Paramagnon fractionalization theory of the cuprate pseudogap metal

New Directions in Strong Correlation Physics:
From Strange Metals to Topological Superconductivity
Aspen Winter Conference
January 24, 2022

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Talk online: sachdev.physics.harvard.edu

INSTITUTE FOR ADVANCED STUDY

PHYSICS
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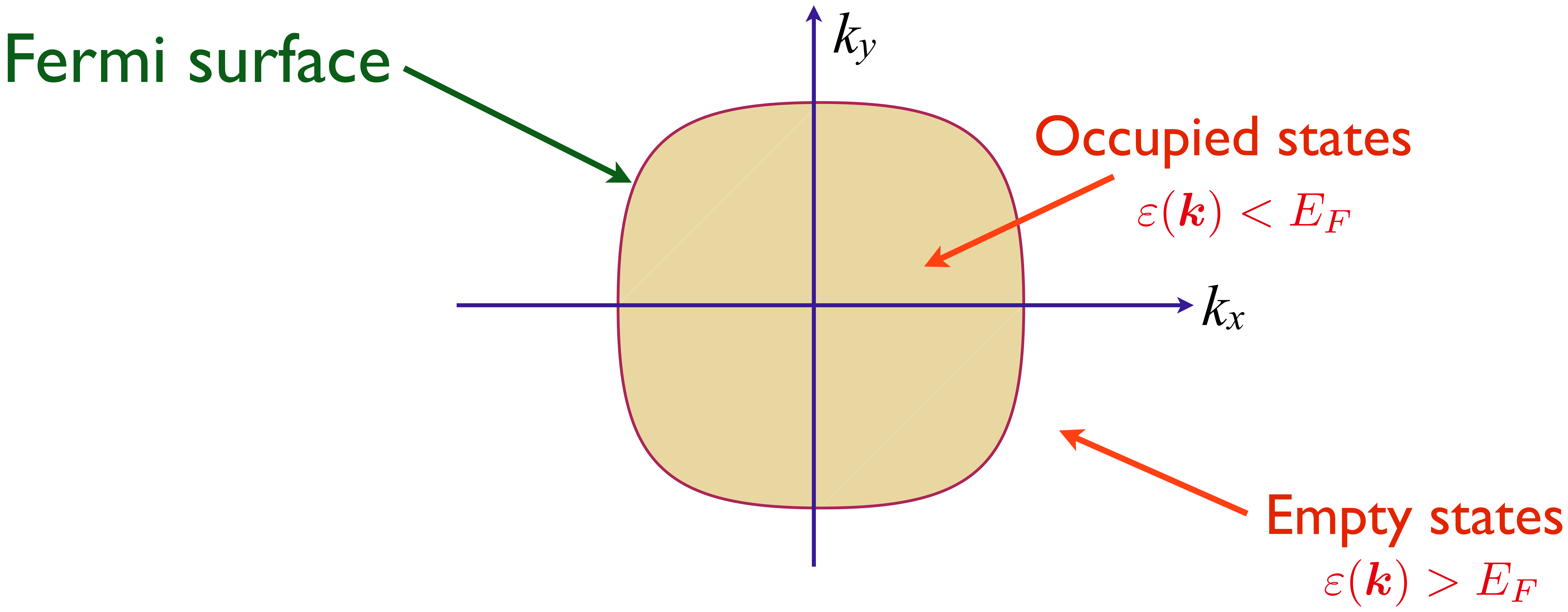
Luttinger relation

Electrons move with momentum **k** through the lattice with dispersion $\varepsilon(\boldsymbol{k})$



$$2 \times \frac{\text{Volume inside Fermi surface}}{(2\pi)^d} = \text{density of electrons (mod 2)}$$

1. Spin liquids and violations of the Luttinger relation: FL* and HFL phases of the Kondo lattice model.

2. Hubbard model - the vanilla FL phase

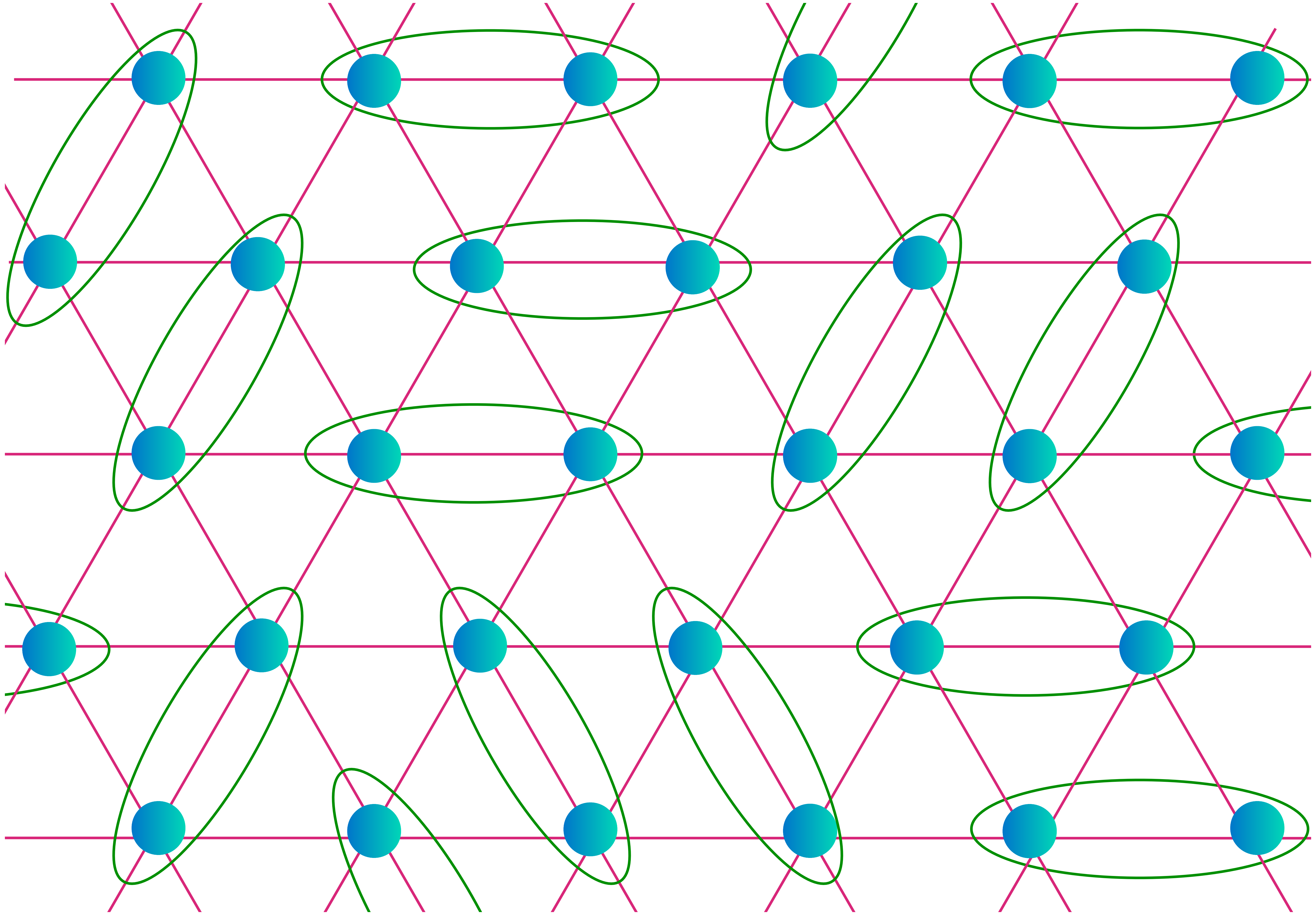
3. Hubbard model - the FL* phase: fractionalizing the paramagnon

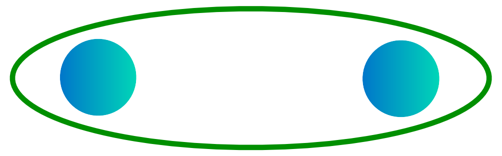
4. Hubbard model - nature of spin liquid

5. Quantum criticality

Mott insulator: Triangular lattice antiferromagnet

Resonating valence bonds:
Z_2 spin liquid





$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

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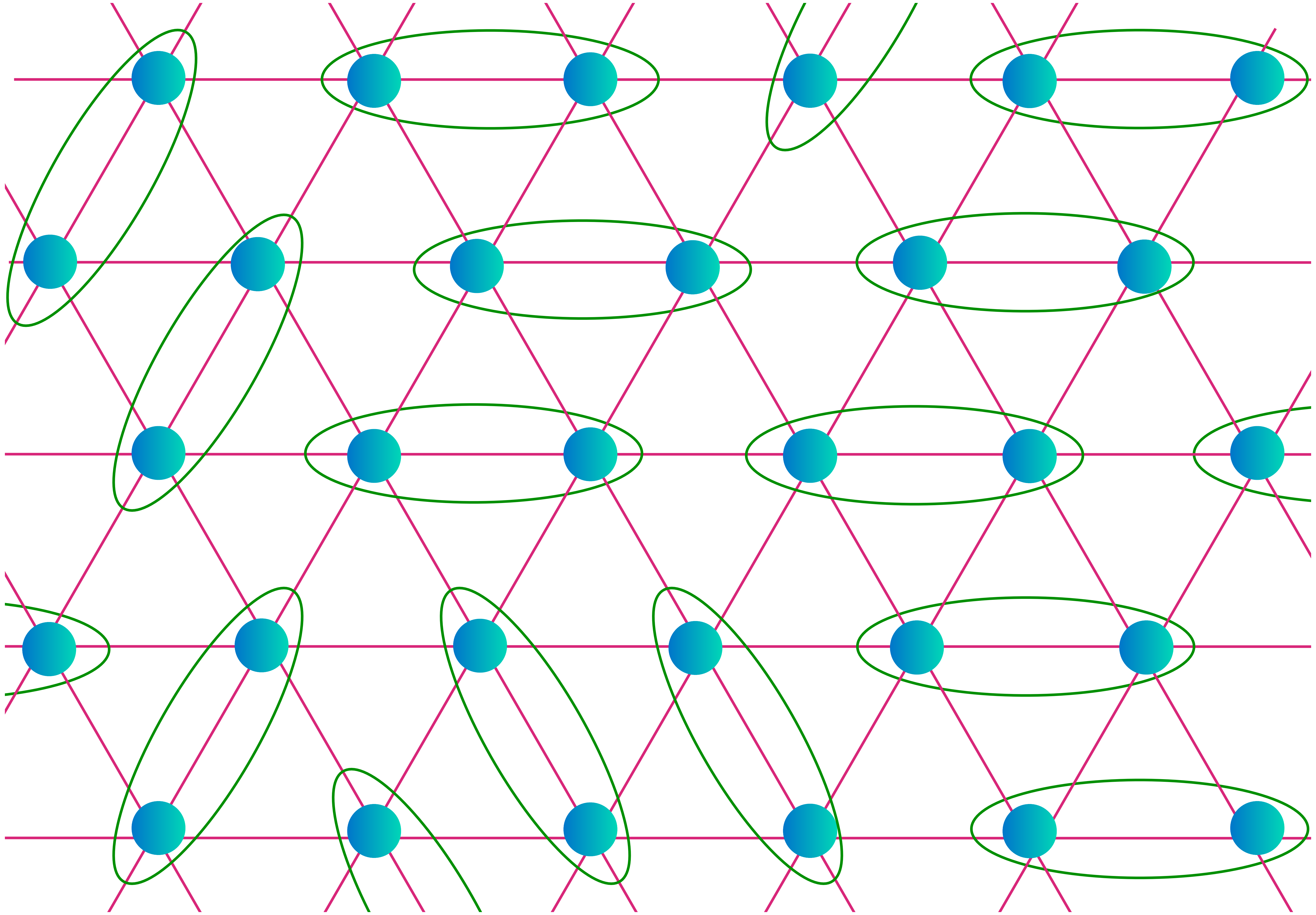

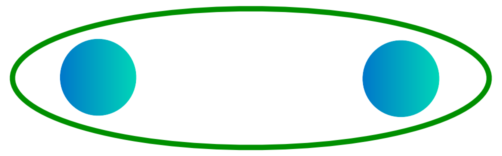

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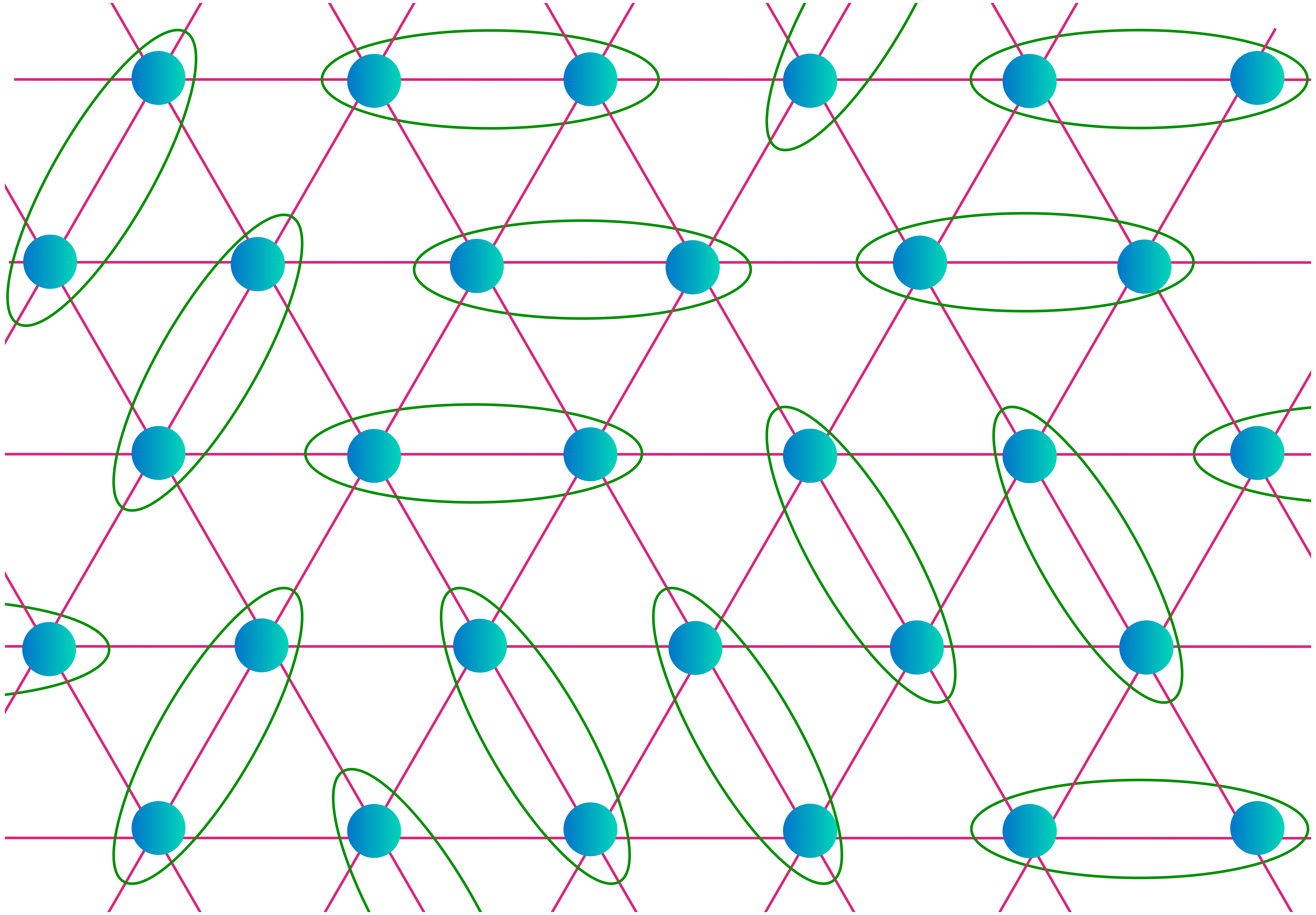

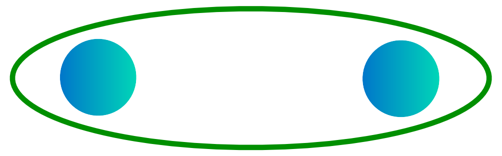

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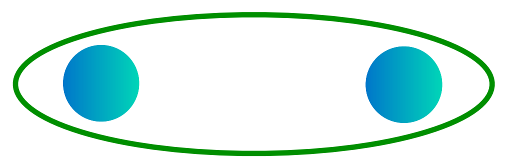
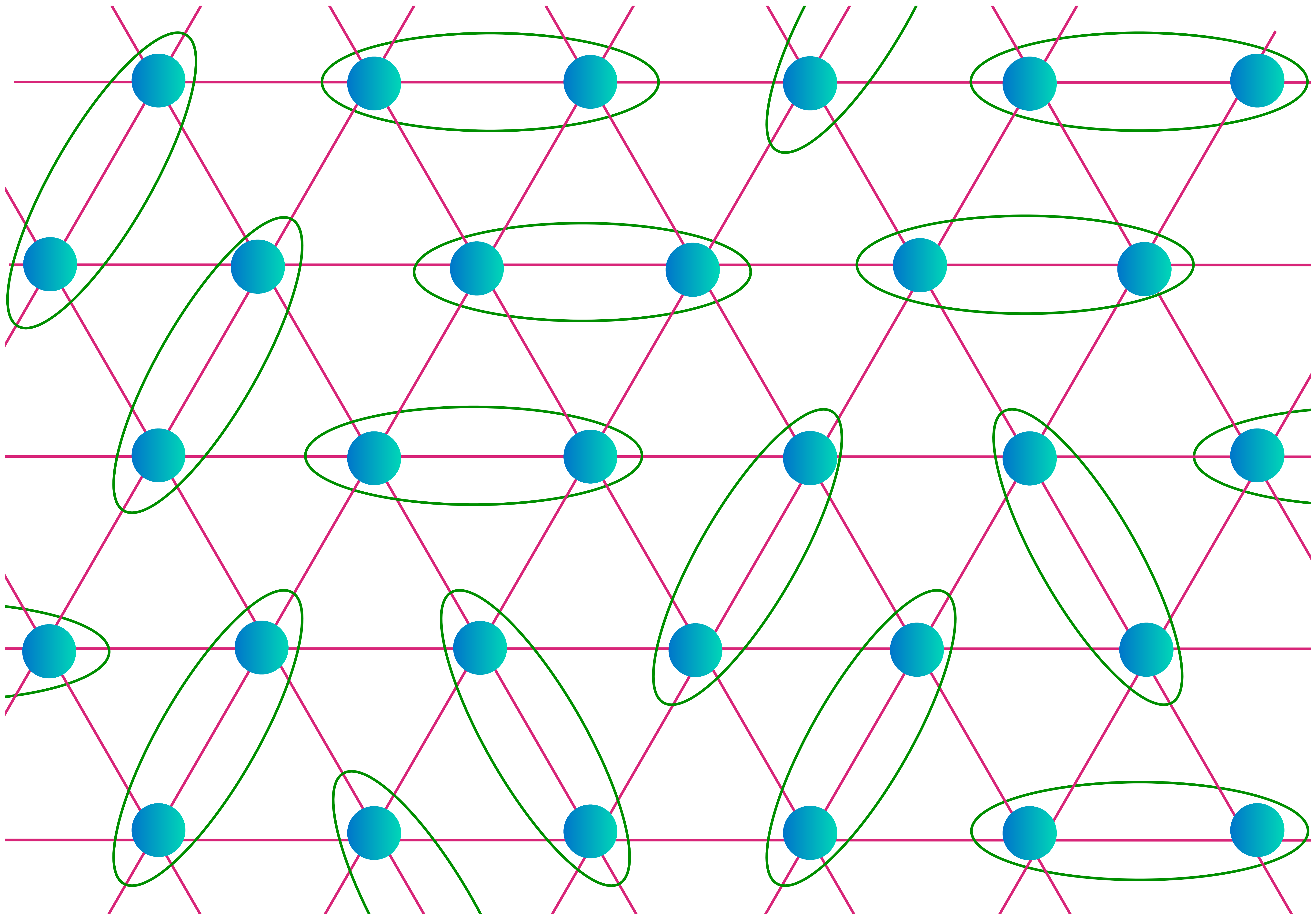


$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



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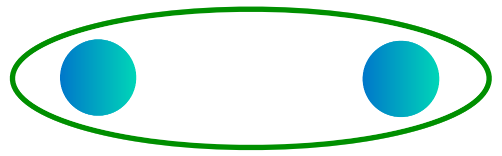

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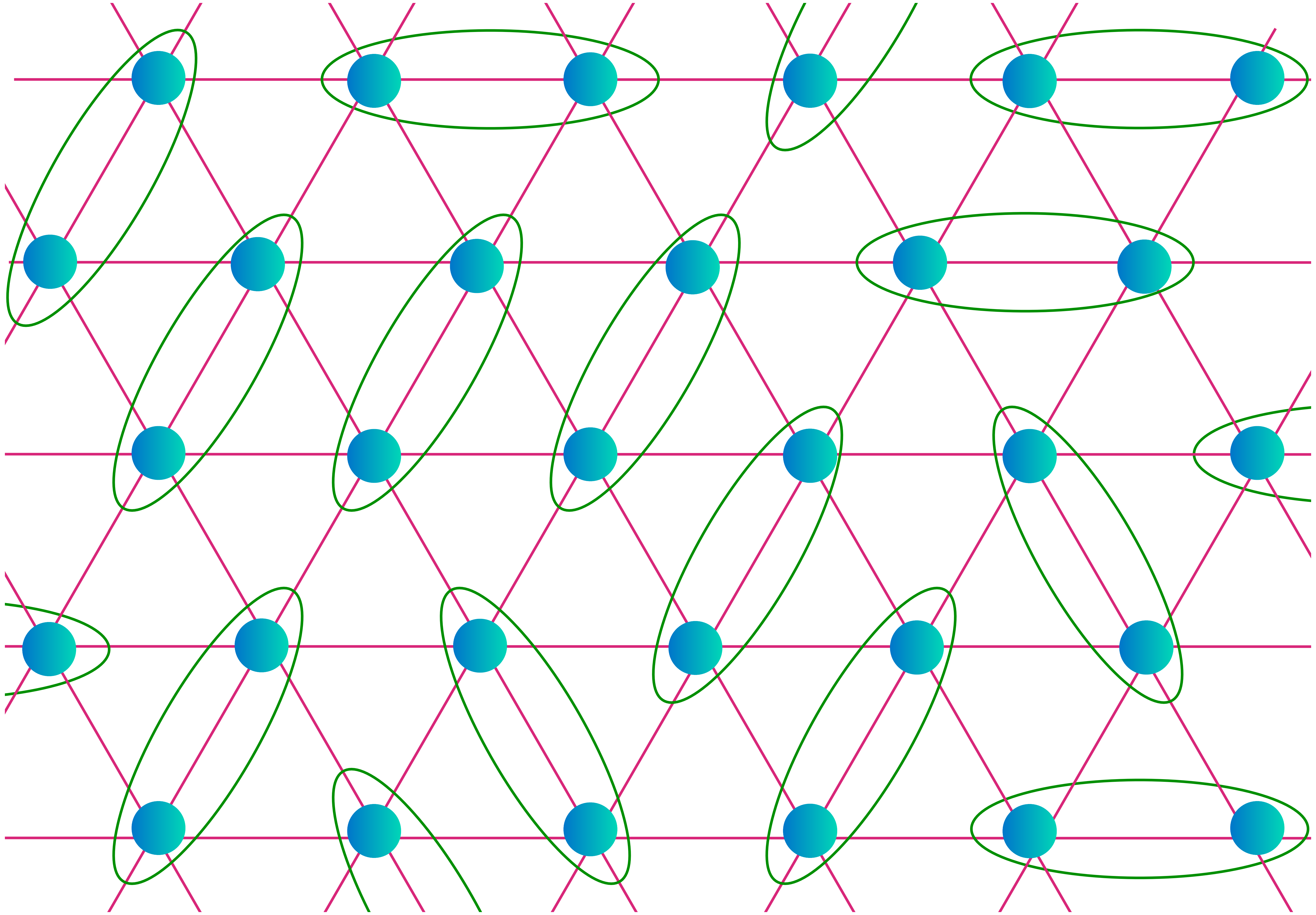

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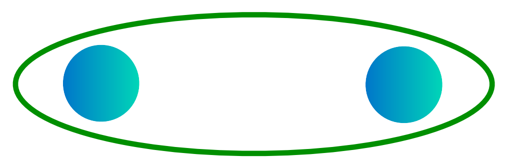

