Fermi surfaces and topological order in lattice fermion systems

BEC 2019

Frontiers in Quantum Gases

Sant Feliu de Guixols, September 8, 2019

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PHYSICS

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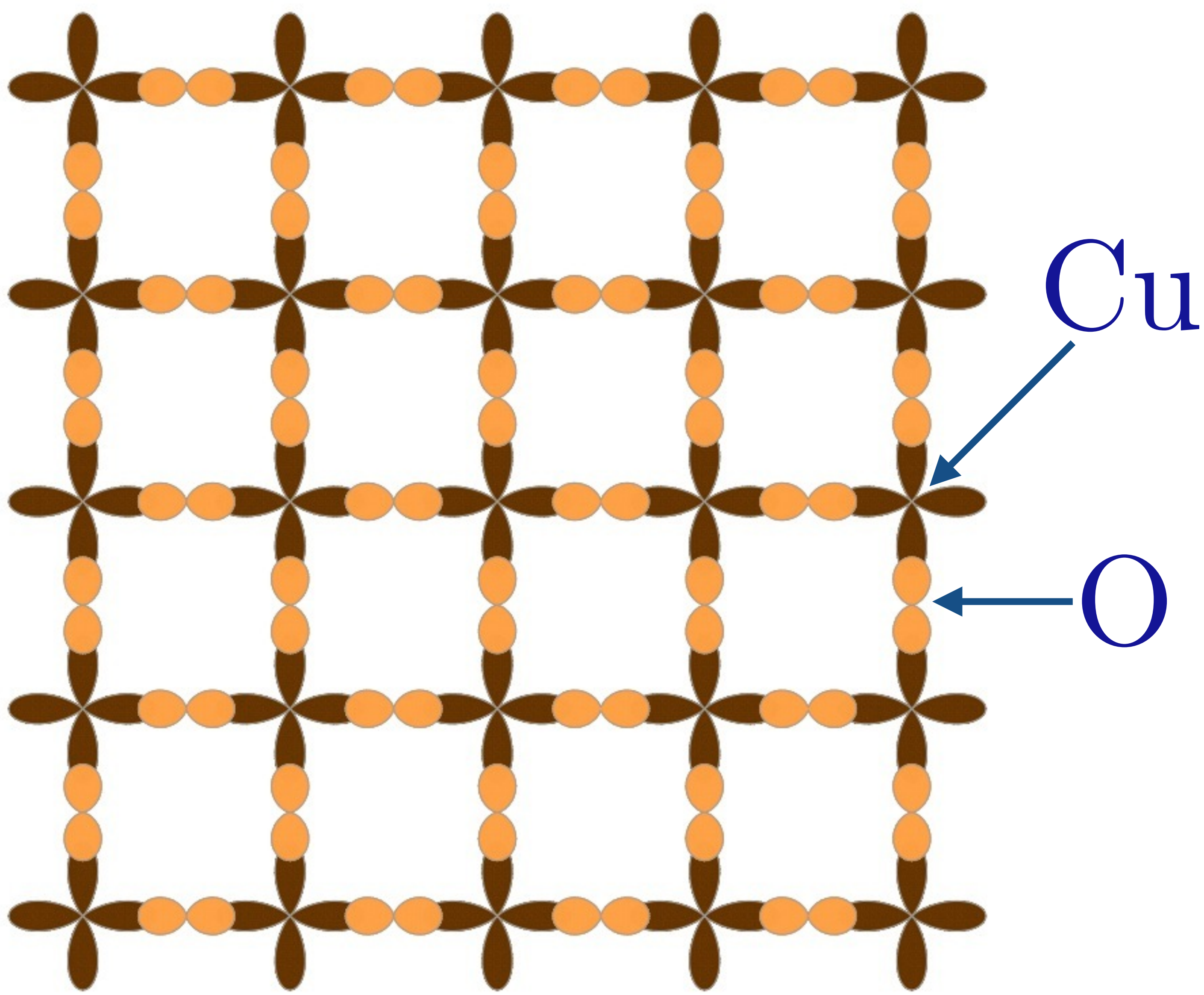
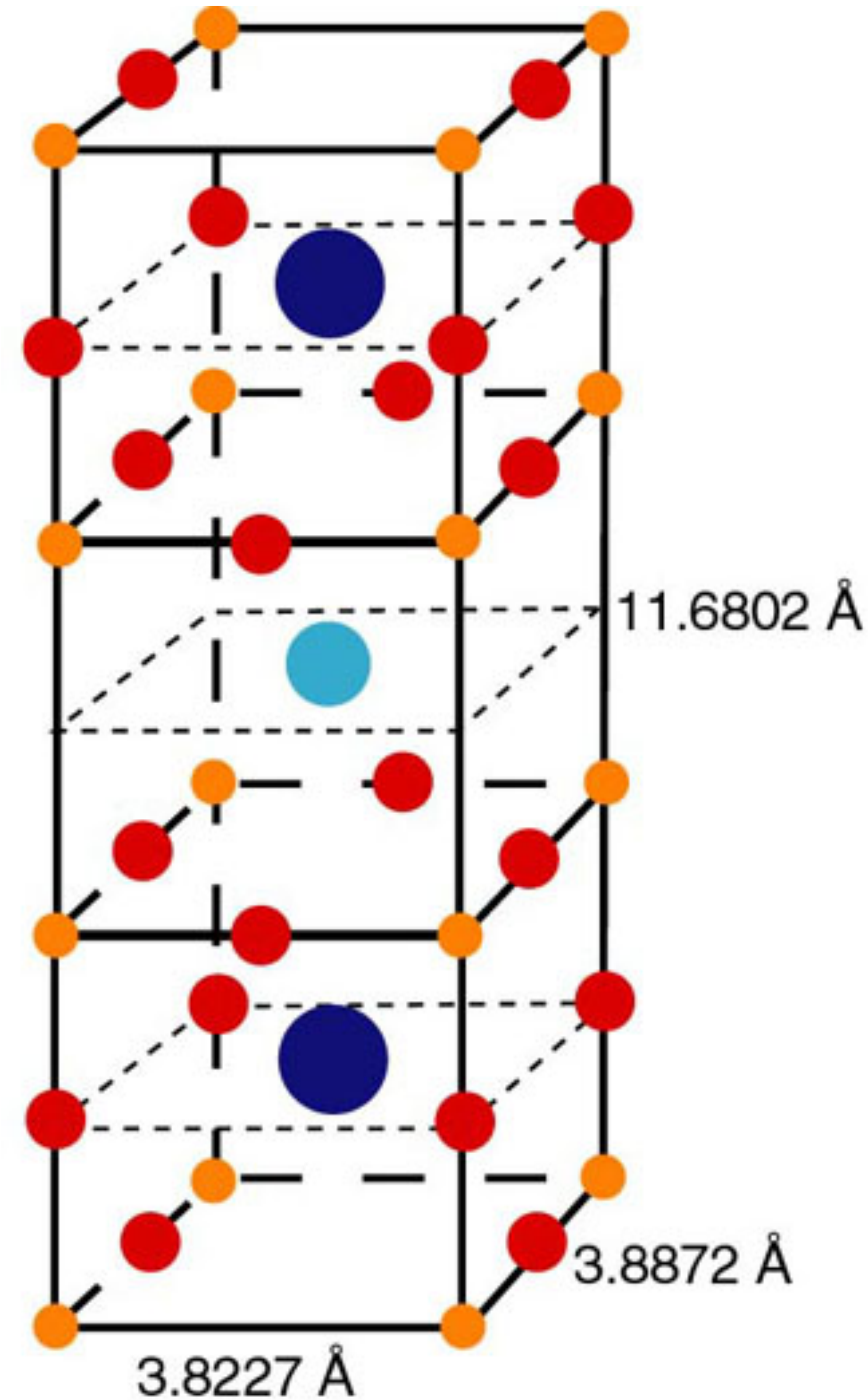
High temperature superconductors



CuO_2 plane



$YBa_2Cu_3O_{6+x}$

The Hubbard Model

$$H = -\sum_{i<j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c^\dagger_{i\alpha} c_{i\alpha}$$

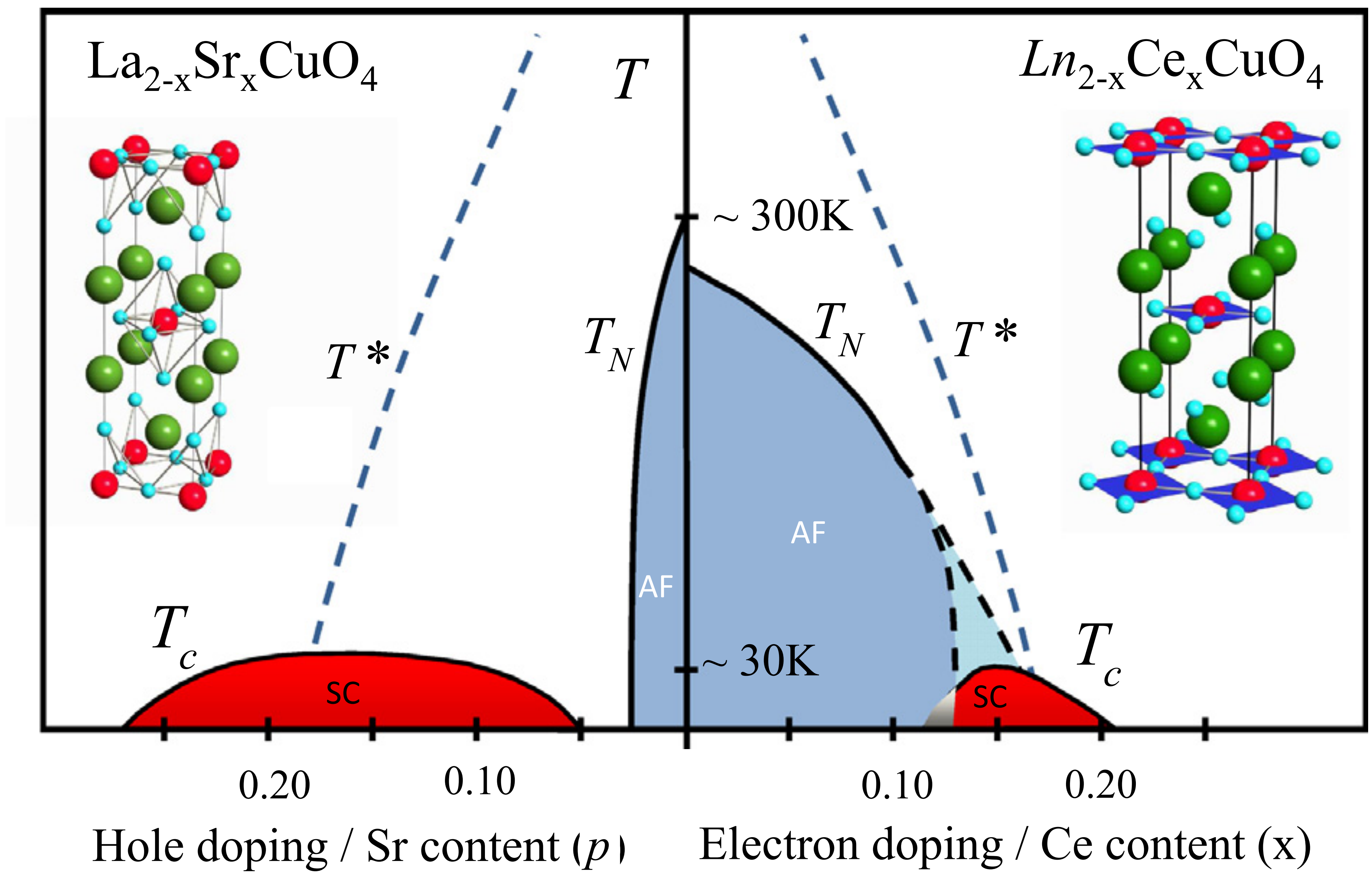
$t_{ij} \rightarrow$ "hopping". $U \rightarrow$ local repulsion, $\mu \rightarrow$ chemical potential

Spin index $\alpha = \uparrow, \downarrow$

$$n_{i\alpha} = c^\dagger_{i\alpha} c_{i\alpha}$$

$$c^\dagger_{i\alpha} c_{j\beta} + c_{j\beta} c^\dagger_{i\alpha} = \delta_{ij}\delta_{\alpha\beta}$$
$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0$$

Will study on the square lattice



$La_{2-x}Sr_xCuO_4$
$Ln_{2-x}Ce_xCuO_4$
T
~ 300K
~ 30K
T_N
T_N
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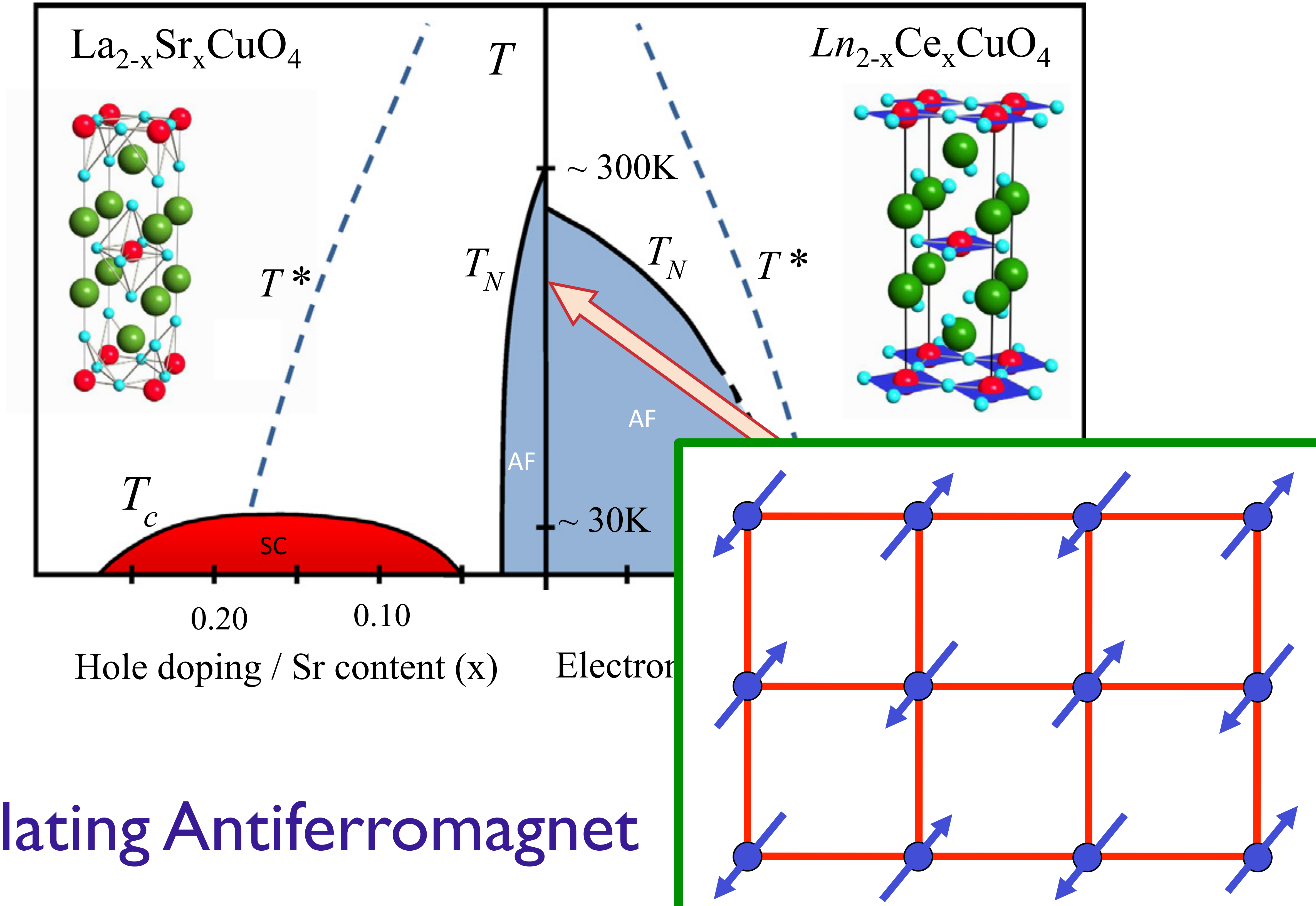
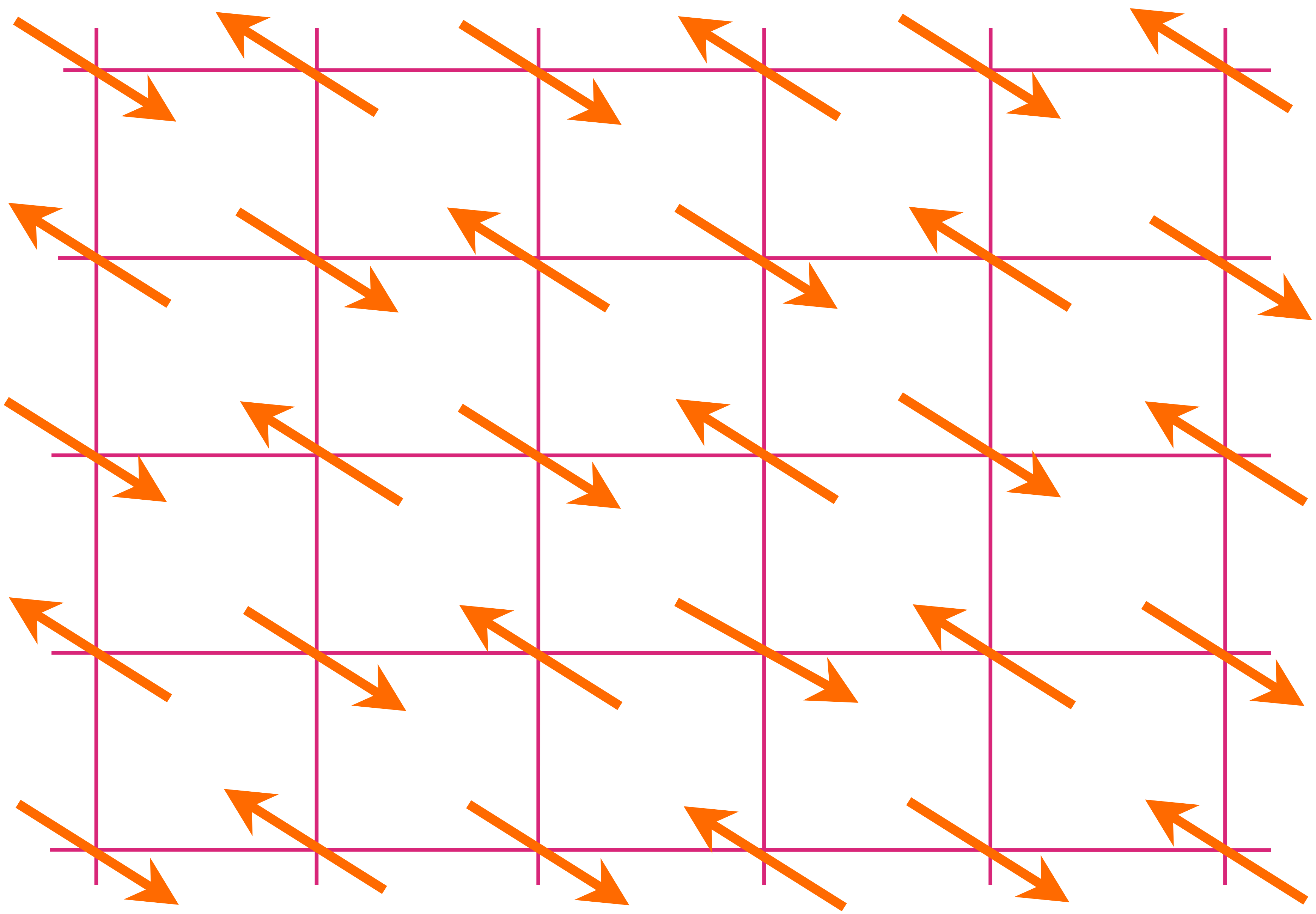
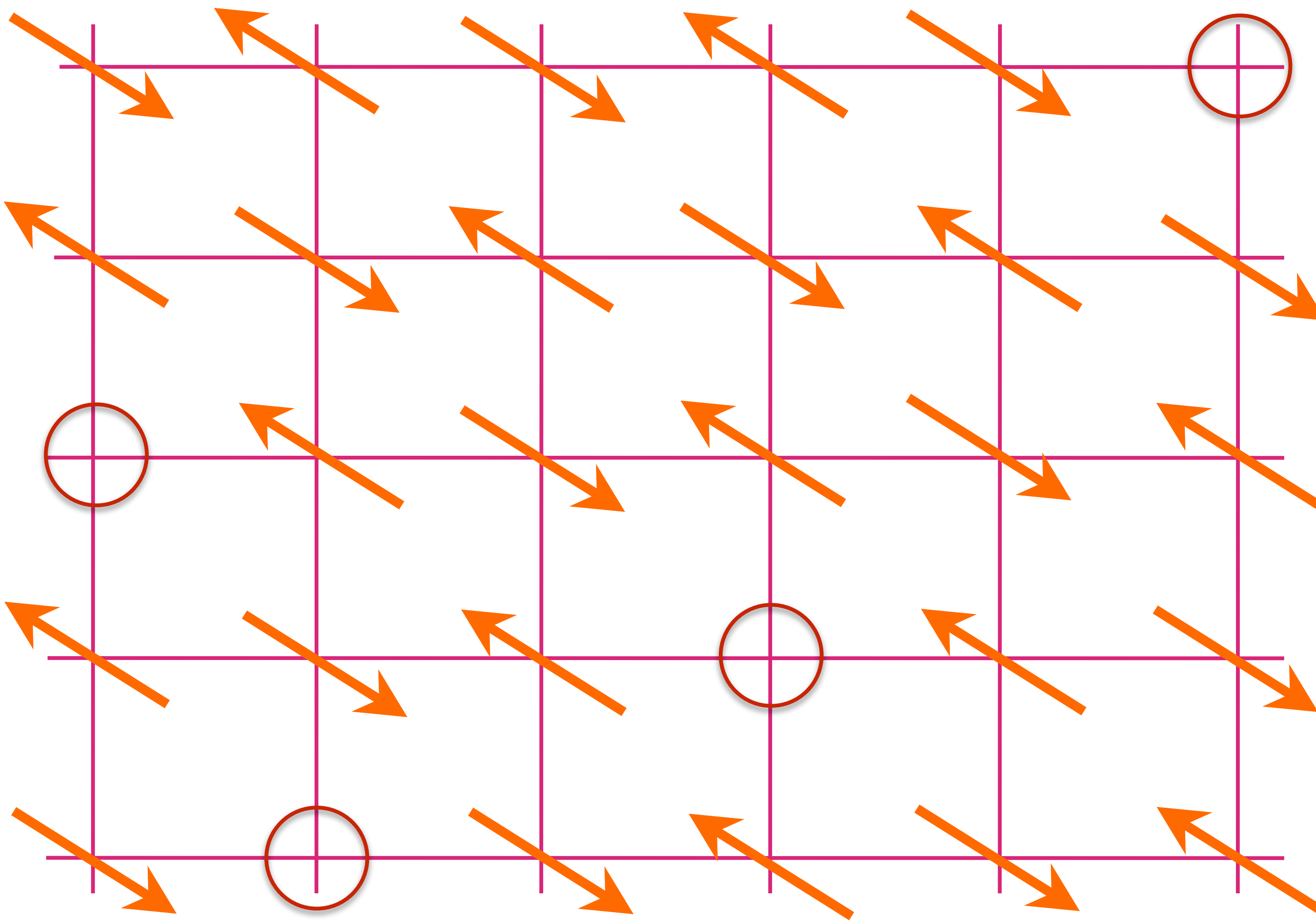
Hole doping / Sr content (p)
Electron doping / Ce content (x)



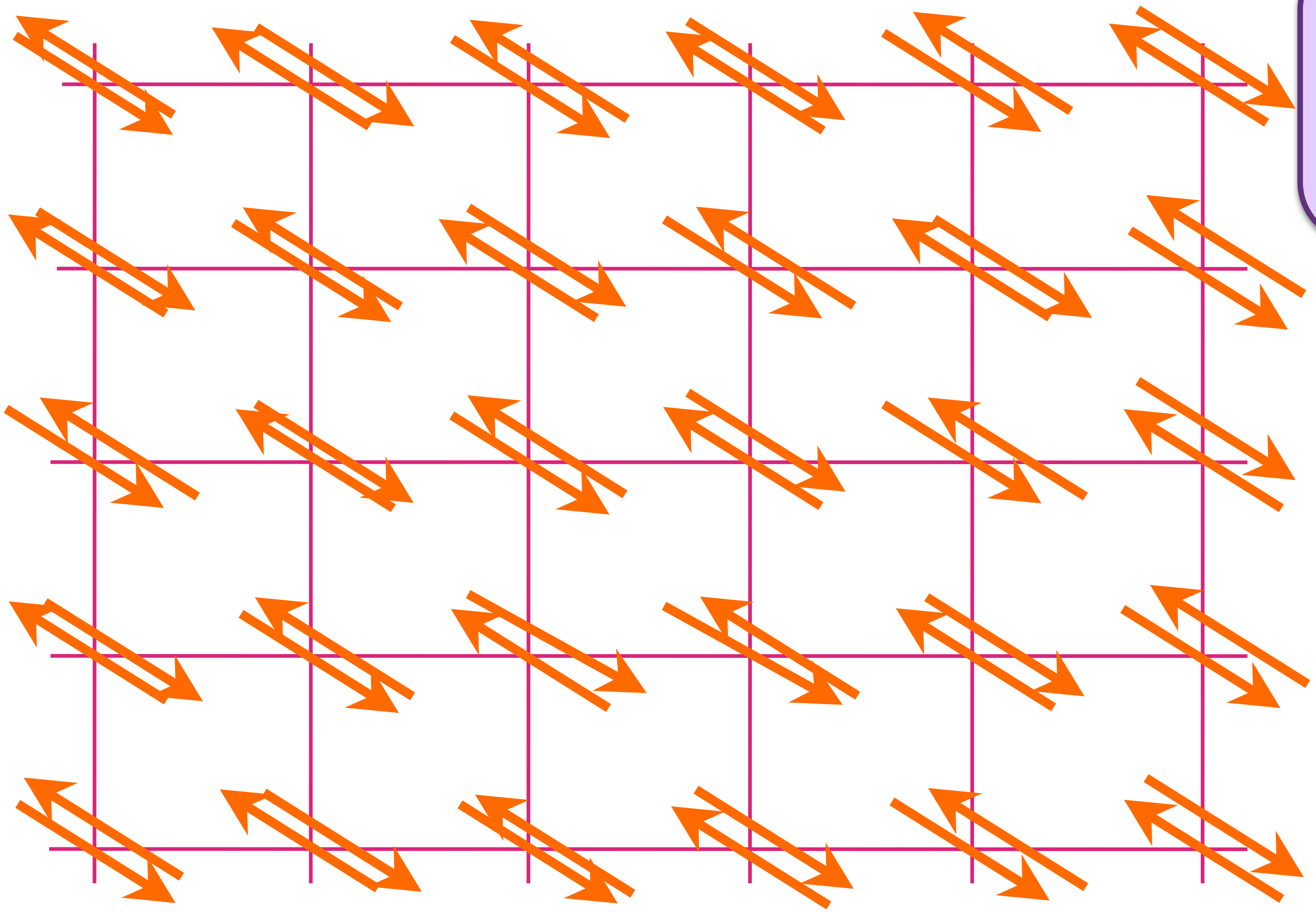
Insulating Antiferromagnet



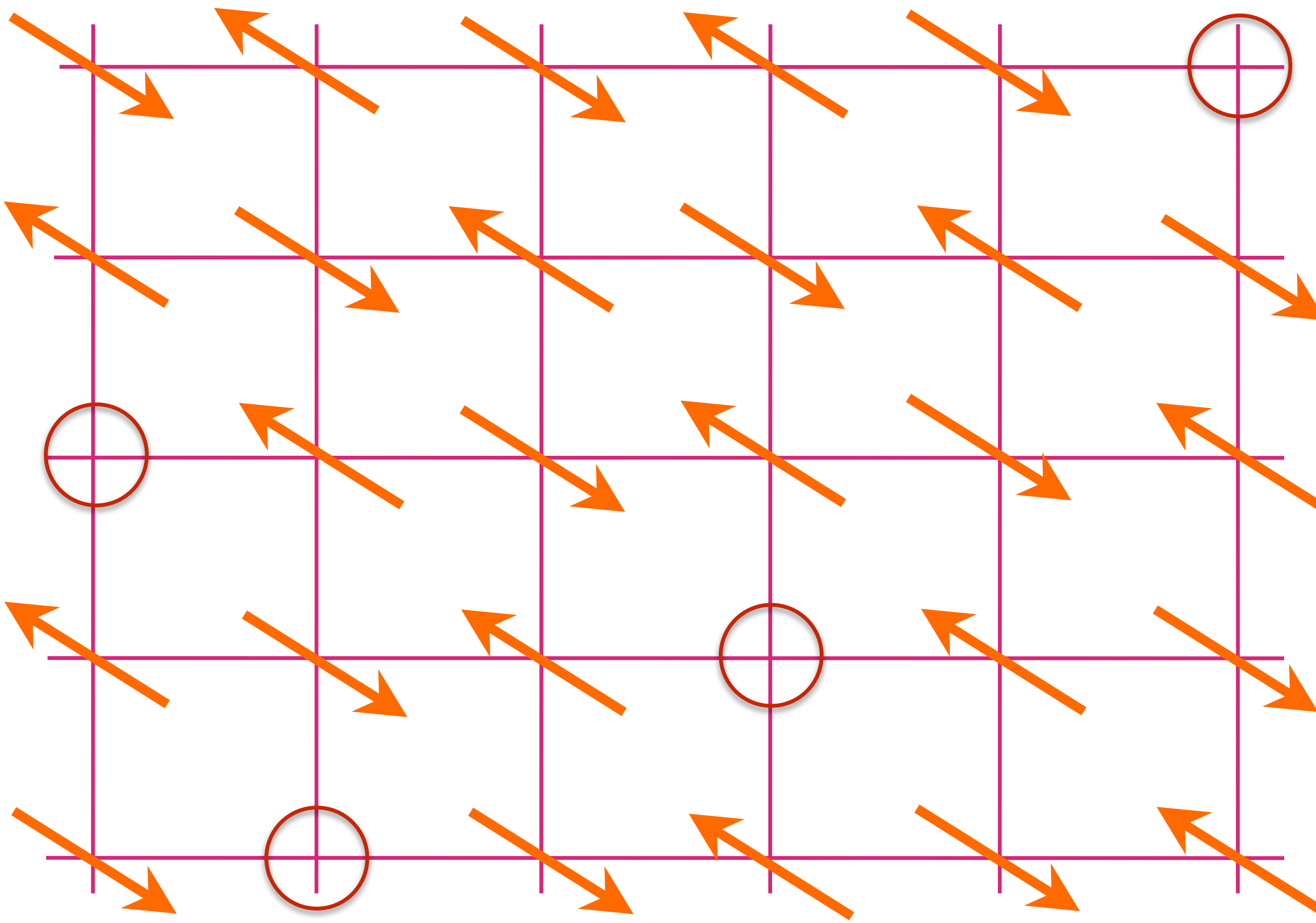
"Undoped" insulating anti-ferromagnet



Anti-
ferromagnet
with p mobile
holes
per square

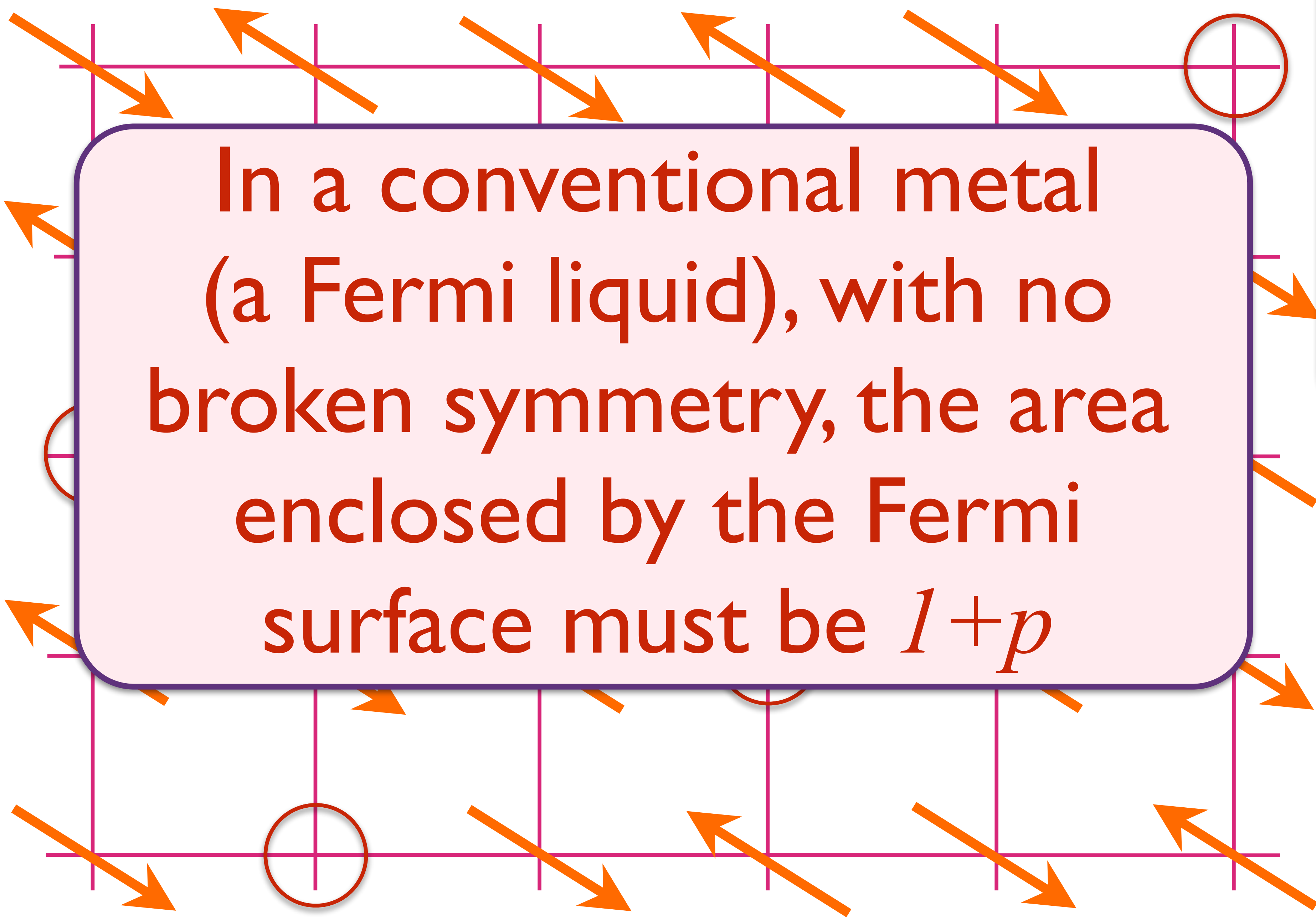


Filled Band



Anti-ferromagnet with p mobile holes per square

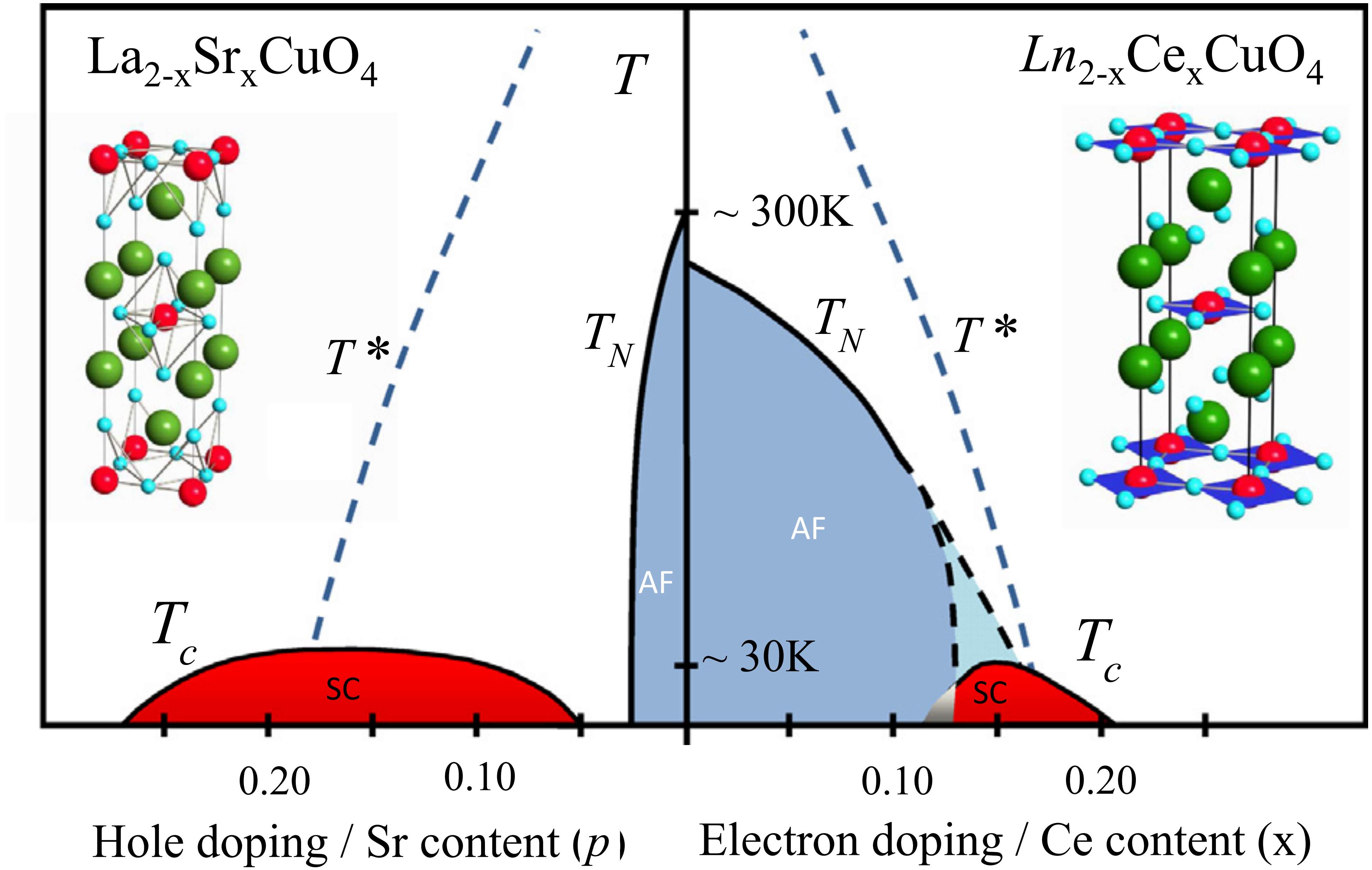
But relative to the band insulator, there are $1+p$ holes per square



In a conventional metal (a Fermi liquid), with no broken symmetry, the area enclosed by the Fermi surface must be $1+p$

