Strange metals and black holes

ICFO

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Talk online: sachdev.physics.harvard.edu

PHYSICS
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Ordinary metals:
quasiparticles

Strange metals:
no quasiparticles

Black
holes

Ordinary metals:
quasiparticles

Strange metals:
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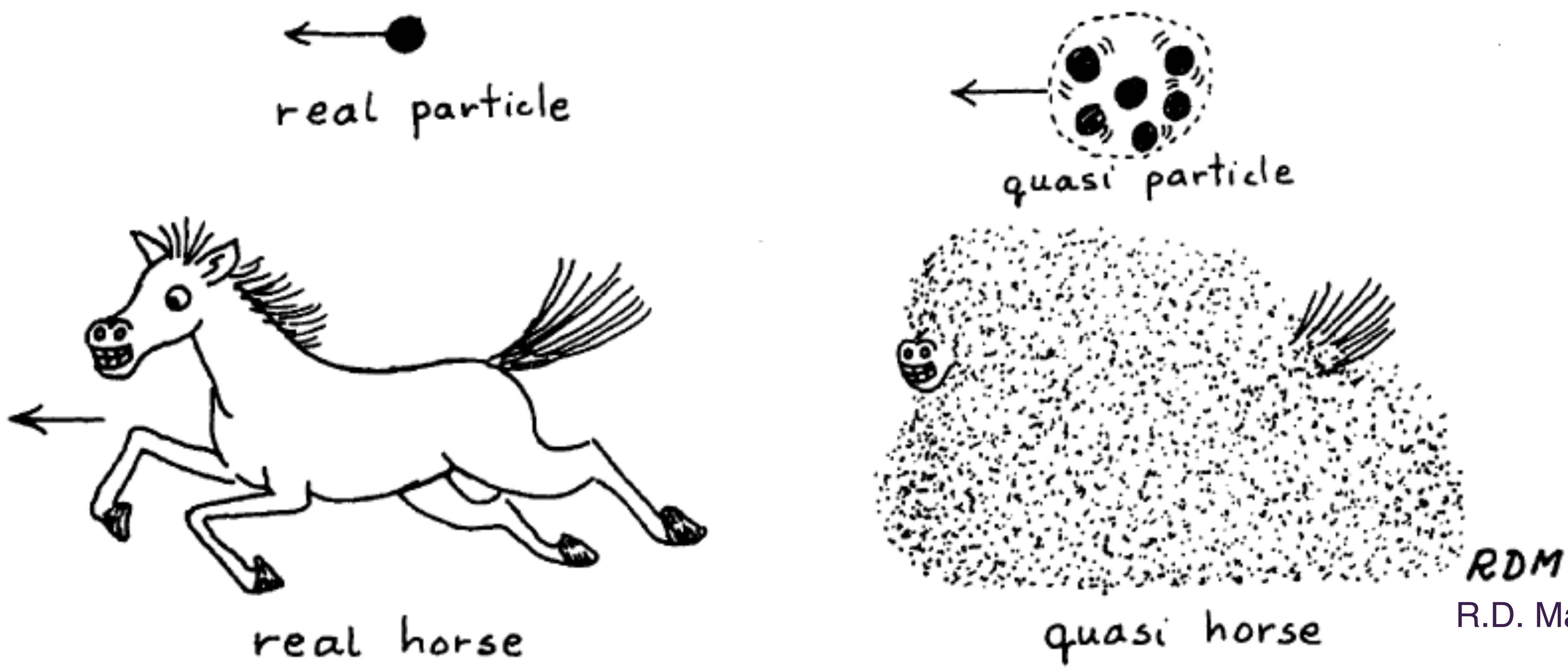
Ordinary metals

Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal

Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle.



R.D. Mattuck

What are quasiparticles ?

- **Quasiparticles are additive excitations:** The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy e_α

$$E = \sum_\alpha n_\alpha e_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \dots$$

In a lattice system of N sites, this parameterizes the energy of $\sim e^{\alpha N}$ states in terms of poly(N) numbers.

What are quasiparticles ?

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{\text{eq}} \sim \frac{\hbar U^2/E_F}{(k_B T)^2} \quad , \quad \text{as } T \to 0,$$

  where U is the strength of interactions and E_F is the Fermi energy.

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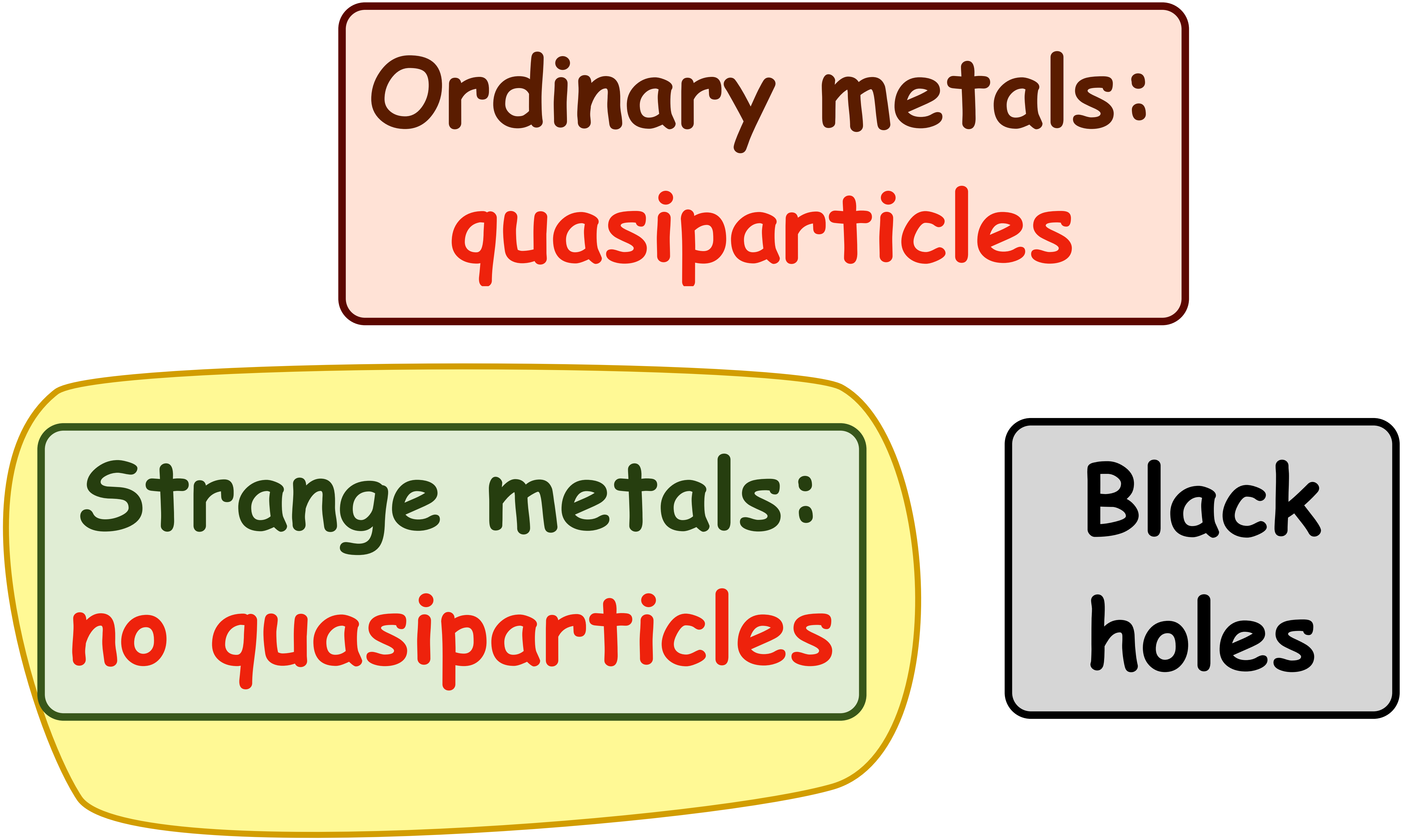
- Similarly, a quasiparticle model implies a resistivity

$$\rho = \frac{m^*}{ne^2}\frac{1}{\tau} \sim T^2 \quad \text{with} \;\; \tau \sim \tau_{\rm eq}$$

What are quasiparticles ?

- These times are much longer than the 'Planckian time' $\hbar/(k_B T)$, which we will find in systems without quasiparticle excitations.

$$\tau \sim \tau_{\text{eq}} \gg \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$



Ordinary metals:
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Strange metals:
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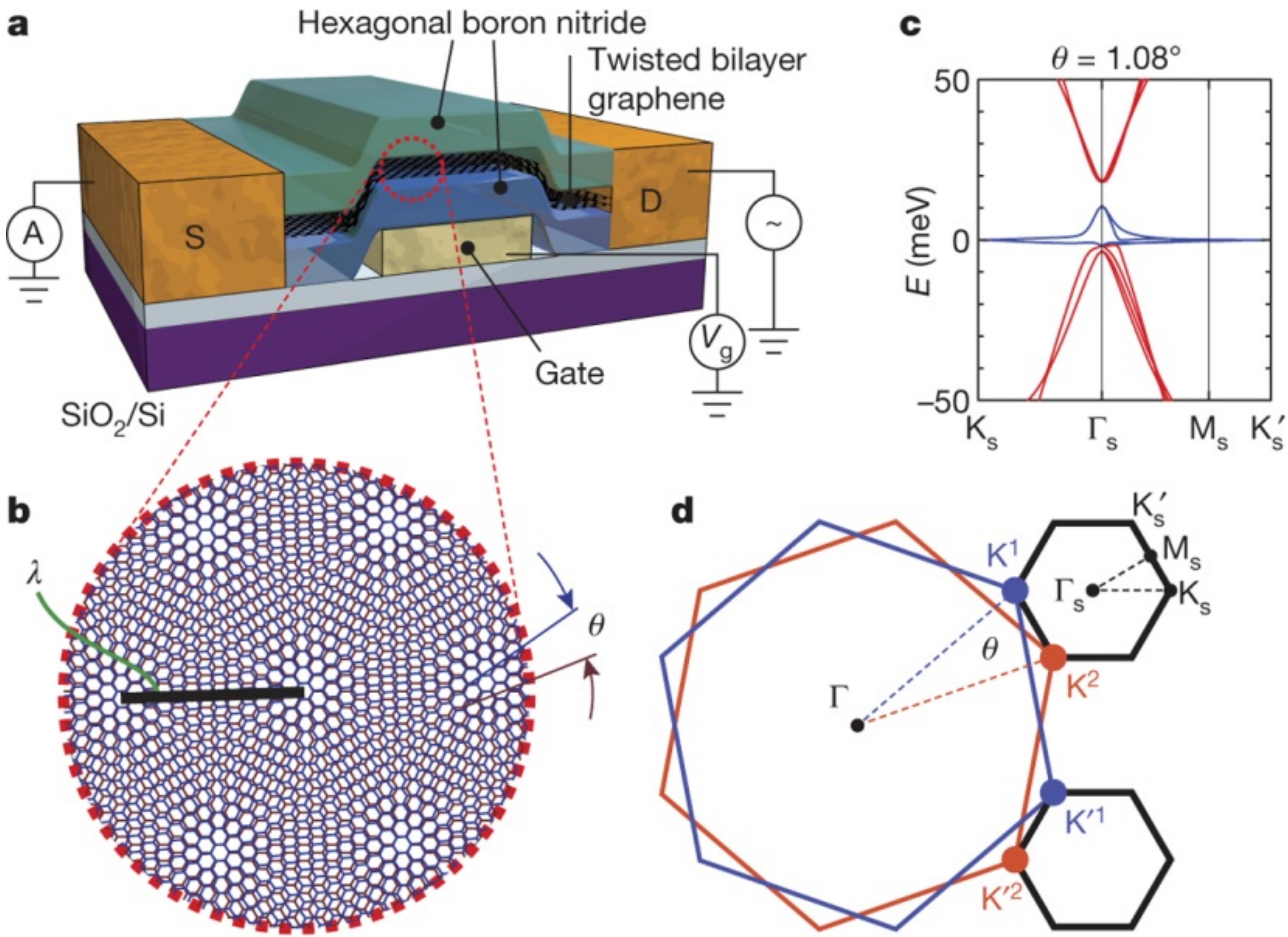
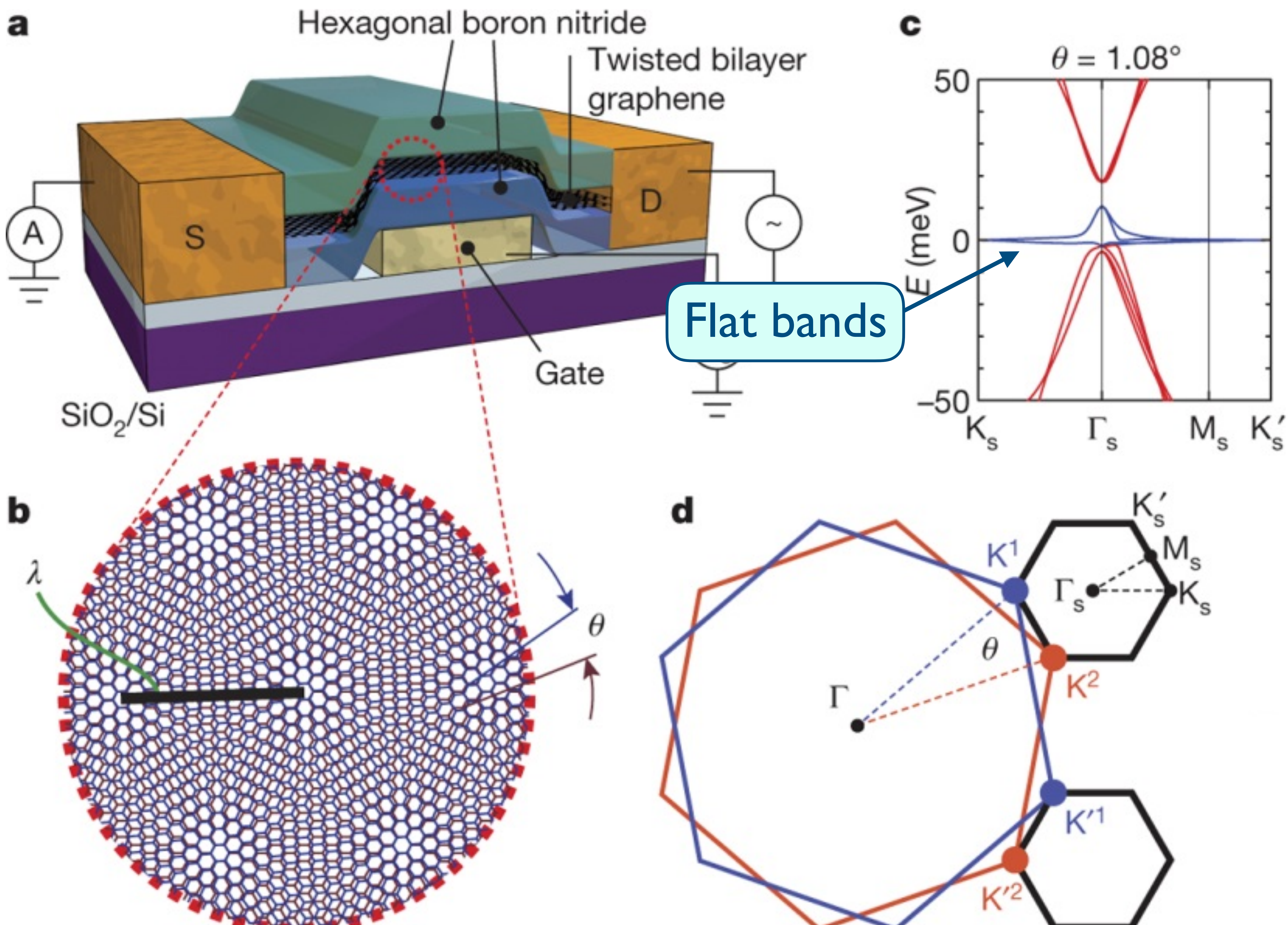
Twisted bilayer graphene



Cao et al., Nature **556**, 80 (2018)

Twisted bilayer graphene



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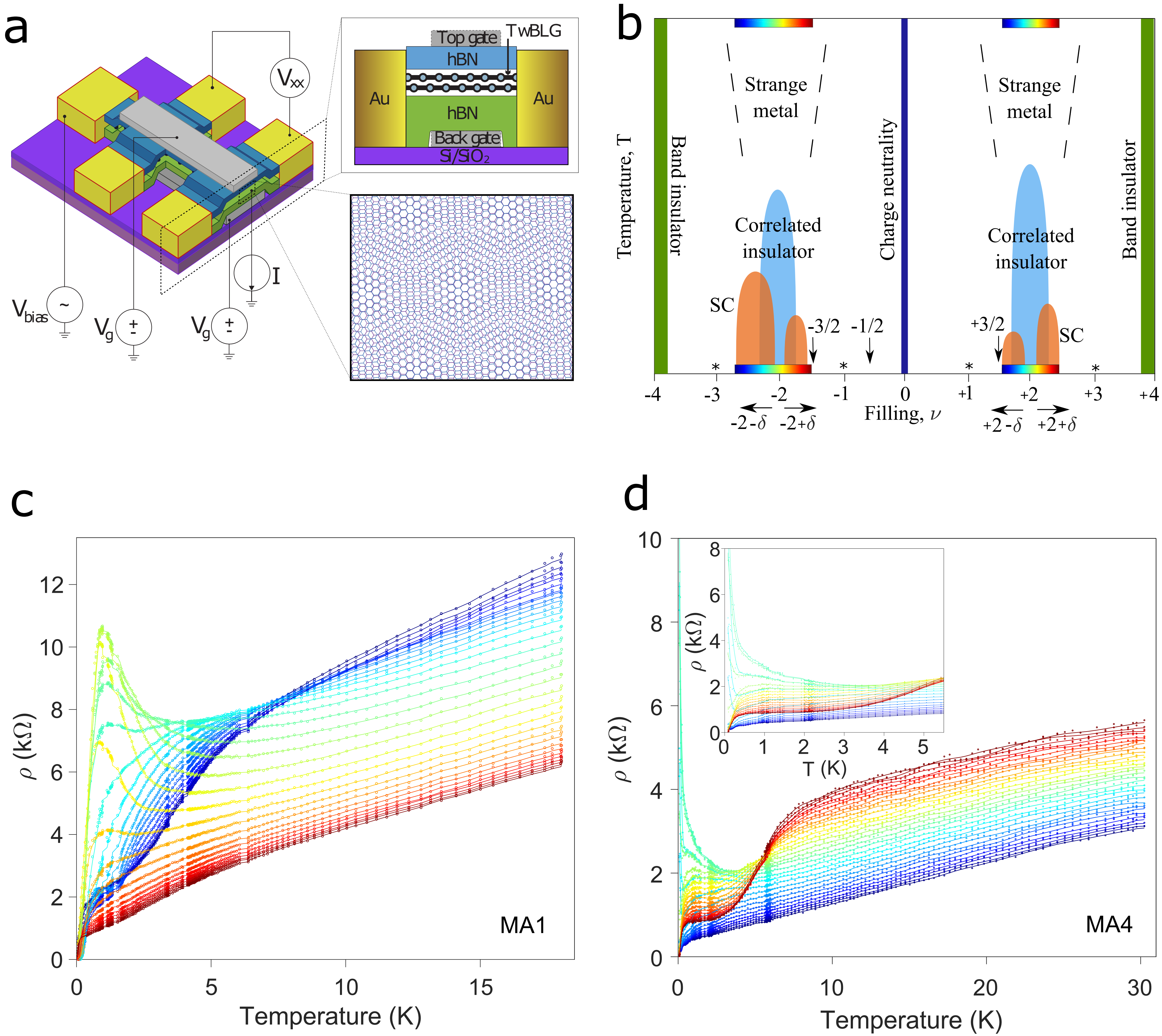
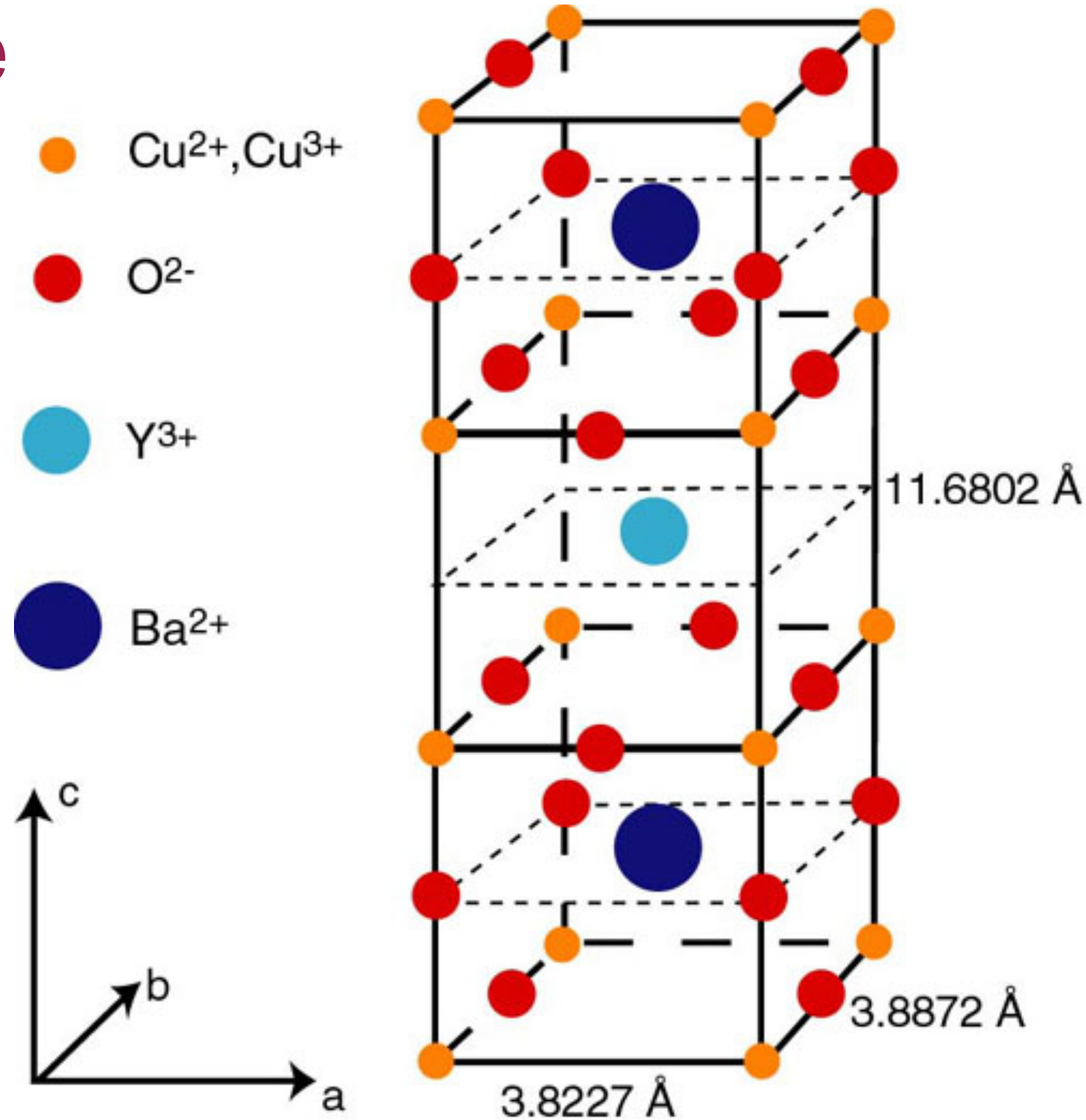


Cao *et al.*, arXiv:1901.03710

High temperature superconductors



$$YBa_2Cu_3O_{6+x}$$

Universal T-linear resistivity and Planckian dissipation in overdoped cuprates

NATURE PHYSICS | VOL 15 | FEBRUARY 2019 | 142–147

A. Legros[1,2], S. Benhabib[3], W. Tabis[3,4], F. Laliberté[1], M. Dion[1], M. Lizaire[1], B. Vignolle[3], D. Vignolles[3], H. Raffy[5], Z. Z. Li[5], P. Auban-Senzier[5], N. Doiron-Leyraud[1], P. Fournier[1,6], D. Colson[2], L. Taillefer[1,6]* and C. Proust[3,6]*

arXiv:1902.01034

Planckian dissipation and scale invariance in a quantum-critical disordered pnictide

Yasuyuki Nakajima,[1,2] Tristin Metz,[2] Christopher Eckberg,[2] Kevin Kirshenbaum,[2] Alex Hughes,[2] Renxiong Wang,[2] Limin Wang,[2] Shanta R. Saha,[2] I-Lin Liu,[2,3,4] Nicholas P. Butch,[2,4] Zhonghao Liu,[5,6] Sergey V. Borisenko,[5] Peter Y. Zavalij,[7] and Johnpierre Paglione[2,8]

Strange metal in magic-angle graphene with near Planckian dissipation

Yuan Cao,[1,*] Debanjan Chowdhury,[1,*] Daniel Rodan-Legrain,[1] Oriol Rubies-Bigordà,[1] Kenji Watanabe,[2] Takashi Taniguchi,[2] T. Senthil,[1,†] and Pablo Jarillo-Herrero[1,†]

arXiv:1901.03710

Bad metallic transport in a cold atom Fermi-Hubbard system

Science **363**, 379–382 (2019)

Peter T. Brown[1], Debayan Mitra[1], Elmer Guardado-Sanchez[1], Reza Nourafkan[2], Alexis Reymbaut[2], Charles-David Hébert[2], Simon Bergeron[2], A.-M. S. Tremblay[2,3], Jure Kokalj[4,5], David A. Huse[1], Peter Schauß[1*], Waseem S. Bakr[1]†

Remarkable recent observation of 'Planckian' strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity, ρ, is

$$\rho = \frac{m^*}{ne^2}\,\frac{1}{\tau}$$

with a universal scattering rate

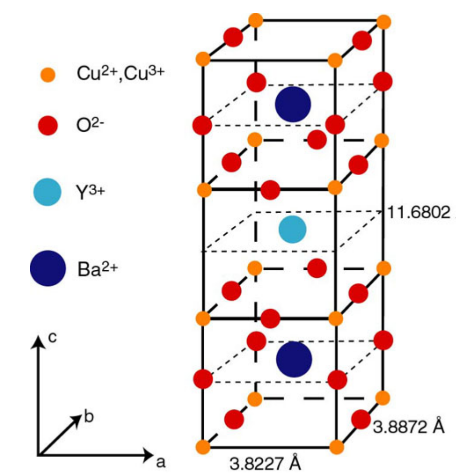
$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar}\,,$$

independent of the strength of interactions!

