Quantum matter without quasiparticles

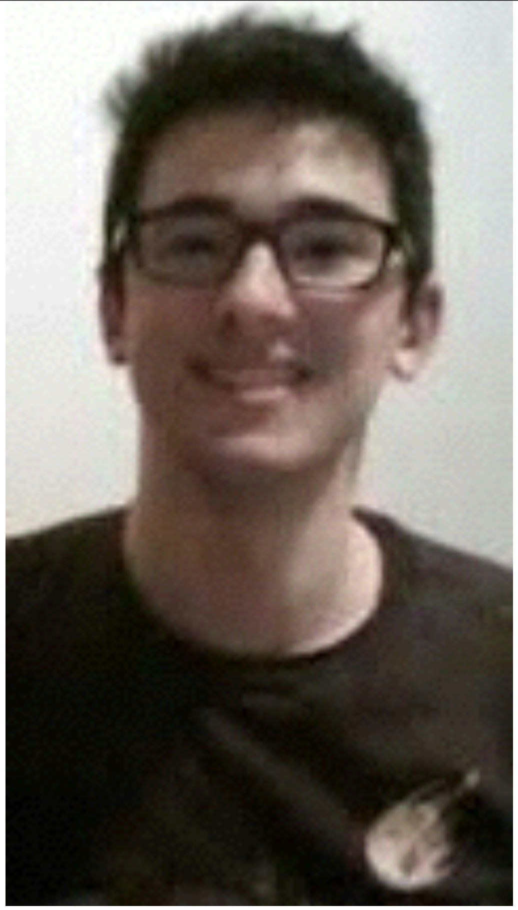
New Perspectives on Thermalization
Aspen Center for Physics
March 17, 2014

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

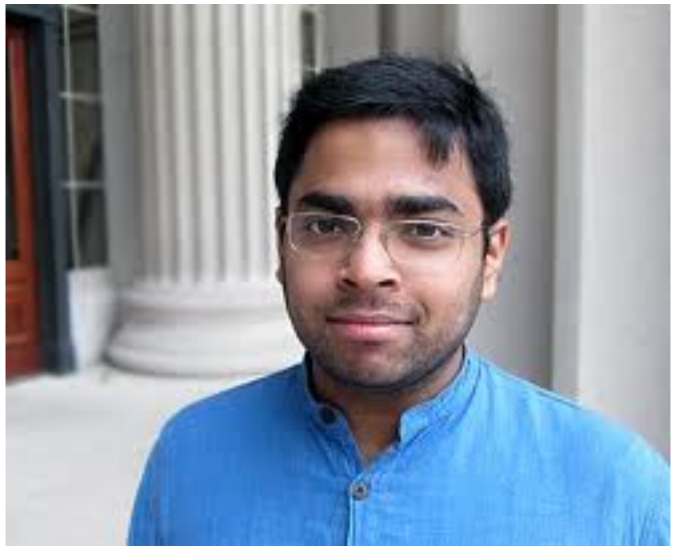
William Witczak-Krempa
Perimeter

Erik Sorensen
McMaster



Andrew Lucas
Harvard

Sean Hartnoll
Stanford



Raghu Mahajan
Stanford

Matthias Punk
Innsbruck

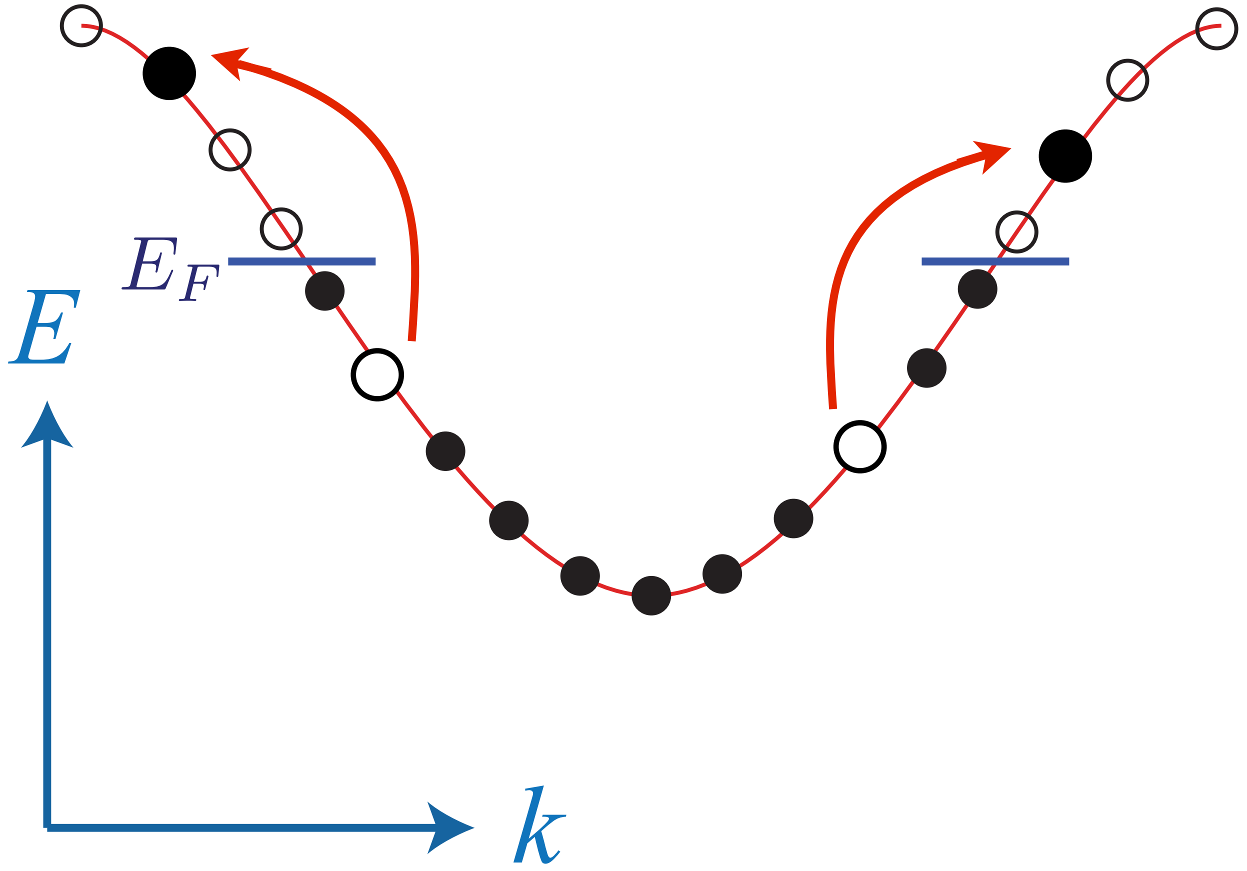
Koenraad Schalm
Leiden

Foundations of quantum many body theory:

1. Ground states connected adiabatically to independent electron states

2. Boltzmann-Landau theory of quasiparticles

Metals



Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. Boltzmann-Landau theory of quasiparticles

Famous examples:

The fractional quantum Hall effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. Boltzmann-Landau theory of quasiparticles

Famous examples:

Electrons in one dimensional wires form the Luttinger liquid. The quanta of density oscillations ("phonons") are a *quasiparticle* basis of the low-energy Hilbert space. Similar comments apply to magnetic insulators in one dimension.

Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. No quasiparticles

Only 2 examples:

1. Conformal field theories in spatial dimension $d > 1$

2. Quantum critical metals in dimension $d=2$

AdS/CFT useful in both cases

Outline

1. The simplest model without quasiparticles

   Superfluid-insulator transition

   of ultracold bosonic atoms in an optical lattice

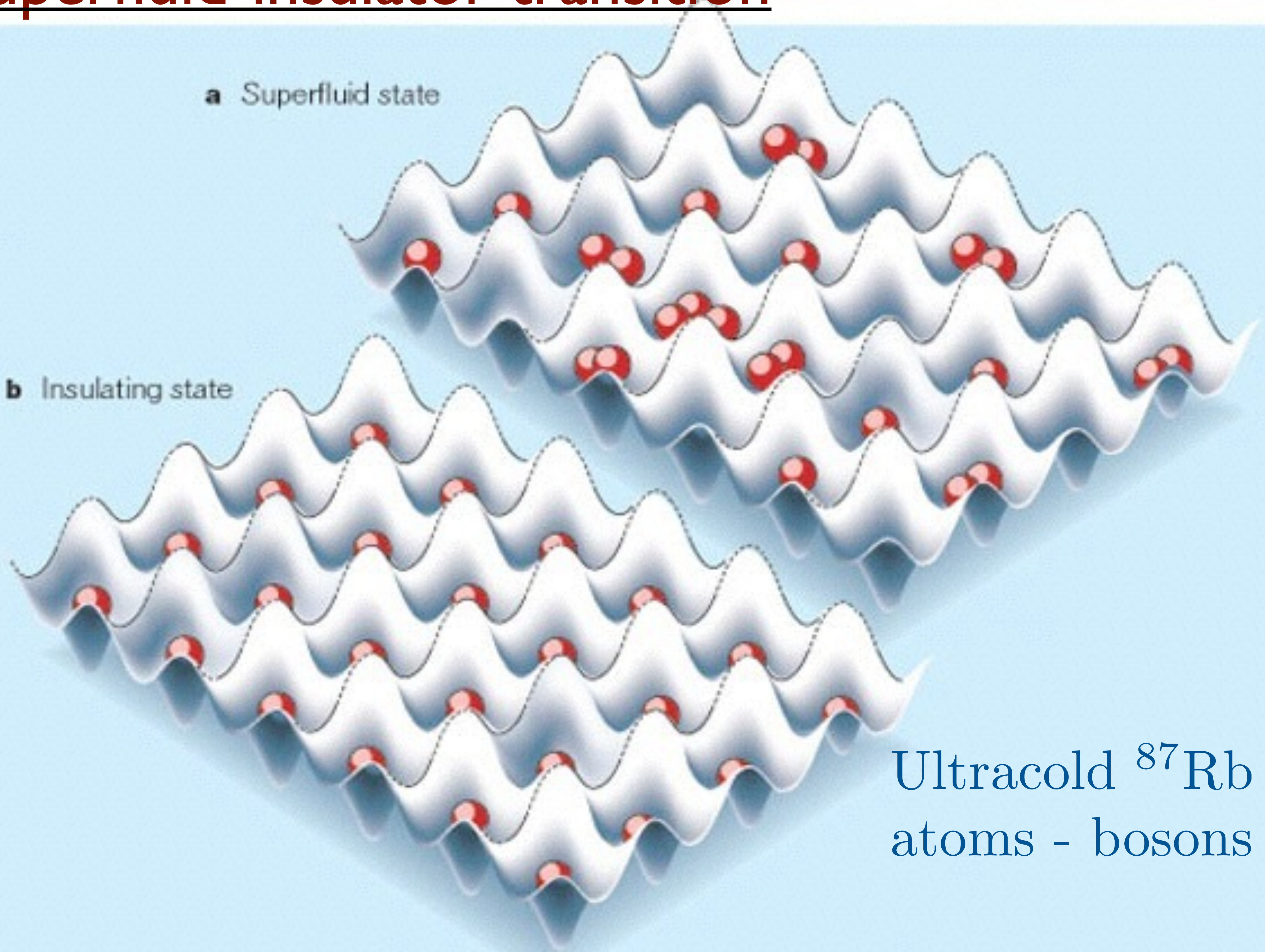
   (Conformal field theories in 2+1 dimensions)

2. Strange metals in the high T_c superconductors

   Non-quasiparticle transport at the

   Ising-nematic quantum critical point

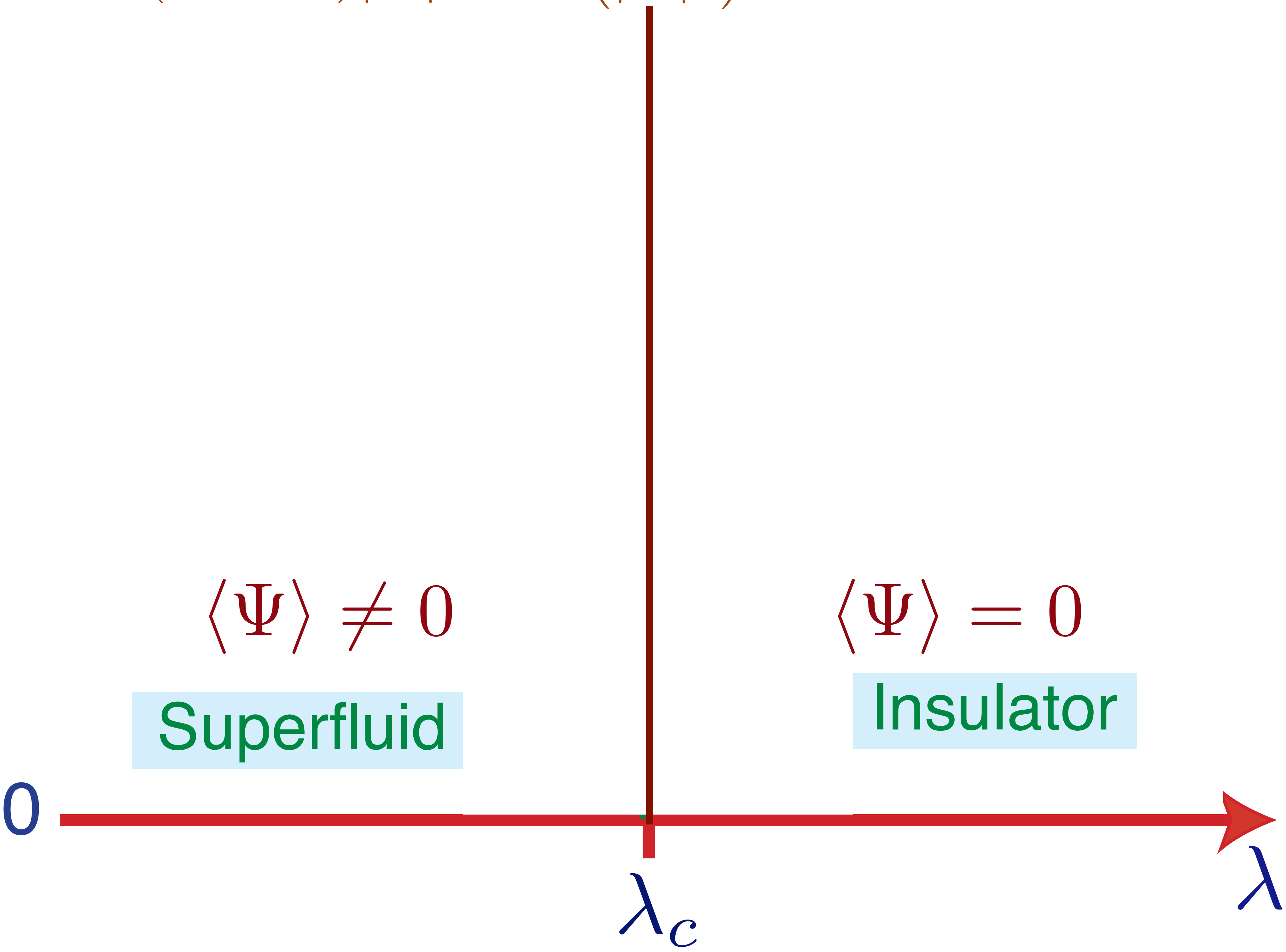
Superfluid-insulator transition



Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

$$\begin{aligned}\mathcal{S} &= \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \\ V(\Psi) &= (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2\end{aligned}$$



$\langle \Psi \rangle \neq 0$

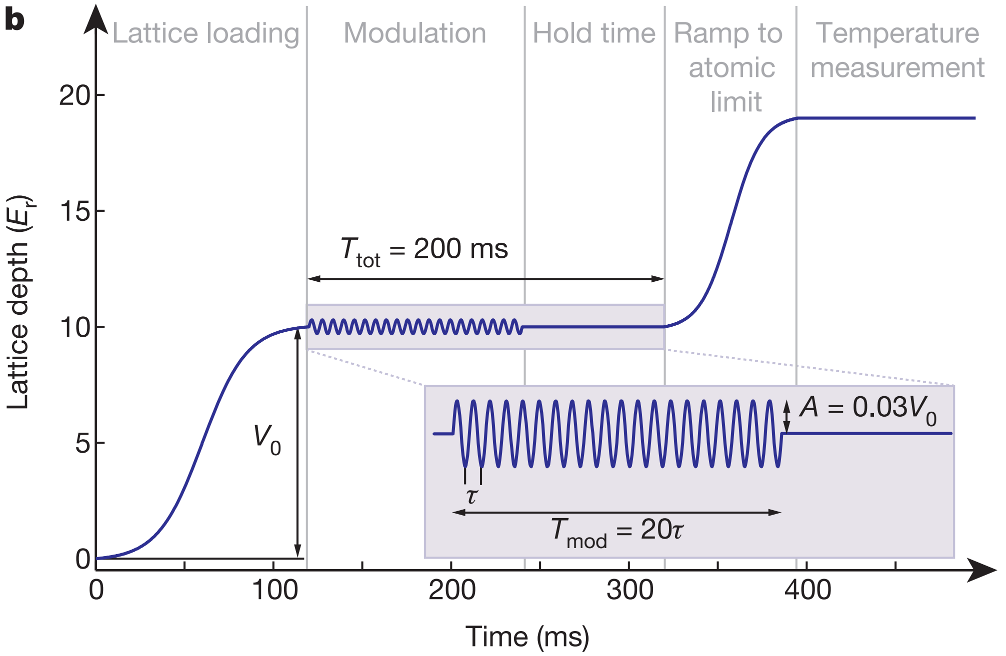
Superfluid

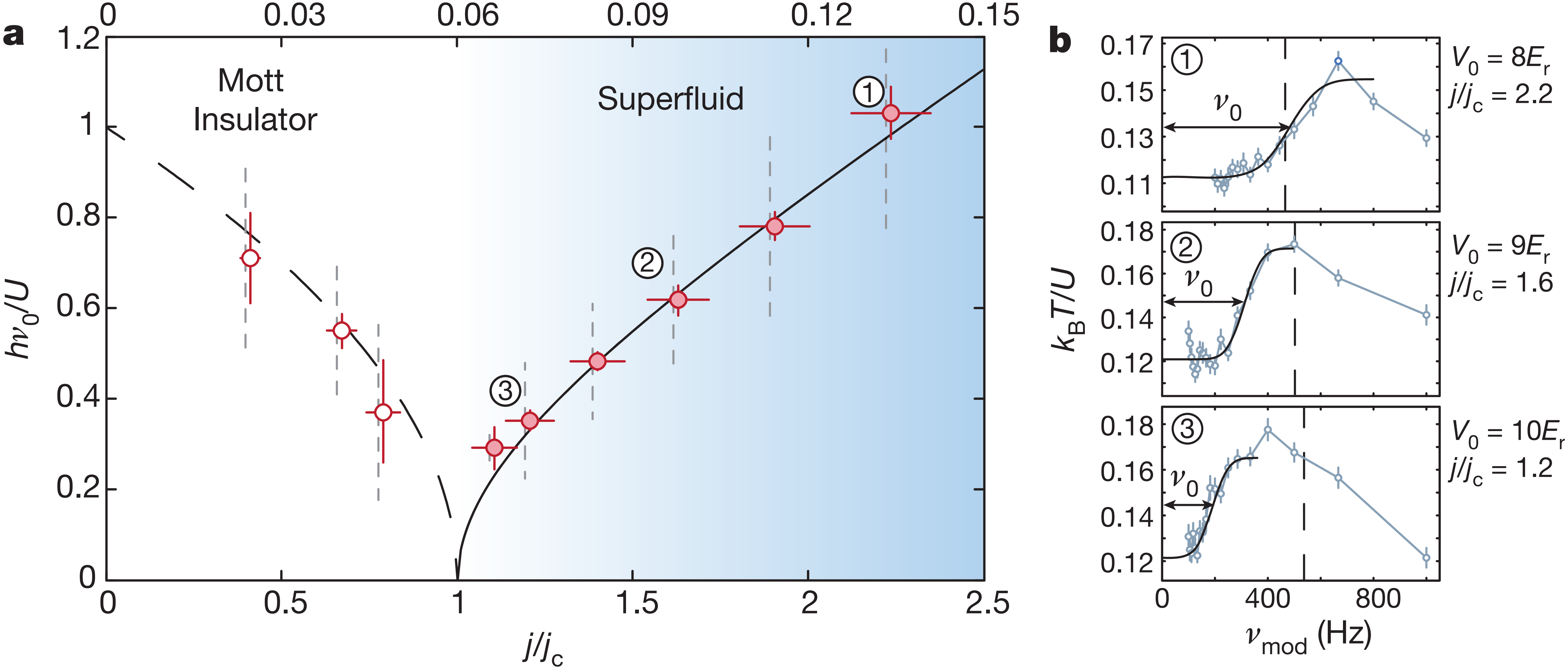
$\langle \Psi \rangle = 0$

Insulator

Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:

Response to modulation of lattice depth scales as expected from the LHP pole

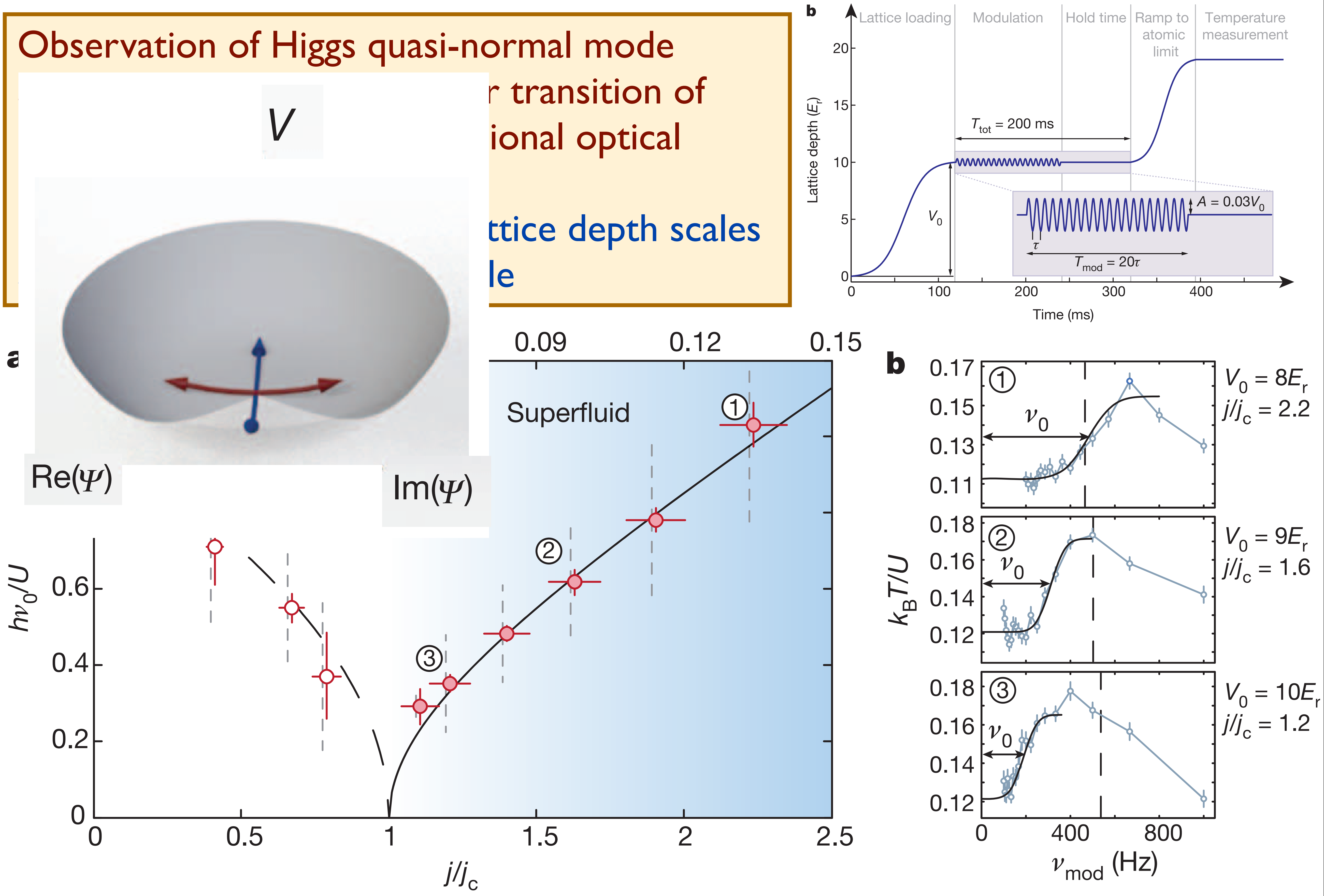




Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

Observation of Higgs quasi-normal mode
r transition of
ional optical

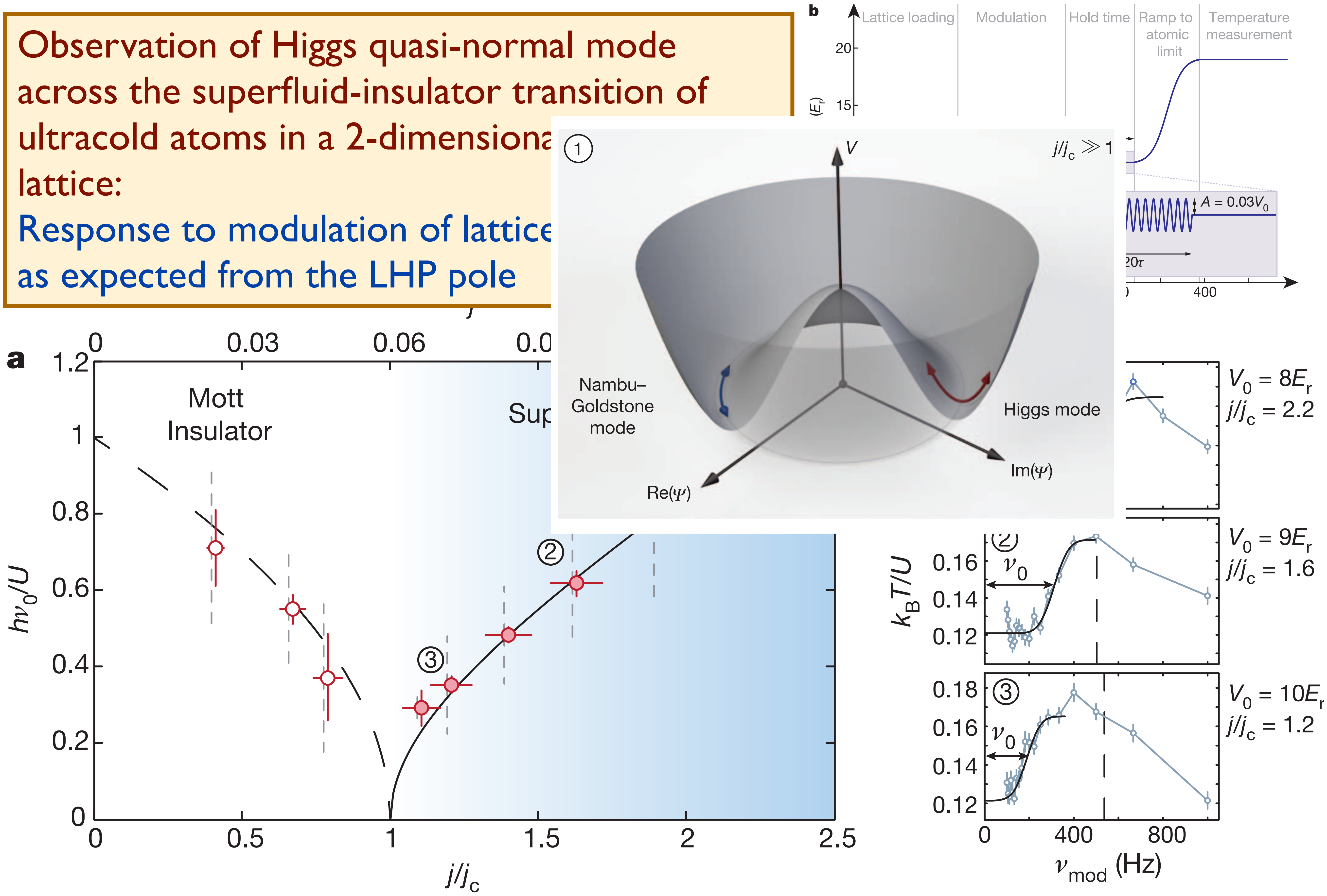
ttice depth scales
le



Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional lattice:

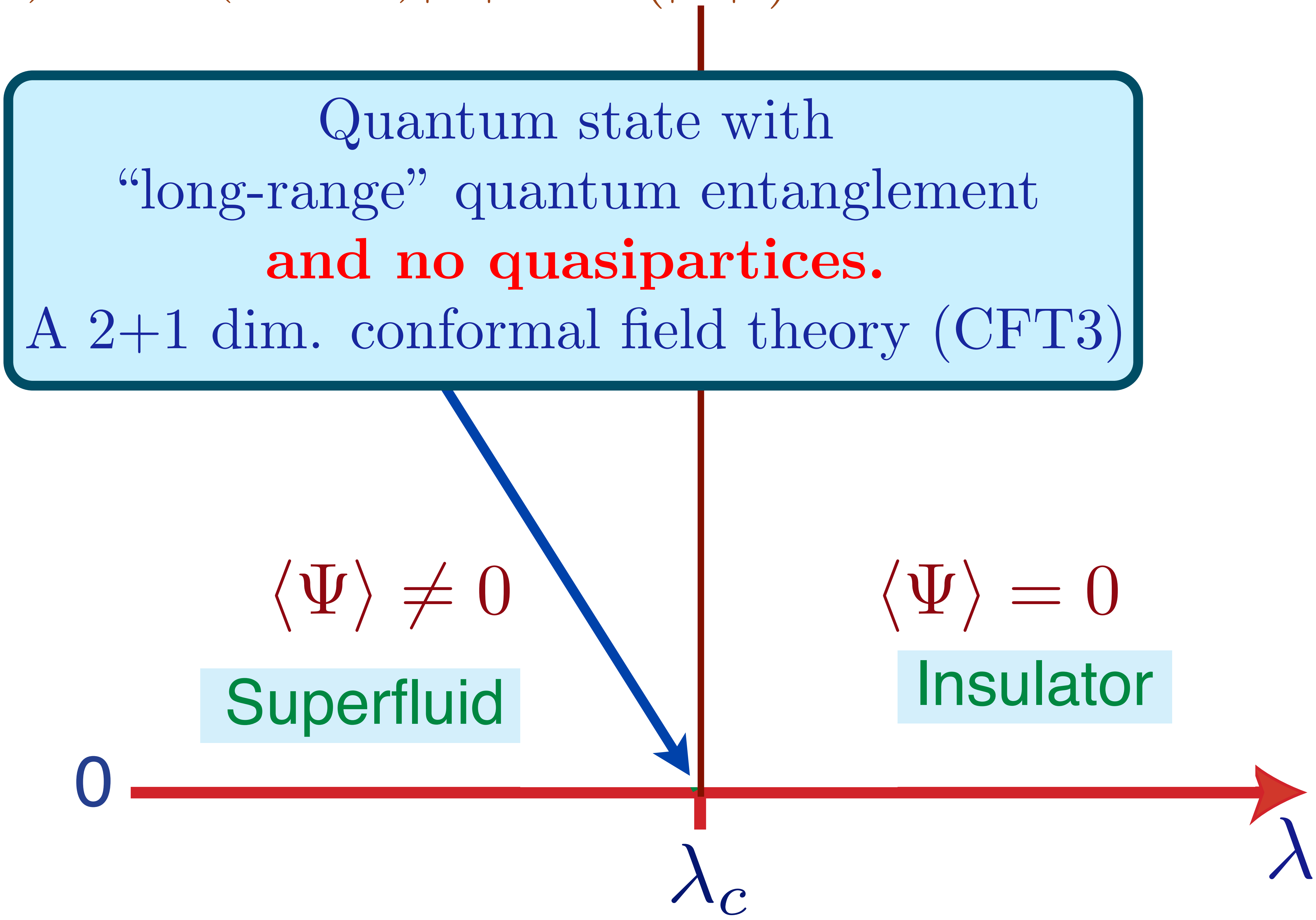
Response to modulation of lattice as expected from the LHP pole



Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$



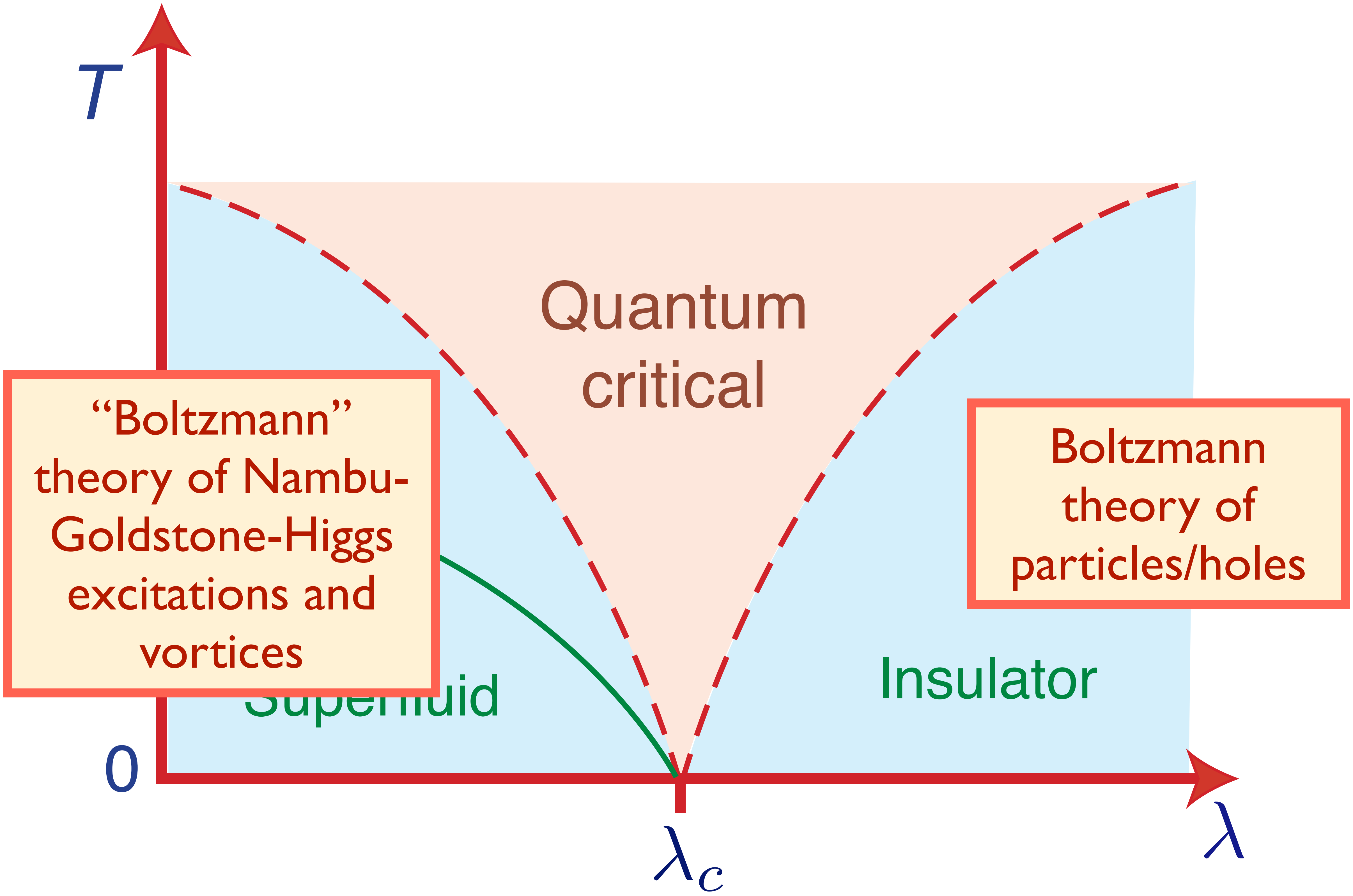
Quantum state with
"long-range" quantum entanglement
and no quasipartices.
A 2+1 dim. conformal field theory (CFT3)

$\langle \Psi \rangle \neq 0$

Superfluid

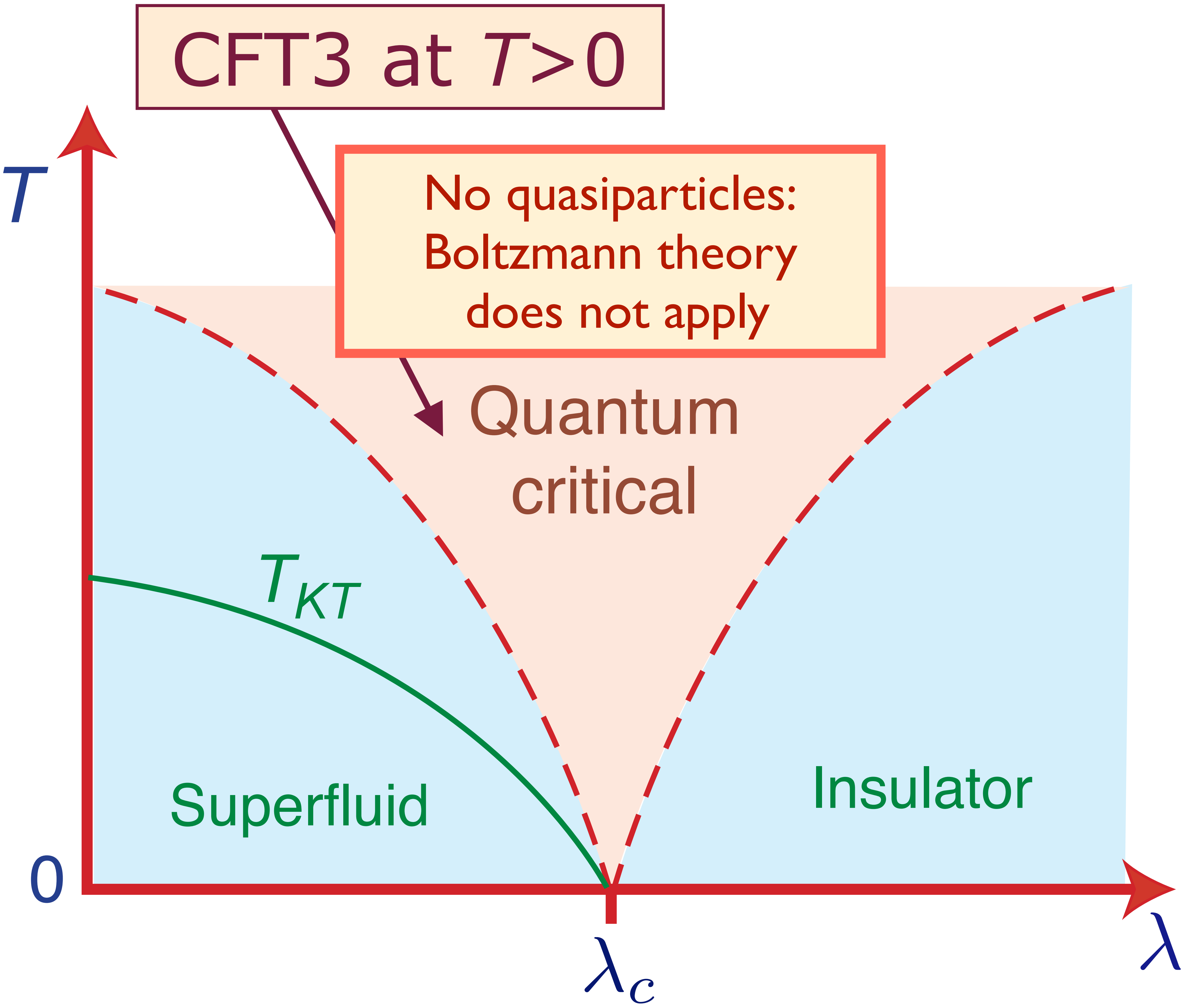
$\langle \Psi \rangle = 0$

Insulator



"Boltzmann" theory of Nambu-Goldstone-Higgs excitations and vortices

Boltzmann theory of particles/holes



CFT3 at $T>0$

No quasiparticles:
Boltzmann theory
does not apply

Traditional CMT

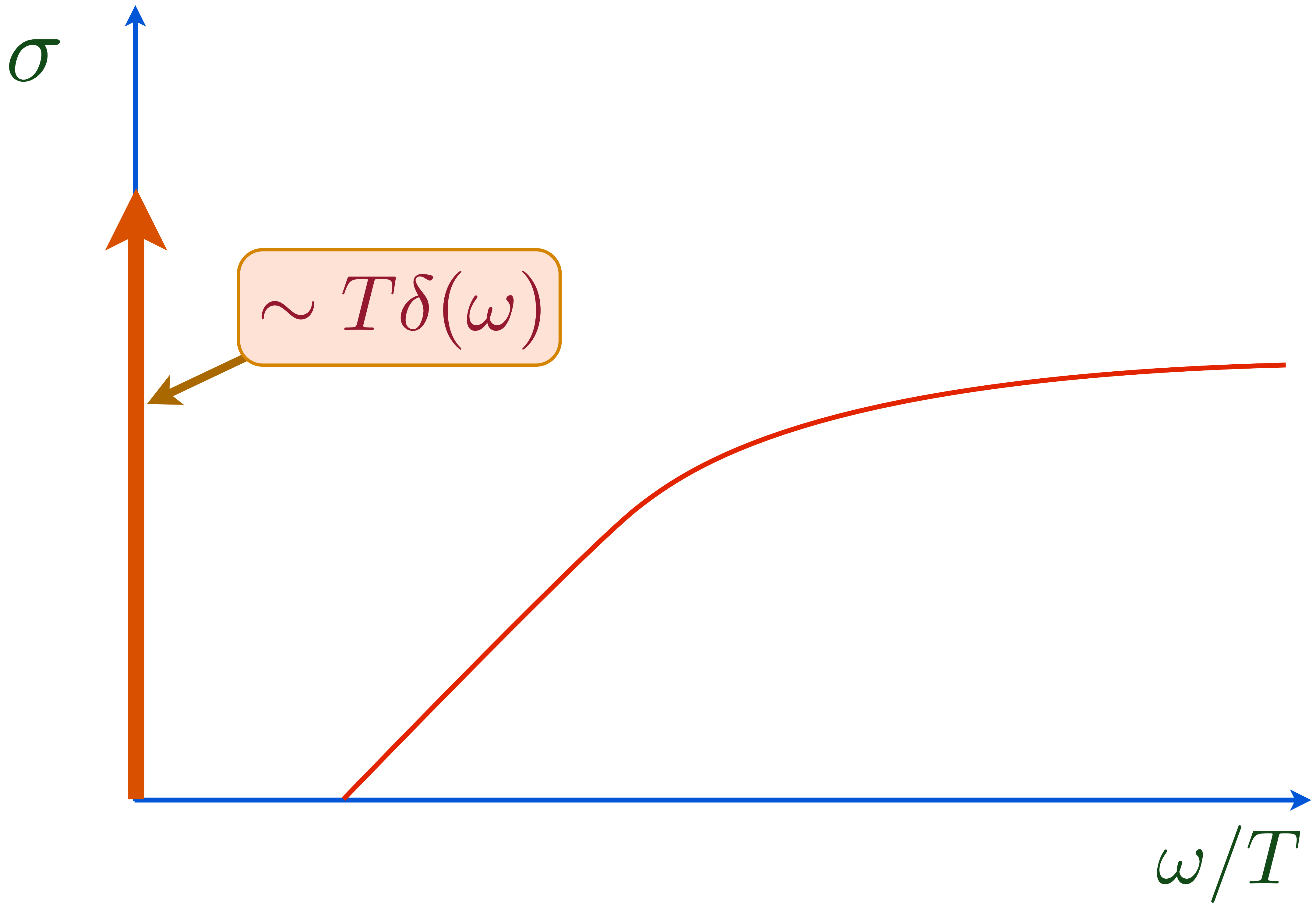
- Identify quasiparticles and their dispersions

- Compute scattering matrix elements of quasiparticles (or of collective modes)

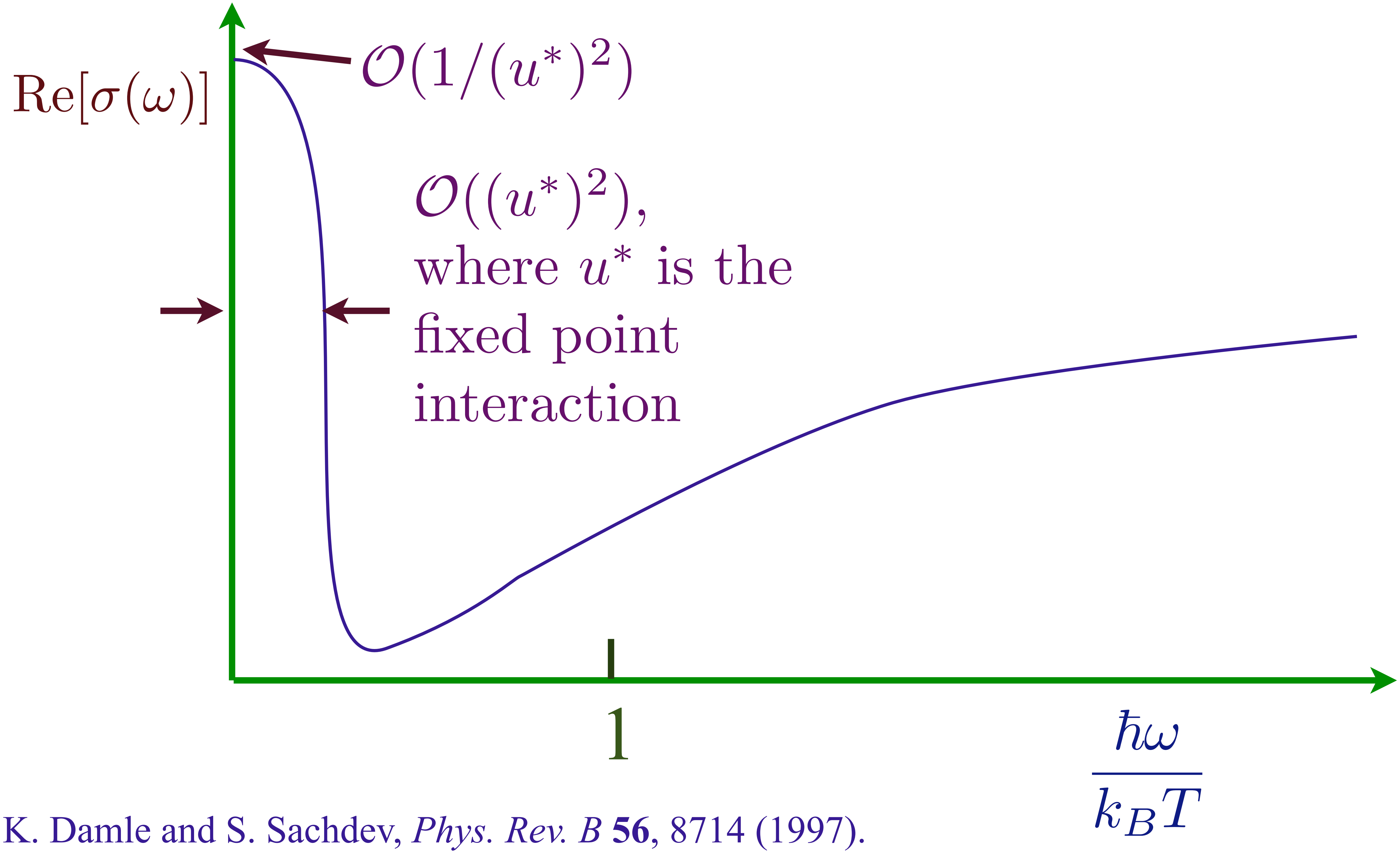
- These parameters are input into a quantum Boltzmann equation

