Entanglement, holography, and the quantum phases of matter

Bethe Colloquium, Bonn, January 14, 2013

Subir Sachdev

PHYSICS

VE RI TAS

HARVARD

Sommerfeld-Bloch theory of
metals, insulators, and superconductors:

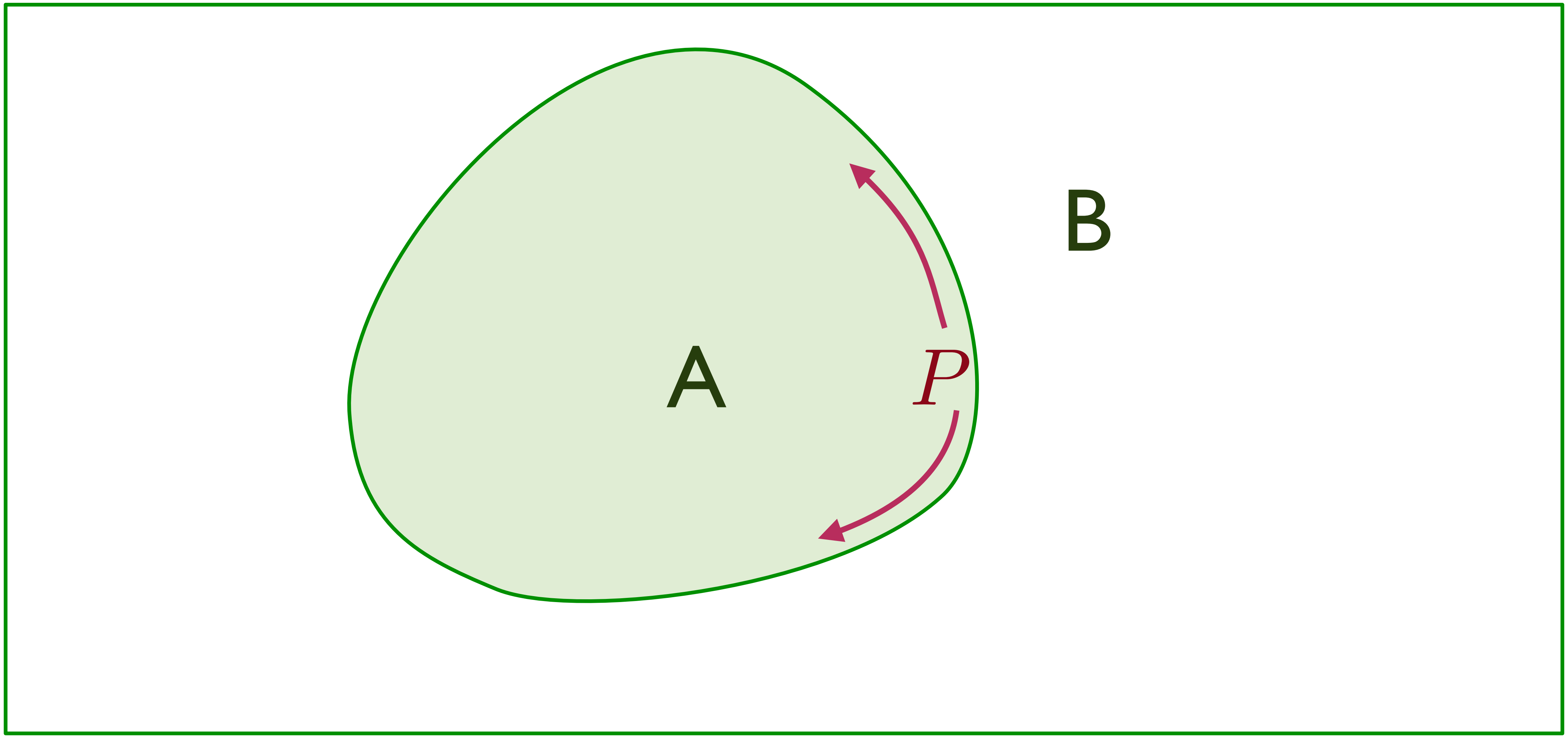
many-electron quantum states are adiabatically
connected to independent electron states

Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle
quantum entanglement

Entanglement entropy



$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

$$\rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_E = -\text{Tr}\left(\rho_A \ln \rho_A\right)$$

Entanglement entropy

$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

$$\text{Take } |\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B\right)$$

$$\text{Then } \rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$$
$$= \frac{1}{2}\left(|\uparrow\rangle_A \langle\uparrow|_A + |\downarrow\rangle_A \langle\downarrow|_A\right)$$

$$\textbf{Entanglement entropy } S_E = -\text{Tr}\left(\rho_A \ln \rho_A\right)$$
$$= \ln 2$$

“Complex entangled” states of quantum matter, *not* adiabatically connected to independent particle states

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene

Compressible quantum matter

Strange metals in high temperature superconductors, Bose metals

S. Sachdev, 100th anniversary Solvay conference,arXiv:1203.4565

“Complex entangled” states of quantum matter, *not* adiabatically connected to independent particle states

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene

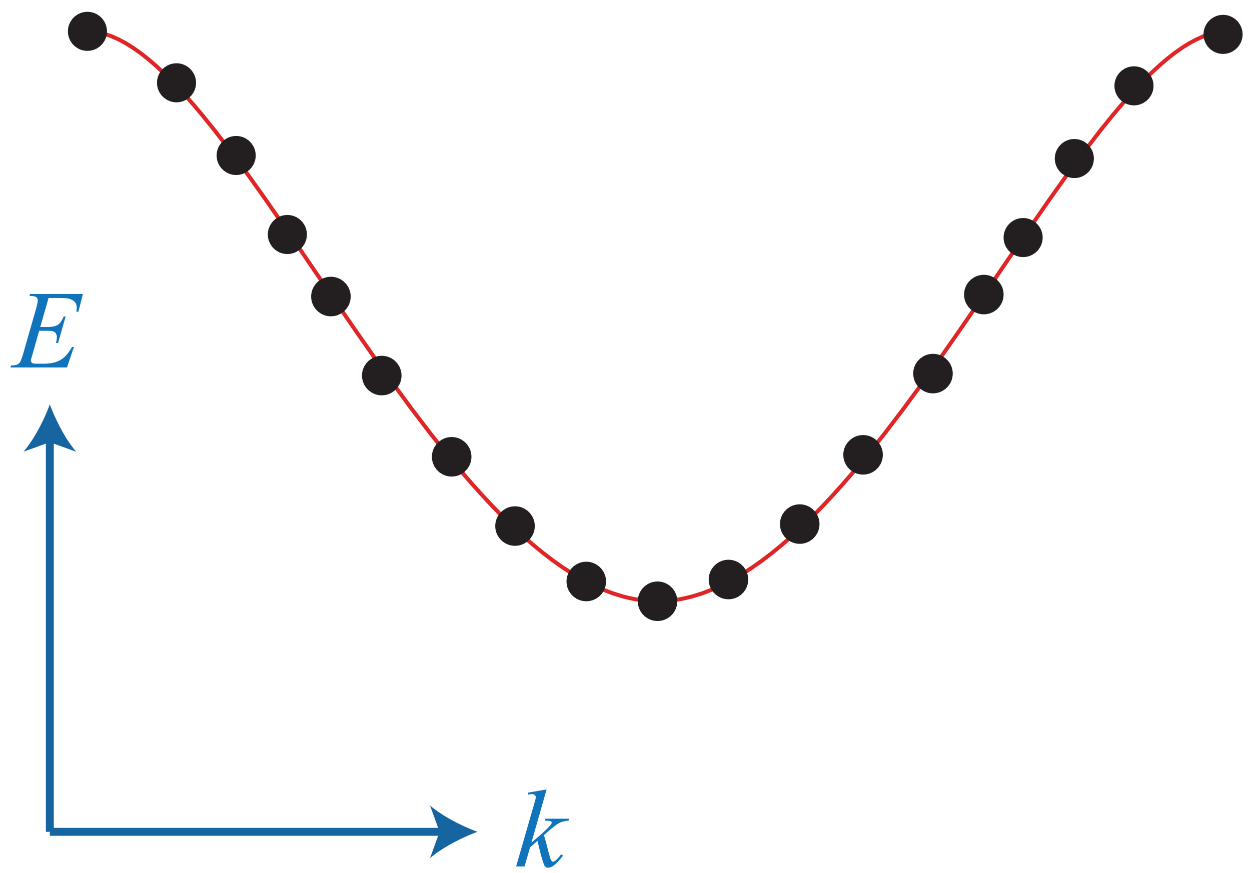
Compressible quantum matter

Strange metals in high temperature superconductors, Bose metals

S. Sachdev, 100th anniversary Solvay conference,arXiv:1203.4565

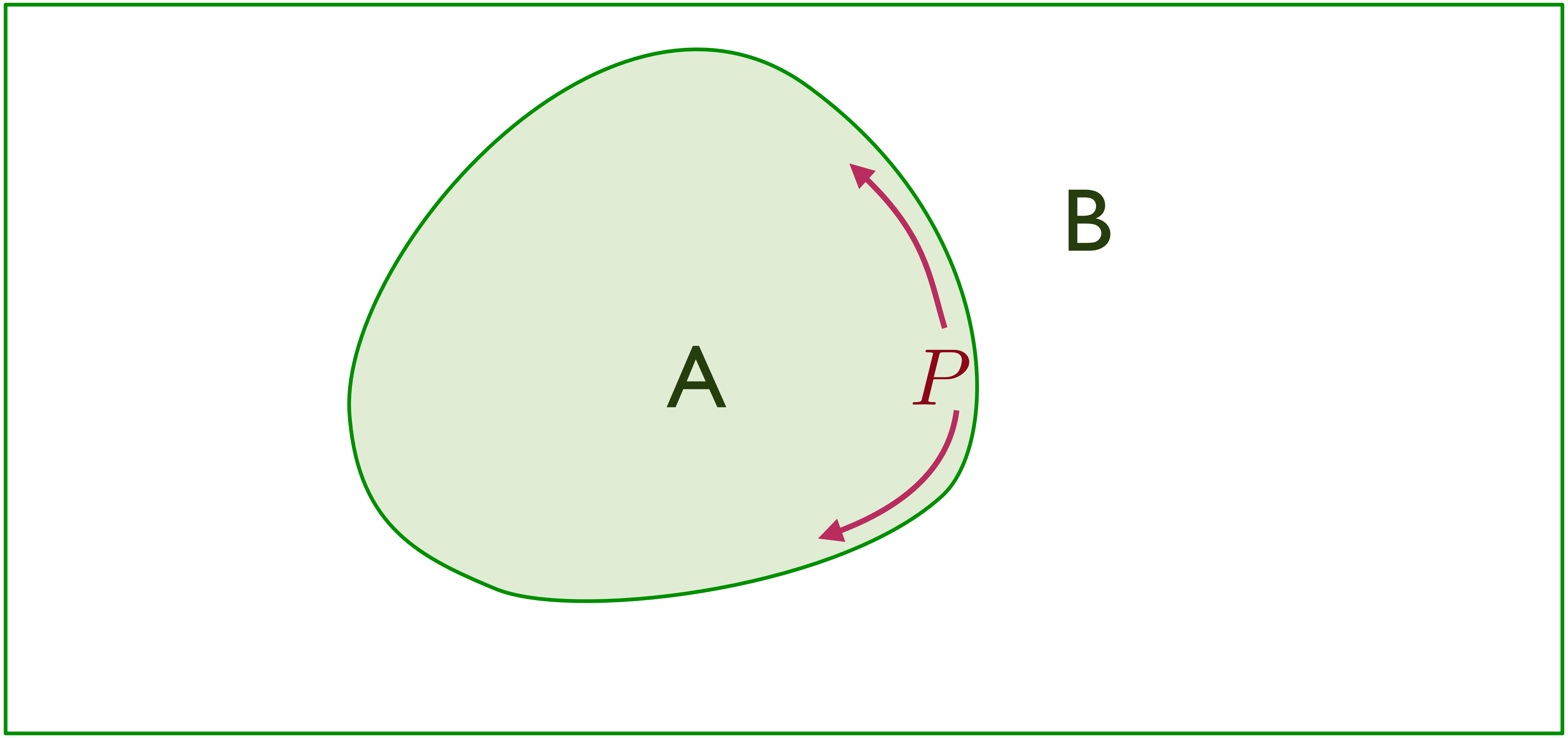
Entanglement entropy of a band insulator

Band insulators



An even number of electrons per unit cell

Entanglement entropy of a band insulator

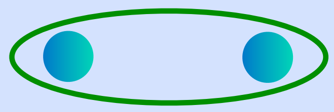


$$S_E = aP - b\exp(-cP)$$

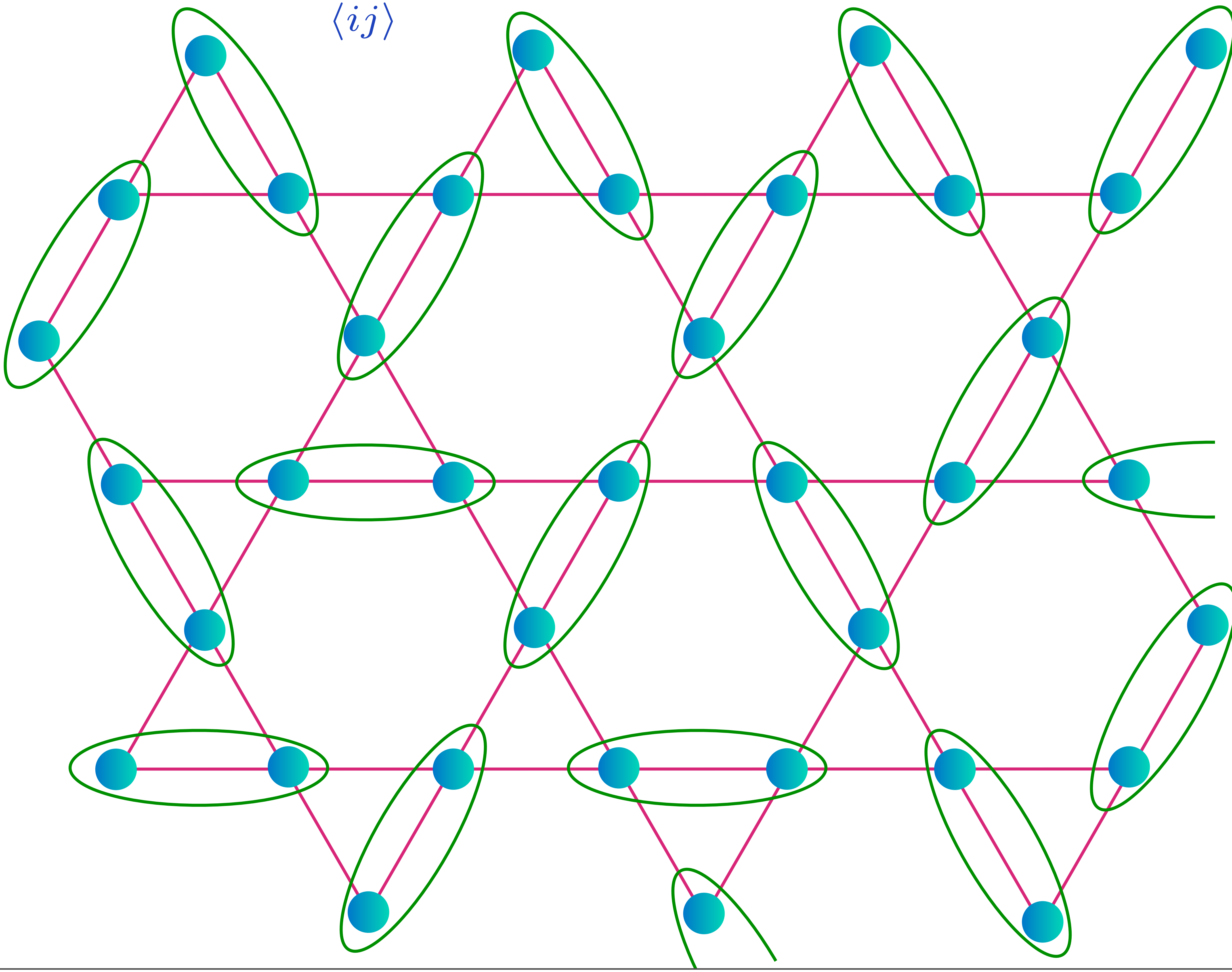
where P is the surface area (perimeter) of the boundary between A and B.

Mott insulator: Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



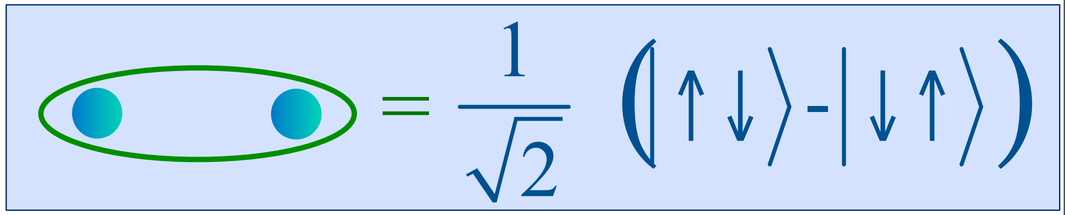
$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

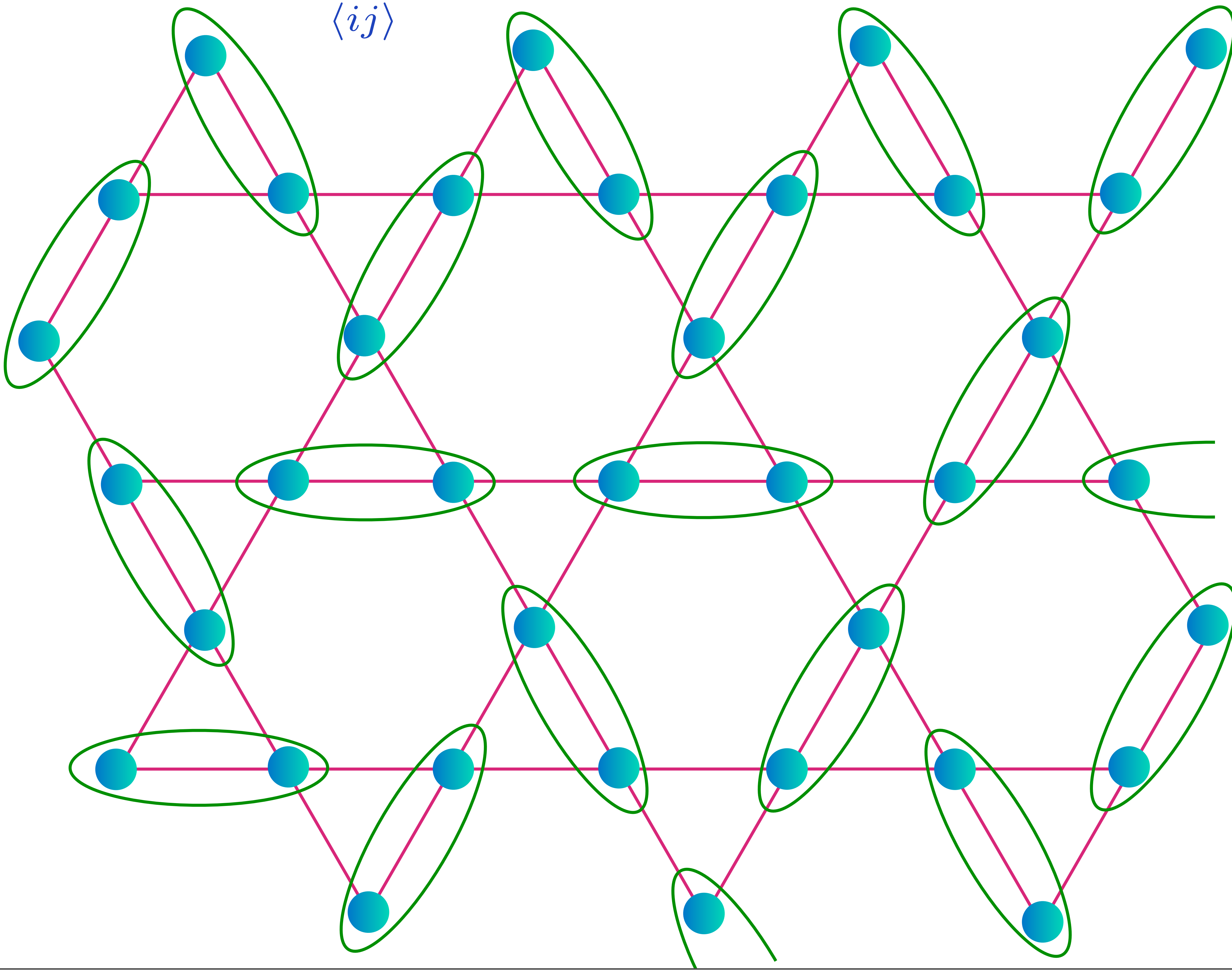


P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

Mott insulator: Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

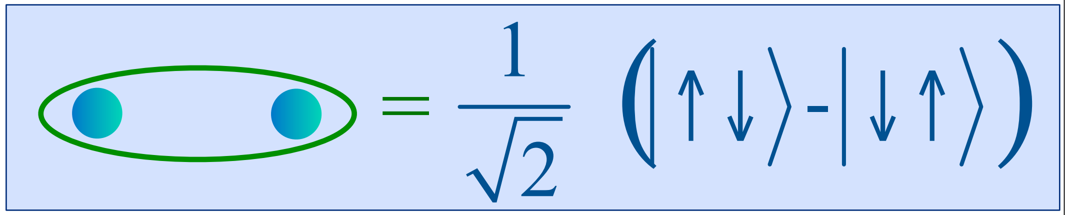


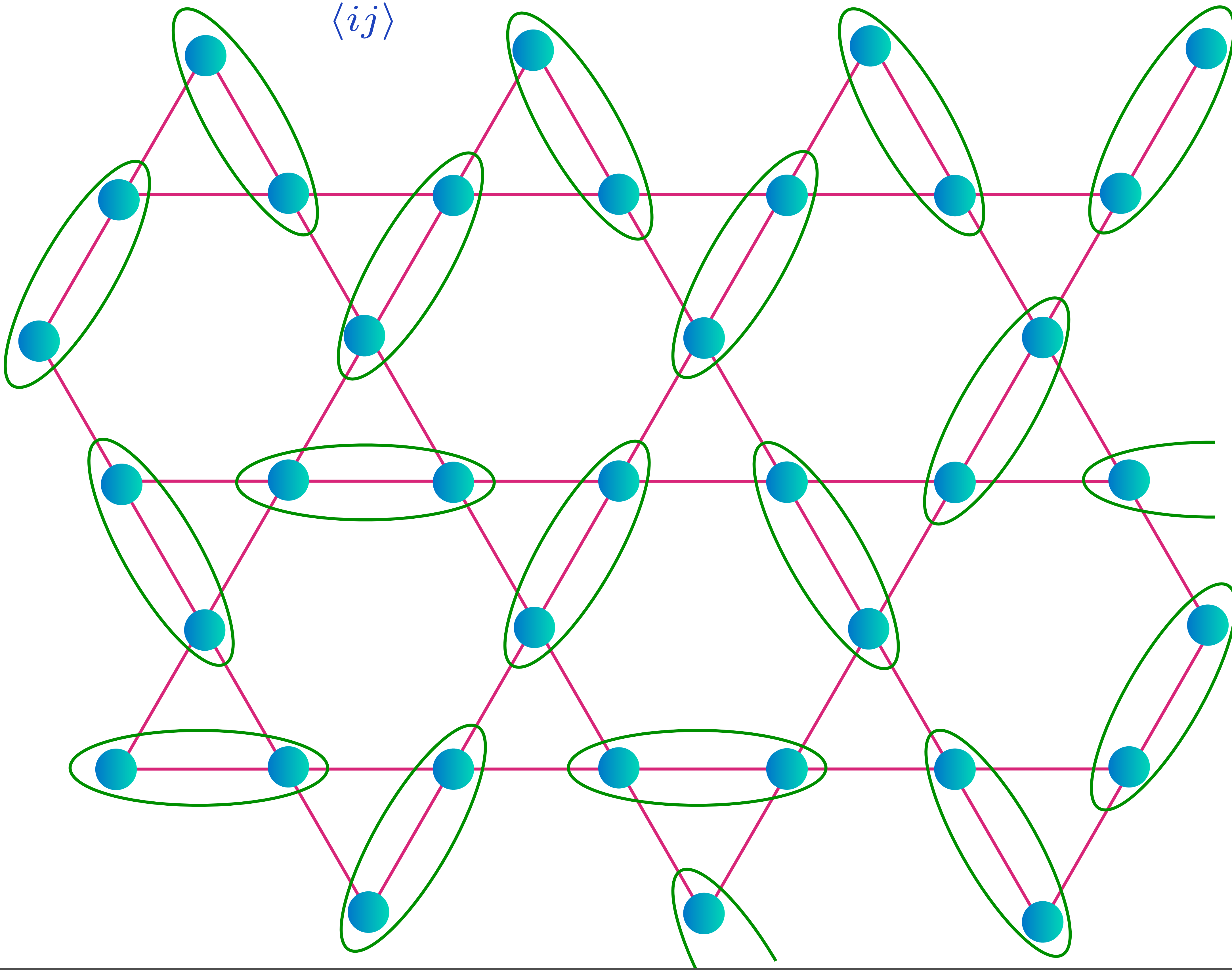


Kekule (1865)

Mott insulator: Kagome antiferromagnet

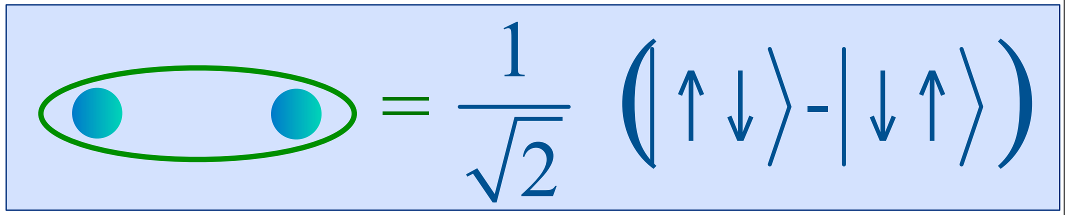
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

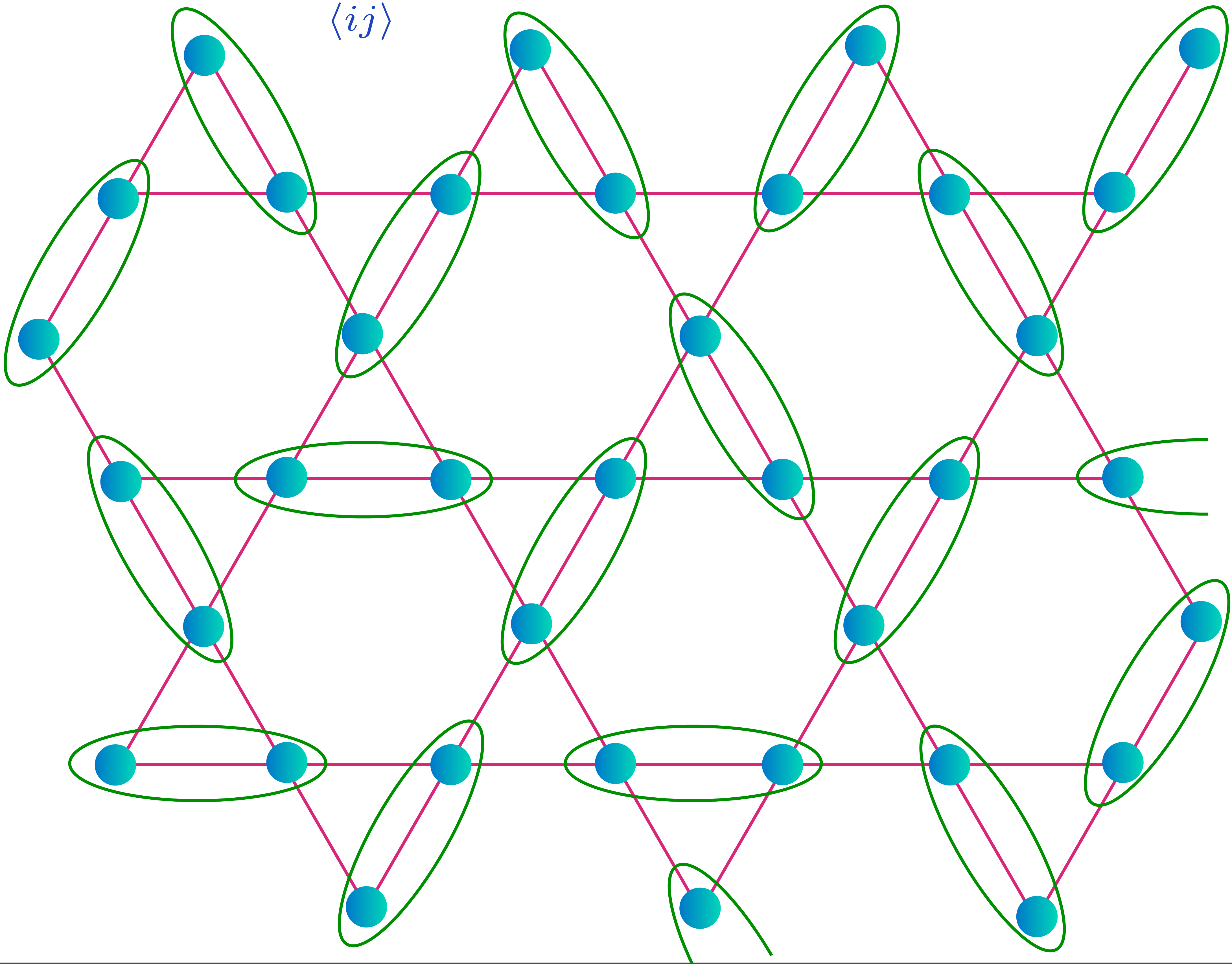




Mott insulator: Kagome antiferromagnet

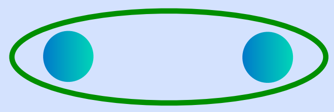
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



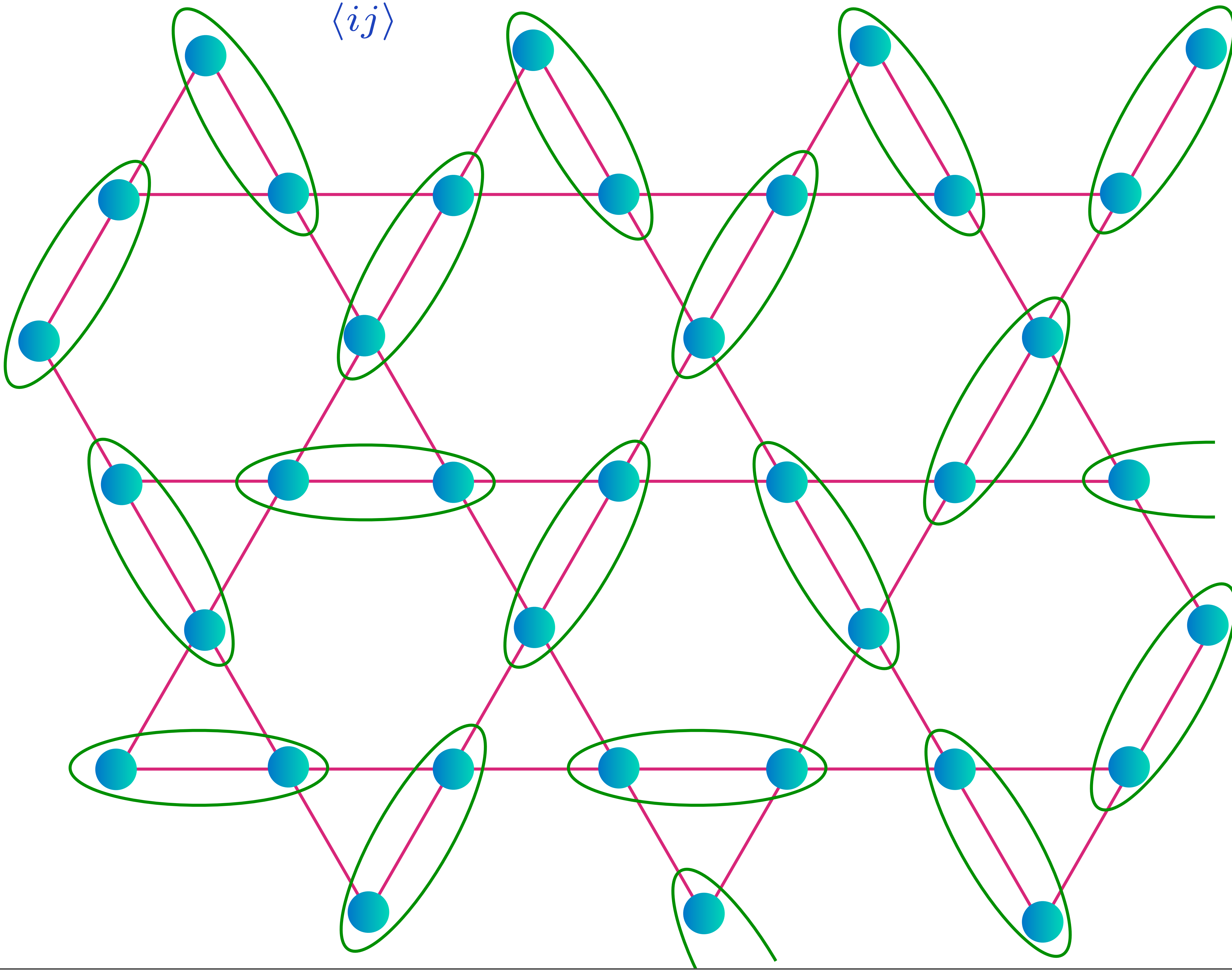


Mott insulator: Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

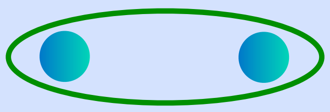


$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

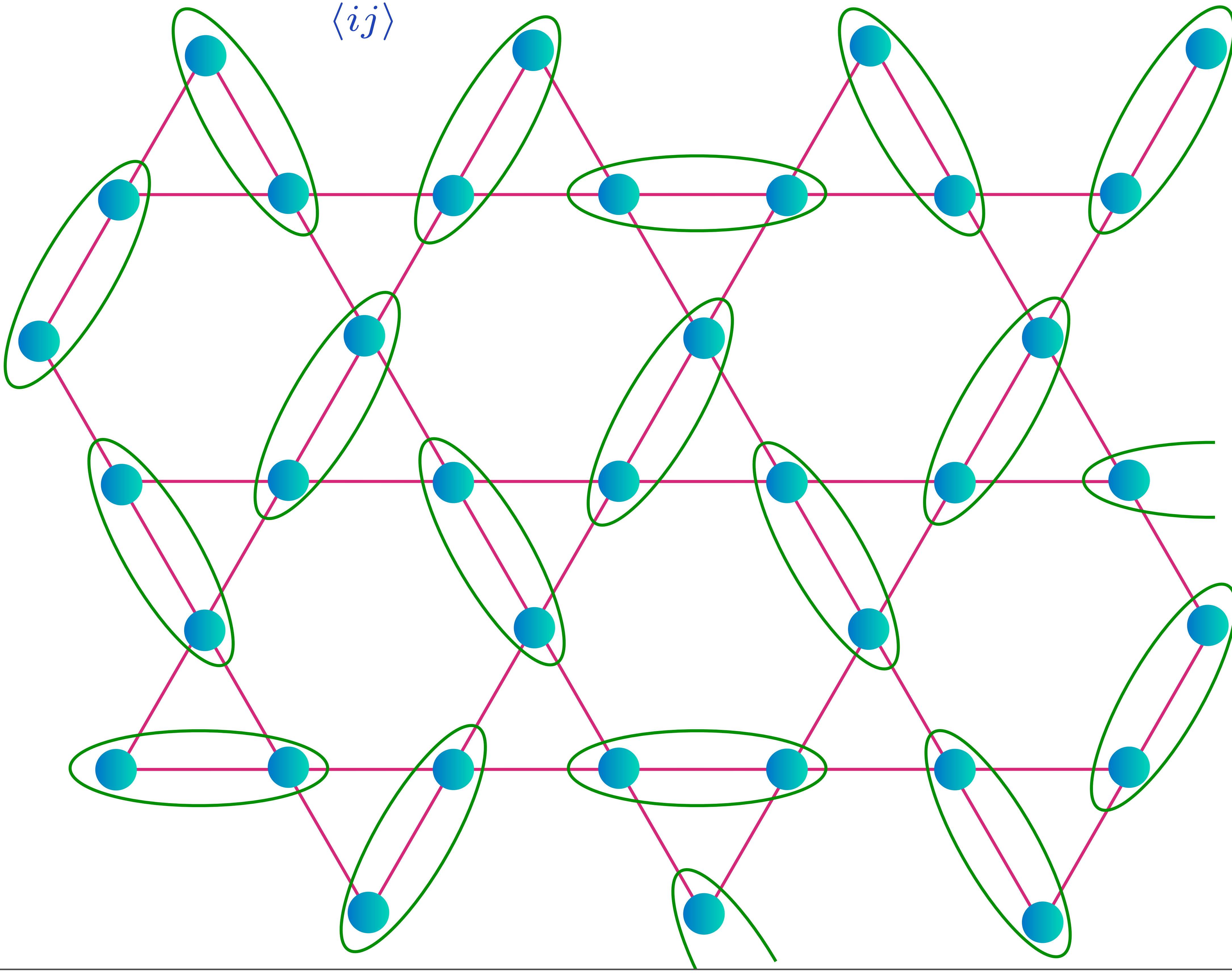


Mott insulator: Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

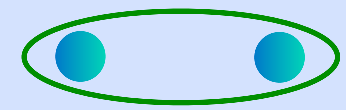


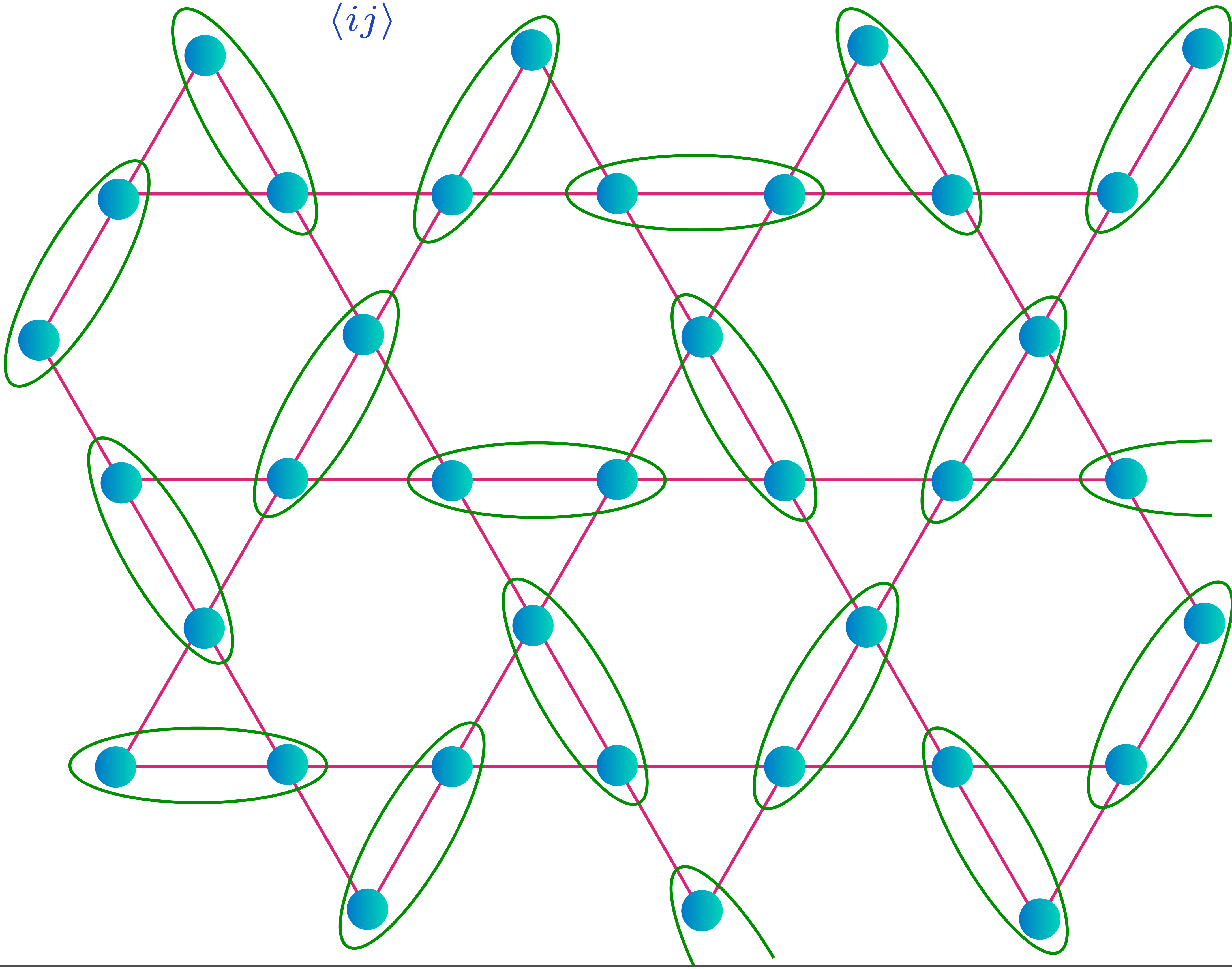
$= \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$