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

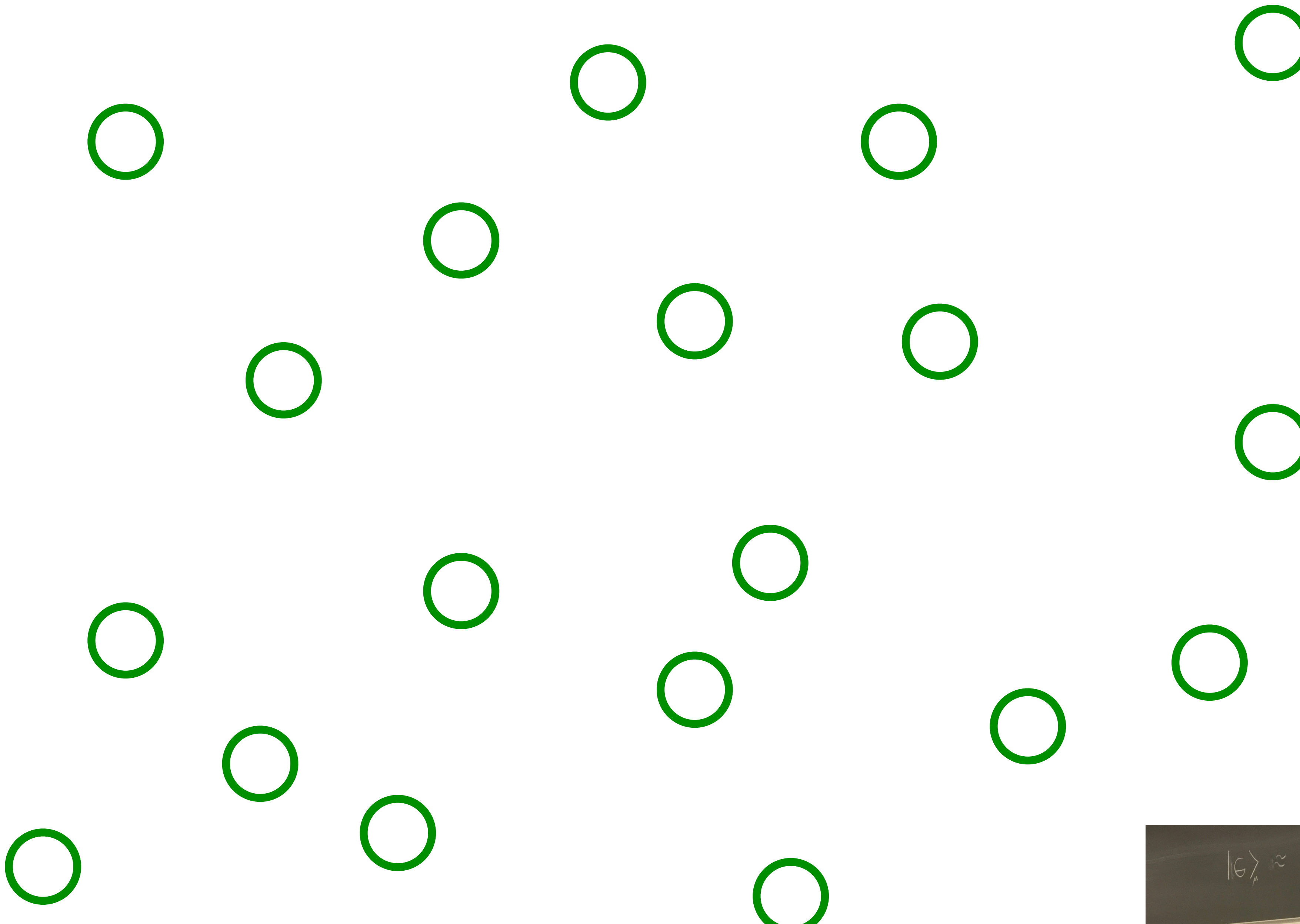
Slope of *T*-linear resistivity vs Planckian limit in seven materials.

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

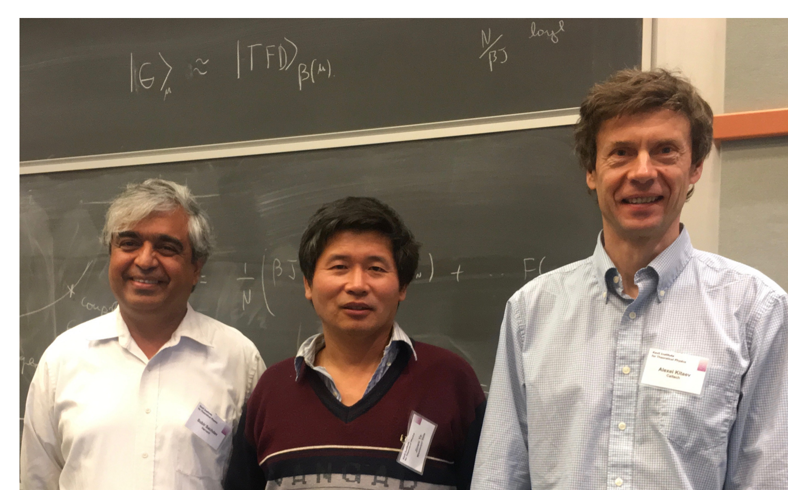


A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, Nature Physics **15**, 142 (2019)

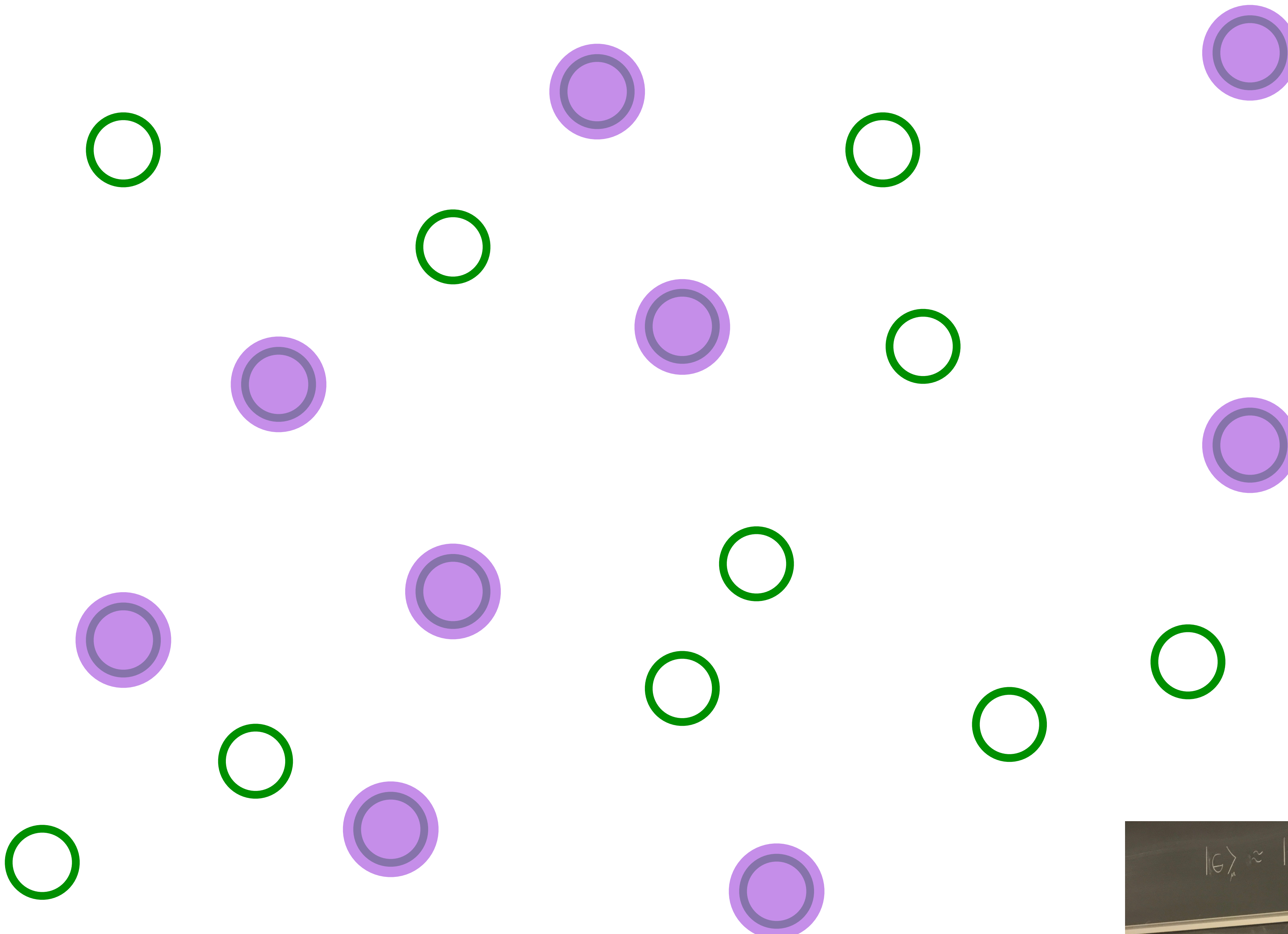
The Sachdev-Ye-Kitaev (SYK) model



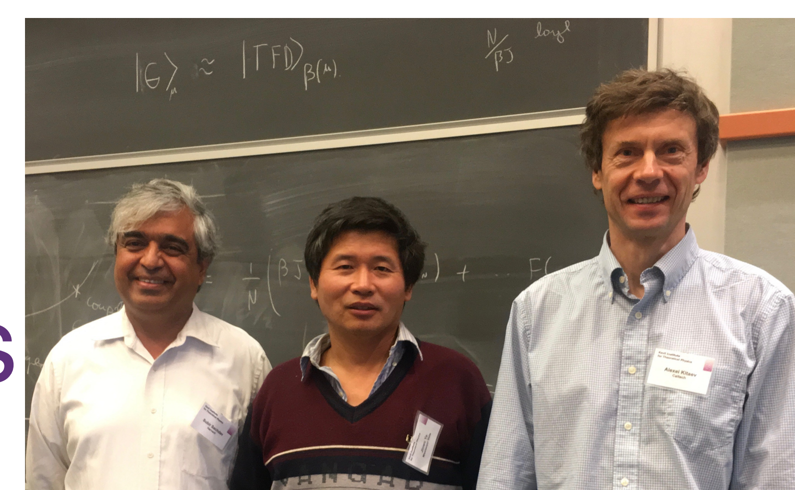
Pick a set of random positions



The SYK model



Place electrons randomly on some sites



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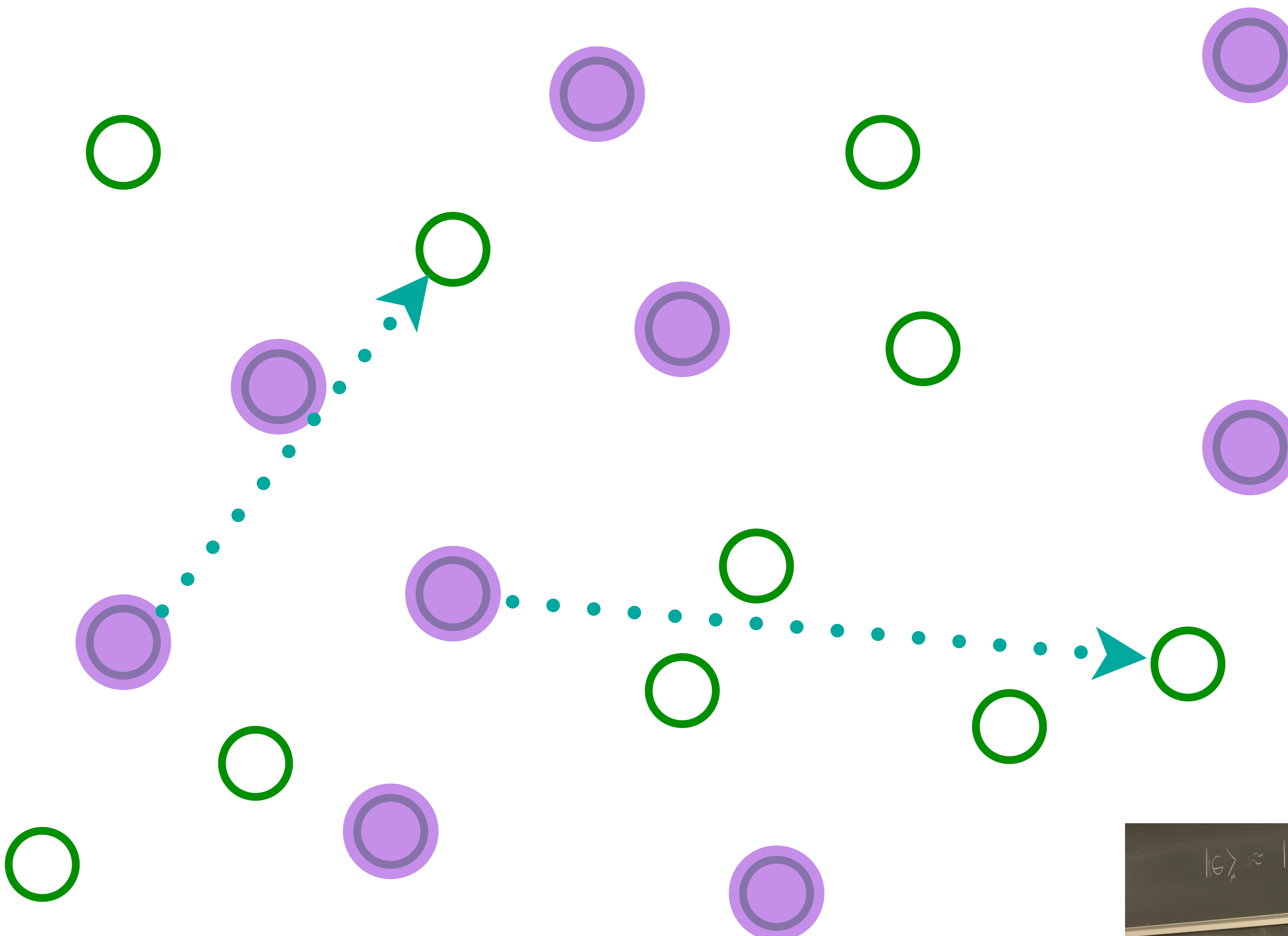

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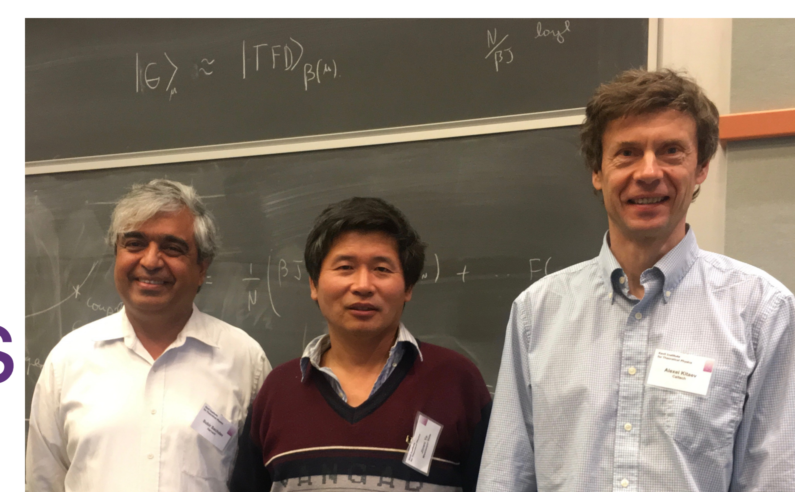

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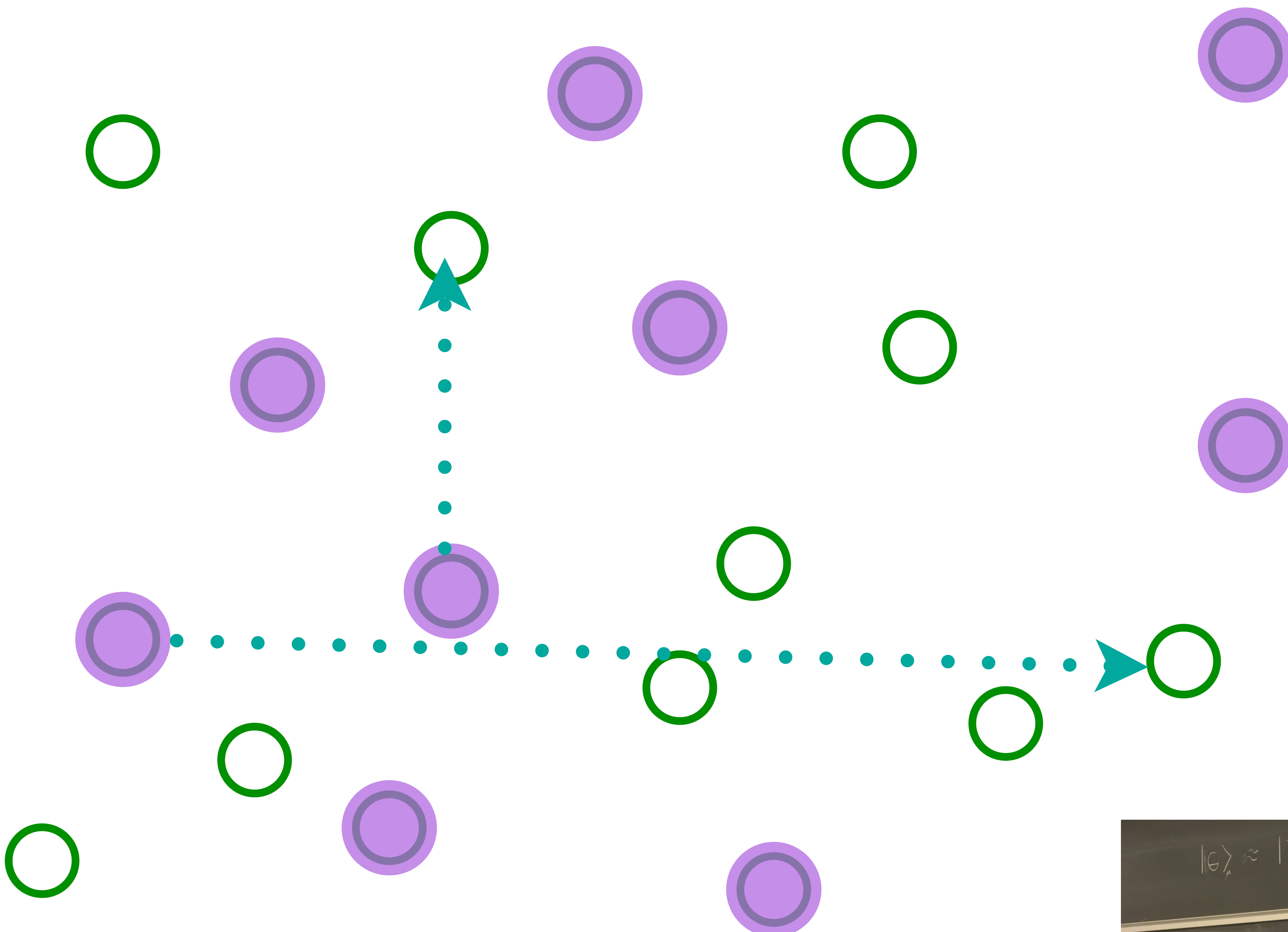

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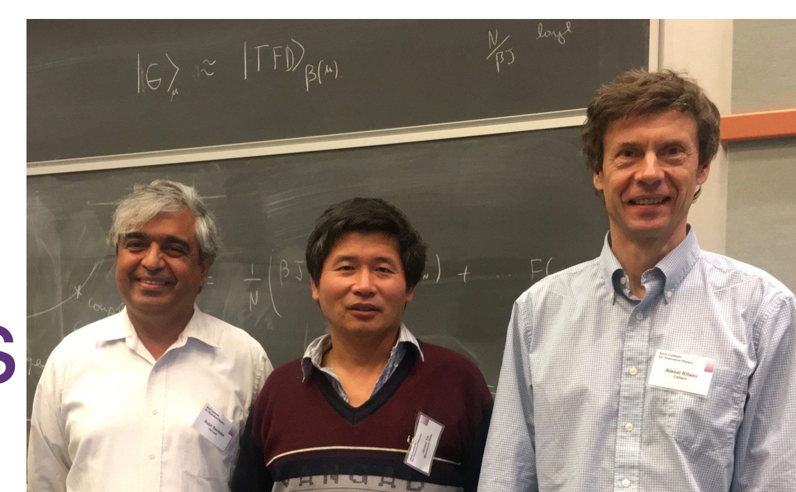

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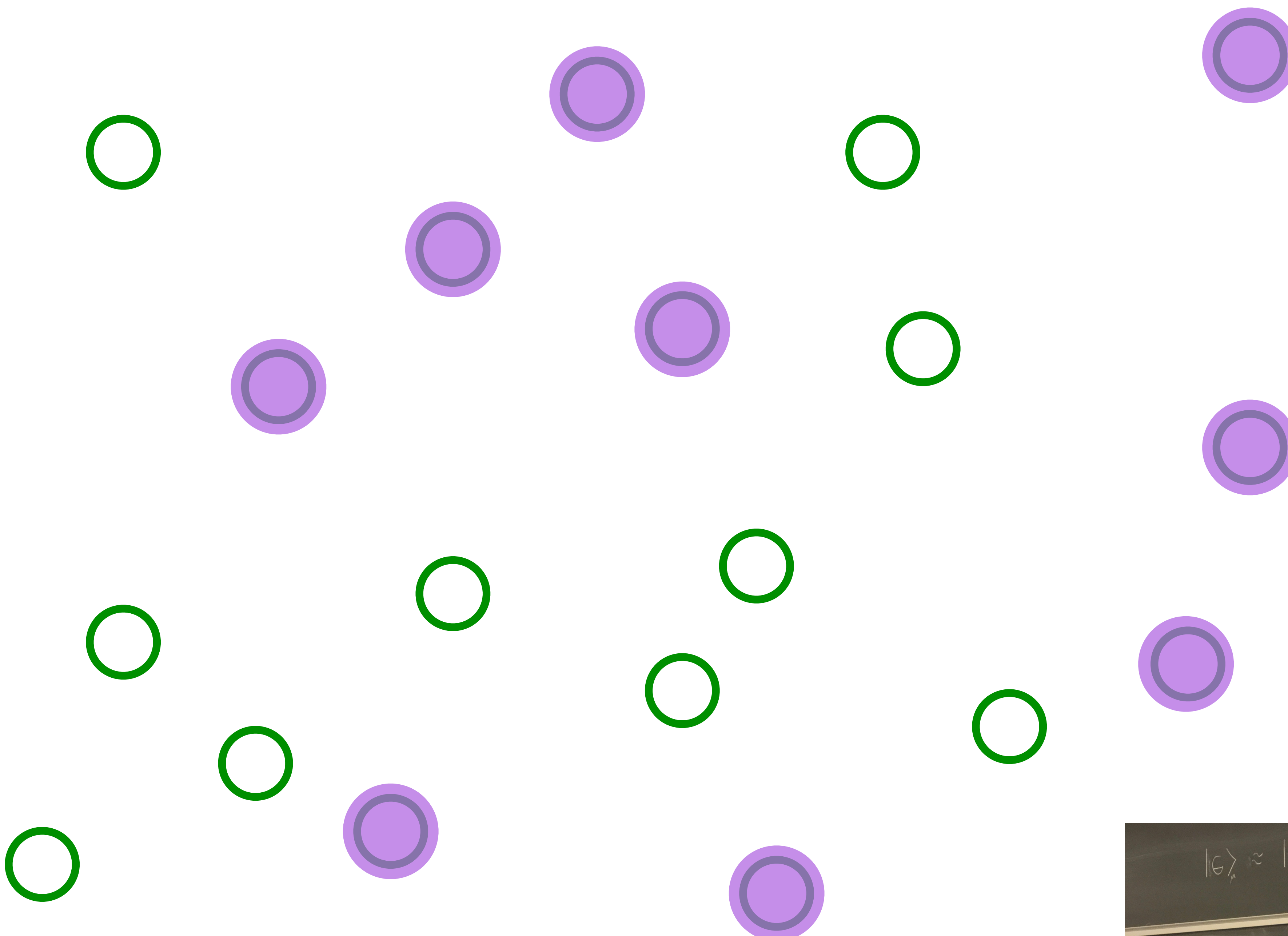
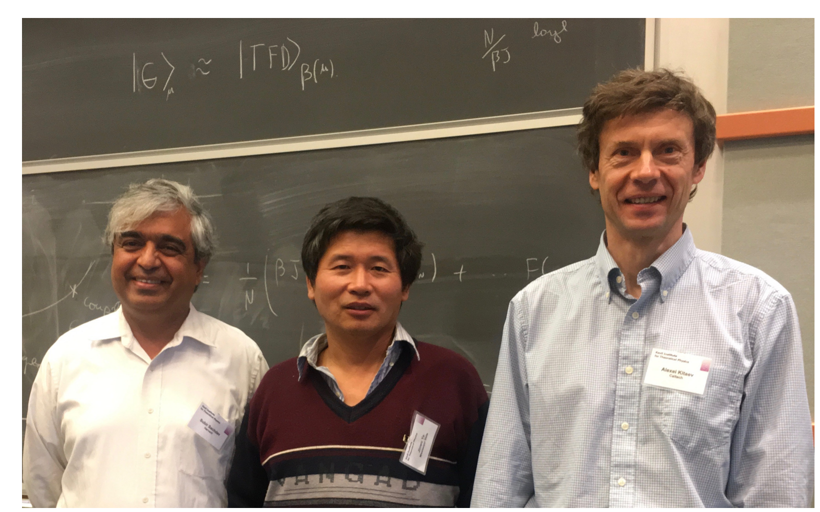


Entangle electrons pairwise randomly



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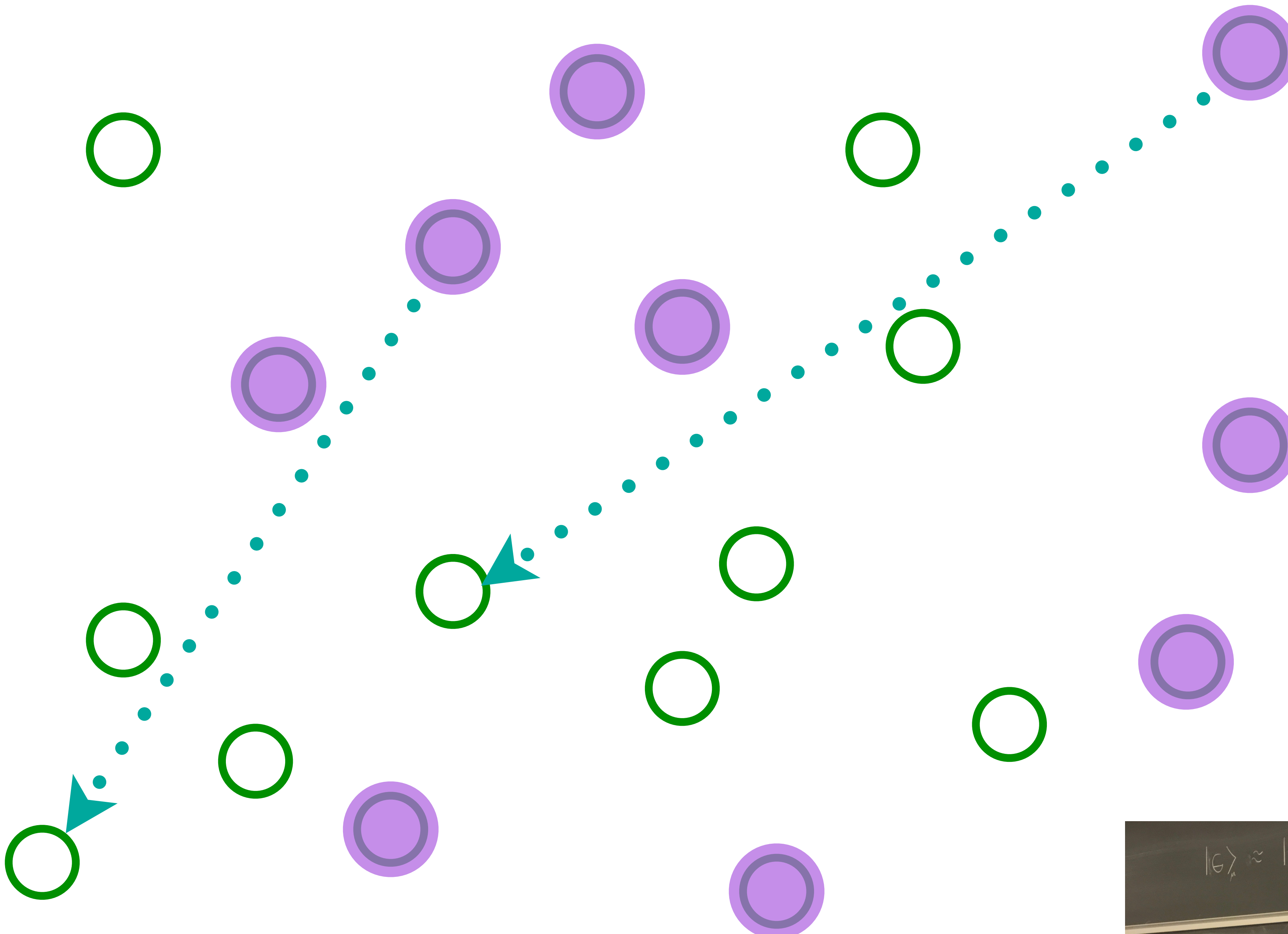

