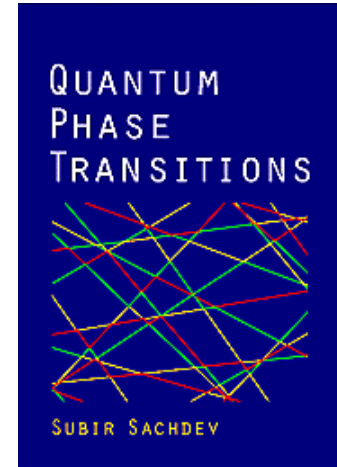


Magnetic phases and critical points of insulators and superconductors

- Colloquium article in *Reviews of Modern Physics*, July 2003, cond-mat/0211005.
- cond-mat/0109419



Quantum Phase Transitions
Cambridge University Press

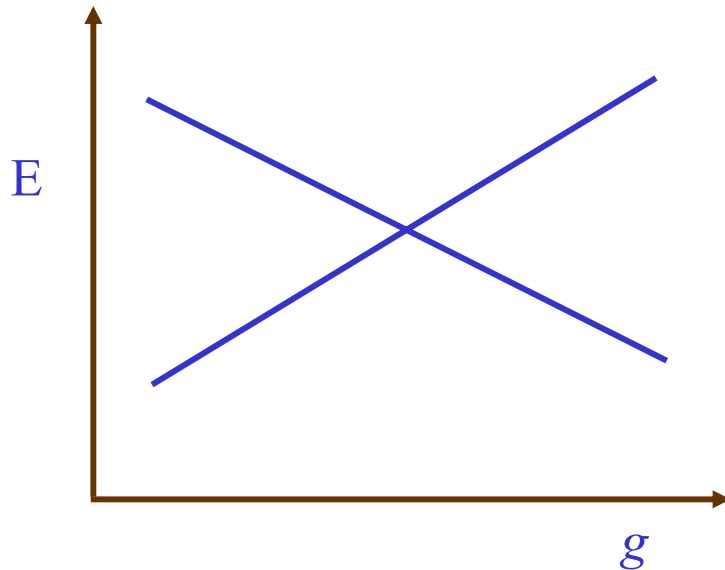


Talks online:
Google™ Sachdev



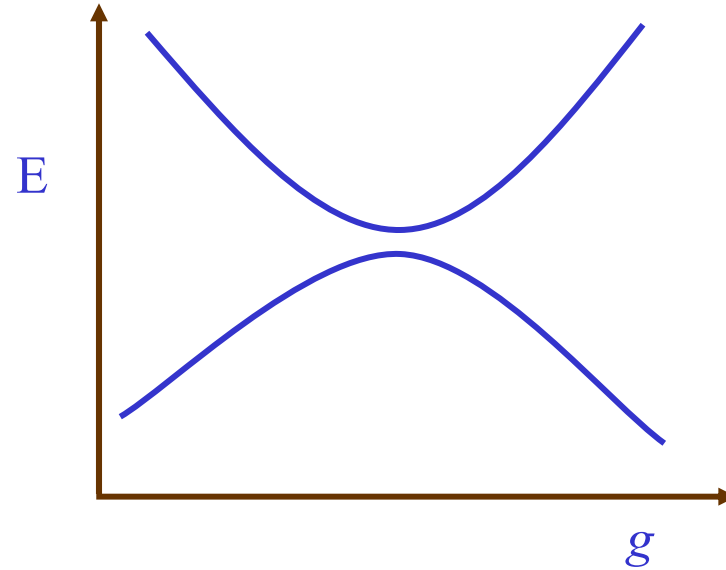
What is a quantum phase transition ?

Non-analyticity in ground state properties as a function of some control parameter g



True level crossing:

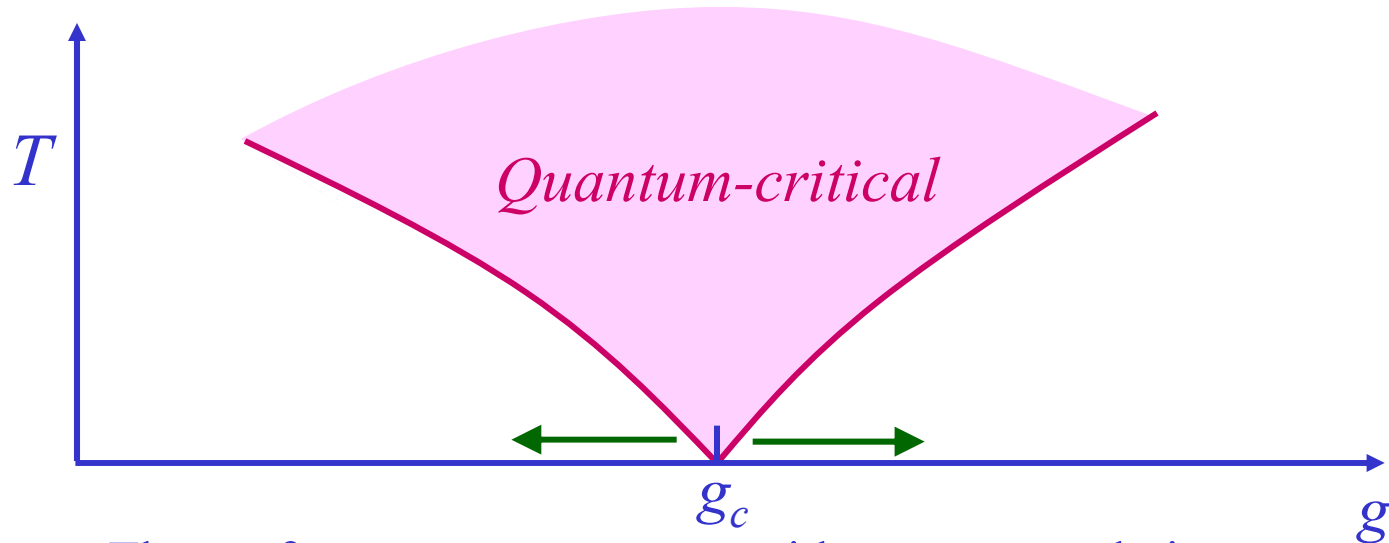
Usually a *first-order* transition



Avoided level crossing which becomes sharp in the infinite volume limit:

second-order transition

Why study quantum phase transitions ?



- Theory for a quantum system with strong correlations: describe phases on either side of g_c by expanding in deviation from the quantum critical point.
- Critical point is a novel state of matter without quasiparticle excitations
- Critical excitations control dynamics in the wide *quantum-critical* region at non-zero temperatures.

Important property of ground state at $g=g_c$:
temporal and spatial scale invariance;
characteristic energy scale at other values of g : $\Delta \sim |g - g_c|^{z\nu}$

Outline

I. Quantum Ising chain

II. Coupled Dimer Antiferromagnet

A. Coherent state path integral

B. Quantum field theory near critical point

III. Coupled dimer antiferromagnet in a magnetic field

Bose condensation of “triplons”

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Quantum phase transition in a background

Abrikosov flux lattice

V. Antiferromagnets with an odd number of $S=1/2$ spins per unit cell.

Class A: Compact $U(1)$ gauge theory: collinear spins, bond order and confined spinons in $d=2$

Class B: Z_2 gauge theory: non-collinear spins, RVB, visons, topological order, and deconfined spinons

VI. Conclusions

Single order parameter.

Multiple order parameters

I. Quantum Ising Chain

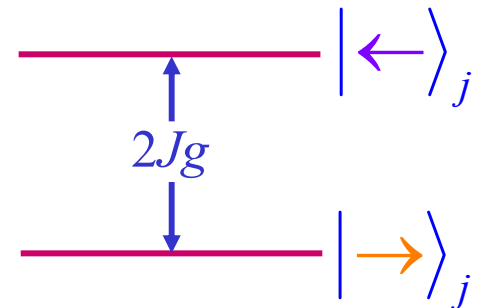
Degrees of freedom: $j = 1 \dots N$ qubits, N "large"

$$|\uparrow\rangle_j, |\downarrow\rangle_j$$

$$\text{or } |\rightarrow\rangle_j = \frac{1}{\sqrt{2}} (|\uparrow\rangle_j + |\downarrow\rangle_j), \quad |\leftarrow\rangle_j = \frac{1}{\sqrt{2}} (|\uparrow\rangle_j - |\downarrow\rangle_j)$$

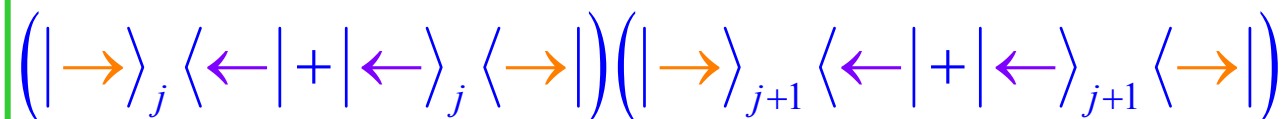
Hamiltonian of decoupled qubits:

$$H_0 = -Jg \sum_j \sigma_j^x$$



Coupling between qubits:

$$H_1 = -J \sum_j \underbrace{\sigma_j^z \sigma_{j+1}^z}$$


$$\left(\left| \rightarrow \right\rangle_j \left\langle \leftarrow \right| + \left| \leftarrow \right\rangle_j \left\langle \rightarrow \right| \right) \left(\left| \rightarrow \right\rangle_{j+1} \left\langle \leftarrow \right| + \left| \leftarrow \right\rangle_{j+1} \left\langle \rightarrow \right| \right)$$

Prefers neighboring qubits

are *either* $\left| \uparrow \right\rangle_j \left| \uparrow \right\rangle_{j+1}$ *or* $\left| \downarrow \right\rangle_j \left| \downarrow \right\rangle_{j+1}$

(not entangled)

Full Hamiltonian

$$H = H_0 + H_1 = -J \sum_j \left(g \sigma_j^x + \sigma_j^z \sigma_{j+1}^z \right)$$

leads to entangled states at g of order unity

Weakly-coupled qubits ($g \gg 1$)

Ground state:

$$|G\rangle = |\cdots \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \cdots\rangle$$

$$-\frac{1}{2g} |\cdots \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \leftarrow \leftarrow \rightarrow \rightarrow \rightarrow \rightarrow \cdots\rangle - \cdots$$

Lowest excited states:

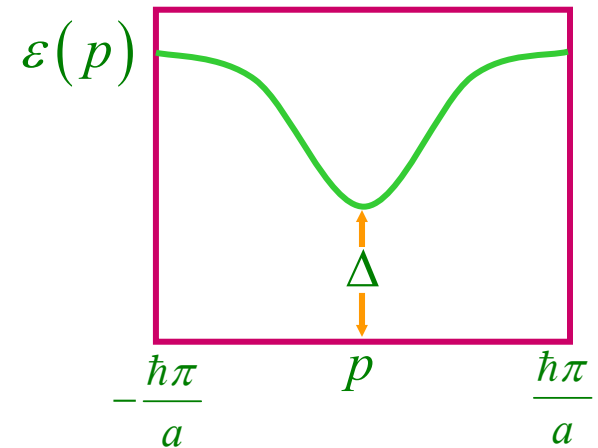
$$|\ell_j\rangle = |\cdots \rightarrow \rightarrow \rightarrow \rightarrow \leftarrow_j \rightarrow \rightarrow \rightarrow \rightarrow \cdots\rangle + \cdots$$

Coupling between qubits creates “flipped-spin” *quasiparticle* states at momentum p

$$|p\rangle = \sum_j e^{ipx_j/\hbar} |\ell_j\rangle$$

$$\text{Excitation energy } \varepsilon(p) = \Delta + 4J \sin^2\left(\frac{pa}{2\hbar}\right) + O(g^{-1})$$

$$\text{Excitation gap } \Delta = 2gJ - 2J + O(g^{-1})$$

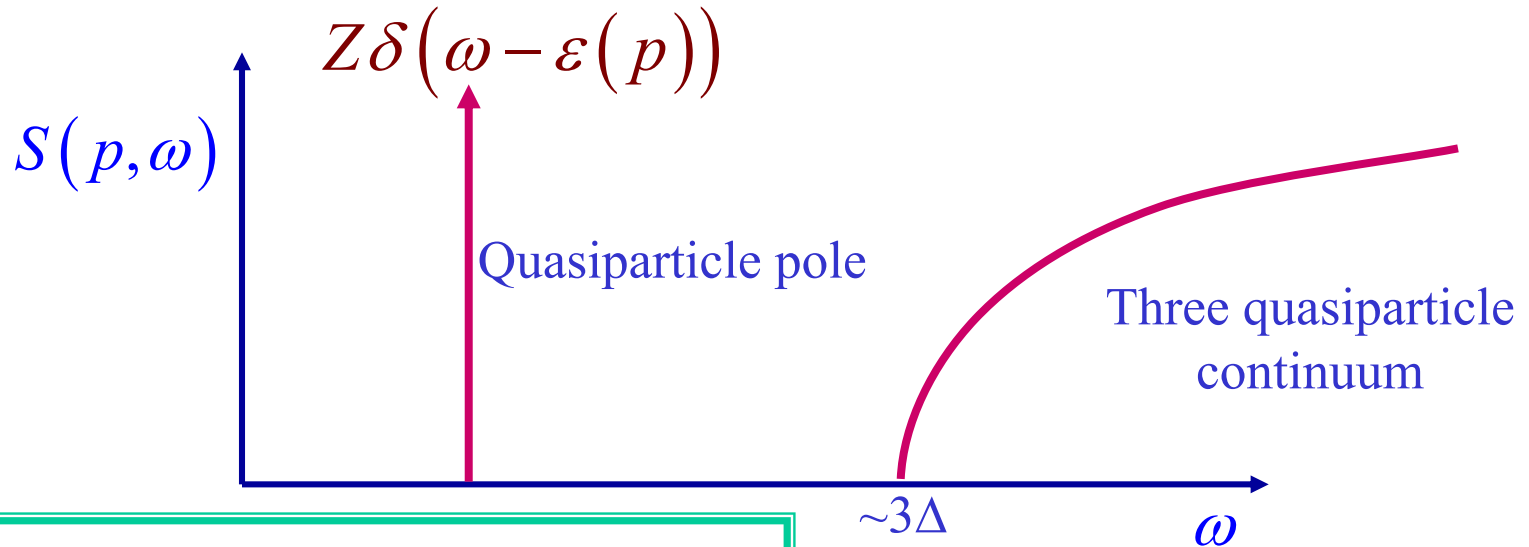


Entire spectrum can be constructed out of multi-quasiparticle states

Dynamic Structure Factor $S(p, \omega)$:

Weakly-coupled qubits ($g \gg 1$)

Cross-section to flip a $|\rightarrow\rangle$ to a $|\leftarrow\rangle$ (or vice versa)
while transferring energy $\hbar\omega$ and momentum p



Structure holds to all orders in $1/g$

At $T > 0$, collisions between quasiparticles broaden pole to a Lorentzian of width $1/\tau_\phi$ where the **phase coherence time** τ_ϕ

is given by

$$\frac{1}{\tau_\phi} = \frac{2k_B T}{\pi \hbar} e^{-\Delta/k_B T}$$

Strongly-coupled qubits ($g \ll 1$)

Ground states:

$$|G \uparrow\rangle = |\dots \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \dots\rangle$$

$$-\frac{g}{2} |\dots \uparrow \uparrow \uparrow \uparrow \downarrow \uparrow \uparrow \uparrow \uparrow \uparrow \dots\rangle - \dots$$

Ferromagnetic moment

$$N_0 = \langle G | \sigma^z | G \rangle \neq 0$$

Second state $|G \downarrow\rangle$ obtained by $\uparrow \Leftrightarrow \downarrow$

$|G \downarrow\rangle$ and $|G \uparrow\rangle$ mix only at order g^N

Lowest excited states: domain walls

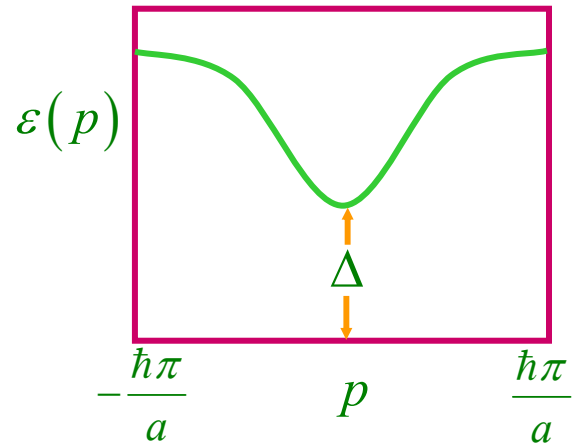
$$|d_j\rangle = |\dots \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow_j \downarrow \downarrow \downarrow \downarrow \downarrow \dots\rangle + \dots$$

Coupling between qubits creates new “domain-wall” *quasiparticle* states at momentum p

$$|p\rangle = \sum_j e^{ipx_j/\hbar} |d_j\rangle$$

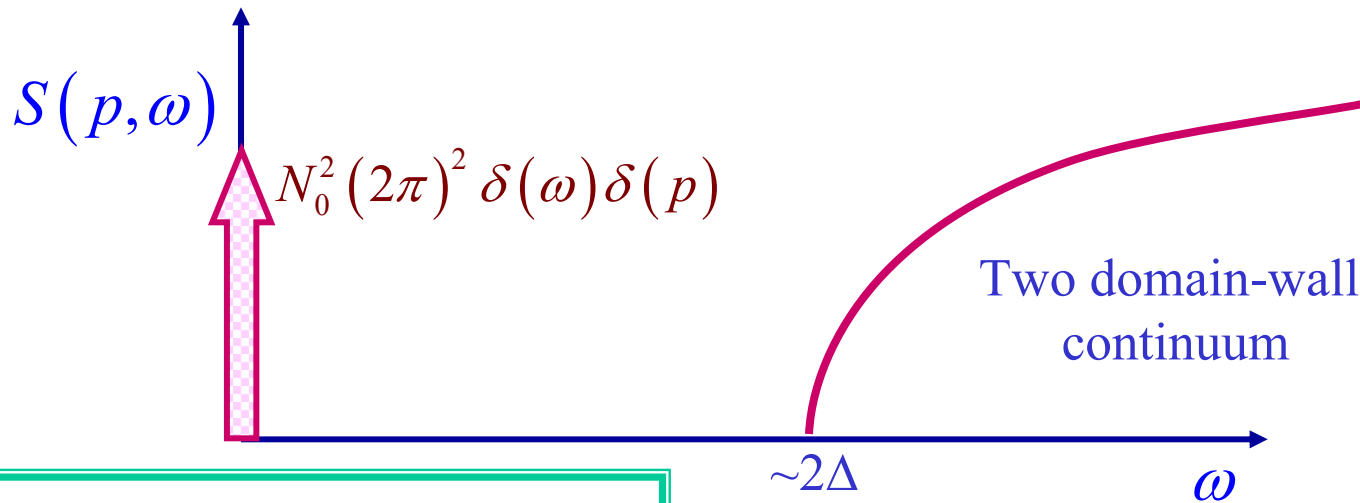
$$\text{Excitation energy } \varepsilon(p) = \Delta + 4Jg \sin^2\left(\frac{pa}{2\hbar}\right) + O(g^2)$$

$$\text{Excitation gap } \Delta = 2J - 2gJ + O(g^2)$$



Dynamic Structure Factor $\mathcal{S}(p, \omega)$: Strongly-coupled qubits ($g \ll 1$)

Cross-section to flip a $|\rightarrow\rangle$ to a $|\leftarrow\rangle$ (or vice versa)
 while transferring energy $\hbar\omega$ and momentum p



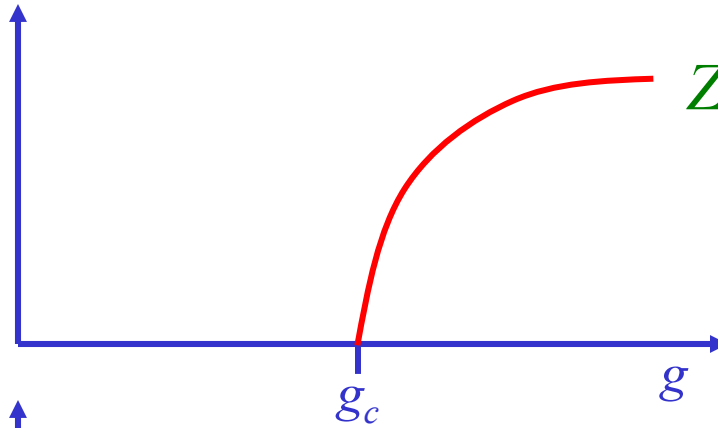
Structure holds to all orders in g

At $T > 0$, motion of domain walls leads to a finite *phase coherence time* τ_ϕ ,

and broadens coherent peak to a width $1/\tau_\phi$ where
$$\frac{1}{\tau_\phi} = \frac{2k_B T}{\pi \hbar} e^{-\Delta/k_B T}$$

Entangled states at g of order unity

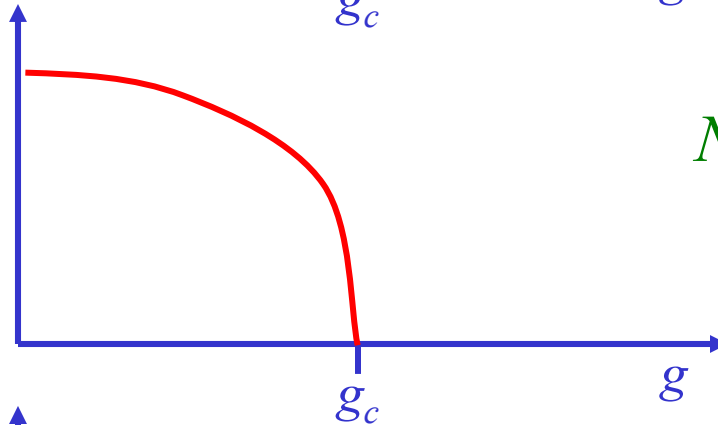
“Flipped-spin”
Quasiparticle
weight Z



$$Z \sim (g - g_c)^{1/4}$$

A.V. Chubukov, S. Sachdev, and J. Ye,
Phys. Rev. B **49**, 11919 (1994)

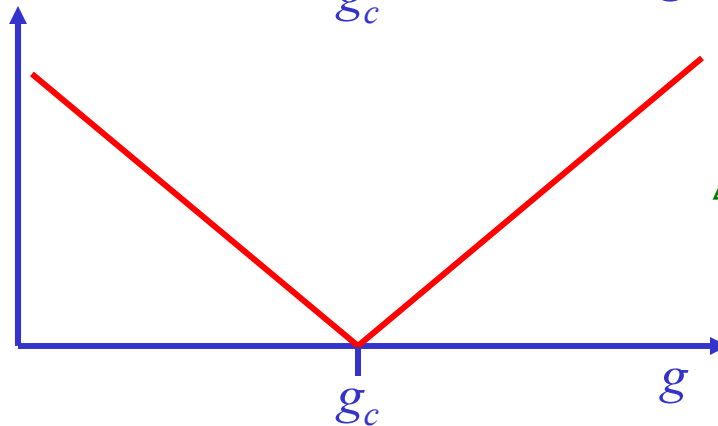
Ferromagnetic
moment N_0



$$N_0 \sim (g_c - g)^{1/8}$$

P. Pfeuty *Annals of Physics*, **57**, 79 (1970)

Excitation
energy gap Δ

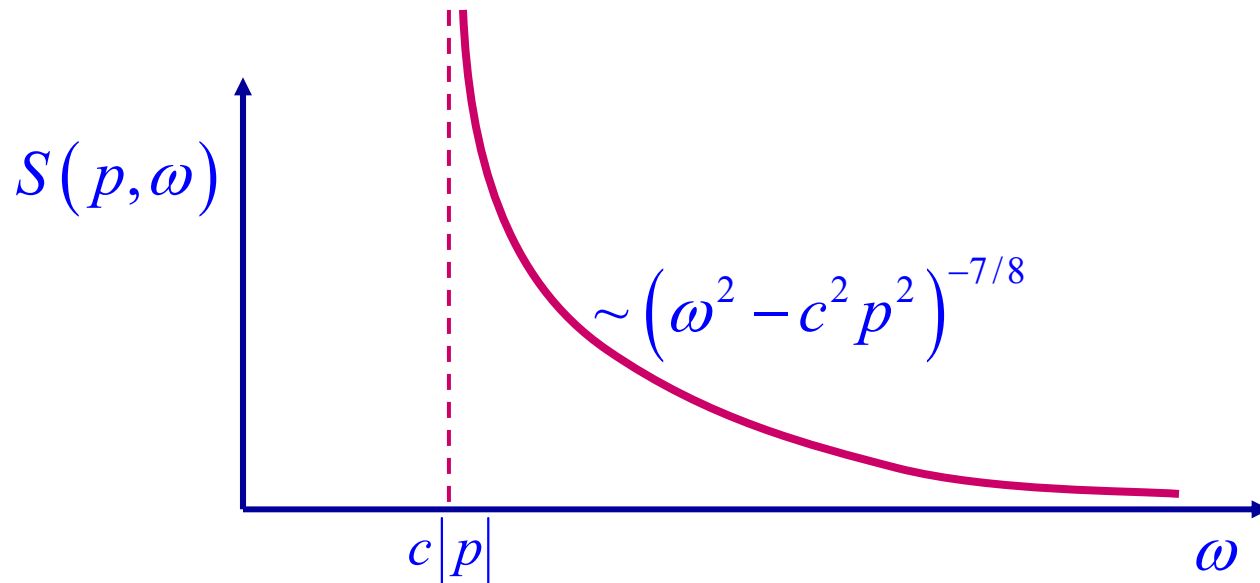


$$\Delta \sim |g - g_c|$$

Dynamic Structure Factor $S(p, \omega)$:

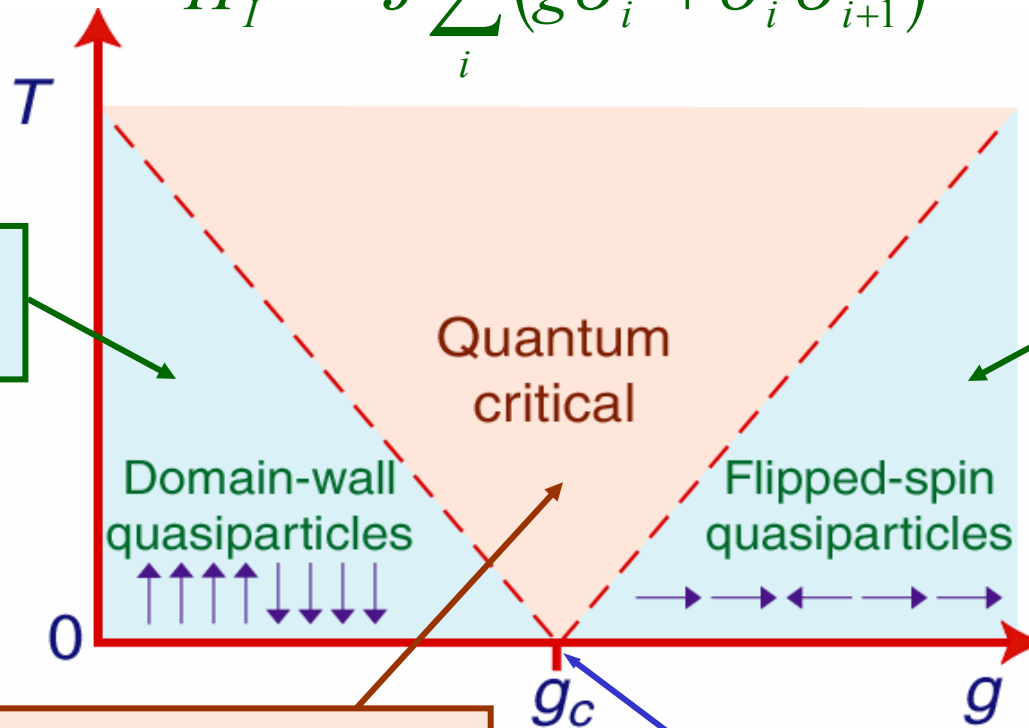
Critical coupling ($g = g_c$)

Cross-section to flip a $|\rightarrow\rangle$ to a $|\leftarrow\rangle$ (or vice versa)
while transferring energy $\hbar\omega$ and momentum p



No quasiparticles --- dissipative critical continuum

$$H_I = -J \sum_i (g \sigma_i^x + \sigma_i^z \sigma_{i+1}^z)$$



Quasiclassical dynamics

Quasiclassical dynamics

$$\chi(\omega) = \frac{i}{\hbar} \sum_k \int_0^\infty dt \langle [\sigma_j^z(t), \sigma_k^z(0)] \rangle e^{i\omega t}$$

$$= \frac{A}{T^{7/4} (1 - i\omega/\Gamma_R + \dots)}$$

$$\Gamma_R = \left(2 \tan \frac{\pi}{16} \right) \frac{k_B T}{\hbar}$$

$$\langle \sigma_j^z \sigma_k^z \rangle \sim \frac{1}{|j-k|^{1/4}}$$

P. Pfeuty *Annals of Physics*, **57**, 79 (1970)

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
 S. Sachdev and A.P. Young, *Phys. Rev. Lett.* **78**, 2220 (1997).

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Multiple order parameters

II. Coupled Dimer Antiferromagnet

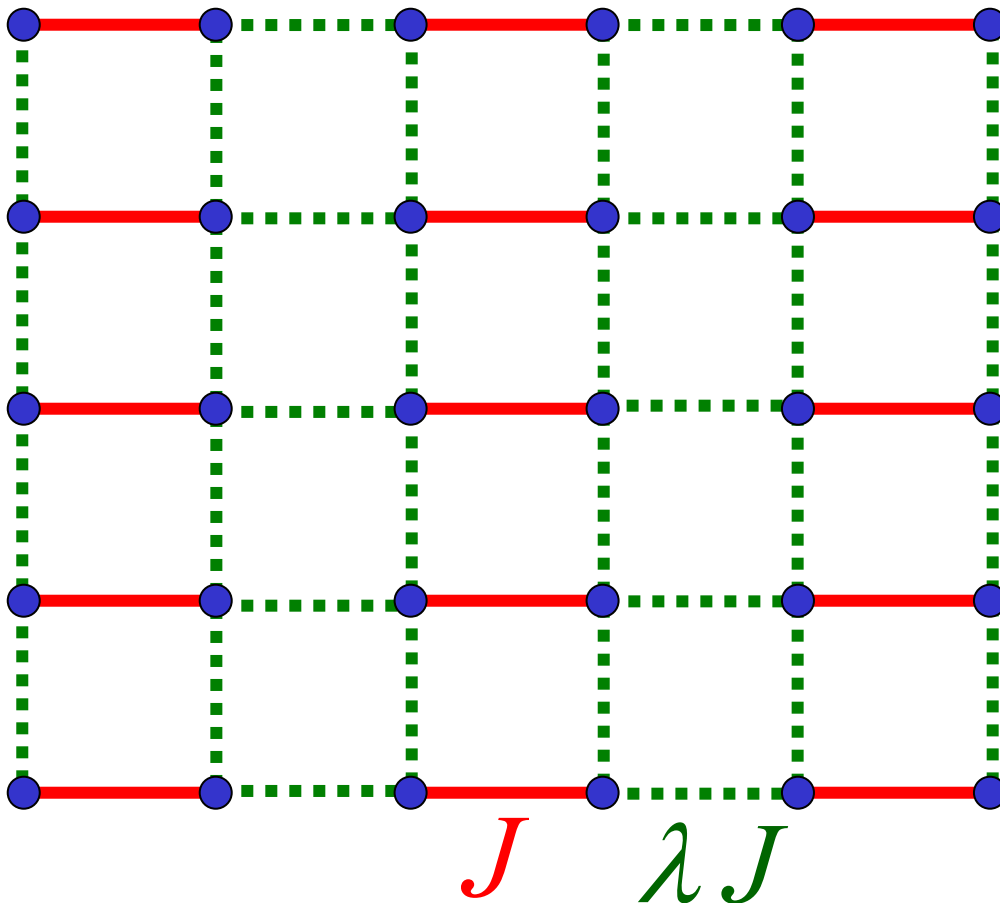
M. P. Gelfand, R. R. P. Singh, and D. A. Huse, *Phys. Rev. B* **40**, 10801-10809 (1989).

