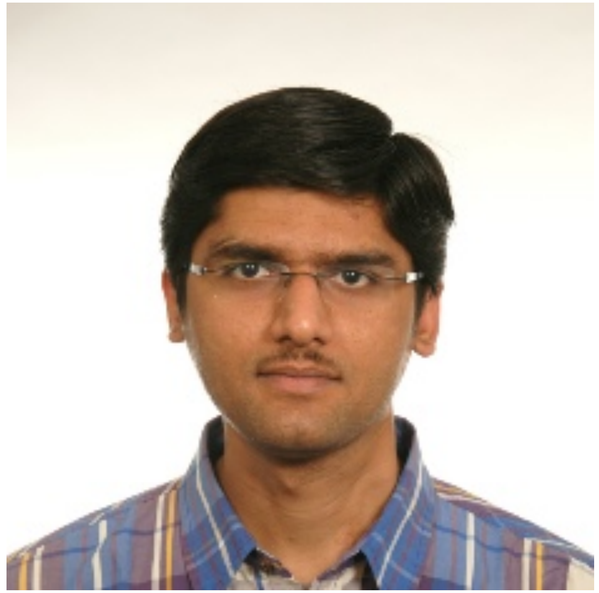
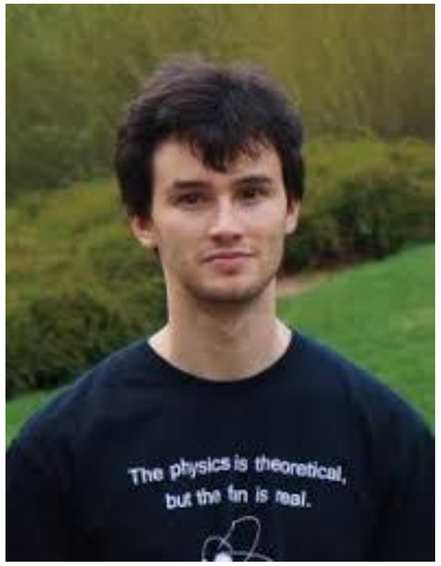
Excitation spectra of quantum matter without quasiparticles

Boston College
October 15, 2020
Subir Sachdev

Talk online: sachdev.physics.harvard.edu



Darshan Joshi



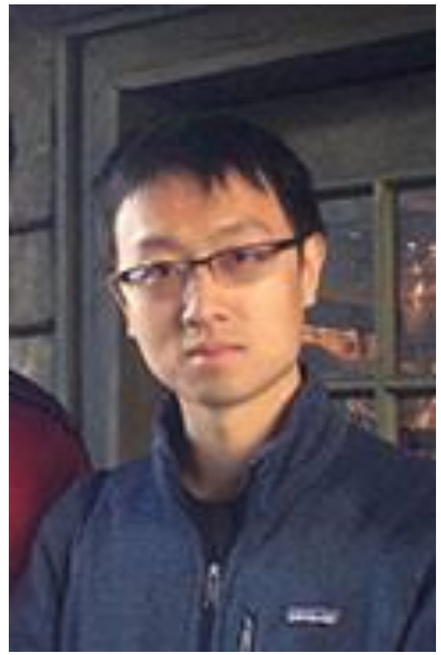
Grigory Tarnopolsky

Physical Review X
10, 021033 (2020)

Chenyuan Li

Antoine Georges

Annals of Physics,
418, 168202 (2020)

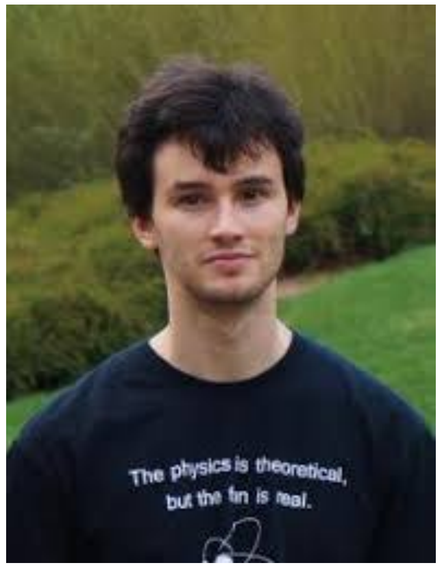


Haoyu Guo

Yingfeu Gu

Maria Tikhanovskaya

Haoyu Guo



Grigory Tarnopolsky

to appear….

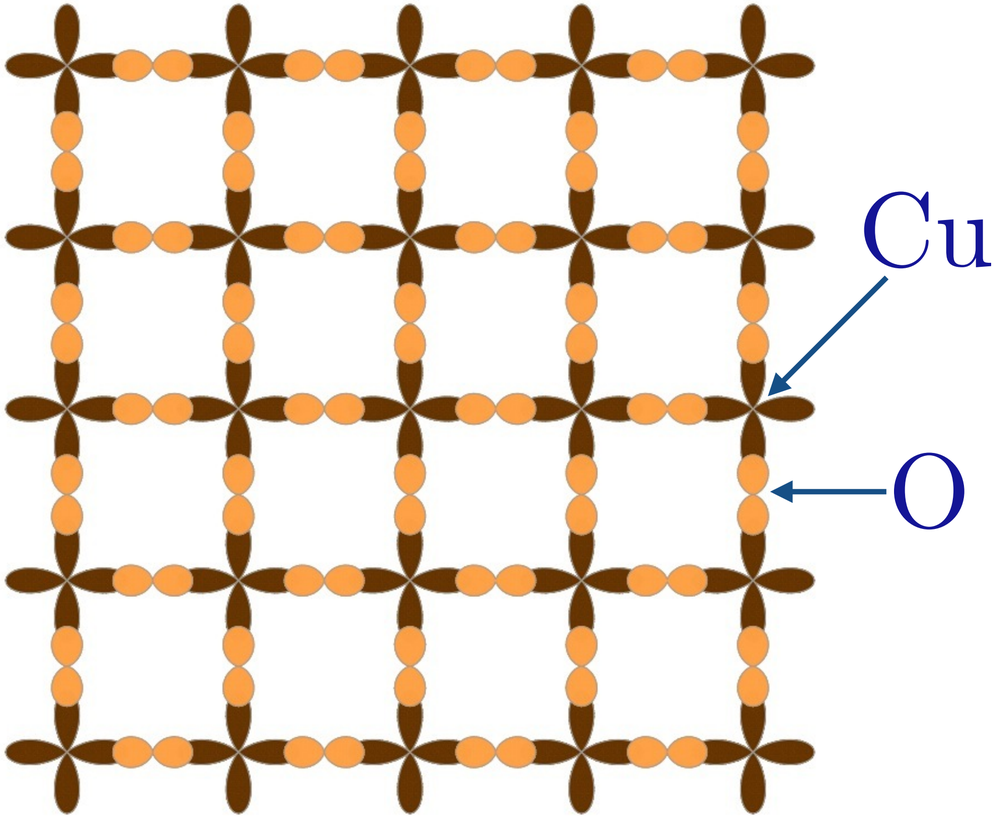
Henry Shackleton

Alexander Wietek

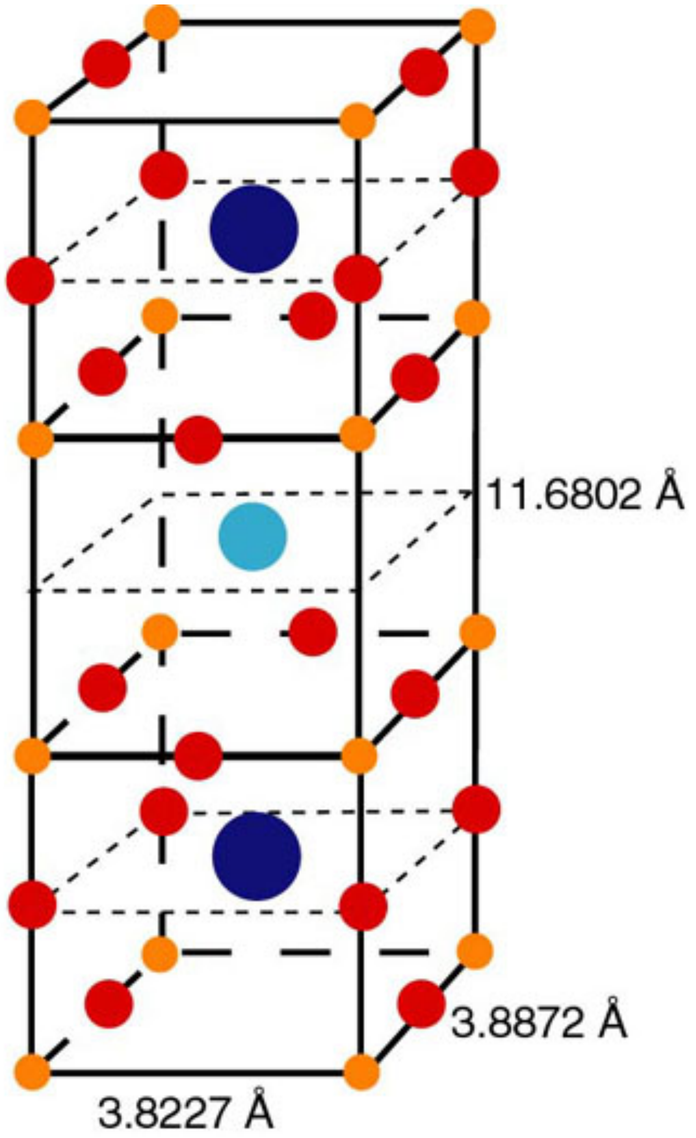
Antoine Georges

to appear….

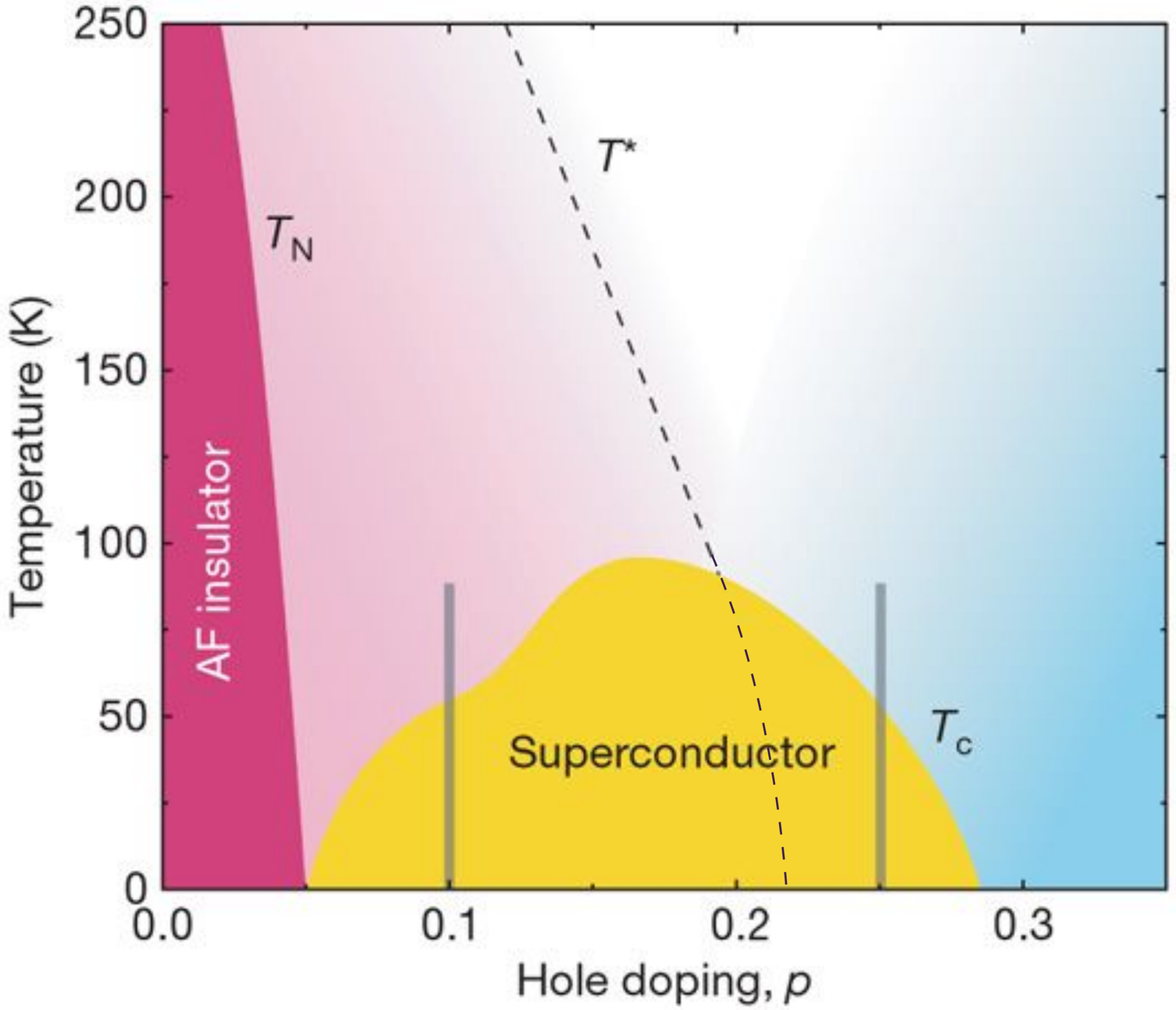
High temperature superconductors

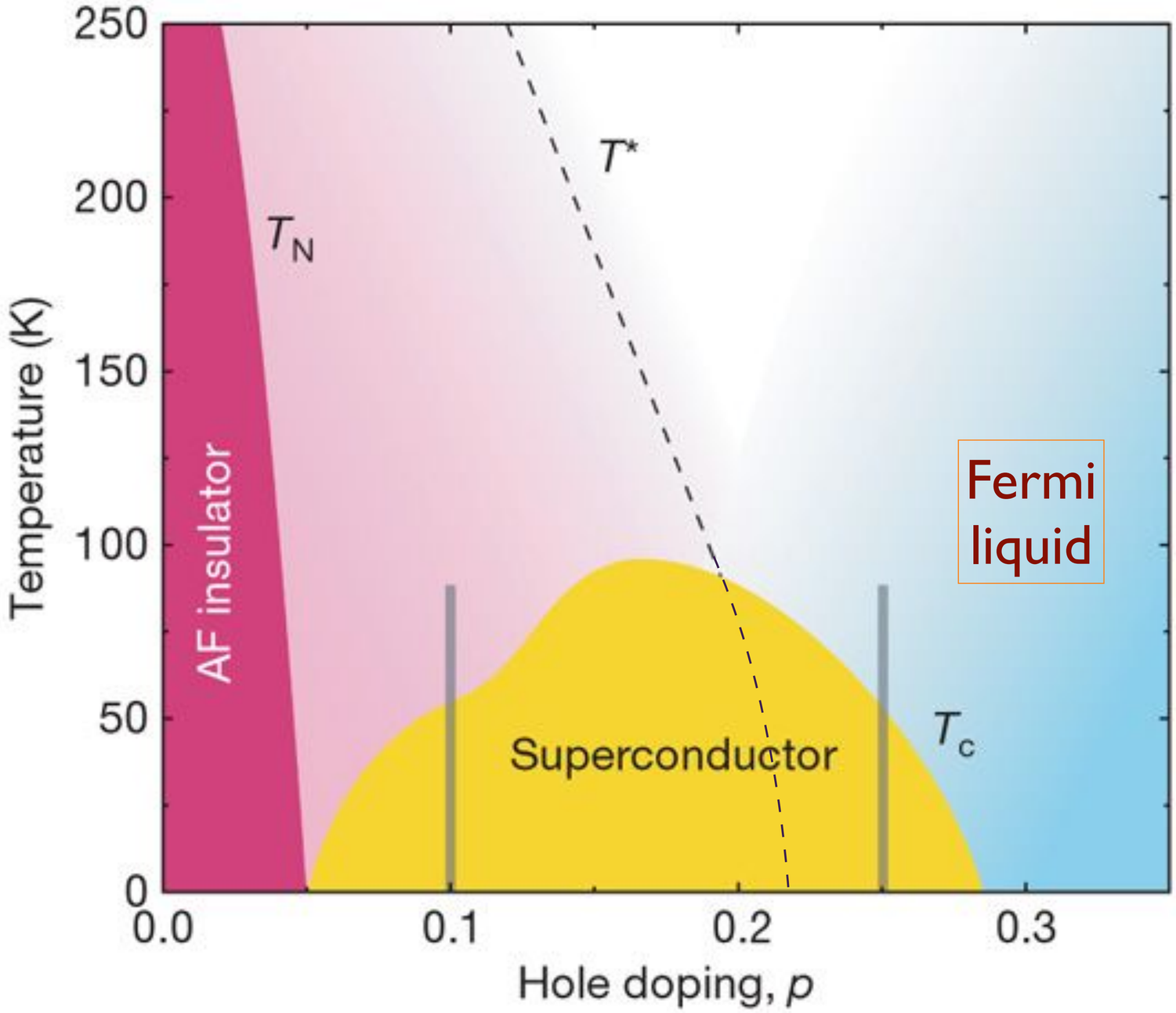


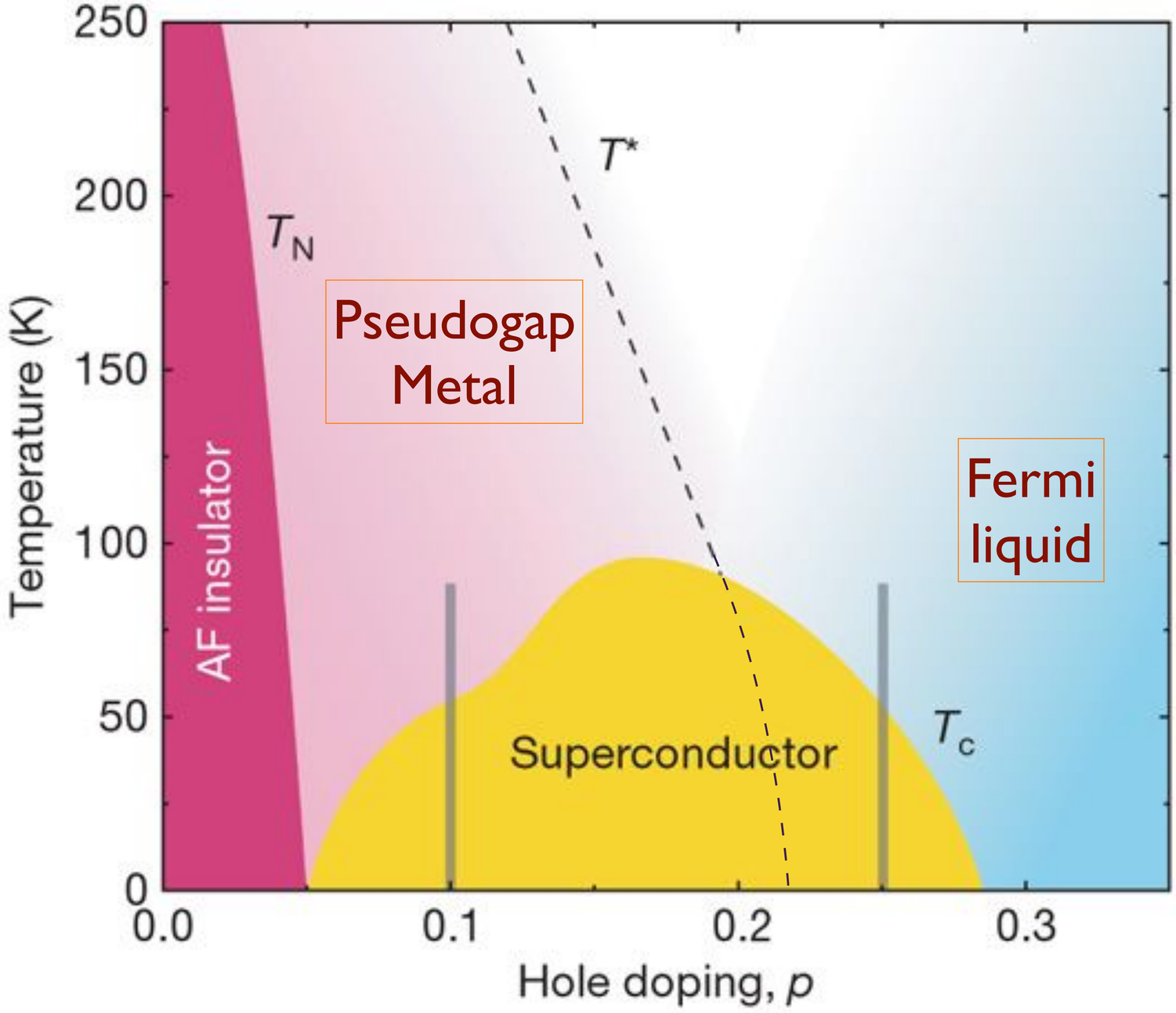
CuO_2 plane



$YBa_2Cu_3O_{6+x}$


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Superconductor
T_c

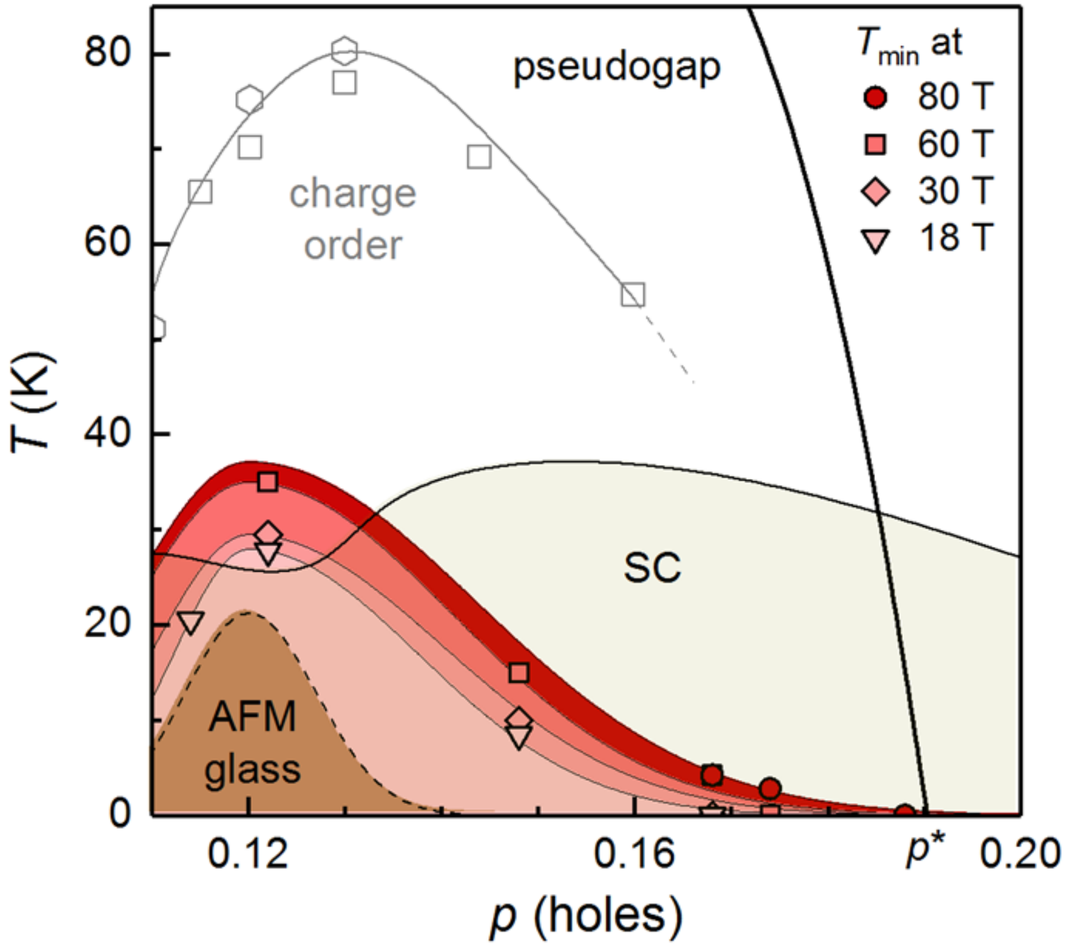

250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Superconductor
T_c
Fermi liquid


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Pseudogap
Metal
Fermi
liquid
Superconductor
T_c

Hidden magnetism at the pseudogap critical point of a high temperature superconductor

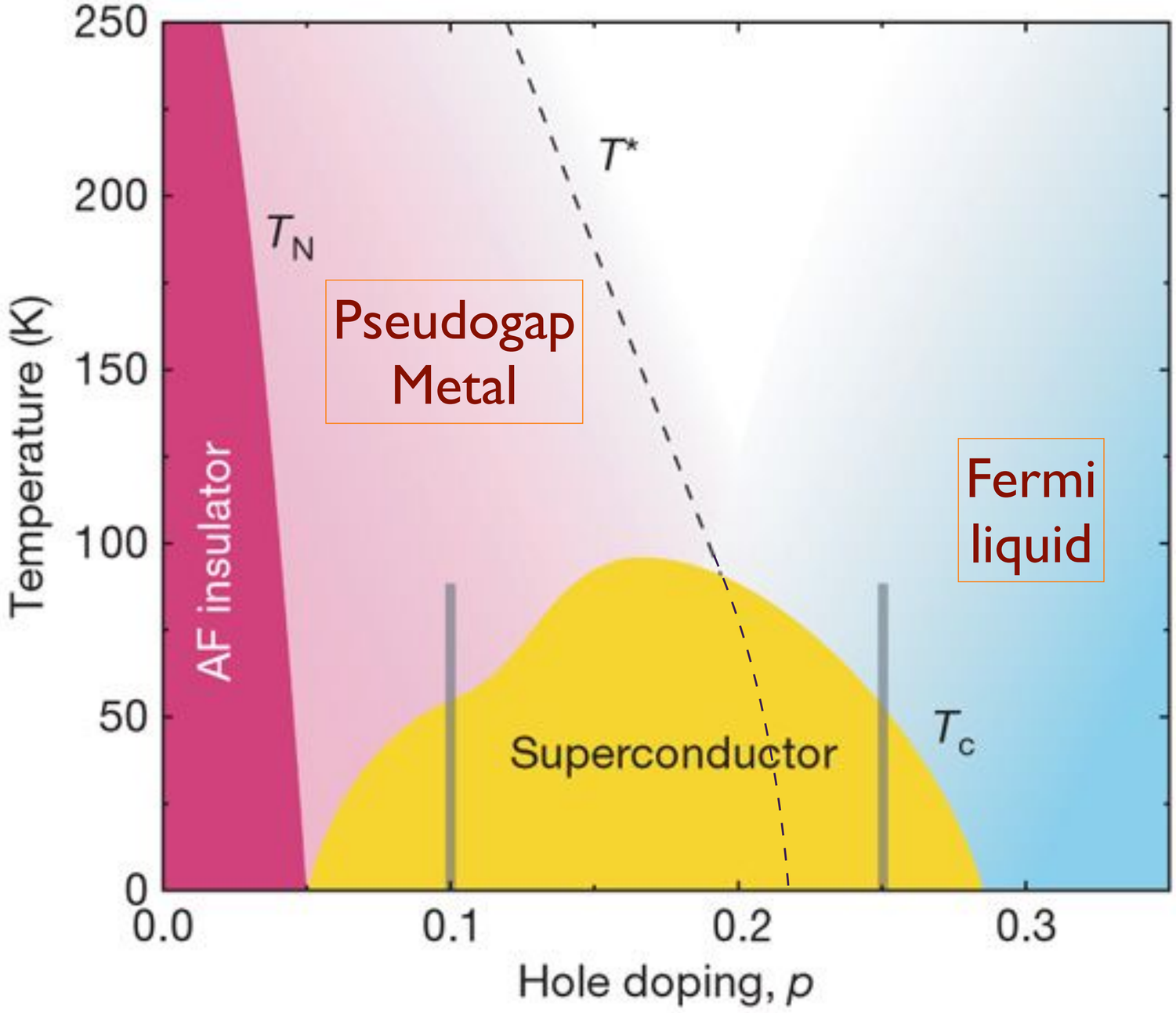
Nature Physics doi: 10.1038/s41567-020-0950-5

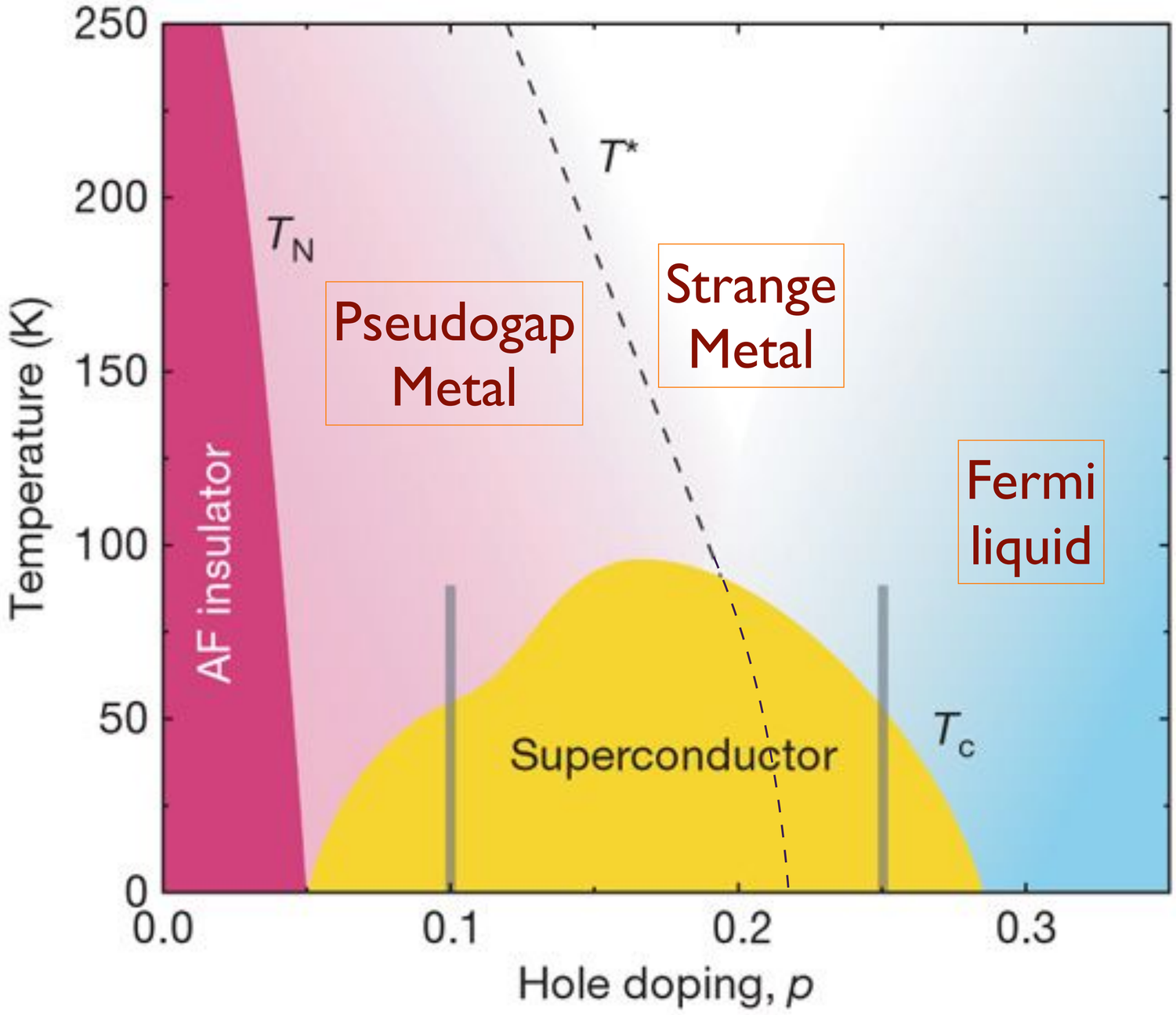
Mehdi Frachet[1]†, Igor Vinograd[1]†, Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1*], Marc-Henri Julien[1*]

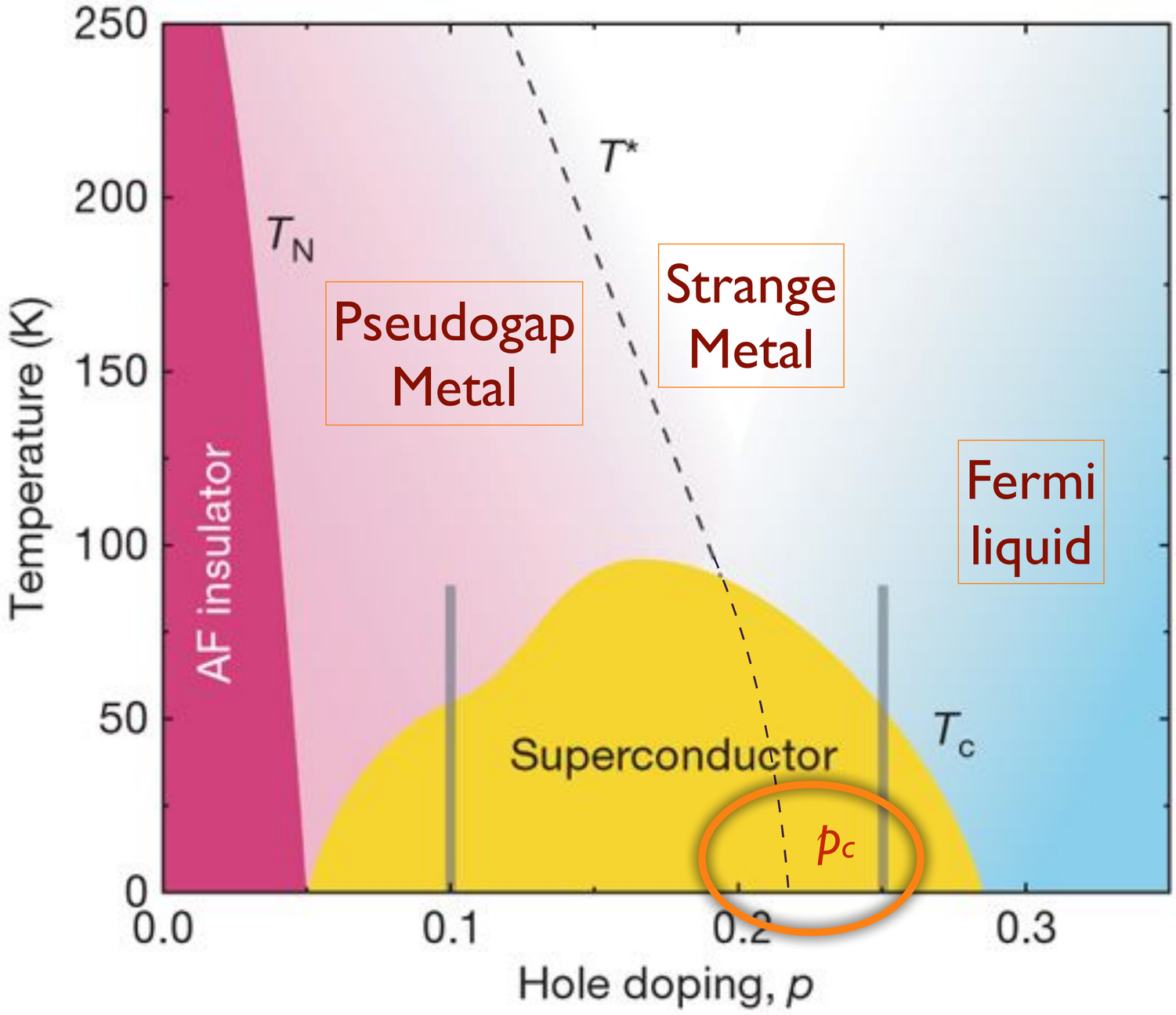


Quasi-static magnetism in the pseudogap state of La2-xSrxCuO4.

