Theory of the Nernst effect near the superfluid-insulator transition

Sean Hartnoll (KITP), Christopher Herzog (Washington),
Pavel Kovtun (KITP), Marcus Mueller (Harvard),
Subir Sachdev (Harvard), Dam Son (Washington)

Outline

1. Superfluid/supersolid/insulator quantum transitions

   Insulators at integer and commensurate densities

2. Theory of quantum-critical transport

   Collisionless-t0-hydrodynamic crossover of conformal field theories

3. Hydrodynamics at incommensurate densities with impurities and a magnetic field

   Exact relations between thermoelectric co-efficients

4. Nernst effect in the cuprate superconductors

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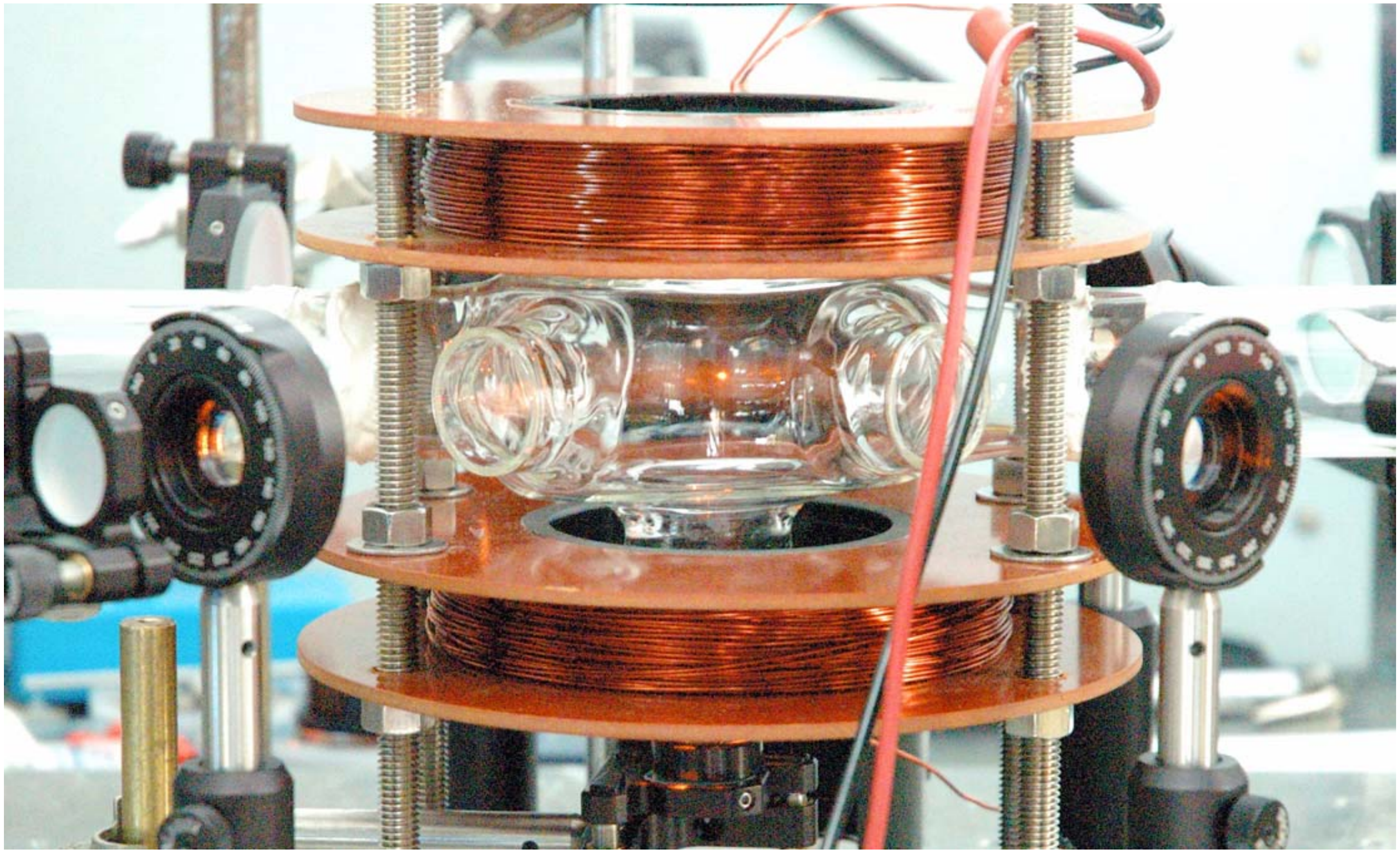
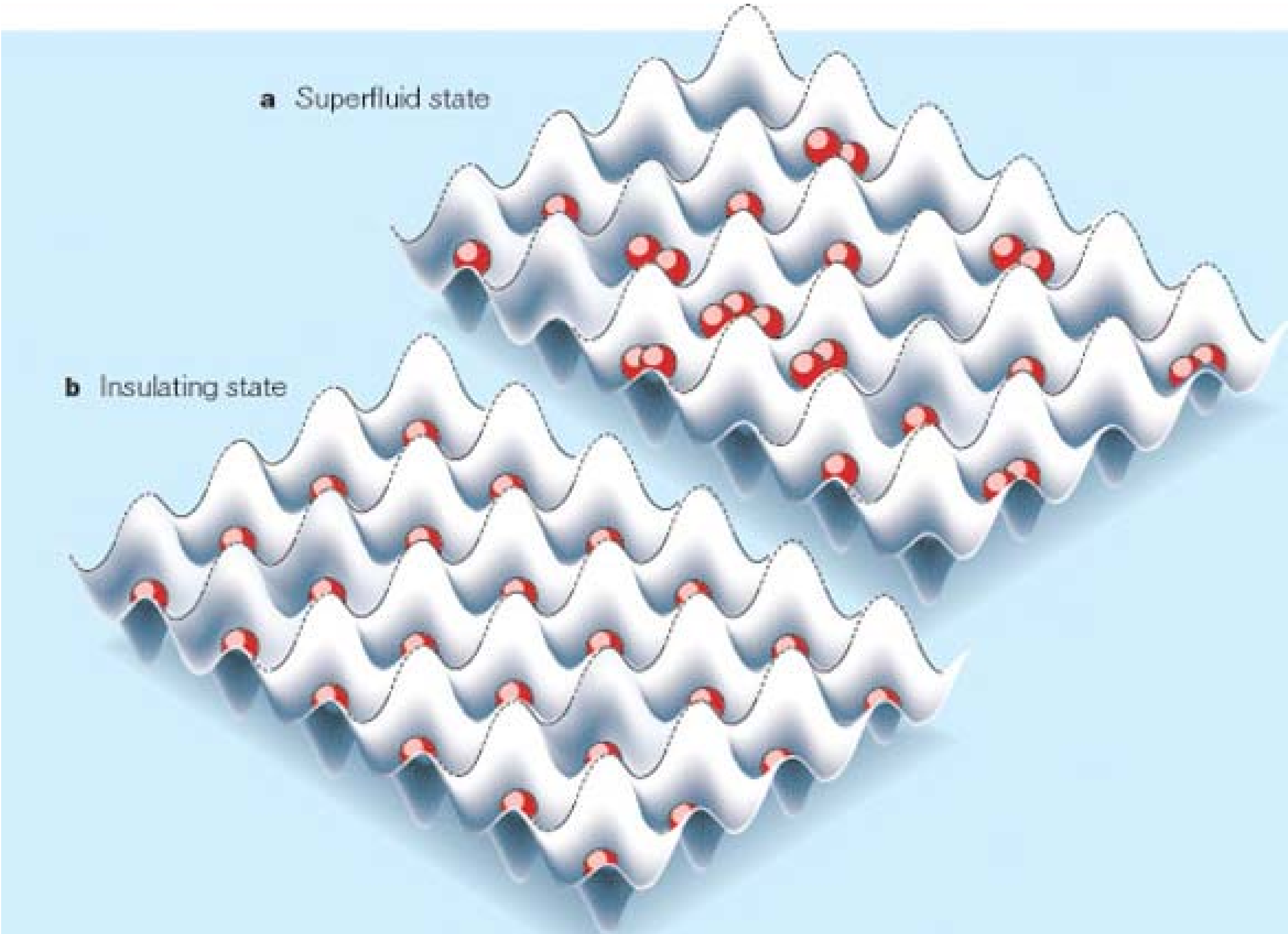


Trap for ultracold ^{87}Rb atoms



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Boson Hubbard model

Degrees of freedom: Bosons, $b_j^\dagger$, hopping between the sites, j, of a lattice, with short-range repulsive interactions.

$$H = -t\sum_{\langle ij\rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2}\sum_j n_j(n_j - 1) + \cdots$$

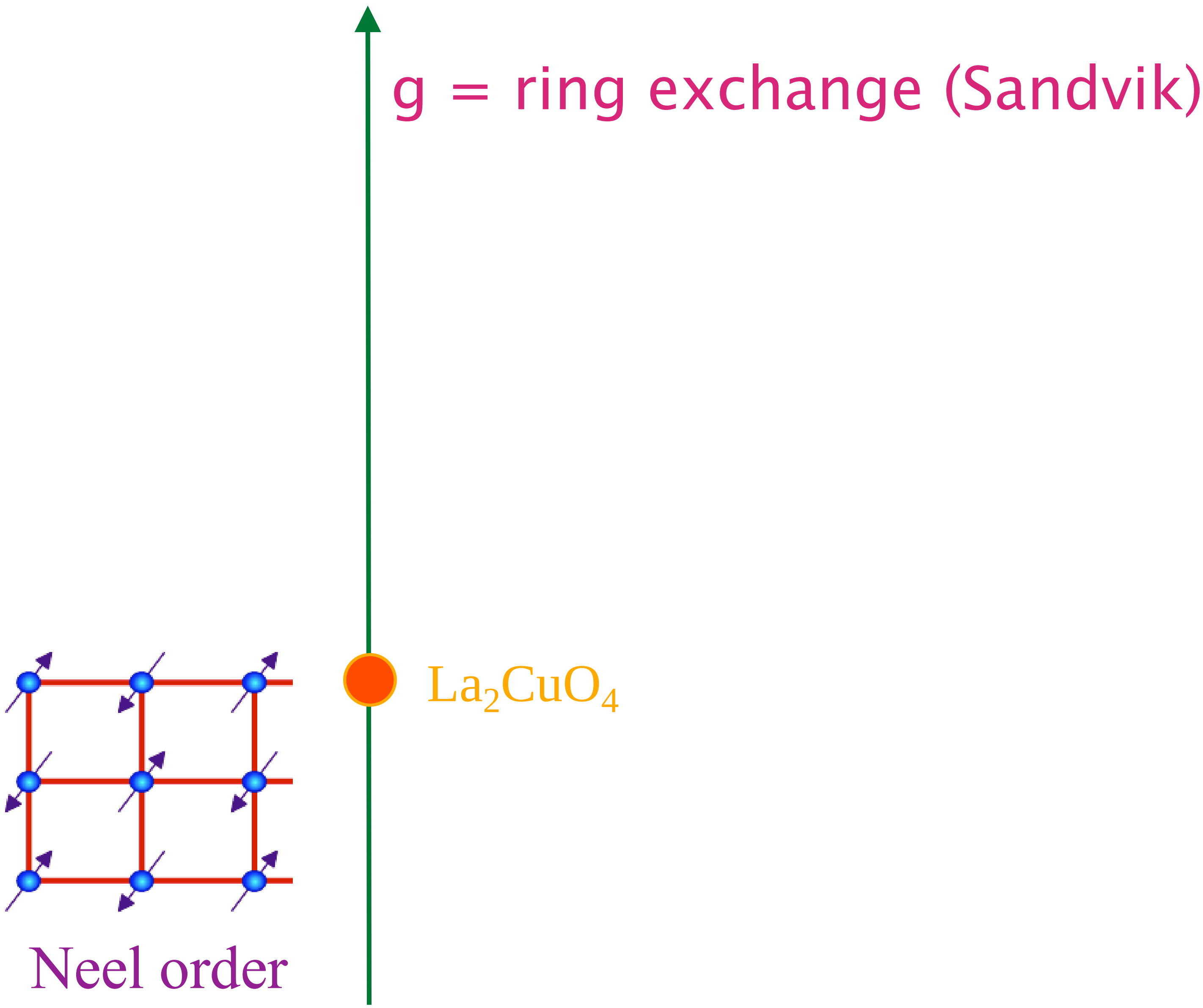
$$n_j \equiv b_j^\dagger b_j$$

M.P.A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher *Phys. Rev.* B **40**, 546 (1989).

For small U/t, superfluid

For large U/t, insulator

Phase diagram of doped antiferromagnets



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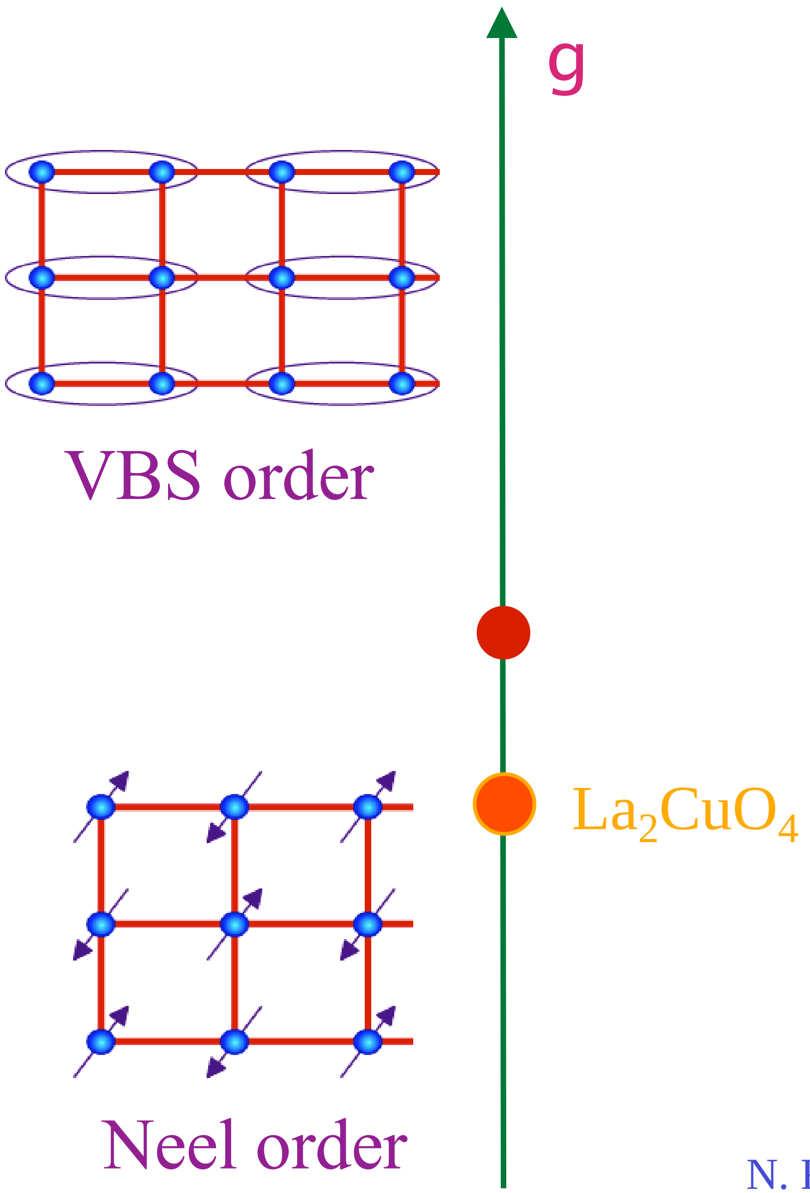

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

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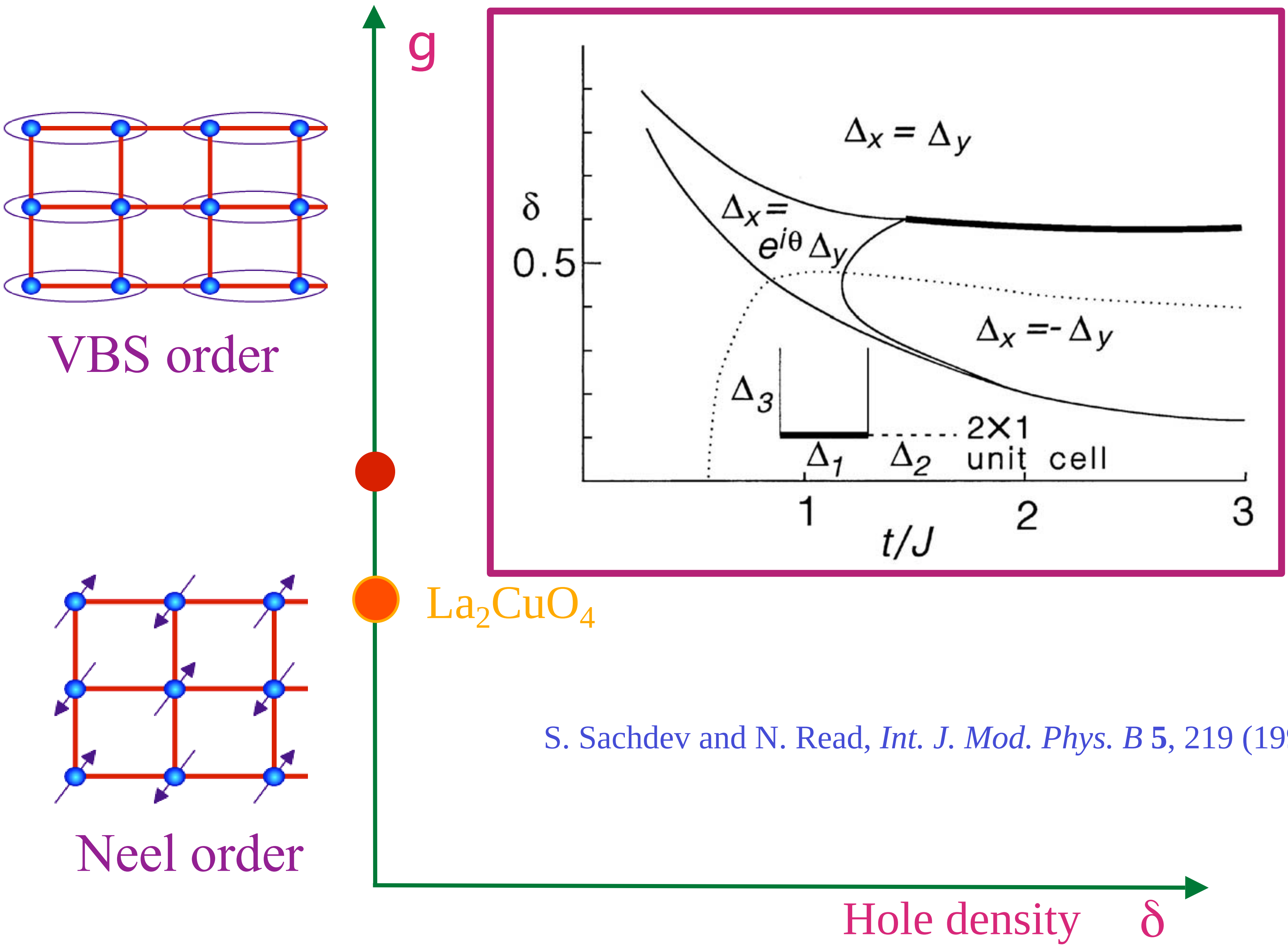