N. Katoh and M. Imada, *J. Phys. Soc. Jpn.* **63**, 4529 (1994).

J. Tworzydło, O. Y. Osman, C. N. A. van Duin, J. Zaanen, *Phys. Rev. B* **59**, 115 (1999).

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev. B* **65**, 014407 (2002).

$S=1/2$ spins on coupled dimers



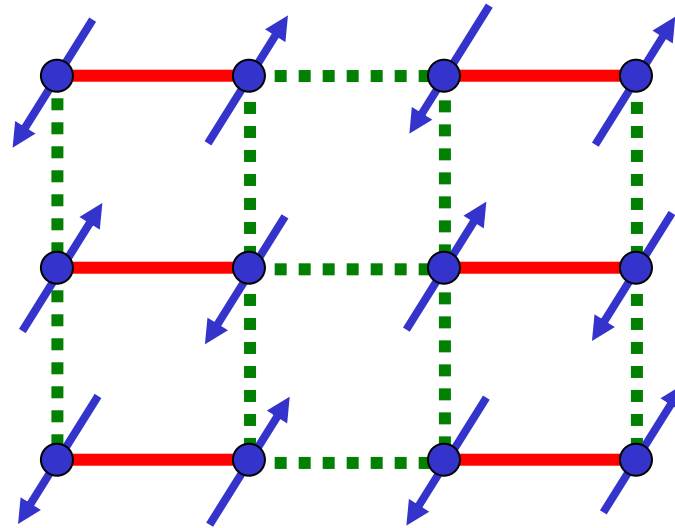
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \leq \lambda \leq 1$$

λ close to 1

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



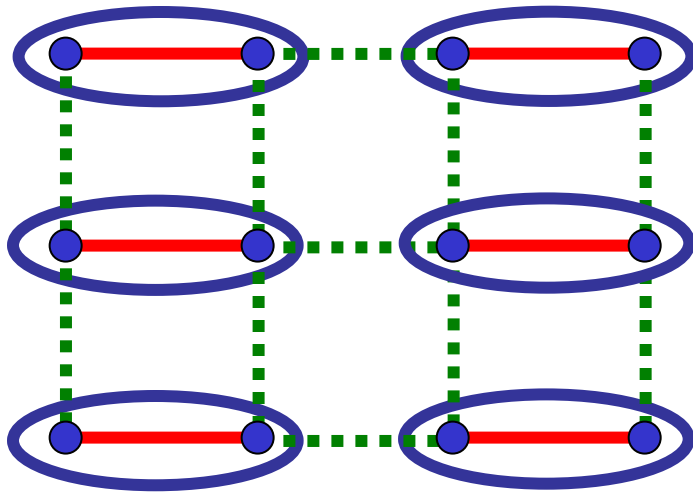
Ground state has long-range magnetic (Neel) order

$$\langle \vec{S}_i \rangle = (-1)^{i_x + i_y} N_0 \neq 0$$

Excitations: 2 spin waves (*magnons*) $\varepsilon_p = \sqrt{c_x^2 p_x^2 + c_y^2 p_y^2}$

λ close to 0

Weakly coupled dimers



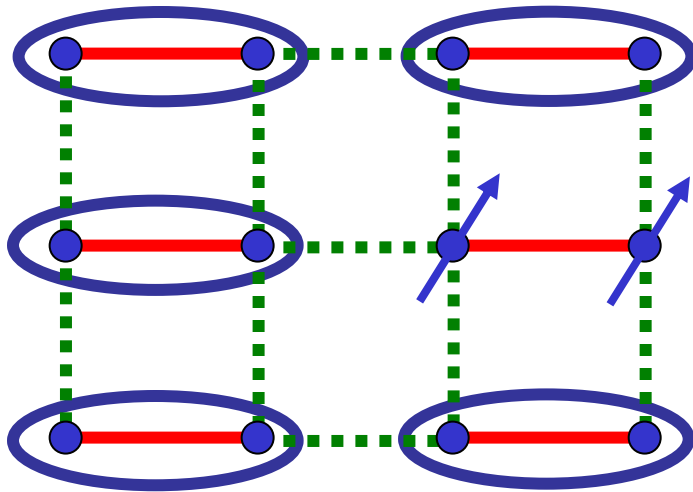
$$\text{dimer} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Paramagnetic ground state

$$\langle \vec{S}_i \rangle = 0$$

λ close to 0

Weakly coupled dimers



$$\text{dimer} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Excitation: $S=1$ *triplon* (*exciton*, spin collective mode)

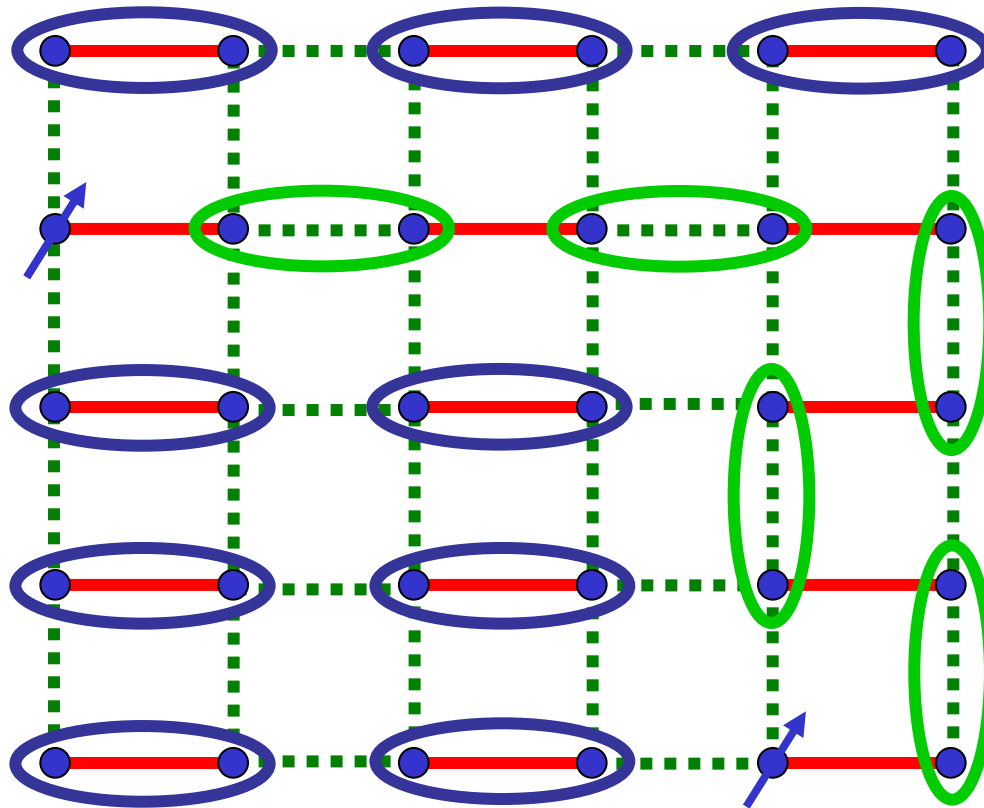
Energy dispersion away from antiferromagnetic wavevector $\varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$

$\Delta \rightarrow$ spin gap

λ close to 0

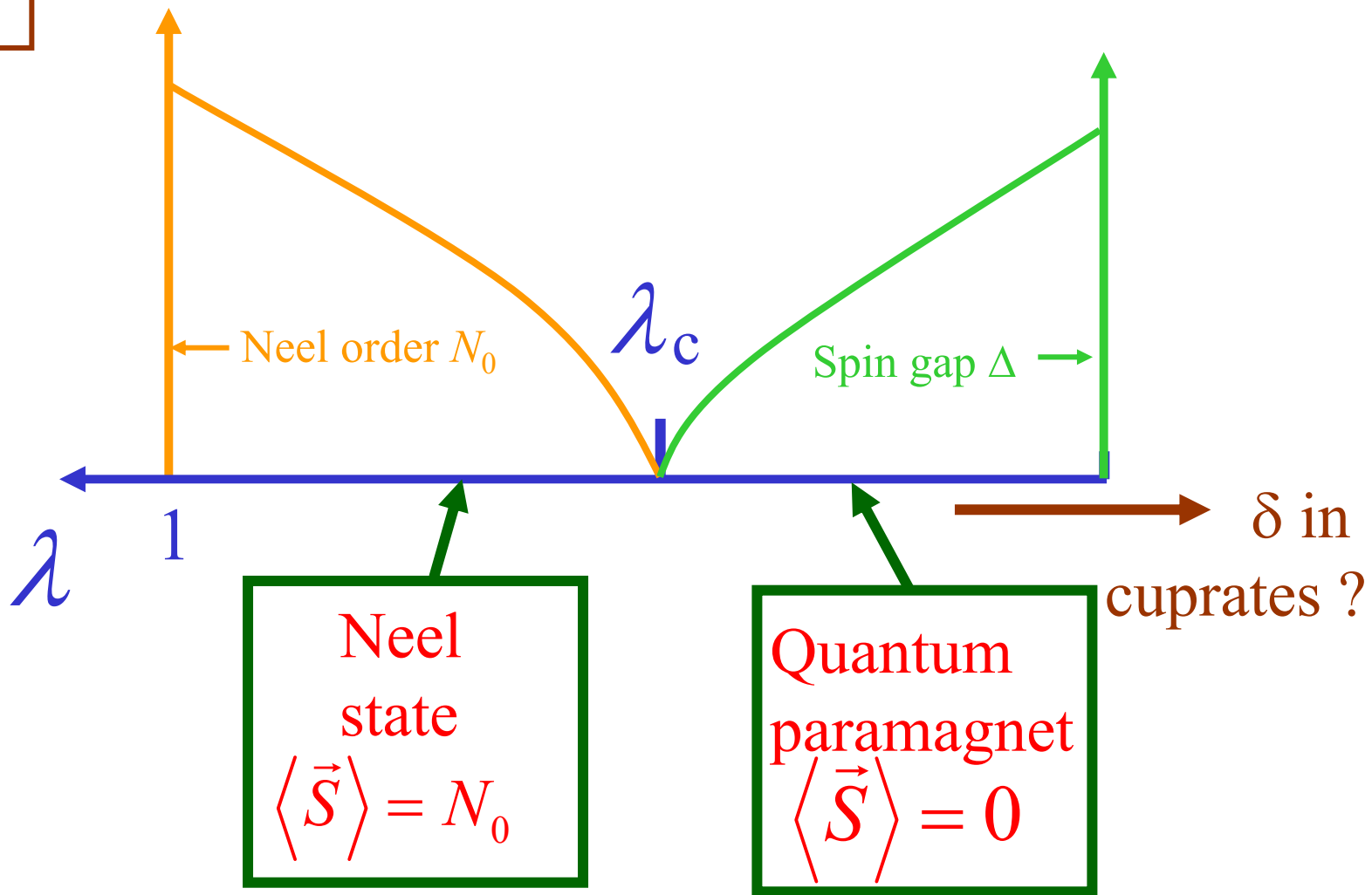
Weakly coupled dimers

$$\text{dimer} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



$S=1/2$ *spinons* are confined by a linear potential into a $S=1$ *triplon*

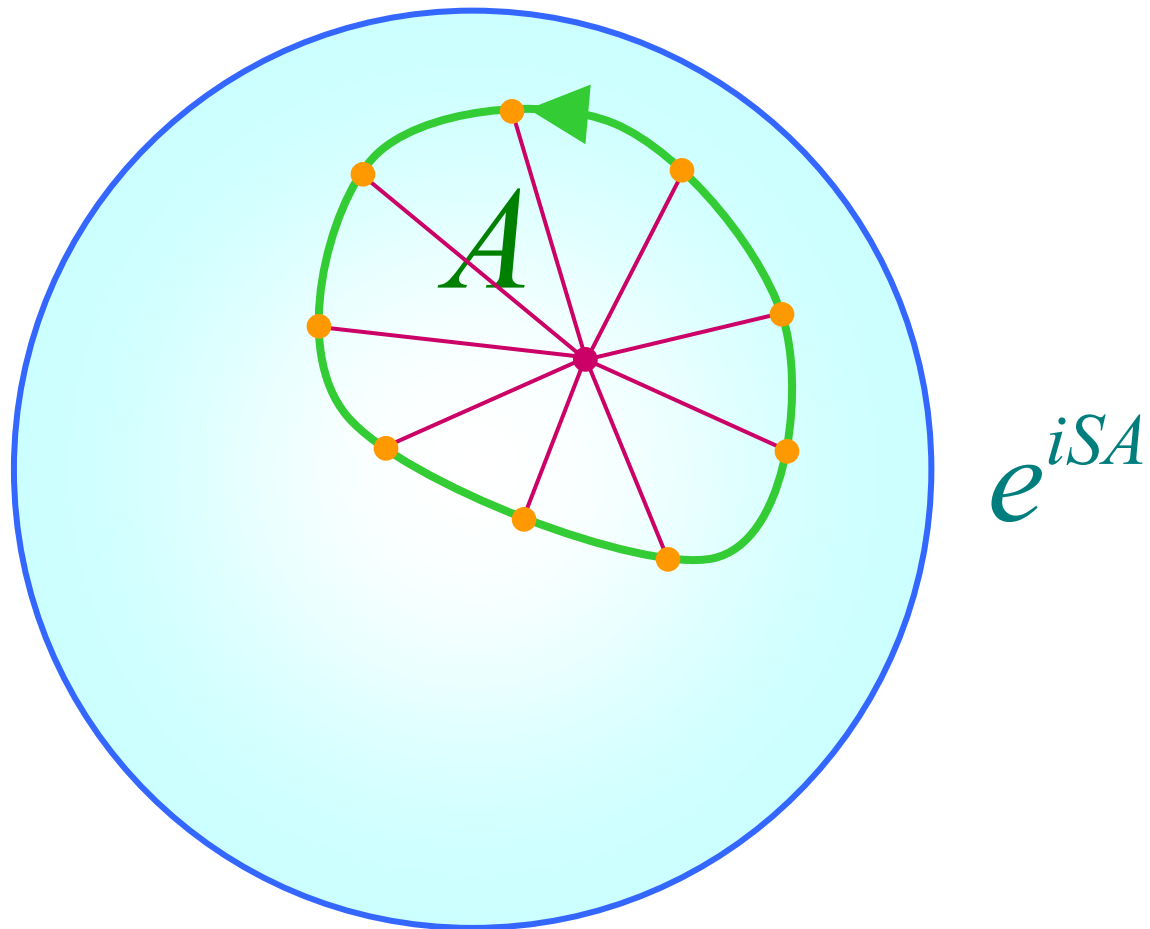
$T=0$



II.A Coherent state path integral

Path integral for quantum spin fluctuations

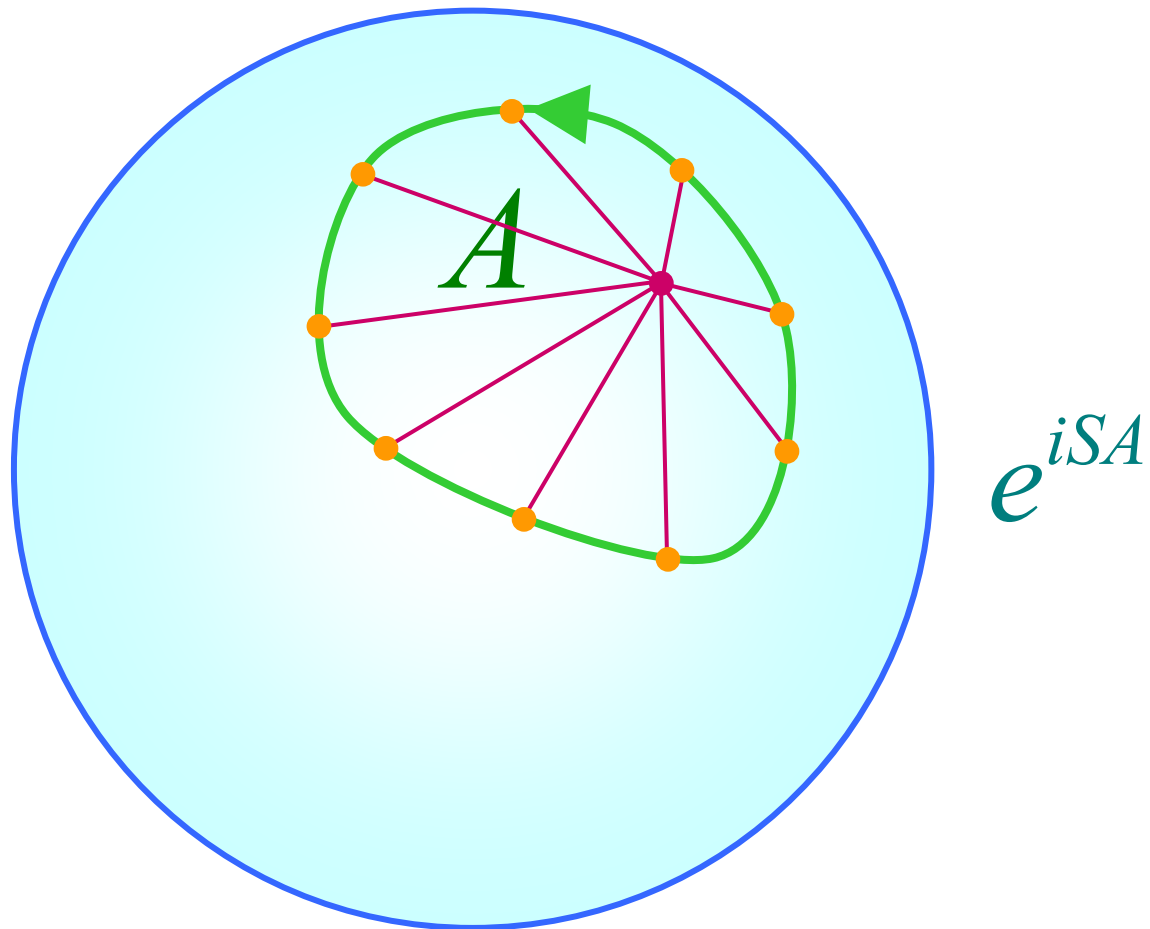
Key ingredient: Spin Berry Phases



II.A Coherent state path integral

Path integral for quantum spin fluctuations

Key ingredient: Spin Berry Phases



II.A Coherent state path integral

See Chapter 13 of *Quantum Phase Transitions*, S. Sachdev, Cambridge University Press (1999).

Path integral for a single spin

$$Z = \text{Tr} \left(e^{-H[S]/T} \right)$$

$$= \int \mathcal{D}N(\tau) \delta(N^2 - 1) \exp \left(-iS \int A_\tau(\tau) d\tau - \int d\tau H[SN(\tau)] \right)$$

$A_\tau(\tau) d\tau$ = Oriented area of triangle on surface of unit sphere bounded by $N(\tau)$, $N(\tau + d\tau)$, and a fixed reference N_0

Action for lattice antiferromagnet

$$N_j(\tau) = \eta_j \mathbf{n}(x_j, \tau) + \mathbf{L}(x_j, \tau)$$

$\eta_j = \pm 1$ identifies sublattices

\mathbf{n} and \mathbf{L} vary slowly in space and time

Integrate out \mathbf{L} and take the continuum limit

$$Z = \int \mathcal{D}\mathbf{n}(x, \tau) \delta(\mathbf{n}^2 - 1) \exp \left(-iS \sum_j \int \eta_j A_\tau(x_j, \tau) d\tau - \frac{1}{2g} \int d^2x d\tau \left((\partial_\tau \mathbf{n})^2 + c^2 (\nabla_x \mathbf{n})^2 \right) \right)$$

$\eta_j = \pm 1$ identifies sublattices

$$\begin{aligned} g < g_c &\Leftrightarrow \lambda > \lambda_c \\ g > g_c &\Leftrightarrow \lambda < \lambda_c \end{aligned}$$

Discretize spacetime into a cubic lattice

$a \rightarrow$ cubic lattice sites;

$$Z = \prod_a \int d\mathbf{n}_a \delta(\mathbf{n}_a^2 - 1) \exp \left(\frac{1}{g} \sum_{a,\mu} \mathbf{n}_a \cdot \mathbf{n}_{a+\mu} - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

$\mu \rightarrow x, y, \tau;$

$A_{a\mu} \rightarrow$ oriented area of spherical triangle

formed by \mathbf{n}_a , $\mathbf{n}_{a+\mu}$, and an arbitrary reference point \mathbf{n}_0

Integrate out L and take the continuum limit

$$Z = \int \mathcal{D}\mathbf{n}(x, \tau) \delta(\mathbf{n}^2 - 1) \exp \left(-iS \sum_j \int \eta_j A_\tau(x_j, \tau) d\tau - \frac{1}{2g} \int d^2x d\tau \left((\partial_\tau \mathbf{n})^2 + c^2 (\nabla_x \mathbf{n})^2 \right) \right)$$

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$a \rightarrow$ cubic lattice sites;
 $\mu \rightarrow x, y, \tau;$

Berry phases can be neglected for coupled dimer antiferromagnet

(justified later)

Quantum path integral for two-dimensional quantum antiferromagnet

\Leftrightarrow Partition function of a classical three-dimensional ferromagnet
at a “temperature” g

Quantum transition at $\lambda = \lambda_c$ is related to classical Curie transition at $g = g_c$

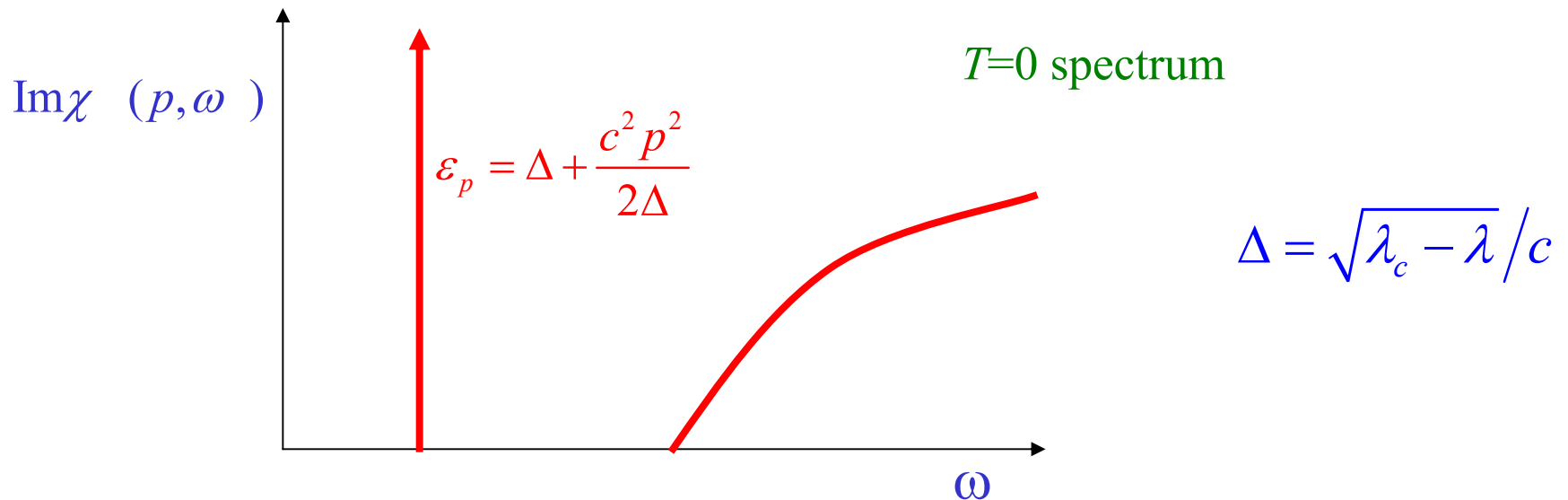
II.B Quantum field theory for critical point

λ close to λ_c : use “soft spin” field

$$\mathcal{S}_b = \int d^2x d\tau \left[\frac{1}{2} \left((\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 + (\lambda_c - \lambda) \phi_\alpha^2 \right) + \frac{u}{4!} (\phi_\alpha^2)^2 \right]$$

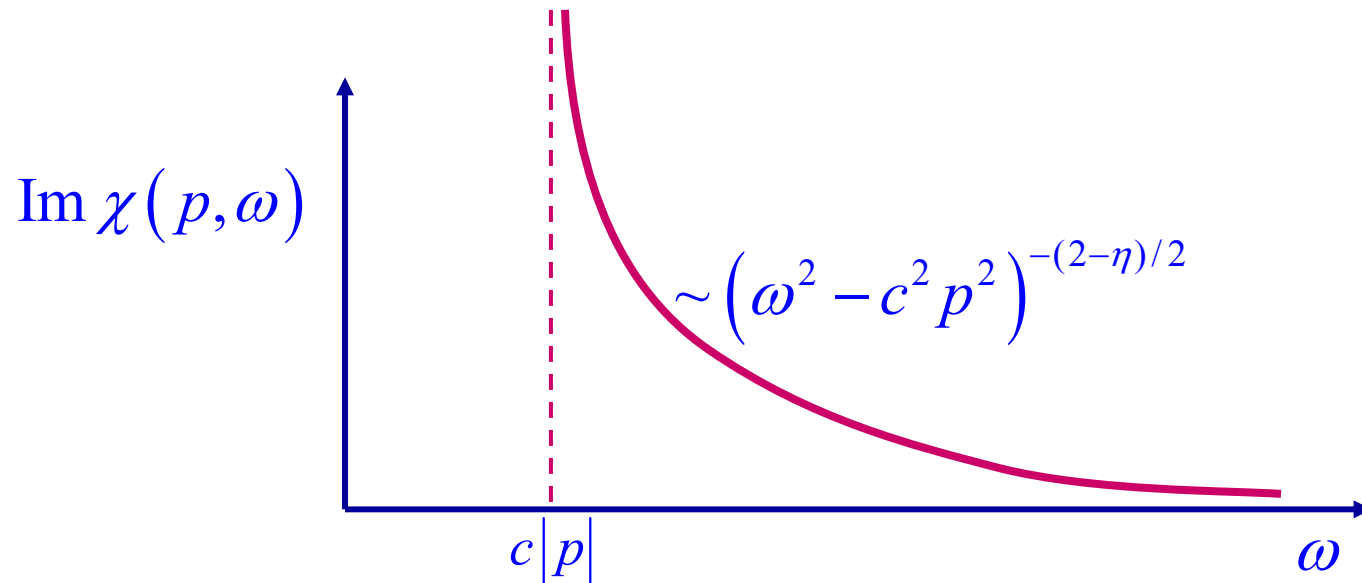
$\phi_\alpha \longrightarrow$ 3-component antiferromagnetic order parameter

Oscillations of ϕ_α about zero (for $\lambda < \lambda_c$)
 \longrightarrow spin-1 collective mode



Critical coupling ($\lambda = \lambda_c$)

Dynamic spectrum at the critical point



No quasiparticles --- dissipative critical continuum

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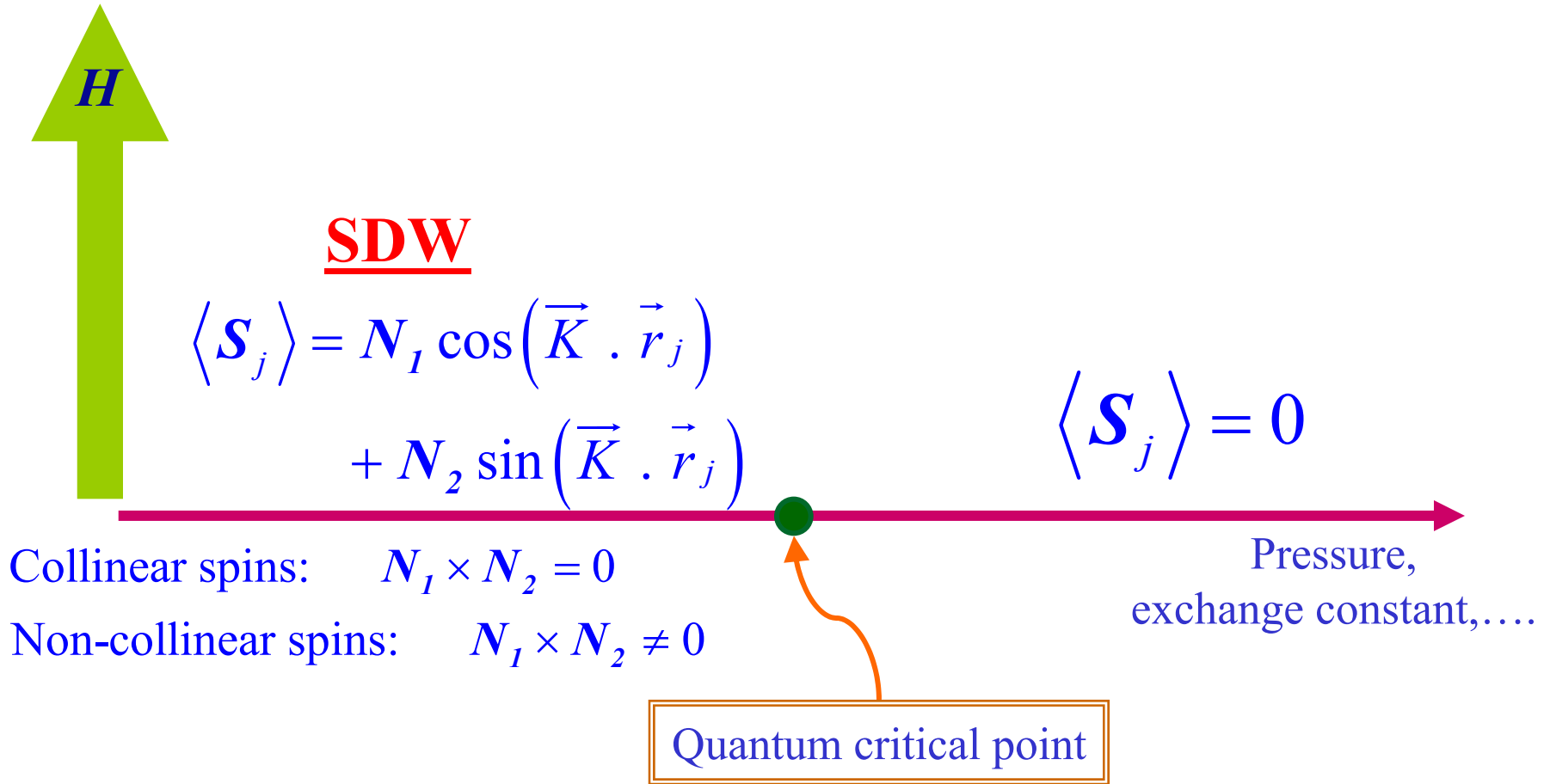
Class B: Z_2 gauge theory: non-collinear spins, RVB, visons, topological order, and deconfined spinons

