

Quantum criticality,
the cuprate superconductors,
and the
AdS/CFT correspondence

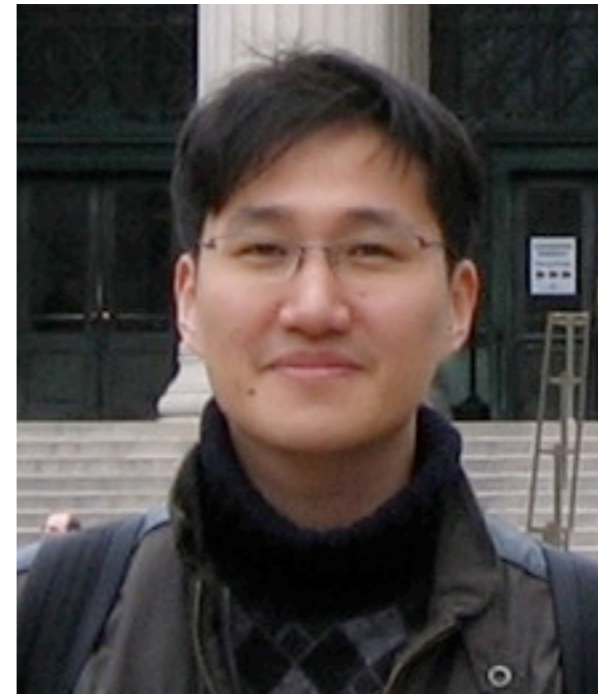
Talk online: sachdev.physics.harvard.edu





Max Metlitski, Harvard

arXiv:1001.1153



Eun Gook Moon, Harvard

Phys. Rev. B 80, 035117 (2009)



Outline

1. Coupled dimer antiferromagnets

Introduction to quantum criticality

2. Theory of Ising-nematic ordering in the cuprate metals

Strongly-coupled field theory

3. The AdS/CFT correspondence

Phases of finite density quantum matter at strong coupling

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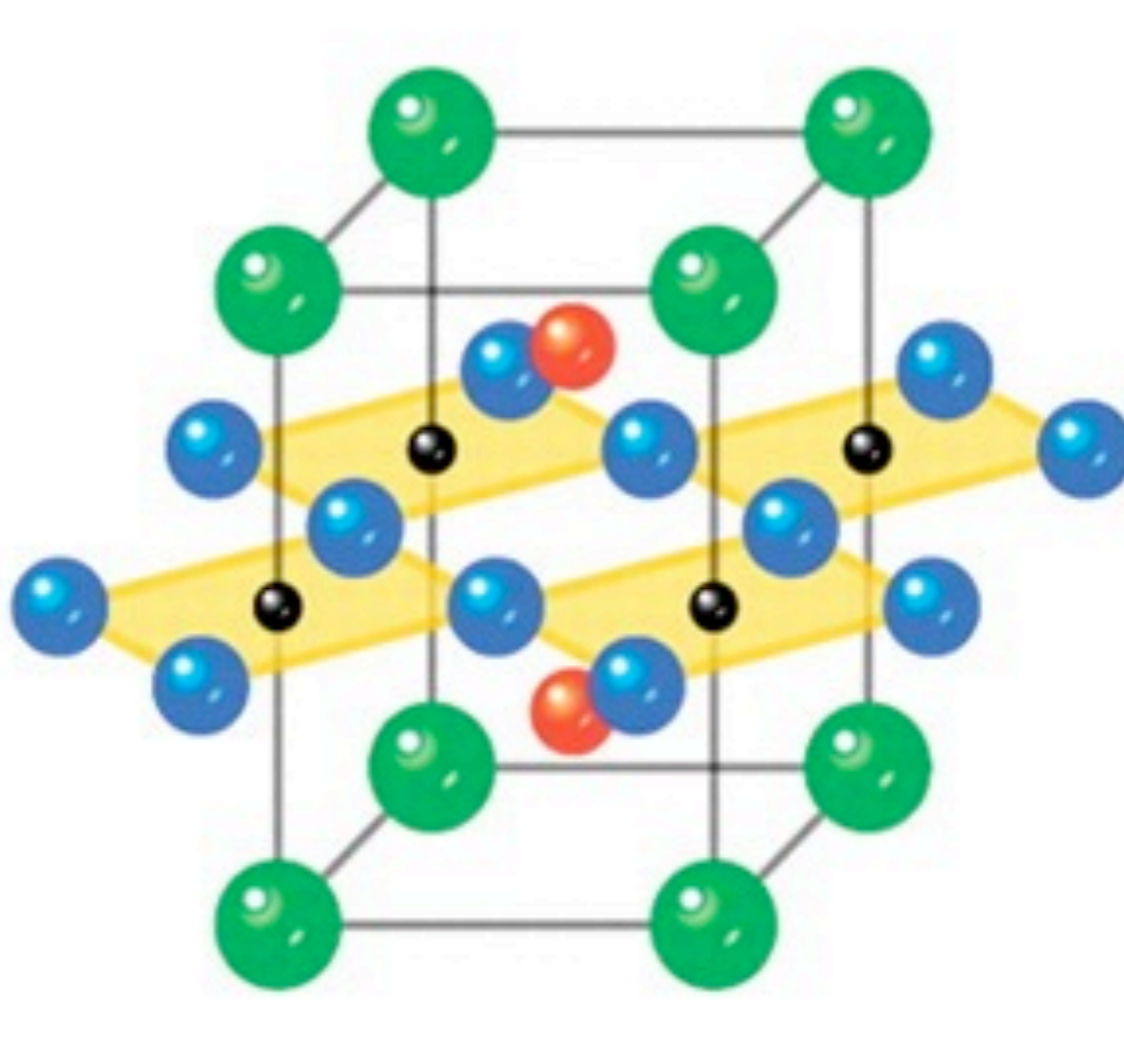
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Phases of finite density quantum matter at strong coupling

The cuprate superconductors

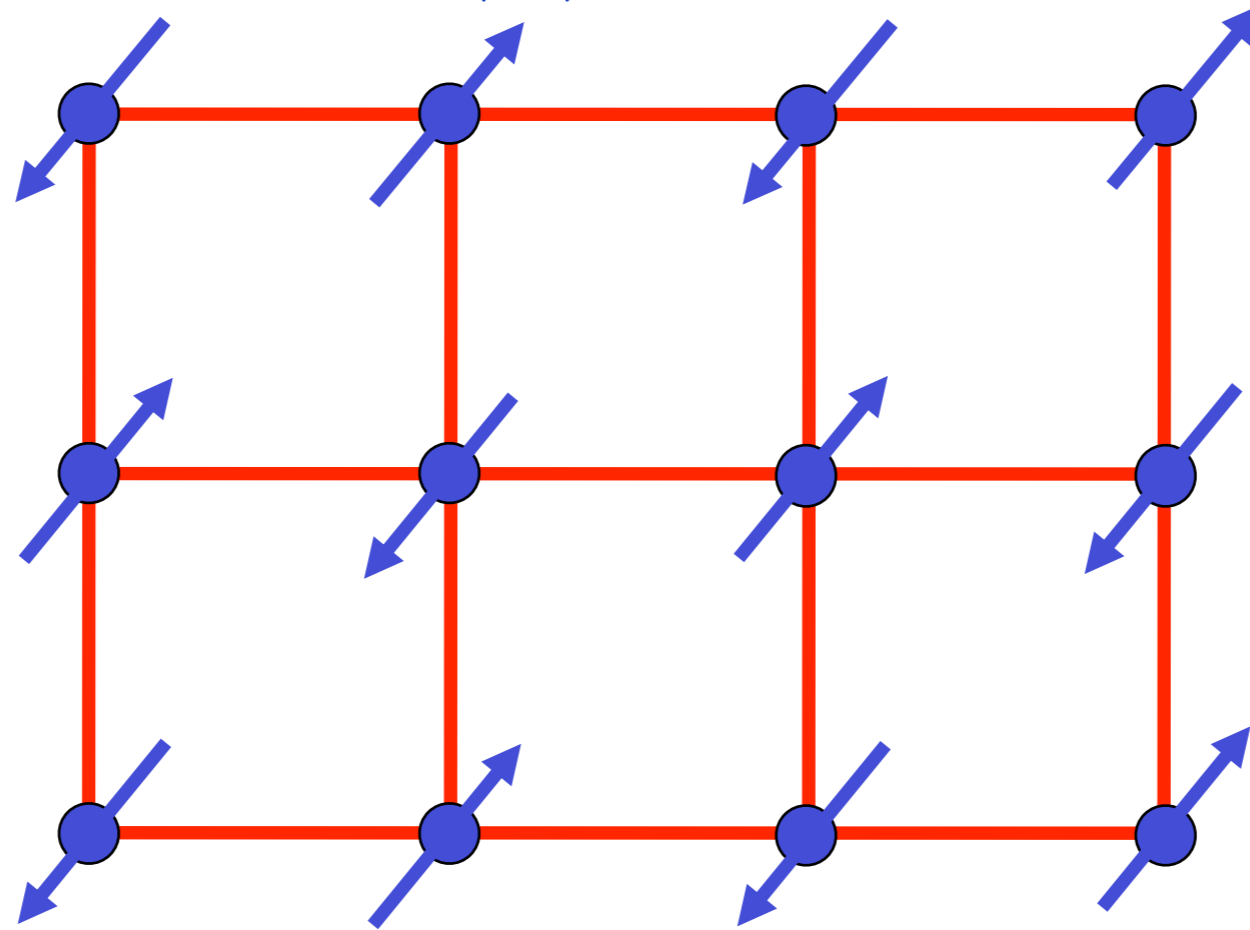
Na-CCOC

- Cu
- Ca/Na
- O
- Cl



Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



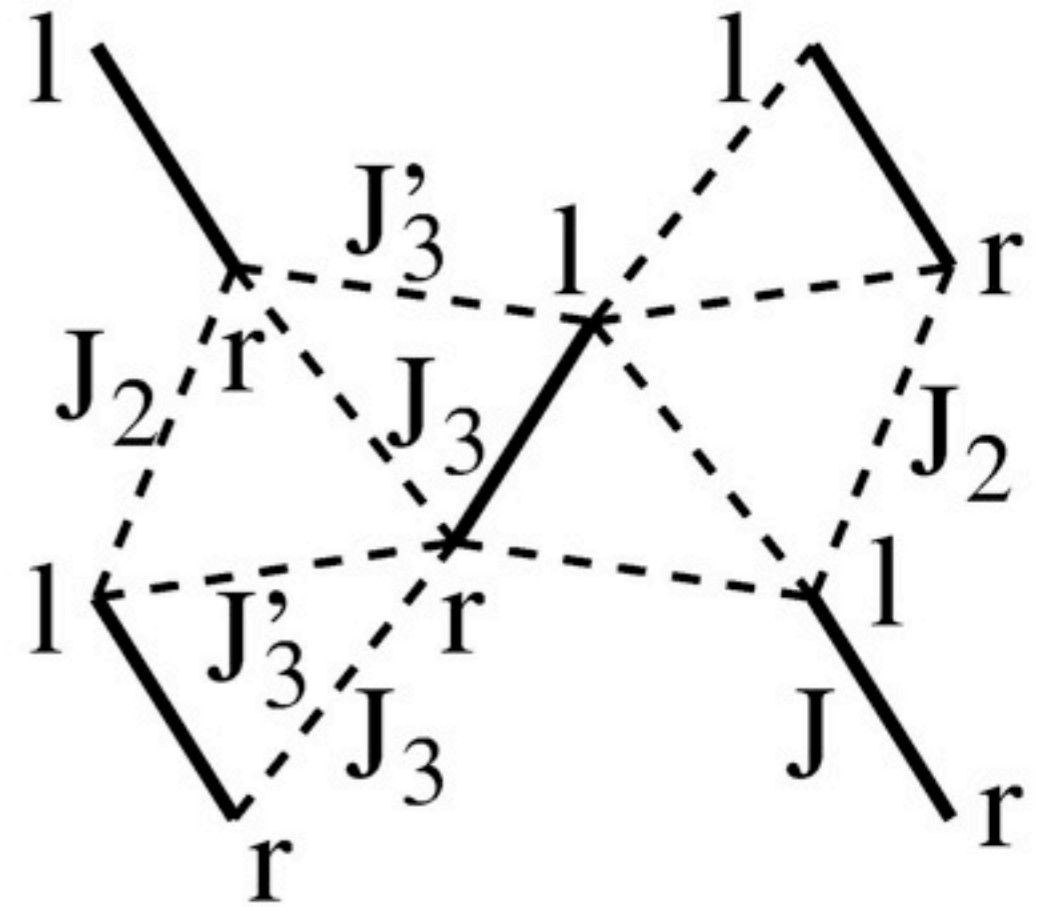
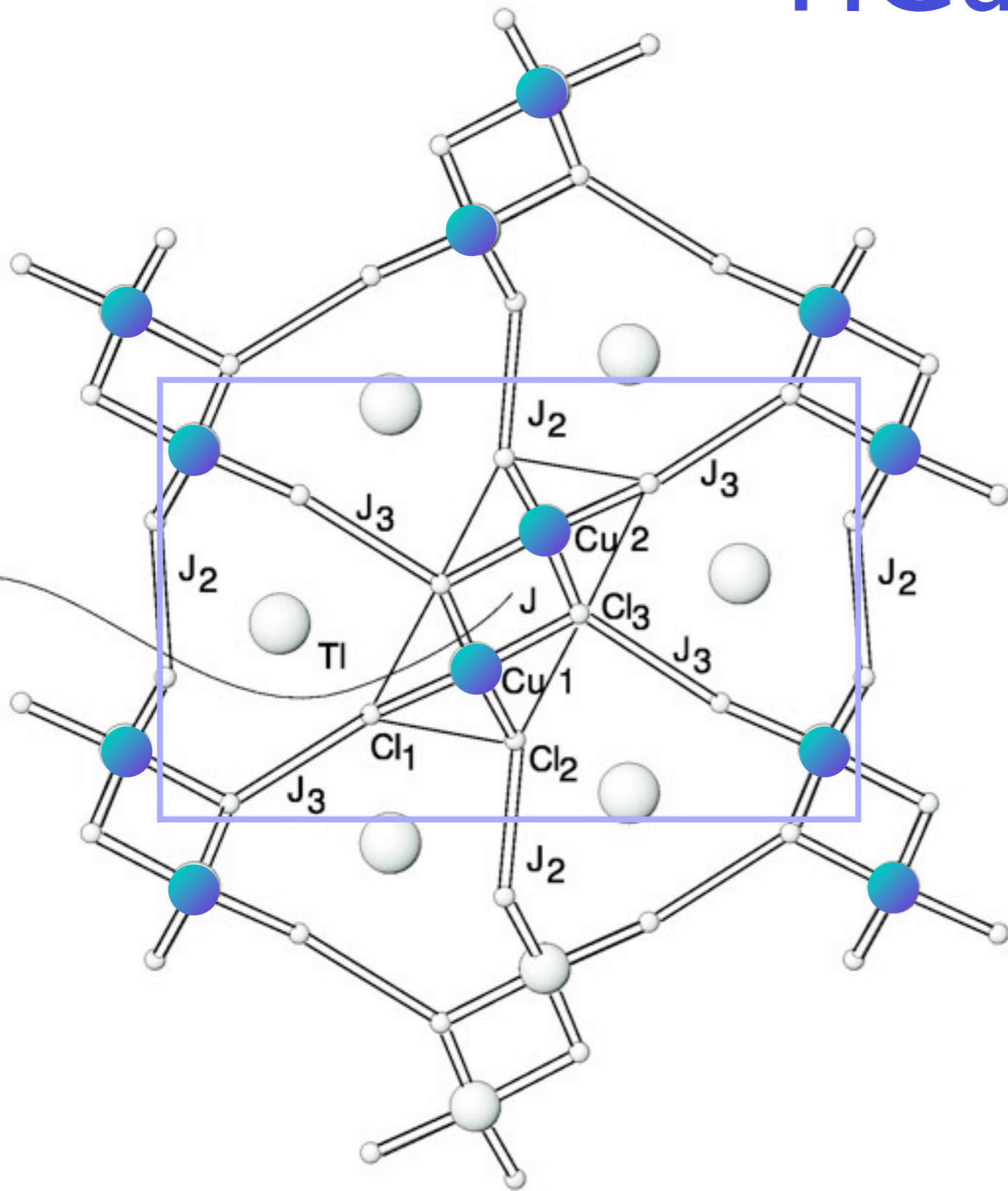
Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

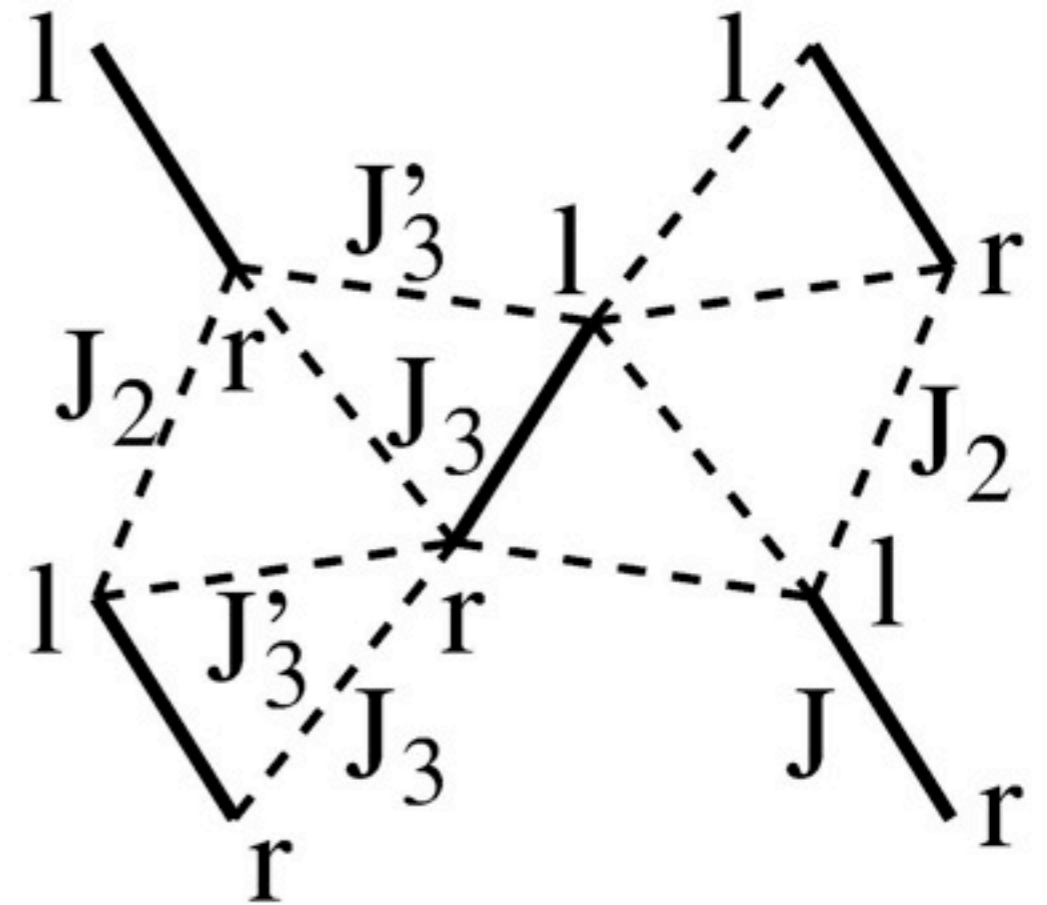
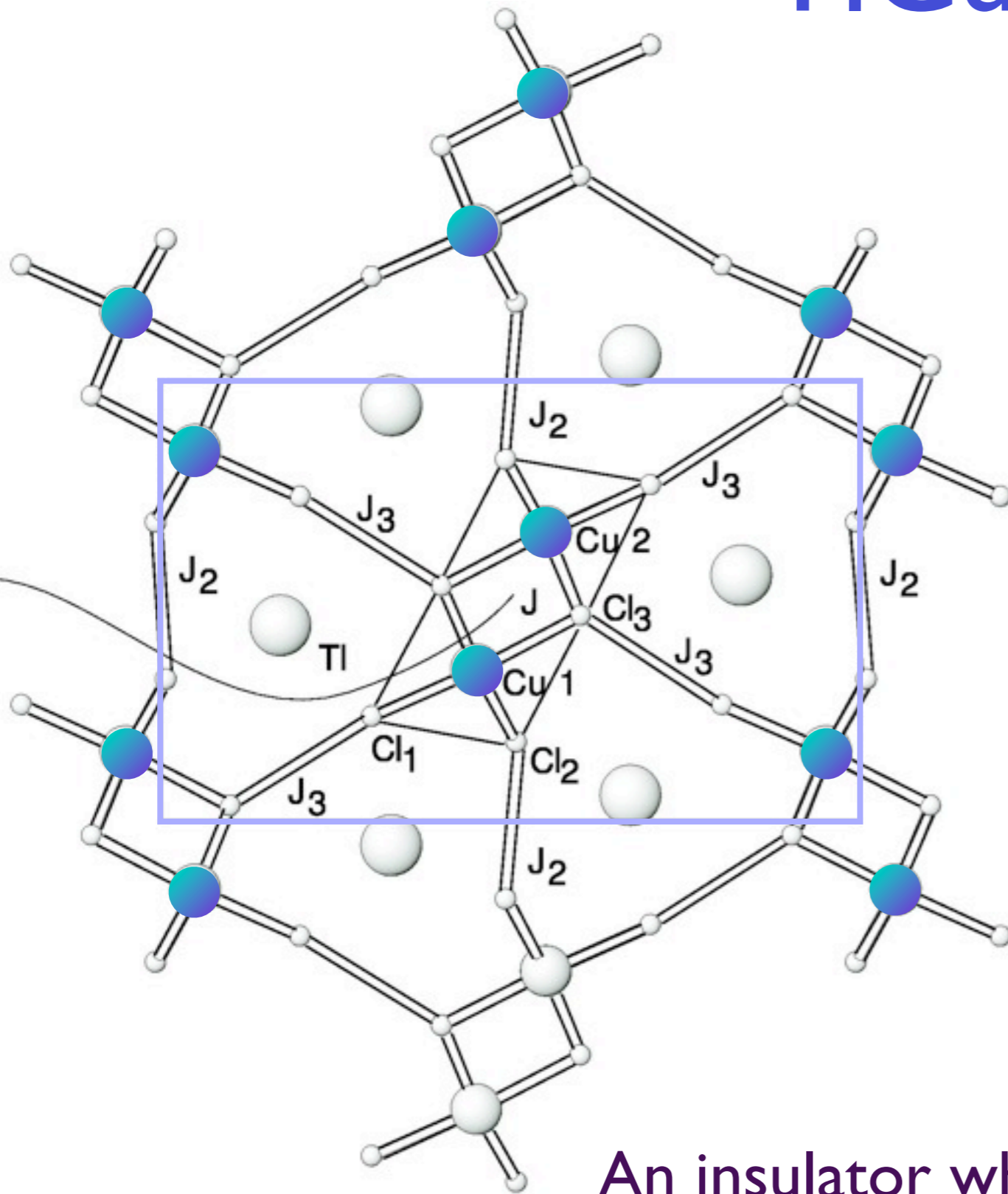
$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

TlCuCl₃



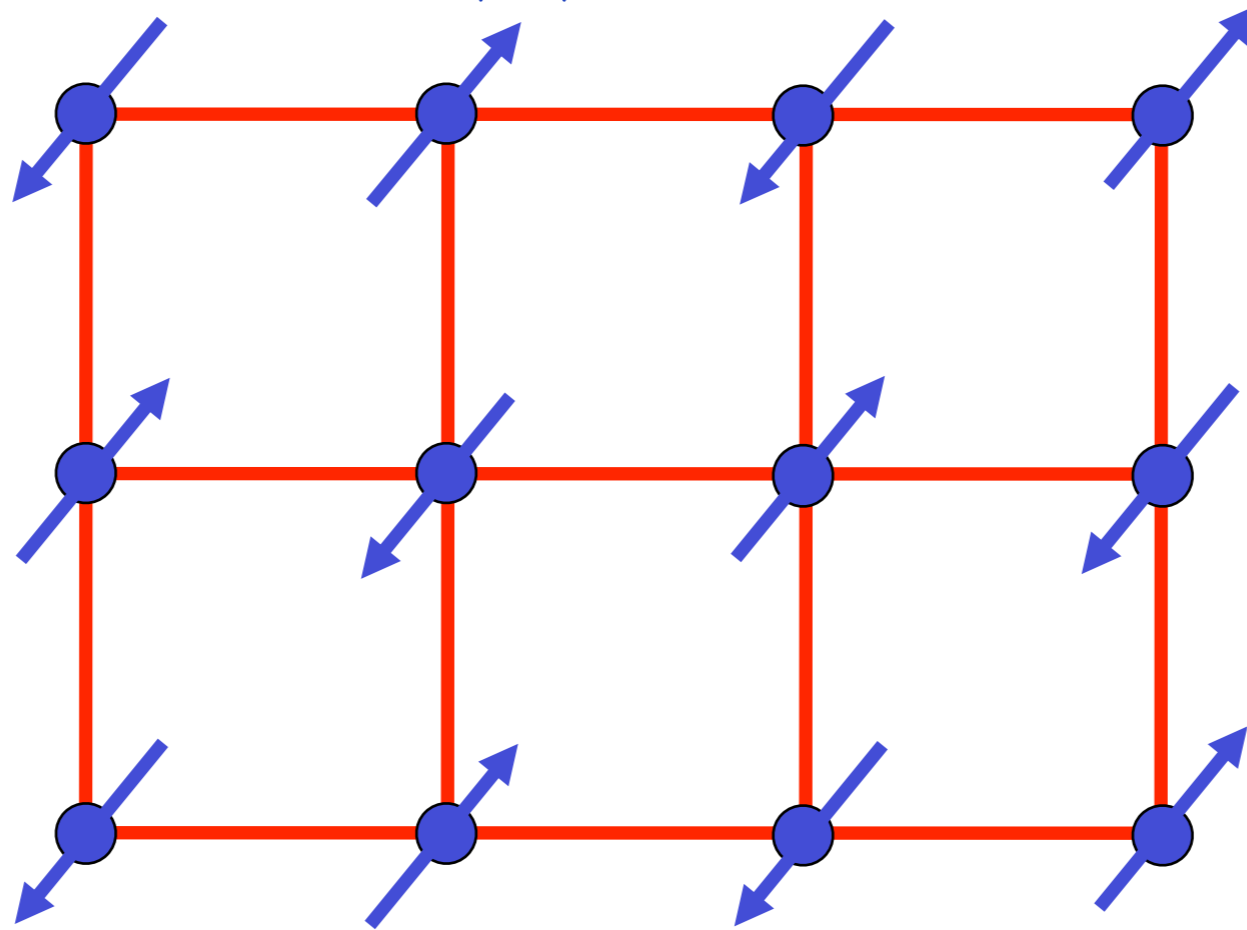
TlCuCl₃



An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

Square lattice antiferromagnet

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Ground state has long-range Néel order

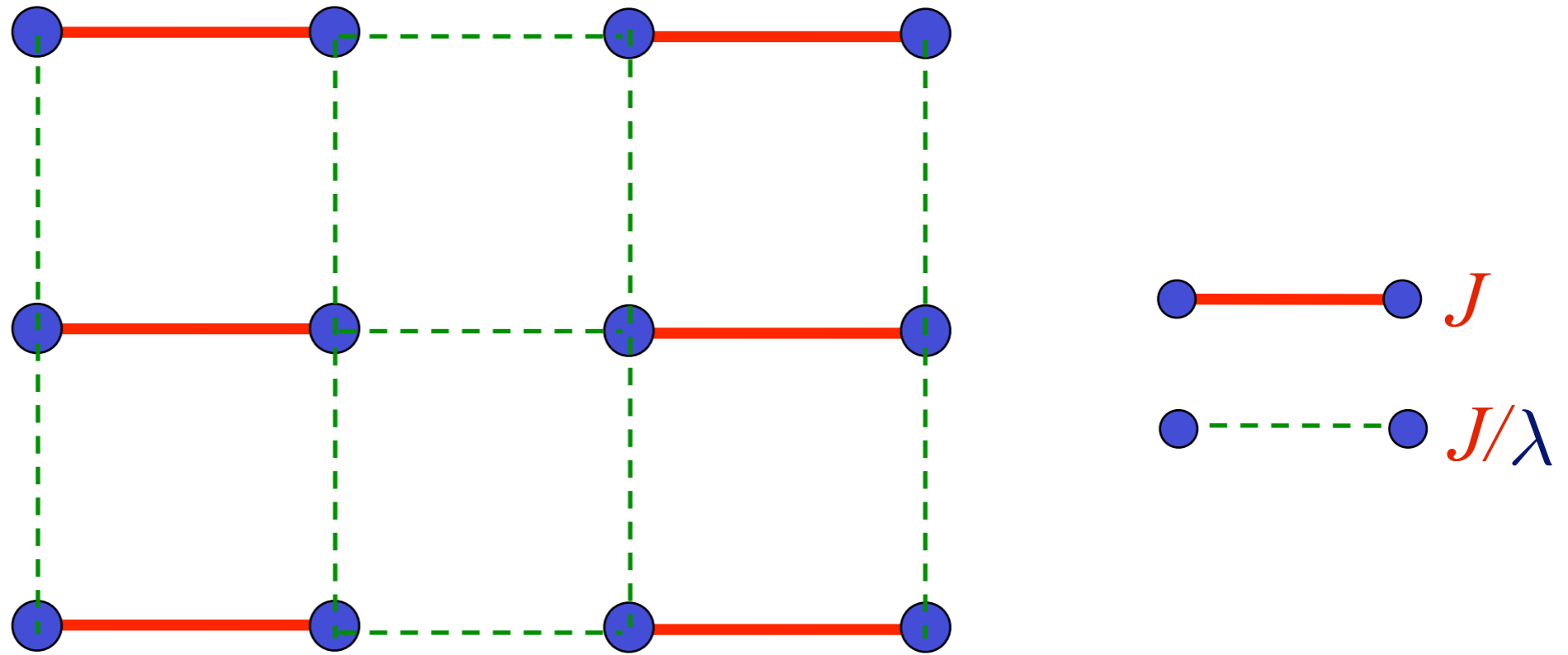
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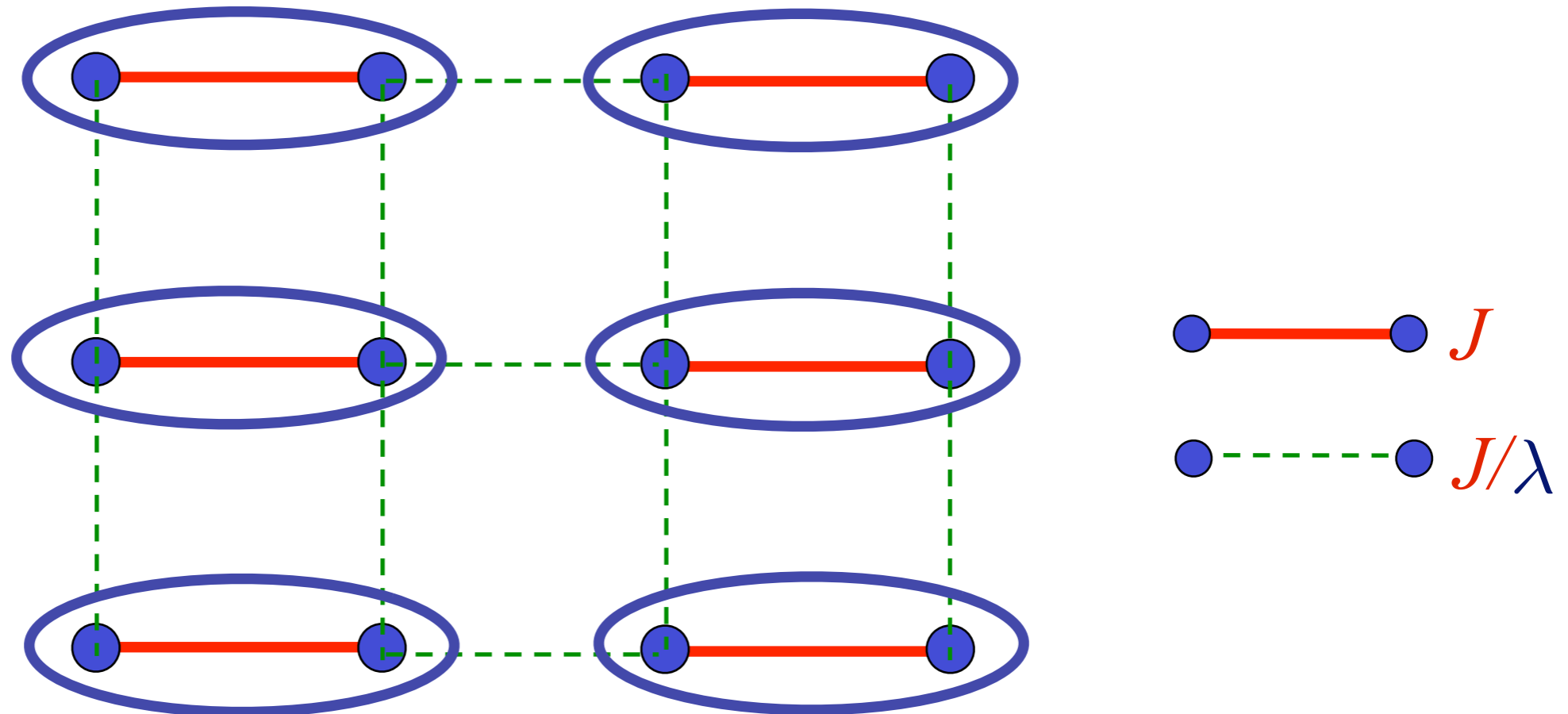
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Weaken some bonds to induce spin entanglement in a new quantum phase

Square lattice antiferromagnet

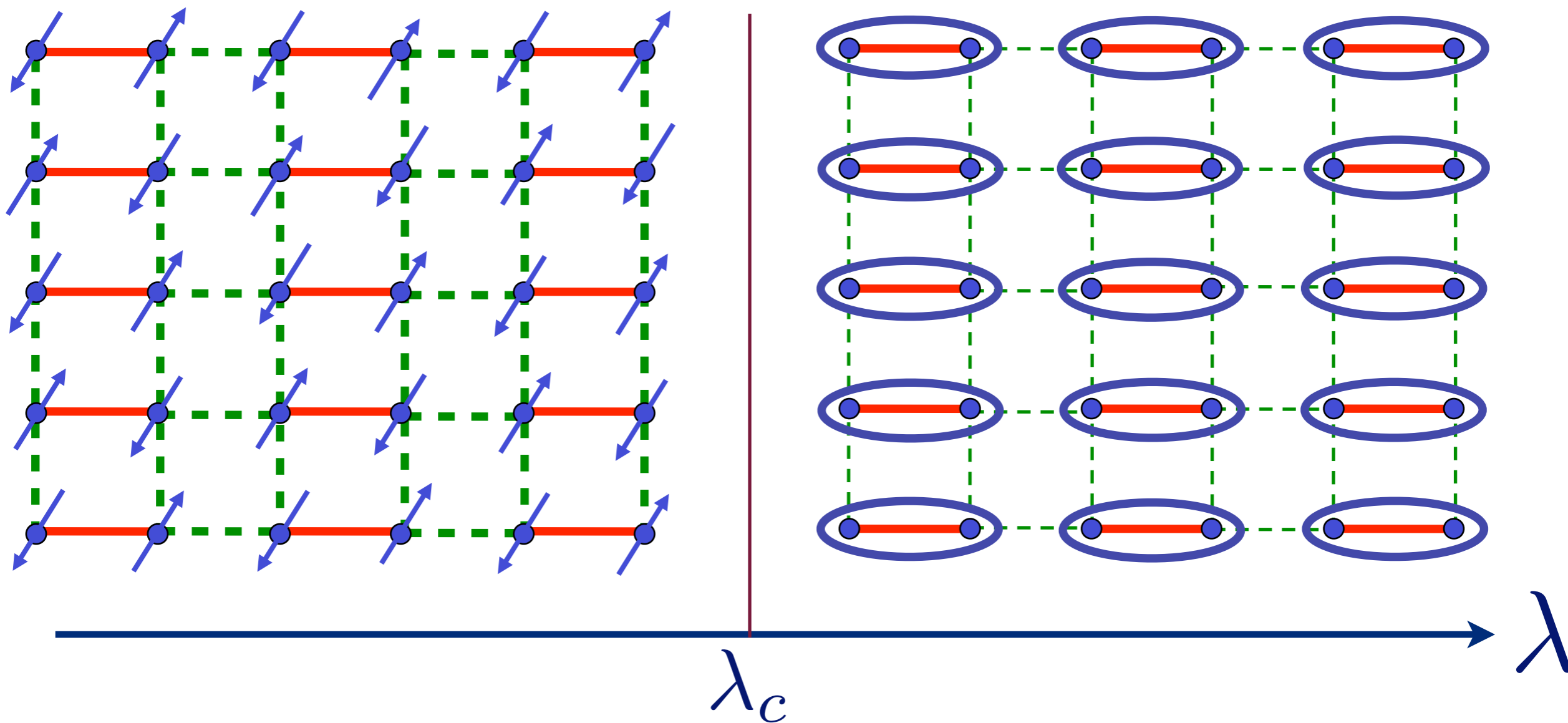
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state is a “quantum paramagnet”
with spins locked in valence bond singlets

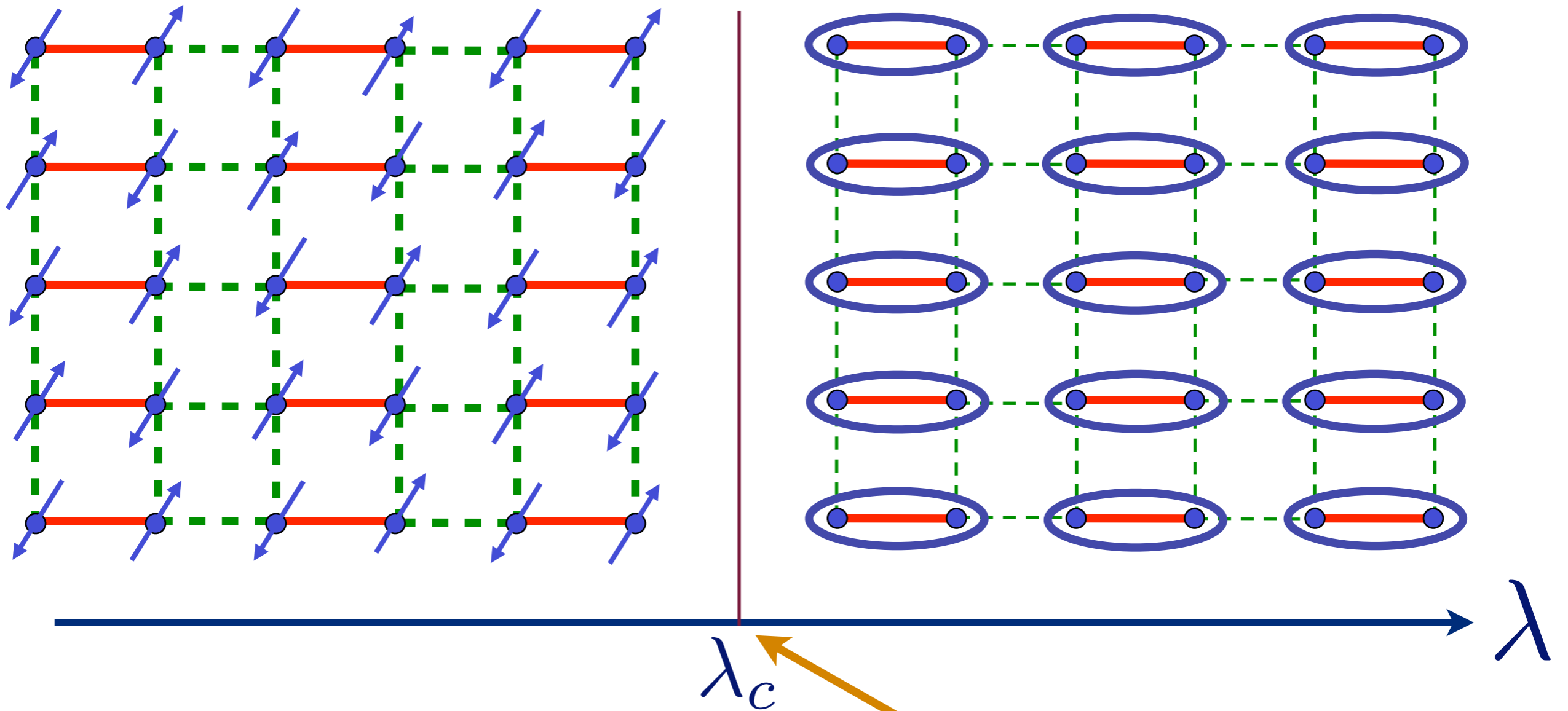
$$\text{[Diagram of a valence bond singlet]} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

$$\text{Diagram of two blue spheres connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



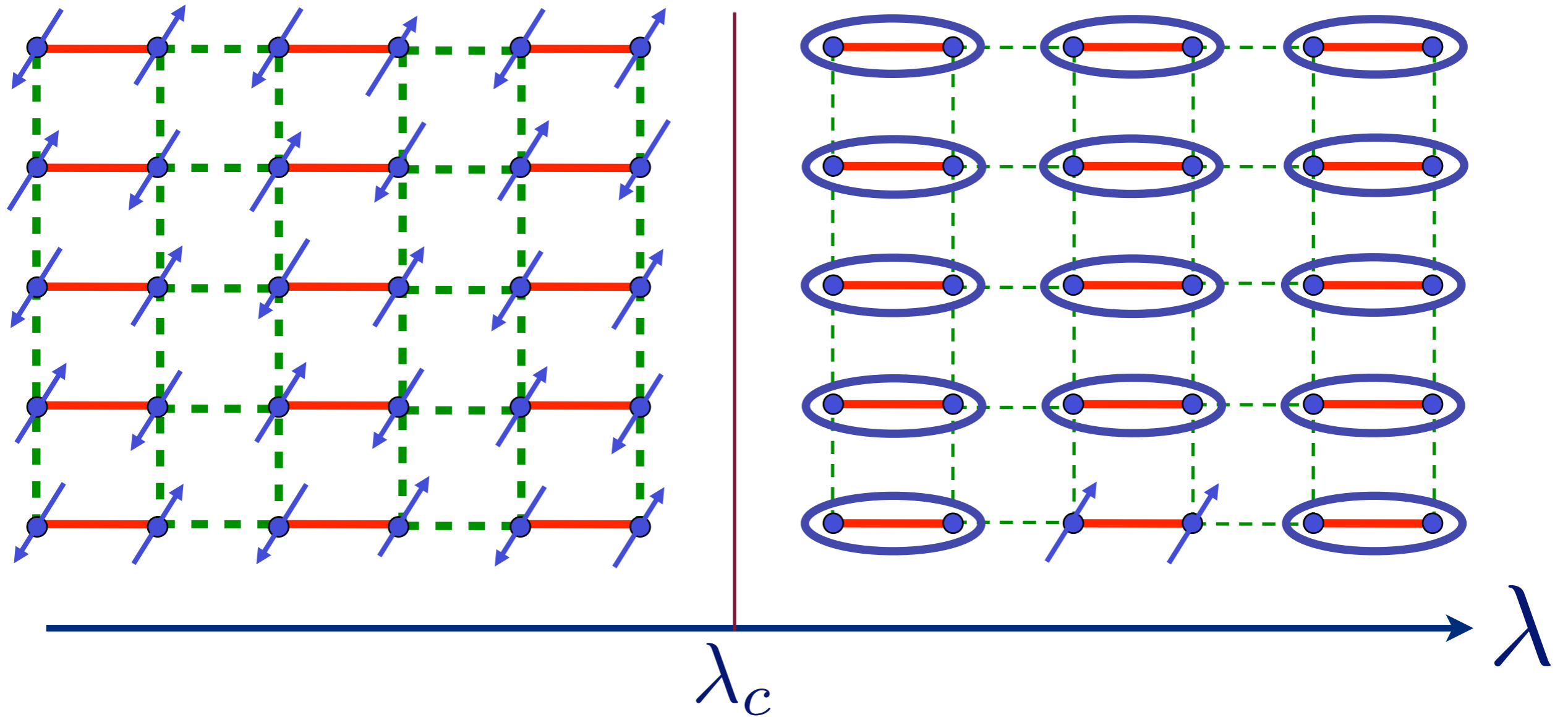
← Pressure in TlCuCl_3

$$\text{Diagram of two blue dots connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