Mott insulator: Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

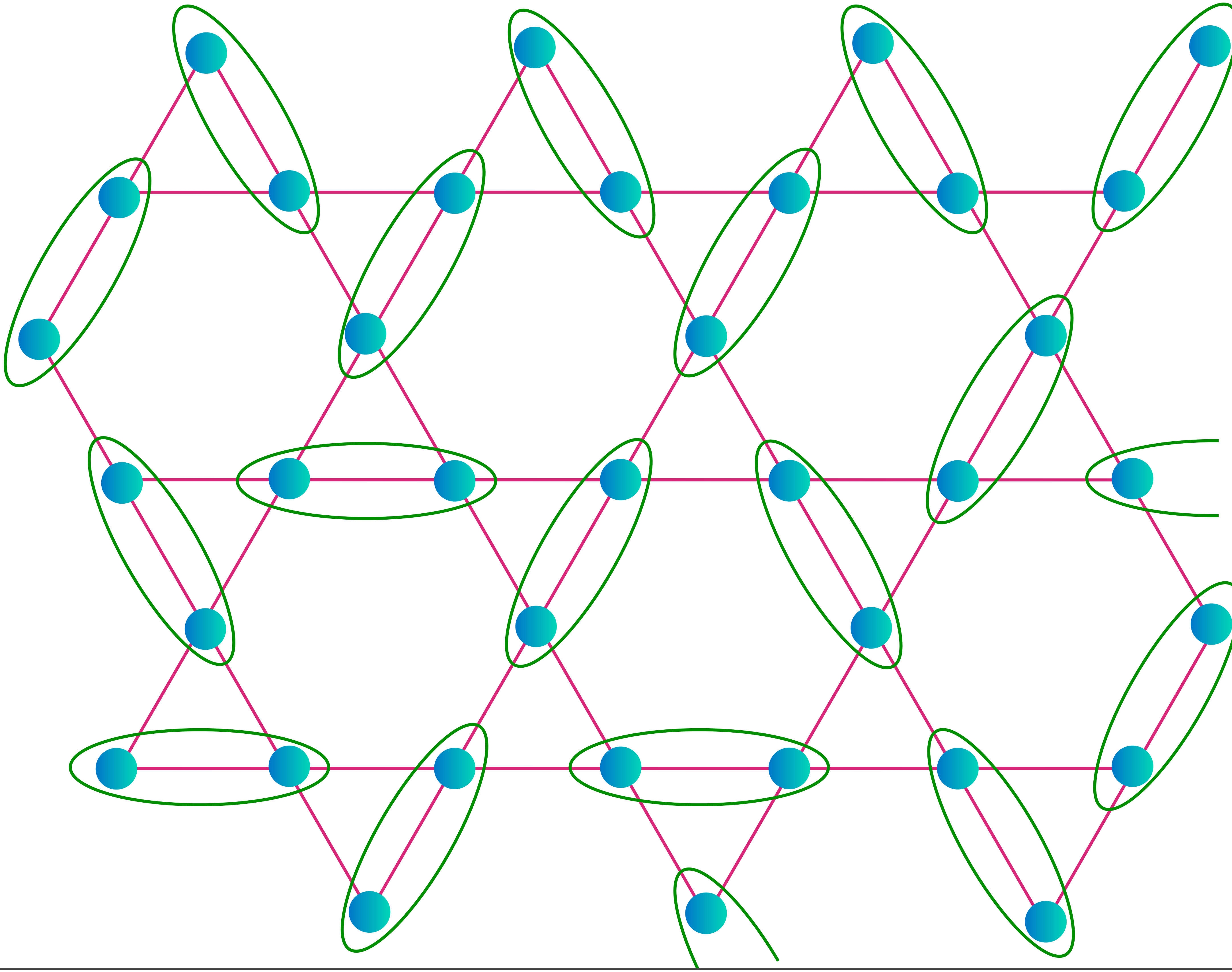

$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



Mott insulator: Kagome antiferromagnet

Alternative view

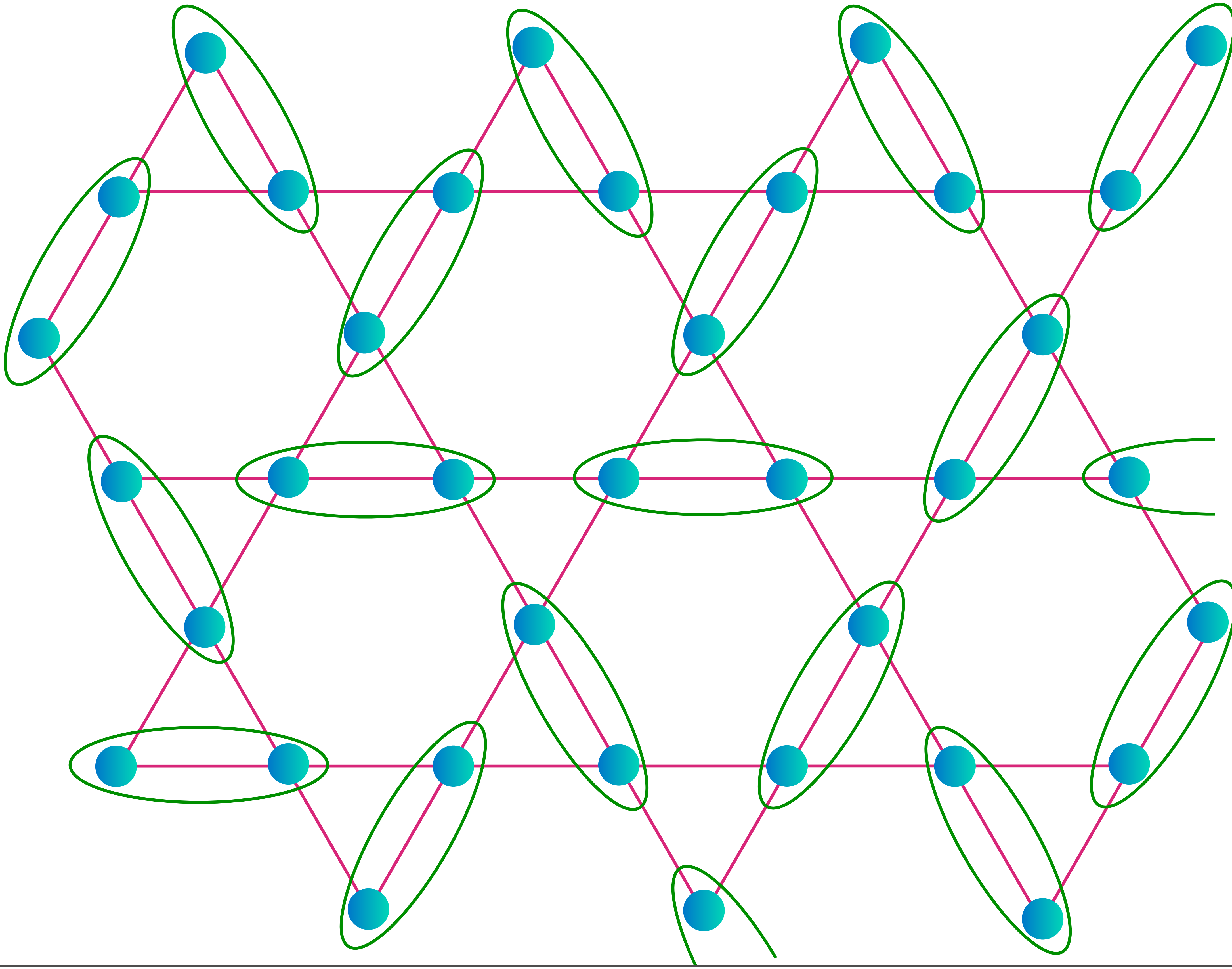
Pick a reference configuration



Mott insulator: Kagome antiferromagnet

Alternative view

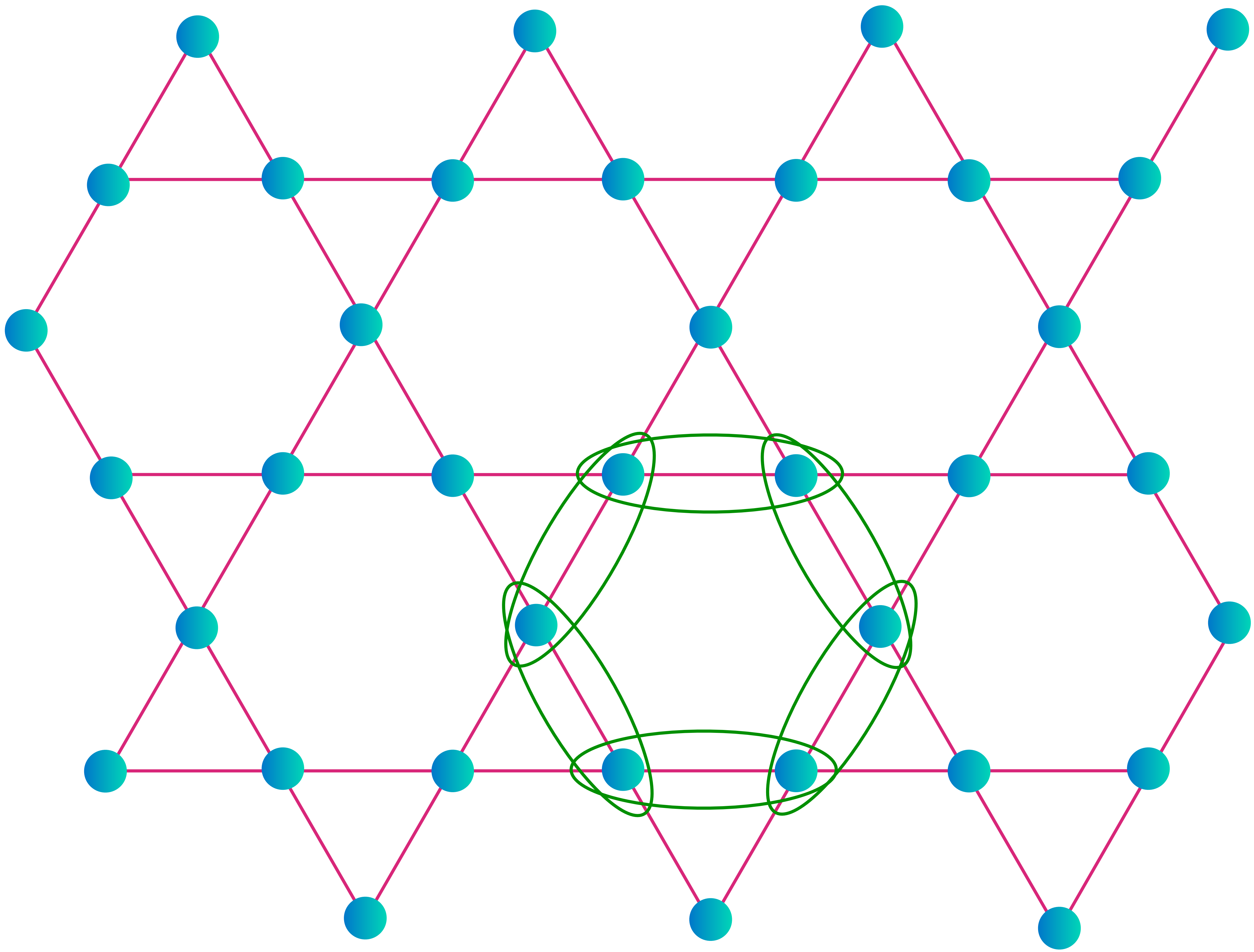
A nearby configuration



Mott insulator: Kagome antiferromagnet

Alternative view

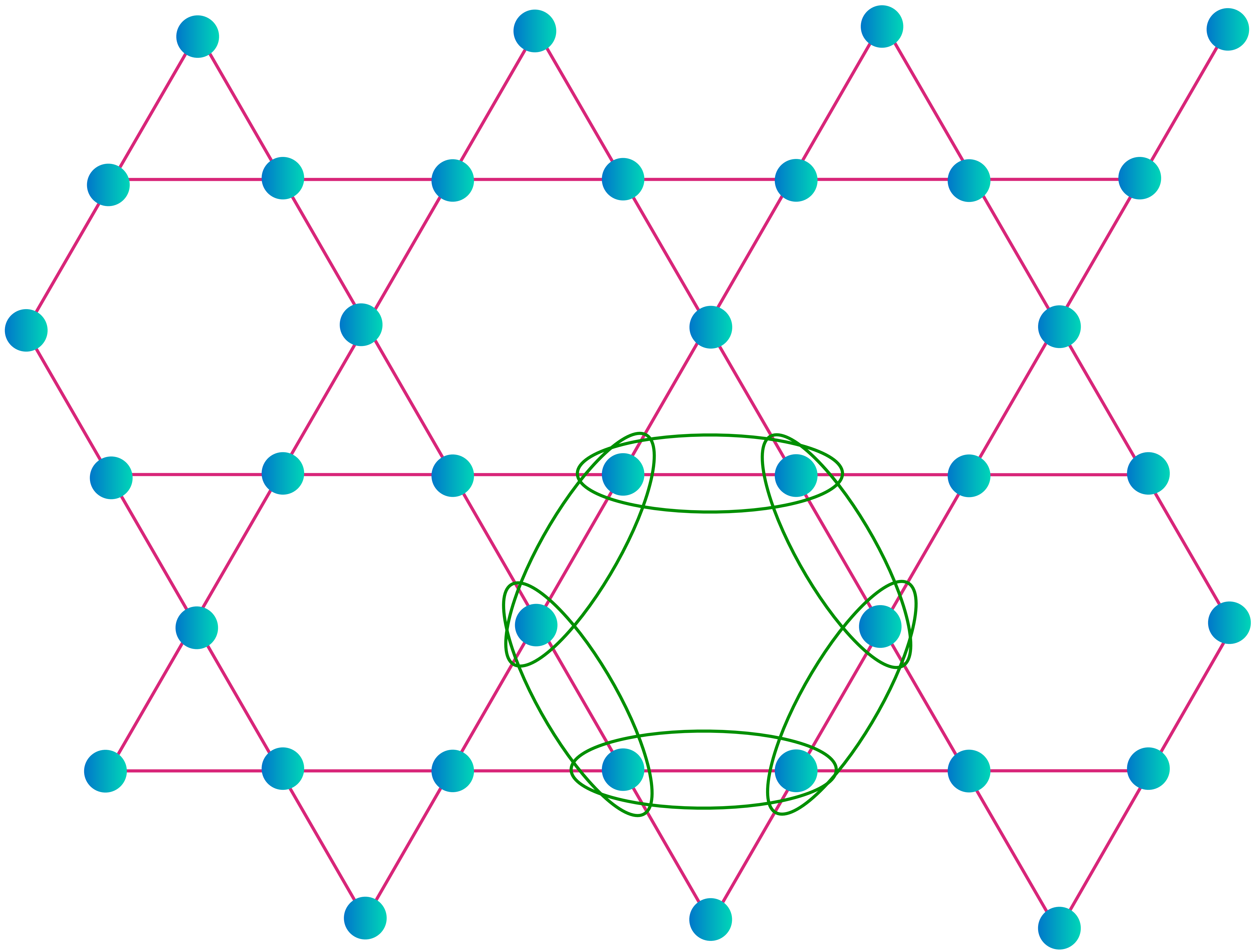
Difference: a closed loop



Mott insulator: Kagome antiferromagnet

Alternative view

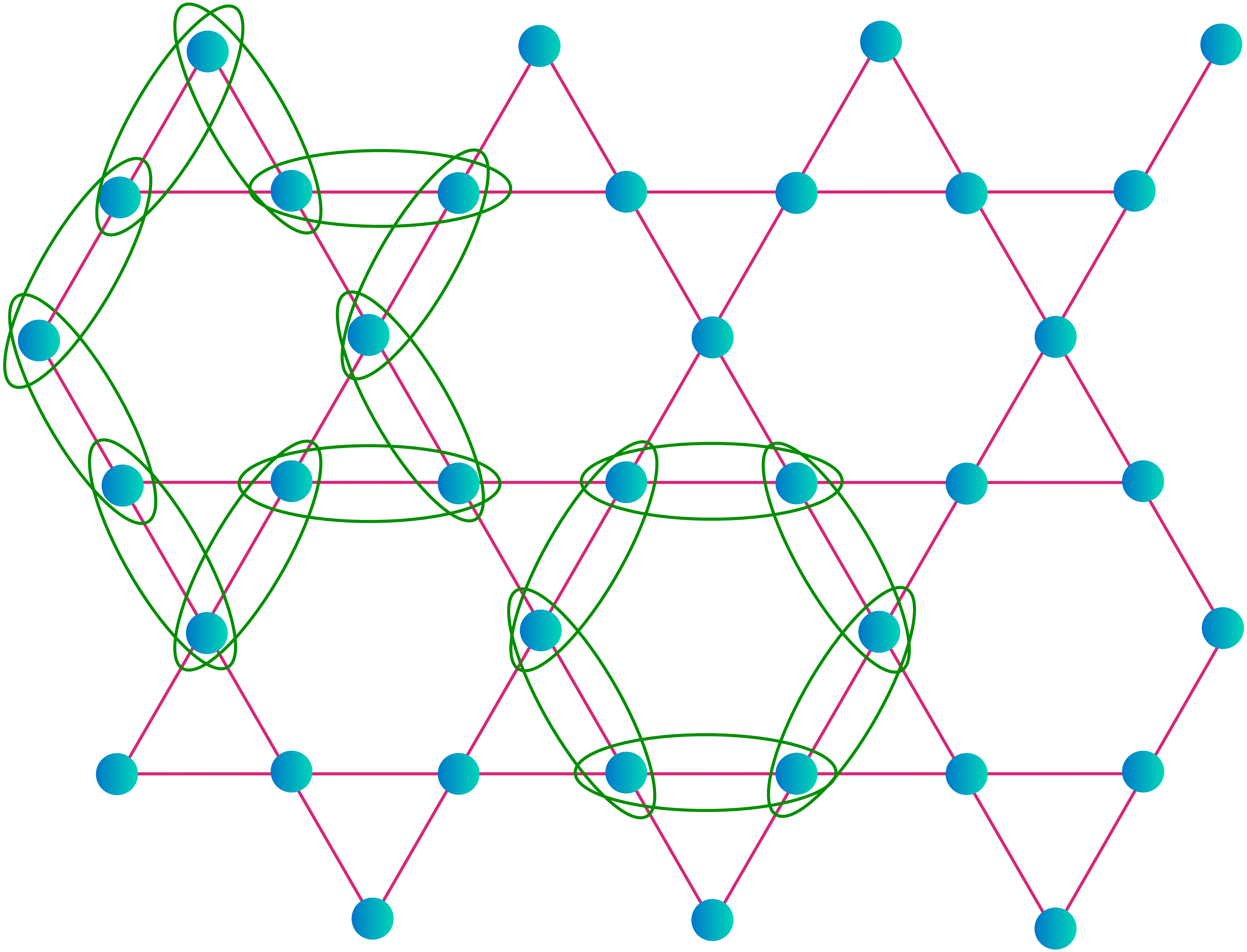
Ground state: sum over closed loops



Mott insulator: Kagome antiferromagnet

Alternative view

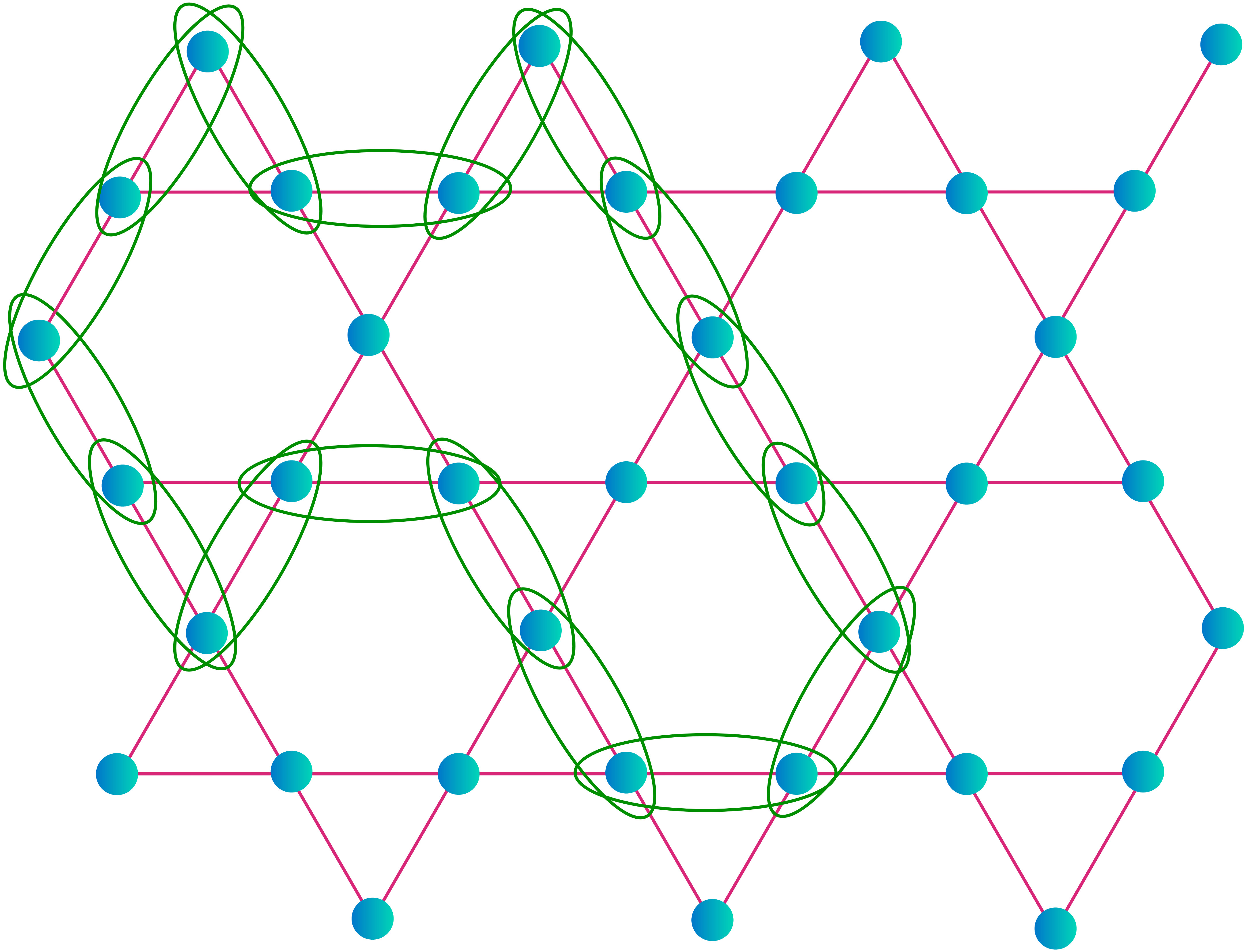
Ground state: sum over closed loops



Mott insulator: Kagome antiferromagnet

Alternative view

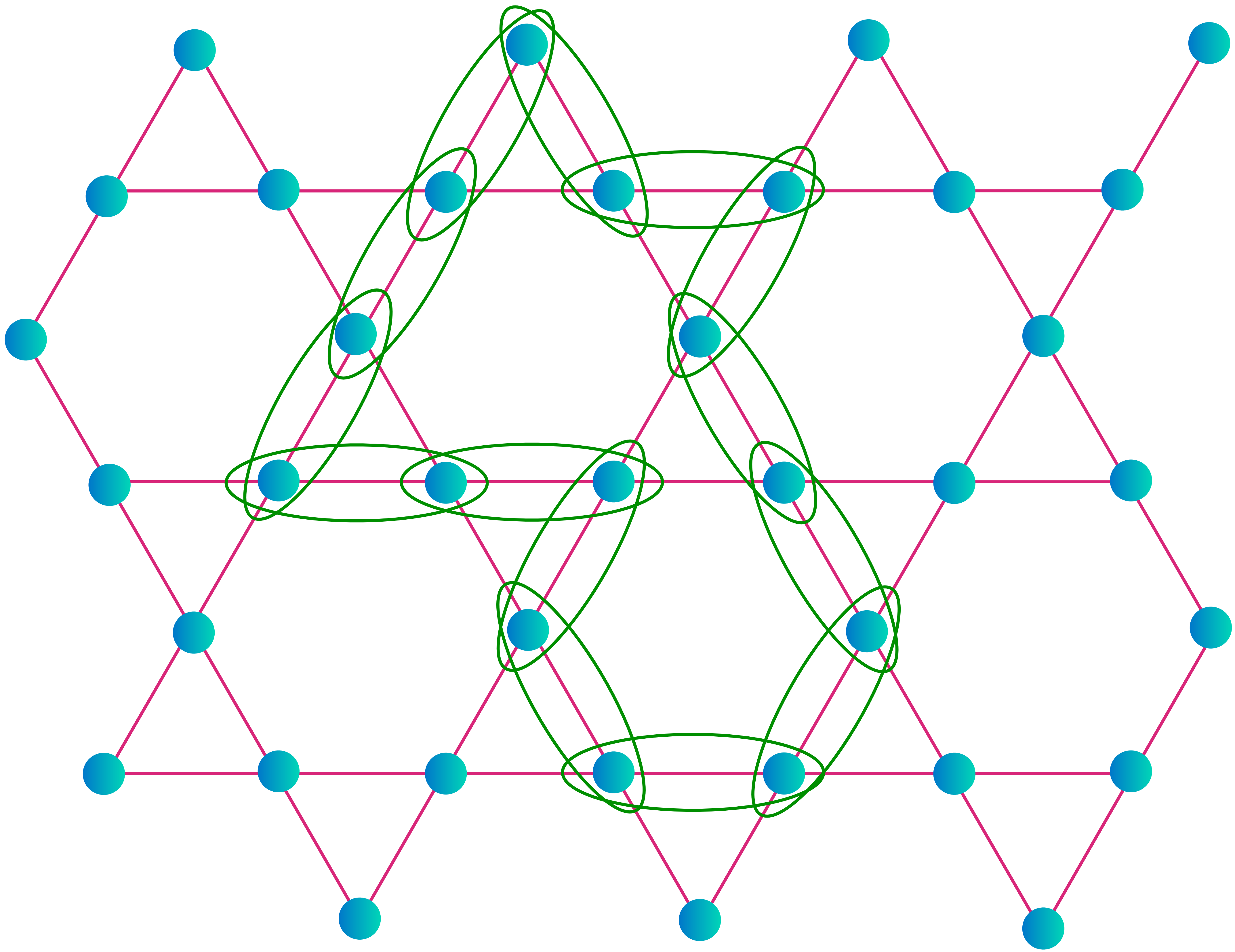
Ground state: sum over closed loops



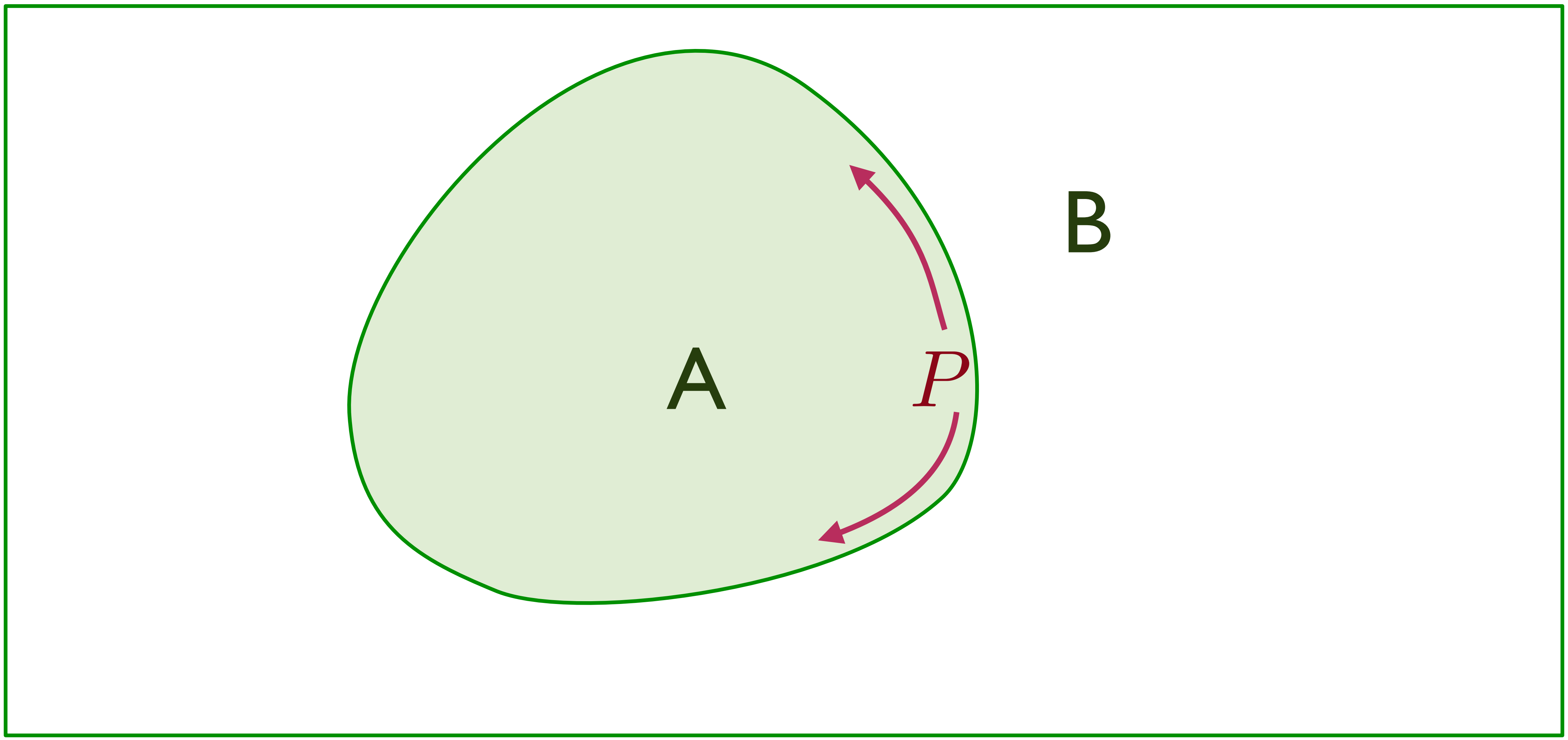
Mott insulator: Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops

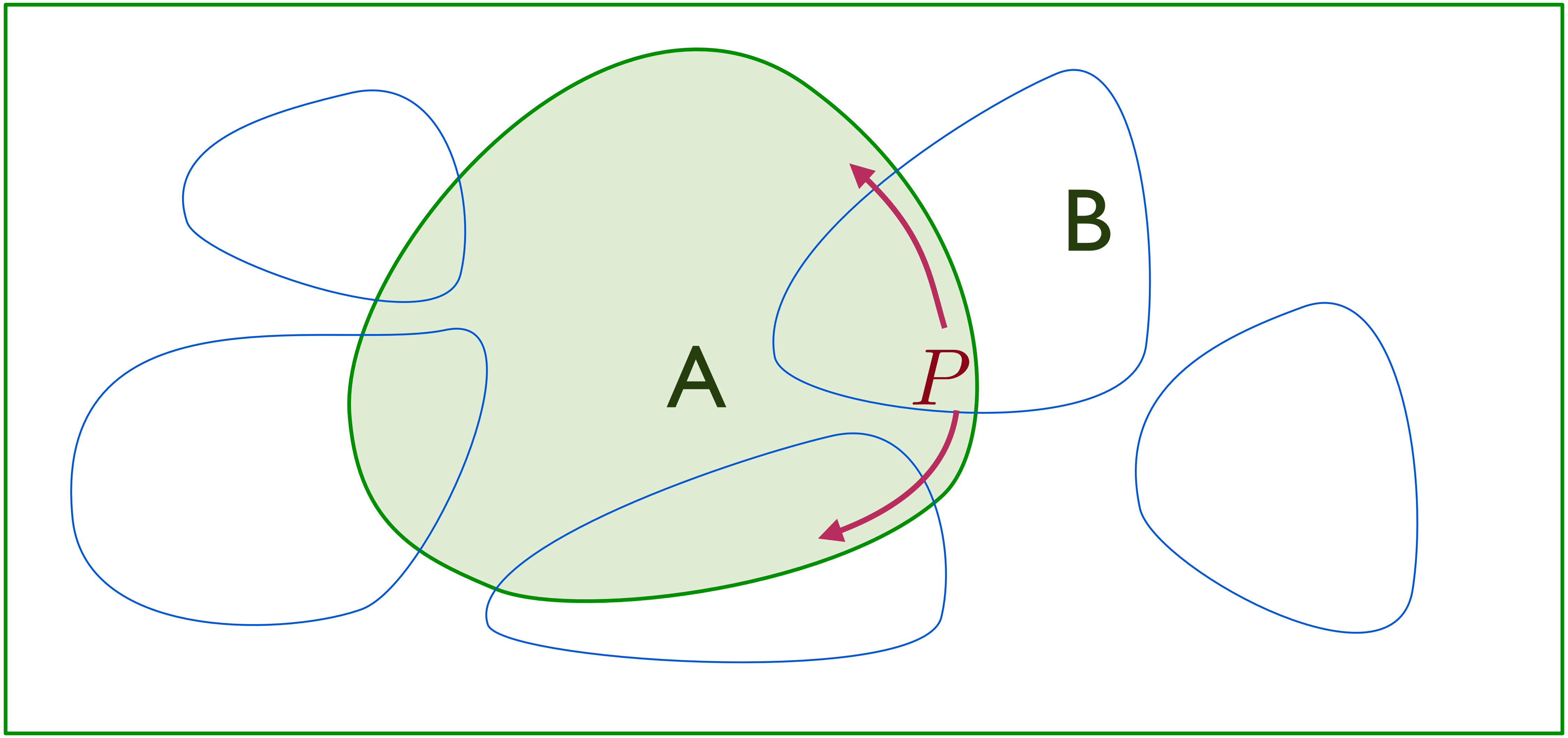


Entanglement in the Z_2 spin liquid



The sum over closed loops is characteristic of the Z_2 spin liquid, introduced in
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991),
X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)

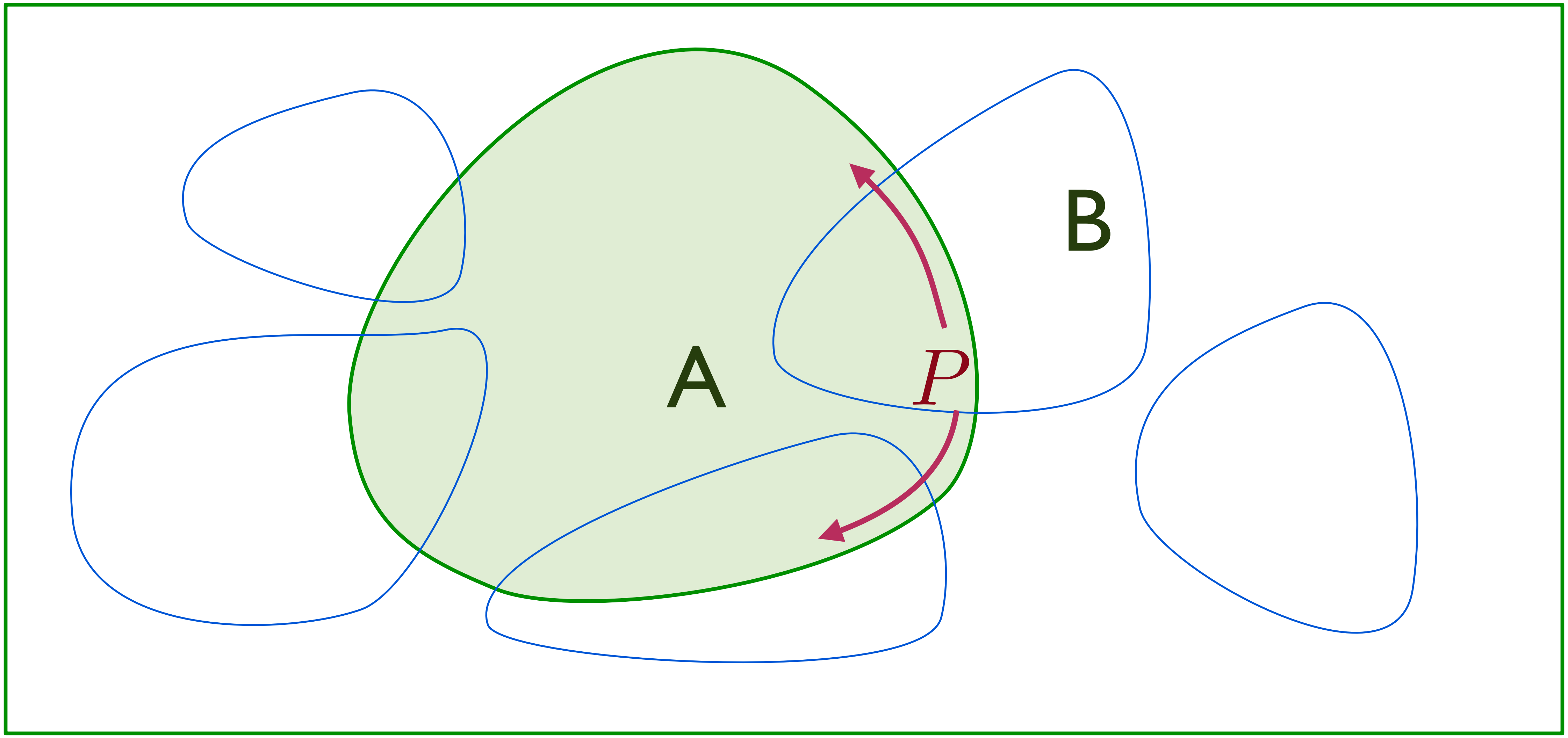
Entanglement in the Z_2 spin liquid



Sum over closed loops: only an even number of links cross the boundary between A and B

The sum over closed loops is characteristic of the Z_2 spin liquid, introduced in
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991),
X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)

Entanglement in the Z_2 spin liquid

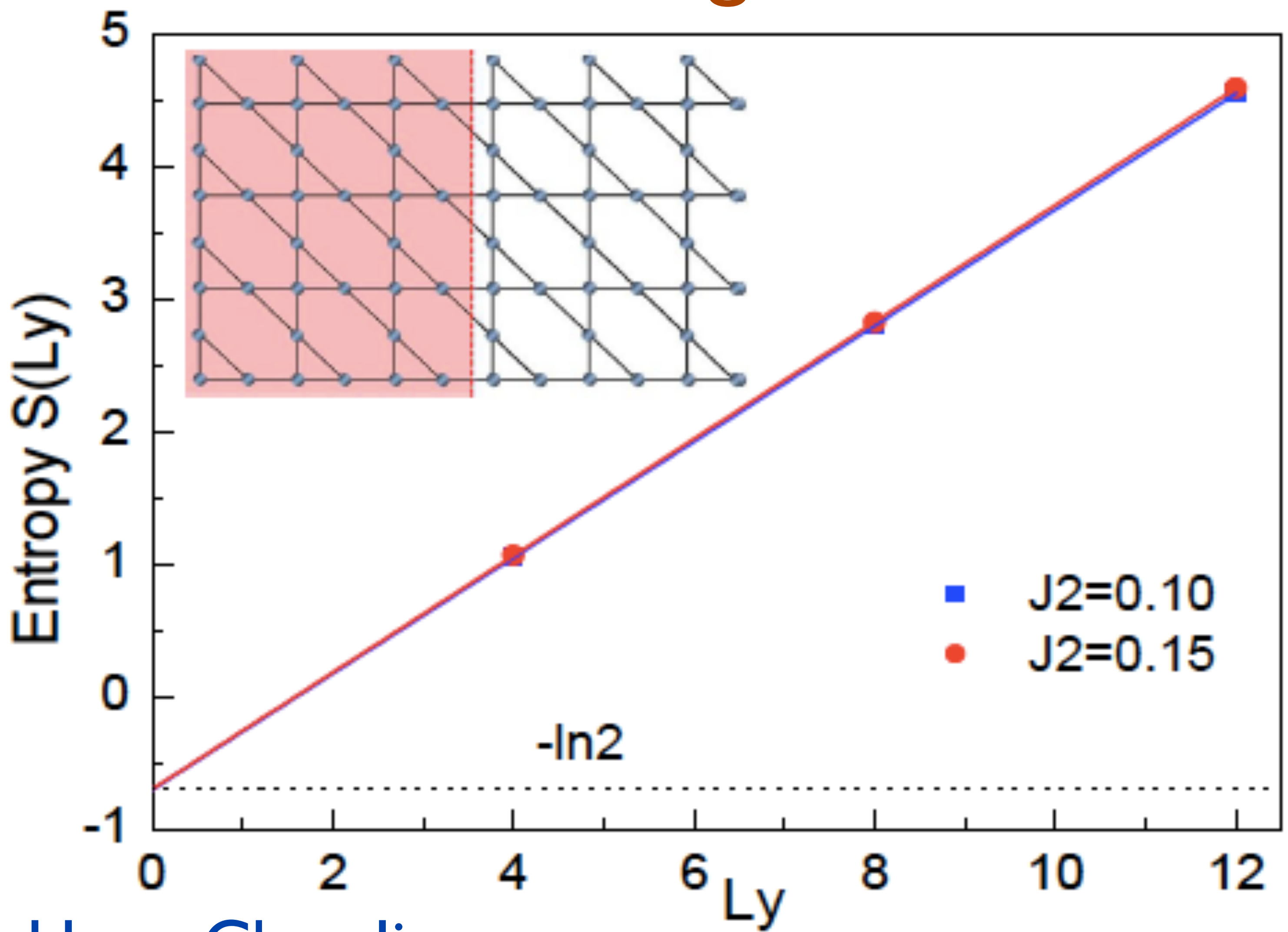


$$S_E = aP - \ln(2)$$

where P is the surface area (perimeter) of the boundary between A and B.

A. Hamma, R. Ionicioiu, and P. Zanardi, Phys. Rev. A **71**, 022315 (2005)
M. Levin and X.-G. Wen, Phys. Rev. Lett. **96**, 110405 (2006); A. Kitaev and J. Preskill, Phys. Rev. Lett. **96**, 110404 (2006)
Y. Zhang, T. Grover, and A. Vishwanath, Phys. Rev. B **84**, 075128 (2011)

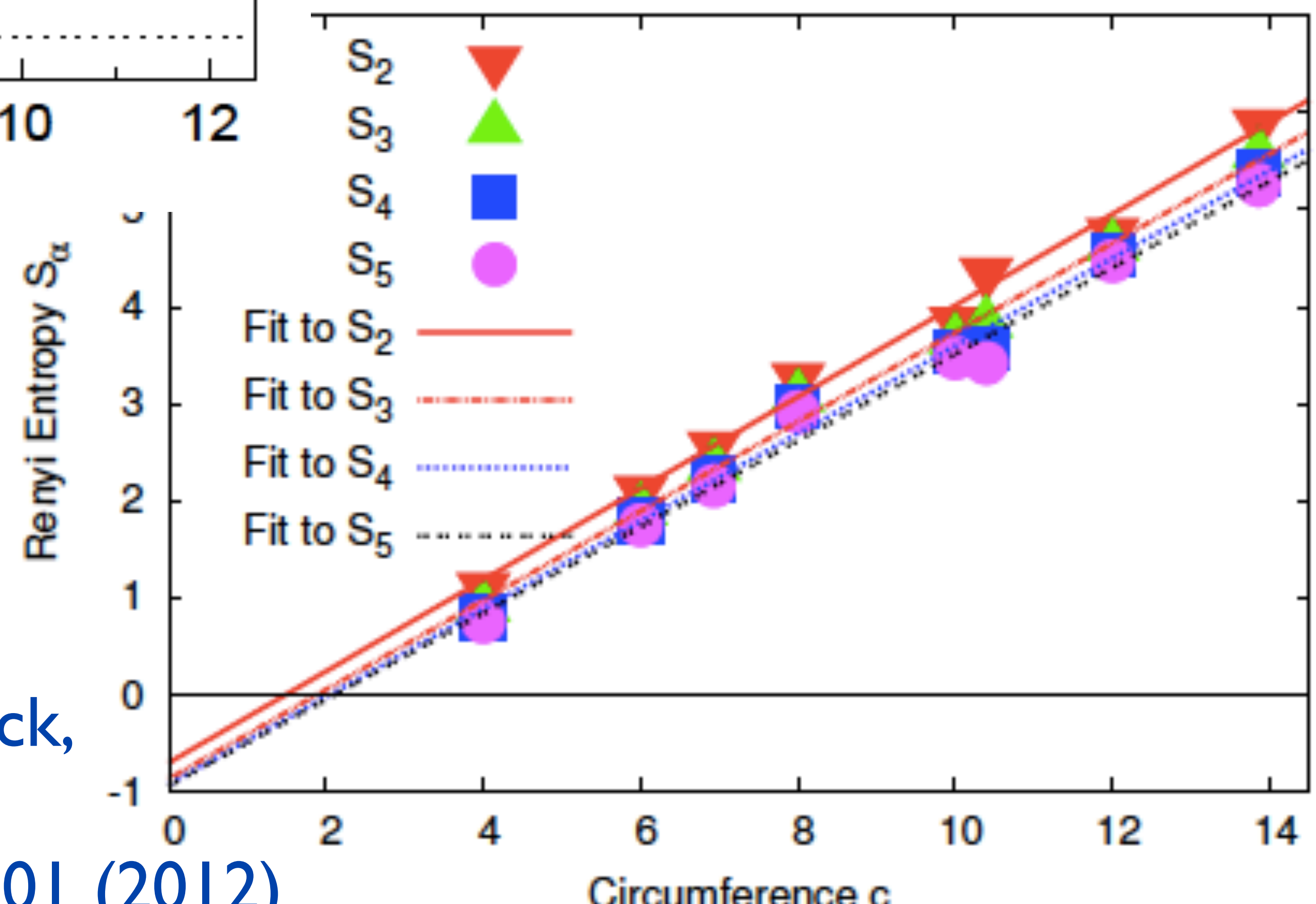
Mott insulator: Kagome antiferromagnet



Strong numerical evidence for a Z_2 spin liquid

Simeng Yan, D. A. Huse, and S. R. White, *Science* **332**, 1173 (2011).