VI. Conclusions

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Multiple order parameters

$$T=0$$

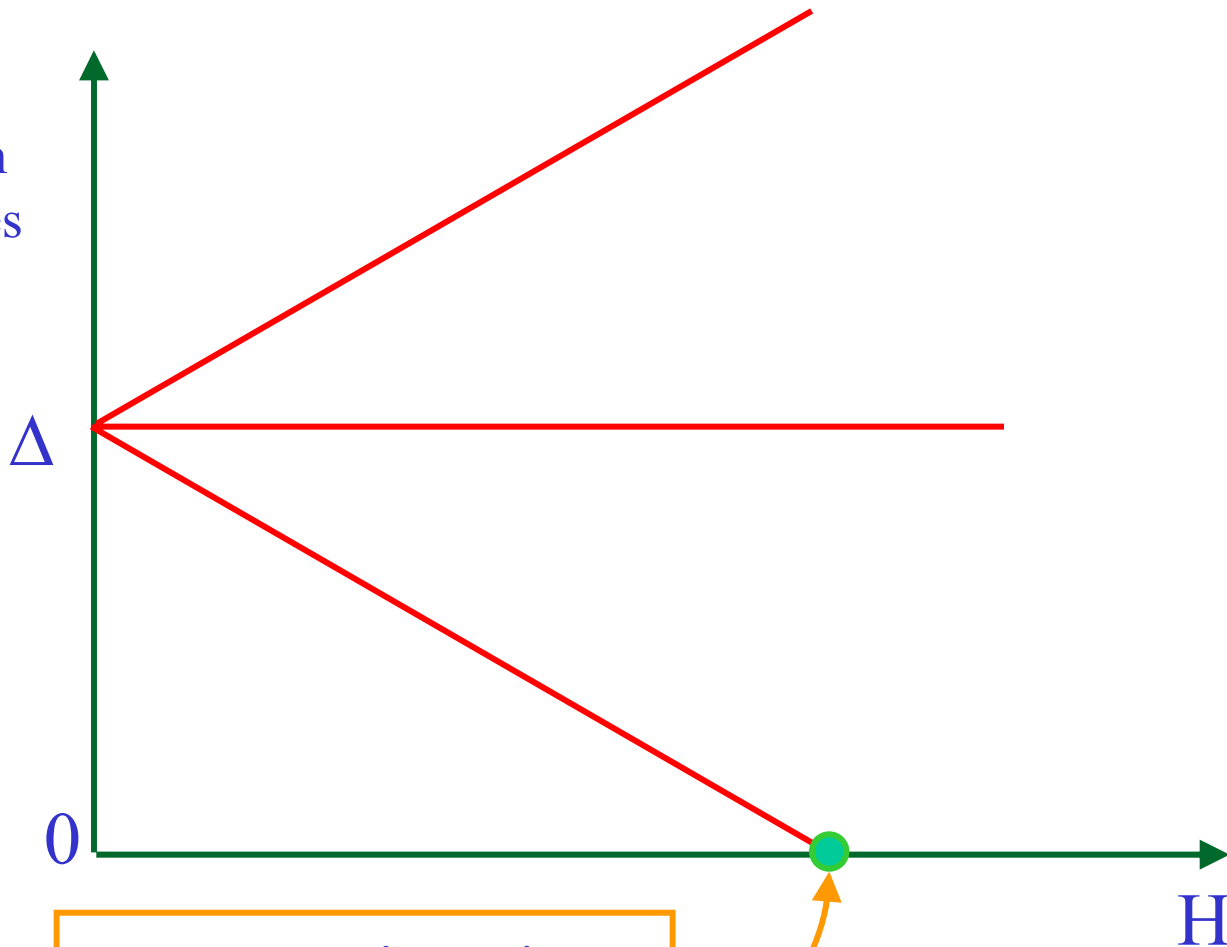


Evolution of phase diagram in a magnetic field

Both states are insulators

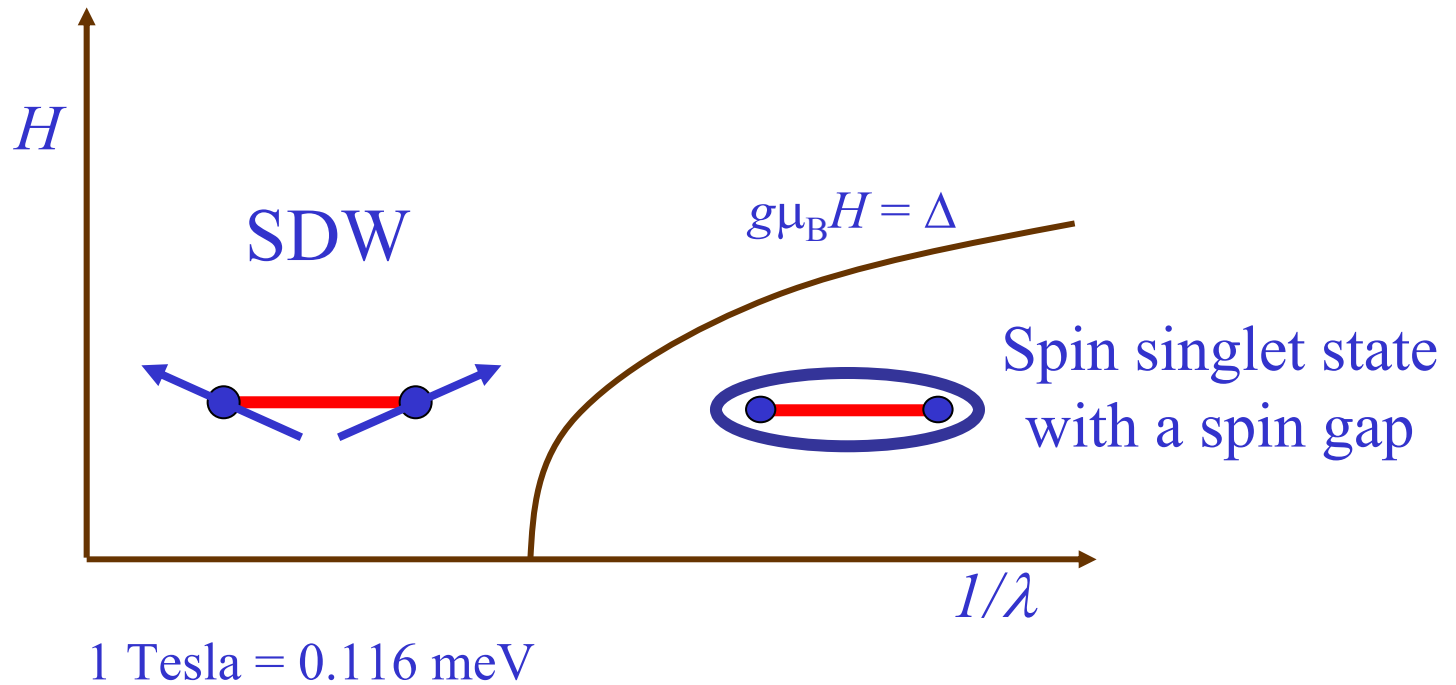
Effect of a field on paramagnet

Energy of
zero
momentum
triplon states



Bose-Einstein
condensation of
 $S_z=1$ triplon

III. Phase diagram in a magnetic field.



Related theory applies to double layer quantum Hall systems at $\nu=2$

III. Phase diagram in a magnetic field.

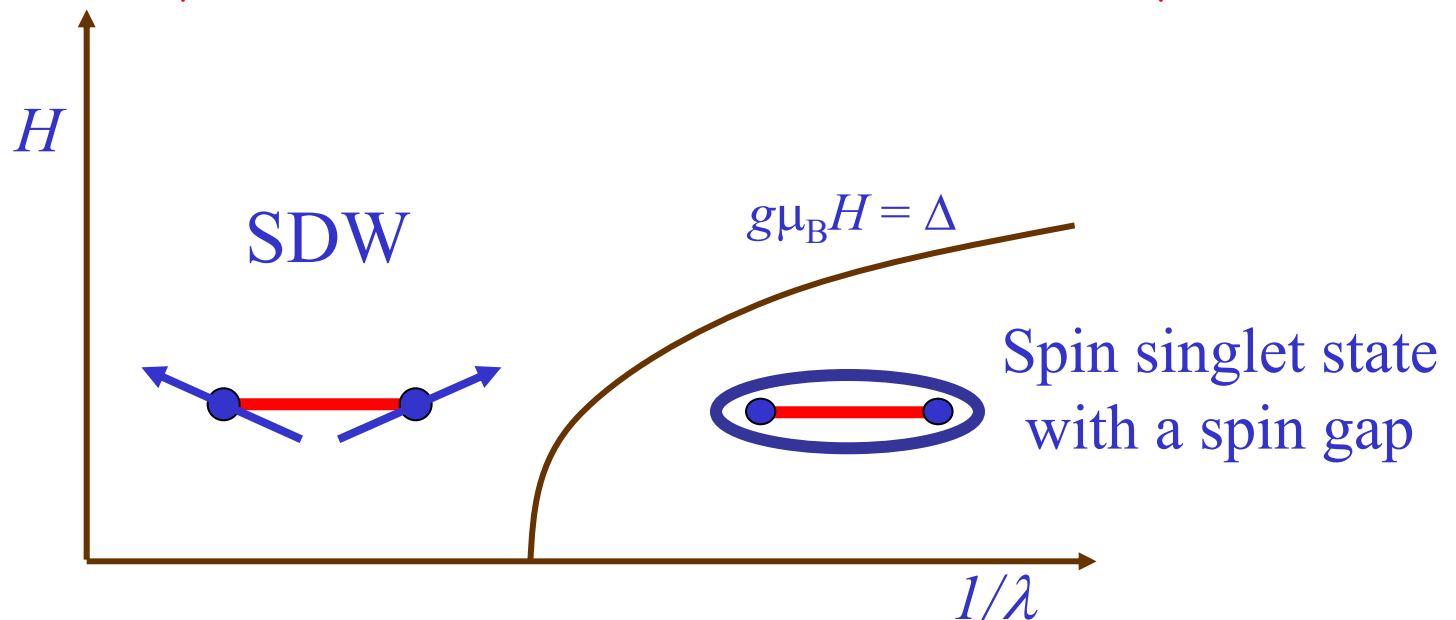
Zeeman term leads to a uniform precession of spins

$$|\partial_\tau \phi_\alpha|^2 \Rightarrow \left(\partial_\tau \phi_\alpha^* - i \varepsilon_{\alpha\sigma\rho} H_\sigma \phi_\rho \right) \left(\partial_\tau \phi_\alpha - i \varepsilon_{\alpha\beta\gamma} H_\beta \phi_\gamma \right)$$

Take H oriented along the z direction. Then

$$(\lambda_c - \lambda)(\phi_x^2 + \phi_y^2) \Rightarrow (\lambda_c - \lambda - H^2)(\phi_x^2 + \phi_y^2).$$

For $\lambda > \lambda_c$, $\phi_x \sim \sqrt{\lambda - \lambda_c + H^2}$, while for $\lambda < \lambda_c$, $H_c = \Delta \sim \sqrt{\lambda_c - \lambda}$



1 Tesla = 0.116 meV

Related theory applies to double layer quantum Hall systems at $\nu=2$

III. Phase diagram in a magnetic field.

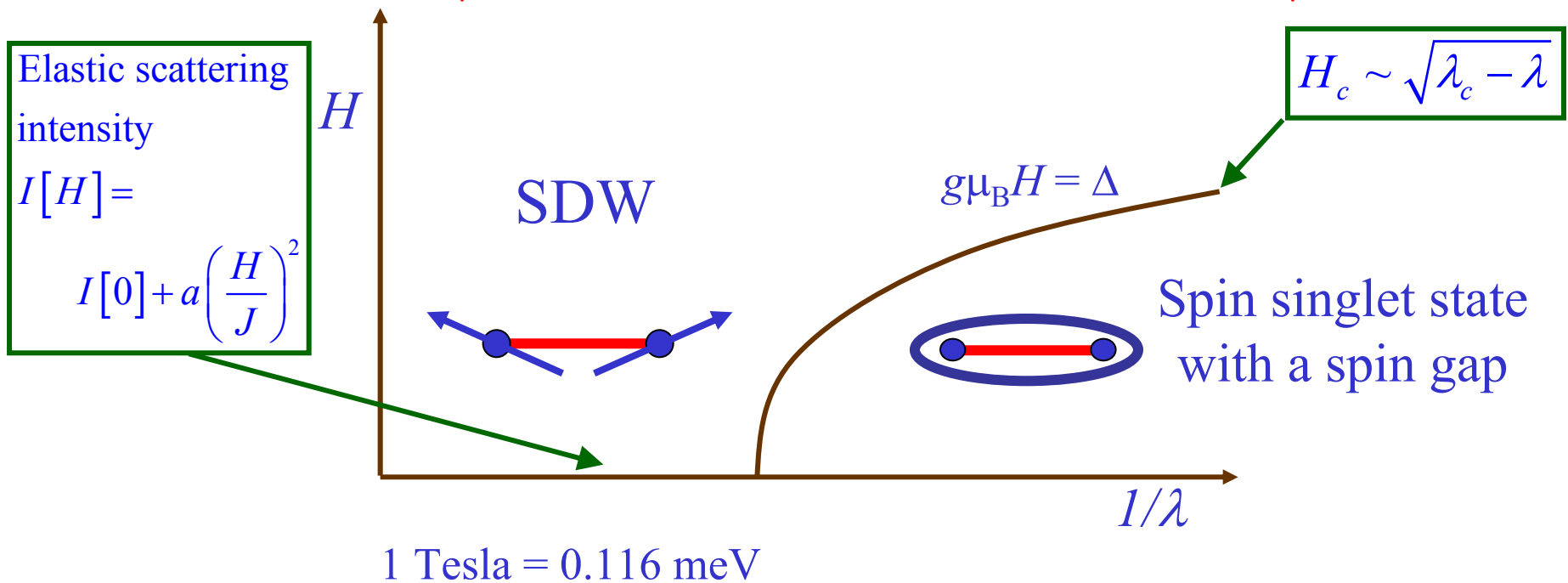
Zeeman term leads to a uniform precession of spins

$$|\partial_\tau \phi_\alpha|^2 \Rightarrow \left(\partial_\tau \phi_\alpha^* - i \varepsilon_{\alpha\sigma\rho} H_\sigma \phi_\rho \right) \left(\partial_\tau \phi_\alpha - i \varepsilon_{\alpha\beta\gamma} H_\beta \phi_\gamma \right)$$

Take H oriented along the z direction. Then

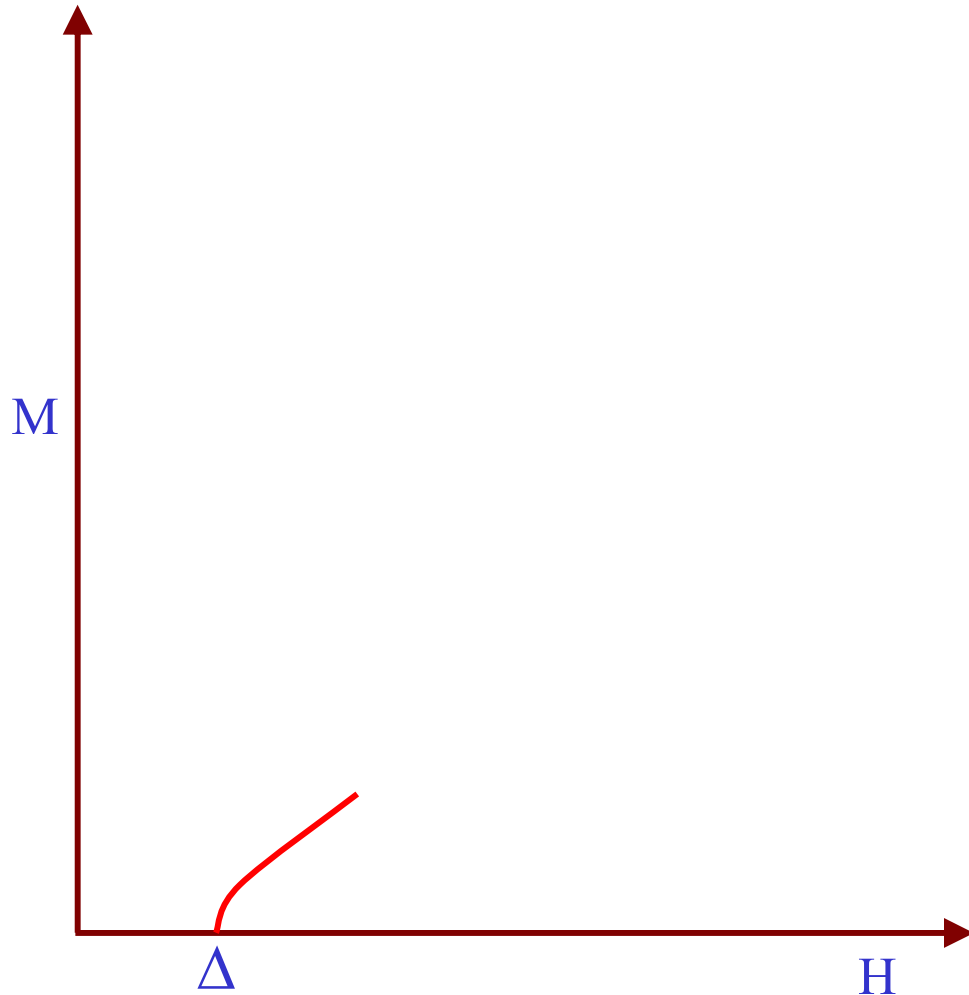
$$(\lambda_c - \lambda)(\phi_x^2 + \phi_y^2) \Rightarrow (\lambda_c - \lambda - H^2)(\phi_x^2 + \phi_y^2).$$

For $\lambda > \lambda_c$, $\phi_x \sim \sqrt{\lambda - \lambda_c + H^2}$, while for $\lambda < \lambda_c$, $H_c = \Delta \sim \sqrt{\lambda_c - \lambda}$

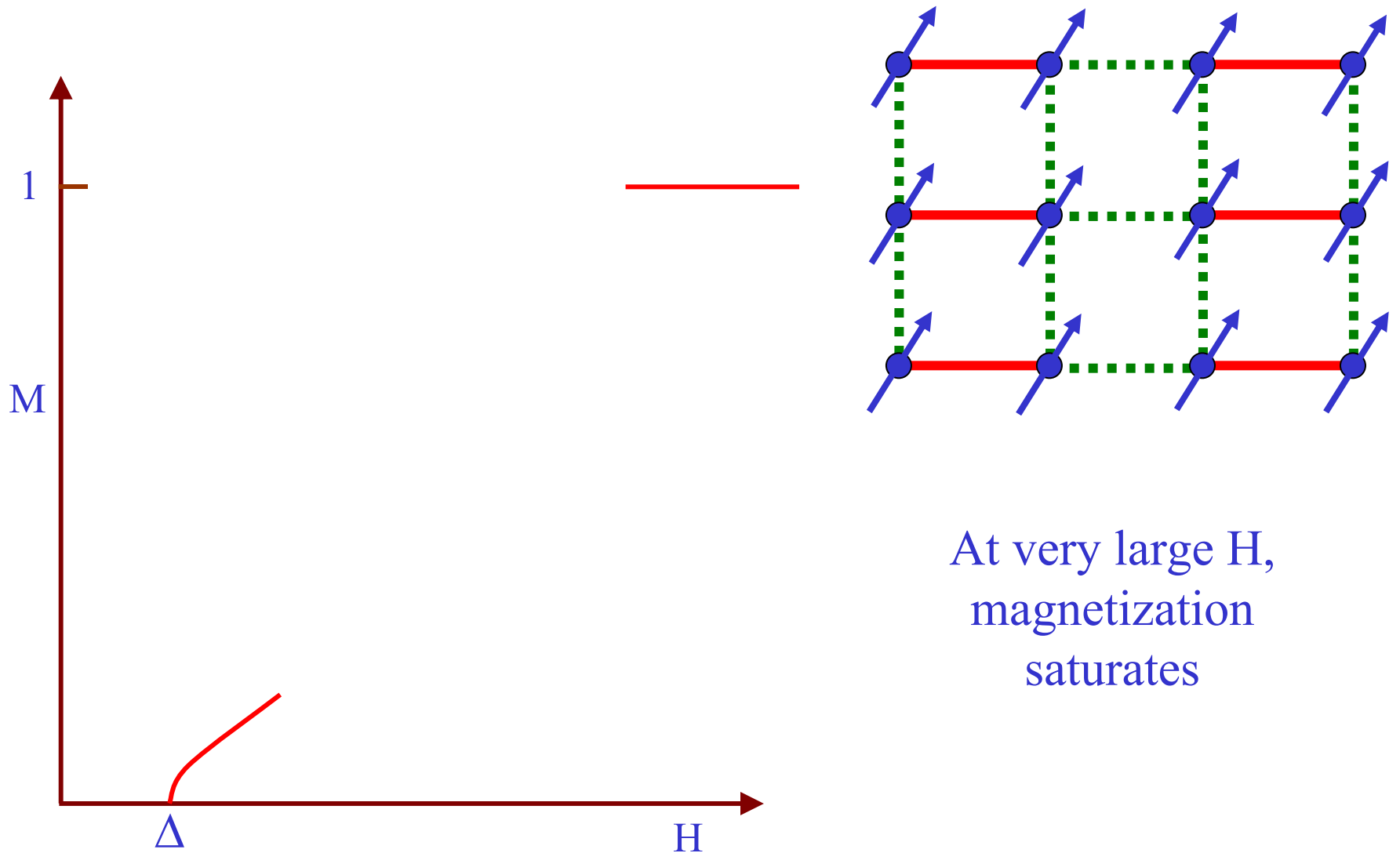


Related theory applies to double layer quantum Hall systems at $\nu=2$

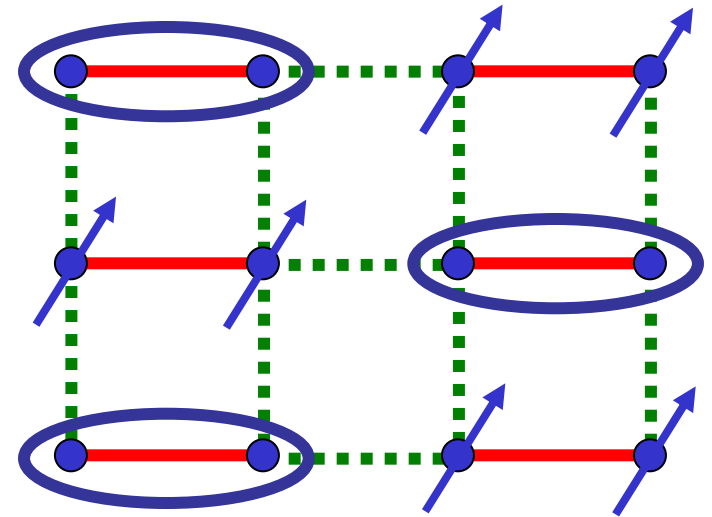
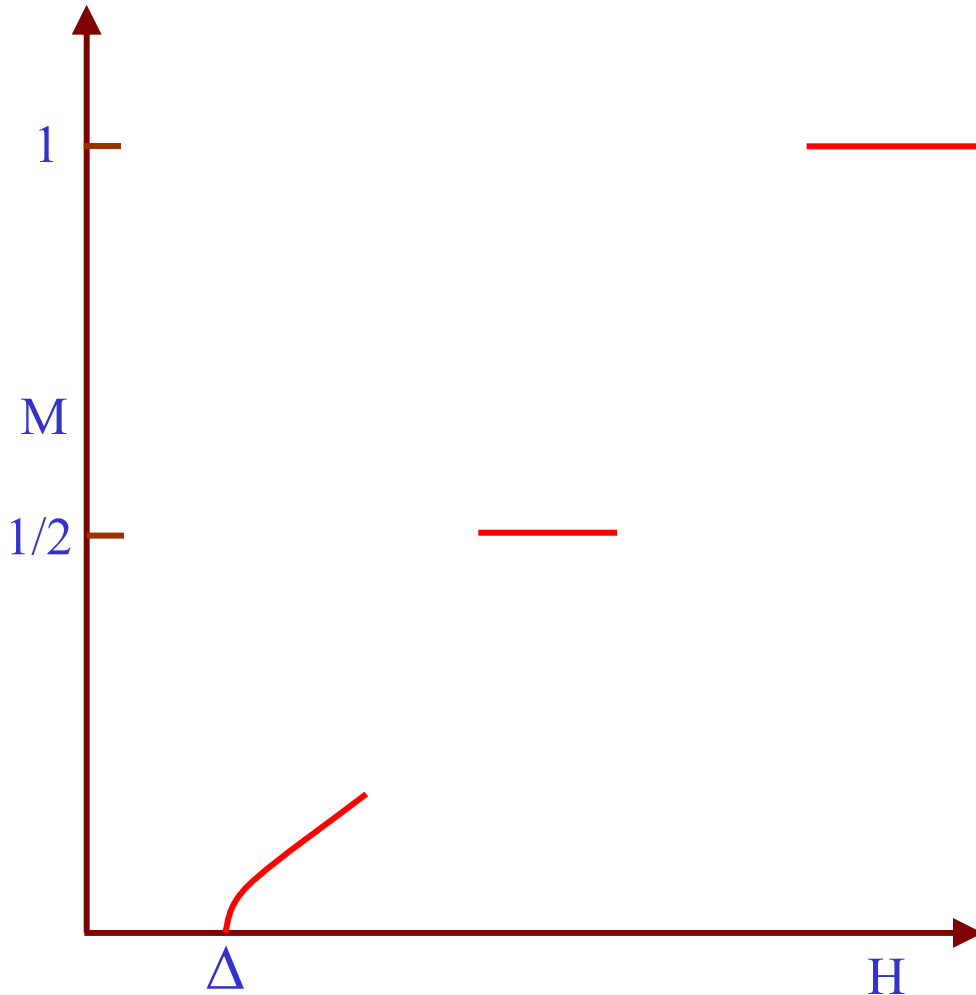
III. Phase diagram in a magnetic field.



III. Phase diagram in a magnetic field.



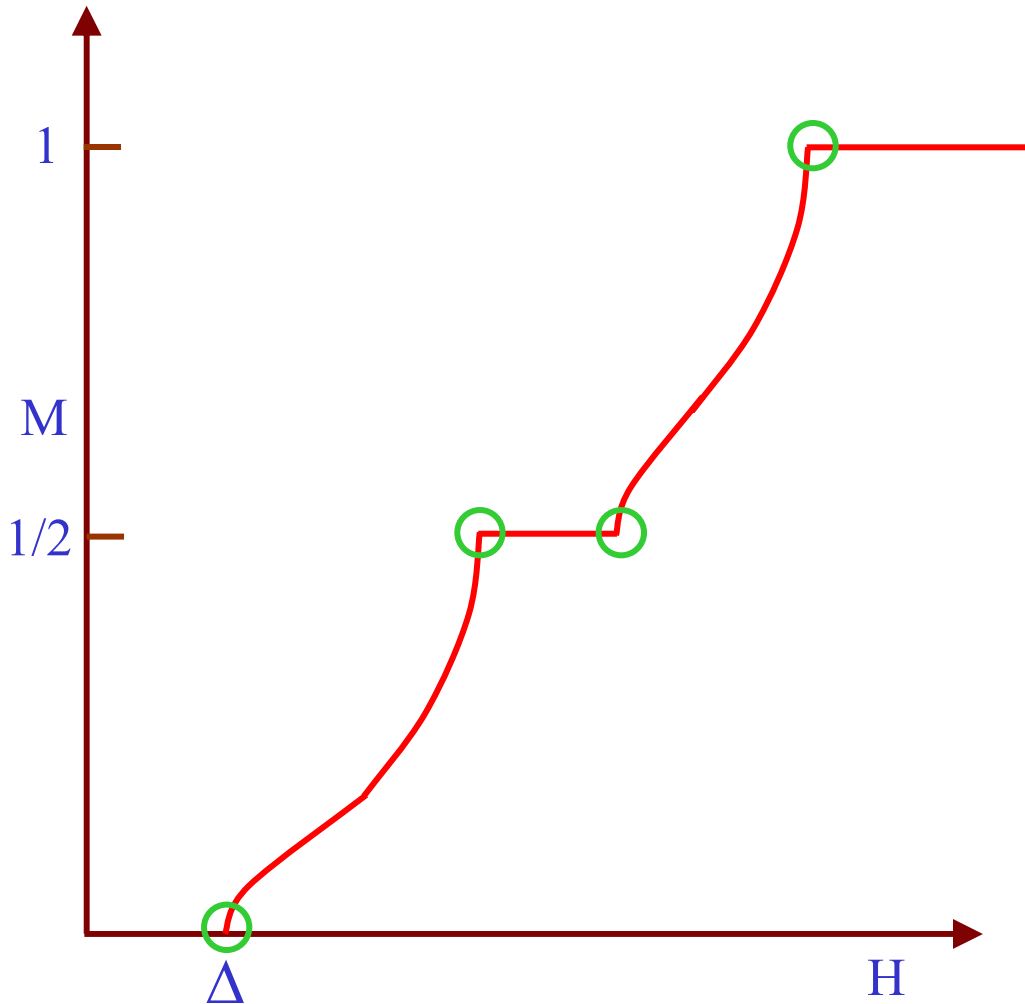
III. Phase diagram in a magnetic field.



$$\sum_{i < j} J_{ij} S_{zi} S_{zj}$$

Repulsive interactions between triplons can lead to magnetization plateau at any rational fraction

III. Phase diagram in a magnetic field.



Quantum transitions in
and out of plateau are
Bose-Einstein
condensations of
“extra/missing”
triplons

Outline

I. Quantum Ising Chain

II. Coupled Dimer Antiferromagnet

A. Coherent state path integral

B. Quantum field theory near critical point

III. Coupled dimer antiferromagnet in a magnetic field

Bose condensation of “triplons”

IV. Magnetic transitions in superconductors

Quantum phase transition in a background

Abrikosov flux lattice

V. Antiferromagnets with an odd number
of $S=1/2$ spins per unit cell.

Class A: Compact $U(1)$ gauge theory: collinear spins,
bond order and confined spinons in $d=2$

Class B: Z_2 gauge theory: non-collinear spins, RVB,
visons, topological order, and deconfined spinons

VI. Conclusions

*Single order
parameter.*

*Multiple
order
parameters*

SDW

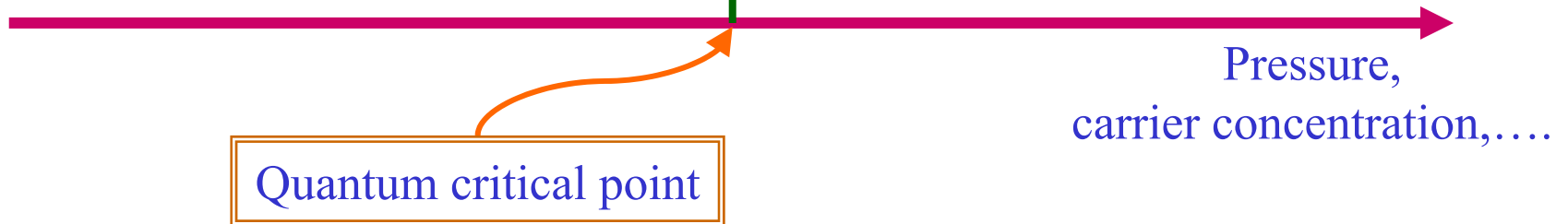
$$T=0$$

$$\begin{aligned}\langle \mathbf{S}_j \rangle &= N_1 \cos(\vec{K} \cdot \vec{r}_j) + N_2 \sin(\vec{K} \cdot \vec{r}_j) \\ &= \text{Re} \left[\Phi e^{i\vec{K} \cdot \vec{r}_j} \right] \\ \Phi &\equiv N_1 - iN_2\end{aligned}$$

$$\langle \mathbf{S}_j \rangle = 0$$

Collinear spins: $N_1 \times N_2 = 0$

Non-collinear spins: $N_1 \times N_2 \neq 0$



We have so far considered the case where
both states are insulators

SC+SDW

$T=0$

SC

$$\begin{aligned}\langle \mathbf{S}_j \rangle &= N_1 \cos(\vec{K} \cdot \vec{r}_j) + N_2 \sin(\vec{K} \cdot \vec{r}_j) \\ &= \text{Re} \left[\Phi e^{i\vec{K} \cdot \vec{r}_j} \right] \\ \Phi &\equiv N_1 - iN_2\end{aligned}$$

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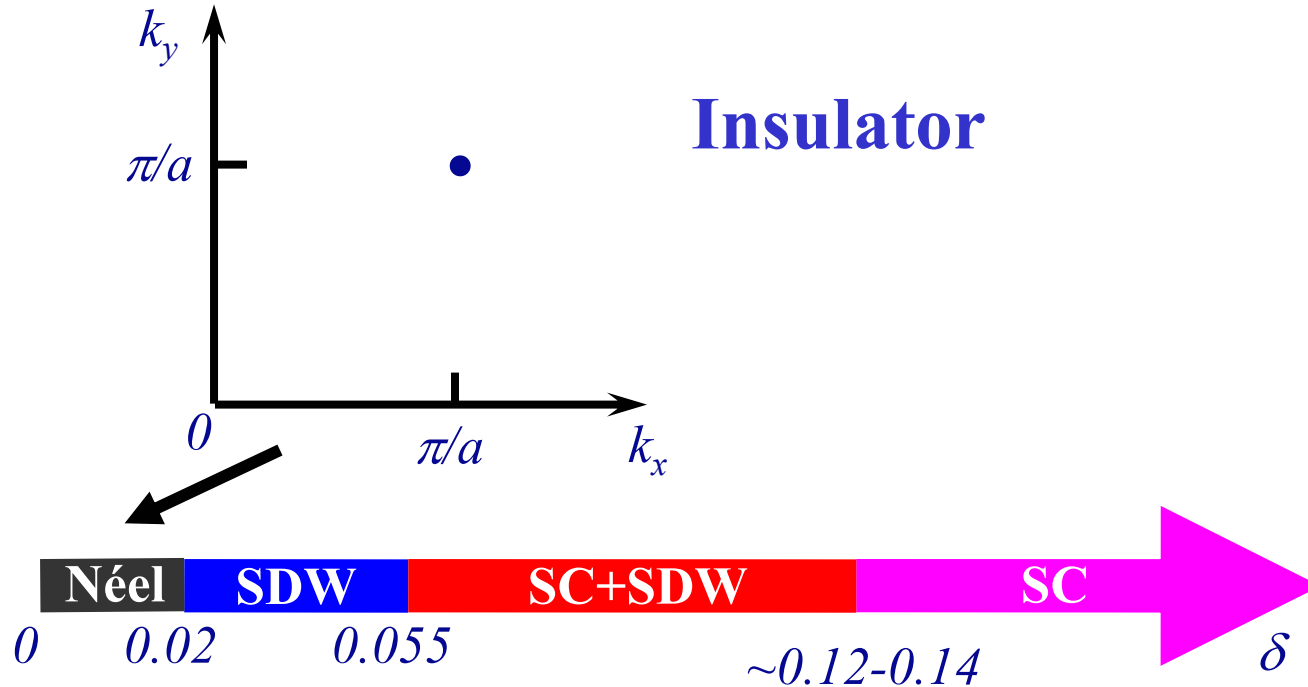
Pressure,
carrier concentration,....

