Planckian metals and black holes

S. N. Bose Colloquium Series
on Quantum Materials and Devices
S. N. Bose National Centre for Basic Sciences
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Talk online: sachdev.physics.harvard.edu

DEPARTMENT OF ENERGY
UNITED STATES OF AMERICA

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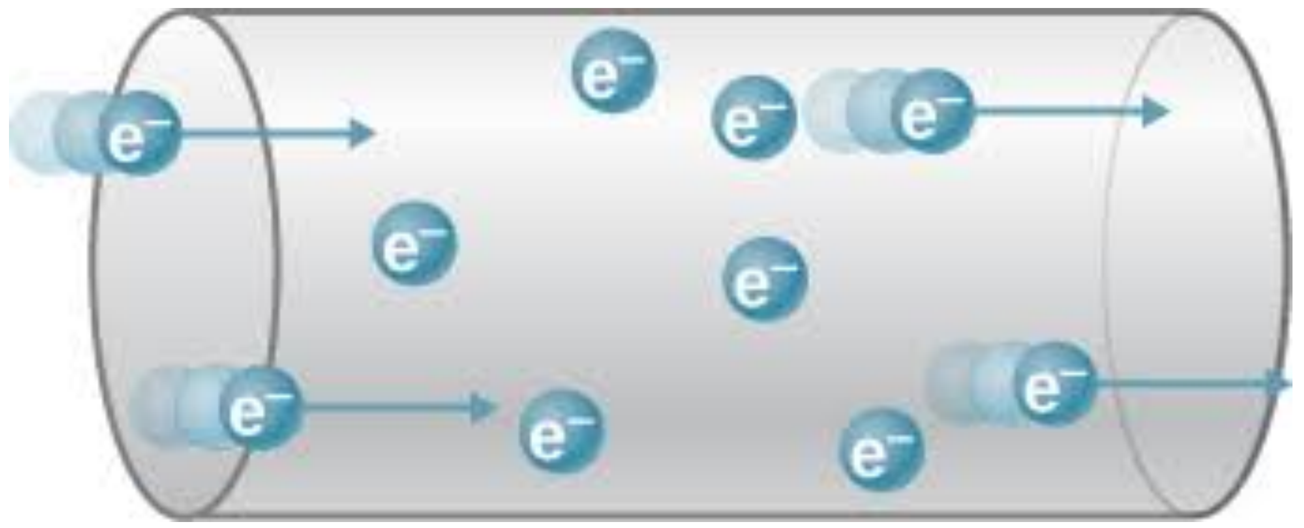
PHYSICS
HARVARD

1. Introduction to Planckian metals
2. Introduction to black holes
3. The SYK model
4. Progress on the theory of black holes
5. Progress on the theory of Planckian metals
    - *A. Random t-J model*
    - *B. Fermi surface coupled to a critical boson*

Current flow with quasiparticles in Copper

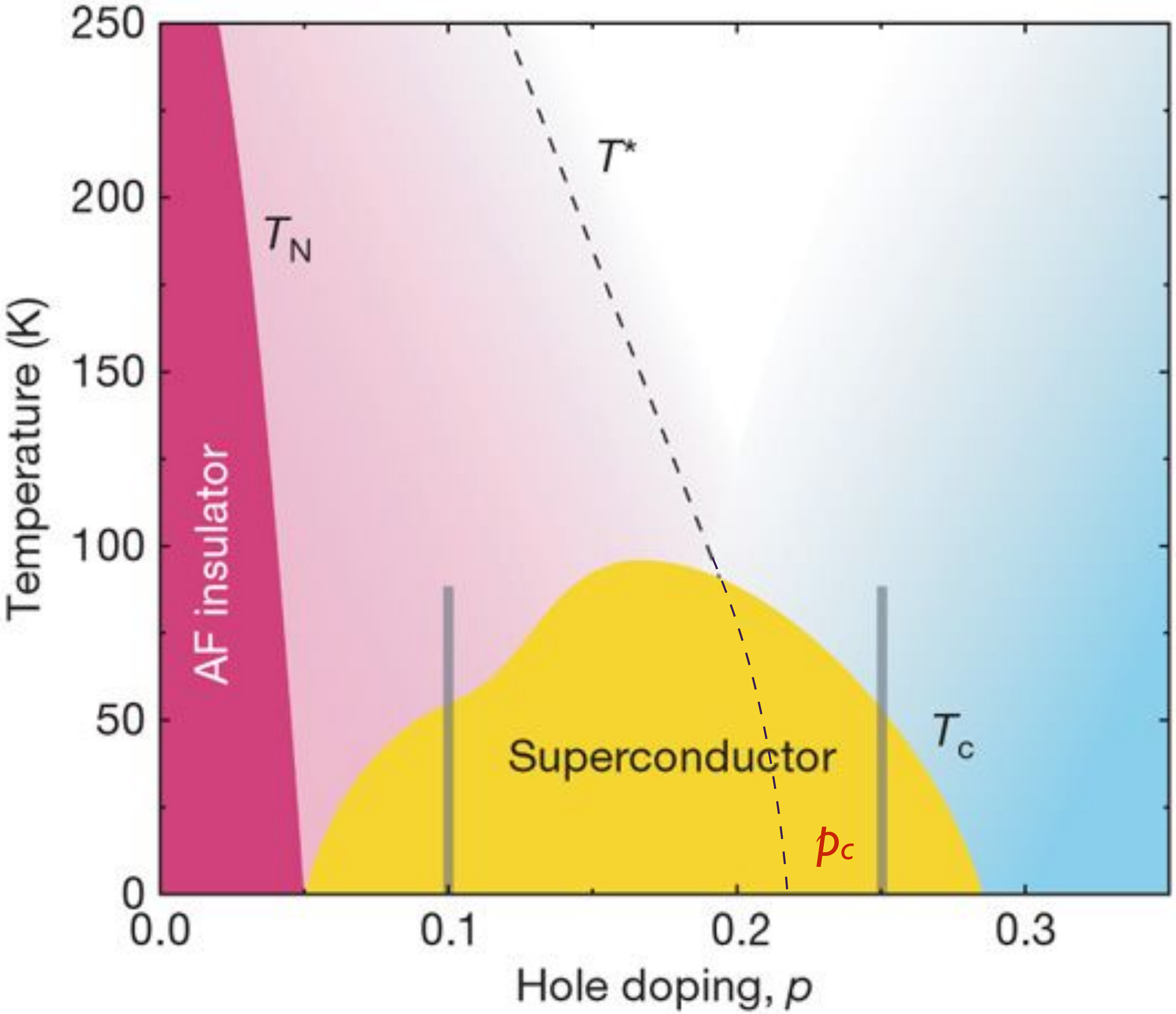


Flowing quasiparticles scatter off each other
in a typical scattering time $\tau \sim 1/T^2$

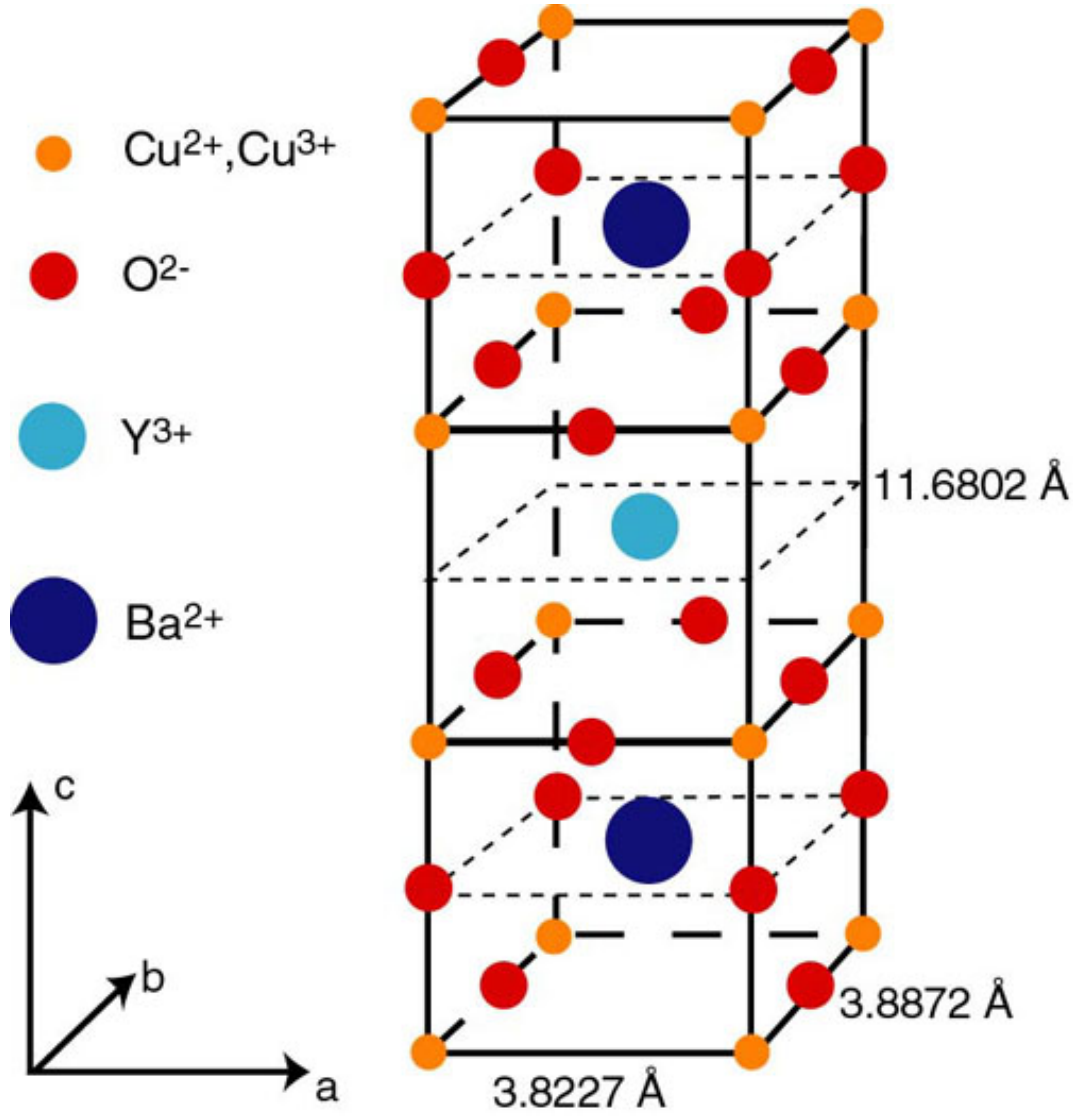
This time is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

The long scattering time implies that quasiparticles are well-defined.

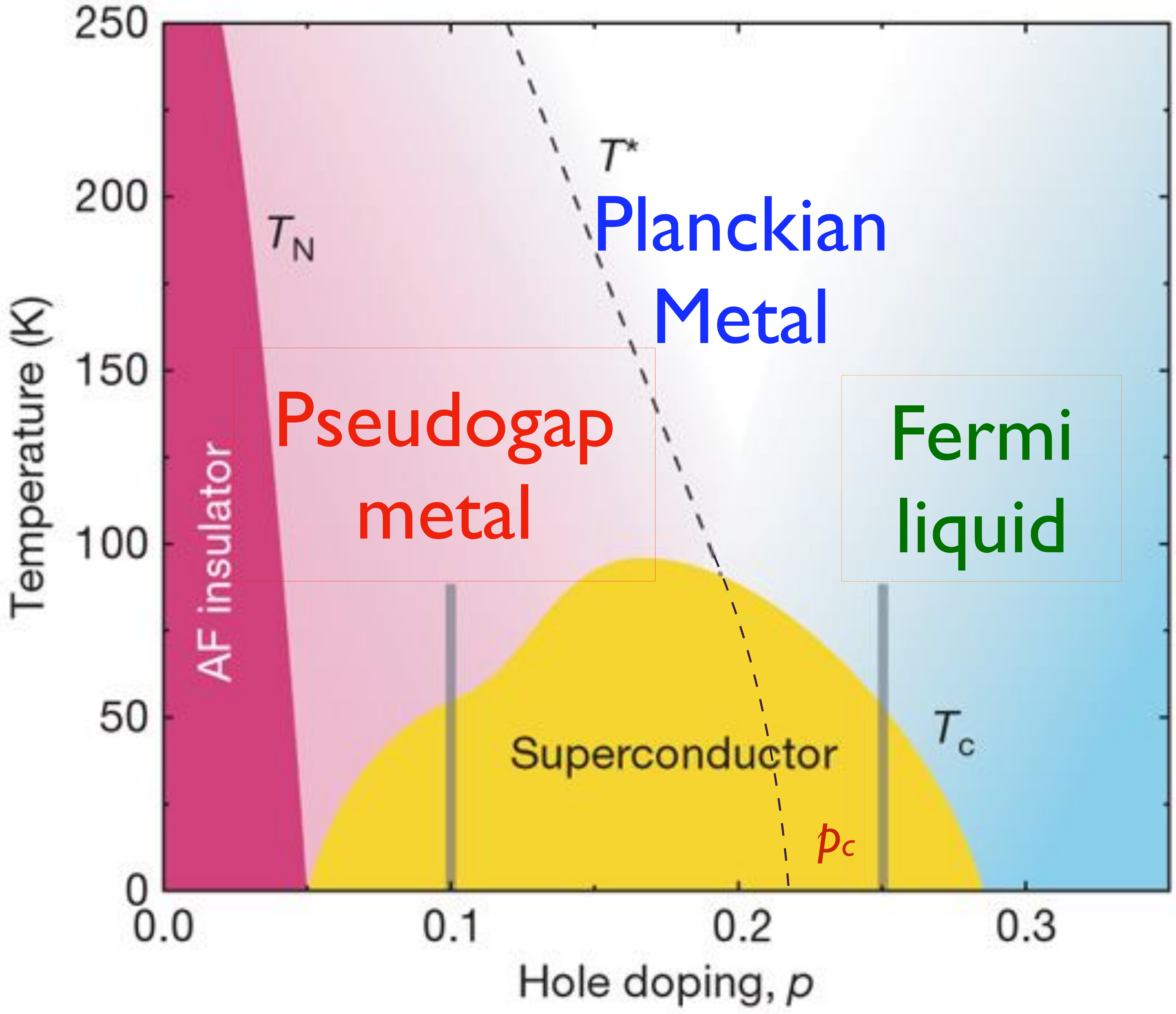
The motion of quasiparticles is 'ballistic' or 'integrable'
up to the long time τ, after which it is chaotic.



250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
T*
T_N
AF insulator
Superconductor
T_c
p_c



Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å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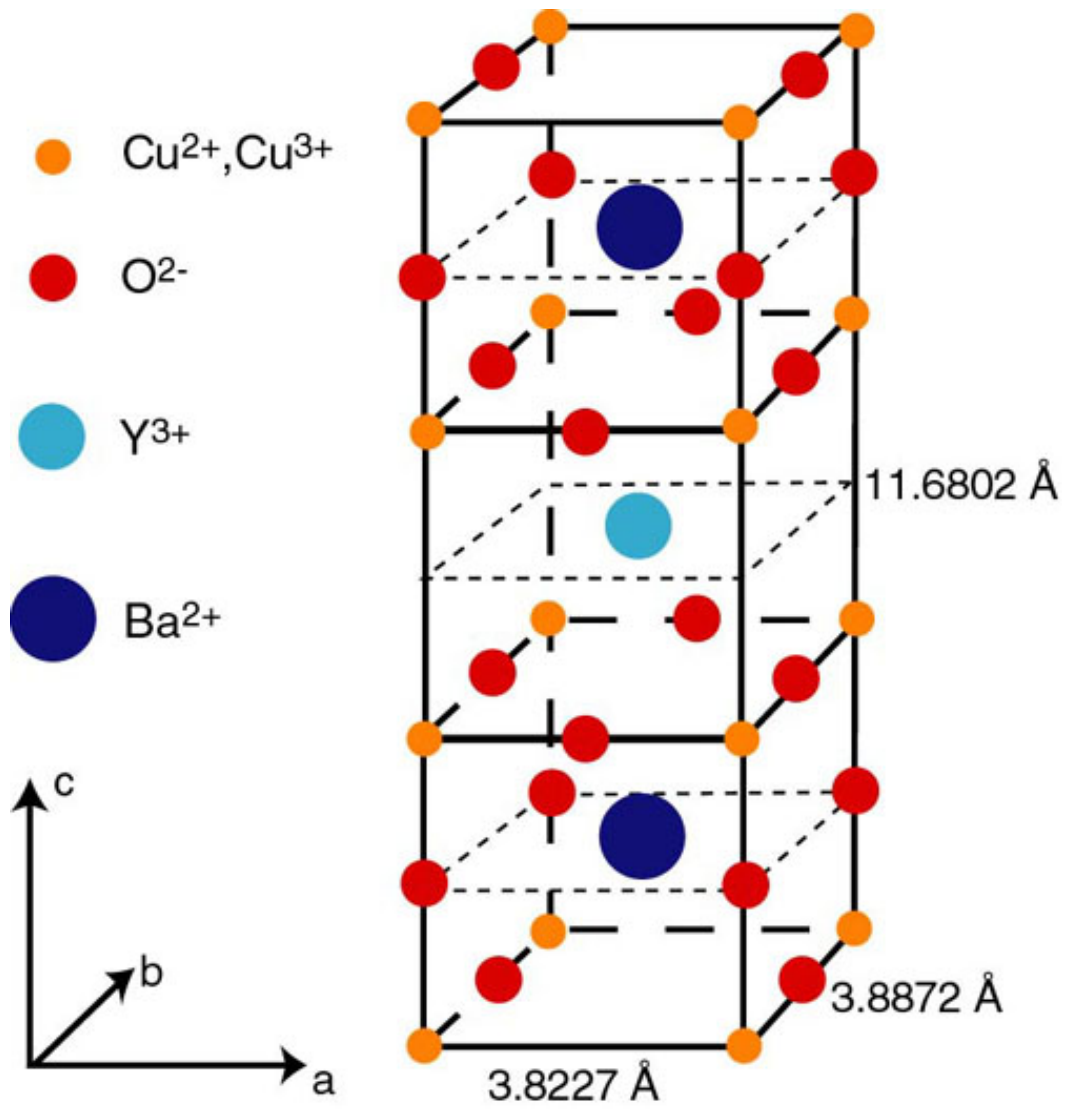
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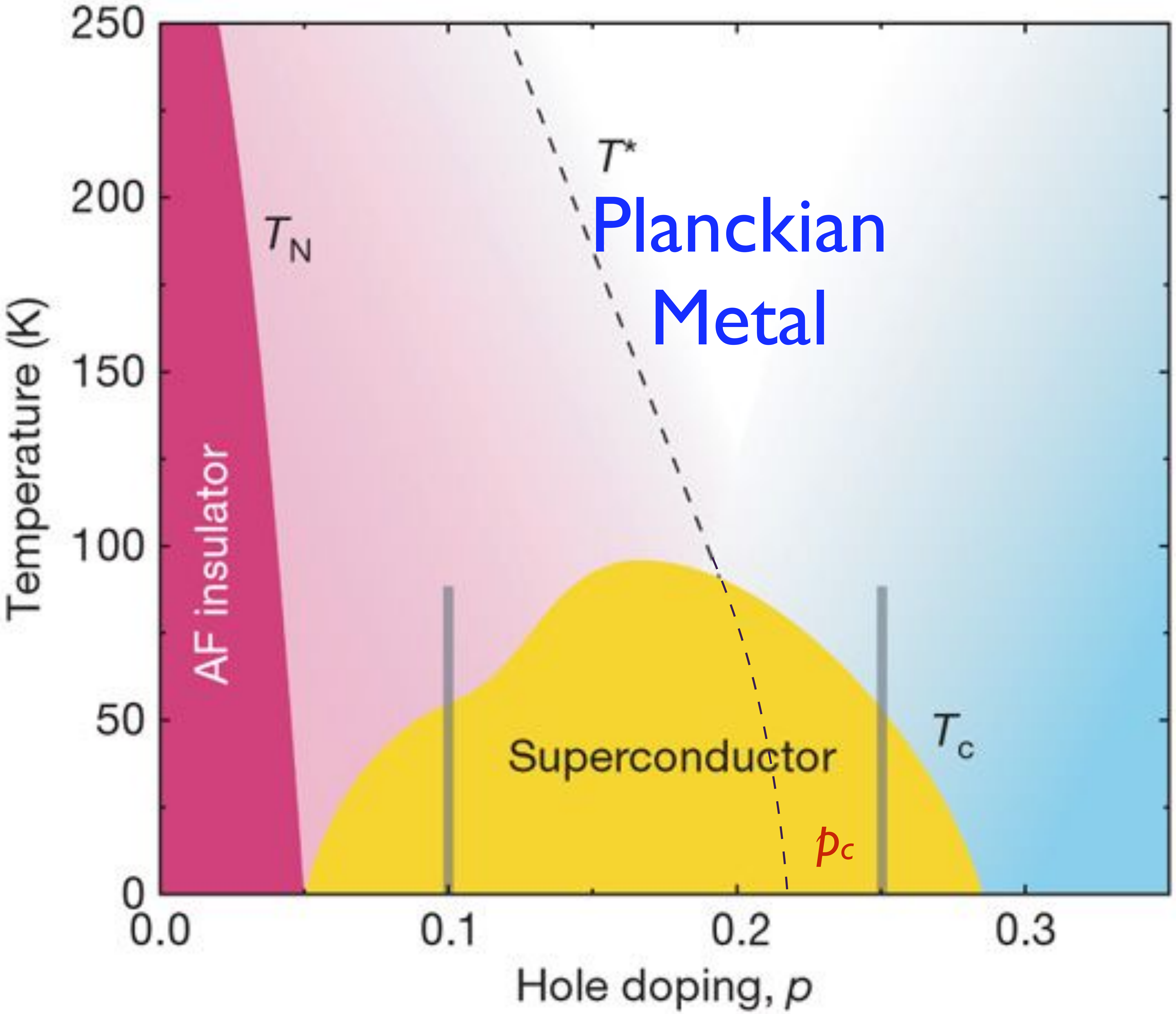
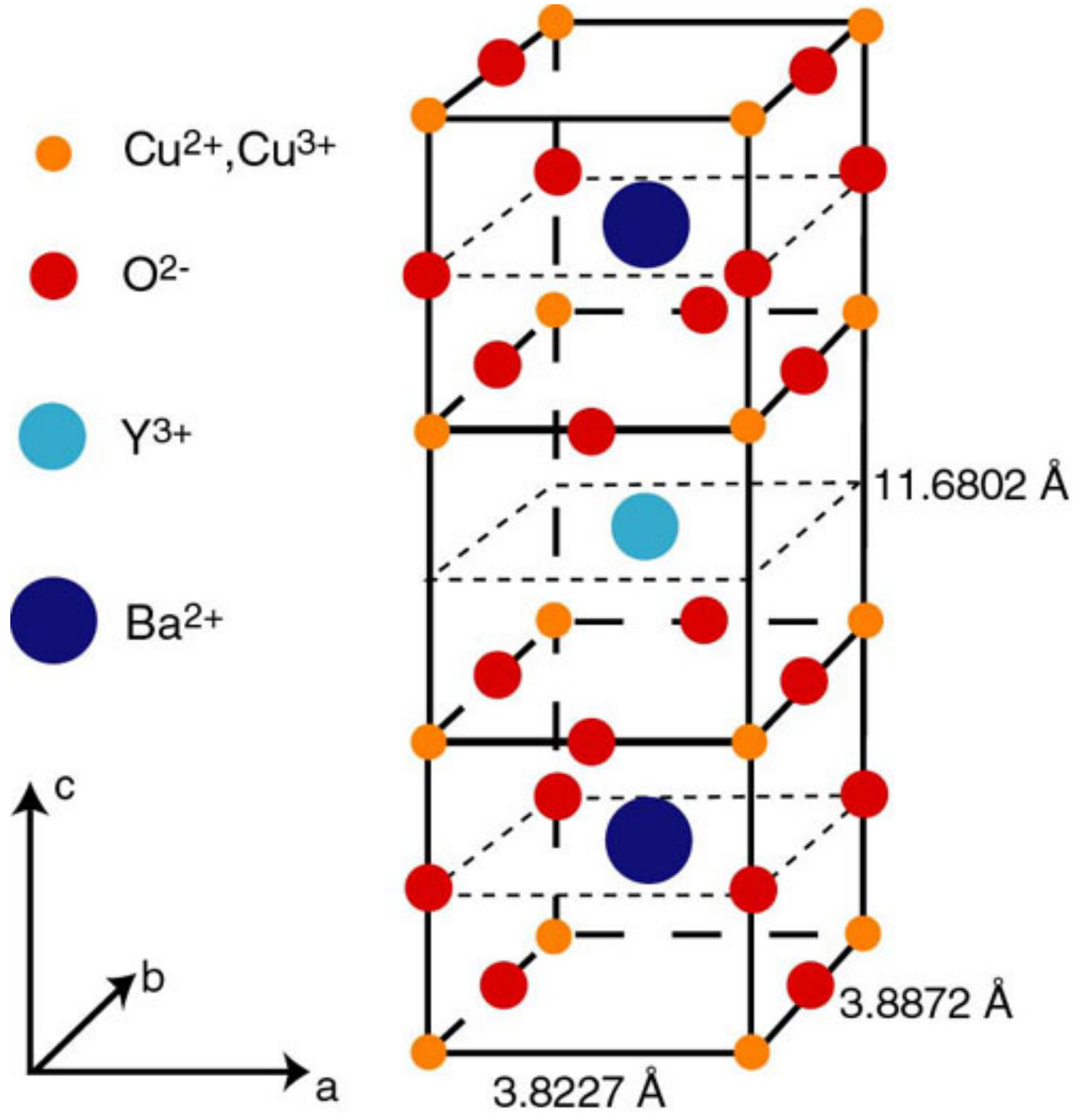
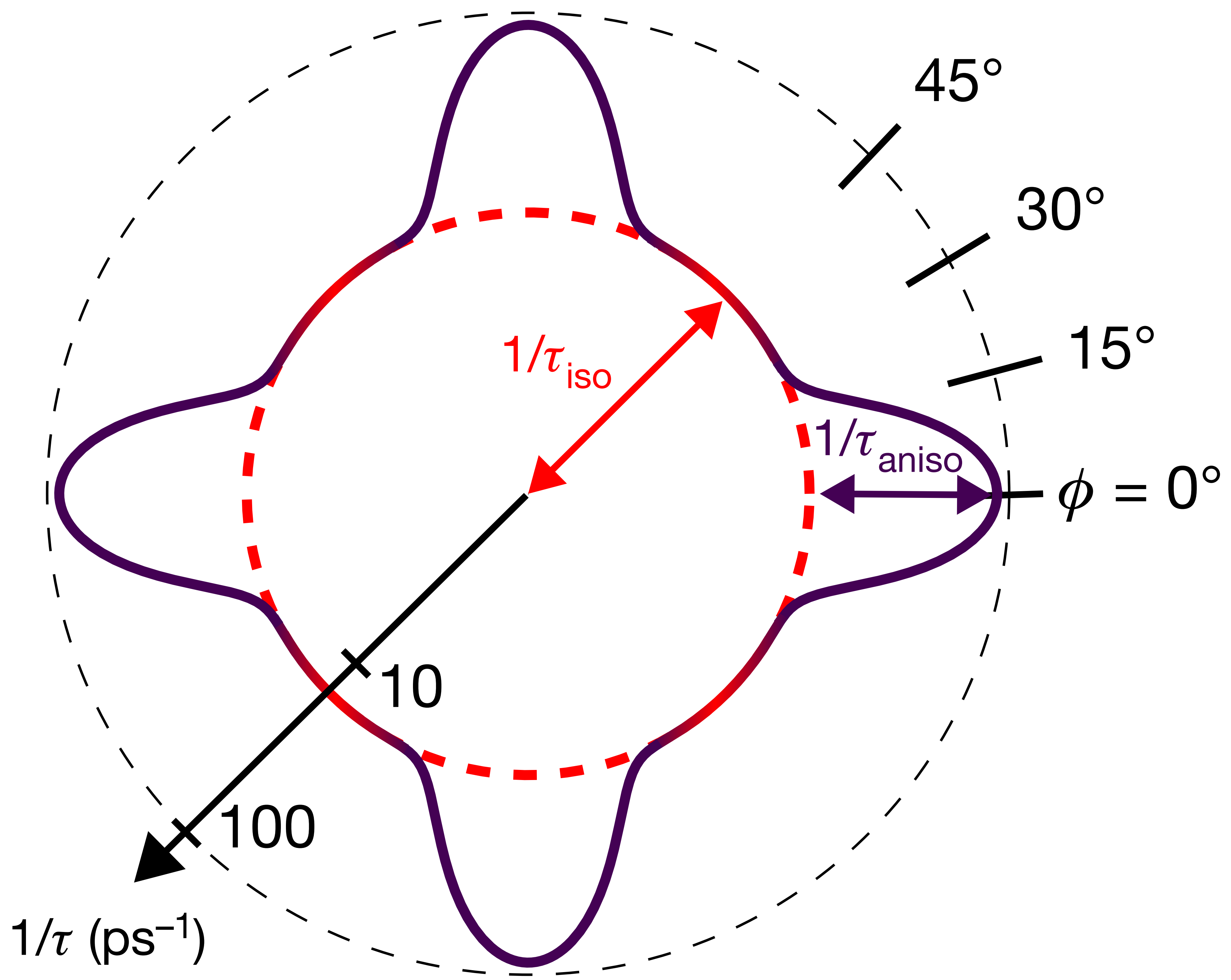

Planckian Metal

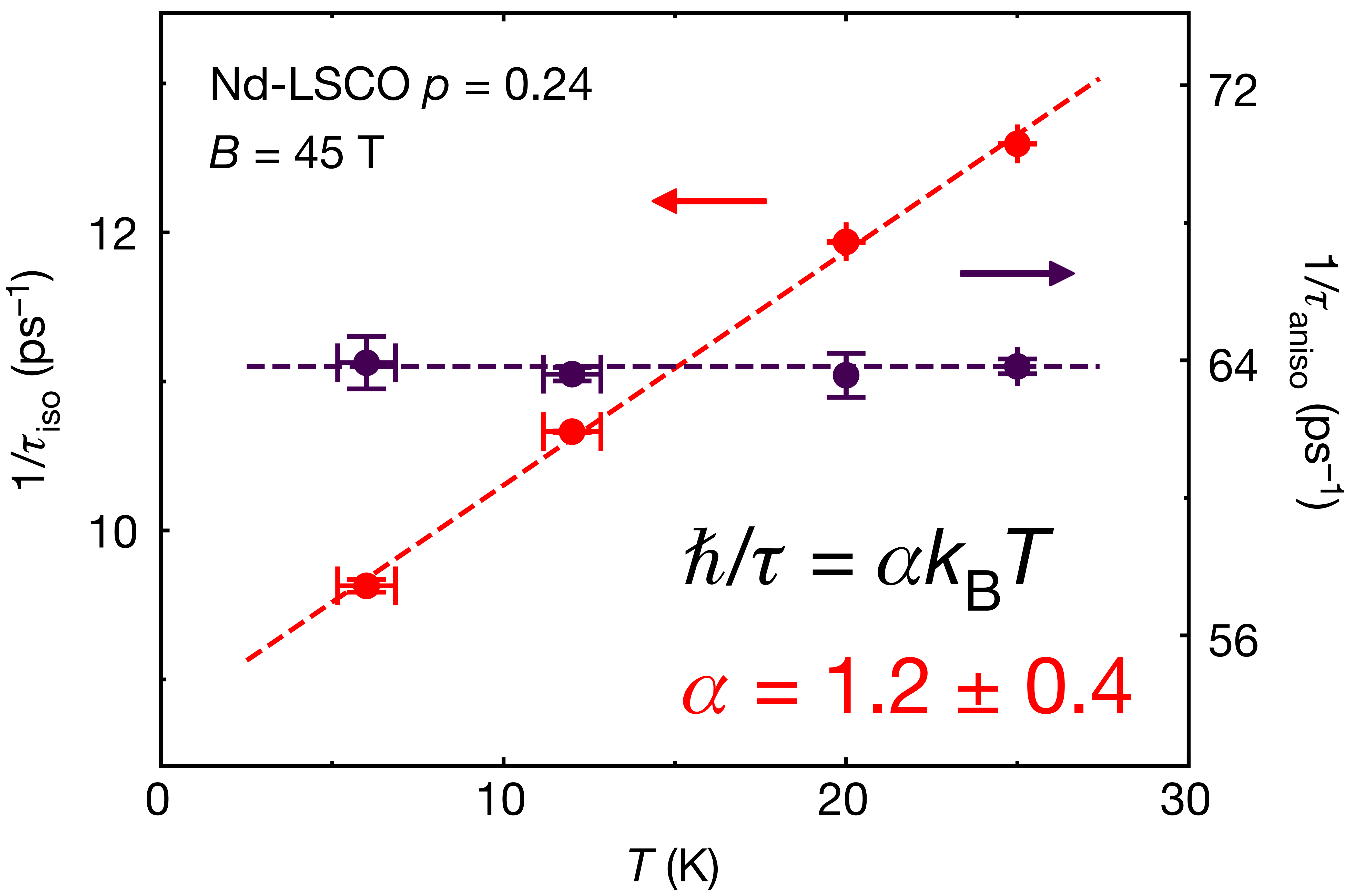


Linear-in temperature resistivity from an isotropic Planckian scattering rate

Nature **595**, 667-672 (2021)

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw





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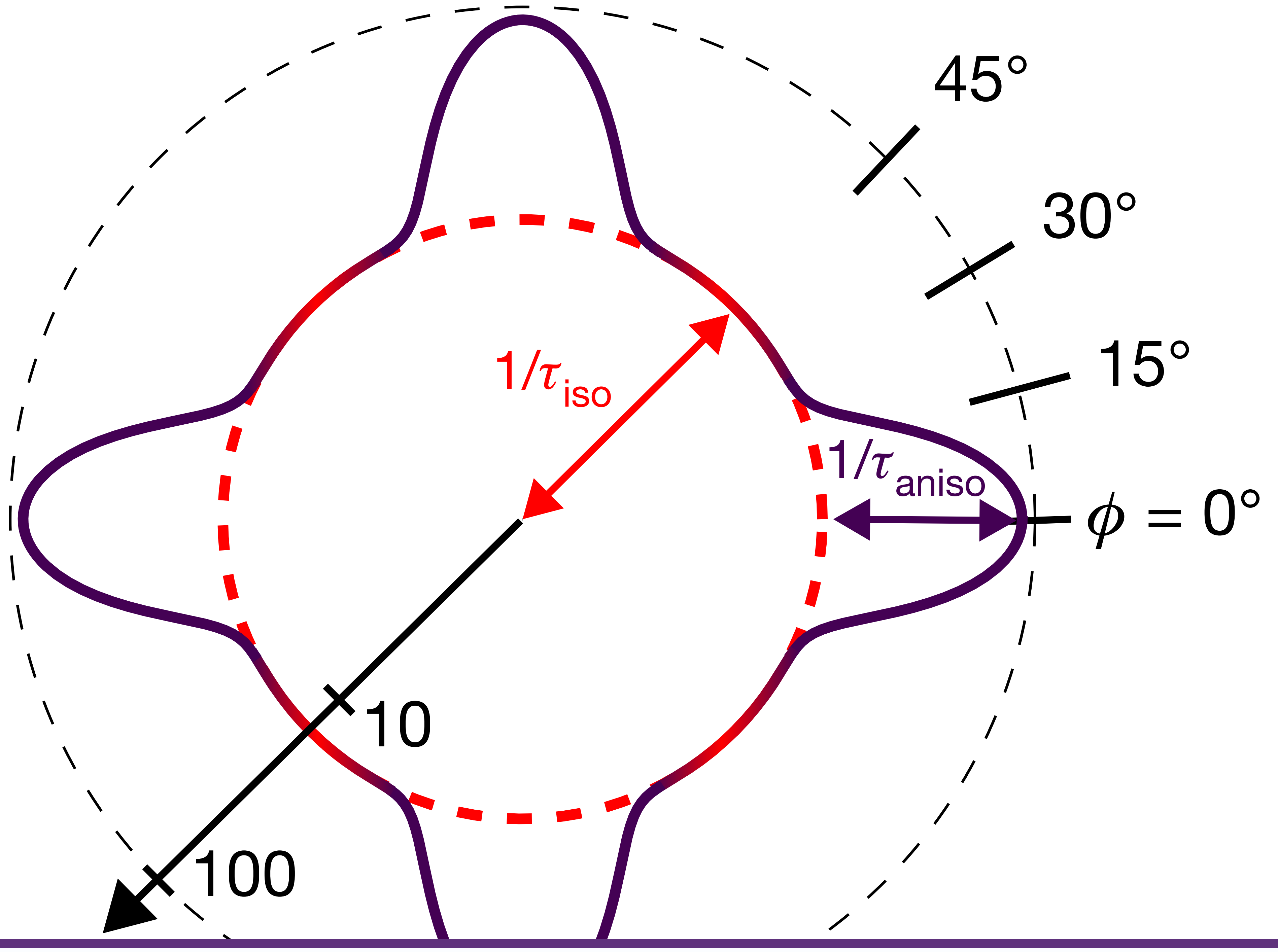
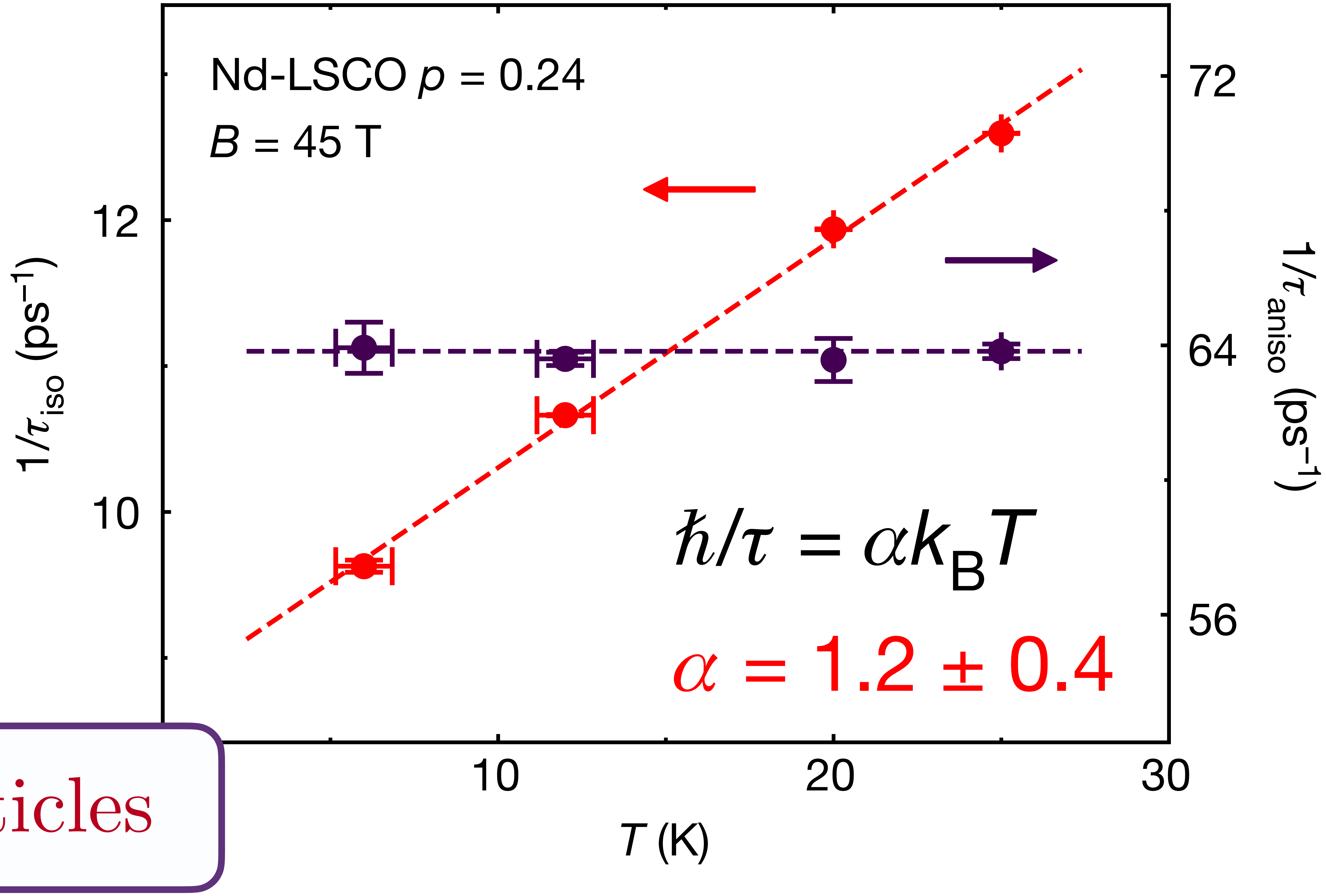
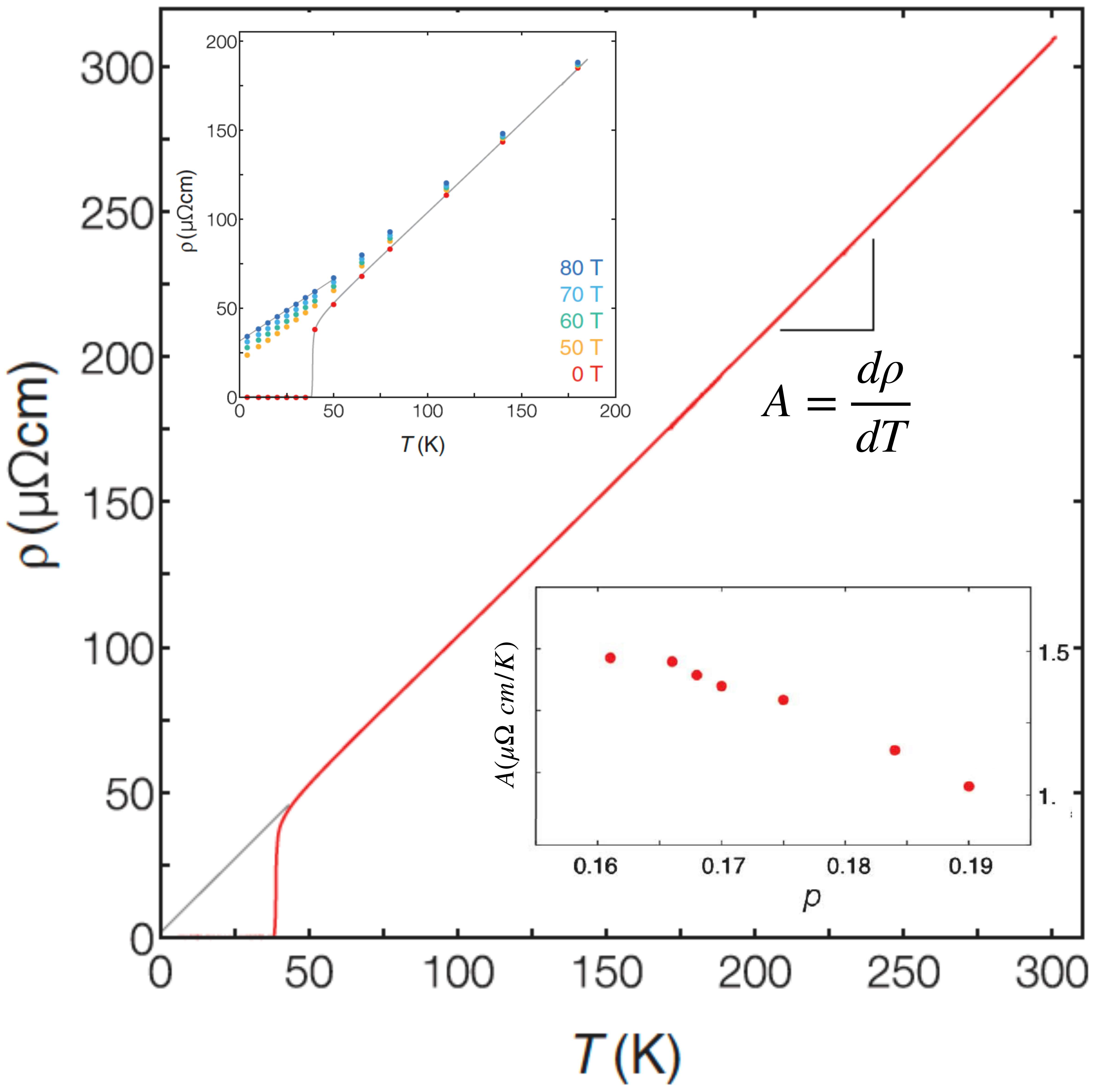
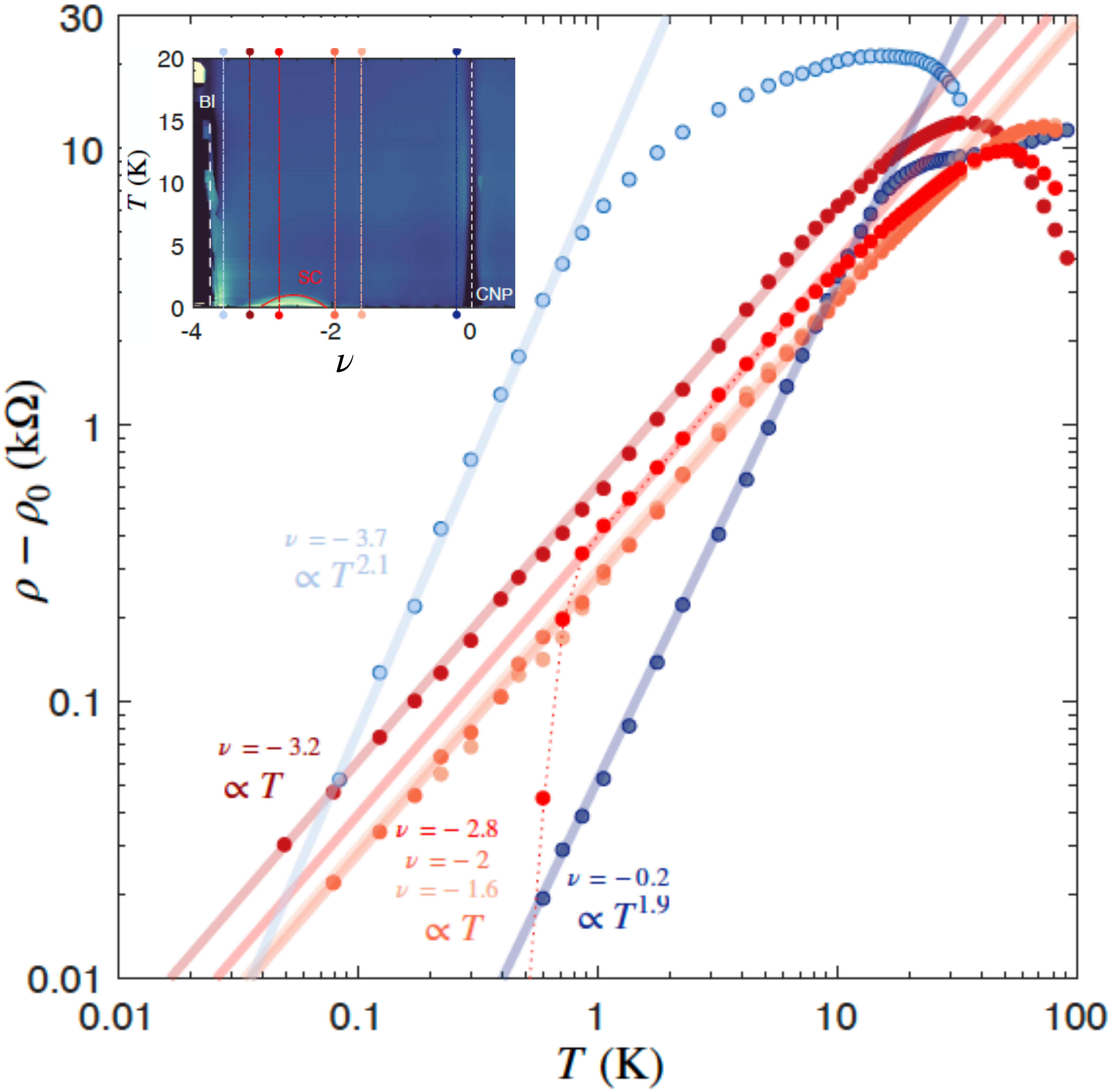