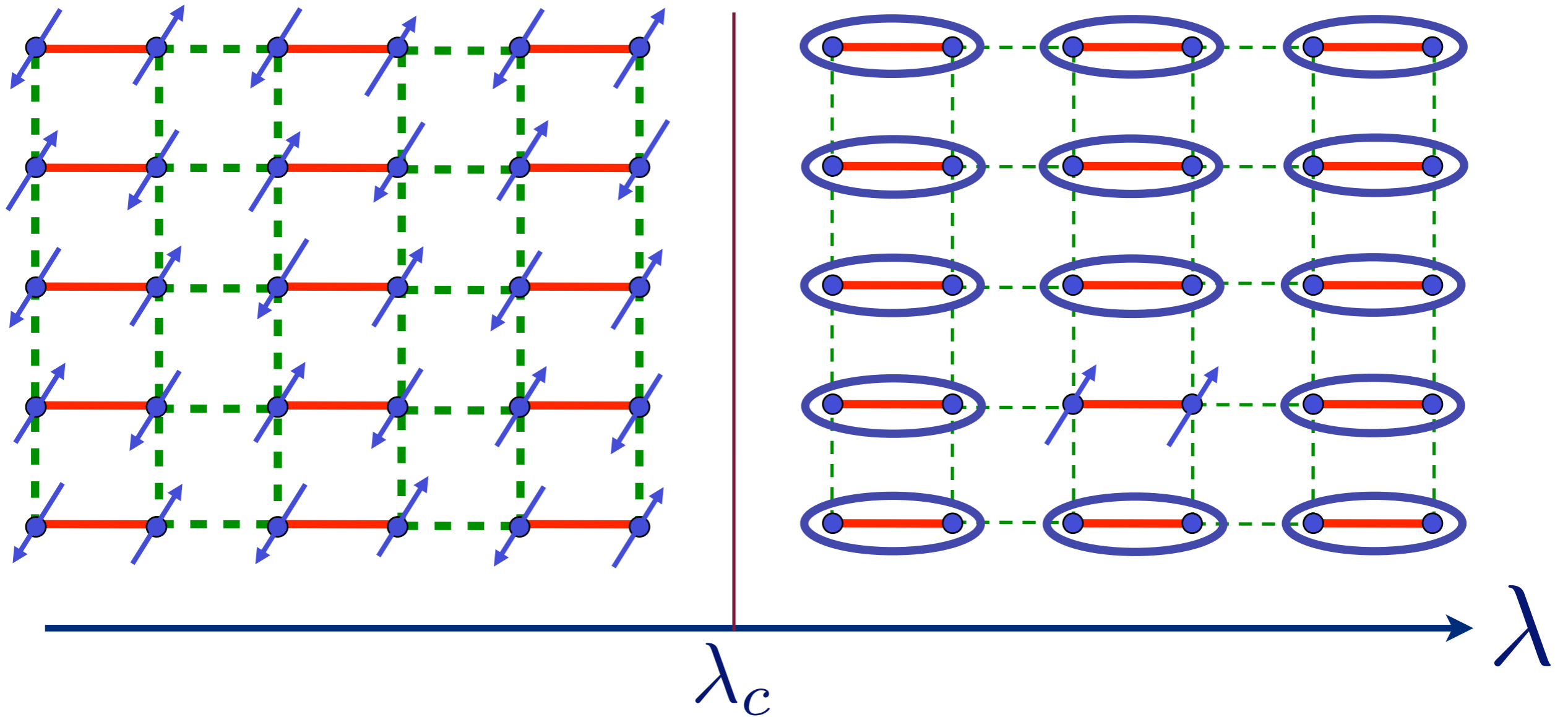


Quantum critical point with non-local entanglement in spin wavefunction

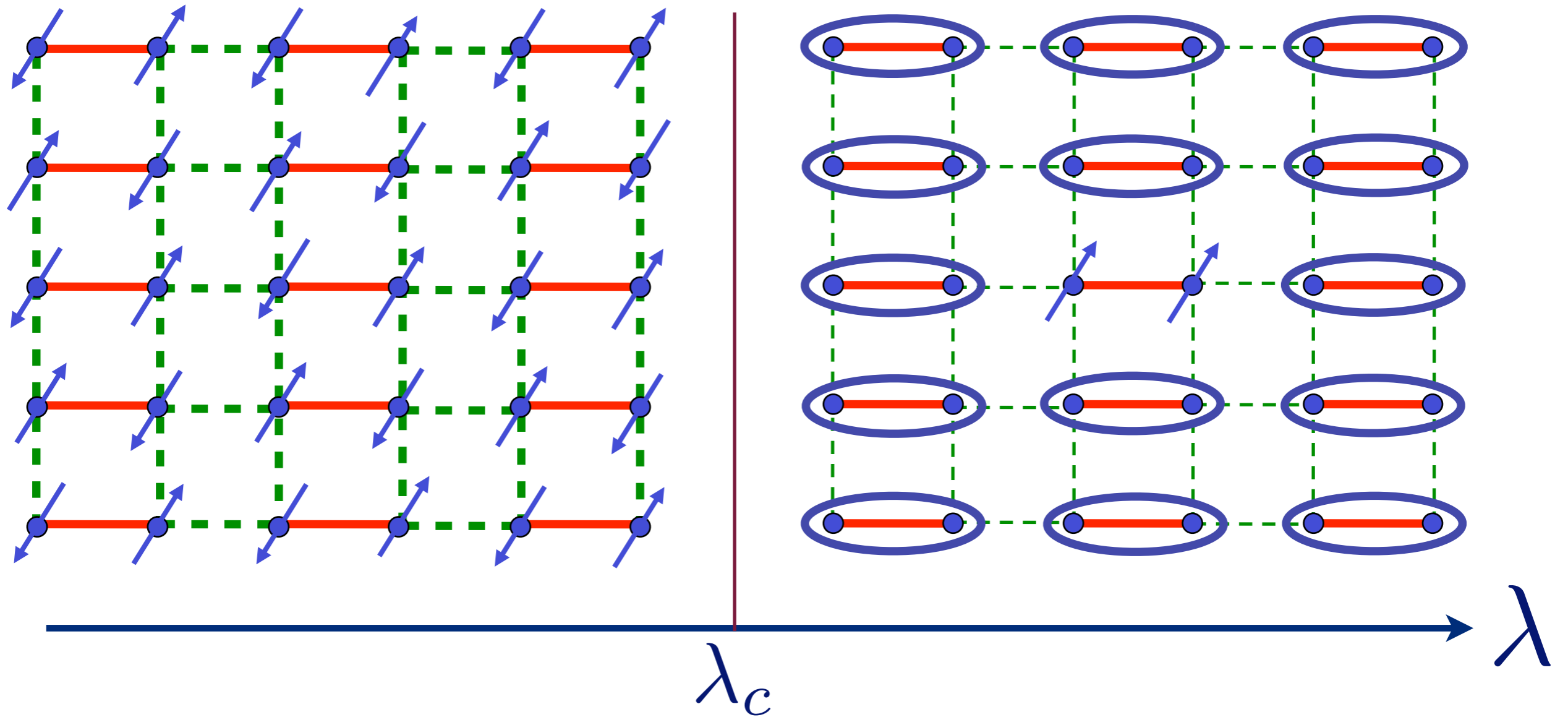
Excitation spectrum in the paramagnetic phase



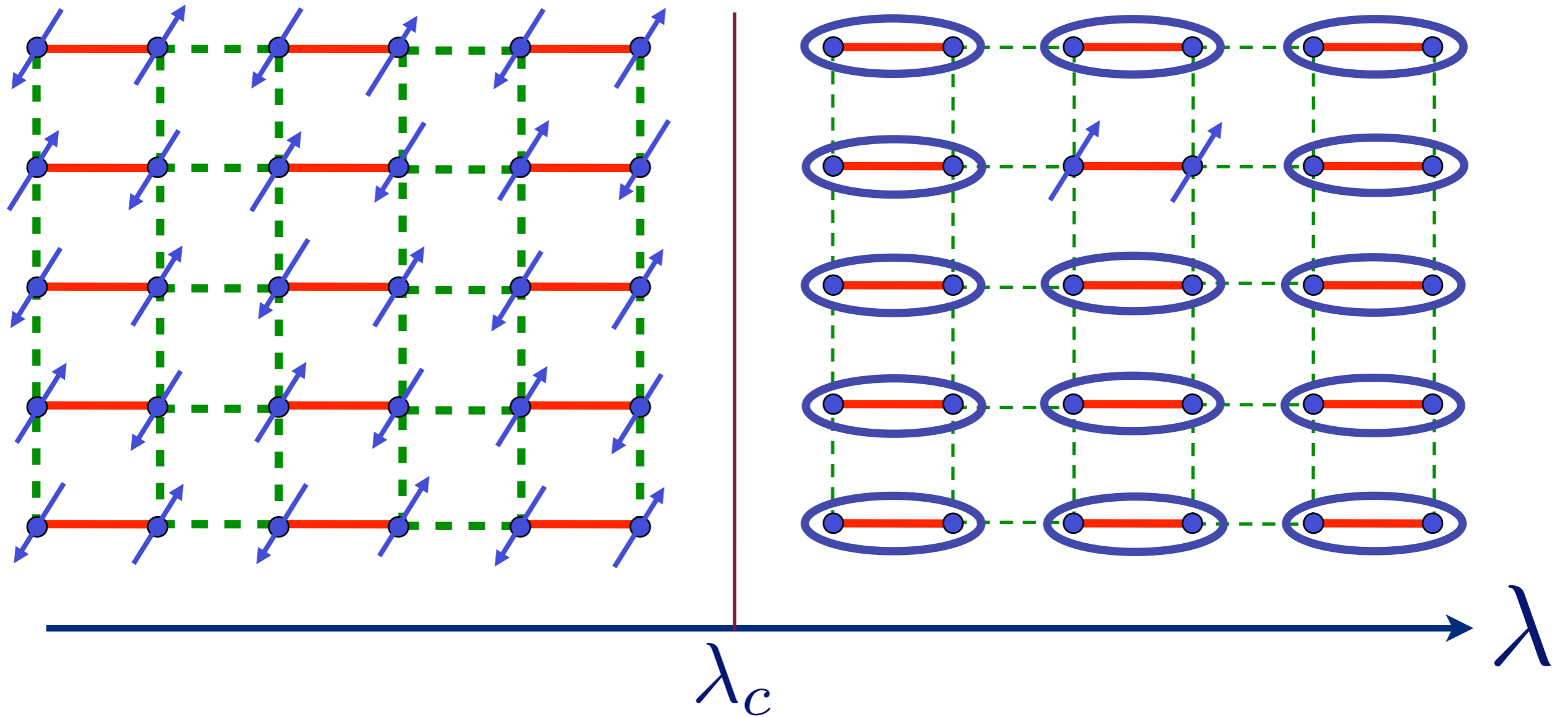
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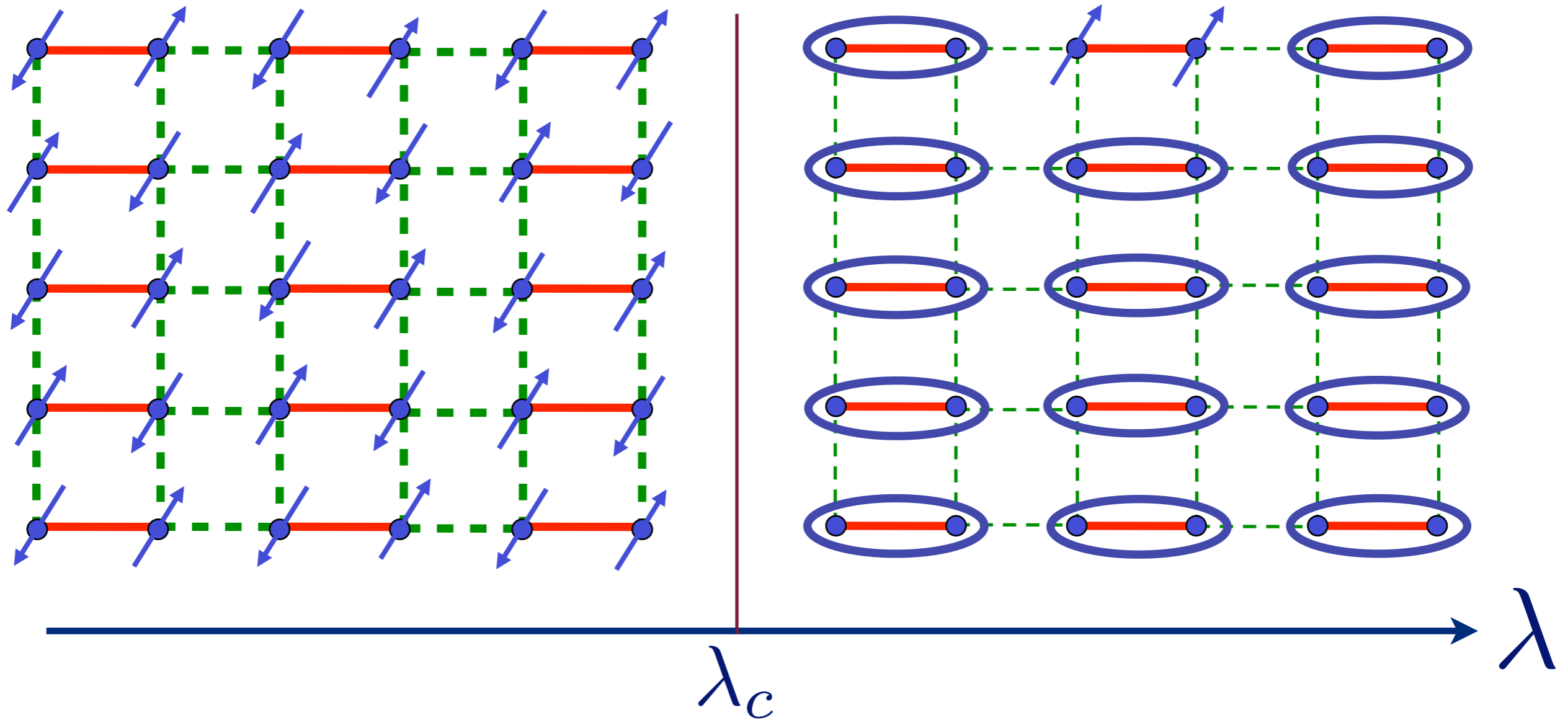
Excitation spectrum in the paramagnetic phase



Excitation spectrum in the paramagnetic phase



Excitation spectrum in the paramagnetic phase



TlCuCl₃ at ambient pressure

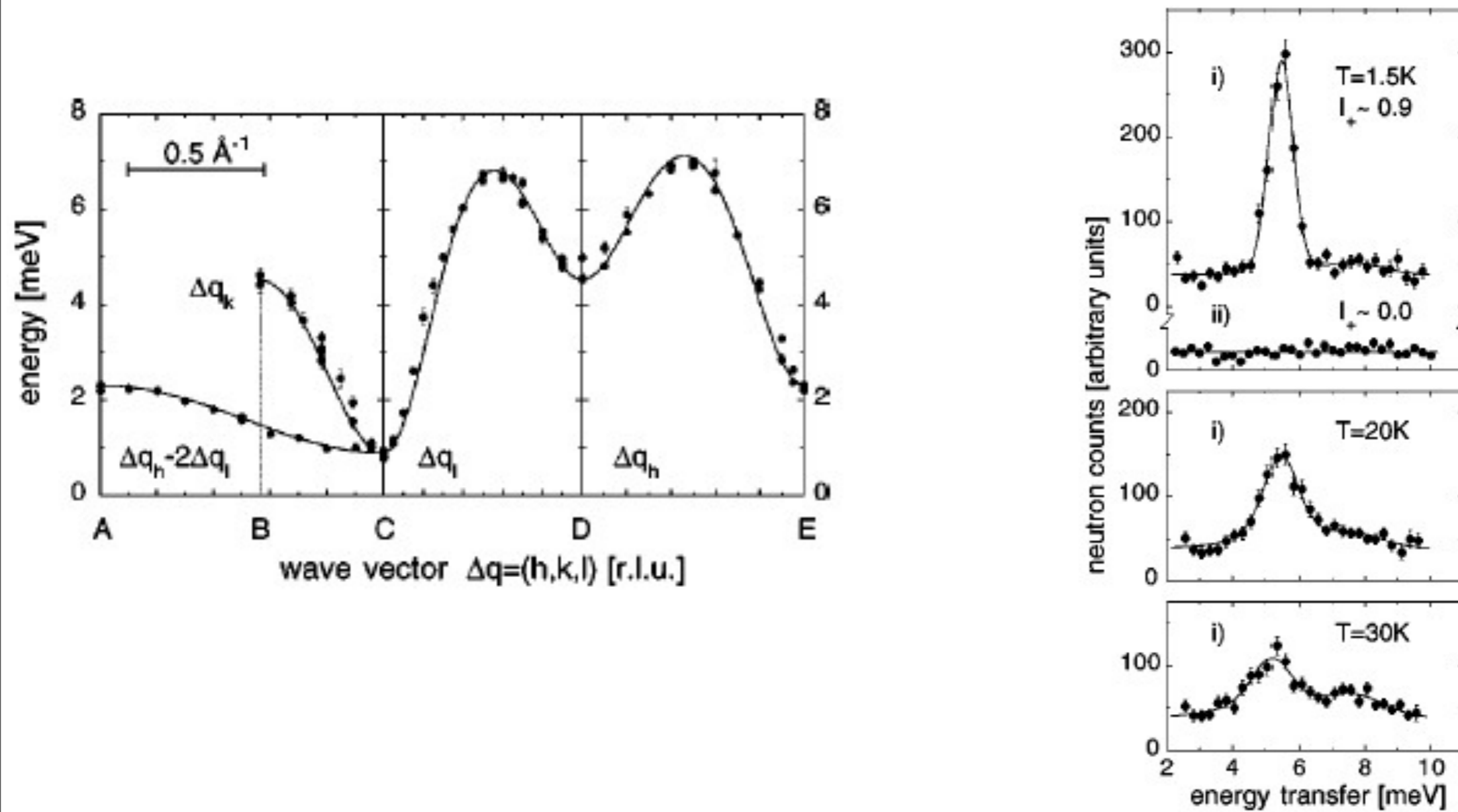
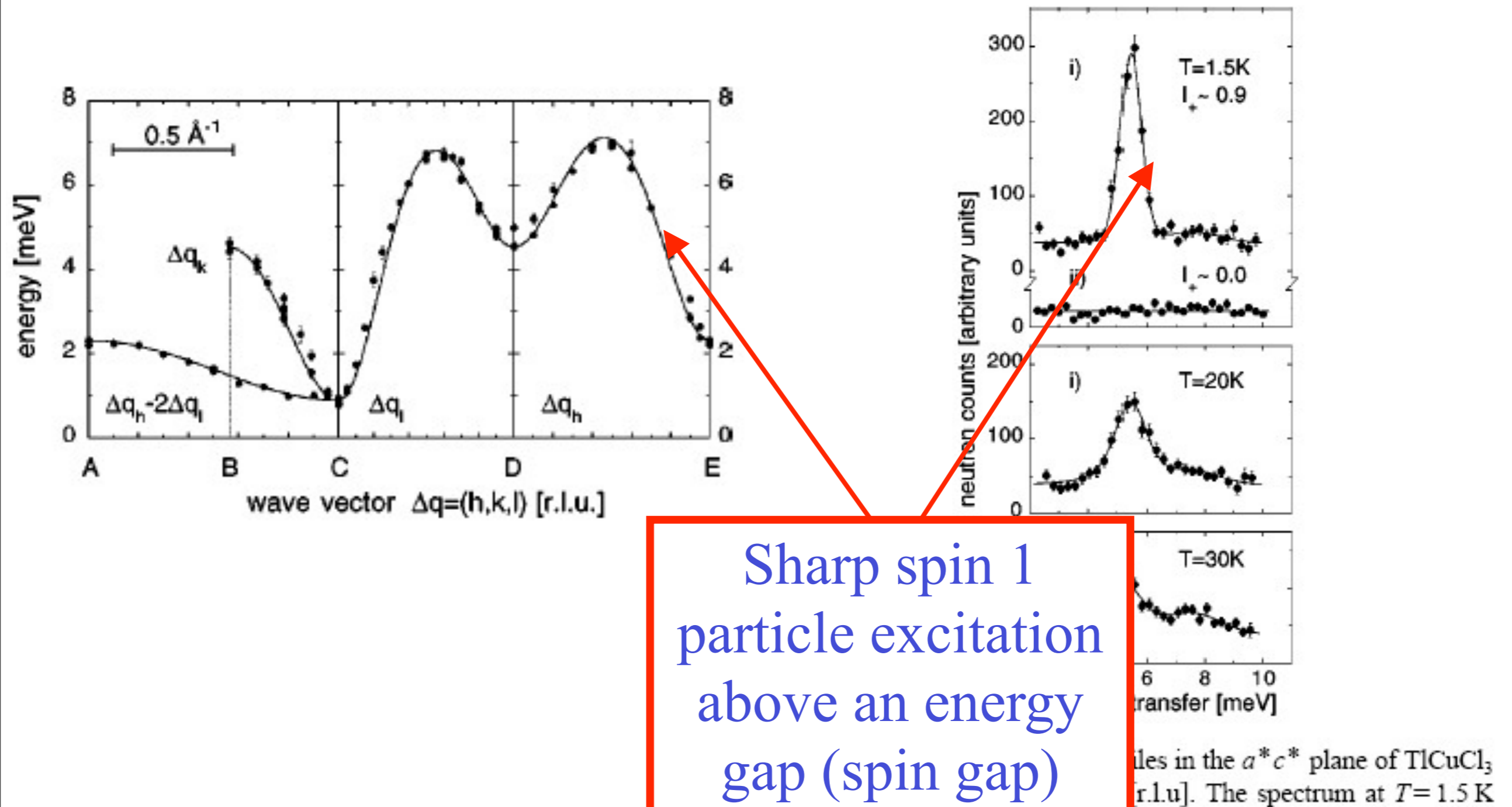


FIG. 1. Measured neutron profiles in the a^*c^* plane of TlCuCl₃ for $i = (1.35, 0, 0)$, $ii = (0, 0, 3.15)$ [r.l.u.]. The spectrum at $T = 1.5 \text{ K}$

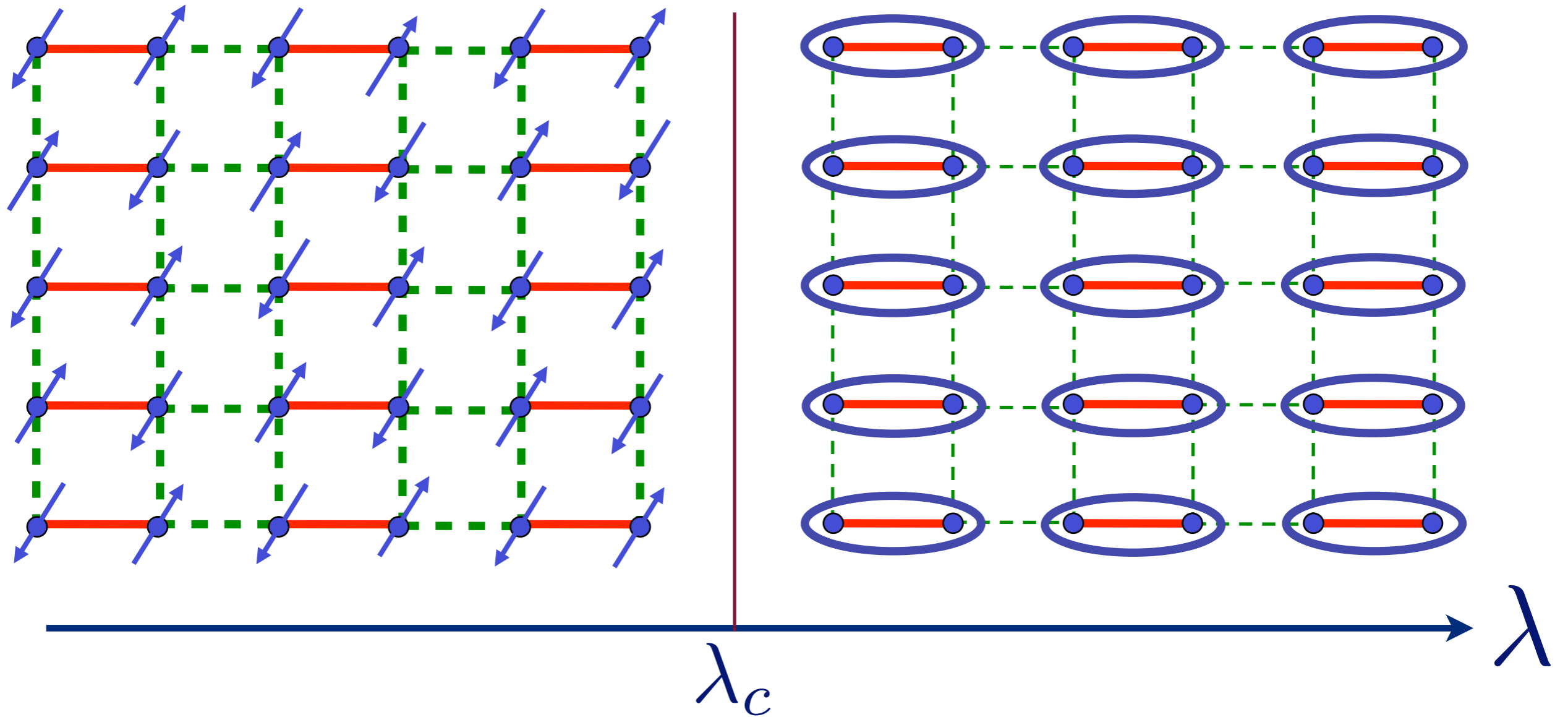
N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev. B* 63 172414 (2001).

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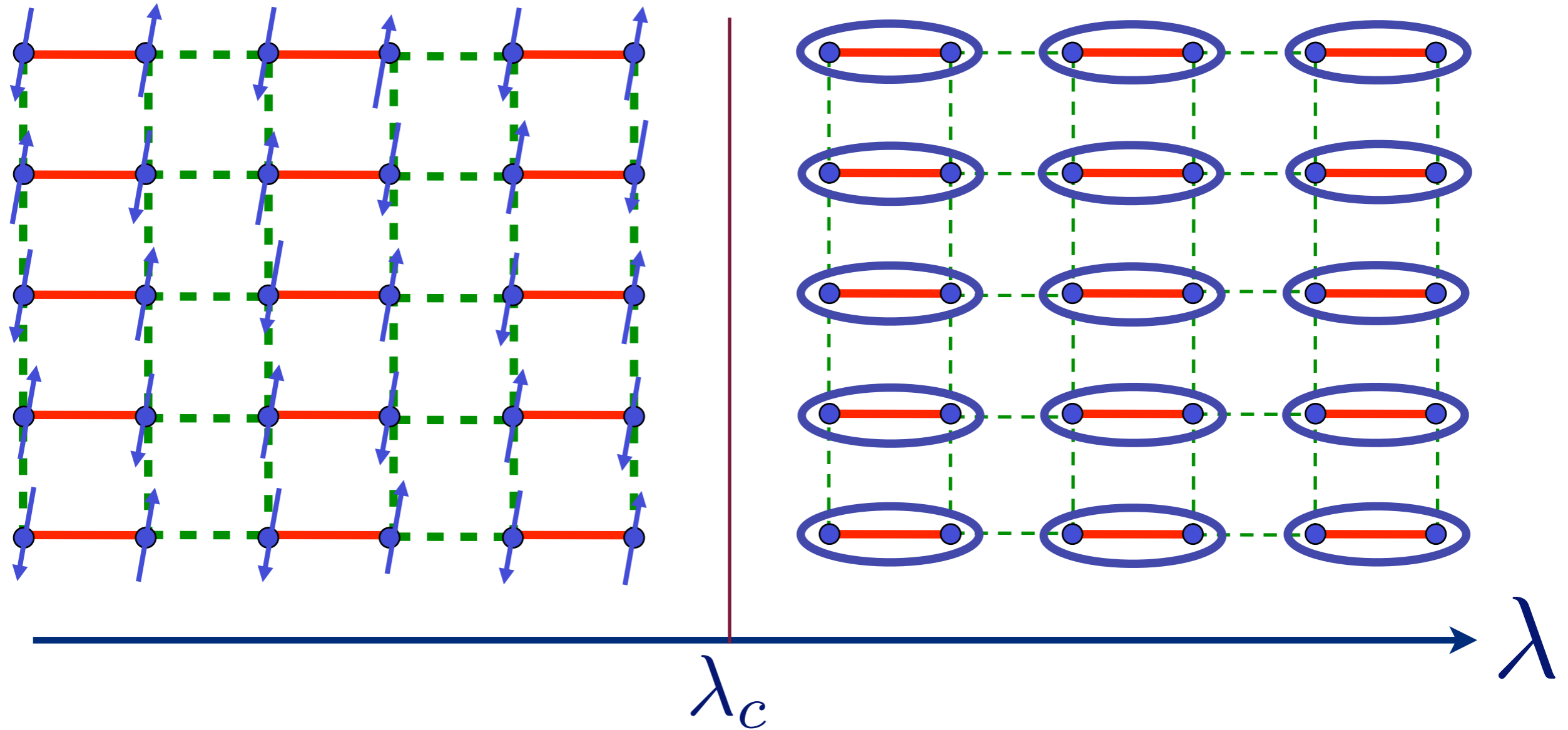


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Excitation spectrum in the Néel phase

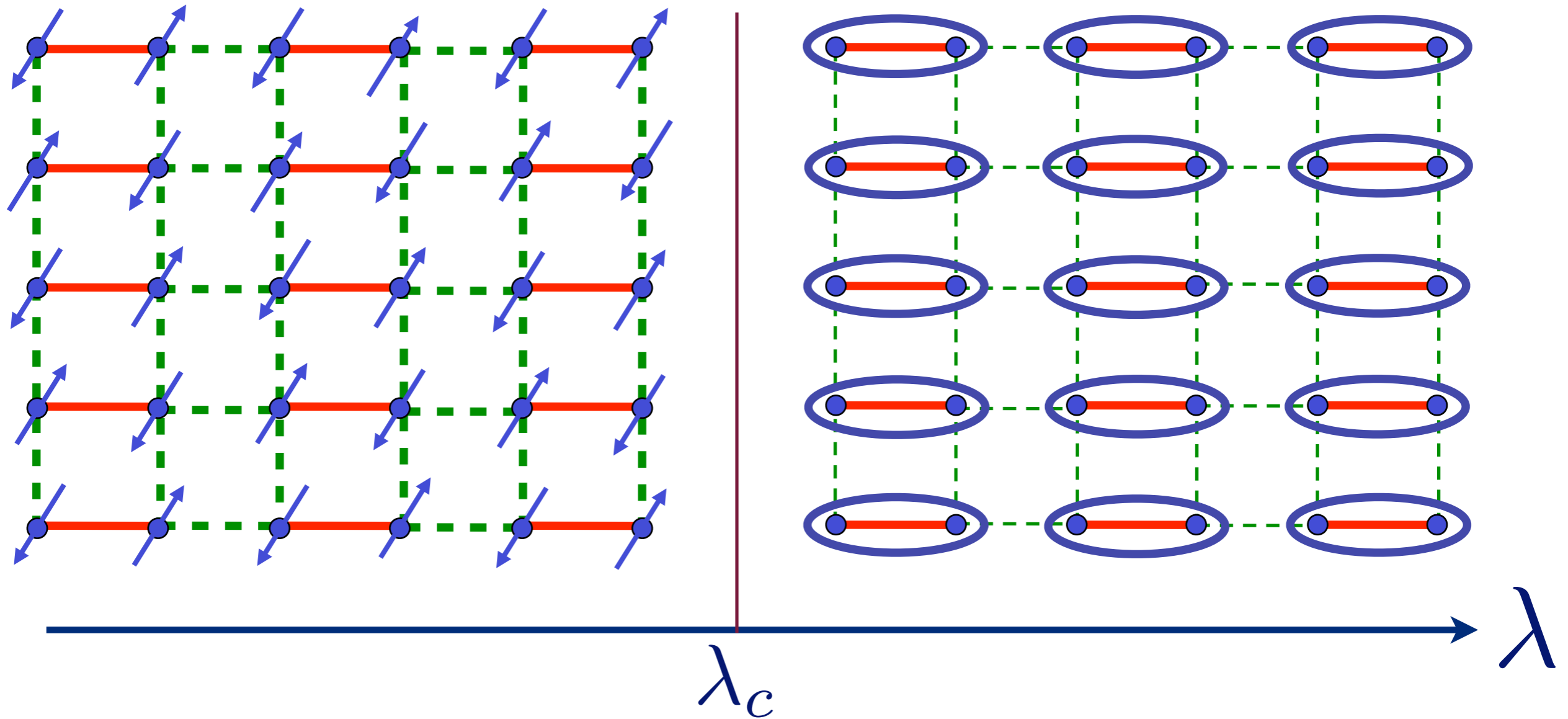


Excitation spectrum in the Néel phase



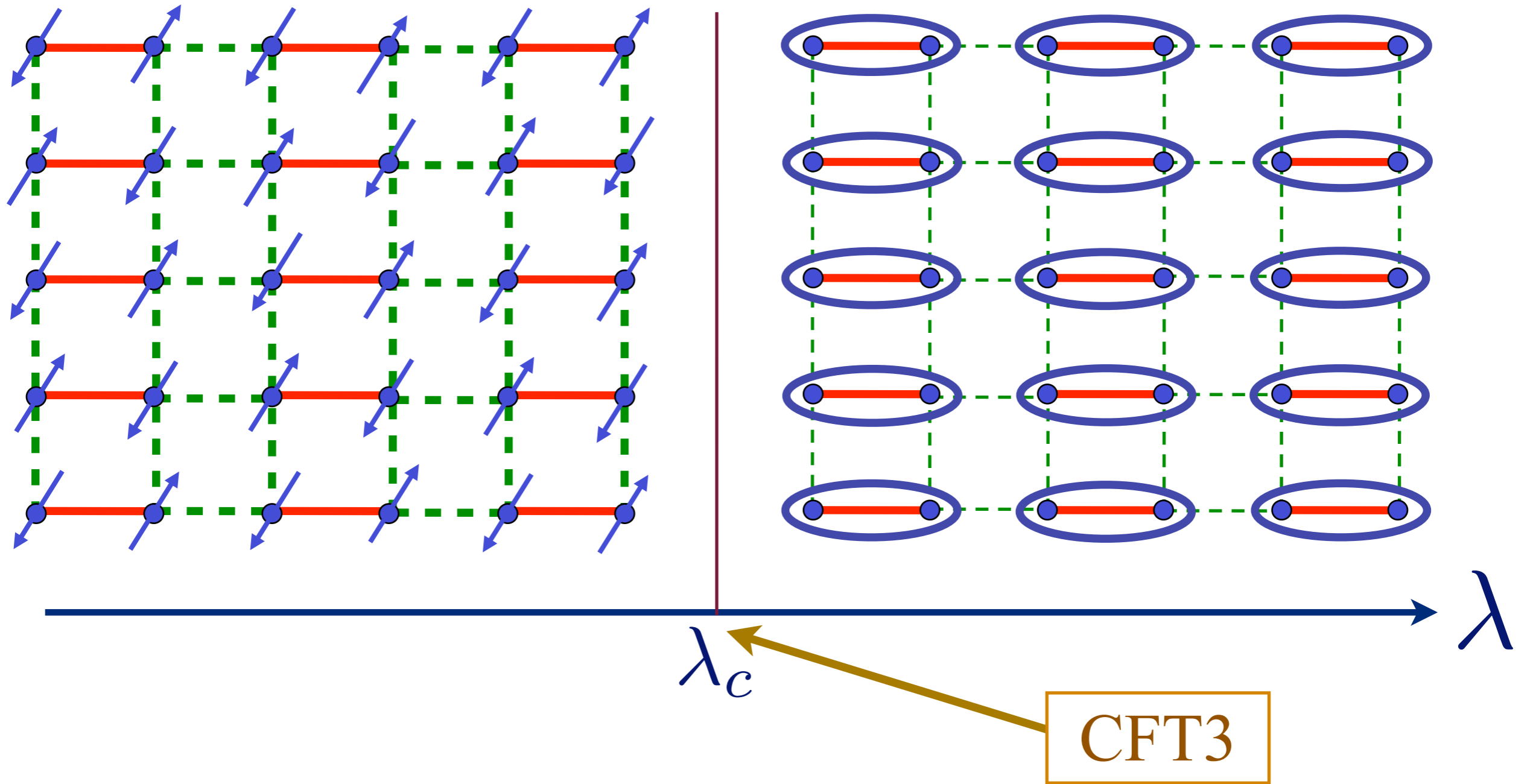
Spin waves

Excitation spectrum in the Néel phase



Spin waves

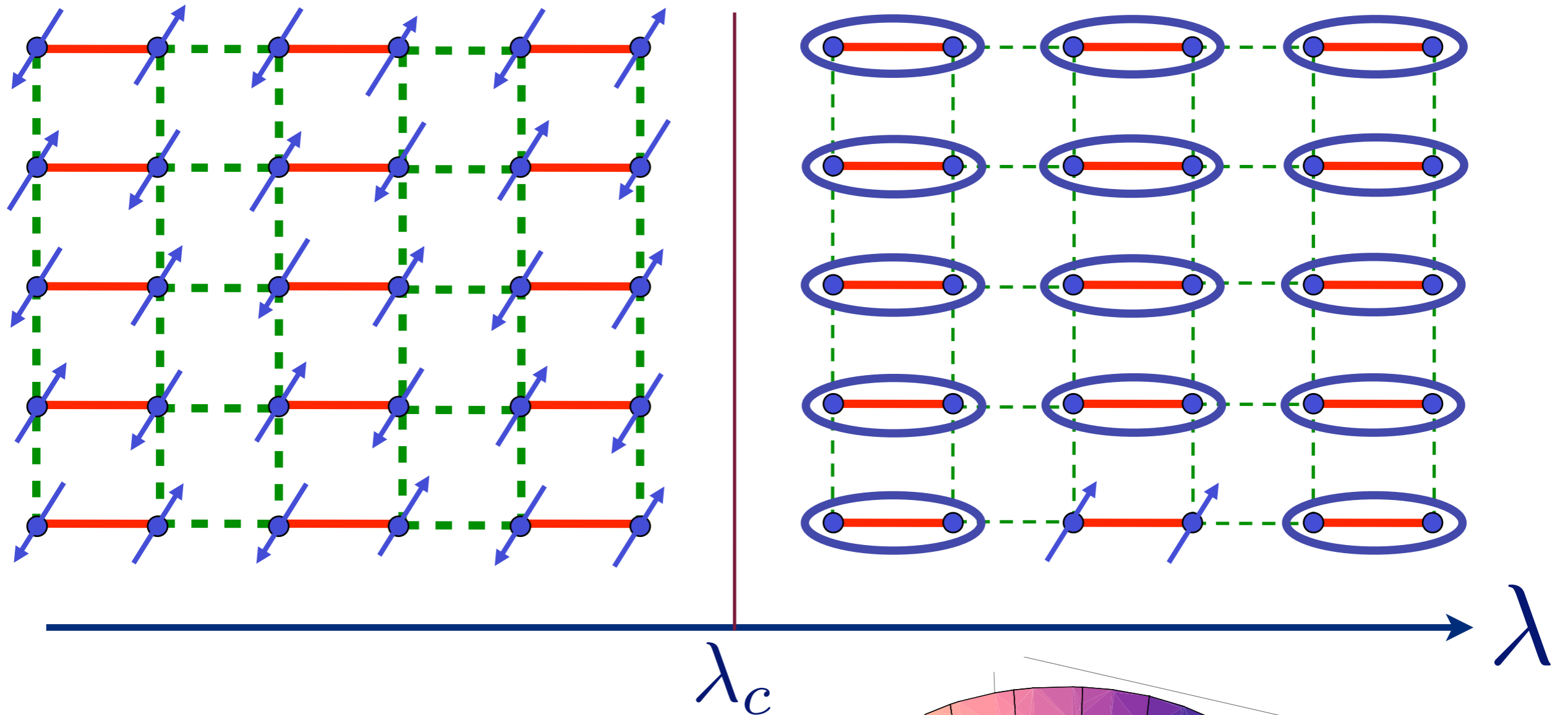
Description using Landau-Ginzburg field theory



$O(3)$ order parameter $\vec{\varphi}$

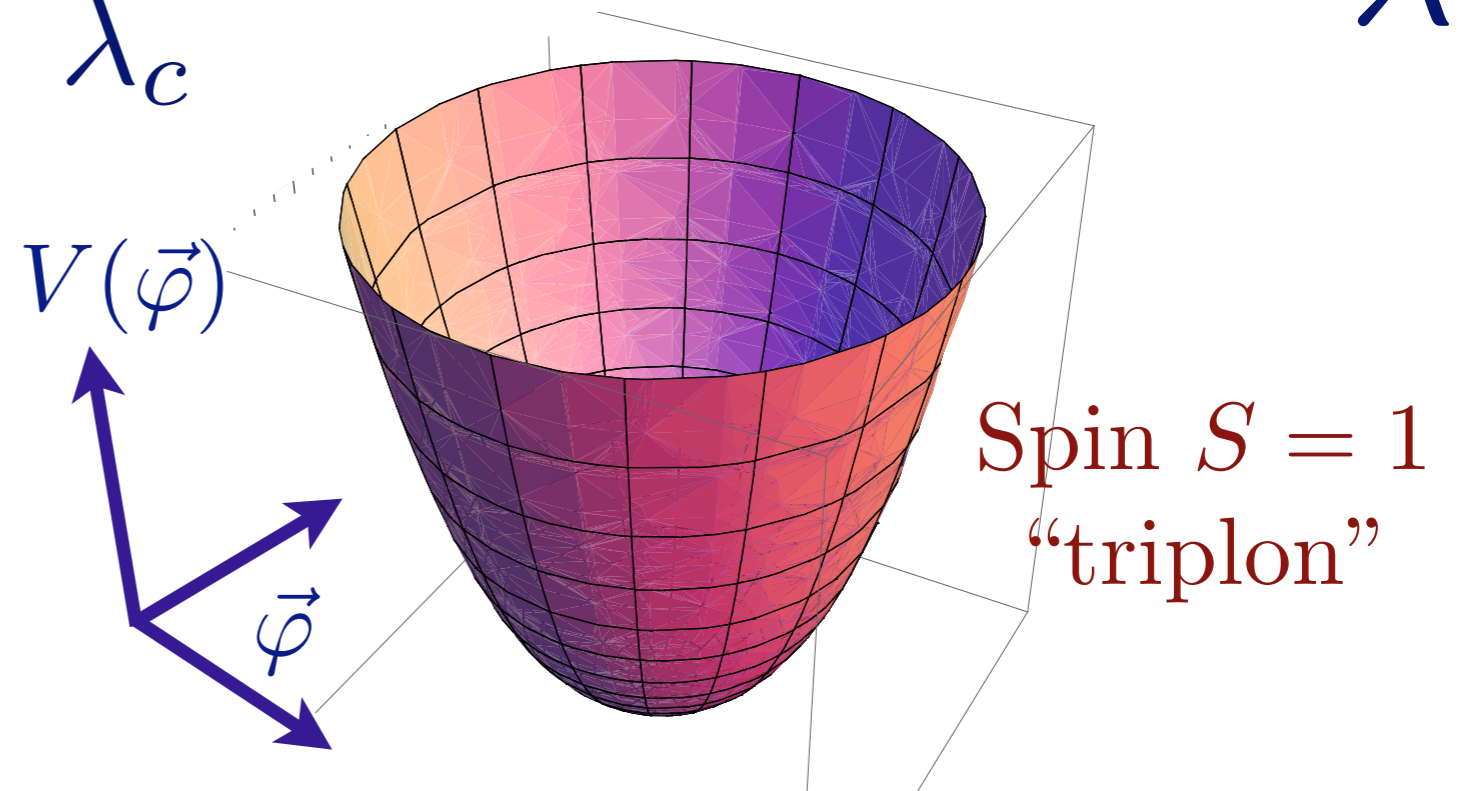
$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \vec{\varphi})^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2 \right]$$

Excitation spectrum in the paramagnetic phase

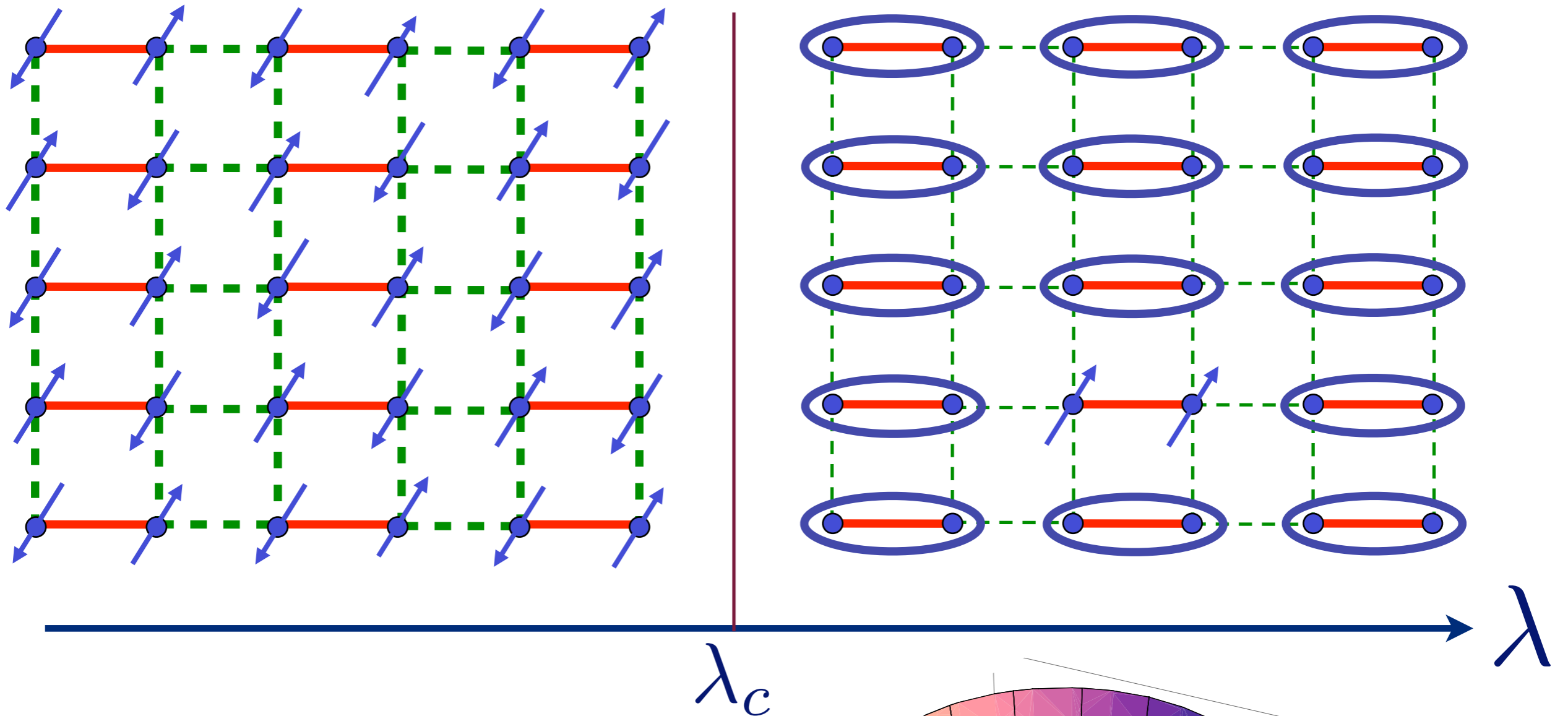


$$V(\vec{\varphi}) = (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2$$

$\lambda > \lambda_c$

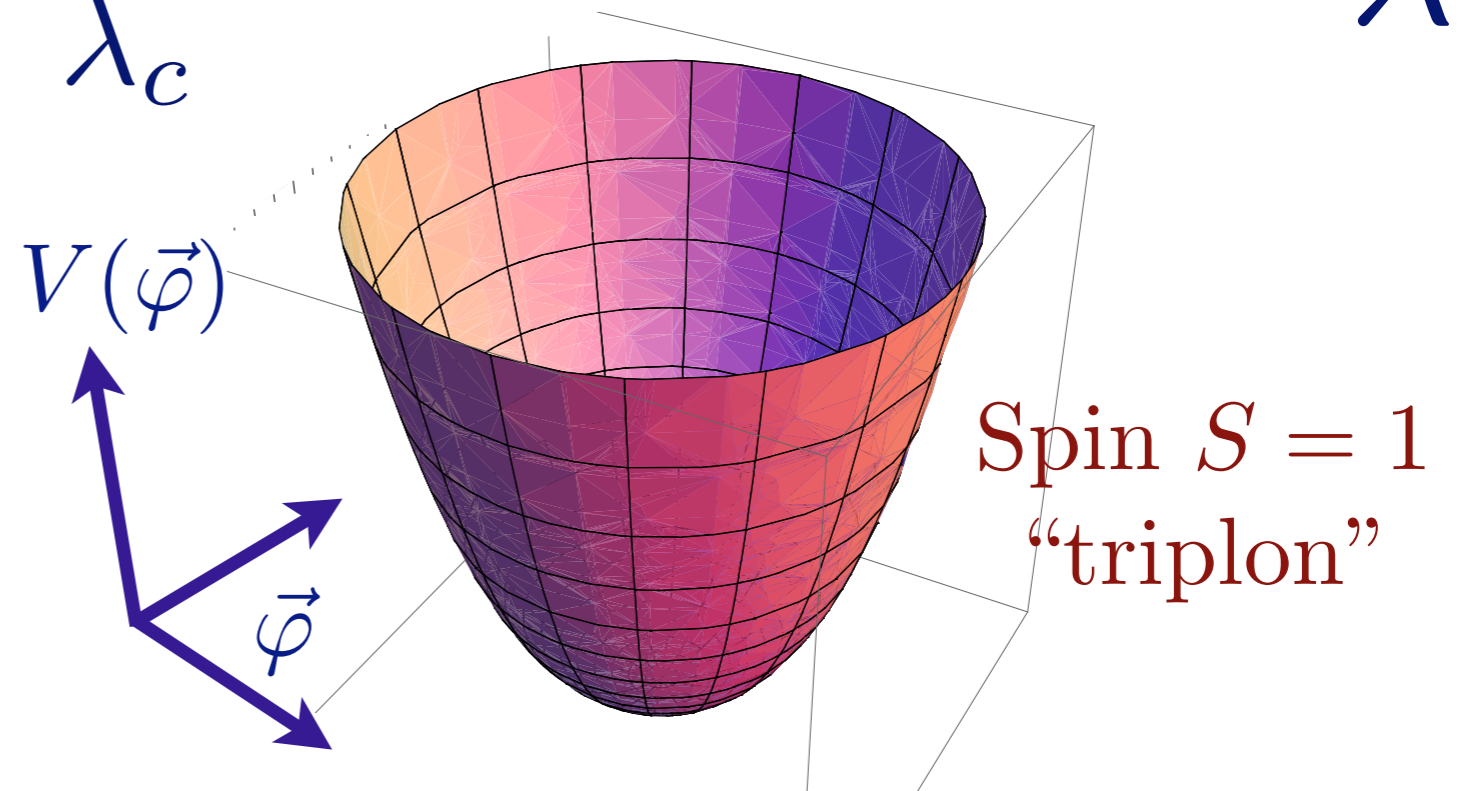


Excitation spectrum in the paramagnetic phase

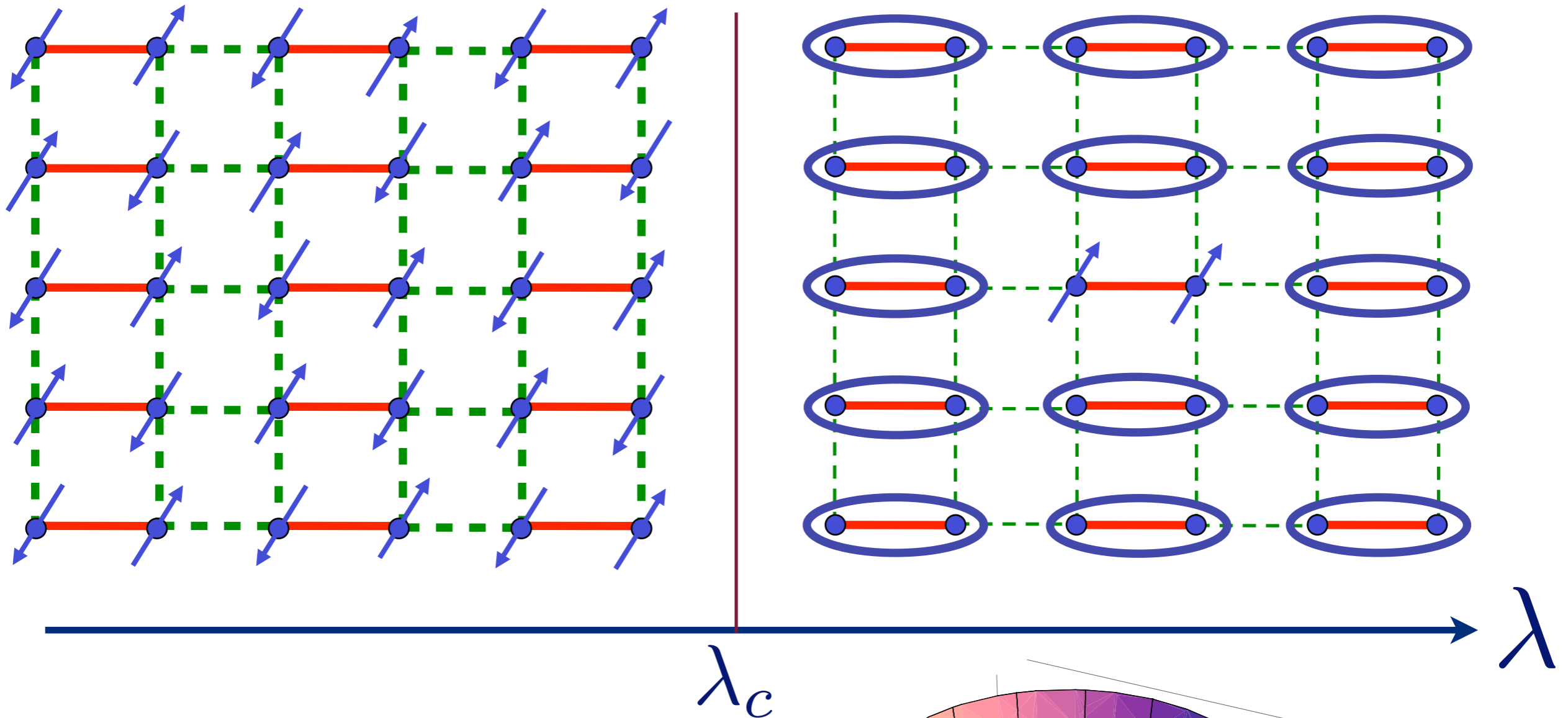


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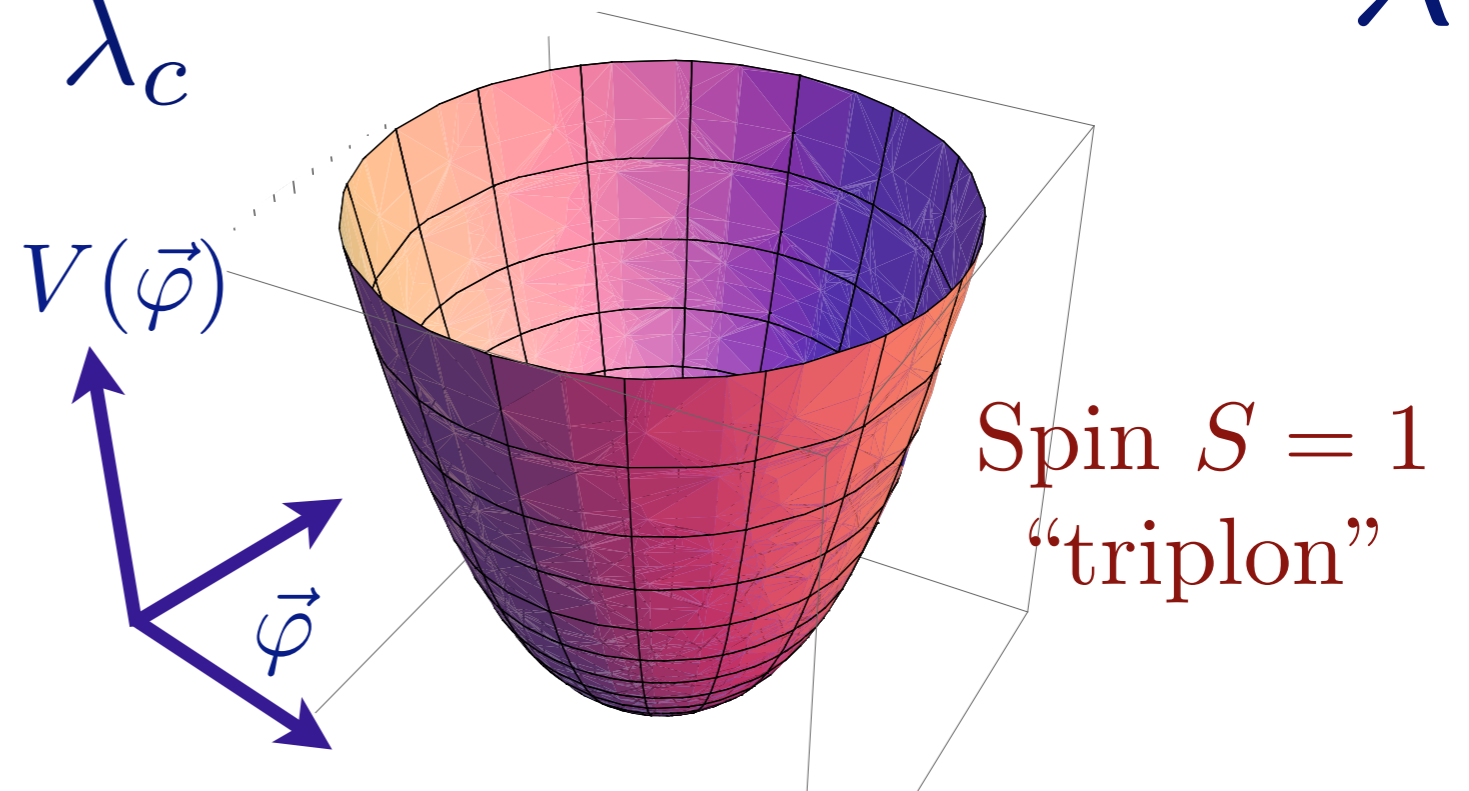