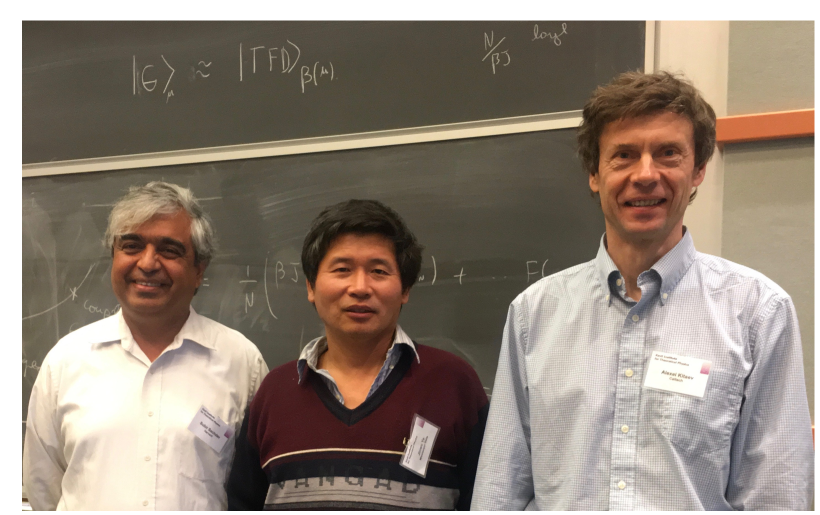

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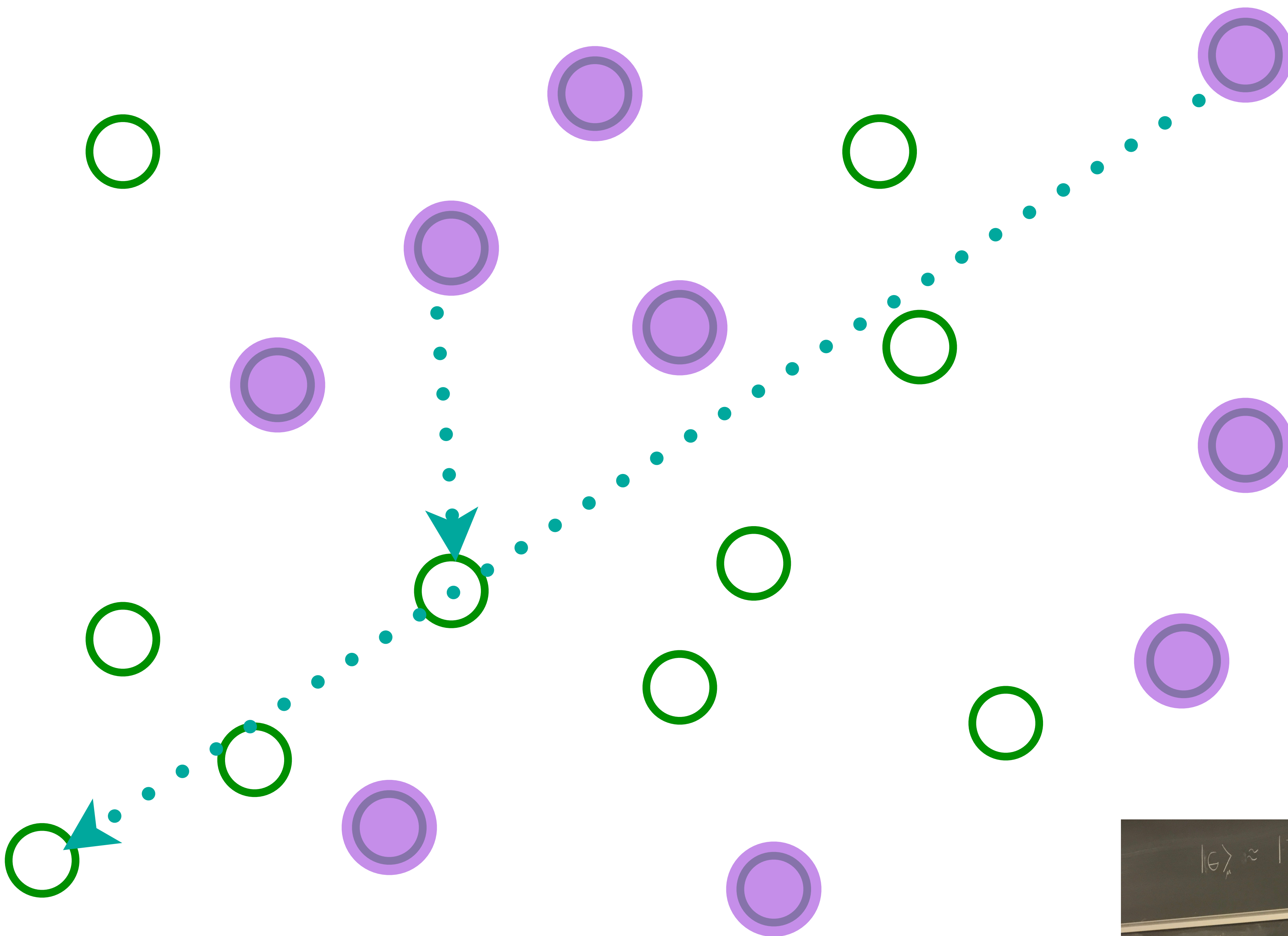

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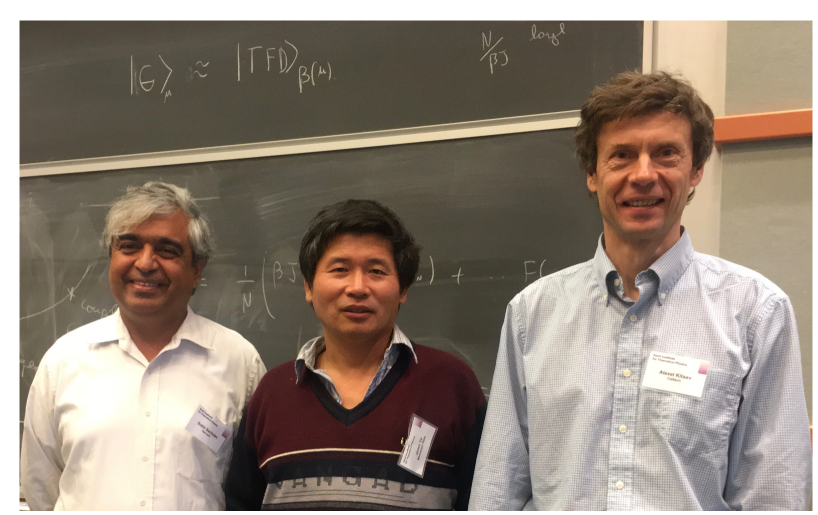

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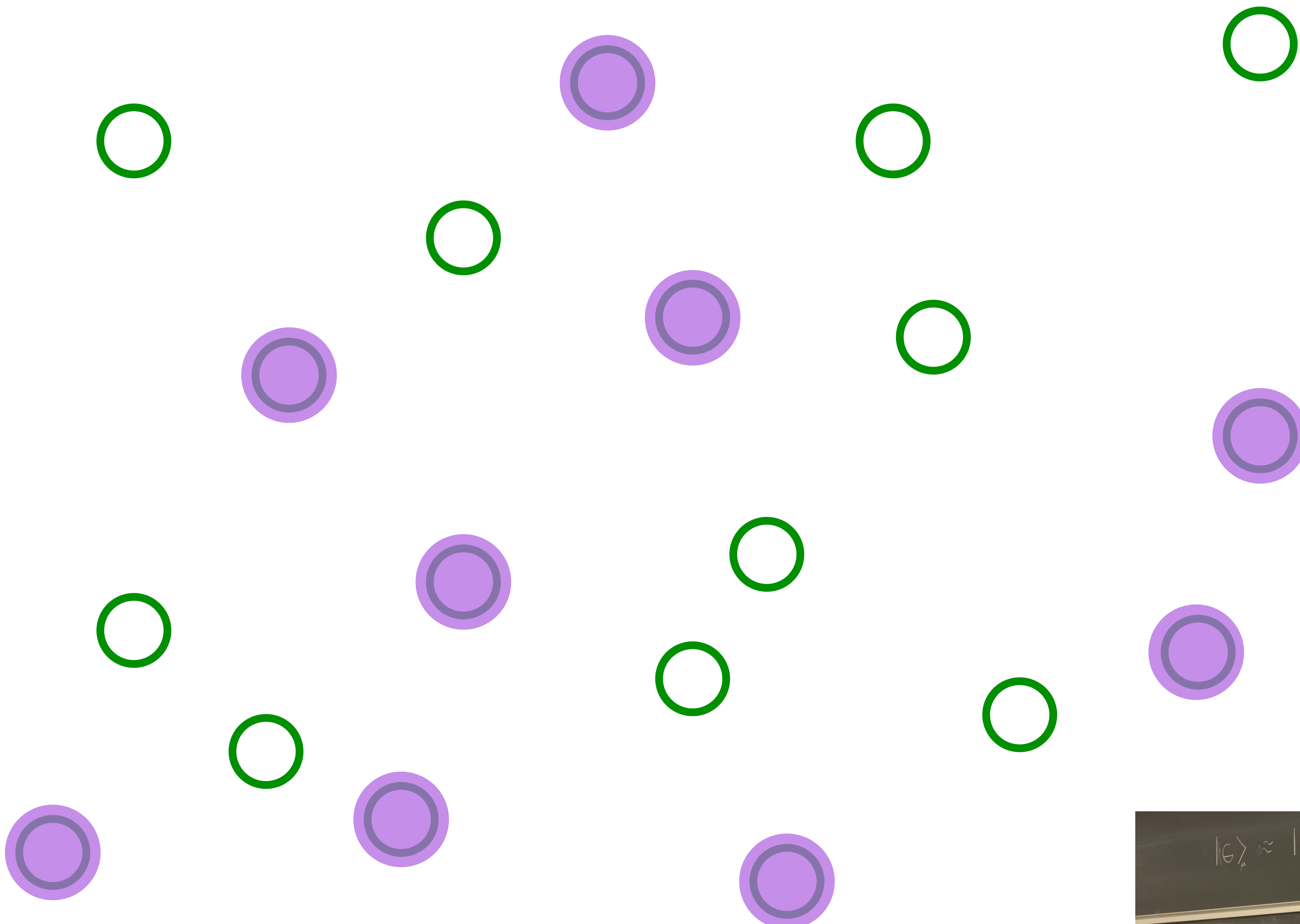

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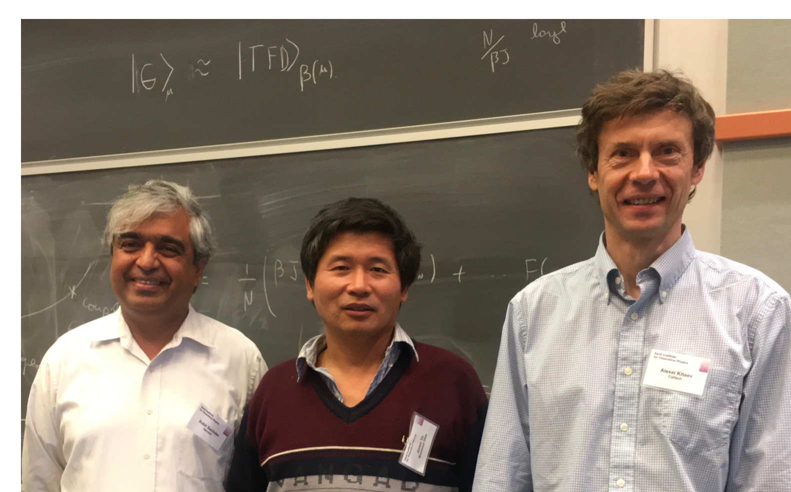

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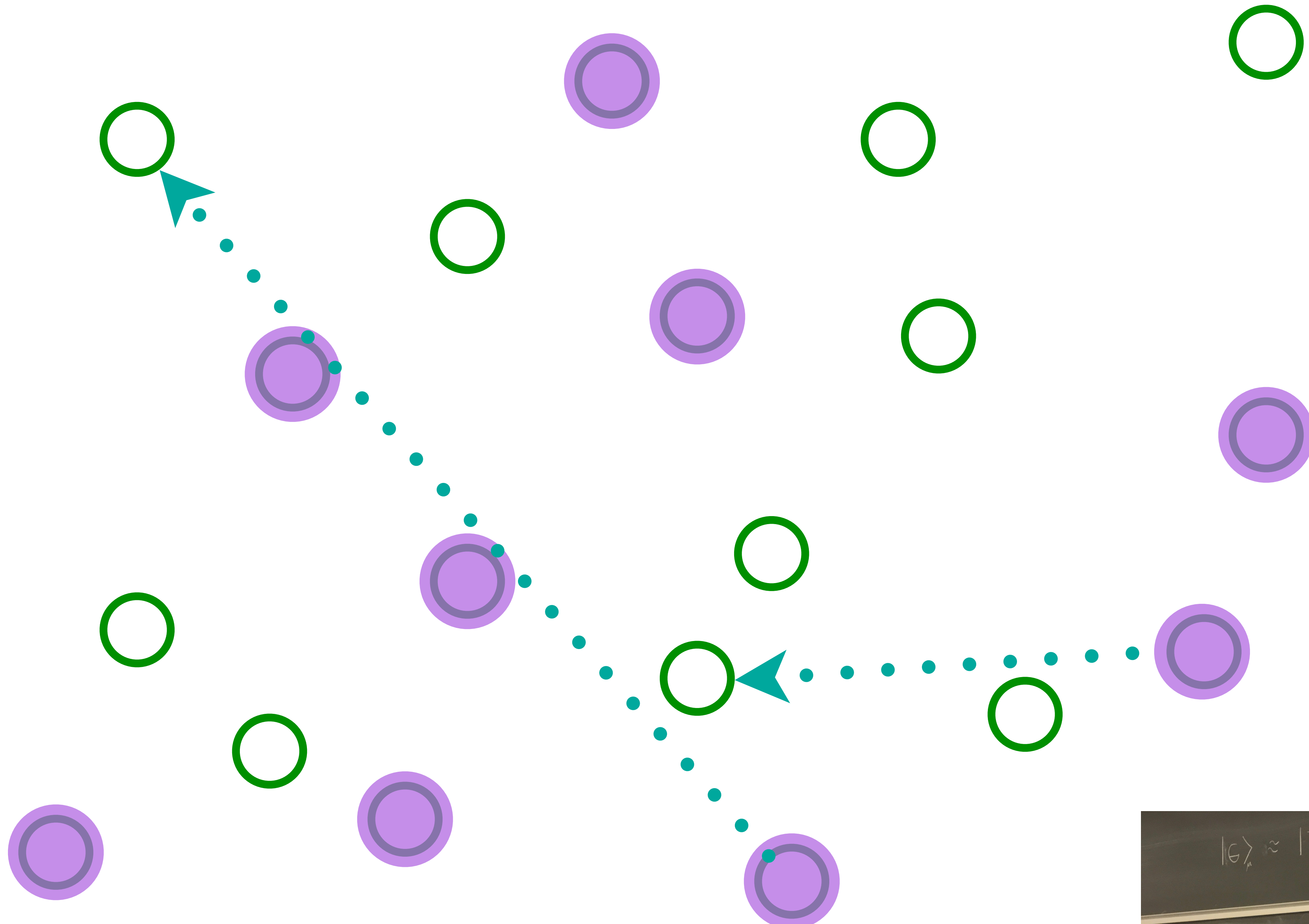

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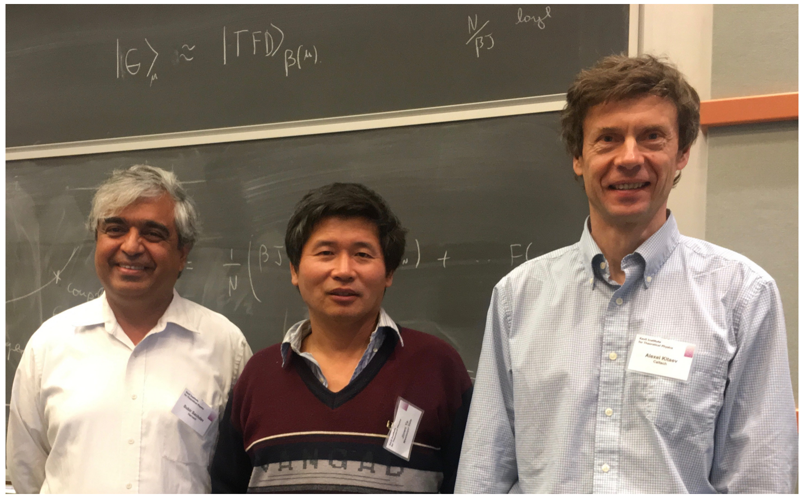

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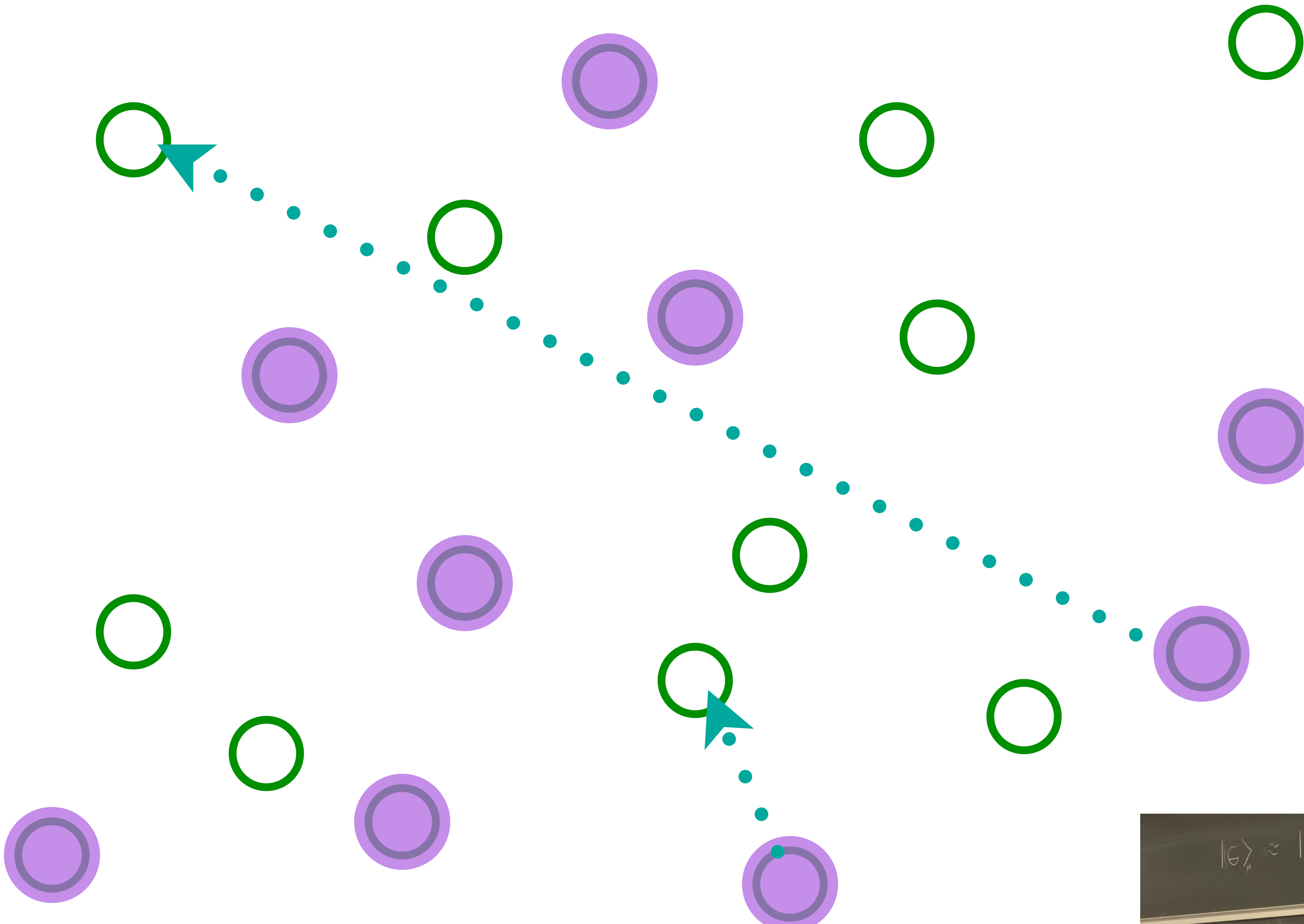

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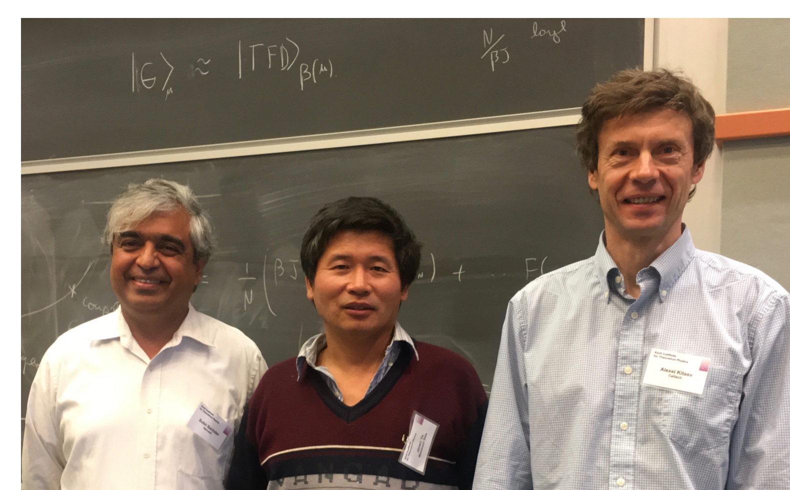

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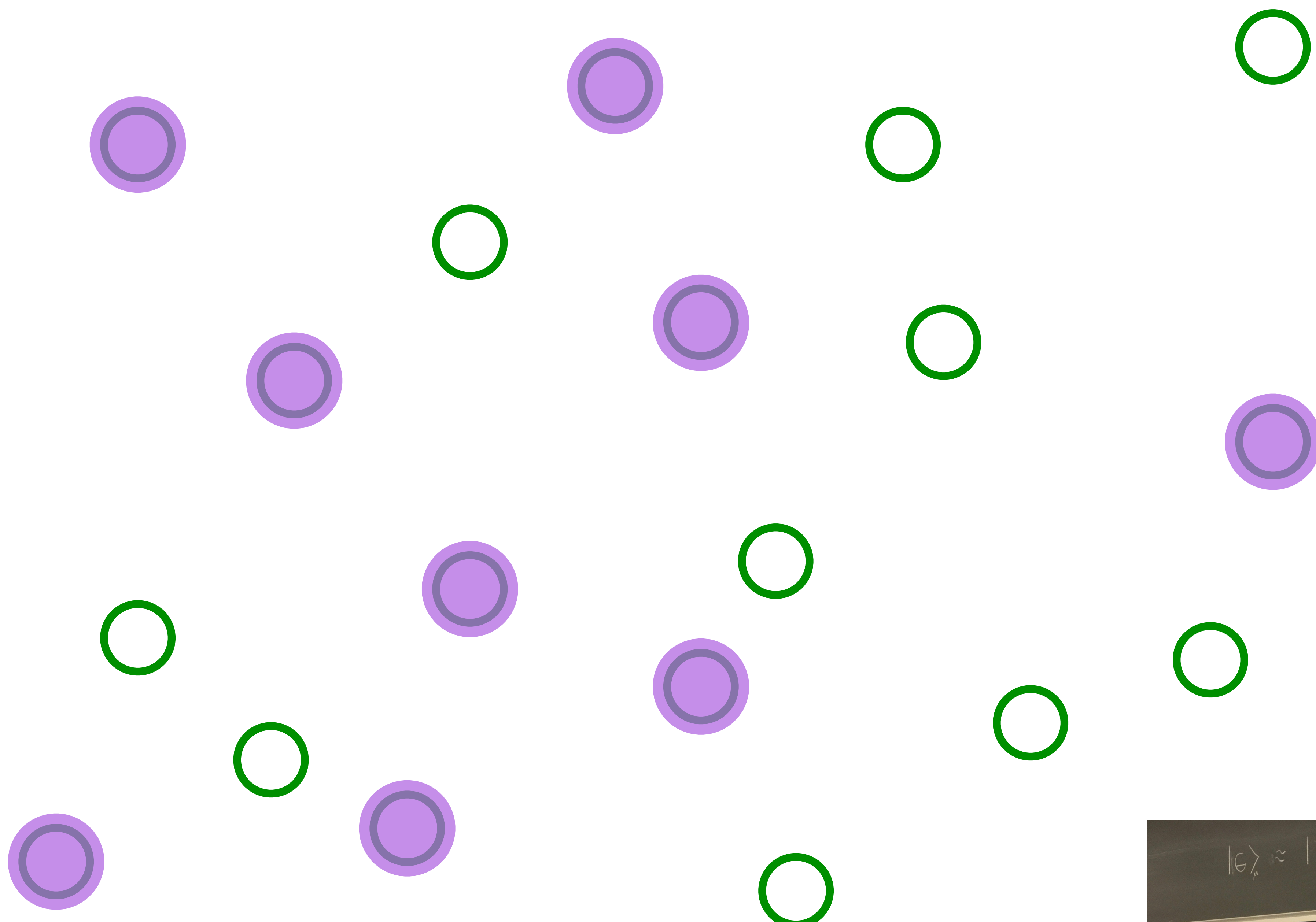

