Topological order in the pseudogap metal

High Temperature Superconductivity – Unifying Themes in Diverse Materials
2018 Aspen Winter Conference
Aspen Center for Physics

Subir Sachdev

January 16, 2018

Review: arXiv:1801.01125

Talk online: sachdev.physics.harvard.edu

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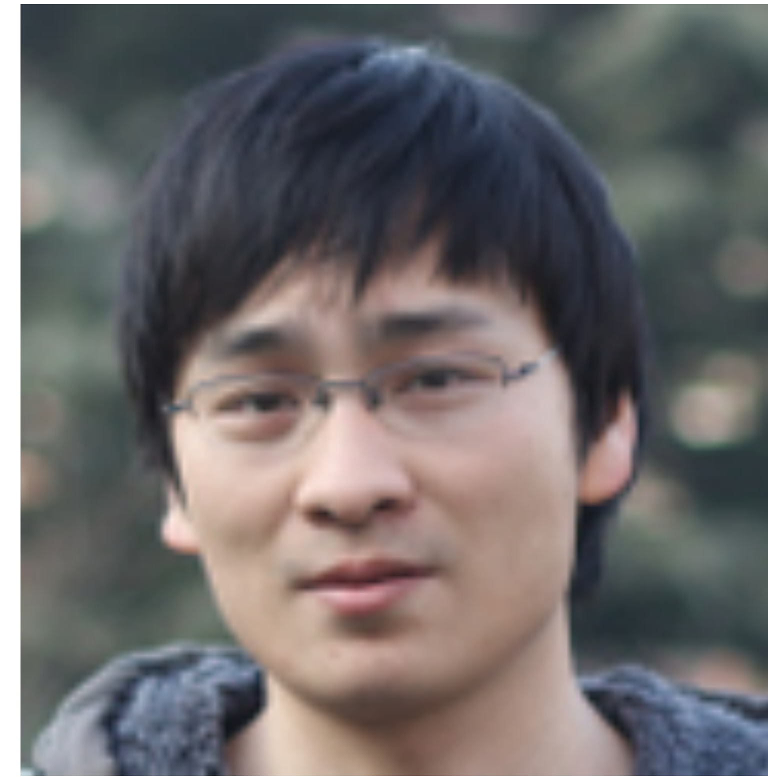


Wei Wu

Michel Ferrero

arXiv:1711.09925

arXiv:1707.06602

Antoine Georges

Topological materials

Descendants of the integer quantum Hall effect

Protected gapless edge states, while bulk excitations are "trivial"

Descendants of the fractional quantum Hall effect

Bulk topological excitations which cannot be created from the ground state by the action of a local operator.

Topological materials

Descendants of the integer quantum Hall effect

Protected gapless edge states, while bulk excitations are "trivial"

Descendants of the fractional quantum Hall effect

Bulk topological excitations which cannot be created from the ground state by the action of a local operator.

Can also appear in gapless metallic states.

Classical XY model

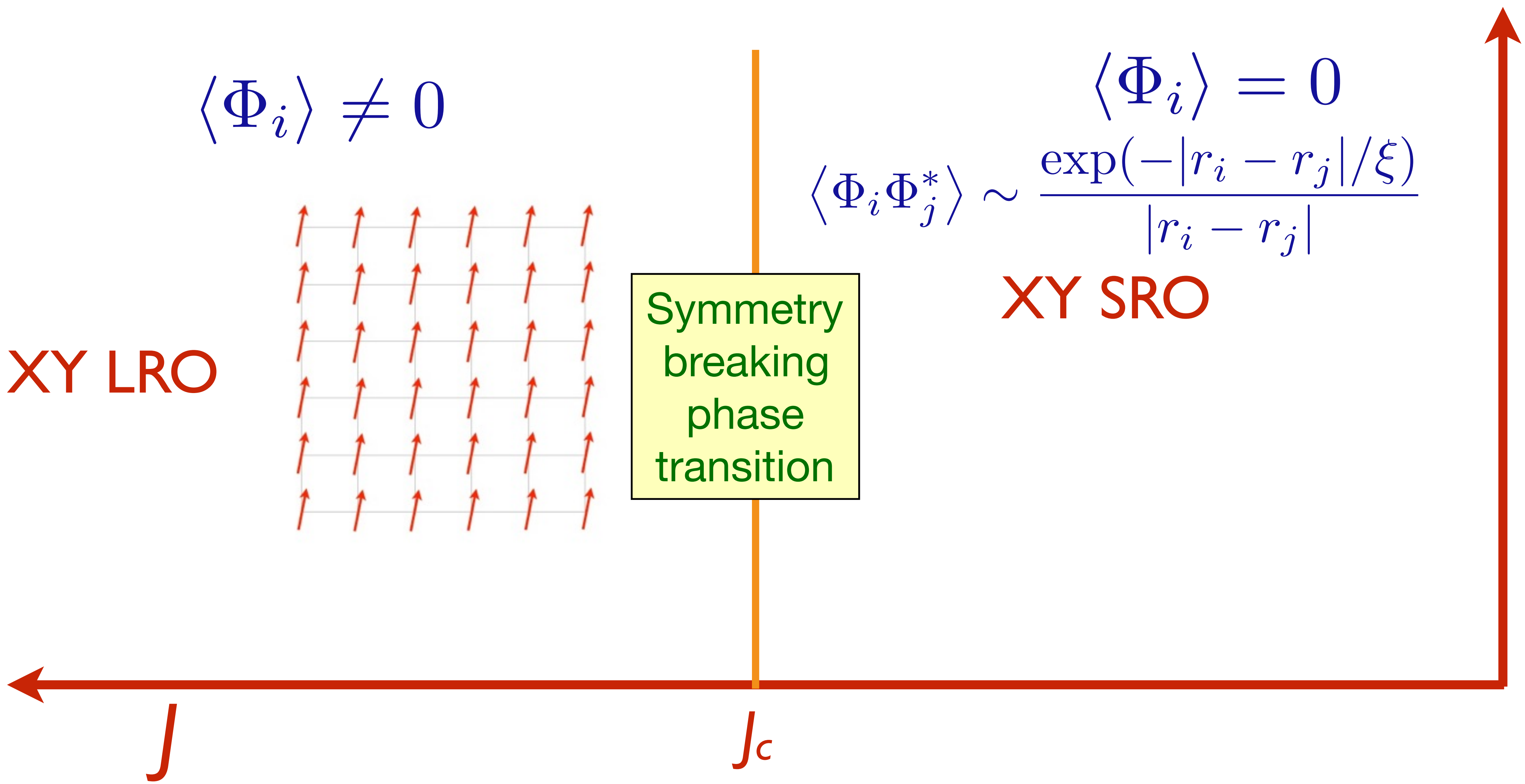
$$\mathcal{Z}_{XY} = \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-H/T\right)$$

$$H = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

$$\Phi_i \equiv e^{i\theta_i}$$

Describes non-zero *T* phase transitions of superfluids, magnets with `easy-plane' spins, ...

Classical XY model in D=3



Classical XY model in D=2

Ordering, metastability and phase transitions in two-dimensional systems

J M Kosterlitz and D J Thouless

J. Phys. C 1973

A new definition of order called topological order is proposed for two-dimensional systems in which no long-range order of the conventional type exists.

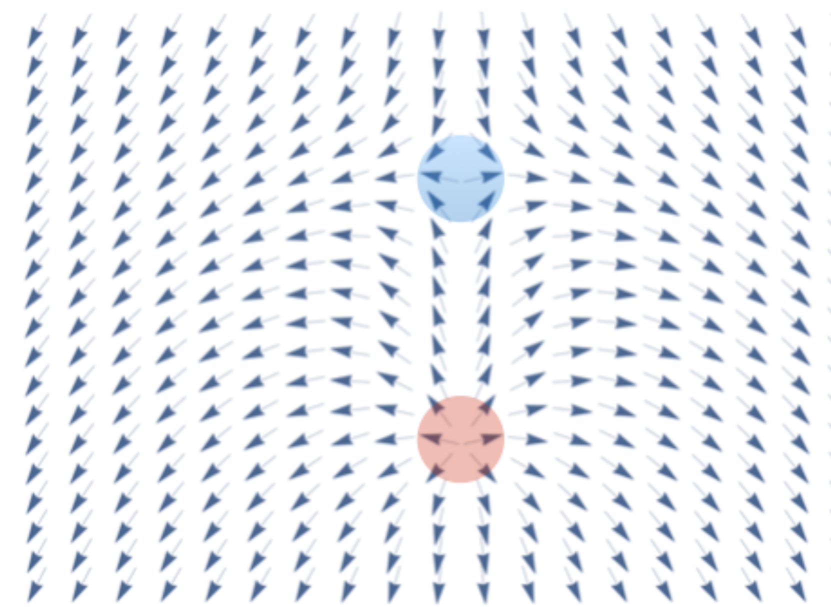
$$\langle \Phi_i \Phi_j^* \rangle \sim \frac{1}{|r_i - r_j|^\alpha}$$

$$\langle \Phi_i \Phi_j^* \rangle \sim \frac{\exp(-|r_i - r_j|/\xi)}{|r_i - r_j|^{1/2}}$$

XY QLRO
Topological order

Topological phase transition: Kosterlitz Thouless

XY SRO
No topological order



Vortices expelled

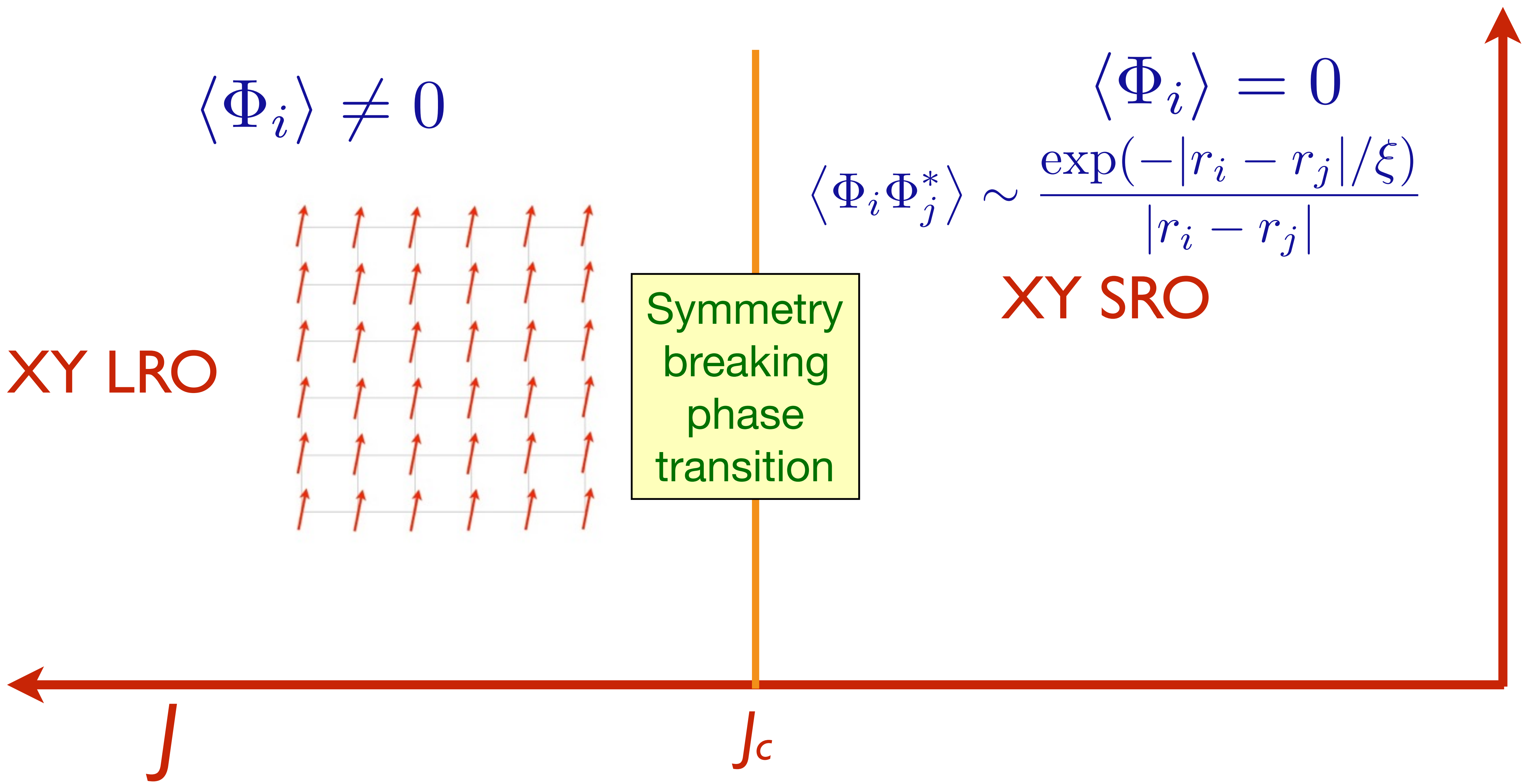
Vortices proliferate

T_{KT}

T

Classical XY model in *D*=3

Can we have a topological phase transition in *D*=3?



$$\widetilde{\mathcal{Z}}_{XY} = \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}/T\right)$$

$$\widetilde{H} = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

$$+ \sum_{ijk\ell} K_{ijk\ell} \cos(\theta_i + \theta_j - \theta_k - \theta_\ell) + \ldots\ldots$$

Add terms which suppress $\pm 2\pi$ but not $\pm 4\pi$ vortices.