Excitation spectrum in the paramagnetic phase

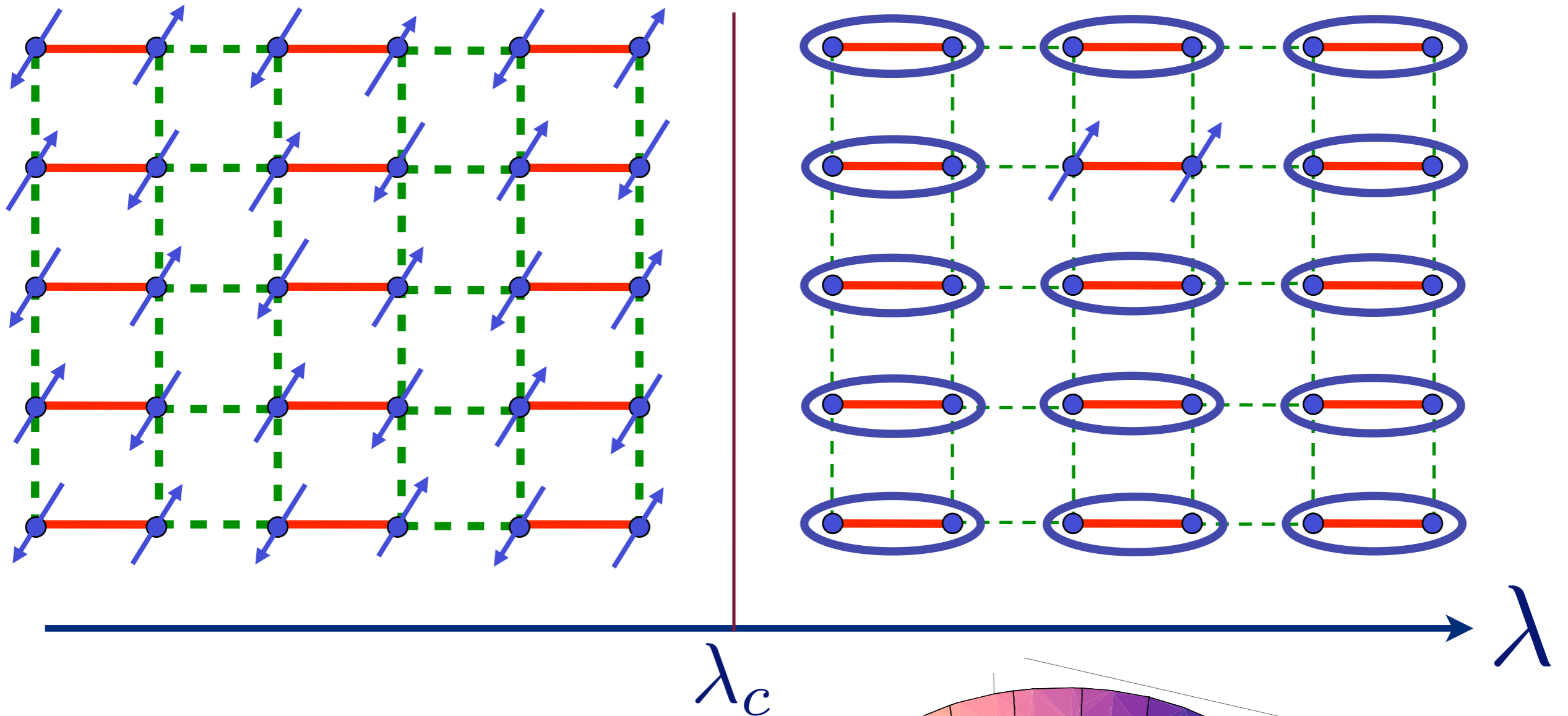


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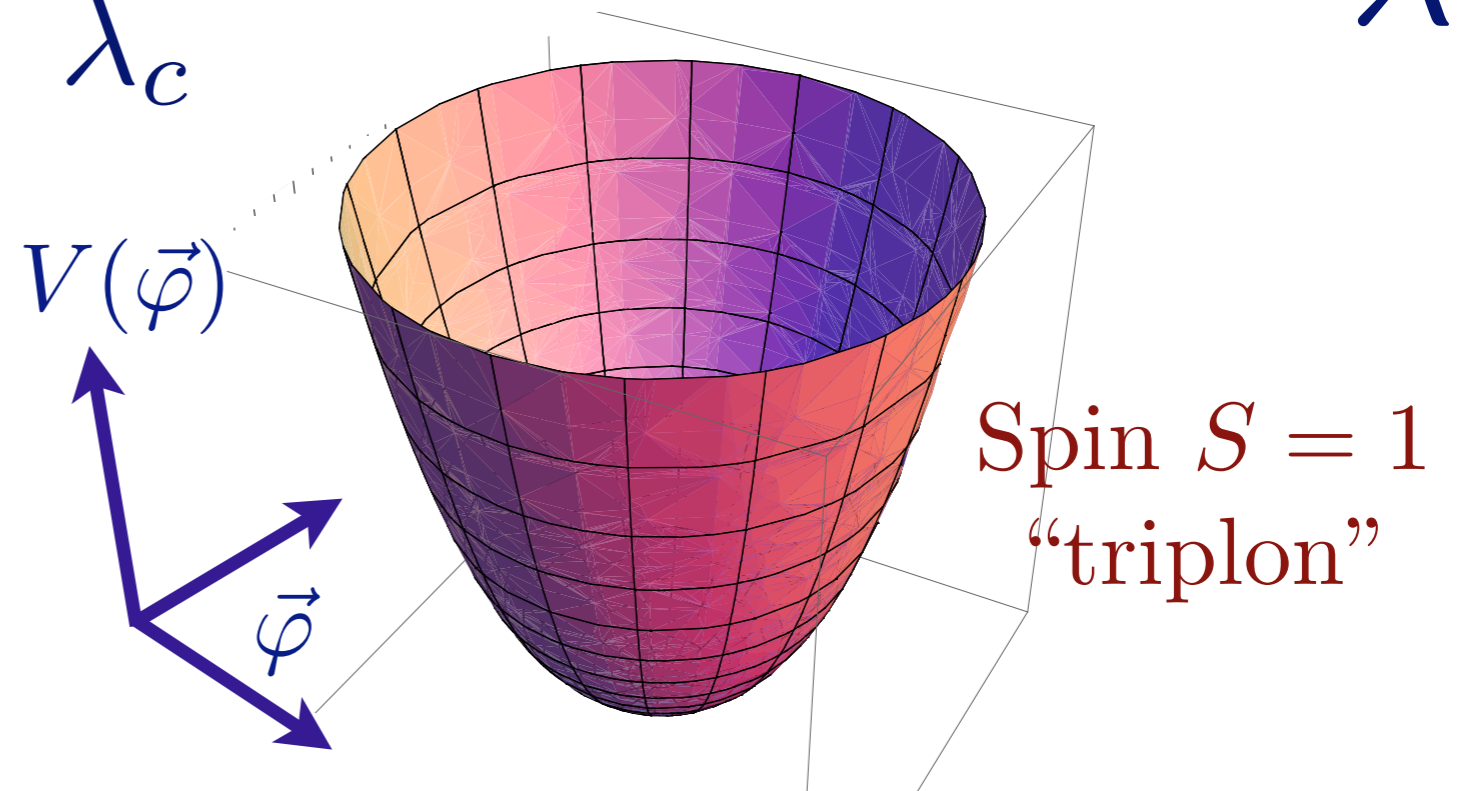


Excitation spectrum in the paramagnetic phase

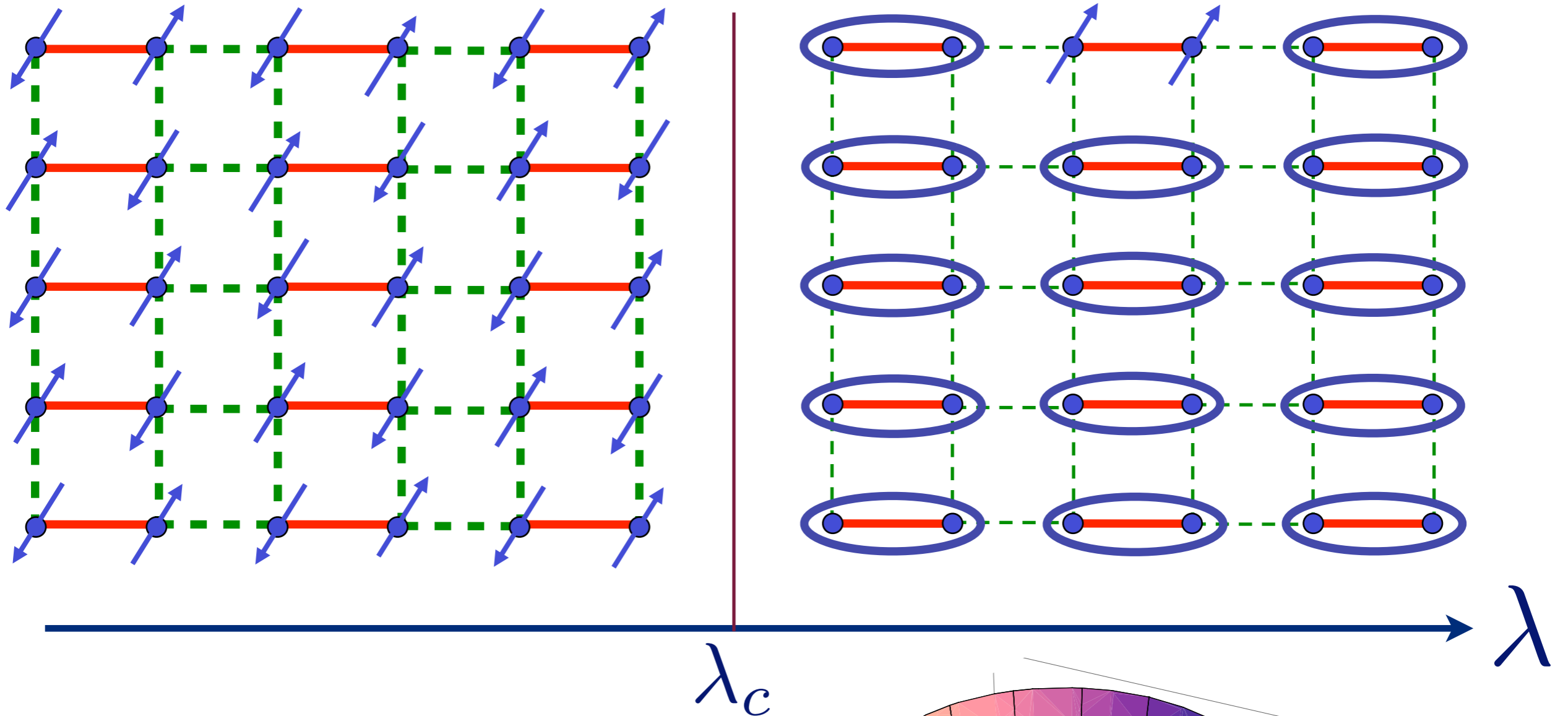


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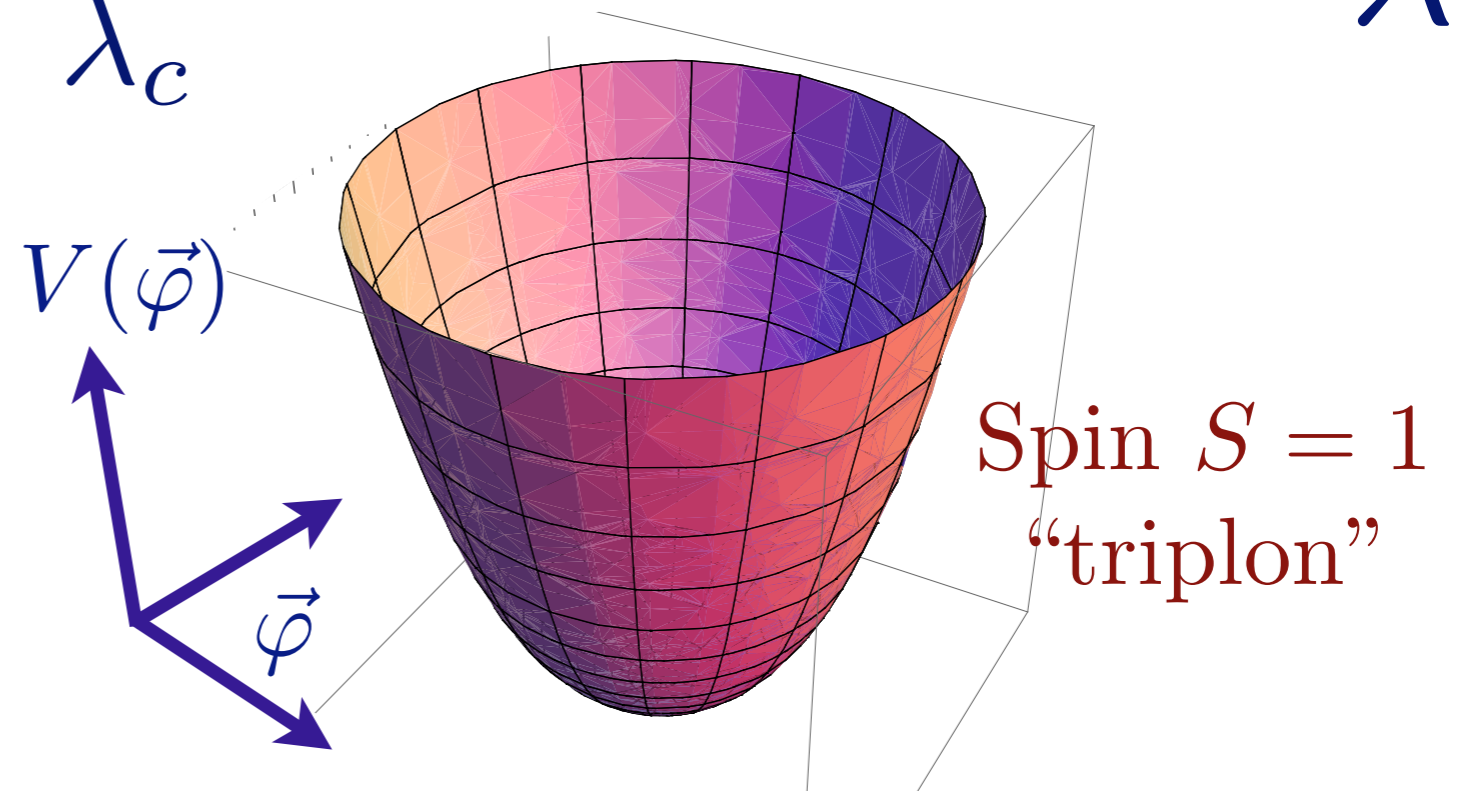


Excitation spectrum in the paramagnetic phase

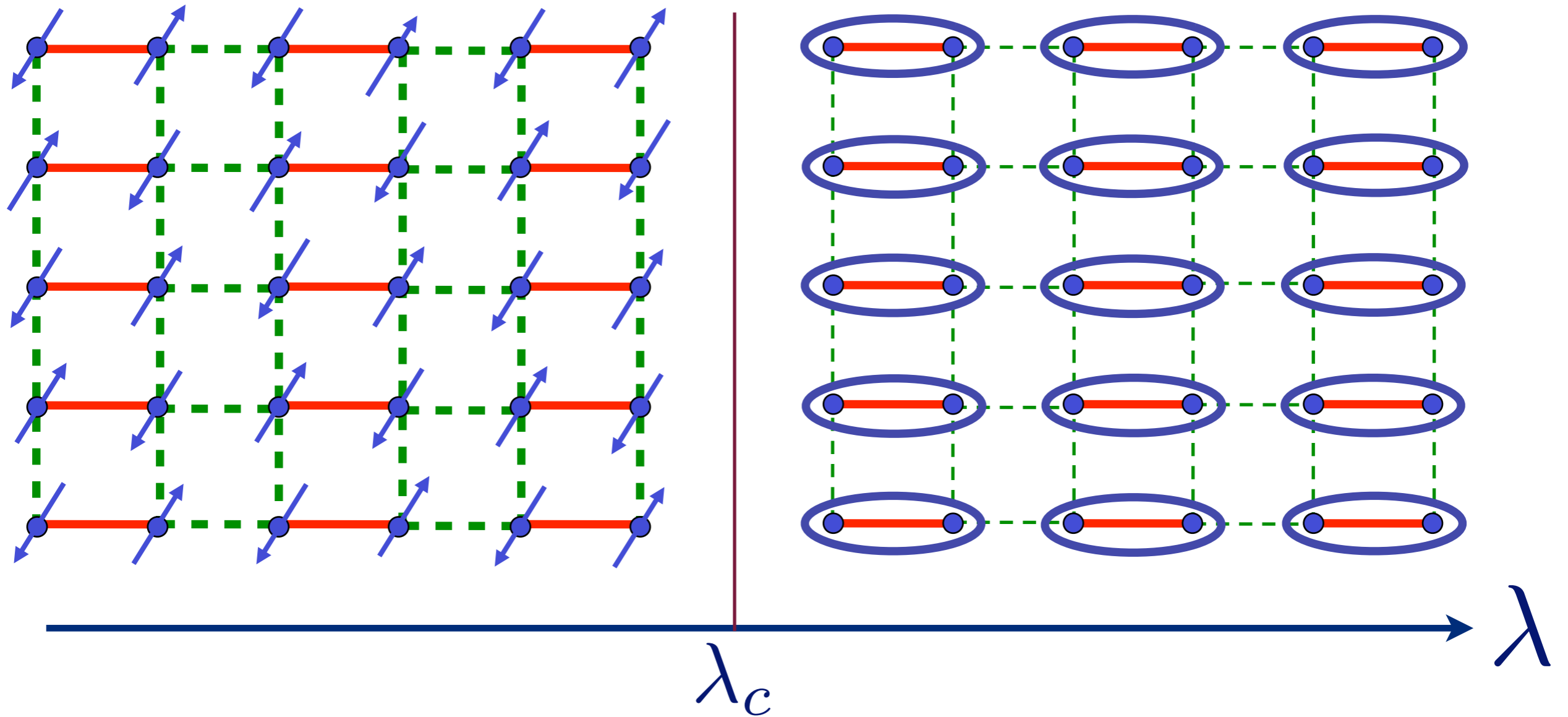


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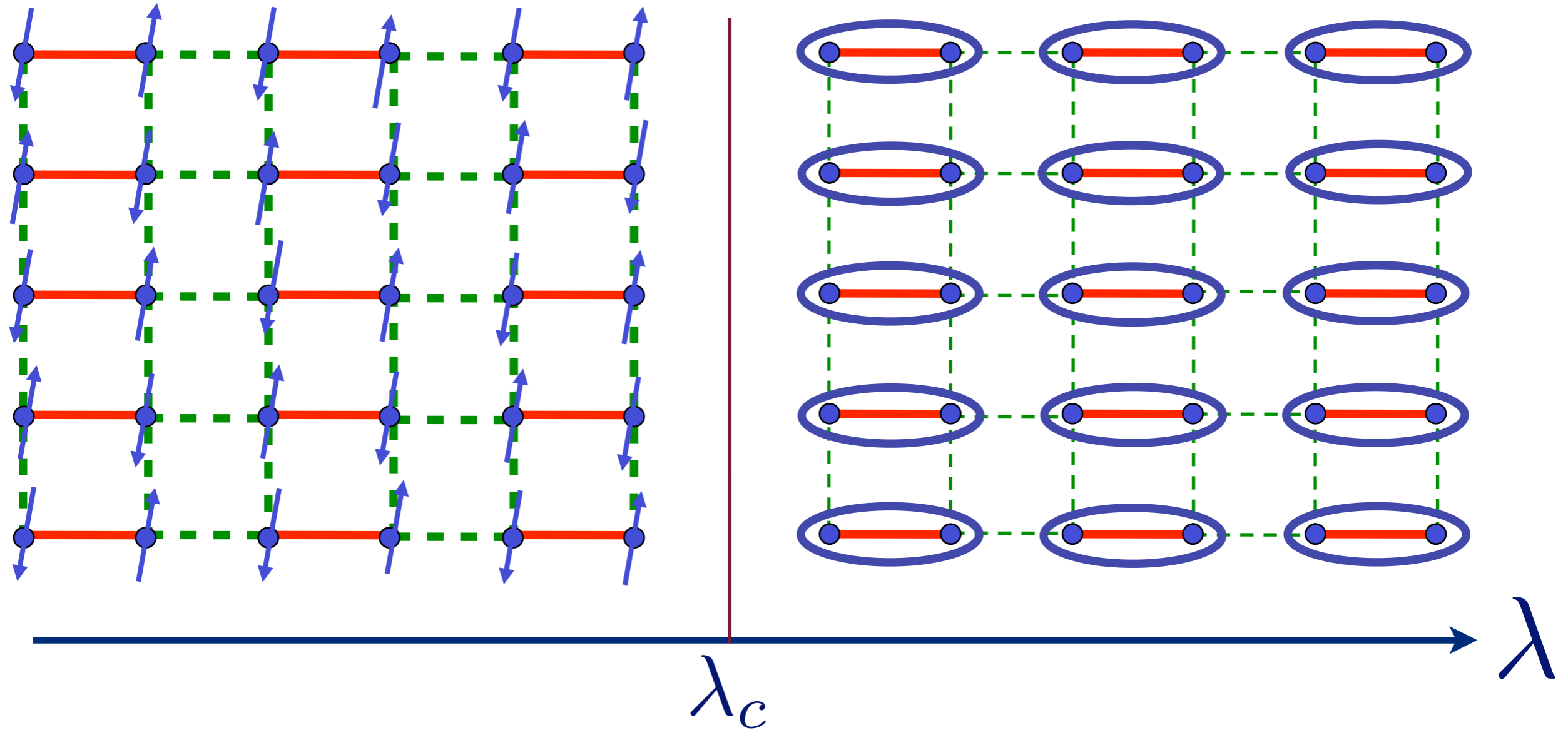
$$\lambda > \lambda_c$$



Excitation spectrum in the Néel phase

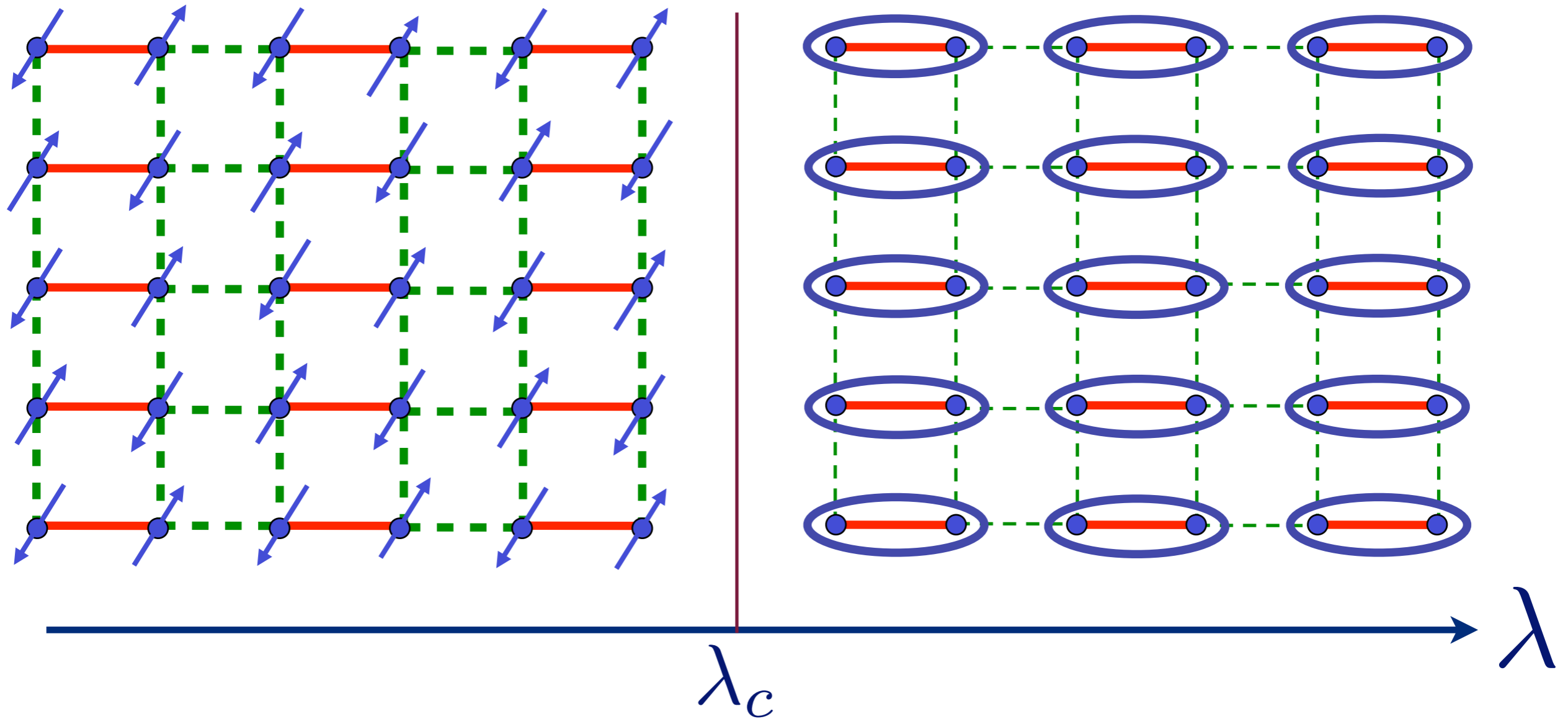


Excitation spectrum in the Néel phase



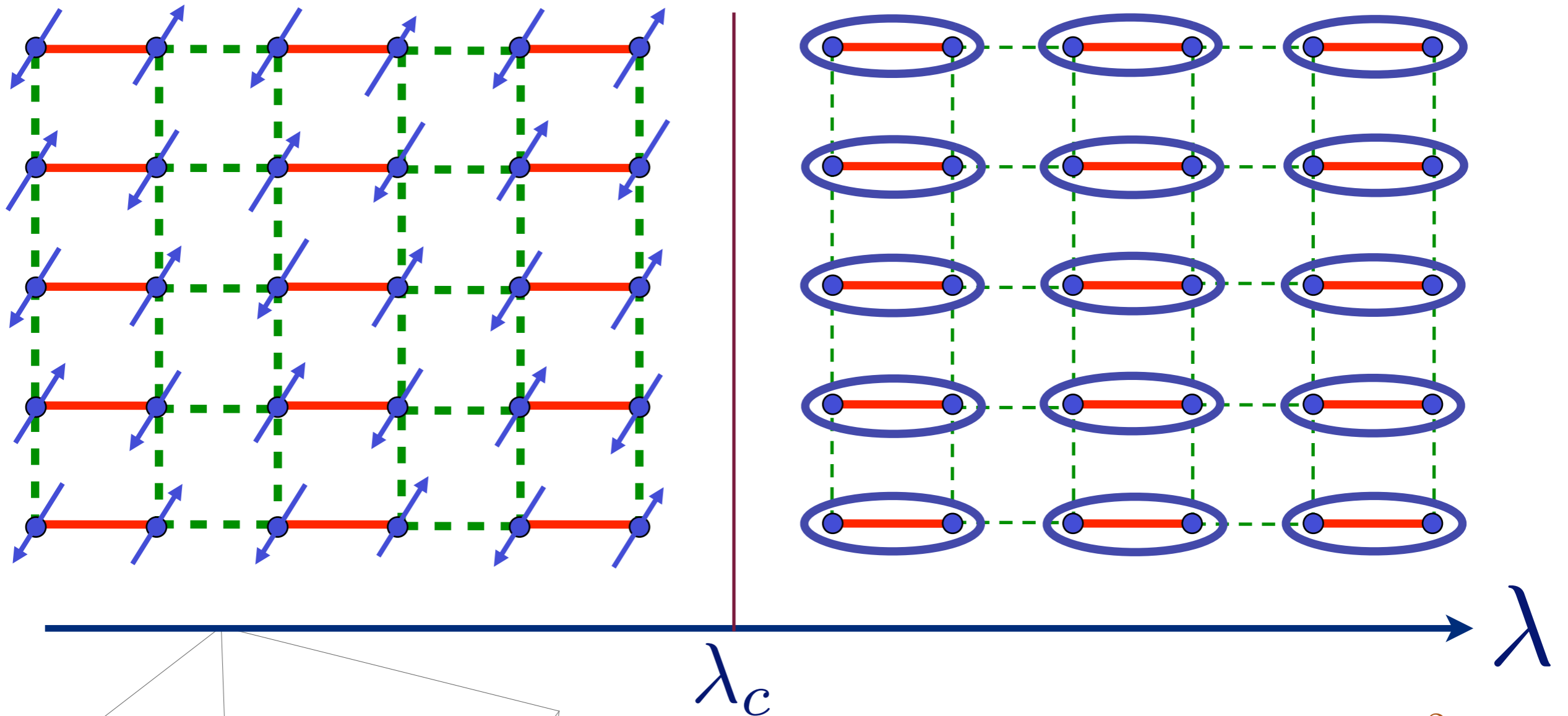
Spin waves

Excitation spectrum in the Néel phase



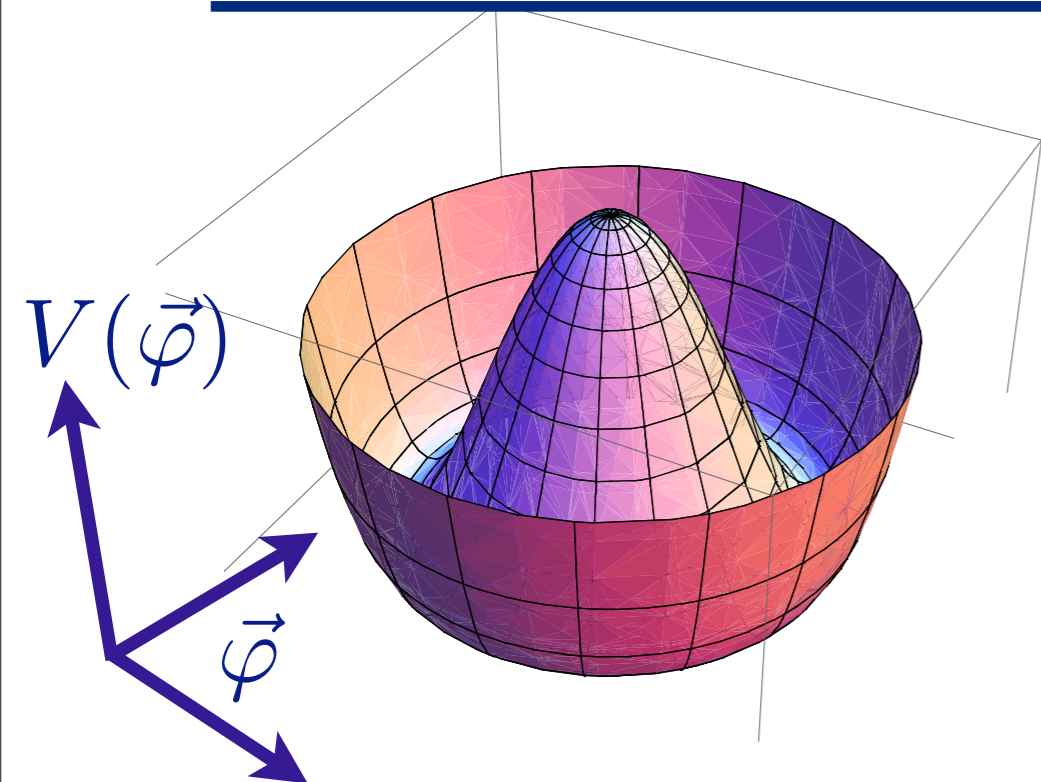
Spin waves

Excitation spectrum in the Néel phase

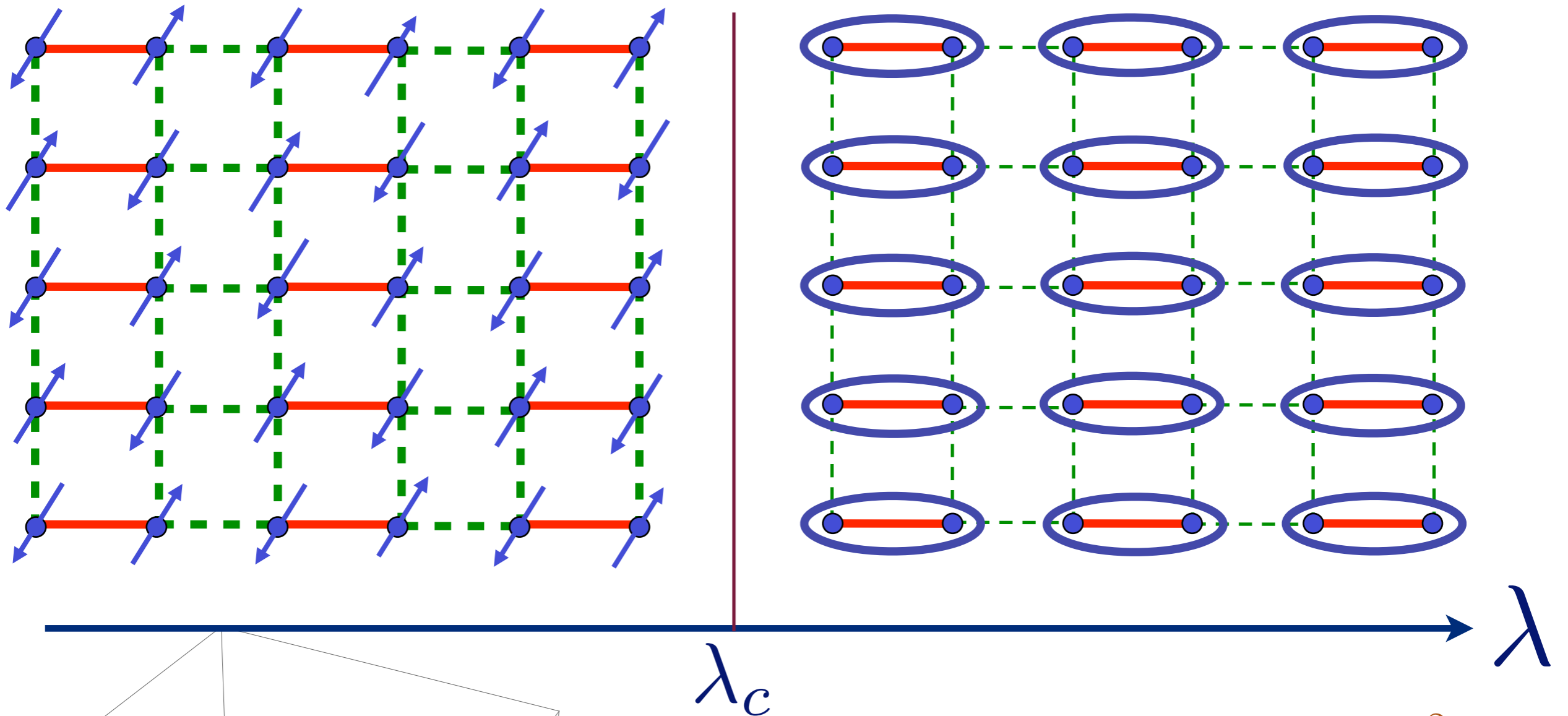


$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u(\vec{\varphi}^2)^2$$

$$\lambda < \lambda_c$$

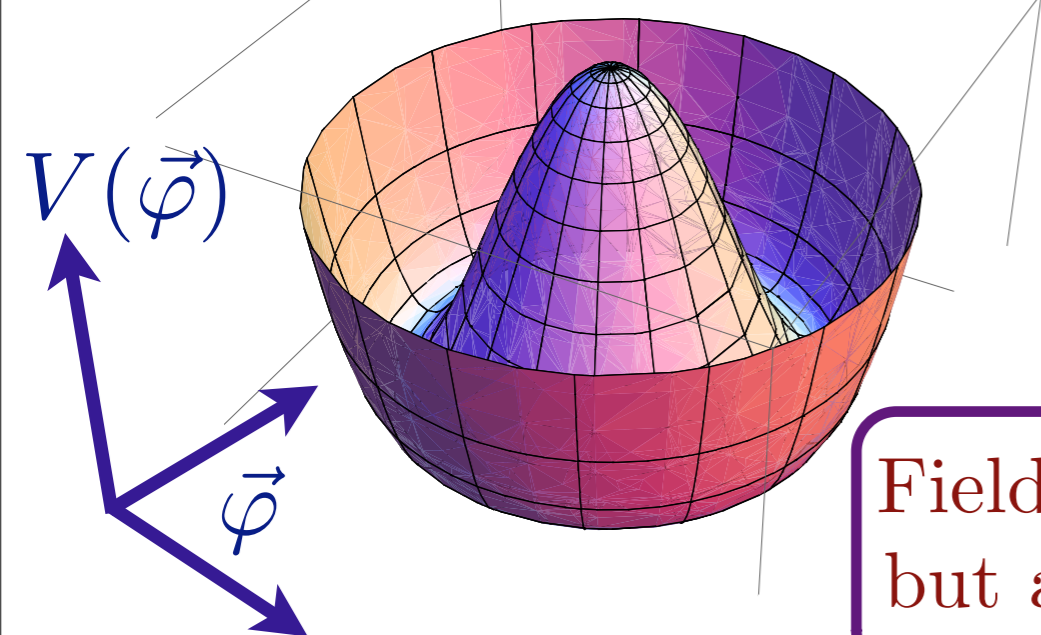


Excitation spectrum in the Néel phase



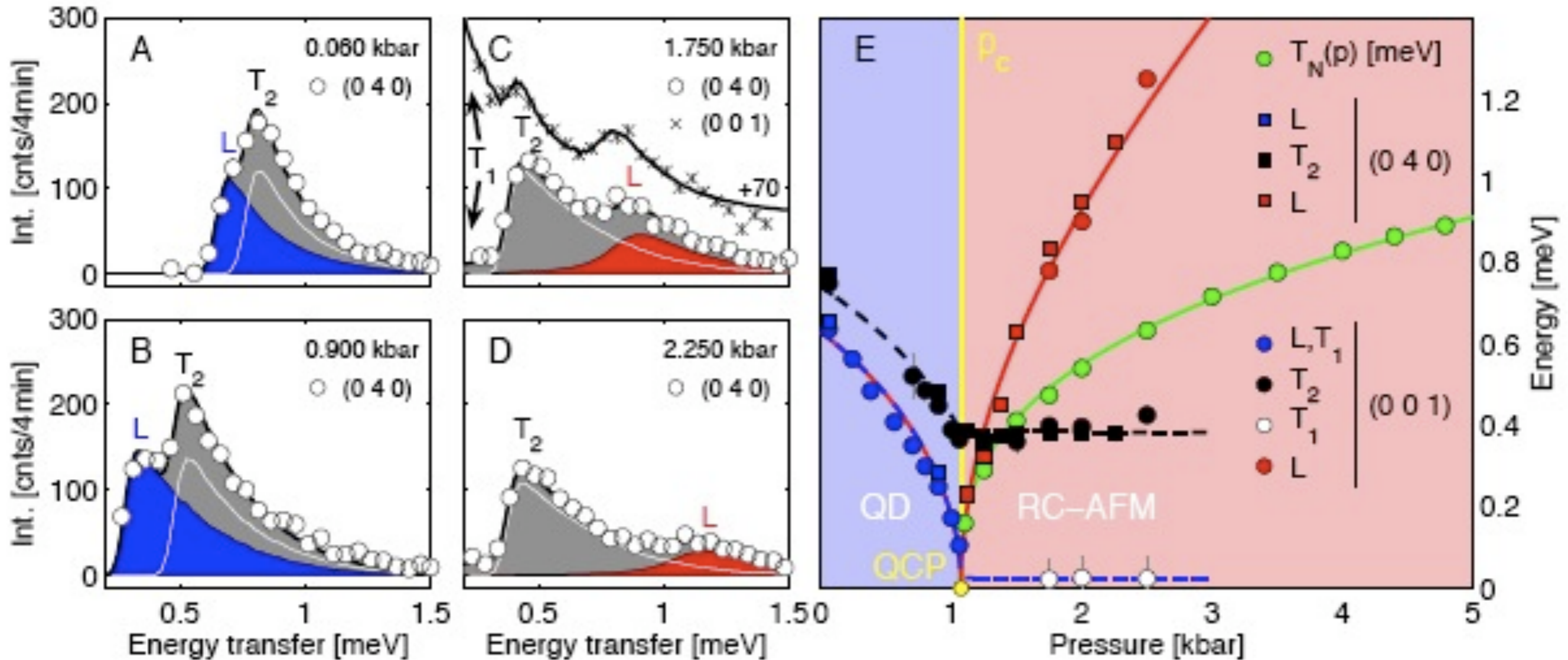
$$V(\vec{\varphi}) = (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2$$

$$\lambda < \lambda_c$$



Field theory yields spin waves (“Goldstone” modes) but also an additional longitudinal “Higgs” particle