Anti-ferromagnet with p mobile holes per square

But relative to the band insulator, there are $1+p$ holes per square



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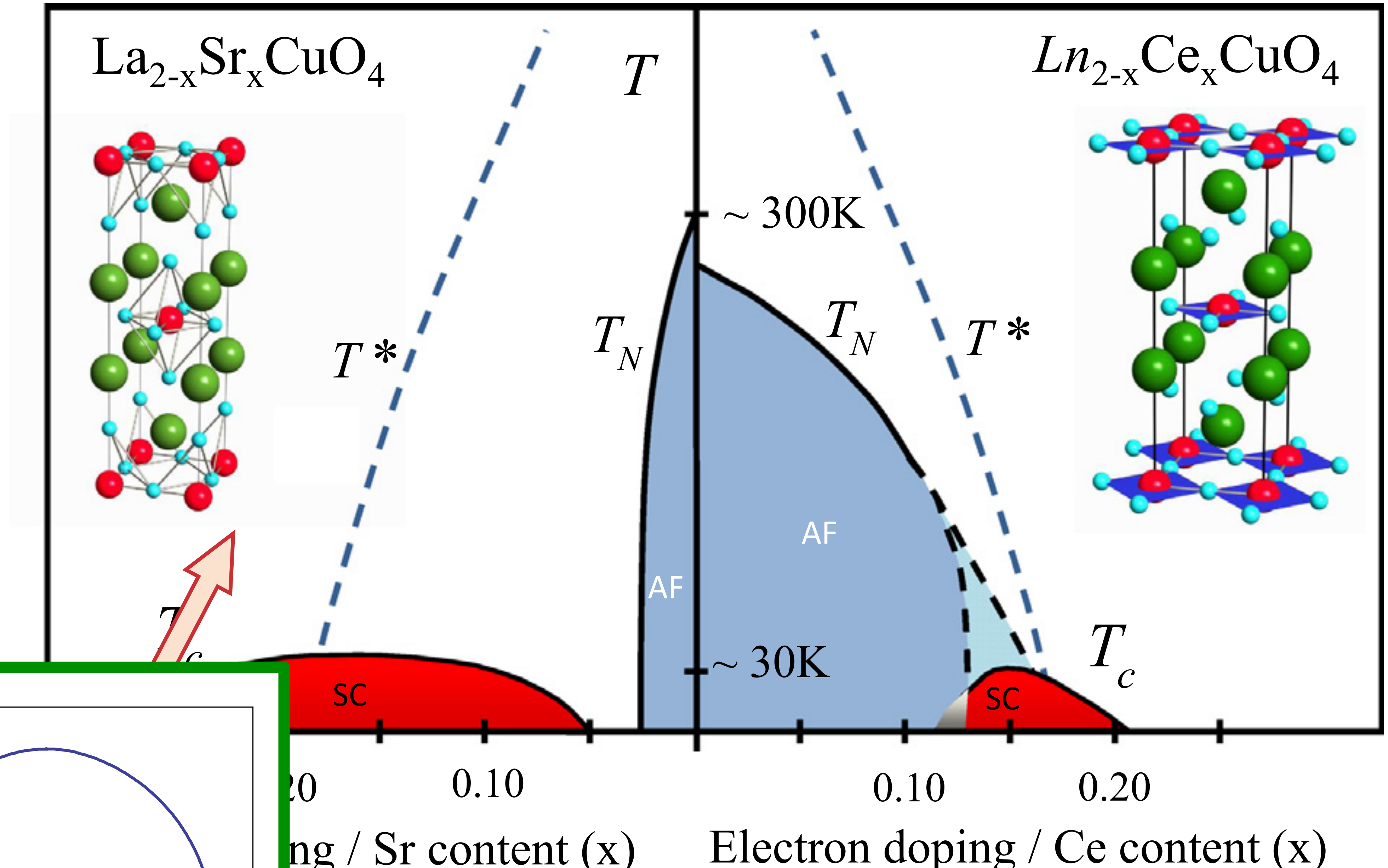
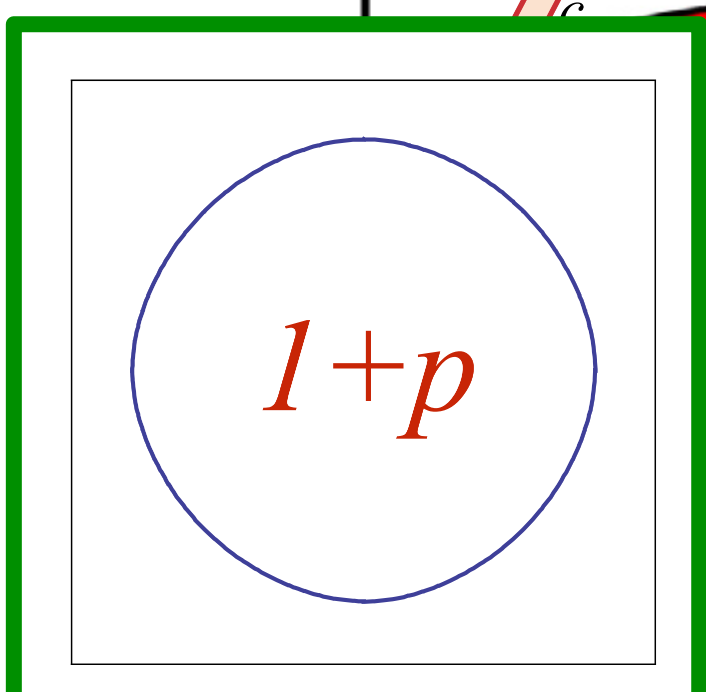
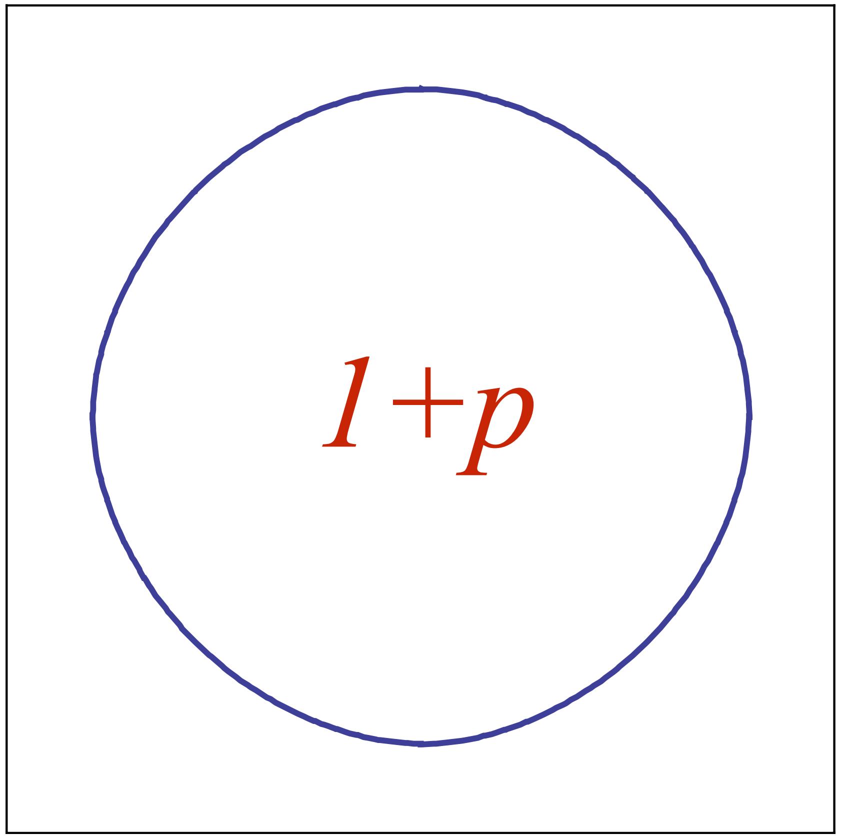


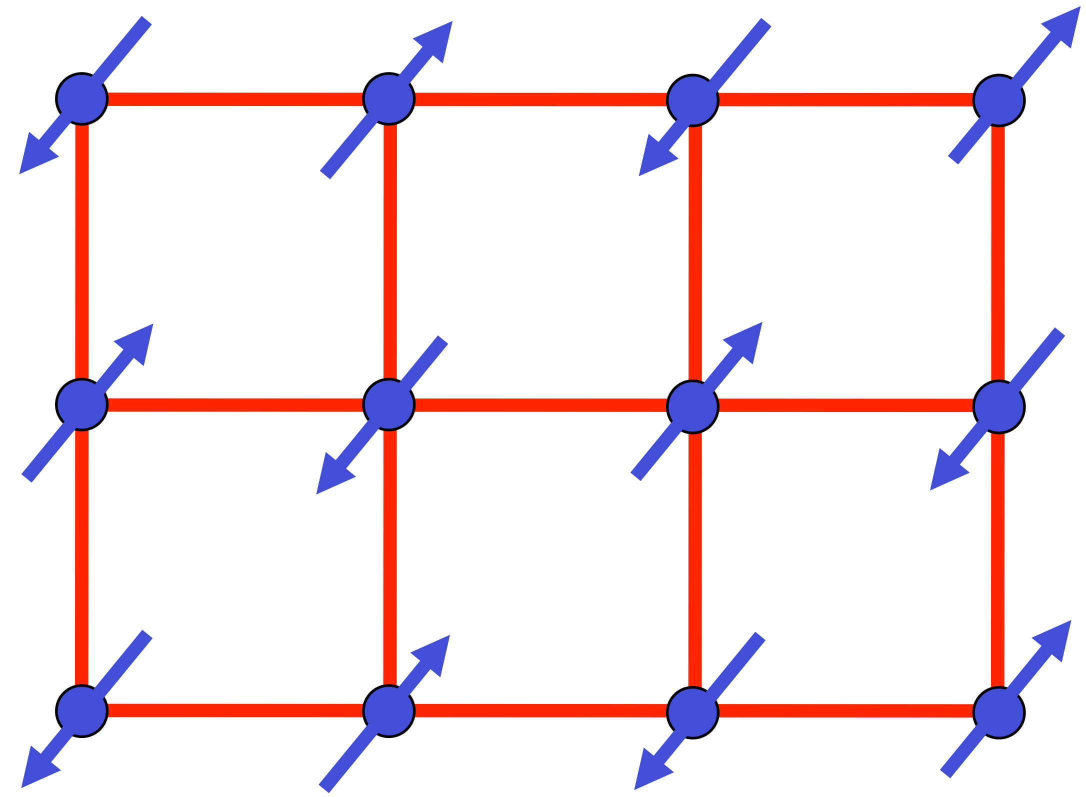


Conventional Fermi liquid

Fermi surface+antiferromagnetism



+

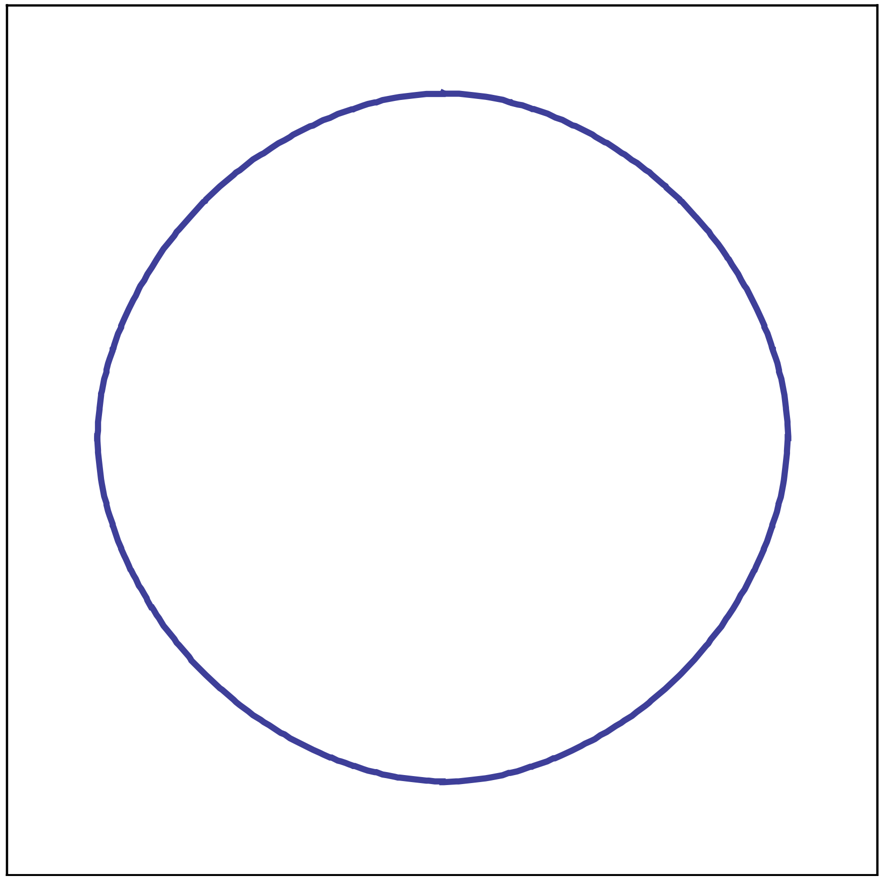


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\Phi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

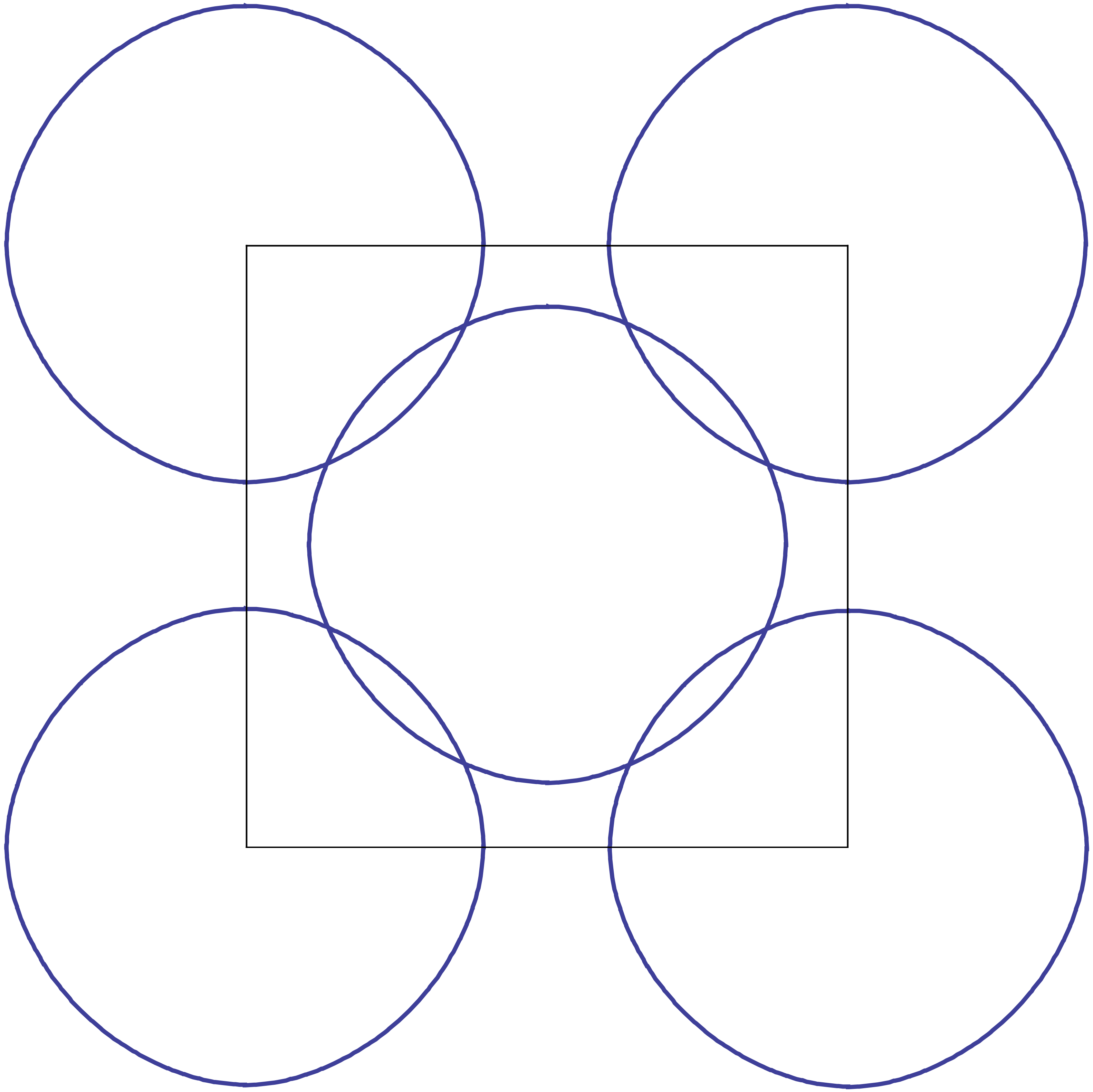
where **K** is the ordering wavevector.

Fermi surface+antiferromagnetism



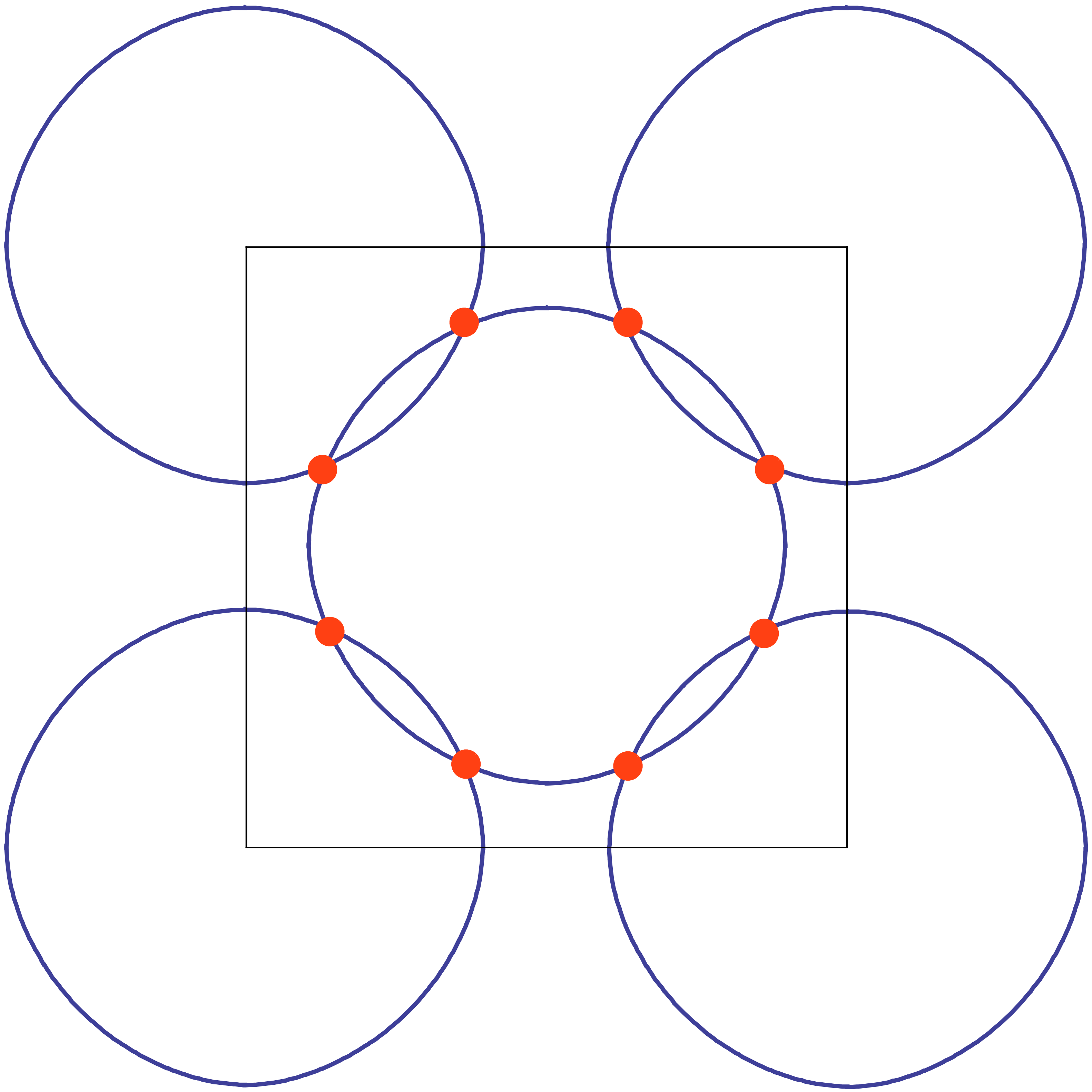
Metal with "large" Fermi surface

Fermi surface+antiferromagnetism



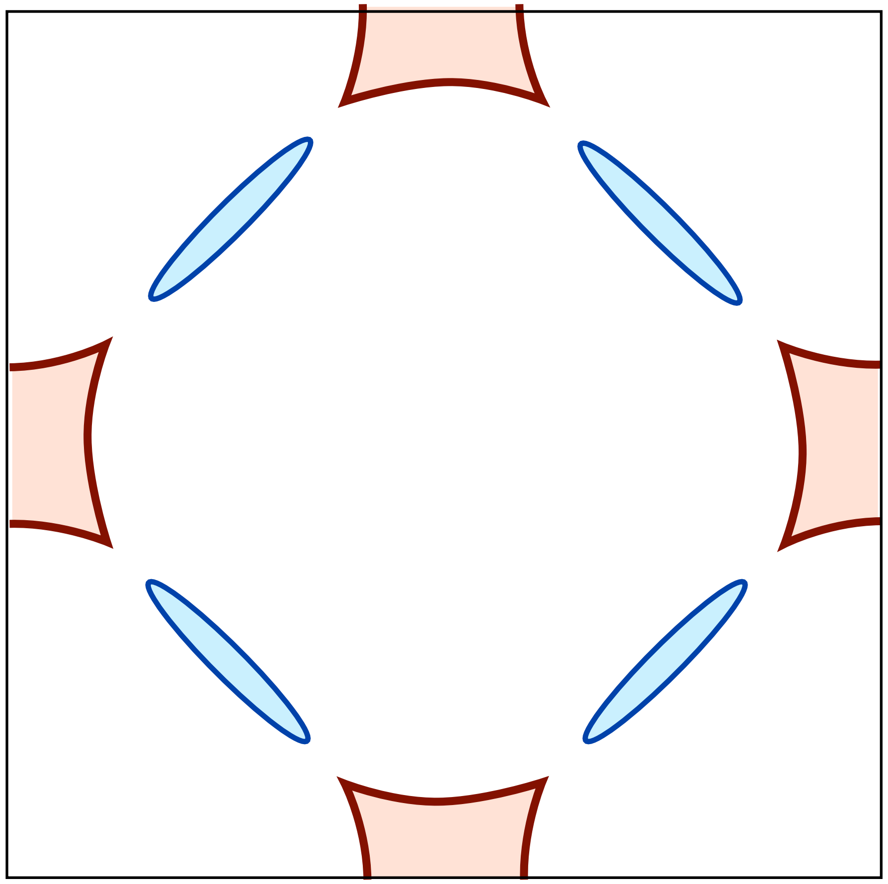
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



"Hot" spots

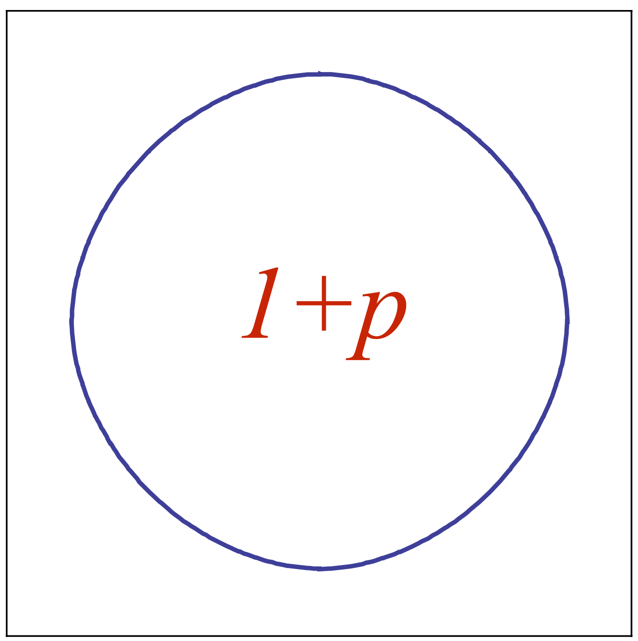
Fermi surface+antiferromagnetism



Electron and hole pockets in antiferromagnetic phase with $\langle \vec{\Phi} \rangle \neq 0$

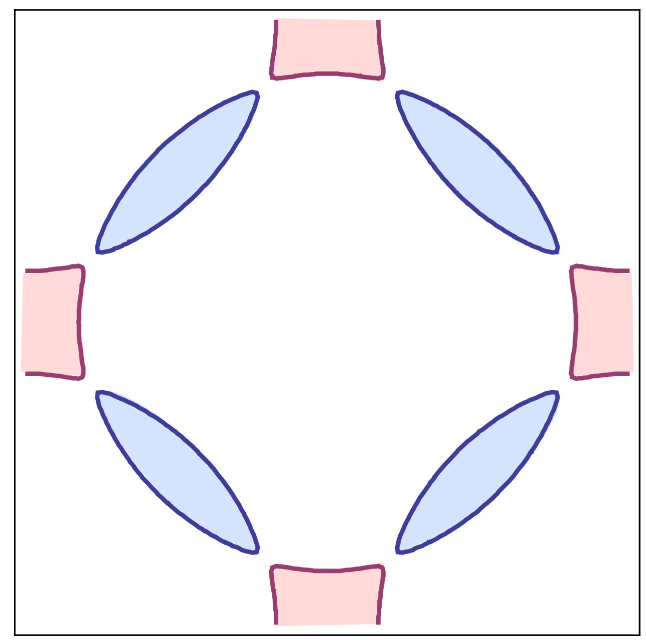
Square lattice Hubbard model with hole doping

$\langle \vec{\Phi} \rangle = 0$



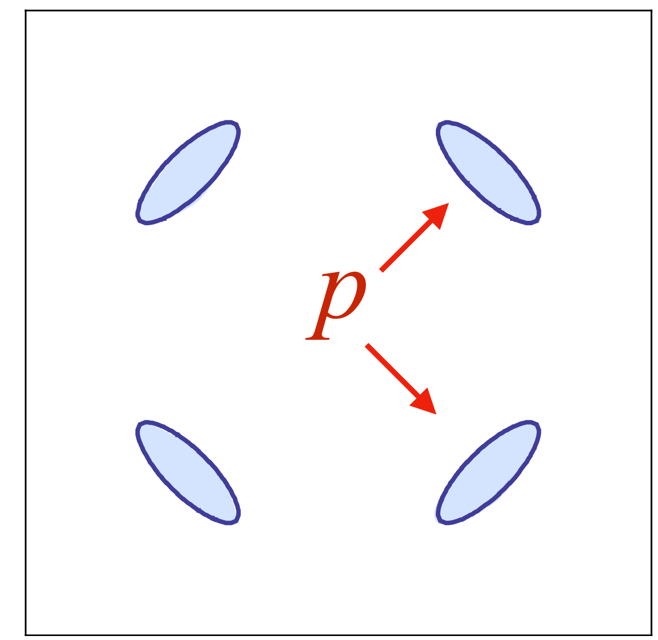
Metal with "large" Fermi surface

$\langle \vec{\Phi} \rangle \neq 0$ and small



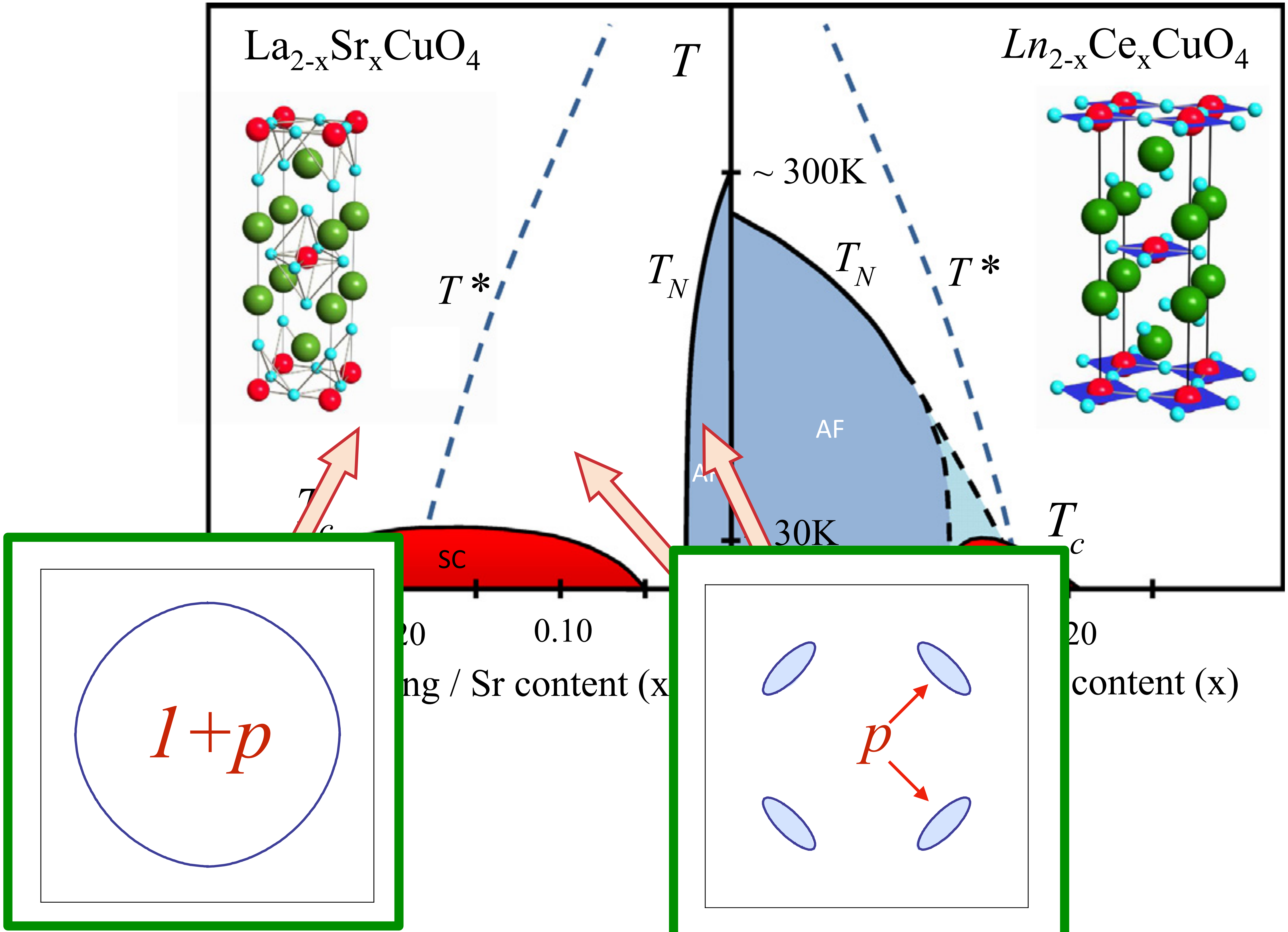
Metal with electron and hole pockets

$\langle \vec{\Phi} \rangle \neq 0$ and large



Metal with hole pockets

p


La2-xSrxCuO4
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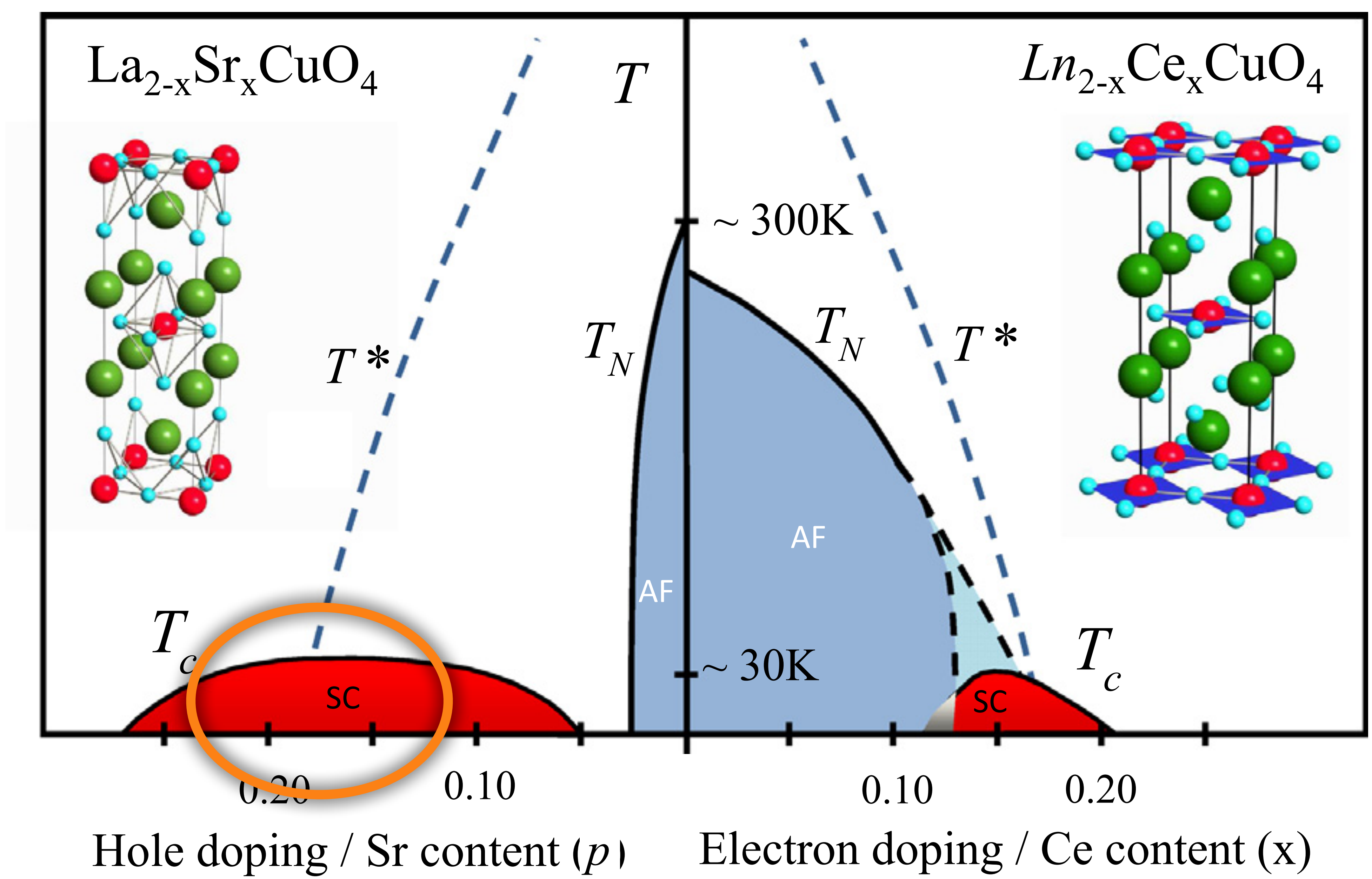

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Hole doping / Sr content (p)
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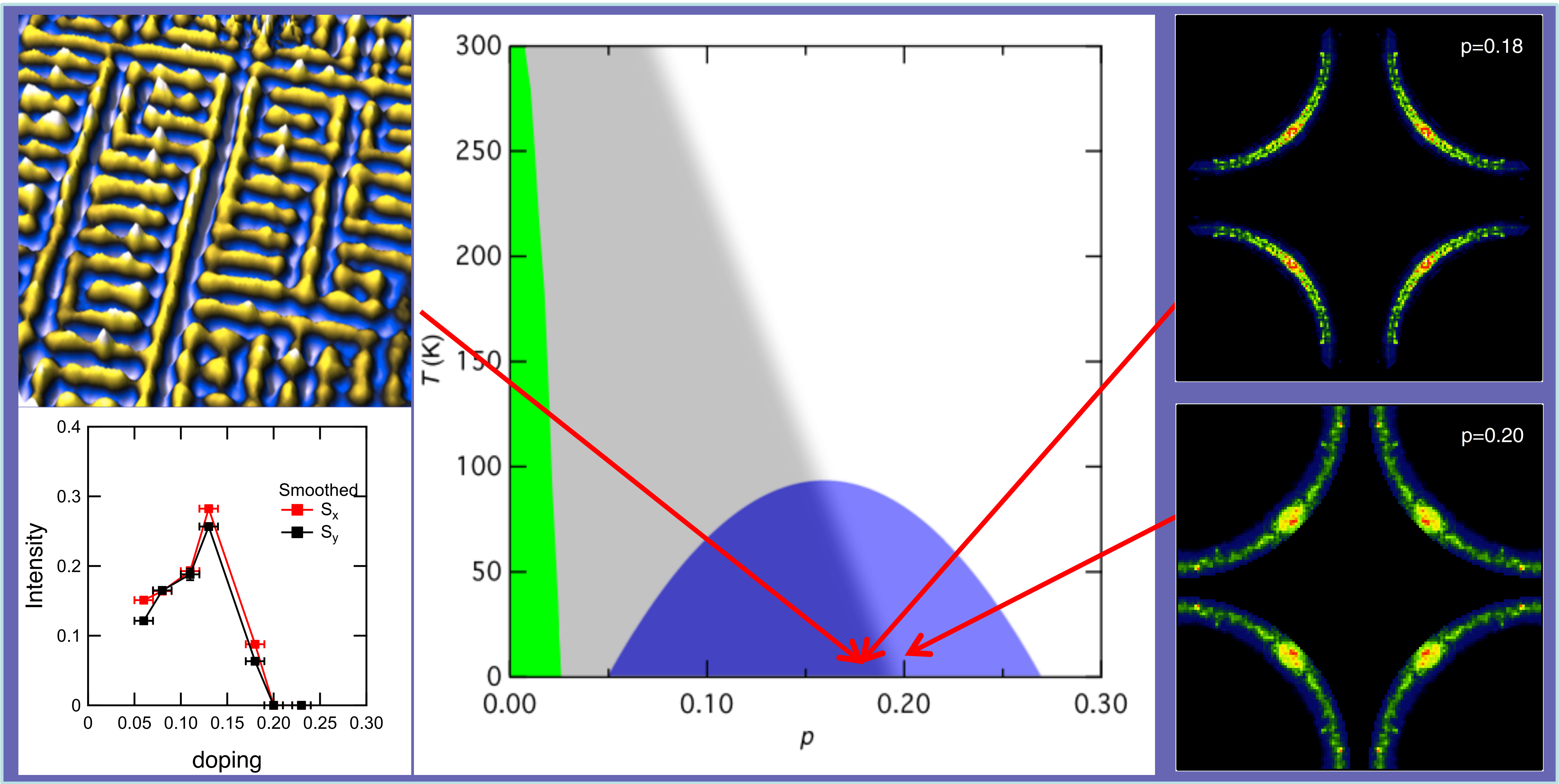
Hole doped cuprates

Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, and J. E. Hoffman, Science **344**, 608 (2014)

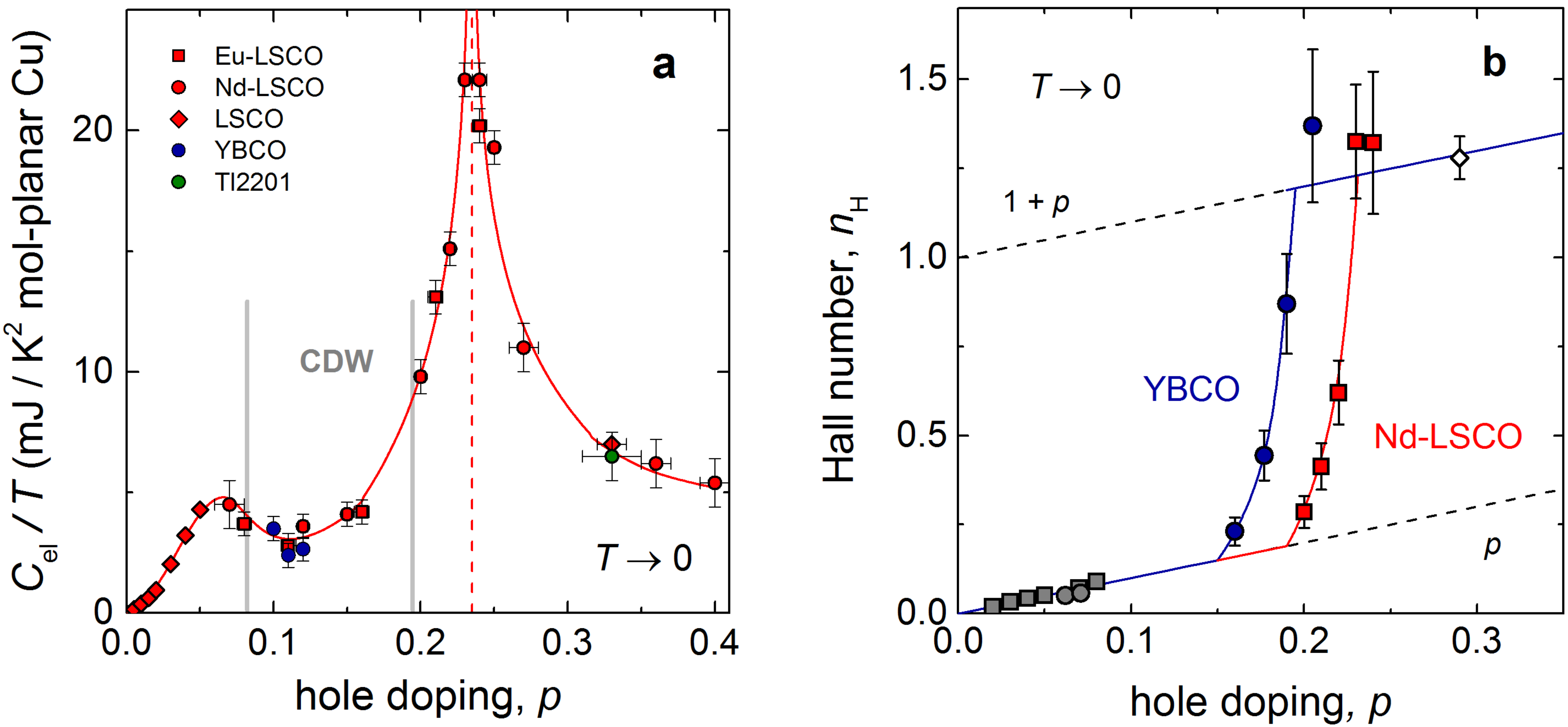
K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, J. C. Davis, Science **344**, 612 (2014)

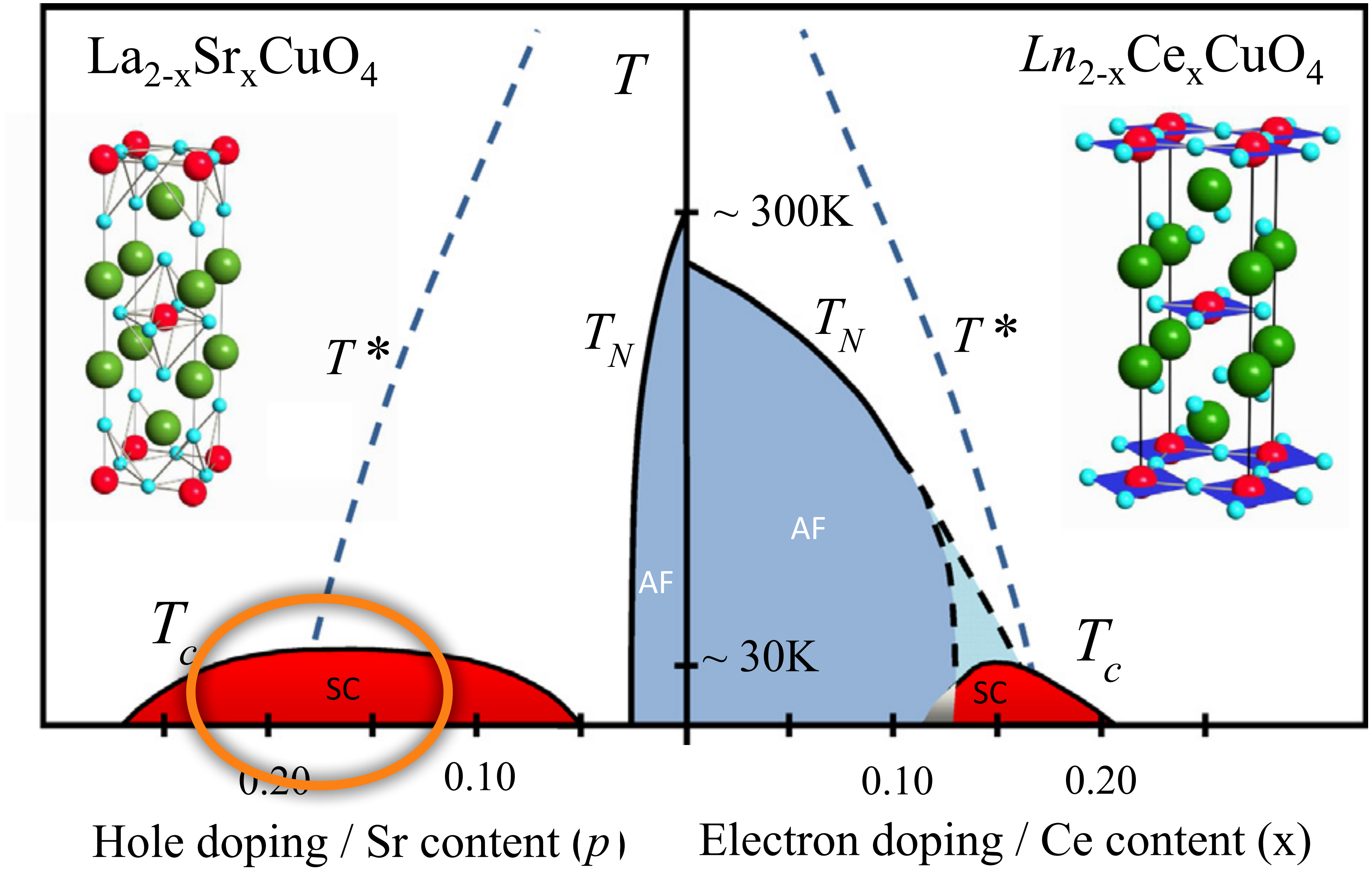


Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

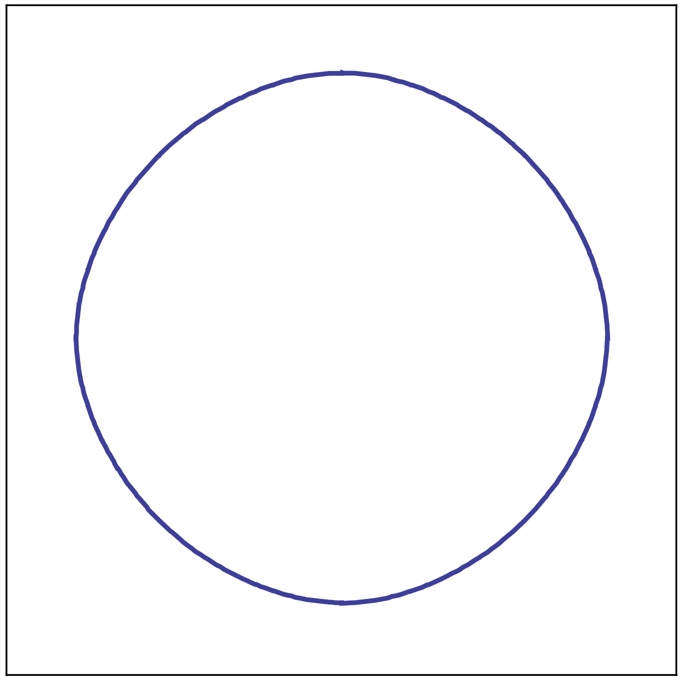
Cyril Proust and Louis Taillefer, arXiv:1807.0507

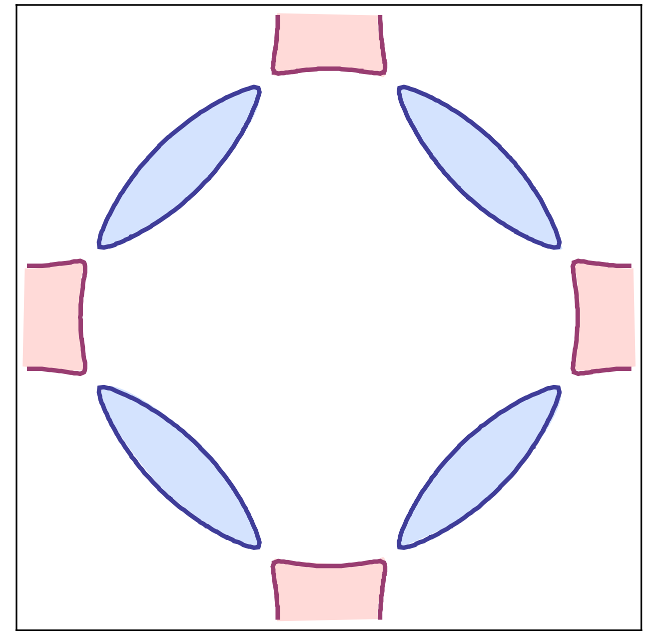




Is there a quantum critical point near optimal doping, not associated with the onset of antiferromagnetism ?

