

# SYK and overdamped-boson-localization regimes in the theory of strange metals

Subir Sachdev

APS March meeting, Minneapolis, March 4, 2024  
B07.00006

See also, talk by Chenyuan Li, A07.00008

Aavishkar A. Patel, Peter Lunts, and S. S., arXiv:2312.06751

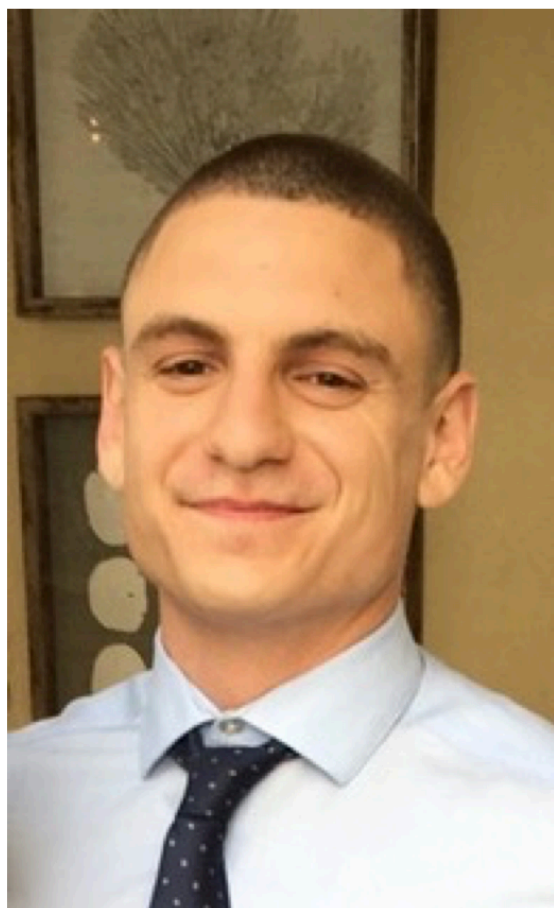
Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, and S. S., Science **381**, 790 (2023)



PHYSICS



HARVARD



Ilya Esterlis  
Wisconsin



Haoyu Guo  
Cornell



Aavishkar Patel  
Flatiron



Chenyuan Li  
Harvard



Davide Valentinis  
KIT



Joerg Schmalian  
KIT

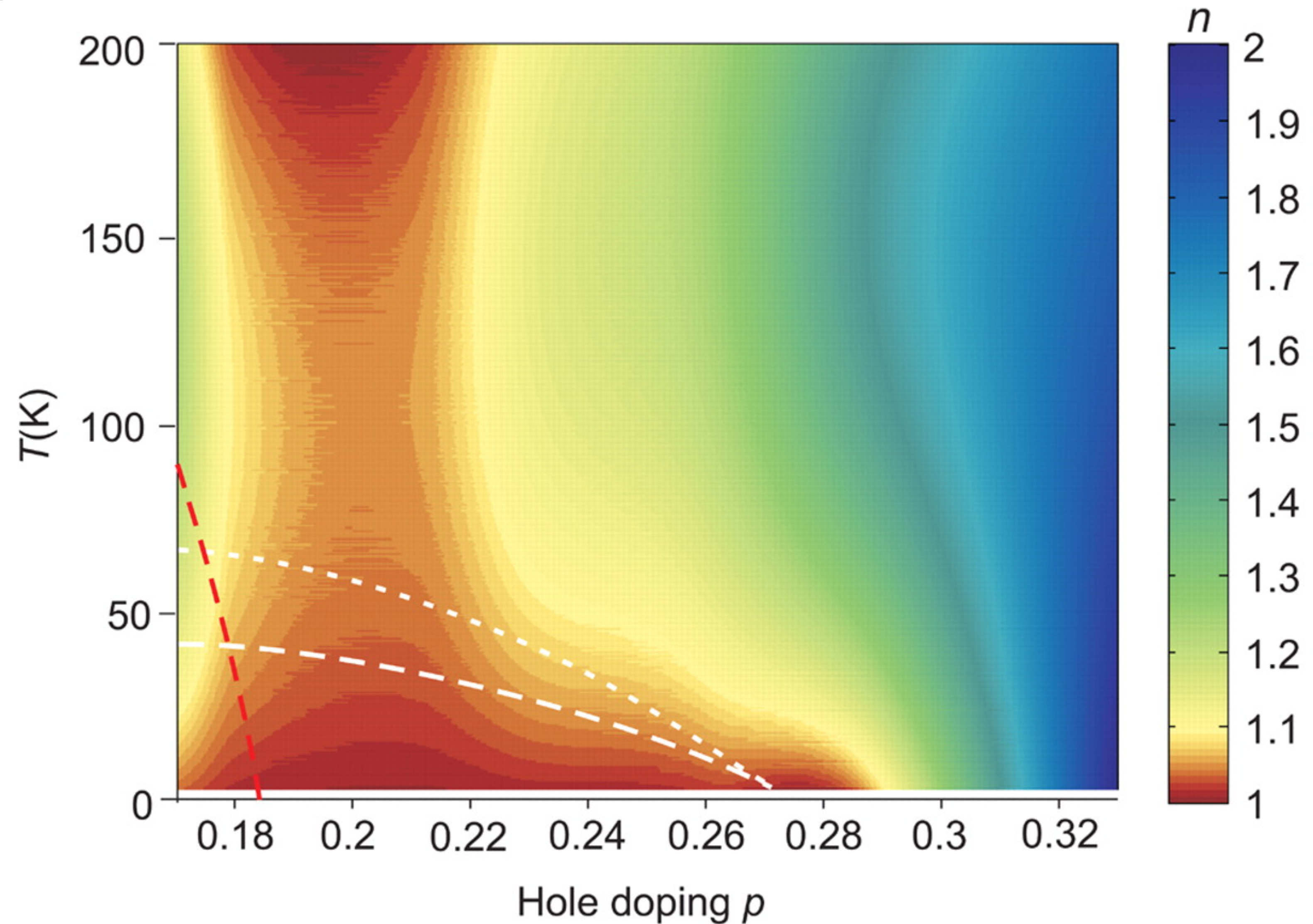
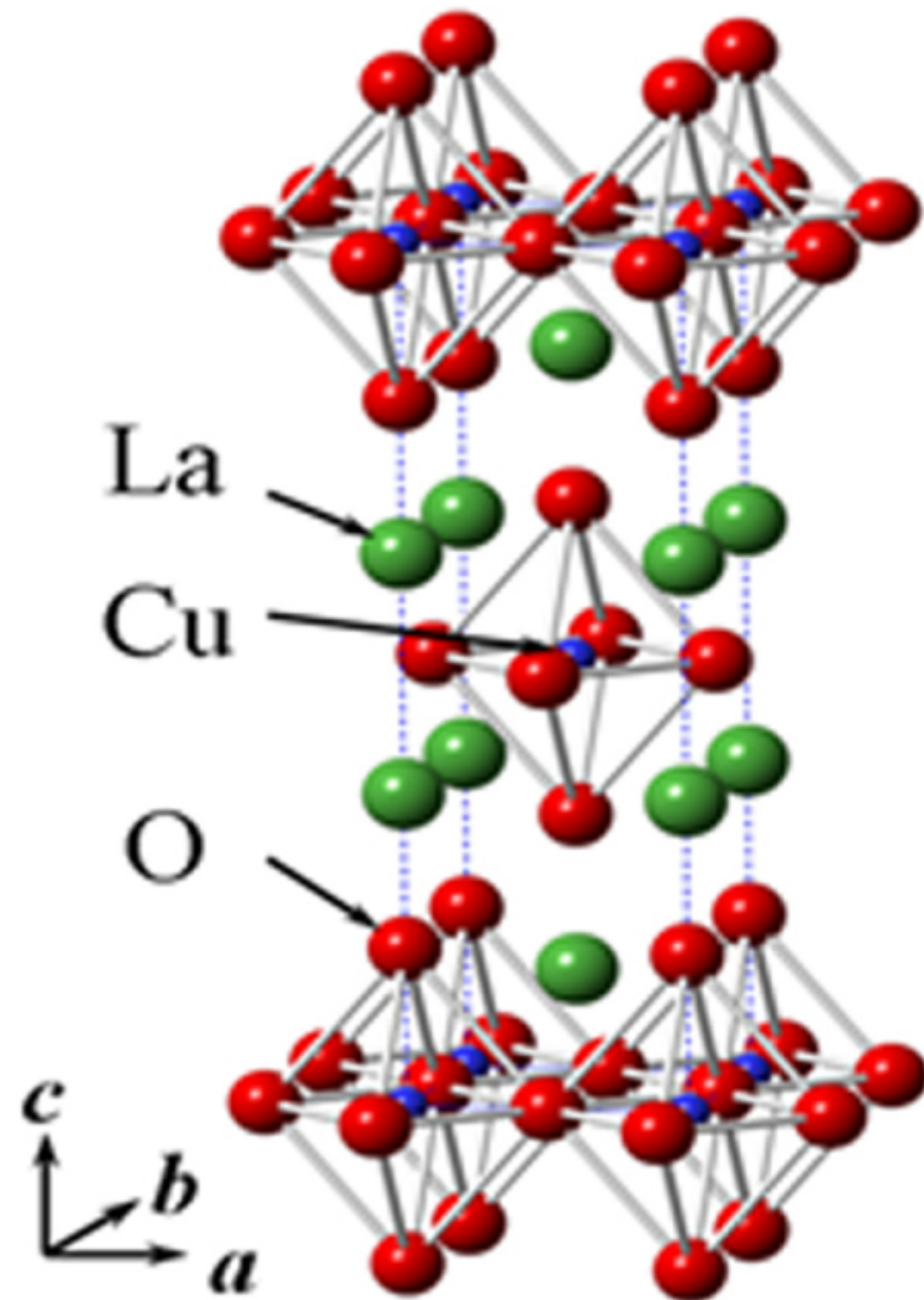


Peter Lunts  
Harvard

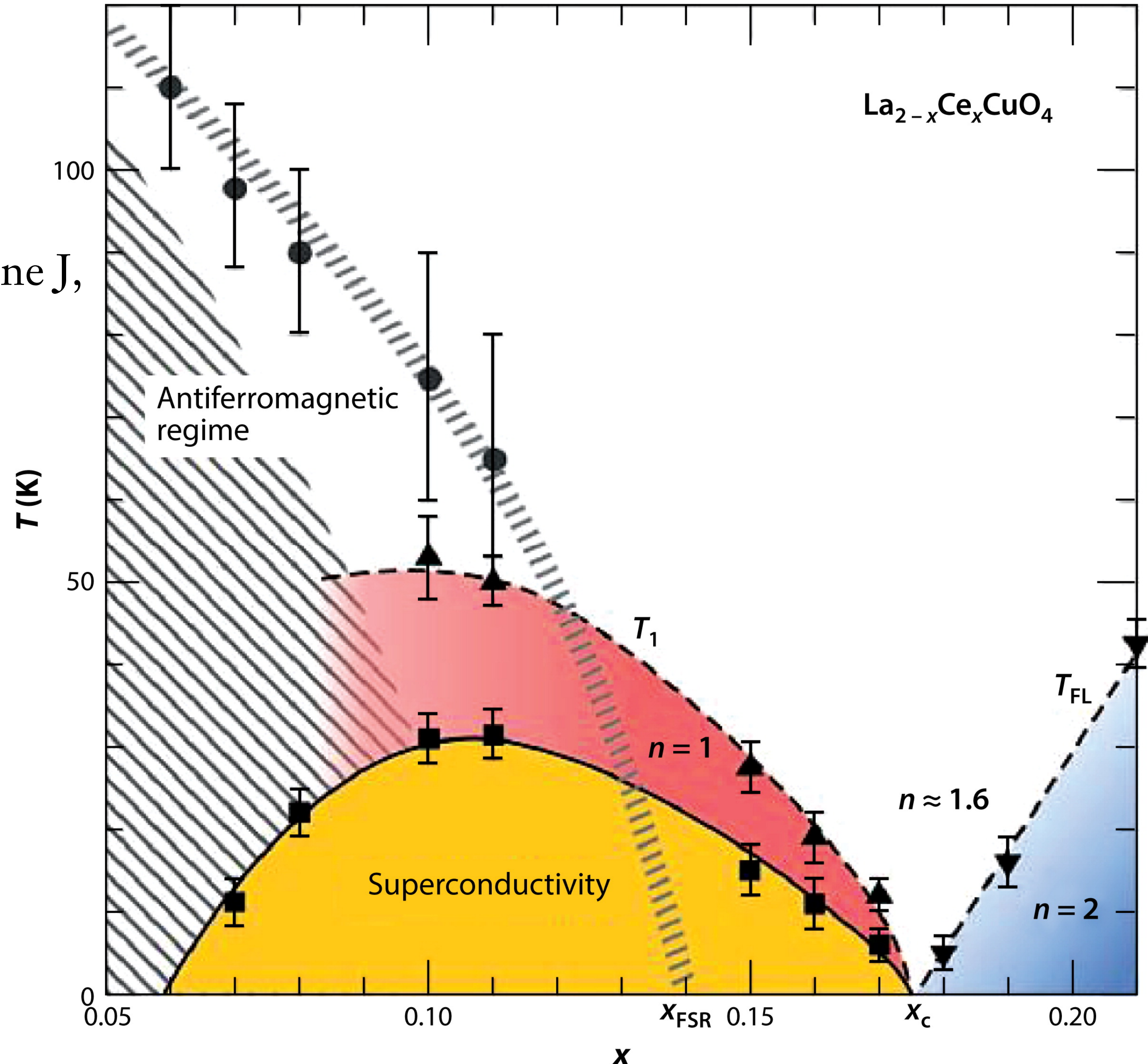
# Anomalous Criticality in the Electrical Resistivity of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

R. A. Cooper,<sup>1</sup> Y. Wang,<sup>1</sup> B. Vignolle,<sup>2</sup> O. J. Lipscombe,<sup>1</sup> S. M. Hayden,<sup>1</sup> Y. Tanabe,<sup>3</sup> T. Adachi,<sup>3</sup> Y. Koike,<sup>3</sup> M. Nohara,<sup>4\*</sup> H. Takagi,<sup>4</sup> Cyril Proust,<sup>2</sup> N. E. Hussey<sup>1†</sup>