Hong-Chen Jiang, Z. Wang, and L. Balents, *Nature Physics* **8**, 902 (2012)



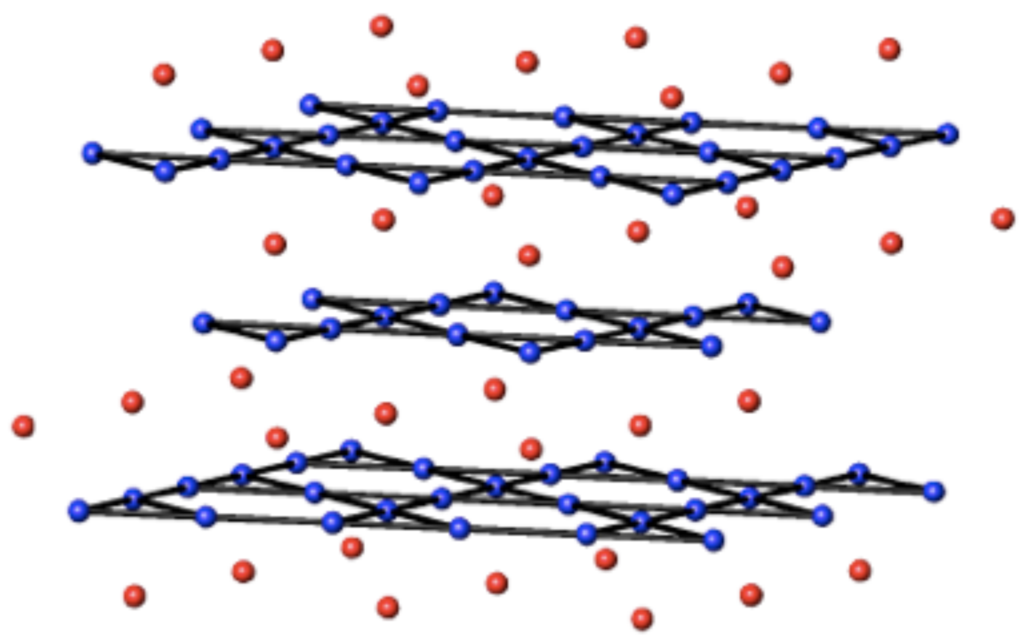
S. Depenbrock, I. P. McCulloch, and U. Schollwoeck, *Physical Review Letters* **109**, 067201 (2012)

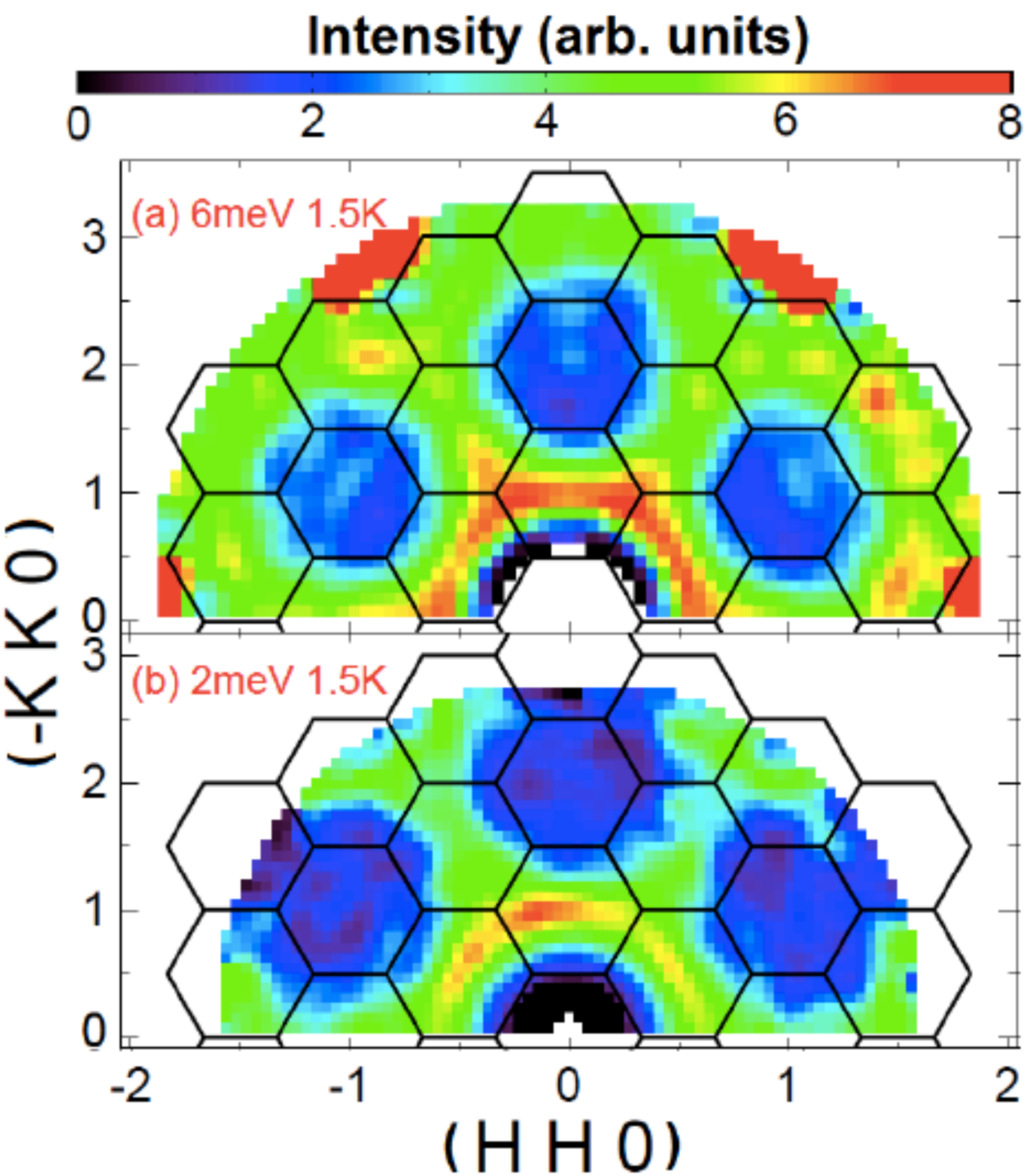
Fractionalized excitations in the spin-liquid state of a kagome-lattice antiferromagnet

Tian-Heng Han[1], Joel S. Helton[2], Shaoyan Chu[3], Daniel G. Nocera[4], Jose A. Rodriguez-Rivera[2,5], Collin Broholm[2,6] & Young S. Lee[1]

Nature **492**, 406 (2012)

$ZnCu_3(OH)_6Cl_2$ (also called Herbertsmithite)





"Complex entangled" states of quantum matter, *not* adiabatically connected to independent particle states

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene

Compressible quantum matter

Strange metals in high temperature superconductors, Bose metals

S. Sachdev, 100th anniversary Solvay conference,arXiv:1203.4565

“Complex entangled” states of
quantum matter,
not adiabatically connected to independent particle states

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene

Compressible quantum matter

Strange metals in high temperature superconductors, Bose metals

S. Sachdev, 100th anniversary Solvay conference,arXiv:1203.4565

Outline

1. Higgs quasi-normal mode near the superfluid-insulator transition in 2 dimensions

2. Quantum criticality and conformal field theories

3. Holography and the quasi-normal modes of black-hole horizons

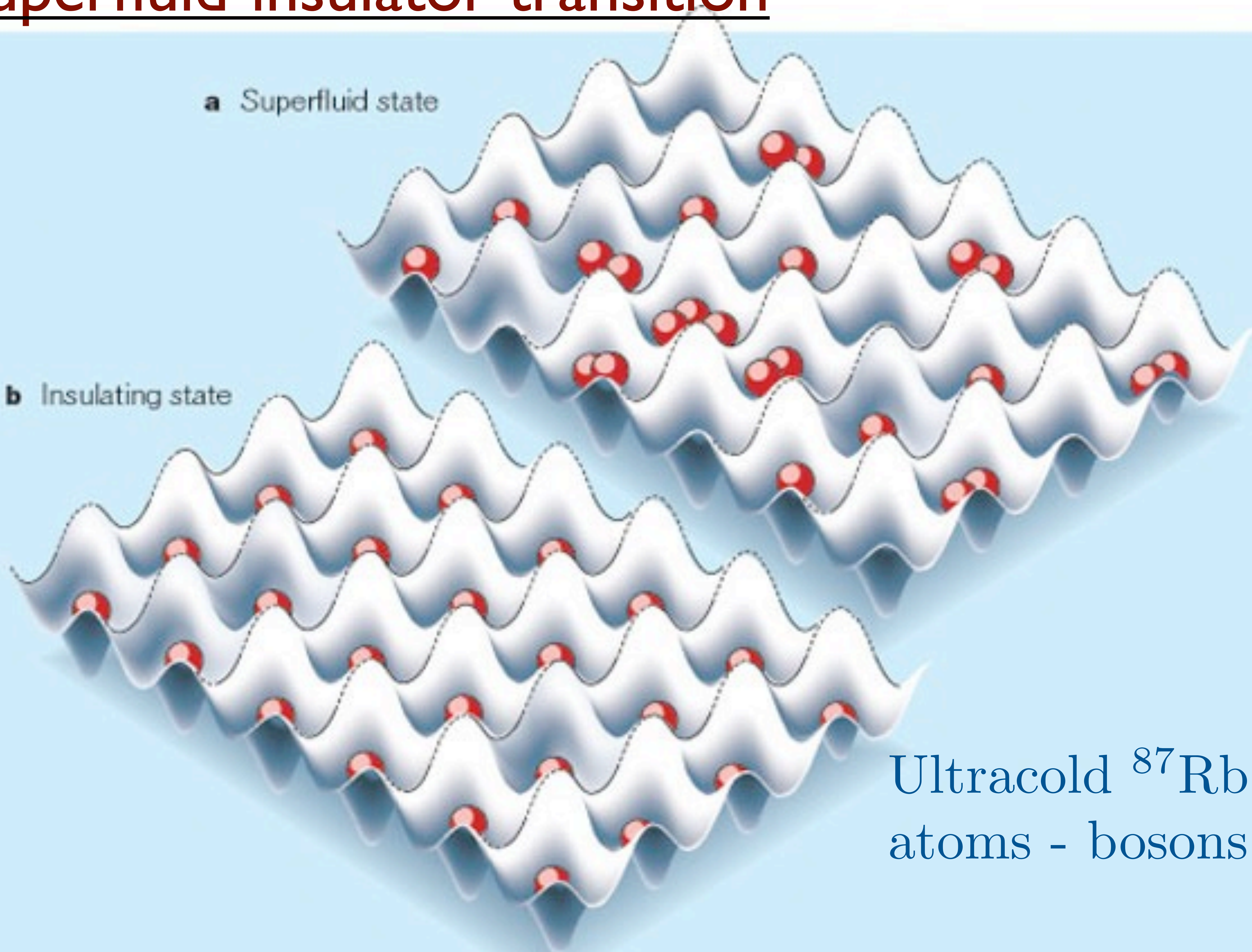
Outline

1. Higgs quasi-normal mode near the superfluid-insulator transition in 2 dimensions

2. Quantum criticality and conformal field theories

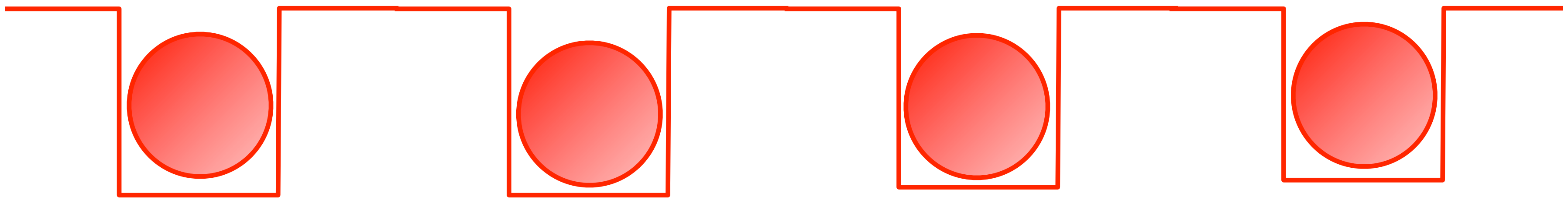
3. Holography and the quasi-normal modes of black-hole horizons

Superfluid-insulator transition



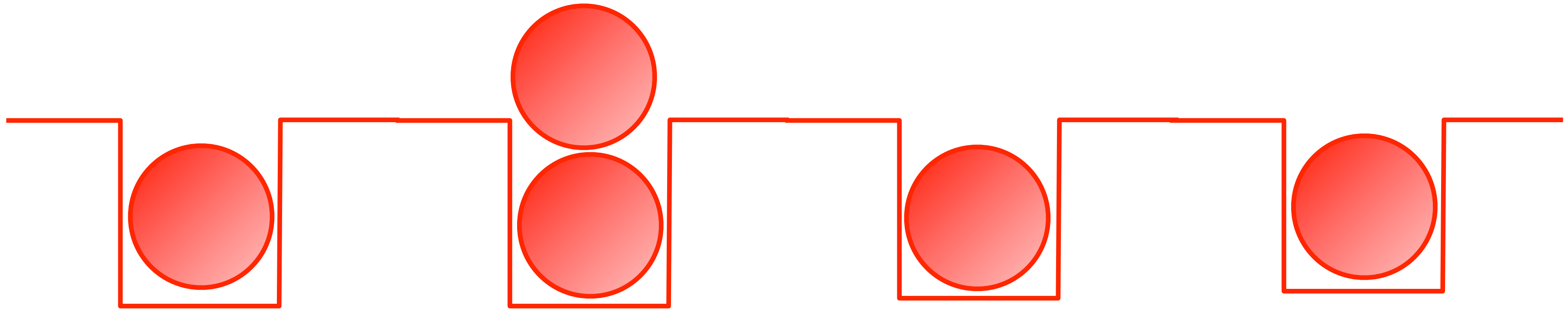
Ultracold ^{87}Rb
atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).



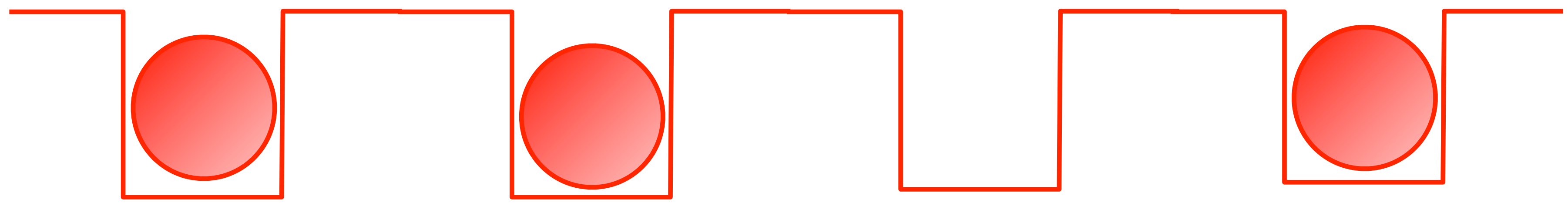
Insulator (the vacuum)
at large repulsion between bosons

Excitations of the insulator:



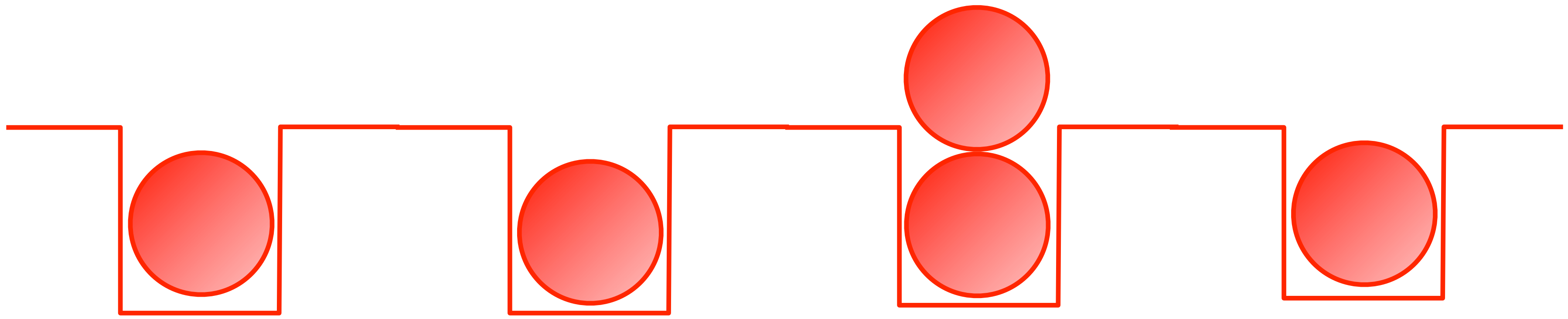
Particles $\sim \Psi^\dagger$

Excitations of the insulator:

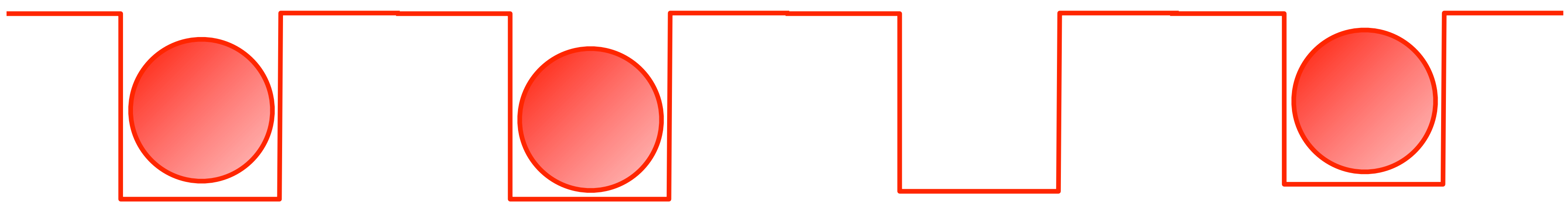


$$\text{Holes} \sim \Psi$$

Excitations of the insulator:



Particles $\sim \Psi^\dagger$



Holes $\sim \Psi$

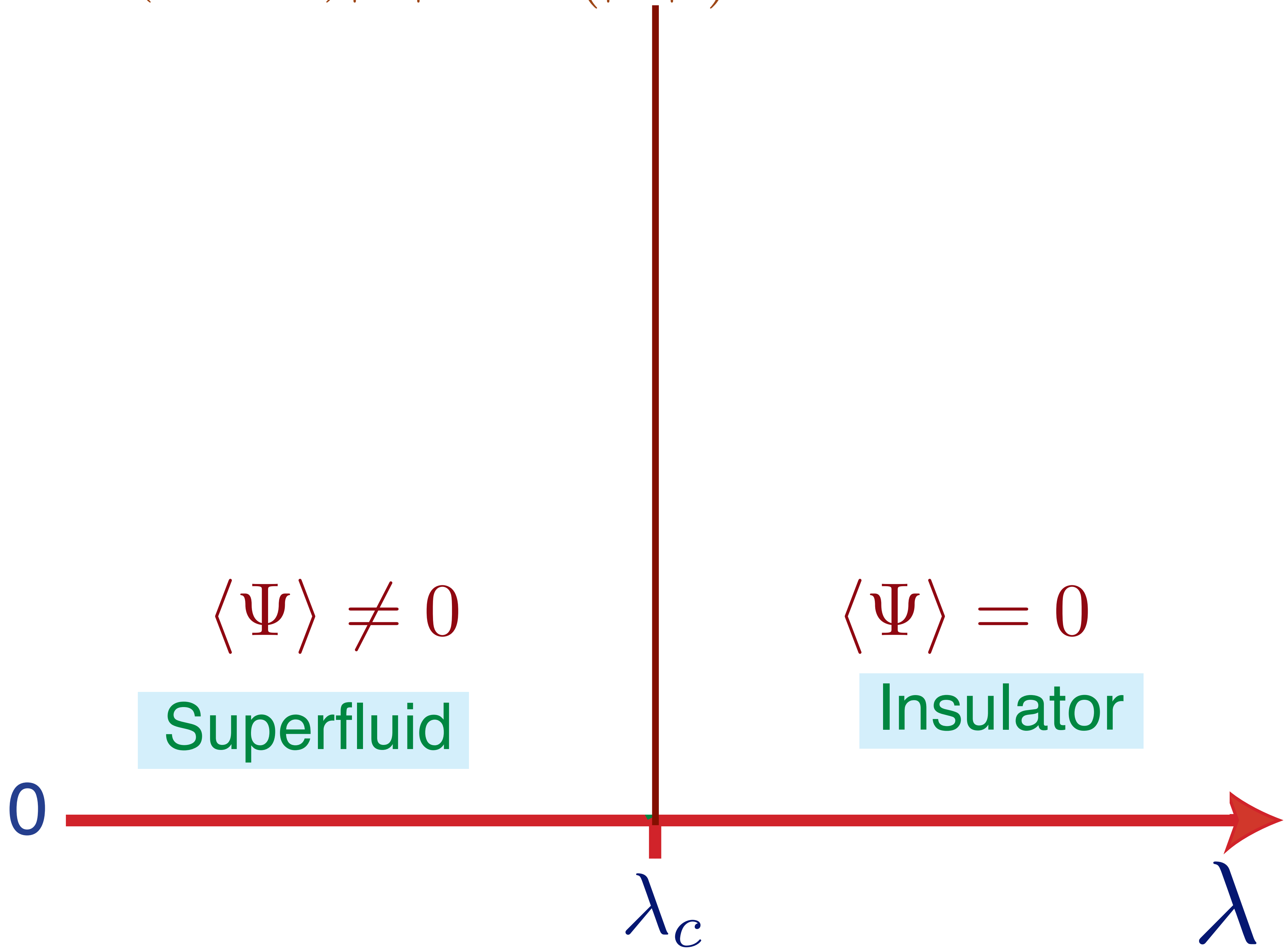
Density of particles = density of holes $\Rightarrow$
"Relativistic" field theory for Ψ:

$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2\right)^2 \right]$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).

$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

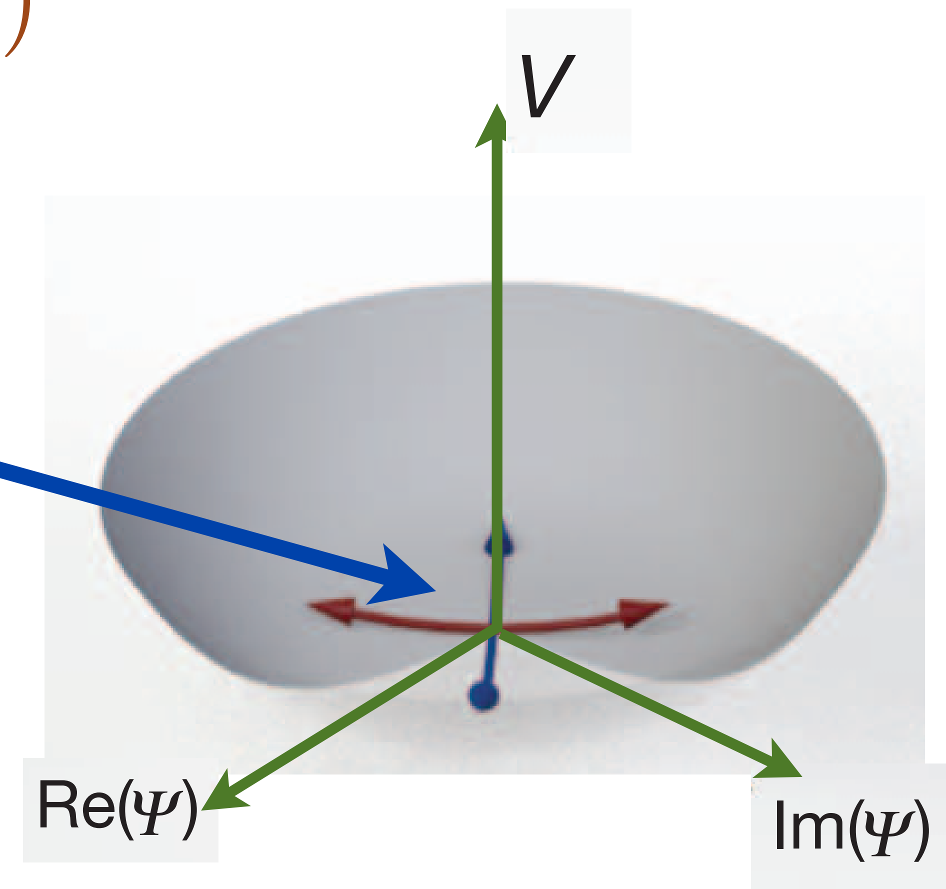
$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$



$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$

Particles and holes correspond to the 2 normal modes in the oscillation of Ψ about $\Psi = 0$.



$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

Insulator

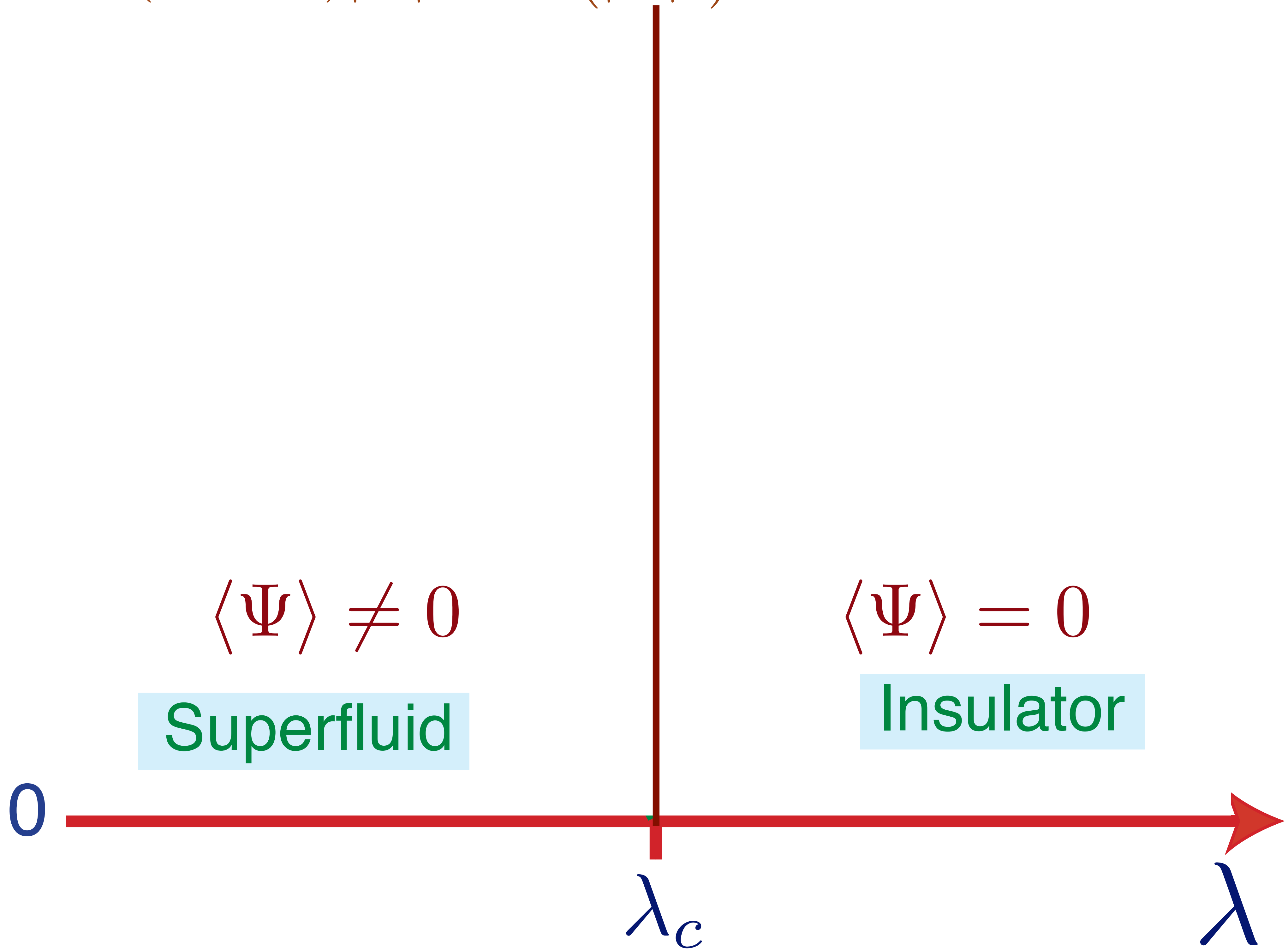
0

λ_c

λ

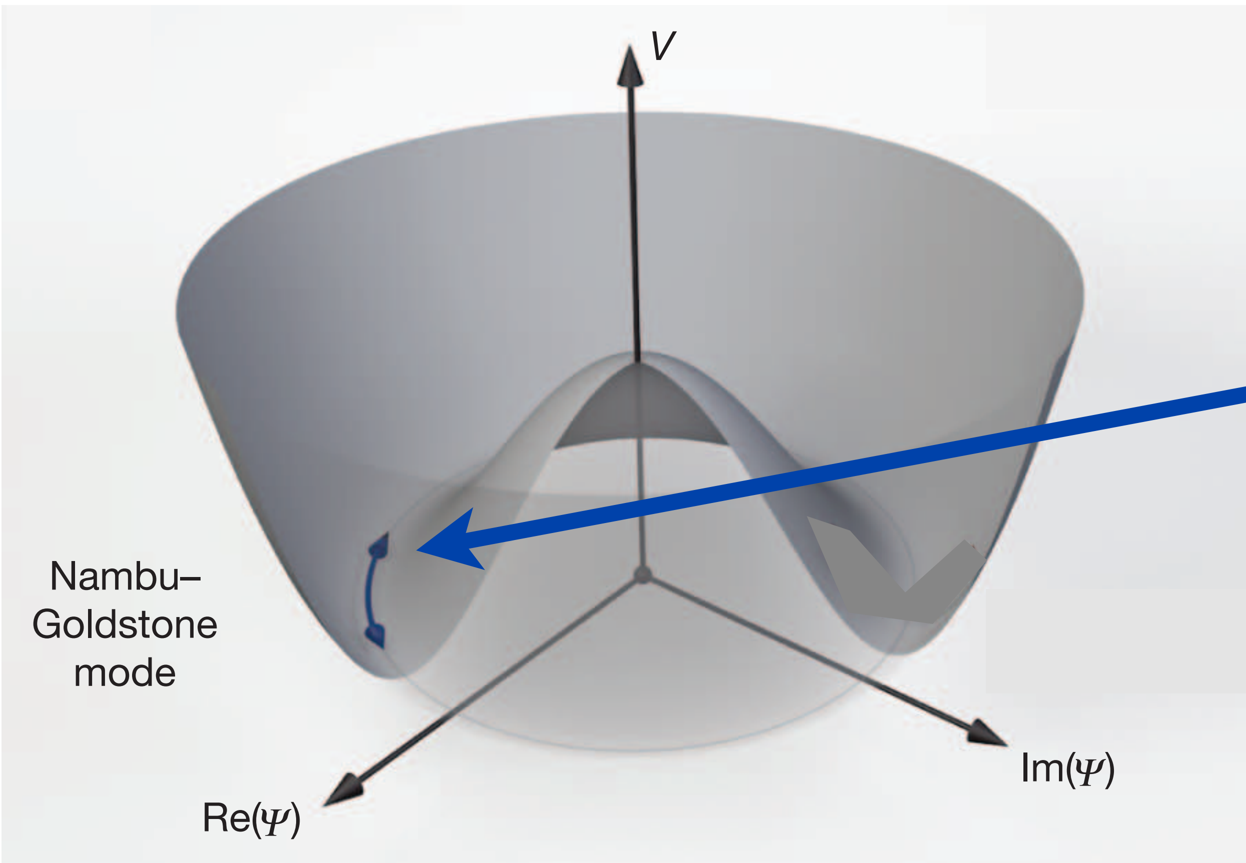
$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$



$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$



Nambu-Goldstone mode is the oscillation in the phase Ψ at a constant non-zero $|\Psi|$.

$\langle \Psi \rangle \neq 0$

Superfluid

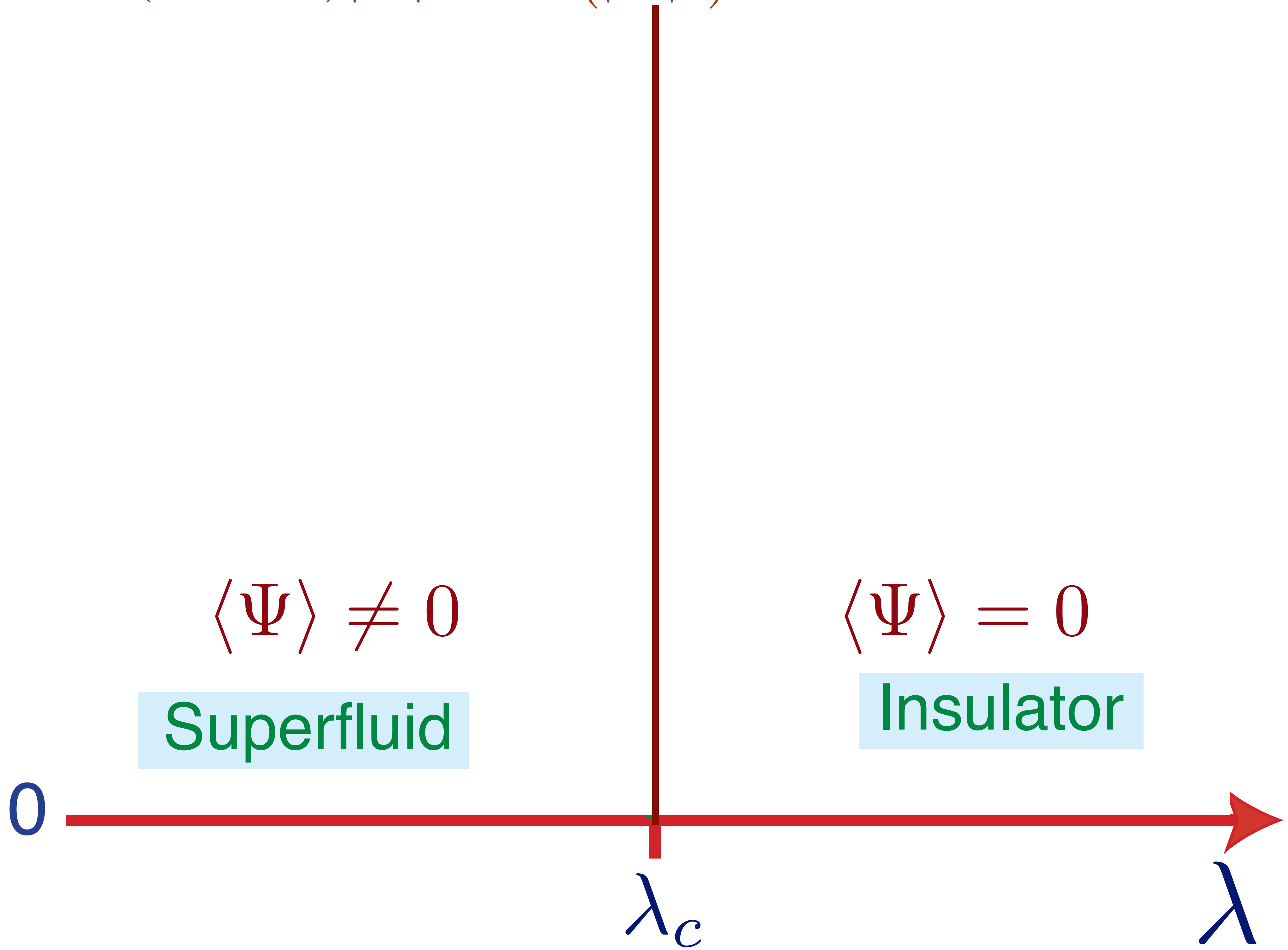
$\langle \Psi \rangle = 0$

Insulator

0 λ_c λ

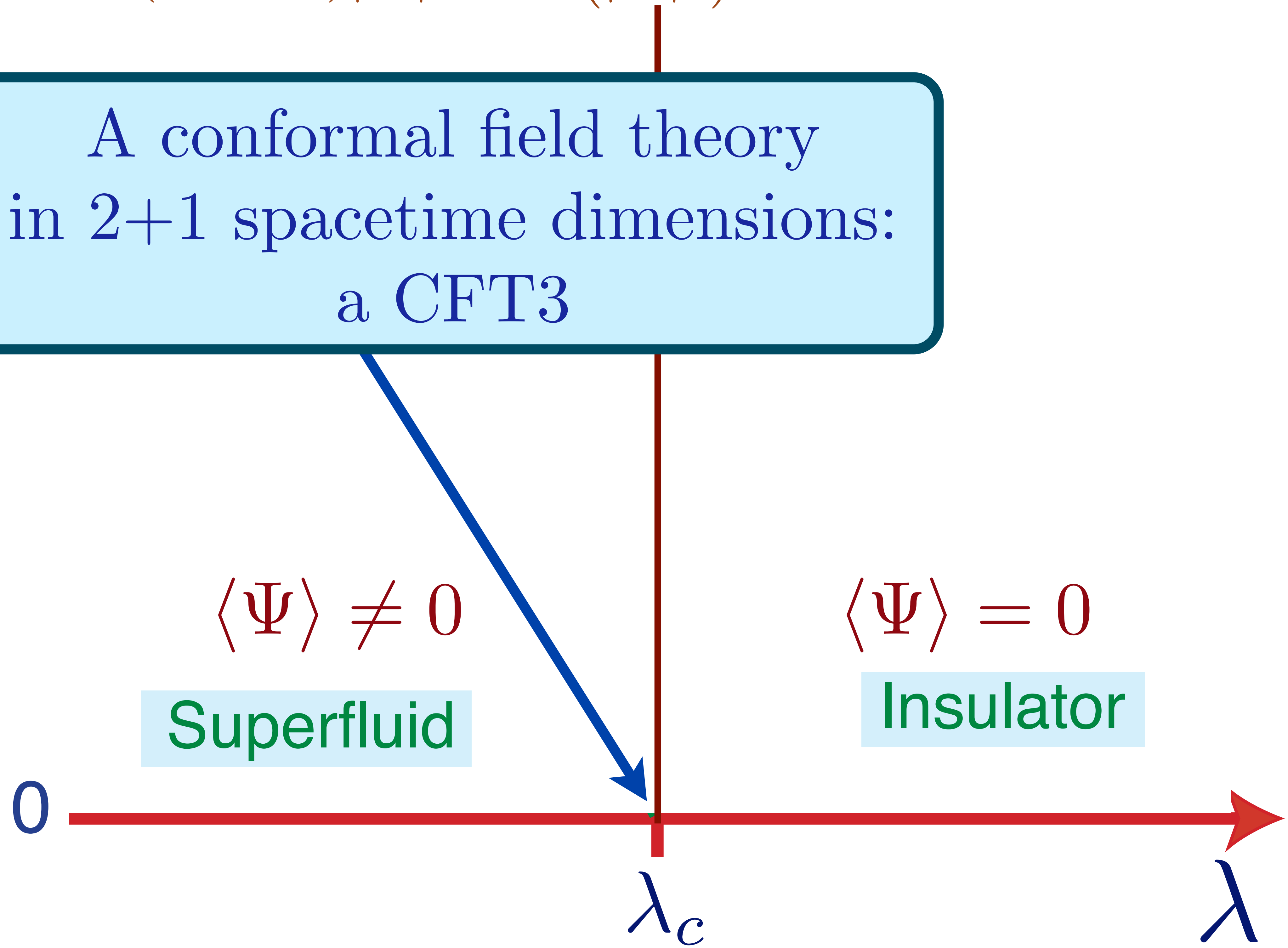
$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$



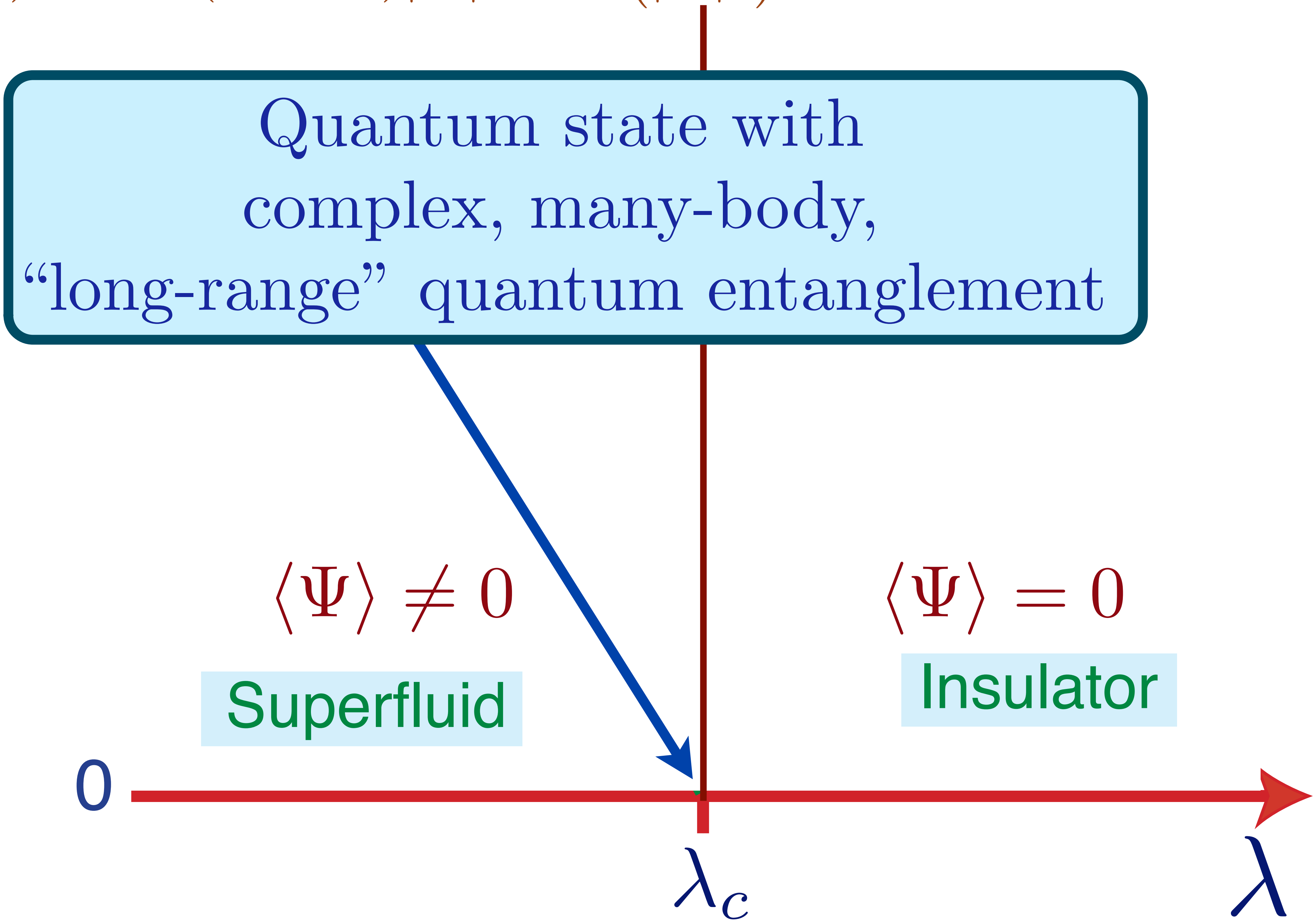
$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$



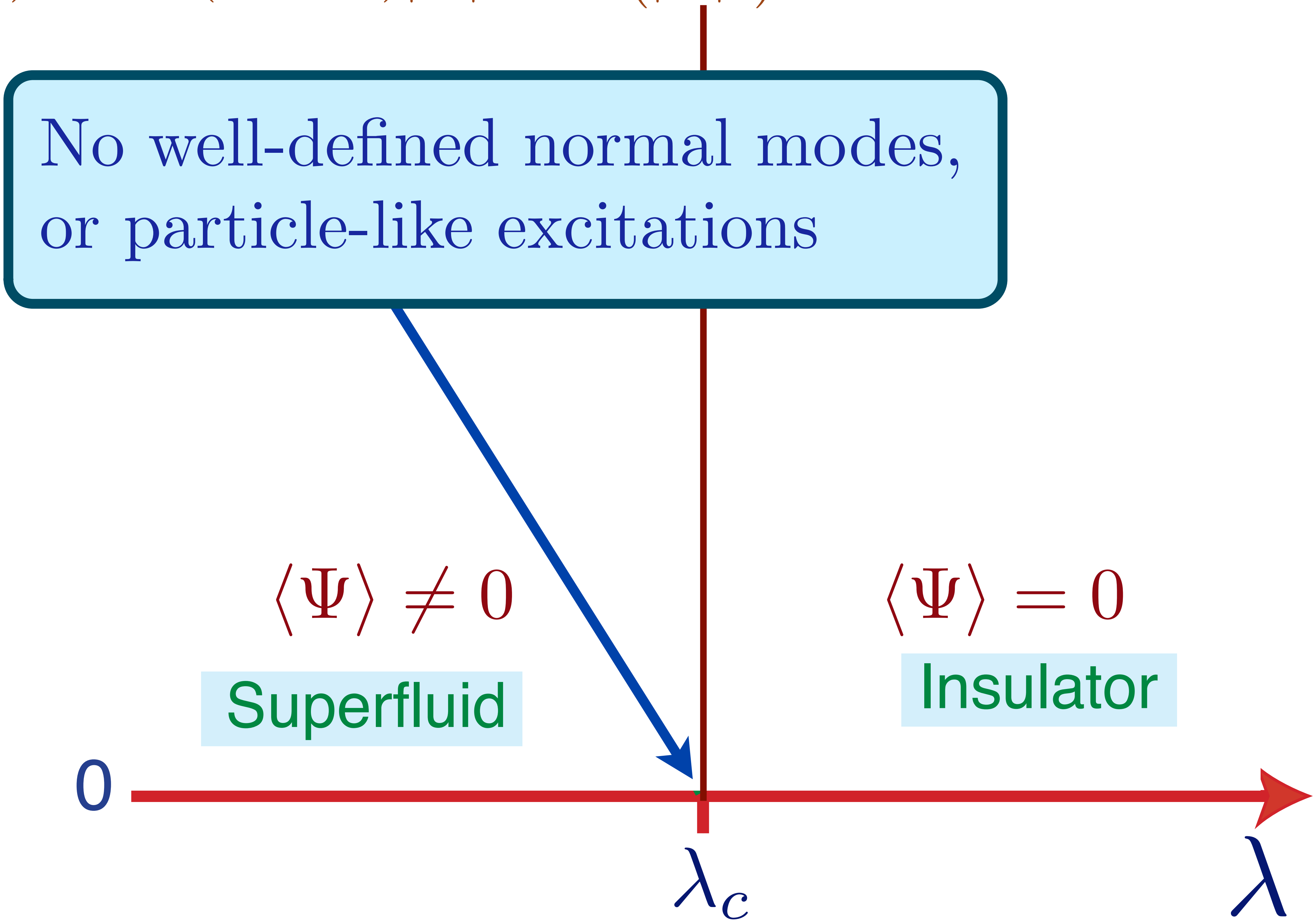
$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$



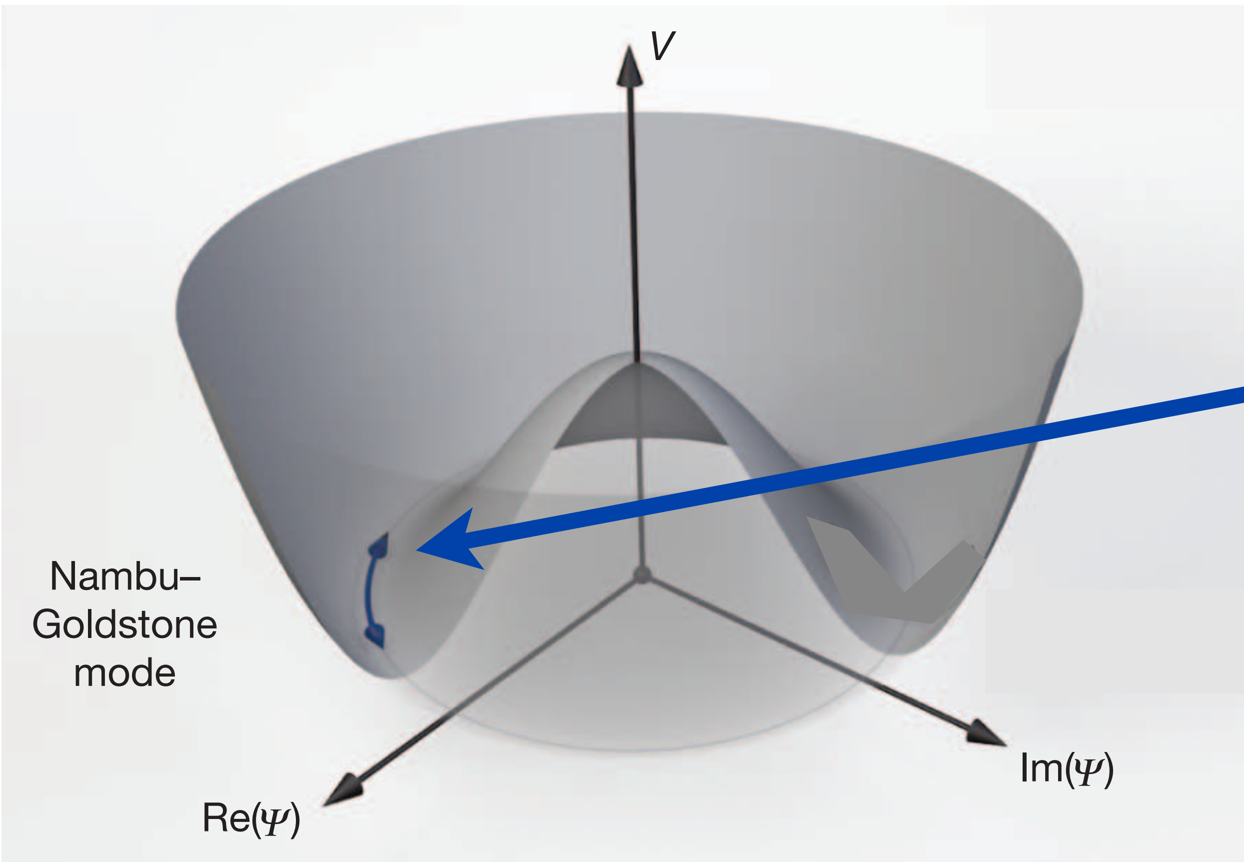
$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$



$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi)\right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$



Nambu-Goldstone mode is the oscillation in the phase Ψ at a constant non-zero $|\Psi|$.