Onset of antiferromagnetism and Fermi surface reconstruction coincide





$$\langle \vec{\Phi} \rangle = 0$$

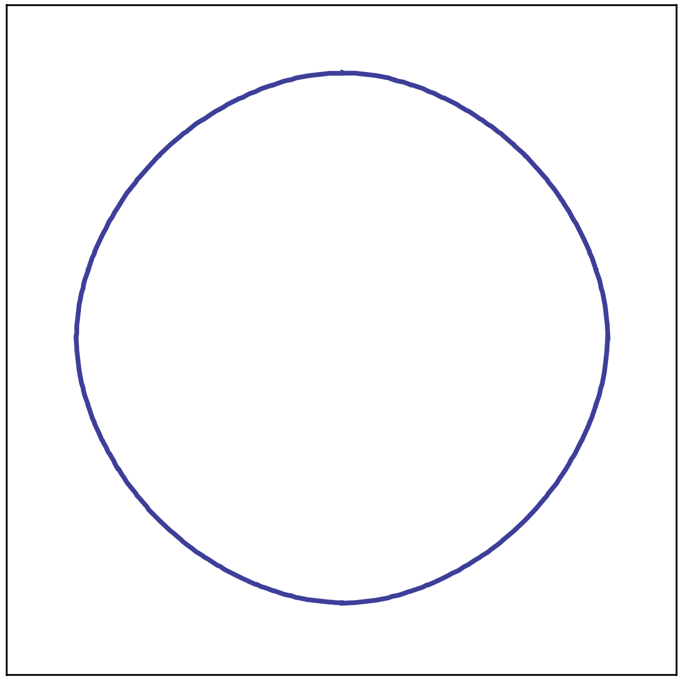
Metal with “large” Fermi surface of size $1+p$

$$\langle \vec{\Phi} \rangle \neq 0$$

Metal with electron and hole pockets of size p

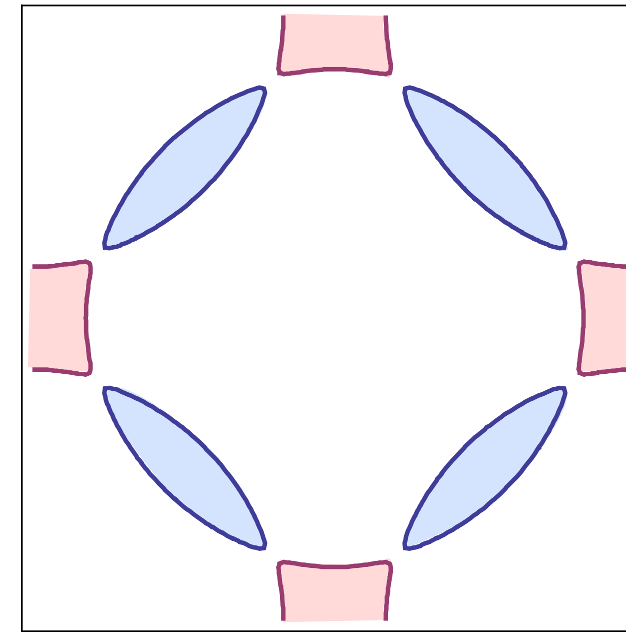
p

Separating onset of antiferromagnetism and Fermi surface reconstruction



$$\langle \vec{\Phi} \rangle = 0$$

Metal with "large" Fermi surface of size *1+p*



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Metal with electron and hole pockets of size p

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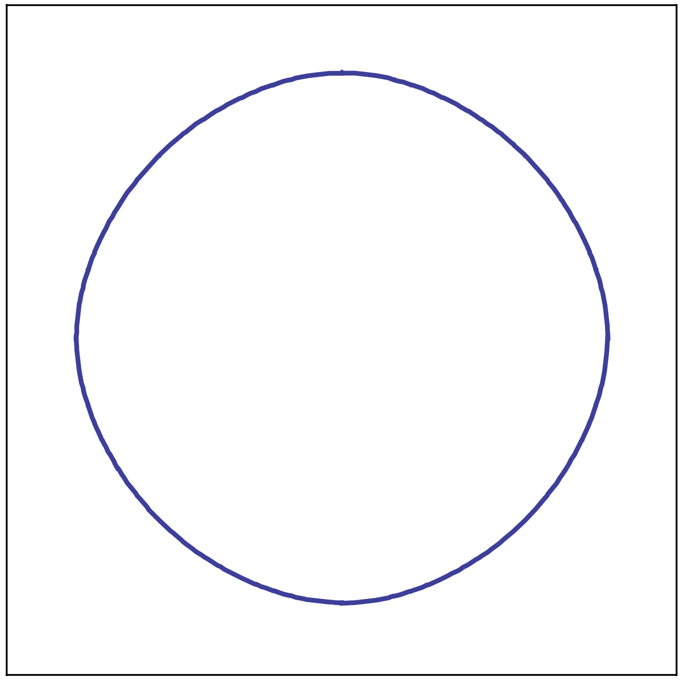

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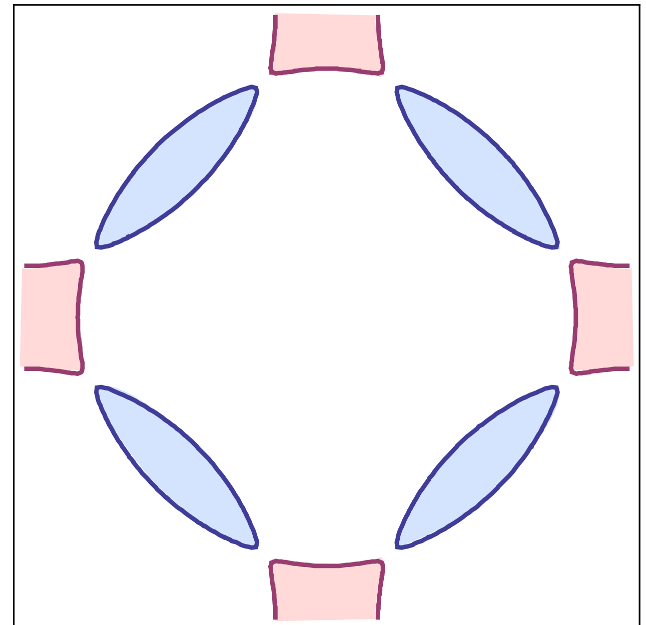
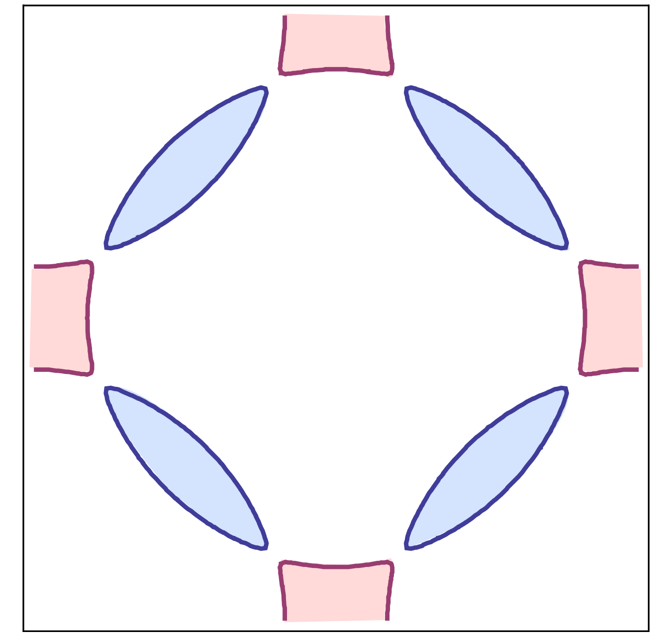
Metal with "large" Fermi surface of size $1+p$



$\langle \vec{\Phi} \rangle = 0$

Metal with electron and/or hole pockets but no antiferromagnetism



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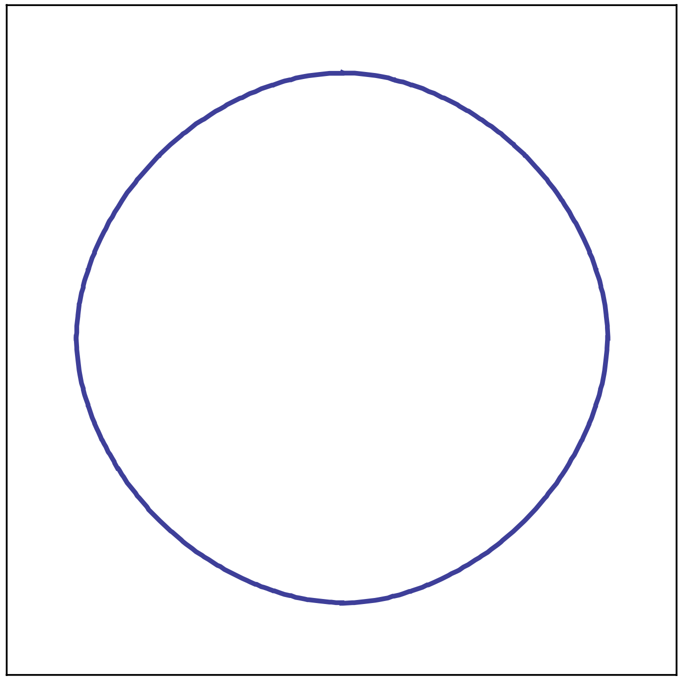
Metal with electron and hole pockets of size p

p

Separating onset of antiferromagnetism and Fermi surface reconstruction



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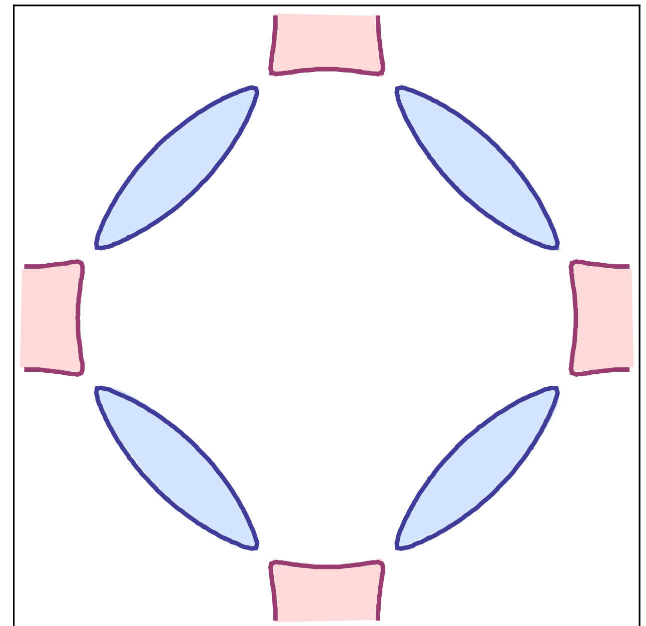
Metal with "large" Fermi surface of size $1+p$



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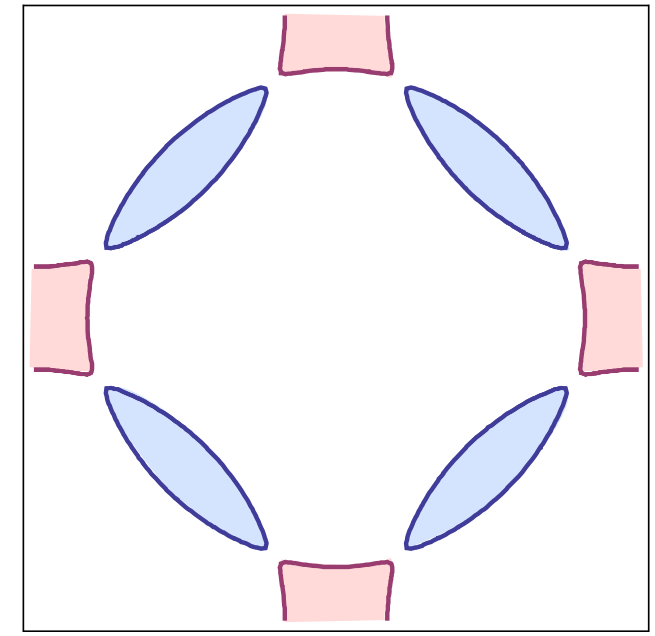
Metal with electron and/or hole pockets but no antiferromagnetism

Such a state must have "topological" order



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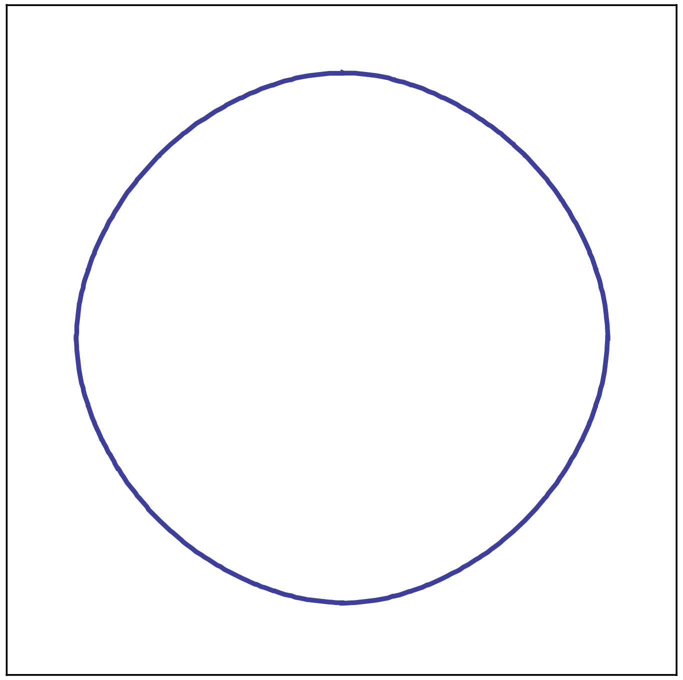

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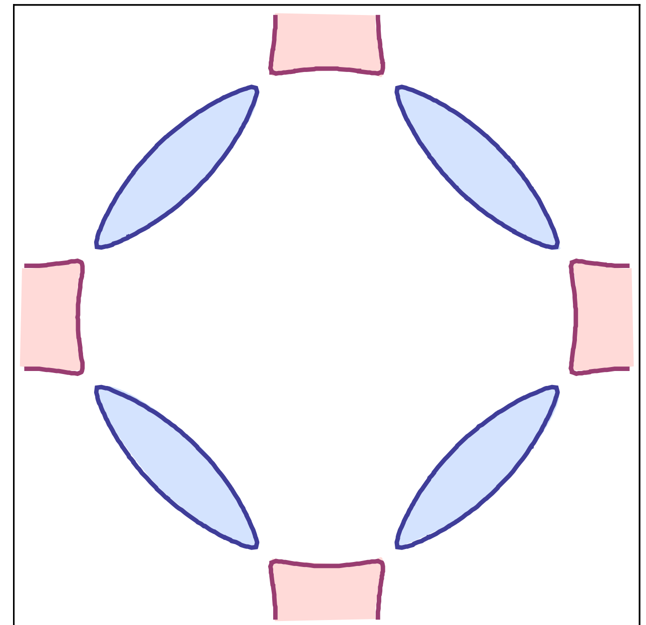

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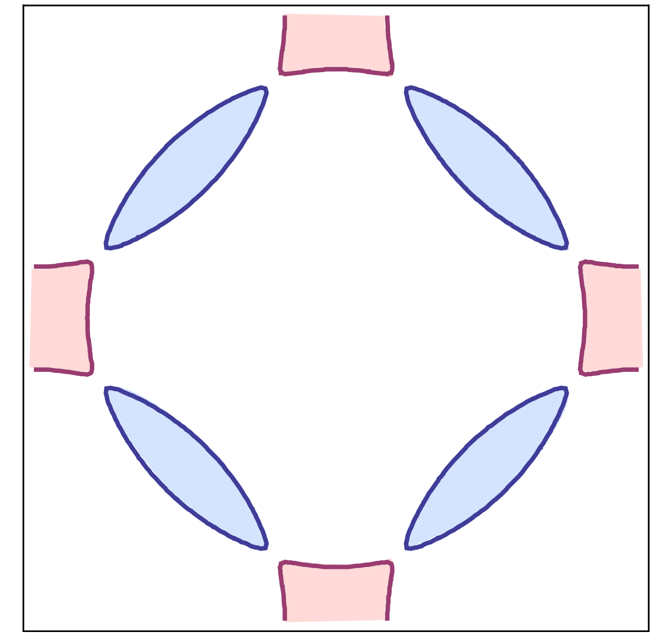

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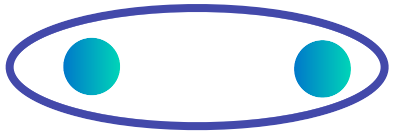
Metal with electron and hole pockets of size p

Topological quantum phase transition

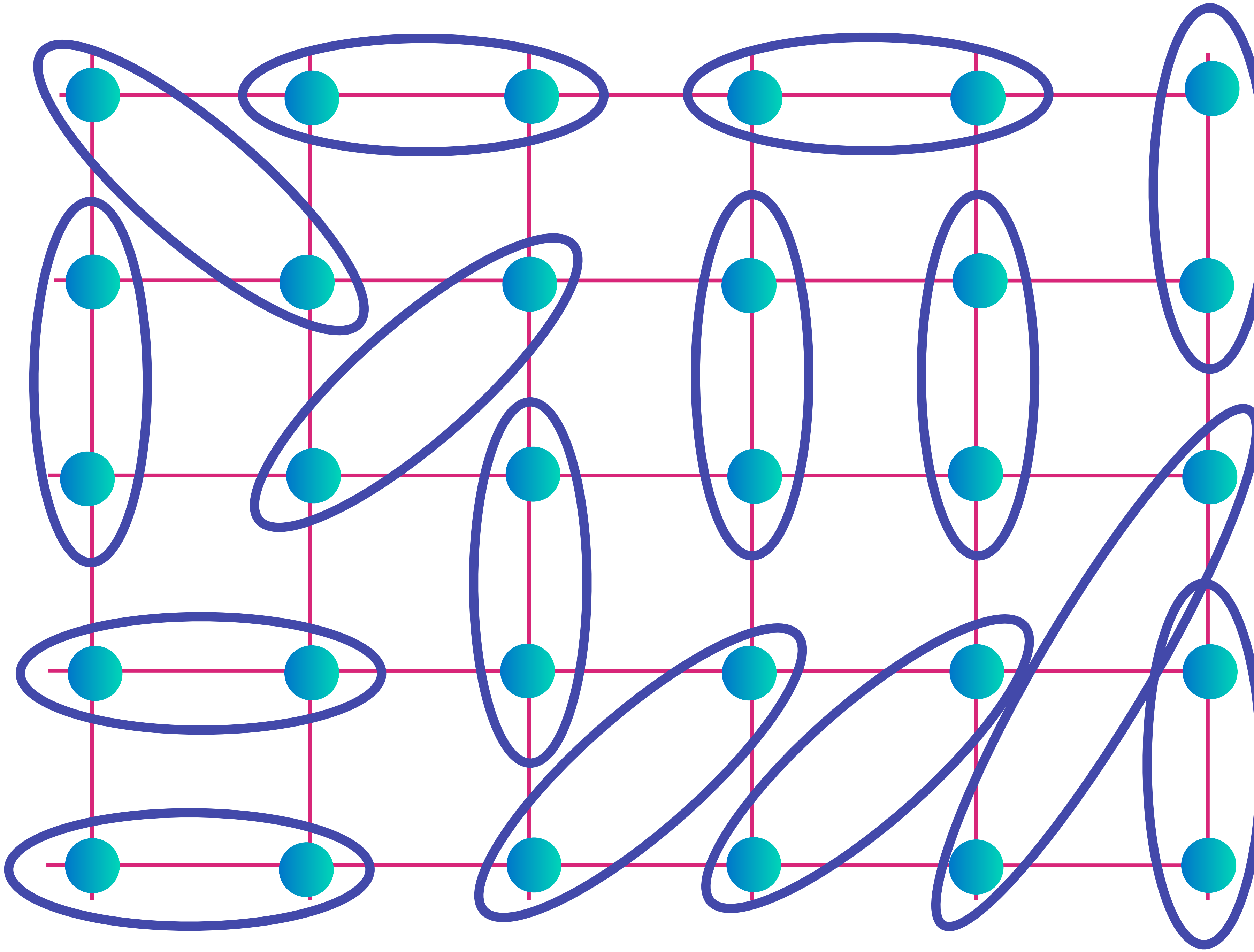
p

S. Sachdev, H. D. Scammell, M. S. Scheurer, and G. Tarnopolsky, PRB **99**, 054516 (2019)

Topological order in a spin liquid



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$



D.J. Thouless, PRB **36**, 7187 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. **6**, 353 (1988)

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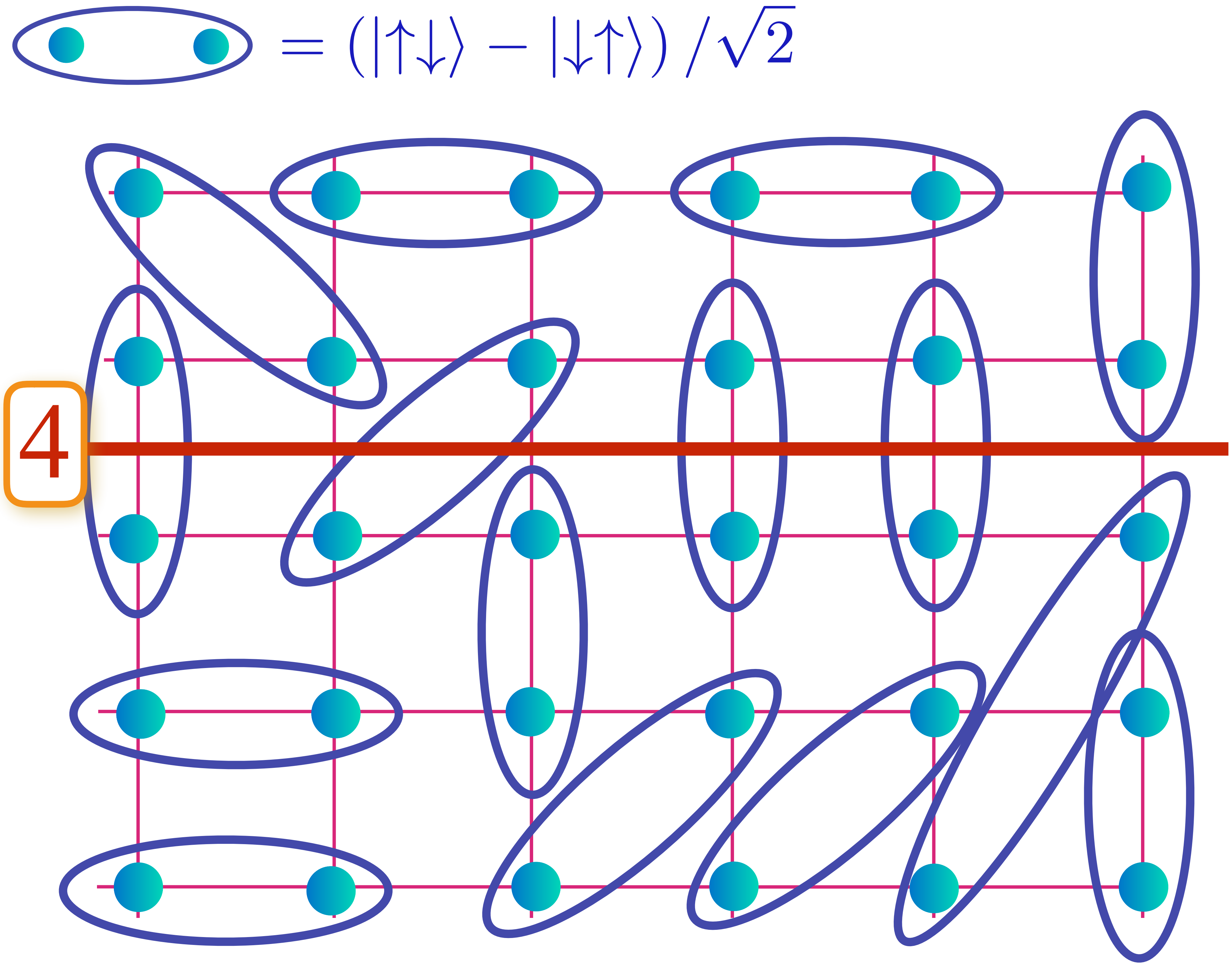

Topological invariant: Number of singlet bonds crossing "branch-cut" is conserved modulo 2

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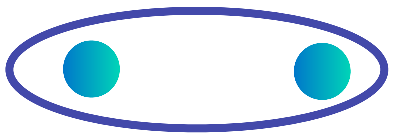
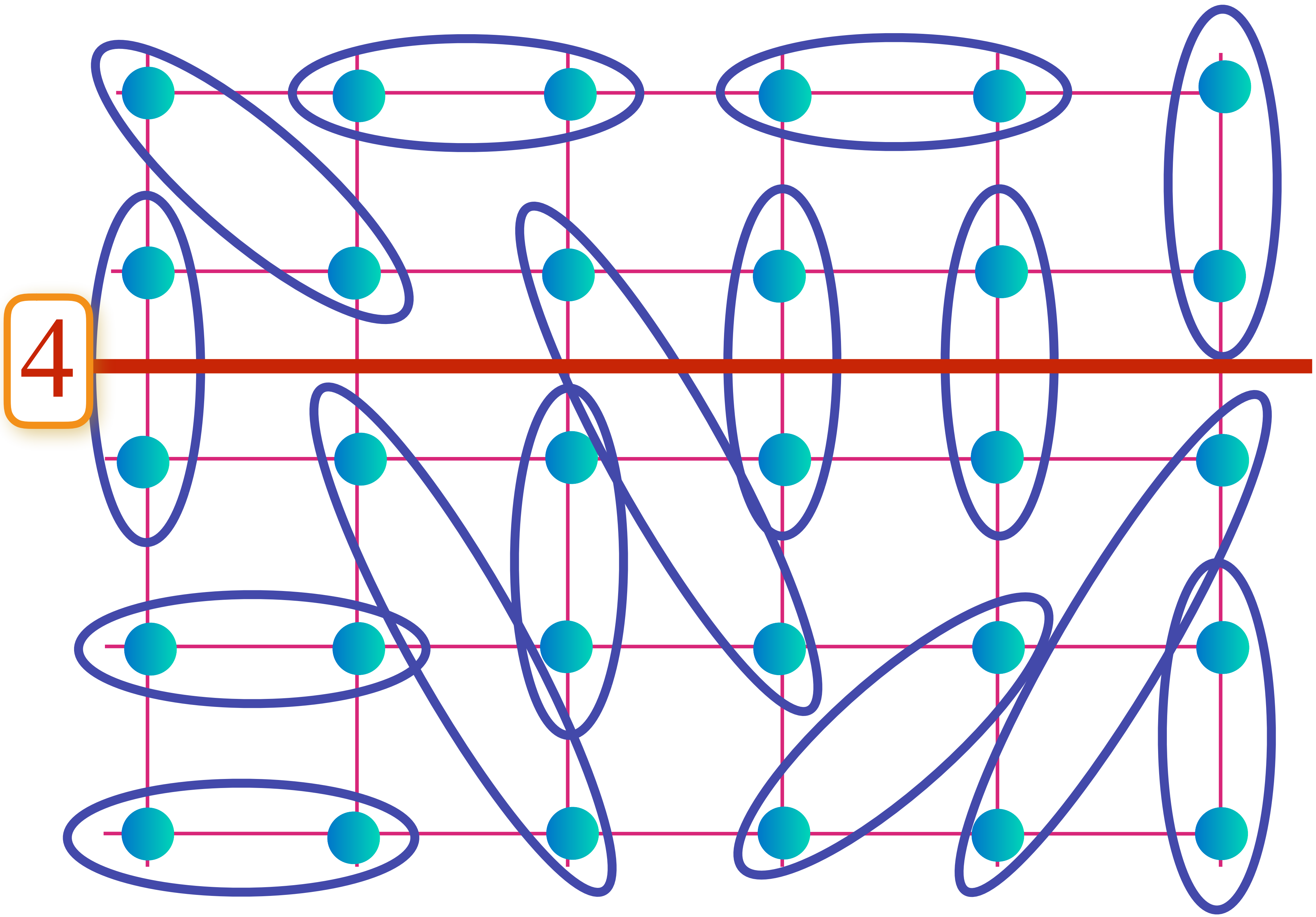
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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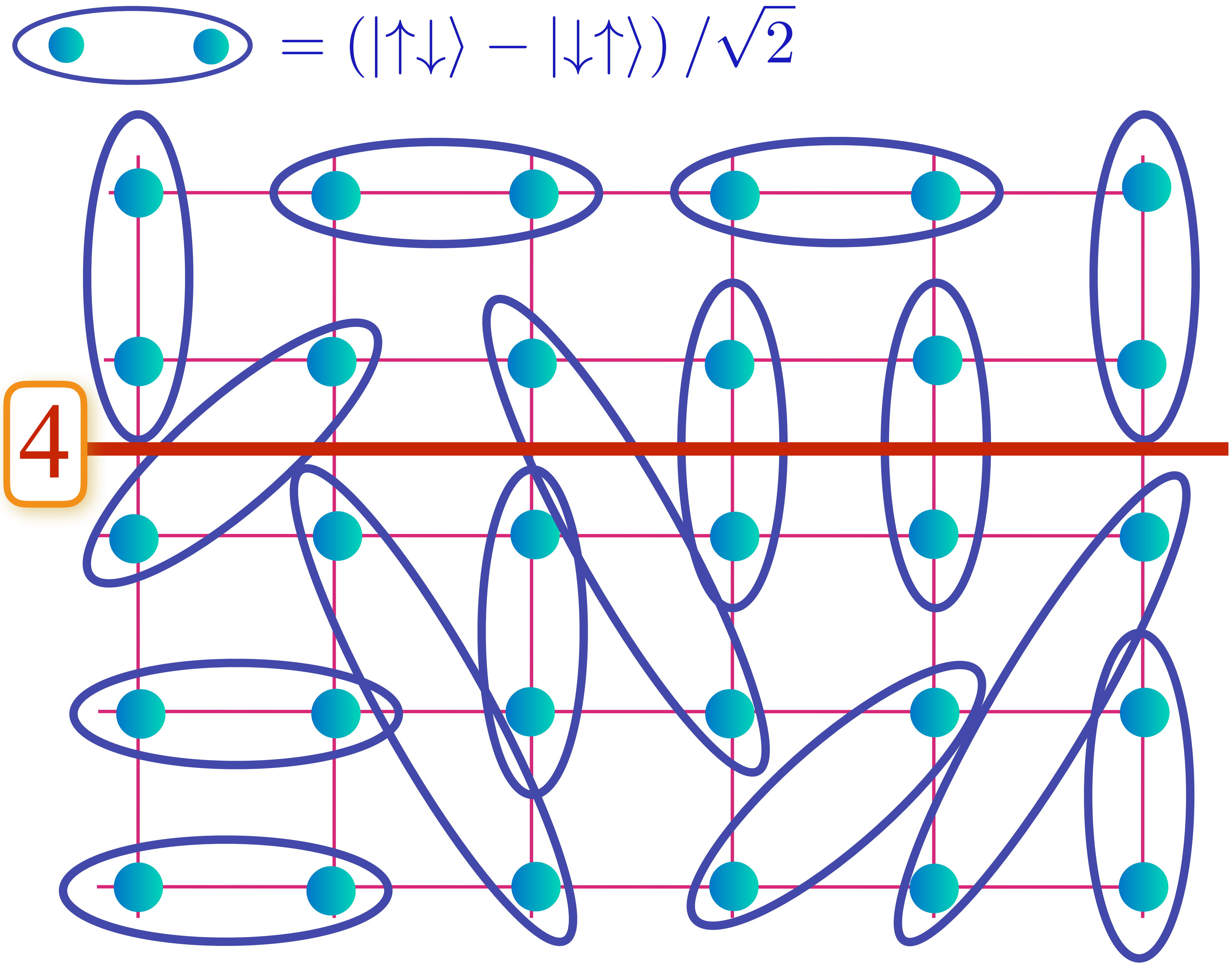

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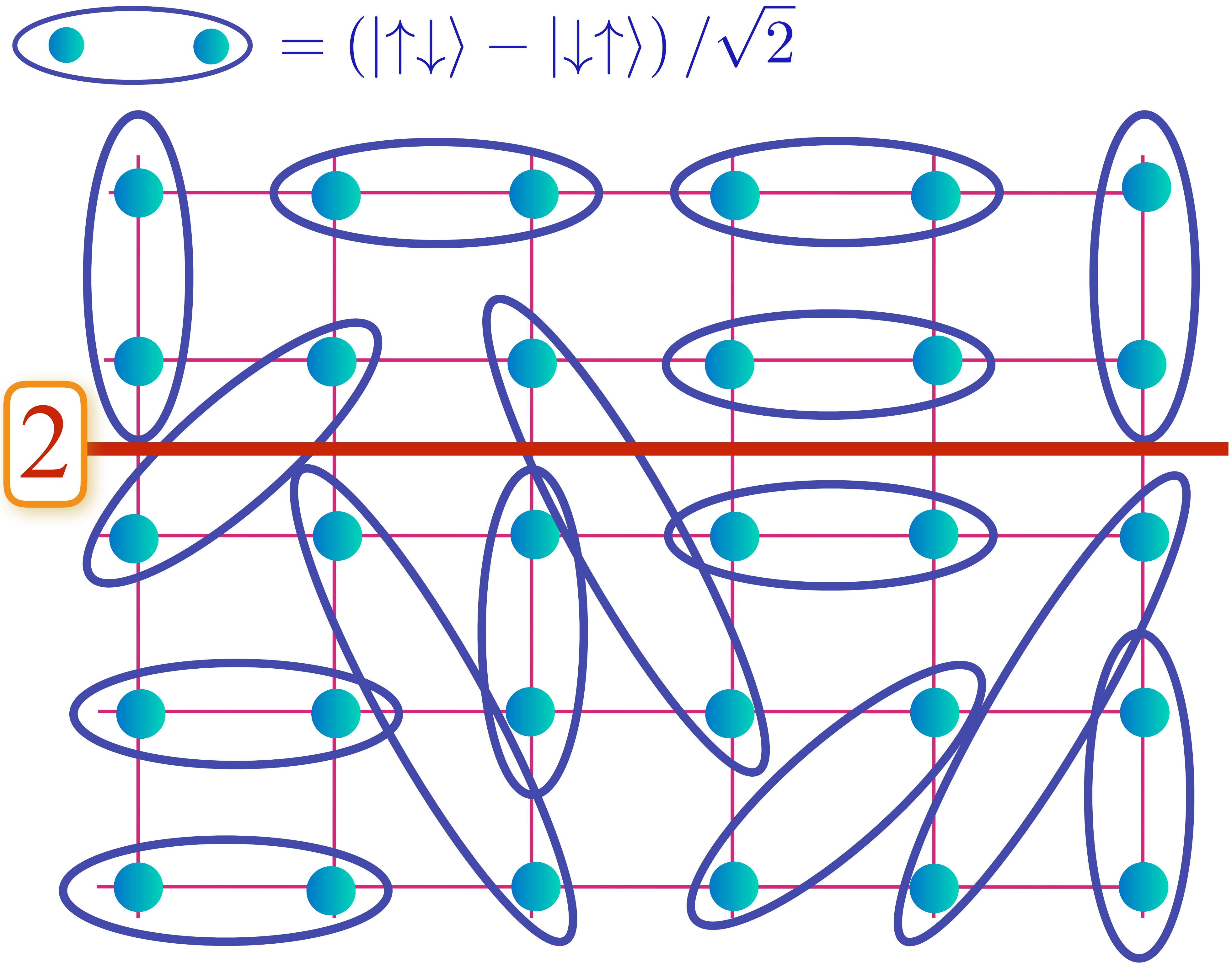
Topological order in a spin liquid