S. Sachdev and N. Read, *Int. J. Mod. Phys. B* **5**, 219 (1991).

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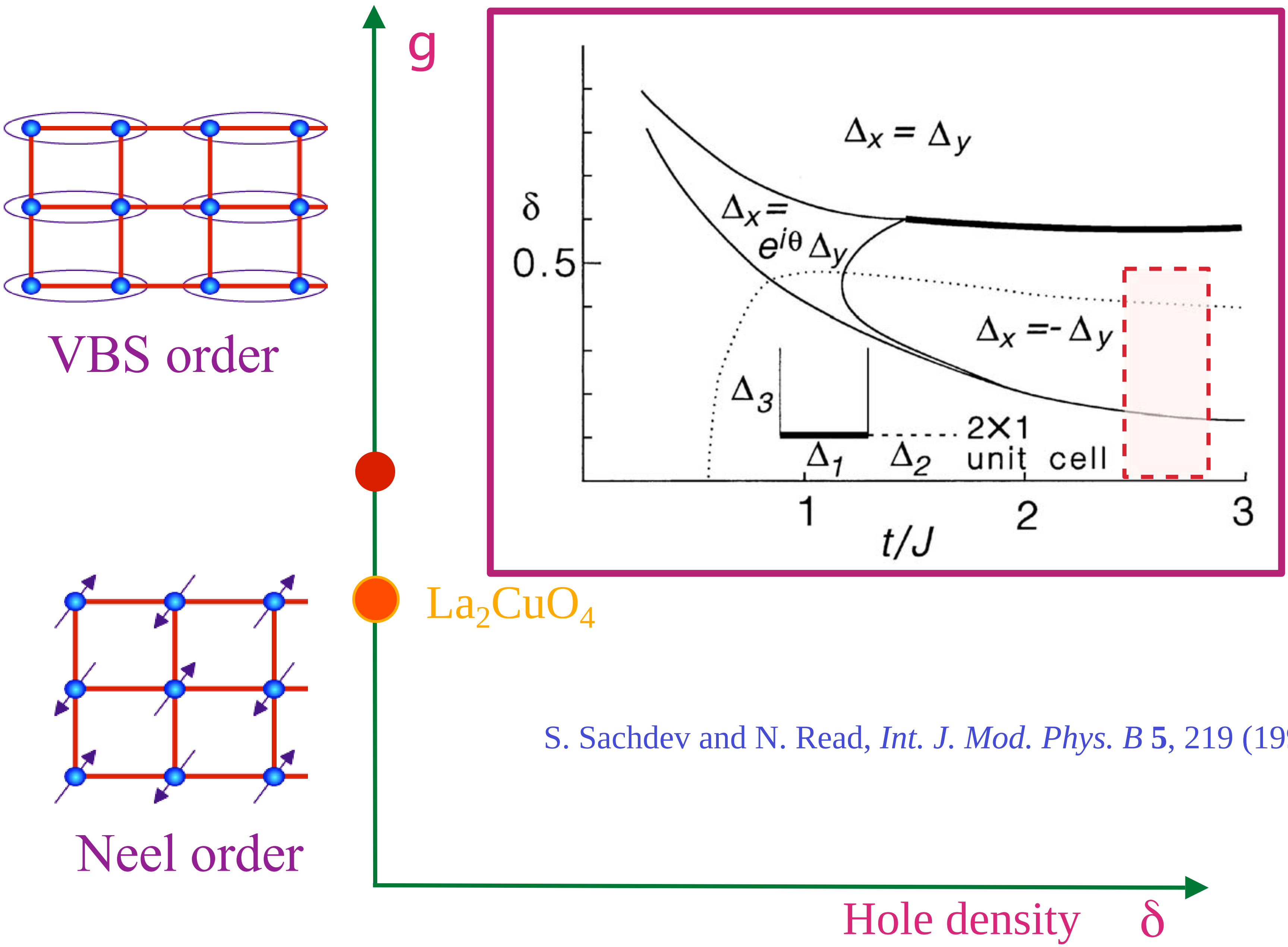

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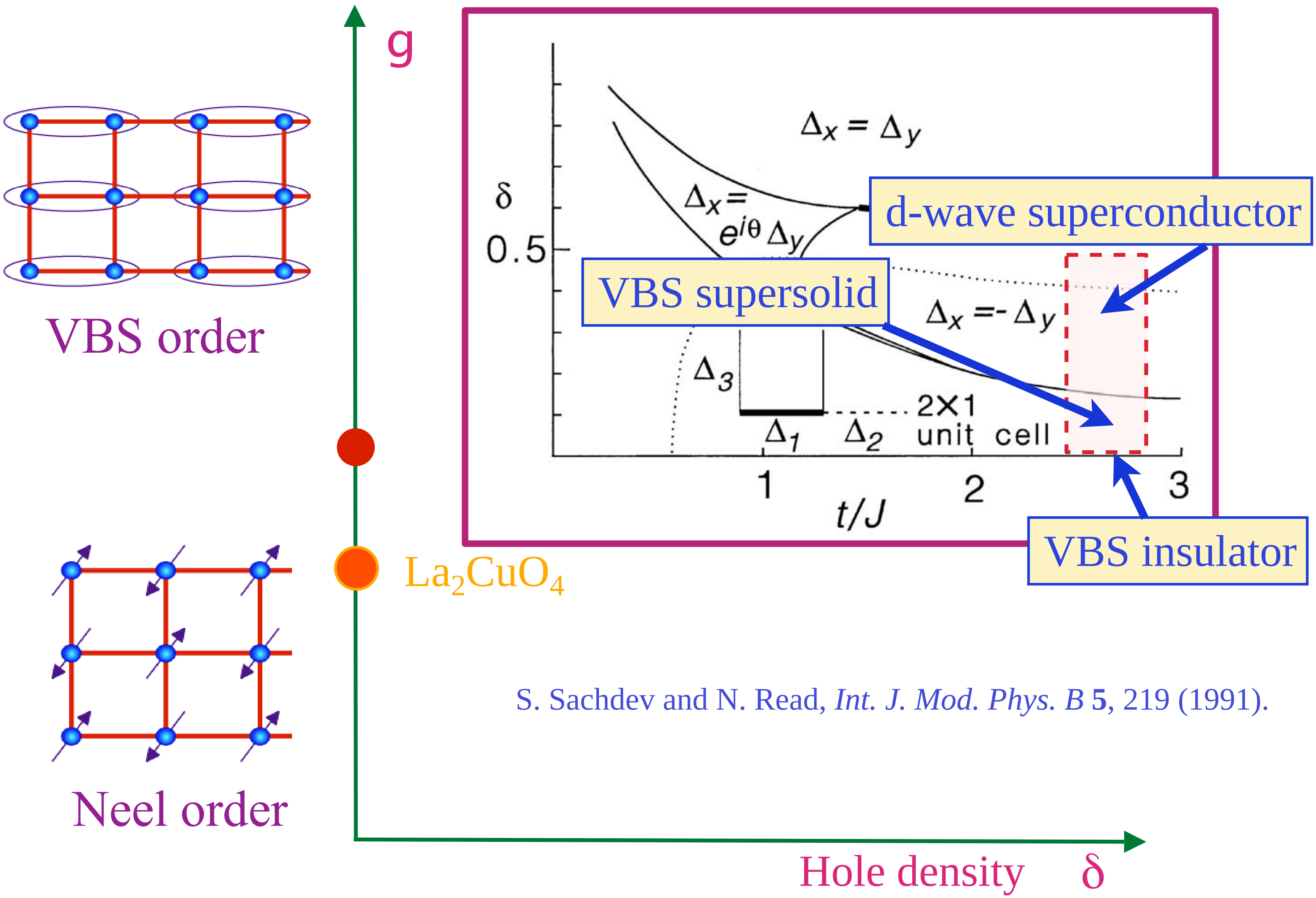

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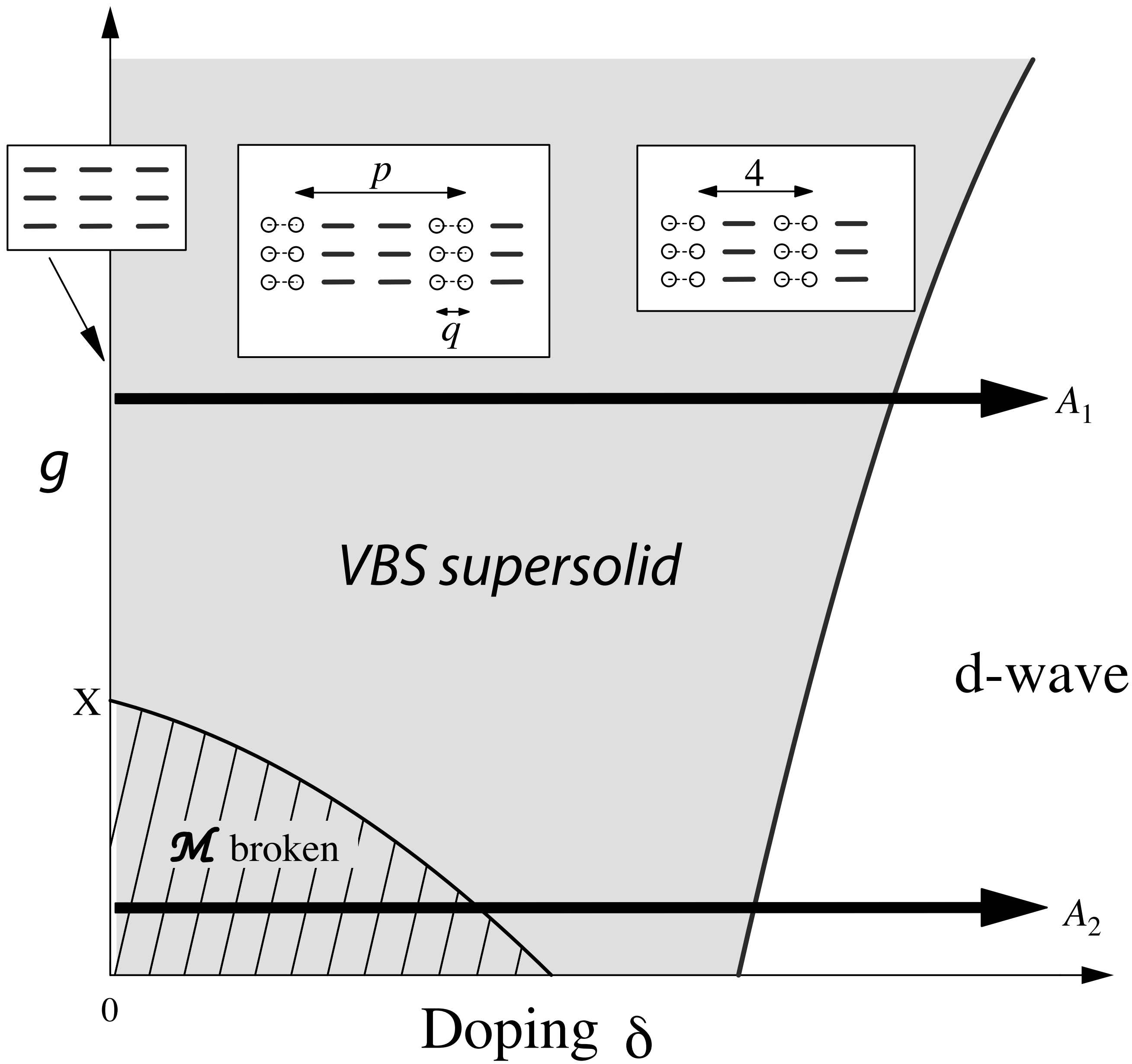
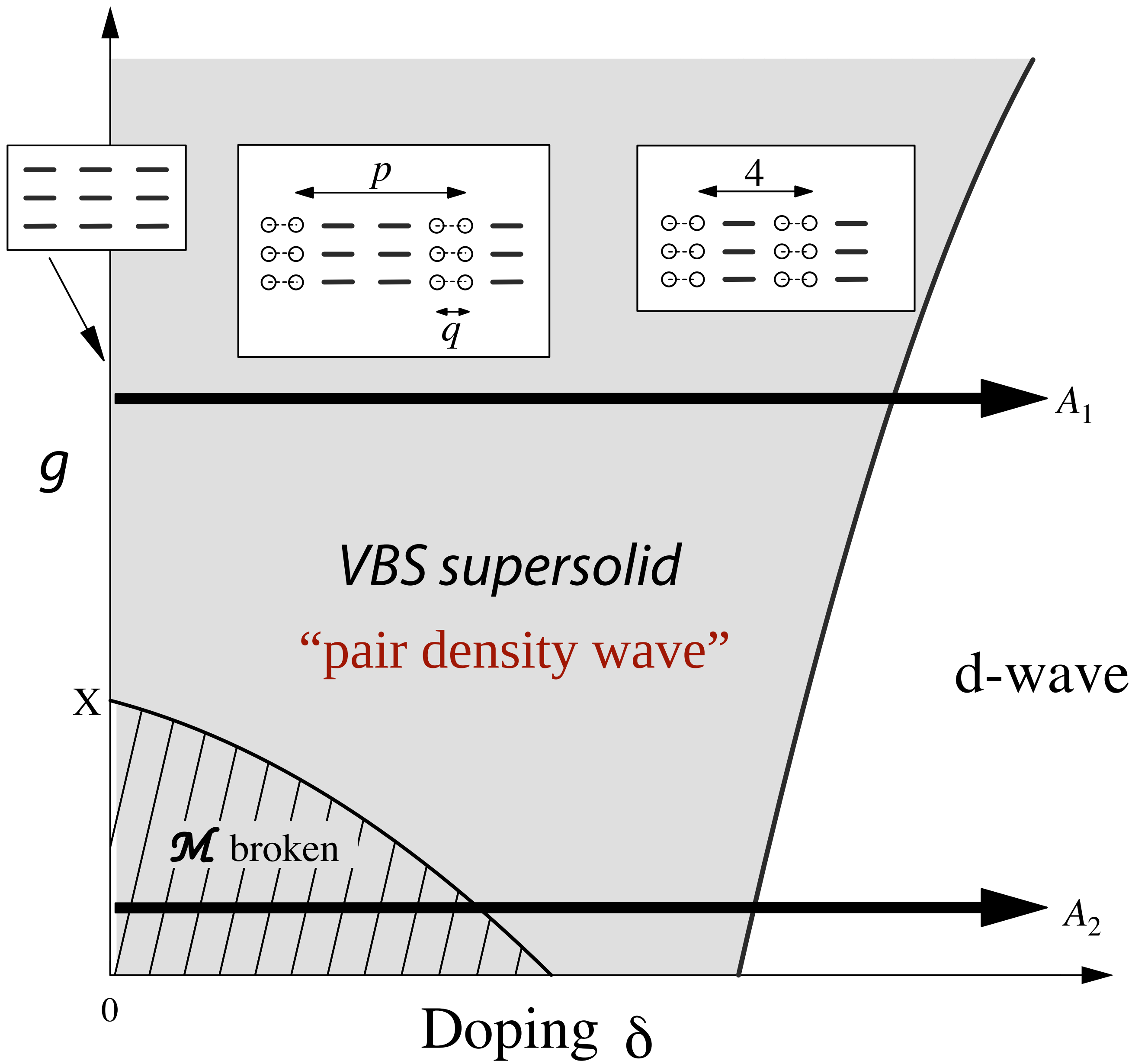


M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999)



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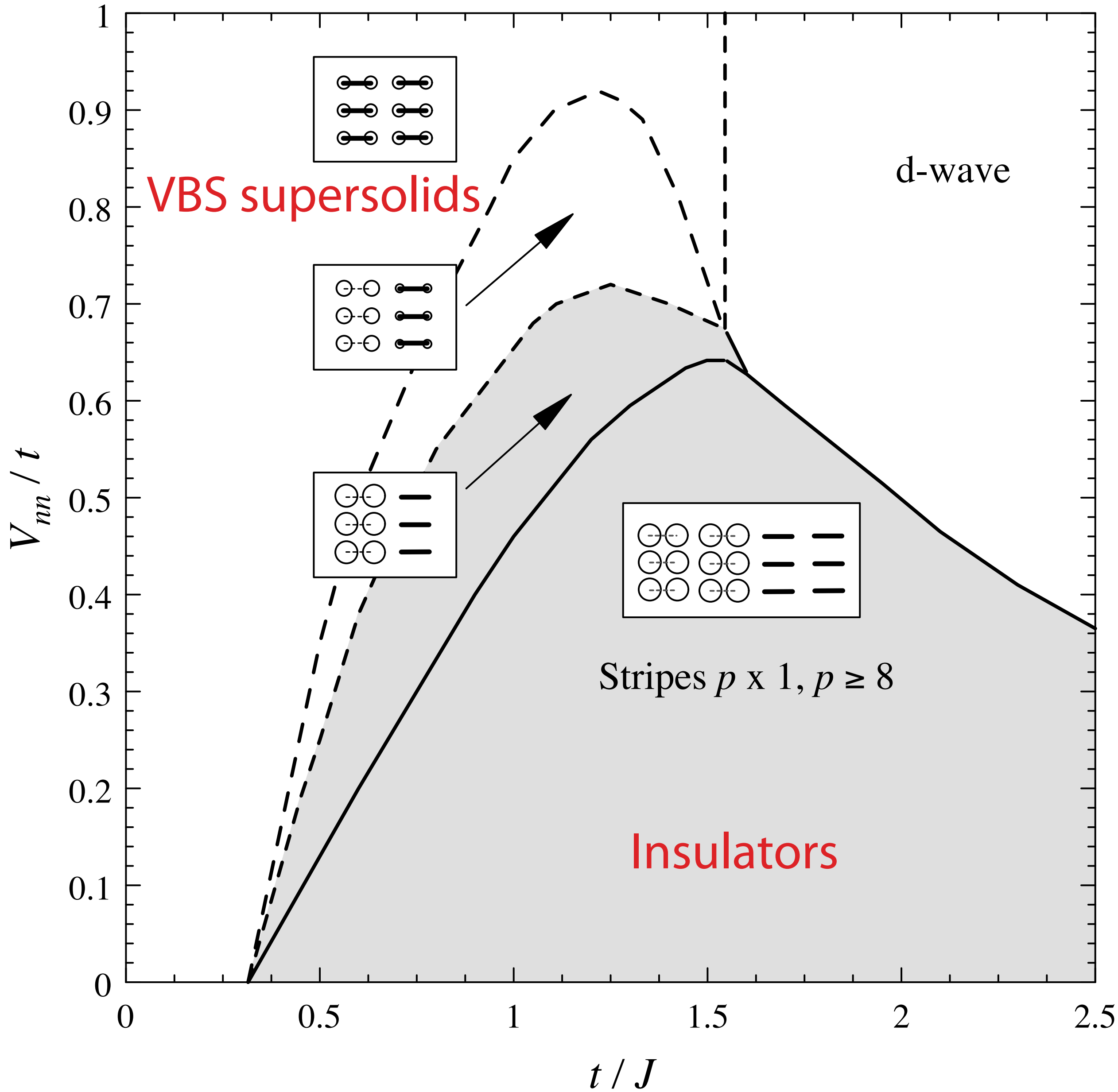