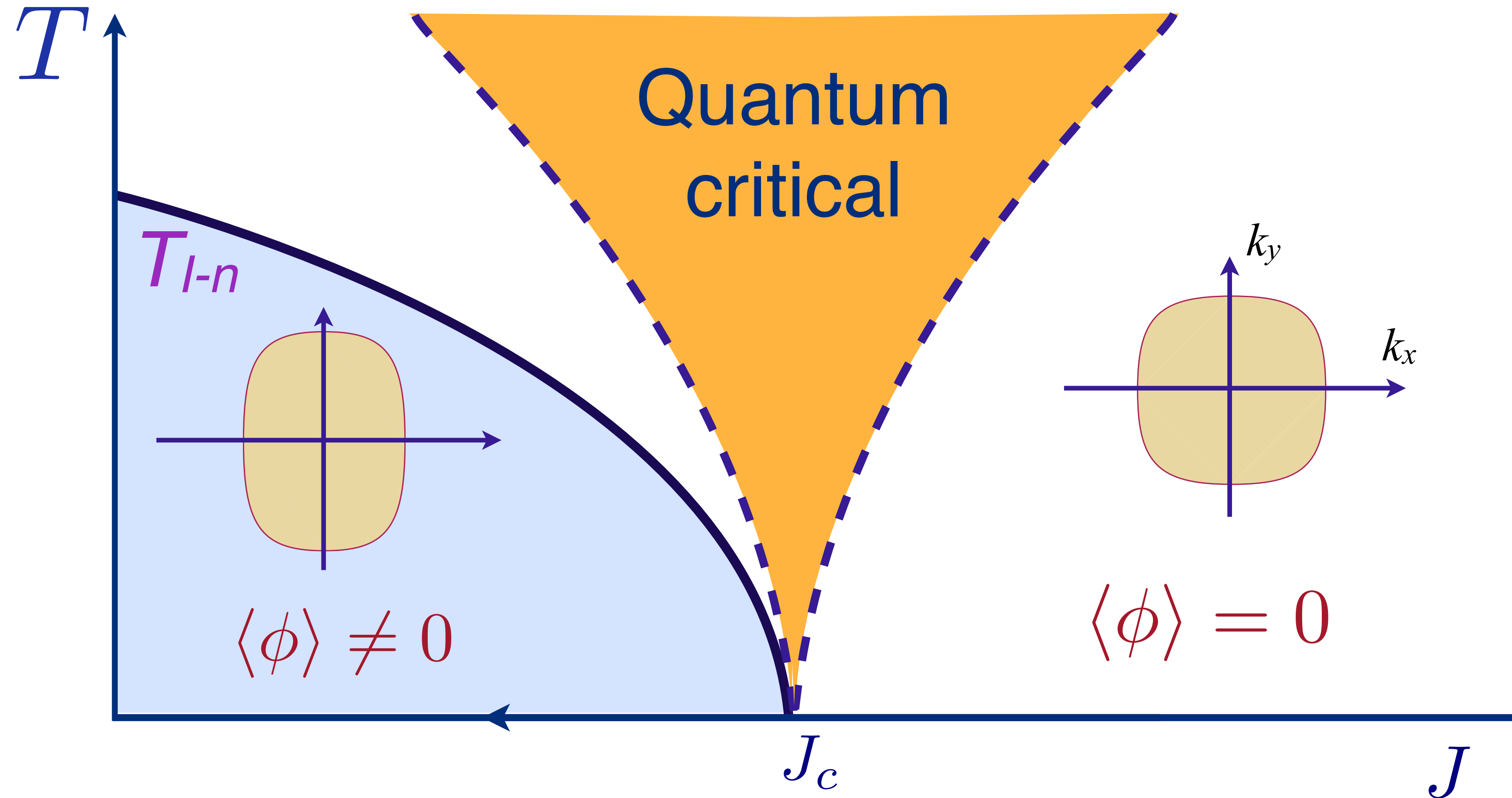
SCIENCE VOL 323 603 2009



Jin K, Butch NP, Kirshenbaum K, Paglione J,  
Greene RL. 2011. *Nature* 476:73–75



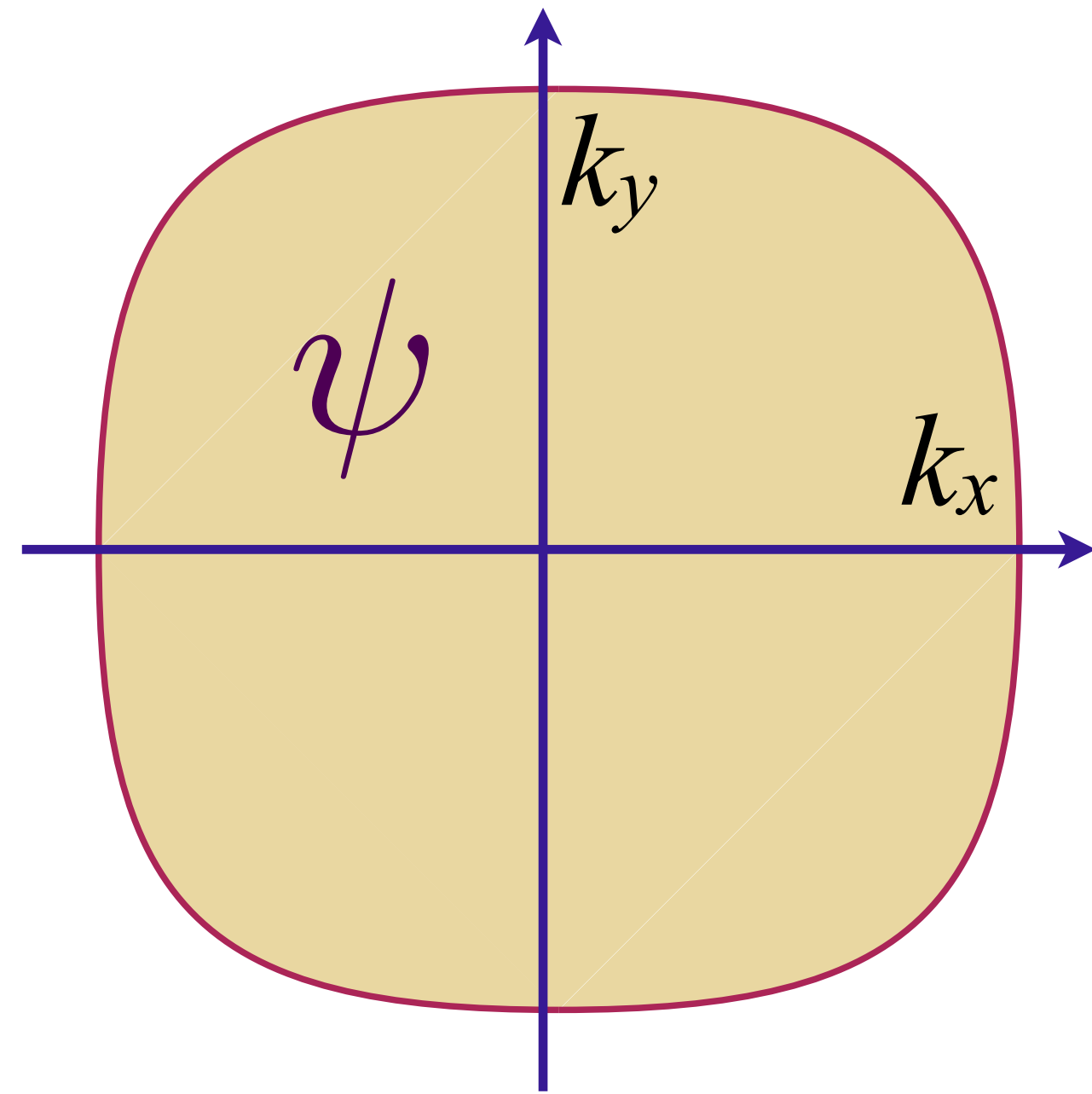
# Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of  $T$  and  $J$

# Fermi surface + critical boson with no spatial disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left( \frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



A critical boson  $\phi$   
e.g. Ising-nematic order,  
spin-density wave order,

Higgs boson for Fermi-volume changing transition

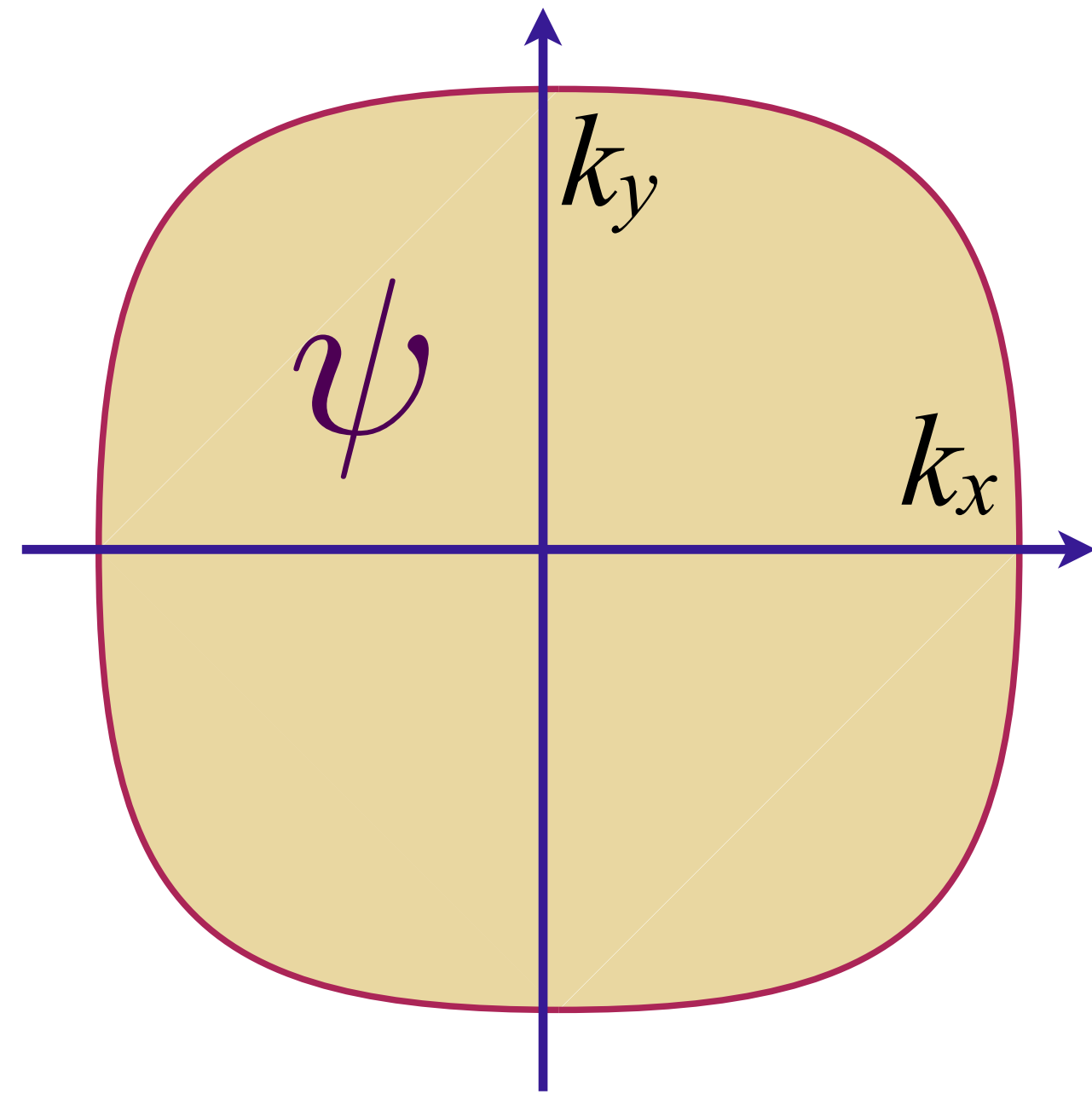
$$+s [\phi(\mathbf{r})]^2$$

$$+g \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r})$$

$$+K [\nabla_{\mathbf{r}} \phi(\mathbf{r})]^2 + u [\phi(\mathbf{r})]^4$$

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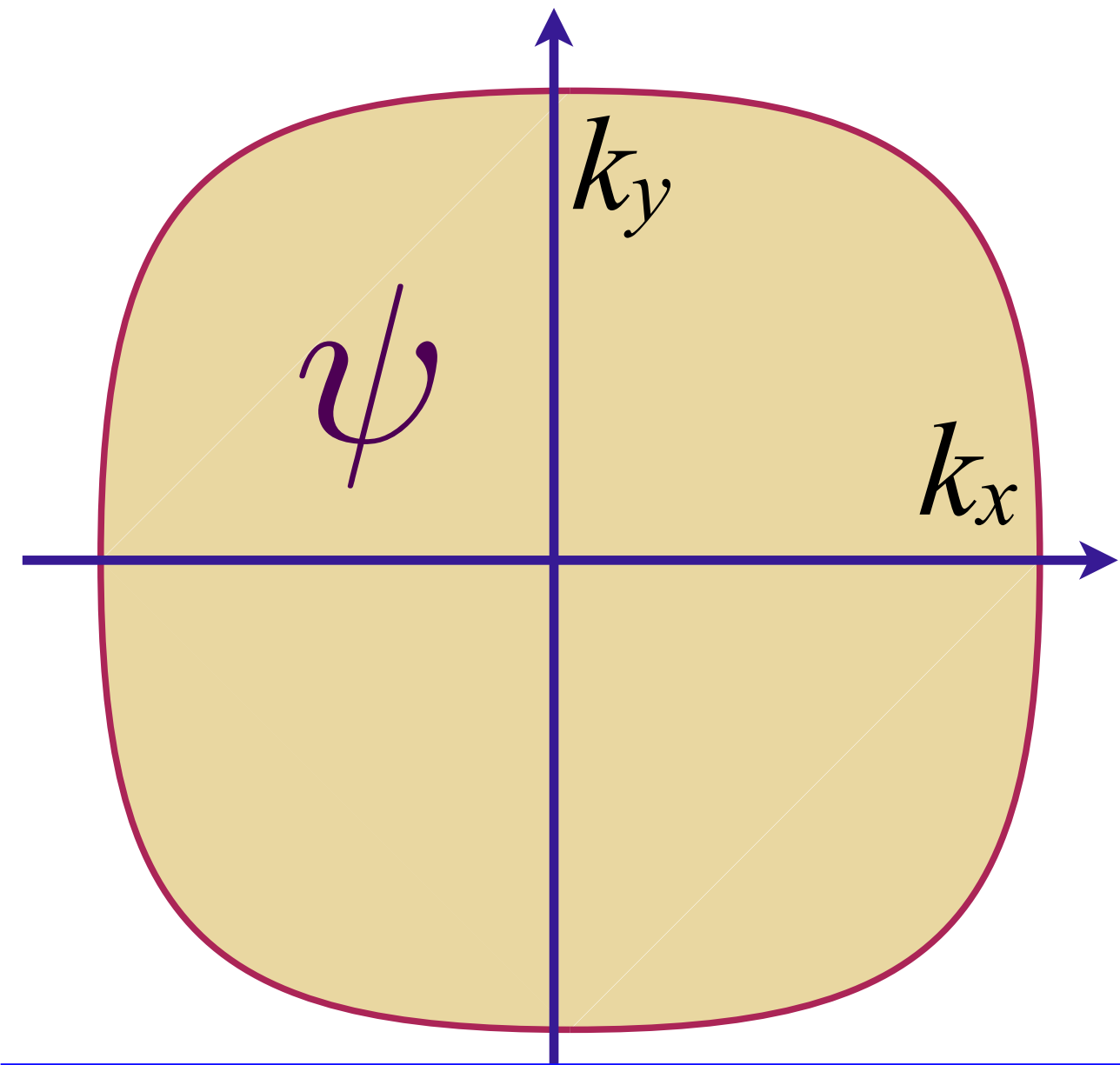


$$\Sigma(\omega) \sim \omega^{2/3}$$
$$\sigma(\omega) = iD/(\omega - \omega_c) + \omega^0 + \dots$$

A non-Fermi liquid in the  
electron spectral function  
but a perfect metal in transport!