Temperature – doping phase diagram representing Tmin, the temperature of the minimum in the sound velocity, at different fields. Since superconductivity precludes the observation of Tmin in zero-field, the dashed line (brown area) represents the extrapolated Tmin(B=0). While not exactly equal to the freezing temperature Tf (see Fig. 2), Tmin is closely tied to Tf and so is expected to have the same doping dependence, including a peak around p = 0.12 in zero/low fields (ref. 2). Onset temperatures of charge order are from ref. 33 (squares) and 35 (hexagons).


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Pseudogap
Metal
Fermi
liquid
Superconductor
T_c


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Pseudogap Metal
Strange Metal
Fermi liquid
Superconductor
T_c


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Pseudogap Metal
Strange Metal
Fermi liquid
Superconductor
T_c
p_c

Remarkable recent observation of
'Planckian' strange metal transport in cuprates,
pnictides, magic-angle graphene, and
ultracold atoms: the resistivity, ρ, is linear
in T down to very low T.
Using the Drude formula

$$\rho = \frac{m^*}{ne^2}\,\frac{1}{\tau_D}$$

a universal Drude scattering rate is observed

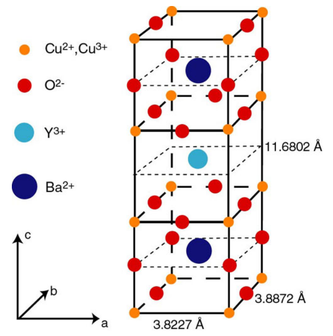
$$\frac{1}{\tau_D} \approx \frac{k_B T}{\hbar}\,,$$

independent of the strength of interactions,
as $T \to 0$

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Slope of *T*-linear resistivity vs Planckian limit in seven materials.

$$\frac{1}{\tau_D} = \alpha \frac{k_B T}{\hbar}$$



A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, Nature Physics **15**, 142 (2019)

Challenge for theory:

A model of a metal in which the resistivity, ρ, obeys

$$\lim_{T \to 0} \frac{d\rho}{dT} \neq 0$$

$$\rho(T) = \rho(0) + AT + \ldots, \quad T \to 0\,.$$

Why random and all-to-all couplings ?

Why random and all-to-all couplings ?

- Randomness is present in the real system.

Why random and all-to-all couplings ?

- Randomness is present in the real system.
- Randomness self-averages (except for certain correlators in spin-glass phase) — Green's functions are the same on every site.

Why random and all-to-all couplings ?

- Randomness is present in the real system.
- Randomness self-averages (except for certain correlators in spin-glass phase) — Green's functions are the same on every site.
- The pseudogap-Fermi liquid transition is primarily a transition in many-body entanglement which survives presence of randomness.

Why random and all-to-all couplings ?

- Randomness is present in the real system.
- Randomness self-averages (except for certain correlators in spin-glass phase) — Green's functions are the same on every site.
- The pseudogap-Fermi liquid transition is primarily a transition in many-body entanglement which survives presence of randomness.
- Introducing randomness removes the "distractions" of other parameters.

Why random and all-to-all couplings ?

- Randomness is present in the real system.
- Randomness self-averages (except for certain correlators in spin-glass phase) — Green's functions are the same on every site.
- The pseudogap-Fermi liquid transition is primarily a transition in many-body entanglement which survives presence of randomness.
- Introducing randomness removes the "distractions" of other parameters.
- The problem maps onto a model of a "quantum impurity" in a self-consistent environment. Closely related models are obtained in non-random models in the limit of large spatial dimension in extended dynamical mean-field theory.

Why random and all-to-all couplings ?

- Randomness is present in the real system.
- Randomness self-averages (except for certain correlators in spin-glass phase) — Green's functions are the same on every site.
- The pseudogap-Fermi liquid transition is primarily a transition in many-body entanglement which survives presence of randomness.
- Introducing randomness removes the "distractions" of other parameters.
- The problem maps onto a model of a "quantum impurity" in a self-consistent environment. Closely related models are obtained in non-random models in the limit of large spatial dimension in extended dynamical mean-field theory.
- Analytic and numeric progress is possible, and we can compare their results !

1. Random *J* model (insulator)

   Operator spectrum and numerics

2. Random t-*J* model (metals)

   Deconfined criticality and numerics

3. Random t-*J*-*U* model

1. Random *J* model (insulator)

   Operator spectrum and numerics

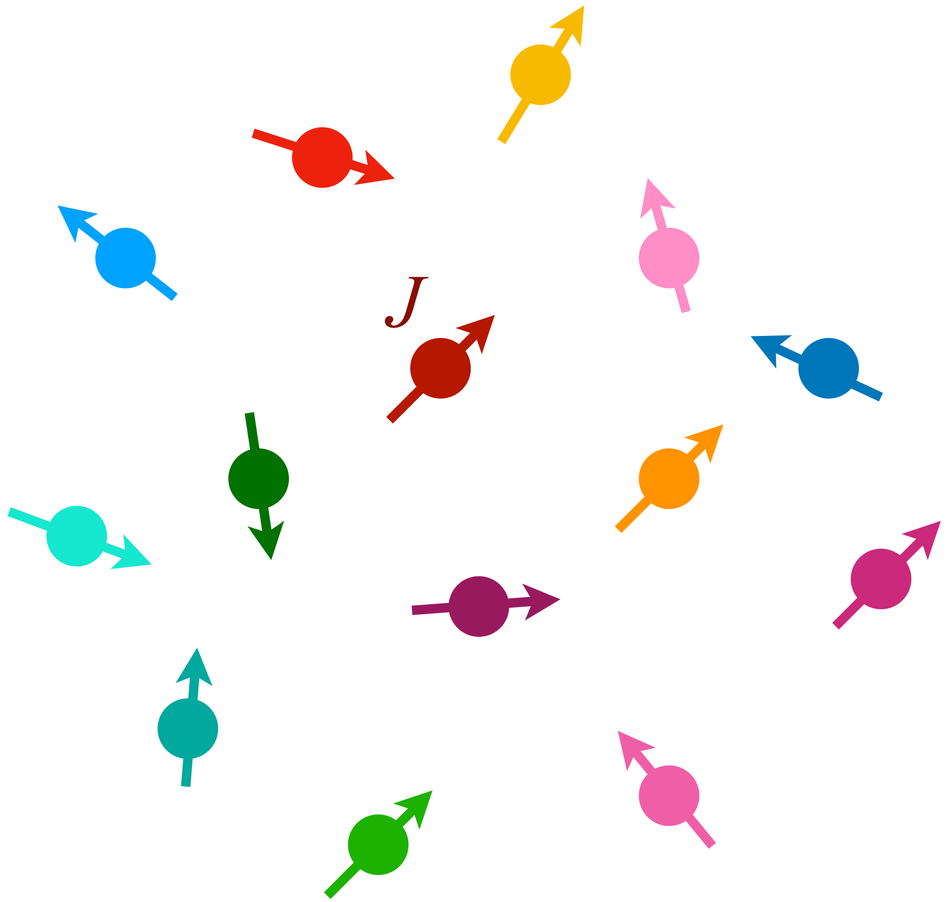
2. Random t-*J* model (metals)

   Deconfined criticality and numerics

3. Random t-*J*-*U* model

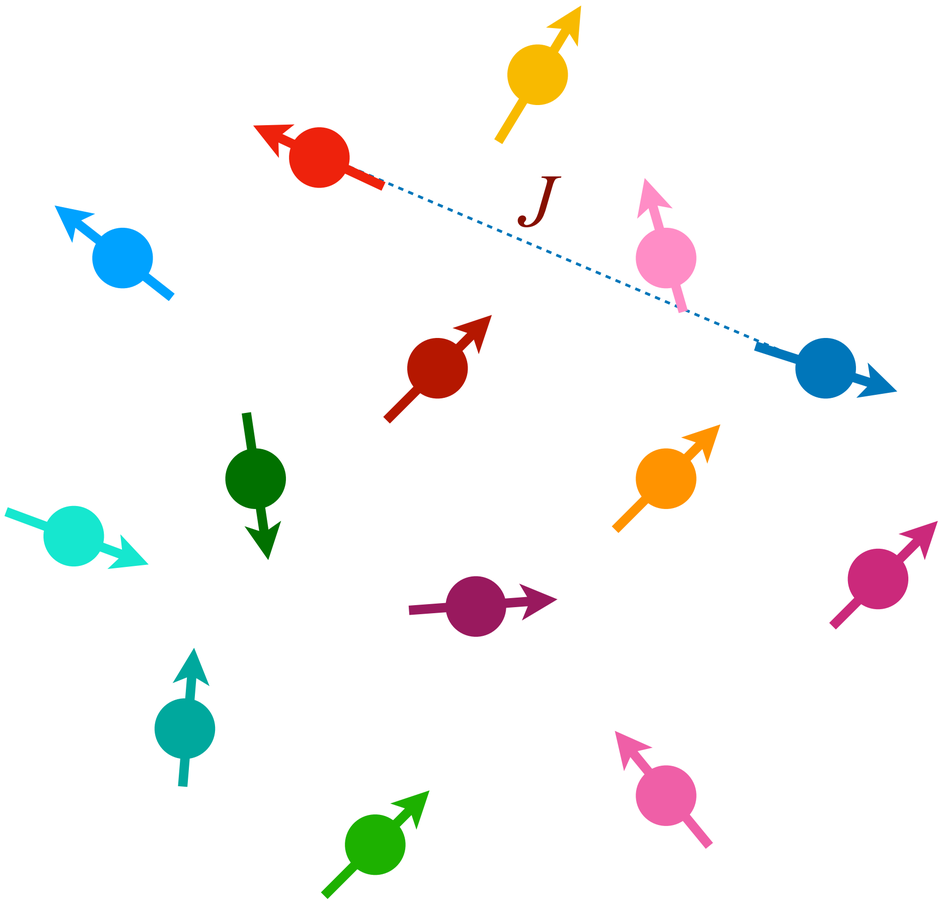
Random J model (insulator)

$$H = \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$



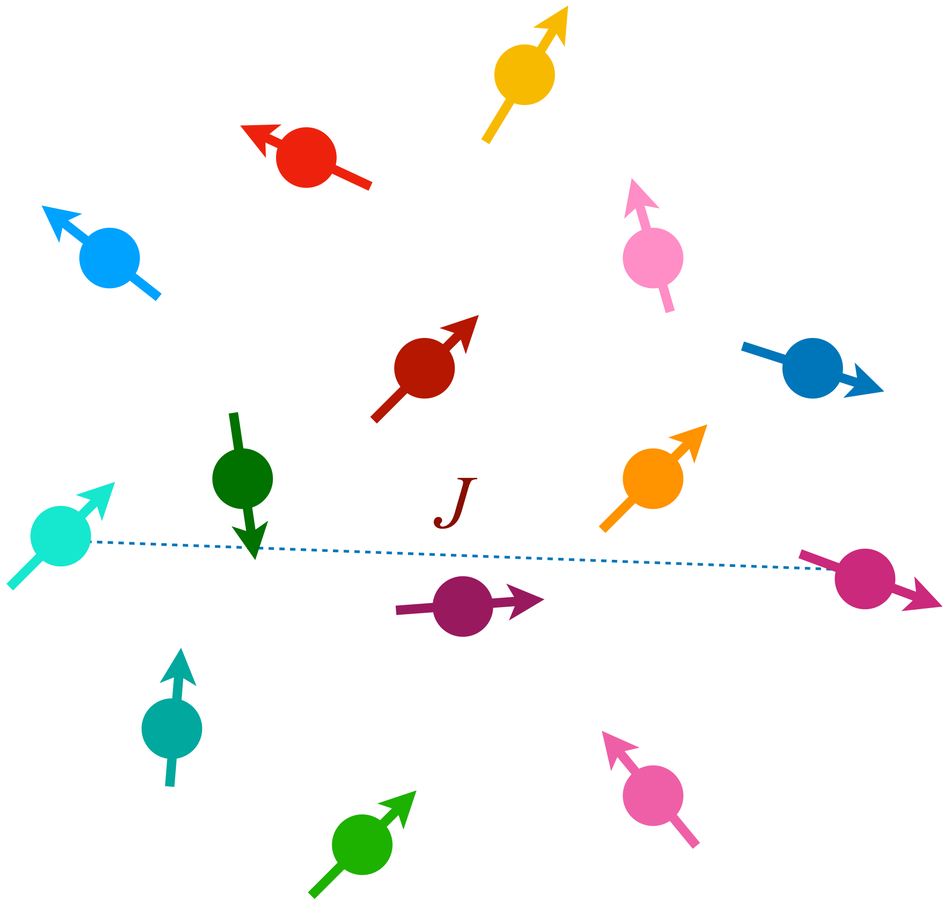
Random J model (insulator)

$$H = \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$



Random J model (insulator)

$$H = \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$



Random J model (insulator)

$$H = \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} f^{\dagger}_{i\alpha} \vec{\sigma}_{\alpha\beta} f_{i\beta}, \quad \sum_{\alpha} f^{\dagger}_{i\alpha} f_{i\alpha} = 1$$

$$J_{ij} \text{ random}, \ \overline{J_{ij}} = 0, \ \overline{J_{ij}^2} = J^2$$

Fermionic spinons

Random J model (insulator): SU(M) symmetry

Express the spin operator in terms of fermionic spinons $\vec{S} = (1/2) f_\alpha^\dagger \vec{\sigma}_{\alpha\beta} f_\beta$, and let $\alpha = 1 \ldots M$. The fermions obey the constraint

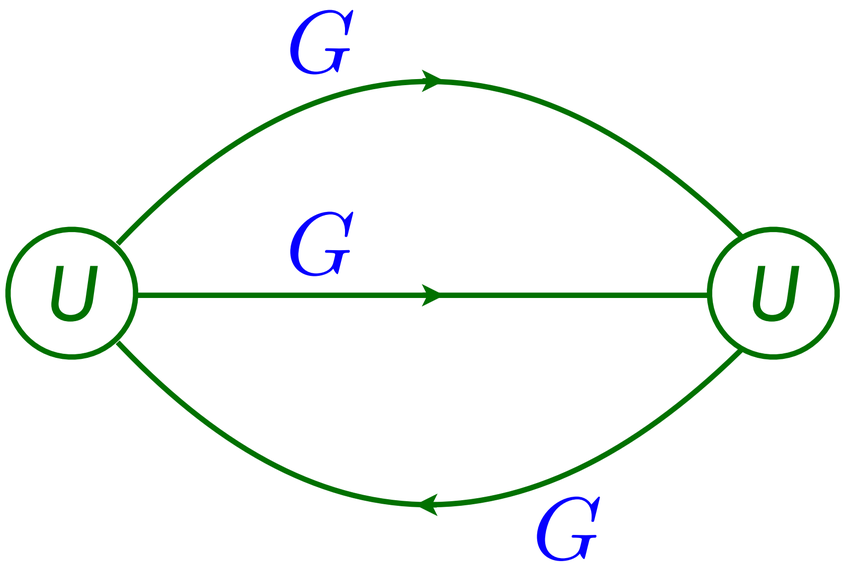
$$\sum_{\alpha=1}^{M} f_\alpha^\dagger f_\alpha = \frac{M}{2}$$

In the limit of large N, *followed* by large M, we obtain the SYK equations for the spinon Green's function G and self energy Σ:

$$G(i\omega) = \frac{1}{i\omega - \Sigma(i\omega)}$$

$$\Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$\Sigma =$



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Random J model (insulator)

$$G(i\omega) = \frac{1}{i\omega - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

The solution at $T = 0$ for $|\tau| \gg 1/J$ is

$$G(\tau) \sim \frac{\mathrm{sgn}(\tau)}{\sqrt{\tau}} \quad , \quad \left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

or for $|z| \ll J$

$$G(z) = \frac{A}{\sqrt{z}} \quad , \quad \Sigma(z) = -\frac{1}{A}\sqrt{z}$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Time reparameterization symmetry

$$G(i\omega) = \frac{1}{i\omega - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

At frequencies $\ll J$, the $i\omega$ can be dropped, and without it the SYK equations are invariant under time reparametrization and gauge transformations. The singular part of the self-energy and the Green's function obey

$$\int_0^\beta d\tau_2 \, \Sigma(\tau_1, \tau_2) G(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)$$

$$\Sigma(\tau_1, \tau_2) = -J^2 G^2(\tau_1, \tau_2) G(\tau_2, \tau_1)$$

These equations can be solved exactly at $T = 0$ and $T > 0$.

A. Kitaev, 2015

S. Sachdev, PRX **5,** 041025 (2015)

Time reparameterization symmetry

$$\int_0^\beta d\tau_2 \, \Sigma(\tau_1, \tau_2) G(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)$$

$$\Sigma(\tau_1, \tau_2) = -J^2 G^2(\tau_1, \tau_2) G(\tau_2, \tau_1)$$

These equations are invariant under

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{G}(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{\Sigma}(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

By using $f(\sigma) = \tan(\pi T \sigma)/(\pi T)$ we can obtain the $T > 0$ solution. A. Kitaev, 2015

S. Sachdev, PRX **5,** 041025 (2015)

Time reparameterization symmetry

Let us write the large N saddle point solutions as

$$\begin{aligned} G_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-1/2} \\ \Sigma_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-3/2}. \end{aligned}$$

and so $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$. Now when we transform to σ co-ordinates, $\widetilde{G}(\sigma_1, \sigma_2)$ will not in general be a function of $\sigma_1 - \sigma_2$: so time reparameterization symmetry is spontaneously broken by the saddle point.

Time reparameterization symmetry

Let us write the large N saddle point solutions as

$$\begin{aligned} G_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-1/2} \\ \Sigma_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-3/2}. \end{aligned}$$

and so $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$. Now when we transform to σ co-ordinates, $\widetilde{G}(\sigma_1, \sigma_2)$ will not in general be a function of $\sigma_1 - \sigma_2$: so time reparameterization symmetry is spontaneously broken by the saddle point.

We search for the transformations under which $\widetilde{G}(\sigma_1, \sigma_2) = G_s(\sigma_1 - \sigma_2)$: this is true only for the SL(2, R) transformations under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

So the (approximate) time reparametrization symmetry is spontaneously broken down to SL(2, R) by the saddle point.

A. Kitaev

Random J model (insulator)

$$G(i\omega) = \frac{1}{\cancel{i\omega} - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

The conformal (SL(2,R) invariant) solution for $|\tau| \gg 1/J$ and $T \ll J$ is

$$G(\tau) \sim \mathrm{sgn}(\tau) \left| \frac{\pi T}{\sin(\pi T \tau)} \right|^{1/2} \quad , \quad G(\omega) \sim \frac{-i}{\sqrt{T}} \frac{\Gamma\left(\frac{1}{4} - \frac{i\hbar\omega}{2\pi T}\right)}{\Gamma\left(\frac{3}{4} - \frac{i\hbar\omega}{2\pi T}\right)}$$

and the local spin susceptibility $\chi_L(i\omega_n) = \int_0^\beta d\tau \left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle$ is

$$\mathrm{Im}\chi_L(\omega) \sim \tanh\left(\frac{\hbar\omega}{2T}\right)$$

A. Georges and O. Parcollet PRB **59**, 5341 (1999)

Corrections to SL(2,R) conformal invariance

The irrelevant $f_\alpha^\dagger \partial_\tau f_\alpha$ term will induce corrections proportional to conformal primaries

$$S_{\rm SYK} = S_{\rm CFT} + \sum_h g_h \int_0^\beta d\tau O_h(\tau)\,,$$

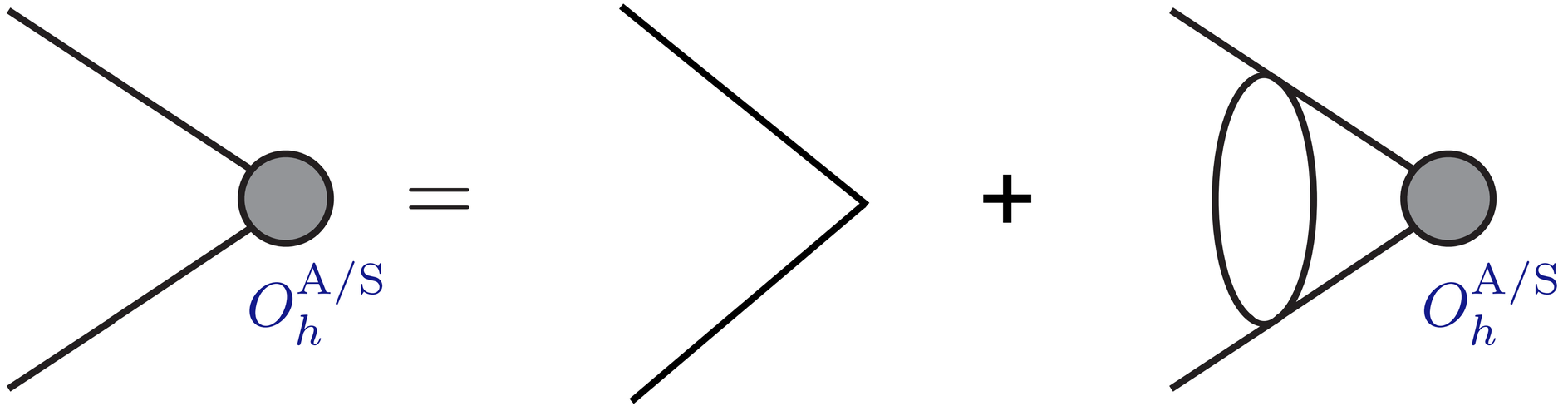
where O_h has scaling dimension h with $\langle O_h(\tau) O_h(0)\rangle \sim |\tau|^{-2h}$. Note that $\dim[g_h] = 1-h$, so $h > 1$ is an irrelevant perturbation. This will induce a correction

$$G(\tau) = G_{\rm conformal}(\tau)\left[1 - \frac{\alpha_h}{|J\tau|^{h-1}} + \ldots\right],$$

where we can relate α_h to g_h by computing the conformal 3-point correlator $\langle f_\alpha^\dagger(\tau_1) f_\alpha(\tau_2) O_h(\tau_3)\rangle$. All expressions can be generalized to $T > 0$ by a conformal mapping.

Corrections to SL(2,R) conformal invariance

There are an infinite set of bilinear primary operators $O_h^{\rm A}(\tau)$ and $O_h^{\rm S}(\tau)$, which descend from the UV operators $O_{h_n}^{\rm A} = f_\alpha^\dagger \partial_\tau^{2n+1} f_\alpha$ and $O_{h_n}^{\rm S} = f_\alpha^\dagger \partial_\tau^{2n} f_\alpha$ for $n = 0, 1, 2, \ldots$. To compute the scaling dimensions of the operators $O_h^{\rm A/S}(\tau)$ we consider three point functions

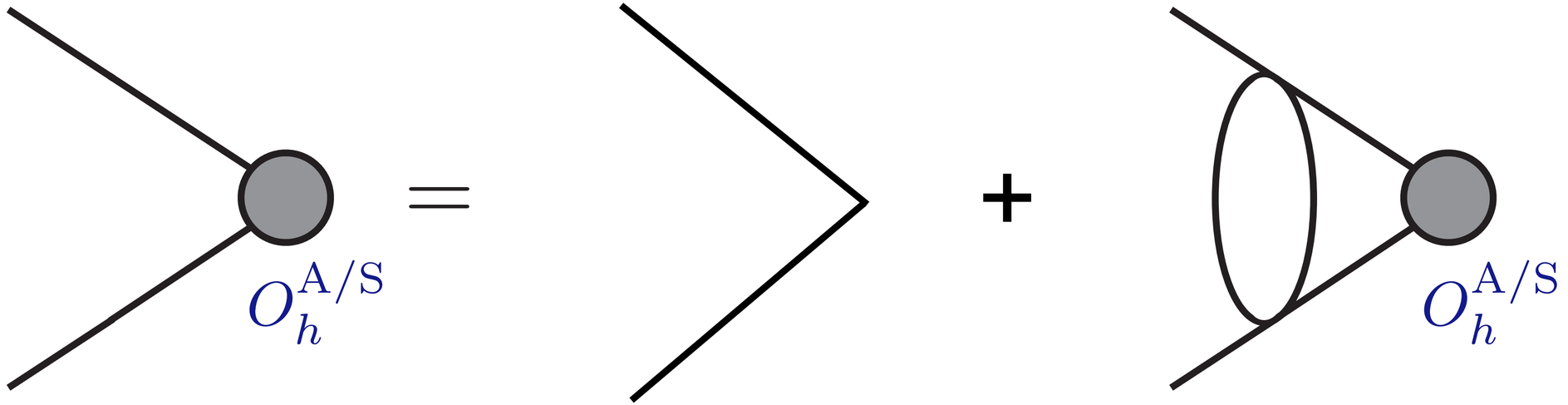


Solution of this integral equation yields the conditions $k_A(h) = 1$ and $k_S(h) = 1$ where

$$k_A(h) = \frac{3(1-\sin(\pi h))}{\cos(\pi h)(2h-1)} \quad , \quad k_S(h) = \frac{(1+\sin(\pi h))}{\cos(\pi h)(1-2h)}$$

Corrections to SL(2,R) conformal invariance

There are an infinite set of bilinear primary operators $O_h^{\rm A}(\tau)$ and $O_h^{\rm S}(\tau)$, which descend from the UV operators $O_{h_n}^{\rm A} = f_\alpha^\dagger \partial_\tau^{2n+1} f_\alpha$ and $O_{h_n}^{\rm S} = f_\alpha^\dagger \partial_\tau^{2n} f_\alpha$ for $n = 0, 1, 2, \ldots$. To compute the scaling dimensions of the operators $O_h^{\rm A/S}(\tau)$ we consider three point functions



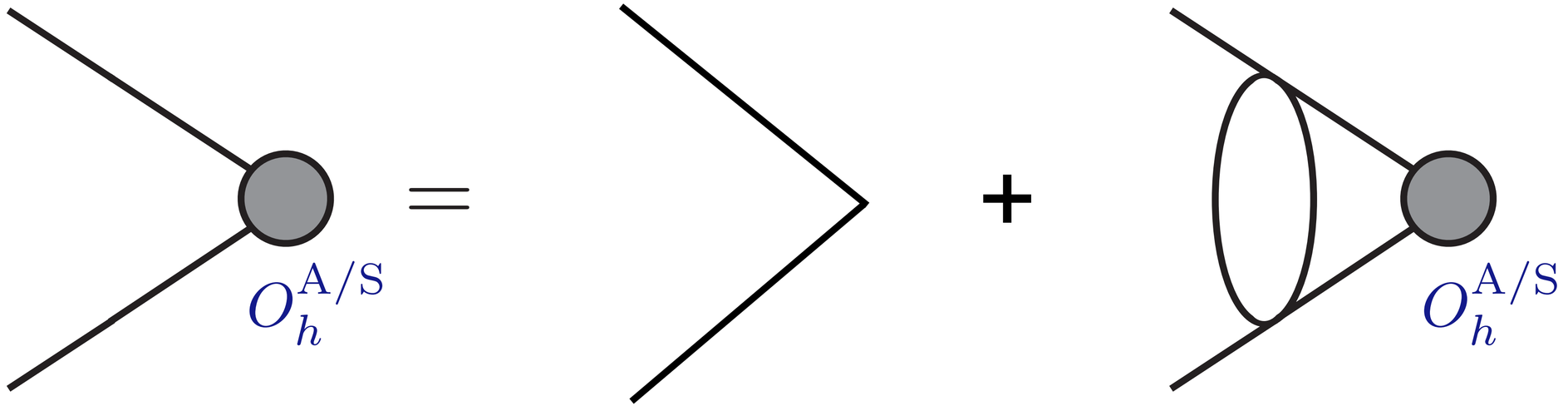
Solution of this integral equation yields the conditions $k_A(h) = 1$ and $k_S(h) = 1$ where

$$k_A(h) = \frac{3(1-\sin(\pi h))}{\cos(\pi h)(2h-1)} \quad , \quad k_S(h) = \frac{(1+\sin(\pi h))}{\cos(\pi h)(1-2h)}$$

'Protected' operators: $k_A(h=2) = 1$: time reparameterization operator

Corrections to SL(2,R) conformal invariance

There are an infinite set of bilinear primary operators $O_h^{\rm A}(\tau)$ and $O_h^{\rm S}(\tau)$, which descend from the UV operators $O_{h_n}^{\rm A} = f_\alpha^\dagger \partial_\tau^{2n+1} f_\alpha$ and $O_{h_n}^{\rm S} = f_\alpha^\dagger \partial_\tau^{2n} f_\alpha$ for $n = 0, 1, 2, \ldots$. To compute the scaling dimensions of the operators $O_h^{\rm A/S}(\tau)$ we consider three point functions



Solution of this integral equation yields the conditions $k_A(h) = 1$ and $k_S(h) = 1$ where

$$k_A(h) = \frac{3(1-\sin(\pi h))}{\cos(\pi h)(2h-1)} \quad , \quad k_S(h) = \frac{(1+\sin(\pi h))}{\cos(\pi h)(1-2h)}$$

'Protected' operators:

$k_A(h=2) = 1$: time reparameterization operator

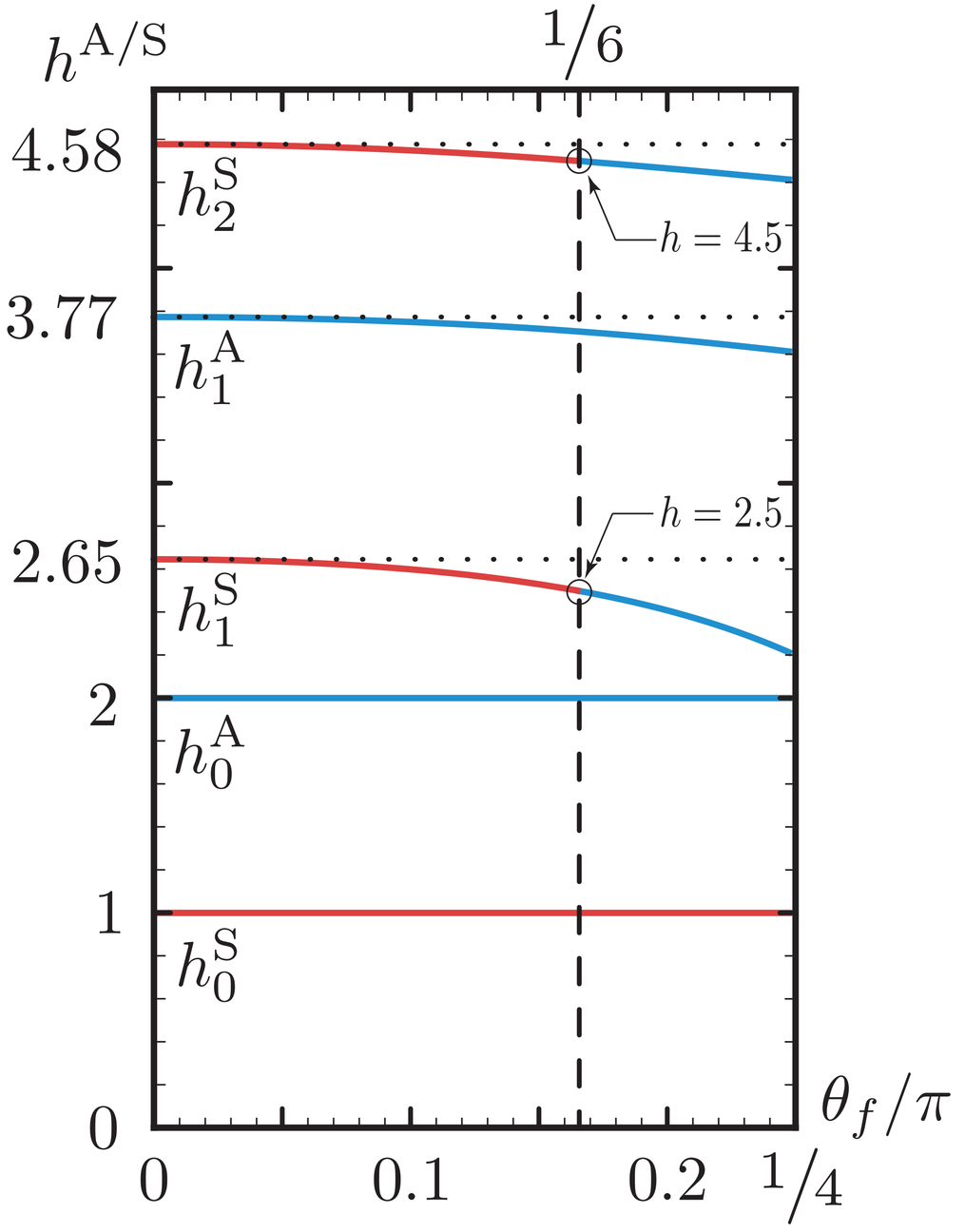
$k_S(h=1) = 1$: emergent U(1) gauge symmetry.

