Quantum Criticality and Black Holes

Talk online: sachdev.physics.harvard.edu

Particle theorists

Sean Hartnoll, KITP

Christopher Herzog, Princeton

Pavel Kovtun, Victoria

Dam Son, Washington

Condensed matter theorists

Markus Mueller, Harvard

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Subir Sachdev, Harvard

1. CFT3s in condensed matter physics and string theory
   Antiferromagnetic ordering transitions, graphene ....

2. Quantum-critical transport
   Collisionless-to-hydrodynamic crossover of CFT3s

3. Black Hole Thermodynamics
   Connections to quantum criticality

4. Generalized magnetohydrodynamics
   Quantum criticality and dyonic black holes

5. Experiments
   The cuprate superconductors and graphene

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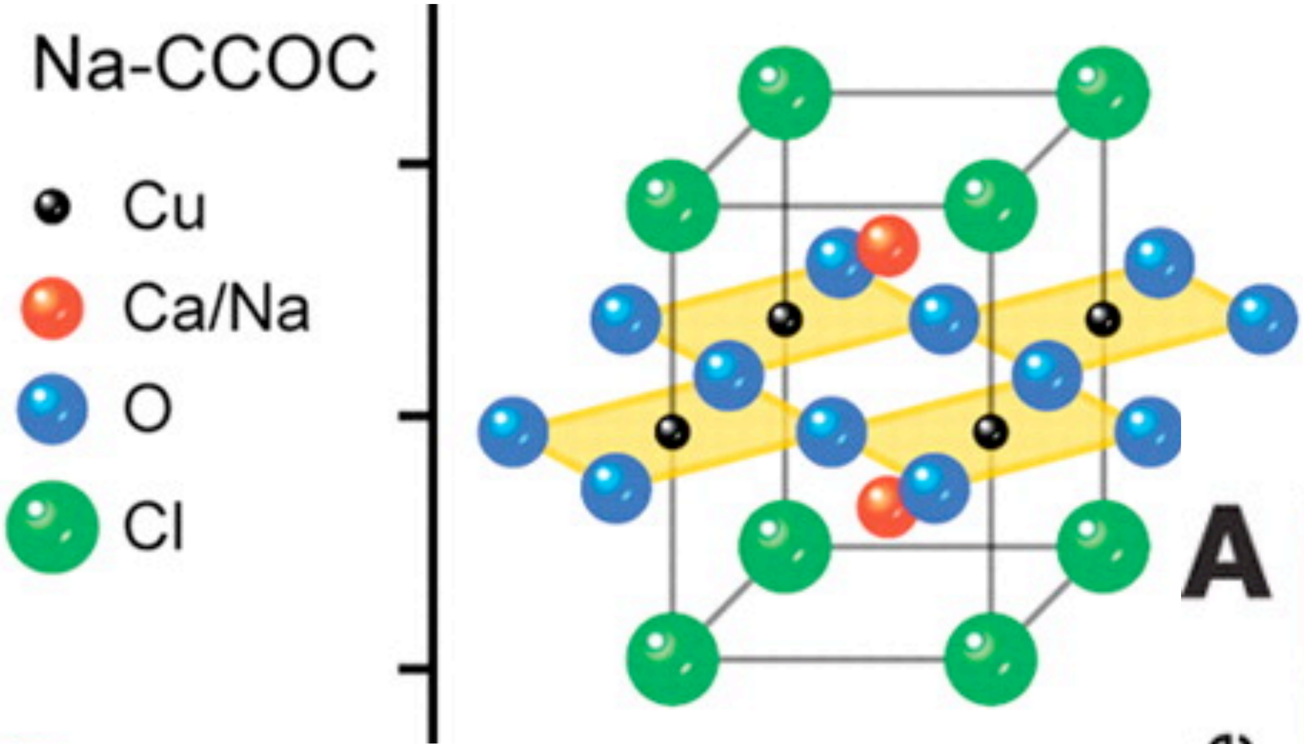
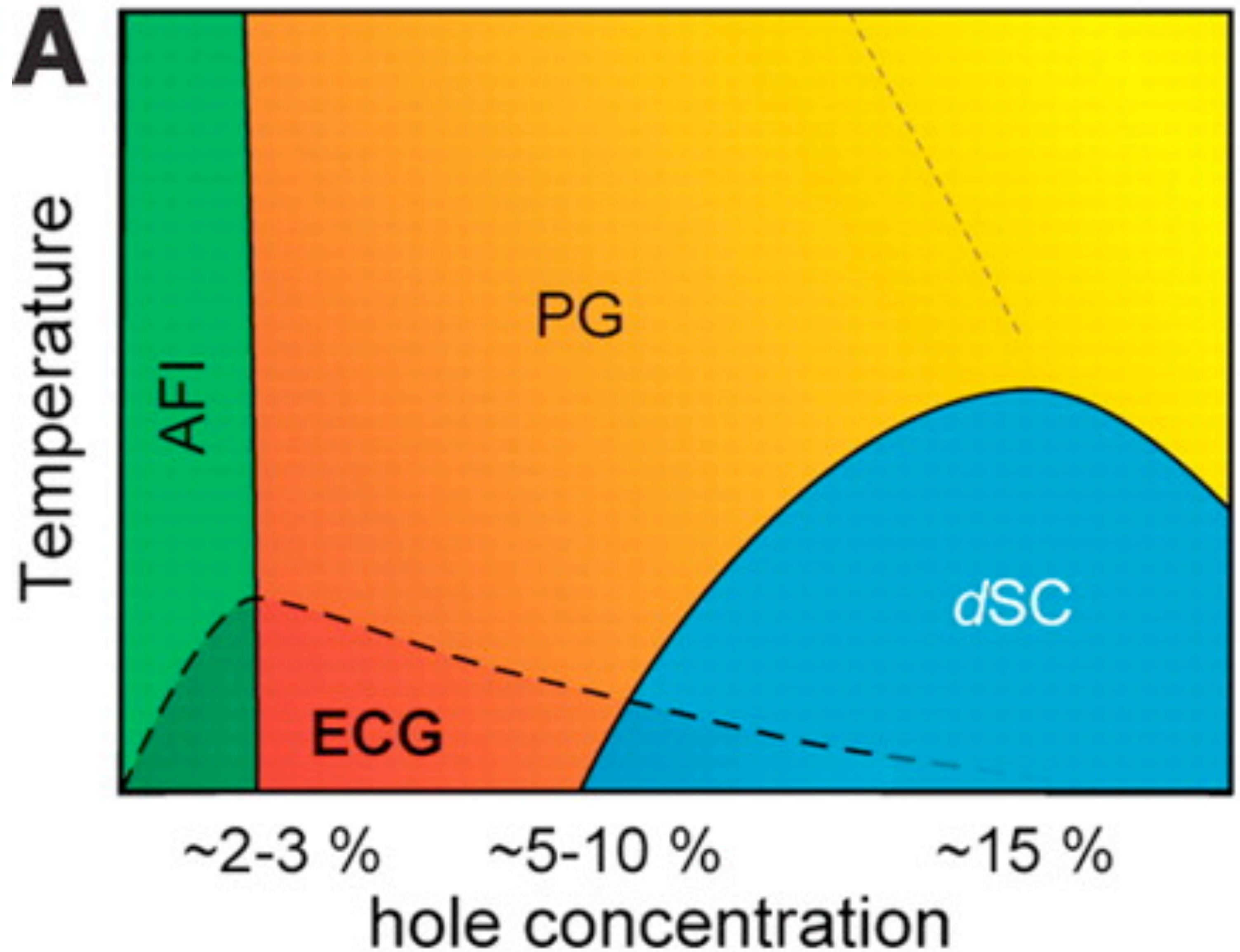
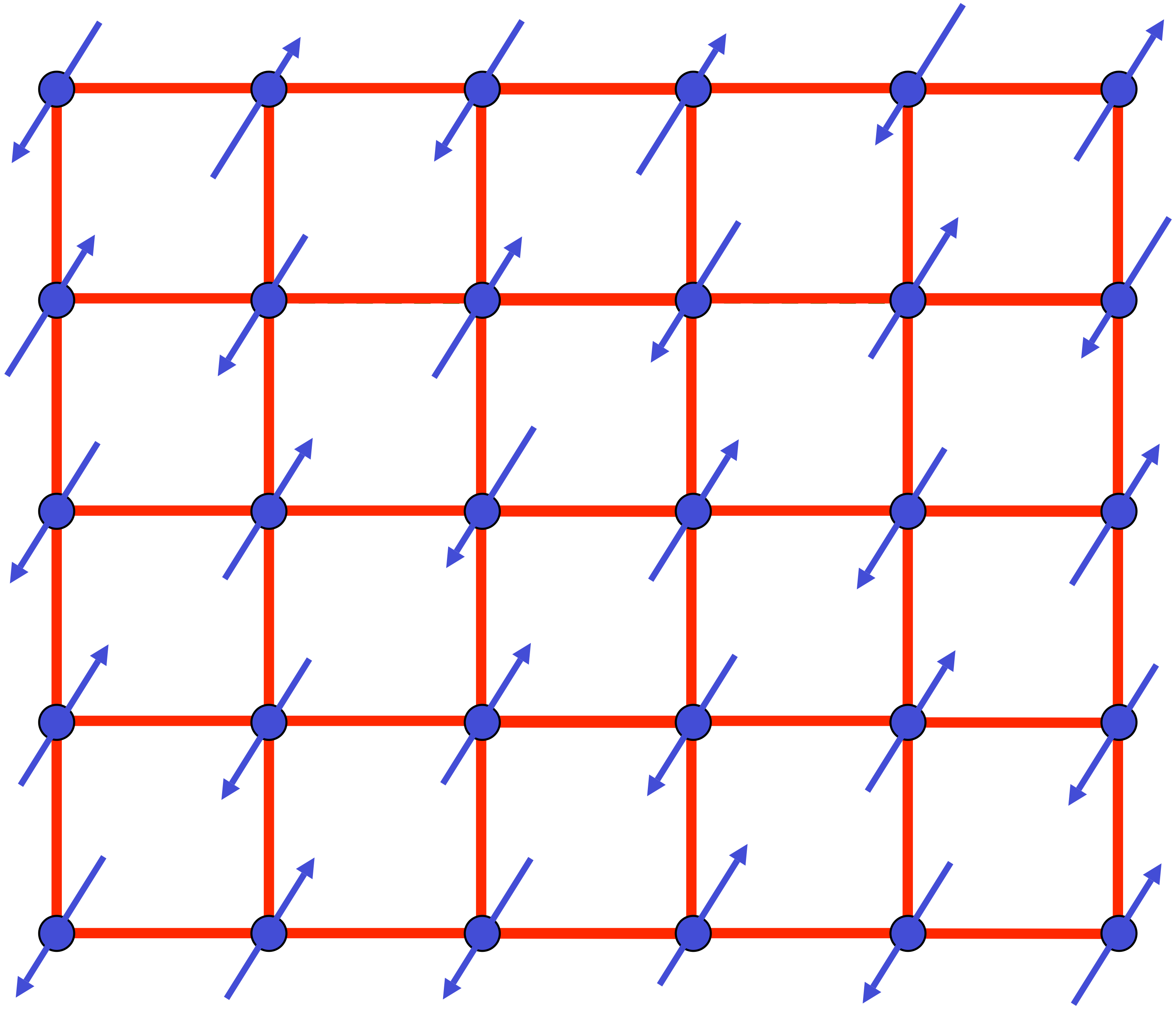
The cuprate superconductors





Antiferromagnetic (Neel) order in the insulator

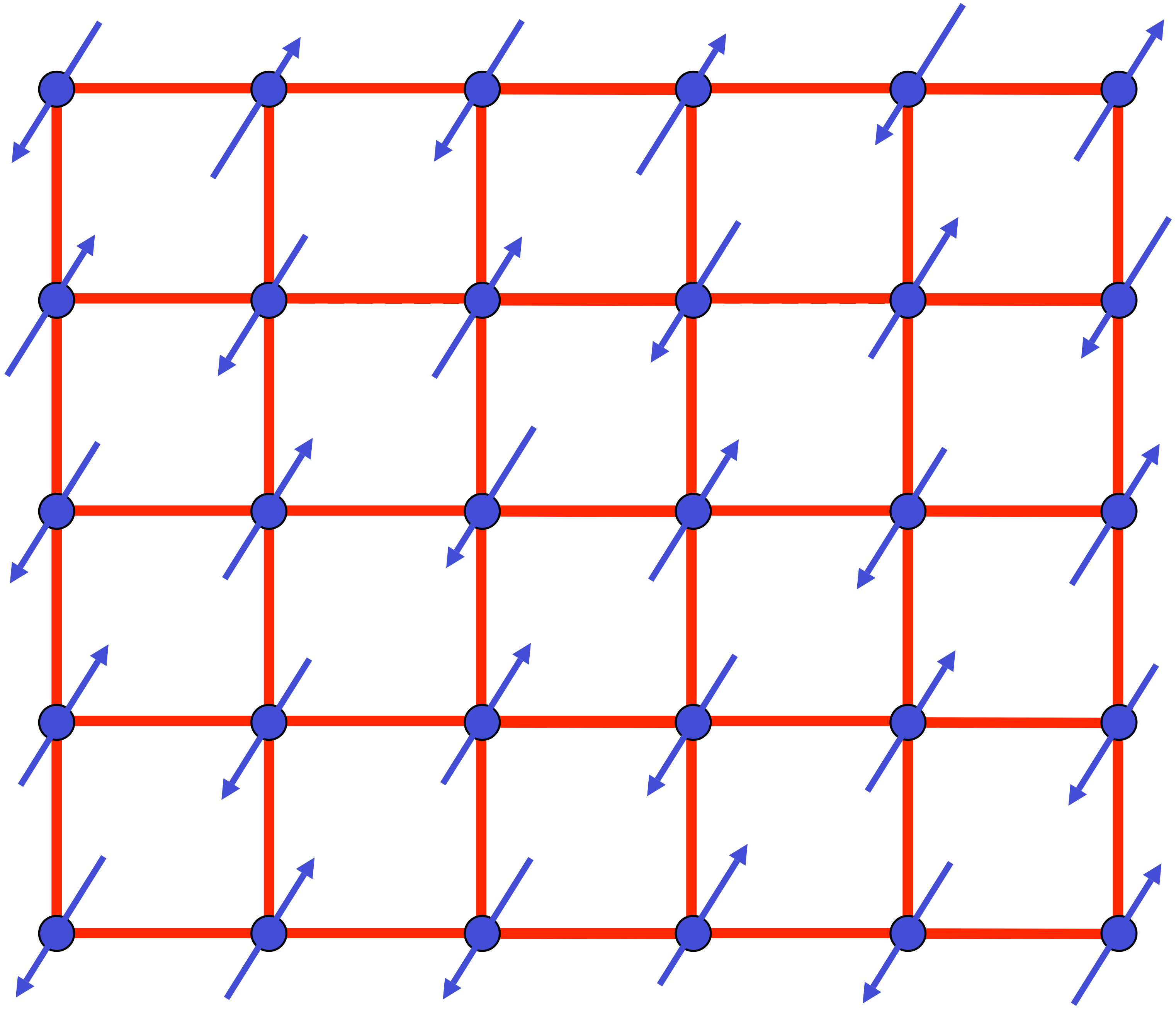


$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

$\vec{S}_i \Rightarrow$ spin operator with $S = 1/2$

No entanglement of spins

Antiferromagnetic (Neel) order in the insulator



$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

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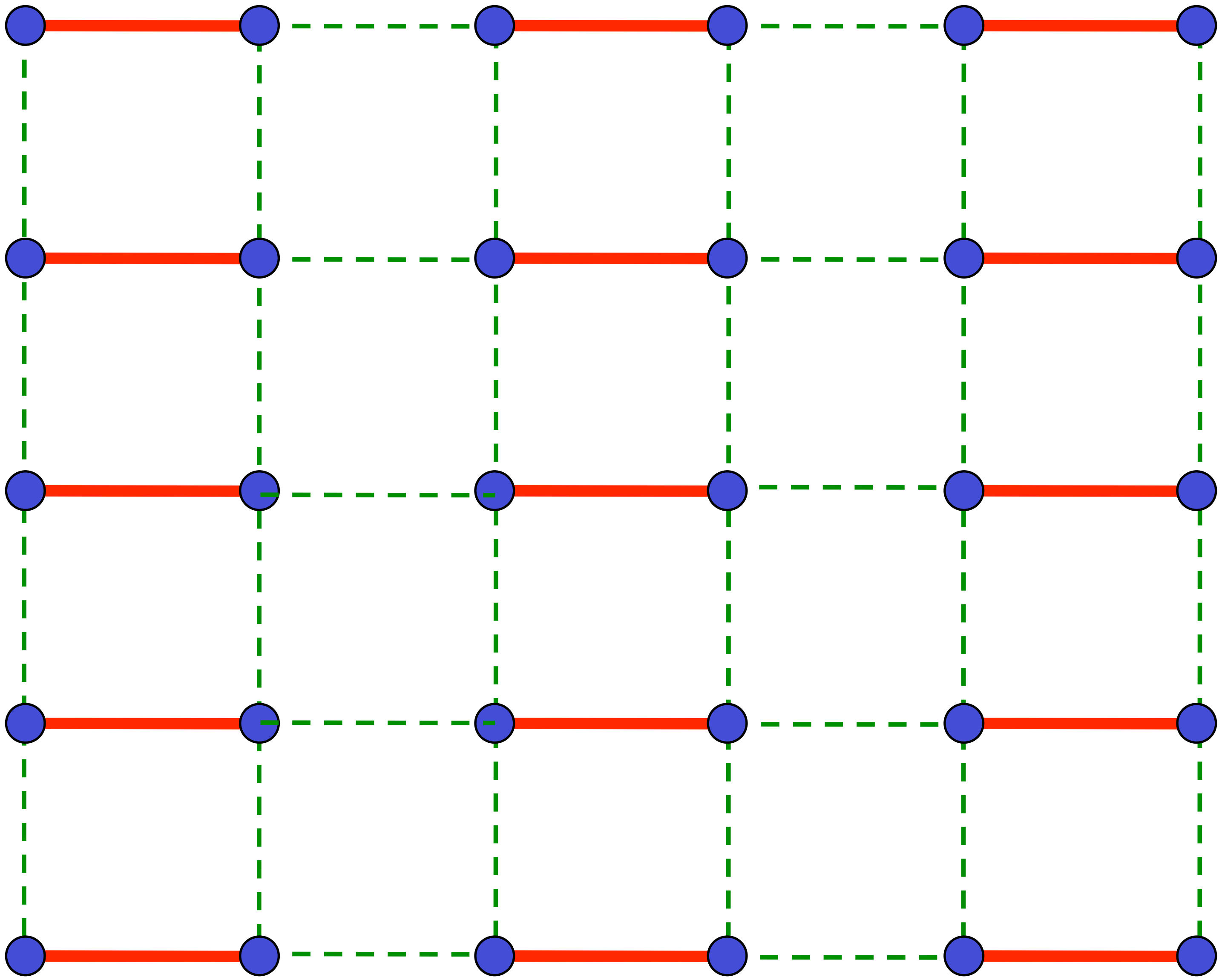
Excitations: 2 spin waves (Goldstone modes)



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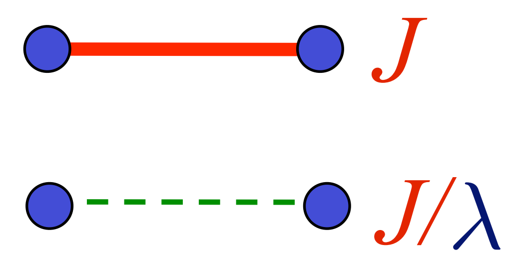
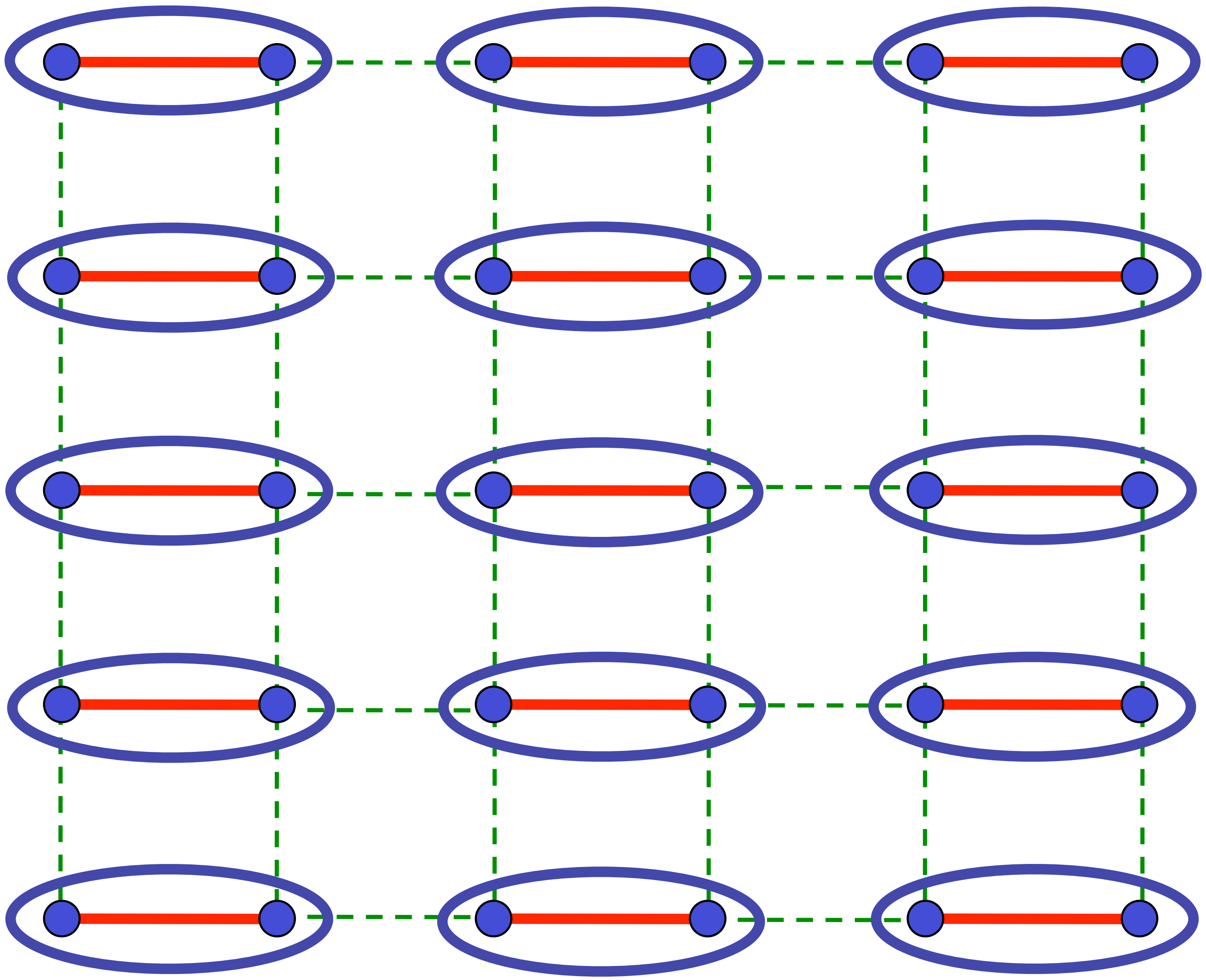


Weaken some bonds to induce spin entanglement in a new quantum phase



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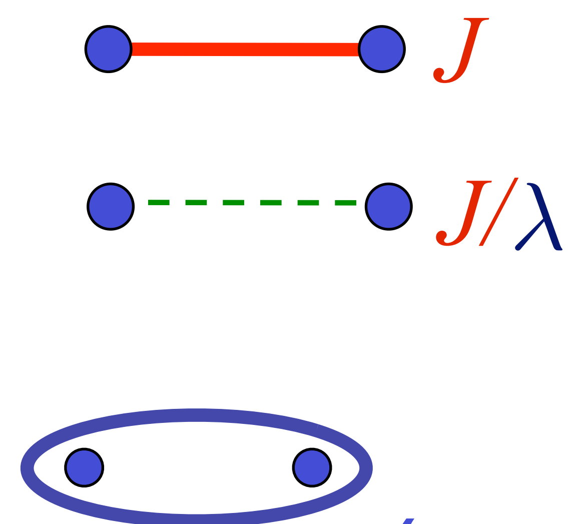
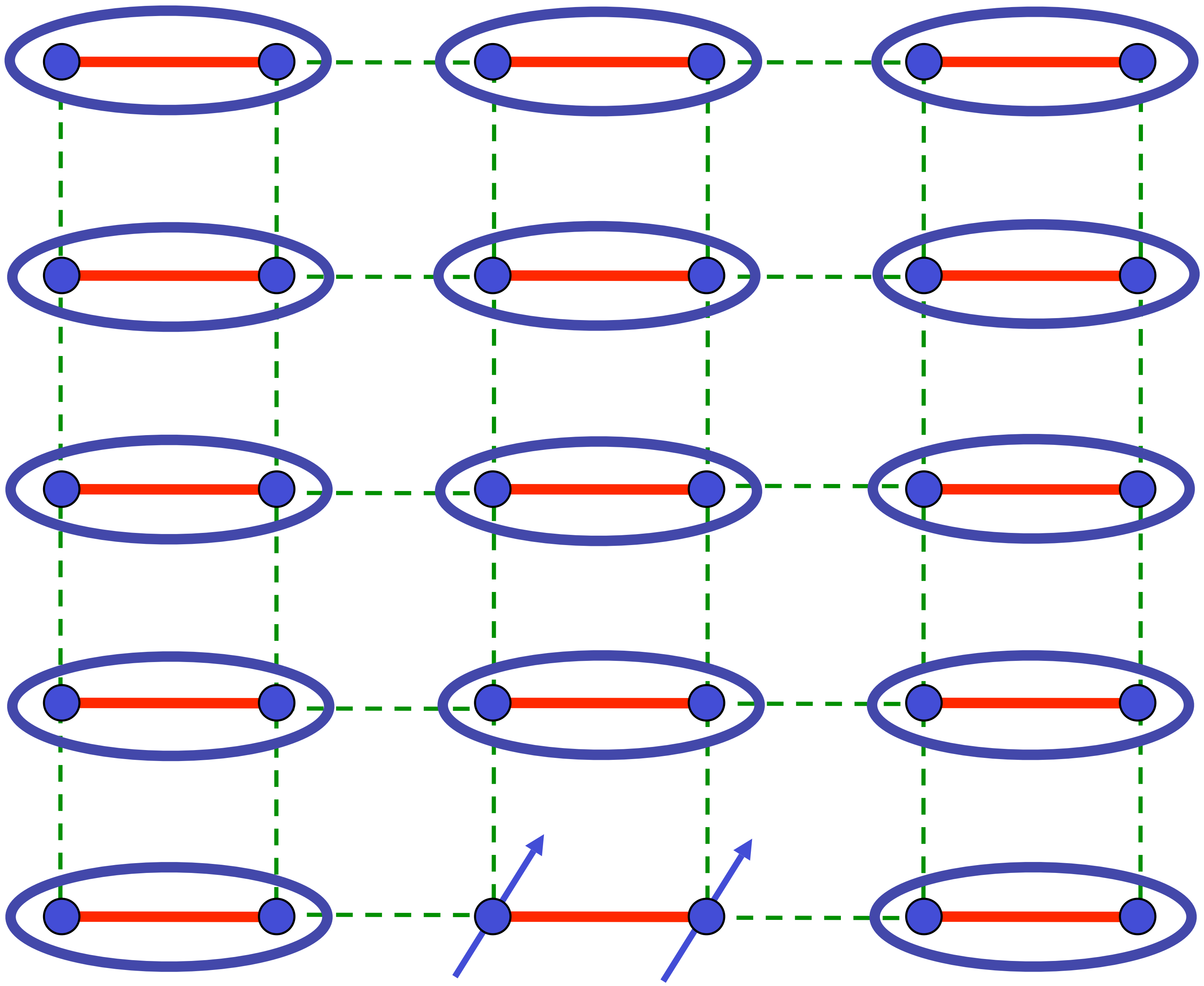


$$= \frac{1}{\sqrt{2}}\Big(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\Big)$$

Ground state is a product of pairs of entangled spins.



