Outline

1. Antiferromagnetism and quantum criticality in insulators

2. Onset of antiferromagnetism in metals, and d-wave superconductivity

3. Competing density wave order, and the pseudogap of the cuprate superconductors

4. Non-Fermi liquids

Pseudospin symmetry of the exchange interaction

$$H_J = \sum_{i<j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

with $\vec{S}_i = \frac{1}{2} c^{\dagger}_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c^{\dagger}_{i\downarrow} \end{pmatrix} \quad , \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c^{\dagger}_{i\uparrow} \end{pmatrix}$$

Then we can write

$$H_J = \frac{1}{8} \sum_{i<j} J_{ij} \left(\Psi^{\dagger}_{i\alpha a} \vec{\sigma}_{\alpha\beta} \Psi_{i\beta a} \right) \cdot \left(\Psi^{\dagger}_{j\gamma b} \vec{\sigma}_{\gamma\delta} \Psi_{j\delta b} \right)$$

where a, b are the Nambu indices. This form makes explicit the symmetry under *independent* SU(2) pseudospin transformations on each site

$$\Psi_{i\alpha a} \to U_{i,ab} \Psi_{i\alpha b}$$

This pseudospin (gauge) symmetry is important in classifying spin liquid ground states of H_J.

I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988)
E. Dagotto, E. Fradkin, and A. Moreo, Phys. Rev. B **38**, 2926 (1988)
P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006)

Pseudospin symmetry of the exchange interaction

$$H_{tJ} = -\sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \sum_{i<j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + V \sum_{\langle ij \rangle} n_i n_j$$

with $\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow}^\dagger \end{pmatrix} \quad , \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c_{i\uparrow}^\dagger \end{pmatrix}$$

Then we can write

$$H_J = \frac{1}{8} \sum_{i<j} J_{ij} \left(\Psi_{i\alpha a}^\dagger \vec{\sigma}_{\alpha\beta} \Psi_{i\beta a} \right) \cdot \left(\Psi_{j\gamma b}^\dagger \vec{\sigma}_{\gamma\delta} \Psi_{j\delta b} \right)$$

where a, b are the Nambu indices. This form makes explicit the symmetry under *independent* SU(2) pseudospin transformations on each site

$$\Psi_{i\alpha a} \rightarrow U_{i,ab} \Psi_{i\alpha b}$$

This pseudospin (gauge) symmetry is important in classifying spin liquid ground states of H_J.

The pseudospin symmetry is fully broken by t_{ij}, V_{ij}, and the chemical potential, and so expected to be unimportant away from half-filling.

$$\mathcal{L}_f = \psi^\dagger_{1\alpha}\left(\partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right)\psi_{1\alpha} + \psi^\dagger_{2\alpha}\left(\partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right)\psi_{2\alpha}$$

Order parameter: $$\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{1}{2}\left(\partial_\tau \vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$$

"Yukawa" coupling: $$\mathcal{L}_c = -\lambda\vec{\varphi} \cdot \left(\psi^\dagger_{1\alpha}\vec{\sigma}_{\alpha\beta}\psi_{2\beta} + \psi^\dagger_{2\alpha}\vec{\sigma}_{\alpha\beta}\psi_{1\beta}\right)$$

The "hot-spot" theory for the onset of antiferromagnetism in metals is exactly invariant under the pseudospin symmetry!

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

$$\mathcal{L}_f = \psi^\dagger_{1\alpha}\left(\partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right)\psi_{1\alpha} + \psi^\dagger_{2\alpha}\left(\partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right)\psi_{2\alpha}$$

Order parameter: $$\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{1}{2}\left(\partial_\tau \vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$$

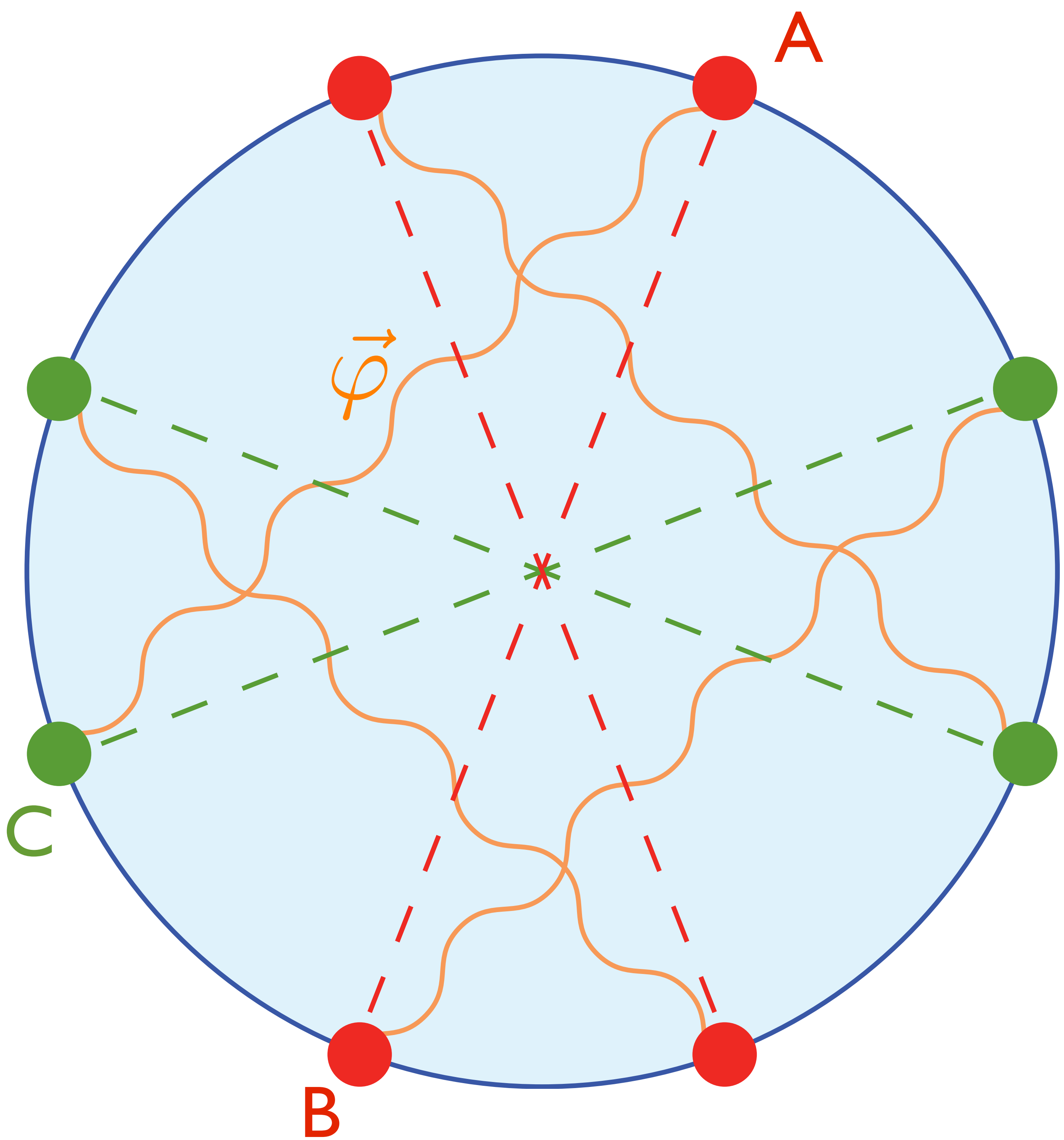
"Yukawa" coupling: $$\mathcal{L}_c = -\lambda \vec{\varphi} \cdot \left(\psi^\dagger_{1\alpha}\vec{\sigma}_{\alpha\beta}\psi_{2\beta} + \psi^\dagger_{2\alpha}\vec{\sigma}_{\alpha\beta}\psi_{1\beta}\right)$$

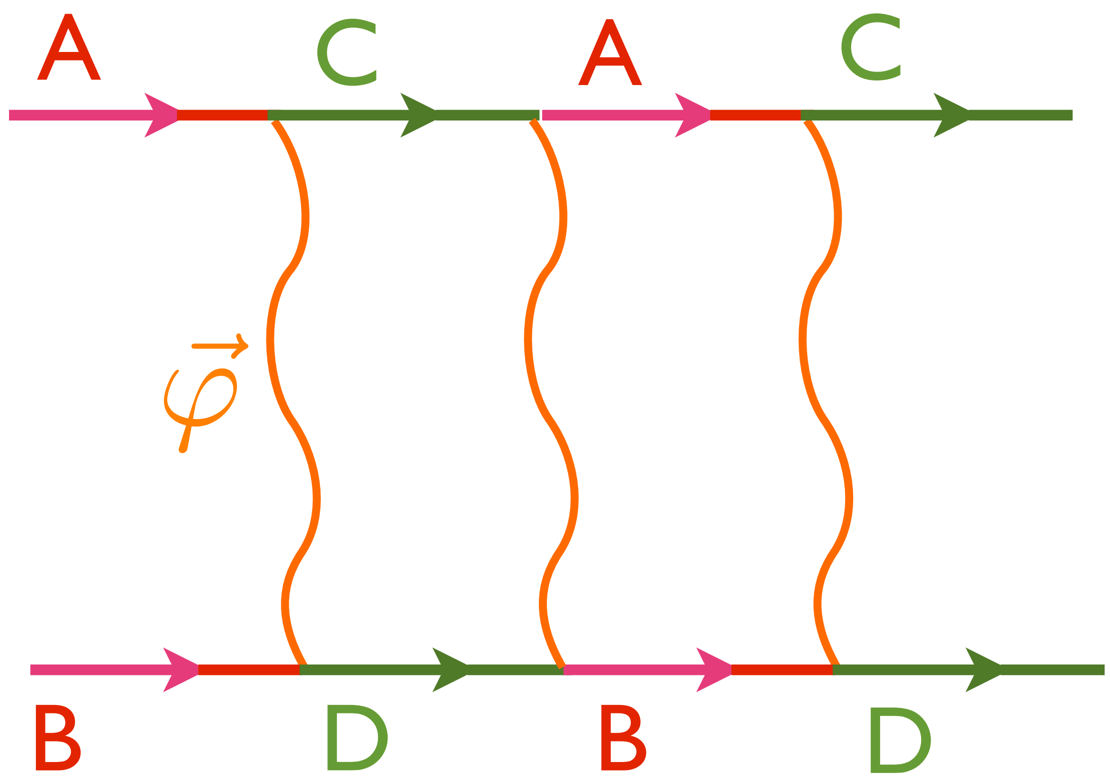
The "hot-spot" theory for the onset of antiferromagnetism in metals is <u>exactly</u> invariant under the pseudospin symmetry!

We will now move away from the antiferromagnetic critical point (and the hot-spot theory) and investigate if there are any remnant consequences of this unexpected symmetry.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

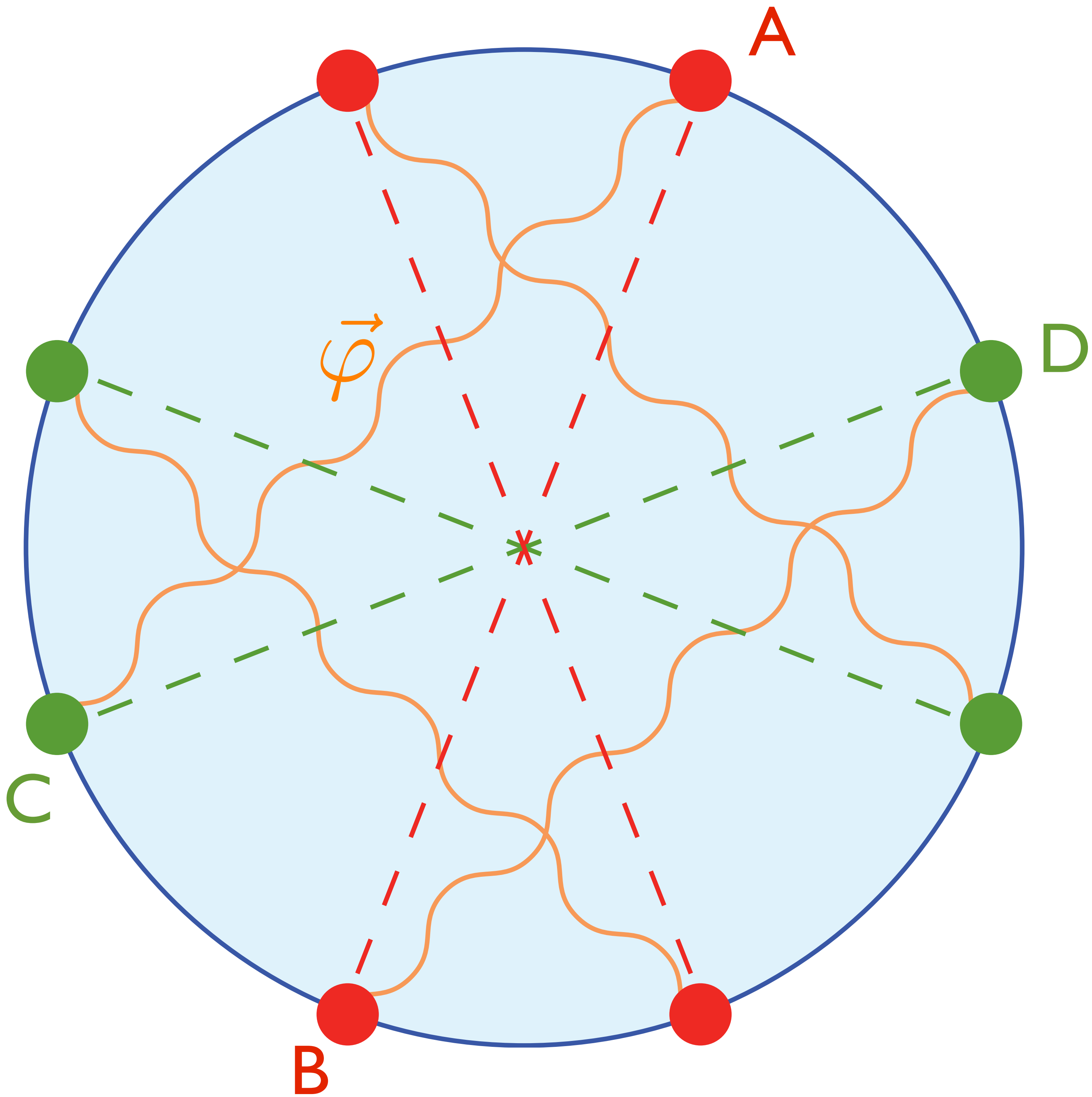
Pairing "glue" from antiferromagnetic fluctuations

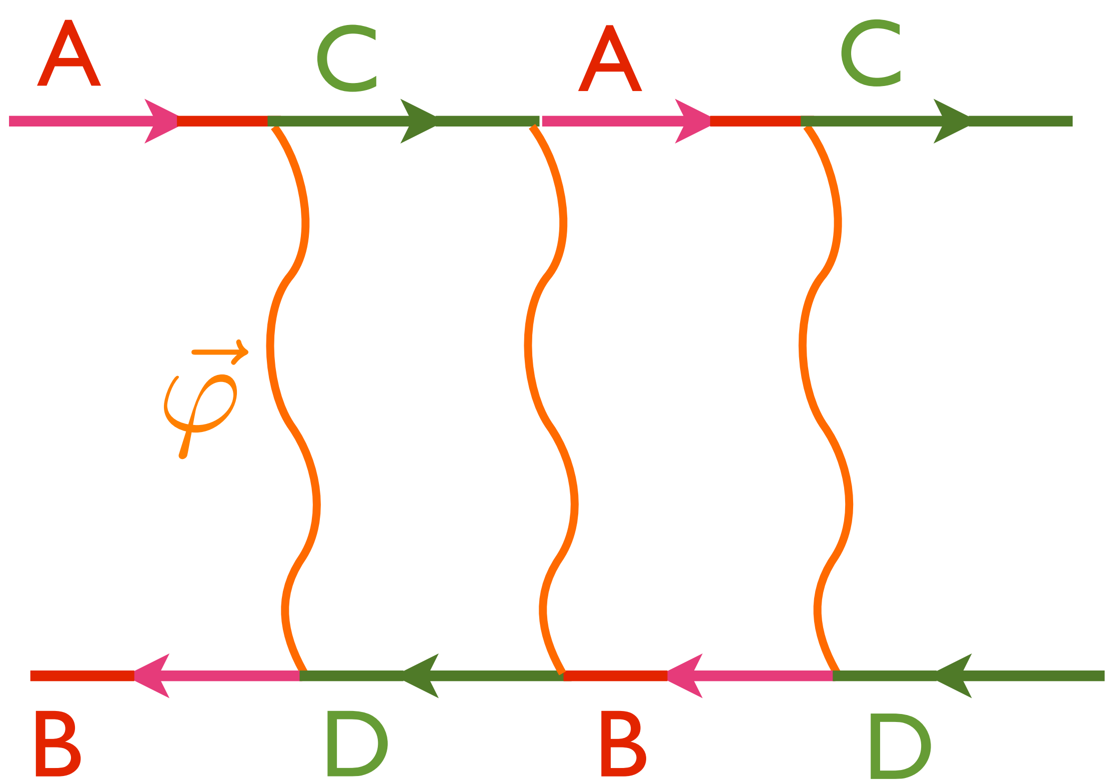




V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

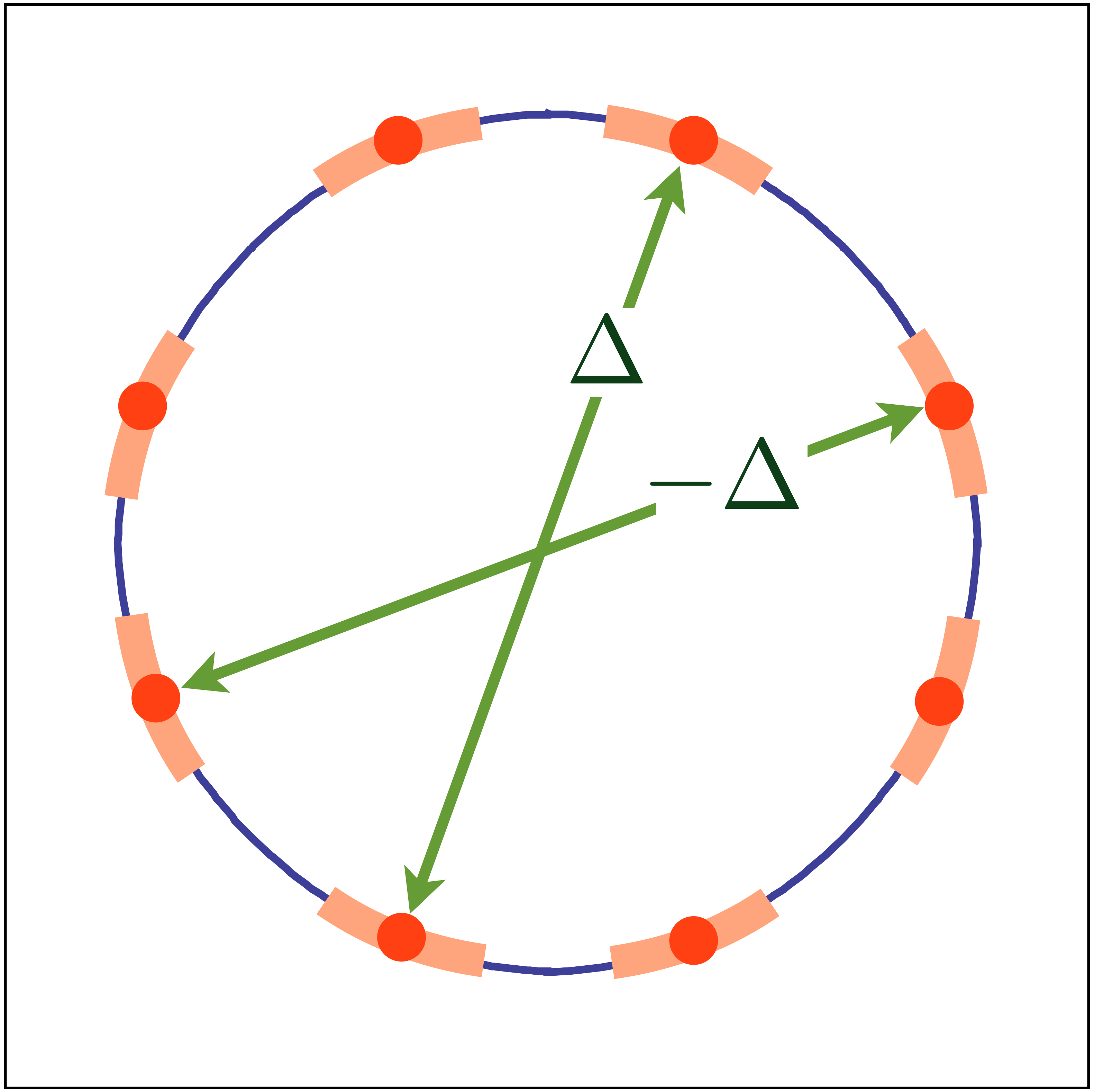
Perform pseudospin rotation on B and D electrons, but not on A and C electrons: *Same* "glue" leads to particle-hole pairing





M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

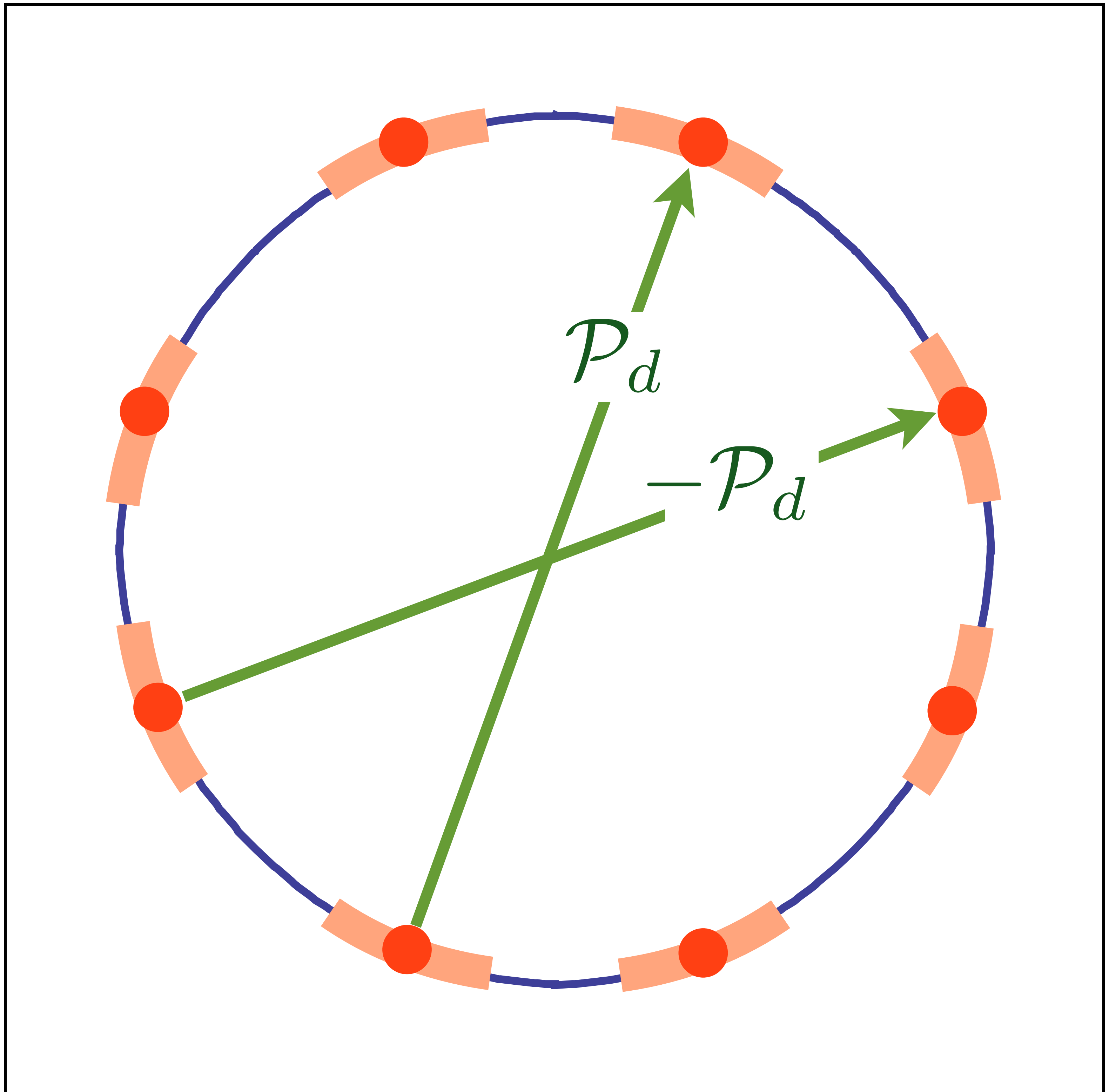
$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$



Unconventional pairing at *and near* hot spots

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \mathcal{P}_d(\cos k_x - \cos k_y)$$

After pseudospin rotation on *half* the hot-spots



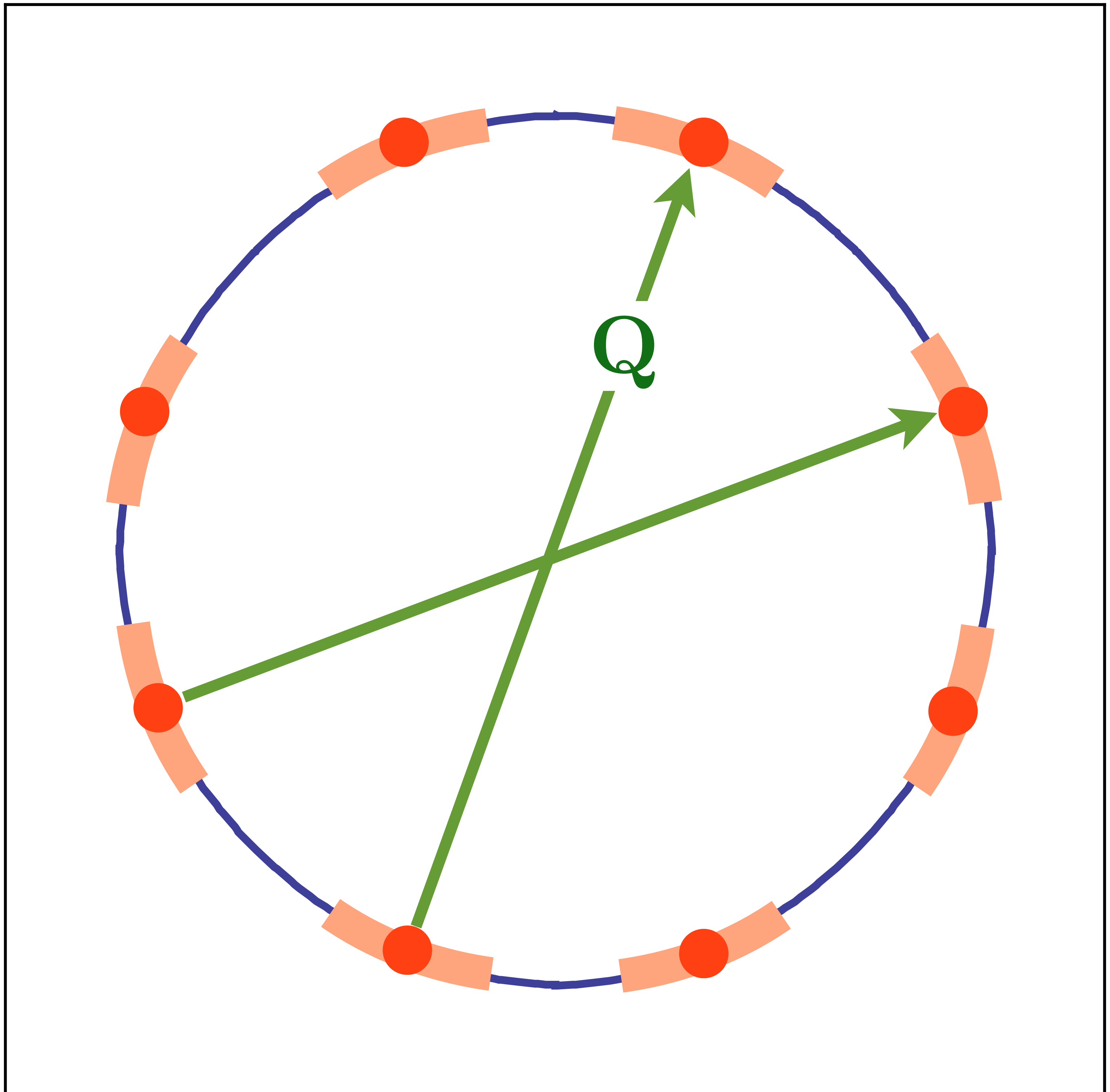
M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

"*d*-form factor" density wave

$$\left\langle c^{\dagger}_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \mathcal{P}_d (\cos k_x - \cos k_y)$$

$\mathbf{Q}$ is '$2k_F$' wavevector

After pseudospin rotation on *half* the hot-spots



M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

"*d*-form factor" density wave

Charge density wave (CDW) order

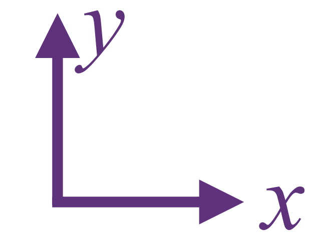
$$\left\langle c_\alpha^\dagger(\mathbf{r}) c_\alpha(\mathbf{r}) \right\rangle = \Psi_{CDW}\left(\mathbf{r}\right) e^{i\mathbf{Q}\cdot\mathbf{r}} + \text{c.c.}$$

Charge density wave (CDW) order

$$\left\langle c_{\alpha}^{\dagger}(\mathbf{r}) c_{\alpha}(\mathbf{r})\right\rangle = \Psi_{CDW}(\mathbf{r})\, e^{i\mathbf{Q}\cdot\mathbf{r}} + \text{c.c.}$$

Nearly constant CDW order parameter

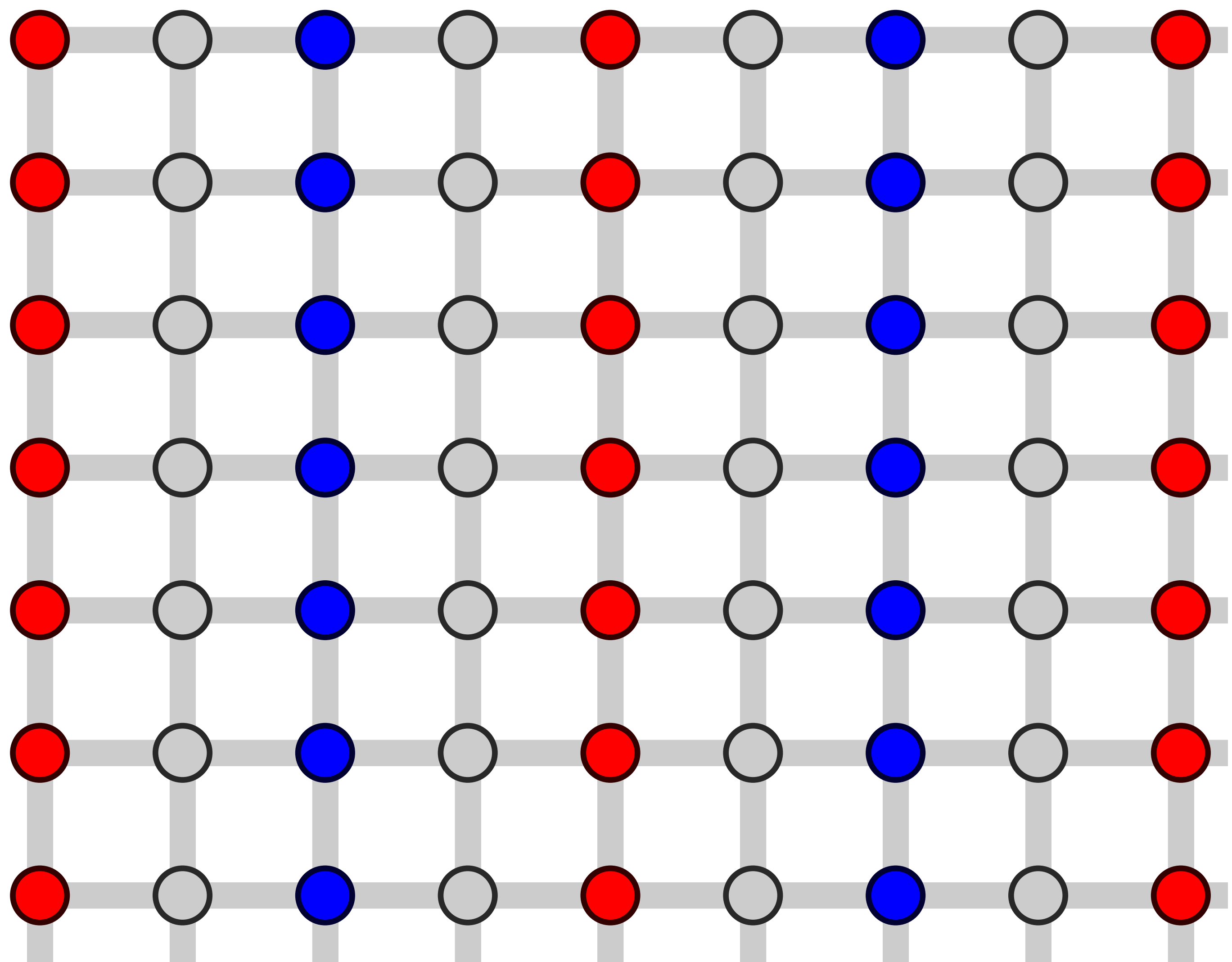
CDW order.

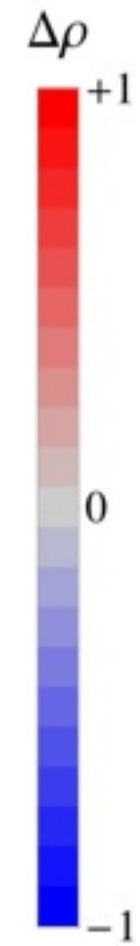


Plot of $P_{ii} = \left\langle c^{\dagger}_{i\alpha} c_{i\alpha} \right\rangle$ with

$$P_{ii} = e^{i\mathbf{Q}\cdot\mathbf{r}_i} + \text{c.c.}$$

$$\text{with} \quad \mathbf{Q} = 2\pi(1/4, 0)$$





Unconventional density wave (DW) : Bose condensation of particle-hole pairs

$$\left\langle c_\alpha^\dagger(\mathbf{r}_1) c_\alpha(\mathbf{r}_2) \right\rangle$$

$$= \Big[\mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2)\Big] \times \Psi_{DW}\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}\right) e^{i\mathbf{Q}\cdot(\mathbf{r}_1+\mathbf{r}_2)/2} + \text{c.c.}$$

Unconventional density wave (DW) : Bose condensation of particle-hole pairs

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Crucial "center-of-mass" co-ordinate.
(Not used in previous work)
Simplifies action of time-reversal

Unconventional density wave (DW) :
Bose condensation of particle-hole pairs

$$\left\langle c_\alpha^\dagger(\mathbf{r}_1) c_\alpha(\mathbf{r}_2) \right\rangle$$

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Nearly constant CDW order parameter

Unconventional density wave (DW) : Bose condensation of particle-hole pairs

$$\left\langle c_\alpha^\dagger(\mathbf{r}_1) c_\alpha(\mathbf{r}_2) \right\rangle$$

$$= \Big[\mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2)\Big] \times \Psi_{DW}\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}\right) e^{i\mathbf{Q}\cdot(\mathbf{r}_1+\mathbf{r}_2)/2} + \text{c.c.}$$

Density wave **form factor** (internal particle-hole pair wavefunction)

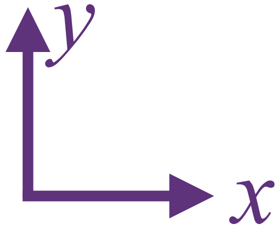
$$\mathcal{P}(\mathbf{r}) = \int \frac{d^2k}{4\pi^2} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}}$$

Time-reversal symmetry requires $\mathcal{P}(\mathbf{k}) = \mathcal{P}(-\mathbf{k})$.