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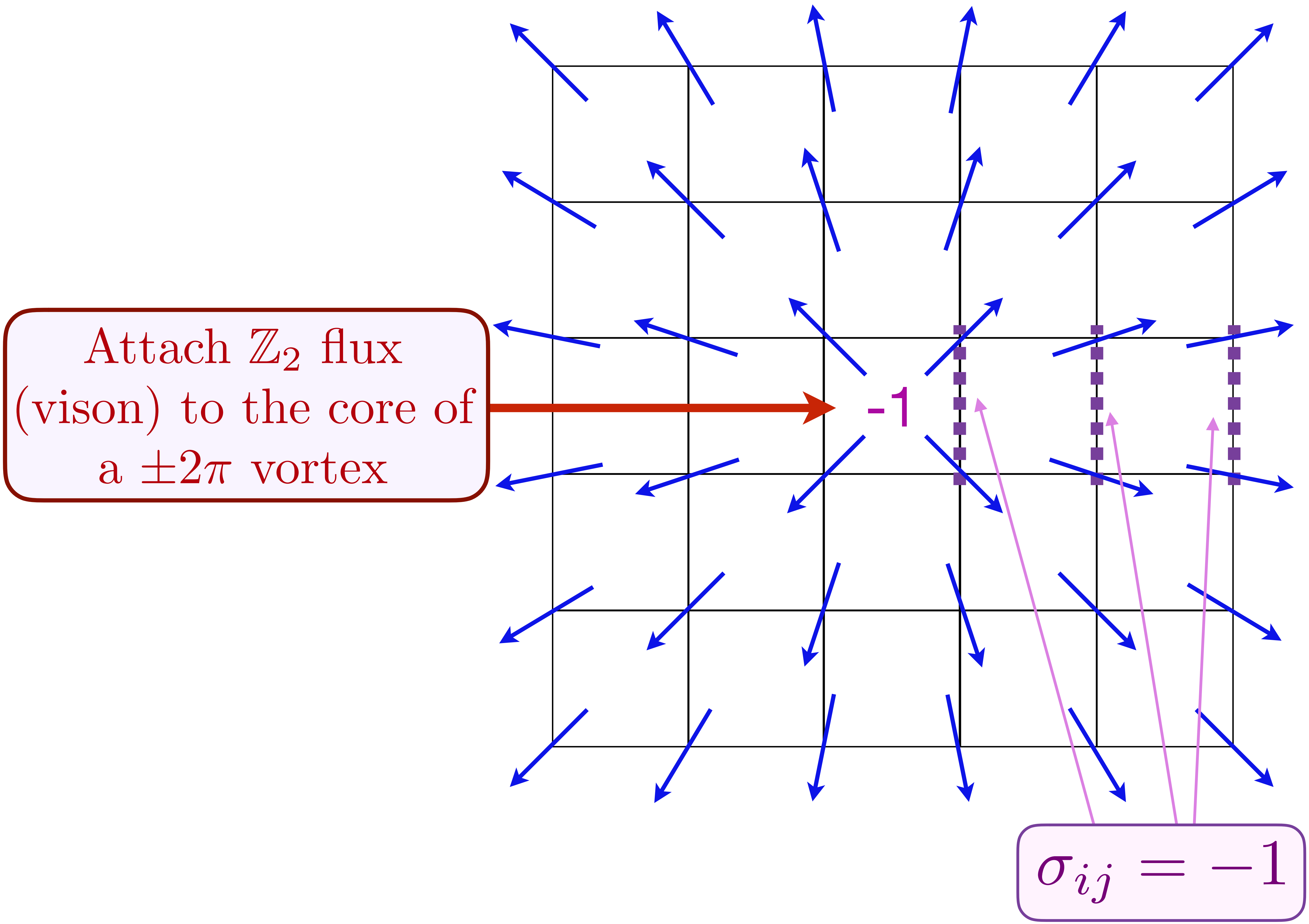
Add terms which suppress $\pm 2\pi$ but not $\pm 4\pi$ vortices.
A convenient form is obtained using an auxiliary variable
$\sigma_{ij} = \pm 1$ on the links of the cubic lattice.

$$\widetilde{\mathcal{Z}}_{XY} = \sum_{\{\sigma_{ij}\}=\pm 1} \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}/T\right)$$

$$\widetilde{H} = -J \sum_{\langle ij \rangle} \sigma_{ij} \cos\left[(\theta_i - \theta_j)/2\right] - K \sum_{\Box} \prod_{(ij)\in\Box} \sigma_{ij}$$



Attach $\mathbb{Z}_2$ flux (vison) to the core of a $\pm 2\pi$ vortex
-1
$\sigma_{ij} = -1$

Classical XY model in *D*=3

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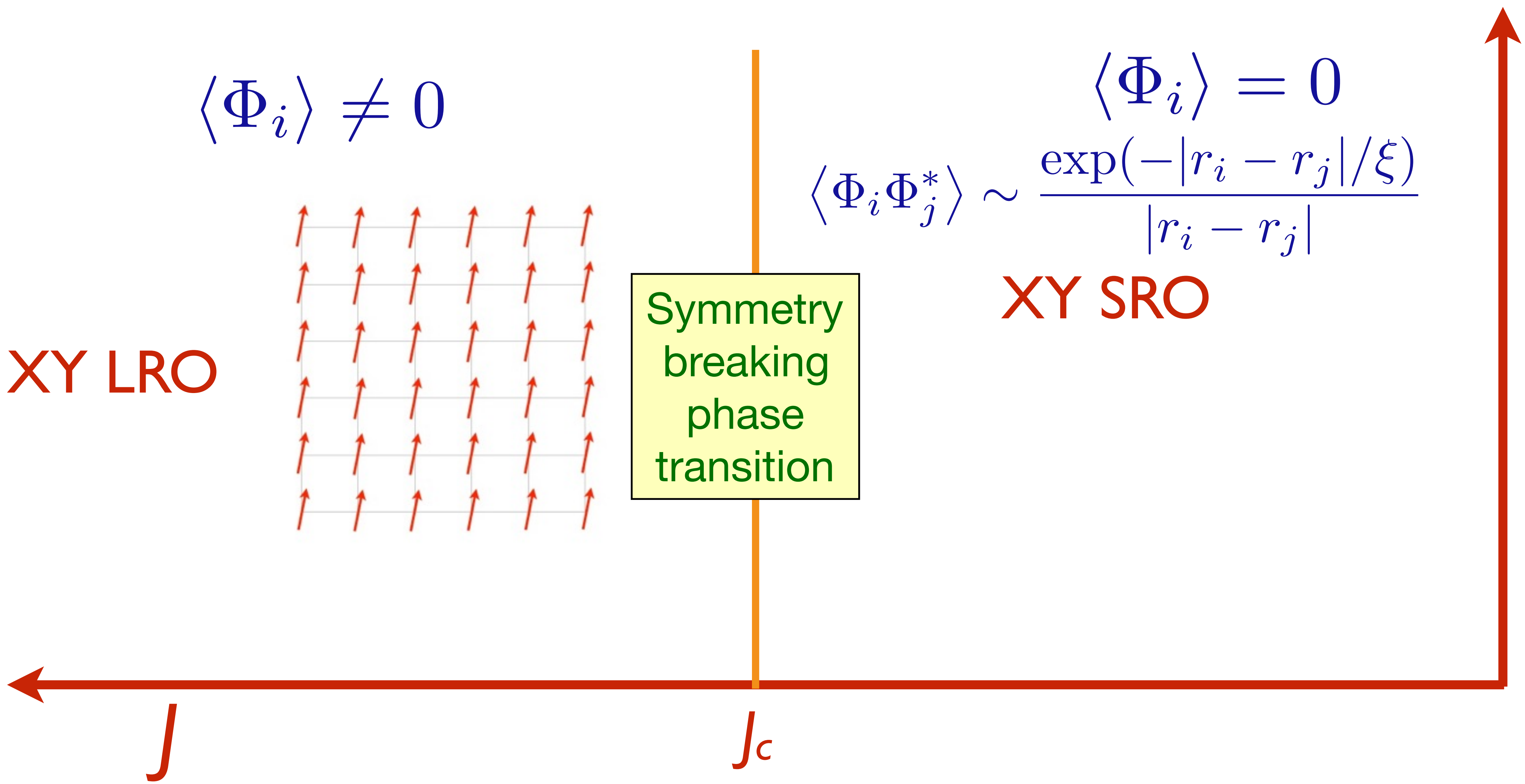
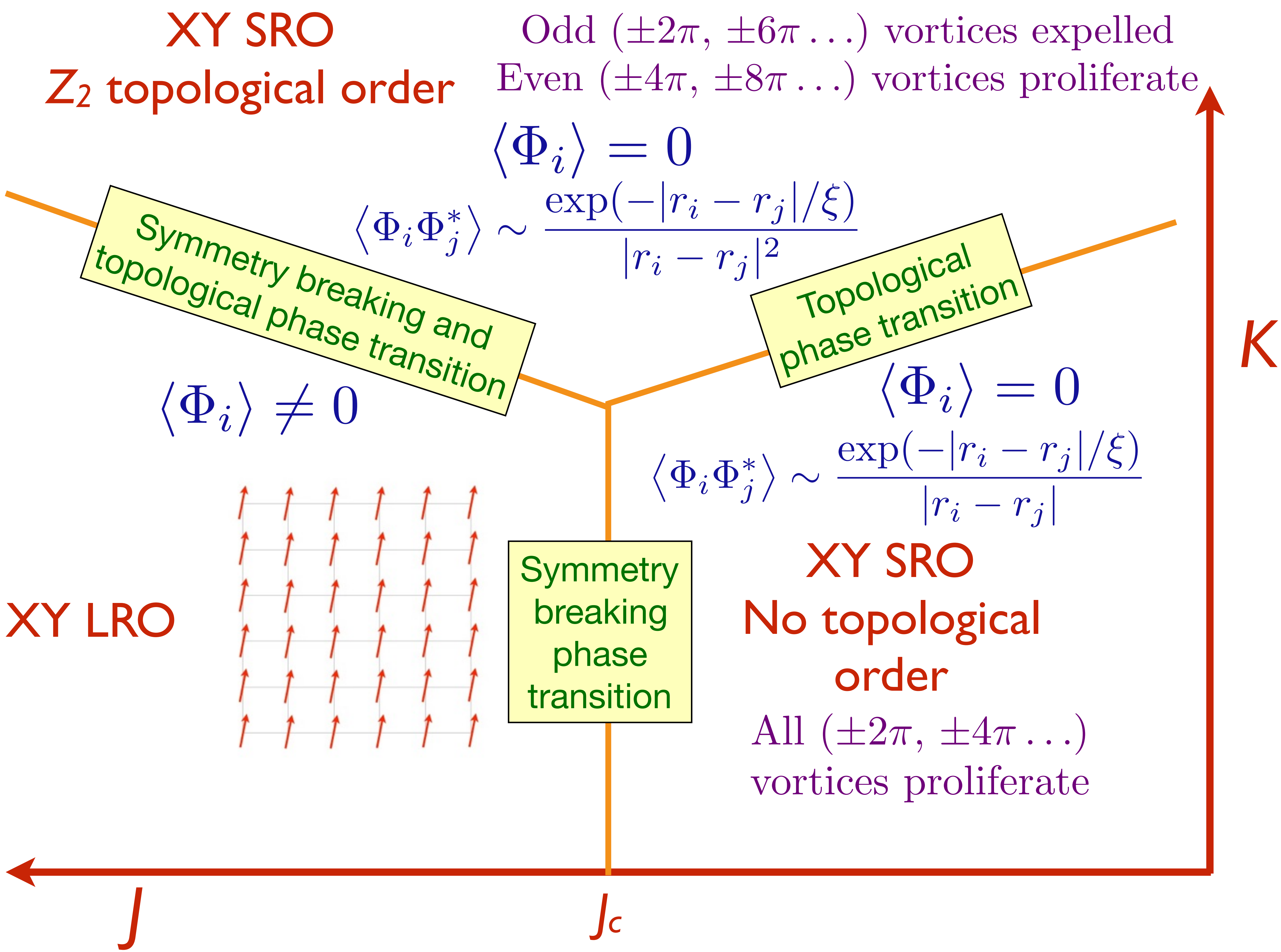
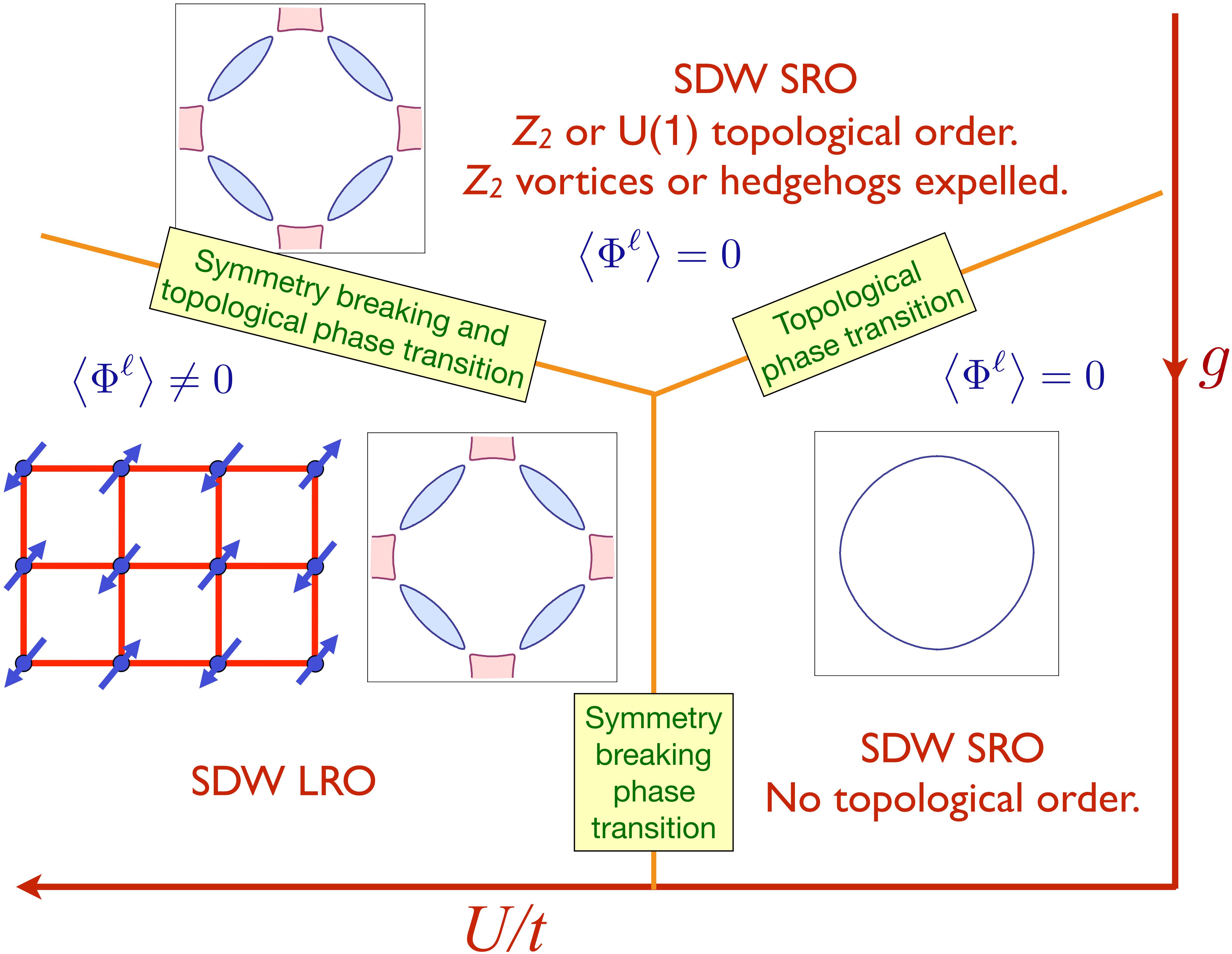