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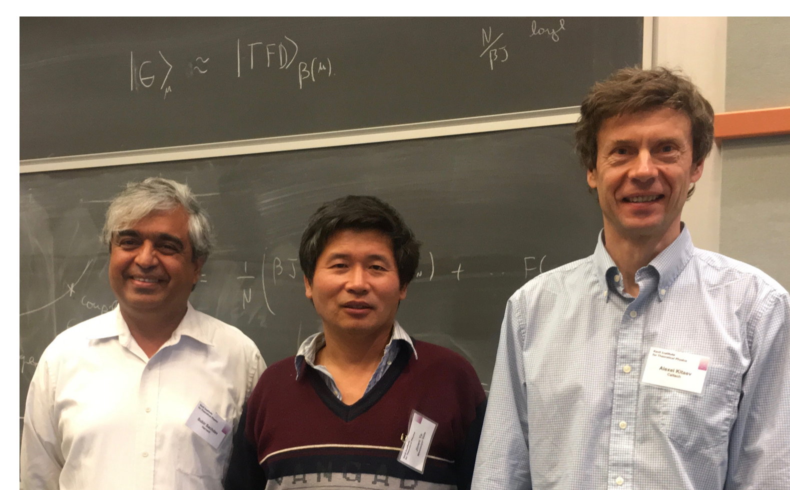

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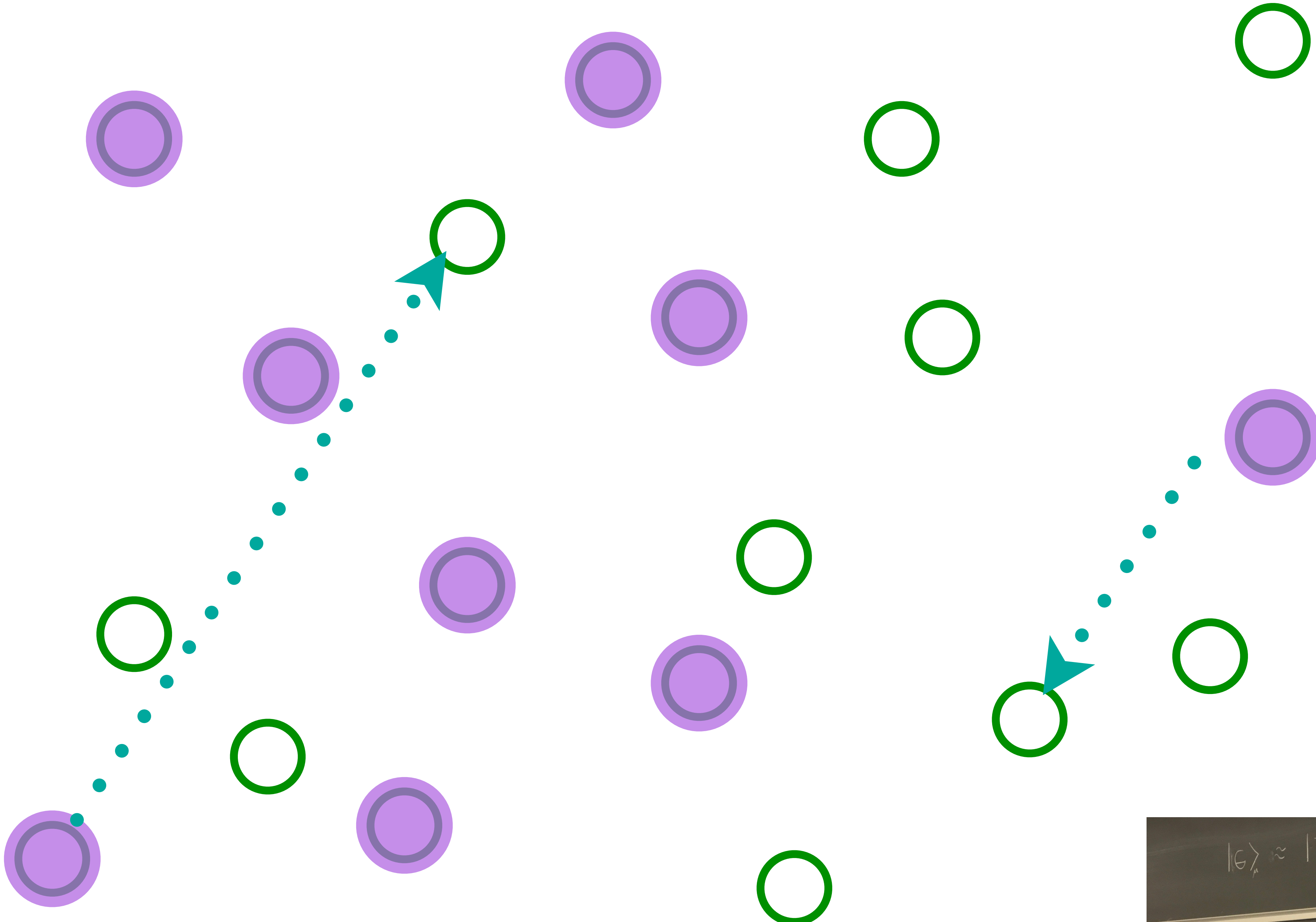

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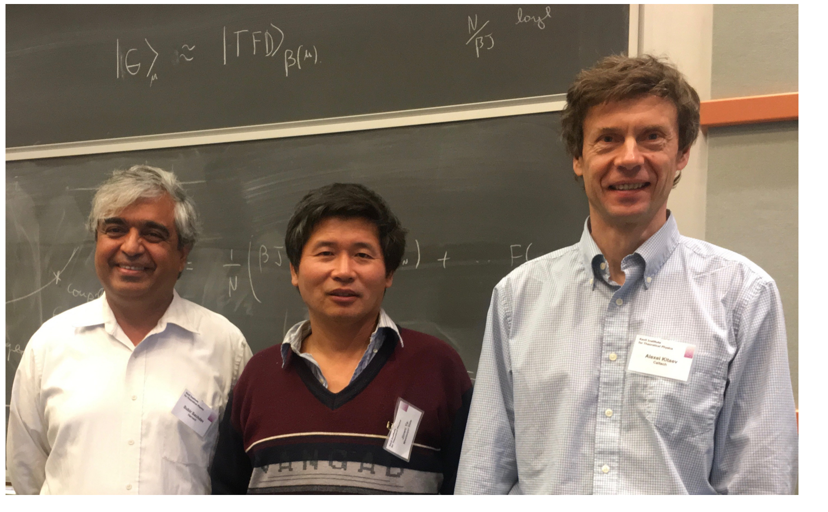

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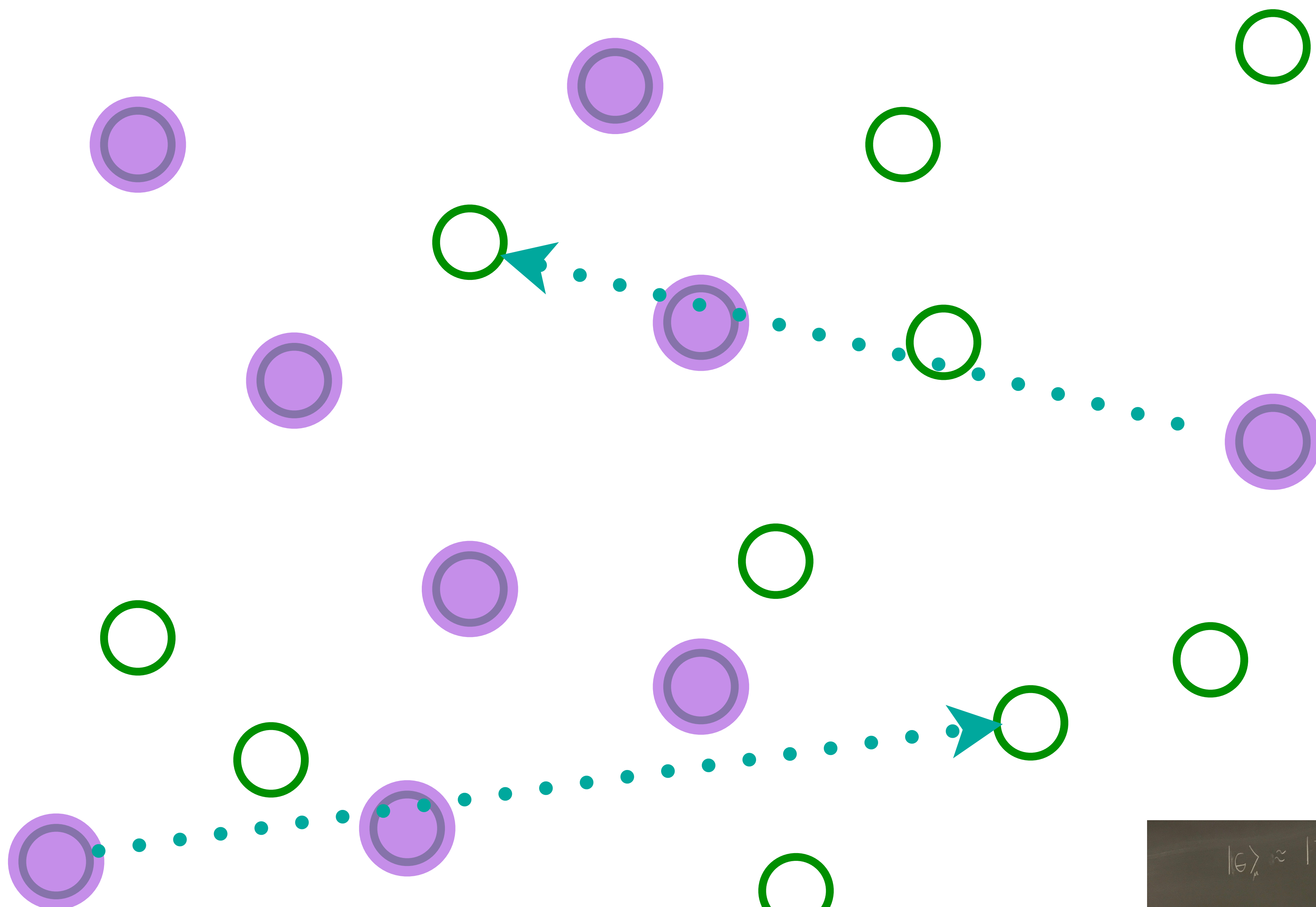

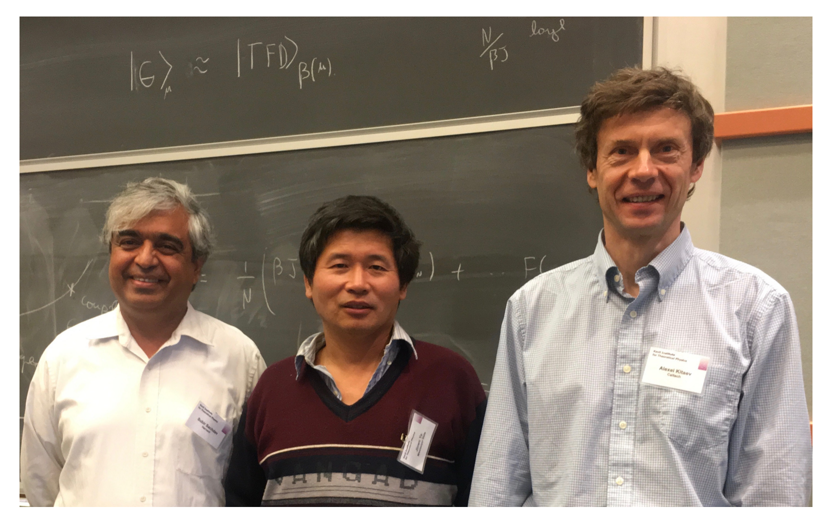

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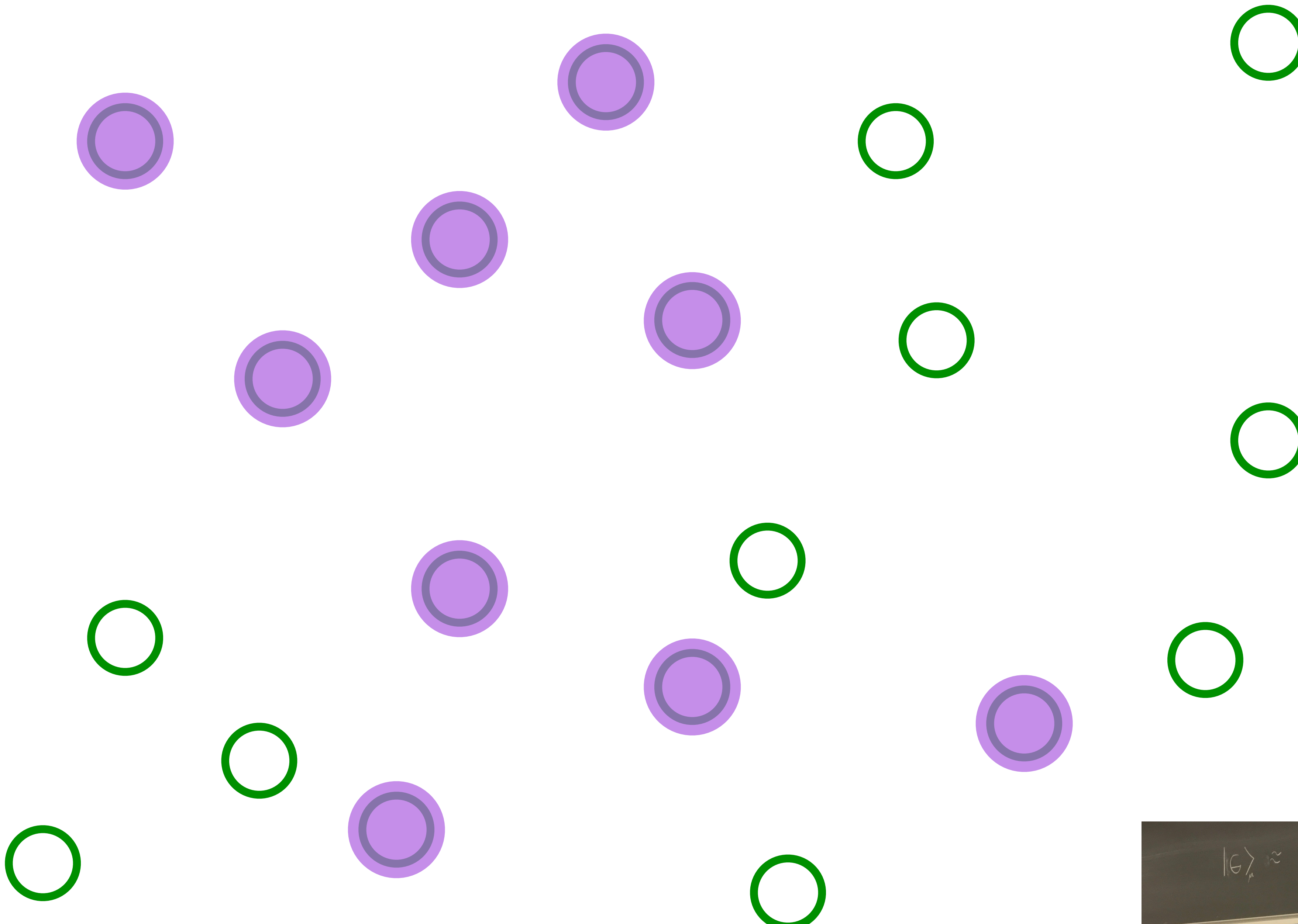

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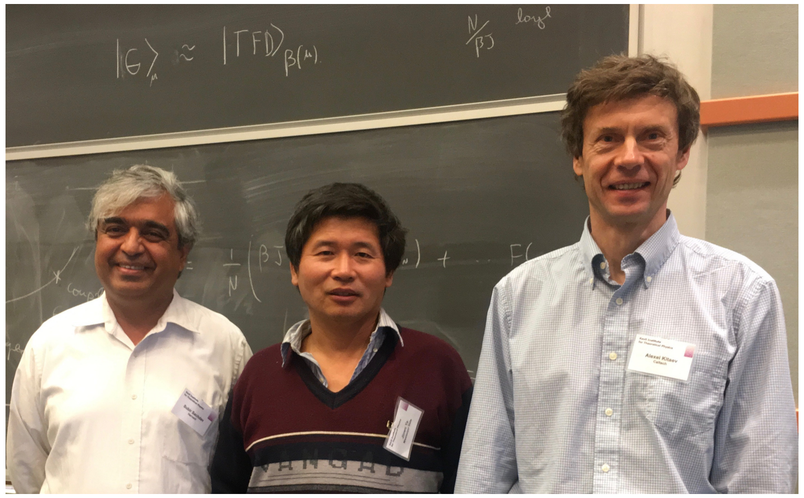

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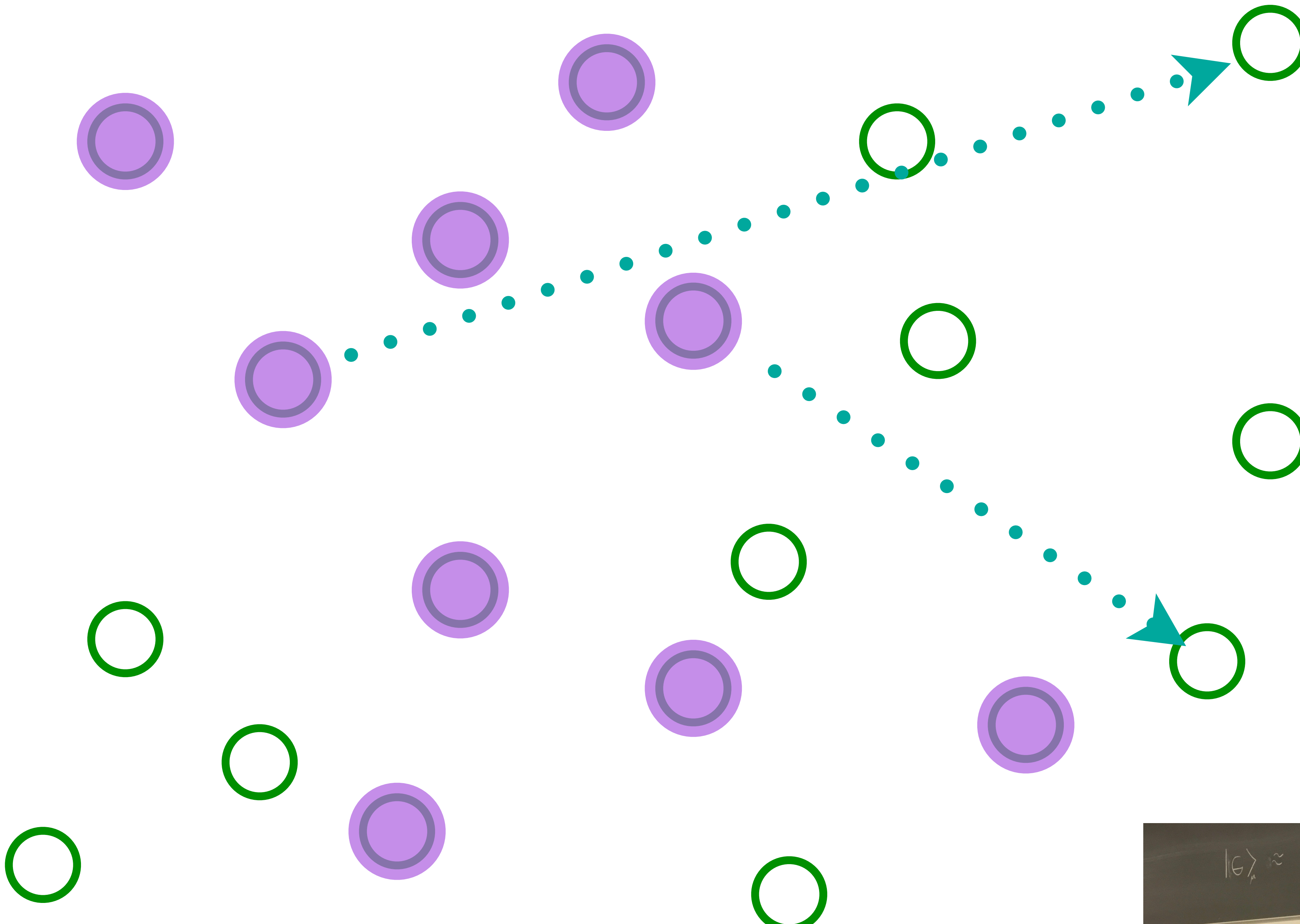

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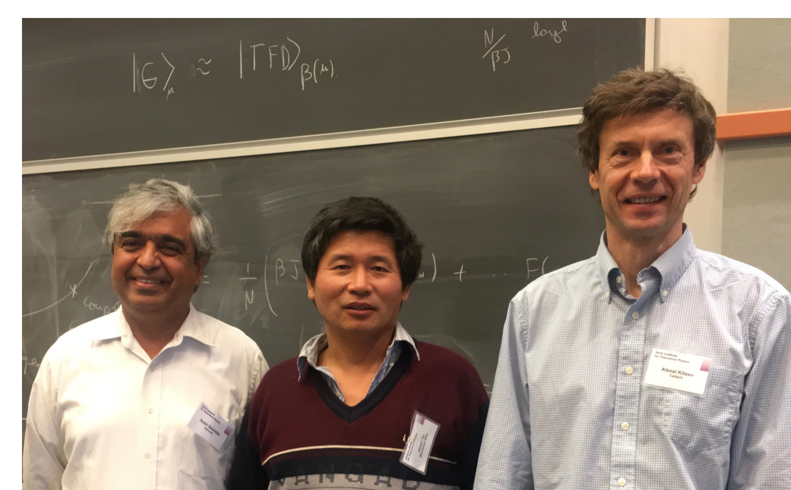

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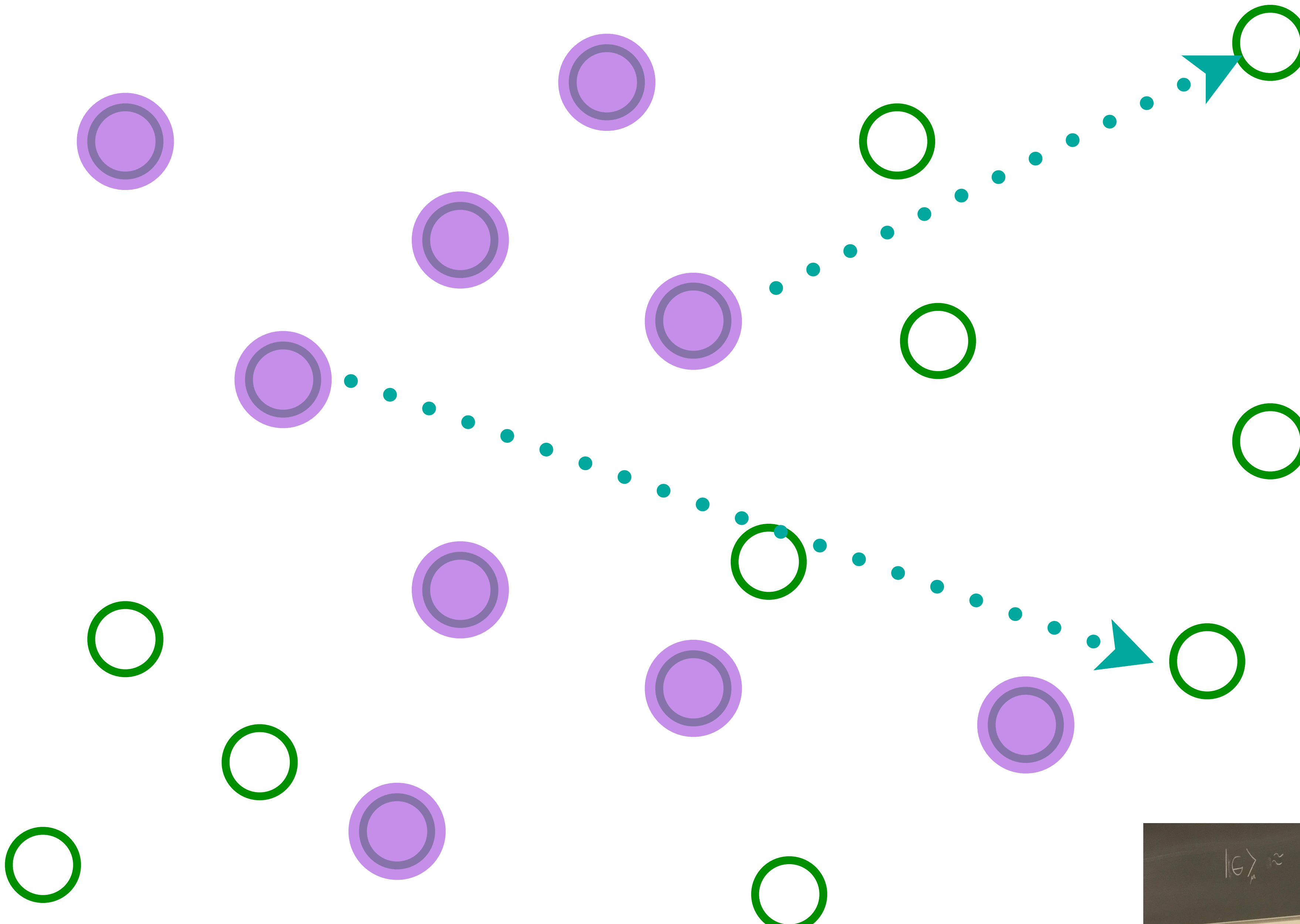

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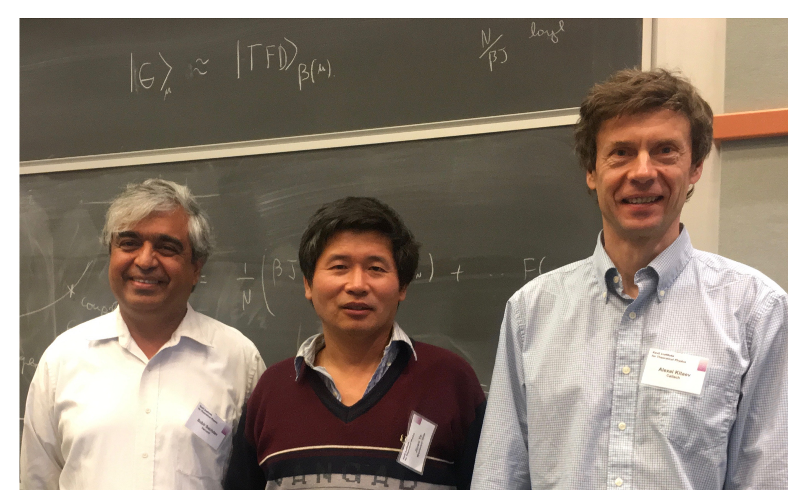

