Transport and chaos from SYK models

Conference on Many-Body Quantum Chaos
Aspen Center for Physics
March 13, 2019

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

Remarkable recent observation of 'Planckian' strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity, ρ, is

$$\rho = \frac{m^*}{ne^2} \frac{1}{\tau}$$

with a universal scattering rate

$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar}$$

Remarkable recent observation of 'Planckian' strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity is associated with a universal scattering time $\approx \hbar/(k_B T)$.

Universal *T*-linear resistivity and Planckian dissipation in overdoped cuprates

NATURE PHYSICS | VOL 15 | FEBRUARY 2019 | 142–147

A. Legros[1,2], S. Benhabib[3], W. Tabis[3,4], F. Laliberté[1], M. Dion[1], M. Lizaire[1], B. Vignolle[3], D. Vignolles[3], H. Raffy[5], Z. Z. Li[5], P. Auban-Senzier[5], N. Doiron-Leyraud[1], P. Fournier[1,6], D. Colson[2], L. Taillefer[1,6*] and C. Proust[3,6*]

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arXiv:1902.01034

Planckian dissipation and scale invariance in a quantum-critical disordered pnictide

Yasuyuki Nakajima,[1,2] Tristin Metz,[2] Christopher Eckberg,[2] Kevin Kirshenbaum,[2] Alex Hughes,[2] Renxiong Wang,[2] Limin Wang,[2] Shanta R. Saha,[2] I-Lin Liu,[2,3,4] Nicholas P. Butch,[2,4] Zhonghao Liu,[5,6] Sergey V. Borisenko,[5] Peter Y. Zavalij,[7] and Johnpierre Paglione[2,8]

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Strange metal in magic-angle graphene with near Planckian dissipation

Yuan Cao,[1,*] Debanjan Chowdhury,[1,*] Daniel Rodan-Legrain,[1] Oriol Rubies-Bigordà,[1] Kenji Watanabe,[2] Takashi Taniguchi,[2] T. Senthil,[1,†] and Pablo Jarillo-Herrero[1,†]

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Bad metallic transport in a cold atom Fermi-Hubbard system

Science **363**, 379–382 (2019)

Peter T. Brown[1], Debayan Mitra[1], Elmer Guardado-Sanchez[1], Reza Nourafkan[2], Alexis Reymbaut[2], Charles-David Hébert[2], Simon Bergeron[2], A.-M. S. Tremblay[2,3], Jure Kokalj[4,5], David A. Huse[1], Peter Schauß[1*], Waseem S. Bakr[1]†

1. The complex SYK model

2. A toy model for Planckian transport

   SYK model in momentum space

3. Coupled SYK Islands

   Transport and many-body chaos

4. Strong electron-phonon and electron-electron interactions

   SYK model with phonons

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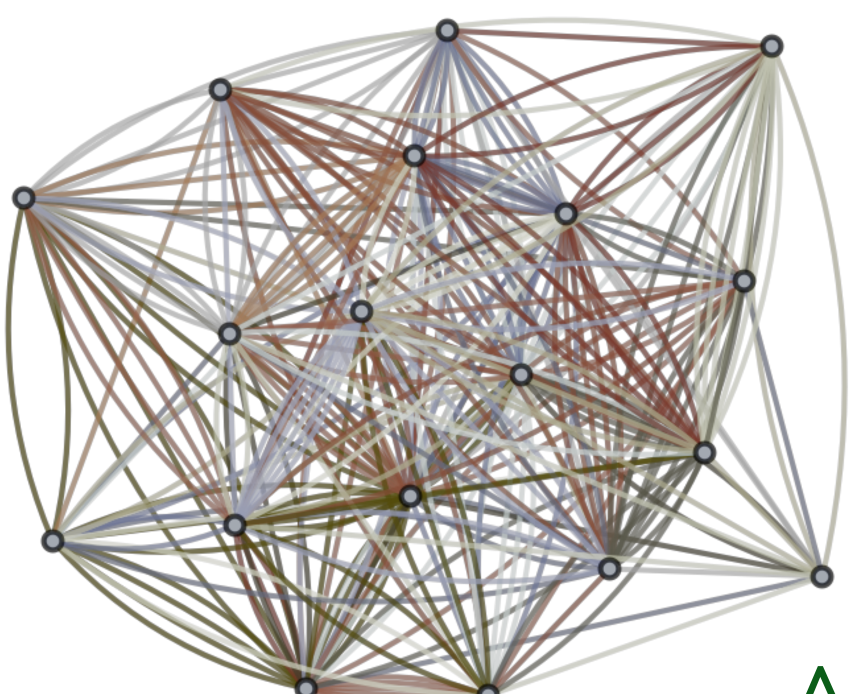
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

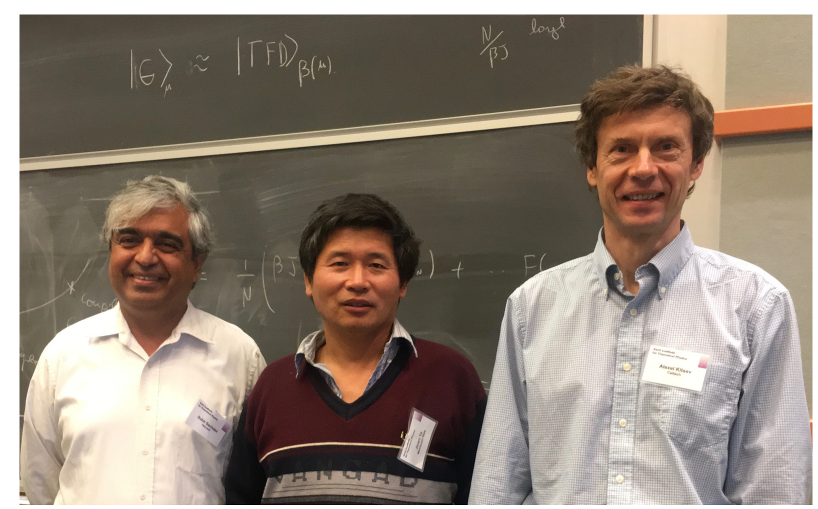
$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
$N \to \infty$ yields critical strange metal.





S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

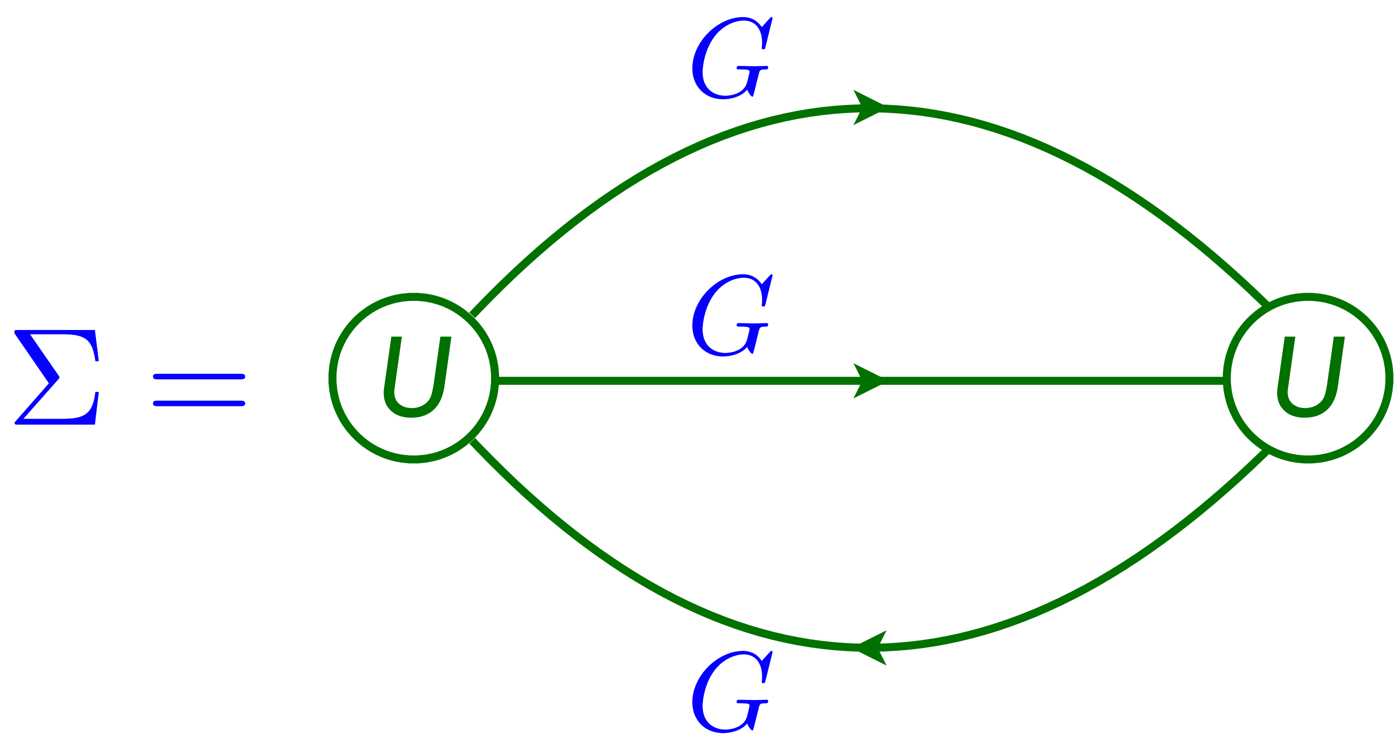
A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The complex SYK model

Feynman graph expansion in $U_{ijk\ell}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$



S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

The complex SYK model

Solution of these equations for the model with a q fermion Hamiltonian yields the following universal results (*i.e.* all results are *quantitatively* unchanged by adding additional higher order fermion terms):

- At long times, and at $T = 0$, $G(\tau) \sim |\tau|^{-2\Delta}$ with $\Delta = 1/q$ ($\Rightarrow$ indication there are no quasiparticles)

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- At general charge $\mathcal{Q}$, there is a spectral symmetry determined by a parameter $\mathcal{E}$:

$$G(\tau) \sim \begin{cases} -\tau^{-2\Delta} & \tau > 0 \\ e^{-2\pi\mathcal{E}}(-\tau)^{-2\Delta} & \tau < 0 \end{cases} \quad , \quad T = 0$$

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- There is a universal 'Luttinger relation' between $-\infty < \mathcal{E} < \infty$ and the total charge $0 < \mathcal{Q} < 1$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$$

$$\mathcal{Q} = \frac{1}{2} - \frac{\theta}{\pi} + \left(\Delta - \frac{1}{2}\right)\frac{\sin(2\theta)}{\sin(2\pi\Delta)}$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, PRB **95**, 155131 (2017)

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- At $T > 0$, we obtain a solution with a conformal structure

$$G(\tau) = -A \frac{e^{-2\pi \mathcal{E} T \tau}}{\sqrt{1 + e^{-4\pi \mathcal{E}}}} \left(\frac{T}{\sin(\pi T \tau)} \right)^{1/2} \quad , \quad 0 < \tau < 1/T \,,$$

where the 'particle-hole asymmetry' is determined by $\mathcal{E}$, and $A = (\pi/(U^2 \cos(2\theta)))^{1/4}$.

