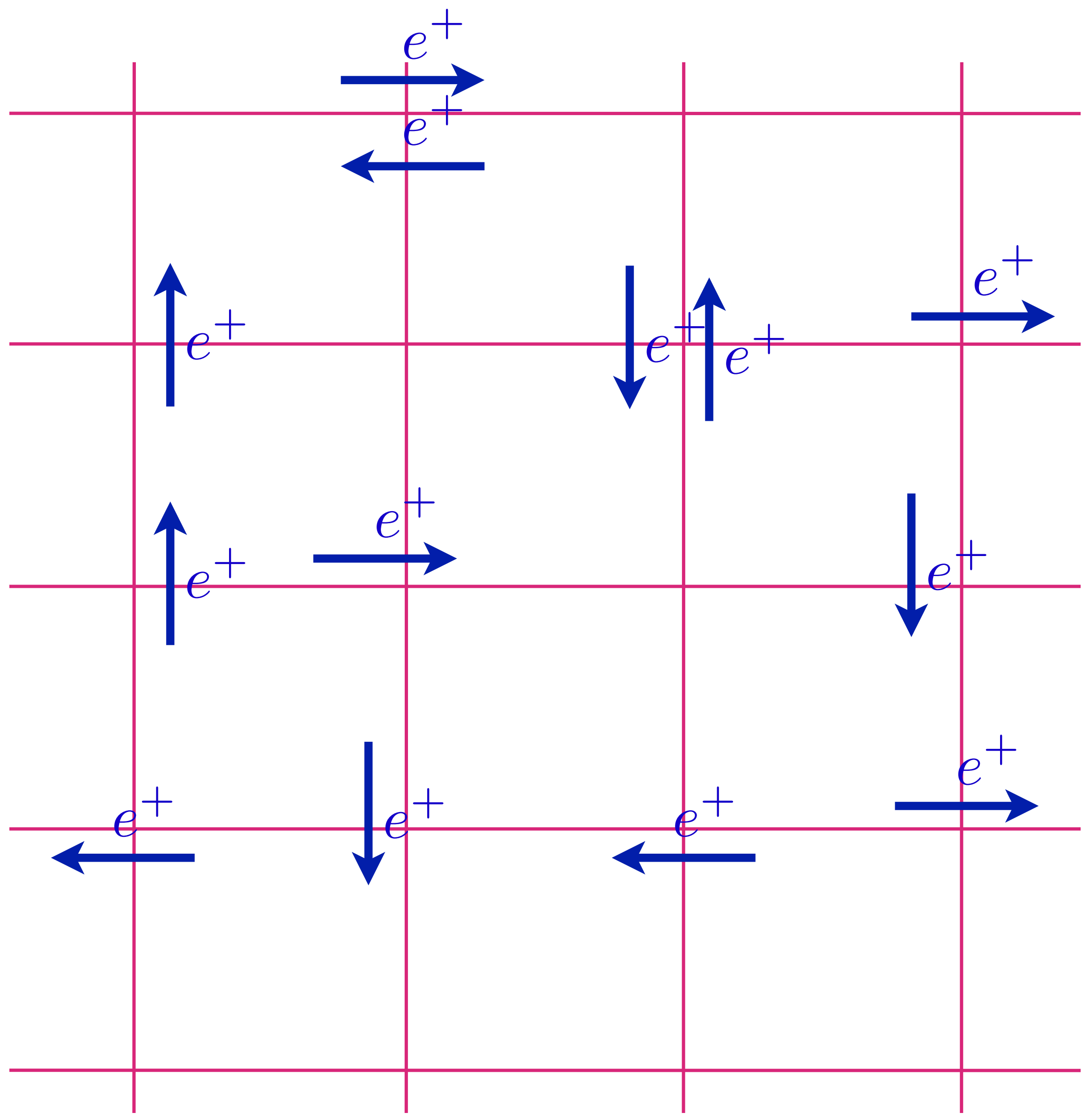
AF Metal

FL*

FL



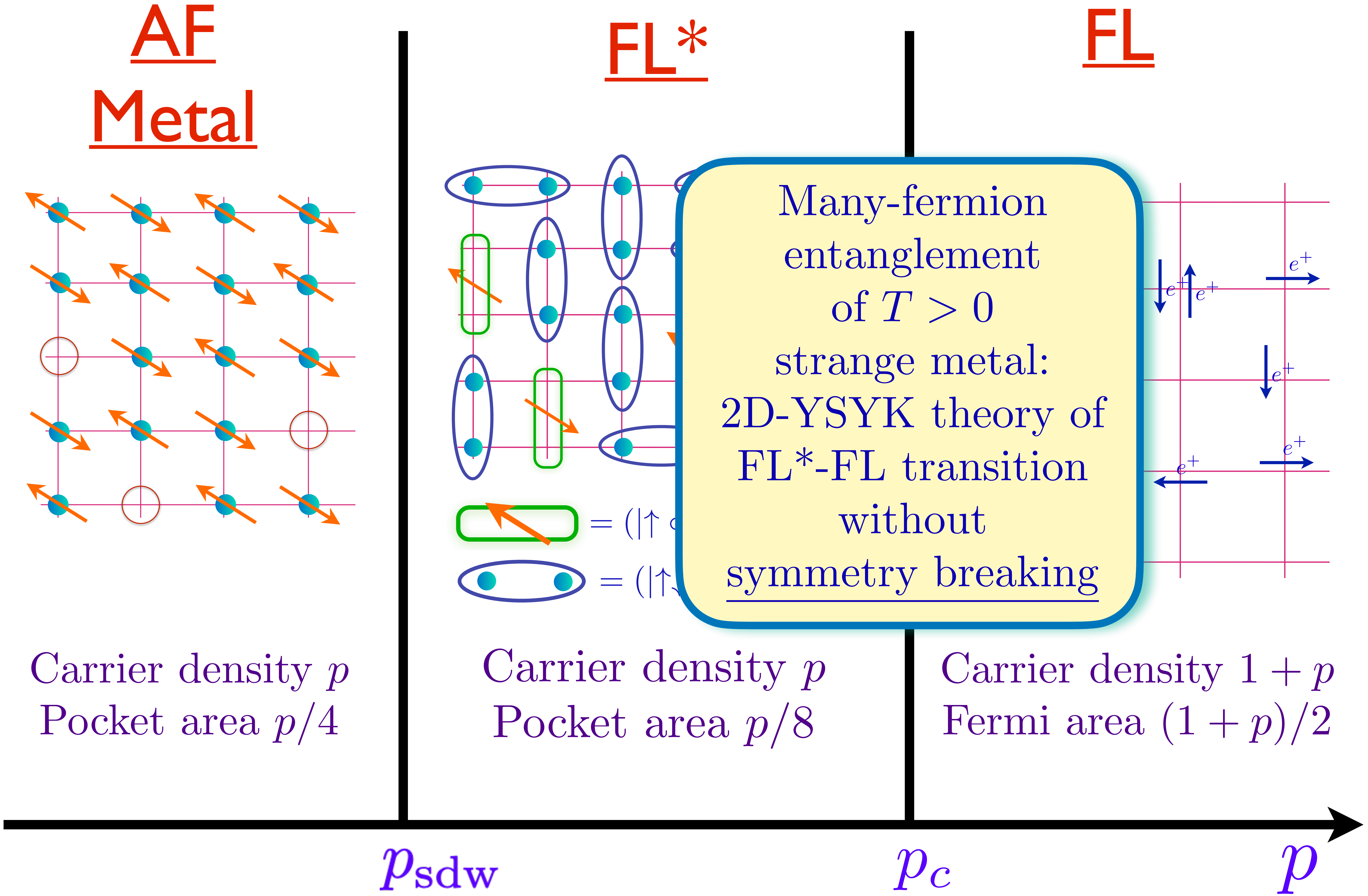




AF Metal	FL*	FL
Carrier density p	Carrier density p	Carrier density $1 + p$
Pocket area $p/4$	Pocket area $p/8$	Fermi area $(1+p)/2$

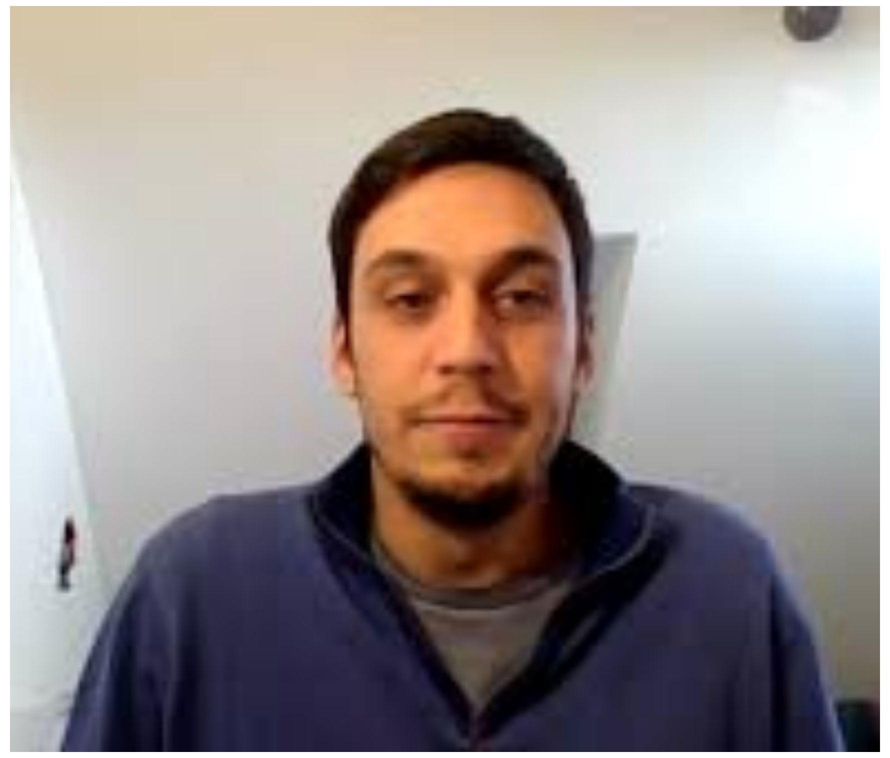
$p_{\rm sdw}$ p_c p

S. Sachdev, M.A. Metlitski and M. Punk, Journal of Physics Condensed Matter **24**, 294205 (2012)



S. Sachdev, M.A. Metlitski and M. Punk, Journal of Physics Condensed Matter **24**, 294205 (2012)

Maine Christos
Caltech



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- *Lectures on insulating and conducting quantum spin liquids*, S. Sachdev, arXiv:2512.23962
- *Fractionalized Fermi liquids and the cuprate phase diagram*, P. M. Bonetti, M. Christos, A. Nikolaenko, A.A. Patel, and S. Sachdev, arXiv:2508.20164
- *Thermal SU(2) lattice gauge theory of the cuprate pseudogap: reconciling Fermi arcs and hole pockets*, H. Pandey, M. Christos, P. M. Bonetti, R. Shanker, A. Nikolaenko, S. Sharma, and S. Sachdev, arXiv:2507.05336