$\langle \Psi \rangle \neq 0$

Superfluid

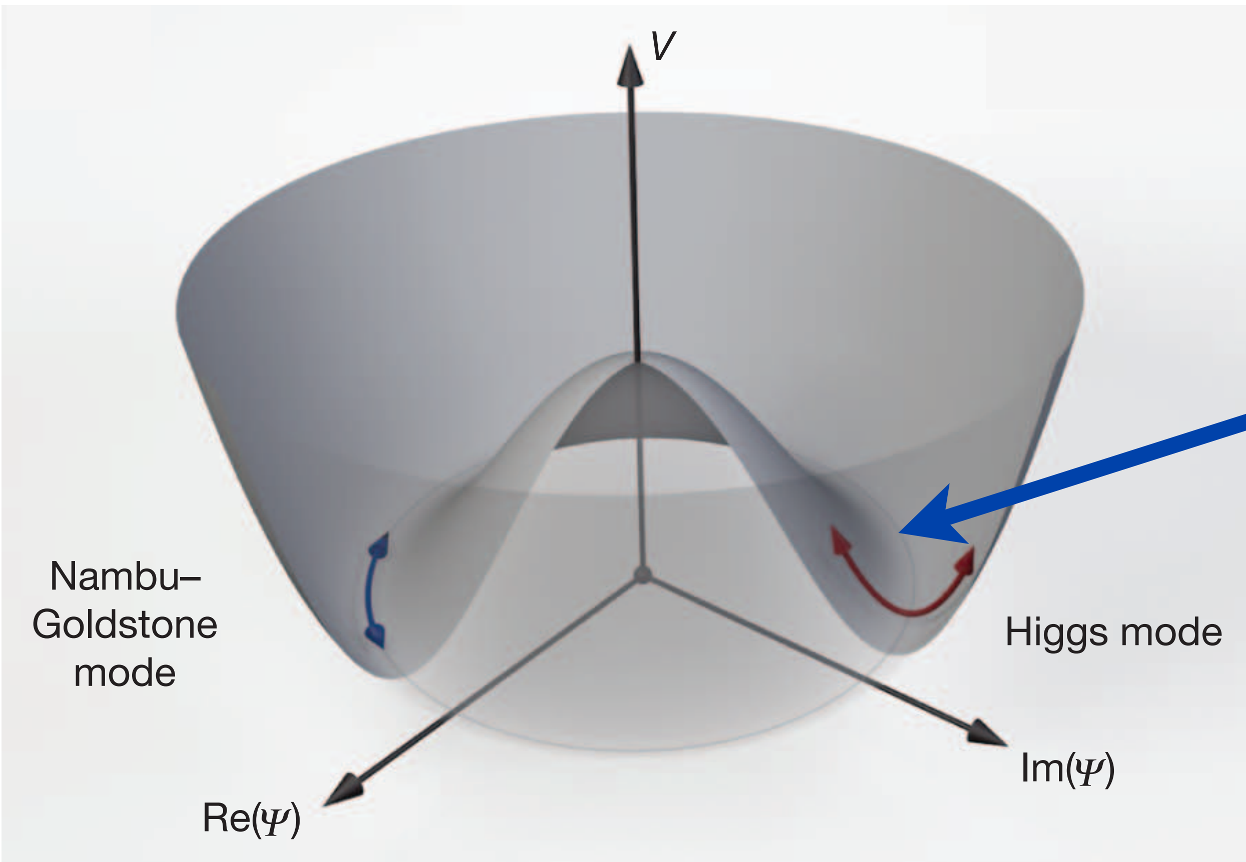
$\langle \Psi \rangle = 0$

Insulator

0 λ_c λ

$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$



Higgs mode is the oscillation in the amplitude $|\Psi|$. This decays rapidly by emitting pairs of Nambu-Goldstone modes.

$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

Insulator

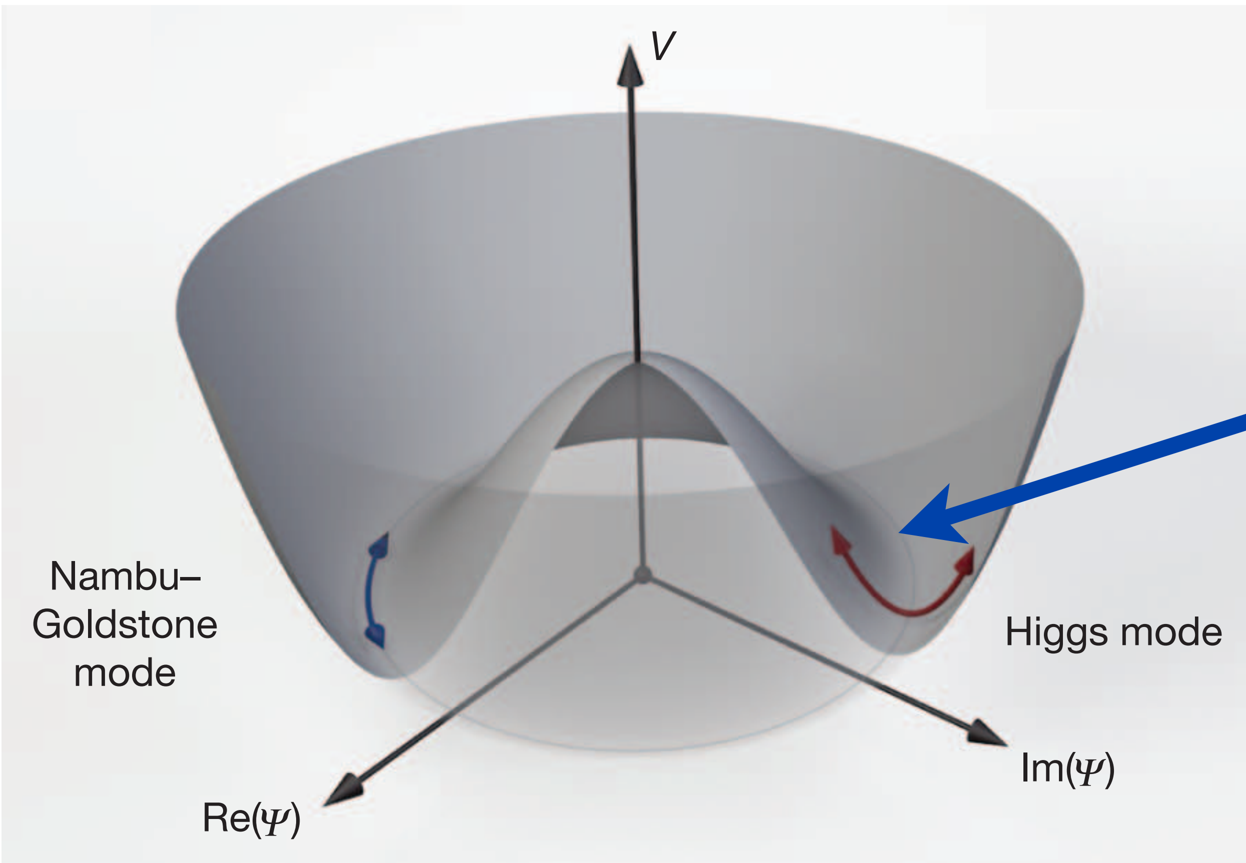
0

λ_c

λ

$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$



Despite rapid decay,
there is a well-defined
Higgs "quasi-normal mode".
This is associated with a pole
in the lower-half
of the complex frequency plane.

$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

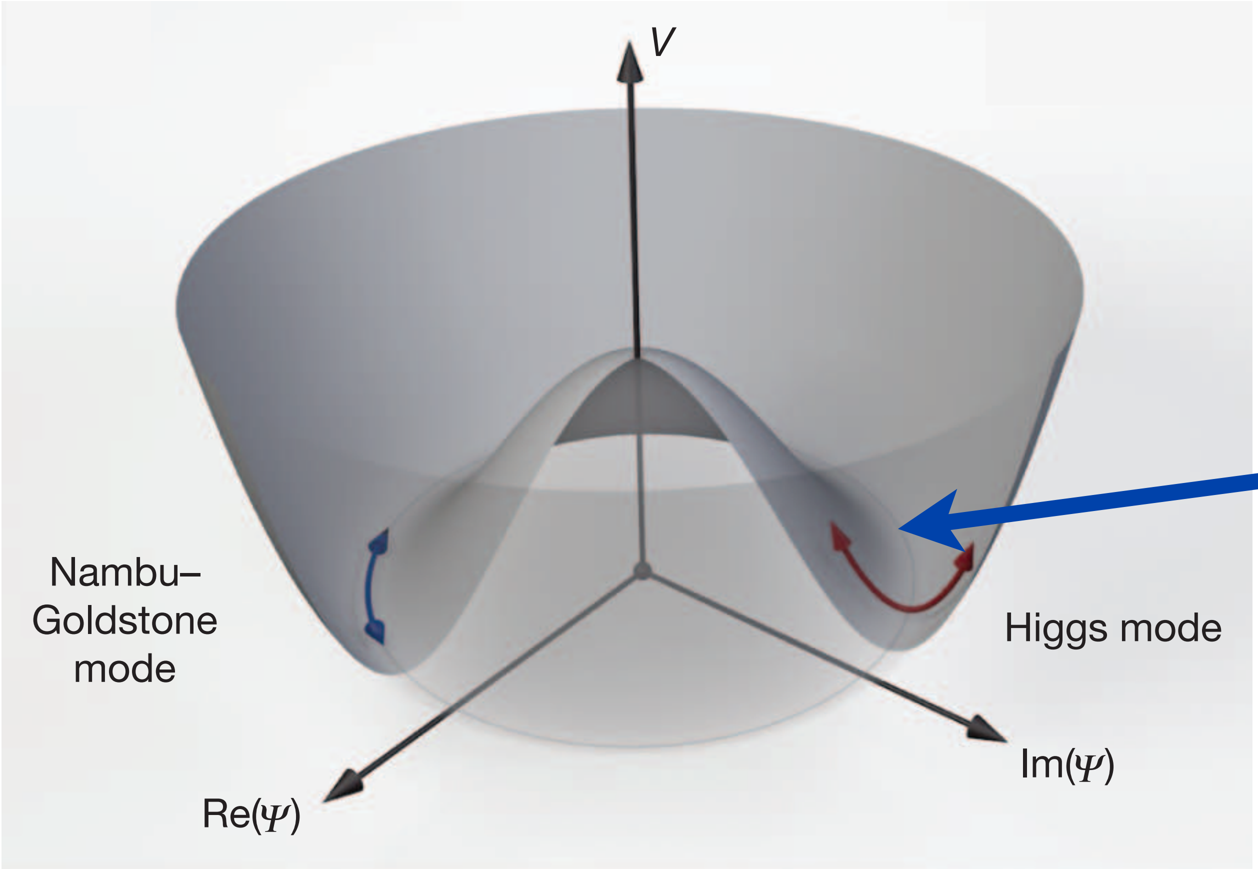
Insulator

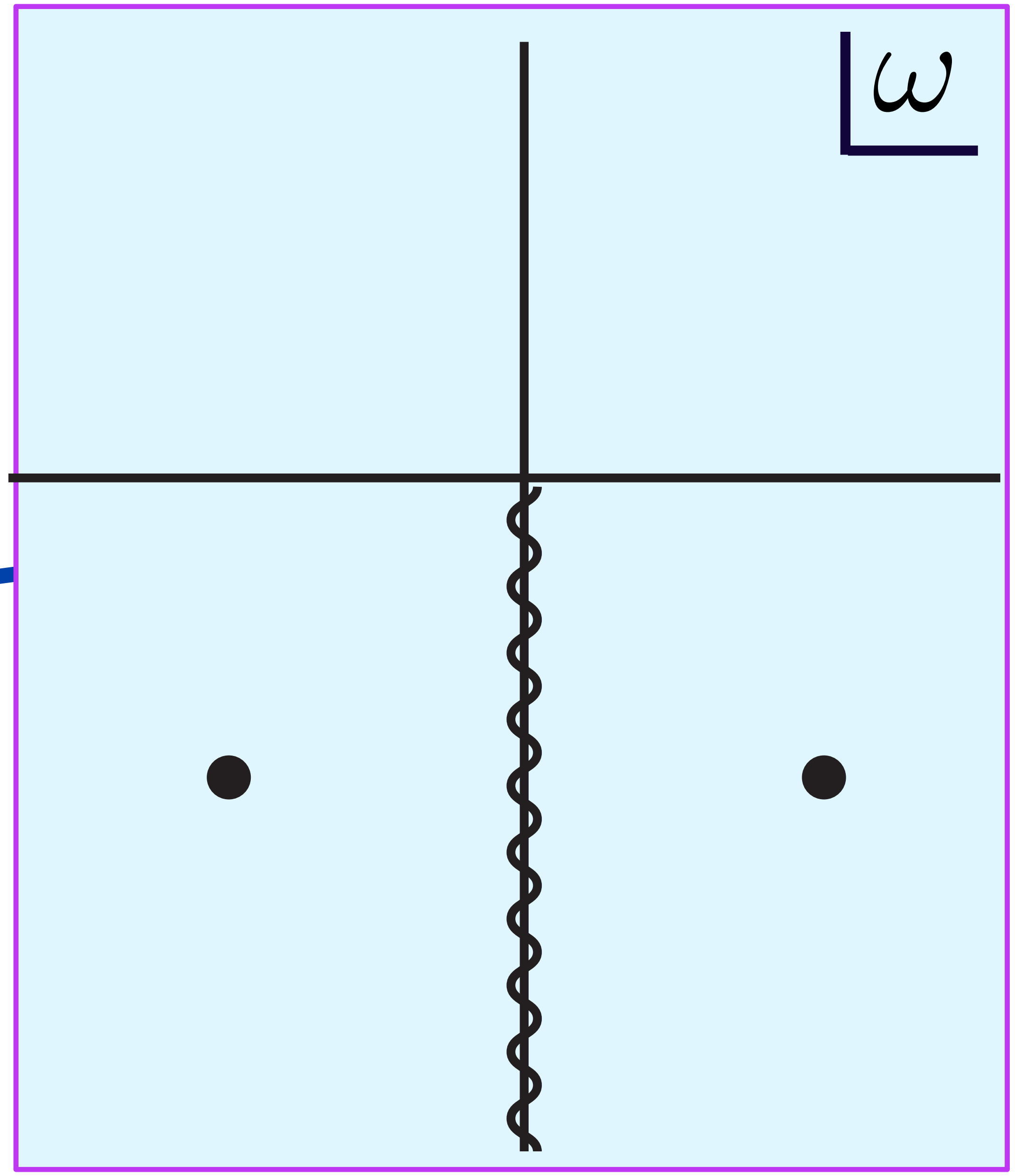
0 λ_c λ

A. V. Chubukov, S. Sachdev, and J. Ye, PRB **49**, 11919 (1994). S. Sachdev, PRB **59**, 14054 (1999).
W. Zwerger, PRL **92**, 027203 (2004).
D. Podolsky, A. Auerbach, and D. P. Arovas, PRB **84**, 174522 (2011).
D. Podolsky and S. Sachdev, PRB **86**, 054508 (2012). L. Pollet and N. Prokof'ev, PRL **109**, 010401 (2012).

$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$





D. Podolsky and S. Sachdev, Phys. Rev. B **86**, 054508 (2012).

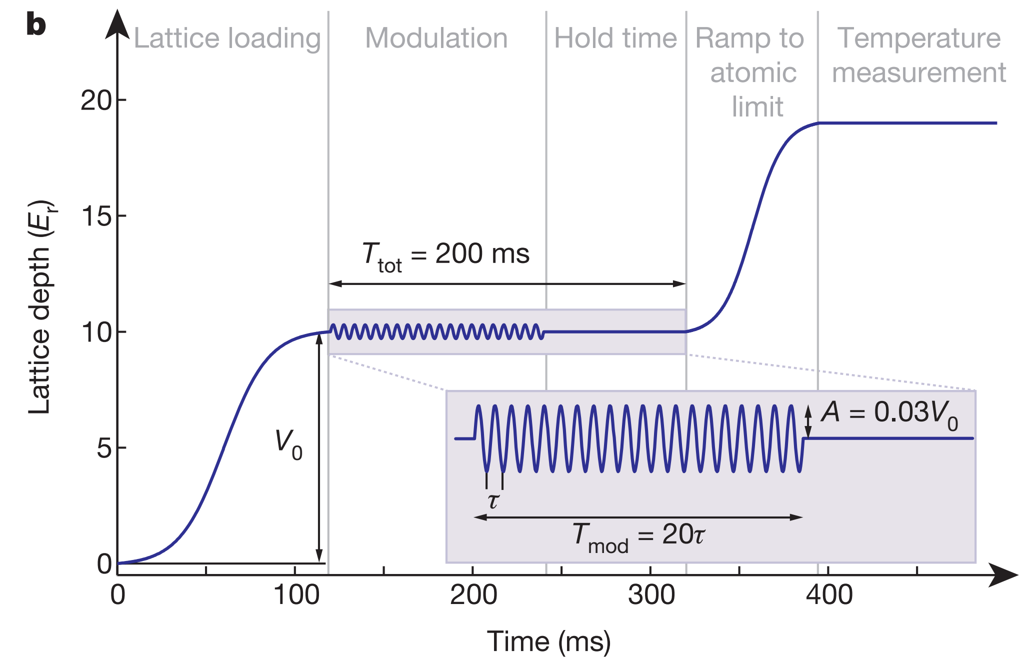
The Higgs quasi-normal mode is at the frequency

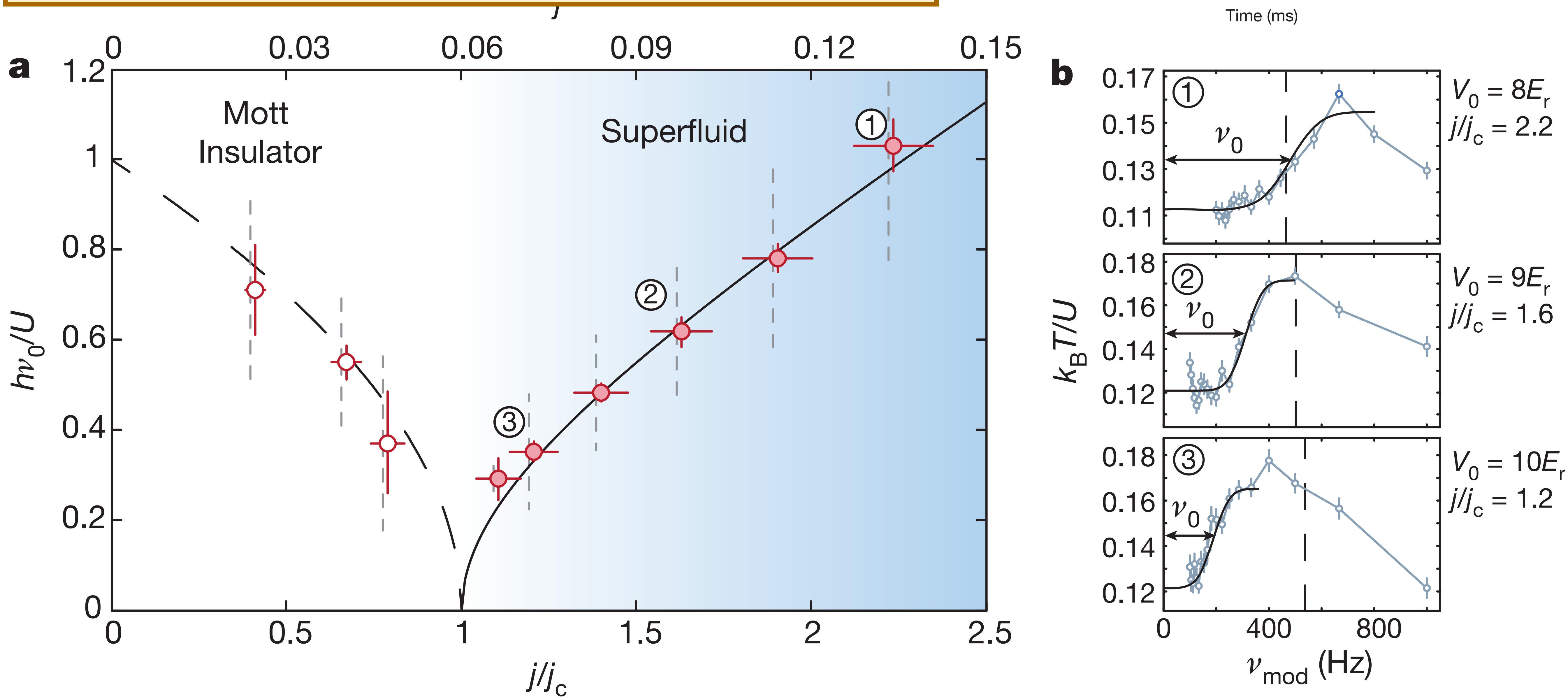
$$\frac{\omega_{\text{pole}}}{\Delta} = -i\,\frac{4}{\pi} + \frac{1}{N}\left(\frac{16\left(4 + \sqrt{2}\log\left(3 - 2\sqrt{2}\right)\right)}{\pi^2} + 2.46531203396\, i \right) + \mathcal{O}\left(\frac{1}{N^2}\right)$$

where Δ is the particle gap at the complementary point in the "paramagnetic" state with the same value of $|\lambda - \lambda_c|$, and $N = 2$ is the number of vector components of Ψ. The universal answer is a consequence of the strong interactions in the CFT3

Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:

Response to modulation of lattice depth scales as expected from the LHP pole





Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:
Response to modulation of lattice depth scales as expected from the LHP pole

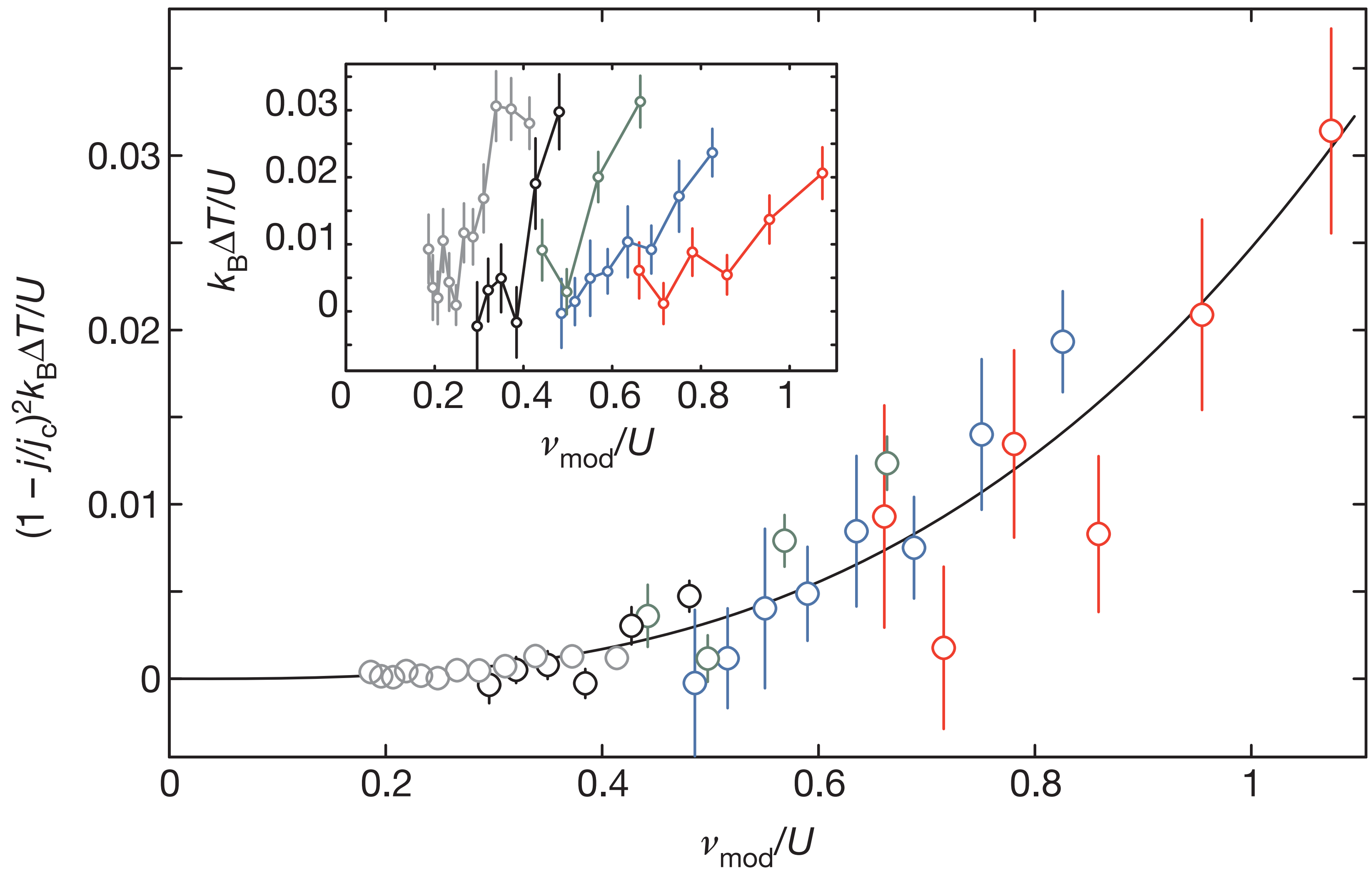


Figure 4 | Scaling of the low-frequency response. The low-frequency response in the superfluid regime shows a scaling compatible with the prediction $(1 - j/j_c)^{-2}\nu^3$ (Methods). Shown is the temperature response rescaled with $(1 - j/j_c)^2$ for $V_0 = 10E_r$ (grey), $9.5E_r$ (black), $9E_r$ (green), $8.5E_r$ (blue) and $8E_r$ (red) as a function of the modulation frequency. The black line is a fit of the form $a\nu^b$ with a fitted exponent $b = 2.9(5)$. The inset shows the same data points without rescaling, for comparison. Error bars, s.e.m.

D. Podolsky and S. Sachdev, Phy. Rev. B **86**, 054508 (2012).

Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

Outline

1. Higgs quasi-normal mode near the superfluid-insulator transition in 2 dimensions

2. Quantum criticality and conformal field theories

3. Holography and the quasi-normal modes of black-hole horizons

Outline

1. Higgs quasi-normal mode near the superfluid-insulator transition in 2 dimensions

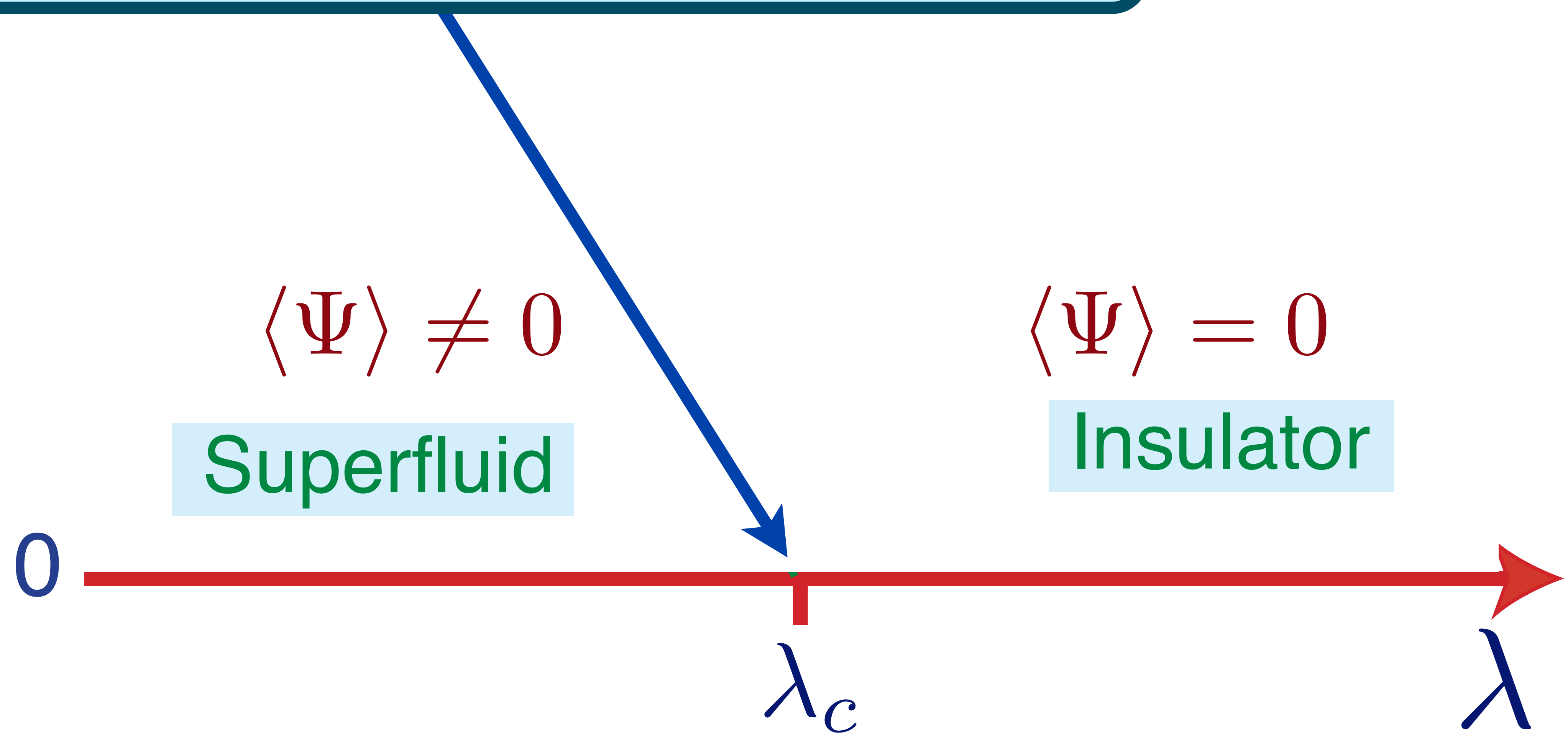
2. Quantum criticality and conformal field theories

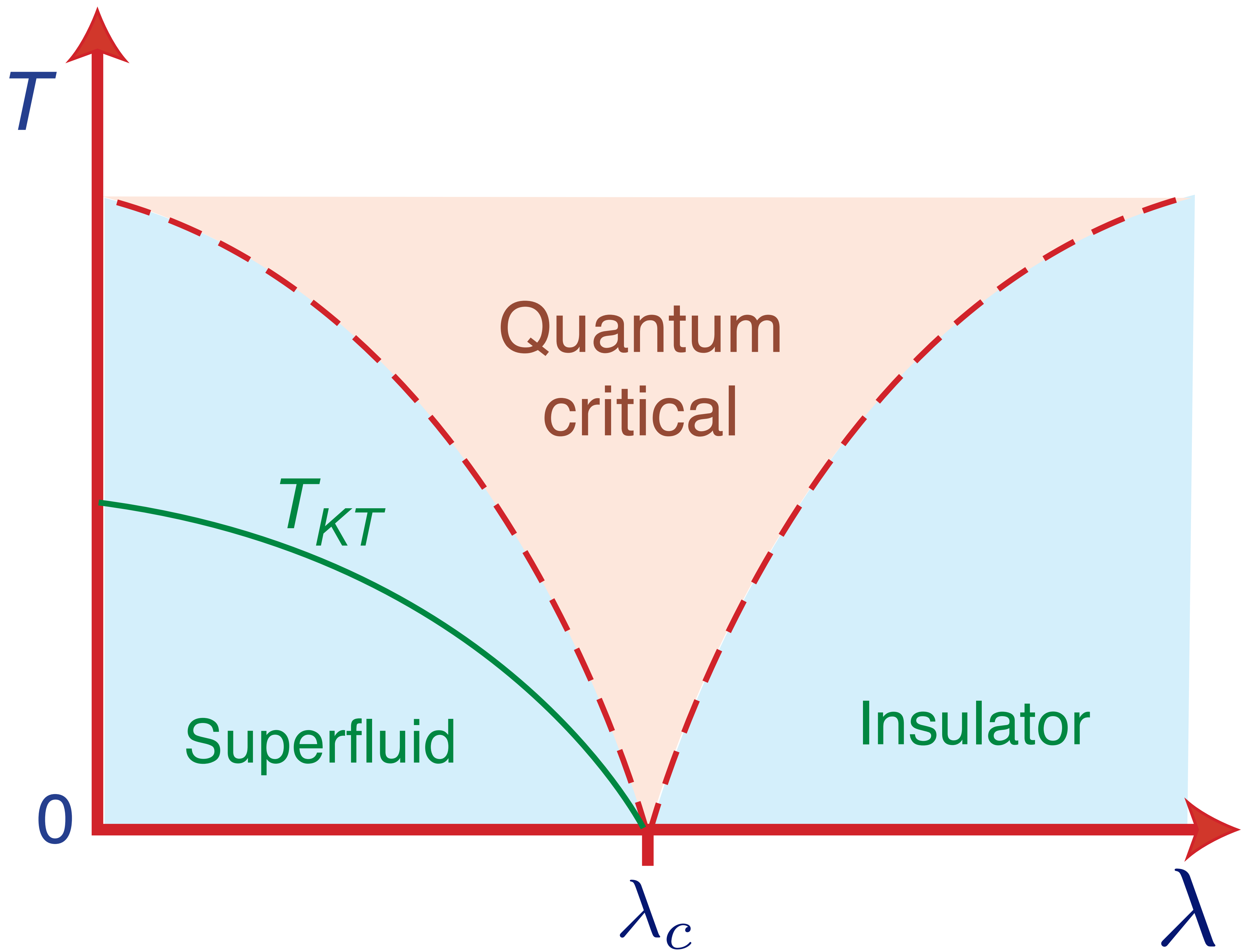
3. Holography and the quasi-normal modes of black-hole horizons

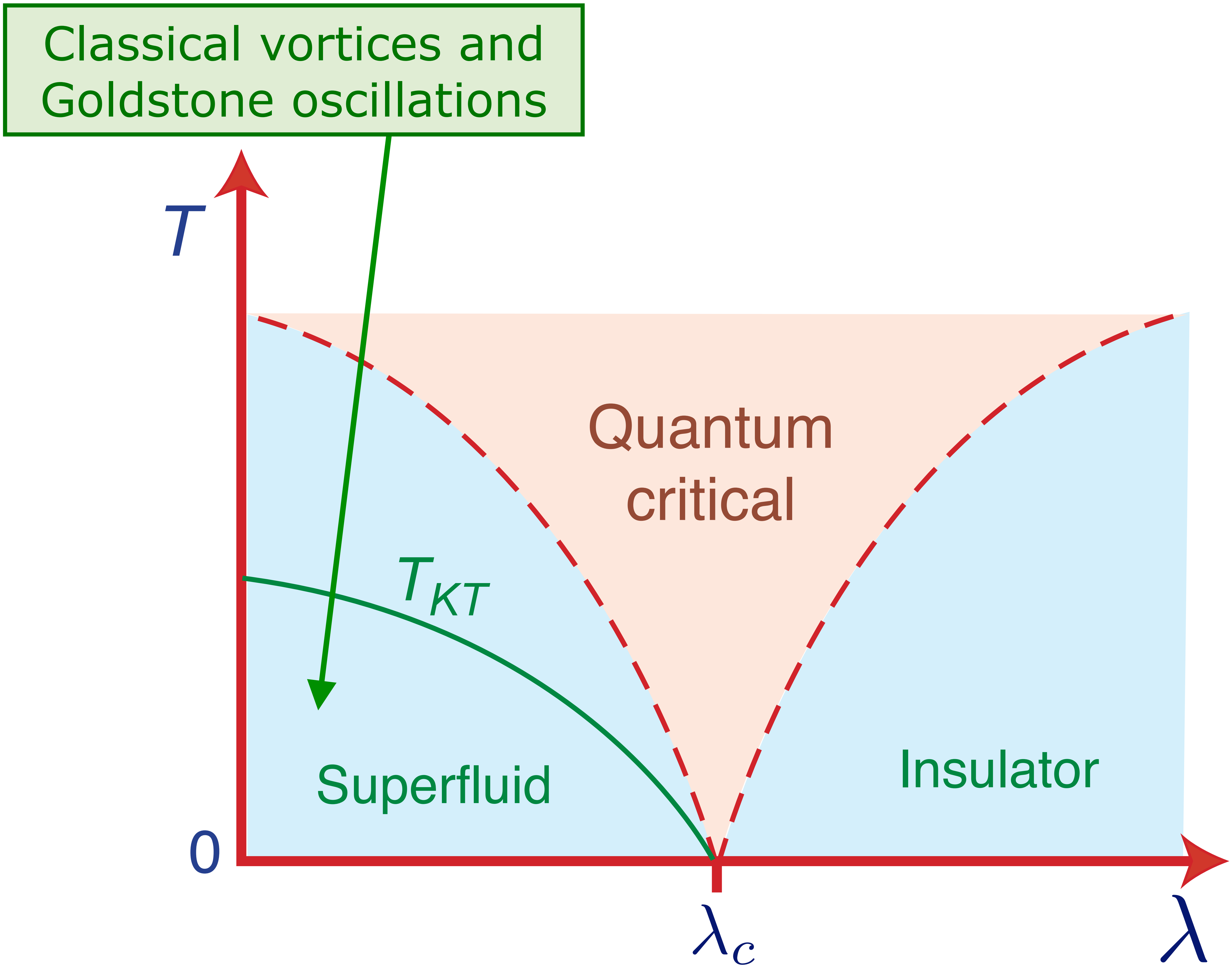
$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$

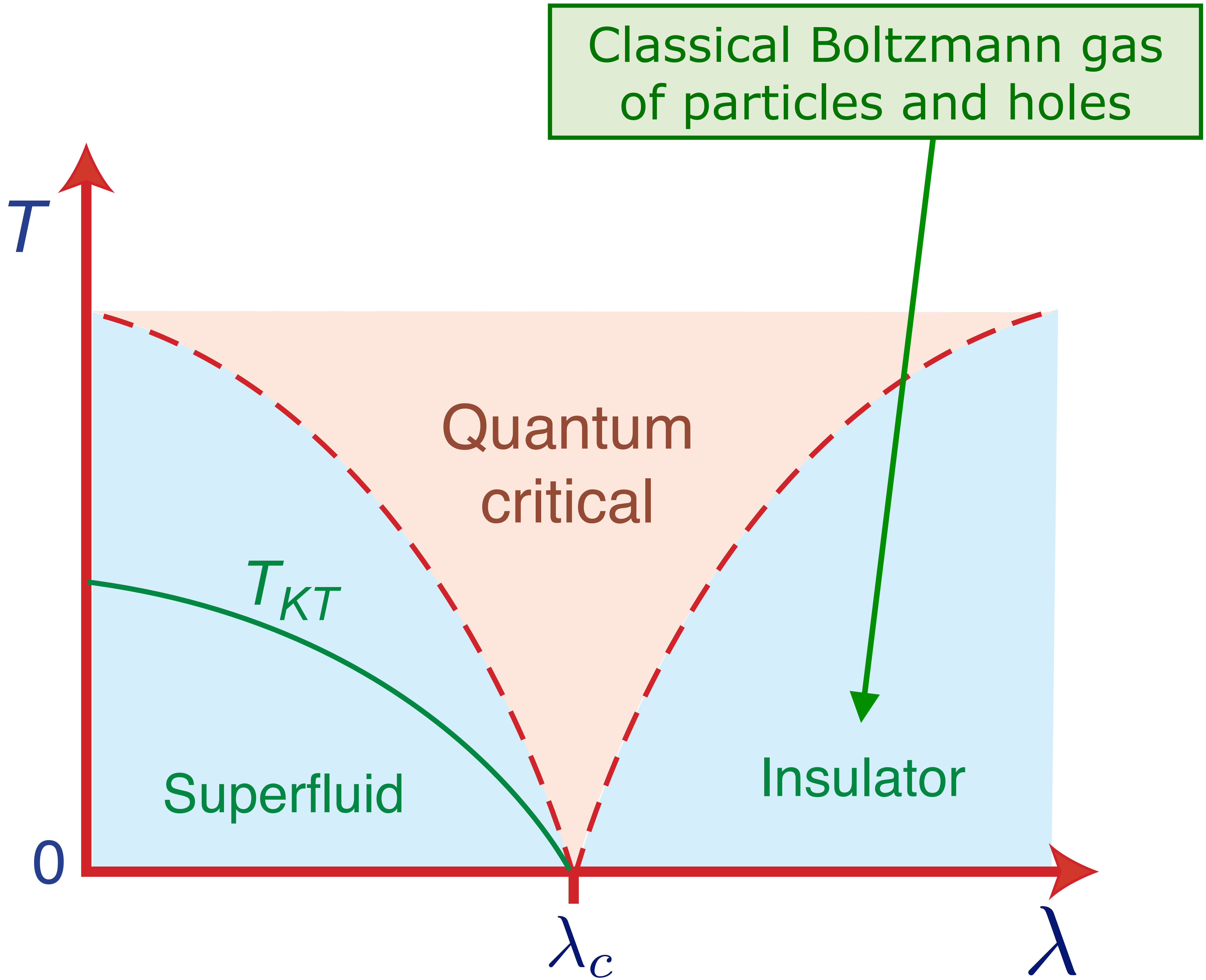
A conformal field theory
in 2+1 spacetime dimensions:
a CFT3



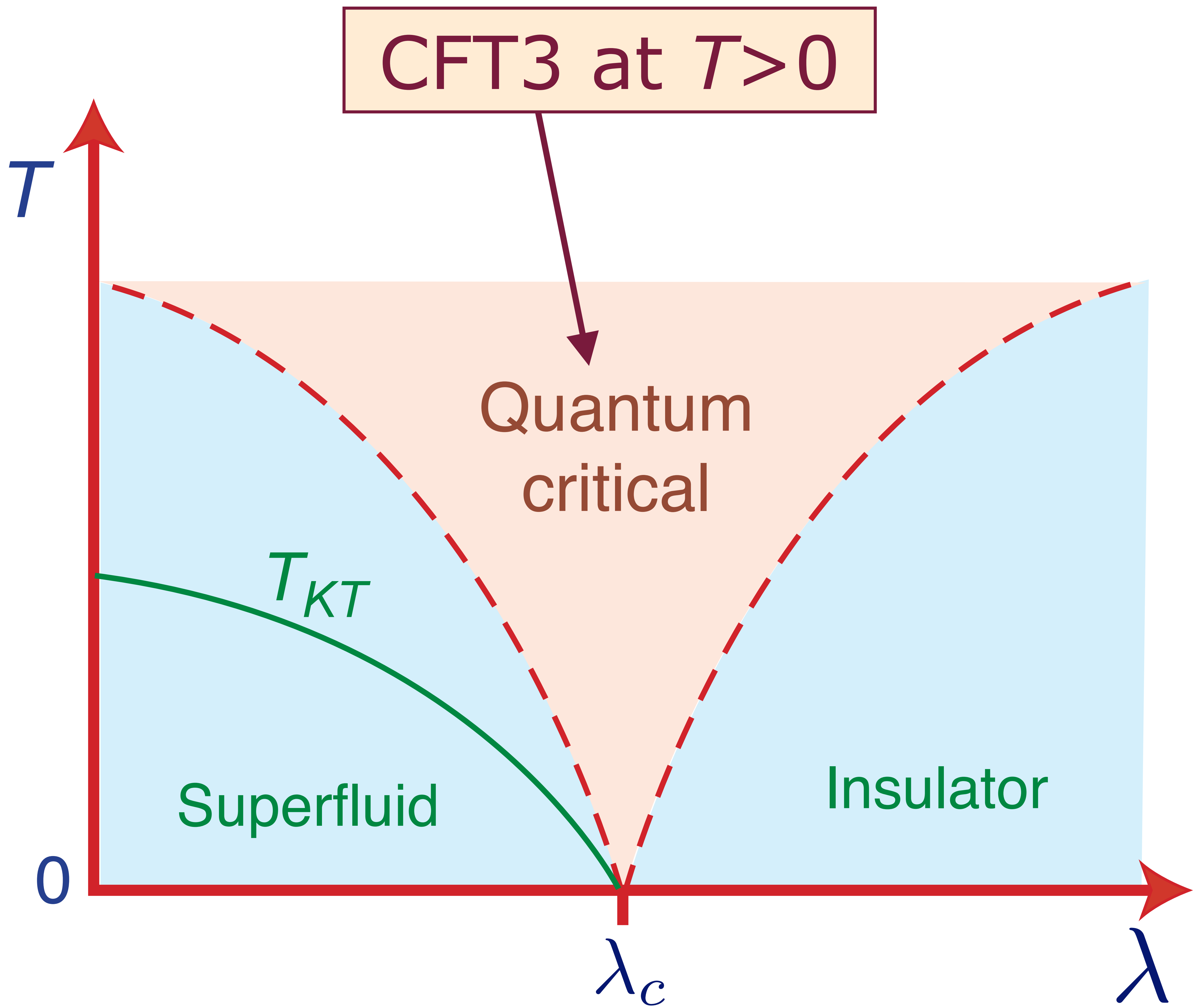

T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
λ_c
λ



Classical vortices and
Goldstone oscillations
T
Quantum
critical
T_KT
Superfluid
Insulator
0
λ_c
λ



Classical Boltzmann gas
of particles and holes
T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
λ_c
λ


CFT3 at $T>0$
T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
λ_c
λ

Quantum critical dynamics

Quantum *"nearly perfect fluid"*

with shortest possible local equilibration time, $\tau_{\rm eq}$

$$\tau_{\rm eq} = \mathcal{C} \frac{\hbar}{k_B T}$$

where $\mathcal{C}$ is a *universal* constant.

