

# Detecting quantum duality in experiments: how superfluids become solids in two dimensions

*Physical Review B* **71**, 144508 and 144509 (2005),  
cond-mat/0502002, cond-mat/0511298

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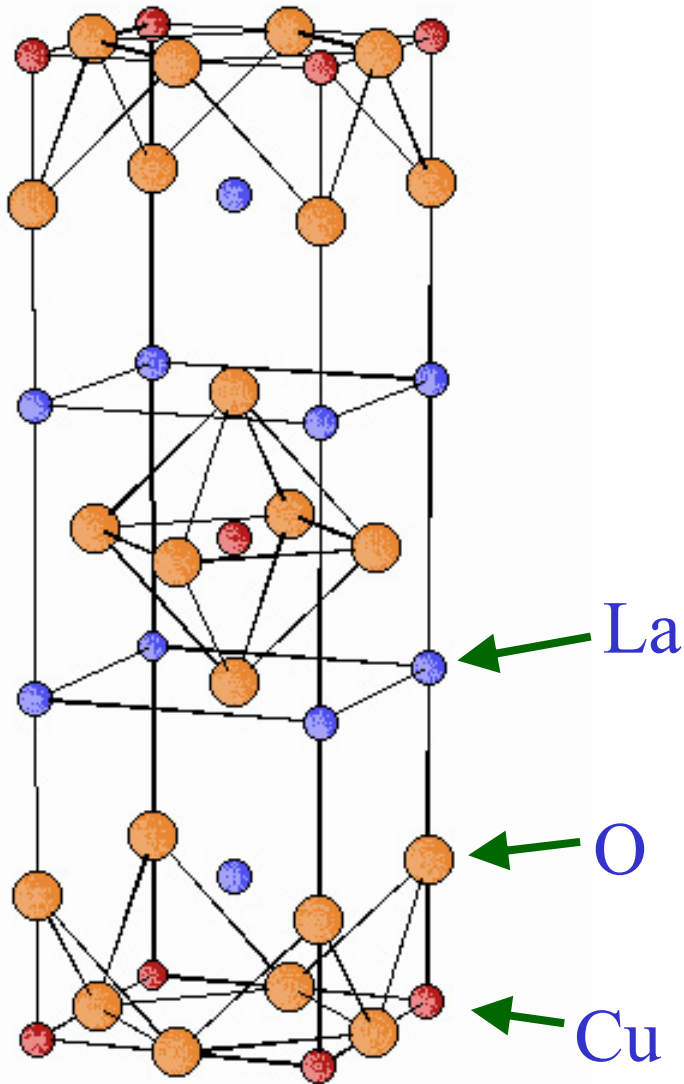
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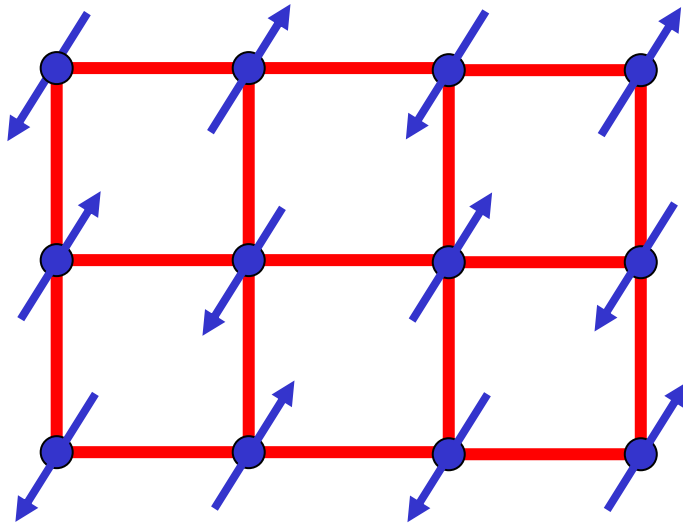
# The cuprate superconductors



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Mott insulator: square lattice antiferromagnet

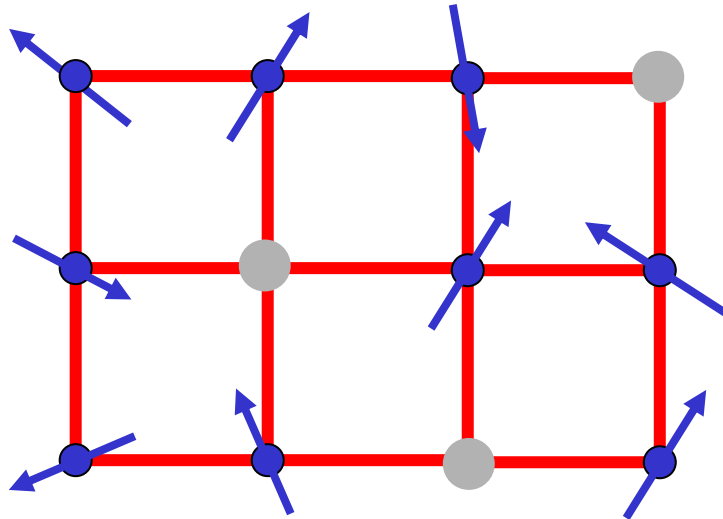


$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

# The cuprate superconductors



Superfluid: condensate of paired holes

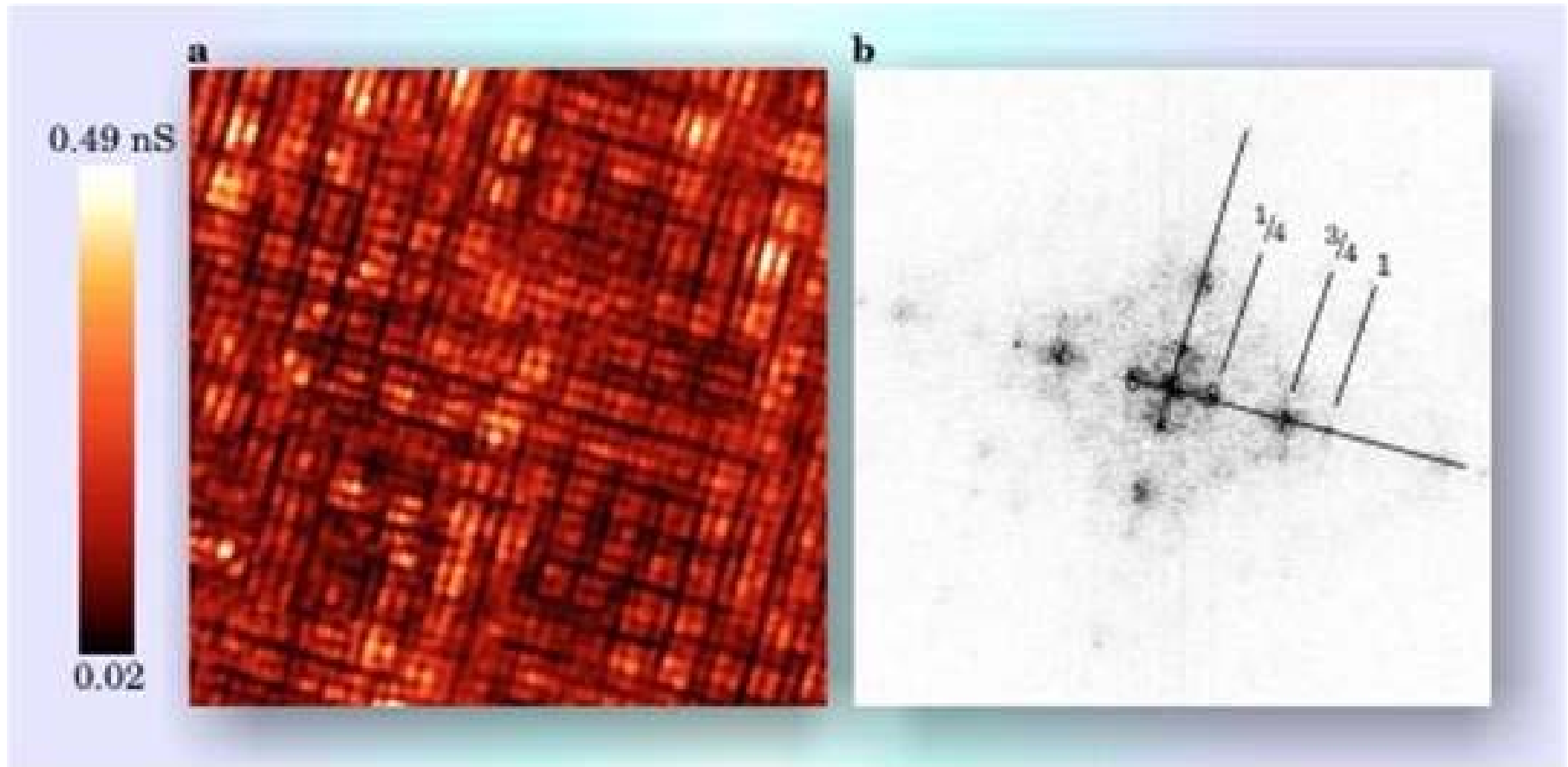


$$\langle \vec{S} \rangle = 0$$

## Experiments on the cuprate superconductors show:

- Proximity to insulating ground states with density wave order at carrier density  $\delta=1/8$
- Vortex/anti-vortex fluctuations for a wide temperature range in the normal state

# The cuprate superconductor $\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$



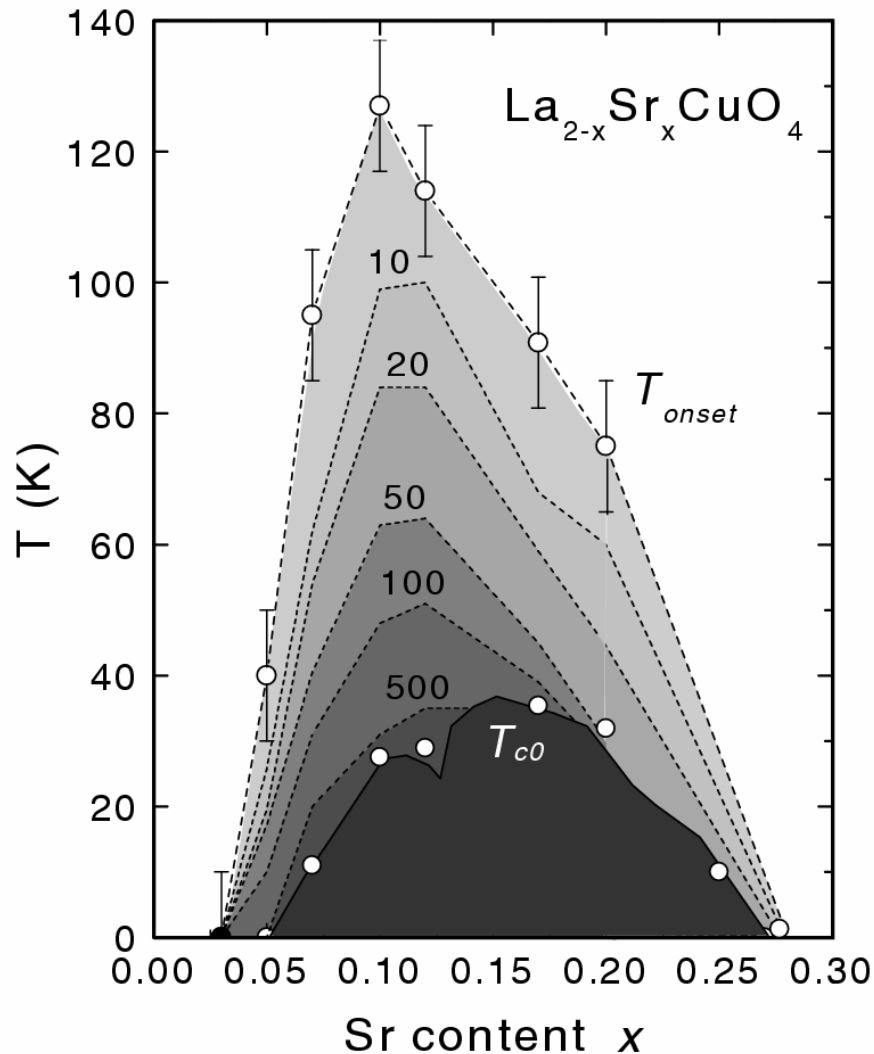
*Multiple order parameters:* superfluidity and density wave.

*Phases:* Superconductors, Mott insulators, and/or supersolids

T. Hanaguri, C. Lupien, Y. Kohsaka, D.-H. Lee, M. Azuma, M. Takano, H. Takagi, and J. C. Davis, *Nature* **430**, 1001 (2004).