- Deduce dissipative and dynamic properties at non-zero temperatures

Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a free CFT3



Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3



$\mathcal{O}(1/(u^*)^2)$

$\mathcal{O}((u^*)^2)$, where u^* is the fixed point interaction

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

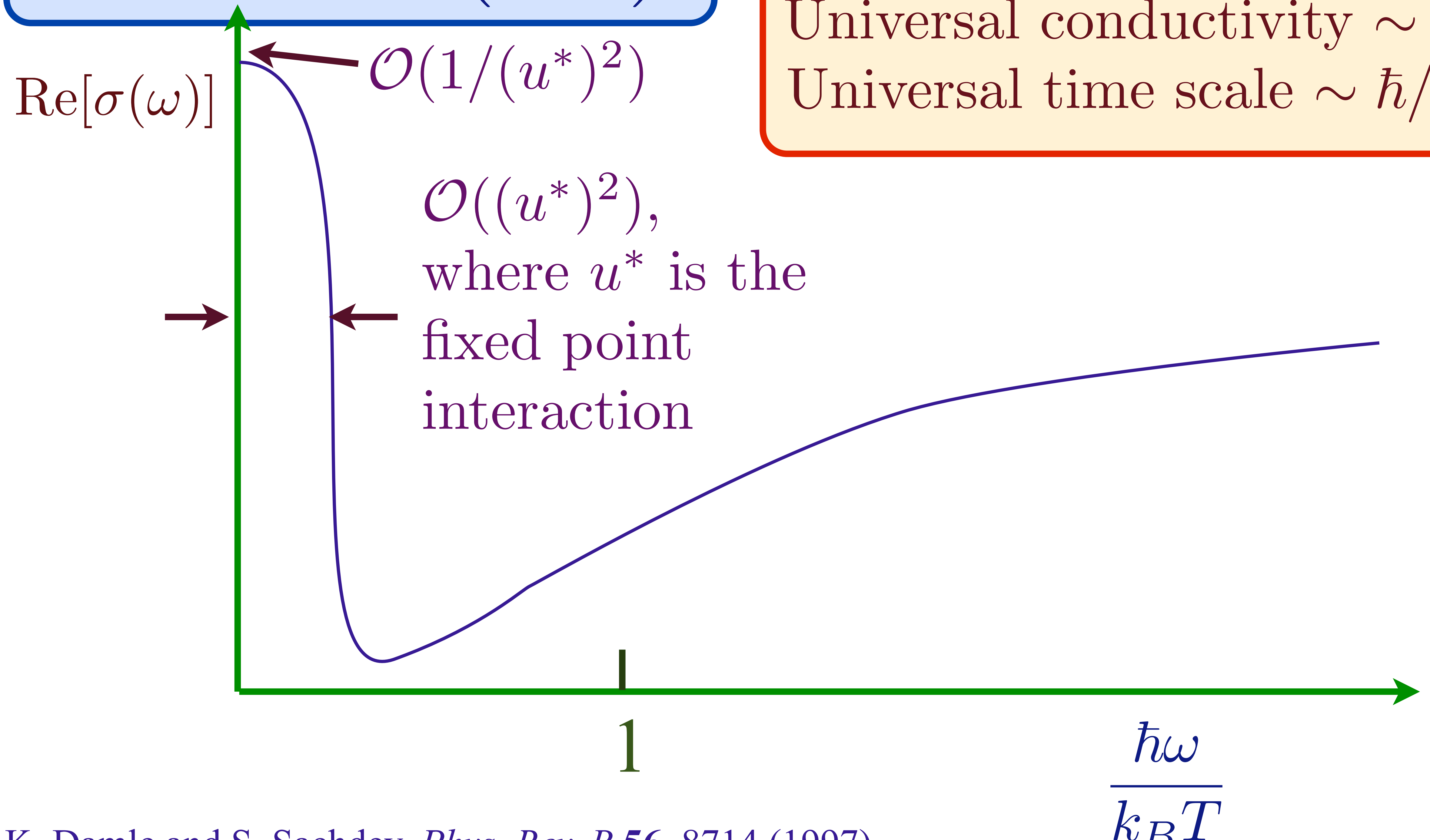
Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3

$$\sigma(\omega, T) = \frac{e^2}{h} \Sigma\left(\frac{\hbar\omega}{k_B T}\right) \; ;$$

$\Sigma \to$ a universal function

Universal conductivity $\sim e^2/h$

Universal time scale $\sim \hbar/k_B T$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Traditional CMT

- Identify quasiparticles and their dispersions
- Compute scattering matrix elements of quasiparticles (or of collective modes)
- These parameters are input into a quantum Boltzmann equation
- Deduce dissipative and dynamic properties at non-zero temperatures

Dynamics without quasiparticles

- Start with strongly interacting CFT without particle- or wave-like excitations
- Compute scaling dimensions and OPE co-efficients of operators of the CFT

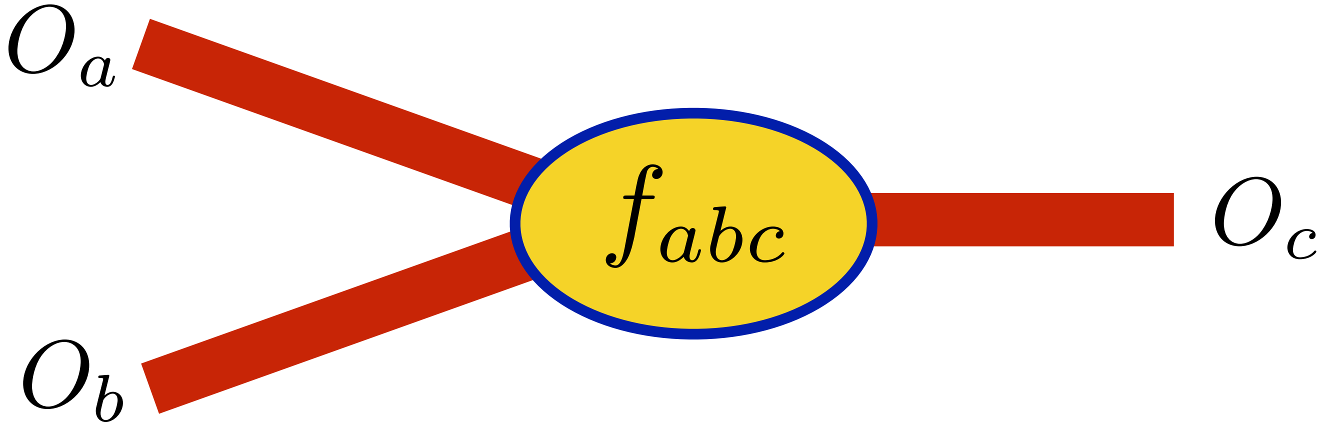
Basic characteristics of CFTs

Primary operators of CFT, $O_a(x)$, obey (at $T = 0$):

$$\boxed{\langle O_a(x) O_b(0) \rangle = \frac{\delta_{ab}}{|x|^{2\Delta_a}}}$$

where Δ_a is their scaling dimension. Their "interactions" are determined by the OPE (considering scalar operators only)

$$\boxed{\lim_{x' \to x} \langle O_a(x') O_b(x) O_c(0) \rangle = \frac{f_{abc}}{|x|^{\Delta_a + \Delta_b + \Delta_c}}}$$



The values of $\boxed{\{\Delta_a, f_{abc}\}}$ determine (in principle) all observable properties of the CFT, as constrained by conformal Ward identities. For the Wilson-Fisher CFT3, systematic methods exist to compute (in principle) all the $\{\Delta_a, f_{abc}\}$, and we will assume this data is *known*. This knowledge will be taken as an *input* to the computation of the finite T dynamics

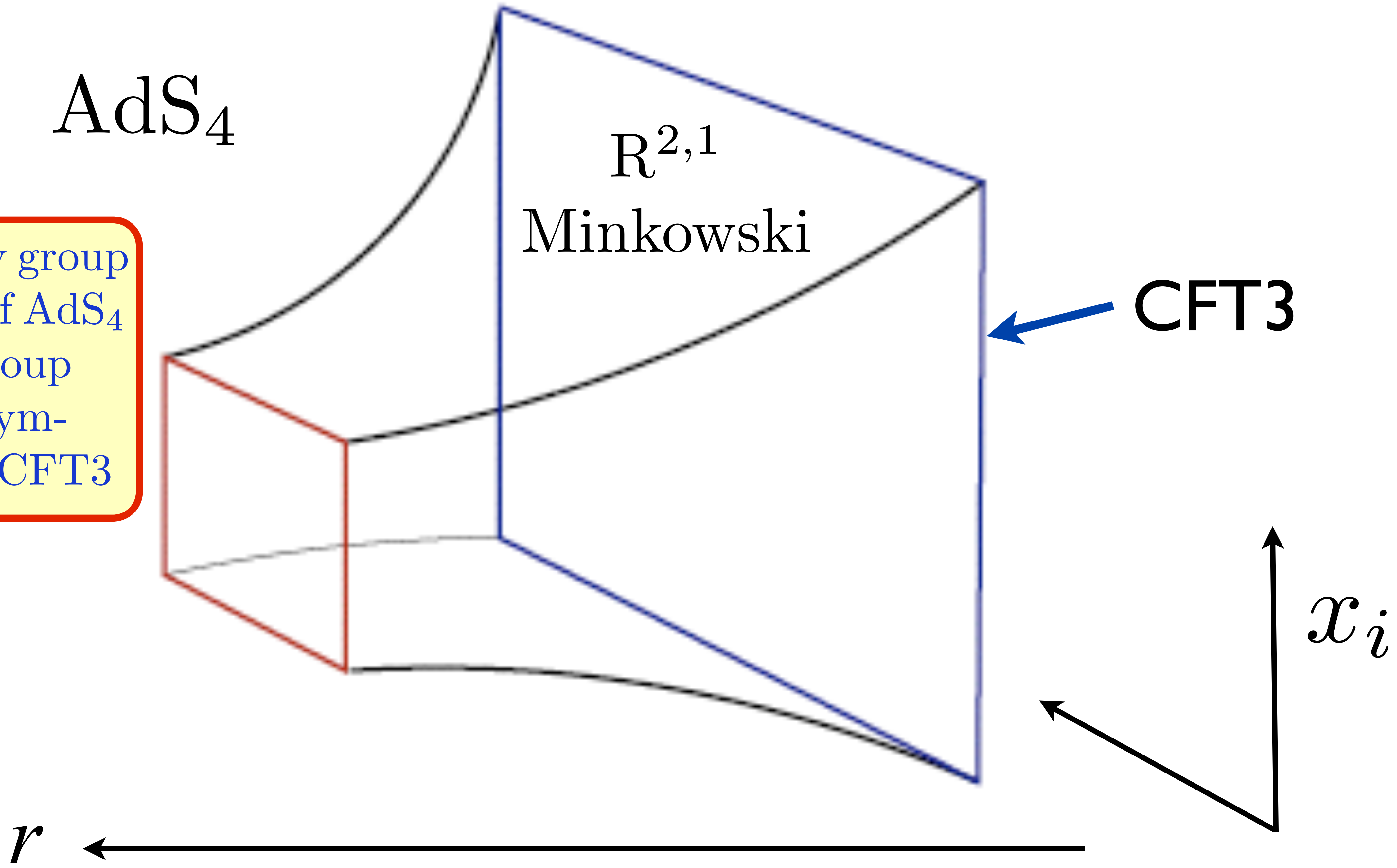
Traditional CMT

- Identify quasiparticles and their dispersions
- Compute scattering matrix elements of quasiparticles (or of collective modes)
- These parameters are input into a quantum Boltzmann equation
- Deduce dissipative and dynamic properties at non-zero temperatures

Dynamics without quasiparticles

- Start with strongly interacting CFT without particle- or wave-like excitations
- Compute scaling dimensions and OPE co-efficients of operators of the CFT
- Relate OPE co-efficients to couplings of an effective gravitational theory on AdS
- Non-zero T dynamics of CFT maps to dynamics of a “horizon” in (Einstein’s) gravitational theory

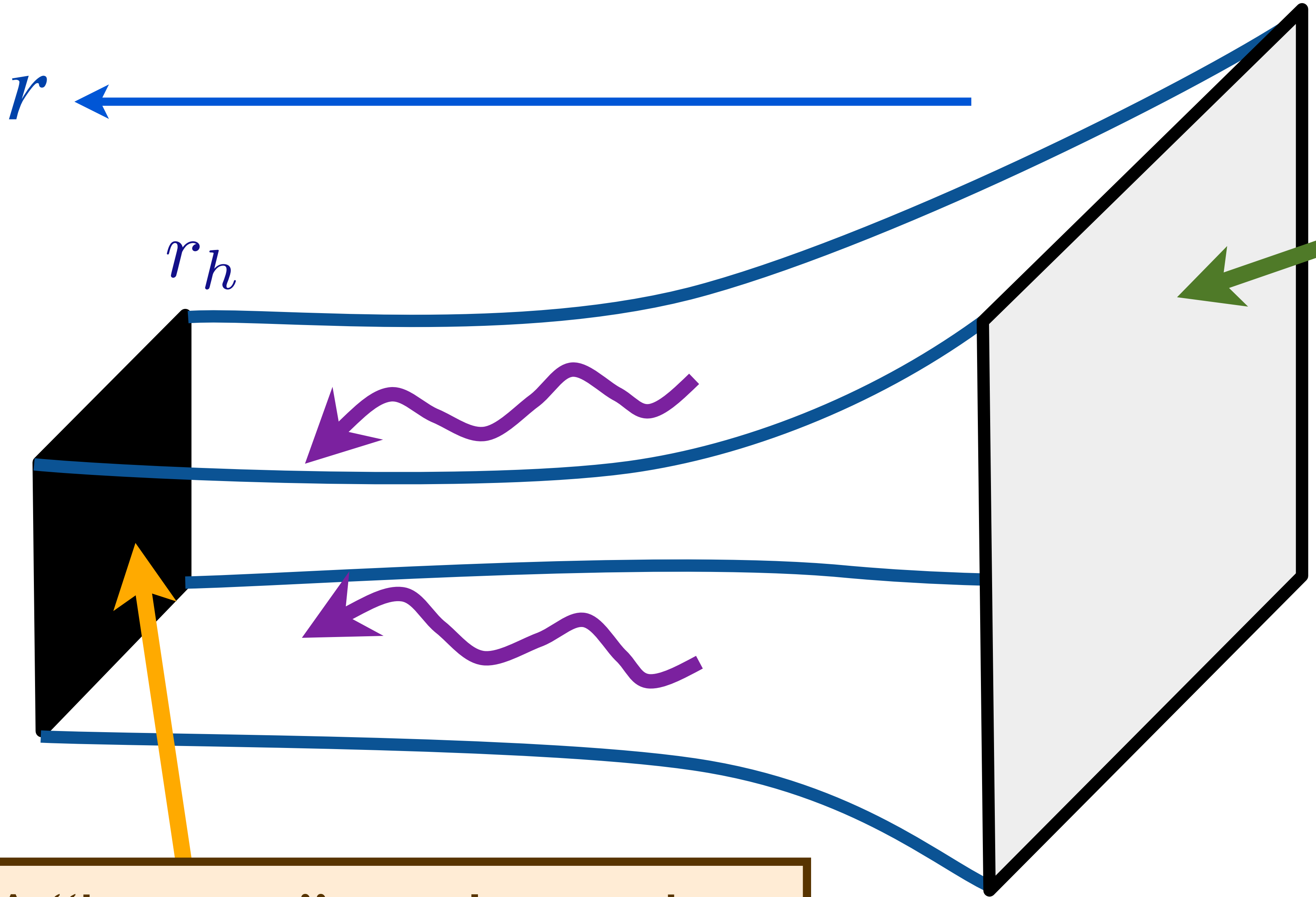
AdS/CFT correspondence at zero temperature



The symmetry group of isometries of AdS_4 maps to the group of conformal symmetries of the CFT3

A classical gravitational theory on AdS_4 encodes the CFT3 data of $\{\Delta_a, f_{abc}\}$, and allows computation of CFT3 correlators consistent with all conformal Ward identities

Gauge-gravity duality at non-zero temperatures

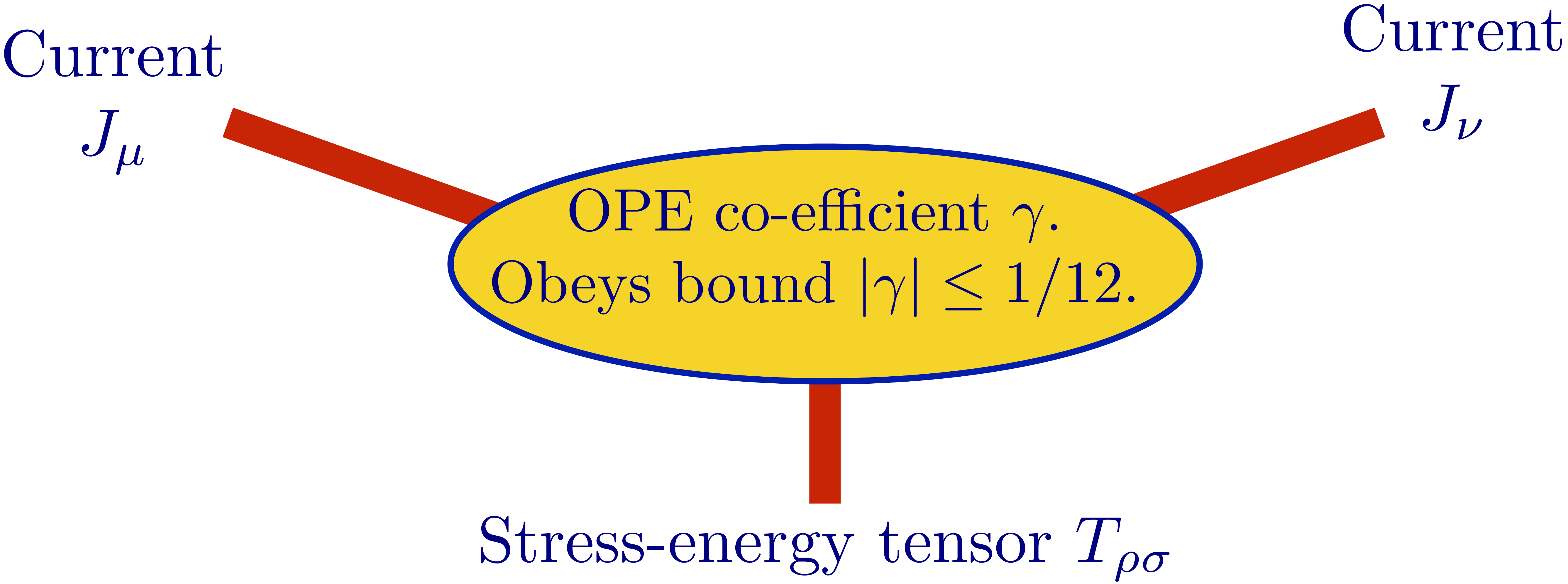


A CFT3 at a temperature $T \sim 1/r_h$ equal to the Hawking temperature of the horizon.

A "horizon", similar to the surface of a black hole !

Dissipation and friction in the CFT3 =
waves falling past the horizon

Physical picture of electrical transport in a CFT3



Current J_μ

Current J_ν

OPE co-efficient γ.
Obeys bound $|\gamma| \leq 1/12$.

Stress-energy tensor $T_{\rho\sigma}$

Conductivity at $T > 0$ determined by
"scattering" of current by
thermal stress-energy tensor.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Physical Review B* **87**, 085138 (2013).

AdS$_4$ theory of quantum criticality

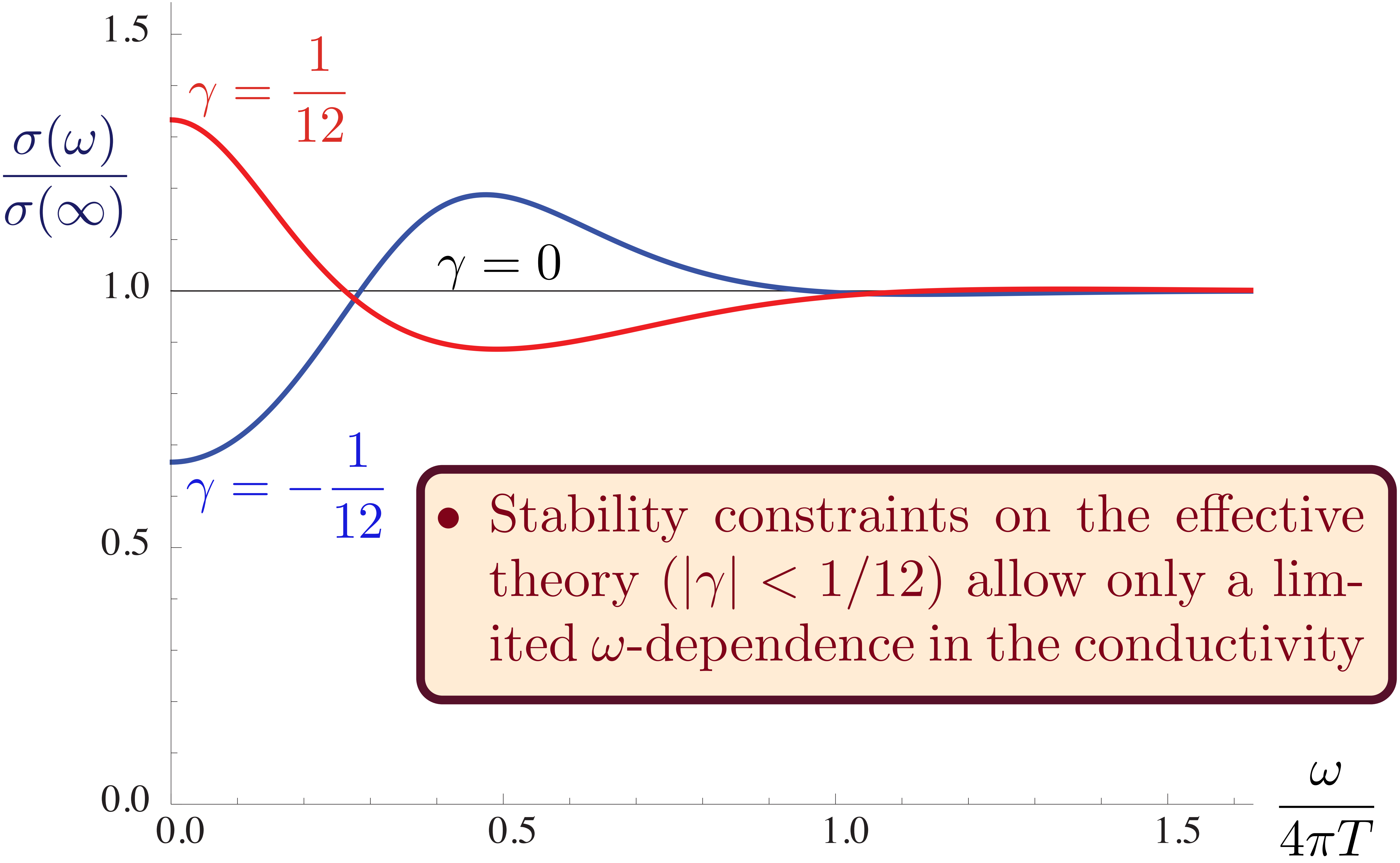
Most general effective holographic theory for linear charge transport with 4 spatial derivatives:

$$\boxed{\begin{aligned}\mathcal{S}_{\text{bulk}} = \frac{1}{g_M^2}\int d^4x\,\sqrt{g}\left[\frac{1}{4}F_{ab}F^{ab} + \gamma L^2 C_{abcd}F^{ab}F^{cd}\right] \\ + \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right],\end{aligned}}$$

This action is characterized by 3 dimensionless parameters, which can be linked to data of the CFT (OPE coefficients): 2-point correlators of the conserved current J_μ and the stress energy tensor $T_{\mu\nu}$, and a 3-point T, J, J correlator. Constraints from both the CFT and the gravitational theory bound $|\gamma| \leq 1/12 = 0.0833..$

R. C. Myers, S. Sachdev, and A. Singh, *Phys. Rev. D* **83**, 066017 (2011)
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Phys. Rev. B* **87**, 085138 (2013)

AdS$_4$ theory of quantum criticality



- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

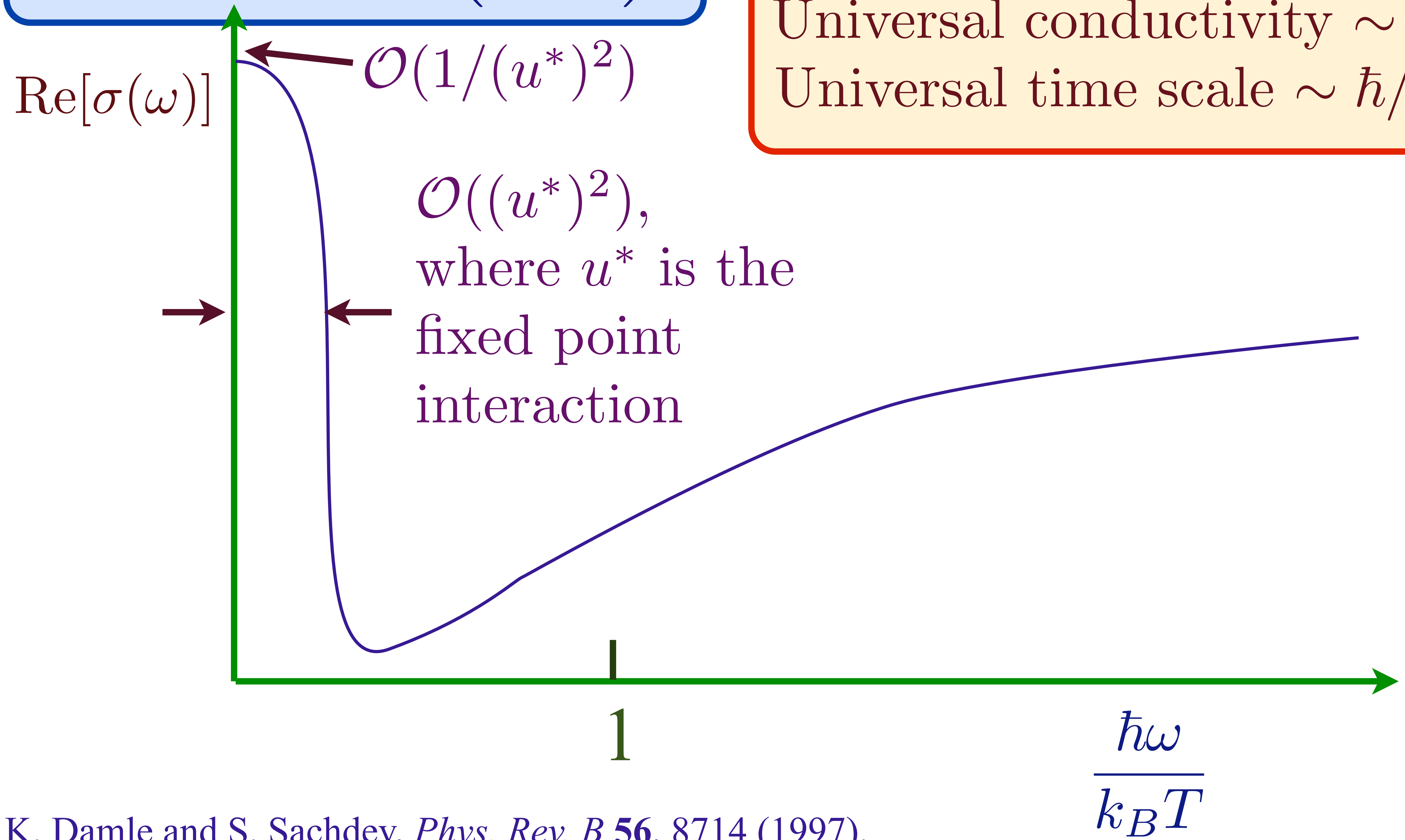
Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3

$$\sigma(\omega, T) = \frac{e^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right) \;;$$