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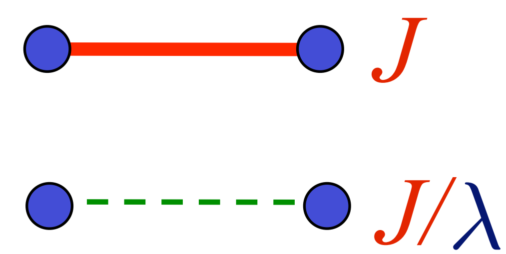
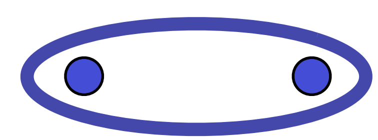
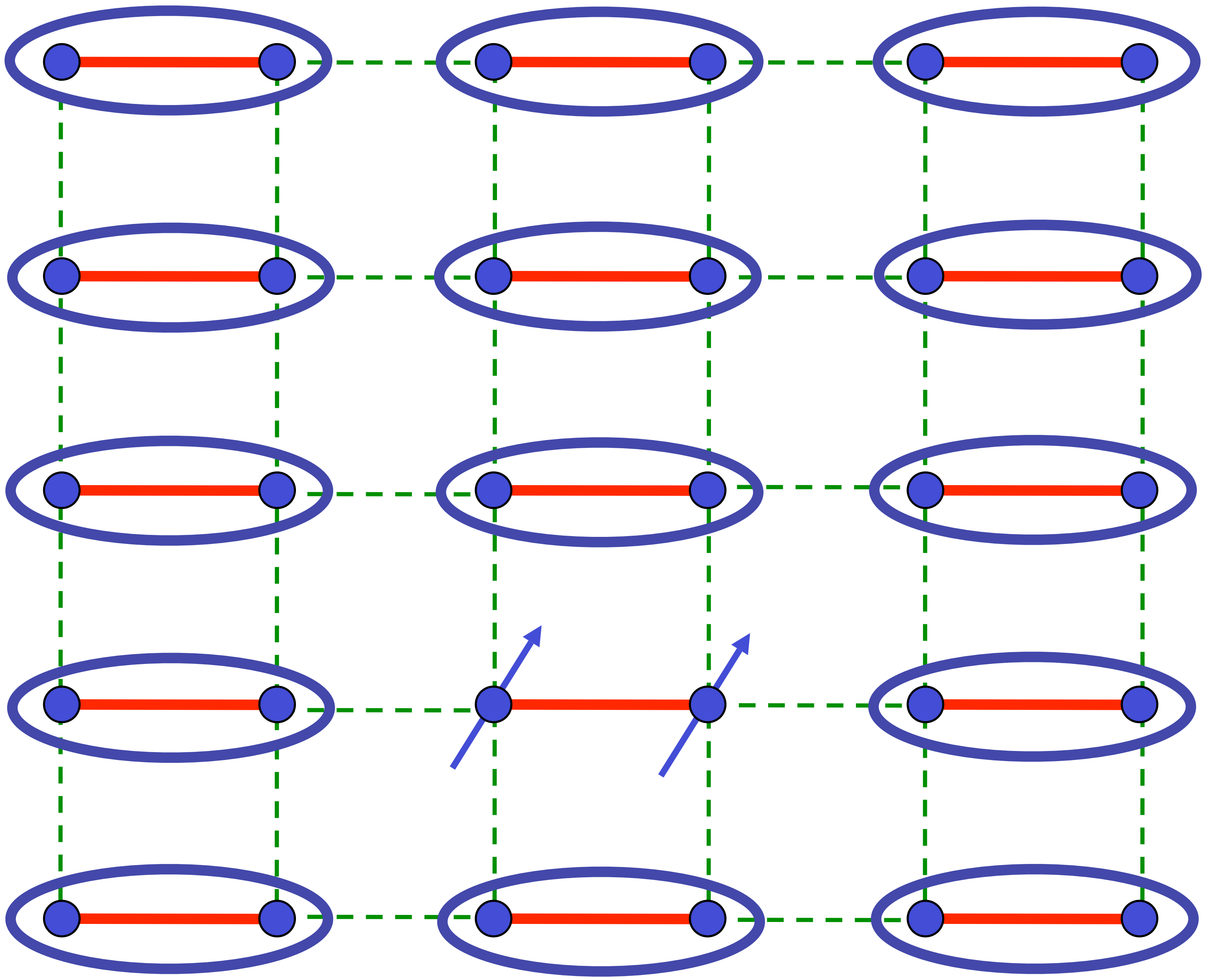




$$= \frac{1}{\sqrt{2}} \Big(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \Big)$$

Excitations: 3 *S=1* triplons



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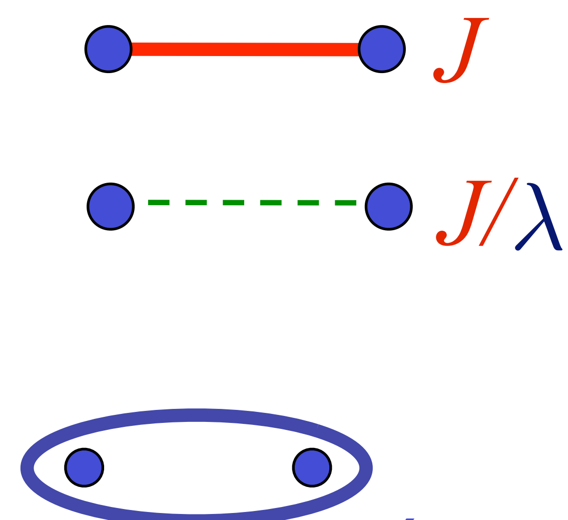

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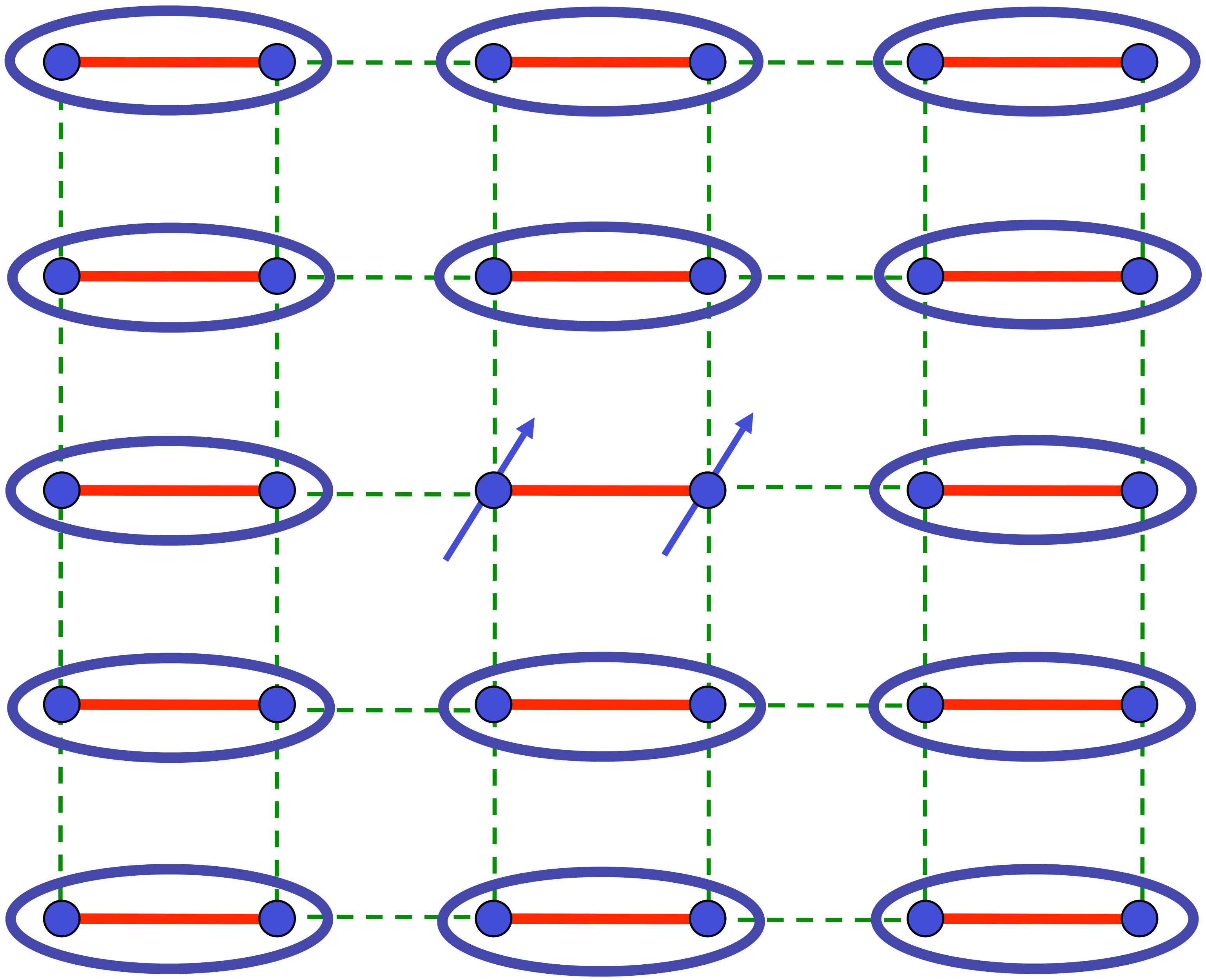

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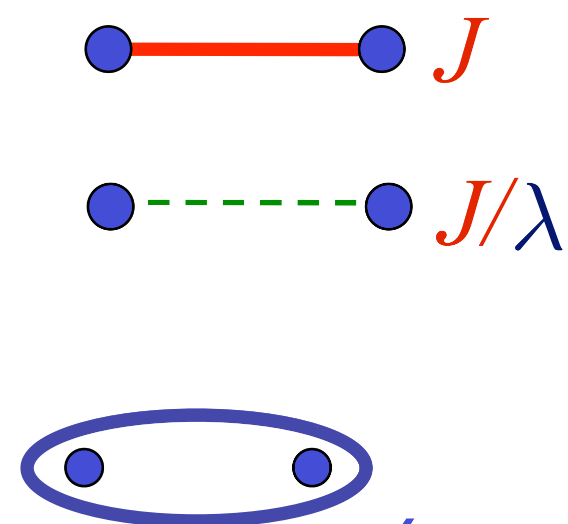

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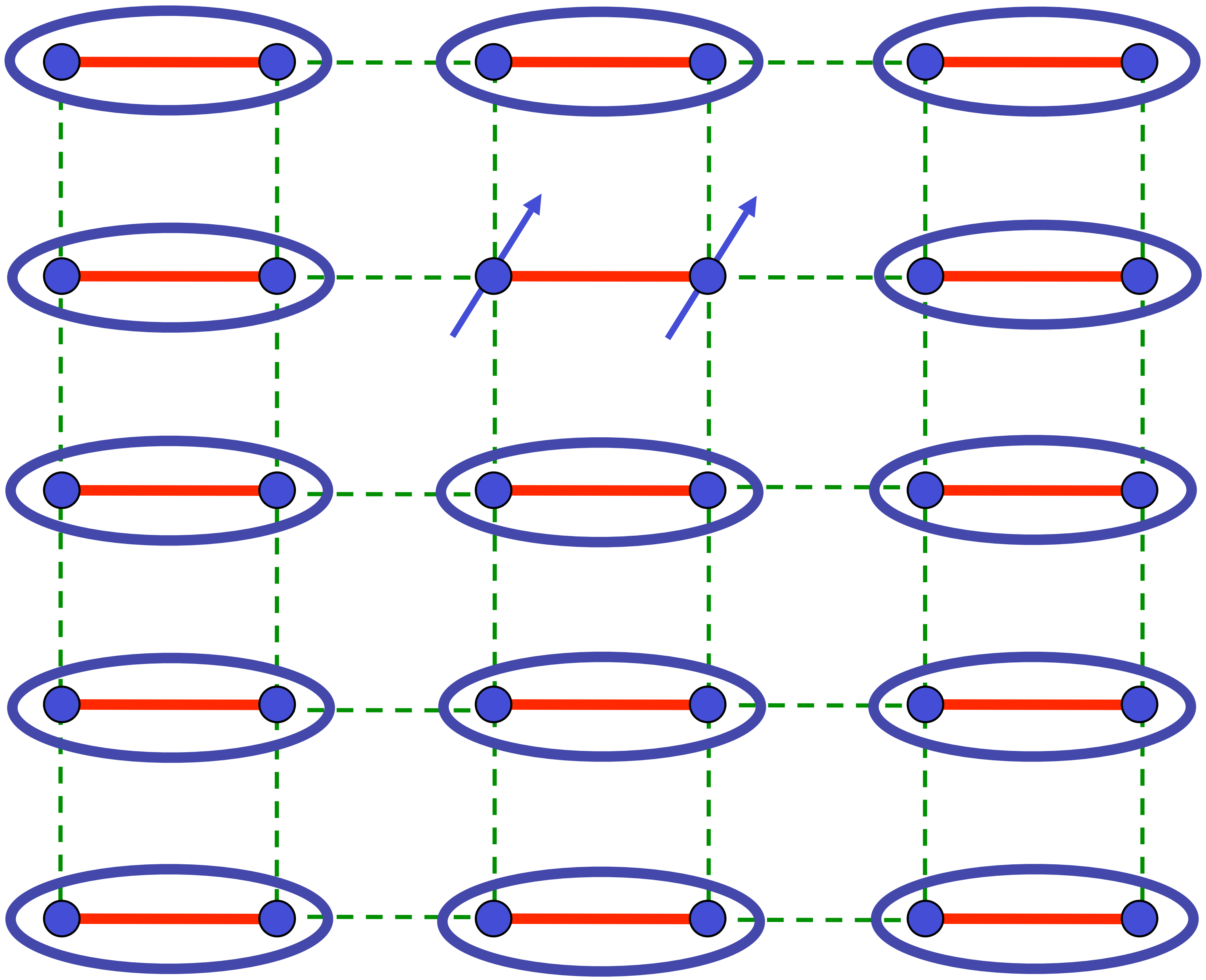

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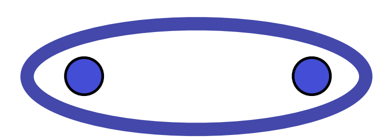
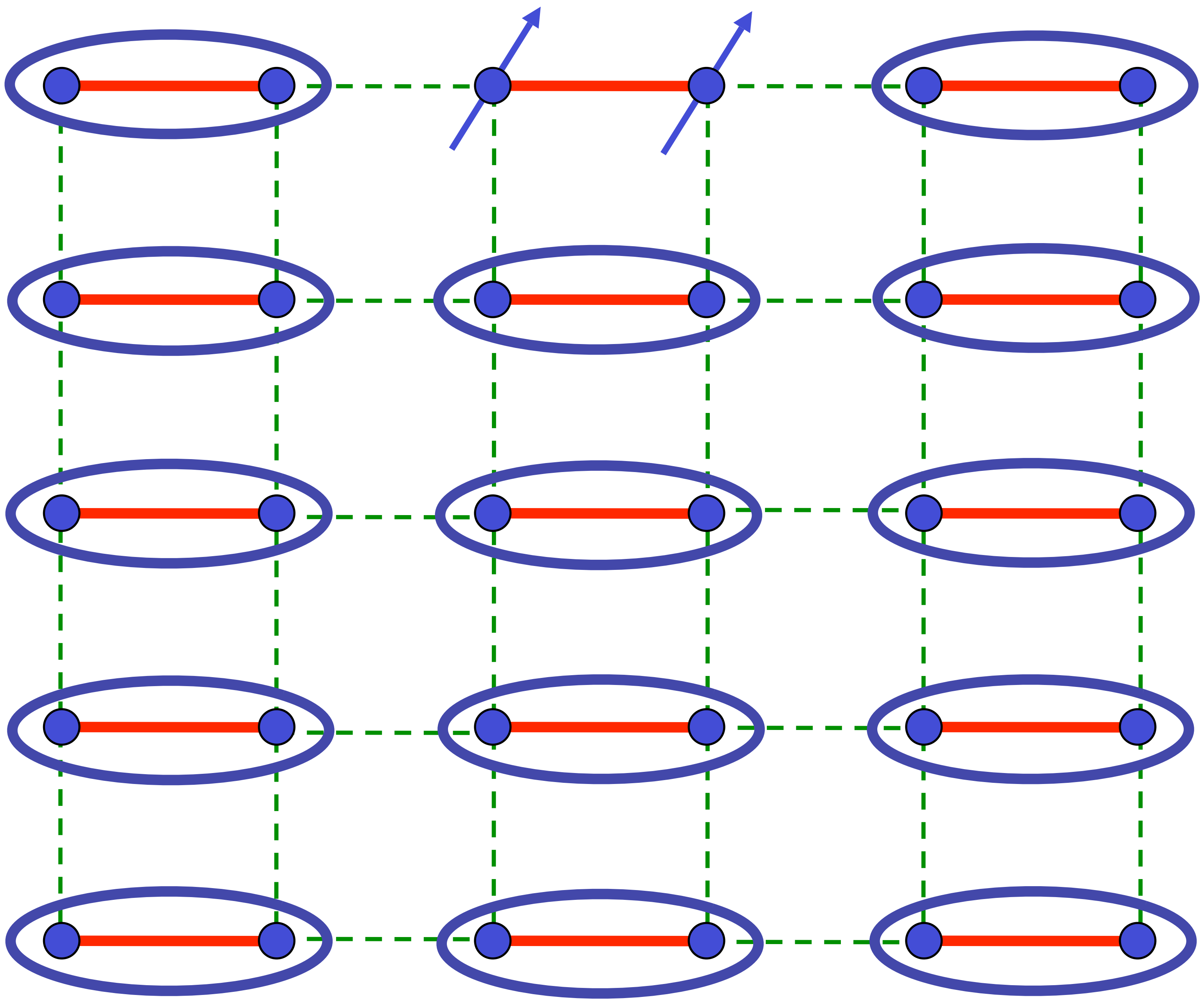
J

J/λ



$= \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$

Excitations: 3 $S=1$ triplons



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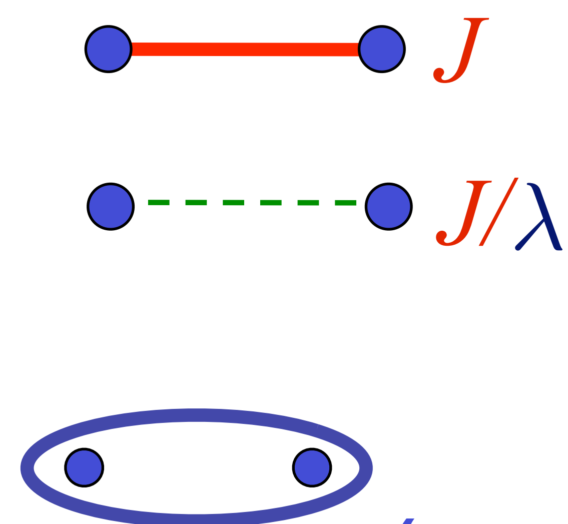

$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$$

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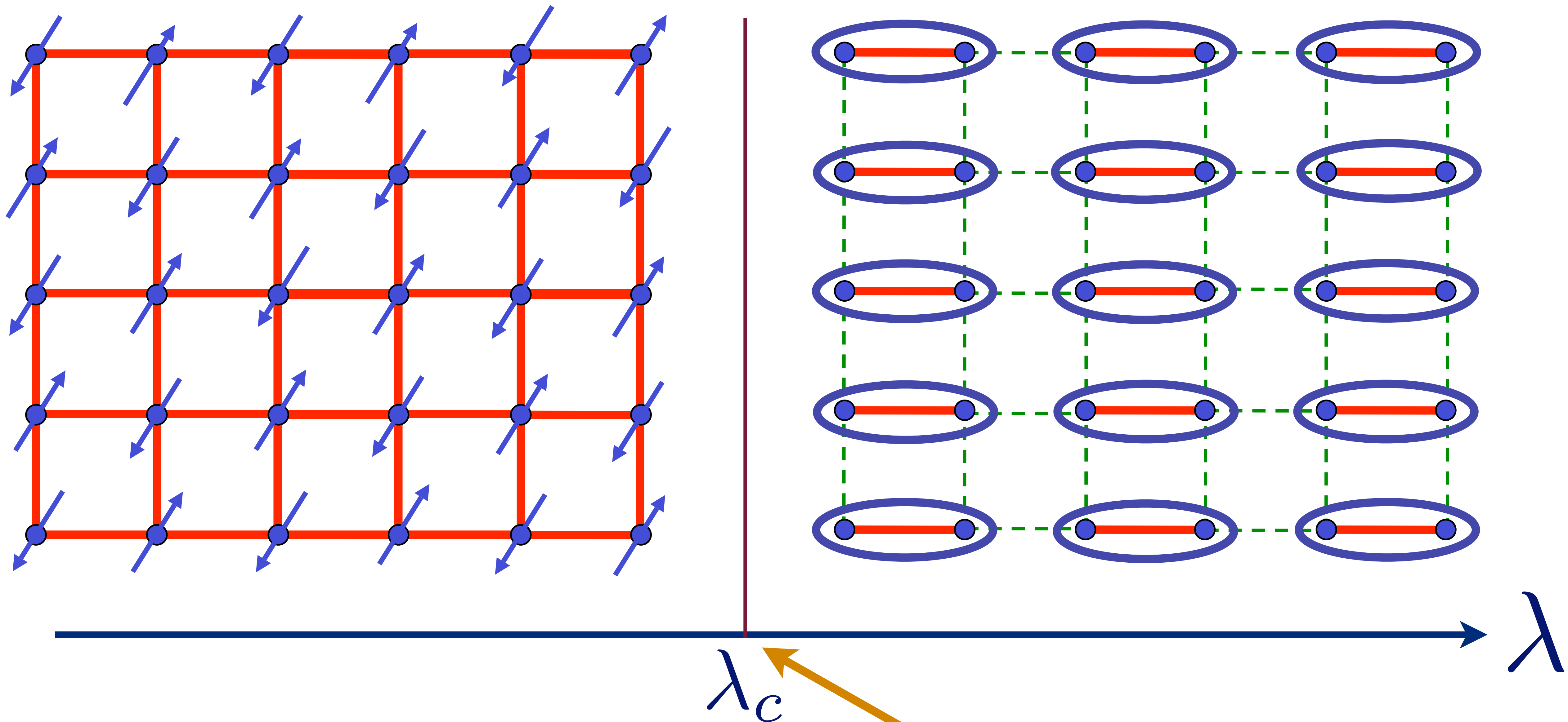
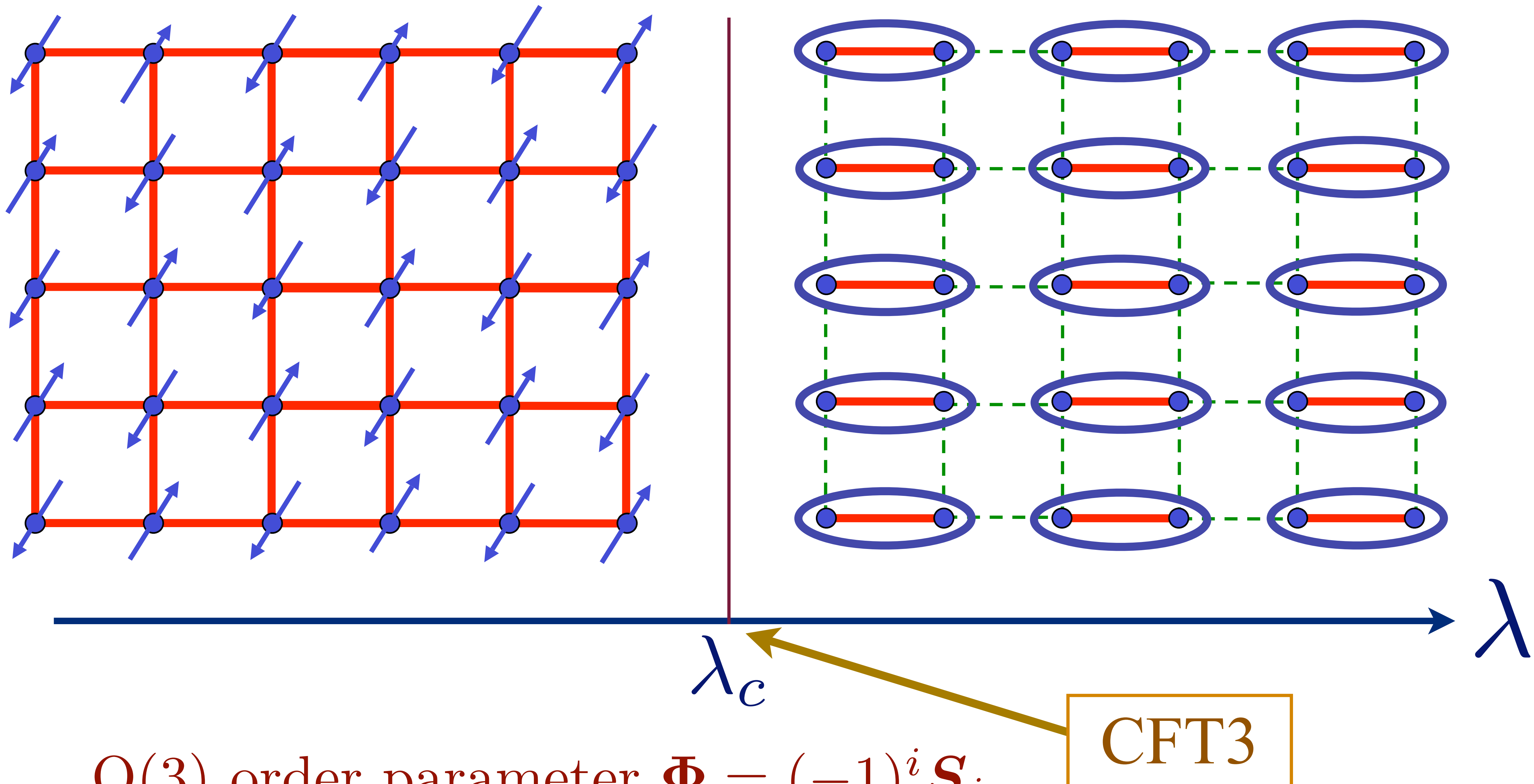
Phase diagram as a function of the ratio of exchange interactions, λ



Quantum critical point with non-local entanglement in spin wavefunction

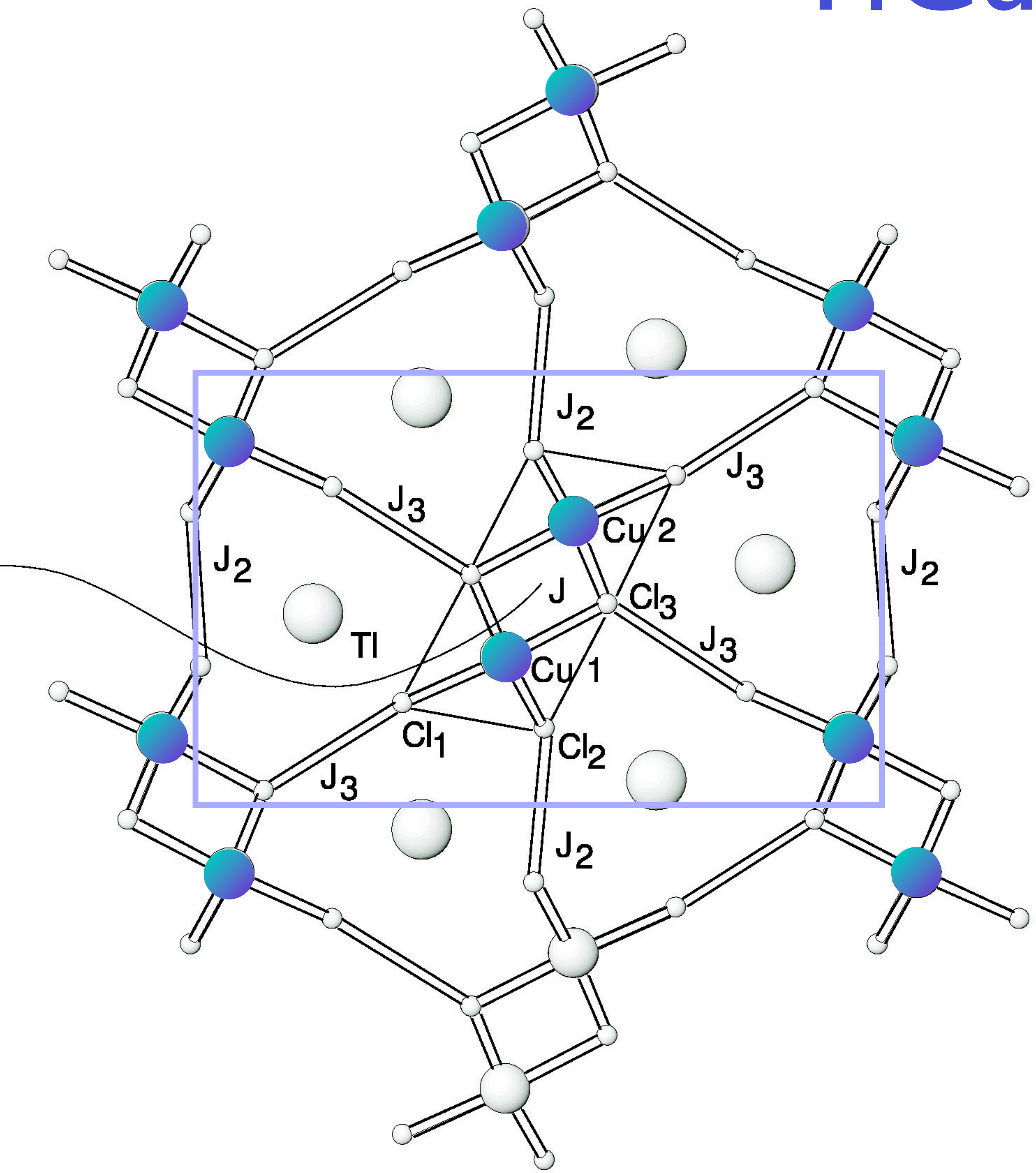
Phase diagram as a function of the ratio of exchange interactions, λ



O(3) order parameter $\mathbf{\Phi} = (-1)^i \boldsymbol{S}_i$

$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \mathbf{\Phi})^2 + c^2 (\vec{\nabla} \mathbf{\Phi})^2 + s \mathbf{\Phi}^2 + u \left(\mathbf{\Phi}^2 \right)^2 \right]$$