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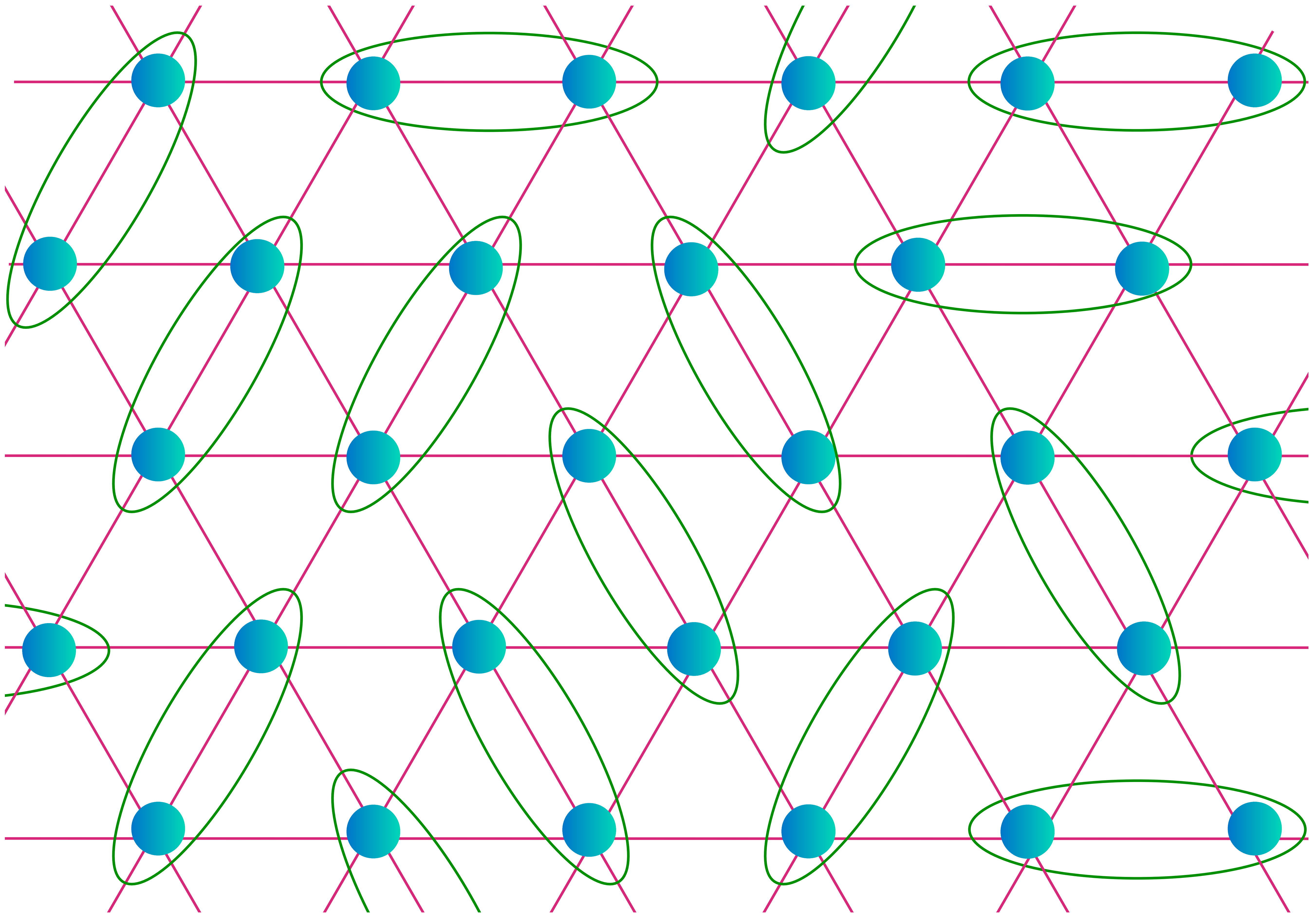
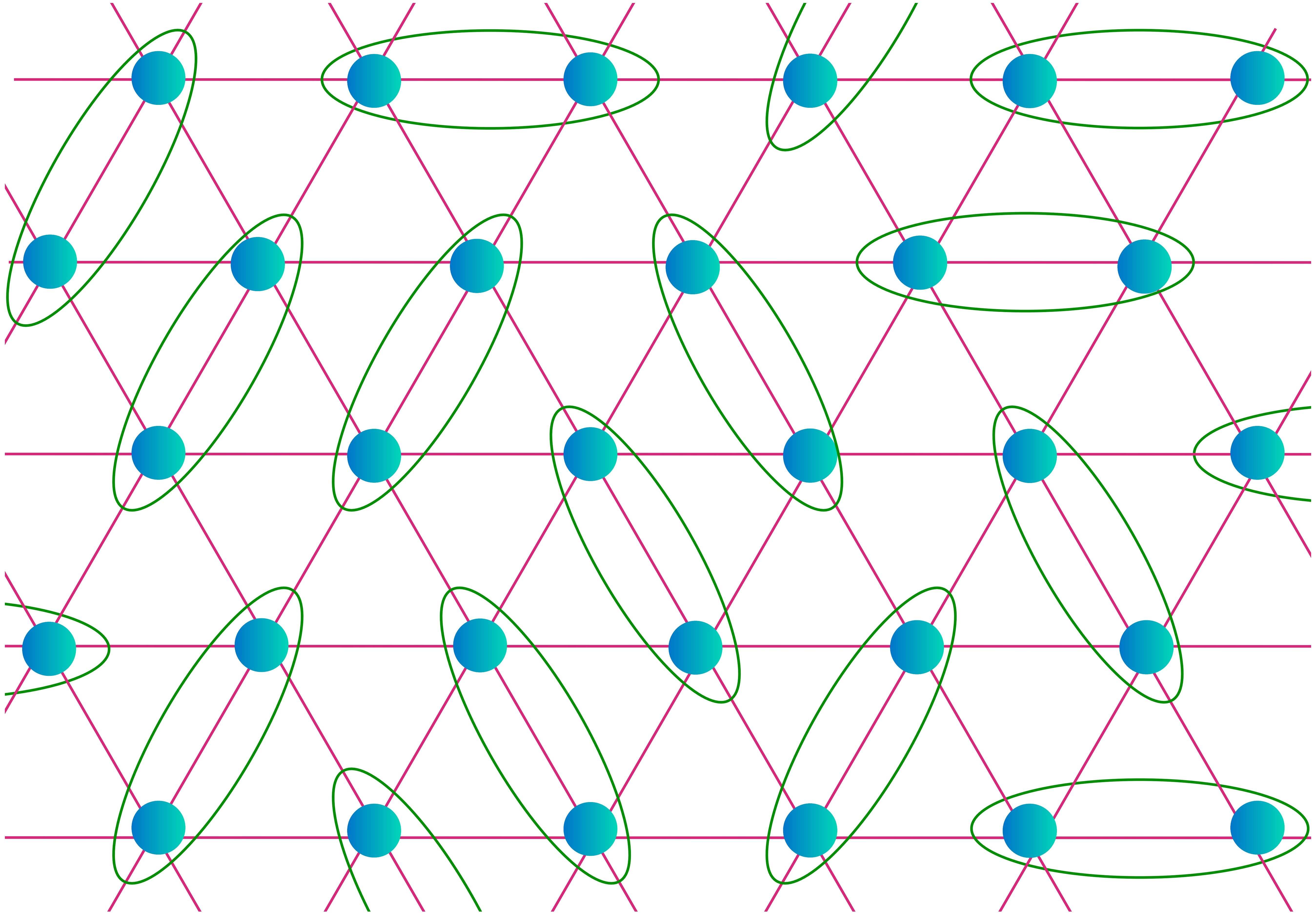


$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} |\mathcal{D}\rangle$$

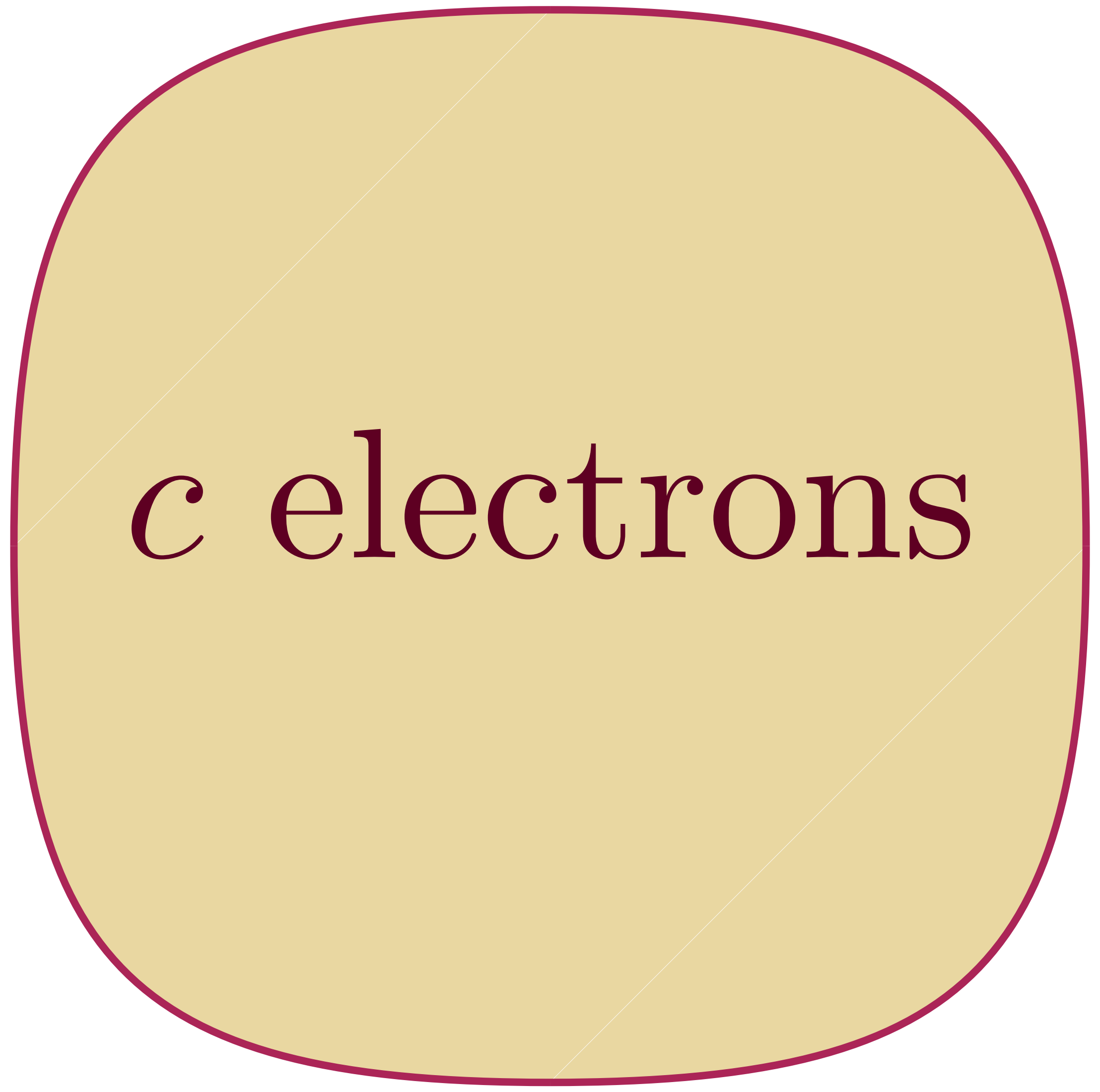
$\mathcal{D} \rightarrow$ dimer covering of lattice

Density of the electrons per unit cell = 1, but no Fermi surface $\Rightarrow$ Violation of Luttinger relation

Kondo lattice



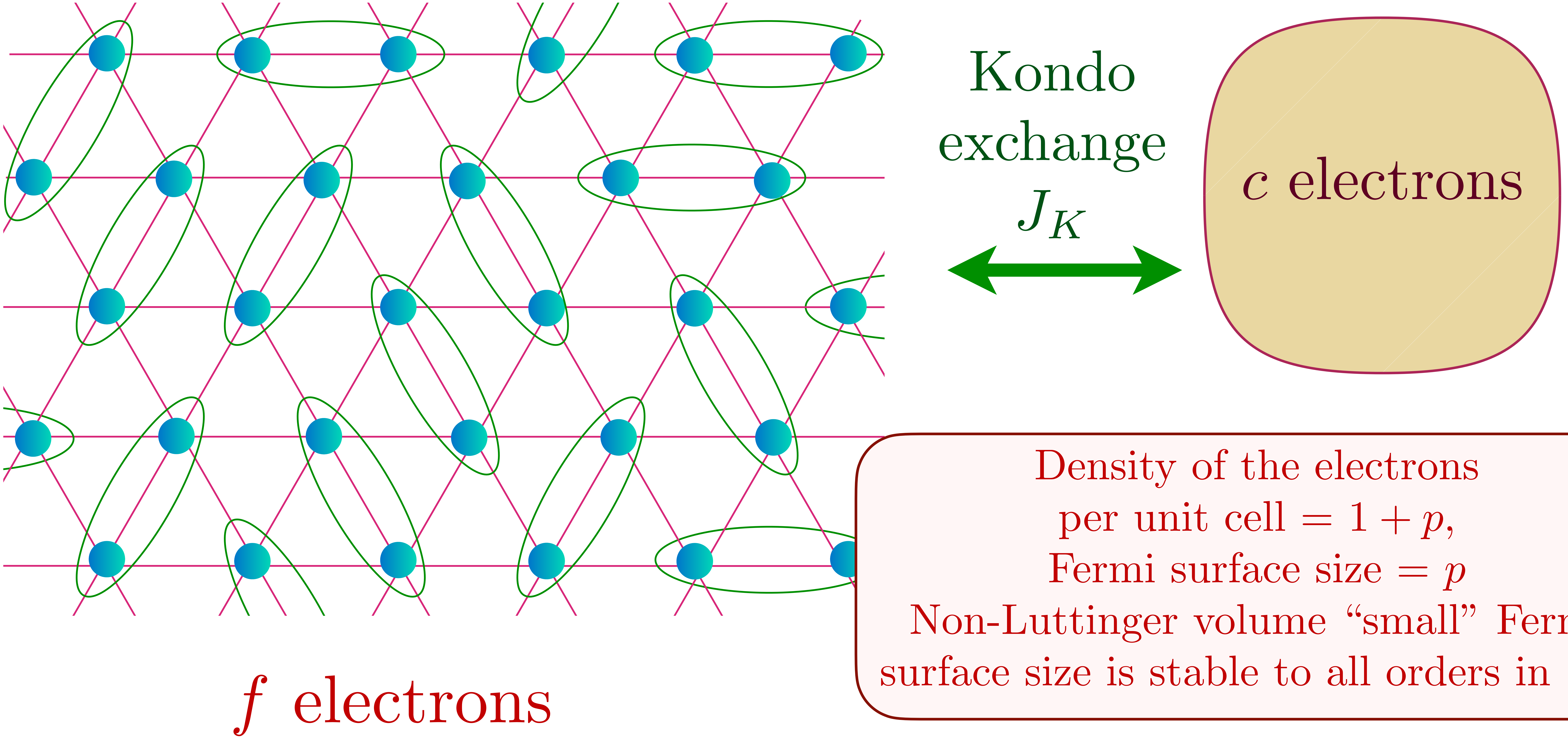
f electrons



Density of the electrons
per unit cell $= 1 + p$,
Fermi surface size $= p$

Kondo lattice: FL* phase

(closely related: OSMT: ignores topological order needed for Luttinger violation)

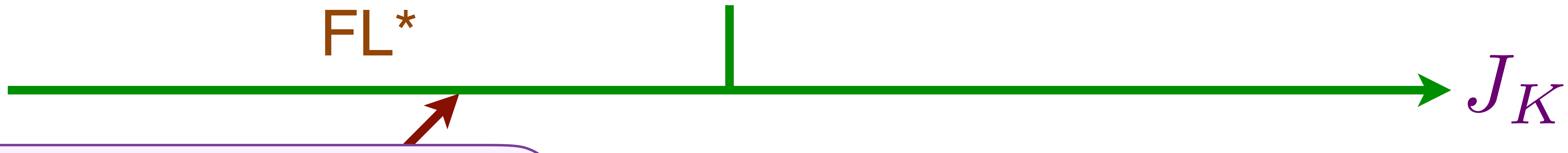


Density of the electrons per unit cell $= 1 + p$,
Fermi surface size $= p$
Non-Luttinger volume "small" Fermi surface size is stable to all orders in J_K.

T. Senthil, M. Vojta, and S. Sachdev, PRL **90**, 216403 (2003)

Kondo lattice

$$\mathcal{H}_{KL} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_i J_K \, c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_i + \sum_{\langle ij \rangle} J_H \, \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



Small Fermi surface of size p

$$|\mathrm{FL}*\rangle = [\text{Projection onto one } f \text{ per site}] \bowtie |\text{Slater determinant of } f\rangle \otimes |\text{Slater determinant of } c\rangle$$

N. Andrei and P. Coleman, PRL **62**, 595 (1989)
S. Burdin, D. R. Grempel, and A. Georges, PRB **66**, 045111 (2002)
T. Senthil, M. Vojta, and S. Sachdev, PRB **69**, 035111 (2004)
A. Paramekanti and A. Vishwanath, PRB **70**, 245118 (2004)

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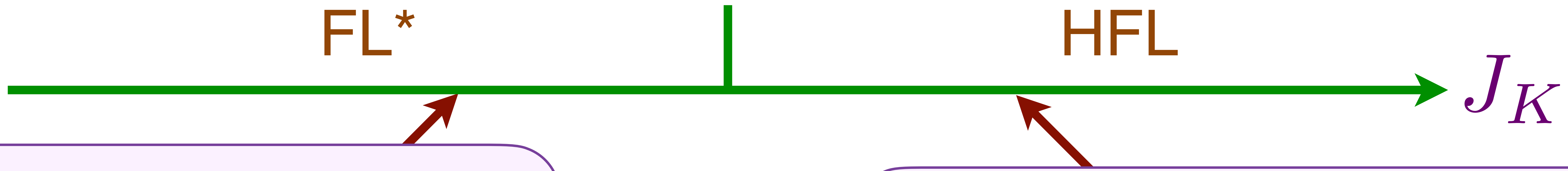

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Large Fermi surface of size $1 + p$

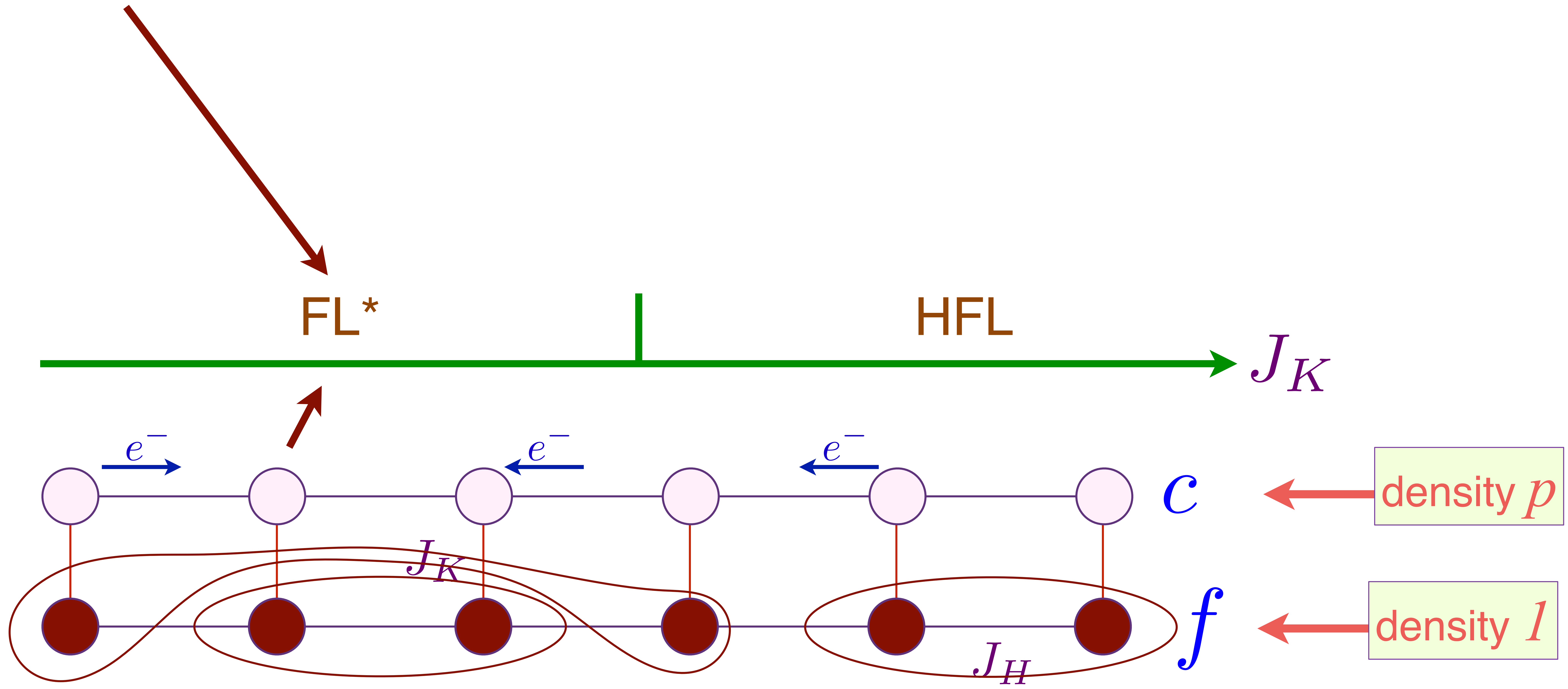
$$|\mathrm{HFL}\rangle = [\text{Projection onto one } f \text{ per site}] \bowtie |\text{Slater determinant of } (c, f)\rangle$$

Coleman, Read, Newns, Millis, P.A. Lee, Auerbach, Levin

Kondo lattice

- Use fermionic spinons $\boldsymbol{S}_i = \frac{1}{2} f_{i\sigma}^\dagger \boldsymbol{\tau}_{\sigma\sigma'} f_{i\sigma'}$, $\sum_\sigma f_{i\sigma}^\dagger f_{i\sigma} = 1$. This introduces a U(1)$_{\text{gauge}}$ gauge symmetry $f_{i\sigma} \to f_{i\sigma} e^{-i\vartheta_i}$ and the total symmetry is U(1)$_{\text{gauge}}\times$ U(1)$_{\text{em}}$.

- In the FL* state, the U(1)$_{\text{gauge}}$ symmetry is at least partially unbroken, and the Luttinger relation applies only to the c fermions carrying U(1)$_{\text{em}}$ charges.



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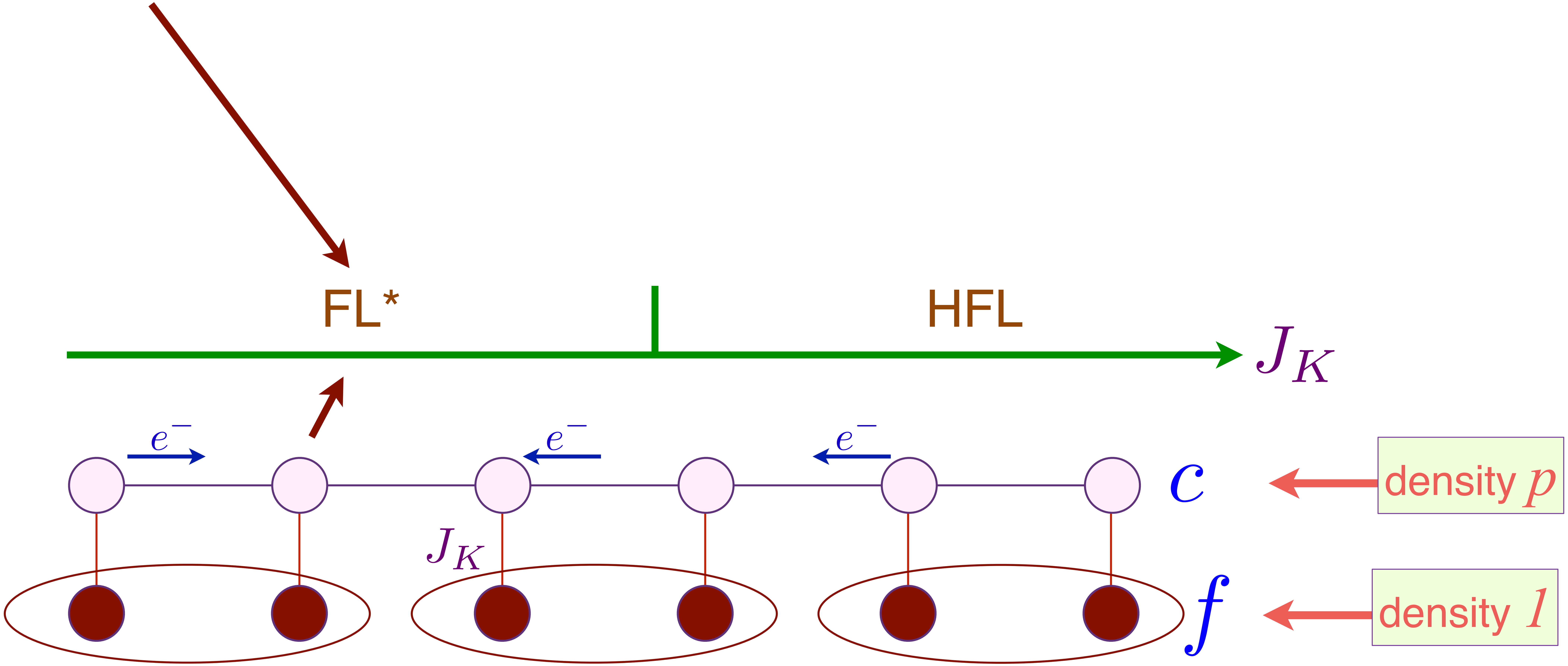

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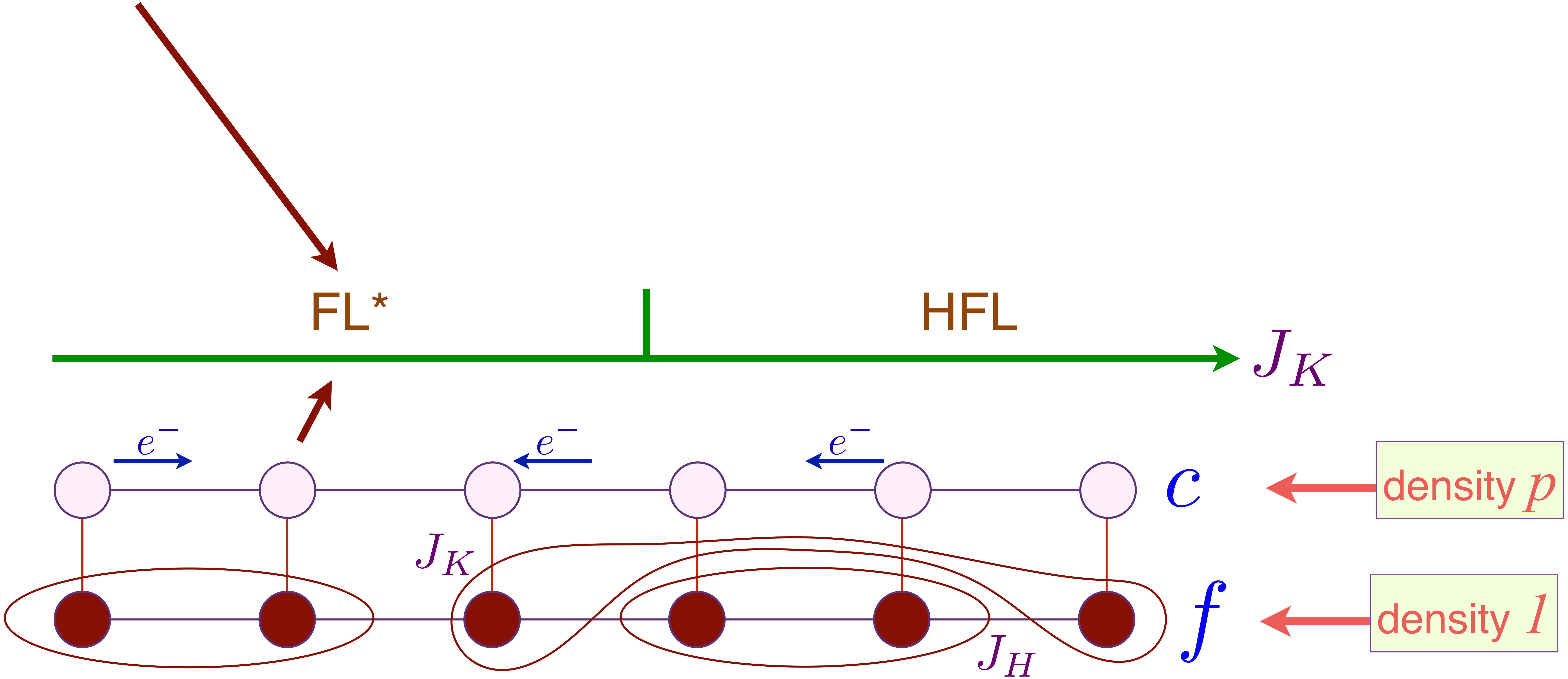