Current flow without quasiparticles



LSCO: Giraldo-Gallo et al. 2018



MATBG: Jaoui et al. 2021

Questions

- Theory for a fermion system with variable density without quasiparticles, and relaxation time $\sim \hbar/(k_B T)$.

- Needed: theory for collision time in resistivity $\sim \hbar/(k_B T)$.

- Needed: theory for the appearance of superconductivity (and other broken symmetries) in such a ‘Planckian metal’.

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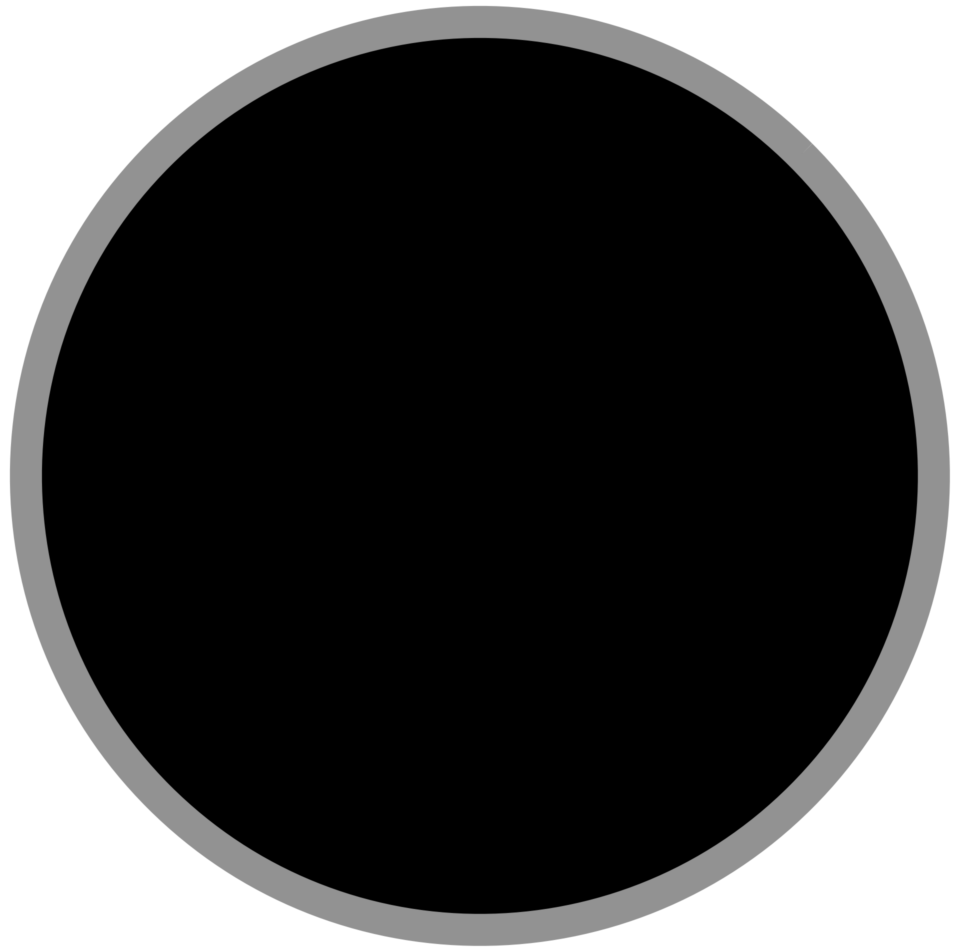
Black Holes

Objects so dense that light is gravitationally bound to them.



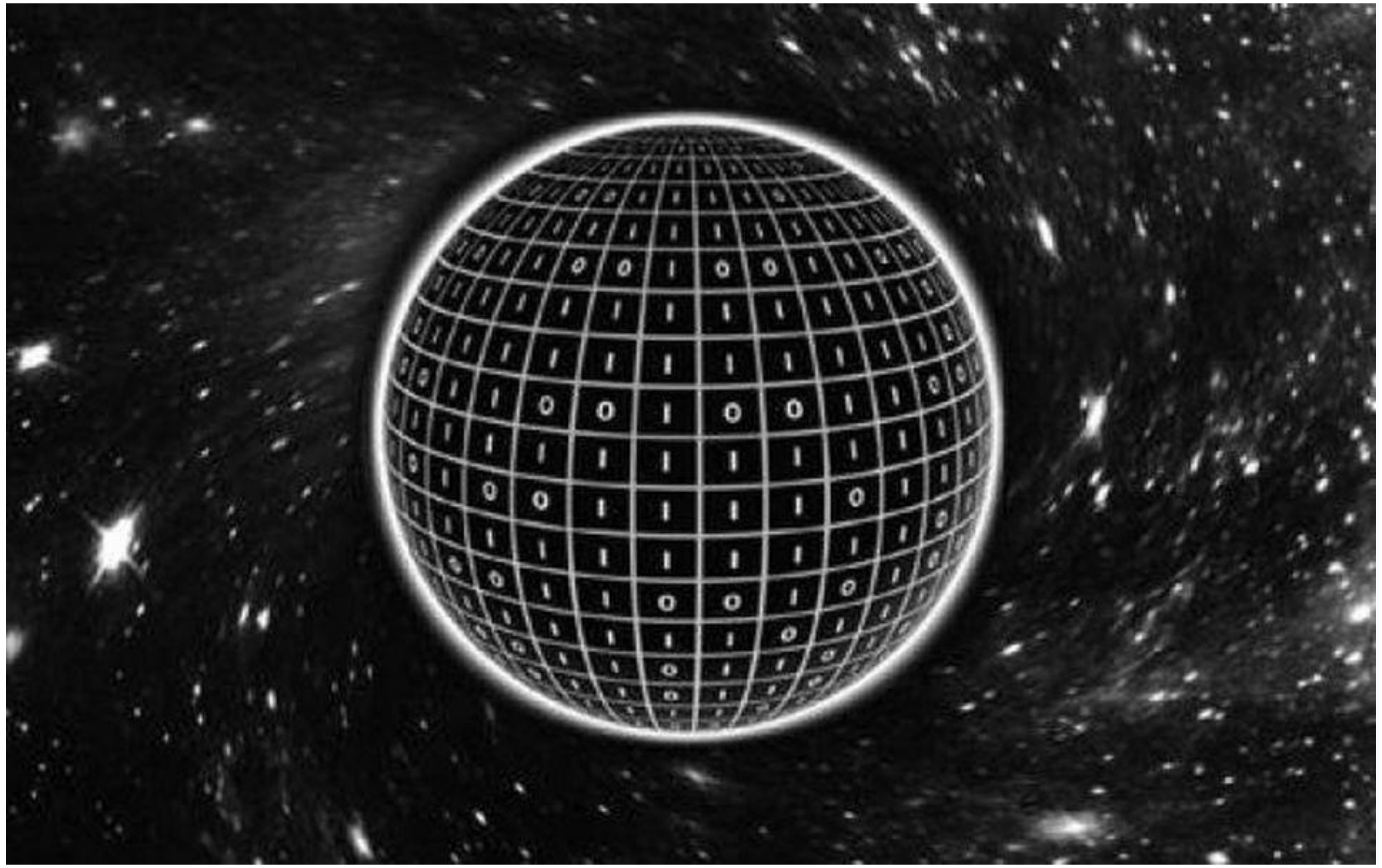
$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

G Newton's constant, c velocity of light, M mass of black hole

For $M =$ earth's mass, $R \approx 9\,mm$!

Quantum Black holes

- Black holes have an entropy and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.
- The entropy is proportional to their surface area.


T.B. BAKKER / DR. J.P. VAN DER SCHAAR

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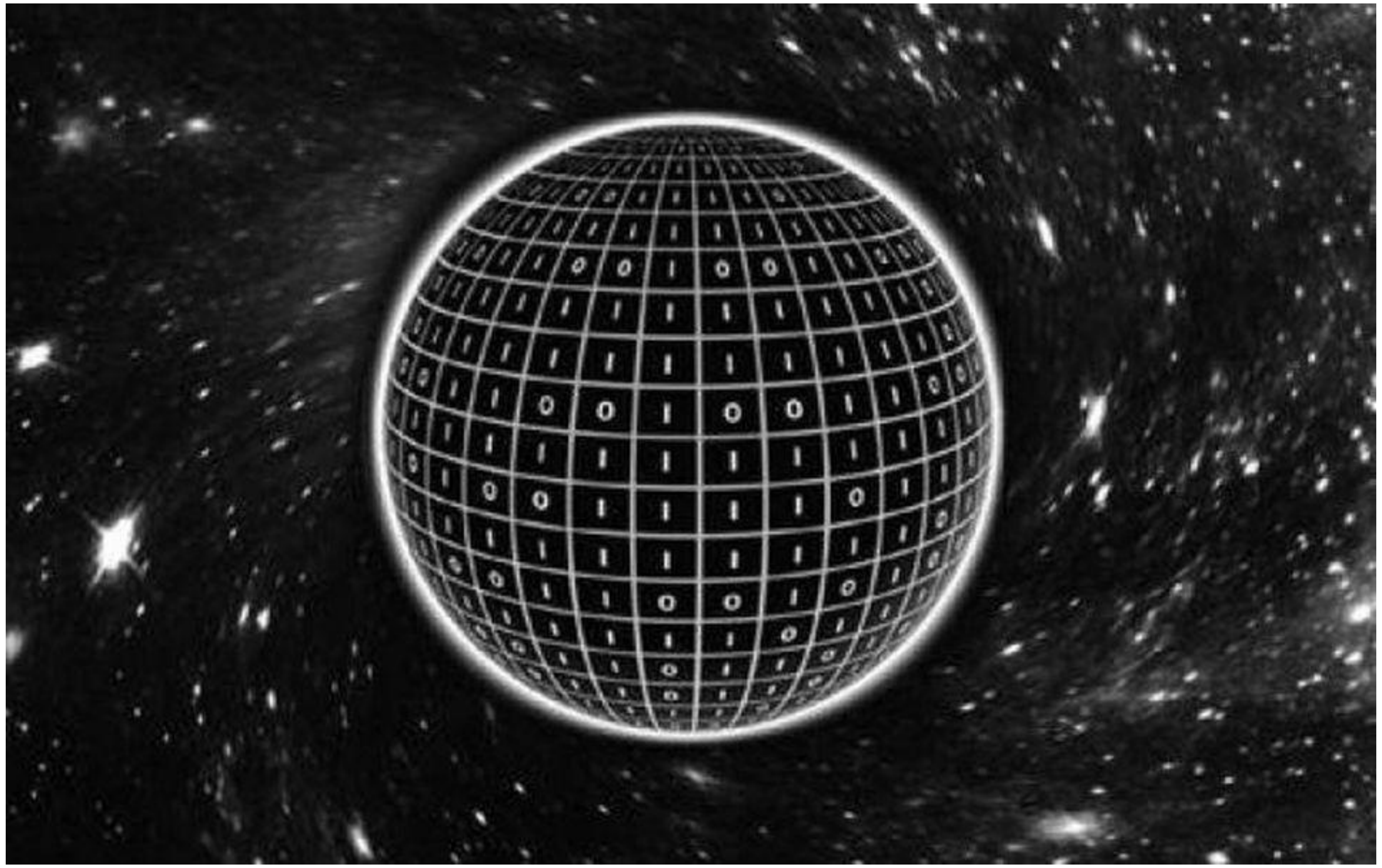

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Remarkable features:

- Entropy is finite.
- Entropy is not proportional to volume

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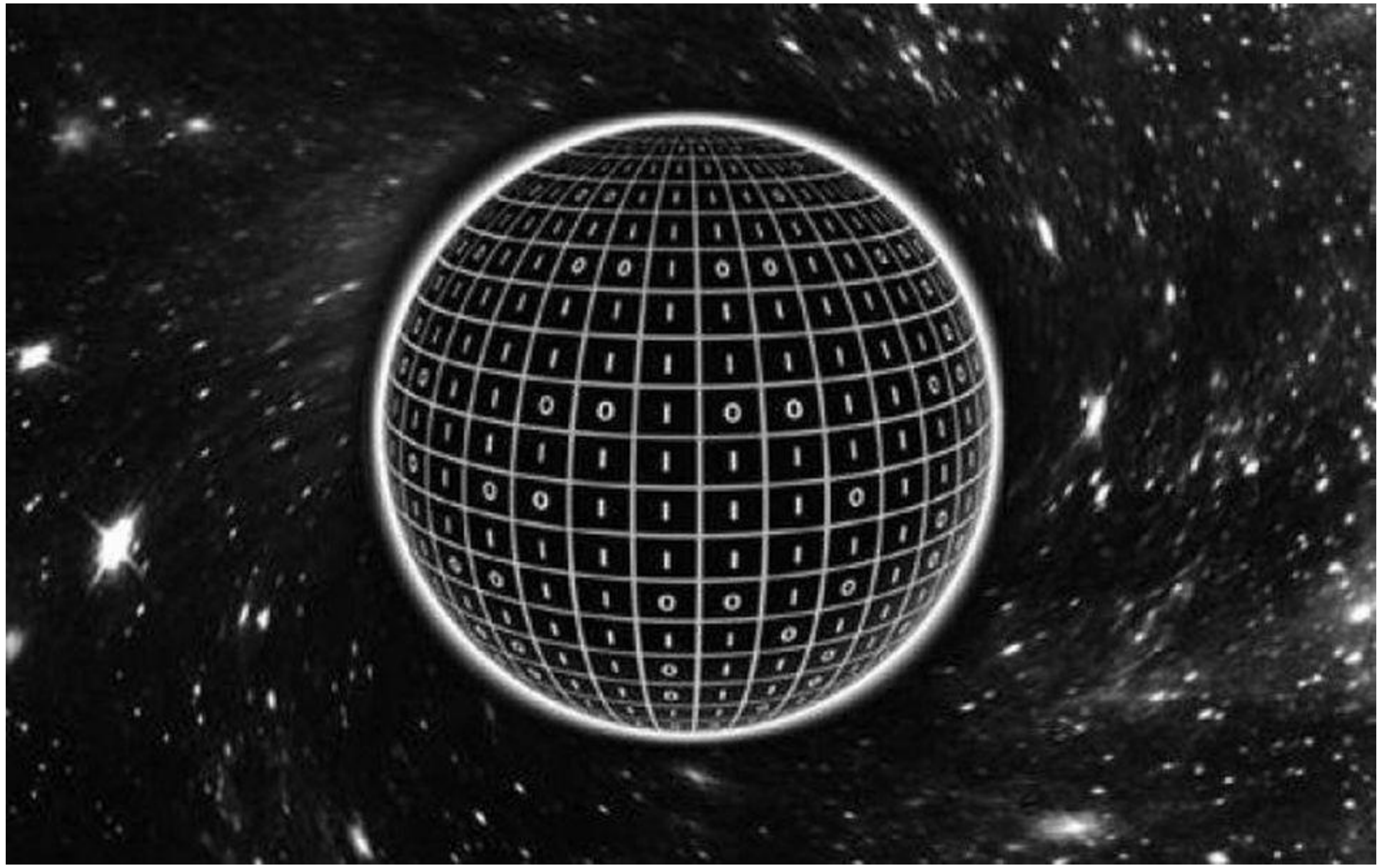
- Black holes have an entropy and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a Planckian time $\sim 8\pi GM/c^3$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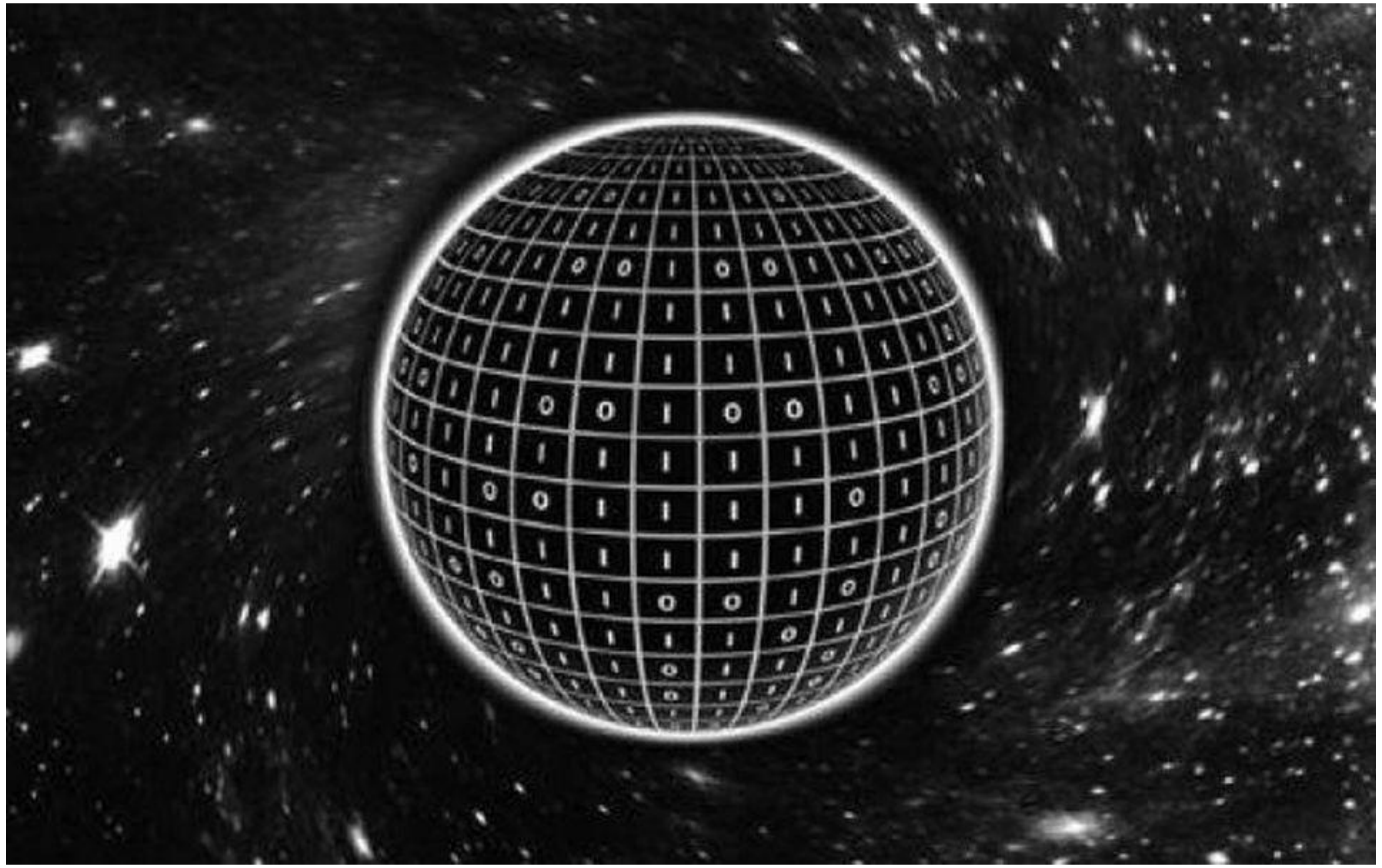

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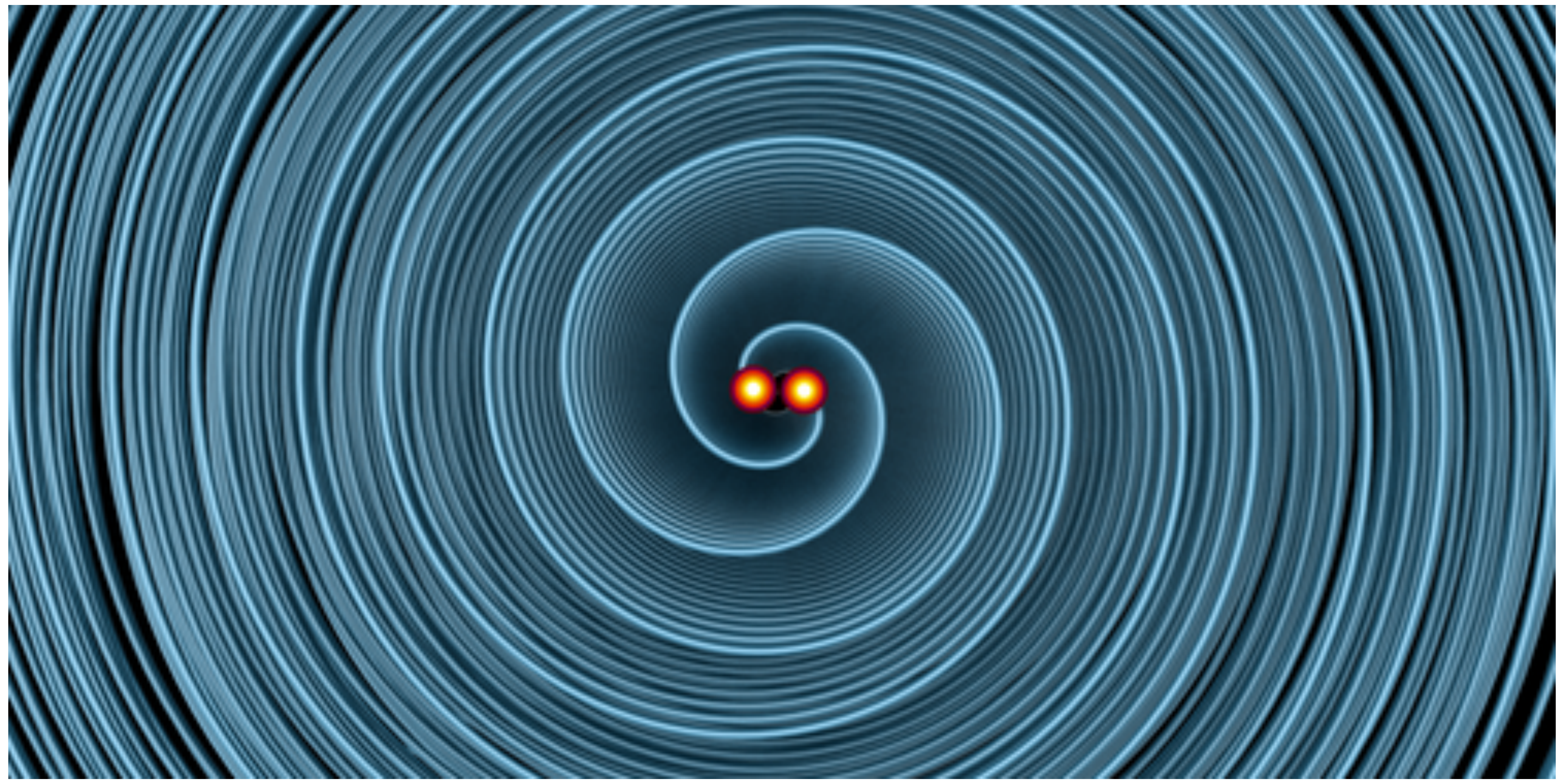
Black Holes Obey Information-Emission Limits

April 22, 2021 • *Physics* 14, s47 –Christopher Crockett

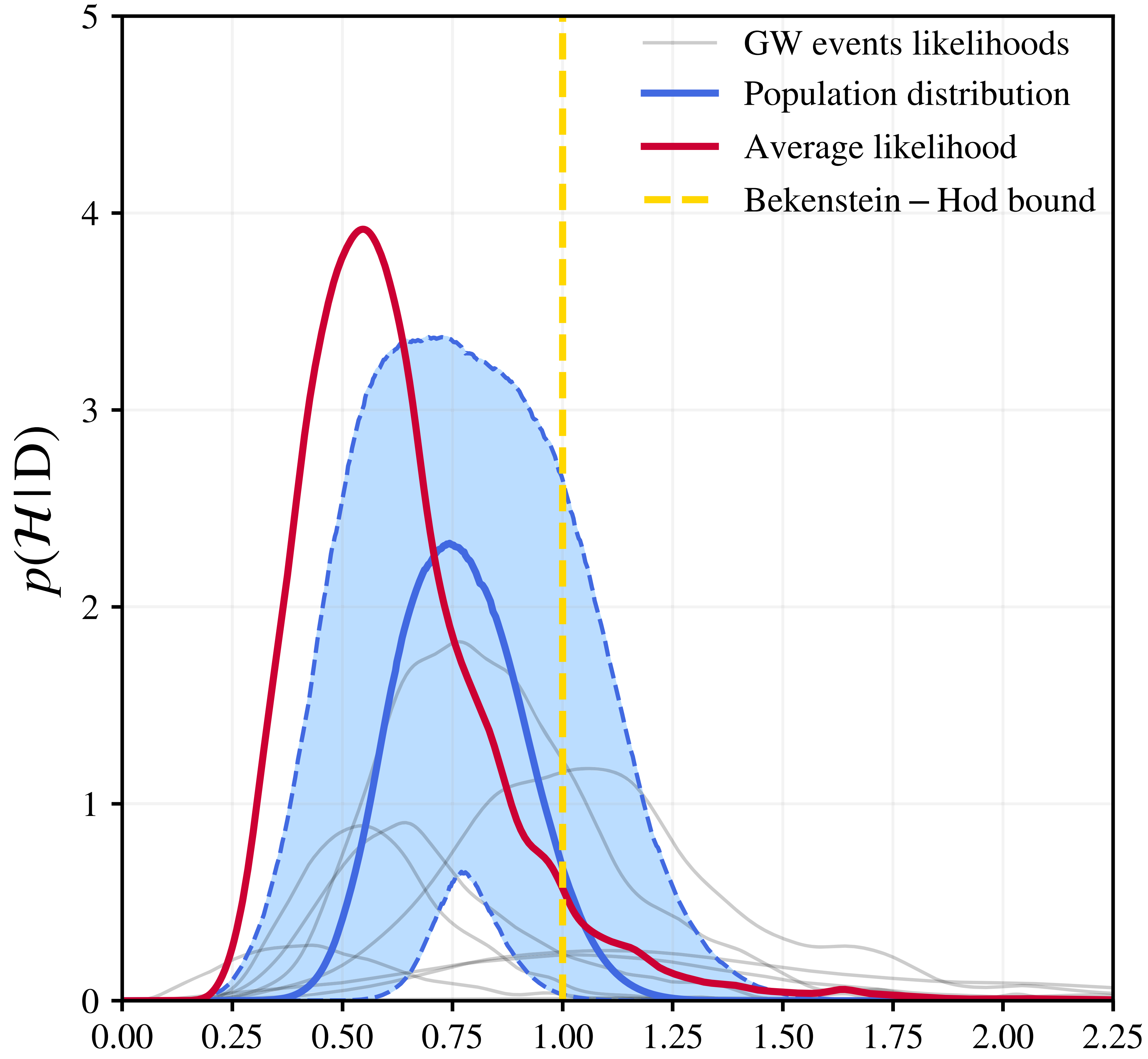
G. Carullo, D. Laghi, J. Veitch, W. Del Pozzo, Phys. Rev. Lett. **126**, 161102 (2021)

An analysis of the gravitational waves emitted from black hole mergers confirms that black holes are the fastest known information dissipaters.



Gravity wave observations of 8 different black holes show a relaxation time

$$\tau \sim \frac{\hbar}{k_B T}$$



$$\mathcal{H} = \frac{1}{\pi}\frac{\hbar/\tau}{k_B T}$$

Thermodynamics of quantum black holes with charge Q:

$$\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu}, A_\mu]\right)$$

Metric of spacetime

Electromagnetic gauge field

In general, this integral is not well defined, because of an uncontrollably large number of spacetime configurations.

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu},A_\mu]\right)$$

$$= \exp\Big(S_{BH}\Big) \times \Bigg(\quad ....????... \quad \Bigg)$$

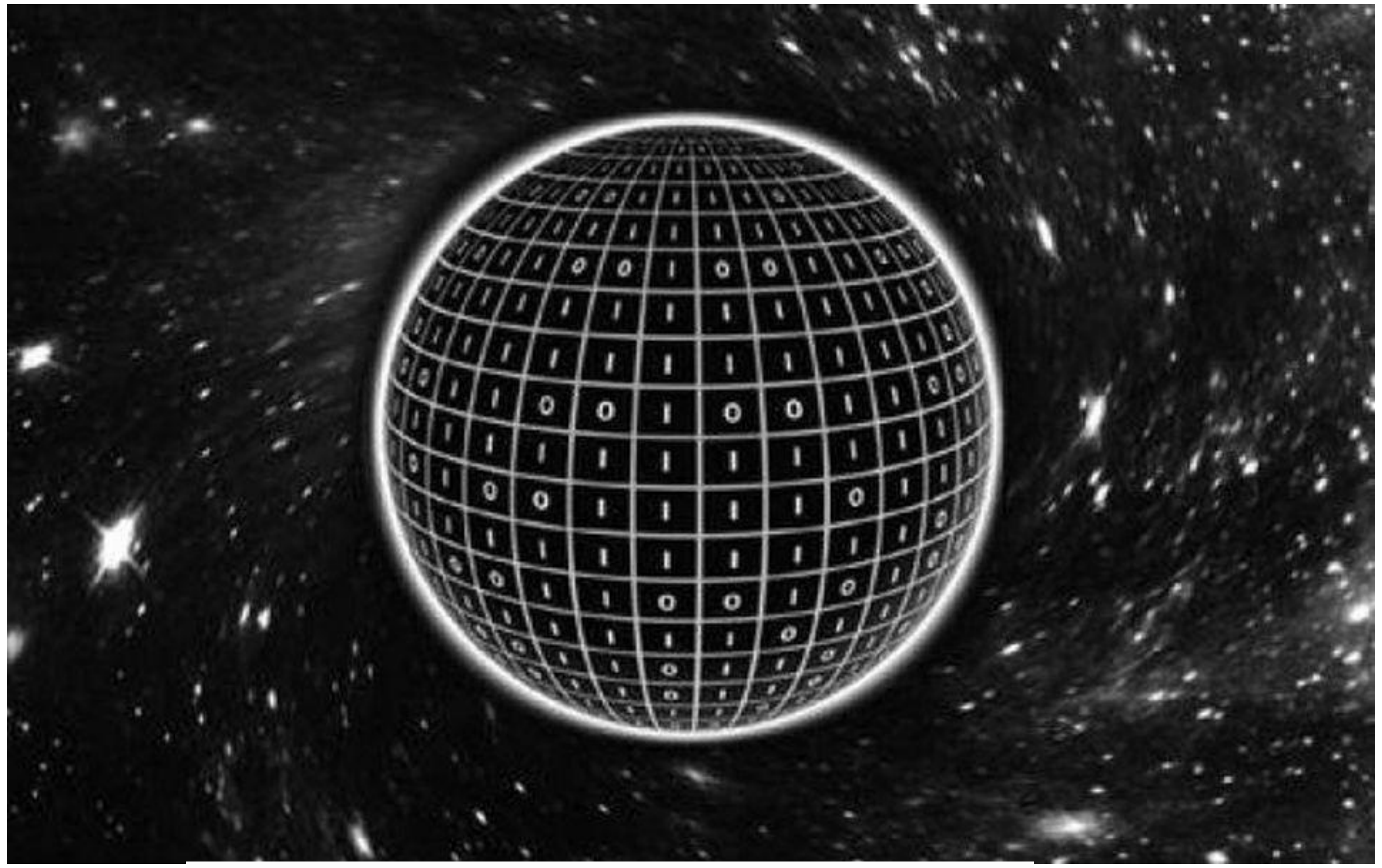
Gibbons, Hawking (1977)
Chambin, Emparan, Johnson, Myers (1999)

$$S_{BH}(T \to 0, \mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1 + \frac{2(\pi A_0)^{1/2}T}{\hbar c}\right)$$

A_0 is the area of the charged black hole horizon at $T = 0$.
$\mathcal{Q}$ is the black hole charge.
A_0 is a function of $\mathcal{Q}$.

Questions

- Is Einstein-Maxwell theory meaningful beyond the saddle point, and can we compute quantum fluctuation corrections to S_{BH}?
- Can the resulting entropy be understood as that of a unitary quantum system with a discrete spectrum ?
- Can we compute the evolution of the entropy as the black hole evaporates? Is it that of an evaporating unitary quantum system?