Quantum critical point

Now both sides have a “background”
superconducting (SC) order

Interplay of SDW and SC order in the cuprates

T=0 phases of LSCO



(additional commensurability effects near $\delta=0.125$)

J. M. Tranquada *et al.*, *Phys. Rev. B* **54**, 7489 (1996).

G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, *Science* **278**, 1432 (1997).

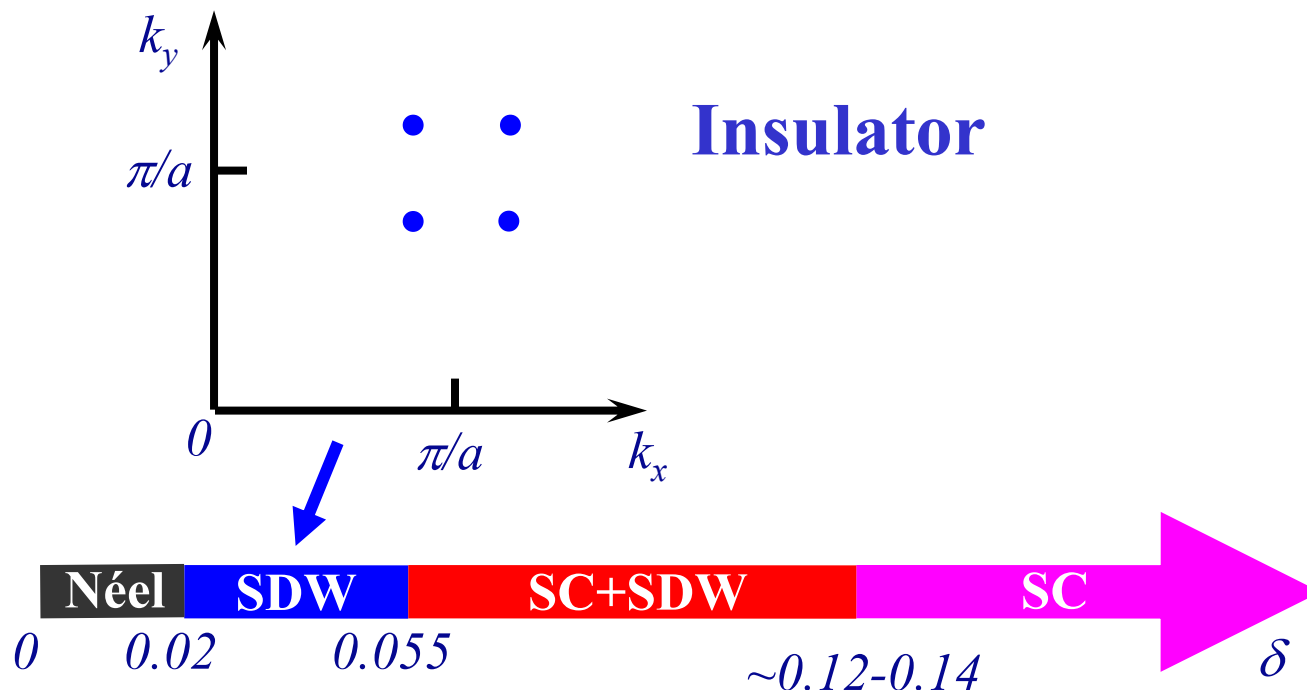
S. Wakimoto, G. Shirane *et al.*, *Phys. Rev. B* **60**, R769 (1999).

Y.S. Lee, R. J. Birgeneau, M. A. Kastner *et al.*, *Phys. Rev. B* **60**, 3643 (1999)

S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys. Rev. B* **63**, 172501 (2001).

Interplay of SDW and SC order in the cuprates

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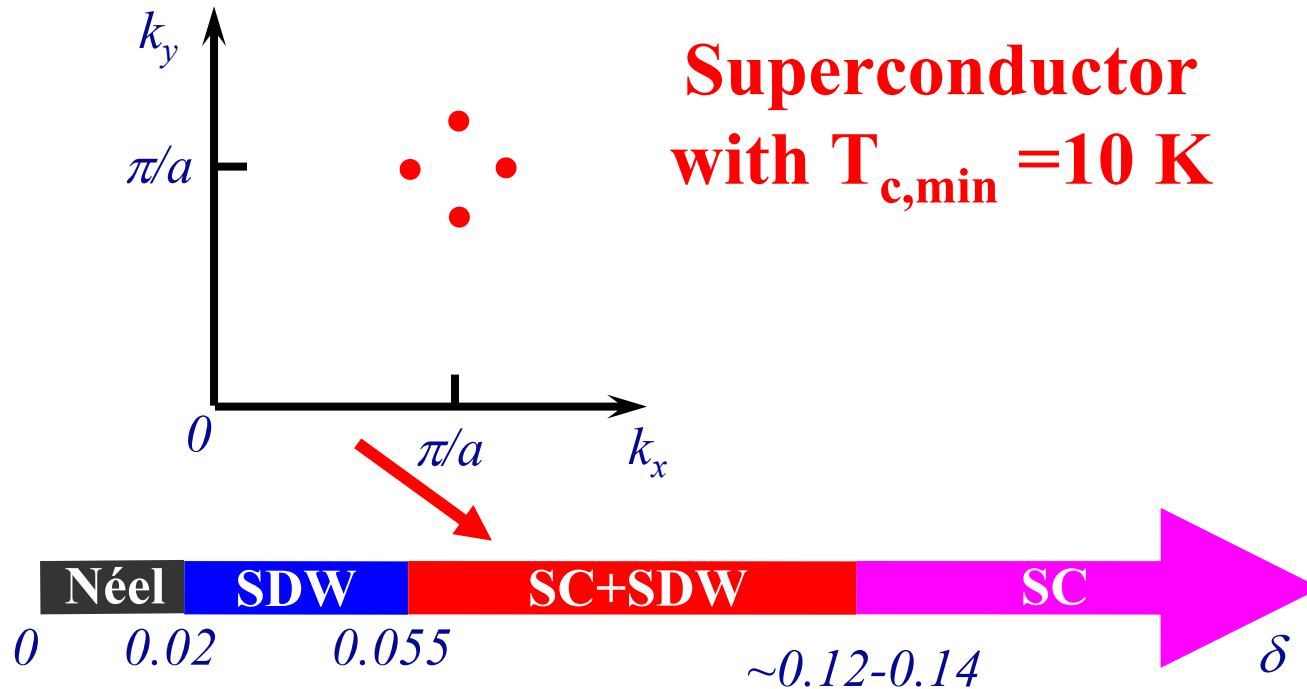
S. Wakimoto, G. Shirane *et al.*, *Phys. Rev. B* **60**, R769 (1999).

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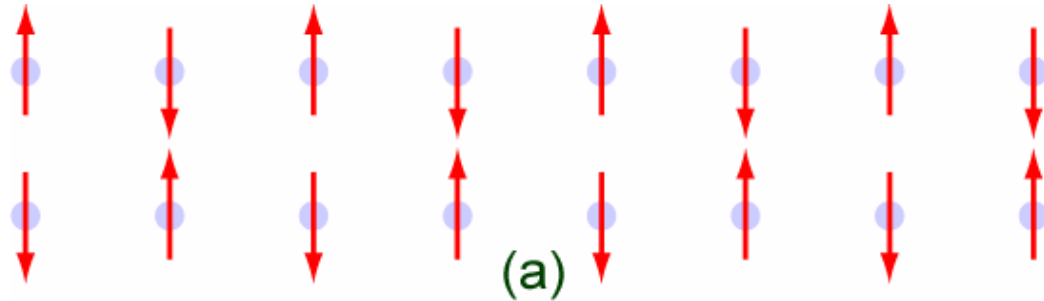
Y.S. Lee, R. J. Birgeneau, M. A. Kastner *et al.*, *Phys. Rev. B* **60**, 3643 (1999)

S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys. Rev. B* **63**, 172501 (2001).

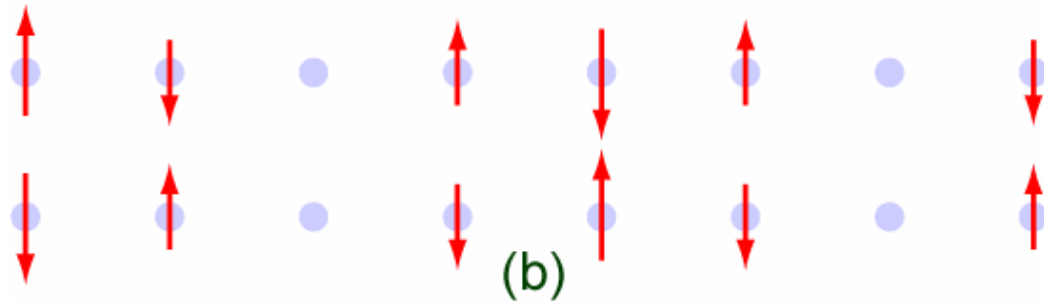
Collinear magnetic (spin density wave) order

$$\langle \mathbf{S}_j \rangle = N_1 \cos(\vec{K} \cdot \vec{r}_j) + N_2 \sin(\vec{K} \cdot \vec{r}_j)$$

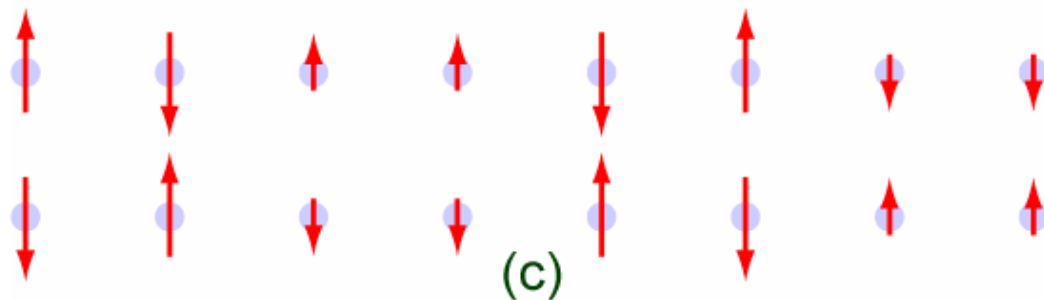
Collinear spins



$$\vec{K} = (\pi, \pi) ; N_2 = 0$$



$$\vec{K} = (3\pi/4, \pi) ; N_2 = 0$$

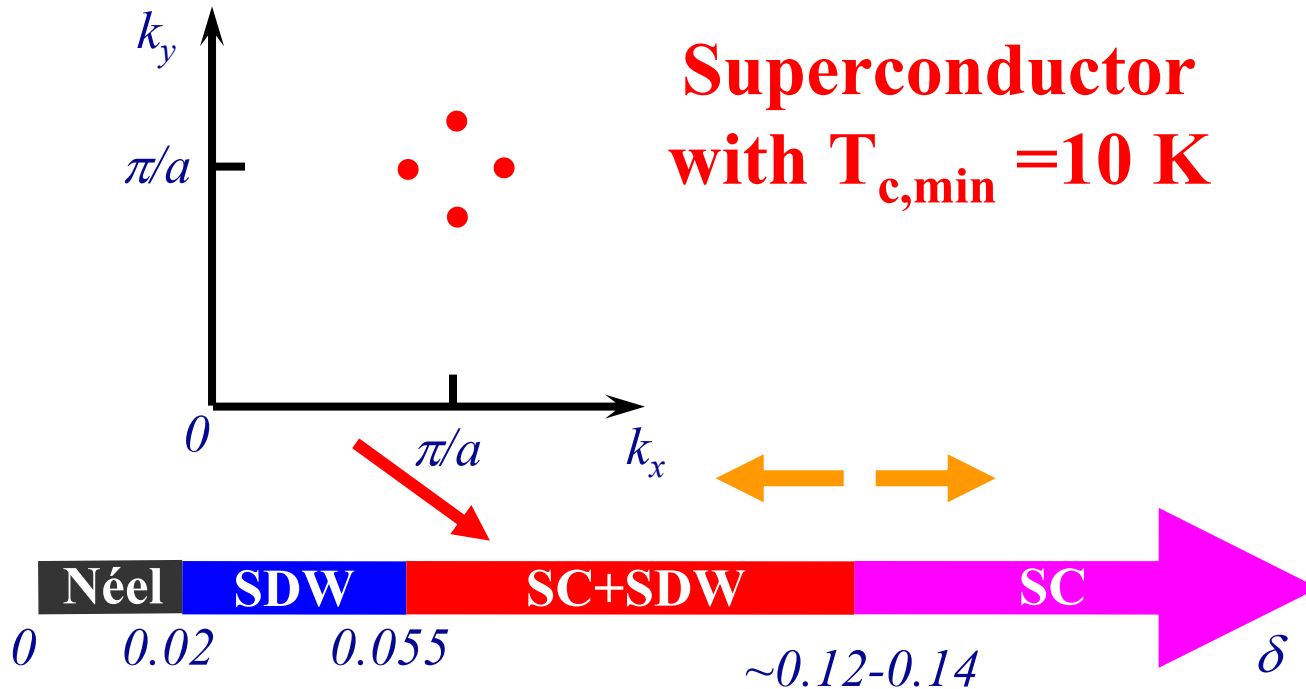


$$\vec{K} = (3\pi/4, \pi) ;$$

$$N_2 = (\sqrt{2} - 1) N_1$$

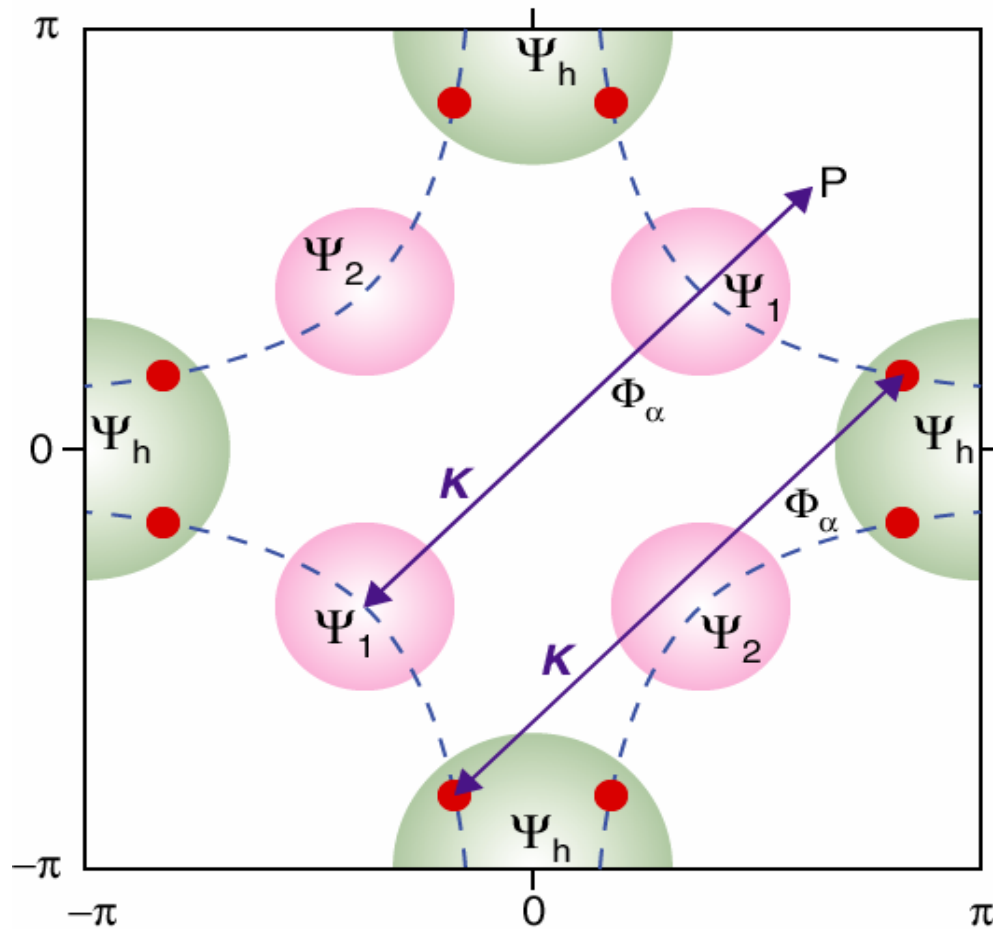
Interplay of SDW and SC order in the cuprates

T=0 phases of LSCO



Use simplest assumption of a direct second-order quantum phase transition between SC and SC+SDW phases

Magnetic transition in a d -wave superconductor



If \vec{K} does not exactly connect two nodal points,
critical theory is as in an insulator

Otherwise, new theory of coupled excitons and nodal quasiparticles

Magnetic transition in a d -wave superconductor

$$S = \int d^2 r d\tau \left[|\nabla_r \Phi_\alpha|^2 + c^2 |\partial_\tau \Phi_\alpha|^2 + V(\Phi_\alpha) \right]$$

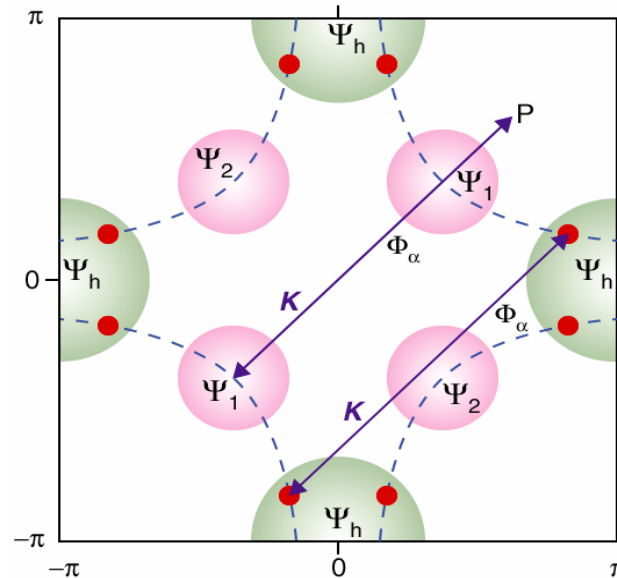
Similar terms present in action for SDW ordering in the insulator

Coupling to the $S=1/2$ Bogoliubov quasiparticles of the d -wave superconductor

Trilinear “Yukawa” coupling

$$\int d^2 r d\tau \Phi_\alpha \Psi \Psi$$

is prohibited unless ordering wavevector is fine-tuned.

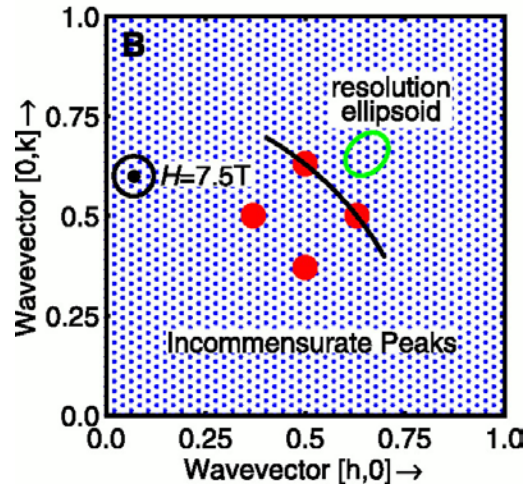


$$\kappa \sum_\alpha \int d^2 r d\tau |\Phi_\alpha|^2 \Psi^\dagger \Psi \text{ is allowed}$$

Scaling dimension of $\kappa = (1/\nu - 2) < 0 \Rightarrow$ irrelevant.

Neutron scattering measurements of dynamic spin correlations of the superconductor (SC) in a magnetic field

B. Lake, G. Aeppli, K. N. Clausen, D. F. McMorrow, K. Lefmann, N. E. Hussey, N. Mangkorntong, M. Nohara, H. Takagi, T. E. Mason, and A. Schröder, *Science* **291**, 1759 (2001).



Peaks at $(0.5, 0.5) \pm (0.125, 0)$

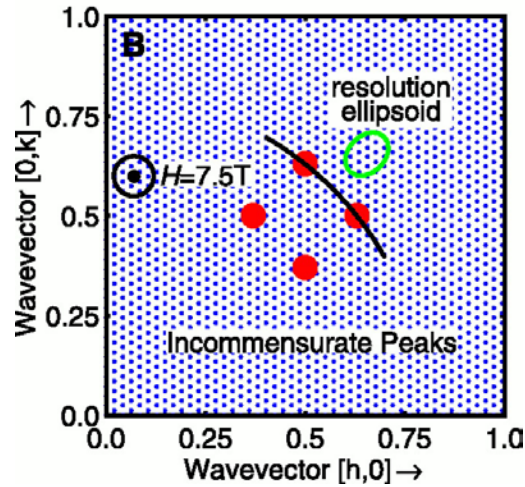
and $(0.5, 0.5) \pm (0, 0.125)$

\Rightarrow dynamic SDW of period 8

Neutron scattering off $\text{La}_{2-\delta}\text{Sr}_\delta\text{CuO}_4$ ($\delta = 0.163$, **SC phase**)
at low temperatures in $H=0$ (red dots) and $H=7.5\text{T}$ (blue dots)

Neutron scattering measurements of dynamic spin correlations of the superconductor (SC) in a magnetic field

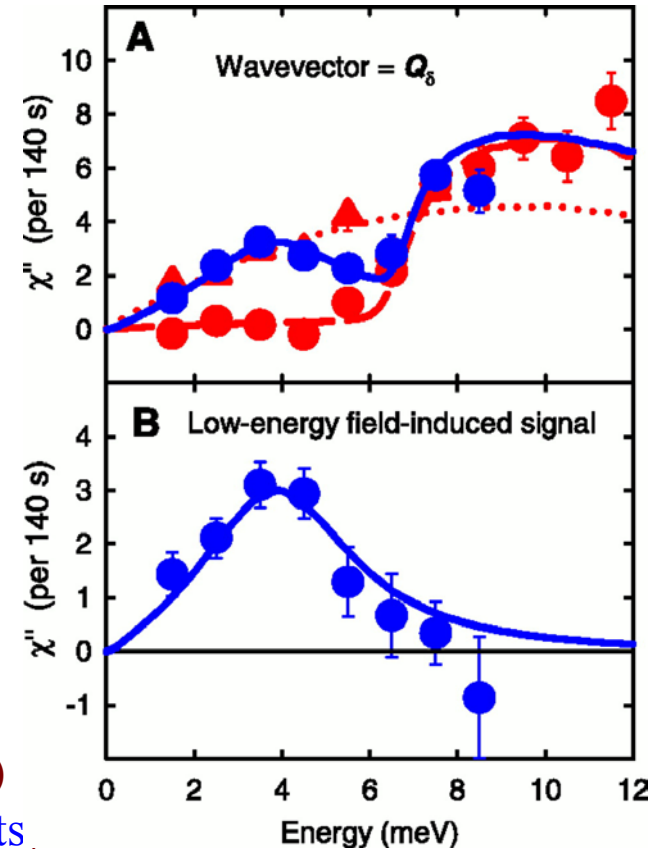
B. Lake, G. Aeppli, K. N. Clausen, D. F. McMorrow, K. Lefmann, N. E. Hussey, N. Mangkorntong, M. Nohara, H. Takagi, T. E. Mason, and A. Schröder, *Science* **291**, 1759 (2001).



Peaks at $(0.5, 0.5) \pm (0.125, 0)$
and $(0.5, 0.5) \pm (0, 0.125)$

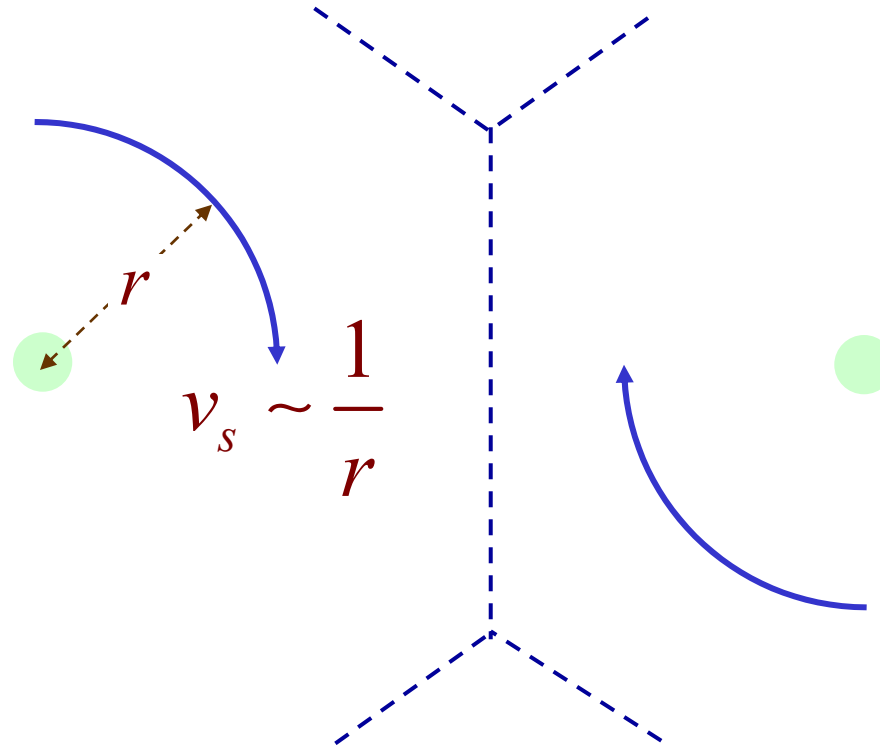
\Rightarrow dynamic SDW of period 8

Neutron scattering off $\text{La}_{2-\delta}\text{Sr}_\delta\text{CuO}_4$ ($\delta = 0.163$, *SC phase*) at low temperatures in $H=0$ (red dots) and $H=7.5\text{T}$ (blue dots),



D. P. Arovas, A. J. Berlinsky, C. Kallin, and S.-C. Zhang, *Phys. Rev. Lett.* **79**, 2871 (1997) proposed static magnetism localized within vortex cores, but signal was much larger than anticipated.