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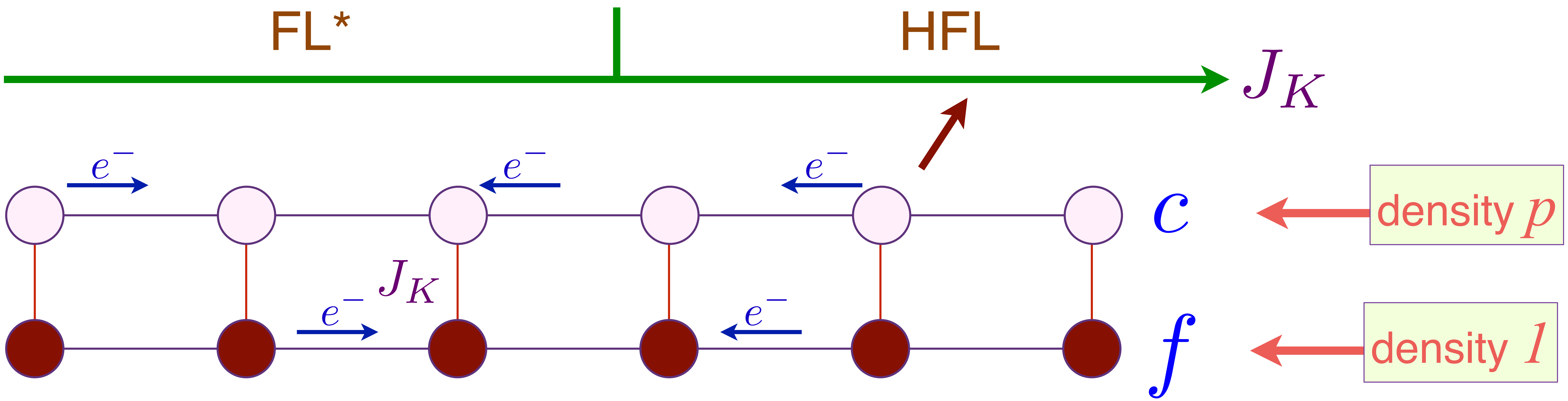
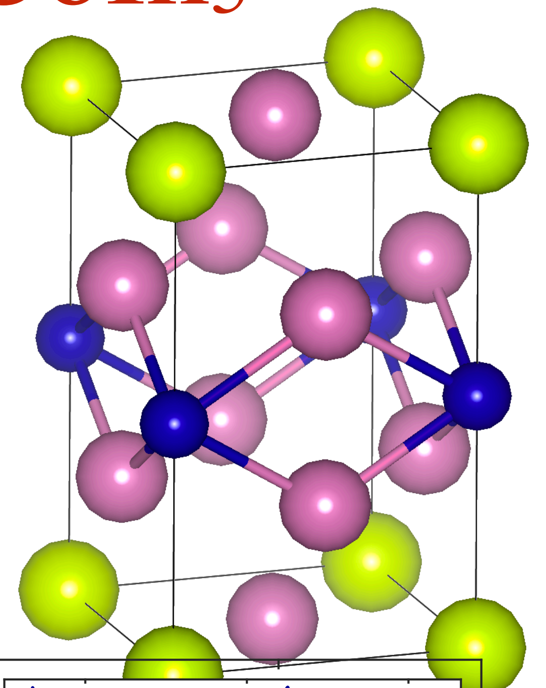
- In the HFL state, the $\mathrm{U}(1)_{\mathrm{gauge}} \times \mathrm{U}(1)_{\mathrm{em}}$ symmetry is 'Higgsed' by the condensation of the hybridization boson $B_i \sim f_{i\sigma}^\dagger c_{i\sigma}$ to a diagonal $\mathrm{U}(1)_{\mathrm{diag}}$ symmetry. The Luttinger arguments can only be applied to the unbroken $\mathrm{U}(1)_{\mathrm{diag}}$ symmetry, which counts *both* c and f fermions, and so the Fermi surface is *large*.

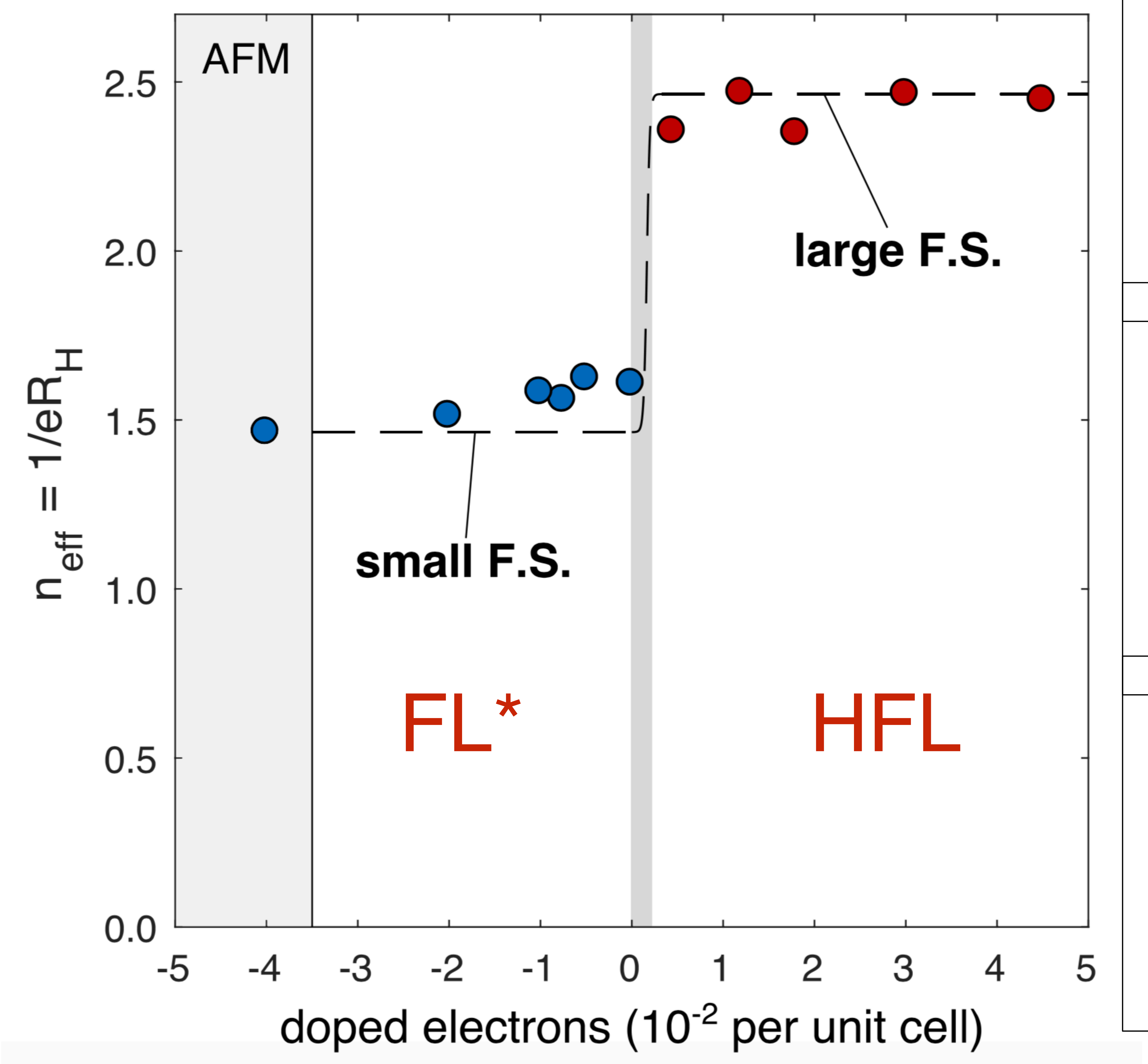
$$H_{\mathrm{HFL}} = -\sum_{i,j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_{i,j} t_{1,ij} f_{i\sigma}^\dagger f_{j\sigma} + \sum_i B(c_{i\sigma}^\dagger f_{i\sigma} + f_{i\sigma}^\dagger c_{i\sigma})$$

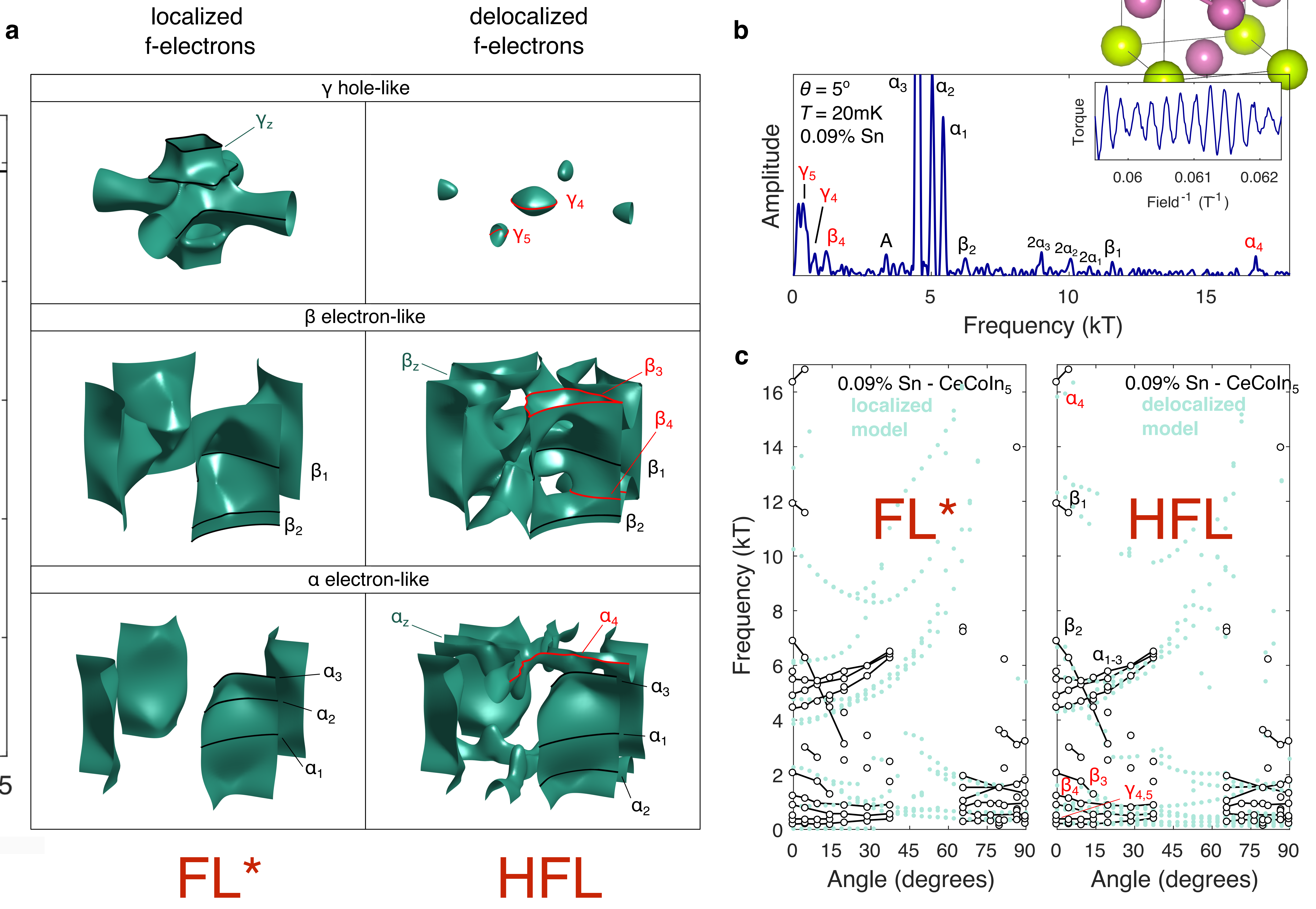


Evidence for a delocalization quantum phase transition without symmetry breaking in $CeCoIn_5$

Nikola Maksimovic, Daniel H. Eilbott, Tessa Cookmeyer, Fanghui Wan, Jan Rusz, Vikram Nagarajan, Shannon C. Haley, Eran Maniv, Amanda Gong, Stefano Faubel, Ian M. Hayes, Ali Bangura, John Singleton, Johanna C. Palmstrom, Laurel Winter, Ross McDonald, Sooyoung Jang, Ping Ai, Yi Lin, Samuel Ciocys, Jacob Gobbo, Yochai Werman, Peter M. Oppeneer, Ehud Altman, Alessandra Lanzara, James G. Analytis, Science **375**, 76 (2021).







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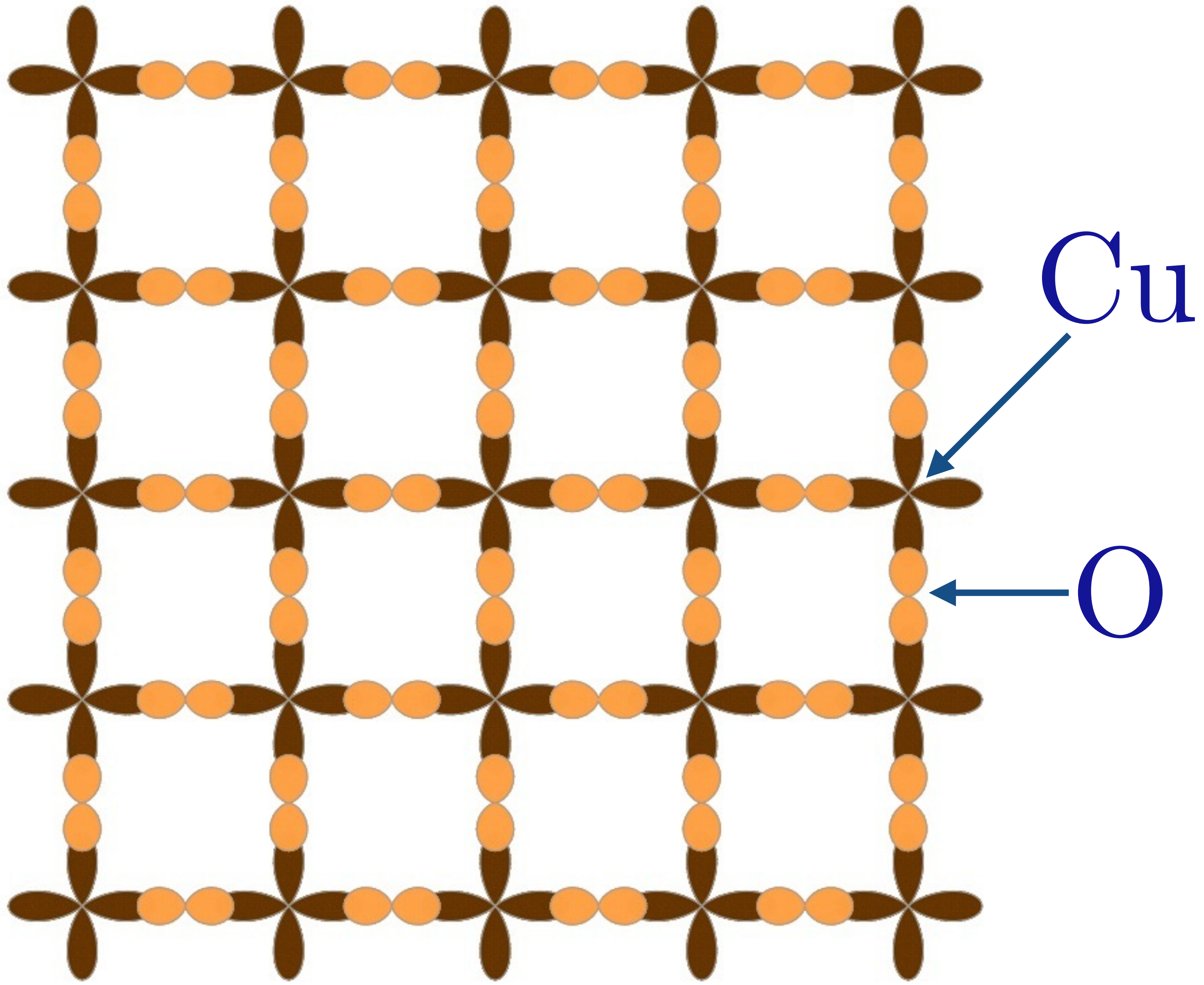
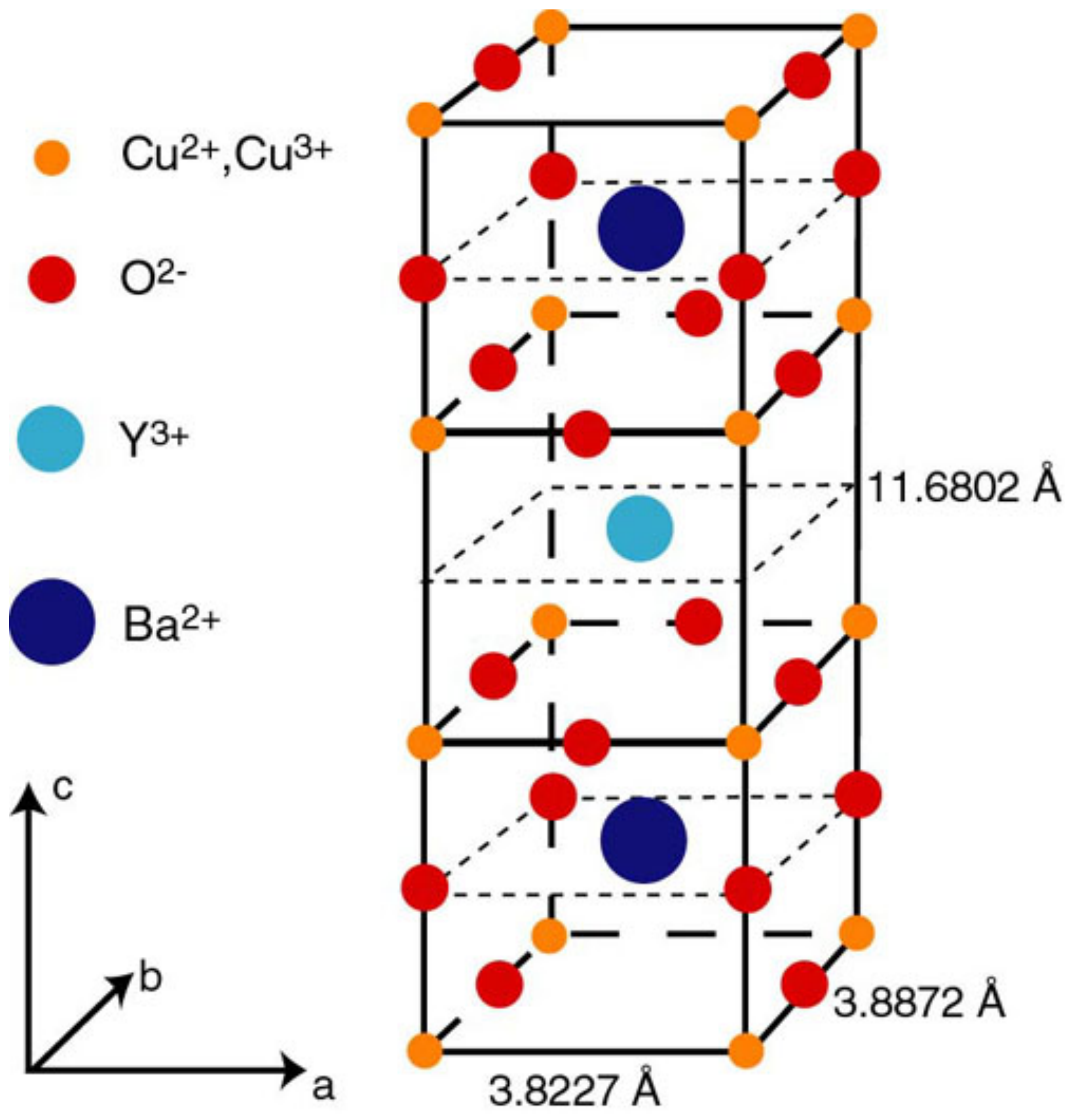
High temperature superconductors



CuO_2 plane



$YBa_2Cu_3O_{6+x}$

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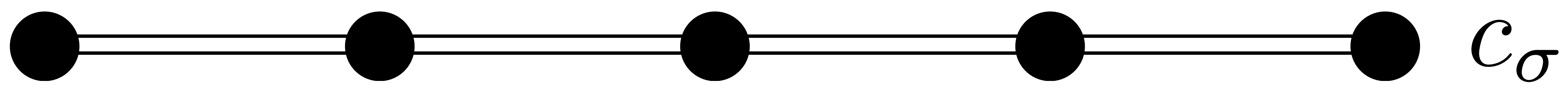
- The Luttinger theorem implies a FL phase with ‘large’ Fermi surface of size $1+p$ holes (or $1-p$ electrons) for all U and all p.



density
1+p

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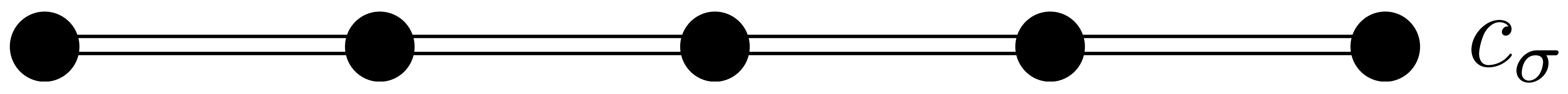

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P.W. Anderson, P.A. Lee, M. Randeria, T. M. Rice, N. Trivedi, and F. C. Zhang, Journal of Physics Condensed Matter **16**, R755 (2004)

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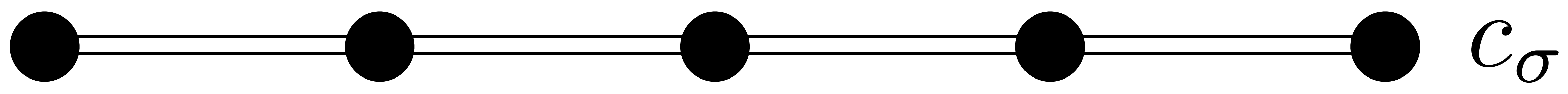
- The main effect of the projection is a 'Brinkman-Rice' enhancement of the quasiparticle mass as $p \to 0$, with $m^*/m \sim 1/p$.