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
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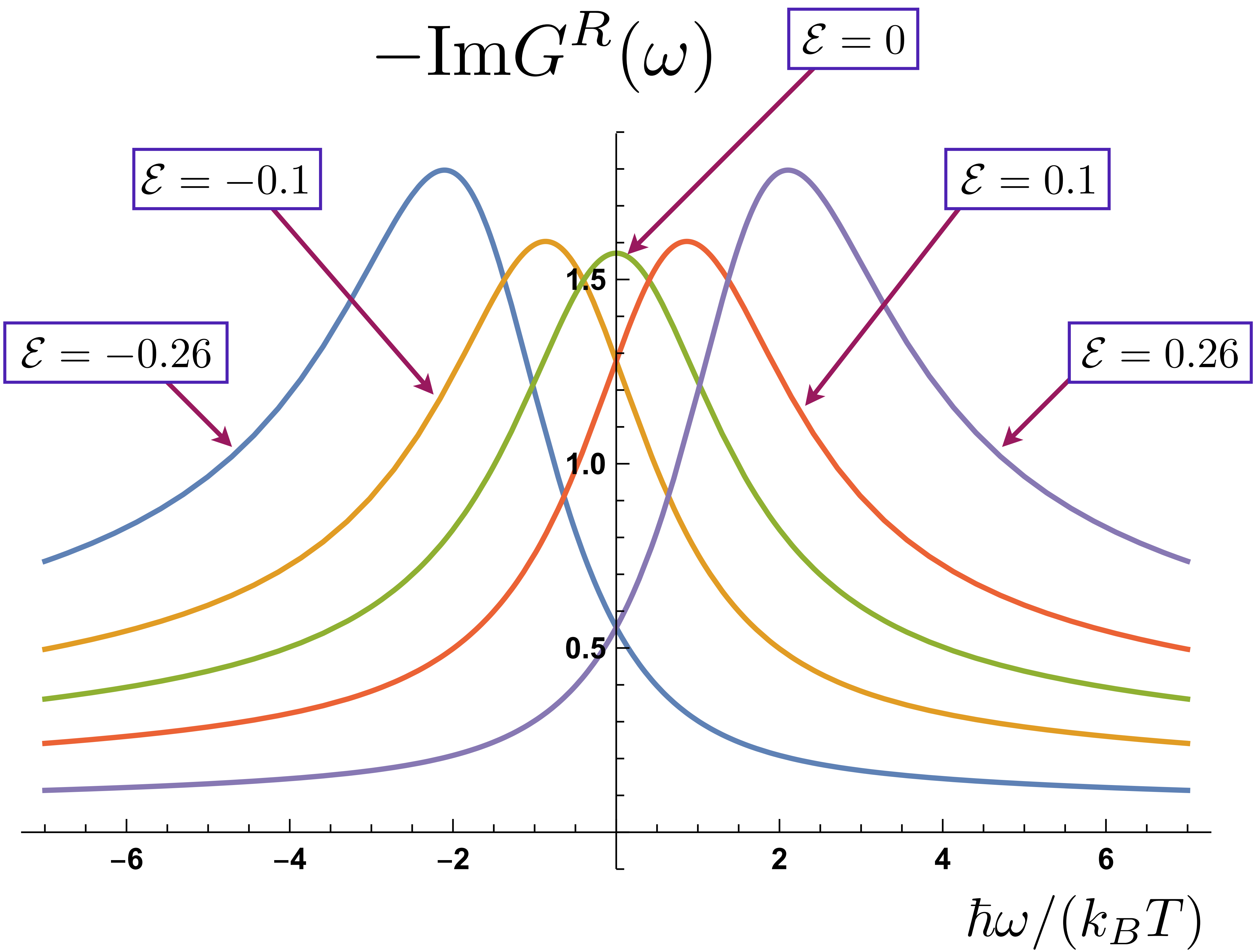
The complex SYK model



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- The dimensionless parameter $\mathcal{E}$ depends upon the ratio of the two dimensionful parameters, μ and U, in a non-universal manner: we have $\mathcal{E} = f(\mu/U)$ for some function f. For small μ/U, $\mathcal{E} = -0.41\mu/U$.

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- Note the non-universal U appears both in the pre-factor A, and in the μ dependence of $\mathcal{E}$.

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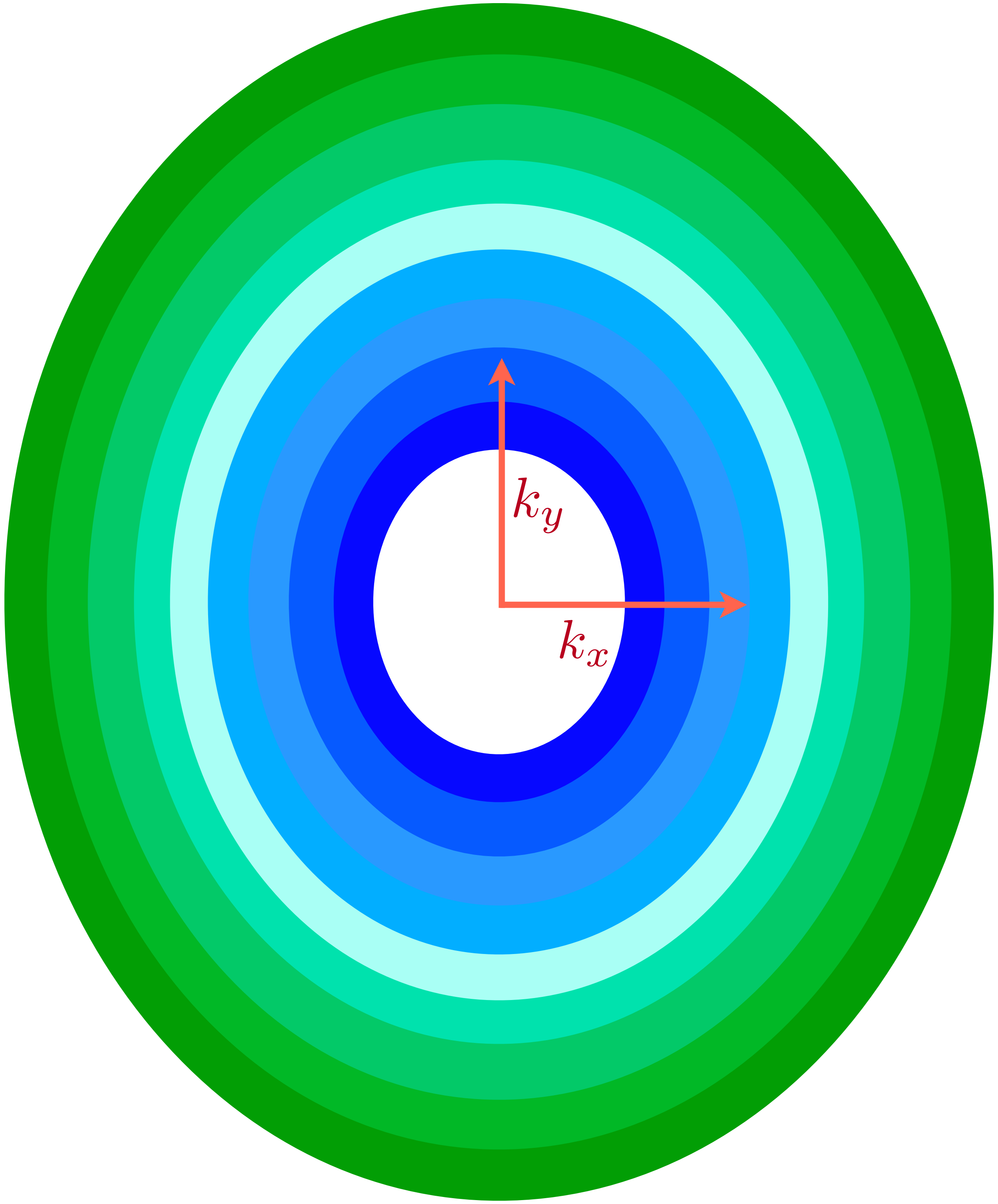


Aavishkar Patel

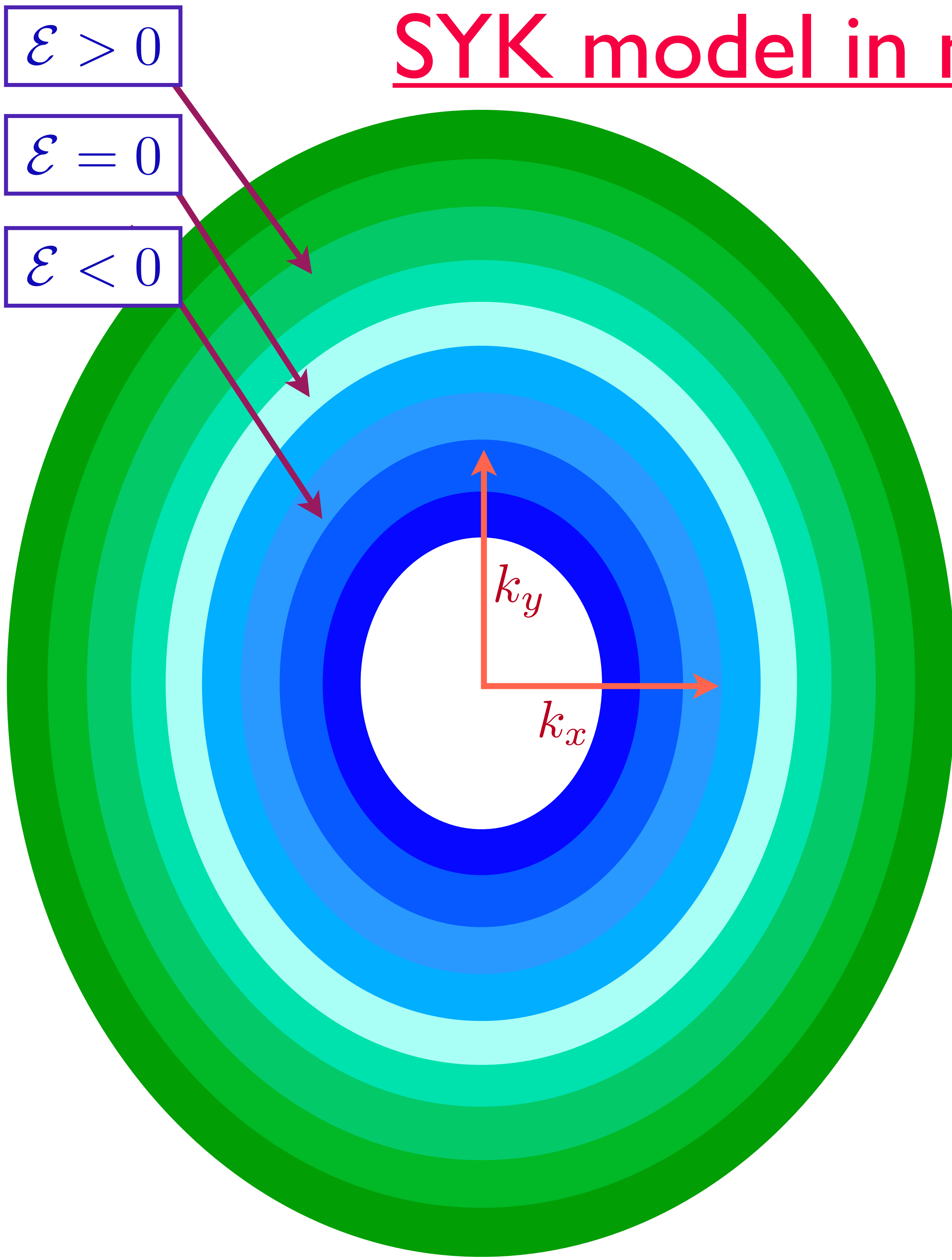
Random SYK interactions between each shell in momentum space with dispersion $\varepsilon_{\boldsymbol{k}}$ imply a Green's function

$$G(\omega, \boldsymbol{k}) = G_{\text{SYK}}(\omega, \mu = -\varepsilon_{\boldsymbol{k}})$$

We define the Fermi velocity $\boldsymbol{v}_F = \partial \varepsilon_{\boldsymbol{k}} / \partial \boldsymbol{k}$ on the Fermi surface where $\varepsilon_{\boldsymbol{k}} = 0$.