We expand (using reflection symmetry for $\mathbf{Q}$ along axes or diagonals)

$$\mathcal{P}(\mathbf{k}) = \mathcal{P}_s + \mathcal{P}_{s'}(\cos k_x + \cos k_y) + \mathcal{P}_d(\cos k_x - \cos k_y)$$

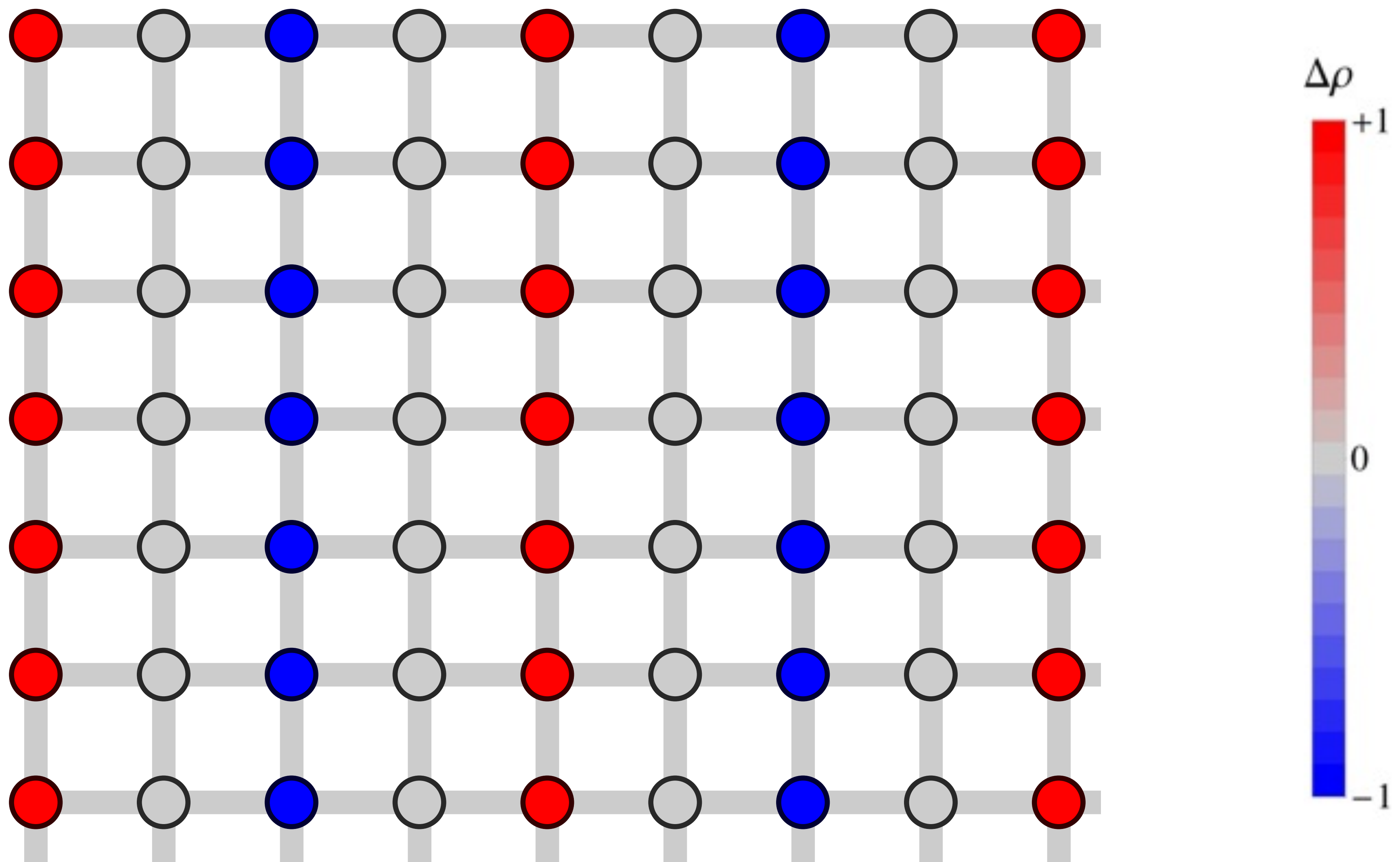
Conventional CDW order: s-form factor



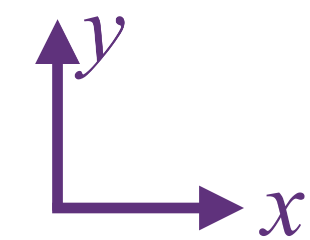
Plot of $P_{ij} = \left\langle c^{\dagger}_{i\alpha} c_{j\alpha} \right\rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = 1 \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$



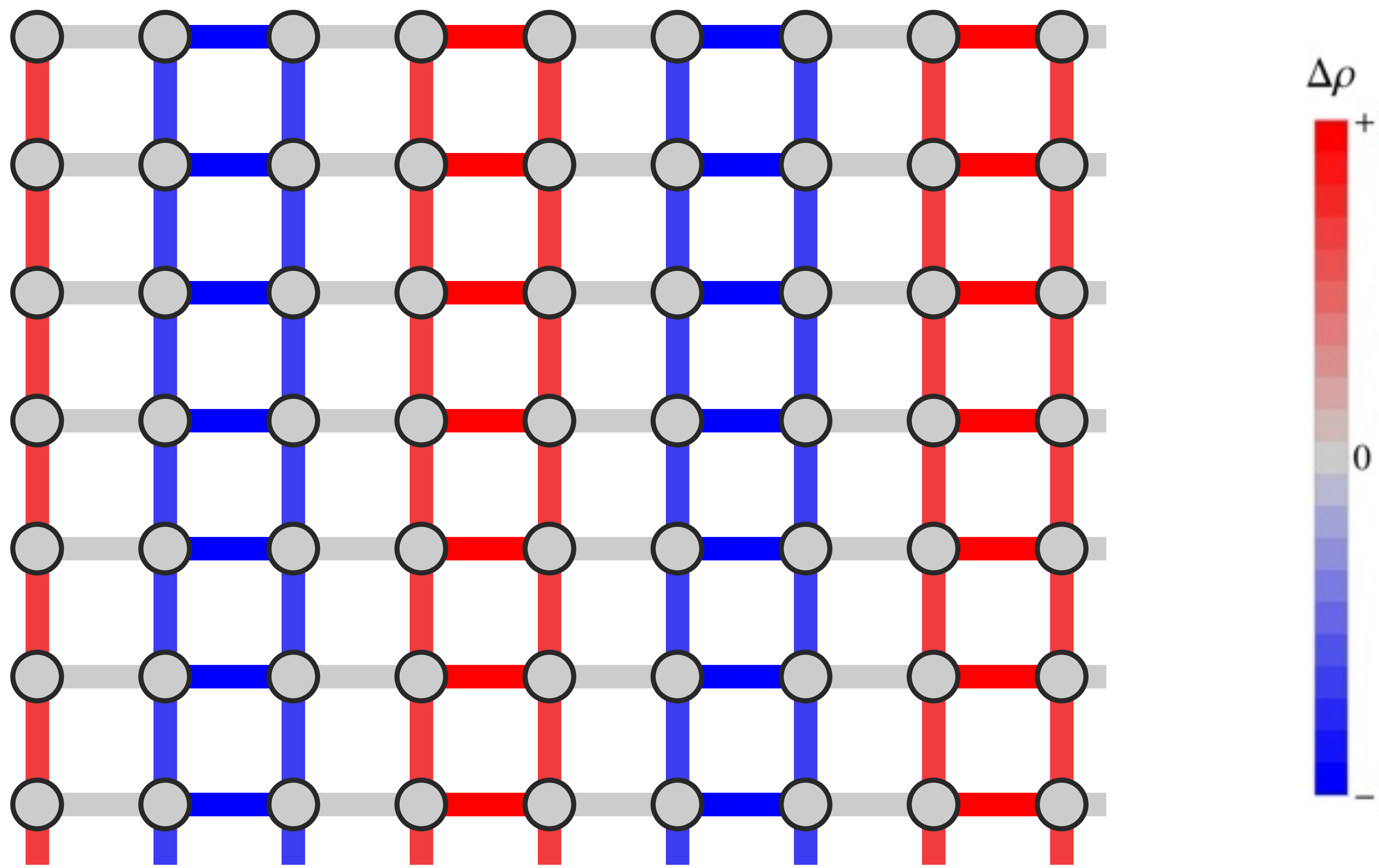
Unconventional DW order: s'-form factor



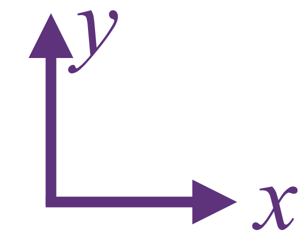
Plot of $P_{ij} = \left\langle c^{\dagger}_{i\alpha} c_{j\alpha} \right\rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} \left[\cos(k_x) + \cos(k_y) \right] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$



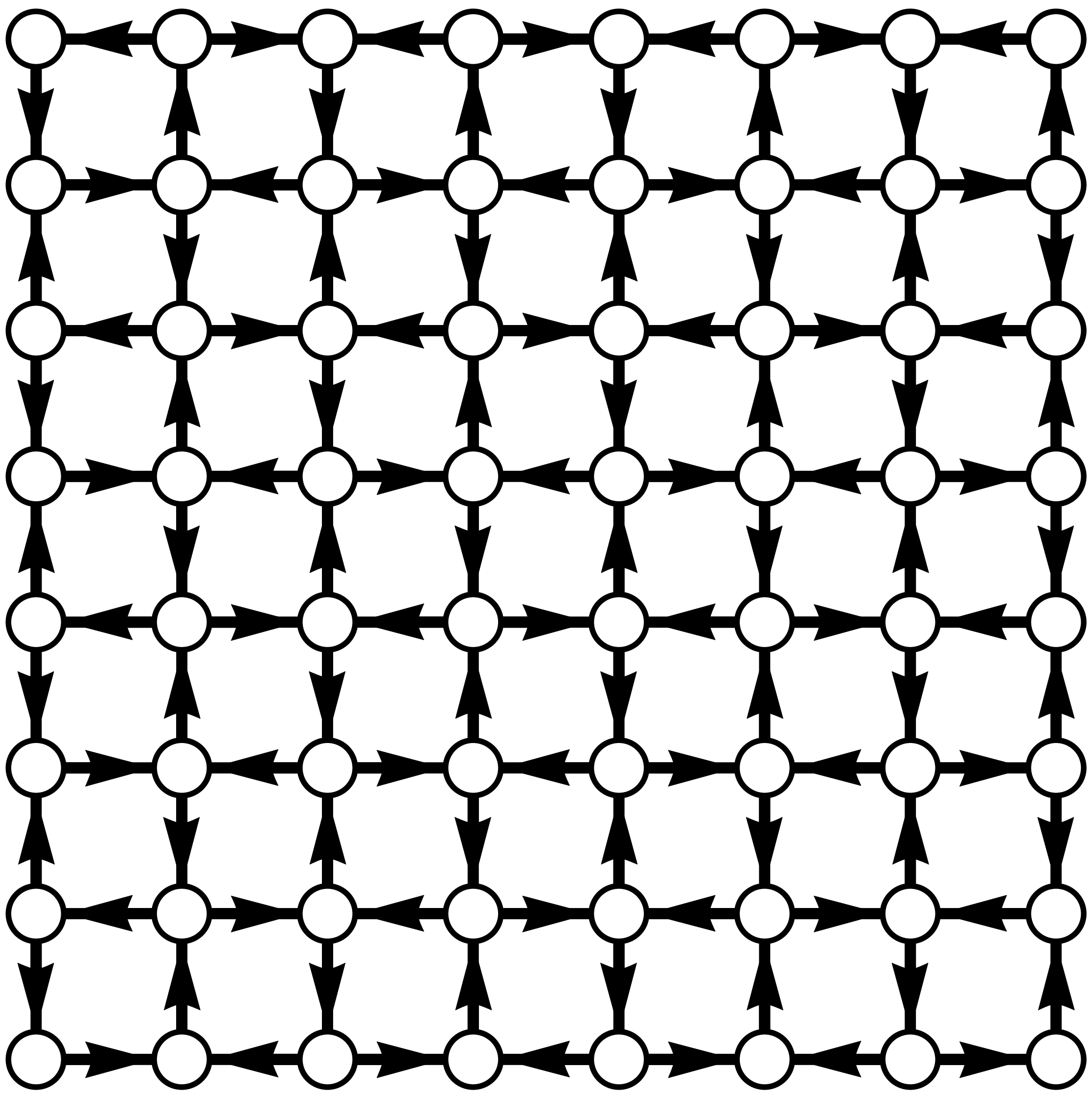
Current order: *p*-form factor



Plot of $P_{ij} = \left\langle c^\dagger_{i\alpha} c_{j\alpha} \right\rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\sum_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$$

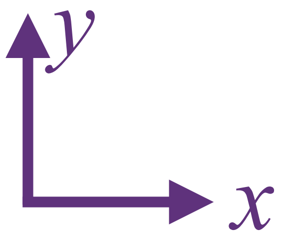
$$\mathcal{P}(\mathbf{k}) = \sin(k_x) - \sin(k_y) \quad \text{and} \quad \mathbf{Q} = (\pi, \pi)$$



This state breaks time-reversal and is also known as "*d*-density wave" (*but is p-form factor in our notation*), and "staggered-flux (SF)". (Similar comments apply to "loop" orders of Varma and others.)

S. Chakravarty, R. B. Laughlin, D. K. Morr, and C. Nayak, Phys. Rev. B **63**, 094503 (2001).
P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006).
R. B. Laughlin, Phys. Rev. B **89**, 035134 (2014).

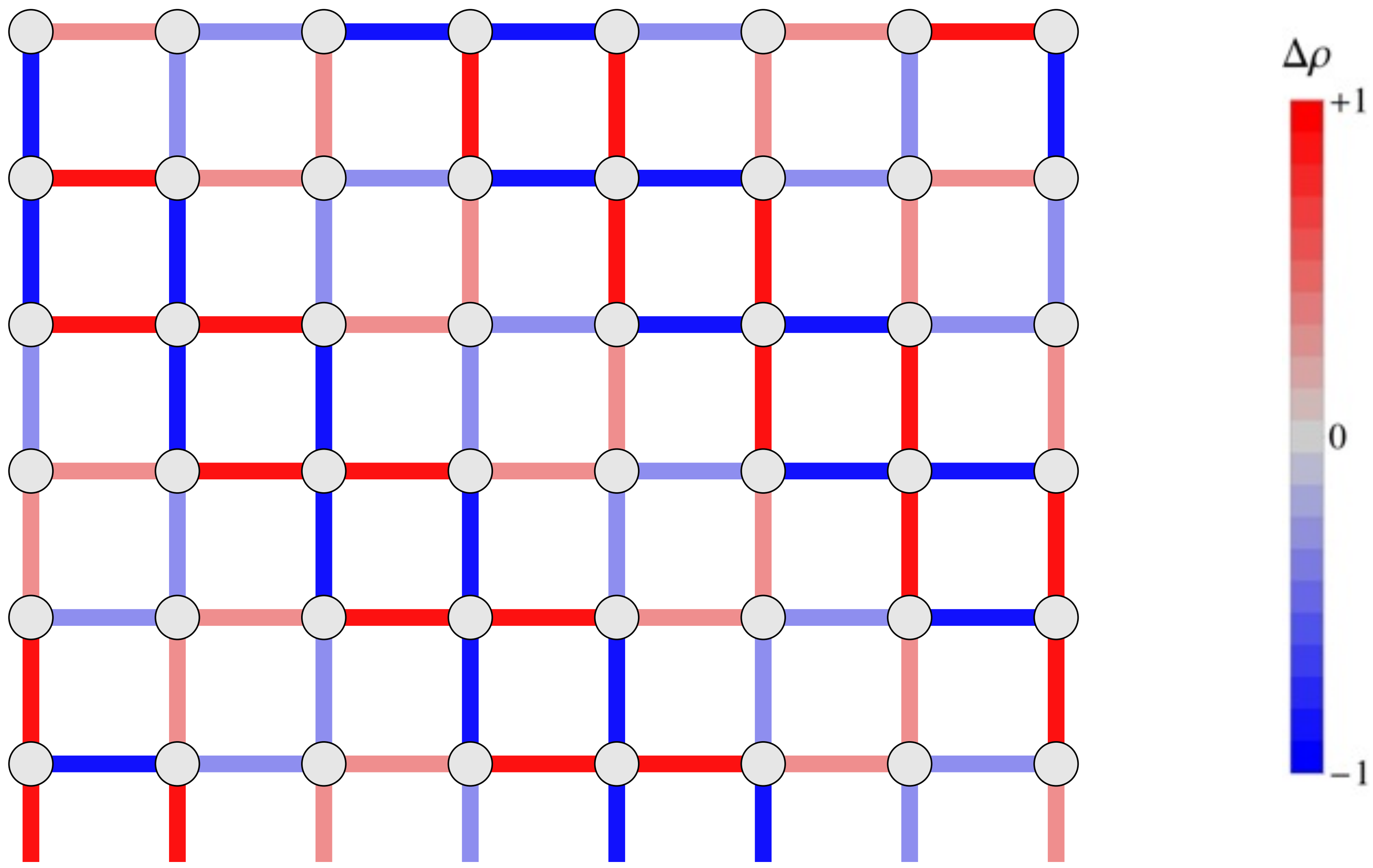
Unconventional DW order: d-form factor



Plot of $P_{ij} = \left\langle c^\dagger_{i\alpha} c_{j\alpha} \right\rangle$ for $i=j$, and i,j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}\right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i+\mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi}\left[\cos(k_x) - \cos(k_y)\right] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 1/4)$$



Density wave predicted by hot-spot theory

Unconventional DW order: d-form factor

Plot of $P_{ij} = \left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

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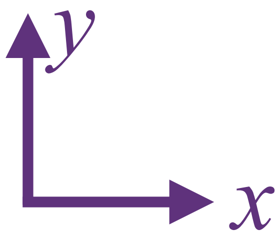
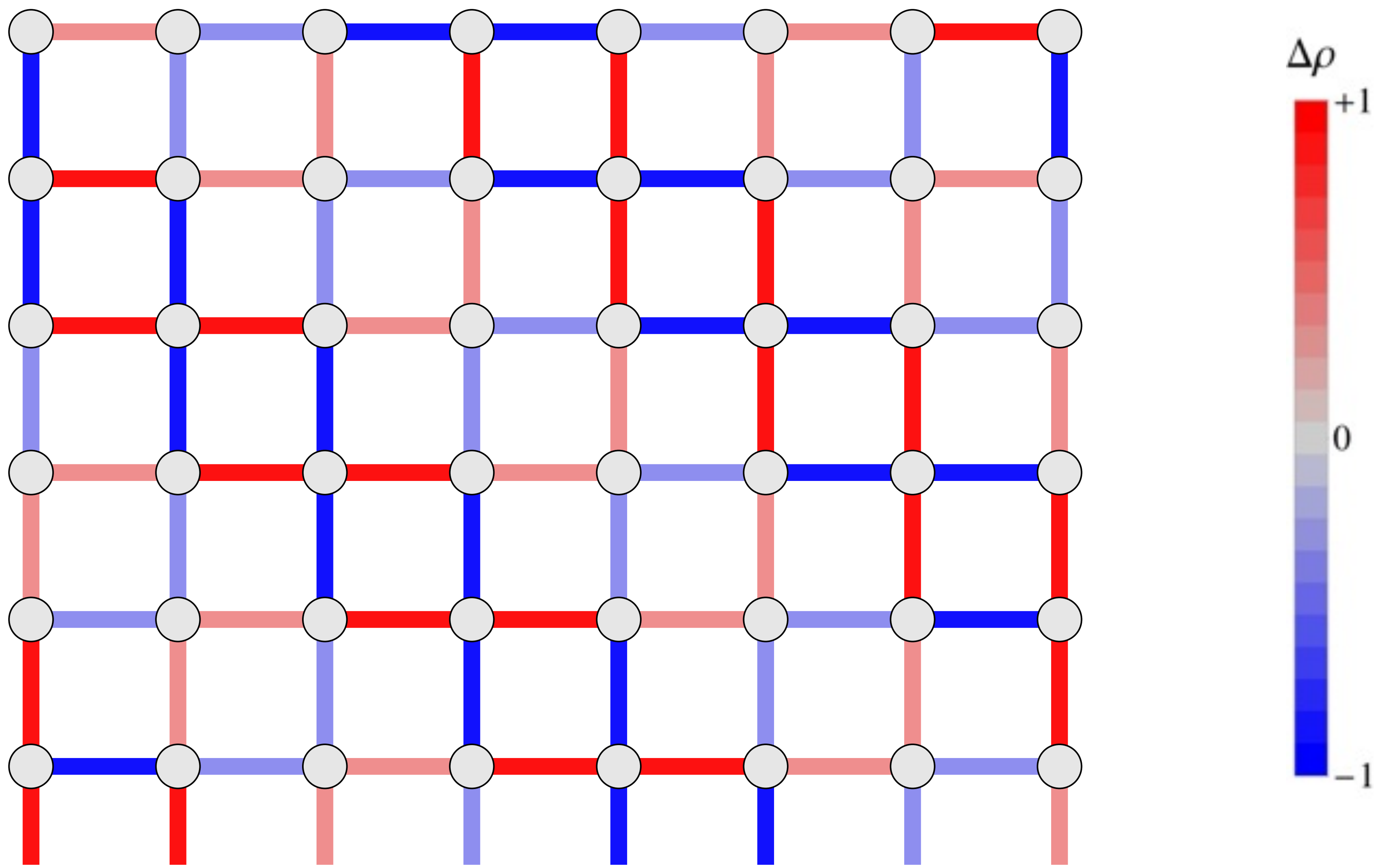
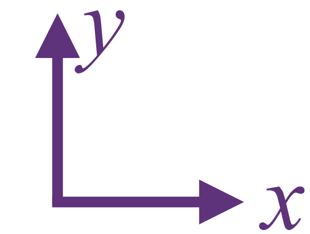


Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



Density wave predicted by hot-spot theory

Unconventional DW order: d-form factor



Plot of $P_{ij} = \left\langle c_{i\alpha}^\dagger c_{j\alpha} \right\rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} \left[\cos(k_x) - \cos(k_y) \right] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

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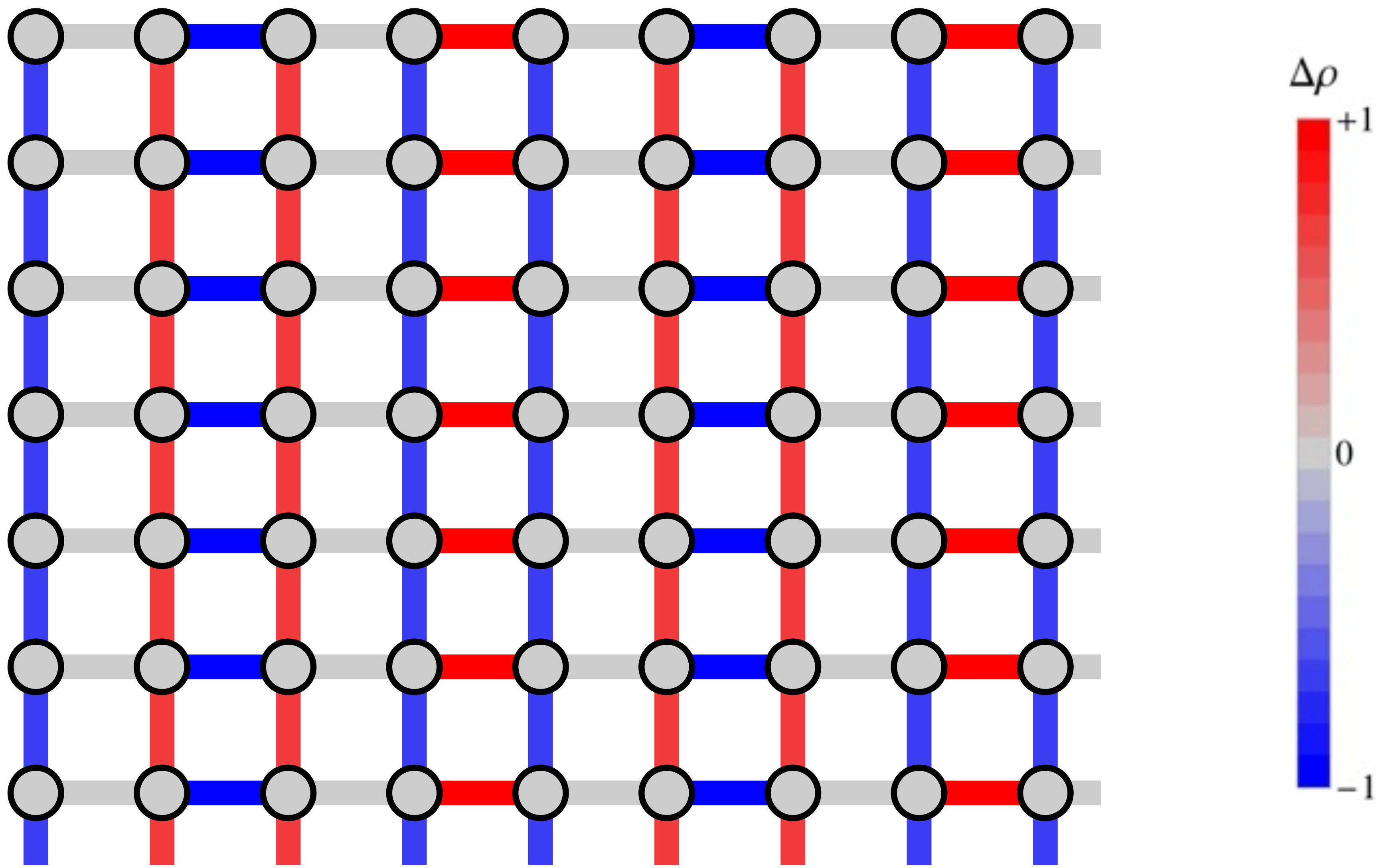
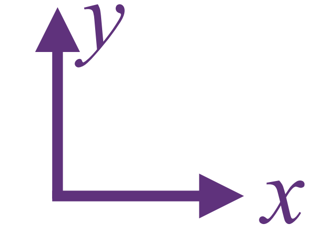


This specific d-form factor density wave order (with $\mathbf{Q}$ along the axes) was first predicted in S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

Unconventional DW order: d-form factor



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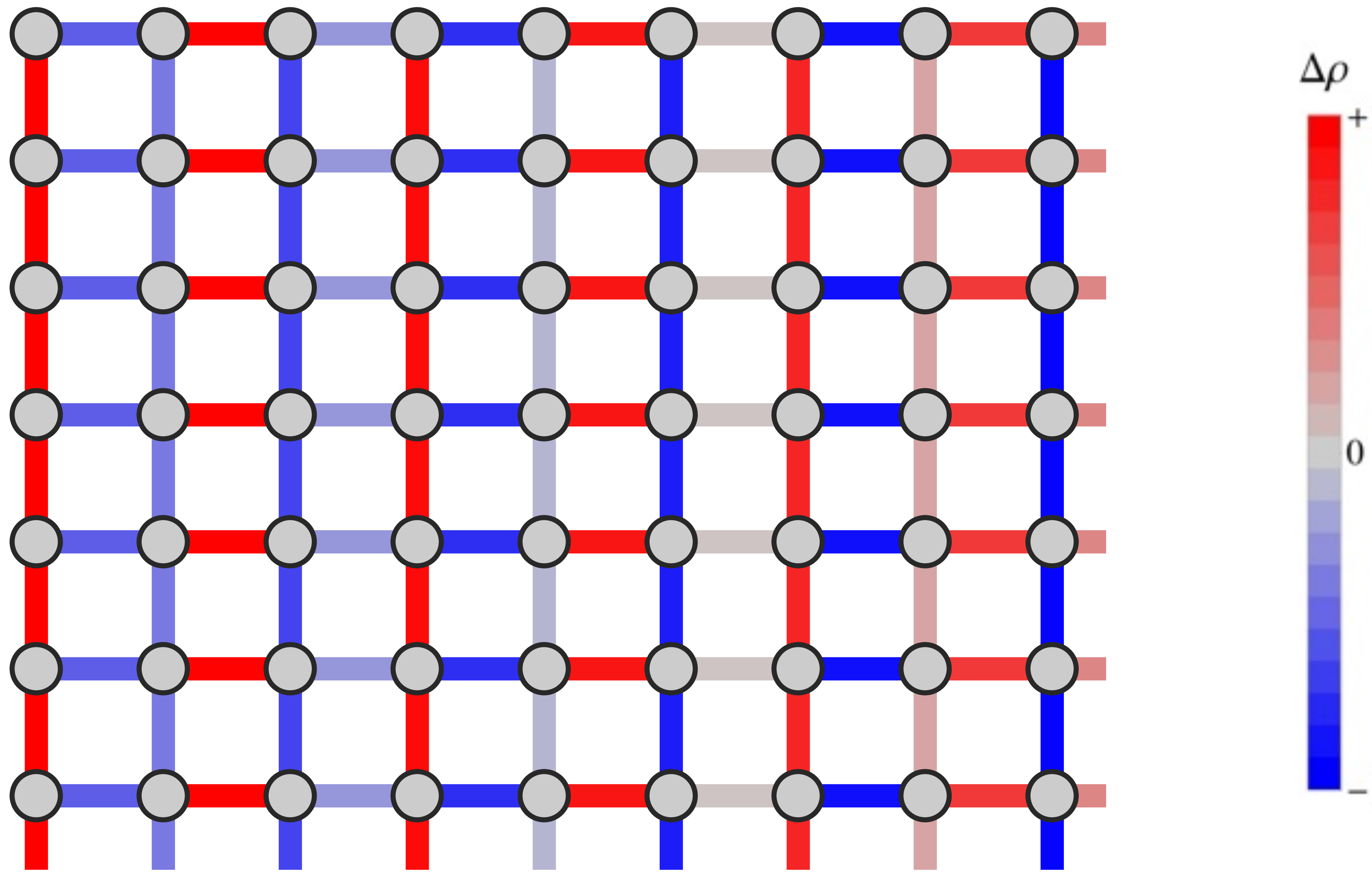
$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}\right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi}\left[\cos(k_x) - \cos(k_y)\right] \quad \text{and} \quad \mathbf{Q} = 2\pi(0.317, 0)$$

Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



This specific d-form factor density wave order (with $\mathbf{Q}$ along the axes) was first predicted in S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

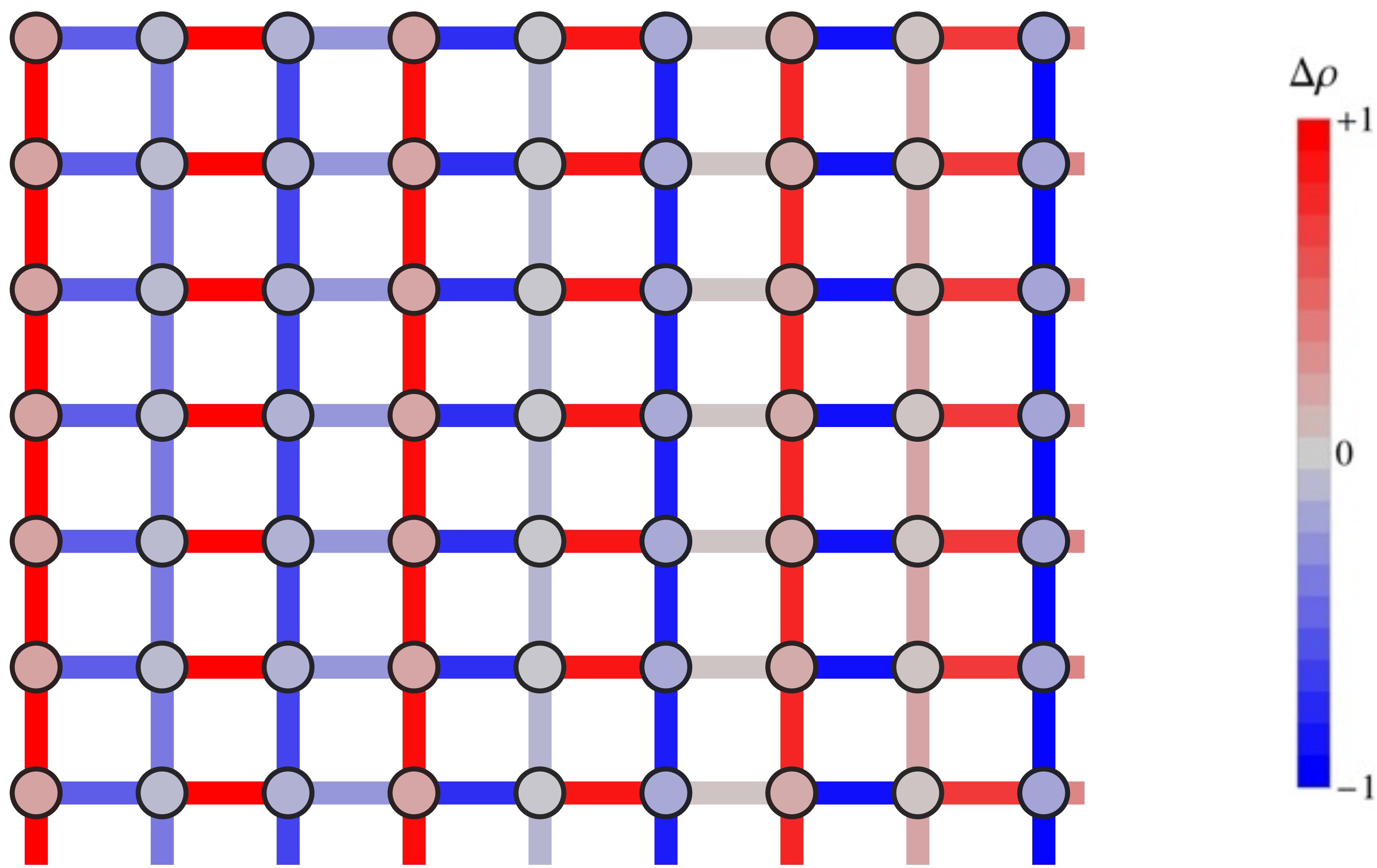
Unconventional DW order: $(d+s)$-form factor

Plot of $P_{ij} = \left\langle c^\dagger_{i\alpha} c_{j\alpha} \right\rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} \left[0.2 + \cos(k_x) - \cos(k_y) \right] \quad \text{and} \quad \mathbf{Q} = 2\pi(0.317, 0)$$

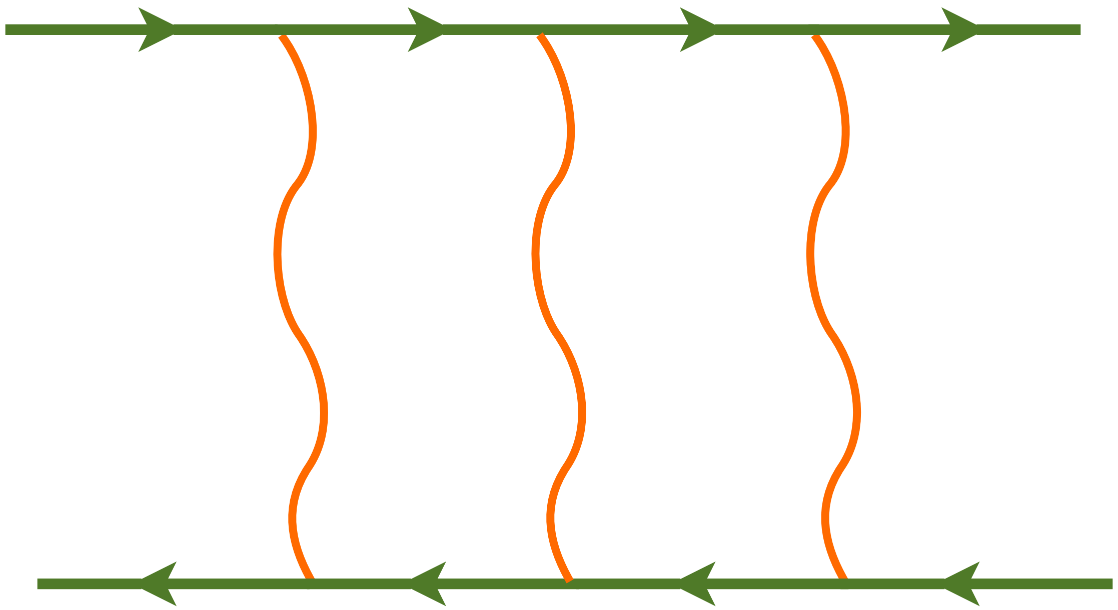
Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



This specific d-form factor density wave order (with $\mathbf{Q}$ along the axes) was first predicted in S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

Determination of form factor of density wave by solution of Bethe-Salpeter equation for the particle-hole pair about the entire Fermi surface

$$H = -\sum_{i,j} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + V \sum_{\langle ij \rangle} n_i n_j + \ldots$$



S. Sachdev and R. LaPlaca *Phys. Rev. Lett.* **111**, 027202 (2013)
J. C. Davis and Dung-Hai Lee, Proc. Natl. Acad. Sci. **110**, 17623 (2013)
J. D. Sau and S. Sachdev, *Phys. Rev. B* **89**, 075129 (2014)
A. Allais, J. Bauer, and S. Sachdev, arXiv:1402.6311