Corrections to SL(2,R) conformal invariance

We can also consider more general models with $\sum_\alpha f^\dagger_{i\alpha} f_{i\alpha} = \kappa M$ (the particle-hole symmetric case is $\kappa = 1/2$. These are characterized by

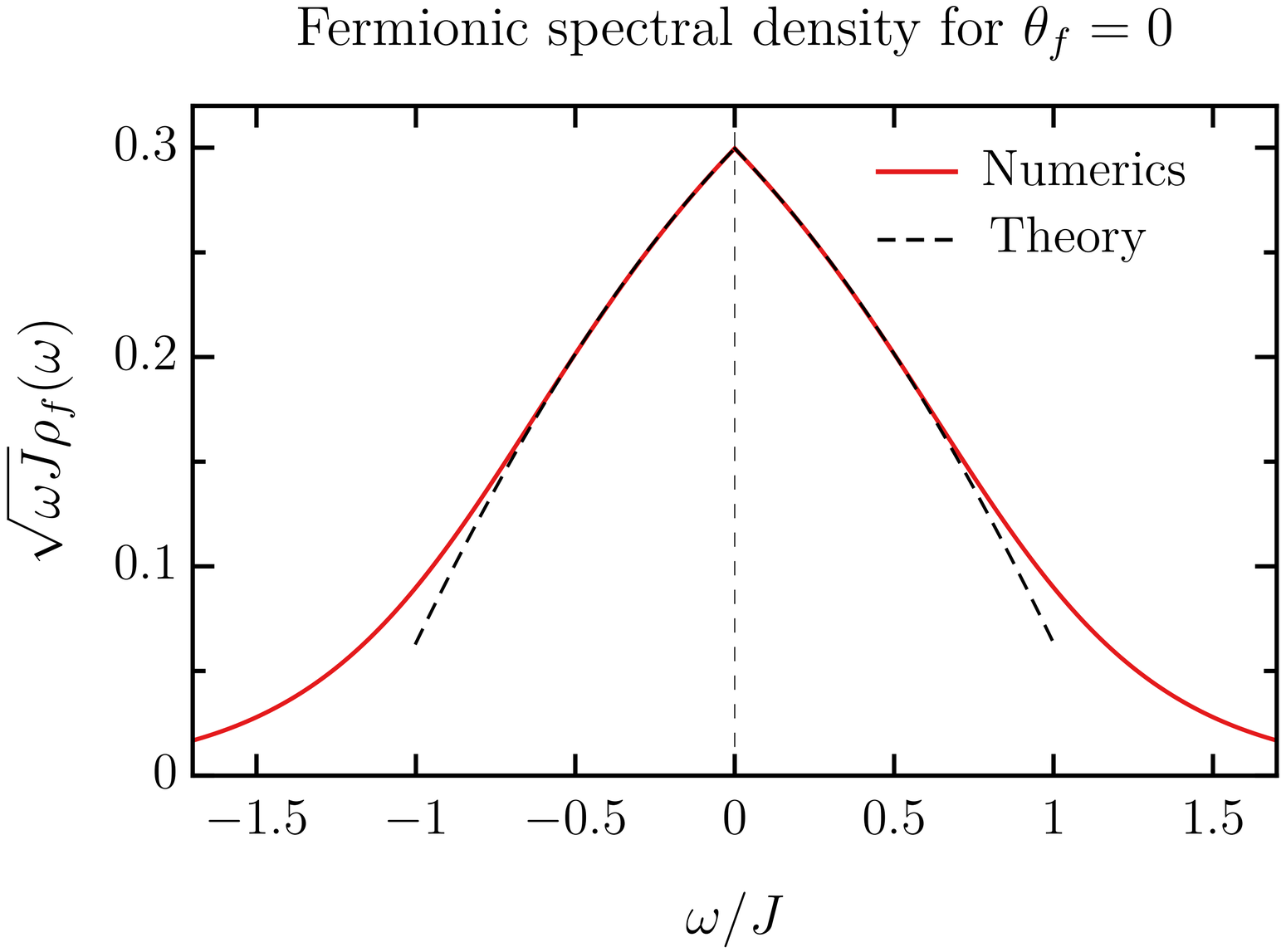
$$G(\omega) = \begin{cases} \dfrac{C e^{-i\pi/4 - i\theta_f}}{\sqrt{\omega}}, & \omega > 0 \\ \dfrac{-C e^{i\pi/4 - i\theta_f}}{\sqrt{|\omega|}}, & \omega < 0 \end{cases}$$

with θ_f determined exactly from κ via a Luttinger relation ($\theta_f = 0$ at $\kappa = 1/2$).



Comparison of $T = 0$ spectral density with numerical solution of SYK equations

$$g_f(\omega) = \sqrt{|\omega|}\,\mathrm{Im}\,G(\omega)$$

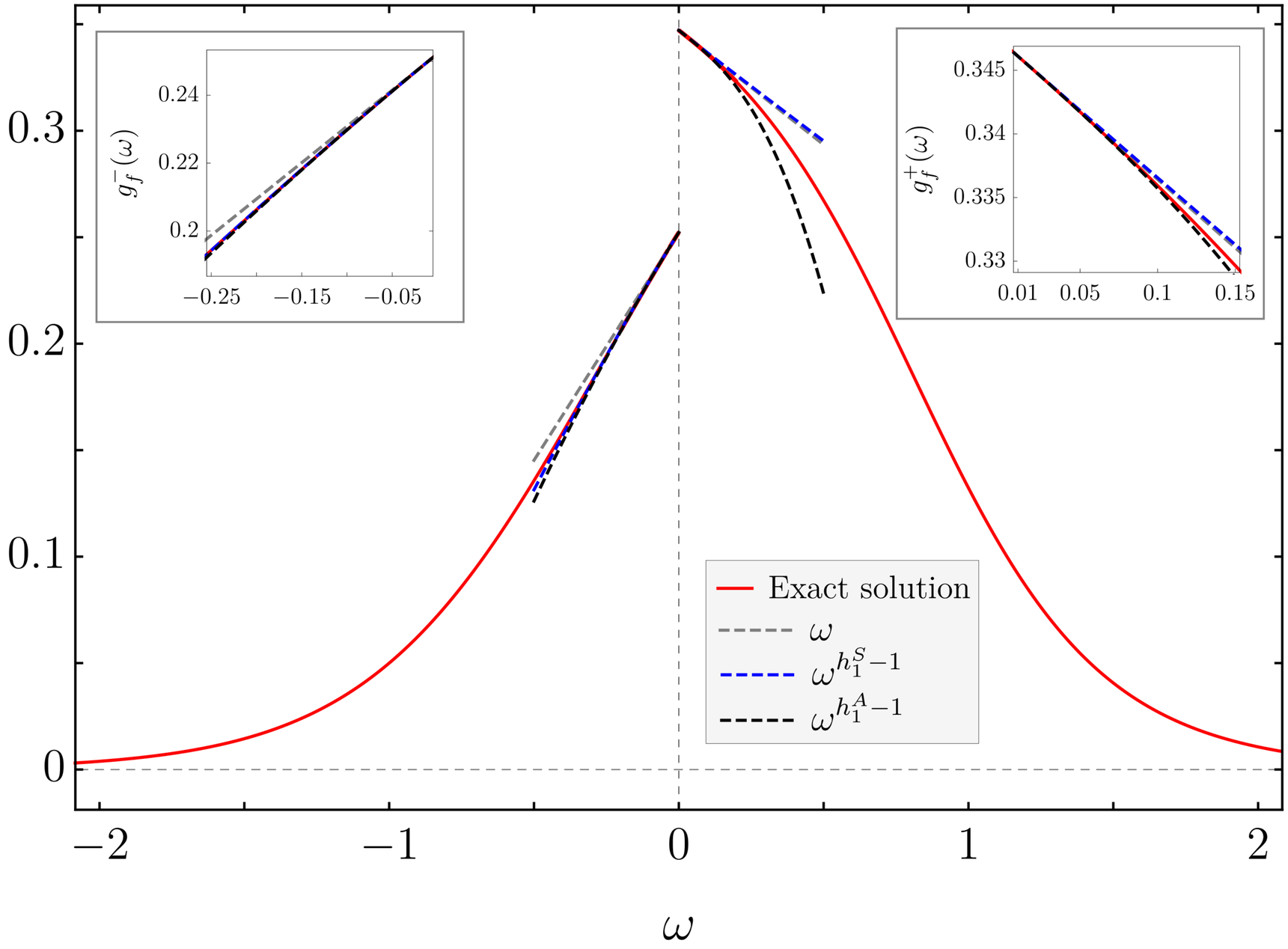


$$g(\omega) = \frac{1}{(4\pi^3)^{\frac{1}{4}} J}\left(1 - 2\alpha_0^{\mathrm{A}}\frac{\omega}{J} - 3(\alpha_0^{\mathrm{A}})^2\left(\frac{\omega}{J}\right)^2 - 0.68\alpha_1^{\mathrm{A}}\left(\frac{\omega}{J}\right)^{2.77} + \frac{26}{3}(\alpha_0^{\mathrm{A}})^3\left(\frac{\omega}{J}\right)^3 - \ldots\right)$$

Comparison of $T = 0$ spectral density with numerical solution of SYK equations

$$g_f(\omega) = \sqrt{|\omega|}\,\mathrm{Im}\,G(\omega)$$

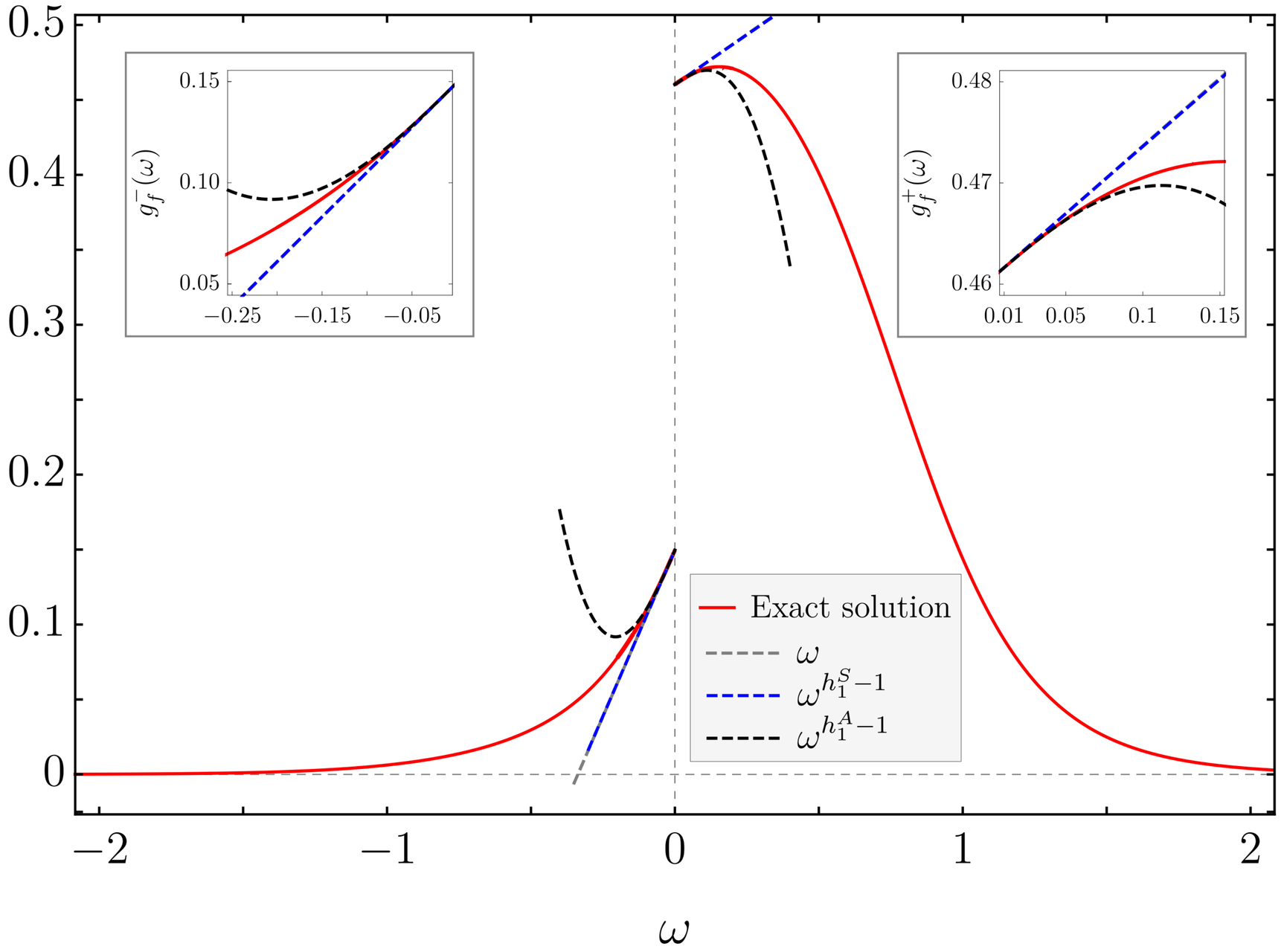
$g_f(\omega) = \sqrt{\omega}\rho_f$ as a function of ω at $\theta_f = 0.05\pi$



Comparison of $T = 0$ spectral density with numerical solution of SYK equations

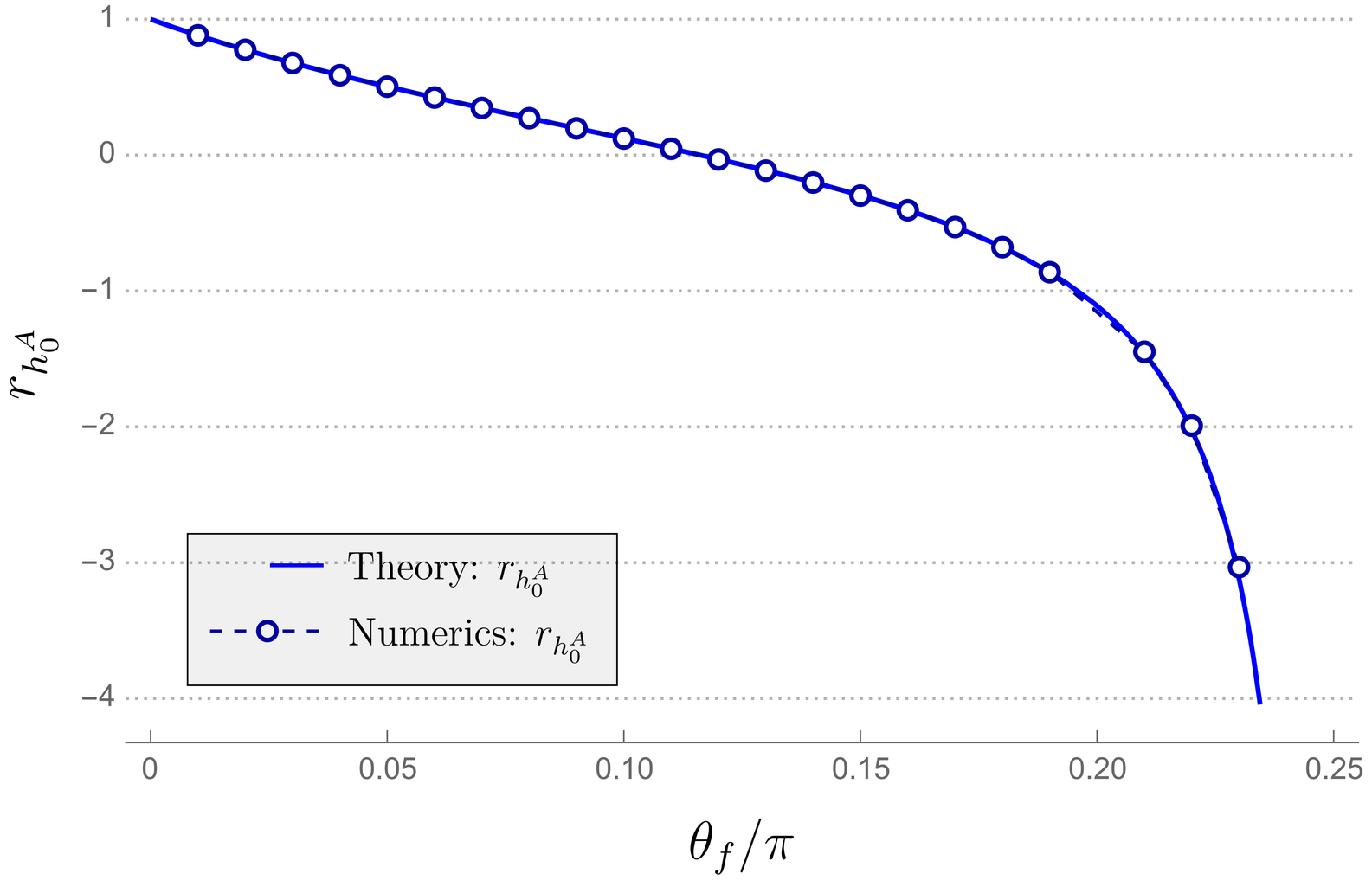
$$g_f(\omega) = \sqrt{|\omega|}\,\mathrm{Im}\,G(\omega)$$

$g_f(\omega) = \sqrt{\omega}\rho_f$ as a function of ω at $\theta_f = 0.15\pi$



Comparison of $T = 0$ spectral density with numerical solution of SYK equations

$$\mathrm{Im}\, G(\omega) = \frac{g_f(\omega)}{\sqrt{|\omega|}}$$



Ratios of $h = 2$ corrections at $\omega > 0$ and $\omega < 0$ compared with conformal perturbation theory

Corrections to SL(2,R) conformal invariance

The $h = 2$ correction to the local spin susceptibility is

$$\mathrm{Im}\chi_L(\omega) \sim \tanh\left(\frac{\hbar\omega}{2T}\right)\left[1 - \mathcal{C}\gamma\,\hbar\omega\tanh\left(\frac{\hbar\omega}{2T}\right) + \ldots\right]$$

where the specific heat per spin component $= \gamma T$, and

$$\mathcal{C} = \frac{24}{\pi(2+3\pi)}$$

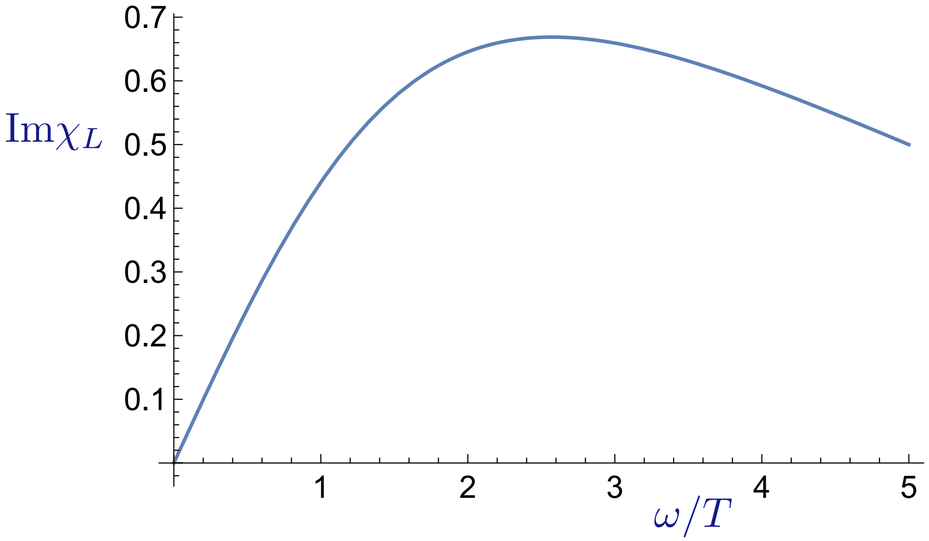
At $T = 0$,

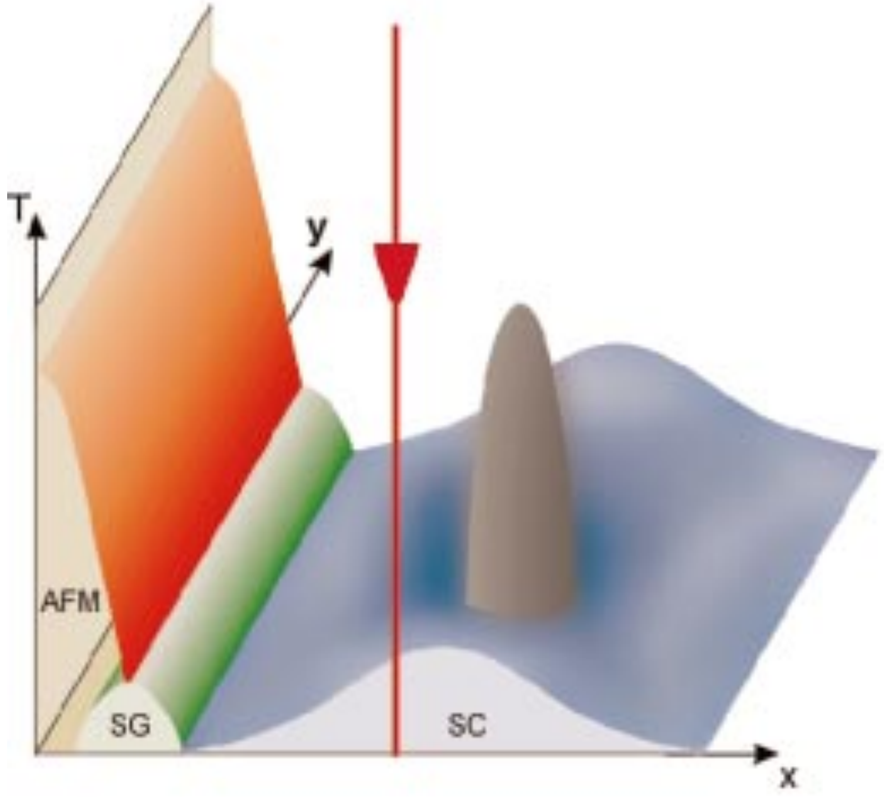
$$\mathrm{Im}\chi_L(\omega) \sim \mathrm{sgn}(\omega)\left[1 - \mathcal{C}\gamma|\omega| - \frac{7}{16}(\mathcal{C}\gamma)^2|\omega|^2 - \mathcal{C}'|\omega|^{2.77354\ldots} + \frac{37}{48}(\mathcal{C}\gamma)^3|\omega|^3 - \ldots\right]$$

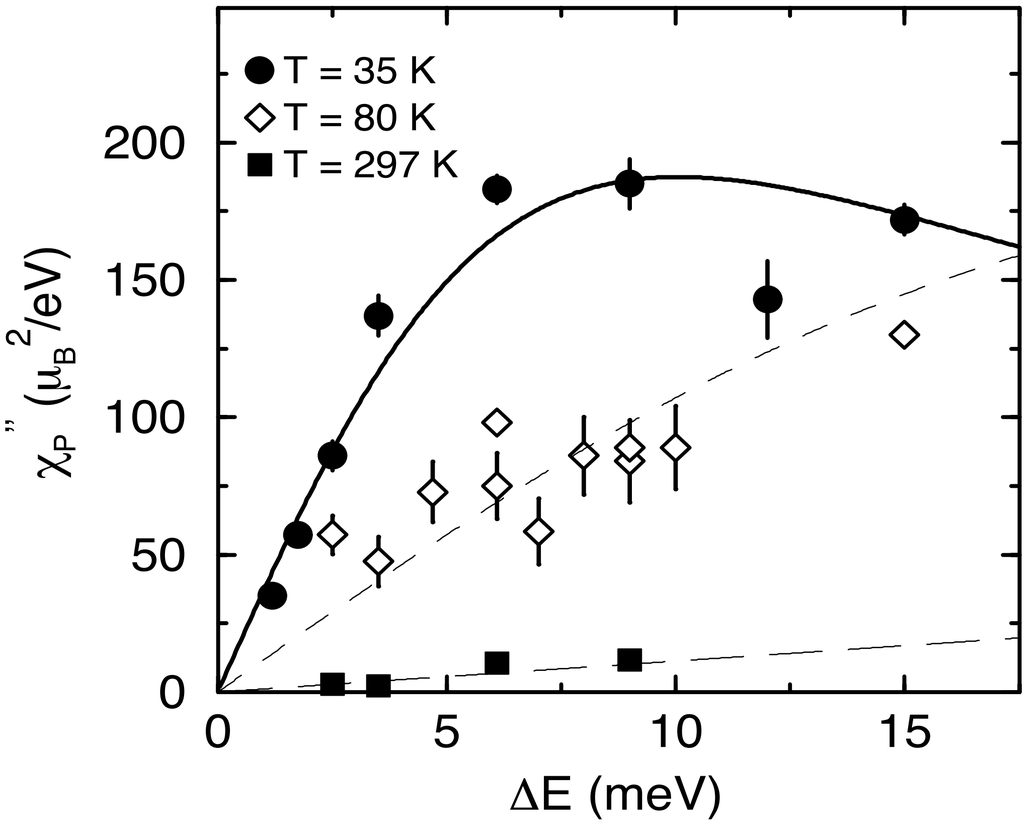
Nearly Singular Magnetic Fluctuations in the Normal State of a High-T_c Cuprate Superconductor

G. Aeppli, T. E. Mason,* S. M. Hayden, H. A. Mook, J. Kulda

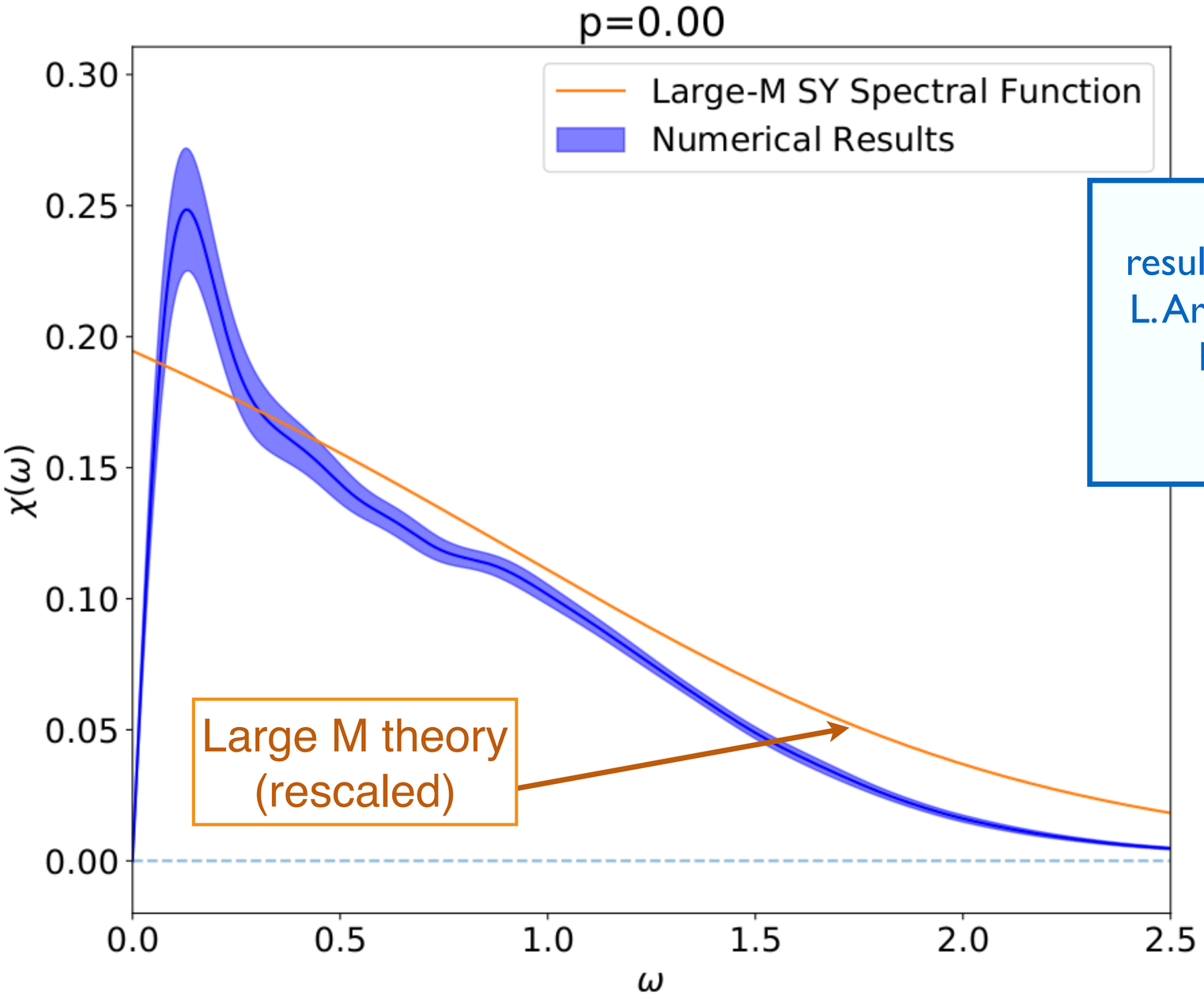
Science **278**, 1432 (1997)