# Distinct experimental characteristics of underdoped cuprates at $T > T_c$

Measurements of Nernst effect are well explained by a model of a liquid of vortices and anti-vortices

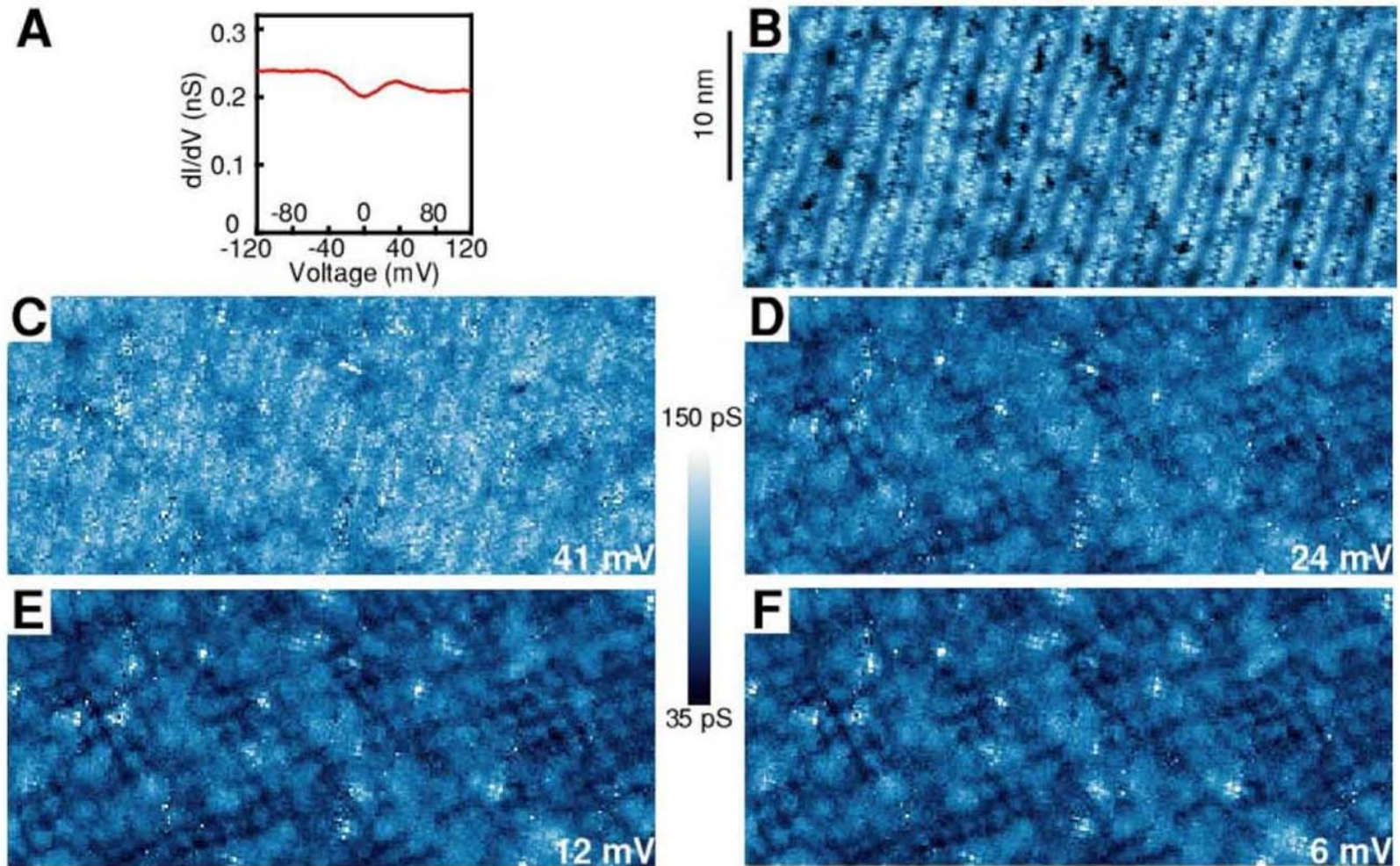


N. P. Ong, Y. Wang, S. Ono, Y. Ando, and S. Uchida, *Annalen der Physik* **13**, 9 (2004).

Y. Wang, S. Ono, Y. Onose, G. Gu, Y. Ando, Y. Tokura, S. Uchida, and N. P. Ong, *Science* **299**, 86 (2003).

# Distinct experimental characteristics of underdoped cuprates at $T > T_c$

STM measurements observe “density” modulations with a period of  $\approx 4$  lattice spacings



LDOS of  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$  at 100 K.

M. Vershinin, S. Misra, S. Ono, Y. Abe, Y. Ando, and A. Yazdani, *Science*, **303**, 1995 (2004).

## Experiments on the cuprate superconductors show:

- Proximity to insulating ground states with density wave order at carrier density  $\delta=1/8$
- Vortex/anti-vortex fluctuations for a wide temperature range in the normal state

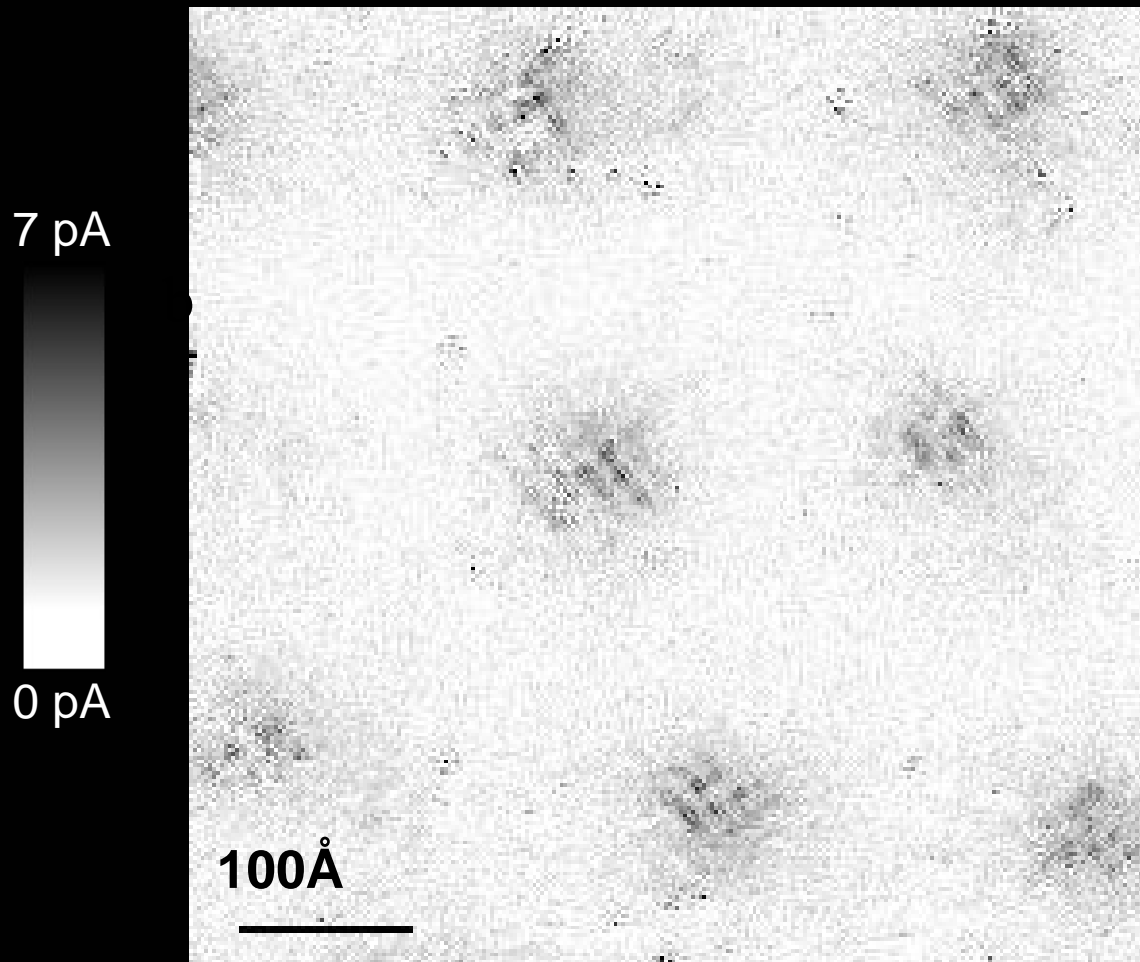
*Needed: A quantum theory of transitions between superfluid/supersolid/insulating phases at fractional filling, and a deeper understanding of the role of vortices*

## Superfluids near Mott insulators

*The Mott insulator has average Cooper pair density,  $f = p/q$  per site, while the density of the superfluid is close (but need not be identical) to this value*

- Vortices with flux  $h/(2e)$  come in multiple (usually  $q$ ) “flavors”
- The lattice space group acts in a projective representation on the vortex flavor space.
- Any pinned vortex must choose an orientation in flavor space. This necessarily leads to modulations in the local density of states over the spatial region where the vortex executes its quantum zero point motion.
- These modulations may be viewed as strong-coupling analogs of Friedel oscillations in a Fermi liquid.

# Vortex-induced LDOS of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ integrated from 1meV to 12meV at 4K



Vortices have halos with LDOS modulations at a period  $\approx 4$  lattice spacings

J. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).

Prediction of VBS order near vortices: K. Park and S. Sachdev, *Phys. Rev. B* **64**, 184510 (2001).

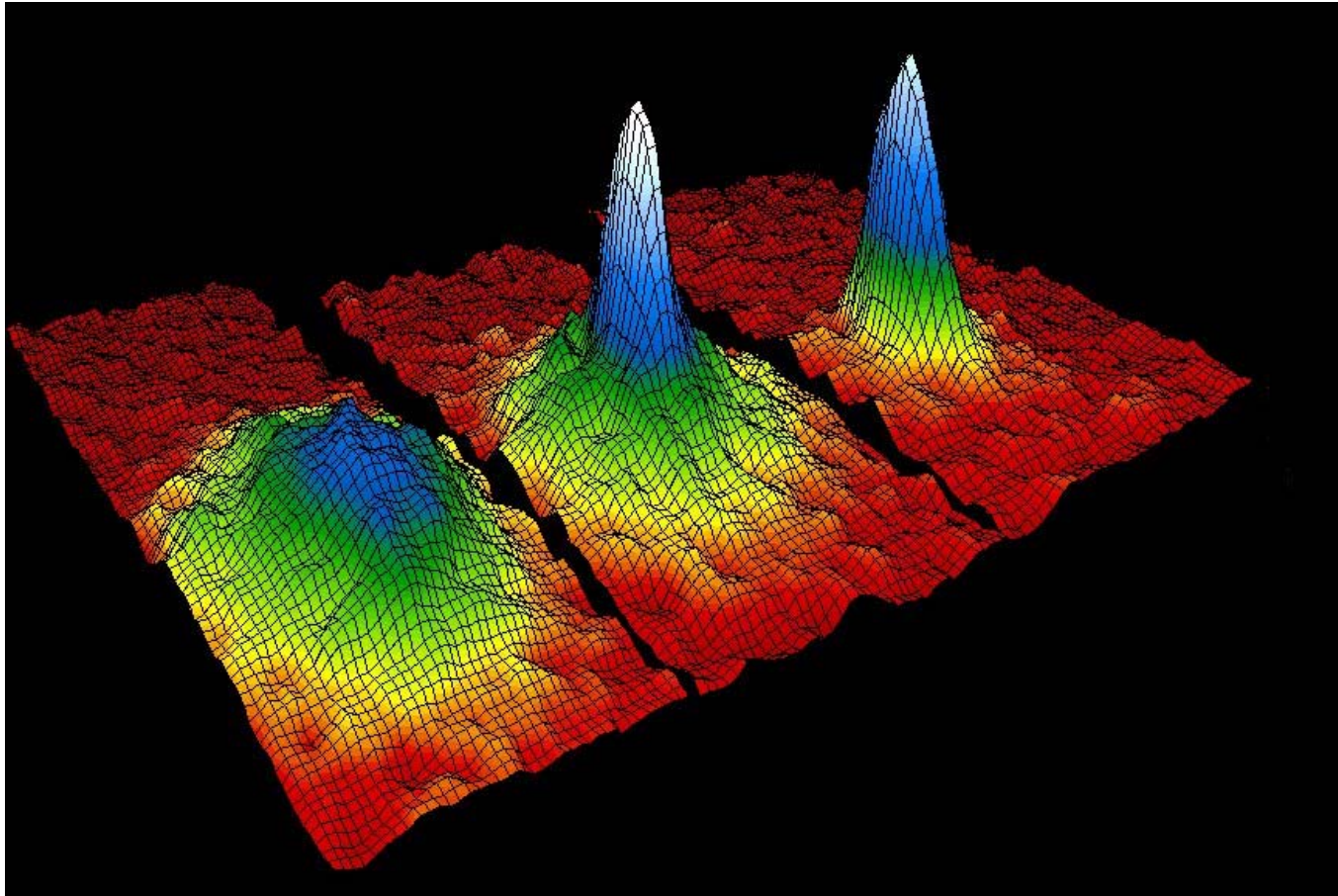
# Outline

- I. Superfluid-insulator transition of bosons
- II. Vortices as elementary quasiparticle excitations of the superfluid
- III. The vortex “flavor” space
- IV. Theory of doped antiferromagnets:  
doping a VBS insulator
- V. Influence of nodal quasiparticles on vortex dynamics in a  $d$ -wave superconductor
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# I. Superfluid-insulator transition of bosons