Solution of Bethe-Salpeter equation

We expand the order parameter $\mathcal{P}(\mathbf{k})$ as

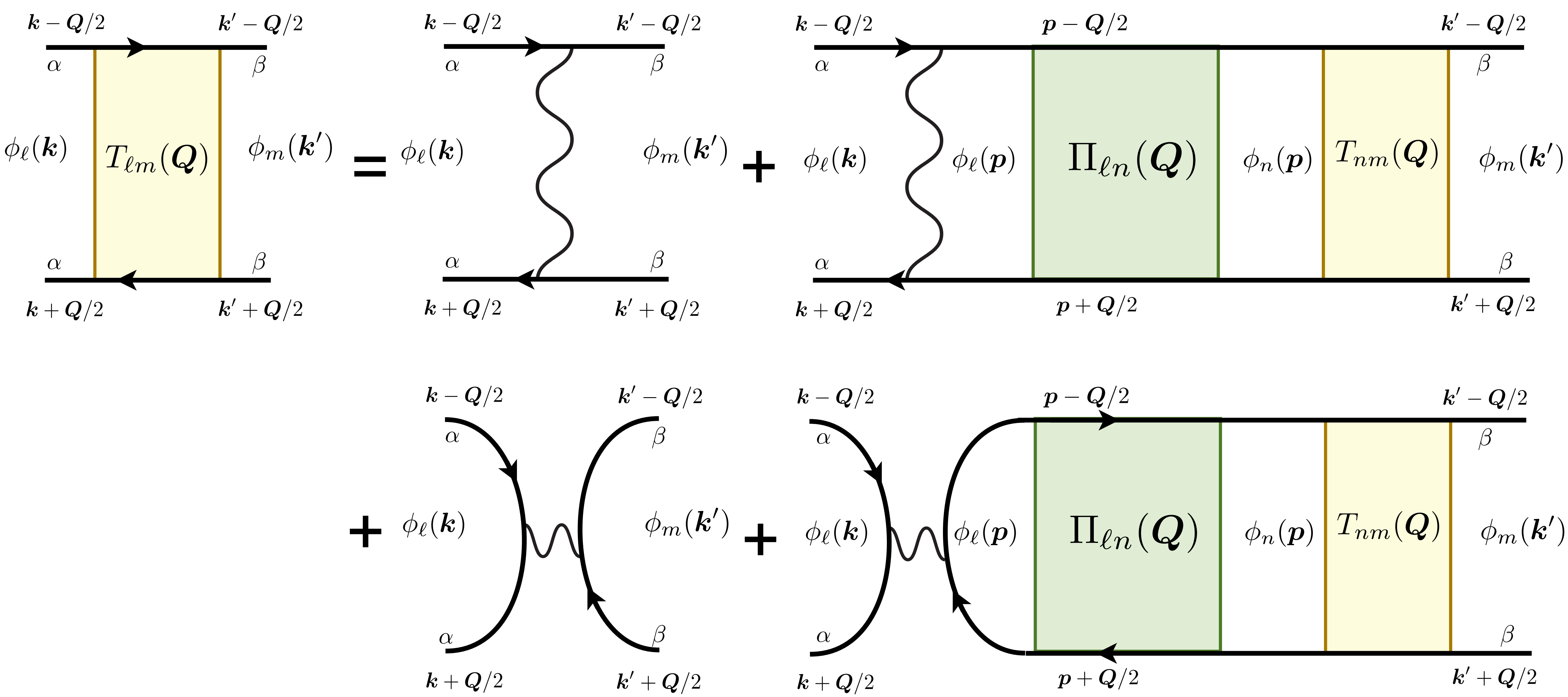
$$\begin{aligned}\mathcal{P}(\mathbf{k}) &= \sum_\ell \mathcal{P}_\ell \, \phi_\ell(\mathbf{k}) \\ \phi_\ell(\mathbf{k}) &= \{1, \cos k_x + \cos k_y, \cos k_x - \cos k_y, \sin k_x - \sin k_y, \ldots\}\end{aligned}$$

We can also write the electron-electron interaction using these basis functions

$$\begin{aligned}H_U + H_J + H_V = \sum_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{q}} \sum_{\ell=0}^{12} \phi_\ell(\boldsymbol{k})\phi_\ell(\boldsymbol{k}') \Bigg[\sum_a \frac{\mathcal{J}_\ell}{8} c^\dagger_{\boldsymbol{k}'-\boldsymbol{q}/2,\alpha} \, \sigma^a_{\alpha\beta} \, c_{\boldsymbol{k}-\boldsymbol{q}/2,\beta} \, c^\dagger_{\boldsymbol{k}+\boldsymbol{q}/2,\gamma} \, \sigma^a_{\gamma\delta} \, c_{\boldsymbol{k}'+\boldsymbol{q}/2,\delta} \\ + \frac{\mathcal{V}_\ell}{2} c^\dagger_{\boldsymbol{k}'-\boldsymbol{q}/2,\alpha} \, c_{\boldsymbol{k}-\boldsymbol{q}/2,\alpha} \, c^\dagger_{\boldsymbol{k}+\boldsymbol{q}/2,\beta} \, c_{\boldsymbol{k}'+\boldsymbol{q}/2,\beta} \Bigg]\end{aligned}$$

ℓ	$\phi_\ell(\boldsymbol{k})$	$\mathcal{J}_\ell$	$\mathcal{V}_\ell$	ℓ	$\phi_\ell(\boldsymbol{k})$	$\mathcal{J}_\ell$	$\mathcal{V}_\ell$
0	1	0	U				
1	$\cos k_x - \cos k_y$	J_1	V_1	7	$\sin k_x - \sin k_y$	J_1	V_1
2	$\cos k_x + \cos k_y$	J_1	V_1	8	$\sin k_x + \sin k_y$	J_1	V_1
3	$2 \sin k_x \sin k_y$	J_2	V_2	9	$2 \cos k_x \sin k_y$	J_2	V_2
4	$2 \cos k_x \cos k_y$	J_2	V_2	10	$2 \sin k_x \cos k_y$	J_2	V_2
5	$\cos(2k_x) - \cos(2k_y)$	J_3	V_3	11	$\sin(2k_x) - \sin(2k_y)$	J_3	V_3
6	$\cos(2k_x) + \cos(2k_y)$	J_3	V_3	12	$\sin(2k_x) + \sin(2k_y)$	J_3	V_3

Solution of Bethe-Salpeter equation



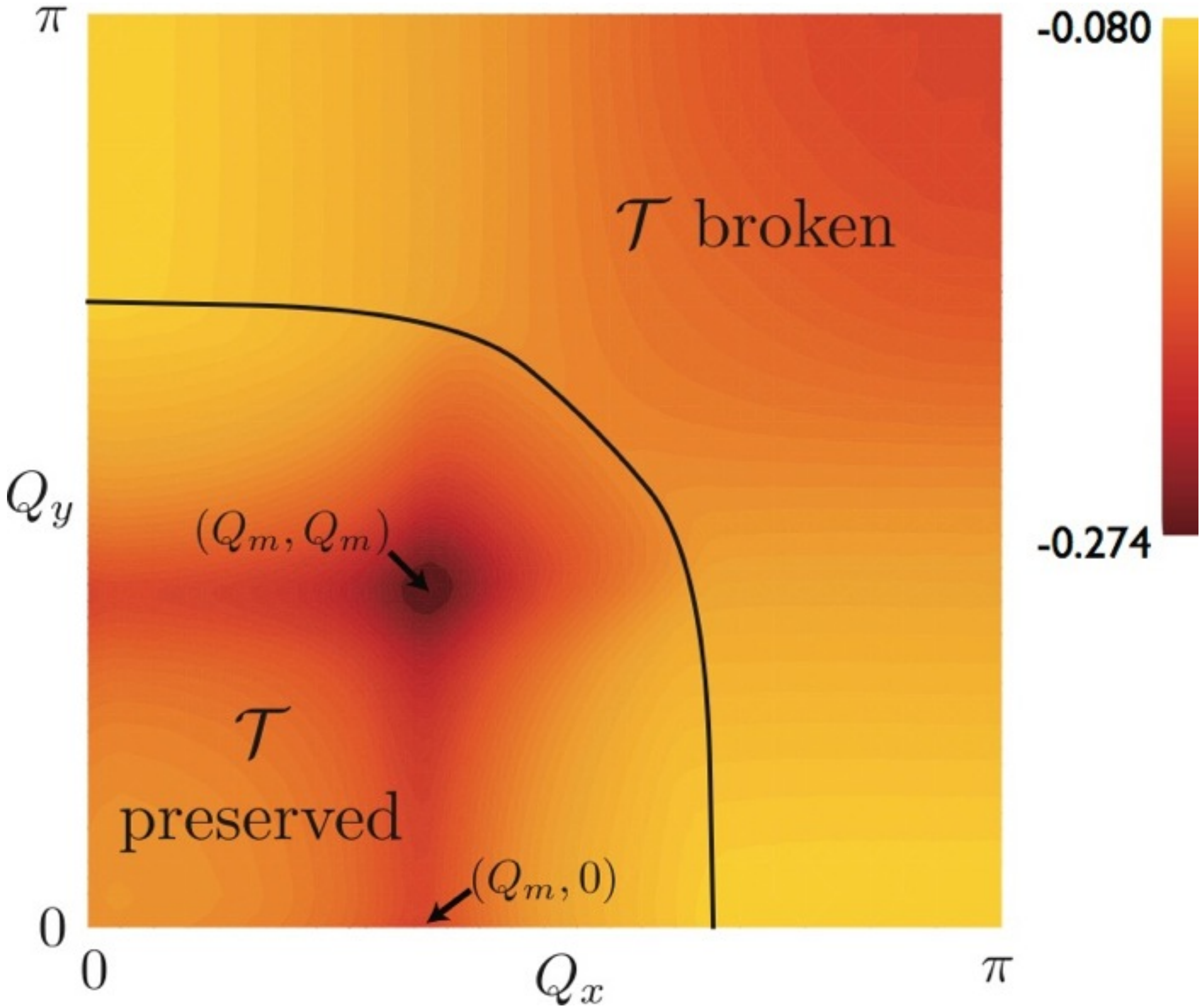
Find the lowest eigenvalues $\lambda(\mathbf{Q})$, and corresponding eigenvectors, of the matrix

$$\delta_{\ell m} - \frac{1}{2}\left(\frac{3}{4}\mathcal{J}_\ell + \mathcal{V}_\ell\right)\Pi_{\ell m}(\mathbf{Q}) + \delta_{\ell,0} W(\mathbf{Q})\Pi_{0m}(\mathbf{Q}), \text{ where}$$

$$\Pi_{\ell m}(\mathbf{Q}) = 2\sum_{\mathbf{k}} \phi_\ell(\mathbf{k})\phi_m(\mathbf{k}) \frac{f(\varepsilon(\mathbf{k}-\mathbf{Q}/2)) - f(\varepsilon(\mathbf{k}+\mathbf{Q}/2))}{\varepsilon(\mathbf{k}+\mathbf{Q}/2) - \varepsilon(\mathbf{k}-\mathbf{Q}/2)} \text{ and}$$

$$W(\mathbf{Q}) \equiv \sum_{\ell} \mathcal{V}_\ell \phi_\ell(0)\phi_\ell(\mathbf{Q})$$

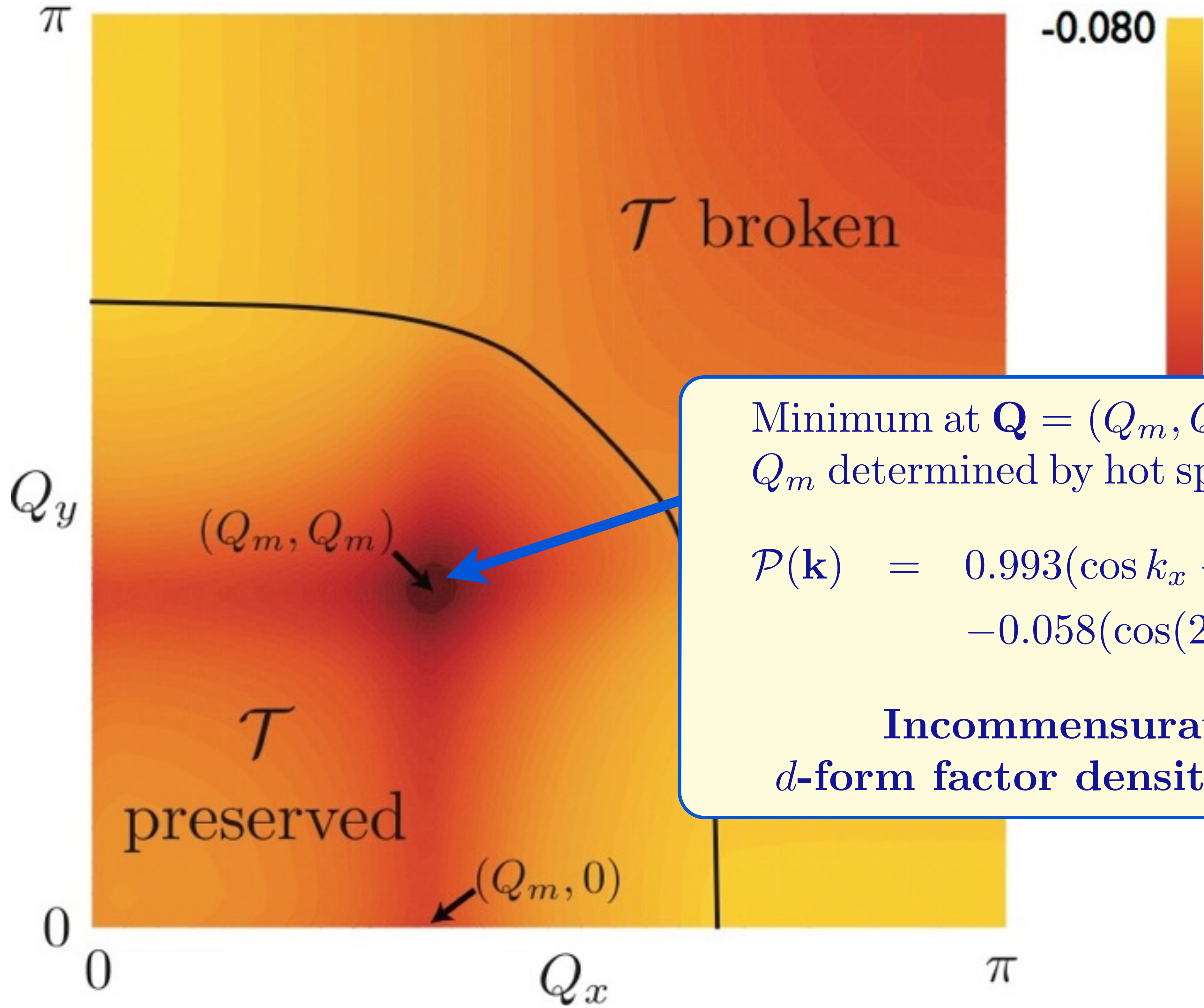
S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013); A. Allais, J. Bauer, and S. Sachdev, arXiv:1402.6311



Eigenvalues, $\lambda(\mathbf{Q})$, of the spin-singlet, particle-hole propagator. The corresponding eigenvector is $\mathcal{P}(\mathbf{k})$ and this leads to the order

$$\left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$$

S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013); A. Allais, J. Bauer, and S. Sachdev, arXiv:1402.6311



Minimum at $\mathbf{Q} = (Q_m, Q_m)$ (with Q_m determined by hot spots) with

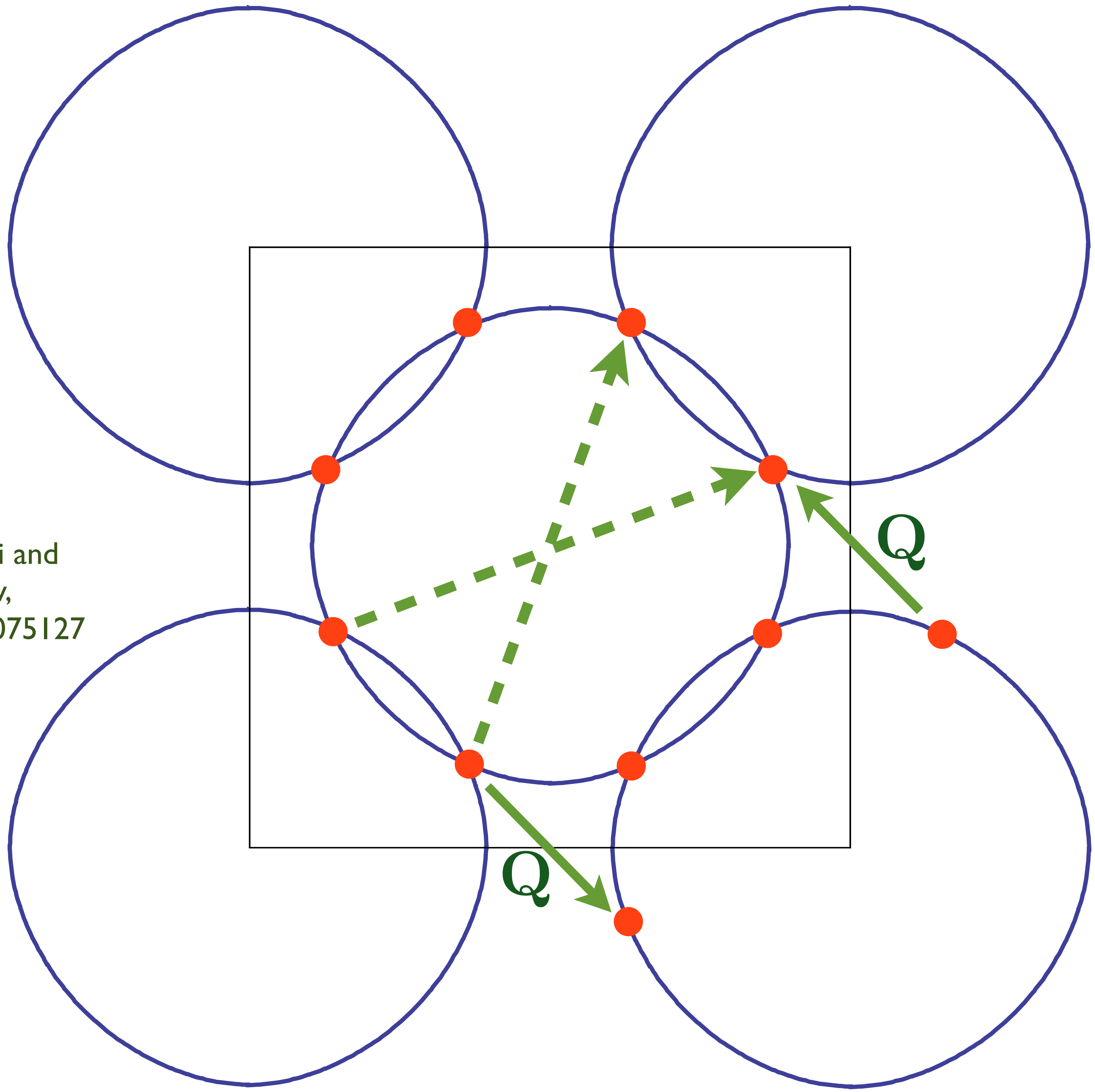
$$\mathcal{P}(\mathbf{k}) = 0.993(\cos k_x - \cos k_y) - 0.058(\cos(2k_x) - \cos(2k_y))$$

Incommensurate
***d*-form factor density wave**

Eigenvalues, $\lambda(\mathbf{Q})$, of the spin-singlet, particle-hole propagator. The corresponding eigenvector is $\mathcal{P}(\mathbf{k})$ and this leads to the order

$$\left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$$

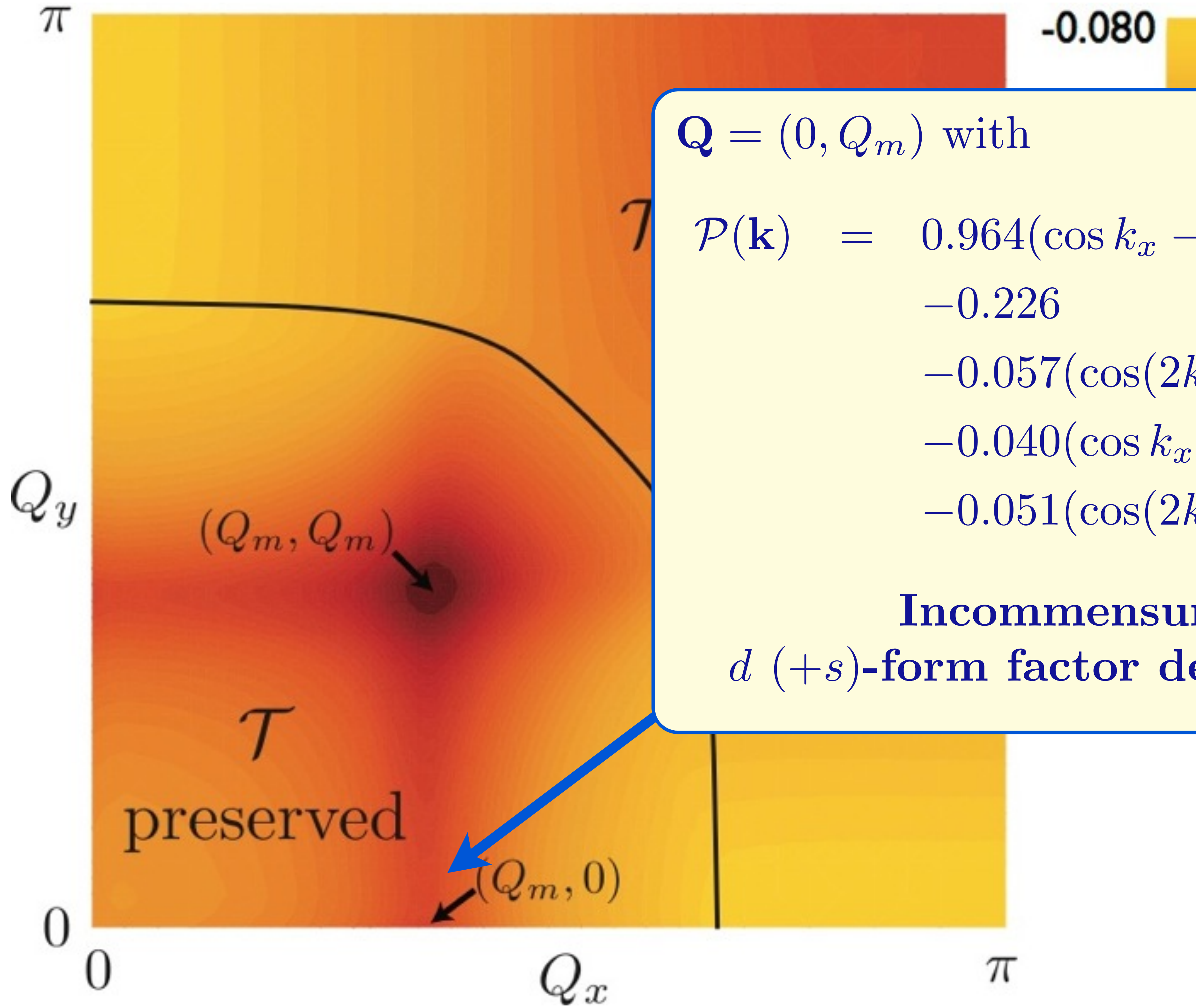
Incommensurate *d*-form factor density wave



M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \mathcal{P}_d (\cos k_x - \cos k_y)$$

S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013); A. Allais, J. Bauer, and S. Sachdev, arXiv:1402.6311



$\mathbf{Q} = (0, Q_m)$ with

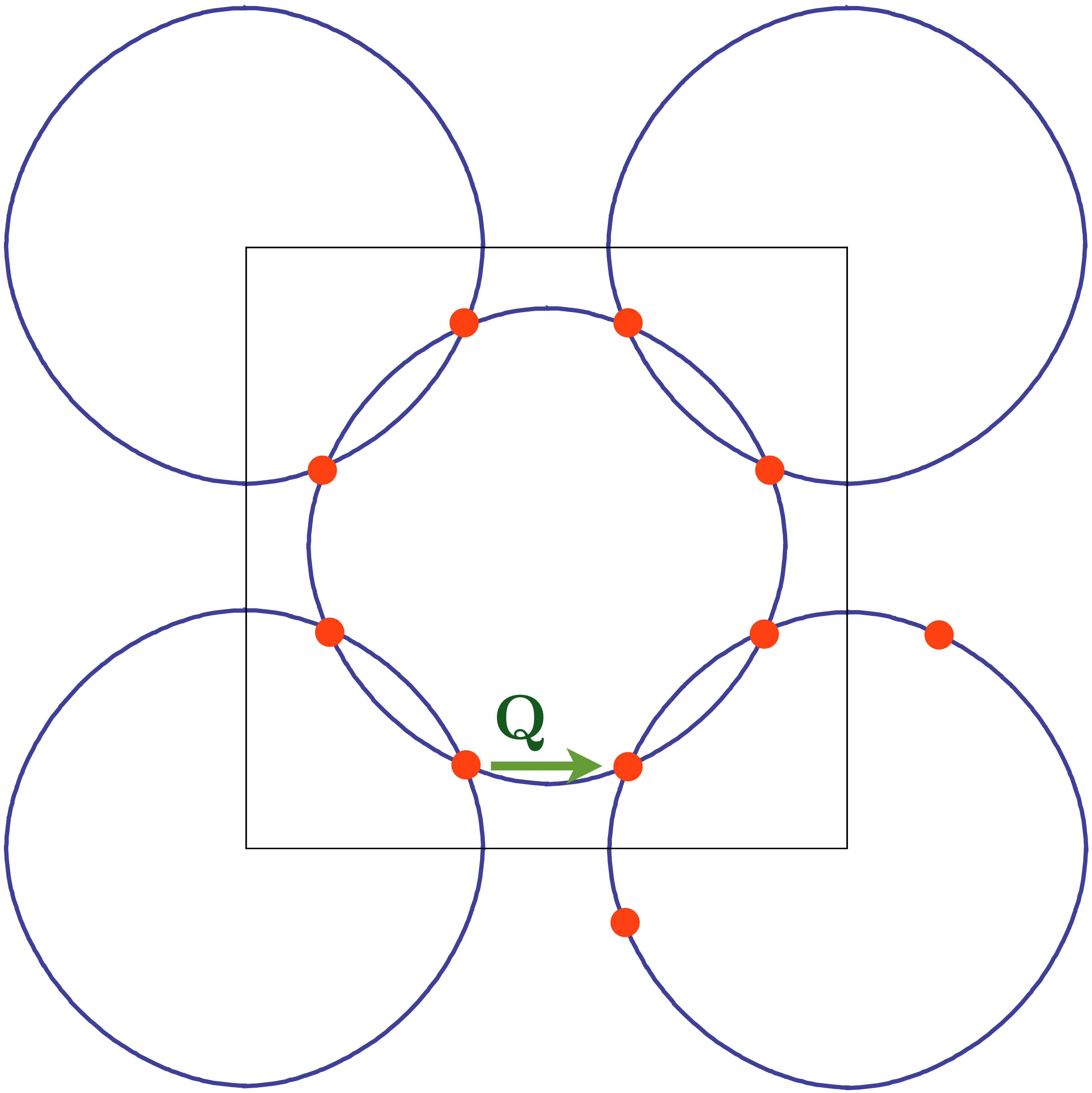
$$\begin{aligned}\mathcal{P}(\mathbf{k}) \;=\; & 0.964(\cos k_x - \cos k_y) \\ & -0.226 \\ & -0.057(\cos(2k_x) - \cos(2k_y)) \\ & -0.040(\cos k_x + \cos k_y) \\ & -0.051(\cos(2k_x) + \cos(2k_y))\end{aligned}$$

Incommensurate
d $(+s)$**-form factor density wave**

Eigenvalues, $\lambda(\mathbf{Q})$, of the spin-singlet, particle-hole propagator. The corresponding eigenvector is $\mathcal{P}(\mathbf{k})$ and this leads to the order

$$\left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}\right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$$

Incommensurate *d*-form factor density wave



$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle \sim -0.226 + 0.964(\cos k_x - \cos k_y)$$

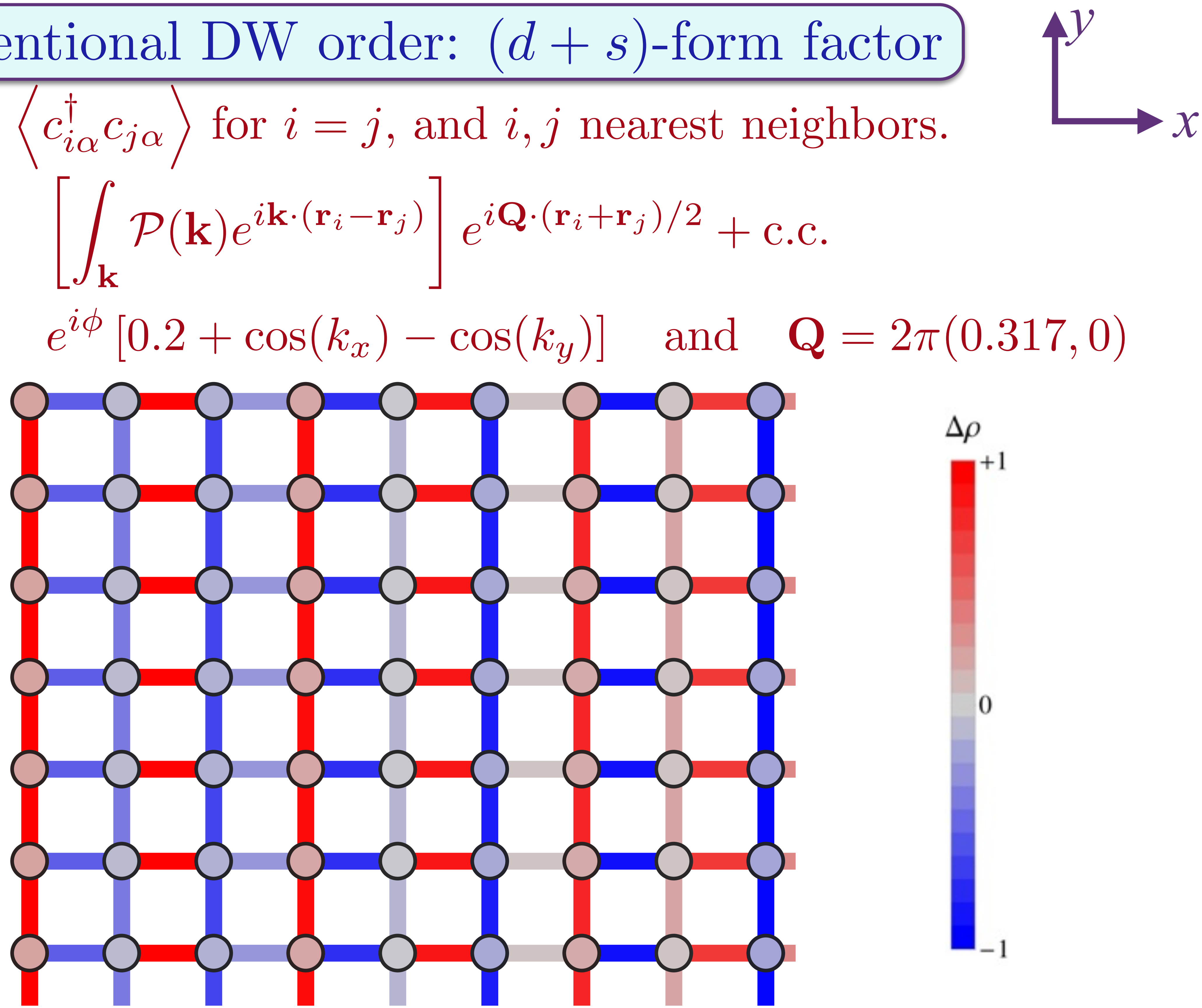
Unconventional DW order: $(d+s)$-form factor

Plot of $P_{ij} = \left\langle c_{i\alpha}^\dagger c_{j\alpha} \right\rangle$ for $i=j$, and i,j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}\right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i+\mathbf{r}_j)/2} + \text{c.c.}$$

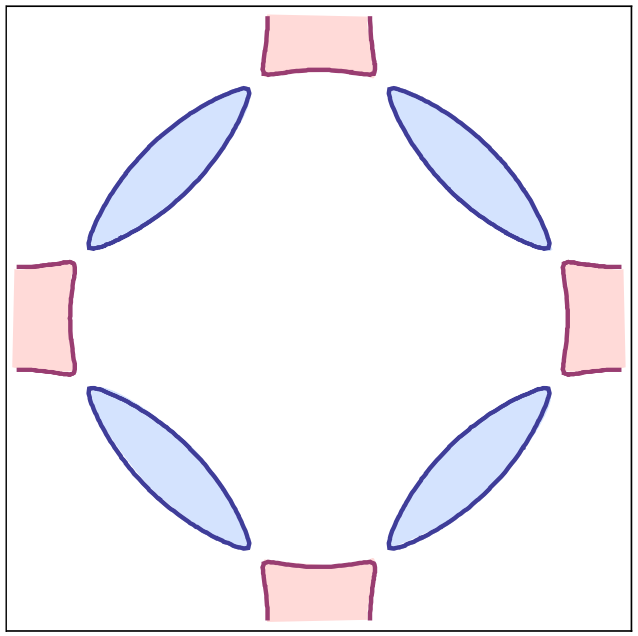
$$\mathcal{P}(\mathbf{k}) = e^{i\phi}\left[0.2 + \cos(k_x) - \cos(k_y)\right] \quad \text{and} \quad \mathbf{Q} = 2\pi(0.317, 0)$$

Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



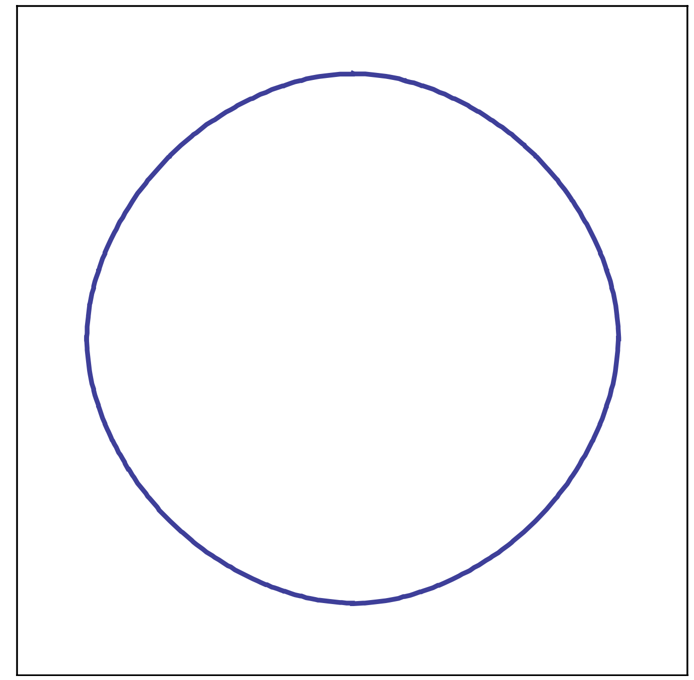
This specific d-form factor density wave order (with $\mathbf{Q}$ along the axes) was first predicted in S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