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$



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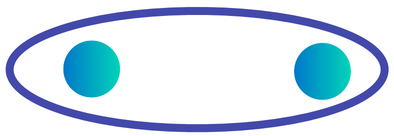
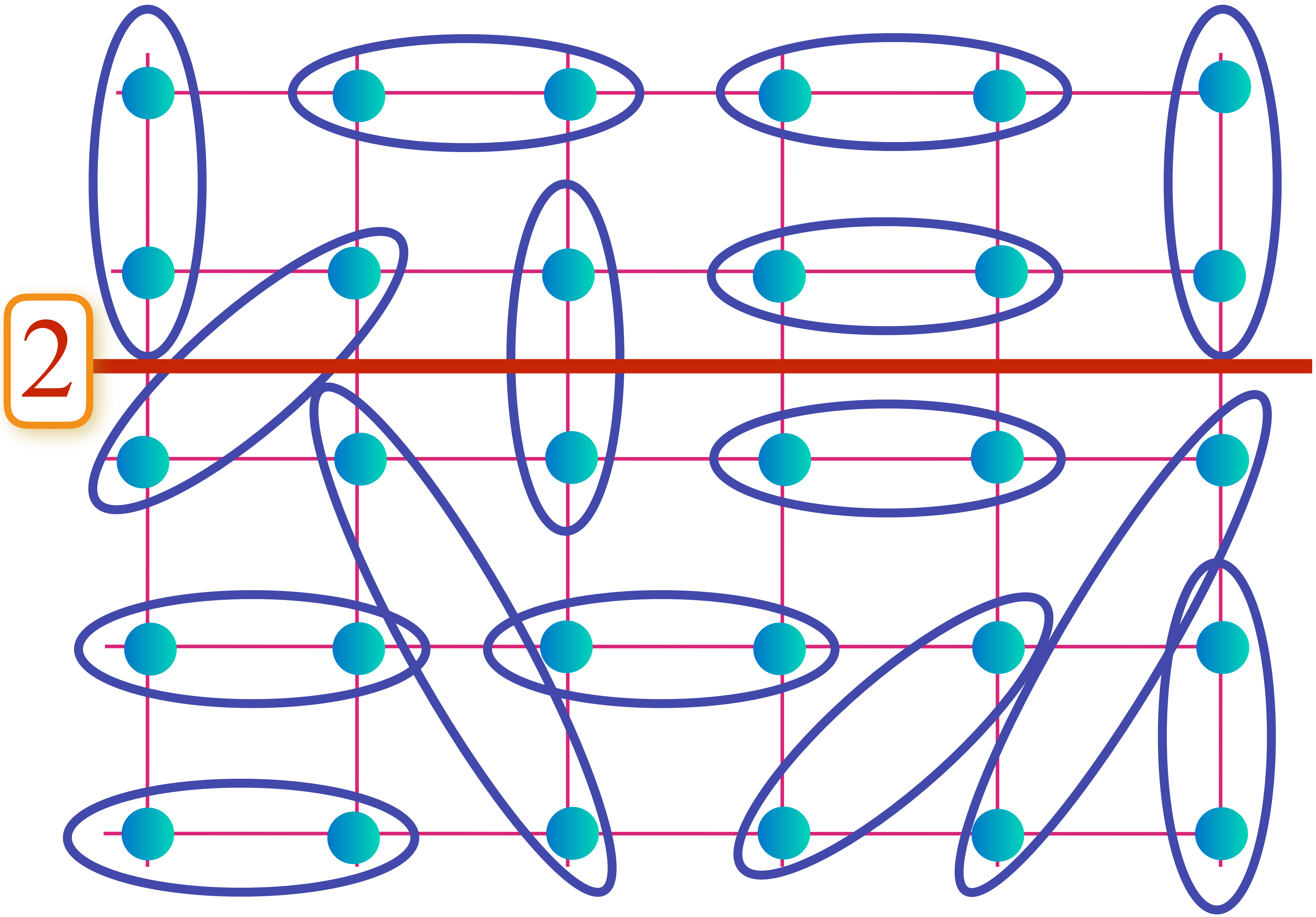
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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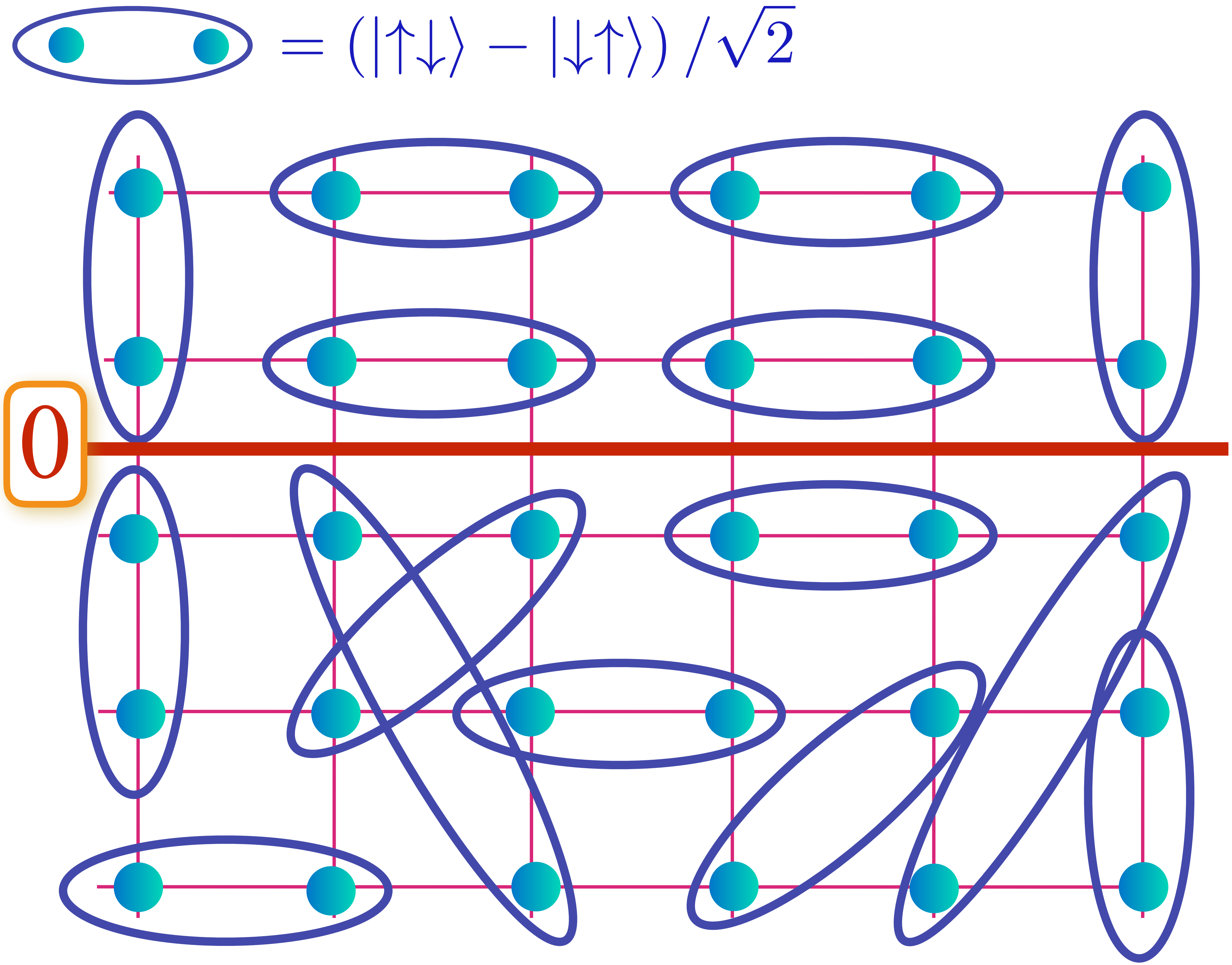

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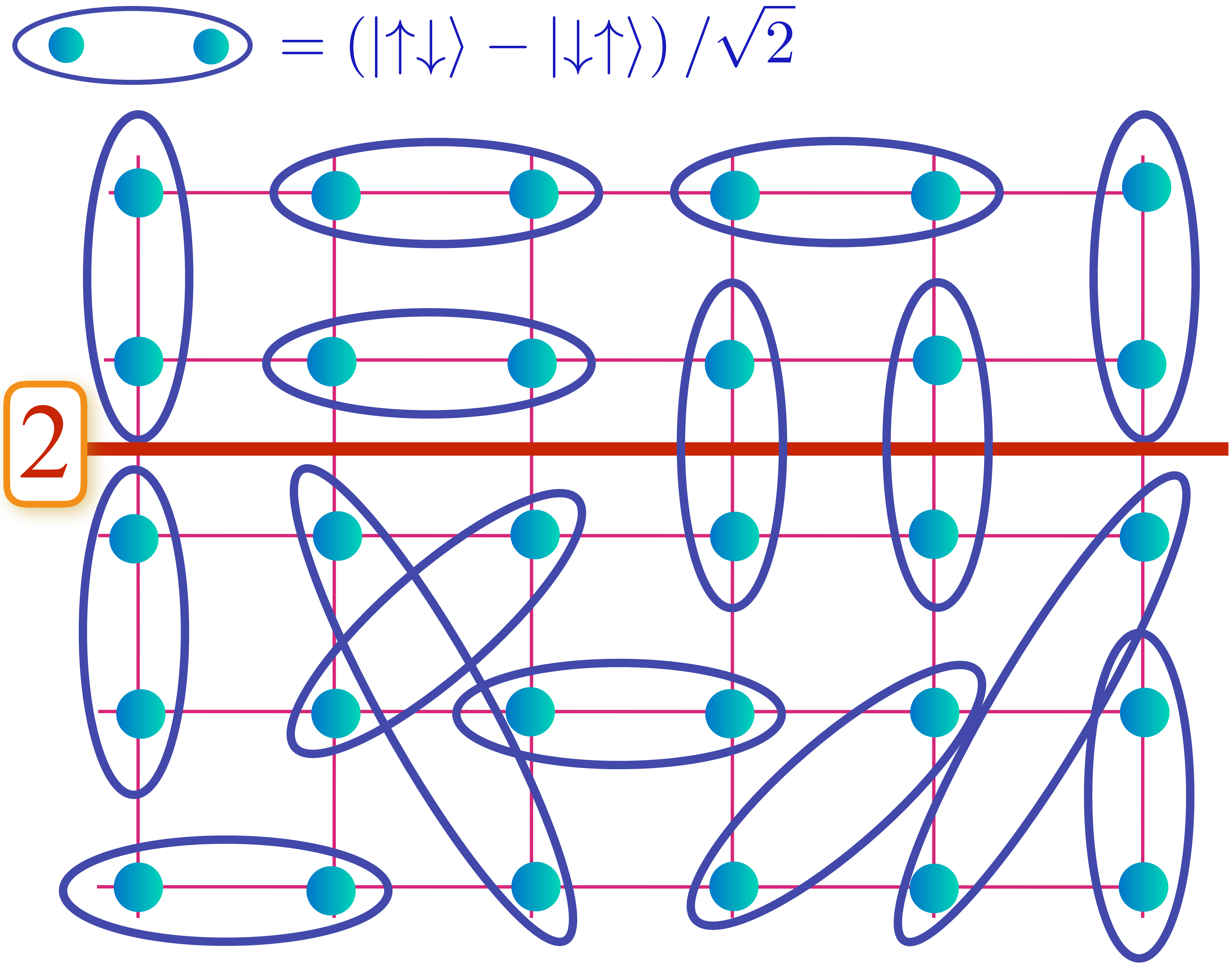

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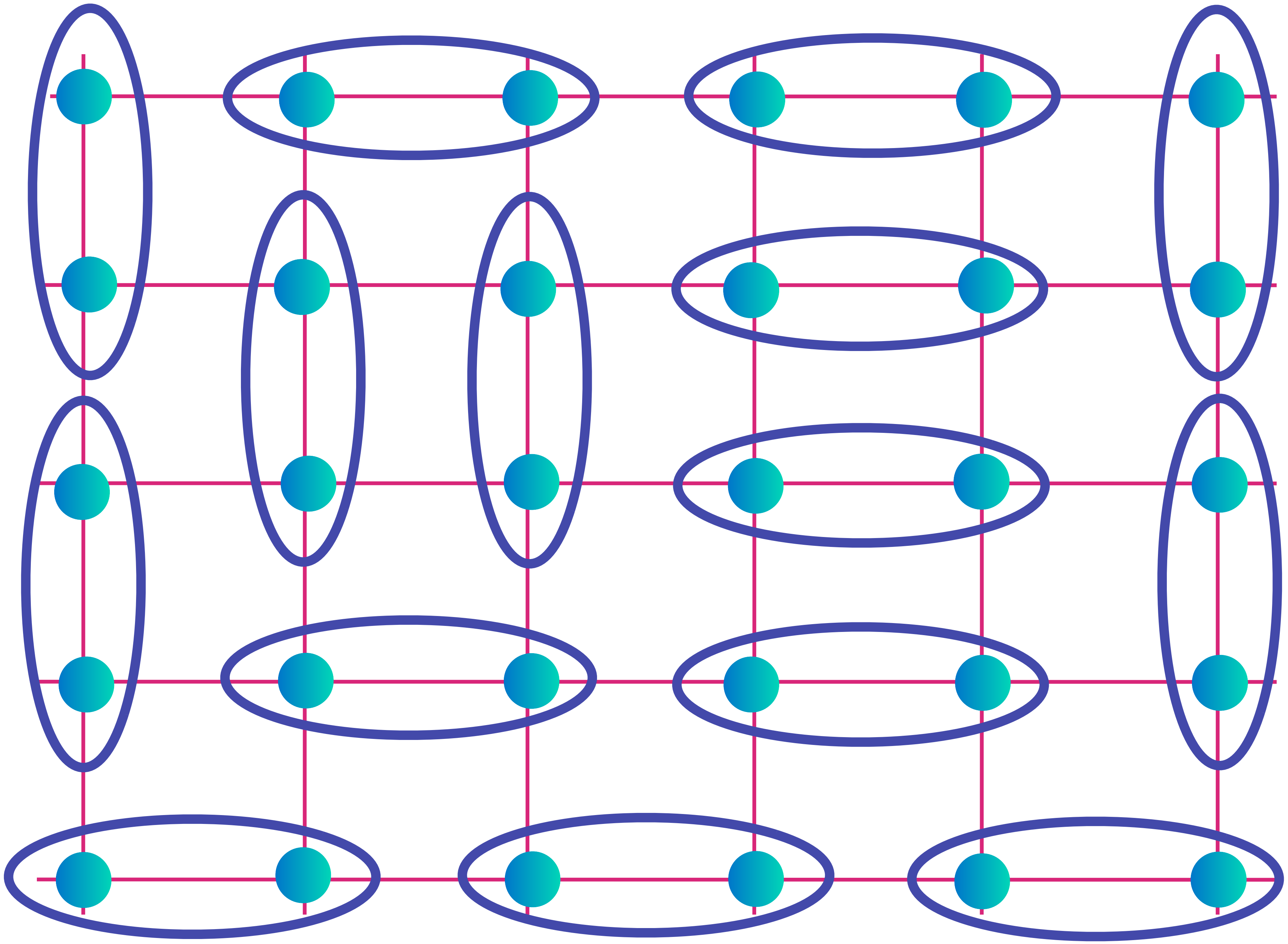
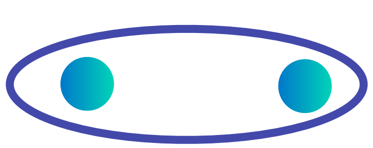
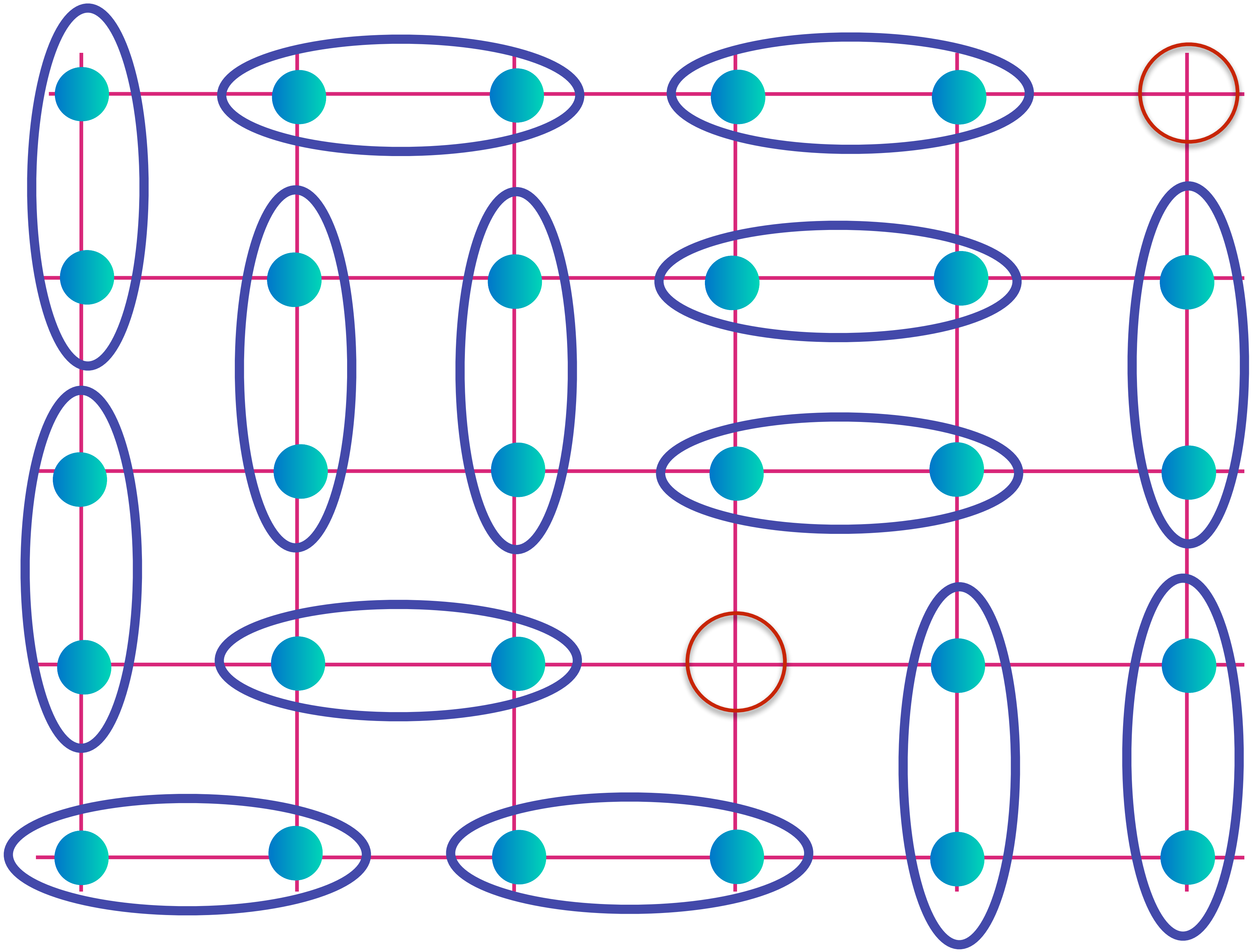
Adding holes to a spin liquid



 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

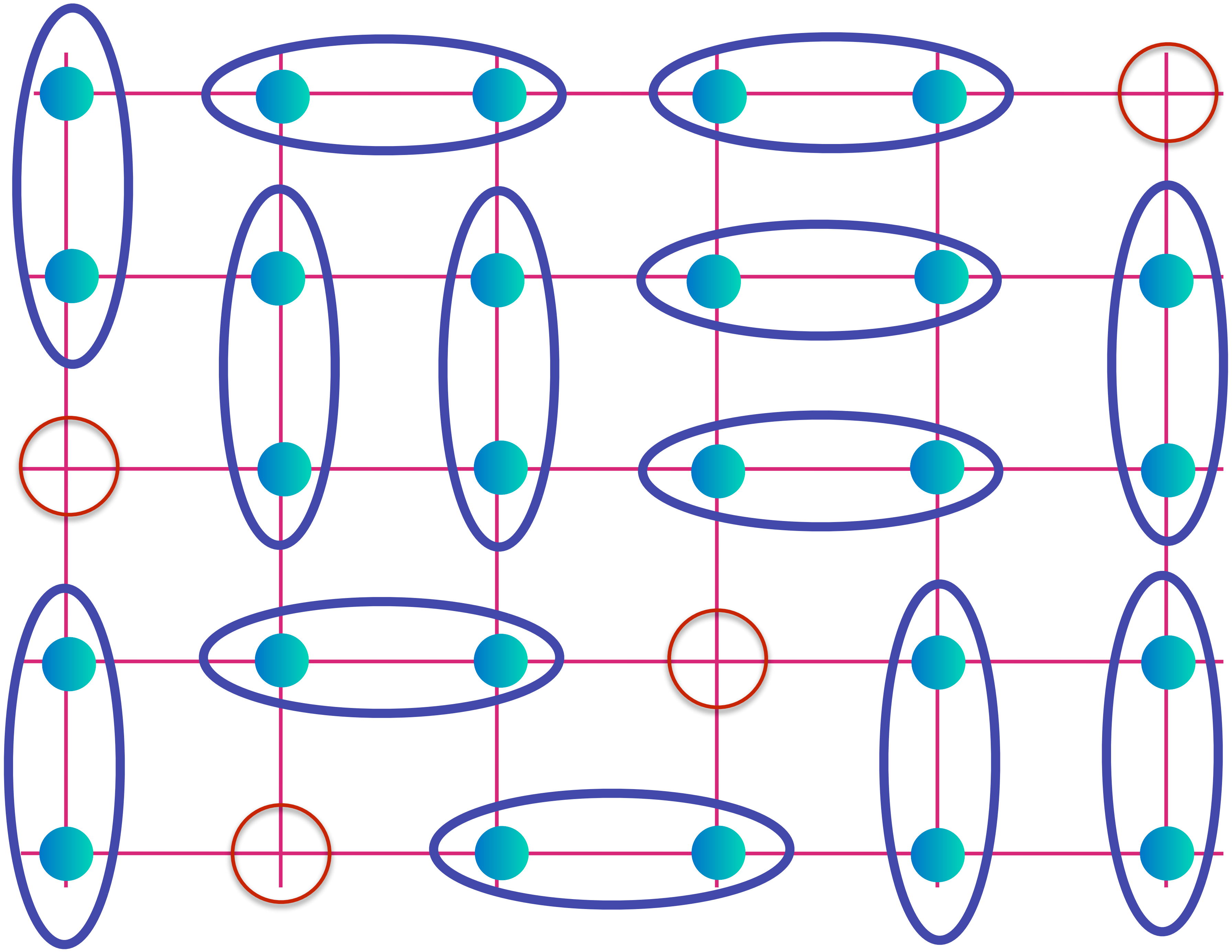
Adding holes to a spin liquid

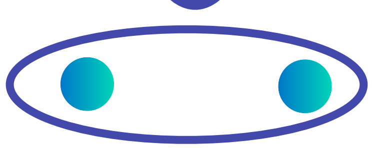


Start with a spin liquid and then remove electrons

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

Adding holes to a spin liquid



 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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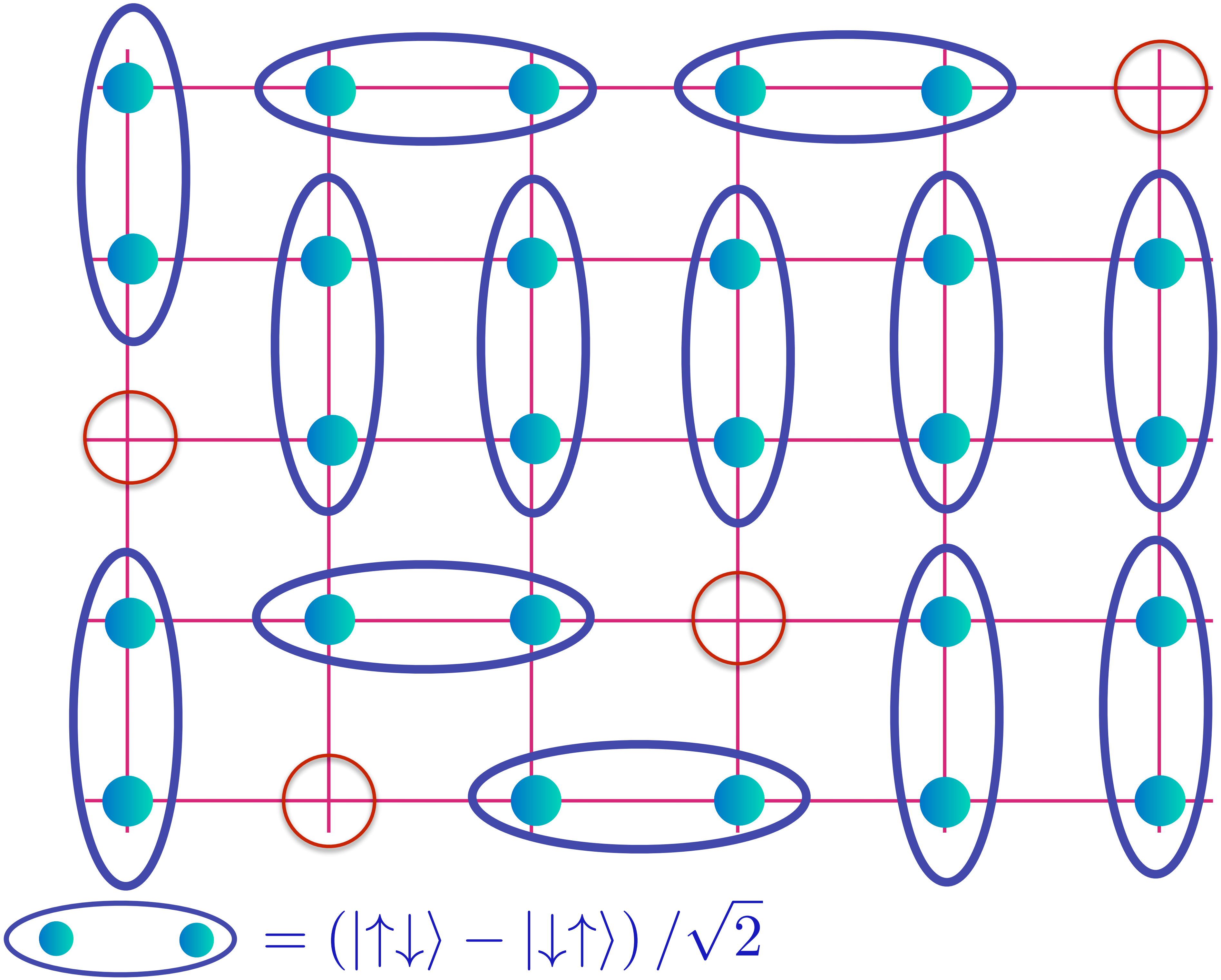
Adding holes to a spin liquid



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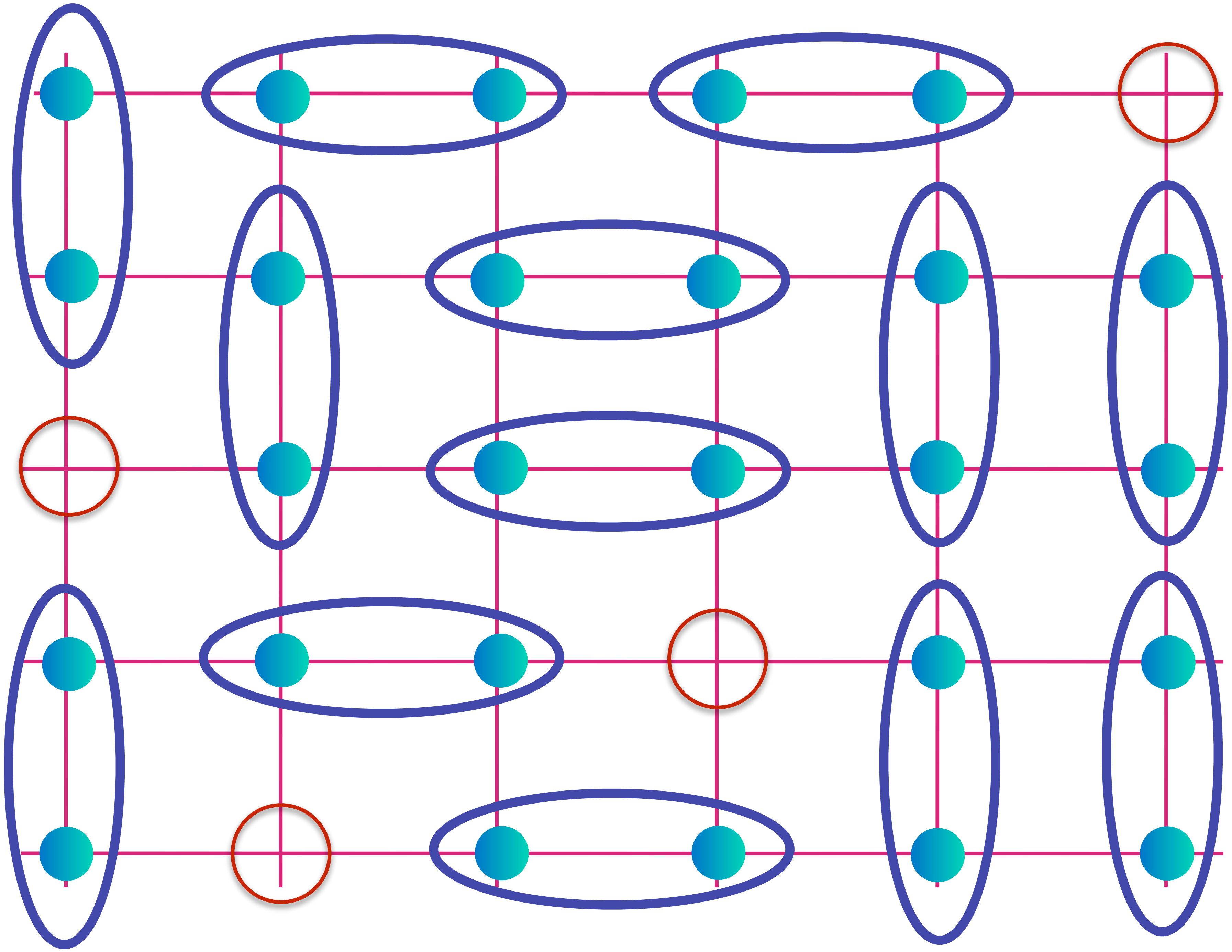

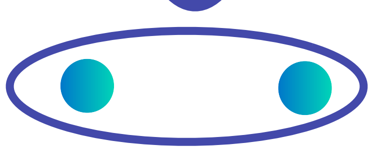
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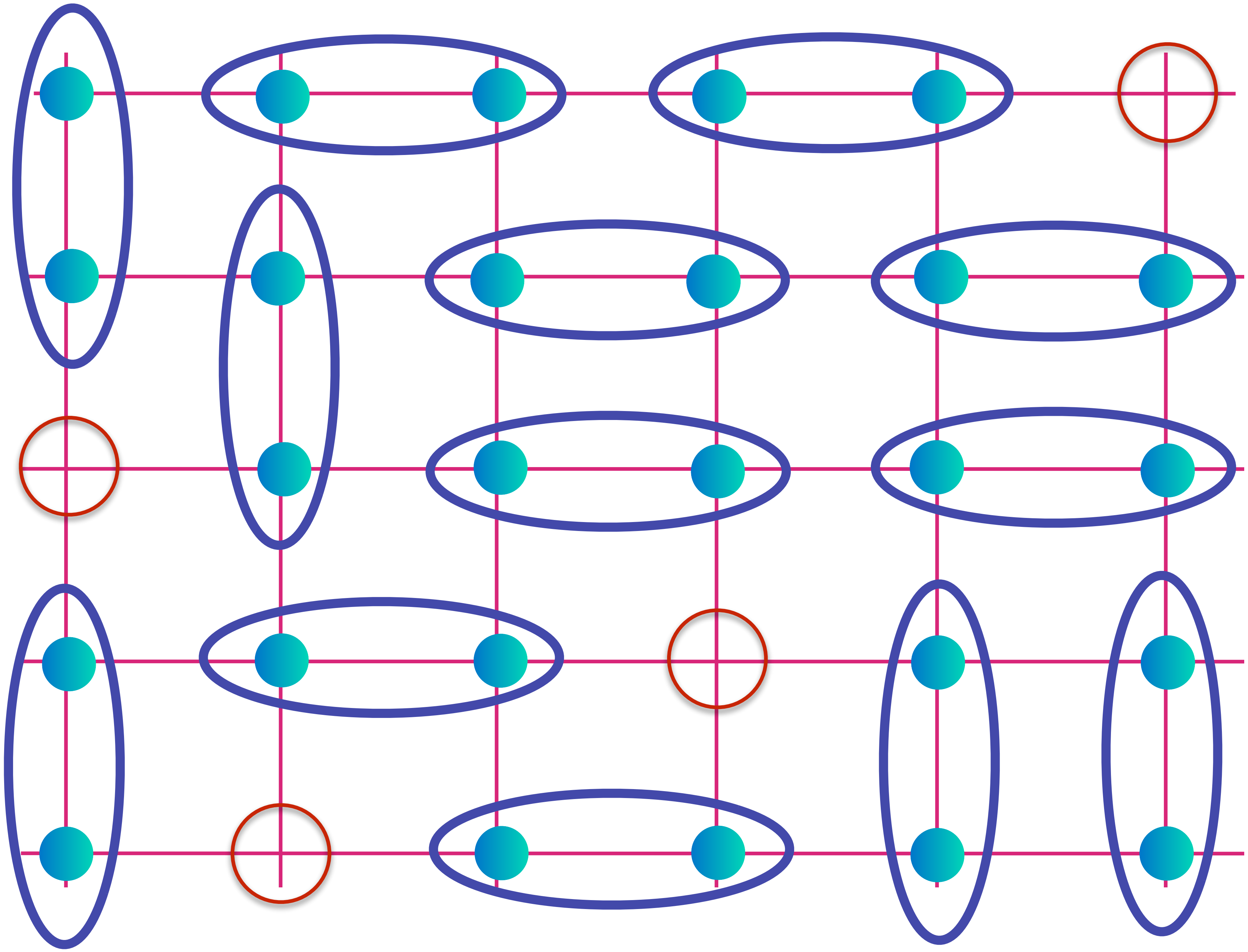
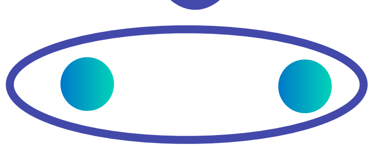
Adding holes to a spin liquid



 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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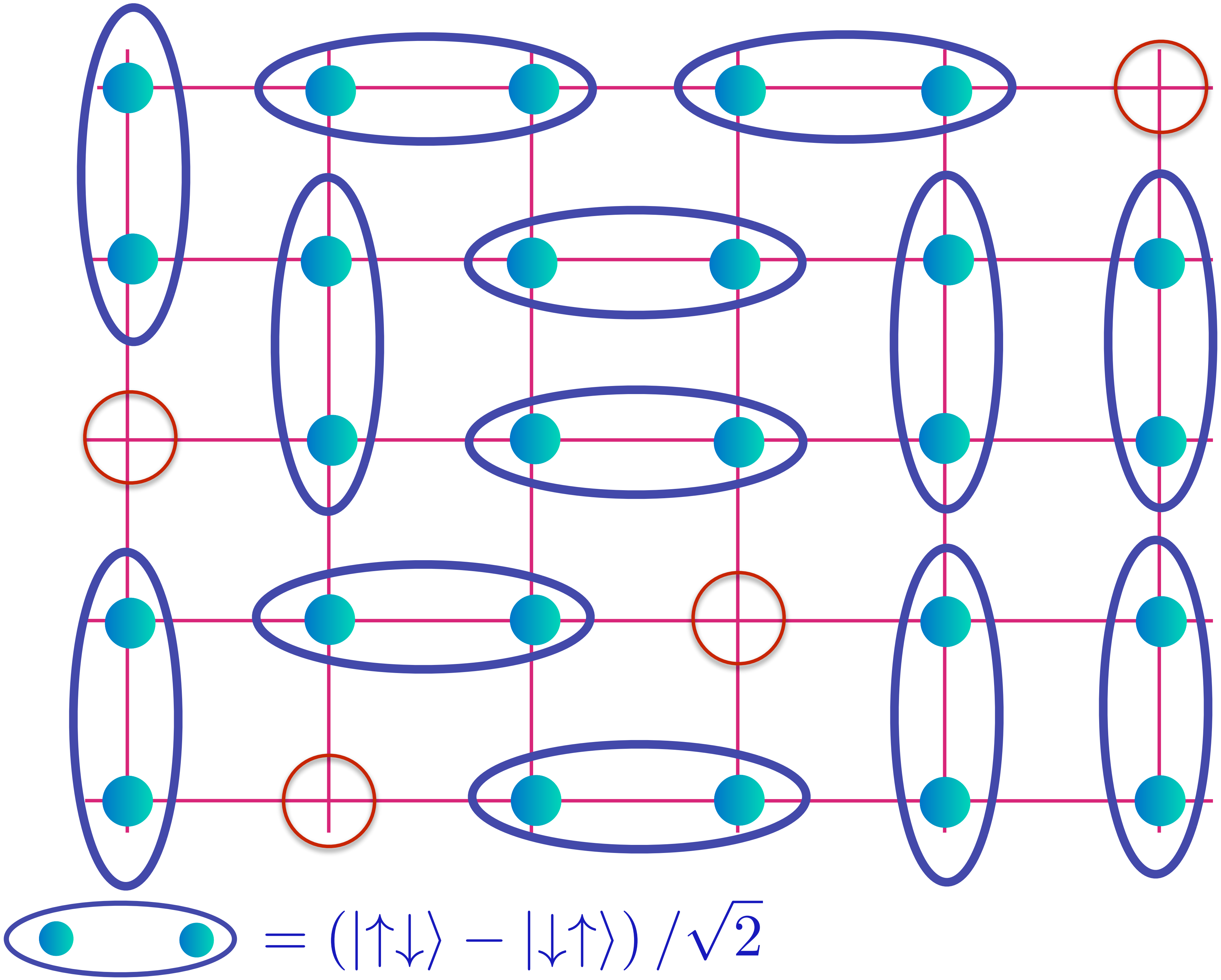

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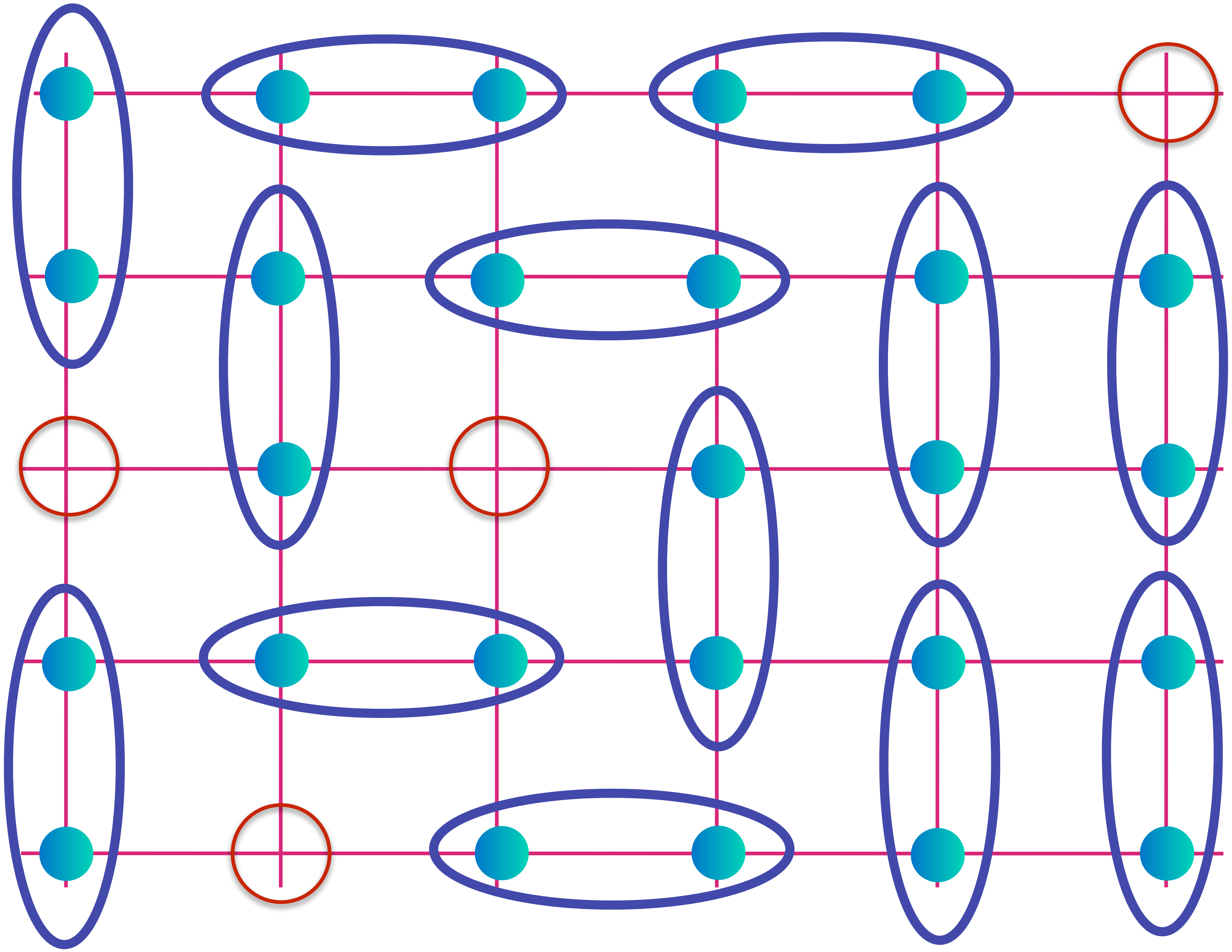
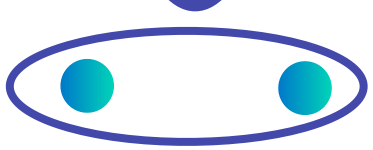
Start with a spin liquid and then remove electrons