TiCuCl₃ with varying pressure



Observation of $3 \rightarrow 2$ low energy modes,
 emergence of new Higgs particle in the Néel phase,
 and vanishing of Néel temperature at the quantum critical point

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer,
 Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya,
 Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

Prediction of quantum field theory

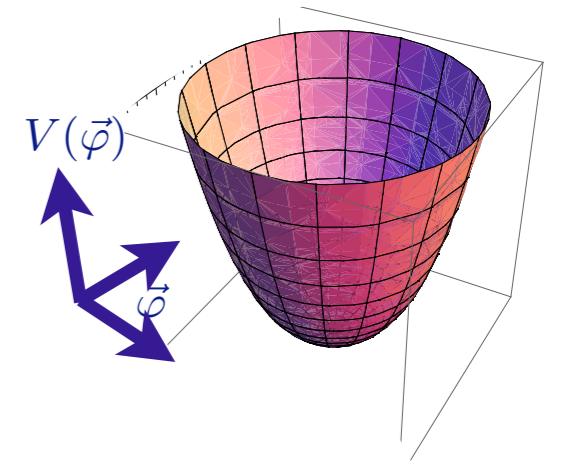
Potential for $\vec{\varphi}$ fluctuations: $V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u(\vec{\varphi}^2)^2$

Paramagnetic phase, $\lambda > \lambda_c$

Expand about $\vec{\varphi} = 0$:

$$V(\vec{\varphi}) \approx (\lambda - \lambda_c)\vec{\varphi}^2$$

Yields 3 particles with energy gap $\sim \sqrt{(\lambda - \lambda_c)}$



Prediction of quantum field theory

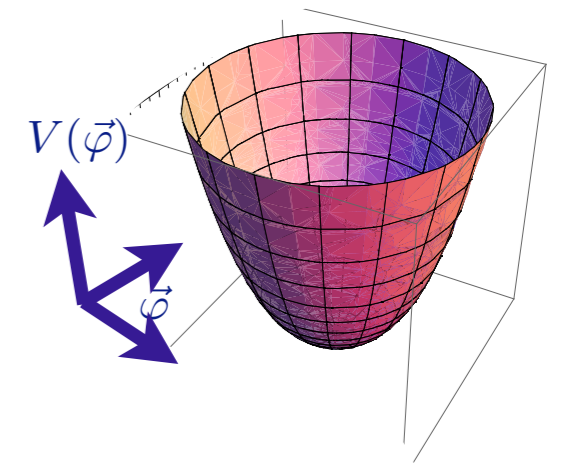
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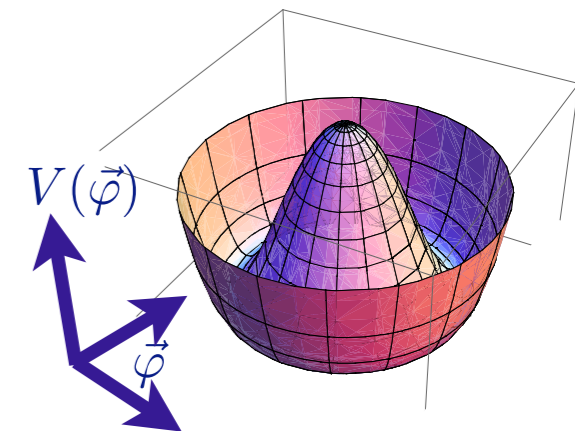


Néel phase, $\lambda < \lambda_c$

Expand $\vec{\varphi} = (0, 0, \sqrt{(\lambda_c - \lambda)/(2u)}) + \vec{\varphi}_1$:

$$V(\vec{\varphi}) \approx 2(\lambda_c - \lambda)\varphi_{1z}^2$$

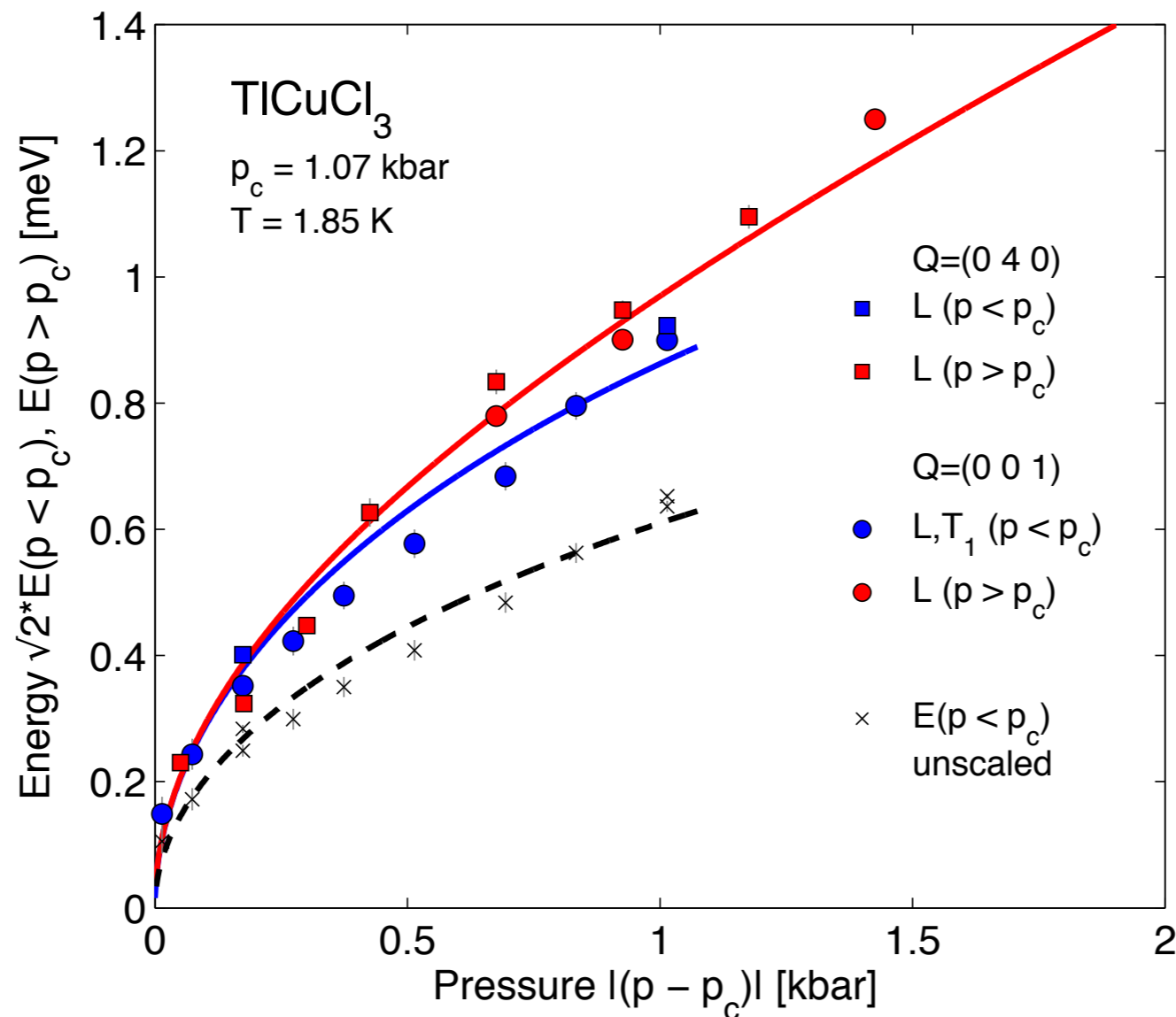
Yields 2 gapless spin waves and one Higgs particle with energy gap $\sim \sqrt{2(\lambda_c - \lambda)}$



Prediction of quantum field theory

$$\frac{\text{Energy of Higgs particle}}{\text{Energy of triplon}} = \sqrt{2}$$

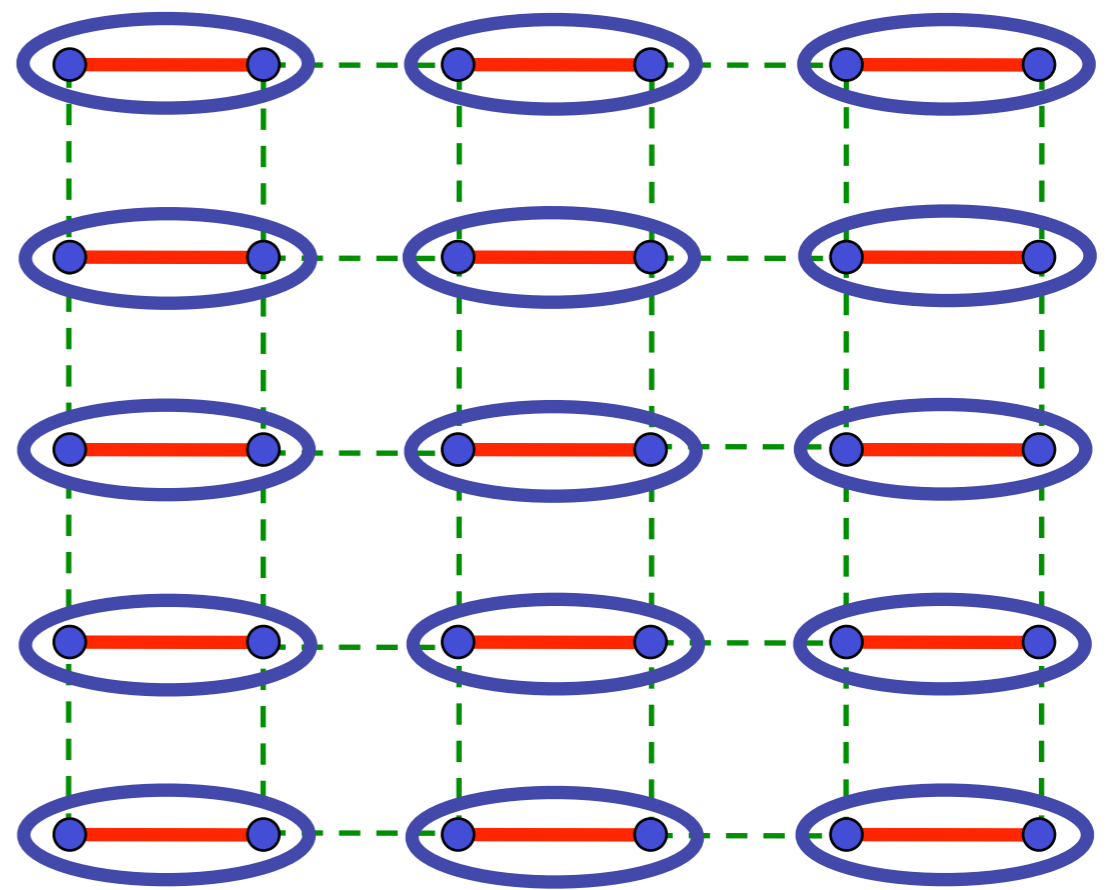
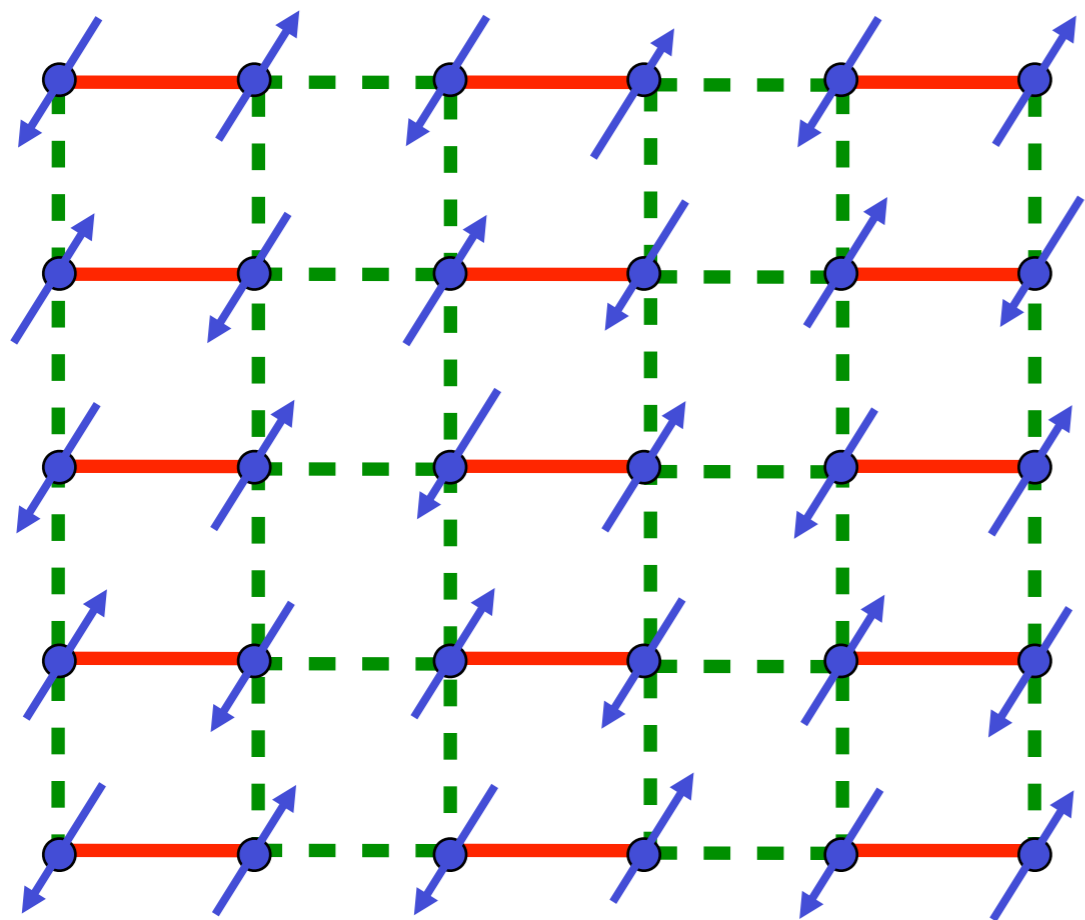
$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u(\vec{\varphi}^2)^2$$



S. Sachdev, arXiv:0901.4103



$$= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



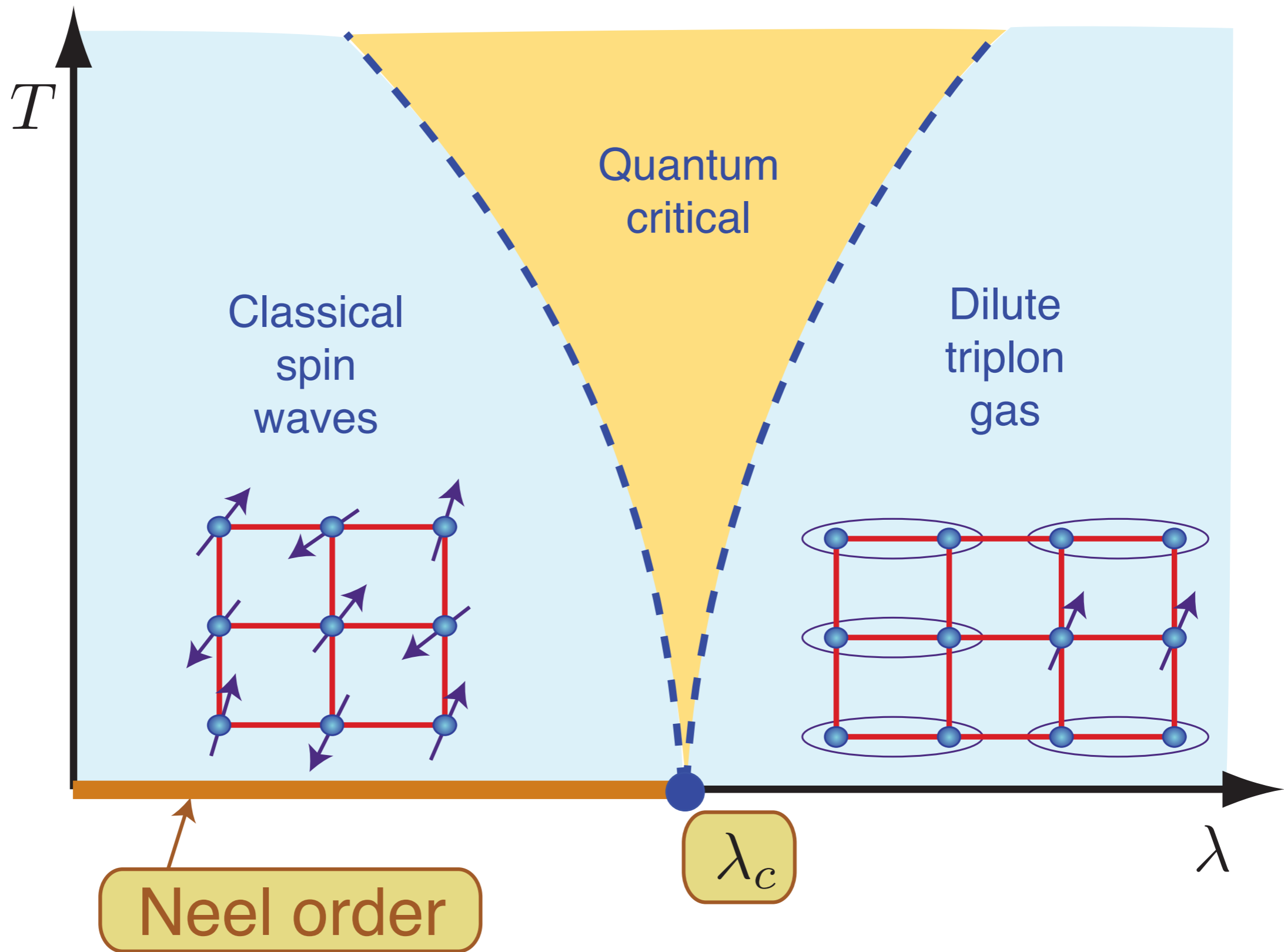
λ_c

λ

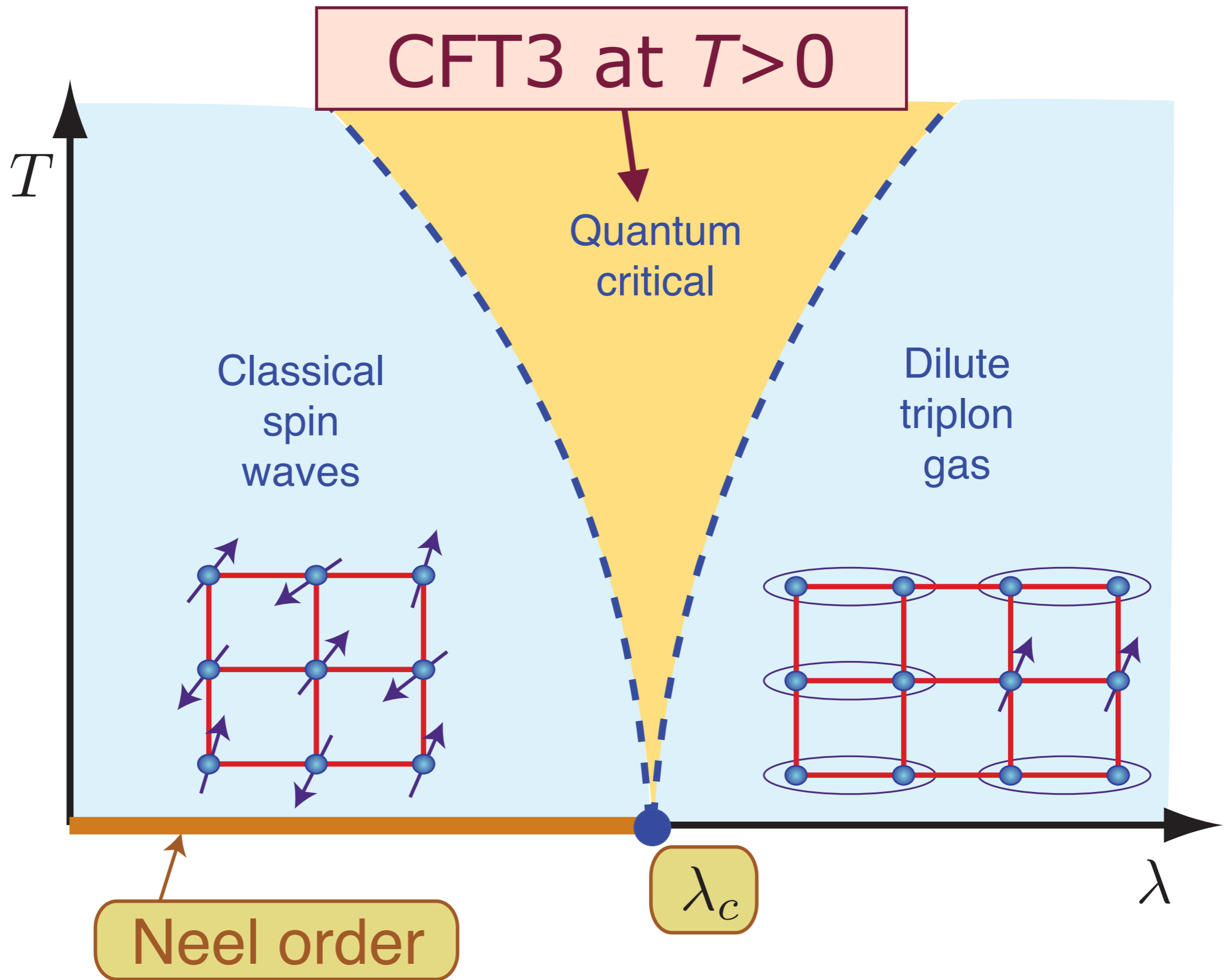
CFT3

$O(3)$ order parameter $\vec{\varphi}$

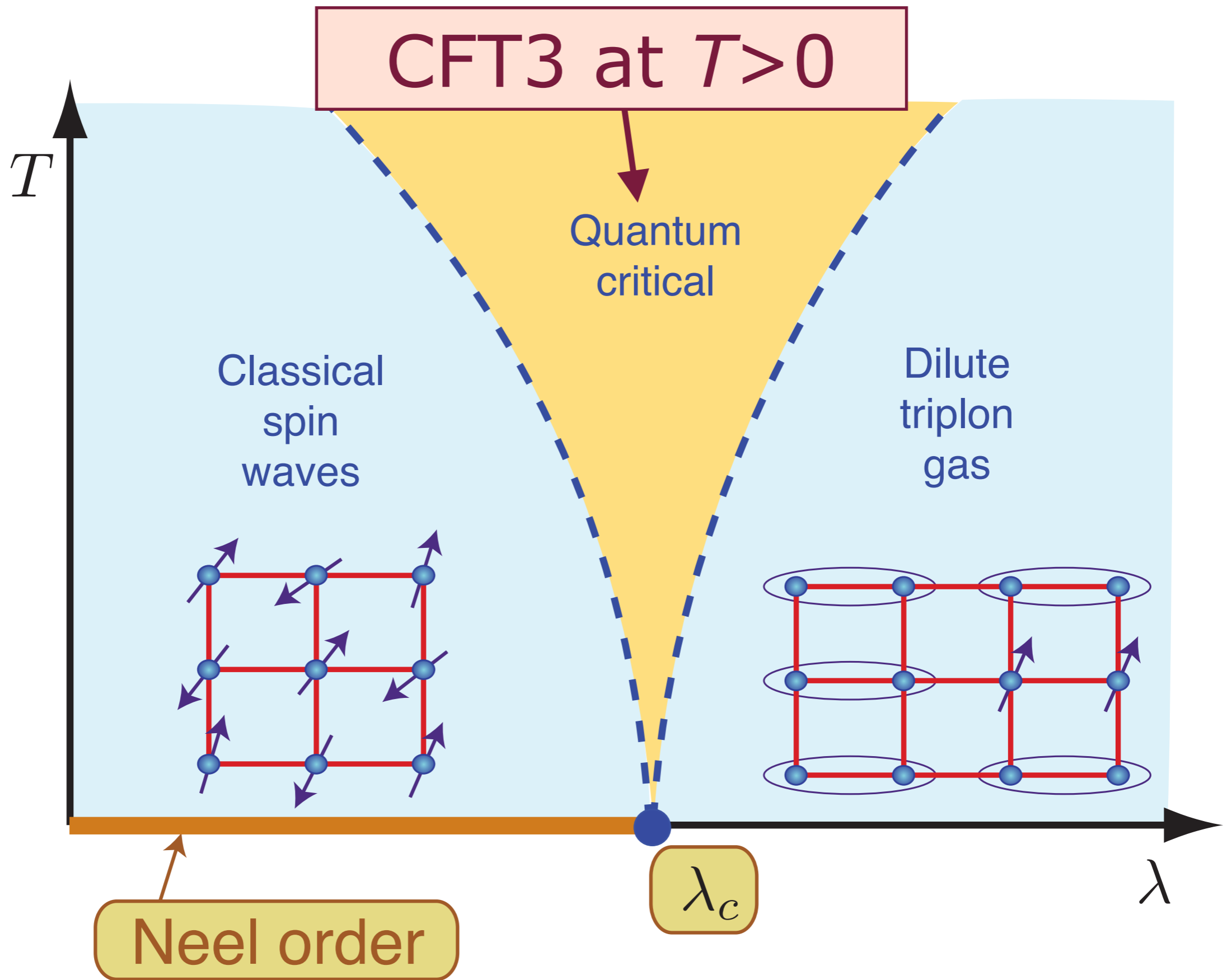
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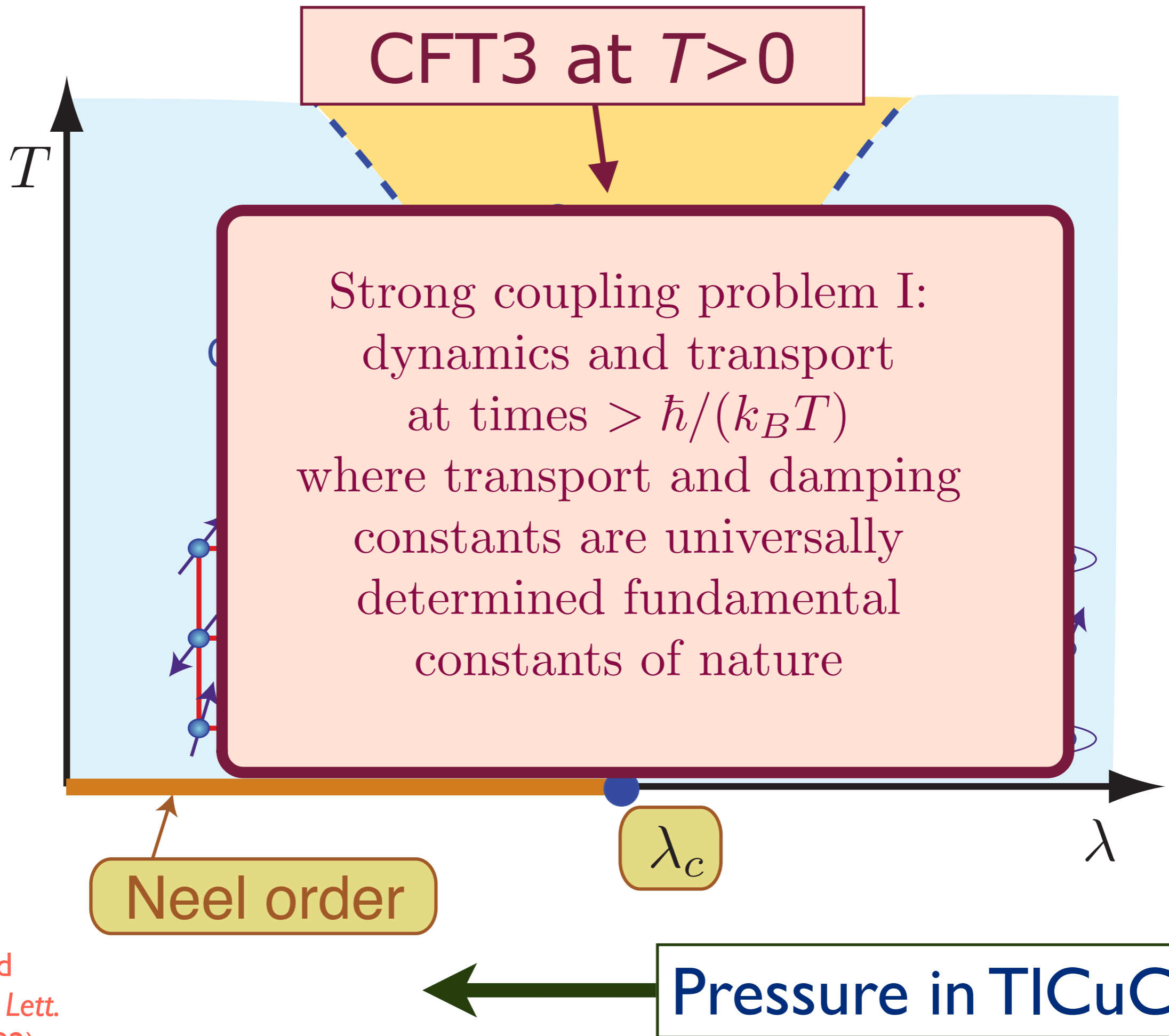
S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).



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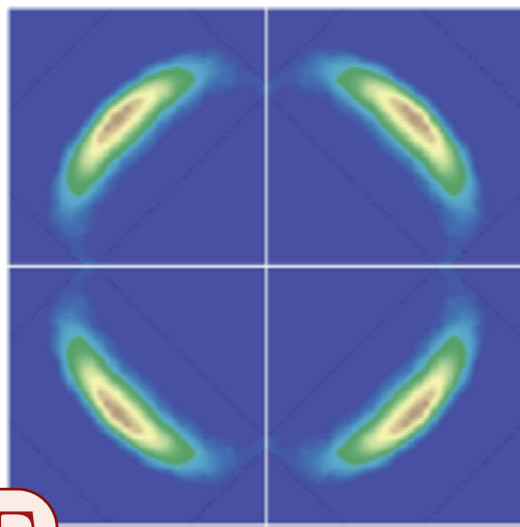
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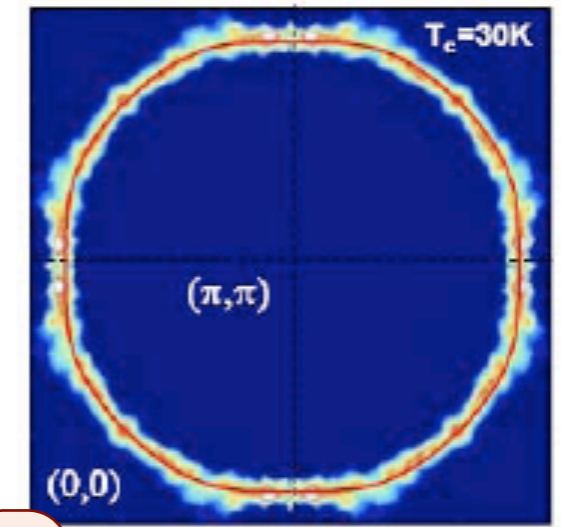
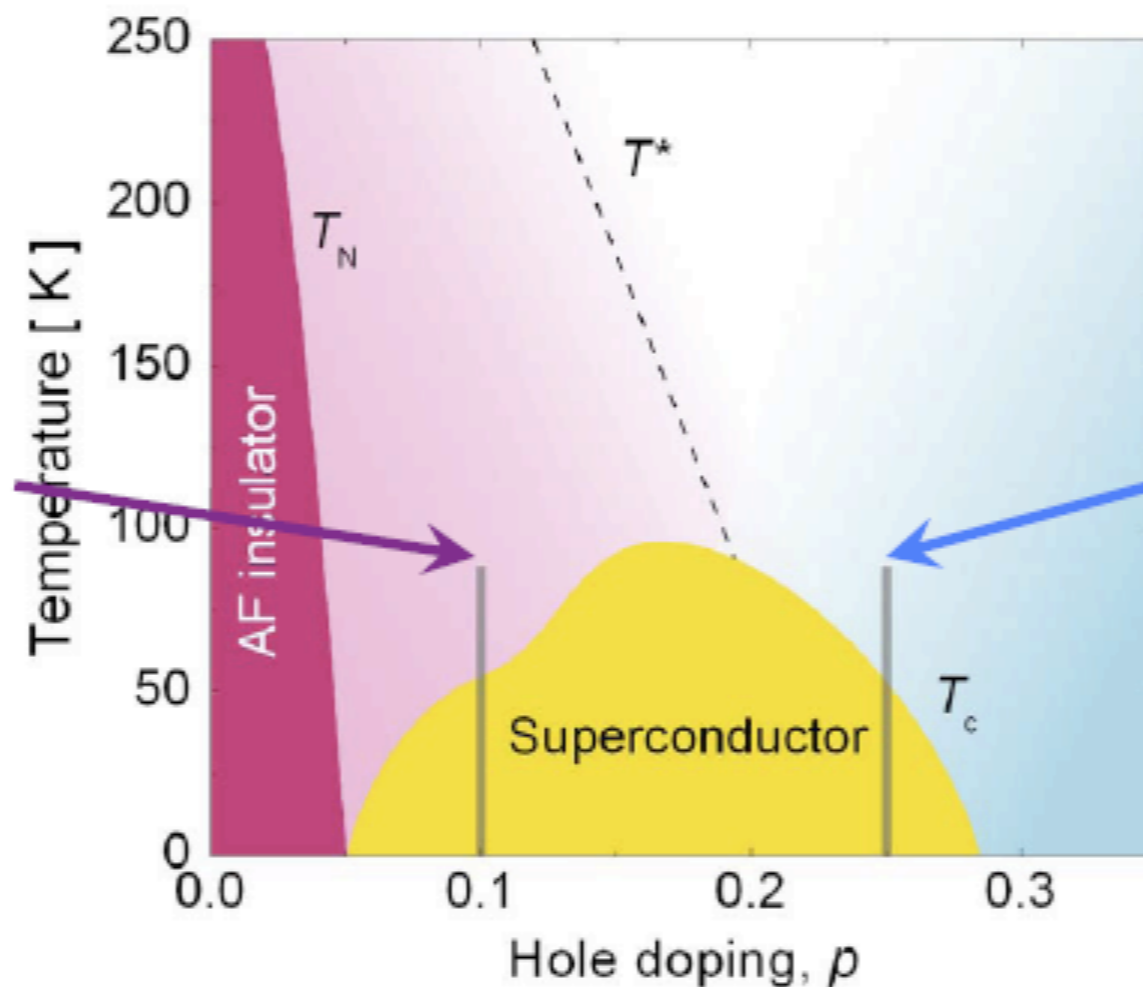
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Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



Γ

K.M. Shen et al., Science 2005



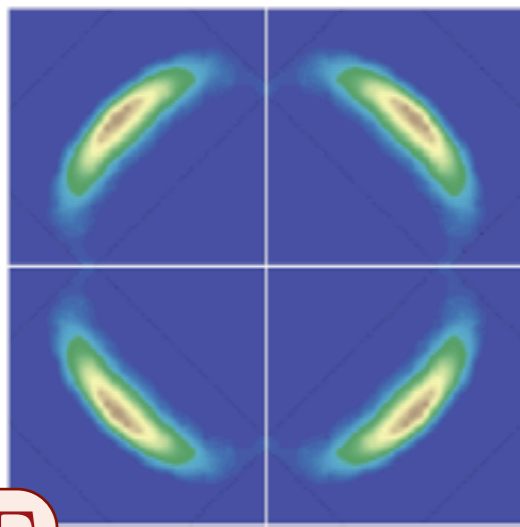
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M. Platé et al., PRL 2005

Smaller hole
Fermi-pockets

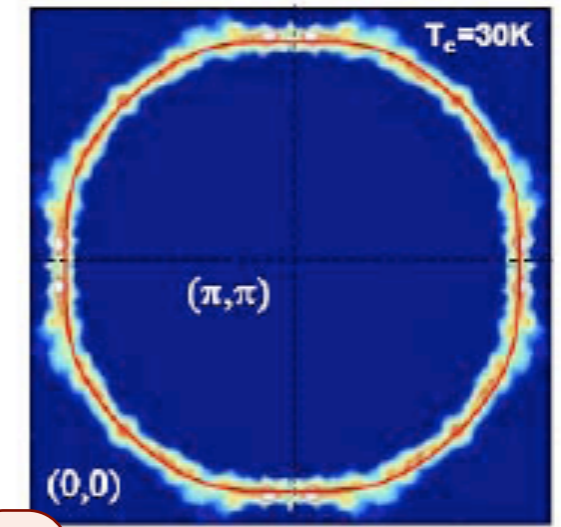
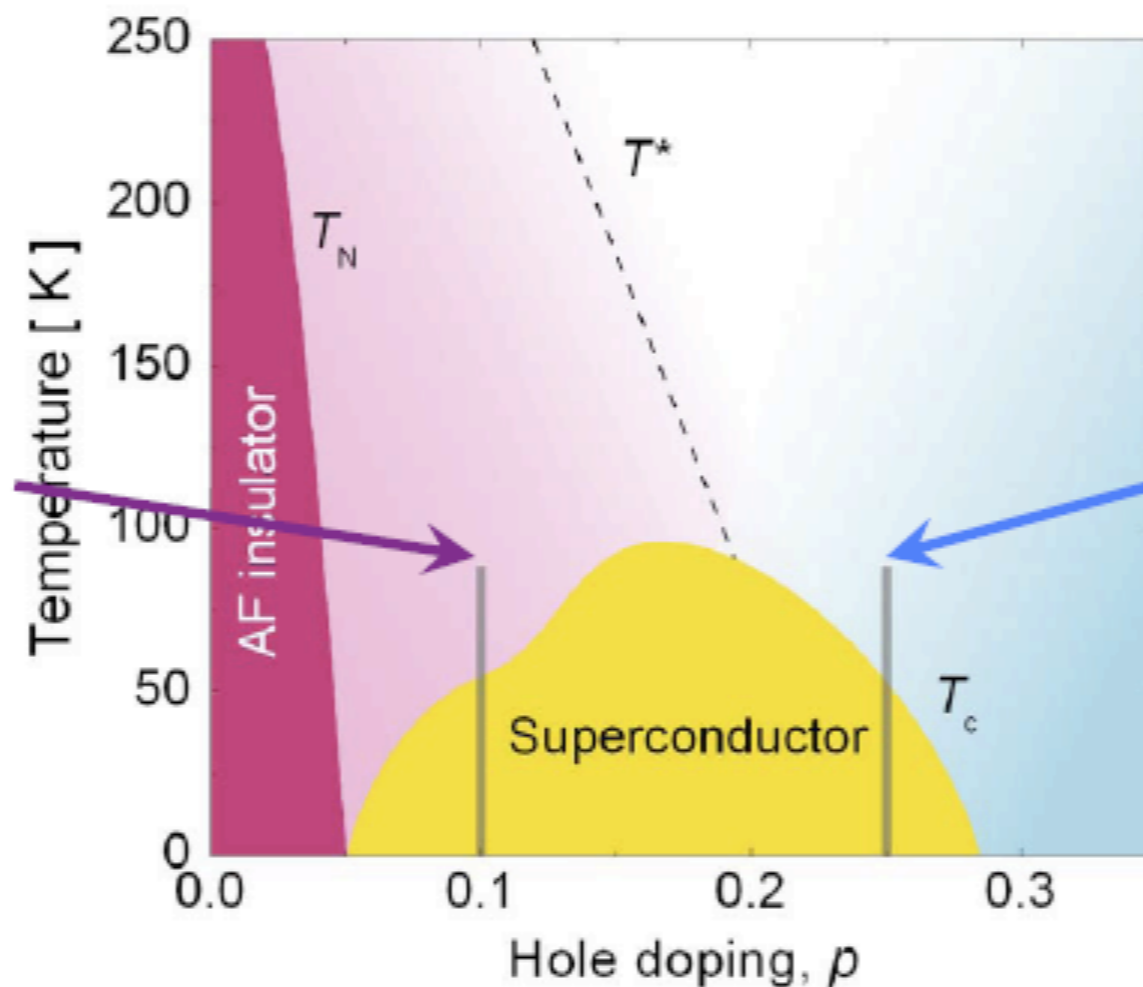
Large hole
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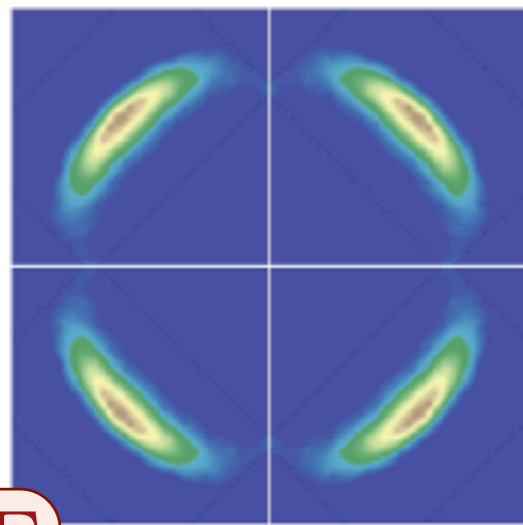
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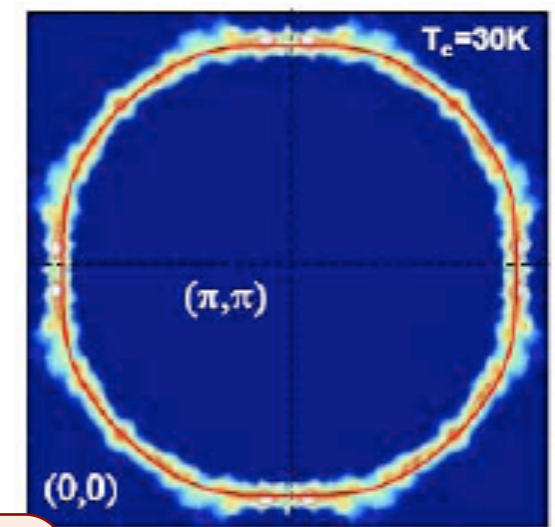
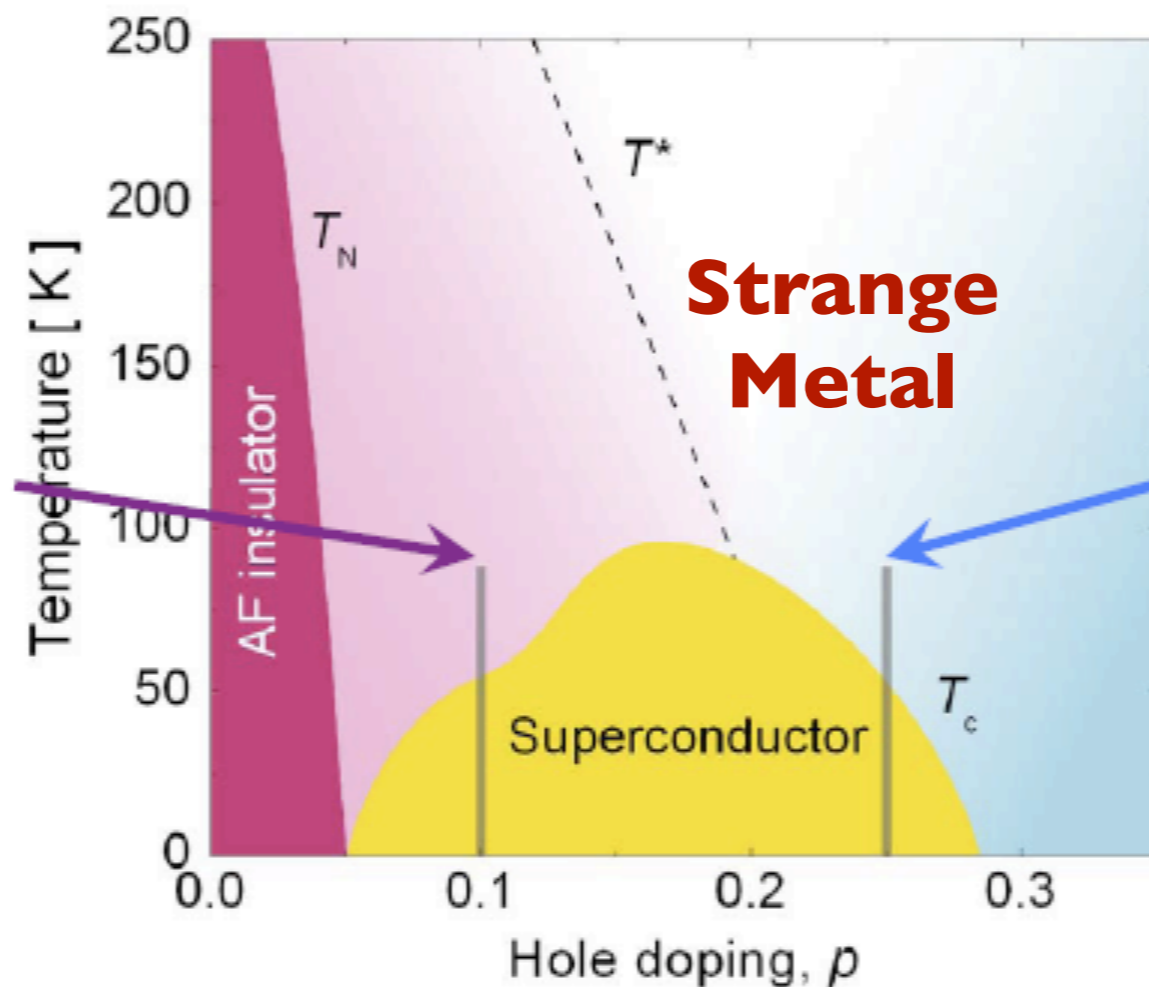
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K.M. Shen et al., Science 2005