Classical XY model in D=3

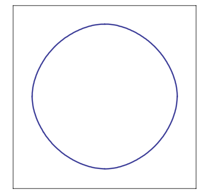


Square lattice Hubbard model at generic density



We can (exactly) transform the Hubbard model to the "spin-fermion" model: **electrons** $c_{i\alpha}$ on the square lattice with dispersion

$$\mathcal{H}_c = -\sum_{i,\rho} t_\rho \left(c^\dagger_{i,\alpha} c_{i+\boldsymbol{v}_\rho,\alpha} + c^\dagger_{i+\boldsymbol{v}_\rho,\alpha} c_{i,\alpha} \right) - \mu \sum_i c^\dagger_{i,\alpha} c_{i,\alpha} + \mathcal{H}_{\rm int}$$



are coupled to an **antiferromagnetic SDW order parameter** $\Phi^\ell(i)$**,** $\ell = x, y, z$

$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i \Phi^\ell(i) c^\dagger_{i,\alpha} \sigma^\ell_{\alpha\beta} c_{i,\beta} + V_\Phi$$

where $\eta_i = \pm 1$ on the two sublattices. (For suitable V_Φ, integrating out the Φ^ℓ yields back the Hubbard model).

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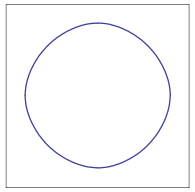

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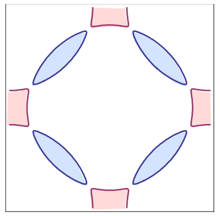
When $\Phi^\ell(i) =$ (non-zero constant) independent of i, we have long-range SDW order, which transforms the Fermi surfaces from large to small.



For (fluctuating) SDW SRO, we transform to a rotating reference frame using the SU(2) rotation R_i

$$\begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{pmatrix} = R_i \begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix},$$

in terms of fermionic "chargons" ψ_s and a **Higgs field** $H^a(i)$

$$\sigma^\ell \Phi^\ell(i) = R_i \, \sigma^a H^a(i) \, R_i^\dagger$$

The Higgs field is the SDW order in the rotating reference frame.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

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The Higgs field is the SDW order in the rotating reference frame. Note that this representation is ambiguous up to a SU(2) gauge transformation, V_i

$$\begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix} \to V_i \begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix}$$

$$R_i \to R_i V_i^\dagger$$

$$\sigma^a H^a(i) \to V_i \, \sigma^b H^b(i) \, V_i^\dagger .$$

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Fluctuating SDW

The simplest effective Hamiltonian for the fermionic chargons is the same as that for the electrons, with the SDW order replaced by the Higgs field.

$$\mathcal{H}_\psi = -\sum_{i,\rho} t_\rho \left(\psi^\dagger_{i,s} \psi_{i+\boldsymbol{v}_\rho, s} + \psi^\dagger_{i+\boldsymbol{v}_\rho, s} \psi_{i,s} \right) - \mu \sum_i \psi^\dagger_{i,s} \psi_{i,s} + \mathcal{H}_{\rm int}$$

$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i H^a(i) \psi^\dagger_{i,s} \sigma^a_{ss'} \psi_{i,s'} + V_H$$

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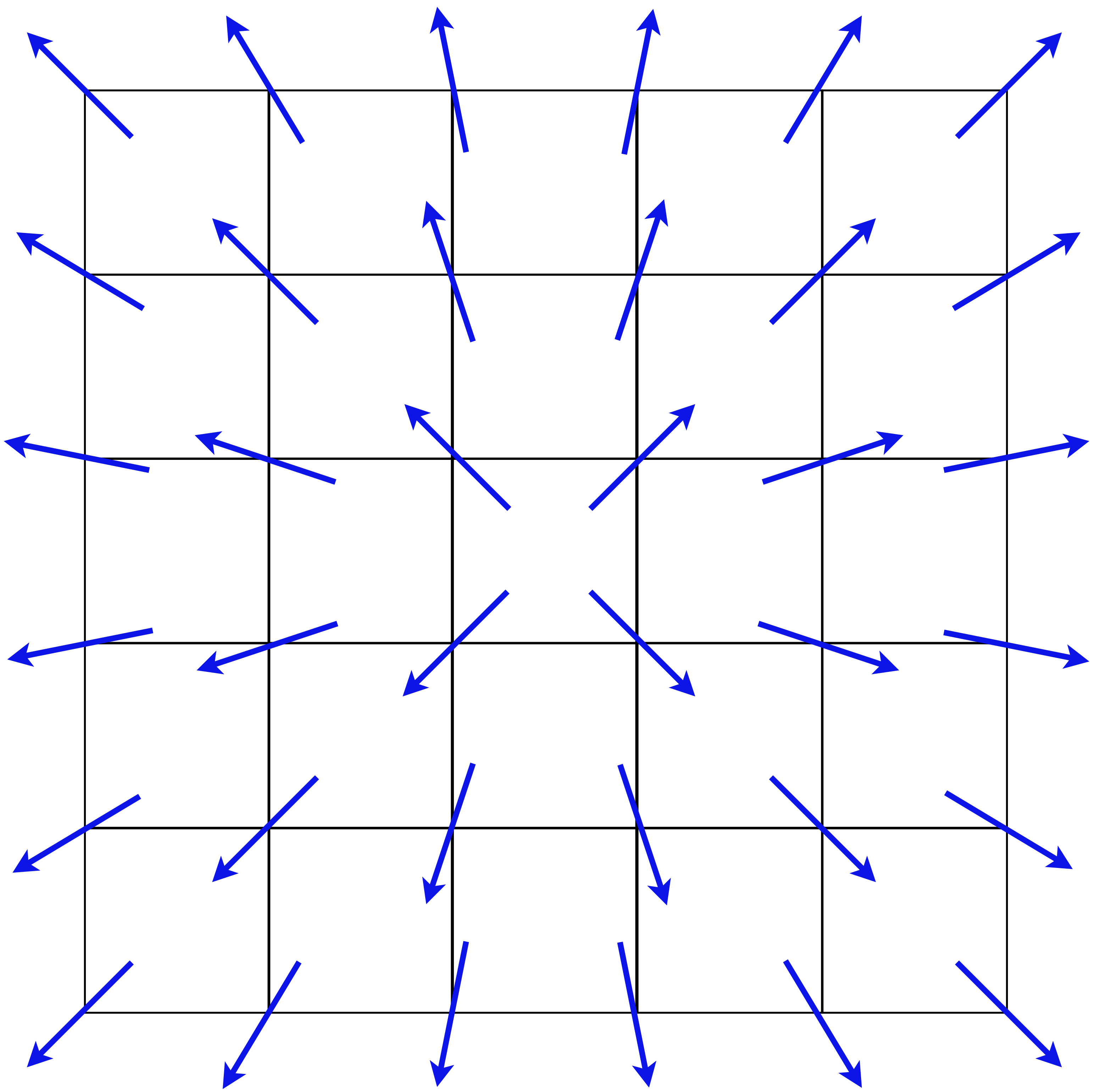
$$\mathcal{H}_{\text{int}} = -\lambda \sum_i \eta_i H^a(i) \psi^\dagger_{i,s} \sigma^a_{ss'} \psi_{i,s'} + V_H$$

IF we can transform to a rotating reference frame in which $H^a(i) =$ a constant independent of i and time, **THEN** the ψ fermions in the presence of (fluctuating) SDW SRO will inherit the small Fermi surfaces of the electrons in the presence of SDW LRO.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Fluctuating SDW

We cannot always find a single-valued SU(2) rotation R_i to make the Higgs field $H^a(i)$ a constant !



vortex in AFM order

(assume easy-plane AFM for simplicity)