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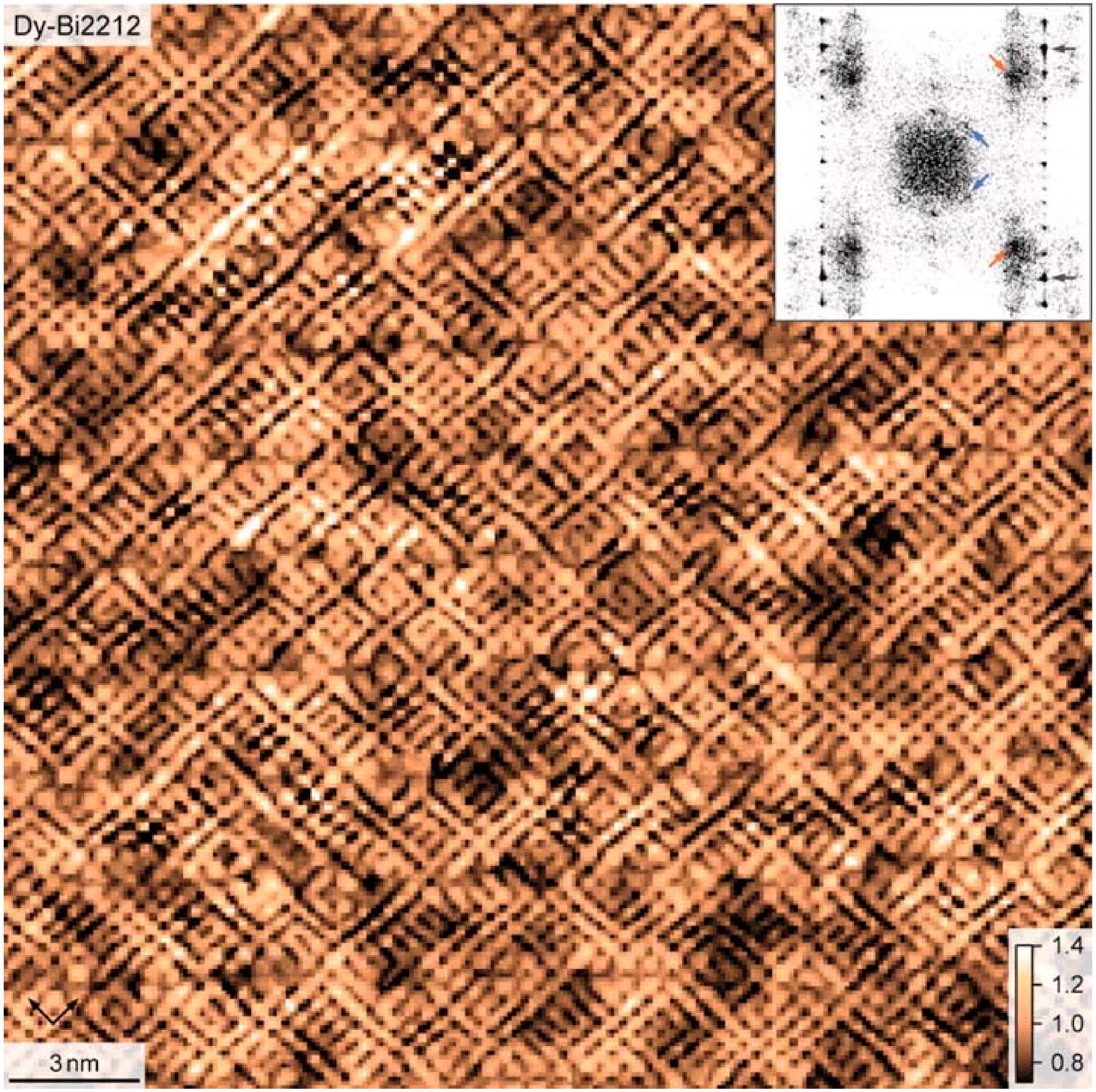

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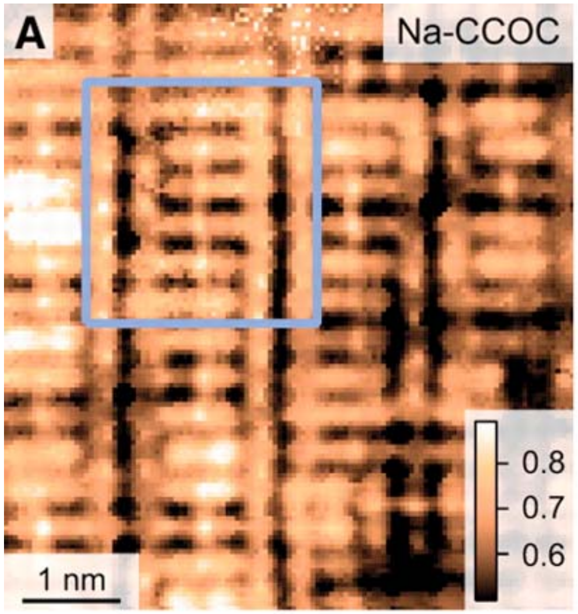

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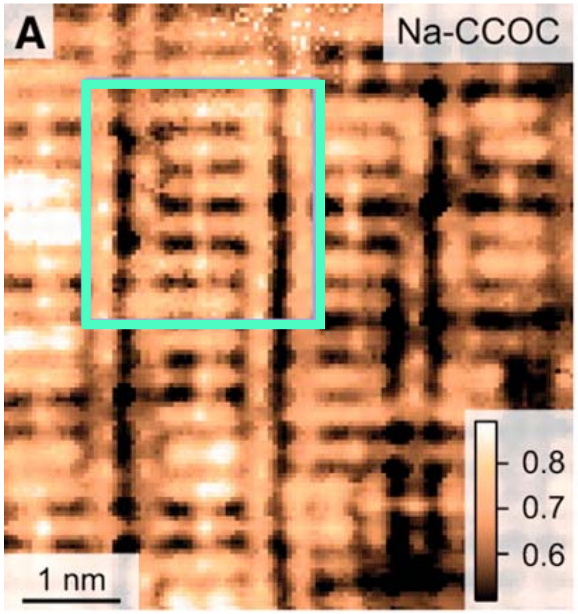

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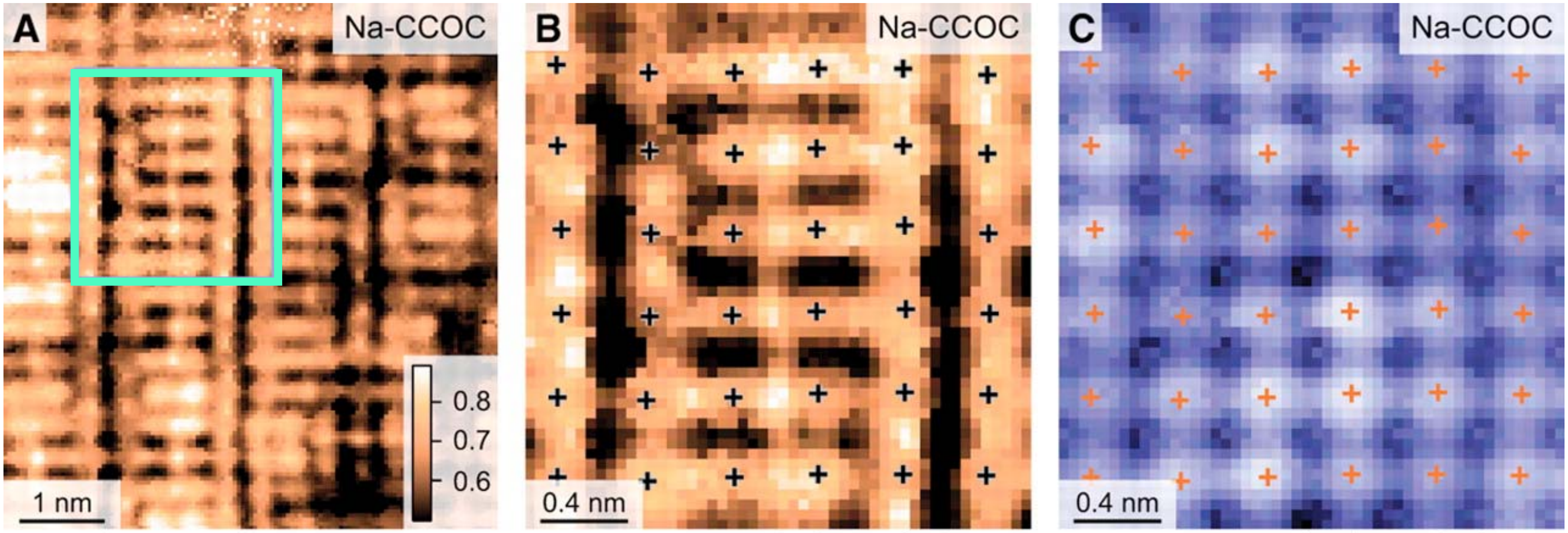

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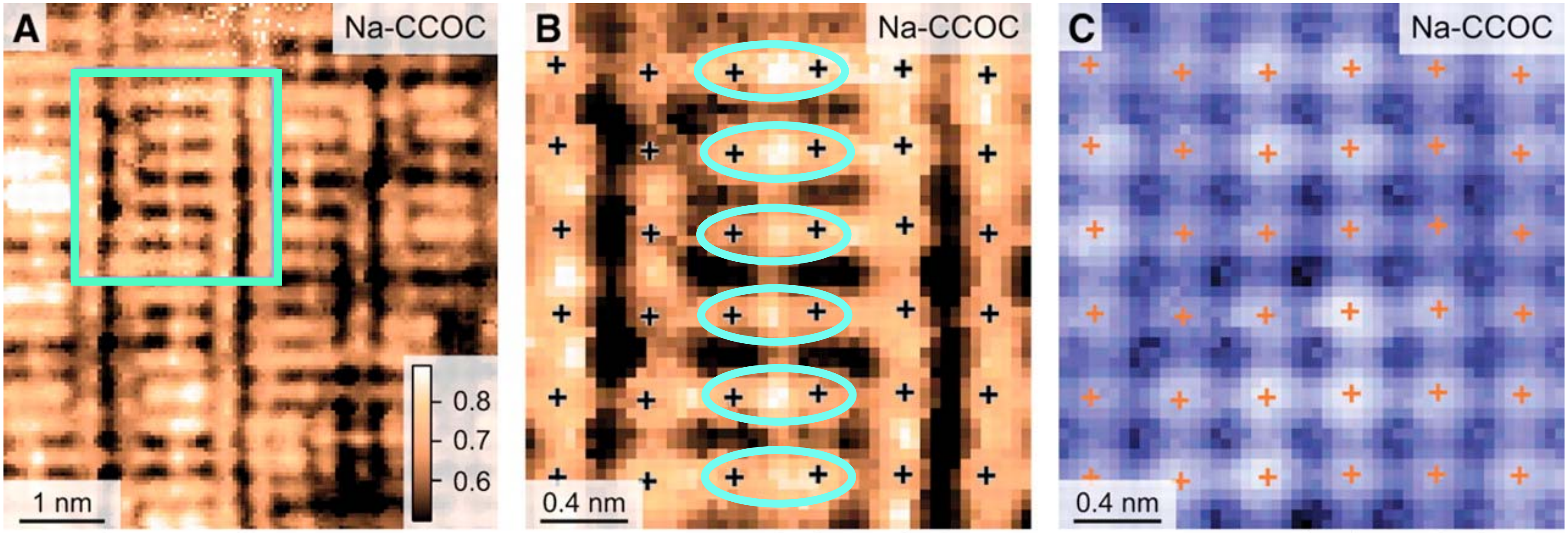

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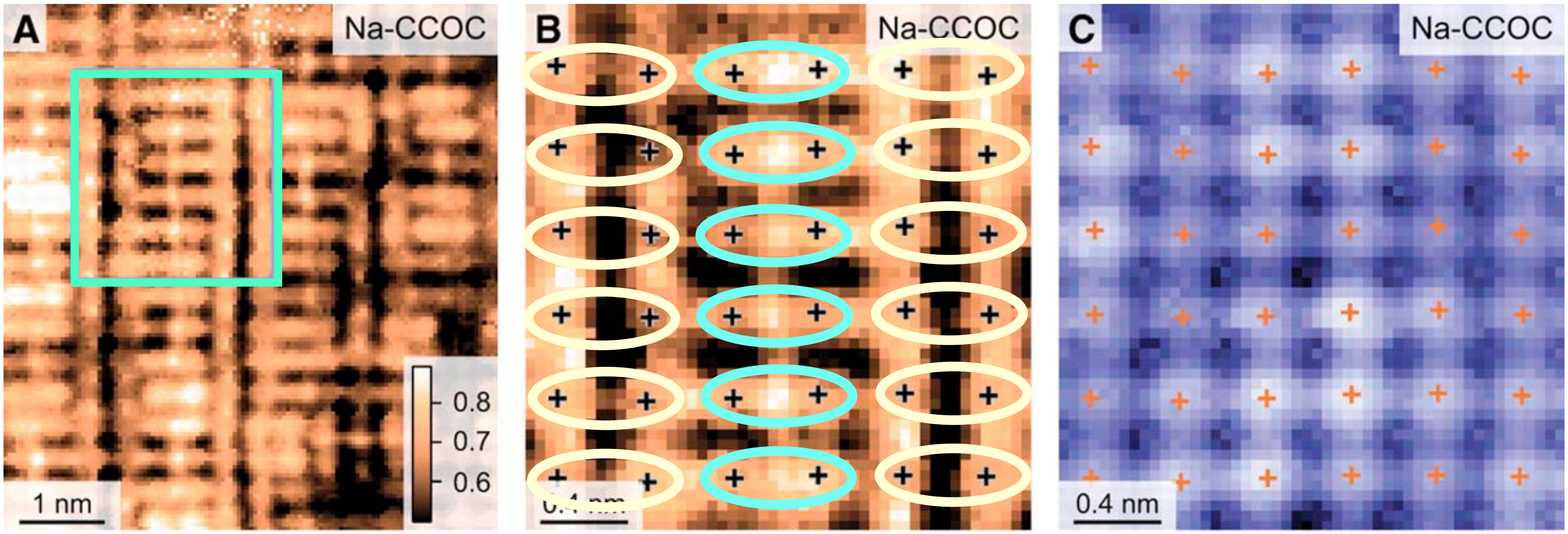

“Glassy” Valence Bond Supersolid

Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007)

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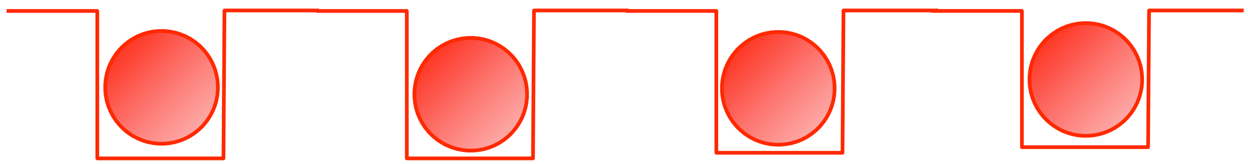
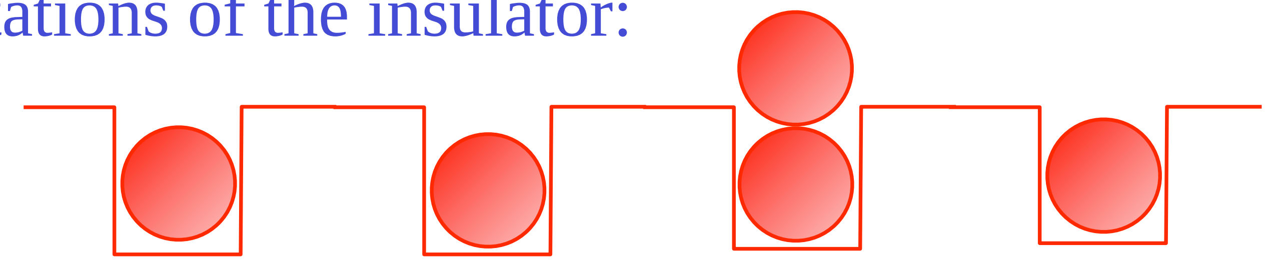
The insulator:



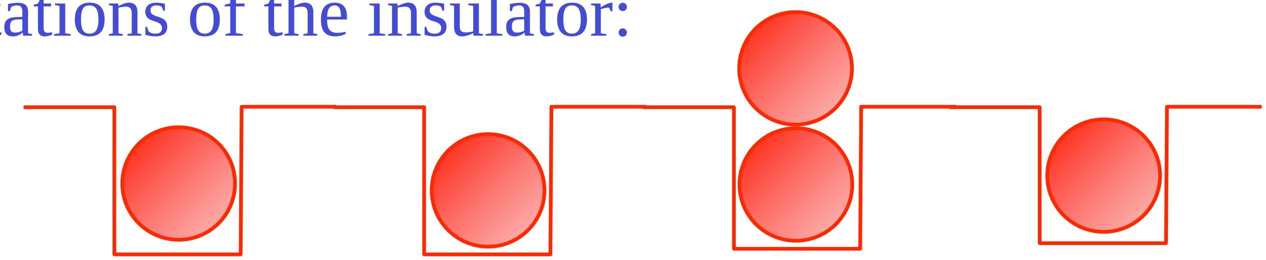
Excitations of the insulator:



Particles $\sim \psi^\dagger$

Holes $\sim \psi$

Excitations of the insulator:



Particles ~ $\psi^\dagger$

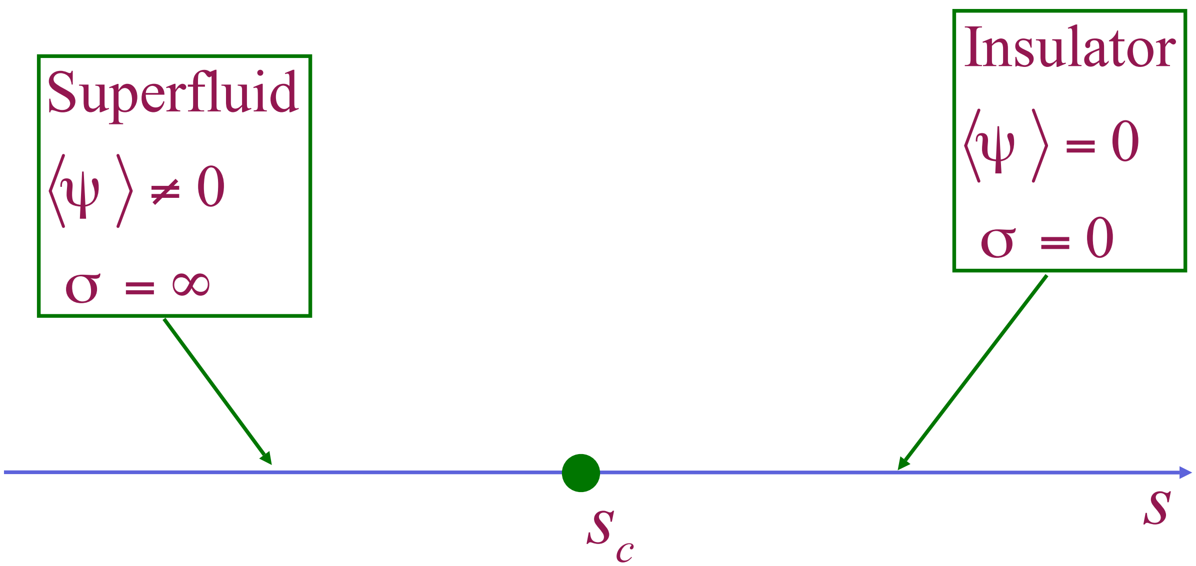
Holes ~ ψ

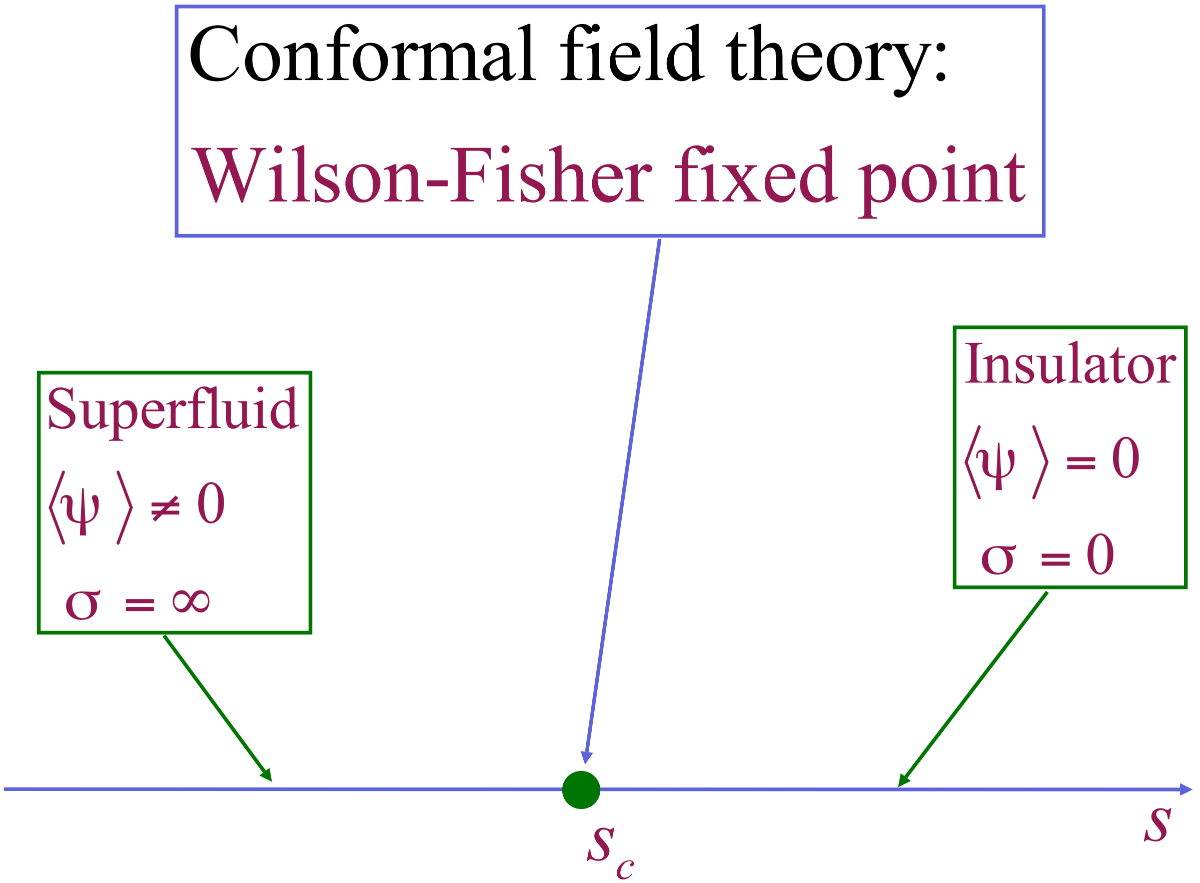
Density of particles = density of holes $\Rightarrow$
"relativistic" field theory for ψ:

$$\mathcal{S} = \int d^3x \left[|\partial_\mu \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\text{Insulator} \Leftrightarrow \langle \psi \rangle = 0$$

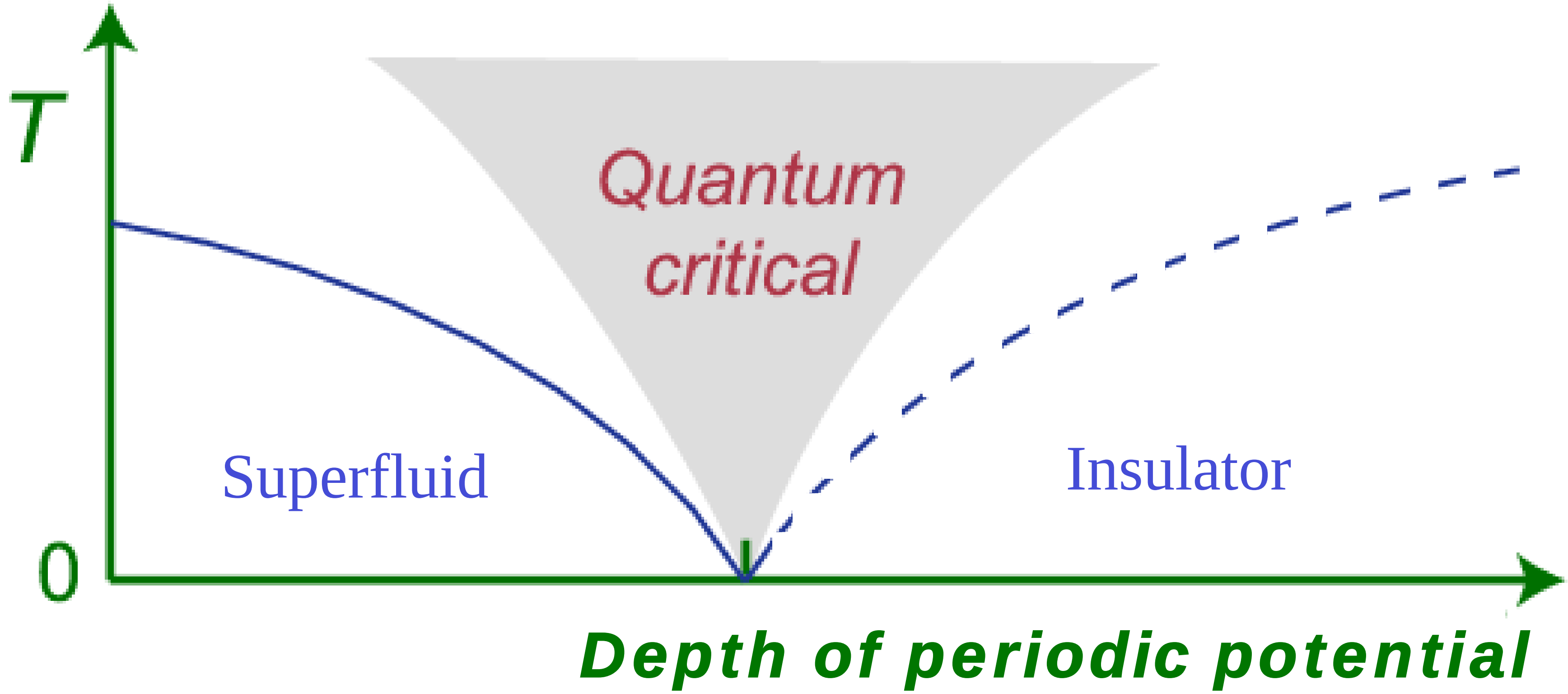
$$\text{Superfluid} \Leftrightarrow \langle \psi \rangle \neq 0$$


Superfluid
⟨ψ⟩ ≠ 0
σ = ∞
Insulator
⟨ψ⟩ = 0
σ = 0
s_c
s



Conformal field theory:
Wilson-Fisher fixed point
Superfluid
$\langle \psi \rangle \neq 0$
$\sigma = \infty$
Insulator
$\langle \psi \rangle = 0$
$\sigma = 0$
s_c
s

Non-zero temperature phase diagram



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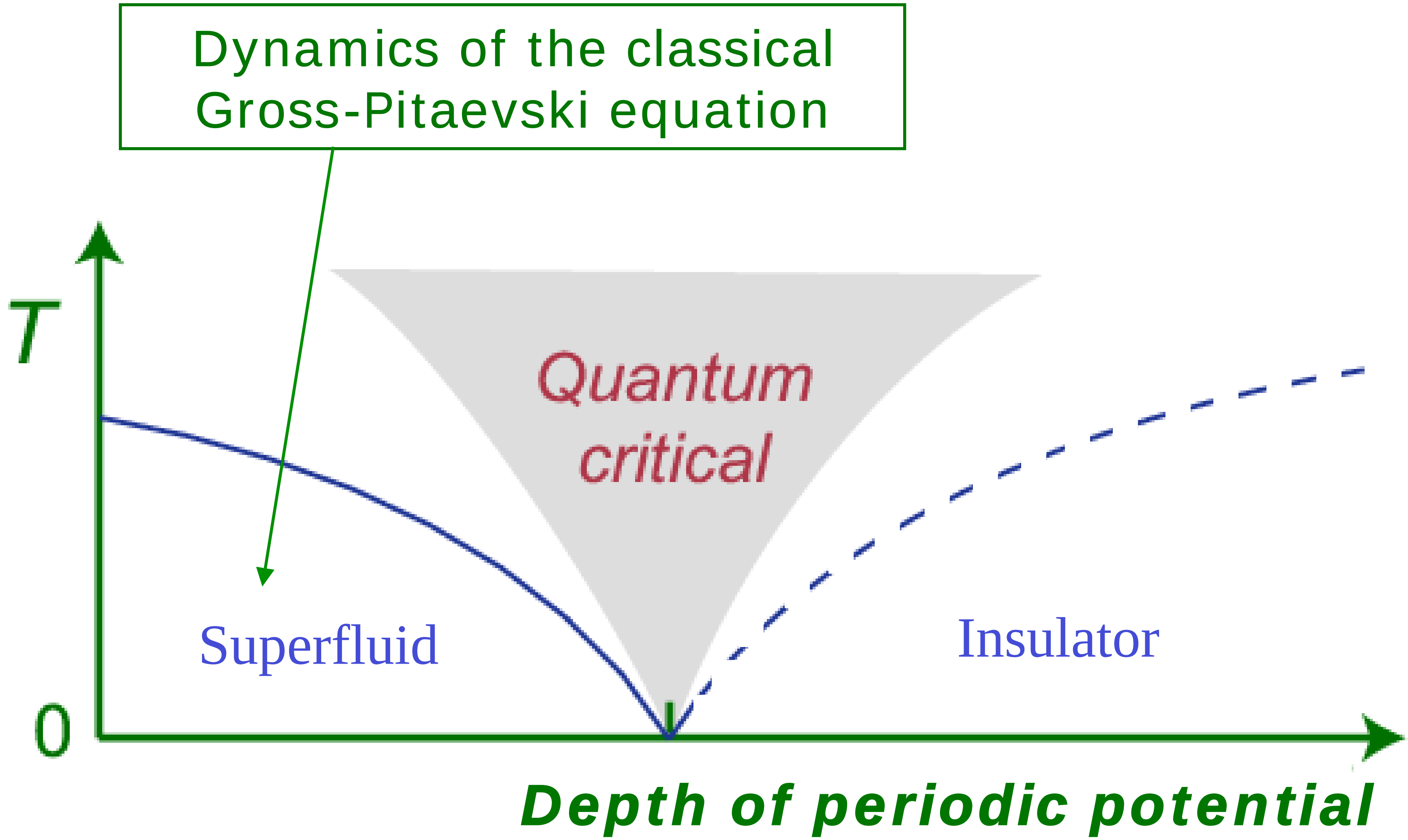

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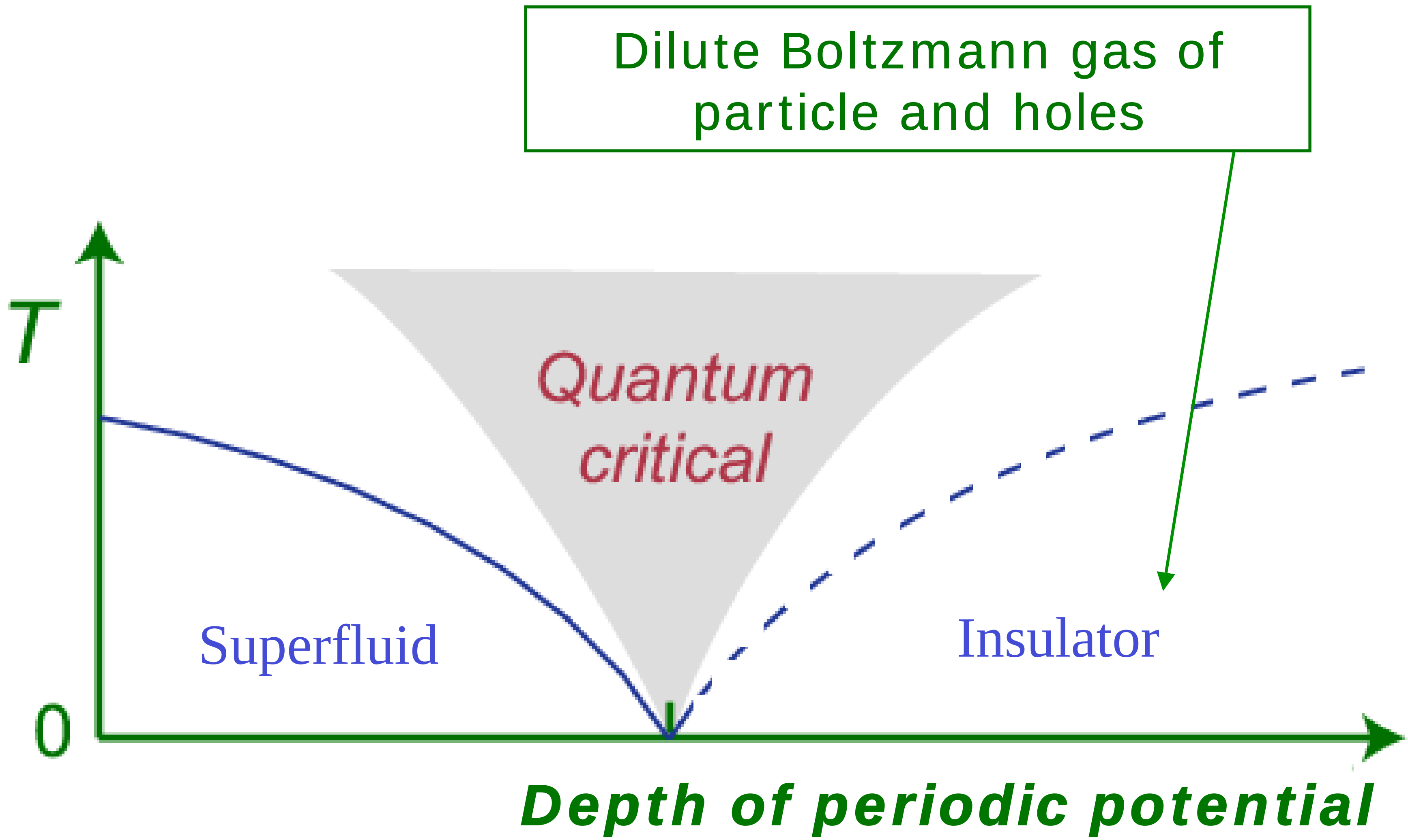

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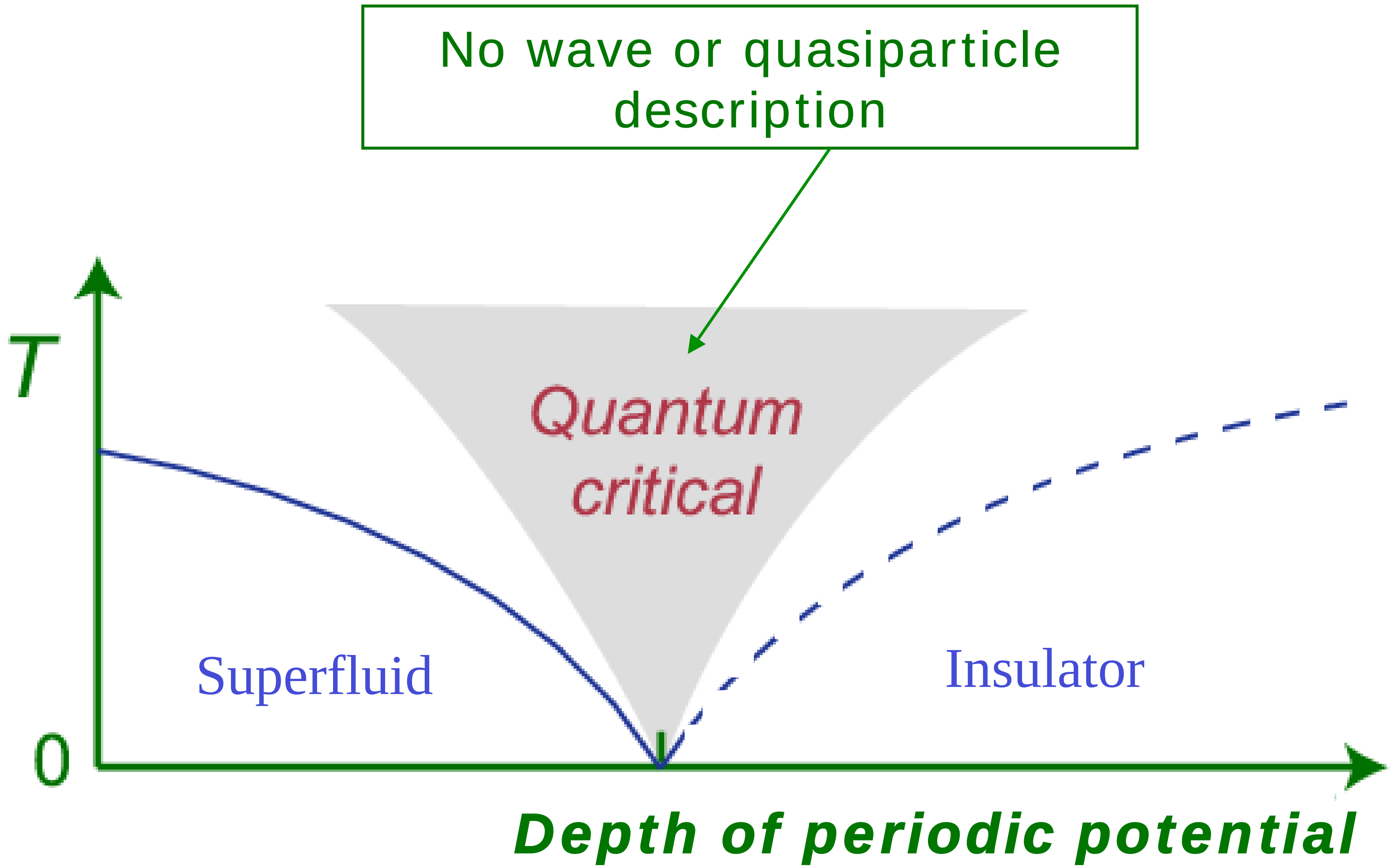