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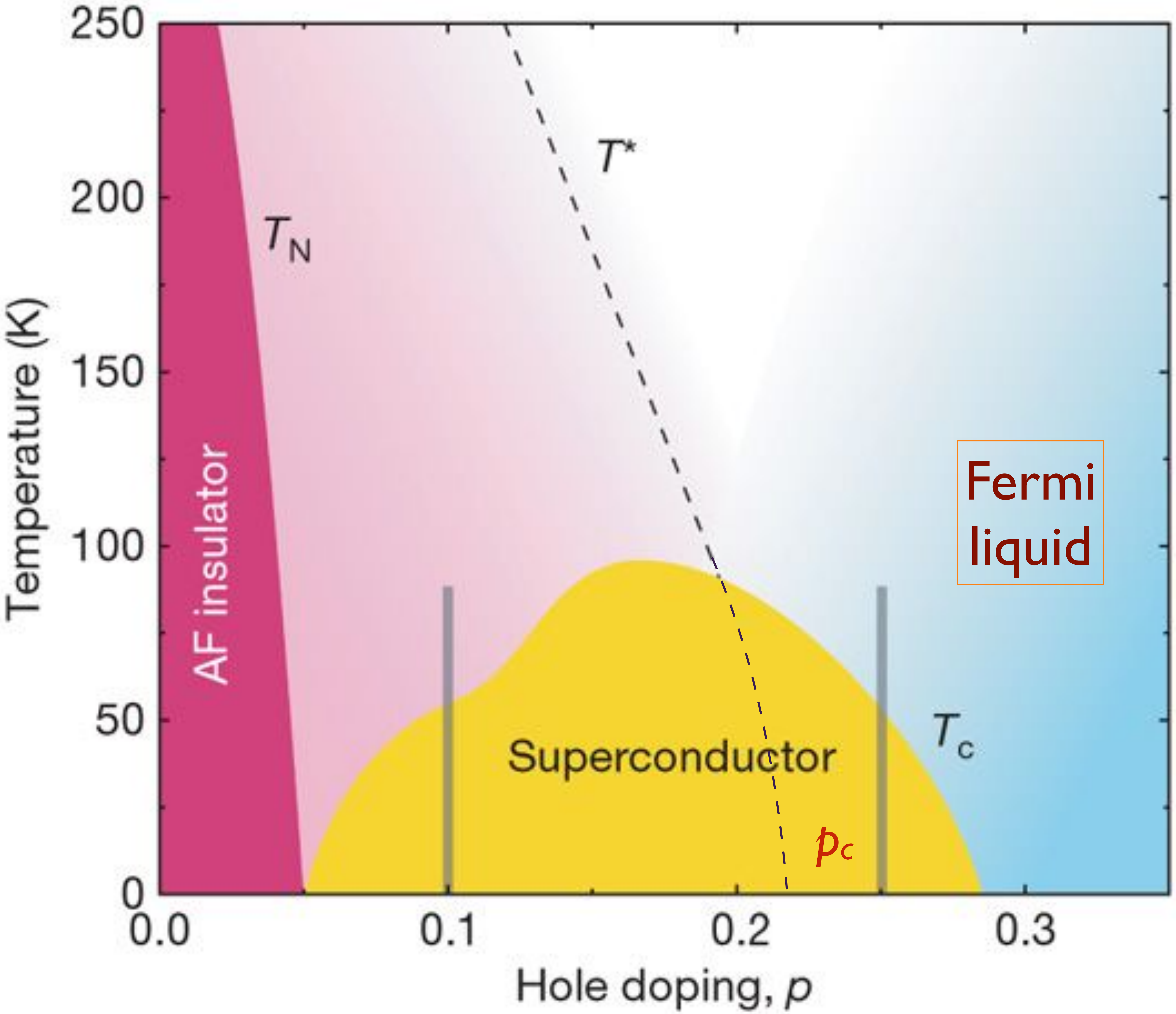
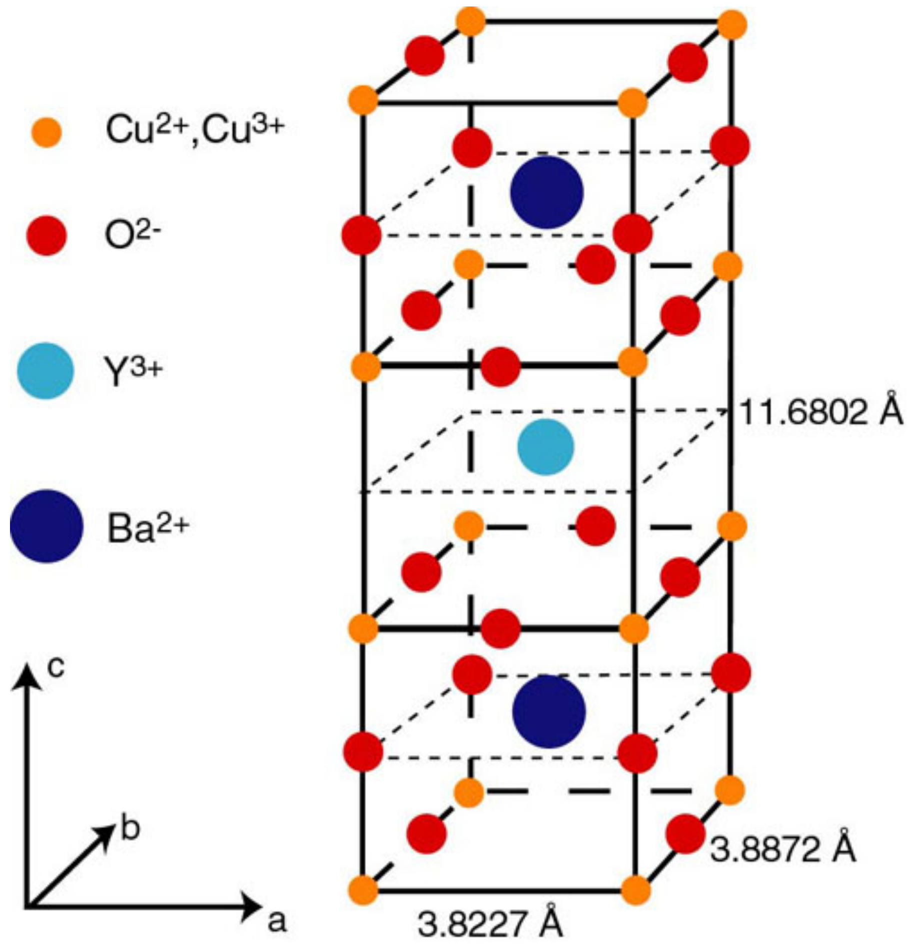
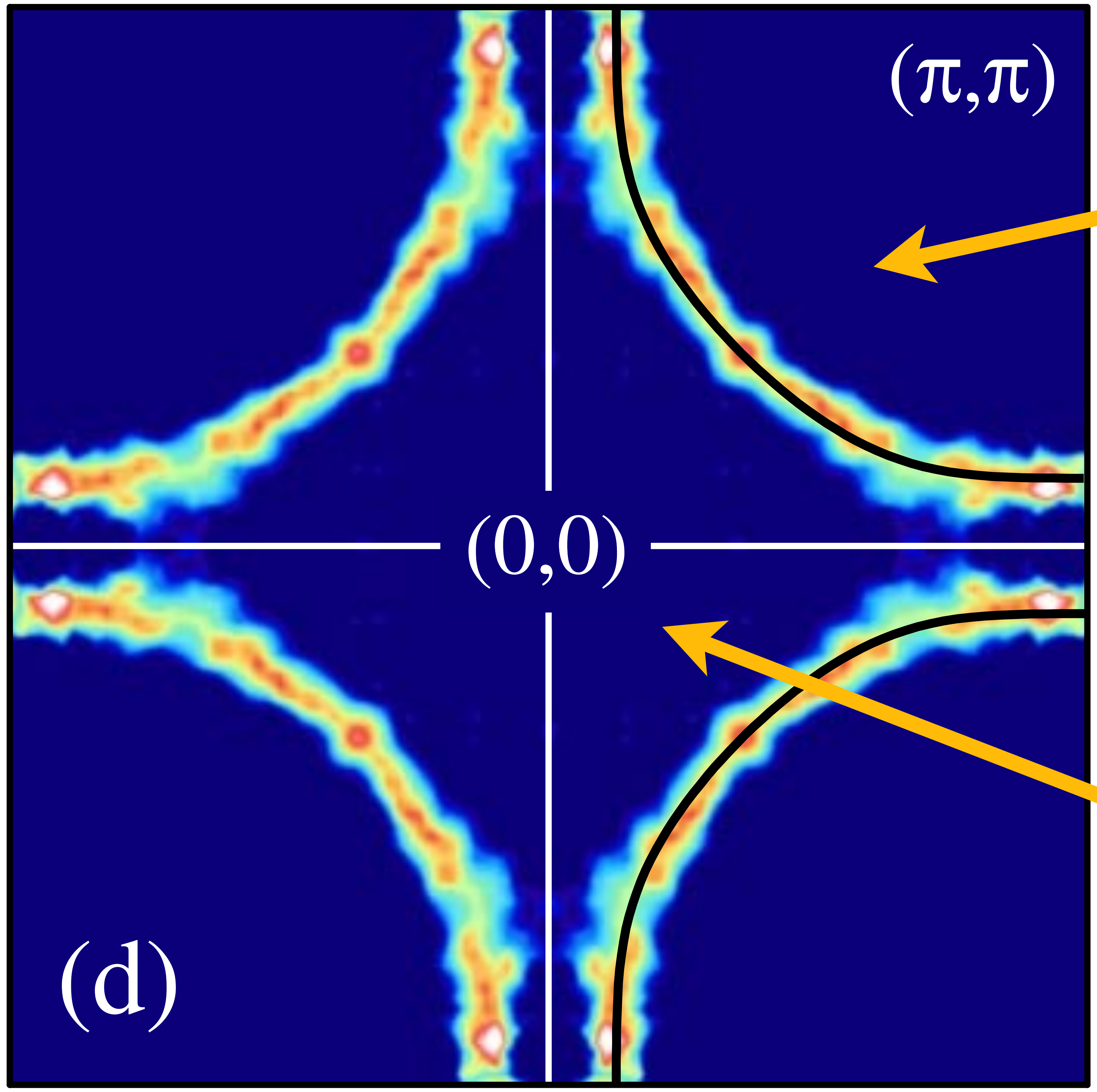

250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Fermi liquid
Superconductor
T_c
p_c


Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

Photoemission at large p



$1+p$ holes

1-p electrons

Overdoped $Tl_2Ba_2CuO_{6+\delta}$
$T_c = 30K$

$1+p$ mobile holes in a filled band

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

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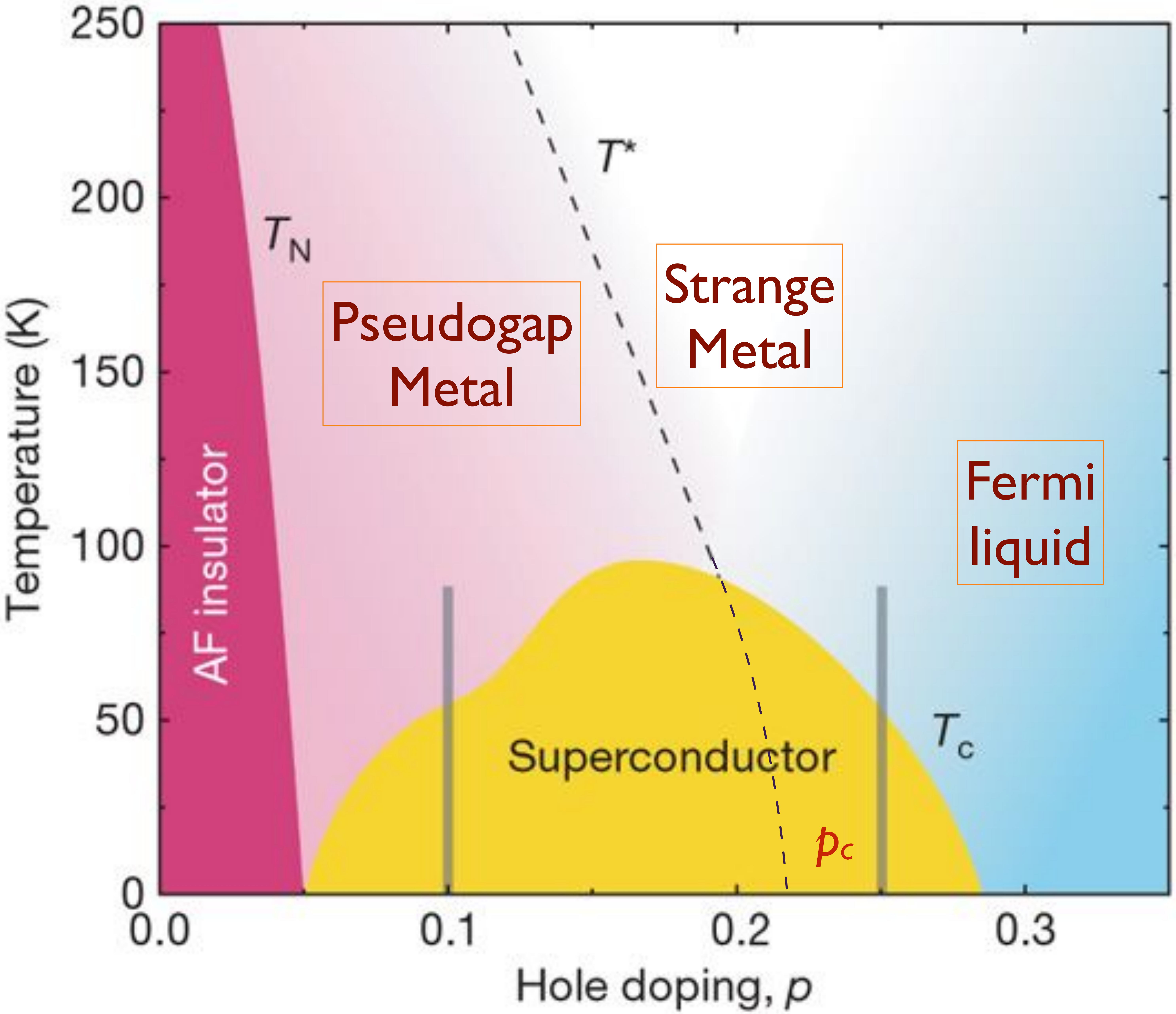
250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
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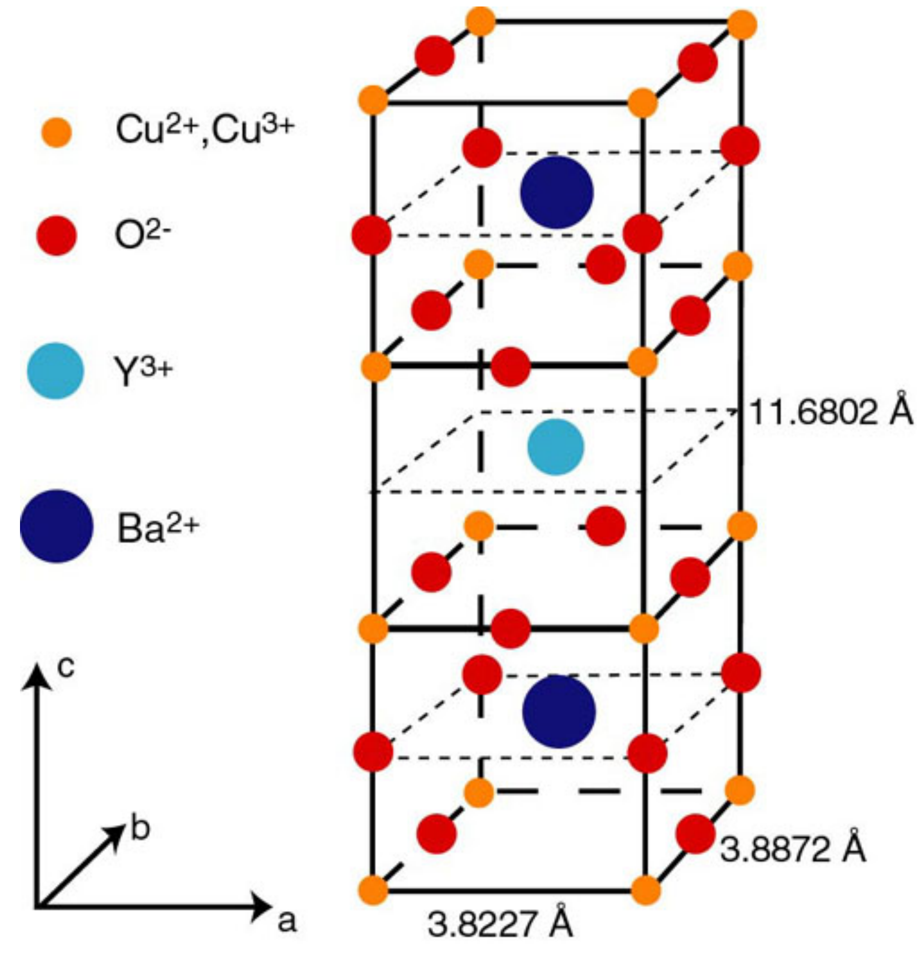
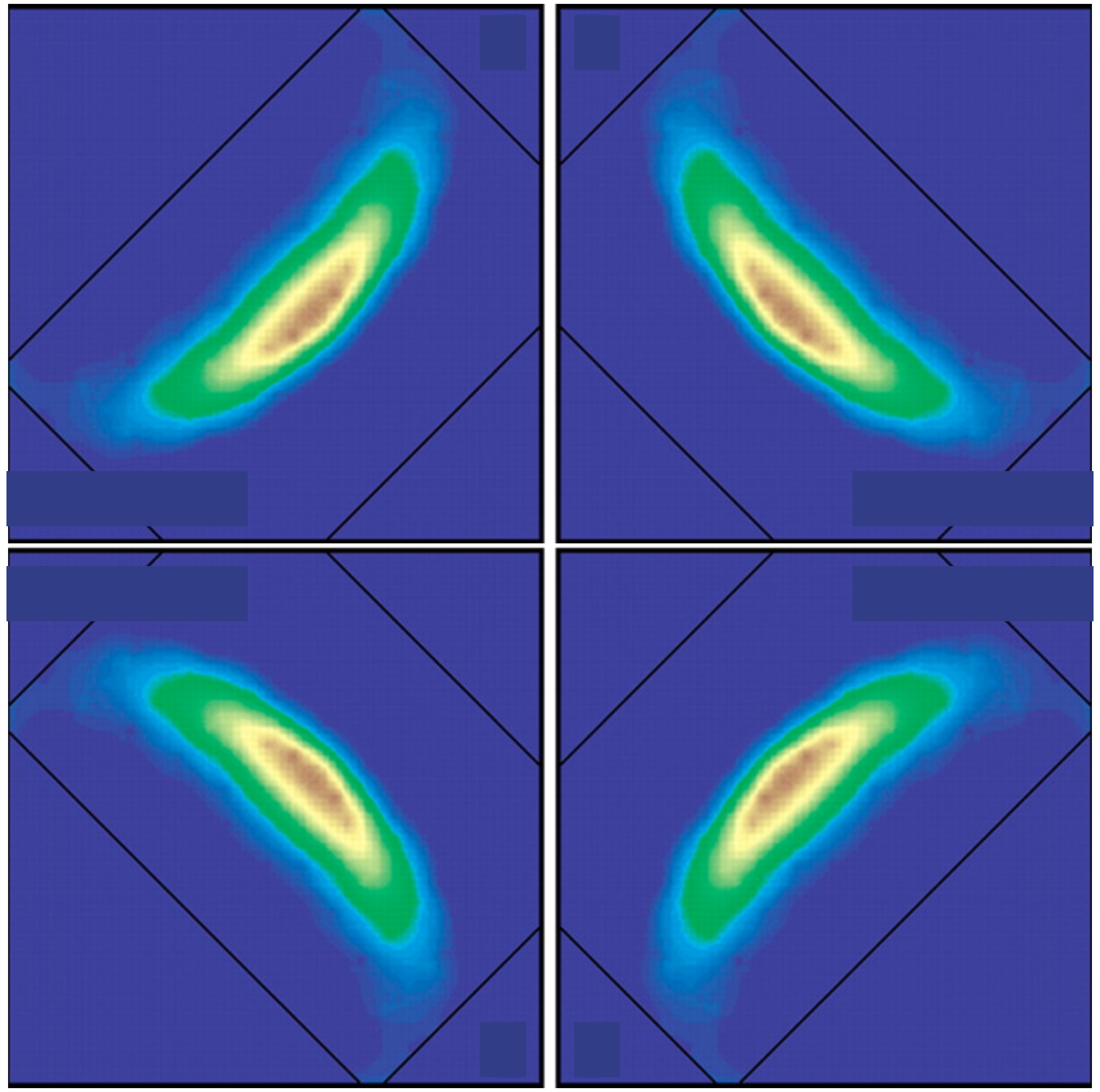
Strange Metal
Fermi liquid
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

Photoemission at small p



$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$

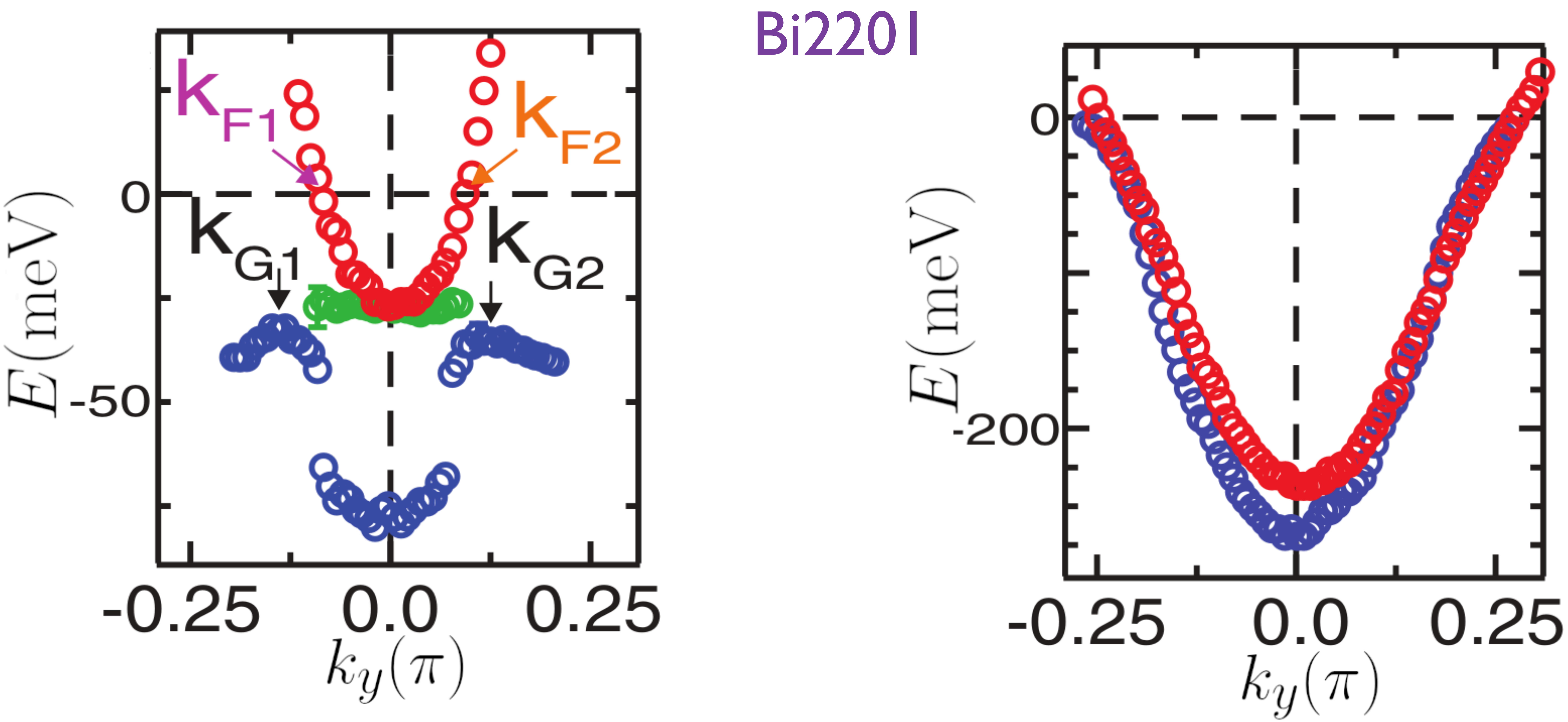
"Fermi arcs"

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

Photoemission at small and large p

Anti-node: $k_x = \pi$

Node: $k_x = 2$

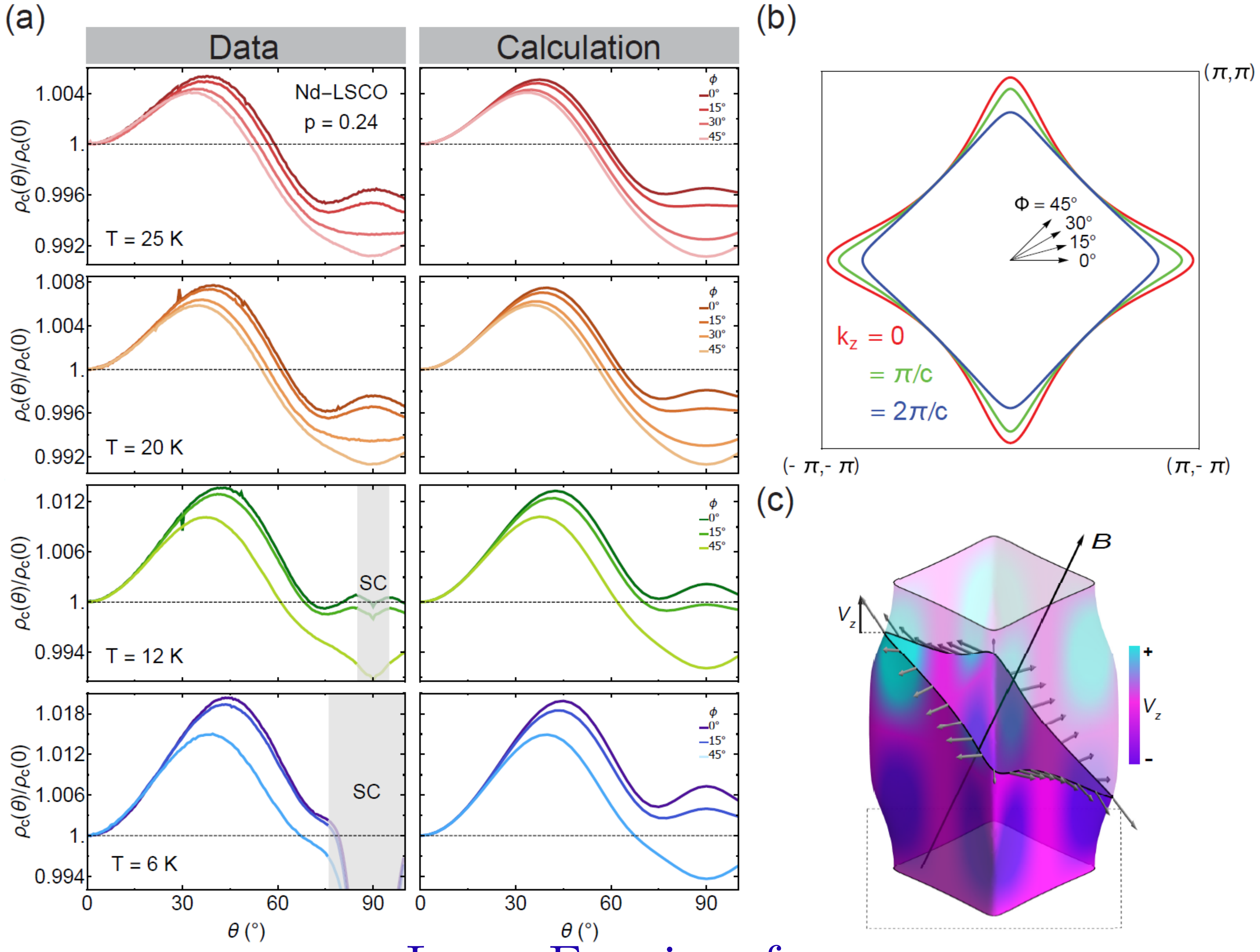


R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

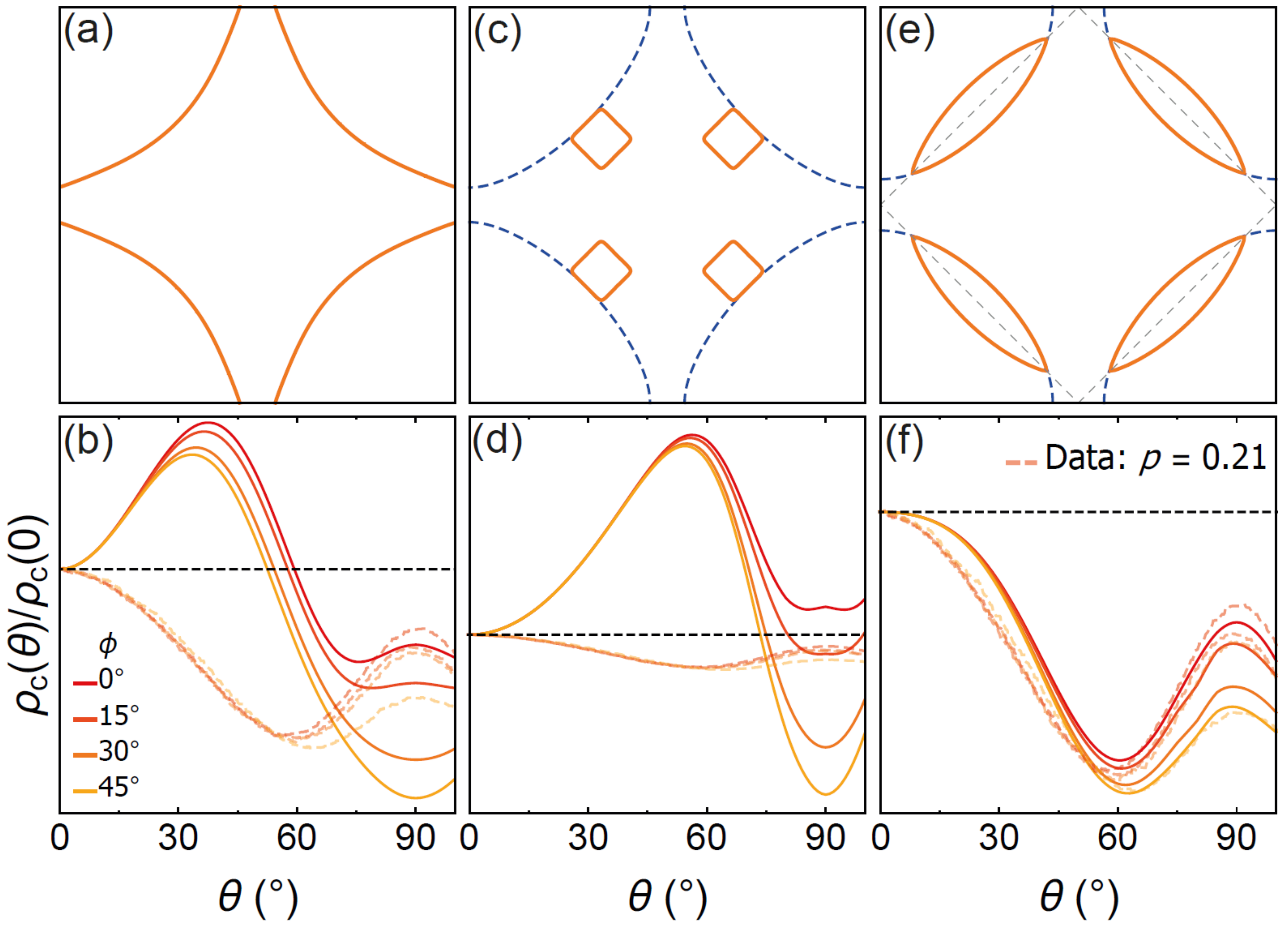
Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, Nature Physics, in press