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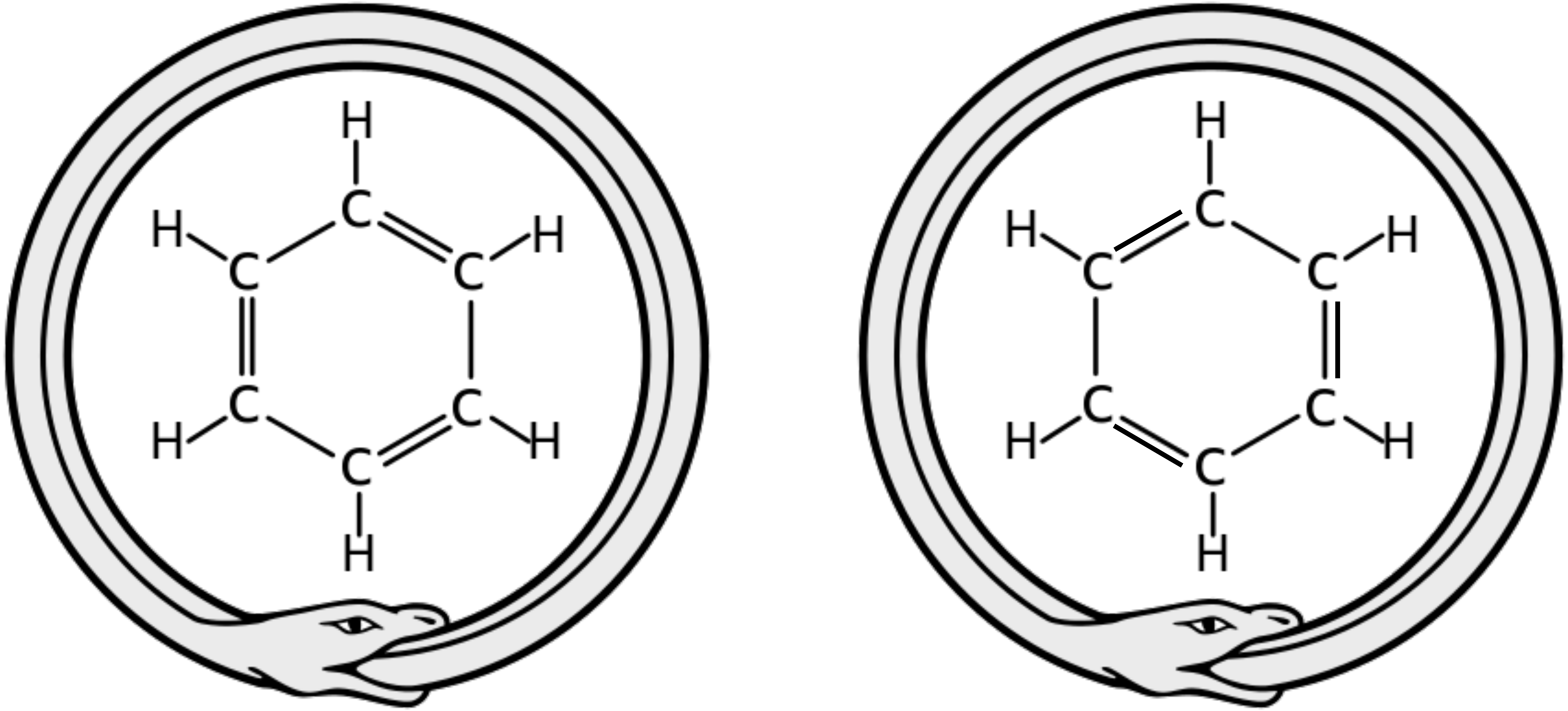
Needed:

A solvable model of multi-particle entanglement leading to a compressible quantum state with no quasiparticle excitations

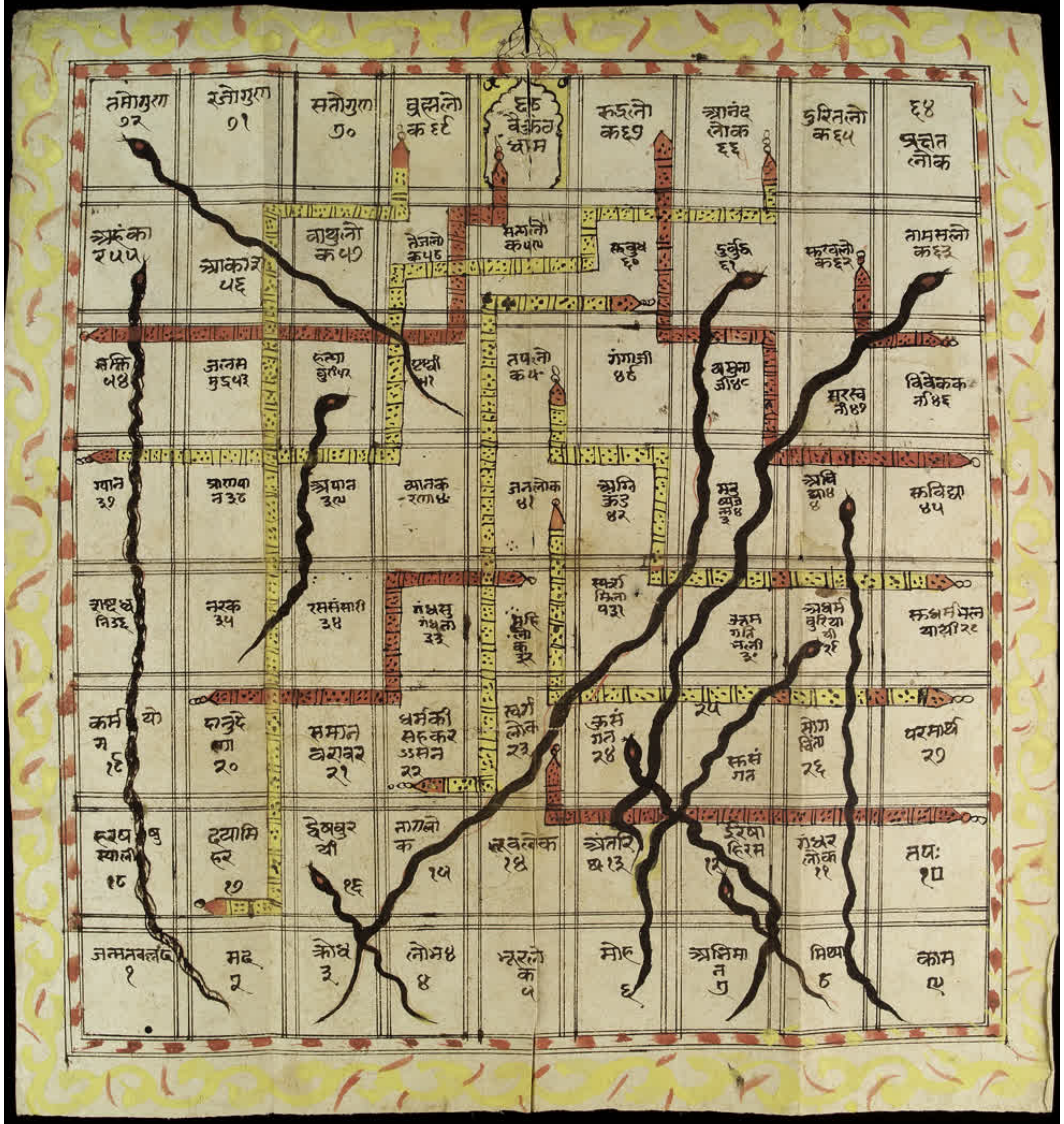
August Kekule, theory of the benzene molecule, 1865

Kekule's dream

Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail[*]



[*] Wikipedia



My dream*

Snakes and ladders

*Not true

The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

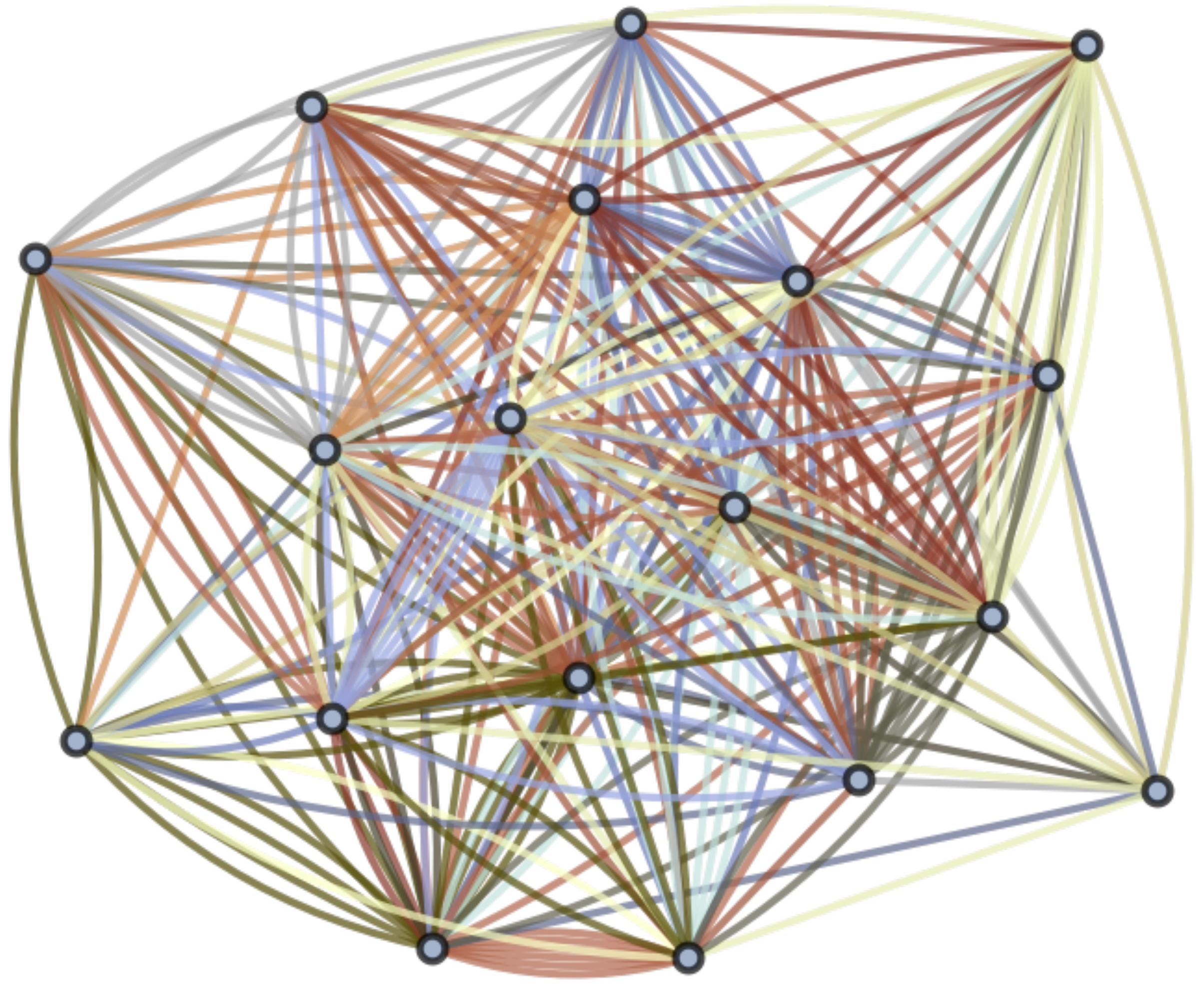
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

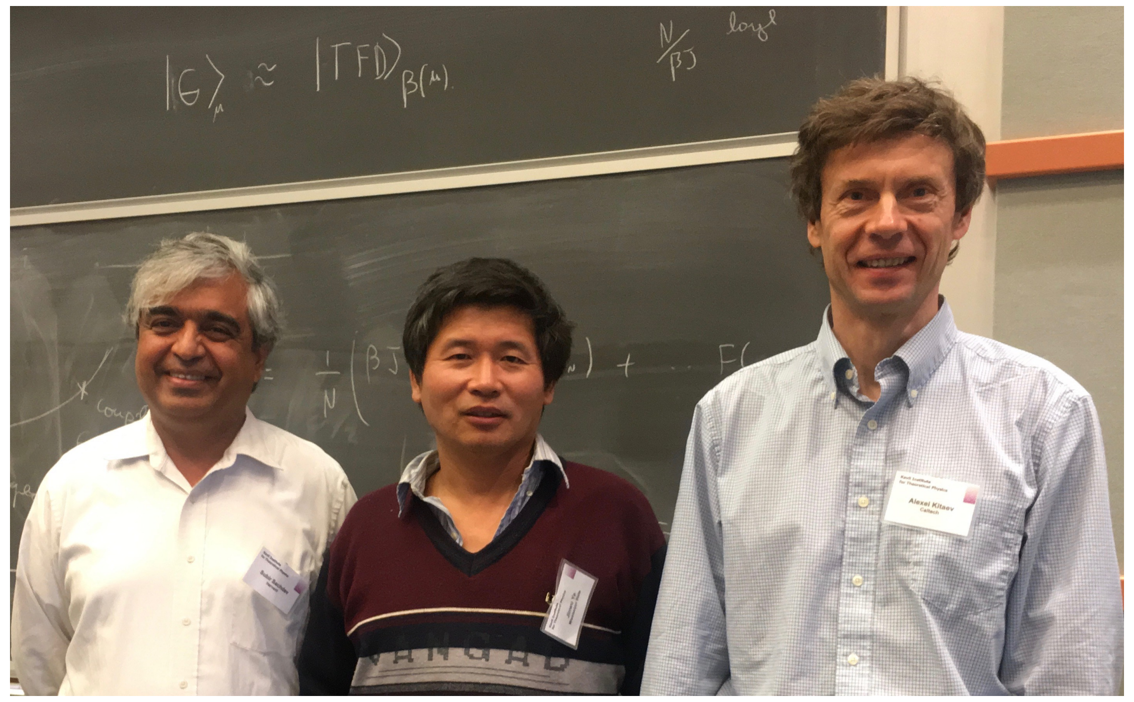
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

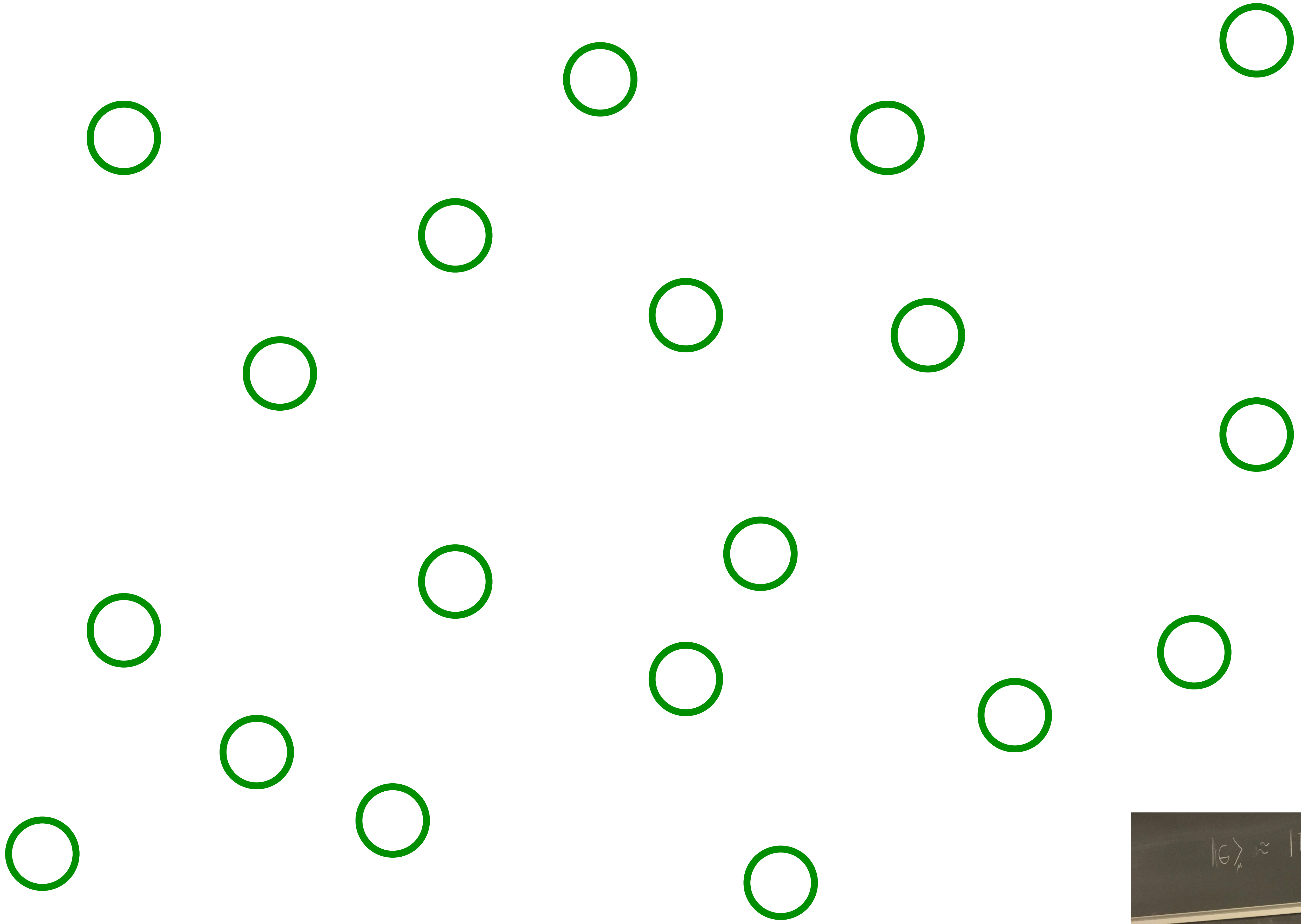
A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)



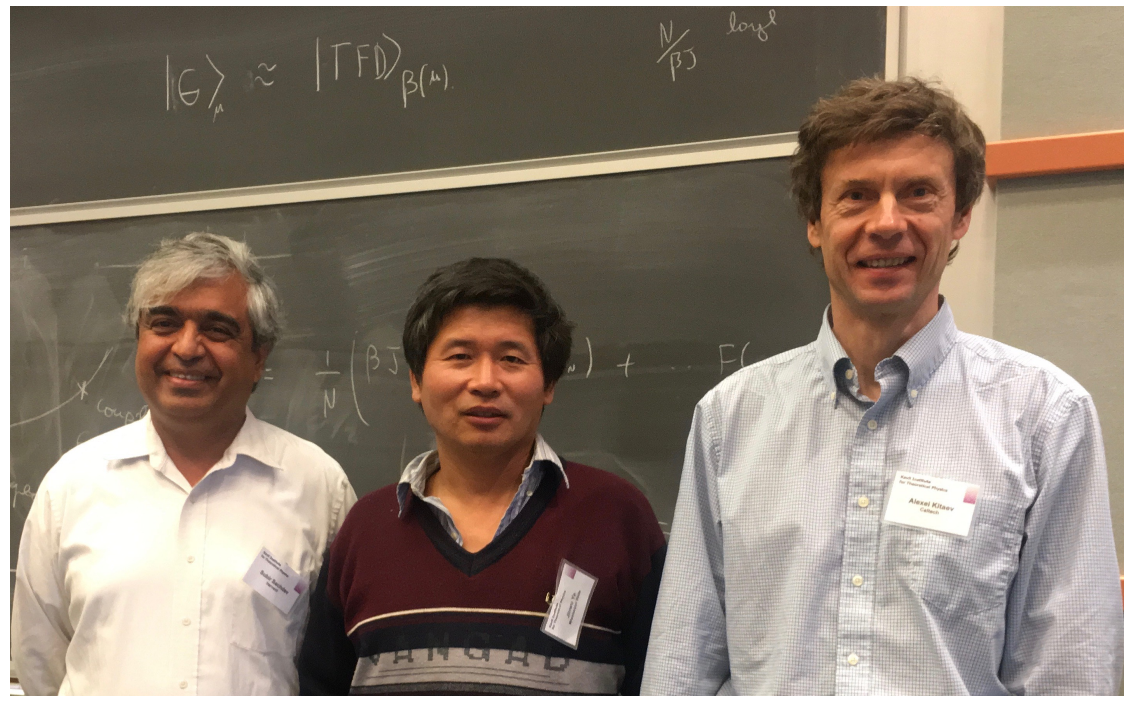


The SYK model

Sachdev, Ye (1993); Kitaev (2015)



Pick a set of random positions



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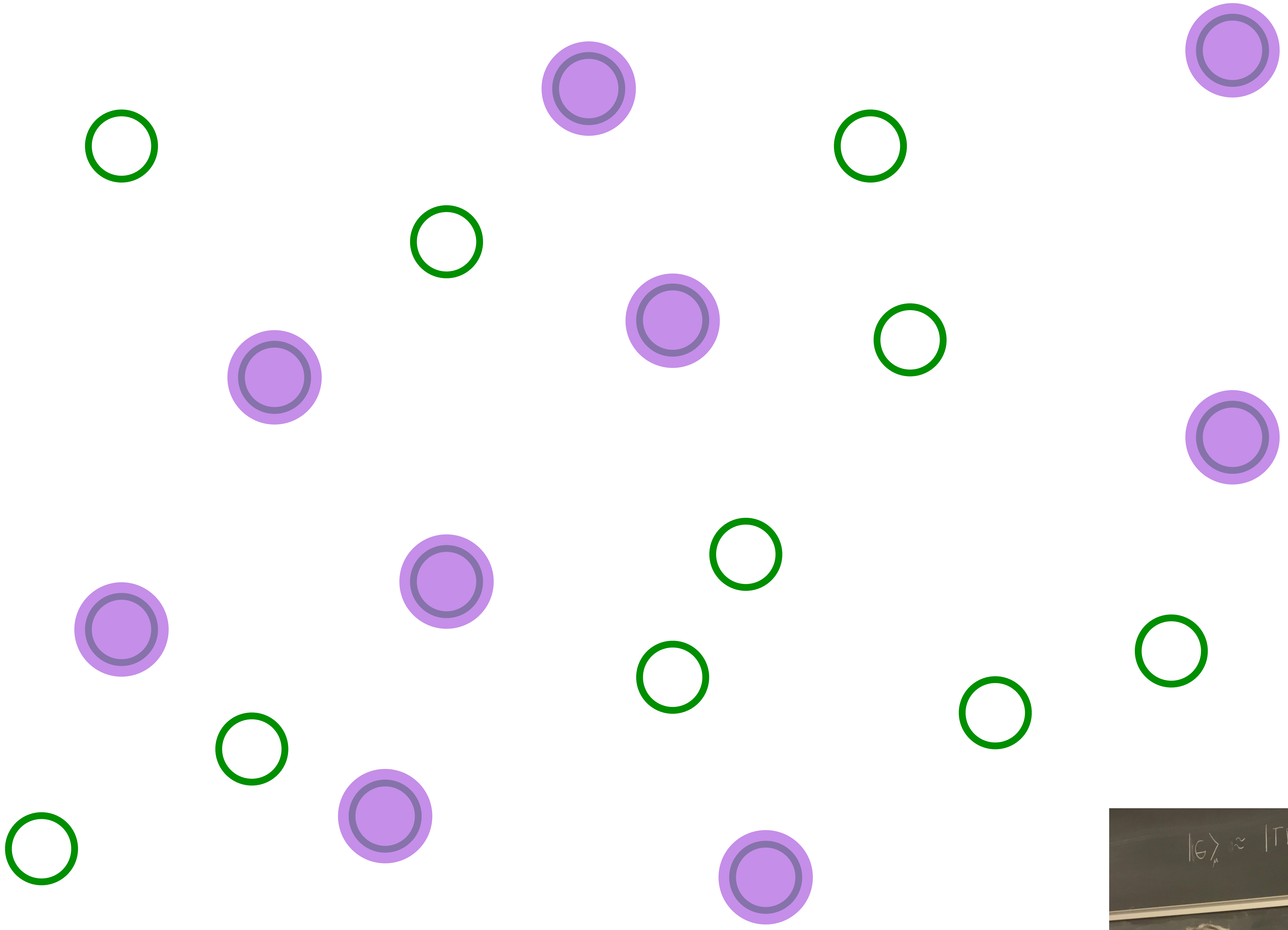
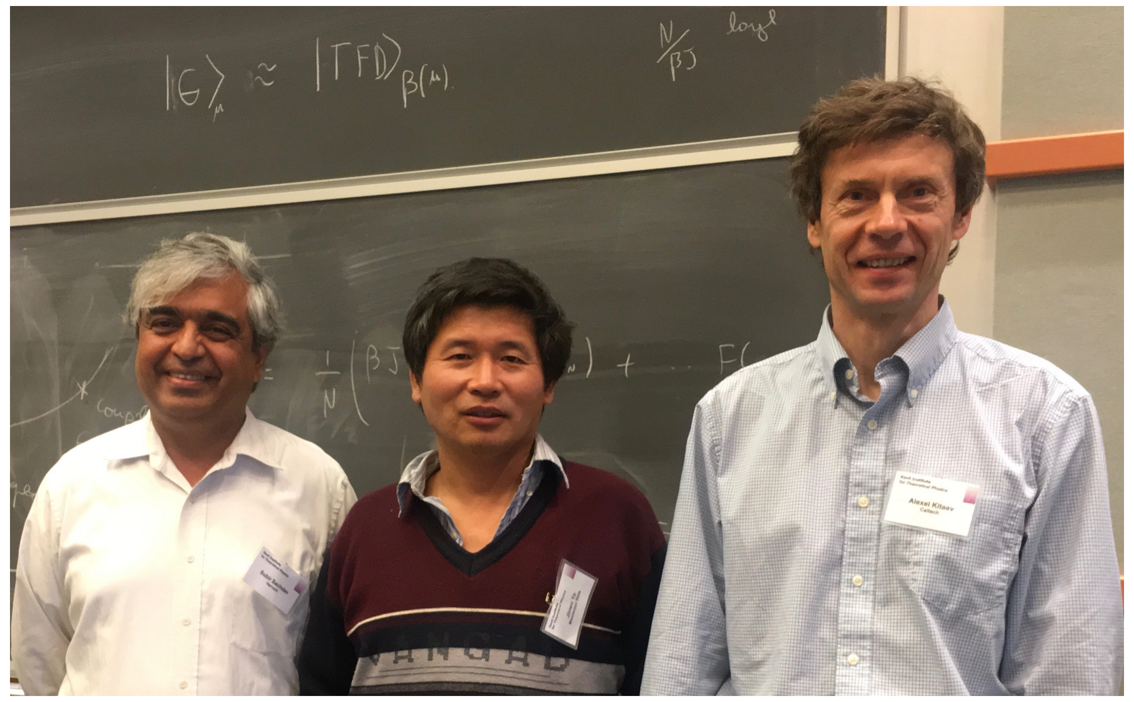


Place electrons randomly on some sites



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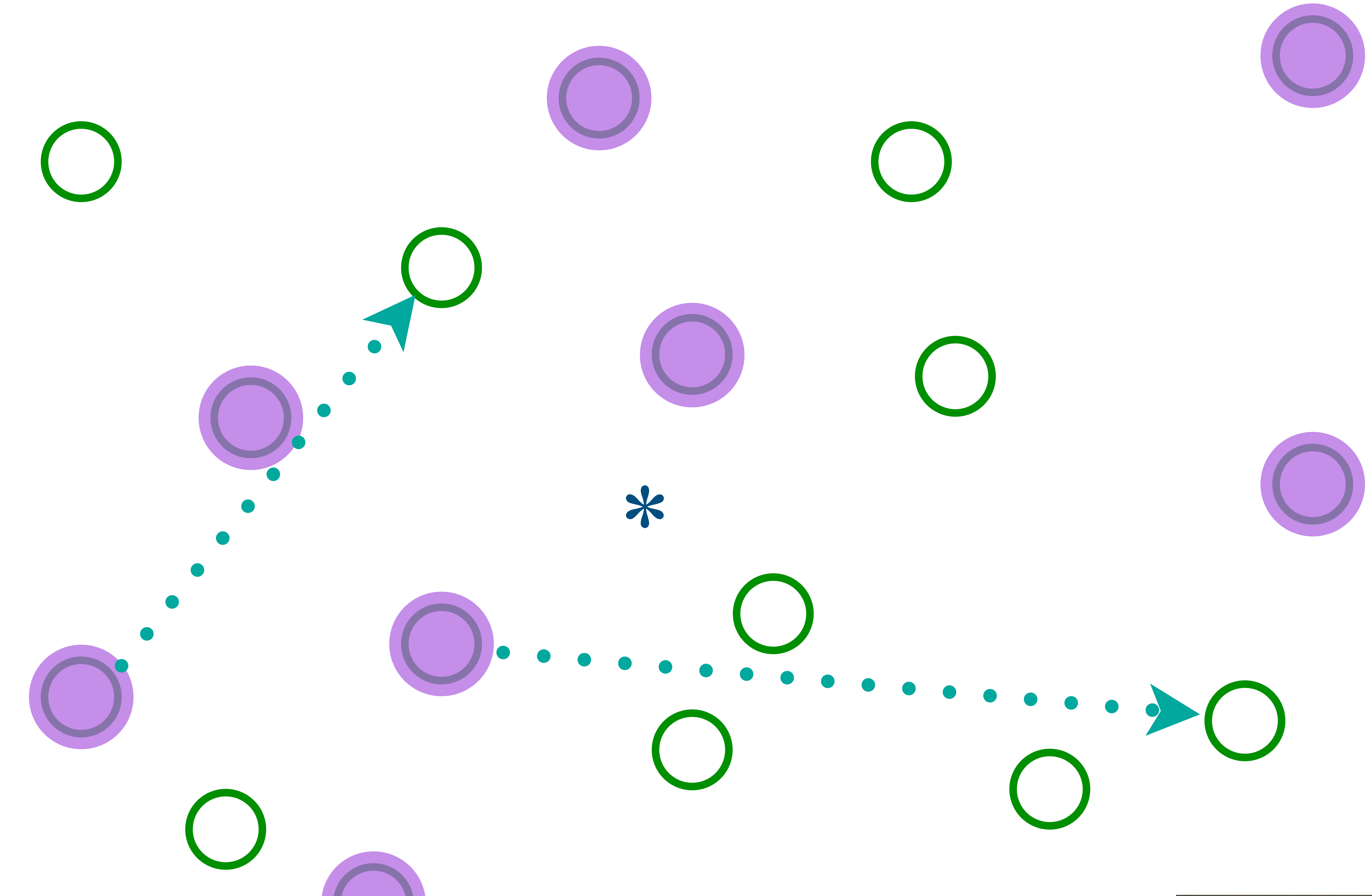

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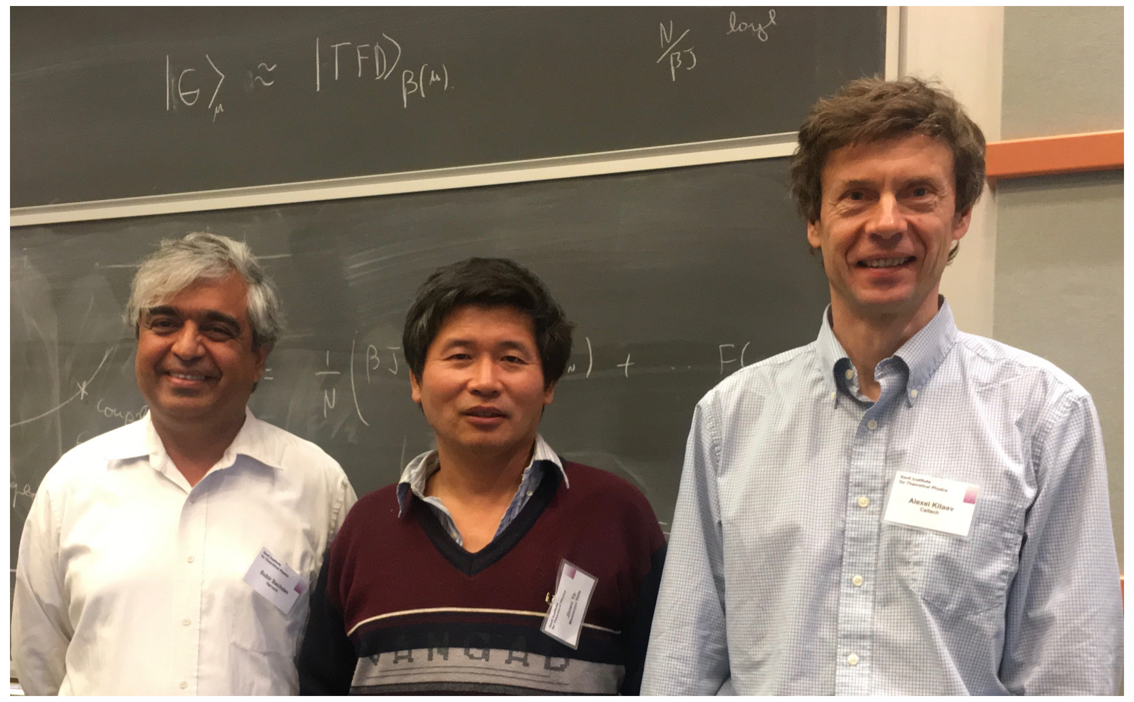

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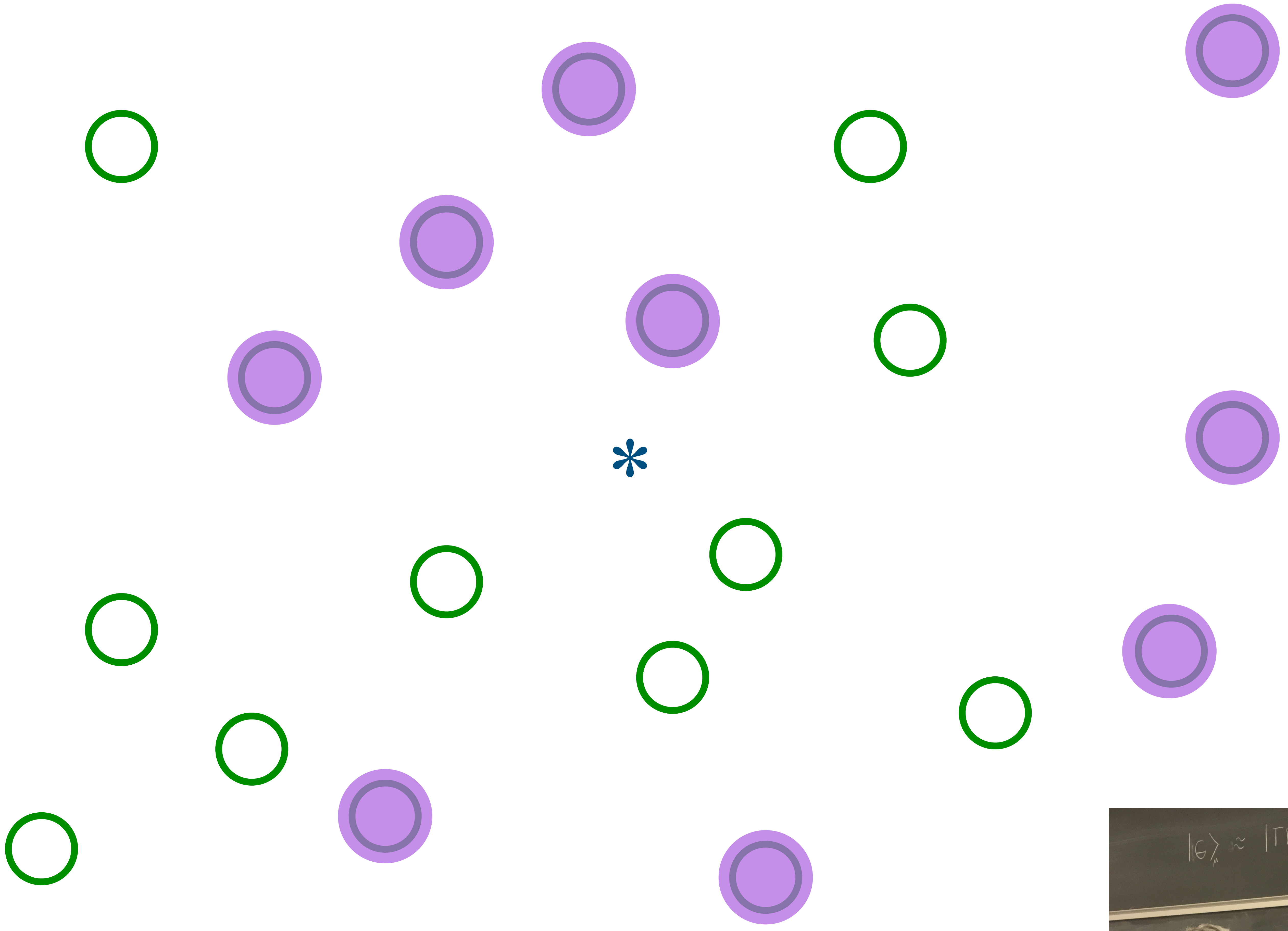
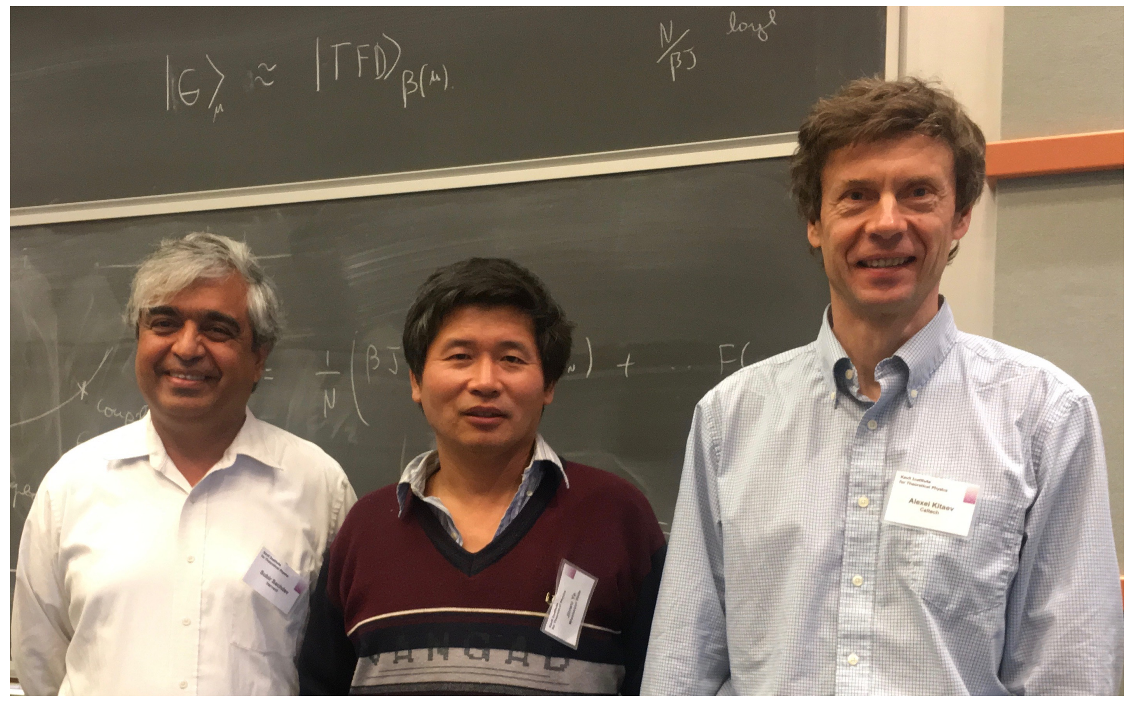


Entangle electrons pairwise randomly



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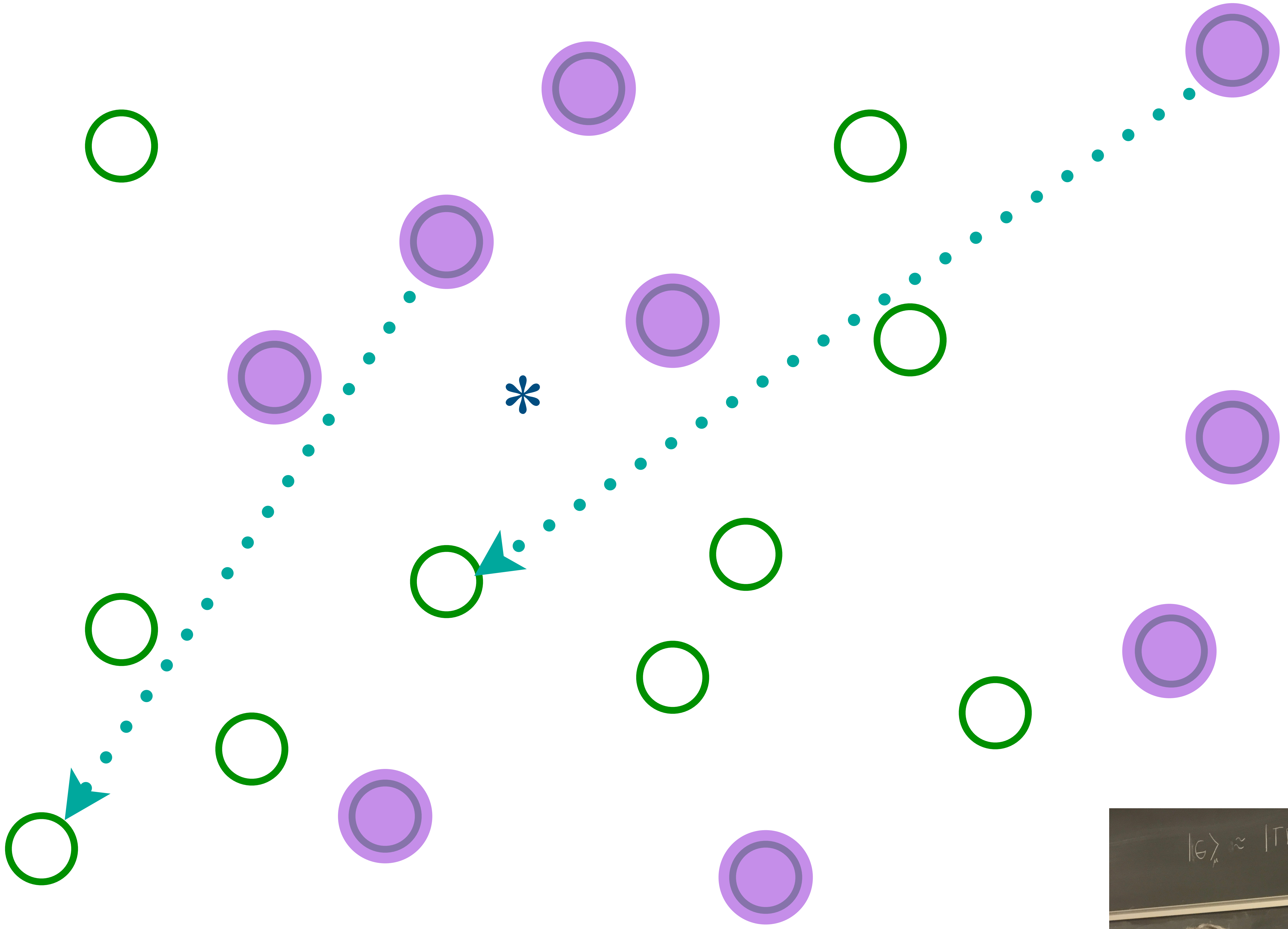

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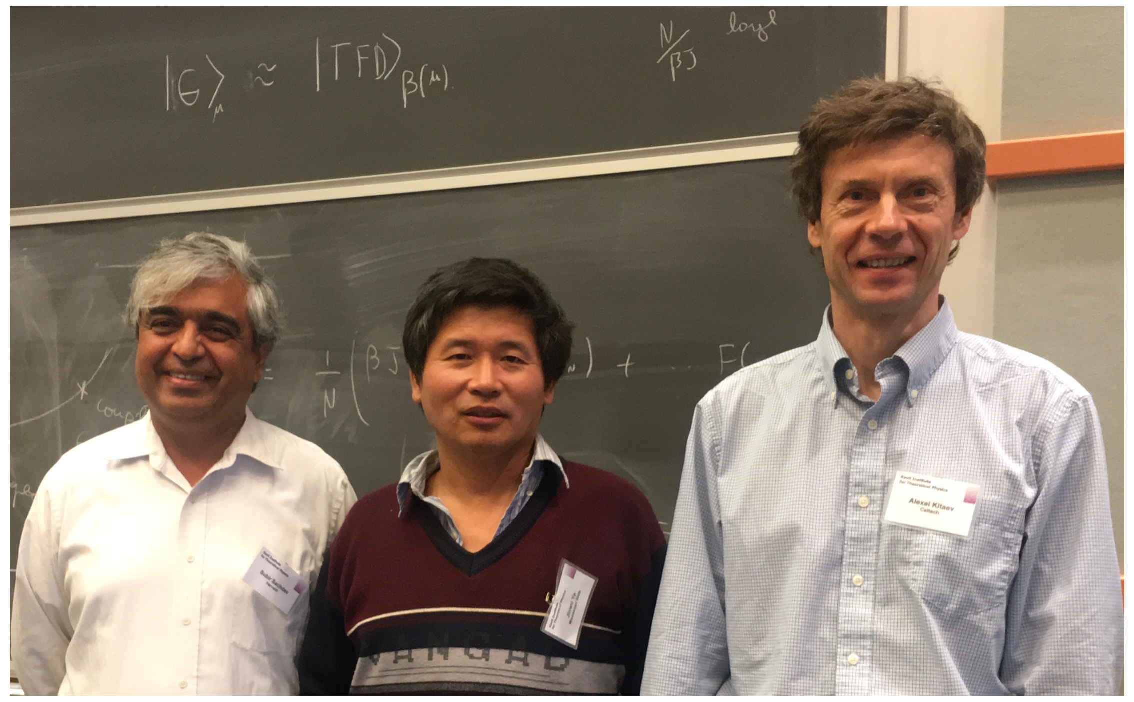