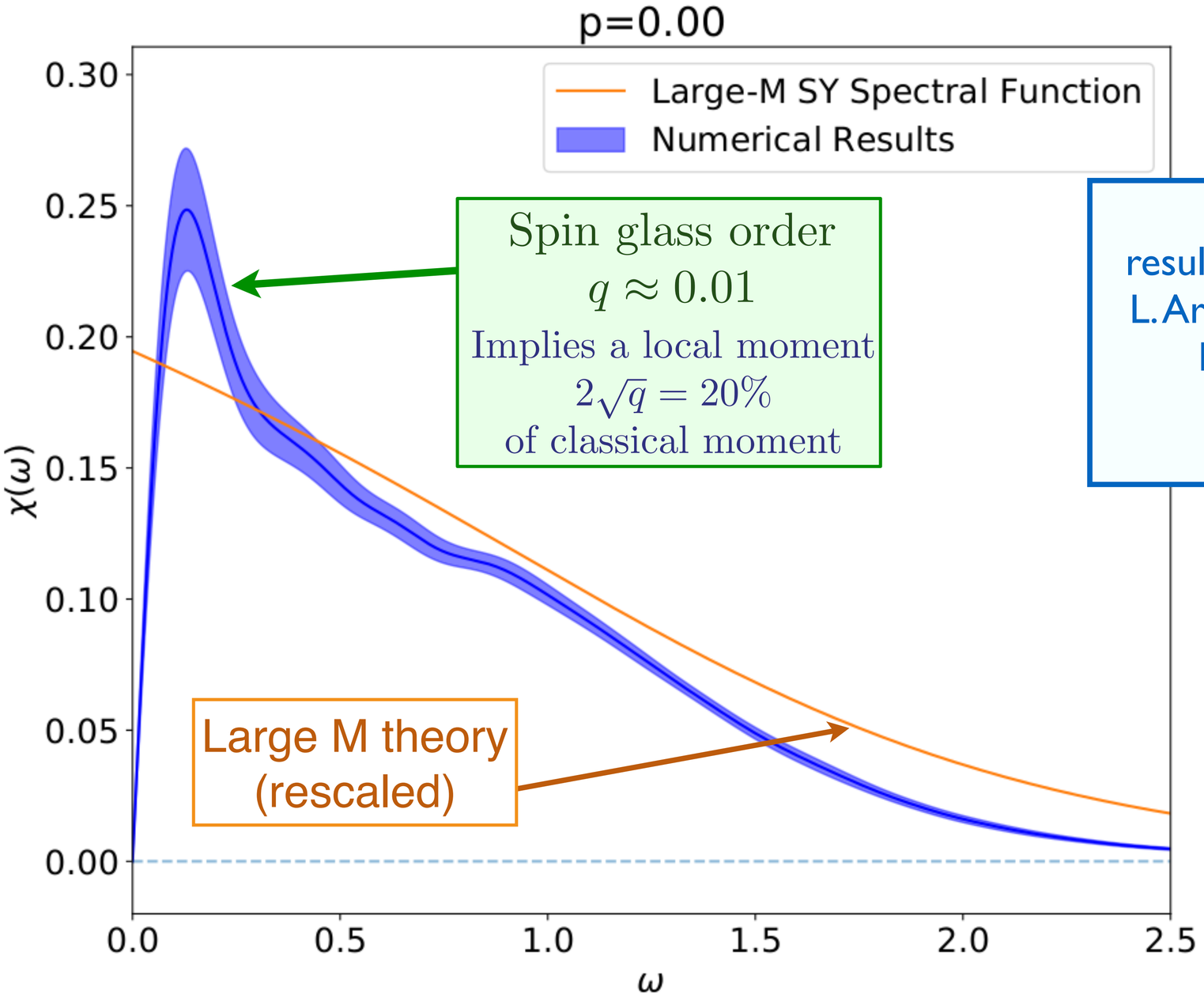


Comparison with exact diagonalization on SU(2) random-J model



Exact diagonalization
results of SU(2) random magnet
L. Arrachea and M. J. Rozenberg,
PRB **65**, 224430 (2002)
(recomputed by
Henry Shackleton)

Comparison with exact diagonalization on SU(2) random-J model

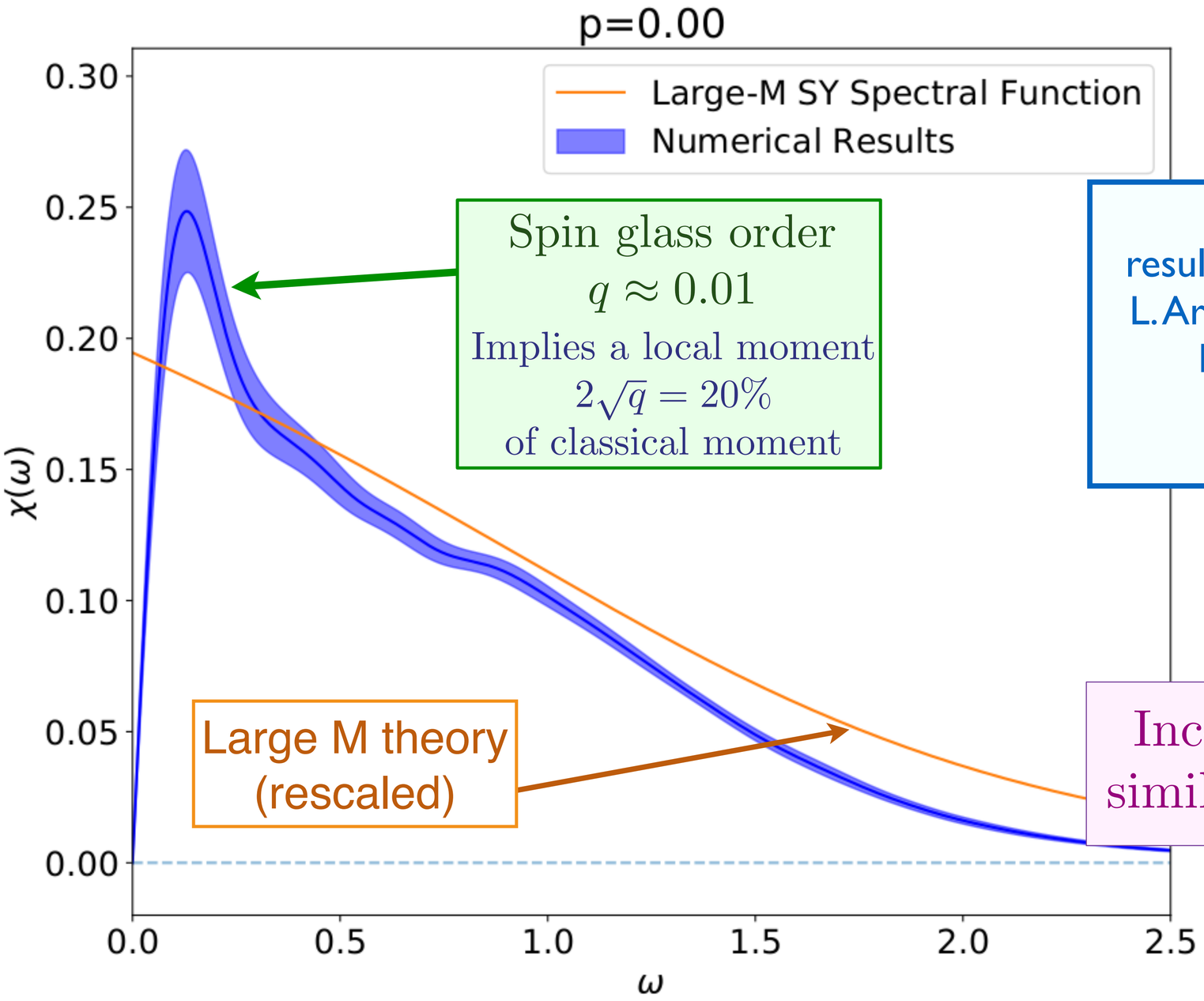


Spin glass order
$q \approx 0.01$
Implies a local moment
$2\sqrt{q} = 20\%$
of classical moment

Large M theory
(rescaled)

Exact diagonalization
results of SU(2) random magnet
L. Arrachea and M. J. Rozenberg,
PRB **65**, 224430 (2002)
(recomputed by
Henry Shackleton)

Comparison with exact diagonalization on SU(2) random-J model



Spin glass order
$q \approx 0.01$
Implies a local moment
$2\sqrt{q} = 20\%$
of classical moment

Large M theory
(rescaled)

Exact diagonalization
results of SU(2) random magnet
L. Arrachea and M. J. Rozenberg,
PRB **65**, 224430 (2002)
(recomputed by
Henry Shackleton)

Incoherent contribution
similar to SY spin liquid !

1. Random *J* model (insulator)

   Operator spectrum and numerics

2. Random t-*J* model (metals)

   Deconfined criticality and numerics

3. Random t-*J*-*U* model

Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

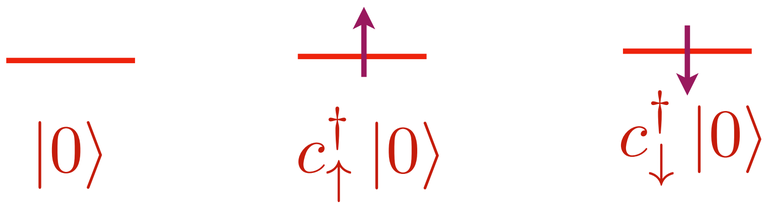
We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c^{\dagger}_{j\beta}\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c^{\dagger}_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c^{\dagger}_{i\alpha} c_{i\alpha} \leq 1, \quad \frac{1}{N} \sum_{i\alpha} c^{\dagger}_{i\alpha} c_{i\alpha} = 1 - p$$

$$J_{ij} \text{ random}, \ \overline{J_{ij}} = 0, \ \overline{J^2_{ij}} = J^2$$

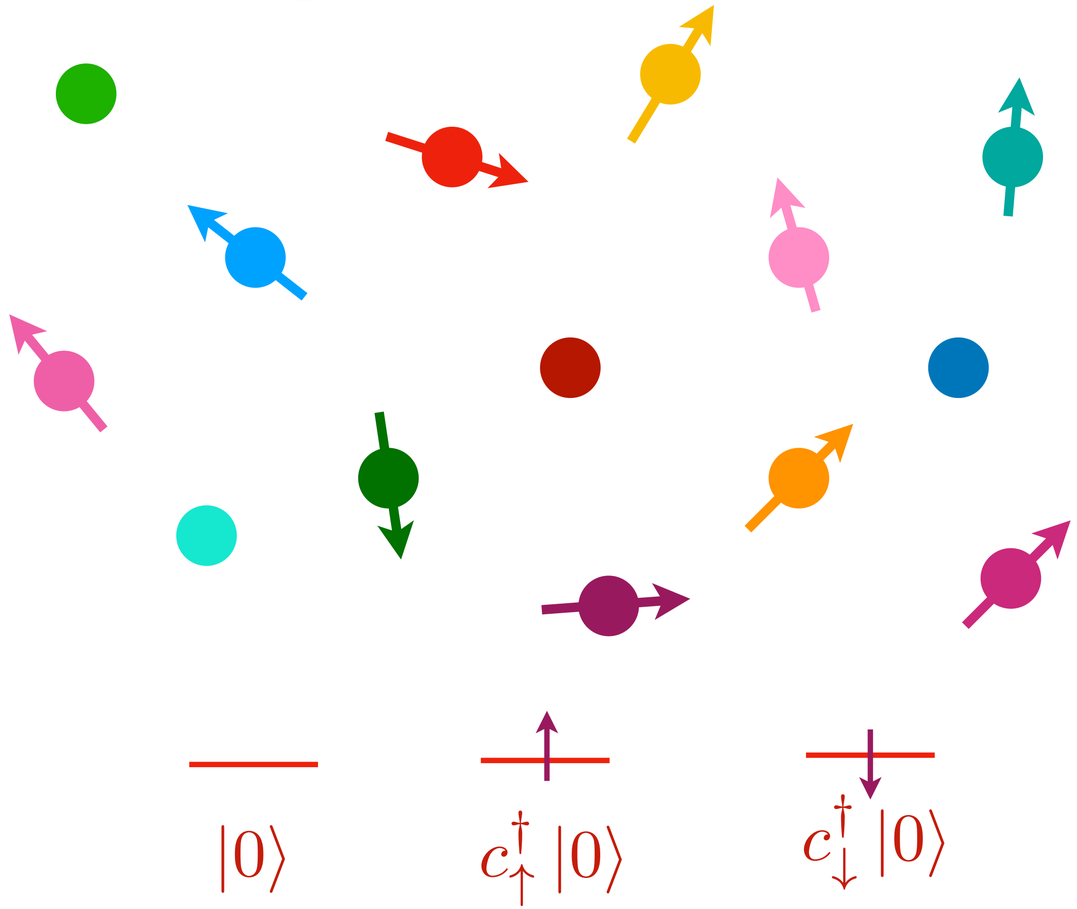
$$t_{ij} \text{ random}, \ \overline{t_{ij}} = 0, \ \overline{t^2_{ij}} = t^2$$



Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

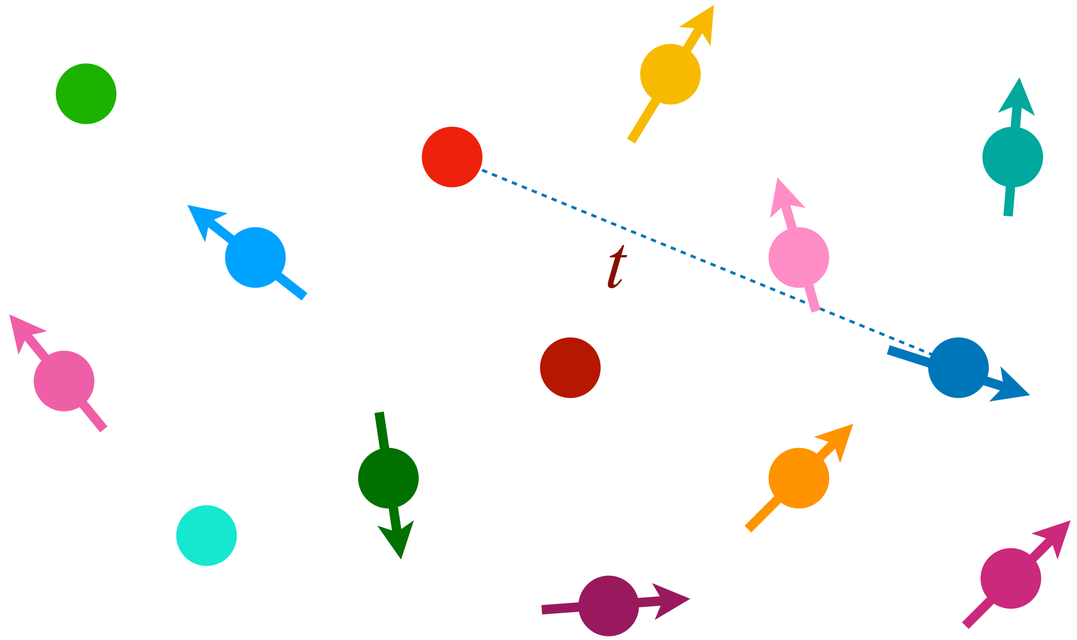
We consider the hole-doped case, with no double occupancy.

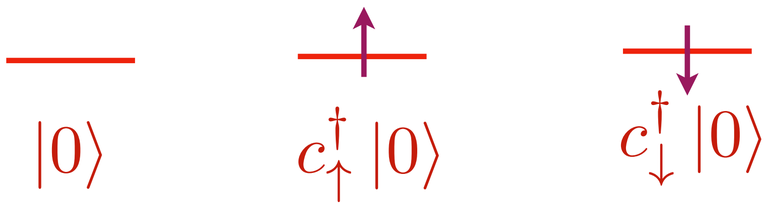


Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

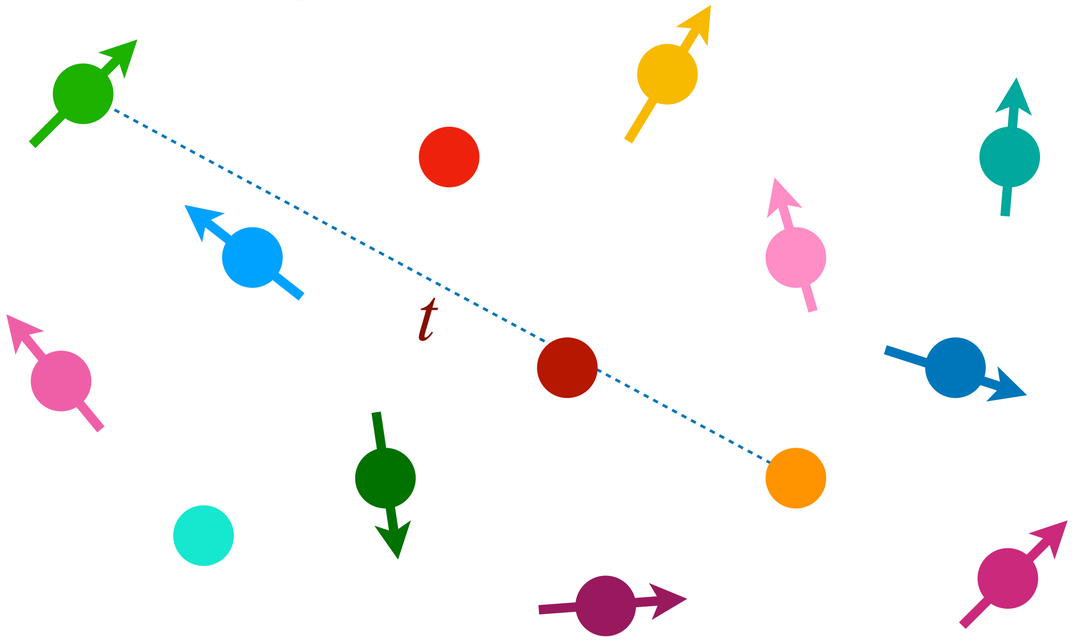


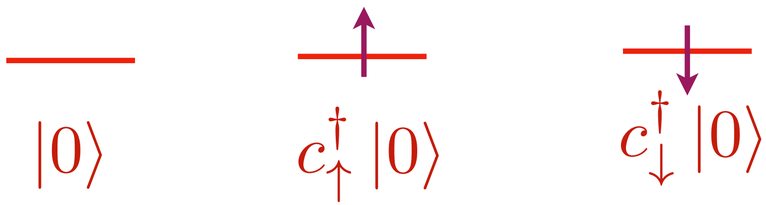


Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

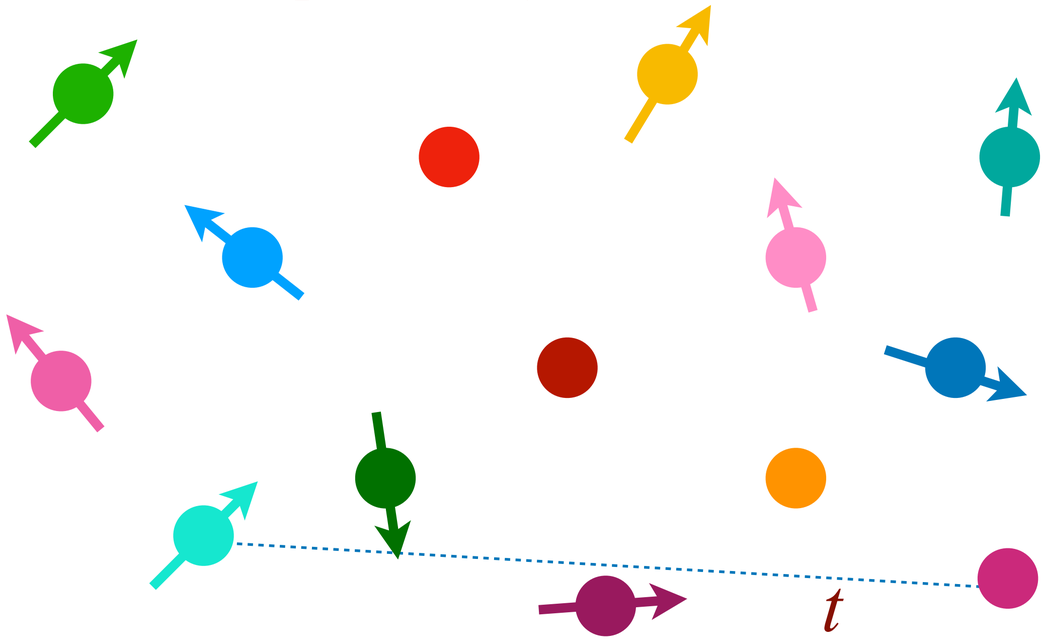


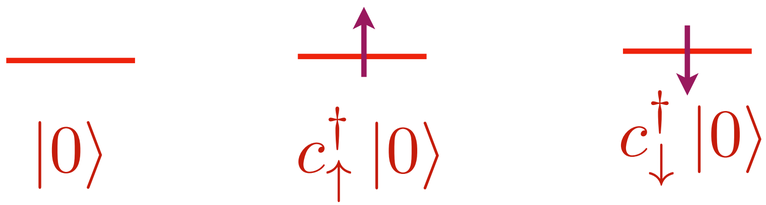


Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

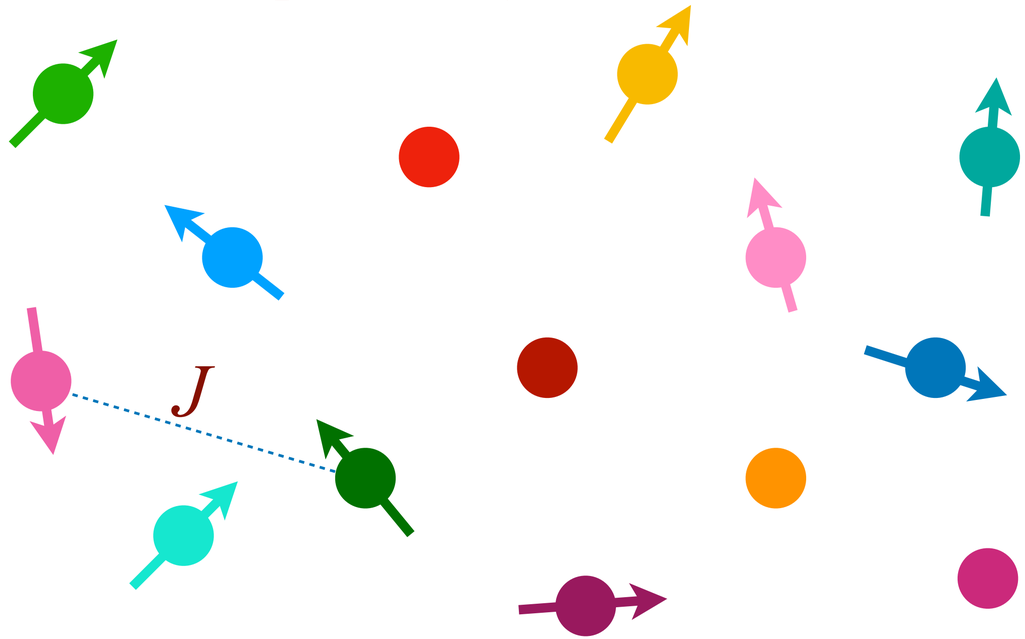


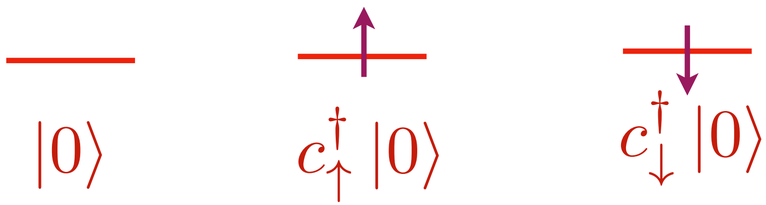


Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

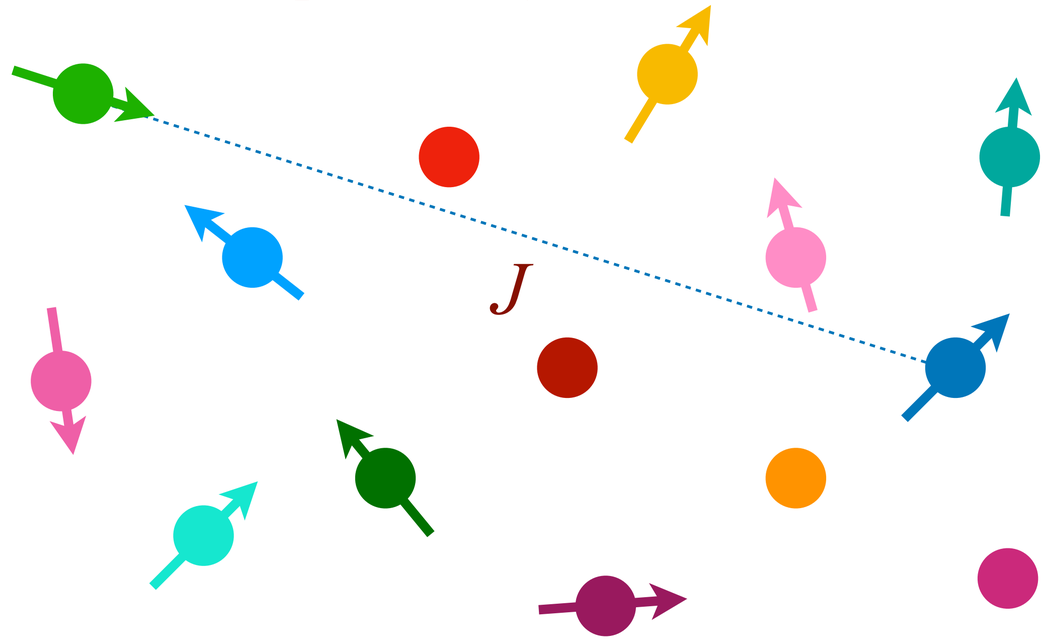


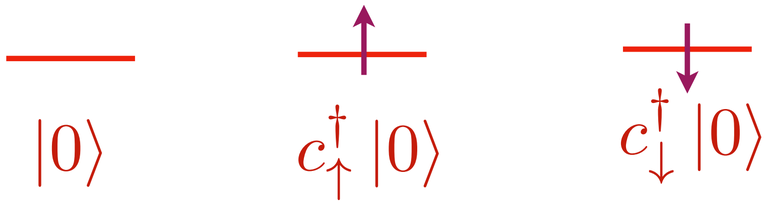


Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

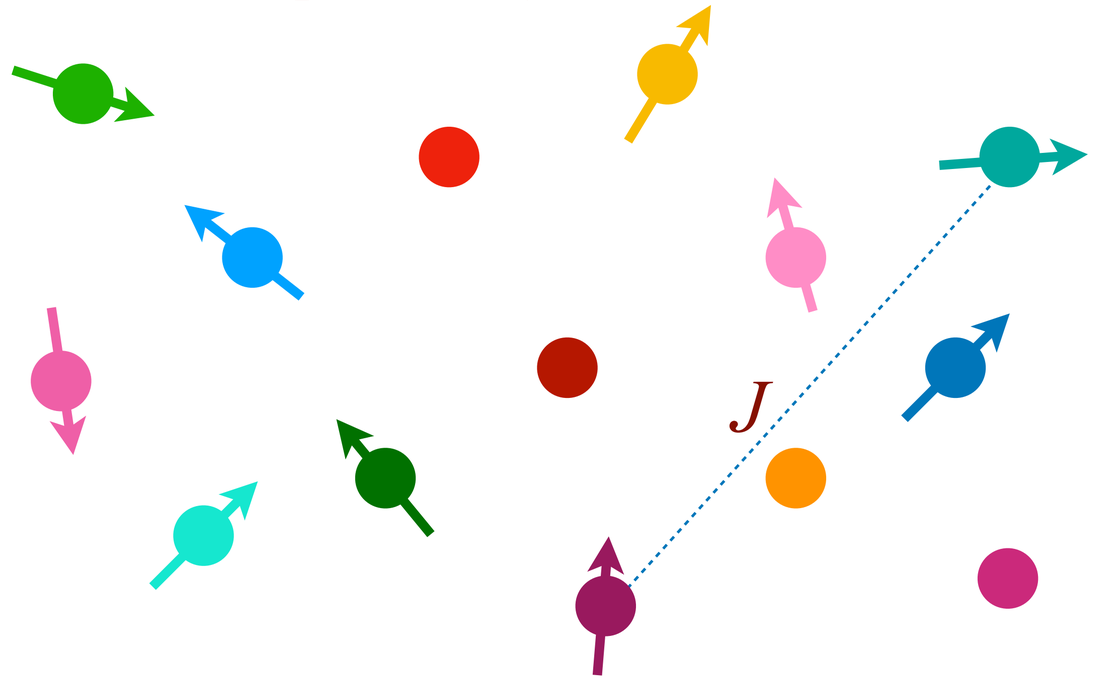




Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.



$|0\rangle$ $c_{\uparrow}^{\dagger}|0\rangle$ $c_{\downarrow}^{\dagger}|0\rangle$

Fractionalization in the t-J model

Large M limit of SU($M'|M$) theory

Each site has 3 states which we map to the space of a boson b (the holon) and a fermion f_α (the spinon):

$$|0\rangle \Rightarrow b^\dagger |v\rangle \qquad , \qquad c_\alpha^\dagger |0\rangle \Rightarrow f_\alpha^\dagger |v\rangle$$
$$c_\alpha = f_\alpha b^\dagger \qquad , \qquad f_\alpha^\dagger f_\alpha + b^\dagger b = 1$$

To obtain a large M limit, let $\alpha = 1 \ldots M$, endow the boson with an 'orbital' index $a = 1 \ldots M'$ and send $M \to \infty$ at fixed $k = M'/M$. Then

$$c_{a\alpha} = f_\alpha b_a^\dagger \qquad , \qquad f_\alpha^\dagger f_\alpha + b_a^\dagger b_a = \kappa M$$

Fractionalization in the t-J model

Large M limit of SU($M'|M$) theory

Assuming the bosons are not condensed, we obtain SYK-like equations for the boson and fermion Green's functions:

$$
\begin{aligned}
G_b(i\omega_n) &= \frac{1}{i\omega_n + \mu_b - \Sigma_b(i\omega_n)} \\
\Sigma_b(\tau) &= -t^2 G_f(\tau) G_f(-\tau) G_b(\tau) \\
G_f(i\omega_n) &= \frac{1}{i\omega_n + \mu_f - \Sigma_f(i\omega_n)} \\
\Sigma_f(\tau) &= -J^2 G_f^2(\tau) G_f(-\tau) + k\, t^2 G_f(\tau) G_b(\tau) G_b(-\tau)
\end{aligned}
$$

Here μ_f and μ_b are chemical potentials chosen to satisfy

$$
\left\langle f^\dagger f \right\rangle = \kappa - kp \quad , \quad \left\langle b^\dagger b \right\rangle = p \, .
$$

Fractionalization in the t-J model

Large M limit of SU($M'|M$) theory

These equations have critical solutions with

$$G_f(z) = C_f \frac{e^{-i(\pi\Delta_f+\theta_f)}}{z^{1-2\Delta_f}}, \quad G_b(z) = C_b \frac{e^{-i(\pi\Delta_b+\theta_b)}}{z^{1-2\Delta_b}}, \quad \mathrm{Im}(z) > 0$$

$$\Delta_f + \Delta_b = \frac{1}{2}$$

$$\frac{\theta_f}{\pi} + \left(\frac{1}{2} - \Delta_f\right) \frac{\sin(2\theta_f)}{\sin(2\pi\Delta_f)} = kp$$

$$\frac{\theta_b}{\pi} + \left(\frac{1}{2} - \Delta_b\right) \frac{\sin(2\theta_b)}{\sin(2\pi\Delta_b)} = \kappa + p\,.$$

The last two are analogs of 'Luttinger' relations

Fractionalization in the t-J model

Large M limit of SU($M'|M$) theory

The critical solution which is self-consistent in both the t and J terms has $\Delta_b = \Delta_f = 1/4$, implying

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle \sim \begin{cases} \dfrac{A_+}{|\tau|} & , \quad \tau > 0 \\ \\ -\dfrac{A_-}{|\tau|} & , \quad \tau < 0 \end{cases} \qquad , \quad \left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|} \, .$$

RG computations show that these results for the exponents of gauge-invariant operators are expected to be exact beyond the large M limit.

Proposed phase diagram of random *t-J* model

Metallic
spin glass.
Carrier density p

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

$$\left\langle c_{i\alpha}(\tau) c^\dagger_{i\alpha}(0) \right\rangle \sim \frac{1}{\tau}$$

Deconfined
quantum
critical
point/phase

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

$$\left\langle c_{i\alpha}(\tau) c^\dagger_{i\alpha}(0) \right\rangle \sim \frac{1}{\tau}$$

Disordered
Fermi liquid.
Carrier density $1 + p$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

$$\left\langle c_{i\alpha}(\tau) c^\dagger_{i\alpha}(0) \right\rangle \sim \frac{1}{\tau}$$

RG flow

p_c

p

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

At the critical point/phase of the t-J model, the Fermi liquid-like behavior of the electron Green's function

$$\left\langle c_{i\alpha}(\tau) c^{\dagger}_{i\alpha}(0) \right\rangle \sim \frac{1}{\tau}$$

leads to a non-zero *residual resitivity*, $\rho(0) \neq 0$.