We use angle-dependent magnetoresistance (ADMR) to measure the Fermi surface of the cuprate Nd-LSCO. Above the critical doping $p^\star$ we extract a Fermi surface geometry that is in quantitative agreement with angle-resolved photoemission. Below $p^\star$ the ADMR is qualitatively different, revealing a clear transformation of the Fermi surface. Changes in the quasiparticle lifetime across $p^\star$ are ruled out as the cause of this transformation. Instead we find that our data are most consistent with a reconstruction of the Fermi surface by a $Q = (\pi, \pi)$ wavevector.

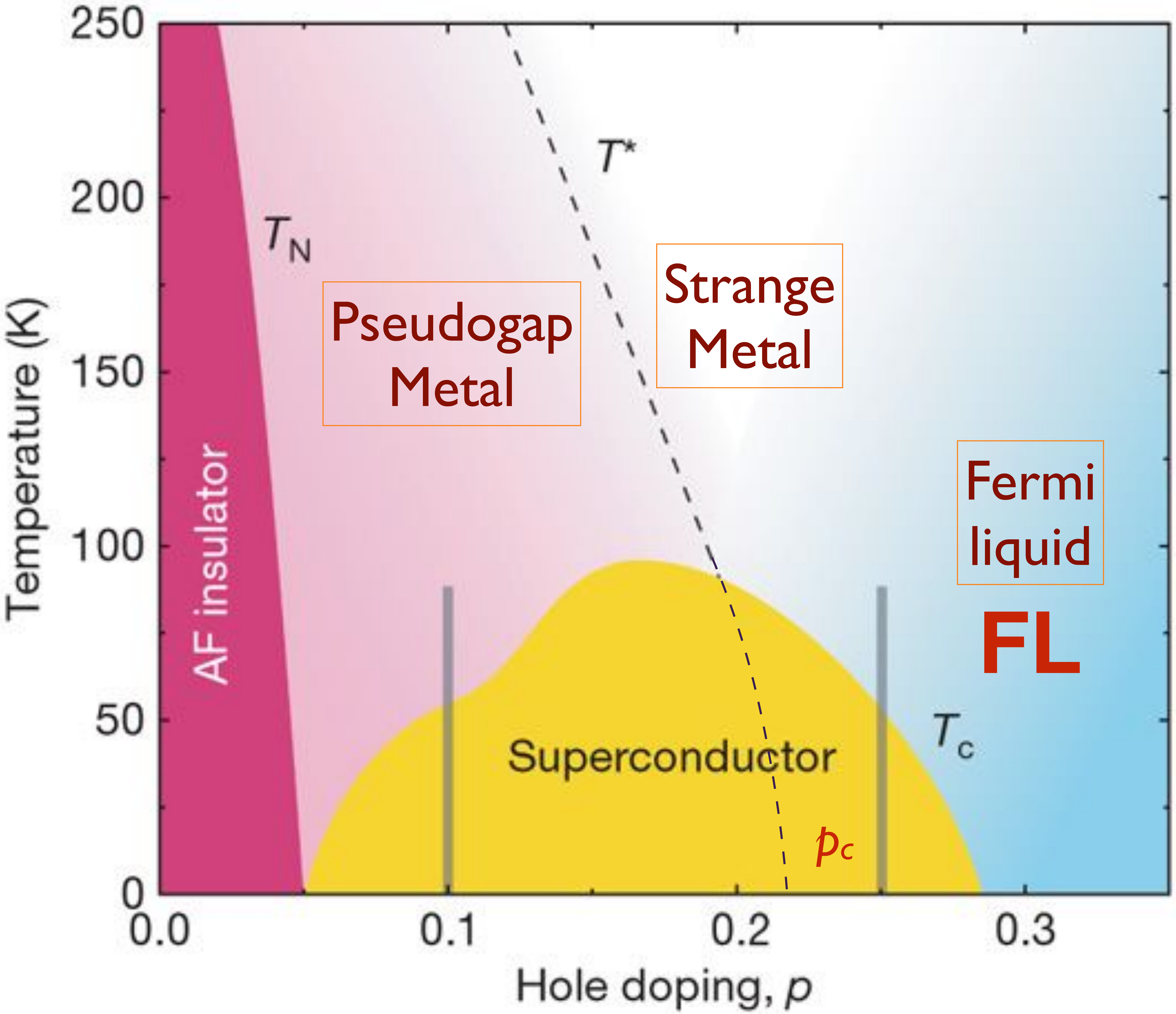


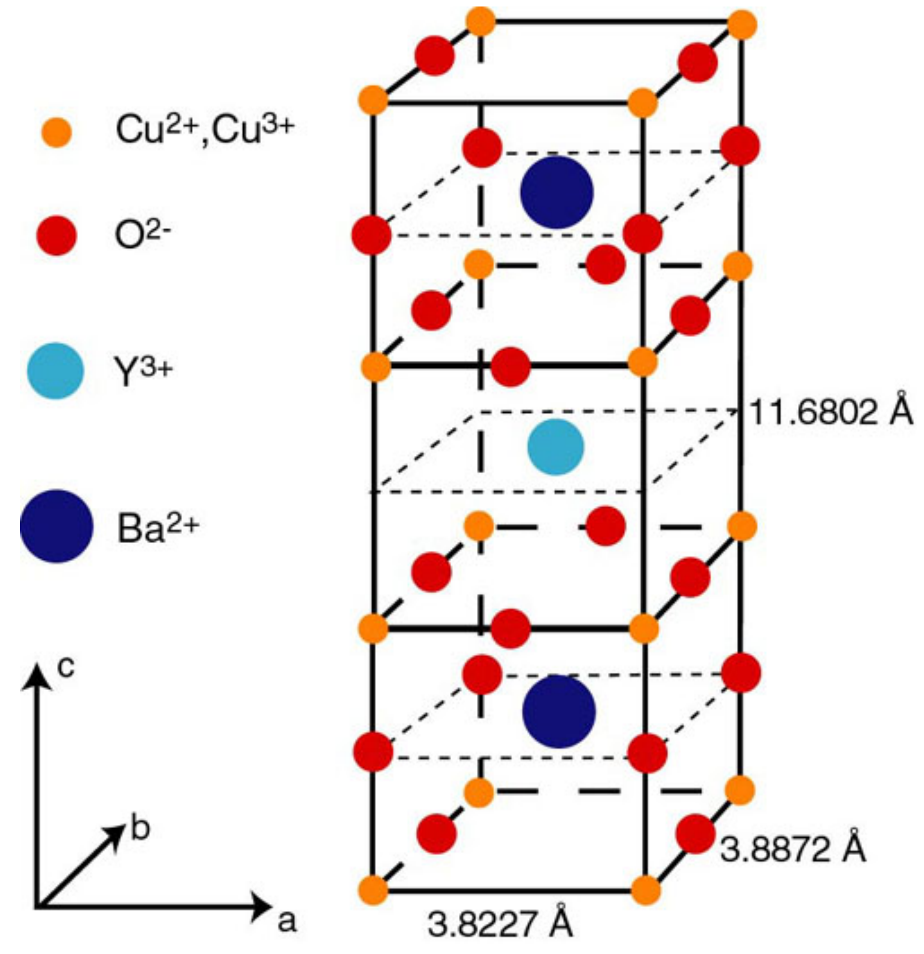
$p > p_c$ Large Fermi surface

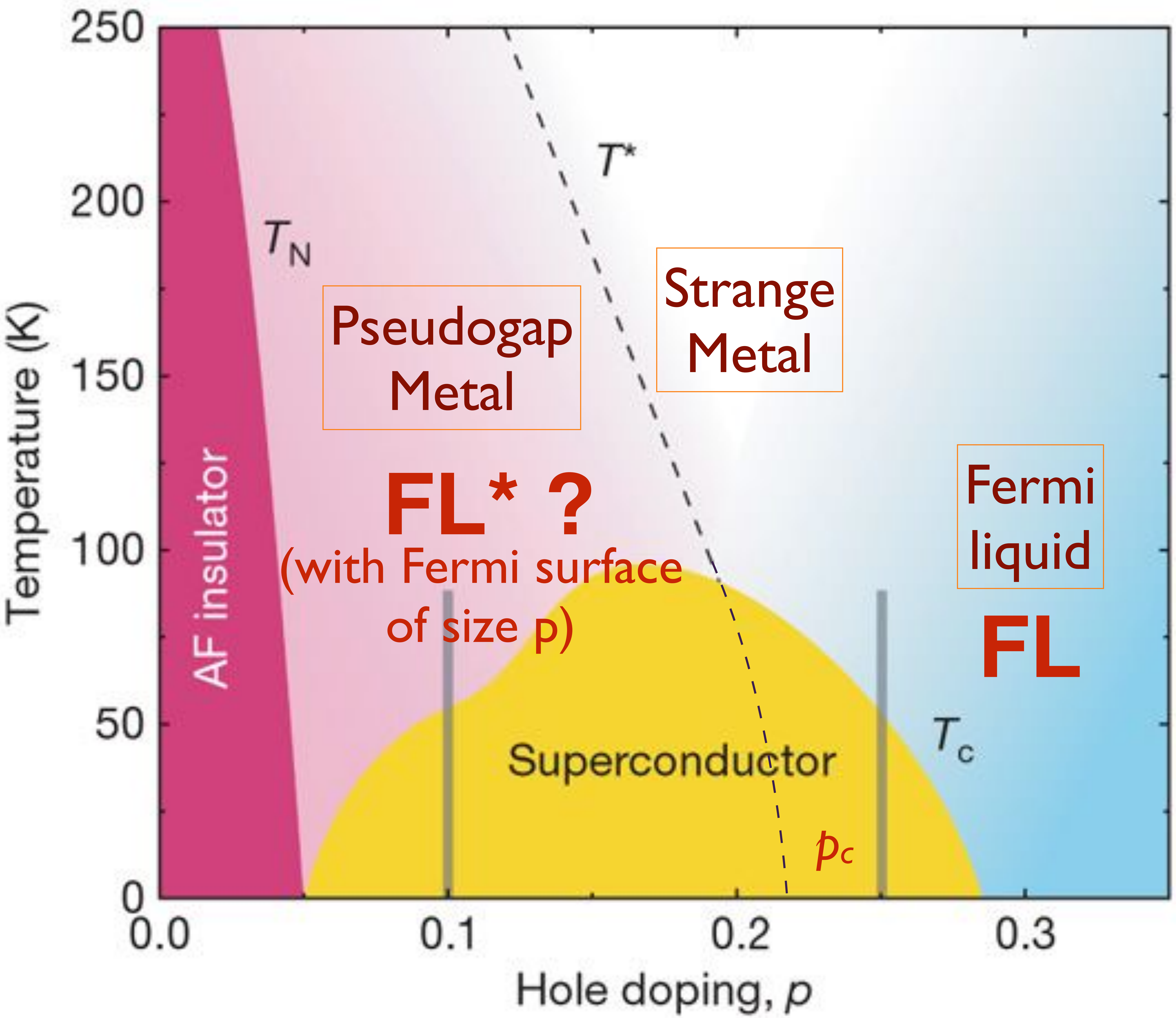


$p < p_c$ Reconstructed Fermi surface

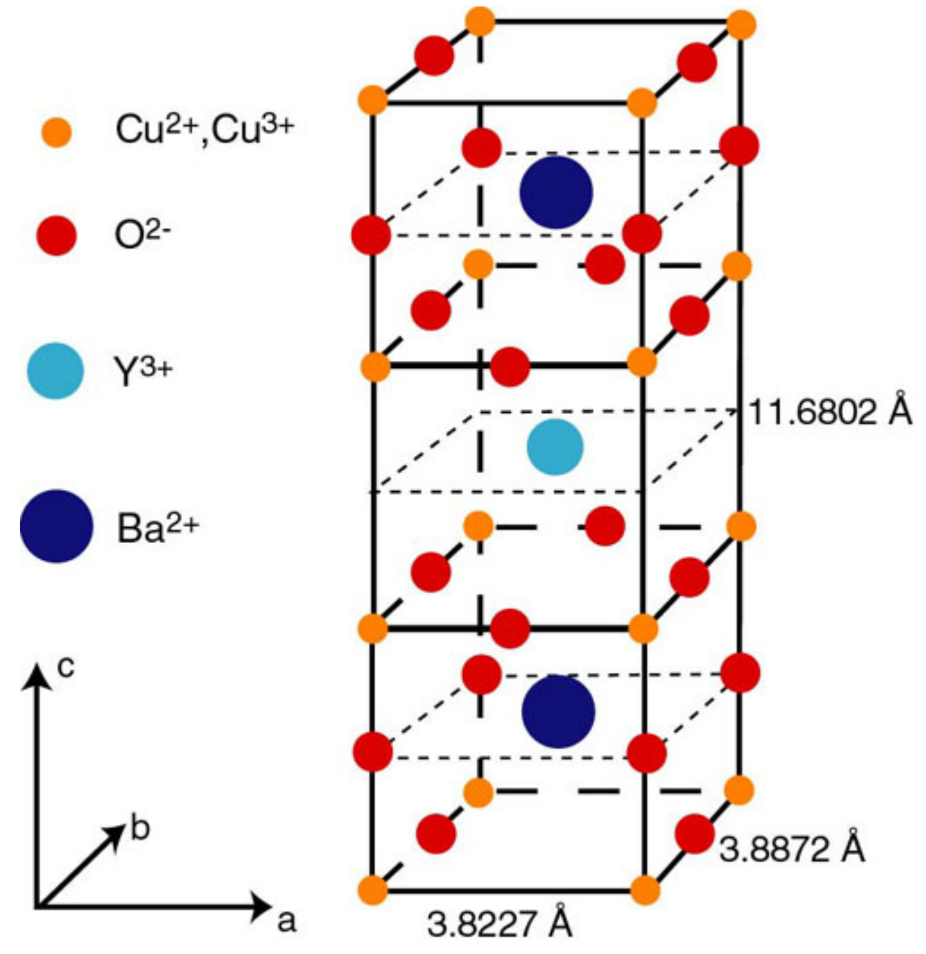
See however, S. Musser, D. Chowdhury, P. A. Lee, T. Senthil, arXiv:2111.08740


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T^*
Pseudogap Metal
Strange Metal
Fermi liquid
FL
Superconductor
T_c
p_c


Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å



250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Pseudogap Metal
Strange Metal
FL* ?
(with Fermi surface of size p)
Fermi liquid
FL
Superconductor
T_c
p_c



Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
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3.8227 Å

The pseudogap metal ≈ FL* (these papers fractionalize the mobile electron)

X.-G. Wen and P. A. Lee, "Theory of Underdoped Cuprates," Phys. Rev. Lett. **76**, 503 (1996), arXiv:cond-mat/9506065 [cond-mat].

J.-W. Mei, S. Kawasaki, G.-Q. Zheng, Z.-Y. Weng, and X.-G. Wen, "Luttinger-volume violating Fermi liquid in the pseudogap phase of the cuprate superconductors," Phys. Rev. B **85**, 134519 (2012), arXiv:1109.0406 [cond-mat.supr-con].

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The pseudogap metal = FL* (these papers fractionalize the mobile electron)

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R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, "Algebraic charge liquids," Nature Physics **4**, 28 (2008), arXiv:0706.2187 [cond-mat.str-el].

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M. Punk and S. Sachdev, "Fermi surface reconstruction in hole-doped t-J models without long-range antiferromagnetic order," Phys. Rev. B **85**, 195123 (2012), arXiv:1202.4023 [cond-mat.str-el].

M. Punk, A. Allais, and S. Sachdev, "A quantum dimer model for the pseudogap metal," Proc. Nat. Acad. Sci. **112**, 9552 (2015), arXiv:1501.00978 [cond-mat.str-el].

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S. Sachdev, H. D. Scammell, M. S. Scheurer, and G. Tarnopolsky, "Gauge theory for the cuprates near optimal doping," Phys. Rev. B **99**, 054516 (2019), arXiv:1811.04930 [cond-mat.str-el].

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The pseudogap metal = FL*

Main lesson from the Kondo lattice

Do not fractionalize the mobile electron, $c_{i\sigma} \neq f_{i\sigma} b_i^\dagger$ or $f_i b_{i\sigma}^\dagger$.

- The J interactions fractionalize *all* electrons into spinons and holons
- The t hopping is an attractive interaction between holons and spinons, and so forms electron-like bound states of density p
- This two-step process is present for $t \gg J$, and has not been completely described.

The pseudogap metal = FL*

Main lesson from the Kondo lattice

Do not fractionalize the mobile electron, $c_{i\sigma} \neq f_{i\sigma} b_i^\dagger$ or $f_i b_{i\sigma}^\dagger$.

Fractionalize the paramagnon instead!

- The J interactions fractionalize *all* electrons into spinons and holons
- The t hopping is an attractive interaction between holons and spinons, and so forms electron-like bound states of density p
- This two-step process is present for $t \gg J$, and has not been completely described.

Yahui Zhang

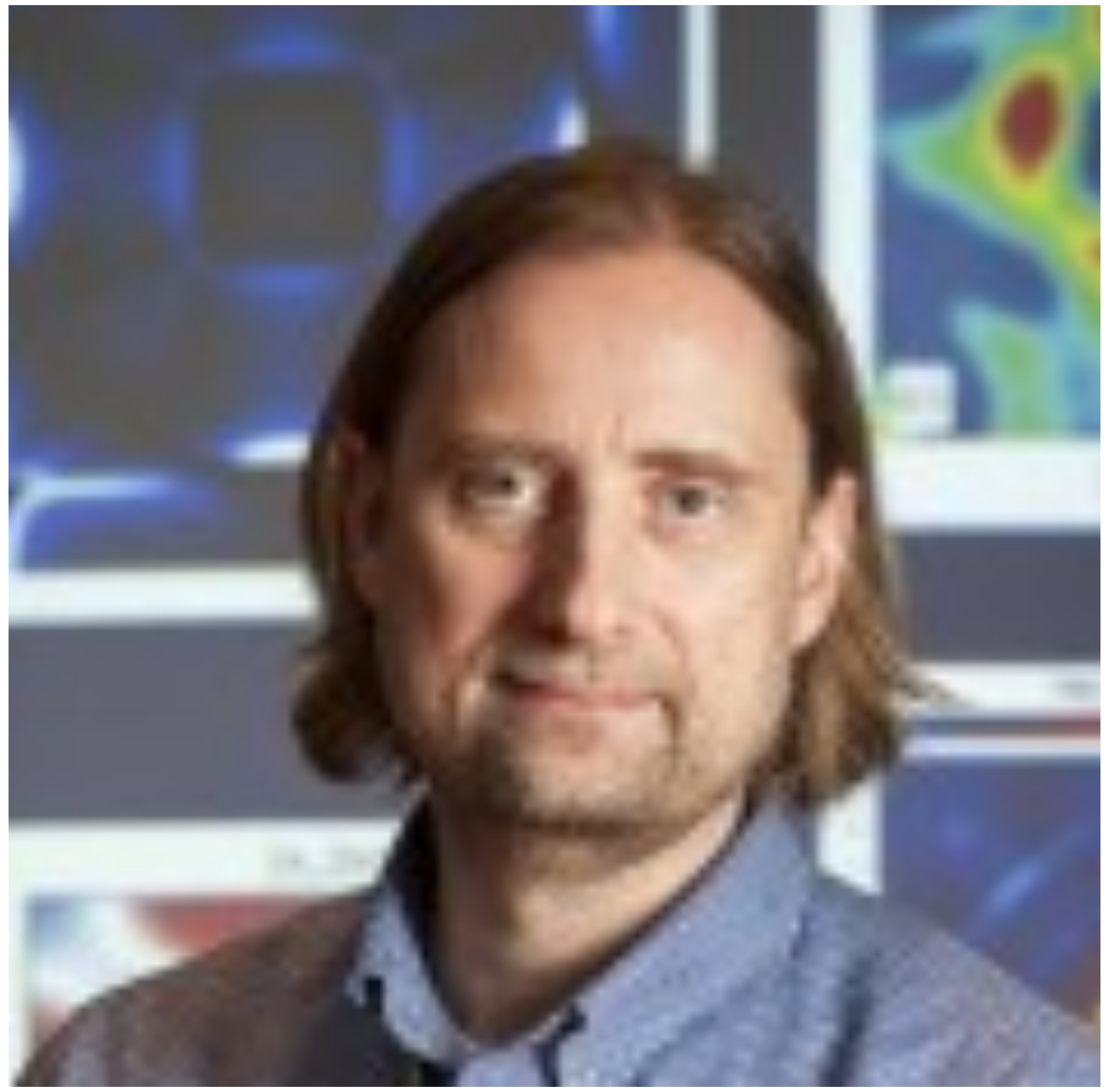
Alexander Nikolaenko

Maria Tikhanovskaya

Eric Mascot

arXiv: 2001.09159
arXiv: 2103.05009

arXiv: 2006.01140
arXiv: 2111.13703



Dirk Morr

Paramagnon theory of the Hubbard model

$$H = -\sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) - \mu \sum_i c_{i\sigma}^\dagger c_{i\sigma}$$

We use the operator equation (valid on each site i):

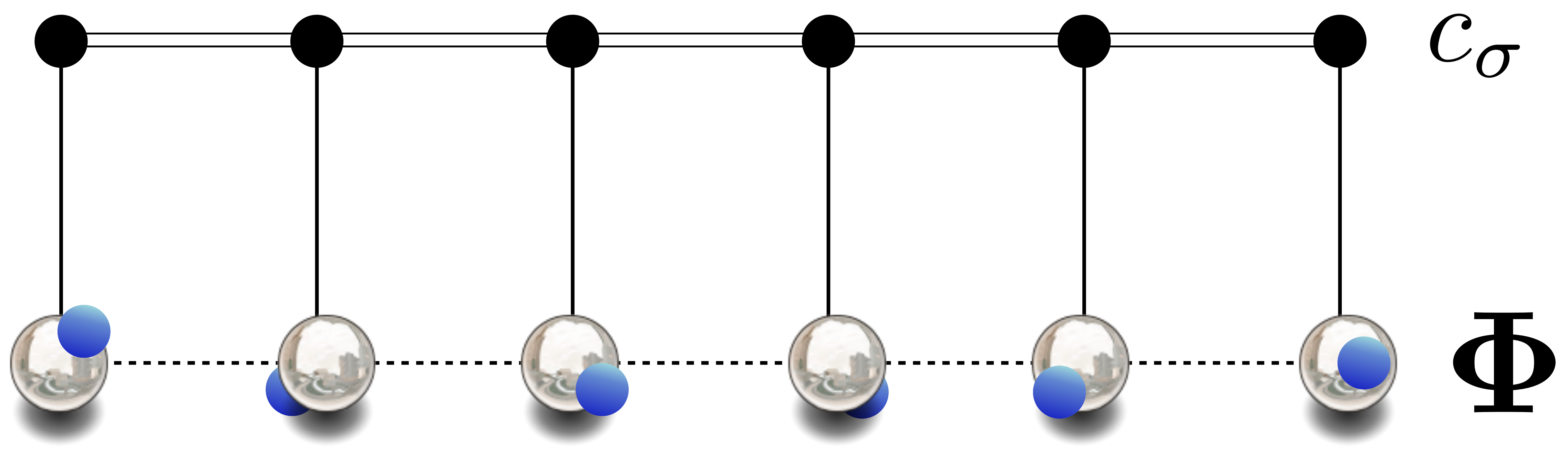
$$U\left(n_{\uparrow} - \frac{1}{2}\right)\left(n_{\downarrow} - \frac{1}{2}\right) = -\frac{2U}{3}\boldsymbol{S}^2 + \frac{U}{4}$$

Then we decouple the interaction via

$$\exp\left(\frac{2U}{3}\sum_i \int d\tau \boldsymbol{S}_i^2\right) = \int \mathcal{D}\boldsymbol{\Phi}_i(\tau) \exp\left(-\sum_i \int d\tau \left[\frac{3}{8U}\boldsymbol{\Phi}_i^2 - \boldsymbol{\Phi}_i \cdot c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'}\right]\right)$$

This yields the 'Scalapino-Pines-Chubukov-Schmalian...' theory for a 'paramagnon quantum rotor' $\boldsymbol{\Phi}_i$ coupled to otherwise free fermions $c_{i\sigma}$.