$TlCuCl_3$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Cu 1
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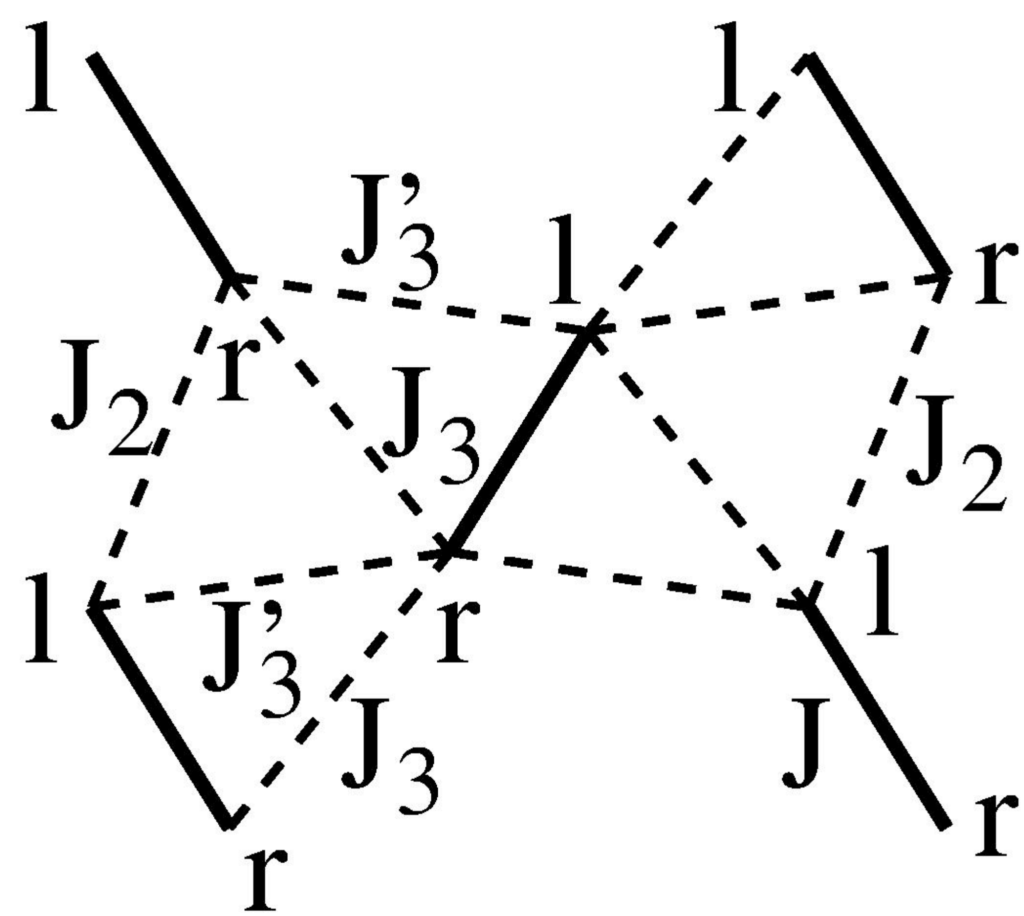
J2


l
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J'3
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J2
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J3
J2
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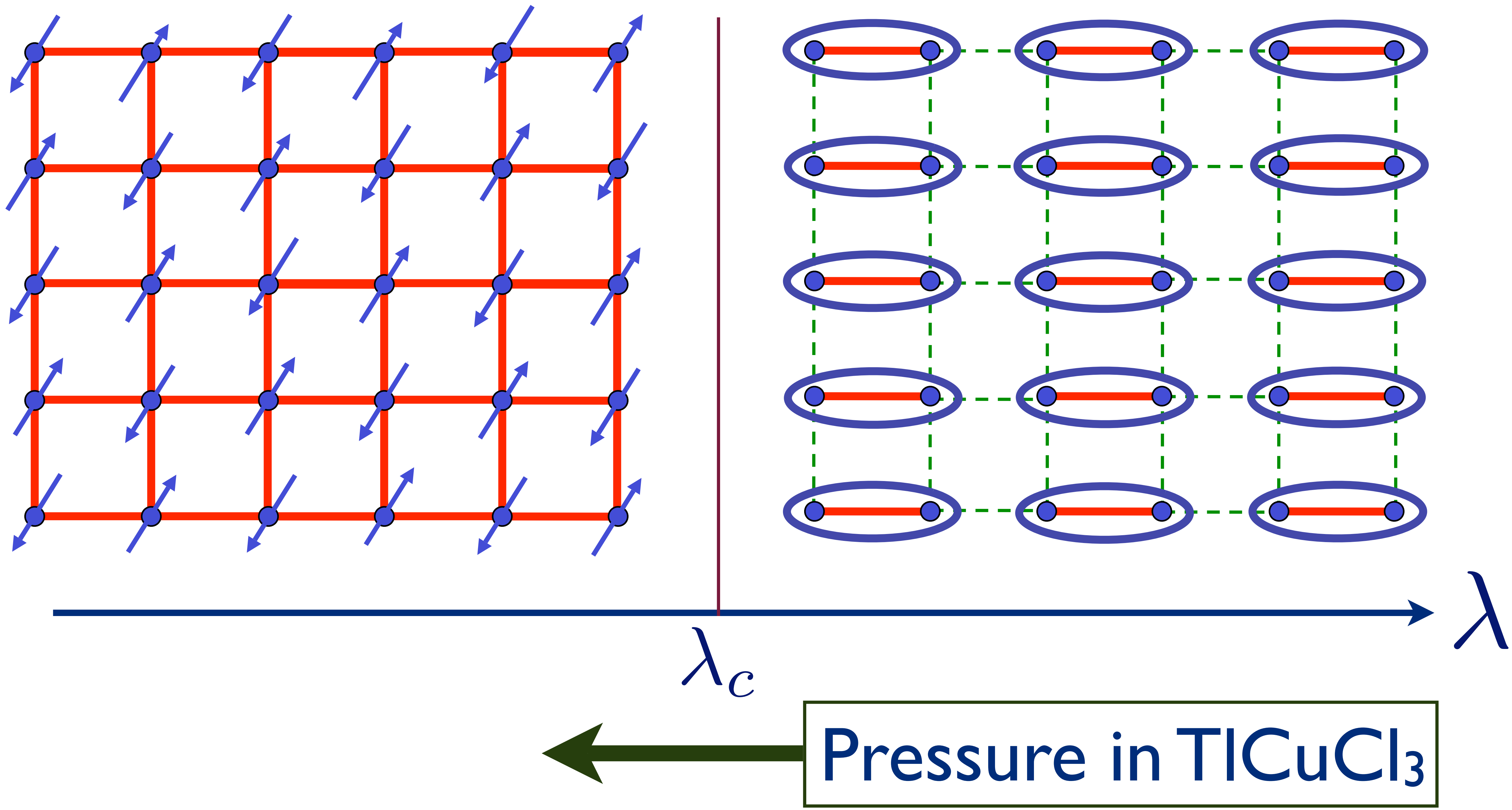
J'3
J3
J
r
r

Phase diagram as a function of the ratio of exchange interactions, λ



$TlCuCl_3$ at ambient pressure

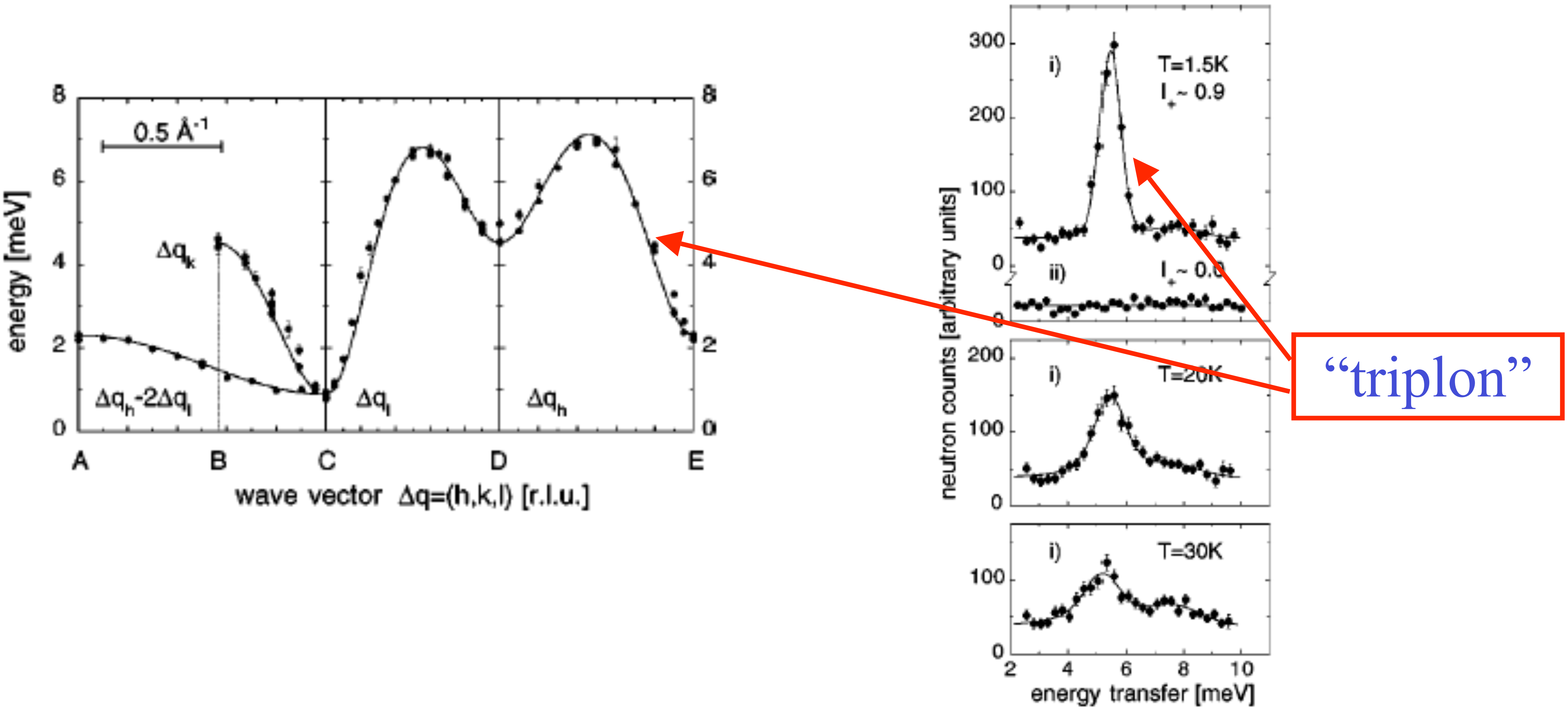
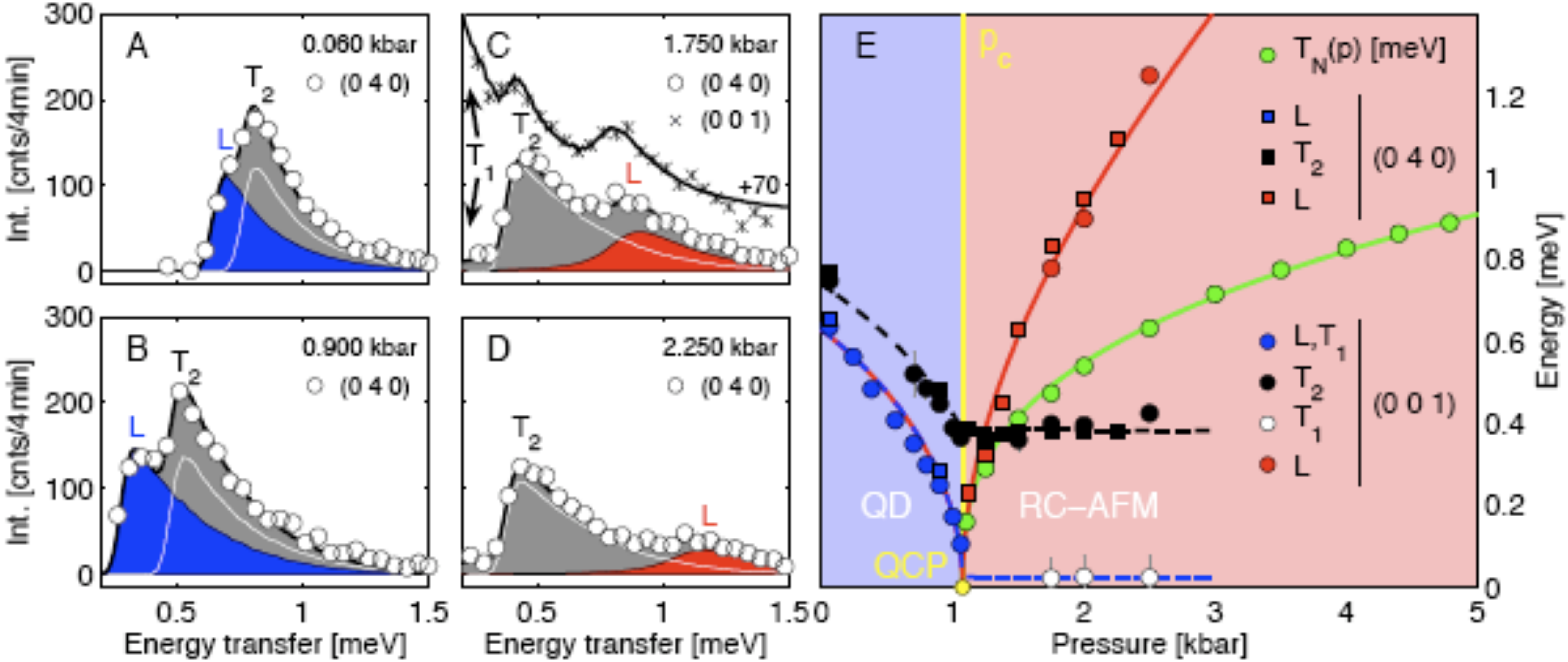


FIG. 1. Measured neutron profiles in the a^*c^* plane of $TlCuCl_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

$TlCuCl_3$ with varying pressure



Observation of 3 → 2 low energy modes, emergence of new longitudinal mode in Néel phase, and vanishing of Néel temperature at the quantum critical point

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, arXiv:0803.3720

S=1/2 insulator on the square lattice

$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} \right) \left(\mathbf{S}_k \cdot \mathbf{S}_l - \frac{1}{4} \right)$$

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 227202 (2007).
R.G. Melko and R.K. Kaul, *Phys. Rev. Lett.* **100**, 017203 (2008).

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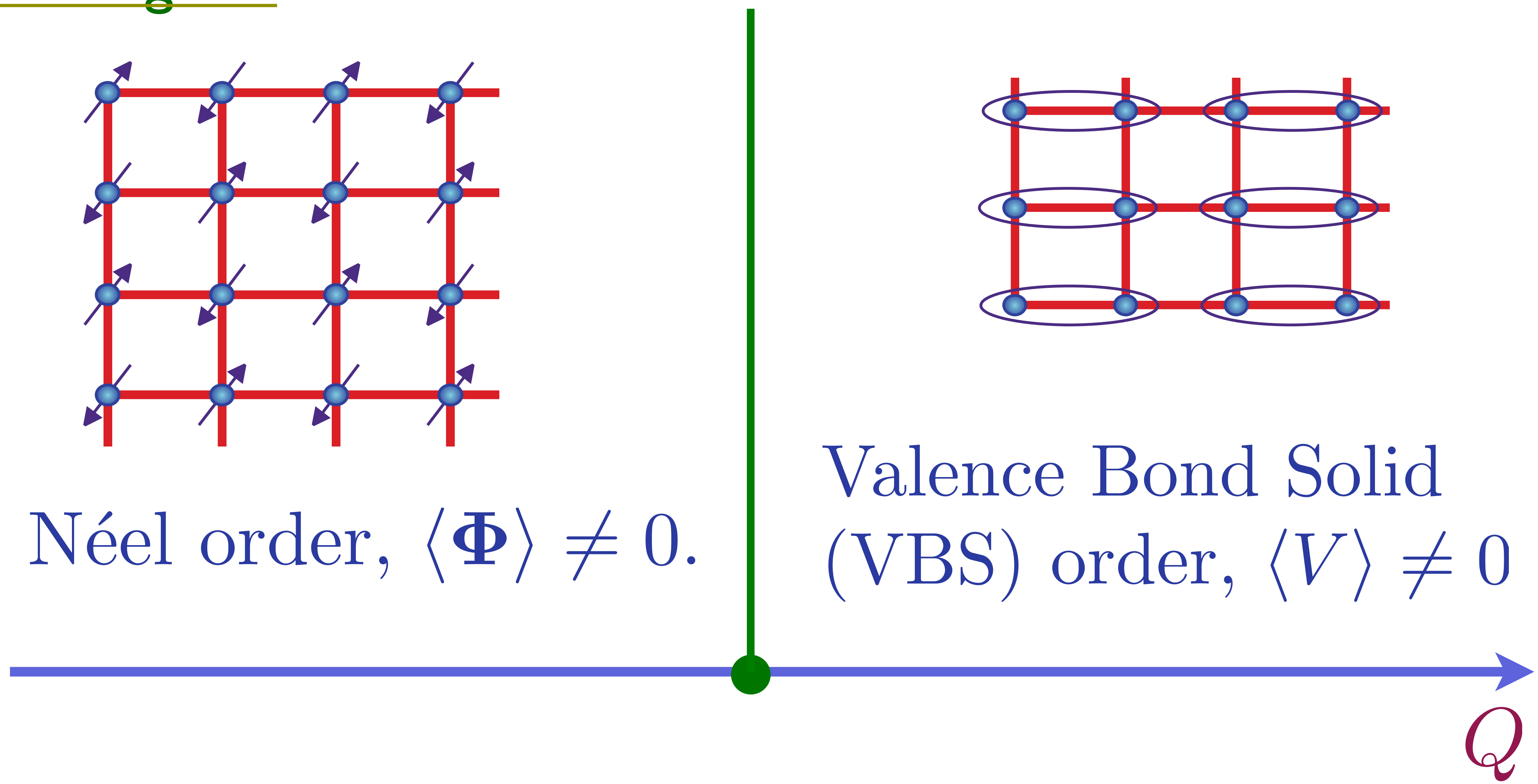
Phase diagram



A.W. Sandvik, *Phys. Rev. Lett.* **98**, 227202 (2007).
R.G. Melko and R.K. Kaul, *Phys. Rev. Lett.* **100**, 017203 (2008).
N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Theory for loss of Neel order

Decompose $\mathbf{\Phi}$ in terms of a complex scalar (a spinon) z_α, $\alpha = \uparrow, \downarrow$:

$$\mathbf{\Phi} = z_\alpha^\dagger \vec{\sigma}_{\alpha\beta} z_\beta$$

where $\vec{\sigma}$ are Pauli matrices. Theory must be invariant under the U(1) gauge transformation

$$z_\alpha \rightarrow e^{i\theta} z_\alpha$$

Low energy spinon theory for loss of Néel order is the CP^1 model

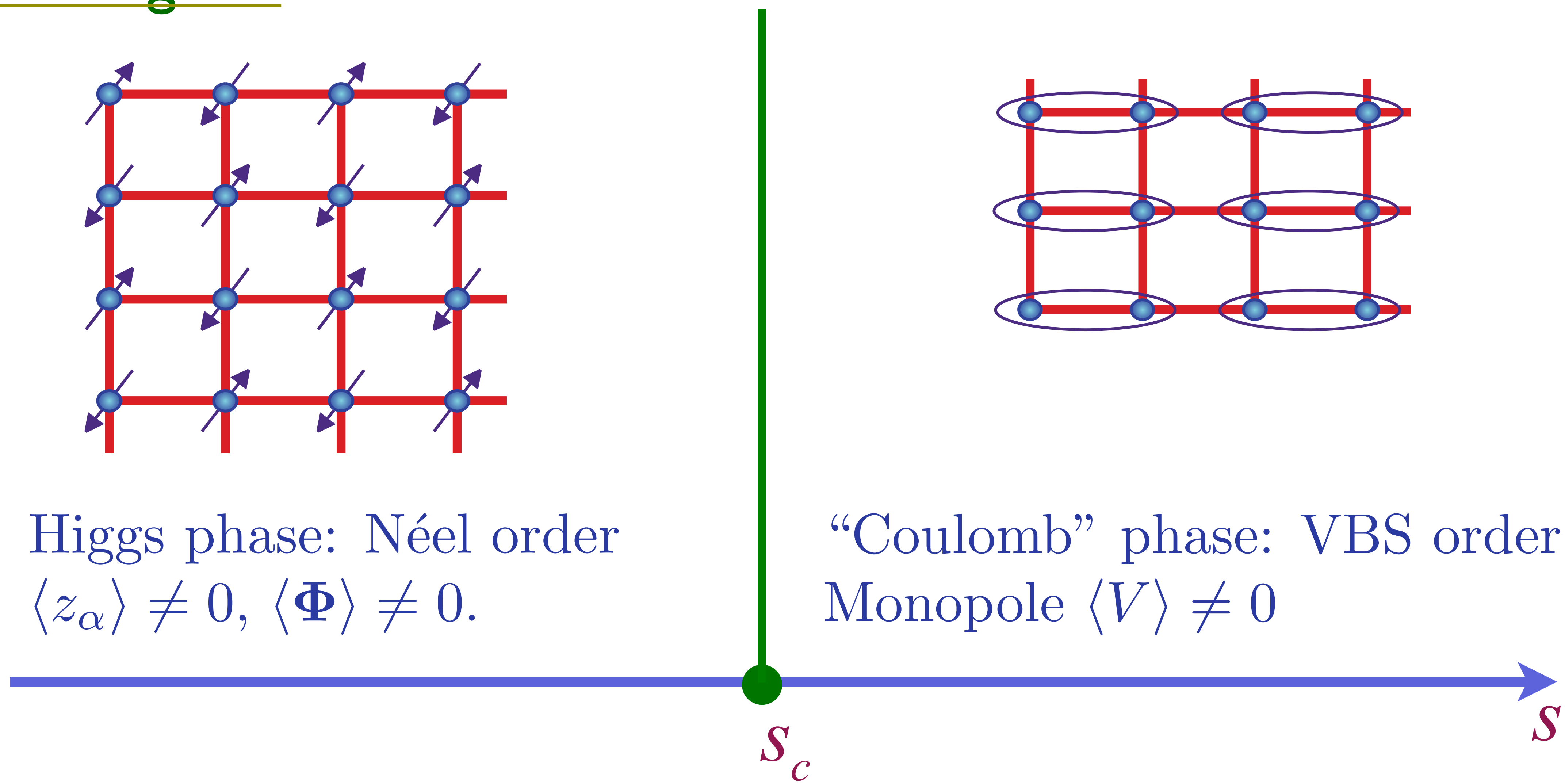
$$\begin{aligned}\mathcal{S}_z &= \int d^2x d\tau \Big[c^2 \left|(\nabla_x - iA_x) z_\alpha\right|^2 + \left|(\partial_\tau - iA_\tau) z_\alpha\right|^2 + s \left|z_\alpha\right|^2 \\ &\qquad + u \left(\left|z_\alpha\right|^2\right)^2 + \frac{1}{4e^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \Big]\end{aligned}$$

where A_μ is an emergent U(1) gauge field.
The monopole creation operator V is identical to the VBS order parameter (the global "shift" symmetry of the dual photon is an enlargement of the lattice rotation symmetry).

S=1/2 insulator on the square lattice

$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \tfrac{1}{4} \right) \left(\mathbf{S}_k \cdot \mathbf{S}_l - \tfrac{1}{4} \right)$$

Phase diagram

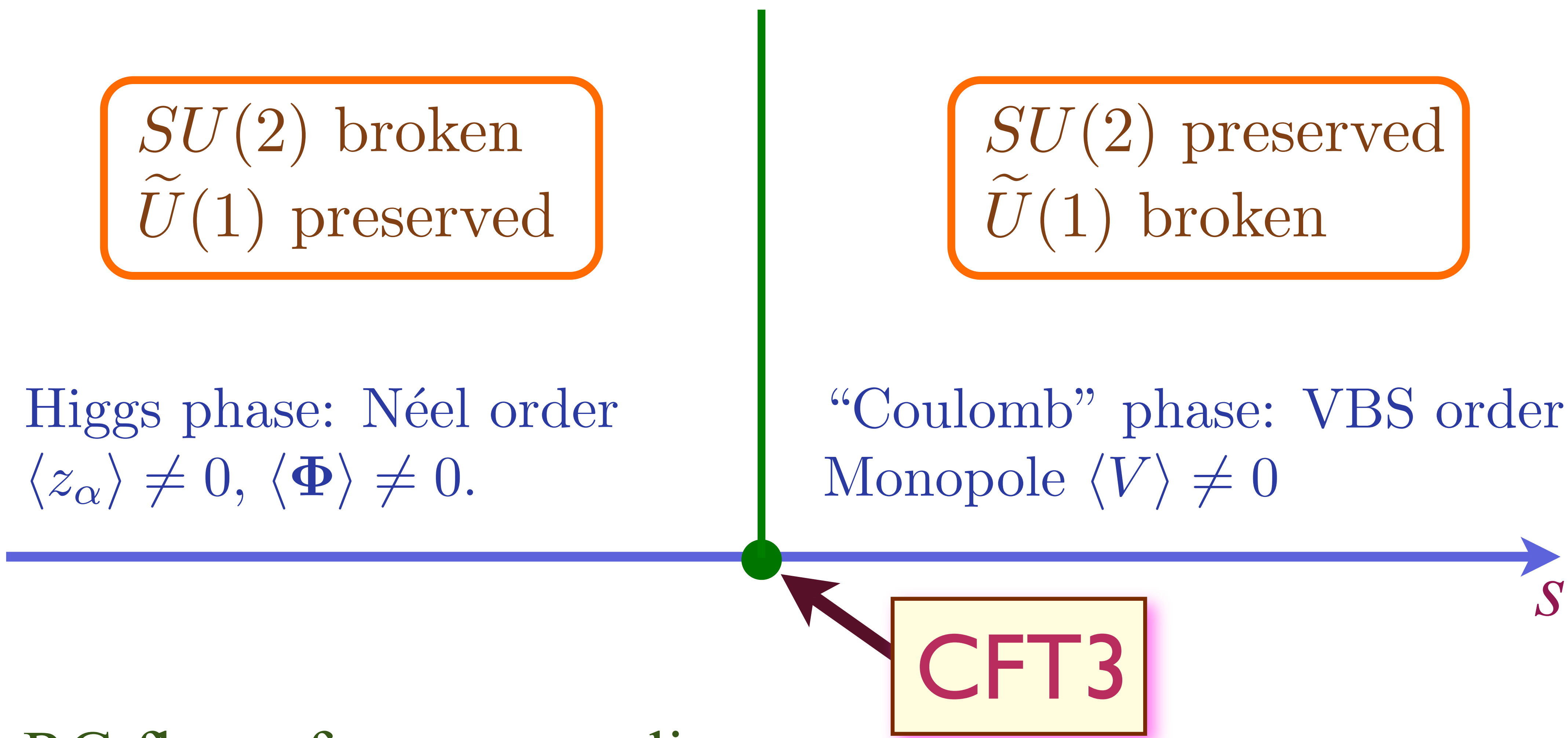


$$\mathcal{S}_z = \int d^2 r d\tau \left[|(\partial_\mu - i A_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right]$$

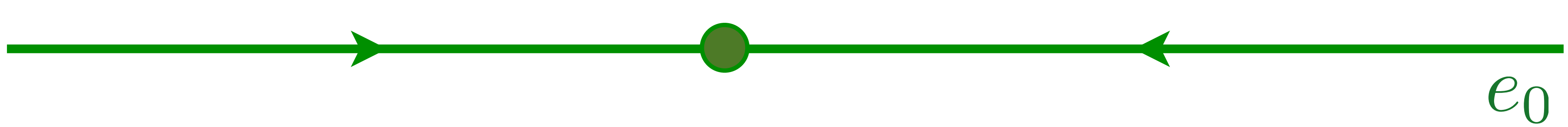
S=1/2 insulator on the square lattice

$$\mathcal{S}_z = \int d^2 r d\tau \left[|(\partial_\mu - i A_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \right]$$

Global symmetry: $SU(2) \times \widetilde{U}(1)$ (the $\widetilde{U}(1)$ is the dual photon shift)



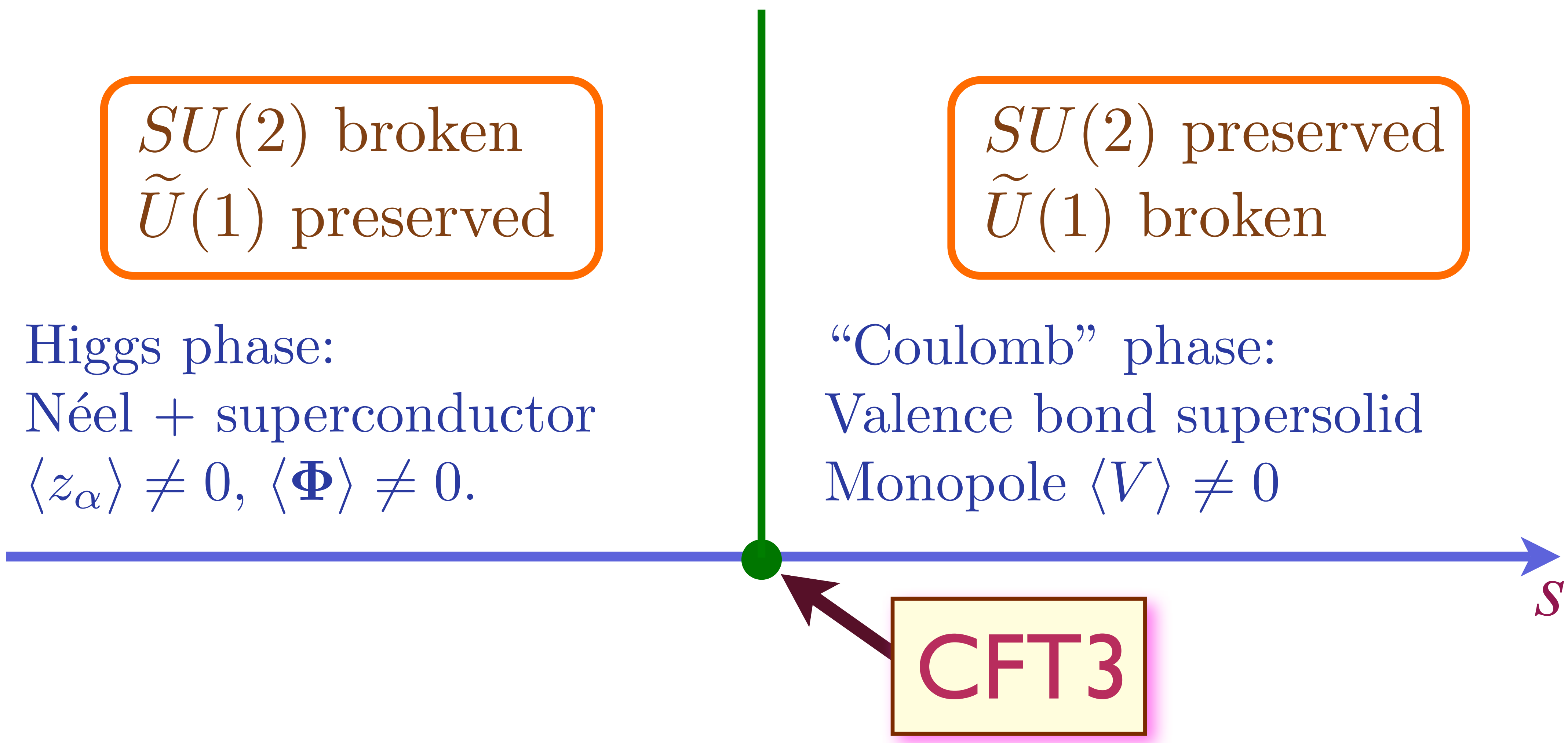
RG flow of gauge coupling:



d-wave superconductor on the square lattice

$$\mathcal{S}_z = \int d^2 r d\tau \left[|(\partial_\mu - iA_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 + \overline{\psi}_a \gamma^\mu \partial_\mu \psi_a \right]$$