We assume $T \ll U \ll E_F$

SYK model in momentum space



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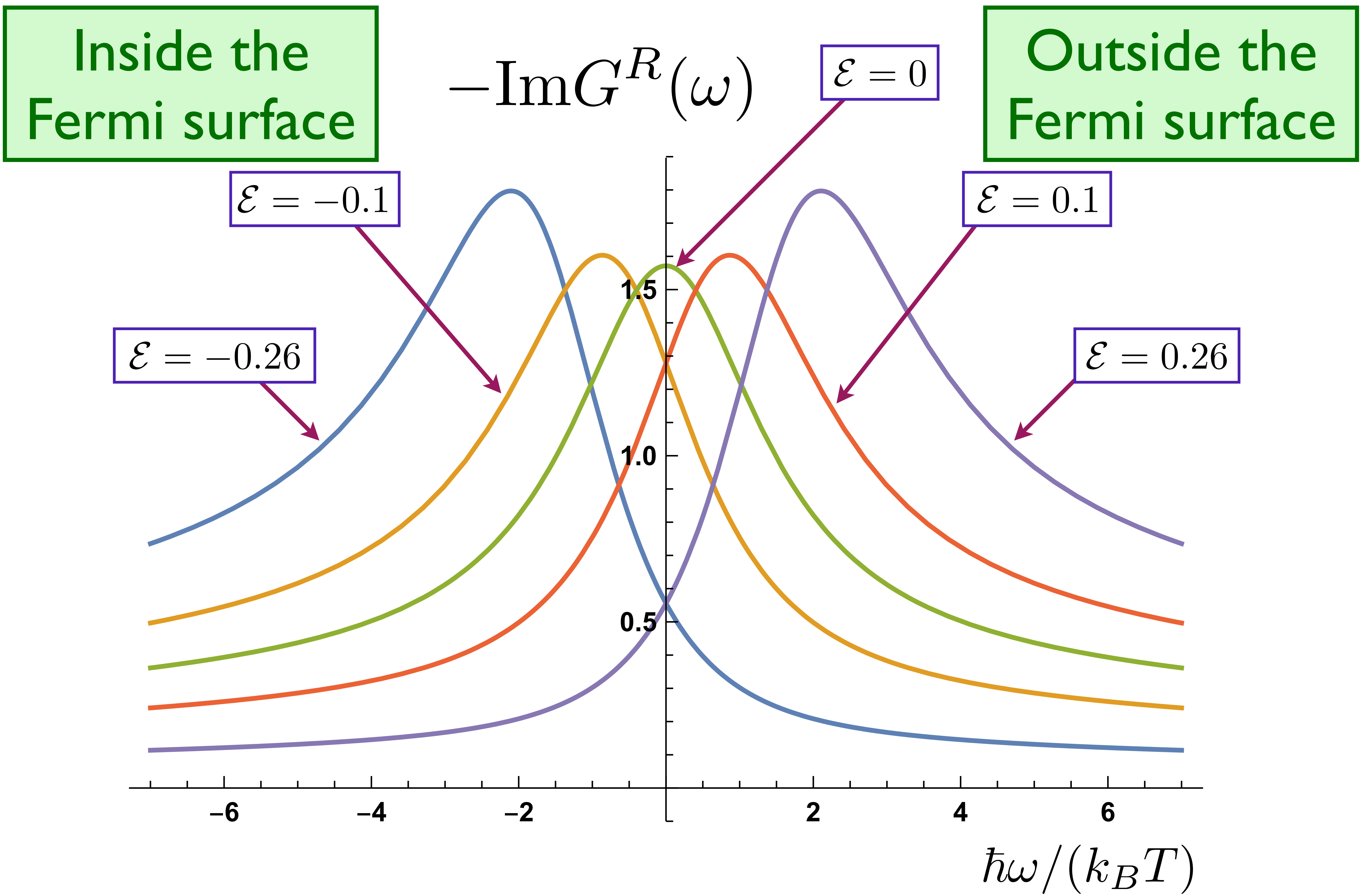
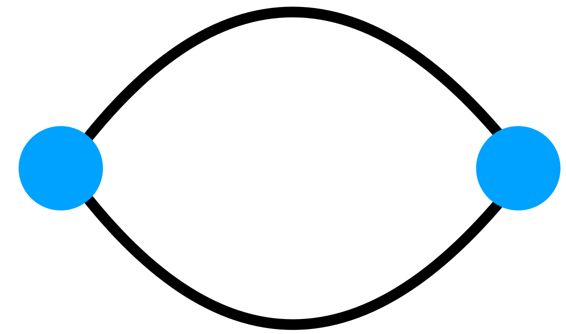
Each shell has fixed value of $\mathcal{E}$
which changes sign across the "Fermi surface"

The complex SYK model



SYK model in momentum space

$$\sigma = $$ 

Aavishkar Patel 

Computation of the resistivity from the Green's function yields the resistivity

$$\rho = \frac{m^*}{ne^2}\left(1.11\frac{k_B T}{\hbar}\right)$$

where the effective mass is

$$m^* = \frac{d\,V_{FS}}{\oint_{FS} |\boldsymbol{v}_F|}$$

where d is spatial dimensionality and V_{FS} is the volume enclosed by the Fermi surface. For a circular Fermi surface, this is the usual m^*.

The U dependencies in the prefactor of G, and from the variation in $\mathcal{E}$ with μ cancel with each other!

This computation ignores the contribution to the current from the momentum-dependent interaction, which is only permissible for the conductivity at a small non-zero momentum.

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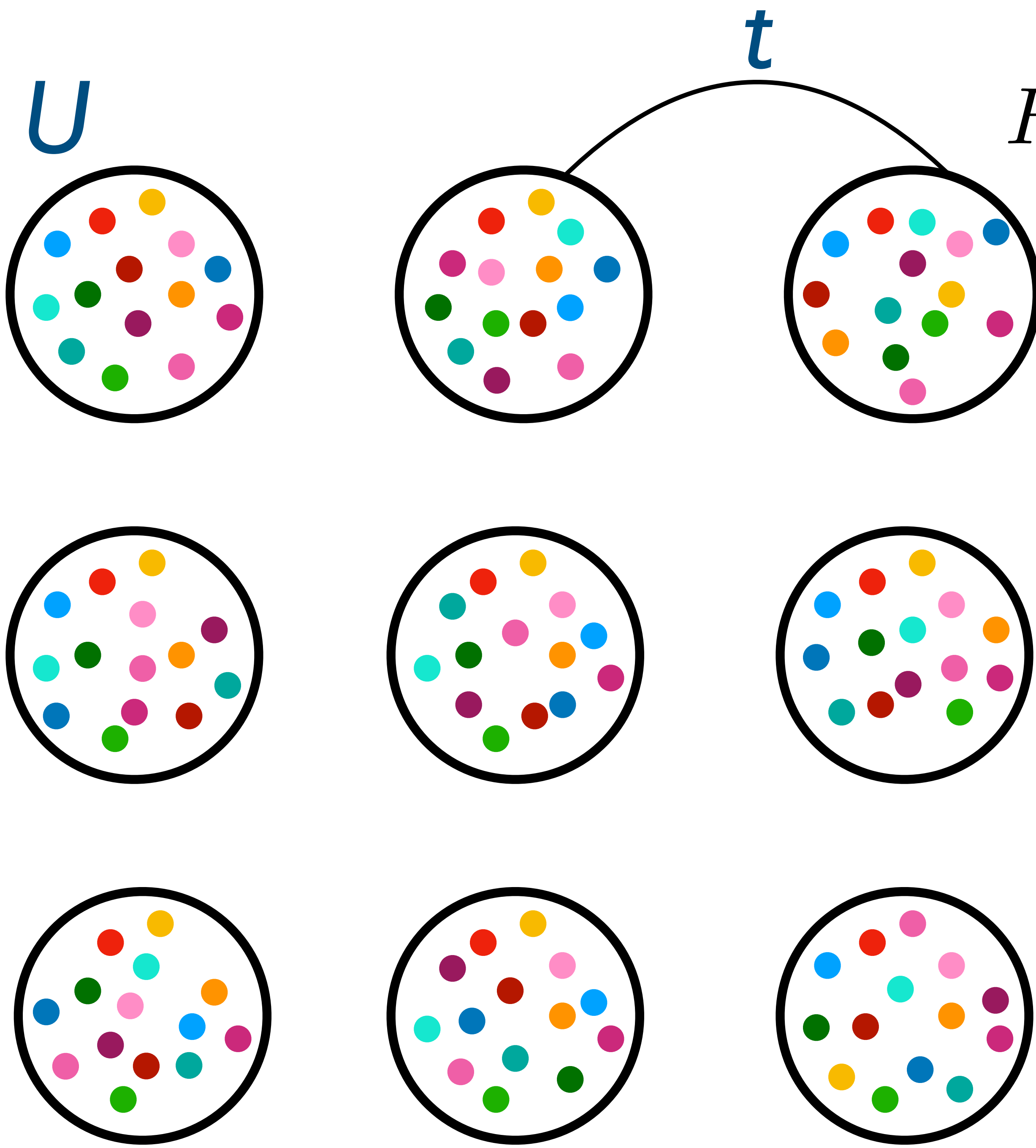
Coupled SYK Islands

Xue-Yang Song

SYK quantum islands of electrons with random or regular hopping between them.