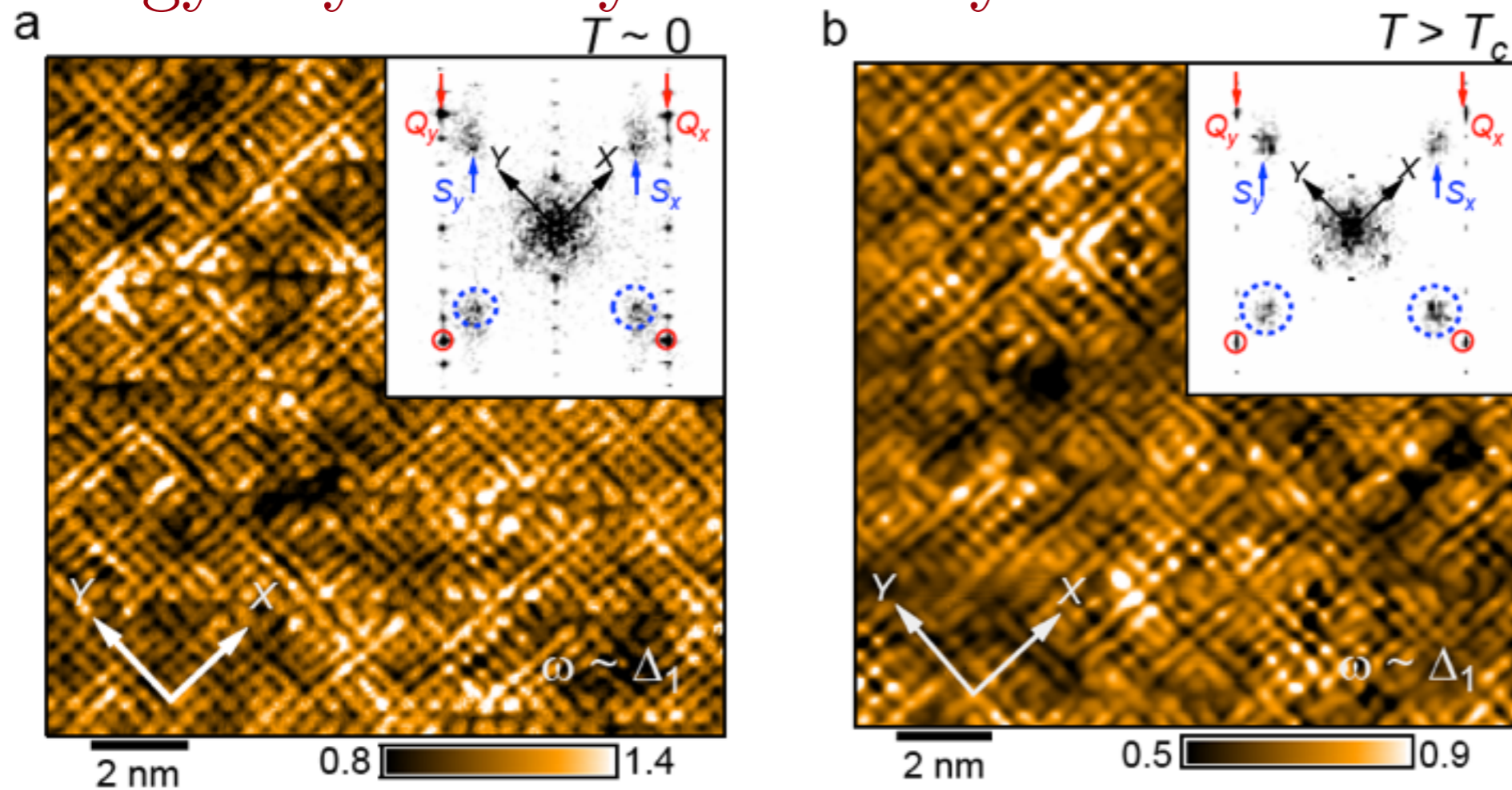
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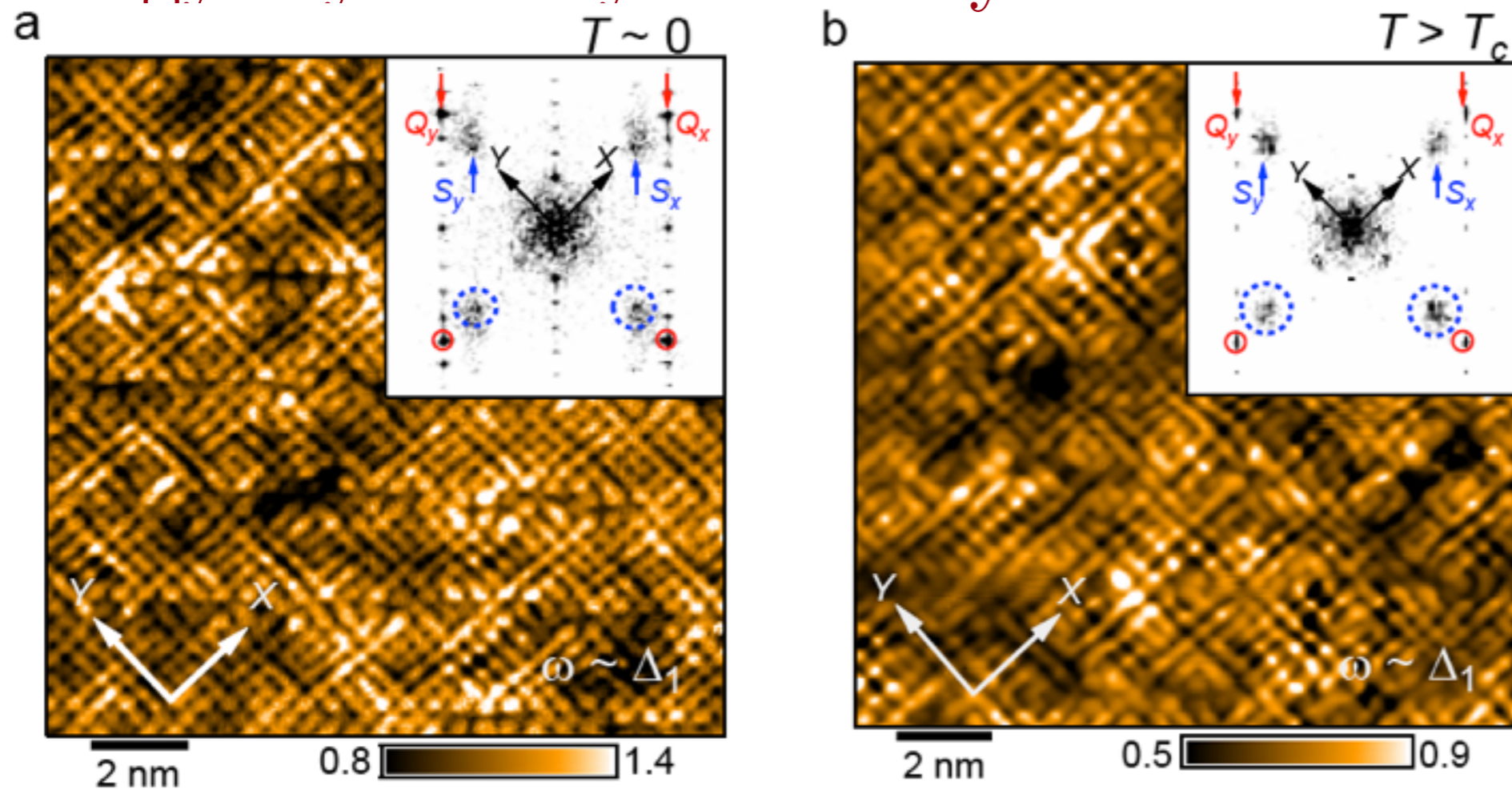
Large hole
Fermi surface

STM measurements of $Z(r)$, energy asymmetry in density of states

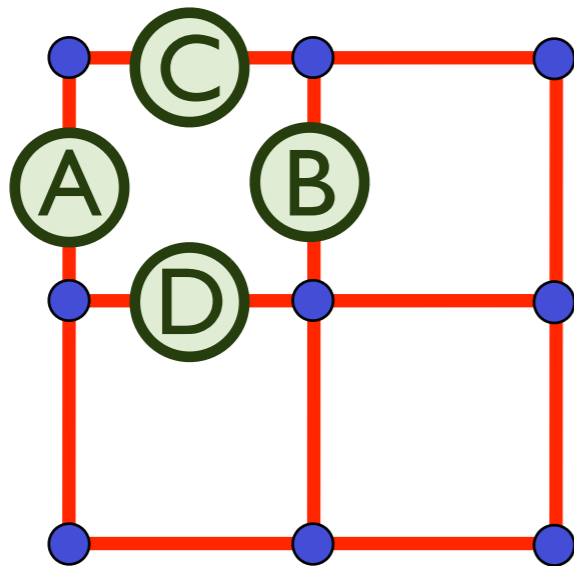


M. J. Lawler, K. Fujita,
Jinhwan Lee,
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S. Uchida, J. C. Davis,
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Eun-Ah Kim, preprint

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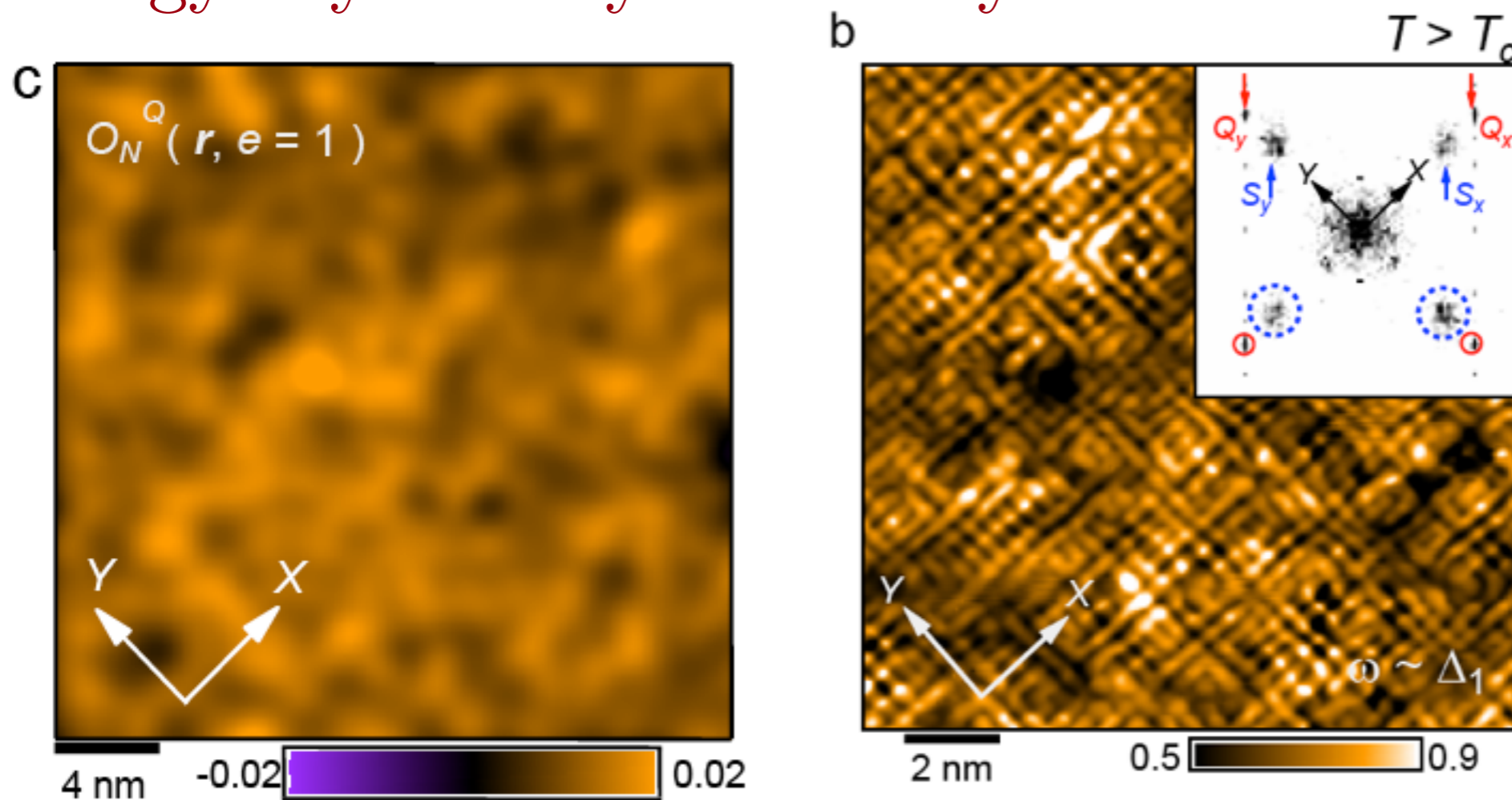


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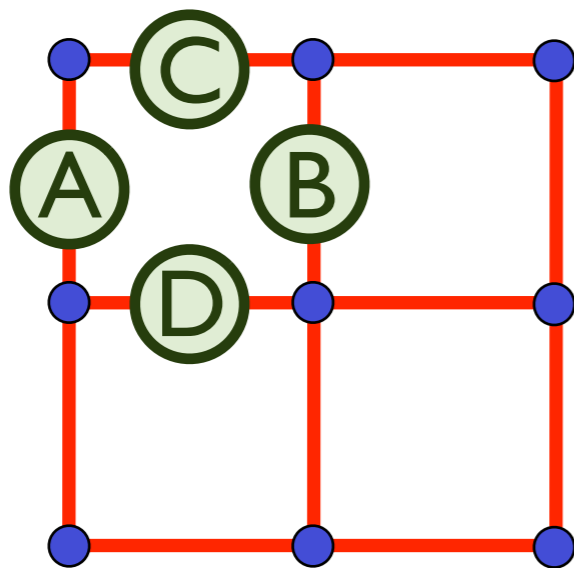


$$O_N = Z_A + Z_B - Z_C - Z_D$$

STM measurements of $Z(r)$, energy asymmetry in density of states



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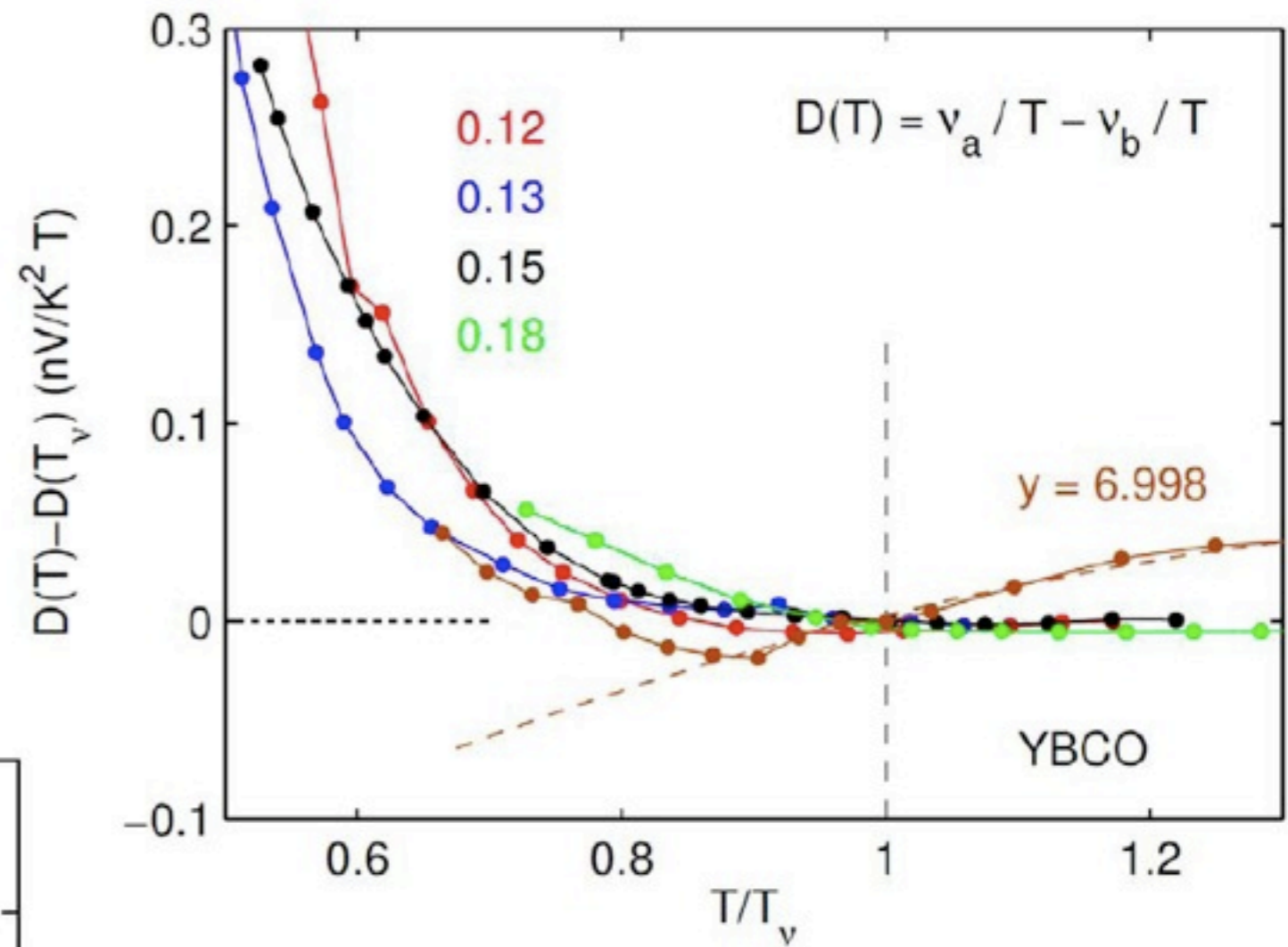
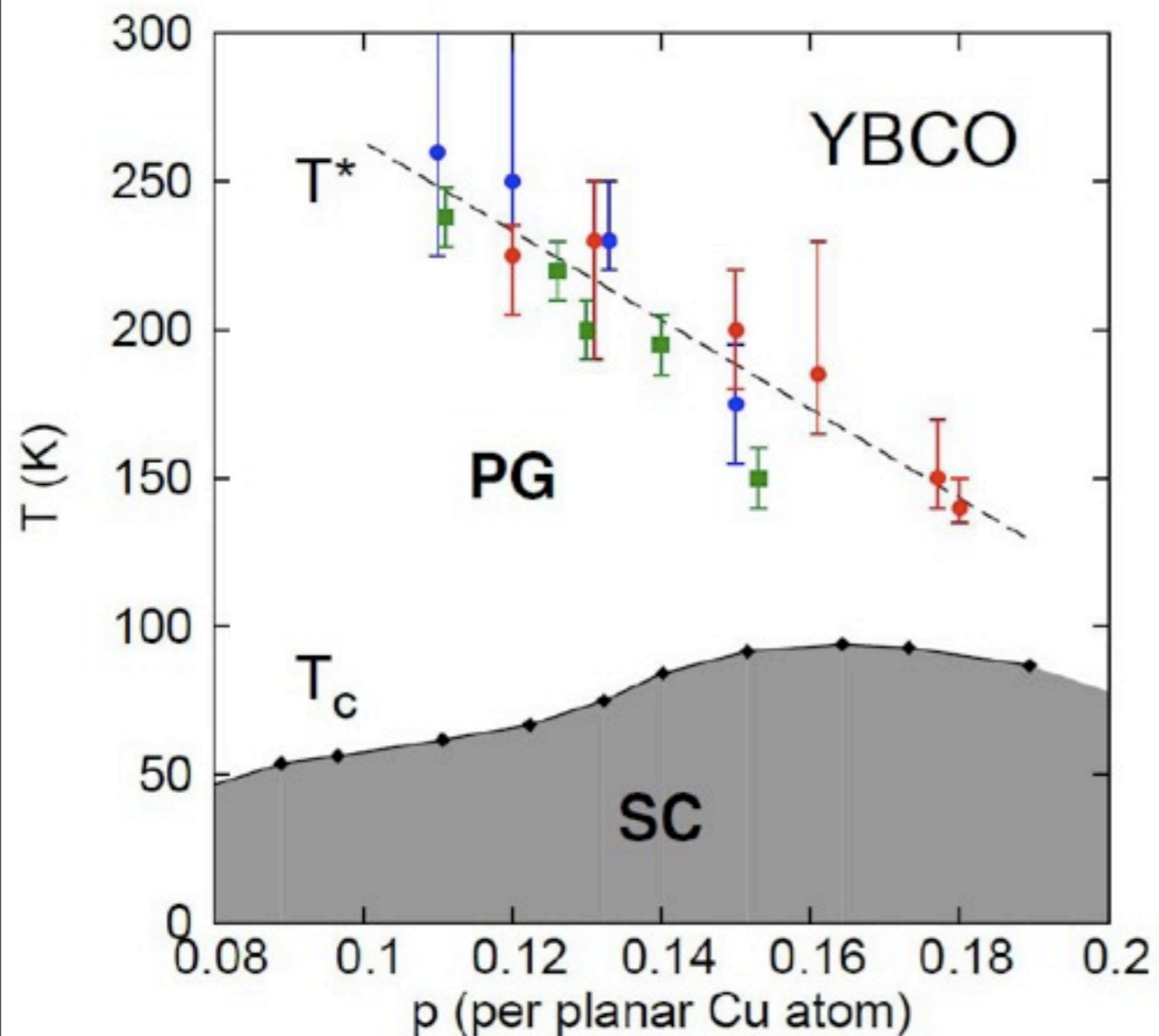


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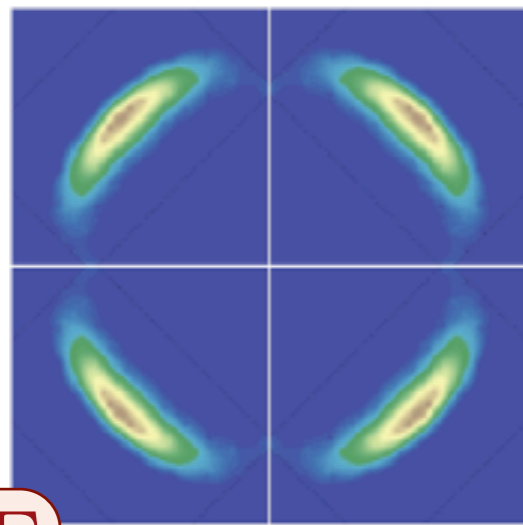
Strong anisotropy of
electronic states between
 x and y directions:
Electronic
“Ising-nematic” order

Broken rotational symmetry in the pseudogap phase of a high- T_c superconductor

R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D.A. Bonn, W. N. Hardy, and Louis Taillefer
Nature, **463**, 519 (2010).

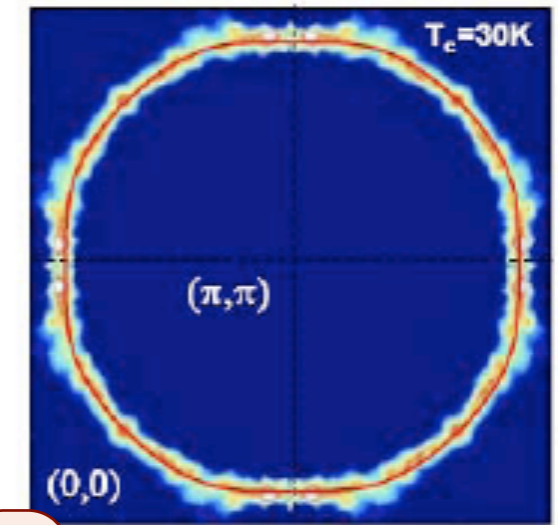
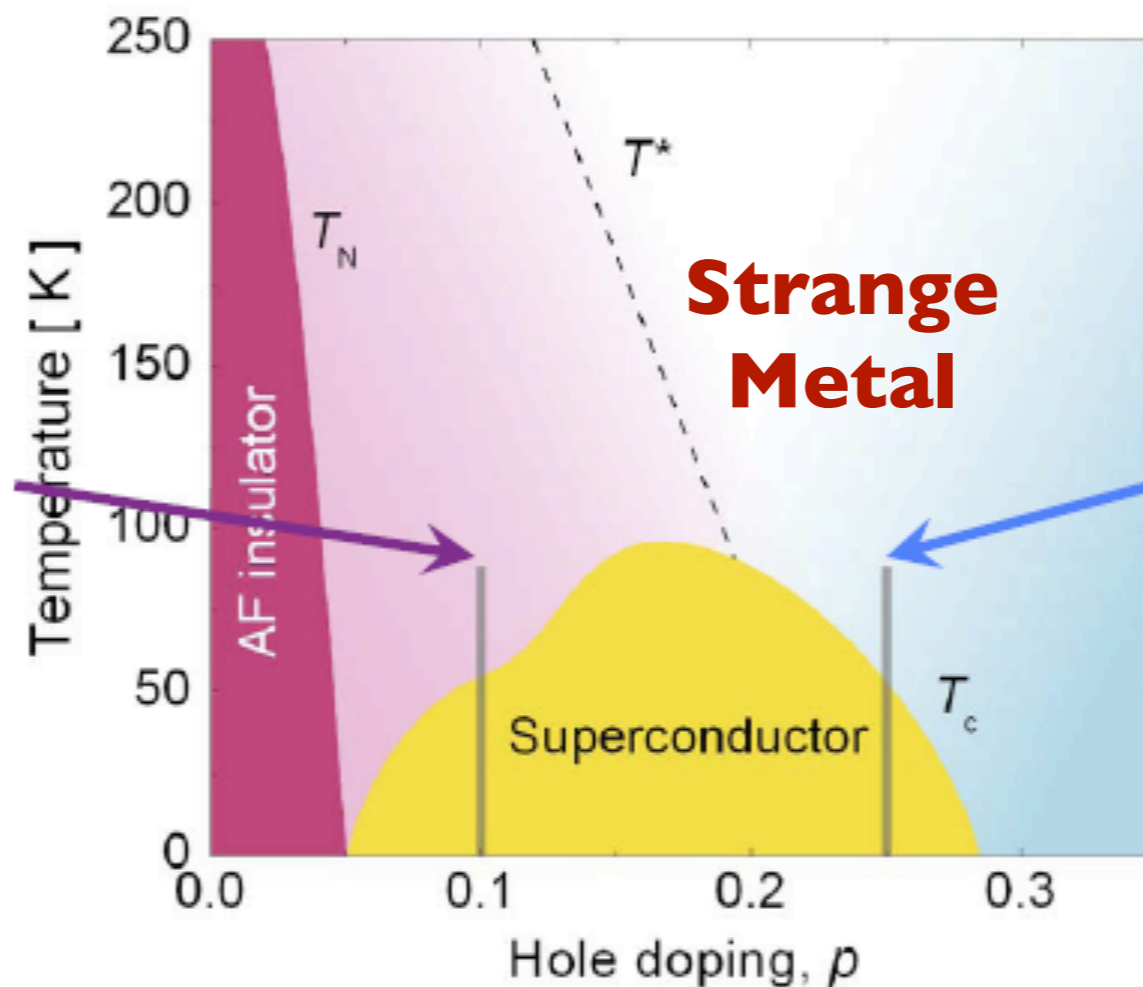


Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



Γ

K.M. Shen et al., Science 2005



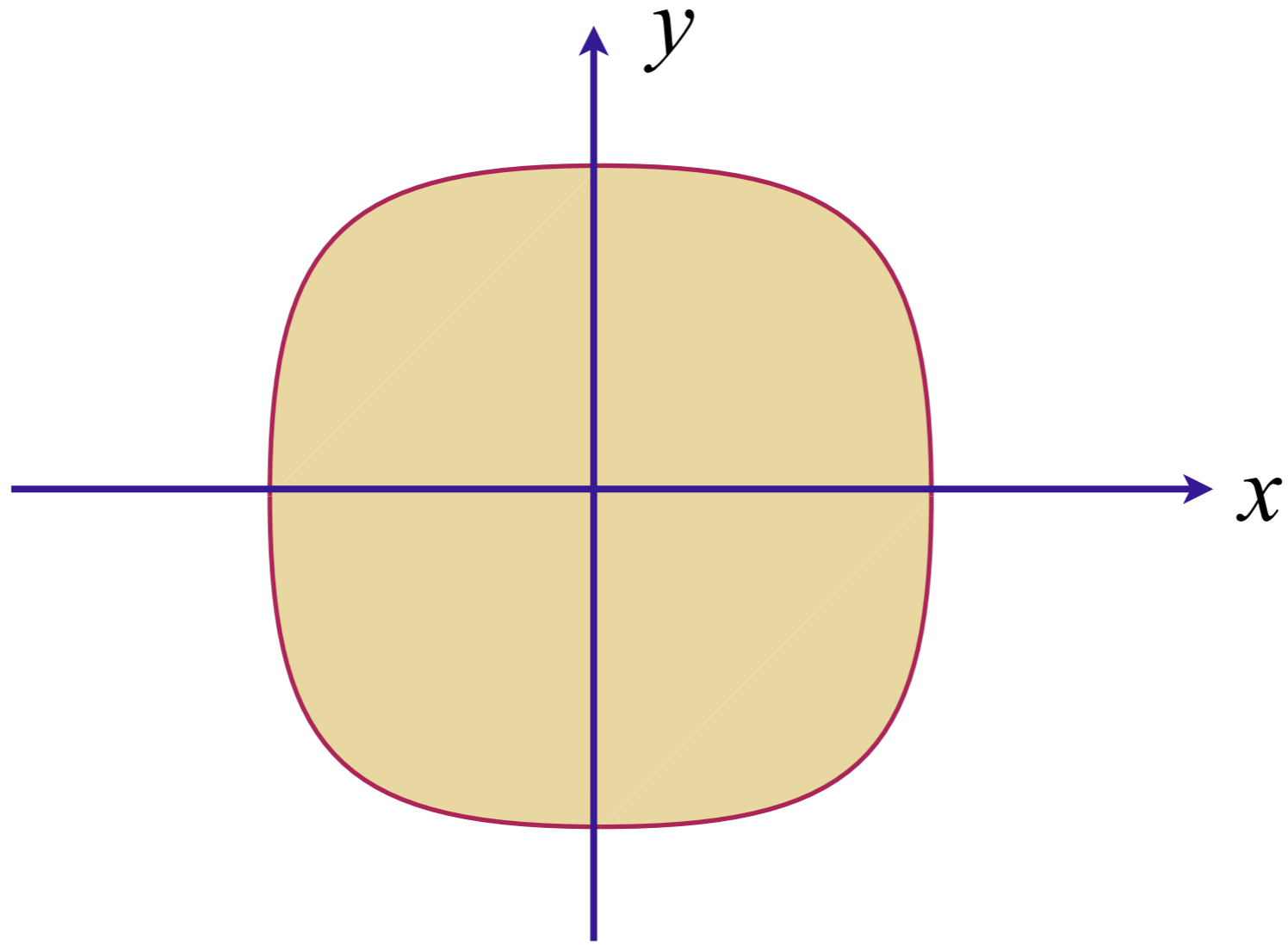
Γ

M. Platé et al., PRL 2005

Smaller hole
Fermi-pockets

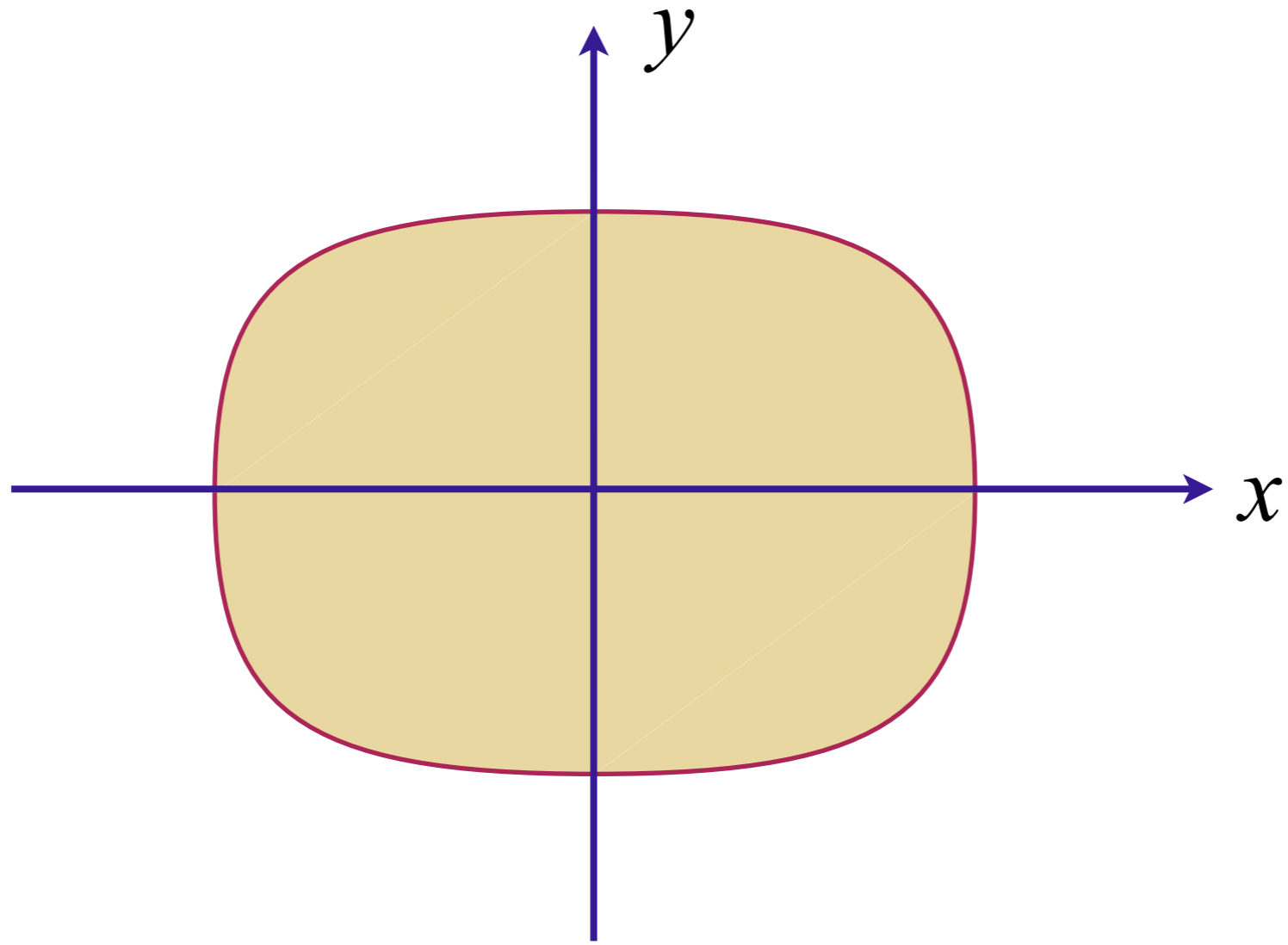
Large hole
Fermi surface

Quantum criticality of Ising-nematic ordering



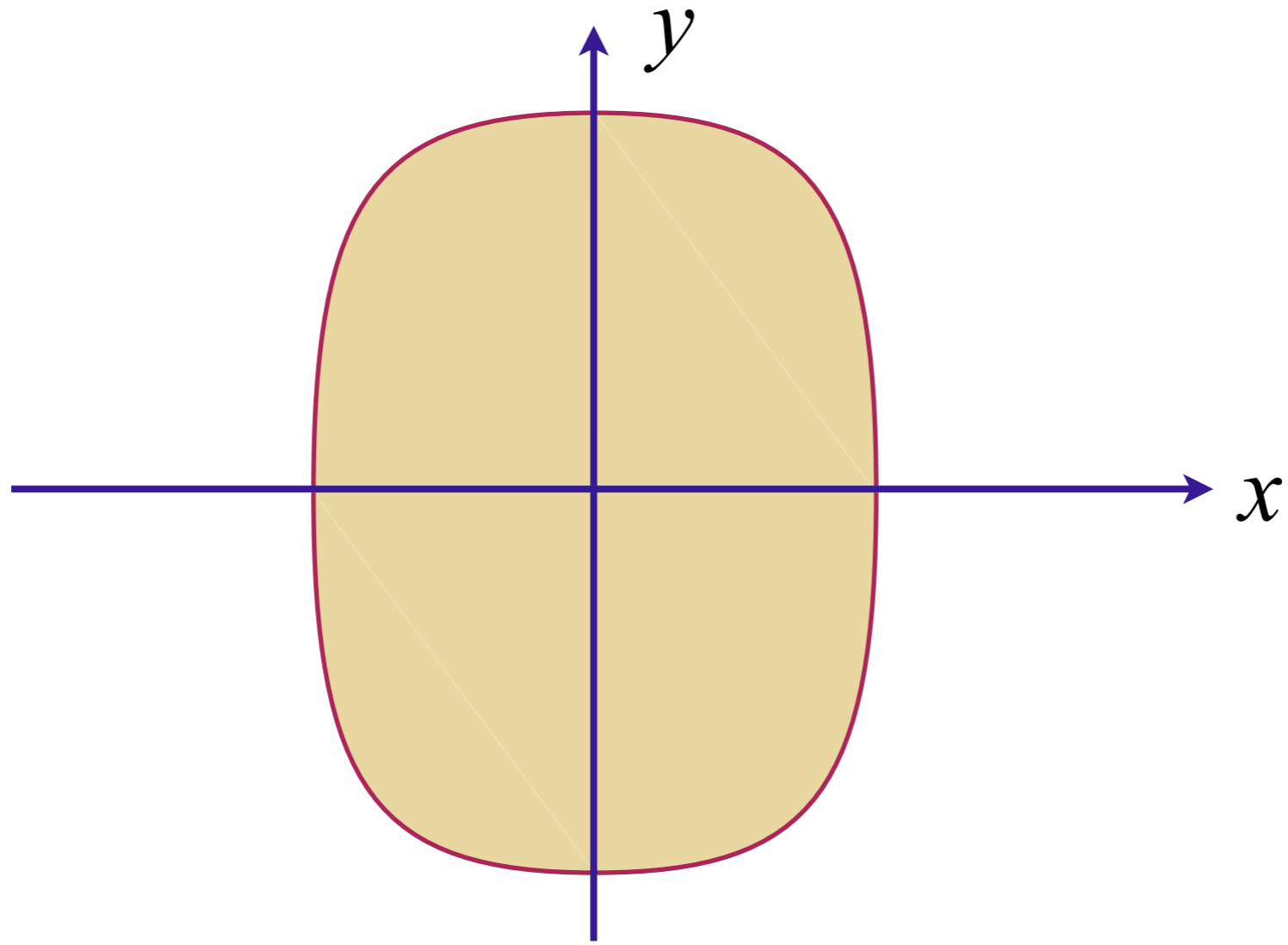
Fermi surface with full square lattice symmetry

Quantum criticality of Ising-nematic ordering



Spontaneous elongation along x direction:

Quantum criticality of Ising-nematic ordering



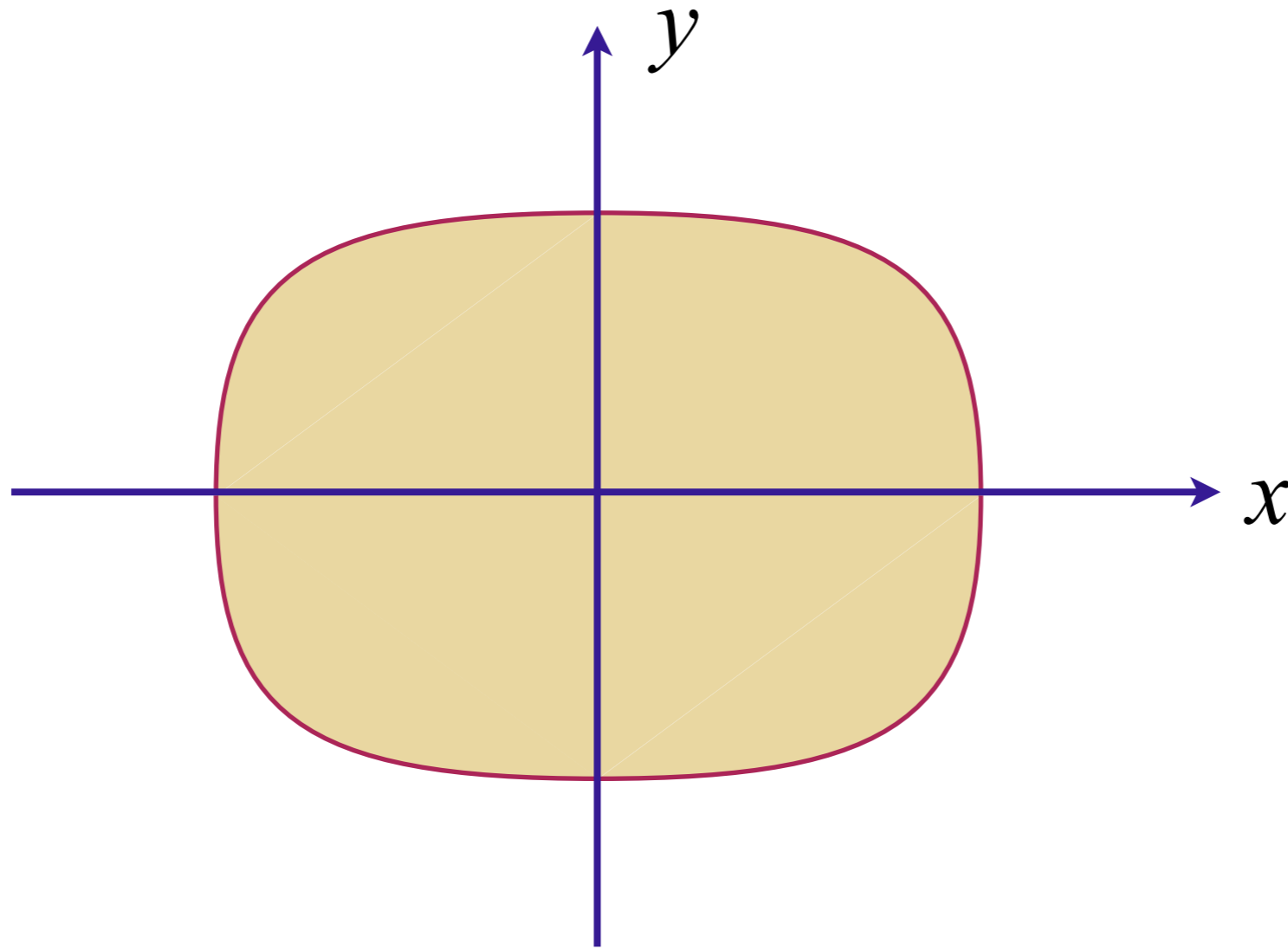
Spontaneous elongation along y direction:

Ising-nematic order parameter

$$\phi \sim \int d^2k (\cos k_x - \cos k_y) c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}$$

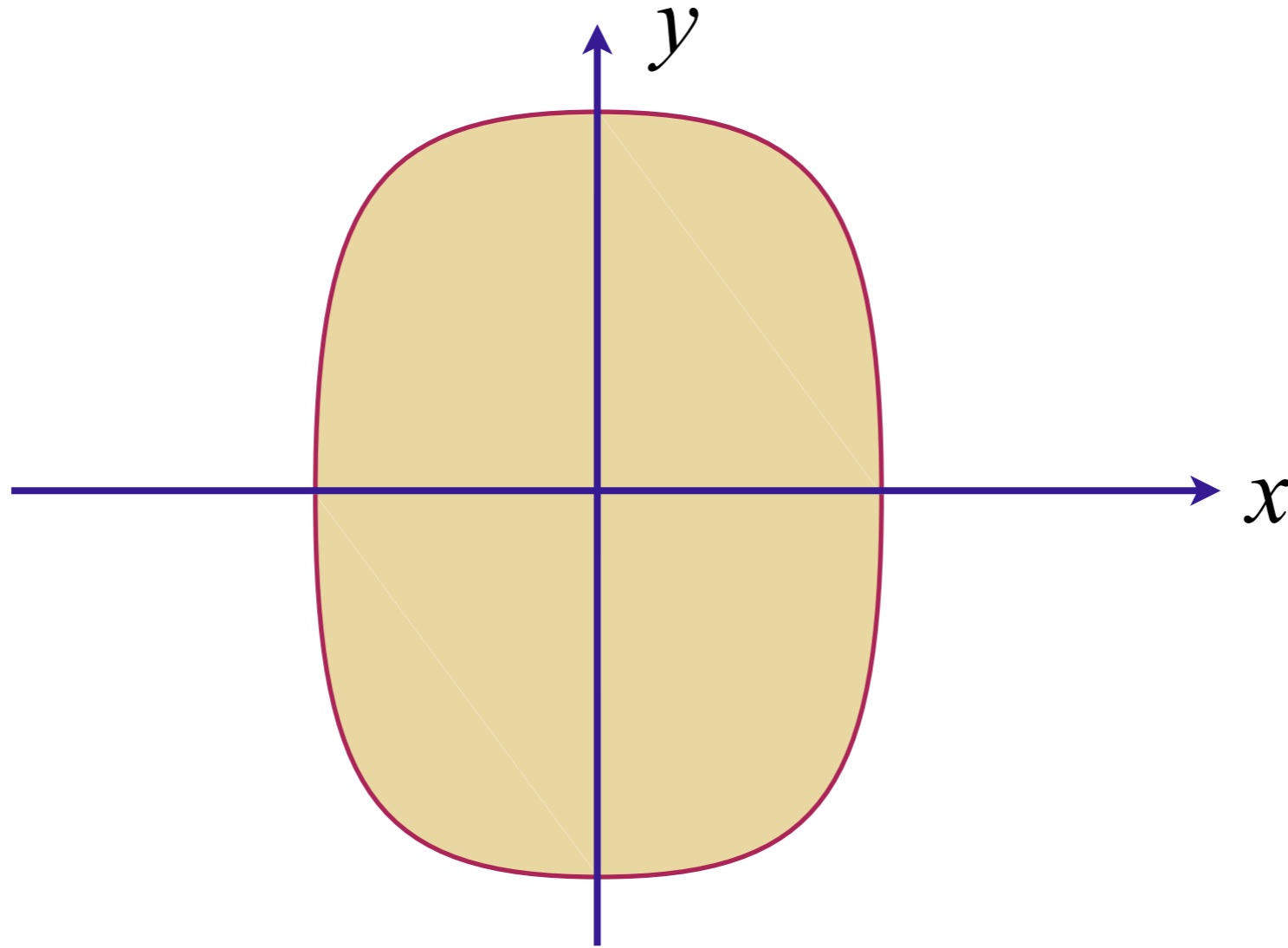
Measures spontaneous breaking of square lattice point-group symmetry of underlying Hamiltonian