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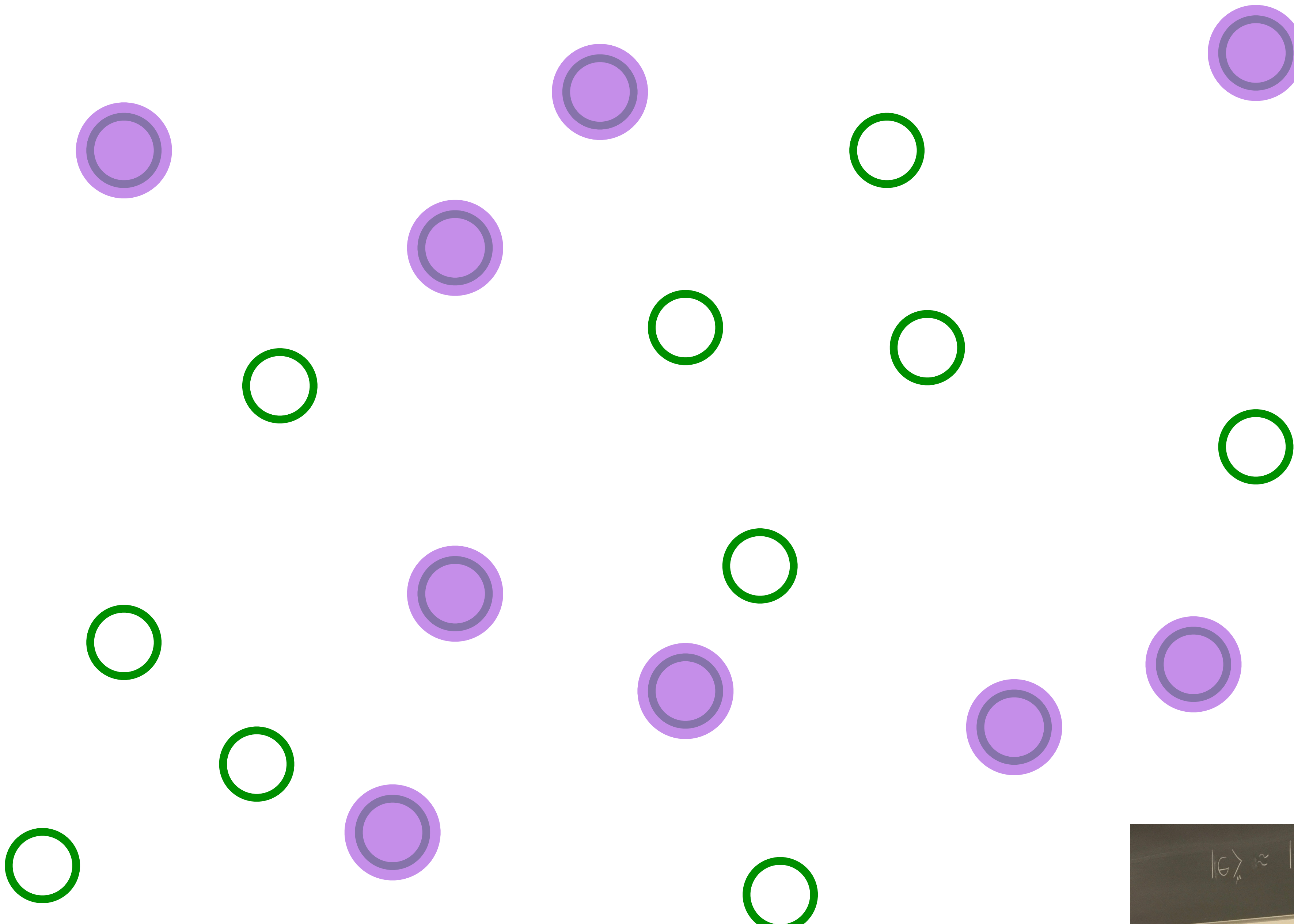

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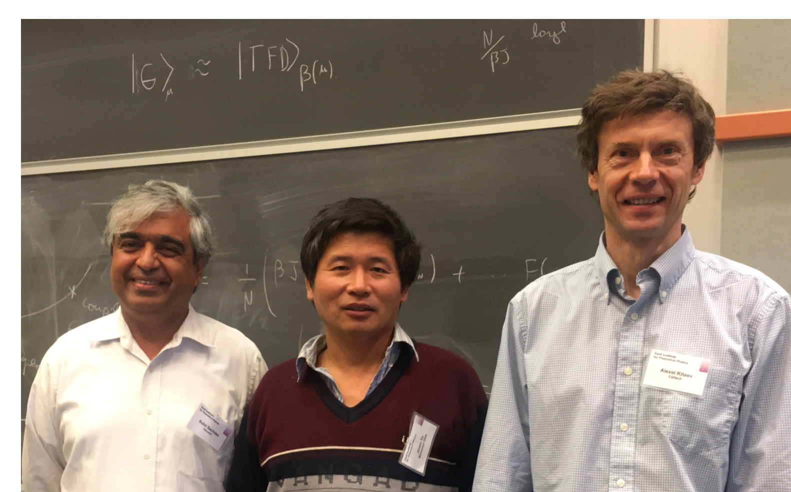

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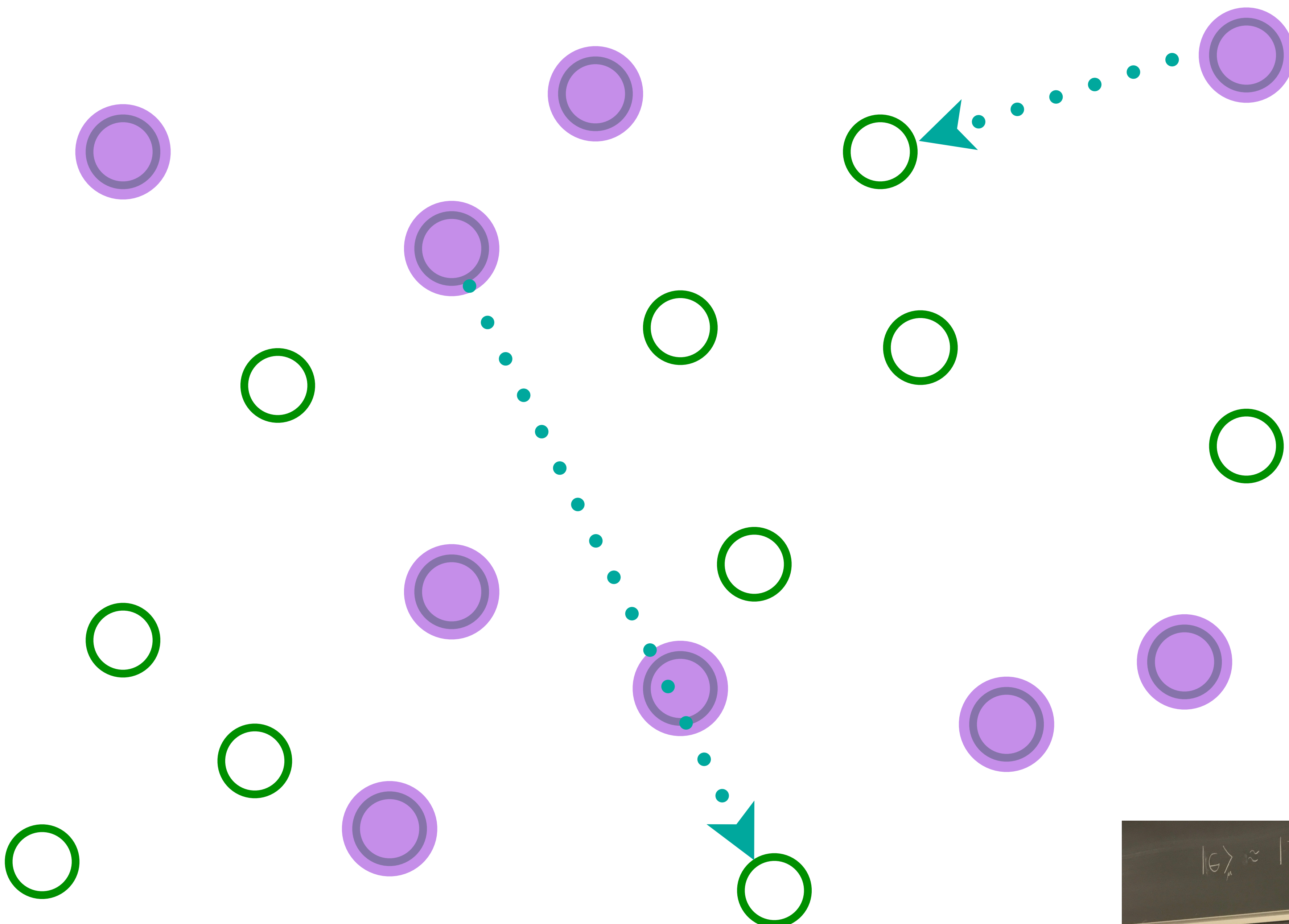

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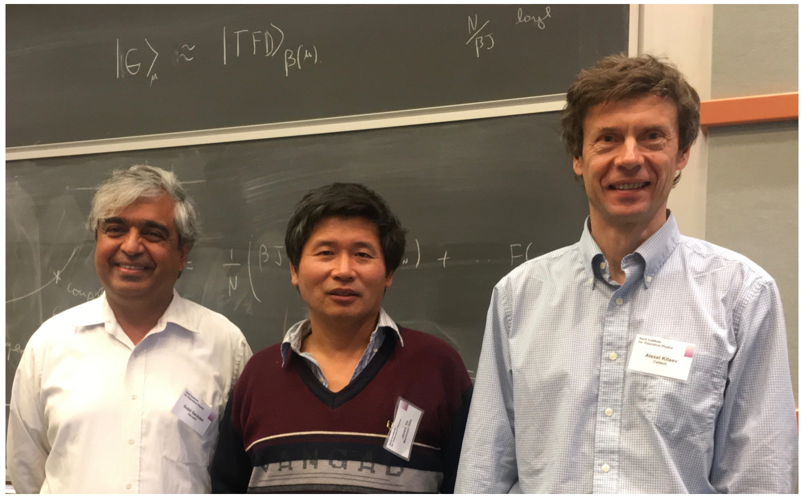

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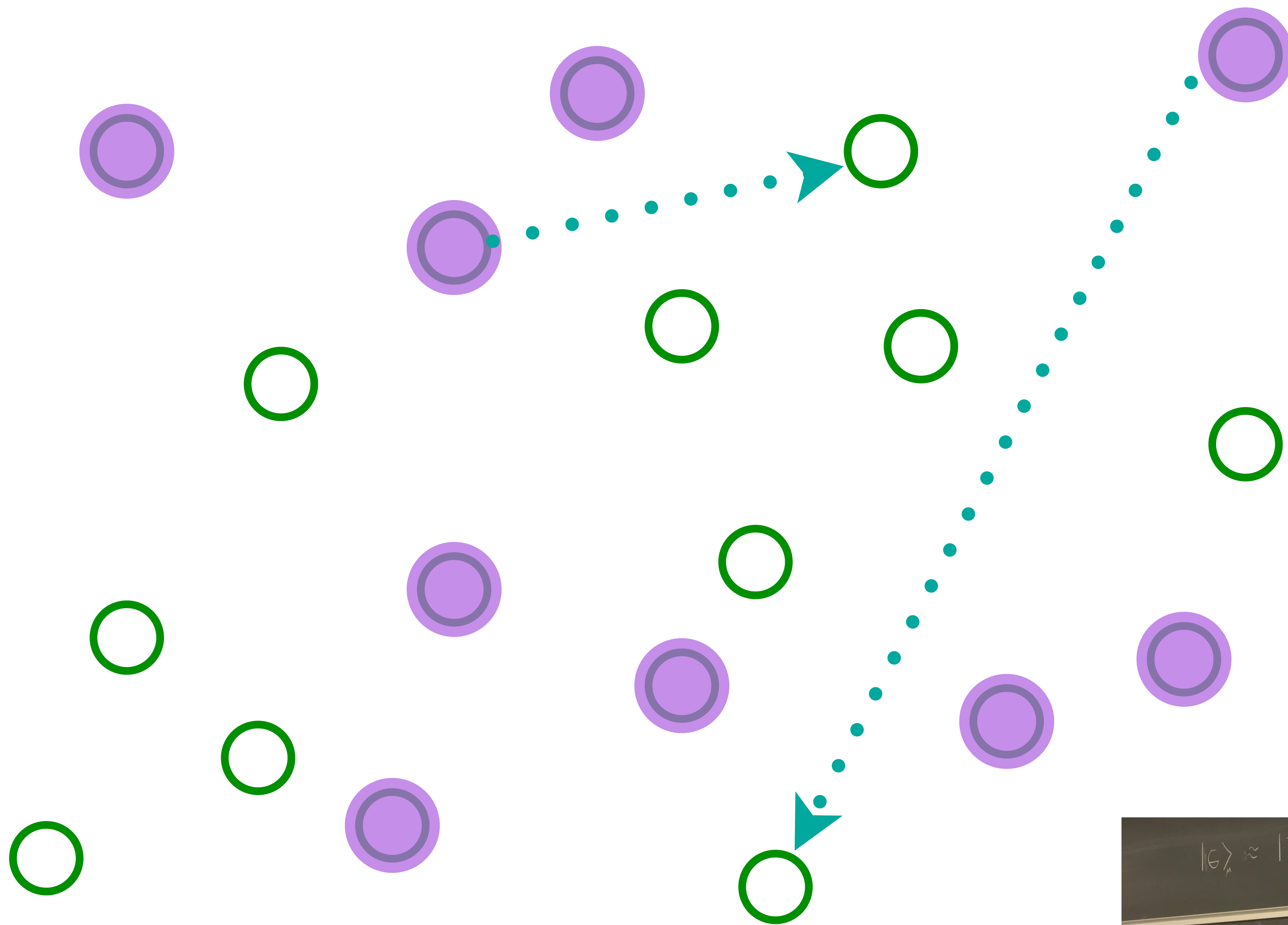

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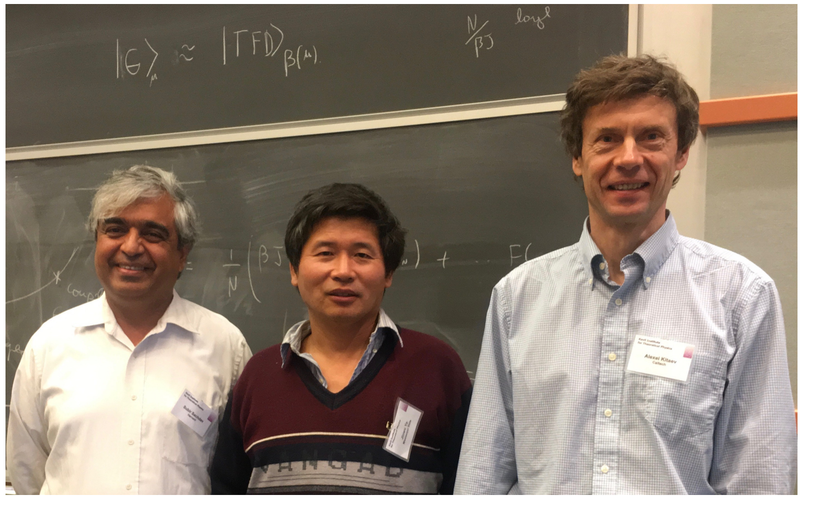

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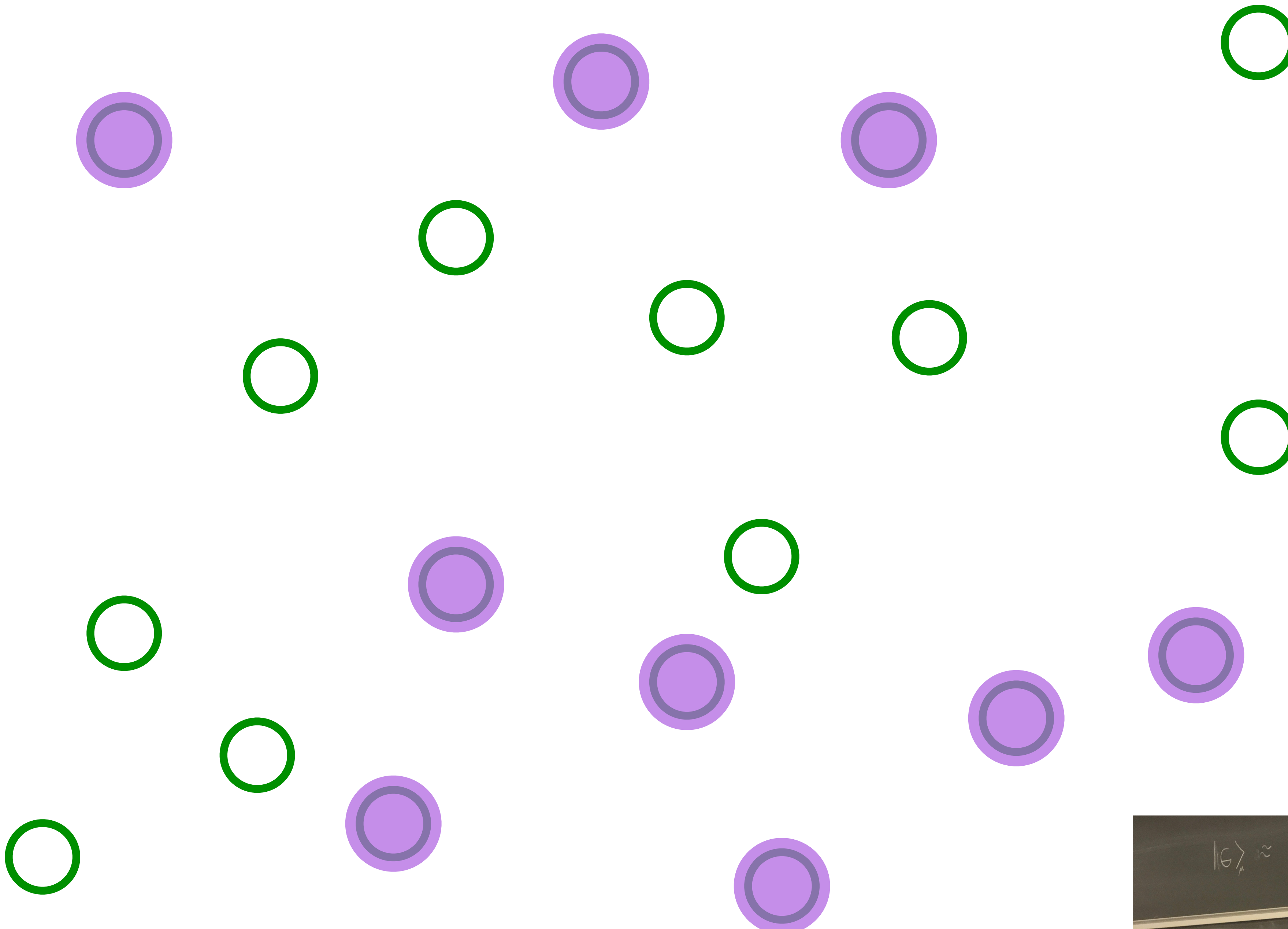

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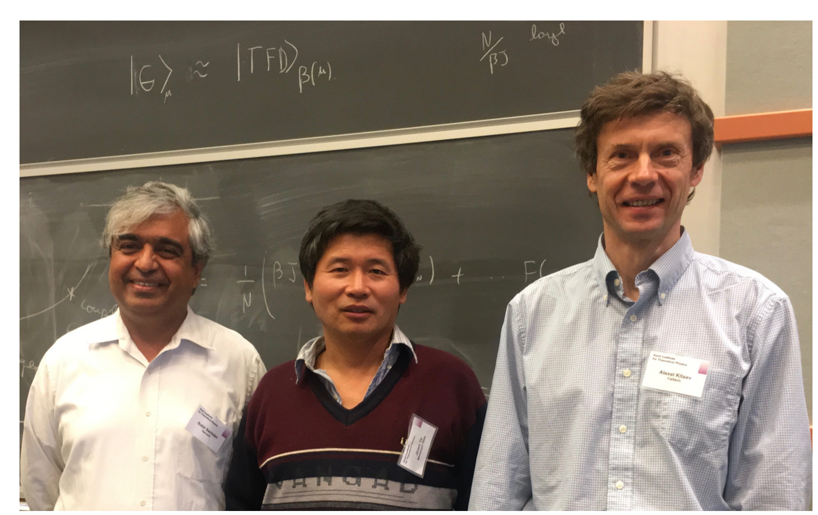

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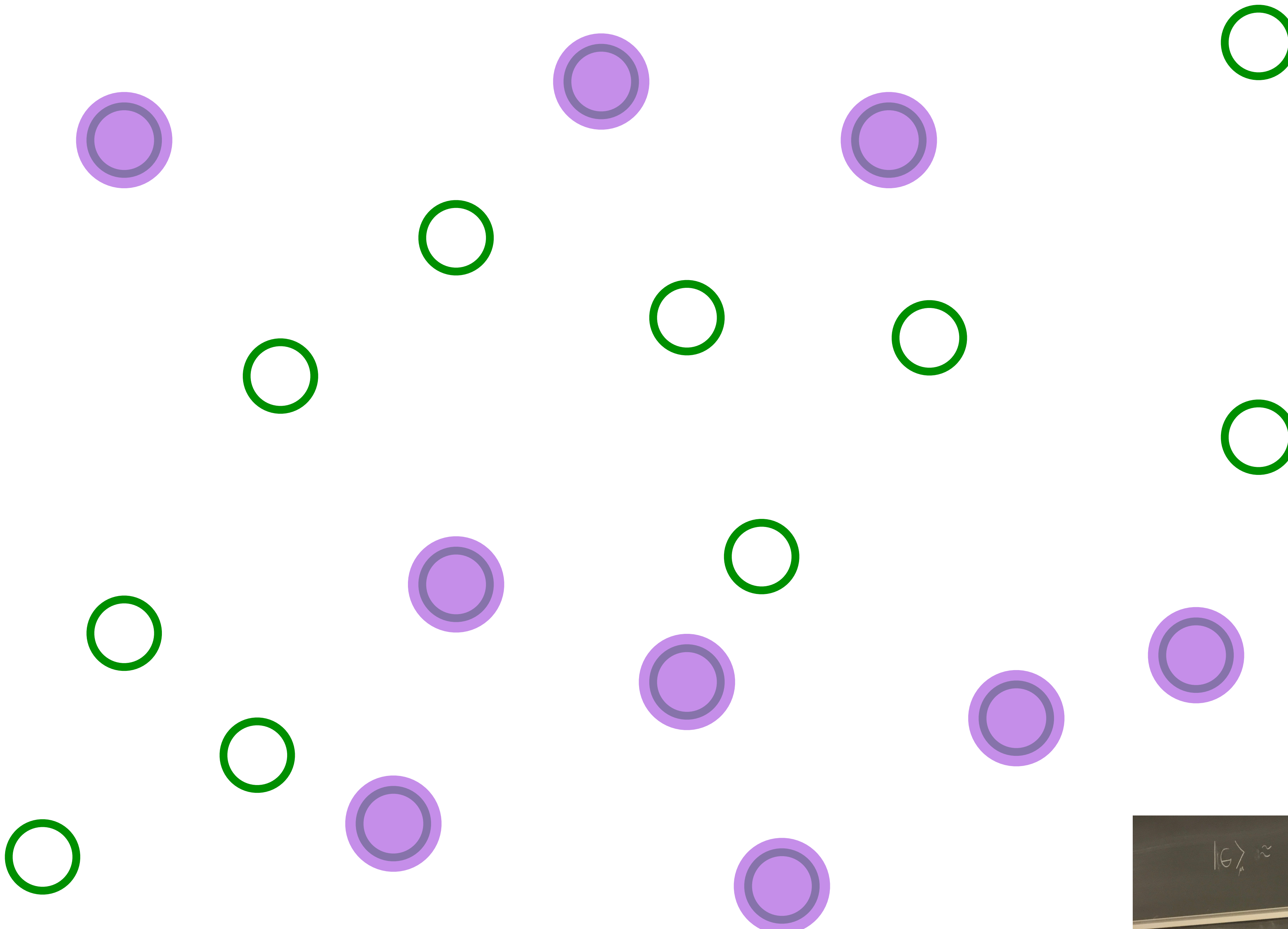
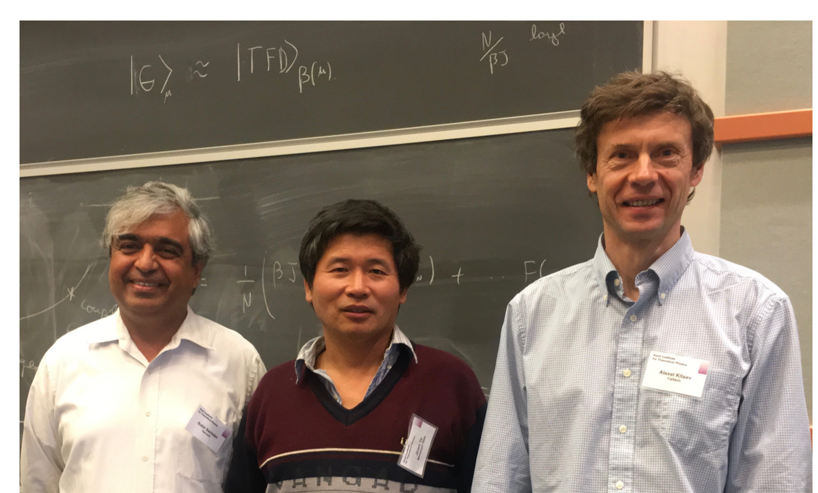


This describes both a strange metal and a black hole!



The SYK model

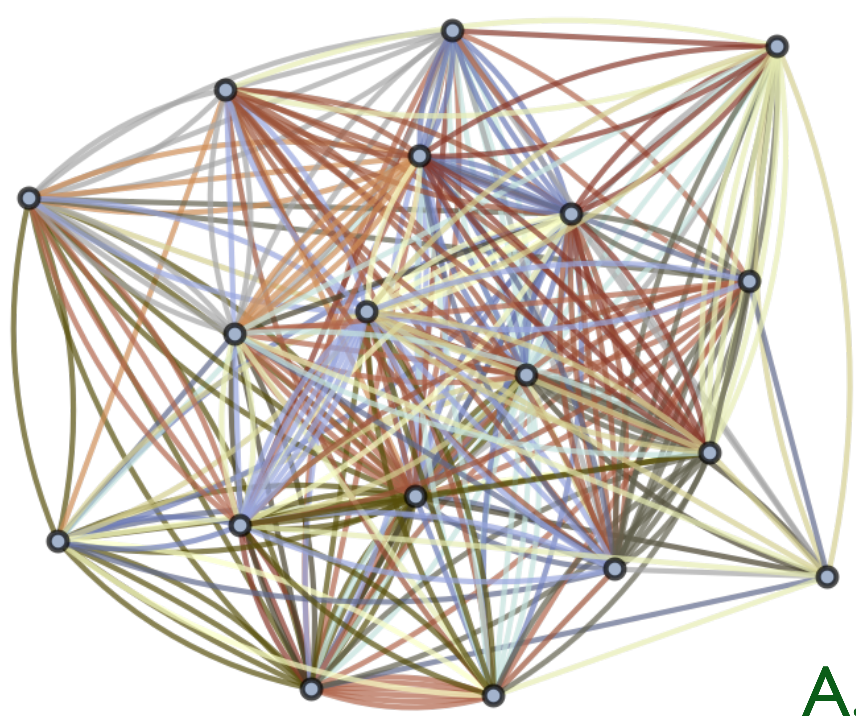
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell + e \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
$N \to \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The SYK model

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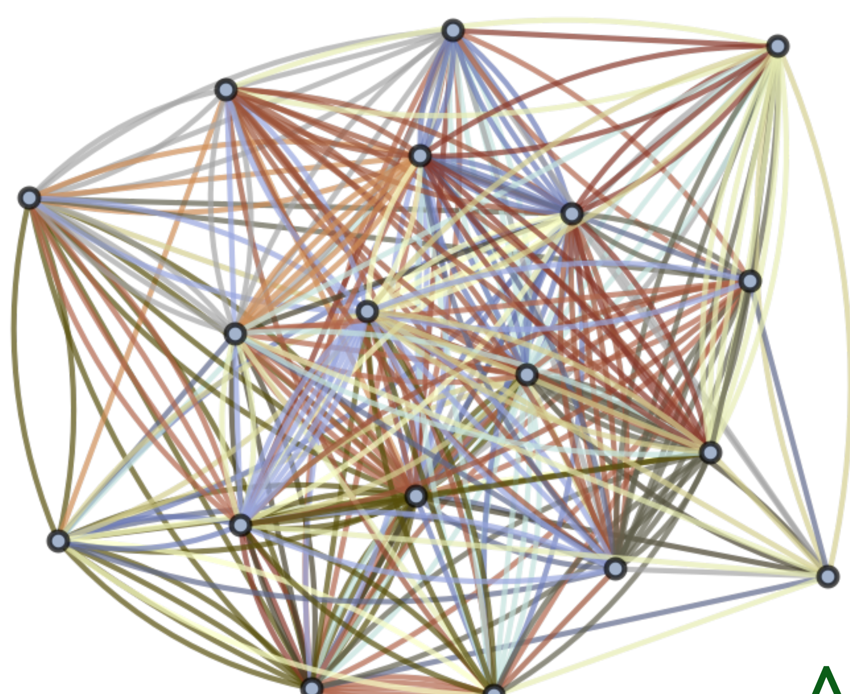
Flat band

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$

$N \to \infty$ yields critical strange metal.