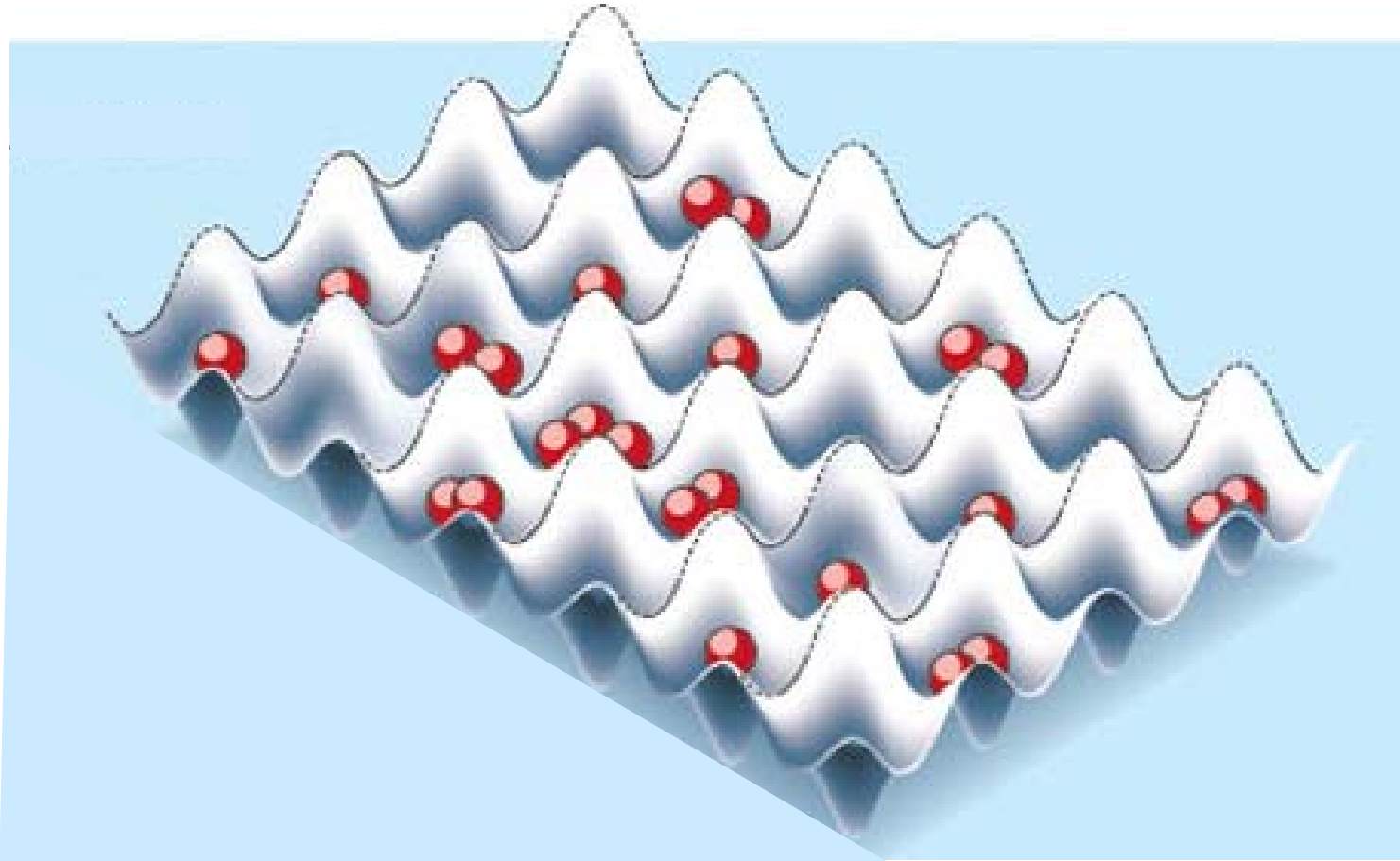
## Bose condensation

Velocity distribution function of ultracold  $^{87}\text{Rb}$  atoms

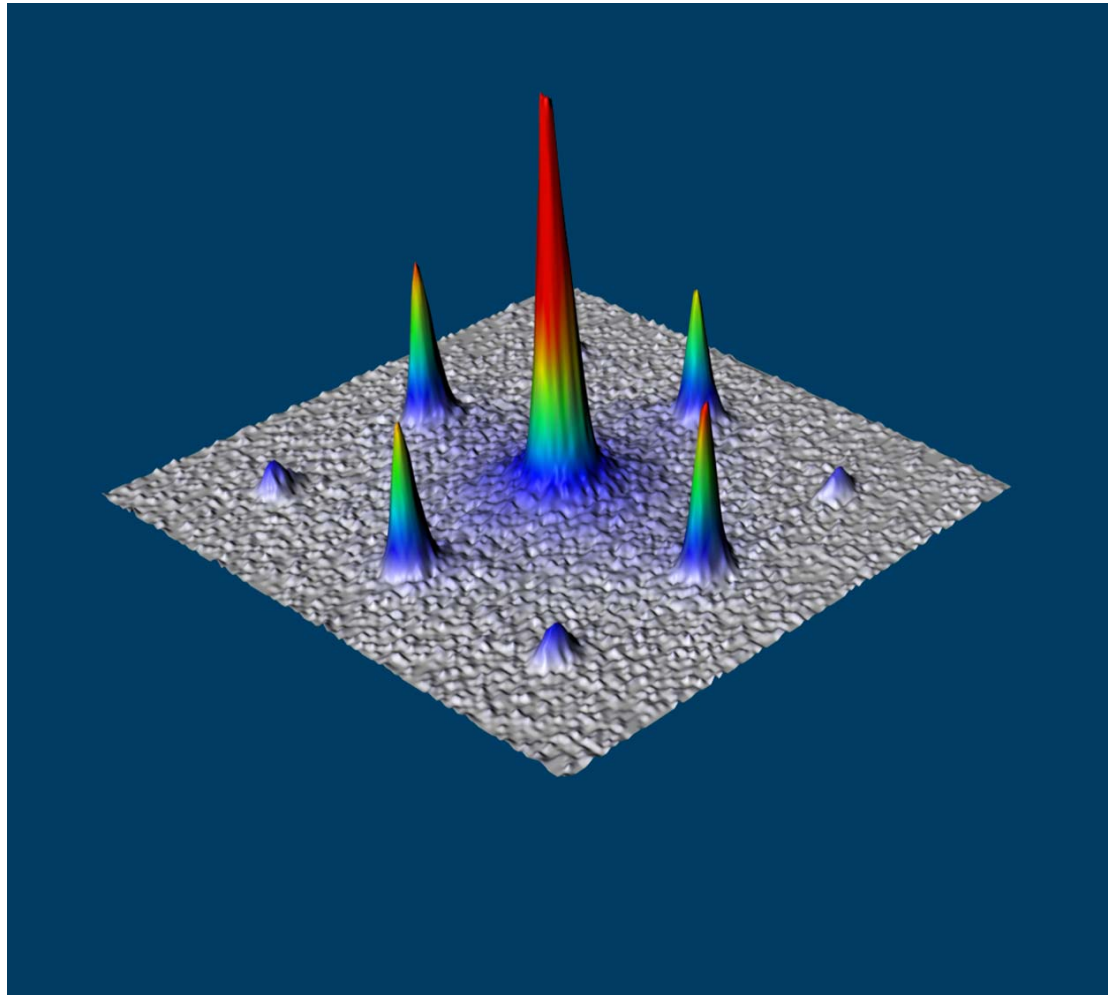


M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman  
and E. A. Cornell, *Science* **269**, 198 (1995)

Apply a periodic potential (standing laser beams)  
to trapped ultracold bosons ( $^{87}\text{Rb}$ )

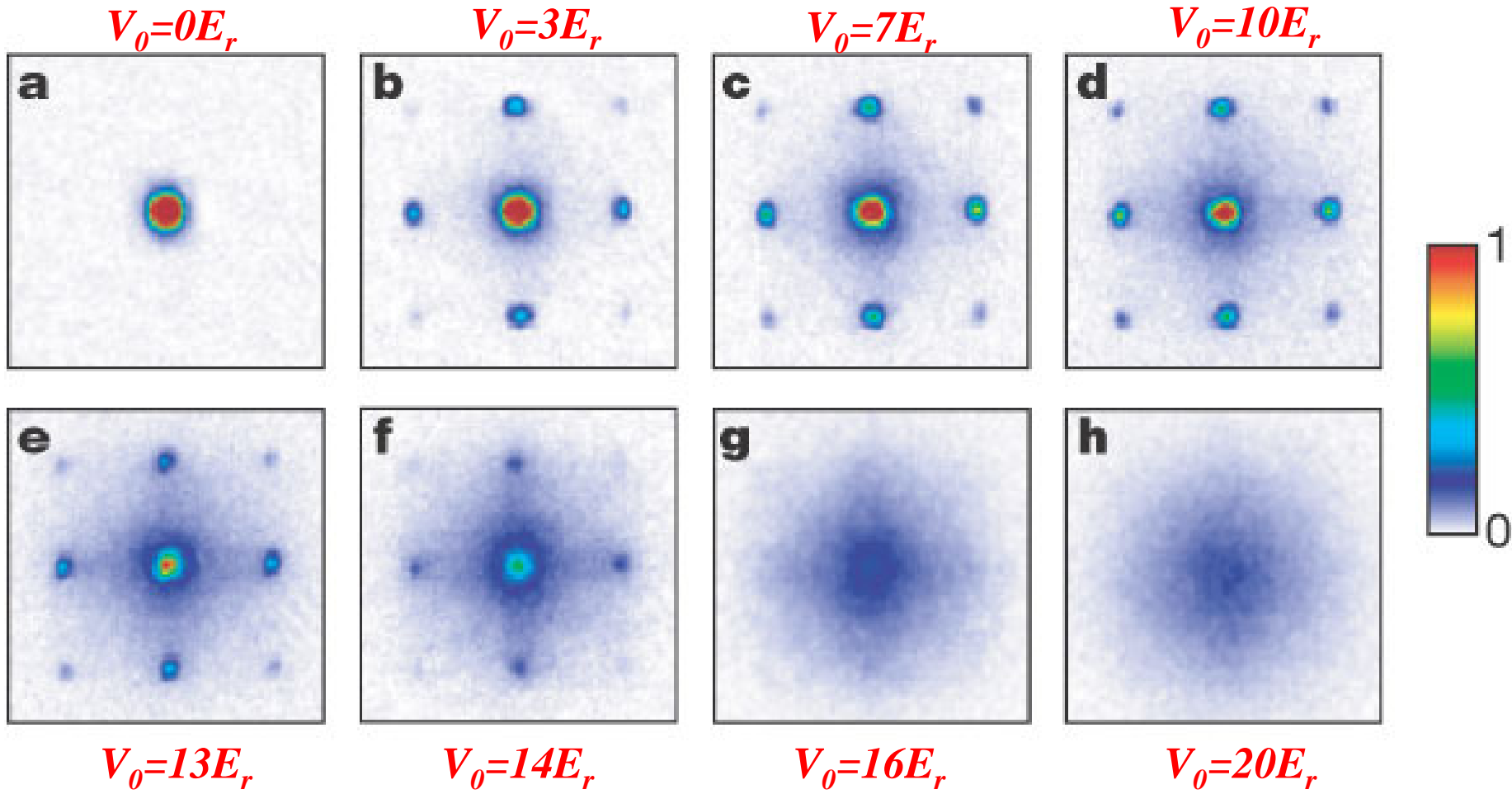
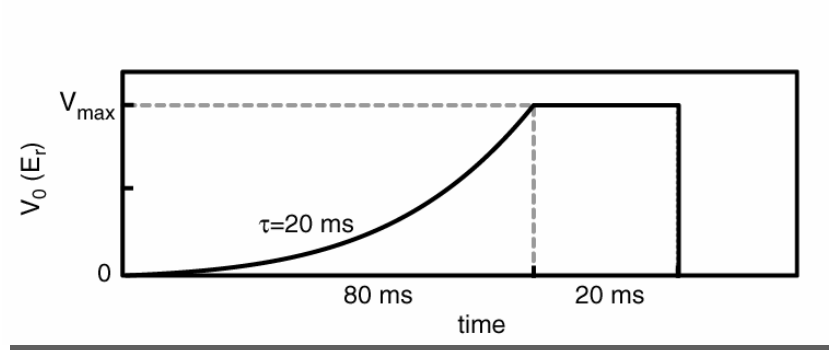


# Momentum distribution function of bosons



Bragg reflections of condensate at reciprocal lattice vectors

# Superfluid-insulator quantum phase transition at $T=0$



# Bosons at filling fraction $f = 1$

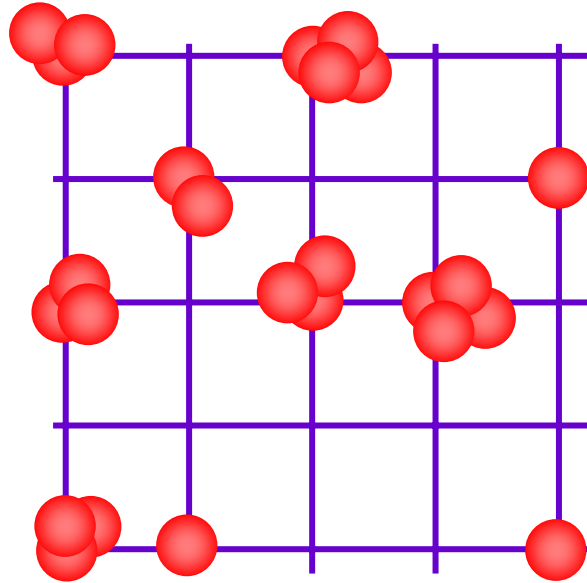
Weak interactions:  
superfluidity

**a** Superfluid state

**b** Insulating state

Strong interactions:  
Mott insulator which  
preserves all lattice  
symmetries

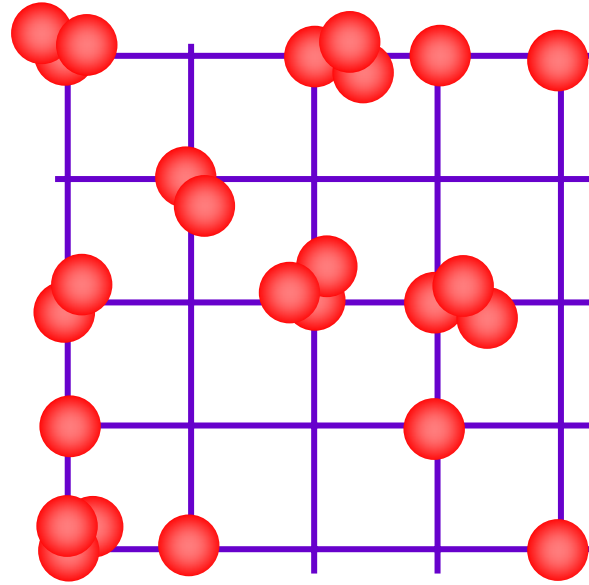
# Bosons at filling fraction $f = 1$



$$\langle \Psi \rangle \neq 0$$

Weak interactions: superfluidity

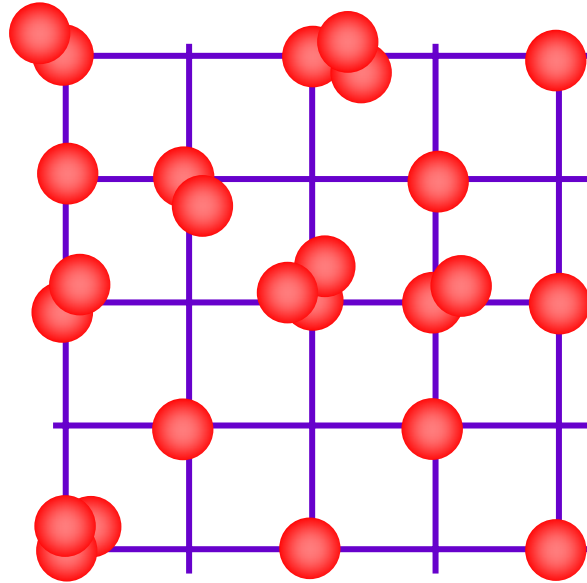
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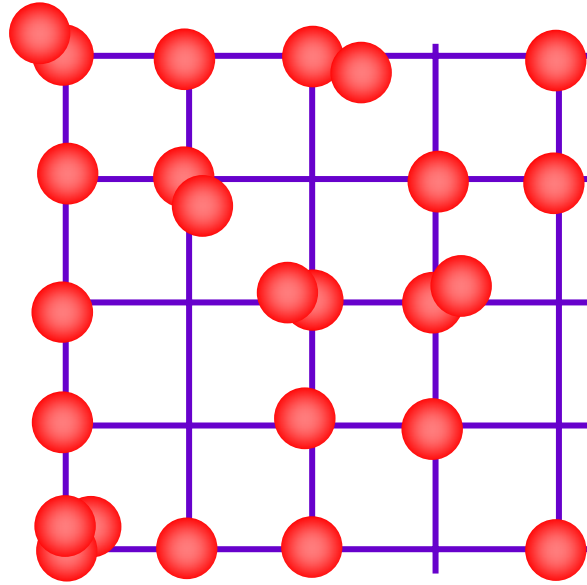
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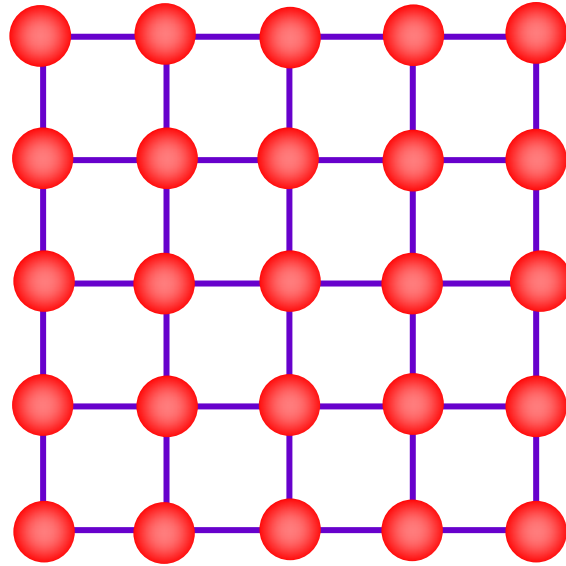
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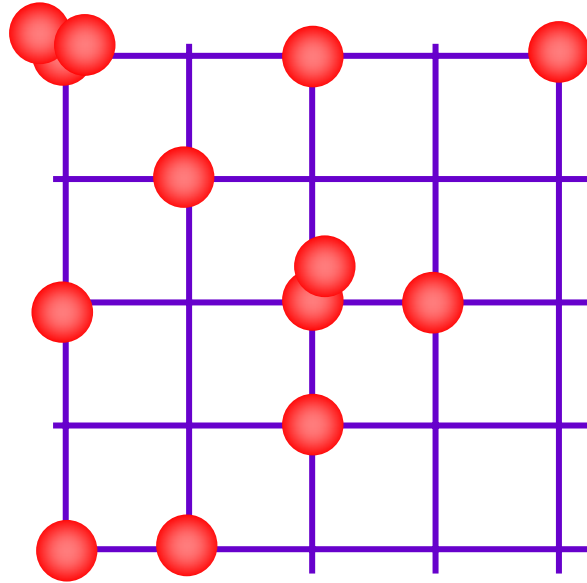
# Bosons at filling fraction $f = 1$