Global symmetry: $SU(2) \times \widetilde{U}(1)$ (the $\widetilde{U}(1)$ is the dual photon shift)

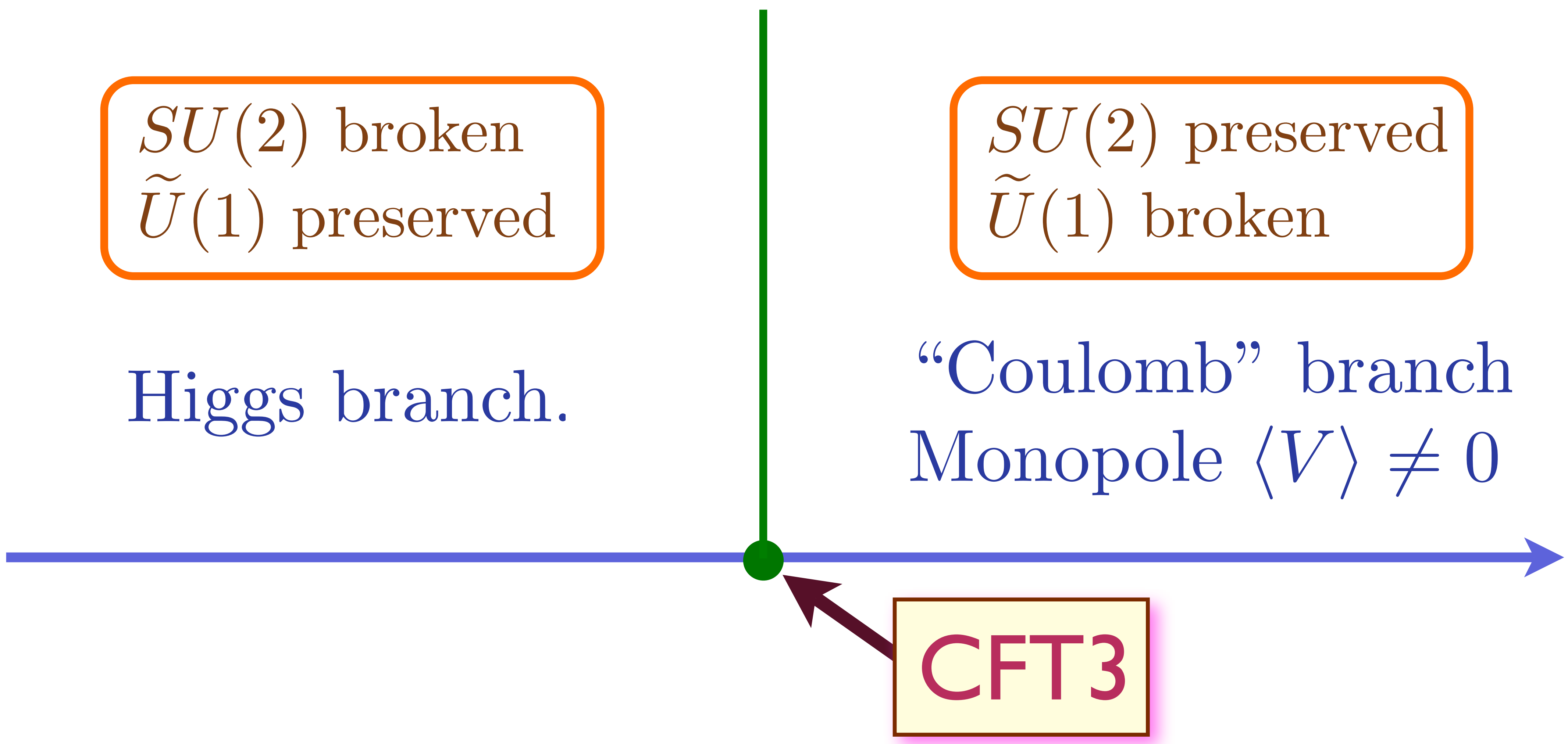


R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, *Nature Physics* **4**, 28 (2008)
R. K. Kaul, M.A. Metlitski, S. Sachdev, and C. Xu, arXiv:0804.1794

U(1) gauge theory with $\mathcal{N}$=4 supersymmetry

Theory with a U(1) vector multiplet $\mathcal{V}$ and 2 hypermultiplets $\mathcal{Q}_i$.

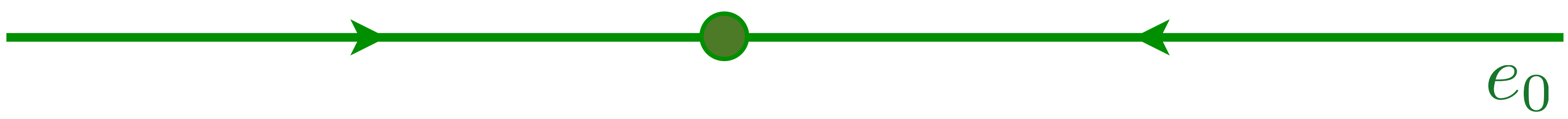
Global symmetry: $SU(2) \times \widetilde{U}(1) \times SO(4)_R$



N. Seiberg and E. Witten, hep-th/9607163
K.A. Intriligator and N. Seiberg, hep-th/9607207
A. Kapustin and M. J. Strassler, hep-th/9902033

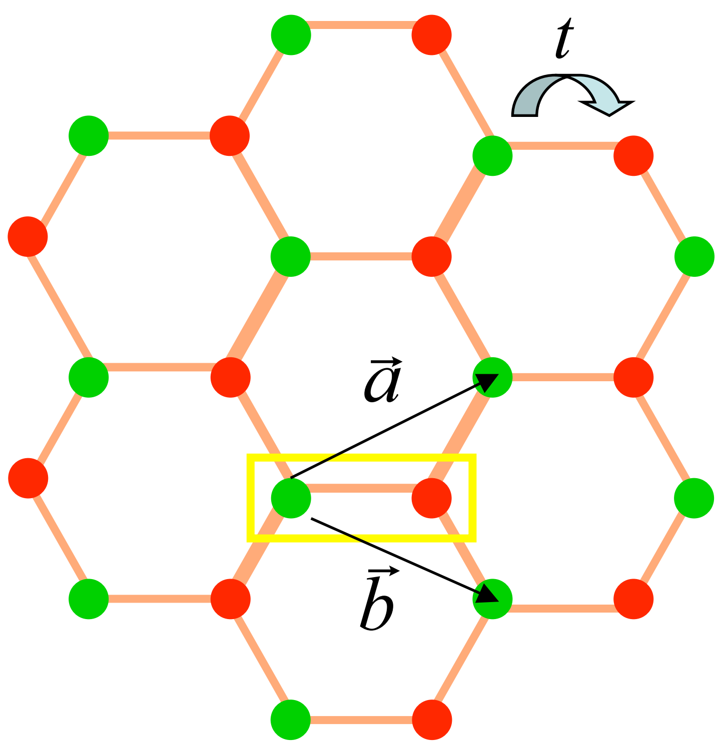
SU(N) gauge theory with $\mathcal{N}$=8 supersymmetry (SYM3)

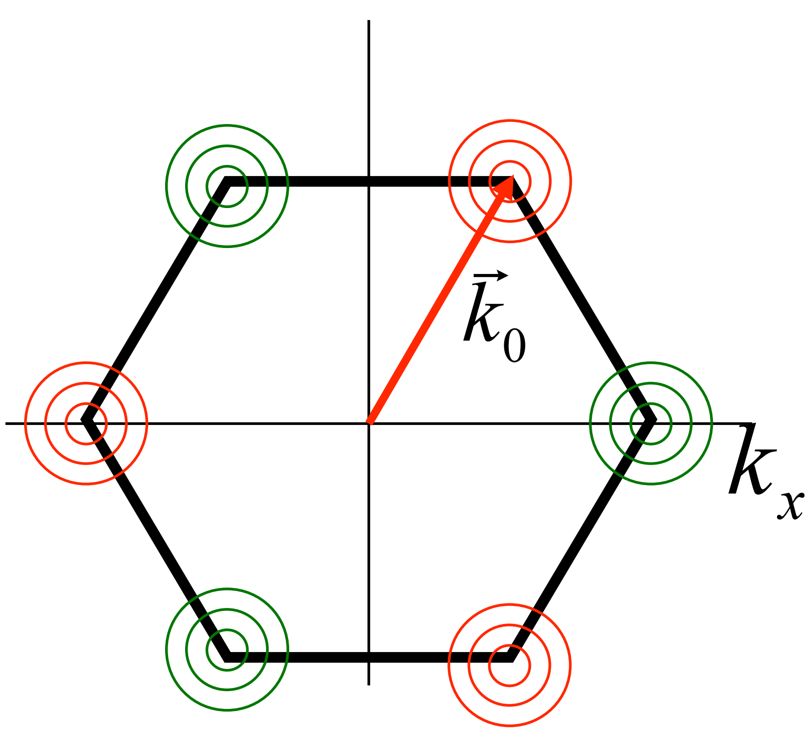
Unique theory with a single gauge coupling constant e_0.

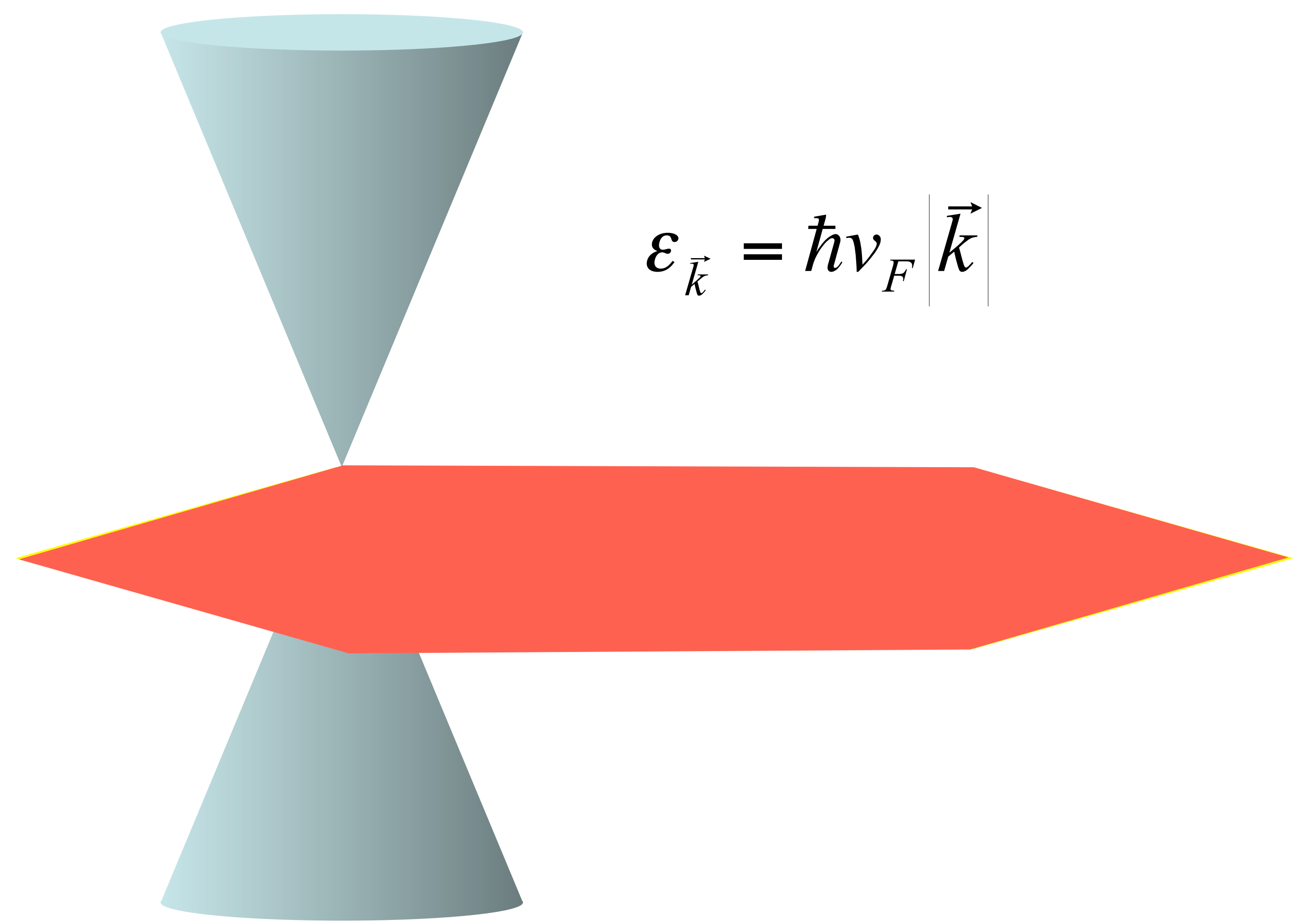


RG flow to an attractive fixed point

Graphene







$$\varepsilon_{\vec{k}} = \hbar v_F \left|\vec{k}\right|$$

Graphene

Low energy theory has 4 two-component Dirac fermions, ψ_α, $\alpha = 1 \ldots 4$, interacting with a $1/r$ Coulomb interaction

$$\begin{aligned} \mathcal{S} &= \int d^2 r d\tau \psi_\alpha^\dagger \left(\partial_\tau - i v_F \vec{\sigma} \cdot \vec{\nabla} \right) \psi_\alpha \\ &\quad + \frac{e^2}{2} \int d^2 r d^2 r' d\tau \psi_\alpha^\dagger \psi_\alpha(r) \frac{1}{|r - r'|} \psi_\beta^\dagger \psi_\beta(r') \end{aligned}$$

Dimensionless "fine-structure" constant $\alpha = e^2/(4\hbar v_F)$.
RG flow of α:

$$\frac{d\alpha}{d\ell} = -\alpha^2 + \ldots$$

Behavior is similar to a CFT3 with $\alpha \sim 1/\ln(\textbf{scale})$.

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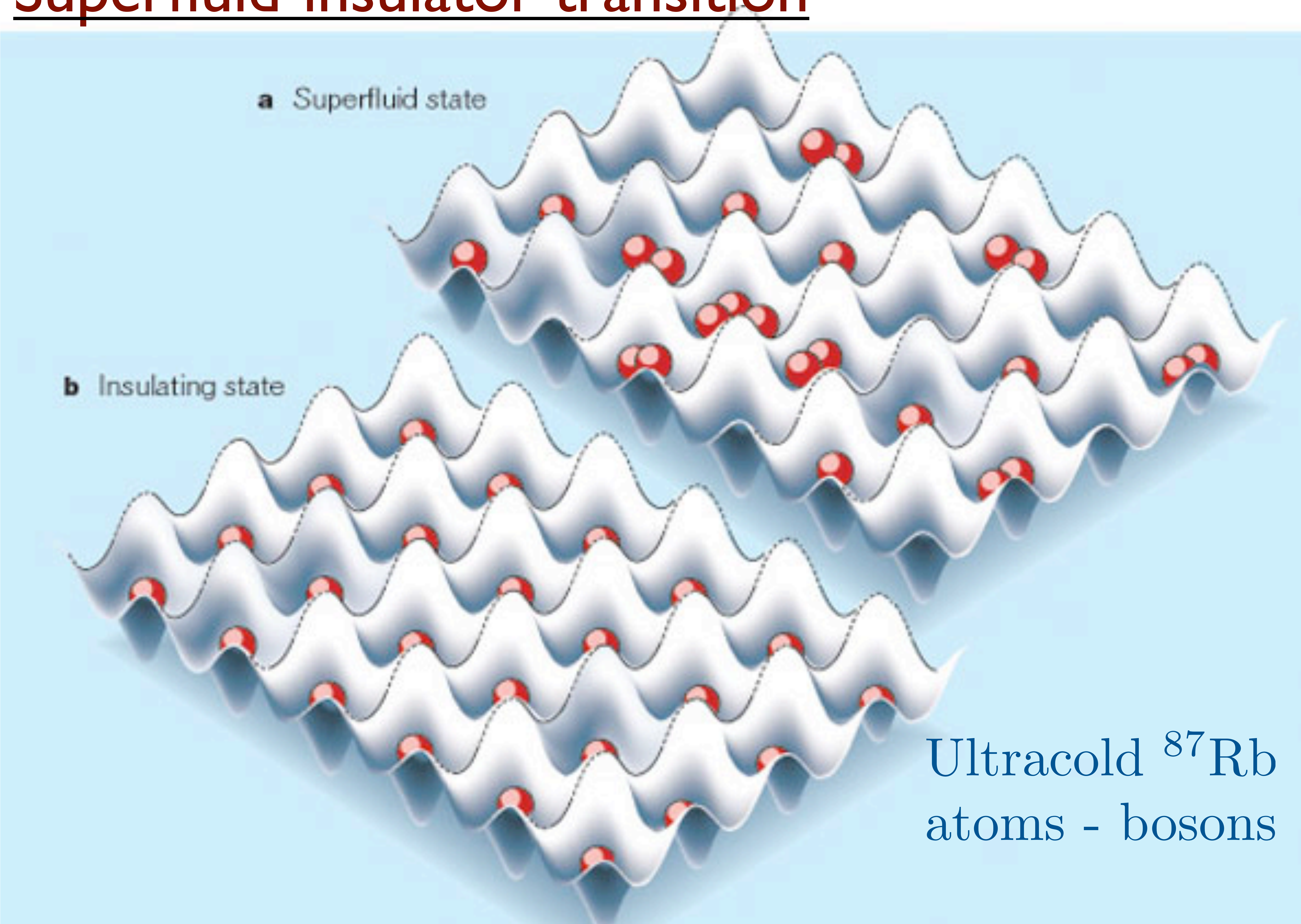
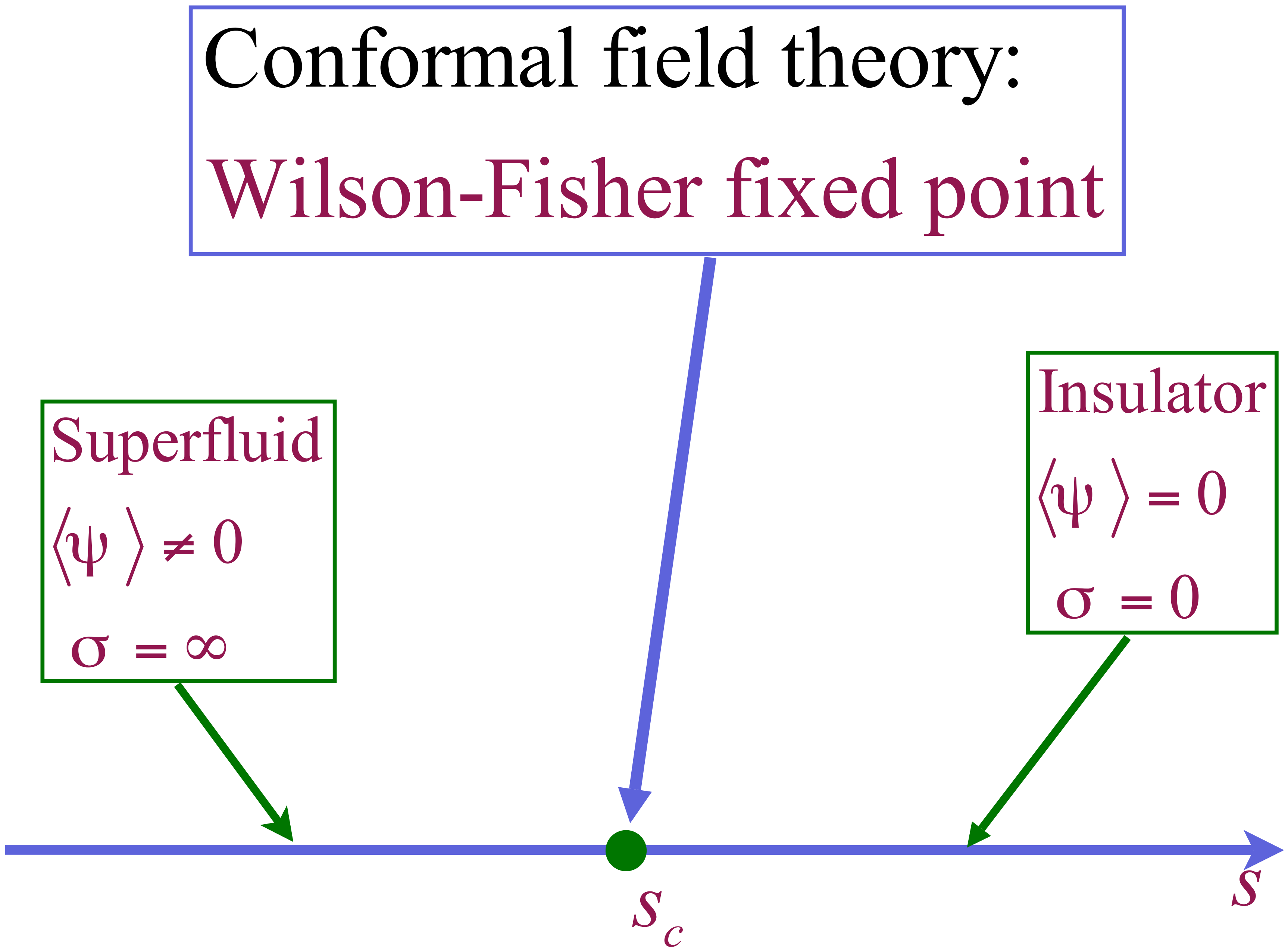
Superfluid-insulator transition

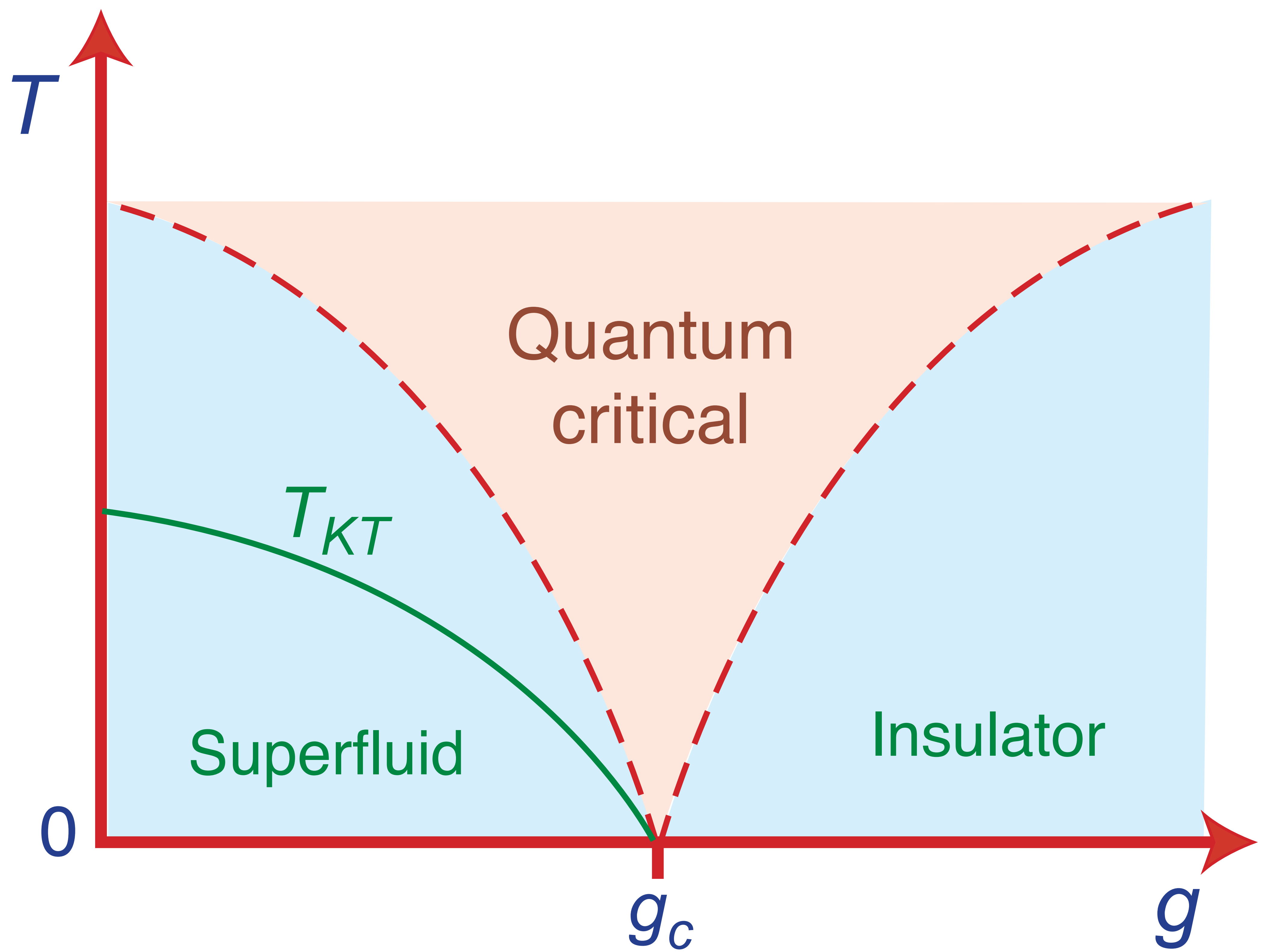


Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).