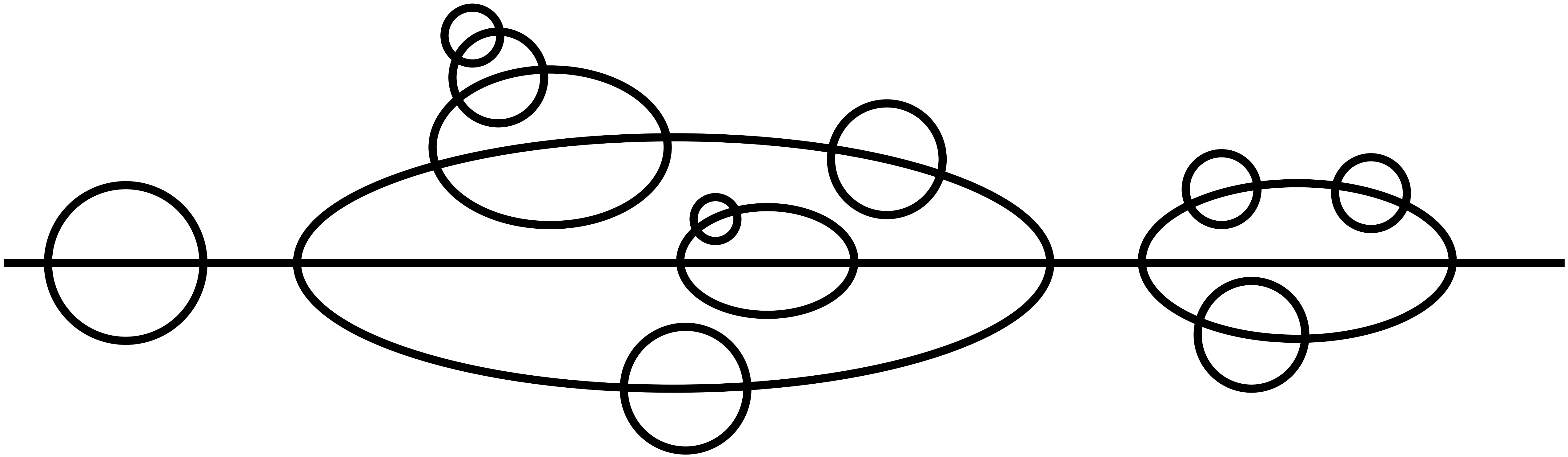


S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The SYK model

The large N limit is given by the sum of "melon" Feynman graphs



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

The complex SYK model

There is a one-parameter family of critical solutions with varying e/U, yielding different $0 < \mathcal{Q} < 1$.

For long (imaginary) times $\tau > 0$

$$\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim e^{-2\pi \mathcal{E} T \tau} \times \left(\frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

In a Fermi liquid,

$$\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \frac{T}{\sin(\pi T \tau)}$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Georges and O. Parcollet PRB **59**, 5341 (1999)

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Characteristic Planckian time scale $\sim \hbar/(k_B T)$ for 'dissipation' of excitations.

A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, PRB **96**, 205123 (2017)

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Georges and O. Parcollet PRB **59**, 5341 (1999)

The complex SYK model

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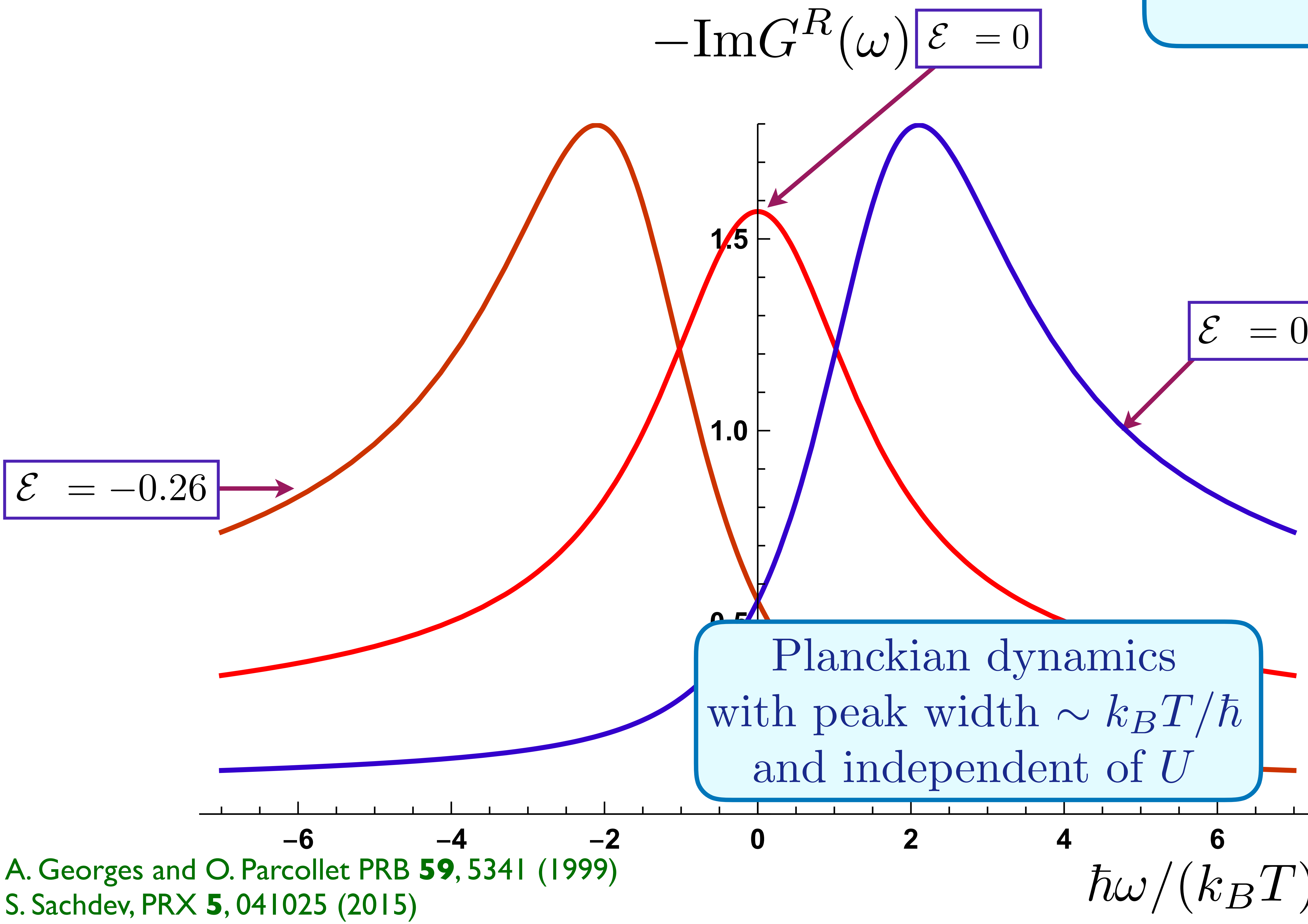
The exponential pre-factor determines the particle-hole asymmetry, and $\mathcal{E} = 0.41(e/U)$ from a numerical solution.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Georges and O. Parcollet PRB **59**, 5341 (1999)

The SYK model

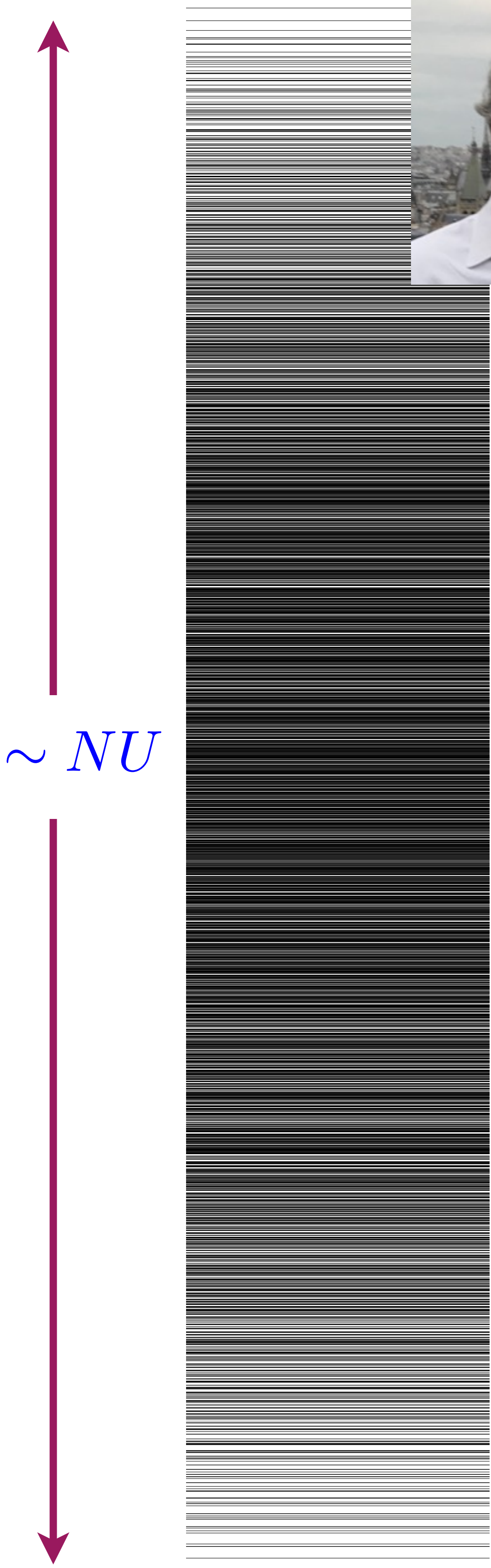
$$\mathcal{E} = \mathbb{C}\frac{e}{U}$$



Planckian dynamics
with peak width $\sim k_B T/\hbar$
and independent of U

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

The SYK model



GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

There are 2^N many body levels with energy E. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$, where $s_0 < \ln 2$ is determined by integrating

$$\frac{ds_0}{d\mathcal{Q}} = 2\pi\mathcal{E}\,.$$

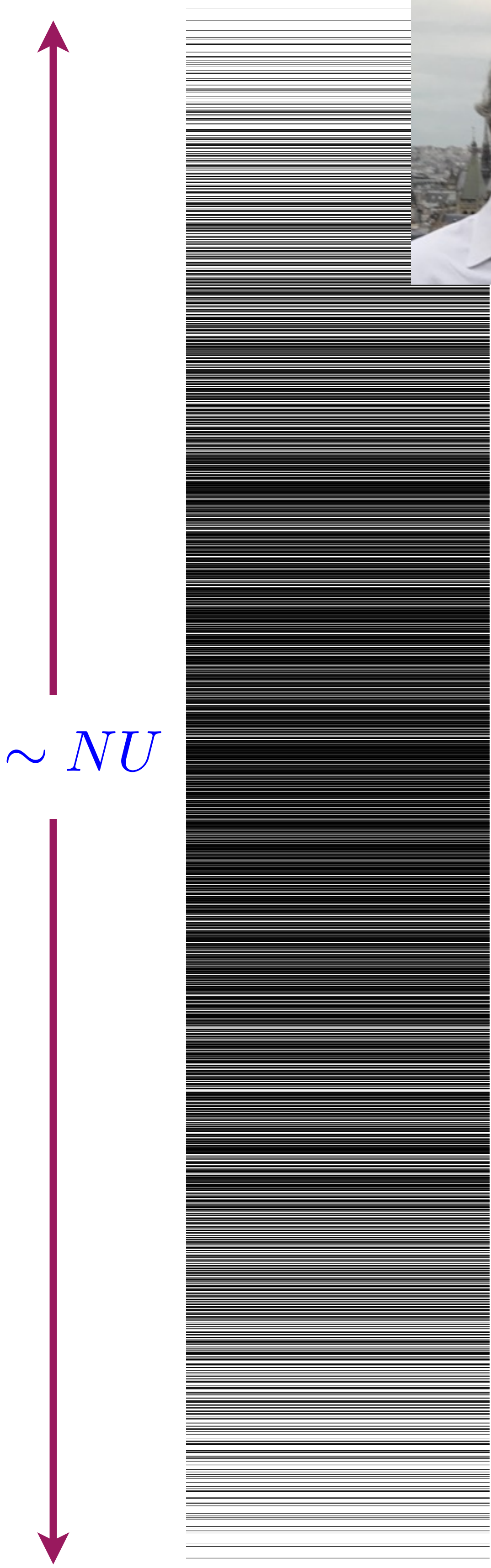
At $\mathcal{Q} = 1/2$,

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$

where G is Catalan's constant.

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

The SYK model



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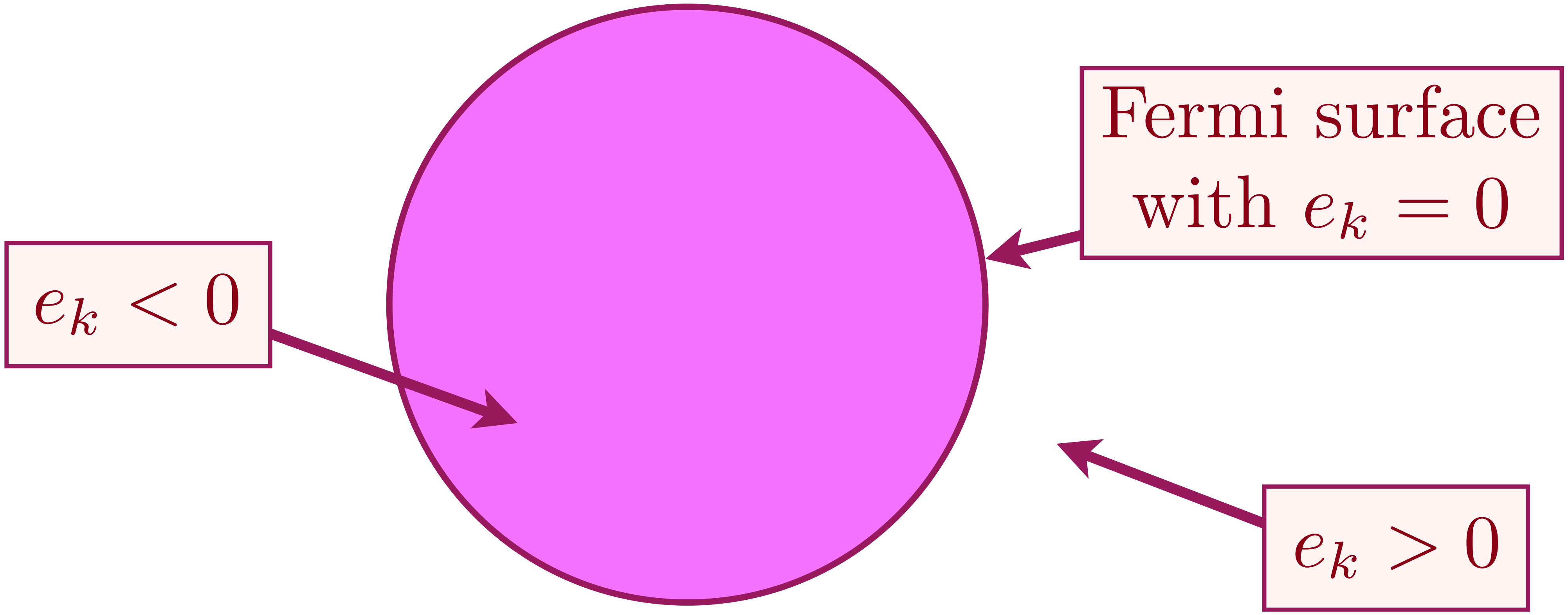
Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Cannot be obtained by quasiparticles, $E = \sum_\alpha n_\alpha e_\alpha$, with $e_\alpha \sim 1/N$

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

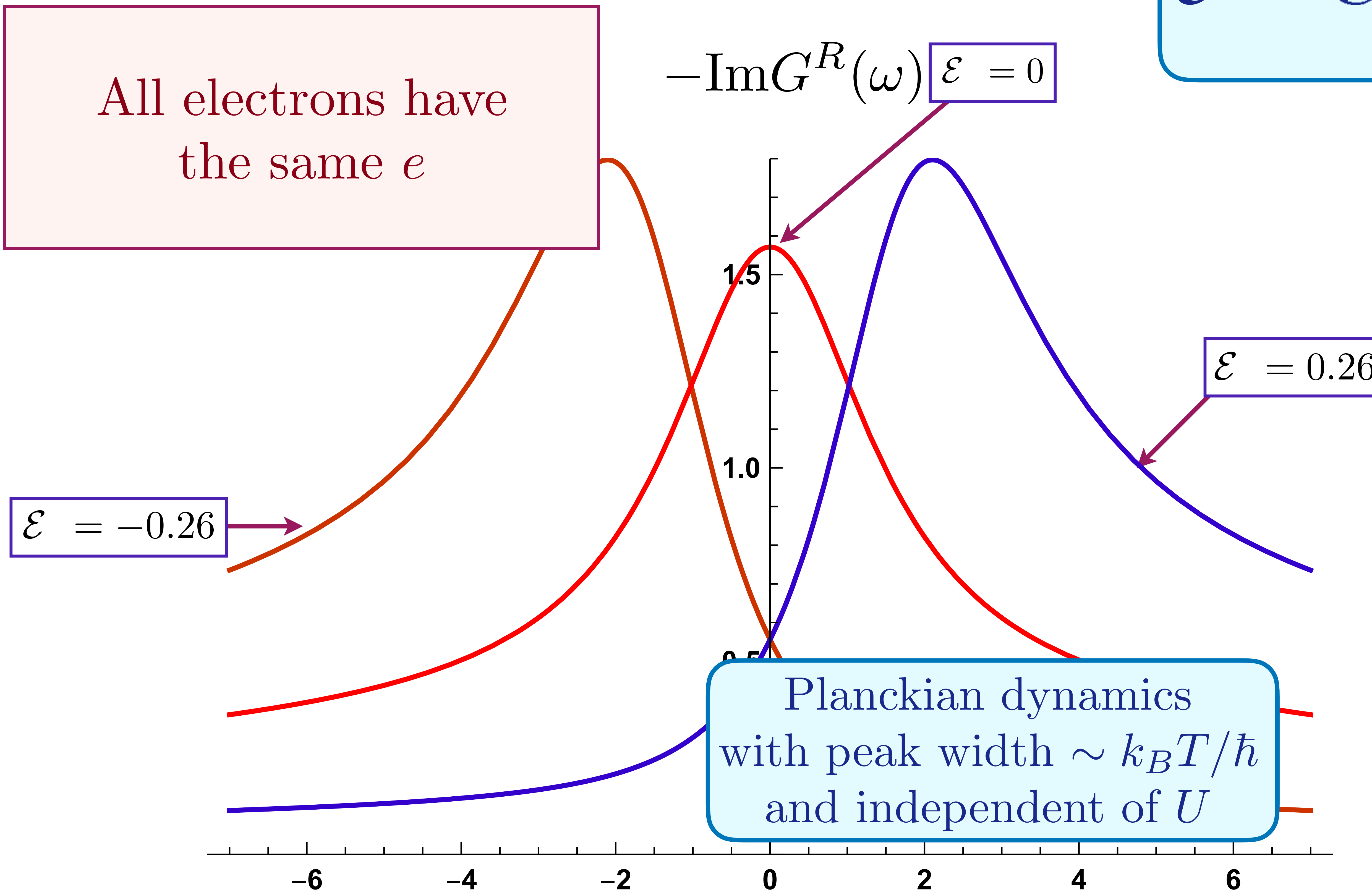
Adding dispersion



- All electrons in the (flat band) SYK model have the same e
- In a more realistic metal, the electrons have a dispersion e_k (k is momentum), and $e_k = 0$ is the Fermi surface.

Flat band metal

$$\mathcal{E} = \mathbb{C}\frac{e}{U}$$

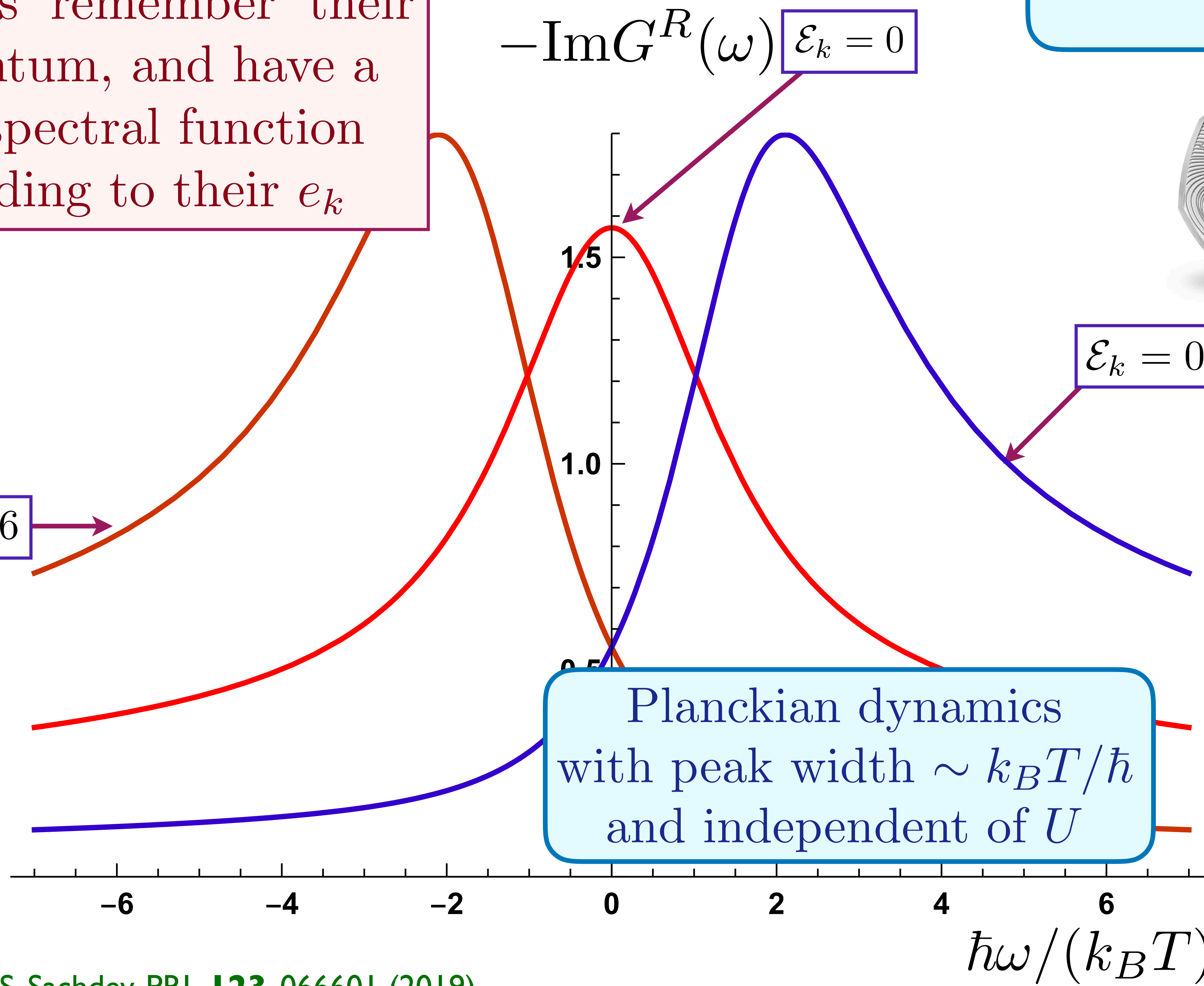


A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

Planckian metal ansatz with some dispersion

$$\mathcal{E}_k = \mathbb{C}\frac{e_k}{U}$$

Electrons 'remember' their momentum, and have a SYK spectral function according to their e_k



Planckian dynamics with peak width $\sim k_B T/\hbar$ and independent of U

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Planckian metal ansatz with some dispersion

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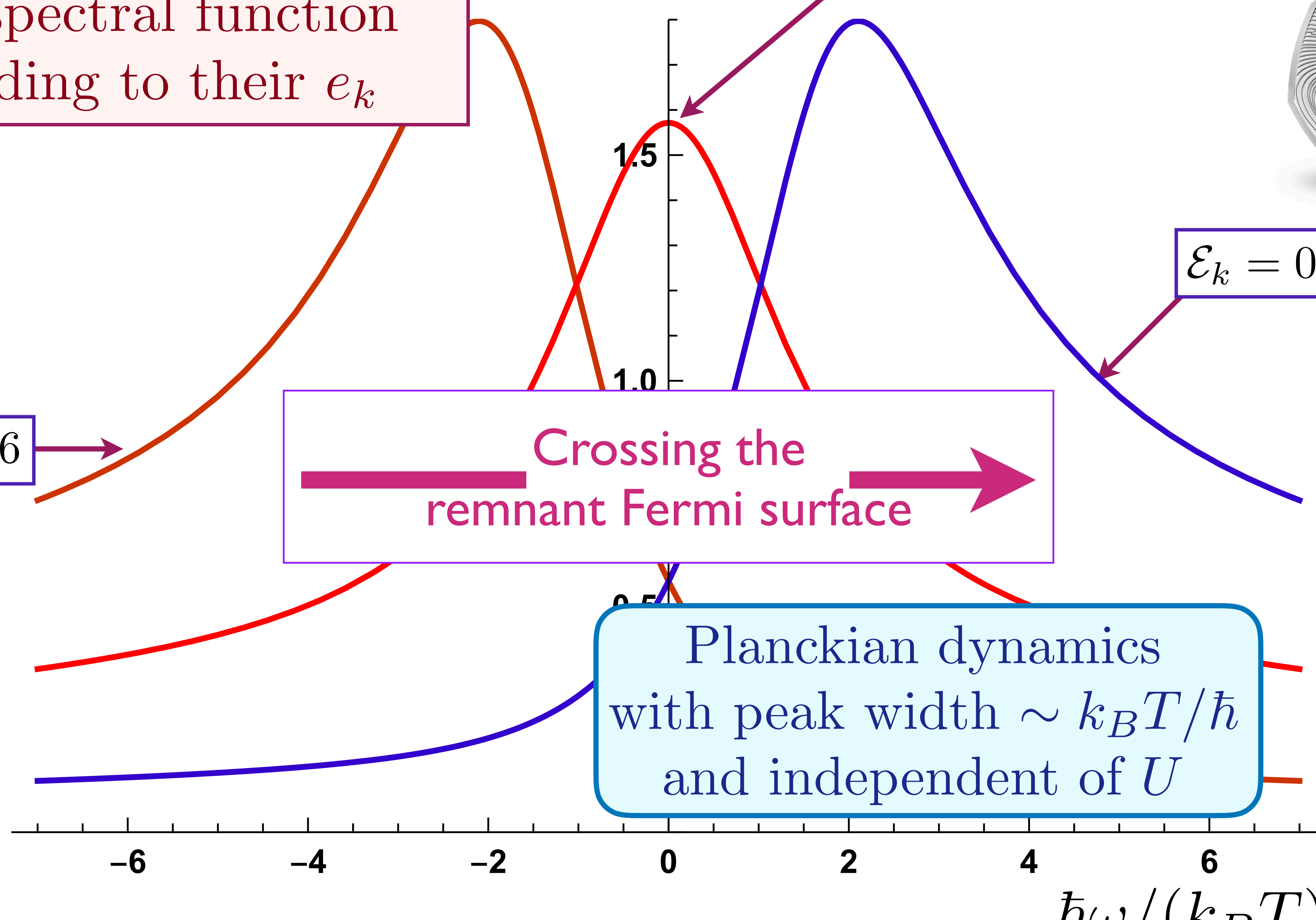

Crossing the remnant Fermi surface

Planckian dynamics with peak width $\sim k_B T/\hbar$ and independent of U

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Flat band metal

For a dispersionless SYK model

$$\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim e^{-(e/U)2\pi\mathbb{C}T\tau} \times \left(\frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)

Planckian metal ansatz with some dispersion

For a strongly-interacting metal with underlying quasiparticle dispersion e_k (k is the momentum)

$$\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle \sim e^{-(e_k/U) 2\pi \mathbb{C} T \tau} \times \left(\frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

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At $e_k = 0$ we have a ‘remnant Fermi surface’ with a particle-hole symmetric spectral function.