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Strong interactions: insulator

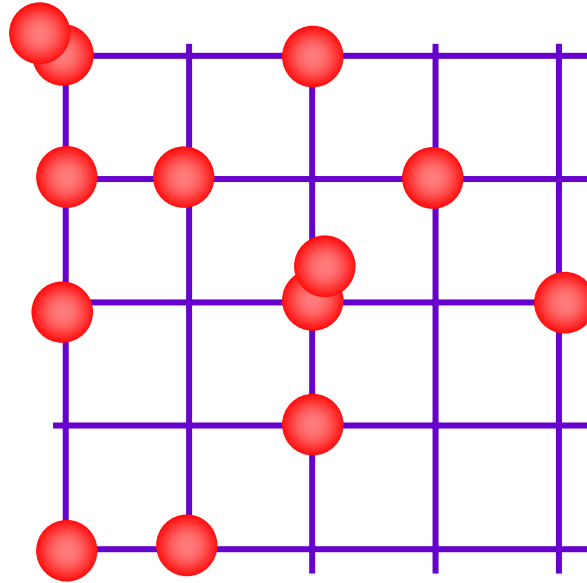
# Bosons at filling fraction $f = 1/2$



$$\langle \Psi \rangle \neq 0$$

Weak interactions: superfluidity

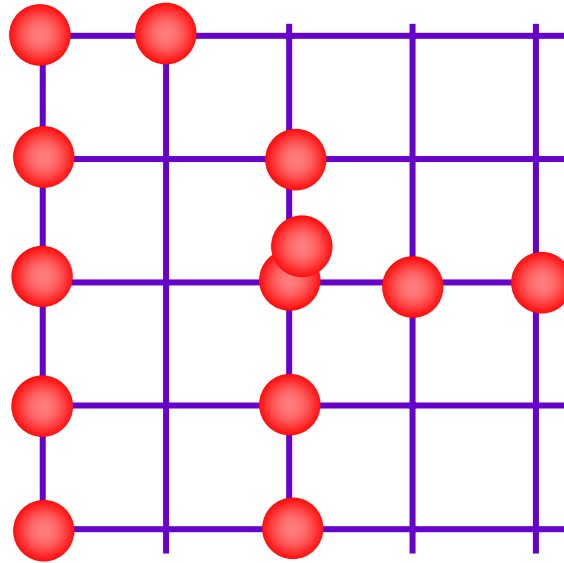
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$$\langle \Psi \rangle \neq 0$$

Weak interactions: superfluidity

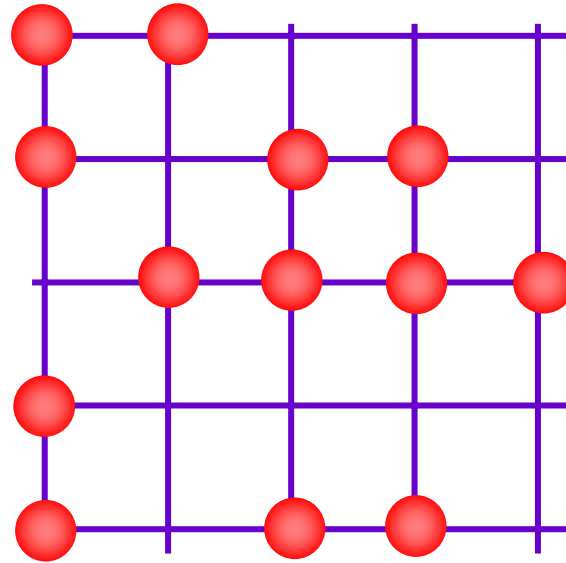
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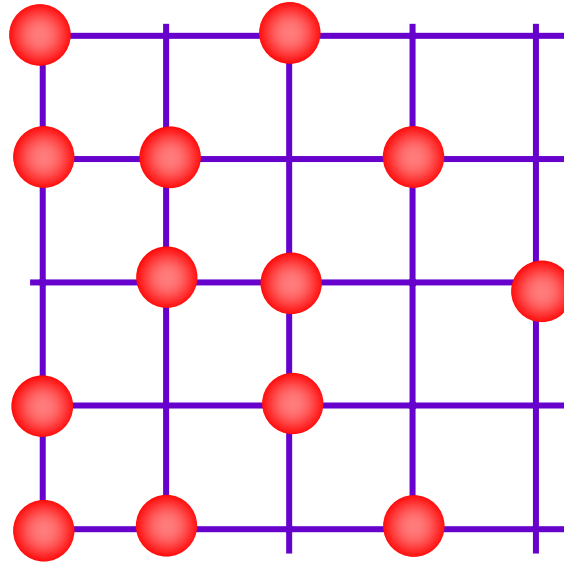
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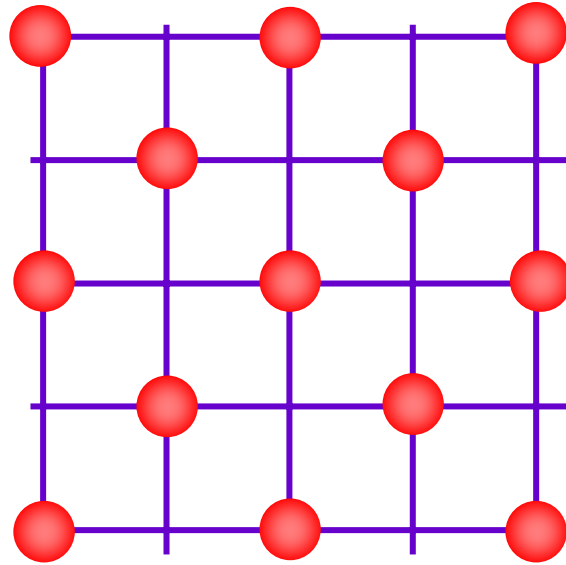
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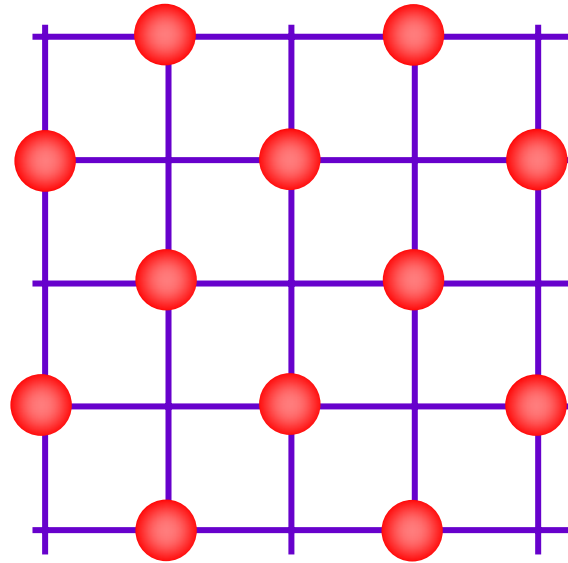
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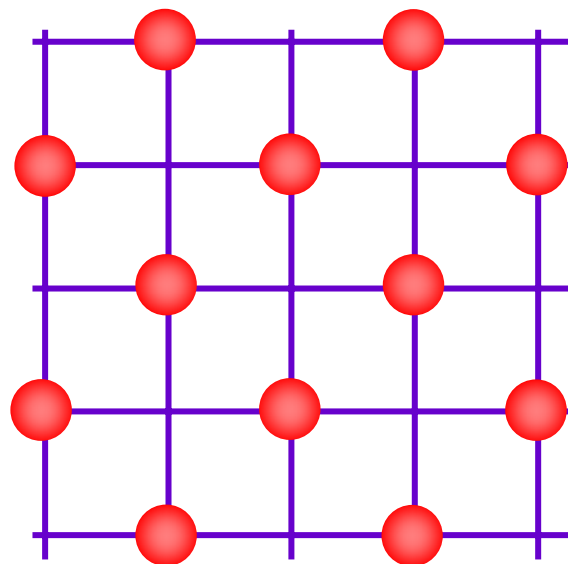
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Strong interactions: insulator

## Bosons at filling fraction $f = 1/2$



$$\langle \Psi \rangle = 0$$

Strong interactions: insulator

Insulator has “density wave” order

*Superfluid-insulator transition of bosons at  
generic filling fraction  $f$*

The transition is characterized by multiple distinct order parameters (boson condensate, density-wave order.....)

*Traditional (Landau-Ginzburg-Wilson) view:*

Such a transition is first order, and there are no precursor fluctuations of the order of the insulator in the superfluid.

## *Superfluid-insulator transition of bosons at generic filling fraction $f$*

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### *Traditional (Landau-Ginzburg-Wilson) view:*

Such a transition is first order, and there are no precursor fluctuations of the order of the insulator in the superfluid.

### *Recent theories:*

Quantum interference effects can render such transitions second order, and the superfluid does contain precursor CDW fluctuations.

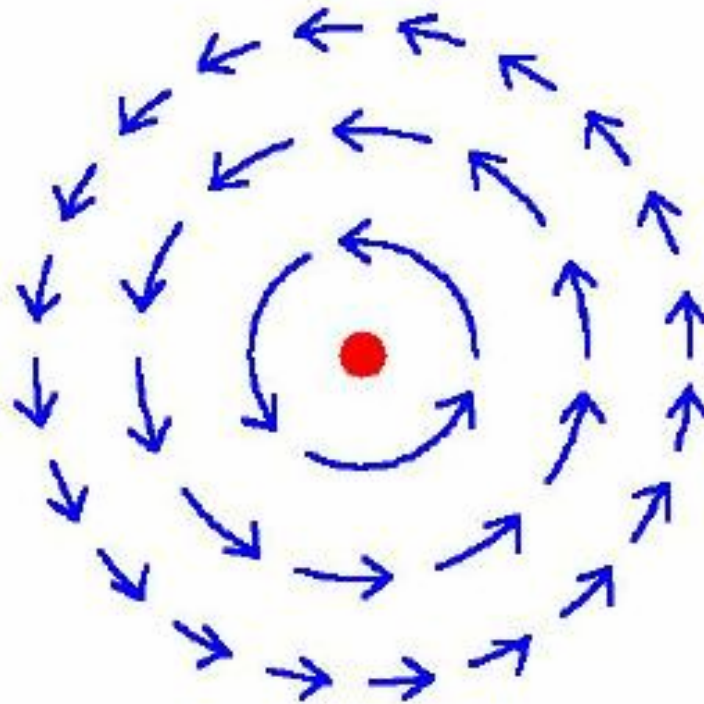
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## II. Vortices as elementary quasiparticle excitations of the superfluid

*Magnus forces, duality, and point vortices as dual “electric” charges*

## Excitations of the superfluid: **Vortices**

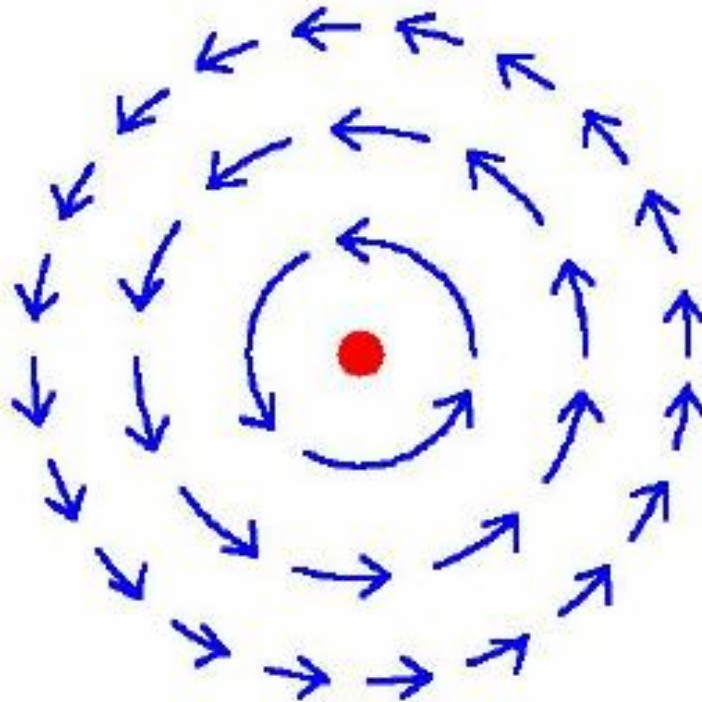


The circulation of a vortex is quantized:

$$\oint \mathbf{v}_s \cdot d\mathbf{r} = \frac{\hbar}{m} \oint \nabla\theta \cdot d\mathbf{r} = n \frac{h}{m}$$

where  $n$  is an integer.