$\Sigma \to$ a universal function

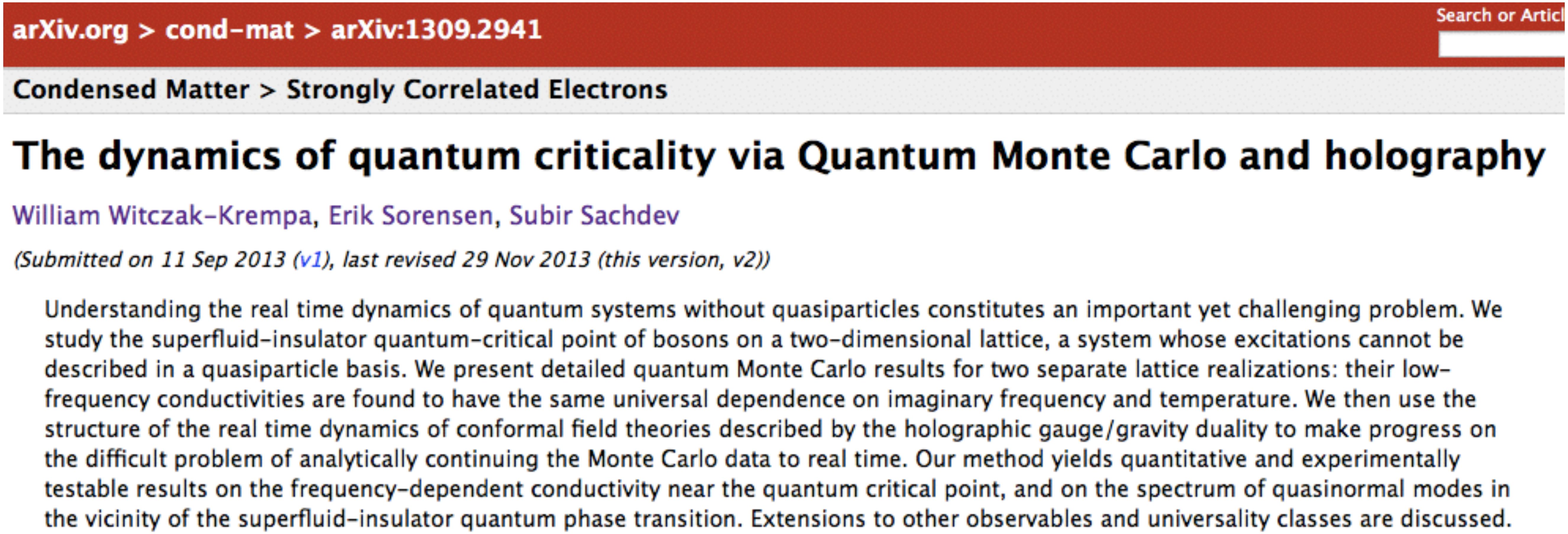
Universal conductivity $\sim e^2/h$

Universal time scale $\sim \hbar/k_B T$



$\mathcal{O}((u^*)^2)$, where u^* is the fixed point interaction

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).



arXiv.org > cond-mat > arXiv:1309.2941

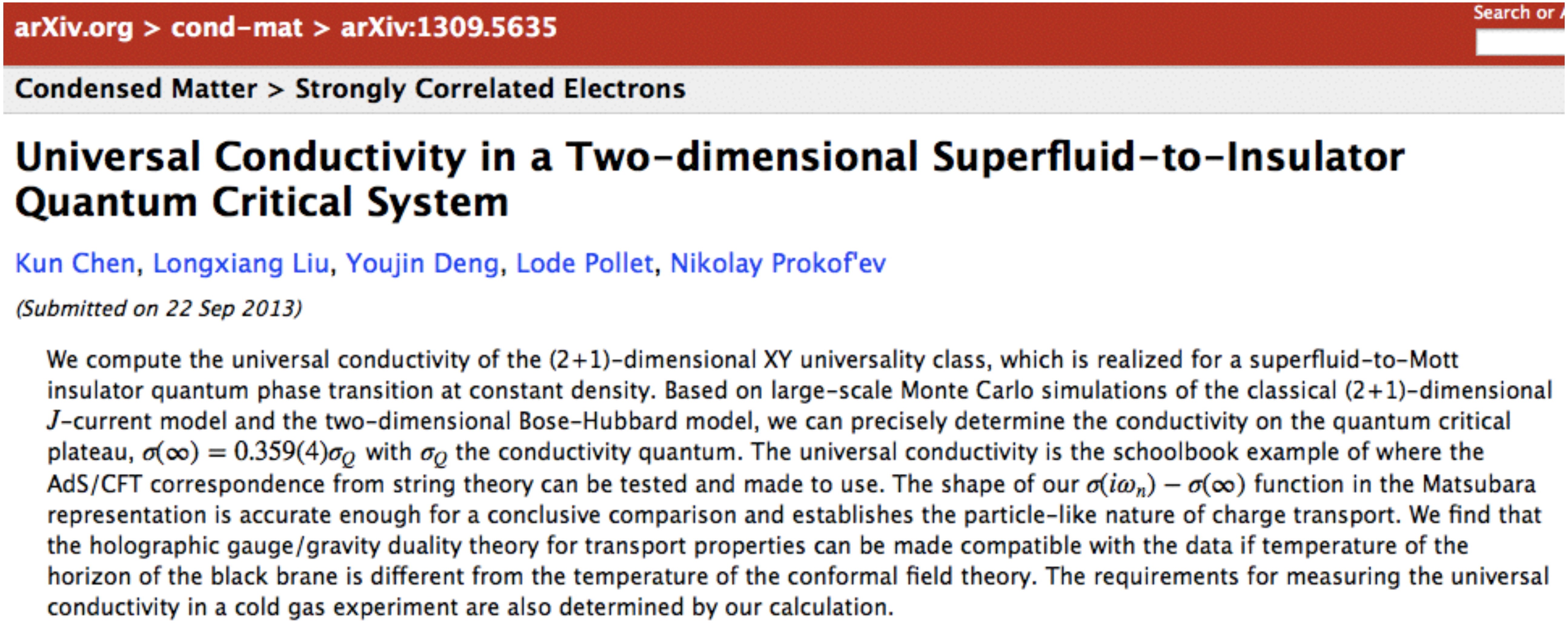
Condensed Matter > Strongly Correlated Electrons

The dynamics of quantum criticality via Quantum Monte Carlo and holography

William Witczak-Krempa, Erik Sorensen, Subir Sachdev

(Submitted on 11 Sep 2013 (v1), last revised 29 Nov 2013 (this version, v2))

Understanding the real time dynamics of quantum systems without quasiparticles constitutes an important yet challenging problem. We study the superfluid-insulator quantum-critical point of bosons on a two-dimensional lattice, a system whose excitations cannot be described in a quasiparticle basis. We present detailed quantum Monte Carlo results for two separate lattice realizations: their low-frequency conductivities are found to have the same universal dependence on imaginary frequency and temperature. We then use the structure of the real time dynamics of conformal field theories described by the holographic gauge/gravity duality to make progress on the difficult problem of analytically continuing the Monte Carlo data to real time. Our method yields quantitative and experimentally testable results on the frequency-dependent conductivity near the quantum critical point, and on the spectrum of quasinormal modes in the vicinity of the superfluid-insulator quantum phase transition. Extensions to other observables and universality classes are discussed.



arXiv.org > cond-mat > arXiv:1309.5635

Condensed Matter > Strongly Correlated Electrons

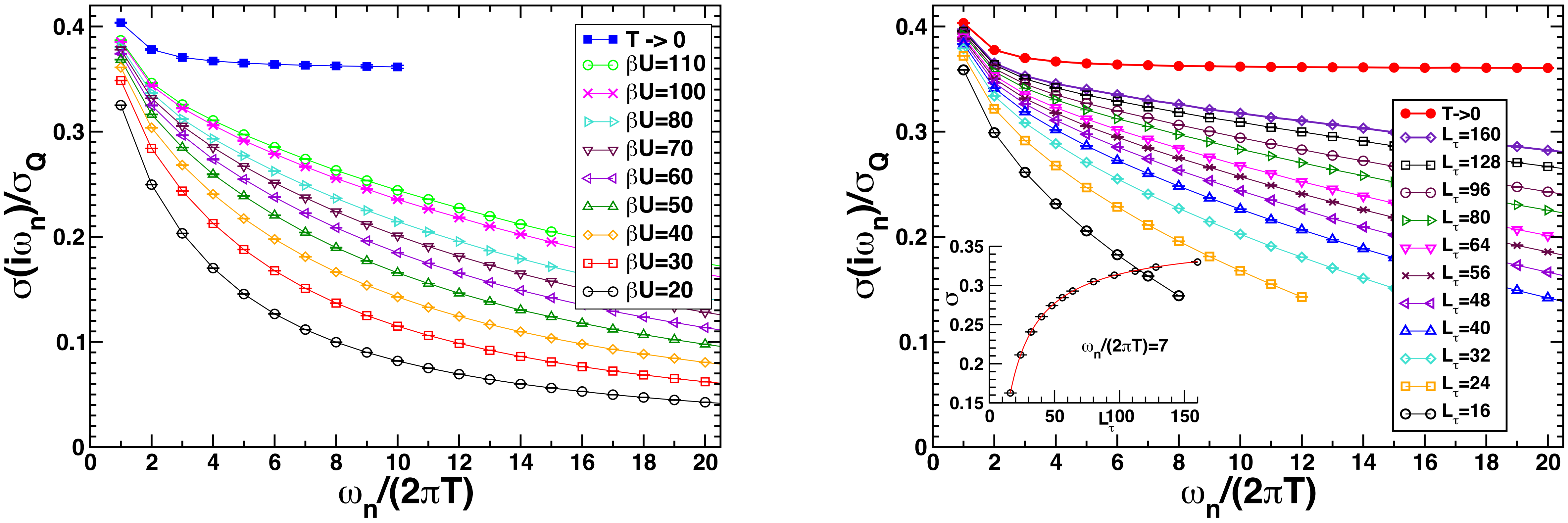
Universal Conductivity in a Two-dimensional Superfluid-to-Insulator Quantum Critical System

Kun Chen, Longxiang Liu, Youjin Deng, Lode Pollet, Nikolay Prokof'ev

(Submitted on 22 Sep 2013)

We compute the universal conductivity of the (2+1)-dimensional XY universality class, which is realized for a superfluid-to-Mott insulator quantum phase transition at constant density. Based on large-scale Monte Carlo simulations of the classical (2+1)-dimensional J-current model and the two-dimensional Bose-Hubbard model, we can precisely determine the conductivity on the quantum critical plateau, $\sigma(\infty) = 0.359(4)\sigma_Q$ with σ_Q the conductivity quantum. The universal conductivity is the schoolbook example of where the AdS/CFT correspondence from string theory can be tested and made to use. The shape of our $\sigma(i\omega_n) - \sigma(\infty)$ function in the Matsubara representation is accurate enough for a conclusive comparison and establishes the particle-like nature of charge transport. We find that the holographic gauge/gravity duality theory for transport properties can be made compatible with the data if temperature of the horizon of the black brane is different from the temperature of the conformal field theory. The requirements for measuring the universal conductivity in a cold gas experiment are also determined by our calculation.

Quantum Monte Carlo for lattice bosons



(a)

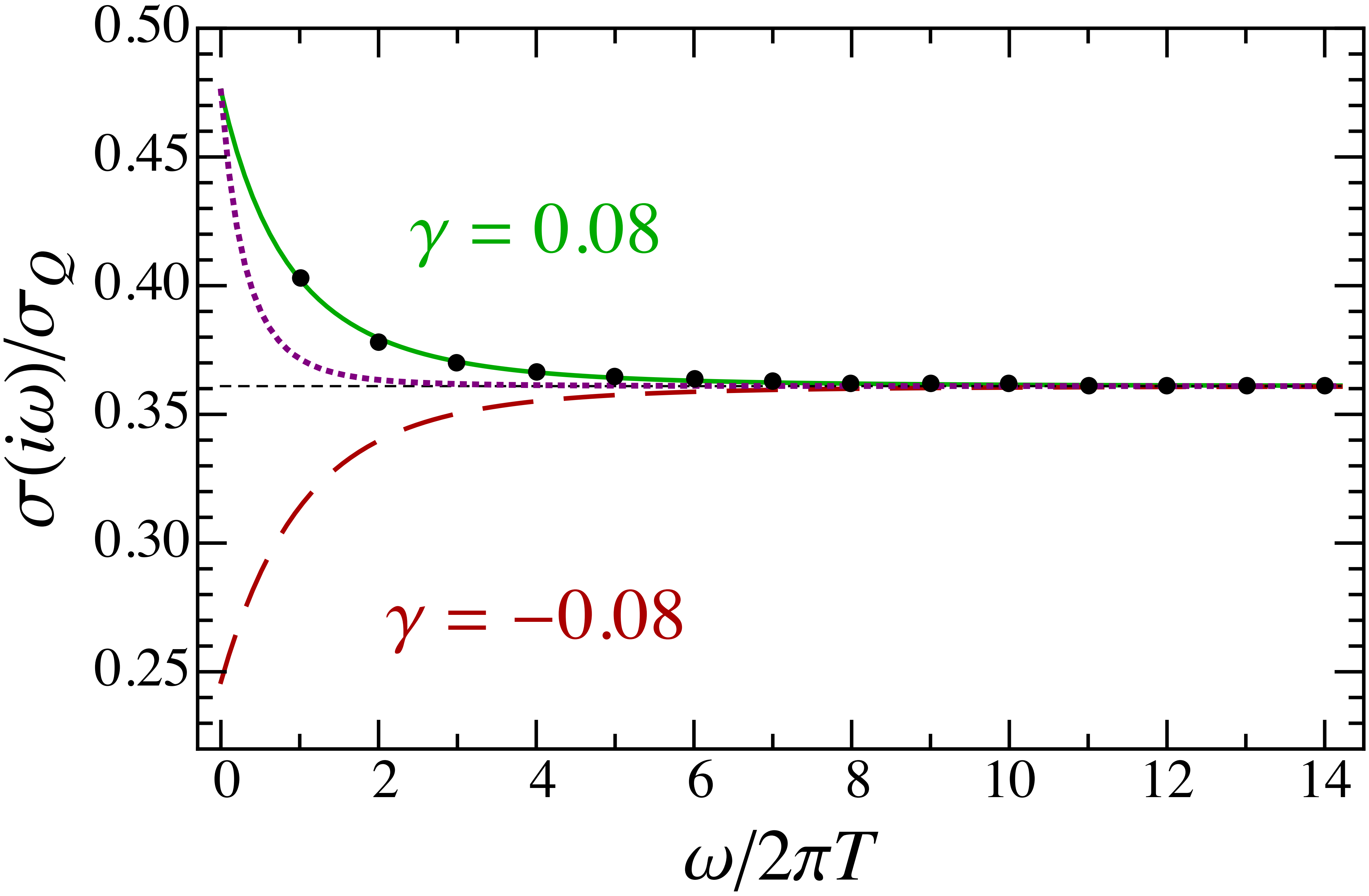
(b)

FIG. 2. **Quantum Monte Carlo data** (a) Finite-temperature conductivity for a range of βU in the $L \to \infty$ limit for the quantum rotor model at $(t/U)_c$. The solid blue squares indicate the final $T \to 0$ extrapolated data. (b) Finite-temperature conductivity in the $L \to \infty$ limit for a range of L_τ for the Villain model at the QCP. The solid red circles indicate the final $T \to 0$ extrapolated data. The inset illustrates the extrapolation to $T = 0$ for $\omega_n/(2\pi T) = 7$. The error bars are statistical for both a) and b).

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941

See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

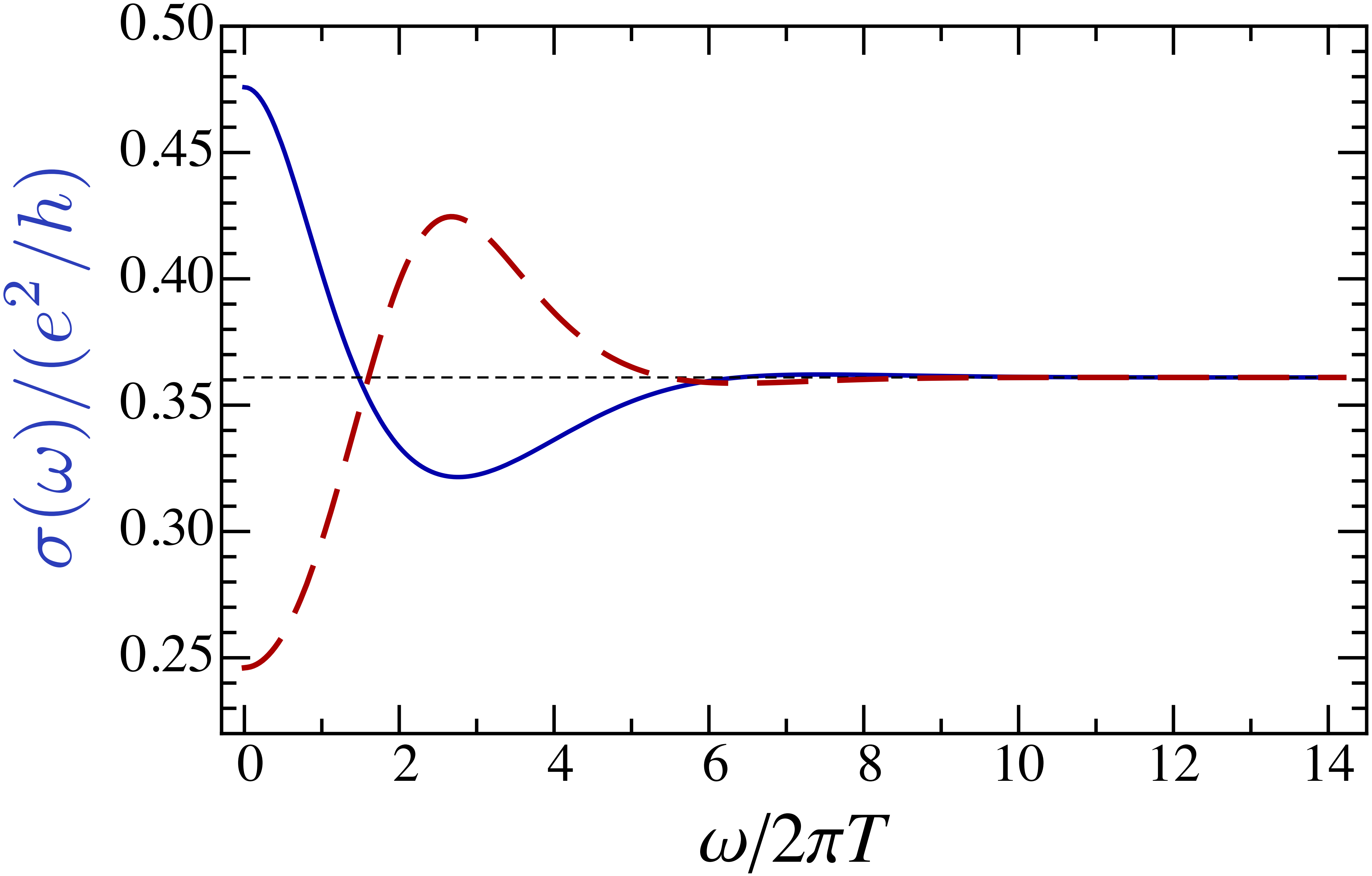
AdS$_4$ theory of quantum criticality



Good agreement between high precision Monte Carlo for imaginary frequencies, and holographic theory after rescaling effective T and taking $\sigma_Q = 1/g_M^2$.