Resistivity of Bi films

$$\sigma_{\text{Superconductor}}(T \to 0) = \infty$$

$$\sigma_{\text{Insulator}}(T \to 0) = 0$$

$$\sigma_{\text{Quantum critical point}}(T \to 0) \approx \frac{4e^2}{h}$$

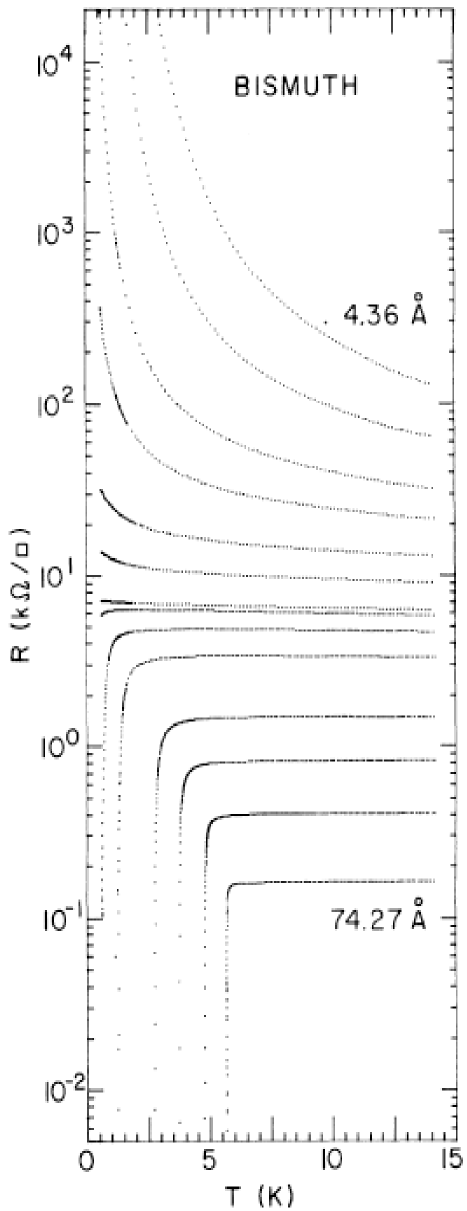

FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

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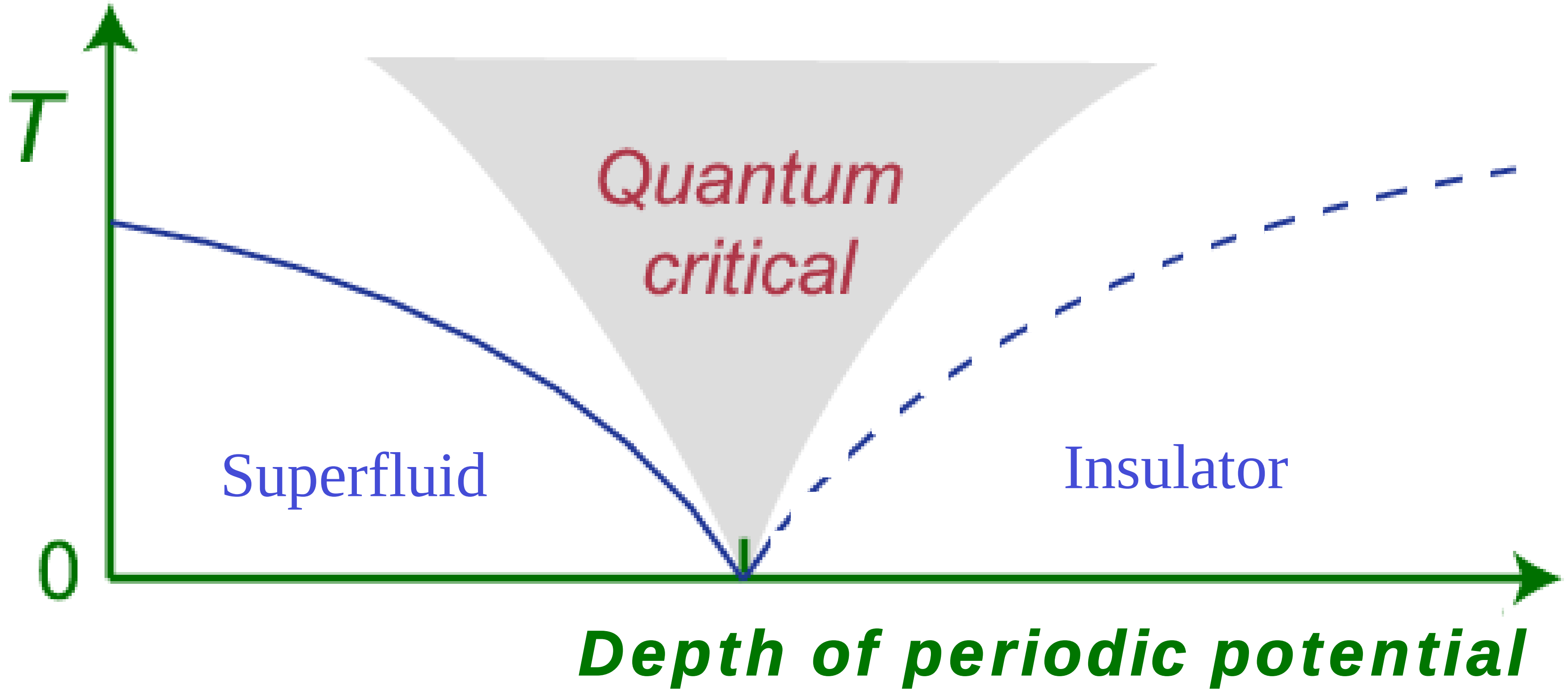

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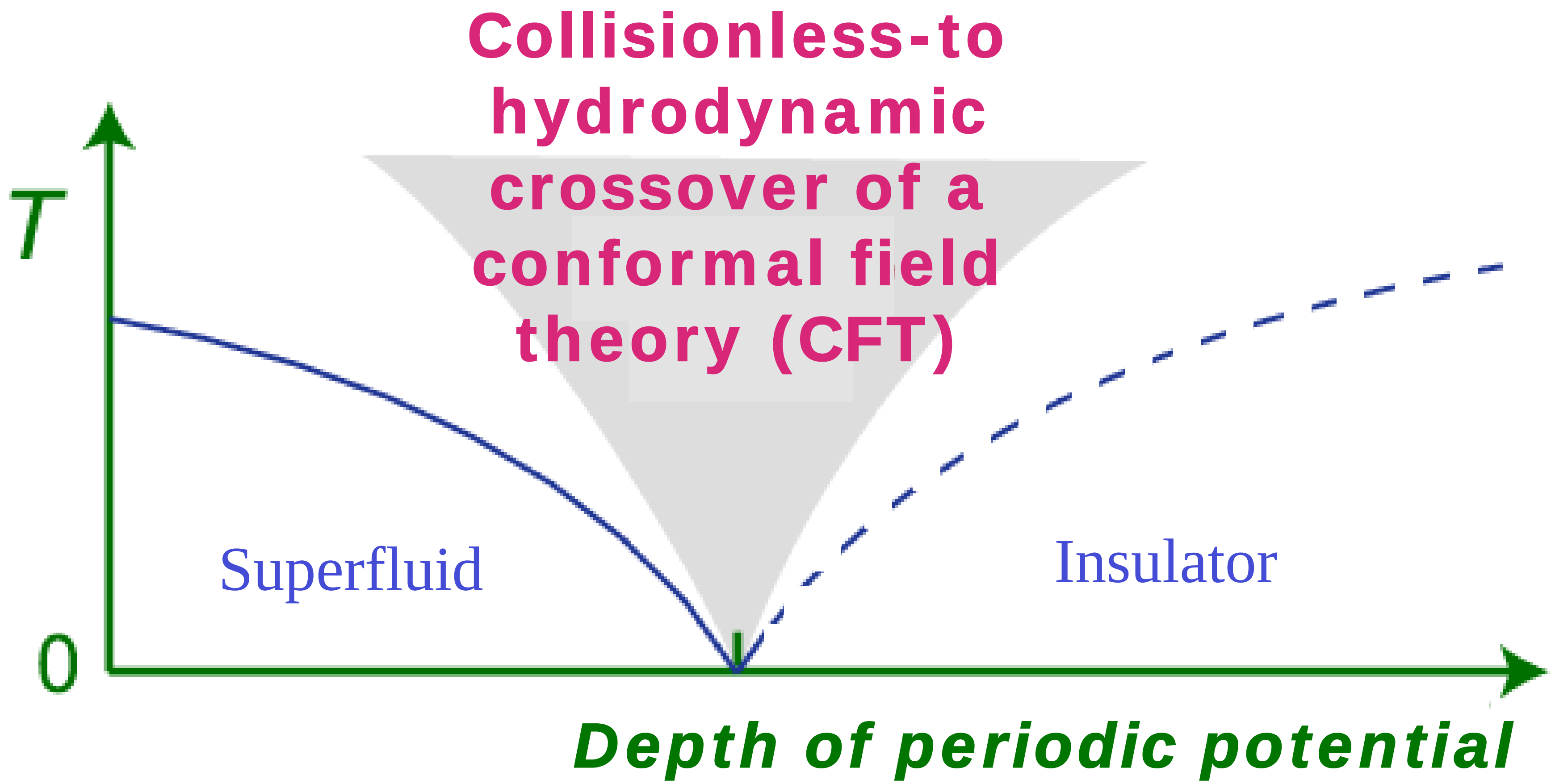

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Collisionless-to-hydrodynamic crossover of a CFT in 2+1 dimensions

Consider the retarded density-density correlation function

$$C(k,\omega) = \langle \rho(k,\omega)\rho(-k,-\omega)\rangle_{\text{ret}}$$

The characteristic collision time for excitations of the CFT is $\hbar/k_B T$. So, for $|\omega - k| \gg T$ we have the collisionless conformal behavior

$$C(k,\omega) = K \frac{k^2}{\sqrt{k^2-\omega^2}}$$

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$$C(k,\omega) = K\frac{k^2}{\sqrt{k^2-\omega^2}}$$

while for $\omega, k \ll T$, we have the hydrodynamic behavior

$$C(k,\omega) = \chi\frac{D_c k^2}{-i\omega + D_c k^2}.$$

So the high frequency conductivity $\sigma(\omega \gg T) = K$, and the hydrodynamic conductivity $\sigma(\omega \ll T) = D_c\chi$, and in general $K \neq D_c\chi$.

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Hydrodynamics of a conformal field theory (CFT)

The scattering cross-section of the thermal excitations is universal and so transport co-efficients are universally determined by $k_B T$

Charge diffusion constant $$D_c = \Theta \frac{c^2}{k_B T}$$

Conductivity $$\sigma_Q = D_c \chi = \Theta \frac{4e^2}{h}$$

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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The AdS/CFT correspondence (Maldacena, Polyakov) relates the hydrodynamics of CFTs to the quantum gravity theory of the horizon of a black hole in Anti-de Sitter space.

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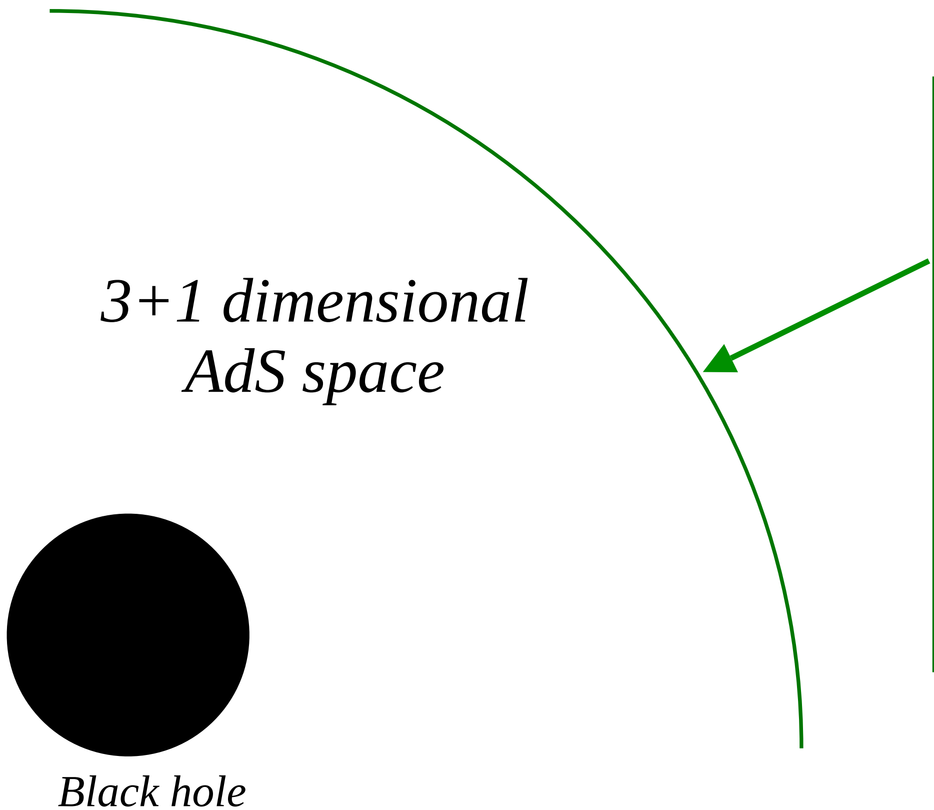
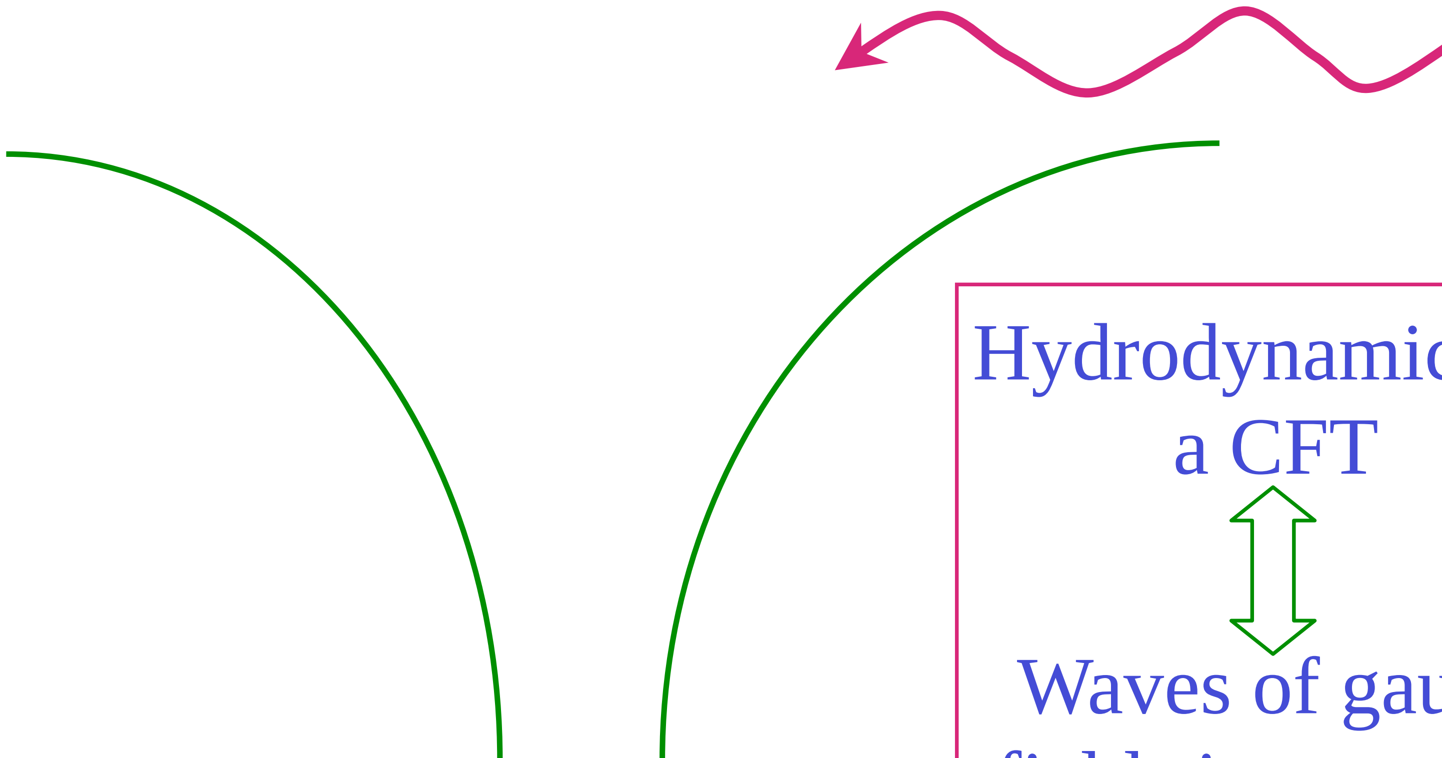


Holographic representation of black hole physics in a 2+1 dimensional CFT at a temperature equal to the Hawking temperature of the black hole.