Response functions are characterized by poles in LHP

with $\omega \sim k_B T/\hbar$

(analogs of Higgs quasi-normal mode.)

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical dynamics

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

(Q is the "charge" of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett*. **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical dynamics

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

Quantum critical dynamics

Describe charge transport using Boltzmann theory of interacting bosons:

$$\frac{dv}{dt} + \frac{v}{\tau_c} = F.$$

This gives a frequency (ω) dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\,\omega\,\tau_c}$$

where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

Quantum critical dynamics

Describe charge transport using Boltzmann theory of interacting bosons:

$$\frac{dv}{dt} + \frac{v}{\tau_c} = F.$$

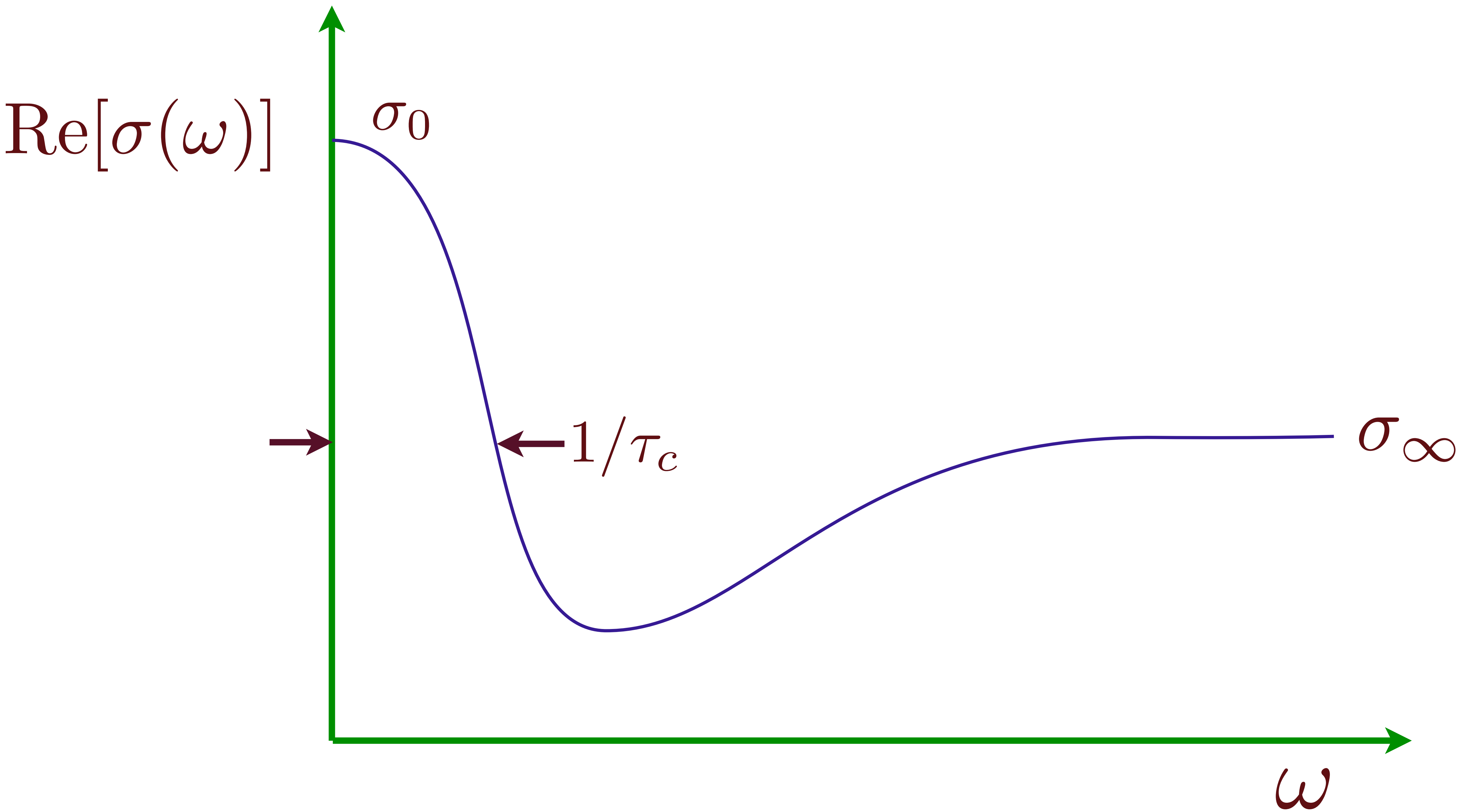
This gives a frequency (ω) dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\,\omega\,\tau_c}$$

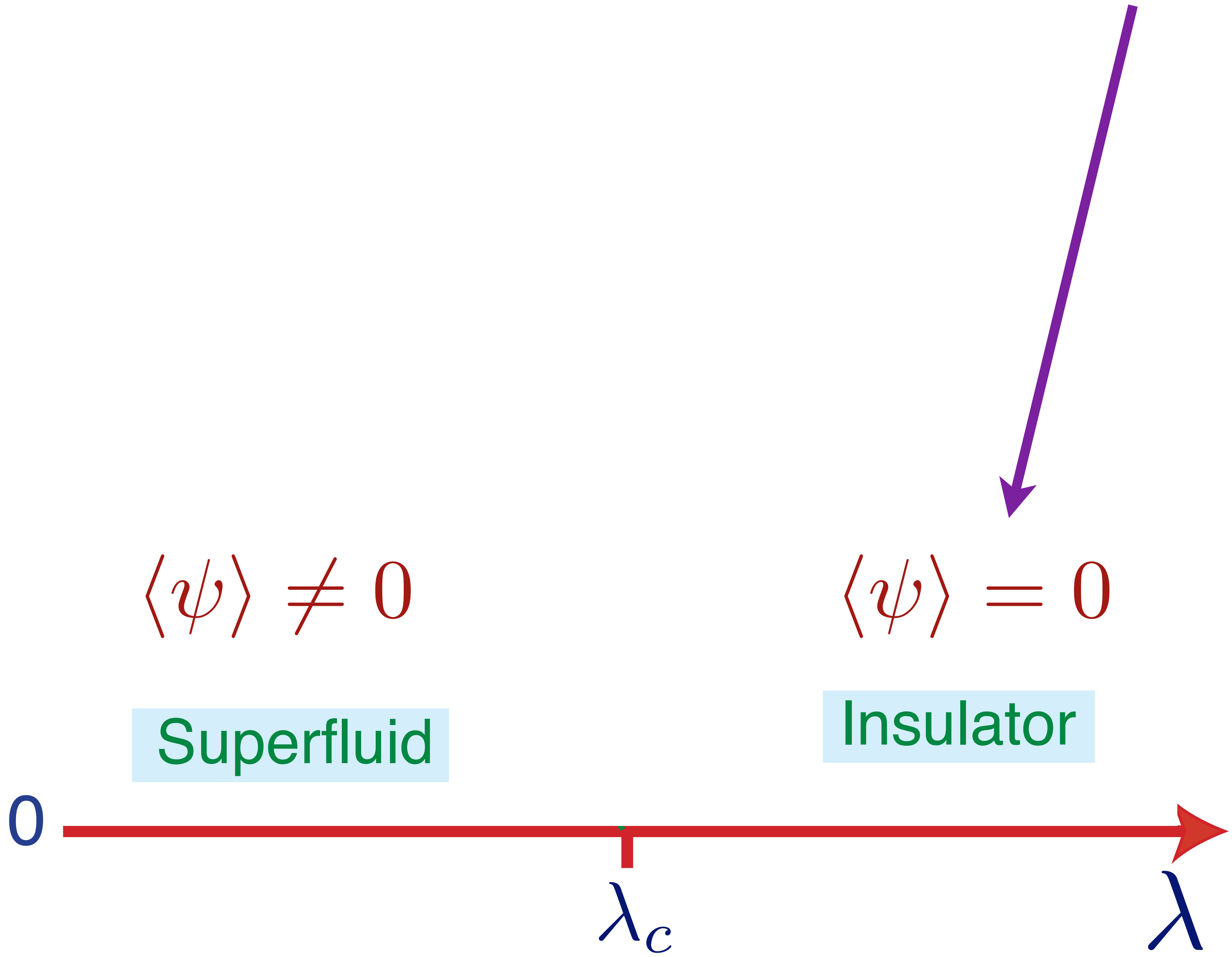
where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

Also, we have $\sigma(\omega \to \infty) = \sigma_\infty$, associated with the density of states for particle-hole creation (the "optical conductivity") in the CFT3.

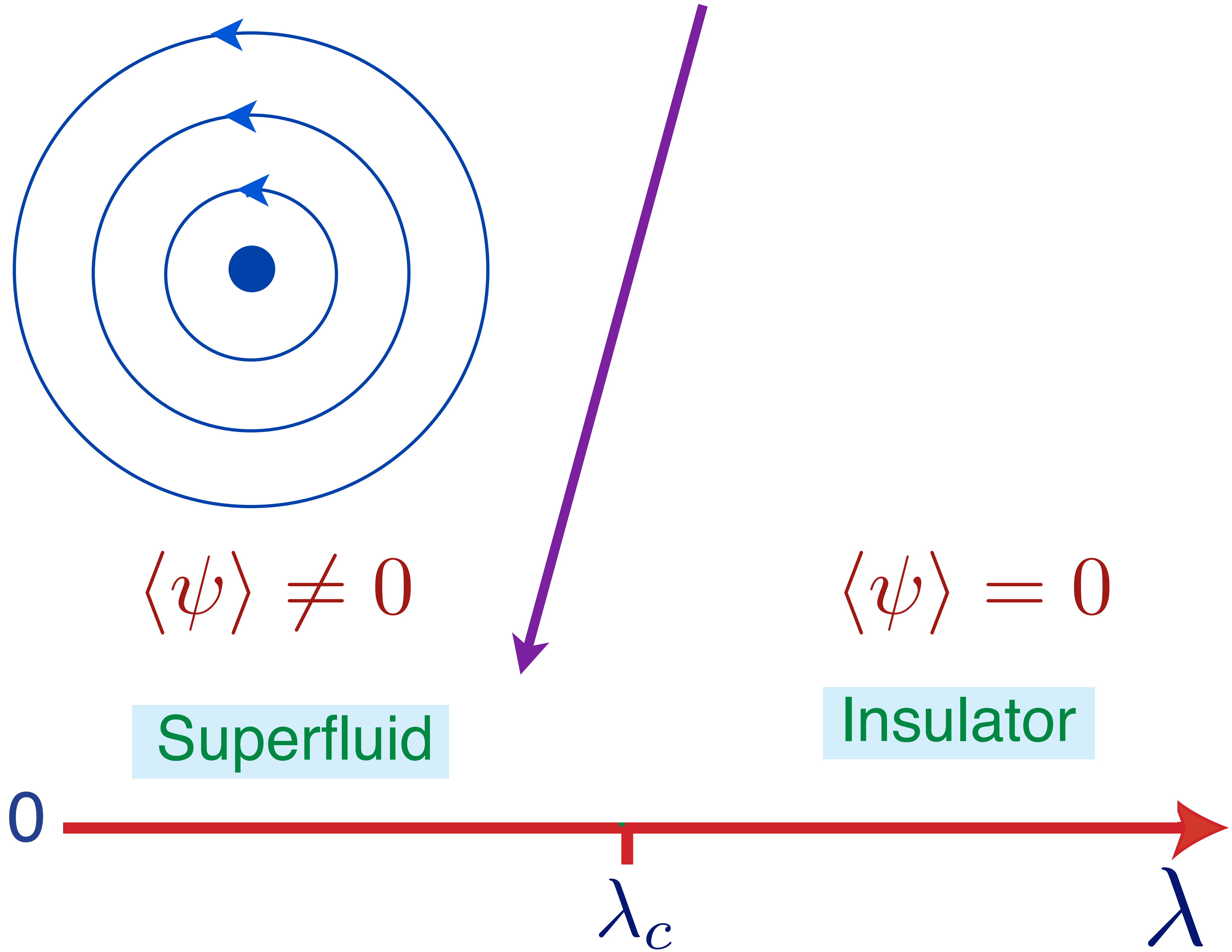
Boltzmann theory of bosons



So far, we have described the quantum critical point using the boson particle and hole excitations of the insulator.



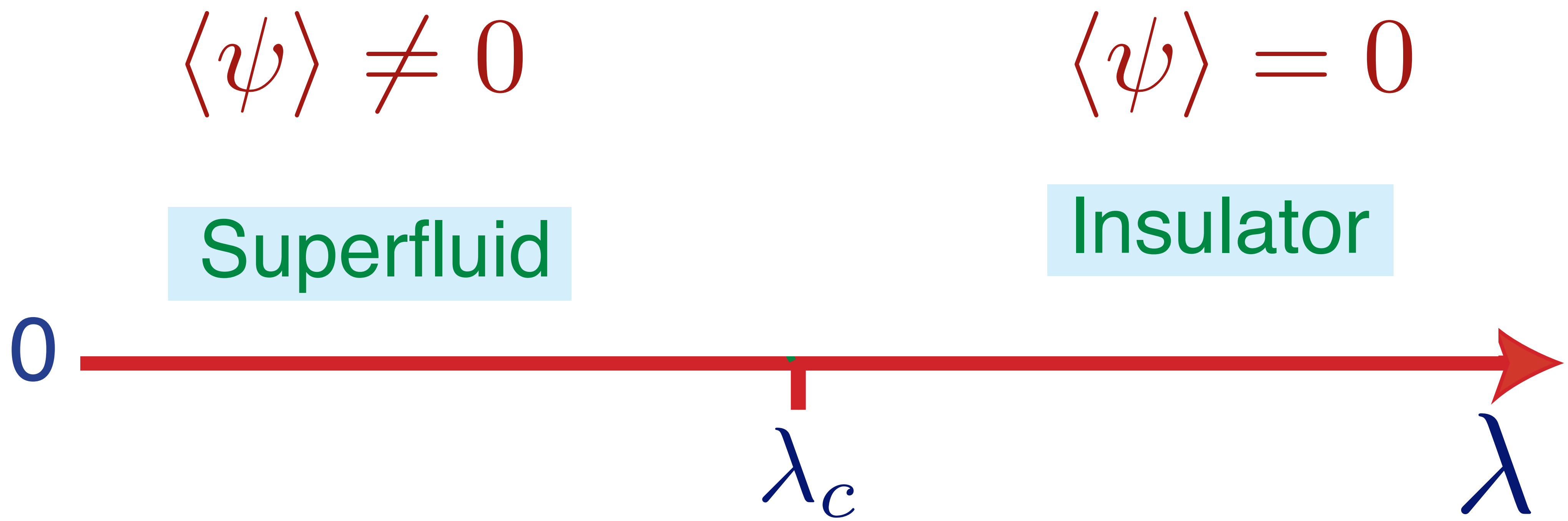
However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.



However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.

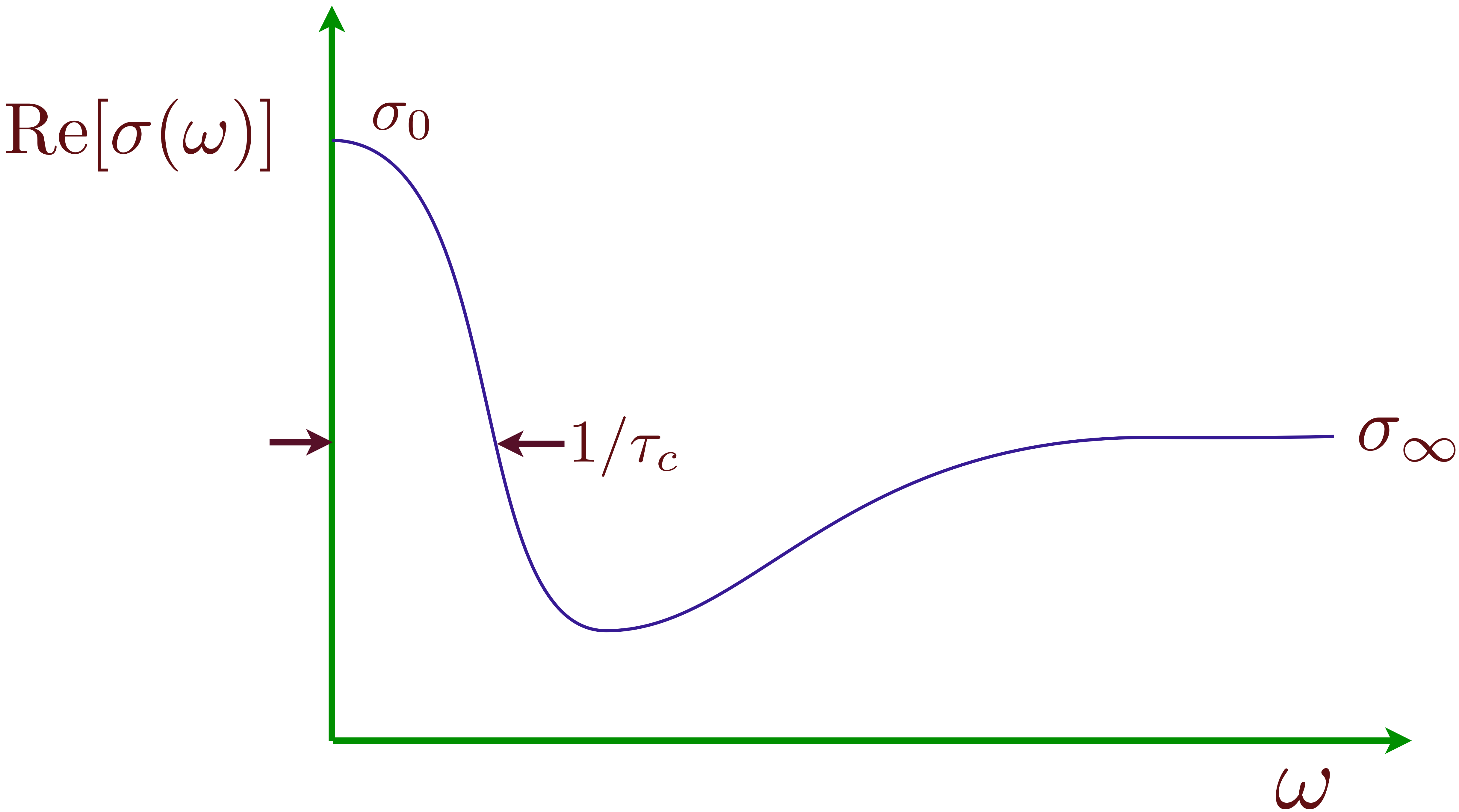
These are quantum particles (in 2+1 dimensions) which described by a (mirror/e.m.) "dual" CFT3 with an emergent U(1) gauge field. Their $T > 0$ dynamics can also be described by a Boltzmann equation:

$$\text{Conductivity} = \text{Resistivity of vortices}$$

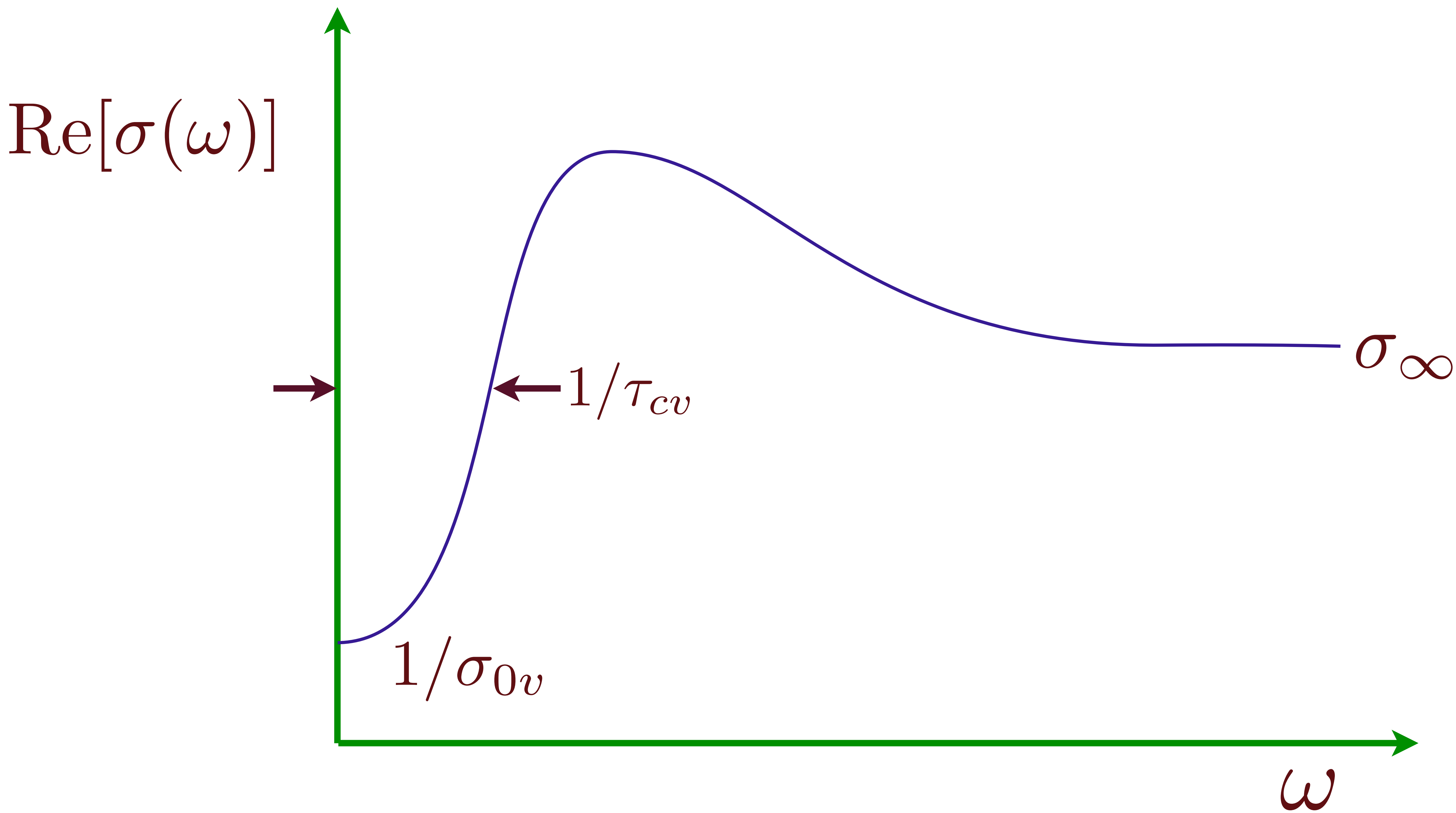


M.P.A. Fisher, *Physical Review Letters* **65**, 923 (1990)

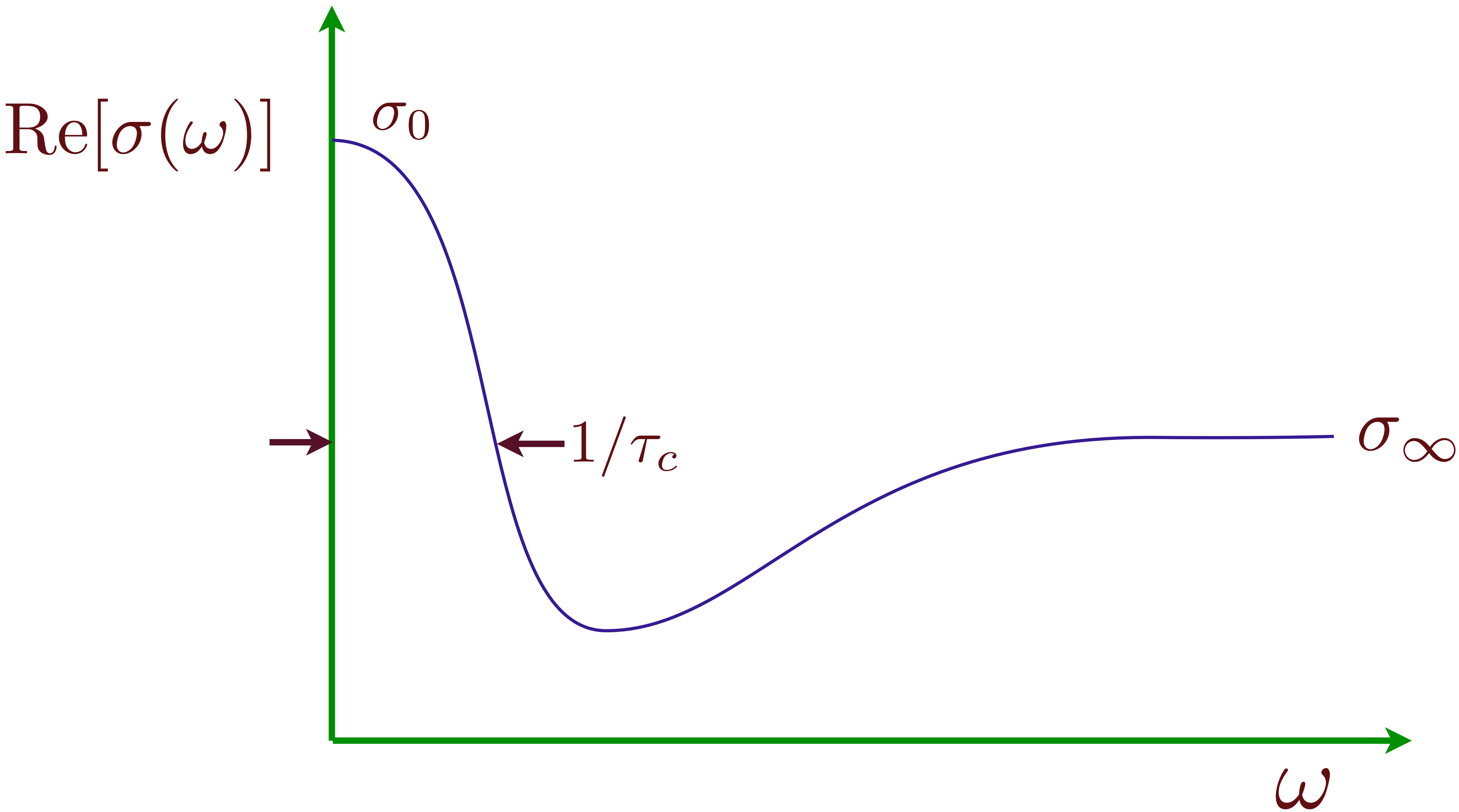
Boltzmann theory of bosons



Boltzmann theory of vortices

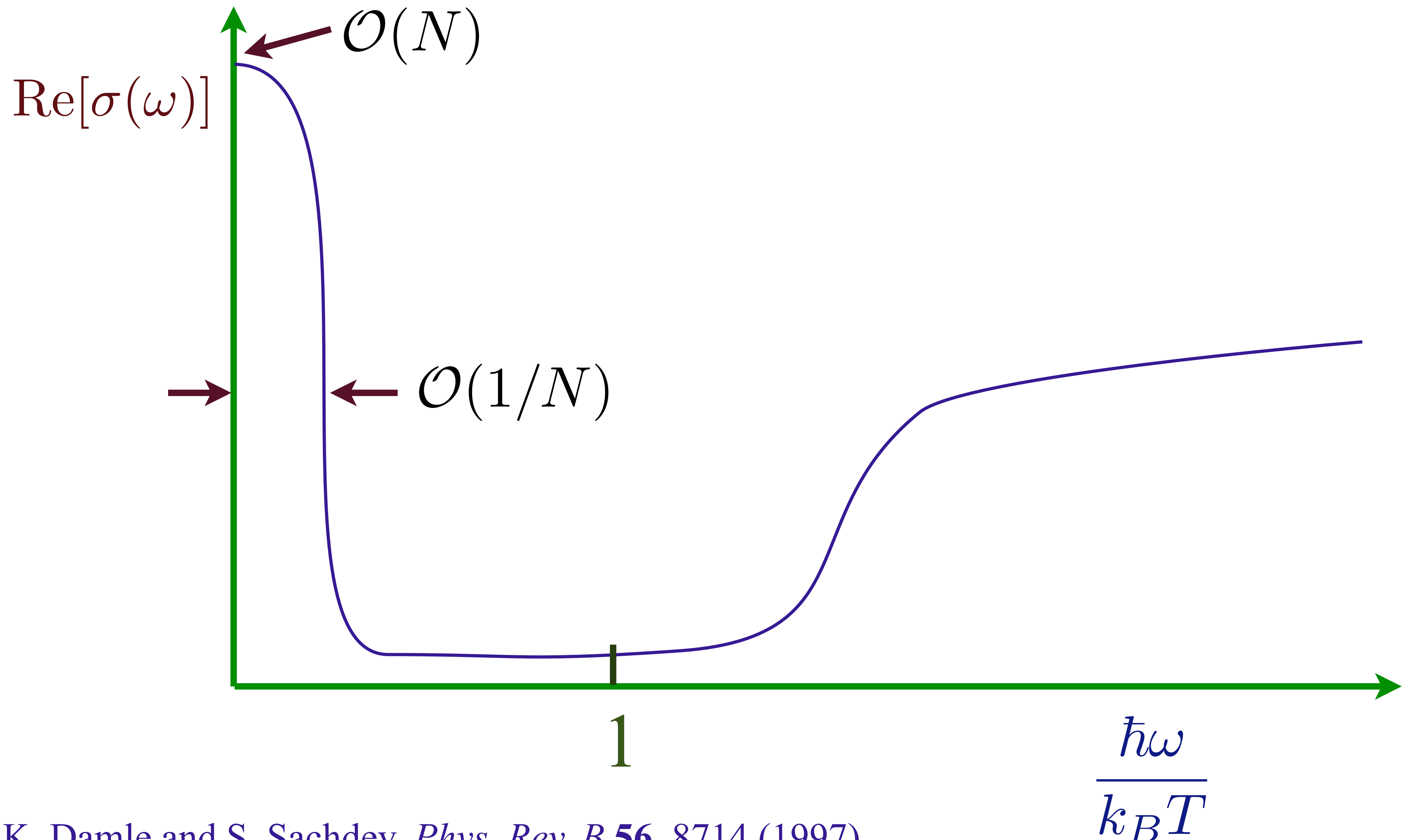


Boltzmann theory of bosons



Vector large *N* expansion for CFT3

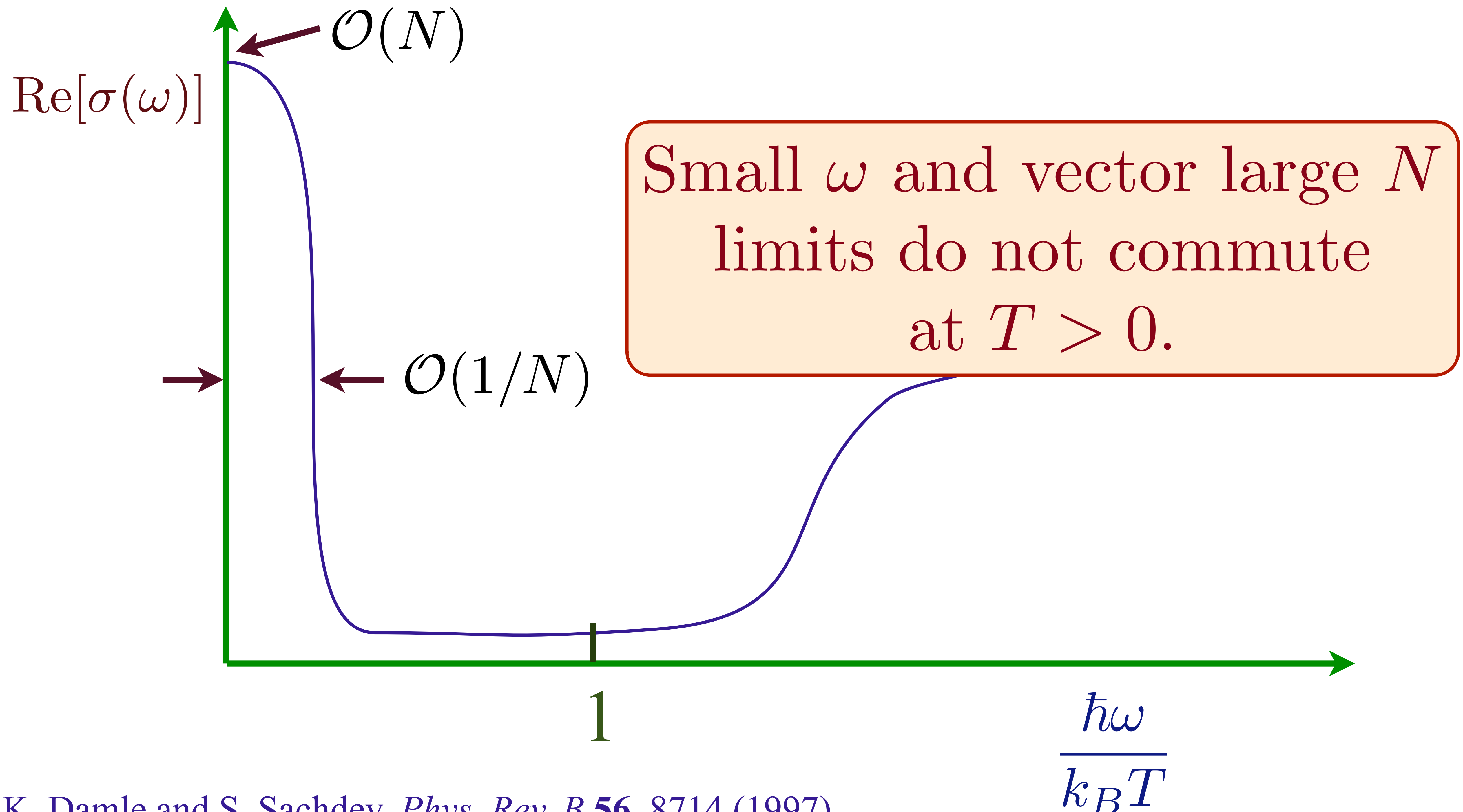
$$\sigma = \frac{Q^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right); \quad \Sigma \to \text{a universal function}$$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Vector large N expansion for CFT3

$$\sigma = \frac{Q^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right); \quad \Sigma \to \text{a universal function}$$

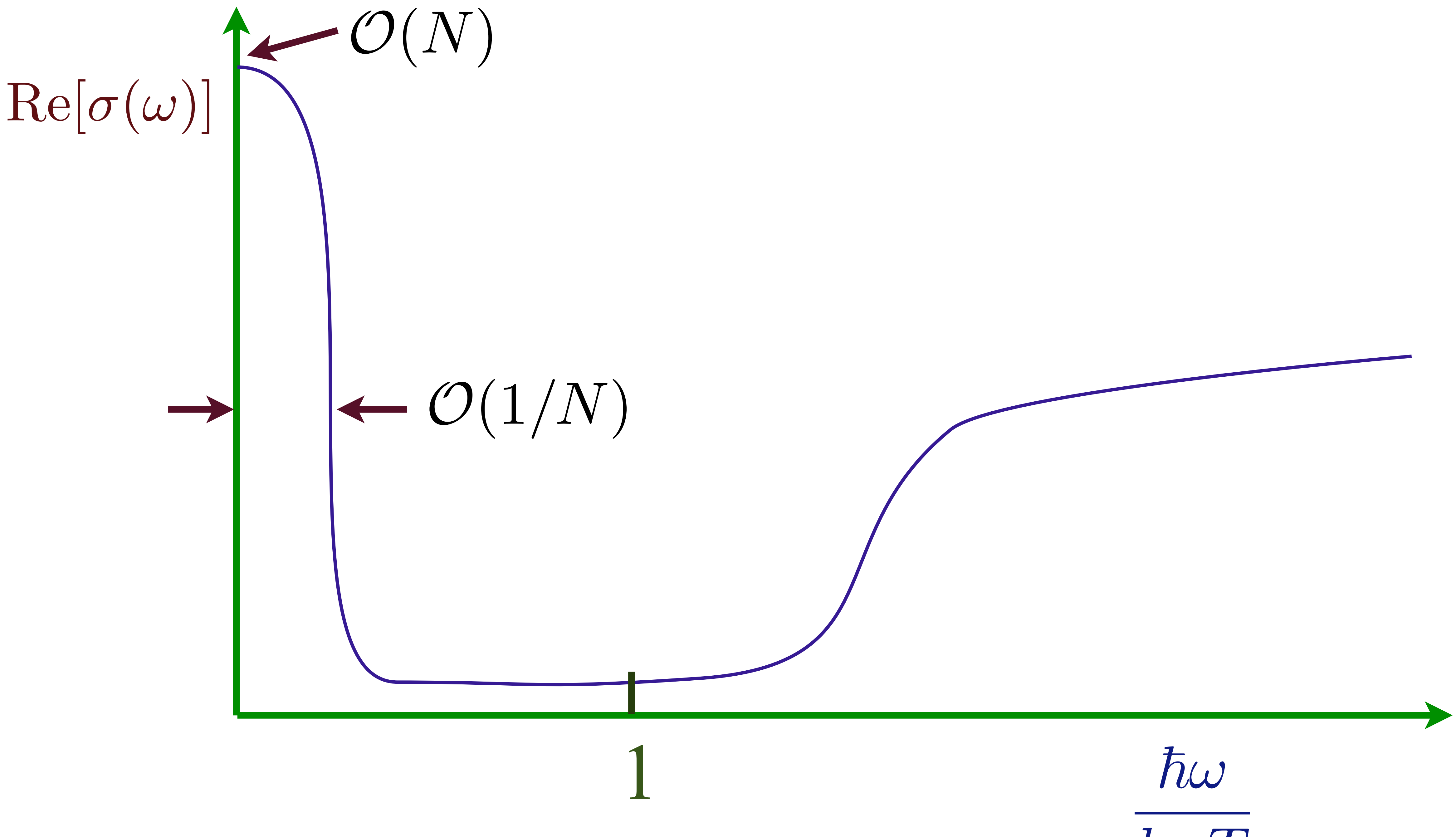


K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Wednesday, January 16, 13

Vector large N expansion for CFT3

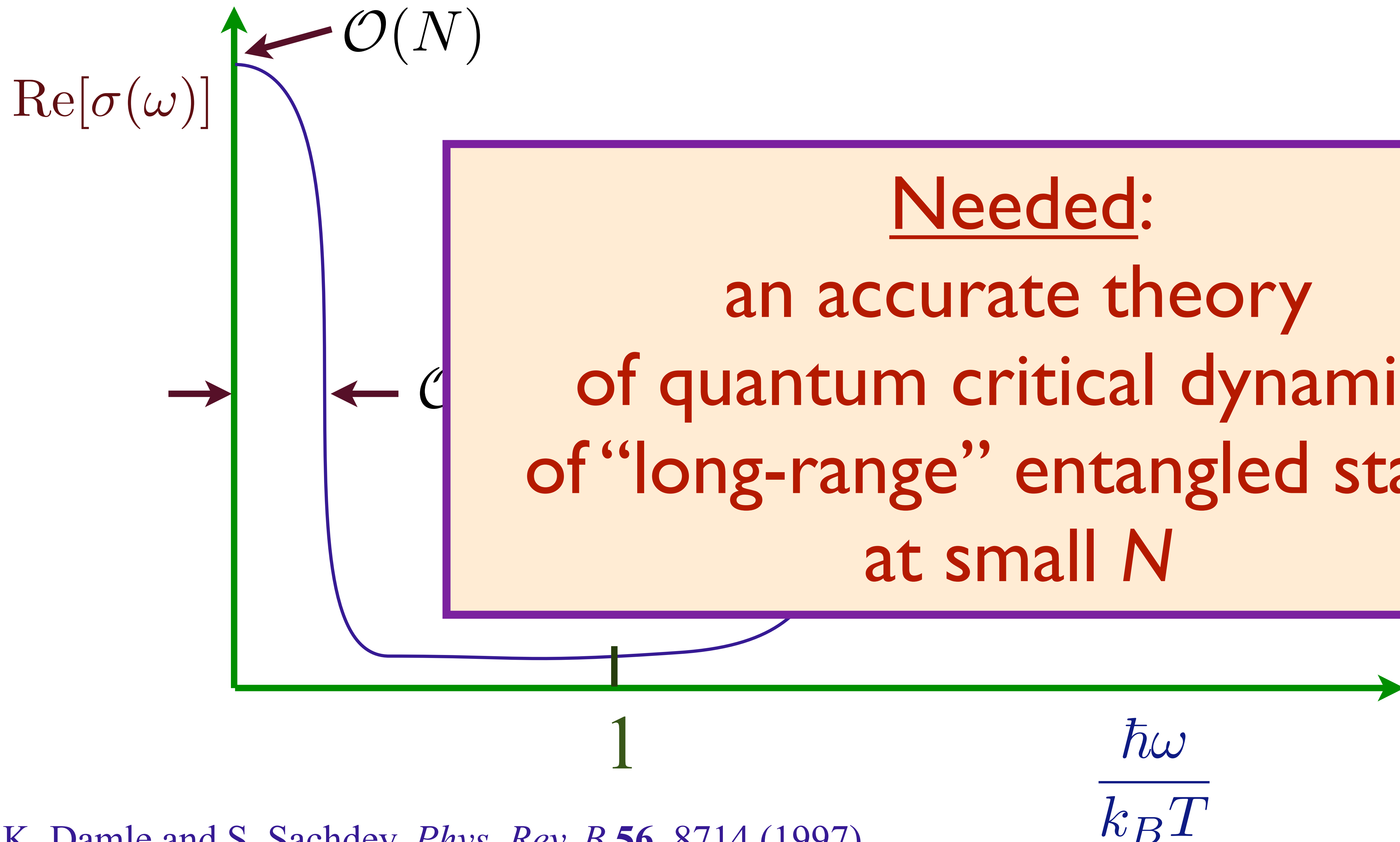
$$\sigma = \frac{Q^2}{h} \Sigma \left(\frac{\hbar \omega}{k_B T} \right); \quad \Sigma \rightarrow \text{a universal function}$$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Vector large N expansion for CFT3

$$\sigma = \frac{Q^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right); \quad \Sigma \to \text{a universal function}$$



Needed:
an accurate theory
of quantum critical dynamics
of "long-range" entangled states
at small N

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Outline

1. Higgs quasi-normal mode near the superfluid-insulator transition in 2 dimensions

2. Quantum criticality and conformal field theories

3. Holography and the quasi-normal modes of black-hole horizons

Outline

1. Higgs quasi-normal mode near the superfluid-insulator transition in 2 dimensions

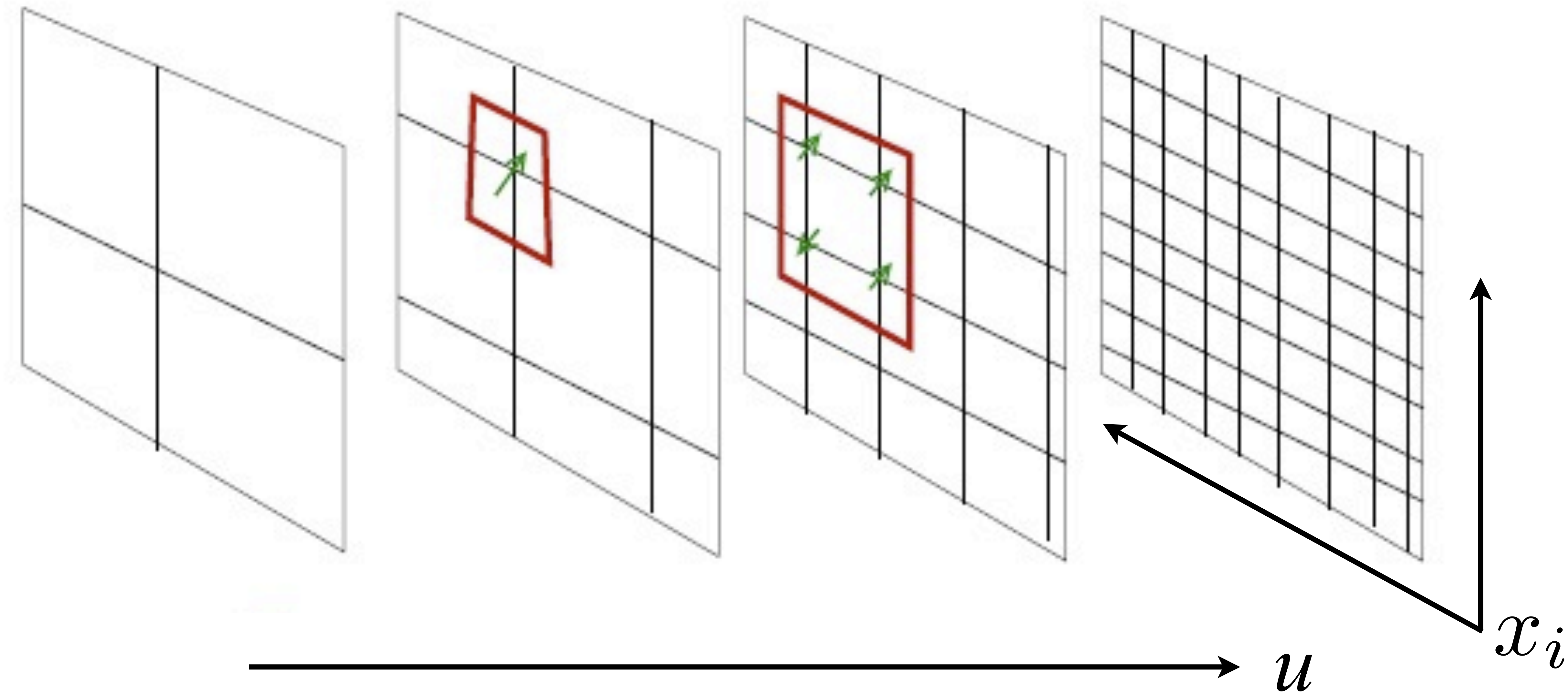
2. Quantum criticality and conformal field theories

3. Holography and the quasi-normal modes of black-hole horizons

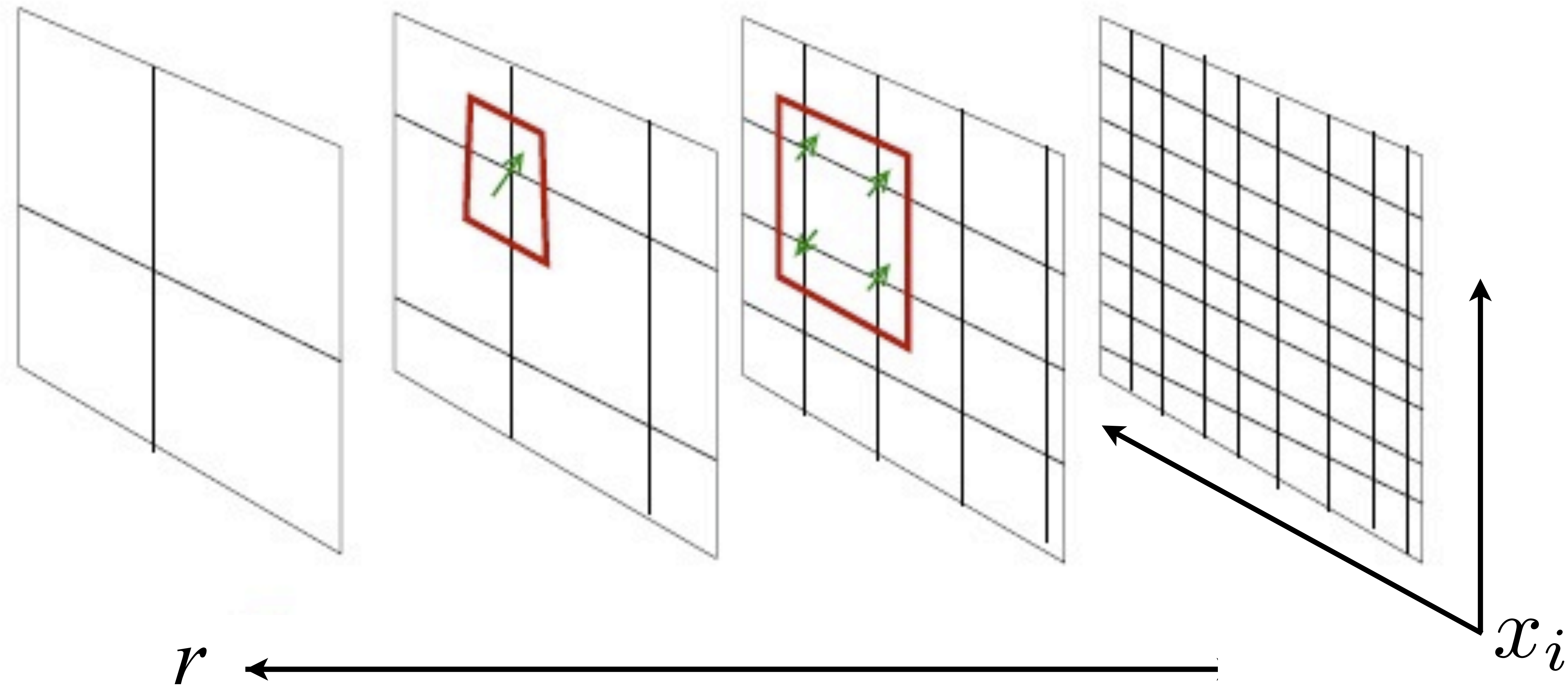
Field theories in $d+1$ spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u\frac{dg}{du} = \beta(g)$$

where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u.