Paramagnon theory of the Hubbard model

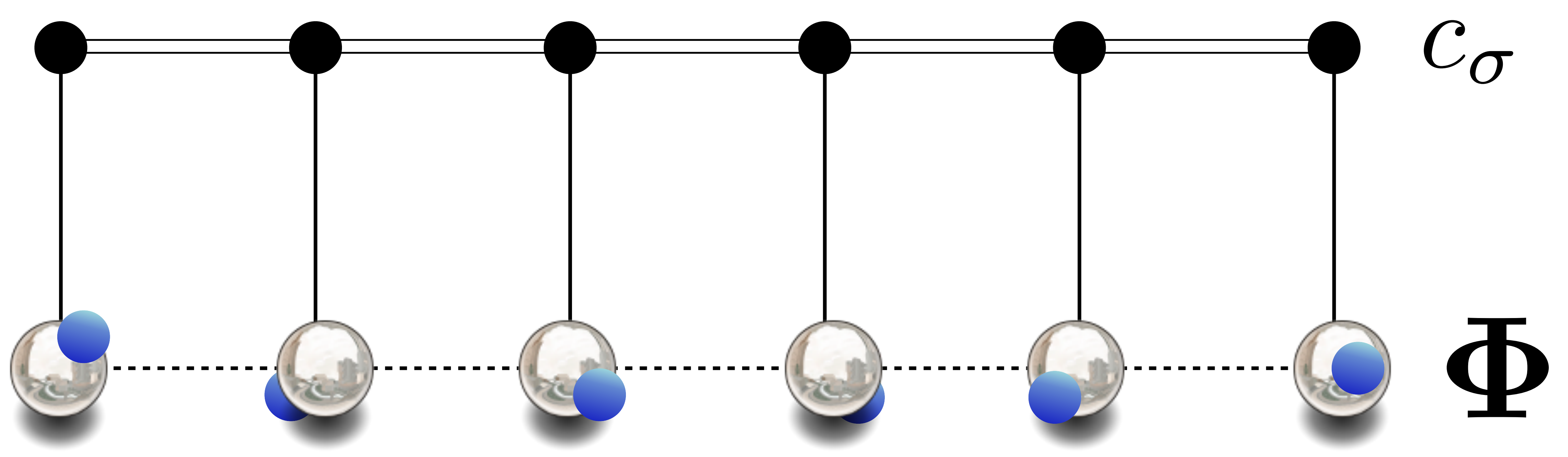


Quantum rotors
$|\mathbf{\Phi}_i| = 1$

$$\mathcal{H}_{\rm rotor} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + \sum_i \frac{g}{2} \boldsymbol{L}_i^2 - \sum_i c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \mathbf{\Phi}_i$$

$$[L_{i\alpha}, L_{j\beta}] = i\epsilon_{\alpha\beta\gamma}\delta_{ij}L_{i\gamma}, \quad [L_{i\alpha}, \Phi_{j\beta}] = i\epsilon_{\alpha\beta\gamma}\delta_{ij}\Phi_{i\gamma}, \quad [\Phi_{i\alpha}, \Phi_{j\beta}] = 0$$

Paramagnon theory of the Hubbard model



Quantum rotors $|\boldsymbol{\Phi}_i| = 1$

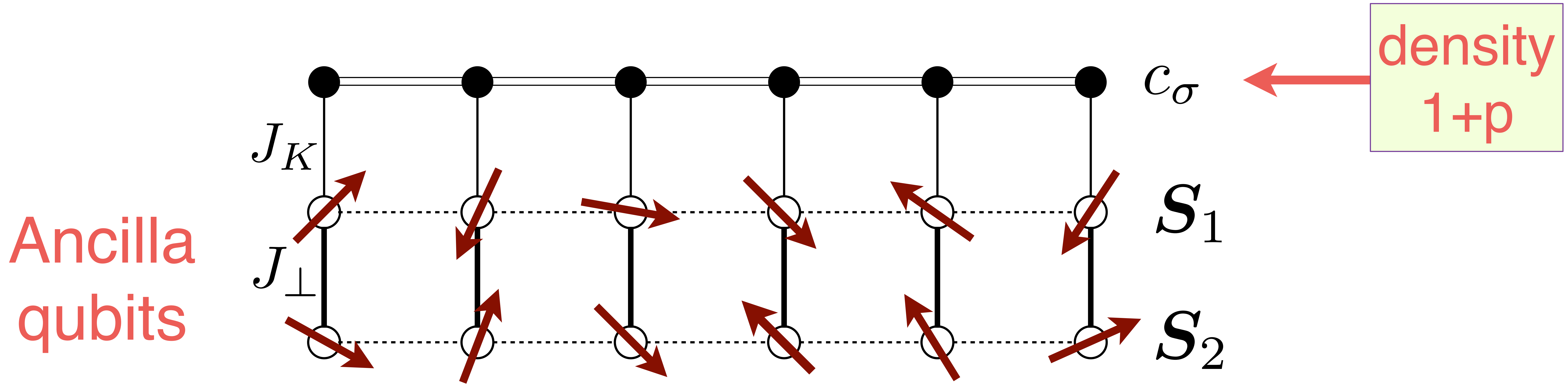
$$\mathcal{H}_{\text{rotor}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_i \frac{g}{2} \boldsymbol{L}_i^2 - \sum_i c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{\Phi}_i$$

Each rotor has eigenvalues $g\ell(\ell+1)/2$, degeneracy $2\ell+1$, $\ell = 0, 1, 2 \ldots$
Restrict to the $\ell = 0, 1$ states, and represent each rotor by 2 "ancilla qubits", $S = 1/2$ spins $\boldsymbol{S}_{1i}$ and $\boldsymbol{S}_{2i}$, with an antiferromagnetic coupling $J_\perp = g$

$$\boldsymbol{L}_i = \boldsymbol{S}_{1i} + \boldsymbol{S}_{2i}$$

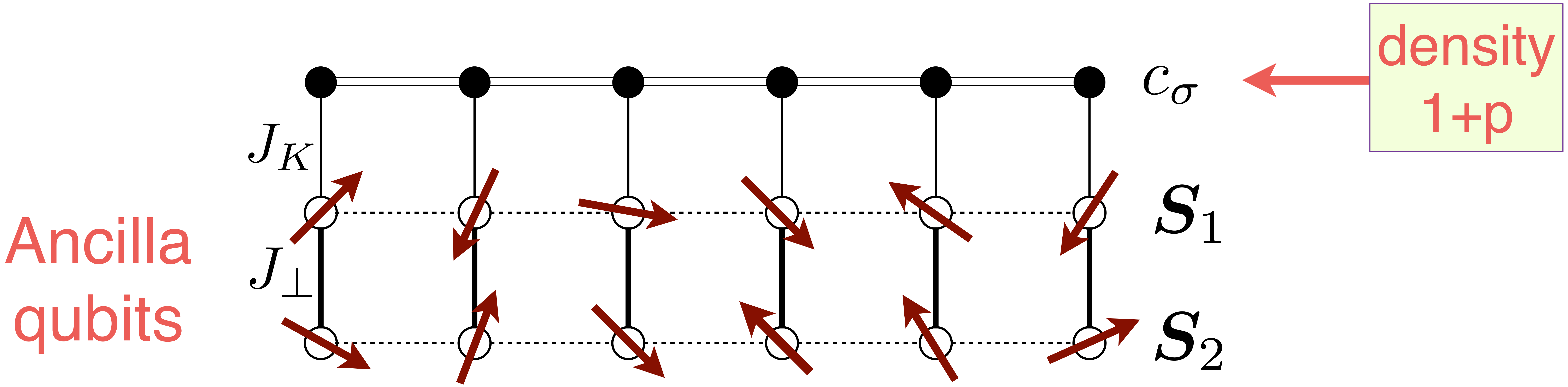
$$\boldsymbol{\Phi}_i = \frac{1}{\sqrt{3}} \left(\boldsymbol{S}_{2i} - \boldsymbol{S}_{1i} \right)$$

Ancilla theory of the Hubbard model



$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_i \left[J_K \, c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_{1i} + J_\perp \, \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i} \right] + \sum_{\langle ij \rangle} \left[J_1 \, \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{1j} + J_2 \, \boldsymbol{S}_{2i} \cdot \boldsymbol{S}_{2j} \right]$$

Ancilla theory of the Hubbard model



$$\mathcal{H}_{\rm ancilla} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + \sum_i \left[J_K \, c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_{1i} + J_\perp \, \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i} \right] + \sum_{\langle ij \rangle} \left[J_1 \, \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{1j} + J_2 \, \boldsymbol{S}_{2i} \cdot \boldsymbol{S}_{2j} \right]$$

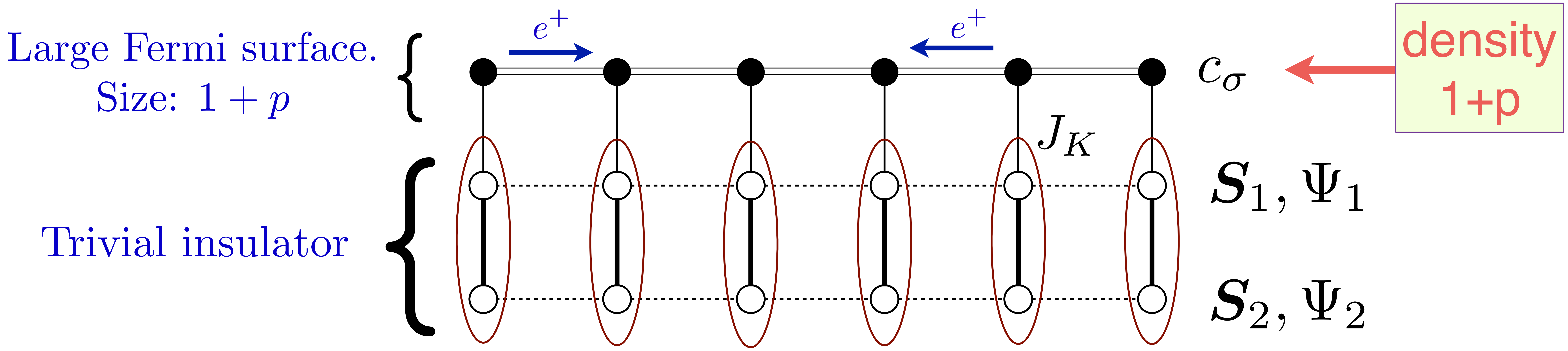
Performing a Schrieffer-Wolff transformation in powers of $1/J_\perp$, we obtain

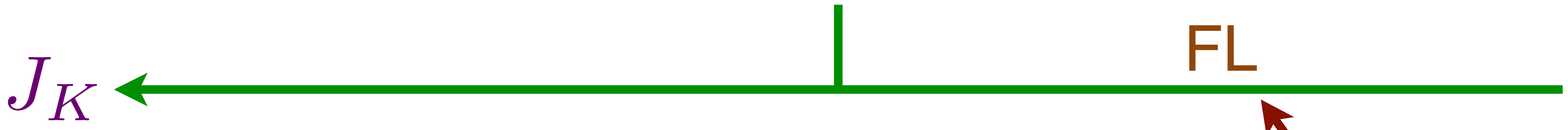
$$\mathcal{H} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + U \sum_i \left[c_{i\uparrow}^\dagger c_{i\uparrow} \right] \left[c_{i\downarrow}^\dagger c_{i\downarrow} \right] + J \sum_{\langle ij \rangle} \left[c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \right] \cdot \left[c_{j\rho}^\dagger \frac{\boldsymbol{\tau}_{\rho\rho'}}{2} c_{j\rho'} \right]$$

i.e. we recover a Hubbard-Heisenberg model with *no ancillas* and

$$U = \frac{3J_K^2}{8J_\perp} + \frac{3J_K^3}{16J_\perp^2} + \ldots \quad , \quad J = \frac{J_K^2 (J_1 + J_2)}{4J_\perp^2}$$

Ancilla theory of the Hubbard model

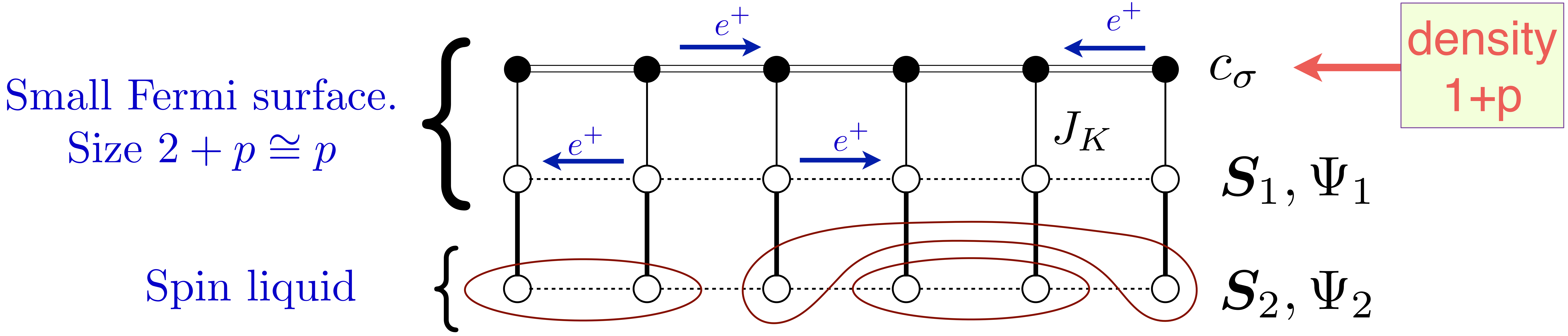


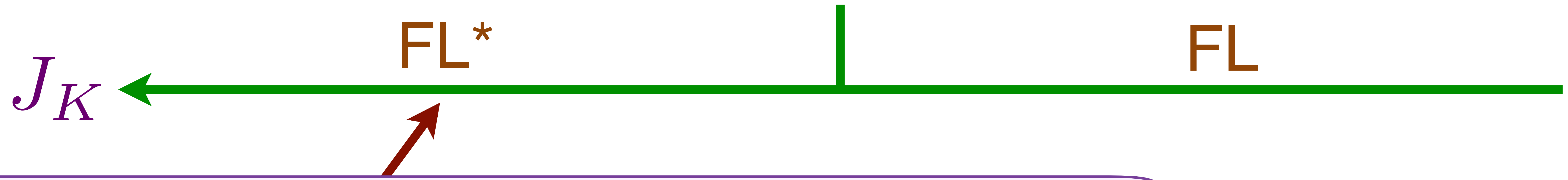


Large Fermi surface of size $1+p$

$$|\mathrm{FL}\rangle = |\text{Rung singlets of } \Psi_1, \Psi_2\rangle \otimes |\text{Slater determinant of } c\rangle$$

Ancilla theory of the Hubbard model

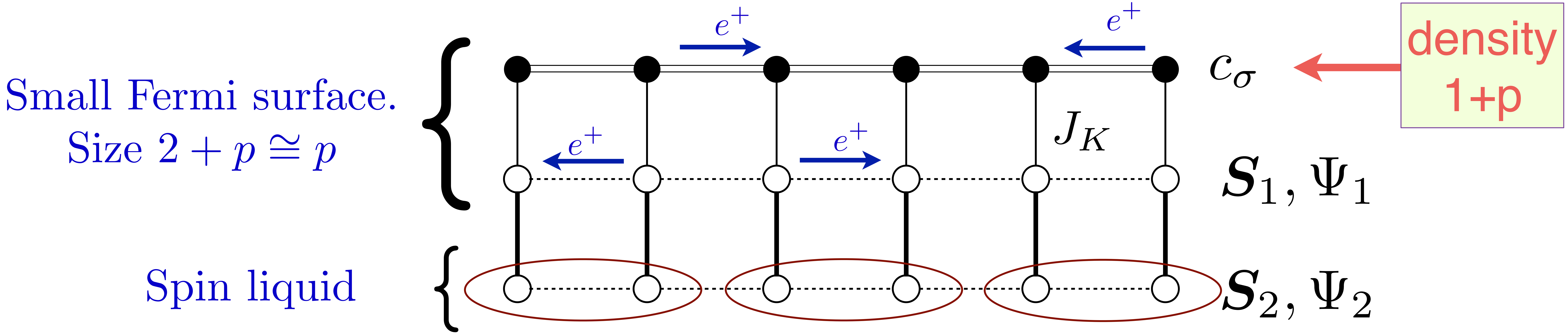


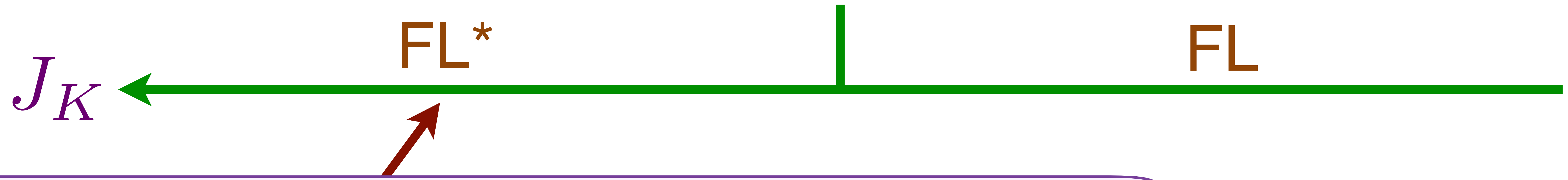


Small Fermi surface of size p

$$|\text{FL}*\rangle = [\text{Projection onto rung singlets of } \Psi_1, \Psi_2]$$
$$\bowtie |\text{Slater determinant of } (c, \Psi_1)\rangle$$
$$\otimes |\text{Slater determinant of } \Psi_2\rangle$$

Ancilla theory of the Hubbard model





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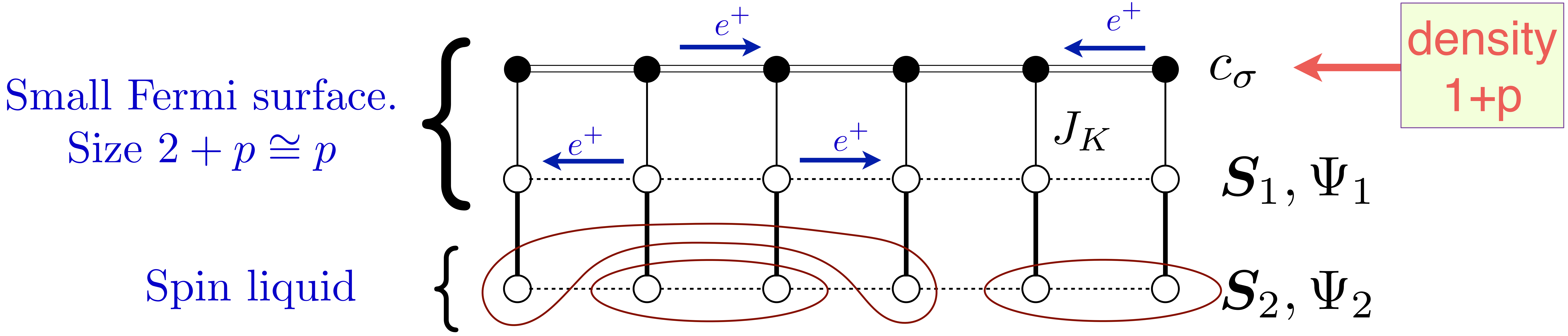
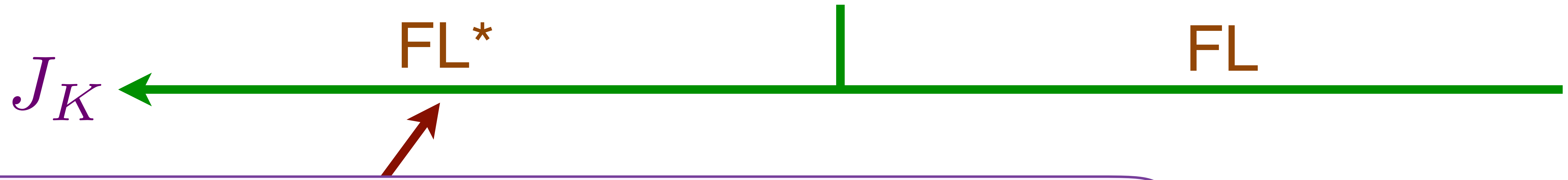
Ancilla theory of the Hubbard model





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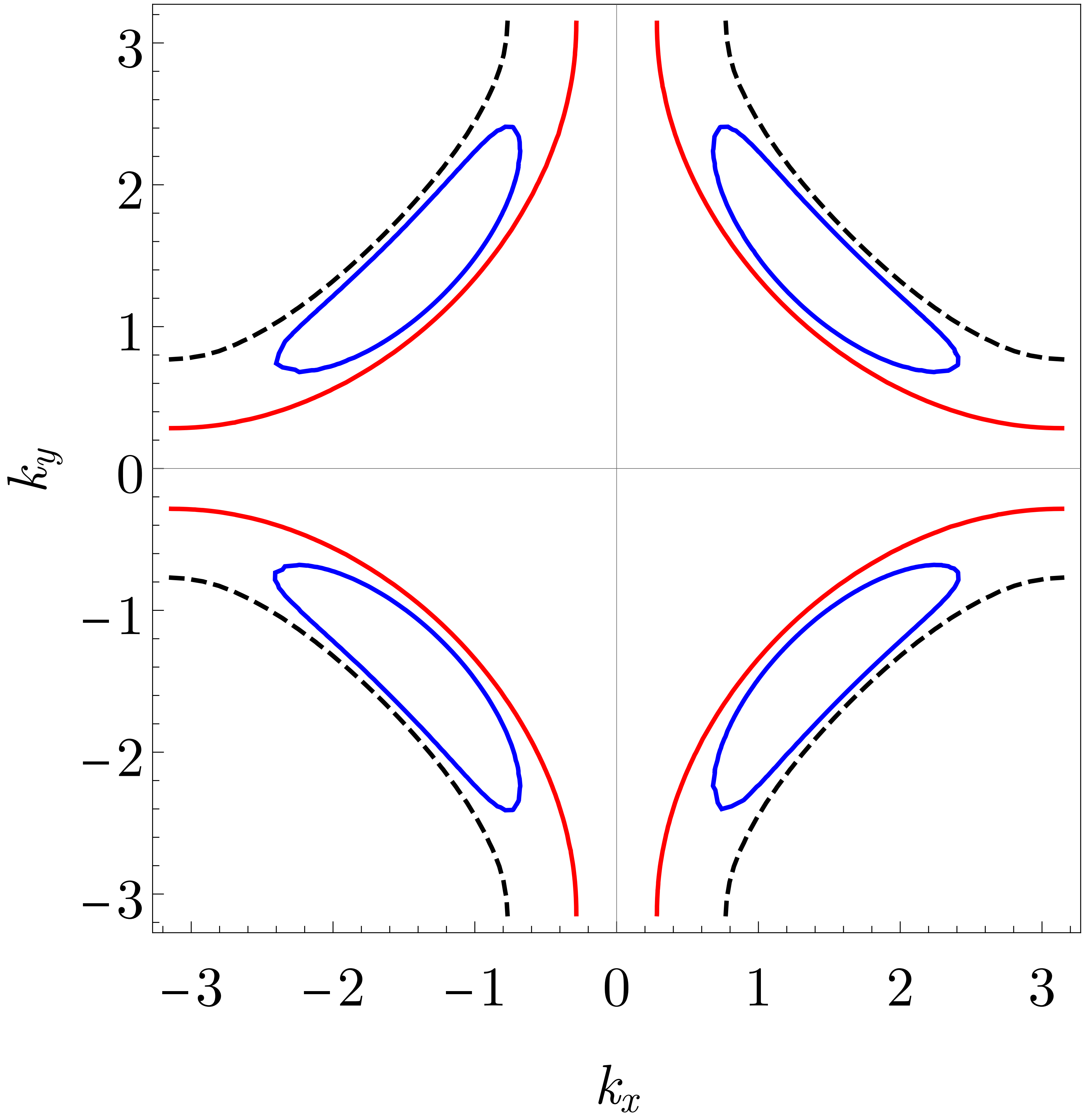
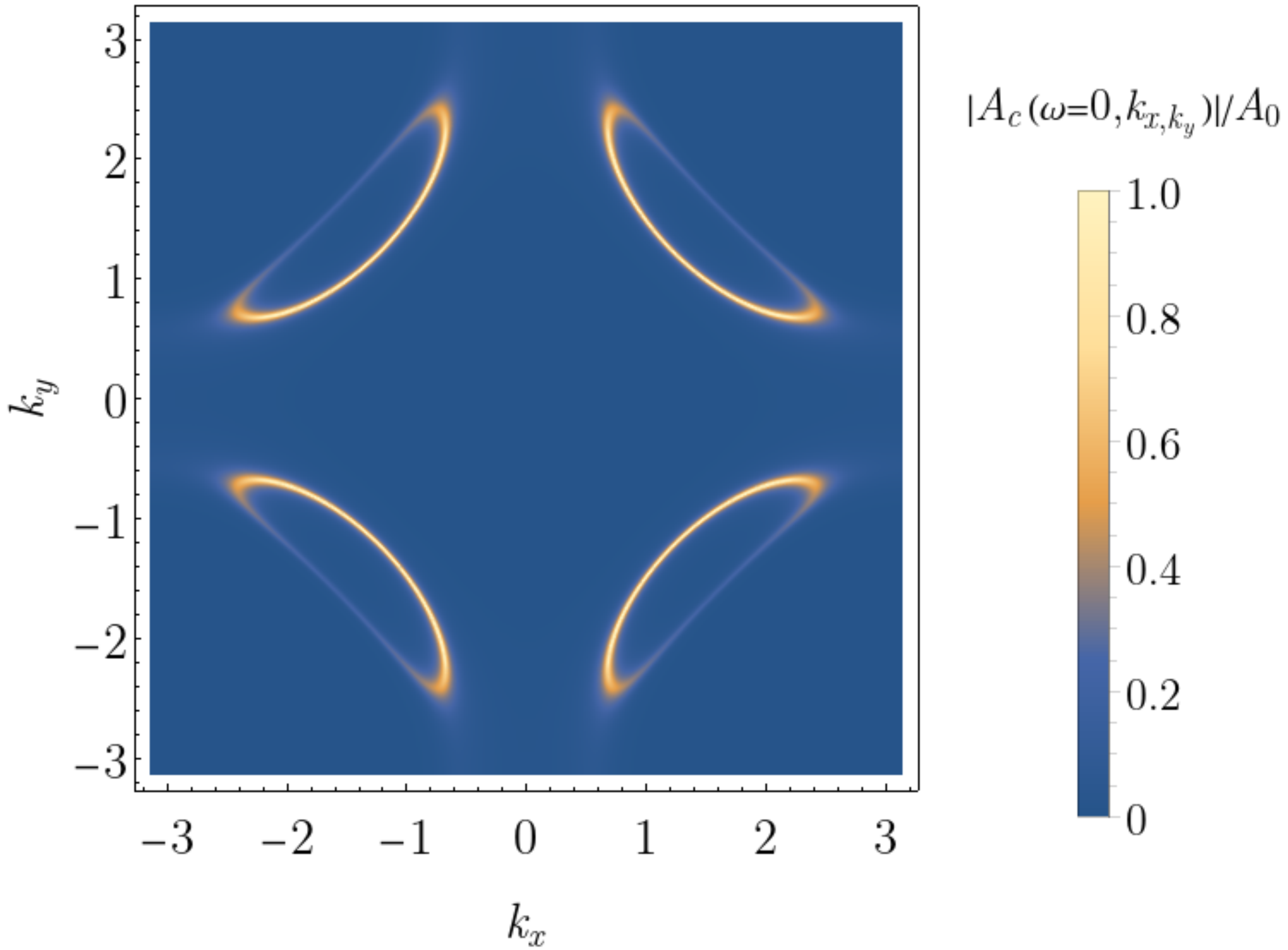
$$\otimes |\text{Slater determinant of } \Psi_2\rangle$$