S. Sachdev, E. Berg, S. Chatterjee, and Y. Schattner, PRB **94**, 115147 (2016)

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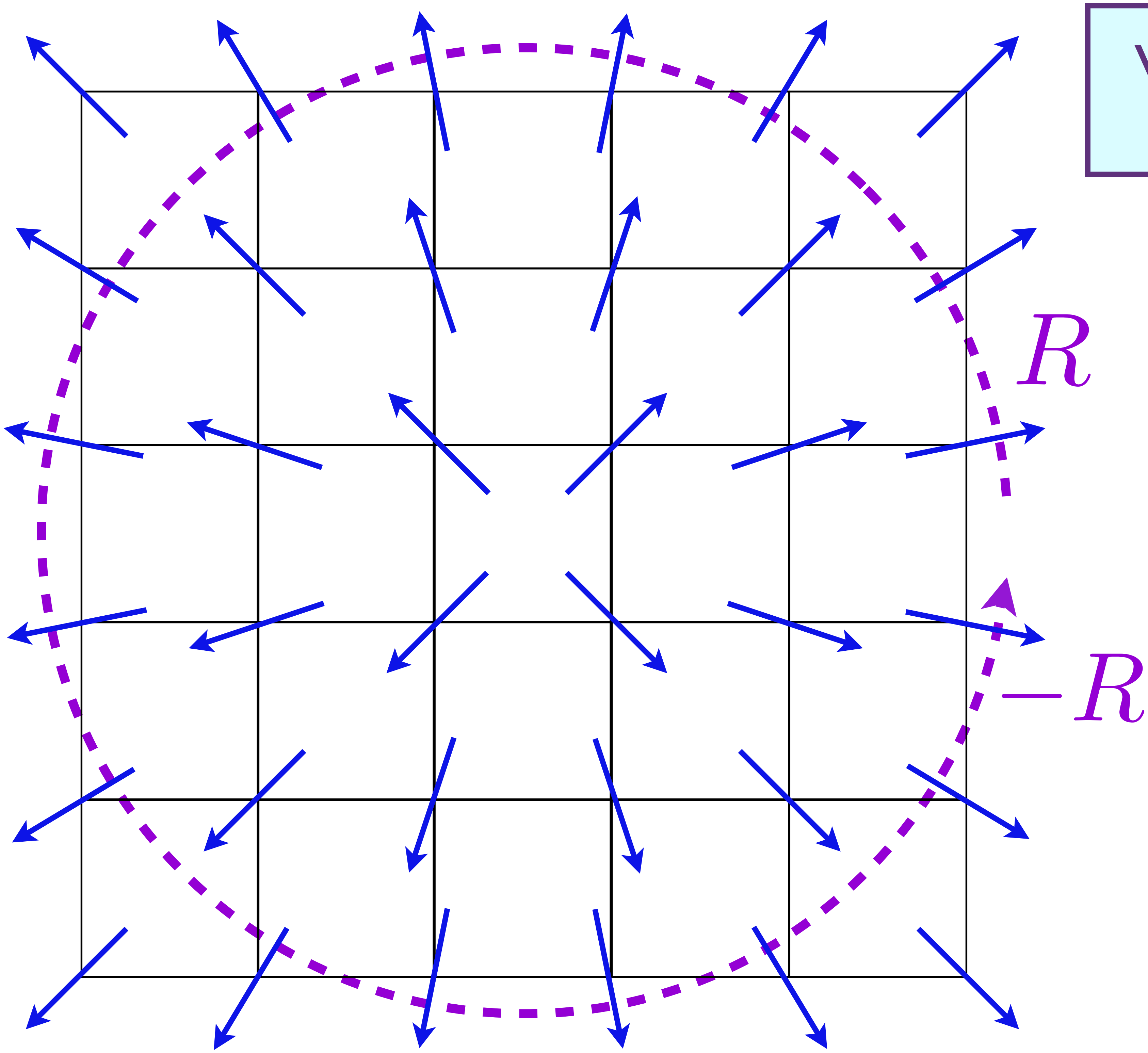

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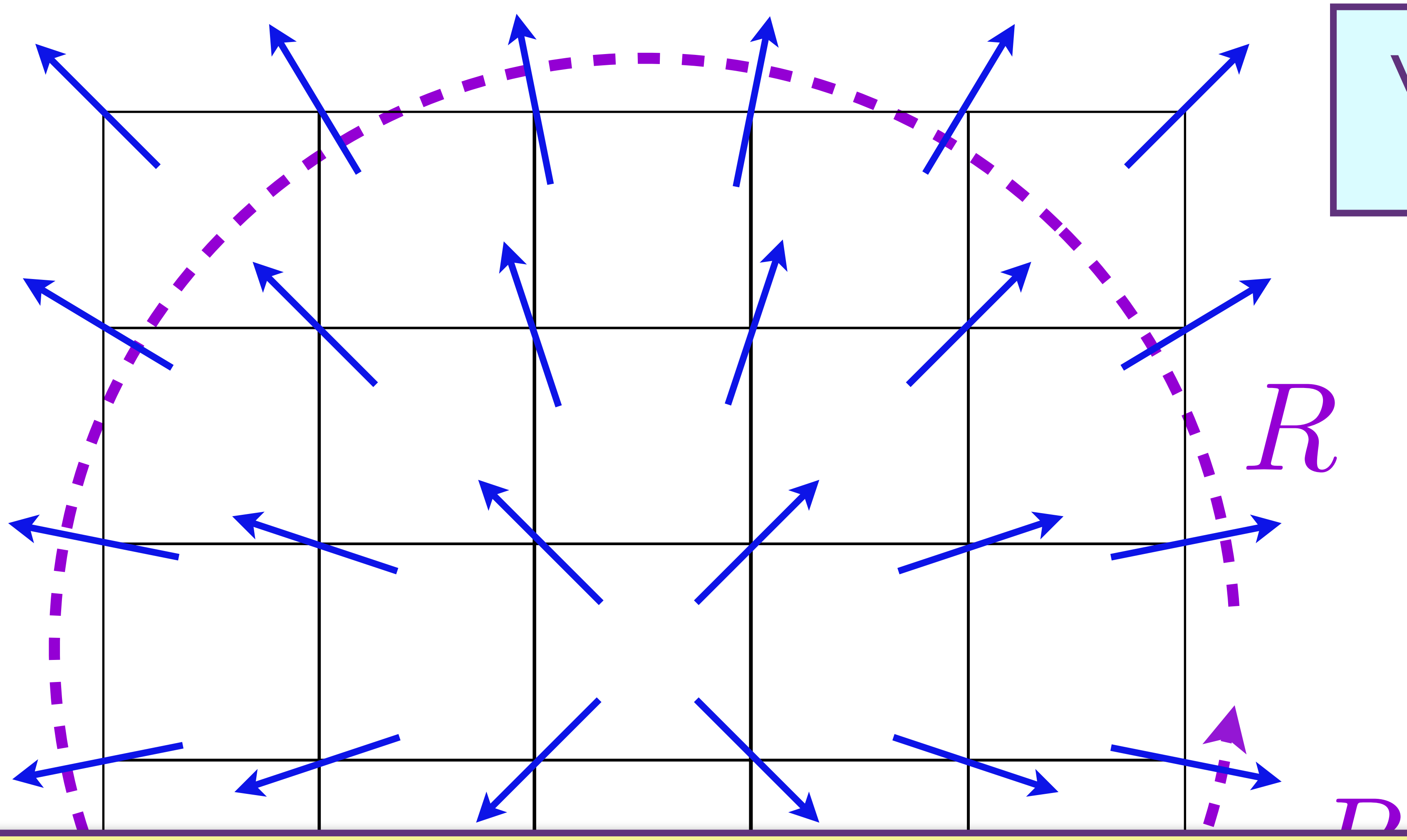
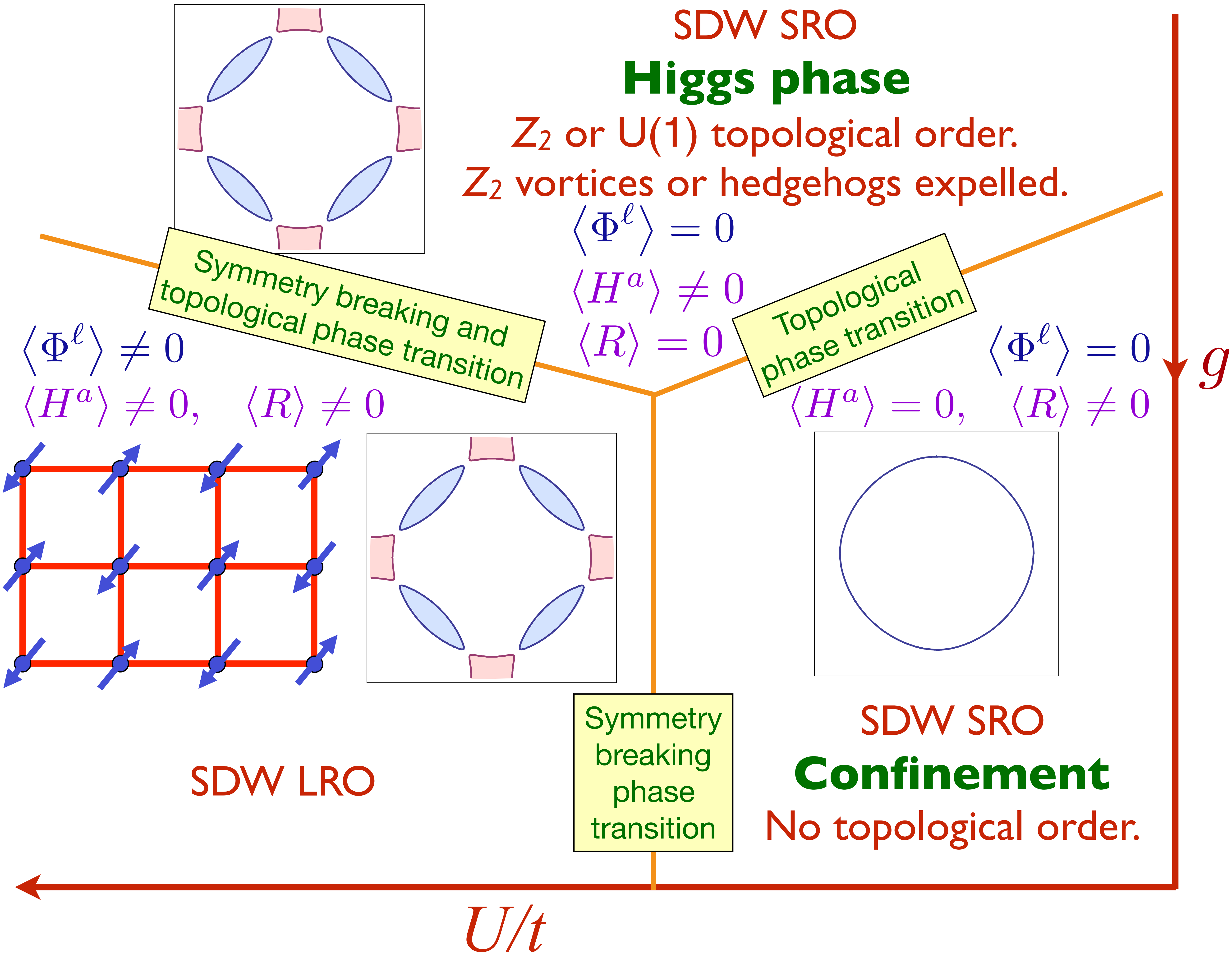


vortex in AFM order

(assume easy-plane AFM for simplicity)

The HIGGS PHASE, with H^a condensed, has fluctuating R and SDW SRO with odd vortices expelled (for easy-plane SDW). Such a metal has topological order and the fermions which inherit the small Fermi surfaces of the metal with SDW LRO.

jee, and T. Schattner, PRB **94**, 115147 (2016)



SDW SRO
Higgs phase
Z_2 or U(1) topological order.
Z_2 vortices or hedgehogs expelled.
$\langle \Phi^\ell \rangle = 0$
$\langle H^a \rangle \neq 0$
$\langle R \rangle = 0$
Symmetry breaking and topological phase transition
Topological phase transition
$\langle \Phi^\ell \rangle \neq 0$
$\langle H^a \rangle \neq 0, \quad \langle R \rangle \neq 0$
$\langle \Phi^\ell \rangle = 0$
$\langle H^a \rangle = 0, \quad \langle R \rangle \neq 0$
g
SDW LRO
Symmetry breaking phase transition
SDW SRO
Confinement
No topological order.
U/t

Electron Green's function in Higgs phase of SU(2) gauge theory

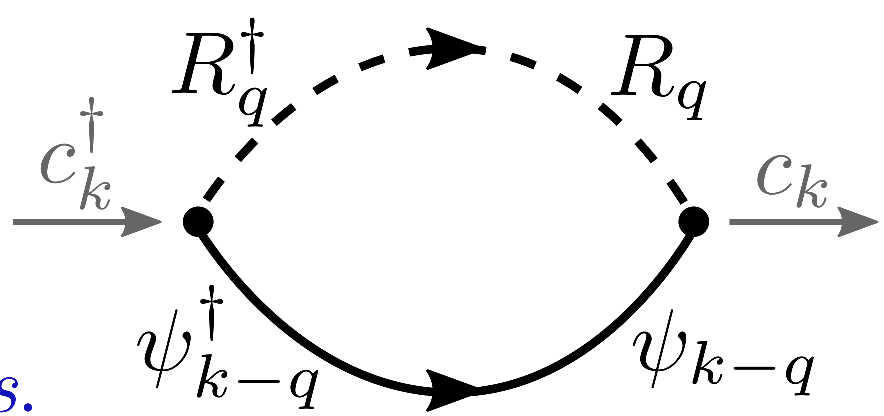
The effective Hamiltonian of the chargons in a constant Higgs potential $\langle H^a \rangle = H_0^a$ is (the hoppings have been renormalized by $\langle R_i^\dagger R_j \rangle$):

$$\mathcal{H}_\psi = -\sum_{i,\rho} \tilde{t}_\rho \left(\psi_{i,s}^\dagger \psi_{i+\boldsymbol{v}_\rho,s} + \psi_{i+\boldsymbol{v}_\rho,s}^\dagger \psi_{i,s} \right) - \mu \sum_i \psi_{i,s}^\dagger \psi_{i,s} - \lambda \sum_i (-1)^{i_x+i_y} H_0^a \psi_{i,s}^\dagger \sigma_{ss'}^a \psi_{i,s'}$$

The chargon Fermi surface reconstructs into "small pockets", even though translational and spin rotation symmetries remain *unbroken*. The diagonal chargon Green's function is

$$G_\psi(\omega, \vec{k}) = \frac{1}{\omega - \varepsilon_{\vec{k}} - \Sigma_\psi(\omega, \vec{k})}, \quad \Sigma_\psi(\omega, \vec{k}) = \frac{H_0^2}{\omega - \varepsilon_{\vec{k}+\vec{Q}}}, \quad \vec{Q} = (\pi, \pi).$$

This has poles at the pocket Fermi surfaces, and zeros at $\varepsilon_{\vec{k}+\vec{Q}}$. The electron Green's function is computed via a convolution with the spinons (R), and then the zeros are smeared to *approximate zeros*.