$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + c^2 |\vec{\nabla} \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$



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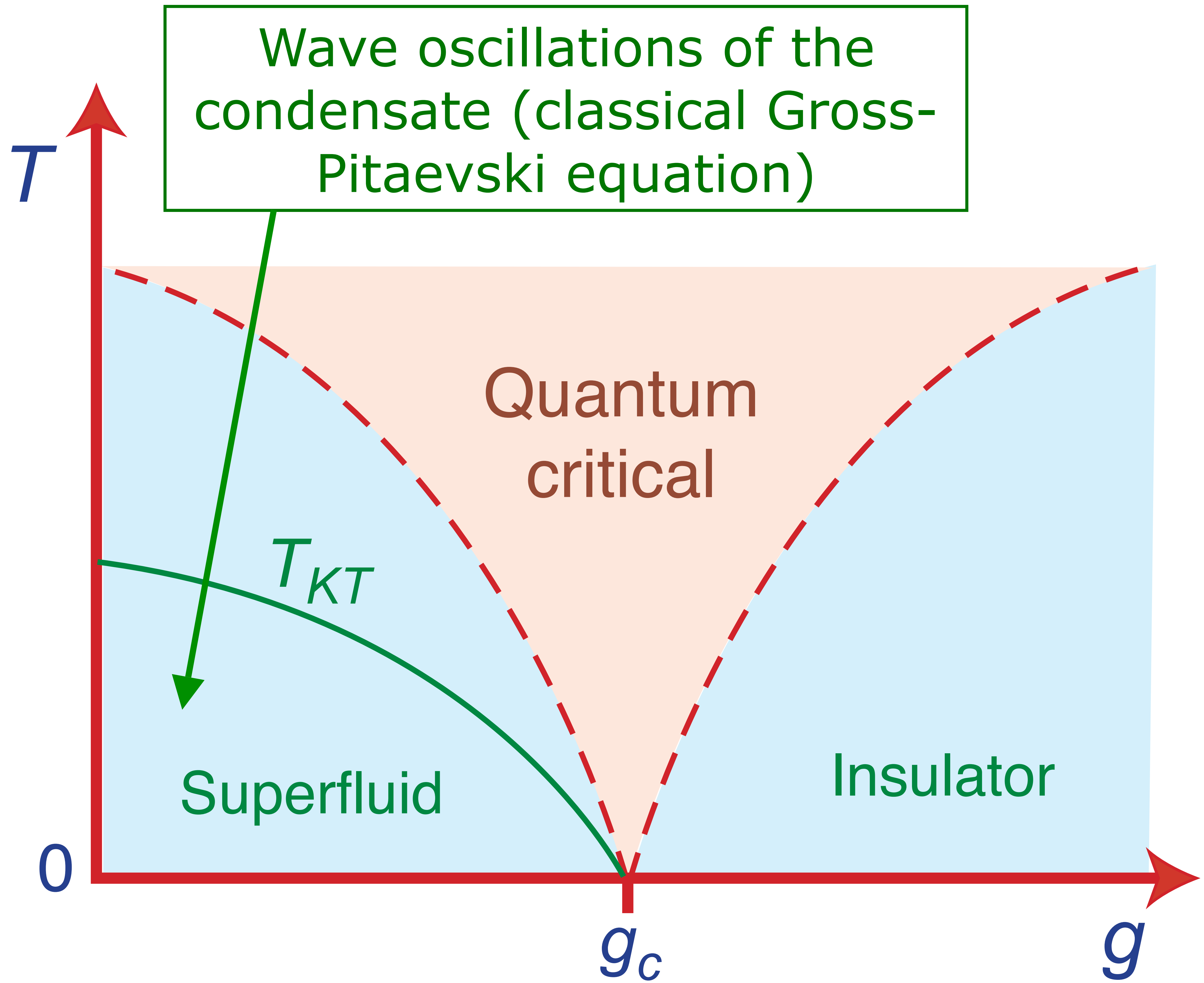
Wave oscillations of the condensate (classical Gross-Pitaevski equation)
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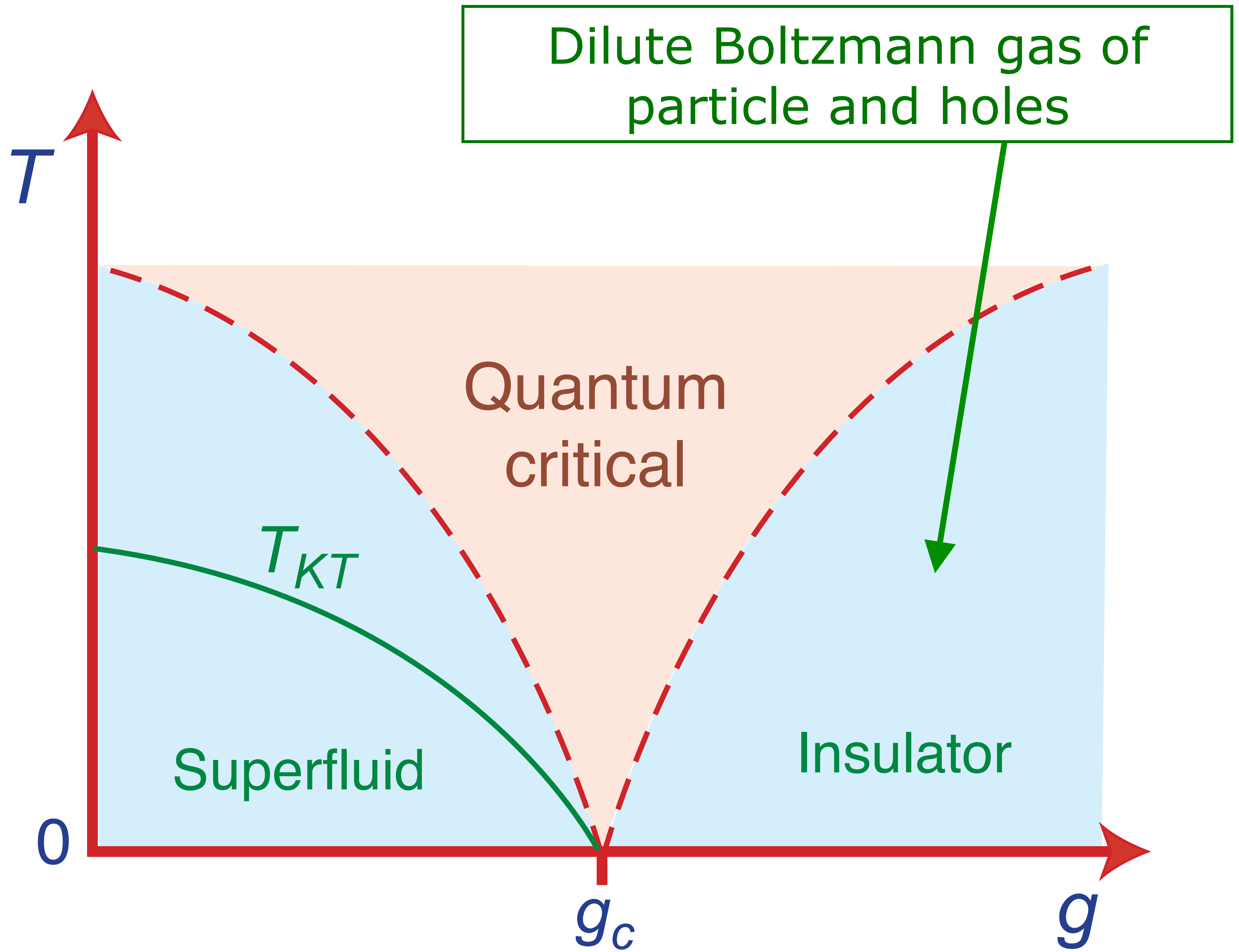
Dilute Boltzmann gas of
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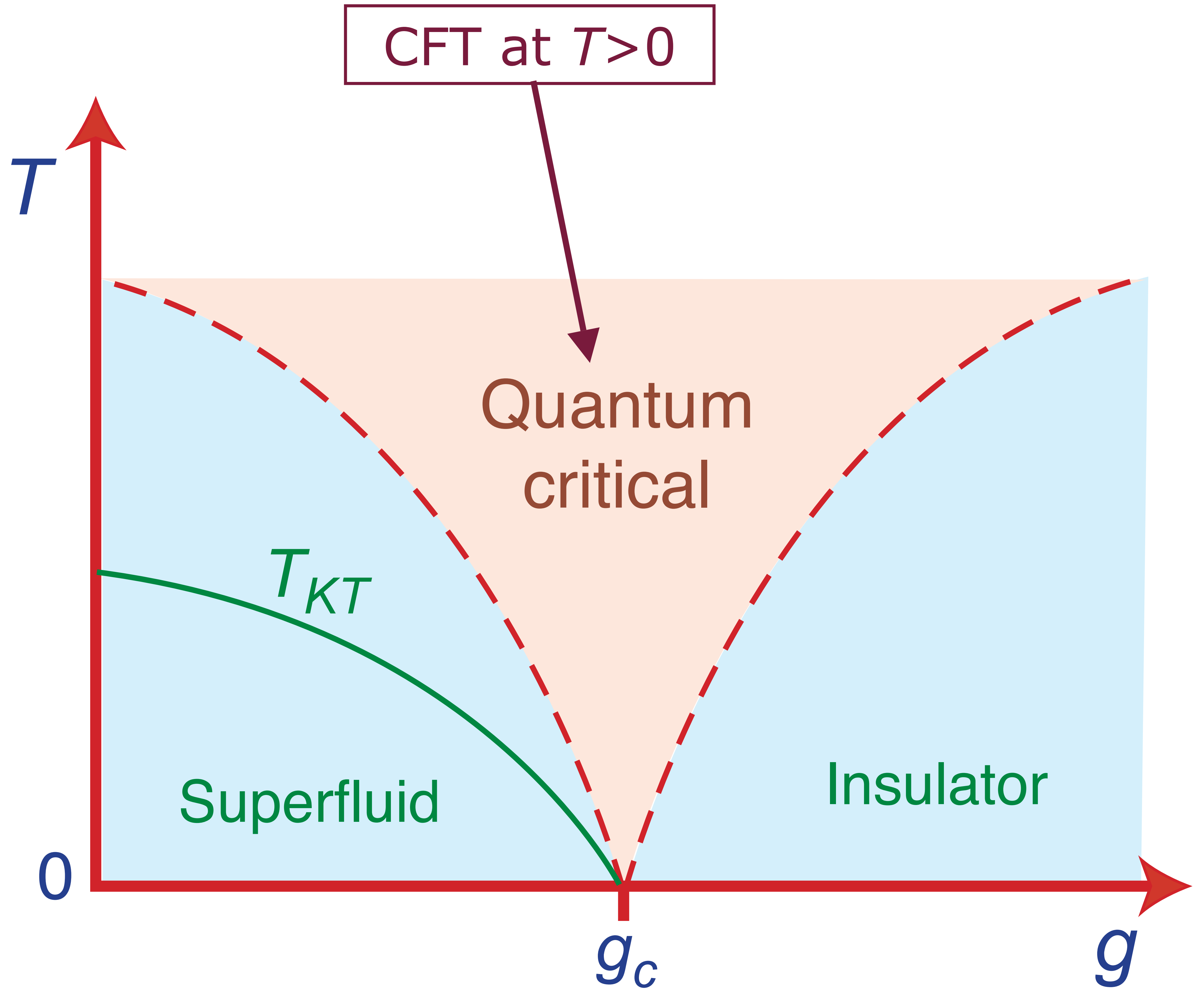
g_c
g


CFT at T>0
T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g

Resistivity of Bi films

Conductivity σ

$$\sigma_{\text{Superconductor}}(T \to 0) = \infty$$

$$\sigma_{\text{Insulator}}(T \to 0) = 0$$

$$\sigma_{\text{Quantum critical point}}(T \to 0) \approx \frac{4e^2}{h}$$

D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

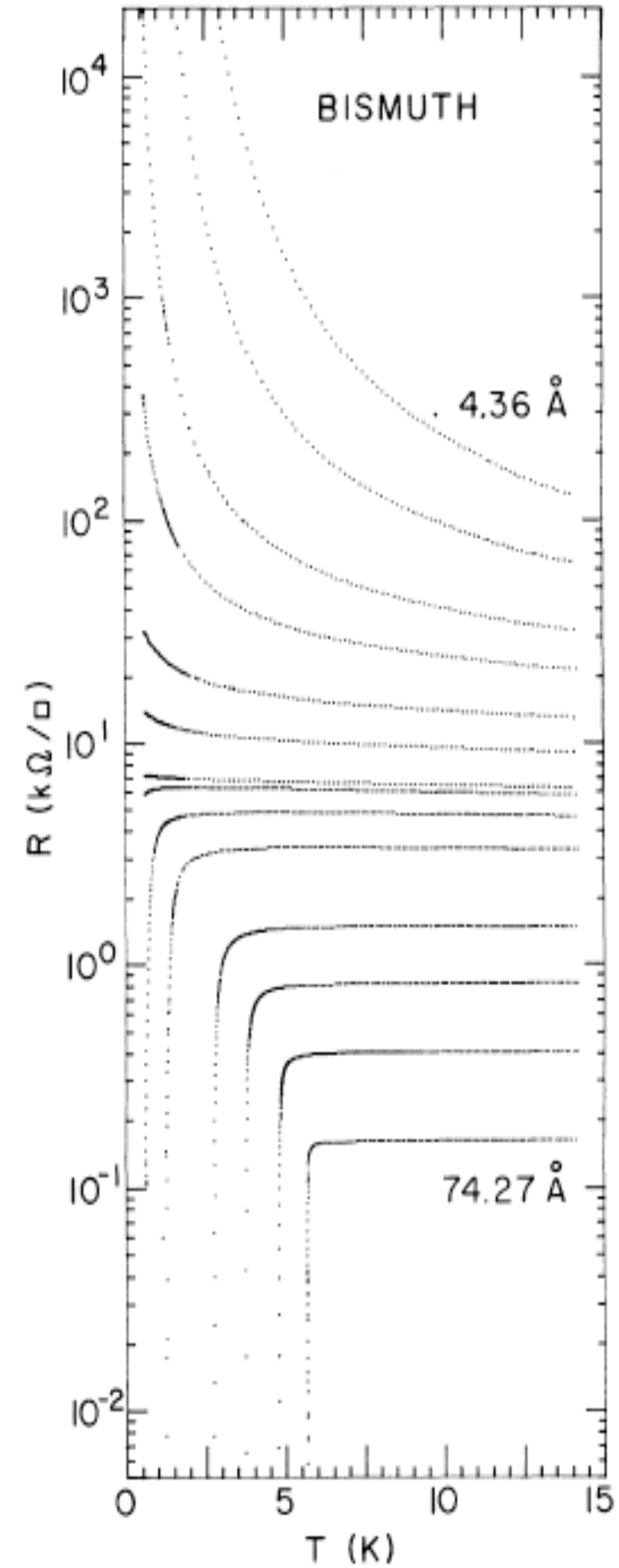


FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k, \omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

For *all* CFT3s, at $\hbar\omega \gg k_B T$

$$\chi(k, \omega) \quad = \quad \frac{4e^2}{h} K \frac{k^2}{\sqrt{v^2 k^2 - \omega^2}} \quad ; \quad \sigma(\omega) = \frac{4e^2}{h} K$$

where K is a universal number characterizing the CFT3, and v is the velocity of "light".

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k\to 0} \frac{-i\omega}{k^2}\chi(k,\omega)$

However, for *all* CFT3s, at $\underline{\hbar\omega \ll k_B T}$, we have the Einstein relation

$$\chi(k,\omega) = 4e^2\chi_c \frac{Dk^2}{Dk^2 - i\omega} \quad ; \quad \sigma(\omega) = 4e^2 D\chi_c = \frac{4e^2}{h}\Theta_1\Theta_2$$

where the **compressibility**, χ_c, and the **diffusion constant** D obey

$$\chi = \frac{k_B T}{(hv)^2}\Theta_1 \quad ; \quad D = \frac{hv^2}{k_B T}\Theta_2$$

with Θ_1 and Θ_2 universal numbers characteristic of the CFT3

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Density correlations in CFTs at $T > 0$

In CFT3s collisions are "phase" randomizing, and lead to relaxation to local thermodynamic equilibrium. So there is a crossover from collisionless behavior for $\hbar\omega \gg k_B T$, to hydrodynamic behavior for $\hbar\omega \ll k_B T$.

$$\sigma(\omega) = \begin{cases} \dfrac{4e^2}{h} K & , \quad \hbar\omega \gg k_B T \\ \dfrac{4e^2}{h} \Theta_1 \Theta_2 & , \quad \hbar\omega \ll k_B T \end{cases}$$

and in general we expect $K \neq \Theta_1 \Theta_2$ (verified for Wilson-Fisher fixed point).

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Density correlations in CFTs at $T > 0$

The transport is due to carriers which have a collision time $\tau_c \sim \hbar/(k_B T)$, independent of the strength of the microscopic collision cross-section. This leads to a "minimal conductivity" which is universal and of order e^2/h. More generally, away from the conformal point, this is the smallest possible collision time:

$$\tau_c \gtrsim \frac{\hbar}{k_B T}$$

For momentum transport, these arguments lead to

$$\frac{\text{Viscosity}}{\text{entropy density}} = \text{Universal number } \mathcal{O}(1)$$

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

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3. Black Hole Thermodynamics

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Black Holes

Objects so massive that light is gravitationally bound to them.

Black Holes

Objects so massive that light is gravitationally bound to them.

The region inside the black hole **horizon** is causally disconnected from the rest of the universe.

$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

Black Hole Thermodynamics

Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics

Entropy of a black hole $S = \dfrac{k_B A}{4\ell_P^2}$

where A is the area of the horizon, and

$\ell_P = \sqrt{\dfrac{G\hbar}{c^3}}$ is the Planck length.

The Second Law: $dA \geq 0$

Black Hole Thermodynamics

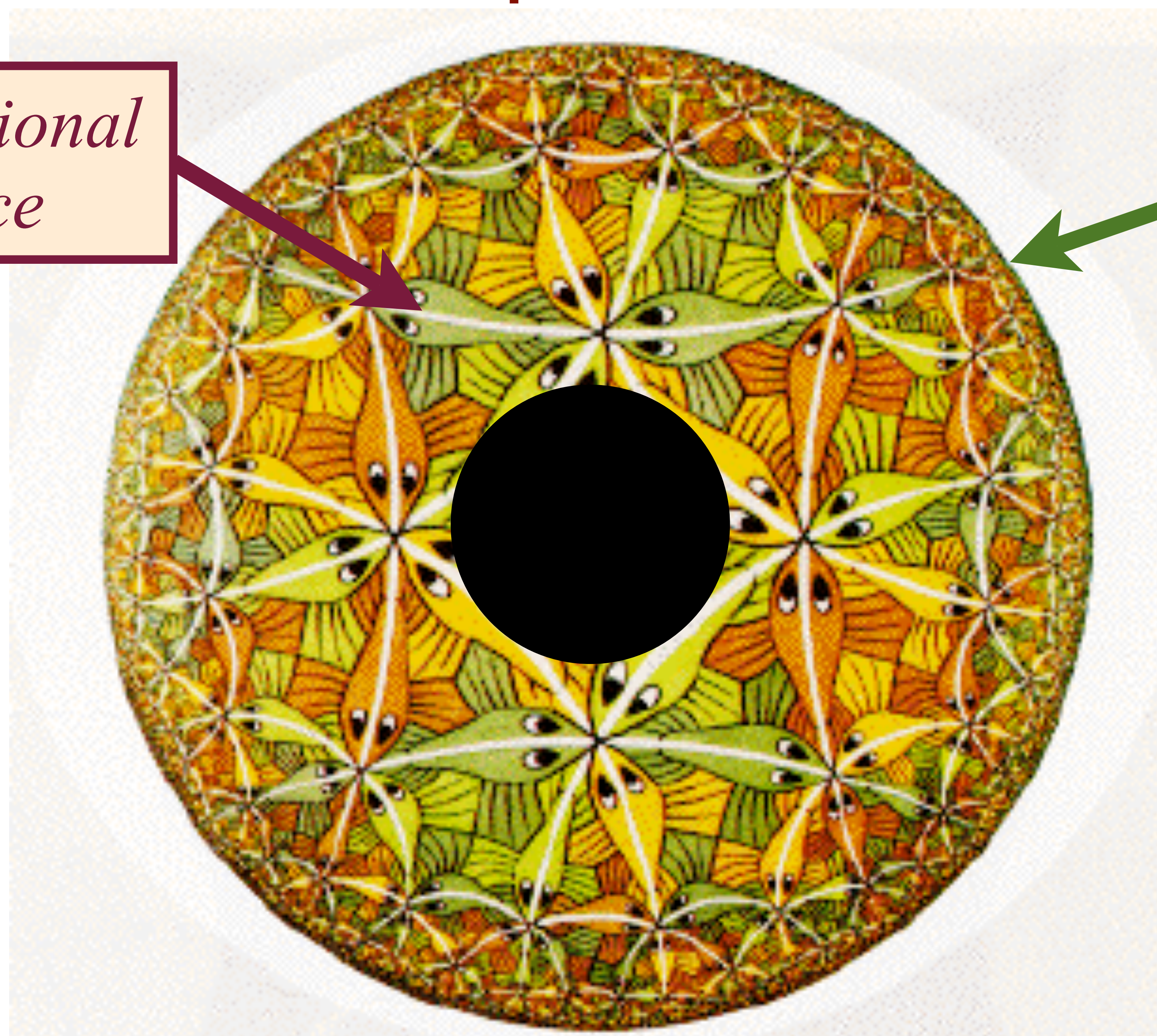
Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics

$$\text{Horizon temperature: } 4\pi k_B T = \frac{\hbar^2}{2M\ell_P^2}$$

AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions

3+1 dimensional AdS space



A 2+1 dimensional system at its quantum critical point

Maldacena, Gubser, Klebanov, Polyakov, Witten

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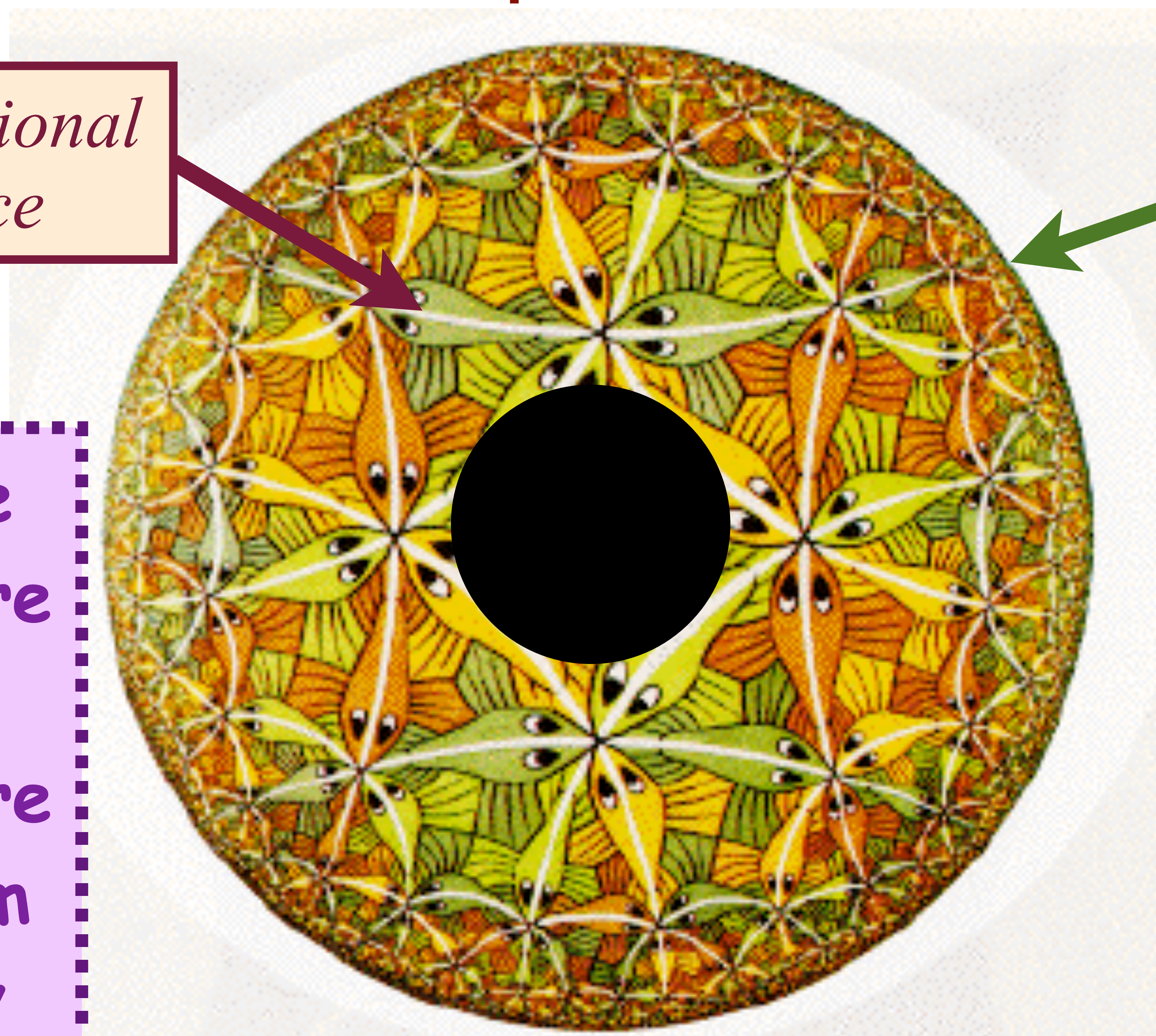
3+1 dimensional AdS space



Quantum criticality in 2+1 dimensions

Black hole temperature = temperature of quantum criticality

Maldacena, Gubser, Klebanov, Polyakov, Witten

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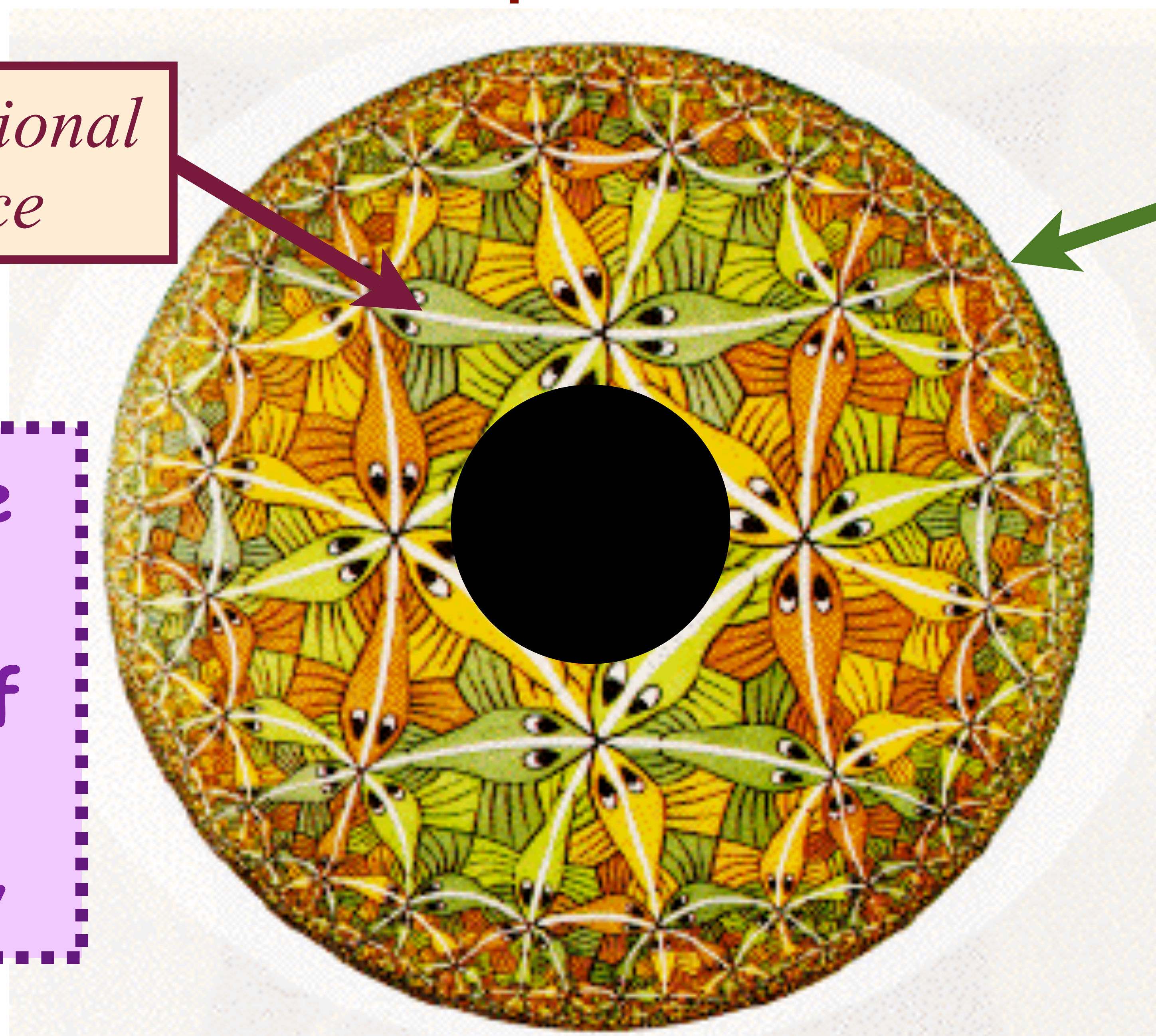

Quantum criticality in 2+1 dimensions

Black hole entropy = entropy of quantum criticality

Strominger, Vafa

AdS/CFT correspondence

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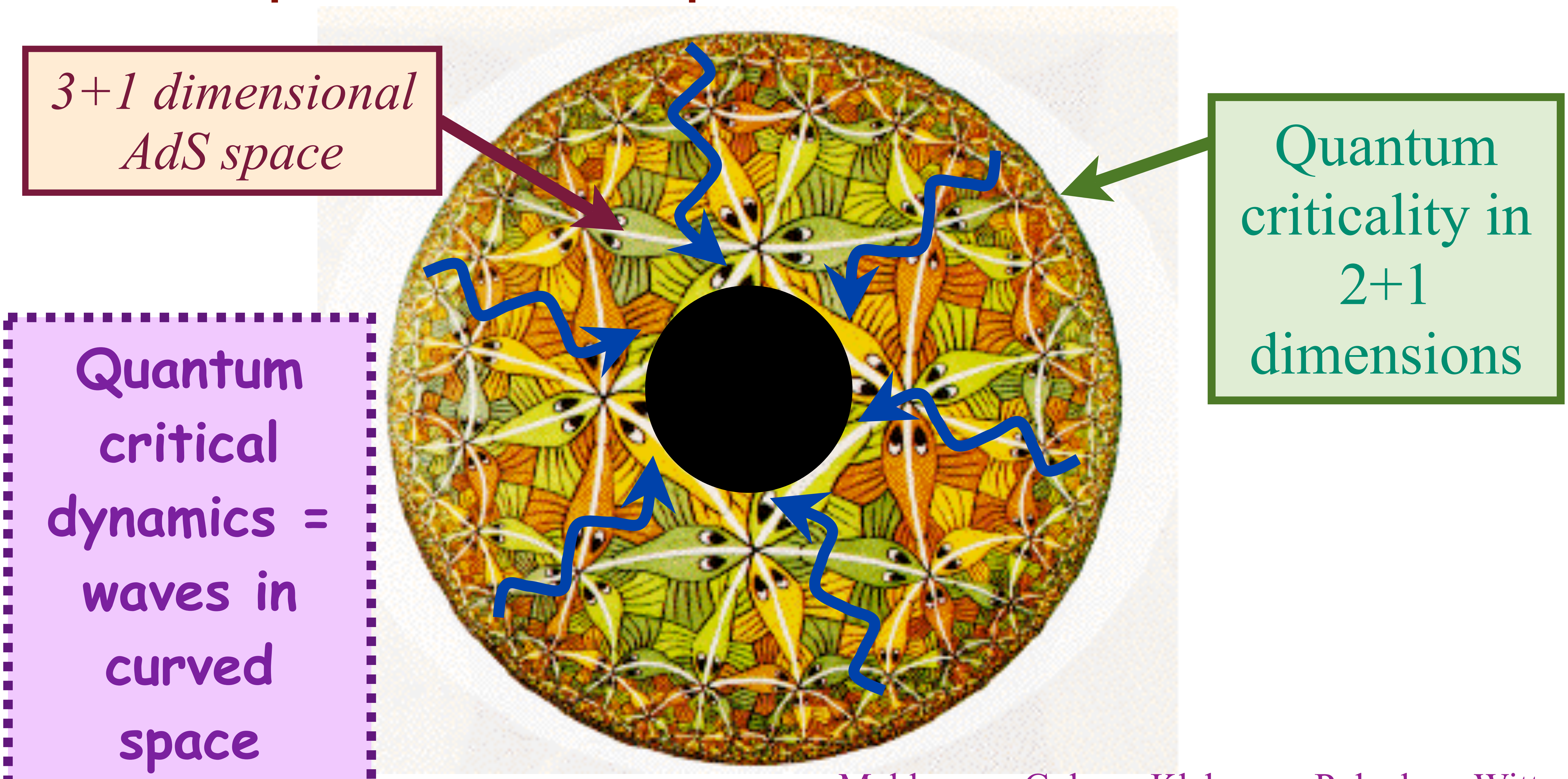