Dominant effect of magnetic field:
Abrikosov flux lattice



Spatially averaged superflow kinetic energy

$$\sim \langle v_s^2 \rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

Effect of magnetic field on SDW+SC to SC transition

(extreme Type II superconductivity)

$$\Phi_\alpha = N_{1\alpha} - iN_{2\alpha}$$

Quantum theory for dynamic and critical spin fluctuations

$$\mathcal{S}_b = \int d^2r \int_0^{1/T} d\tau \left[|\nabla_r \Phi_\alpha|^2 + c^2 |\partial_\tau \Phi_\alpha|^2 + s |\Phi_\alpha|^2 + \frac{g_1}{2} (|\Phi_\alpha|^2)^2 + \frac{g_2}{2} |\Phi_\alpha^2|^2 \right]$$

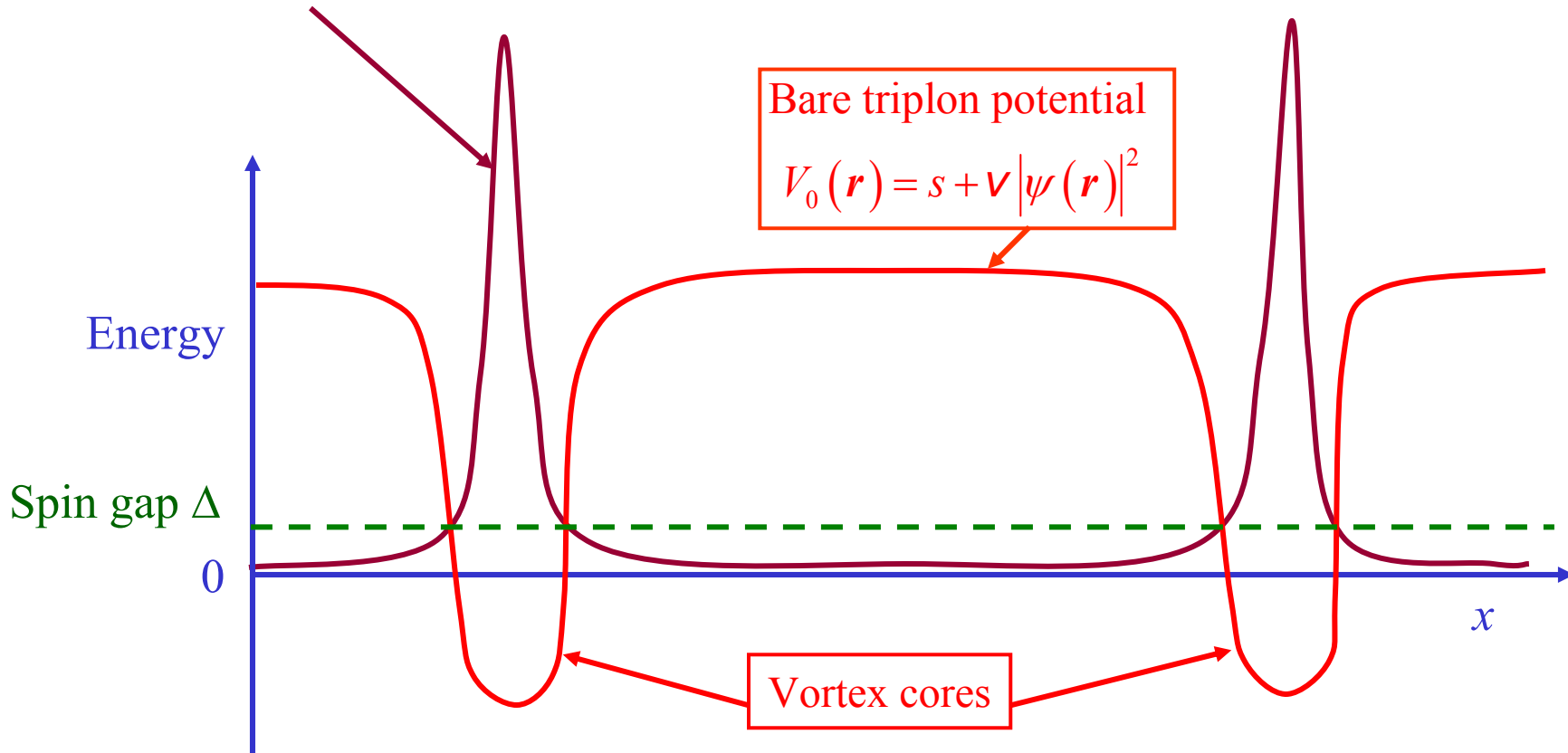
$$\mathcal{S}_c = \int d^2r d\tau \left[\frac{v}{2} |\Phi_\alpha|^2 |\psi|^2 \right]$$

$$Z[\psi(r)] = \int D\Phi(r, \tau) e^{-F_{GL} - \mathcal{S}_b - \mathcal{S}_c}$$
$$\frac{\delta \ln Z[\psi(r)]}{\delta \psi(r)} = 0$$

$$F_{GL} = \int d^2r \left[-|\psi|^2 + \frac{|\psi|^4}{2} + |(\nabla_r - iA)\psi|^2 \right]$$

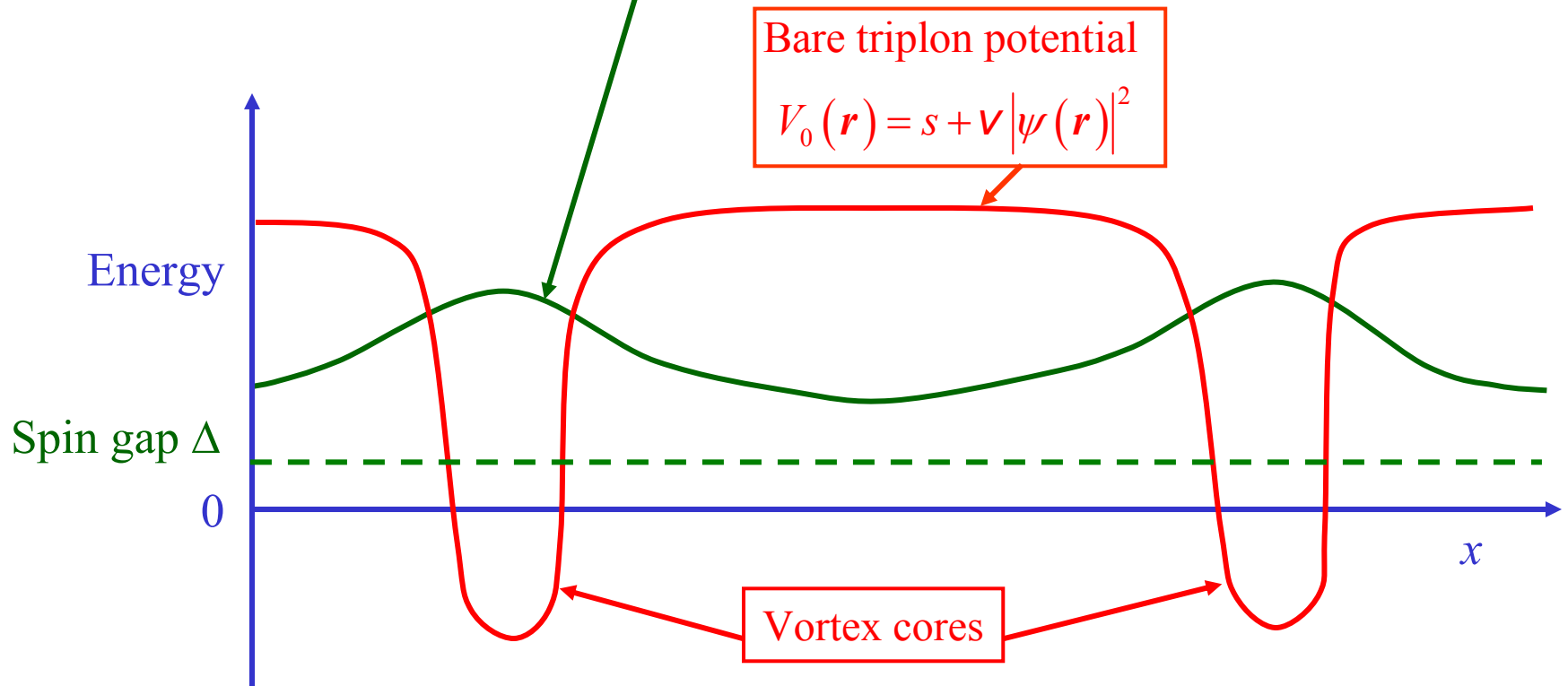
Static Ginzburg-Landau theory for non-critical superconductivity

Triplon wavefunction in
bare potential $V_0(x)$



Wavefunction of lowest energy triplon Φ_α

after including triplon interactions: $V(\mathbf{r}) = V_0(\mathbf{r}) + g \langle |\Phi_\alpha(\mathbf{r})|^2 \rangle$



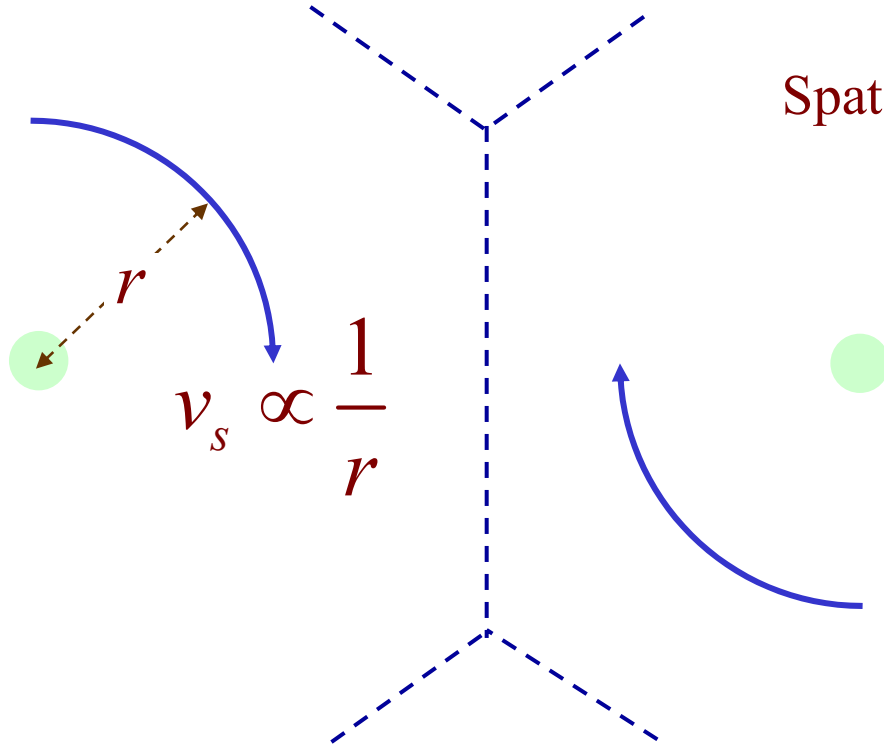
Strongly relevant repulsive interactions between excitons imply that triplons must be extended as $\Delta \rightarrow 0$.

E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

A.J. Bray and M.A. Moore, *J. Phys. C* **15**, L7 65 (1982).

J.A. Hertz, A. Fleishman, and P.W. Anderson, *Phys. Rev. Lett.* **43**, 942 (1979).

Phase diagram of SC and SDW order in a magnetic field



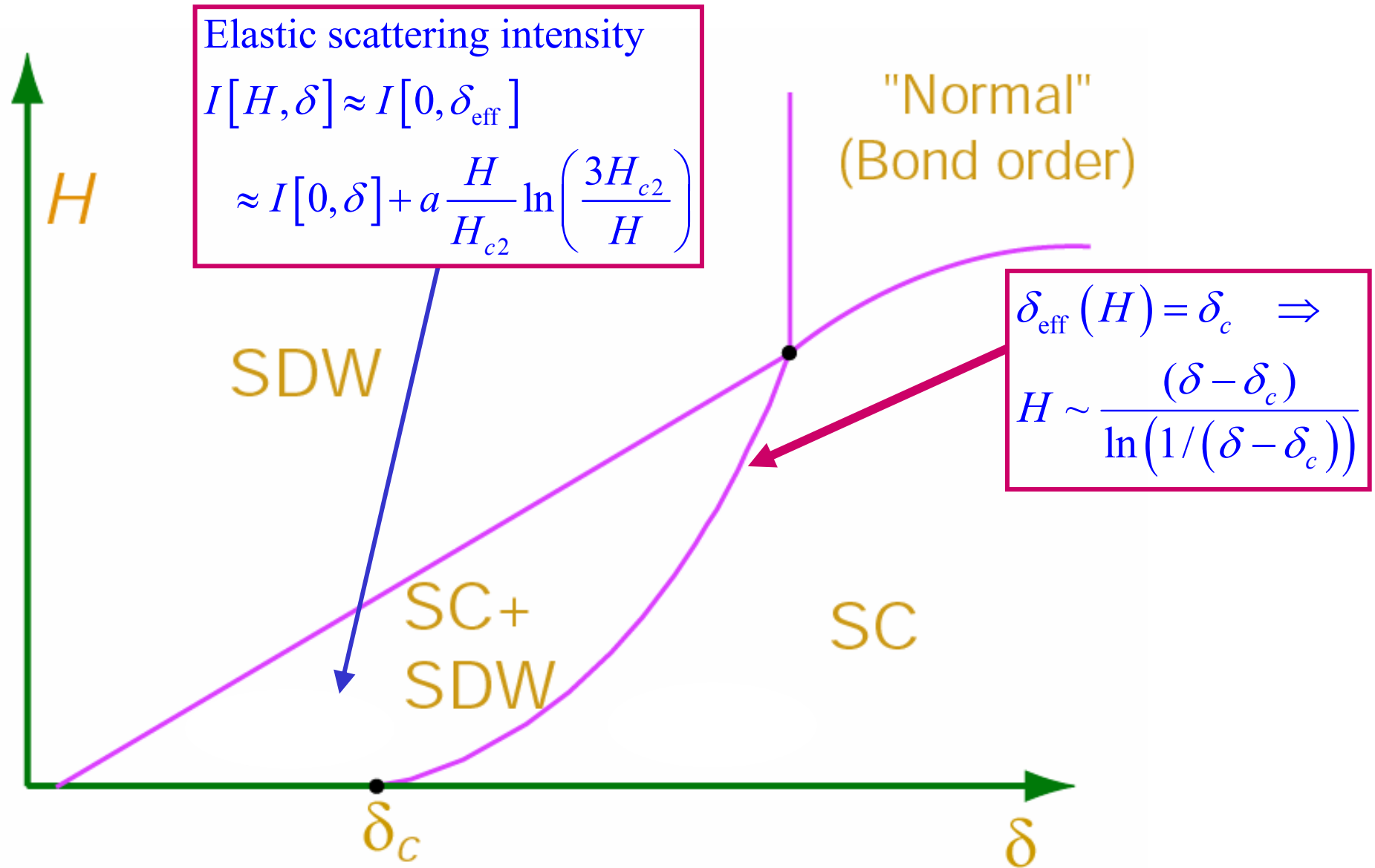
Spatially averaged superflow kinetic energy

$$\langle v_s^2 \rangle \propto \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

The suppression of SC order appears to the SDW order as a **uniform** effective "doping" δ :

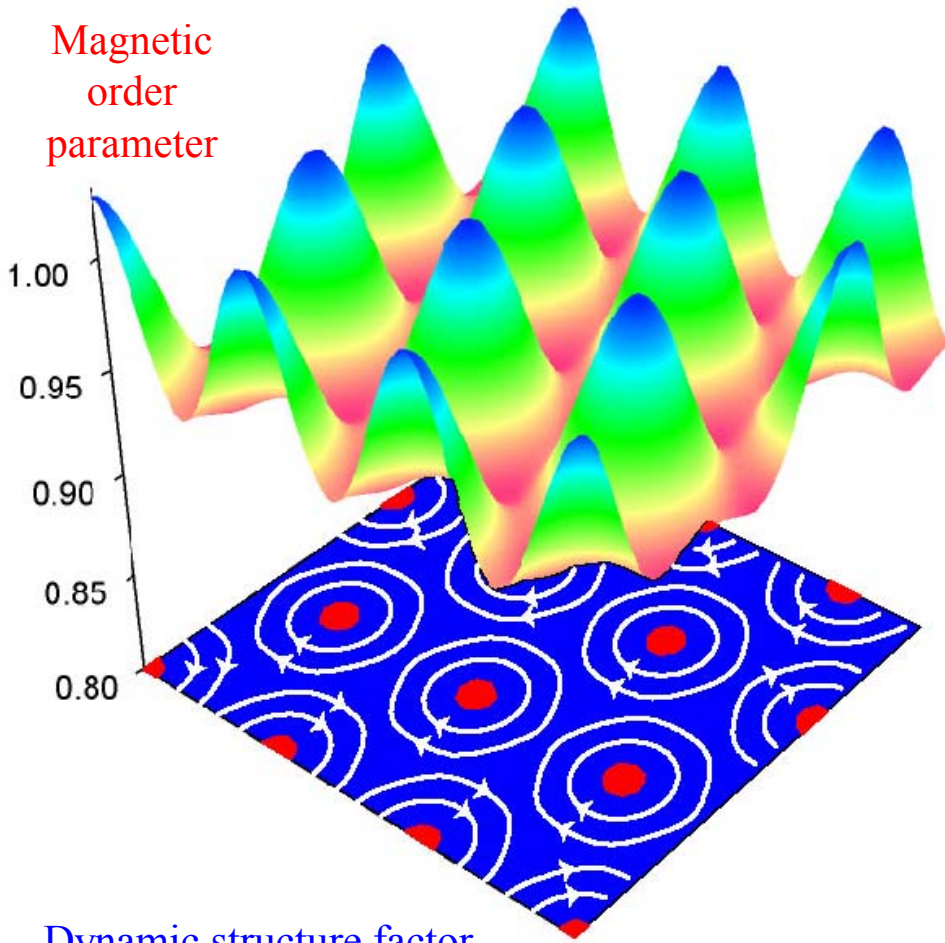
$$\delta_{\text{eff}}(H) = \delta - C \frac{H}{H_{c2}} \ln \left(\frac{3H_{c2}}{H} \right)$$

Phase diagram of SC and SDW order in a magnetic field

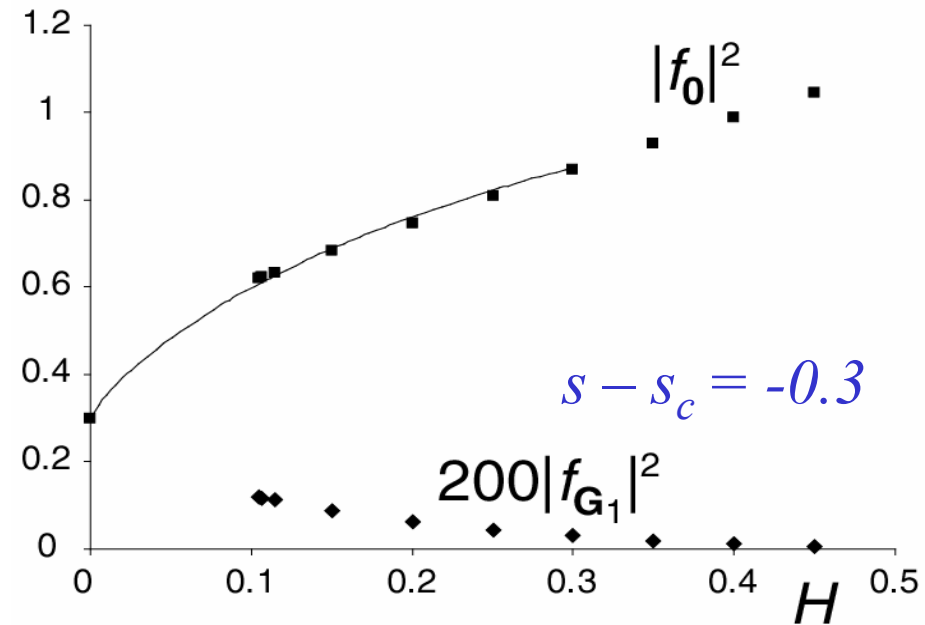


Structure of *long-range* SDW order in SC+SDW phase

E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

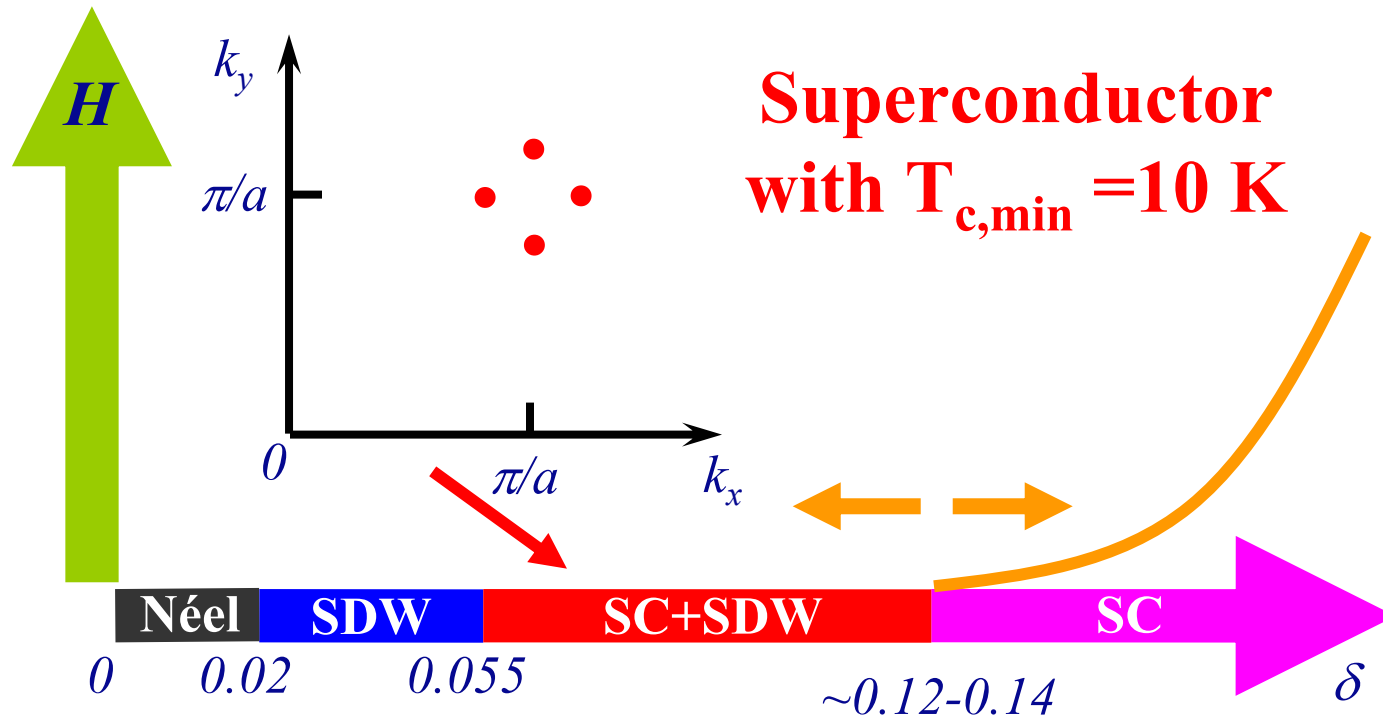


$$\delta |f_0|^2 \propto H \ln(1/H)$$



Interplay of SDW and SC order in the cuprates

T=0 phases of LSCO

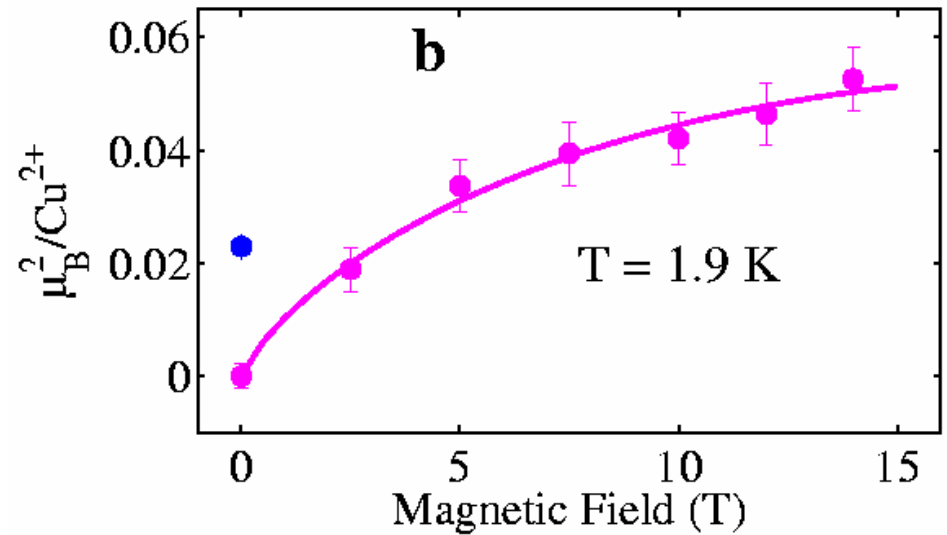
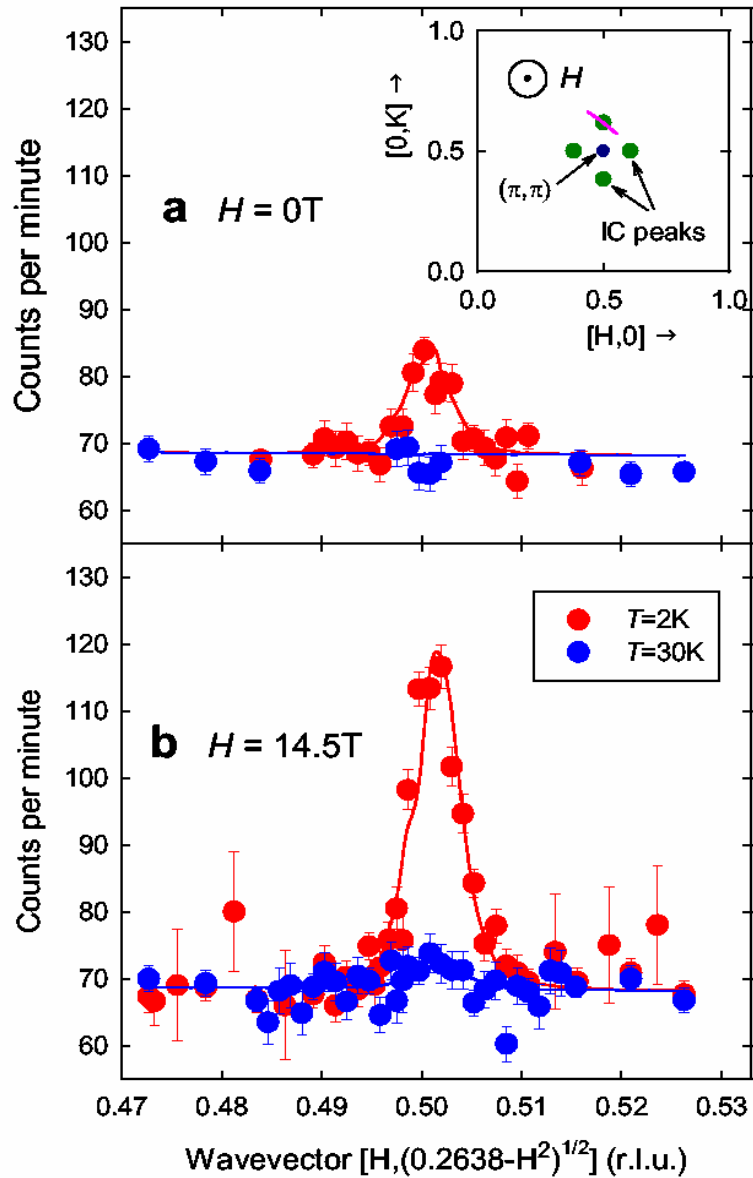


Use simplest assumption of a direct second-order quantum phase transition between SC and SC+SDW phases

Follow intensity of elastic Bragg spots in a magnetic field

Neutron scattering of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ at $x=0.1$

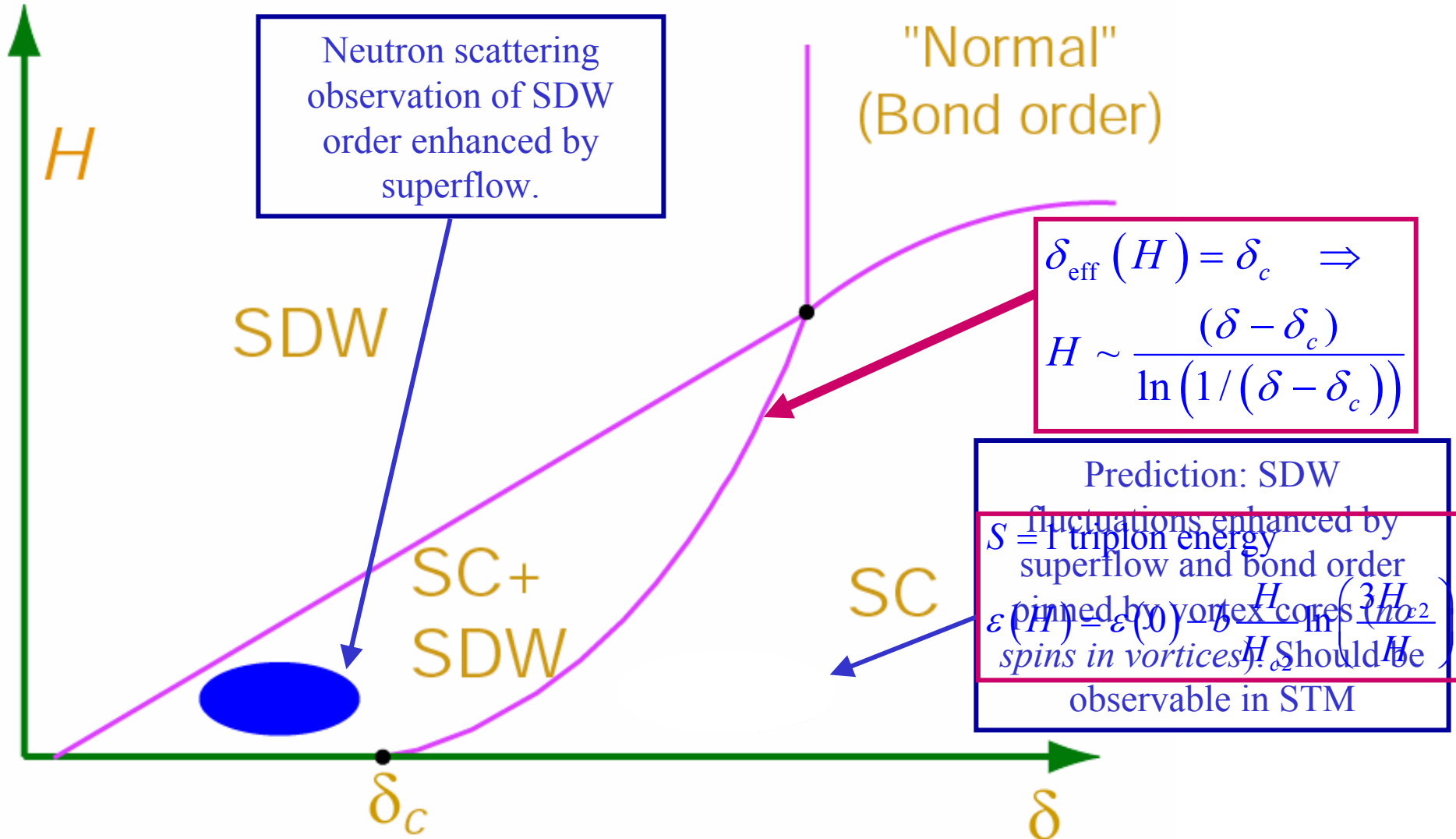
B. Lake, H. M. Rønnow, N. B. Christensen, G. Aeppli, K. Lefmann, D. F. McMorrow, P. Vorderwisch, P. Smeibidl, N. Mangkorntong, T. Sasagawa, M. Nohara, H. Takagi, T. E. Mason, *Nature*, **415**, 299 (2002).



Solid line - fit to : $I(H) = a \frac{H}{H_{c2}} \ln \left(\frac{H_{c2}}{H} \right)$

See also S. Katano, M. Sato, K. Yamada, T. Suzuki, and T. Fukase, *Phys. Rev. B* **62**, R14677 (2000).