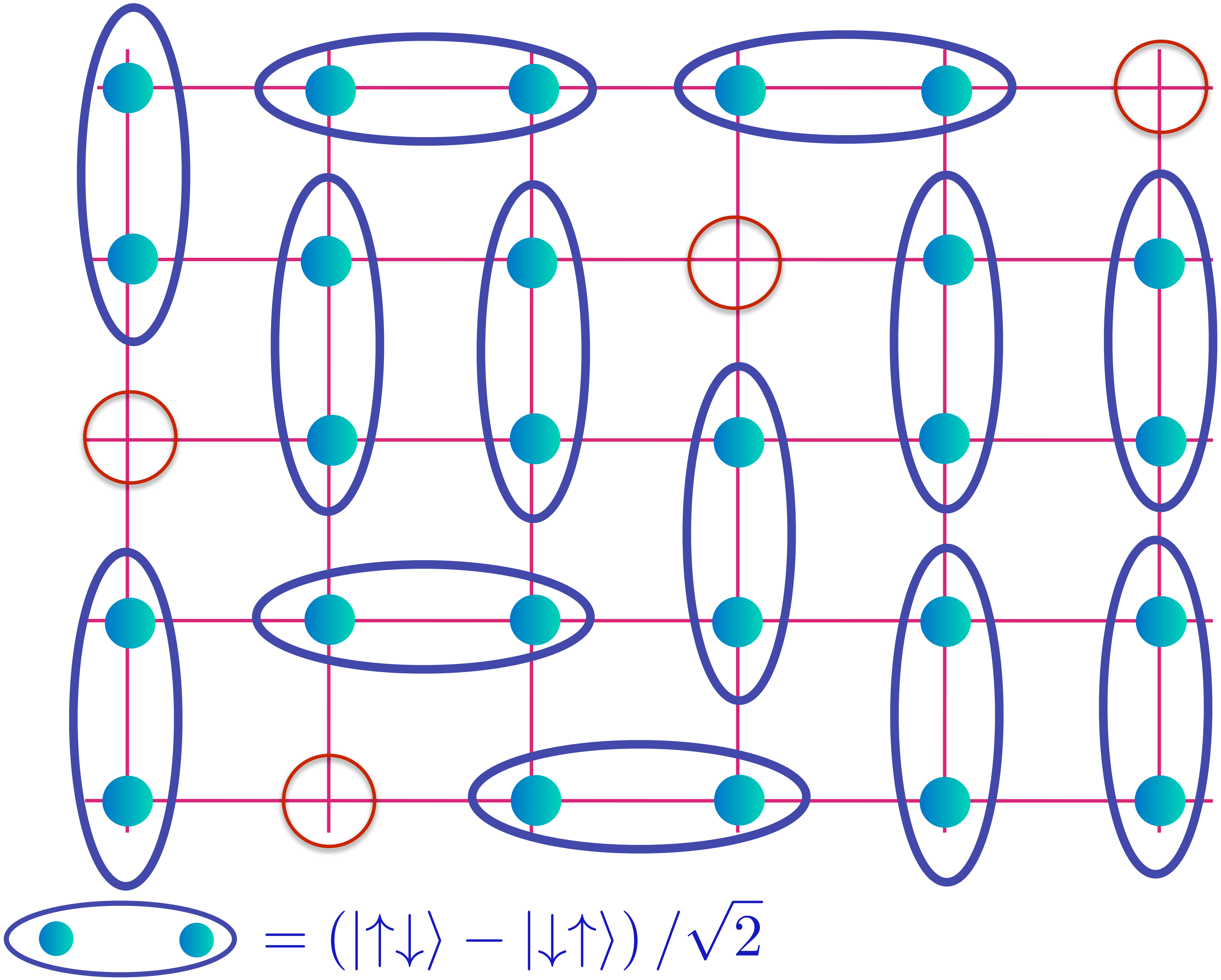
Adding holes to a spin liquid



 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

A mobile charge +e, but carrying no spin

Adding holes to a spin liquid



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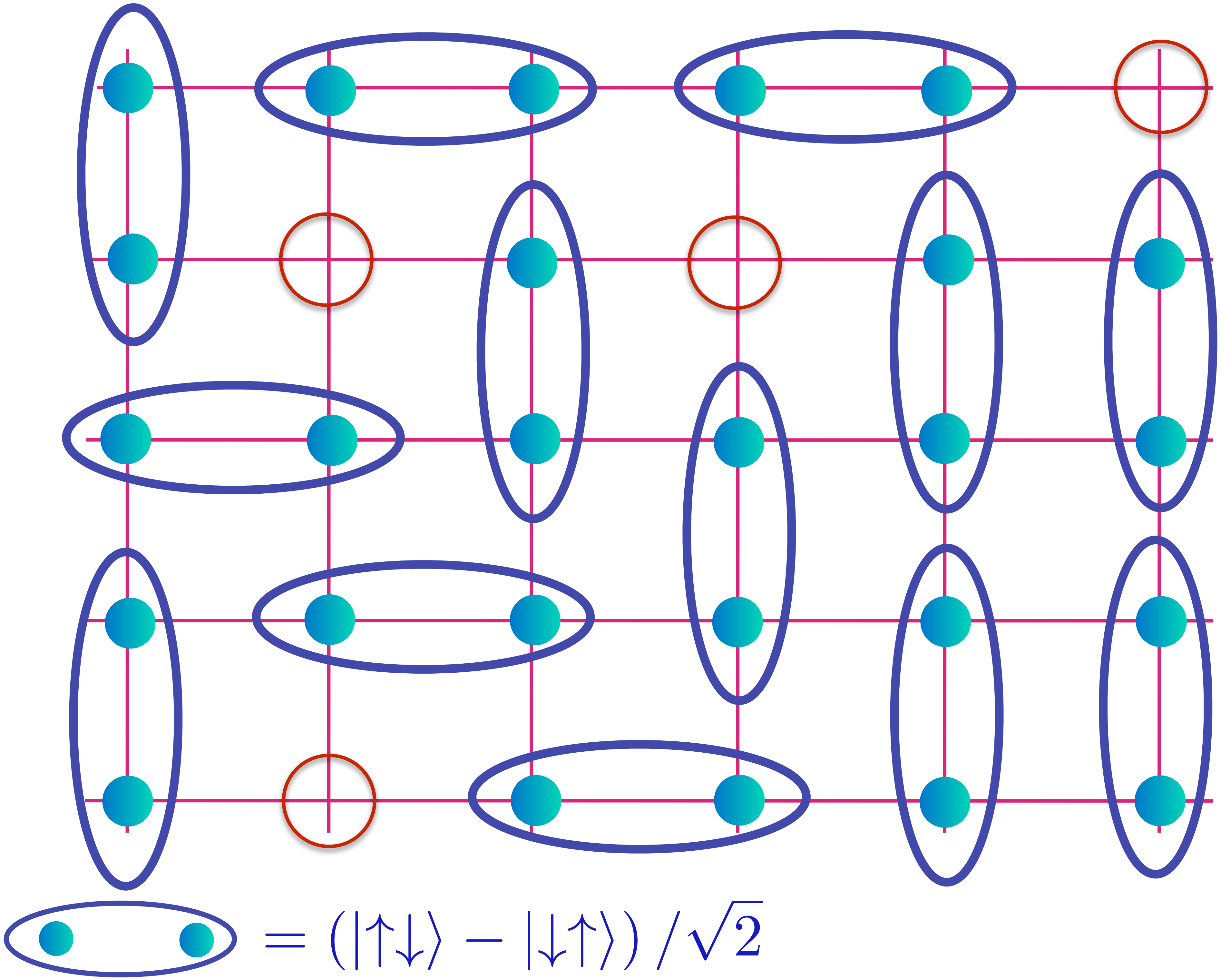
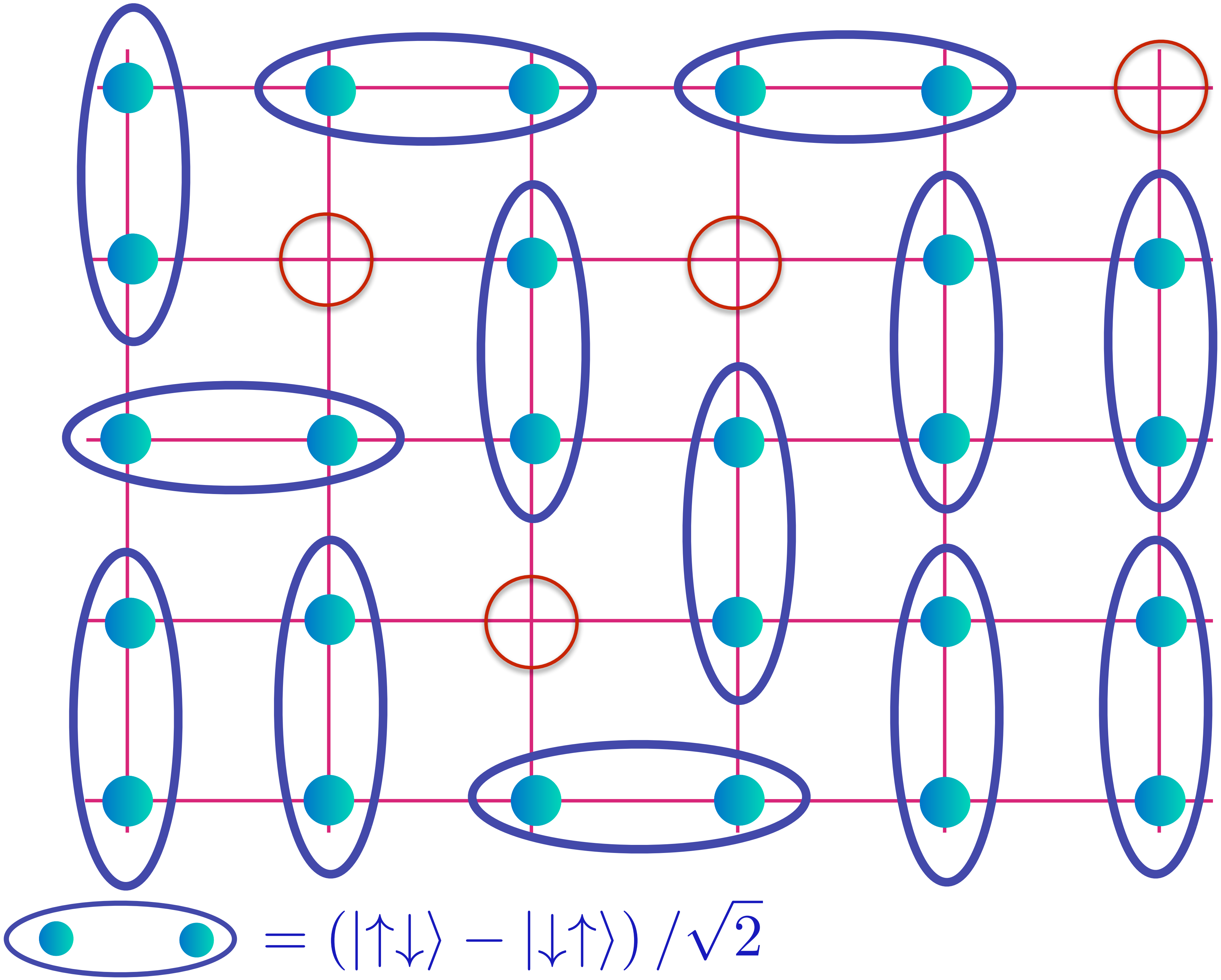


Spin liquid with density p of spinless, charge +e "holons". These can form a Fermi surface of size p, but not of electrons

Adding holes to a spin liquid



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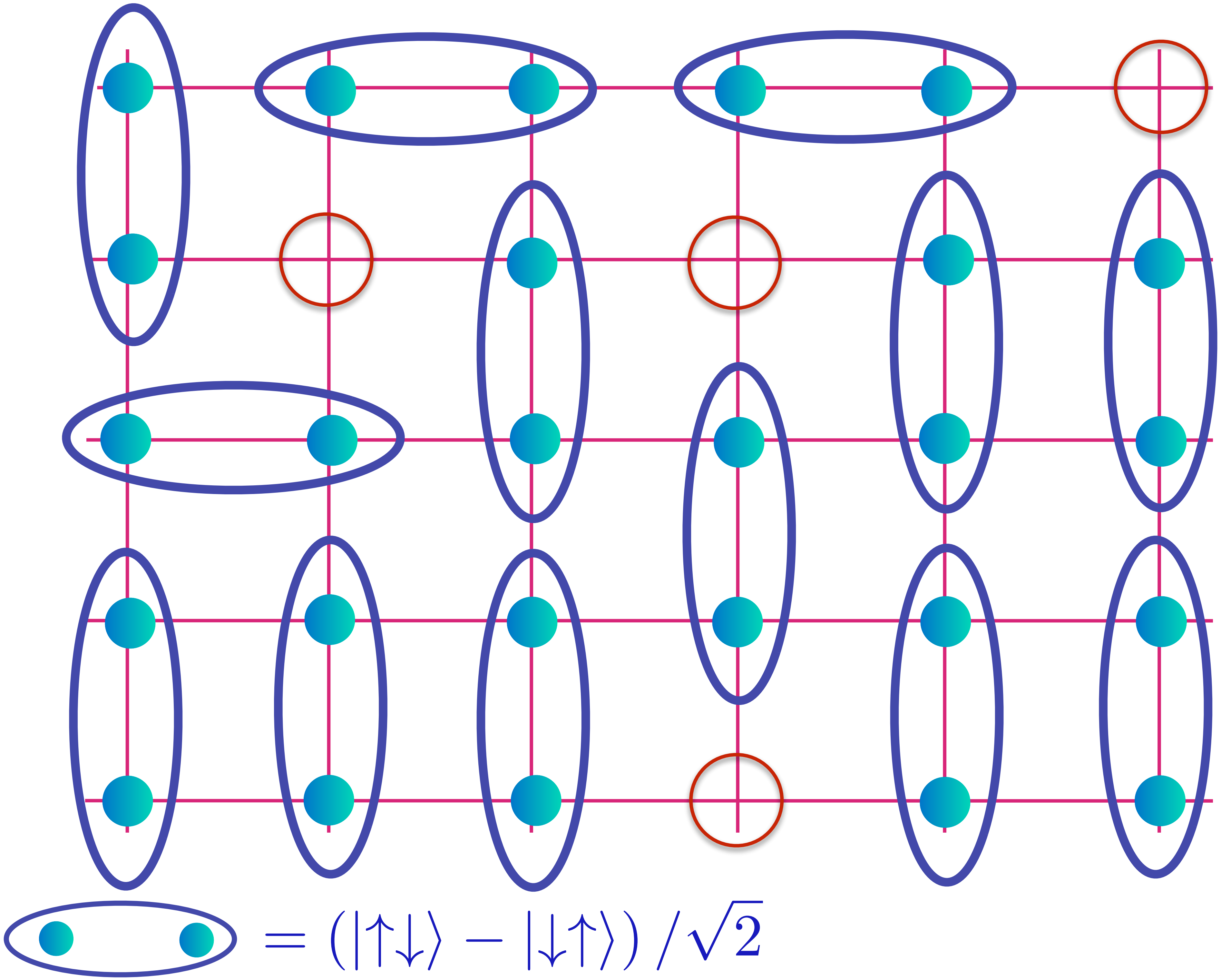
Spin liquid with density p of spinless, charge +e "holons". These can form a Fermi surface of size p, but not of electrons

Adding holes to a spin liquid



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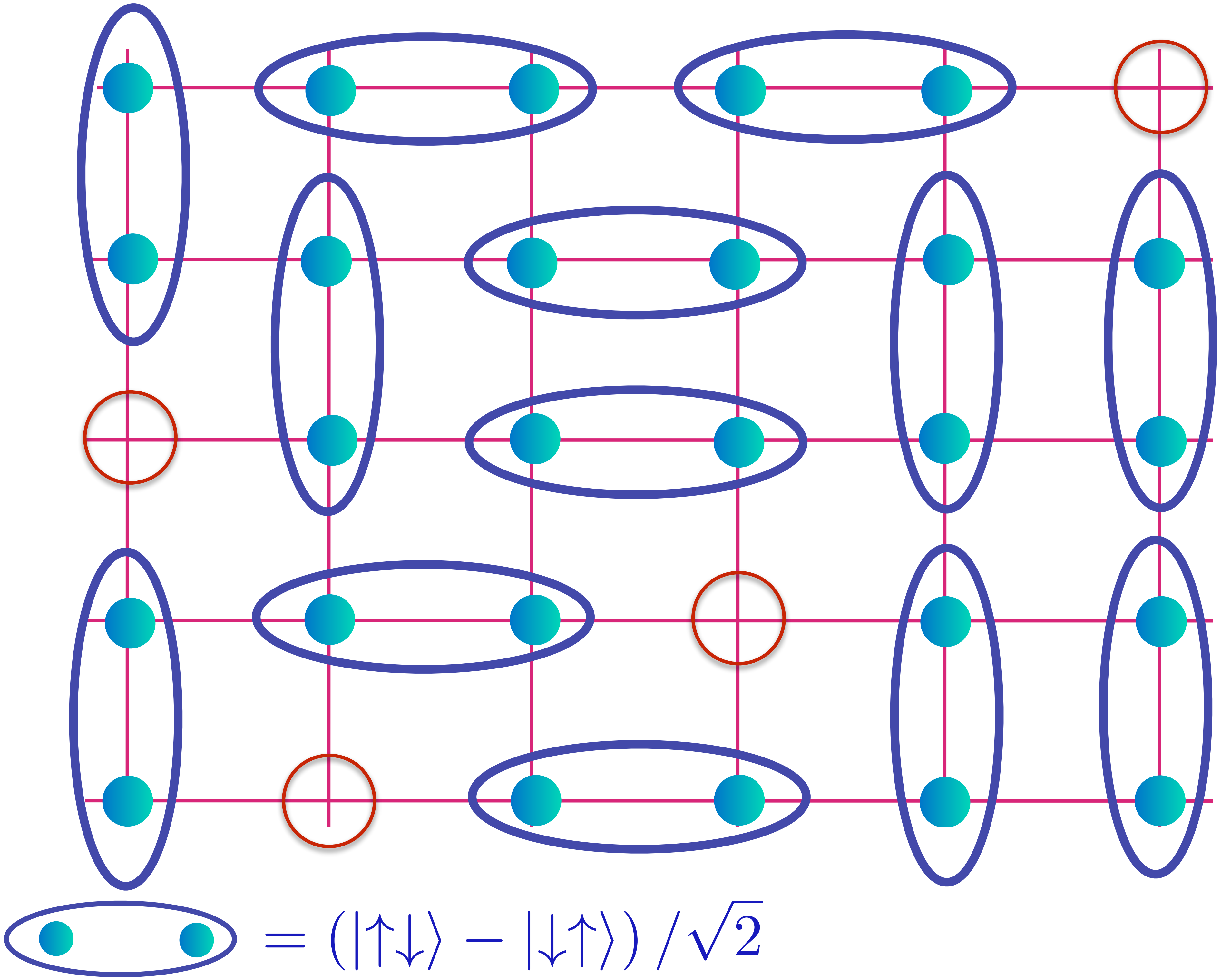
Spin liquid with density *p* of spinless, charge +e "holons". These can form a Fermi surface of size p, but not of electrons

Adding holes to a spin liquid



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Spin liquid with density p of spinless, charge +e "holons". These can form a Fermi surface of size p, but not of electrons

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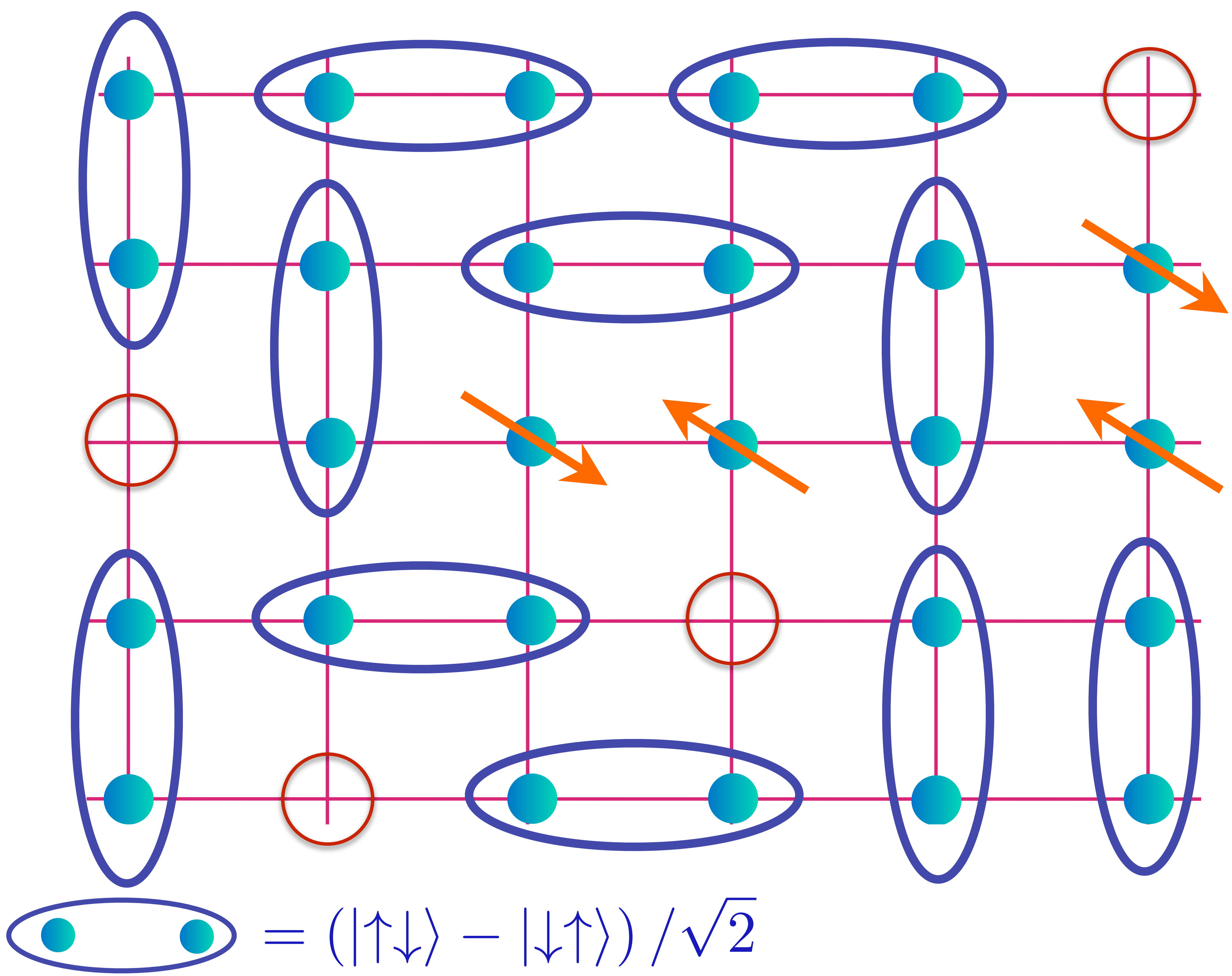
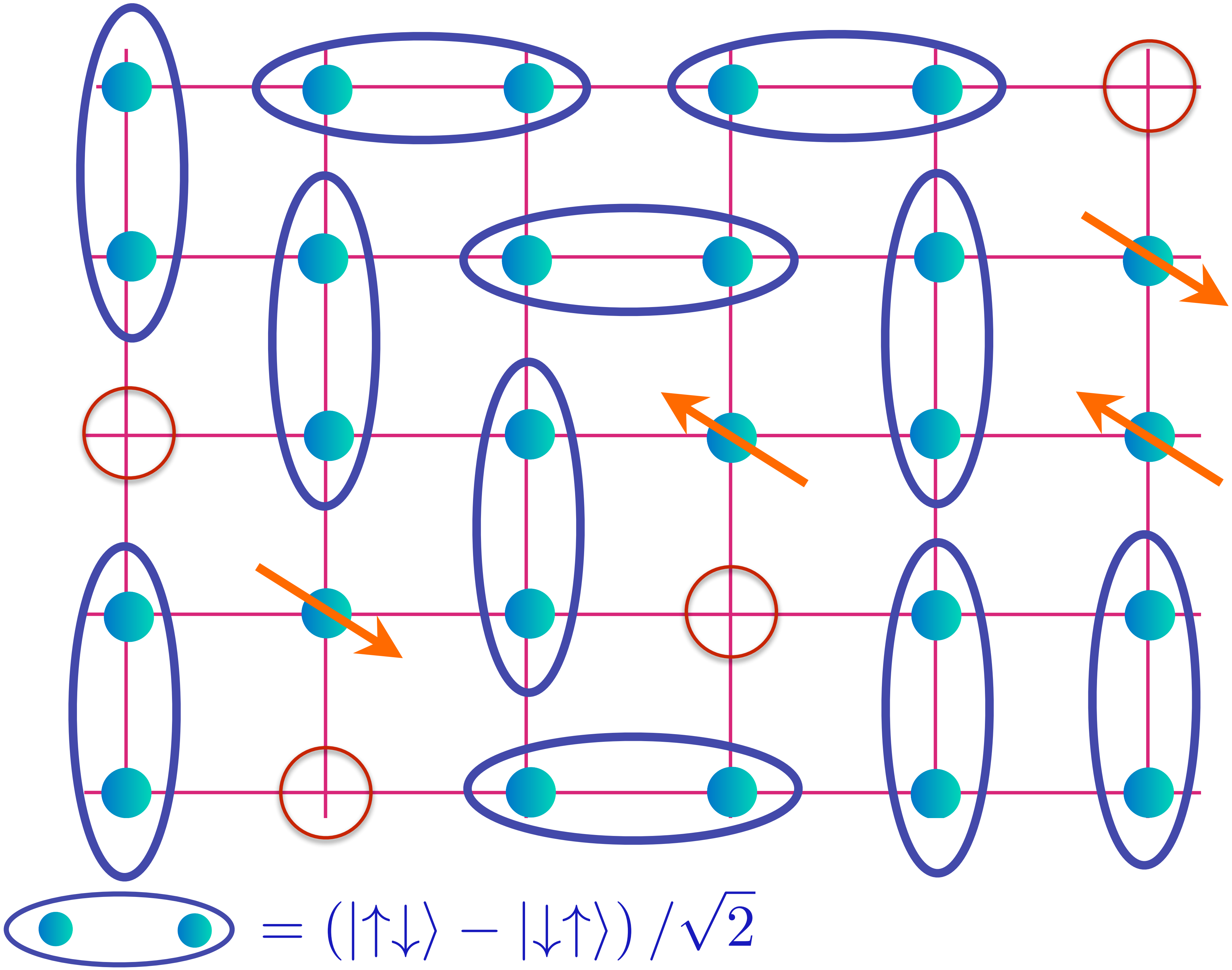


A spin liquid also has mobile spin-1/2 excitations which are charge neutral excitations: "spinons"

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

Holons and spinons a spin liquid



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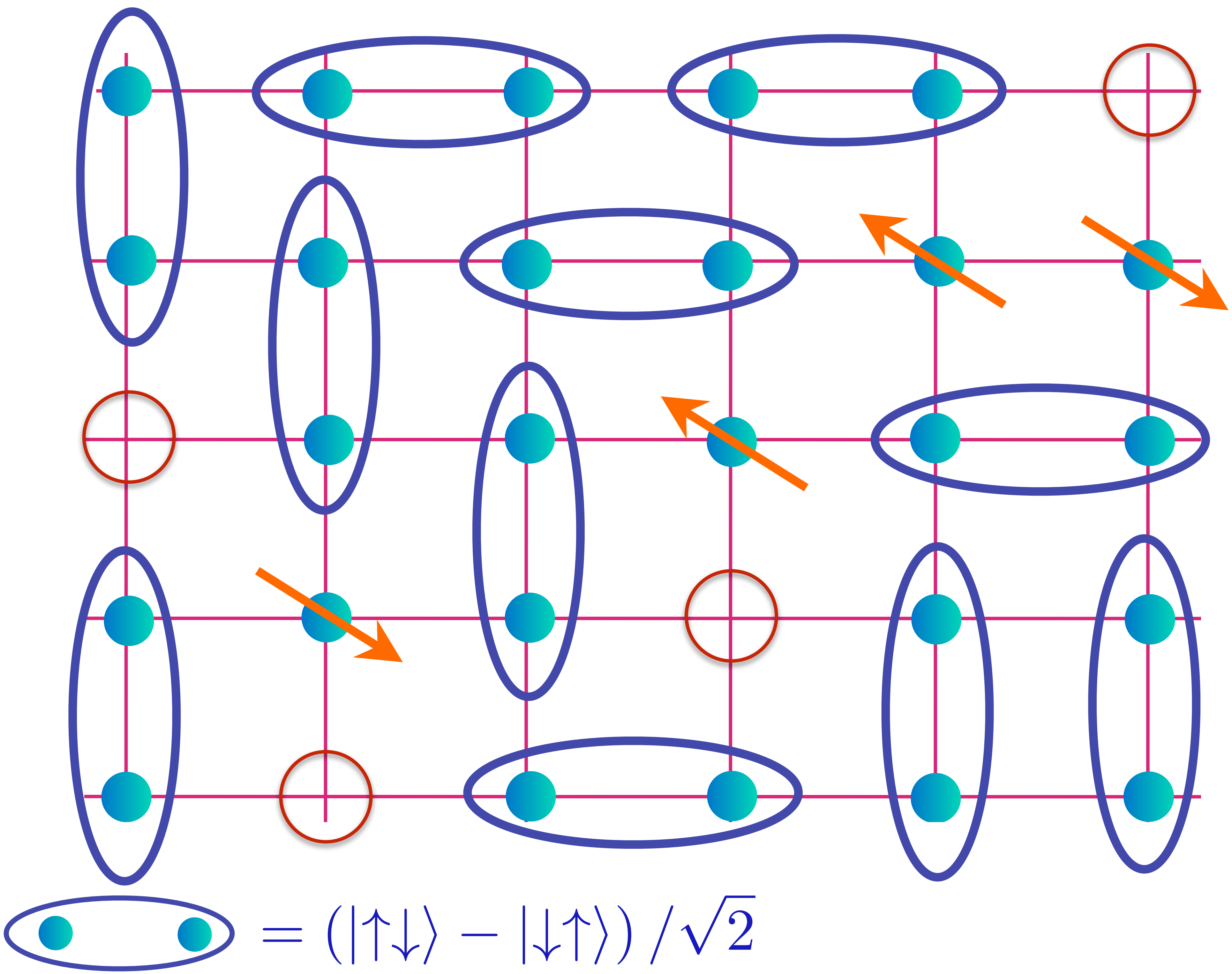
Holons and spinons a spin liquid



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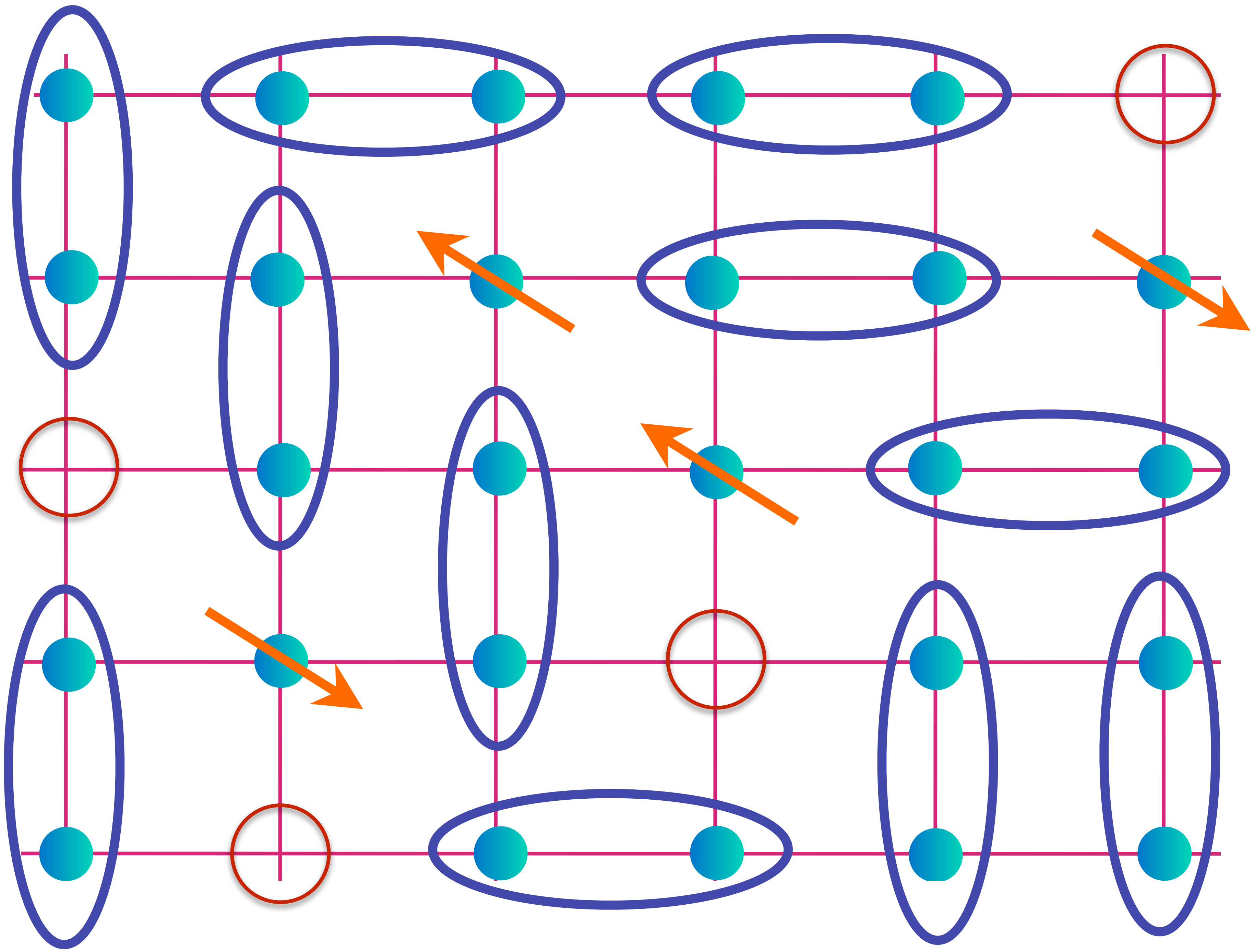

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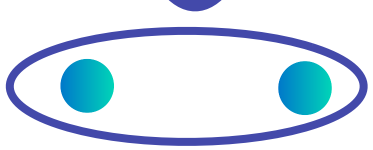
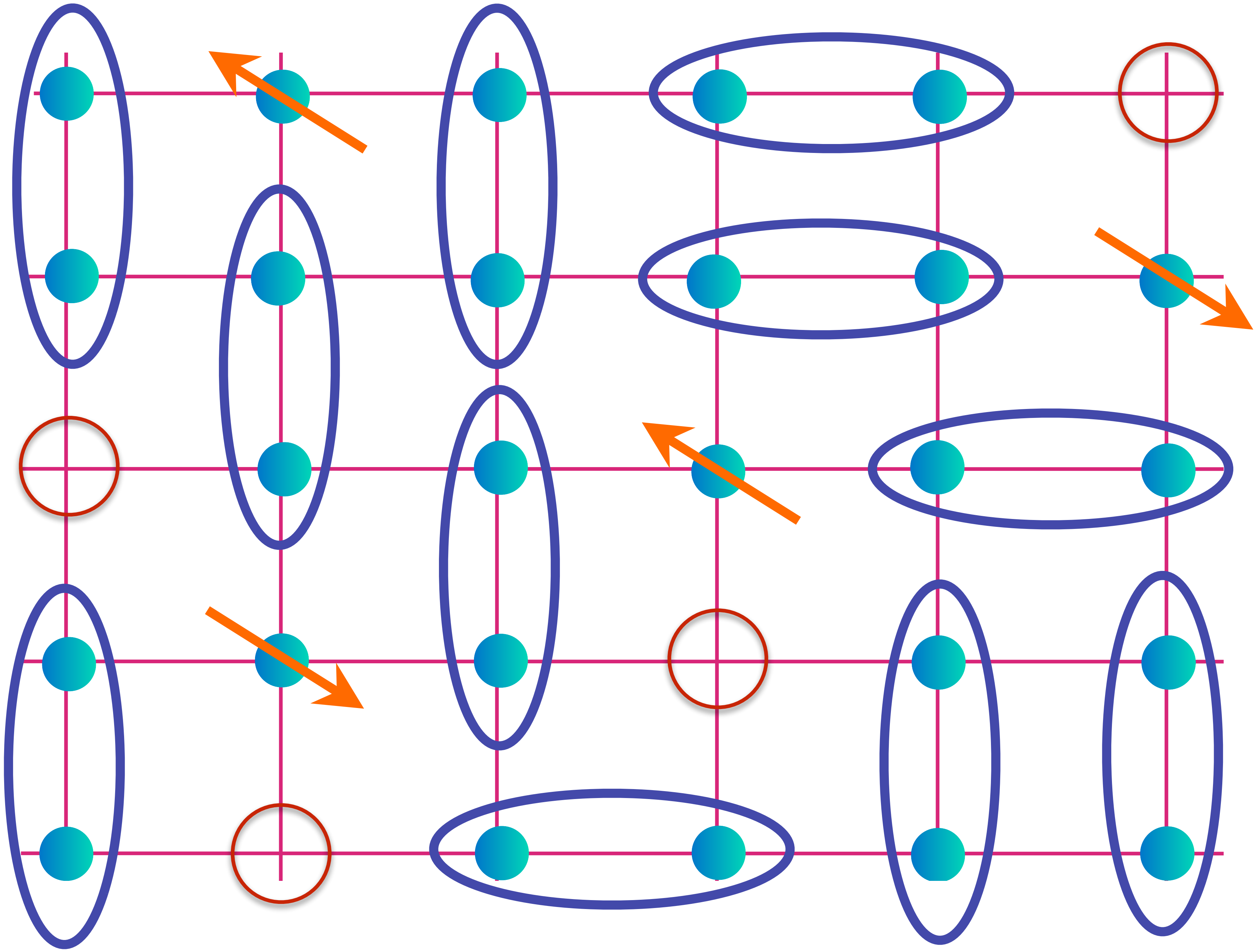
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

Holons and spinons a spin liquid



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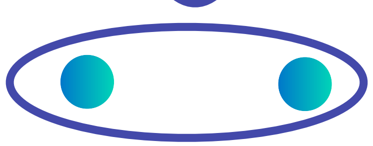

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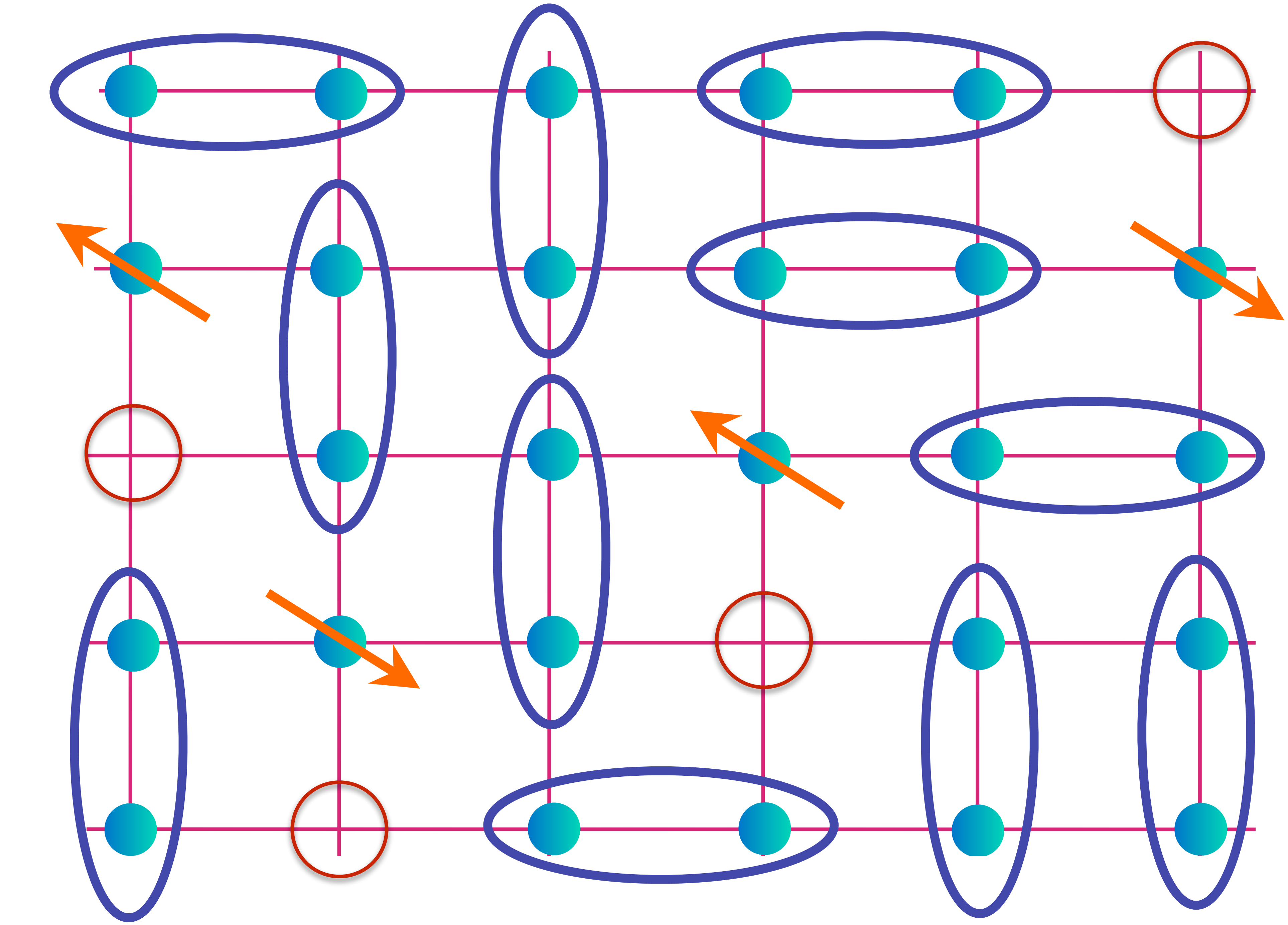
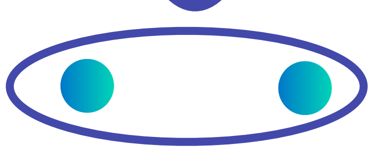
Holons and spinons a spin liquid