FL* in a ***one-band*** model



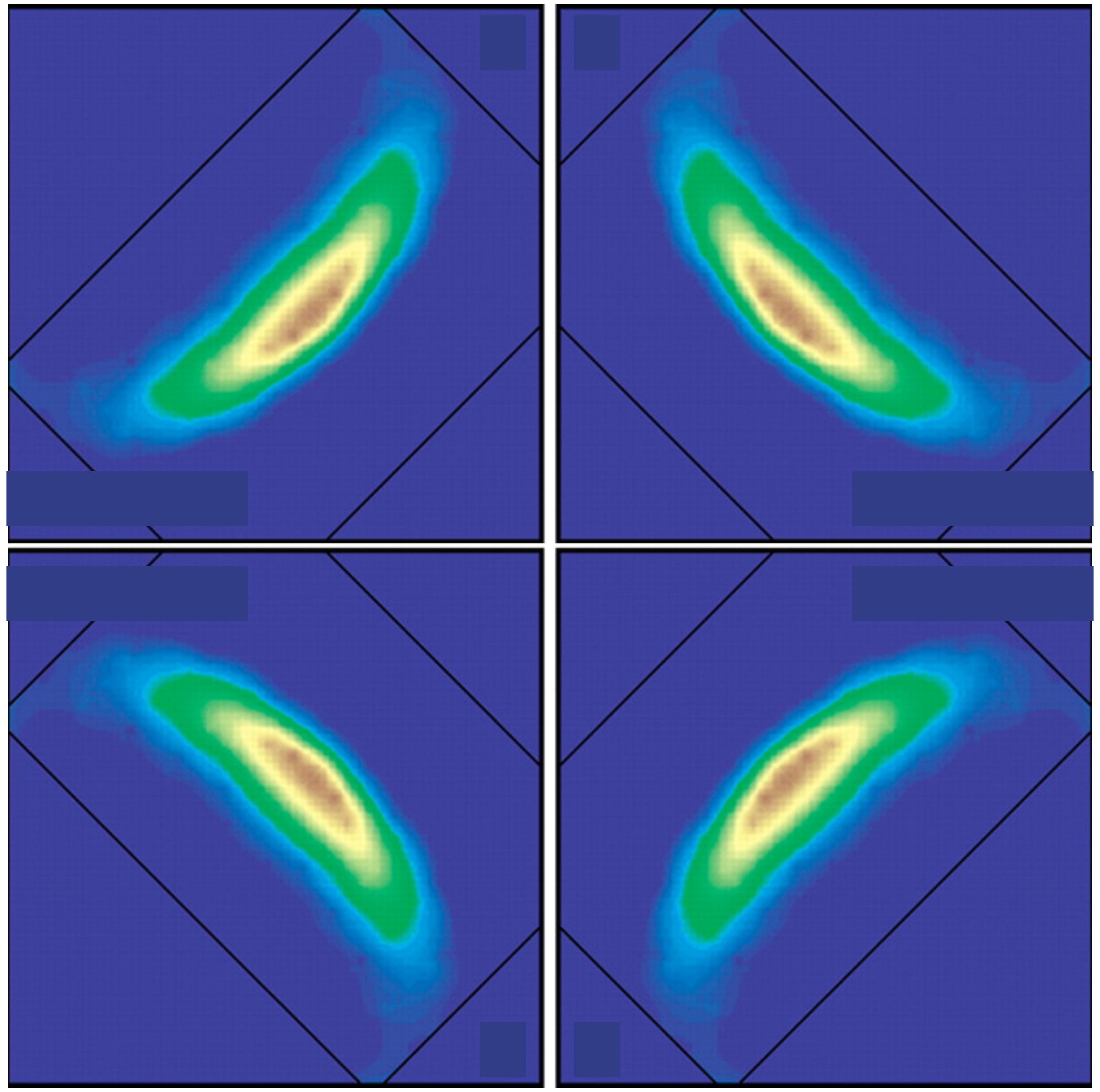
"Fermi arc" spectral functions



FL* Hamiltonian: $[(\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_S)/\mathbb{Z}_2] \times \mathrm{U}(1)_{\mathrm{em}}$ is broken to $\mathrm{U}(1)_{\mathrm{diag}}$ by Higgs condensate Φ:

$$H = -\sum_{i,j} t_{ij} c^\dagger_{i;\alpha} c_{j;\alpha} + \sum_{i,j} t_{1,ij} \Psi^\dagger_{1i;\alpha} \Psi_{1j;\alpha} + \sum_i \Phi (c^\dagger_{i;\alpha} \Psi_{1i;\alpha} + \Psi^\dagger_{1i;\alpha} c_{i;\alpha})$$

Photoemission at small p



$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$

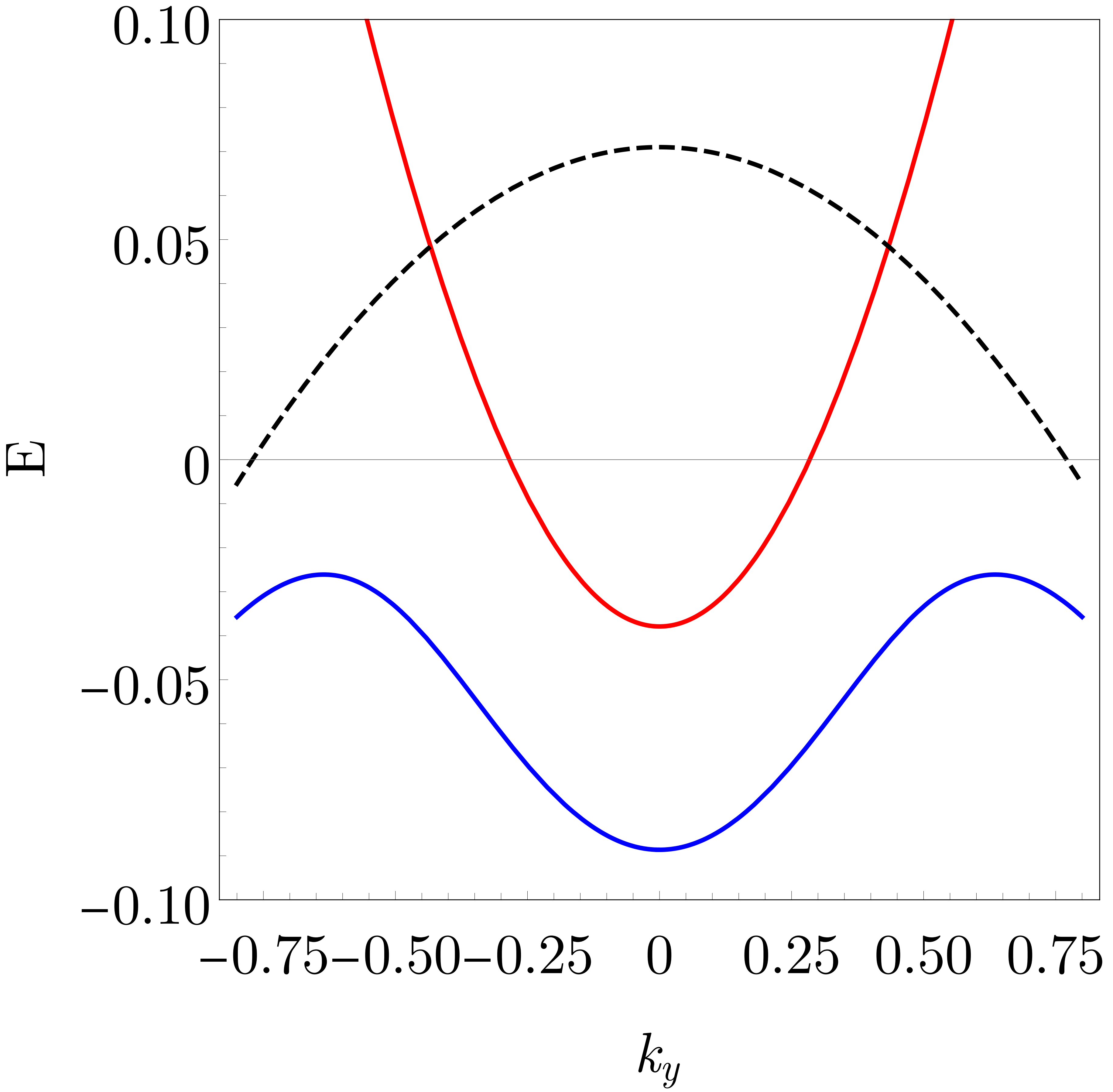
"Fermi arcs"

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

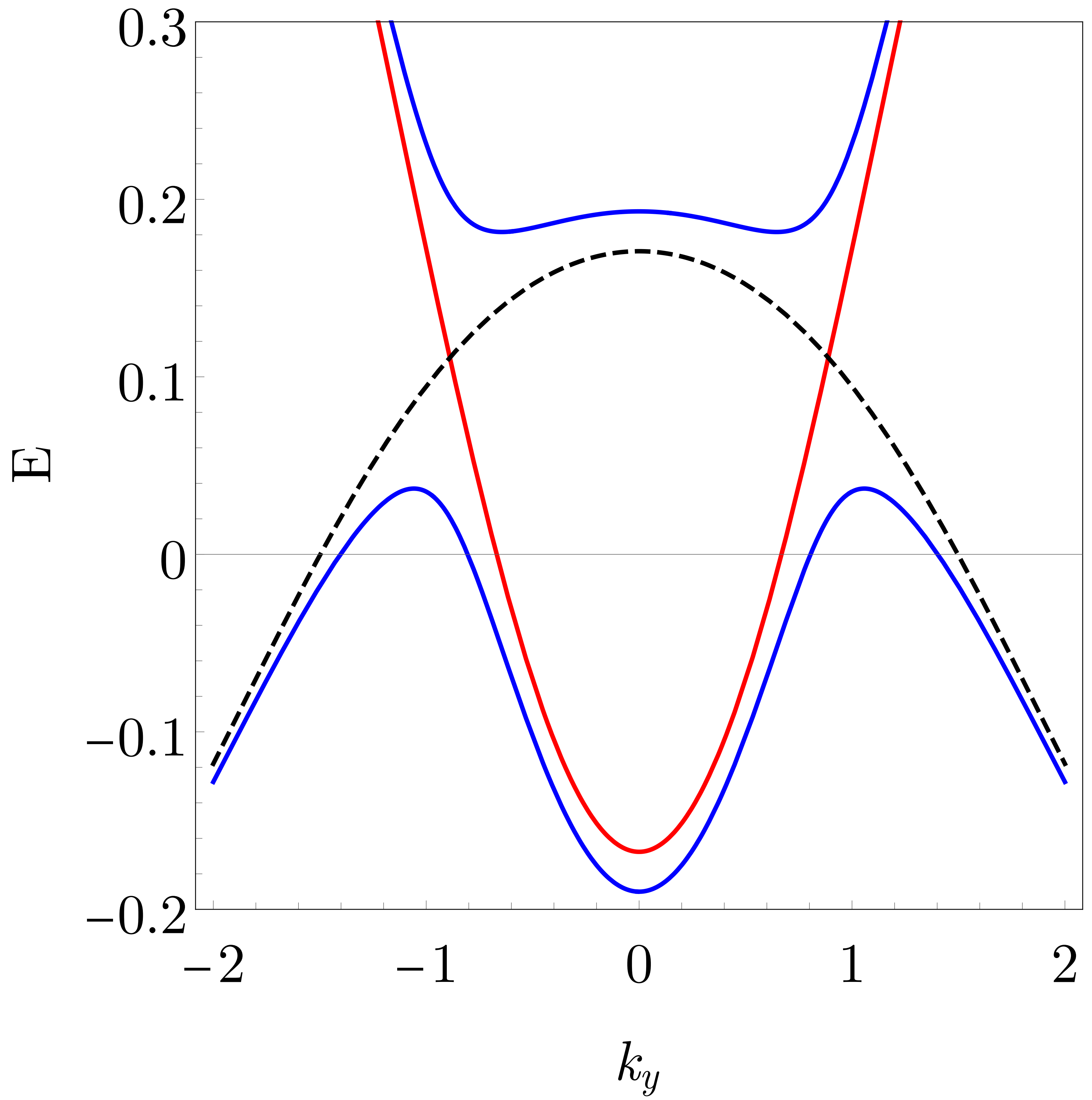
FL* in a ***one-band*** model

Electronic dispersion

Anti-node: $k_x = \pi$



Node: $k_x = 2$



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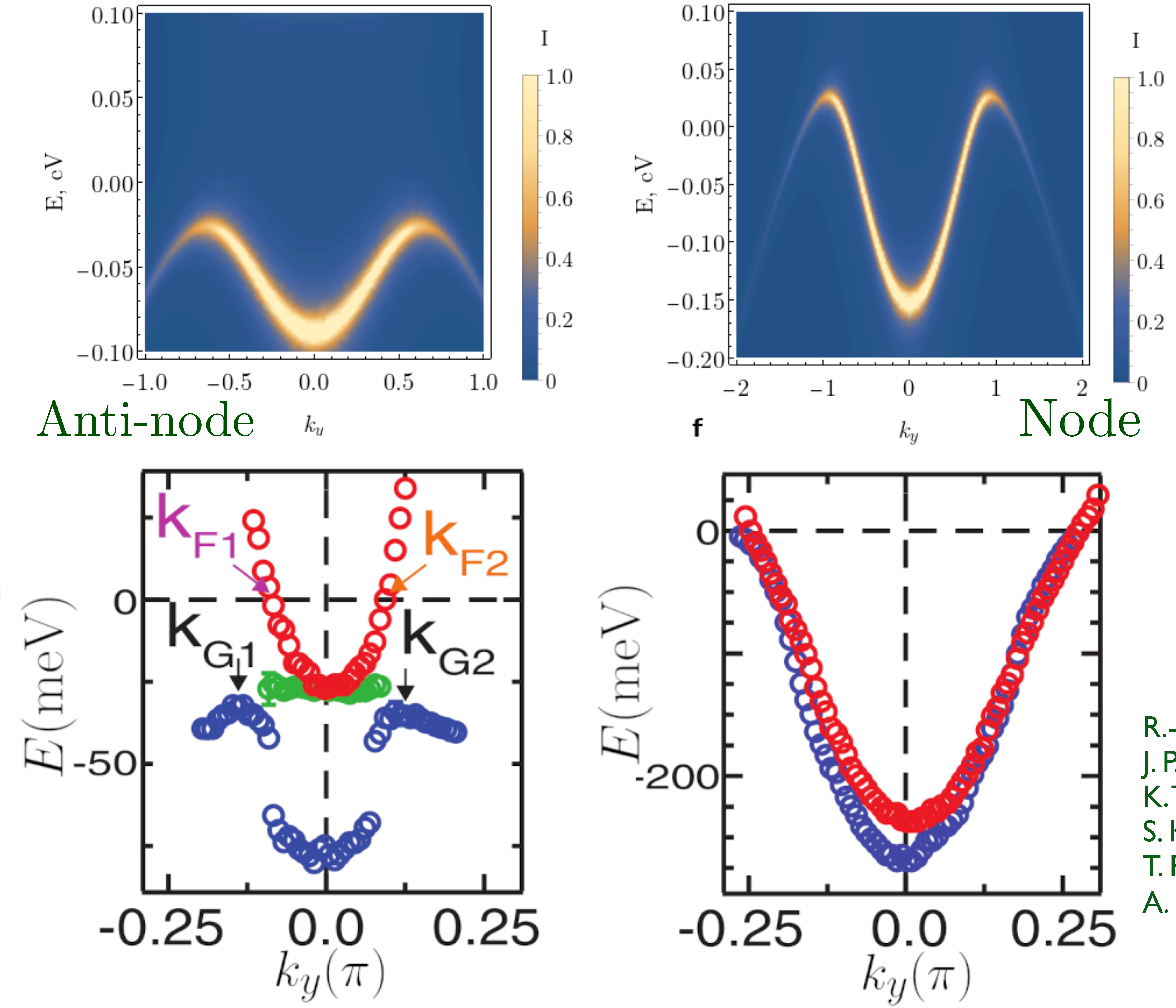
FL* in a ***one-band*** model



ARPES on Bi2201

R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

1. Spin liquids and violations of the Luttinger relation: FL* and HFL phases of the Kondo lattice model.

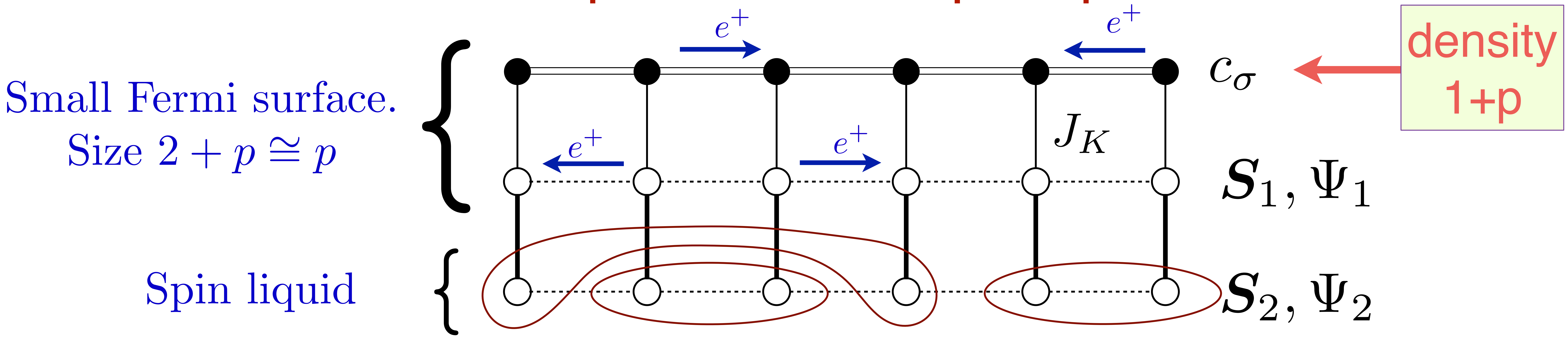
2. Hubbard model - the vanilla FL phase

3. Hubbard model - the FL* phase: fractionalizing the paramagnon

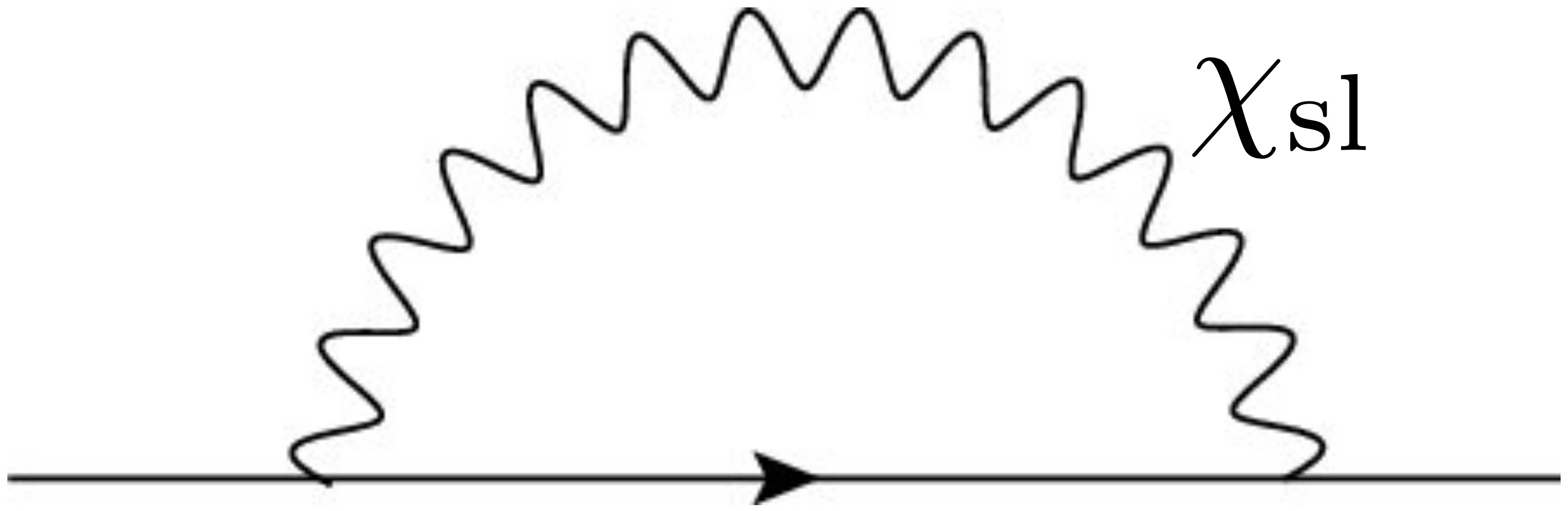
4. Hubbard model - nature of spin liquid

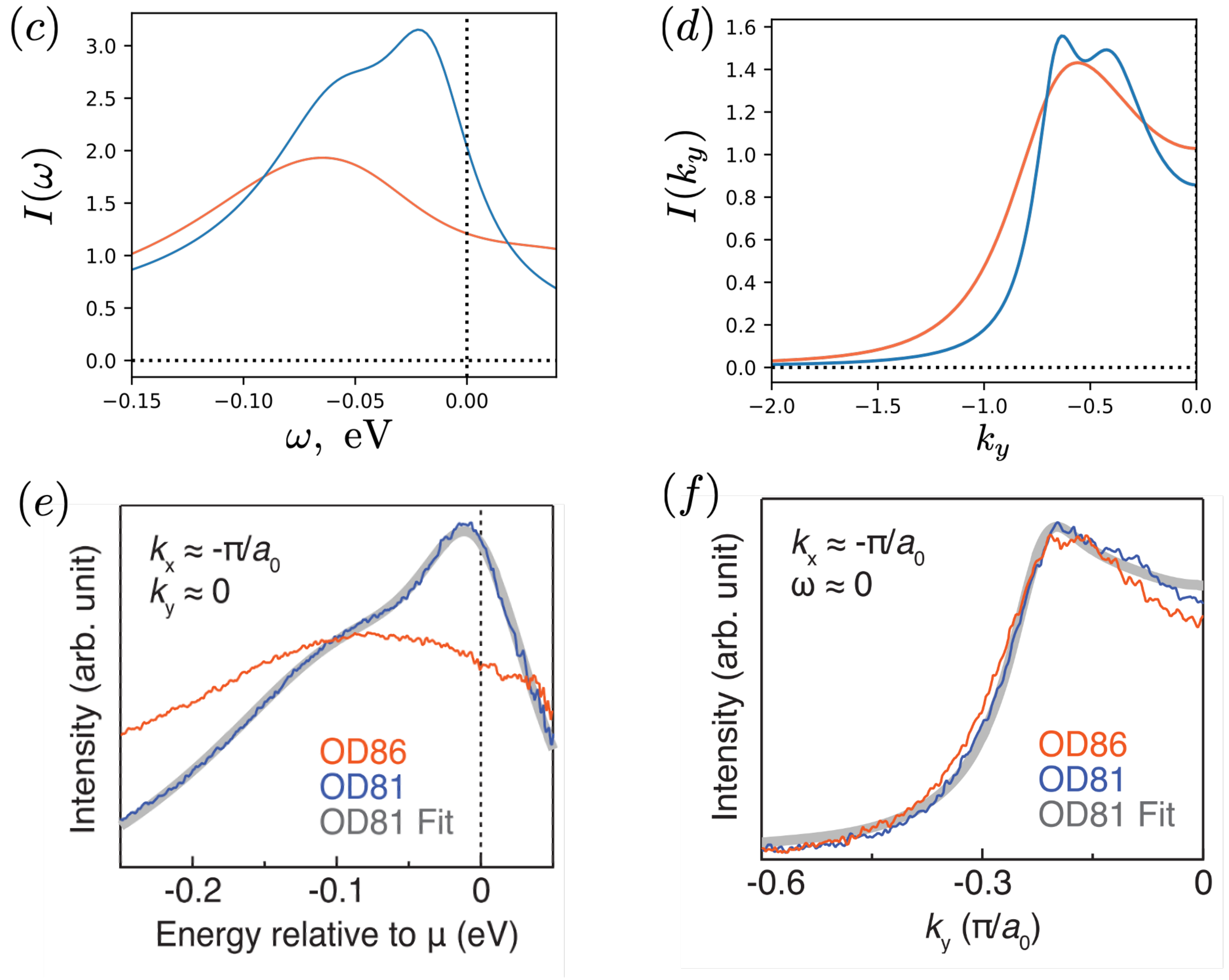
5. Quantum criticality

Consequences of the spin liquid



The only singular gauge fluctuations are those in the spin liquid of the Ψ_2. We can compute their influence on the electronic spectrum perturbatively in the exchange couplings in terms of the dynamic spin susceptibility $\chi_{\rm sl}$.