Quantum phase transition with onset of antiferromagnetism in a metal



$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets

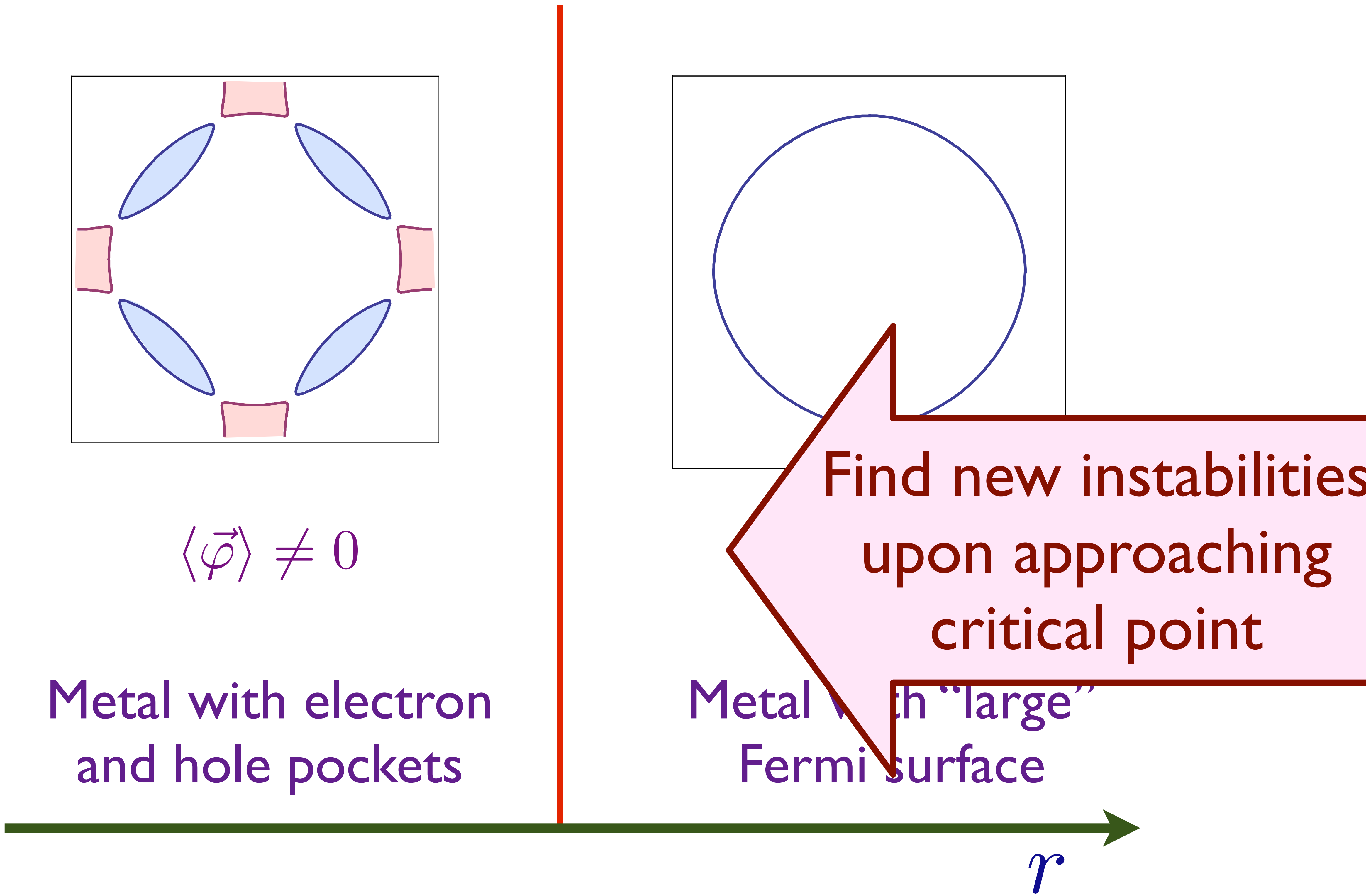


$\langle \vec{\varphi} \rangle = 0$

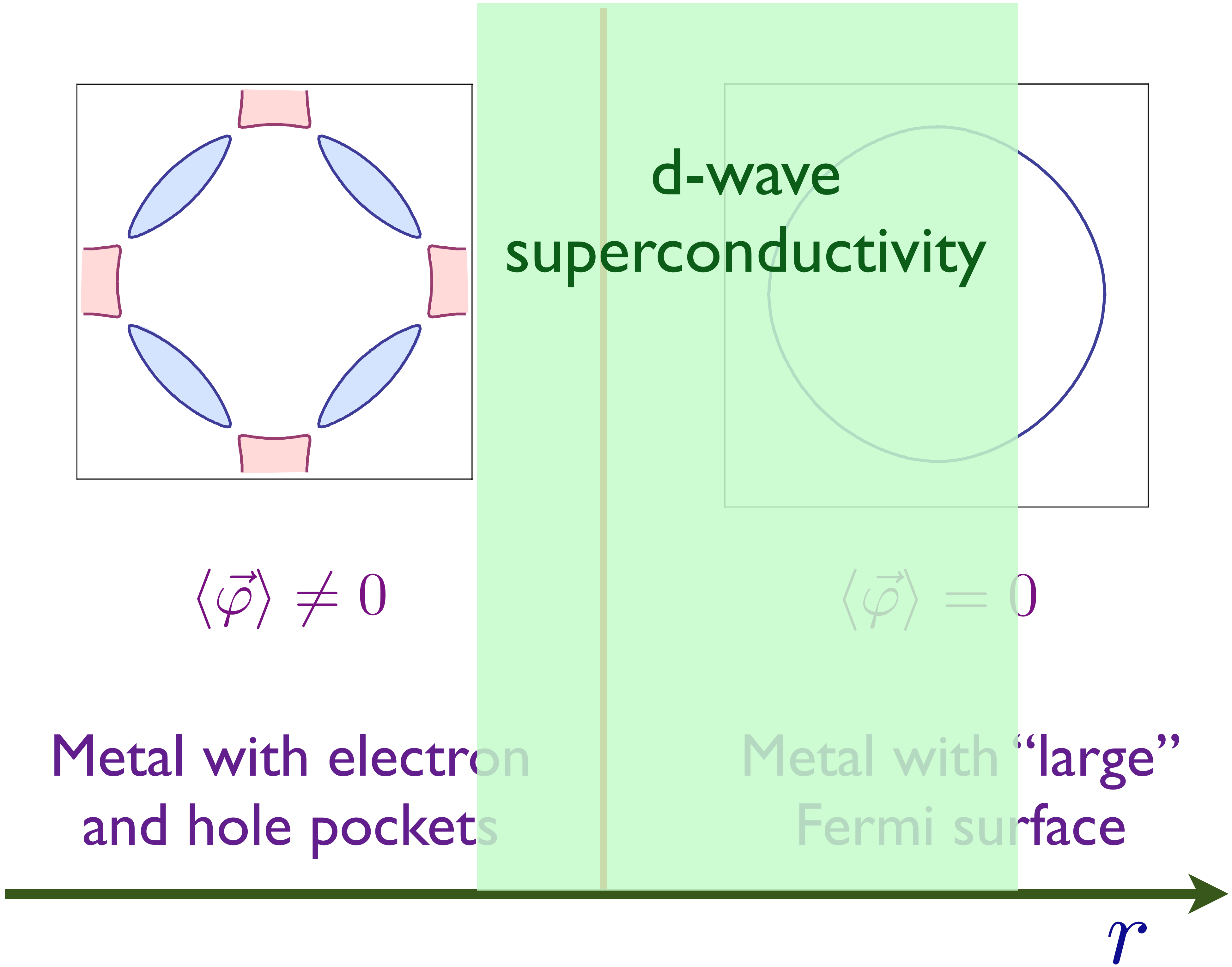
Metal with "large" Fermi surface

r

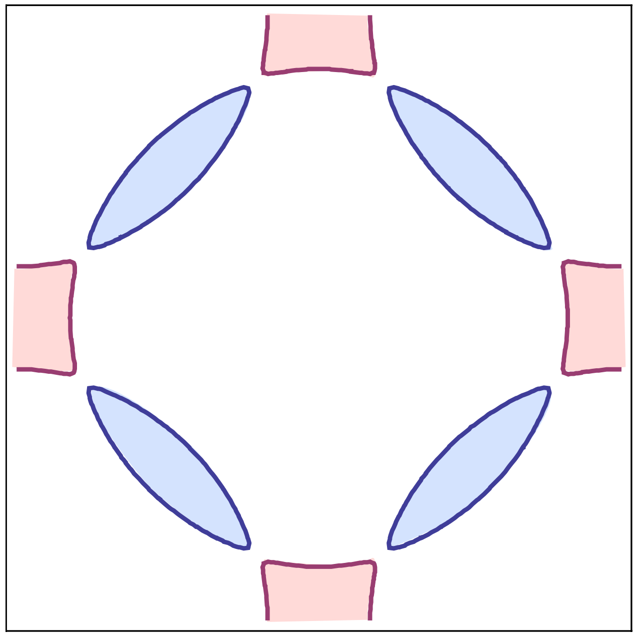
Quantum phase transition with onset of antiferromagnetism in a metal

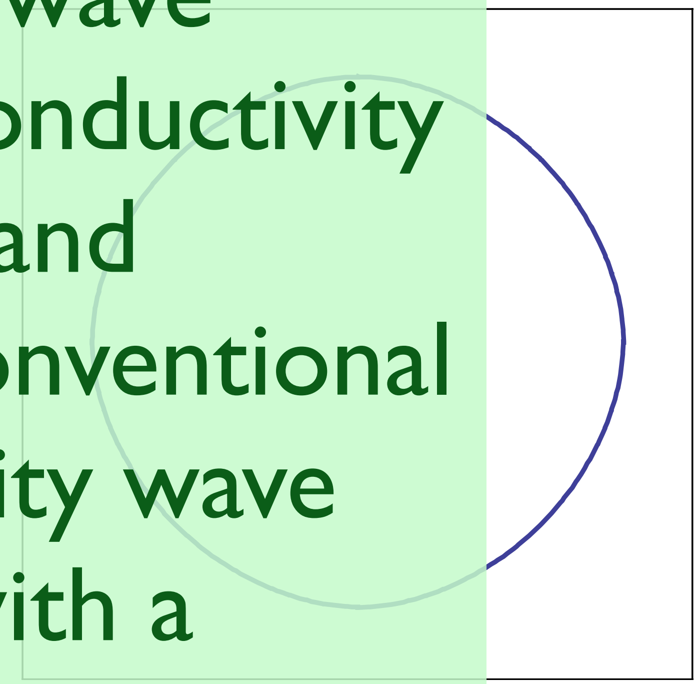


Quantum phase transition with onset of antiferromagnetism in a metal



Quantum phase transition with onset of antiferromagnetism in a metal





d-wave
superconductivity
and
an unconventional
density wave
with a
d-form factor

$\langle \vec{\varphi} \rangle \neq 0$

$\langle \vec{\varphi} \rangle = 0$

Metal with electron and hole pockets

Metal with "large" Fermi surface

r

Direct phase-sensitive identification of a *d*-form factor density wave in underdoped cuprates

Kazuhiro Fujita[a,b,c,1], Mohammad H. Hamidian[a,b,1], Stephen D. Edkins[b,d], Chung Koo Kim[a], Yuhki Kohsaka[e], Masaki Azuma[f], Mikio Takano[g], Hidenori Takagi[c,h,i], Hiroshi Eisaki[j], Shin-ichi Uchida[c], Andrea Allais[k], Michael J. Lawler[b,l], Eun-Ah Kim[b], Subir Sachdev[k,m], and J. C. Séamus Davis[a,b,d,2]

The identity of the fundamental broken symmetry (if any) in the underdoped cuprates is unresolved. However, evidence has been accumulating that this state may be an unconventional density wave. Here we carry out site-specific measurements within each CuO_2 unit cell, segregating the results into three separate electronic structure images containing only the Cu sites [$Cu(r)$] and only the x/y axis O sites [$O_x(r)$ and $O_y(r)$]. Phase-resolved Fourier analysis reveals directly that the modulations in the $O_x(r)$ and $O_y(r)$ sublattice images consistently exhibit a relative phase of π. We confirm this discovery on two highly distinct cuprate compounds, ruling out tunnel matrix-element and materials-specific systematics. These observations demonstrate by direct sublattice phase-resolved visualization that the density wave found in underdoped cuprates consists of modulations of the intraunit-cell states that exhibit a predominantly *d*-symmetry form factor.

Direct phase-sensitive visualization of the *d*-form factor density wave in underdoped cuprates arXiv:1404.0362, PNAS to appear.

J. C. Seamus Davis
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Mohammad Hamidian
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Stephen Edkins
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Yuhki Kohsaka
RIKEN - Tokyo

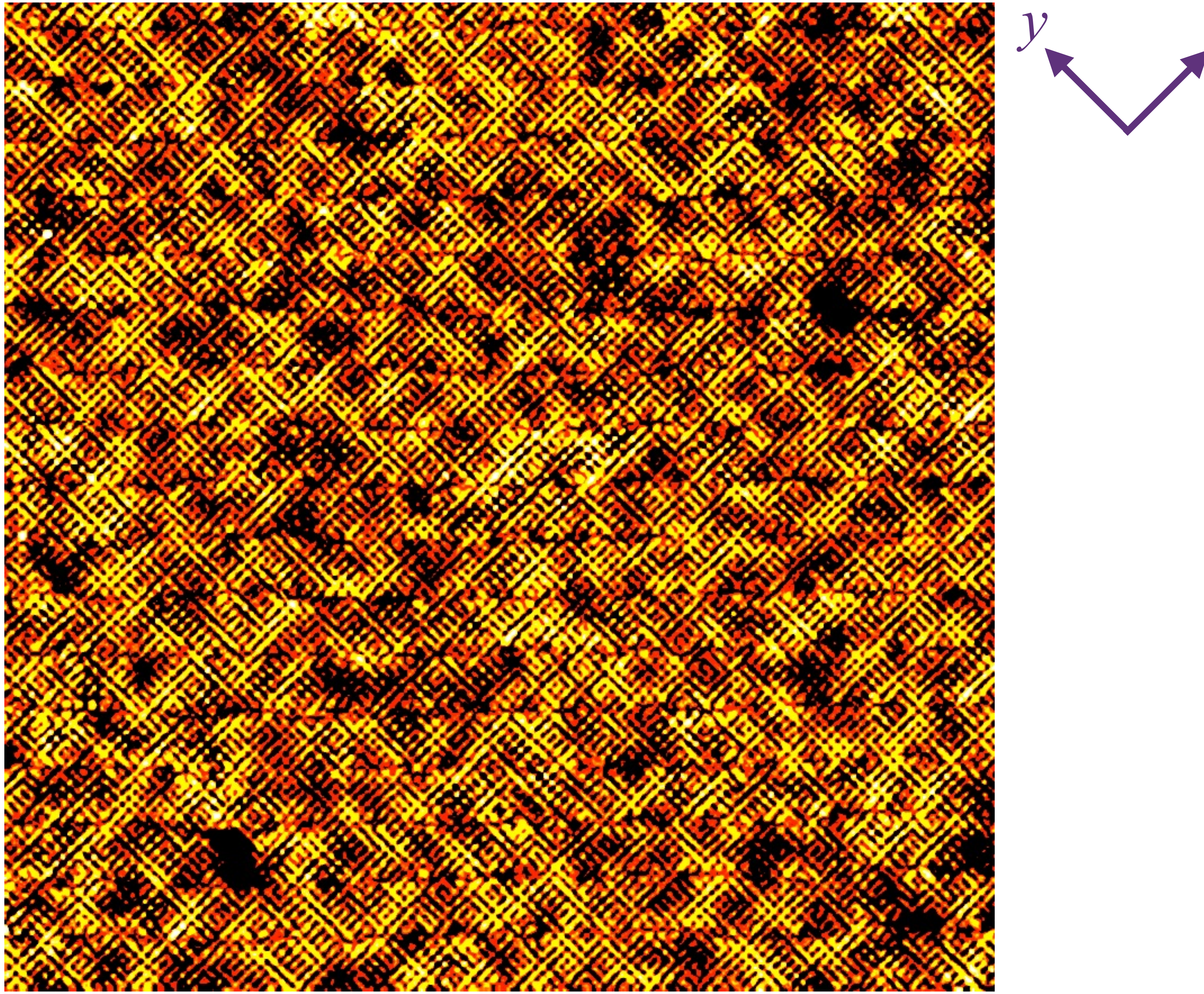
Michael Lawler
Cornell / Binghamton

Eun-Ah Kim
Cornell

See also
C. Howald, H. Eisaki, N. Kaneko, M. Greven, and A. Kapitulnik, Phys. Rev. B **67**, 014533 (2003);

M. Vershinin, S. Misra, S. Ono, Y. Abe, Yoichi Ando, and A. Yazdani, Science **303**, 1995 (2004).

W. D. Wise, M. C. Boyer, K. Chatterjee, T. Kondo, T. Takeuchi, H. Ikuta, Y. Wang, and E. W. Hudson, Nature Phys. **4**, 696 (2008).



"R-map" of BSCCO in zero magnetic field, similar to those published in Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007). Davis group has sub-angstrom resolution capabilities, with lattice drift corrections, which make sublattice phase-resolved STM possible.

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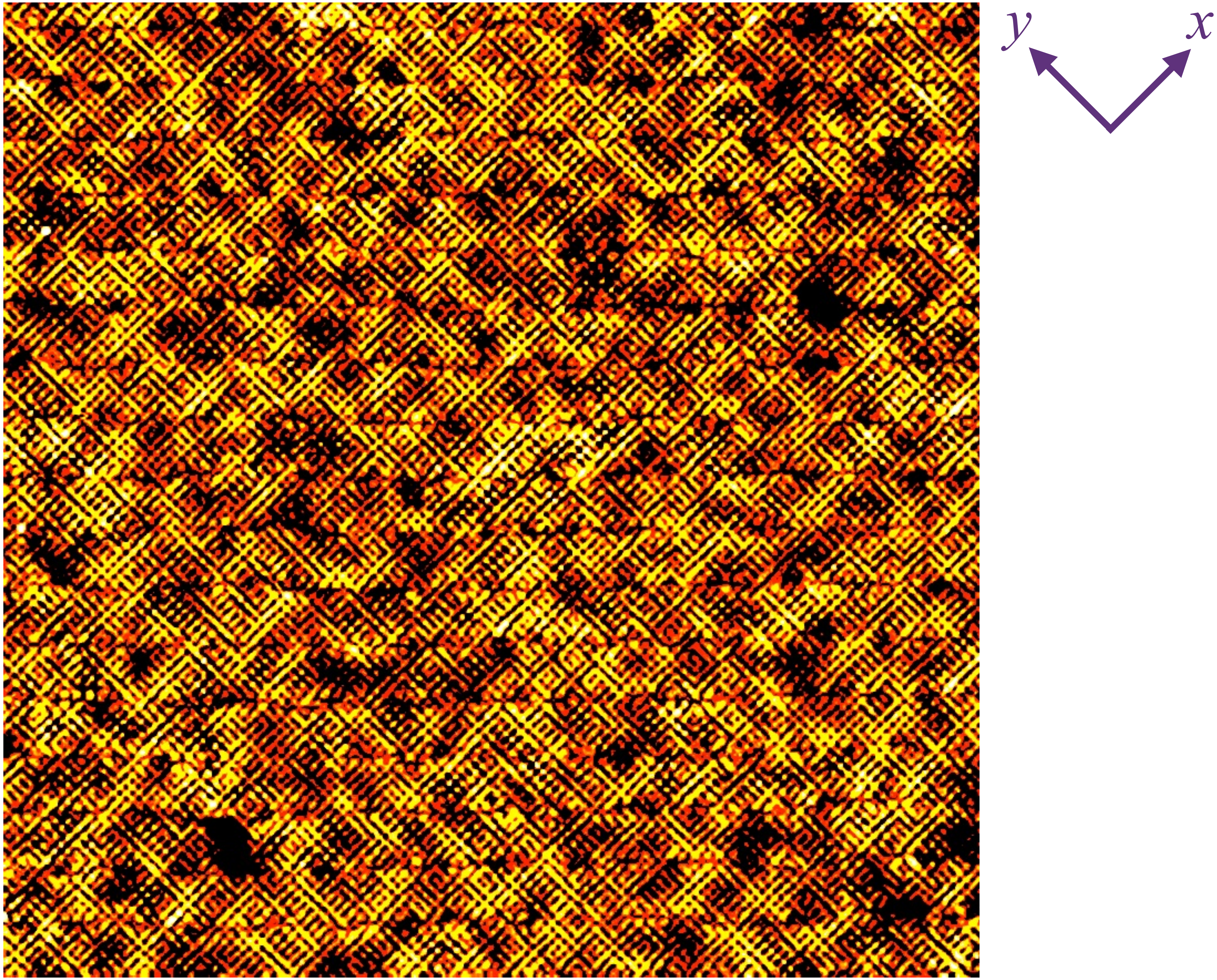

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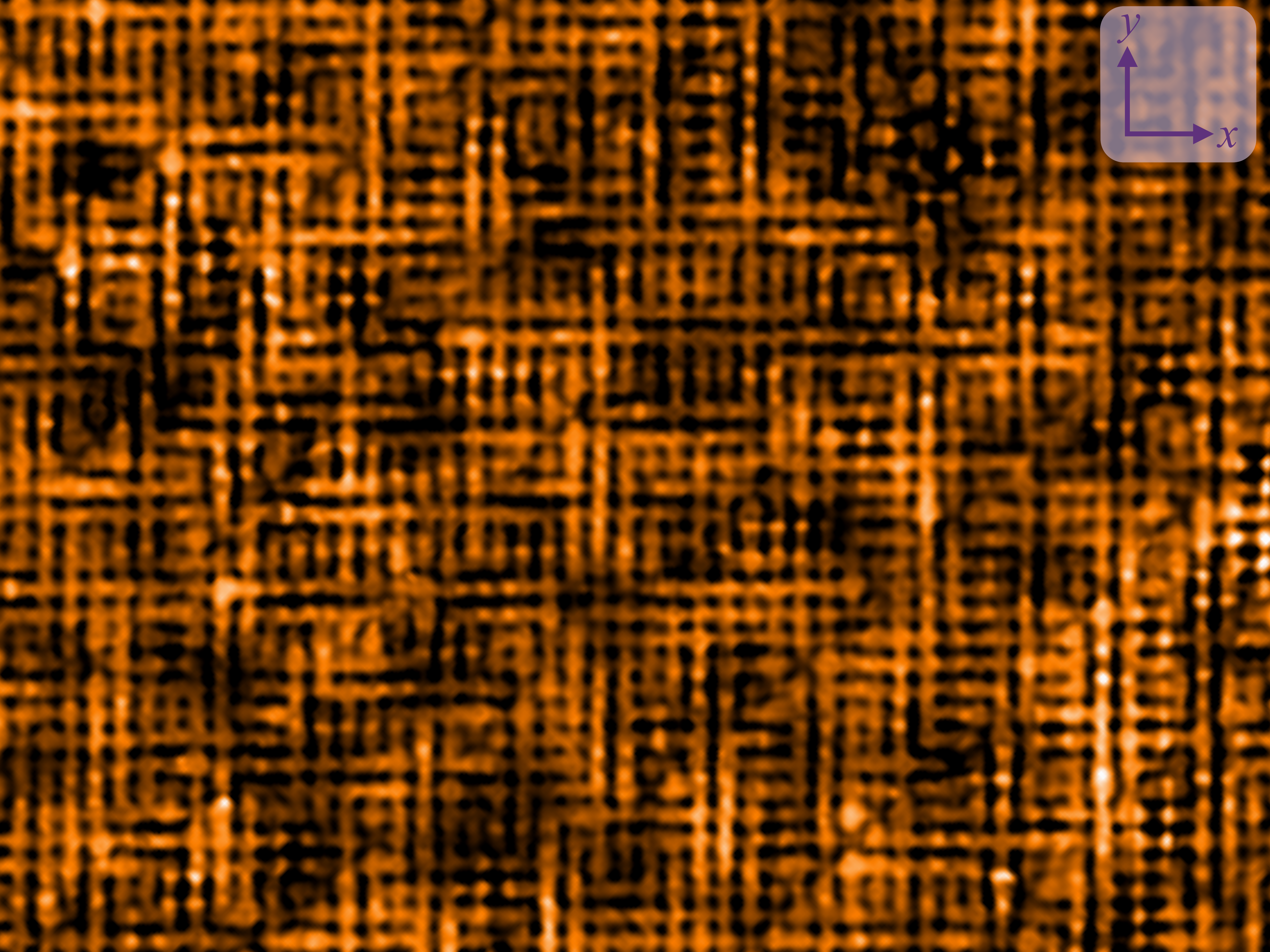
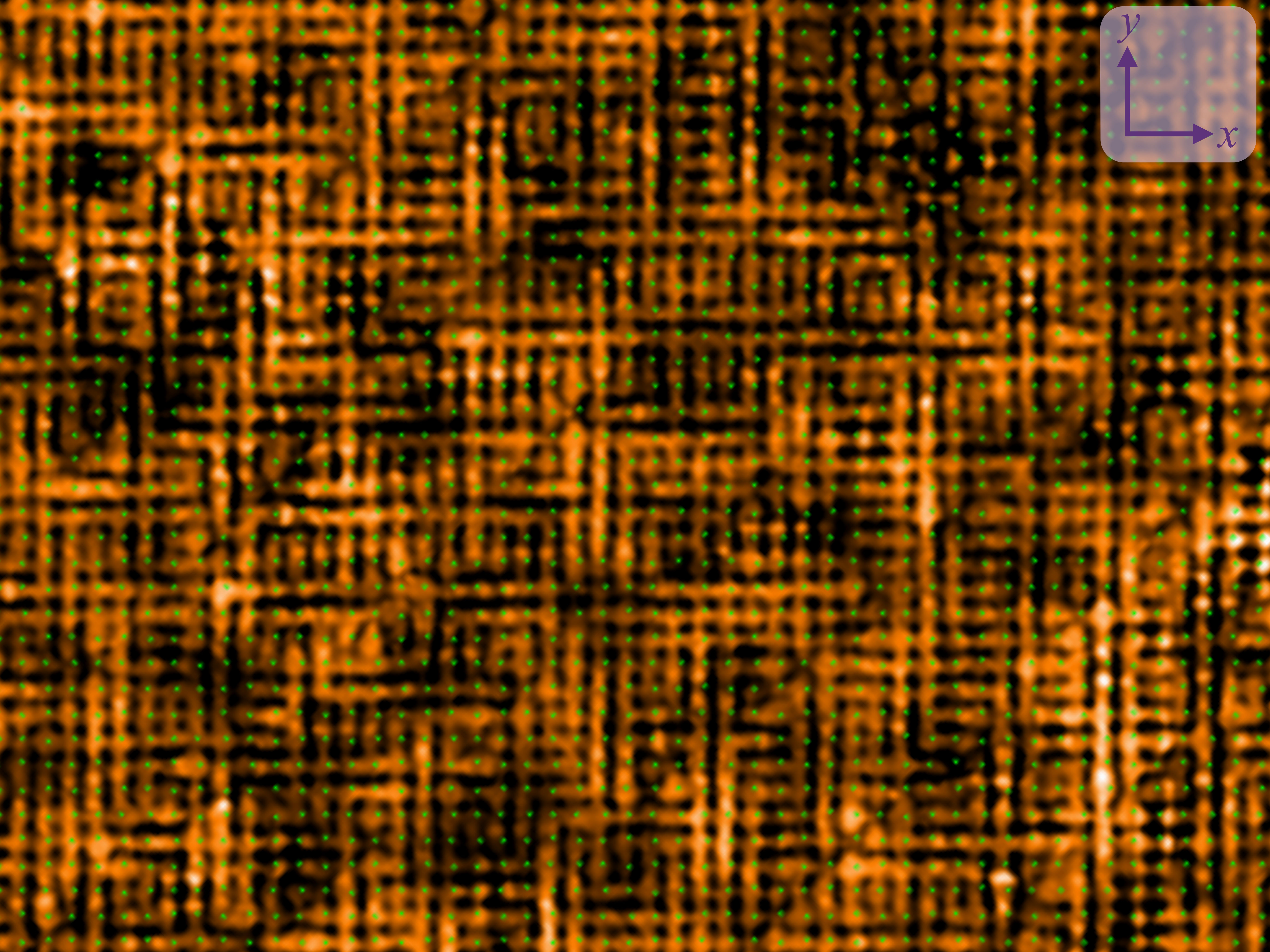
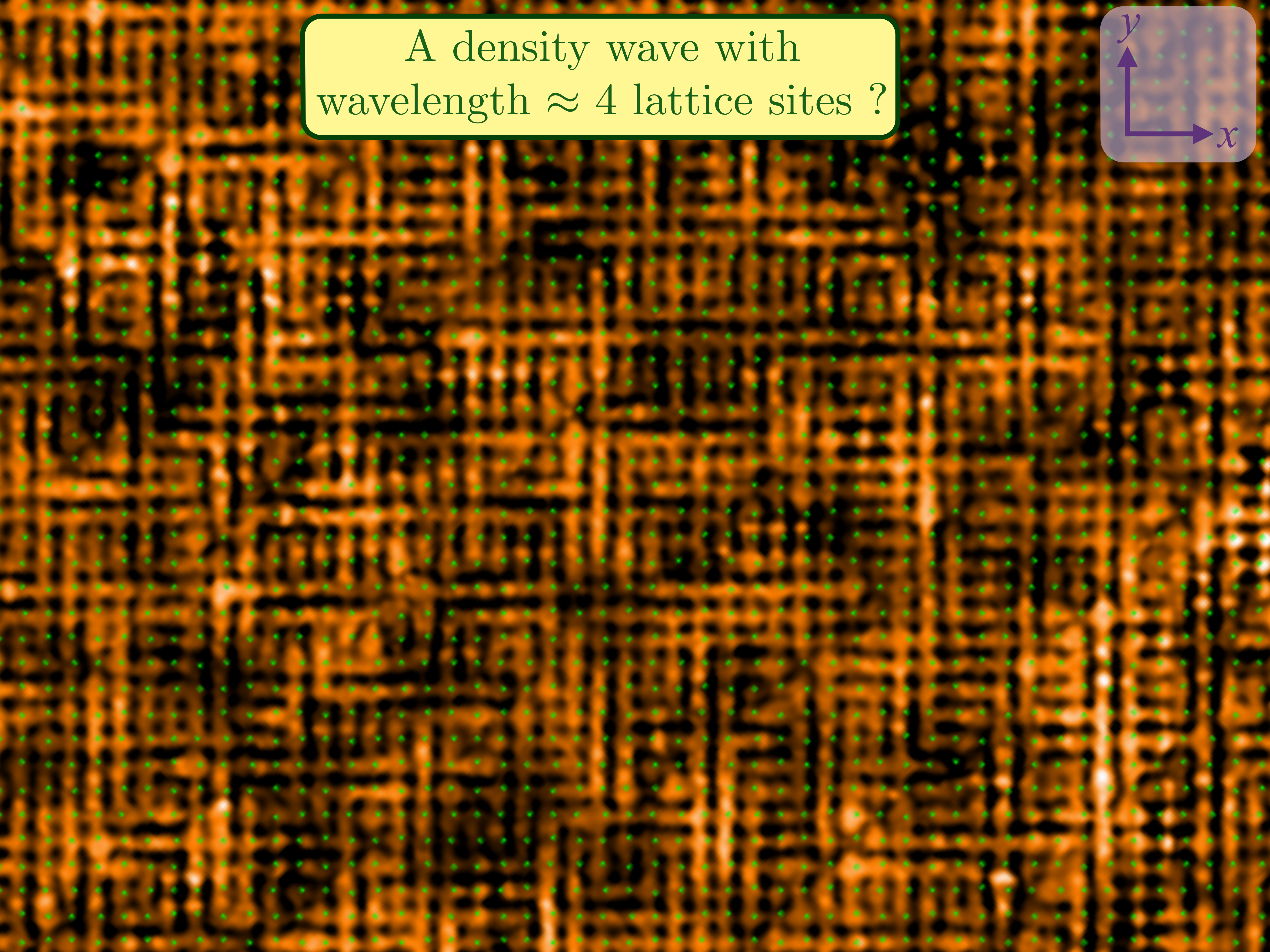

y
x


y
x


A density wave with
wavelength ≈ 4 lattice sites ?
y
x

See also
C. Howald, H. Eisaki, N. Kaneko, M. Greven, and A. Kapitulnik, Phys. Rev. B **67**, 014533 (2003);

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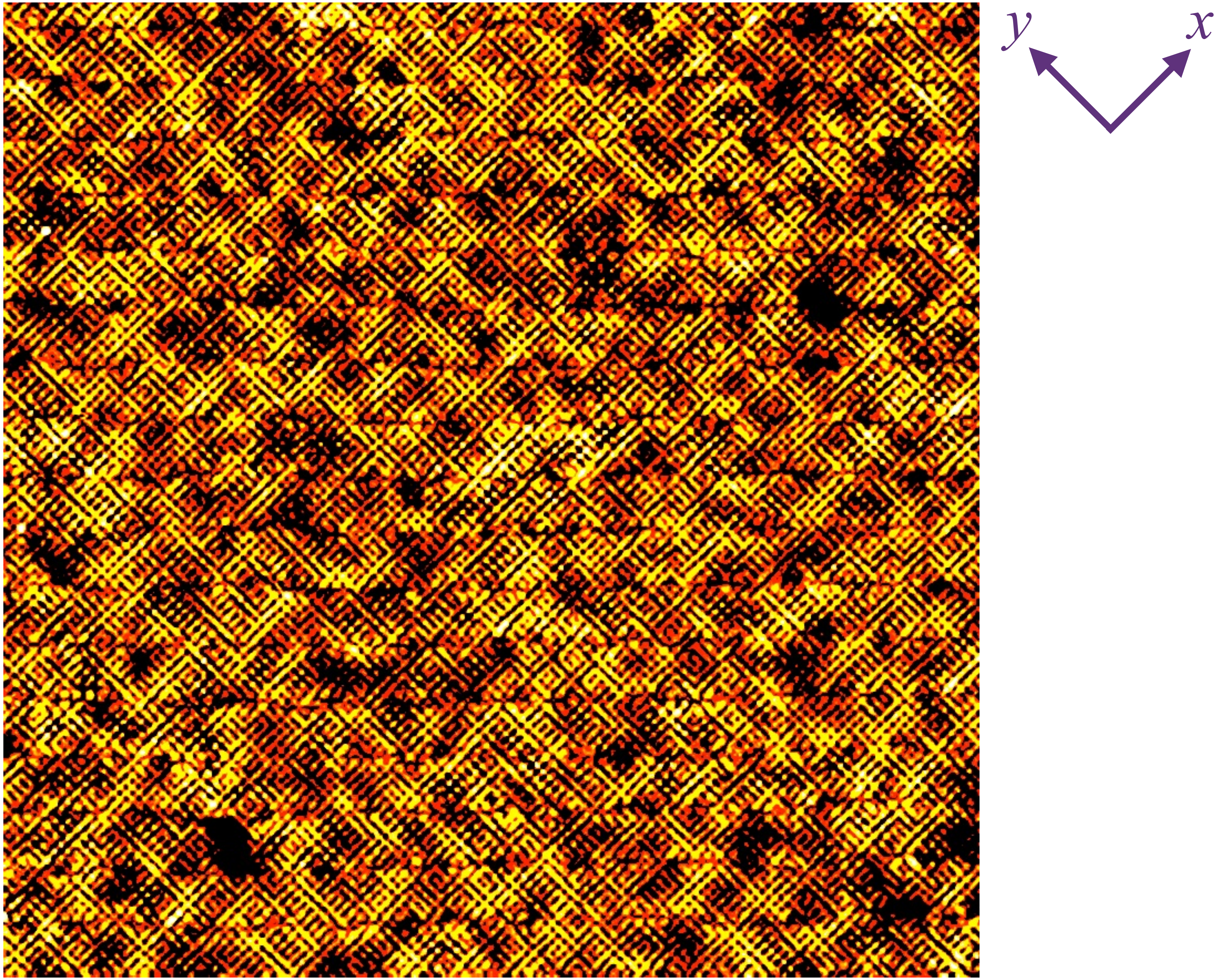

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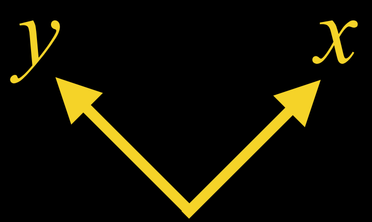
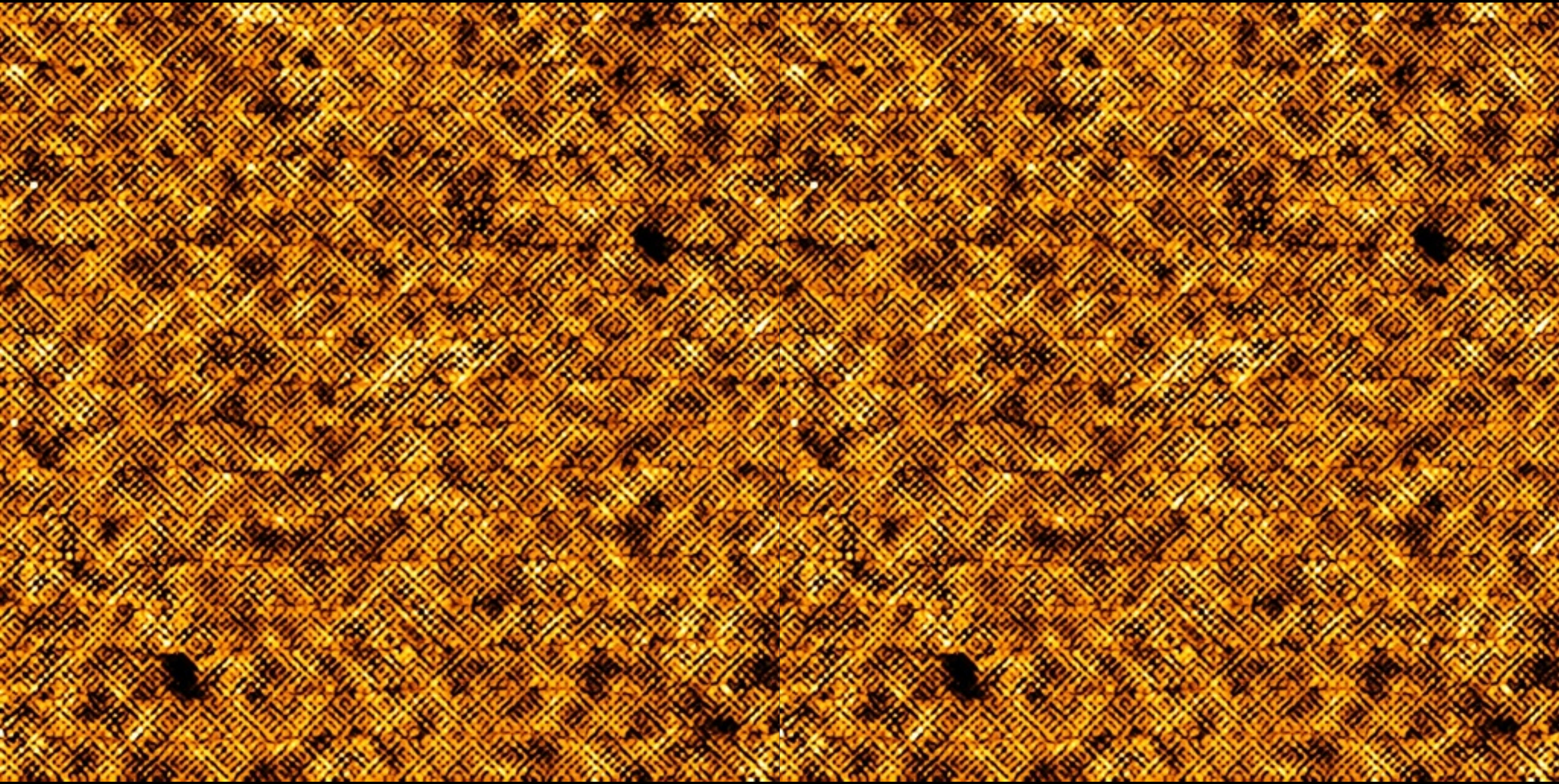
UD45K
BSCCO

R(r,150mV)

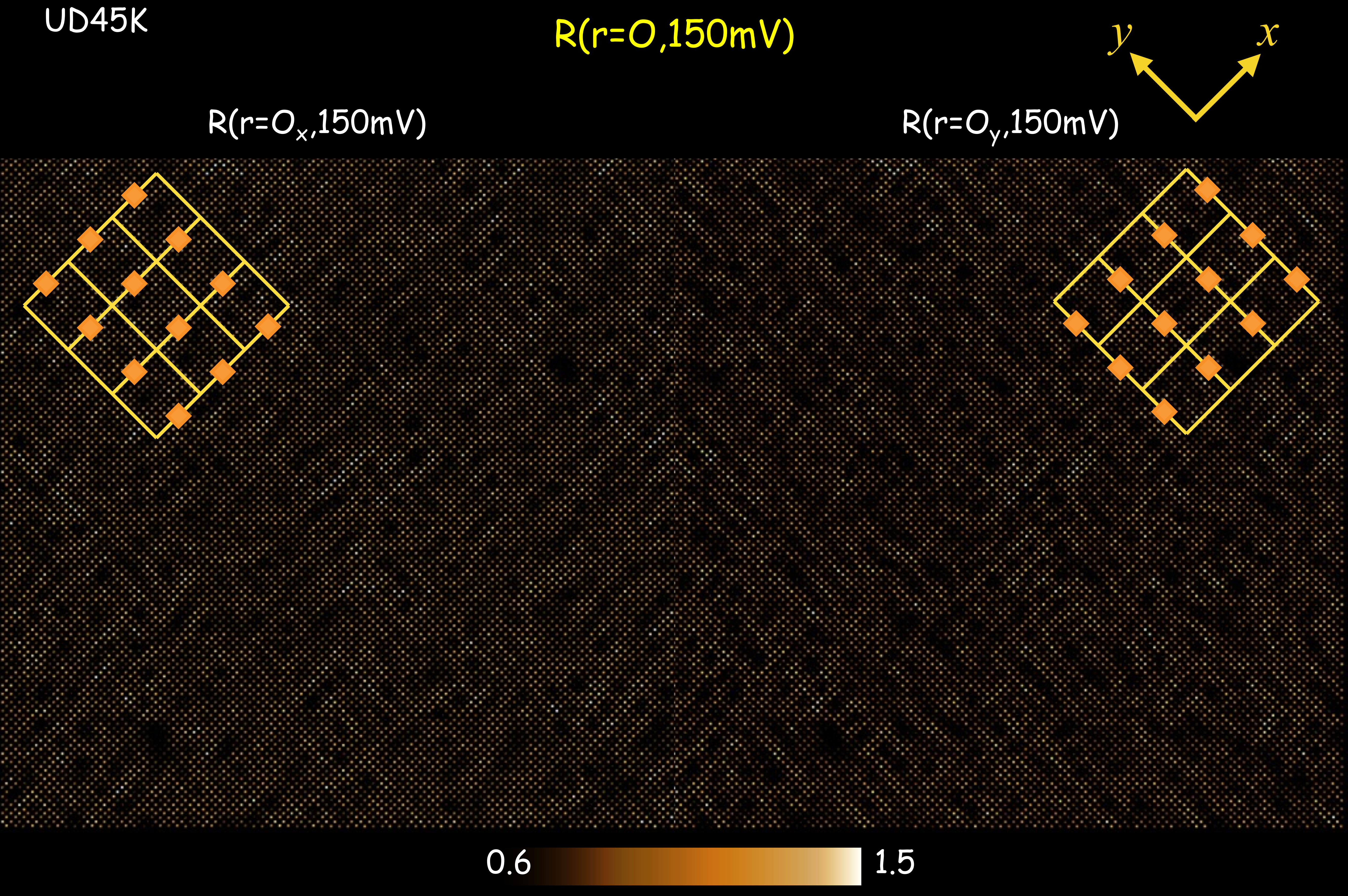


R(r,150mV) R(r,150mV)



Note that these are identical images.