Common features of many cluster-DMFT computations of pseudogap metal:

- Momentum-space differentiation: electron self-energy is enhanced at low frequencies in the anti-nodal region, and vanishes in the nodal region.
- Gapped spectrum in the anti-nodal region
- Fermi arcs in the nodal region
- Apparent zero of Green's function on a "Luttinger surface".

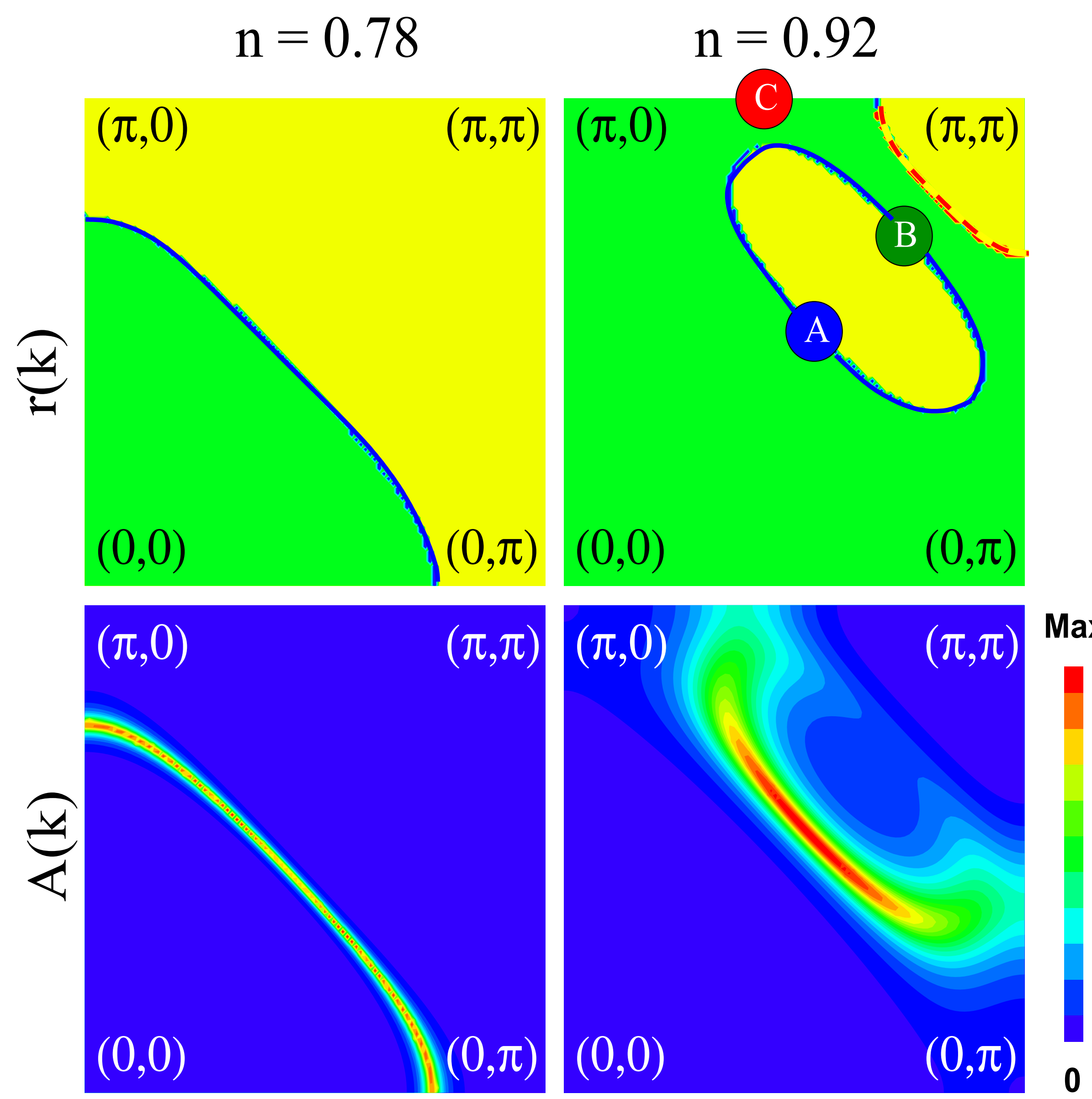
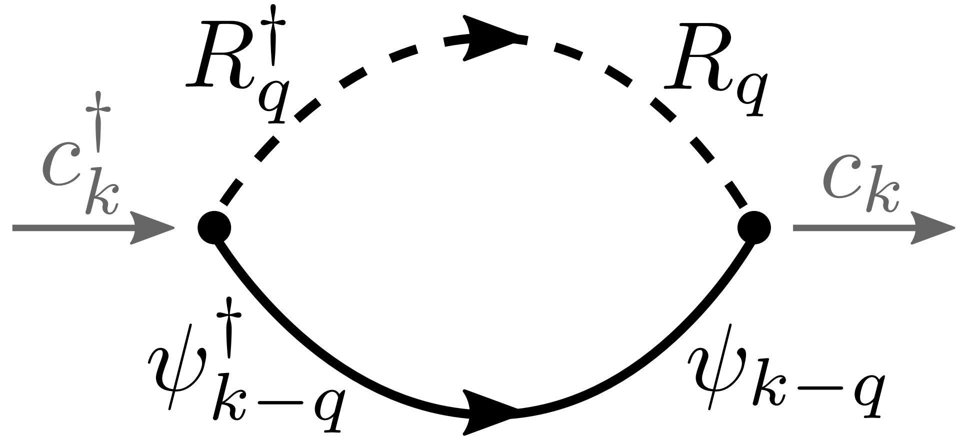
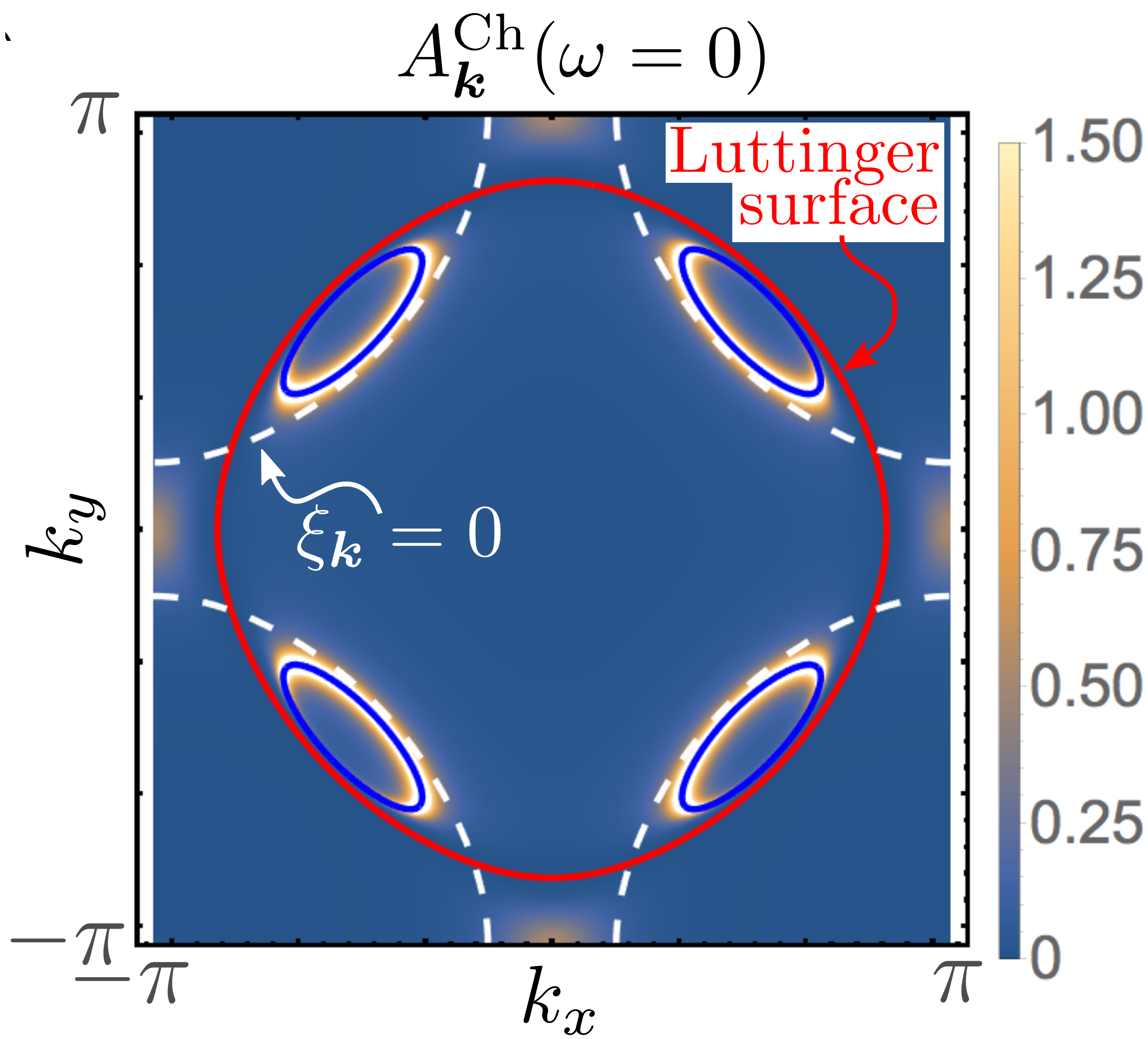


FIG. 4. (Color online) Renormalized energy $r(\mathbf{k})$ (upper panels) and spectral function $A(\mathbf{k})$ (lower panels) for the 2D Hubbard model with $U=8t$ and $T=0$. The color code for the upper panels is green/gray ($r<0$), blue/dark gray line ($r=0$), yellow/light gray ($r>0$), red dashed line ($r\rightarrow\infty$).