 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

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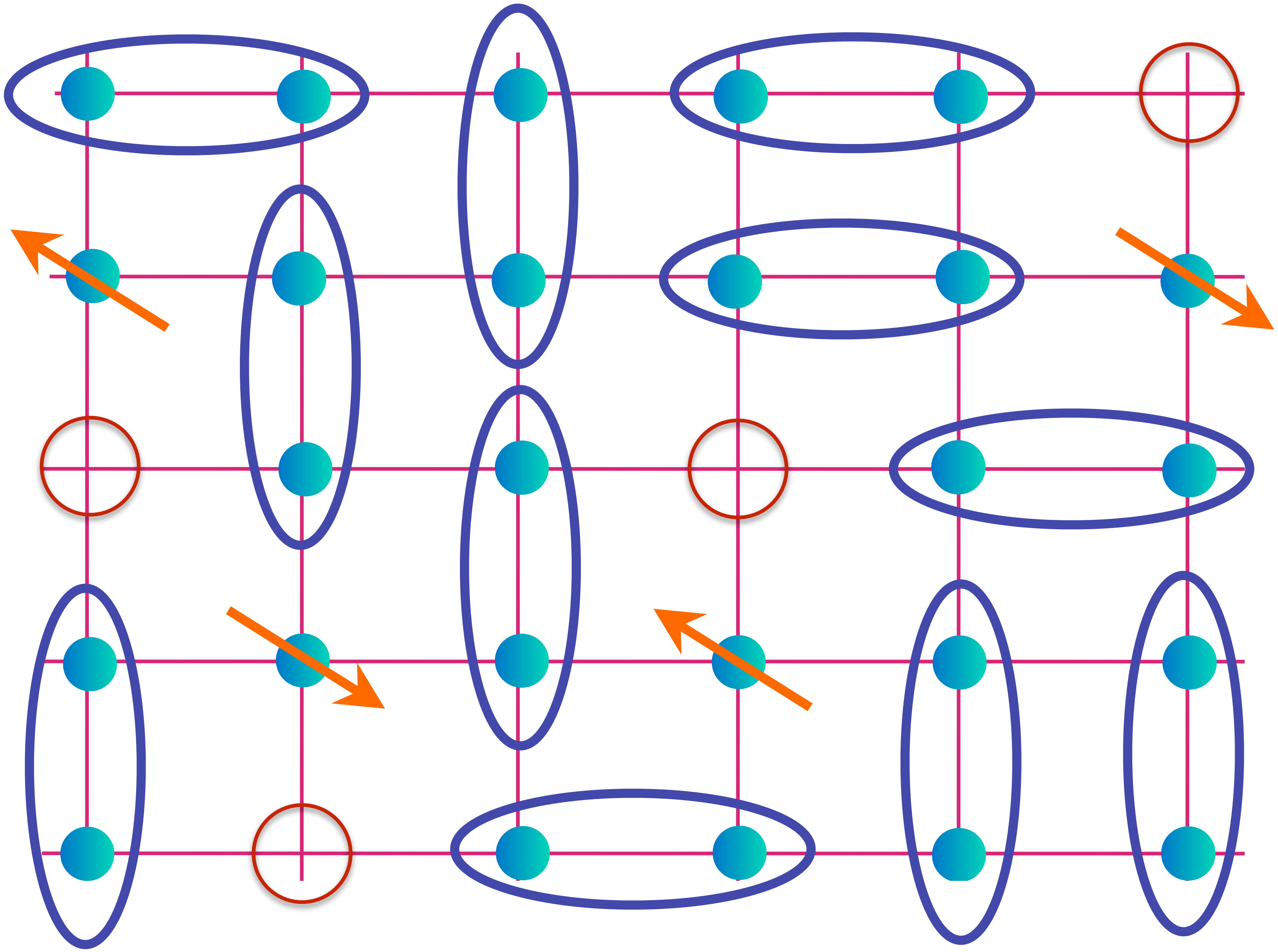
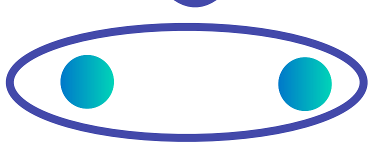
Holons and spinons a spin liquid





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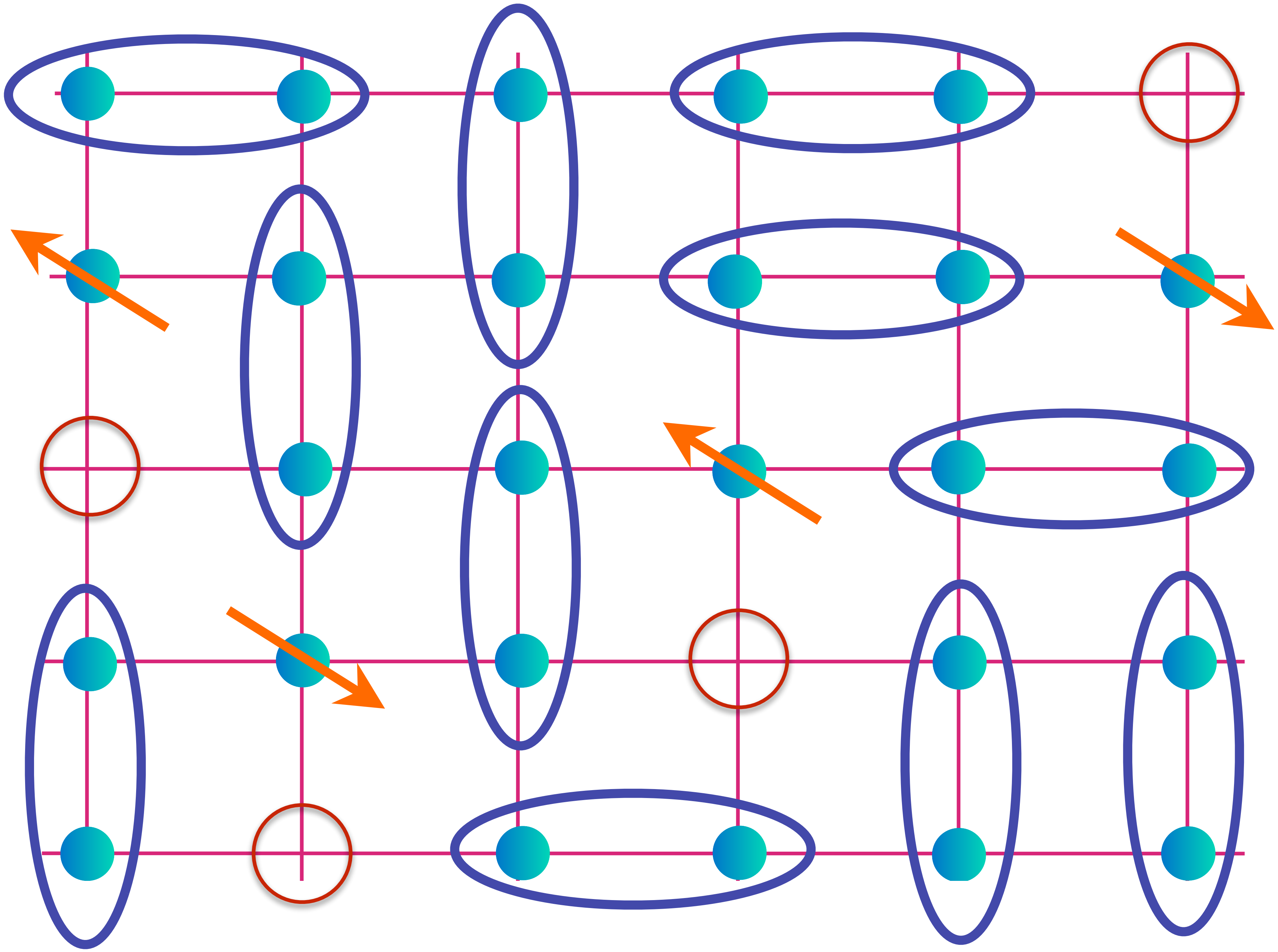
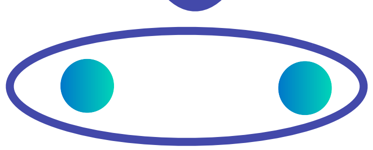
Holons and spinons a spin liquid

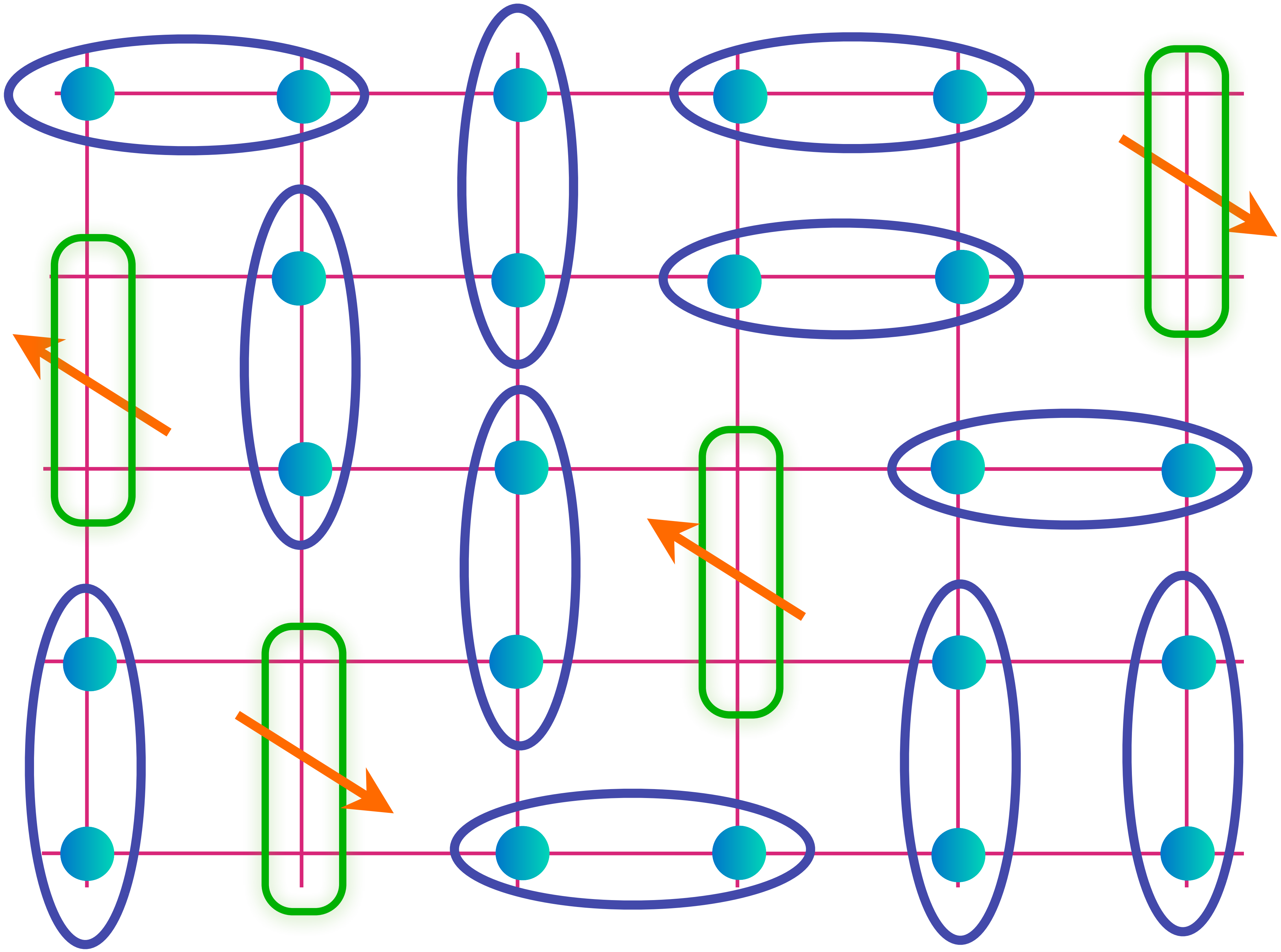


 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

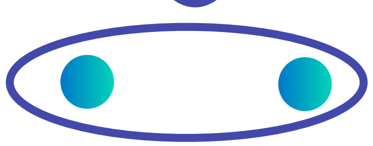
A spin liquid also has mobile spin-1/2 excitations which are charge neutral excitations: “spinons”

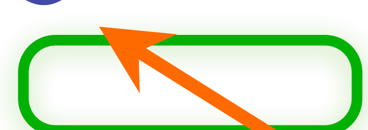
FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Green dimers: Mobile S=1/2, charge +e fermions: form a Fermi surface of size p of electrons

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*

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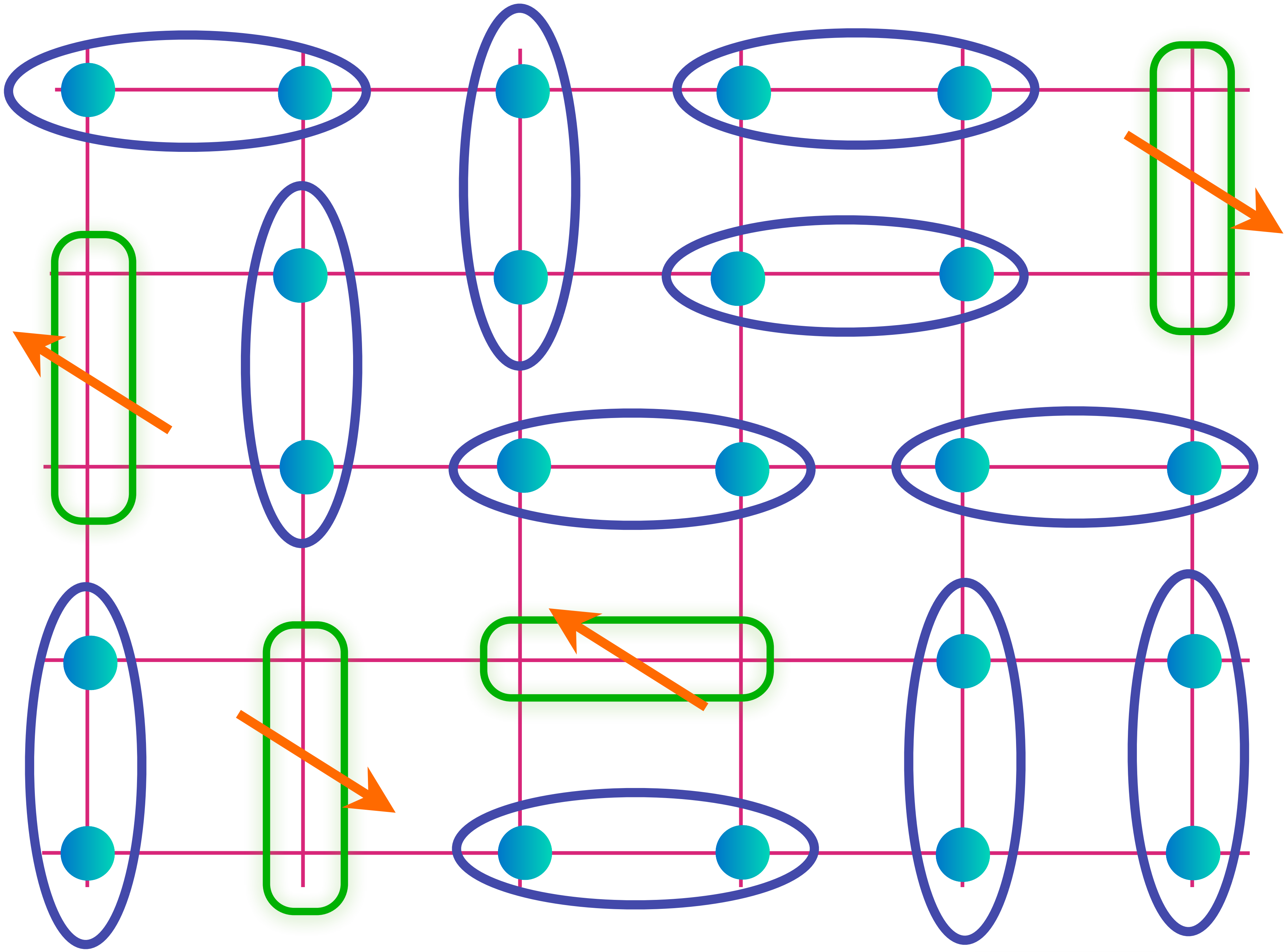

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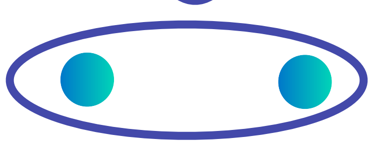
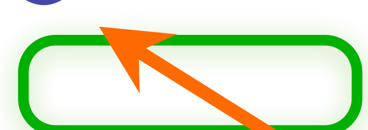
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

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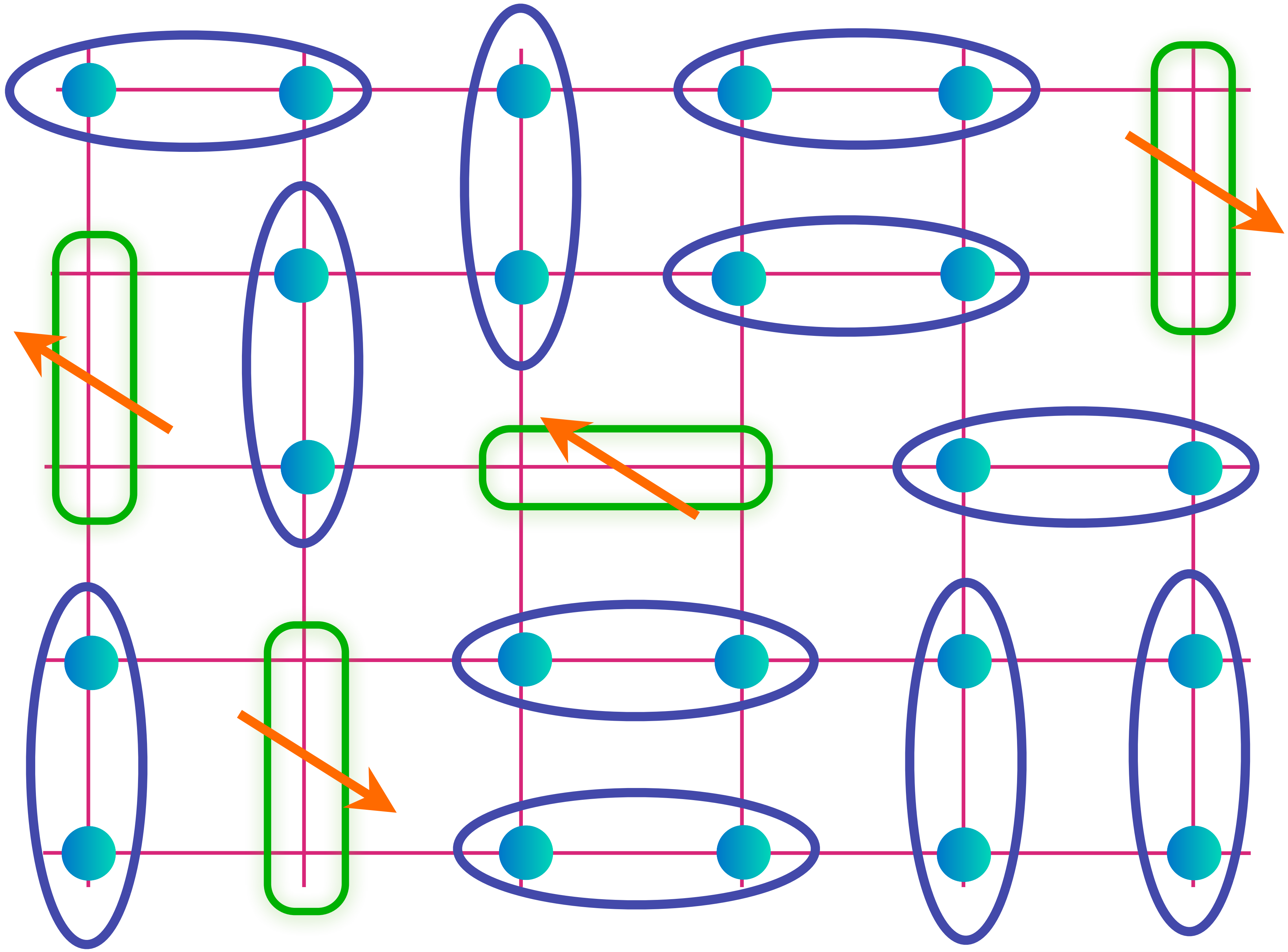

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$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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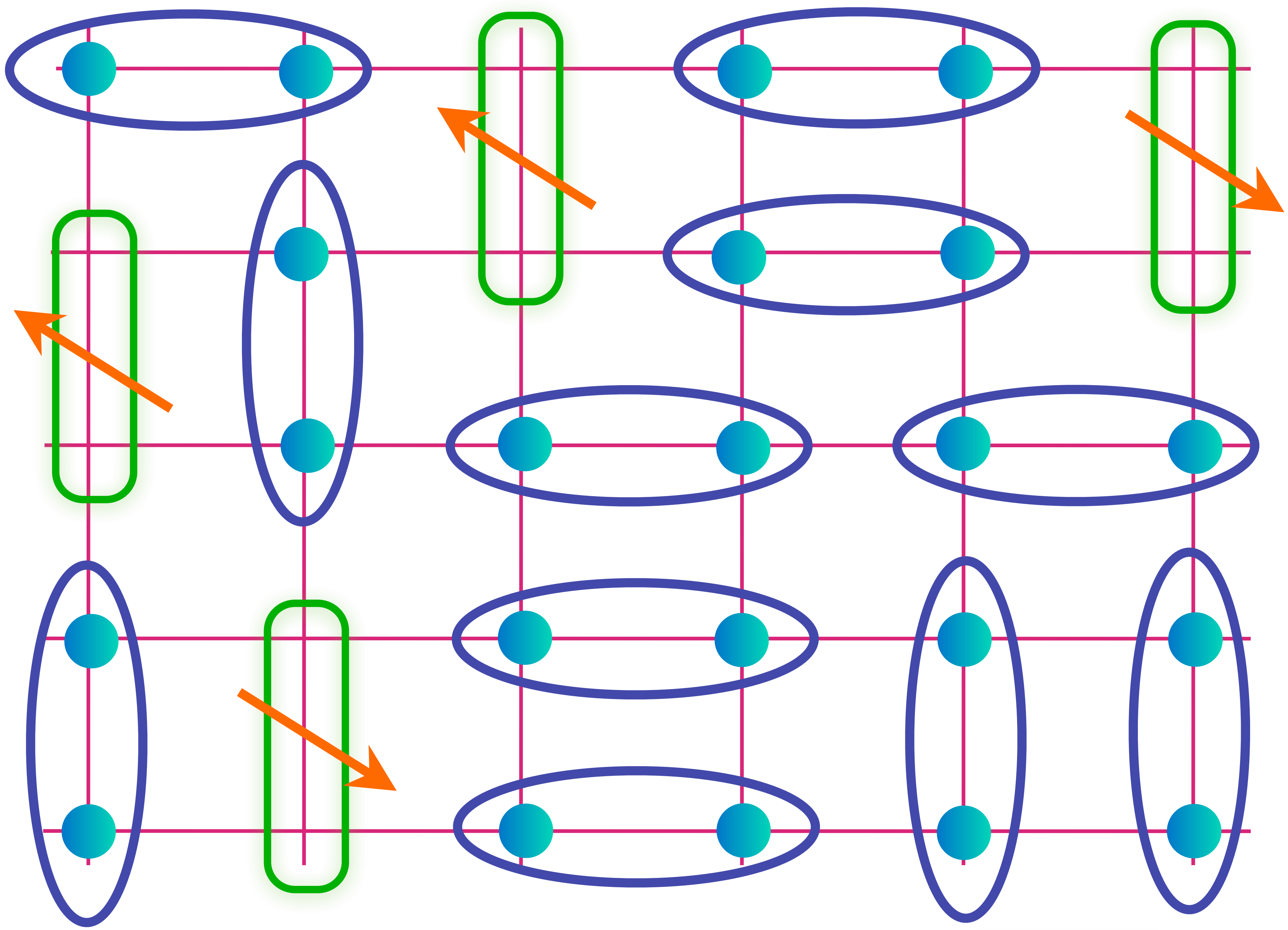

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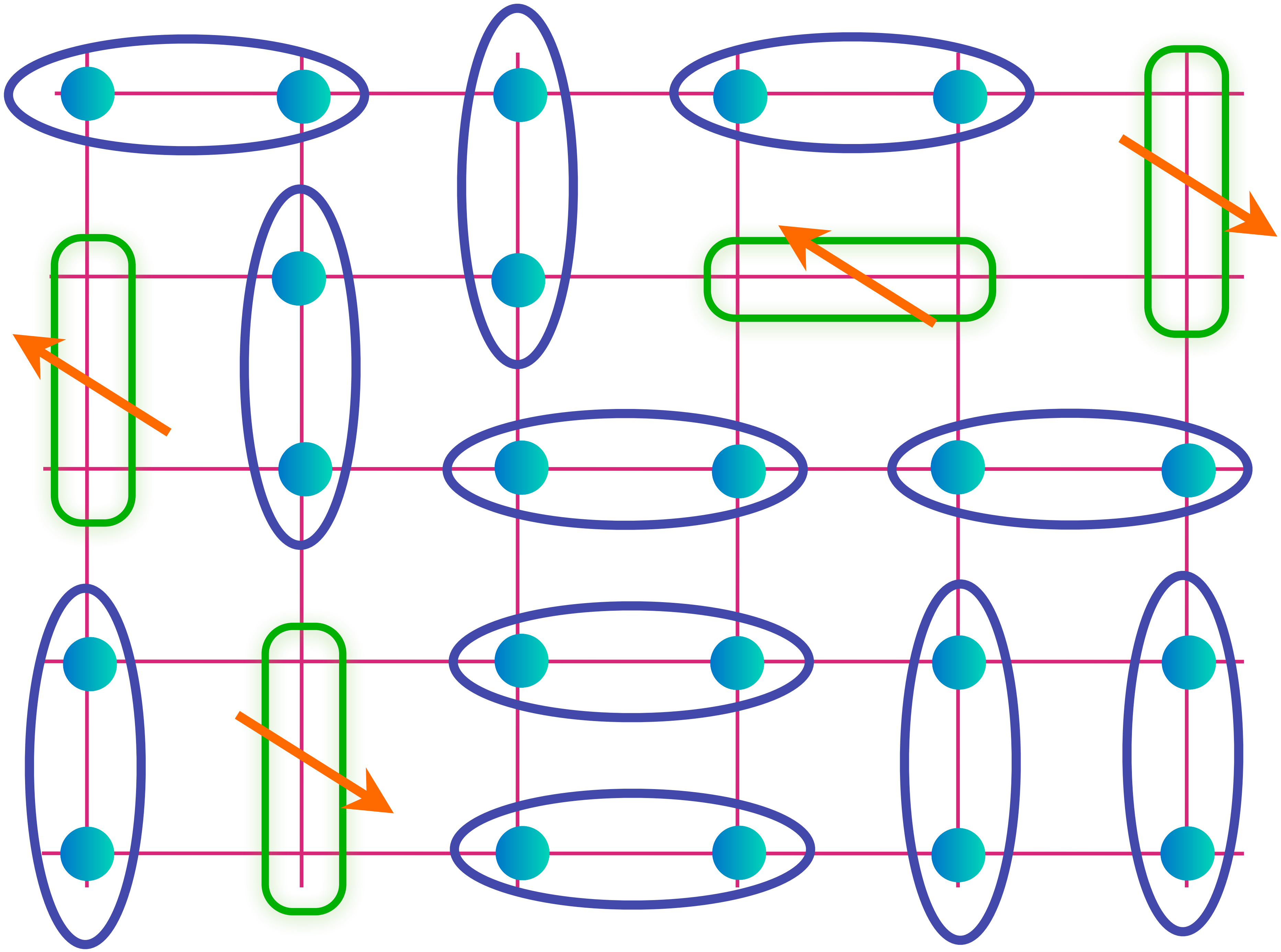

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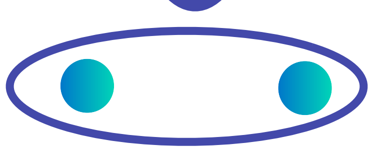
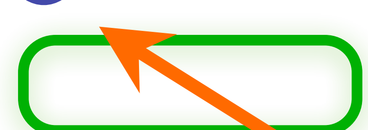
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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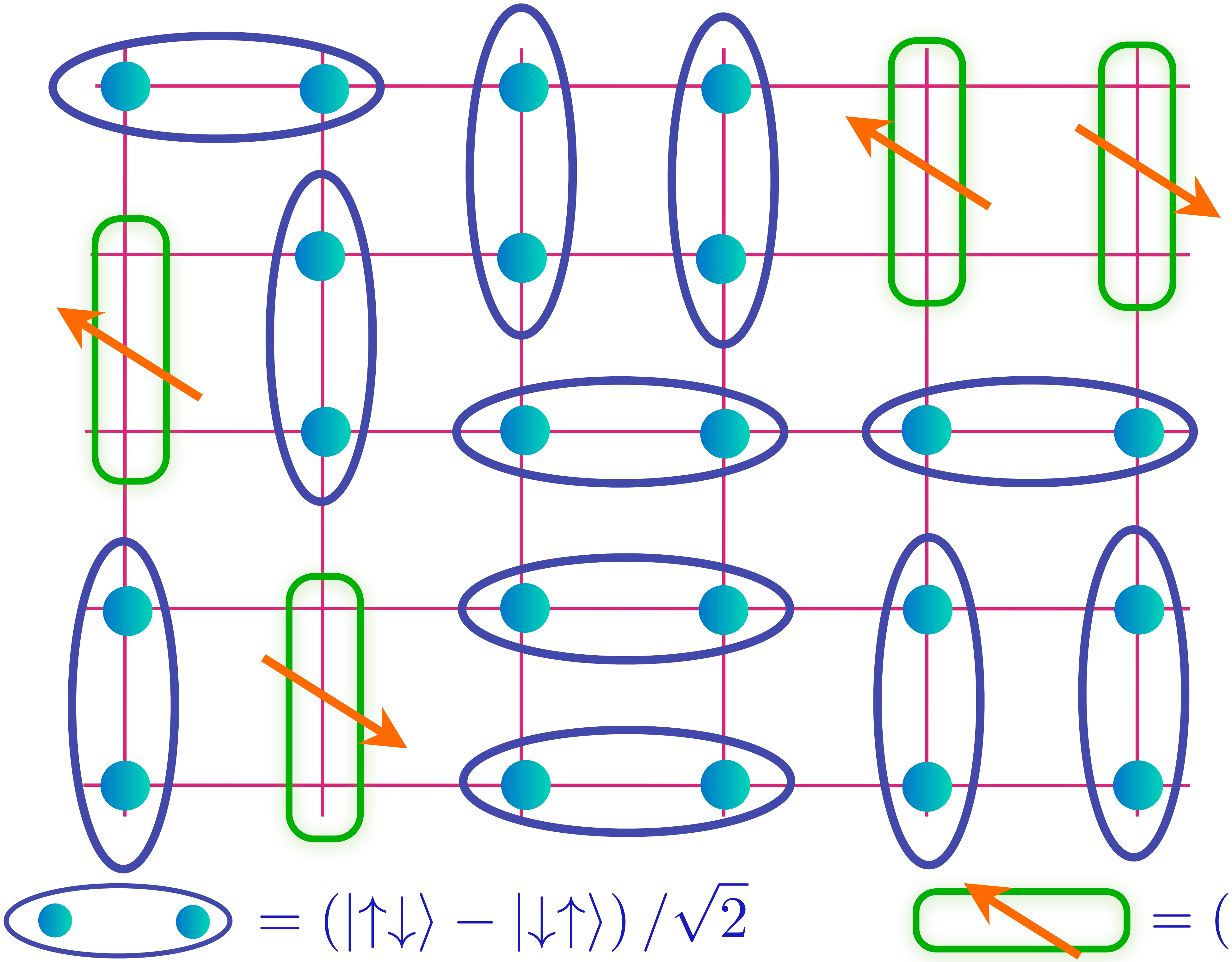

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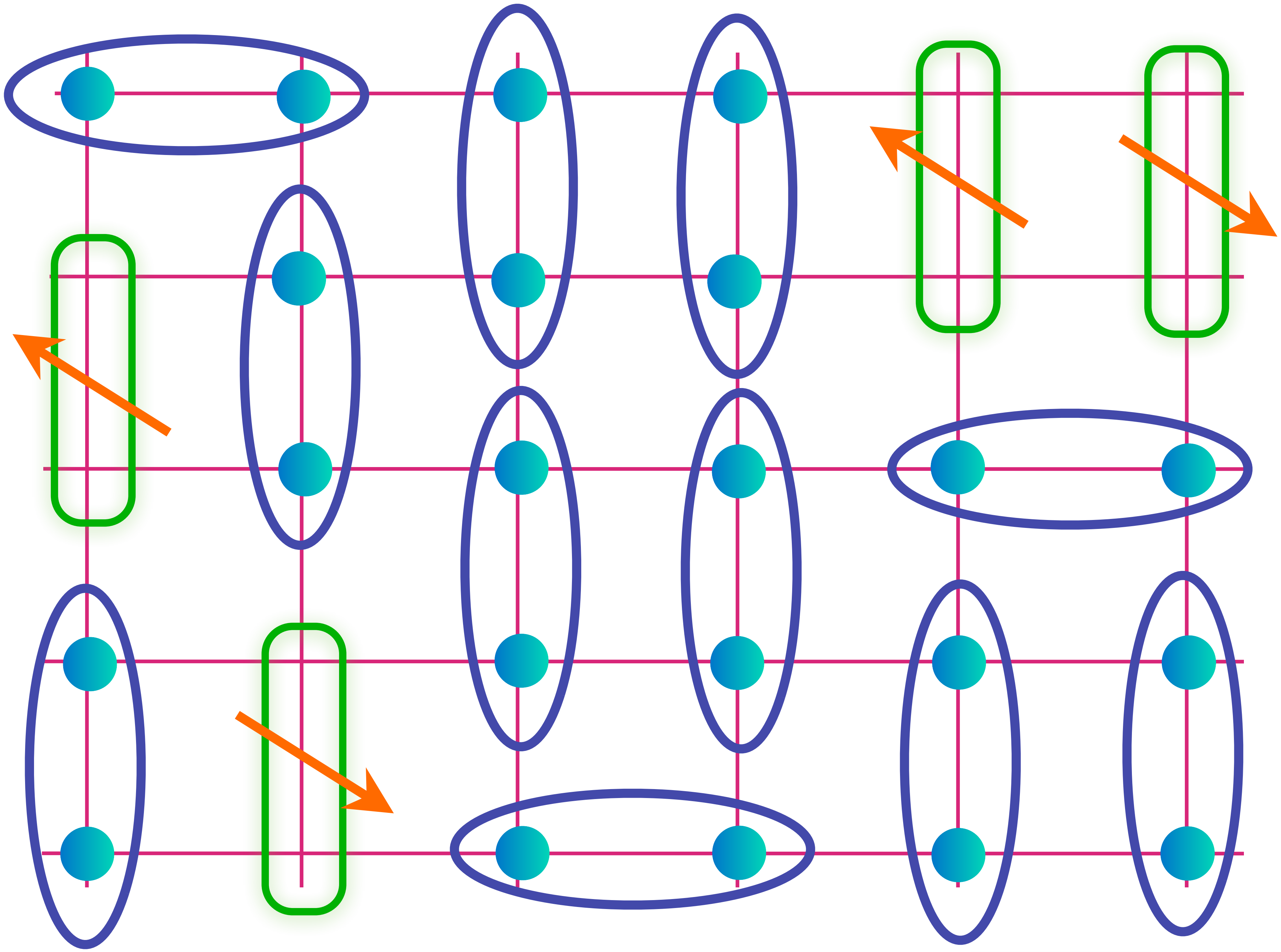

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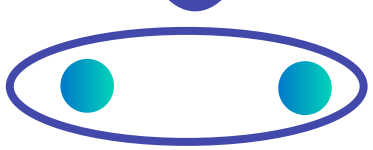
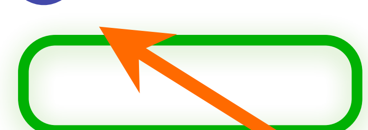
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

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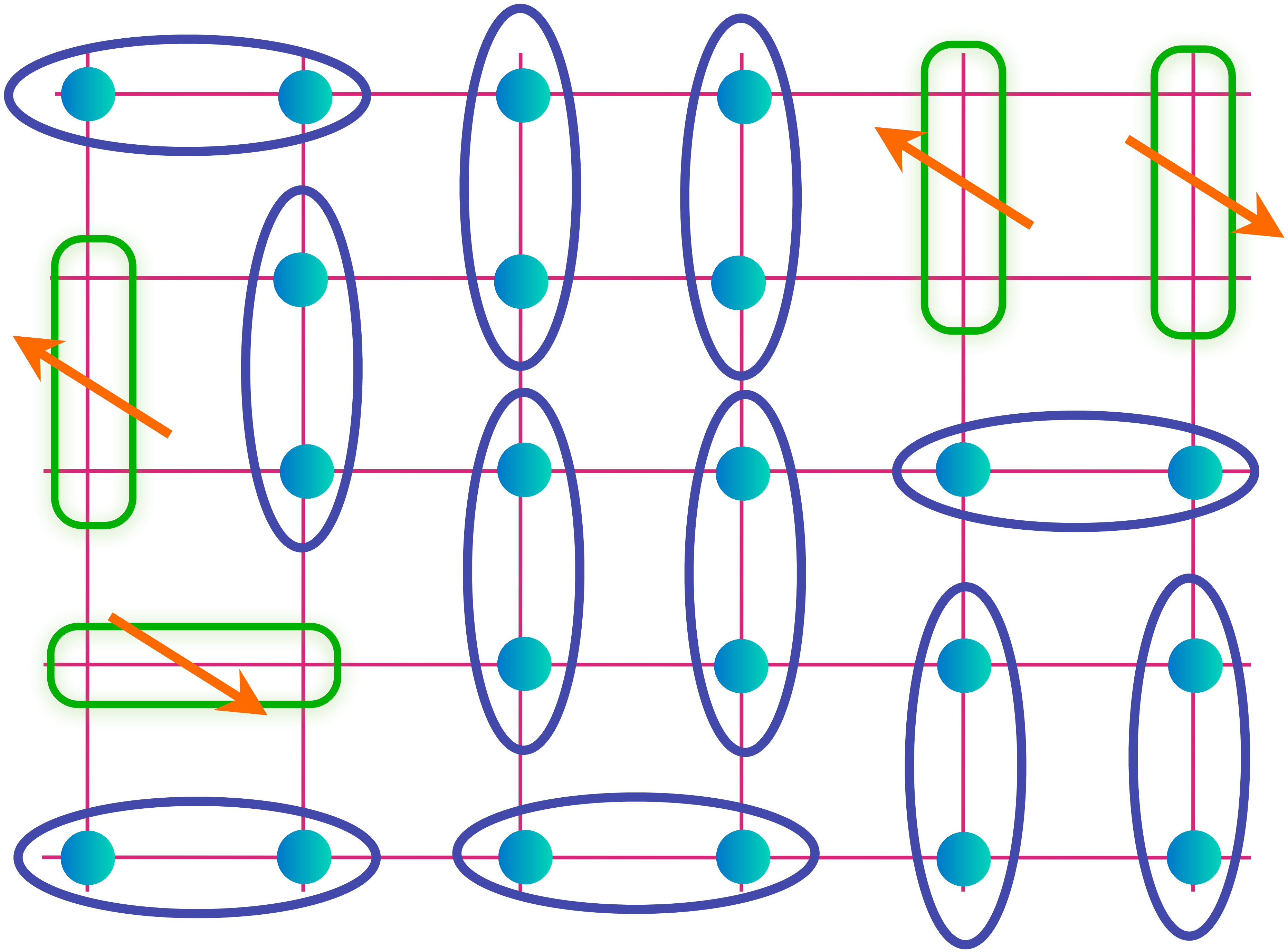