Aavishkar Patel



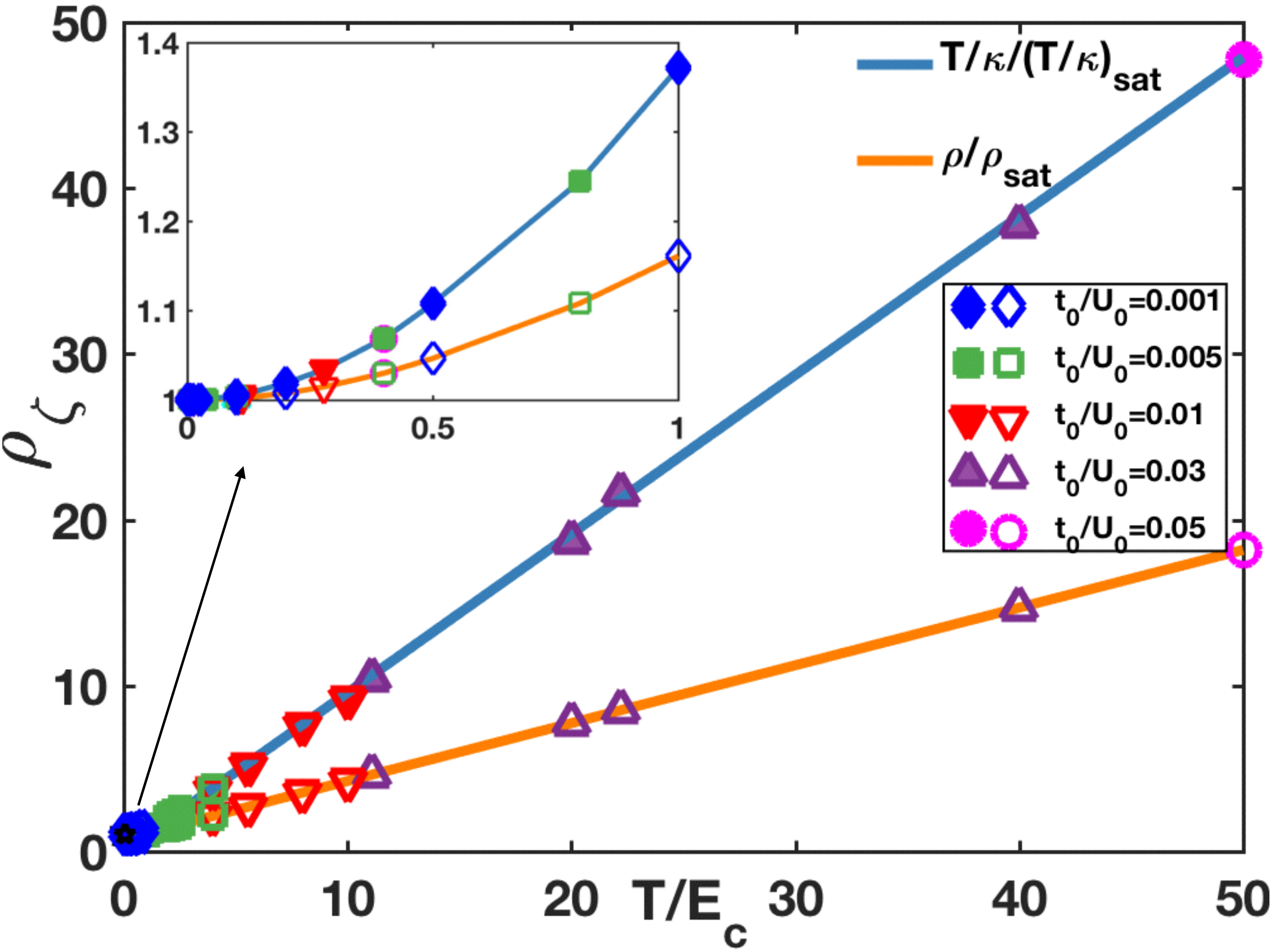
$$H = \sum_{x} \sum_{i<j,k<l} U_{ijkl,x} c^{\dagger}_{ix} c^{\dagger}_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^{\dagger}_{i,x} c_{j,x'}$$

$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3} \qquad \overline{|t_{ij,x,x'}|^2} = t_0^2/N$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)
Pengfei Zhang, PRB **96**, 205138 (2017)
Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 021049 (2018)
Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)
See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

Coupled SYK Islands

Low 'coherence' scale



$$E_c \sim \frac{t_0^2}{U}$$

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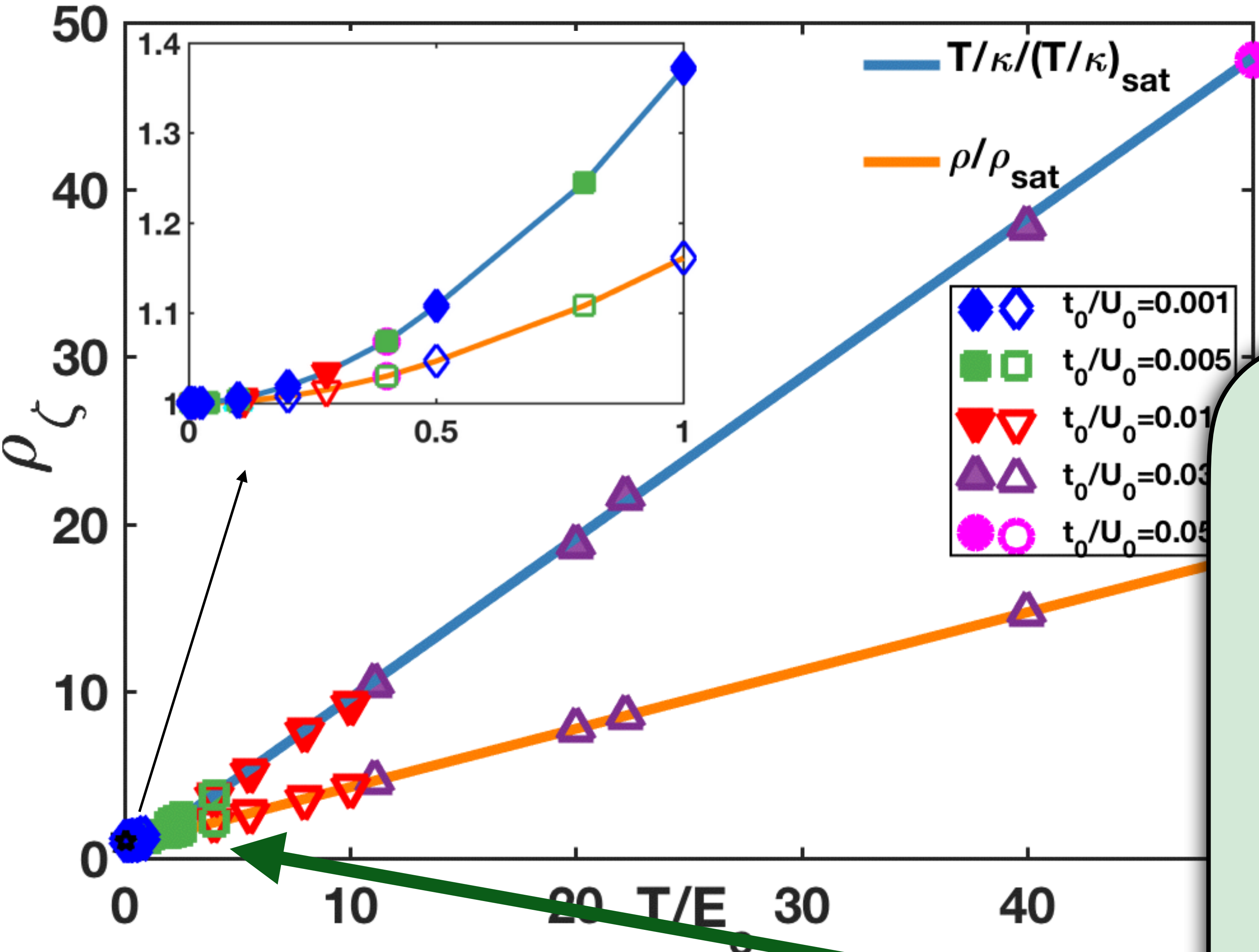

Low 'coherence' scale

$$E_c \sim \frac{t_0^2}{U}$$

For $T < E_c$, the resistivity, ρ, and entropy density, s, are

$$\rho = \frac{h}{e^2}\left[c_1 + c_2\left(\frac{T}{E_c}\right)^2\right]$$

$$s \sim s_0\left(\frac{T}{E_c}\right)$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

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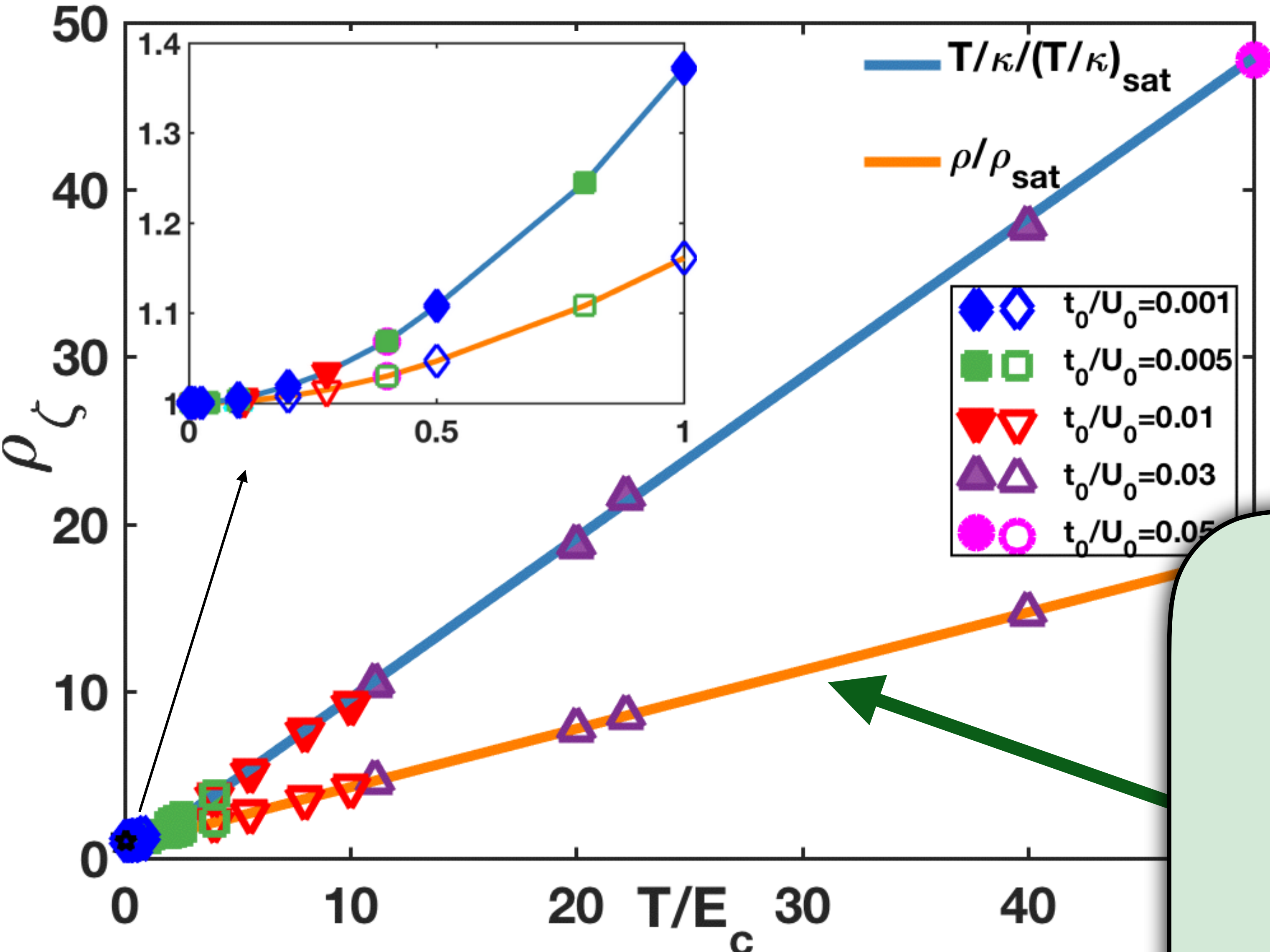