Hydrodynamics of a conformal field theory (CFT)



Hydrodynamics of a CFT

⇕

Waves of gauge fields in a curved background

Hydrodynamics of a conformal field theory (CFT)

For the (unique) CFT with a SU(*N*) gauge field and 16 supercharges, we know the exact diffusion constant associated with a global SO(8) symmetry:

Spin diffusion constant $$D_c = \frac{3}{4\pi}\frac{c^2}{k_B T}$$

Spin conductivity $$\sigma_Q = \frac{N^{3/2}}{3\sqrt{2}\pi}$$

P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

Collisionless-to-hydrodynamic crossover of solvable SYM_3

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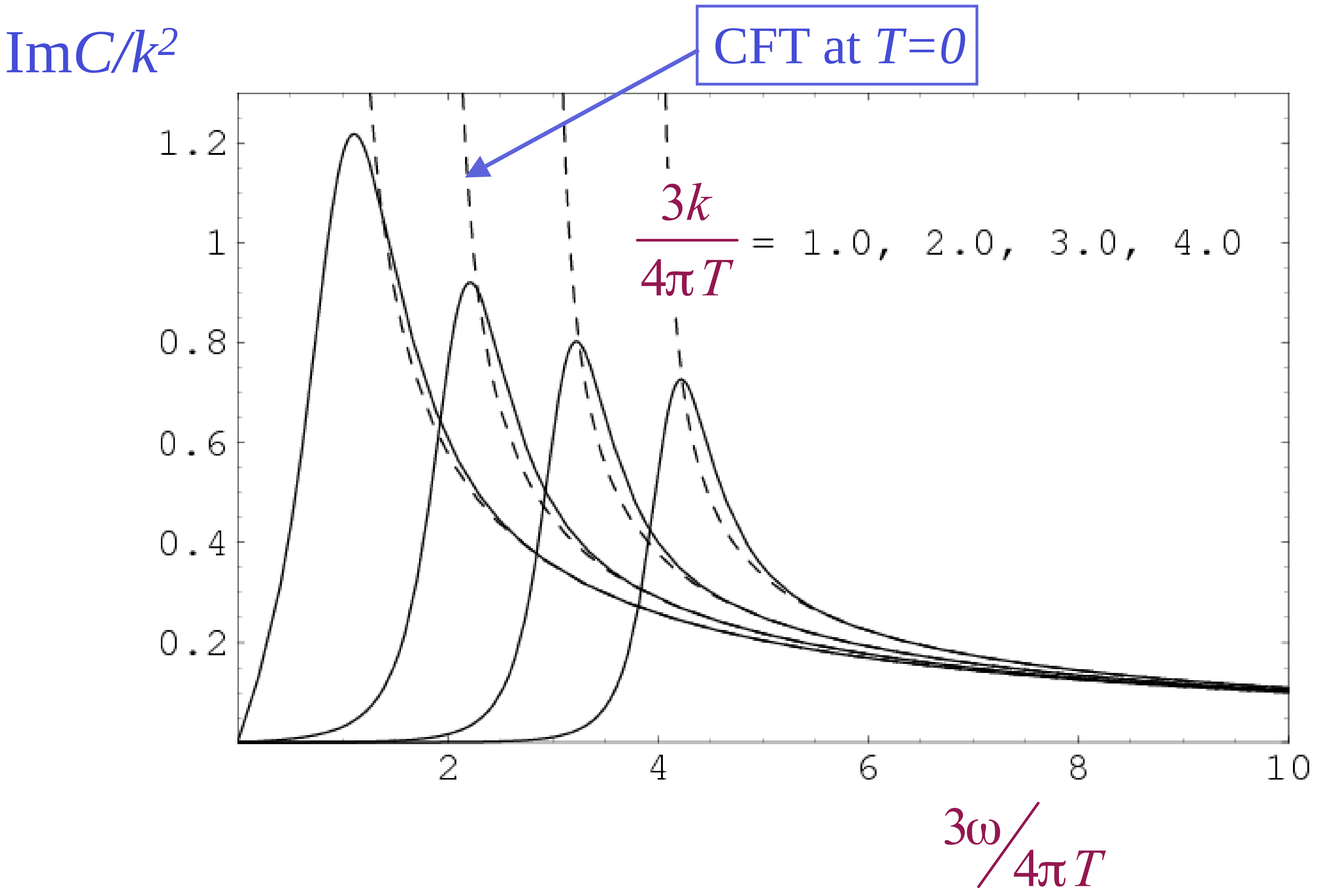

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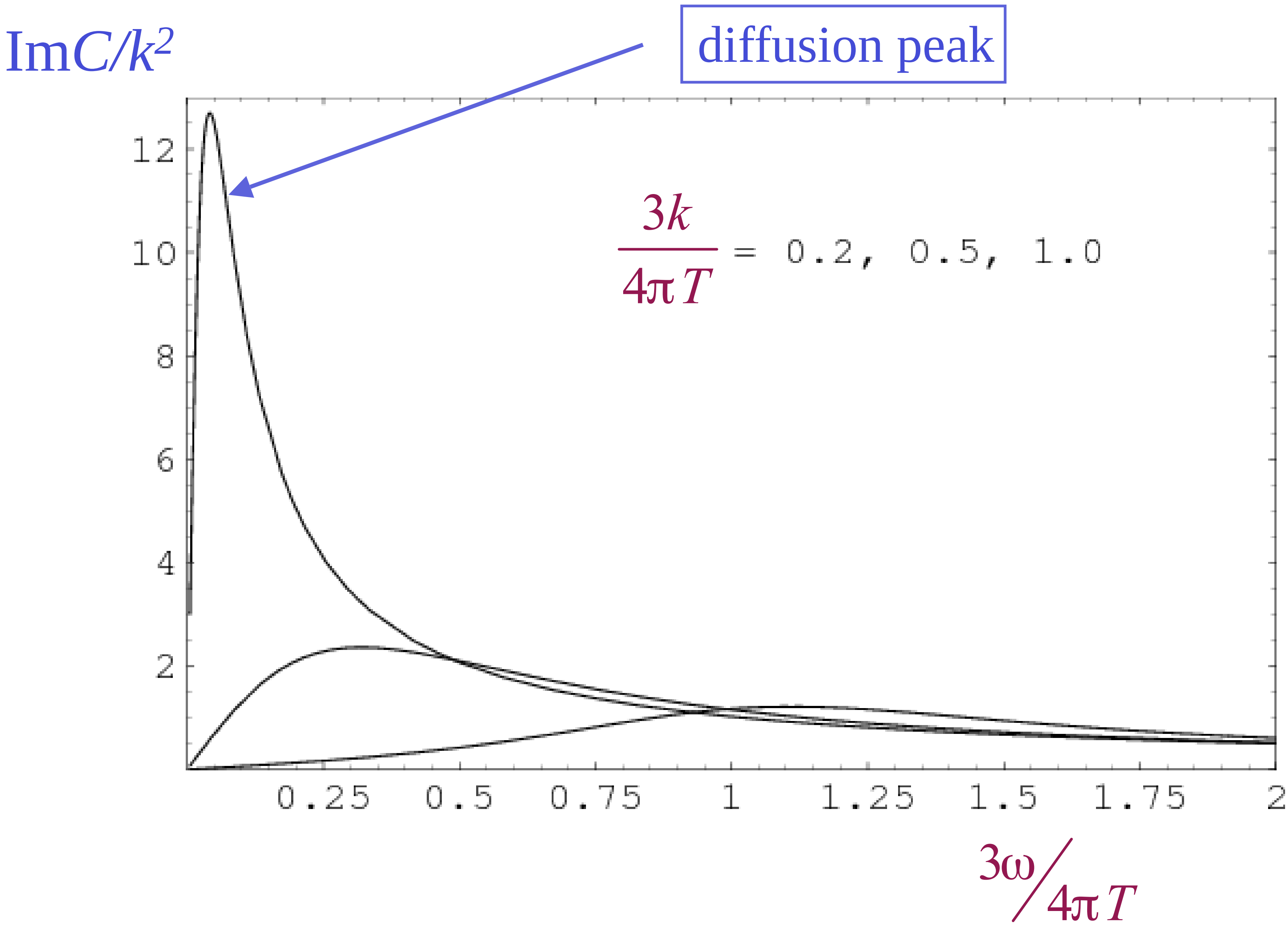

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For experimental applications, we must move away from the ideal CFT

e.g. $$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 \left|\vec{\nabla}\psi\right|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

For experimental applications, we must move away from the ideal CFT

- A chemical potential μ

e.g. $$\mathcal{S} = \int d^2r d\tau \left[|(\partial_\tau - \mu)\psi|^2 + v^2 \left|\vec{\nabla}\psi\right|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

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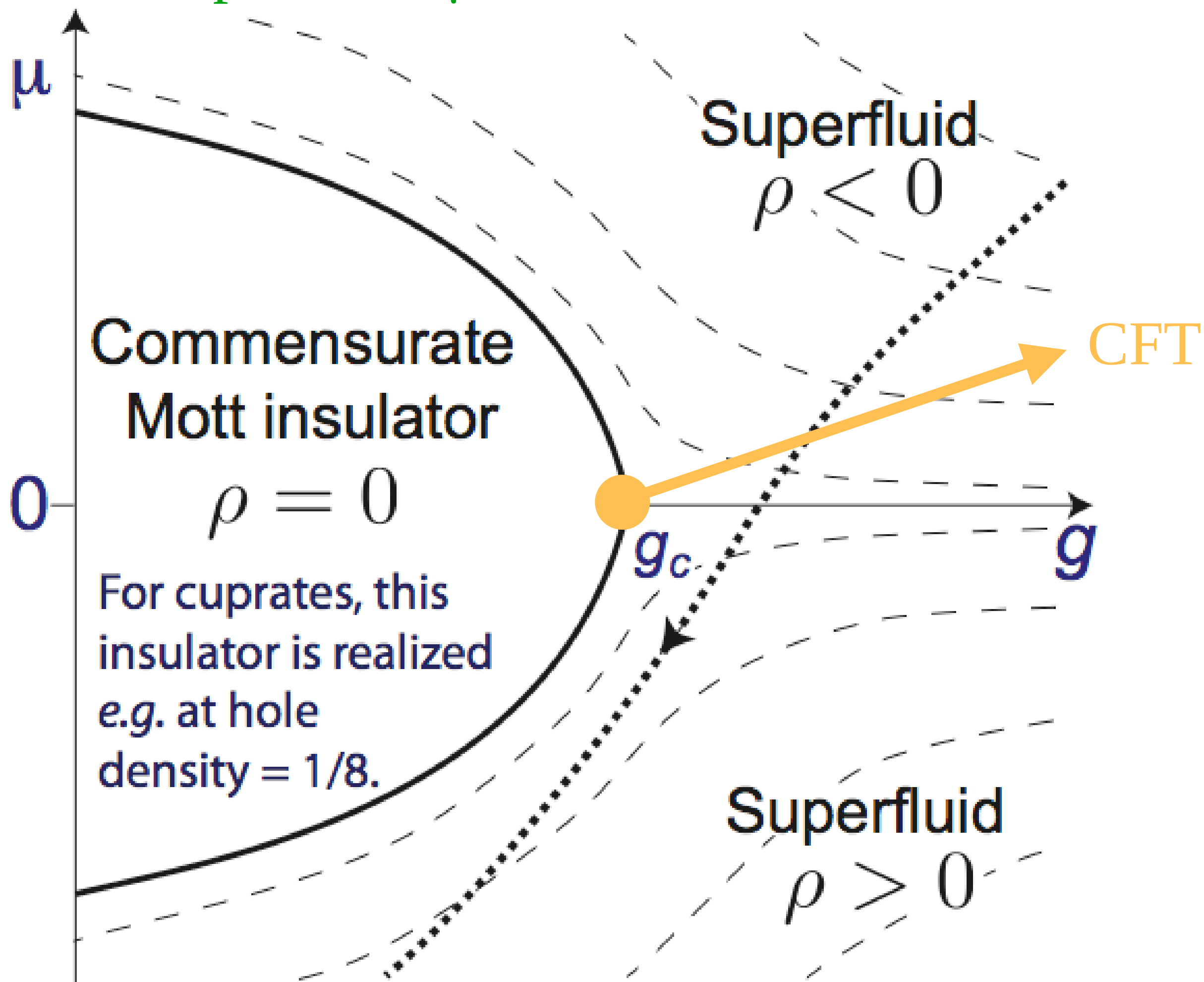

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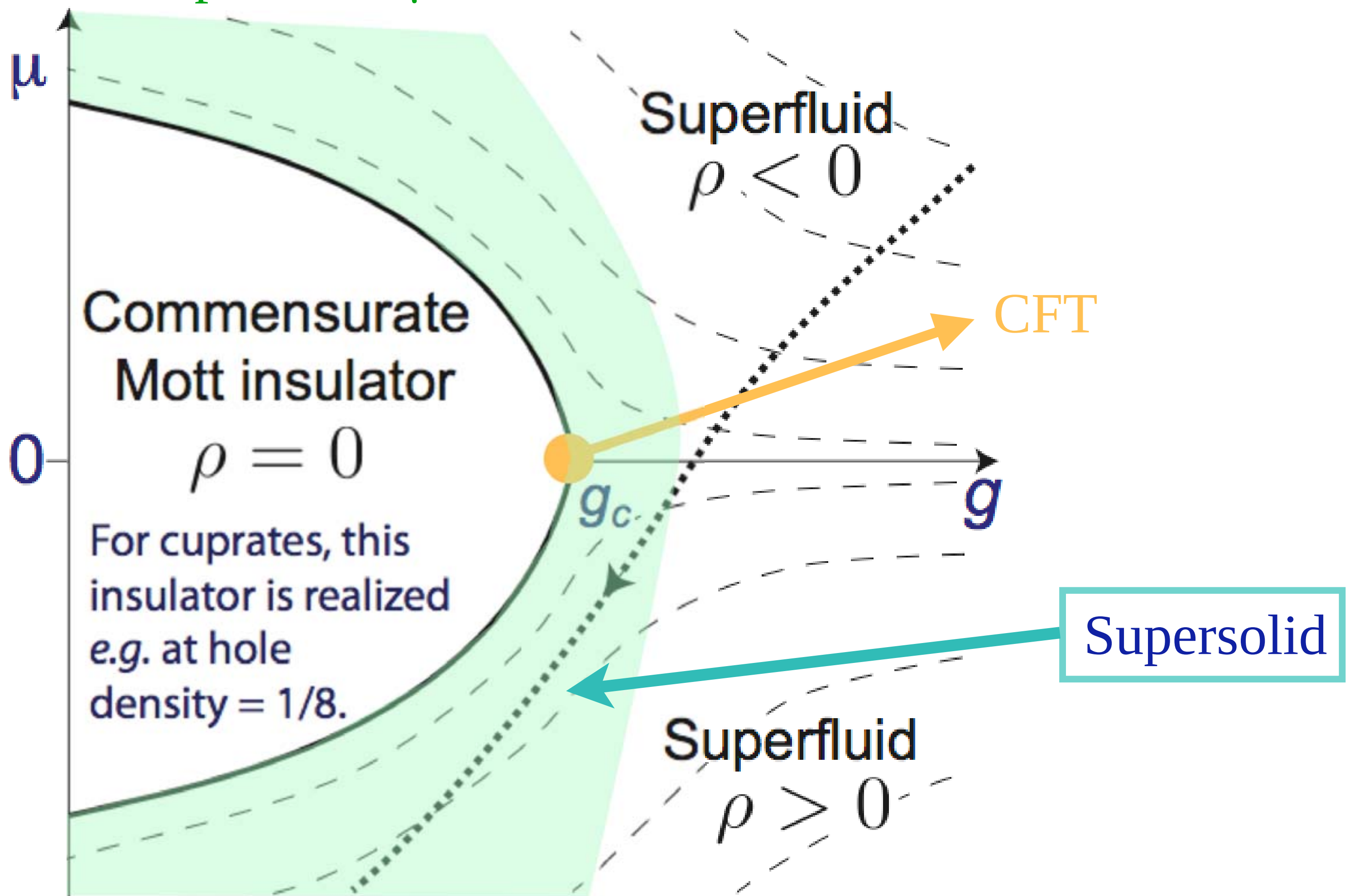

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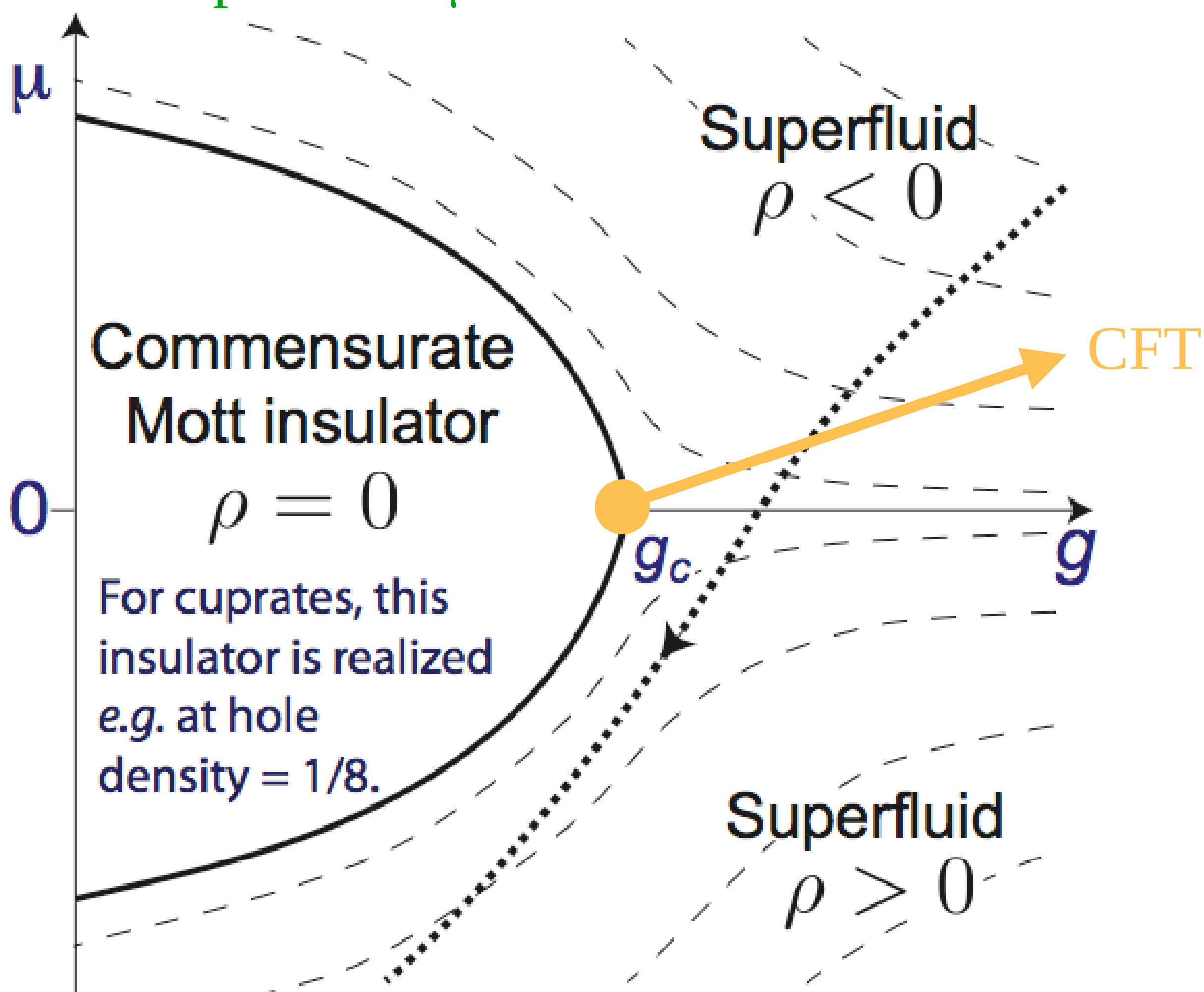