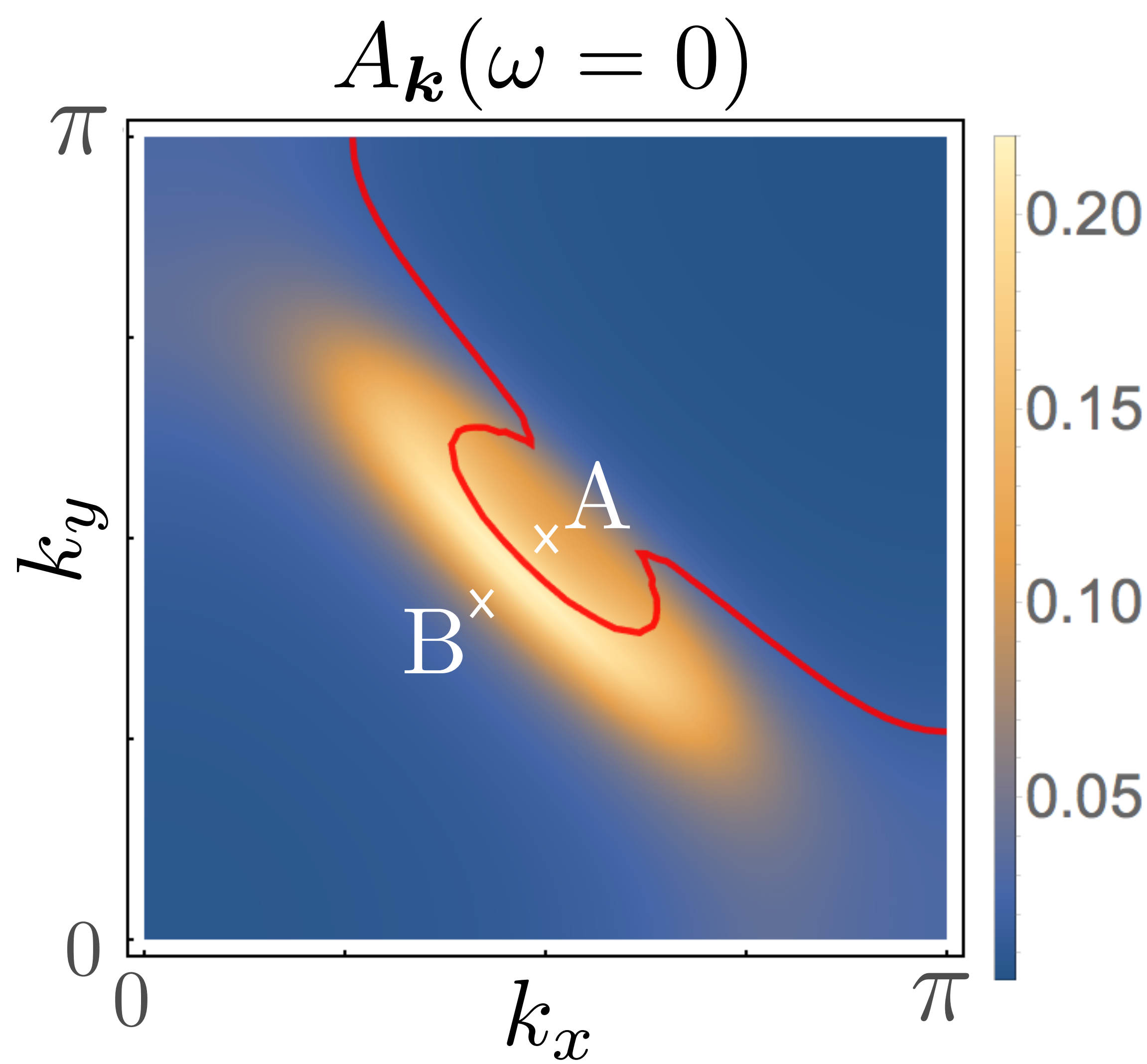
T.D. Stanescu and G. Kotliar,
PRB **74**, 125110 (2006)

Electron Green's function in Higgs phase of SU(2) gauge theory





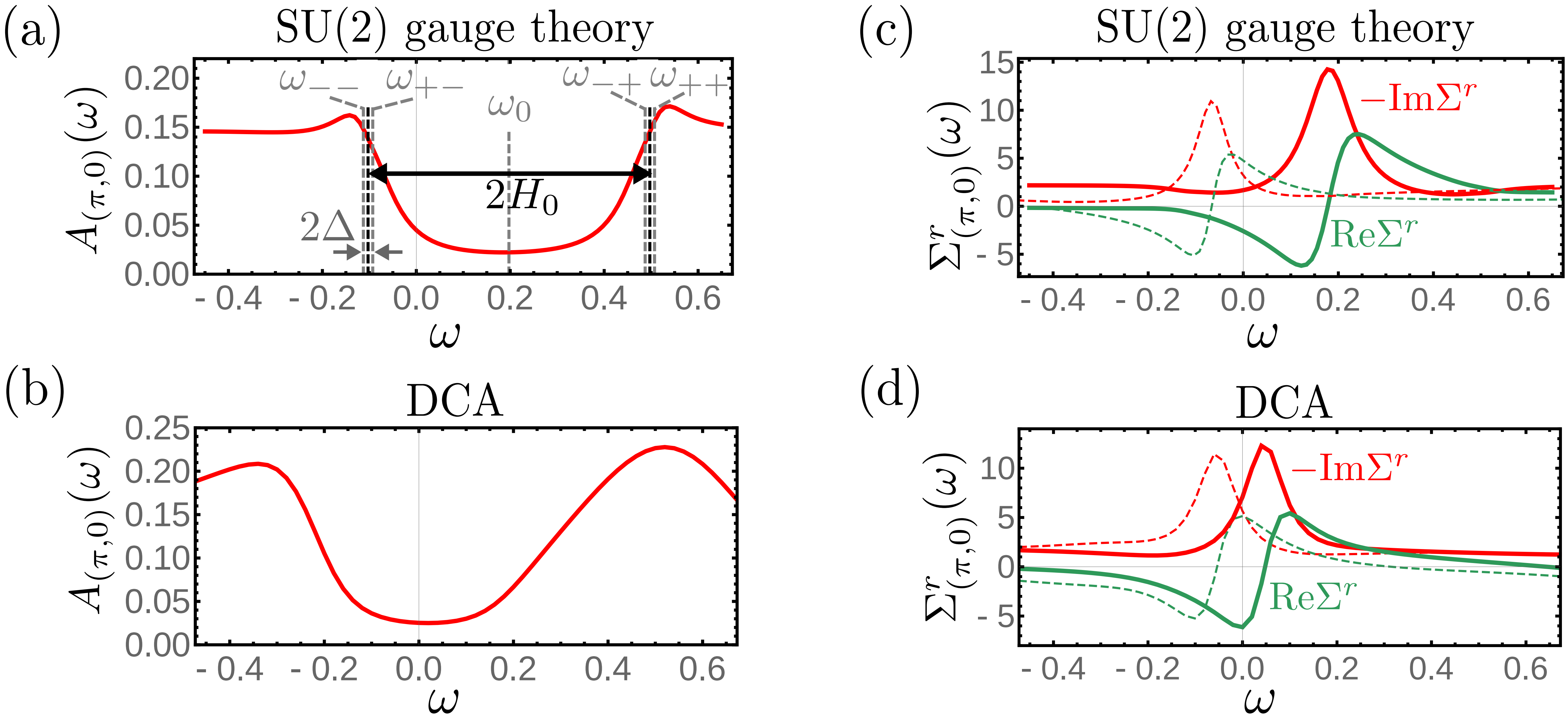
Red line indicates the locus of $G(\mathbf{k}, \omega = 0) = 0$



Red line indicates the locus of $\mathrm{Re}\, G(\mathbf{k}, \omega = 0) = 0$

Full Brillouin zone spectra of chargons (ψ) and electrons (c)

Electron Green's function in Higgs phase of SU(2) gauge theory

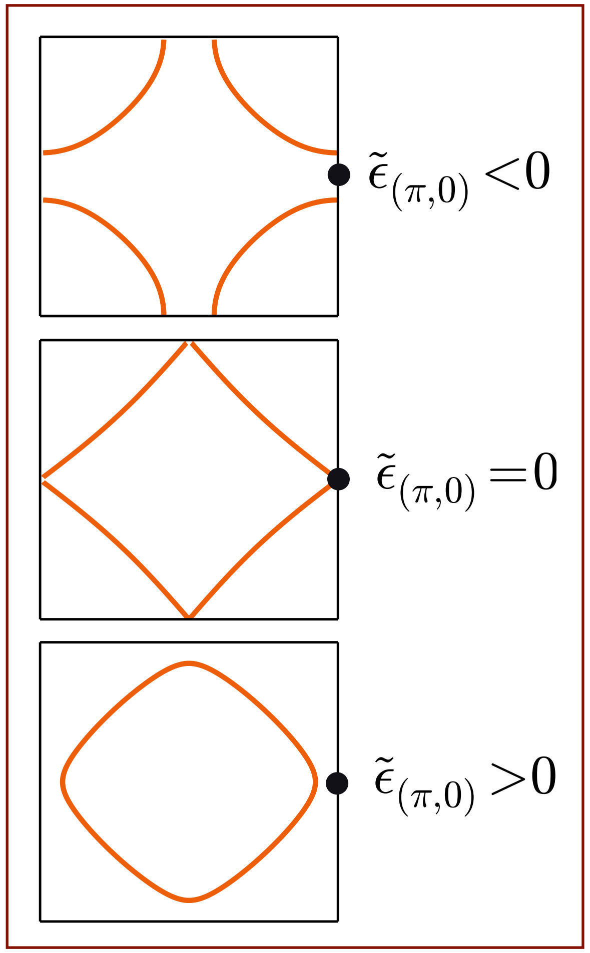


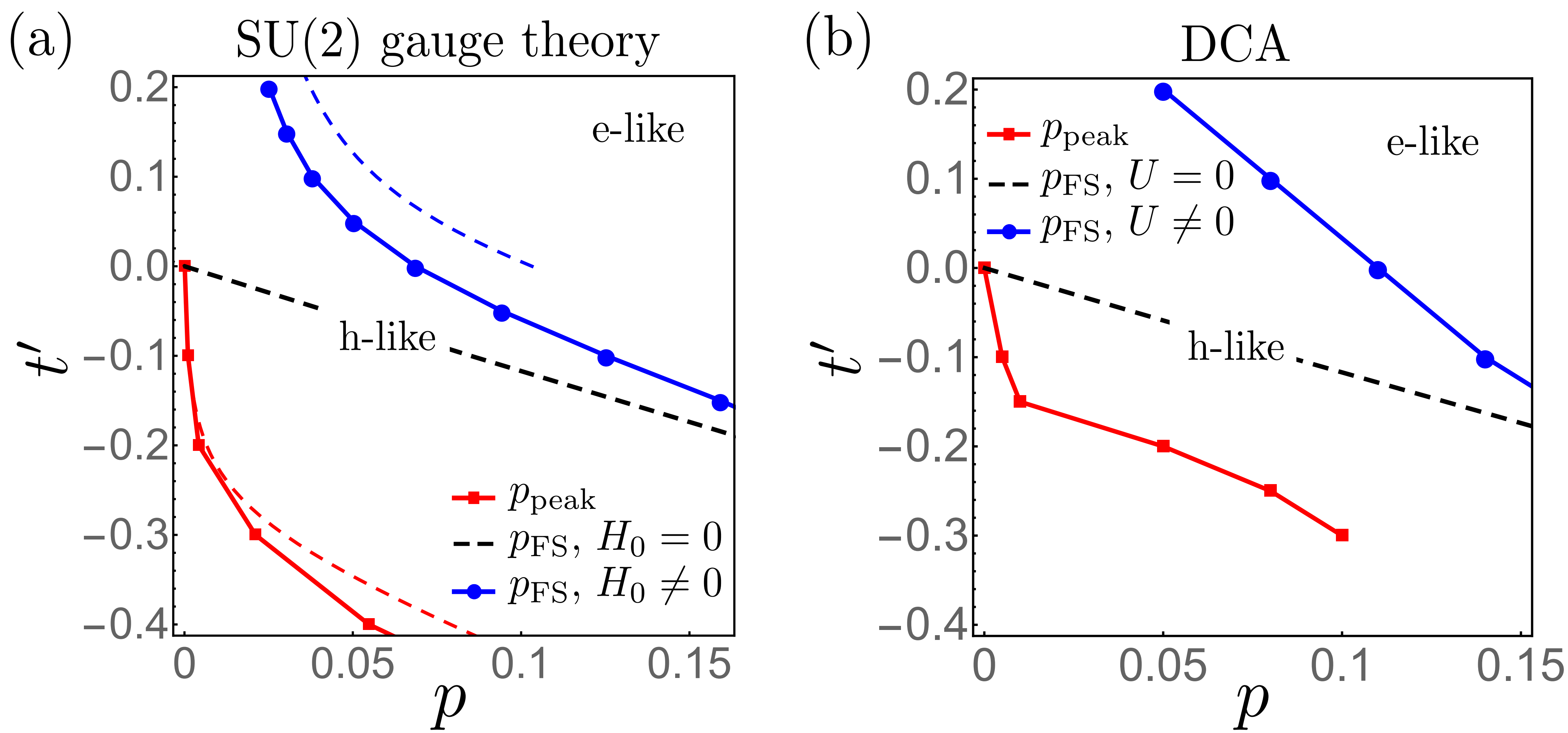
$$T = t/30 \quad , \quad U = 7t \quad , \quad p = 0.05$$