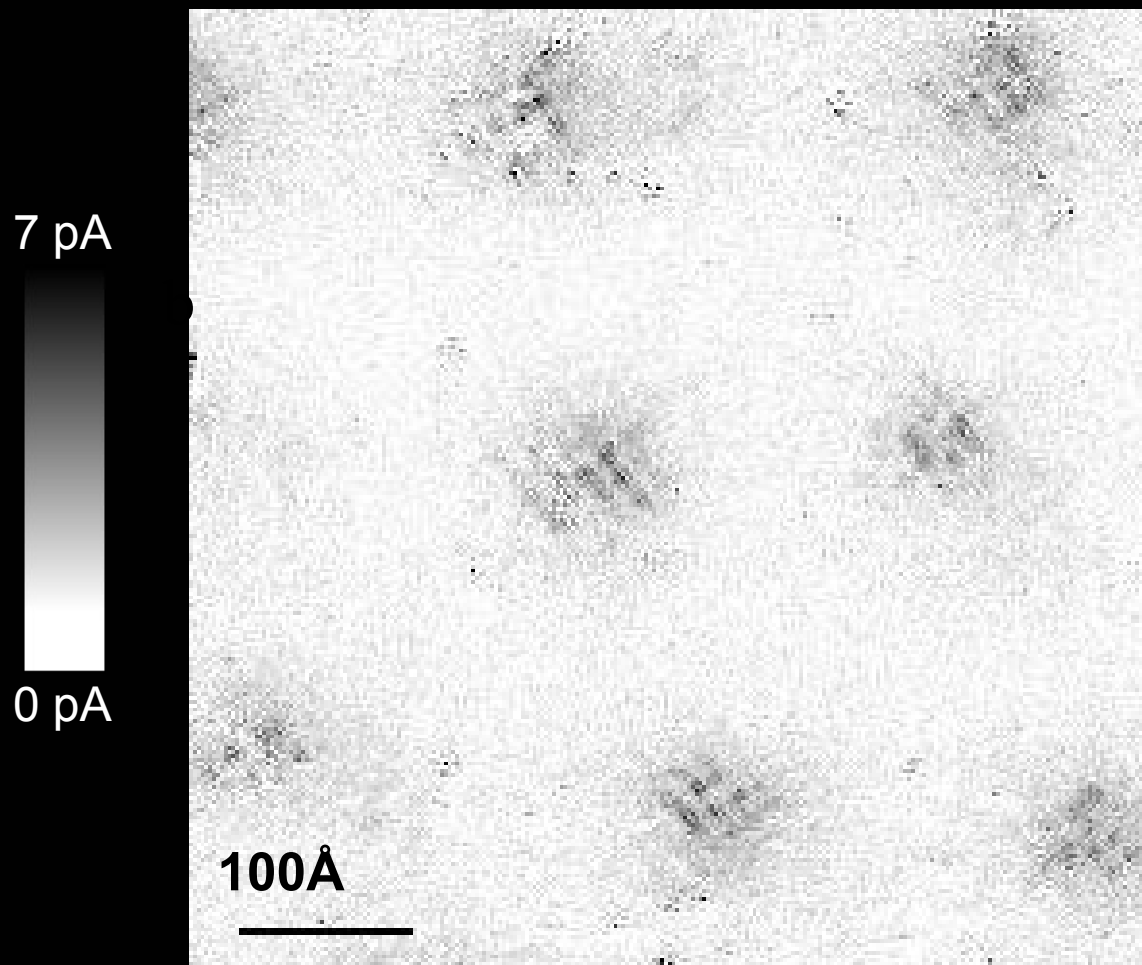
Phase diagram of a superconductor in a magnetic field



K. Park and S. Sachdev *Physical Review B* **64**, 184510 (2001);

E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067209 (2001)
 Y. Zhang, E. Demler and S. Sachdev, *Physical Review B* **66**, 094501 (2002).

Vortex-induced LDOS of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ integrated from 1meV to 12meV



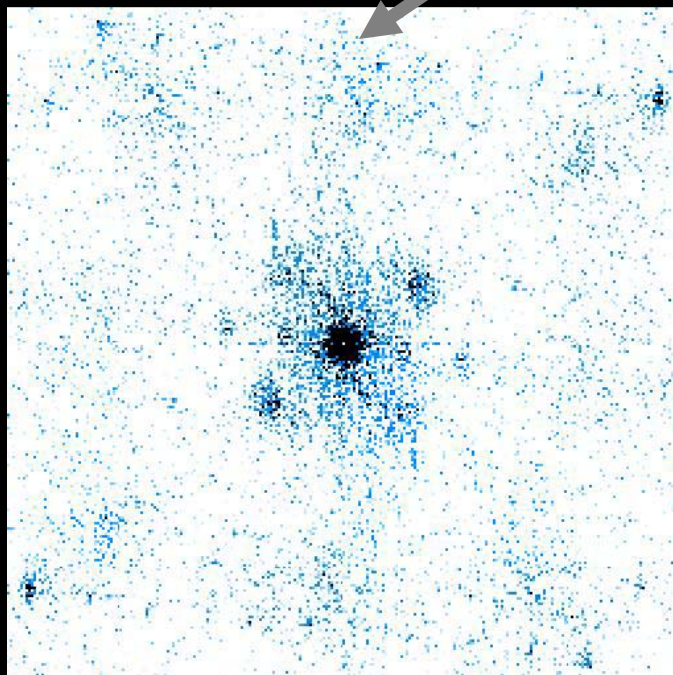
Our interpretation:
LDOS modulations are
signals of bond order of
period 4 revealed in
vortex halo

See also:

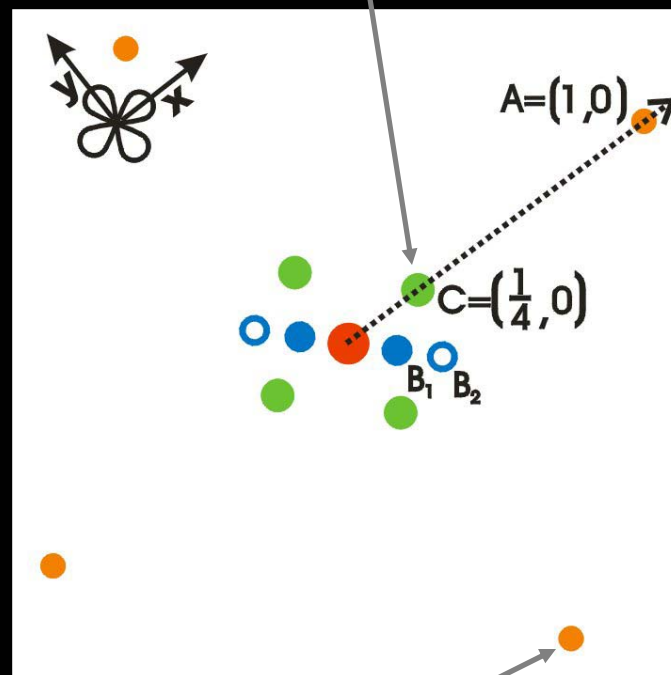
S. A. Kivelson, E. Fradkin,
V. Oganesyan, I. P. Bindloss,
J. M. Tranquada,
A. Kapitulnik, and
C. Howald,
cond-mat/0210683.

J. Hoffman E. W. Hudson, K. M. Lang,
V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida,
and J. C. Davis, *Science* 295, 466 (2002).

Fourier Transform of Vortex-Induced LDOS map



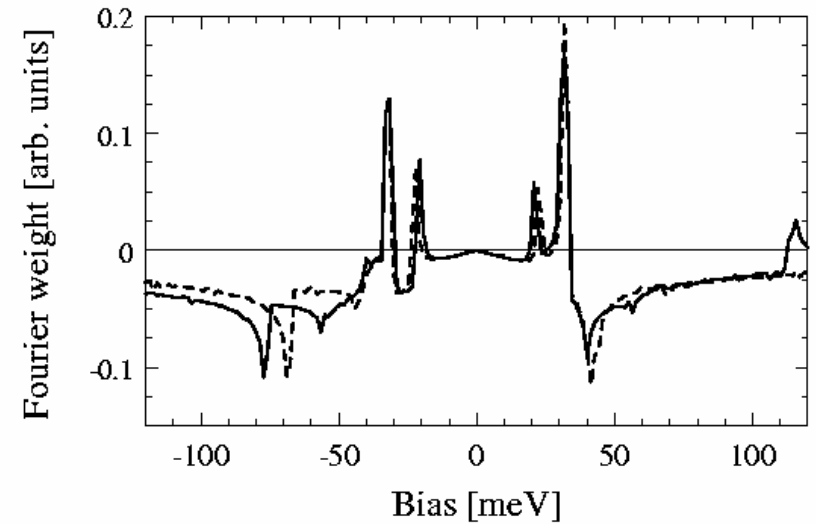
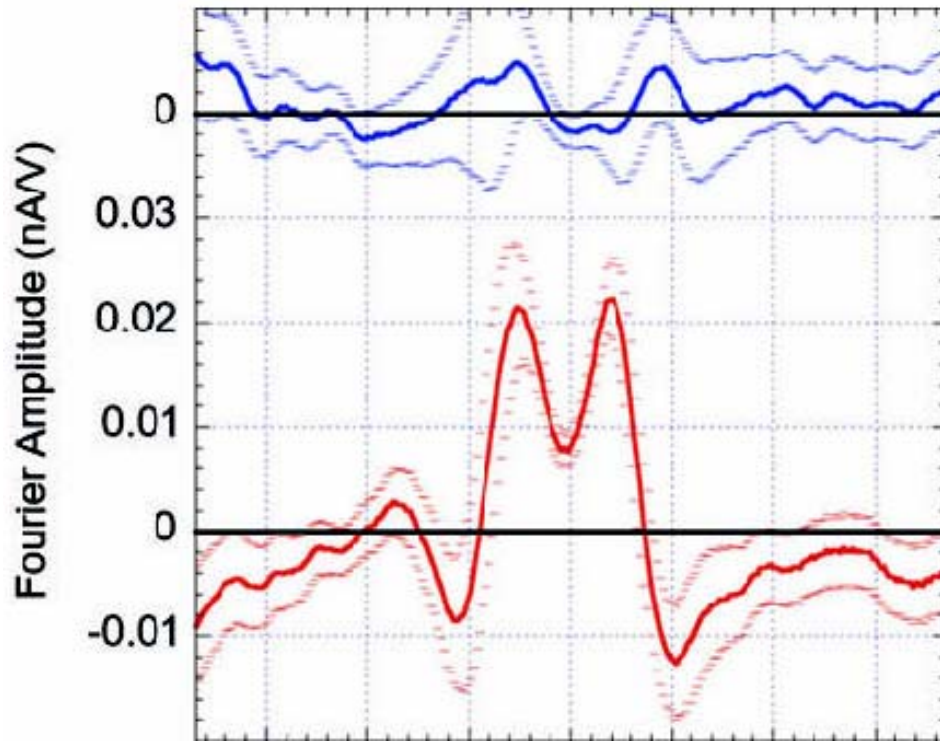
K-space locations of vortex induced LDOS



K-space locations of Bi and Cu atoms

Distances in k-space have units of $2\pi/a_0$
 $a_0 = 3.83 \text{ \AA}$ is Cu-Cu distance

Spectral properties of the STM signal are sensitive to the microstructure of the charge order



Theoretical modeling shows that this spectrum is best obtained by a modulation of bond variables, such as the exchange, kinetic or pairing energies.

Measured energy dependence of the Fourier component of the density of states which modulates with a period of 4 lattice spacings

C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, *Phys. Rev. B* **67**, 014533 (2003).

M. Vojta, *Phys. Rev. B* **66**, 104505 (2002);
D. Podolsky, E. Demler, K. Damle, and B.I. Halperin, *Phys. Rev. B* in press, cond-mat/0204011

Outline

- I. Quantum Ising Chain
- II. Coupled Dimer Antiferromagnet
 - A. Coherent state path integral
 - B. Quantum field theory near critical point
- III. Coupled dimer antiferromagnet in a magnetic field
 - Bose condensation of “triplons”

Single order parameter.

- IV. Magnetic transitions in superconductors
 - Quantum phase transition in a background
 - Abrikosov flux lattice

Multiple order parameters

V. Antiferromagnets with an odd number of $S=1/2$ spins per unit cell

Class A : Compact $U(1)$ gauge theory: collinear spins, bond order and confined spinons in $d=2$

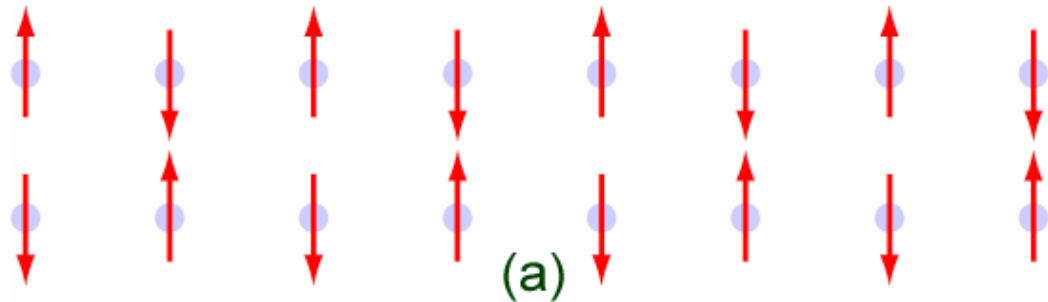
Class B: Z_2 gauge theory: non-collinear spins, RVB, visons, topological order, and deconfined spinons

- VI. Conclusions

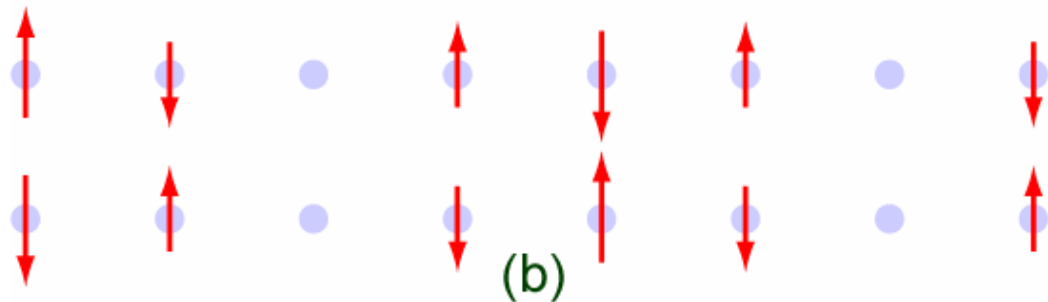
V. Order in Mott insulators

Magnetic order $\langle \mathbf{S}_j \rangle = N_1 \cos(\vec{K} \cdot \vec{r}_j) + N_2 \sin(\vec{K} \cdot \vec{r}_j)$

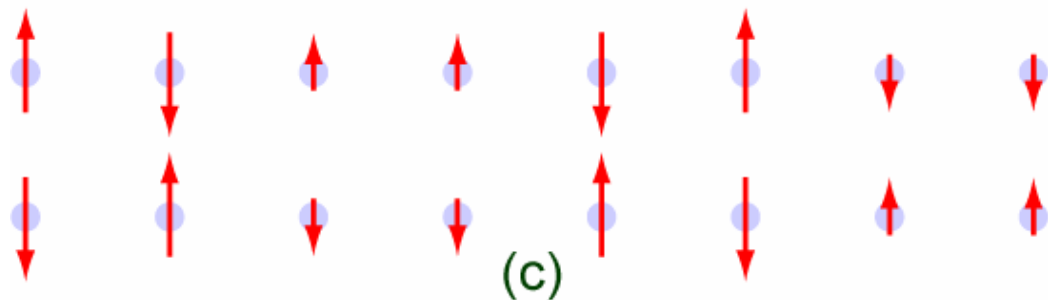
Class A. Collinear spins



$$\vec{K} = (\pi, \pi) ; N_2 = 0$$



$$\vec{K} = (3\pi/4, \pi) ; N_2 = 0$$



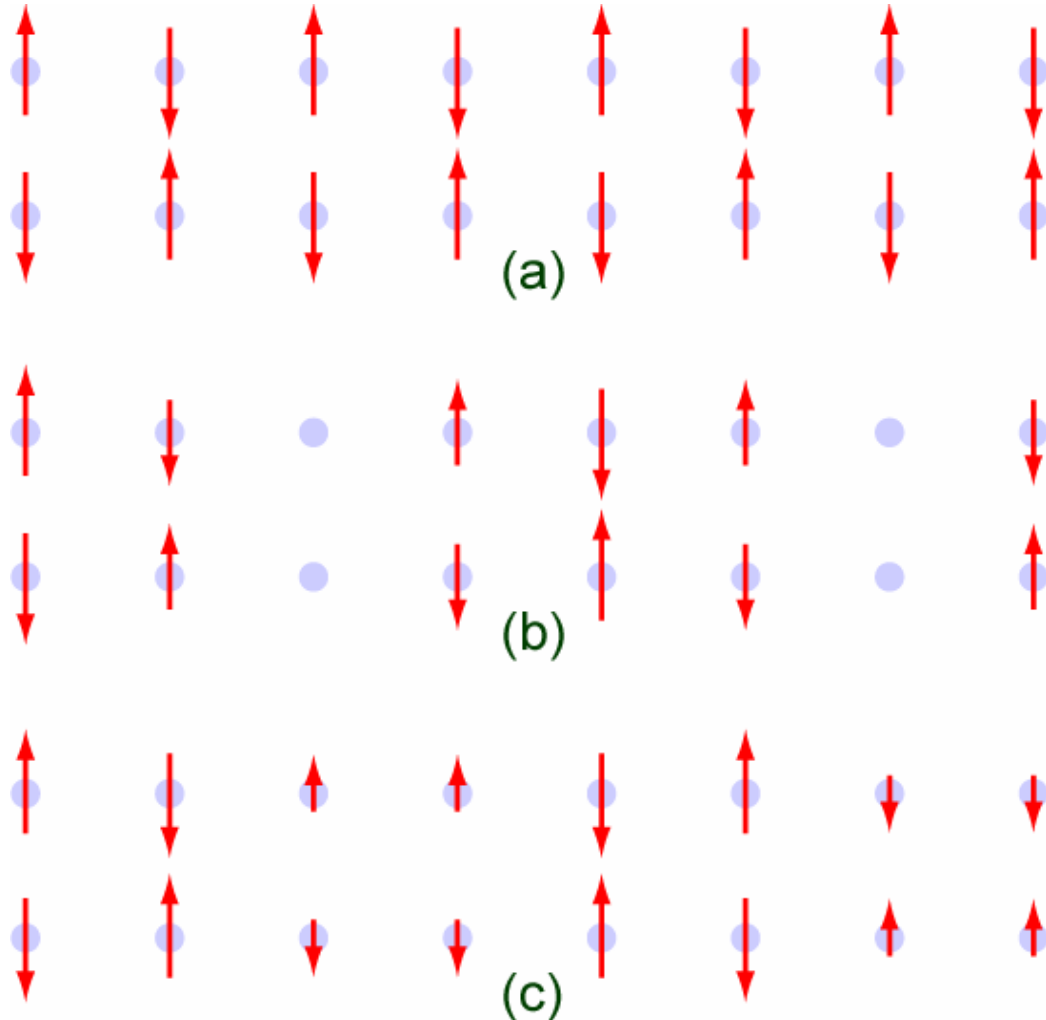
$$\vec{K} = (3\pi/4, \pi) ;$$

$$N_2 = (\sqrt{2} - 1) N_1$$

V. Order in Mott insulators

Magnetic order $\langle \mathbf{S}_j \rangle = N_1 \cos(\vec{K} \cdot \vec{r}_j) + N_2 \sin(\vec{K} \cdot \vec{r}_j)$

Class A. Collinear spins



Key property

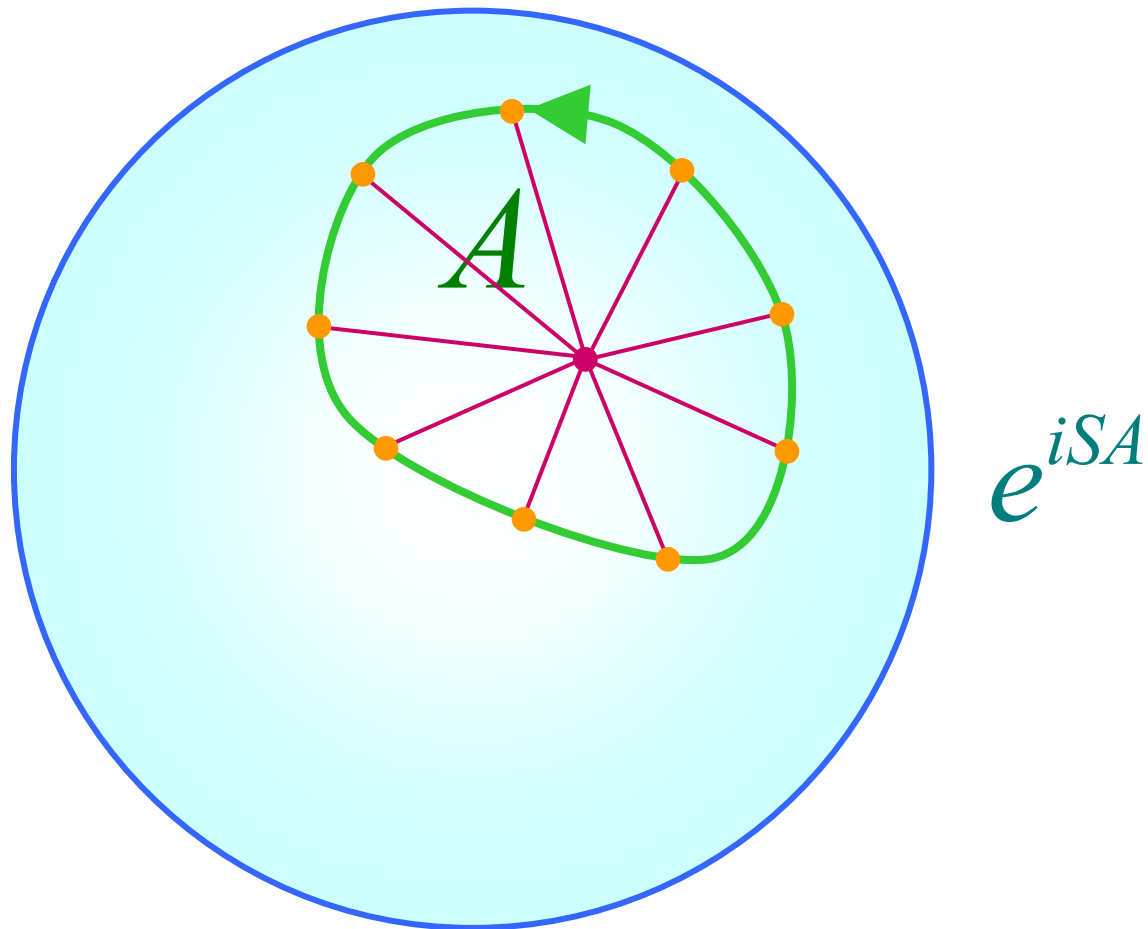
Order specified by a single vector N .

Quantum fluctuations leading to loss of magnetic order should produce a paramagnetic state with a vector ($S=1$) quasiparticle excitation.

Class A: Collinear spins and compact U(1) gauge theory

Write down path integral for quantum spin fluctuations

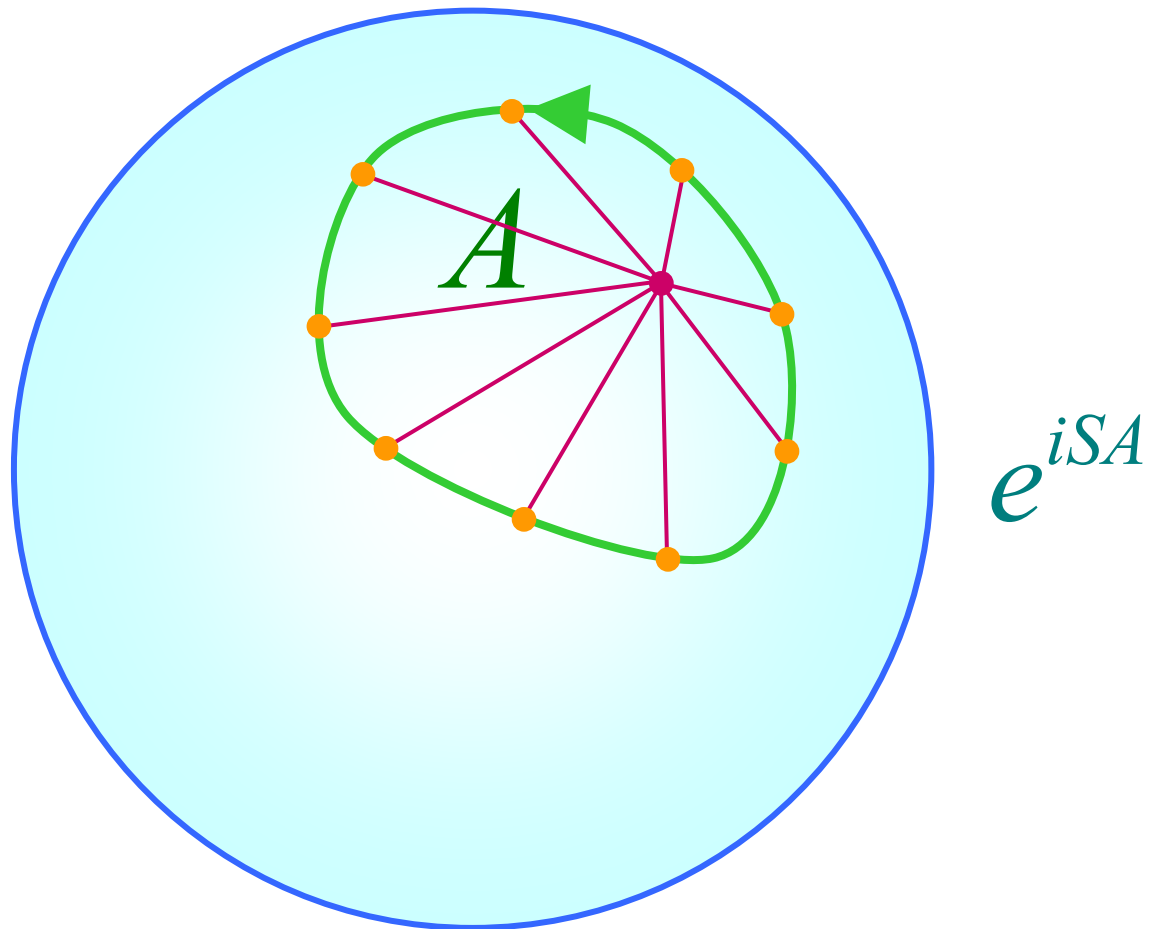
Key ingredient: Spin Berry Phases



Class A: Collinear spins and compact U(1) gauge theory

Write down path integral for quantum spin fluctuations

Key ingredient: Spin Berry Phases



Class A: Collinear spins and compact U(1) gauge theory

$S=1/2$ square lattice antiferromagnet with non-nearest neighbor exchange

$$H = \sum_{i < j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Include Berry phases after discretizing coherent state path integral on a cubic lattice in spacetime

$$Z = \prod_a \int d\mathbf{n}_a \delta(\mathbf{n}_a^2 - 1) \exp\left(\frac{1}{g} \sum_{a,\mu} \mathbf{n}_a \cdot \mathbf{n}_{a+\mu} - \frac{i}{2} \sum_a \eta_a A_{a\tau}\right)$$

$\eta_a \rightarrow \pm 1$ on two square sublattices ;

$\mathbf{n}_a \sim \eta_a \vec{S}_a \rightarrow$ Neel order parameter;

$A_{a\mu} \rightarrow$ oriented area of spherical triangle

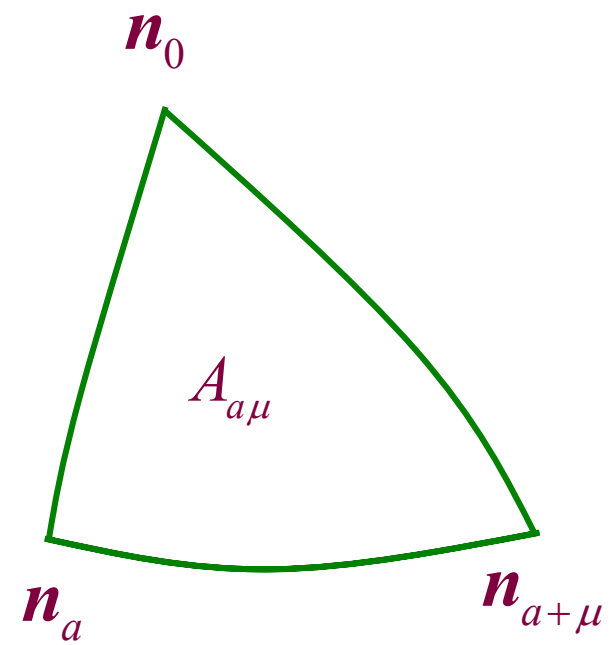
formed by \mathbf{n}_a , $\mathbf{n}_{a+\mu}$, and an arbitrary reference point \mathbf{n}_0

$$Z = \prod_a \int d\mathbf{n}_a \delta(\mathbf{n}_a^2 - 1) \exp\left(\frac{1}{g} \sum_{a,\mu} \mathbf{n}_a \cdot \mathbf{n}_{a+\mu} - \frac{i}{2} \sum_a \eta_a A_{a\tau}\right)$$

Small g \rightarrow Spin-wave theory about Neel state receives minor modifications from Berry phases.

Large g \rightarrow Berry phases are crucial in determining structure of "quantum-disordered" phase with $\langle \mathbf{n}_a \rangle = 0$

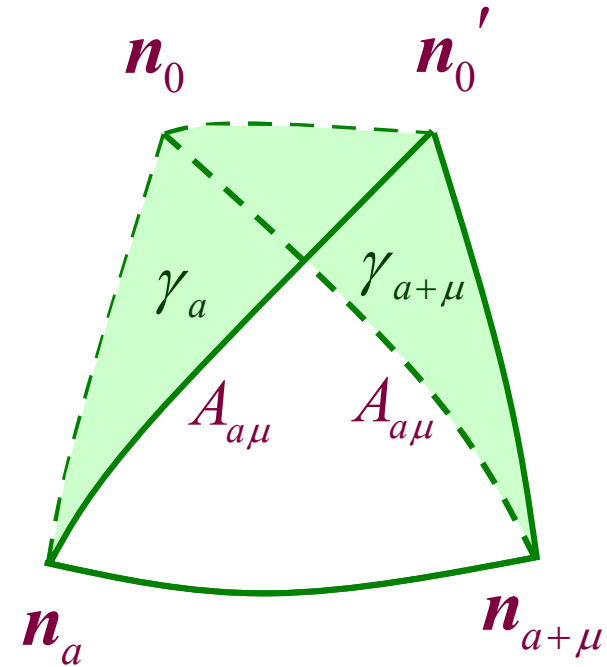
Integrate out n_a *to obtain effective action for* $A_{a\mu}$



Change in choice of \mathbf{n}_0 is like a “gauge transformation”

$$A_{a\mu} \rightarrow A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

(γ_a is the oriented area of the spherical triangle formed by \mathbf{n}_a and the two choices for \mathbf{n}_0).



The area of the triangle is uncertain modulo 4π , and the action is invariant under

$$A_{a\mu} \rightarrow A_{a\mu} + 4\pi$$

These principles strongly constrain the effective action for $A_{a\mu}$ which provides description of the large g phase

Simplest large g effective action for the $A_{a\mu}$

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp \left(\frac{1}{2e^2} \sum_{\square} \cos \left(\frac{1}{2} (\Delta_{\mu} A_{a\nu} - \Delta_{\nu} A_{a\mu}) \right) - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

This is compact QED in $d+1$ dimensions with static charges ± 1 on two sublattices.

This theory can be reliably analyzed by a duality mapping.

$d=2$: The gauge theory is *always* in a *confining* phase and there is bond order in the ground state.

$d=3$: A deconfined phase with a gapless “photon” is possible.

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

S. Sachdev and R. Jalabert, *Mod. Phys. Lett. B* **4**, 1043 (1990).

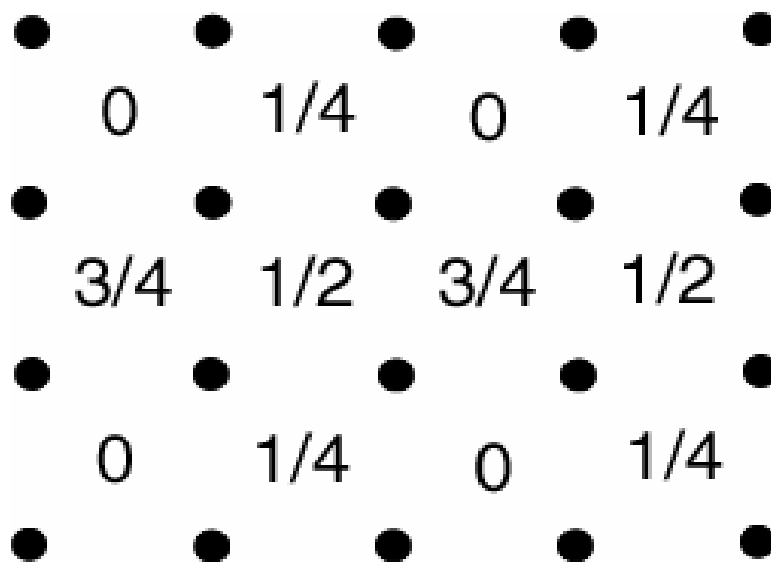
K. Park and S. Sachdev, *Phys. Rev. B* **65**, 220405 (2002).