Maldacena, Gubser, Klebanov, Polyakov, Witten

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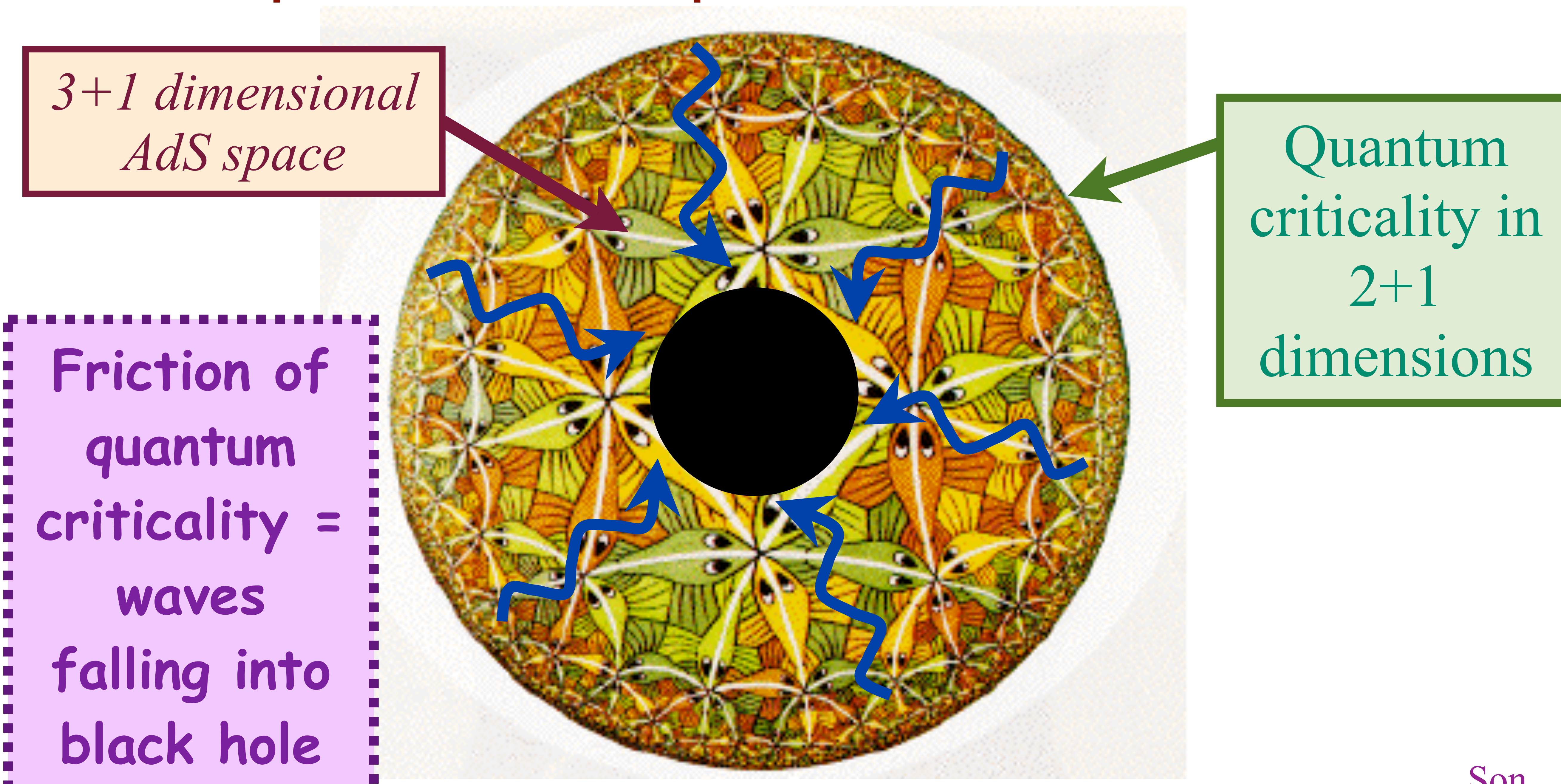
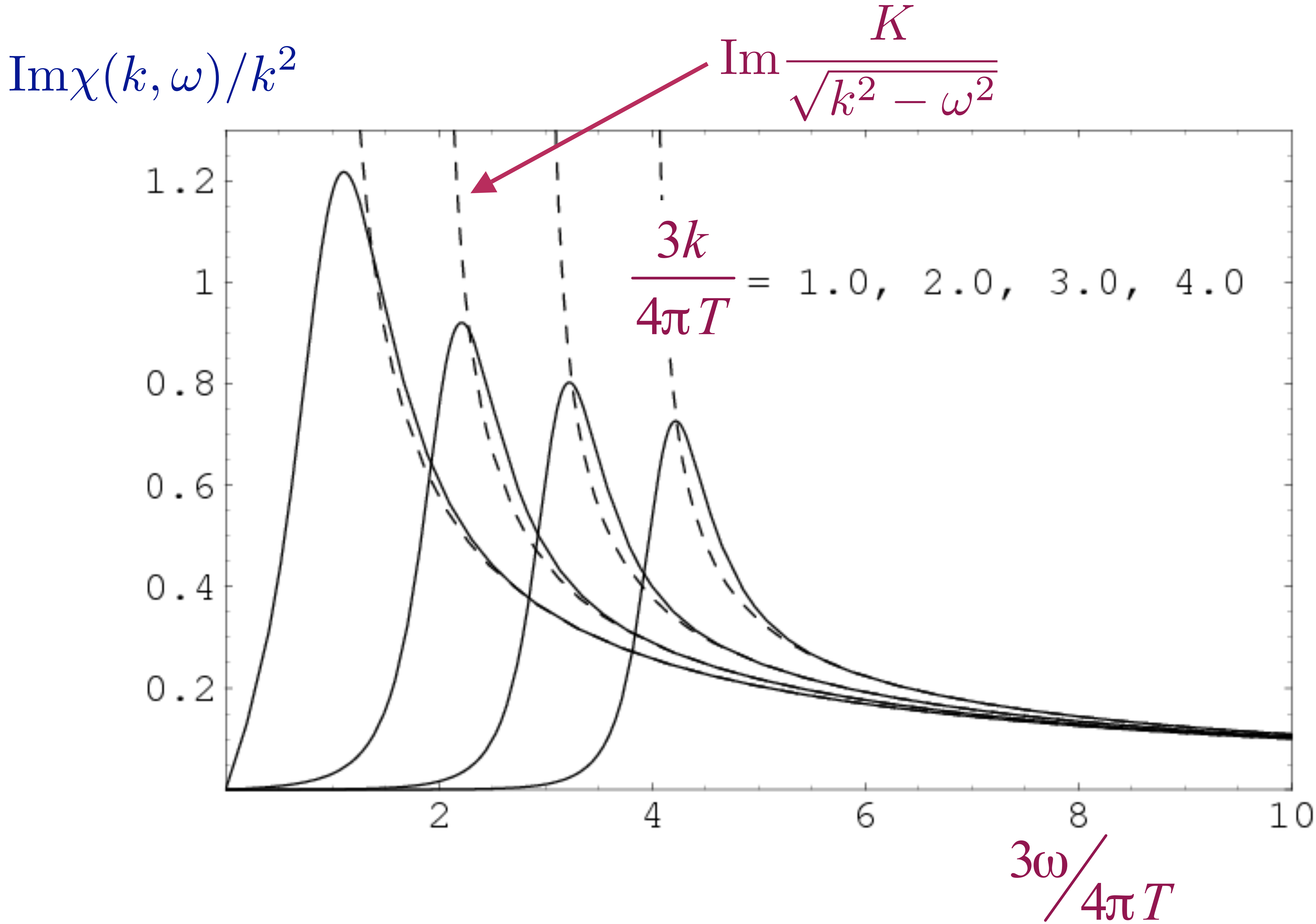


Son

Collisionless to hydrodynamic crossover of SYM3



P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

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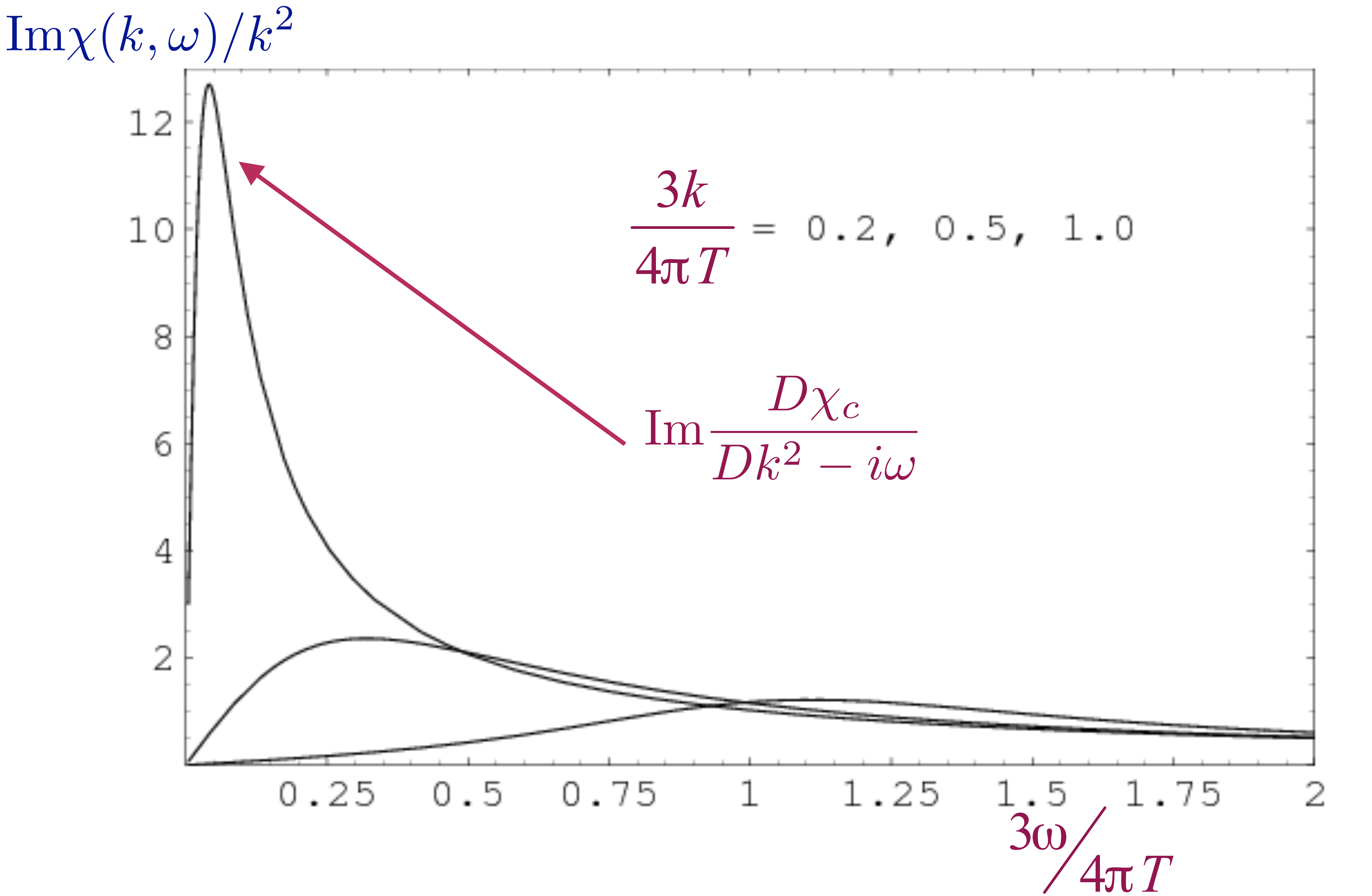

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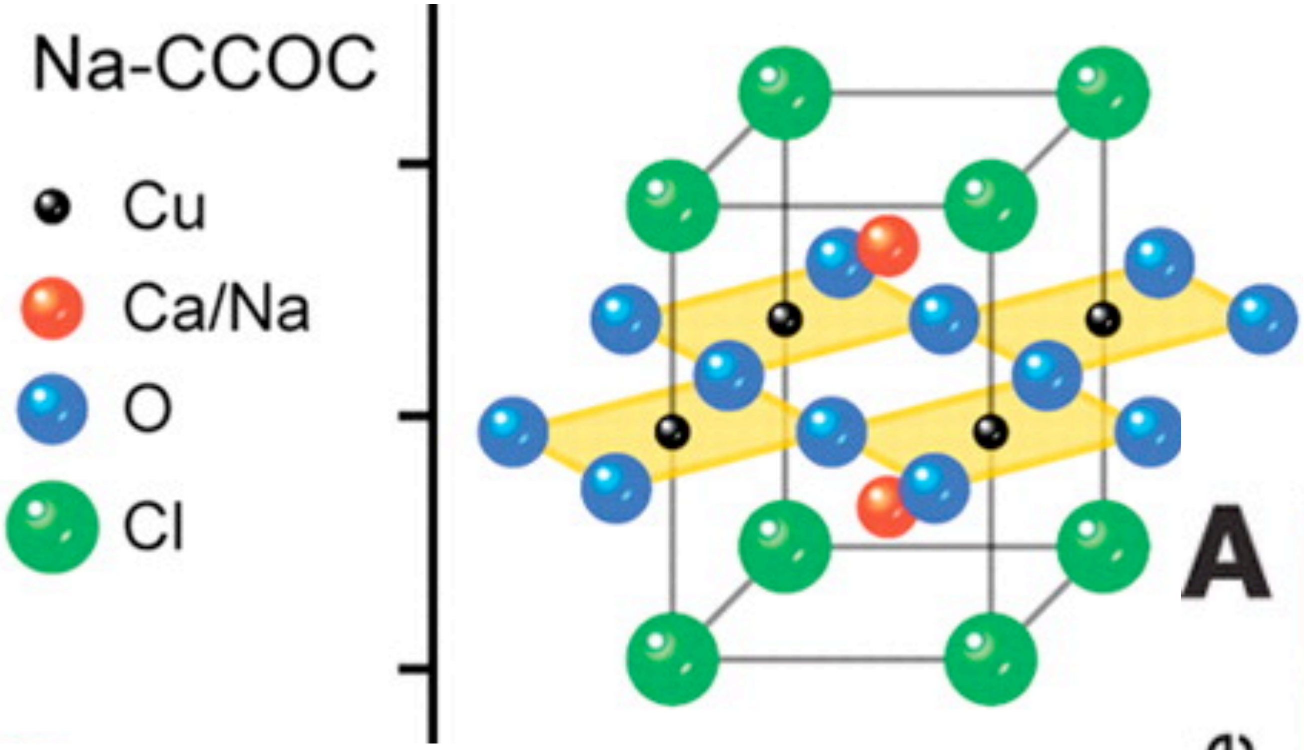
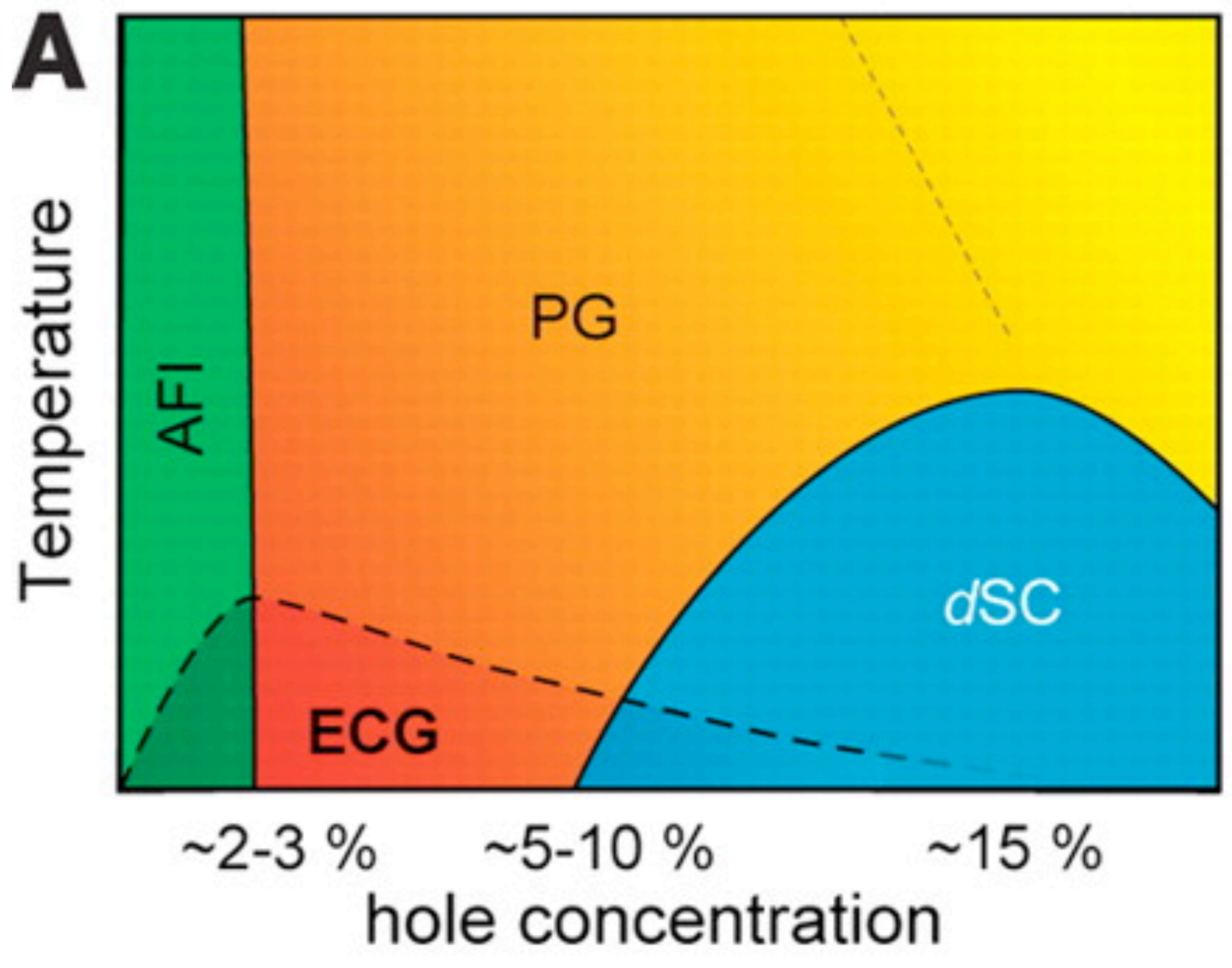
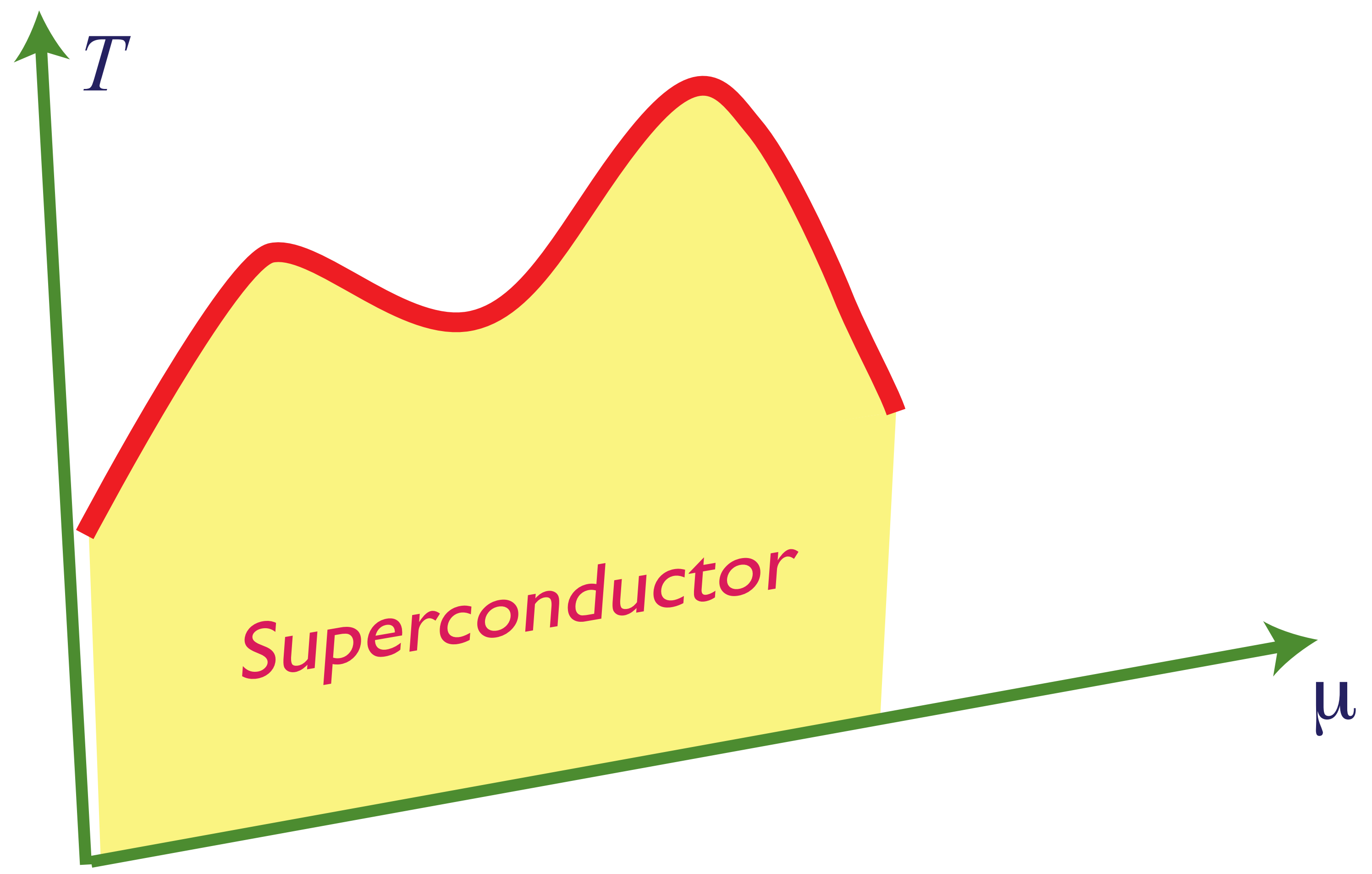
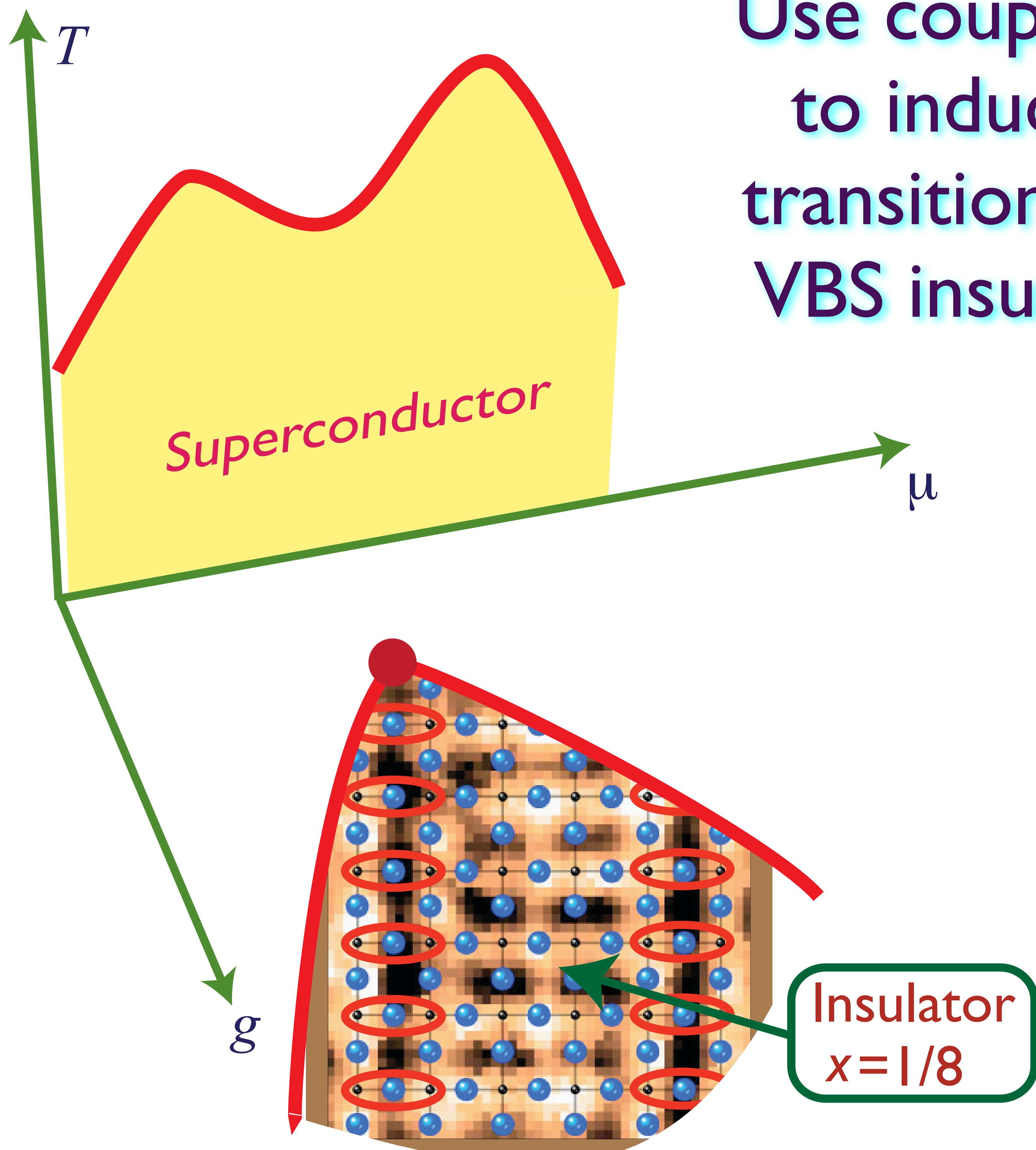
The cuprate superconductors