t' takes different negative values

Anti-nodal spectra compared to cluster DMFT

Lifshitz transition compared to cluster DMFT

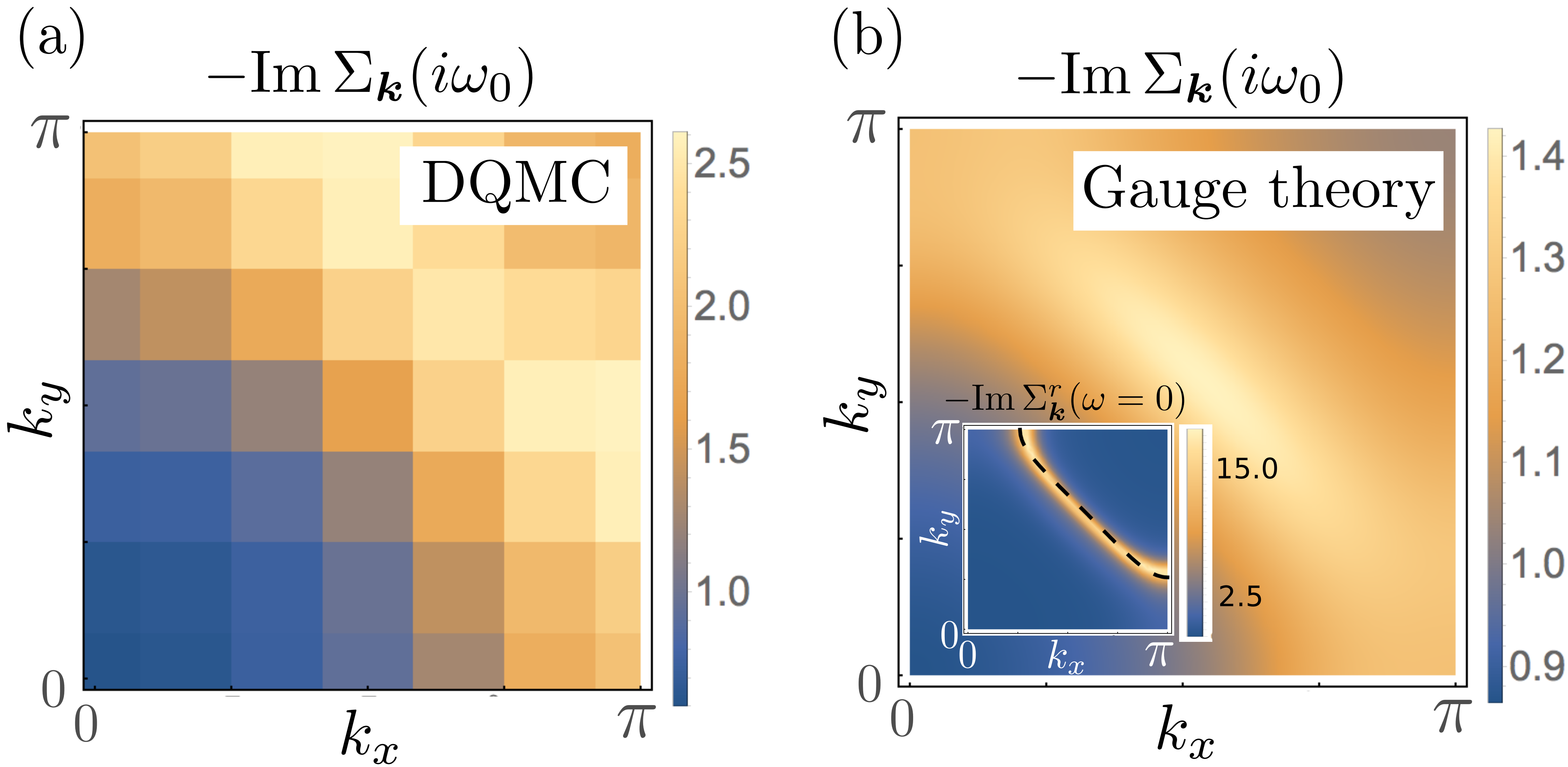




$$\tilde{\epsilon}_{\vec{k}} = \epsilon_{\vec{k}} + \mathrm{Re}\,\Sigma_{\vec{k}}(\omega = 0) = -\mathrm{Re}\left(G_c(\omega = 0, \vec{k})\right)^{-1}$$

The p-t' dependence of the "interacting Lifshitz transition", defined by the sign change of the renormalized quasiparticle energy $\tilde{\epsilon}_{(\pi,0)}$ at $\omega_{\rm peak} > 0$, is shown as solid blue lines calculated from the SU(2) gauge theory, part (a), and DCA, part (b). The black dashed lines show the location of the same transition for non-interacting electrons. The red lines indicate where the particle-hole asymmetry of the self-energy changes, *i.e.*, where the peak position $\omega_{\rm peak}$ of the anti-nodal Im(self-energy) changes sign.

Electron Green's function in Higgs phase of SU(2) gauge theory



The imaginary part of the self-energy at the lowest Matsubara frequency $\omega_0 = \pi T$ determined from DQMC on the Hubbard model ($U = 7t$, $t' = -0.1t$, $T = 0.25t$, $p = 0.042$) and from the SU(2) gauge theory is shown in (a) and (b), respectively. To avoid too much broadening, we have applied a slightly smaller temperature of $T = 0.15t$ for the gauge theory. The inset in (b) shows the gauge theory prediction at zero frequency and low temperature (as before $T = t/30$). The black dashed line corresponds to the position of the Luttinger surface of the chargons.

- New classes of quantum states with topological order

arXiv:1707.06602   **arXiv:1711.09925**

- New classes of quantum states with topological order

- Can be understood as:
  (a) defect suppression in states with fluctuating order associated with broken symmetries
  (b) Higgs phases of emergent gauge fields

arXiv:1707.06602 **arXiv:1711.09925**

- New classes of quantum states with topological order

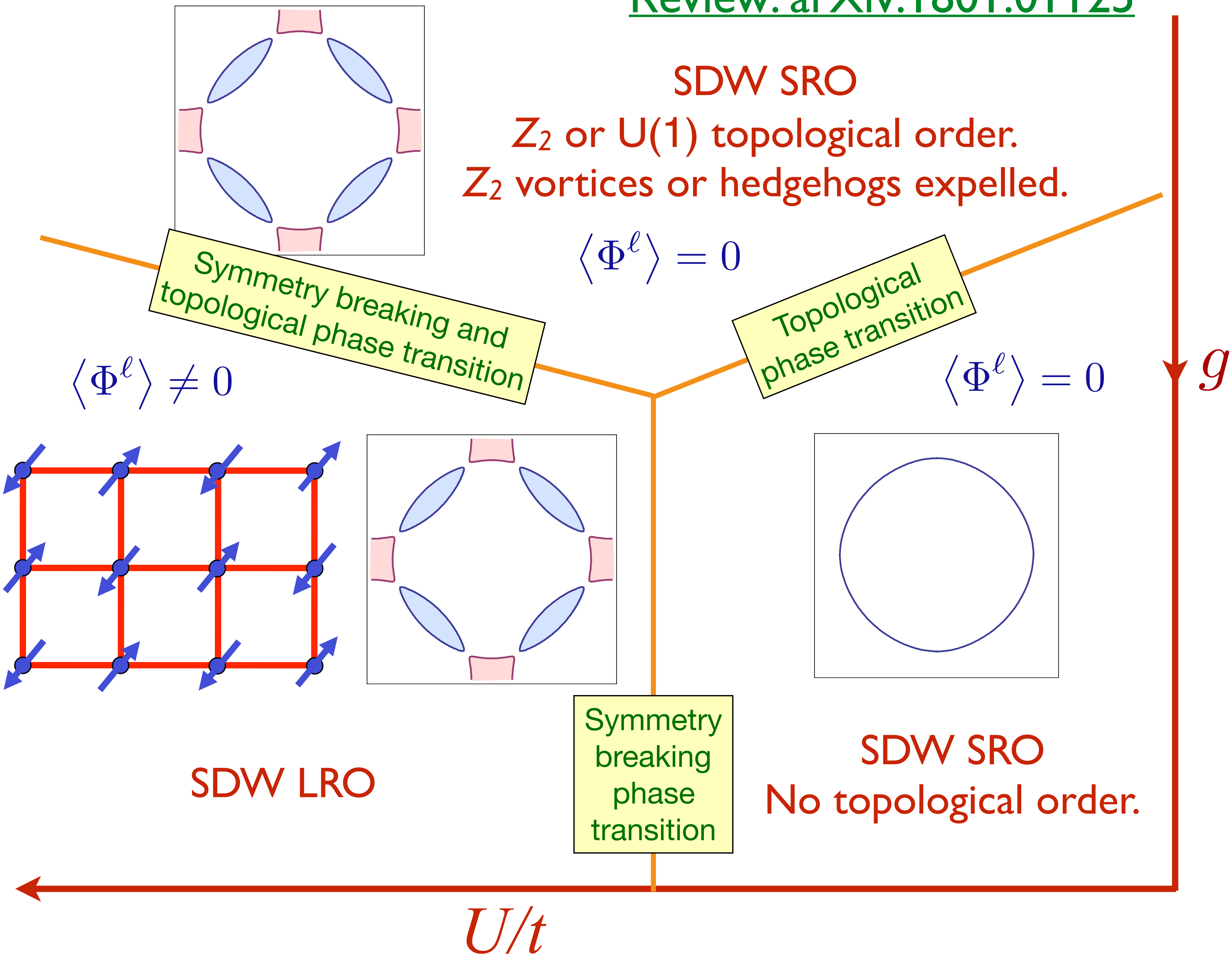
- Can be understood as:
  (a) defect suppression in states with fluctuating order associated with broken symmetries
  (b) Higgs phases of emergent gauge fields

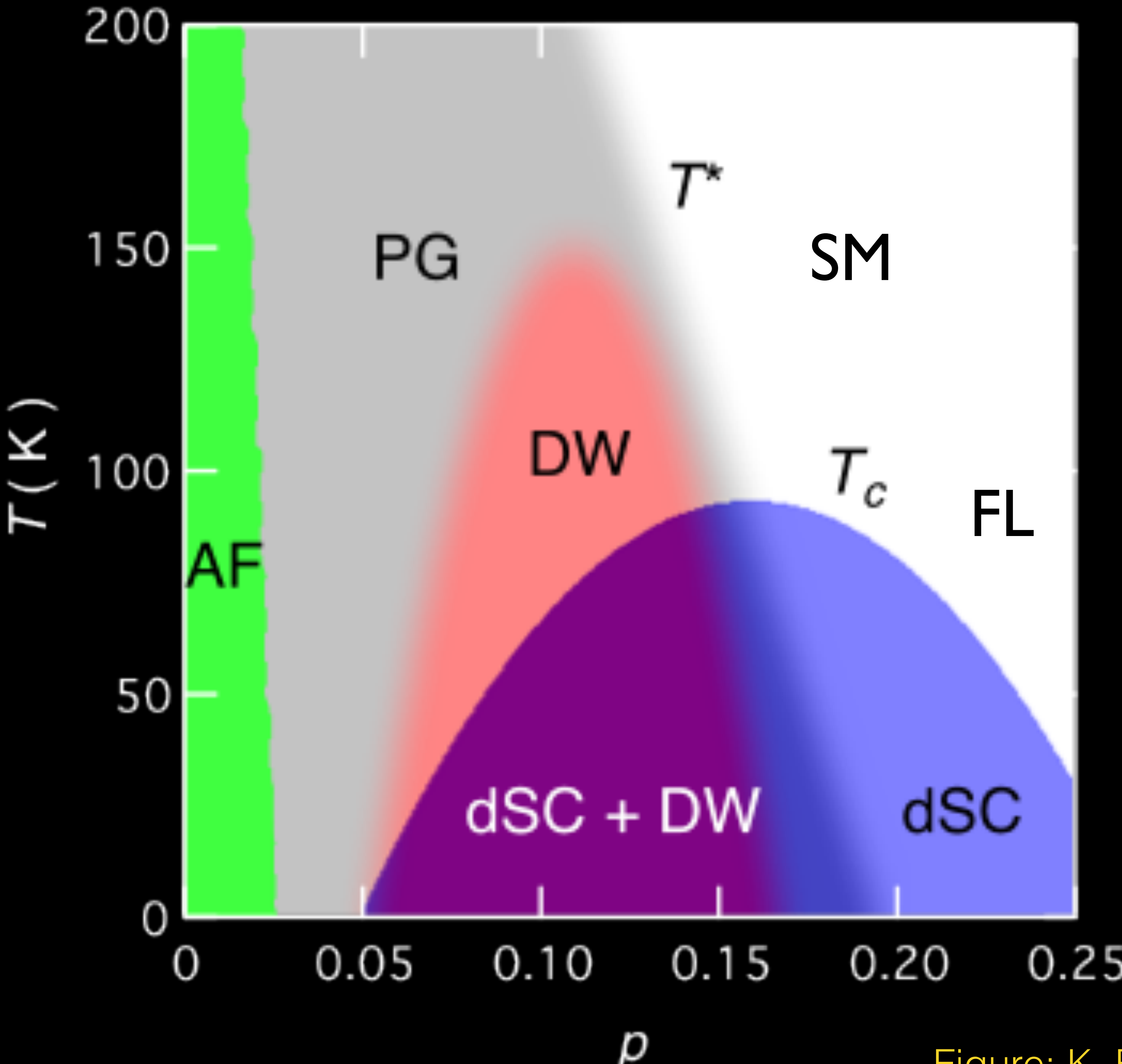
- A metal with bulk topological order (*i.e.* long-range quantum entanglement) can explain existing experiments in cuprates, and agrees well with cluster-DMFT