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Planckian metal ansatz with some dispersion

For a strongly-interacting metal with underlying quasiparticle dispersion e_k (k is the momentum)

$$\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle \sim e^{-(e_k/U)2\pi\mathbb{C}T\tau} \times \left(\frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

No free parameters—everything is determined by the (underlying) quasiparticle dispersion e_k, and the interaction strength U.

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Resistivity of a Planckian metal as $T \to 0$

From the Kubo formula,

$$\sigma = \frac{e^2 m^* v_F^2}{2T} \int_{-\infty}^{\infty} \frac{de}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{4\pi} \left[\text{Im}\, G^R_{\text{SYK}} \left(e, \frac{\omega}{T} \right) \right]^2 \text{sech}^2 \left(\frac{\omega}{2T} \right)$$

where the Fermi surface is defined by $e_k = 0$, $\boldsymbol{v}_F = \nabla_{\boldsymbol{k}} e_k$ on the Fermi surface, and

$$m^* = \frac{d\, V_{FS}}{\oint_{FS} |\boldsymbol{v}_F|}\,,$$

with d the spatial dimensionality, and V_{FS} is the volume enclosed by the Fermi surface. For a circular Fermi surface, this is the usual m^*.

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

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Evaluating the integrals, we find

$$\rho = \frac{m^*}{ne^2}\, 2.71 \mathbb{C}\, \frac{k_B T}{\hbar} \quad , \quad \text{using } \mathcal{E} = \mathbb{C} e/U \,,$$

where $n = V_{FS}/(2\pi)^d$ is the density.

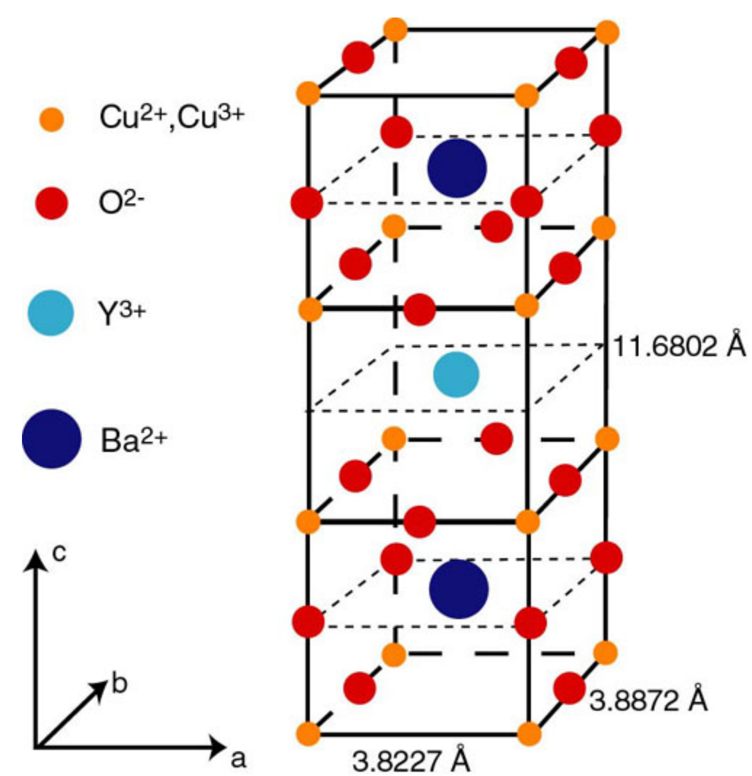
A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Resistivity of a Planckian metal as $T \to 0$

$$\rho = \frac{m^*}{ne^2}\, 2.71\mathbb{C}\, \frac{k_B T}{\hbar}$$

Note that all explicit dependence on U has cancelled out!

Choosing $\mathbb{C} = 0.41$ as in the SYK model, we have the prefactor $2.71\mathbb{C} = 1.11$.



Aavishkar Patel

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Ordinary metals:
quasiparticles

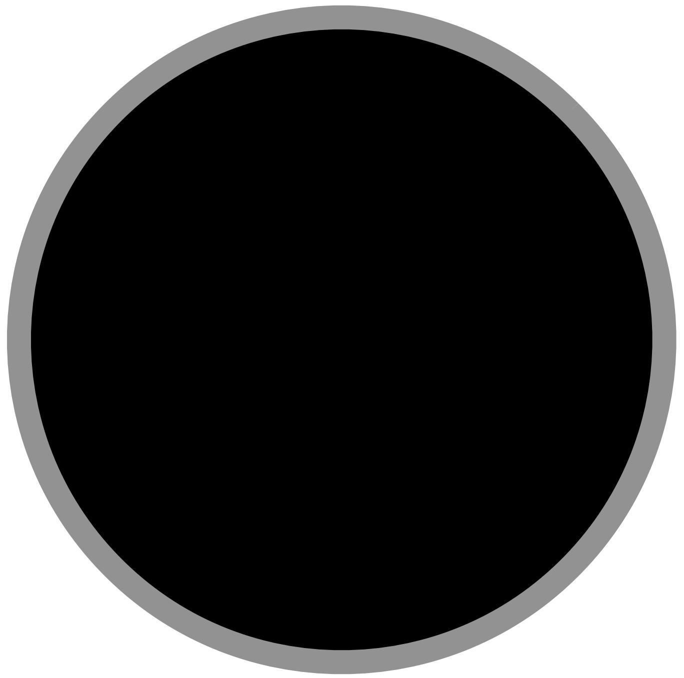
Strange metals:
no quasiparticles

Black
holes

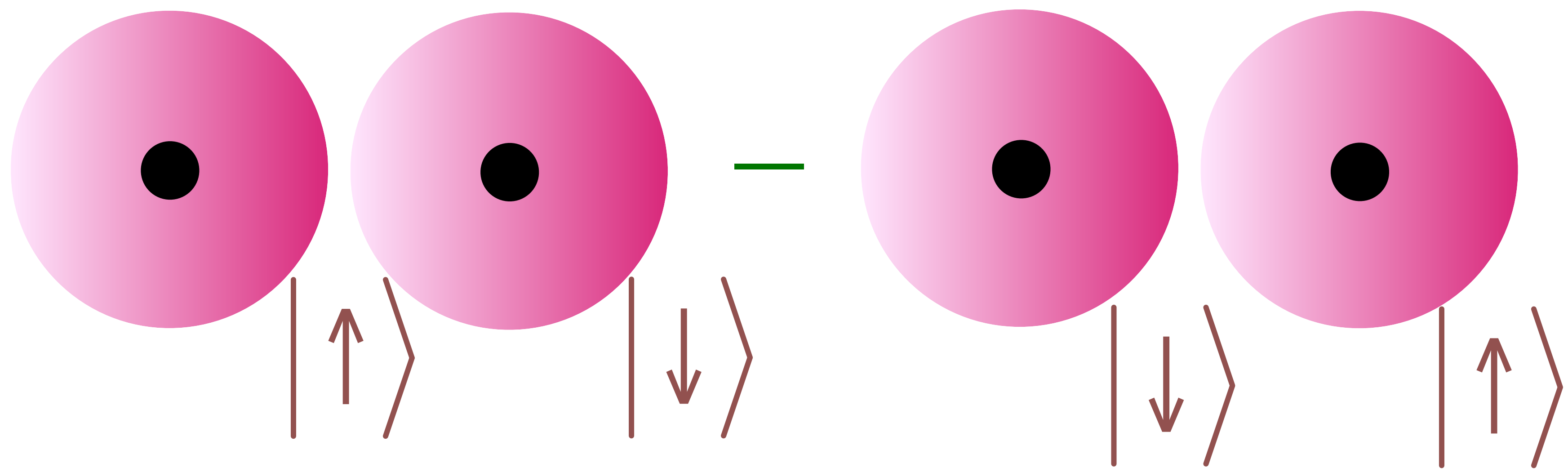
Black Holes

Objects so dense that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole horizon is disconnected from the rest of the universe.



Quantum Entanglement across a black hole horizon



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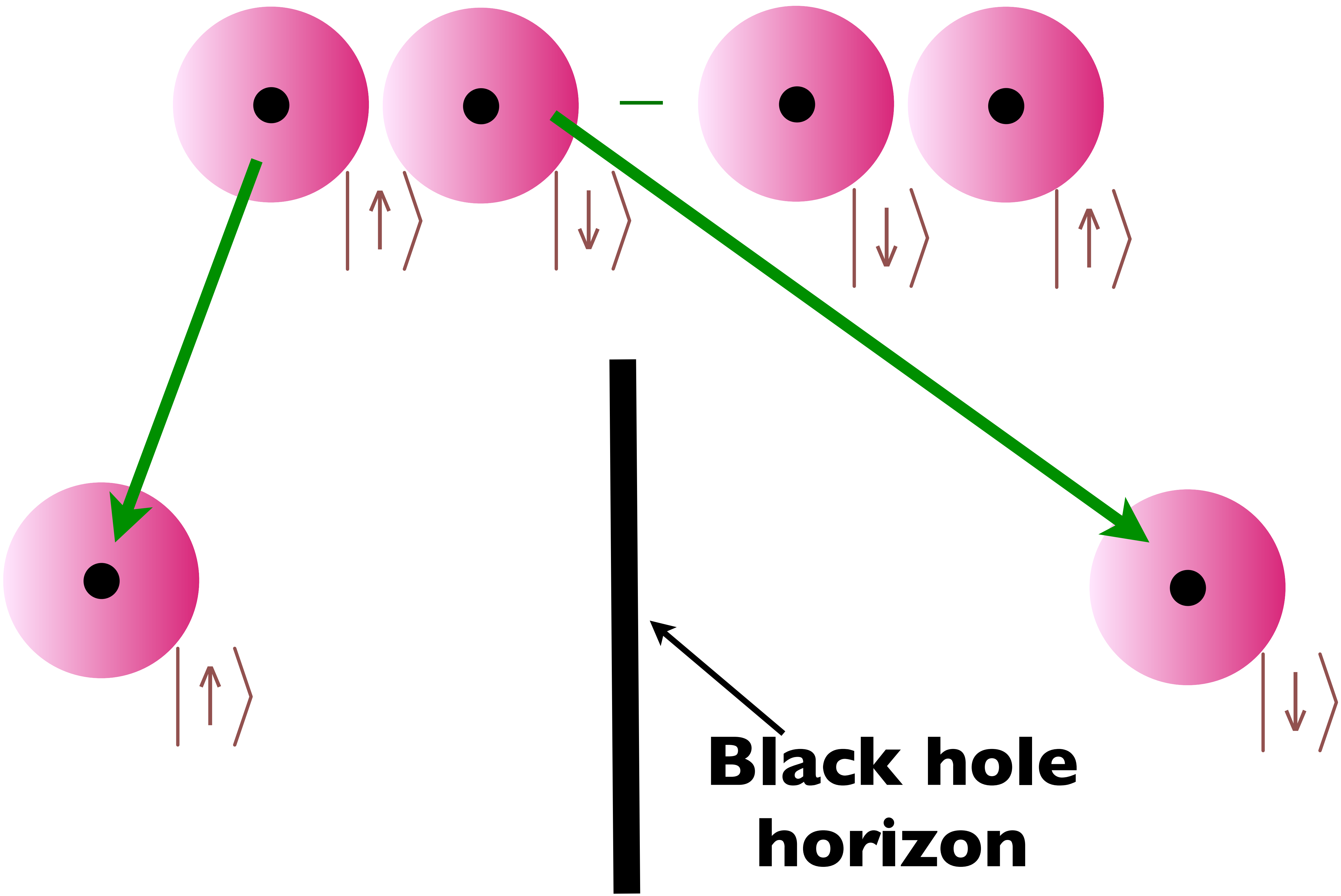
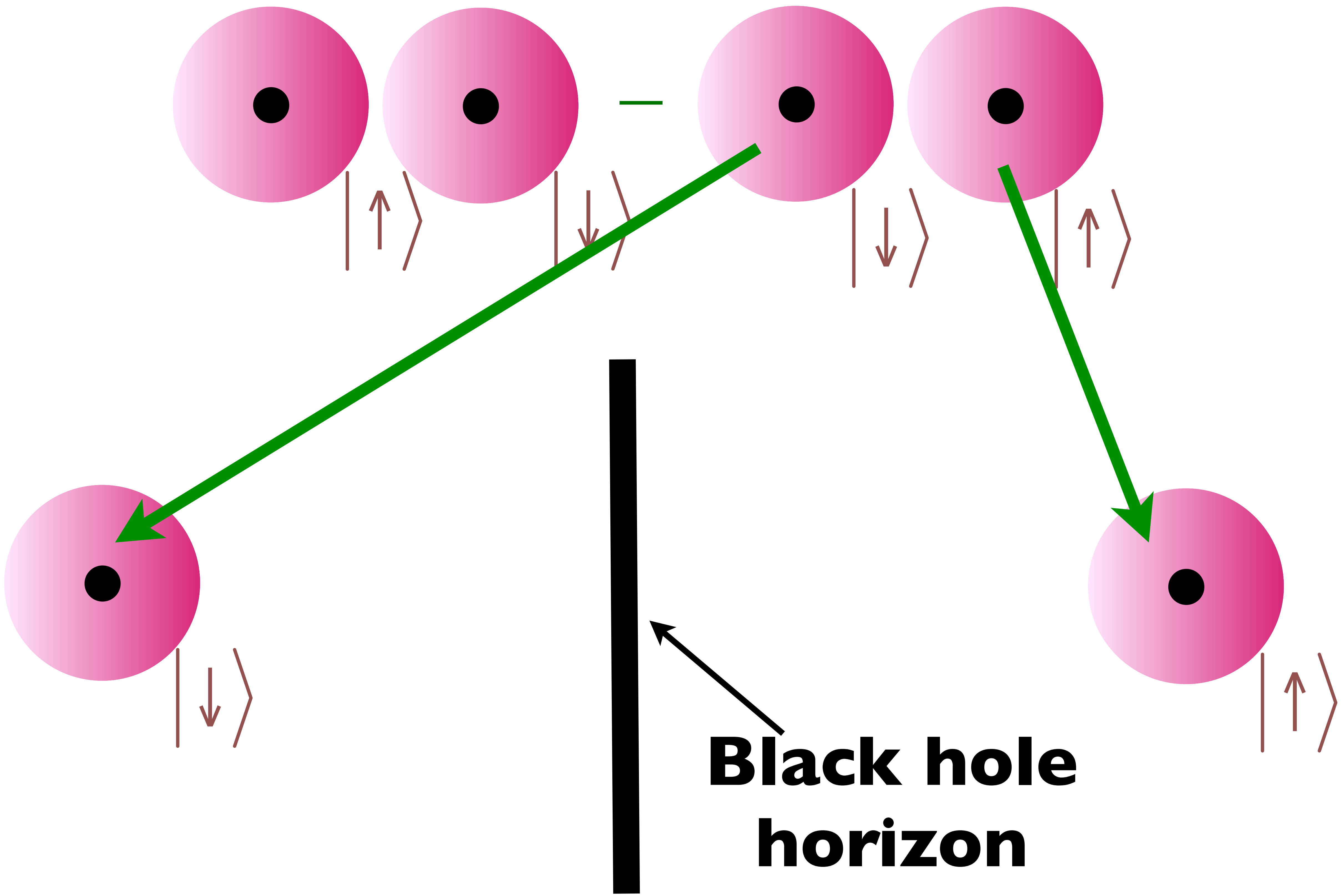
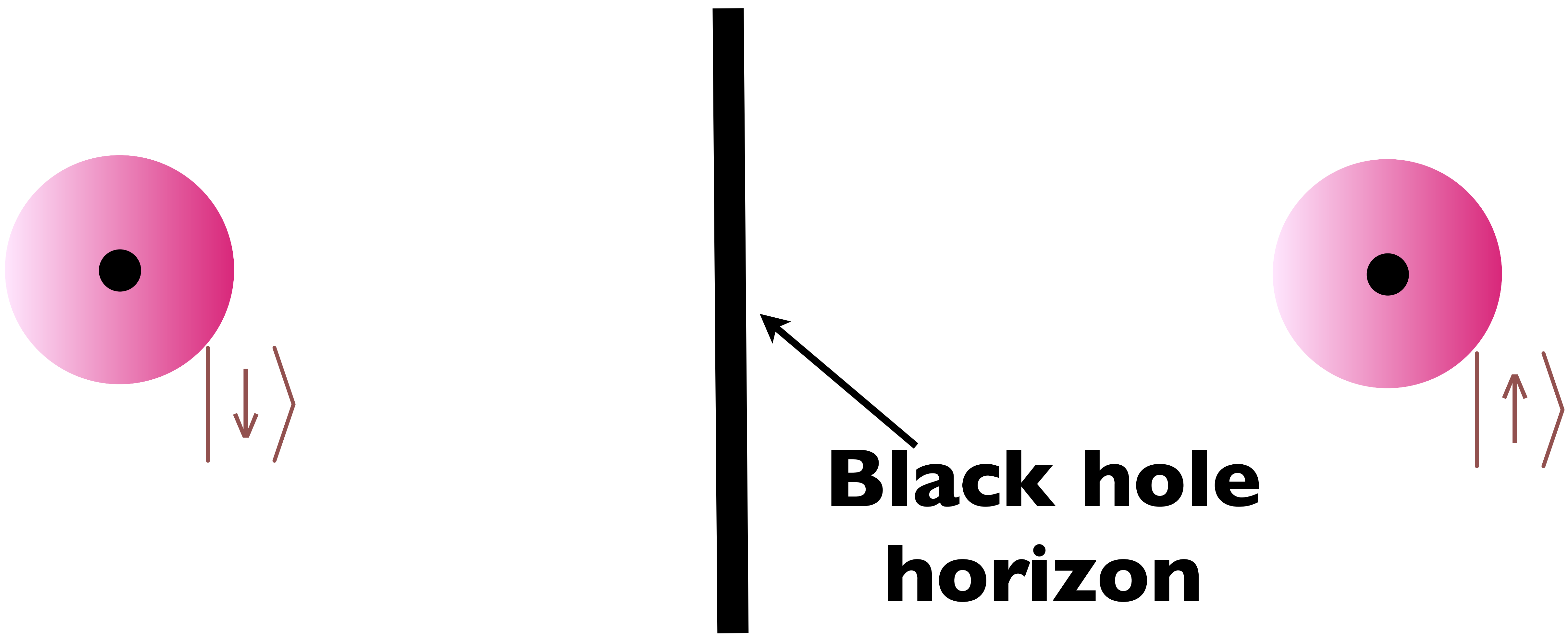


Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

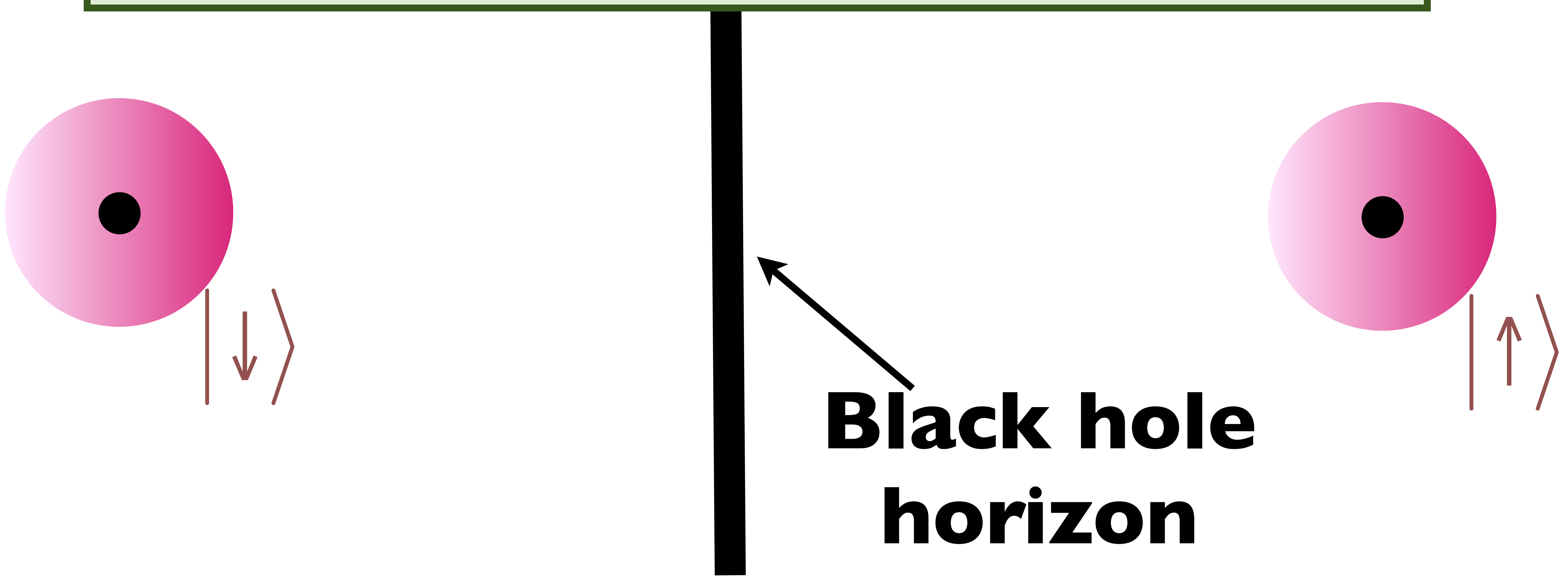
There is quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature

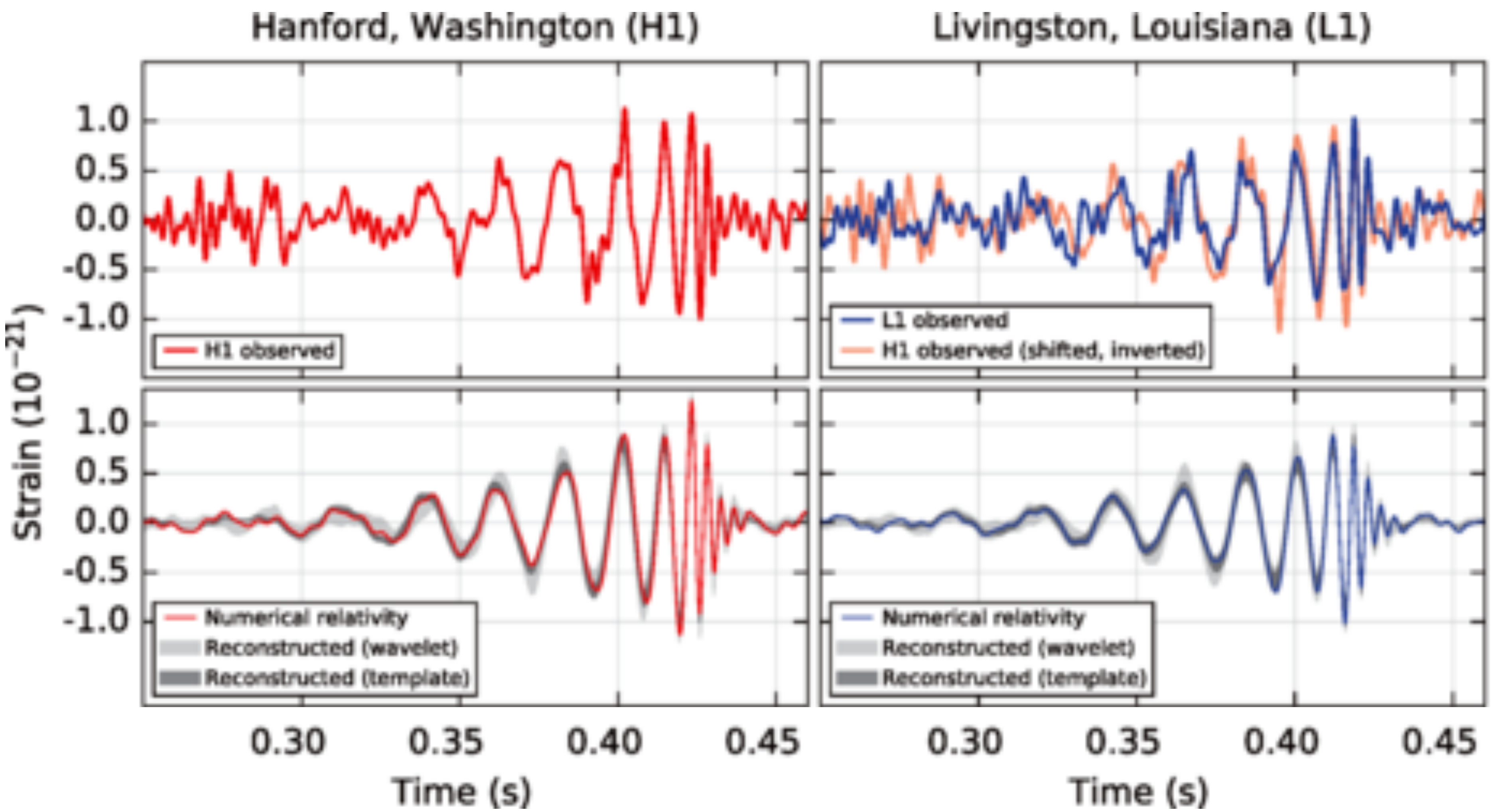
(because to an outside observer, the state of the electron inside the black hole is an unknown)

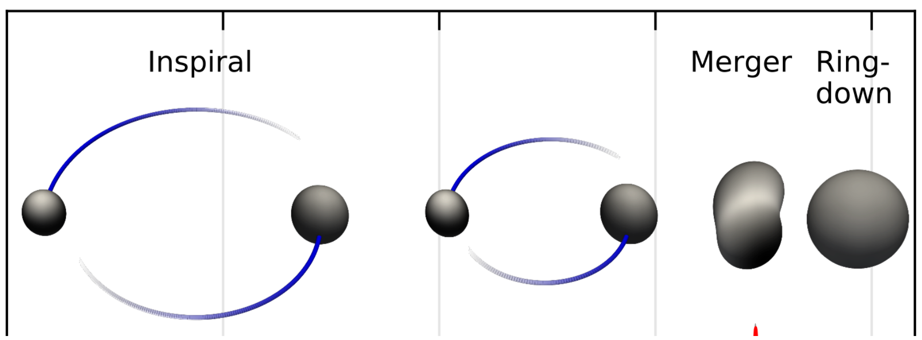


Quantum Black holes

- Black holes have an entropy and a temperature, T_H
- The entropy is proportional to their surface area.