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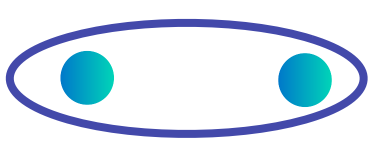
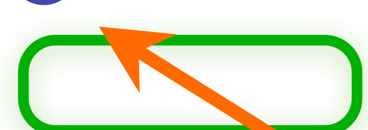
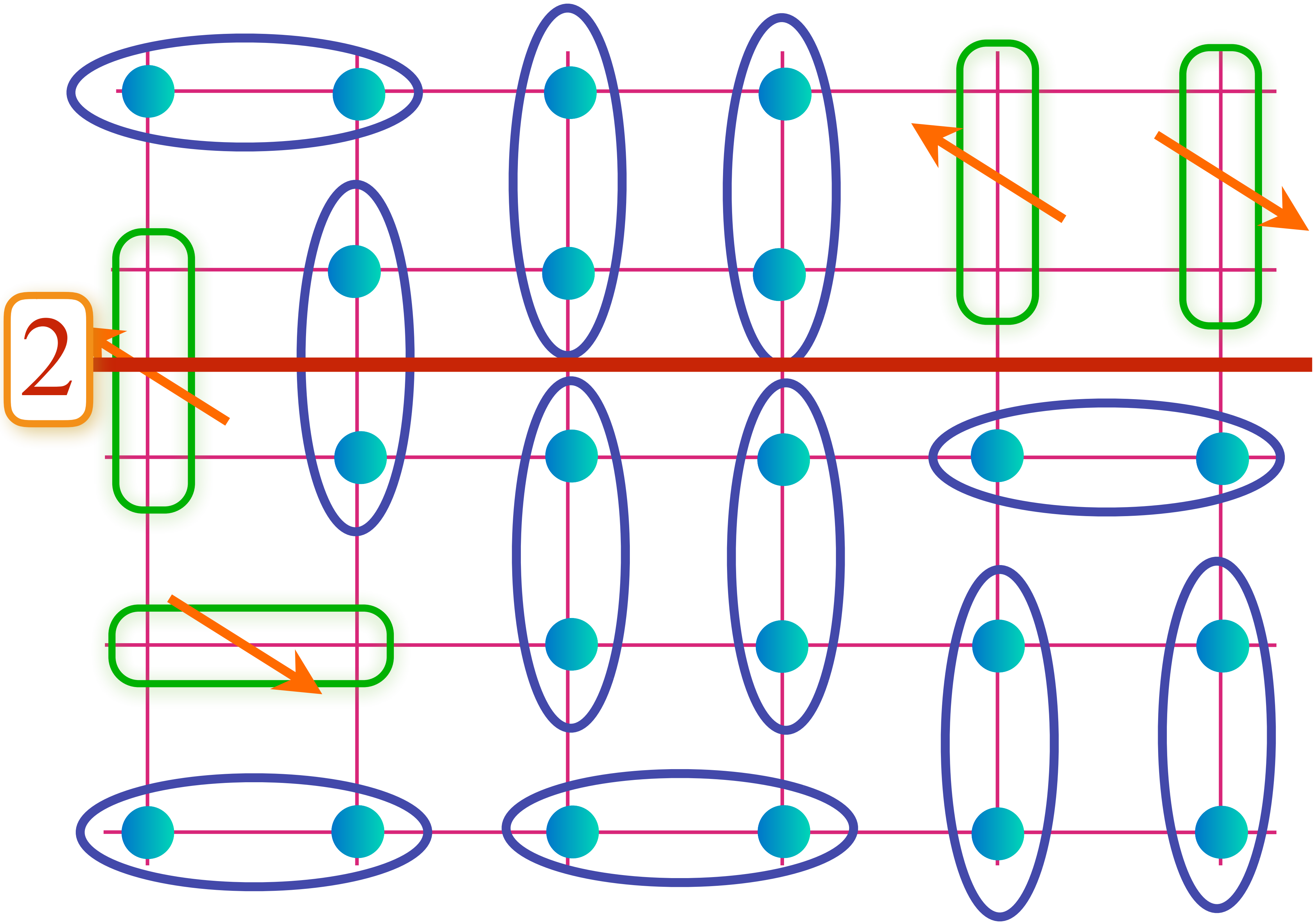
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*



The FL* state also has topological order:

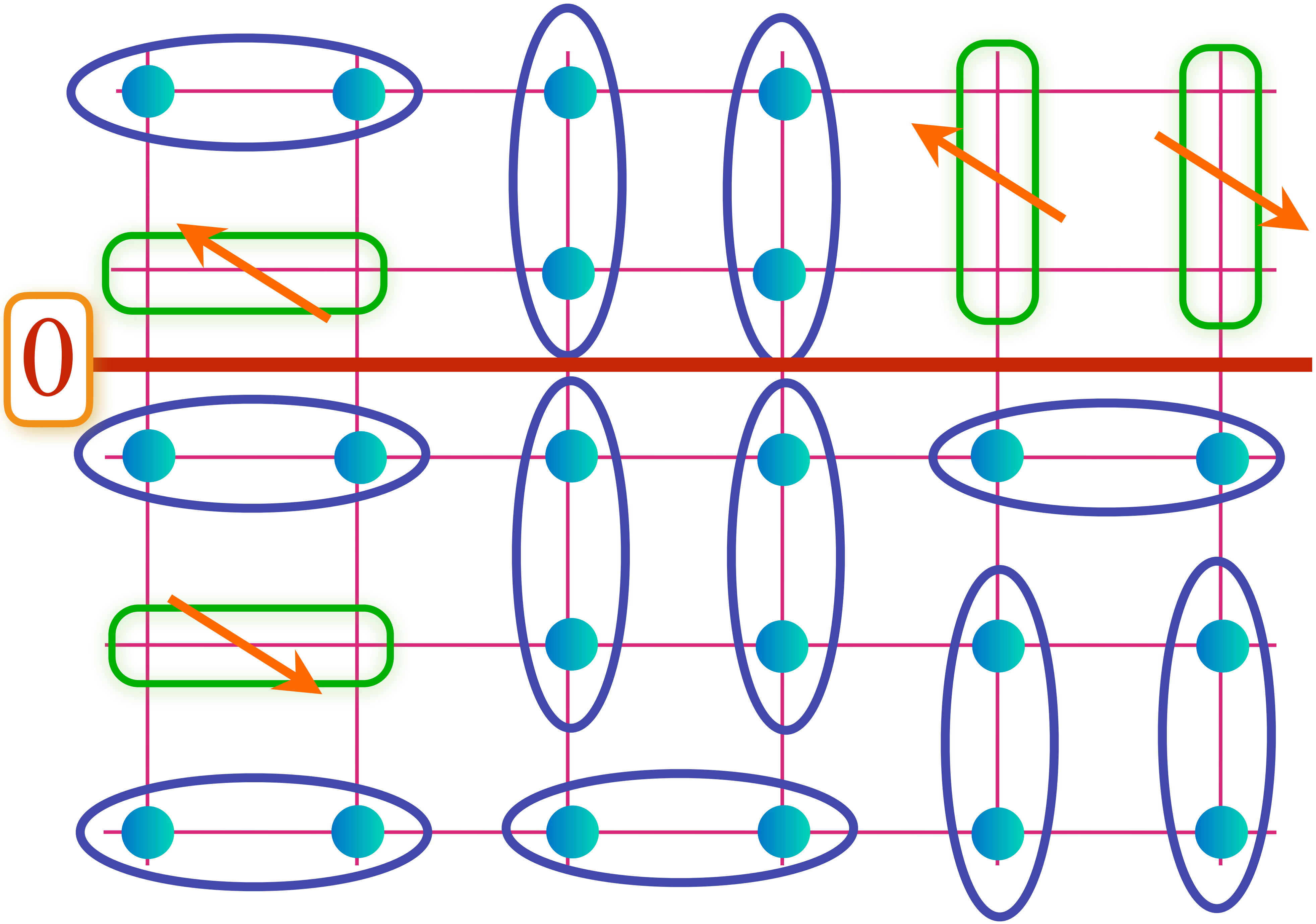
the number of dimers crossing red line is conserved modulo 2

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

$$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$$

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

FL*



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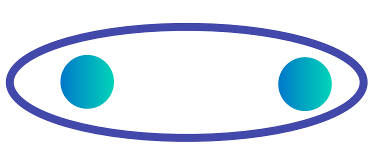
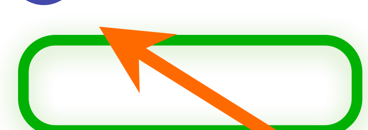
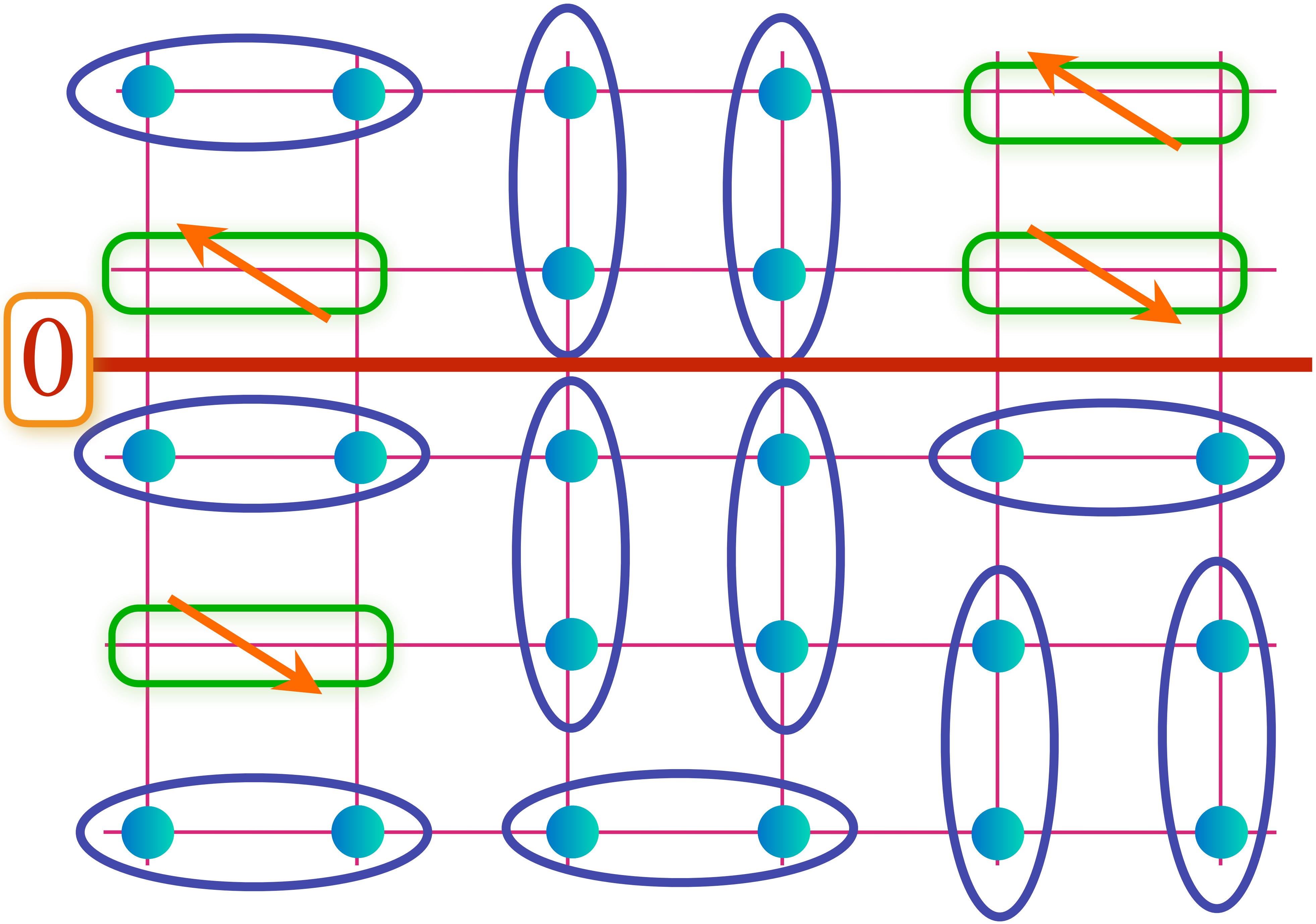
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

FL*



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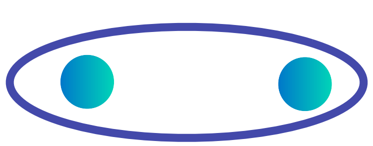
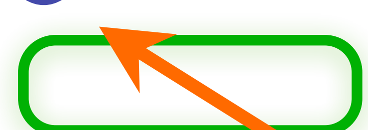
 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$

 $= (|\uparrow \circ\rangle + |\circ \uparrow\rangle) / \sqrt{2}$

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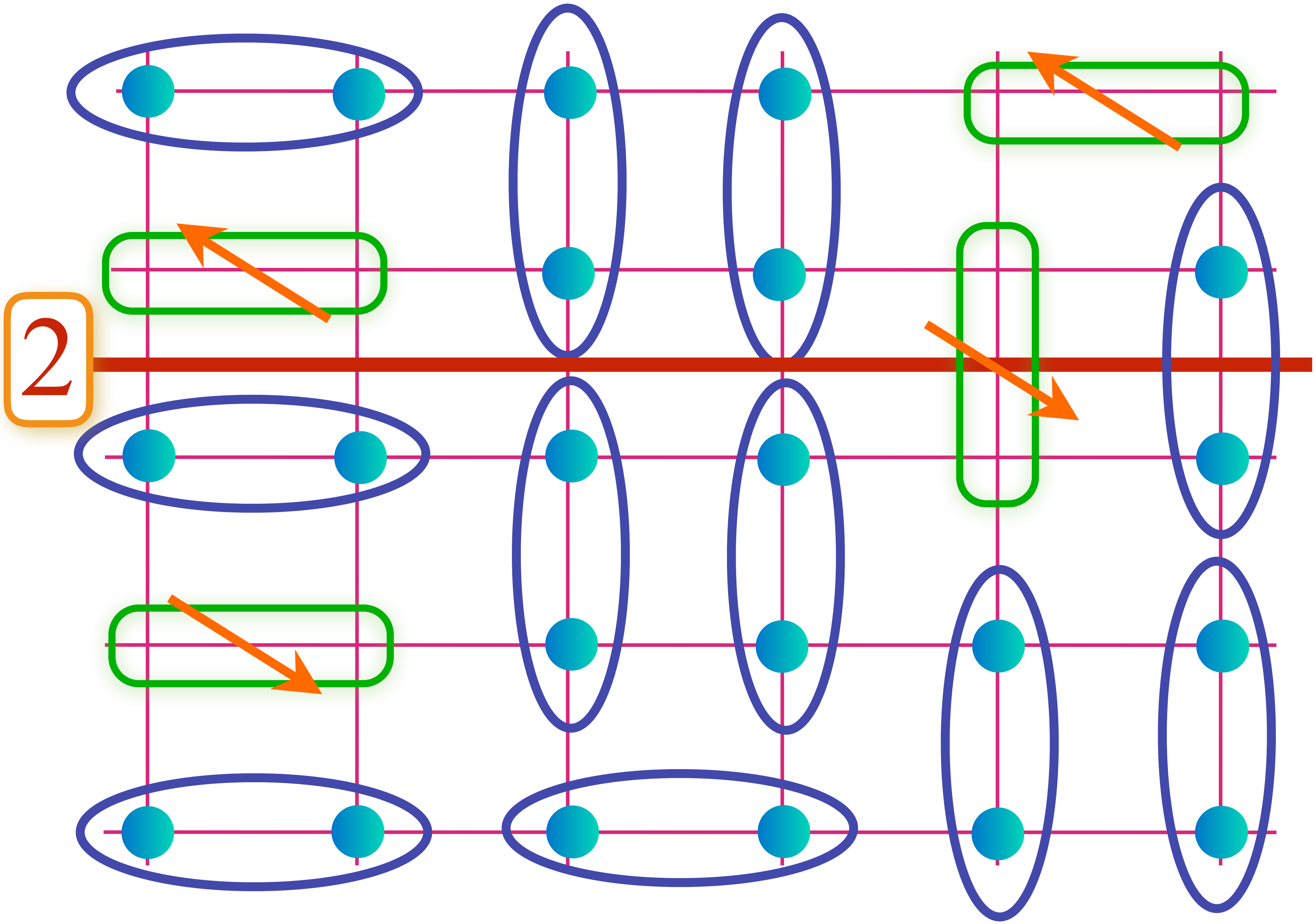

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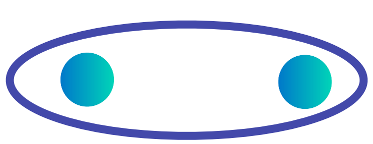
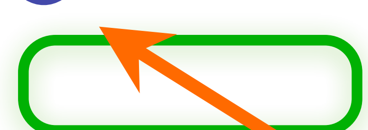
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

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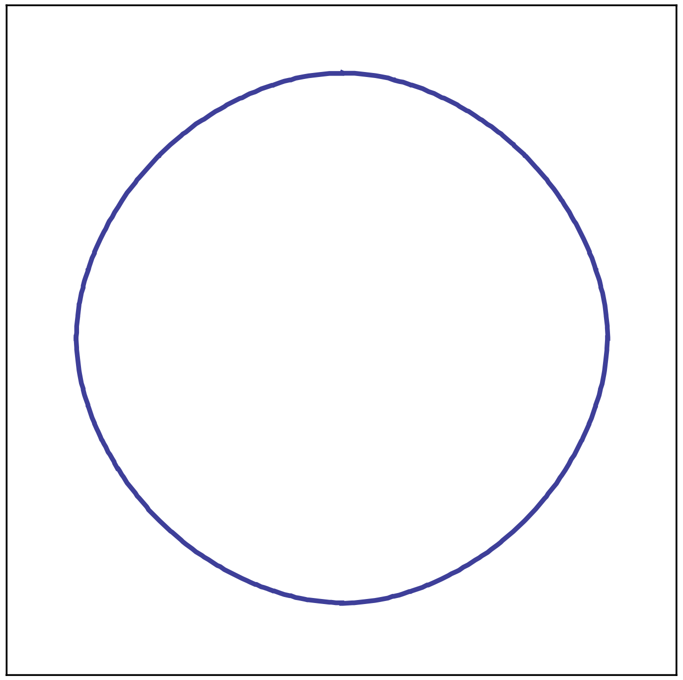
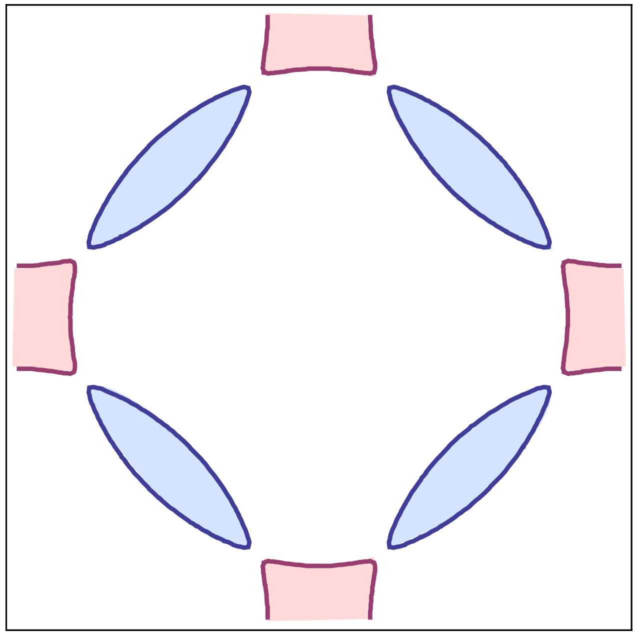
Separating onset of antiferromagnetism and Fermi surface reconstruction



$\langle \vec{\Phi} \rangle = 0$

Metal with "large" Fermi surface of size $1+p$



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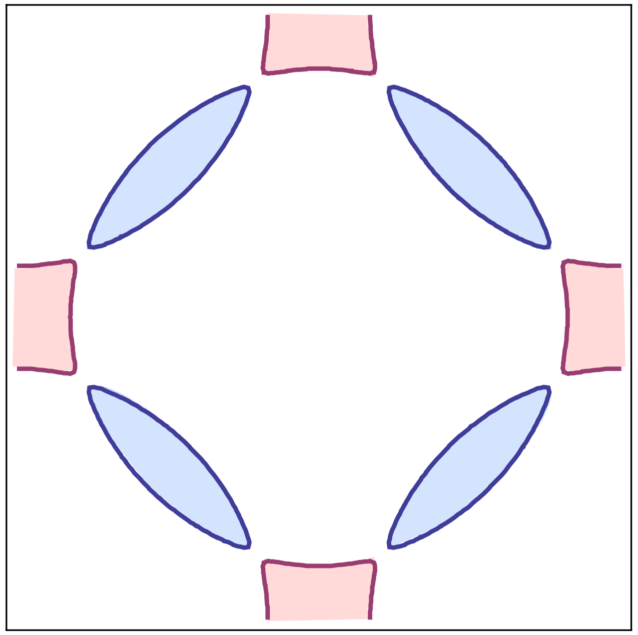
FL* Metal with electron and/or hole pockets of size p, **topological order** but no antiferromagnetism



$\langle \vec{\Phi} \rangle \neq 0$

Metal with electron and hole pockets of size p

p

Topological quantum phase transition

S. Sachdev, H. D. Scammell, M. S. Scheurer, and G. Tarnopolsky, PRB **99**, 054516 (2019)

1. SU(2) gauge theory of fluctuating antiferromagnetism in metals

2. Electron-doped cuprates

Higgs phase with topological order: Fermi surface reconstruction without translational symmetry breaking

Transforming to a rotating reference frame

We can (exactly) transform the Hubbard model to the "spin-fermion" model:
electrons $c_{i\alpha}$ on the square lattice with dispersion

$$\begin{aligned}
\mathcal{H}_c &= -\sum_{i,\rho} t_\rho \left(c^\dagger_{i,\alpha} c_{i+\boldsymbol{v}_\rho,\alpha} + c^\dagger_{i+\boldsymbol{v}_\rho,\alpha} c_{i,\alpha} \right) \\
&\quad -\mu \sum_i c^\dagger_{i,\alpha} c_{i,\alpha} + \mathcal{H}_{\rm int}
\end{aligned}$$

are coupled to a magnetic moment order parameter $\Phi^a(i)$, $a = x, y, z$

$$\begin{aligned}
\mathcal{H}_{\rm int} &= -\lambda \sum_i \Phi^a(i) c^\dagger_{i,\alpha} \sigma^a_{\alpha\beta} c_{i,\beta} + V(\Phi^a) \\
V(\Phi^a) &= s\Phi^a\Phi^a + u\Phi^a\Phi^a\Phi^b\Phi^b
\end{aligned}$$

Transforming to a rotating reference frame

For fluctuating antiferromagnetism (spin density waves (SDW)), we transform to a rotating reference frame using the SU(2) rotation R_i

$$\begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{pmatrix} = R_i \begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix},$$

in terms of fermionic "chargons" ψ_s and a **Higgs field** $H^a(i)$

$$\sigma^a \Phi^a(i) = R_i \, \sigma^a H^a(i) \, R_i^\dagger$$

The Higgs field is the SDW order in the rotating reference frame.

Transforming to a rotating reference frame

The simplest effective Hamiltonian for the fermionic chargons is the same as that for the electrons, with the SDW order replaced by the Higgs field.

$$\mathcal{H}_\psi = -\sum_{i,\rho} t_\rho \left(\psi_{i,s}^\dagger \psi_{i+\boldsymbol{v}_\rho,s} + \psi_{i+\boldsymbol{v}_\rho,s}^\dagger \psi_{i,s} \right) - \mu \sum_i \psi_{i,s}^\dagger \psi_{i,s} + \mathcal{H}_{\rm int}$$

$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i H^a(i) \psi_{i,s}^\dagger \sigma_{ss'}^a \psi_{i,s'} + V(H^a)$$

IF we can transform to a rotating reference frame in which $H^a(i) =$ a constant independent of i and time, **THEN** the ψ fermions in the presence of (fluctuating) SDW SRO will inherit the small Fermi surfaces of the electrons in the presence of SDW LRO.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Gauge theory of fluctuating antiferromagnetism

Field	Symbol	Statistics	$SU(2)_{\rm gauge}$	$SU(2)_{\rm spin}$	$U(1)_{\rm e.m.charge}$
Electron	c	fermion	$\mathbf{1}$	$\mathbf{2}$	-1
AF order	Φ	boson	$\mathbf{1}$	$\mathbf{3}$	0
Chargon	ψ	fermion	$\mathbf{2}$	$\mathbf{1}$	-1
Spinon	R or z	boson	$\bar{\mathbf{2}}$	$\mathbf{2}$	0
Higgs	H	boson	$\mathbf{3}$	$\mathbf{1}$	0

Note that the transformation to a rotating reference frame is ambiguous up to a SU(2) gauge transformation, V_i

$$\begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix} \rightarrow V_i \begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix}$$

$$R_i \rightarrow R_i V_i^\dagger$$

$$\sigma^a H^a(i) \rightarrow V_i \, \sigma^b H^b(i) \, V_i^\dagger .$$

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SU(2) gauge theory: fractionalize the SDW order parameter into the Higgs field (H) and the spinons (R); fractionalize the electron (c) into chargons (ψ) and spinons (R). When the Higgs field is condensed, we obtain metallic states with **topological order and small Fermi surfaces**.

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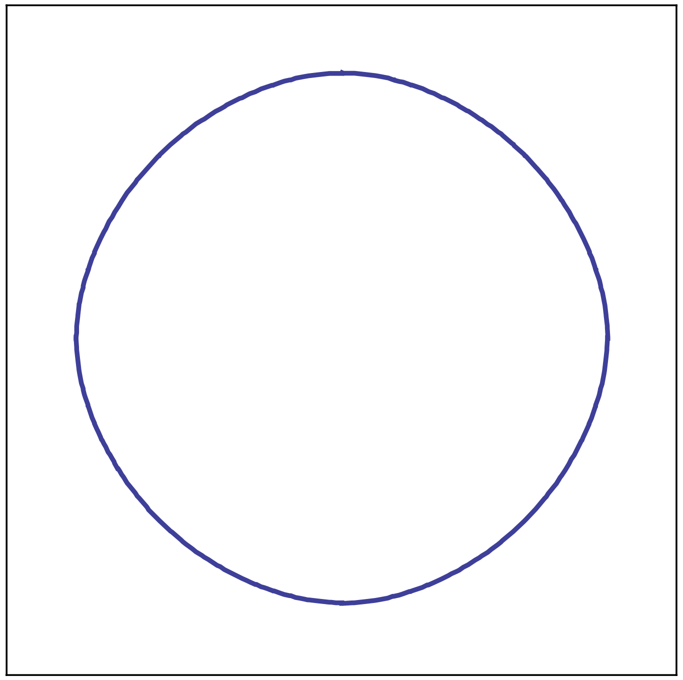
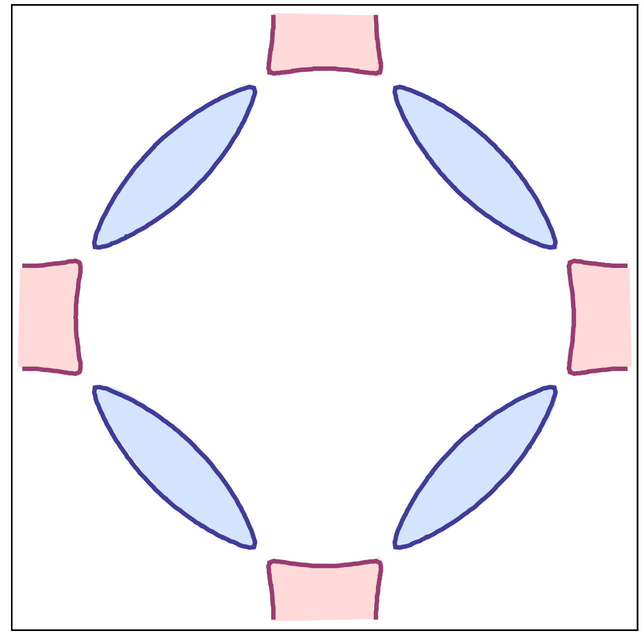
Separating onset of antiferromagnetism and Fermi surface reconstruction



$\langle \vec{\Phi} \rangle = 0$

Metal with "large" Fermi surface of size $1+p$



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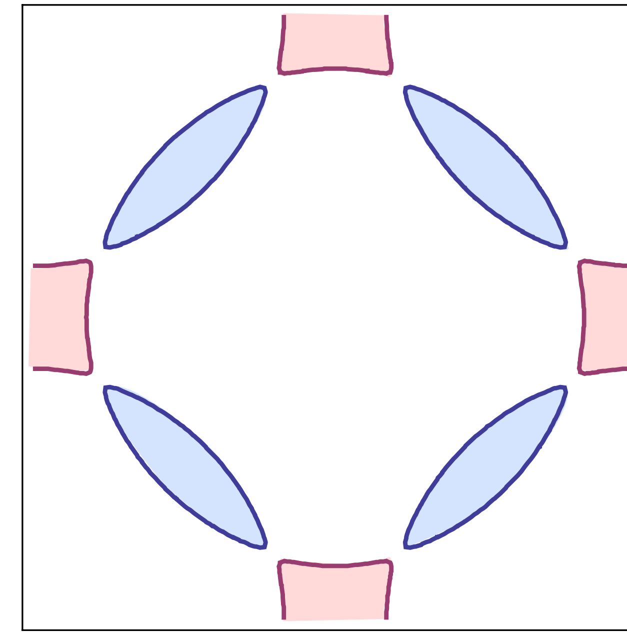

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Topological quantum phase transition

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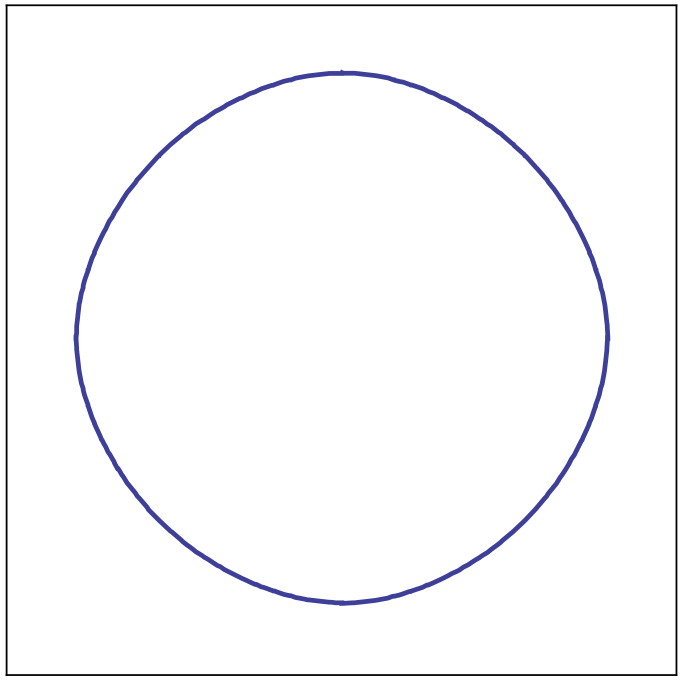

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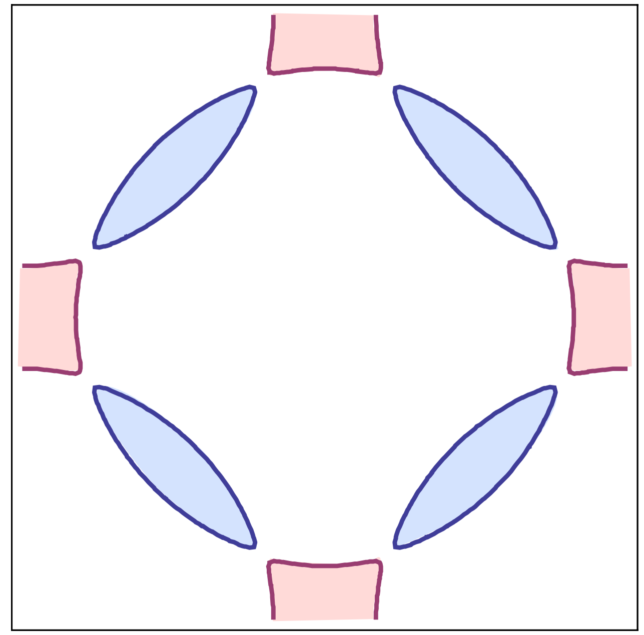
Confinement.
Metal with “large” Fermi surface of size $1+p$



$\langle H \rangle \neq 0; \langle R \rangle = 0$

FL* Higgs phase
with electron and/or hole pockets of size p,
topological order
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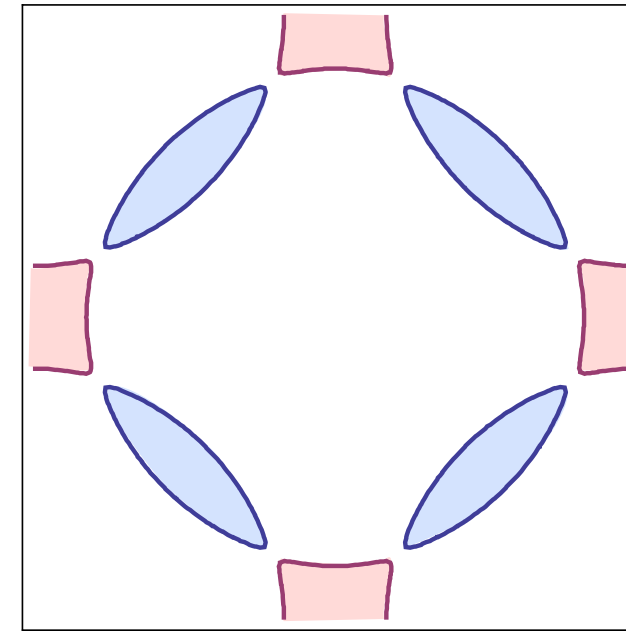

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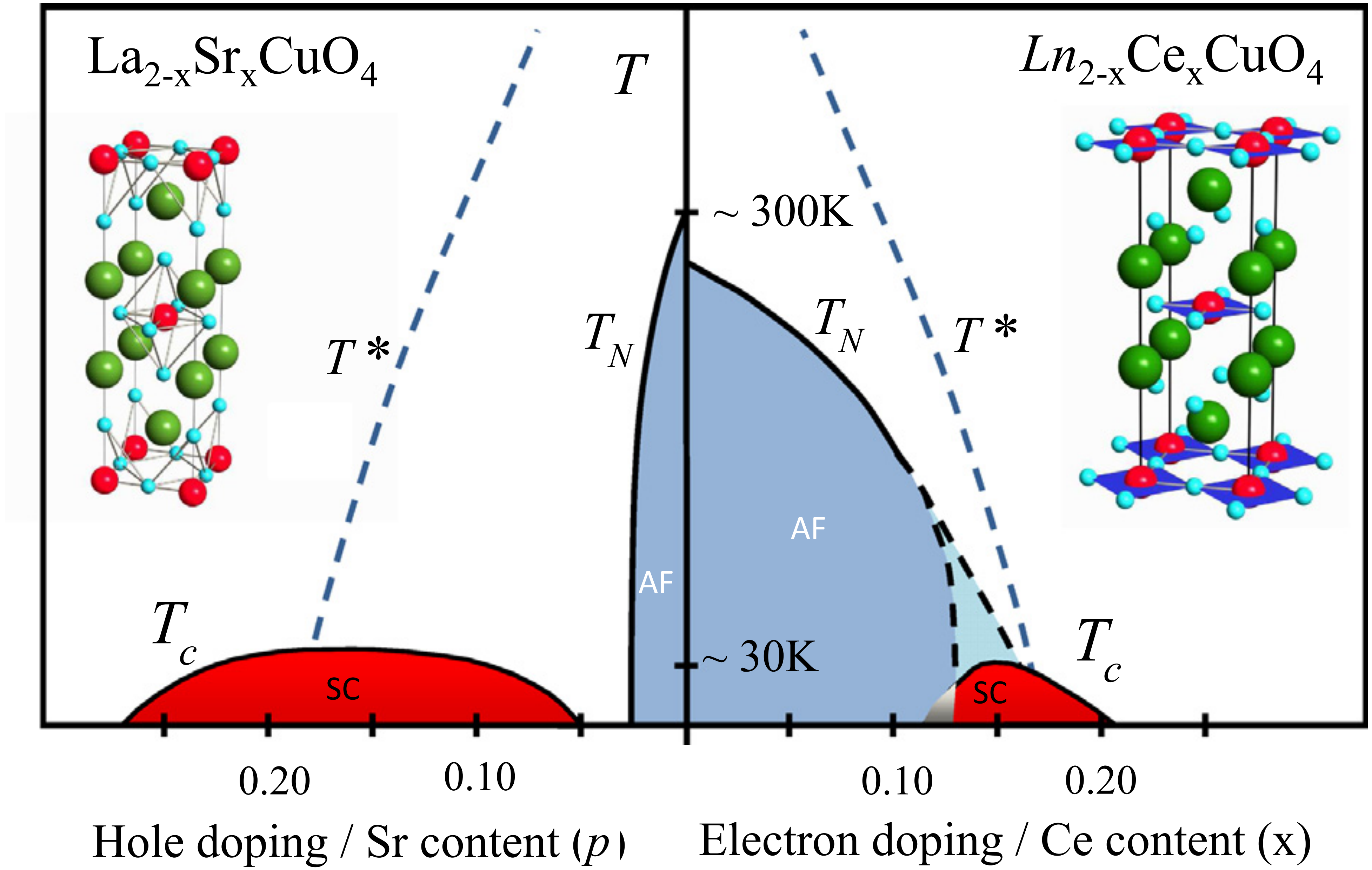
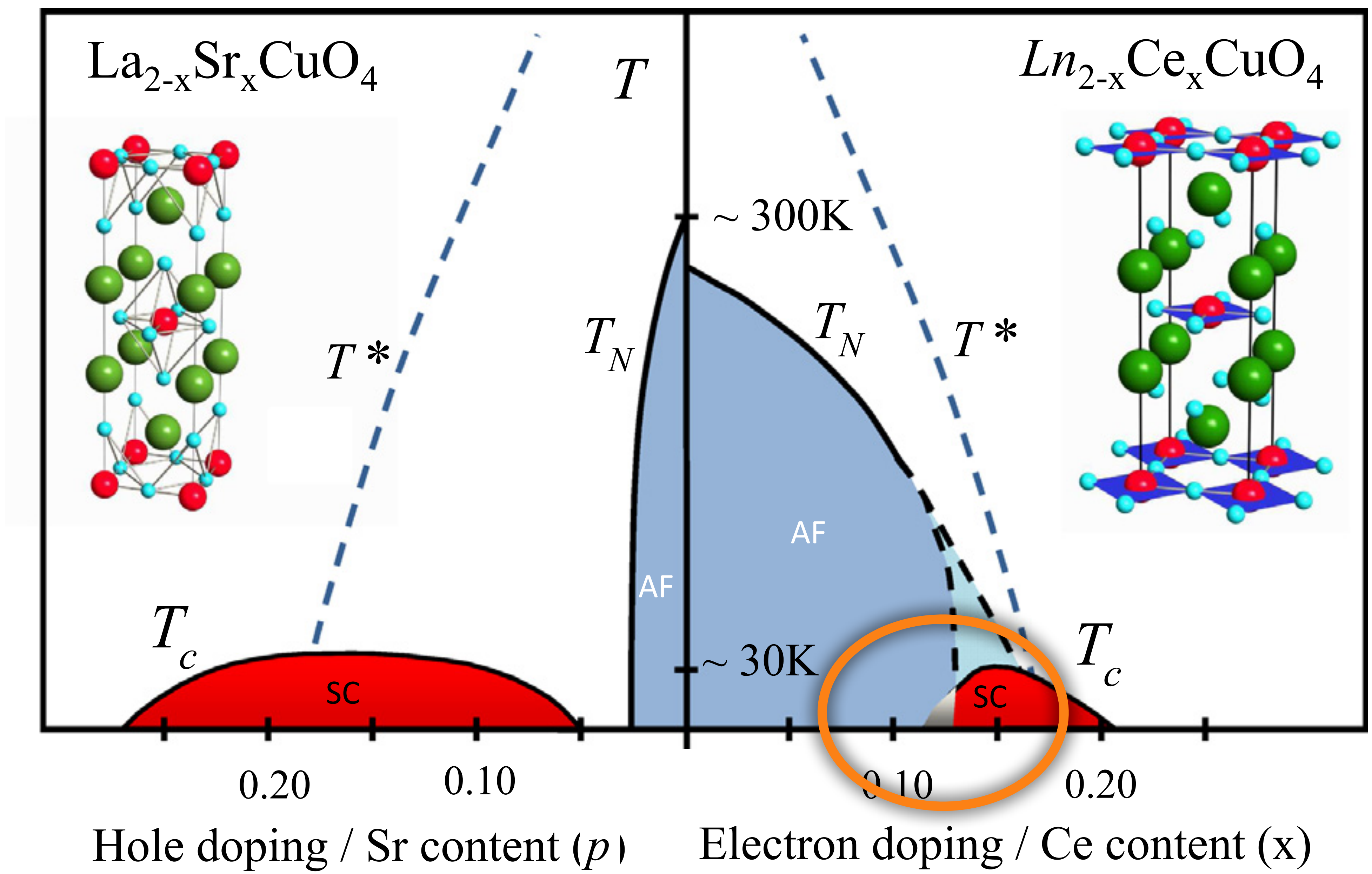

$La_{2-x}Sr_xCuO_4$
$Ln_{2-x}Ce_xCuO_4$
T
~ 300K
~ 30K
T_N
T_N
T^*
T^*
AF
AF
T_c
T_c
SC
SC
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)



La2-xSrxCuO4
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T
~ 300K
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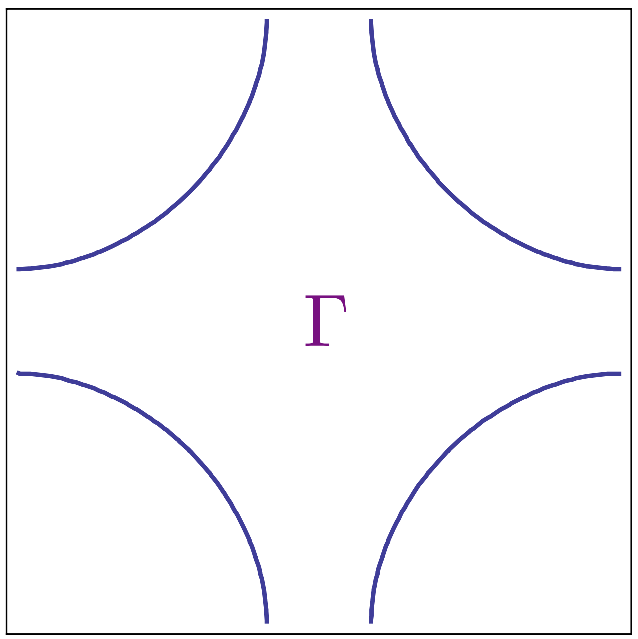
Hole doping / Sr content (p)
Electron doping / Ce content (x)