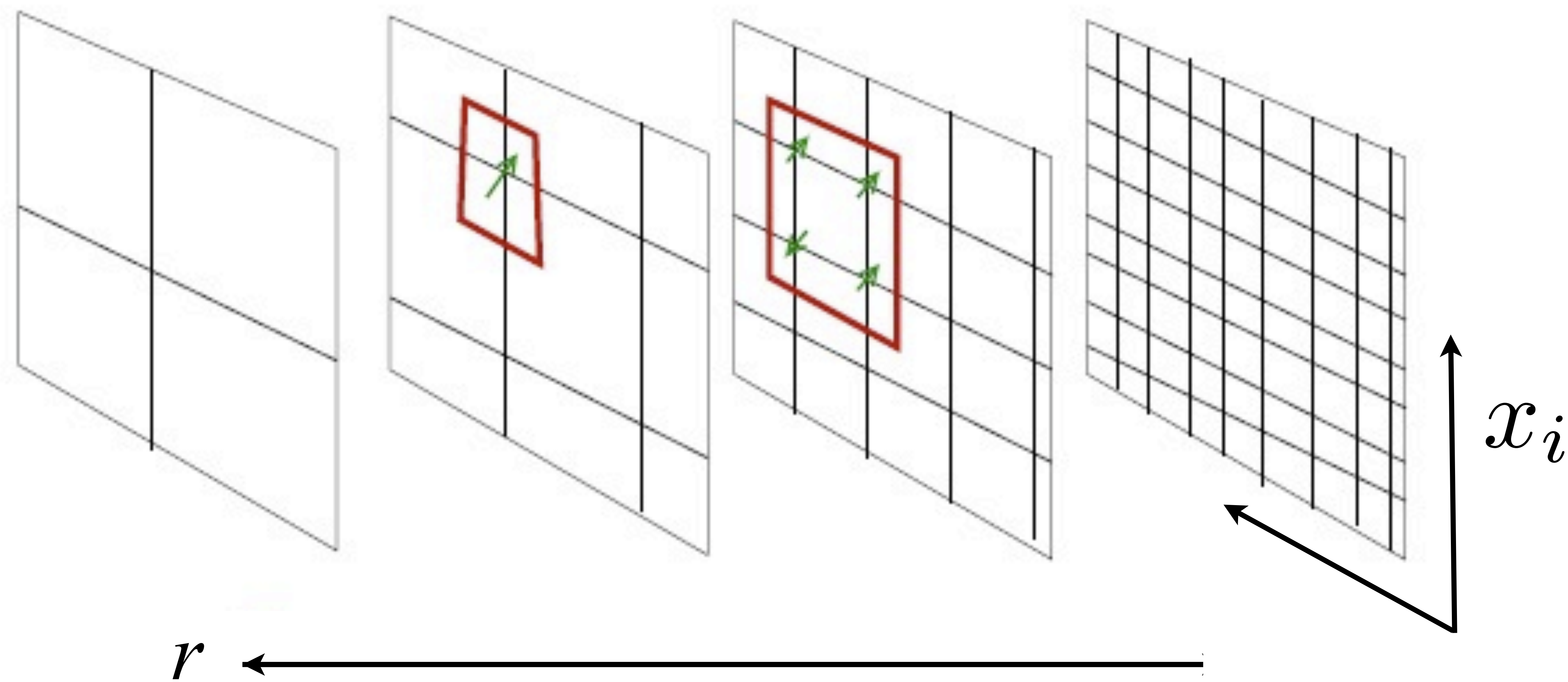


J. McGreevy, arXiv0909.0518



Key idea: $\Rightarrow$ Implement r as an extra dimension, and map to a local theory in $d+2$ spacetime dimensions.

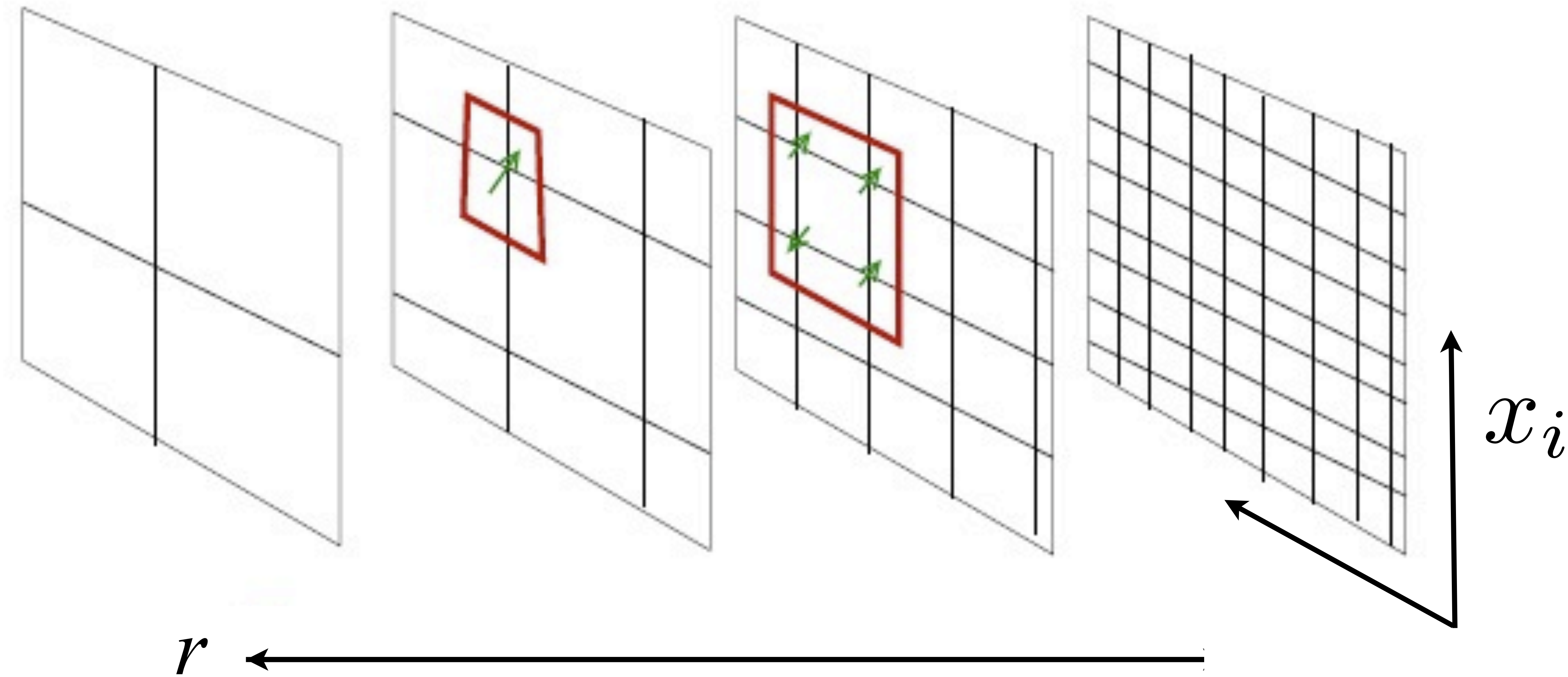
Holography



For a relativistic CFT in d spatial dimensions, the metric in the holographic space is fixed by demanding the scale transformation ($i = 1 \ldots d$)

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

Holography

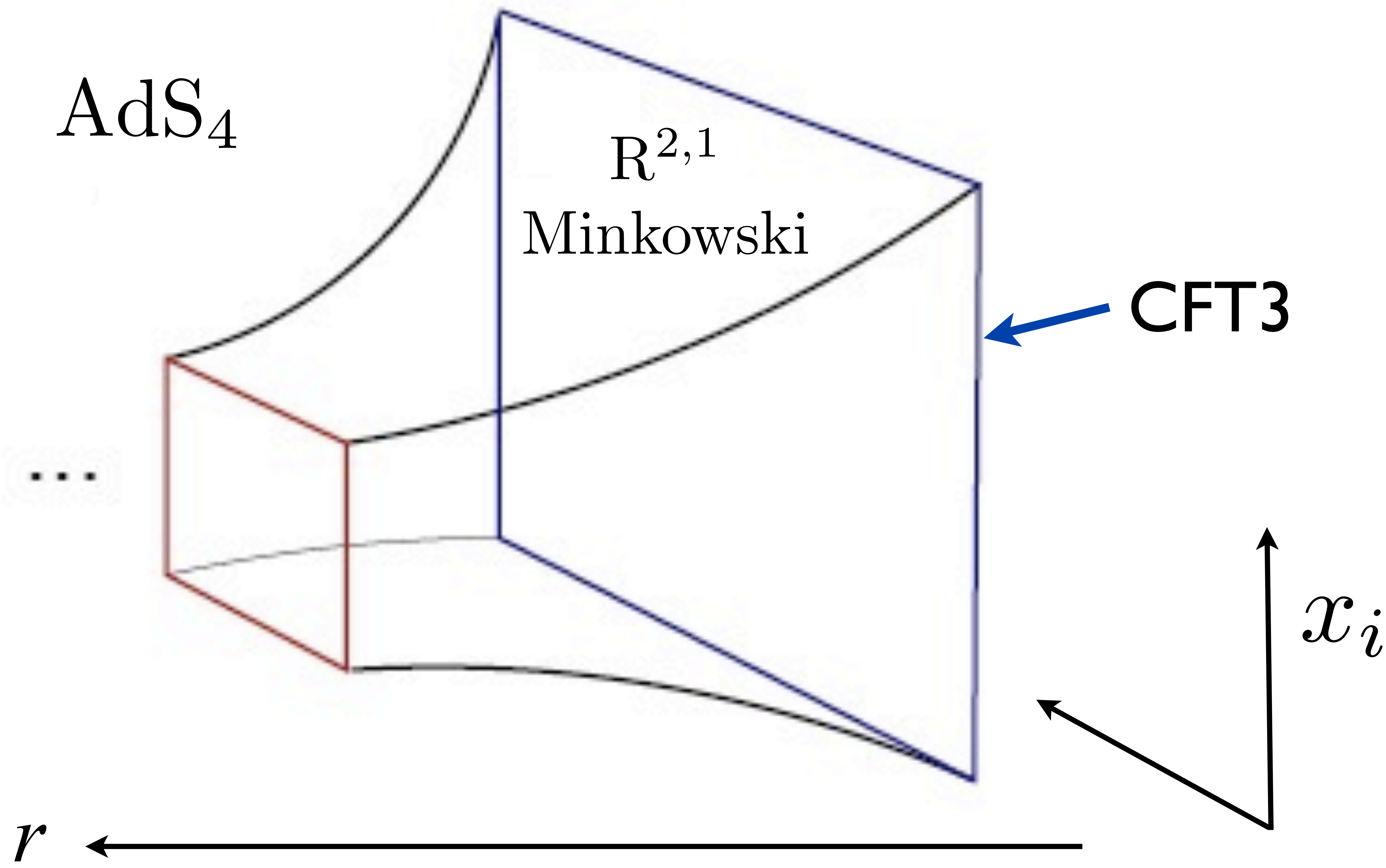


This gives the unique metric

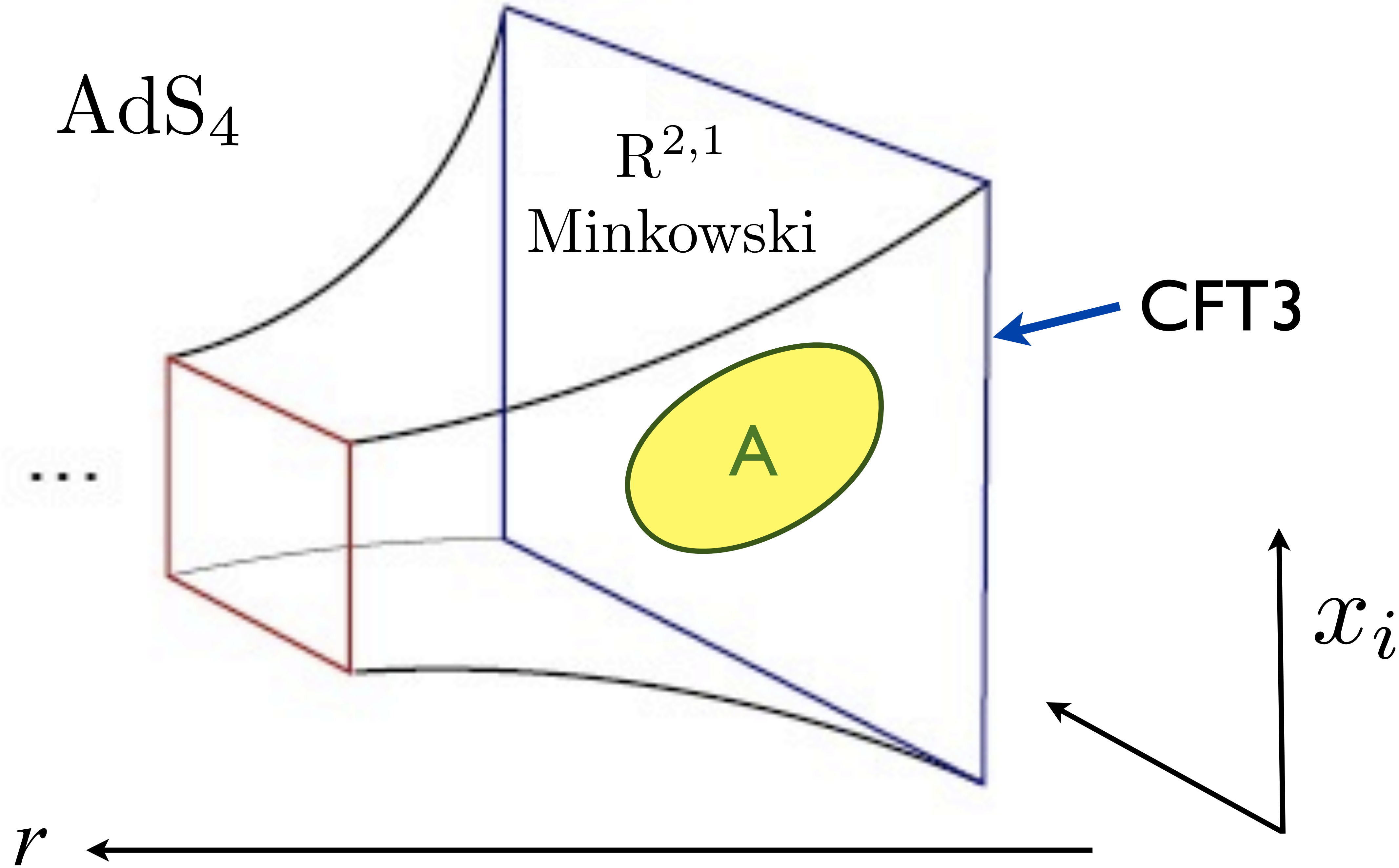
$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

This is the metric of anti-de Sitter space AdS_{d+2}.

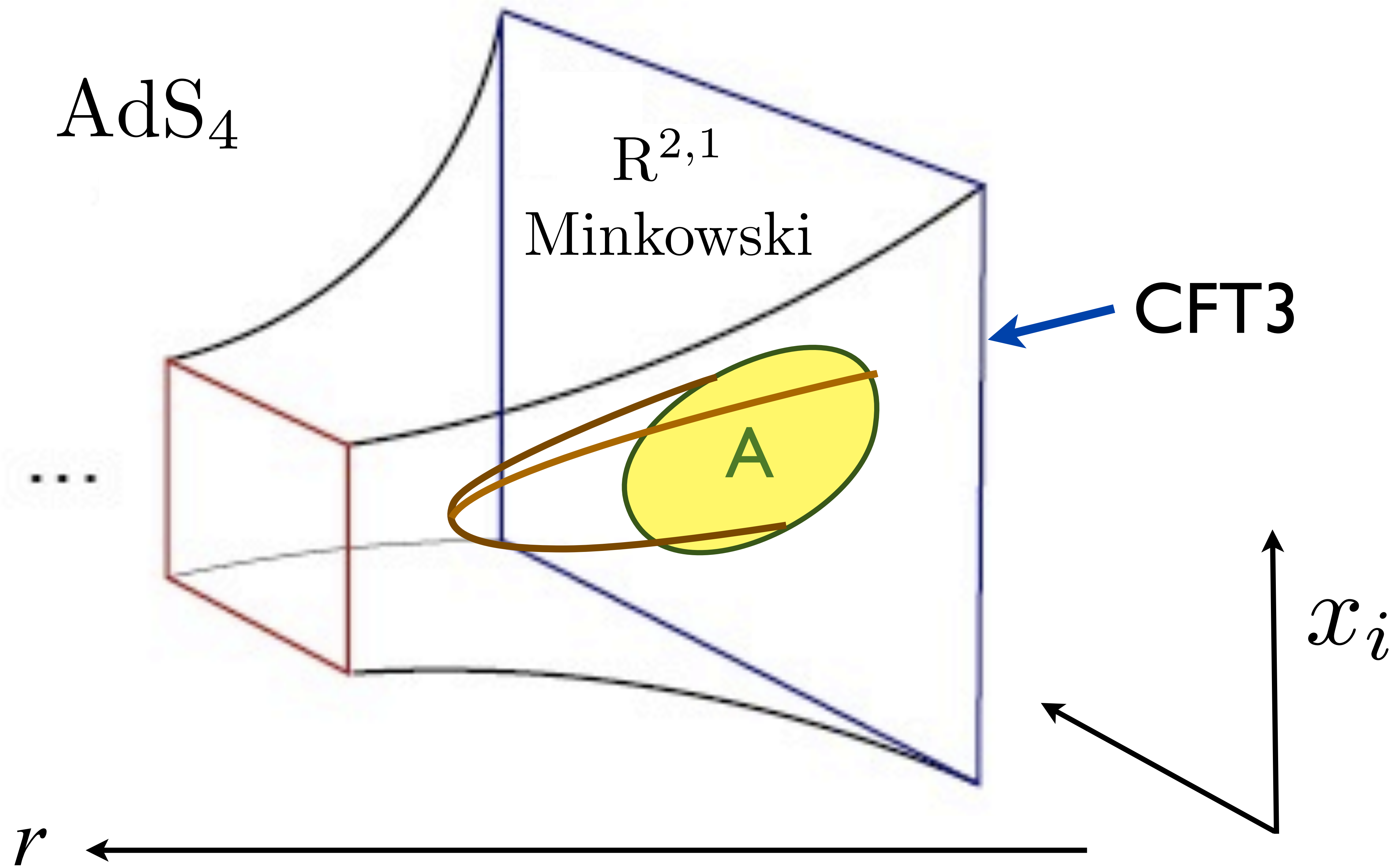
AdS/CFT correspondence



AdS/CFT correspondence



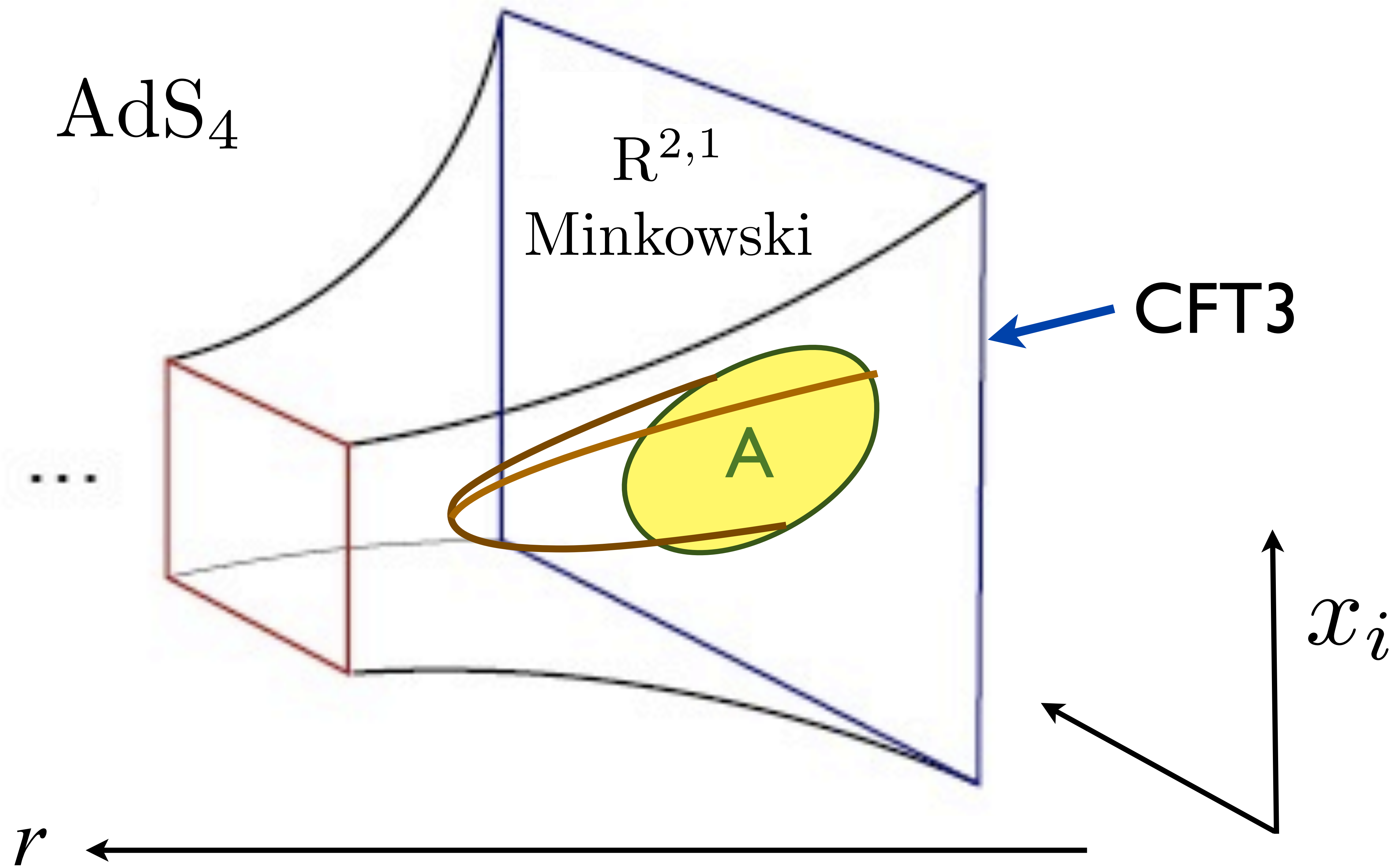
AdS/CFT correspondence



Associate entanglement entropy with an observer in the enclosed spacetime region, who cannot observe "outside" : *i.e.* the region is surrounded by an imaginary horizon.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

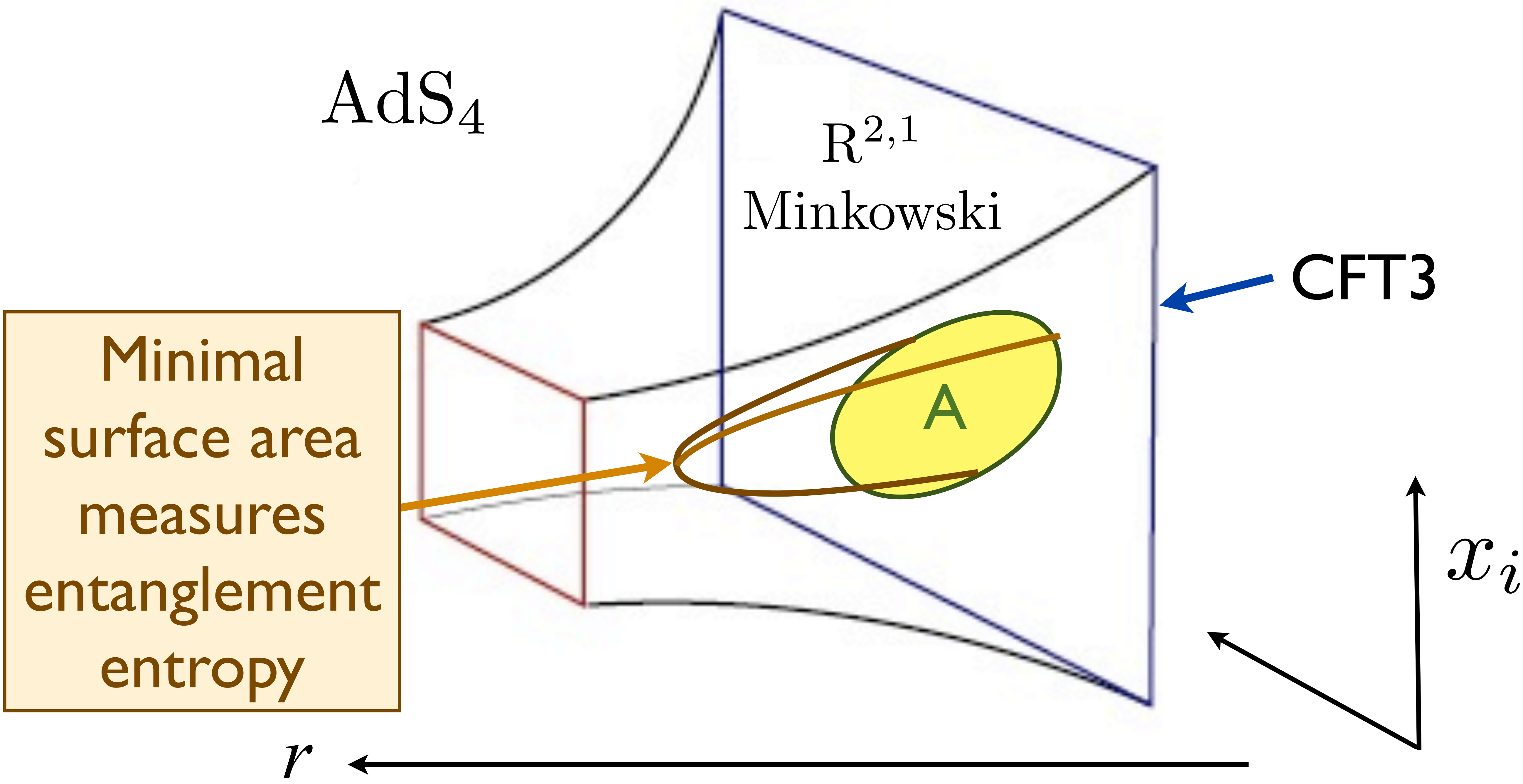
AdS/CFT correspondence



The entropy of this region is bounded by its surface area
(Bekenstein-Hawking-'t Hooft-Susskind)

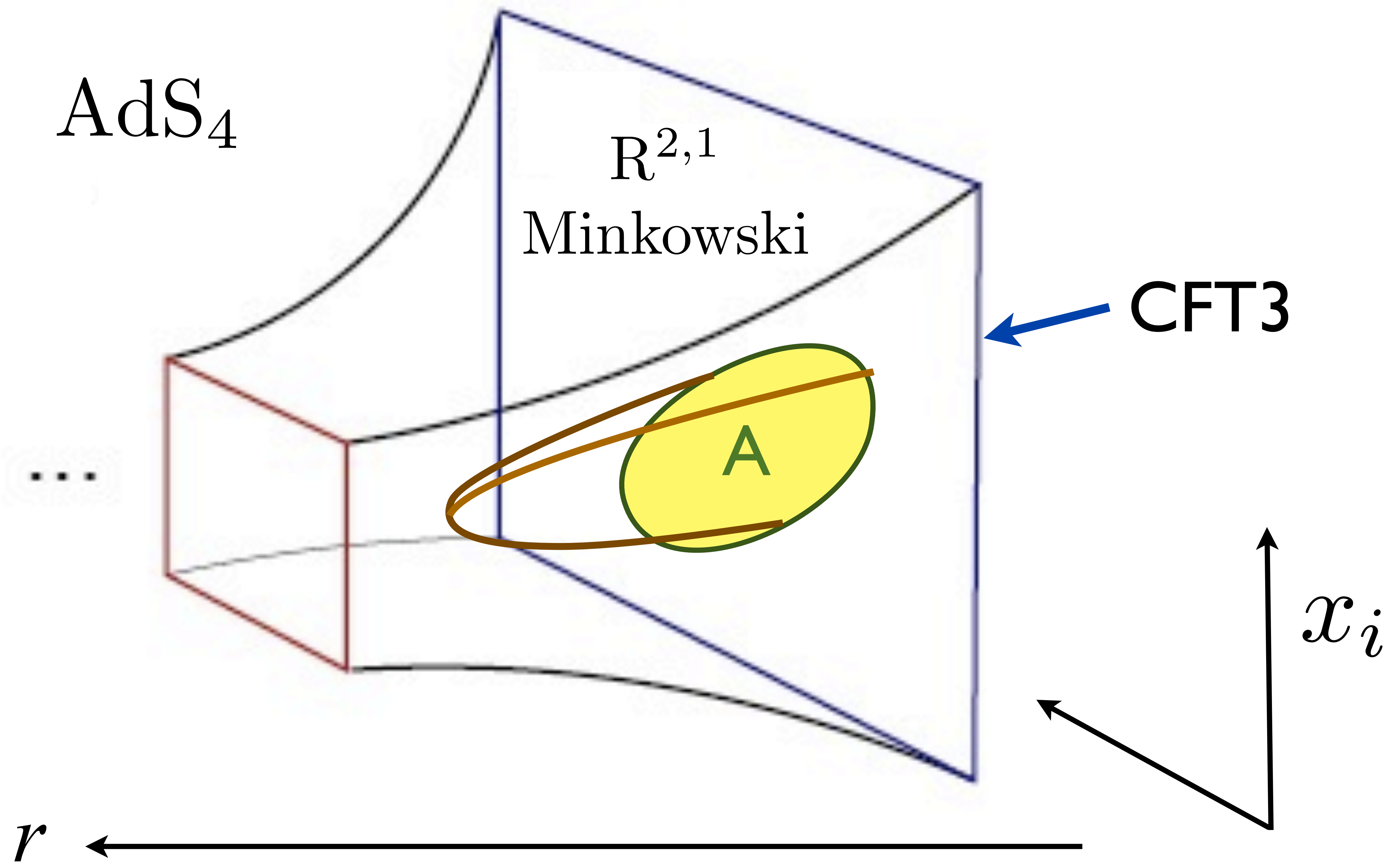
S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

AdS/CFT correspondence



S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

AdS/CFT correspondence



- Computation of minimal surface area yields

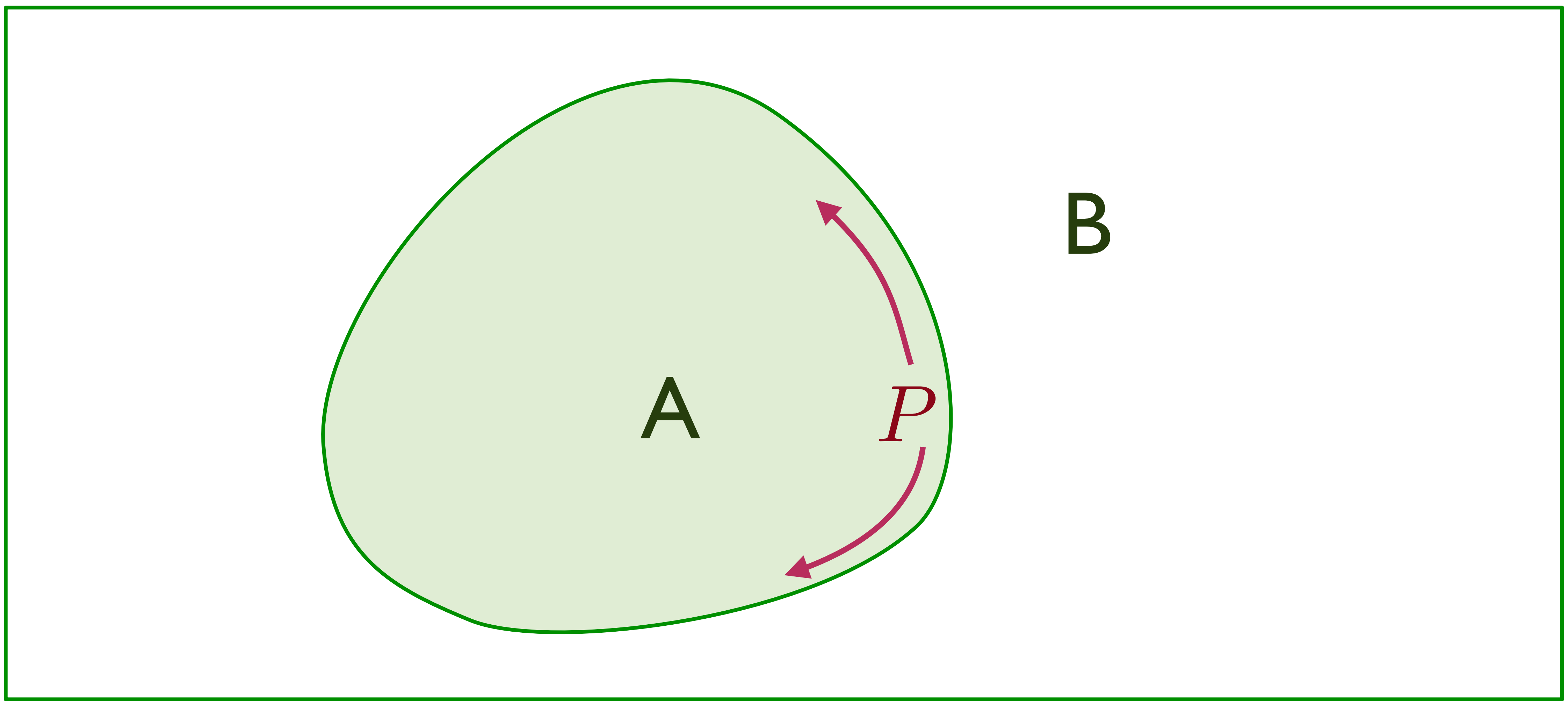
$$S_E = aP - \gamma,$$

where γ is a shape-dependent universal number.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

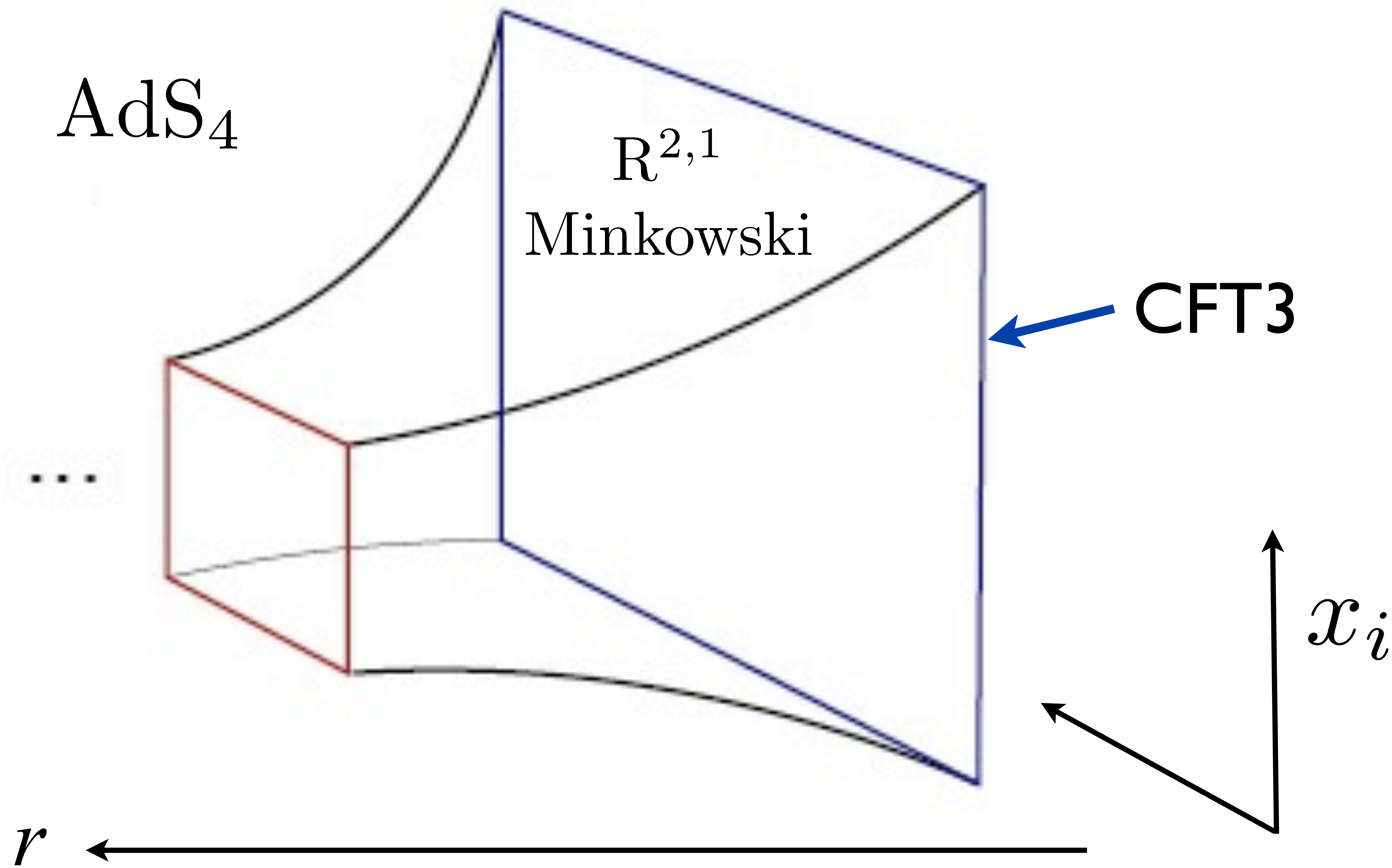
Entanglement entropy from field theory of CFT3

- Entanglement entropy obeys $S_E = aP - \gamma$, where γ is a shape-dependent universal number associated with the CFT3.