J. D. Bekenstein, PRD **7**, 2333 (1973)
S. W. Hawking, Nature **248**, 30 (1974)

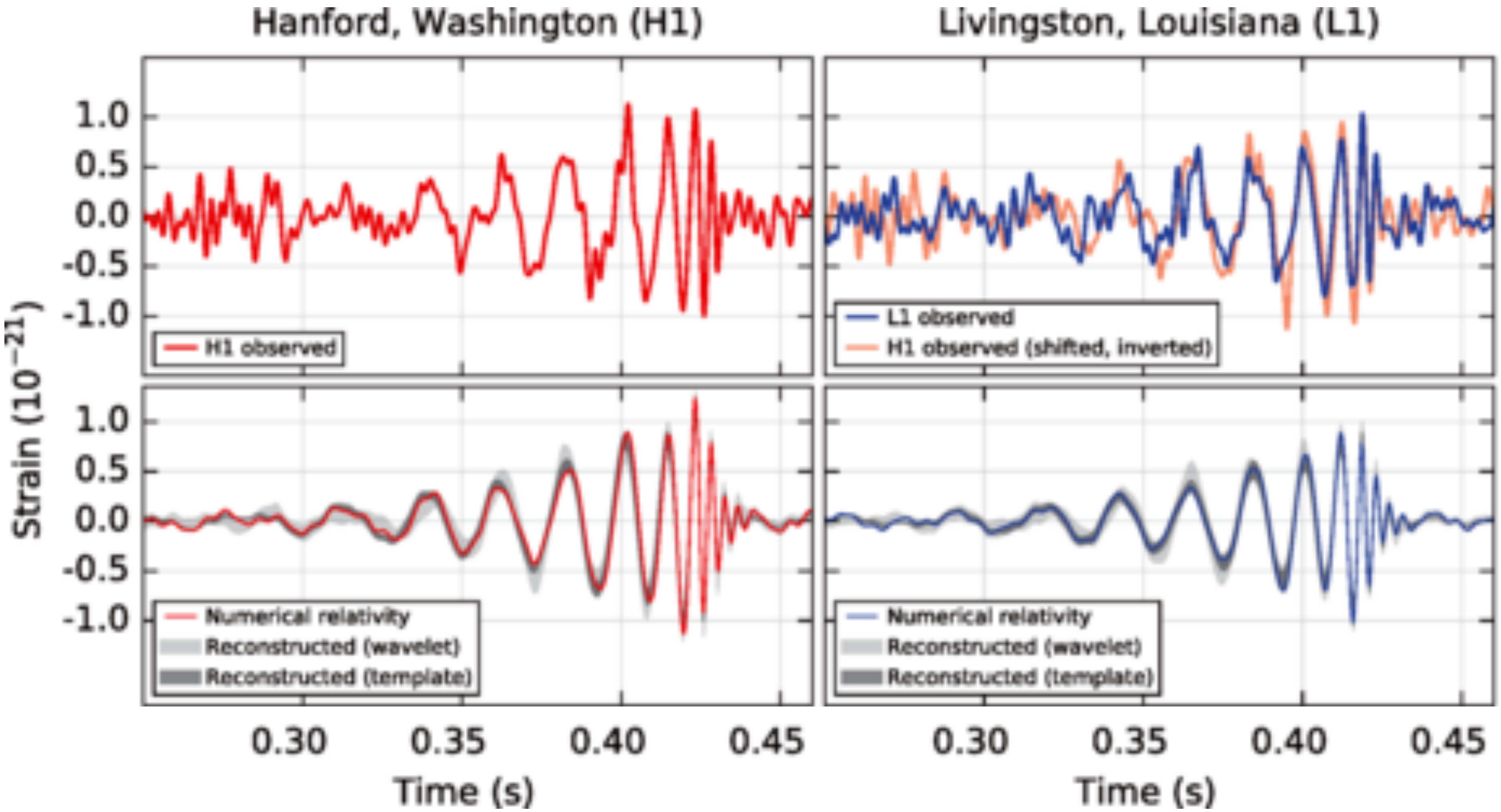


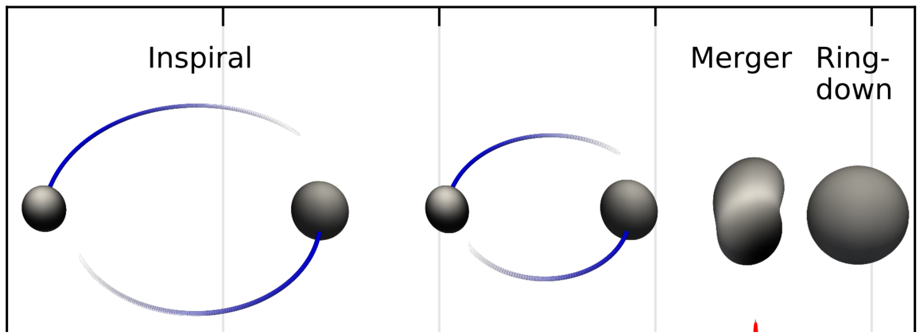


LIGO
September 14, 2015

- The ring-down is predicted by General Relativity to happen in a time $\frac{8\pi GM}{c^3} \sim 8$ milliseconds.

C. V. Vishveshwara, Nature **227**, 936 (1970)





LIGO
September 14, 2015

- The ring-down is predicted by General Relativity to happen in a time $\frac{8\pi GM}{c^3} \sim 8$ milliseconds. Curiously this happens to equal $\frac{\hbar}{k_B T_H}$: so the ring down can also be viewed as the approach of a quantum system to thermal equilibrium at the fastest possible rate!

Quantum Black holes

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- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.

Quantum Black holes

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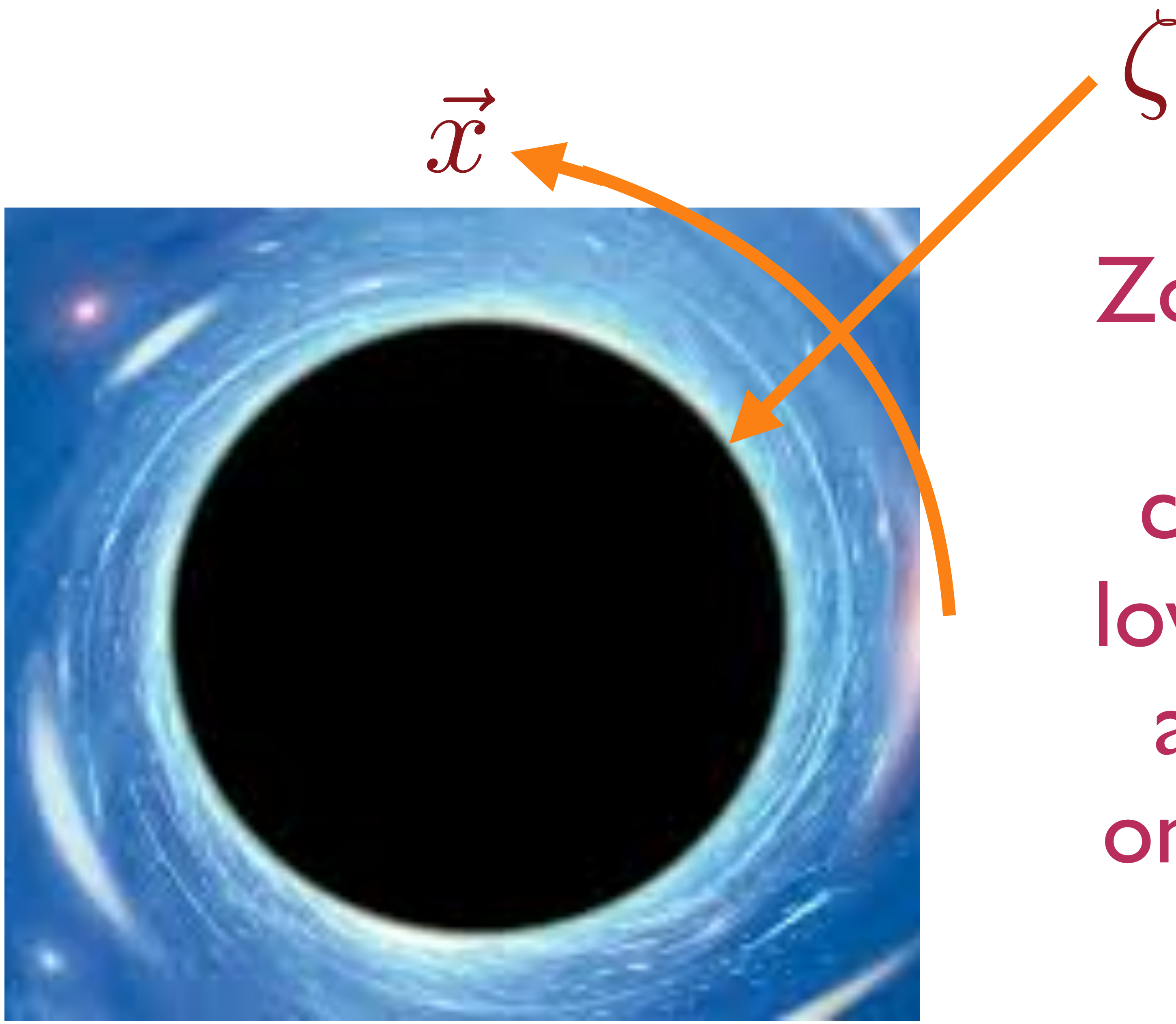
- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.

Holography:
Quantum black holes "look like" quantum many-particle systems without quasiparticle excitations, residing "on" the surface of the black hole

J. Maldacena, IJTP **38**, 1113 (1999); S.S. Gubser, I.R. Klebanov, and A.M. Polyakov Phys. Lett. B **428**, 105 (1998); E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998)

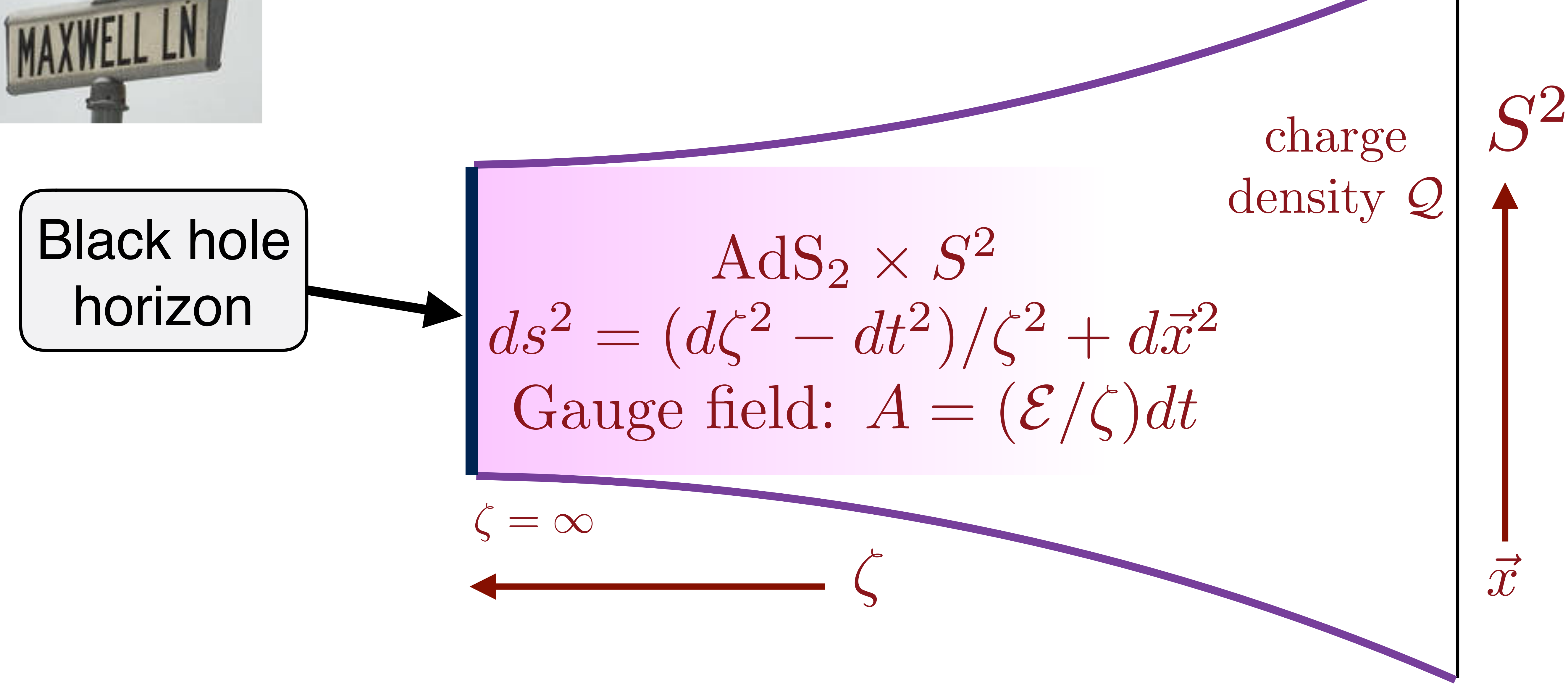
Work with a theory of Maxwell's electromagnetism and Einstein's general relativity. Include a negative cosmological constant, and examine black hole solutions with a net charge

Work with a theory of Maxwell's electromagnetism and Einstein's general relativity. Include a negative cosmological constant, and examine black hole solutions with a net charge



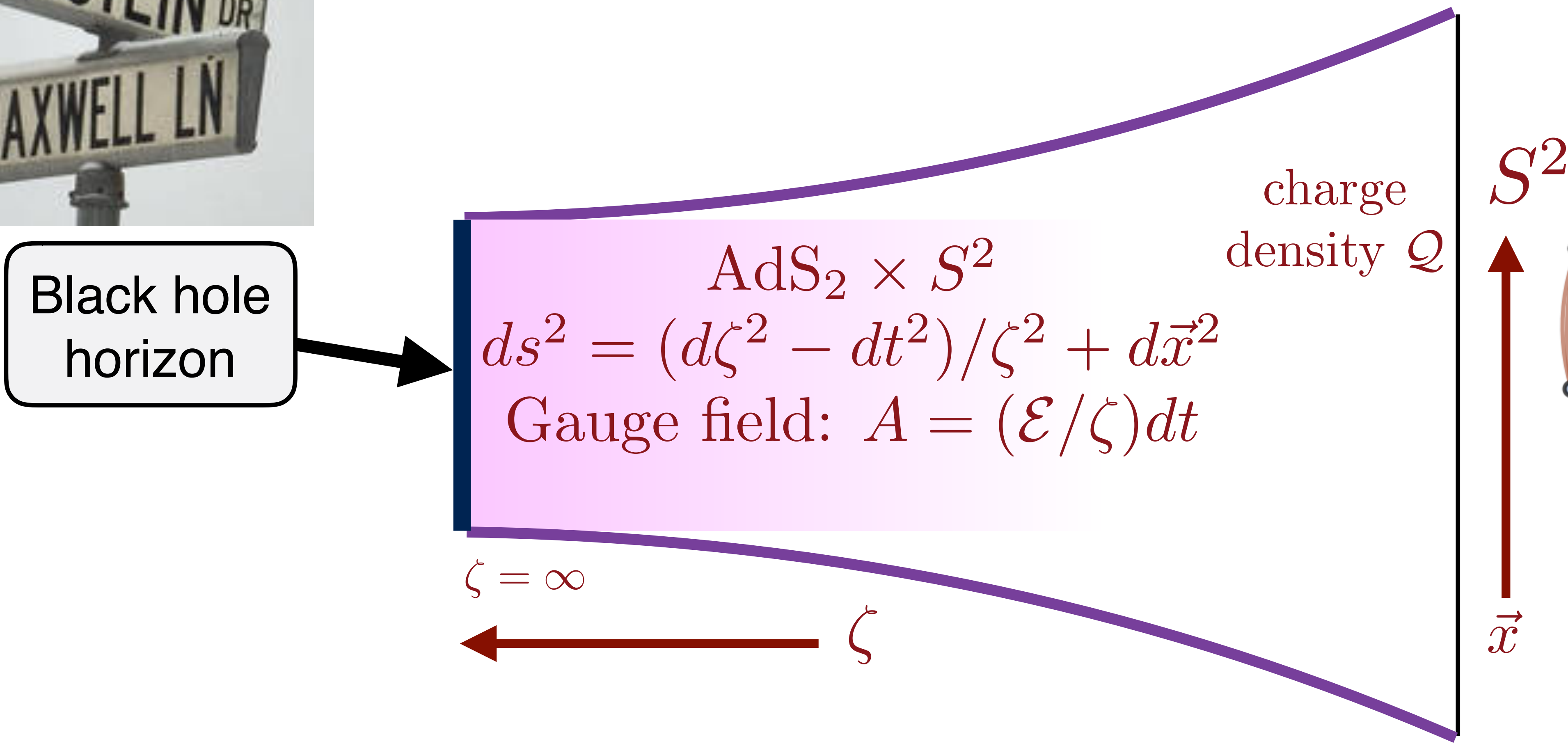
Zooming into the near-horizon region of a charged black hole at low temperature, yields a quantum theory in one space (ζ) and one time dimension

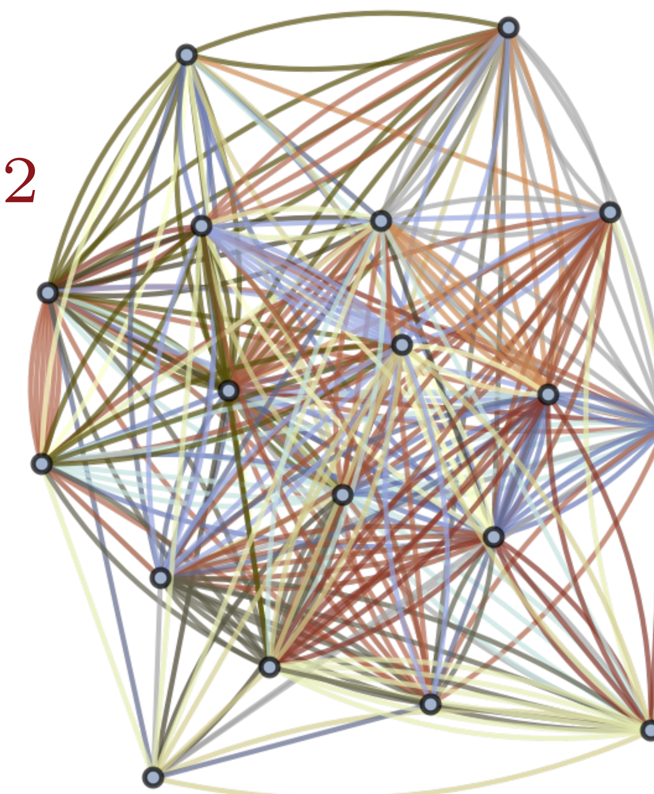
SYK model and charged black holes



The near-horizon region of a charged black hole has the geometry of (1+1)-dimensional anti-de Sitter spacetime. By holography, this should map to a zero-dimensional quantum system: this turns out to be the SYK model

SYK model and charged black holes





Bekenstein-Hawking entropy of AdS_2 horizon at $T = 0 \Leftrightarrow N s_0$ entropy of SYK model.

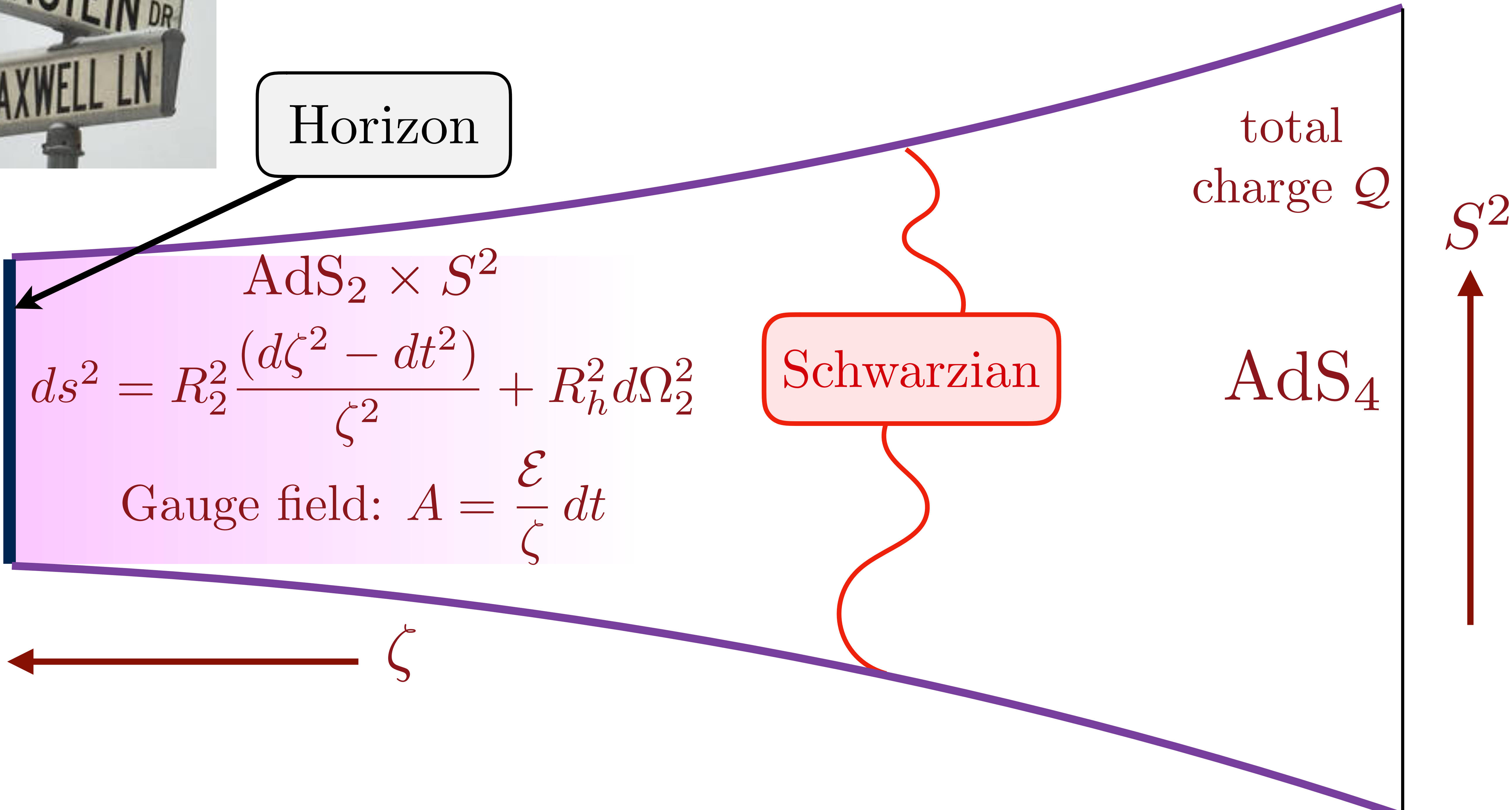
$\frac{ds_0}{d\mathcal{Q}} = 2\pi\mathcal{E}$ can be obtained from the Einstein equations for the black hole,

and the quantum theory of the SYK model,

and $\mathcal{E}$ determines identical fermion spectral functions.

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010); PRX **5,** 041025 (2015)

SYK model and charged black holes



Remarkably, the correspondence between charged black holes and the SYK model also holds for the leading fluctuations at higher temperatures: both are described by a 'Schwarzian' theory with emergent SL(2,R) and U(1) gauge symmetries. For the black hole, the Schwarzian describes the fluctuations of the boundary between AdS_2 and AdS_4.

Main result

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

A. Kitaev (2015)

S. Sachdev, Phys. Rev. X **5**, 041025 (2015)

J. Maldacena and D. Stanford, Phys. Rev. D **94,** 106002 (2016)

J. Maldacena, D. Stanford, and Zhenbin Yang, PTEP 12C104 (2016)

K. Jensen, Phys. Rev. Lett. **117**, 111601 (2016)

J. Engelsoy, T.G. Mertens, and H. Verlinde, JHEP 1607 (2016) 139

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev,
Phys. Rev. B **95**, 155131 (2017)

A. Gaikwad, L.K. Joshi, G. Mandal, and S.R. Wadia, arXiv:1802.07746 P. Nayak, A. Shukla,

R.M. Soni, S.P. Trivedi, and V. Vishal, arXiv:1802.09547

P. Chaturvedi, Yingfei Gu, Wei Song, Boyang Yu, arXiv:1808.08062

U. Moitra, S. P. Trivedi, and V. Vishal, arXiv:1808.08239

S. Sachdev, arXiv:1902.04078

Main result

SYK model of fermions with random interactions of mean-square-value U, with total fermion number $\mathcal{Q}$, at temperatures $T \ll U$

Main result

SYK model of fermions with random interactions of mean-square-value U, with total fermion number $\mathcal{Q}$, at temperatures $T \ll U$

and

Charged black holes in 3+1 dimensions of radius R_h, with total charge $\mathcal{Q}$, at temperatures $T \ll 1/R_h$

are described by a common low energy quantum theory in $0+1$ dimensions

Main result

The common low T path integral is $\mathcal{Z} = \int \mathcal{D}f\, \mathcal{D}\phi\, e^{-I}$. This can be exactly evaluated, and the action is

$$I = -s_0 + \int_0^{1/T} d\tau \left\{ \frac{K}{2} \left(\frac{\partial \phi}{\partial \tau} + i(2\pi \mathcal{E} T) \frac{\partial f}{\partial \tau} \right)^2 - \frac{\gamma}{4\pi^2} \mathrm{Sch}[\tan(\pi T f(\tau)), \tau] \right\},$$

where $f(\tau)$ is a monotonic reparameterization of the temporal circle with

$f(\tau + 1/T) = f(\tau) + 1/T$,

ϕ is a phase conjugate to the charge density with

$\phi(\tau + 1/T) = \phi(\tau) + 2\pi n$, n integer,

$\mathrm{Sch}[g[\tau], \tau]$ is the Schwarzian derivative of $g(\tau)$.

The couplings are related to the entropy $S(T, \mathcal{Q})$ and the chemical potential μ via

$$S(T \to 0, \mathcal{Q}) = s_0 + \gamma T, \quad K = \left(\frac{d\mathcal{Q}}{d\mu} \right)_{T \to 0}, \quad 2\pi \mathcal{E} = \frac{ds_0}{d\mathcal{Q}}$$

Main result

- Closely related to, but not the usual AdS/CFT correspondence, which involves only neutral black holes at $T > 0$.

- Unlike the AdS/CFT correspondence, *both* sides of the duality are fully solvable. This has enabled numerous recent studies of black holes quantum information.

Quantum matter without quasiparticles

- Planckian dynamics (*i.e.* fastest possible local thermalization in a time $\hbar/(k_B T)$) is realized in the 'solvable' SYK models.

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- A Schwarzian theory of a time reparameterization mode, with SL(2,R) symmetry, (along with a phase fluctuating mode) describes the quantum dynamics of
  - the SYK models
  - black holes with near-extremal AdS_2 horizons