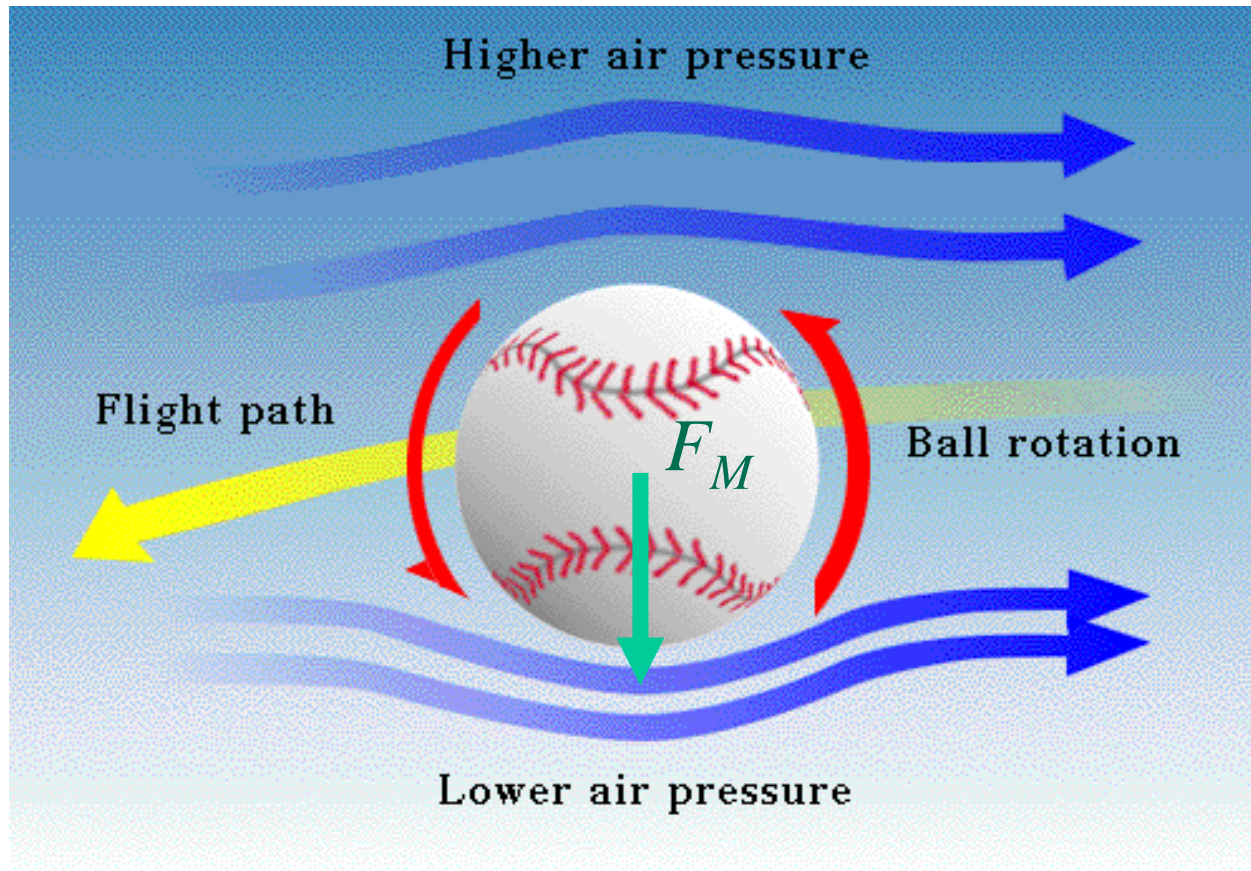
## Excitations of the superfluid: **Vortices**



### Central question:

In two dimensions, we can view the vortices as point particle excitations of the superfluid. What is the quantum mechanics of these “particles” ?

## In ordinary fluids, vortices experience the Magnus Force



$$F_M = (\text{mass density of air}) \cdot (\text{velocity of ball}) \cdot (\text{circulation})$$

For a vortex in a superfluid, this is

$$\begin{aligned}\mathbf{F}_M &= (m\rho) \left( \left( \mathbf{v}_s - \frac{d\mathbf{r}_v}{dt} \right) \times \hat{\mathbf{z}} \right) \left( \oint \mathbf{v}_s \cdot d\mathbf{r} \right) \\ &= nh\rho \left( \mathbf{v}_s - \frac{d\mathbf{r}_v}{dt} \right) \times \hat{\mathbf{z}}\end{aligned}$$

where  $\rho$  = number density of bosons

$\mathbf{v}_s$  = local velocity of superfluid

$\mathbf{r}_v$  = position of vortex

For a vortex in a superfluid, this is

$$\begin{aligned}\mathbf{F}_M &= (m\rho) \left( \left( \mathbf{v}_s - \frac{d\mathbf{r}_v}{dt} \right) \times \hat{\mathbf{z}} \right) \left( \oint \mathbf{v}_s \cdot d\mathbf{r} \right) \\ &= nh\rho \left( \mathbf{v}_s - \frac{d\mathbf{r}_v}{dt} \right) \times \hat{\mathbf{z}} \\ &= n \left( \mathbf{E} + \frac{d\mathbf{r}_v}{dt} \times \mathbf{B} \right)\end{aligned}$$

where  $\mathbf{E} = \rho\mathbf{v}_s \times \hat{\mathbf{z}}$  and  $\mathbf{B} = -h\rho\hat{\mathbf{z}}$

### Dual picture:

The vortex is a quantum particle with dual “electric” charge  $n$ , moving in a dual “magnetic” field of strength =  $h \times$  (number density of Bose particles)

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### III. The vortex “flavor” space

*Vortices carry a “flavor” index which encodes the density-wave order of the proximate insulator*

- The vortices are quantum particles moving in a periodic potential with the symmetry of the square lattice, and in the presence of a dual “magnetic” field of strength  $= h\rho$ , where  $\rho$  is the number density of bosons per unit cell.
- The dual “magnetic” flux per unit cell is  $2\pi f$ , where  $f$  is the filling fraction of the bosons.
- The vortex Hamiltonian commutes with all the space group operations of the square lattice. We focus on
  - $T_x$ , translation by one lattice spacing in the  $x$  direction
  - $T_y$ , translation by one lattice spacing in the  $y$  direction
  - $R$ , rotation by 90 degrees about a dual lattice point

## Bosons at rational filling fraction $f=p/q$

Quantum mechanics of the vortex “particle” in a periodic potential with  $f$  flux quanta per unit cell

Space group symmetries of vortex Hamiltonian:

Magnetic space group:

$$T_x T_y = e^{2\pi i f} T_y T_x \ ;$$

$$R^{-1} T_y R = T_x \ ; \ R^{-1} T_x R = T_y^{-1} \ ; \ R^4 = 1$$

The low energy vortex states must form a representation of this algebra

**For  $f = p/q$ , the representations of this algebra are necessarily at least  $q$  dimensional.**

*Proof:* Let  $|\lambda\rangle$  be an eigenvector of  $T_x$  with eigenvalue  $\lambda$ . Then it follows that  $T_y|\lambda\rangle$  is also an eigenvector of  $T_x$  with eigenvalue  $\lambda e^{2\pi i f}$ . As long as  $e^{2\pi i f} \neq 1$ , this is a distinct eigenvalue, and so  $|\lambda\rangle$  and  $T_y|\lambda\rangle$  are orthogonal states. Repeating this argument, we can generate  $q$  orthogonal states  $(T_y)^m|\lambda\rangle$ , with  $m = 0, 1, \dots, (q - 1)$ . These states form a  $q$  dimensional representation of the magnetic space group.

# Vortices in a superfluid near a Mott insulator at filling $f=p/q$

Simplest representation of magnetic space group by the quantum vortex “particle” with field operator  $\varphi$

At filling  $f=p/q$ , there are  $q$  species of vortices,  $\varphi_\ell$  (with  $\ell=1\dots q$ ), associated with  $q$  degenerate minima in the vortex spectrum. These vortices realize the smallest,  $q$ -dimensional, representation of the magnetic algebra.

$$T_x : \varphi_\ell \rightarrow \varphi_{\ell+1} \quad ; \quad T_y : \varphi_\ell \rightarrow e^{2\pi i \ell f} \varphi_\ell$$

$$R : \varphi_\ell \rightarrow \frac{1}{\sqrt{q}} \sum_{m=1}^q \varphi_m e^{2\pi i \ell m f}$$

# Vortices in a superfluid near a Mott insulator at filling $f=p/q$

The wavefunction of the  $\varphi_\ell$  vortices in flavor space characterizes the density-wave order

Density-wave order:

Status of space group symmetry determined by

density operators  $\rho_{\mathbf{Q}}$  at wavevectors  $\mathbf{Q}_{mn} = \frac{2\pi p}{q} (m, n)$

$$\rho_{mn} = e^{i\pi mnf} \sum_{\ell=1}^q \varphi_\ell^* \varphi_{\ell+n} e^{2\pi i \ell m f}$$

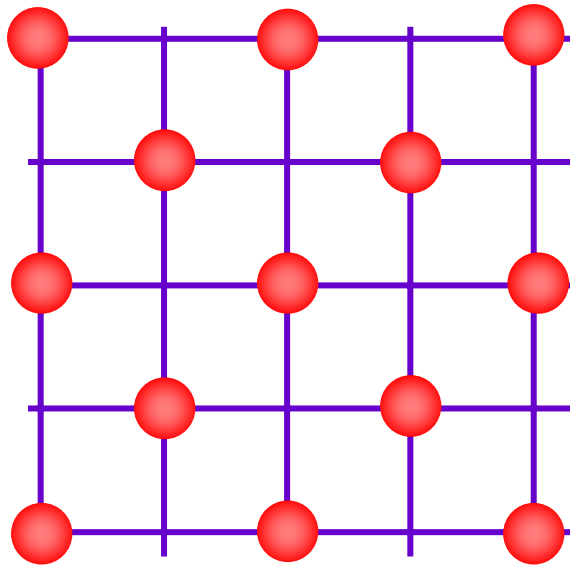
$$T_x : \rho_{\mathbf{Q}} \rightarrow \rho_{\mathbf{Q}} e^{i\mathbf{Q} \cdot \hat{x}} \quad ; \quad T_y : \rho_{\mathbf{Q}} \rightarrow \rho_{\mathbf{Q}} e^{i\mathbf{Q} \cdot \hat{y}}$$

$$R : \rho(\mathbf{Q}) \rightarrow \rho(R\mathbf{Q})$$

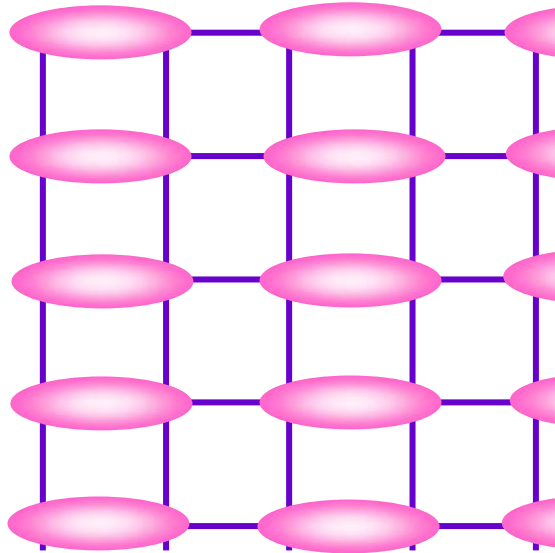
## Vortices in a superfluid near a Mott insulator at filling $f=p/q$

- The excitations of the superfluid are described by the quantum mechanics of  $q$  flavors of low energy vortices moving in zero dual "magnetic" field.
- The orientation of the vortex in flavor space implies a particular configuration of density-wave order in its vicinity.

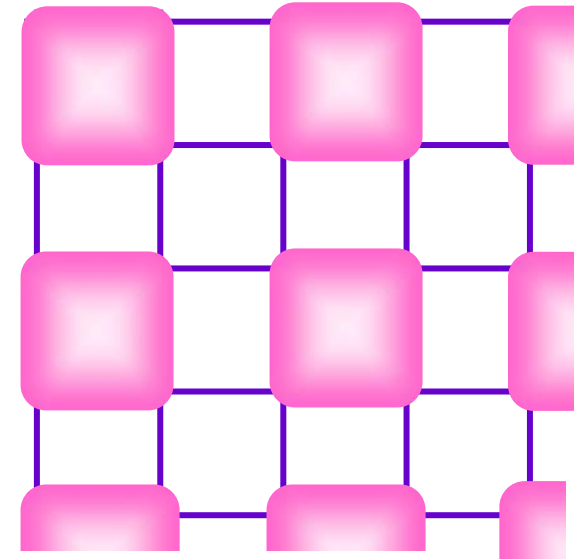
# Mott insulators obtained by condensing vortices at $f = 1/2$



Charge density wave (CDW) order



Valence bond solid (VBS) order



Valence bond solid (VBS) order

$$\text{pink oval} = \frac{1}{\sqrt{2}} \left( \text{red sphere} - \text{bond} + \text{bond} - \text{bond} + \text{red sphere} \right)$$