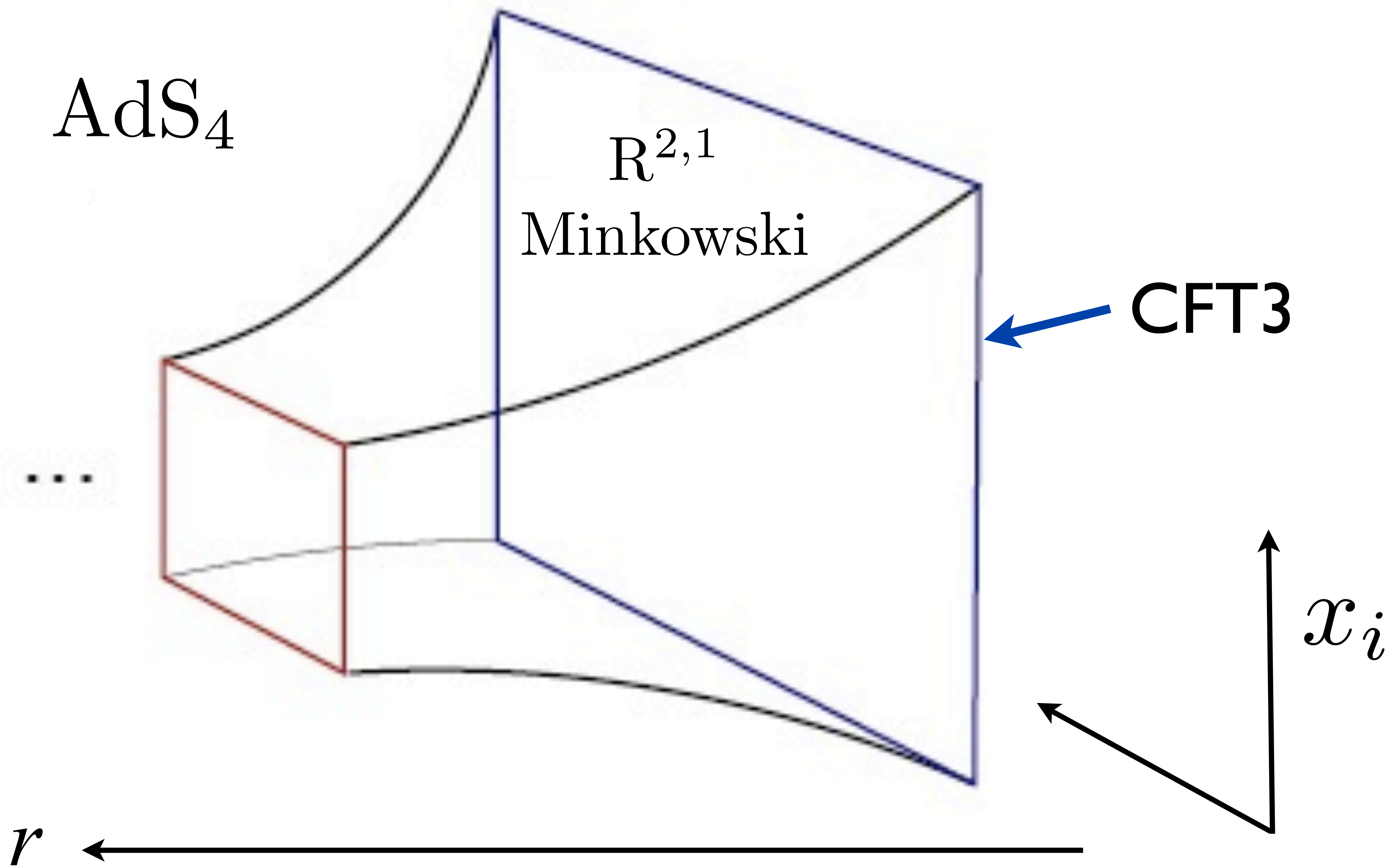


M. A. Metlitski, C. A. Fuertes, and S. Sachdev, Physical Review B 80, 115122 (2009).
H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011)
I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

AdS/CFT correspondence



AdS/CFT correspondence

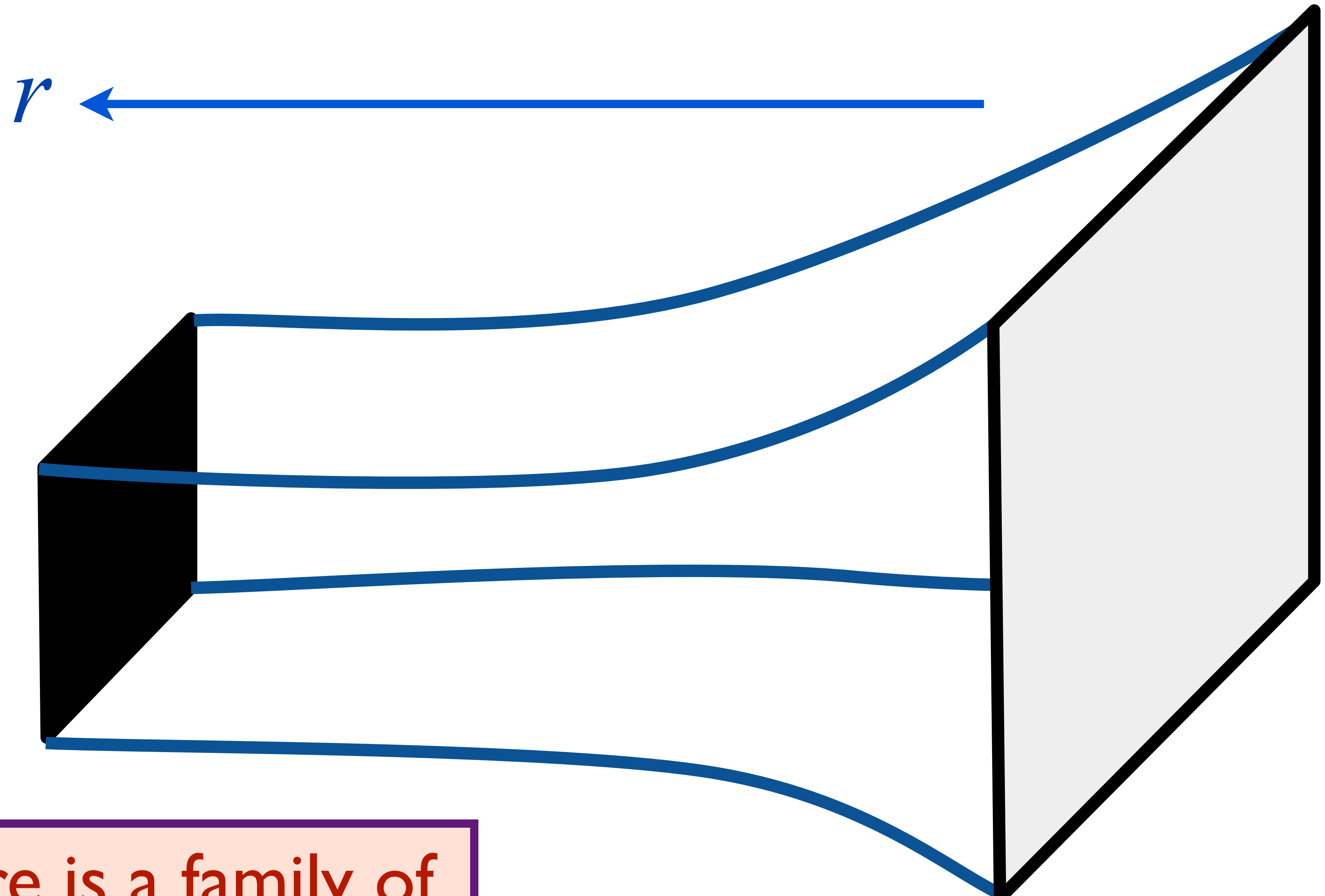


This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S}_E = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane

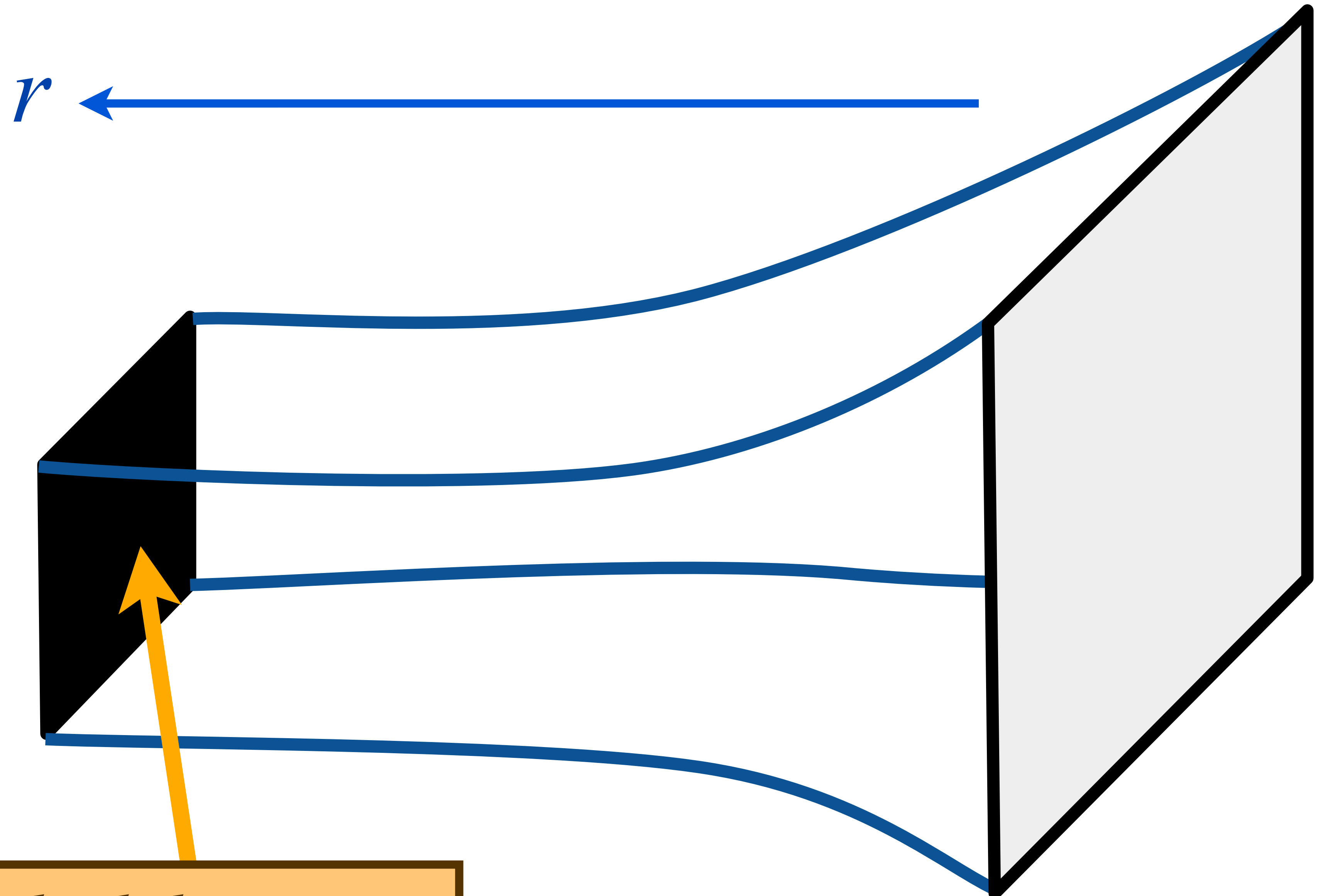


There is a family of solutions of Einstein gravity which describe non-zero temperatures

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane



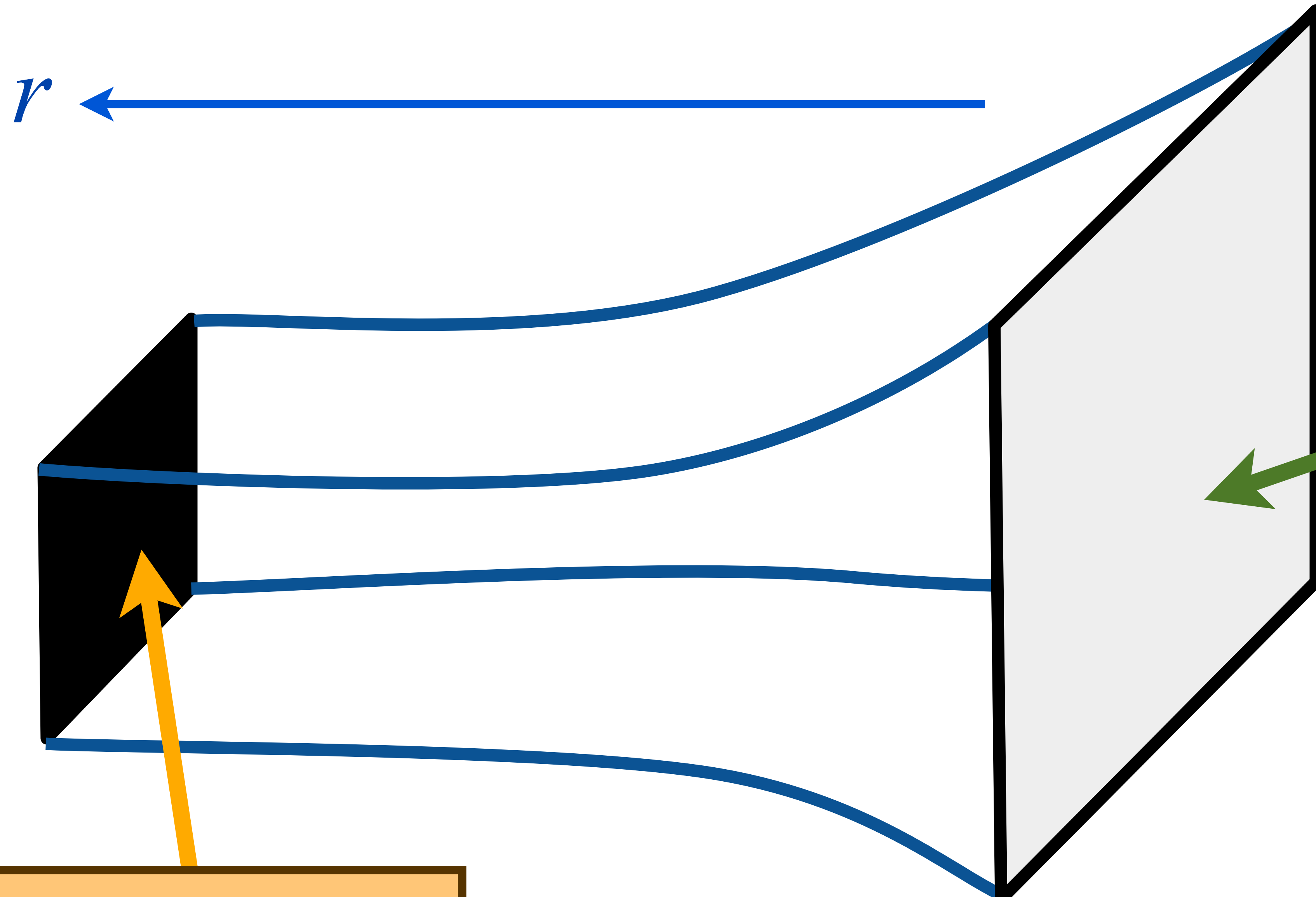
Black-brane at temperature of 2+1 dimensional quantum critical system

$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = 1 - (r/R)^3$

AdS/CFT correspondence at non-zero temperatures

AdS_4-Schwarzschild black-brane



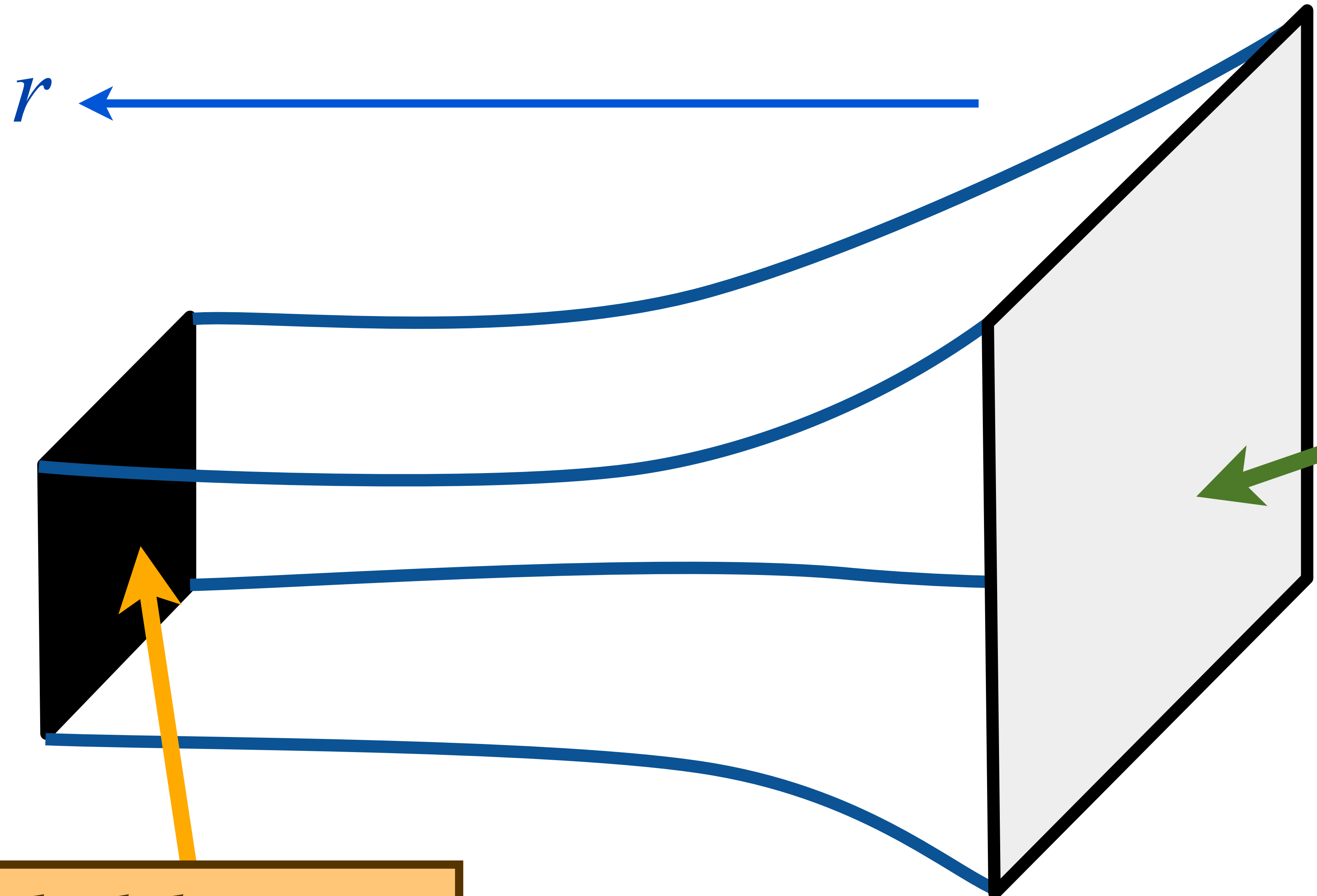
A 2+1 dimensional system at its quantum critical point: $k_B T = \frac{3\hbar}{4\pi R}$.

Black-brane at temperature of 2+1 dimensional quantum critical system

$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = 1 - (r/R)^3$

AdS$_4$-Schwarzschild black-brane



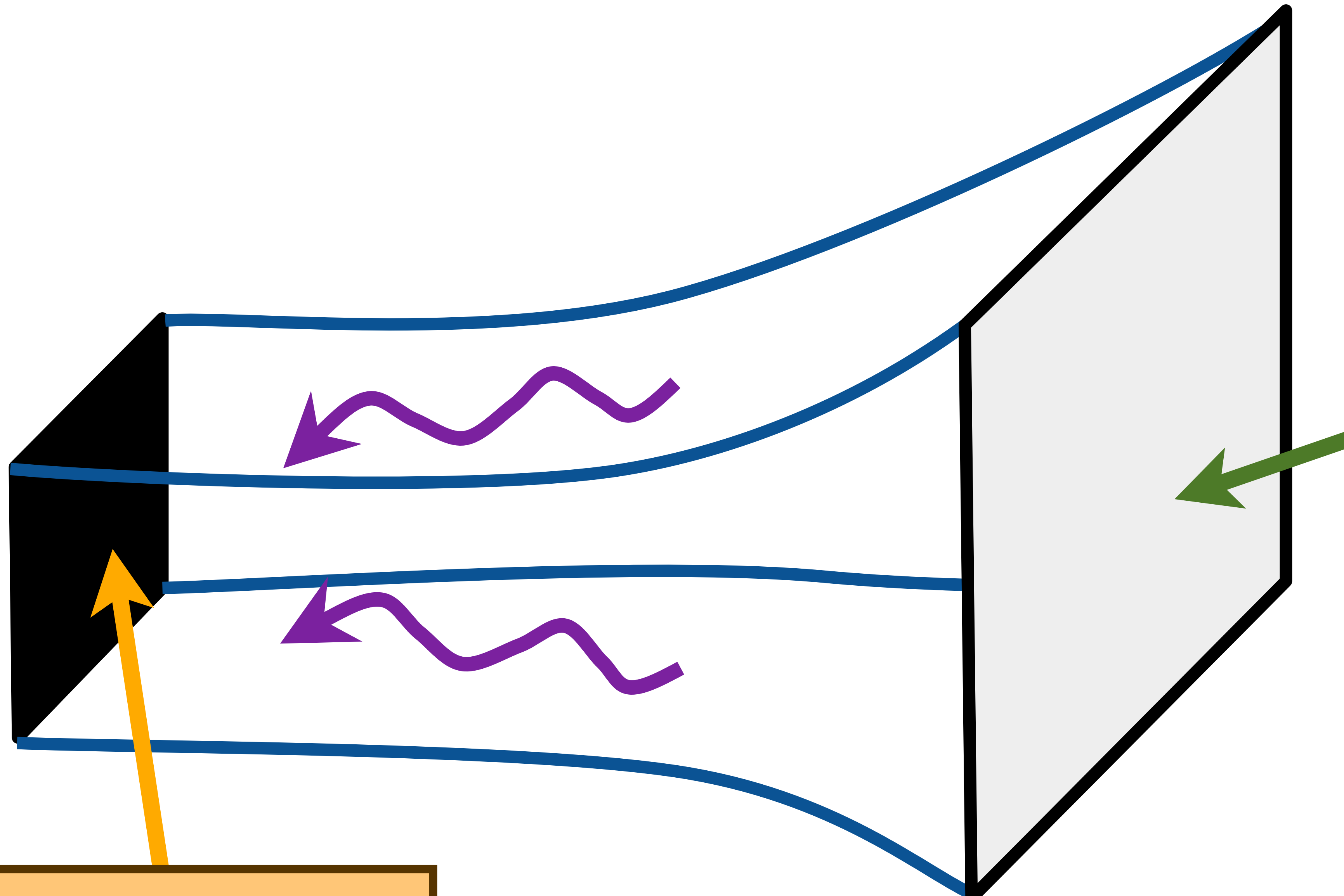
A 2+1 dimensional system at its quantum critical point:

$$k_B T = \frac{3\hbar}{4\pi R}.$$

Black-brane at temperature of 2+1 dimensional quantum critical system

Beckenstein-Hawking entropy of black brane = entropy of CFT3

AdS$_4$-Schwarzschild black-brane



A 2+1 dimensional system at its quantum critical point:
$$k_B T = \frac{3\hbar}{4\pi R}.$$

Black-brane at temperature of 2+1 dimensional quantum critical system

Friction of quantum criticality = waves falling into black brane

D. T. Son

AdS4 theory of quantum criticality

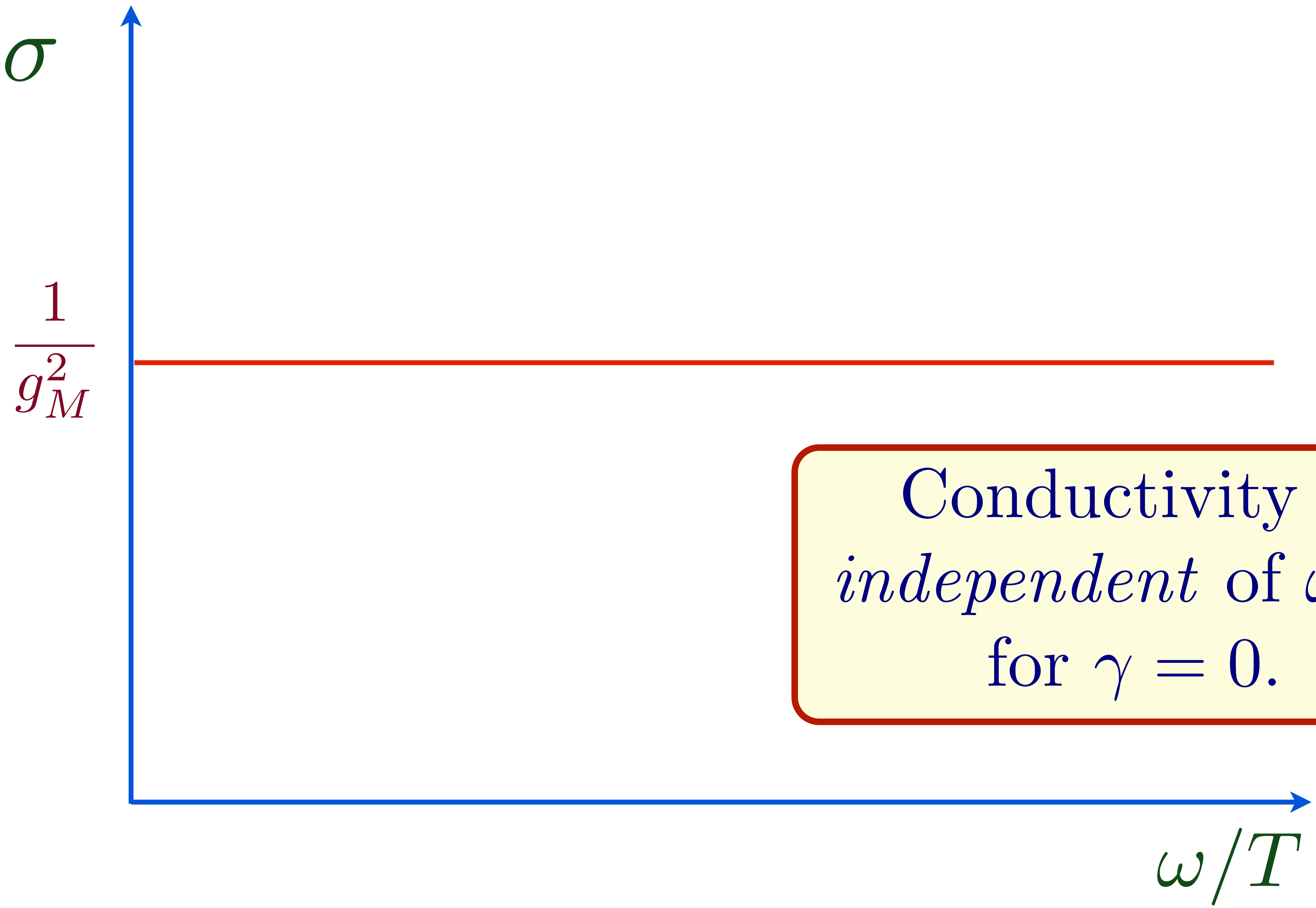
Most general effective holographic theory for linear charge transport with 4 spatial derivatives:

$$\mathcal{S}_{\text{bulk}} = \frac{1}{g_M^2} \int d^4x \sqrt{g} \left[\frac{1}{4} F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right] + \int d^4x \sqrt{g} \left[-\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right],$$

where C_{abcd} is the Weyl tensor, and F_{ab} is the bulk gauge flux dual to the conserved current of the CFT. Three dimensionless parameters, g_M, L^2/κ^2, and γ, are related to CFT3 correlators: σ_∞, the central charge, and to 3-point correlators of current and stress energy tensors. Boundary and bulk methods show that $|\gamma| \leq 1/12$, and the bound is saturated by free fields.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, arXiv:1210.5247

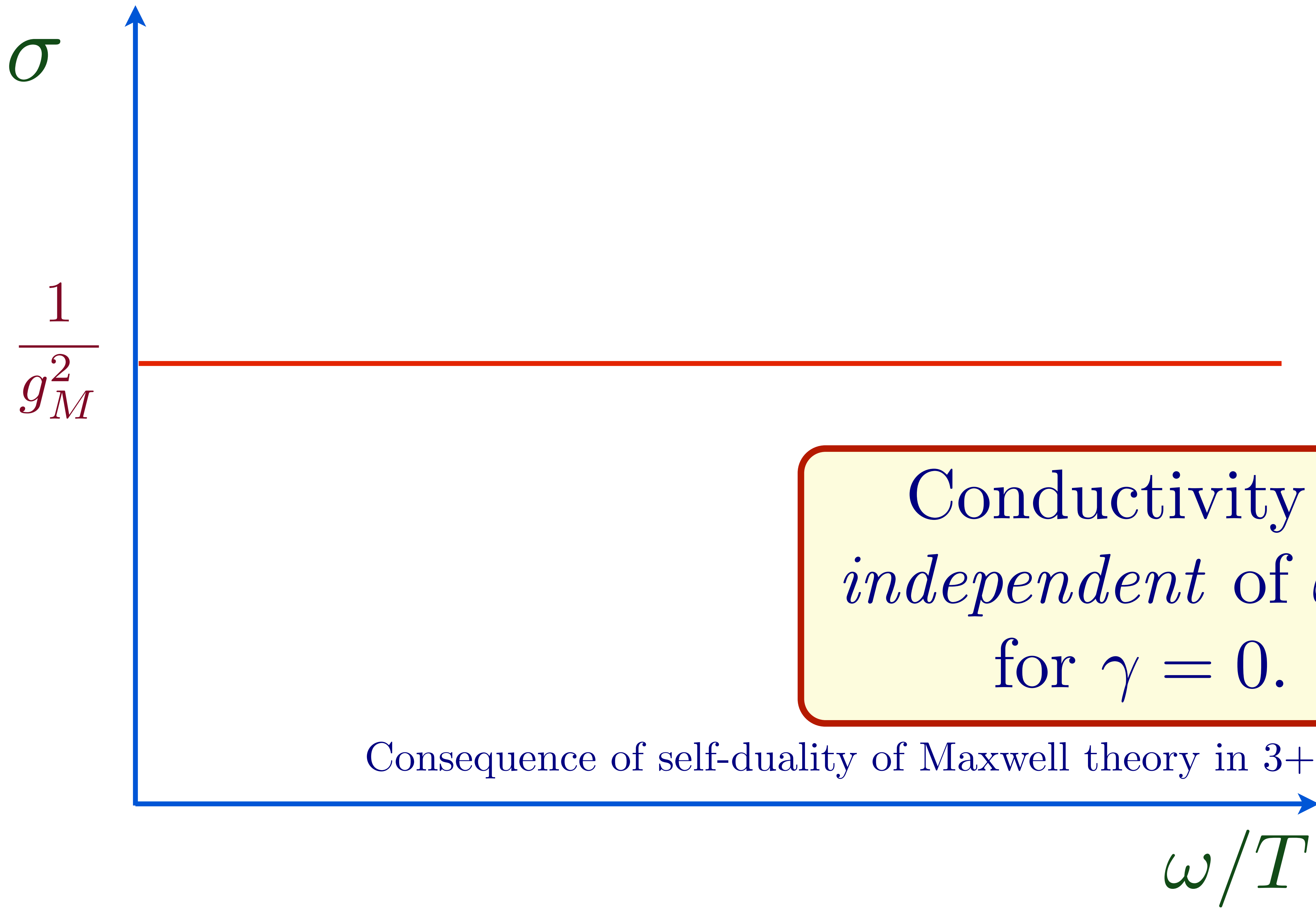
AdS_4 theory of quantum criticality



Conductivity is *independent* of ω/T for $\gamma = 0$.

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

AdS$_4$ theory of quantum criticality

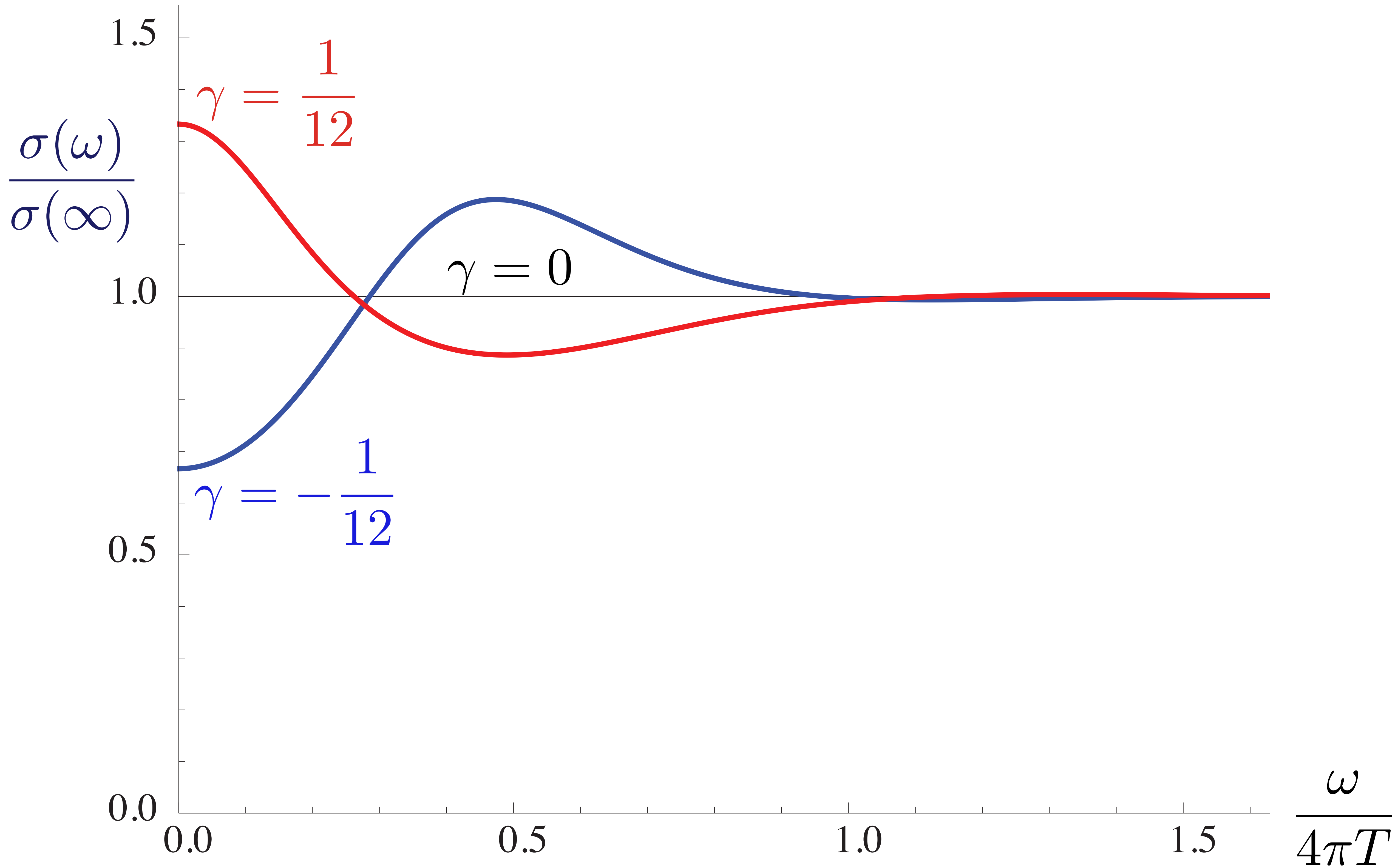


Conductivity is *independent* of ω/T for $\gamma = 0$.

Consequence of self-duality of Maxwell theory in 3+1 dimensions

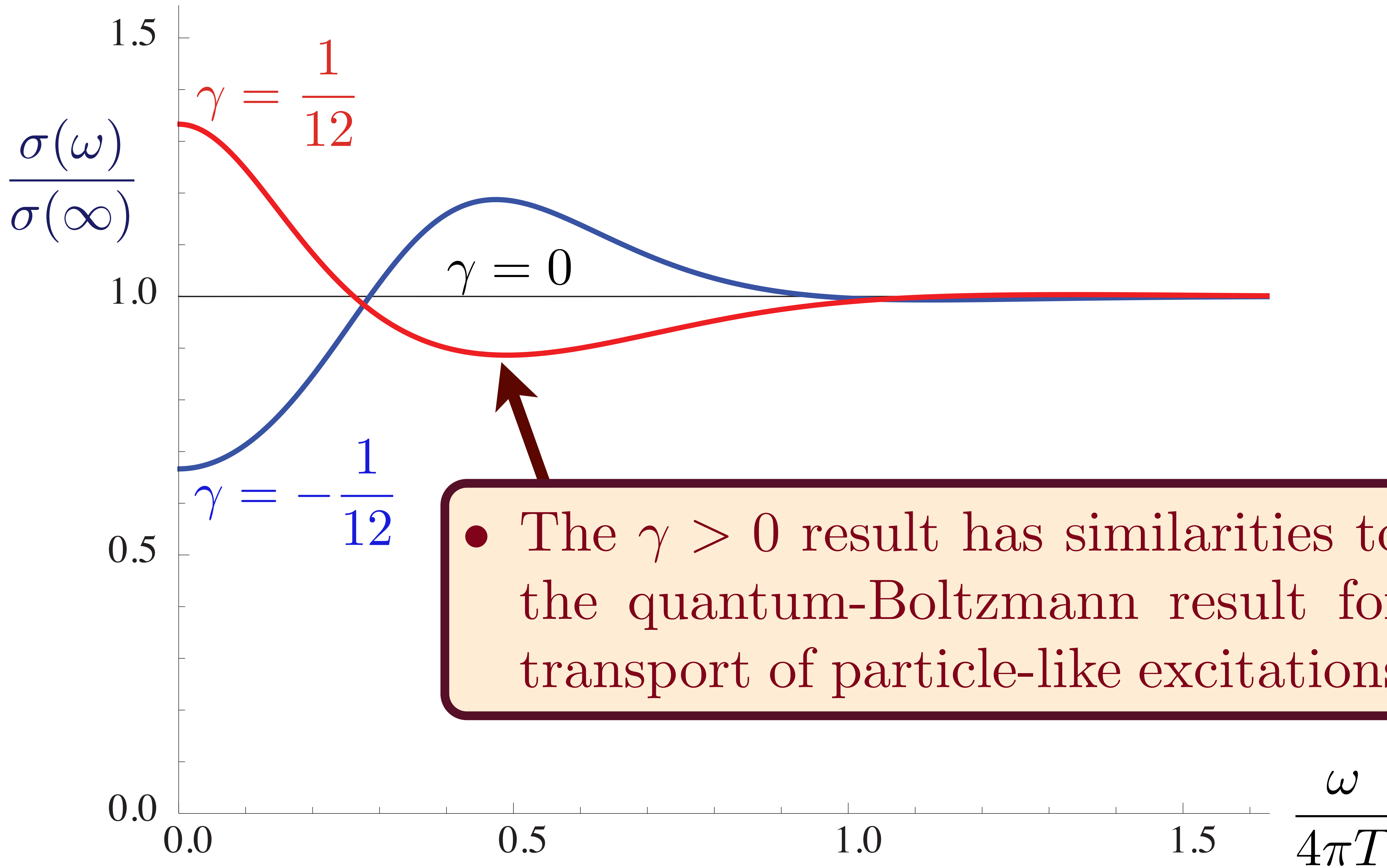
C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