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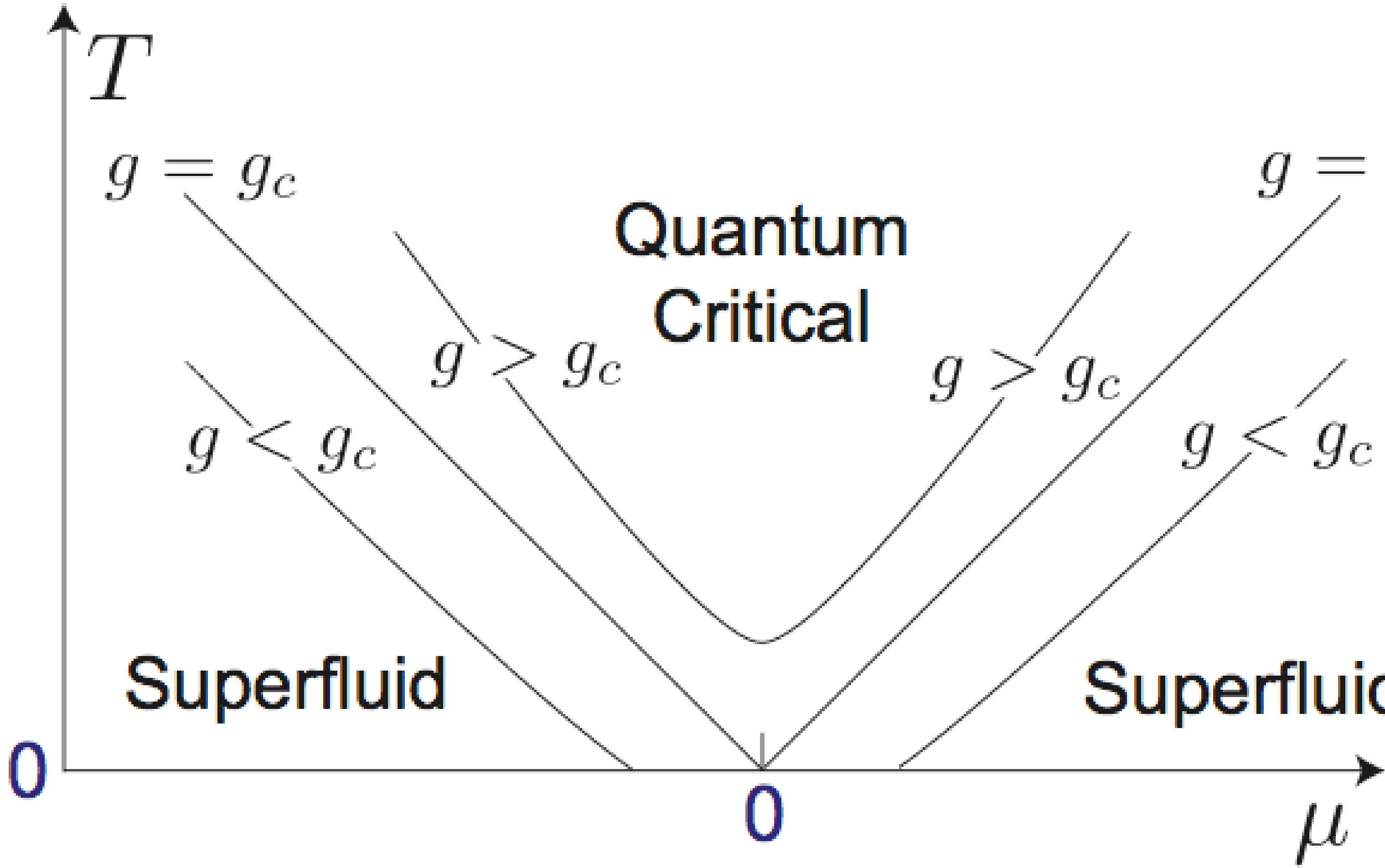

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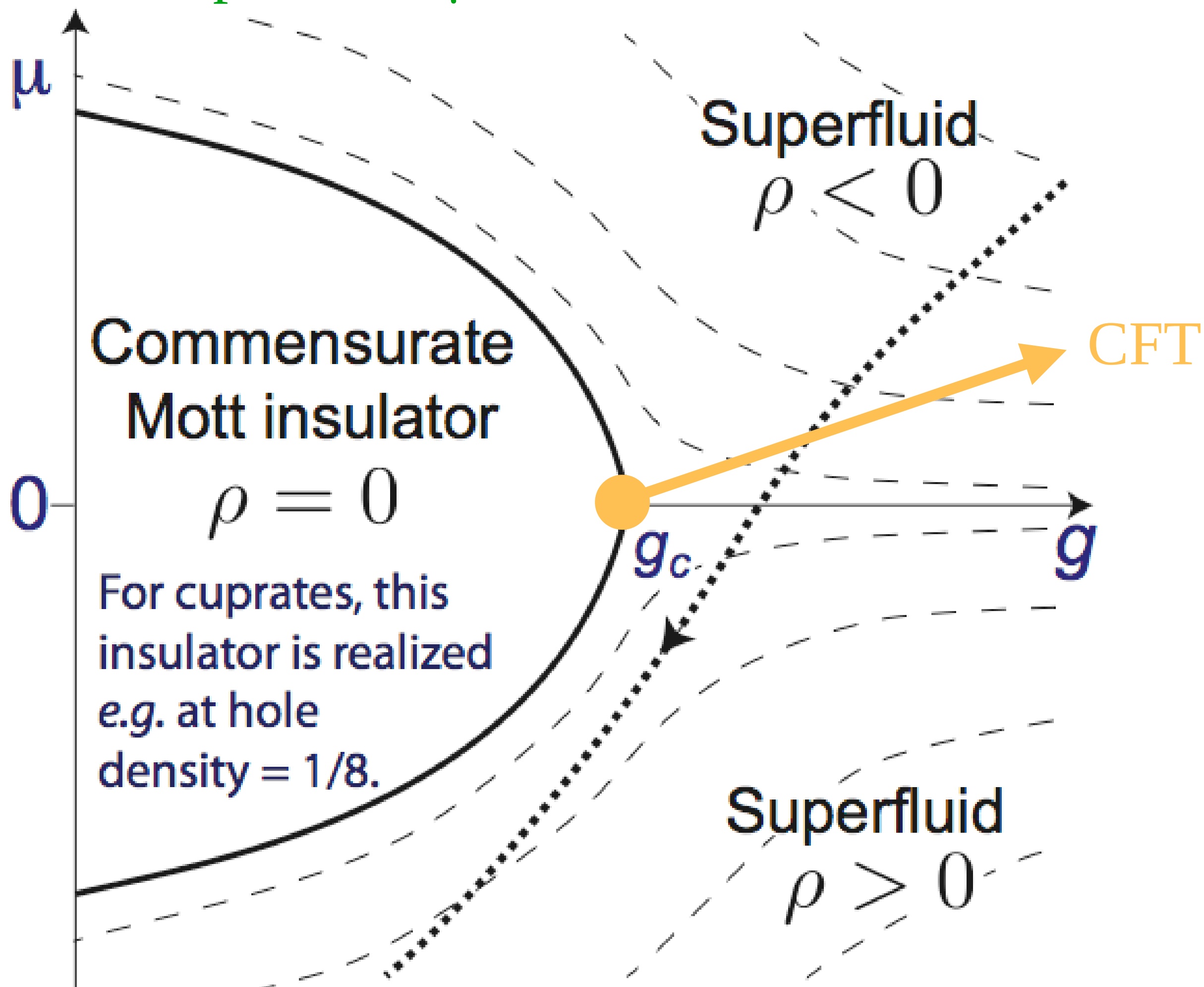

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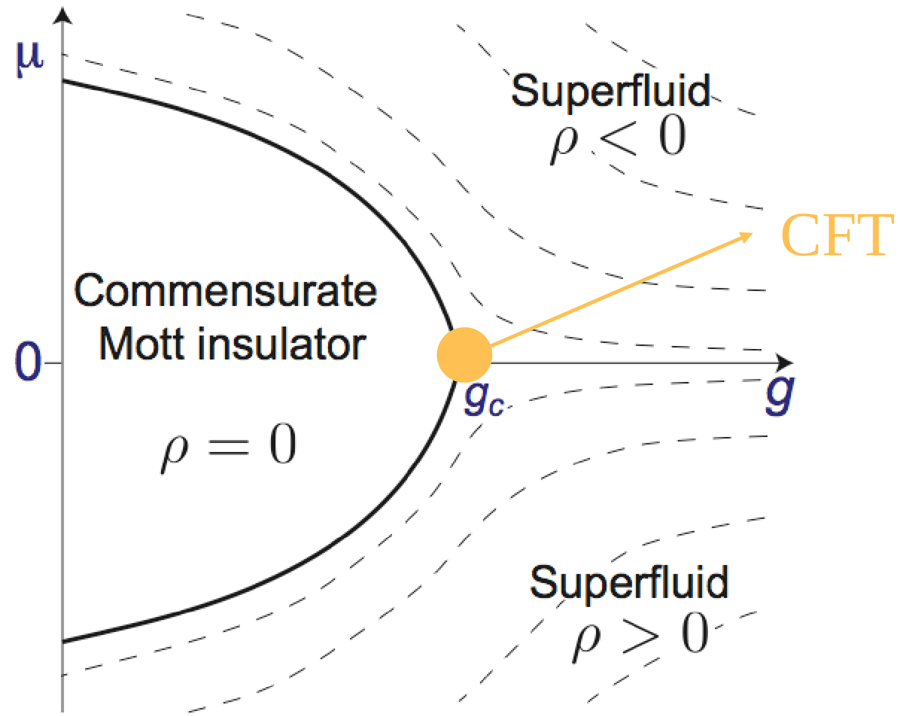
For experimental applications, we must move away from the ideal CFT

- A chemical potential μ
- A magnetic field B



e.g.

$$\mathcal{S} = \int d^2r d\tau \left[|(\partial_\tau - \mu)\psi|^2 + v^2 \left|(\vec{\nabla} - i\vec{A})\psi\right|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\nabla \times \vec{A} = B$$

In the hydrodynamic regime, $\hbar\omega \ll k_B T$, we can use classical principles involving relaxation to local equilibrium to understand these perturbations.
The variables entering the hydrodynamic theory are

- the external magnetic field $F^{\mu\nu}$,

$$F^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & -B & 0 \end{pmatrix},$$

- $T^{\mu\nu}$, the stress energy tensor,
- J^{μ}, the current,
- ρ, the local number density,
- ε, the local energy density,
- P, the local pressure,
- u^{μ}, the local velocity, and
- σ_Q, a universal conductivity, which is the **single transport co-efficient**.

The dependence of ε, P, σ_Q on T and v follows from simple scaling arguments

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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$$\begin{aligned} \partial_\mu J^\mu &= 0 \\ \partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \end{aligned}$$

Conservation laws/equations of motion

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

$$
\begin{aligned}
\partial_\mu J^\mu &= 0 \\
\partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \\
T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\
J^\mu &= \rho u^\mu
\end{aligned}
$$

Constitutive relations which follow from Lorentz transformation to moving frame

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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\partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \\
T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\
J^\mu &= \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[\left(-\partial_\nu \mu + F_{\nu\lambda} u^\lambda \right) + \mu \frac{\partial_\mu T}{T} \right]
\end{aligned}$$

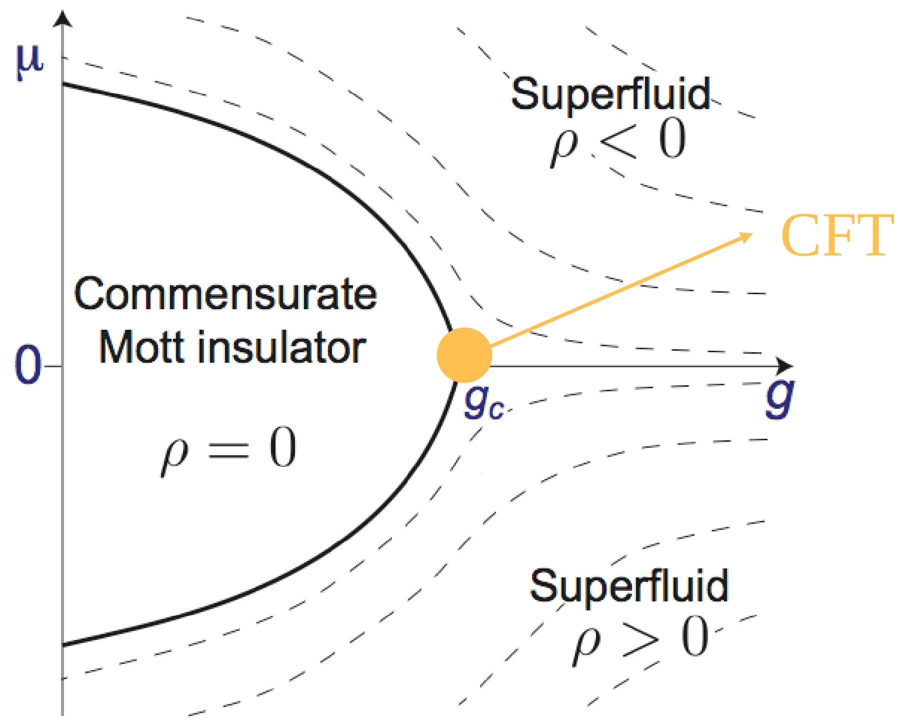
Single dissipative term allowed by requirement of positive entropy production. There is only one independent transport co-efficient

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

For experimental applications, we must move away from the ideal CFT

- A chemical potential μ

- A magnetic field *B*



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$$\nabla \times \vec{A} = B$$

For experimental applications, we must move away from the ideal CFT

- A chemical potential μ

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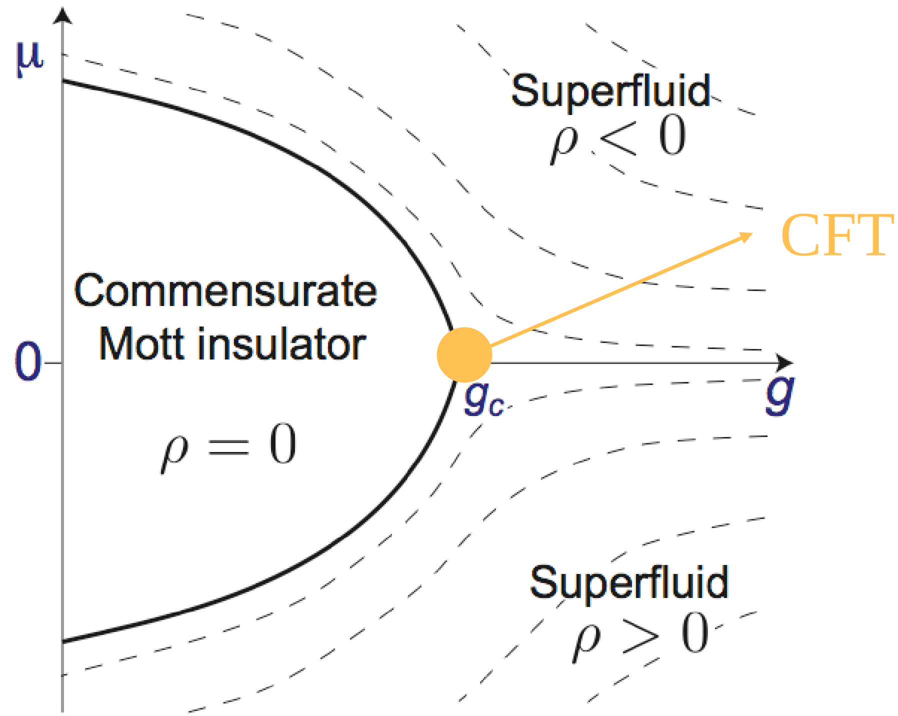
- An impurity scattering rate $1/\tau_{imp}$ (its T dependence follows from scaling arguments)



e.g.

$$\mathcal{S} = \int d^2 r d\tau \left[|(\partial_\tau - \mu)\psi|^2 + v^2 \left|(\vec{\nabla} - i\vec{A})\psi\right|^2 - g|\psi|^2 + V(r)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\nabla \times \vec{A} = B \quad , \quad \overline{V(r)} = 0 \quad , \quad \overline{V(r)V(r')} = V_{\text{imp}}^2 \delta^2(r - r')$$

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

$$\begin{aligned}
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\partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu + \frac{1}{\tau_{\text{imp}}} \left(\delta^\mu_\nu + u^\mu u_\nu\right) T^{\nu\gamma} u_\gamma \\
T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\
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S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

From these relations, we obtained results for the transport co-efficients, expressed in terms of a "cyclotron" frequency and damping:

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Longitudinal conductivity

$$\sigma_{xx} = \sigma_Q \left[\frac{(\omega + i/\tau_{\text{imp}})(\omega + i\gamma + i\omega_c^2/\gamma + i/\tau_{\text{imp}})}{(\omega + i\gamma + i/\tau_{\text{imp}})^2 - \omega_c^2} \right] .$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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$$= \sigma_Q + \frac{4e^2\rho^2 v^2}{(\varepsilon + P)} \frac{1}{(-i\omega + 1/\tau_{\rm imp})} \quad \text{as } B \to 0$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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Hall conductivity

$$\begin{aligned} \sigma_{xy} &= -\frac{2e\rho c}{B}\left[\frac{\gamma^2 + \omega_c^2 - 2i\gamma\omega + 2\gamma/\tau_{\rm imp}}{(\omega + i\gamma + i/\tau_{\rm imp})^2 - \omega_c^2}\right] \\ &= B\left[\sigma_Q \frac{4e\rho v^2}{(\varepsilon + P)(1/\tau_{\rm imp} - i\omega)} + \frac{8e^3\rho^3 v^4}{(\varepsilon + P)^2(1/\tau_{\rm imp} - i\omega)^2}\right] \\ & \qquad \text{as } B \to 0 \end{aligned}$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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Thermal conductivity

$$\begin{aligned}\kappa_{xx} &= \sigma_Q \left(\frac{k_B^2 T}{4e^2}\right)\left(\frac{\varepsilon + P}{k_B T \rho}\right)^2 \left[\frac{(\omega_c^2/\gamma)(\omega_c^2/\gamma + 1/\tau_{\text{imp}})}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2}\right] \\ &= \frac{1}{\sigma_Q} k_B^2 T \left(\frac{c(\varepsilon + P)}{k_B T B}\right)^2 \left[\frac{\gamma(\omega_c^2/\gamma + 1/\tau_{\text{imp}})}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2}\right]\end{aligned}$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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Thermal conductivity

$$\kappa_{xx} = \sigma_Q \left(\frac{k_B^2 T}{4e^2}\right)\left(\frac{\varepsilon + P}{k_B T \rho}\right)^2 \quad \boxed{\rightarrow 1 \text{ as } B \rightarrow 0}$$

$$= \frac{1}{\sigma_Q} k_B^2 T \left(\frac{c(\varepsilon + P)}{k_B T B}\right)^2 \left[\frac{\gamma(\omega_c^2/\gamma + 1/\tau_{\text{imp}})}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2}\right]$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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$\rightarrow 1$ as $\rho \rightarrow 0$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

Outline

1. Superfluid/supersolid/insulator quantum transitions

   Insulators at integer and commensurate densities

2. Theory of quantum-critical transport

   Collisionless-t0-hydrodynamic crossover of conformal field theories

3. Hydrodynamics at incommensurate densities with impurities and a magnetic field

   Exact relations between thermoelectric co-efficients

4. Nernst effect in the cuprate superconductors

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Nernst signal

$$e_N = \left(\frac{k_B}{2e}\right)\left(\frac{\varepsilon + P}{k_B T \rho}\right)\left[\frac{\omega_c/\tau_{\rm imp}}{(\omega_c^2/\gamma + 1/\tau_{\rm imp})^2 + \omega_c^2}\right]$$

$$\frac{k_B}{2e} = 43.086\mu\mathrm{V/K}$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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Transverse thermoelectric co-efficient