# Fermi surface + critical boson with potential disorder

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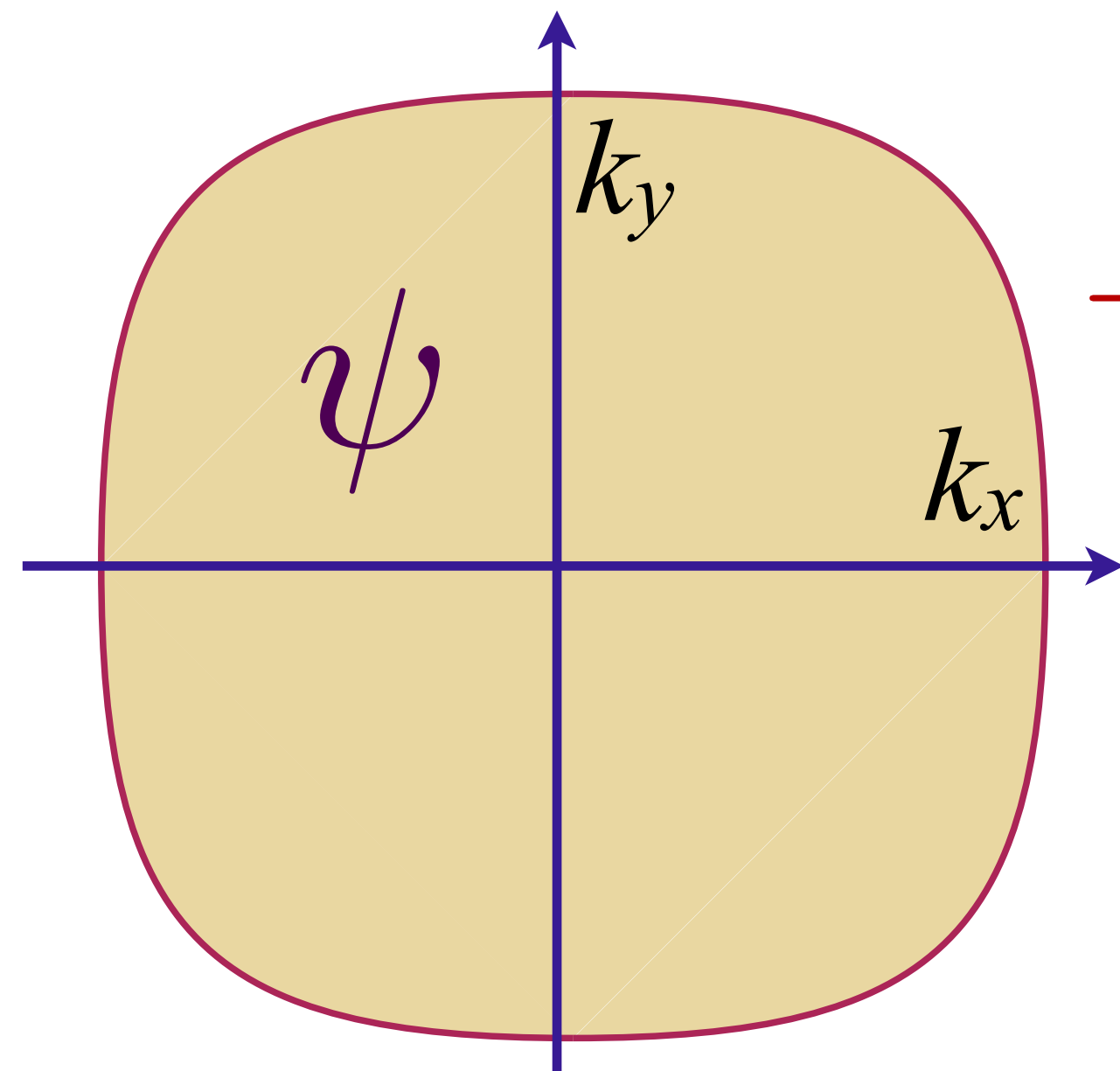
$$+g \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r})$$

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Spatially random potential  $v(\mathbf{r})$  with  $\overline{v(\mathbf{r})} = 0$ ,  $\overline{v(\mathbf{r})v(\mathbf{r}')} = v^2 \delta(\mathbf{r} - \mathbf{r}')$

# Fermi surface + critical boson with potential and interaction disorder

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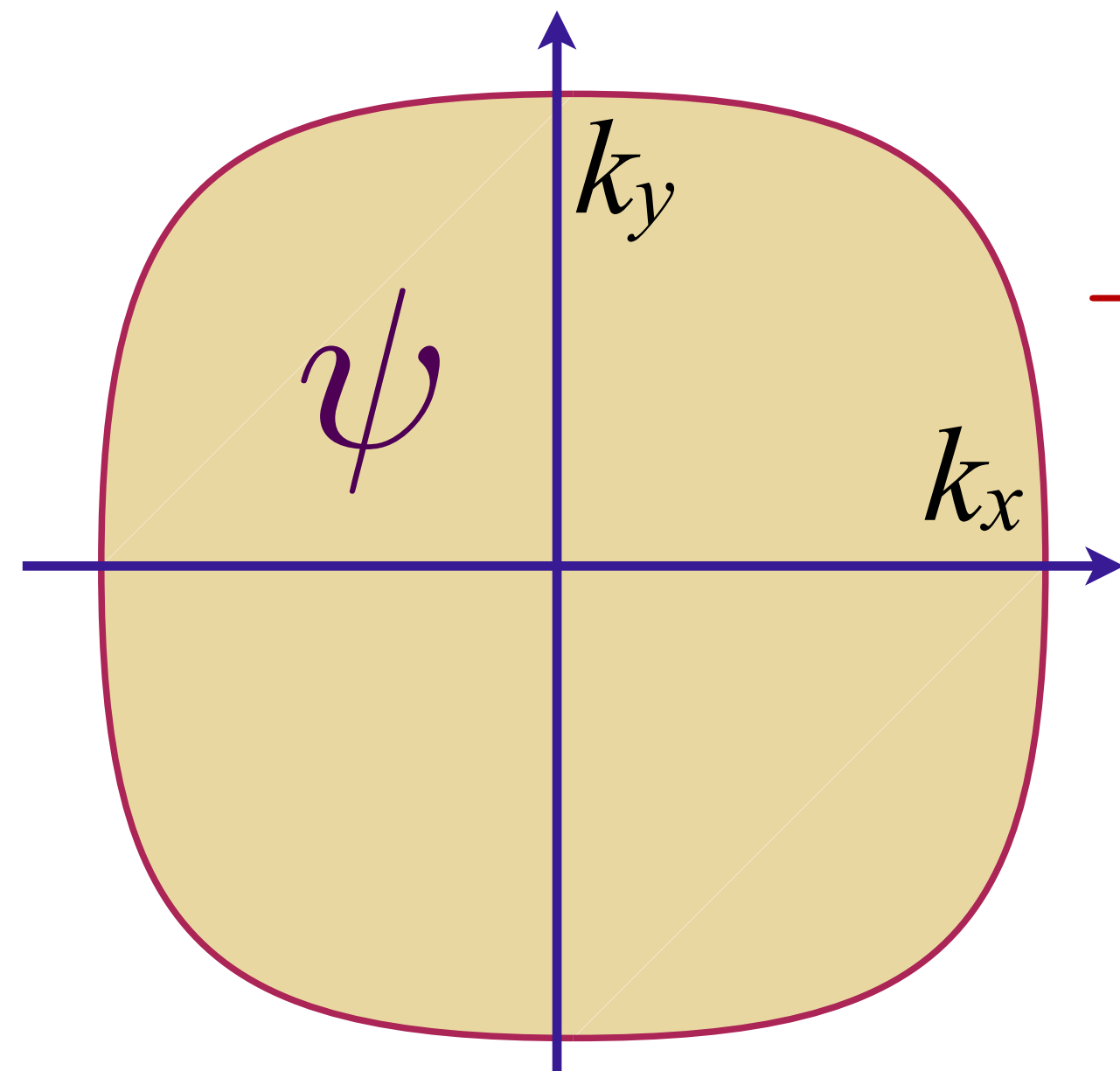
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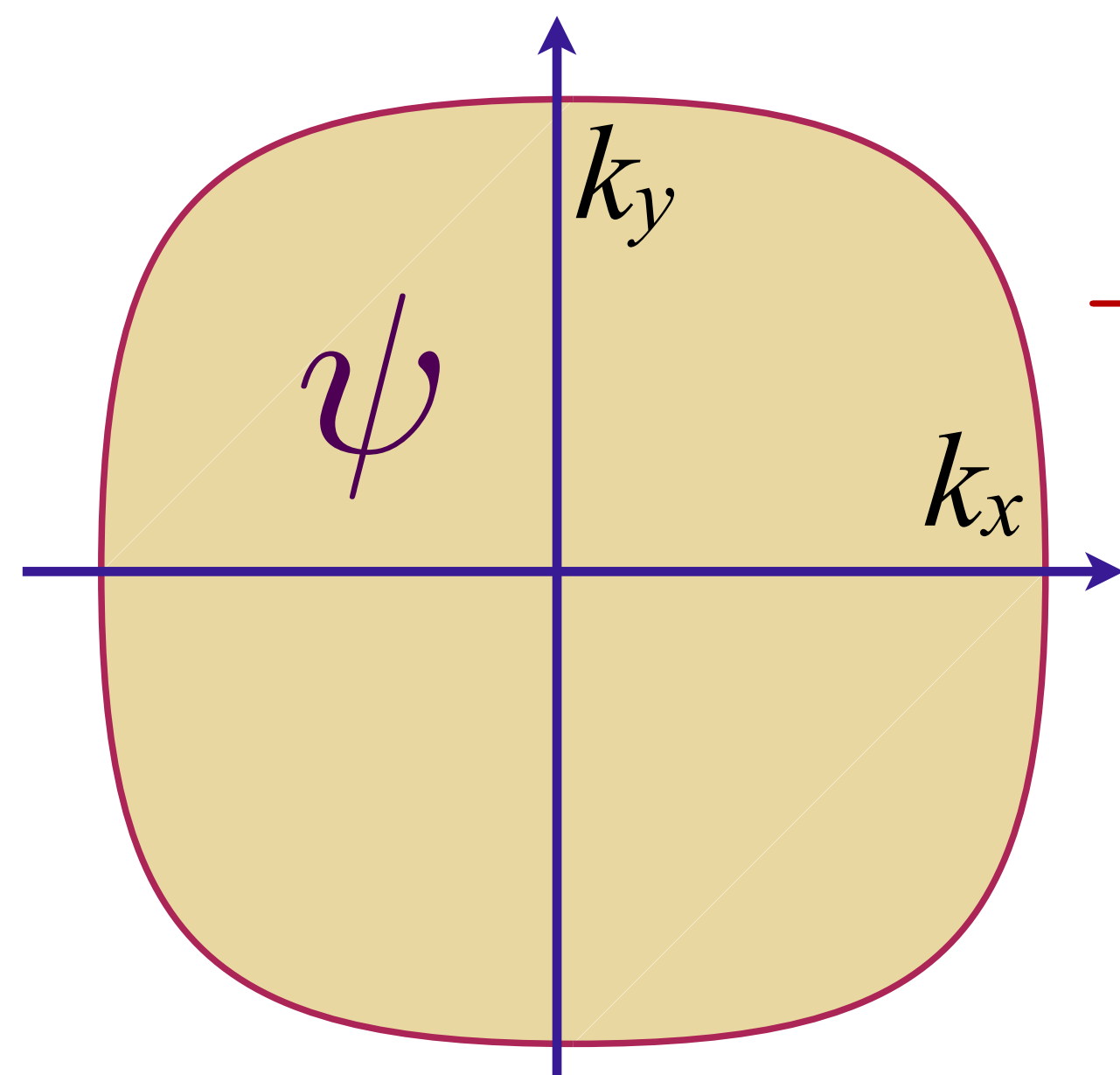
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Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

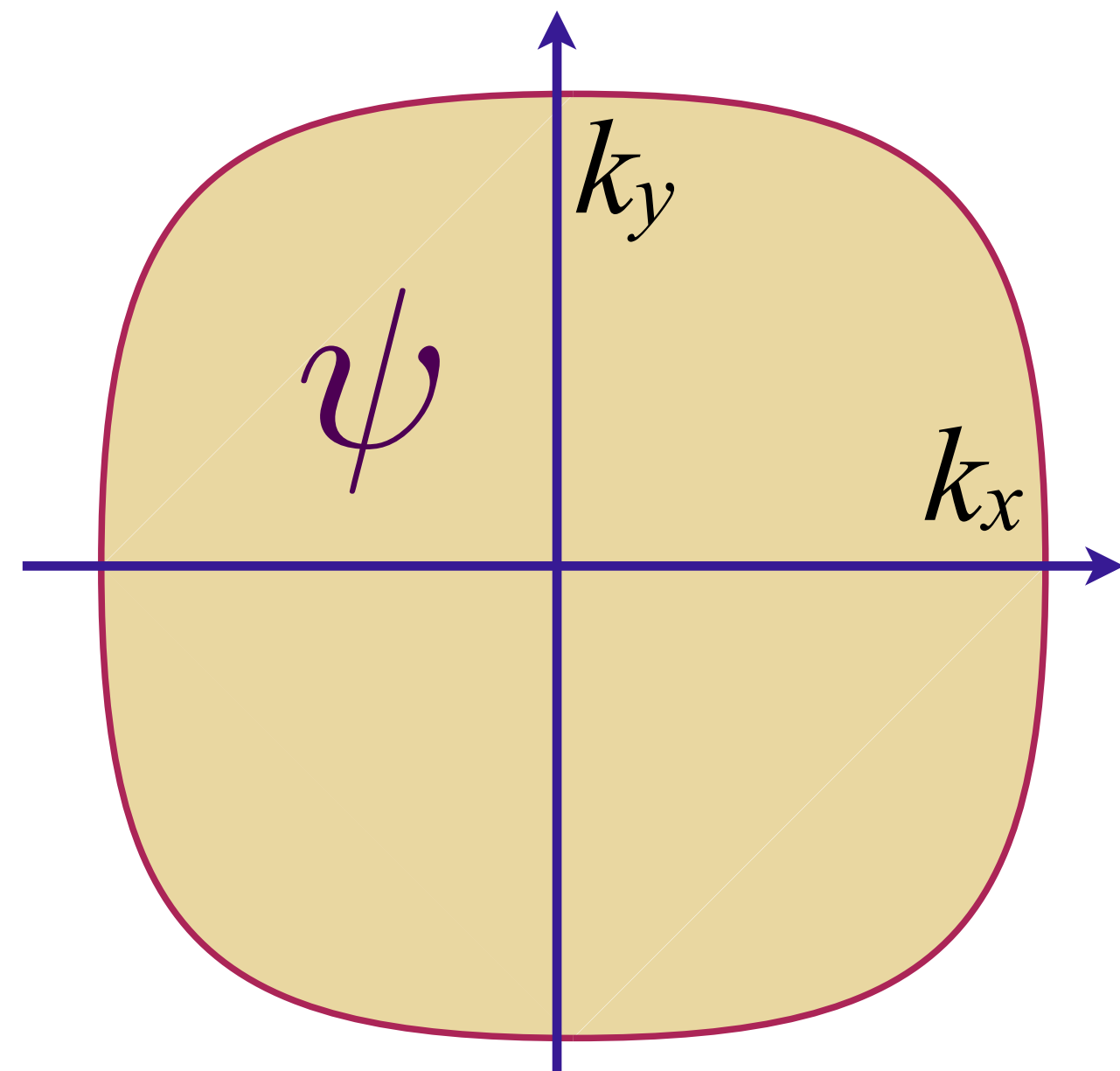
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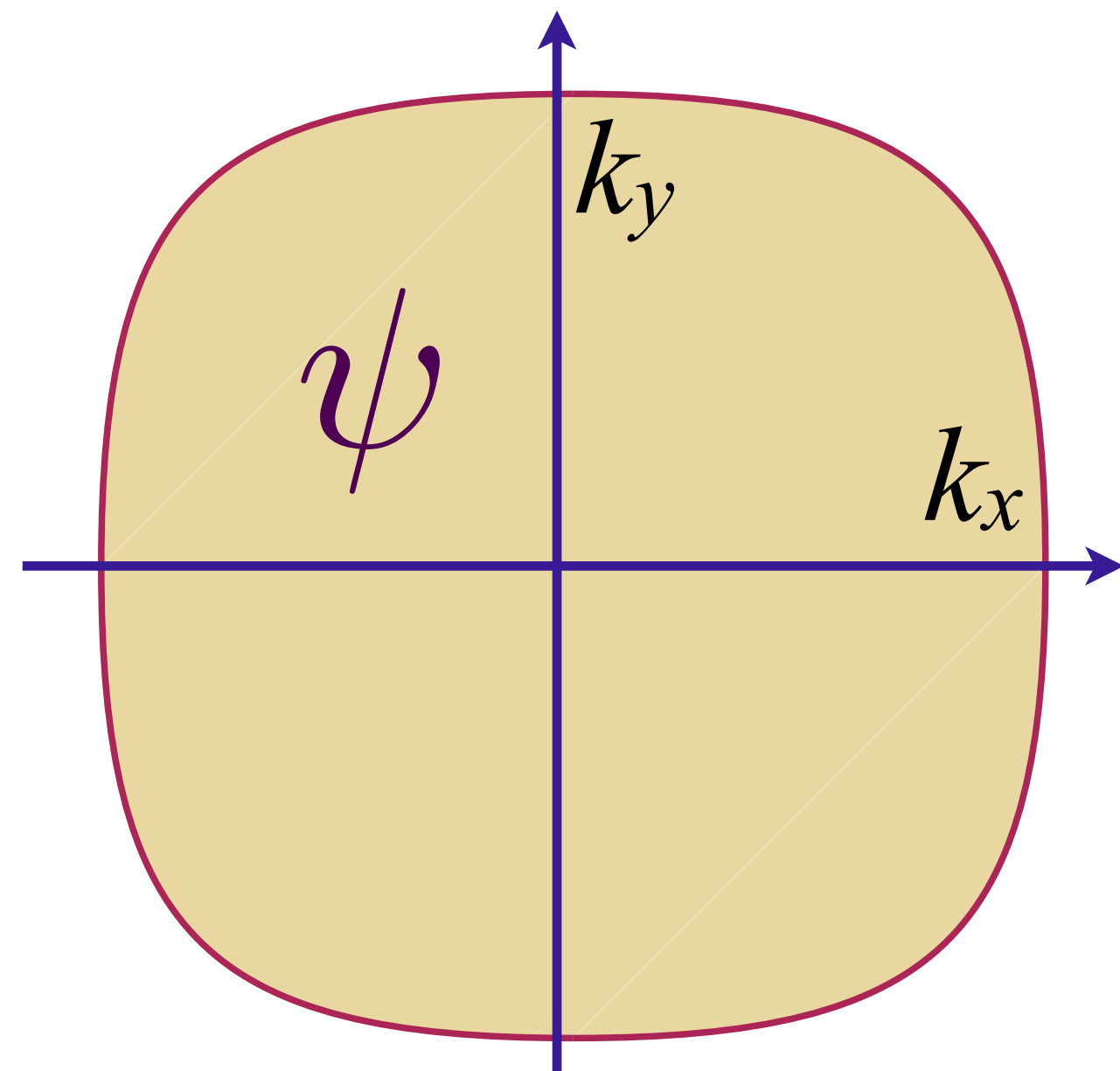
Spatially random Yukawa coupling  $g'(\mathbf{r})$  with  $\overline{g'(\mathbf{r})} = 0$ ,  $\overline{g'(\mathbf{r})g'(\mathbf{r}')} = g'^2 \delta(\mathbf{r} - \mathbf{r}')$