K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS to appear, arXiv:1404.0362



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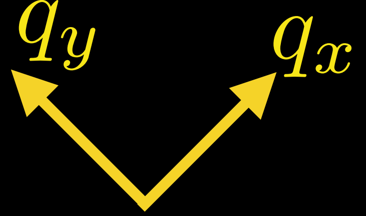
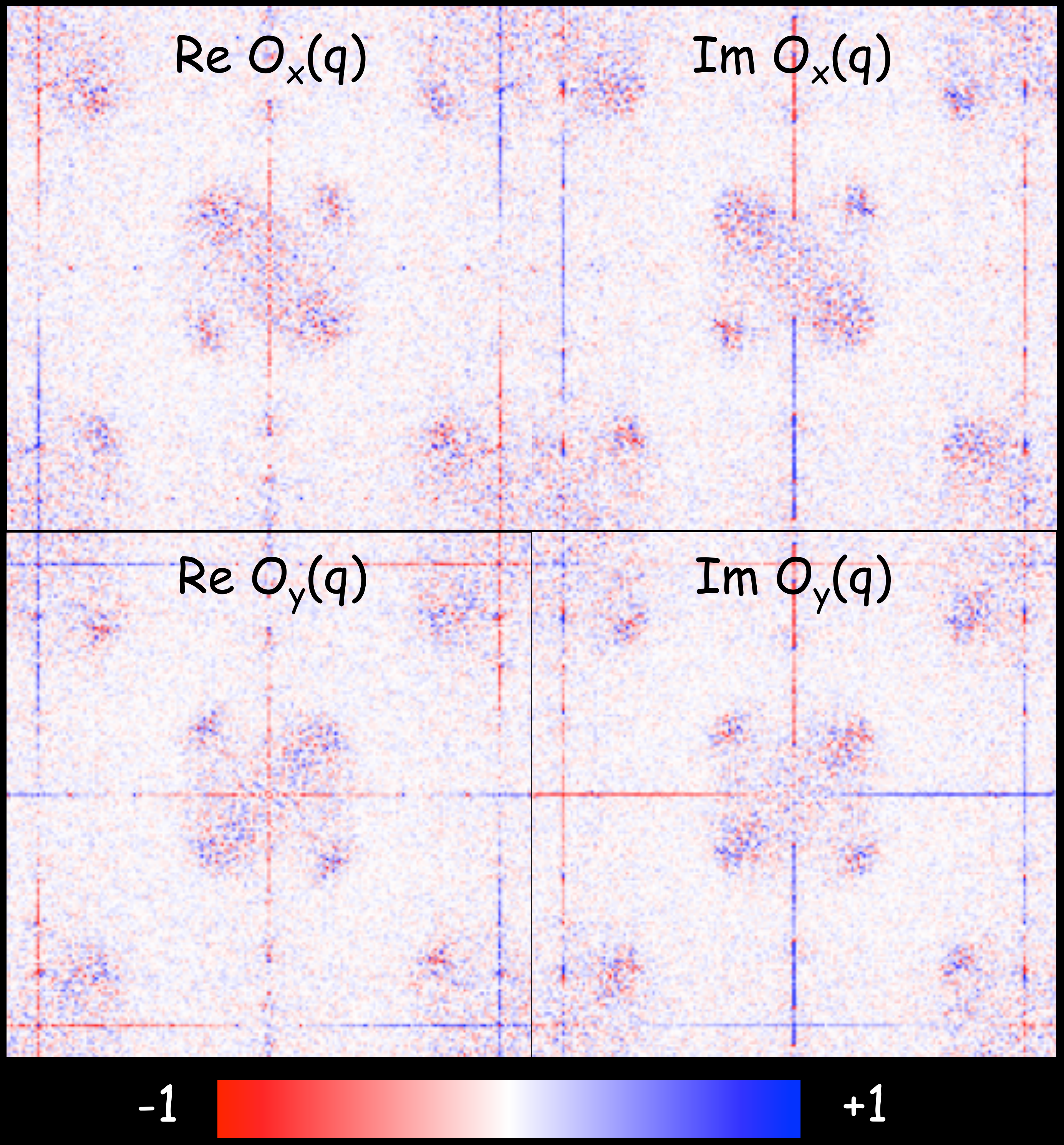
UD45K

Broad (0,Q) and (Q,0) DW Features





K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS to appear, arXiv:1404.0362

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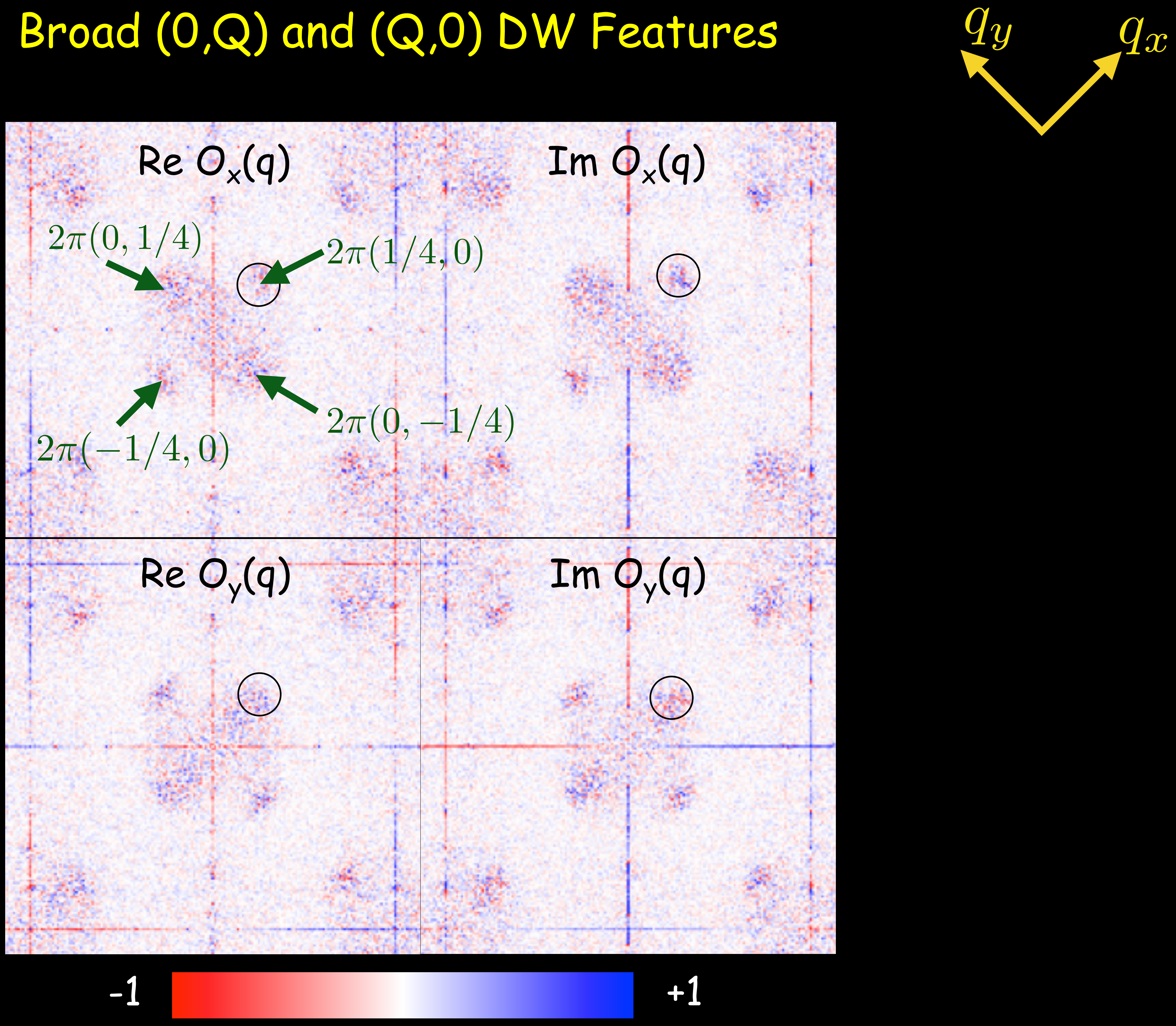

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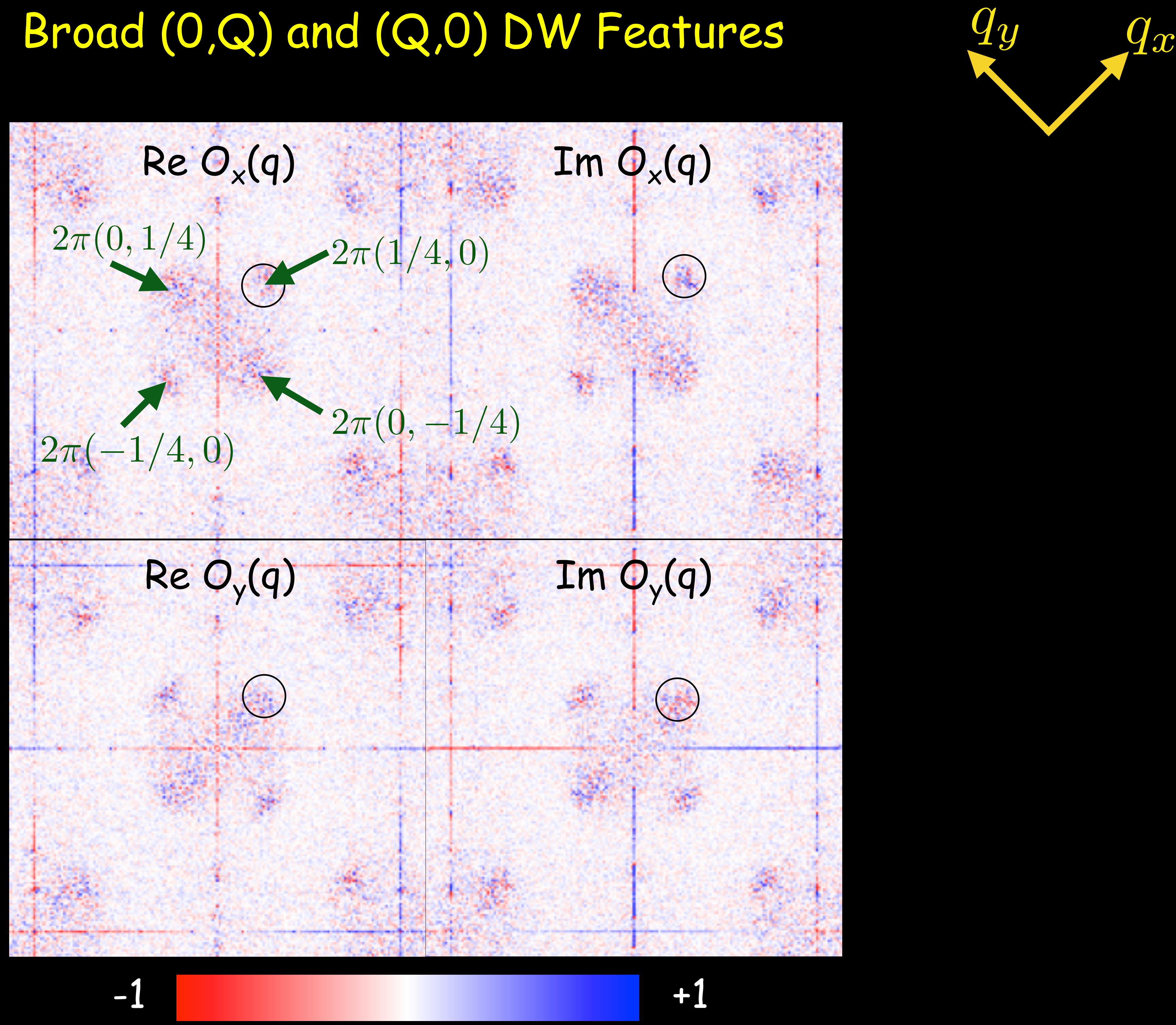
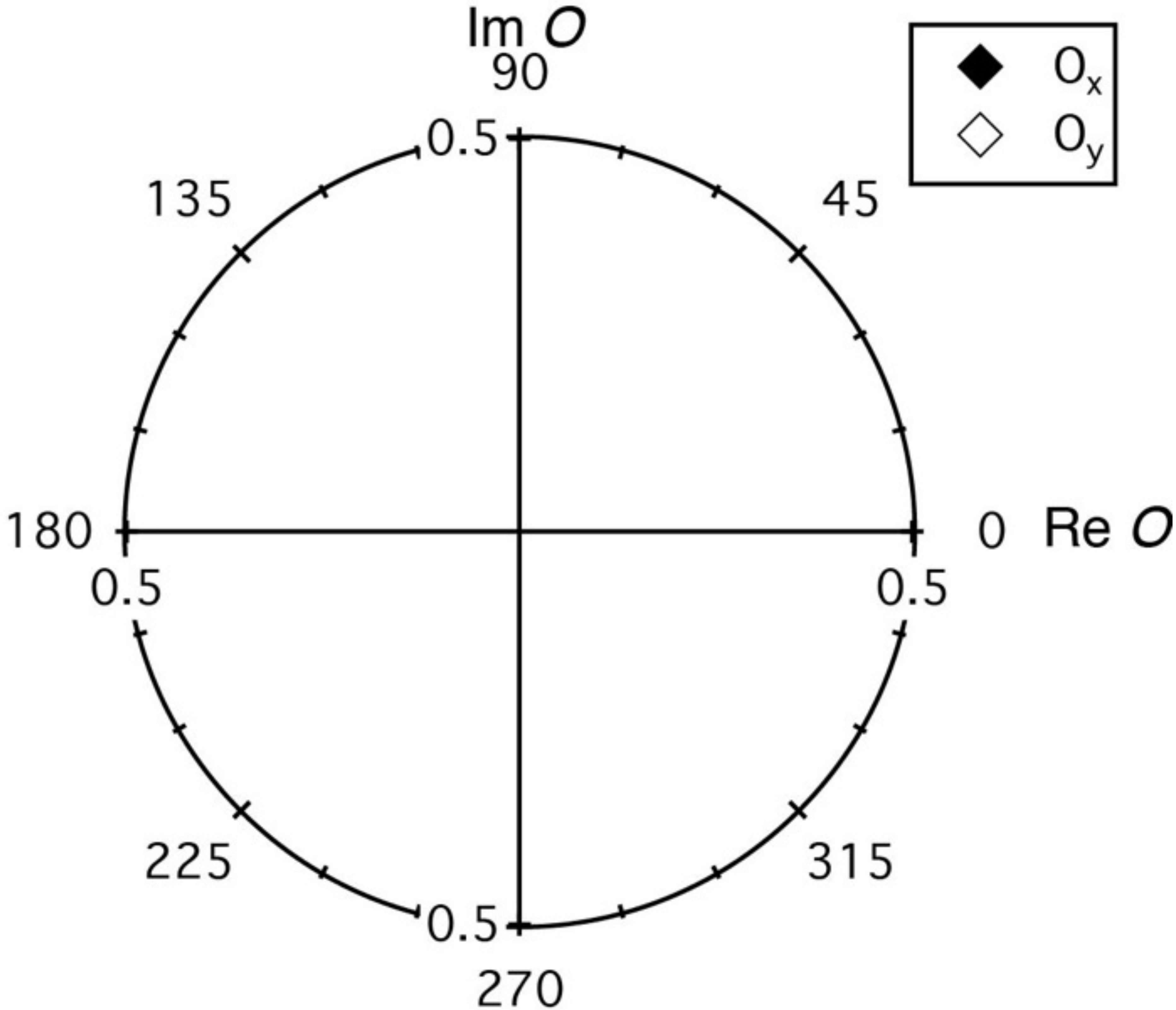


For each pixel in the circles, we obtain 2 complex numbers, $O_x(q)$ and $O_y(q)$.

K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS to appear, arXiv:1404.0362



Phase-sensitive measurement of the *d*-form factor of density wave order

K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS to appear, arXiv:1404.0362

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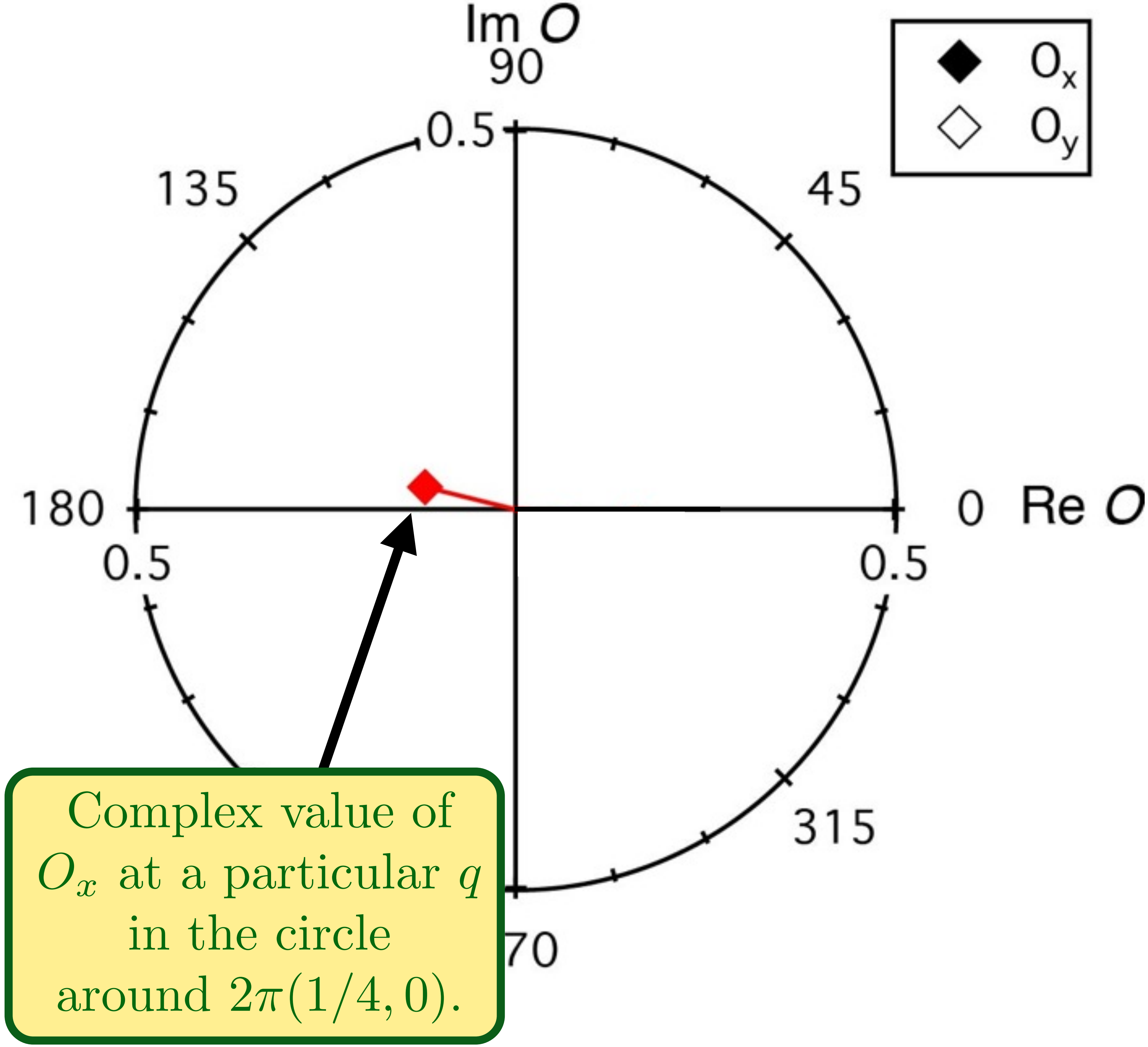

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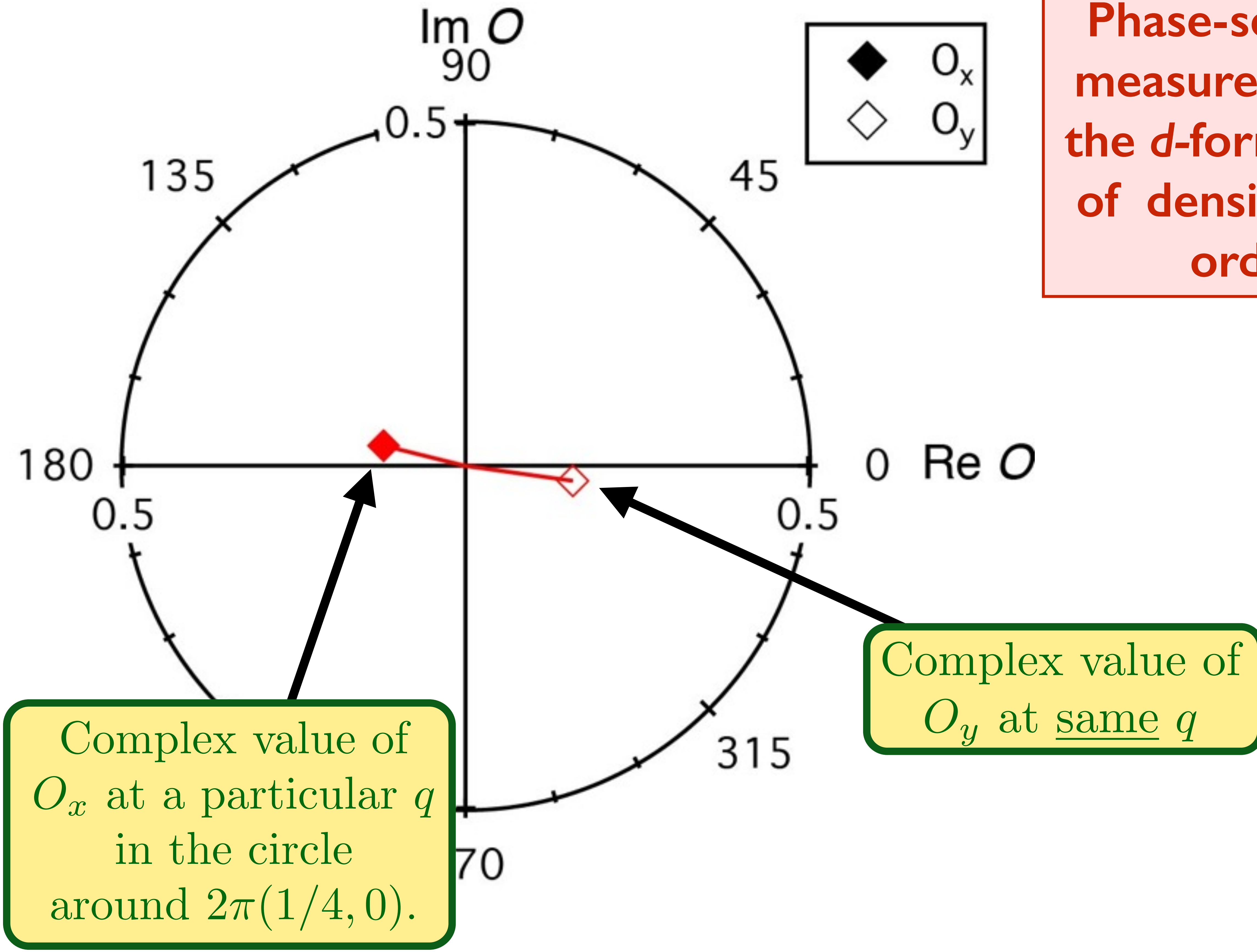

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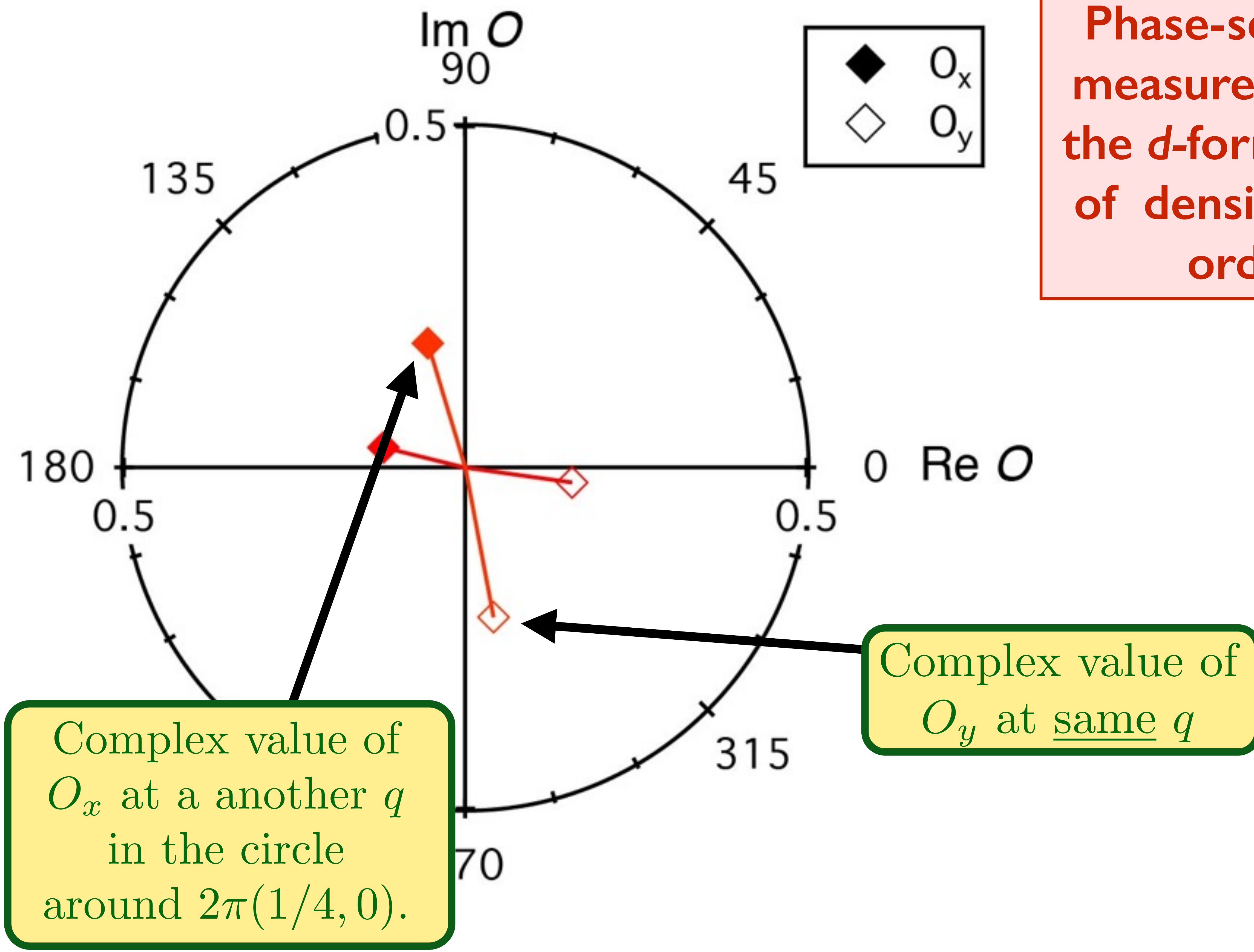

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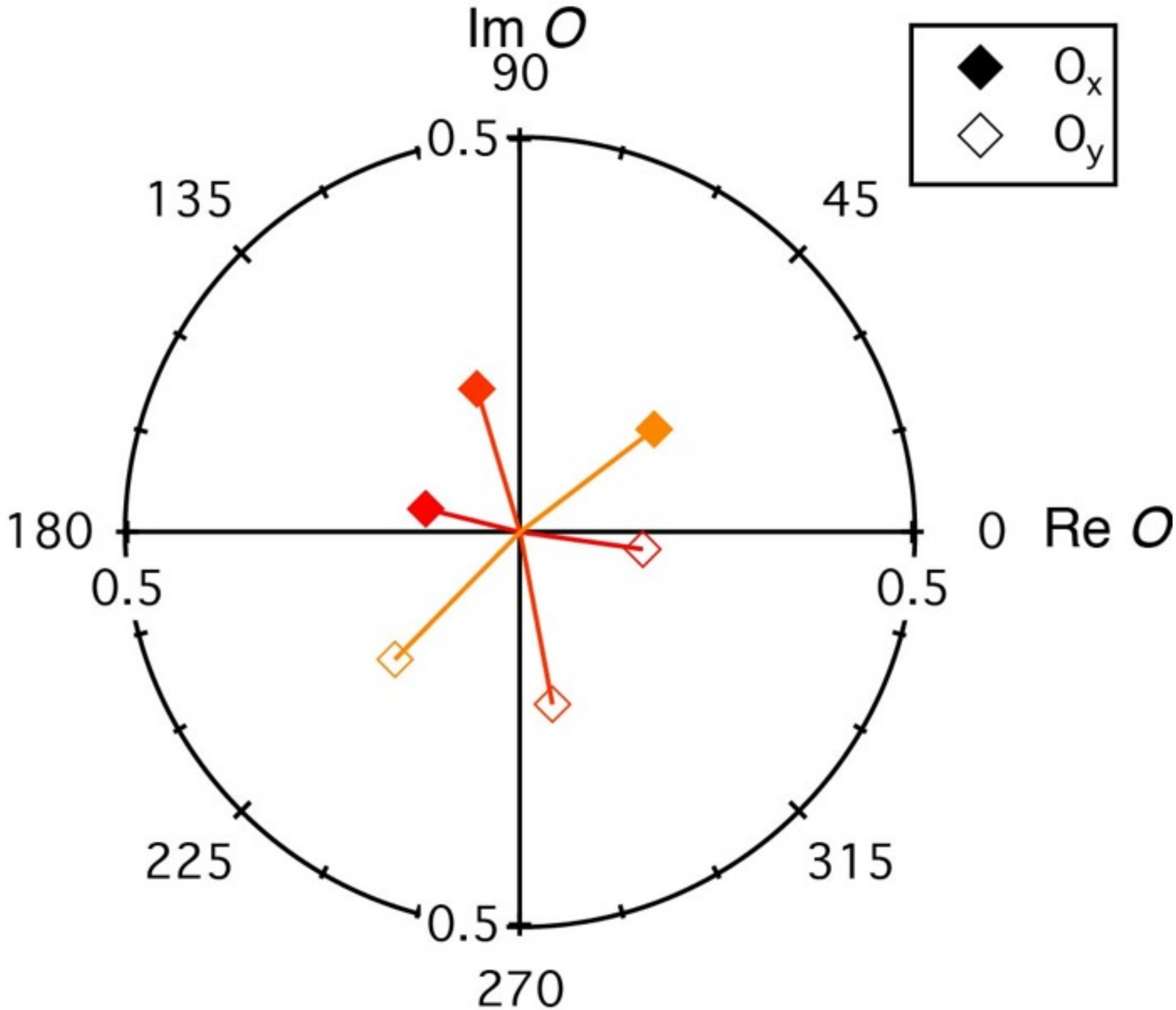

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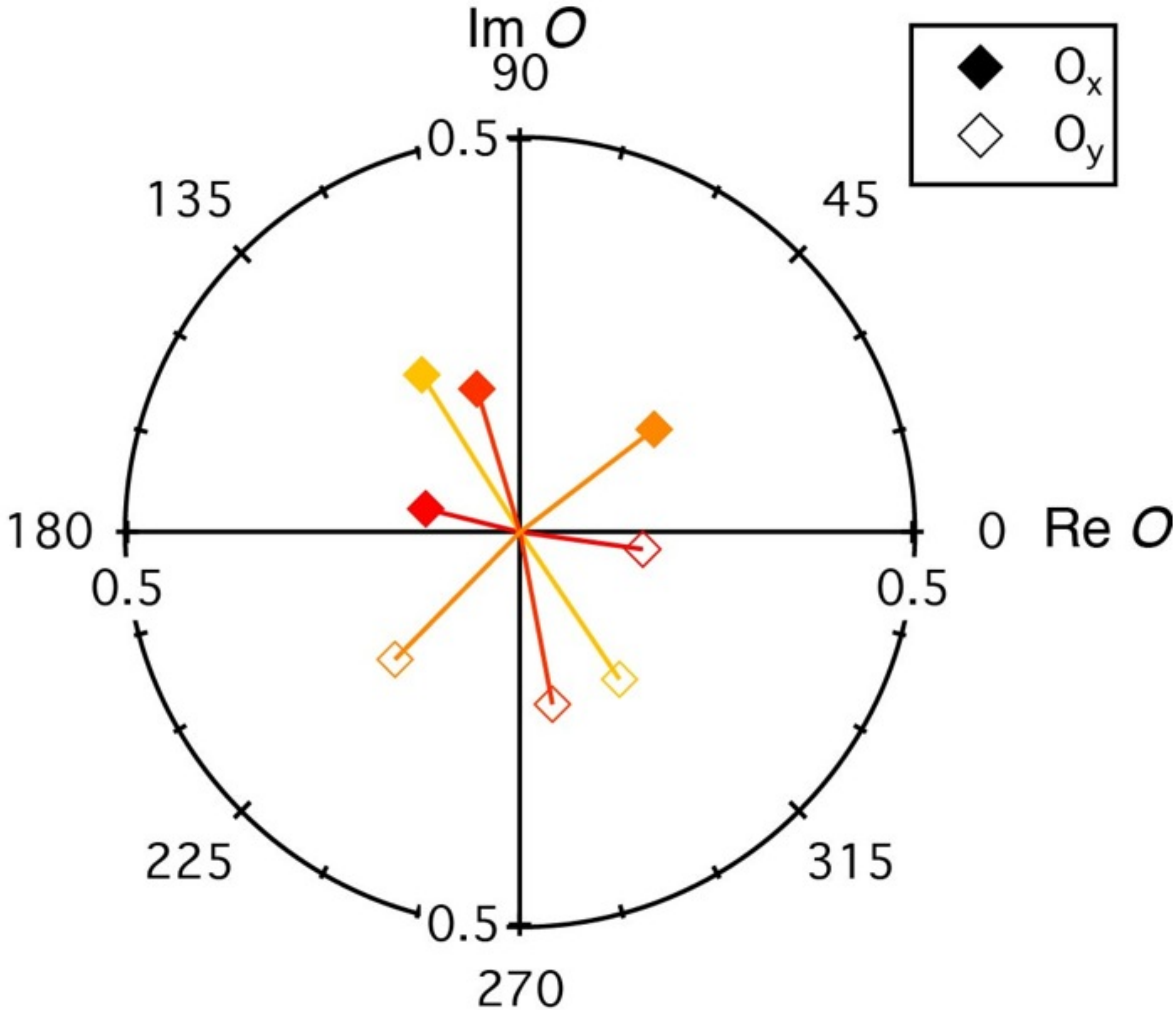