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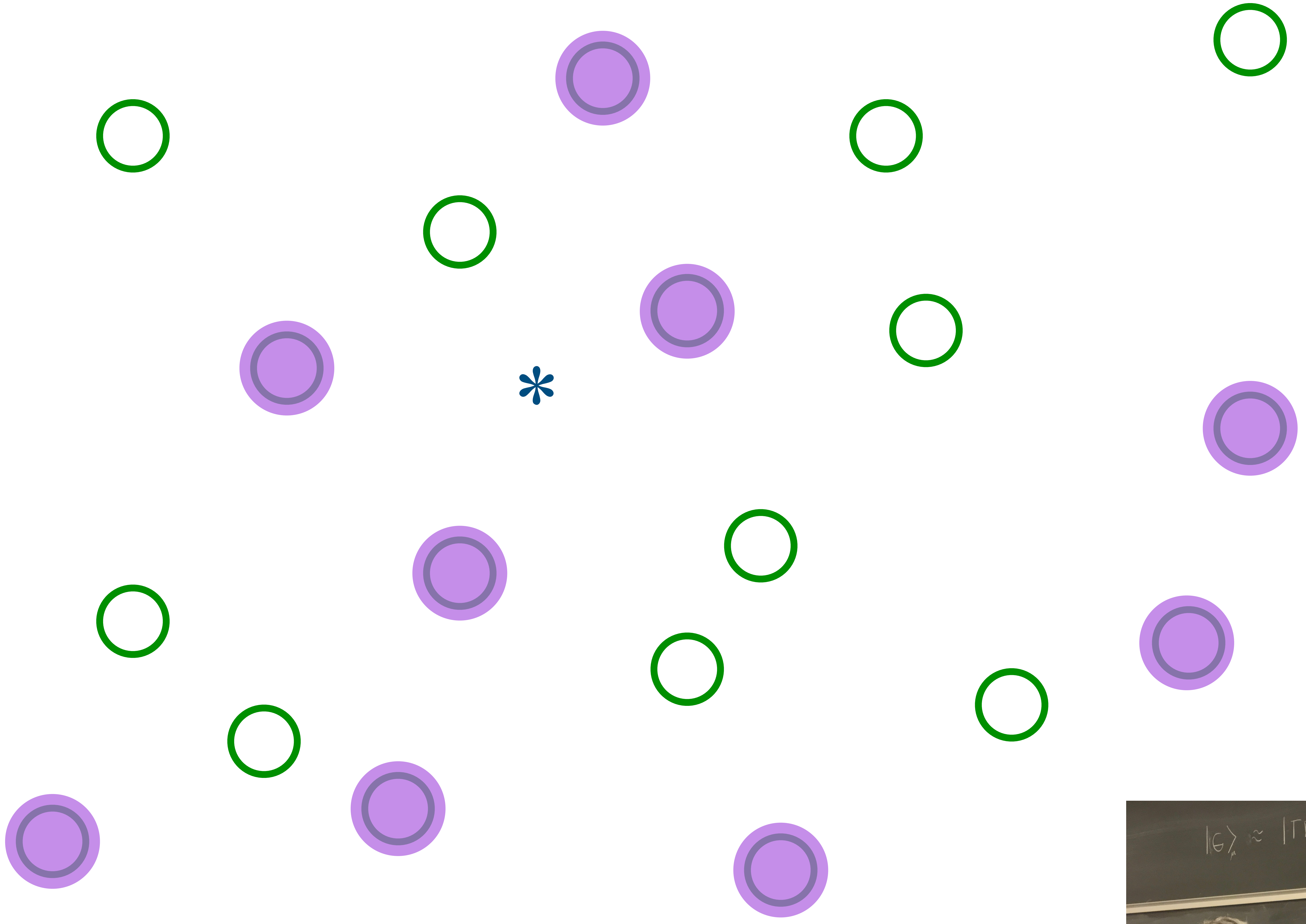

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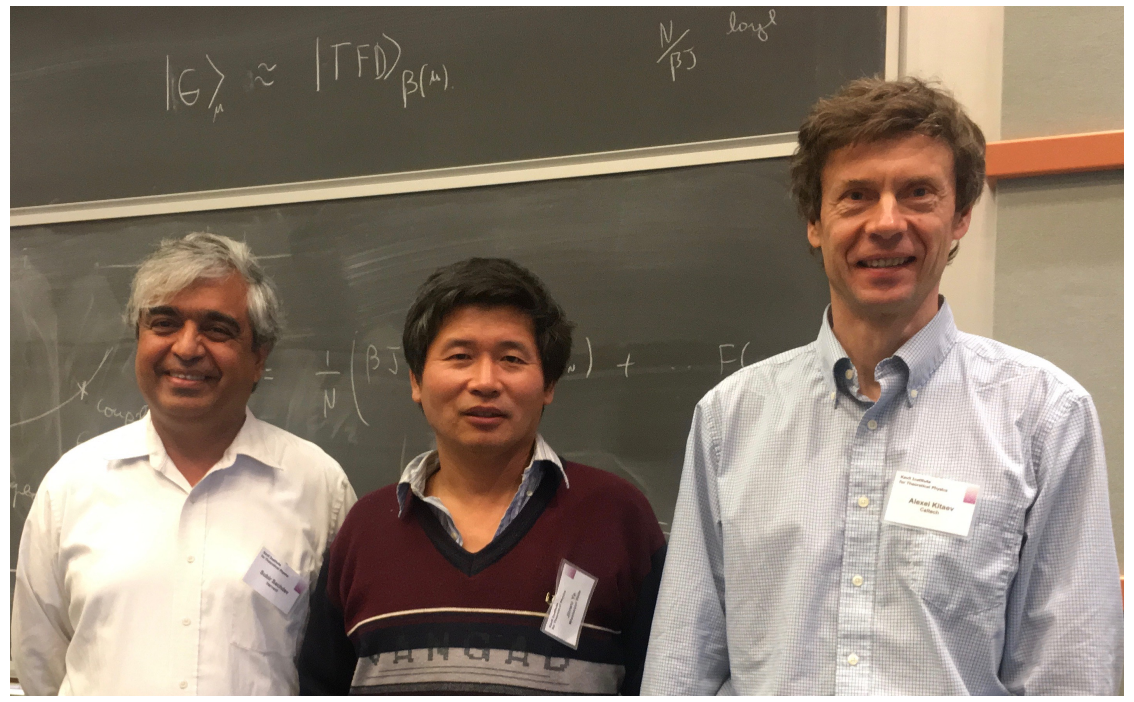

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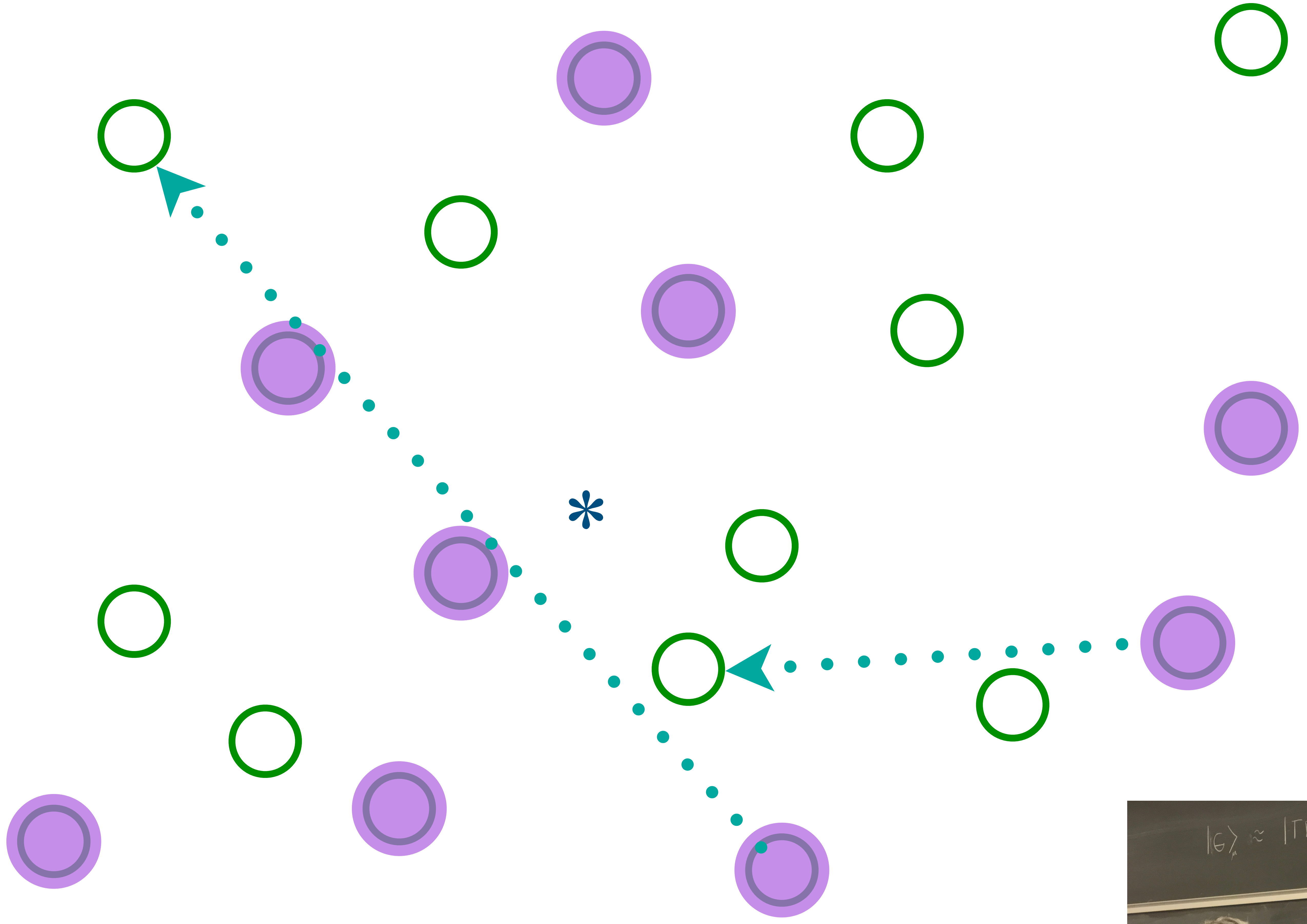

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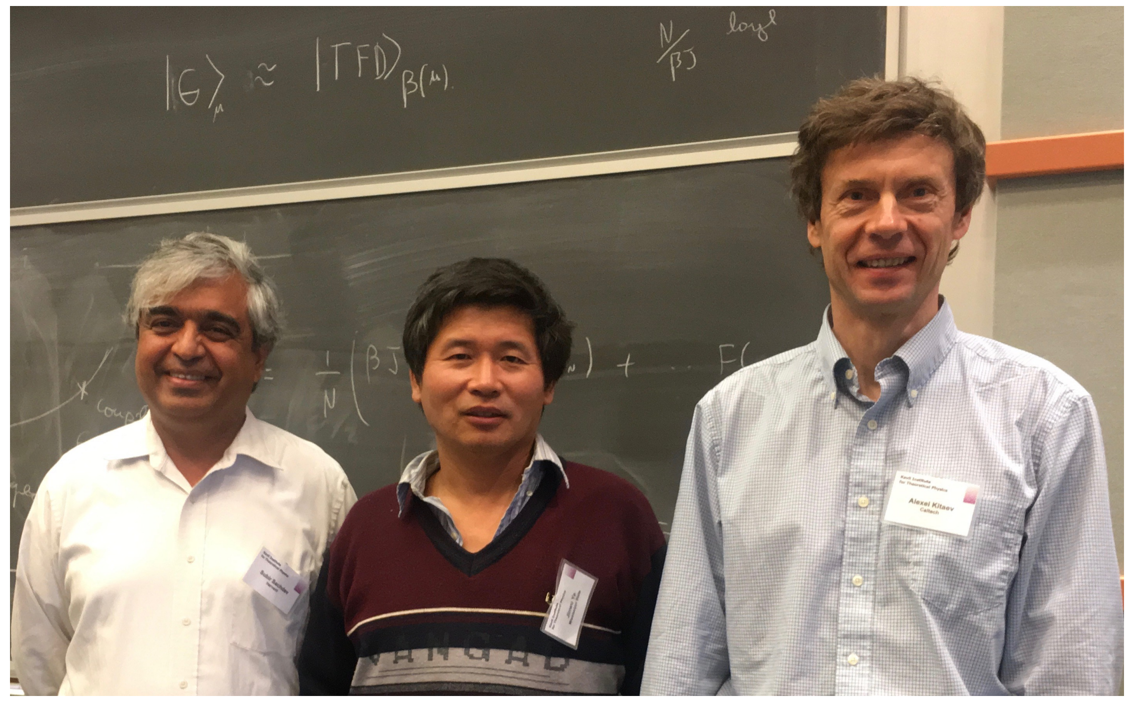

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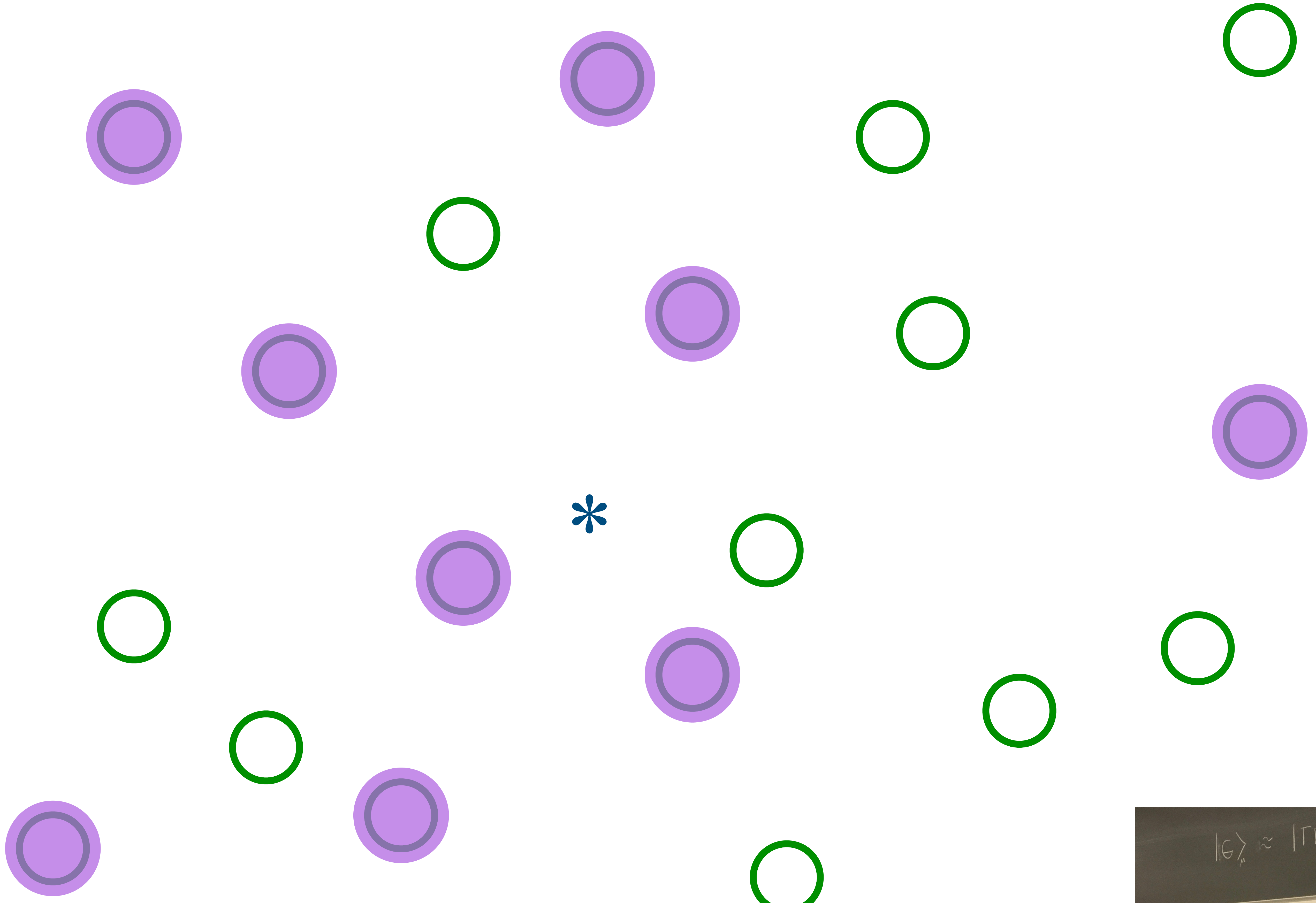

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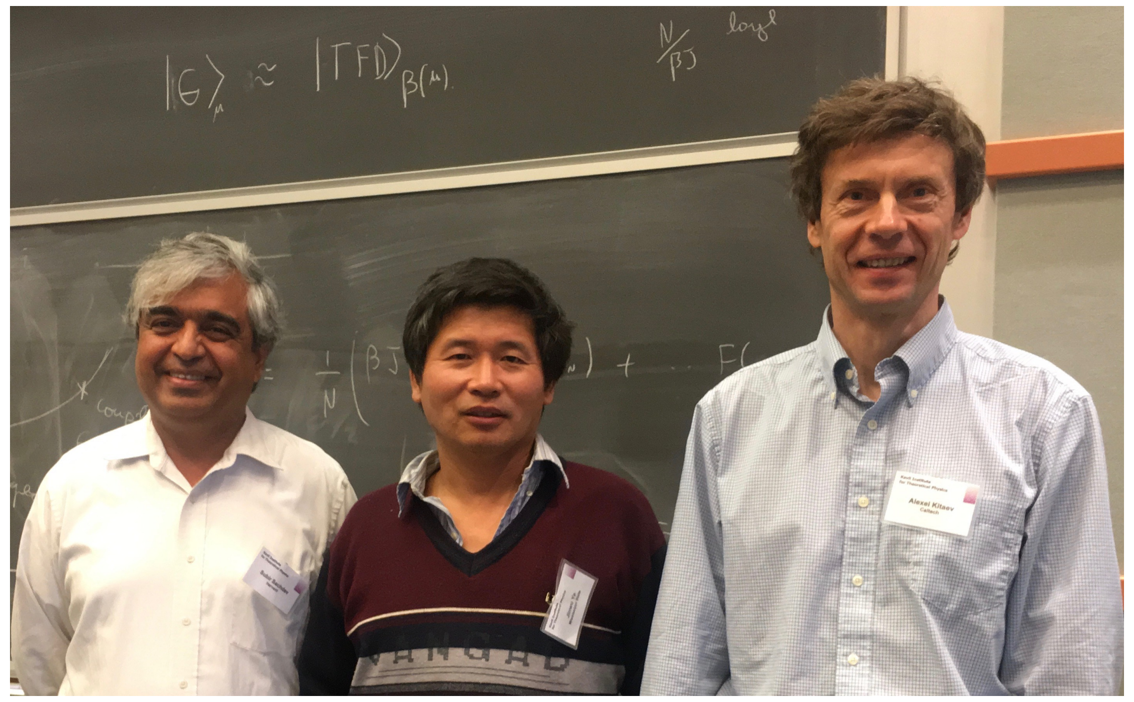

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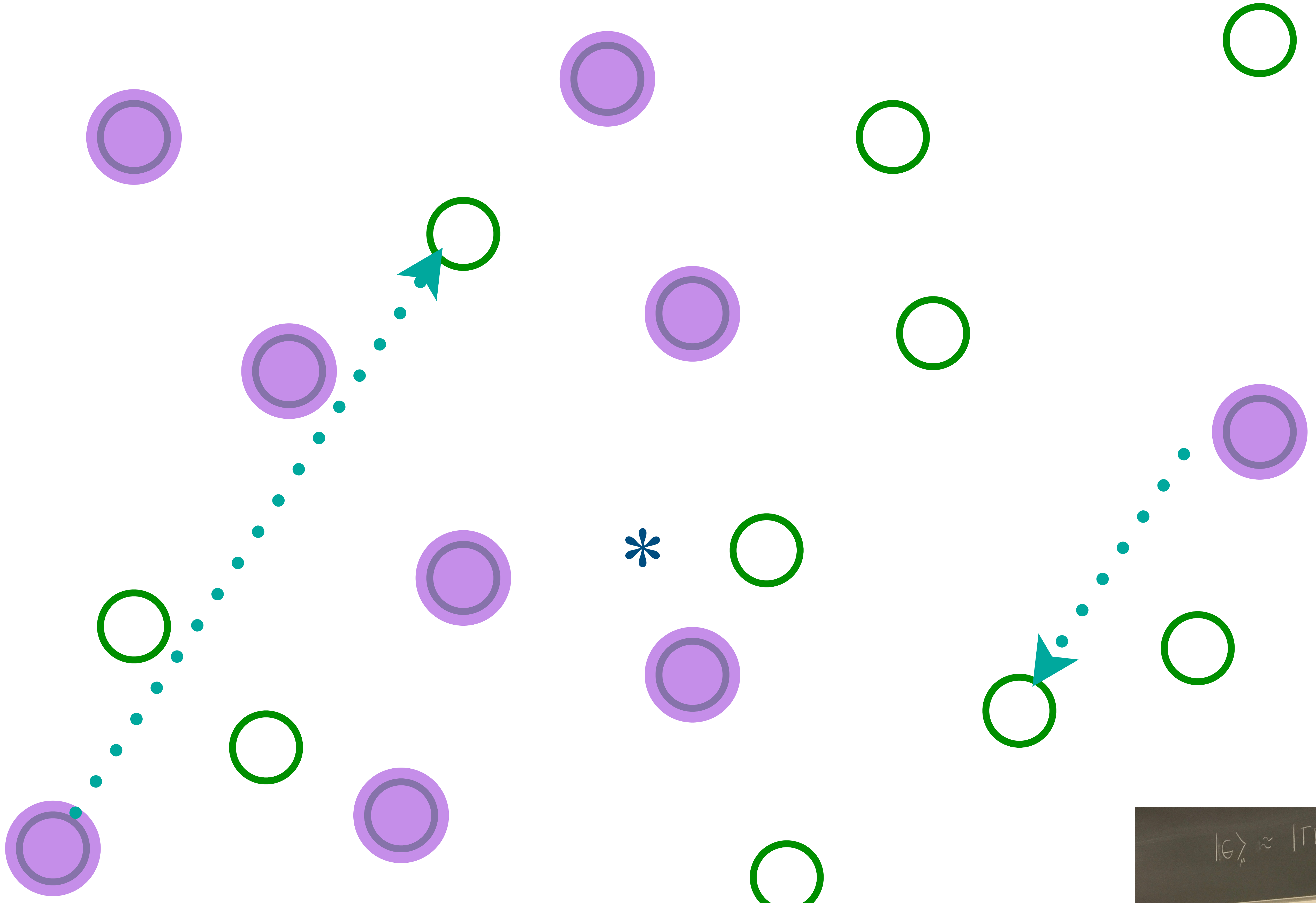

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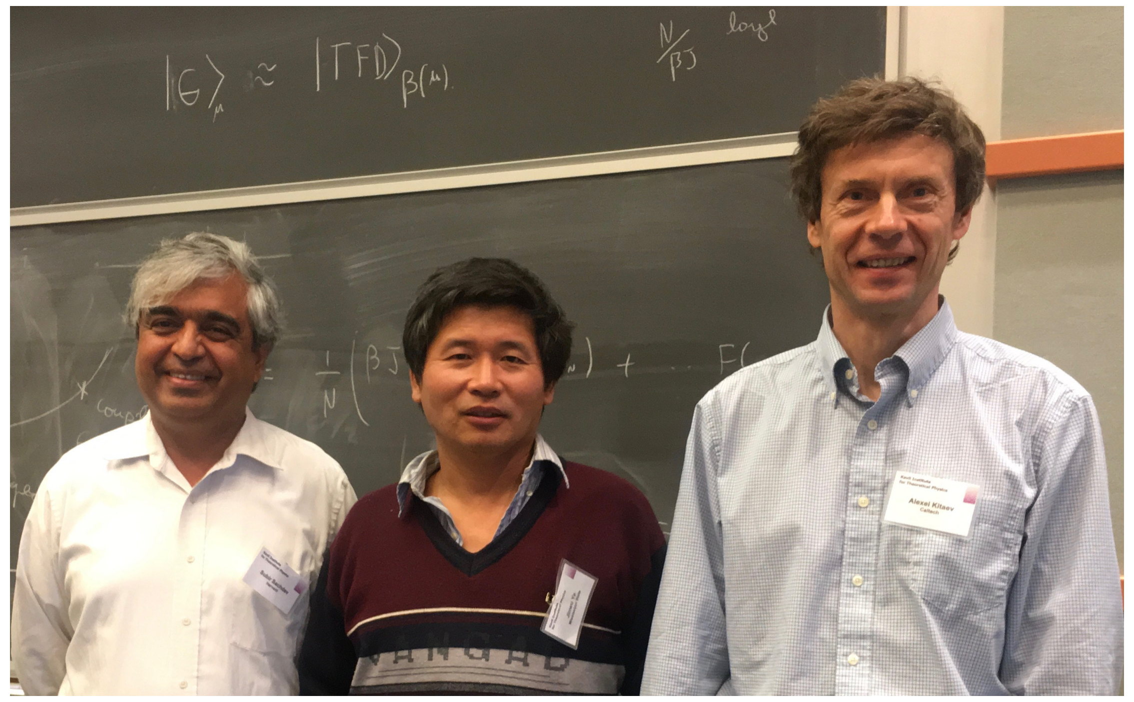

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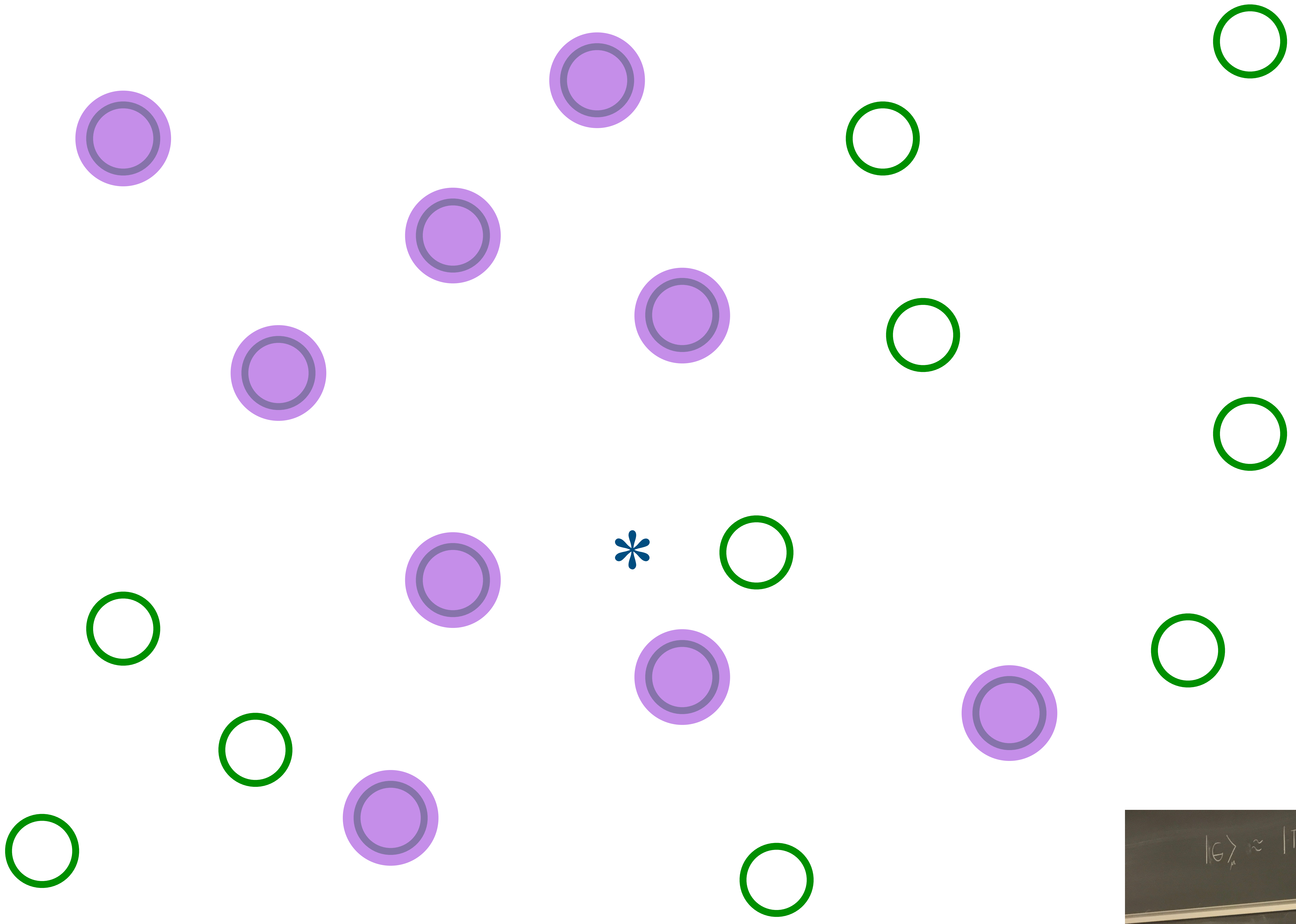

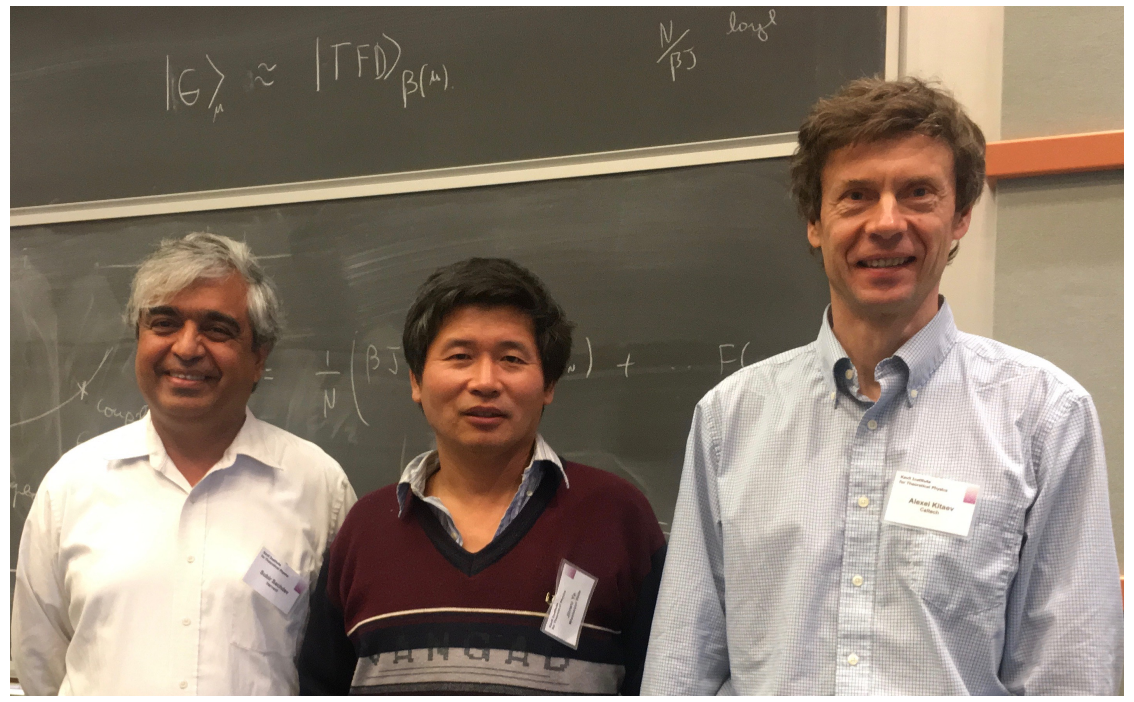

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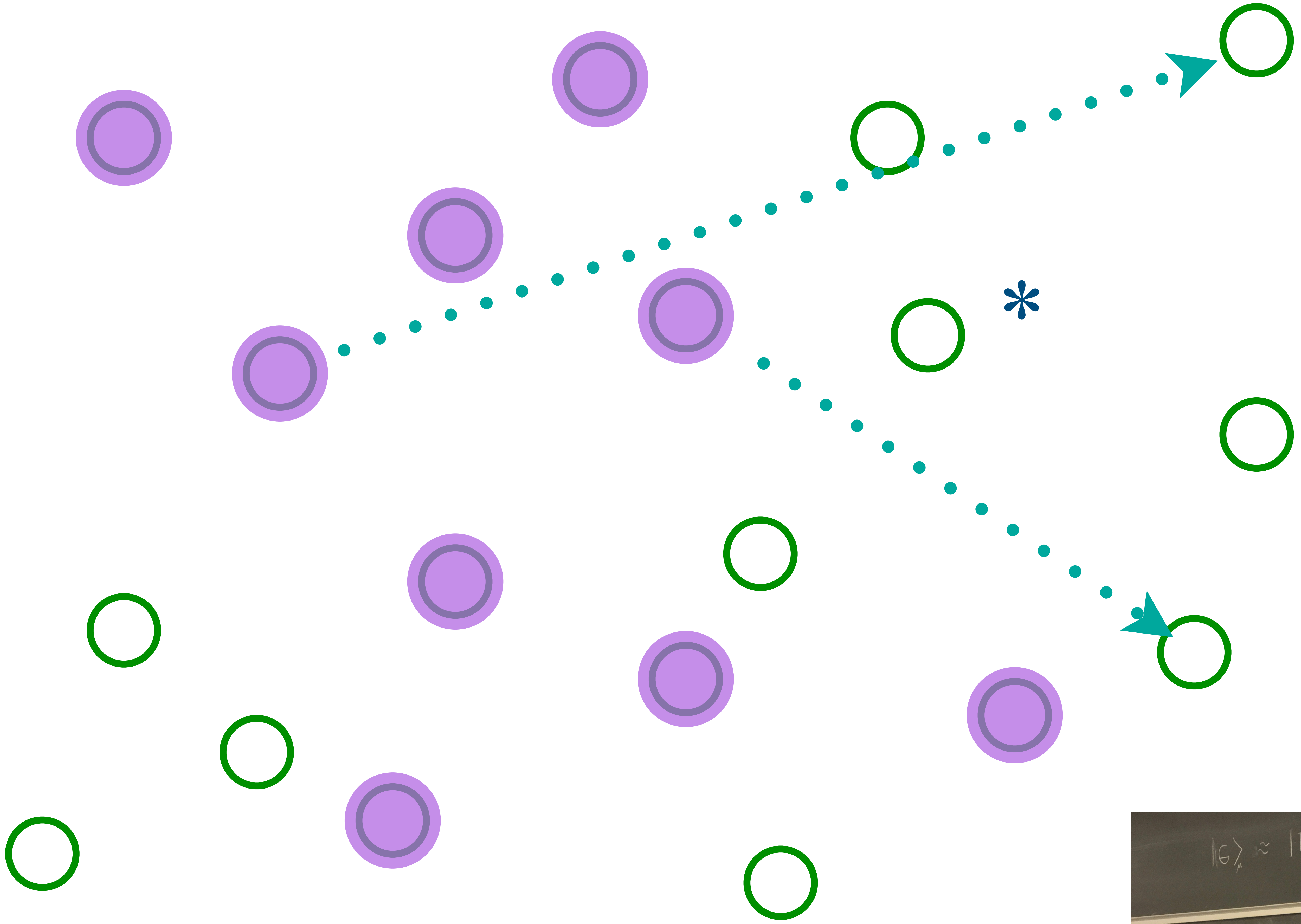

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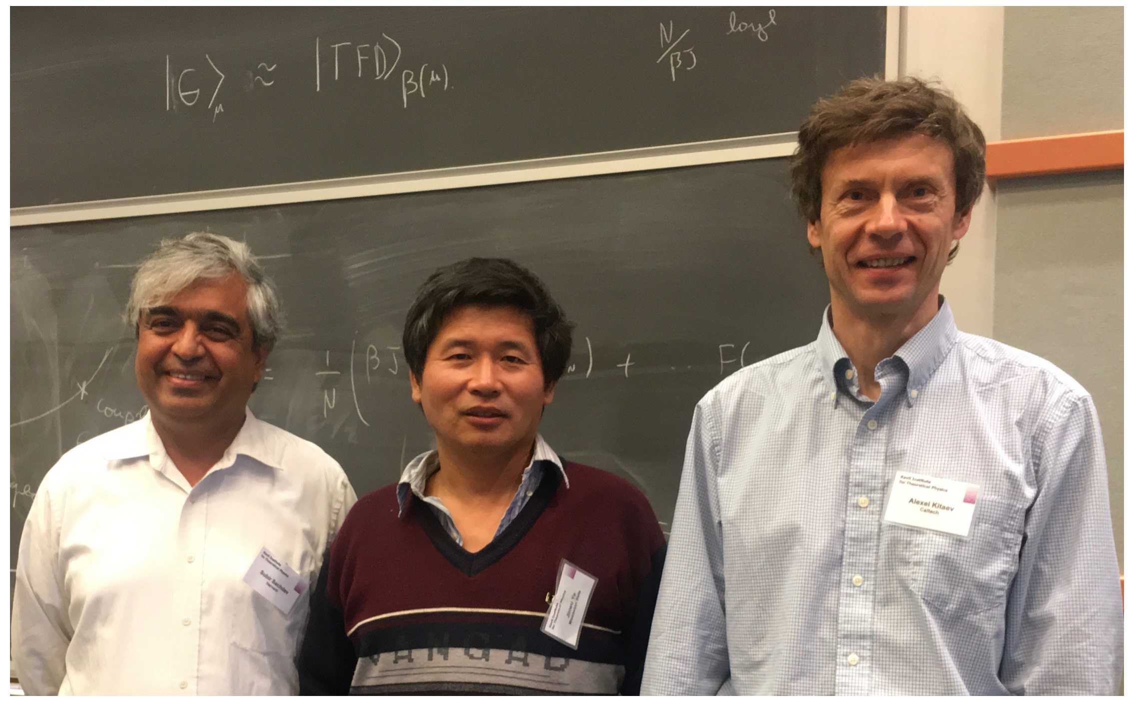

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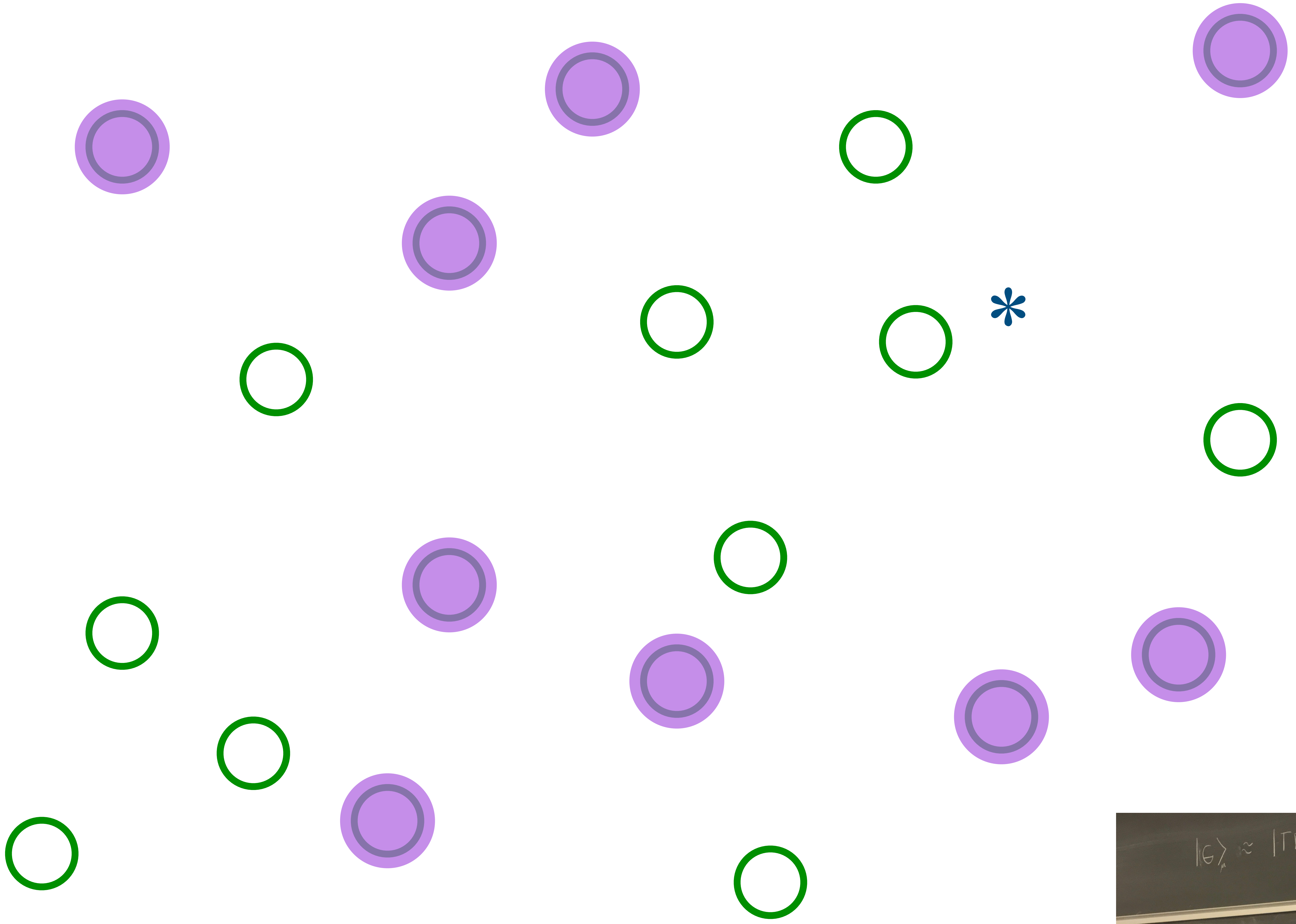
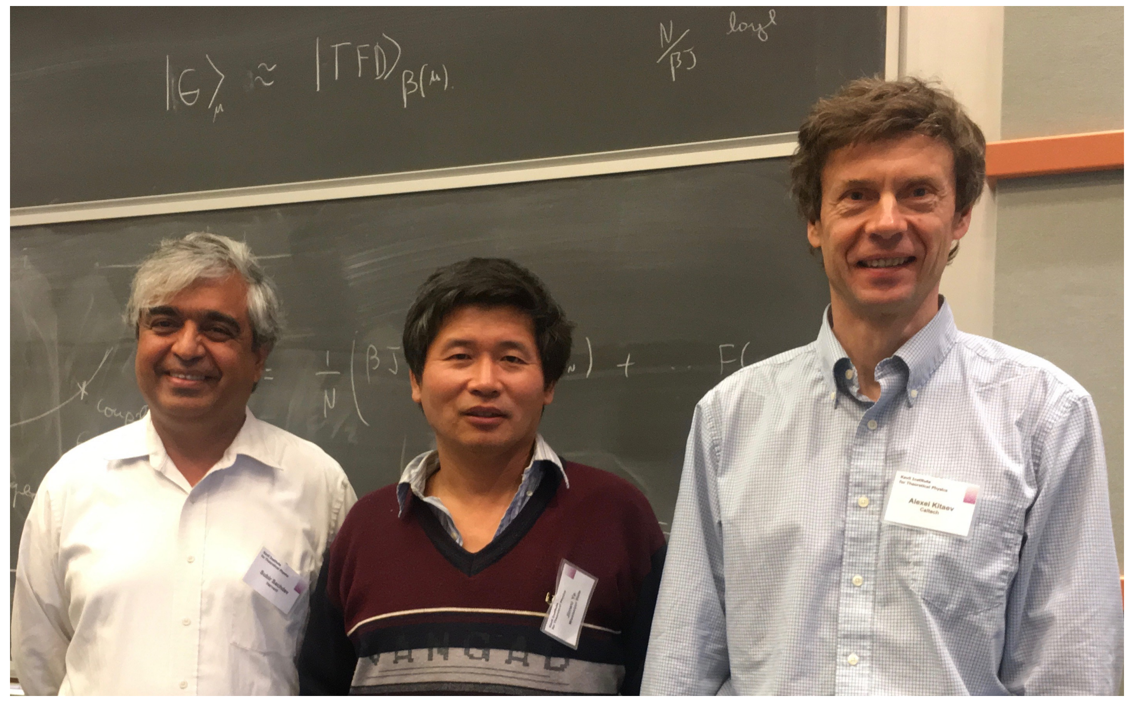