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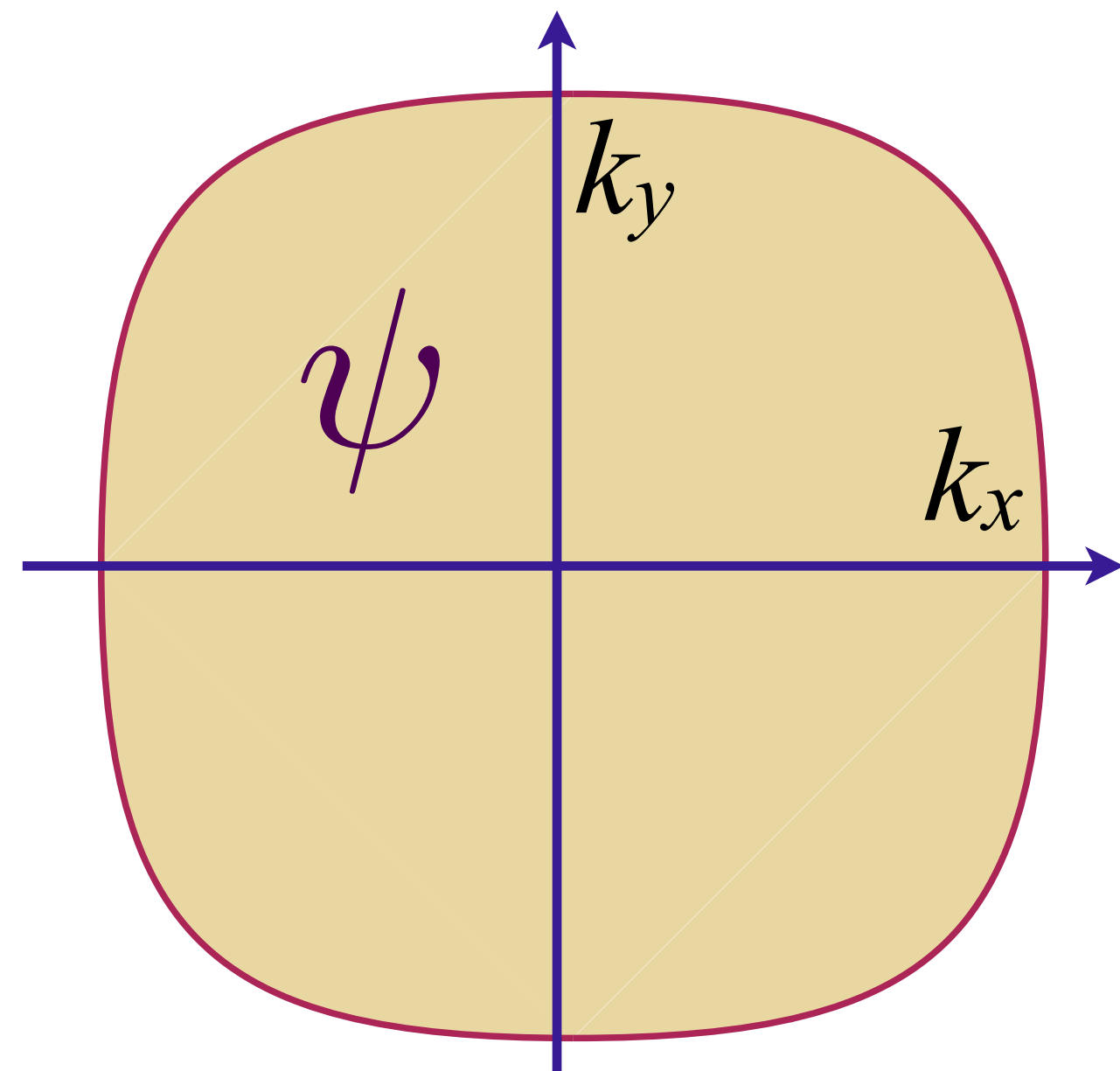
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Analyze such a theory in a self-averaging manner as in the Yukawa-SYK model.

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Analyze such a theory in a self-averaging manner as in the Yukawa-SYK model.  
 Should be applicable as long as eigenmodes of  $\phi(\mathbf{r})$  are extended.

# Fermi surface + critical boson with potential and interaction disorder

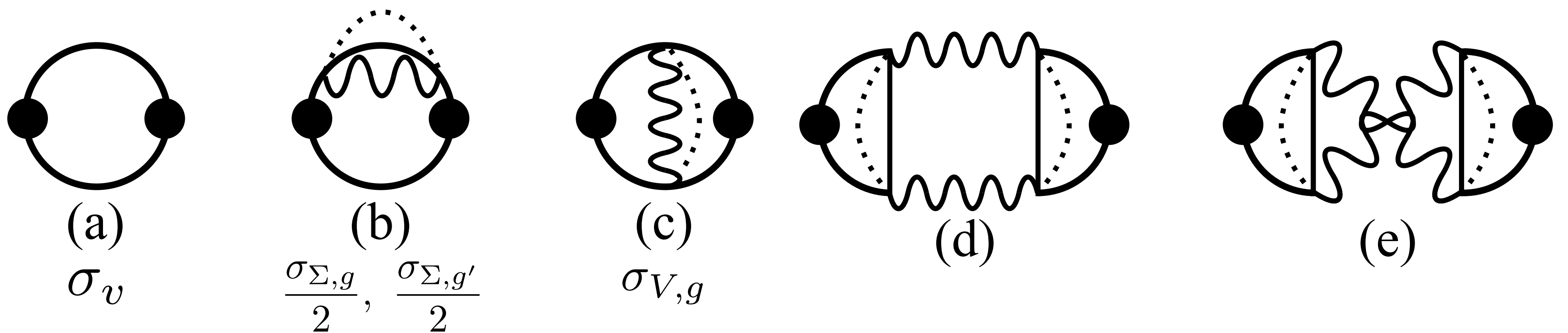
SYK-type self-consistent equations

$$\Sigma(\tau, \mathbf{r}) = g^2 D(\tau, \mathbf{r}) G(\tau, \mathbf{r}) + v^2 G(\tau, \mathbf{r}) \delta^2(\mathbf{r}) + g'^2 G(\tau, \mathbf{r}) D(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

$$\Pi(\tau, \mathbf{r}) = -g^2 G(-\tau, -\mathbf{r}) G(\tau, \mathbf{r}) - g'^2 G(-\tau, \mathbf{r}) G(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

$$G(i\omega, \mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega, \mathbf{k})},$$

$$D(i\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega, \mathbf{q})}.$$



+ all ladders and bubbles.....

Conductivity:

# Fermi surface + critical boson with potential and interaction disorder

Electron Green's function:  $G(\omega) \sim \frac{1}{\omega \frac{m^*(\omega)}{m} - \varepsilon(\mathbf{k}) + i \left( \frac{1}{\tau_e} + \frac{1}{\tau_{\text{in}}(\omega)} \right) \text{sgn}(\omega)}$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\text{in}}(\omega)} \sim \left( \frac{g^2}{v^2} + g'^2 \right) |\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi} \left( \frac{g^2}{v^2} + g'^2 \right) \ln(\Lambda/\omega)$$

T.J. Reber....D. Dessau, Nature Communications **10**, 5737 (2019)

Marginal Fermi liquid self energy and  $T \ln(1/T)$  specific heat.

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Conductivity: 
$$\sigma(\omega) \sim \frac{1}{\frac{1}{\tau_{\text{trans}}(\omega)} - i\omega \frac{m_{\text{trans}}^*(\omega)}{m}}$$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2 |\omega| \quad ; \quad \frac{m_{\text{trans}}^*(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

B. Michon.....A. Georges, Nat. Commun. **14**, 3033 (2023)

Marginal Fermi liquid self energy and  $T \ln(1/T)$  specific heat.

Residual resistivity is determined by  $v^2$ ; Linear-in- $T$  resistivity determined by  $g'^2$ ;

Transport insensitive to  $g$ ;

# Quantum Interference of Hydrodynamic Modes in a Dirty Marginal Fermi Liquid

Tsz Chun Wu, Yunxiang Liao, Matthew S. Foster  
Phys. Rev. B **106**, 155108 (2022)

Diffusion (Altshuler-Aronov) corrections to conductivity are singular  $\sim -1/T$ .

Our interpretation:

need to consider the feedback on the boson propagator from stronger disorder, where the dominant effect is the localization of overdamped bosonic modes ...

# Landau-damped bosonic eigenmodes with random mass

Integrate out the fermions (assuming fermionic eigenmodes remain extended), and considering the Landau-damped Hertz theory for the boson alone, in the presence of a random mass.

$$\mathcal{S}_\phi = \int d\tau \left[ \frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \left( \frac{\lambda + \lambda'_j}{2} \phi_{ja}^2 + \frac{u}{4M} (\phi_{ja}^2)^2 \right) \right]$$

$$\mathcal{S}_{\phi d} = \frac{T}{2} \sum_{\Omega} \sum_j (\gamma |\Omega| + \Omega^2 / c^2) |\phi_{ja}(i\Omega)|^2,$$

where  $a = 1 \dots M$  is a flavor index for an order parameter with  $O(M)$  symmetry. Analyze in a self-consistent quadratic theory, treating disorder numerically exactly

$$\bar{\mathcal{S}}_\phi = \int d\tau \left[ \frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \frac{\bar{\lambda}'_j}{2} \phi_{ja}^2 \right]$$

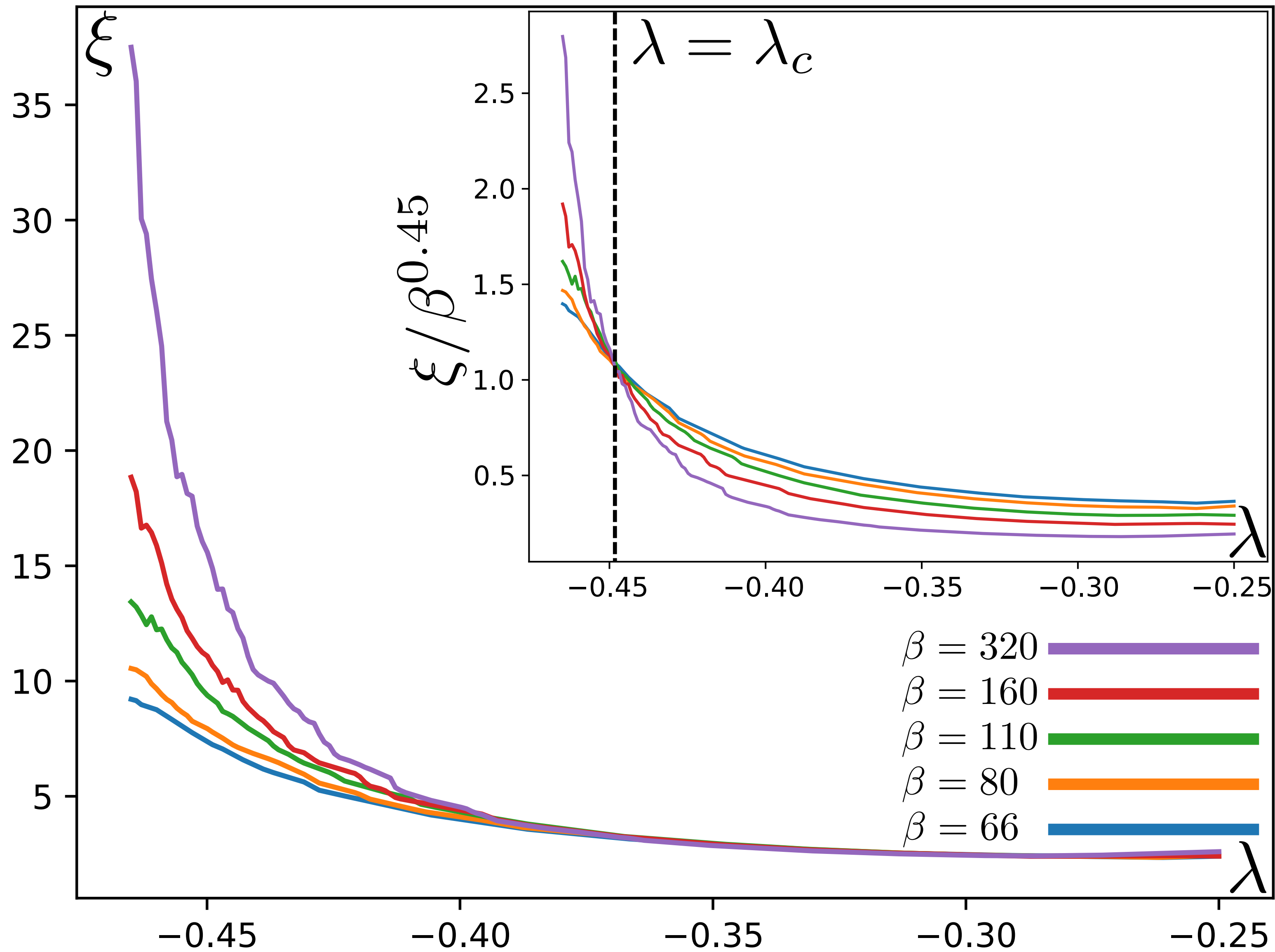
Similar analysis in  $d = 1$  works very well  
A. Del Maestro, B. Rosenow, M. Müller and S. Sachdev,  
Phys. Rev. Lett. **101**, 035701 (2008).