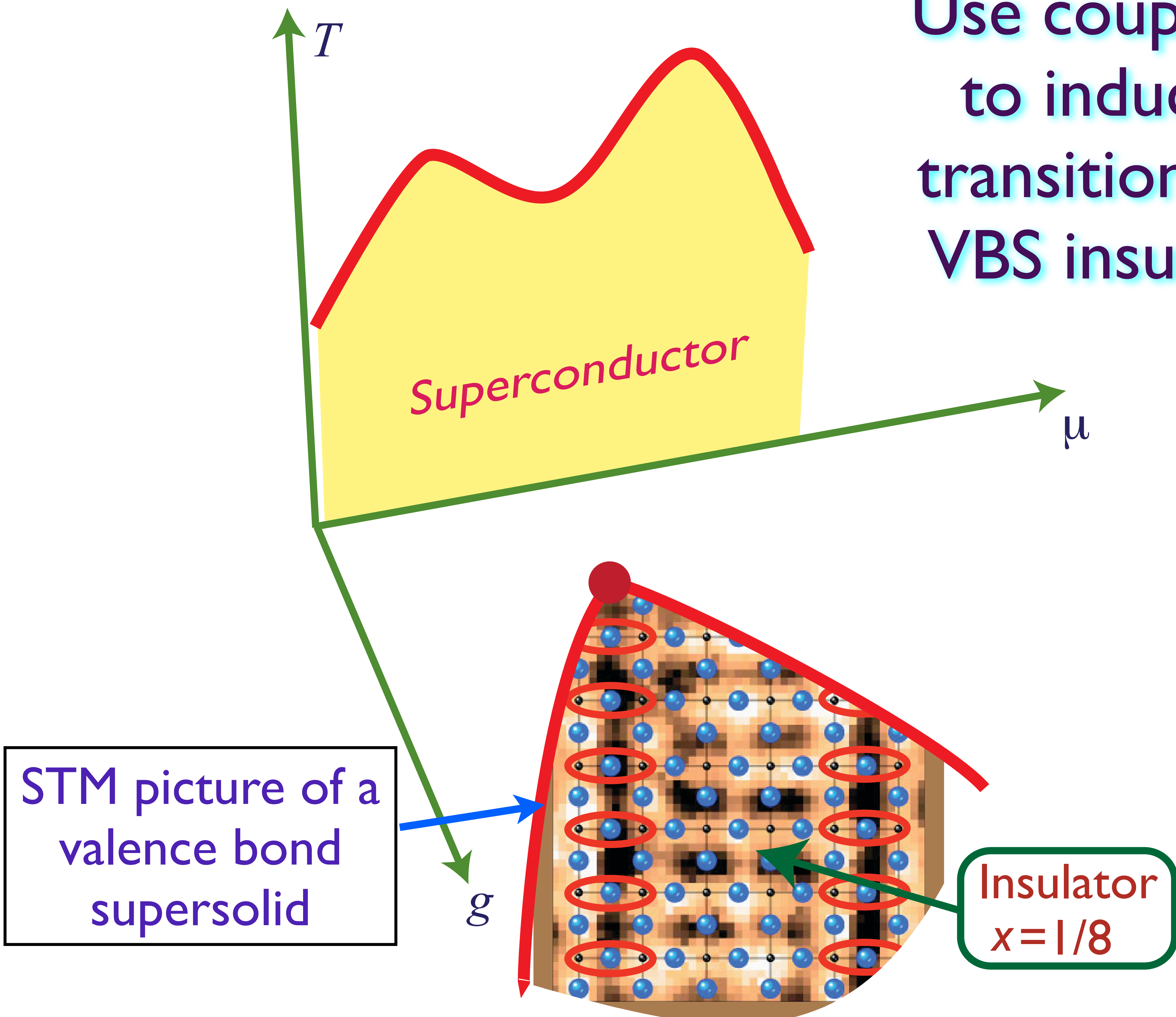



T
Superconductor
μ

Use coupling *g* to induce a transition to a VBS insulator



Use coupling g to induce a transition to a VBS insulator



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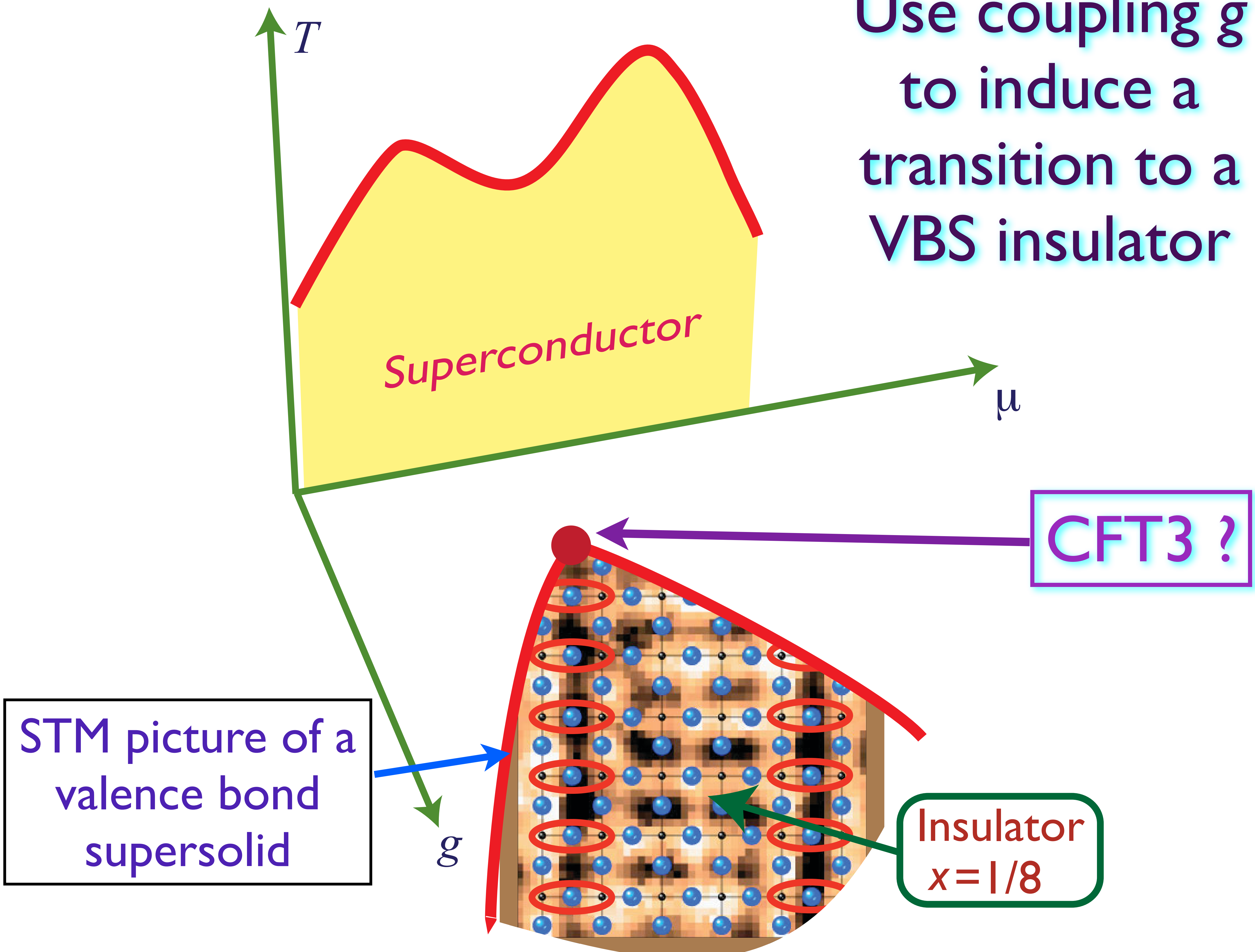
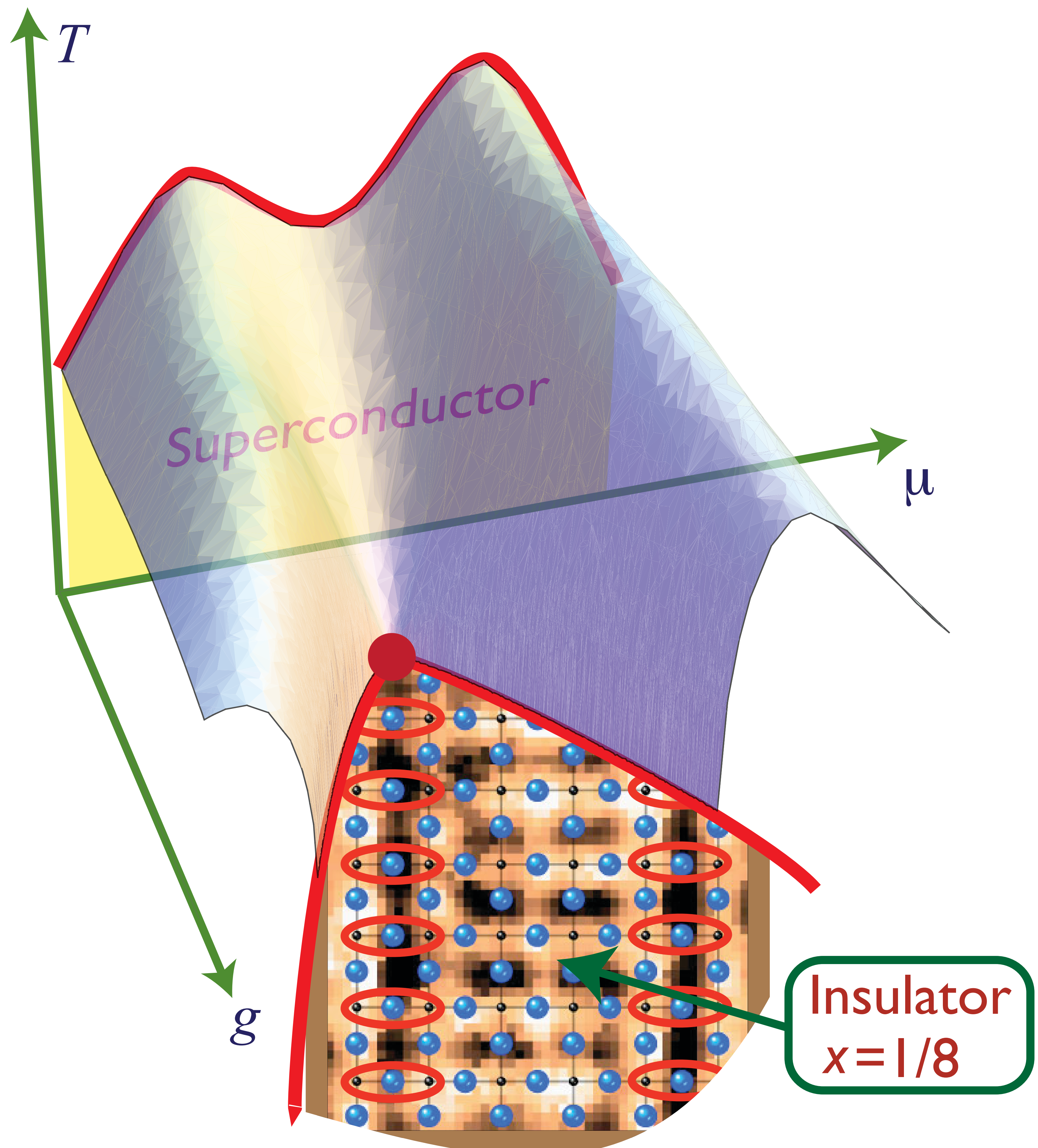
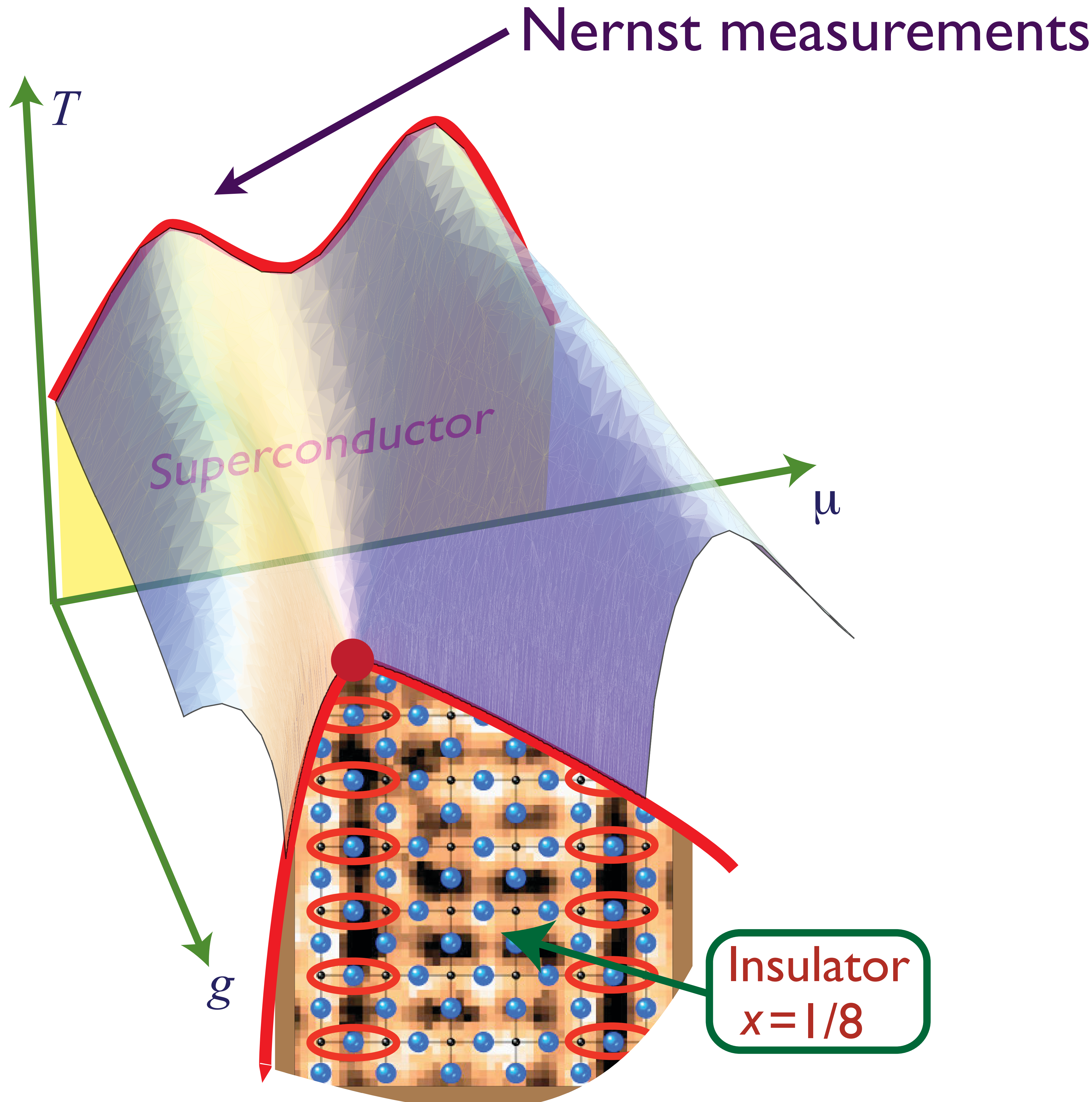
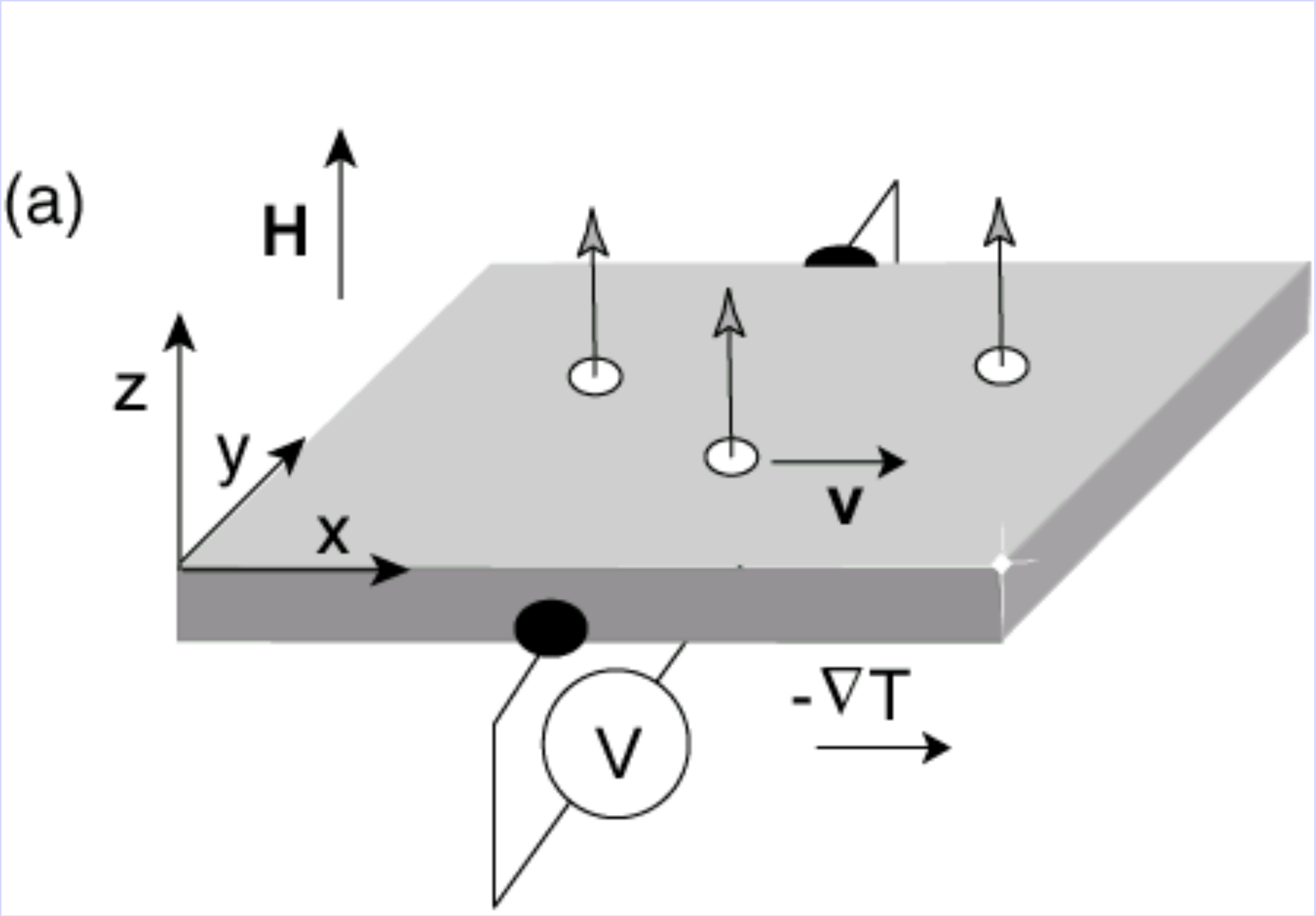


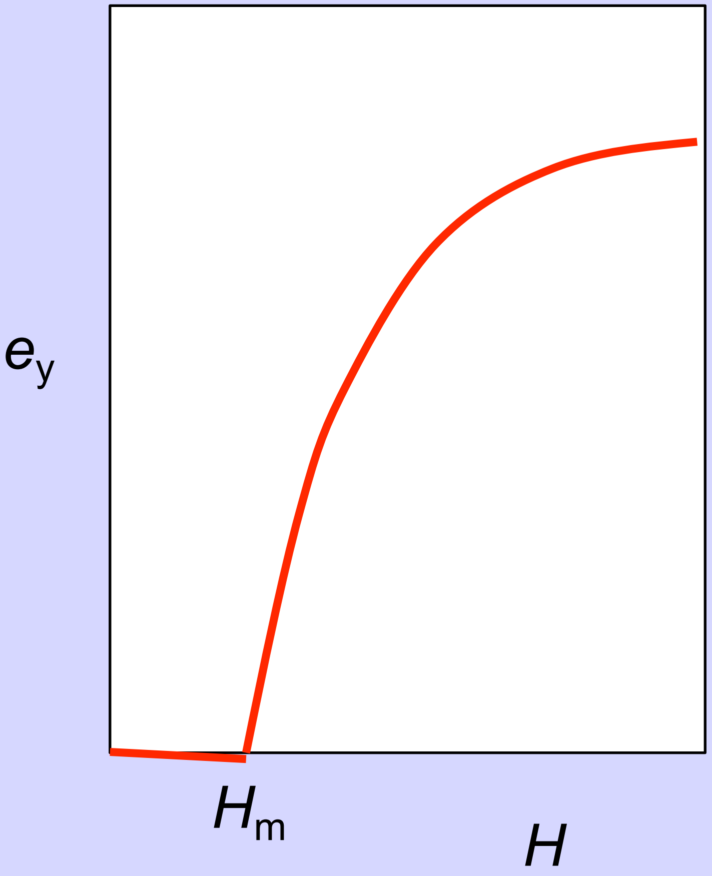
Proposed generalized phase diagram

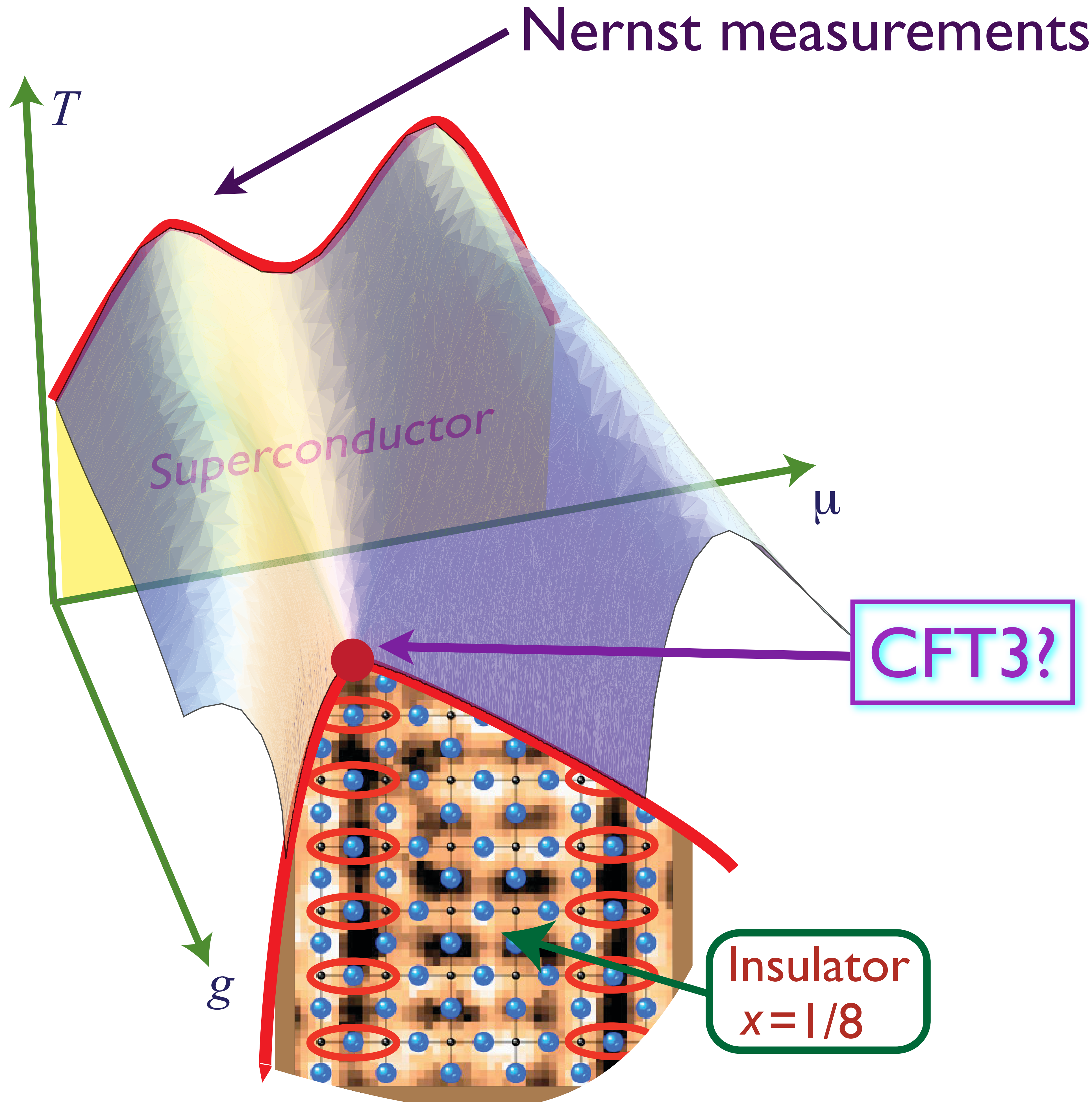



Nernst measurements
T
Superconductor
μ
g
Insulator
x=1/8

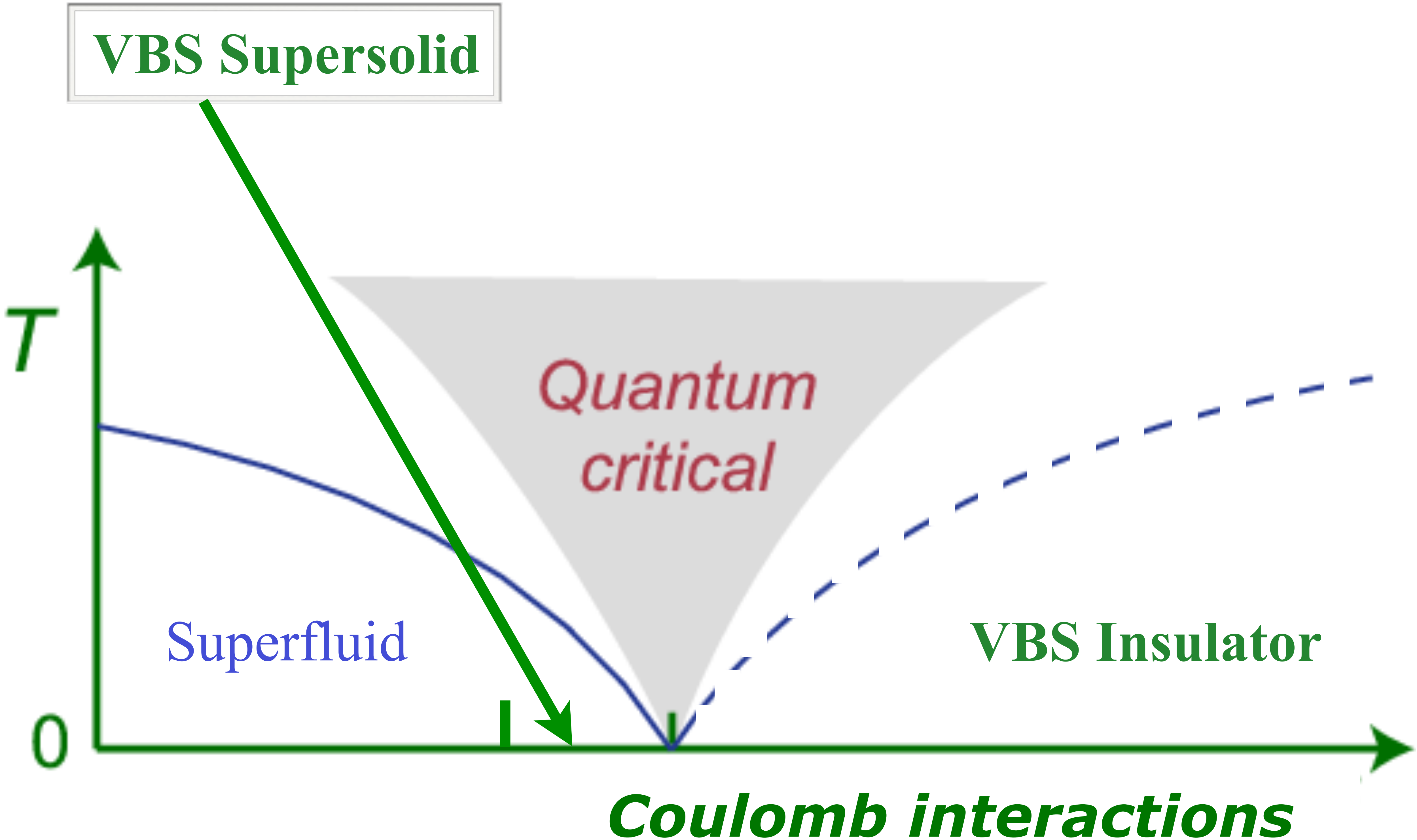
Nernst experiment



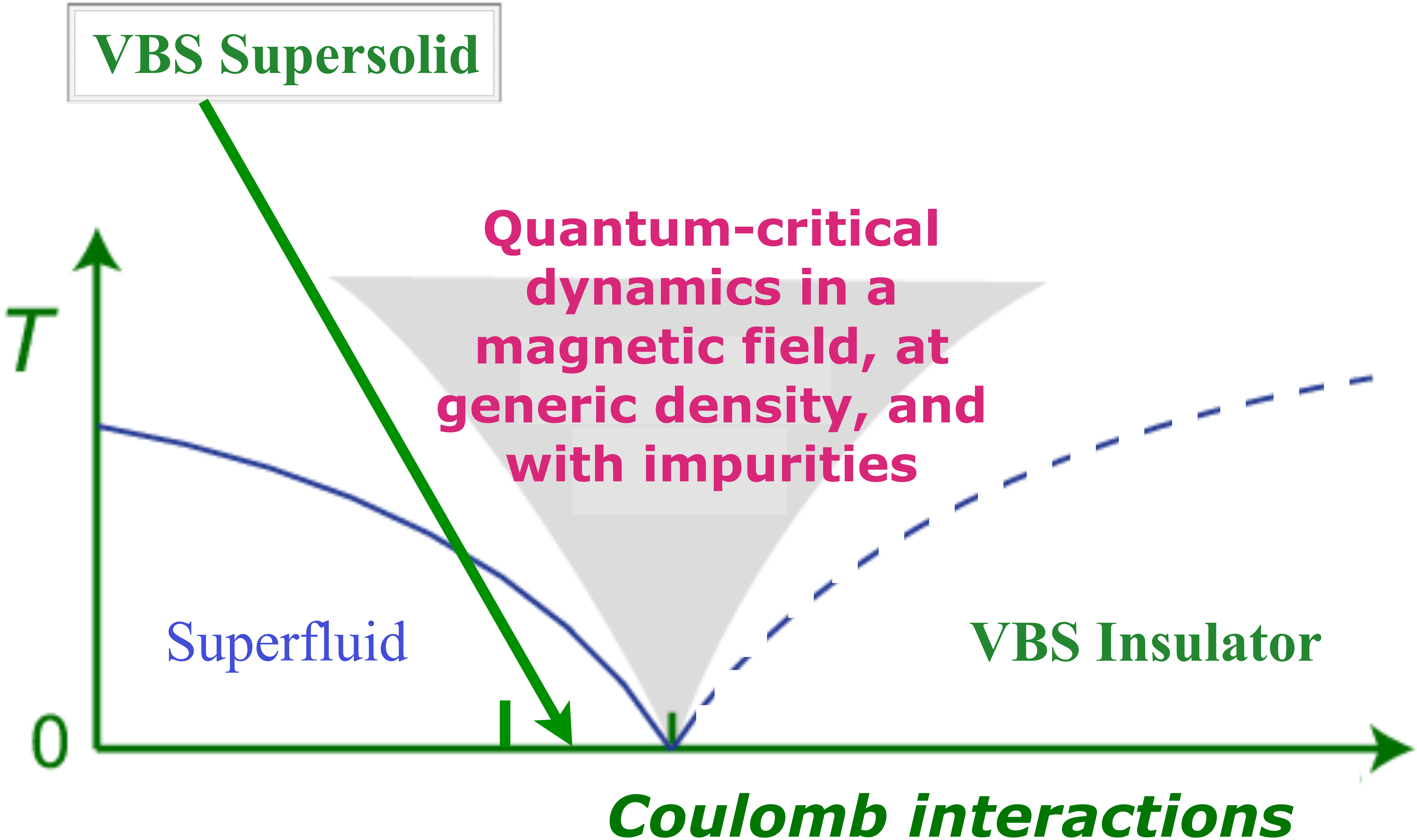



Nernst measurements
T
Superconductor
μ
CFT3?
g
Insulator
x=1/8

Non-zero temperature phase diagram



Non-zero temperature phase diagram



To the CFT of the quantum critical point, we add

- A chemical potential μ
- A magnetic field B

After the AdS/CFT mapping, we obtain the Einstein-Maxwell theory of a black hole with

- An electric charge
- A magnetic charge

A precise correspondence is found between general hydrodynamics of vortices near quantum critical points and solvable models of black holes with electric and magnetic charges

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, Phys. Rev. B (2007)

From these relations, we obtained results for the transport co-efficients, expressed in terms of a "cyclotron" frequency and damping:

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Nernst signal

$$e_N = \left(\frac{k_B}{2e}\right)\left(\frac{\varepsilon + P}{k_B T \rho}\right)\left[\frac{\omega_c/\tau_{\rm imp}}{(\omega_c^2/\gamma + 1/\tau_{\rm imp})^2 + \omega_c^2}\right]$$

$$\frac{k_B}{2e} = 43.086 \mu\text{V}/\text{K}$$

where ρ is the charge density, ε is the energy density, and P is the pressure.