However, the critical state is *not* a Fermi liquid, as indicated by the slow decay of the spin correlations

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

Moreover, in a Fermi liquid, we expect $\rho(T) - \rho(0) \sim T^2$, which also does not hold here.

Time reparameterization soft mode

Corrections to the critical spinon Green's function arising from the time reparameterization soft mode take the form

$$\left\langle f_\alpha(\tau) f_\alpha^\dagger(0)\right\rangle \sim \left[\frac{\pi T}{\sin(\pi T\tau)}\right]^{1/2} \left(1 + \mathcal{C}_f \gamma\, \Phi_{\rm non-conformal}(T\tau)\right)$$

where $\Phi_{\rm non-conformal}(T\tau)$ is a computable (in the large M limit) scaling function, γT is the specific heat per spin component, and $\mathcal{C}_f$ is a UV-insensitive number

J. Maldacena and D. Stanford, PRD **94**, 106002 (2016)
A. Kitaev and J. Suh, JHEP 183 (2018)

Time reparameterization soft mode

Corrections to the critical electron Green's function arising from the time reparameterization soft mode take the form

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle \sim \left[\frac{\pi T}{\sin(\pi T \tau)} \right] \quad (1 + \mathcal{C}_c \gamma \, \Phi_{\rm non-conformal}(T\tau))$$

where $\Phi_{\rm non-conformal}(T\tau)$ is a computable (in the large M limit) scaling function, γT is the specific heat per spin component, and $\mathcal{C}_c$ is a UV-insensitive number

Haoyu Guo, Yingfei Guo, S. Sachdev, Annals of Physics **418**, 168202 (2020)

Time reparameterization soft mode

Computing the resistivity from this Green's function via the Kubo formula, we find

$$\rho(T) = \rho(0)\left(1 + \mathcal{C}_\rho \gamma T + \ldots\right)$$

Haoyu Guo, Yingfei Guo, S. Sachdev, Annals of Physics **418**, 168202 (2020)

Time reparameterization soft mode

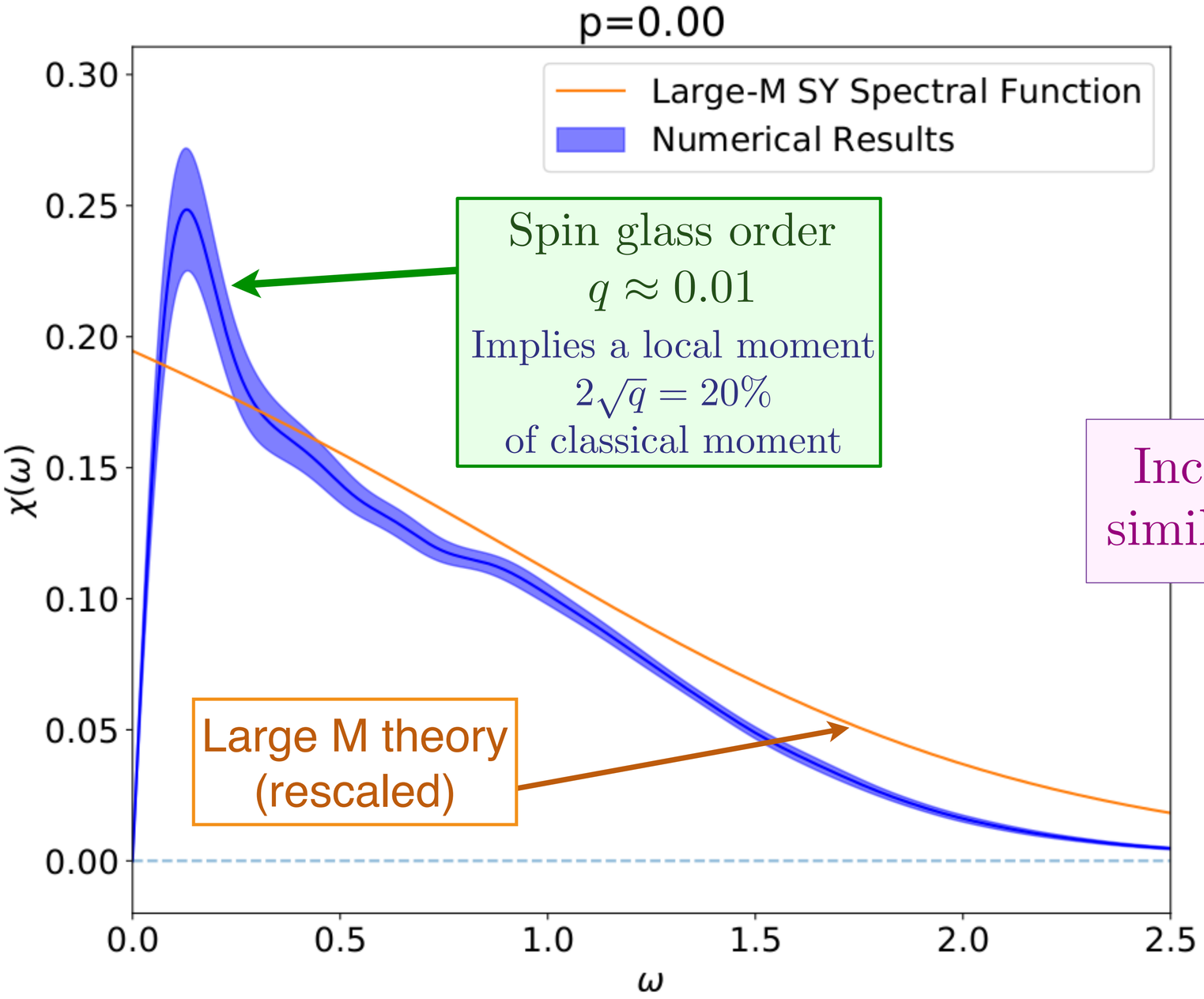
Computing the resistivity from this Green's function via the Kubo formula, we find

$$\rho(T) = \rho(0)\left(1 + \mathcal{C}_\rho \gamma T + \ldots\right)$$

In general, an operator with scaling dimension h, yield a resistivity $\rho(T) \sim T^{h-1}$. In the large M solution of the t-J model, we do find an operator with $1 < h < 2$, but with a smaller prefactor.

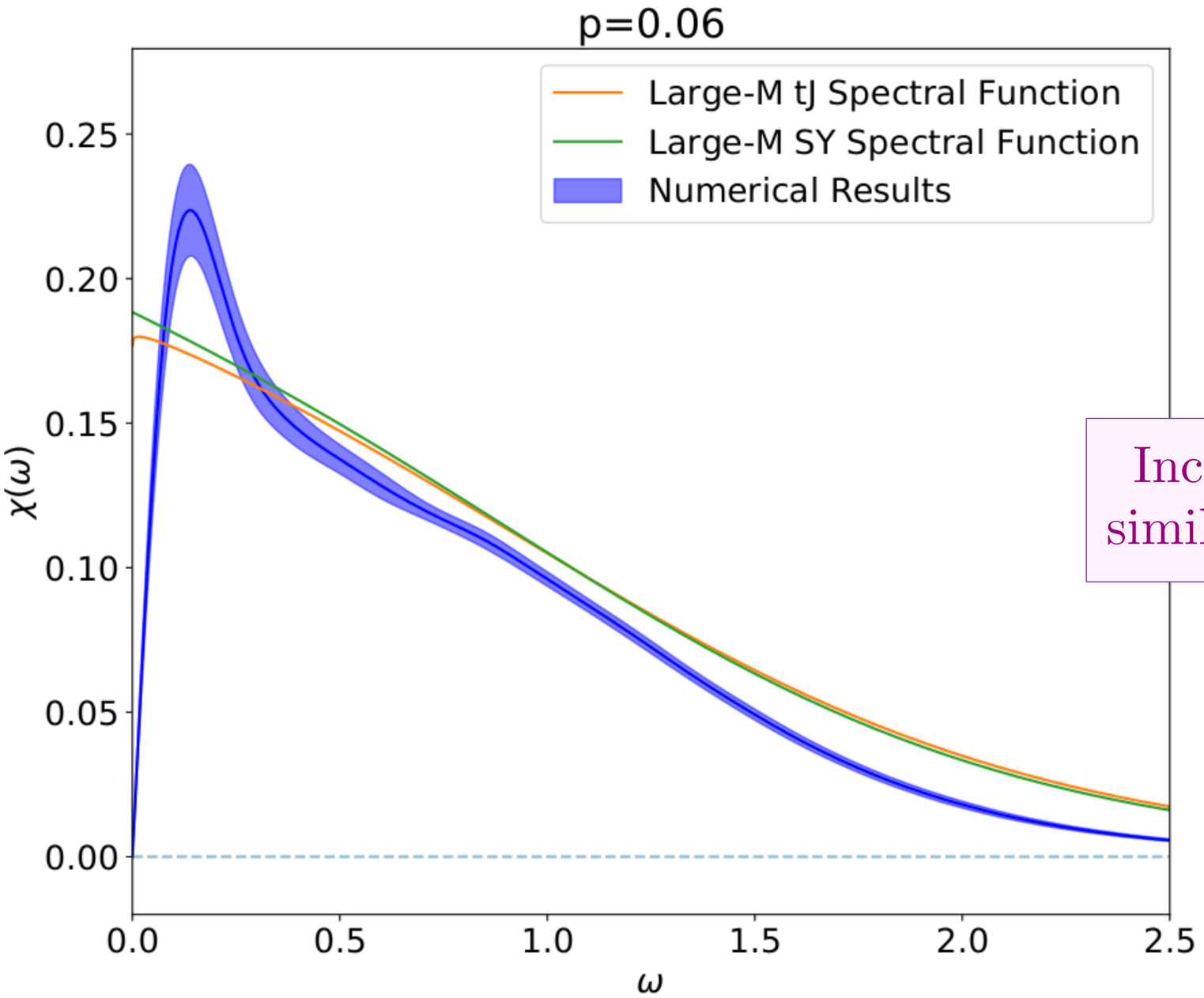
Haoyu Guo, Yingfei Guo, S. Sachdev, Annals of Physics **418**, 168202 (2020)
M. Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, to appear

Comparison with exact diagonalization on SU(2) random-t-J model



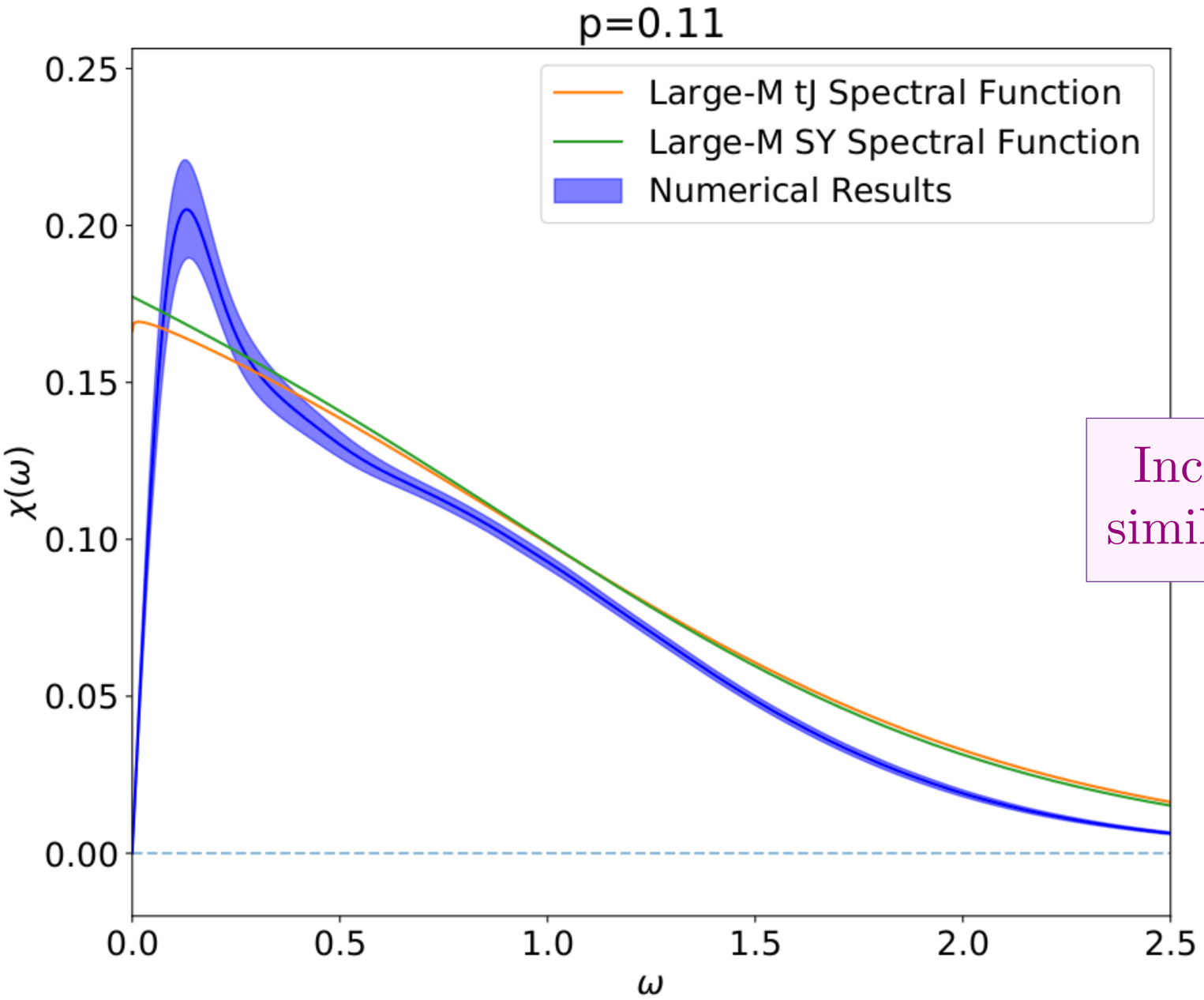
Incoherent contribution similar to SY spin liquid !

Comparison with exact diagonalization on SU(2) random-t-J model



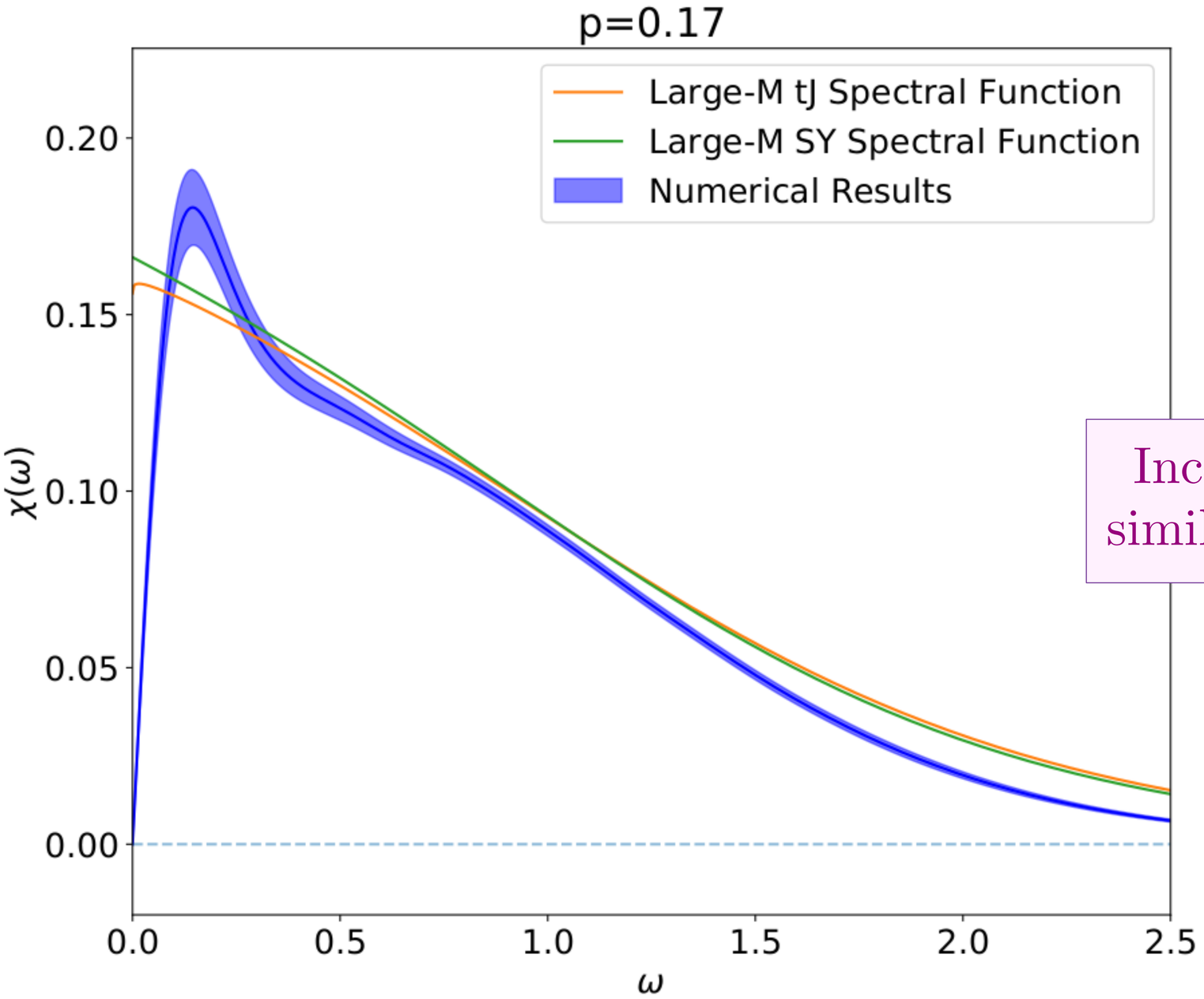
Incoherent contribution similar to SY spin liquid !

Comparison with exact diagonalization on SU(2) random-t-J model



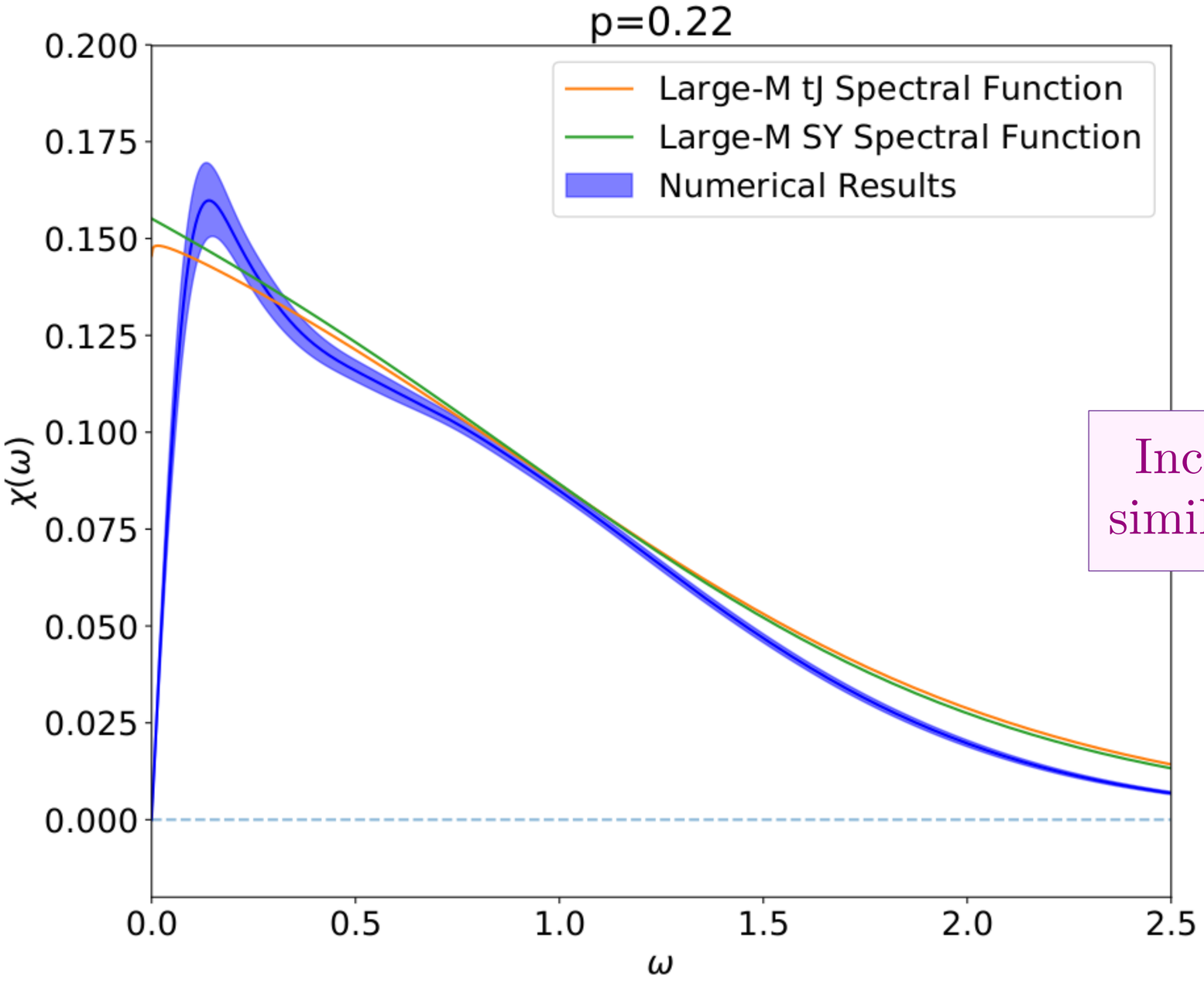
Incoherent contribution similar to SY spin liquid !

Comparison with exact diagonalization on SU(2) random-t-J model



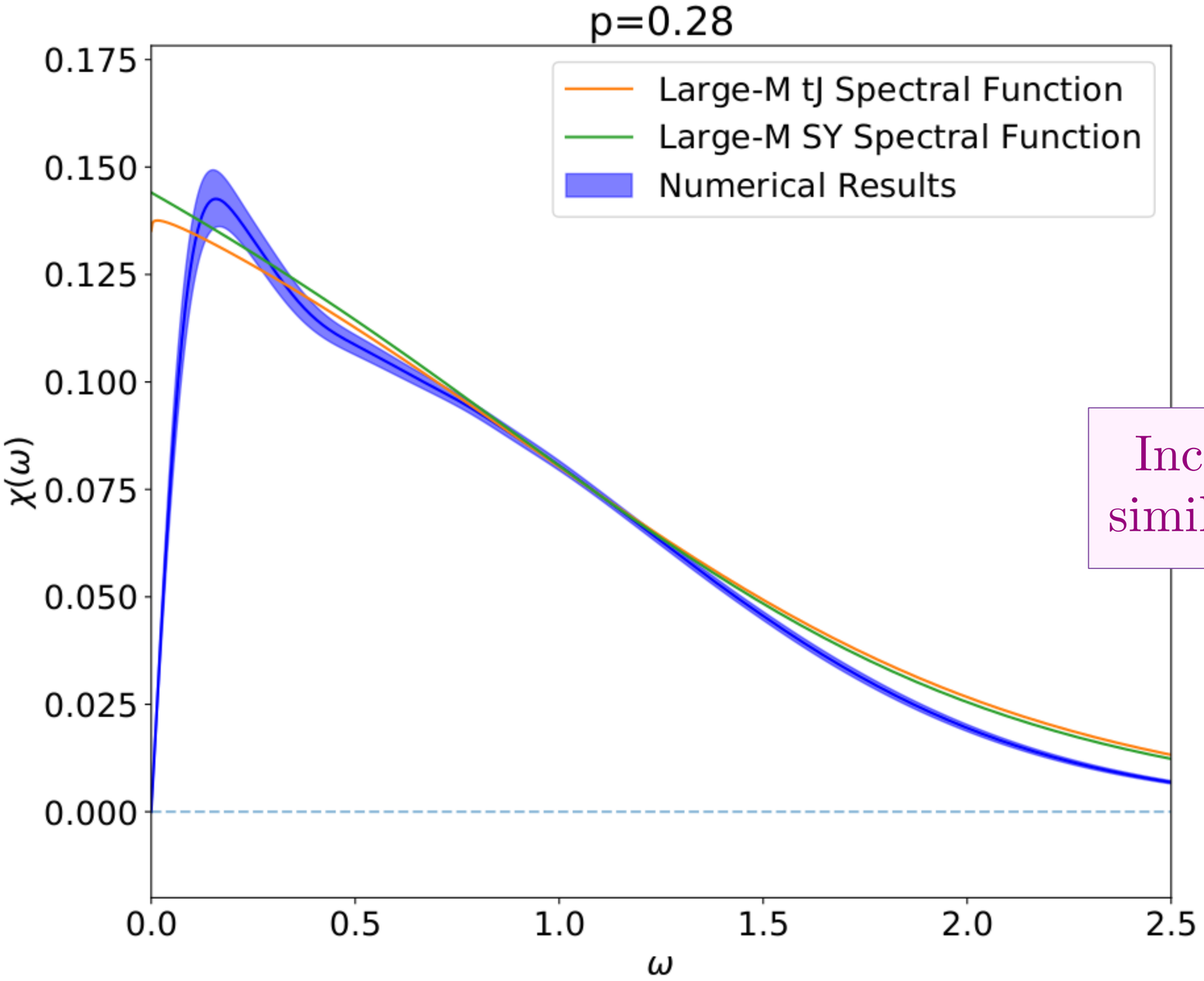
Incoherent contribution similar to SY spin liquid !

Comparison with exact diagonalization on SU(2) random-t-J model



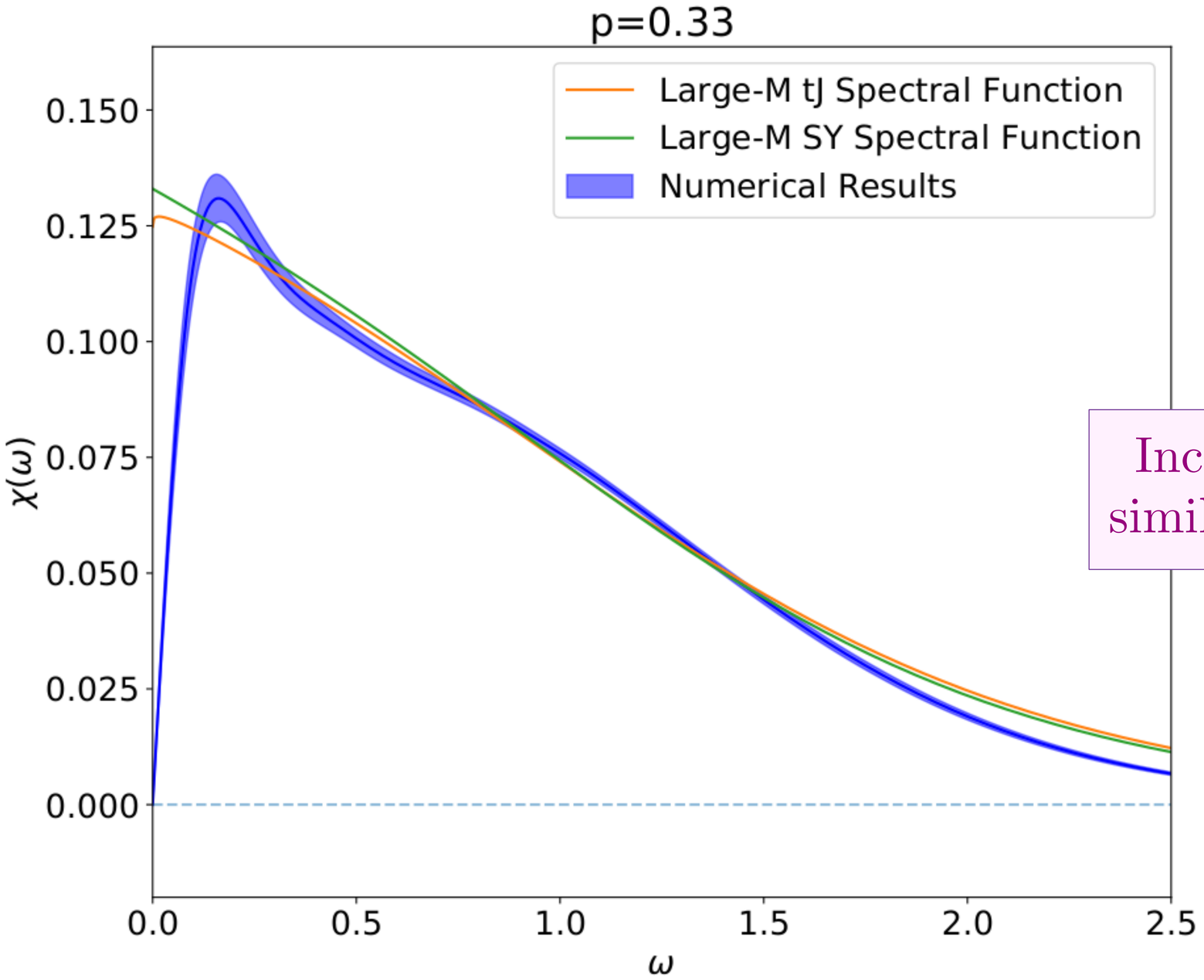
Incoherent contribution similar to SY spin liquid !

Comparison with exact diagonalization on SU(2) random-t-J model



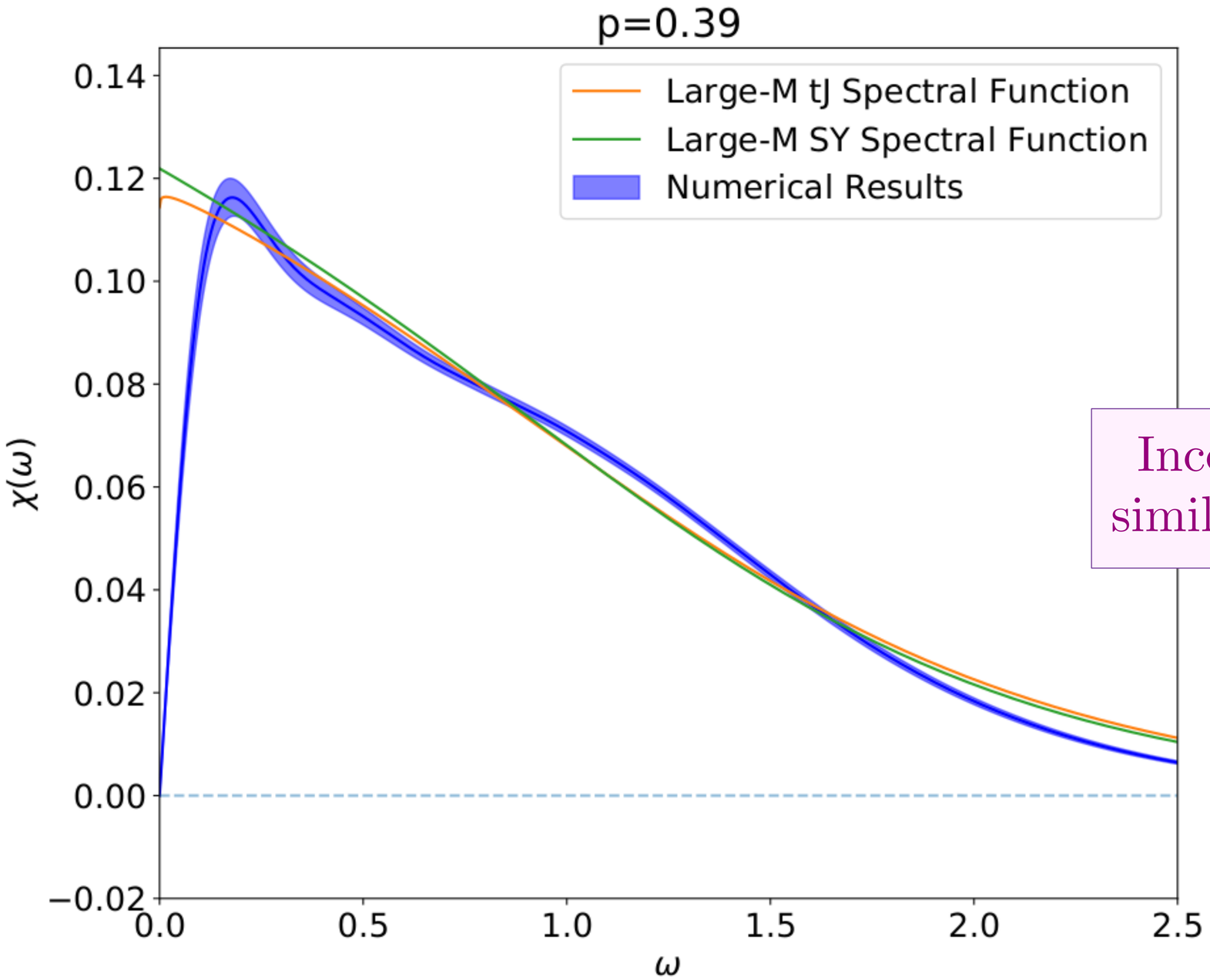
Incoherent contribution similar to SY spin liquid !