Exact duality transform on periodic Gaussian (“Villain”) action for compact QED yields

$$Z = \sum_{\{h_{\bar{j}}\}} \exp \left(-\frac{e^2}{2} \sum_{\bar{j}} (\Delta_{\mu} h_{\bar{j}} - \Delta_{\mu} \mathcal{X}_{\bar{j}})^2 \right)$$

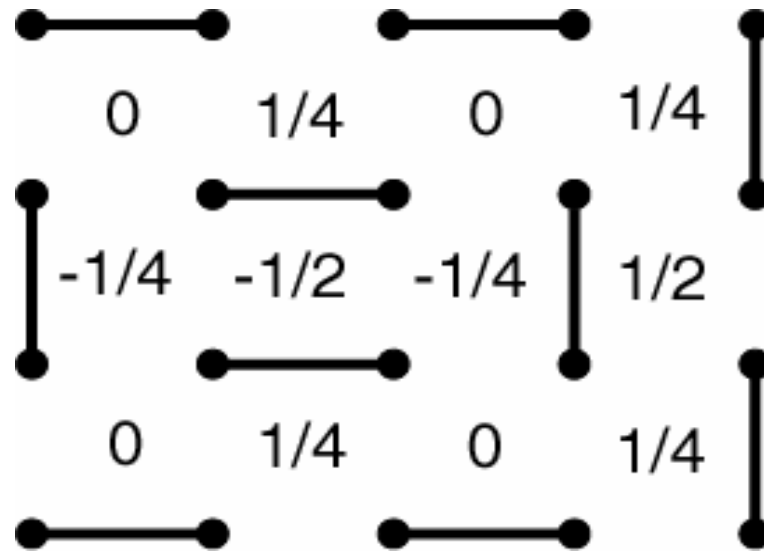
with $h_{\bar{j}}$ integer.

Height model in 2+1 dimensions with ‘offsets’ $\mathcal{X}_{\bar{j}} = 0, 1/4, 1/2, 3/4$ on the four dual sublattices.



For large e^2 , low energy height configurations are in exact one-to-one correspondence with dimer coverings of the square lattice

⇒ 2+1 dimensional height model is the path integral of the Quantum Dimer Model



There is no roughening transition for three dimensional interfaces, which are smooth for all couplings

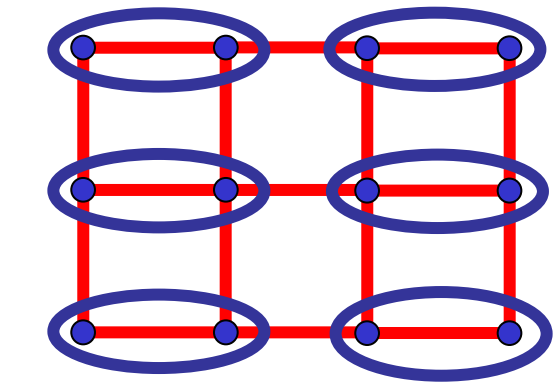
⇒ There is a definite average height of the interface

⇒ Ground state has bond order.

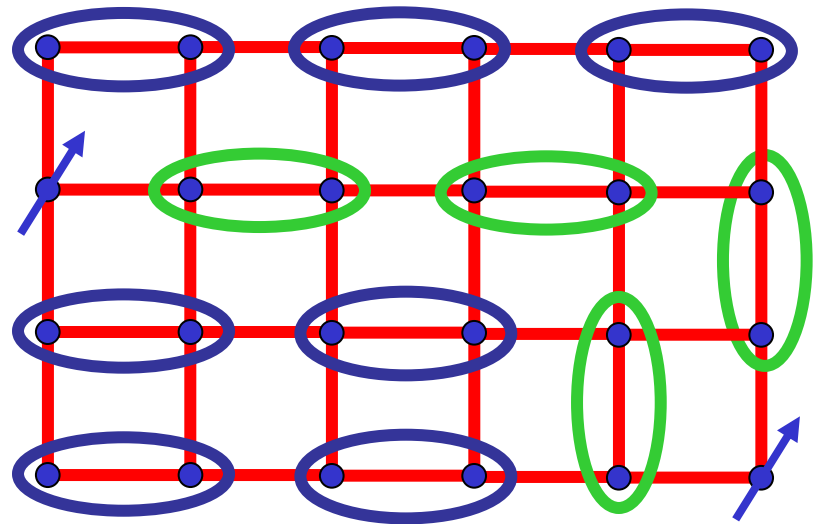
V. Order in Mott insulators

Paramagnetic states $\langle \mathbf{S}_j \rangle = 0$

Class A. Bond order and spin excitons in $d=2$



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



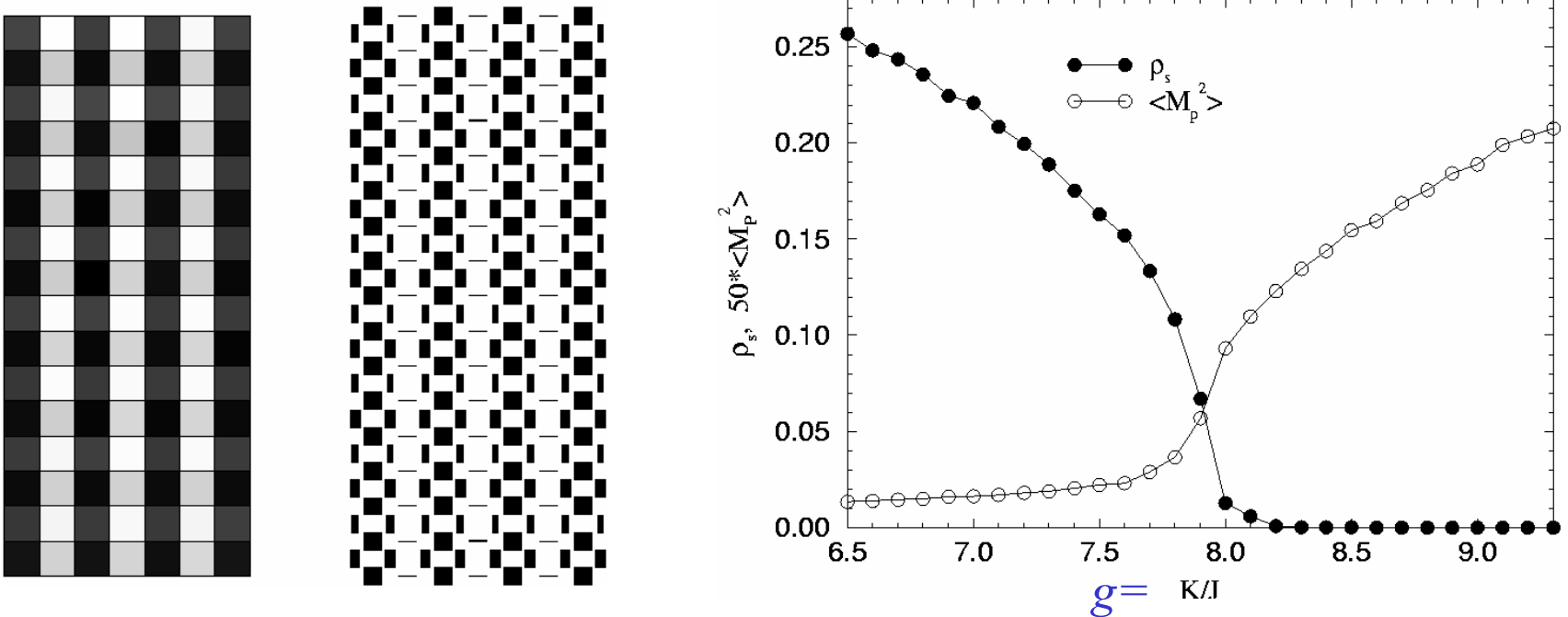
$S=1/2$ *spinons* are confined
by a linear potential into a
 $S=1$ spin *triplon*

Spontaneous bond-order leads to vector $S=1$ spin excitations

Bond order in a frustrated $S=1/2$ XY magnet

A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, *Phys. Rev. Lett.* **89**, 247201 (2002)

First *large scale* numerical study of the destruction of Neel order in a $S=1/2$ antiferromagnet with full square lattice symmetry



$$H = 2J \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y) - K \sum_{\langle ijkl \rangle \square} (S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+)$$

Outline

I. Quantum Ising Chain

II. Coupled Dimer Antiferromagnet

A. Coherent state path integral

B. Quantum field theory near critical point

III. Coupled dimer antiferromagnet in a magnetic field

Bose condensation of “triplons”

IV. Magnetic transitions in superconductors

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Abrikosov flux lattice

V. Antiferromagnets with an odd number of $S=1/2$ spins per unit cell

Class A: Compact $U(1)$ gauge theory: collinear spins, bond order and confined spinons in $d=2$

Class B Z_2 gauge theory: non-collinear spins, RVB, visons, topological order, and deconfined spinons

VI. Conclusions

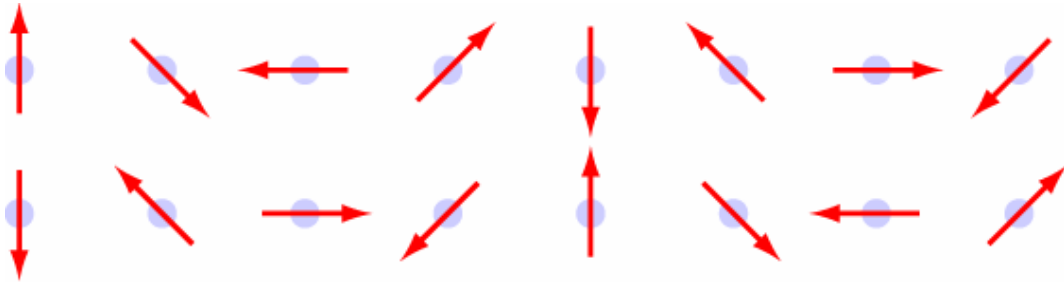
Single order parameter.

Multiple order parameters

V.B Order in Mott insulators

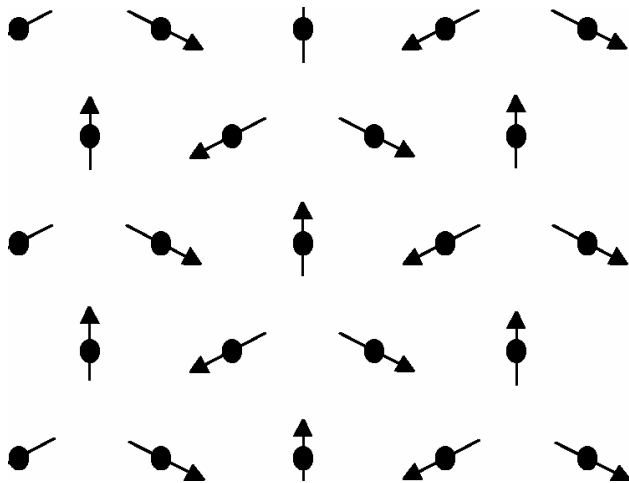
Magnetic order $\langle \mathbf{S}_j \rangle = N_1 \cos(\vec{K} \cdot \vec{r}_j) + N_2 \sin(\vec{K} \cdot \vec{r}_j)$

Class B. Noncollinear spins



$$\vec{K} = (3\pi/4, \pi)$$

(B.I. Shraiman and E.D. Siggia,
Phys. Rev. Lett. **61**, 467 (1988))



$$\vec{K} = (4\pi/3, 4\pi/\sqrt{3})$$

$$N_2^2 = N_1^2, N_1 \cdot N_2 = 0$$

V.B Order in Mott insulators

Magnetic order $\langle \mathbf{S}_j \rangle = N_1 \cos(\vec{K} \cdot \vec{r}_j) + N_2 \sin(\vec{K} \cdot \vec{r}_j)$

Class B. Noncollinear spins

$$N_2^2 = N_1^2, N_1 \cdot N_2 = 0$$

Solve constraints by expressing $N_{1,2}$ in terms of two complex numbers z_\uparrow, z_\downarrow

$$N_1 + iN_2 = \begin{pmatrix} z_\downarrow^2 - z_\uparrow^2 \\ i(z_\downarrow^2 + z_\uparrow^2) \\ 2z_\uparrow z_\downarrow \end{pmatrix}$$

Order in ground state specified by a spinor $(z_\uparrow, z_\downarrow)$ (modulo an overall sign).

This spinor can become a $S=1/2$ spinon in paramagnetic state.

Order parameter space: S_3/Z_2

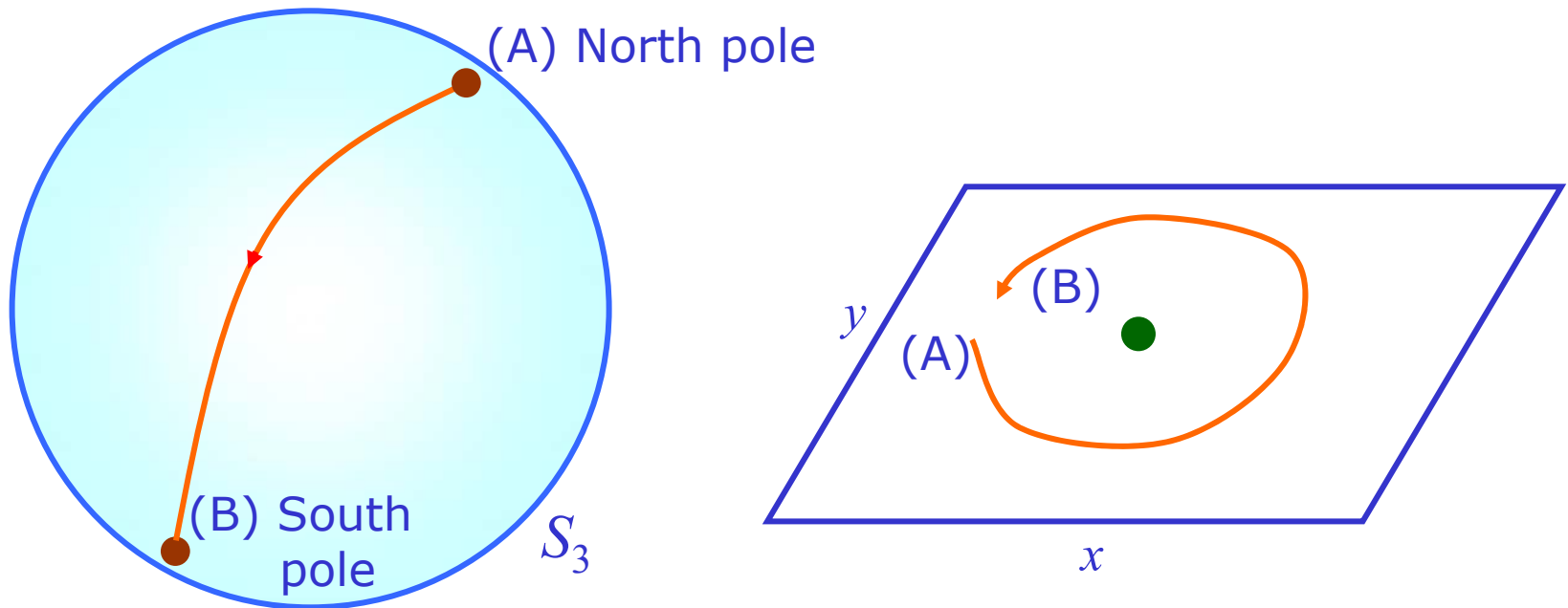
Physical observables are invariant under the Z_2 gauge transformation $z_a \rightarrow \pm z_a$

V.B Order in Mott insulators

Magnetic order $\langle \mathbf{S}_j \rangle = N_1 \cos(\vec{K} \cdot \vec{r}_j) + N_2 \sin(\vec{K} \cdot \vec{r}_j)$

Class B. Noncollinear spins

Vortices associated with $\pi_1(S_3/Z_2)=Z_2$ (*visons*)



Such vortices (visons) can also be defined in the phase in which spins are “quantum disordered”. A Z_2 spin liquid with deconfined spinons must have *visons suppressed*

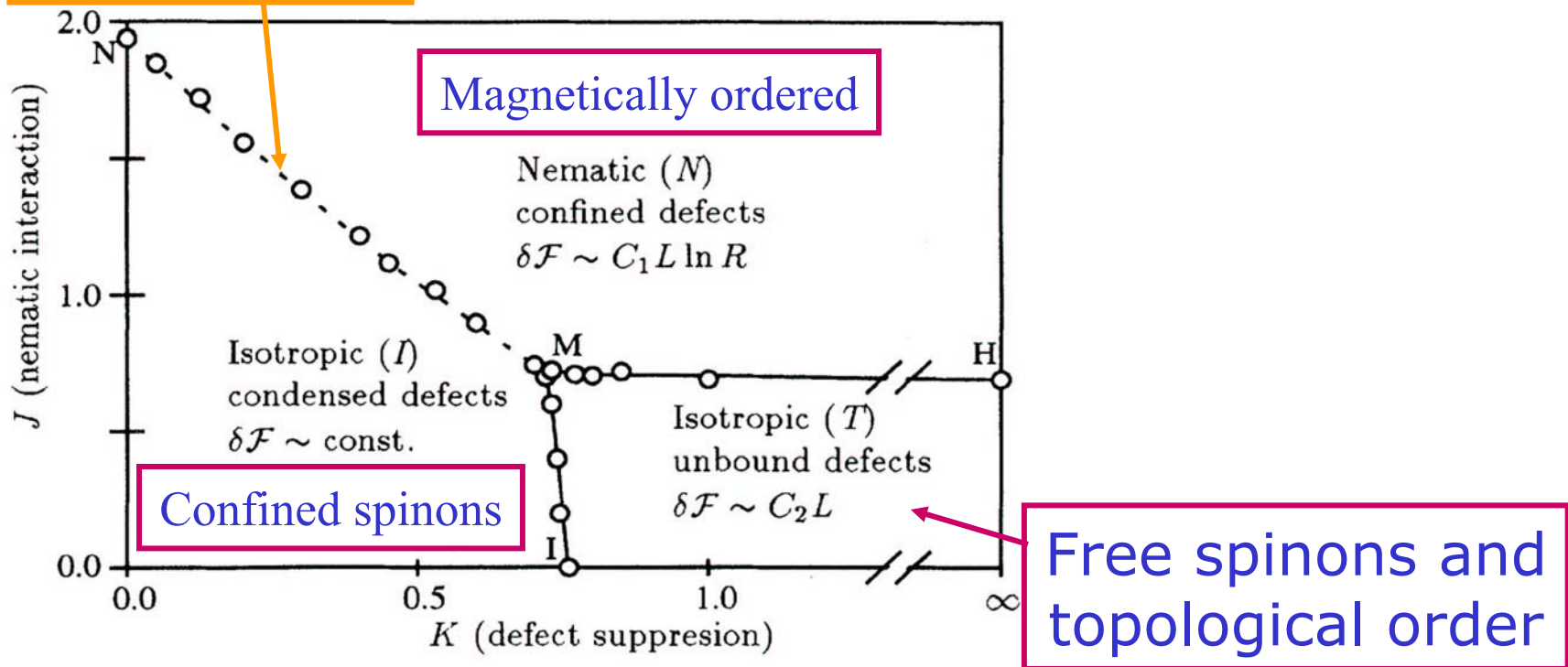
Model effective action and phase diagram

$$\mathcal{S} = -J \sum_{\langle ij \rangle} \sigma_{ij} z_{\alpha i}^* z_{\alpha j} + \text{h.c.} - K \sum_{\square} \prod_{\square} \sigma_{ij}$$

(Derivation using Schwinger bosons on a quantum antiferromagnet: S. Sachdev and N. Read, *Int. J. Mod. Phys. B* **5**, 219 (1991)).

$\sigma_{ij} \rightarrow Z_2$ gauge field

First order transition



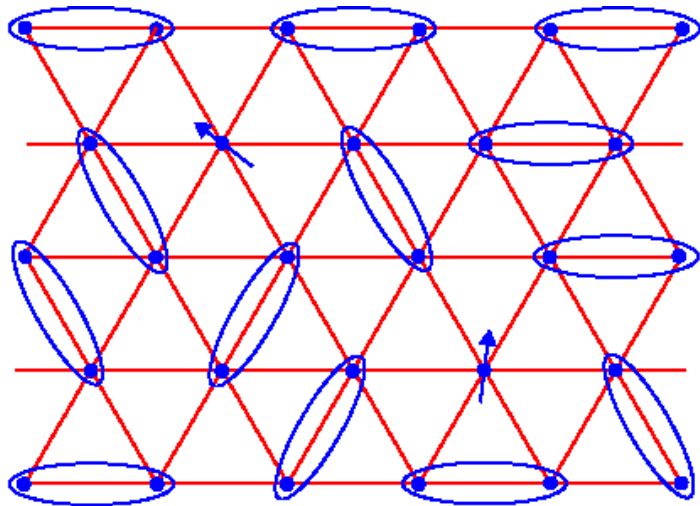
P. E. Lammert, D. S. Rokhsar, and J. Toner, *Phys. Rev. Lett.* **70**, 1650 (1993) ; *Phys. Rev. E* **52**, 1778 (1995). (For nematic liquid crystals)

V.B Order in Mott insulators

Paramagnetic states $\langle \mathbf{S}_j \rangle = 0$

Class B. Topological order and deconfined spinons

A topologically ordered state in which vortices associated with $\pi_1(S_3/Z_2)=Z_2$ [“visons”] are gapped out. This is an RVB state with deconfined $S=1/2$ spinons z_a



Spinons are deconfined

N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991).

X. G. Wen, *Phys. Rev. B* **44**, 2664 (1991).

A.V. Chubukov, T. Senthil and S. S., *Phys. Rev. Lett.* **72**, 2089 (1994).

T. Senthil and M.P.A. Fisher, *Phys. Rev. B* **62**, 7850 (2000).

P. Fazekas and P.W. Anderson, *Phil Mag* **30**, 23 (1974).

G. Misguich and C. Lhuillier, *Eur. Phys. J. B* **26**, 167 (2002).

R. Moessner and S.L. Sondhi, *Phys. Rev. Lett.* **86**, 1881 (2001).

Recent experimental realization: Cs_2CuCl_4

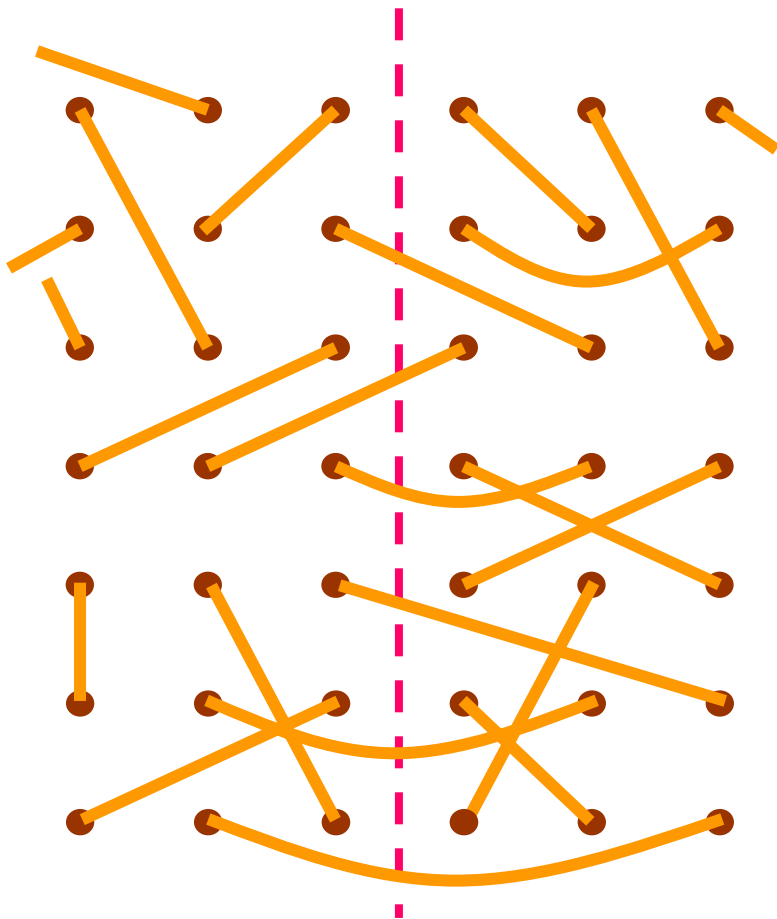
R. Coldea, D.A. Tennant, A.M. Tsvelik, and Z. Tylczynski, *Phys. Rev. Lett.* **86**, 1335 (2001).

V.B Order in Mott insulators

Paramagnetic states $\langle \mathbf{S}_j \rangle = 0$

Class B. Topological order and deconfined spinons

Direct description of topological order with valence bonds



Number of valence bonds cutting line is conserved modulo 2. Changing sign of each such bond does not modify state. This is equivalent to a Z_2 gauge transformation with $z_a \rightarrow -z_a$ on sites to the right of dashed line.

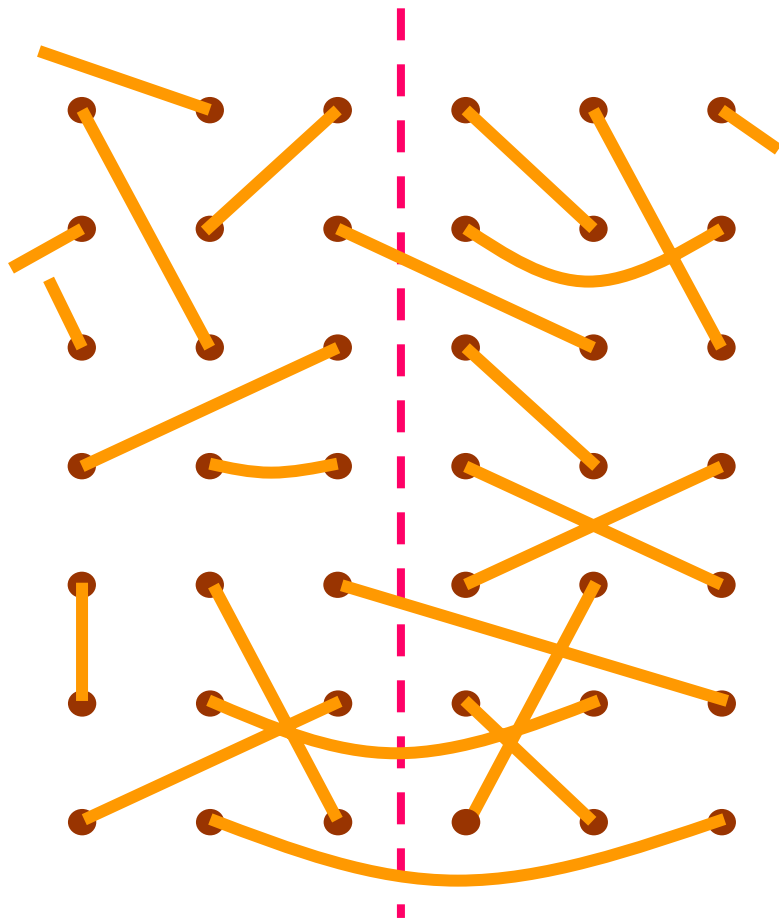
D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988); N. Read and B. Chakraborty, *Phys. Rev. B* **40**, 7133 (1989).

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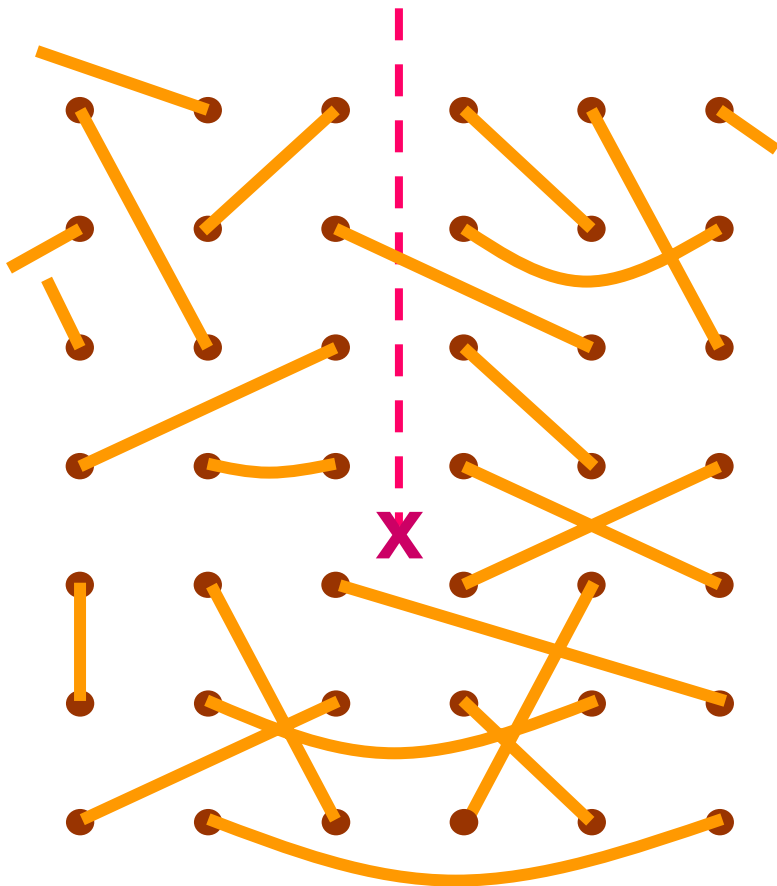
D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988); N. Read and B. Chakraborty, *Phys. Rev. B* **40**, 7133 (1989).

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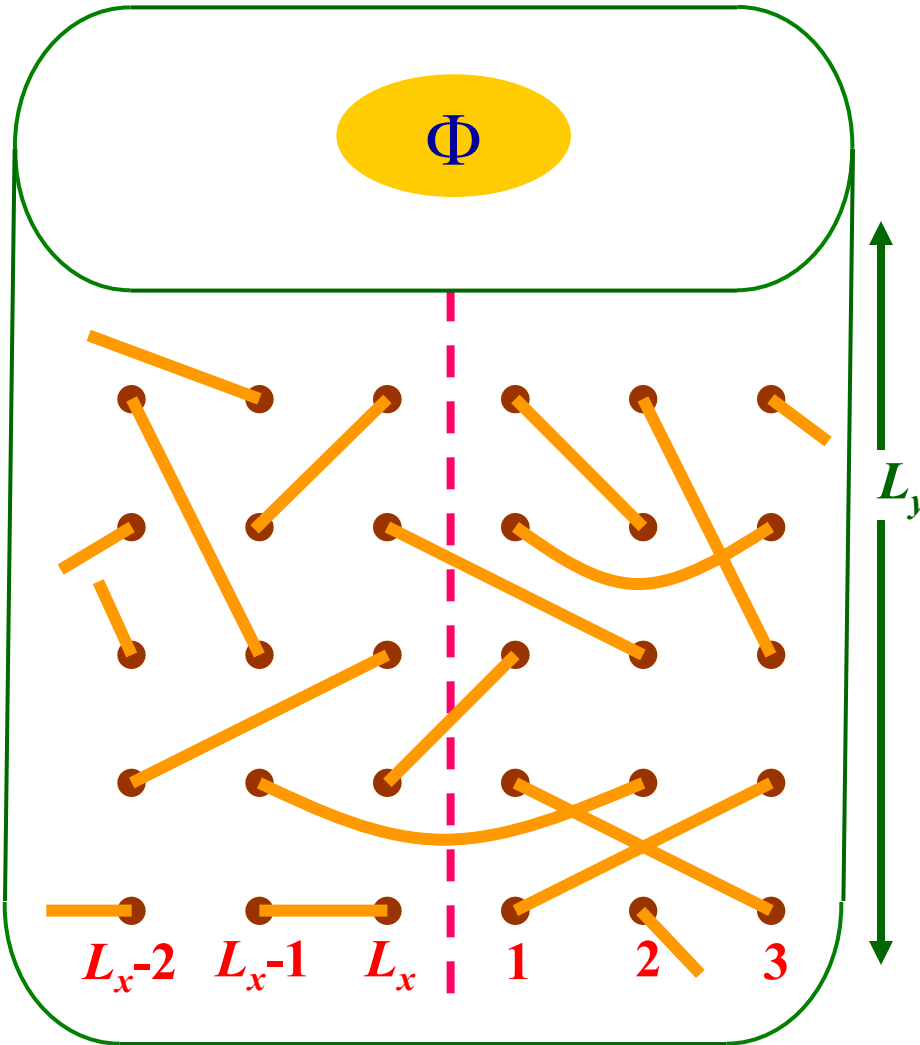
Terminating the line creates a
plaquette with Z_2 flux at the X
--- a *vison*.