Quantum criticality of Ising-nematic ordering



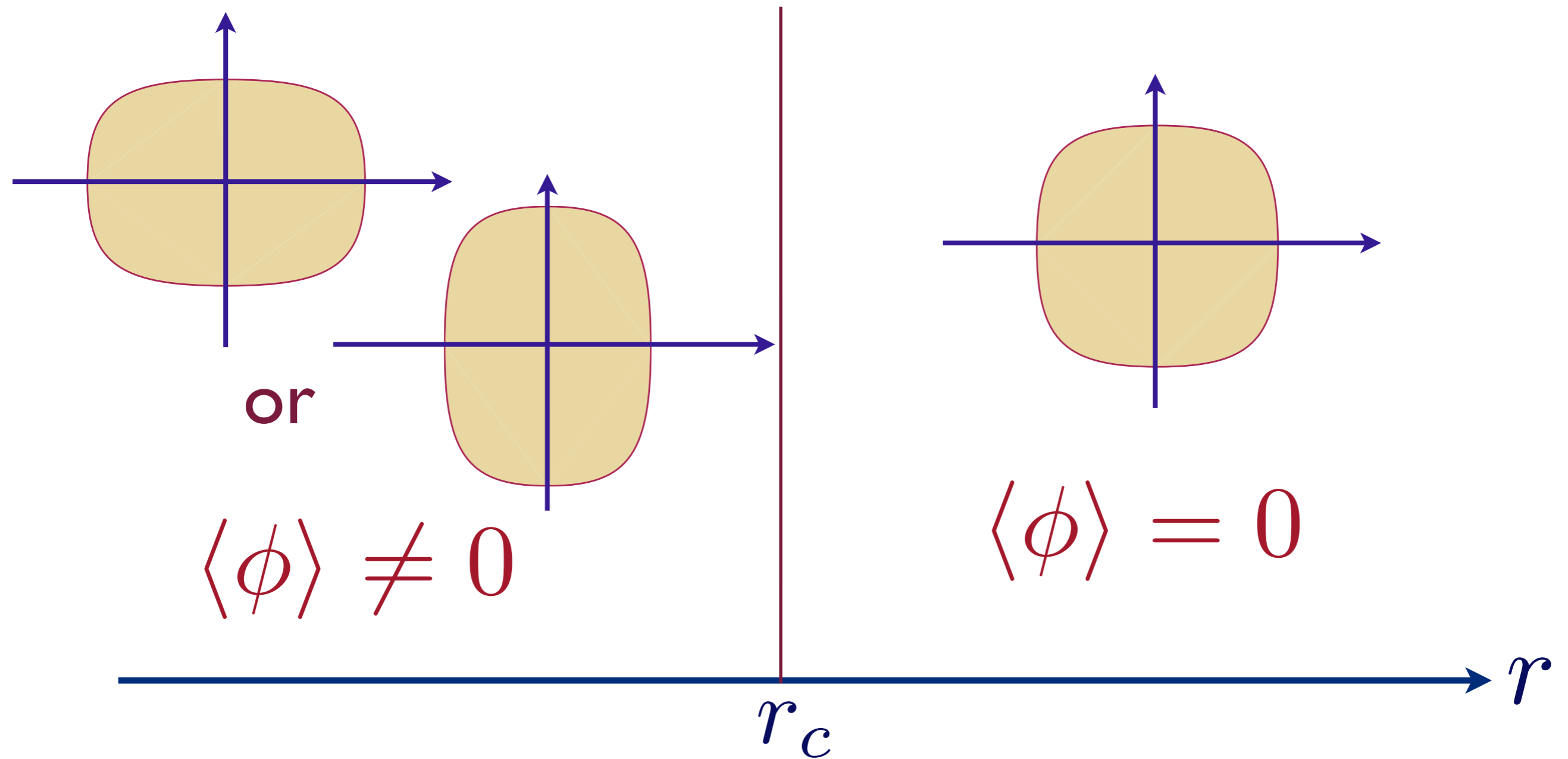
Spontaneous elongation along x direction:
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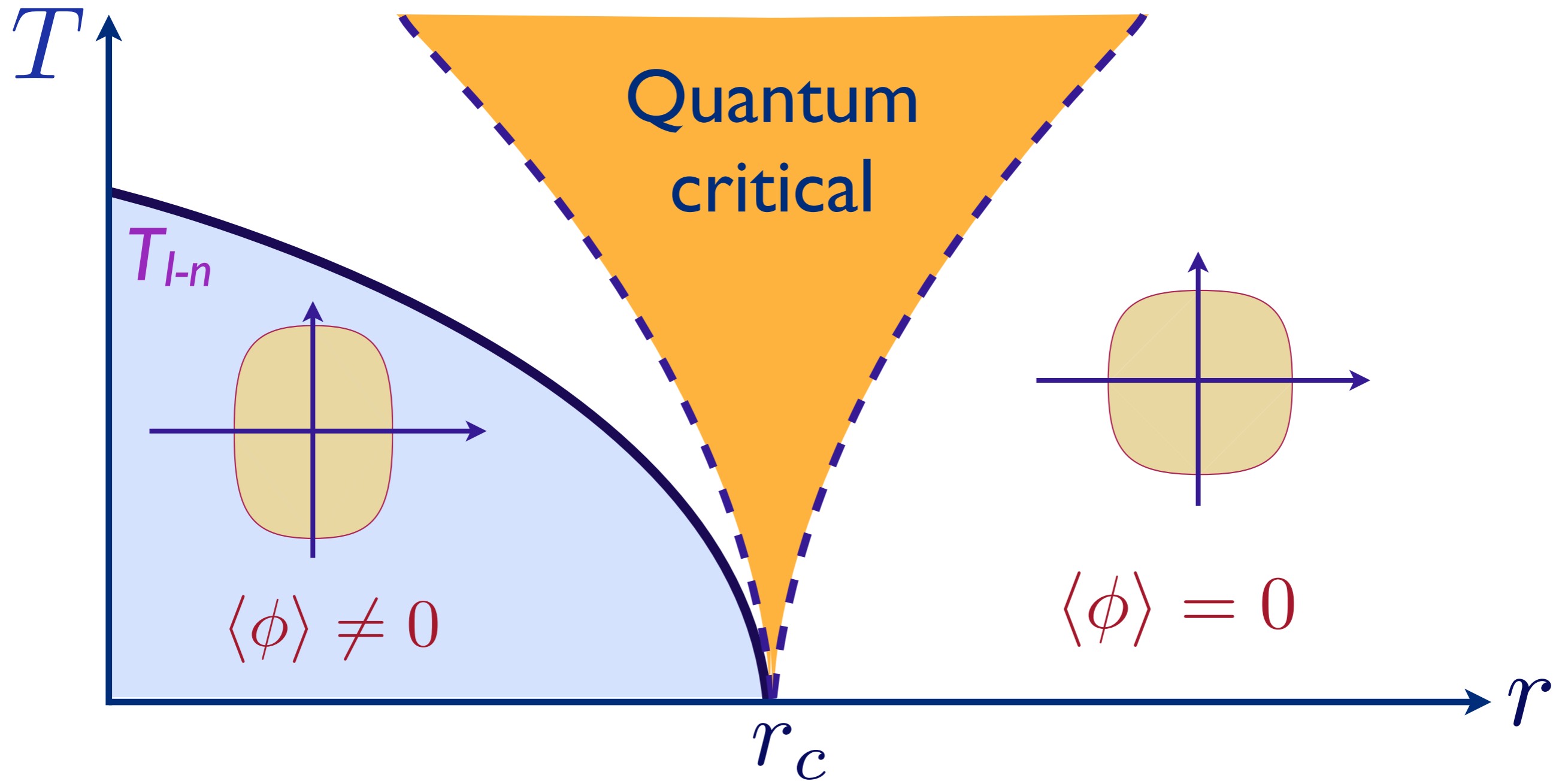
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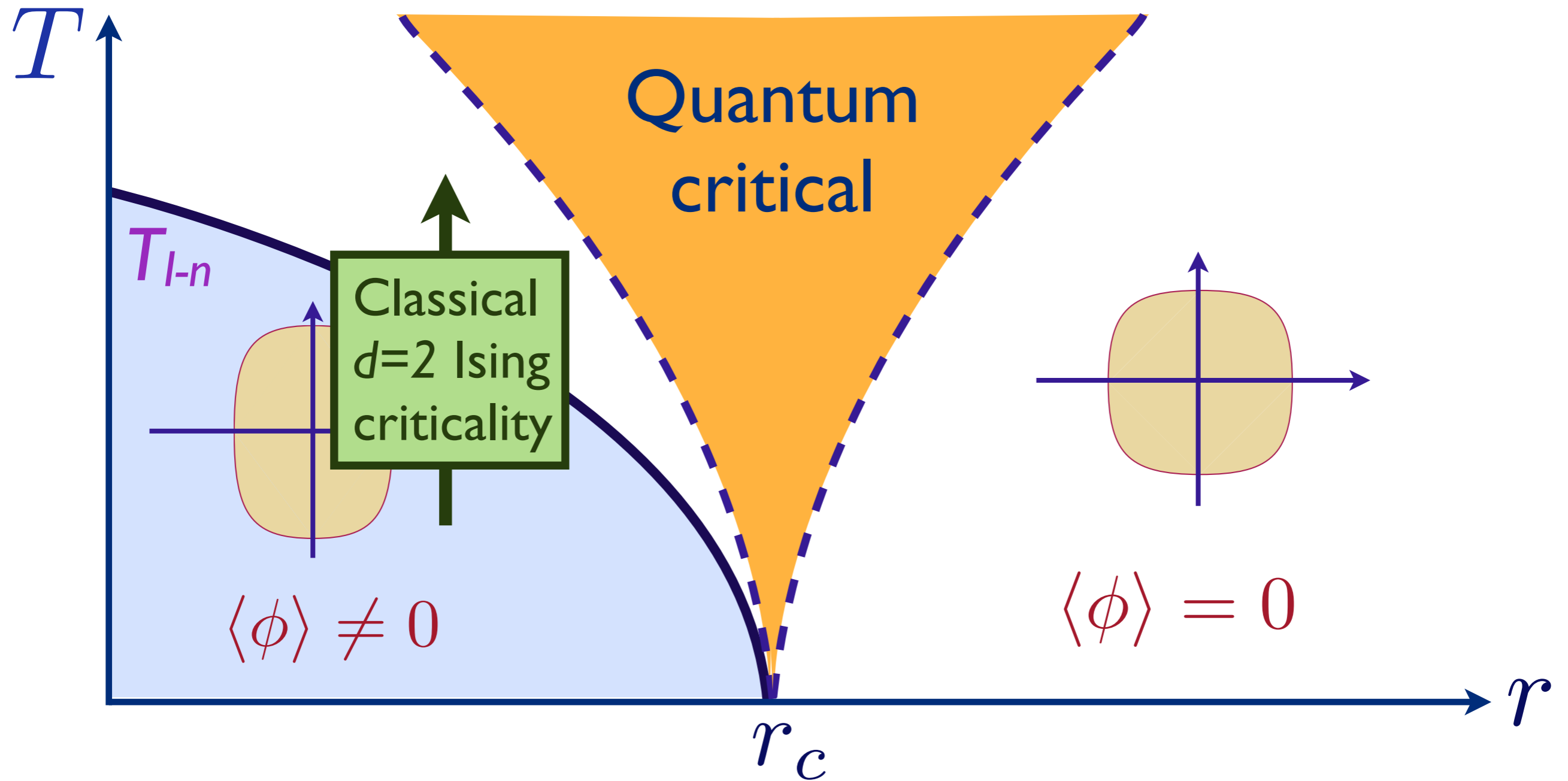
Pomeranchuk instability as a function of coupling r

Quantum criticality of Ising-nematic ordering



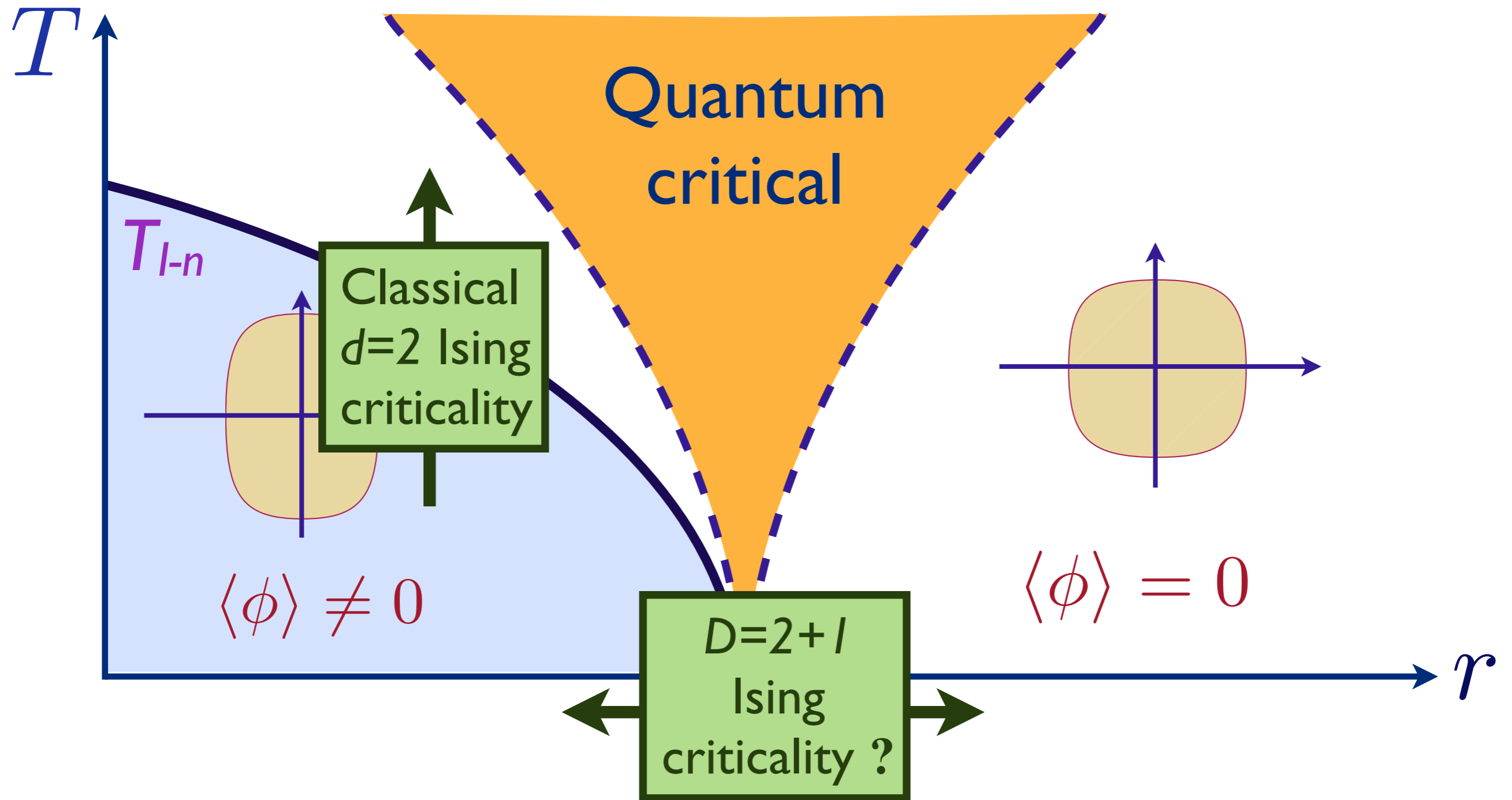
Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering



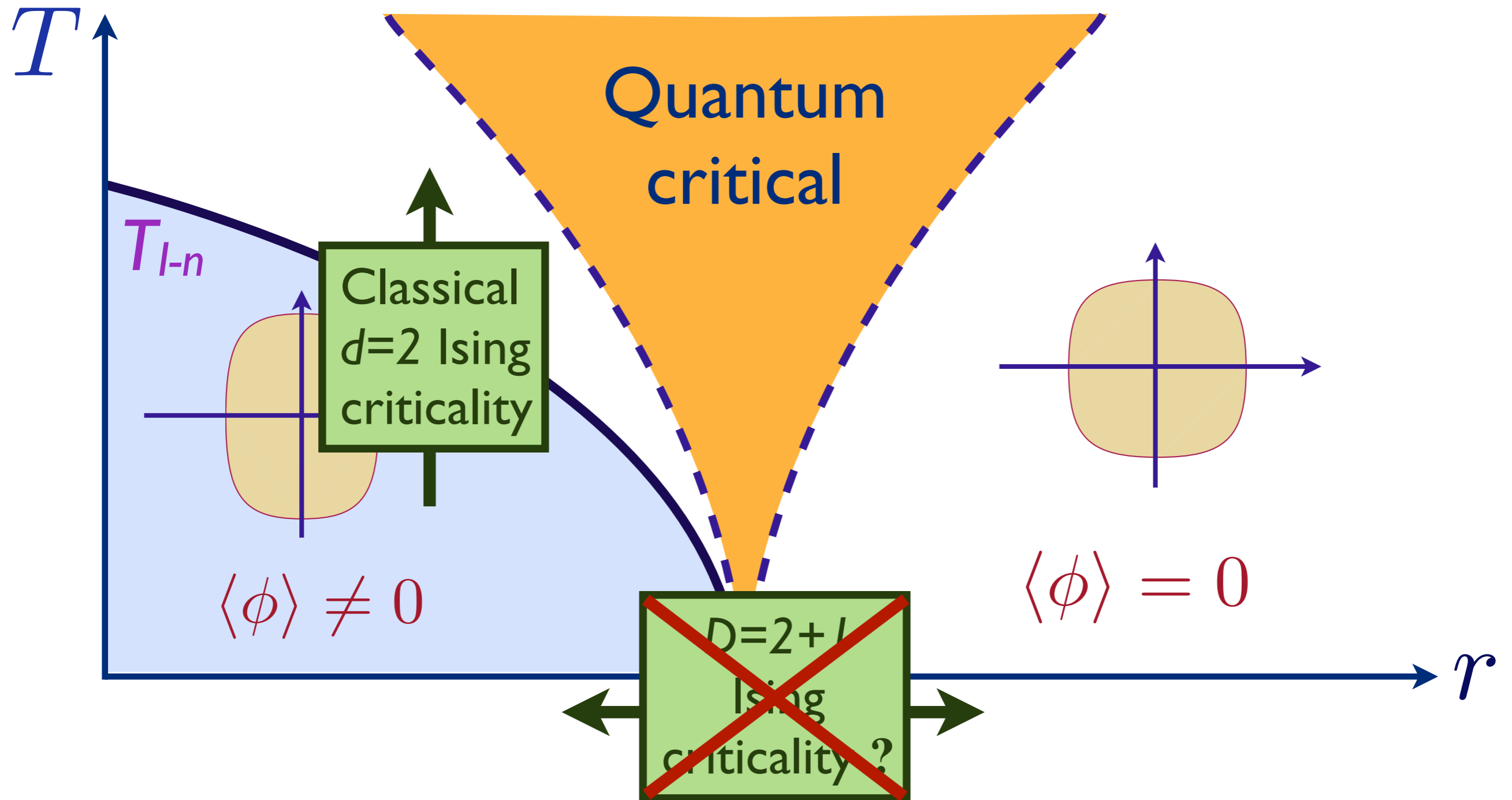
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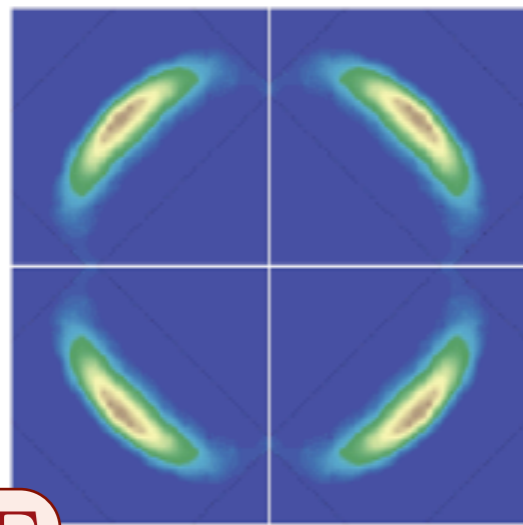
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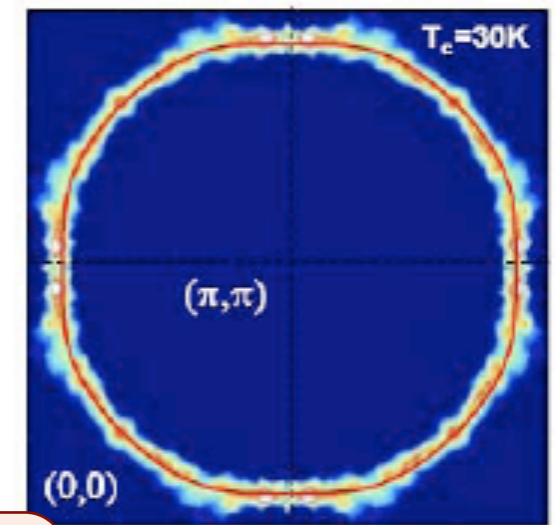
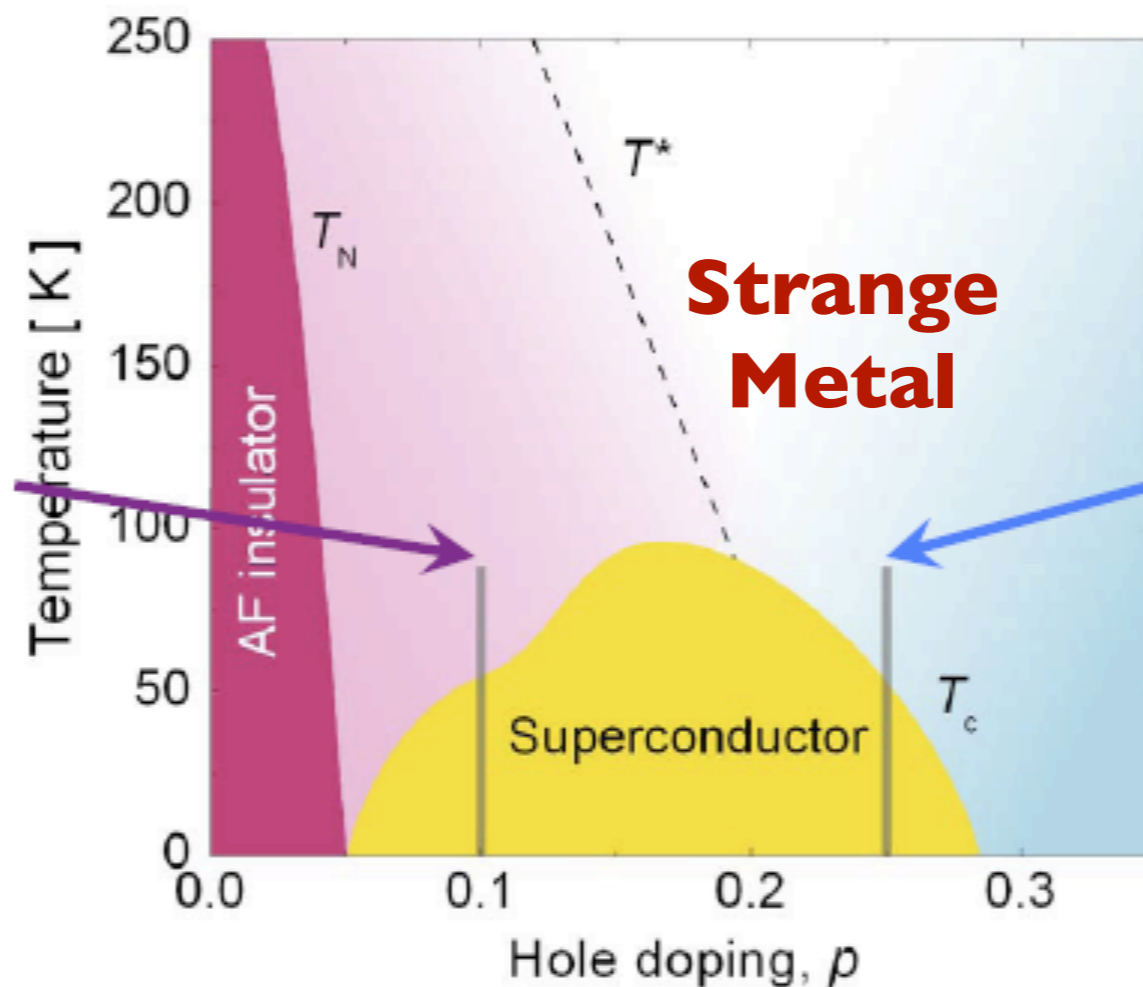
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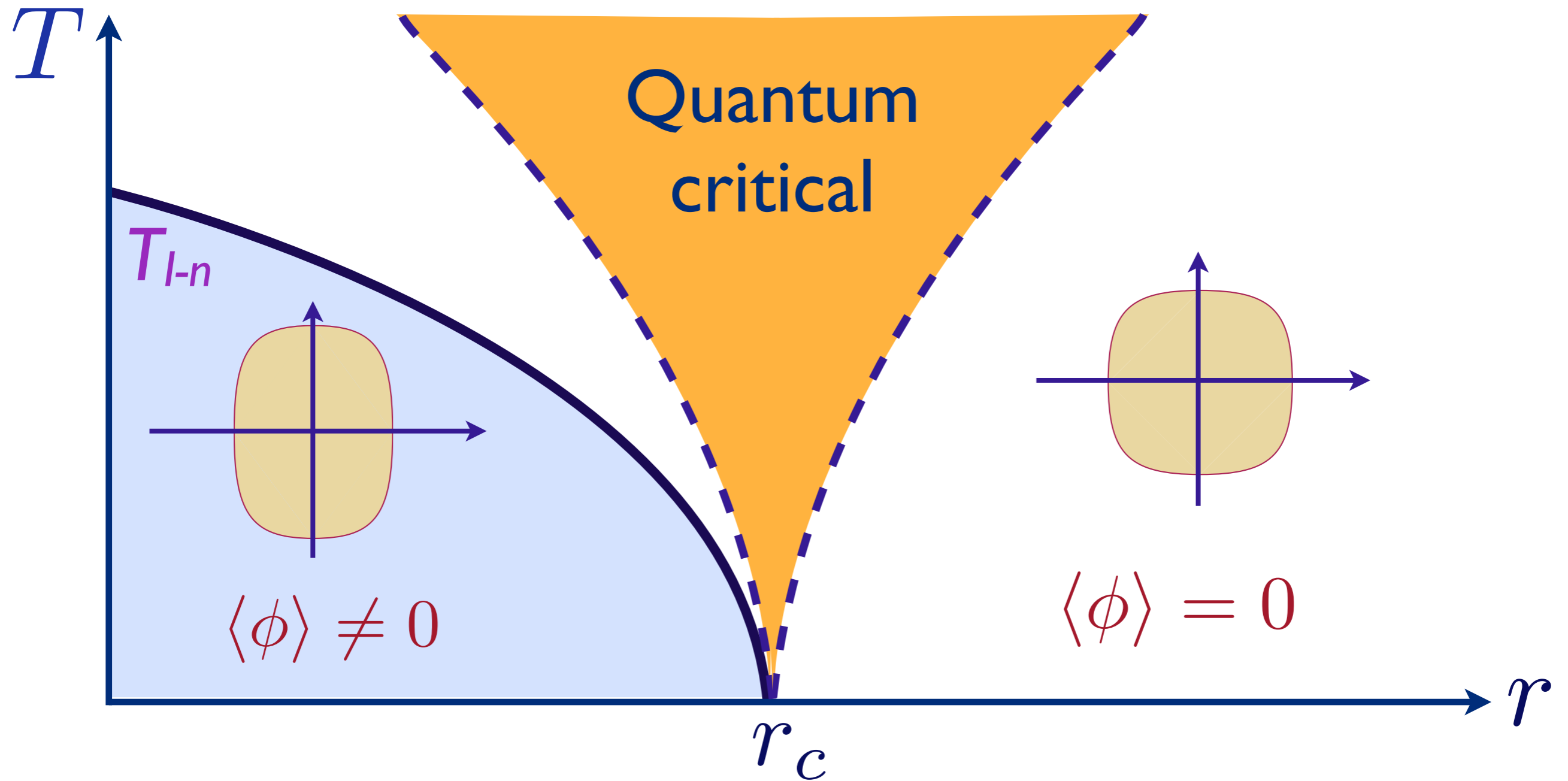
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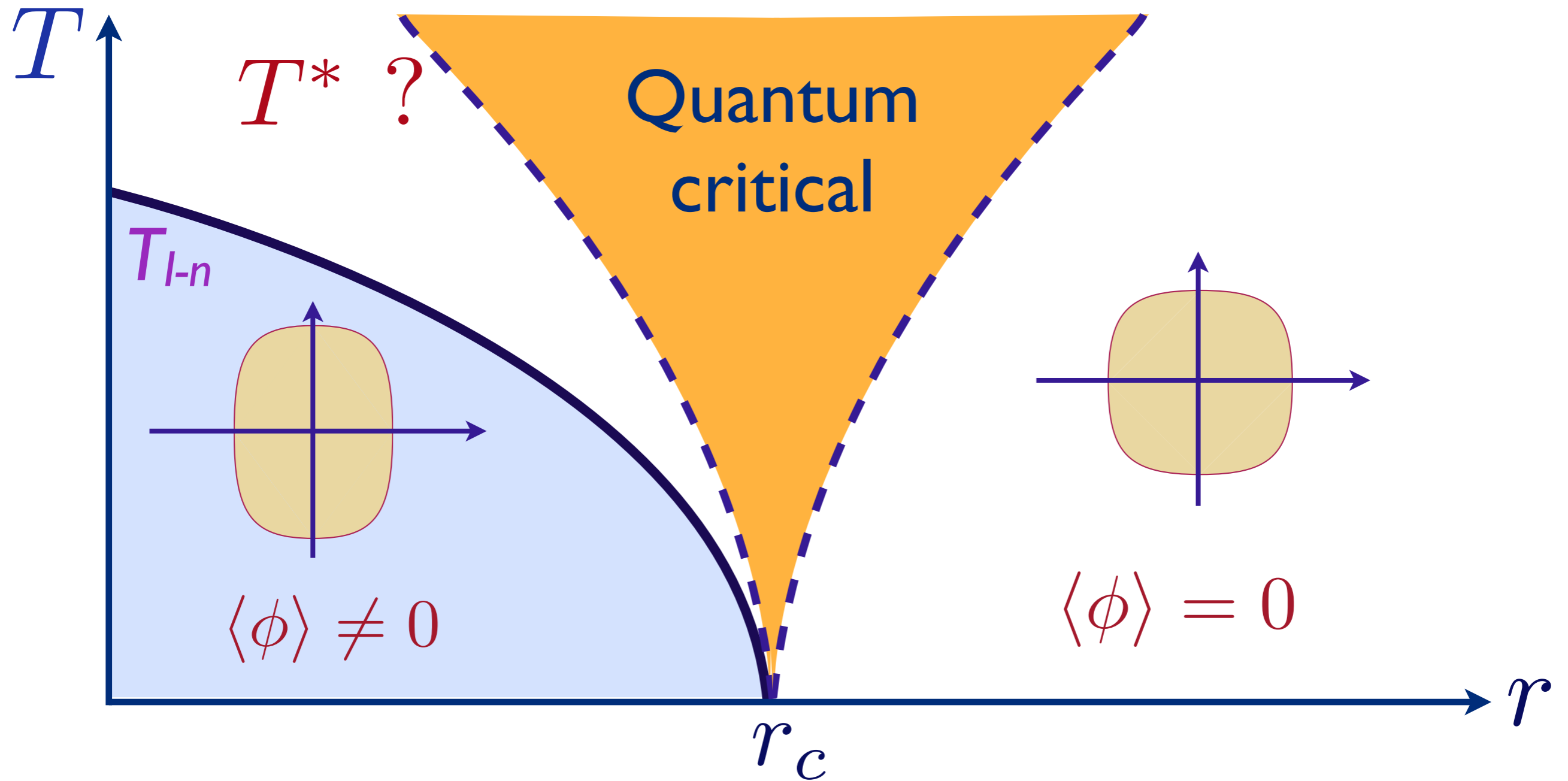
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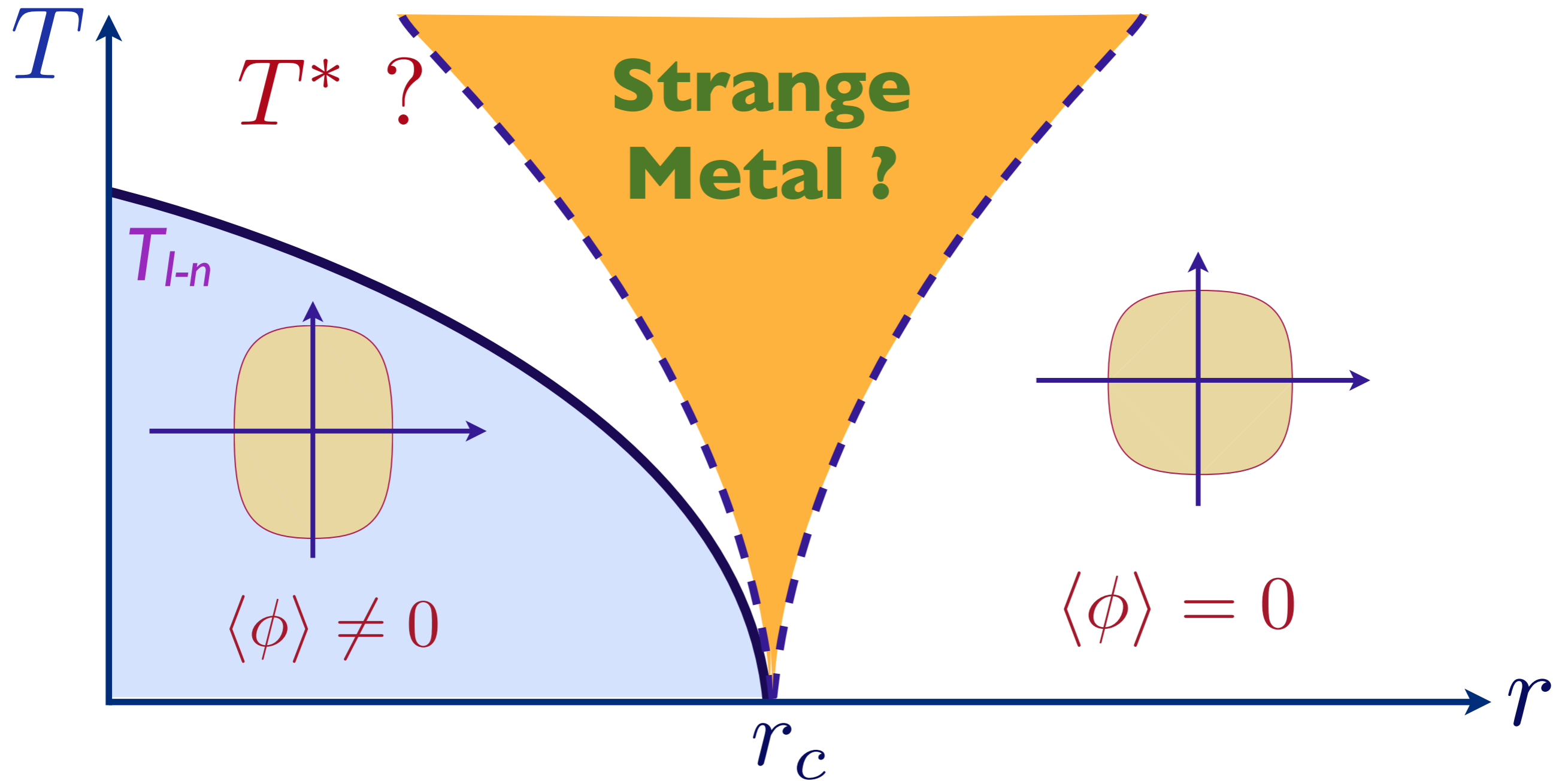
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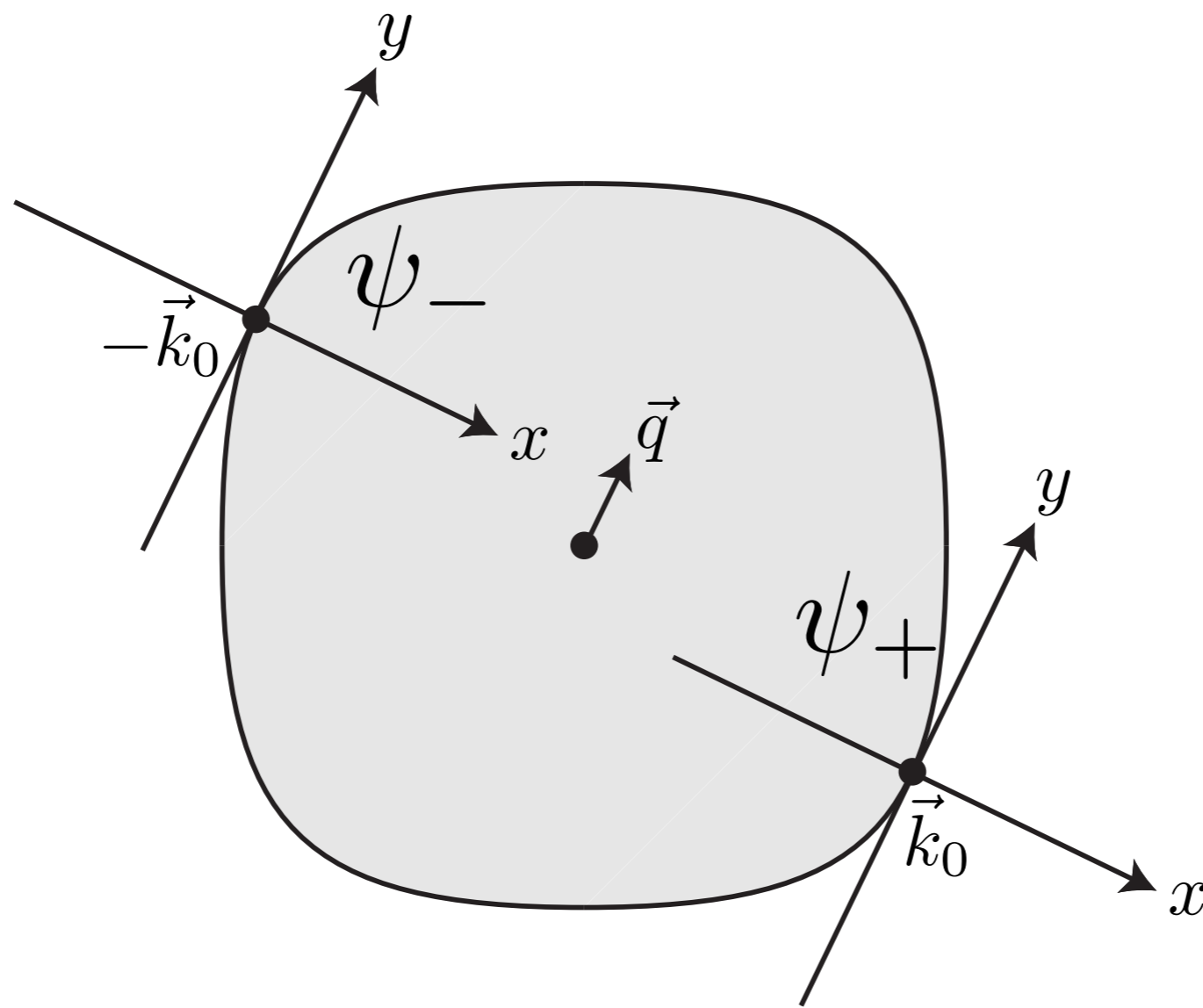


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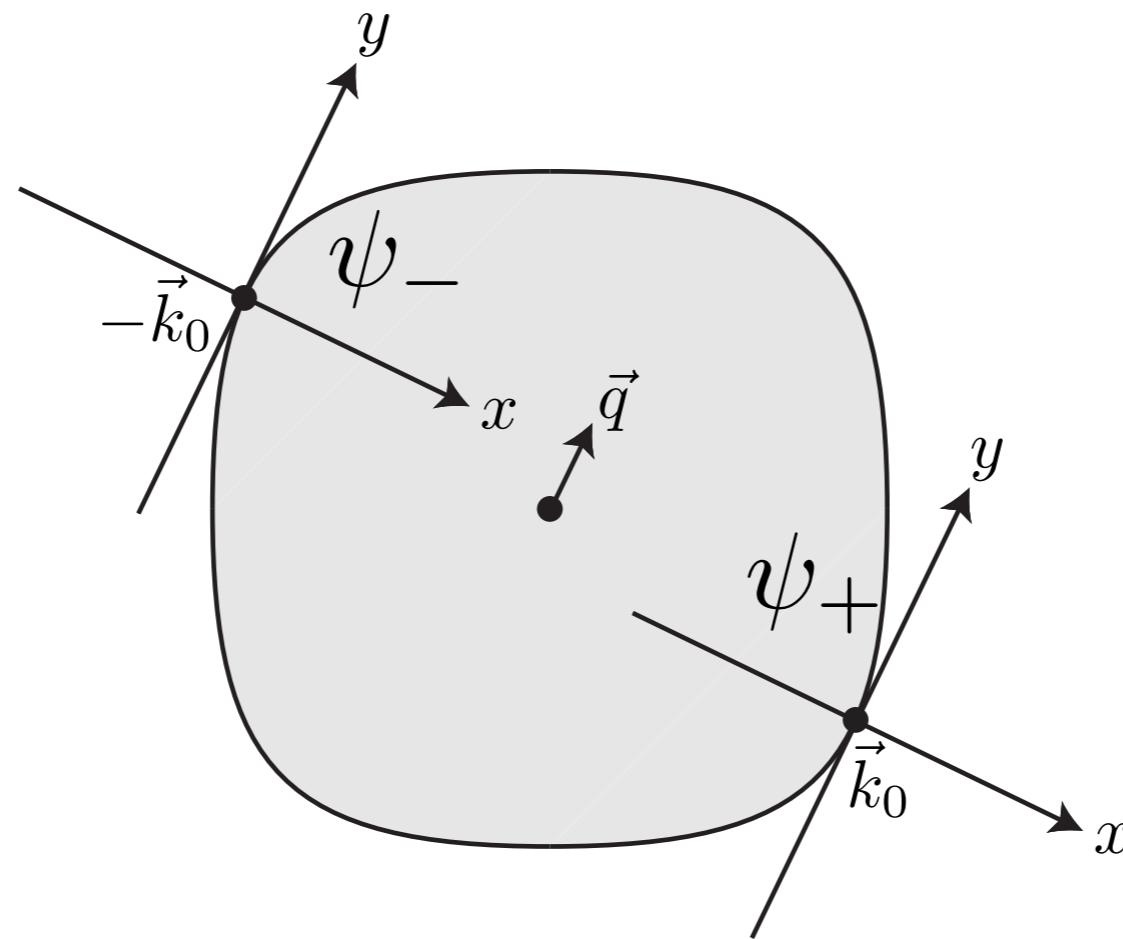
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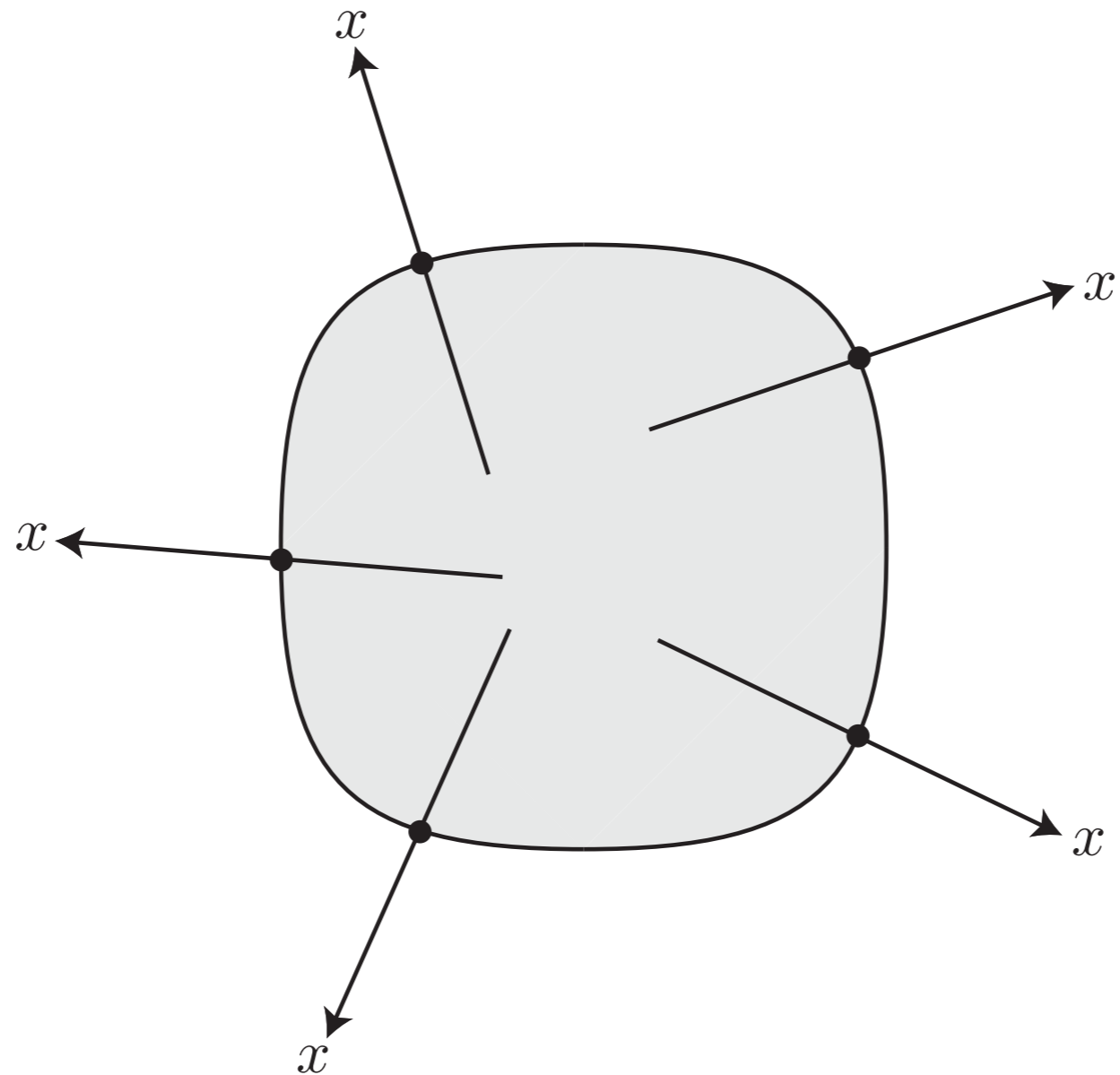
A ϕ fluctuation at wavevector \vec{q} couples most efficiently to fermions near $\pm\vec{k}_0$.