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941
See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

AdS$_4$ theory of quantum criticality



Predictions of holographic theory,
after analytic continuation to real frequencies

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941
See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

Outline

1. The simplest model without quasiparticles

   Superfluid-insulator transition

   of ultracold bosonic atoms in an optical lattice

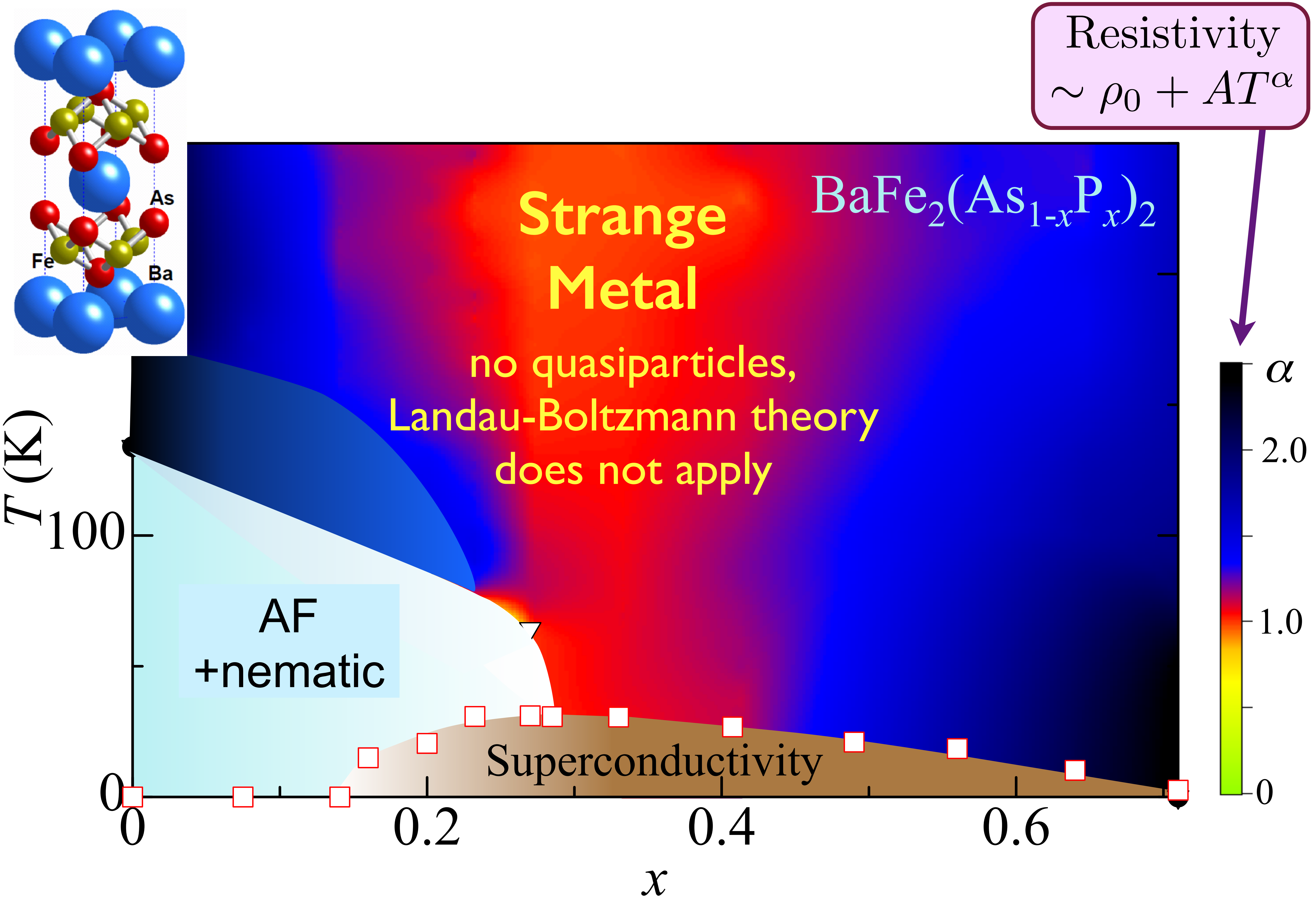
   (Conformal field theories in 2+1 dimensions)

2. Strange metals in the high T_c superconductors

   Non-quasiparticle transport at the

   Ising-nematic quantum critical point

Resistivity
$\sim \rho_0 + AT^{\alpha}$



Strange Metal

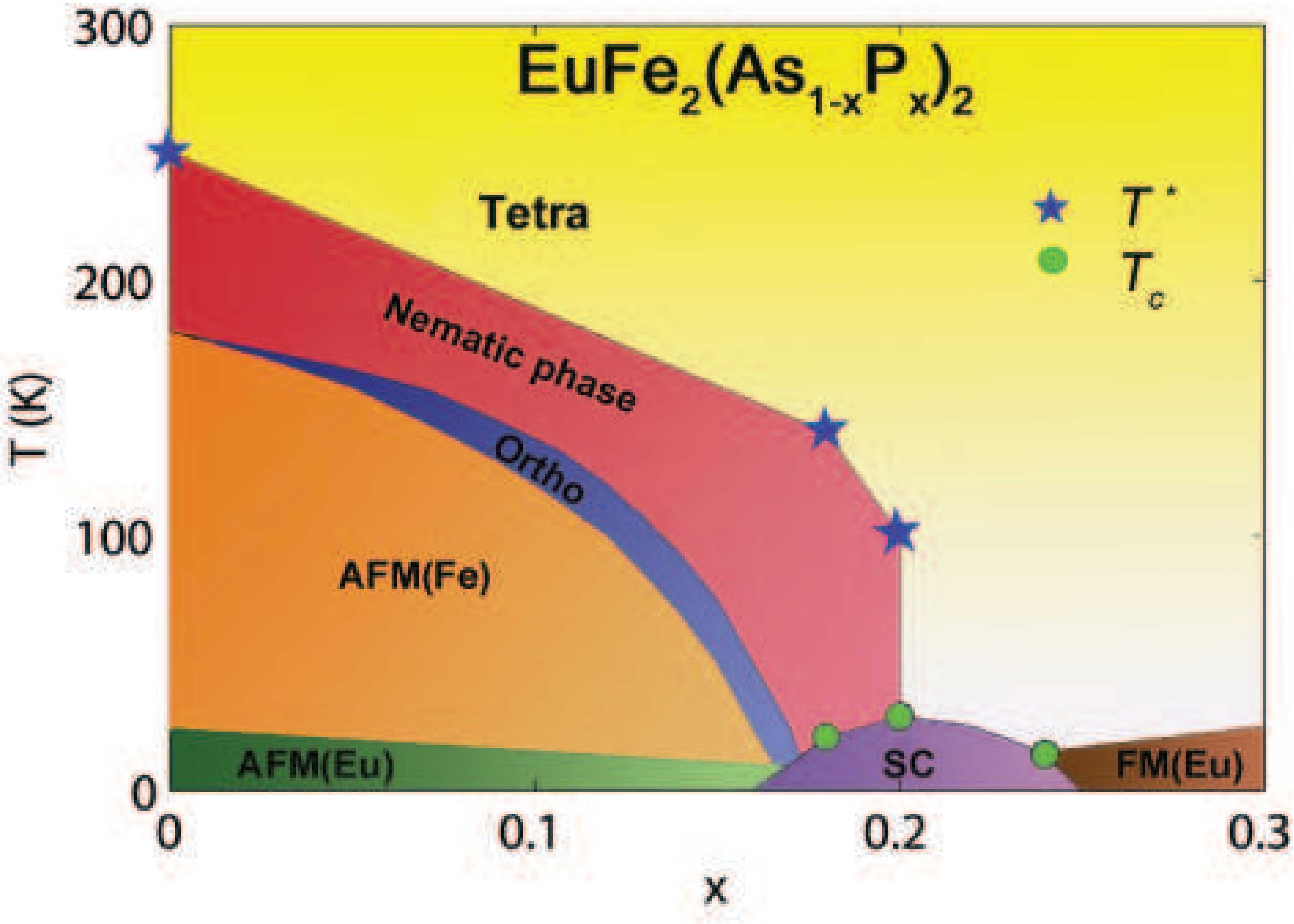
no quasiparticles,
Landau-Boltzmann theory
does not apply

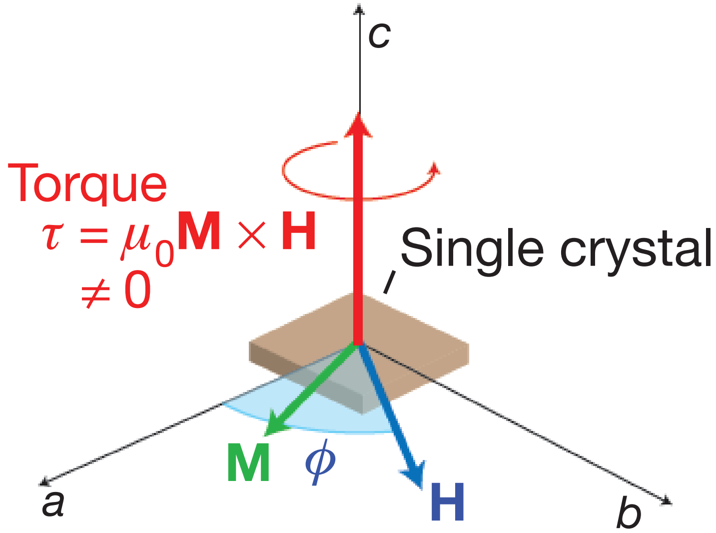
AF
+nematic

Superconductivity

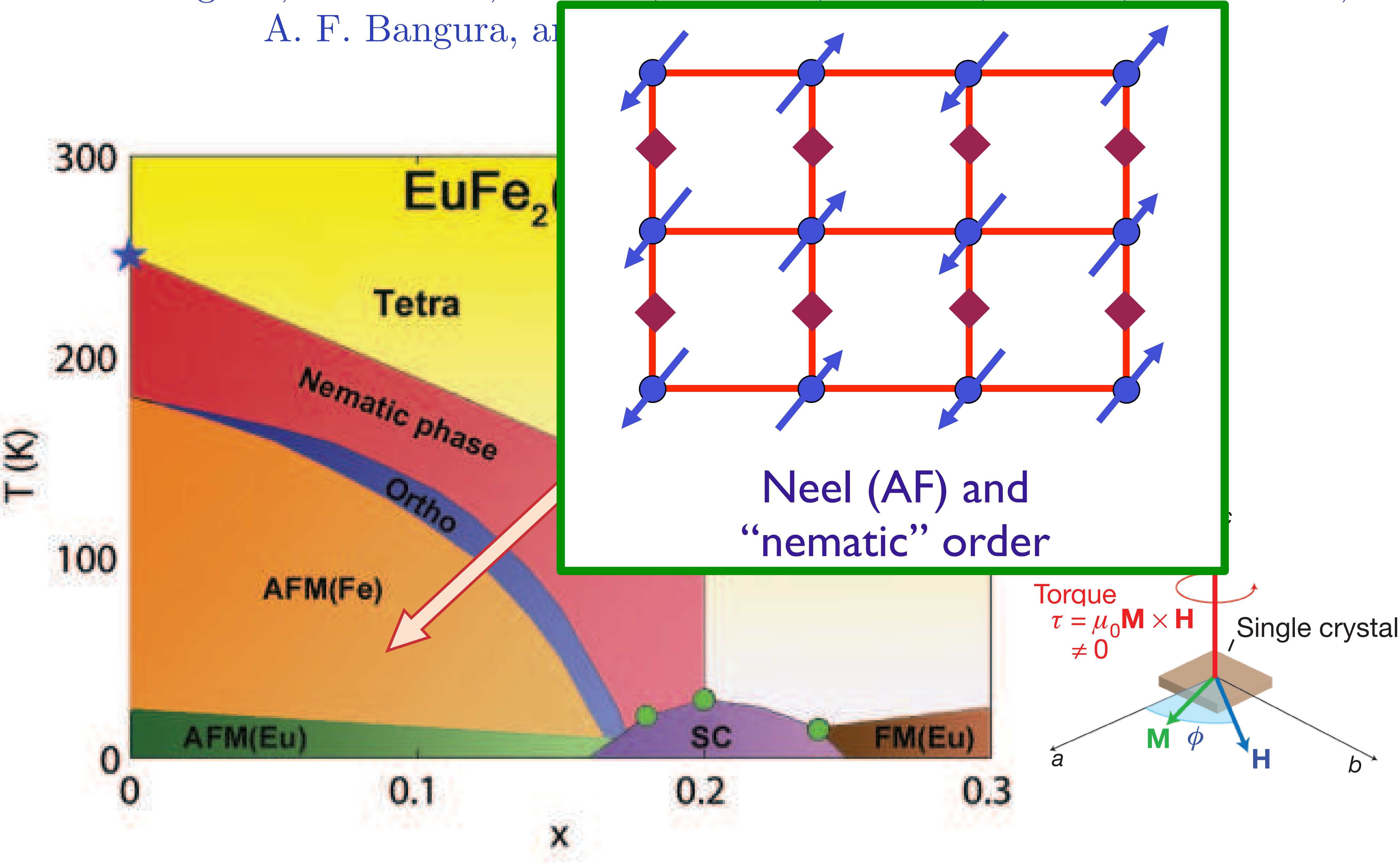
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido,
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,
Physical Review B **81**, 184519 (2010)

Xiaofeng Xu, W. H. Jiao, N. Zhou, Y. K. Li, B. Chen, C. Cao, Jianhui Dai, A. F. Bangura, and Guanghan Cao, arXiv:1402.4124



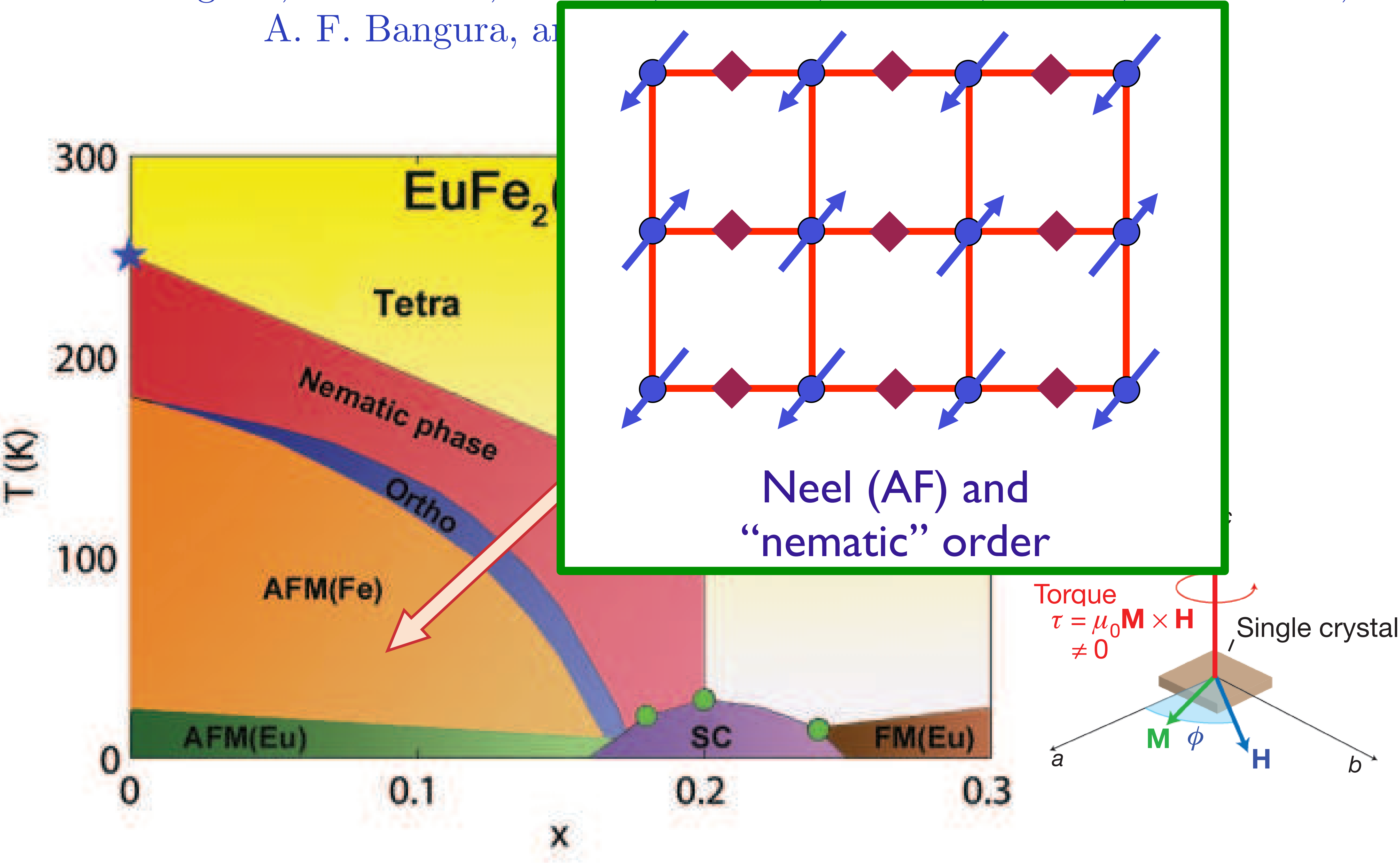


Xiaofeng Xu, W. H. Jiao, N. Zhou, Y. K. Li, B. Chen, C. Cao, Jianhui Dai,
A. F. Bangura, ar



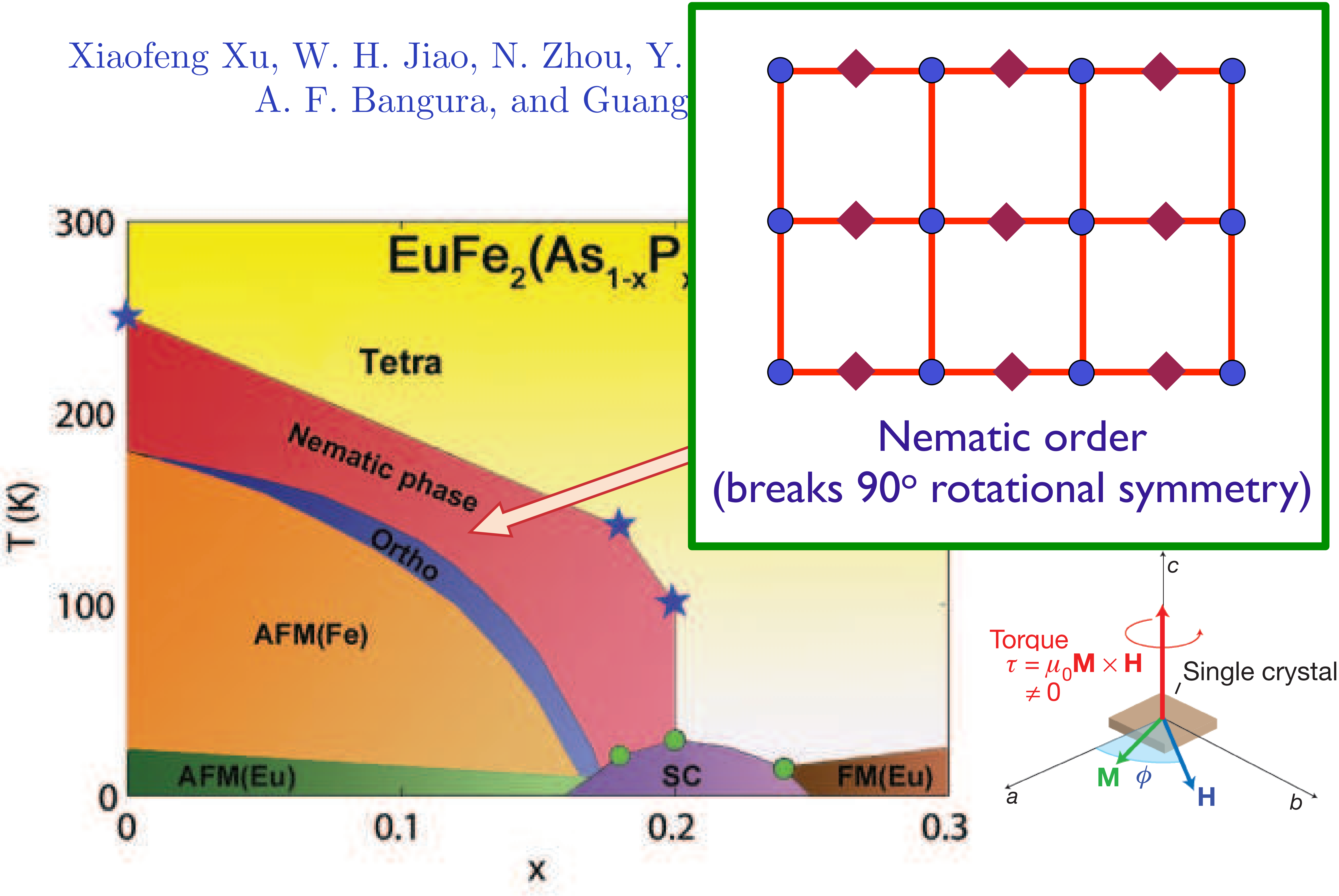
Neel (AF) and
“nematic” order

Xiaofeng Xu, W. H. Jiao, N. Zhou, Y. K. Li, B. Chen, C. Cao, Jianhui Dai,
A. F. Bangura, a



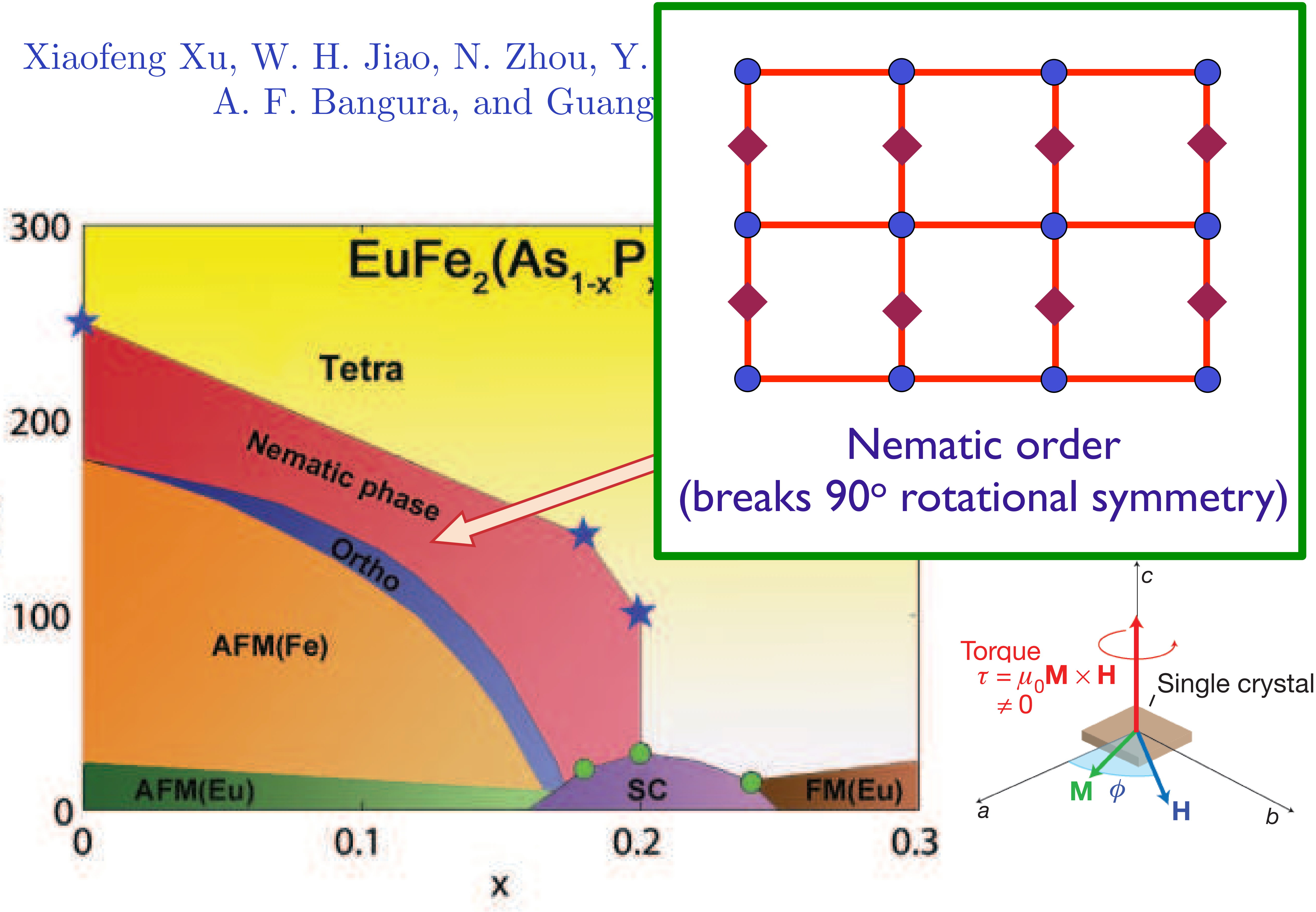
Neel (AF) and
“nematic” order

Xiaofeng Xu, W. H. Jiao, N. Zhou, Y.
A. F. Bangura, and Guang



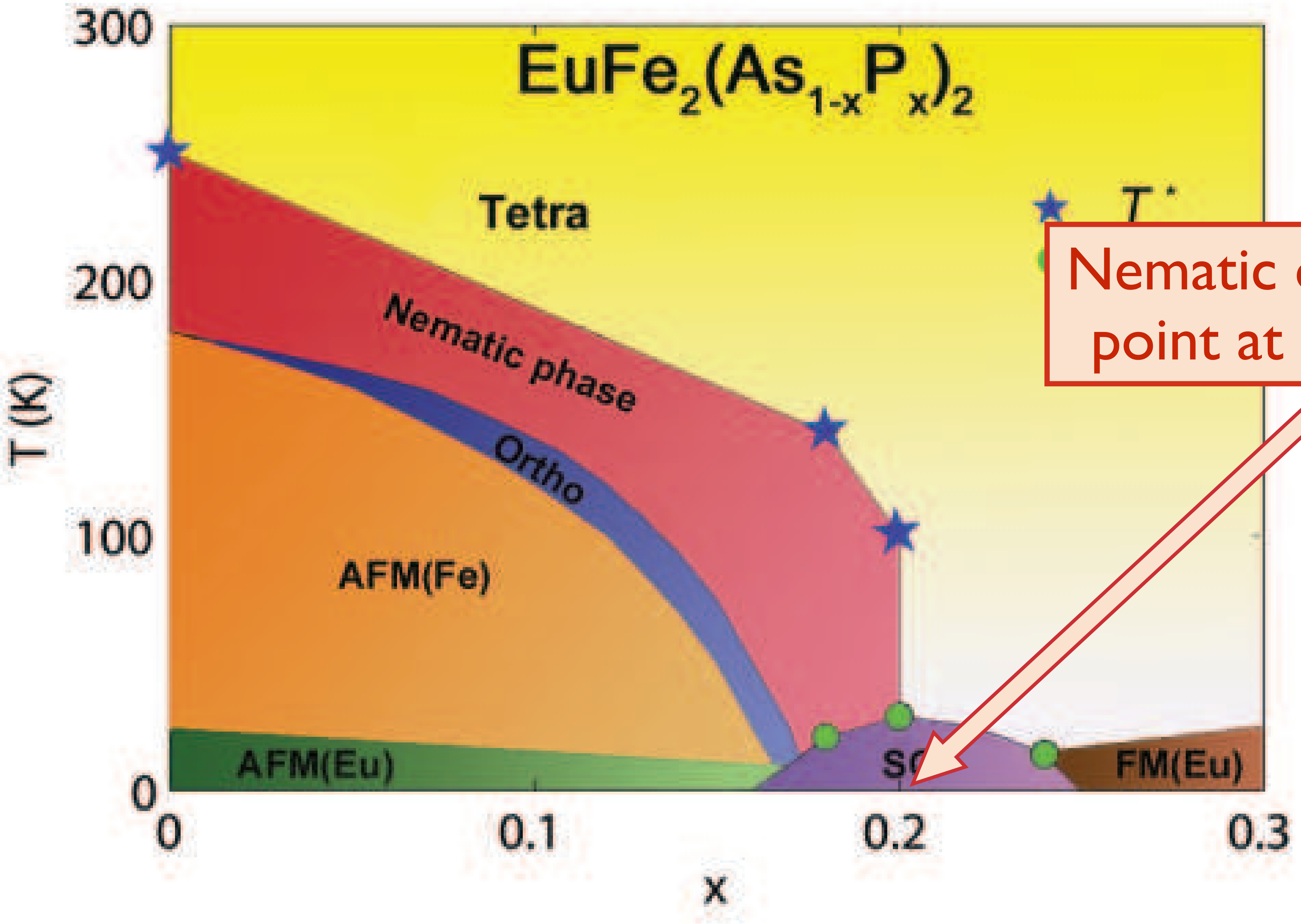
Nematic order
(breaks 90° rotational symmetry)