Comparison with exact diagonalization on SU(2) random-t-J model



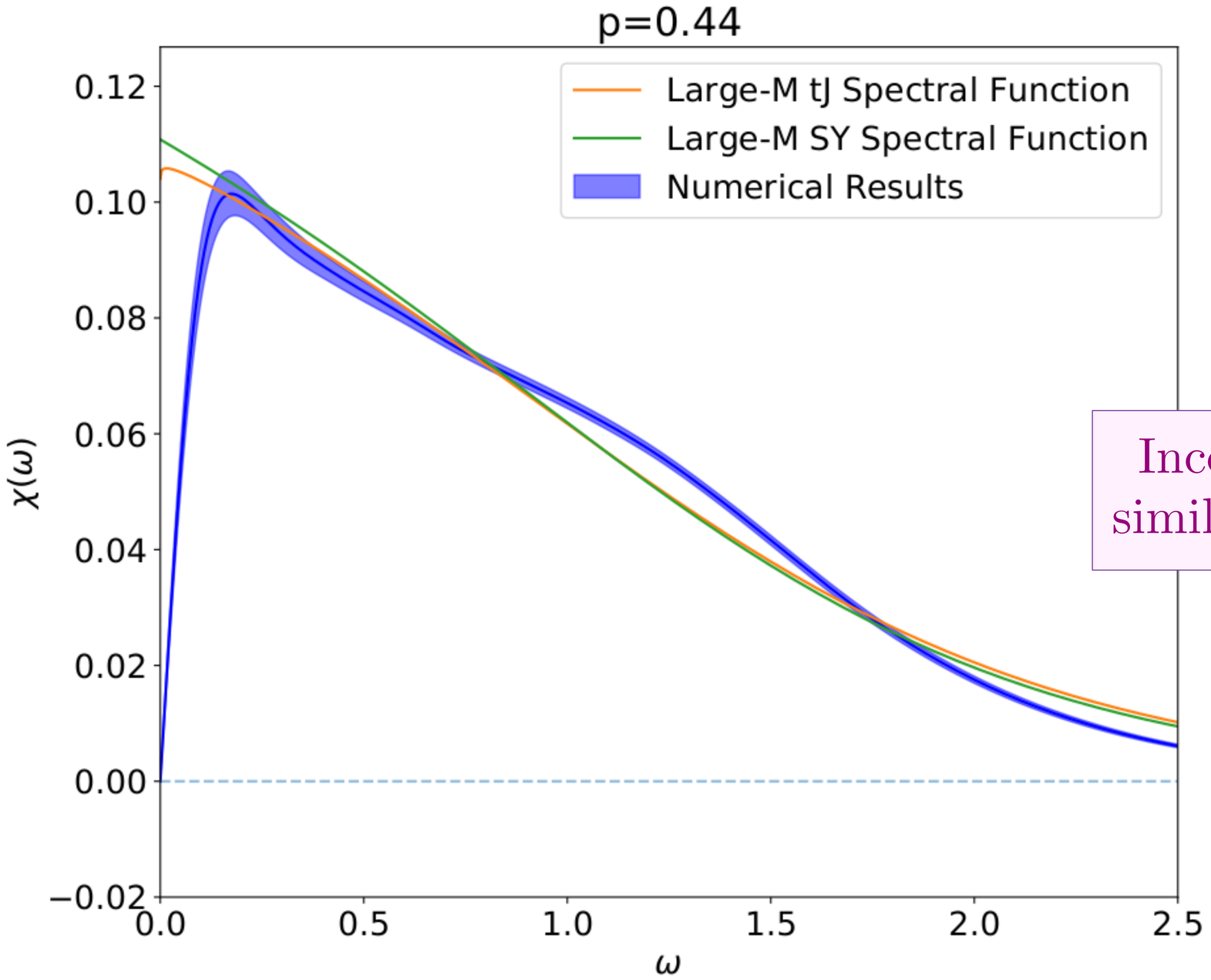
Incoherent contribution similar to SY spin liquid !

Comparison with exact diagonalization on SU(2) random-t-J model



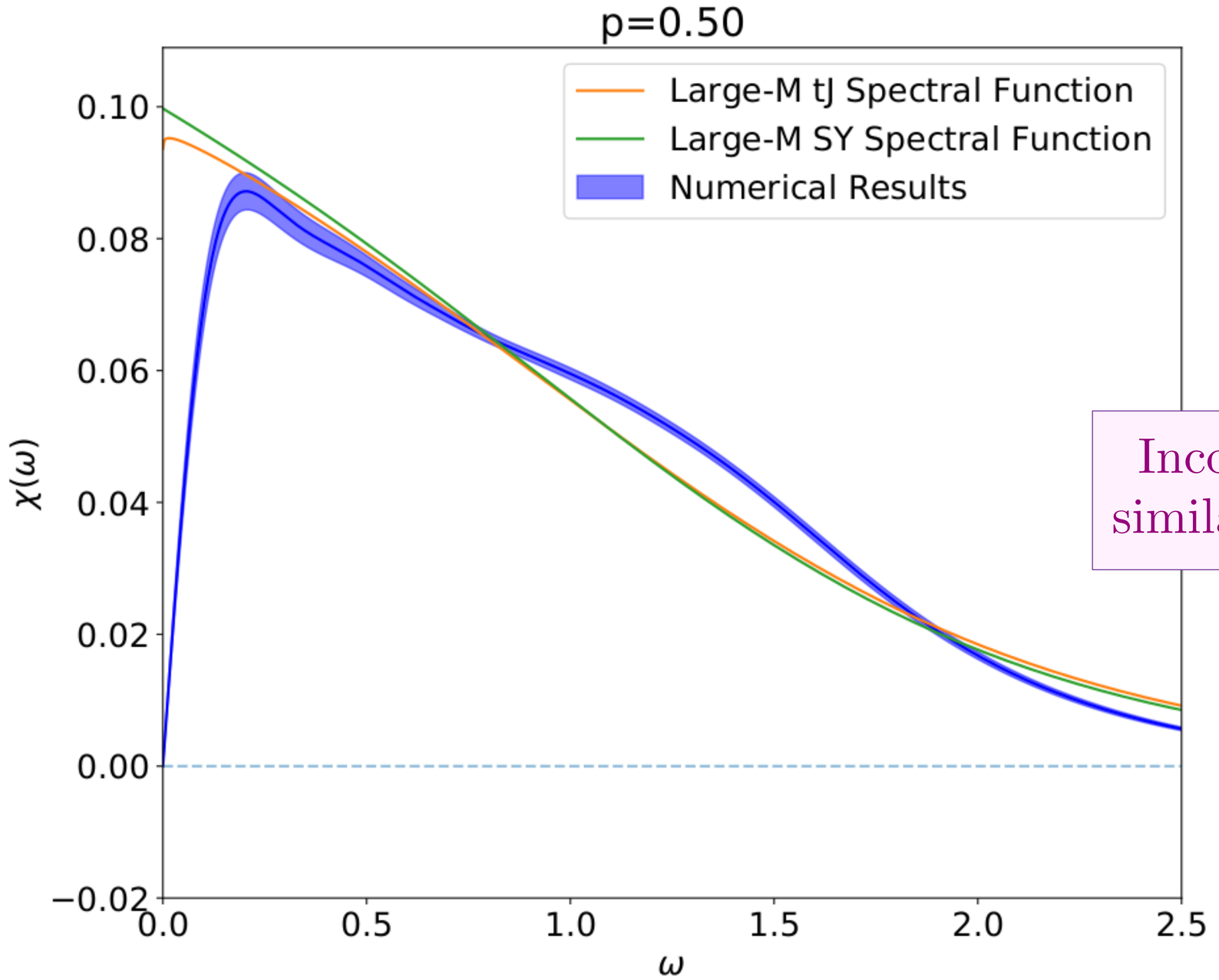
Incoherent contribution similar to SY spin liquid !

Comparison with exact diagonalization on SU(2) random-t-J model



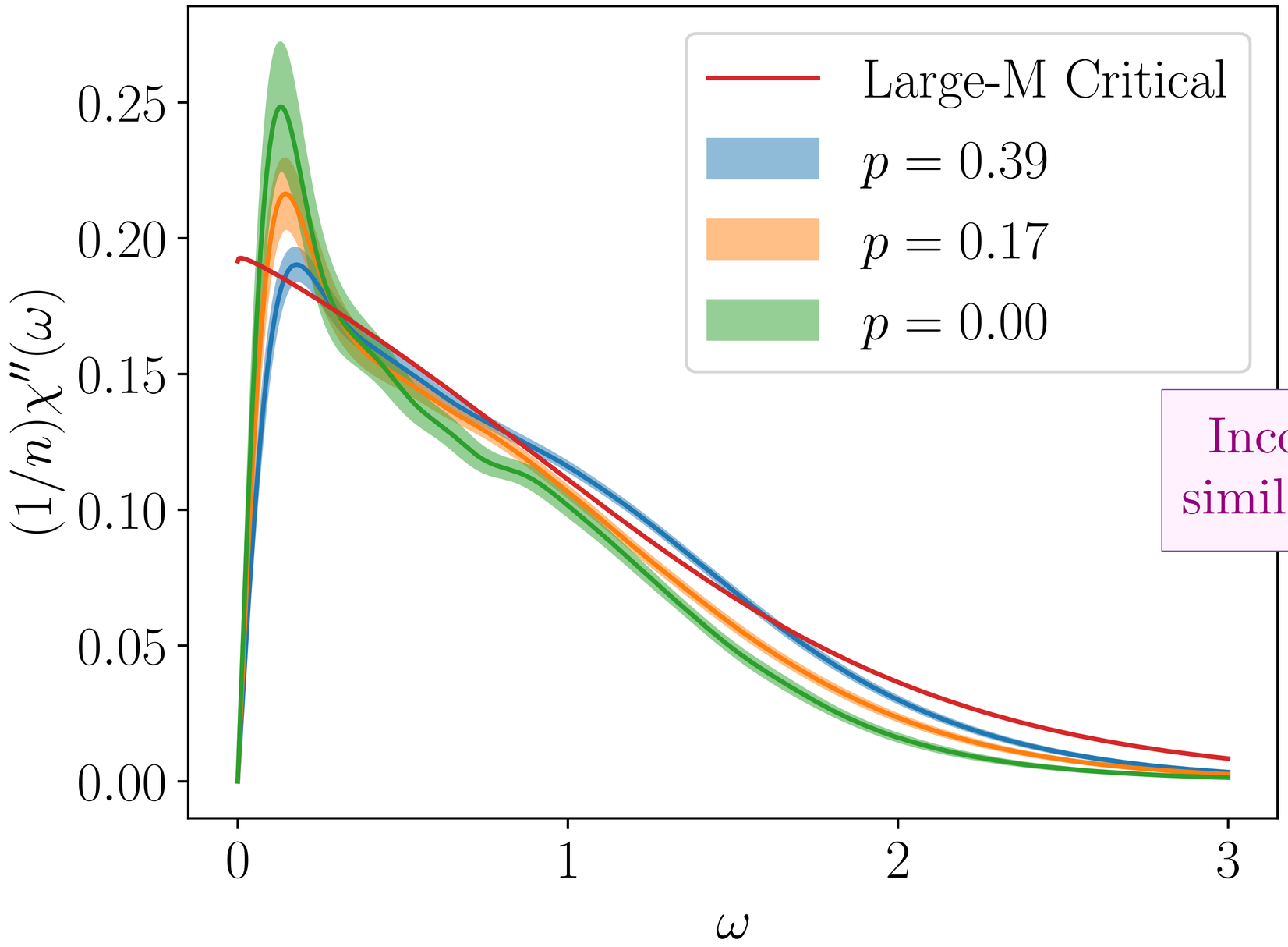
Incoherent contribution similar to SY spin liquid !

Comparison with exact diagonalization on SU(2) random-t-J model



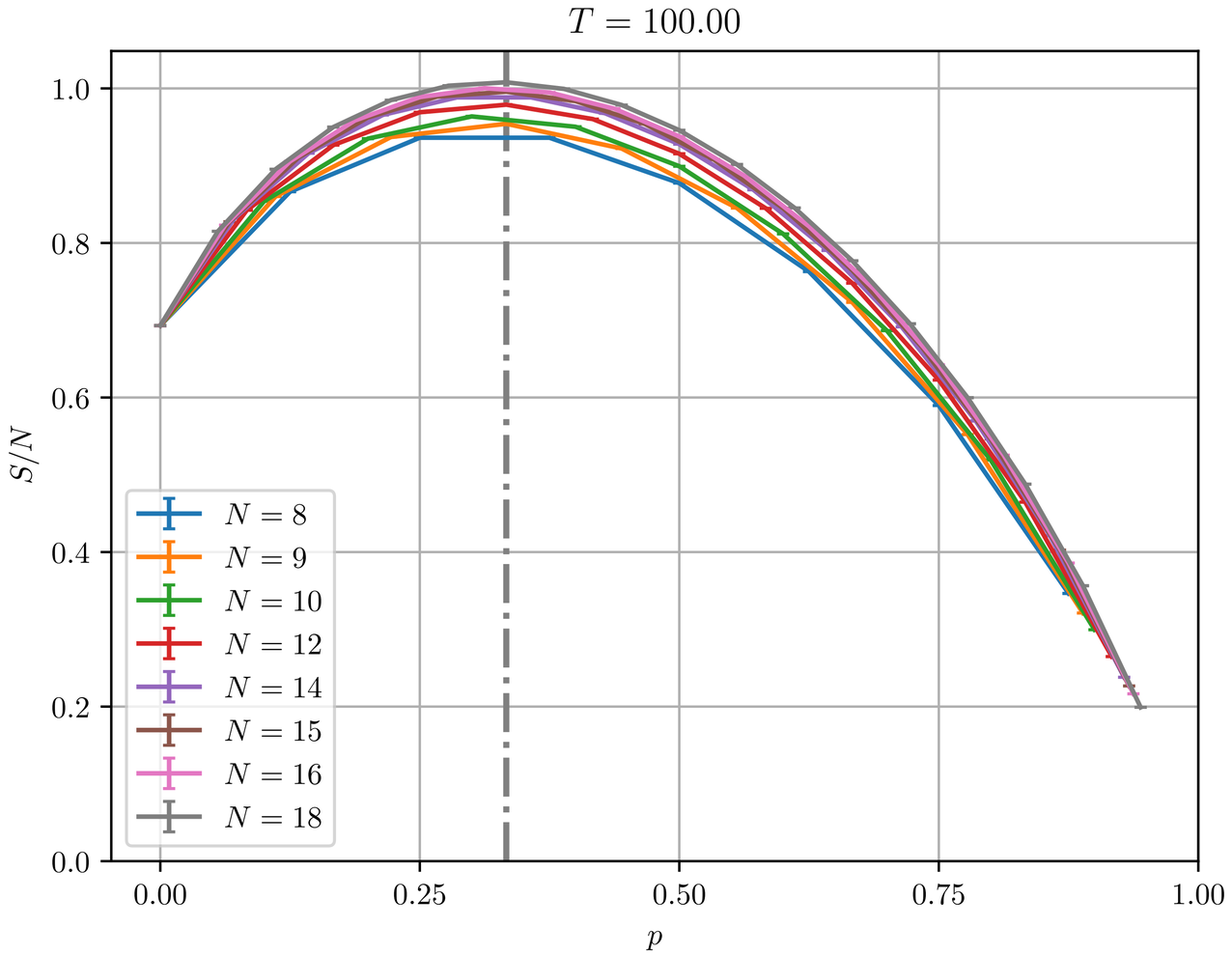
Incoherent contribution similar to SY spin liquid !

Comparison with exact diagonalization on SU(2) random-t-J model



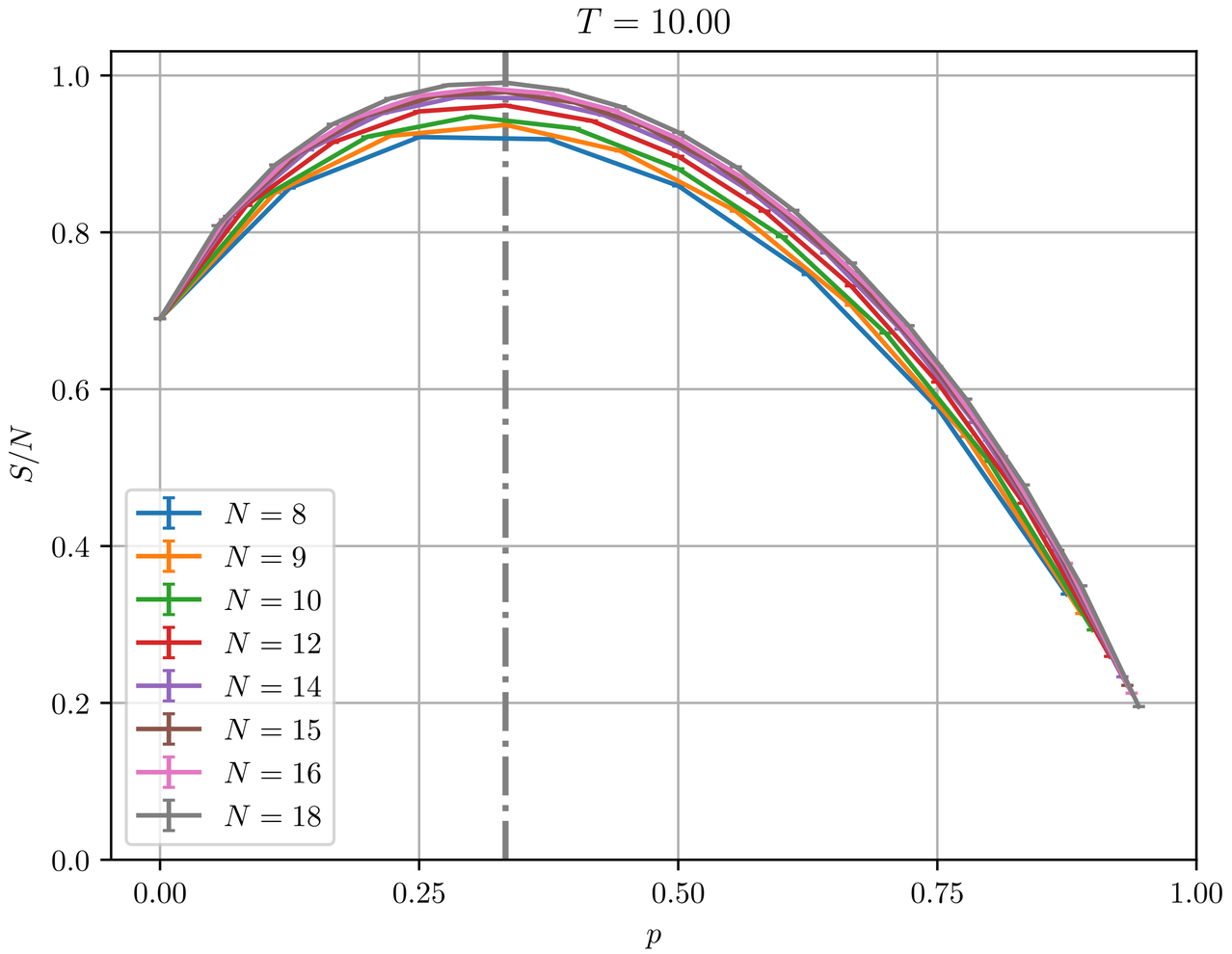
Incoherent contribution similar to SY spin liquid !

Entropy at $T \gg 1$ determined by $\dim(\mathcal{H})$



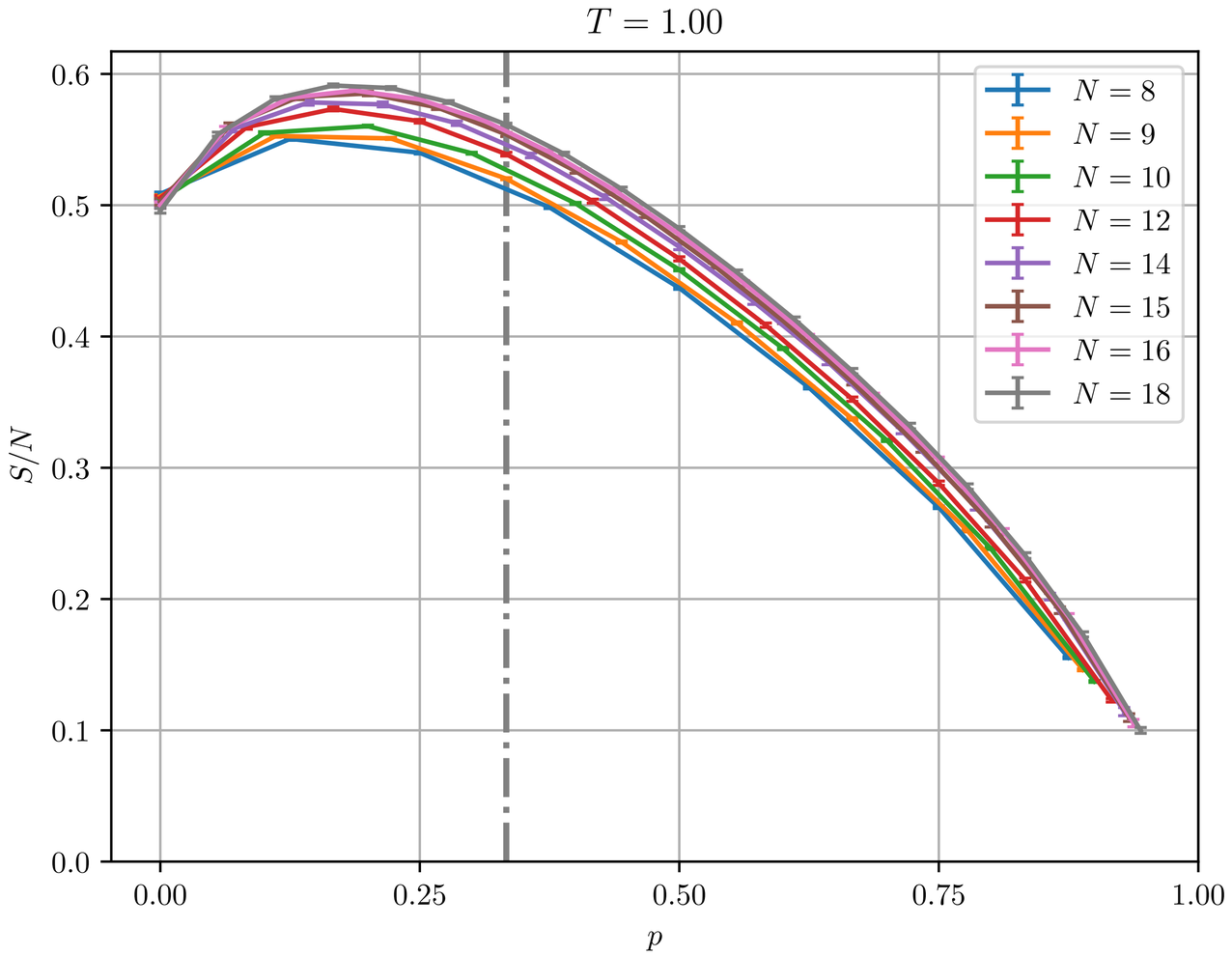
Thermodynamics from exact diagonalization of SU(2) random-t-J model

Maximum entropy shifts at lower temperature



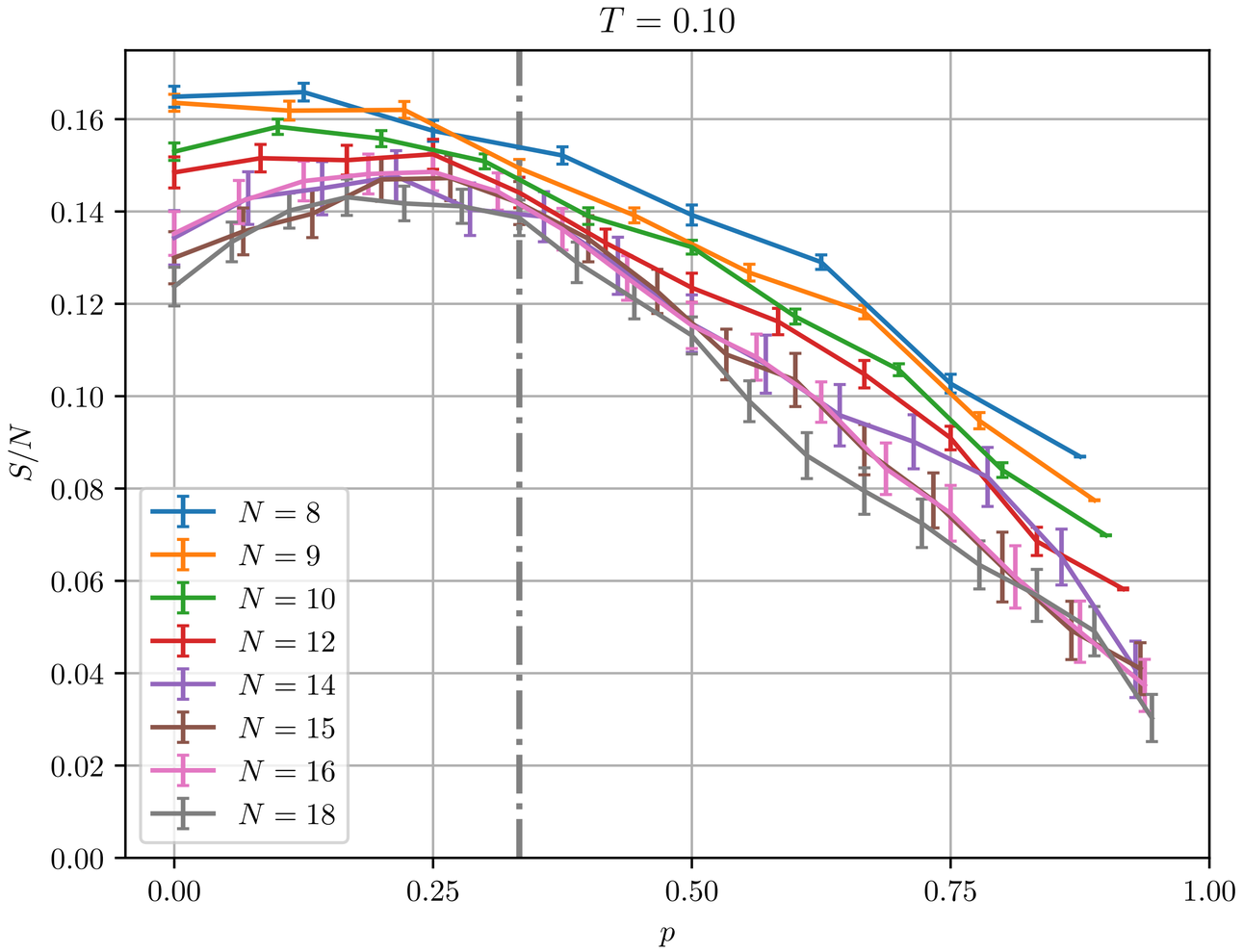
Thermodynamics from exact diagonalization of SU(2) random-t-J model

Maximum entropy shifts at lower temperature



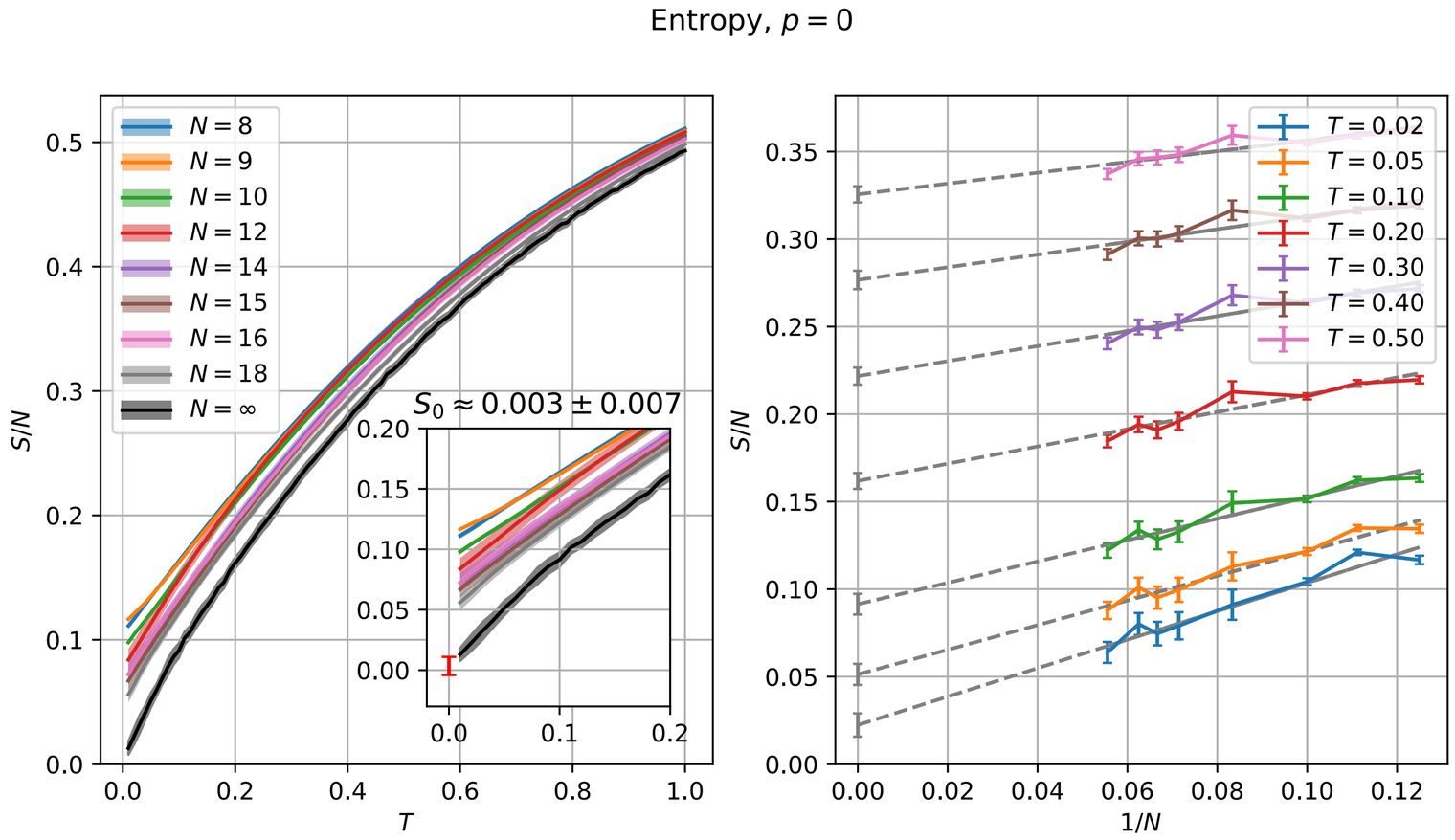
Thermodynamics from exact diagonalization of SU(2) random-t-J model