Entangle electrons pairwise randomly



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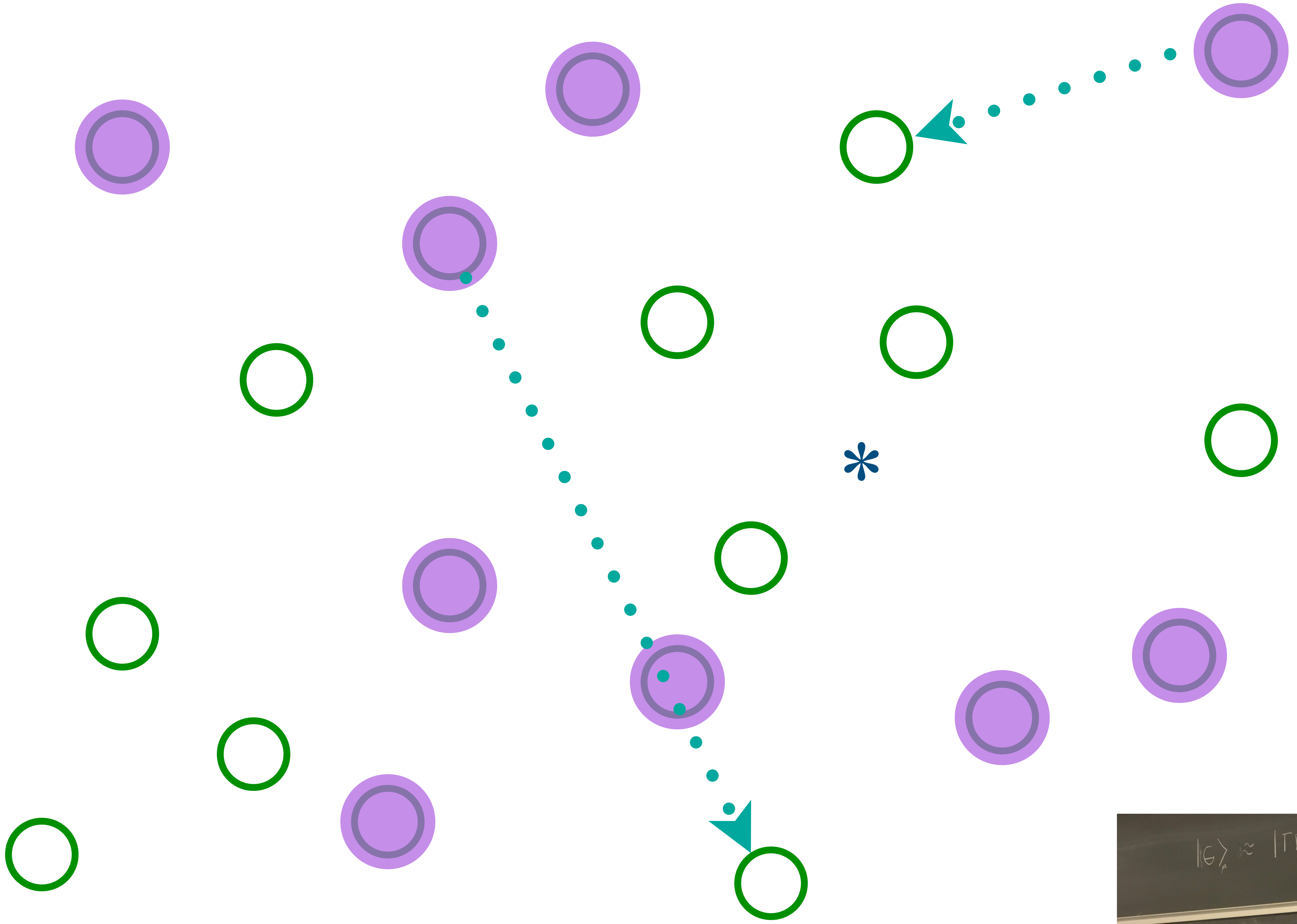

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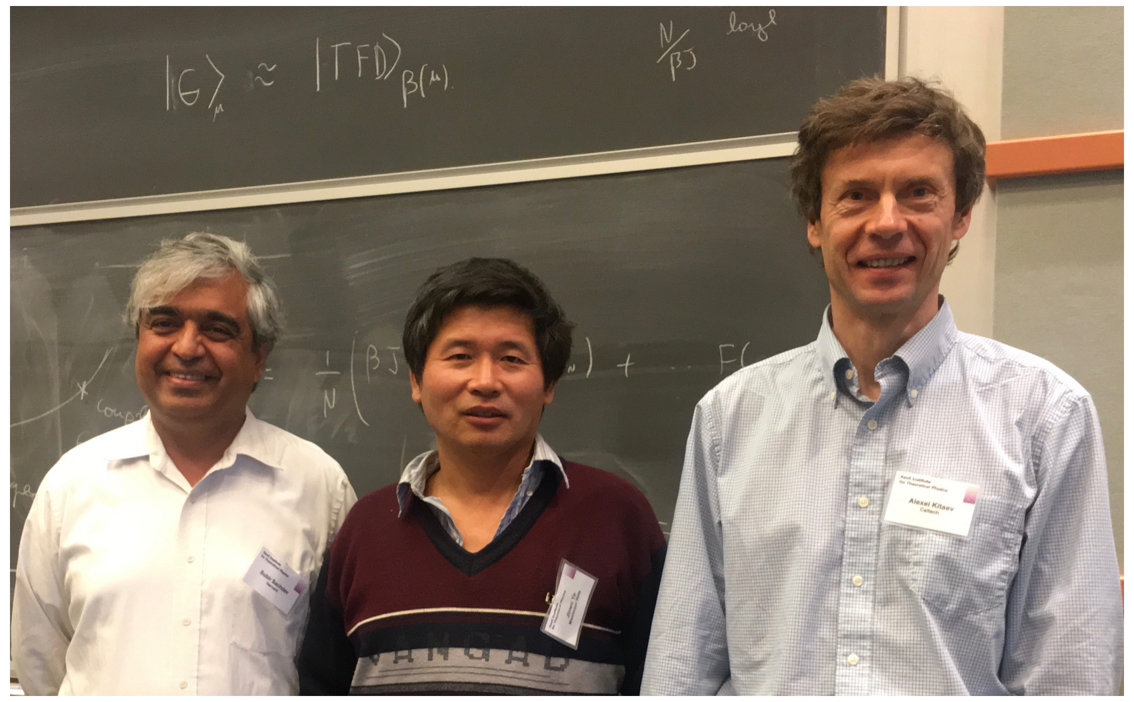

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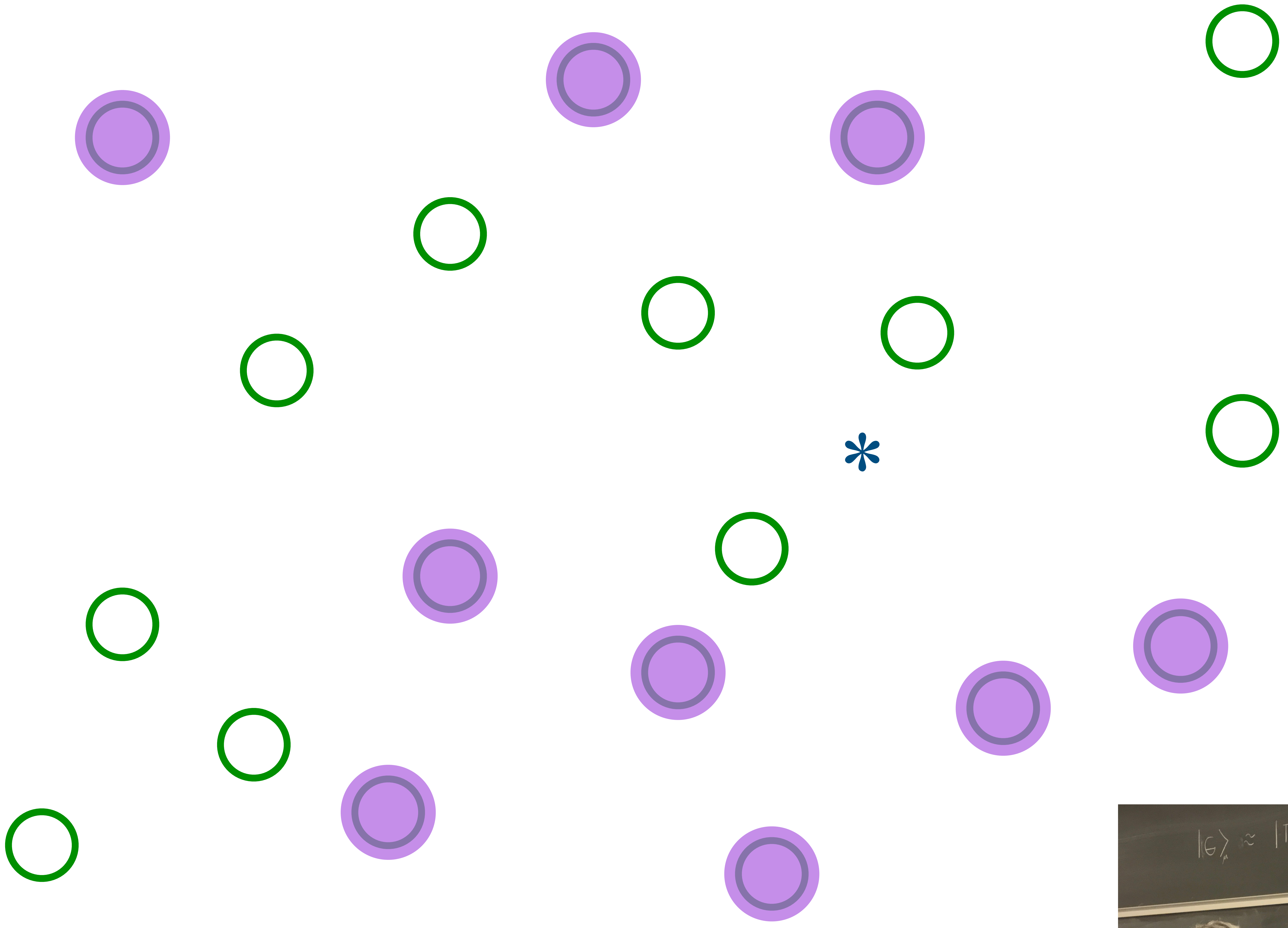
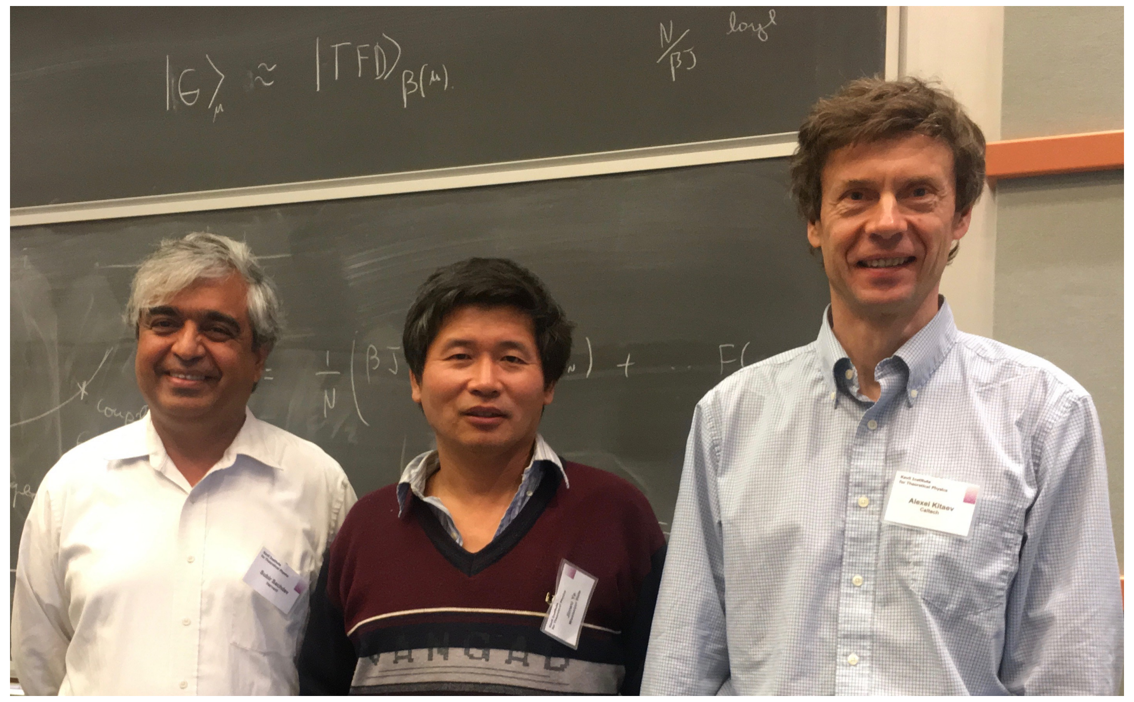




Entangle electrons pairwise randomly

The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

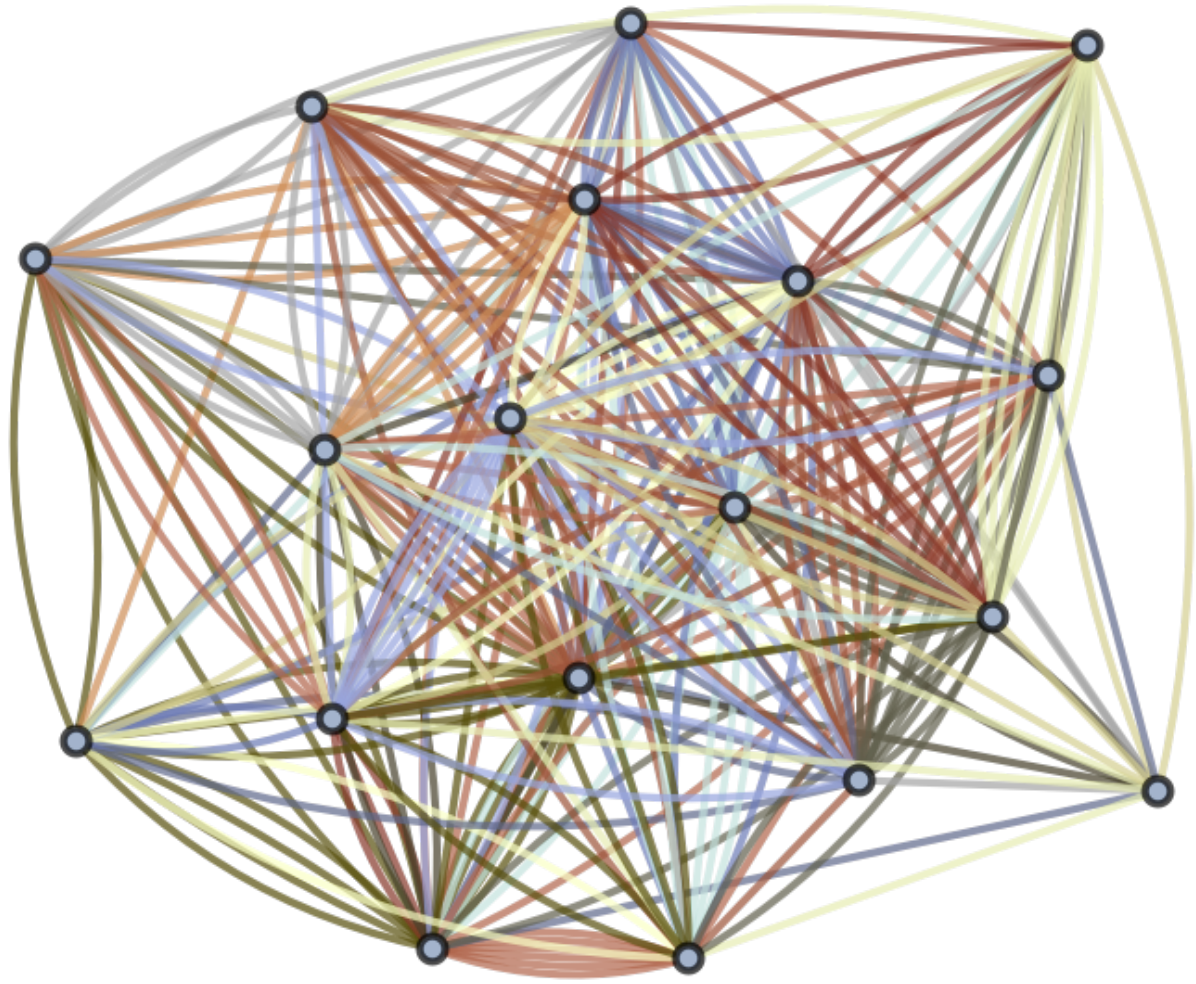
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

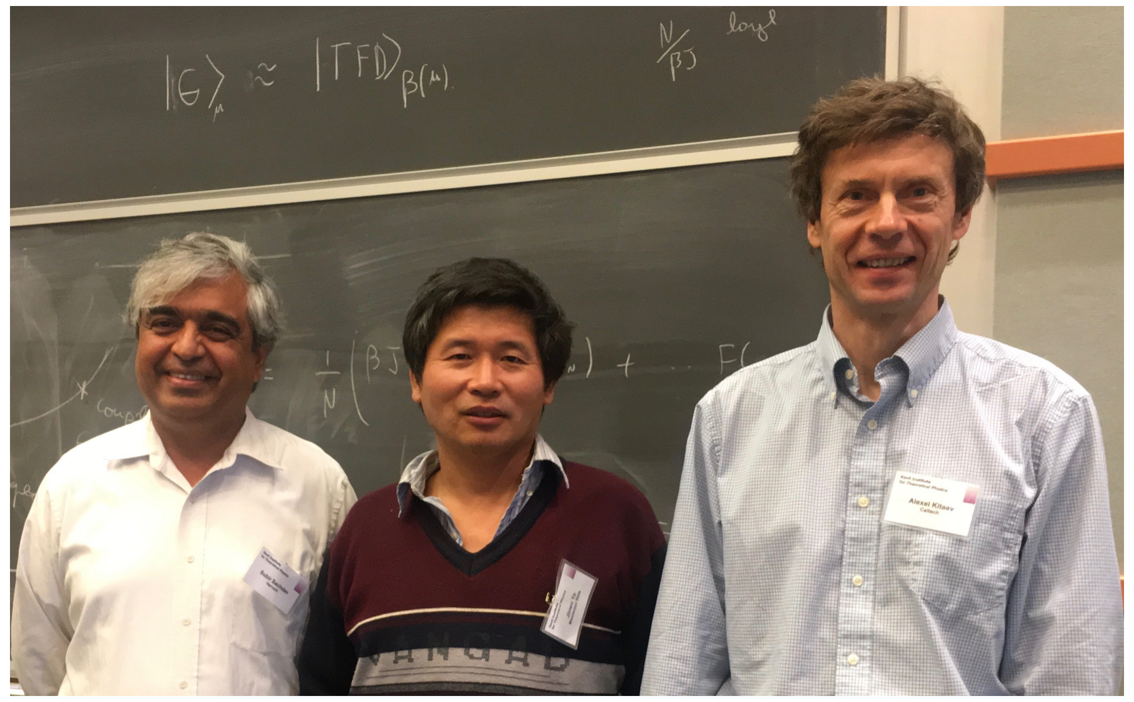
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)





A simple model of a metal with quasiparticles

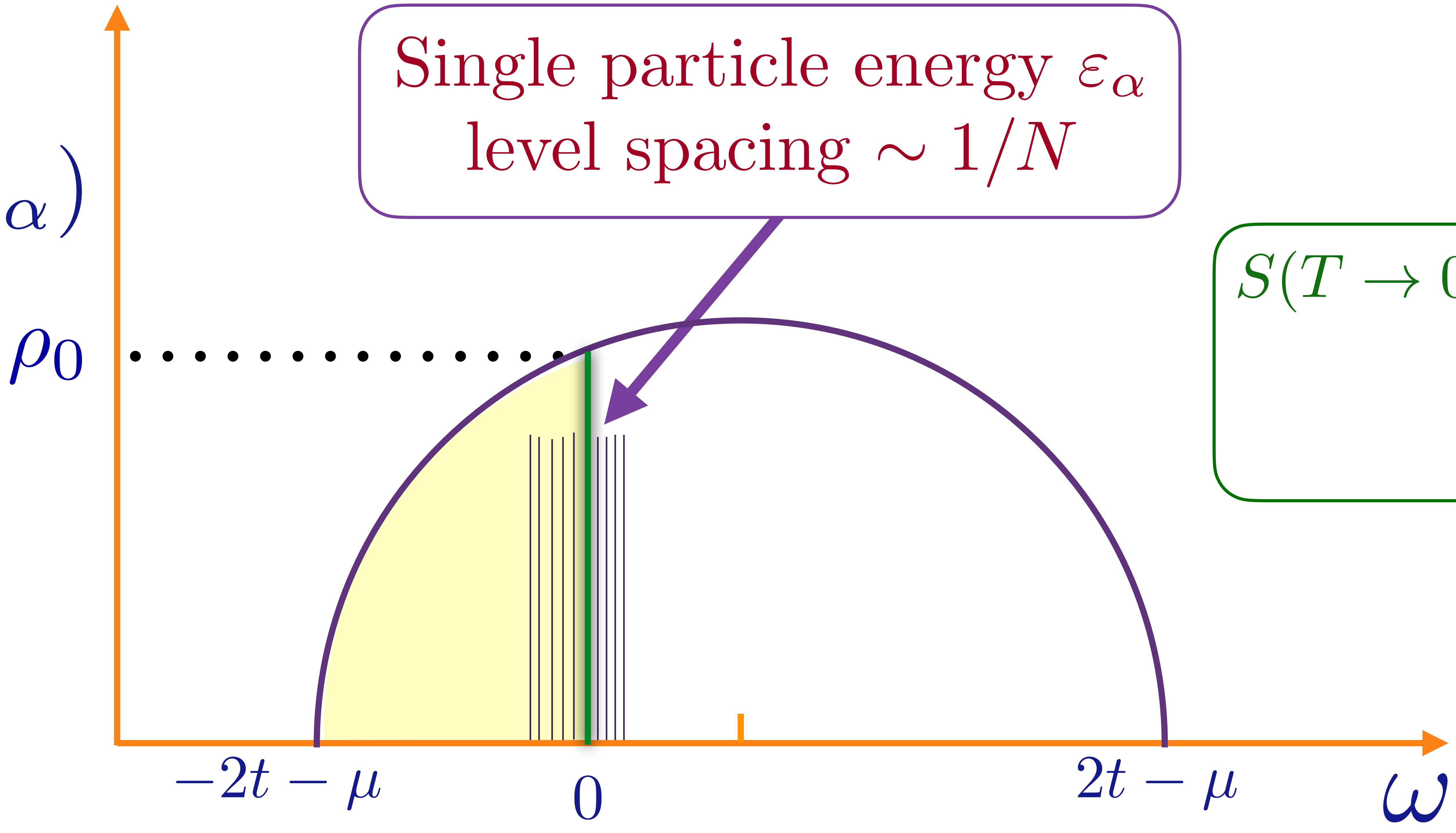
$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

$$\rho(\omega) = \frac{1}{N} \sum_\alpha \delta(\omega - \varepsilon_\alpha)$$



Single particle energy ε_α
level spacing $\sim 1/N$

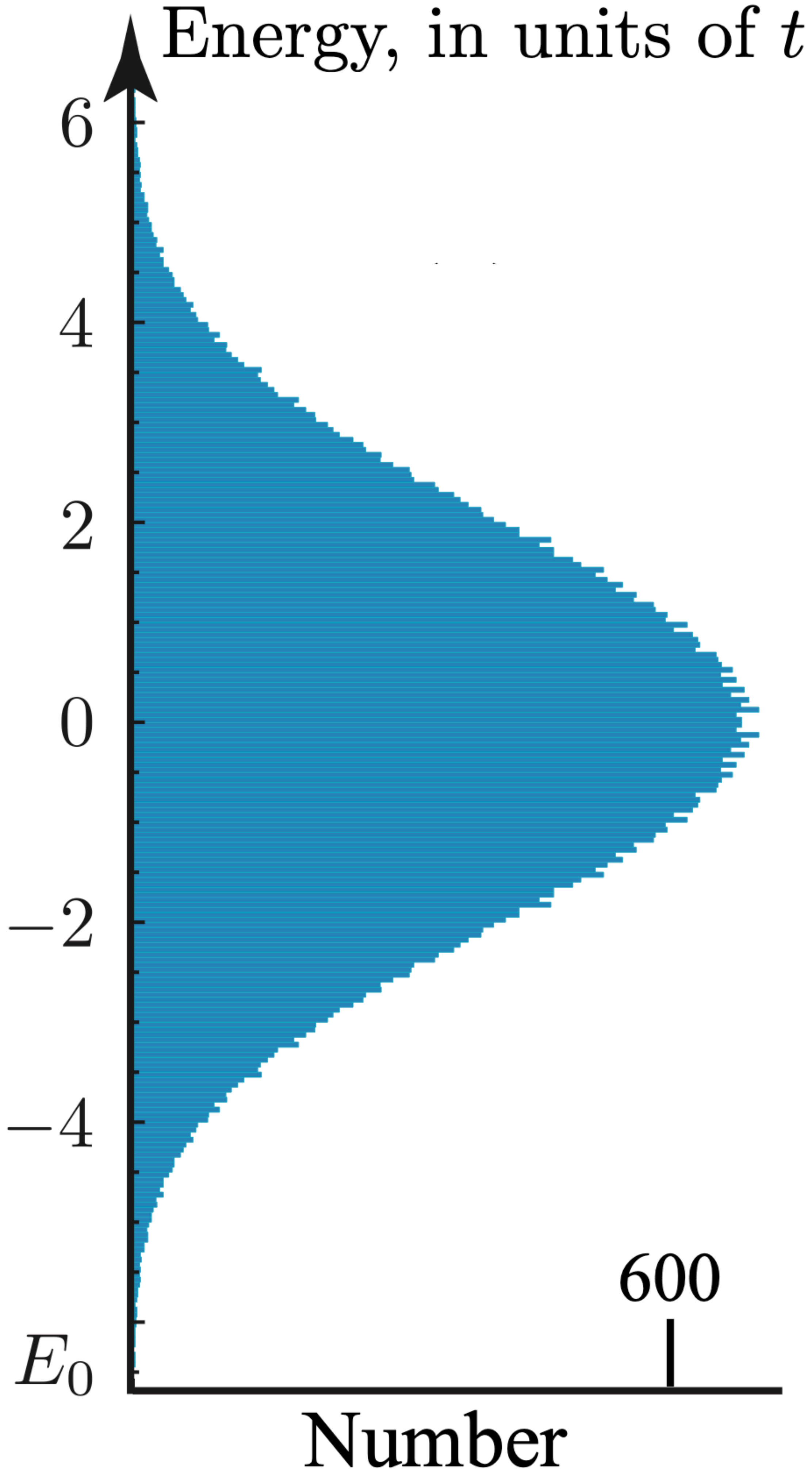
$$S(T \to 0) = N\gamma T$$

$$\gamma = \frac{\pi^2}{3} \rho_0$$

Random matrix model

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

For random matrix model: $E_0 + E_i = \sum_\alpha n_\alpha \varepsilon_\alpha$ $n_\alpha = 0, 1,$ occupation number



Many-body density of states

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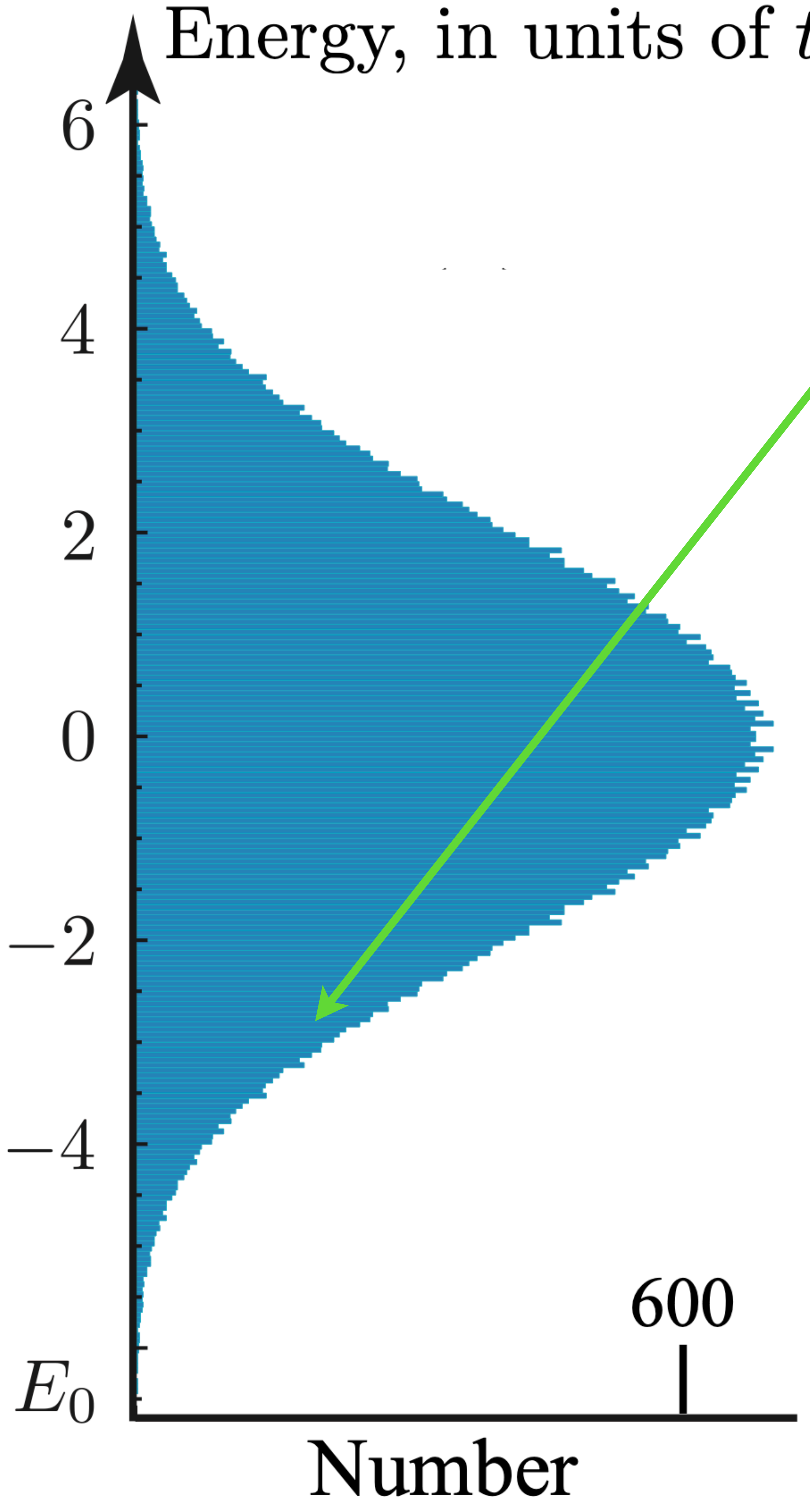

$$D(E) \sim e^{S(E)}$$

$$= e^{\sqrt{2N\gamma E}}$$

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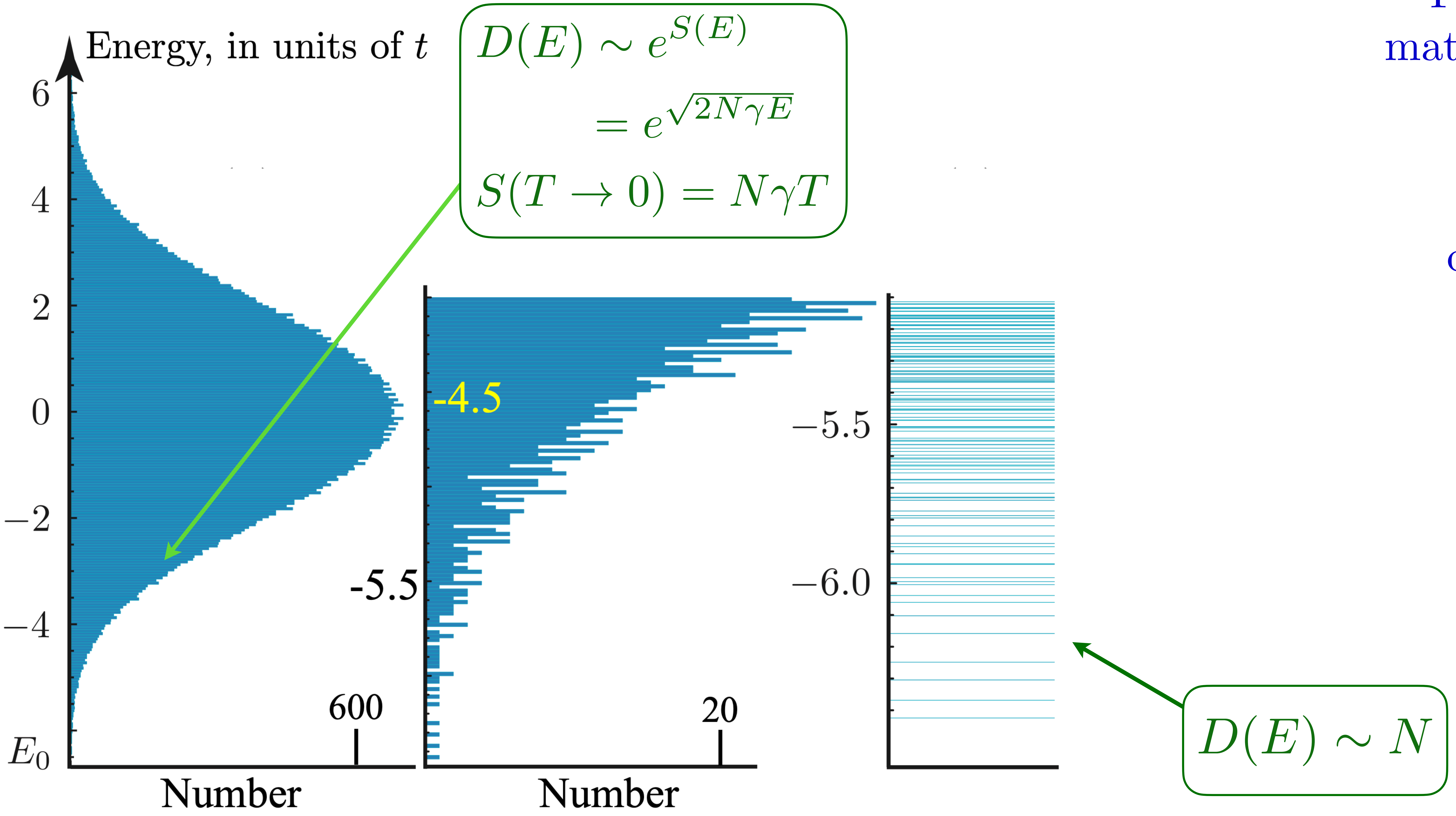

$D(E) \sim e^{S(E)}$

$= e^{\sqrt{2N\gamma E}}$

$S(T \to 0) = N\gamma T$

$D(E) \sim N$

Many-body density of states

The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
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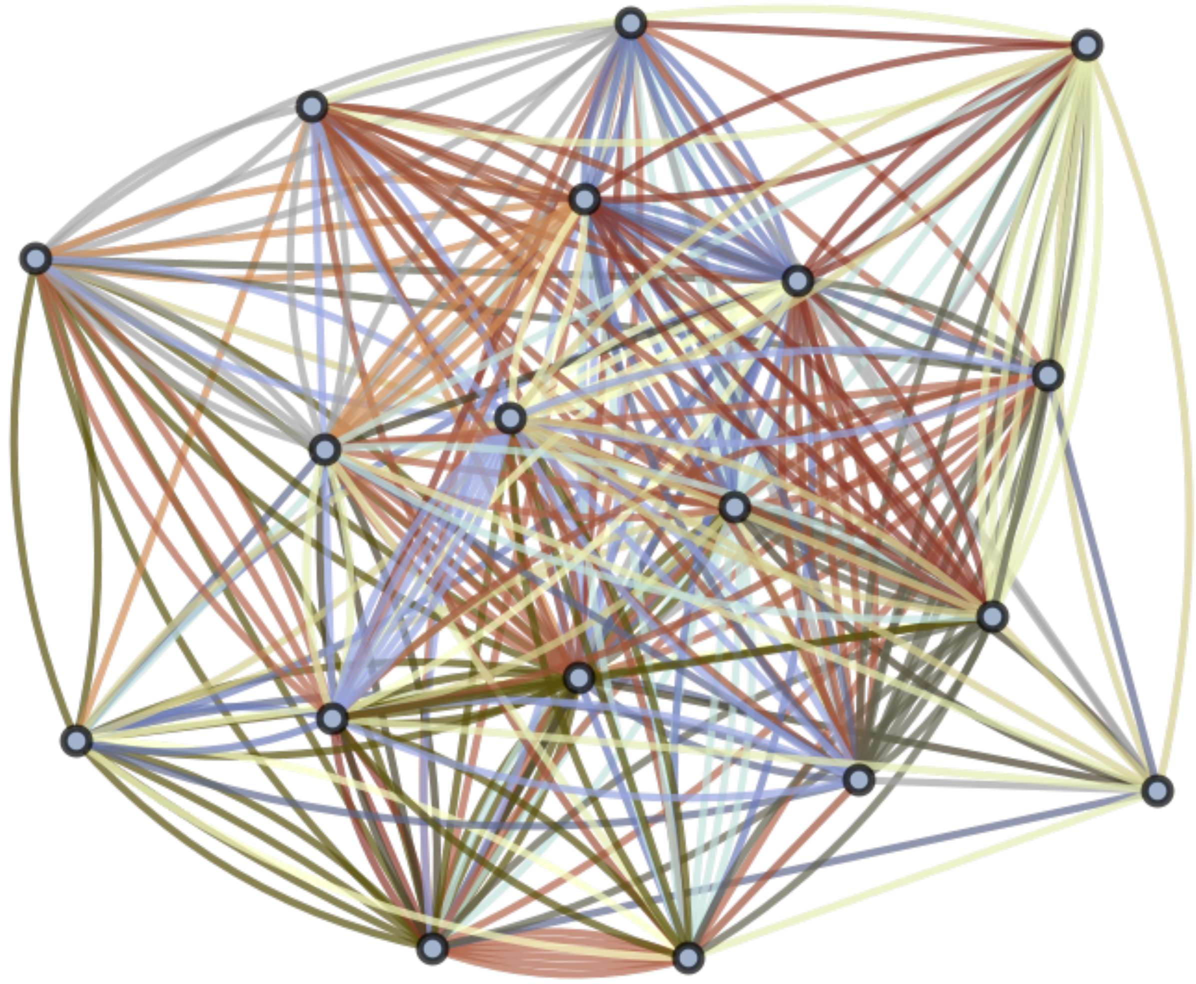