For $E_c < T < U$, the resistivity, ρ, and entropy density, s, are

$$\rho \sim \frac{h}{e^2}\left(\frac{T}{E_c}\right) \quad , \quad s = s_0$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

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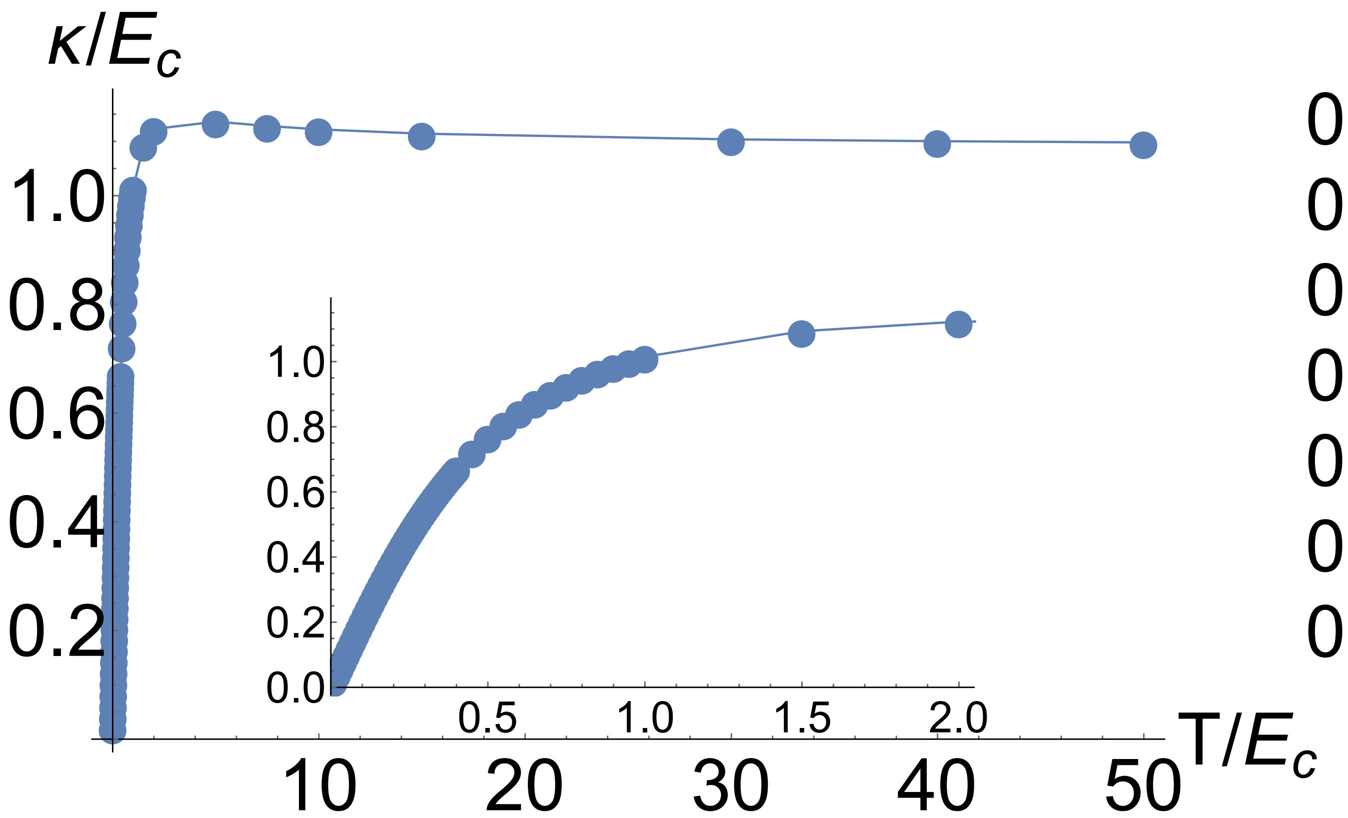
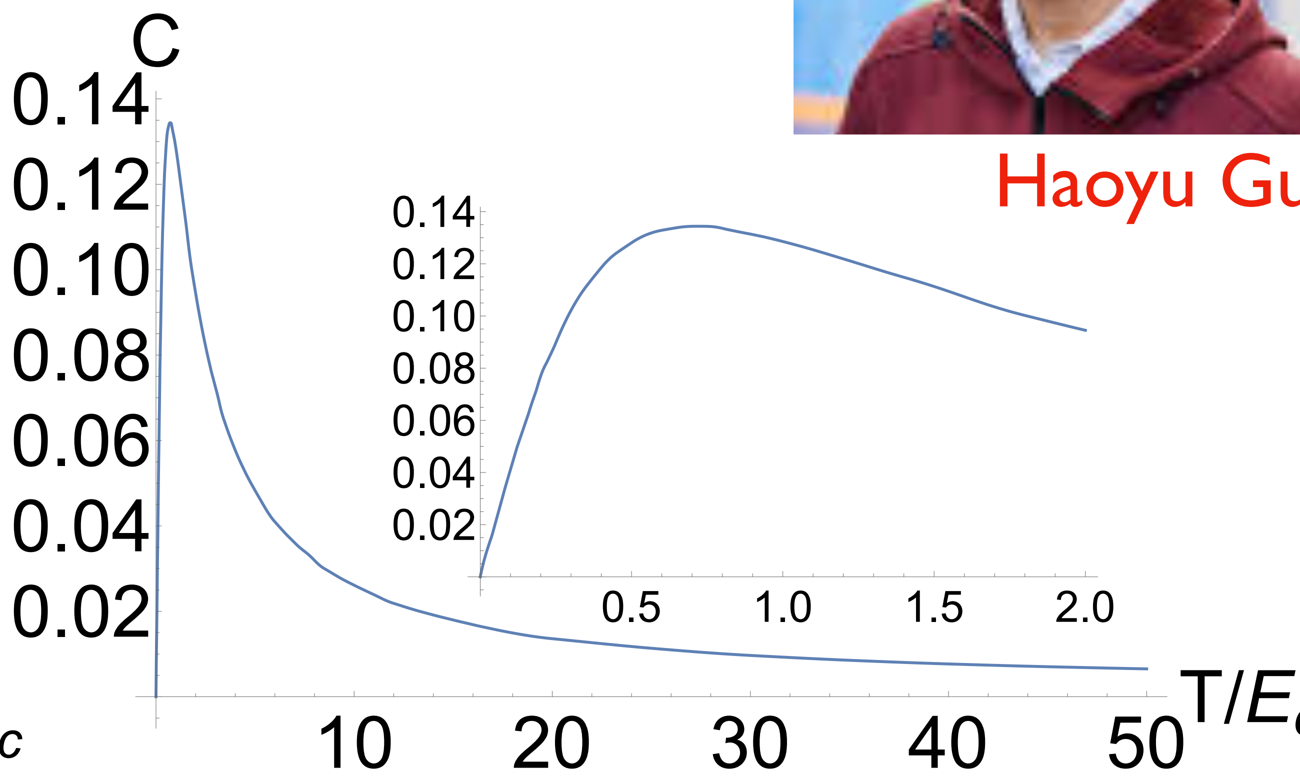
Coupled SYK Islands

Haoyu Guo

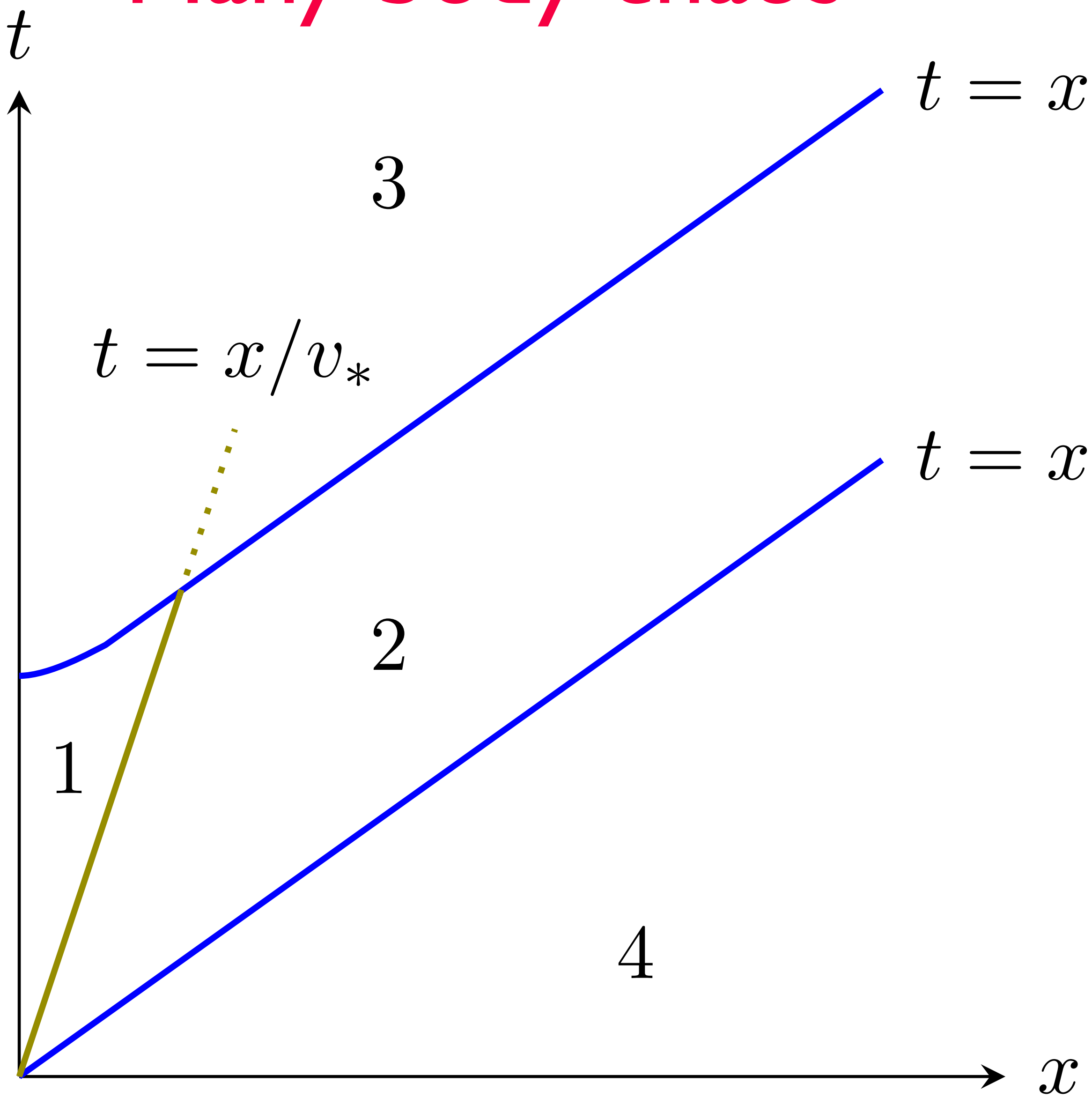




The thermal conductivity κ roughly scales $\sim T\sigma$, and the specific heat $C \sim T$ at $T \ll E_c$.