$$\bar{\lambda}'_j = \lambda + \lambda'_j + \frac{u}{M} \sum_a \langle \phi_{ja}^2 \rangle_{\bar{\mathcal{S}}_\phi + \mathcal{S}_{\phi d}} = \lambda + \lambda'_j + uT \sum_{\Omega} \sum_{\alpha} \frac{\psi_{\alpha i} \psi_{\alpha j}}{\gamma |\Omega| + \Omega^2 / c^2 + e_{\alpha}}$$

where  $e_{\alpha}$  and  $\psi_{\alpha j}$  are eigenvalues and eigenfunctions of the  $\phi$  quadratic form in  $\bar{\mathcal{S}}_\phi$ , labeled by the index  $\alpha = 1 \dots L^2$  for a  $L \times L$  sample.

# Landau-damped bosonic eigenmodes with random mass

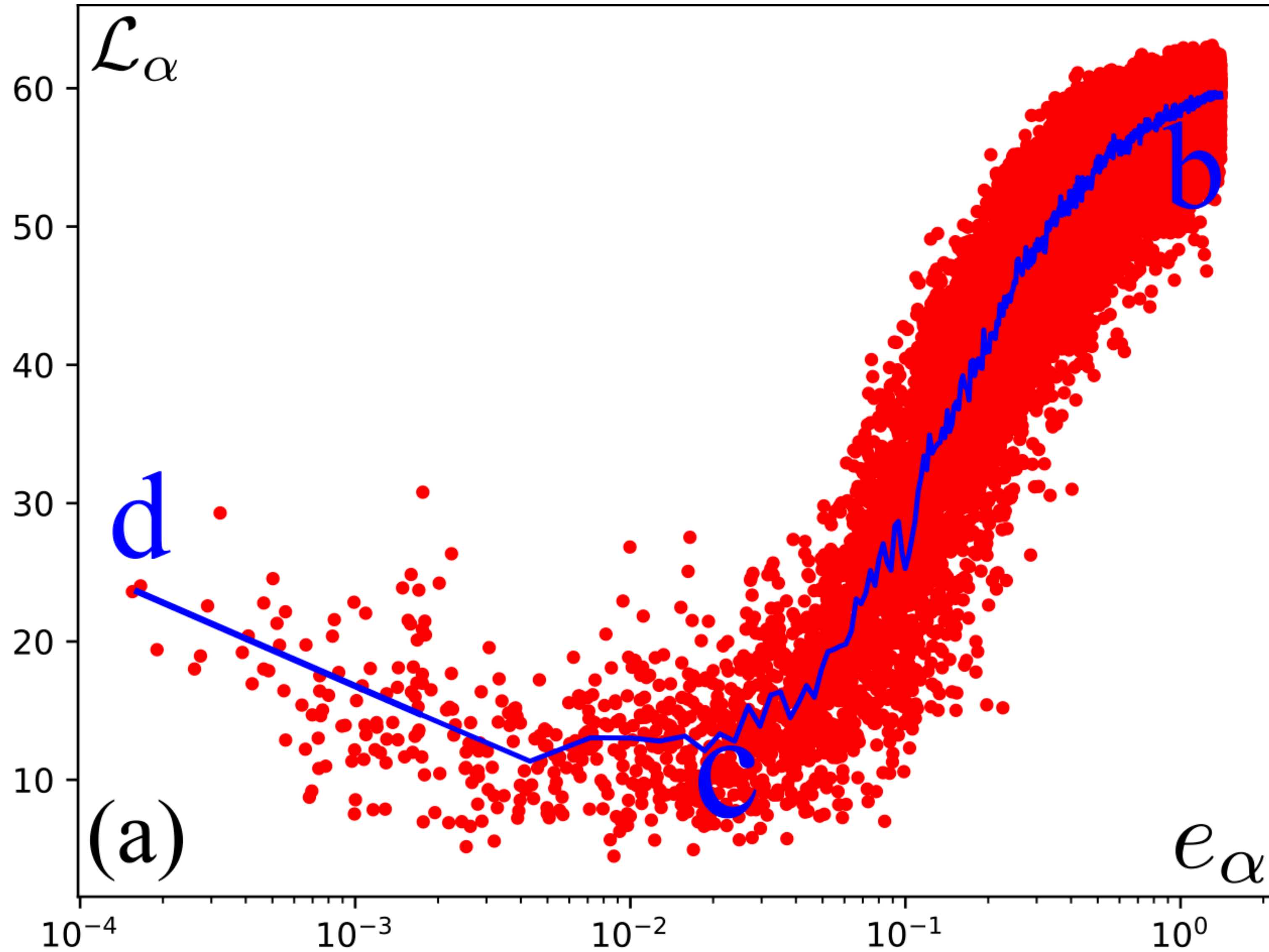
$\phi$  correlation length  $\xi$



Aavishkar A. Patel,  
Peter Lunts, S.S.,  
[arXiv:2312.06751](https://arxiv.org/abs/2312.06751)

# Landau-damped bosonic eigenmodes with random mass

$\phi$  eigenmodes localization length  $\mathcal{L}_\alpha$

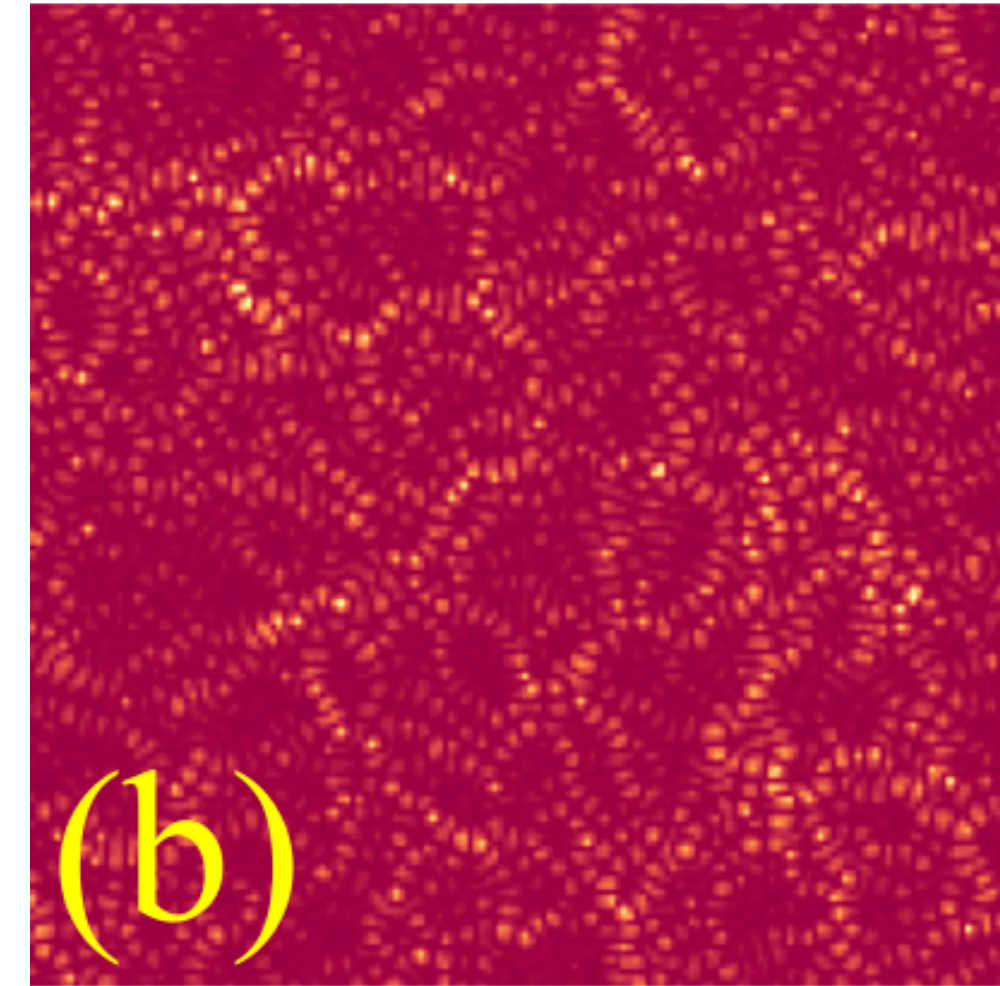
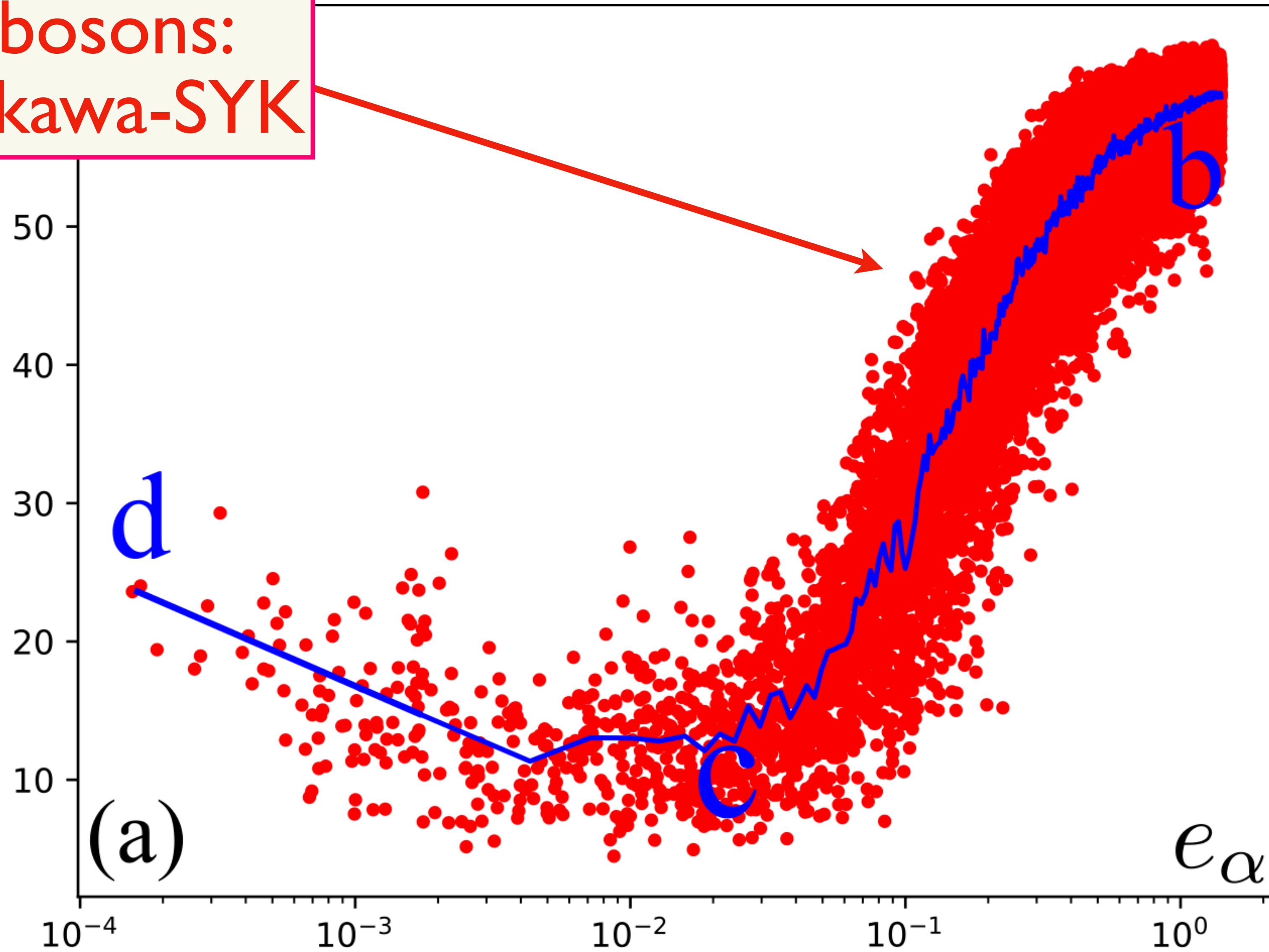