AdS_4 theory of quantum criticality



R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

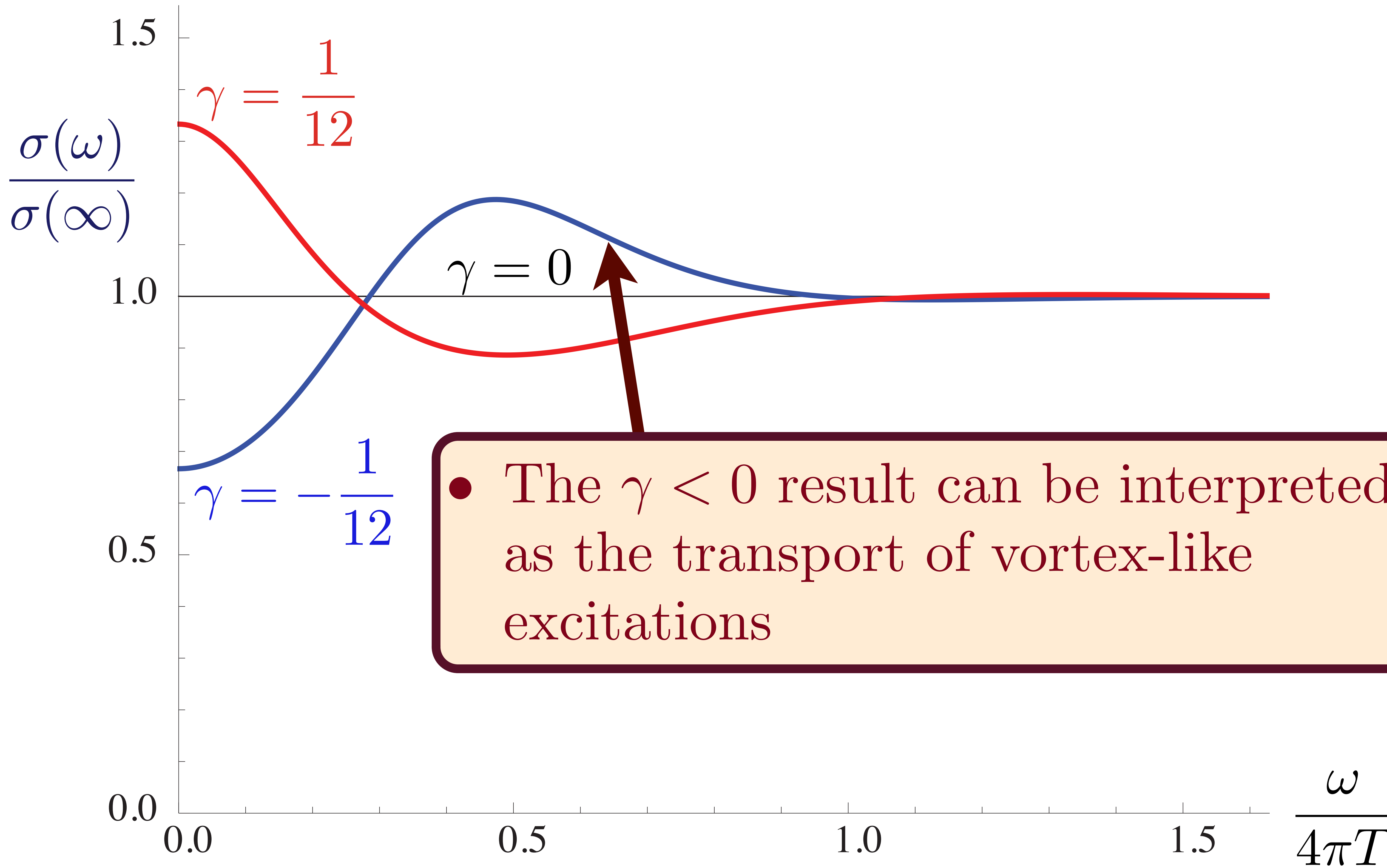
AdS$_4$ theory of quantum criticality



- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

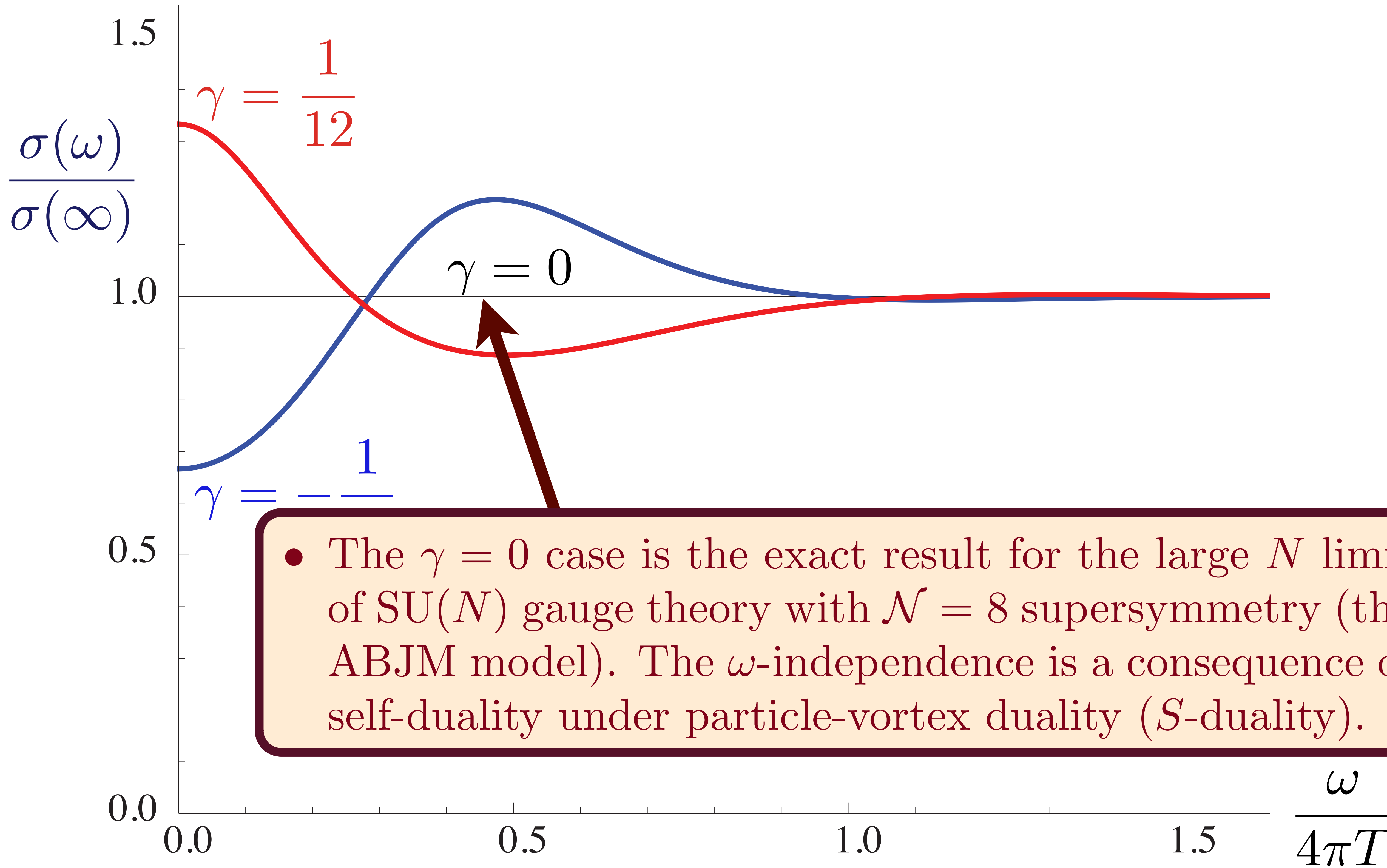
AdS_4 theory of quantum criticality



- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

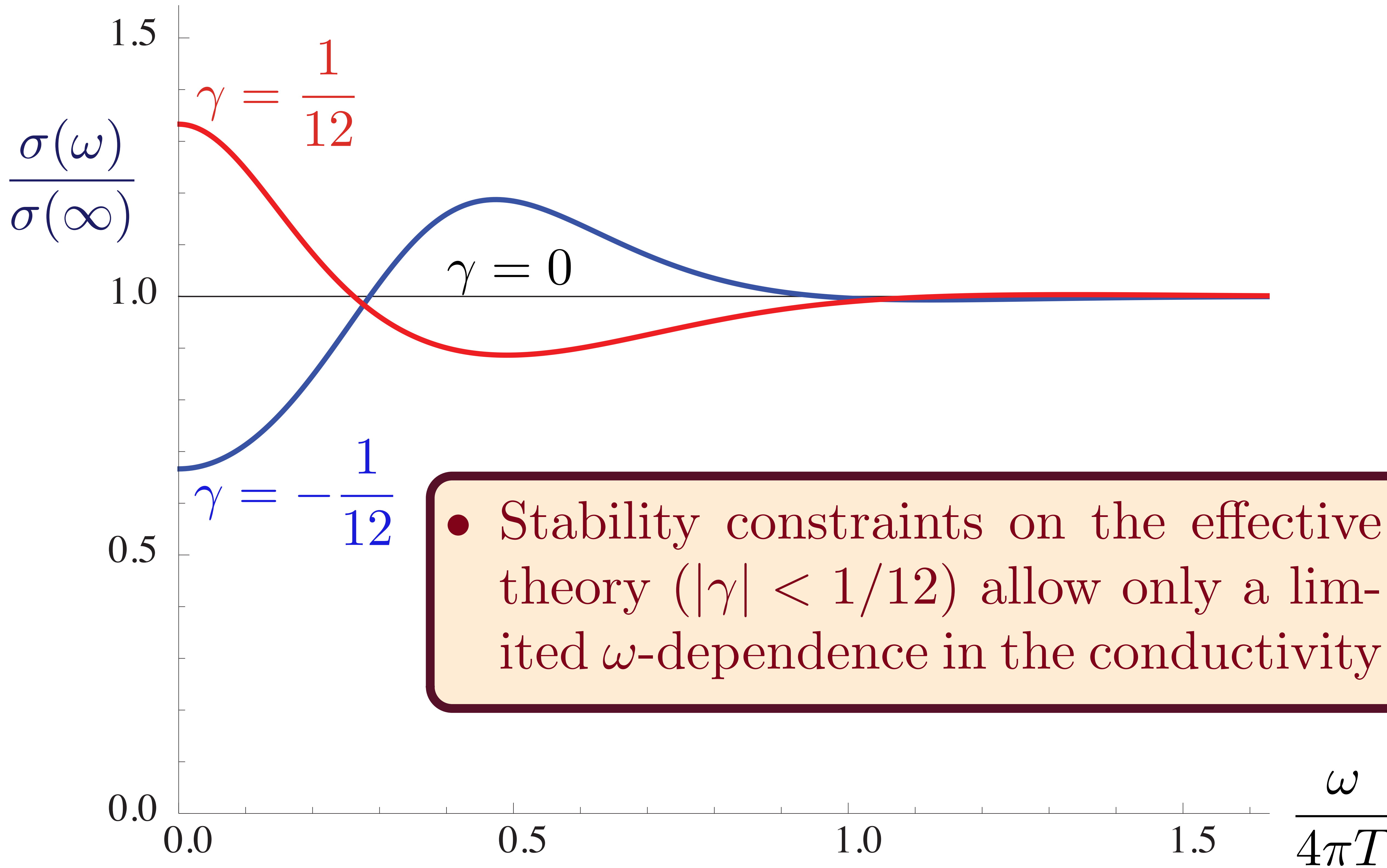
AdS$_4$ theory of quantum criticality



- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of quantum criticality

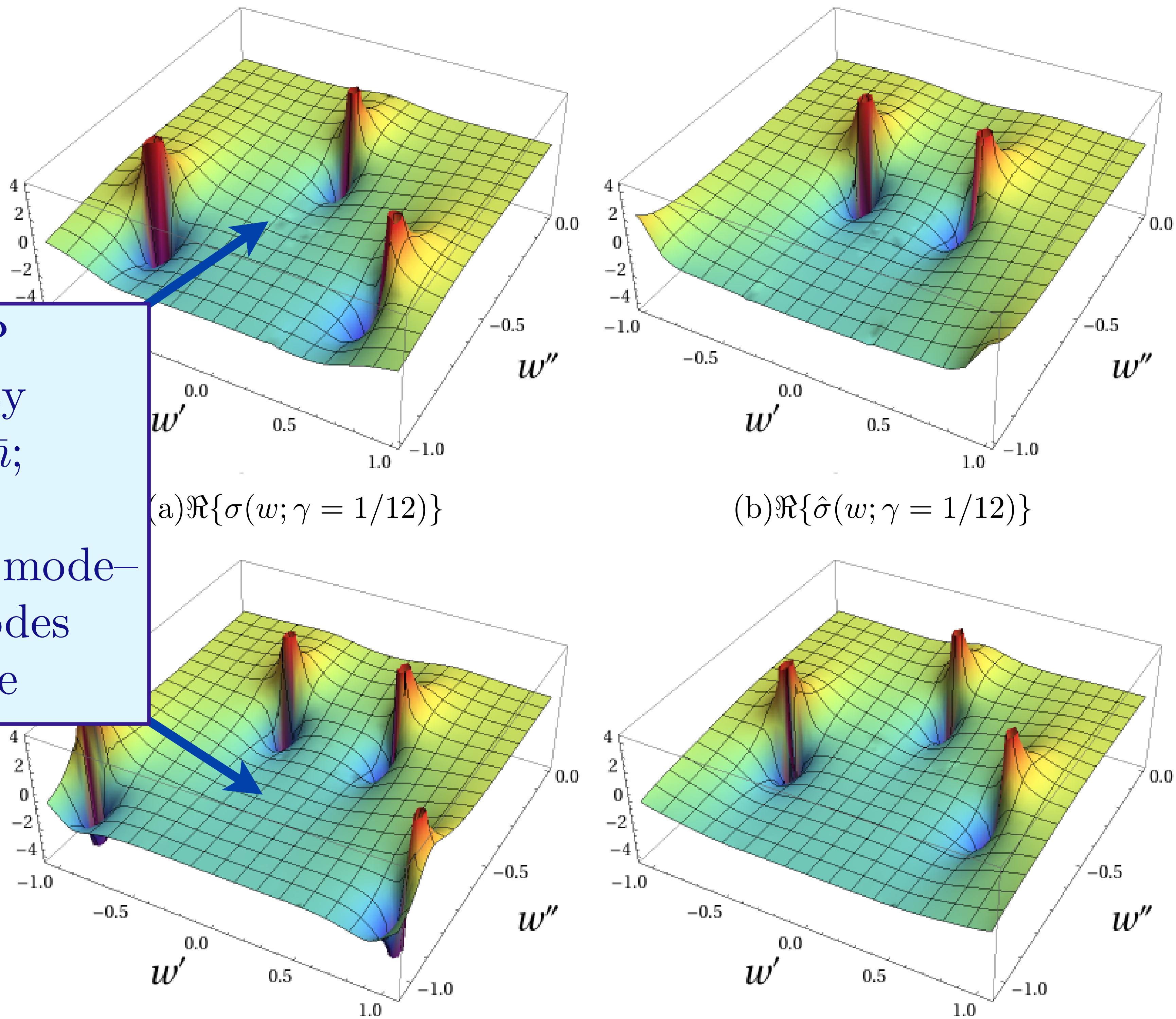


- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of quantum criticality

Poles in LHP
of conductivity
at $\omega \sim k_B T/\hbar$;
analog of
Higgs quasinormal mode–
quasinormal modes
of black brane



(a) $\Re\{\sigma(w; \gamma = 1/12)\}$

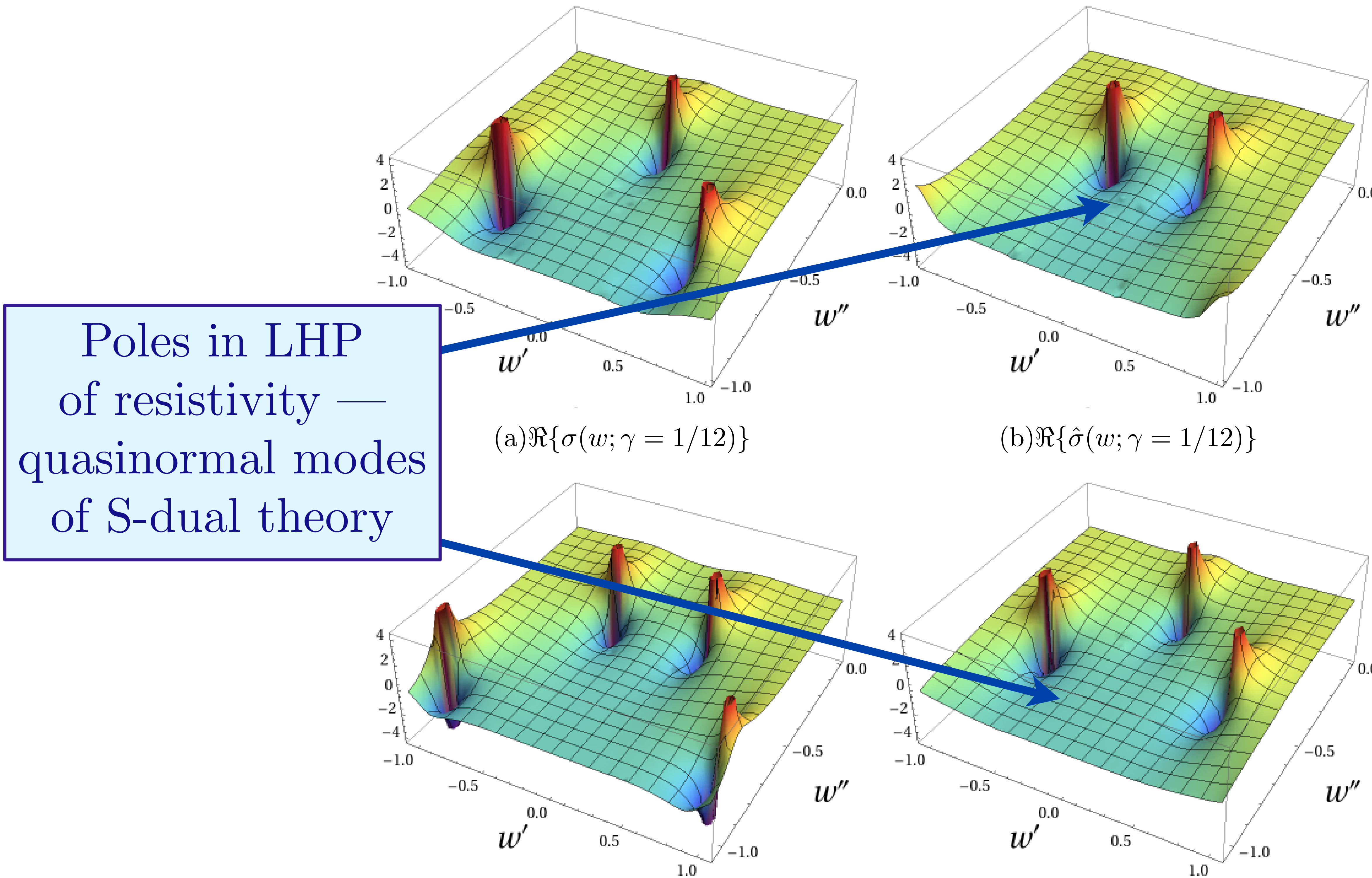
(b) $\Re\{\hat{\sigma}(w; \gamma = 1/12)\}$

(c) $\Re\{\sigma(w; \gamma = -1/12)\}$

(d) $\Re\{\hat{\sigma}(w; \gamma = -1/12)\}$

W. Witzack-Krempa and S. Sachdev, *Physical Review D* **86**, 235115 (2012)

AdS4 theory of quantum criticality



(a) $\Re\{\sigma(w;\gamma=1/12)\}$

(b) $\Re\{\hat{\sigma}(w;\gamma=1/12)\}$

(c) $\Re\{\sigma(w;\gamma=-1/12)\}$

(d) $\Re\{\hat{\sigma}(w;\gamma=-1/12)\}$

W. Witzack-Krempa and S. Sachdev, *Physical Review D* **86**, 235115 (2012)

AdS$_4$ theory of quantum criticality

The holographic solutions for the conductivity satisfy two sum rules, valid for all CFT3s. (W. Witzack-Krempa and S. Sachdev, Phys. Rev. B **86**, 235115 (2012))

$$\int_0^\infty d\omega \mathrm{Re}\left[\sigma(\omega) - \sigma(\infty)\right] = 0$$

$$\int_0^\infty d\omega \mathrm{Re}\left[\frac{1}{\sigma(\omega)} - \frac{1}{\sigma(\infty)}\right] = 0$$

The second rule follows from the existence of a EM-dual CFT3.

Boltzmann theory chooses a "particle" basis: this satisfies only *one* sum rule but not the other.

Holographic theory satisfies both sum rules.

Traditional CMT

- Identify quasiparticles and their dispersions

Traditional CMT

- Identify quasiparticles and their dispersions

- Compute scattering matrix elements of quasiparticles (or of collective modes)

Traditional CMT

- Identify quasiparticles and their dispersions

- Compute scattering matrix elements of quasiparticles (or of collective modes)

- These parameters are input into a quantum Boltzmann equation

Traditional CMT

- Identify quasiparticles and their dispersions

- Compute scattering matrix elements of quasiparticles (or of collective modes)

- These parameters are input into a quantum Boltzmann equation

- Deduce dissipative and dynamic properties at non-zero temperatures

Traditional CMT

- Identify quasiparticles and their dispersions
- Compute scattering matrix elements of quasiparticles (or of collective modes)
- These parameters are input into a quantum Boltzmann equation
- Deduce dissipative and dynamic properties at non-zero temperatures

Holography and black-branes

- Start with strongly interacting CFT without particle- or wave-like excitations

Traditional CMT

- Identify quasiparticles and their dispersions
- Compute scattering matrix elements of quasiparticles (or of collective modes)
- These parameters are input into a quantum Boltzmann equation
- Deduce dissipative and dynamic properties at non-zero temperatures

Holography and black-branes

- Start with strongly interacting CFT without particle- or wave-like excitations
- Compute OPE co-efficients of operators of the CFT

Traditional CMT

- Identify quasiparticles and their dispersions
- Compute scattering matrix elements of quasiparticles (or of collective modes)
- These parameters are input into a quantum Boltzmann equation
- Deduce dissipative and dynamic properties at non-zero temperatures

Holography and black-branes

- Start with strongly interacting CFT without particle- or wave-like excitations
- Compute OPE co-efficients of operators of the CFT
- Relate OPE co-efficients to couplings of an effective graviational theory on AdS

Traditional CMT

- Identify quasiparticles and their dispersions
- Compute scattering matrix elements of quasiparticles (or of collective modes)
- These parameters are input into a quantum Boltzmann equation
- Deduce dissipative and dynamic properties at non-zero temperatures

Holography and black-branes

- Start with strongly interacting CFT without particle- or wave-like excitations
- Compute OPE co-efficients of operators of the CFT
- Relate OPE co-efficients to couplings of an effective graviational theory on AdS
- Solve Einsten-Maxwell equations. Dynamics of quasi-normal modes of black branes.

"Complex entangled" states of quantum matter, *not* adiabatically connected to independent particle states

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene

Compressible quantum matter

Strange metals in high temperature superconductors, Bose metals

S. Sachdev, 100th anniversary Solvay conference,arXiv:1203.4565

"Complex entangled" states of quantum matter, *not* adiabatically connected to independent particle states

Gapped quantum matter

Spin liquids, quantum Hall states

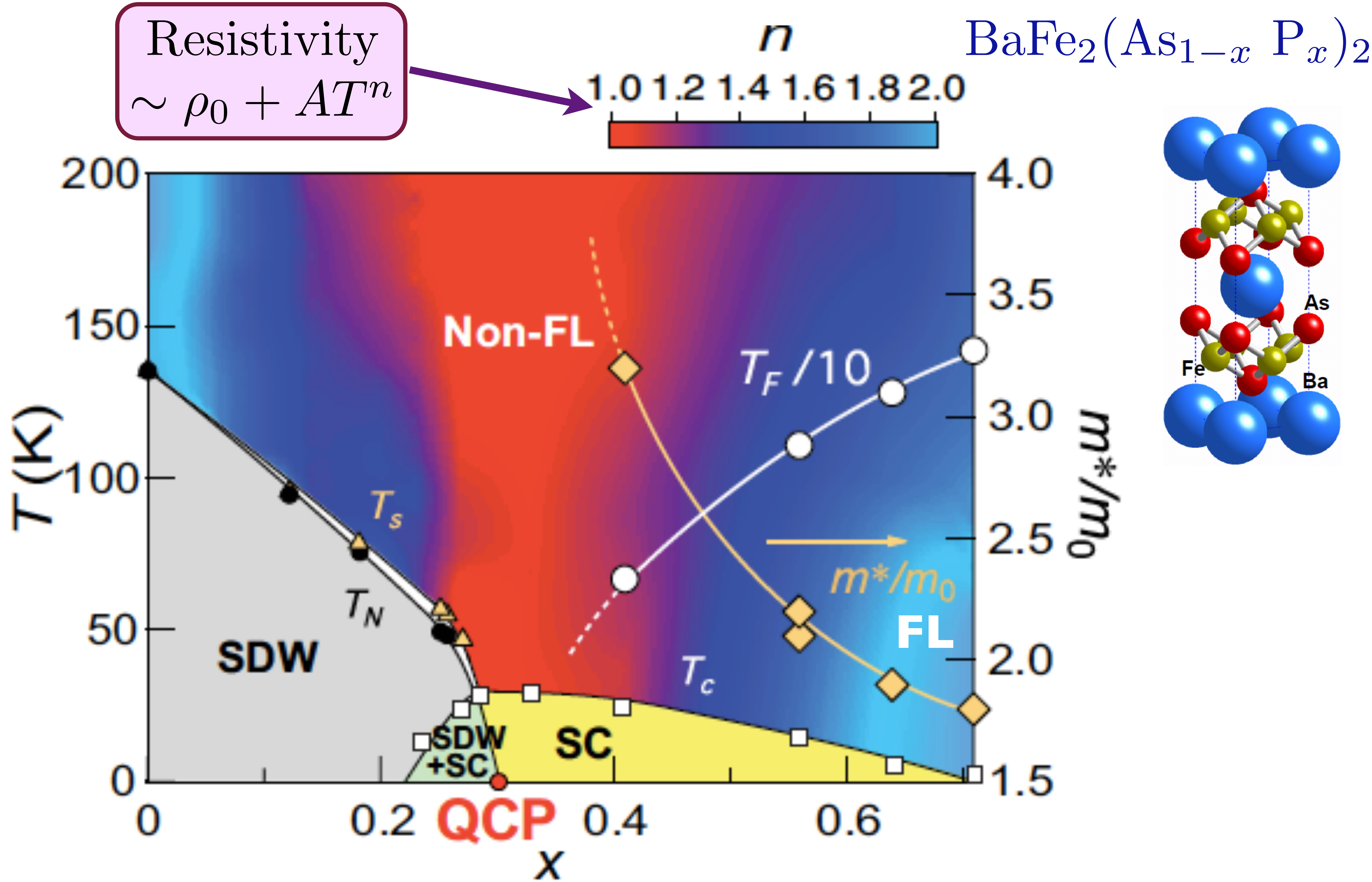
Conformal quantum matter

Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene

Compressible quantum matter

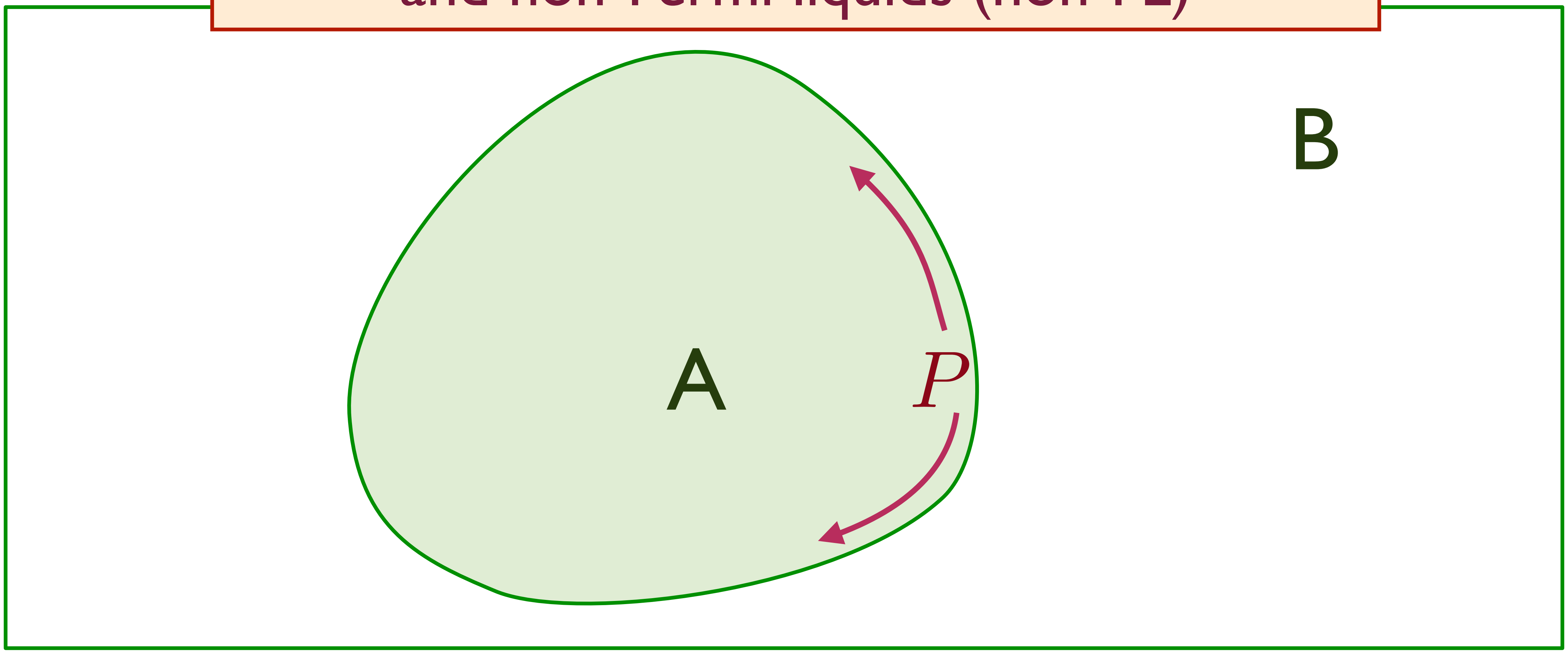
Strange metals in high temperature superconductors, Bose metals

S. Sachdev, 100th anniversary Solvay conference,arXiv:1203.4565



K. Hashimoto, K. Cho, T. Shibauchi, S. Kasahara, Y. Mizukami, R. Katsumata, Y. Tsuruhara, T. Terashima, H. Ikeda, M. A. Tanatar, H. Kitano, N. Salovich, R. W. Giannetta, P. Walmsley, A. Carrington, R. Prozorov, and Y. Matsuda, *Science* **336**, 1554 (2012).

Entanglement entropy of Fermi liquids (FL) and non-Fermi liquids (non-FL)

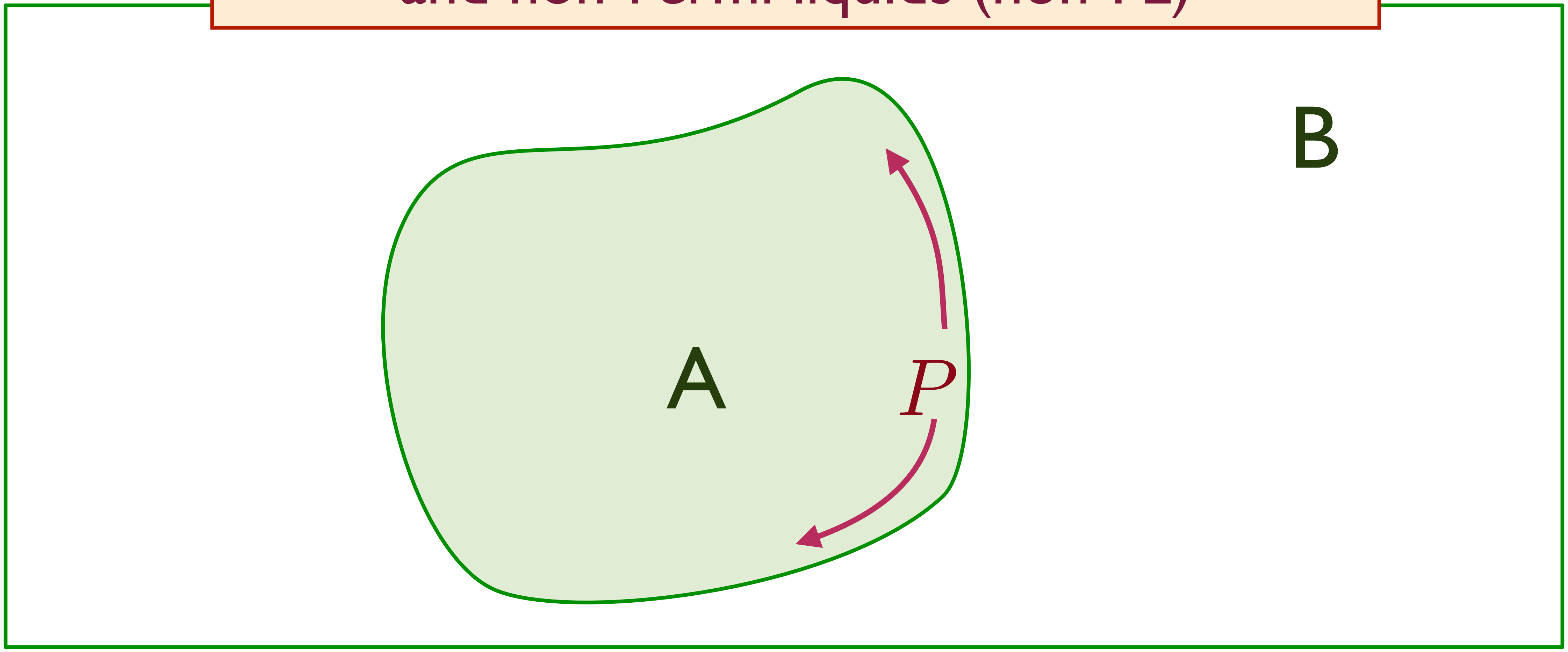


Logarithmic violation of "area law": $S_E \propto (k_F P) \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)
Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

Entanglement entropy of Fermi liquids (FL) and non-Fermi liquids (non-FL)

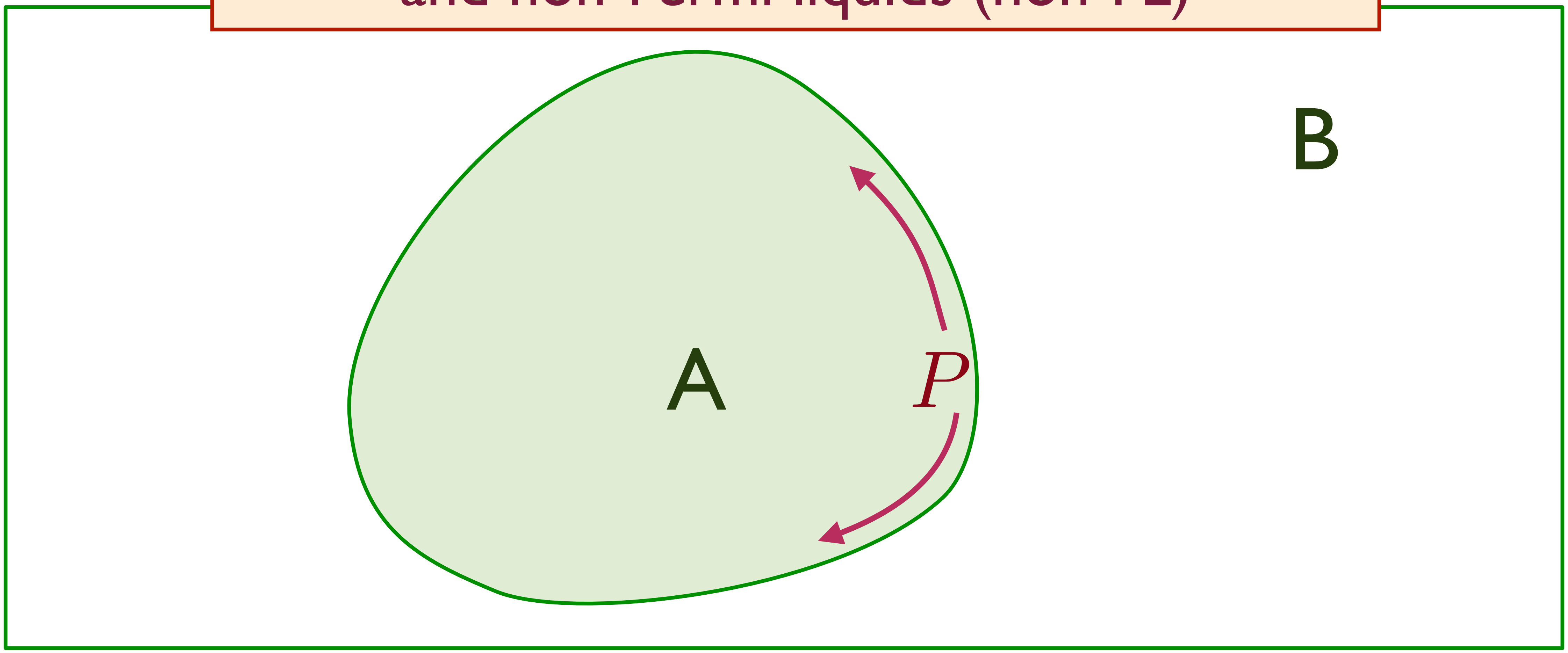


Logarithmic violation of "area law": $S_E \propto (k_F P) \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A.
The coefficient is *independent* of the shape of A.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)
Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

Entanglement entropy of Fermi liquids (FL) and non-Fermi liquids (non-FL)

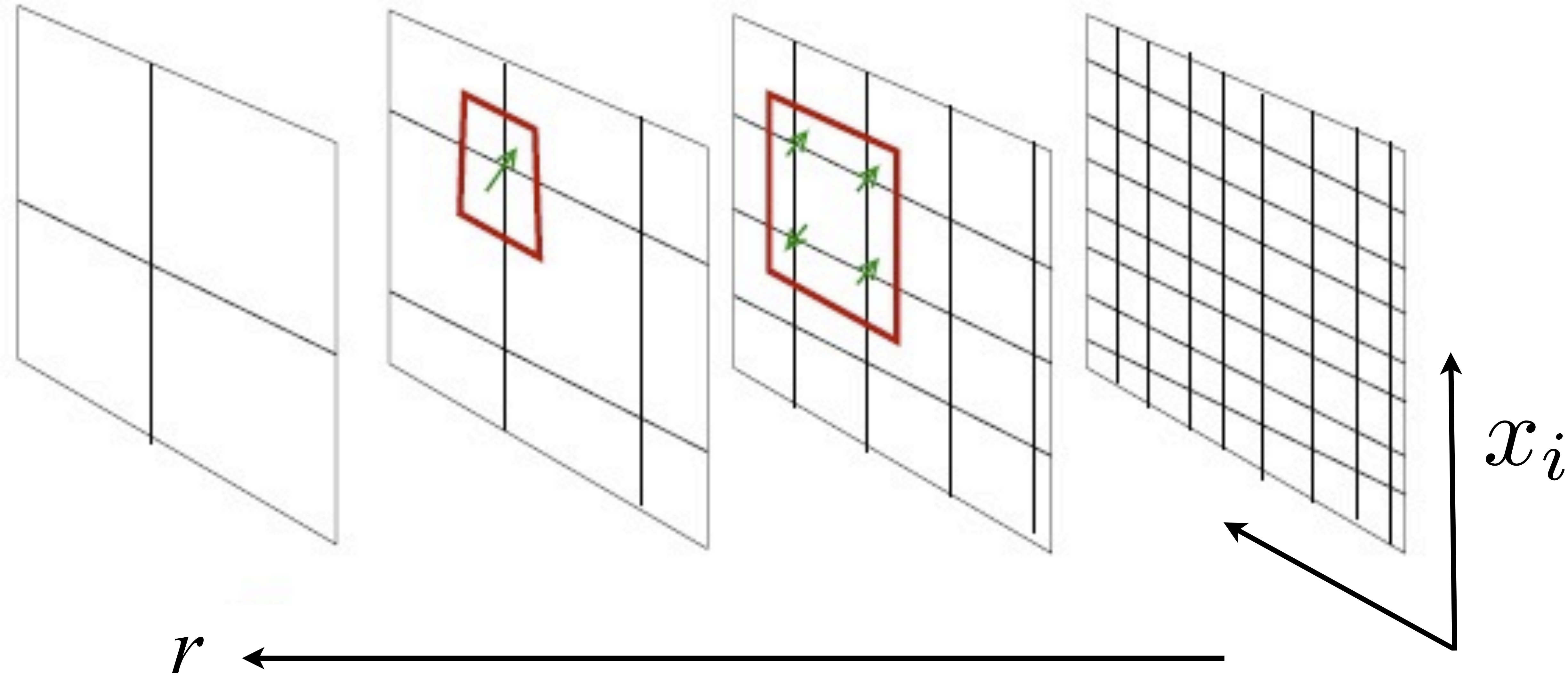


Logarithmic violation of "area law": $S_E \propto (k_F P) \ln(k_F P)$

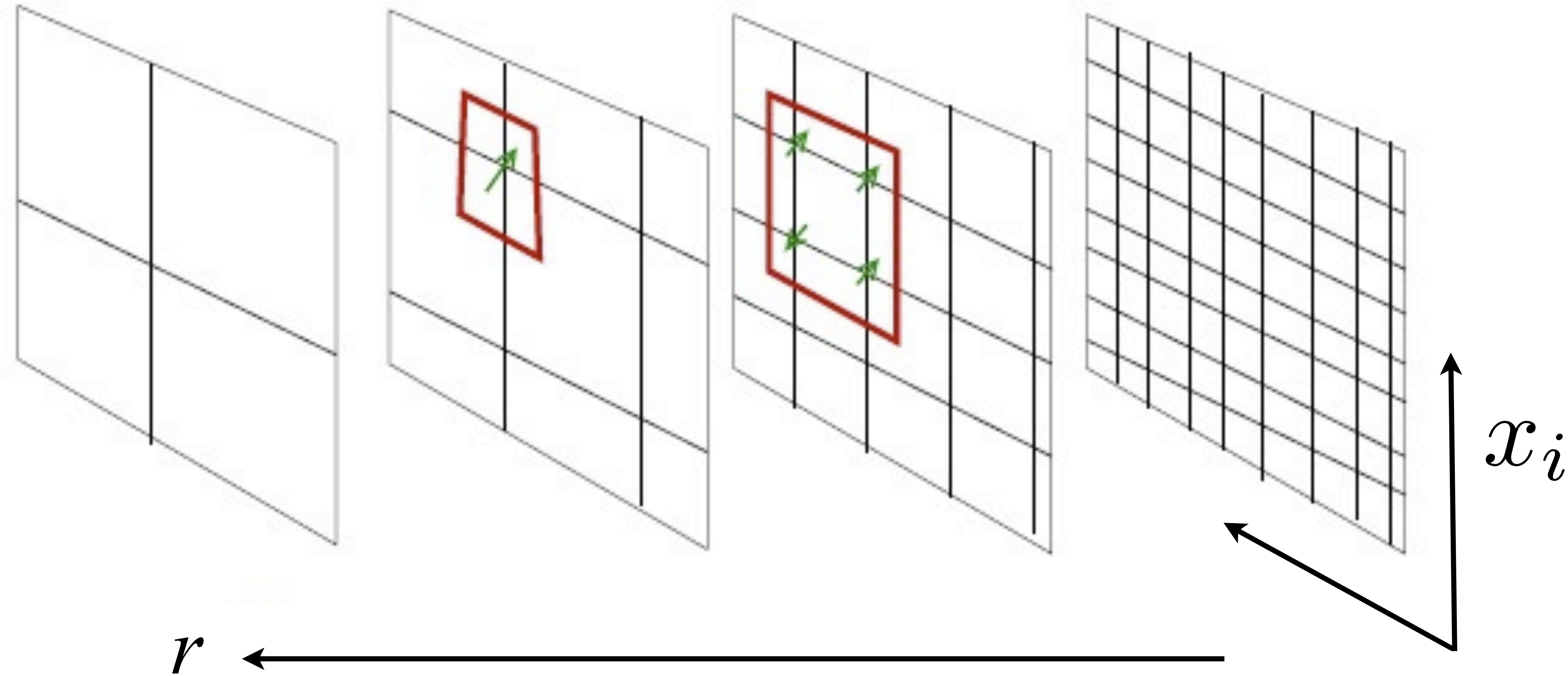
for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A.
The coefficient is *independent* of the shape of A.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)
Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

Holography



Holography

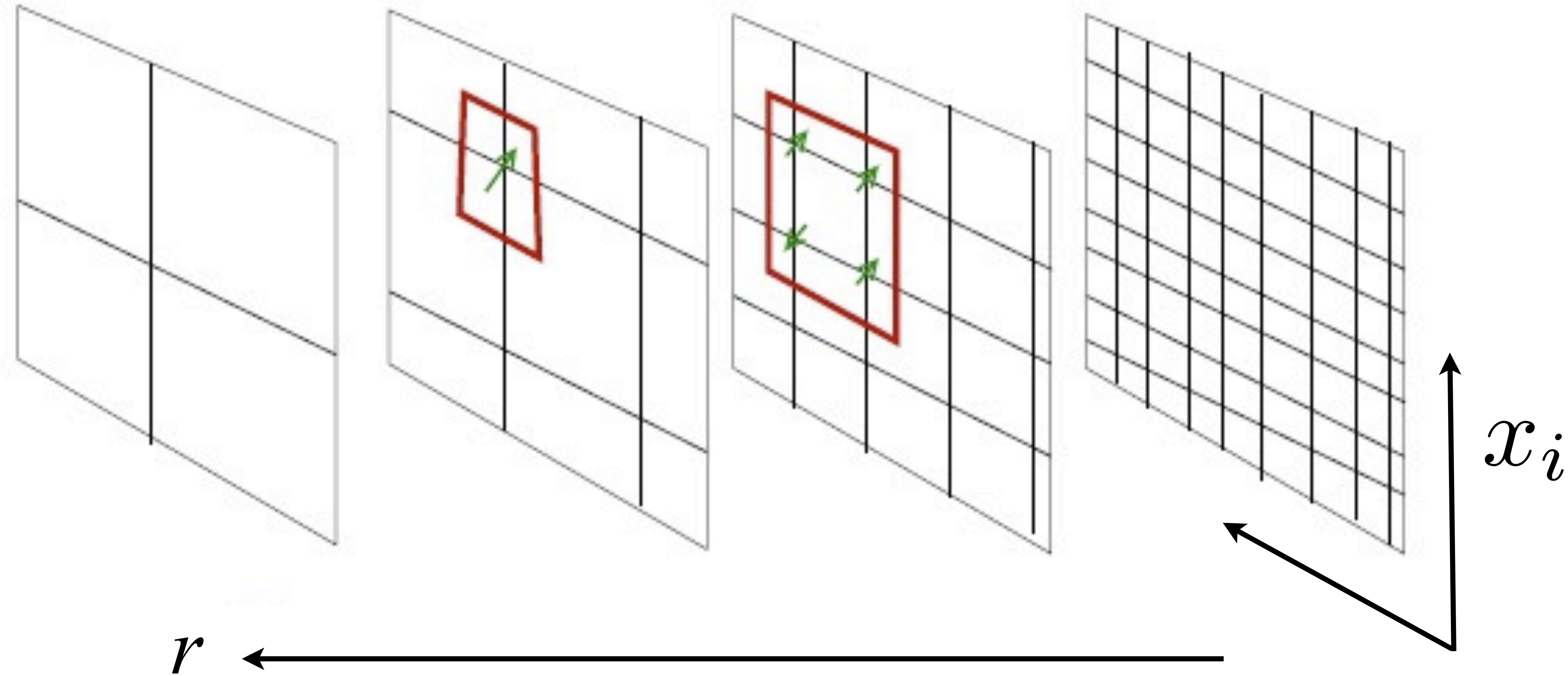


Consider a metric which transforms under rescaling as

$$x_i \to \zeta\, x_i, \quad t \to \zeta^z\, t, \quad ds \to \zeta^{\theta/d}\, ds.$$

Recall: conformal matter has $\theta = 0$, $z = 1$, and the metric is anti-de Sitter

Holography



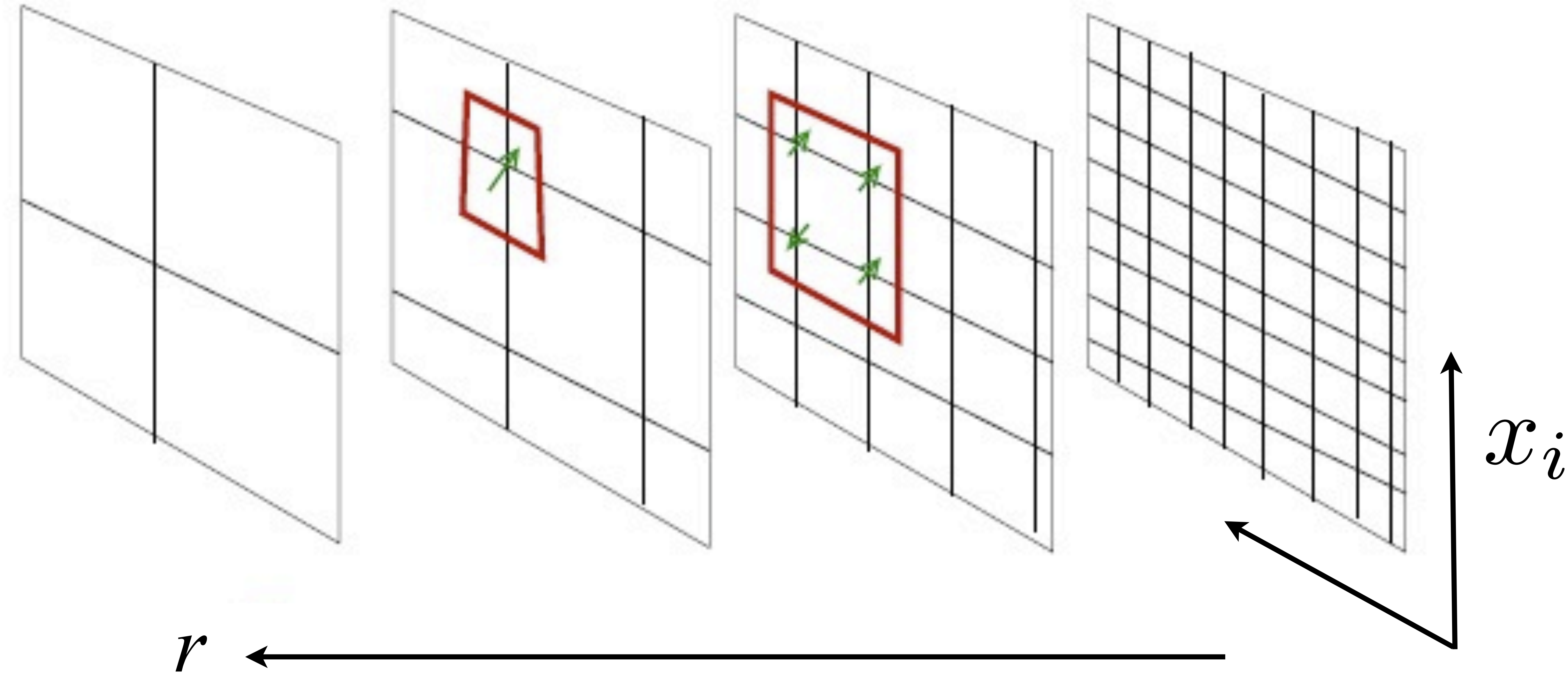
Consider a metric which transforms under rescaling as

$$x_i \to \zeta\, x_i, \quad t \to \zeta^z\, t, \quad ds \to \zeta^{\theta/d}\, ds.$$

The value $\theta = d-1$ reproduces *all* the essential characteristics of the entropy and entanglement entropy of a non-FL.

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography



Consider a metric which transforms under rescaling as

$$x_i \to \zeta\, x_i, \quad t \to \zeta^z\, t, \quad ds \to \zeta^{\theta/d}\, ds.$$

The null-energy condition of gravity yields $z \geq 1 + \theta/d$. In $d = 2$, this leads to $z \geq 3/2$. Field theory on non-FL yields $z = 3/2$ to 3 loops!

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)
P. A. Lee, Phys. Rev. Lett. **63**, 680 (1989)
B. Blok and H. Monien, Phys. Rev. B **47**, 3454 (1993)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Conclusions

Realizations of many-particle entanglement:
Z_2 spin liquids and
conformal quantum critical points

Conclusions

Conformal quantum matter

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.

- Good prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport

Conclusions

More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory

Conclusions

String theory and gravity in emergent dimensions offer a remarkable new approach to describing states with many-particle quantum entanglement.

Much recent progress offers hope of a holographic description of "strange metals"