Maximum entropy shifts at lower temperature



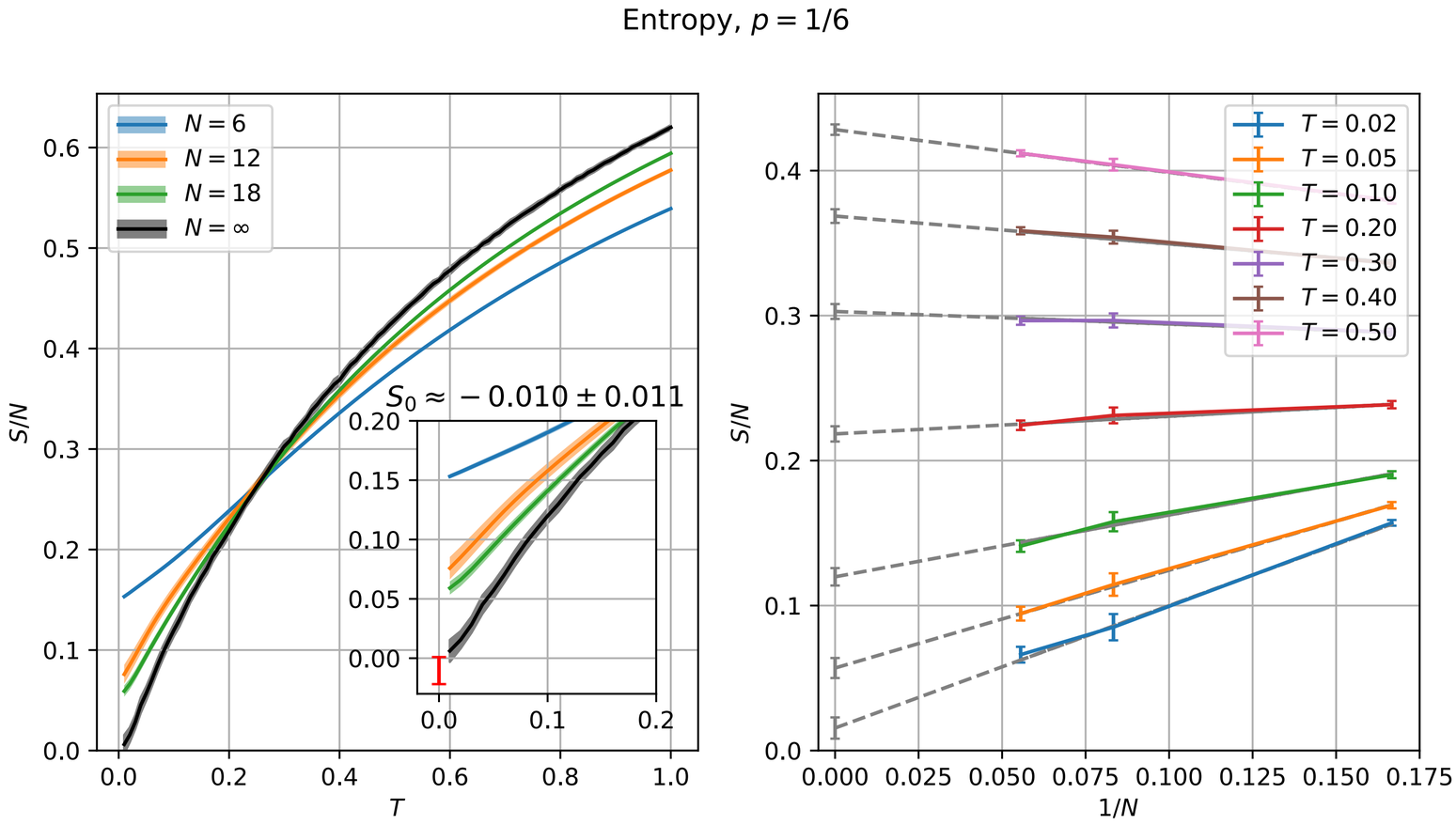
Thermodynamics from exact diagonalization of SU(2) random-t-J model

Large-N extrapolation of entropy density



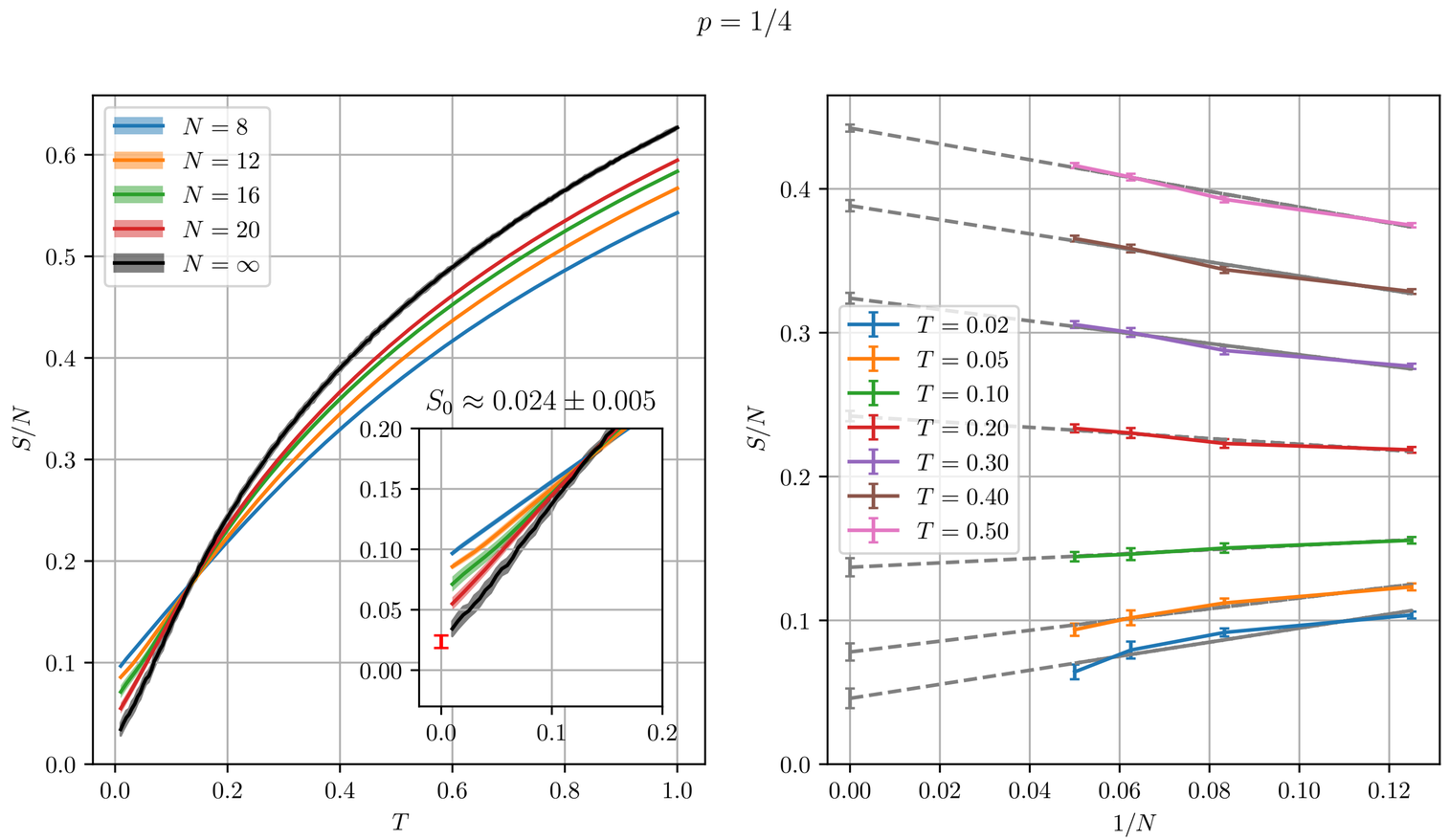
Thermodynamics from exact diagonalization of SU(2) random-t-J model

Large-N extrapolation of entropy density



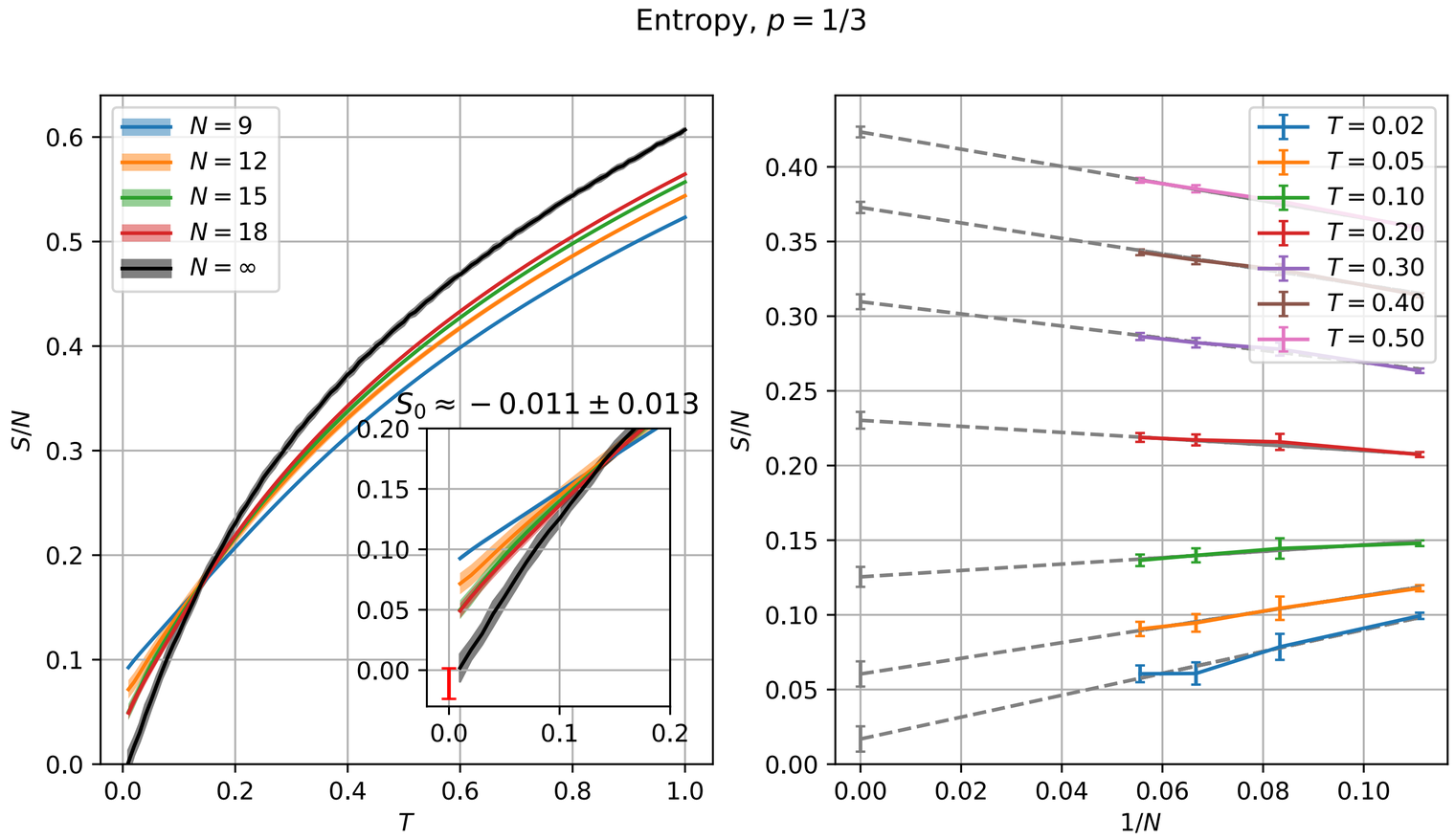
Thermodynamics from exact diagonalization of SU(2) random-t-J model

Large-N extrapolation of entropy density



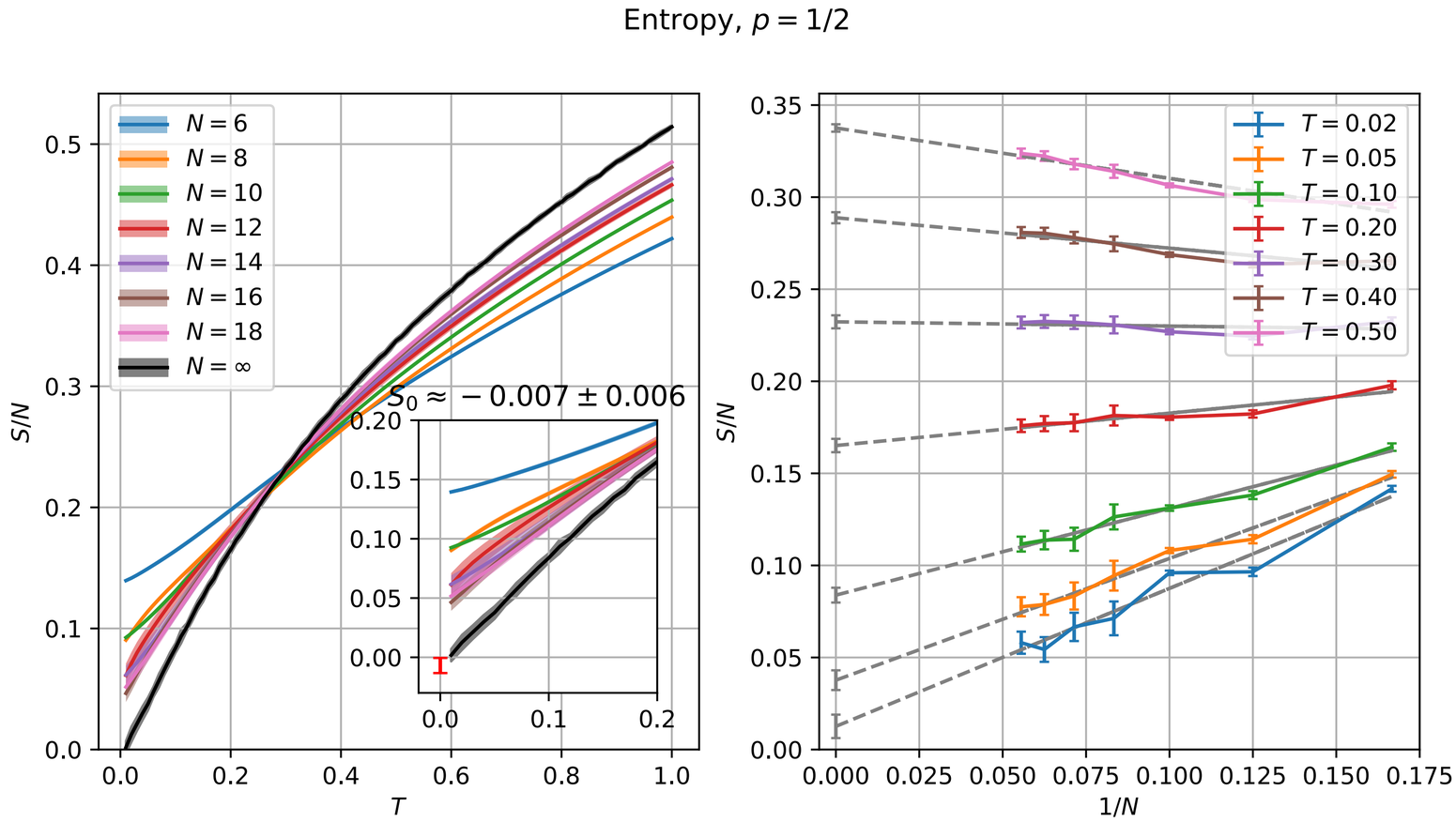
Thermodynamics from exact diagonalization of SU(2) random-t-J model

Large-N extrapolation of entropy density



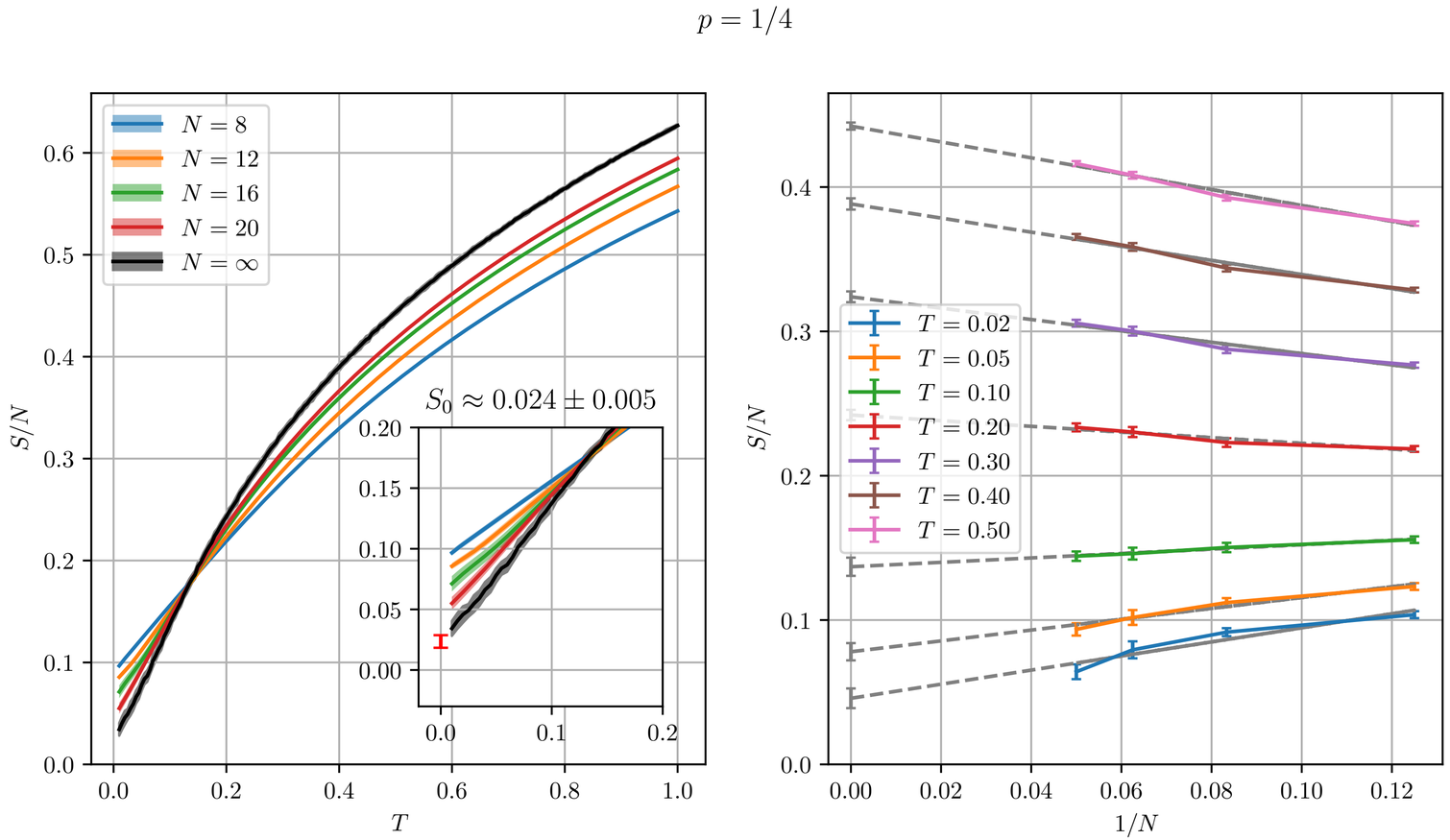
Thermodynamics from exact diagonalization of SU(2) random-t-J model

Large-N extrapolation of entropy density



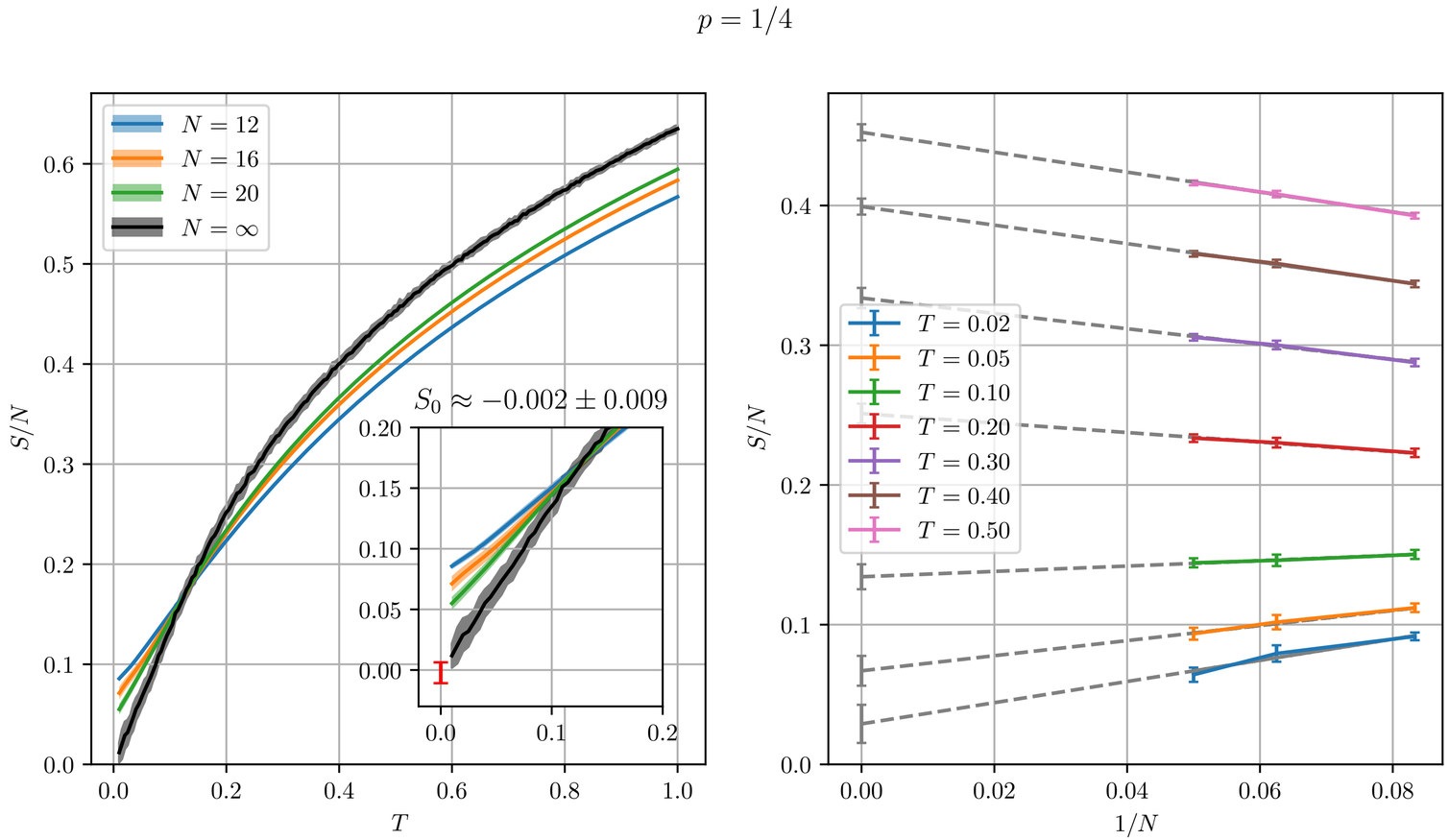
Thermodynamics from exact diagonalization of SU(2) random-t-J model

Non-zero s_0 at $p = 1/4$ is extrapolation-dependent



Thermodynamics from exact diagonalization of SU(2) random-t-J model

Non-zero s_0 at $p = 1/4$ is extrapolation-dependent



Thermodynamics from exact diagonalization of SU(2) random-t-J model

1. Random *J* model (insulator)

Operator spectrum and numerics

2. Random t-*J* model (metals)

Deconfined criticality and numerics

3. Random t-*J*-*U* model

Random *t-J-U*$_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c^{\dagger}_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c^{\dagger}_{i\alpha} c_{i\alpha},$$

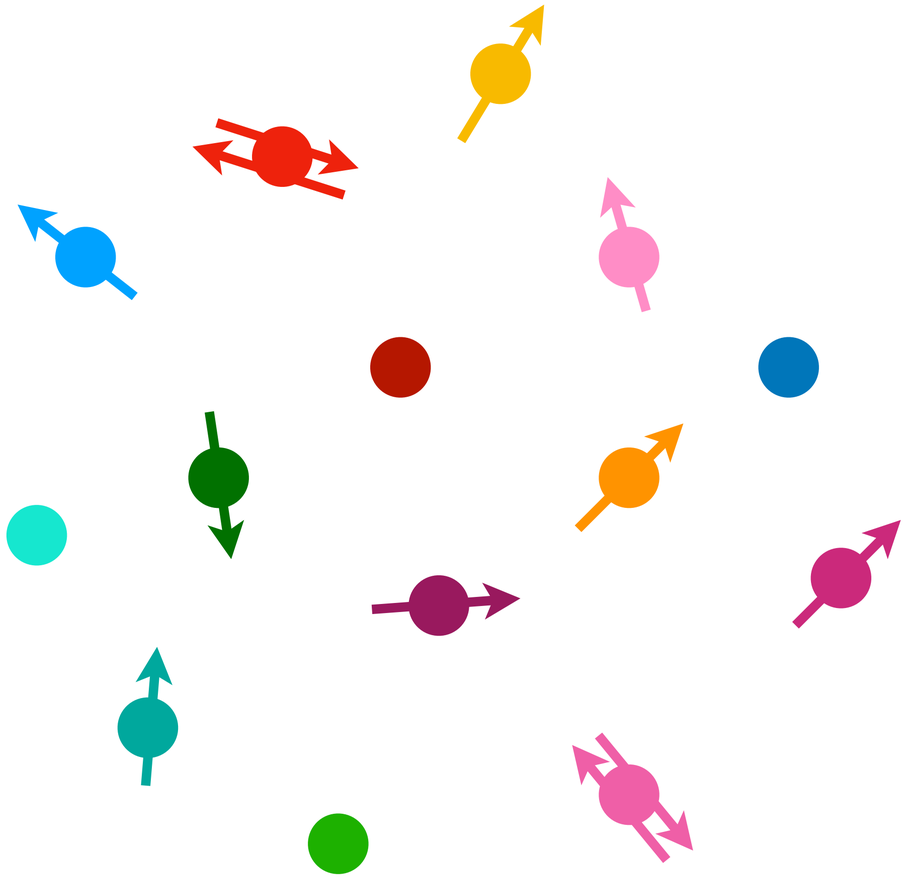
$$J_{ij} \text{ random}, \ \overline{J_{ij}} = 0, \ \overline{J^2_{ij}} = J^2$$

$$t_{ij} \text{ random}, \ \overline{t_{ij}} = 0, \ \overline{t^2_{ij}} = t^2$$

$$U_H > 0 \text{ non-random}$$

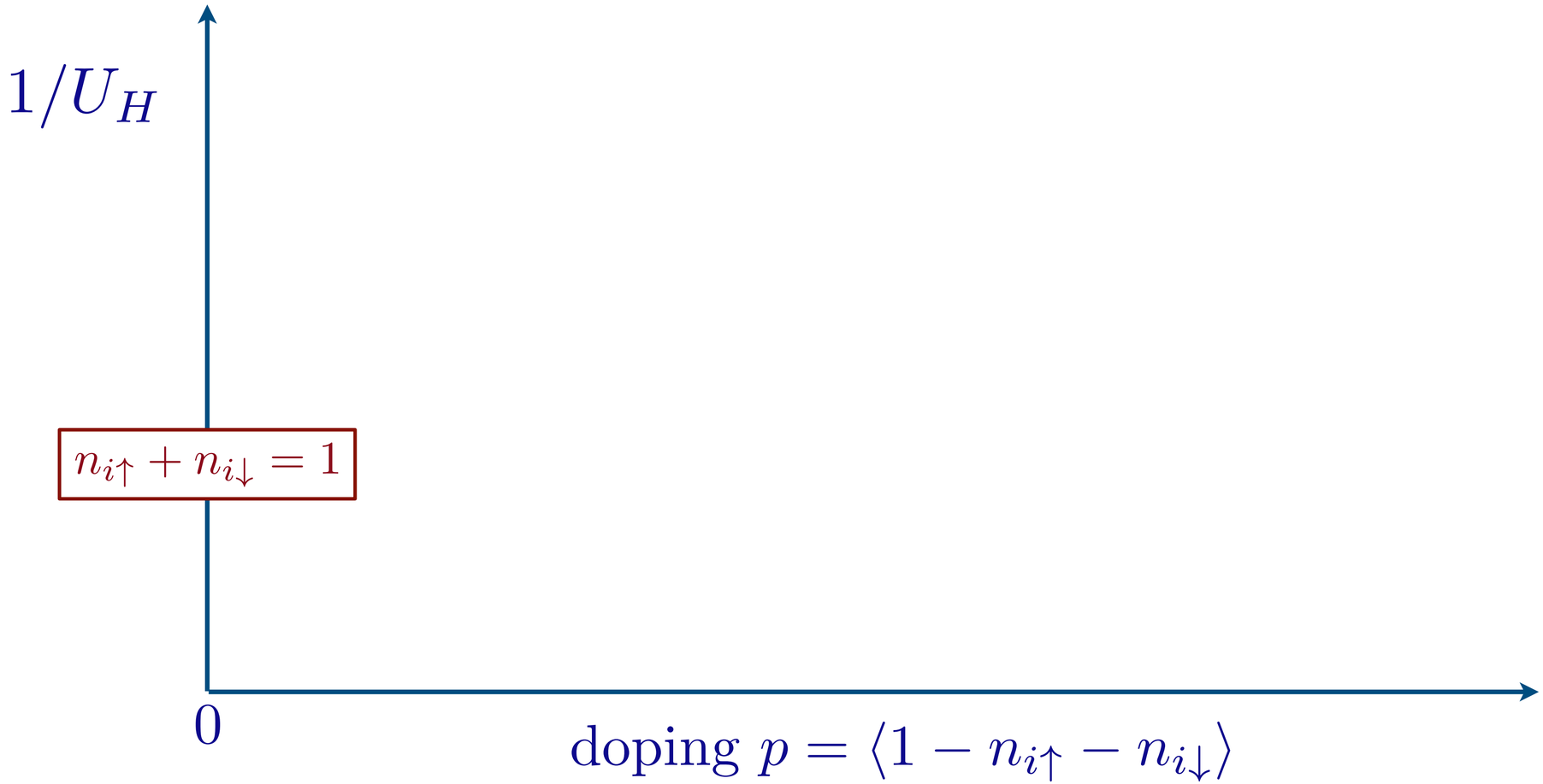
Random *t-J-U*$_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



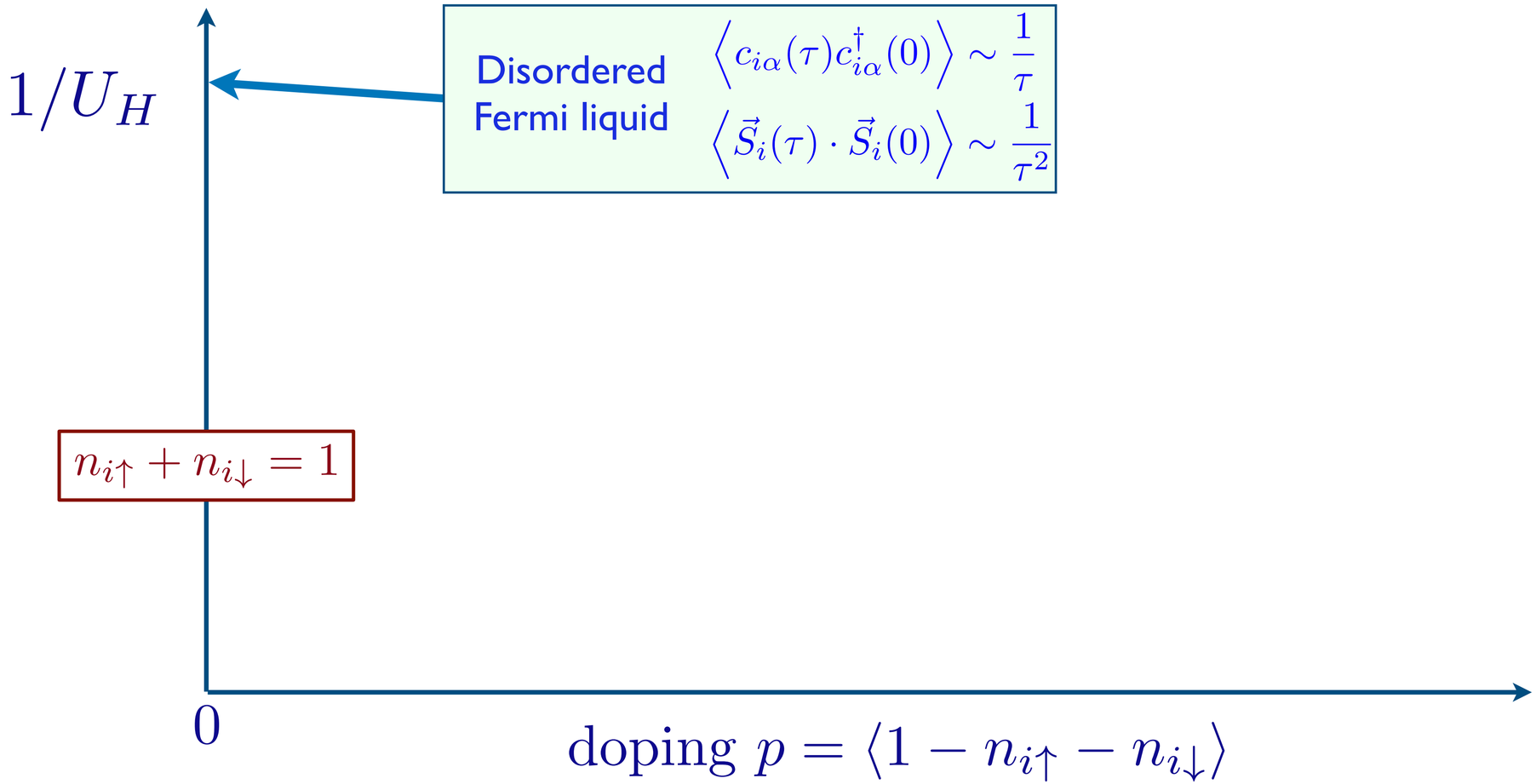
Random *t-J-U*$_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