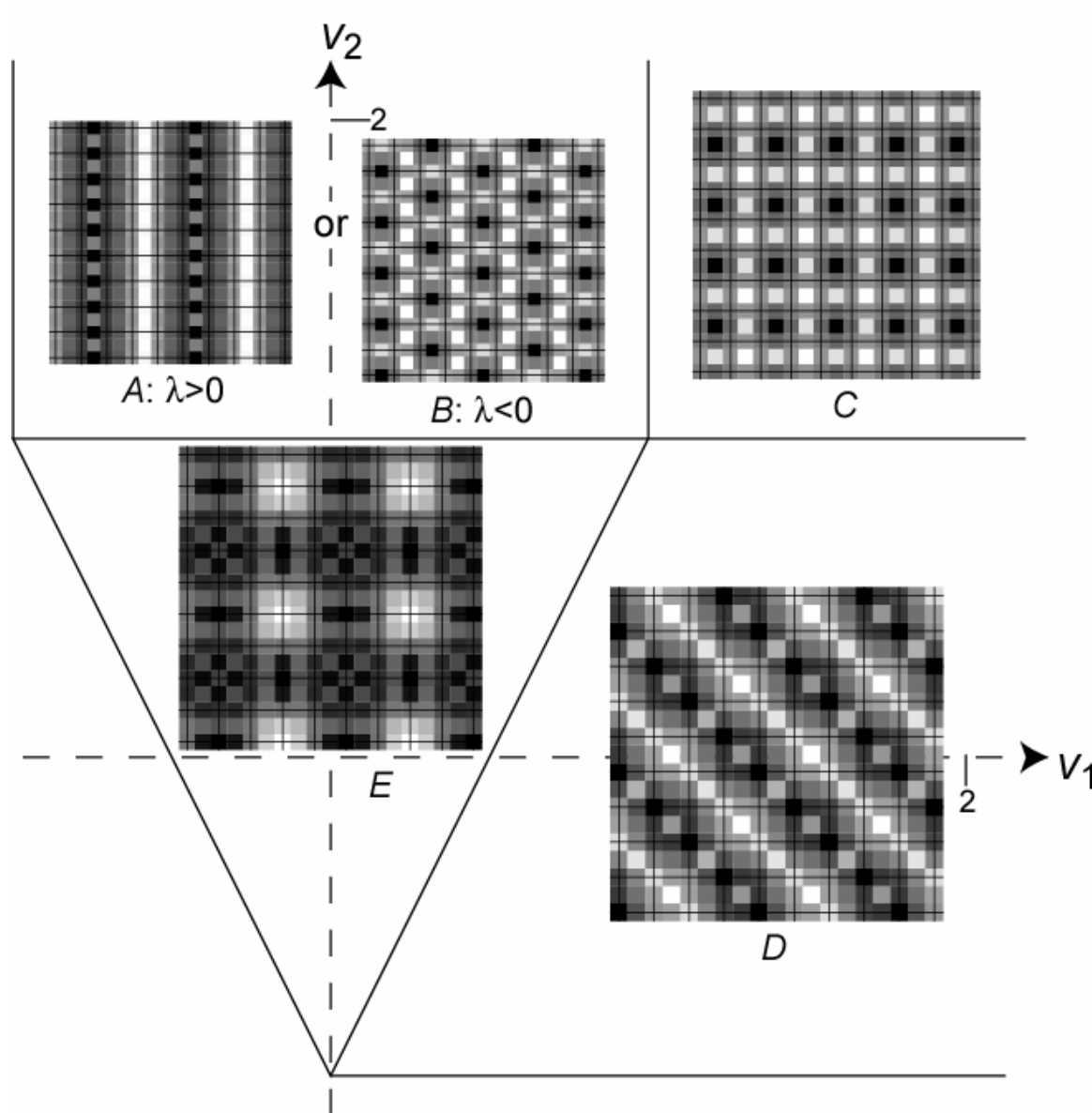
Can define a common CDW/VBS order using a generalized "density"  $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q} \cdot \mathbf{r}}$

All insulators have  $\langle \Psi \rangle = 0$  and  $\langle \rho_{\mathbf{Q}} \rangle \neq 0$  for certain  $\mathbf{Q}$

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev. B* **63**, 134510 (2001)

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Mott insulators obtained by condensing vortices  
at  $f = 1/4, 3/4$



$a \times b$  unit cells;  
 $q/a, q/b, ab/q,$   
 all integers

## Vortices in a superfluid near a Mott insulator at filling $f=p/q$

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- The orientation of the vortex in flavor space implies a particular configuration of VBS order in its vicinity.
- Any pinned vortex must pick an orientation in flavor space: this induces a halo of VBS order in its vicinity

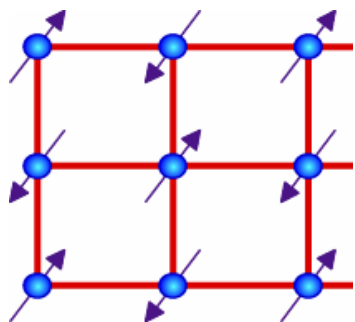
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# IV. Theory of doped antiferromagnets: doping a VBS insulator

# (B.1) Phase diagram of doped antiferromagnets

$g$  = parameter controlling strength of quantum fluctuations in a semiclassical theory of the destruction of Neel order

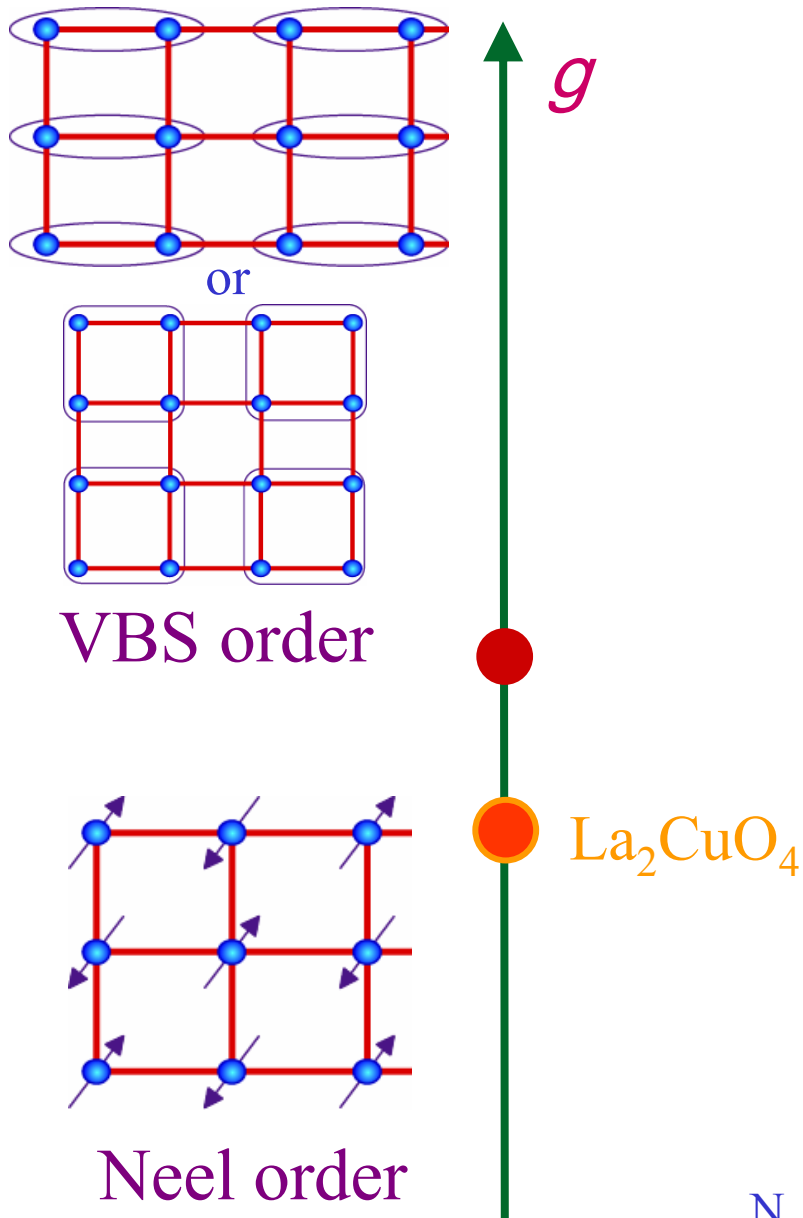


Neel order



$\text{La}_2\text{CuO}_4$

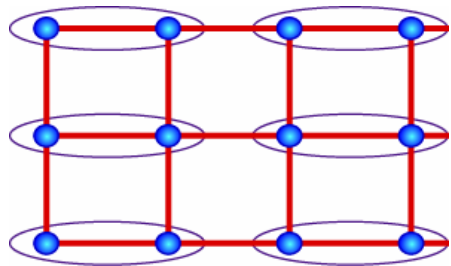
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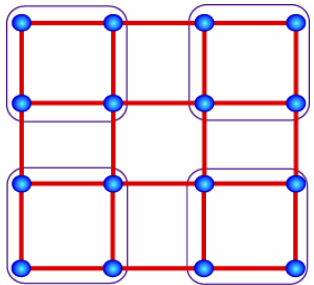
N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

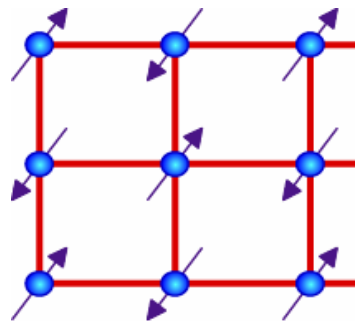
# (B.1) Phase diagram of doped antiferromagnets



or



VBS order



Neel order

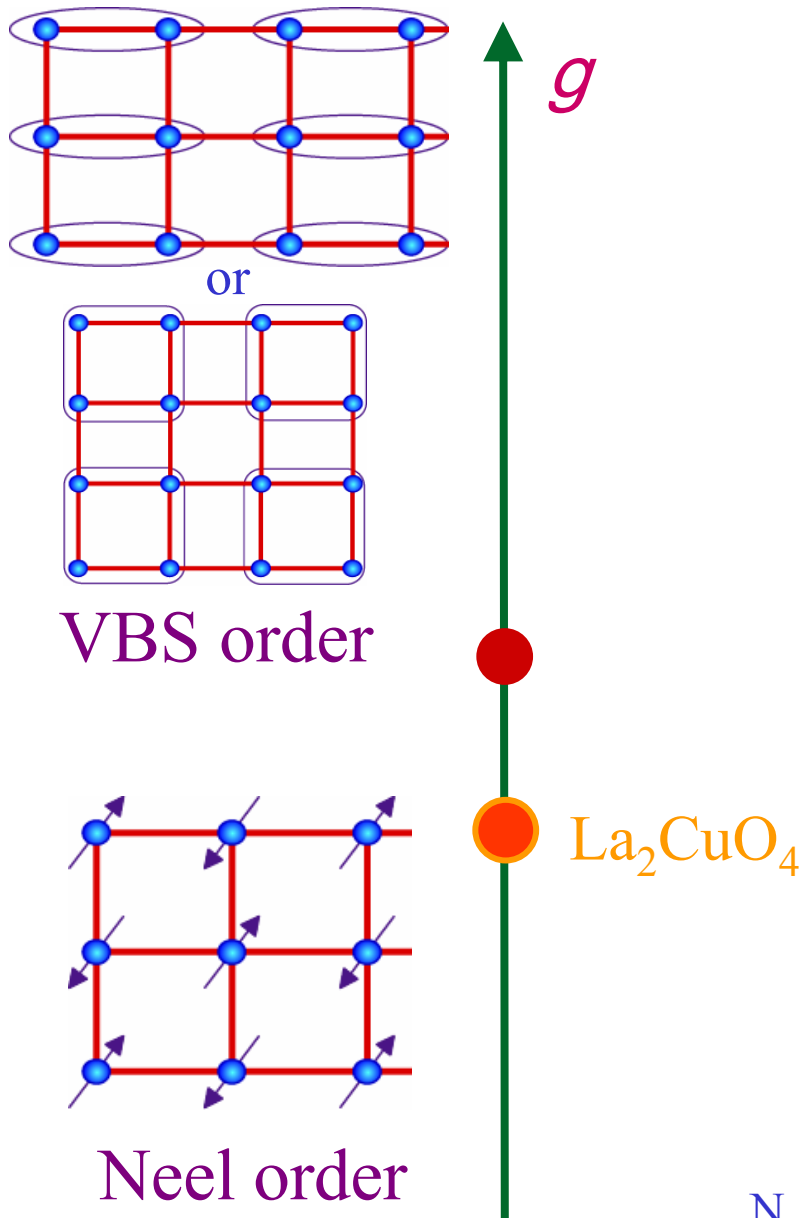
$g$

Upon viewing down spins as hard-core bosons, these phases map onto "superfluid" and VBS phases of bosons at  $f=1/2$  considered earlier

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

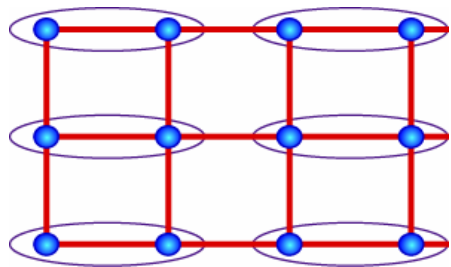
# (B.1) Phase diagram of doped antiferromagnets



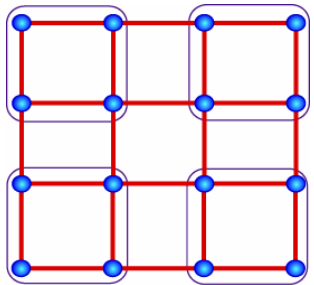
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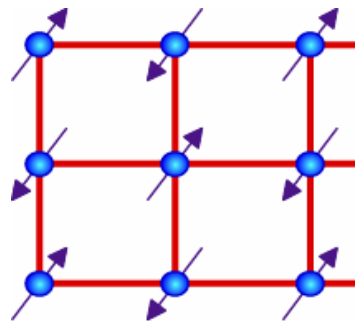
# (B.1) Phase diagram of doped antiferromagnets



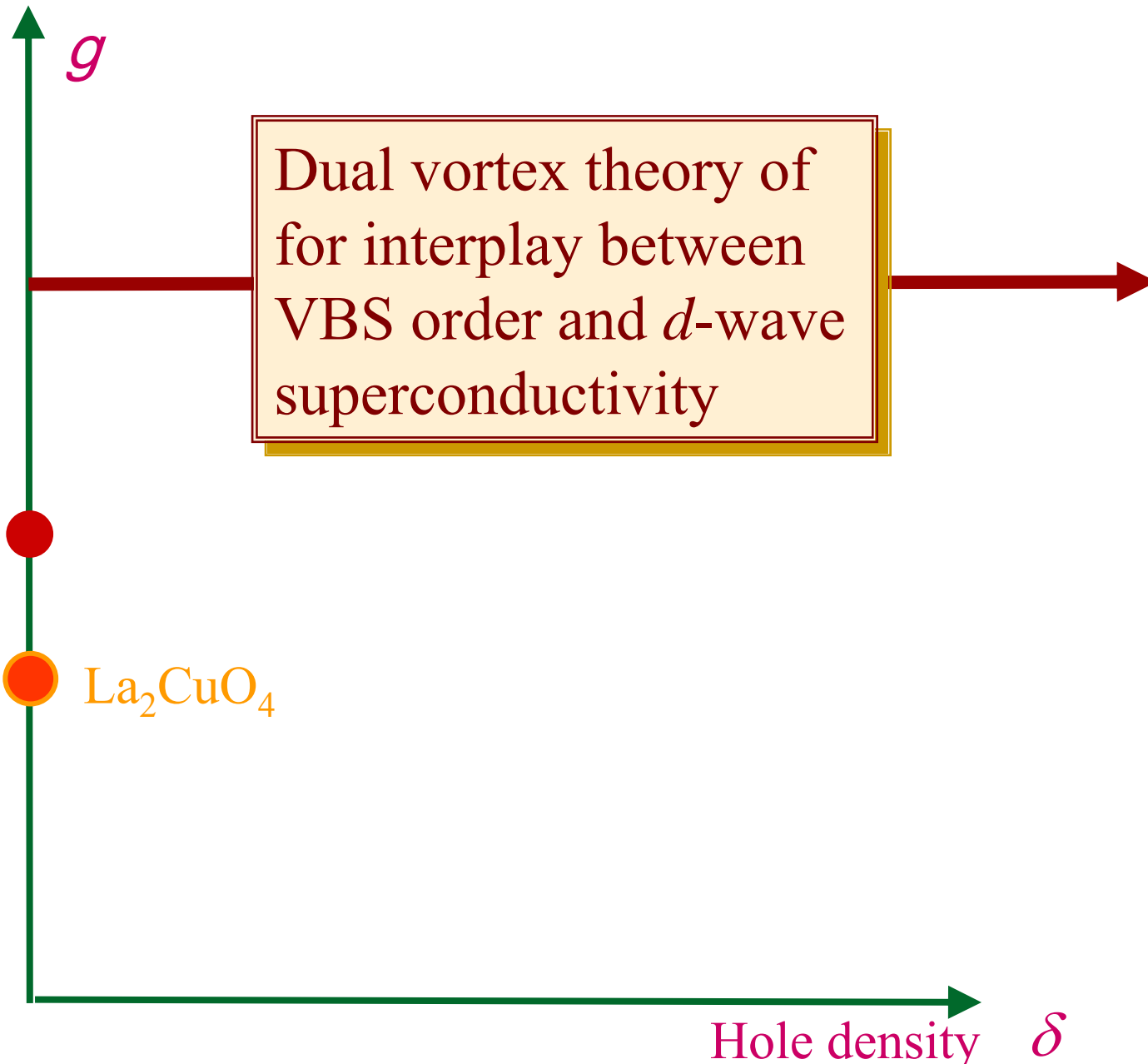
or



VBS order



Neel order



Dual vortex theory of  
for interplay between  
VBS order and  $d$ -wave  
superconductivity