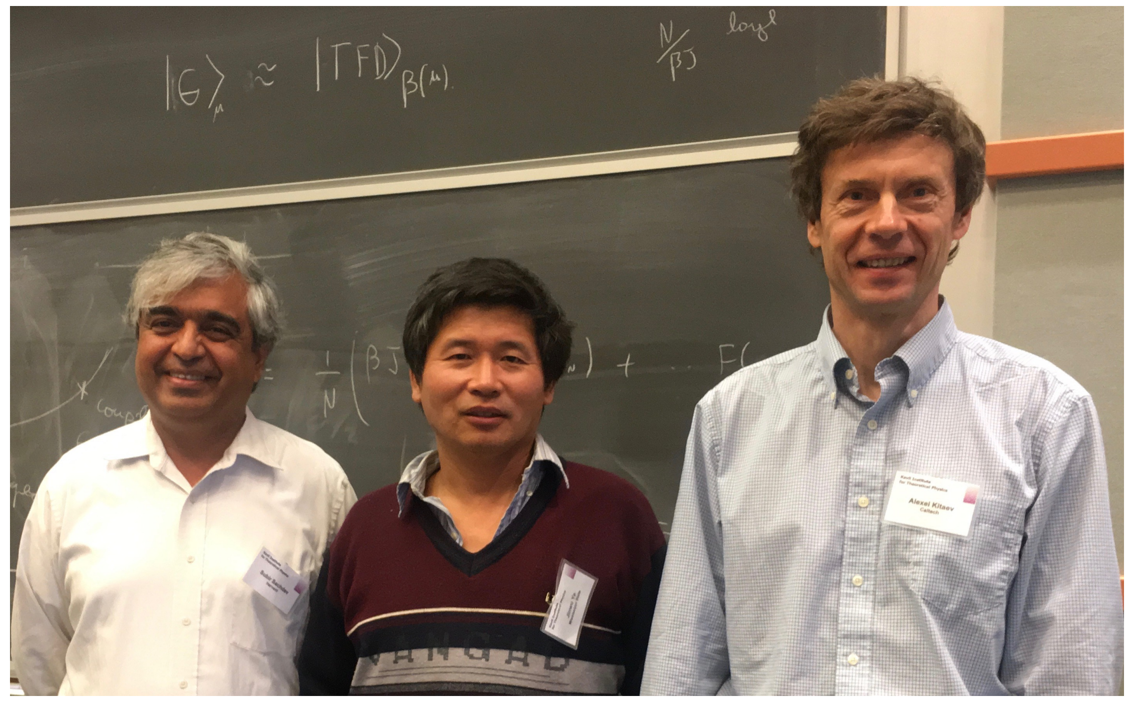

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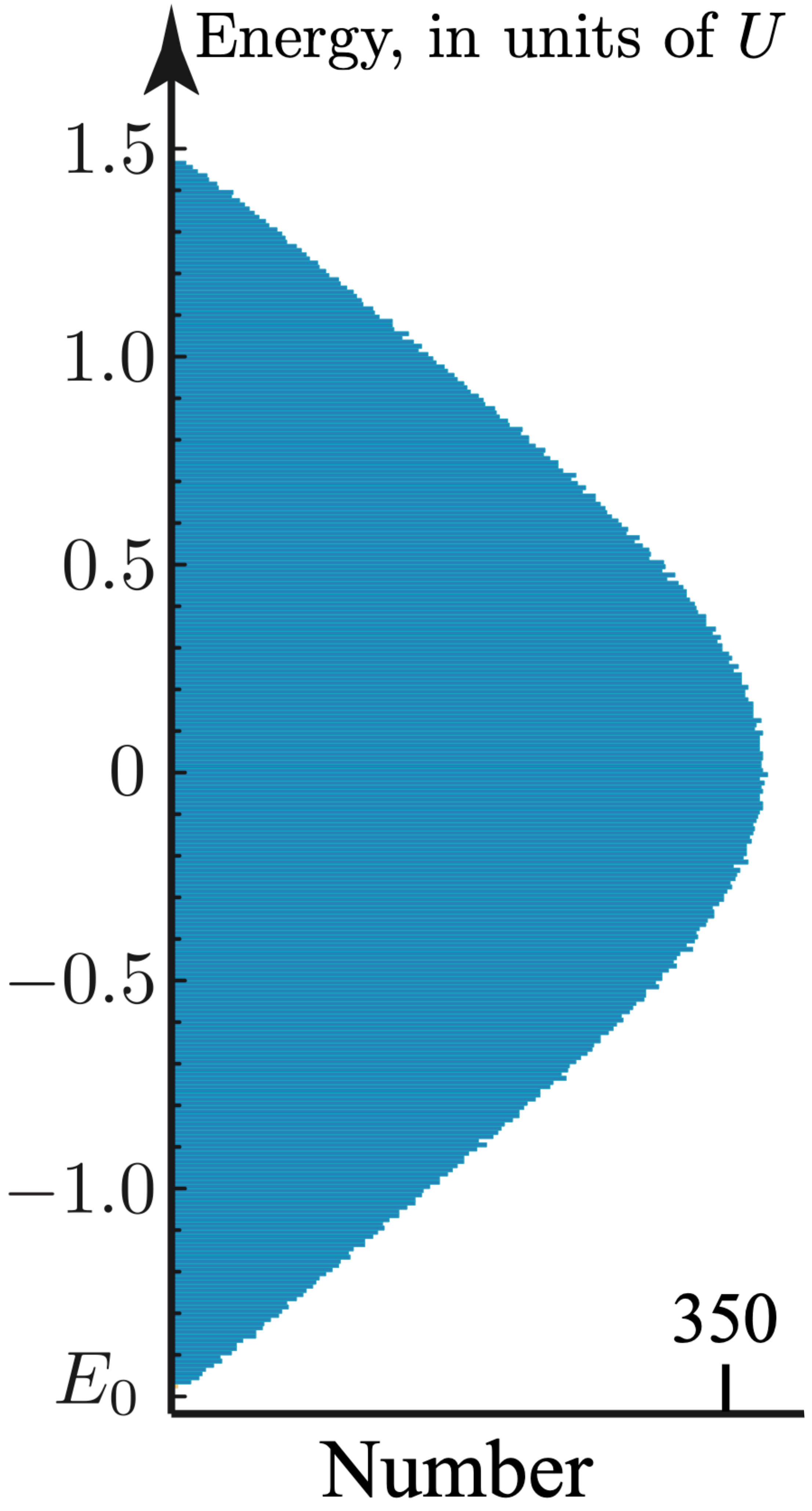

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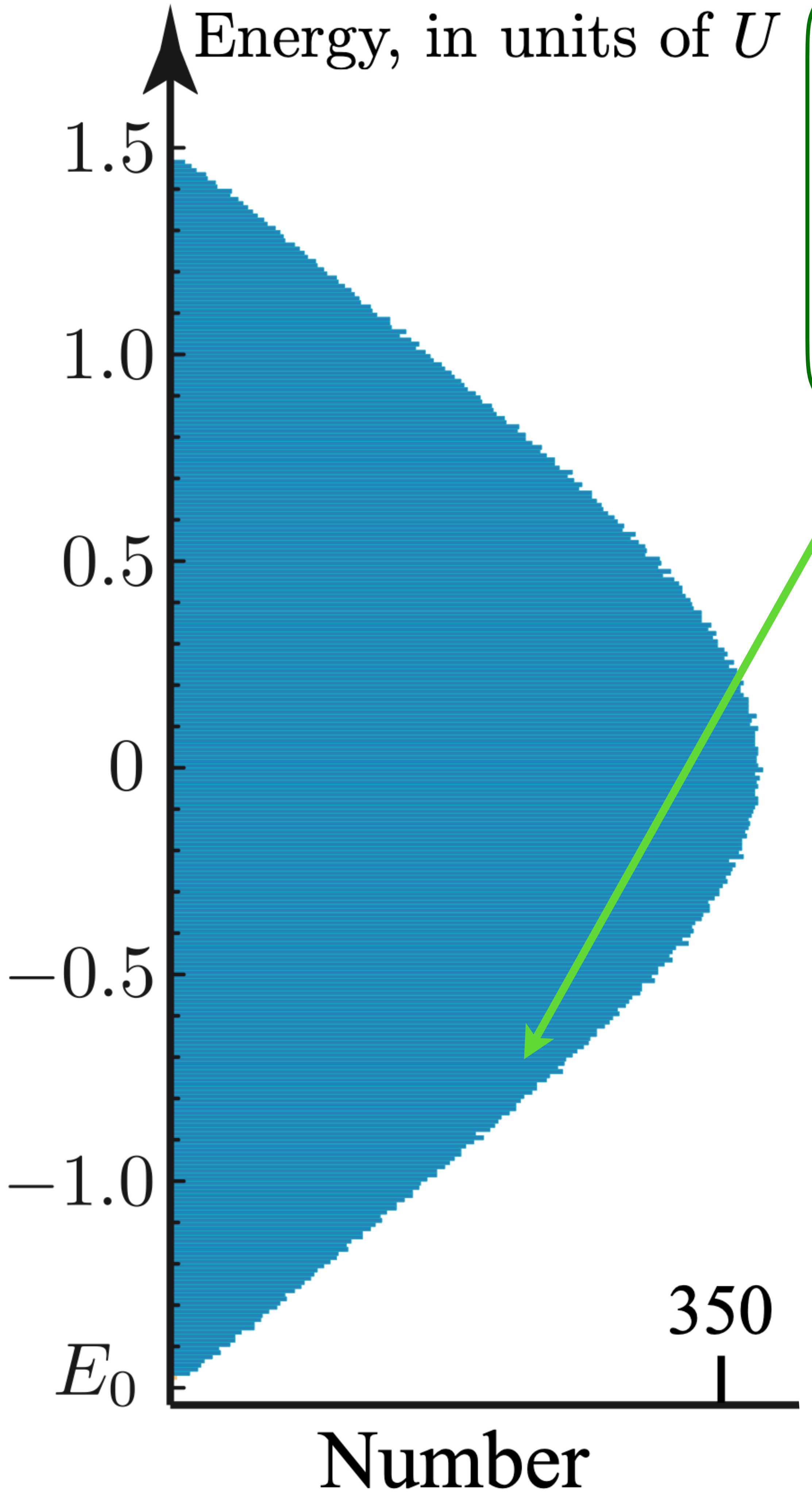

$$D(E) \sim e^{S(E)}$$

$$= e^{N s_0 + \sqrt{2N\gamma E}}$$

$$S(T \to 0) = N(s_0 + \gamma T)$$

$$s_0 = 0.464848\ldots$$

A. Georges, O. Parcollet, and
S. Sachdev,
PRB **63**, 134406 (2001)

Many-body density of states

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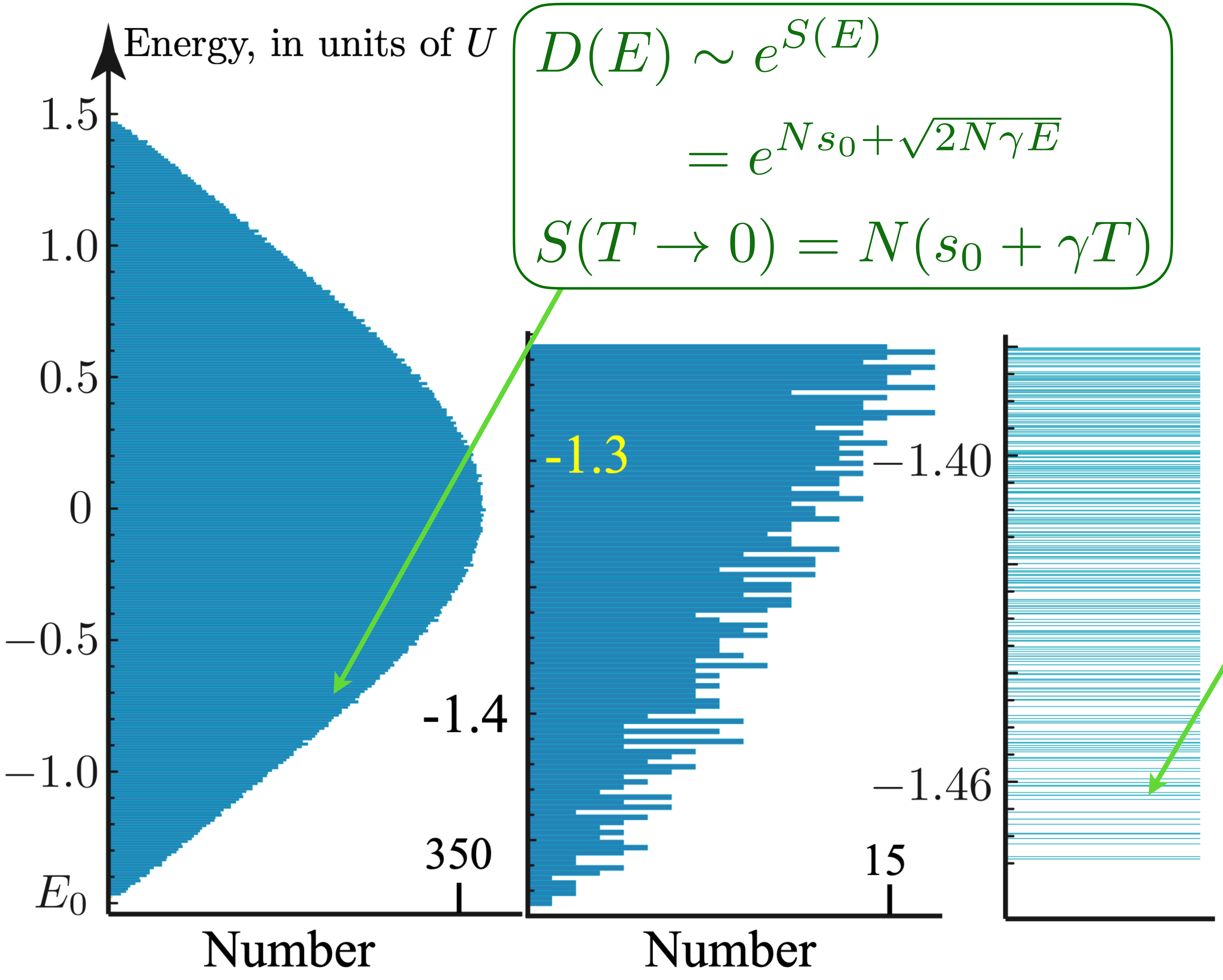

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No quasiparticle decomposition: wavefunctions change chaotically from one state to the next.

$$s_0 = 0.464848\ldots$$

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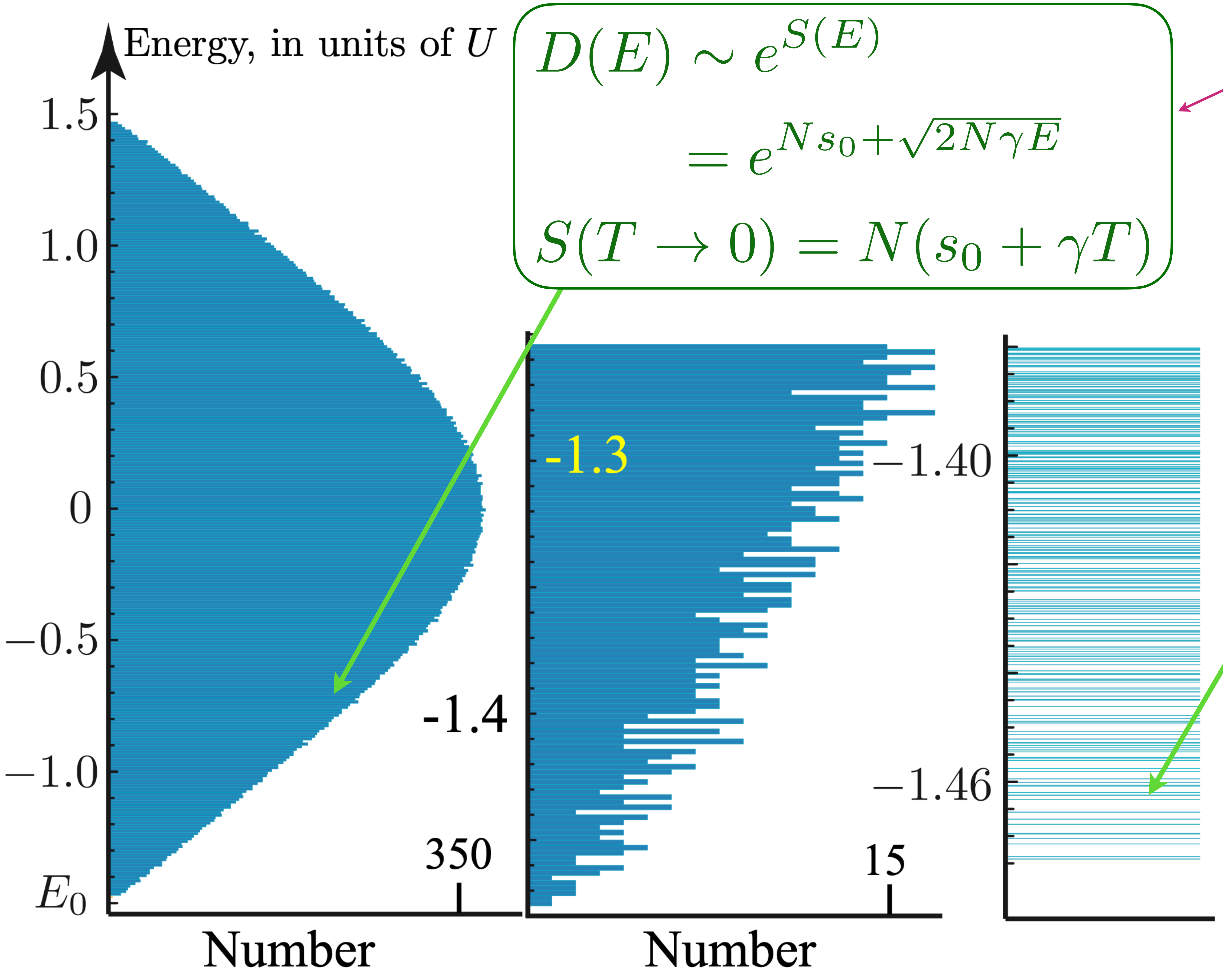

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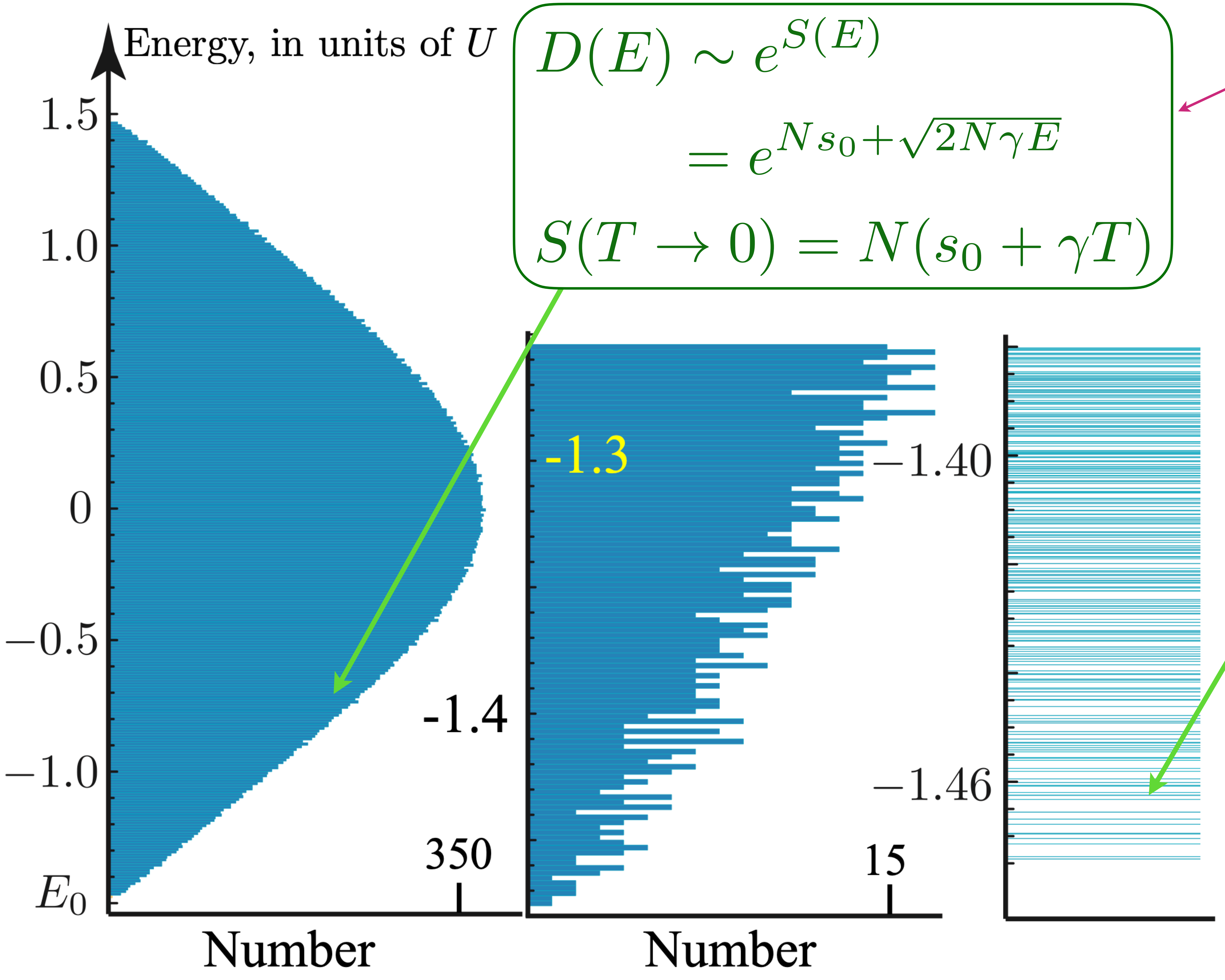

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$$e^{-F(T)/T} = \int_0^\infty dE D(E) e^{-E/T}$$
$$S(T) = -\partial F / \partial T$$

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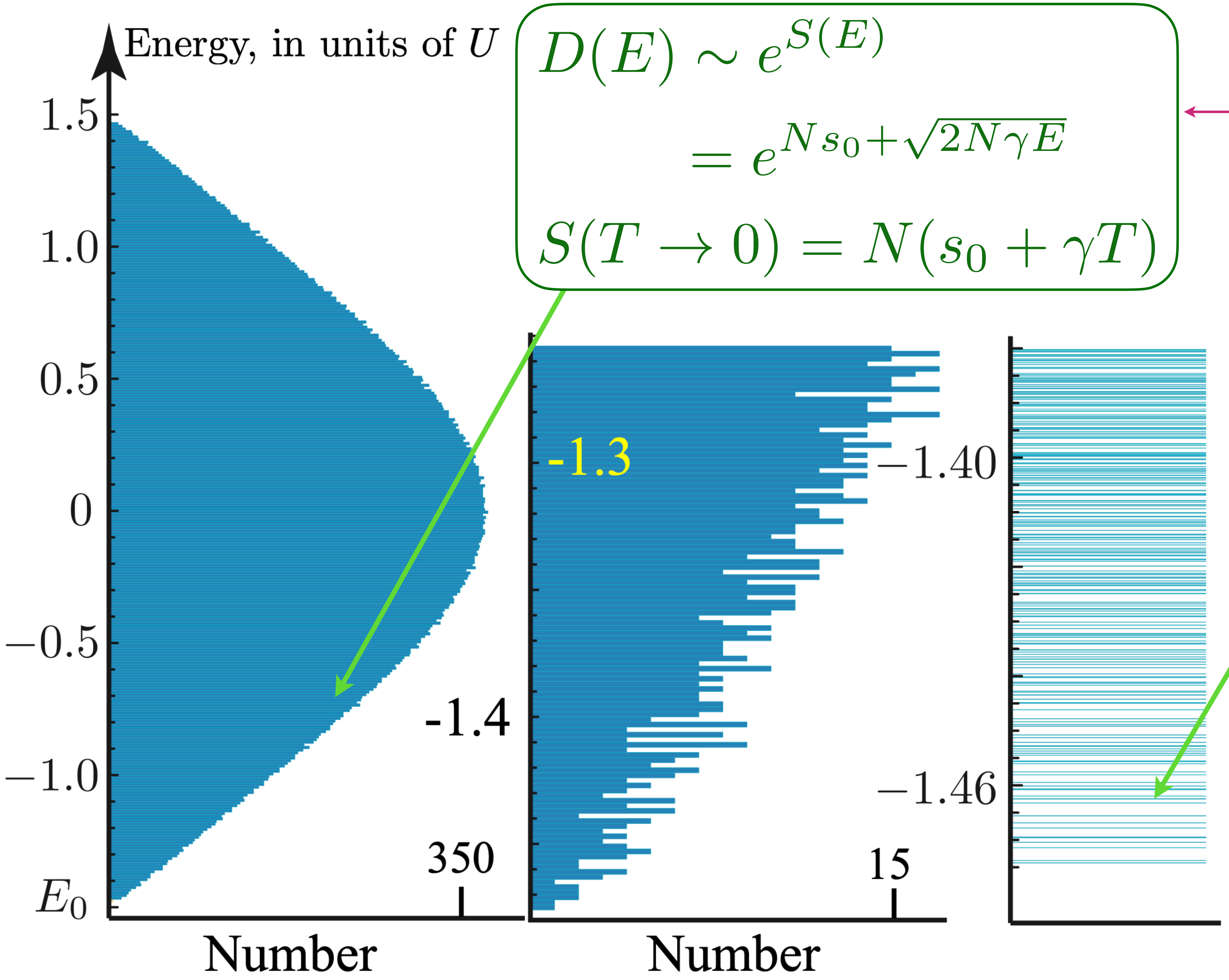

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Many-body density of states

The Sachdev-Ye-Kitaev (SYK) model

Universal Planckian time dynamics

- Green's function has Planckian time scaling $G(\omega, T) \sim \omega^{-1/2} F(\hbar\omega/k_B T)$.
- Leading (dangerously) irrelevant operator is a time reparameterization soft mode $\tau \to f(\tau)$.
- Time reparameterization mode leads to many-body quantum chaos in the out-of-time-order correlator (OTOC) with maximal Lyapunov exponent $\lambda_L = 2\pi k_B T/\hbar$.

Kitaev (2015) Maldacena, Shenker, Stanford (2015) Maldacena Stanford (2016)

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv: 2109.05037, review article

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Kitaev (2015) Maldacena, Shenker, Stanford (2015) Maldacena Stanford (2016)

- The T-dependence of the entropy also arises from the time reparameterization soft mode: $S = N(s_0 + \gamma T) - (3/2)\ln(U/T)$.

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv: 2109.05037, review article

1. Introduction to Planckian metals

2. Introduction to black holes

3. The SYK model

4. Progress on the theory of black holes

5. Progress on the theory of Planckian metals
    A. Random t-J model
    B. Fermi surface coupled to a critical boson

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu}, A_\mu]\right)$$

$$= \exp\Big(S_{BH}\Big) \times \Bigg(\quad ....????... \quad \Bigg)$$

Gibbons, Hawking (1977)
Chambin, Emparan, Johnson, Myers (1999)

$$S_{BH}(T \to 0, \mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1 + \frac{2(\pi A_0)^{1/2}T}{\hbar c}\right)$$

A_0 is the area of the charged black hole horizon at $T = 0$.
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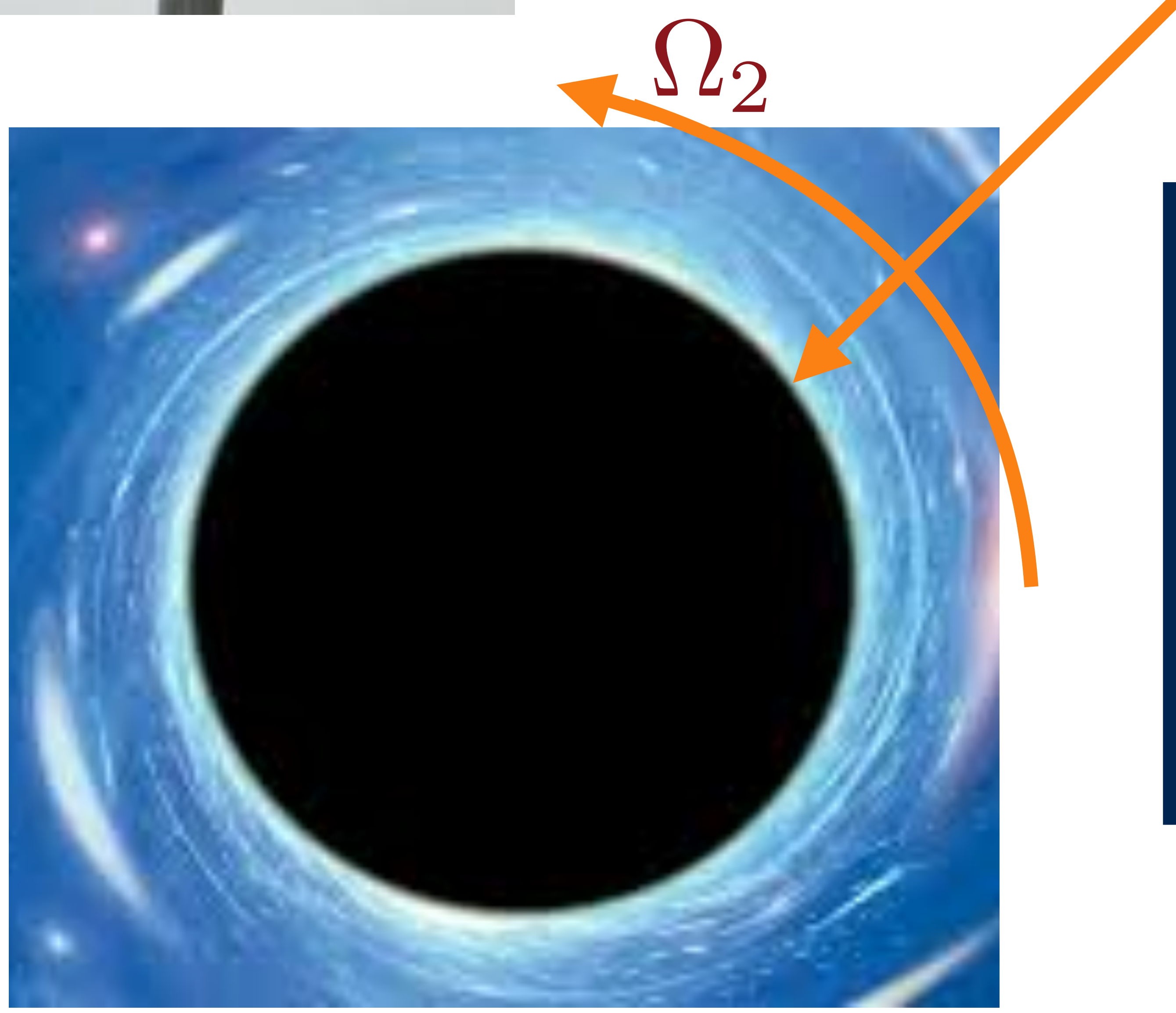
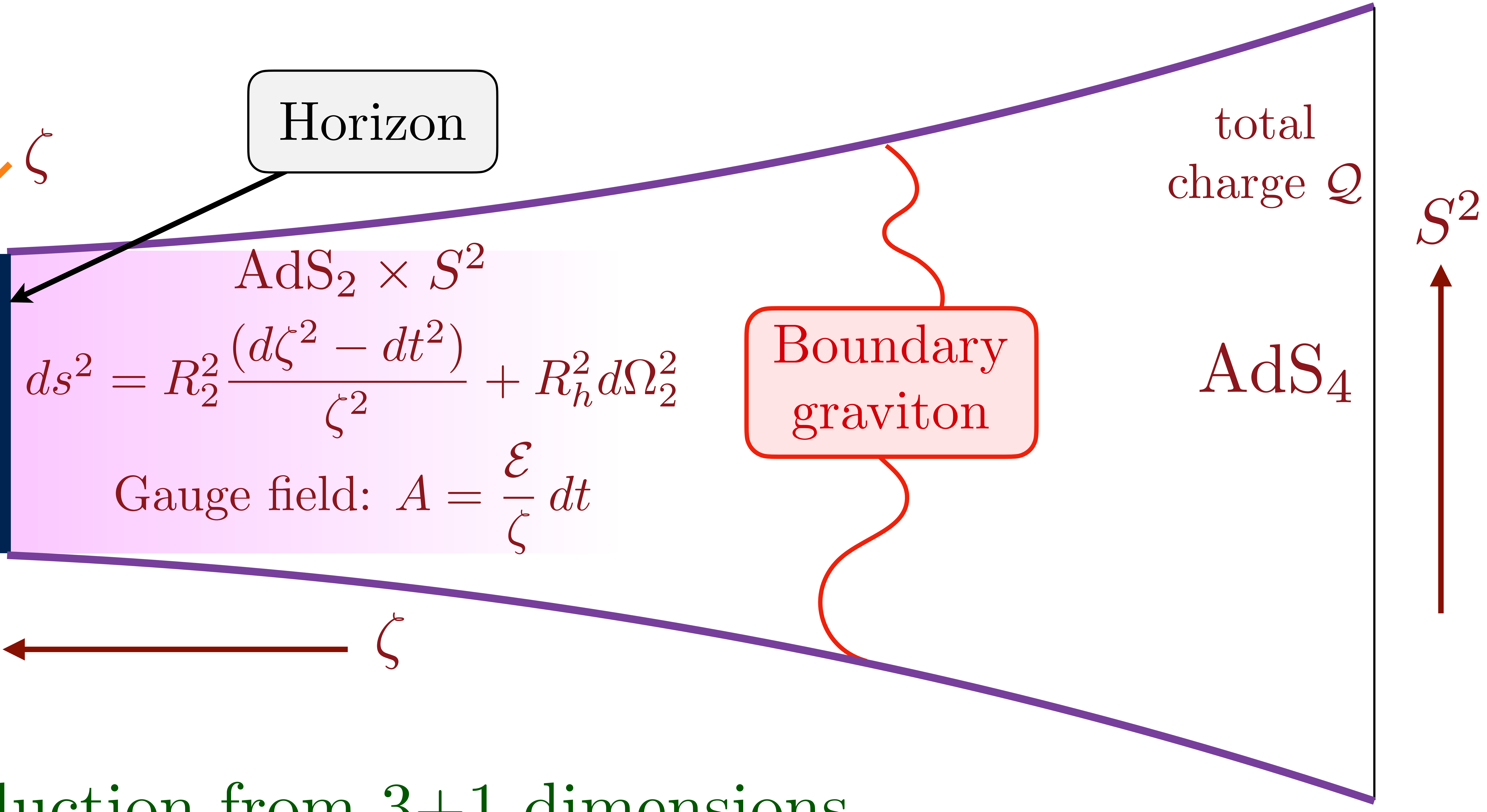
Note the similarity to the large *N* entropy of the SYK model !
(along with other similarities) Sachdev PRL 2010

Reissner-Nordstrom black hole of Einstein-Maxwell theory





$\text{AdS}_2 \times S^2$

$$ds^2 = R_2^2 \frac{(d\zeta^2 - dt^2)}{\zeta^2} + R_h^2 d\Omega_2^2$$

Gauge field: $A = \dfrac{\mathcal{E}}{\zeta}\, dt$

Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

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$$S_{BH}(T\to 0,\mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0c^3}{4G\hbar}\left(1+\frac{2(\pi A_0)^{1/2}T}{\hbar c}\right)$$

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Maldacena, Stanford, Yang (2016)

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$$= \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau) \exp\left(-\frac{1}{\hbar}\mathcal{S}_{\rm SYK}[\text{time reparameterizations } f(\tau), \text{ phase rotations } \phi(\tau)]\right)$$

$$S_{BH}(T \to 0, \mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1 + \frac{2(\pi A_0)^{1/2}T}{\hbar c}\right)$$

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Sachdev (2010); Kitaev (2015); Sachdev (2015); Bagrets, Altland, Kamenev (2016); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Sachdev (2019); Iliesiu, Turaci (2020)

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A_0 is the area of the charged black hole horizon at $T=0$, $\mathcal{Q}$ is the black hole charge. The $\ln T$ term is the contribution of the boundary graviton.

(There is also a $-(241/45)\ln(A_0/G)$ correction at $T=0$ A. Sen 2011)

Sachdev (2010); Kitaev (2015); Sachdev (2015); Bagrets, Altland, Kamenev (2016); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Sachdev (2019); Iliesiu, Turaci (2020)

Many-body density of states

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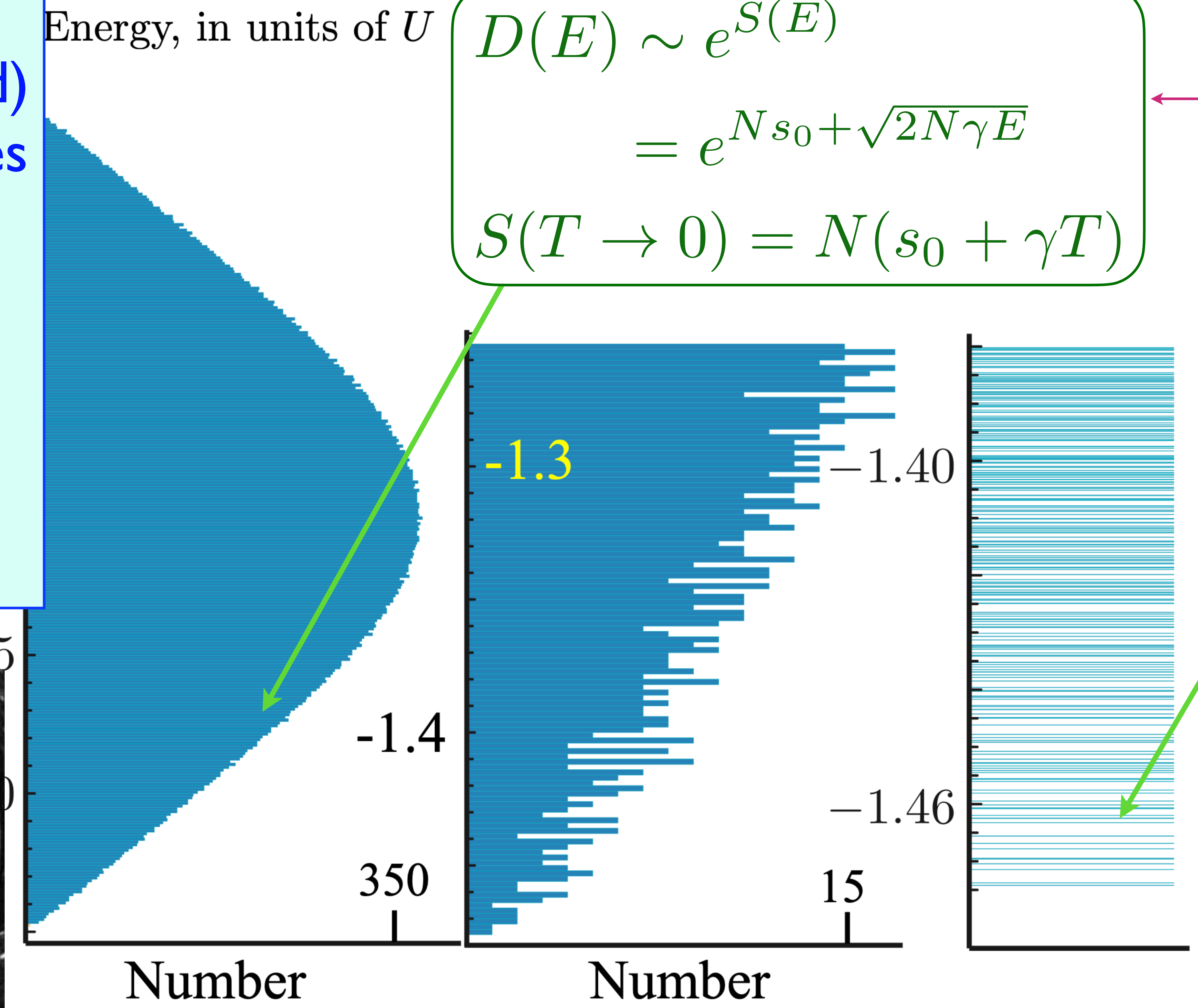
Same entropy and (coarse-grained) density of states in a model of interacting (fermionic) qubits with a discrete spectrum!