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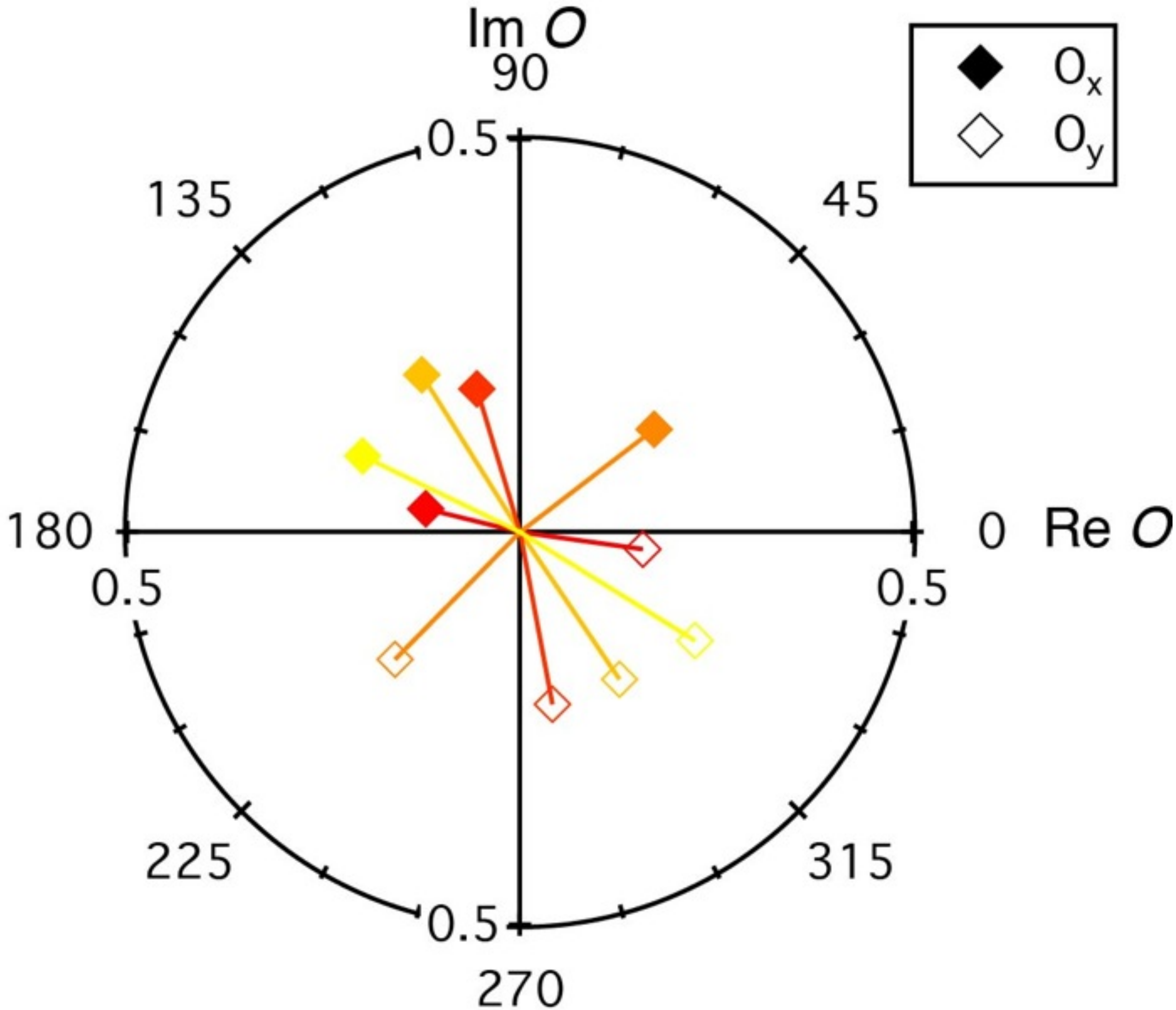

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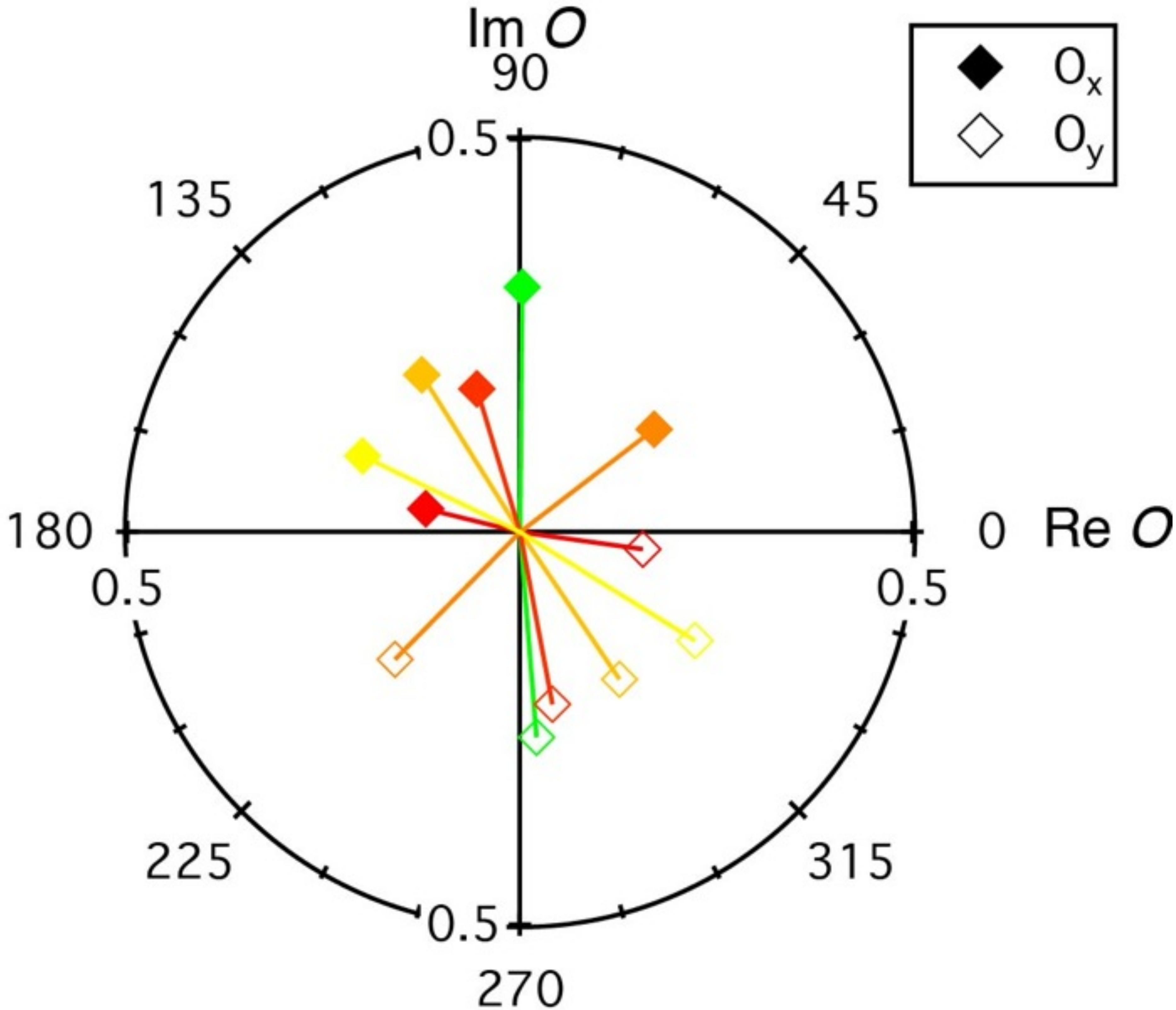

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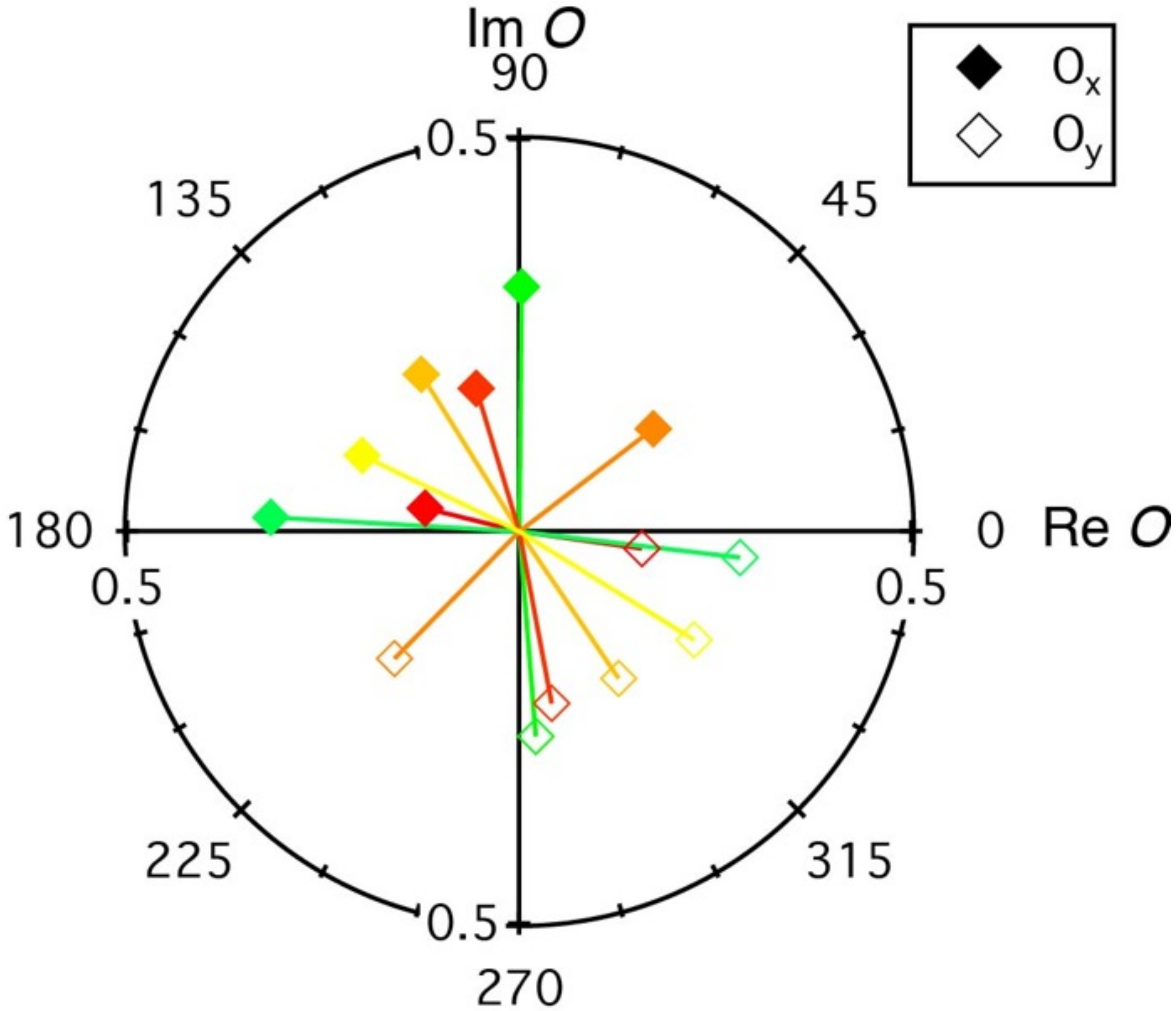

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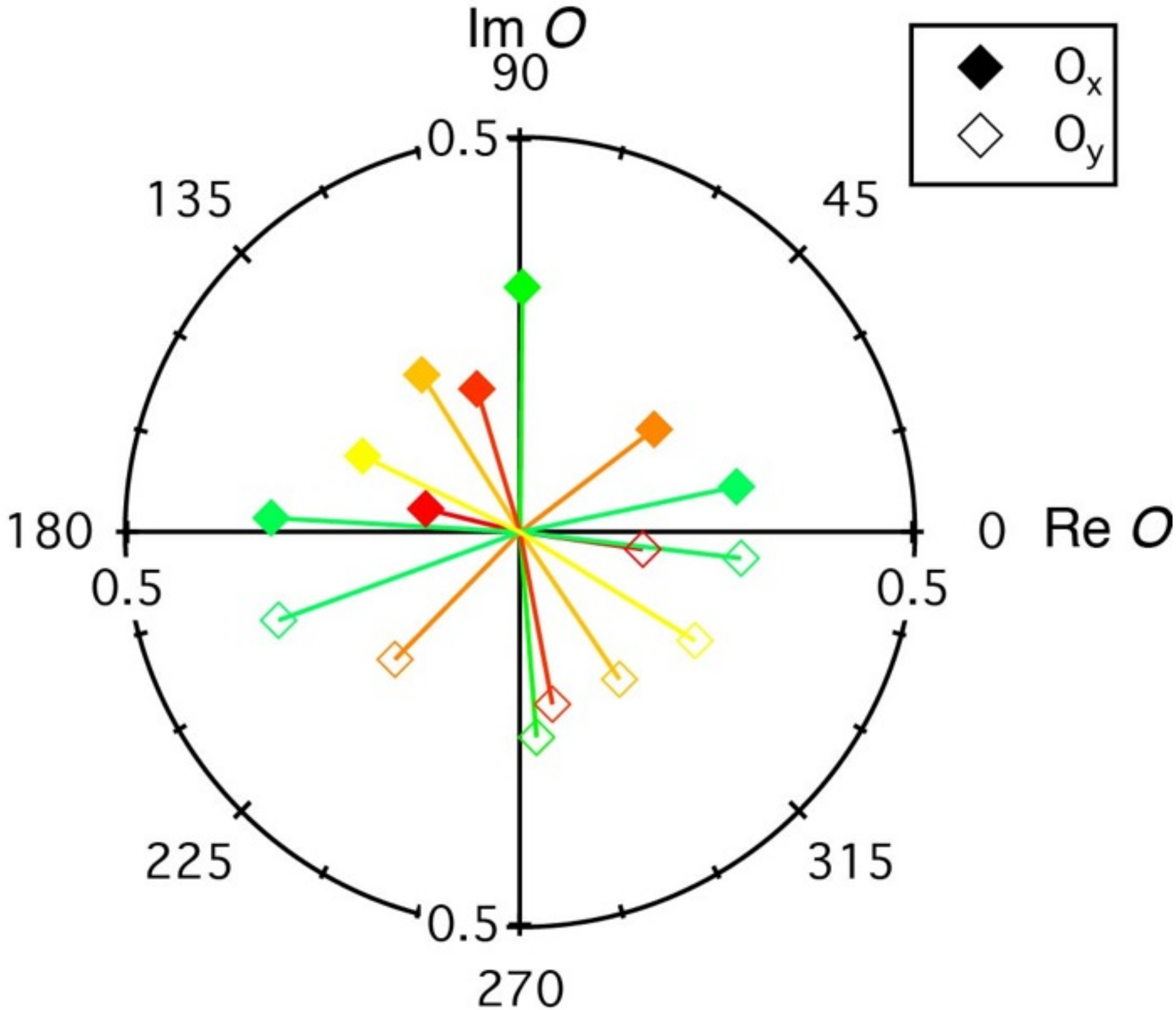

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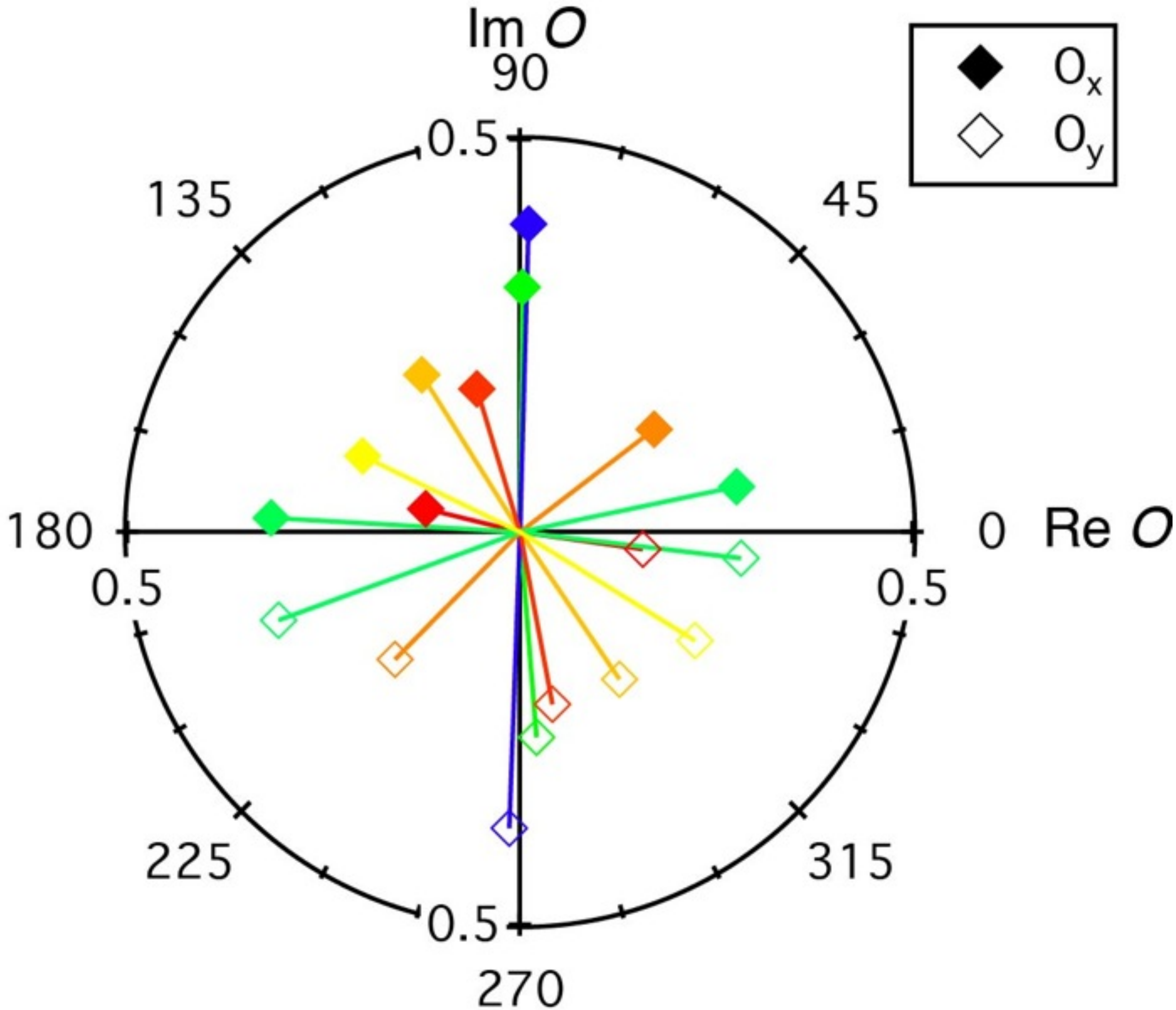

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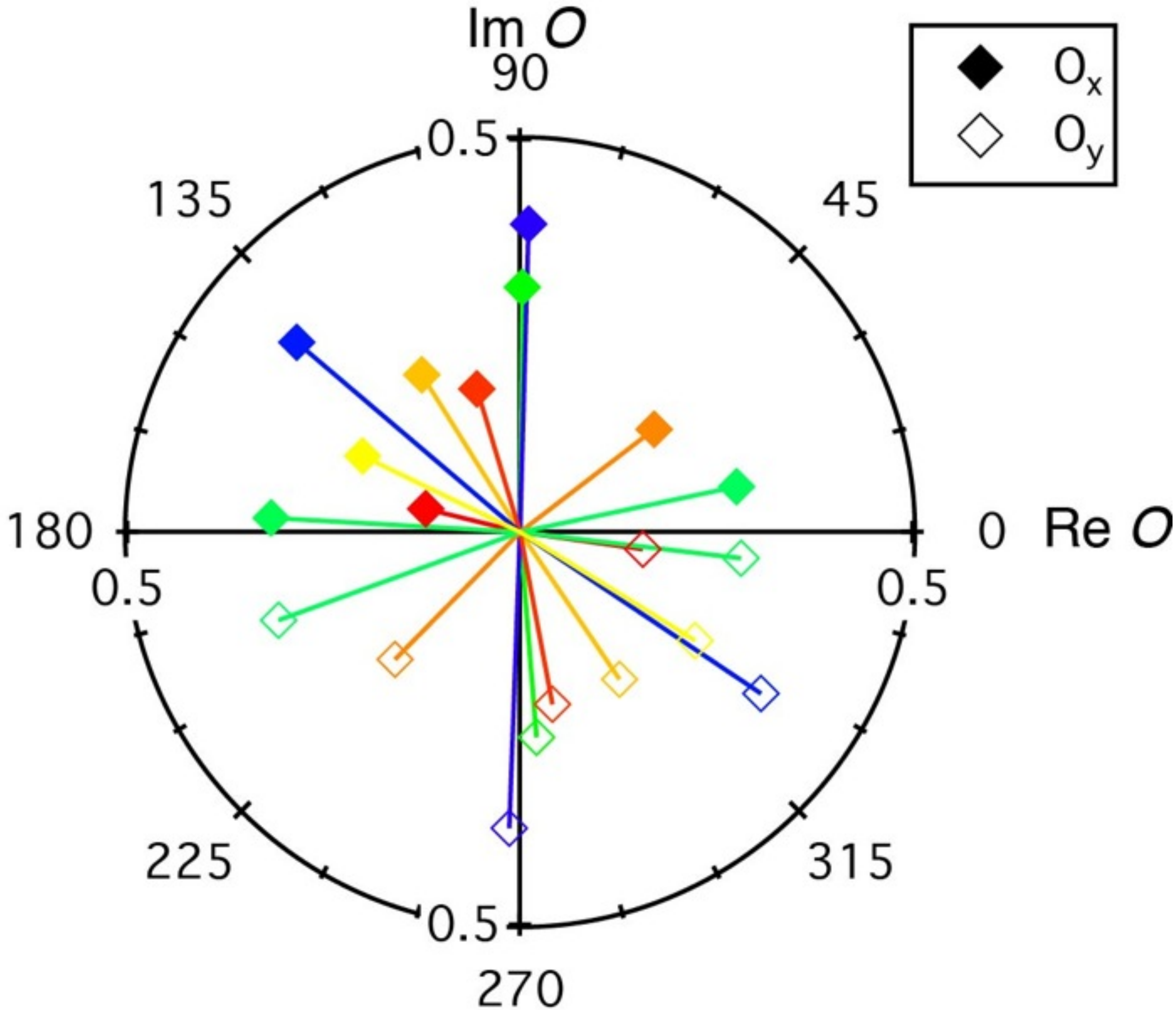

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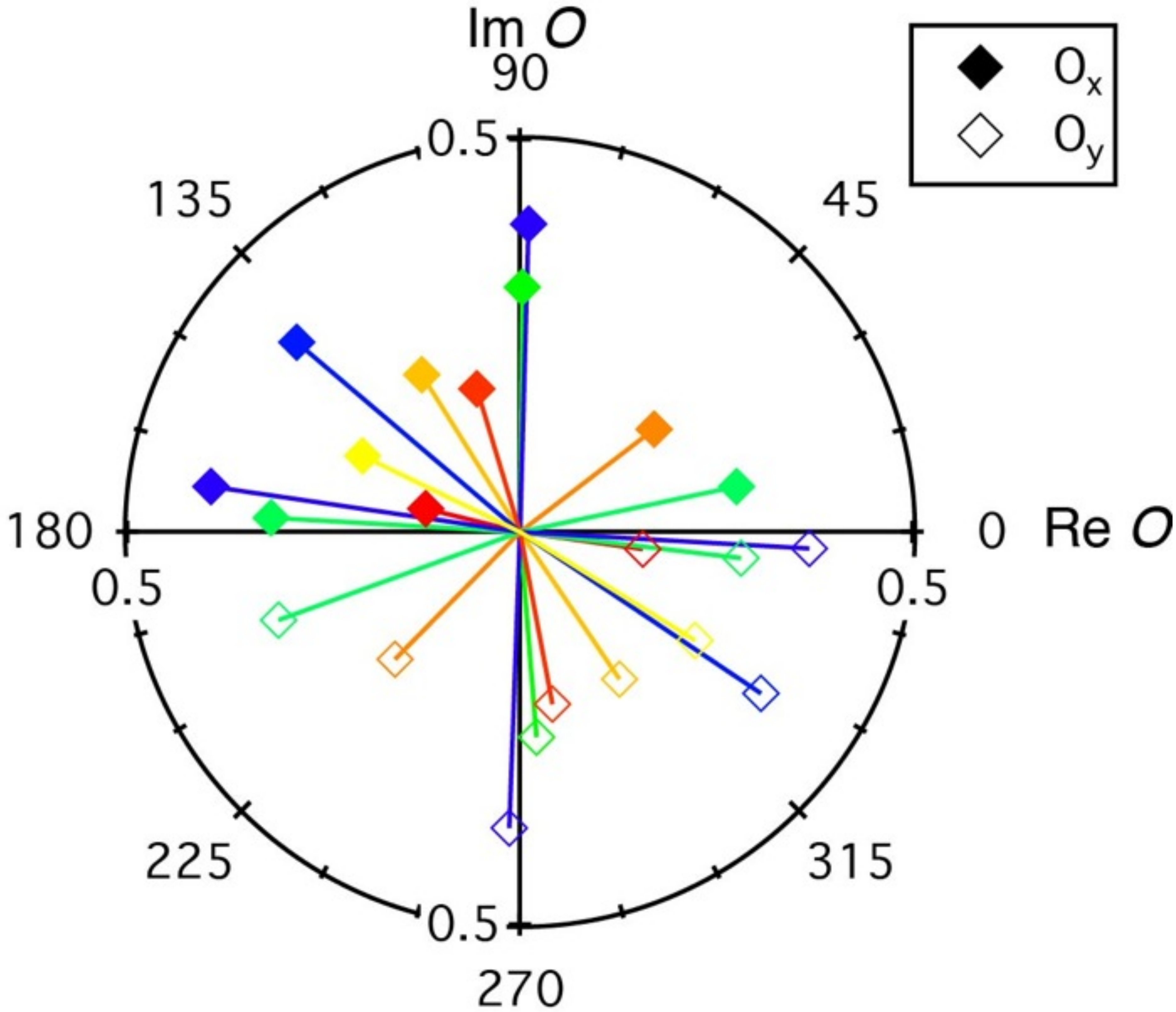

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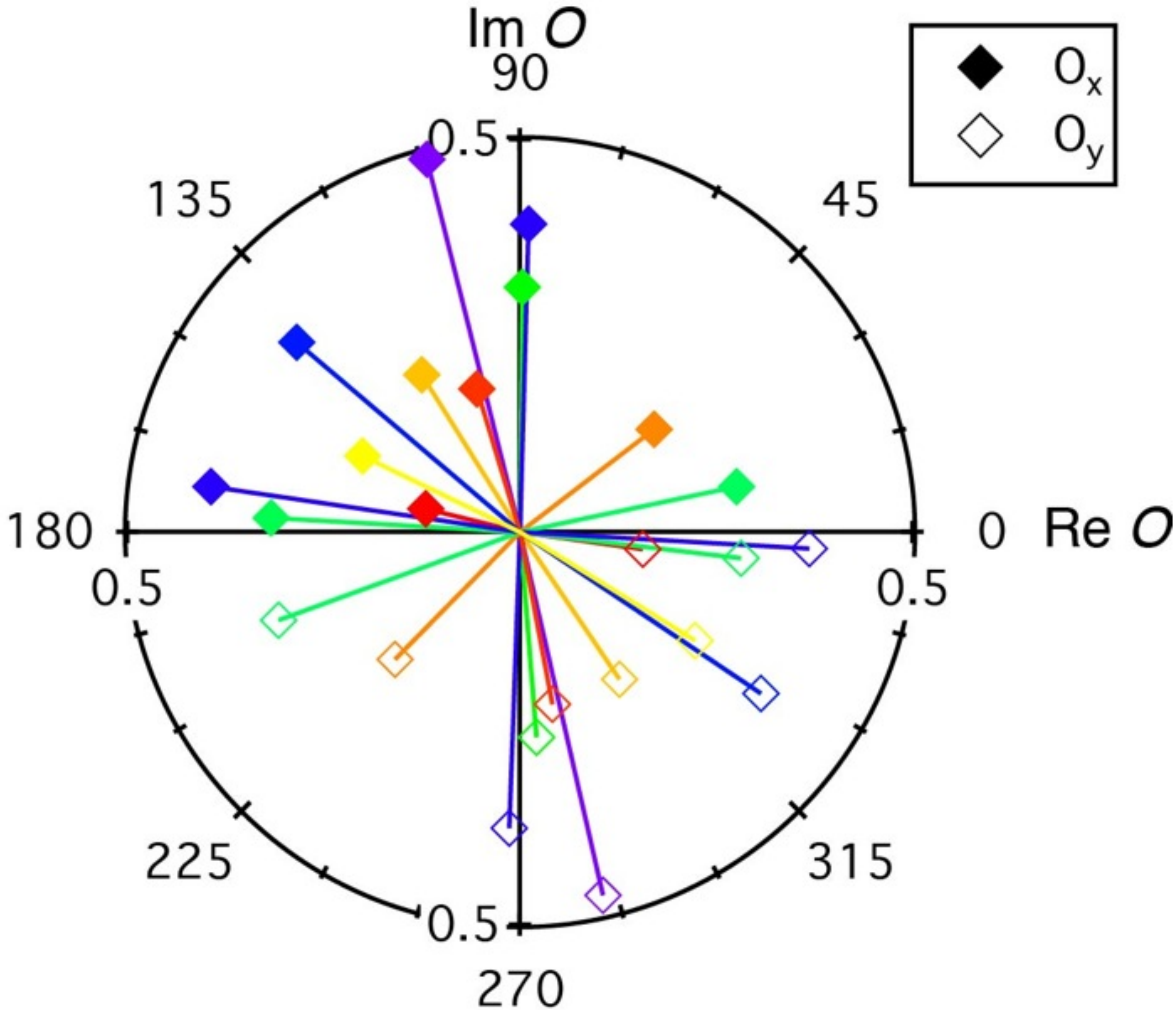

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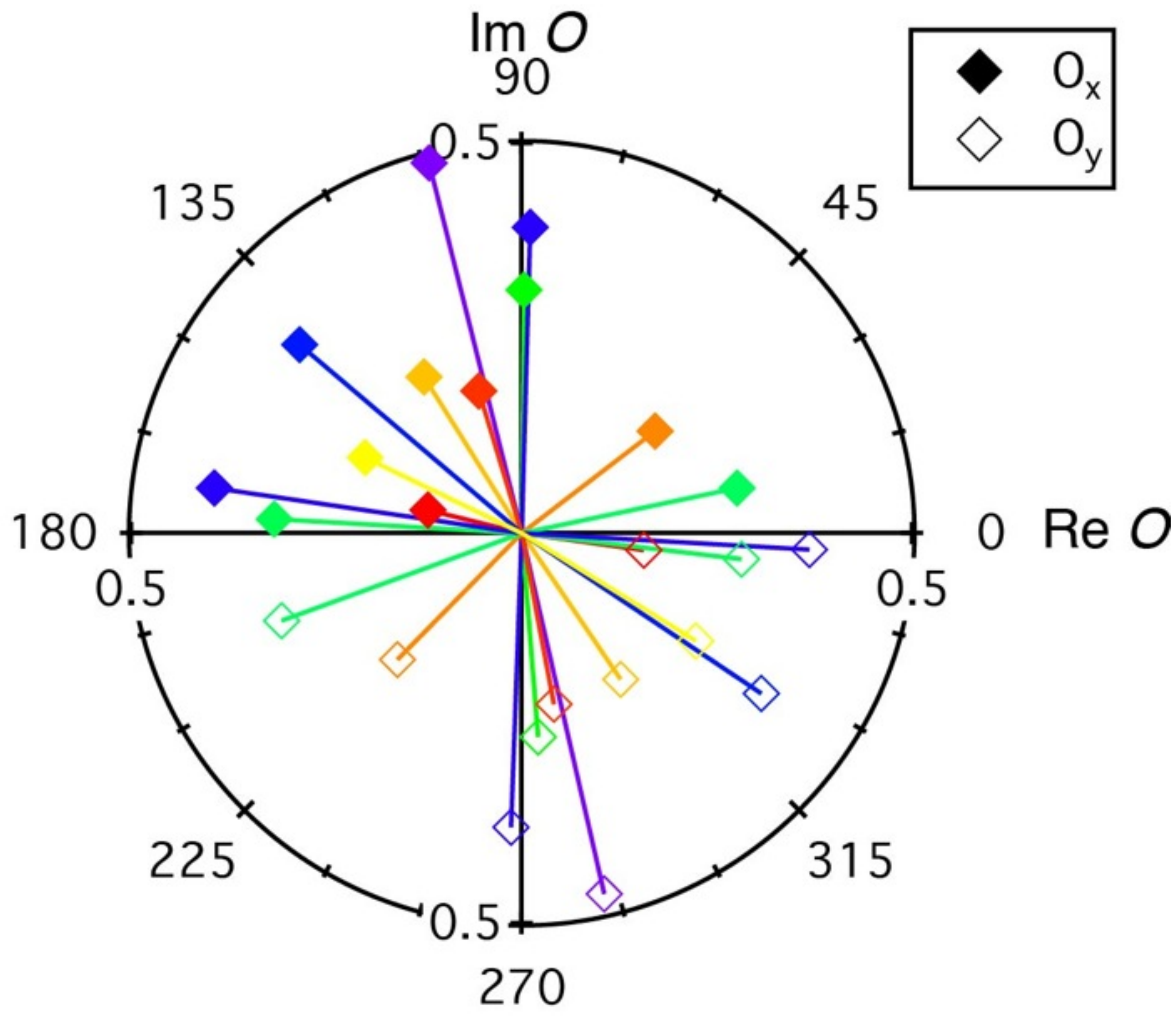