D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**,
2376 (1988); N. Read and B. Chakraborty, *Phys. Rev.*
B **40**, 7133 (1989).

Effect of flux-piercing on a topologically ordered quantum paramagnet

N. E. Bonesteel,
Phys. Rev. B **40**, 8954 (1989).
G. Misguich, C. Lhuillier,
M. Mambrini, and P. Sindzingre,
Eur. Phys. J. B **26**, 167 (2002).

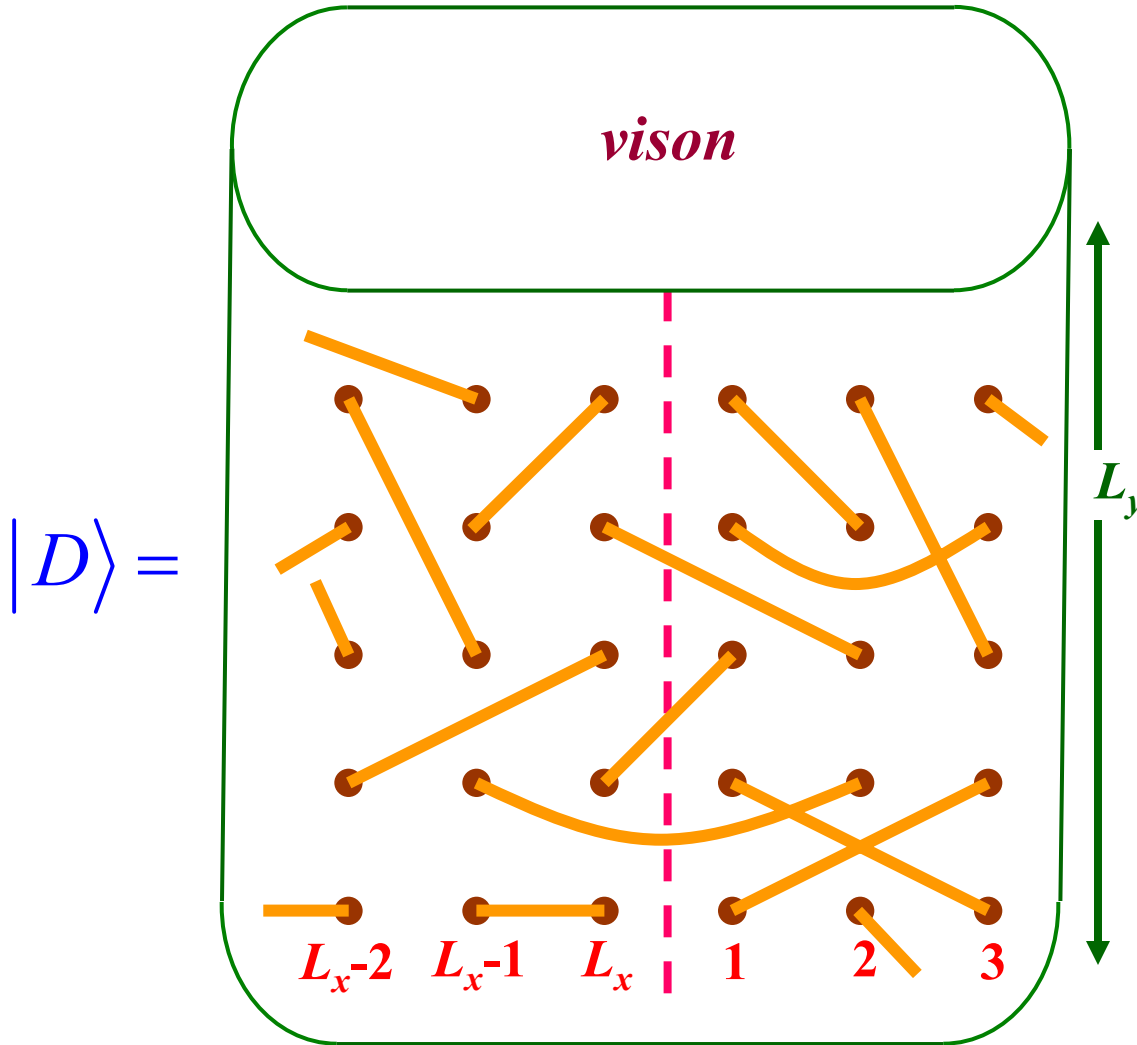
$|D\rangle =$



$$|\Psi\rangle = \sum_D a_D |D\rangle$$

Effect of flux-piercing on a topologically ordered quantum paramagnet

N. E. Bonesteel,
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 G. Misguich, C. Lhuillier,
 M. Mambrini, and P. Sindzingre,
Eur. Phys. J. B **26**, 167 (2002).



$$|\Psi\rangle = \sum_D a_D |D\rangle$$

After flux insertion $|D\rangle \Rightarrow$

$$(-1)^{\text{Number of bonds cutting dashed line}} |D\rangle;$$

Equivalent to inserting a *vison* inside hole of the torus.

This leads to a ground state degeneracy.

VI. Conclusions

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visons, topological order, and deconfined spinons

VI. Cuprates are best understood as doped class A Mott insulators.

*Single order
parameter.*

*Multiple
order
parameters*

Competing order parameters in the cuprate superconductors

1. Pairing order of BCS theory (SC)

(Bose-Einstein) condensation of d -wave Cooper pairs

Orders (possibly fluctuating) associated with proximate Mott insulator in class A

2. Collinear magnetic order (CM)

3. Bond/charge/stripe order (B)

(couples strongly to half-breathing phonons)

S. Sachdev and N. Read, *Int. J. Mod. Phys. B* **5**, 219 (1991).

M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999);

M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev. B* **62**, 6721 (2000);

M. Vojta, *Phys. Rev. B* **66**, 104505 (2002).

Evidence cuprates are in class A

Evidence cuprates are in class A

- Neutron scattering shows collinear magnetic order co-existing with superconductivity

J. M. Tranquada *et al.*, *Phys. Rev. B* **54**, 7489 (1996).

Y.S. Lee, R. J. Birgeneau, M. A. Kastner *et al.*, *Phys. Rev. B* **60**, 3643 (1999).

S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys. Rev. B* **63**, 172501 (2001).

Evidence cuprates are in class A

- Neutron scattering shows collinear magnetic order co-existing with superconductivity
- Proximity of Z_2 Mott insulators requires stable hc/e vortices, vison gap, and Senthil flux memory effect

S. Sachdev, *Physical Review B* **45**, 389 (1992)

N. Nagaosa and P.A. Lee, *Physical Review B* **45**, 966 (1992)

T. Senthil and M. P. A. Fisher, *Phys. Rev. Lett.* **86**, 292 (2001).

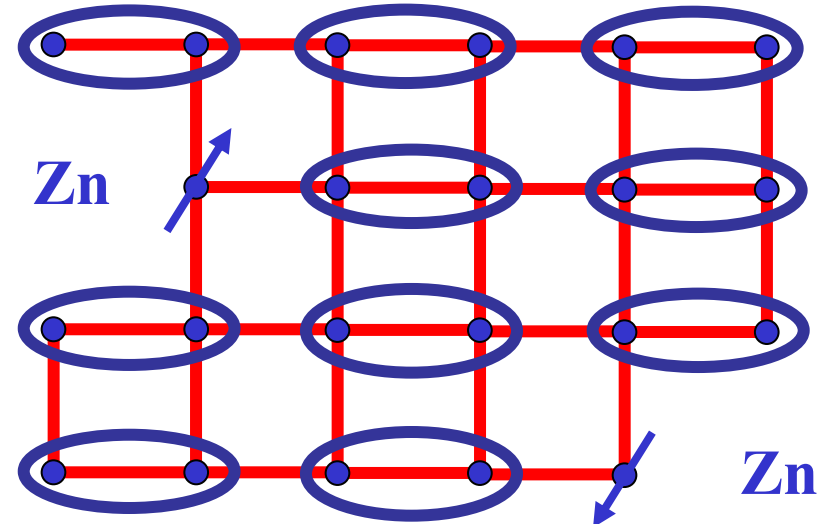
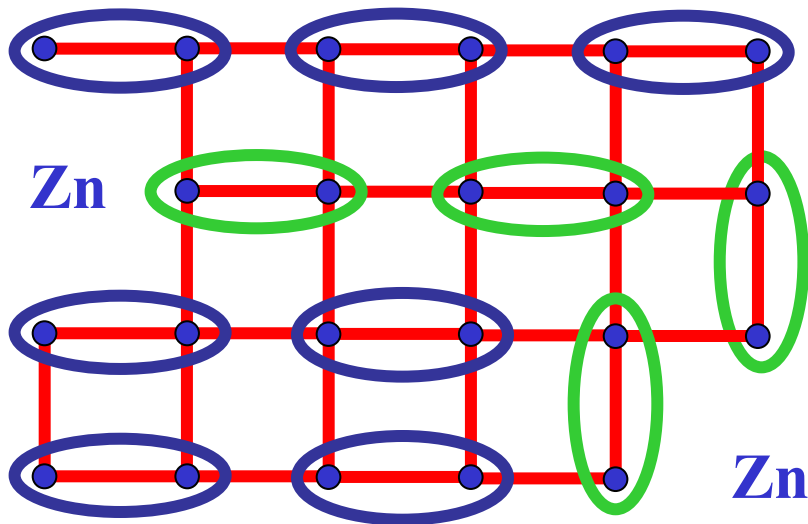
D. A. Bonn, J. C. Wynn, B. W. Gardner, Y.-J. Lin, R. Liang, W. N. Hardy, J. R. Kirtley, and K. A. Moler, *Nature* **414**, 887 (2001).

J. C. Wynn, D. A. Bonn, B. W. Gardner, Y.-J. Lin, R. Liang, W. N. Hardy, J. R. Kirtley, and K. A. Moler, *Phys. Rev. Lett.* **87**, 197002 (2001).

Evidence cuprates are in class A

- Neutron scattering shows collinear magnetic order co-existing with superconductivity
- Proximity of Z_2 Mott insulators requires stable hc/e vortices, vison gap, and Senthil flux memory effect
- Non-magnetic impurities in underdoped cuprates acquire a $S=1/2$ moment

Effect of static non-magnetic impurities (Zn or Li)



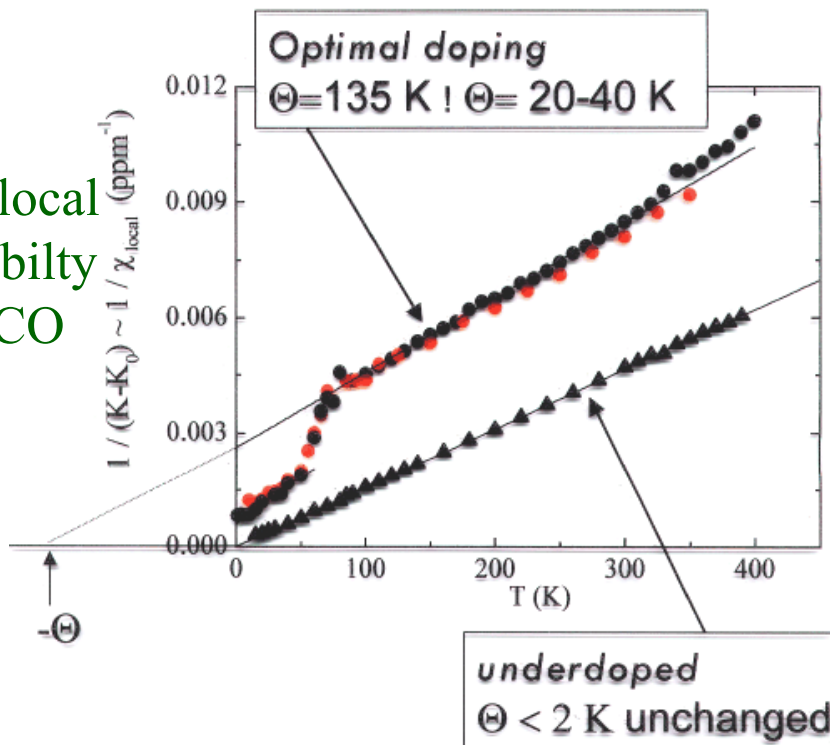
Spinon confinement implies that free $S=1/2$ moments form near each impurity

$$\chi_{\text{impurity}}(T \rightarrow 0) = \frac{S(S+1)}{3k_B T}$$

Spatially resolved NMR of Zn/Li impurities in the superconducting state

^7Li NMR below T_c

J. Bobroff, H. Alloul, W.A. MacFarlane, P. Mendels, N. Blanchard, G. Collin, and J.-F. Marucco, *Phys. Rev. Lett.* **86**, 4116 (2001).



Inverse local susceptibility in YBCO

Measured $\chi_{\text{impurity}}(T \rightarrow 0) = \frac{S(S+1)}{3k_B T}$ with $S = 1/2$ in underdoped sample.

This behavior does not emerge out of BCS theory.

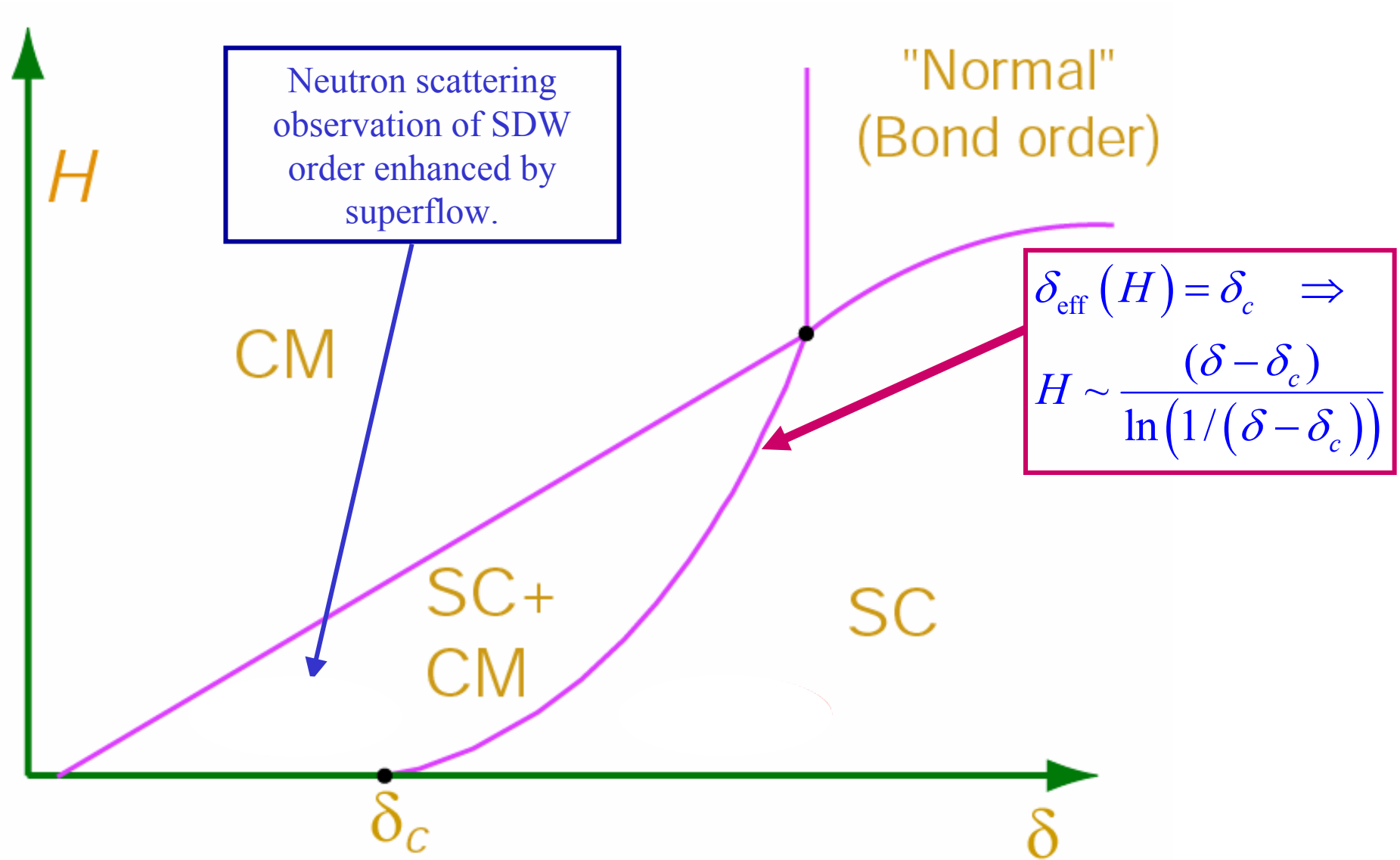
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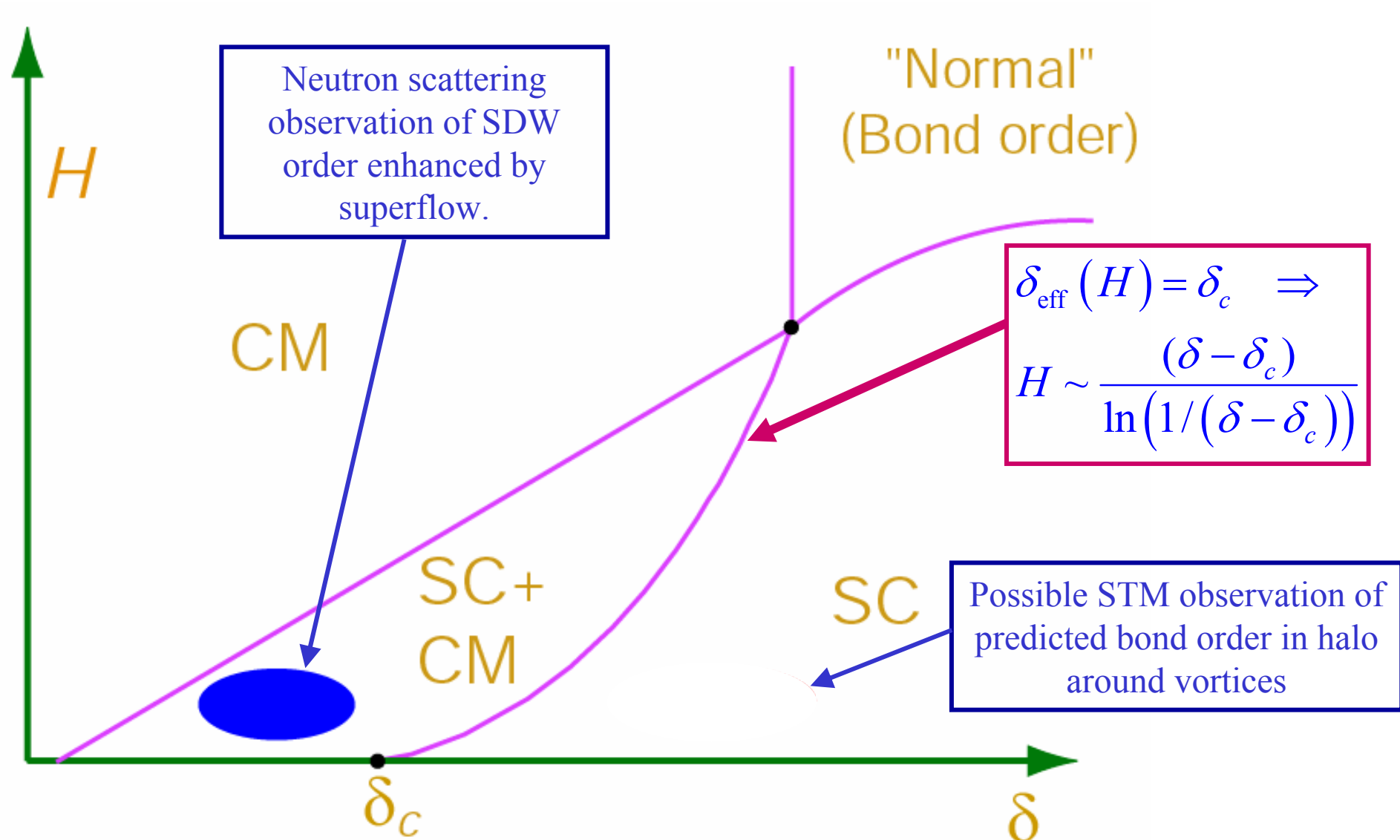
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- Non-magnetic impurities in underdoped cuprates acquire a $S=1/2$ moment
- Tests of phase diagram in a magnetic field

Phase diagram of a superconductor in a magnetic field



Phase diagram of a superconductor in a magnetic field



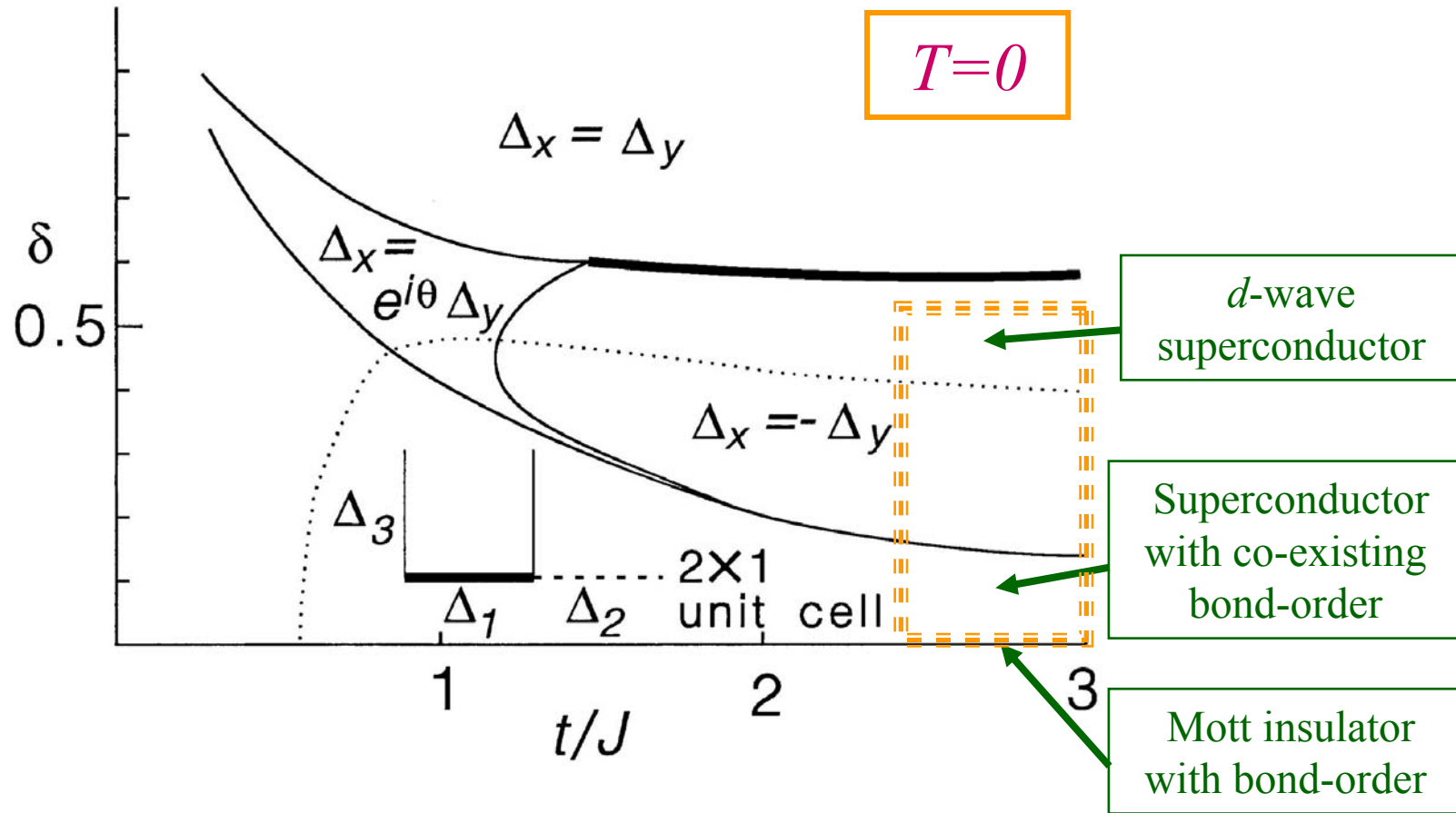
K. Park and S. Sachdev *Physical Review B* **64**, 184510 (2001);

E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001);
 Y. Zhang, E. Demler and S. Sachdev, *Physical Review B* **66**, 094501 (2002).

VI. Doping Class A

Doping a paramagnetic bond-ordered Mott insulator

systematic $Sp(N)$ theory of translational symmetry breaking, while preserving spin rotation invariance.

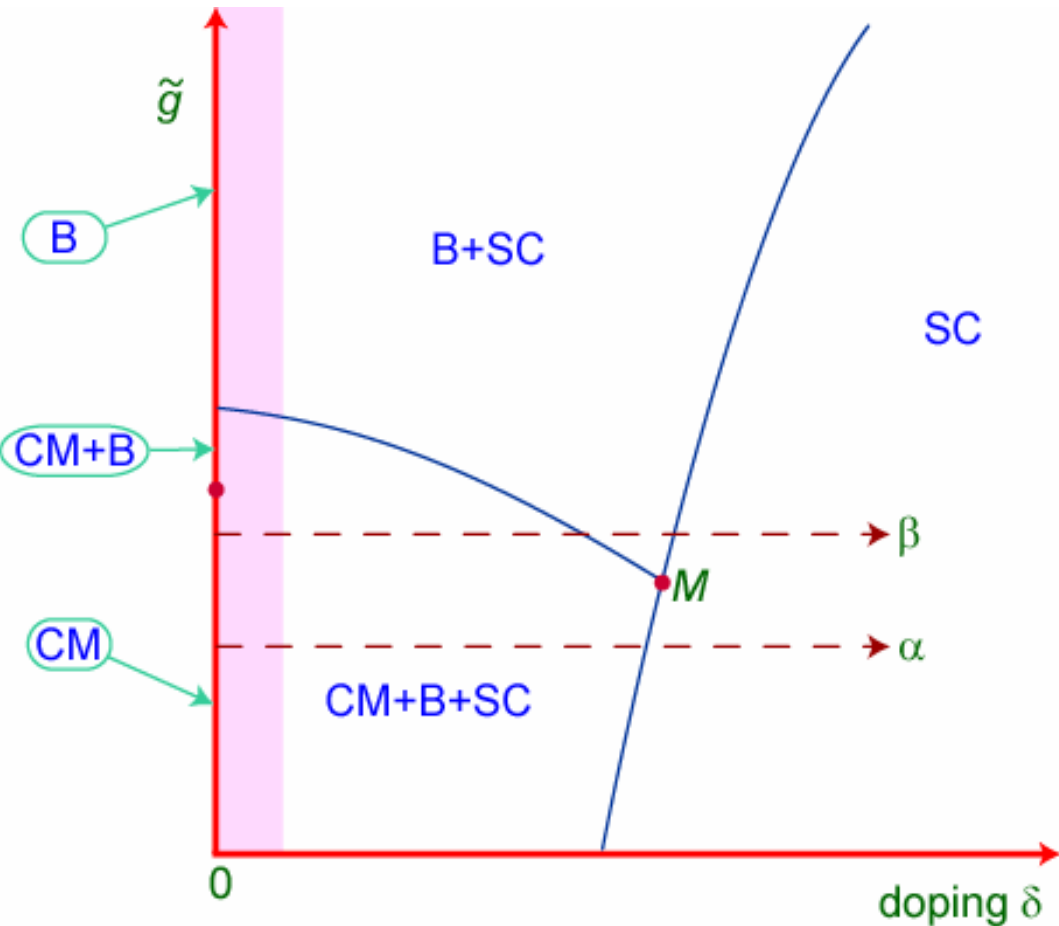


S. Sachdev and N. Read, *Int. J. Mod. Phys. B* **5**, 219 (1991).

A phase diagram

Vertical axis is any microscopic parameter which suppresses

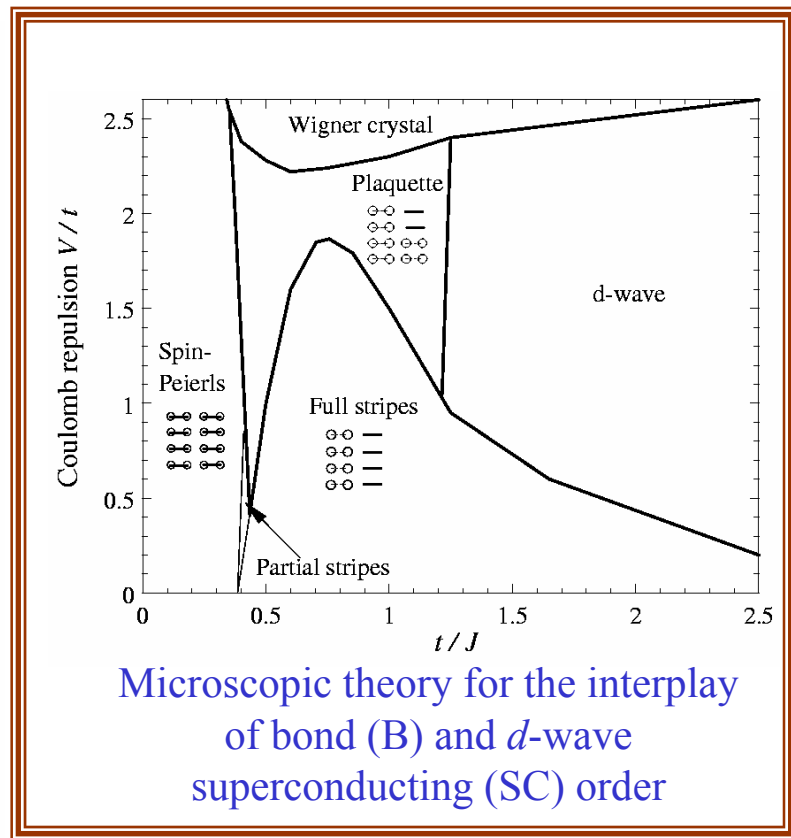
CM order



• Pairing order of BCS theory (SC)

• Collinear magnetic order (CM)

• Bond order (B)



S. Sachdev and N. Read, *Int. J. Mod. Phys. B* **5**, 219 (1991).
 M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999);
 M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev. B* **62**, 6721 (2000);
 M. Vojta, *Phys. Rev. B* **66**, 104505 (2002).