Many-body chaos

Yingfei Gu and A. Kitaev, arXiv:1812.00120



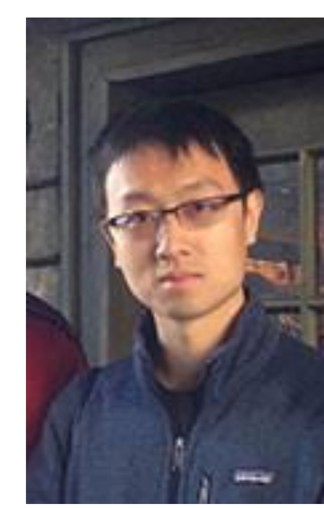
$$\mathrm{OTOC} \sim \frac{1}{N} \frac{1}{\cos(\lambda_L(q)/(4T))} \frac{1}{[\omega - i\lambda_L(q)]}$$

This has a pole at imaginary q at $q_1 = i|q_1|$ where $\lambda_L(q_1) = 2\pi T$.

We can define two characteristic velocities

$$\begin{aligned} v_* &= |\lambda_L'(q_1)| \\ v_B &= 2\pi T/|q_1|\,, \end{aligned}$$

and the scrambling time $t_{\mathrm{scr}} \approx (\ln N)/\lambda_L$



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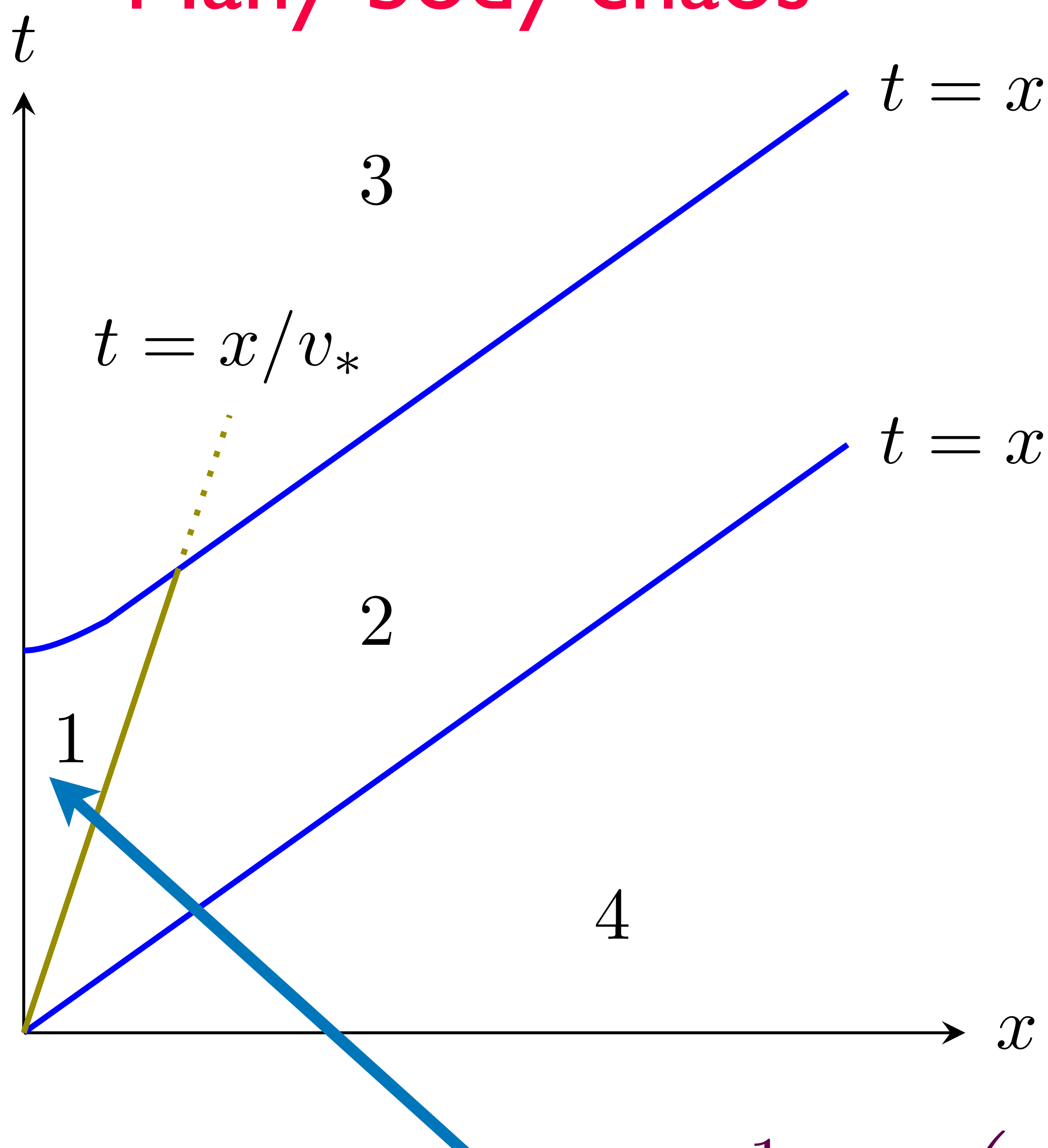

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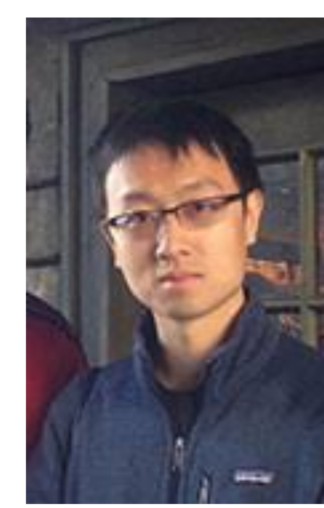
$$\text{OTOC} \sim \frac{1}{N} \exp\left(\lambda_L(0)t - \frac{x^2}{D_* t}\right) \quad \text{in region 1}$$

where the chaos diffusion co-efficient D_* is defined by

$$\lambda_L(q) = \lambda_L(0) - \frac{D_*}{4} q^2 + \ldots$$



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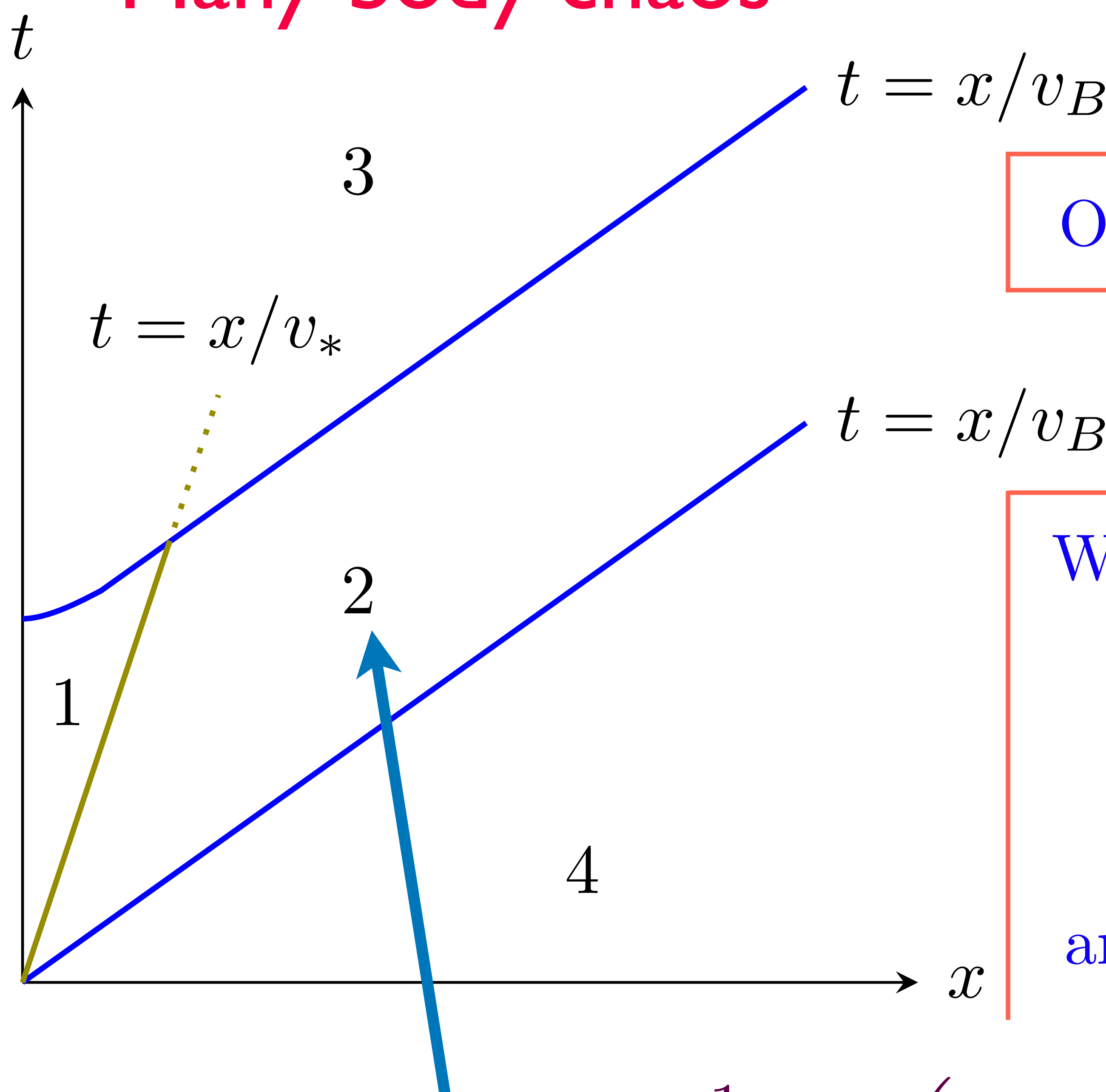


$$\text{OTOC} \sim \frac{1}{N}\frac{1}{\cos(\lambda_L(q)/(4T))}\frac{1}{[\omega - i\lambda_L(q)]}$$

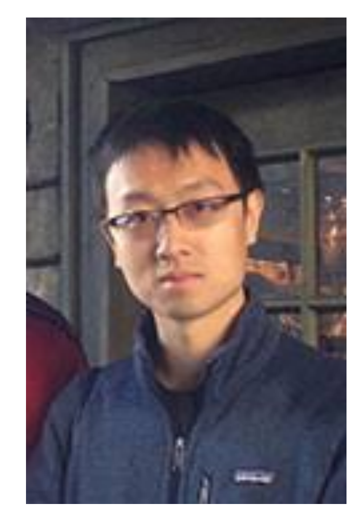
This has a pole at imaginary q at $q_1 = i|q_1|$ where $\lambda_L(q_1) = 2\pi T$.