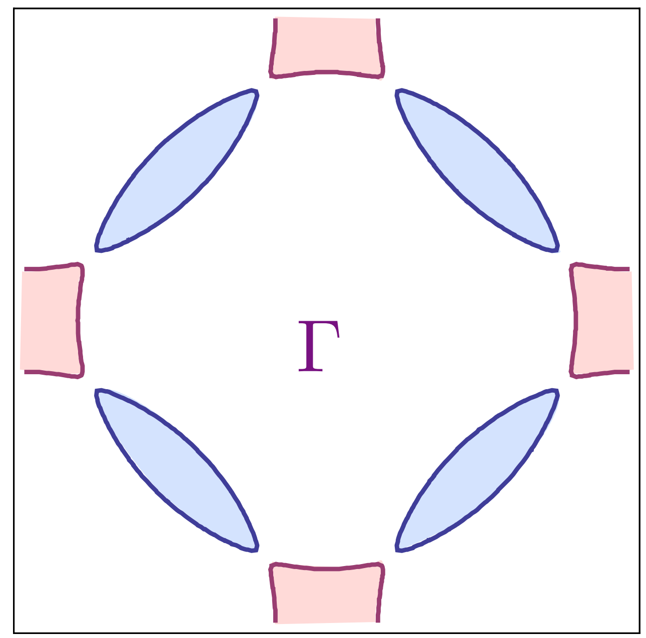
Square lattice Hubbard model with hole doping

$\langle \vec{\Phi} \rangle = 0$



Metal with "large" Fermi surface

$\langle \vec{\Phi} \rangle \neq 0$ and small



Metal with electron and hole pockets

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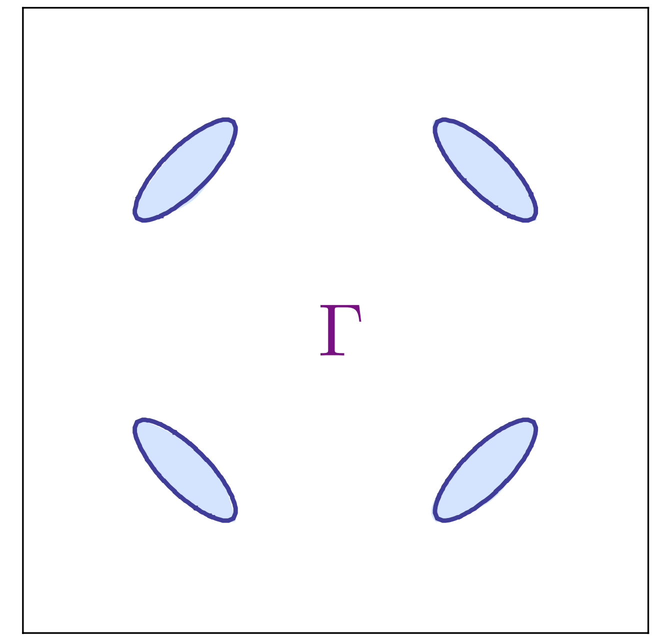

Metal with hole pockets

p

Square lattice Hubbard model with electron doping

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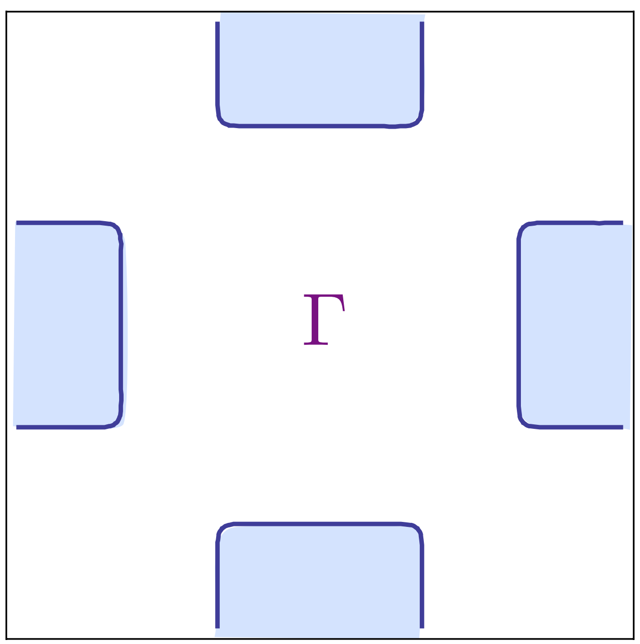

Metal with
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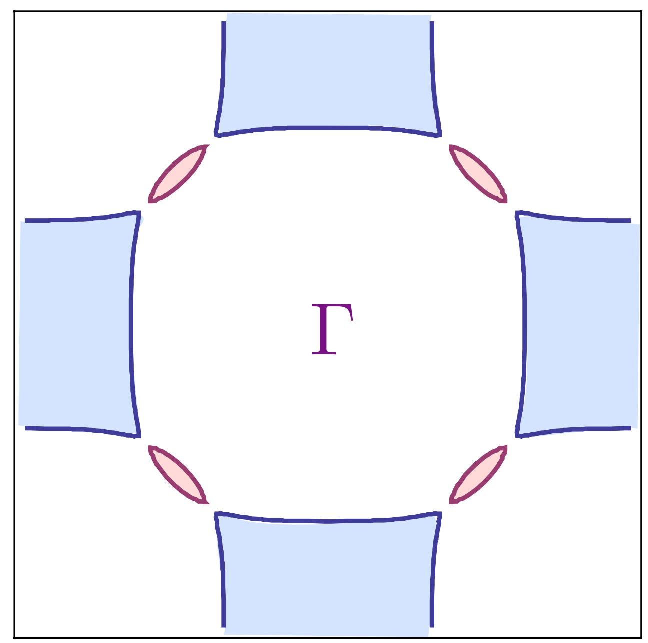

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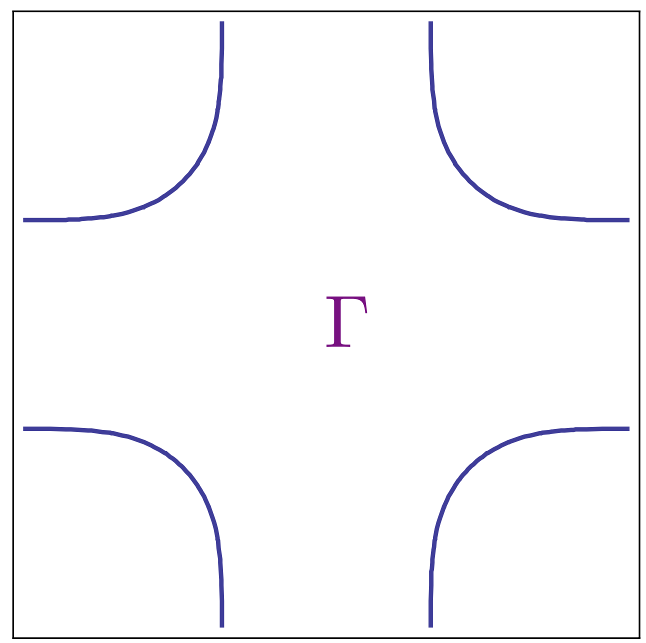
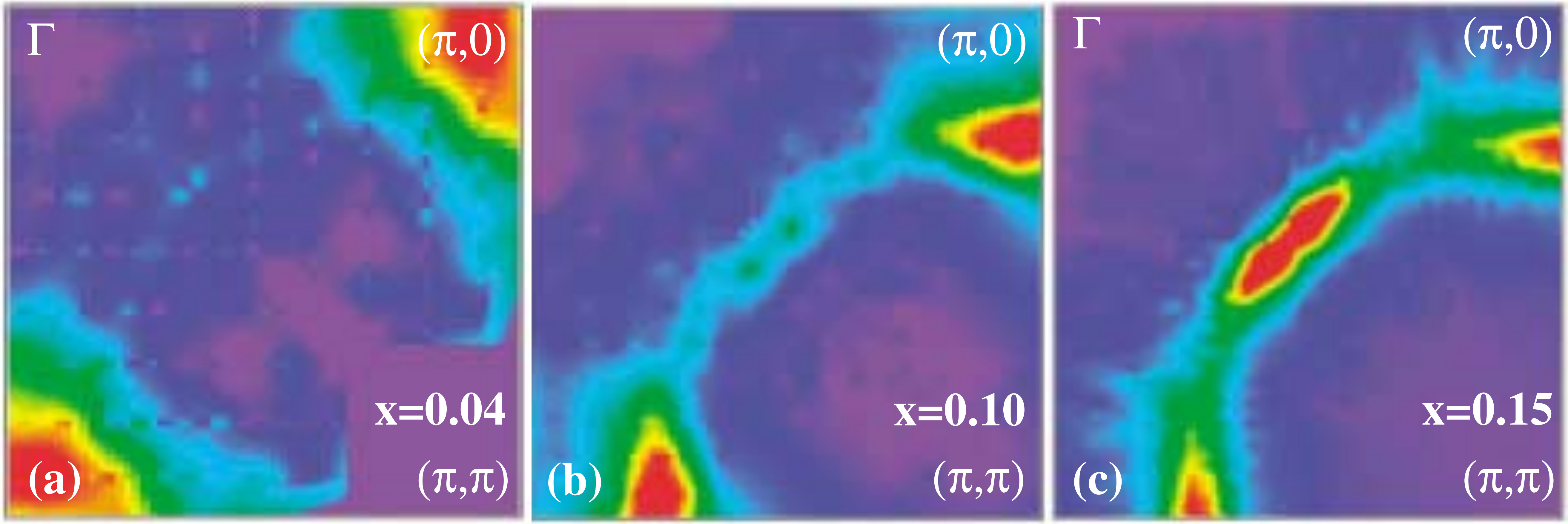


Metal with
“large” Fermi
surface

x

Electron doped cuprates

$Nd_{2-x}Ce_xCuO_{4\pm\delta}$



Doping Dependence of an n-Type Cuprate Superconductor Investigated by Angle-Resolved Photoemission Spectroscopy

N. P. Armitage, F. Ronning, D. H. Lu, C. Kim, A. Damascelli, K. M. Shen, D. L. Feng, H. Eisaki, Z.-X. Shen, P. K. Mang, N. Kaneko, M. Greven, Y. Onose, Y. Taguchi, and Y. Tokura

Phys. Rev. Lett. **88**, 257001 (2002)

PNAS 116, 3449 (2019)

Fermi surface reconstruction in electron-doped cuprates without antiferromagnetic long-range order

Junfeng He, C. R. Rotundu, M. S. Scheurer, Y. He, M. Hashimoto, K. Xu, Y. Wang, E. W. Huang, T. Jia, S.-D. Chen, B. Moritz, D.-H. Lu, Y. S. Lee, T. P. Devereaux and Z.-X. Shen

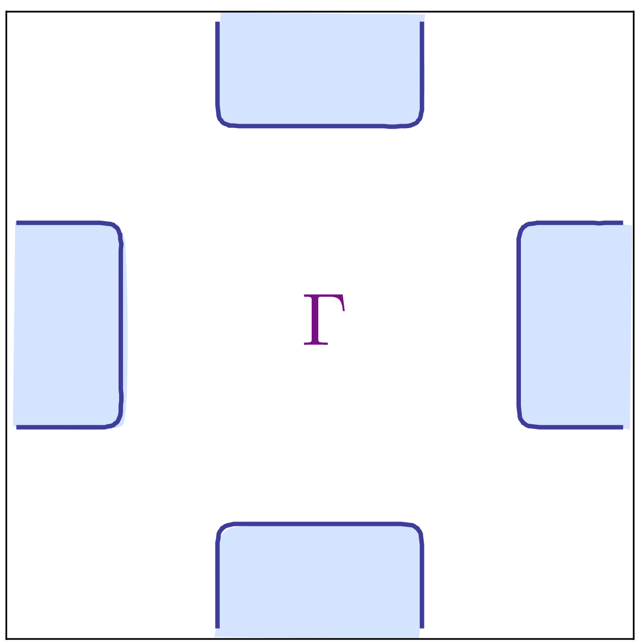
- New photoemission measurements at zero magnetic field show Fermi surfaces in quantitative agreement with quantum oscillation measurements.

- The energy gap between the electron and hole pockets collapses near $x = 0.17$ like an order parameter.

- "The totality of the data points to a mysterious order between $x = 0.14$ and $x = 0.17$, whose appearance favors the FS reconstruction and disappearance defines the quantum critical doping. A recent topological proposal provides an ansatz for its origin."

Square lattice Hubbard model with electron doping

$\langle \vec{\Phi} \rangle \neq 0$
and large



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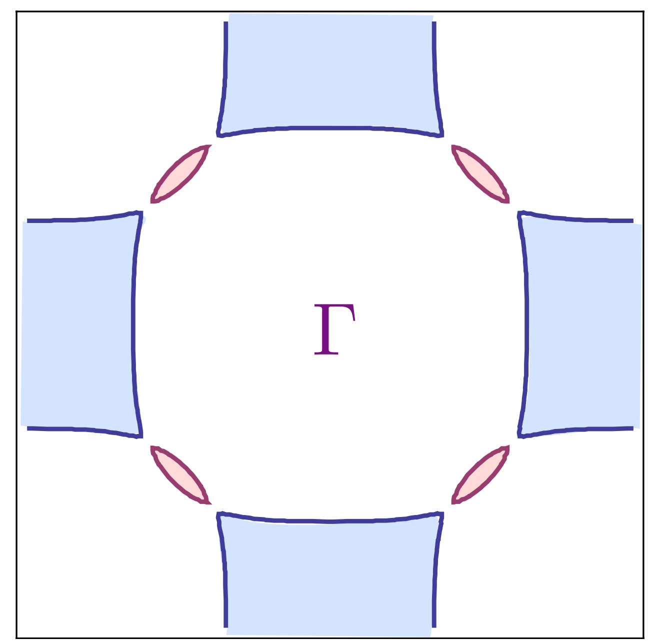

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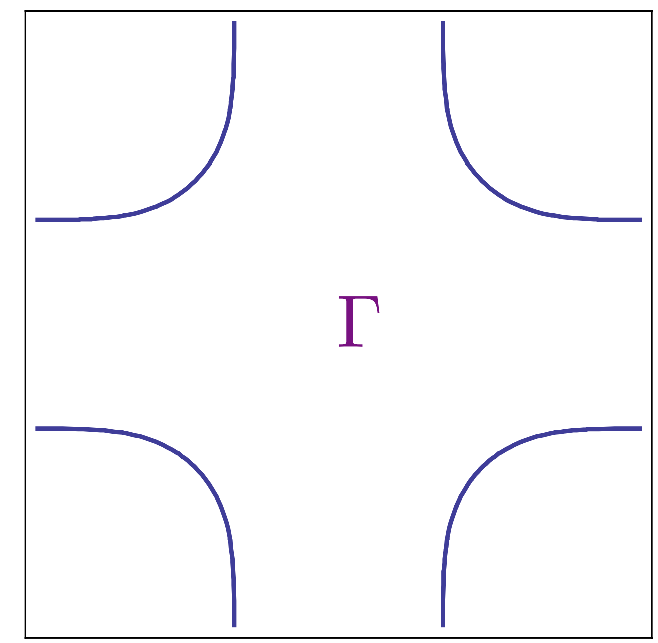

Metal with
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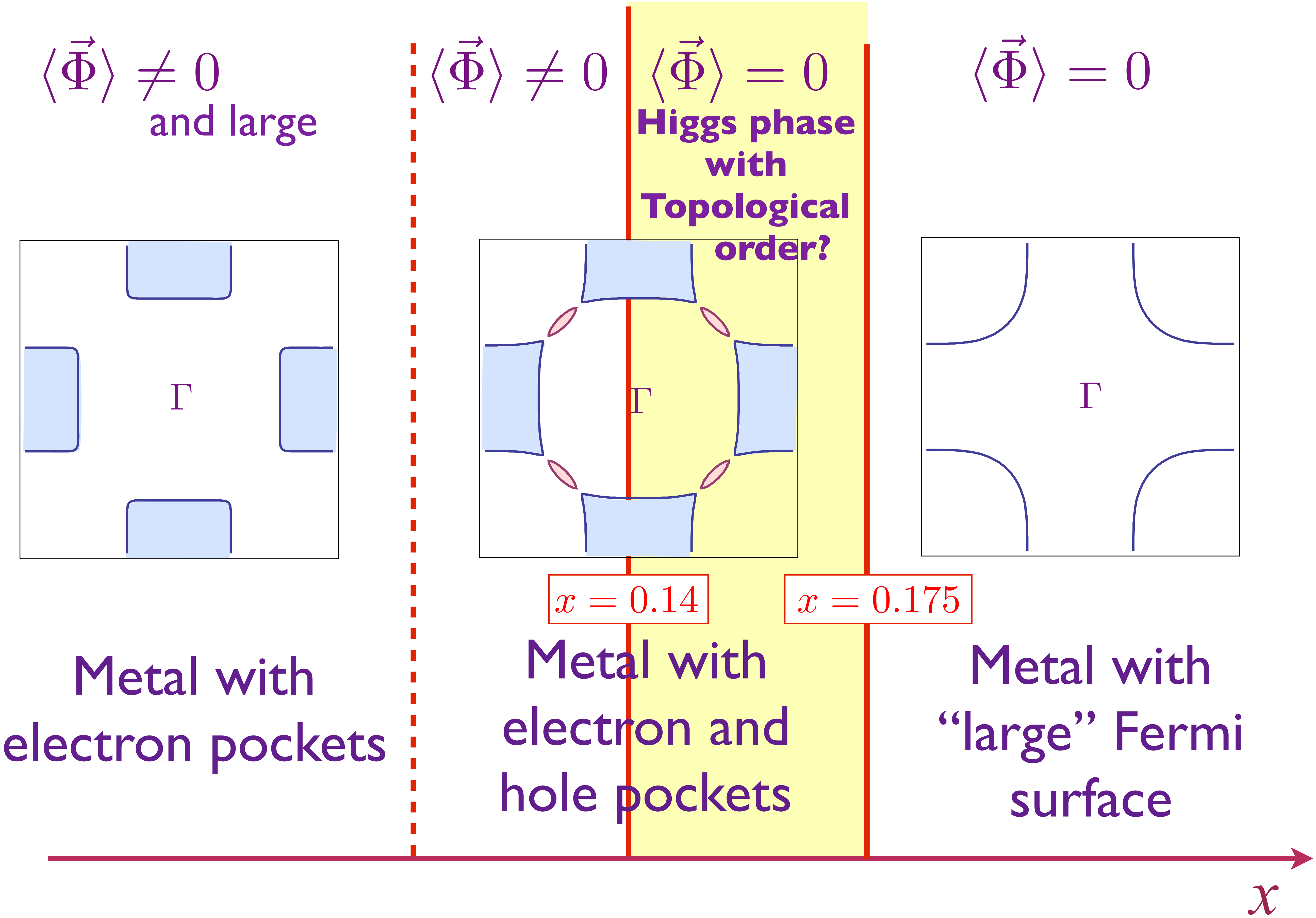

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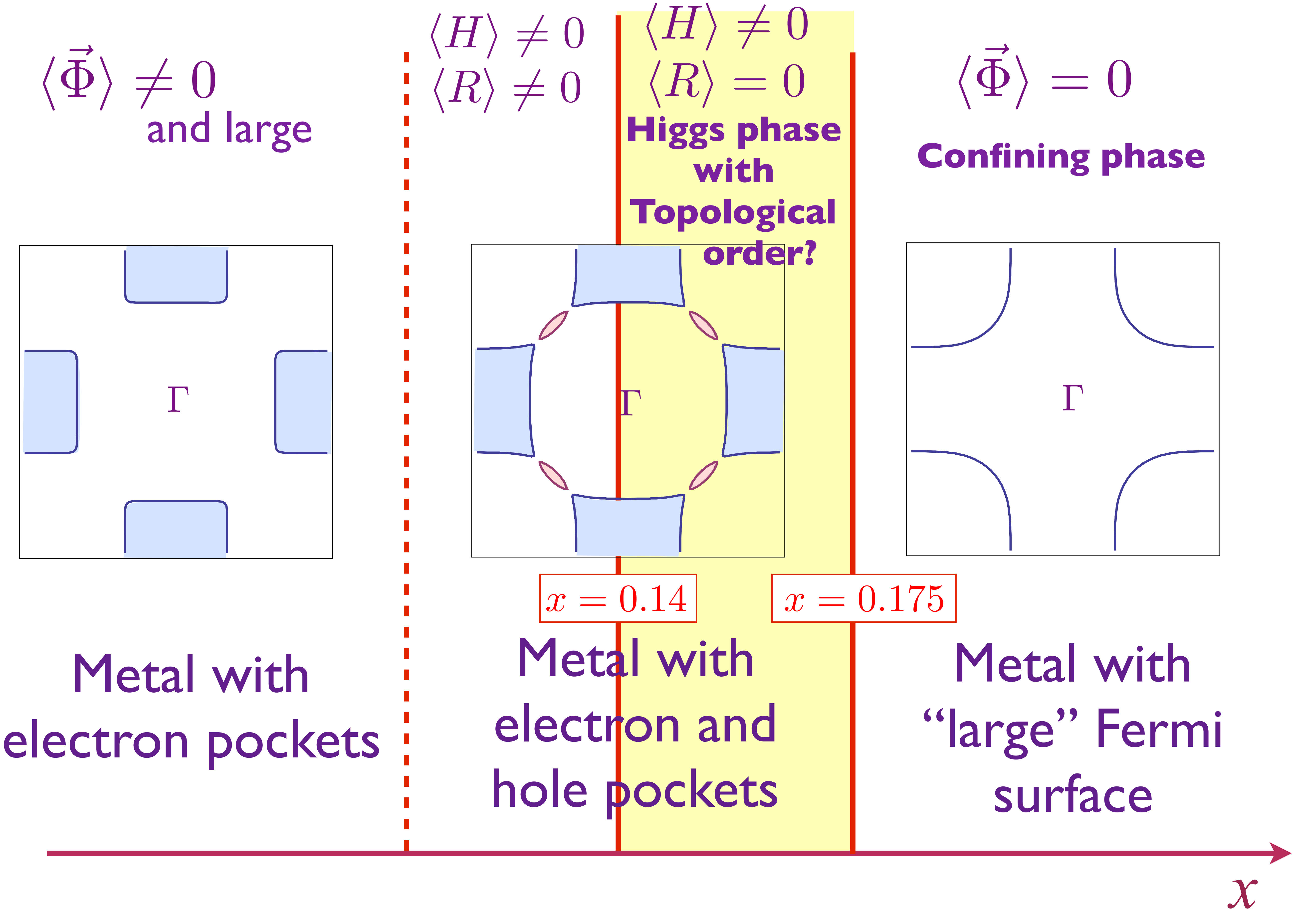

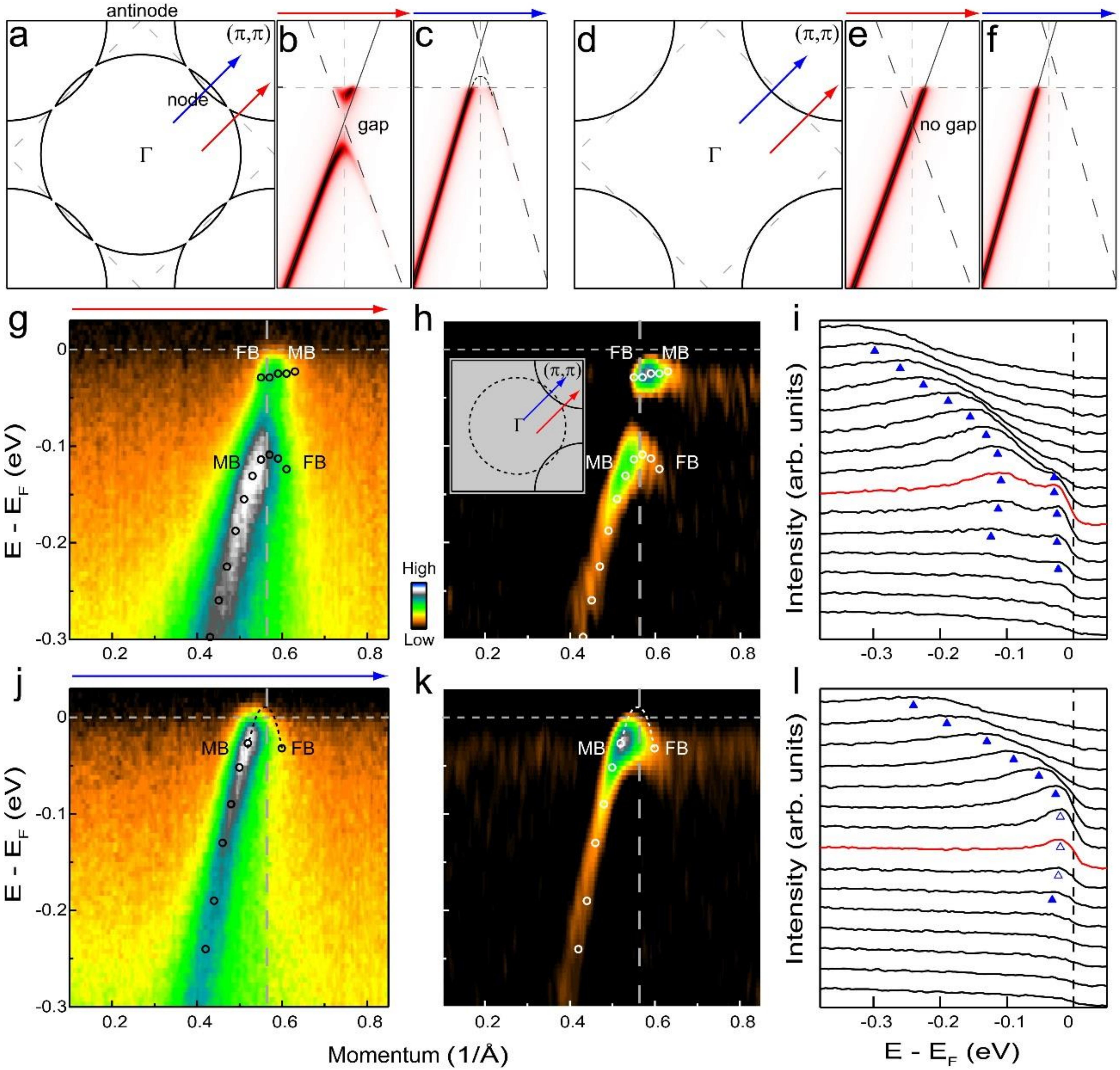

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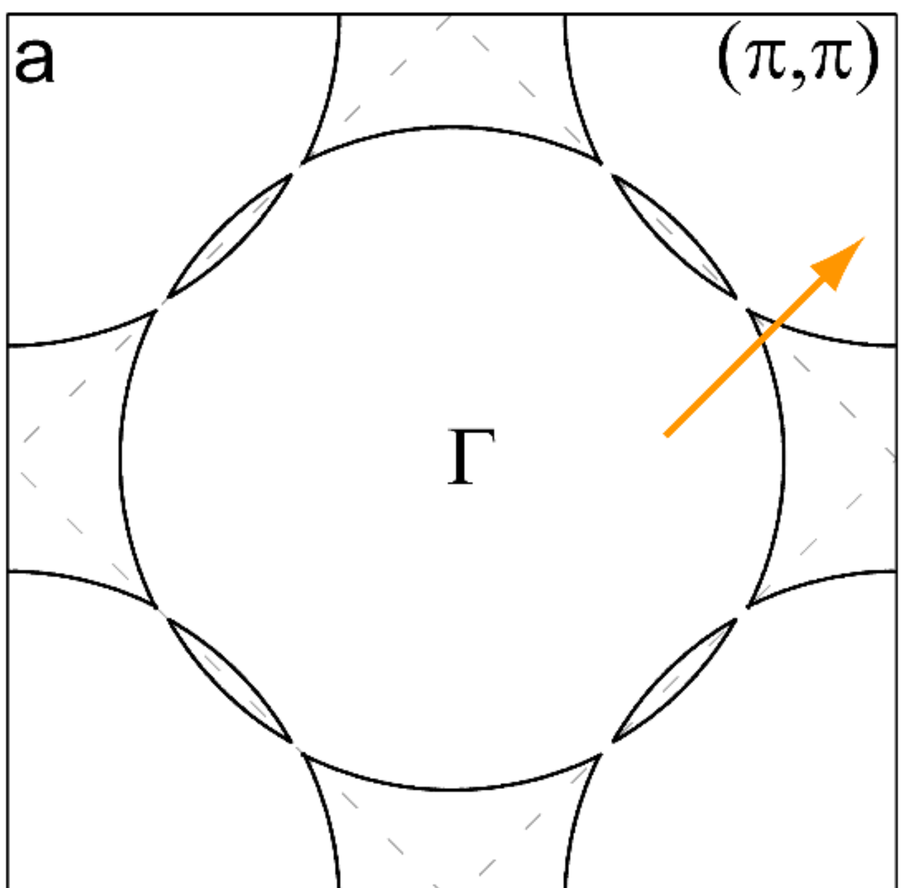
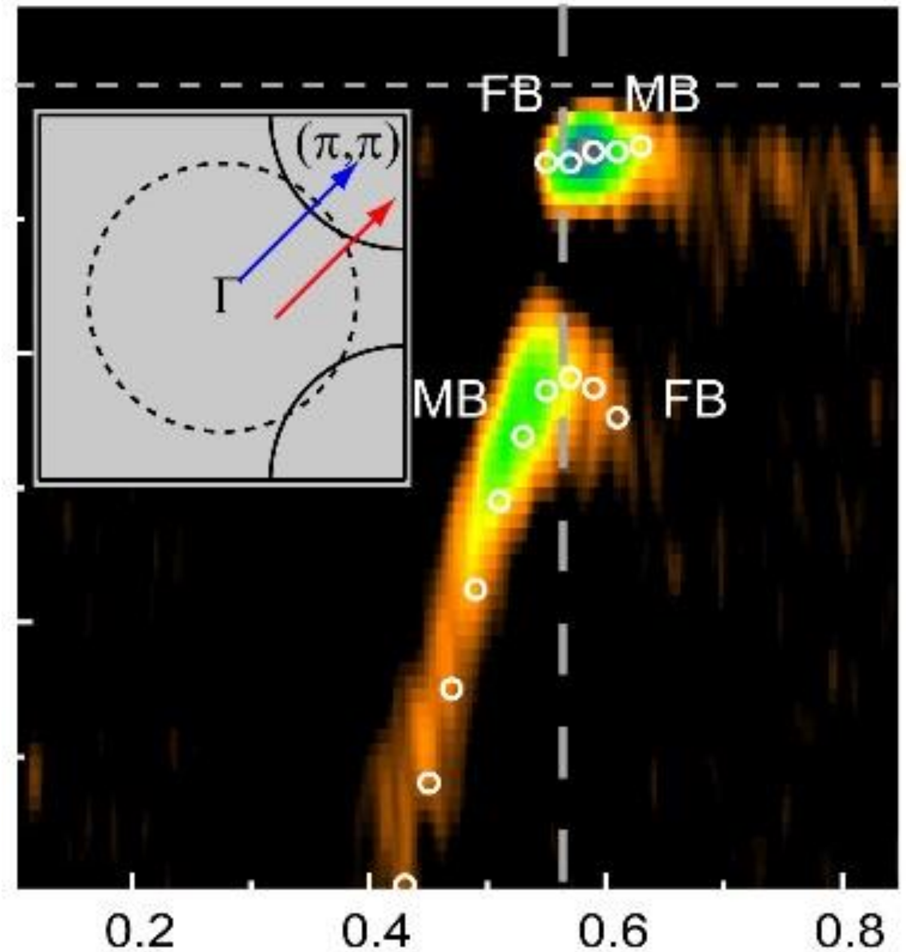
PNAS **116**,
3449 (2019)

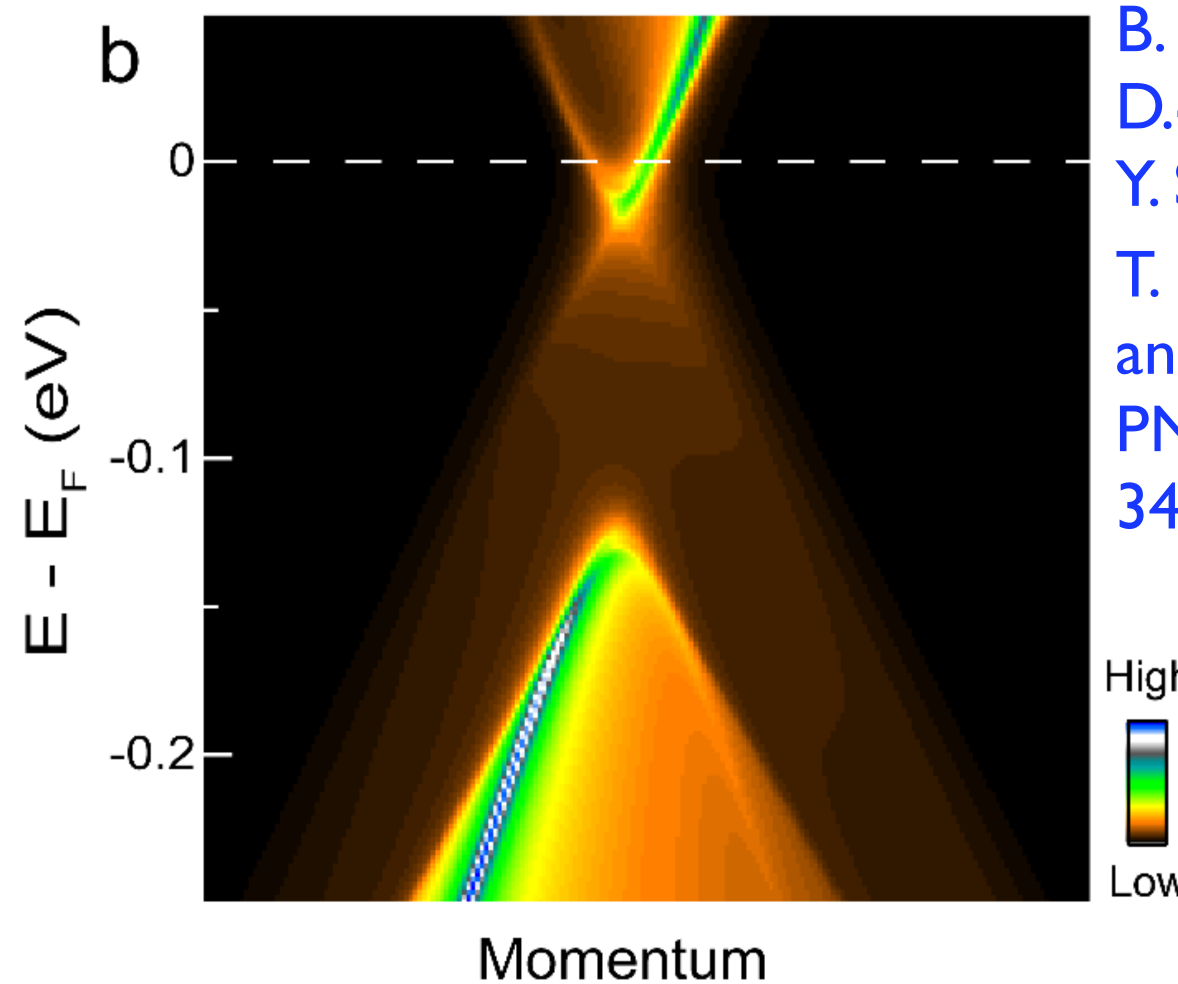
S. Sachdev, Topological order and Fermi surface reconstruction,
Reports on Progress in Physics **82**, 014001 (2019)

M. S. Scheurer, S. Chatterjee, Wei Wu,
M. Ferrero, A. Georges, and S. Sachdev, Proceedings of the National Academy of Sciences **115**, E3665 (2018)

Mathias Scheurer







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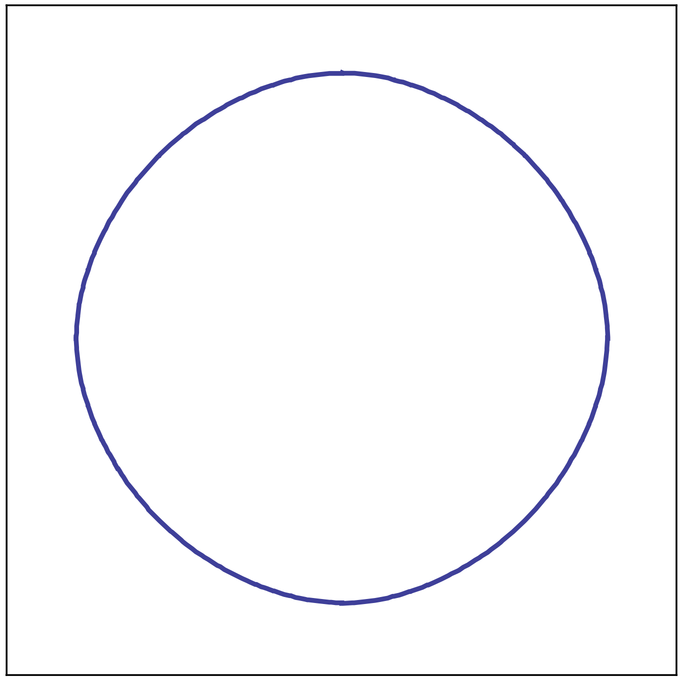
SU(2) gauge theory of fluctuating antiferromagnetism



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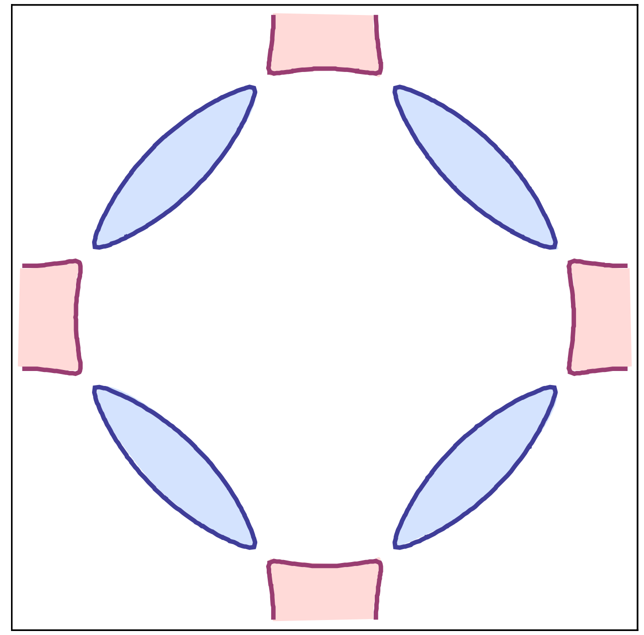

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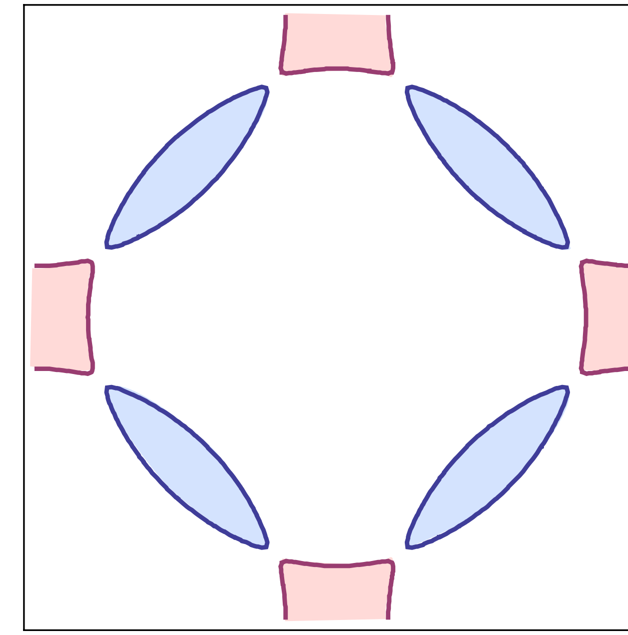

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Topological quantum phase transition

S. Sachdev, H. D. Scammell, M. S. Scheurer, and G. Tarnopolsky, PRB **99**, 054516 (2019)

Metallic quantum matter

- A fundamental property of any metallic state is Luttinger rule: the volume enclosed by the Fermi surface must equal the electron density (mod 2).

- Metals with non-Luttinger volume Fermi surfaces are also possible: they must have "topological order" and "emergent gauge fields".

Topological quantum matter

- Emergent gauge fields are obtained by transformations to a "rotating reference frame".

Topological quantum matter

- Emergent gauge fields are obtained by transformations to a “rotating reference frame”.

- SU(2) gauge theory Higgsed down to Z_2 or U(1) yields quantum phases with Z_2 or U(1) topological order

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- Emergent gauge fields are obtained by transformations to a “rotating reference frame”.

- SU(2) gauge theory Higgsed down to Z_2 or U(1) yields quantum phases with Z_2 or U(1) topological order

- Theory of fluctuating antiferromagnetism in the electron-doped cuprates. Found a metallic state with topological order, reconstructed Fermi surfaces, and violation of the Luttinger theorem. This phase can explain recent photoemission experiments near optimal doping.