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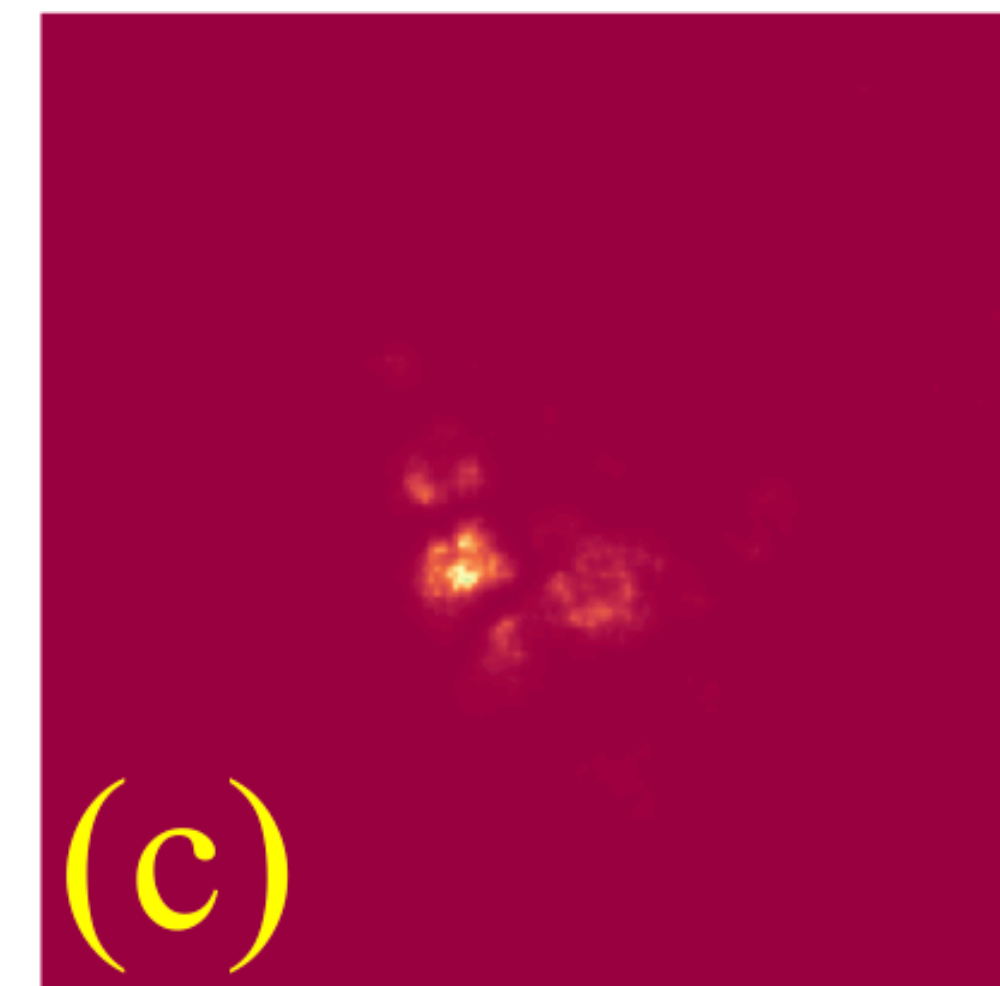
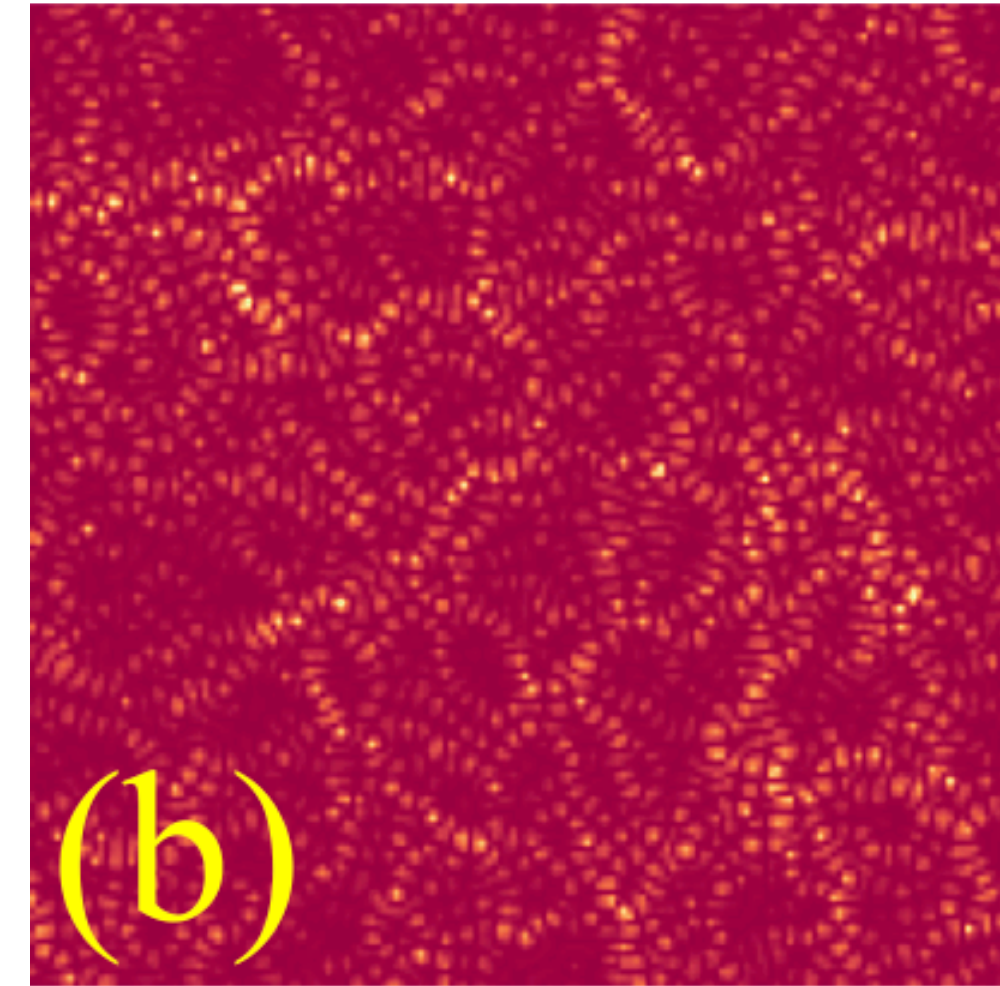
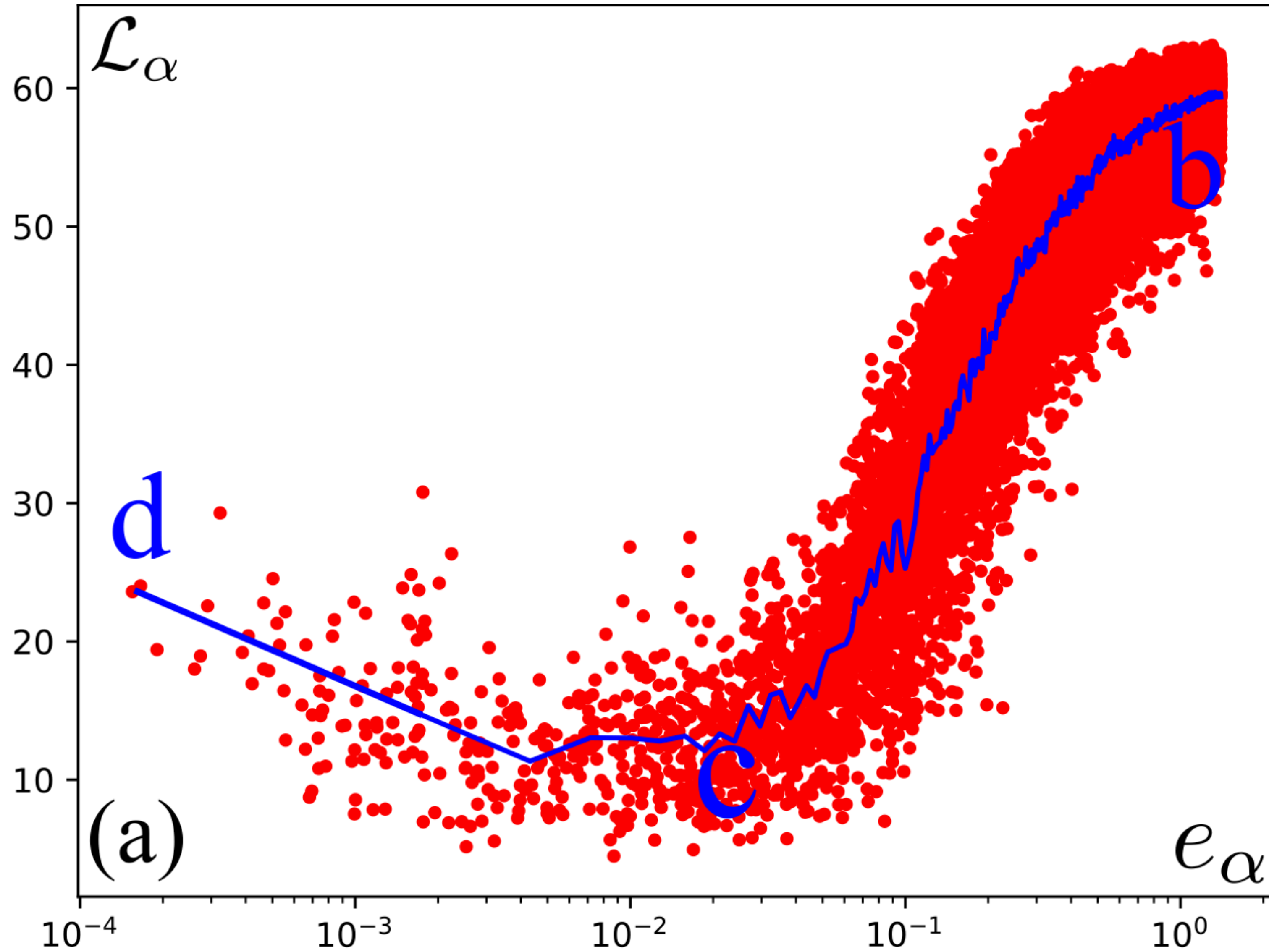
Extended bosons:  
physics of Yukawa-SYK