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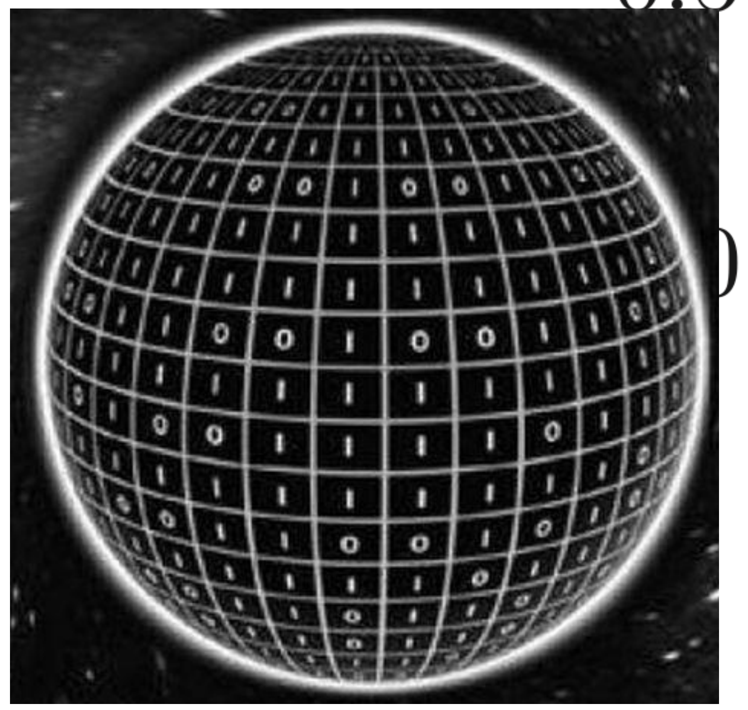
A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)



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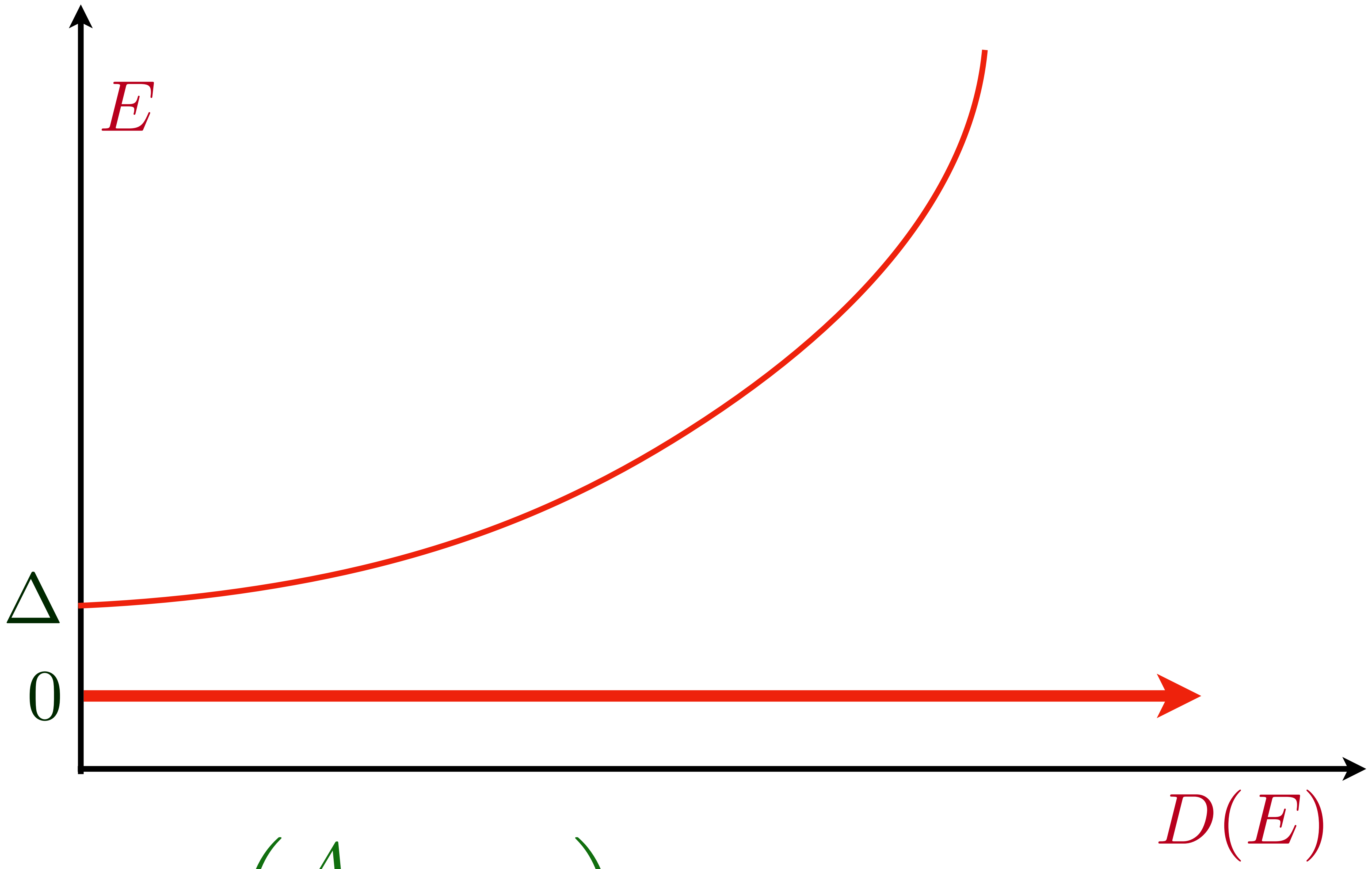

$$D(E) \sim \exp\left(\frac{A_0}{4G} + \ldots\right)\delta(E) + f_{\text{reg}}(E - \Delta), \quad \Delta \sim R_h^{-1}$$

Supersymmetric black holes and SYK models

Fu, Gaiotto, Maldacena, Sachdev (2017); Stanford, Witten (2017); Heydeman, Iliesiu, Turiaci, Zhao (2020)

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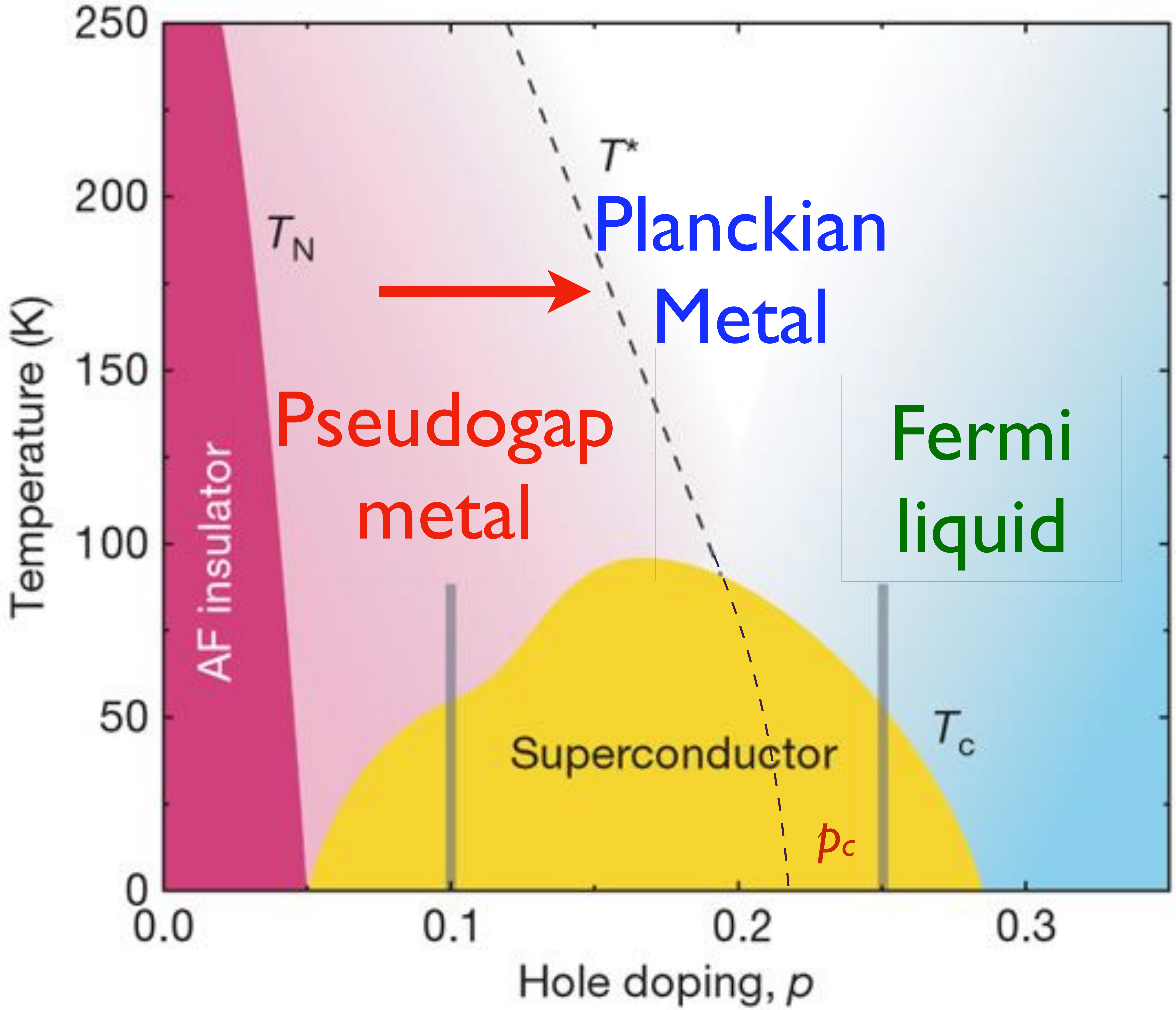
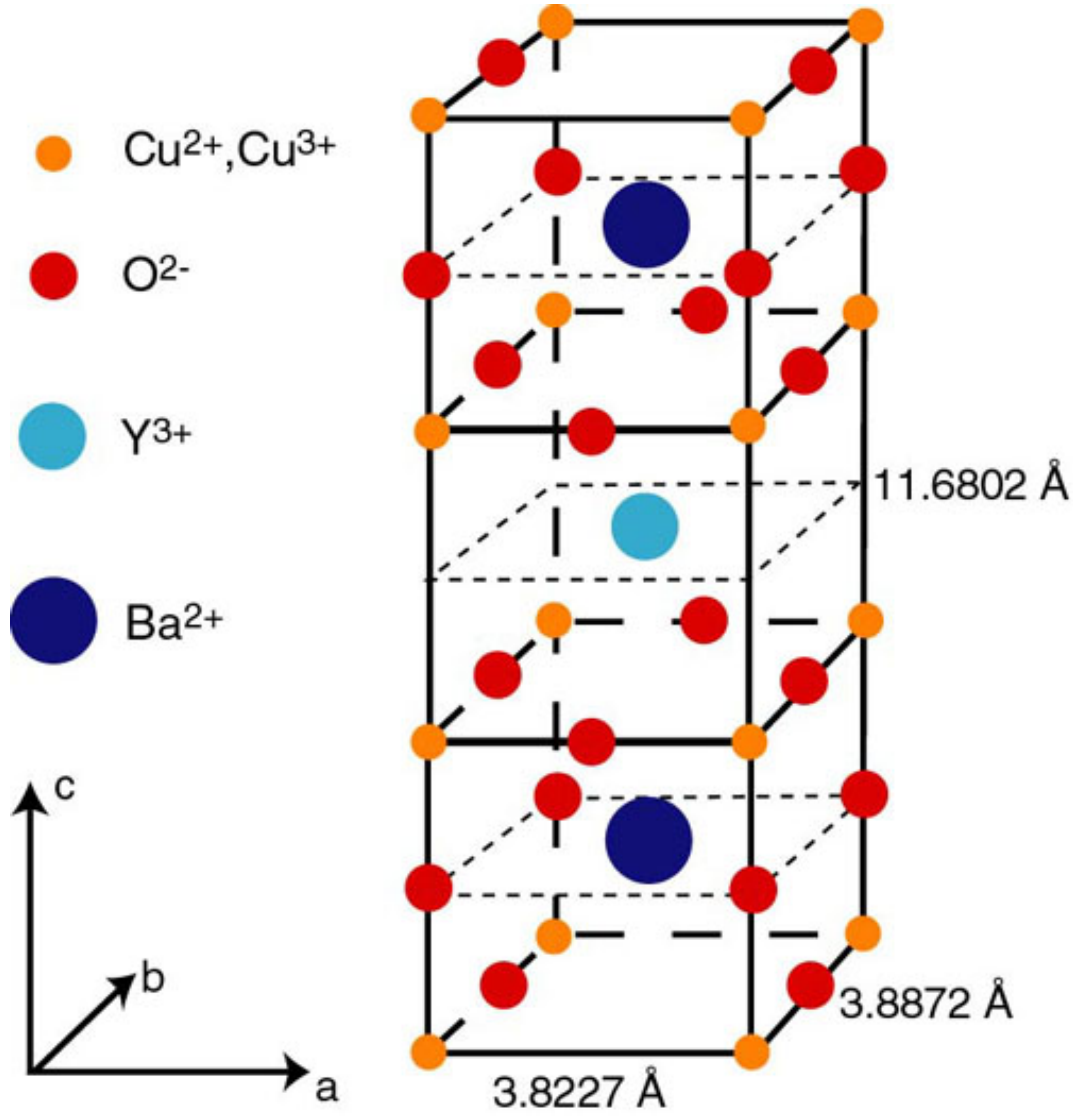


Planckian Metal

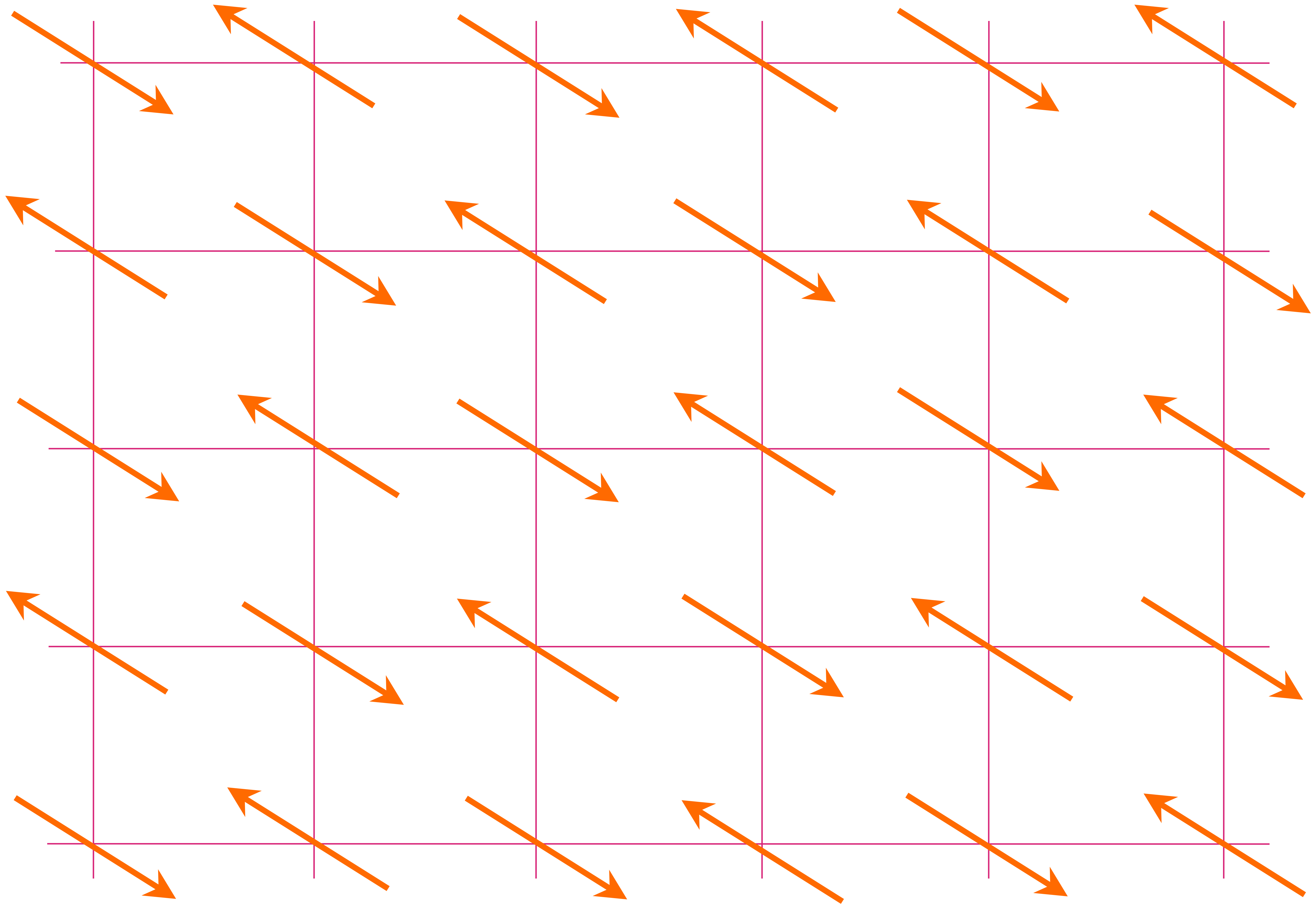
Pseudogap metal

Fermi liquid

p_c



Insulating antiferromagnet



$p=0$

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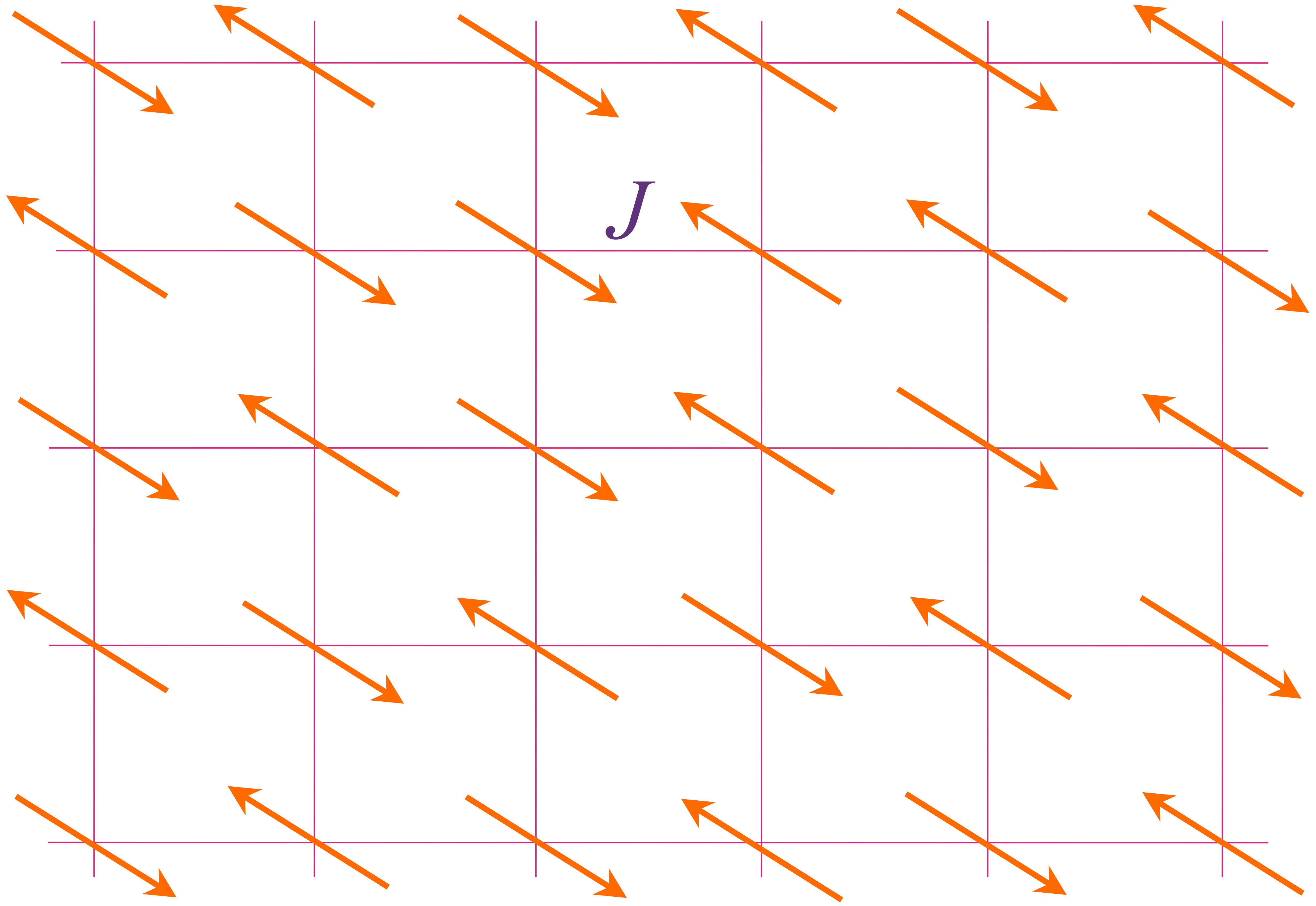

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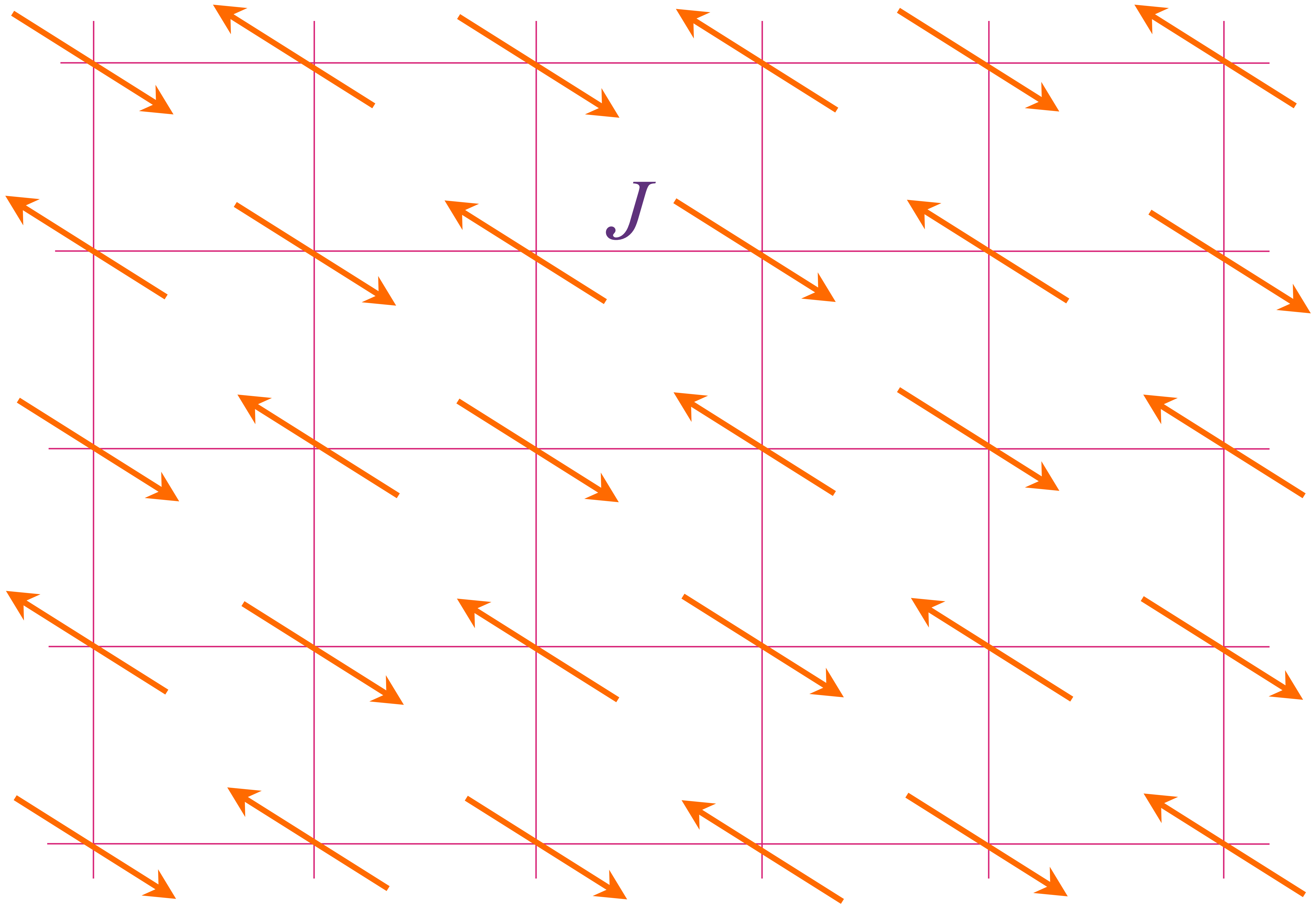

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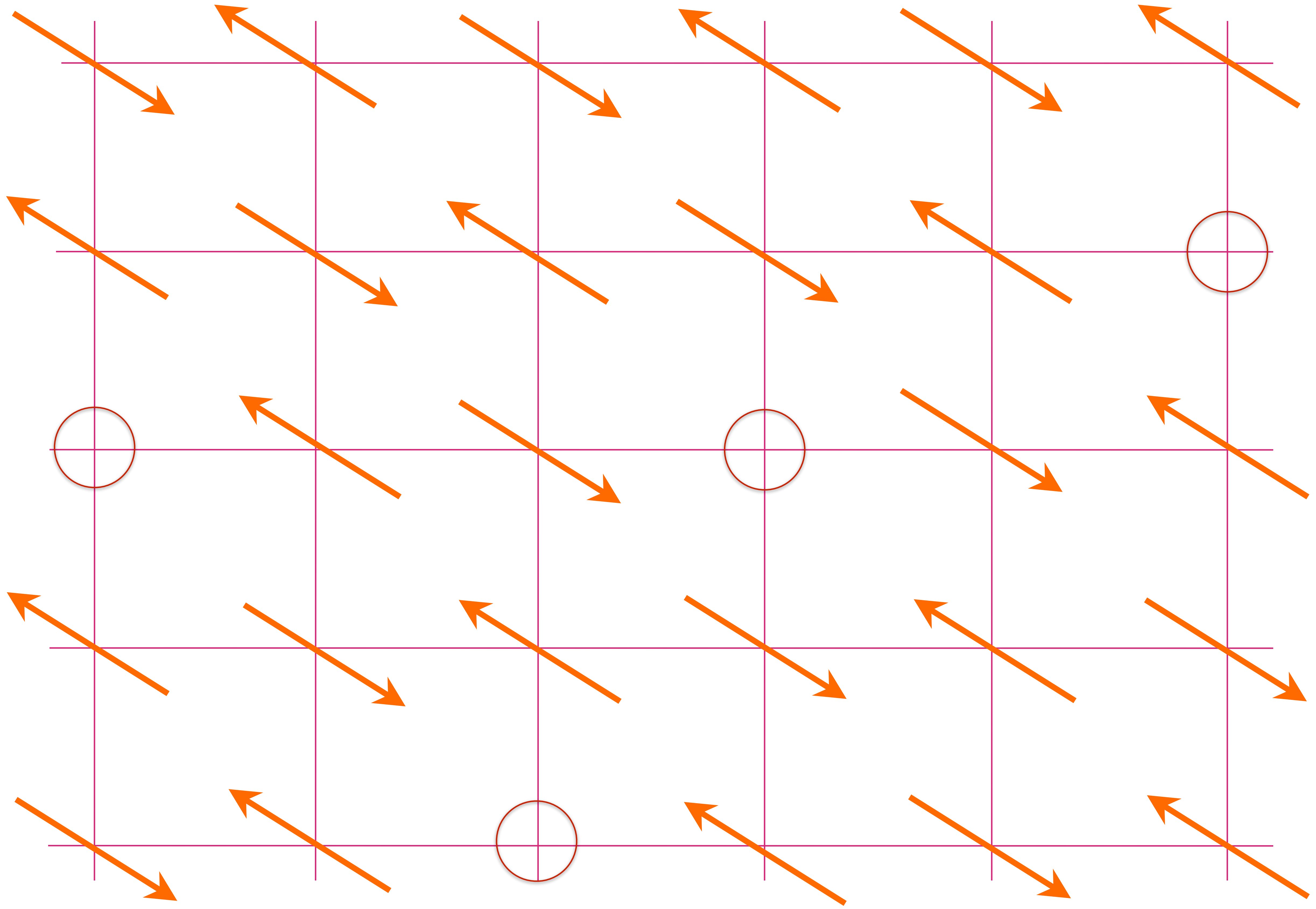
Antiferromagnet doped with hole density p



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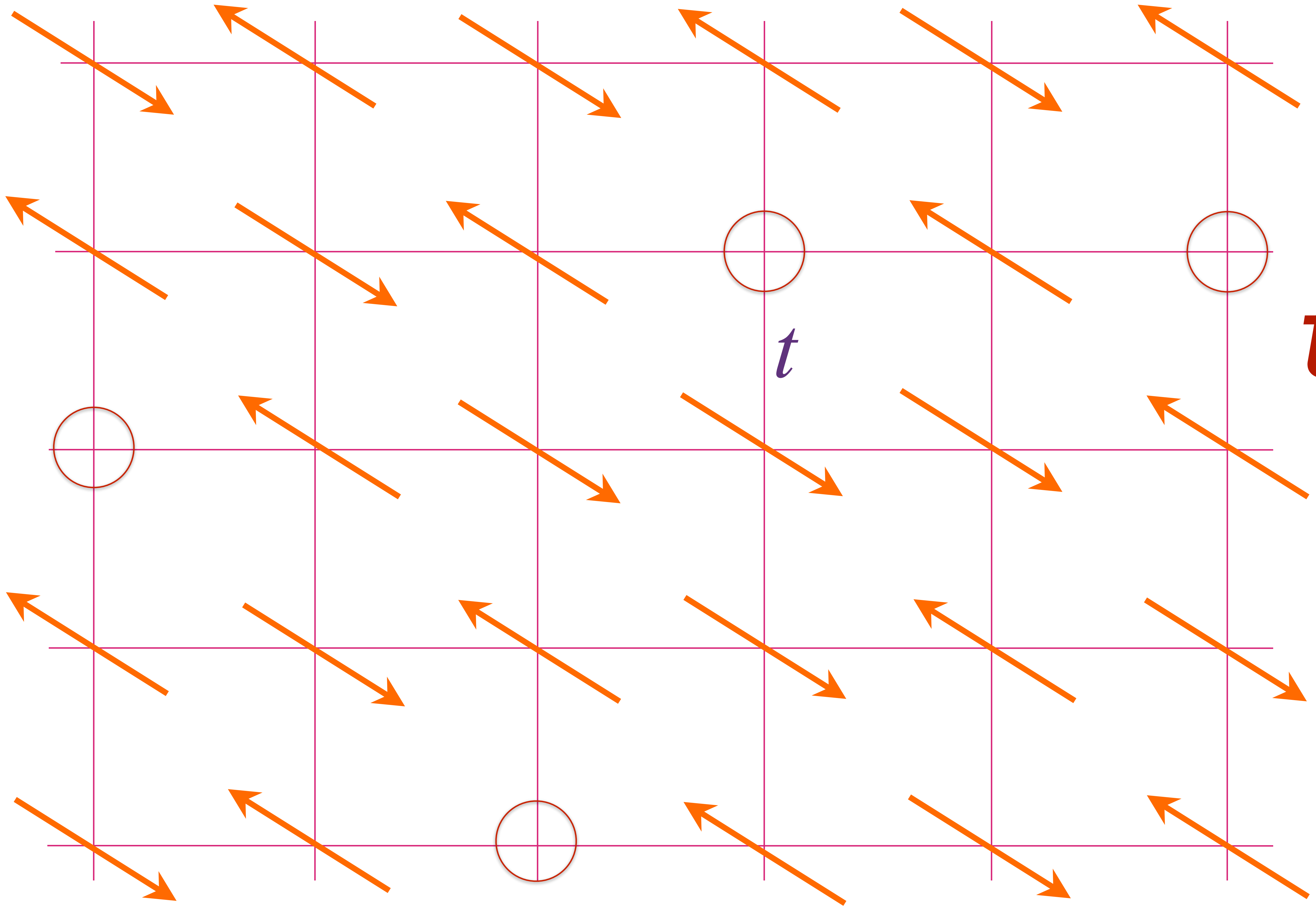
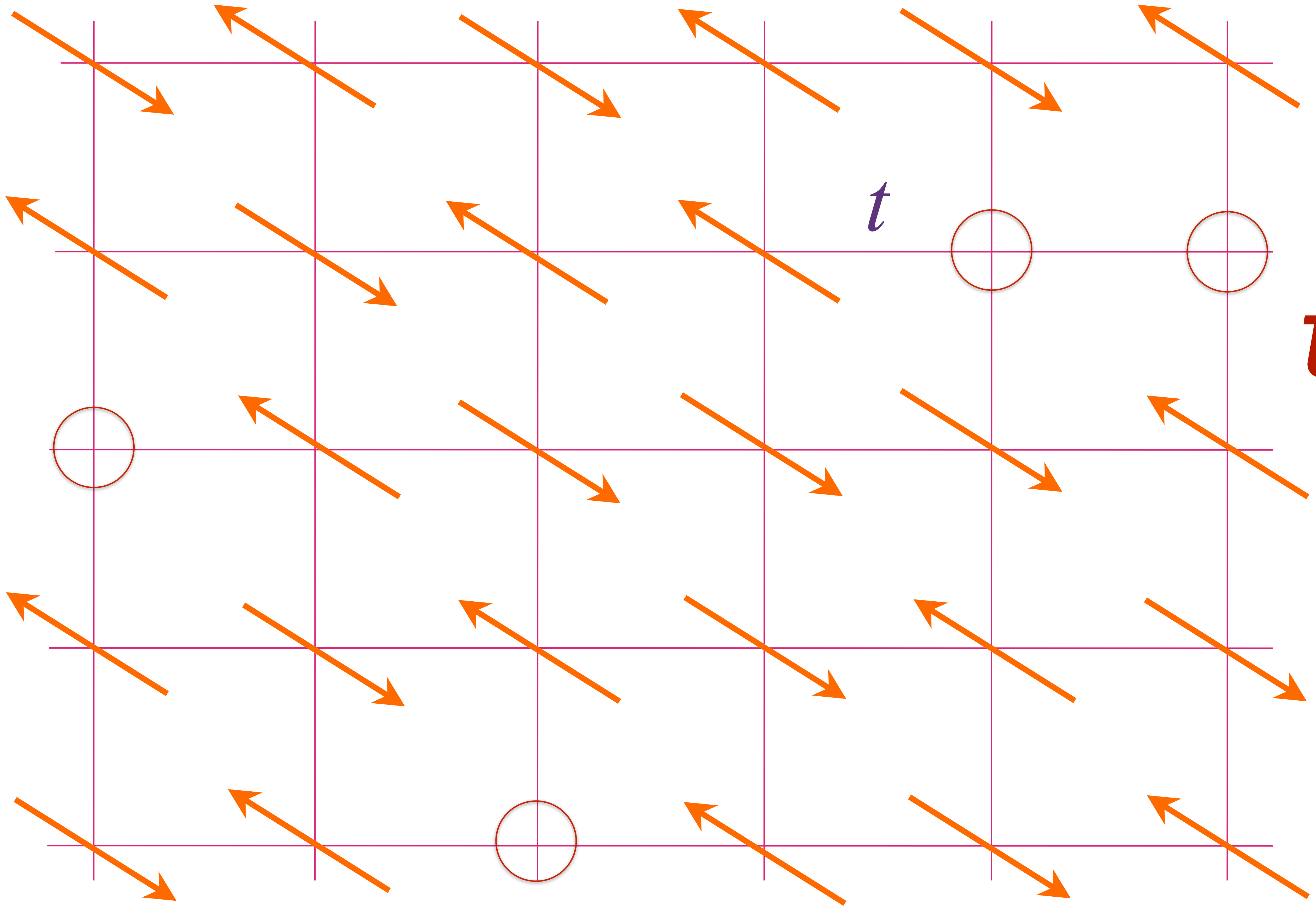


t-*J* model

Antiferromagnet doped with hole density p



***t*-*J* model**

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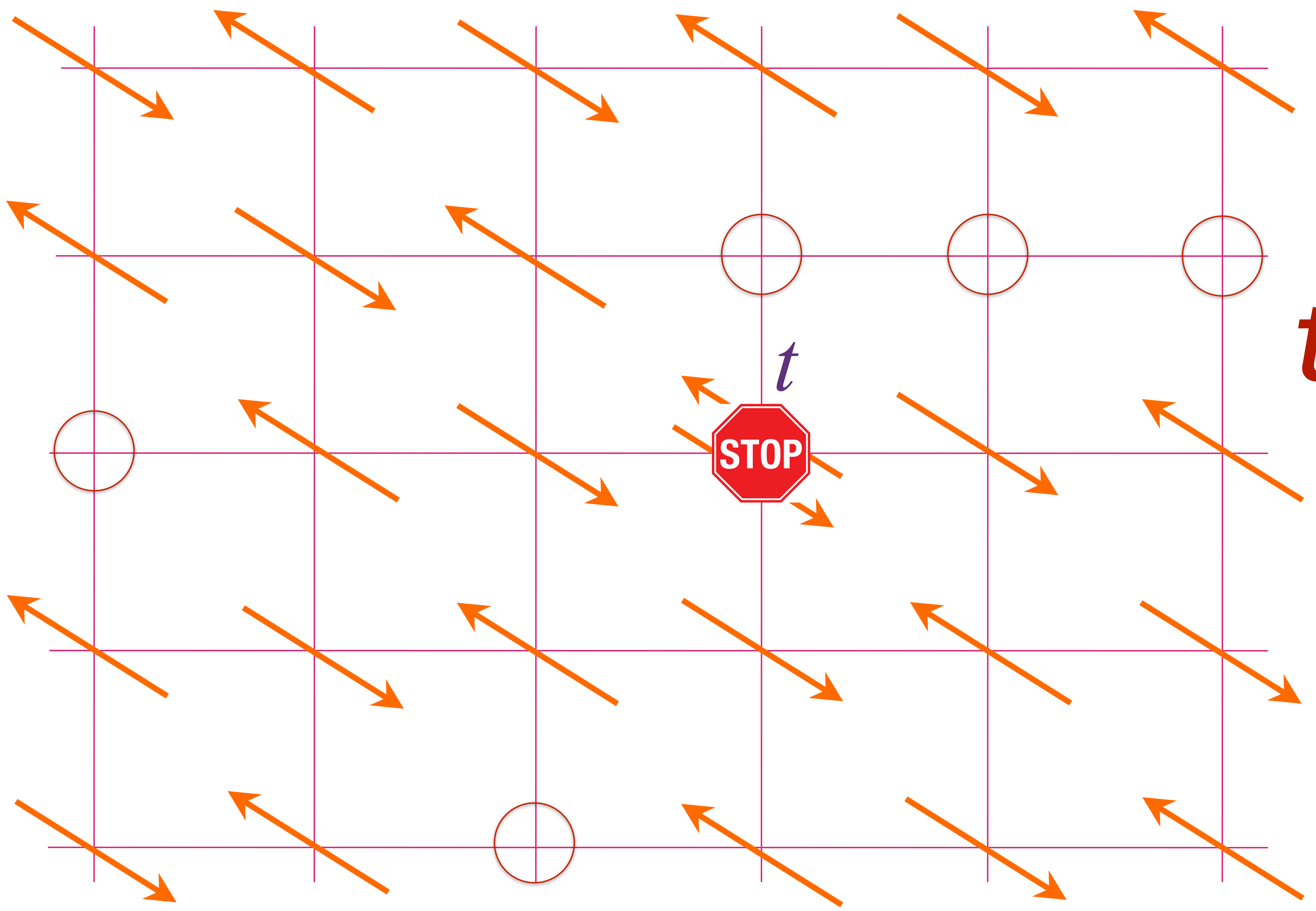

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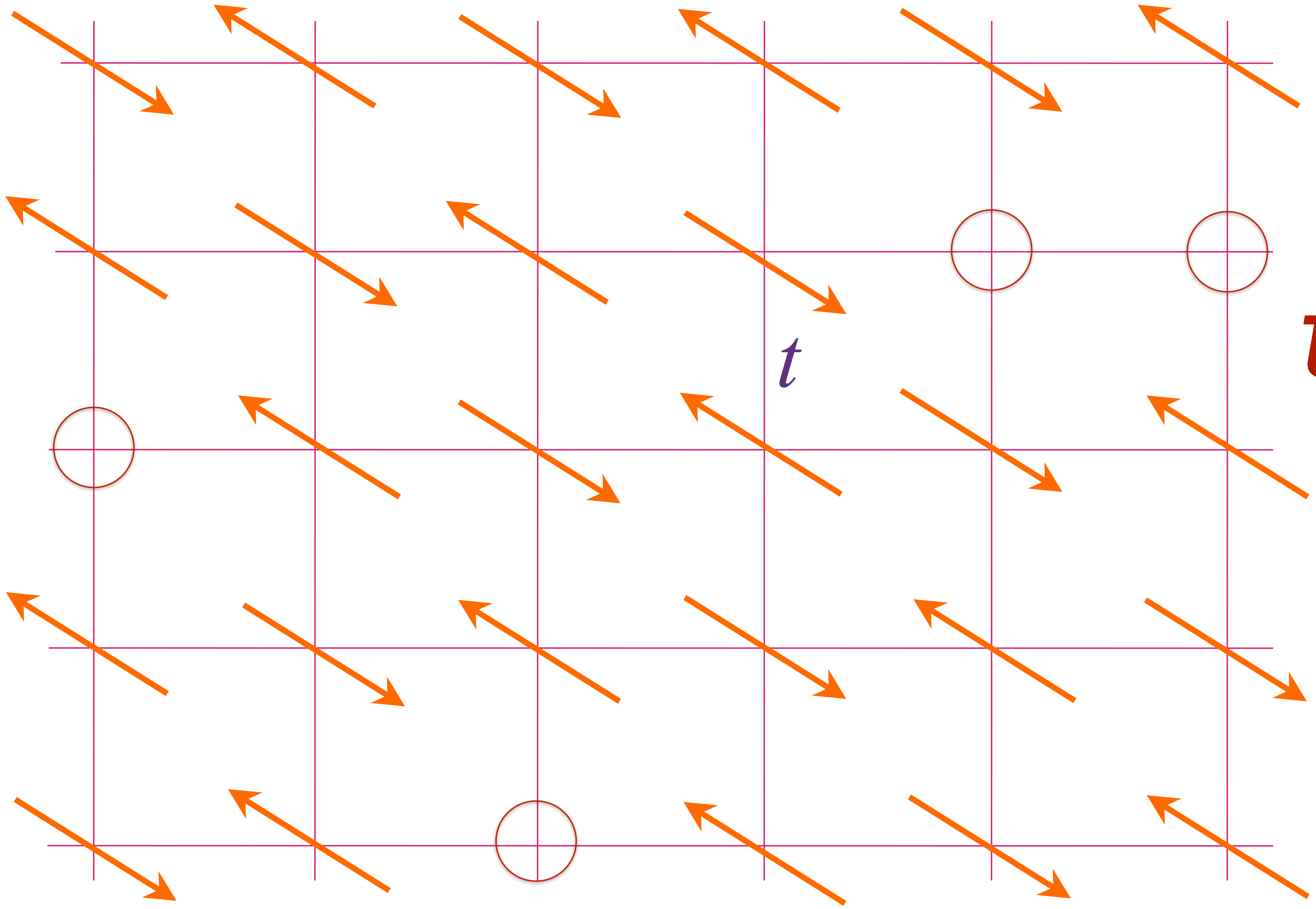

t-*J* model

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv: 2109.05037, review article

Random *t-J* model doped with hole density p

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, \mathcal{P}_d c_{i\alpha}^\dagger c_{j\alpha} \mathcal{P}_d + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma} c_{i\alpha}$$

$\mathcal{P}_d$ projects out doubly-occupied sites.