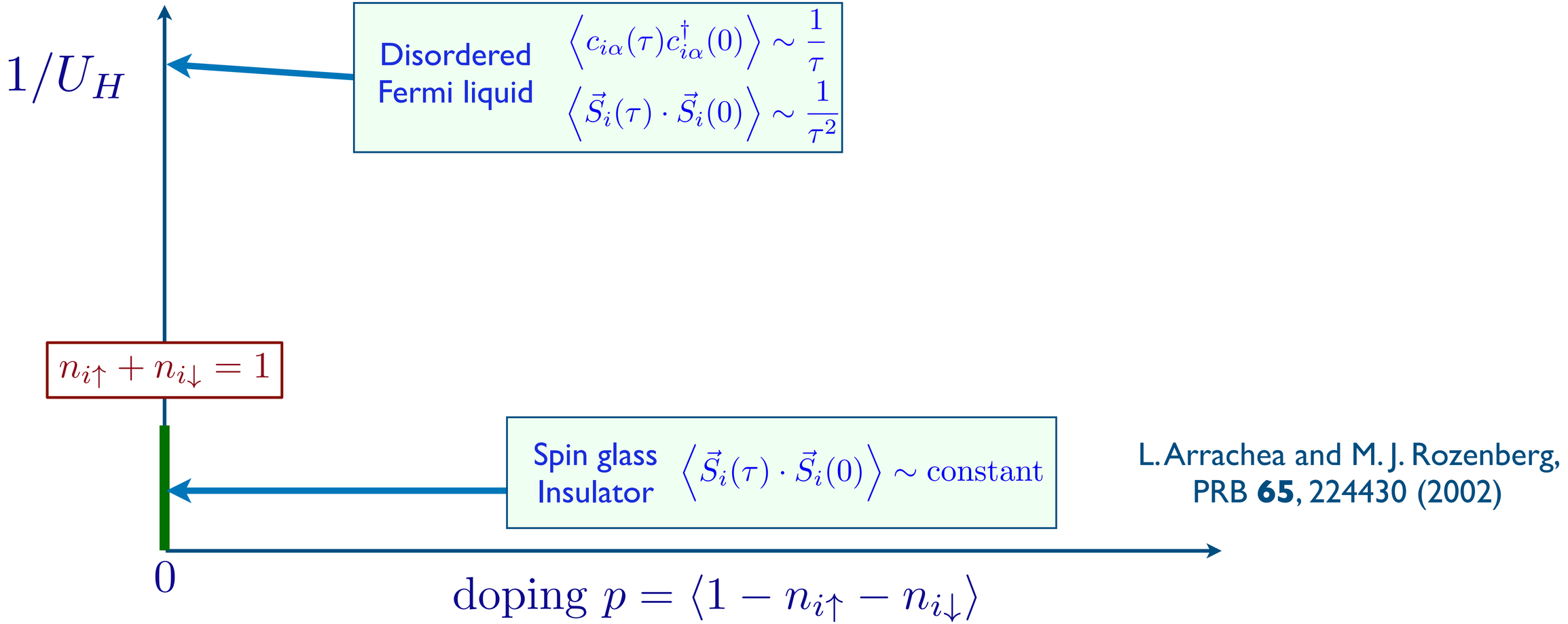
Random *t-J-U*$_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



Random *t-J-U*$_H$ model

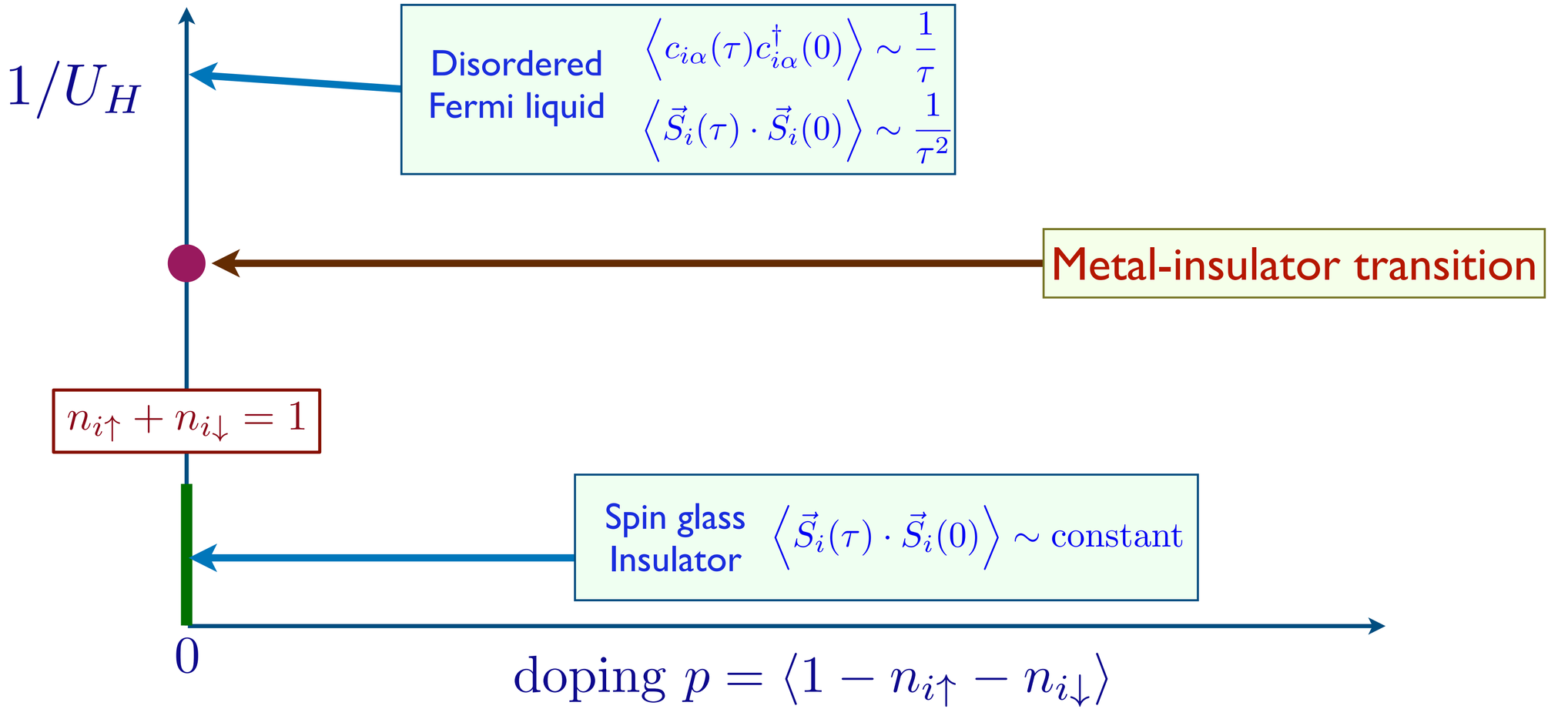
$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



L. Arrachea and M. J. Rozenberg,
PRB **65**, 224430 (2002)

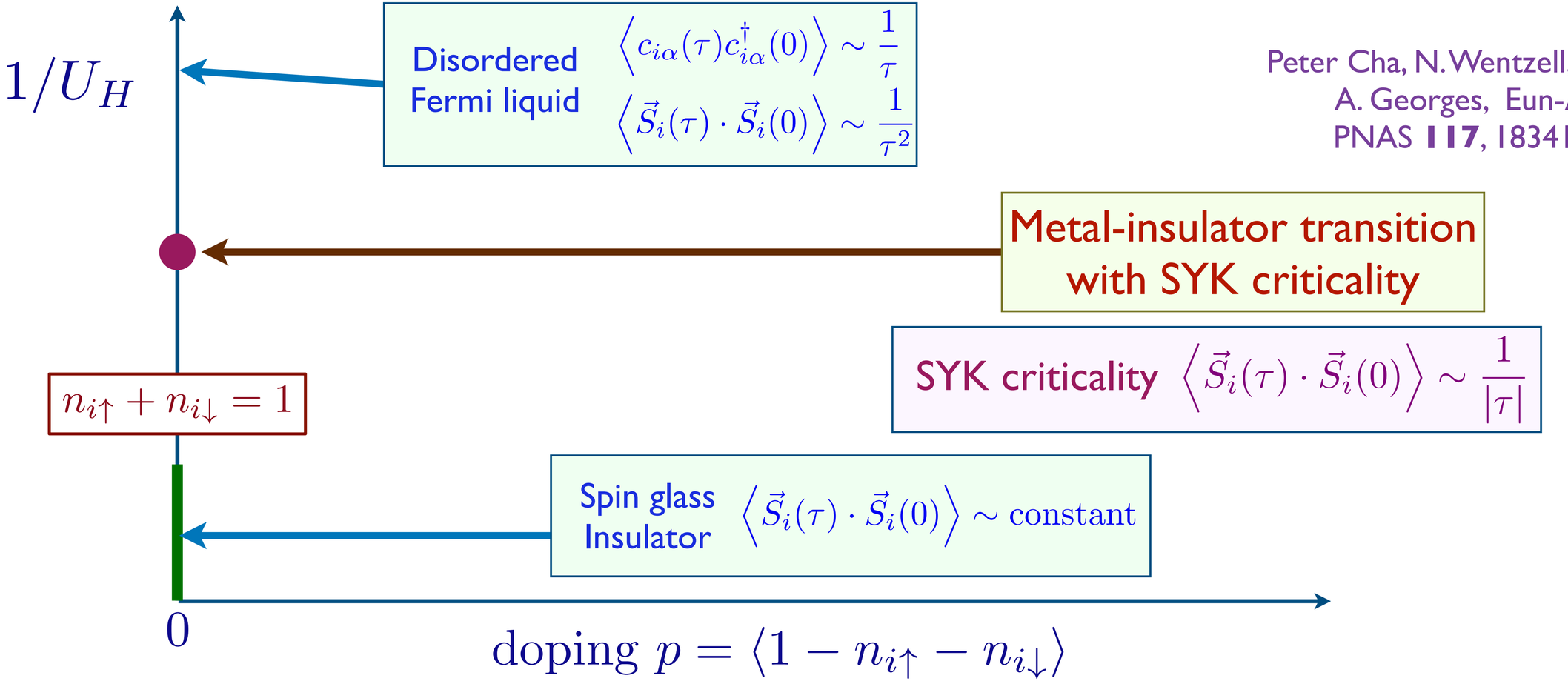
Random *t-J-U*$_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



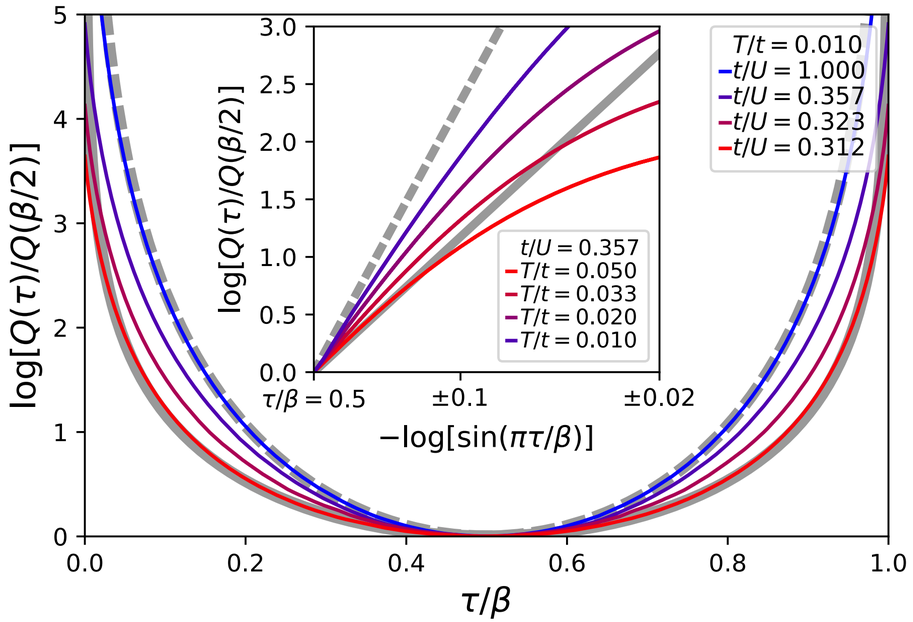
Random *t-J-U$_H$* model

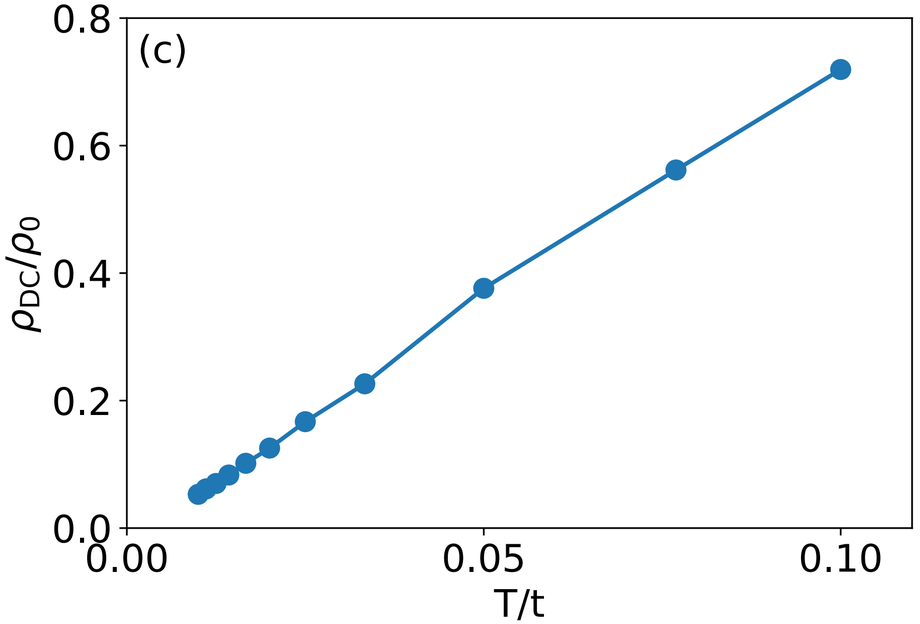
$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



Peter Cha, N. Wentzell, O. Parcollet, A. Georges, Eun-Ah Kim, PNAS **117**, 18341 (2020)

Linear resistivity and Sachdev-Ye-Kitaev (SYK) spin liquid behavior in a quantum critical metal with spin-1/2 fermions





Critical spin correlations:

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

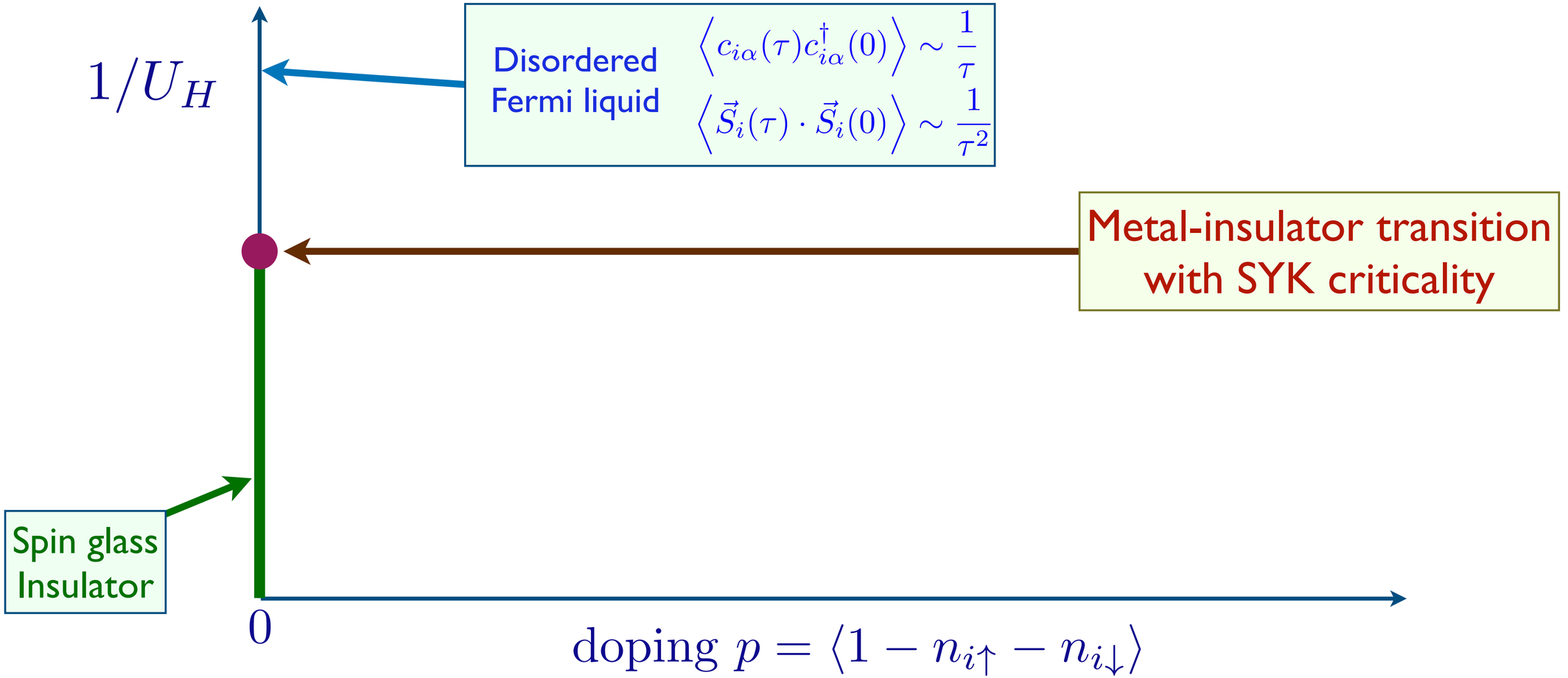
Resisitivity $\rho \sim T$ to the lowest T at the critical point

Onset of insulating gap and spin glass order appear to co-incide.

Peter Cha, N. Wentzell, O. Parcollet, A. Georges, Eun-Ah Kim, PNAS **117**, 18341 (2020)

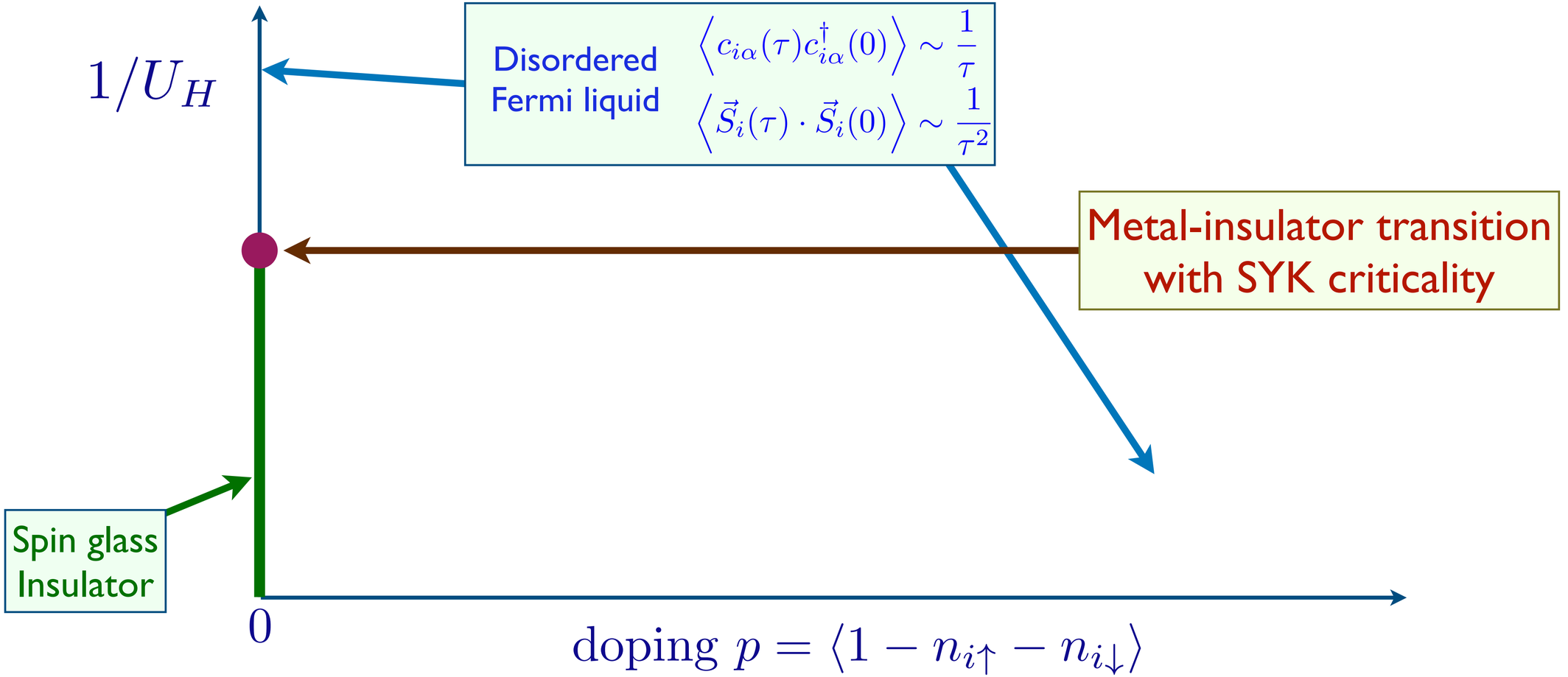
Random *t-J-U*$_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^\dagger_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



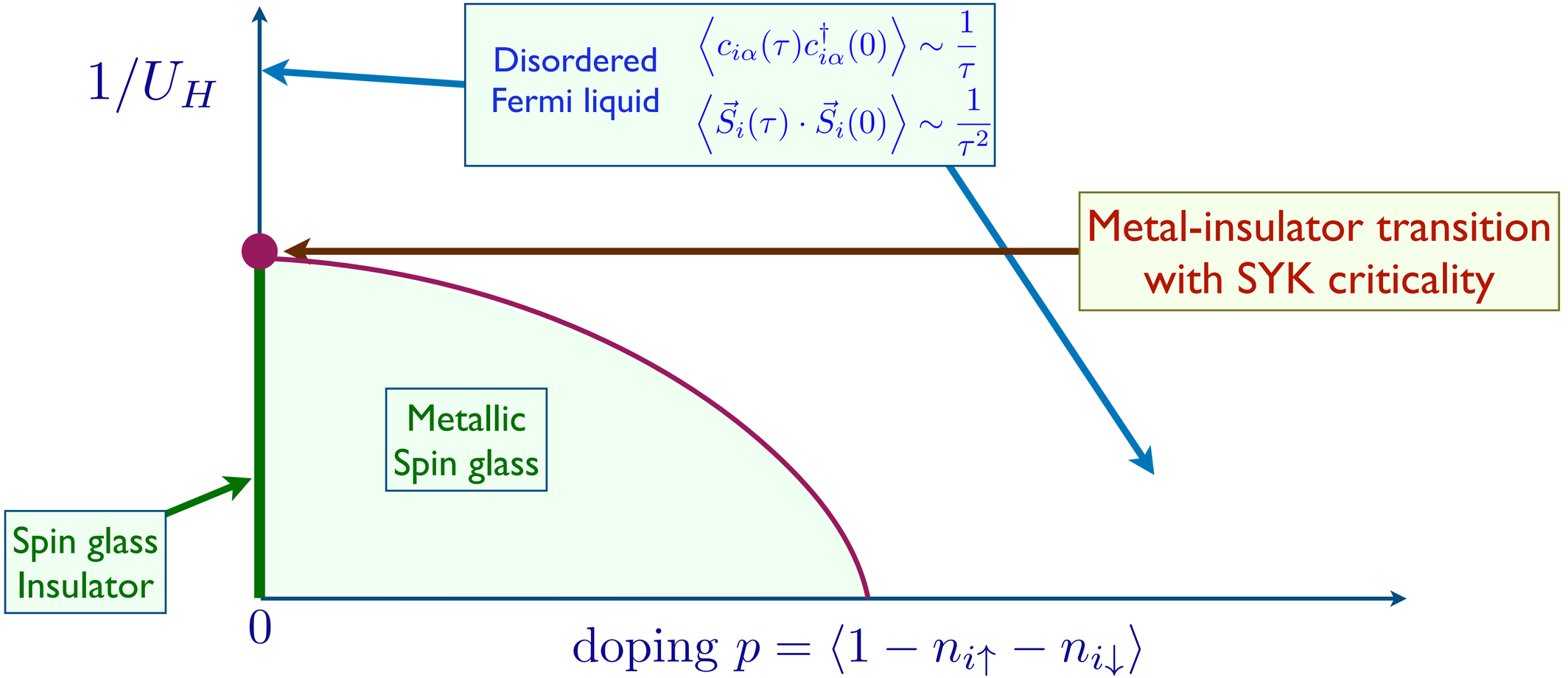
Random *t-J-U*$_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



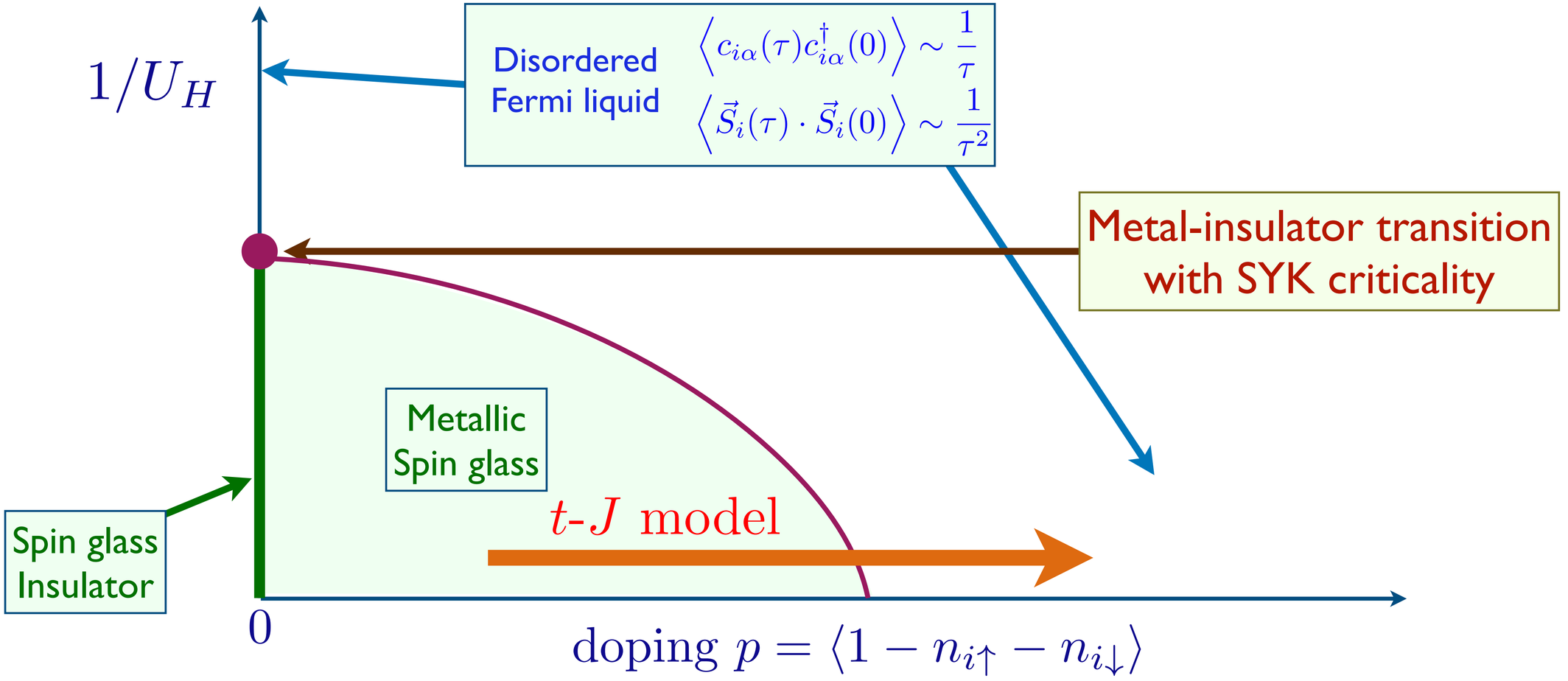
Random *t-J-U*$_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



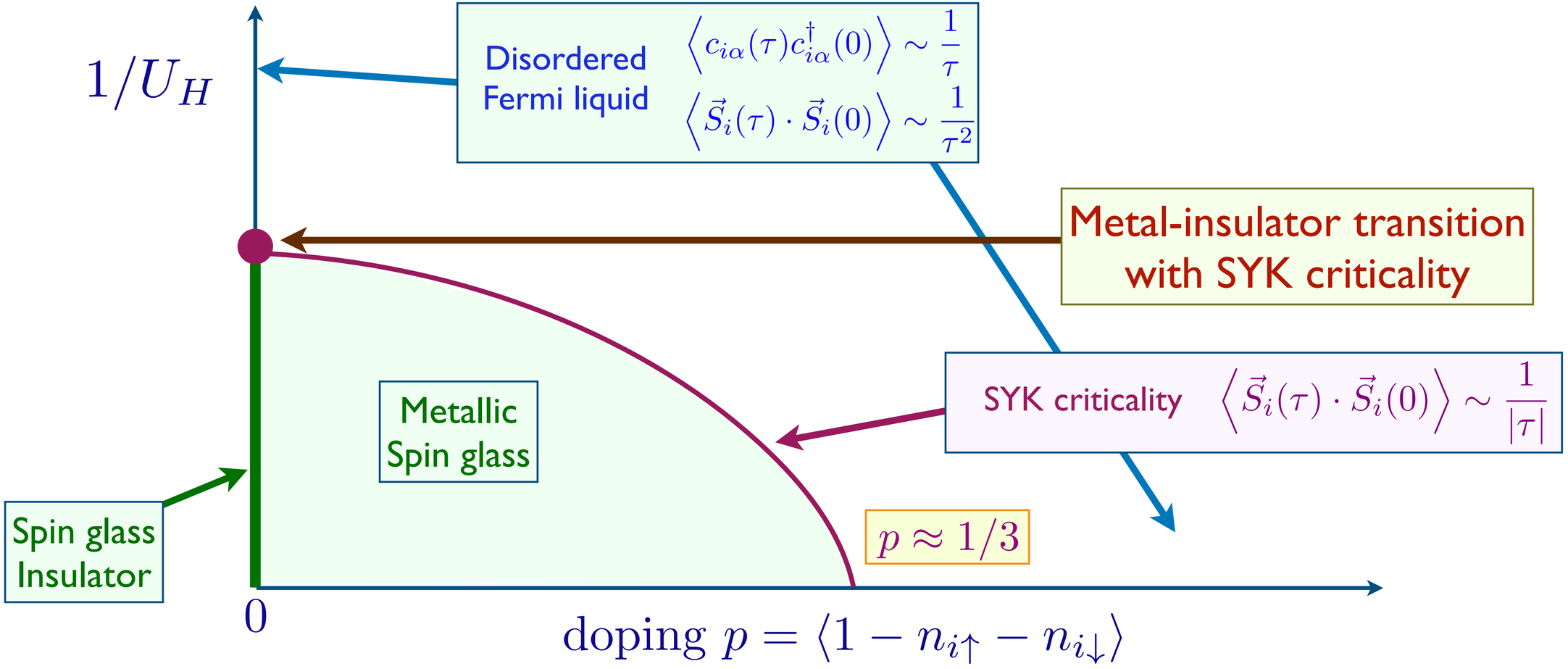
Random t-J-U_H model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



Random *t-J-U*$_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



Random t-J model

- Proposed phase diagram of random t-J model captures key characteristics of cuprates.
- Critical electron Green's function $G_c(\tau) \sim 1/\tau$ and local spin correlator $\chi(\tau) \sim 1/|\tau|$.
- Can be interpreted in terms of fractionalization with spinon and holon correlators $\sim 1/\sqrt{\tau}$ (deconfined criticality).
- Linear-in-T resistivity down to $T = 0$ at the critical point from the time reparameterization soft mode (but there could be other singular modes).
- Carrier density p for $p < p_c$, and $1 + p$ for $p > p_c$.
- Extensive zero temperature entropy $\lim_{T\to 0} \lim_{N\to\infty} S/N > 0$. Related to entropy of extremal black holes.