Hole density  $\delta$

## Theory for the VBS phase at half filling

Expressed as a compact U(1) gauge theory for the field  $\mathcal{A}_\mu$ . The “electric” field is a number operator for singlet valence bonds. The low energy action is

$$\mathcal{S}_A = \frac{1}{2e^2} \int d^2x d\tau (\epsilon_{\mu\nu\lambda} \partial_\nu \mathcal{A}_\lambda)^2$$

The theory also has **monopoles**, with annihilation operator  $m$ . The monopole operator transforms “projectively” under the square lattice space group

$$\begin{aligned} T_x & : m \rightarrow im^\dagger \\ T_y & : m \rightarrow -im^\dagger \\ R & : m \rightarrow m^\dagger \end{aligned}$$

## Theory for the VBS phase at half filling

Monopole-antimonopole configurations have a finite action, and hence

$$\lim_{|r| \rightarrow 0} \langle m(r) m^\dagger(0) \rangle \neq 0$$

This implies that the monopoles have “condensed”, and we can work in a ground state with  $m = \text{constant}$ .

From the symmetry transformation of  $m$  under the square lattice space group, it can be shown that the VBS order parameter is proportional to  $m$ . Hence condensation of  $m$  implies long-range VBS order.

## Doping the VBS phase with holes of density $\delta$

- Holes are represented by charge  $e$  bosons  $b_1$  and  $b_2$  which also carry  $\mathcal{A}_\mu$  gauge charges  $+1$  and  $-1$ .
- The density of each boson species is

$$\frac{\delta}{2} = \frac{p}{q}$$

- Dualize each boson species to vortices  $\varphi_1$  and  $\varphi_2$ . Because of “screening” by the  $\mathcal{A}_\mu$  gauge field, each  $\varphi_{1,2}$  vortex carries magnetic flux  $hc/(2e)$ .
- At this point, it appears that the magnetic space group obeys

$$T_x T_y = T_y T_x e^{2\pi i p/q}$$

and there are  $2q$  flavors of  $hc/2e$  vortices

## Doping the VBS phase with holes of density $\delta$

The Key Step: There are terms combining the *monopoles* of the half-filled antiferromagnet with the *vortices* of the doped holes which are invariant under all space group operations.

For  $q = 2n$  such a term has the structure

$$\mathcal{L} = m^\dagger \sum_{\ell=0}^{q-1} \left[ e^{-i\pi/4} (-1)^\ell \varphi_{1\ell}^* \varphi_{2\ell} + e^{i\pi/4} \varphi_{1\ell}^* \varphi_{2,\ell+q/2} \right] + \text{c.c.}$$

Consequently, the low energy vortices have degeneracy  $q'$ , where

$$q' = \begin{cases} 2q & \text{for } q = 2n + 1 \\ q & \text{for } q = 4n \\ q/2 & \text{for } q = 4n + 2 \end{cases}$$

## Doping the VBS phase with holes of density $\delta$

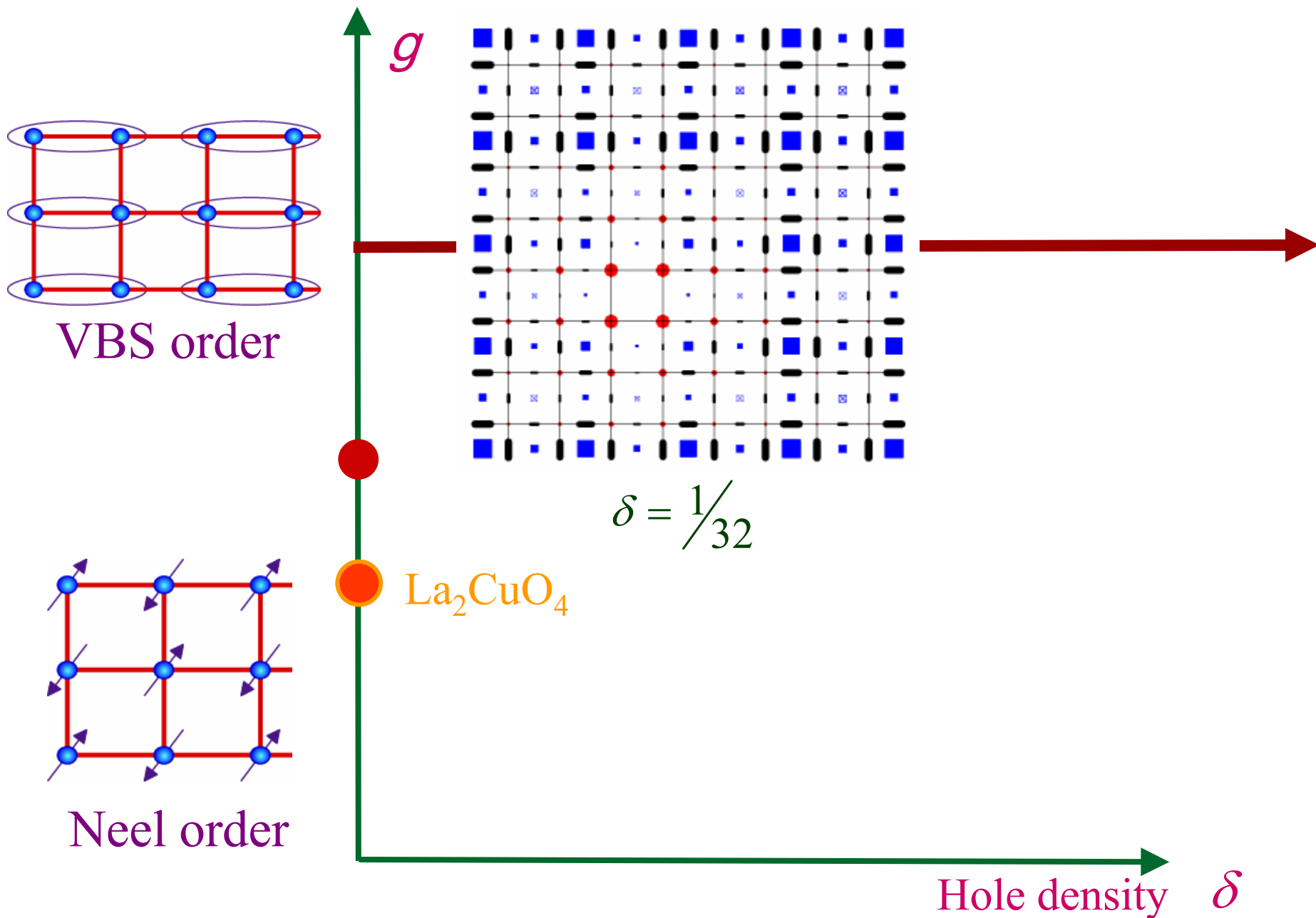
Alternative interpretation of the value of  $q'$ : The same values of  $q'$  are obtained by the relation

$$\frac{p'}{q'} = \frac{1}{2} - \frac{p}{q} = \frac{1 - \delta}{2}$$

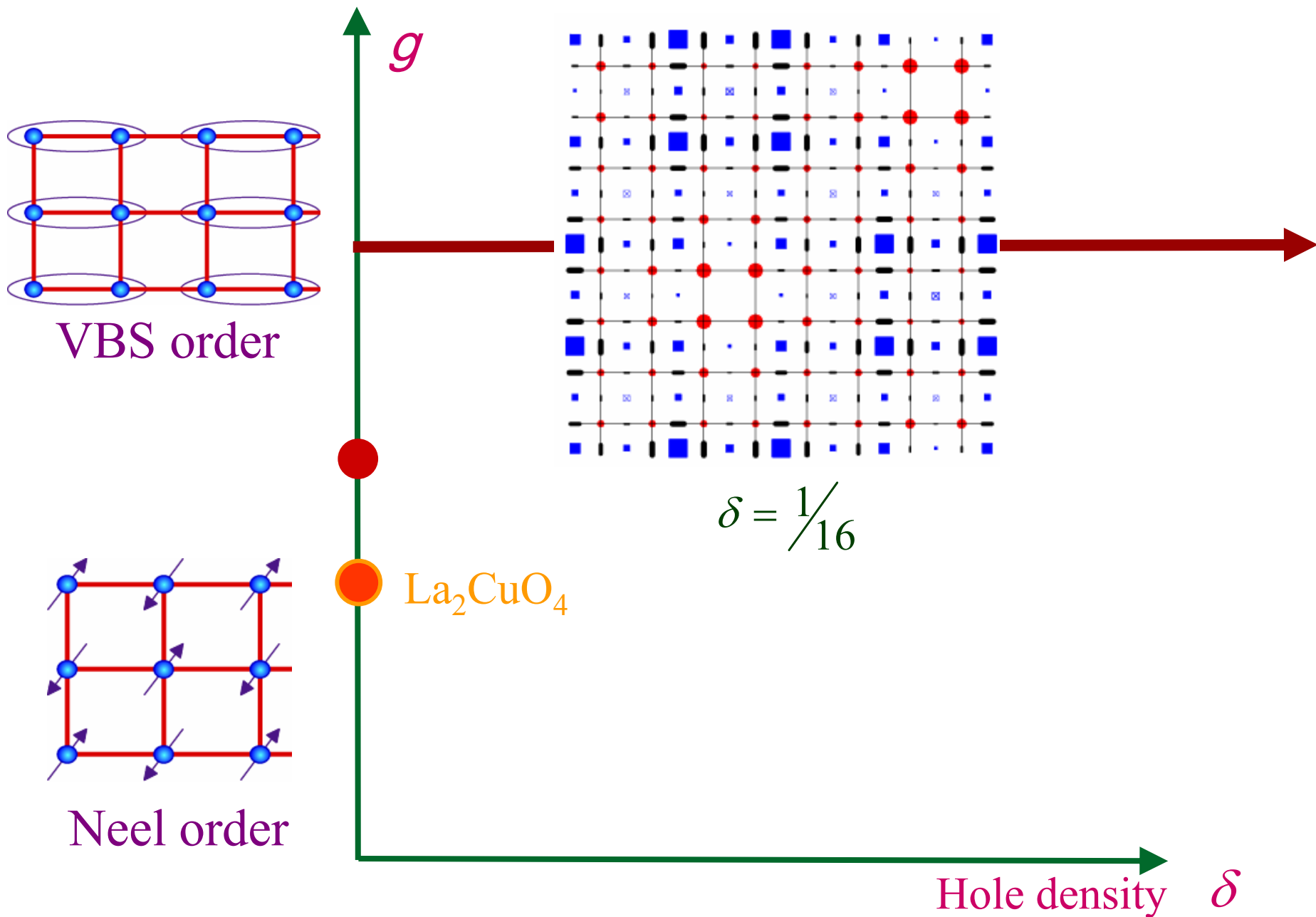
Note that  $(1 - \delta)/2$  is the total number of *pairs of electrons*. So at the end, there are  $q'$  vortices, where  $p'/q'$  is the density of Cooper pairs.