J_{ij} random, $\overline{J_{ij}} = 0$, $\overline{J_{ij}^2} = J^2$

$J \quad \Rightarrow \quad$ two-particle interaction, similar to that in SYK

$t \quad \Rightarrow \quad$ one-particle hopping, can be regular or random

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Parisi solved the model with this term only with $\vec{S}_i$ replaced by classical Ising spins $\sigma_i = \pm 1$

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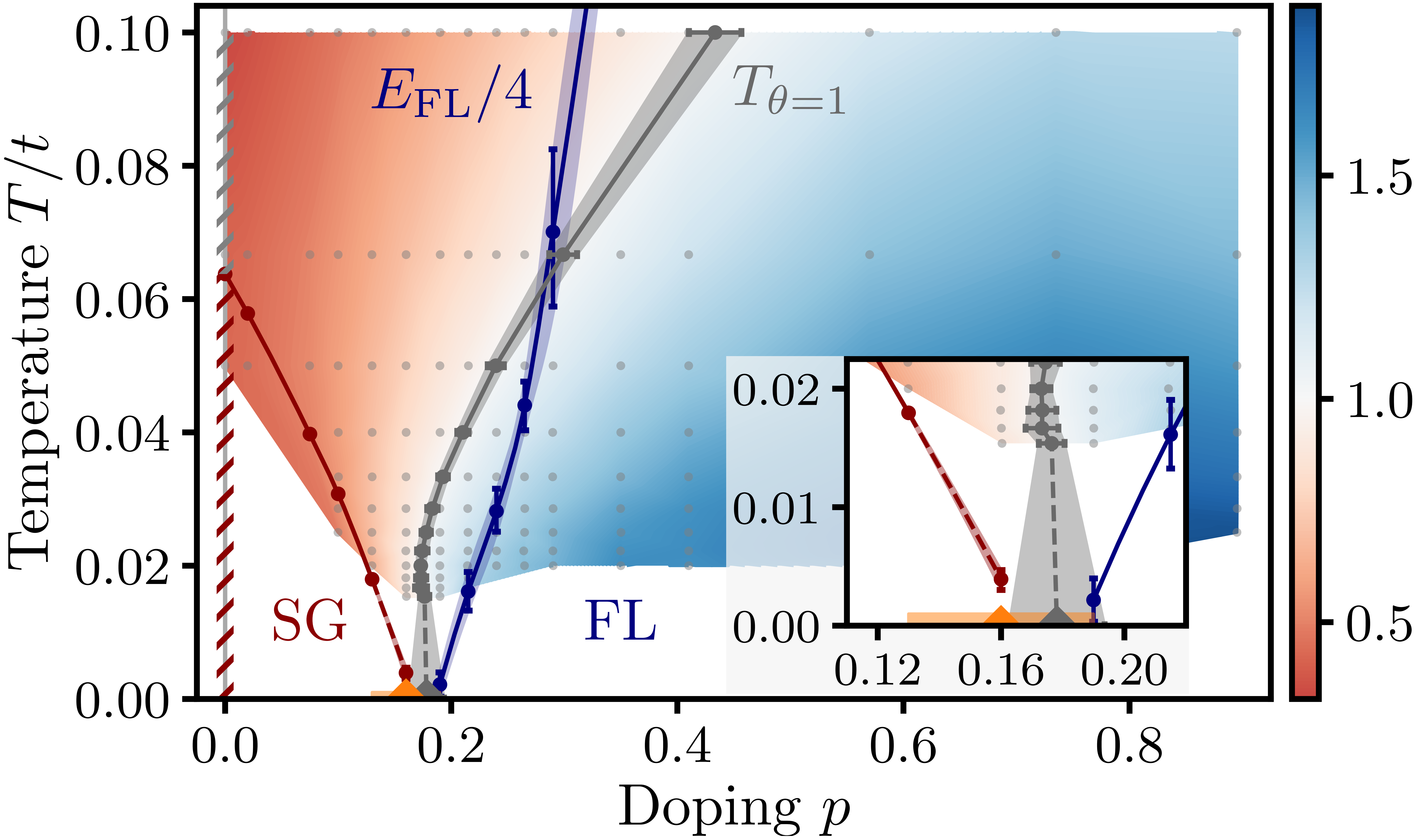
Numerical solution of t-J model on a fully-connected cluster with all-to-all and random t_{ij} and J_{ij}



P. T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet, arXiv:2103.08607

H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

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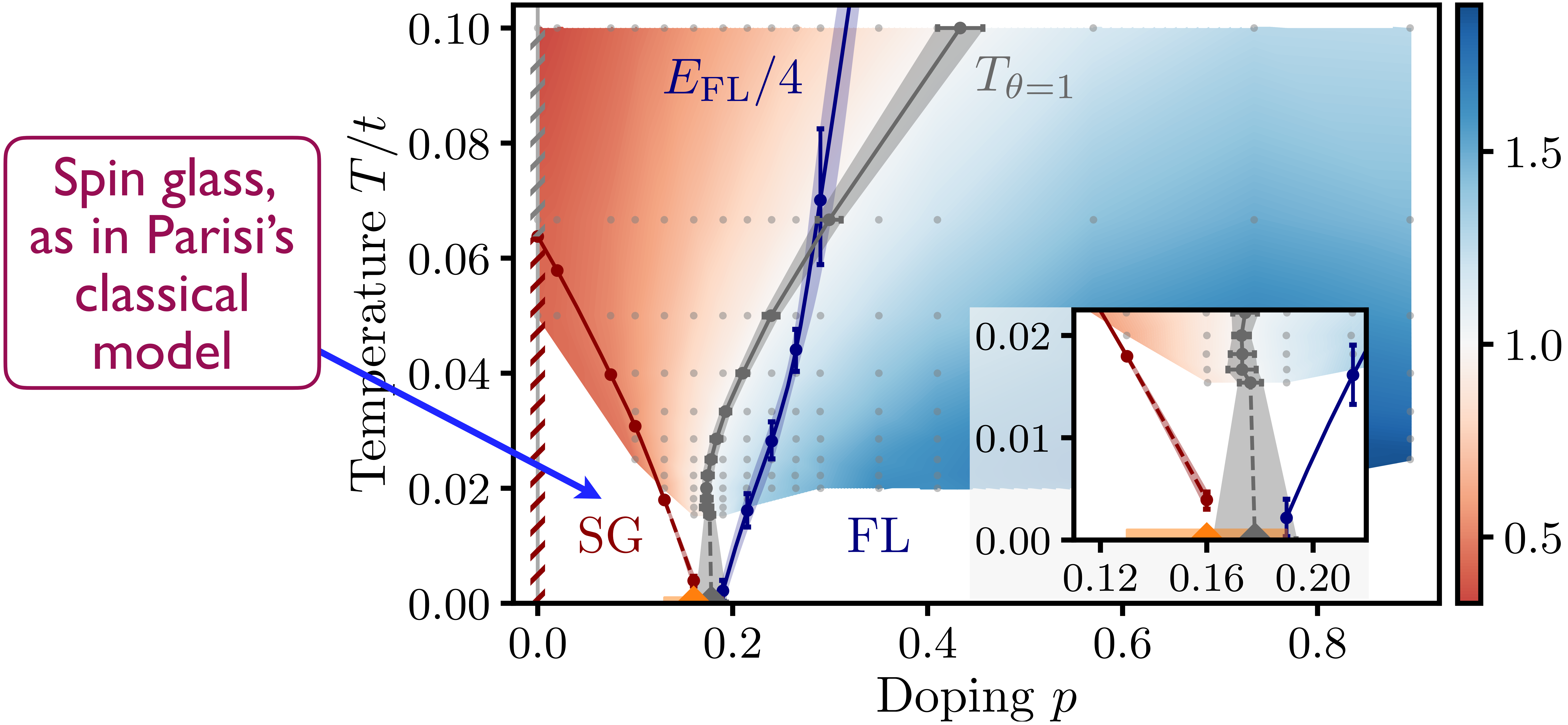
Spin glass, as in Parisi's classical model

P. T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet, arXiv:2103.08607

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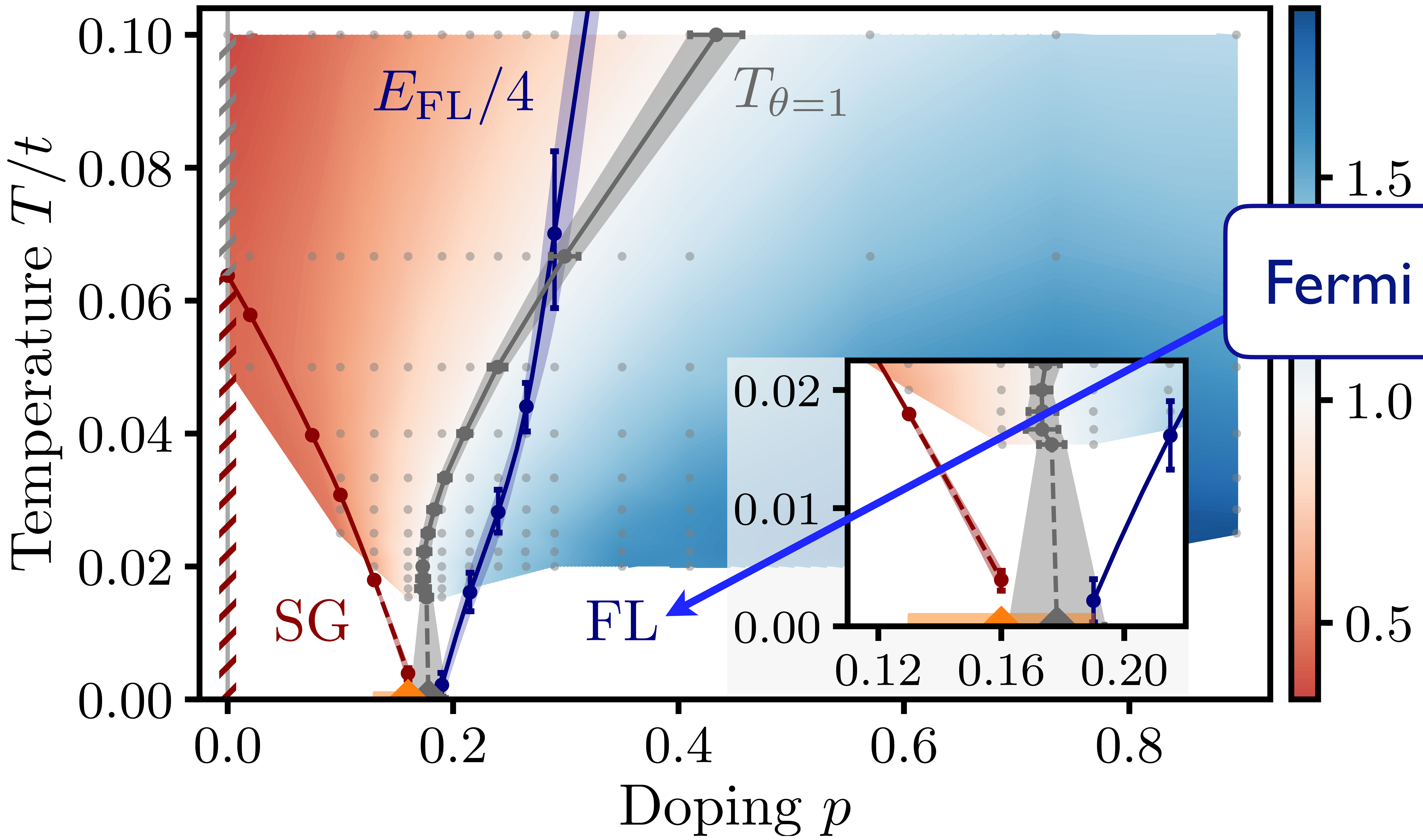

Fermi liquid

P. T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet, arXiv:2103.08607
H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

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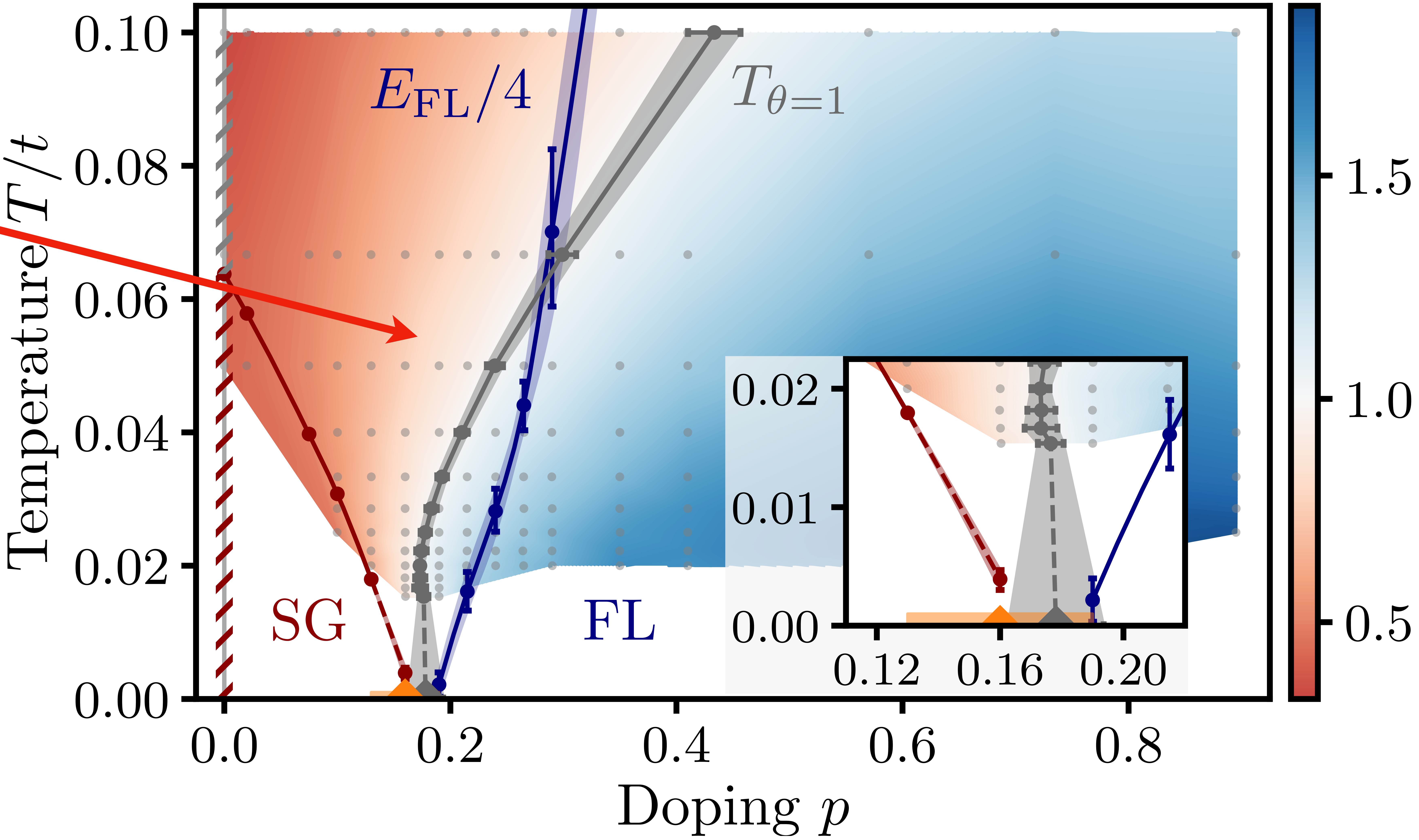
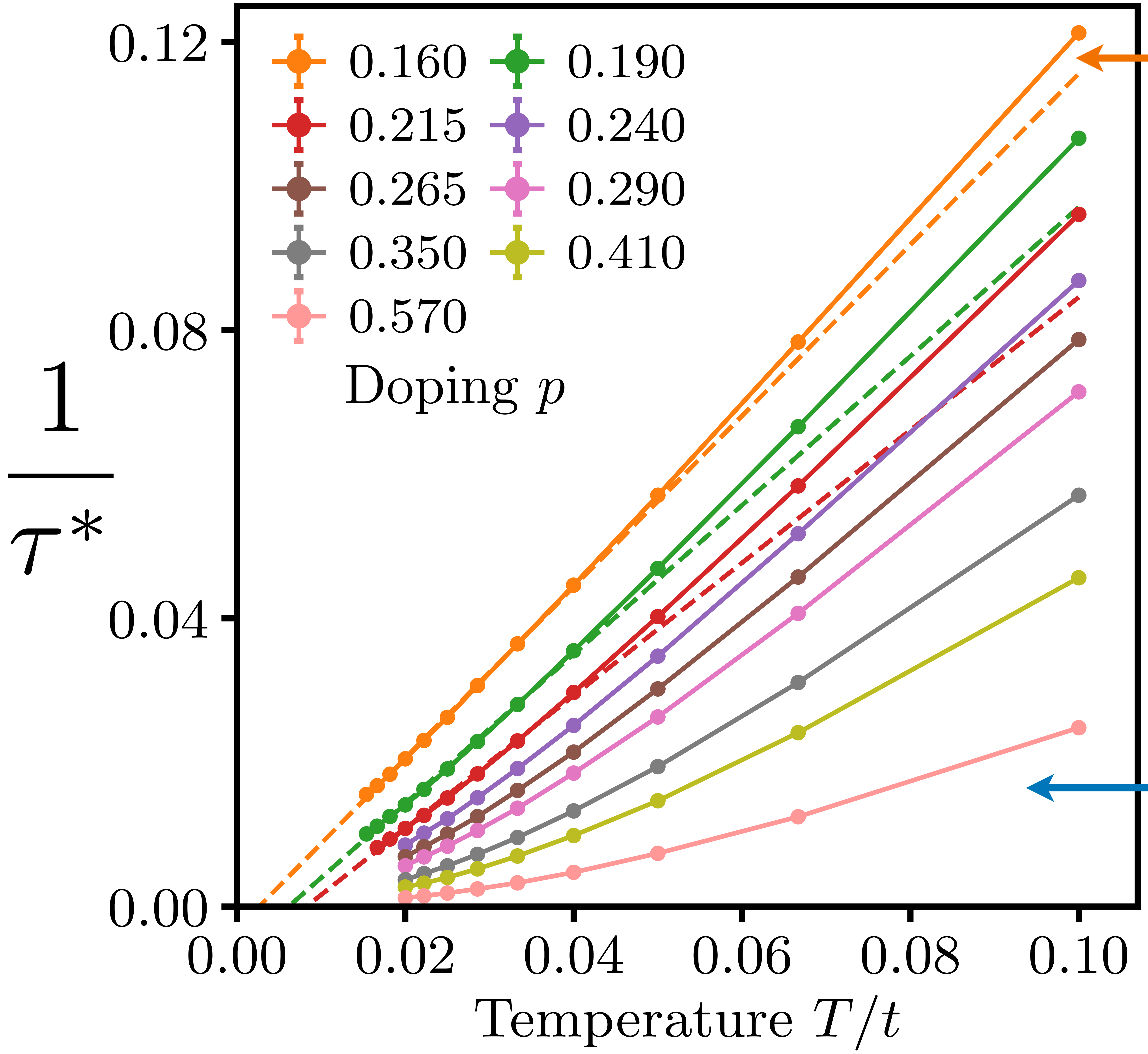
Planckian metal with SYK dynamics of fractionalized spinon excitations



P. T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet, arXiv:2103.08607

H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)



$$\frac{1}{\tau^*} \simeq c\frac{k_B T}{\hbar}$$

$$c \approx 1.0$$

Planckian metal
for $p \approx p_c$

Large M theory
Resistivity: $\rho(T) = \rho(0) + \tilde{c}\,T \ldots$
Linear T term is
correction to scaling
from time reparameterization mode.

Haoyu Guo, Yingfei Gu, and S. Sachdev 2020

$$\frac{1}{\tau^*} \propto T^2$$

Negligible residual resistivity
from random interactions J_{ij}

P. T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet, arXiv:2103.08607

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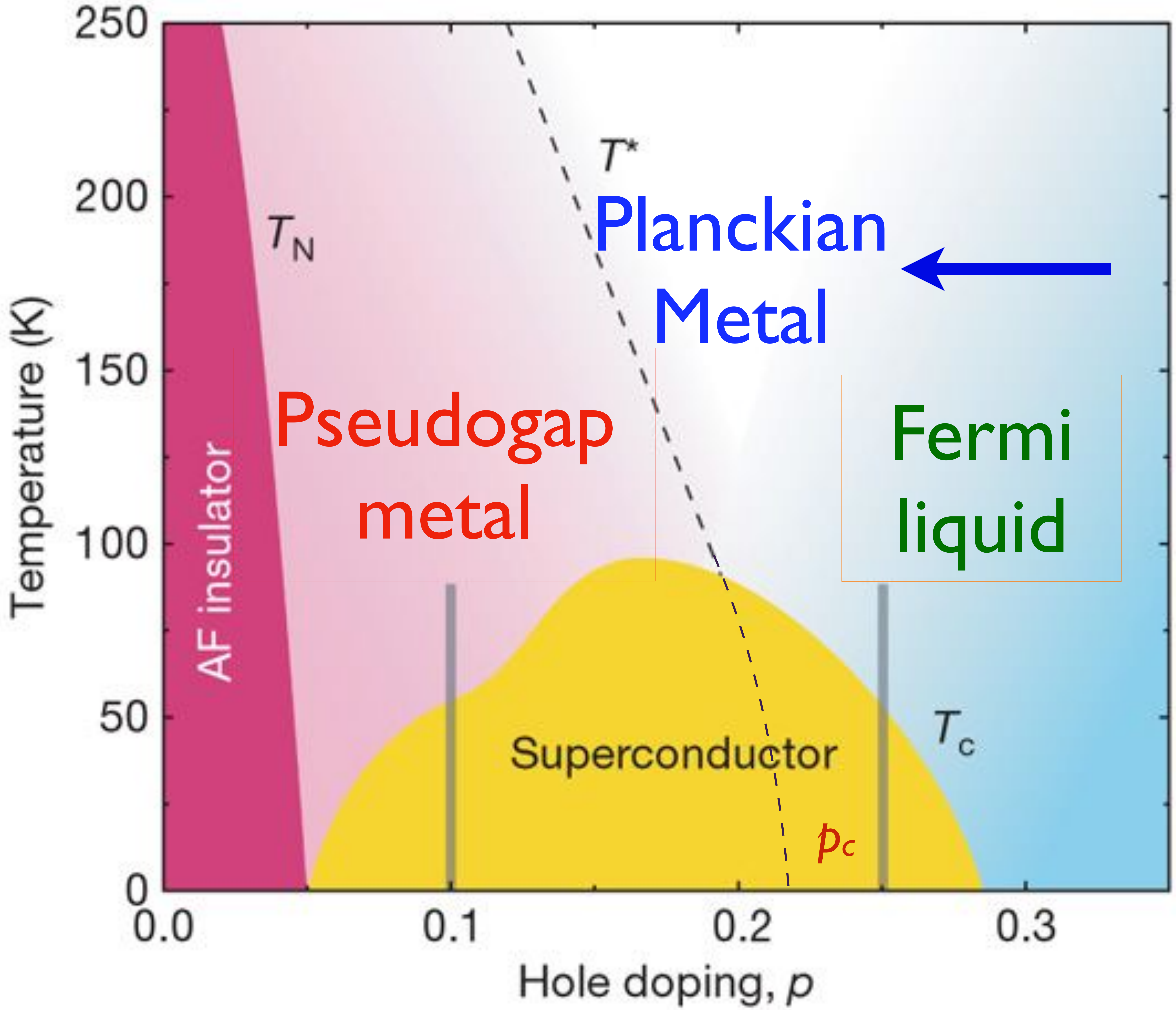
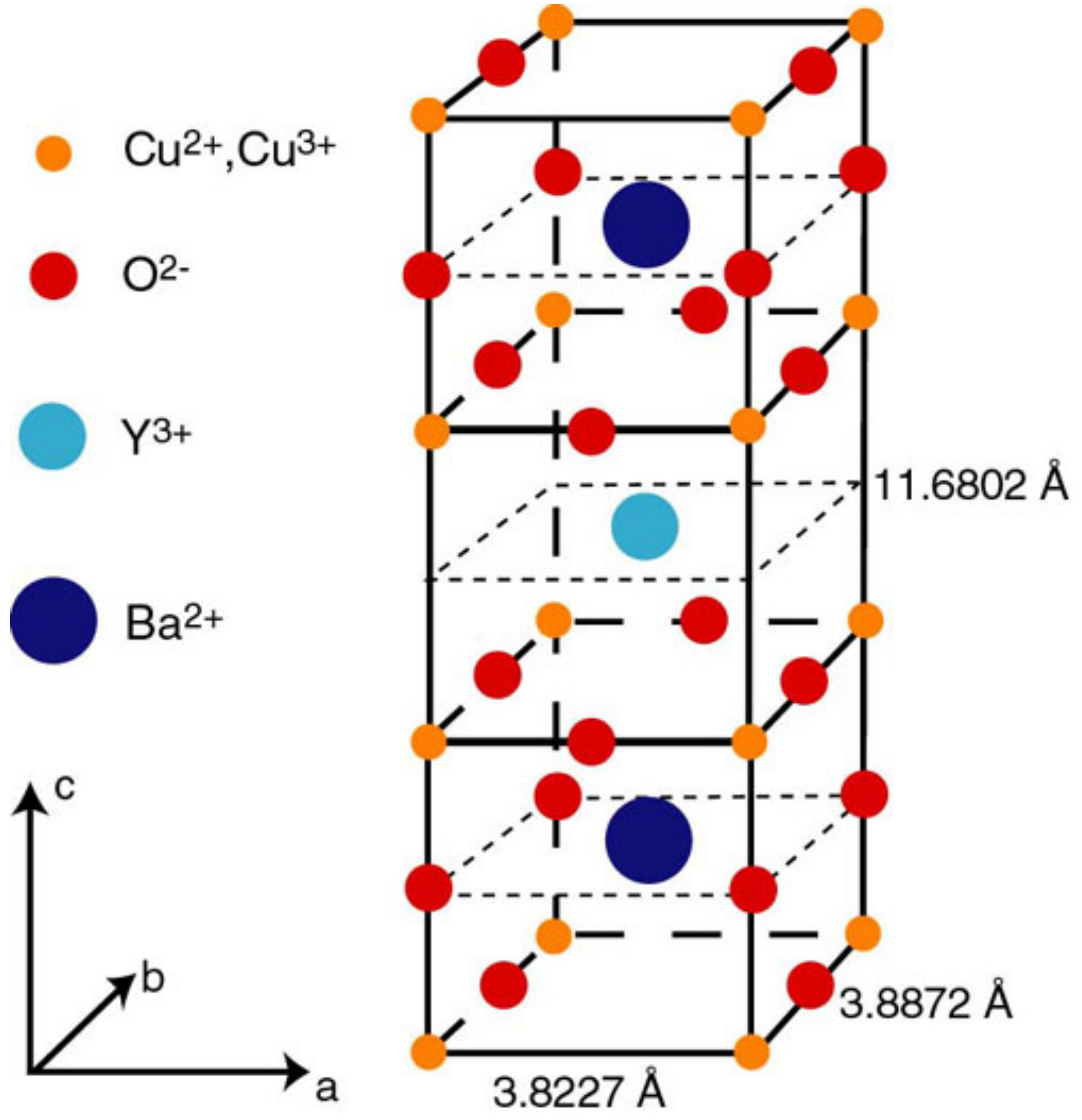
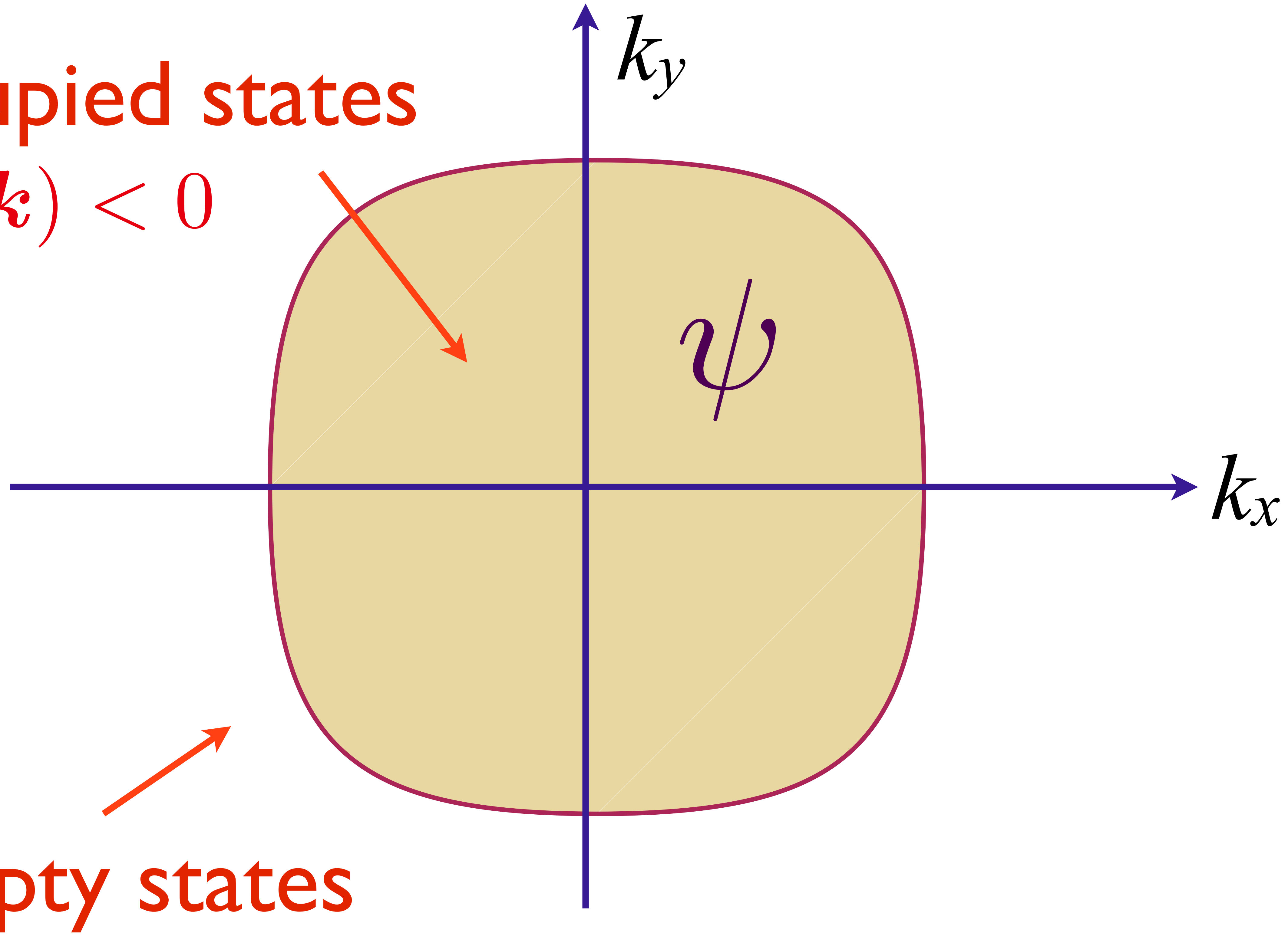

Temperature (K)
250
200
150
100
50
0
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Planckian
Metal
Pseudogap
metal
Fermi
liquid
Superconductor
T_c
p_c


Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

Fermi surface coupled to a critical boson



Occupied states

$\varepsilon(\boldsymbol{k}) < 0$

Empty states

$\varepsilon(\boldsymbol{k}) > 0$

$+$

a
critical
boson
ϕ

Fermi surface coupled to a critical boson

“Yukawa” coupling: $g \int d^2r d\tau \, \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)$

Yields a state without quasiparticle excitations, but the theory is not systematic at large N

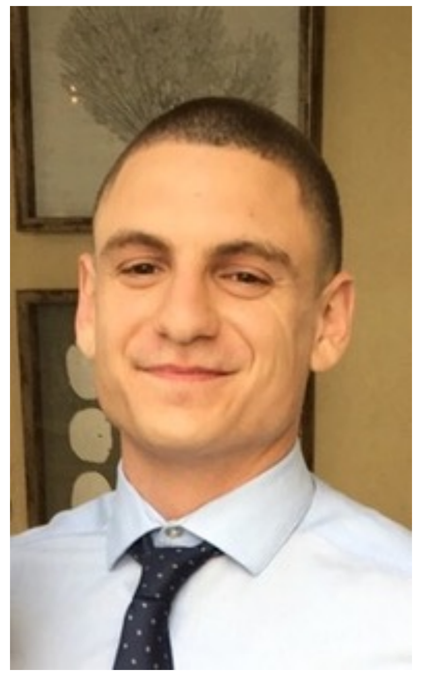
Sung-Sik Lee (2009)

Fermi surface coupled to a critical boson

“Yukawa” coupling: $\dfrac{g_{ij\ell}}{N}\displaystyle\int d^2r d\tau\, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$

$$\overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

Fermi surface coupled to a critical boson

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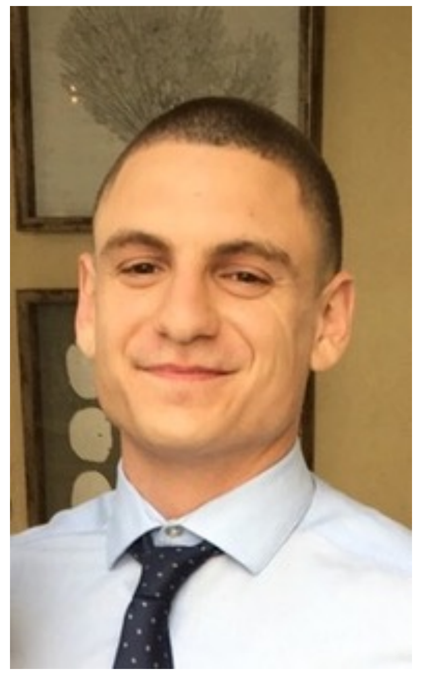
- Yields a systematic large N theory of a “non-Fermi liquid”: a compressible state without quasiparticle excitations. The Fermi surface is sharp in momentum space, but the spectral functions are diffuse in frequency space.
- There is Planckian dynamics at the Fermi surface:

$$G(k = k_F, \omega, T) \sim \omega^{-2/3} F(\hbar\omega/k_B T)$$

- There is many-body quantum chaos in the out-of-time-order correlator (OTOC) with maximal Lyapunov exponent $\lambda_L = 2\pi k_B T/\hbar$.

Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. arXiv: 2103.08615
Maria Tikhanovskaya, Aavishkar Patel, S.S. to appear

Fermi surface coupled to a critical boson



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Conservation of momentum implies the d.c. conductivity is infinite

A. Eberlein, I. Mandal, and S. S., PRB **94**, 045133 (2016)

$$\mathrm{Re}\,\sigma(\omega) = D\delta(\omega) + \mathrm{Re}\,\sigma_{\mathrm{reg}}(\omega)$$

$$\mathrm{Re}\,\sigma_{\mathrm{reg}}(\omega, T=0) \sim \frac{1}{\omega^{2/3}}$$

Yong Baek Kim, A. Furusaki, Xiao-Gang Wen, P. A. Lee, PRB 50, 17917 (1994)

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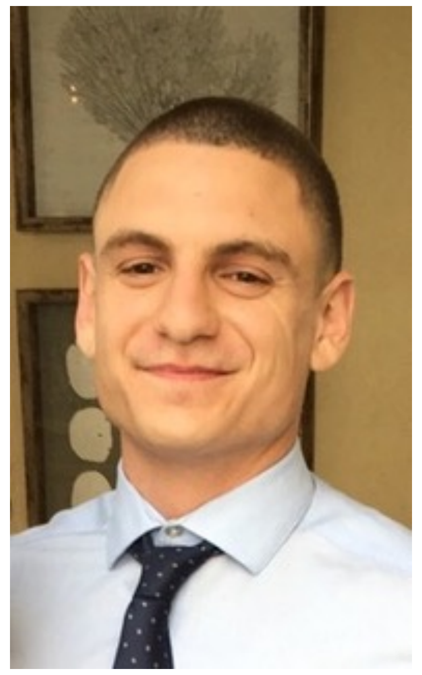

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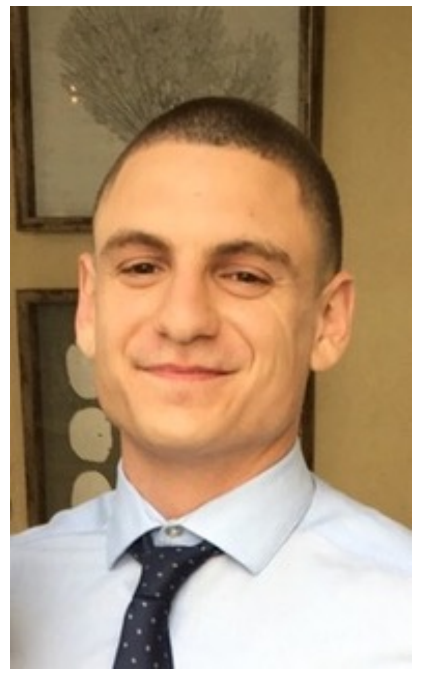
Have to include the effects of spatial disorder or umklapp

Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. to appear

Fermi surface coupled to a critical boson


"Yukawa" coupling: $$\frac{g_{ij\ell}}{N}\int d^2r d\tau\, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

Random potential: $$+\frac{1}{\sqrt{N}}\int d^2r d\tau\, v_{ij}(r)\psi_i^\dagger(r,\tau)\psi_j(r,\tau)$$

Random interactions: $$+\frac{1}{N}\int d^2r d\tau\, g'_{ijl}(r)\psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

$$\overline{g_{ijl}} = 0 \quad , \quad \overline{g^*_{ijl}g_{abc}} = g^2\,\delta_{ia}\delta_{jb}\delta_{lc} \quad , \quad \overline{v_{ij}(r)} = 0 \quad , \quad \overline{v^*_{ij}(r)v_{lm}(r')} = v^2\,\delta(r-r')\delta_{il}\delta_{jm}$$

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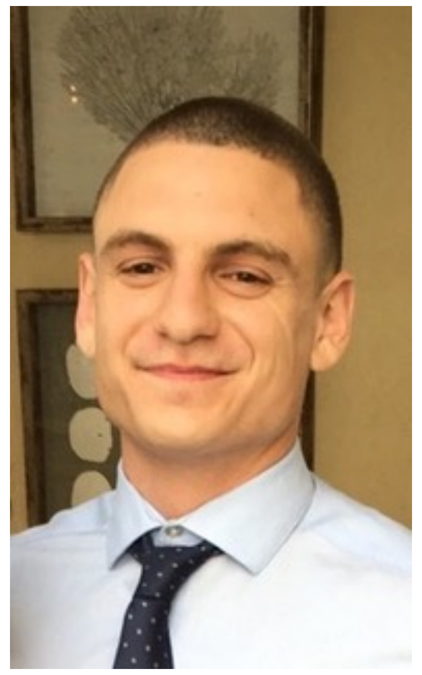
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. arXiv:2103.08615, and to appear

Fermi surface coupled to a critical boson



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With g, v, g' all non-zero, resistivity $\rho(T) = \rho(0) + \tilde{c}\,T \ldots$.
$\rho(0)$ is determined by v,
while $\tilde{c}$ is determined by a subleading random interaction g'.
These features are just as in the random t-J model.

E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. arXiv:2103.08615, and to appear

Summary

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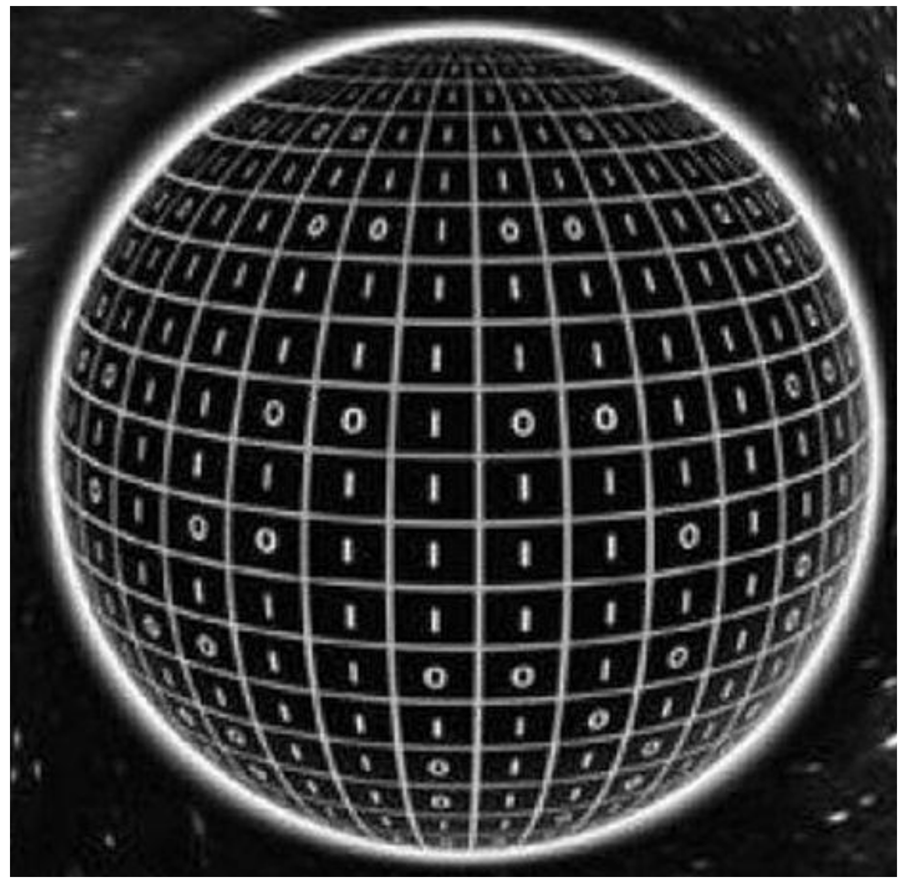
- Low energy theory of time reparameterizations is the theory of the boundary graviton in 2D quantum gravity on AdS_2.

Summary

- Boundary graviton leads to universal $-3/2\ln(1/T)$ correction to Bekenstein-Hawking entropy of low T charged black holes in Einstein gravity, and to the SYK model. So the semiclassical entropy of Einstein gravity is reproduced by a unitary quantum system with a discrete spectrum. Further work along these lines has led to progress on the Page curve describing the time evolution of the entropy of an evaporating black hole.



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