Aavishkar A. Patel,  
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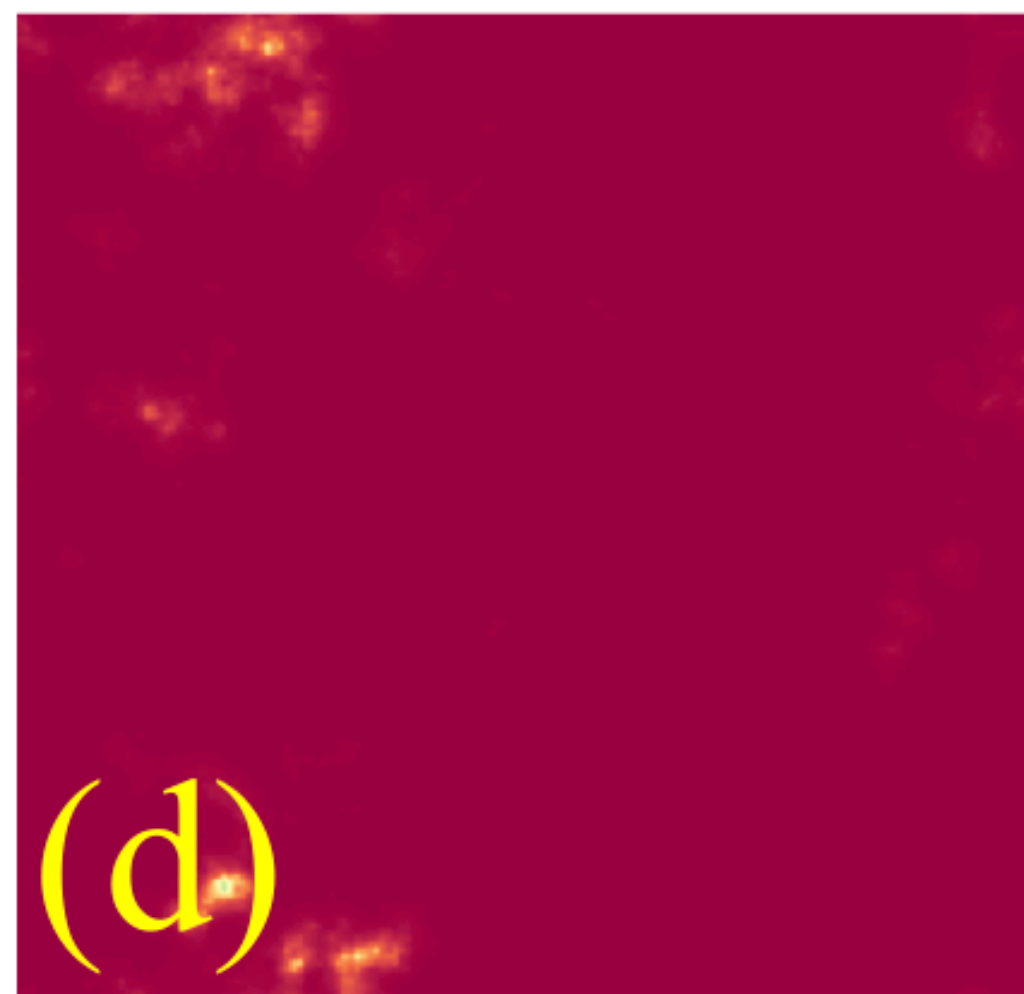
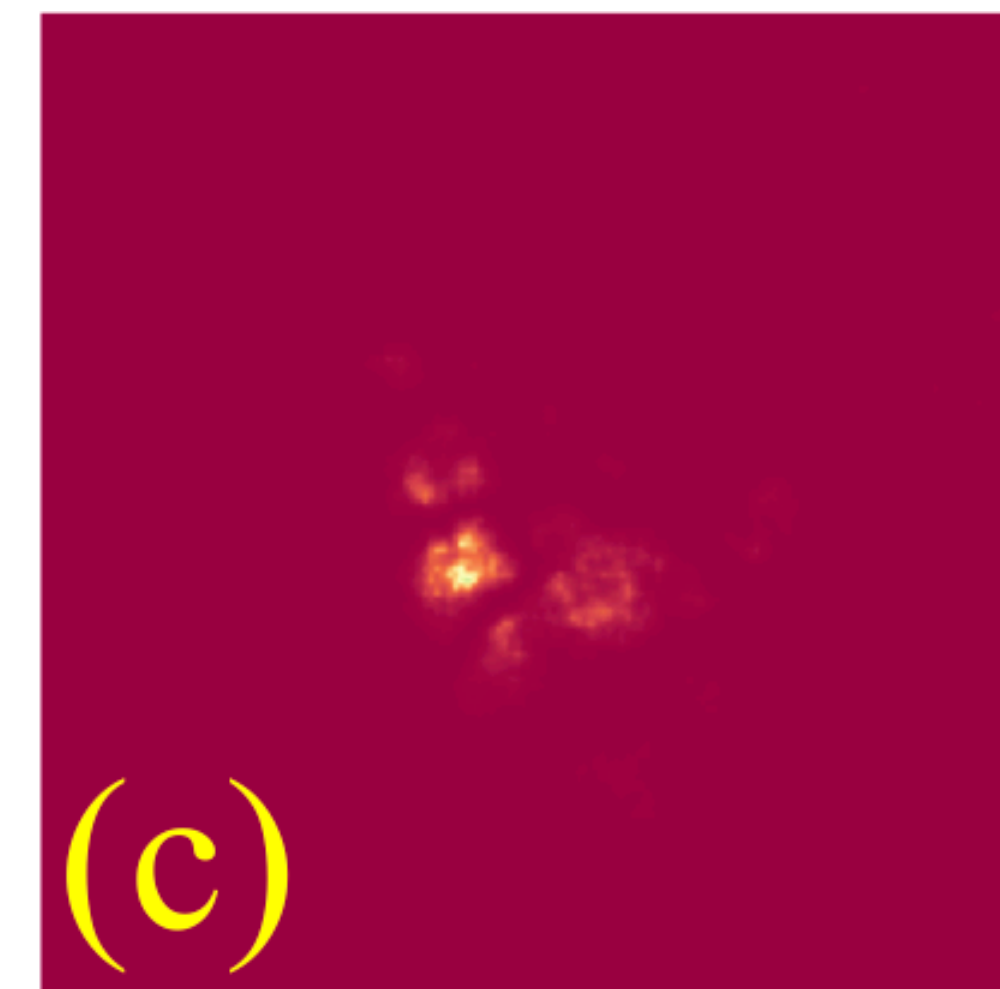
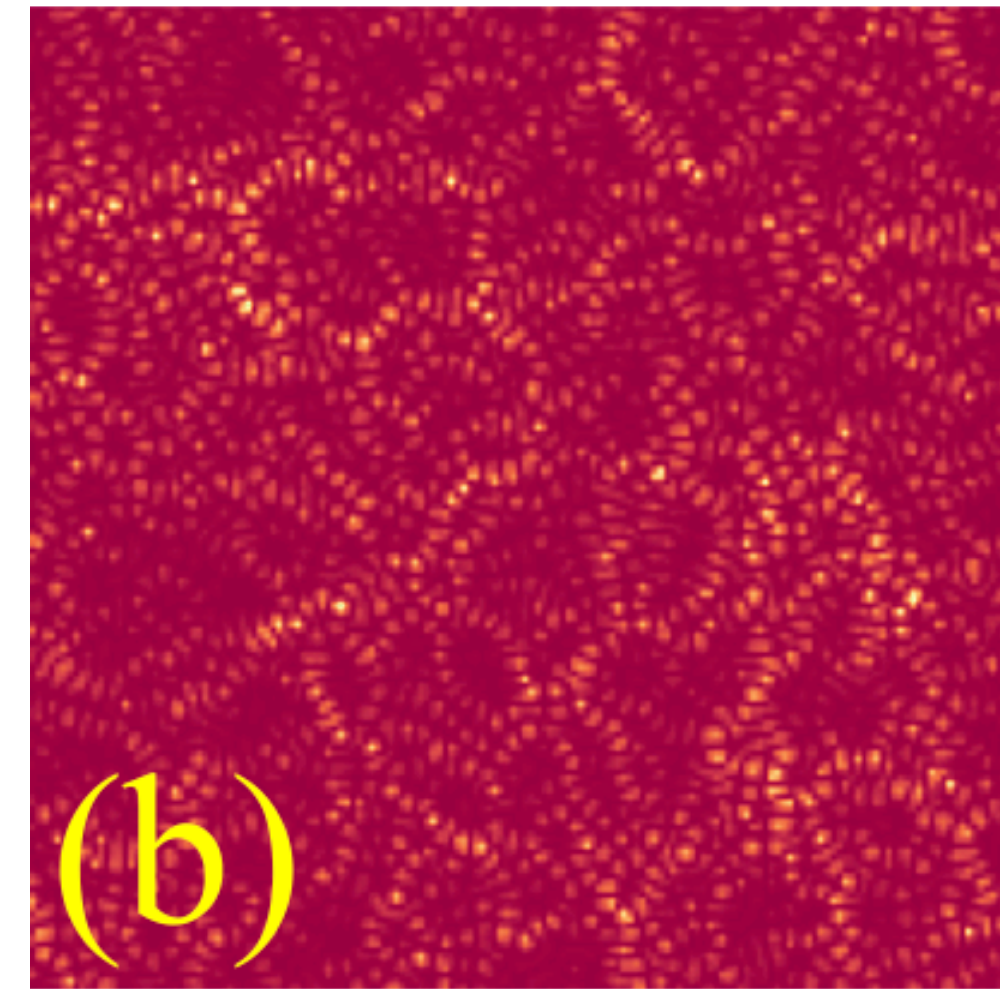
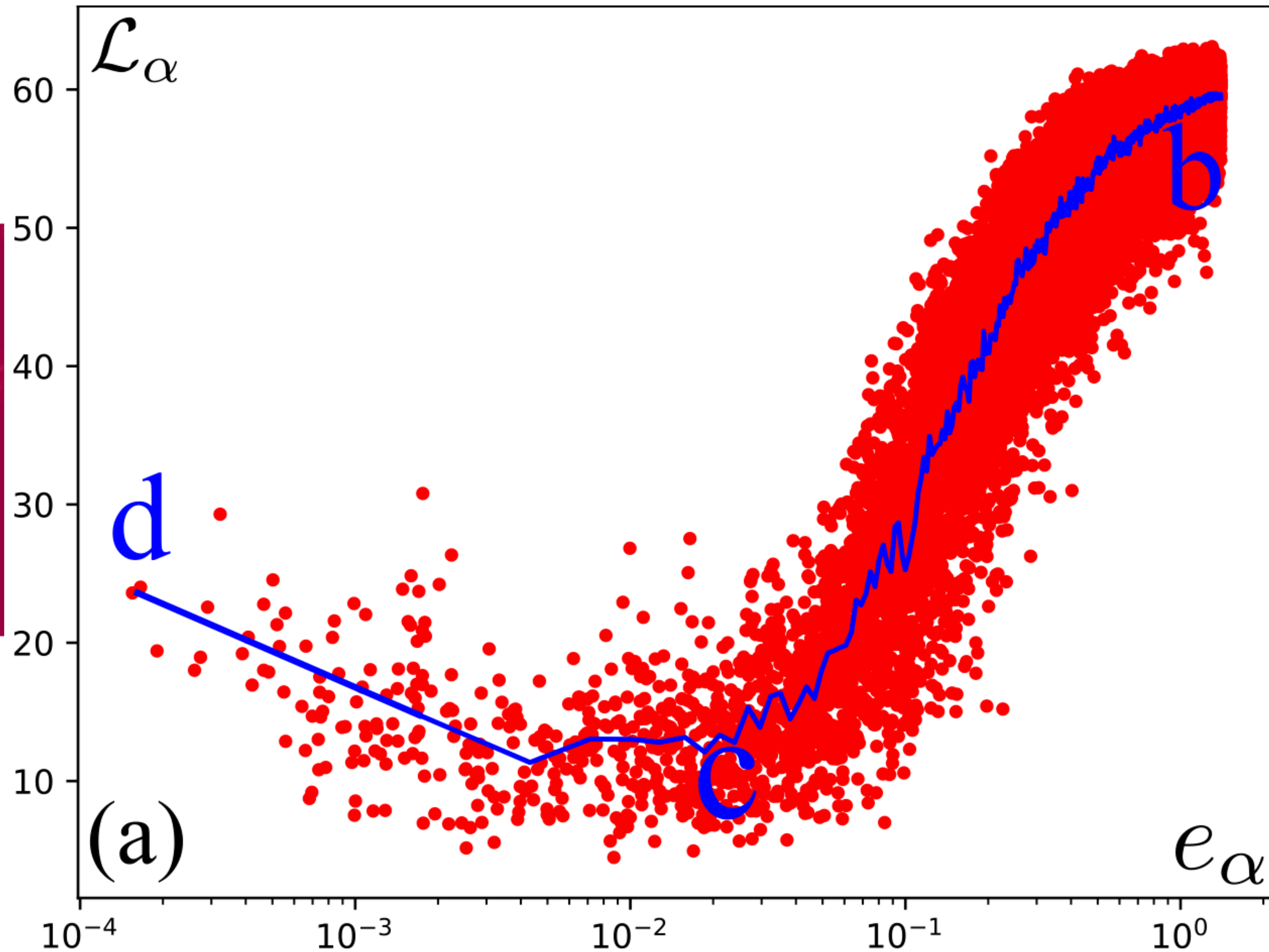
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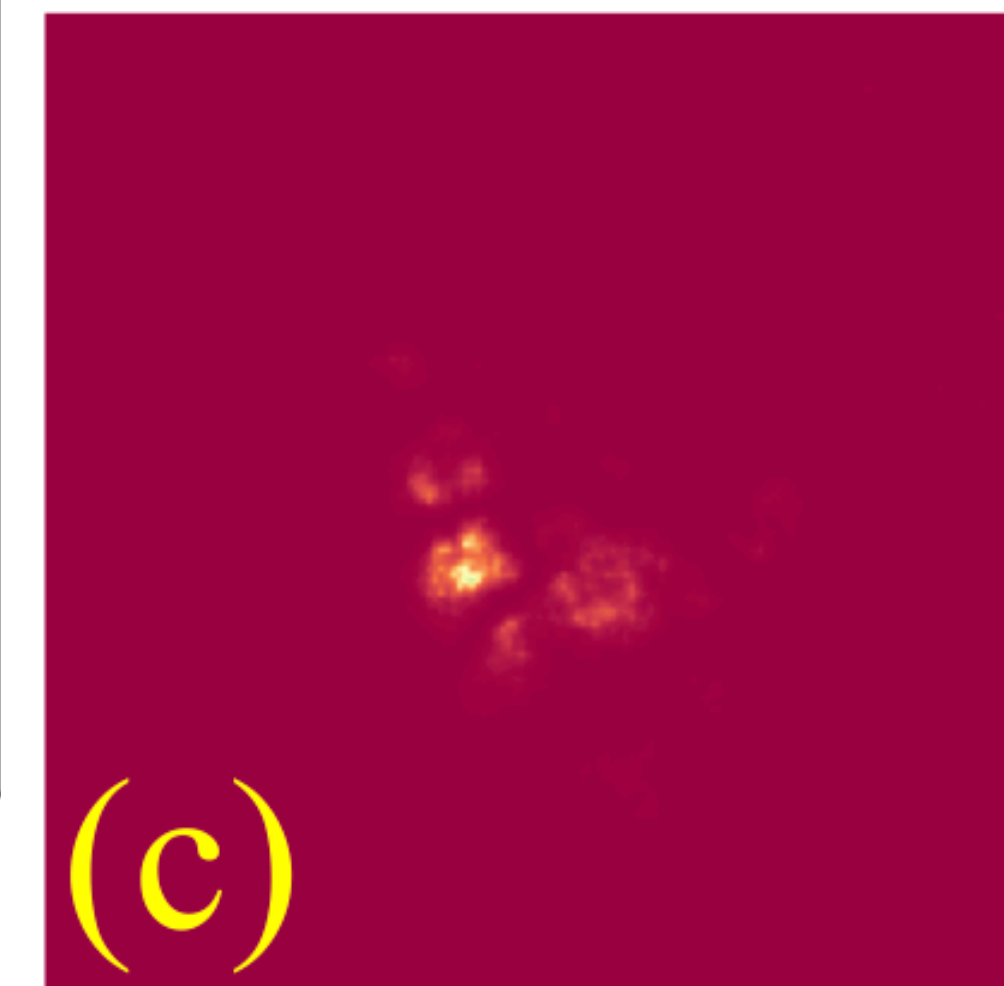
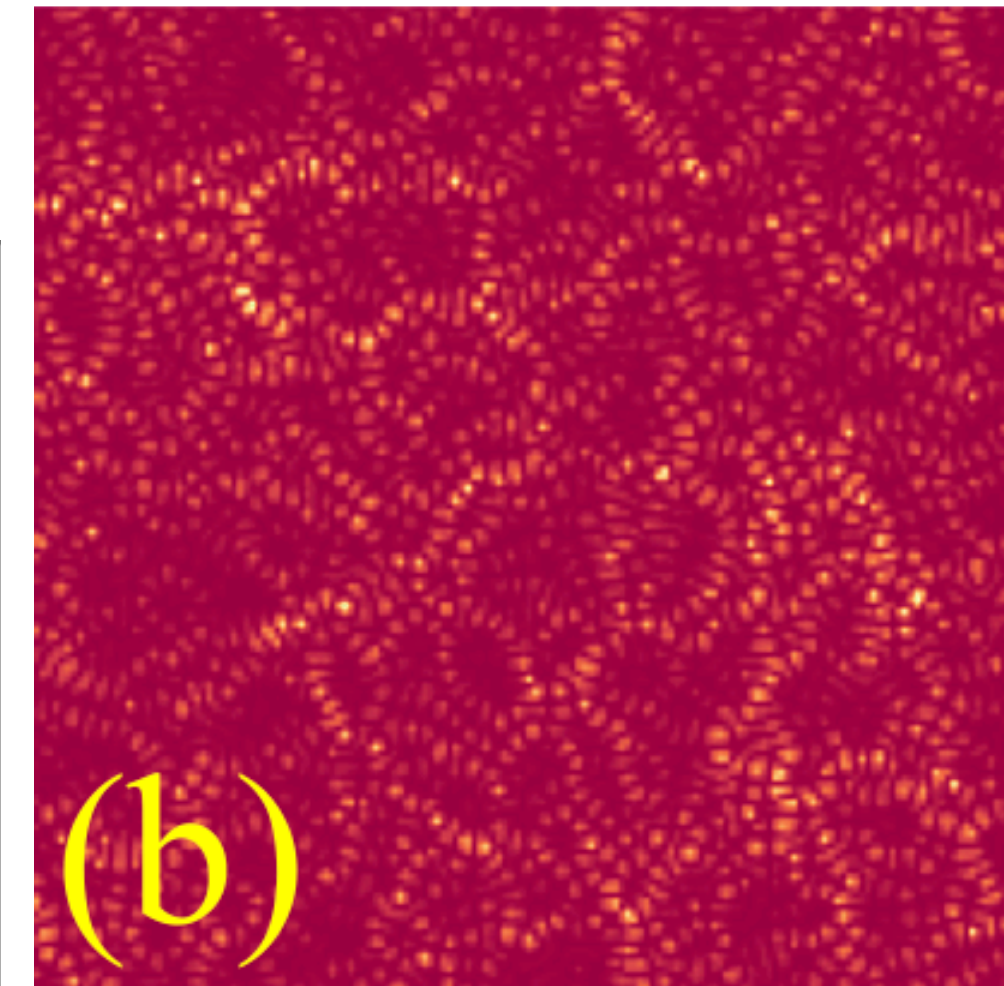
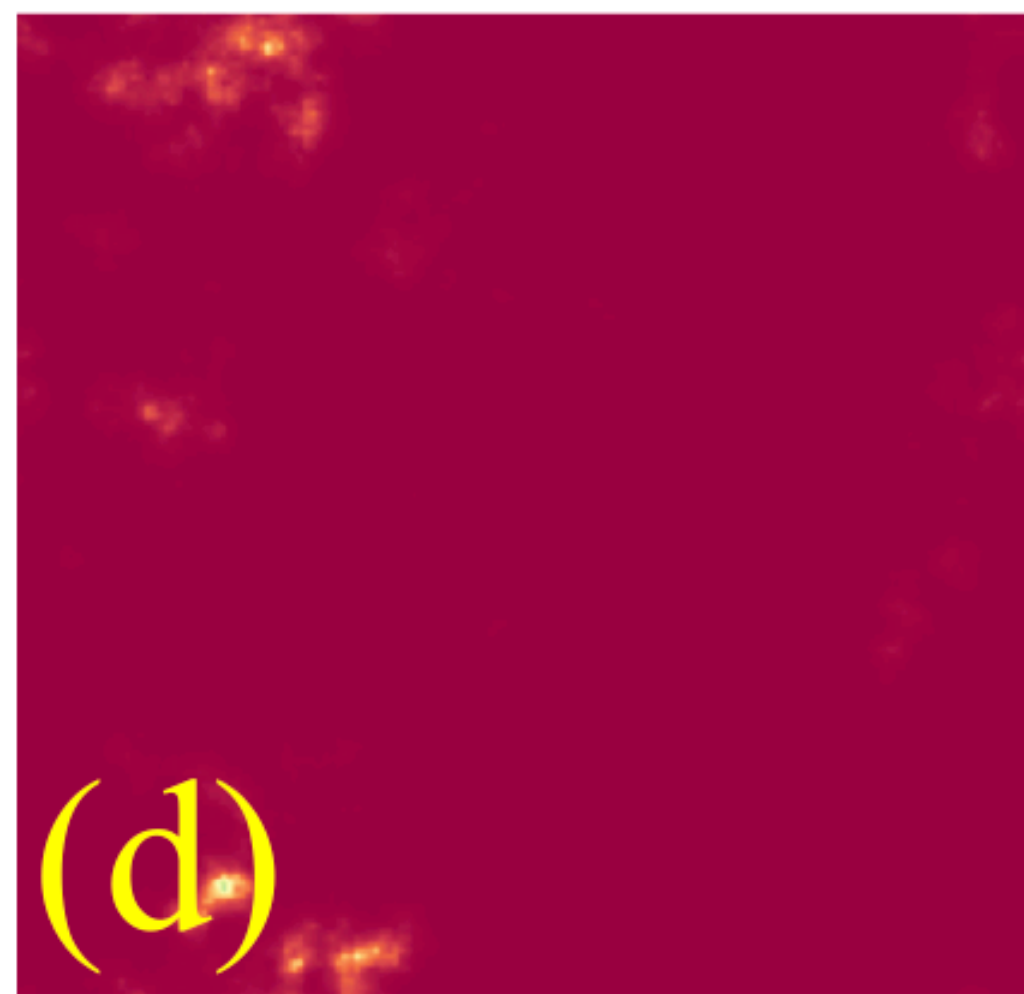
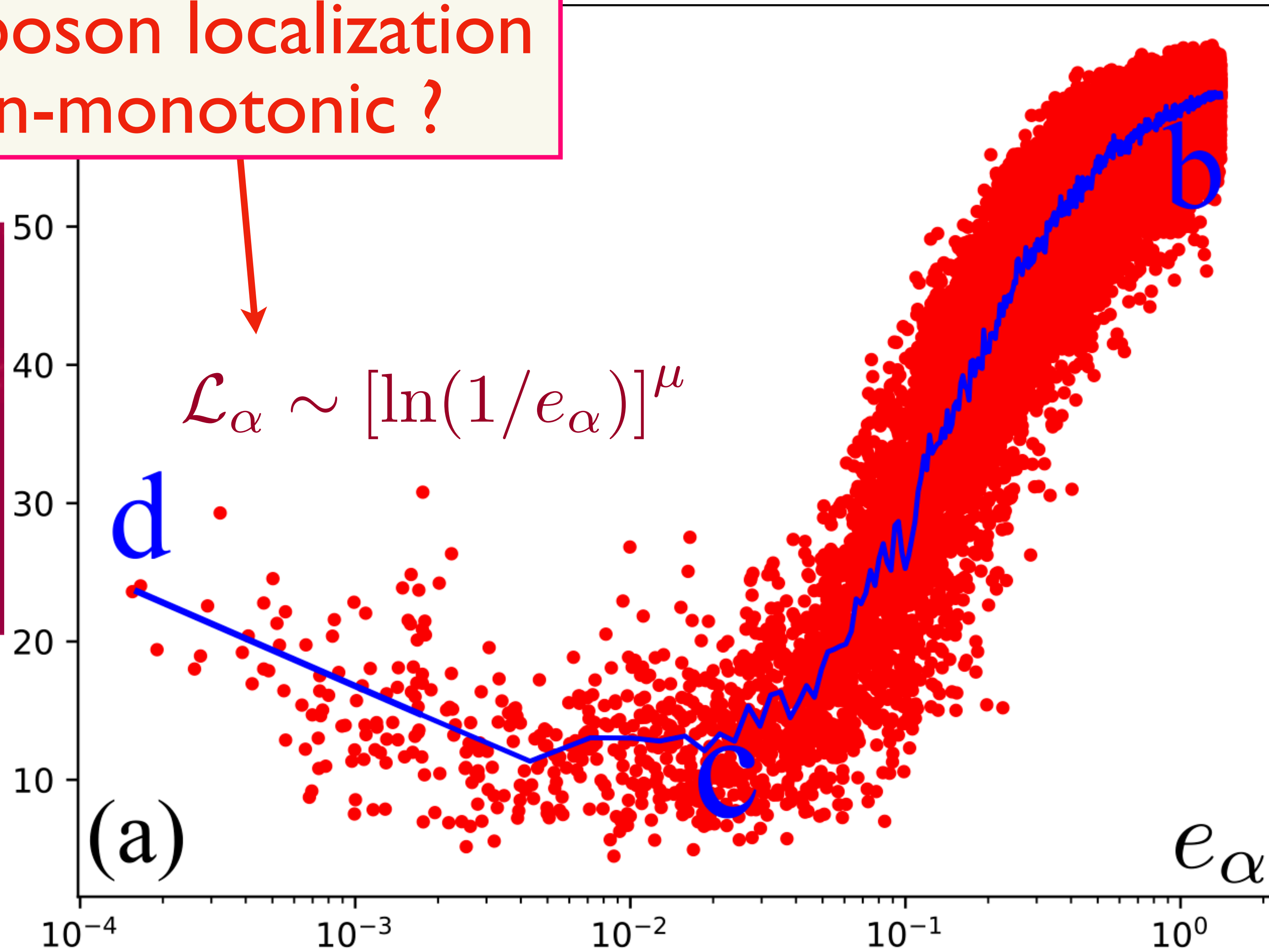