We can define two characteristic velocities

$$\begin{aligned} v_* &= |\lambda_L'(q_1)| \\ v_B &= 2\pi T/|q_1|\,, \end{aligned}$$

and the scrambling time $t_{\rm scr} \approx (\ln N)/\lambda_L$

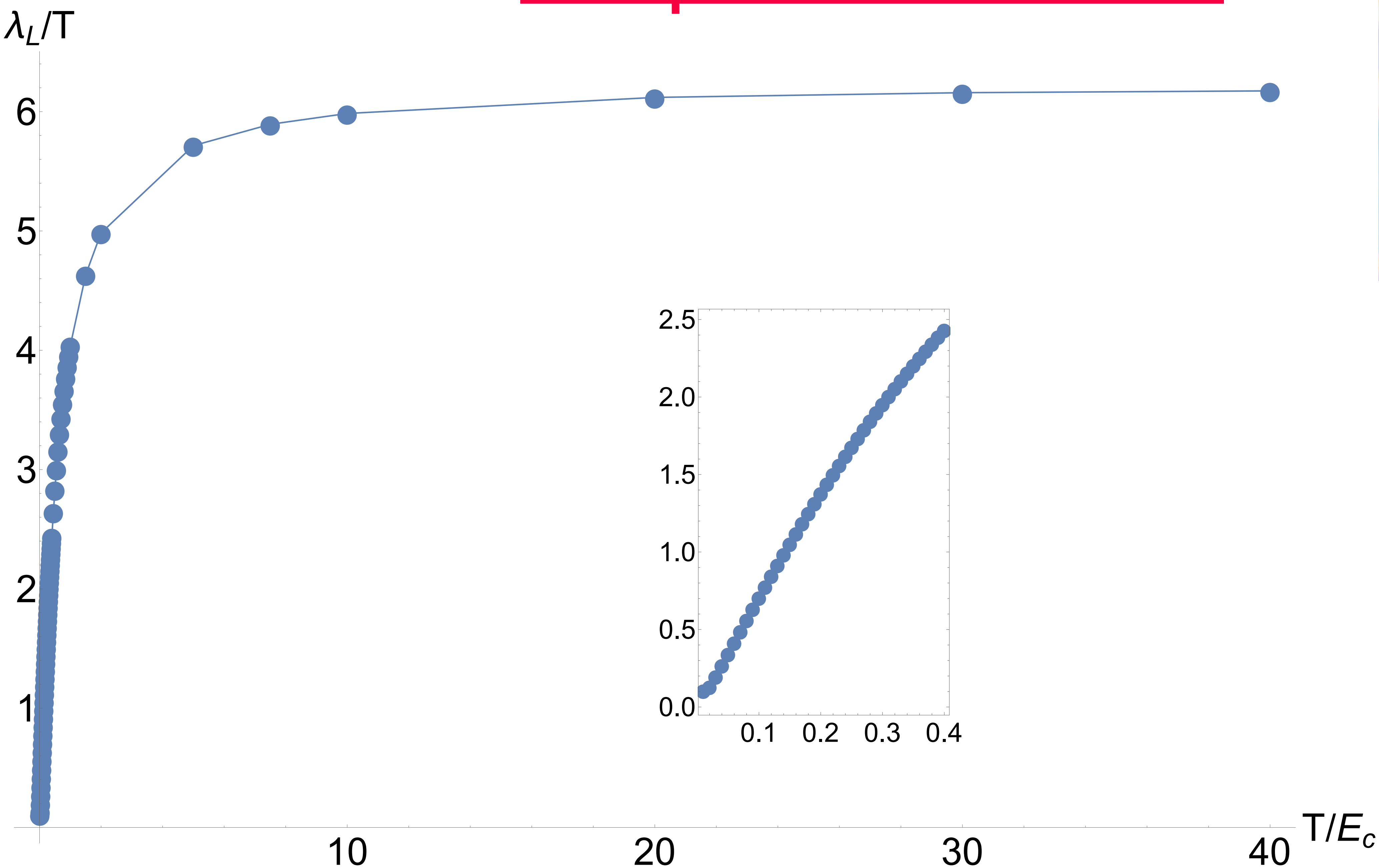
$$\text{OTOC} \sim \frac{1}{N}\exp\left(2\pi T\left[t - \frac{|x|}{v_B}\right]\right) \quad \text{in region 2}$$



Now we define the chaos diffusion co-efficient D_1 by

$$D_1 = \frac{v_B^2}{2\pi T} = \frac{2\pi T}{|q_1|^2}$$

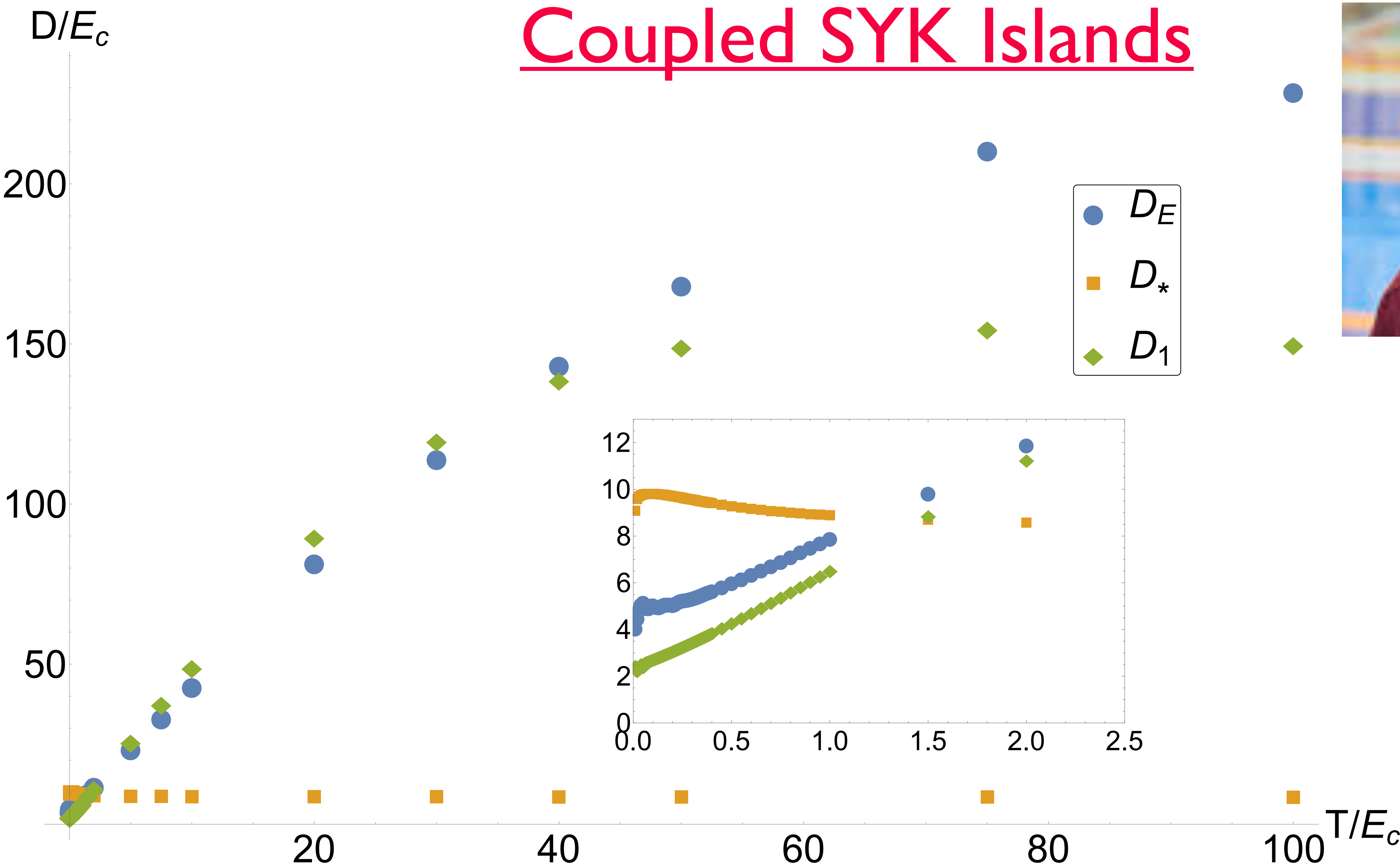
Coupled SYK Islands



Haoyu Guo

The chaos Lyapunov exponent $\lambda_L(0) \sim T^2/E_c$ for $T \ll E_c$, and $\lambda_L(0) \sim 2\pi T$ for $E_c \ll T \ll U$.

Coupled SYK Islands



Haoyu Guo

The energy diffusion constant, $D_E = \kappa/C$, is close to the long-time chaos diffusion constant $D_1 = v_B^2/(2\pi T)$, but not the chaos diffusion constant D_*.

- For a large class of holographic theories with momentum dissipation, we find

$$D_E = \frac{z}{2z-2}\,\frac{v_B^2}{2\pi T}$$

where z is the dynamic critical exponent

M. Blake, R.A. Davison, and S. Sachdev, PRD **96**, 106008 (2017)

- In the large N theory of the Fermi surface coupled to a gauge field, we have

$$D_E = \frac{v_B^2}{5.90T}$$

A.A. Patel and S. Sachdev, PNAS **114**, 1844 (2017)

- Similar relations do *not* apply to the charge diffusion constant.

1. The complex SYK model

2. A toy model for Planckian transport

   SYK model in momentum space

3. Coupled SYK Islands

   Transport and many-body chaos

4. Strong electron-phonon and electron-electron interactions

   SYK model with phonons

$$
\begin{aligned}
H &= H_{\text{hopping}} + H_{\text{SYK}} + H_{\text{ph}} + H_{\text{e-ph}}, \\
H_{\text{hopping}} &= \frac{1}{\sqrt{zN}} \sum_{<xx'>} \sum_{ab} t^{ab}_{xx'} c^{\dagger}_{ax} c_{bx'}, \\
H_{\text{SYK}} &= \frac{1}{(2N)^{3/2}} \sum_{x} U_{abcd,x} c^{\dagger}_{ax} c^{\dagger}_{bx} c_{cx} c_{dx}, \\
H_{\text{ph}} &= \frac{M}{2} \sum_{x} \sum_{ab} \left(|\partial_t X_{abx}|^2 + \omega_0^2 |X_{abx}|^2 \right), \\
H_{\text{e-ph}} &= -\frac{\alpha}{\sqrt{N}} \sum_{x} \sum_{ab} X_{abx} c^{\dagger}_{ax} c_{bx}.
\end{aligned}
$$