In other words, the vortex PSG is just that would have been obtained by considering **a theory of elementary bosons on the square lattice, with density equal to the density of pairs of electrons.**

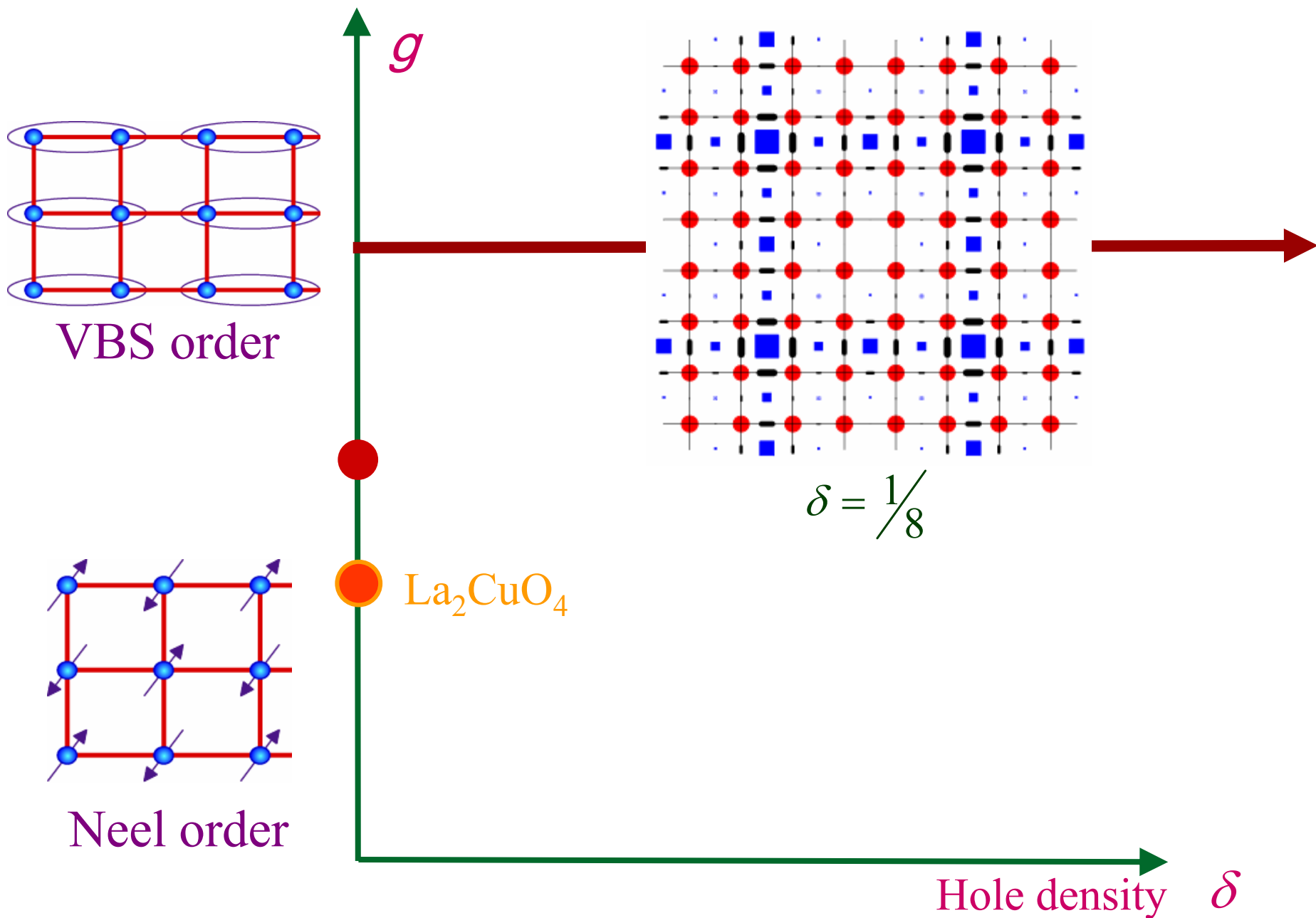
# (B.1) Phase diagram of doped antiferromagnets



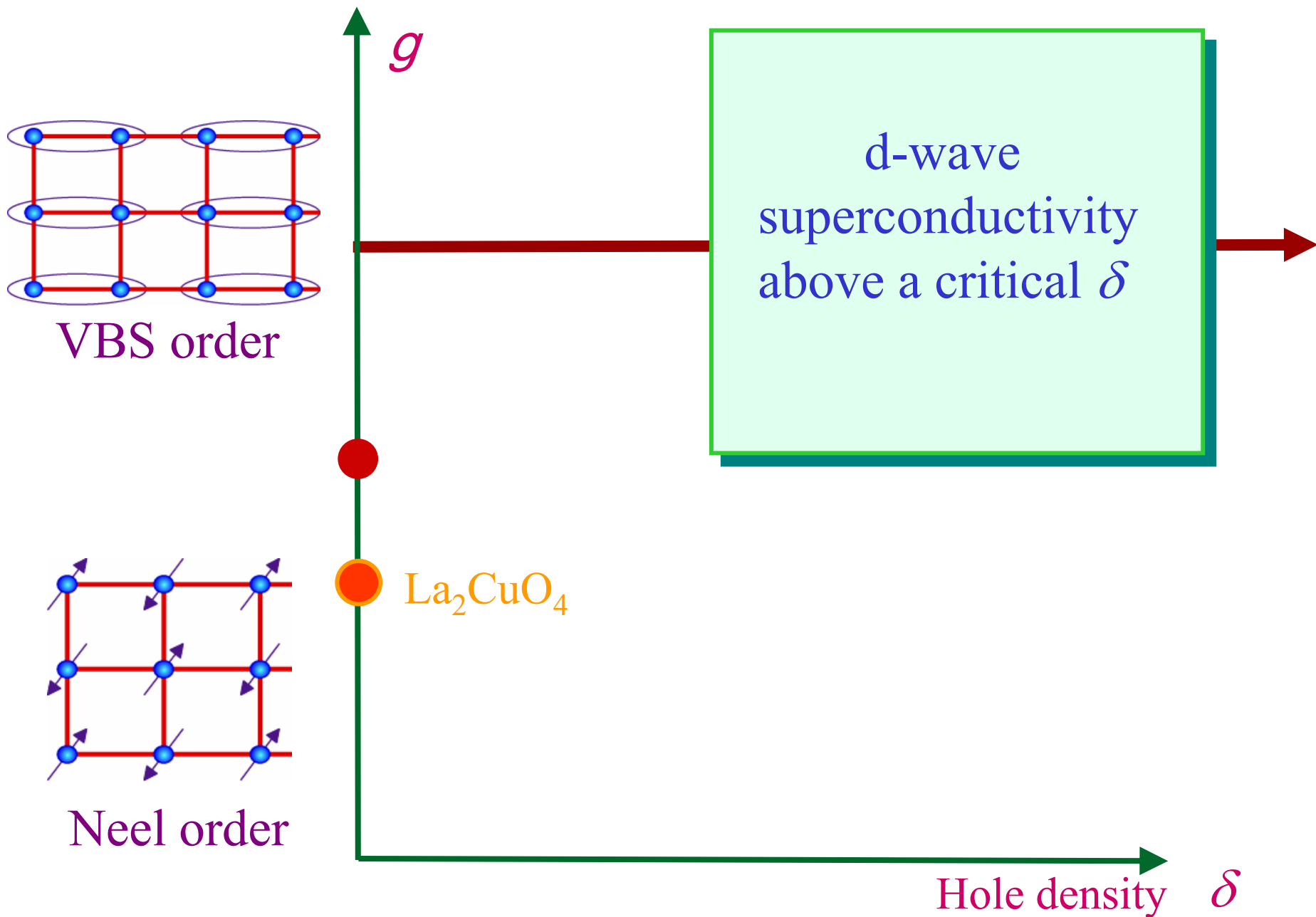
# (B.1) Phase diagram of doped antiferromagnets



# (B.1) Phase diagram of doped antiferromagnets



# (B.1) Phase diagram of doped antiferromagnets



# Outline

- I. Superfluid-insulator transition of bosons
- II. Vortices as elementary quasiparticle excitations of the superfluid
- III. The vortex “flavor” space
- IV. Theory of doped antiferromagnets:  
doping a VBS insulator
- V. Influence of nodal quasiparticles on vortex dynamics in a  $d$ -wave superconductor
- VI. Predictions for experiments on the cuprates

V. Influence of nodal quasiparticles on  
vortex dynamics in a *d*-wave  
superconductor

## A single vortex in a nodal $d$ -wave superconductor

Consider a single vortex at position  $\mathbf{r}_v(\tau)$ . After the Franz-Tesanovic gauge transformation, this vortex appears as a  $\pi$  flux tube to the fermionic quasiparticles. After integrating out these quasiparticles (which have a Dirac spectrum near the nodal points), the effective action of the vortex has the form

$$\begin{aligned} \mathcal{S}[\mathbf{r}_v(\tau)] &= -\text{Tr} \ln (\gamma_\mu (i\partial_\mu - a_\mu)) \\ &+ \text{additional terms from the "Doppler shift"} \end{aligned}$$

where

$$\vec{\nabla} \times \vec{a} = \pi \delta(\mathbf{r} - \mathbf{r}_v(\tau))$$

## A single vortex in a nodal $d$ -wave superconductor

We obtained

$$\mathcal{S}[\mathbf{r}_v(\tau)] = \int \frac{d\omega}{2\pi} |\mathbf{r}_v(\omega)|^2 \left[ \frac{m_v \omega^2}{2} + C_1 |\omega|^3 + C_2 T^2 |\omega| \right] + \dots$$

## A single vortex in a nodal $d$ -wave superconductor

We obtained

$$\mathcal{S}[\mathbf{r}_v(\tau)] = \int \frac{d\omega}{2\pi} |\mathbf{r}_v(\omega)|^2 \left[ \frac{m_v \omega^2}{2} + C_1 |\omega|^3 + C_2 T^2 |\omega| \right] + \dots$$

A *finite* effective mass

$$m_v \sim \frac{\Lambda}{v_F^2}$$

where  $\Lambda \sim \Delta$  is a high energy cutoff

## A single vortex in a nodal $d$ -wave superconductor

We obtained

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sub-Ohmic damping with

$$C_1 = v_F^{-2} \times \left( \text{Universal function of } \frac{v_\Delta}{v_F} \right)$$

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Bardeen-Stephen viscous drag with

$$C_2 = v_F^{-2} \times \left( \text{Universal function of } \frac{v_\Delta}{v_F} \right)$$

## A single vortex in a nodal $d$ -wave superconductor

We obtained

$$\mathcal{S}[\mathbf{r}_v(\tau)] = \int \frac{d\omega}{2\pi} |\mathbf{r}_v(\omega)|^2 \left[ \frac{m_v \omega^2}{2} + C_1 |\omega|^3 + C_2 T^2 |\omega| \right] + \dots$$


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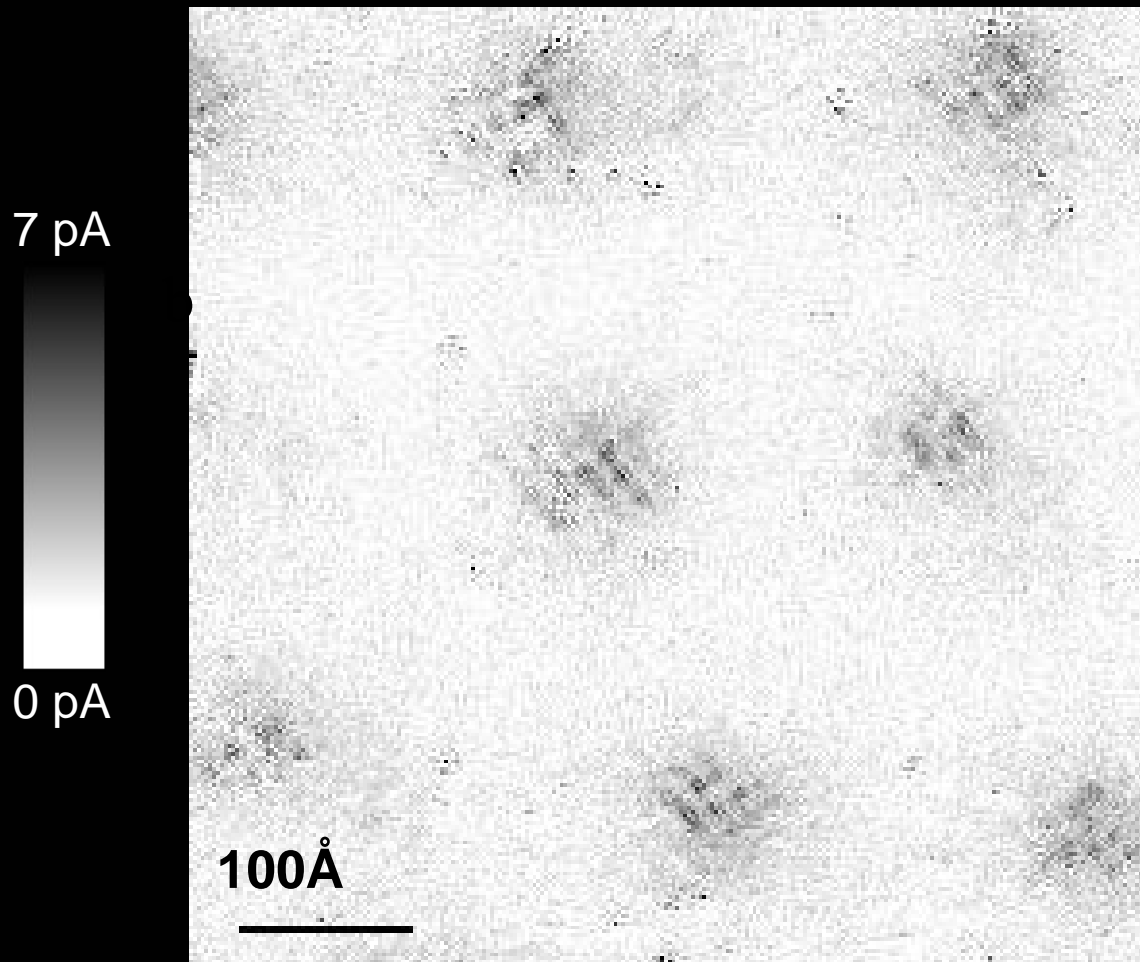
Effect of nodal quasiparticles on vortex dynamics is relatively innocuous.

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# VI. Predictions for experiments on the cuprates

# Vortex-induced LDOS of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ integrated from 1meV to 12meV at 4K



Vortices have halos with LDOS modulations at a period  $\approx 4$  lattice spacings

Prediction of VBS order near vortices: K. Park and S. Sachdev, *Phys. Rev. B* **64**, 184510 (2001).

J. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).

# Measuring the inertial mass of a vortex

The spatial extent of the LDOS modulations measures the region over which the vortex executes its zero-point motion. The size of this region can be determined by solving the equations of motion

$$m_v \frac{d^2 \mathbf{r}}{dt^2} = F_M$$

for a triangular lattice of vortices. Defining

$u_{\text{rms}}$  = rms displacement of vortex from its equilibrium position,

we obtain from the vortex ‘magnetophonon’ spectrum

$$m_v = 0.0419 \frac{\hbar^2 A_0}{\rho_s u_{\text{rms}}^4} F \left( \frac{u_{\text{rms}}^2 B}{\hbar} \right)$$
$$F(x) \approx 0.5039 + \sqrt{0.2461 + 0.4147x^2}$$

where  $A_0$  is the area of a vortex lattice unit cell, and  $B = -h(\rho - \rho_{MI})$ .

# Measuring the inertial mass of a vortex

Preliminary estimates for the BSCCO experiment:

Inertial vortex mass  $m_v \approx 10m_e$

Vortex magnetoplasmon frequency  $\nu_p \approx 1 \text{ THz} = 4 \text{ meV}$

Future experiments can directly detect vortex zero point motion by looking for resonant absorption at this frequency.

Vortex oscillations can also modify the electronic density of states.

## Superfluids near Mott insulators

*The Mott insulator has average Cooper pair density,  $f = p/q$  per site, while the density of the superfluid is close (but need not be identical) to this value*

- Vortices with flux  $h/(2e)$  come in multiple (usually  $q$ ) “flavors”
- The lattice space group acts in a projective representation on the vortex flavor space.
- Any pinned vortex must choose an orientation in flavor space. This necessarily leads to modulations in the local density of states over the spatial region where the vortex executes its quantum zero point motion.
- These modulations may be viewed as strong-coupling analogs of Friedel oscillations in a Fermi liquid.