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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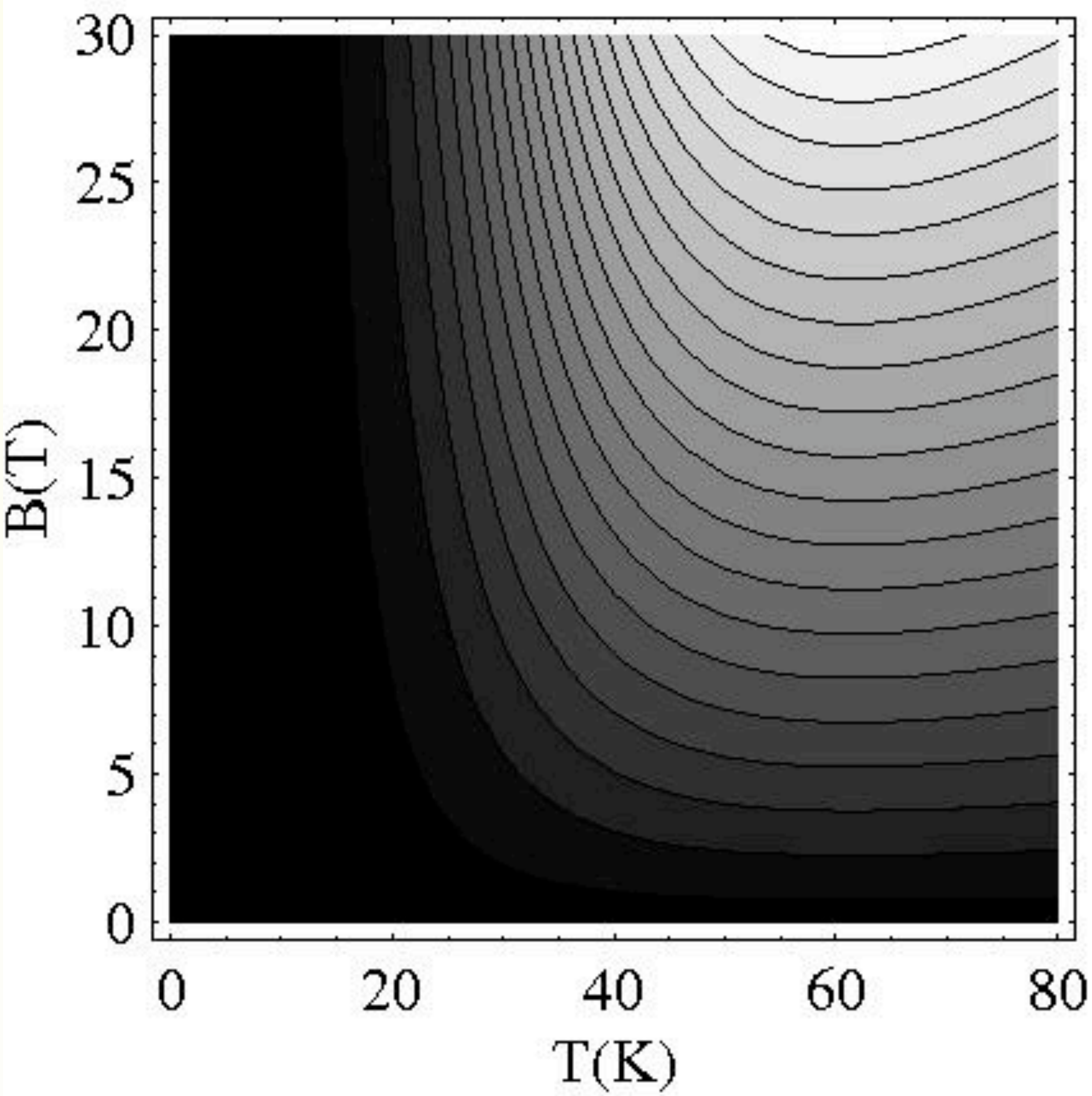
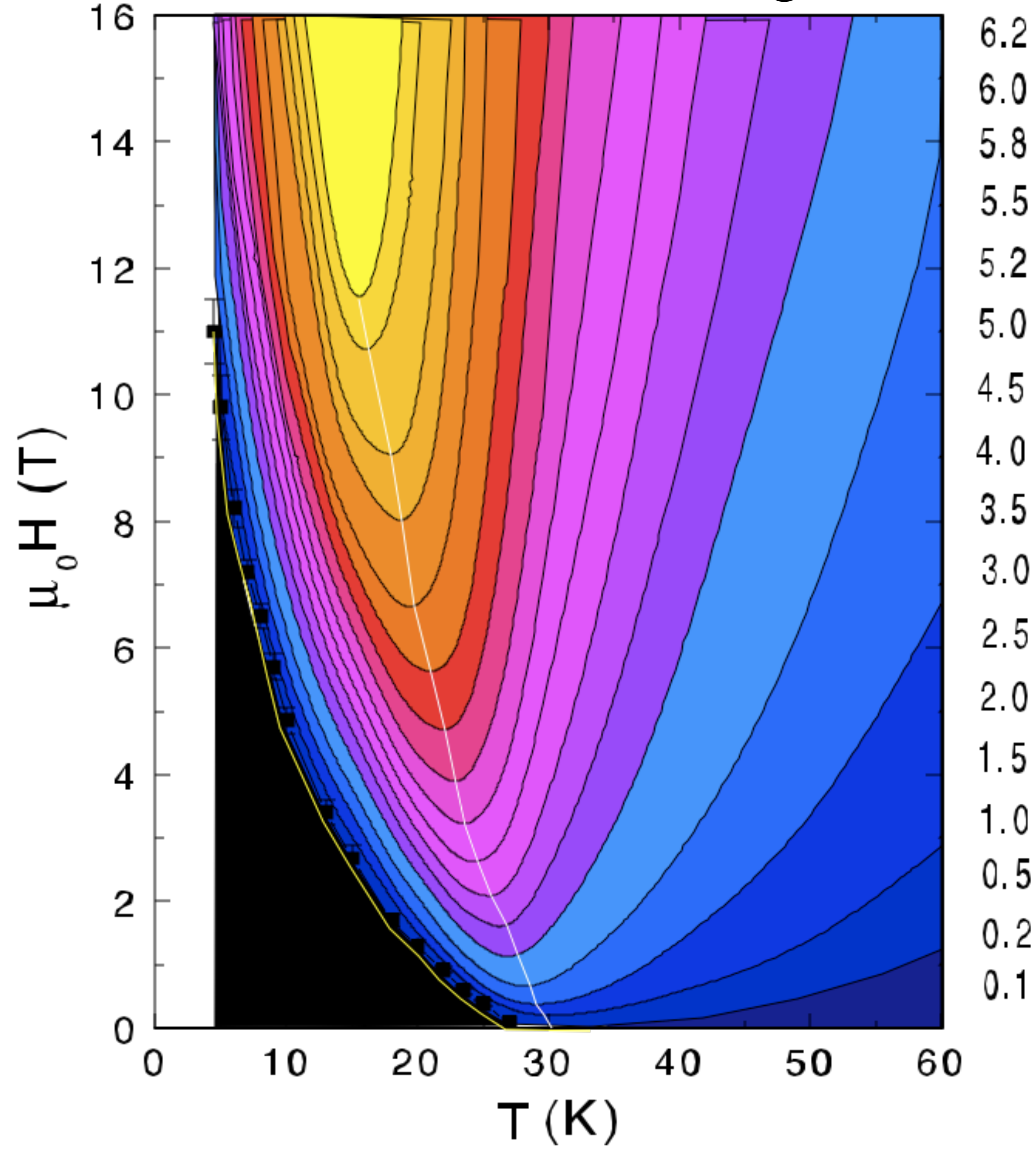
LSCO Experiments

Theory for N



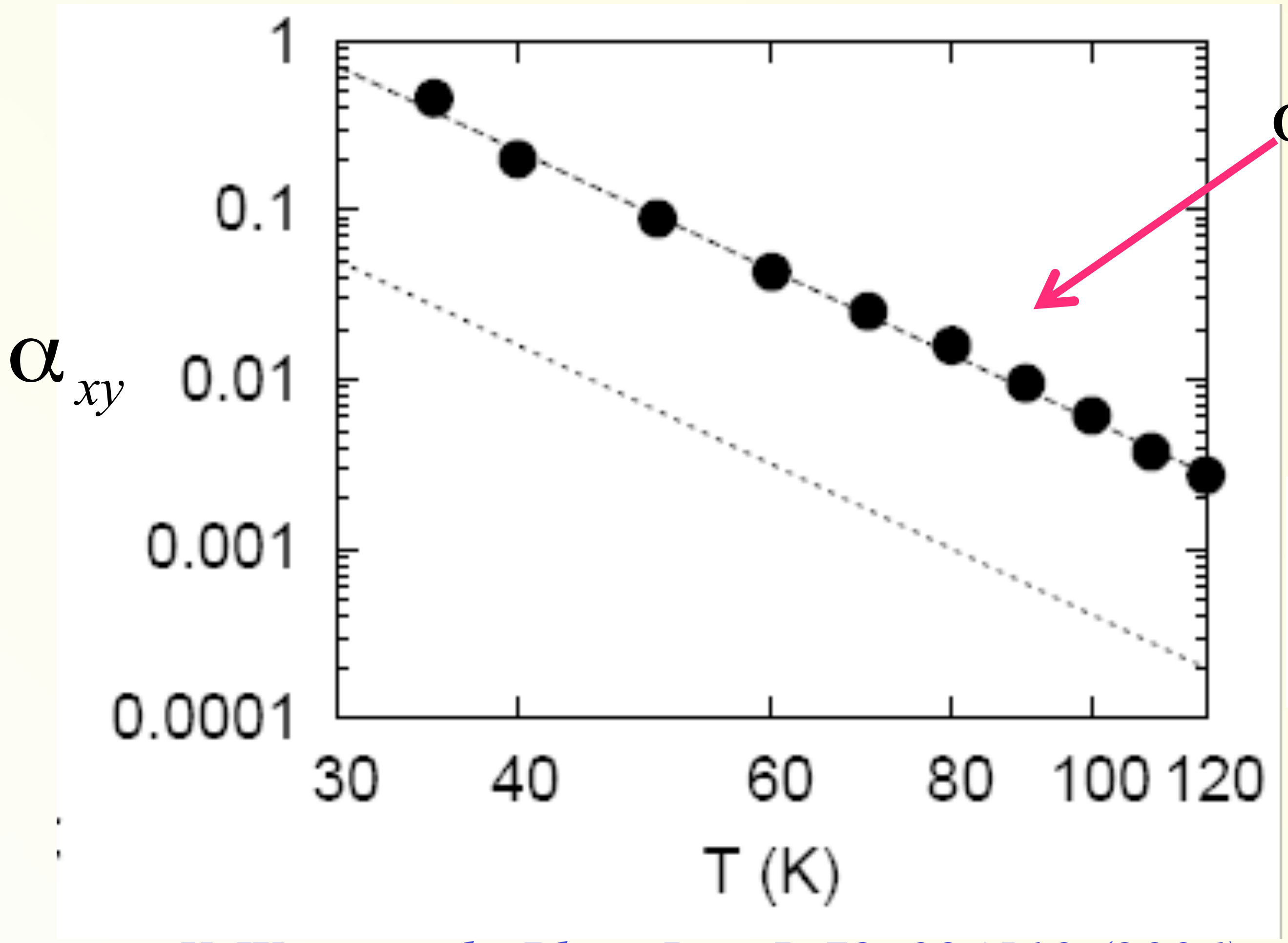
LSCO-0.12 Nernst signal N



Y. Wang, L. Li, and N. P. Ong, Phys. Rev. B 73, 024510 (2006).

LSCO Experiments

Measurement of $\alpha_{xy} \approx \sigma_{xx} e_N$



Y. Wang et al., Phys. Rev. B ***73****, 024510 (2006).*

$$\alpha_{xy} \propto \frac{1}{T^4}$$

$$\alpha_{xy} \propto \frac{BT^2\left(\#\rho^2\tau_{imp} + \#T^3\right)}{T^6 + \#B^2\rho^2\tau_{imp}^2}$$

(T small)

$$\frac{\alpha_{xy}}{B}(B \to 0) \approx \left(\frac{2ek_B}{h\phi_0}\right)\frac{\Phi_s}{\Phi_{\varepsilon+P}^2}\left(\frac{2\pi\tau_{\text{imp}}}{\hbar}\right)^2\frac{\rho^2(\hbar v)^6}{(k_B T)^4}$$

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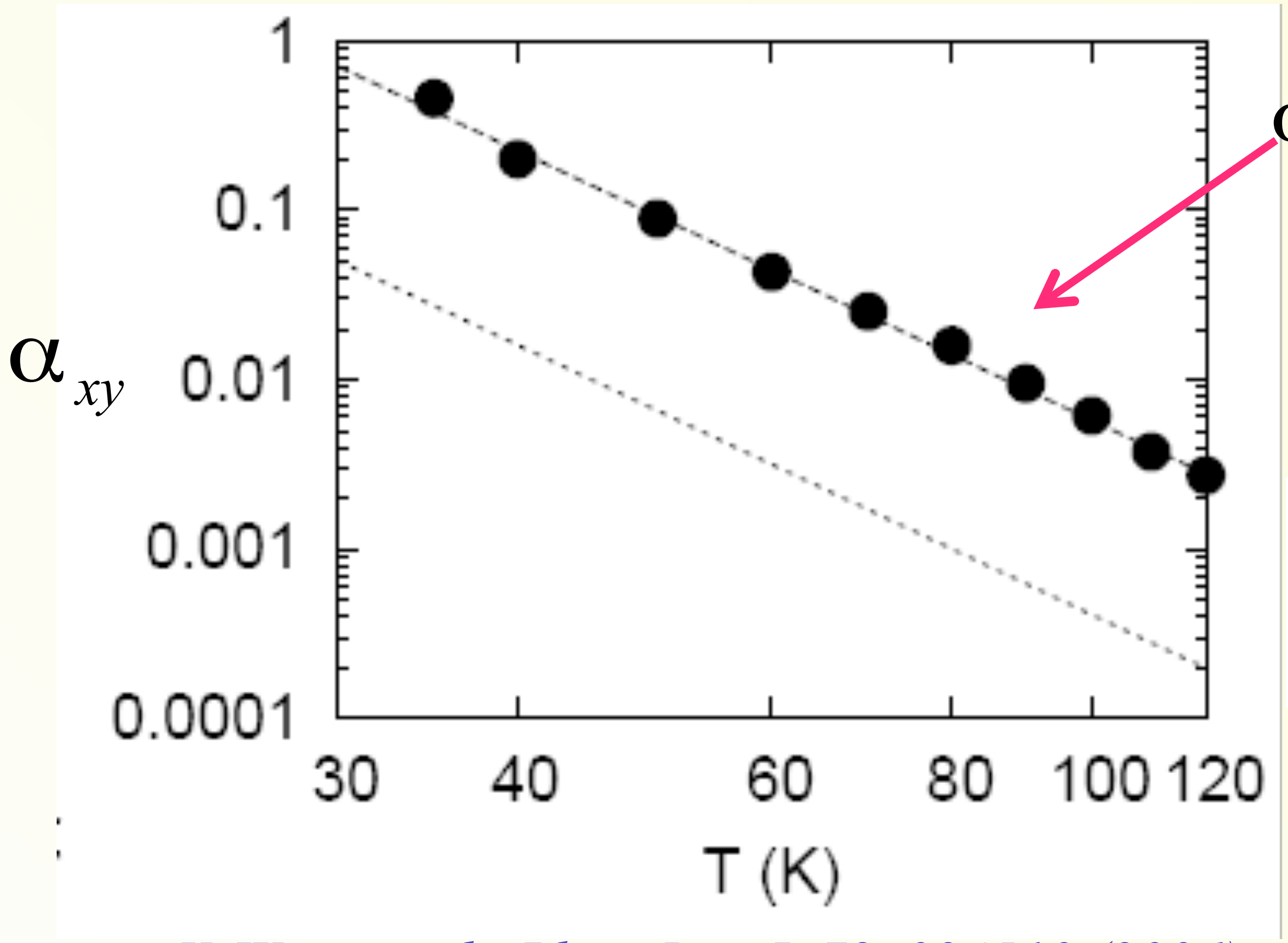

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$\hbar v \approx 47 \text{meV Å}$

$v \approx 2.5 \times 10^{-5} c$

$\tau_{imp} \approx 10^{-12} \text{s}$

LSCO Experiments

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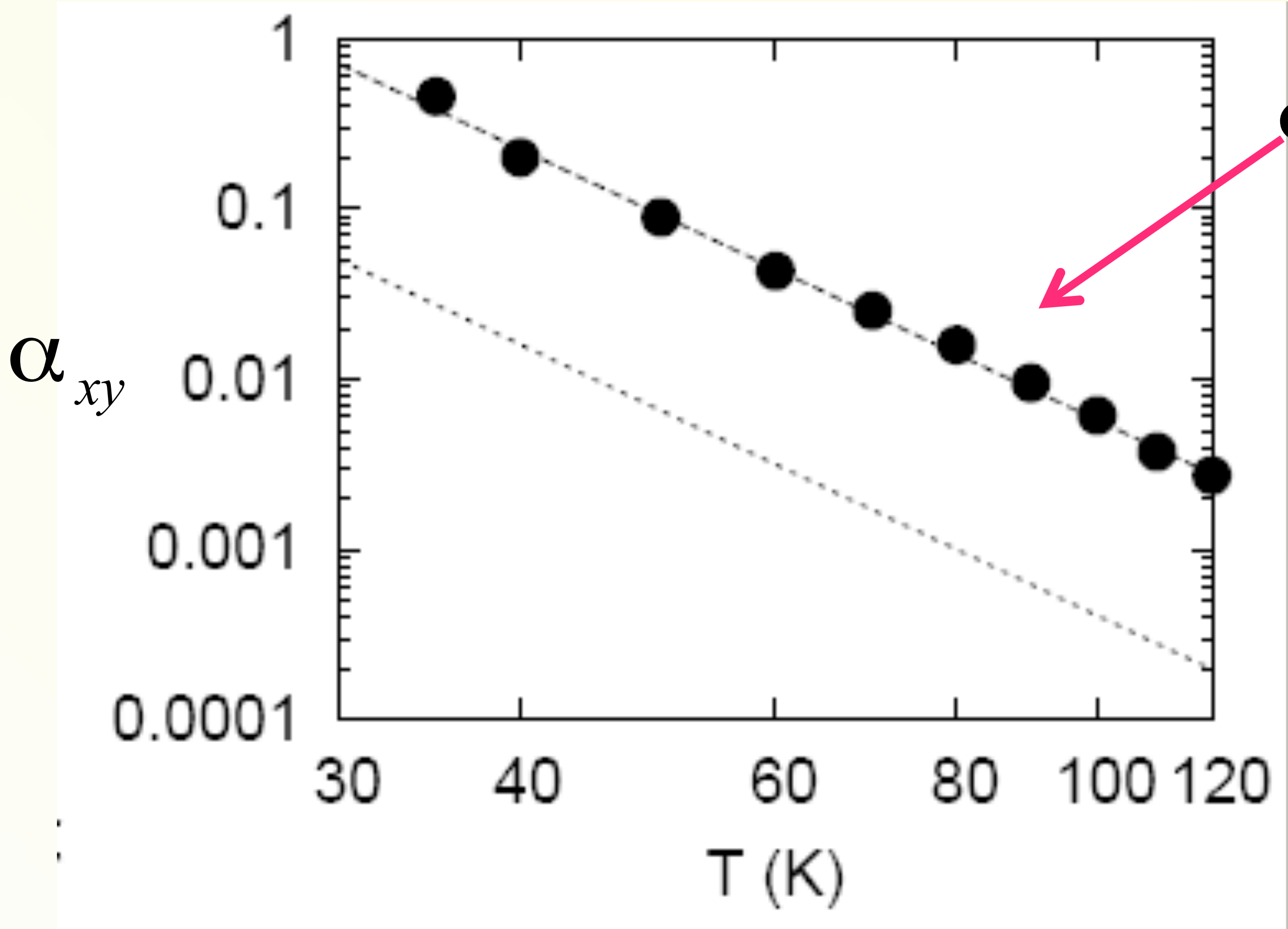

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$\hbar v \approx 47\,\text{meV}\,\mathring{A}$

$v \approx 2.5 \times 10^{-5} c$

$\tau_{imp} \approx 10^{-12}\,\text{s}$

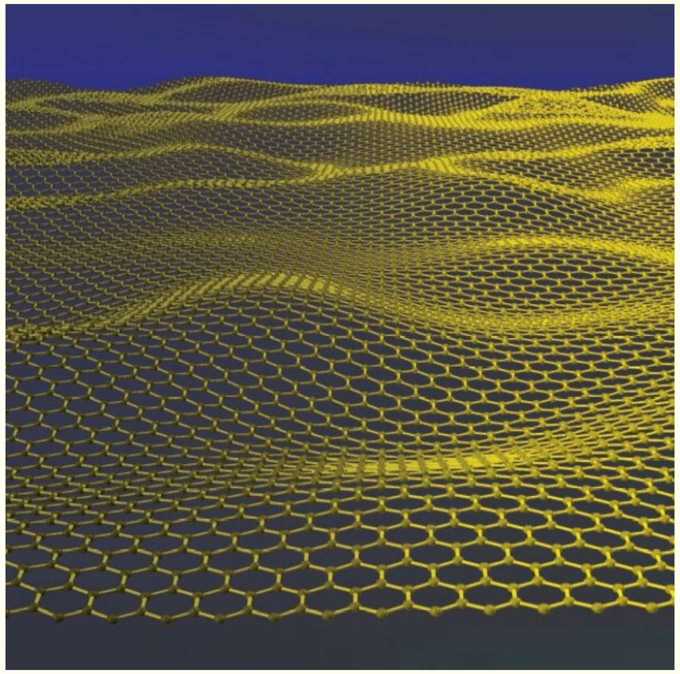
→ Prediction for ω_c:

$$\omega_c = 6.2\ \text{GHz}\frac{B}{1\ \text{T}}\left(\frac{35\ \text{K}}{T}\right)^3$$

- T-dependent cyclotron frequency!
- 0.035 times smaller than the cyclotron frequency of free electrons (at T=35 K)
- Only observable in ultra-pure samples where $\tau_{imp}^{-1} \leq \omega_c$.

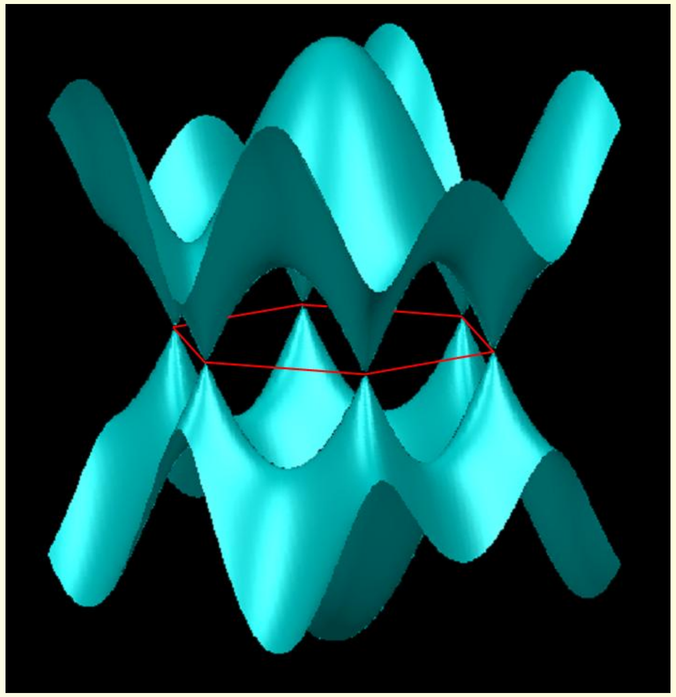
Cyclotron resonance in graphene

M. Mueller, and S. Sachdev, arXiv:0801.2970.



$$\omega = \pm\omega_c^{rel} - i\gamma - i/\tau$$

$$v = 1.1\times10^6\, m/s \approx c/300$$



Conditions to observe resonance

- Negligible Landau quantization $\quad E_{LL} = \hbar v\sqrt{\frac{2eB}{\hbar c}} << k_B T$
- Hydrodynamic, collison-dominated regime $\quad \hbar\omega_c^{rel} << k_B T$
- Negligible broadening $\quad \gamma, \tau^{-1} < \omega_c^{rel}$
- Relativistic, quantum critical regime $\quad \rho \leq \rho_{th} = \frac{(k_B T)^2}{(\hbar v)^2}$

$T \approx 300\,\mathrm{K}$

$B \approx 0.1\,\mathrm{T}$

$\rho \approx 10^{11}\,\mathrm{cm}^{-2}$

$\omega_c \approx 10^{13}\,\mathrm{s}^{-1}$

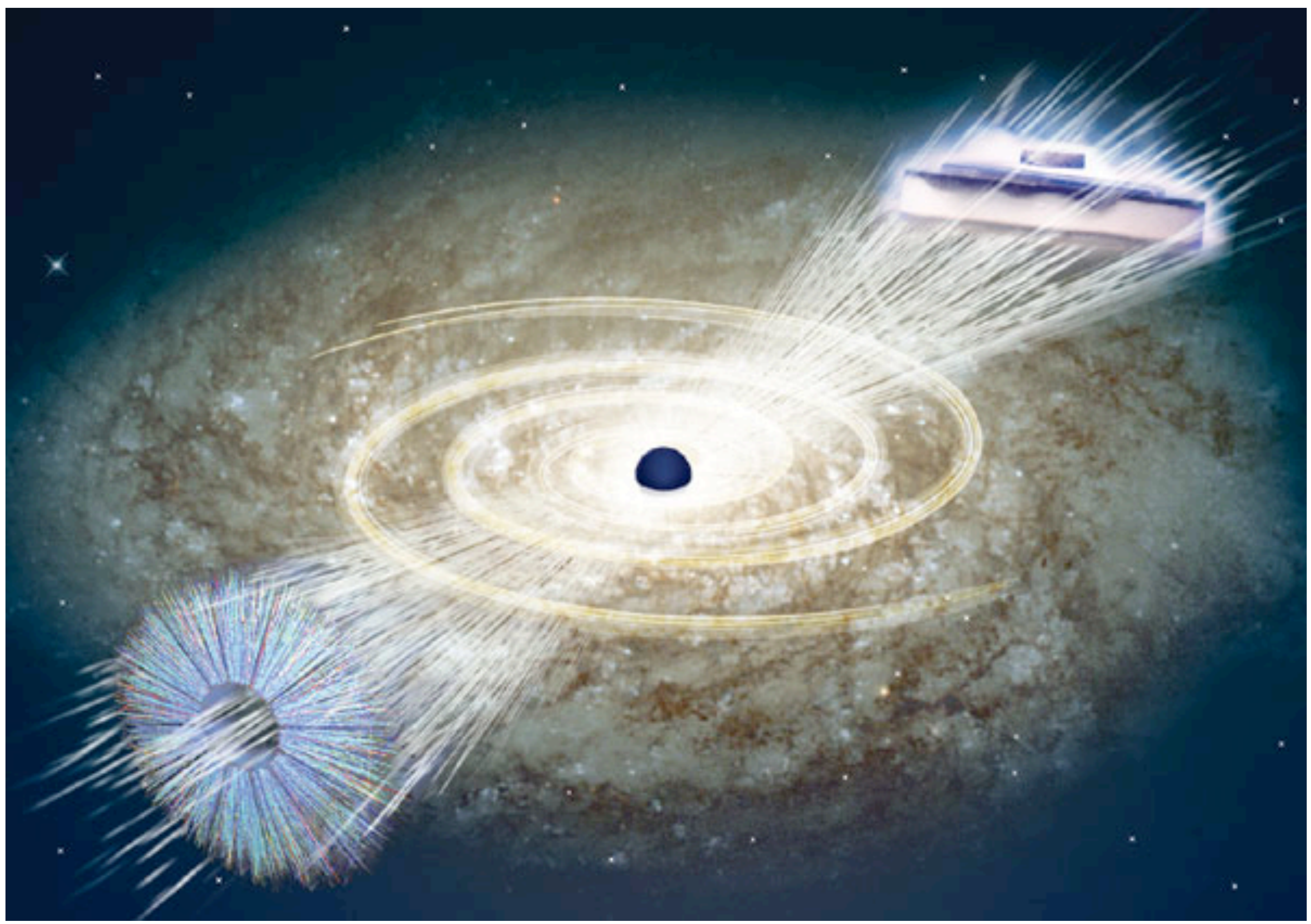
THEORETICAL PHYSICS

A black hole full of answers

Jan Zaanen

A facet of string theory, the currently favoured route to a 'theory of everything', might help to explain some properties of exotic matter phases — such as some peculiarities of high-temperature superconductors.

NATURE|Vol 448|30 August 2007



Conclusions

- Theory for transport near quantum phase transitions in superfluids and antiferromagnets
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.
- Theory of Nernst effect near the superfluid-insulator transition, and connection to cuprates.
- Quantum-critical magnetotransport in graphene.