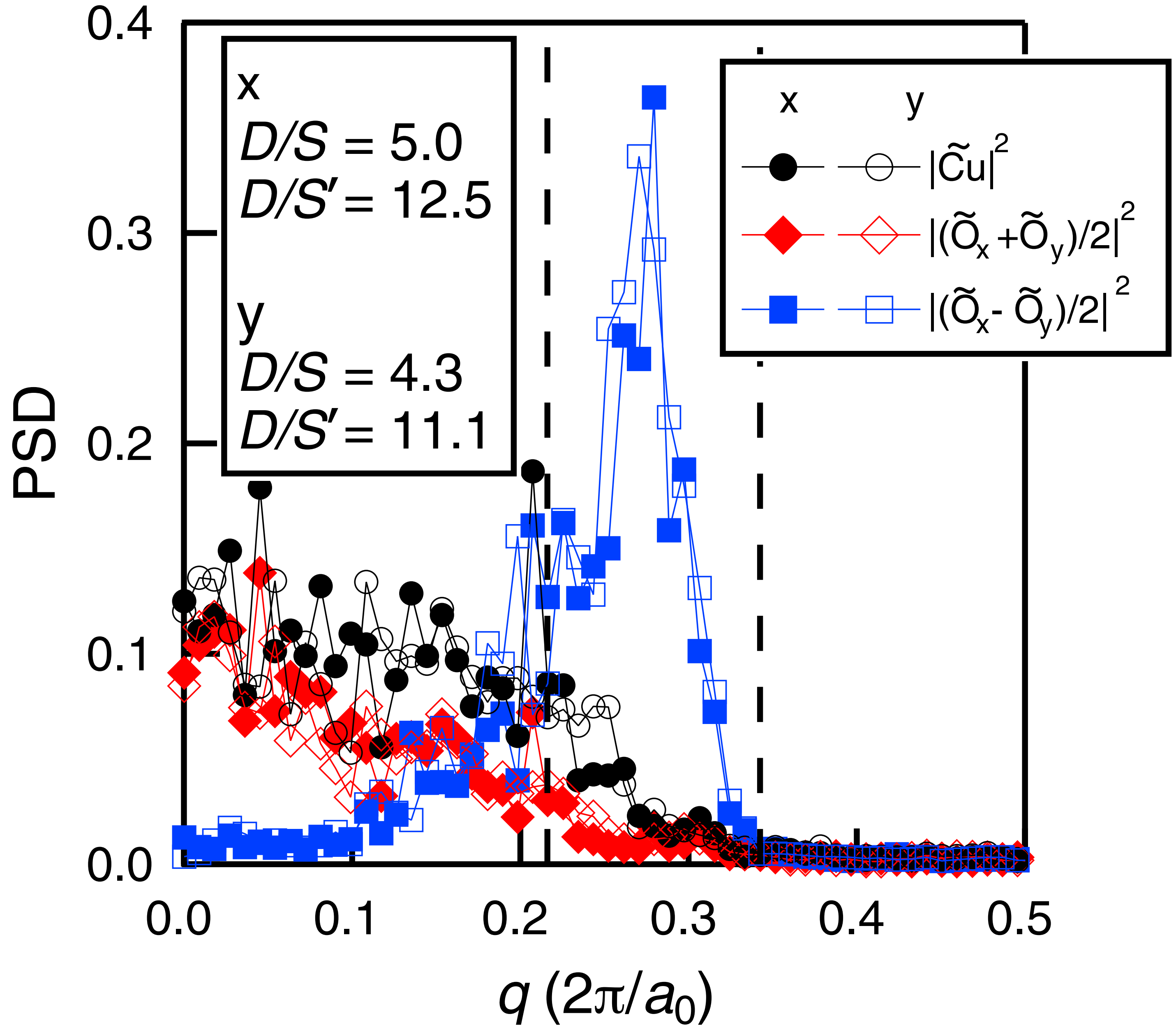

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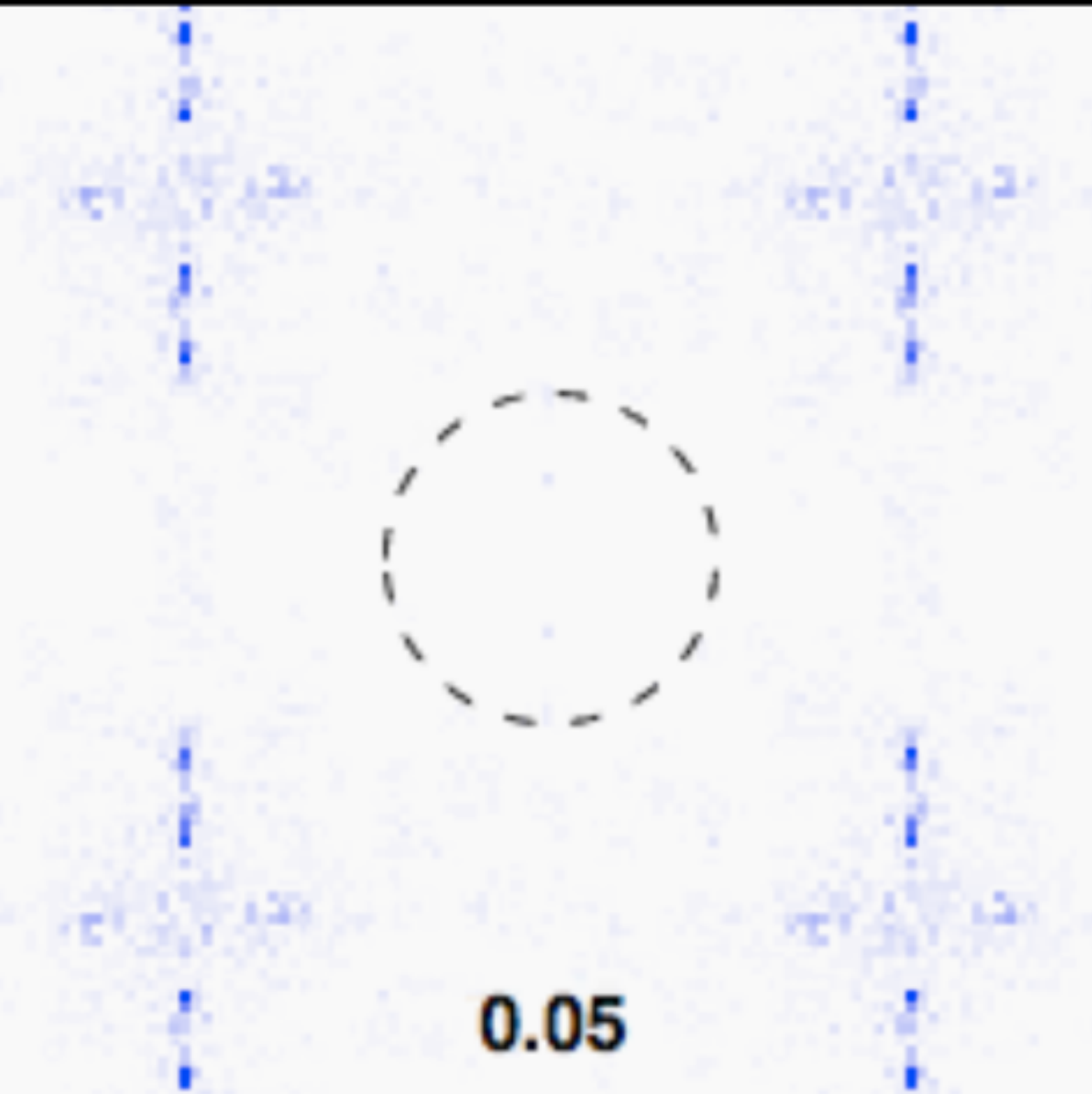
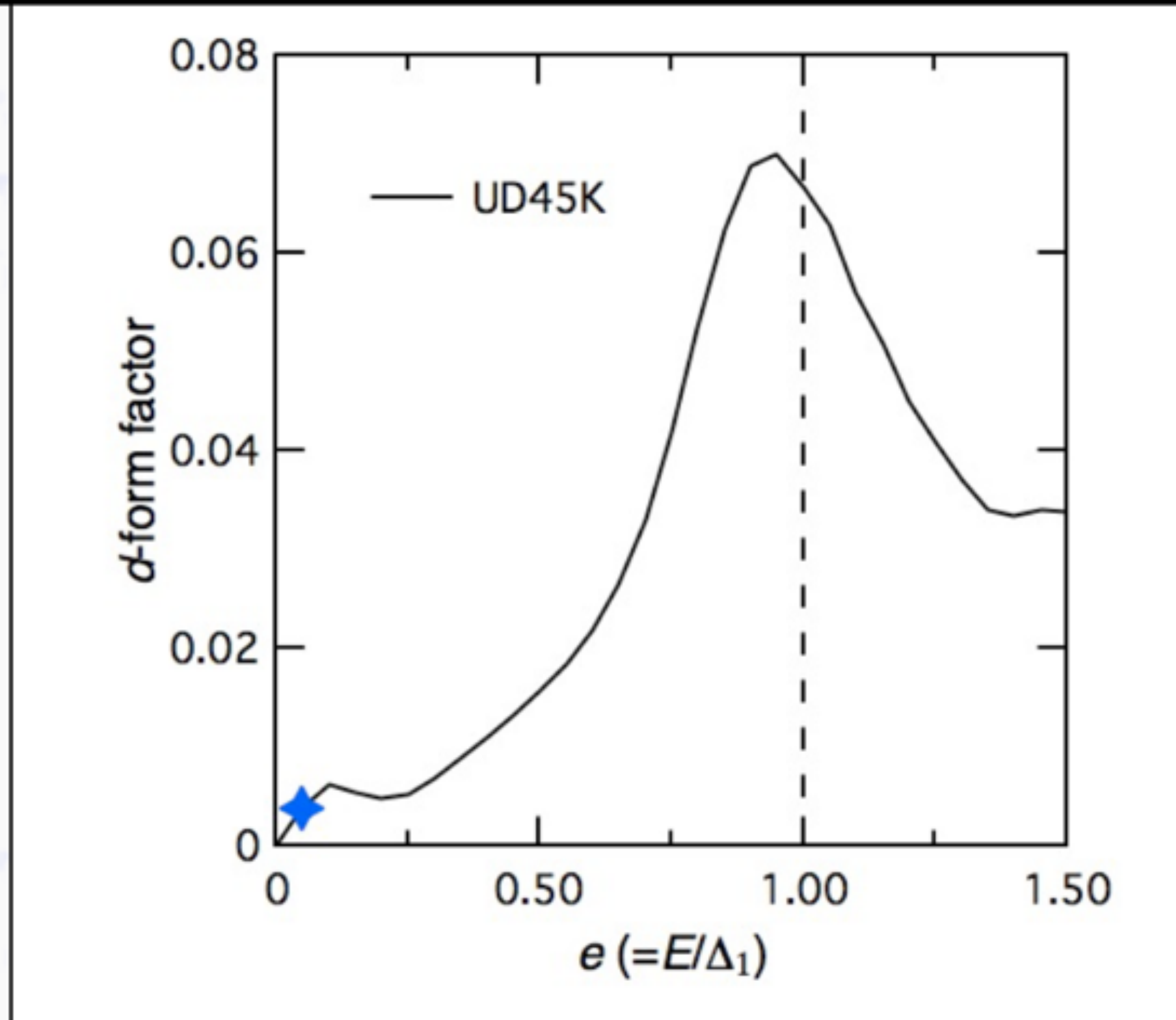
d-form Factor Predominant at Pseudogap Energy

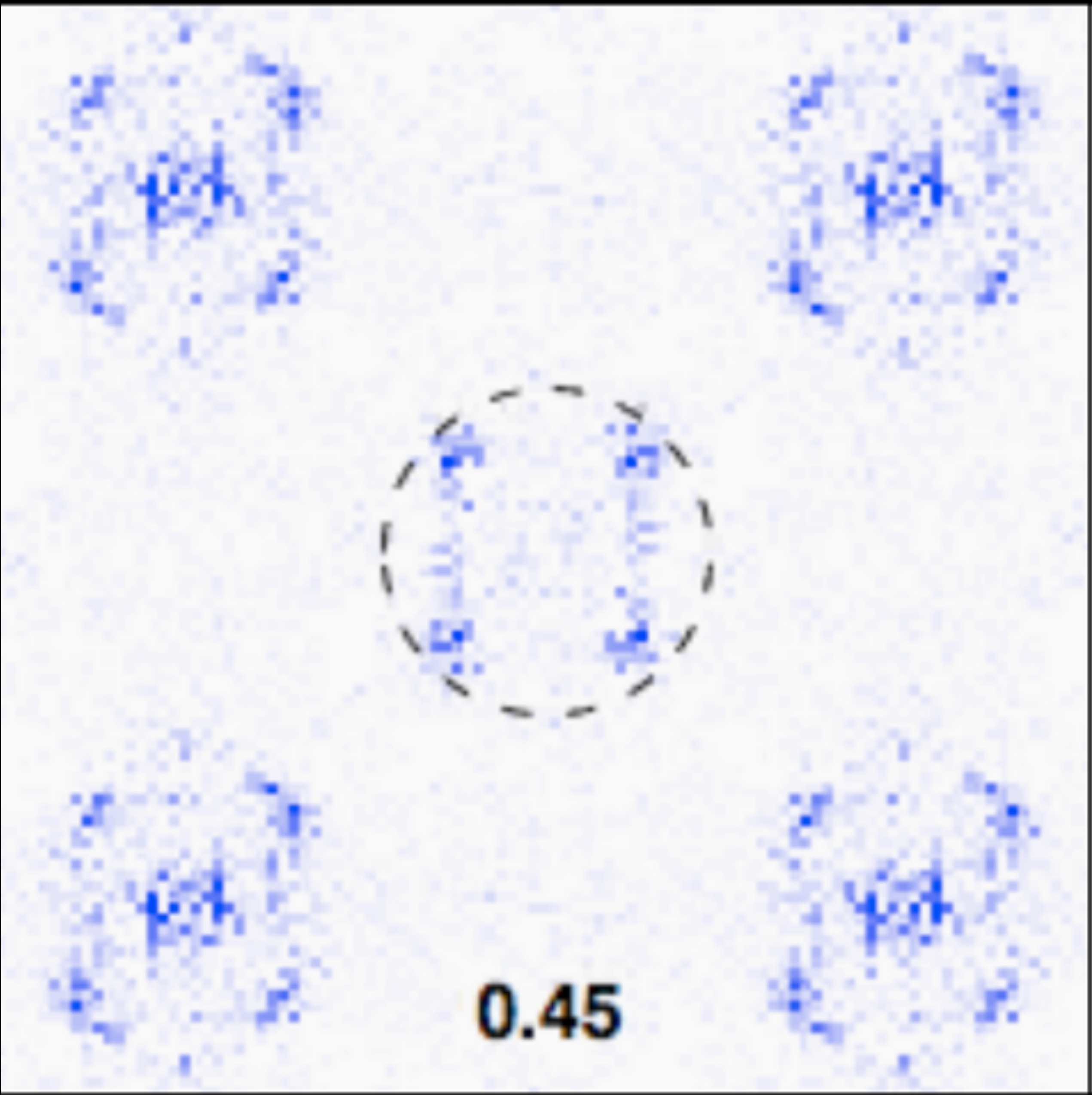


$|(O_x(q,e)-O_y(q,e))/2|^2$



Spectral weight
inside the broken circle

d-form Factor Predominant at Pseudogap Energy



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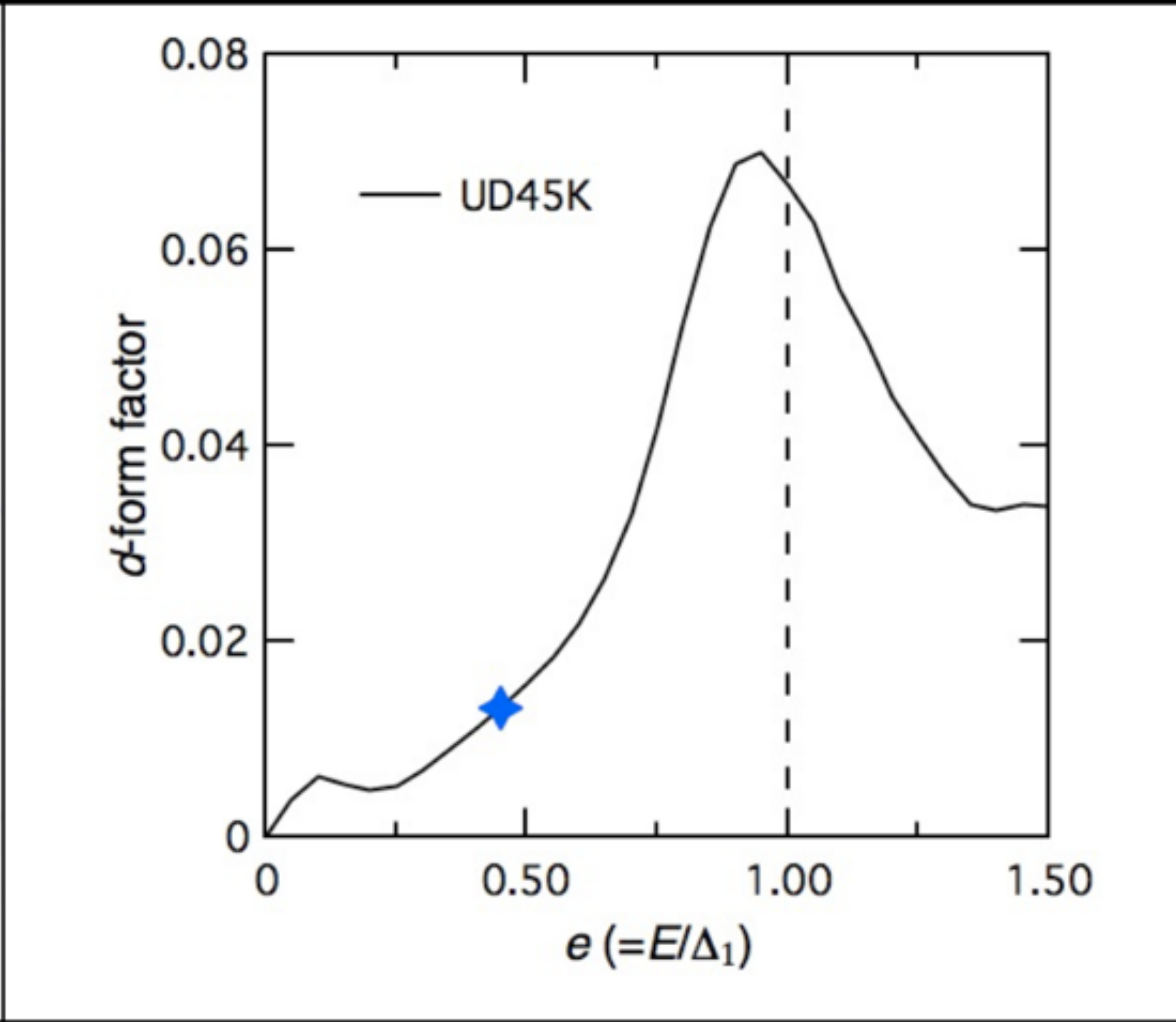

Spectral weight
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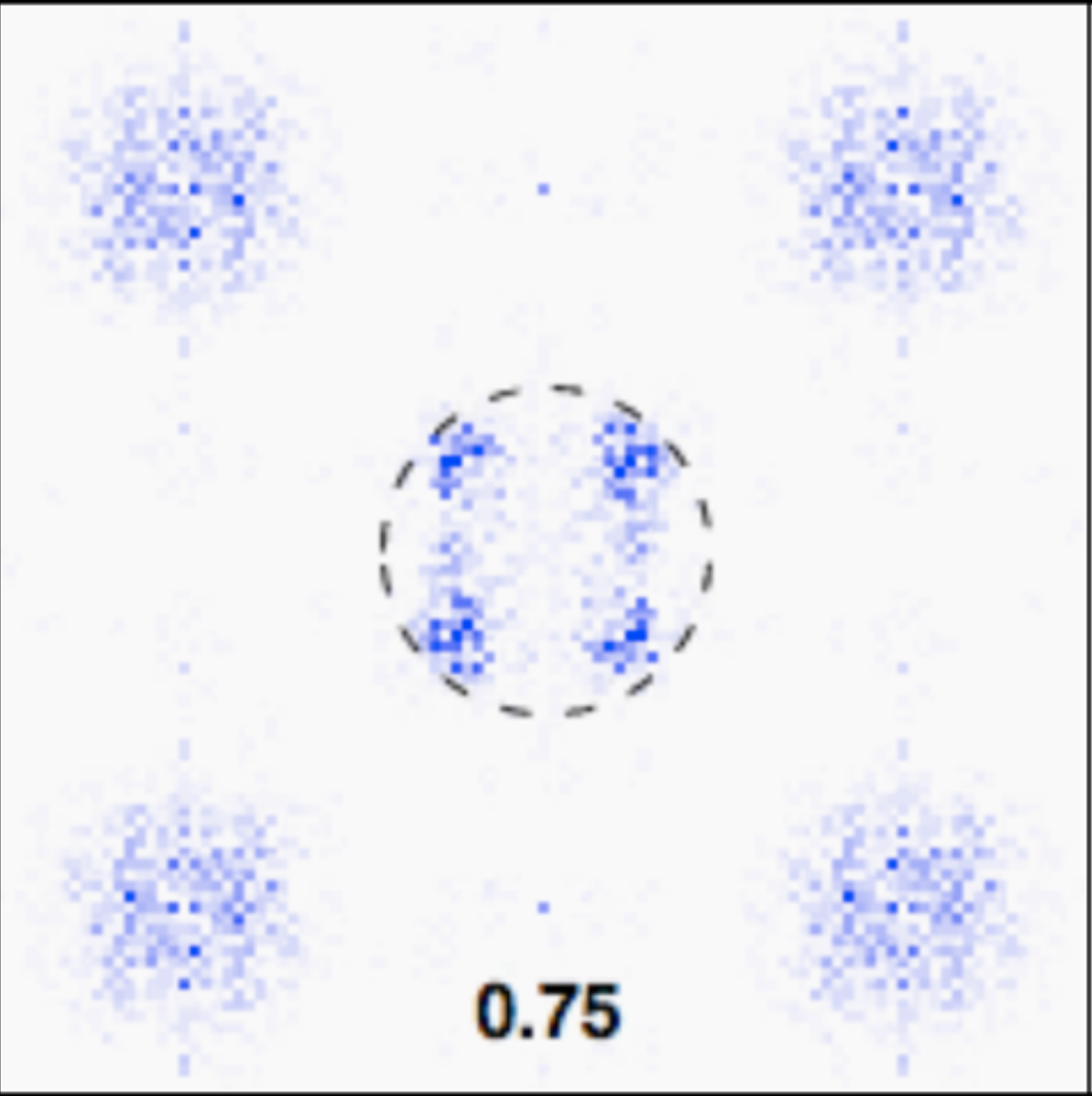

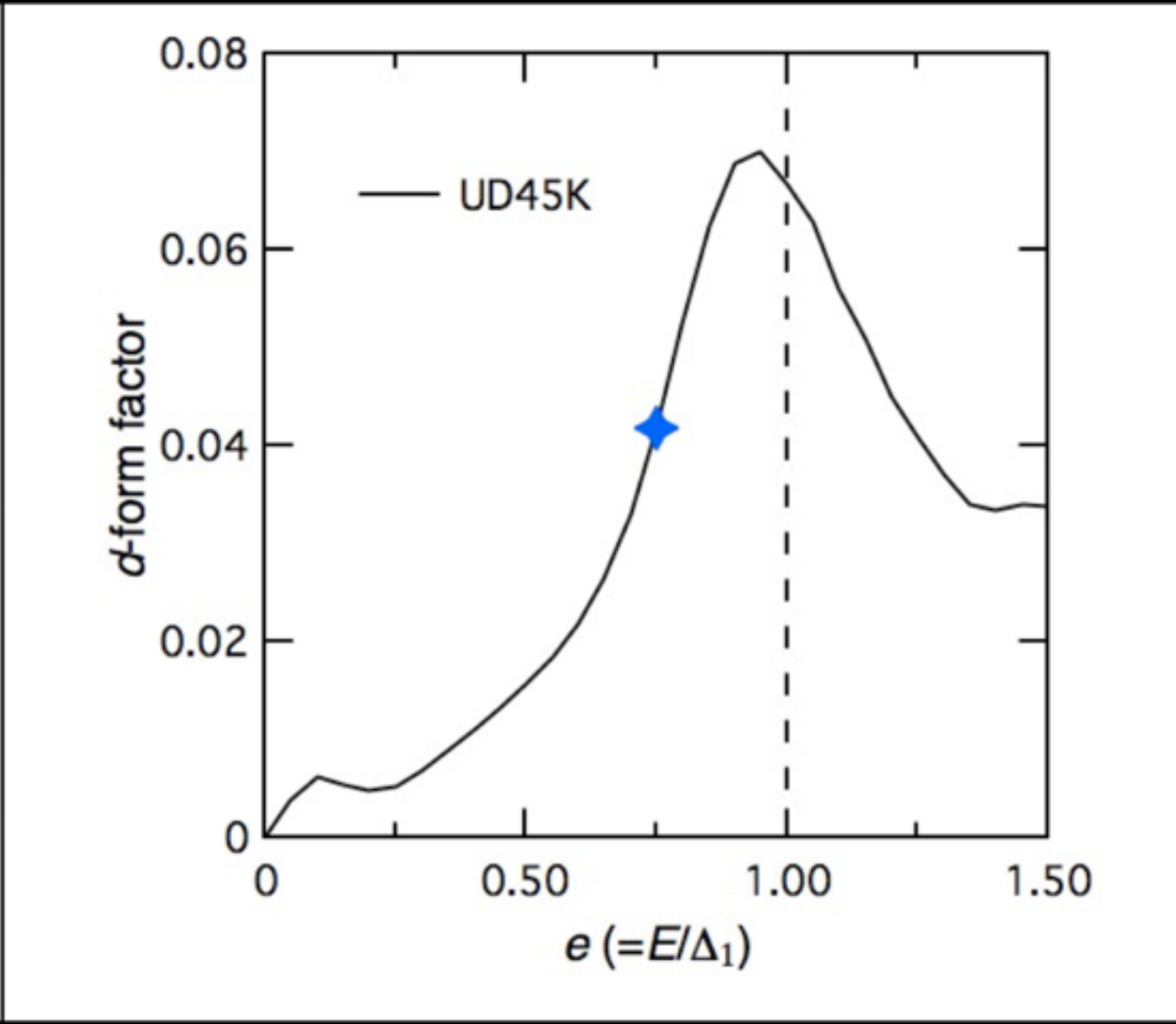

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Spectral weight
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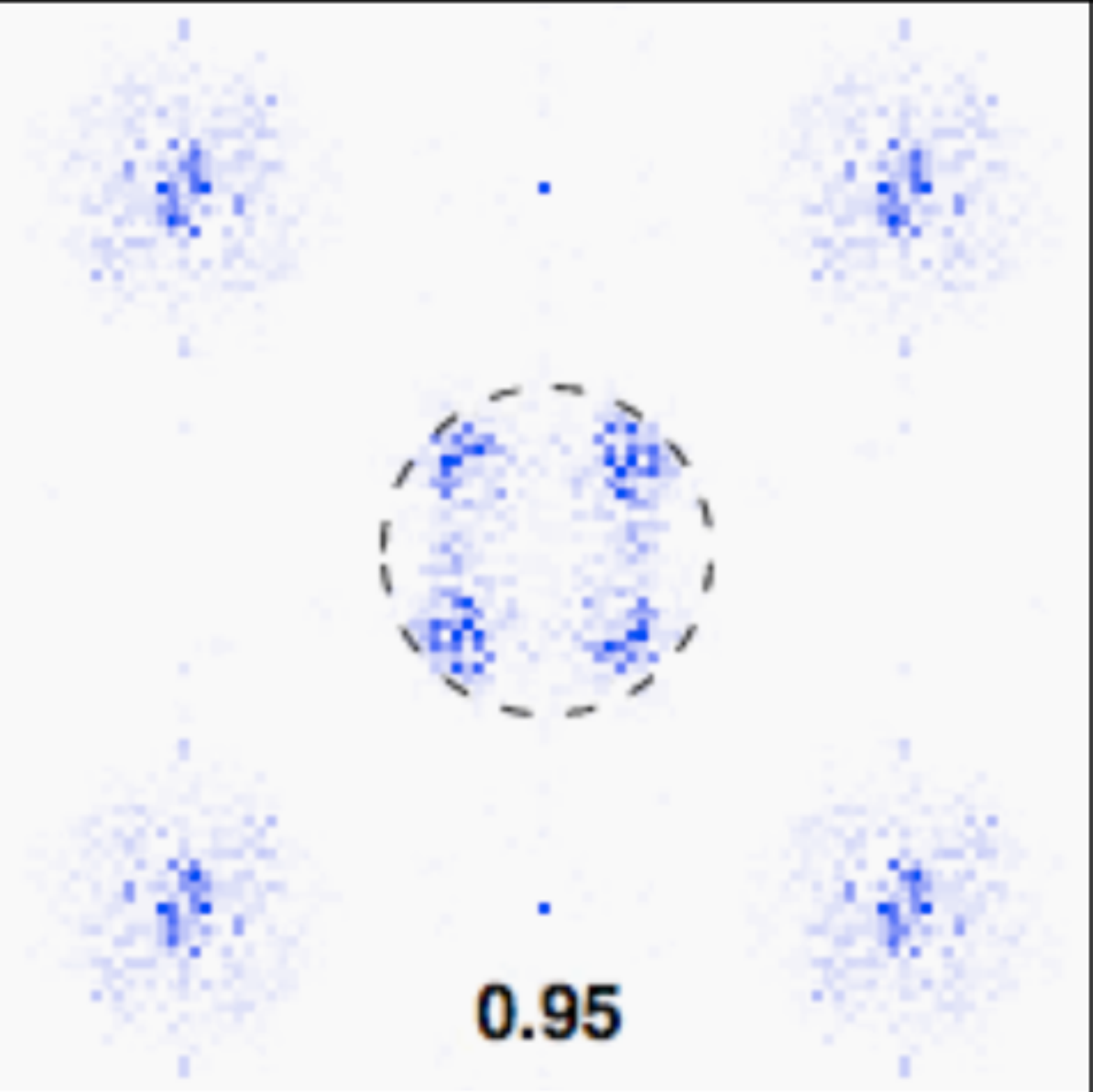

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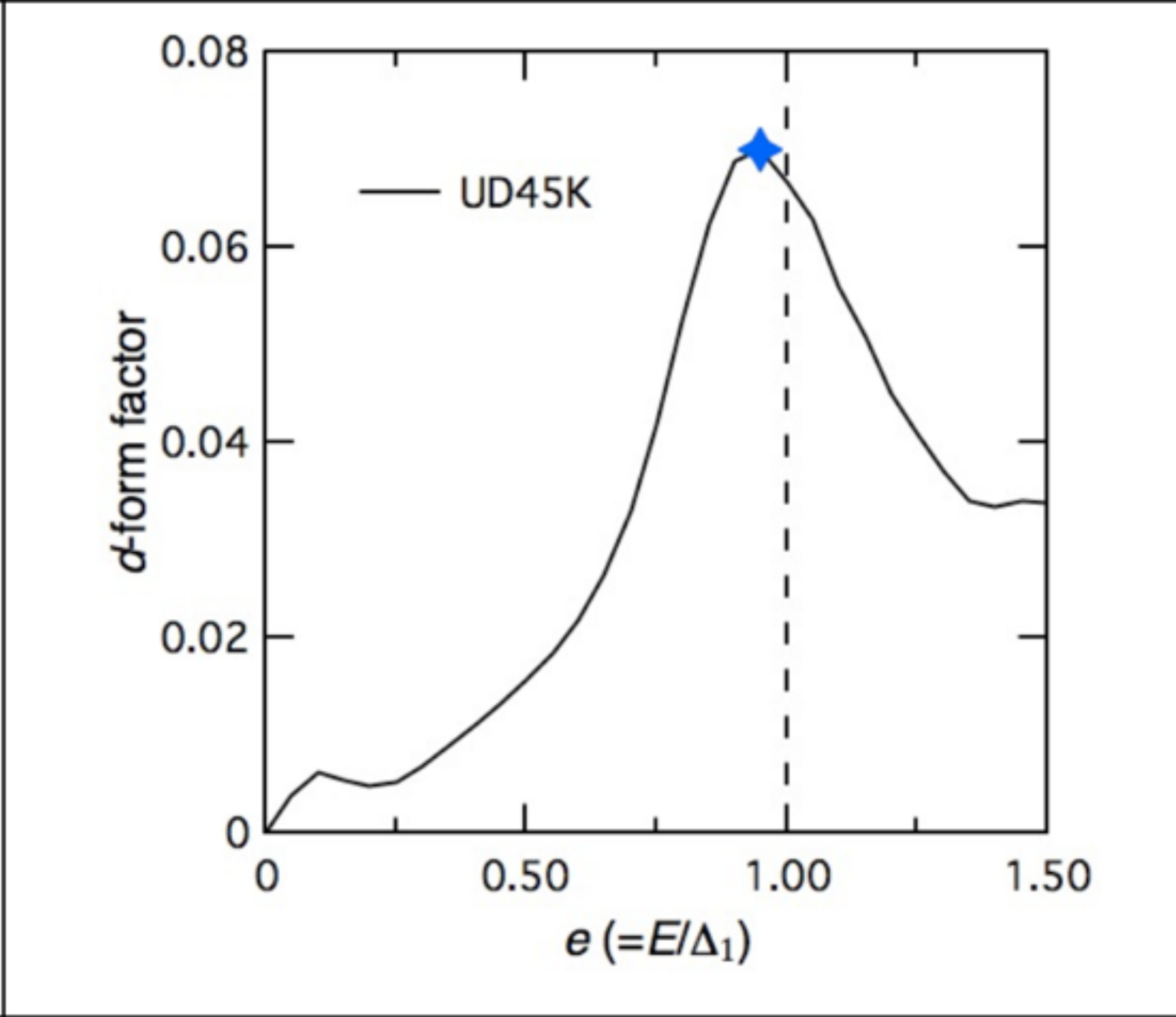

Spectral weight
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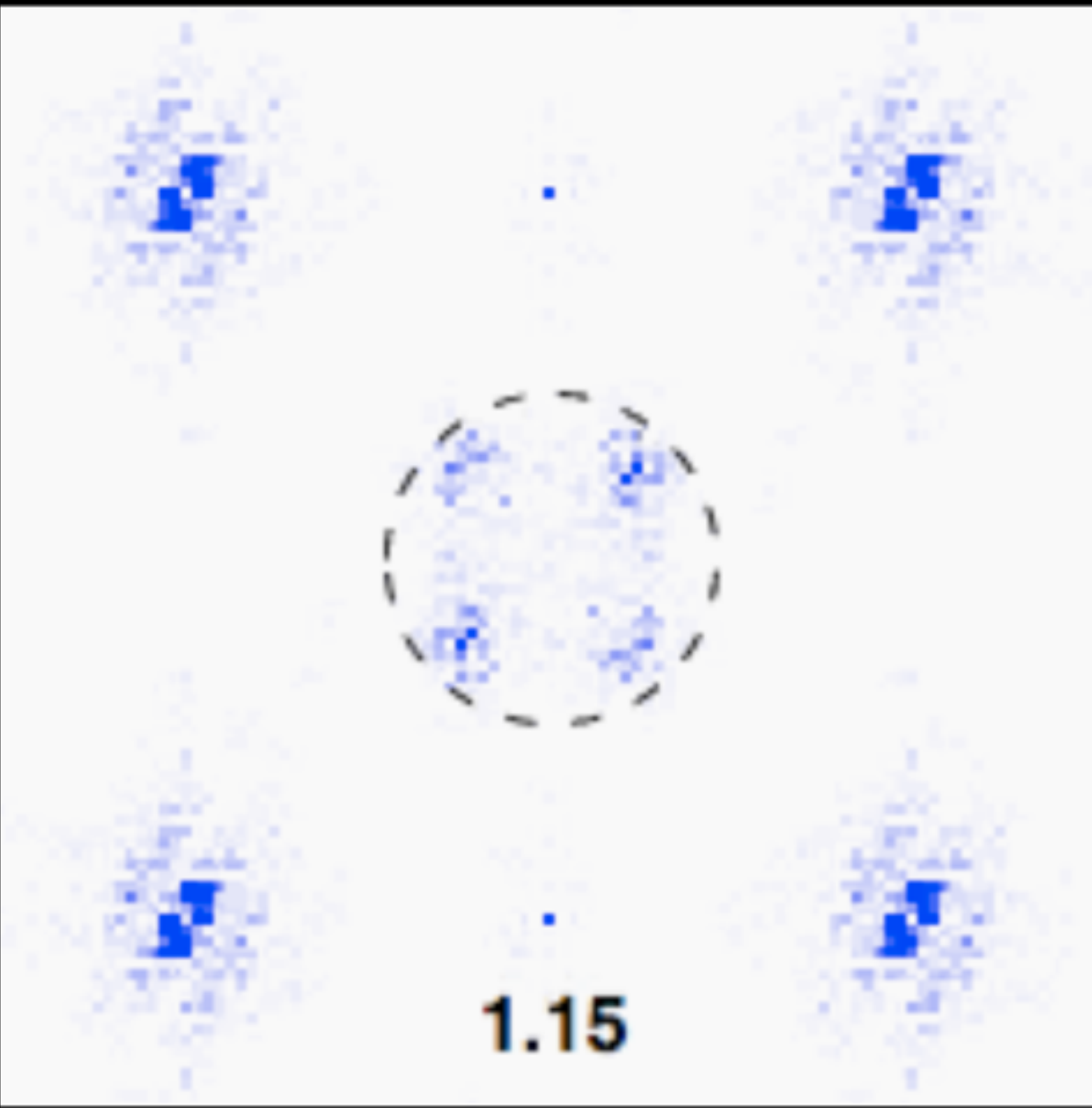