Antinodal EDC and MDC

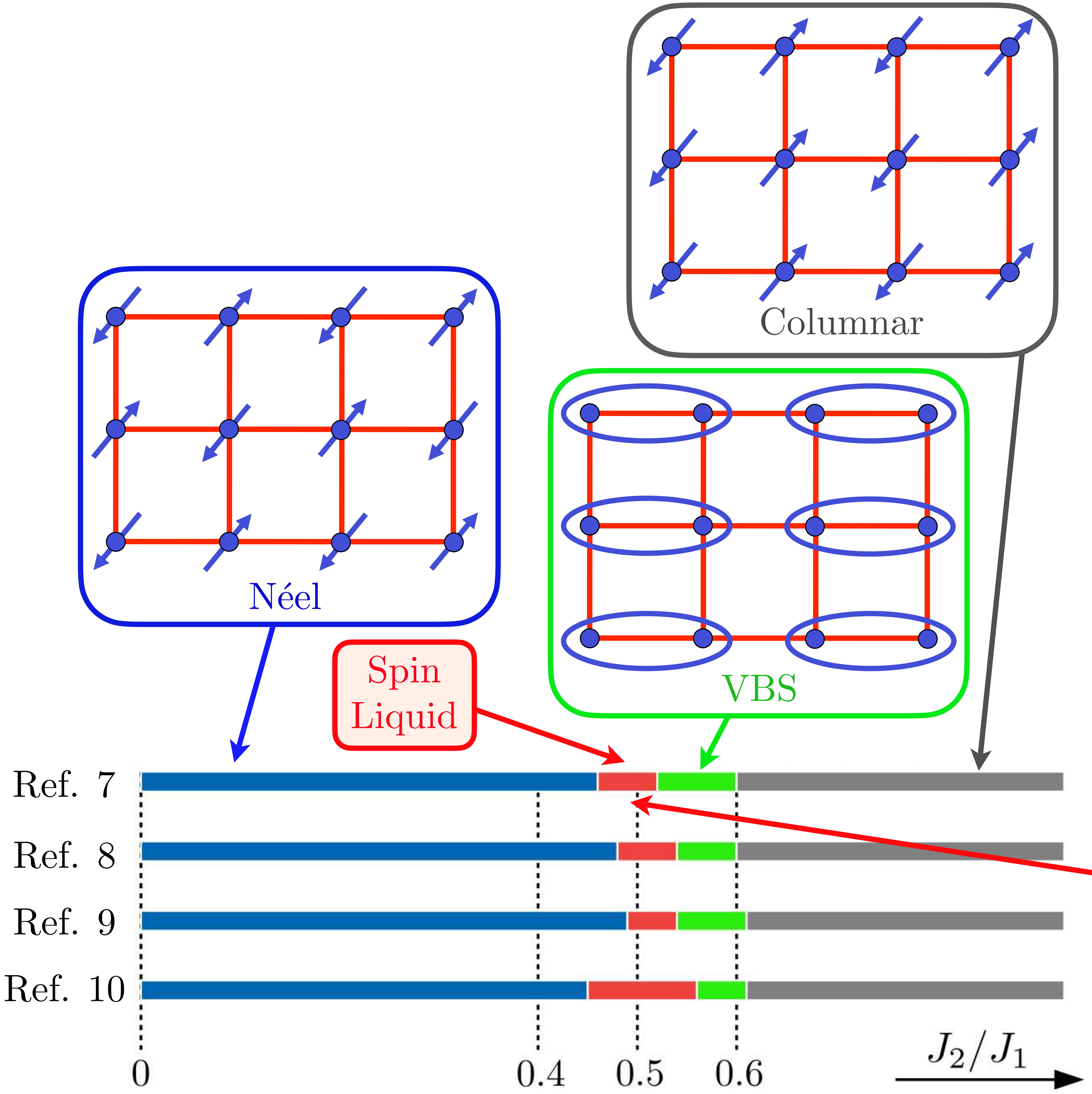
(c,d) Theory with SYK spin liquid in Ψ_2 layer. Similar EDC obtained by gapless $\mathbb{Z}_2$ spin liquid

(e,f) Experiments on Bi2212 by S.-D. Chen, M. Hashimoto, Y. He, D. Song, K.-J. Xu, J.-F. He, T. P. Devereaux, H. Eisaki, D.-H. Lu, J. Zaanen, and Z.-X. Shen, Science **366**, 1099 (2019).

FL*

- Recent evidence for a FL* phase in a Kondo lattice: $CeCoIn_5$ (Maksimovic *et al.*, Science **375**, 76 (2021), and in CePdAl, Zhao *et al.*, Nature Physics **15**, 1261 (2019). And perhaps YbB_{12} (Liu *et al.* arXiv:2102.09545)?

- Ancilla theory of FL* for the pseudogap metal of the cuprates: Don't fractionalize the mobile electron, but fractionalize the 'paramagnon rotor' into 'ancilla qubits'. Predicts electronic spectra in good agreement with observations in *both* nodal and anti-nodal regions.



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Gapless Z_2 spin liquid phase !

Wen-Jun Hu,
Federico Becca,
Alberto Parola,
Sandro Sorella,
PRB **88**, 060402(R) (2013).

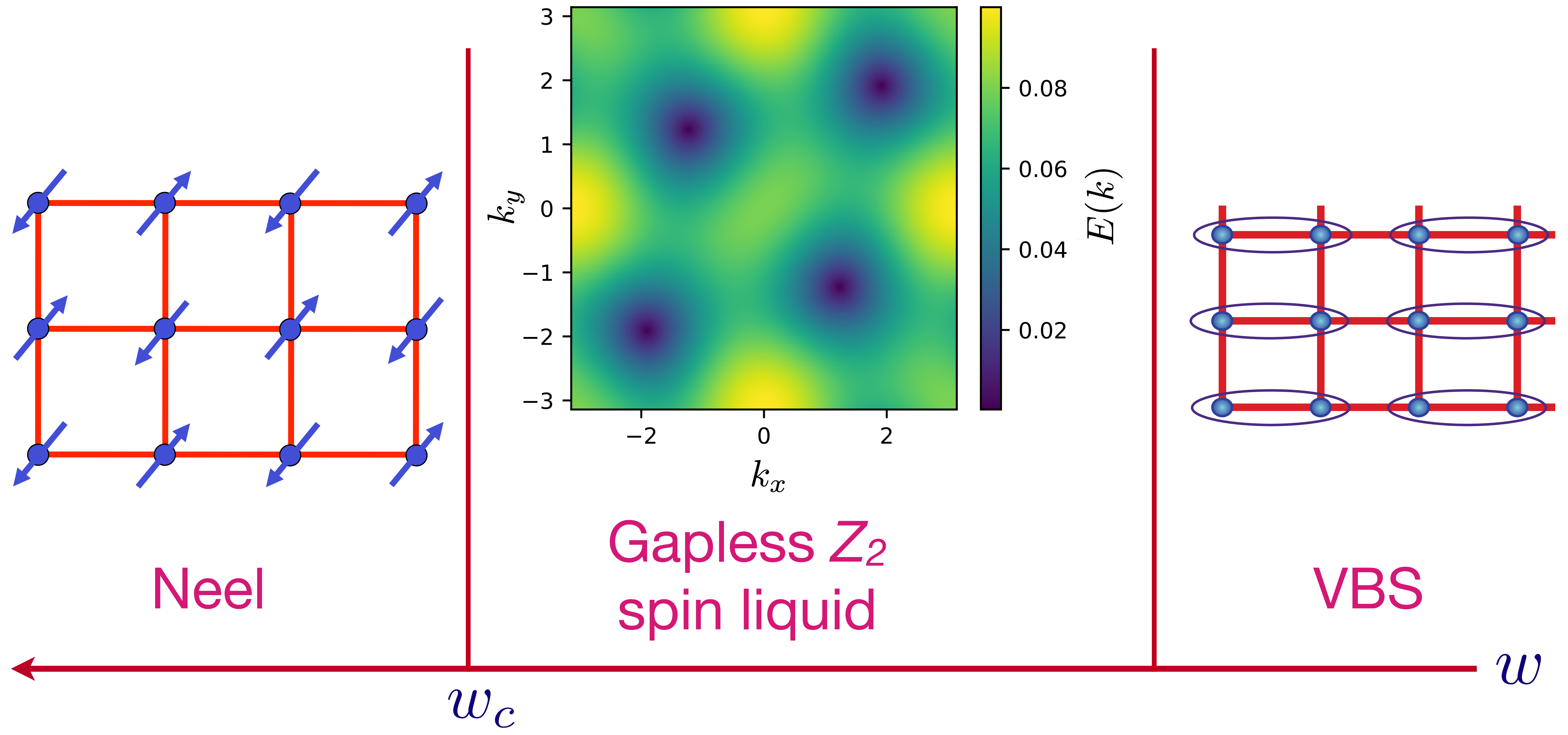
SU(2) gauge theory for transition from π-flux to gapless $\mathbb{Z}_2$ spin liquid:
Condense *two* adjoint Higgs scalars, Φ_1^a, Φ_2^a without breaking any symmetry.

$$\mathcal{S} = \int d^2x d\tau \Big[i\overline{\psi}\gamma_\mu \left(\partial_\mu - i a_\mu^a \sigma^a\right)\psi + \ldots$$
$$+ \lambda \left(\Phi_1^a \, \overline{\psi} \, \mu^z \gamma^x \sigma^a \psi + \Phi_2^a \, \overline{\psi} \, \mu^x \gamma^y \sigma^a \psi \right) + \frac{w}{2}\left(\Phi_1^a \Phi_1^a + \Phi_2^a \Phi_2^a\right) + \ldots \Big]$$

Henry Shackleton

Alex Thomson

PRB **104**, 045110 (2021)



1. Spin liquids and violations of the Luttinger relation:

   FL* and HFL phases of the Kondo lattice model.

2. Hubbard model - the vanilla FL phase

3. Hubbard model - the FL* phase:
   fractionalizing the paramagnon

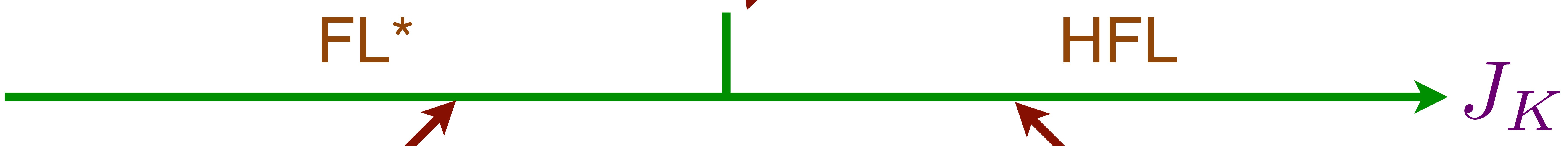
4. Hubbard model - nature of spin liquid

5. Quantum criticality

Kondo lattice

Deconfined criticality of a U(1) gauge theory with a Higgs field, spinons, and a small Fermi surface of electrons. (FL* can be replaced by a confining phase with AFM or VBS order).

T. Senthil, M. Vojta, and S. Sachdev, PRB **69**, 035111 (2004)



Small Fermi surface of size p

$$|\mathrm{FL}*\rangle = [\text{Projection onto one } f \text{ per site}] \bowtie |\text{Slater determinant of } f\rangle \otimes |\text{Slater determinant of } c\rangle$$

Large Fermi surface of size $1 + p$

$$|\mathrm{HFL}\rangle = [\text{Projection onto one } f \text{ per site}] \bowtie |\text{Slater determinant of } (c, f)\rangle$$

$(\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2 \times \mathrm{SU}(2)_S)/\mathbb{Z}_2$ gauge theory of **one-band** model

Write fermion operators as 2×2 matrices

$$\boldsymbol{\Psi}_1 = \begin{pmatrix} \Psi_{1\uparrow} & -\Psi^\dagger_{1\downarrow} \\ \Psi_{1\downarrow} & \Psi^\dagger_{1\uparrow} \end{pmatrix} \quad , \quad \boldsymbol{\Psi}_2 = \begin{pmatrix} \Psi_{2\uparrow} & -\Psi^\dagger_{2\downarrow} \\ \Psi_{2\downarrow} & \Psi^\dagger_{2\uparrow} \end{pmatrix}$$

Constraints $\Psi^\dagger_\alpha \Psi_\alpha = 1$ and $\widetilde{\Psi}^\dagger_\alpha \widetilde{\Psi}_\alpha = 1$ lead to:

P.A. Lee, N. Nagaosa, and X.-G. Wen, RMP **78**, 17 (2006)

$$\begin{aligned} \mathrm{SU}(2)_1 : \quad & \boldsymbol{\Psi}_1 \to \boldsymbol{\Psi}_1 U_1 \quad , \quad \boldsymbol{\Psi}_2 \to \boldsymbol{\Psi}_2 \\ \mathrm{SU}(2)_2 : \quad & \boldsymbol{\Psi}_1 \to \boldsymbol{\Psi}_1 \quad , \quad \boldsymbol{\Psi}_2 \to \boldsymbol{\Psi}_2 U_2 \end{aligned}$$

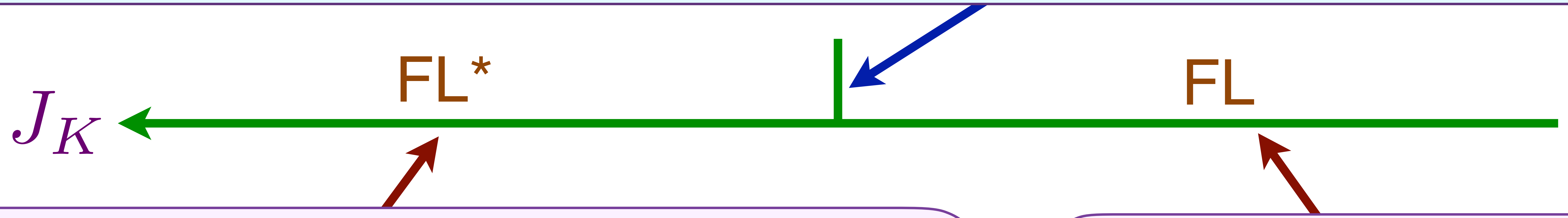
S. Sachdev, M.A. Metlitski, Yang Qi, and Cenke Xu, PRB **80**, 155129 (2009)
S. Sachdev, H. D. Scammell, M. S. Scheurer, and G. Tarnopolsky, PRB **99**, 054516 (2019)

Rung singlet formation $\boldsymbol{S}_1 + \boldsymbol{S}_2 \approx 0$ leads to:

$$\mathrm{SU}(2)_S : \quad \boldsymbol{\Psi}_1 \to U_S \boldsymbol{\Psi}_1 \quad , \quad \boldsymbol{\Psi}_2 \to U_S \boldsymbol{\Psi}_2$$

Ancilla theory of the Hubbard model

- Deconfined criticality of a $(\mathrm{SU}(2)_S \times \mathrm{U}(1)_1)/Z_2$ gauge theory.
- 'Hybridization-Higgs' boson $\sim C_\sigma^\dagger \Psi_a$ which condenses on the FL* side (in Kondo lattice, Higgs boson was condensed on the FL side).
- Gauge-charged 'ghost' Fermi surface of Ψ_1 fermions.
- Large Fermi surface of c_σ gauge-neutral electrons.



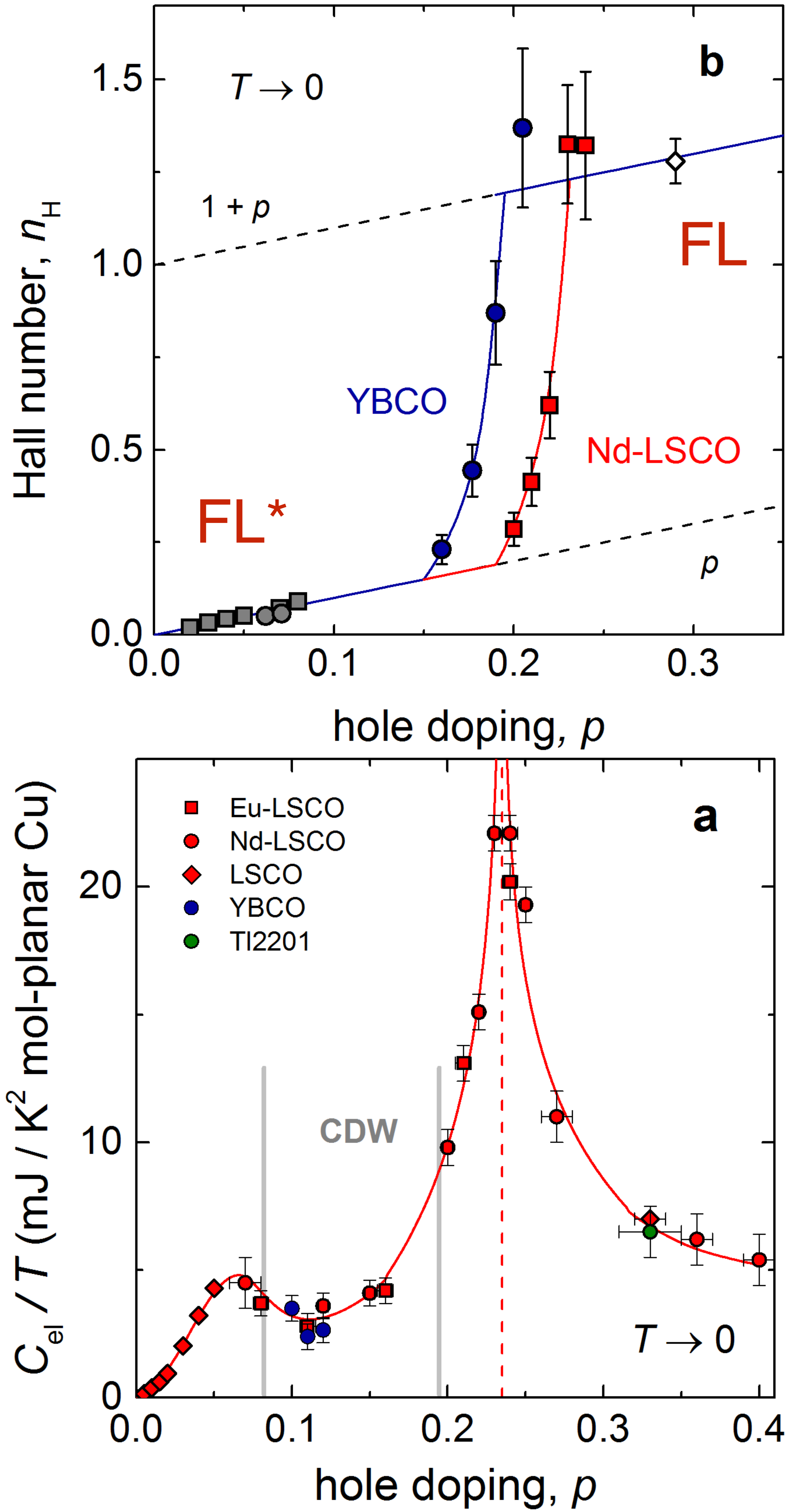
Small Fermi surface of size p

$$|\mathrm{FL}*\rangle = [\text{Projection onto rung singlets of } \Psi_1, \Psi_2] \\ \bowtie |\text{Slater determinant of } (c, \Psi_1)\rangle \\ \otimes |\text{Slater determinant of } \Psi_2\rangle$$

Large Fermi surface of size $1 + p$

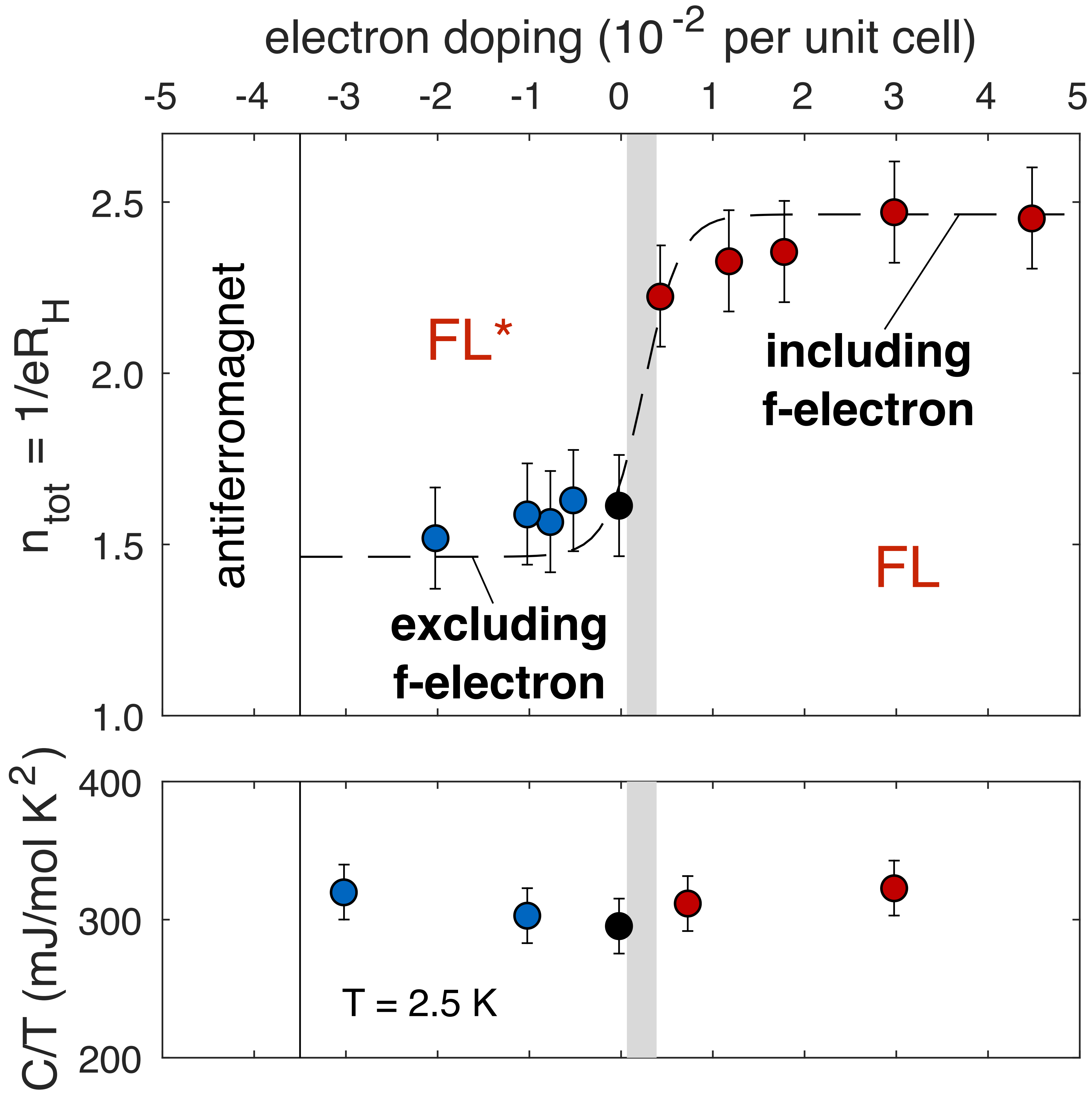
$$|\mathrm{FL}\rangle = |\text{Rung singlets of } \Psi_1, \Psi_2\rangle \\ \otimes |\text{Slater determinant of } c\rangle$$

Cuprates



Evidence for
ghost Fermi surfaces
in the
FL*-FL transition
in a
single-band model ?

$CeCoIn_5$



FL*

- Recent evidence for a FL* phase in a Kondo lattice: $CeCoIn_5$ (Maksimovic *et al.*, Science **375**, 76 (2021), and in CePdAl, Zhao *et al.*, Nature Physics **15**, 1261 (2019). And perhaps YbB_{12} (Liu *et al.* arXiv:2102.09545)?

- Ancilla theory of FL* for the pseudogap metal of the cuprates: Don't fractionalize the mobile electron, but fractionalize the 'paramagnon rotor' into 'ancilla qubits'. Predicts electronic spectra in good agreement with observations in *both* nodal and anti-nodal regions.

- Theory of FL-FL* transition on a single band Hubbard model: $(SU(2)\times U(1))/\mathbb{Z}_2$ gauge theory coupled to hybridization boson, a gauge-neutral *large* Fermi surface of electrons, and a 'ghost' Fermi surface.
  Prediction: critical 'ghost' Fermi surfaces near the transition.