Expand fermion kinetic energy at wavevectors about \vec{k}_0

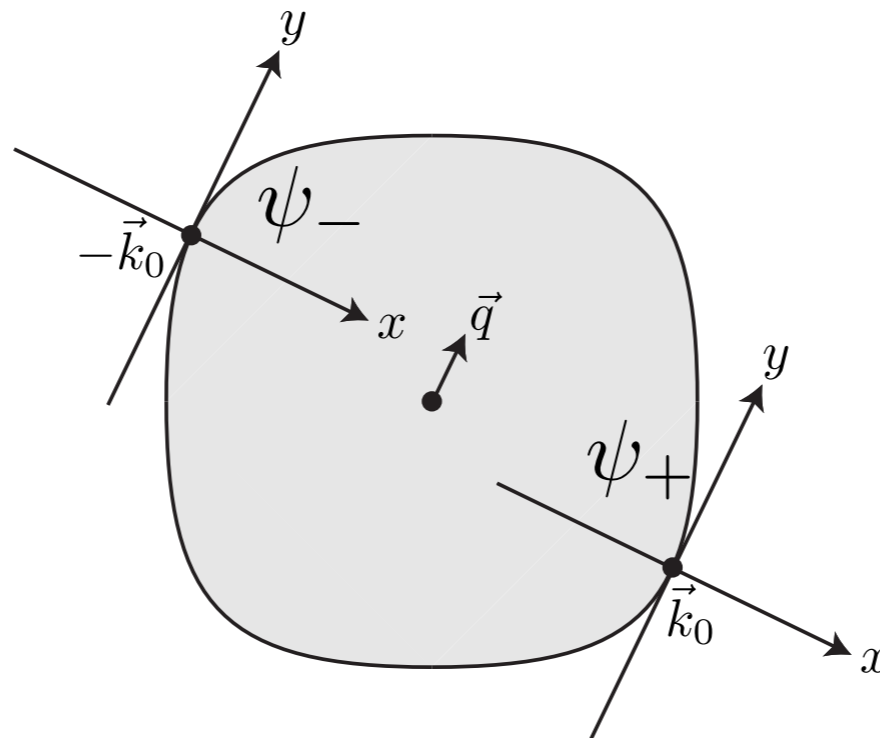
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- Cont... methods
when... nsional
field

Strong coupling problem II:

Infinite number of 2+1

dimensional field theories

at Ising-nematic

quantum critical point

- Our... of un-
derly... metry”
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Outline

1. Coupled dimer antiferromagnets

Introduction to quantum criticality

2. Theory of Ising-nematic ordering in the cuprate metals

Strongly-coupled field theory

3. The AdS/CFT correspondence

Phases of finite density quantum matter at strong coupling

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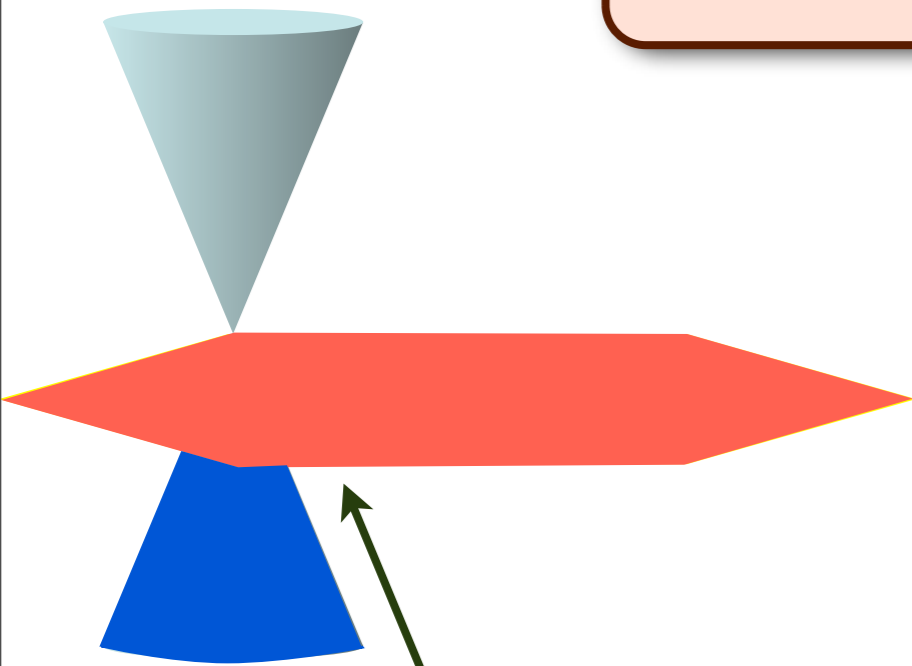
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Conformal field theory
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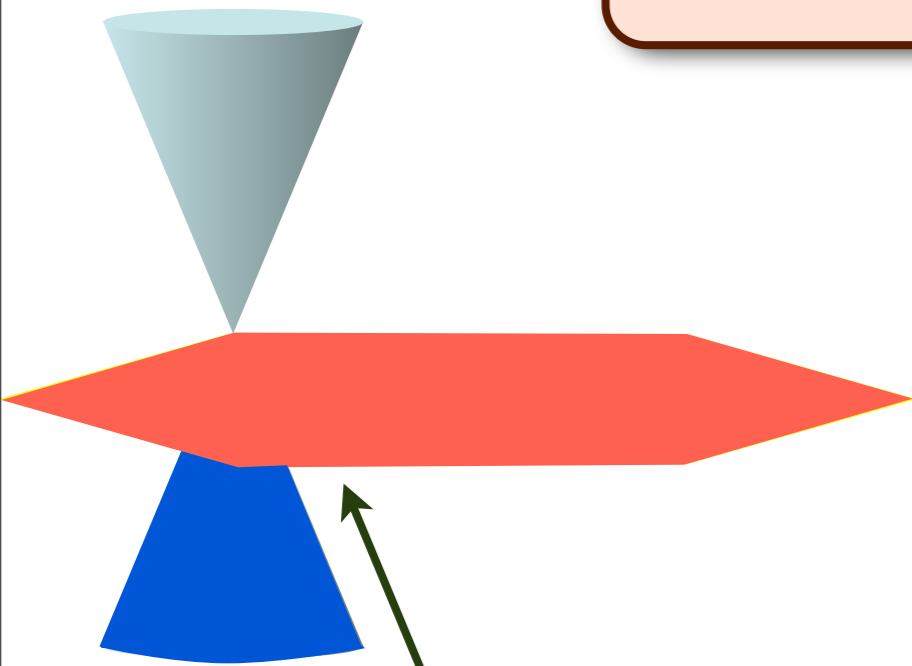


e.g.
Graphene
at zero bias



Einstein gravity
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Conformal field theory
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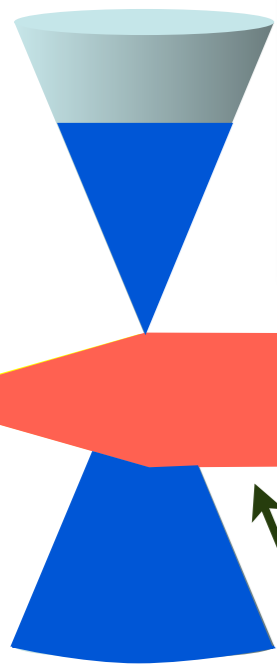


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Einstein gravity on AdS_4
with a Schwarzschild
black hole

Conformal field theory
in $2+1$ dimensions at $T > 0$,
with a non-zero chemical potential, μ
and applied magnetic field, B



e.g.
Graphene
at non-zero
bias

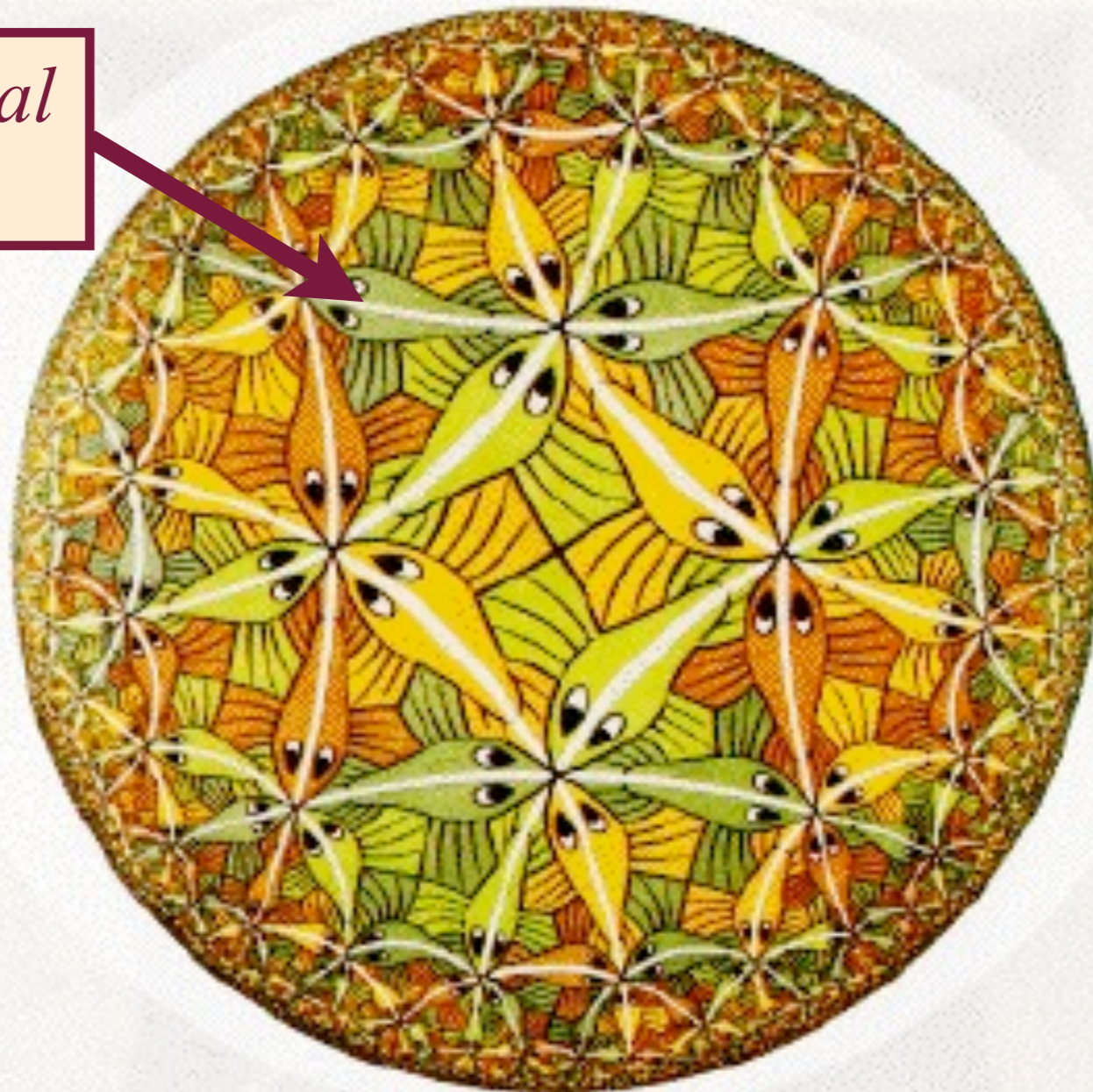
Einstein gravity on AdS_4
with a Reissner-Nordstrom
black hole carrying electric
and magnetic charges



AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions

*3+1 dimensional
AdS space*

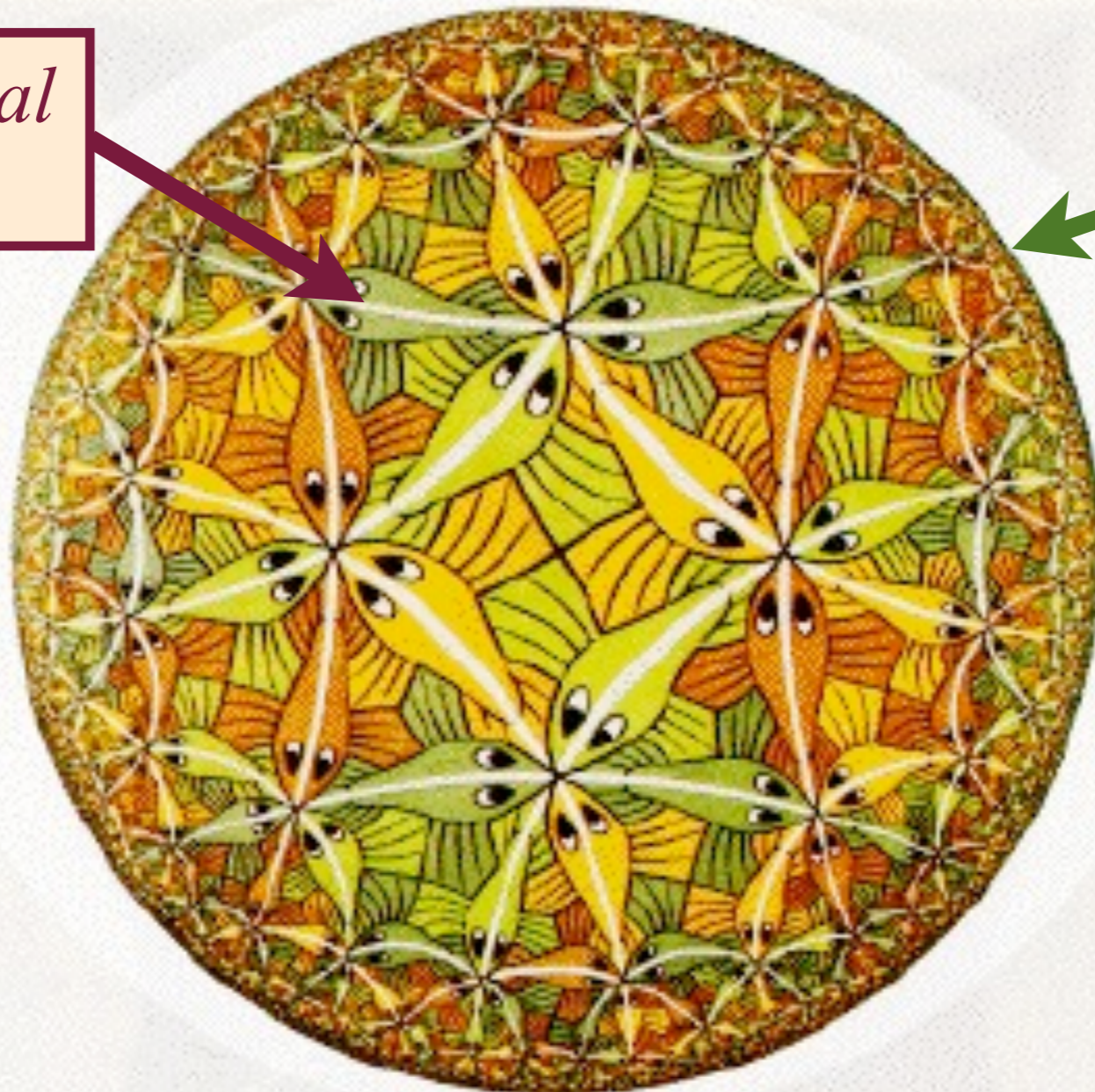


Maldacena, Gubser, Klebanov, Polyakov, Witten

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A 2+1
dimensional
system at its
quantum
critical point

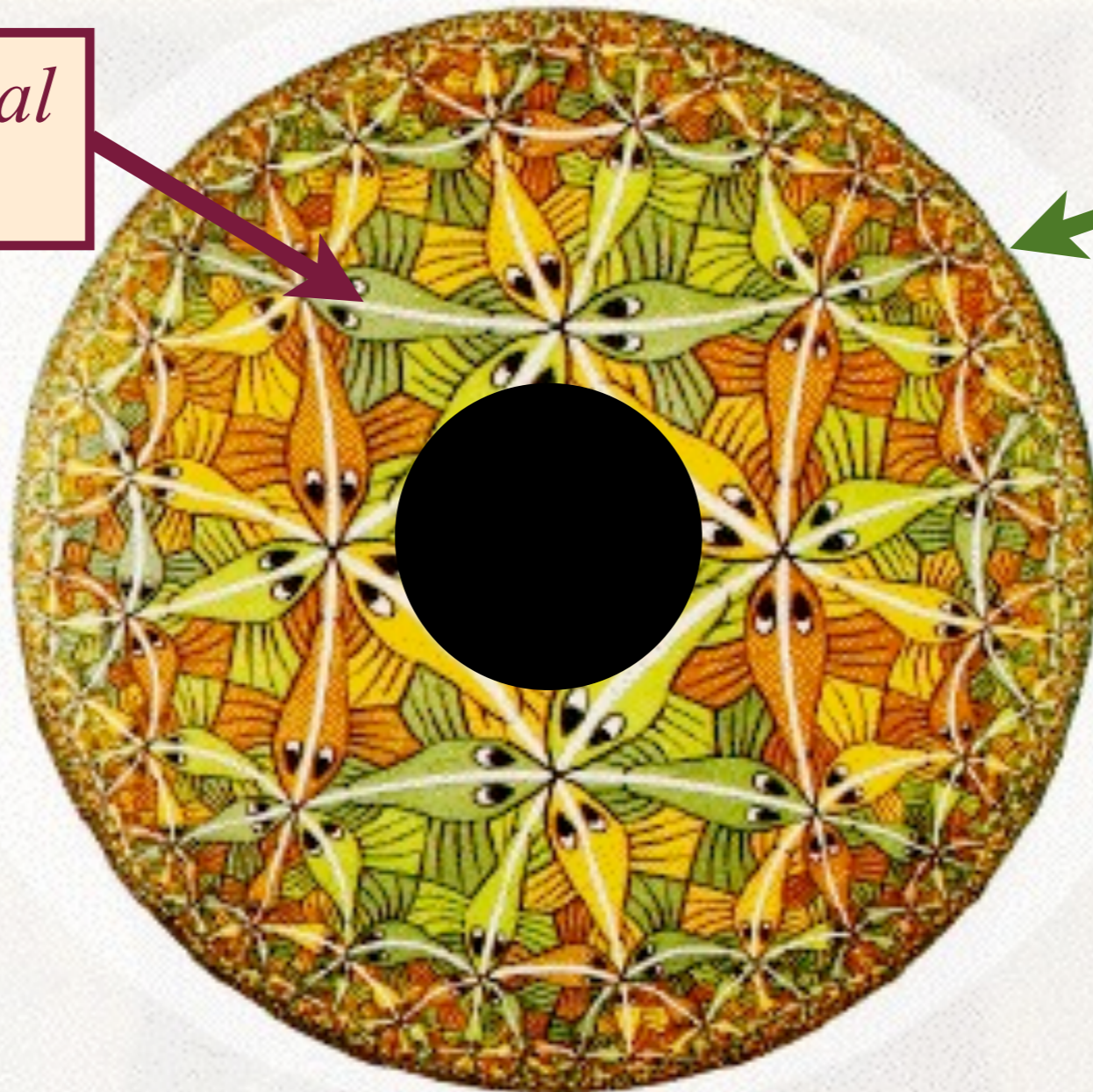
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Quantum
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Black hole
temperature
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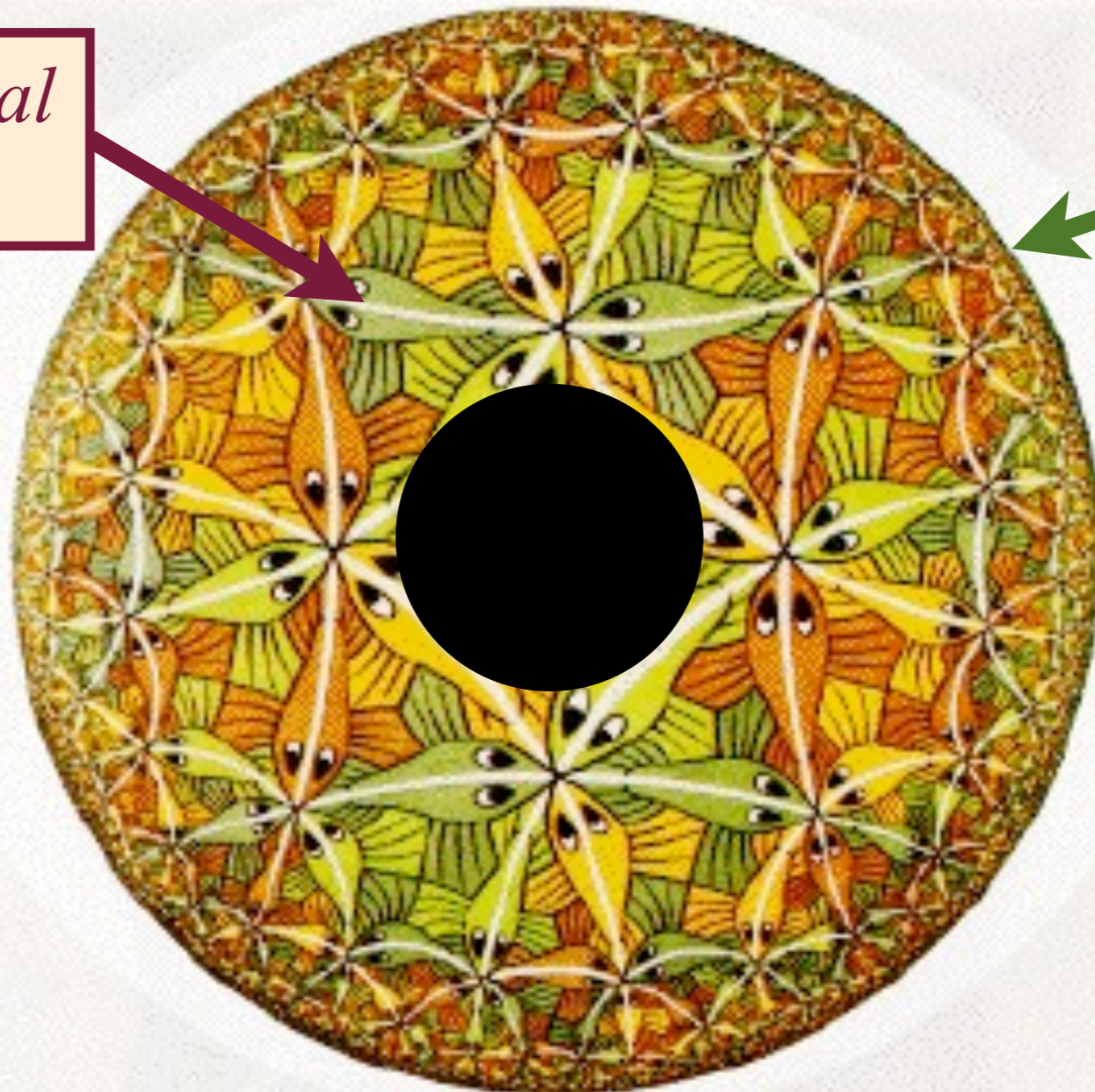
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Black hole
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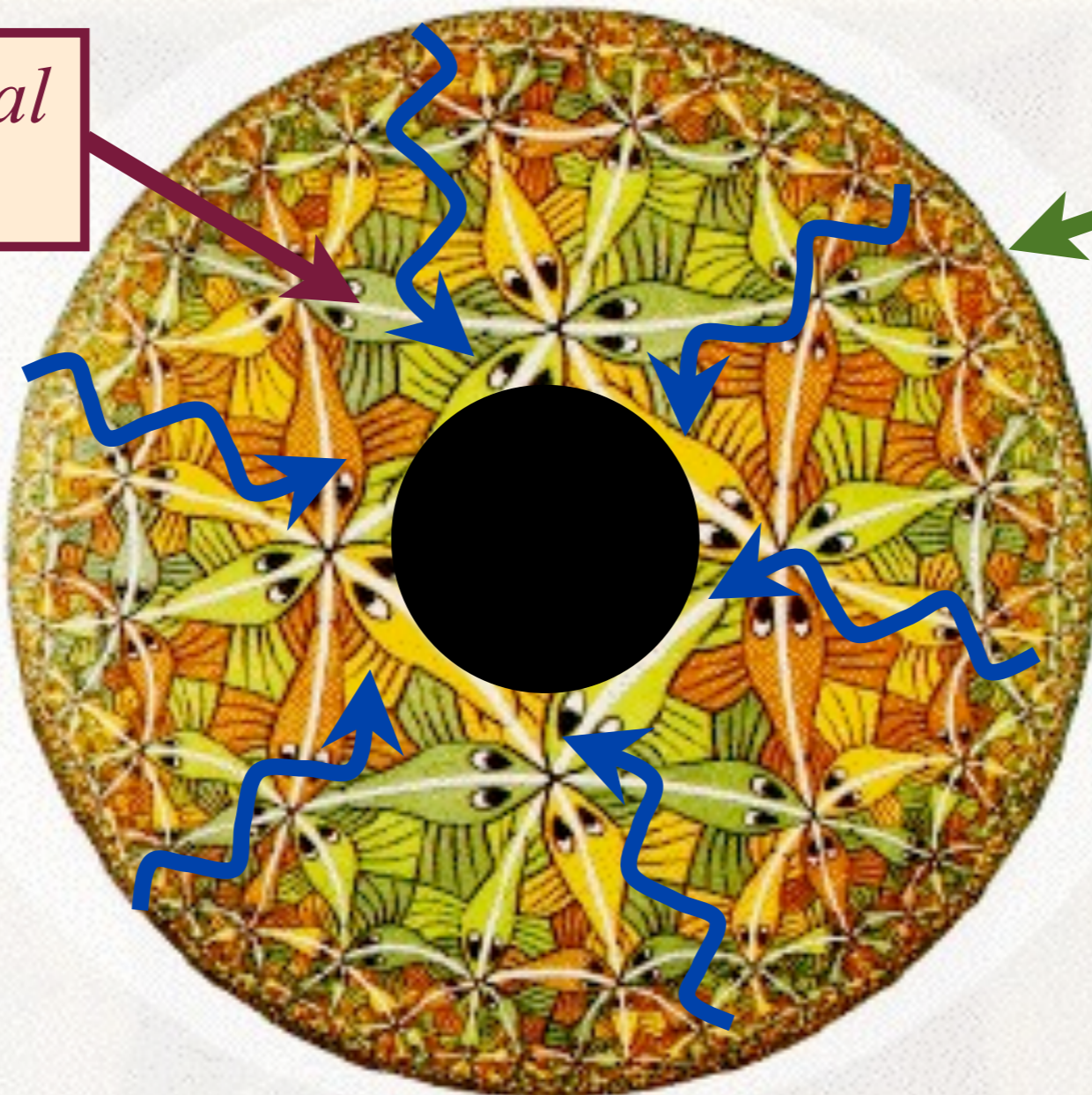
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Quantum
critical
dynamics =
waves in
curved
space

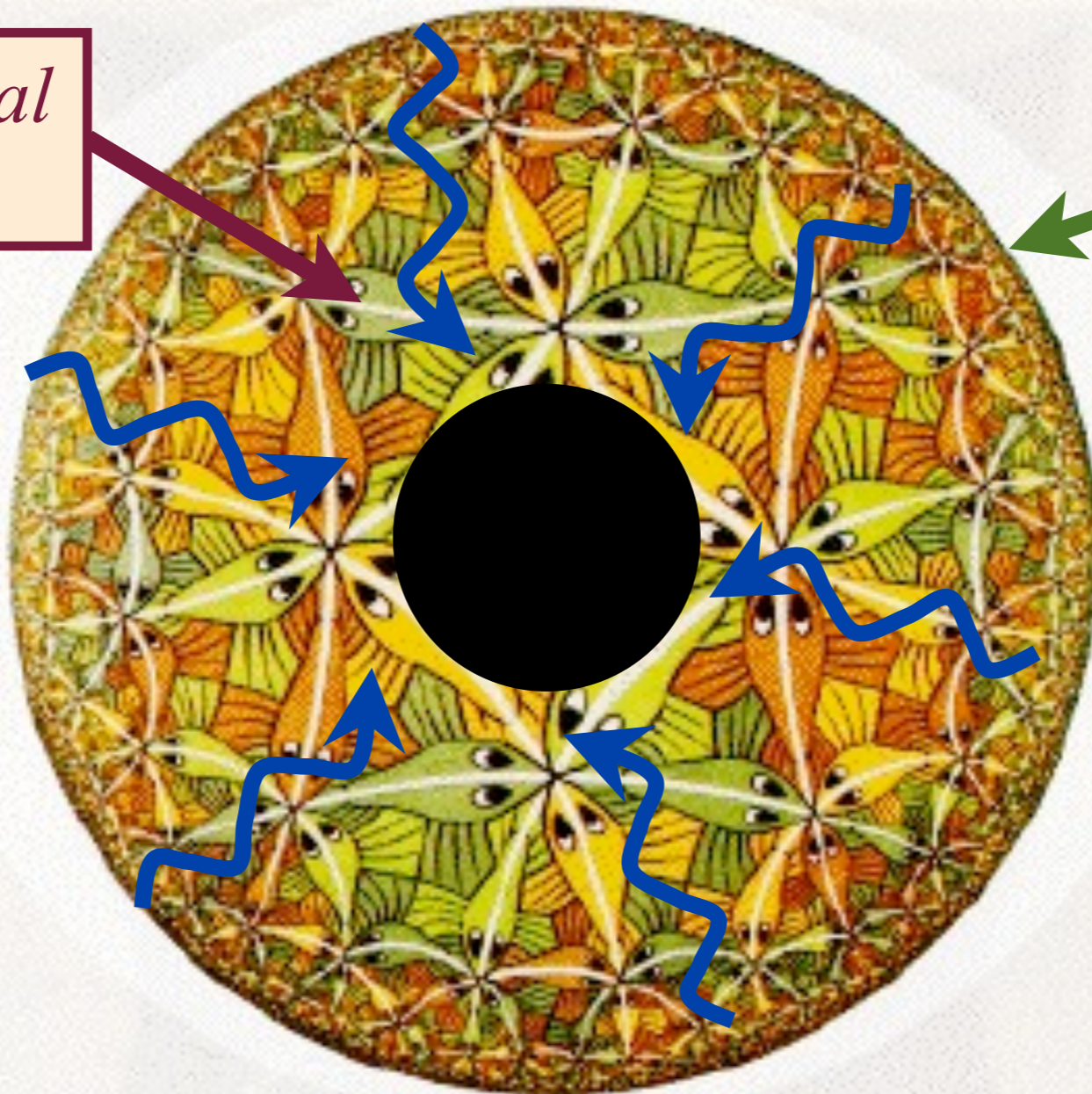


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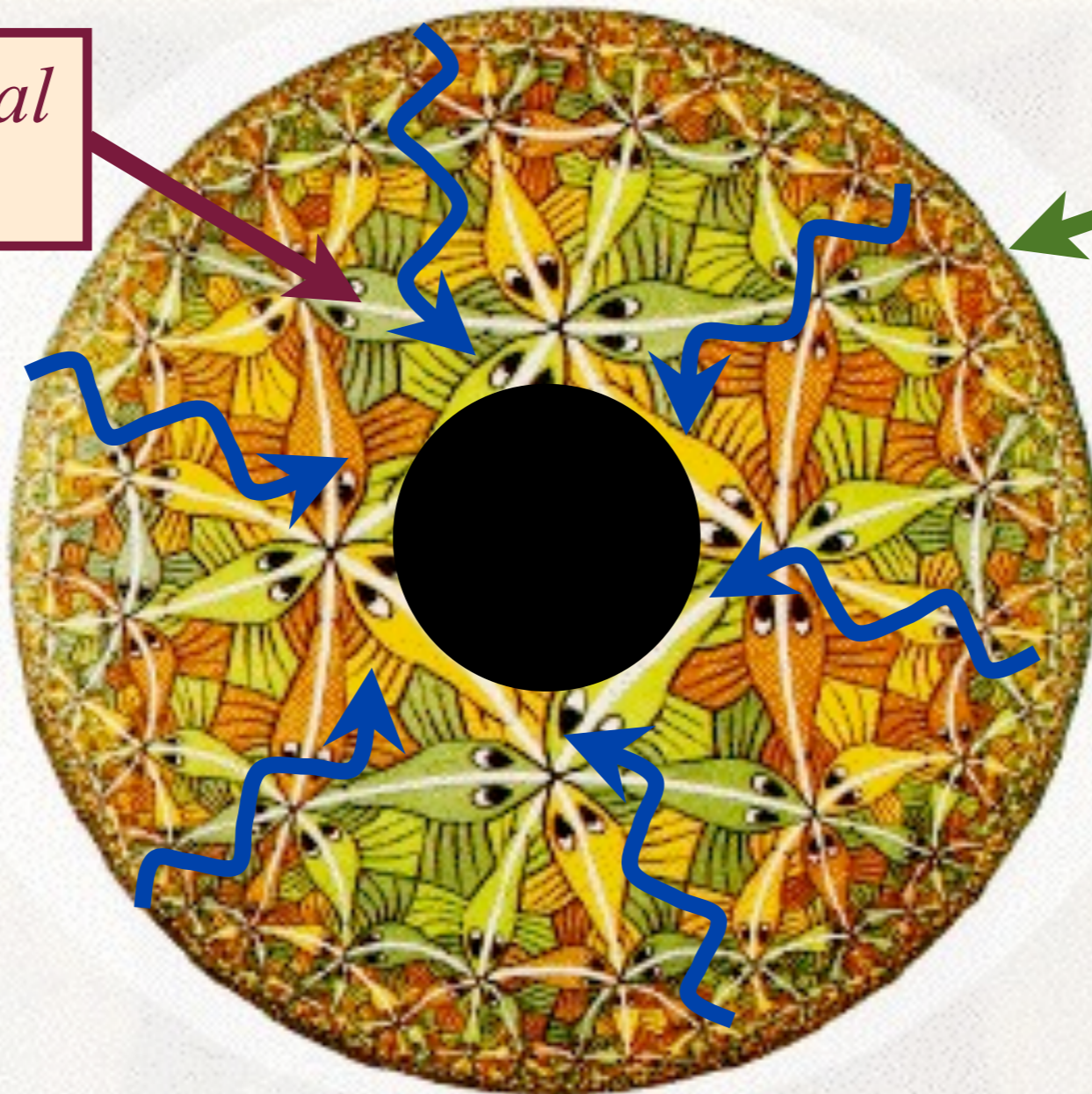
Friction of
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Kovtun, Policastro, Son

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The quantum theory of a black hole in a 3+1-

holog

of a

Strong coupling problem I:
General solution of
magneto-thermo-electric transport
in quantum critical region.

3+1 dim
AdS

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1
dimensions

C. P. Herzog, P. Kovtun, S. Sachdev, and D. T. Son,
Phys. Rev. D **75**, 085020 (2007).

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev,
Phys. Rev. B **76**, 144502 (2007).

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Kovtun, Policastro, Son

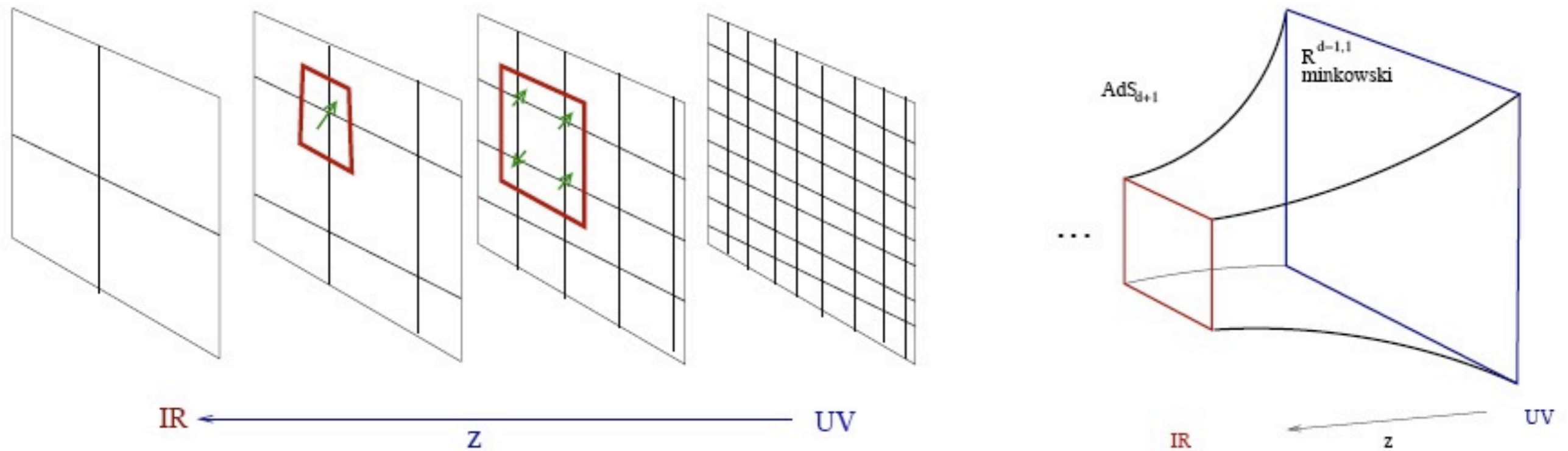
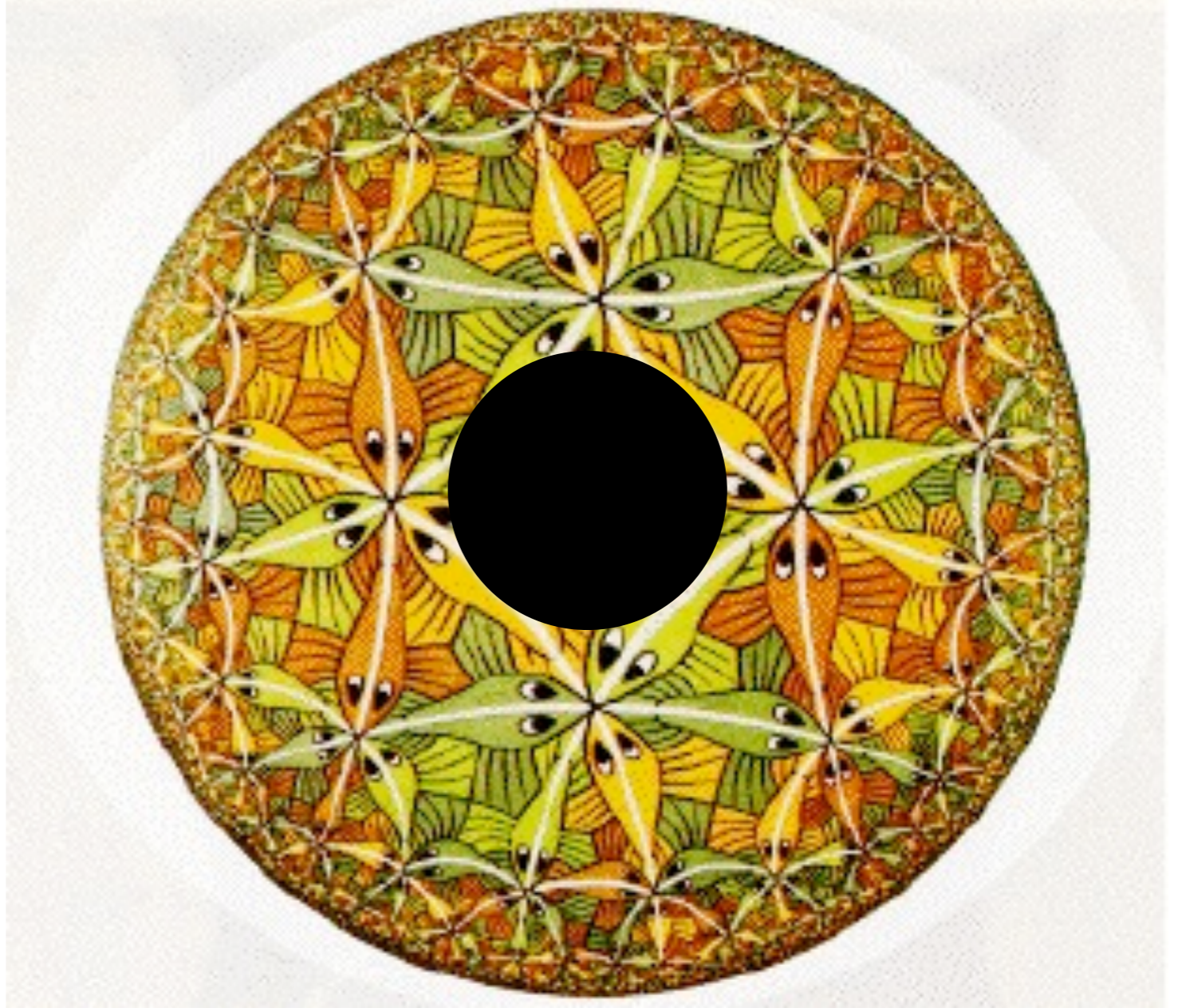
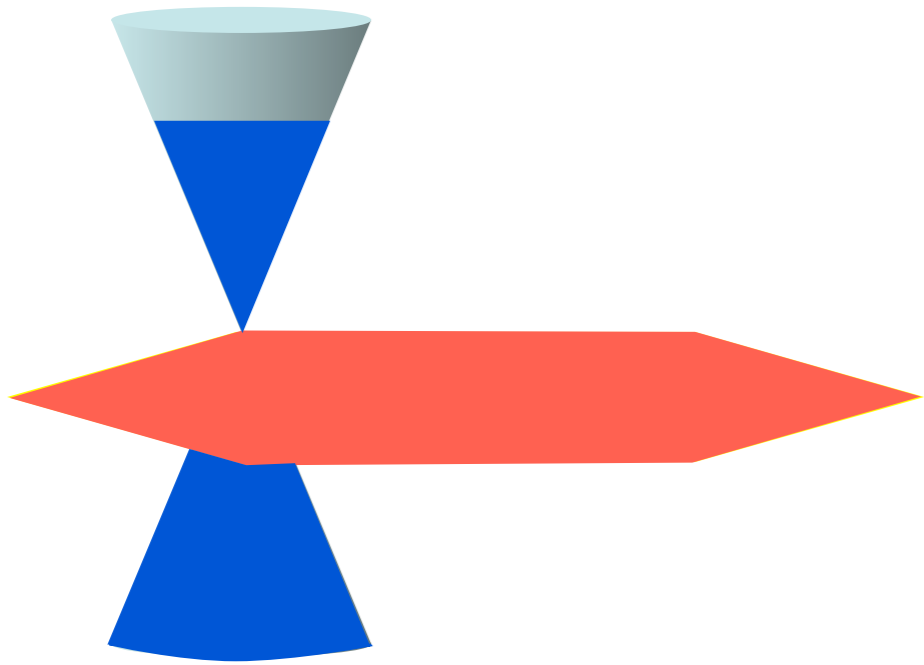
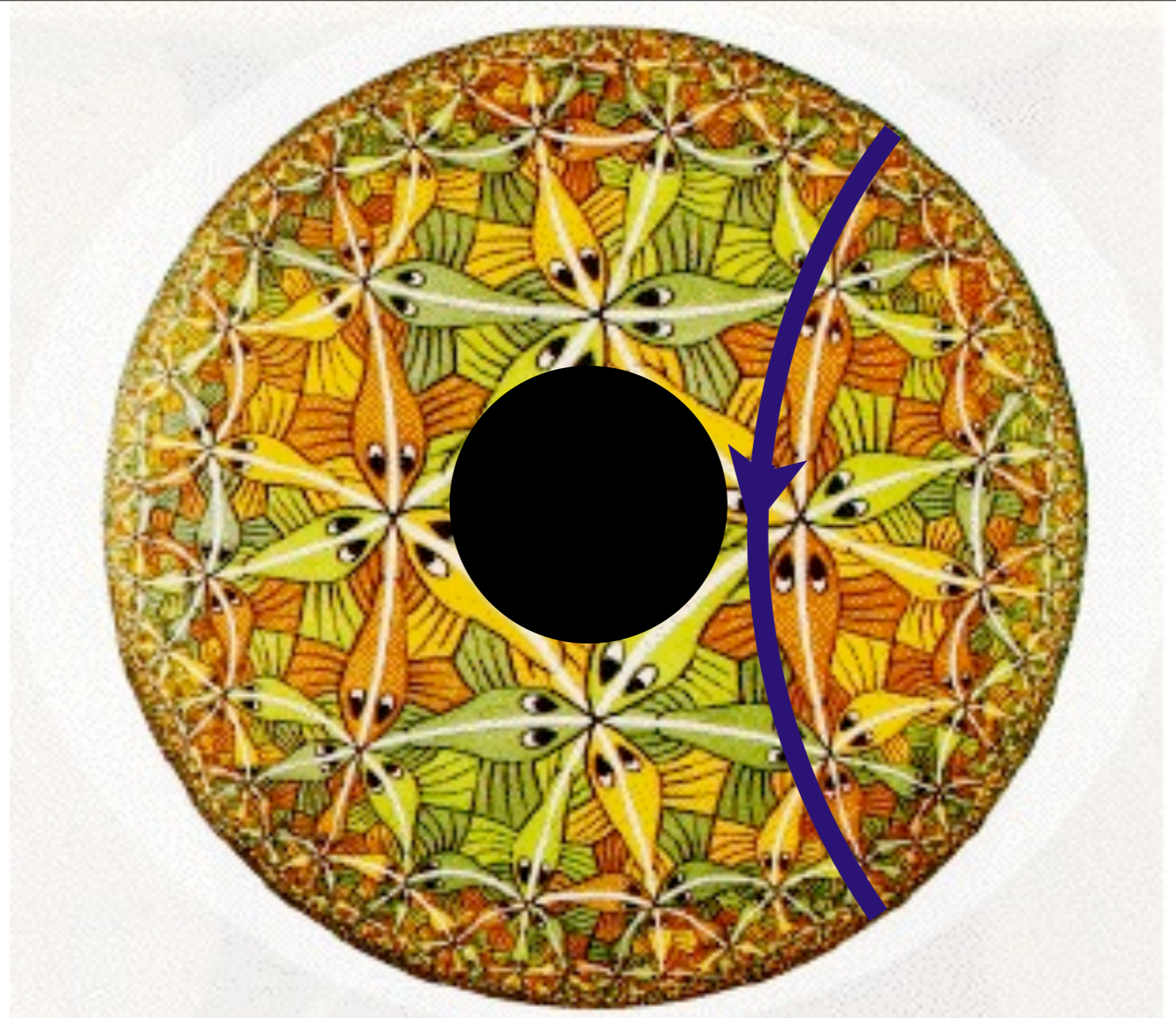
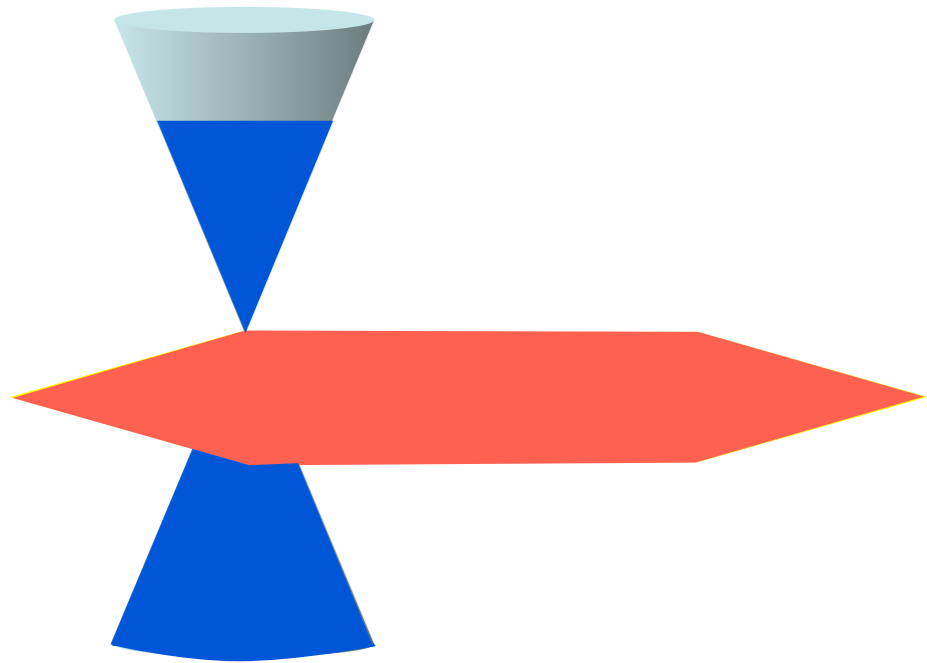


Figure 1: The extra ('radial') dimension of the bulk is the resolution scale of the field theory. The left figure indicates a series of block spin transformations labelled by a parameter z . The right figure is a cartoon of AdS space, which organizes the field theory information in the same way. In this sense, the bulk picture is a hologram: excitations with different wavelengths get put in different places in the bulk image.