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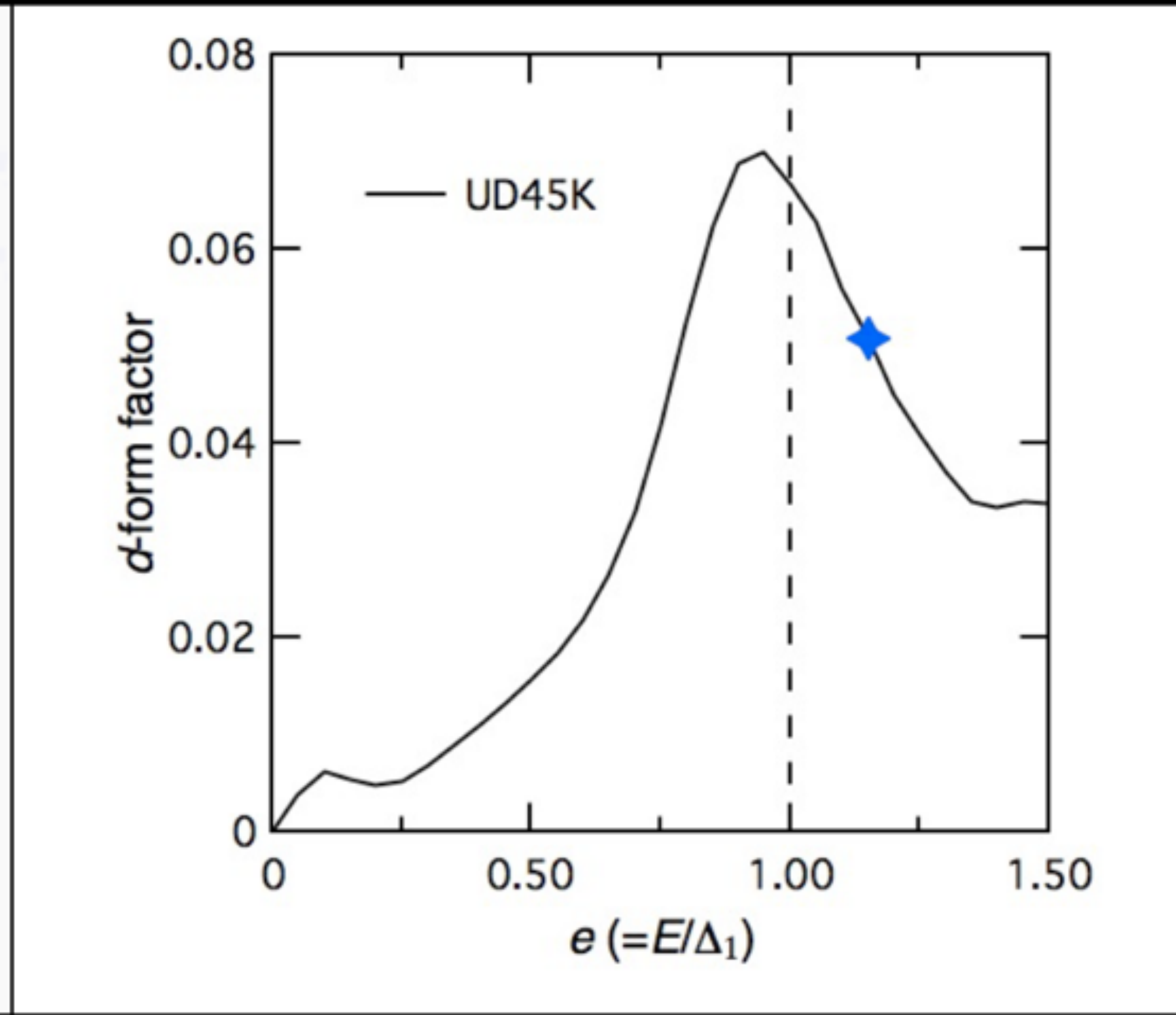

Spectral weight
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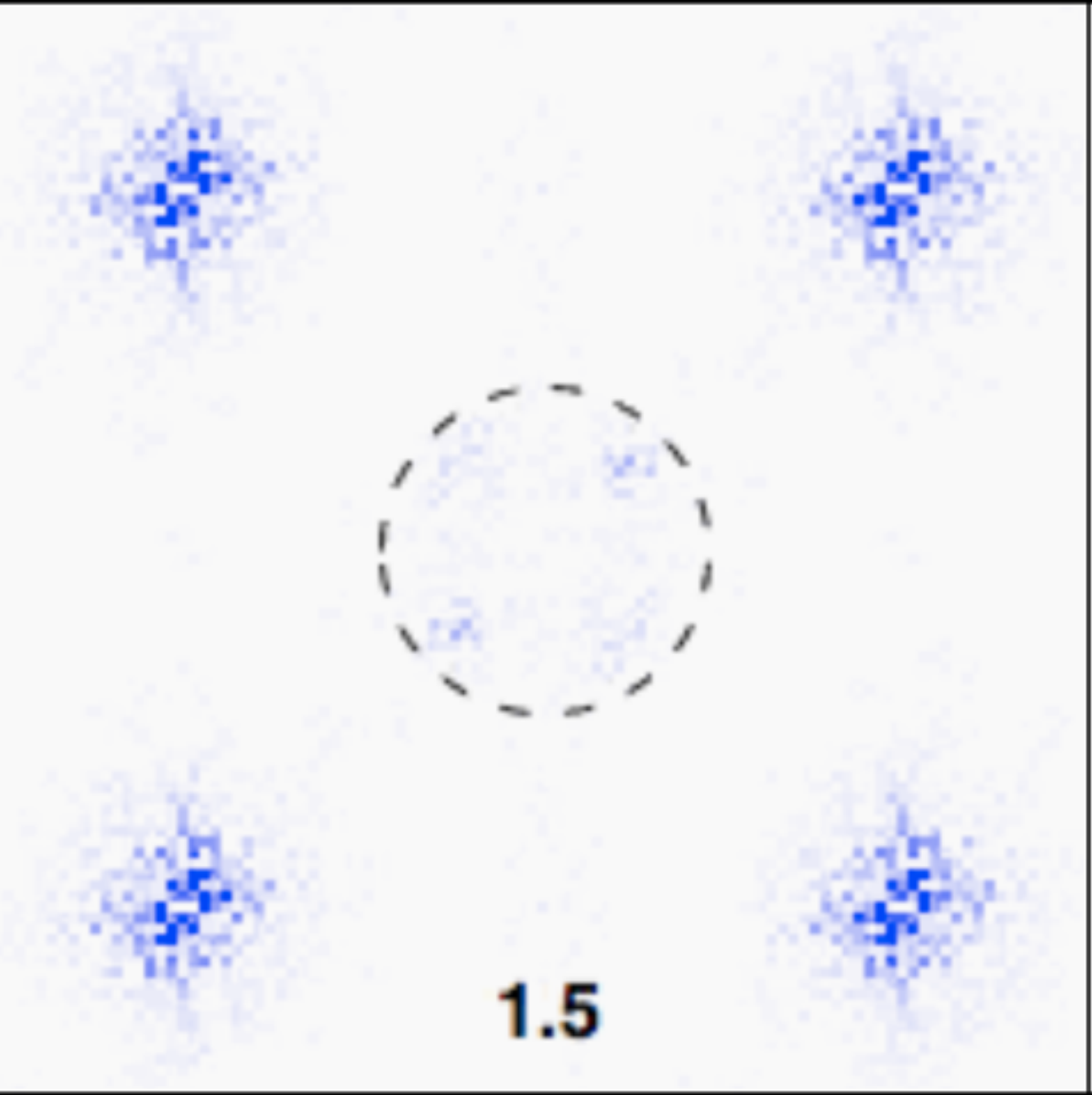

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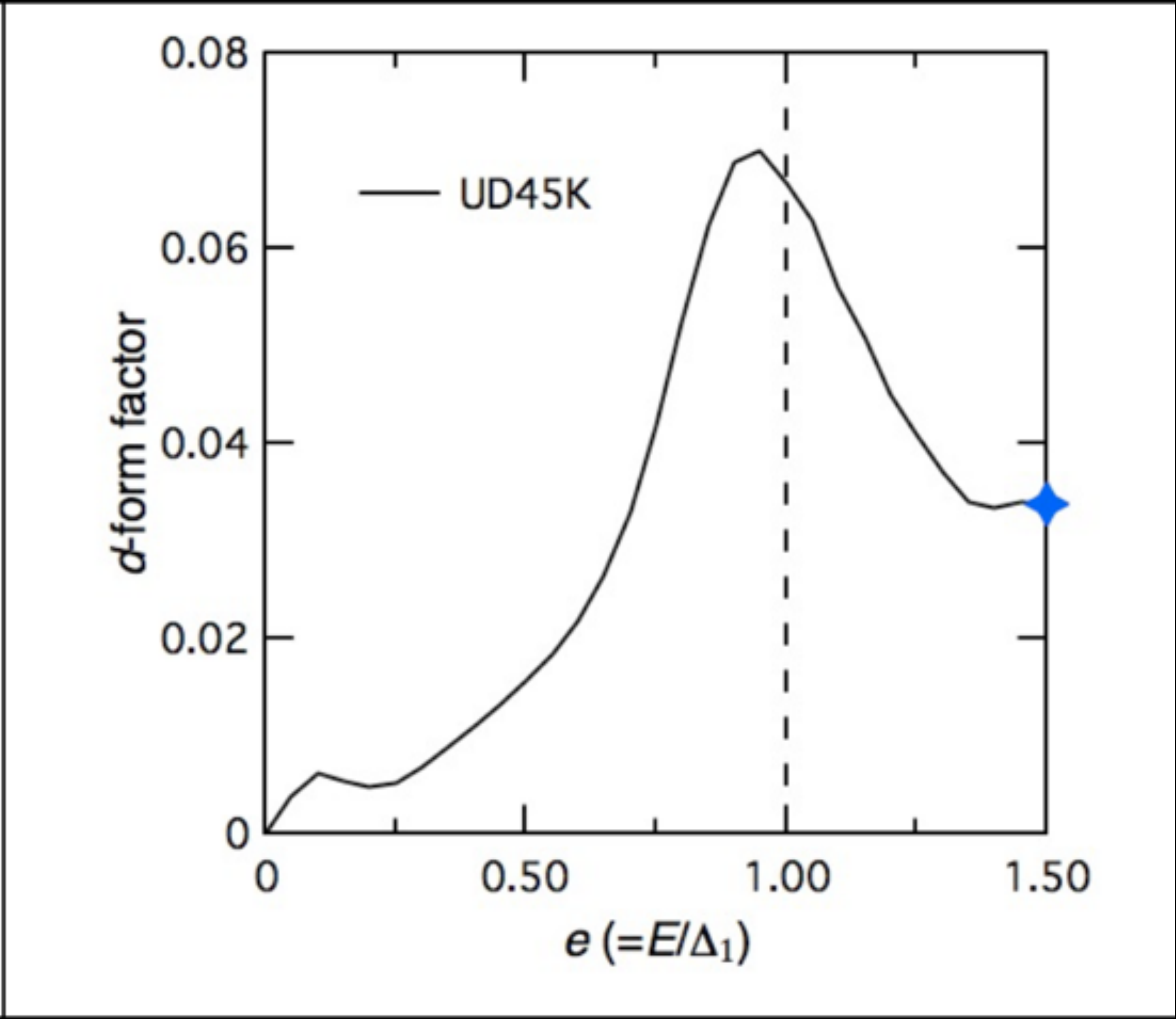

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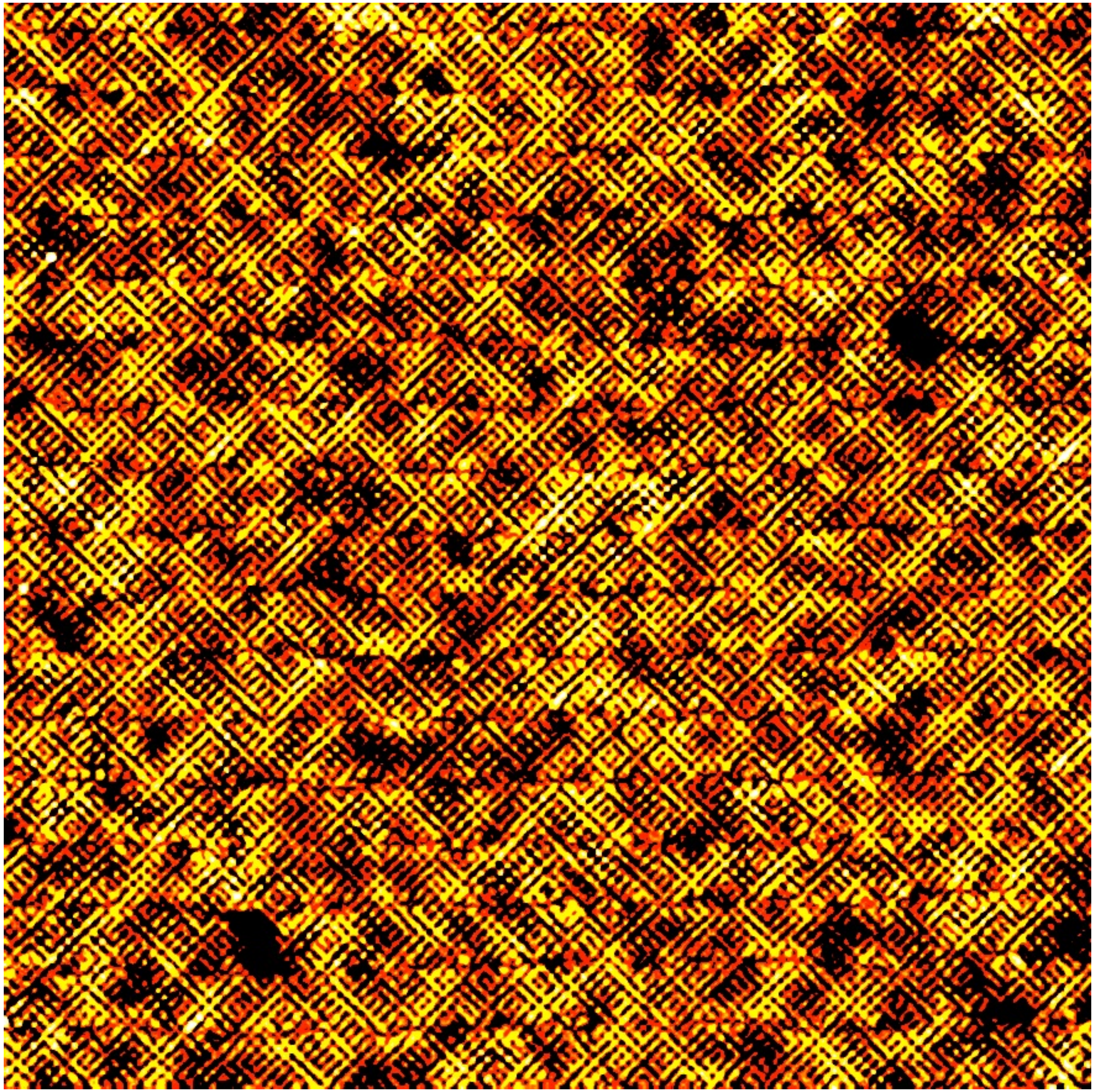

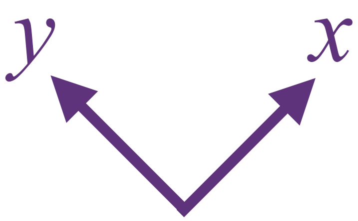

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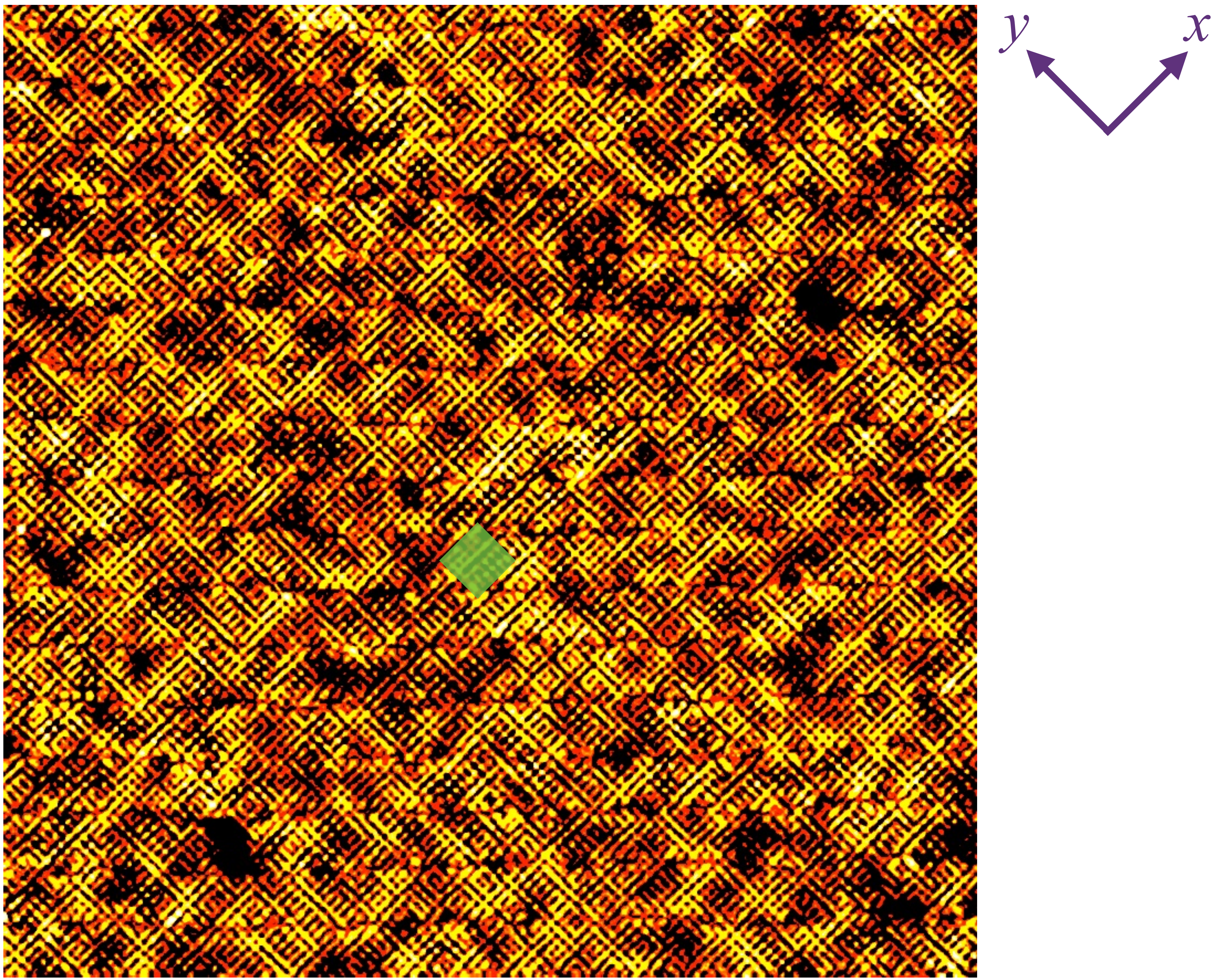

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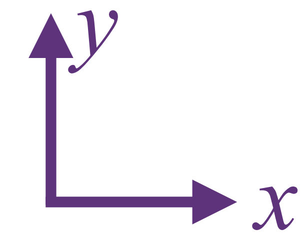

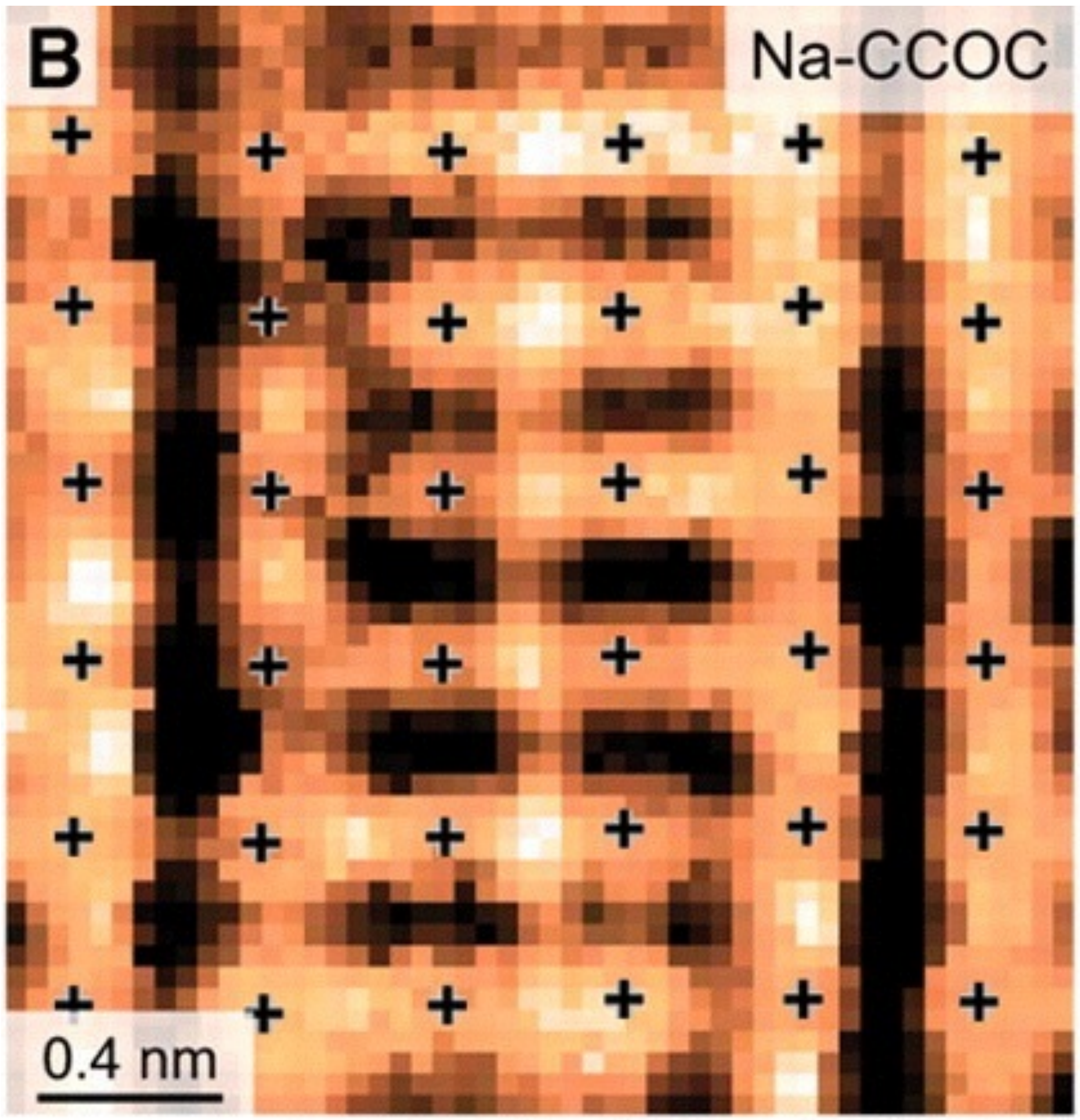

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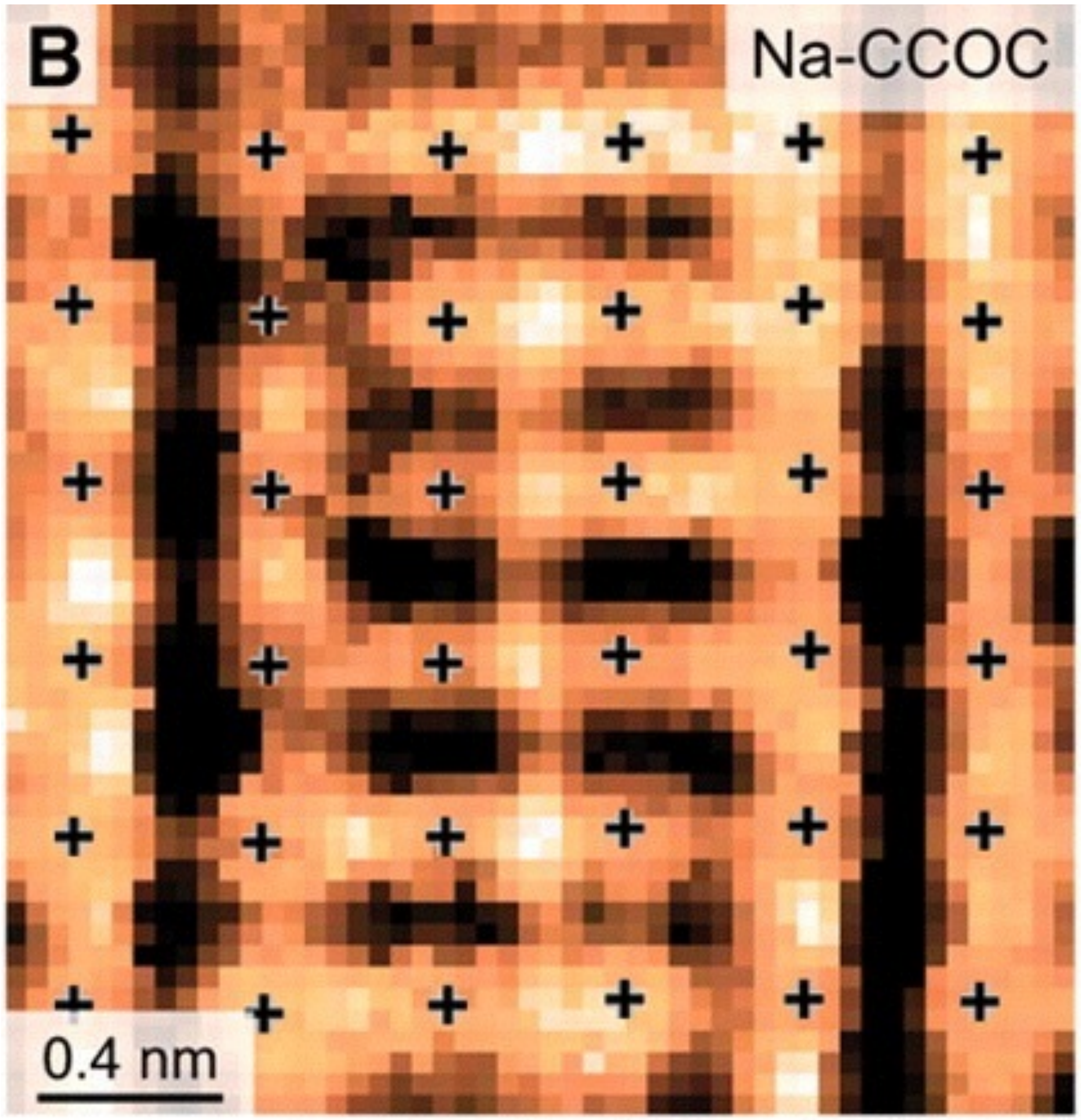

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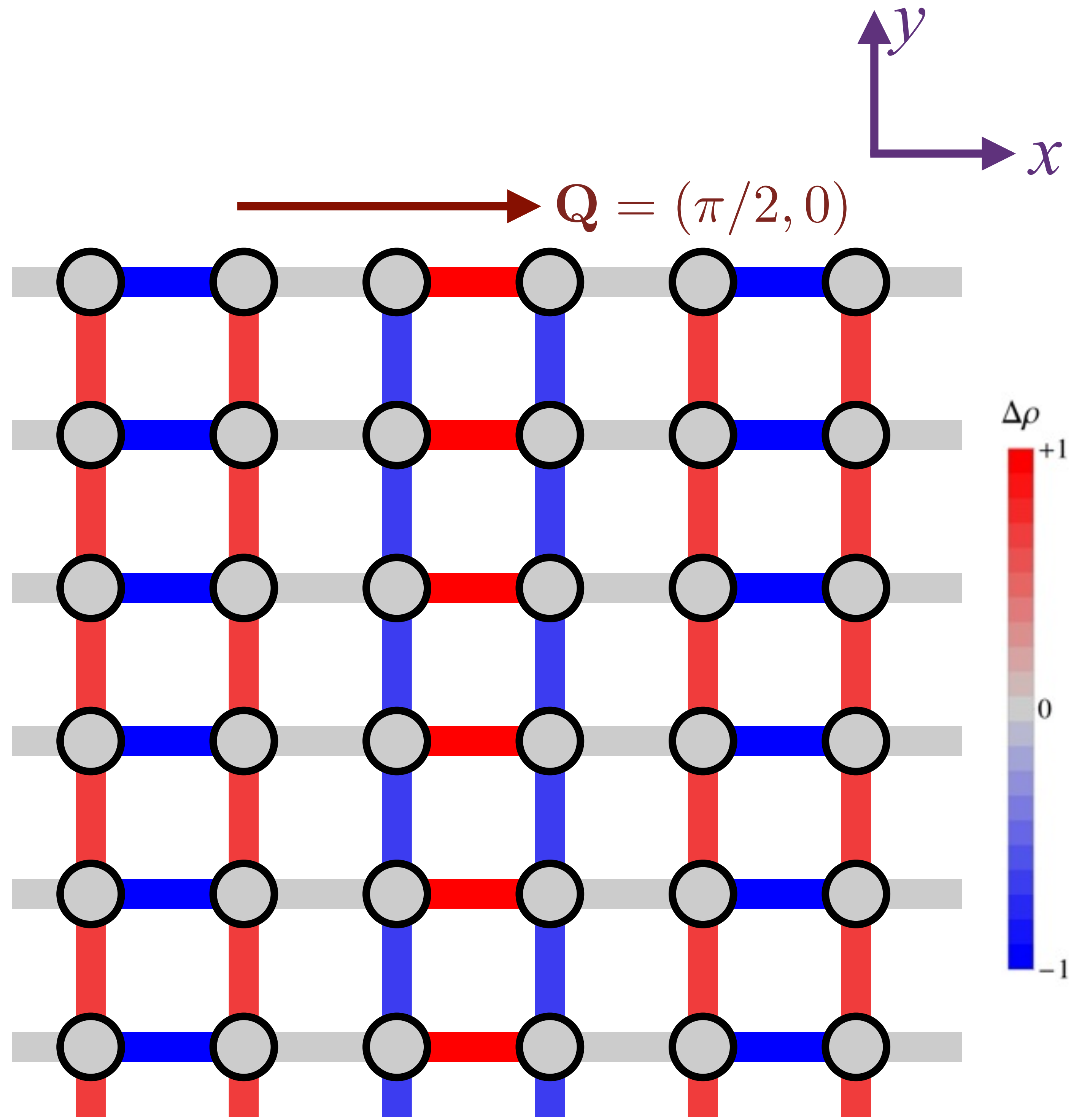

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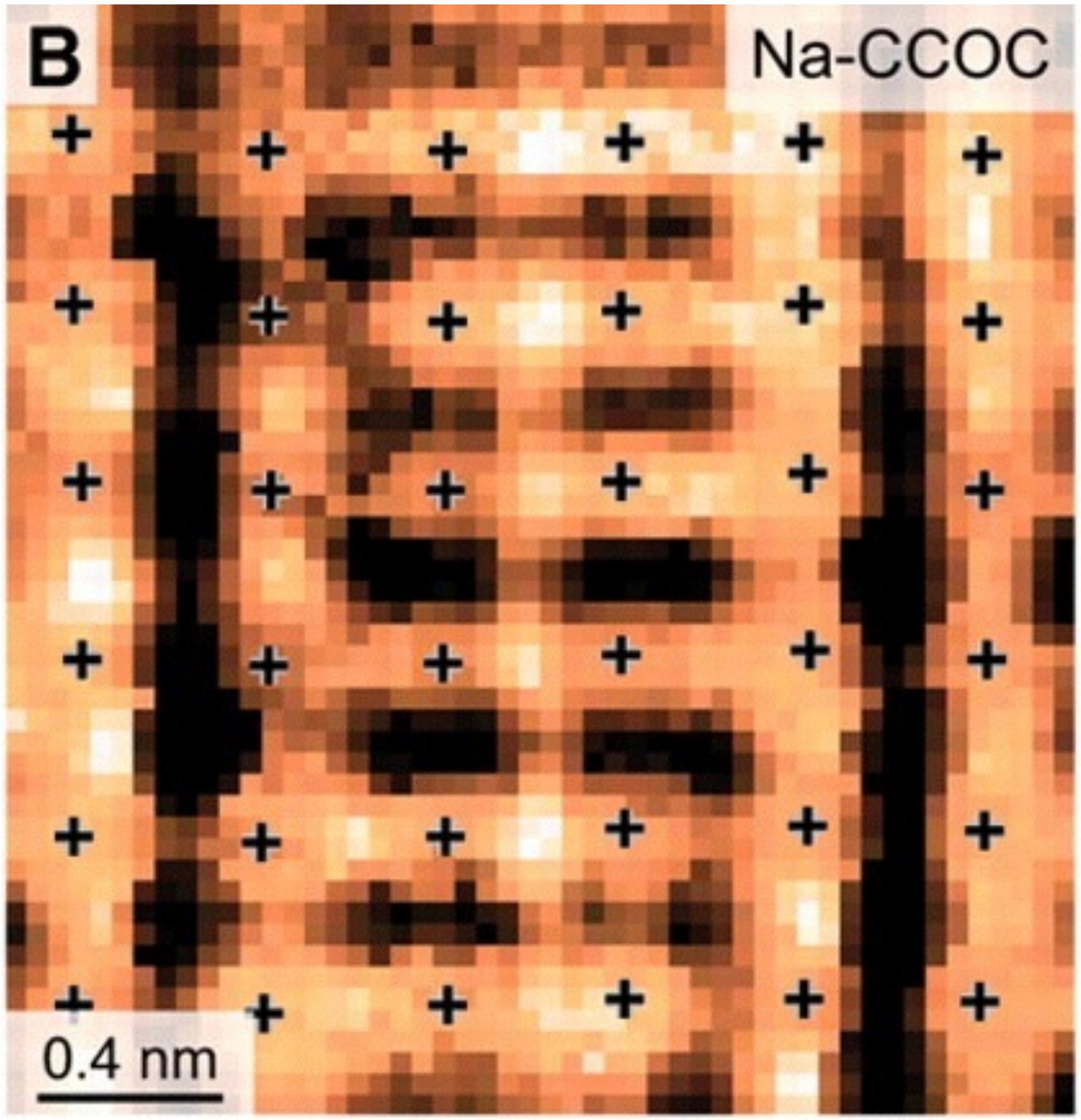

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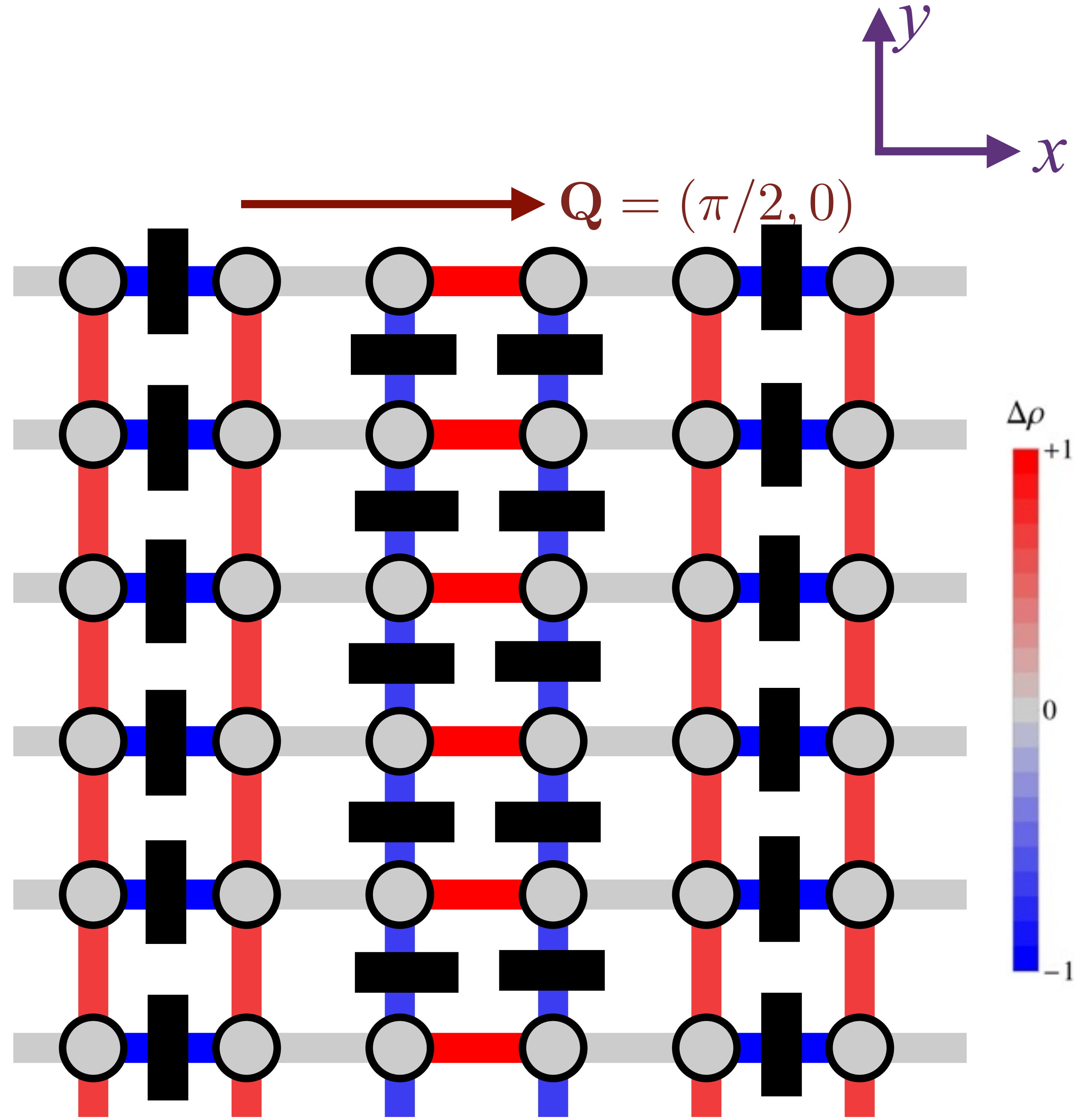

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