Xiaofeng Xu, W. H. Jiao, N. Zhou, Y.
A. F. Bangura, and Guang

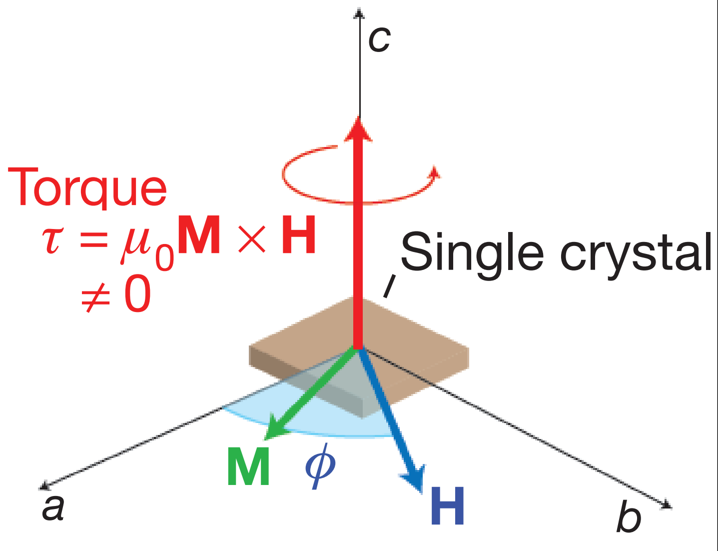


Nematic order
(breaks 90° rotational symmetry)

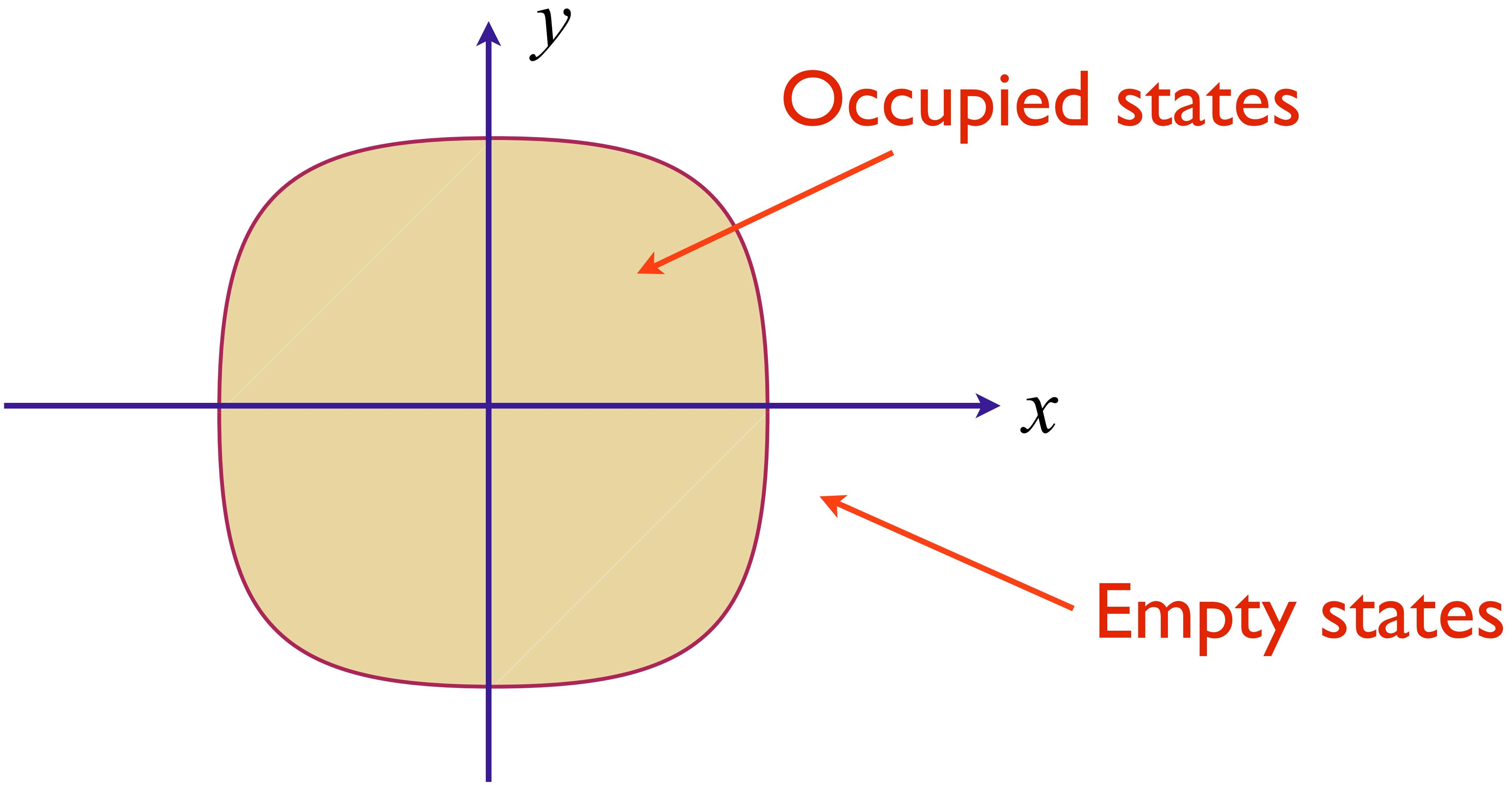
Xiaofeng Xu, W. H. Jiao, N. Zhou, Y. K. Li, B. Chen, C. Cao, Jianhui Dai, A. F. Bangura, and Guanghan Cao, arXiv:1402.4124



Nematic quantum critical point at optimal doping

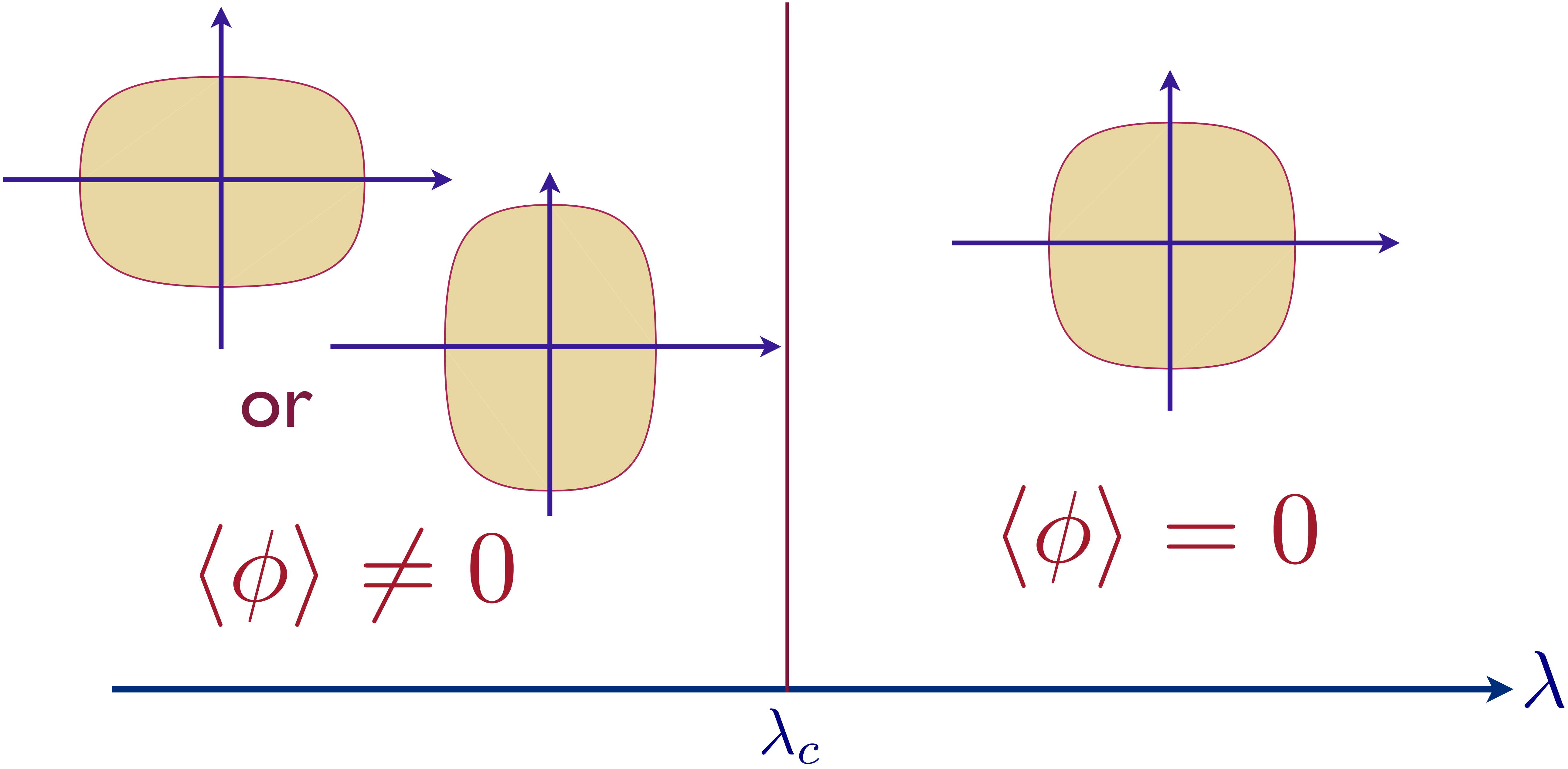


Quantum criticality of Ising-nematic ordering in a metal



A metal with a Fermi surface
with full square lattice symmetry

Quantum criticality of Ising-nematic ordering in a metal



or

$\langle \phi \rangle \neq 0$

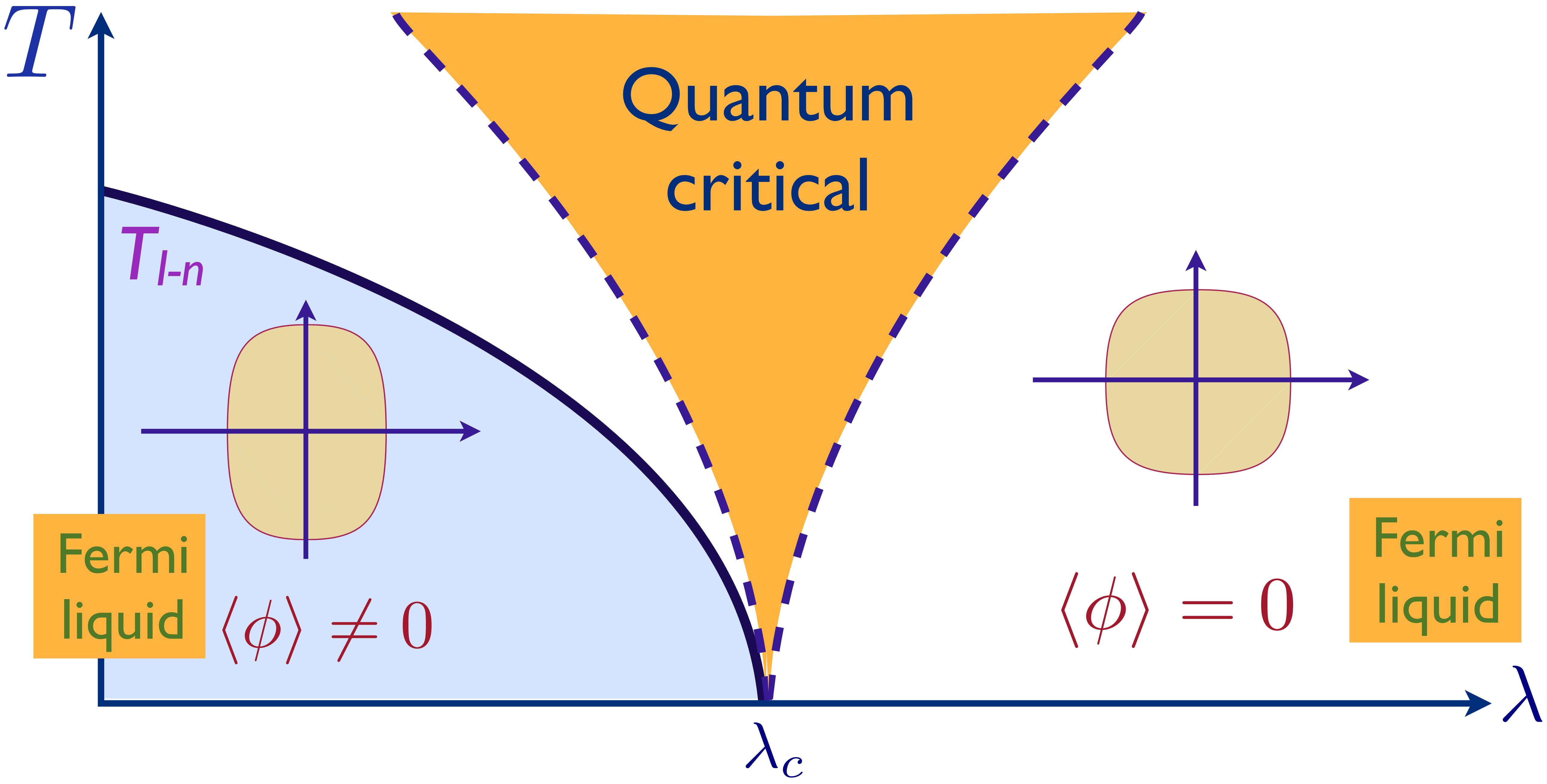
$\langle \phi \rangle = 0$

λ_c

λ

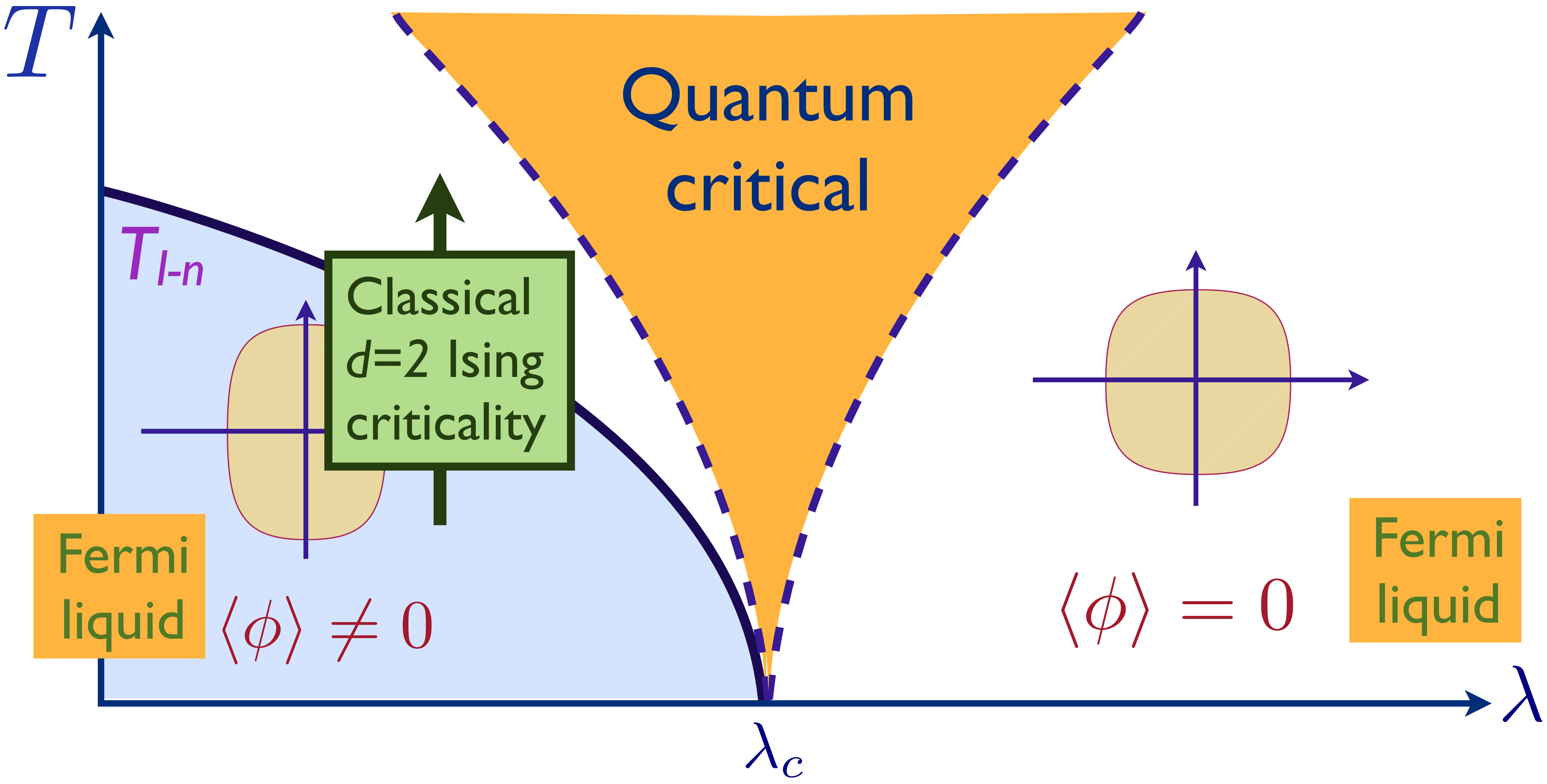
Pomeranchuk instability as a function of coupling λ

Quantum criticality of Ising-nematic ordering in a metal



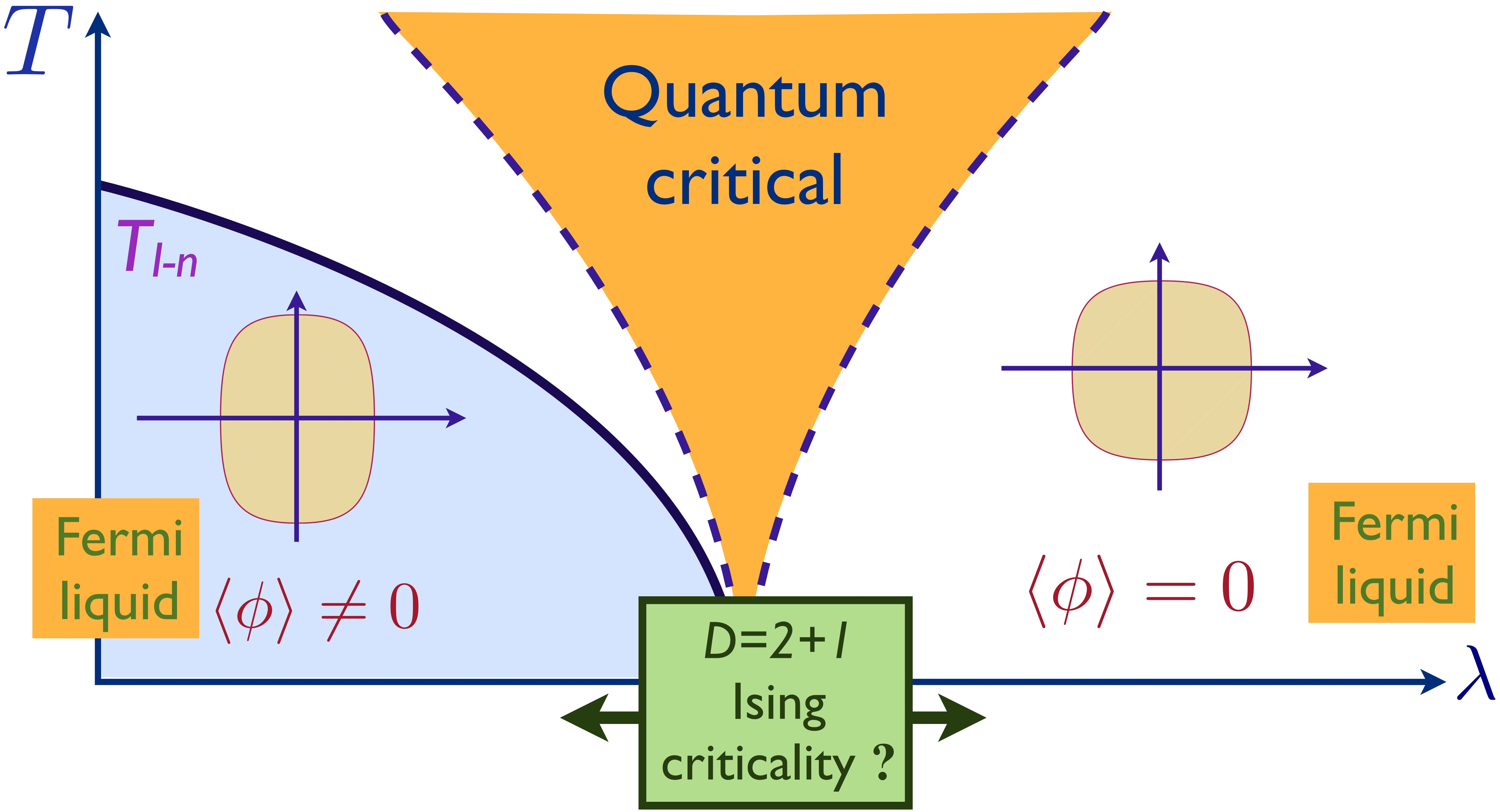
Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



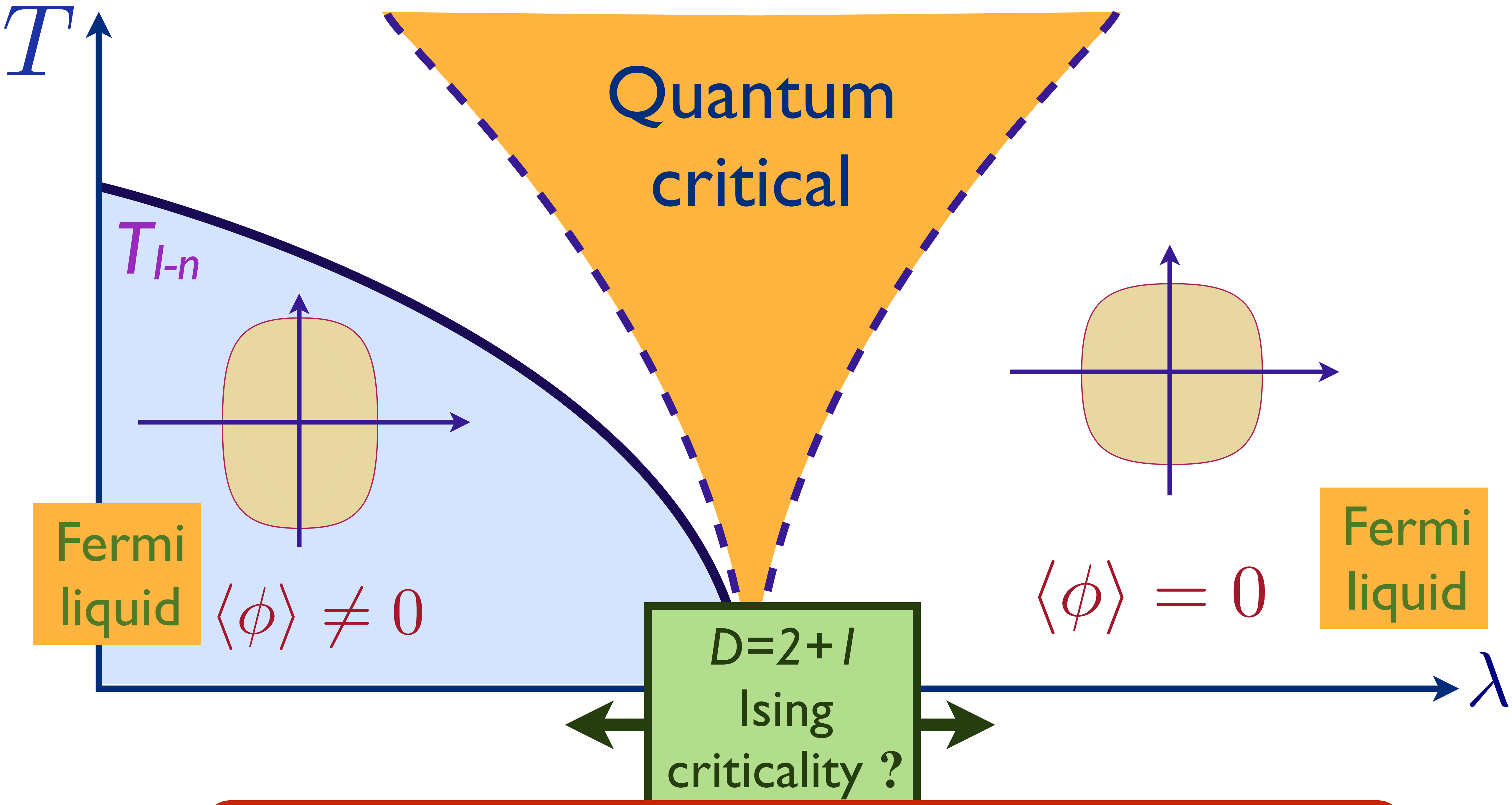
Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



Only at higher energies; at the lowest energy bosonic ϕ fluctuations are strongly coupled to fermionic excitations near Fermi surface.

Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal

The "standard model":

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c^\dagger_{\mathbf{k}\alpha} \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y \right) c^\dagger_{\mathbf{k}+\mathbf{q}/2,\alpha} c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

Quantum criticality of Ising-nematic ordering in a metal

The "standard model":

$$\mathcal{S}_\phi = \int d^2 r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$\mathcal{S}_{\phi c} = - g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k \right.$$

Field theory of bosonic order parameter

Quantum criticality of Ising-nematic ordering in a metal

The "standard model":

$$\mathcal{S}_\phi = \int d^2 r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

Electrons with a Fermi surface: $\varepsilon_{\mathbf{k}} = -2t(\cos k_x + \cos k_y) - \mu \ldots$

$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y \right) c_{\mathbf{k}+\mathbf{q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

Quantum criticality of Ising-nematic ordering in a metal

The "standard model":

$$\mathcal{S}_\phi = \int d^2 r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda -\right.$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y \right) c_{\mathbf{k}+\mathbf{q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

"Yukawa" coupling between bosons and fermions

Quantum criticality of Ising-nematic ordering in a metal

Boltzmann view of electrical transport:

- Identify charge carriers: electrons near the Fermi surface. Compute the scattering rate of these charged excitations off the bosonic ϕ fluctuations.

Quantum criticality of Ising-nematic ordering in a metal

Boltzmann view of electrical transport:

- Identify charge carriers: electrons near the Fermi surface. Compute the scattering rate of these charged excitations off the bosonic ϕ fluctuations.

- Analogous to electron-phonon scattering in metals, where we have "Bloch's law": a resistivity $\rho(T) \sim T^5$.

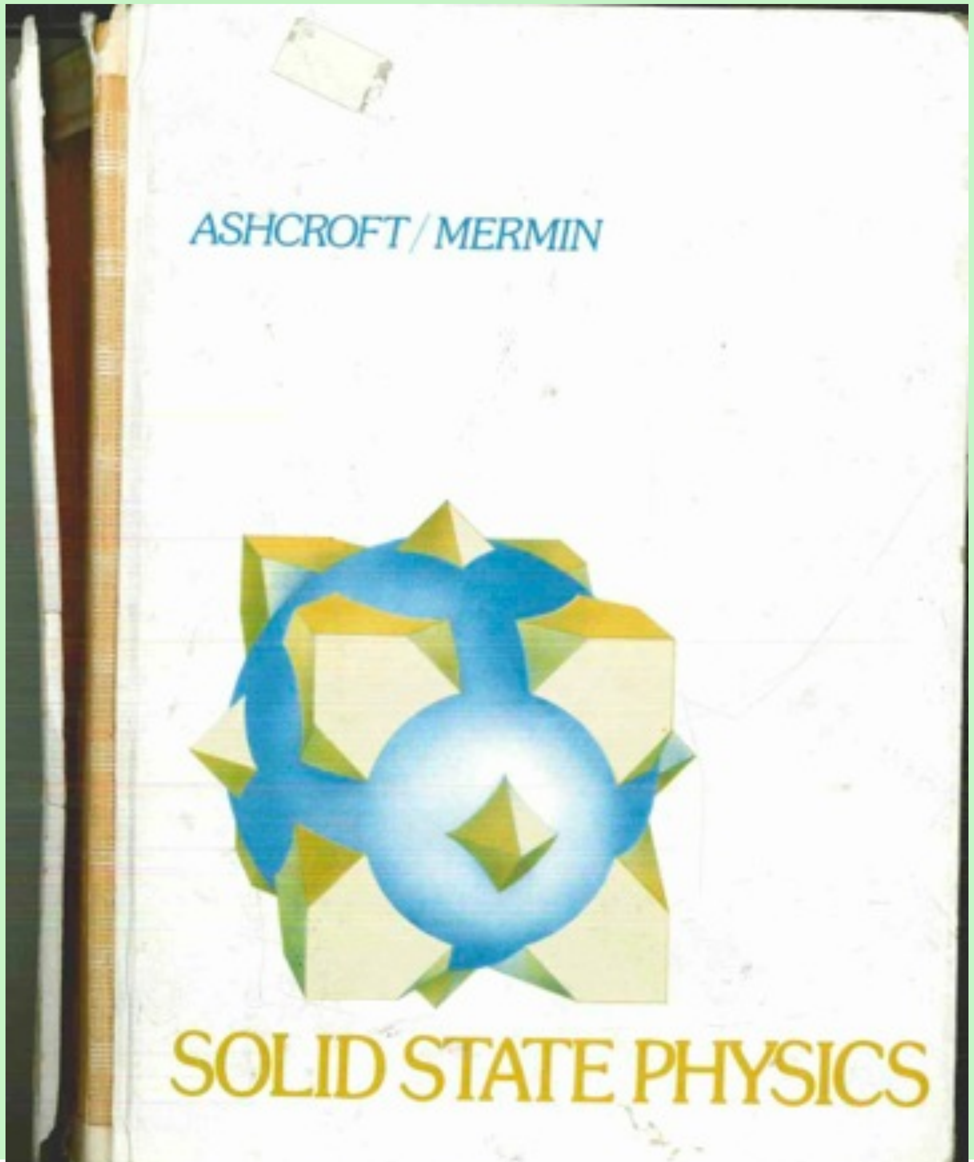
- "Bloch's law" for the Ising-nematic critical point yields $\rho(T) \sim T^{4/3}$.

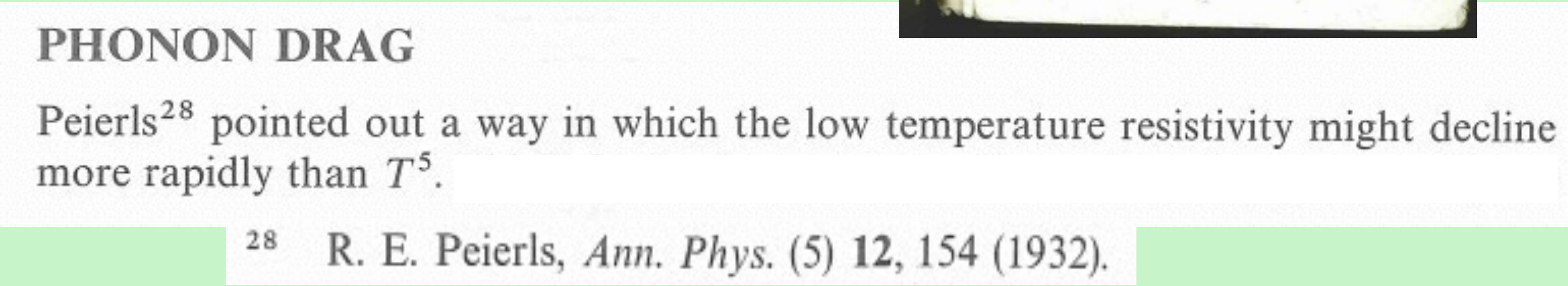
Quantum criticality of Ising-nematic ordering in a metal

Boltzmann view of electrical transport:

- Identify charge carriers: electrons
  pute the scattering rate of these ch
  ϕ fluctuations.
- Analogous to electron-phonon scat
  "Bloch's law": a resistivity $\rho(T) \sim$
- "Bloch's law" for the Ising-nemati
  $\rho(T) \sim T^{4/3}$.

However, this ignores "phonon drag"





PHONON DRAG

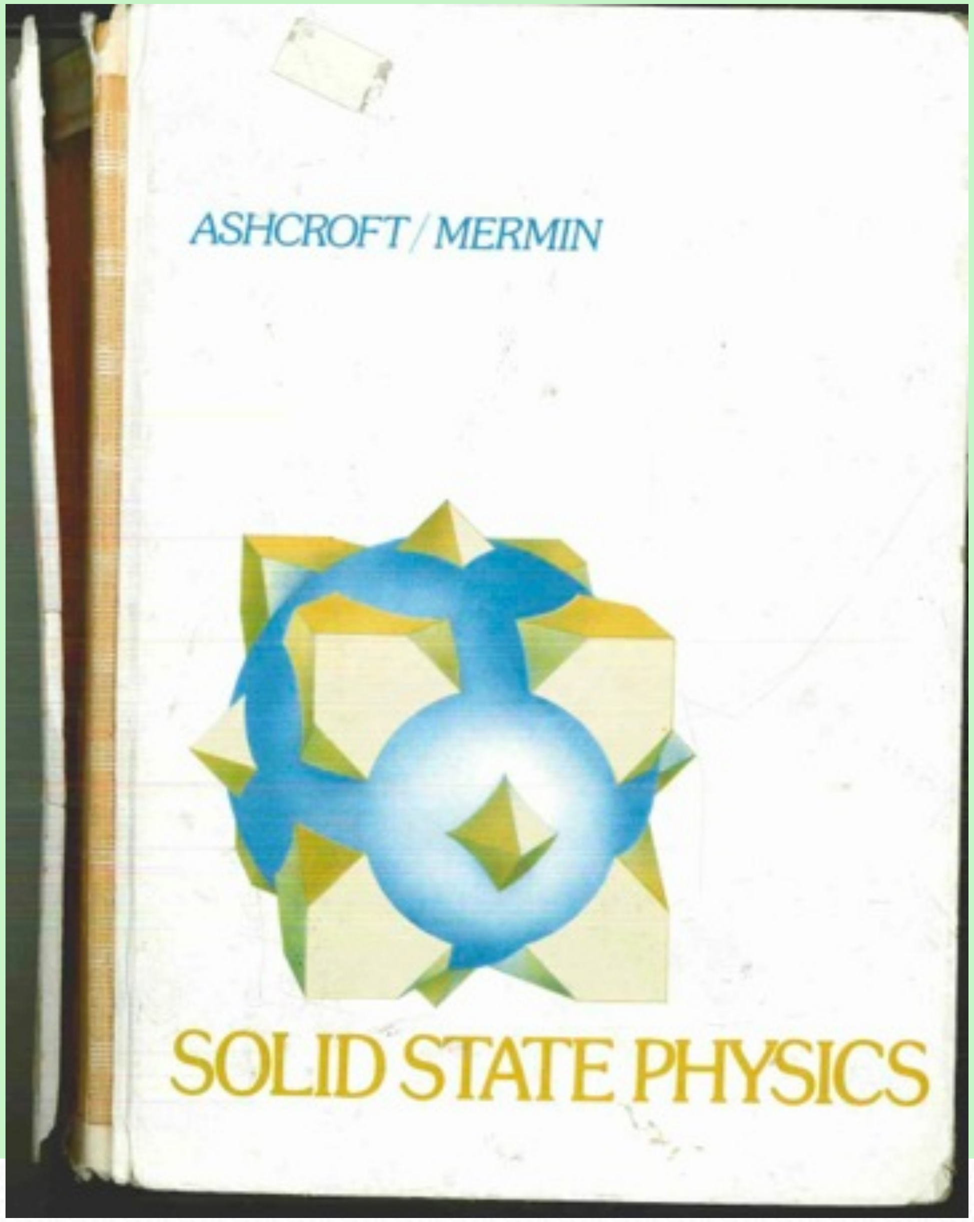
Peierls[28] pointed out a way in which the low temperature resistivity might decline more rapidly than T^5.

[28] R. E. Peierls, *Ann. Phys.* (5) **12**, 154 (1932).

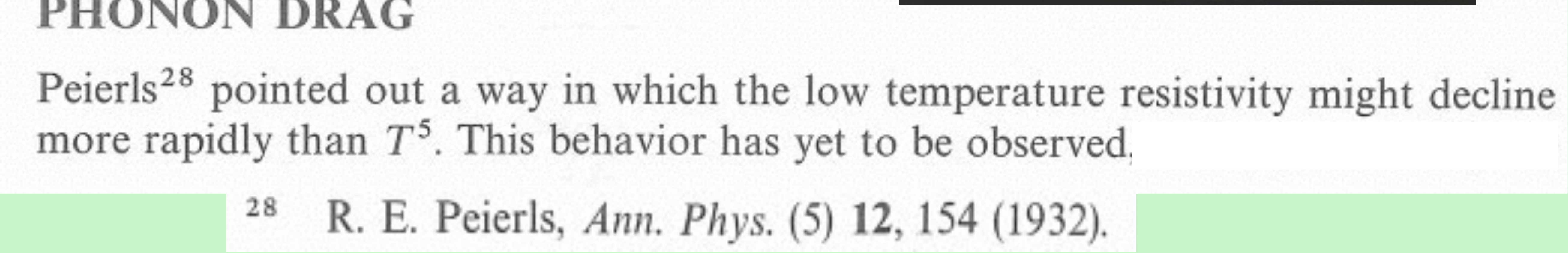
Quantum criticality of Ising-nematic ordering in a metal

Boltzmann view of electrical transport:

- Identify charge carriers: electrons pute the scattering rate of these ch ϕ fluctuations.
- Analogous to electron-phonon scat "Bloch's law": a resistivity $\rho(T) \sim$
- "Bloch's law" for the Ising-nemati $\rho(T) \sim T^{4/3}$.



However, this ignores "phonon drag"



PHONON DRAG

Peierls[28] pointed out a way in which the low temperature resistivity might decline more rapidly than T^5. This behavior has yet to be observed.

[28] R. E. Peierls, *Ann. Phys.* (5) **12**, 154 (1932).

Quantum criticality of Ising-nematic ordering in a metal

The "standard model":

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y \right) c_{\mathbf{k}+\mathbf{q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

Quantum criticality of Ising-nematic ordering in a metal

The "standard model":

$$\mathcal{S}_\phi = \int d^2 r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \int d^2 r d\tau c_\alpha^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} + \frac{\nabla^4}{2m'} + \ldots - \mu \right) c_\alpha$$

$$\mathcal{S}_{\phi c} = -g \int d^2 r d\tau \sum_{\alpha=1}^{N_f} \phi \Big[c_\alpha^\dagger \left\{ (\partial_x^2 - \partial_y^2 + \ldots) c_\alpha \right\} + \left\{ (\partial_x^2 - \partial_y^2 + \ldots) c_\alpha^\dagger \right\} c_\alpha \Big]$$

This continuum theory has has strong electron$-\phi$ scattering, and no quasi-particle excitations. But it has a conserved momentum $\mathbf{P}$, and $\chi_{\mathbf{J},\mathbf{P}} \neq 0$ ("phonon drag"), and so the resistivity $\rho(T) = 0$.

Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:

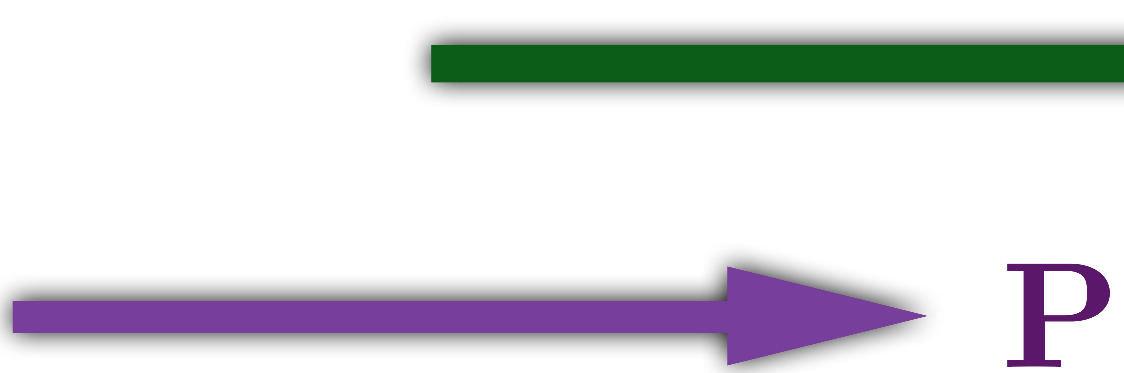
- Focus on the interplay between J_μ and $T_{\mu\nu}$!

The most-probable state with a non-zero current **J**
has a non-zero momentum **P** (and vice versa).
At non-zero density, **J** “drags” **P**.

Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:

- Focus on the interplay between J_μ and $T_{\mu\nu}$!



The most-probable state with a non-zero current **J**
has a non-zero momentum **P** (and vice versa).
At non-zero density, **J** "drags" **P**.

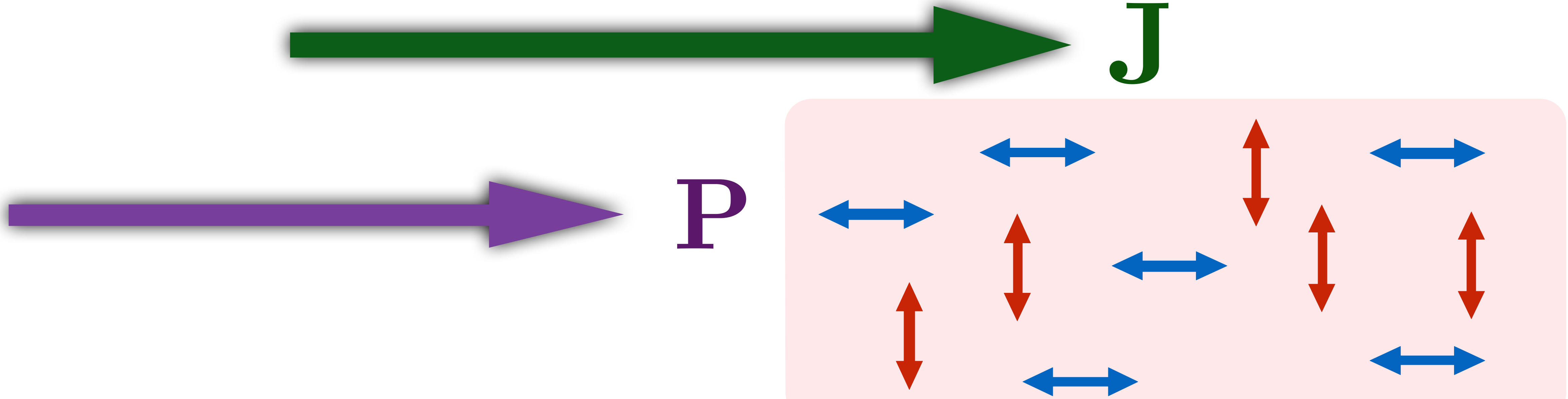
The resistivity of this metal is *not* determined by the scattering rate of charged excitations near the Fermi surface, but by the dominant rate of momentum loss by *any* excitation, whether neutral or charged, or fermionic or bosonic

Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:

S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, `arXiv:1401.7012`.
A. Lucas, S. Sachdev, and K. Schalm, `arXiv:1401.7933`

- Focus on the interplay between J_μ and $T_{\mu\nu}$!



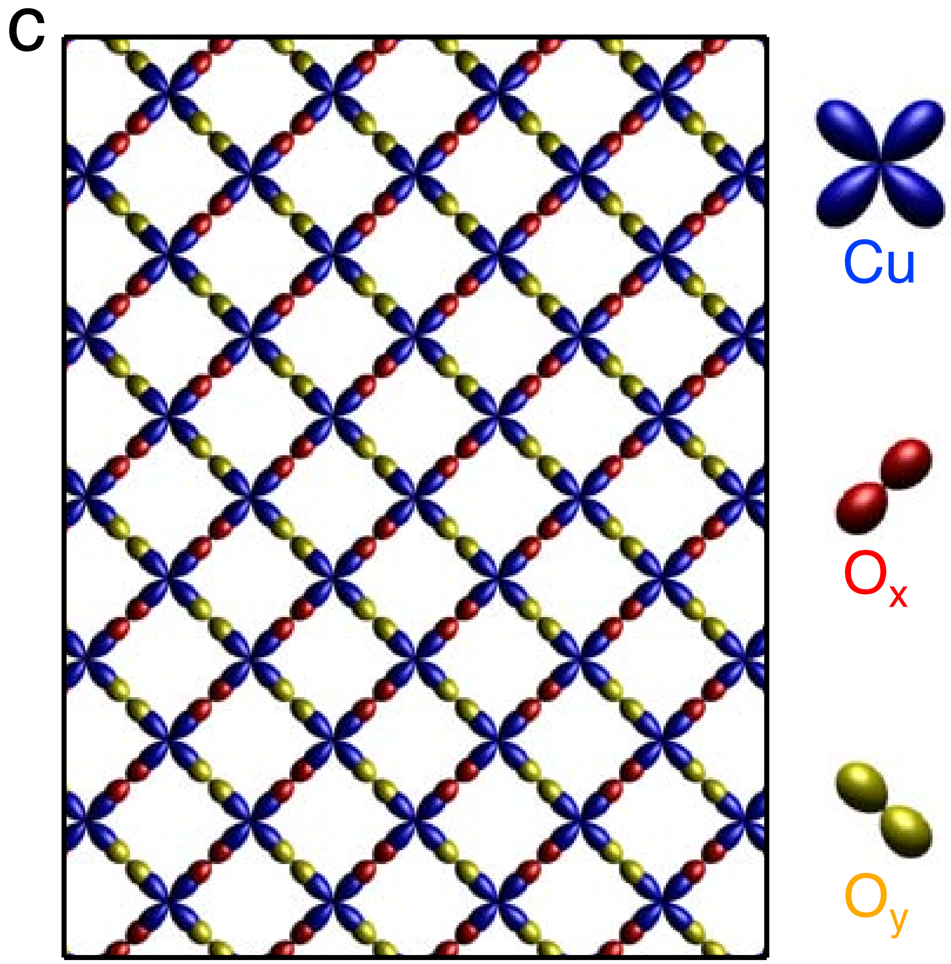
The momentum loss via scattering of charged fermionic excitations near the Fermi surface off impurities yields a resistivity (via the memory function approach) in agreement with earlier computations from the Kubo formula. The dominant momentum loss occurs via the scattering of the neutral bosonic ϕ excitations off **random fields**. This is good news for the AdS/CFT approaches, which do not capture the Fermi surface of most of the charged carriers.