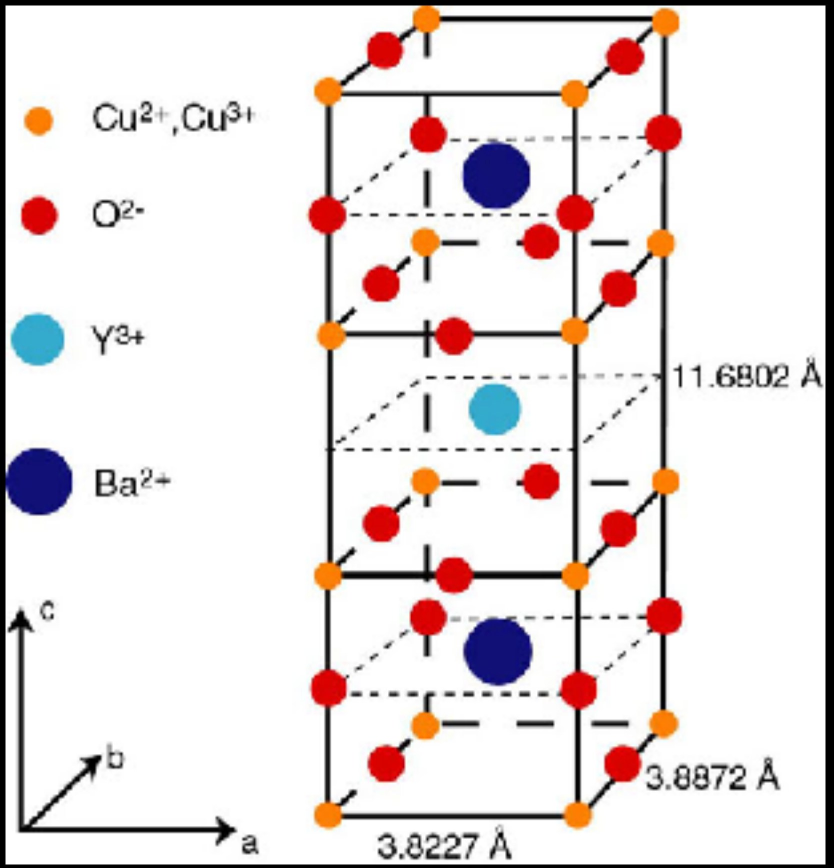
arXiv:1707.06602 arXiv:1711.09925

Square lattice Hubbard model at generic density

Review: arXiv:1801.01125

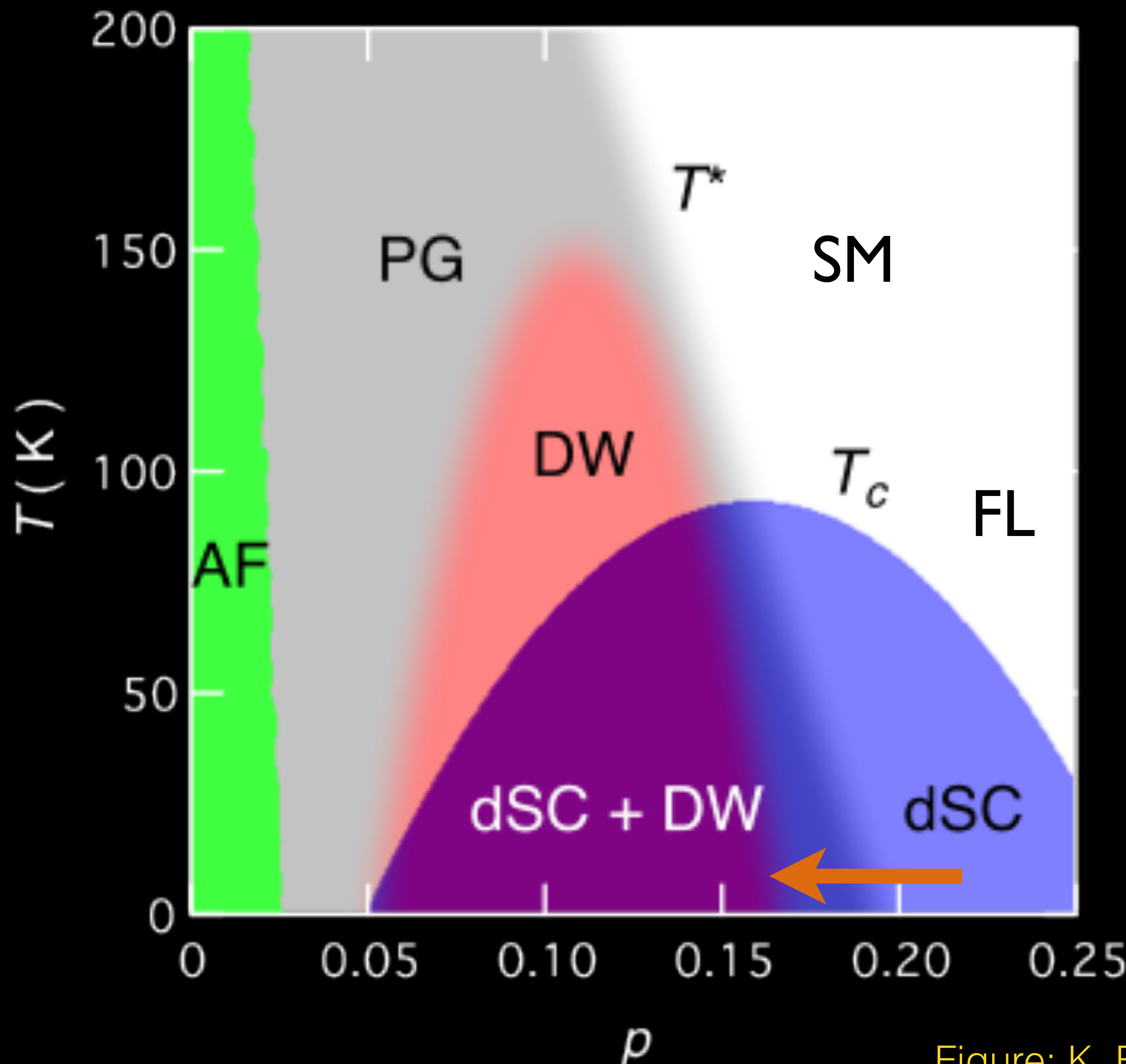






$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis



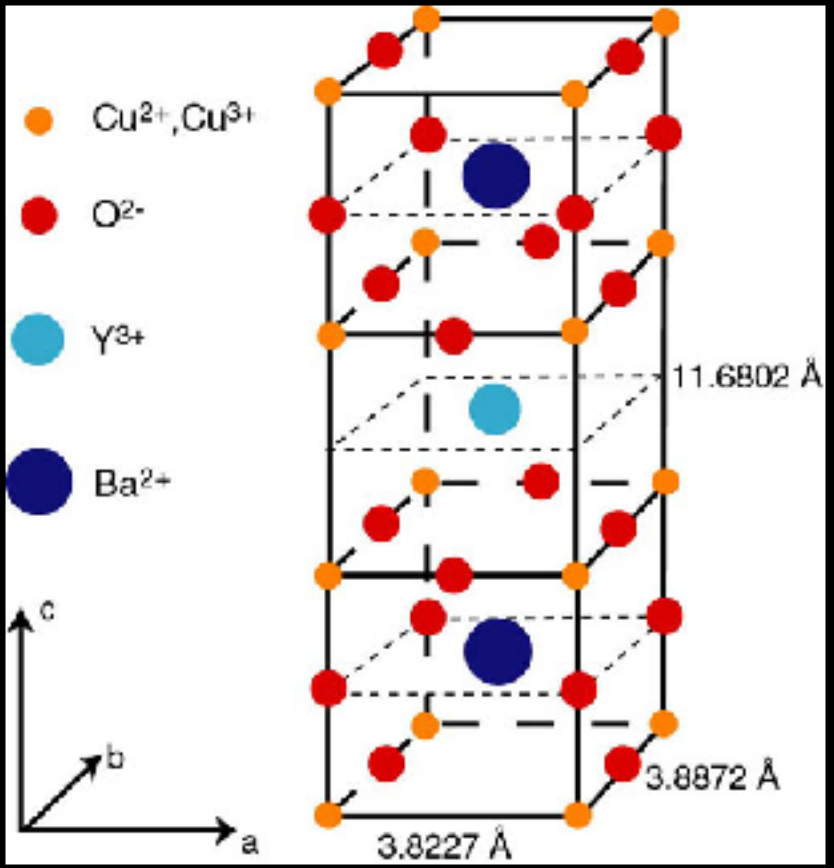


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Figure: K. Fujita and J. C. Seamus Davis

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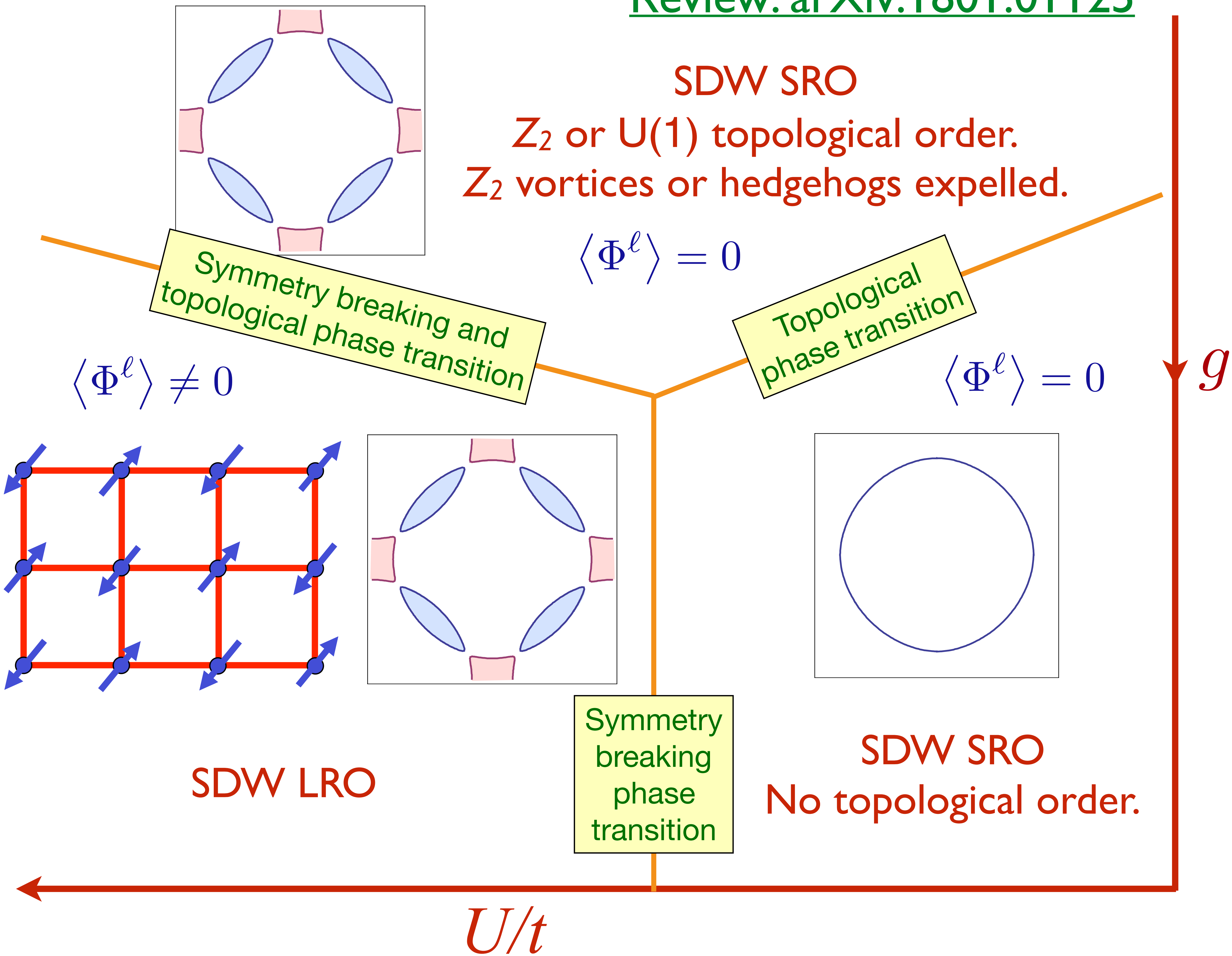

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