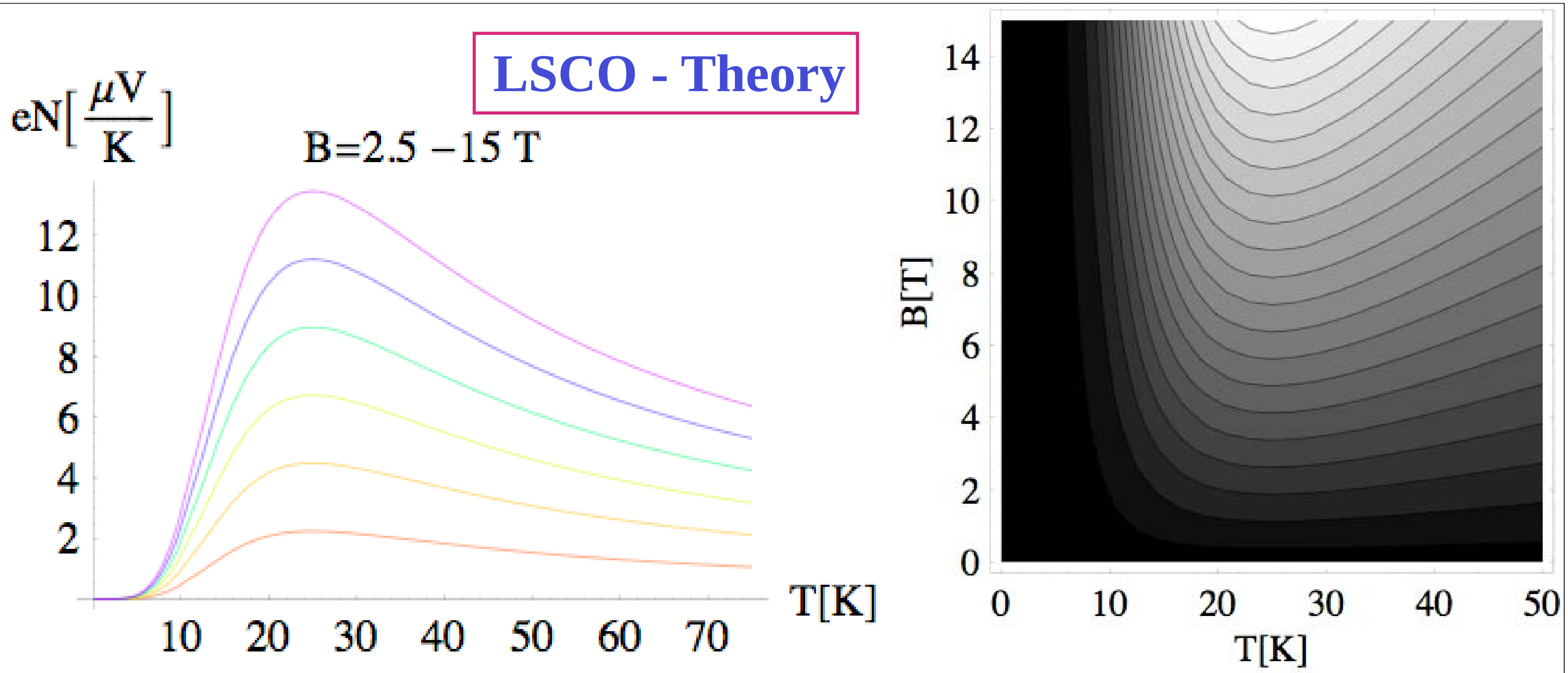
$$\left(\frac{h}{2ek_B}\right)\alpha_{xy} = \Phi_s \overline{B}\,(k_B T)^2 \left(\frac{2\pi\tau_{\rm imp}}{\hbar}\right)^2 \frac{\overline{\rho}^2 + \Phi_\sigma \Phi_{\varepsilon+P} (k_B T)^3\, \hbar/2\pi\tau_{\rm imp}}{\Phi^2_{\varepsilon+P}(k_B T)^6 + \overline{B}^2 \overline{\rho}^2 (2\pi\tau_{\rm imp}/\hbar)^2},$$

where

$$B = \overline{B}\phi_0/(\hbar v)^2 \quad ; \quad \rho = \overline{\rho}/(\hbar v)^2.$$

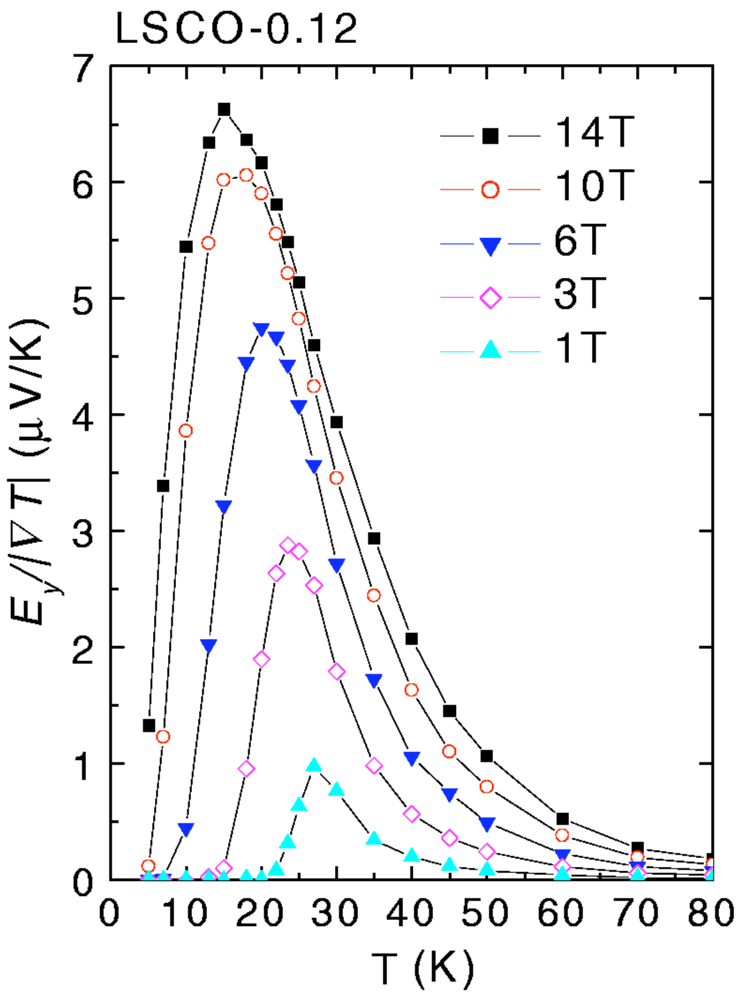
S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

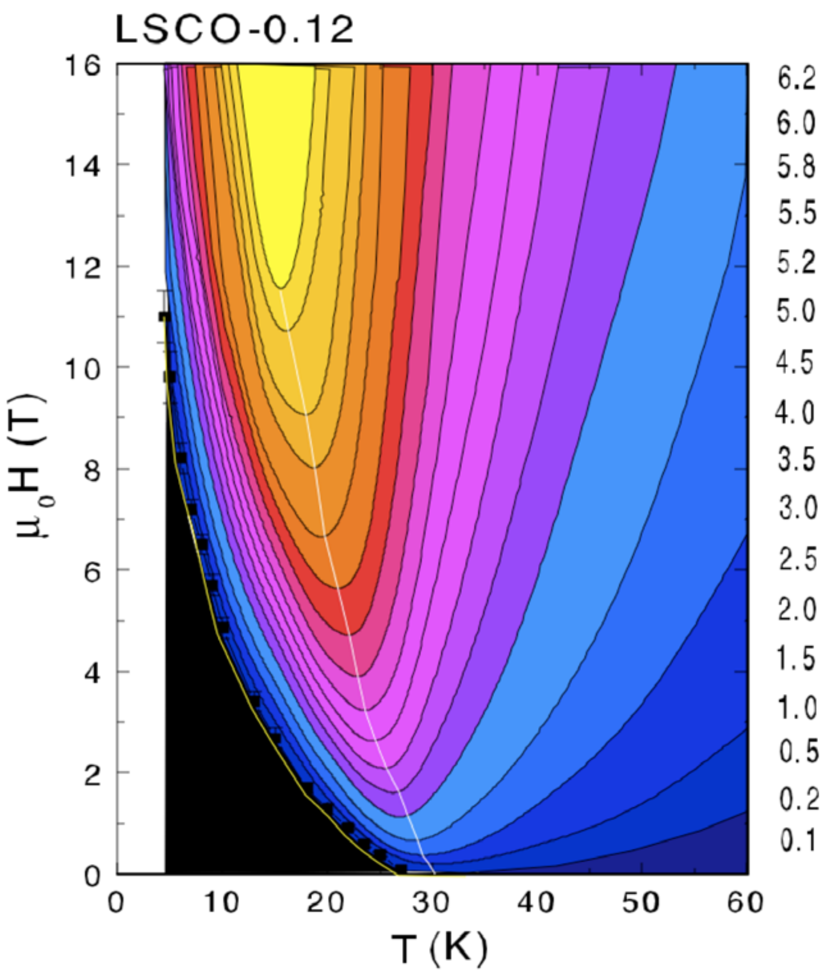
LSCO - Theory



S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

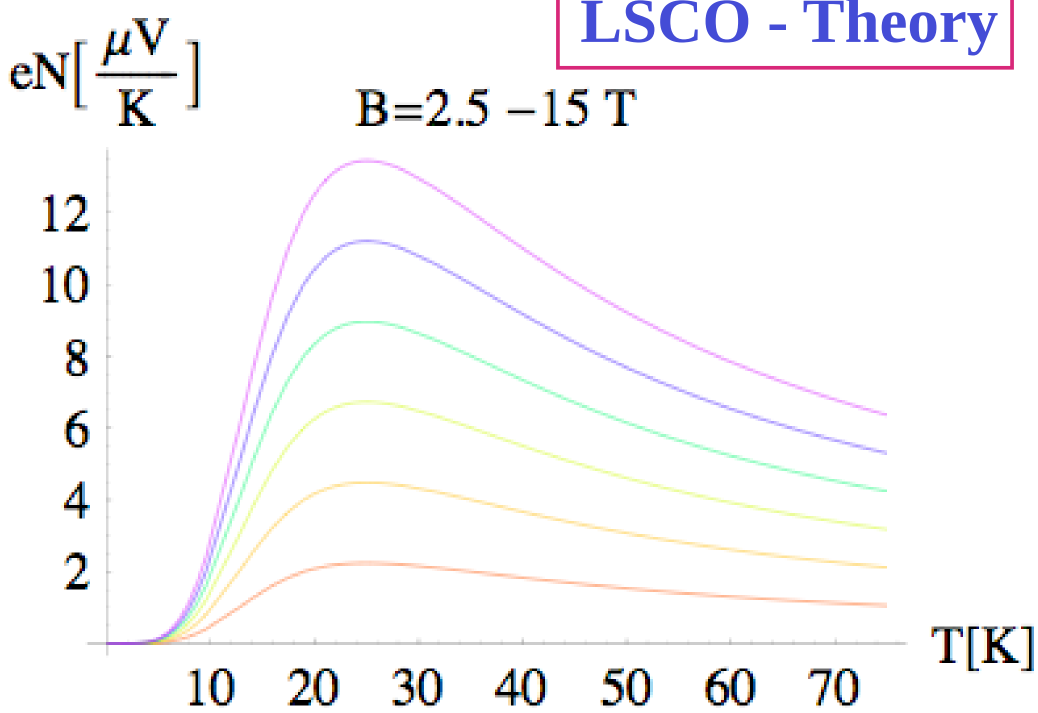
LSCO - Experiments

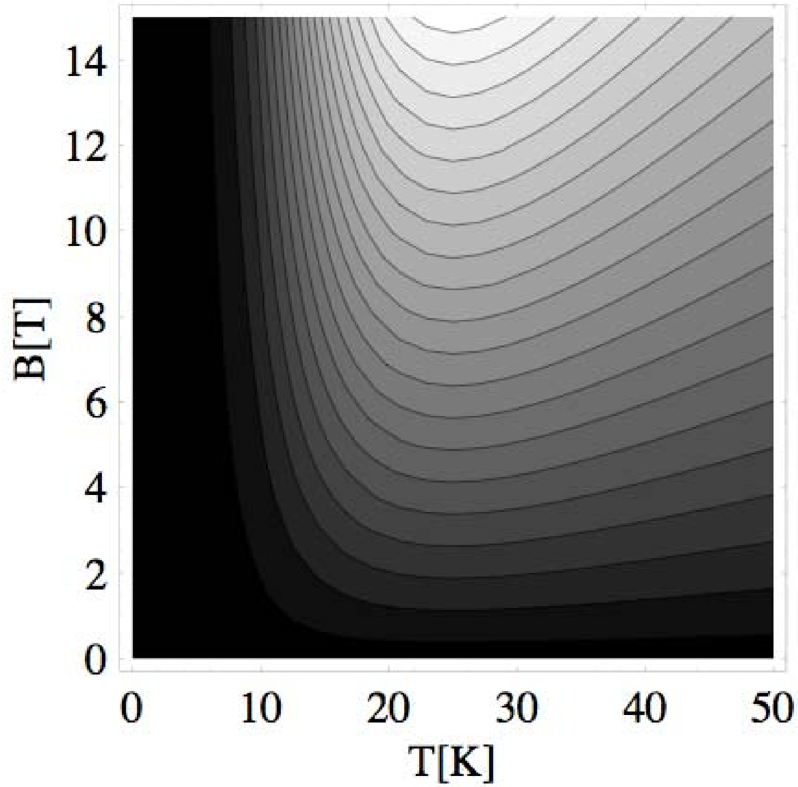




N. P. Ong *et al.*

LSCO - Theory





Only input parameters

$$\hbar v = 47 \text{ meV Å}$$

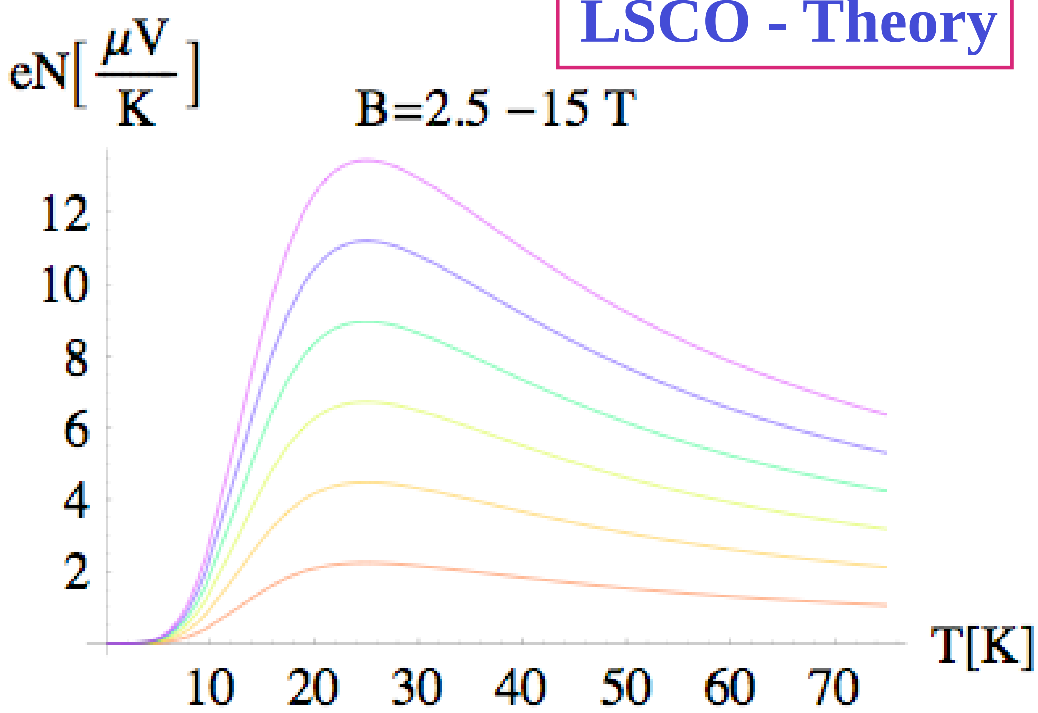
$$\tau_{\text{imp}} \approx 10^{-12} \text{ s}$$

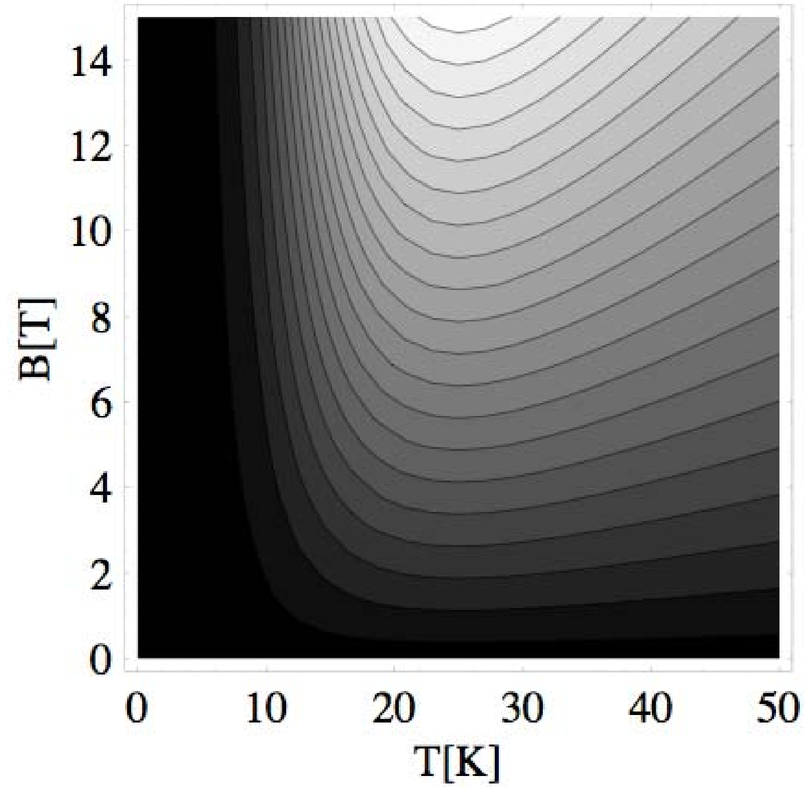
Output

$$\omega_c = 6.2\text{GHz} \cdot \frac{B}{1\text{T}} \left(\frac{35\text{K}}{T}\right)^3$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

LSCO - Theory





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Similar to velocity estimates by
A.V. Balatsky and Z-X. Shen, *Science* **284**, 1137 (1999).

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

To the solvable supersymmetric, Yang-Mills theory CFT, we add

- A chemical potential μ
- A magnetic field B

After the AdS/CFT mapping, we obtain the Einstein-Maxwell theory of a black hole with

- An electric charge
- A magnetic charge

The exact results are found to be in *precise* accord with *all* hydrodynamic results presented earlier

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

Conclusions

- General theory of transport in a weakly disordered ``vortex liquid'' state.
- “Relativistic” magnetohydrodynamics offers an efficient approach to disentangling momentum and charge transport
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.
- Simple model reproduces many trends of the Nernst measurements in cuprates.