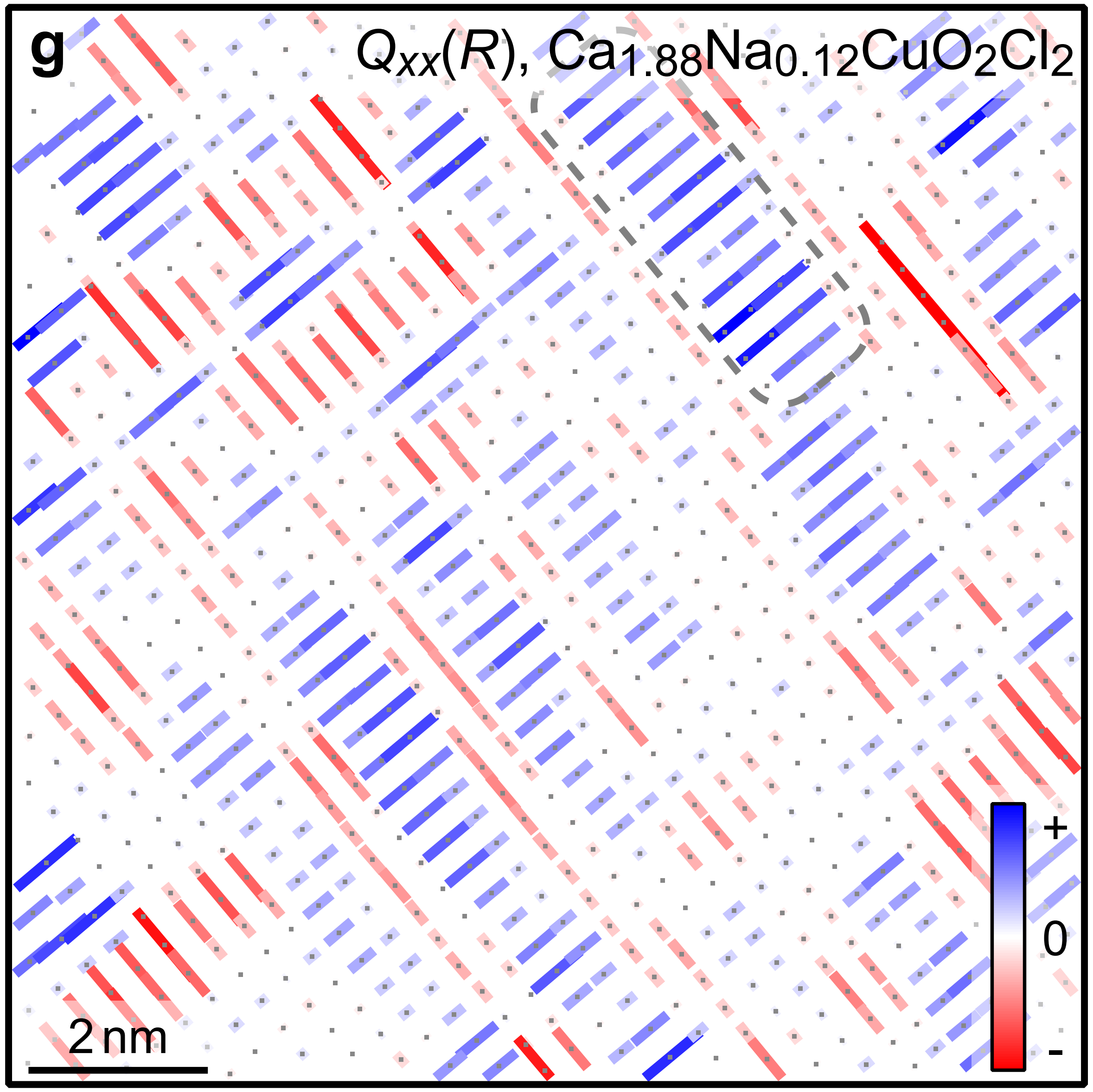
$$\rho(T) \sim h_0^2\, T^{(d-z+\eta)/z}$$

Visualization of the emergence of the pseudogap state and the evolution to superconductivity in a lightly hole-doped Mott insulator

Y. Kohsaka, T. Hanaguri, M. Azuma, M. Takano, J. C. Davis, and H. Takagi

Nature Physics, **8**, 534 (2012).

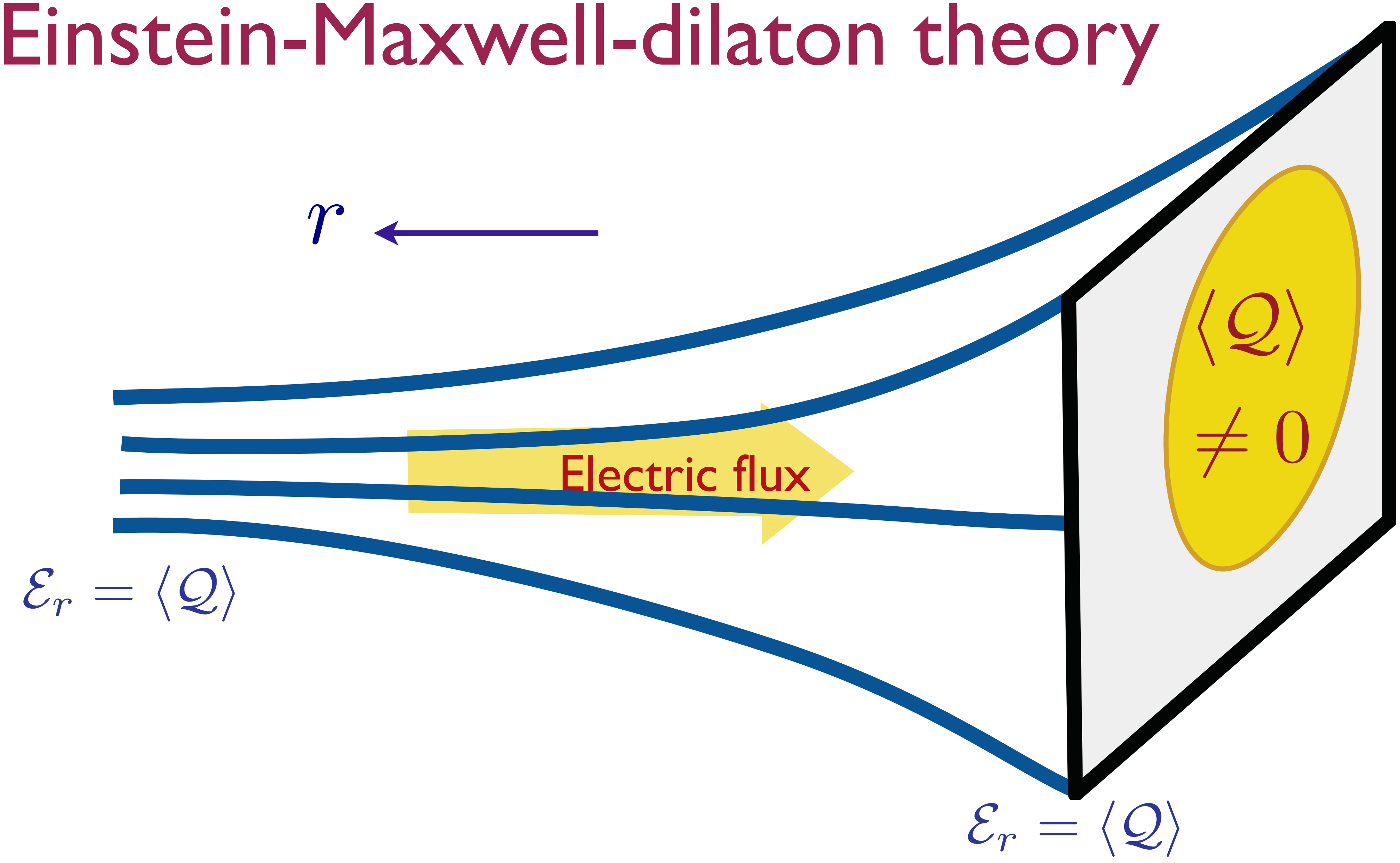




Evidence for "nematic" order
(*i.e.* breaking of 90° rotation symmetry) in $Ca_{1.88}Na_{0.12}CuO_2Cl_2$.

Holography of a non-Fermi liquid

Einstein-Maxwell-dilaton theory



$$\mathcal{S} = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2}\right) - \frac{Z(\Phi)}{4e^2}F_{ab}F^{ab}\right]$$

with $Z(\Phi) = Z_0 e^{\alpha\Phi}$, $V(\Phi) = -V_0 e^{-\beta\Phi}$, as $\Phi \to \infty$.

C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, arXiv:1105.1162 [hep-th].

Holography of a non-Fermi liquid

$$\boxed{ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)}$$

The $r \to \infty$ limit of the metric of the Einstein-Maxwell-dilaton (EMD) theory has the most general form with $\theta = d^2\beta/(\alpha + (d-1)\beta)$ and $z = 1 + \theta/d + 8(d(d-\theta)+\theta)^2/(d^2(d-\theta)\alpha^2)$. To this theory we add a bulk-scalar ψ with action

$$S_\psi = -\int \mathrm{d}^{d+2}x\,\sqrt{-g}\left(\frac{1}{2}(\partial\psi)^2 + \frac{B(\Phi)}{2}\psi^2\right).$$

with $B(\Phi) \sim e^{-\beta\Phi}$ required to obtain a primary operator. Finally we couple a random-field to the boundary operator $\mathcal{O}_\psi$ dual to ψ:

$$\mathcal{S}_{\text{rf}} = \int d\tau d^d\mathbf{x}\; h(\mathbf{x})\mathcal{O}(\mathbf{x},\tau),$$

where $h(\mathbf{x})$ is a time-independent Gaussian-random variable:

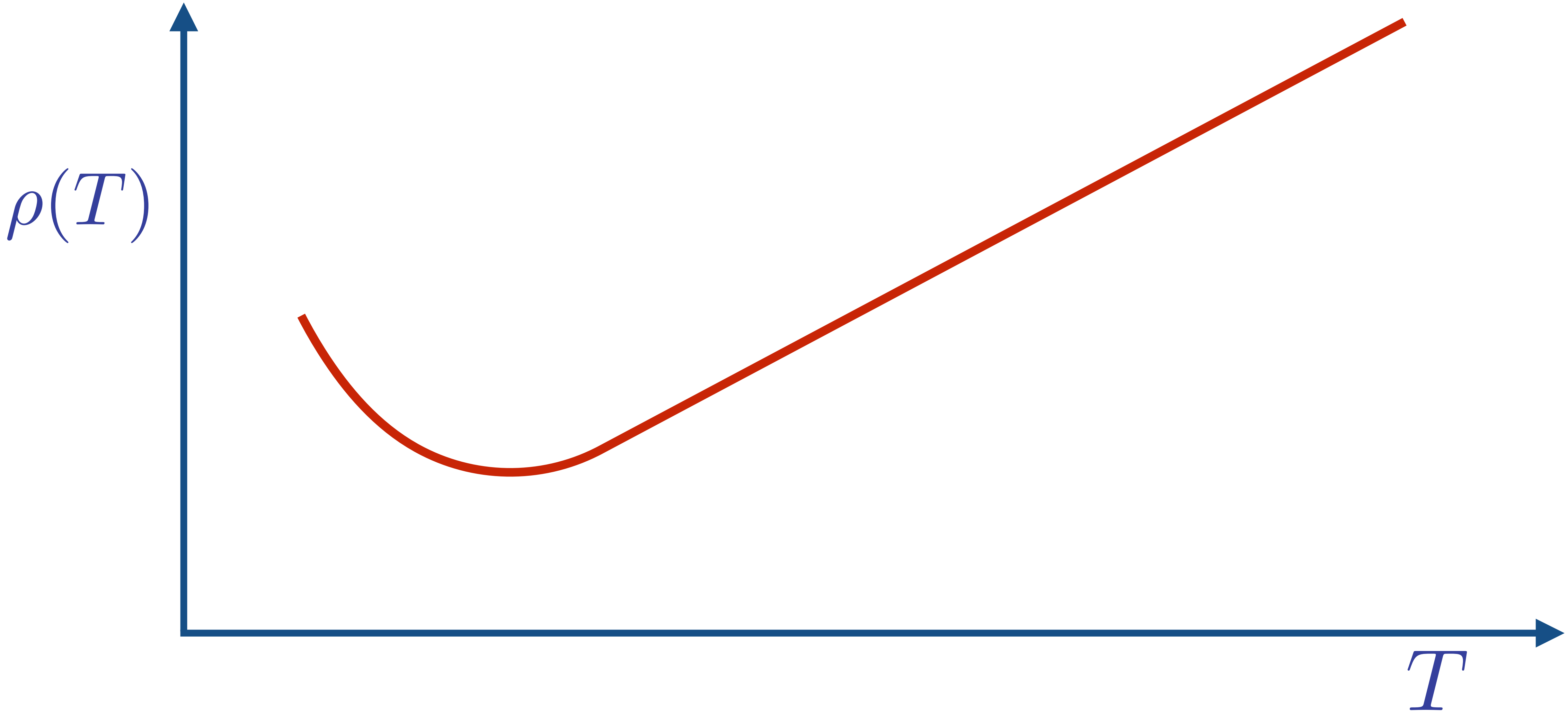
$$\mathbb{E}[h(\mathbf{x})] = 0, \quad \mathbb{E}[h(\mathbf{x})h(\mathbf{x}')] = h_0^2\delta(\mathbf{x}-\mathbf{x}').$$

Then we obtain the resistivity $\boxed{\rho(T) \sim h_0^2\, T^{(d-z+\eta)/z}}$ where $\dim[\mathcal{O}_\psi] = (d+z-2+\eta)/2$.

A. Lucas, S. Sachdev, and K. Schalm, `arXiv:1401.7933`

Quantum criticality of Ising-nematic ordering in a metal

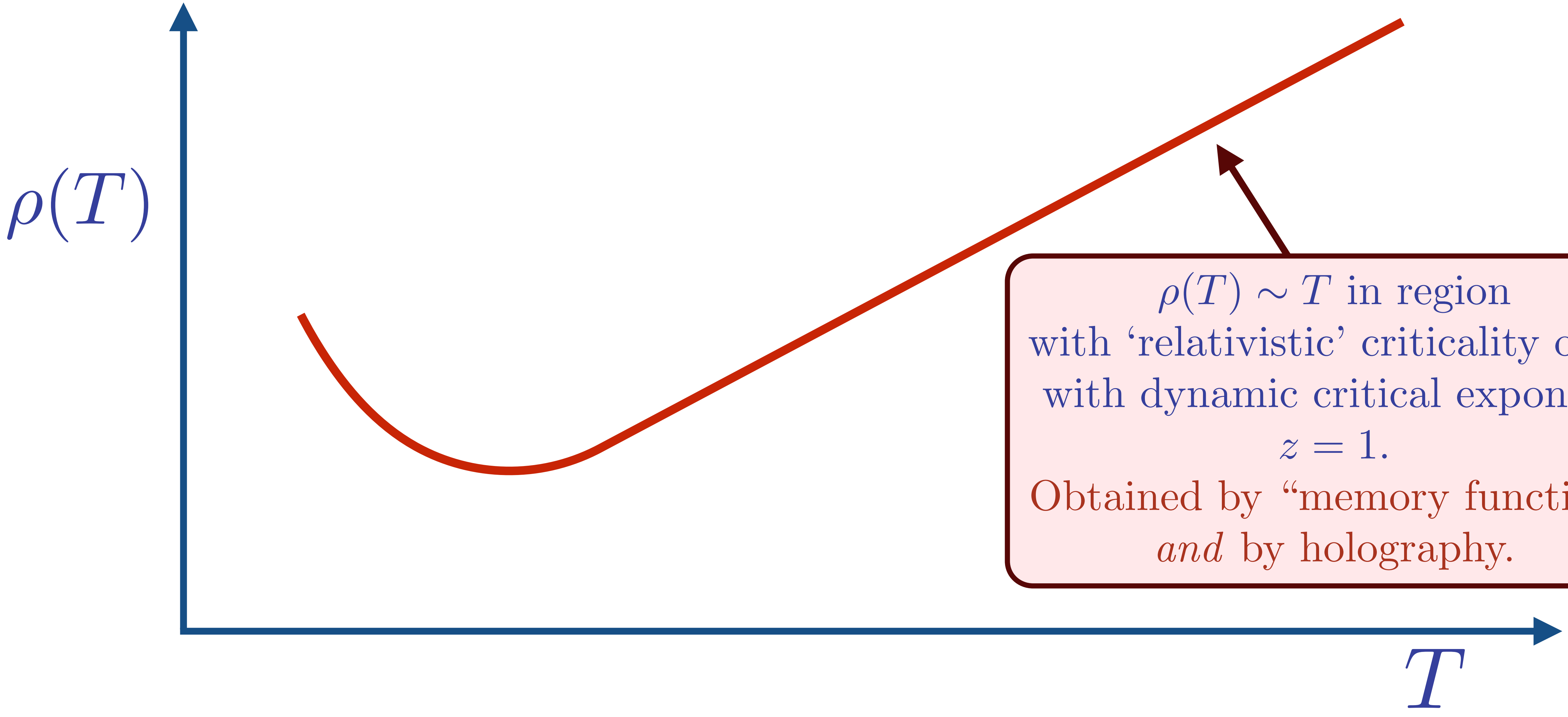
Transport without quasiparticles:



S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, `arXiv:1401.7012`.
A. Lucas, S. Sachdev, and K. Schalm, `arXiv:1401.7933`

Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:

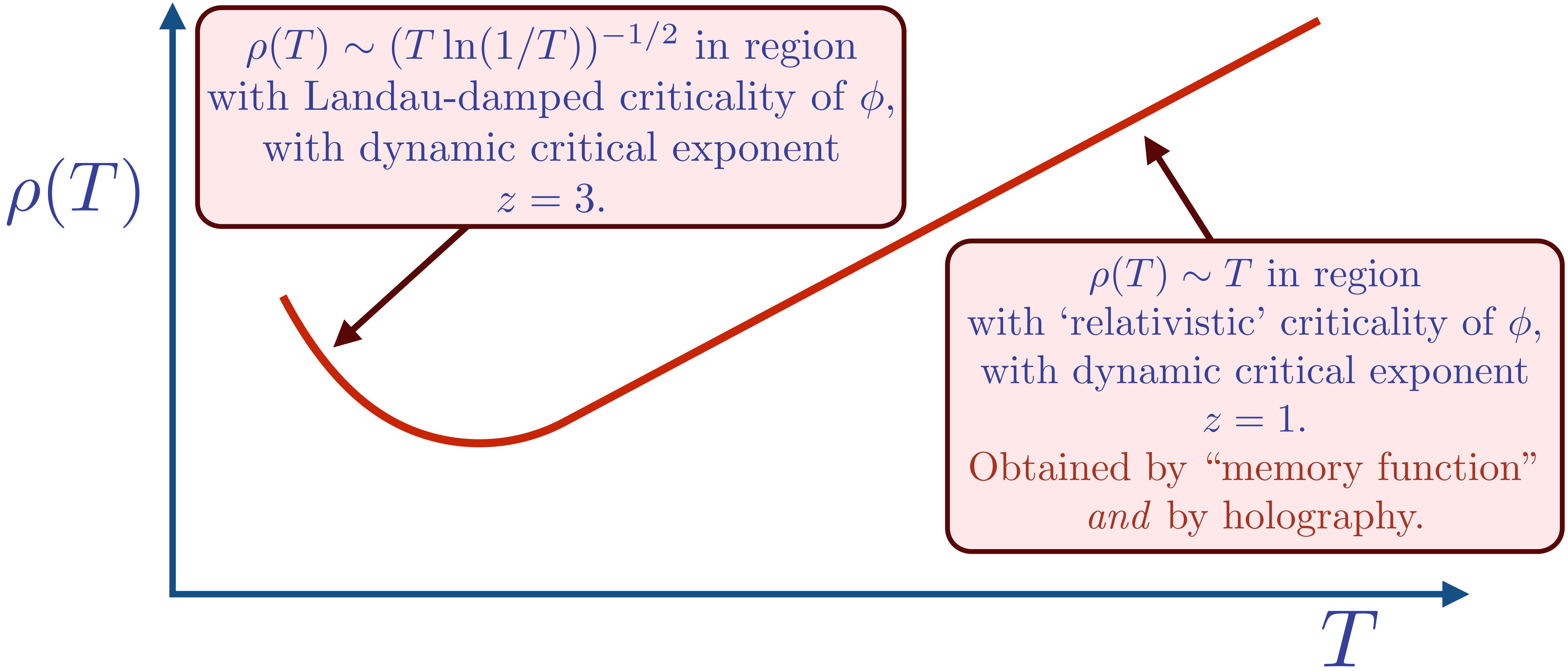


$\rho(T) \sim T$ in region
with 'relativistic' criticality of ϕ,
with dynamic critical exponent
$z = 1$.
Obtained by "memory function"
and by holography.

S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, `arXiv:1401.7012`.
A. Lucas, S. Sachdev, and K. Schalm, `arXiv:1401.7933`

Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:



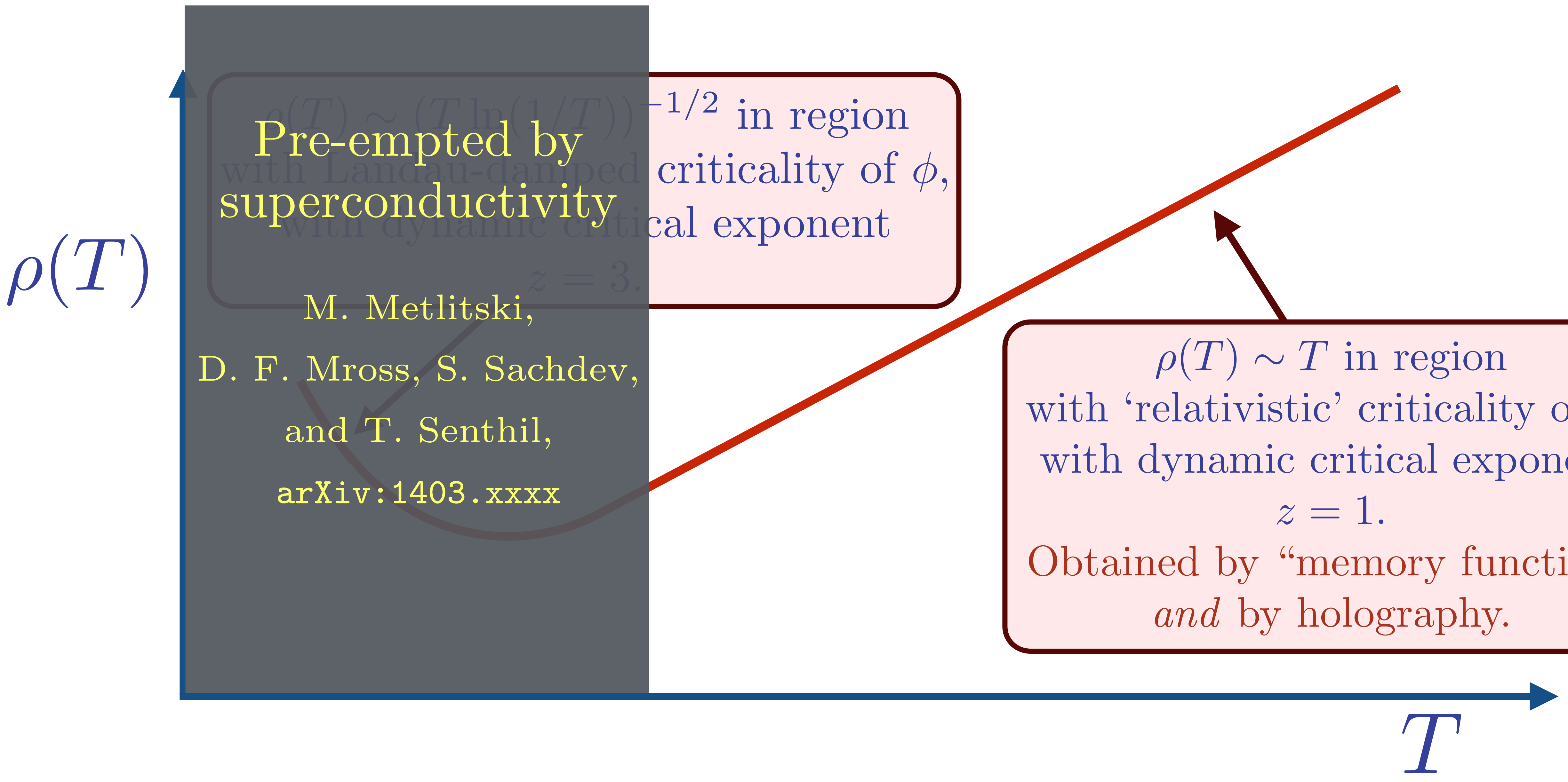
$\rho(T) \sim (T \ln(1/T))^{-1/2}$ in region with Landau-damped criticality of ϕ, with dynamic critical exponent $z = 3$.

$\rho(T) \sim T$ in region with ‘relativistic’ criticality of ϕ, with dynamic critical exponent $z = 1$. Obtained by “memory function” *and* by holography.

S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, `arXiv:1401.7012`.
A. Lucas, S. Sachdev, and K. Schalm, `arXiv:1401.7933`

Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:



$\rho(T) \sim (T \ln(1/T))^{-1/2}$ in region with Landau-damped criticality of ϕ, with dynamic critical exponent $z = 3$.

Pre-empted by superconductivity

M. Metlitski,
D. F. Mross, S. Sachdev,
and T. Senthil,
`arXiv:1403.xxxx`

$\rho(T) \sim T$ in region with 'relativistic' criticality of ϕ, with dynamic critical exponent $z = 1$.
Obtained by "memory function" *and* by holography.

S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, `arXiv:1401.7012`.
A. Lucas, S. Sachdev, and K. Schalm, `arXiv:1401.7933`

- Strongly-coupled quantum criticality leads to a novel regime of quantum dynamics without quasiparticles.

- Strongly-coupled quantum criticality leads to a novel regime of quantum dynamics without quasiparticles.

- The simplest examples are conformal field theories in 2+1 dimensions, realized by ultracold atoms in optical lattices. Quantitative predictions for transport by combining quantum Monte Carlo and holography.

- Strongly-coupled quantum criticality leads to a novel regime of quantum dynamics without quasiparticles.

- The simplest examples are conformal field theories in 2+1 dimensions, realized by ultracold atoms in optical lattices. Quantitative predictions for transport by combining quantum Monte Carlo and holography.

- Exciting recent progress on the description of transport in metallic states without quasiparticles, via field theory and holography.