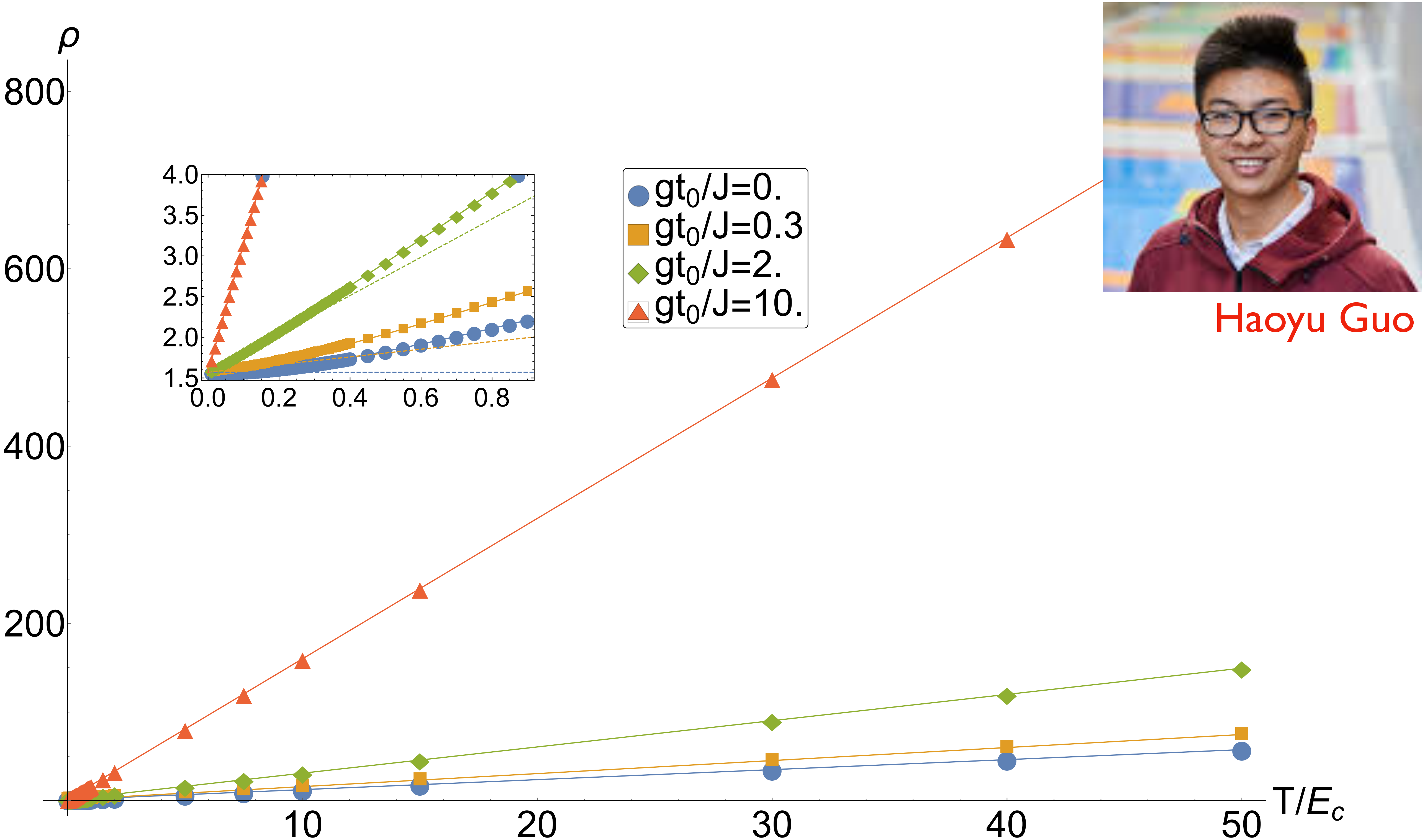
Haoyu Guo

Yochai Werman, Steven A. Kivelson, and Erez Berg, arXiv:1705.07895

Lattice of SYK islands coupled to N^2 local Einstein phonons.

We study the limit $\omega_0 \to 0$ with fixed electron-phonon coupling $g = \alpha^2/(M\omega_0^2 t_0)$. Then all properties are characterized by the energy scale $E_c = t_0^2/U$ and the dimensionless parameter $g t_0/U$

(The following plots use $J \equiv U$)



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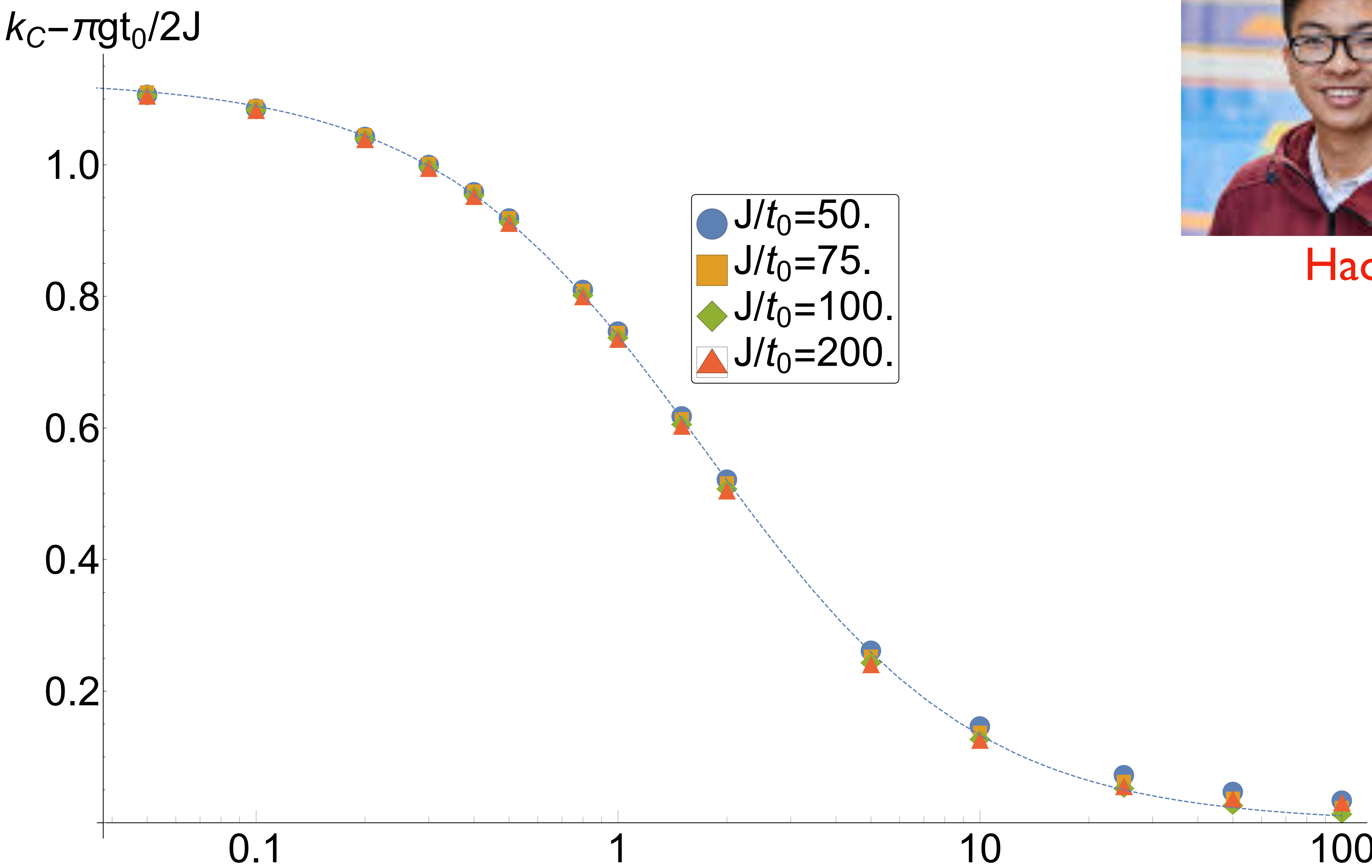
The slope of the linear-in-T resistivity depends upon gt_0/U
(similar results apply to the thermal conductivity)



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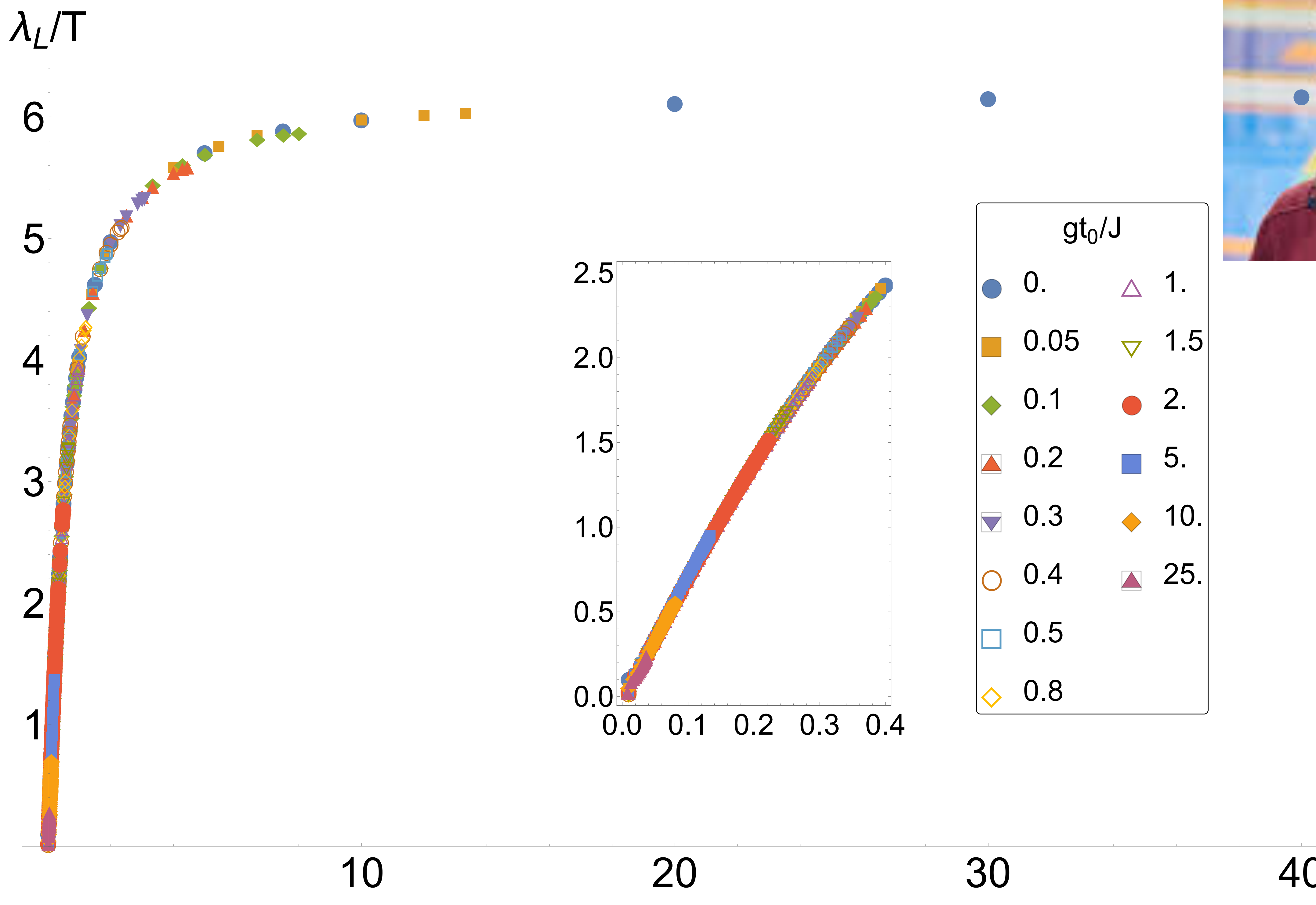
Dependence of the slope $k_C = E_c \dfrac{d\rho}{dT}$ at large T on gt_0/U.



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The electron scrambling rate $\lambda_L(q=0)$:

the ratio λ_L/T depends only on the combination $\dfrac{T/E_c}{1+(T/E_c)(gt_0/U)}$.

Quantum matter without quasiparticles

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Aavishkar Patel

Haoyu Guo