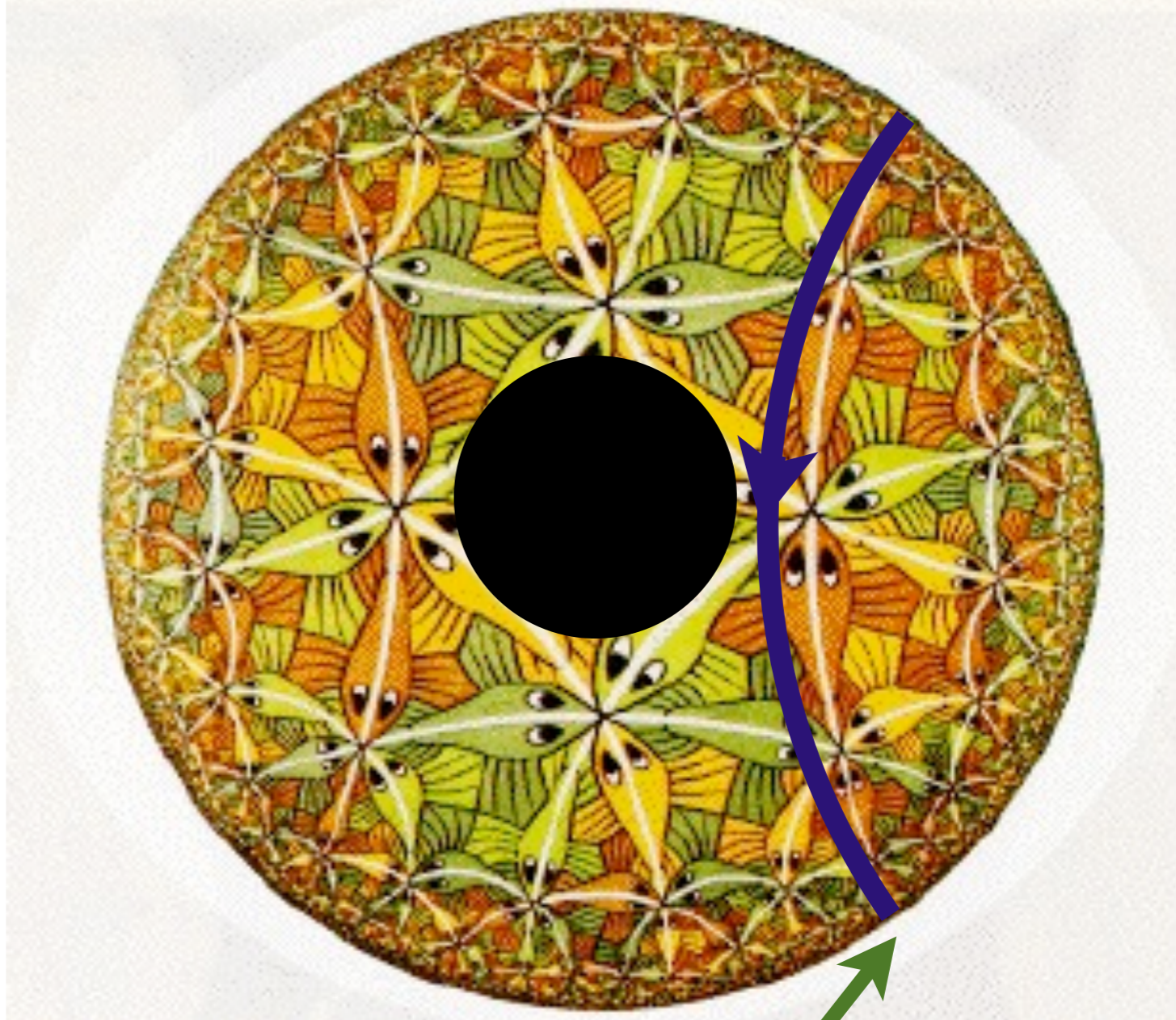
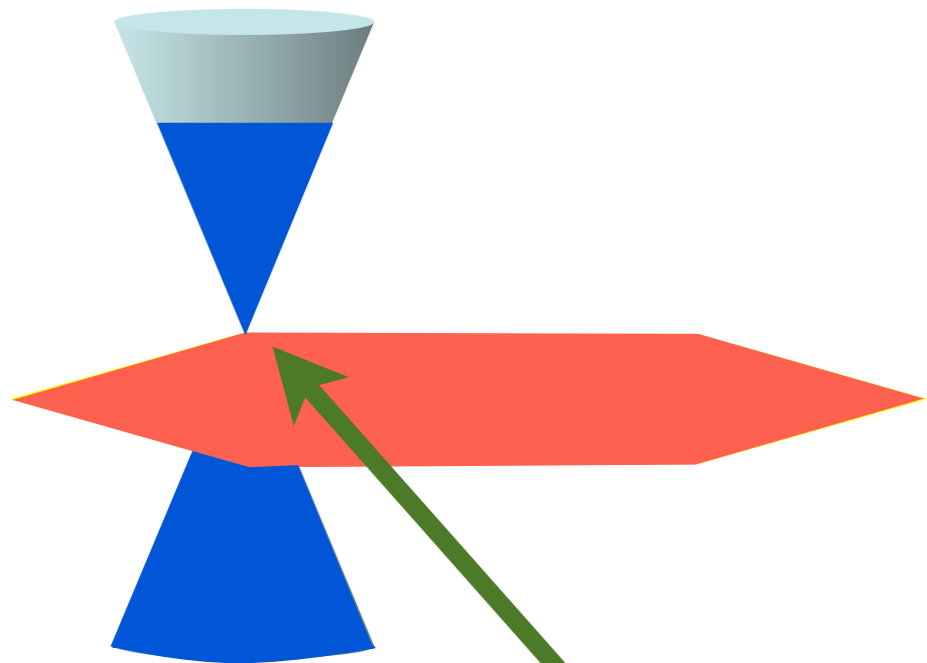
J. McGreevy, arXiv0909.0518





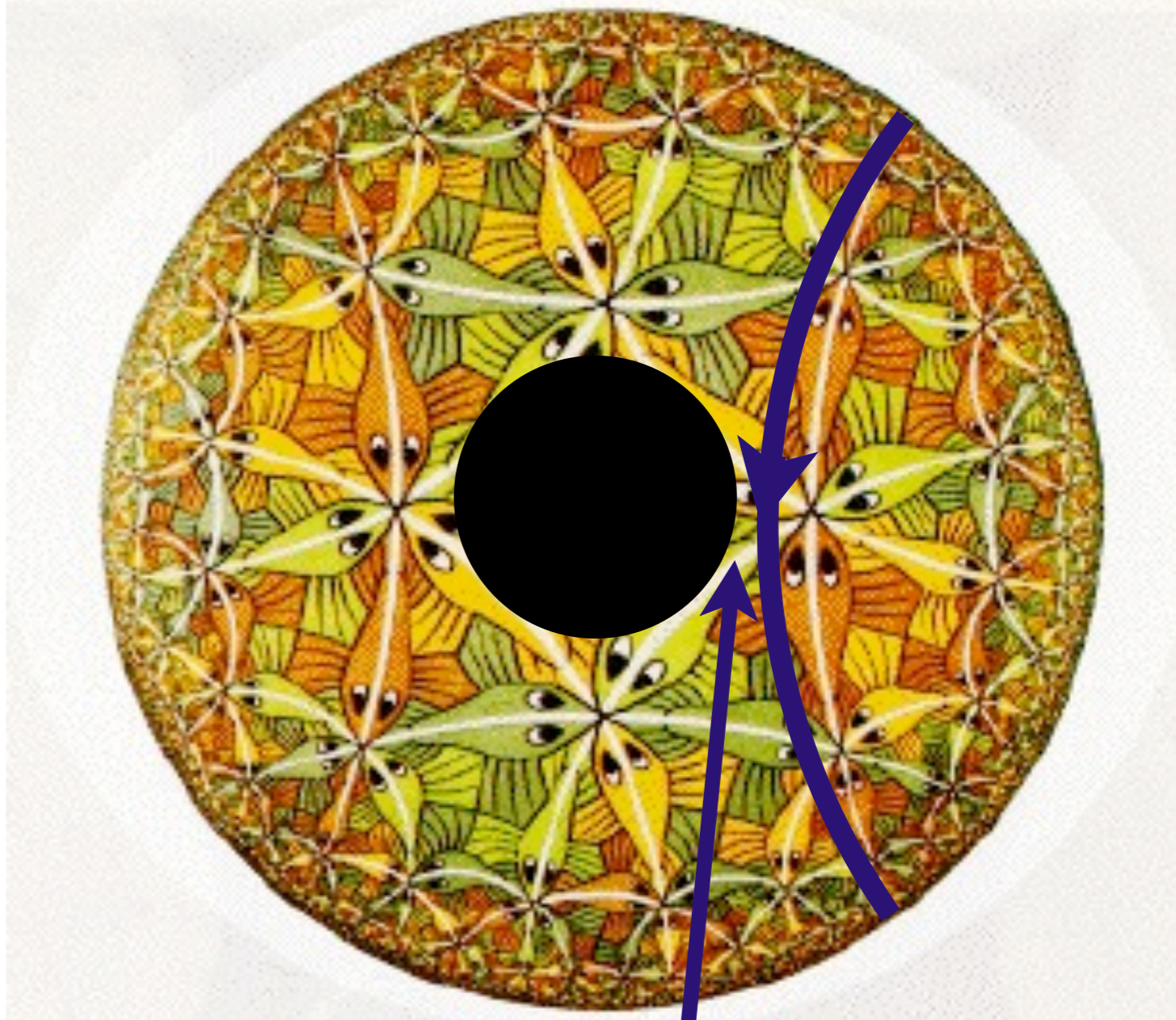
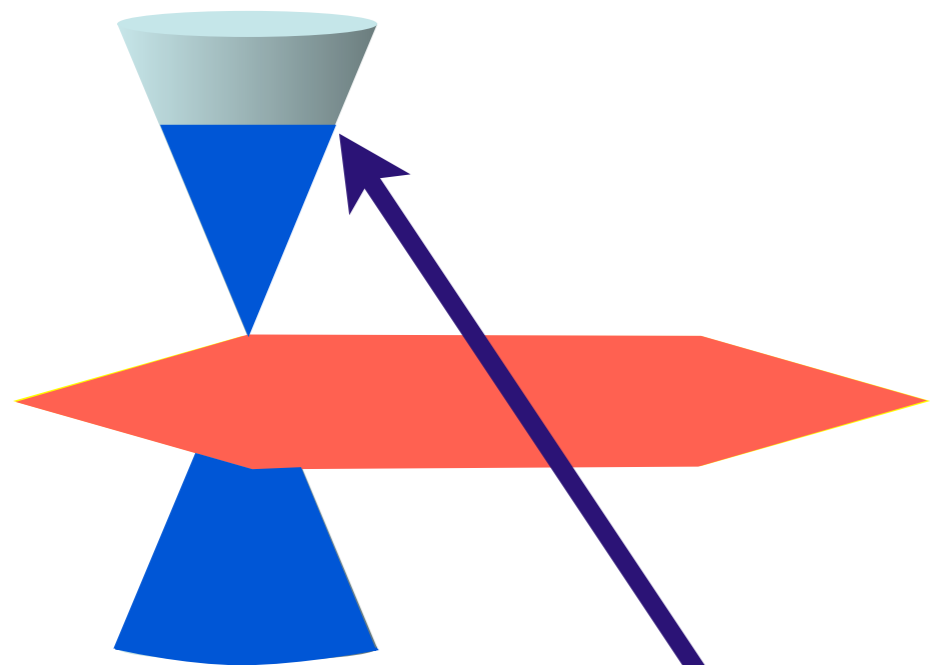
Examine free energy and Green's function
of a probe particle

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788



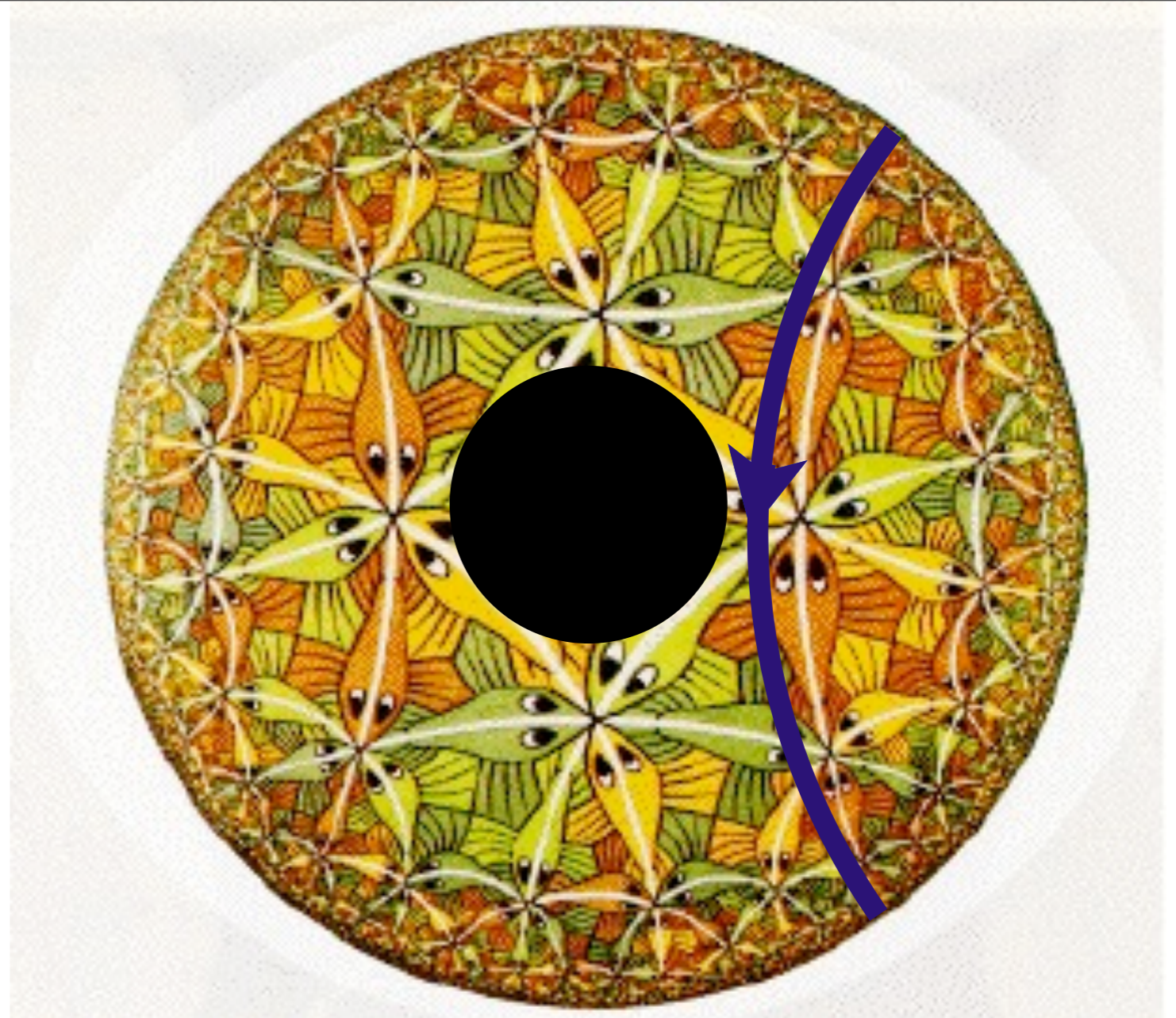
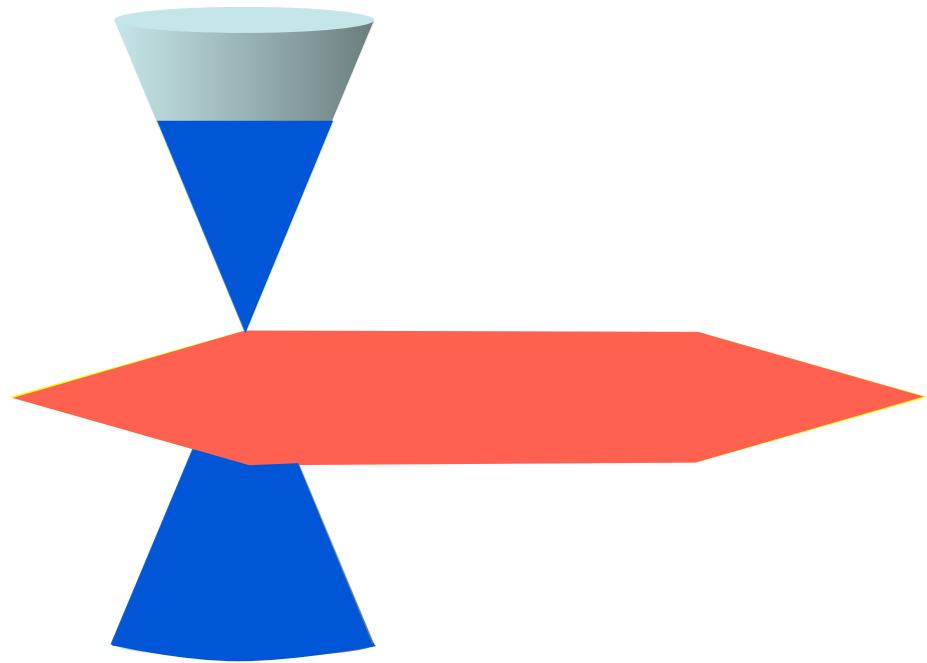
Short time behavior depends upon
conformal AdS_4 geometry near boundary

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788



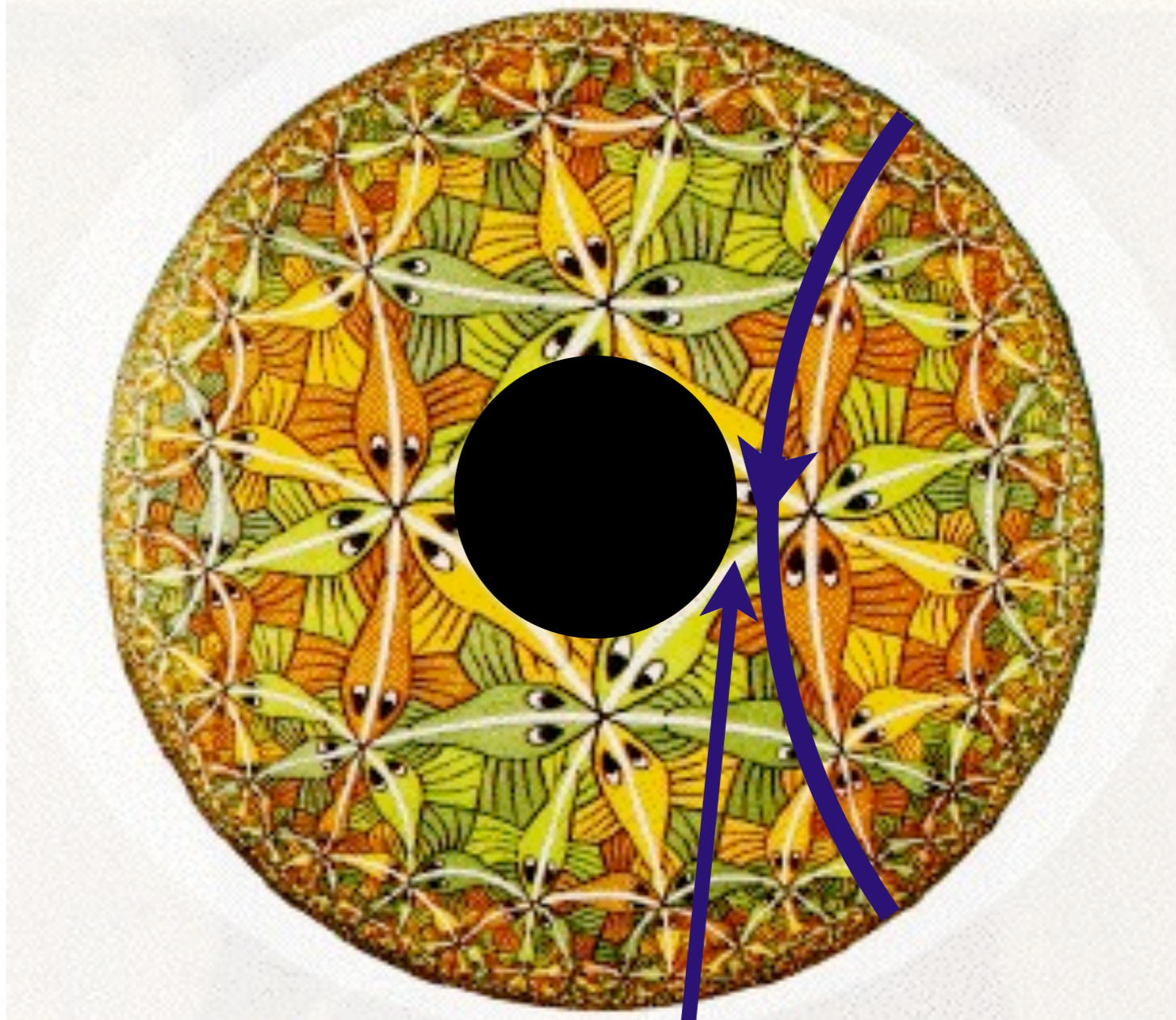
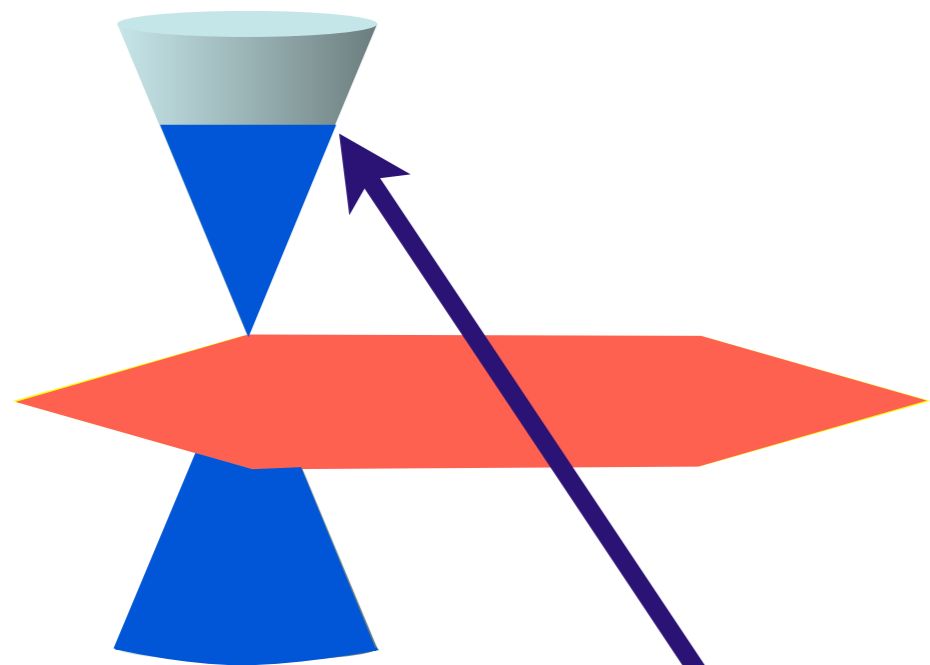
Long time behavior depends upon
near-horizon geometry of black hole

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
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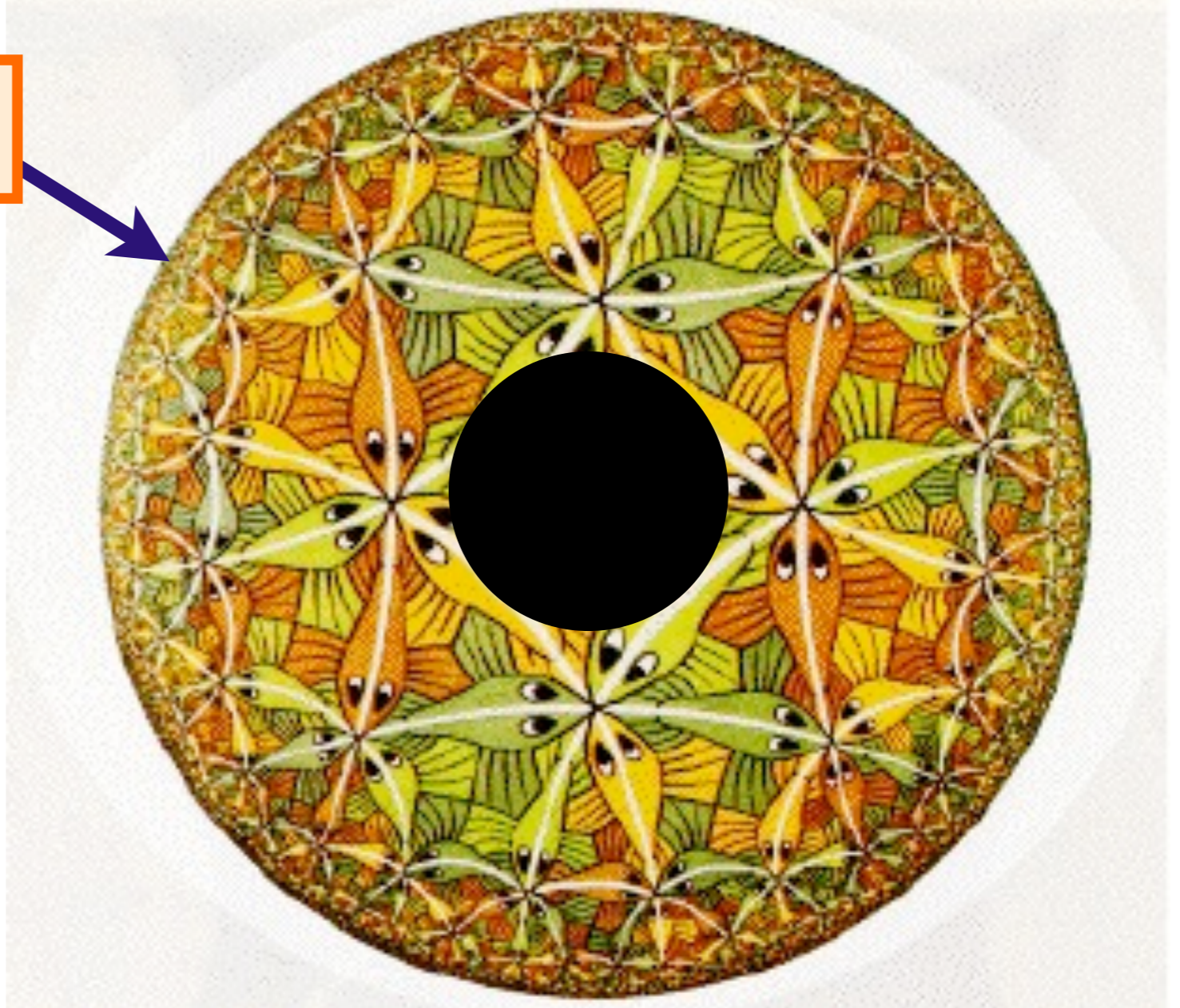
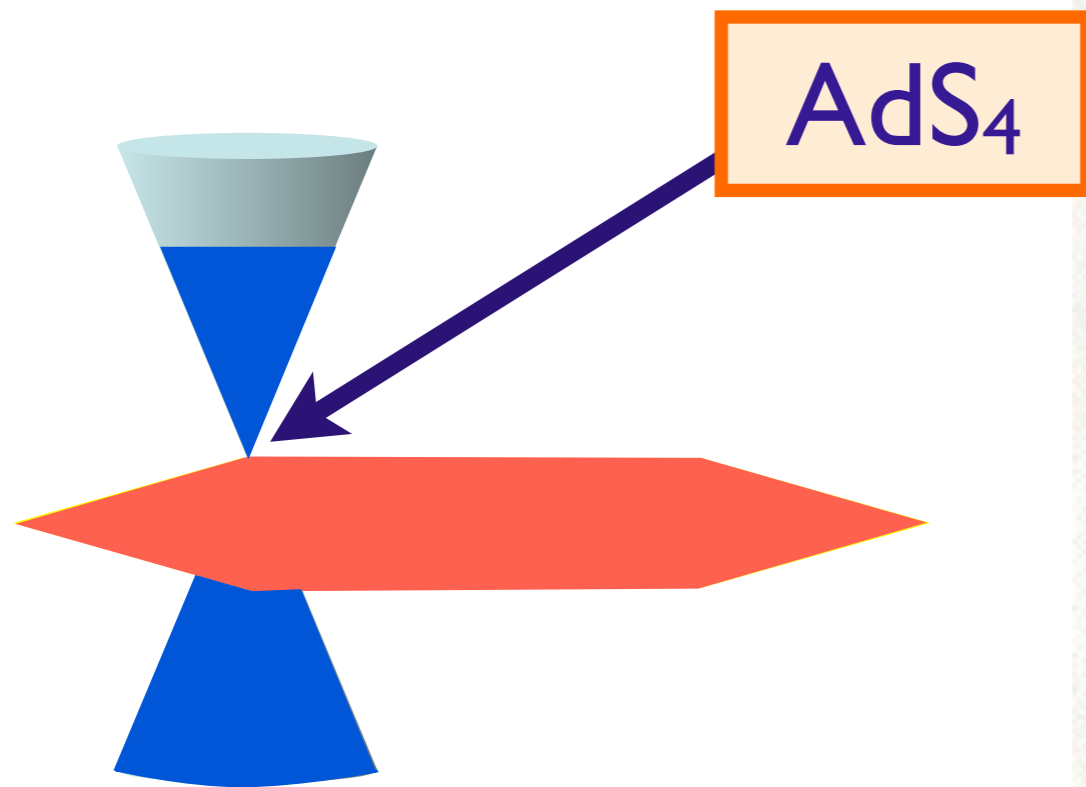
Radial direction of gravity theory is
measure of energy scale in CFT

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788



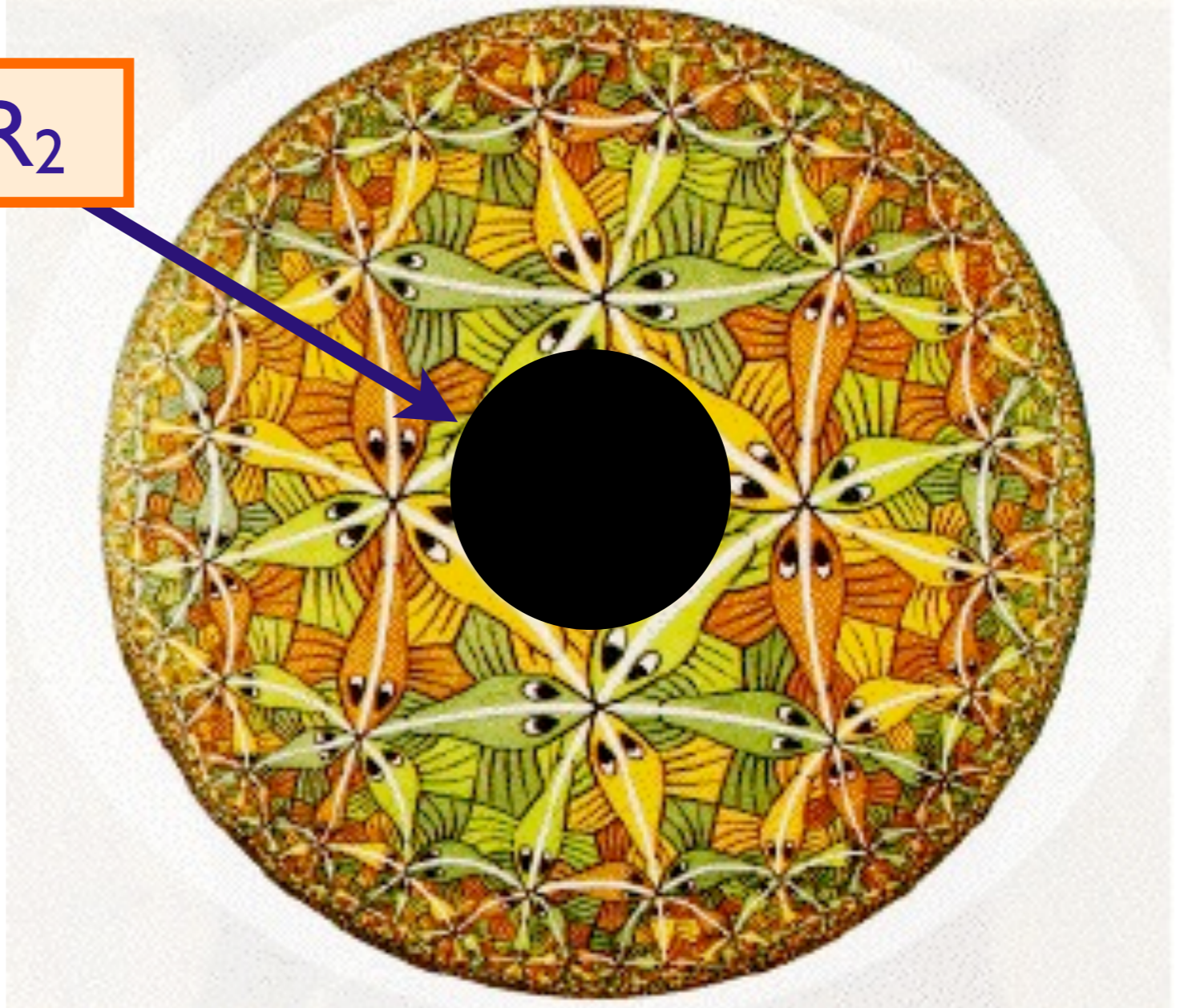
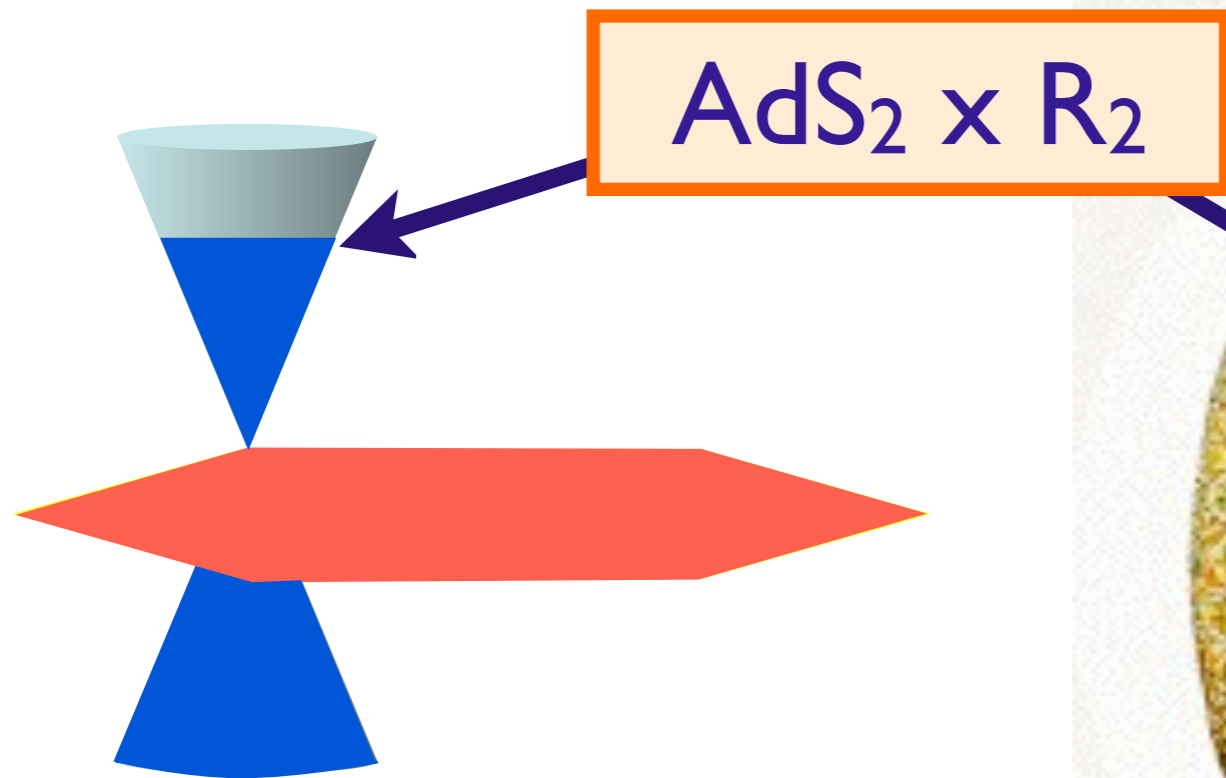
Infrared physics of Fermi surface is linked to the near horizon AdS_2 geometry of Reissner-Nordstrom black hole

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694



Geometric interpretation of RG flow

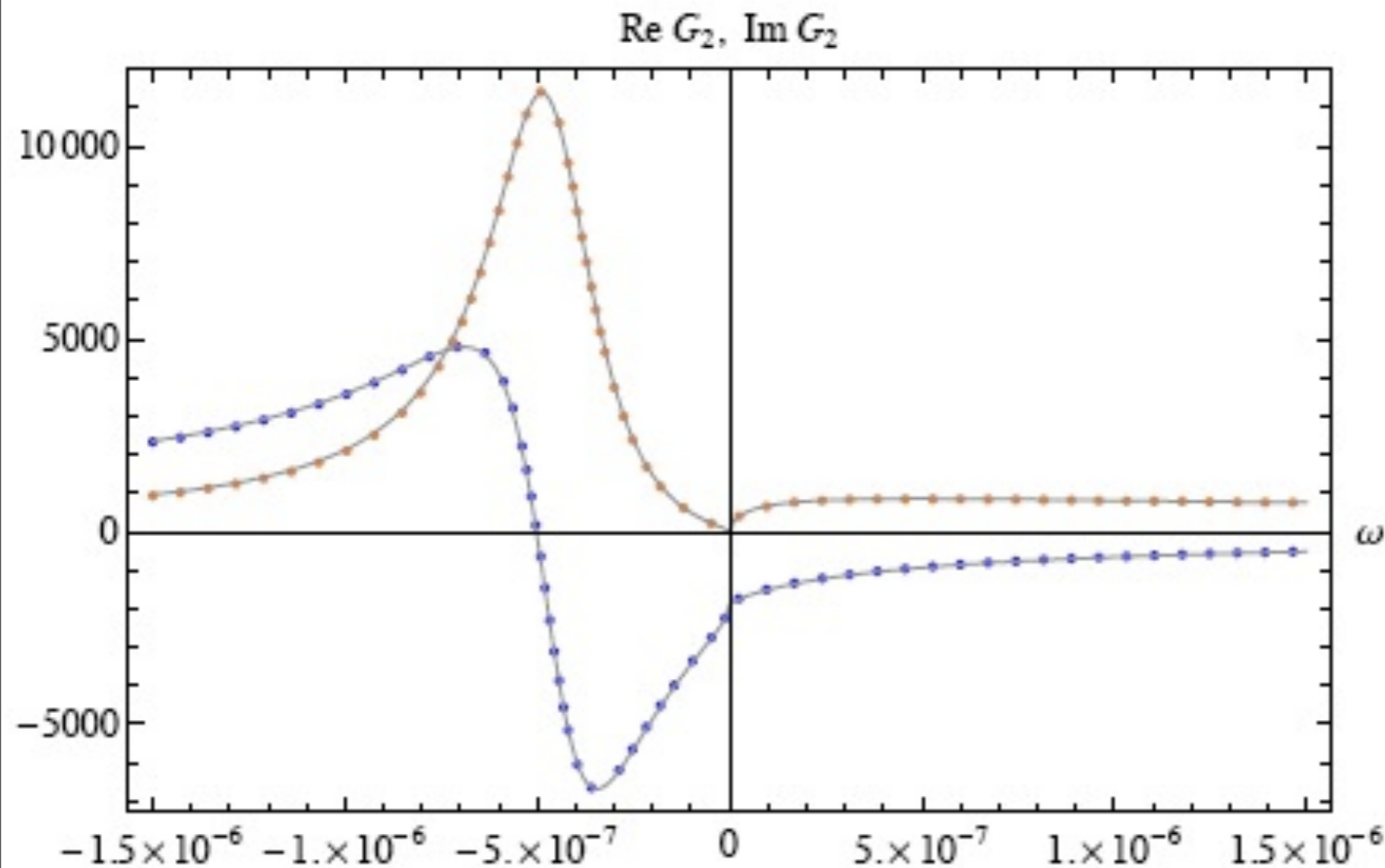
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Green's function of a fermion



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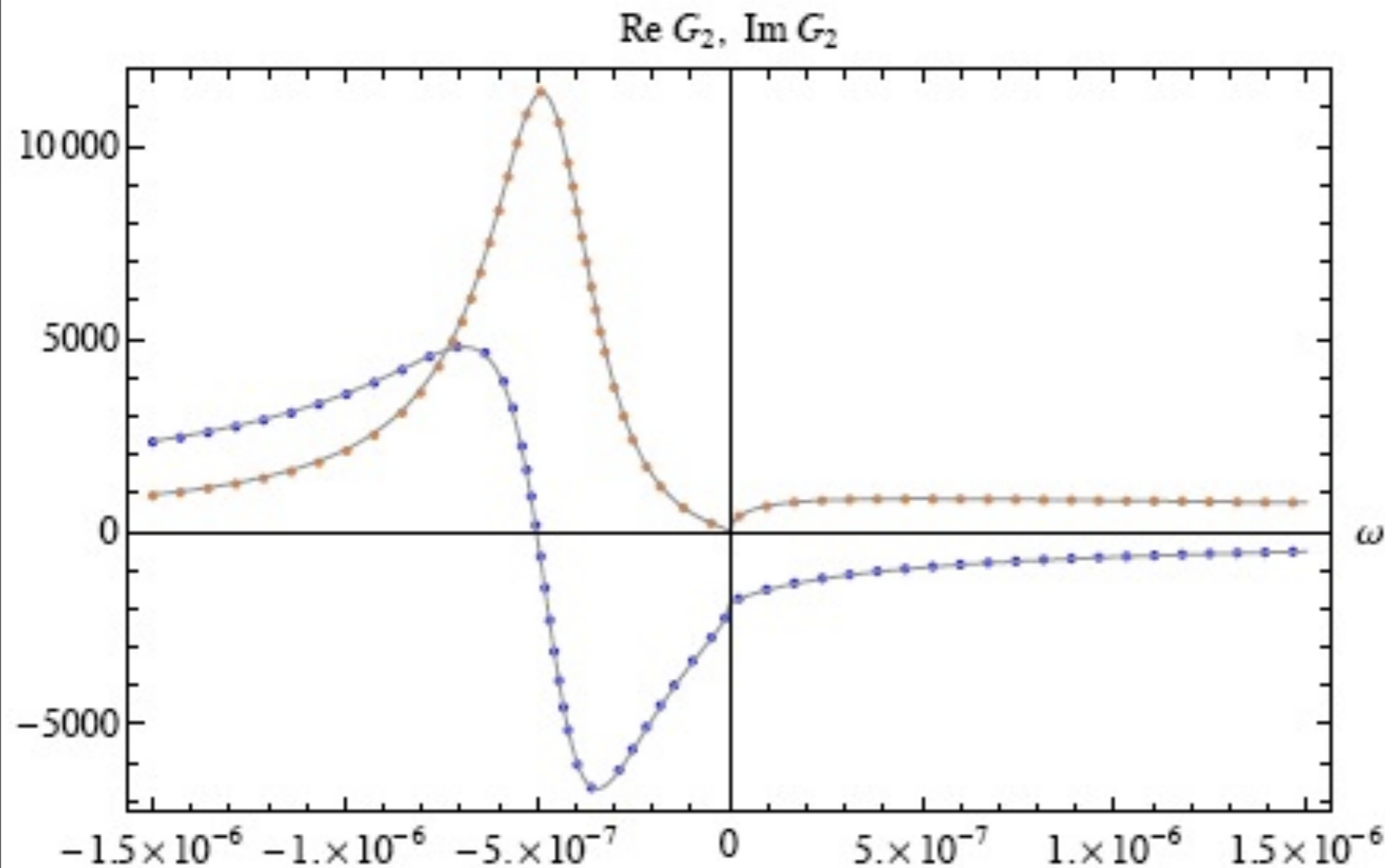
$$G(k, \omega) \approx \frac{1}{\omega - v_F(k - k_F) - i\omega^\theta(k)}$$

See also S.-S. Lee, *Phys. Rev. D* **79**, 086006 (2009);

M. Cubrovic, J. Zaanen, and K. Schalm, *Science* **325**, 439 (2009);

F. Denef, S.A. Hartnoll, and S. Sachdev, *Phys. Rev. D* **80**, 126016 (2009)

Green's function of a fermion

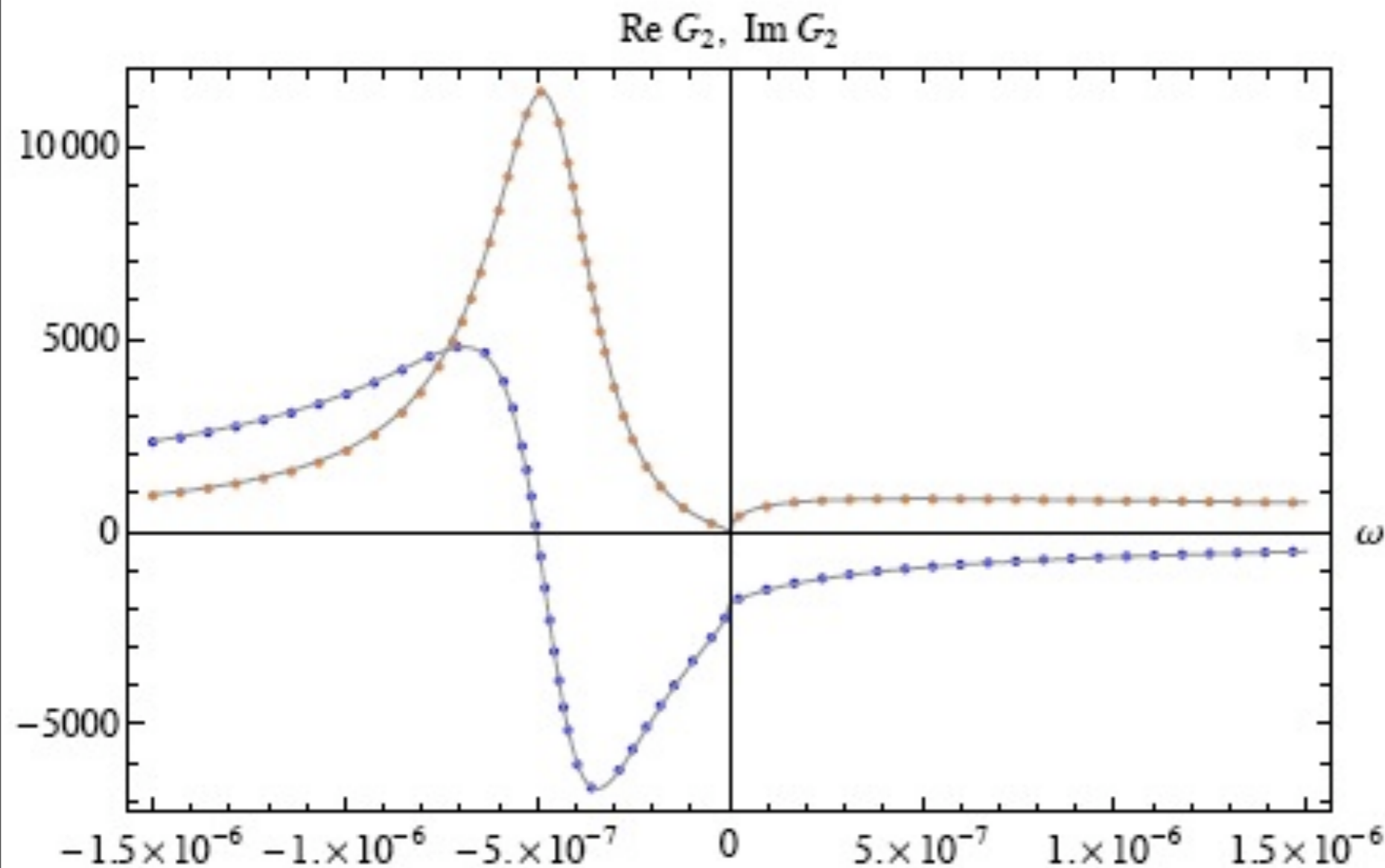


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near the Ising-nematic quantum critical point

Green's function of a fermion

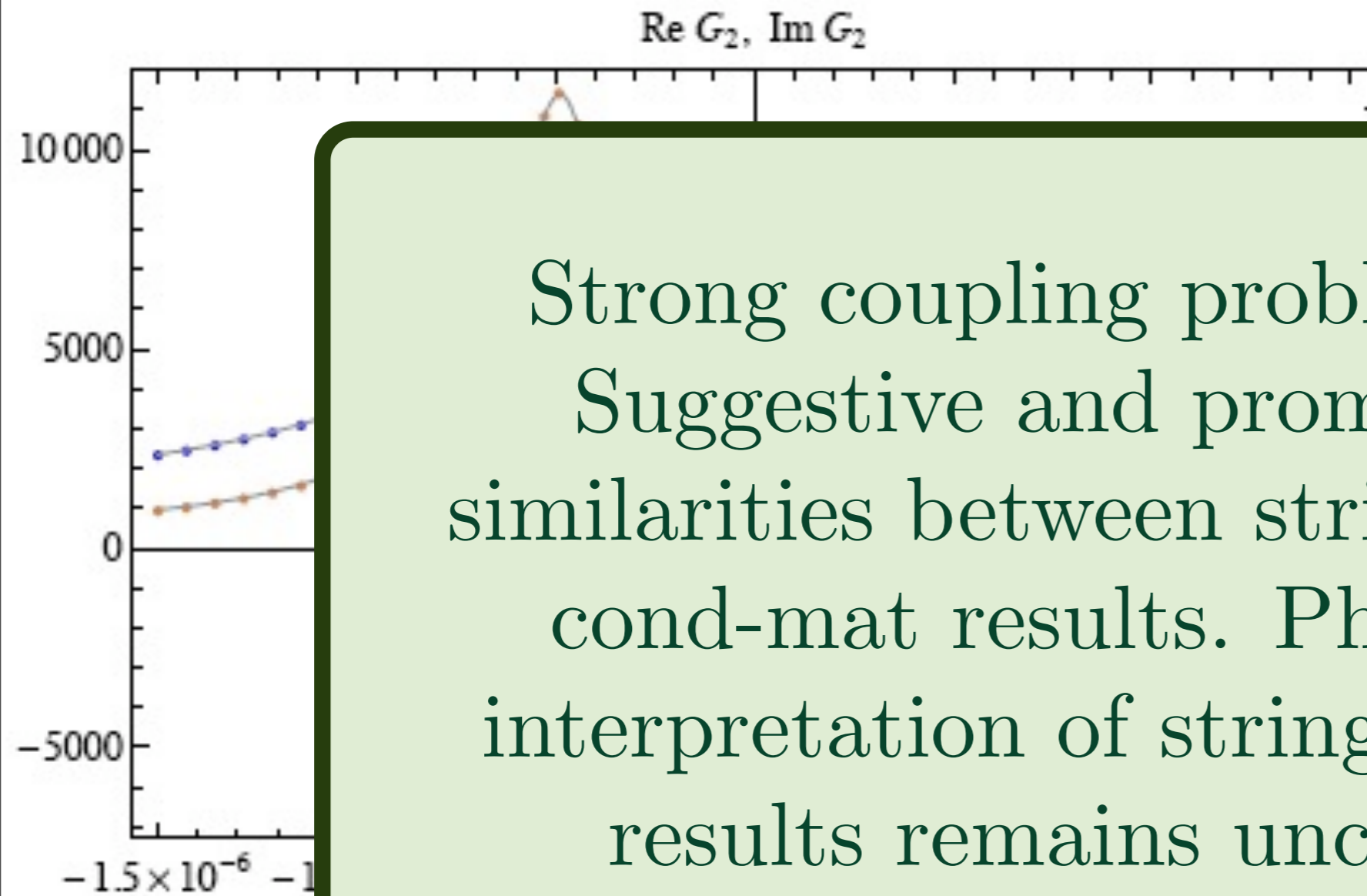


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Strong coupling problem II:
Suggestive and promising similarities between stringy and cond-mat results. Physical interpretation of string theory results remains unclear.

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$$G(k, \omega) \sim \frac{1}{\omega - v_F(k - k_F) - i\omega^\theta(k)}$$

Similar to our theory of the singular Fermi surface near the Ising-nematic quantum critical point

Conclusions

Theories for the onset of Ising-nematic order (and spin density wave order) in metals are strongly coupled in two dimensions

Conclusions

The AdS/CFT offers promise in providing a new understanding of strongly interacting quantum matter at non-zero density