Aavishkar A. Patel,  
Peter Lunts, S.S.,  
[arXiv:2312.06751](https://arxiv.org/abs/2312.06751)

# Landau-damped bosonic eigenmodes with random mass

$\phi$  eigenmodes localization length  $\mathcal{L}_\alpha$

Why is the boson localization length non-monotonic ?



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## Effects of Dissipation on a Quantum Critical Point with Disorder

José A. Hoyos, Chetan Kotabage, and Thomas Vojta

*Department of Physics, University of Missouri-Rolla, Rolla, Missouri 65409, USA*

$$\mathcal{S}_b = \int d\tau \left( - \sum_{\langle ij \rangle} J_{ij} \phi_{ia} \phi_{ja} + \sum_j \left[ \frac{s_j}{2} \phi_{ja}^2 + \frac{u}{4} (\phi_{ja}^2)^2 \right] \right) + \frac{T\gamma}{2} \sum_{\omega_n} \sum_j |\omega_n| |\phi_{ja}(\omega_n)|^2$$

## Effects of Dissipation on a Quantum Critical Point with Disorder

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Strong disorder RG identical to that for the RTFIM

$$\tilde{J}_{ij} = J_{ij} + \frac{J_{i2}J_{2j}}{s_2}$$

$$\tilde{s}_2 = 2 \frac{s_2 s_3}{J_{23}}$$

$$H_{\text{RTFIM}} = - \sum_{\langle ij \rangle} J_{ij} Z_i Z_j - \sum_j s_j X_j$$

Numerically studied in  $d=2$  by  
O. Motrunich, S.-C. Mau, D.A. Huse and D.S. Fisher,  
PRB **61** (2000) 1160

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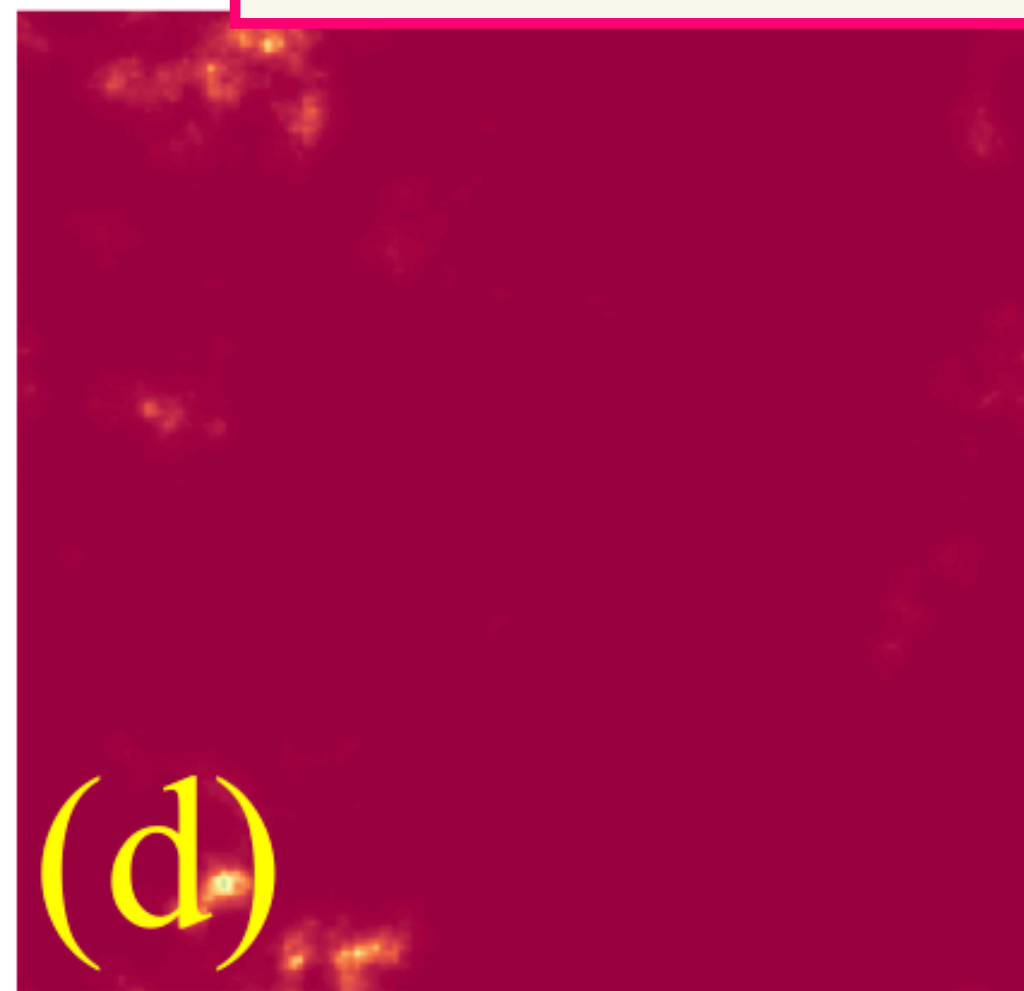
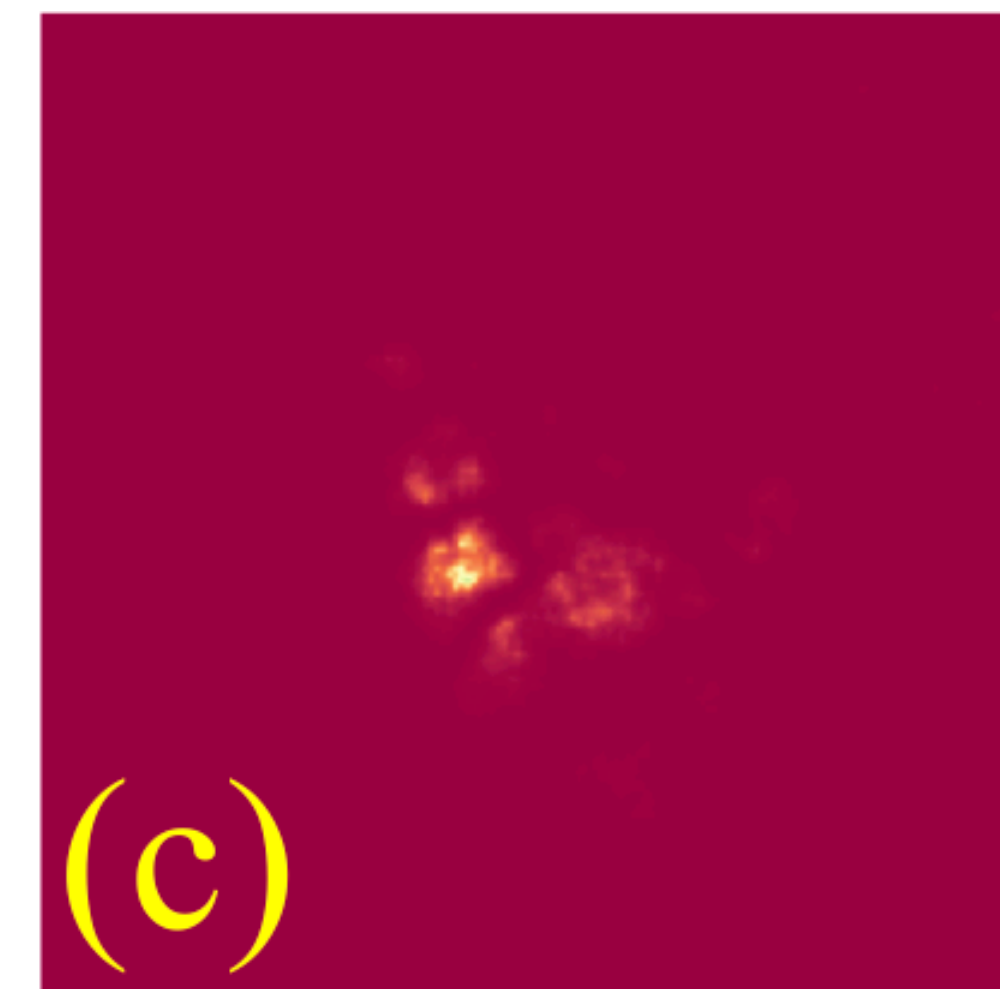
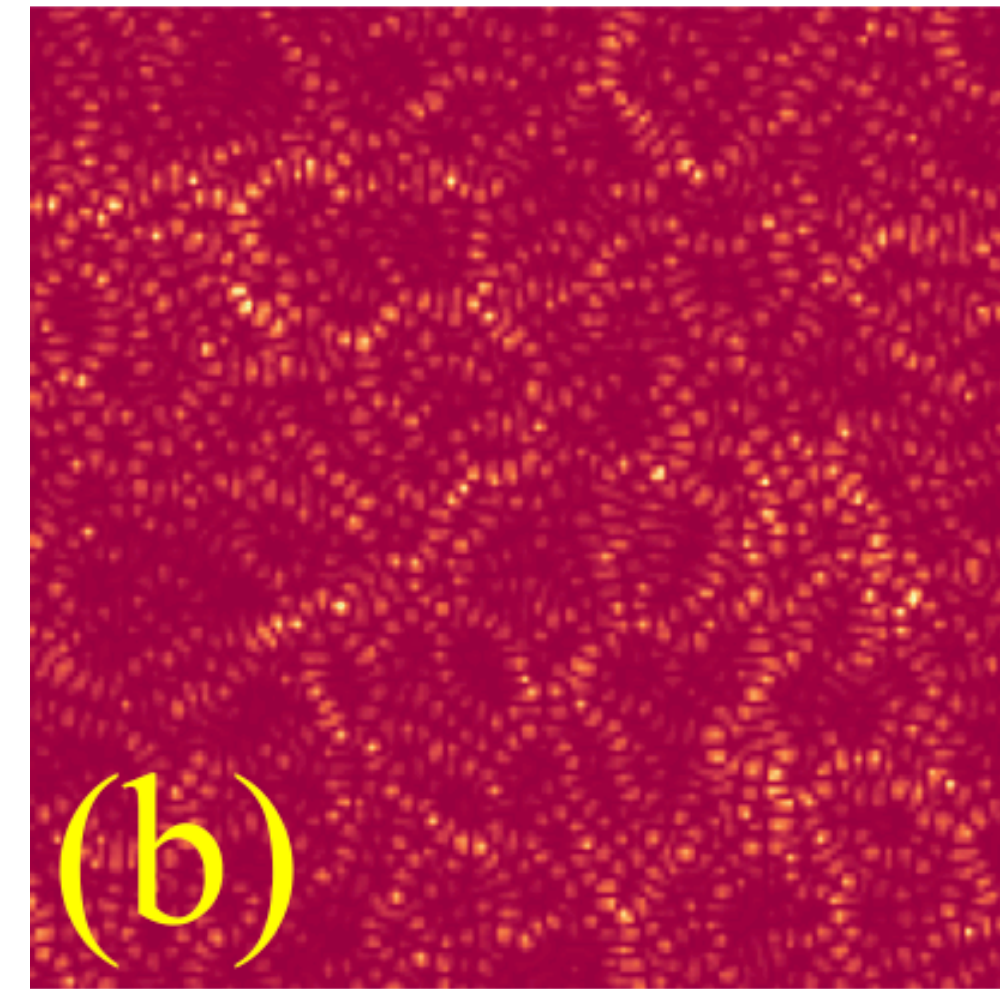
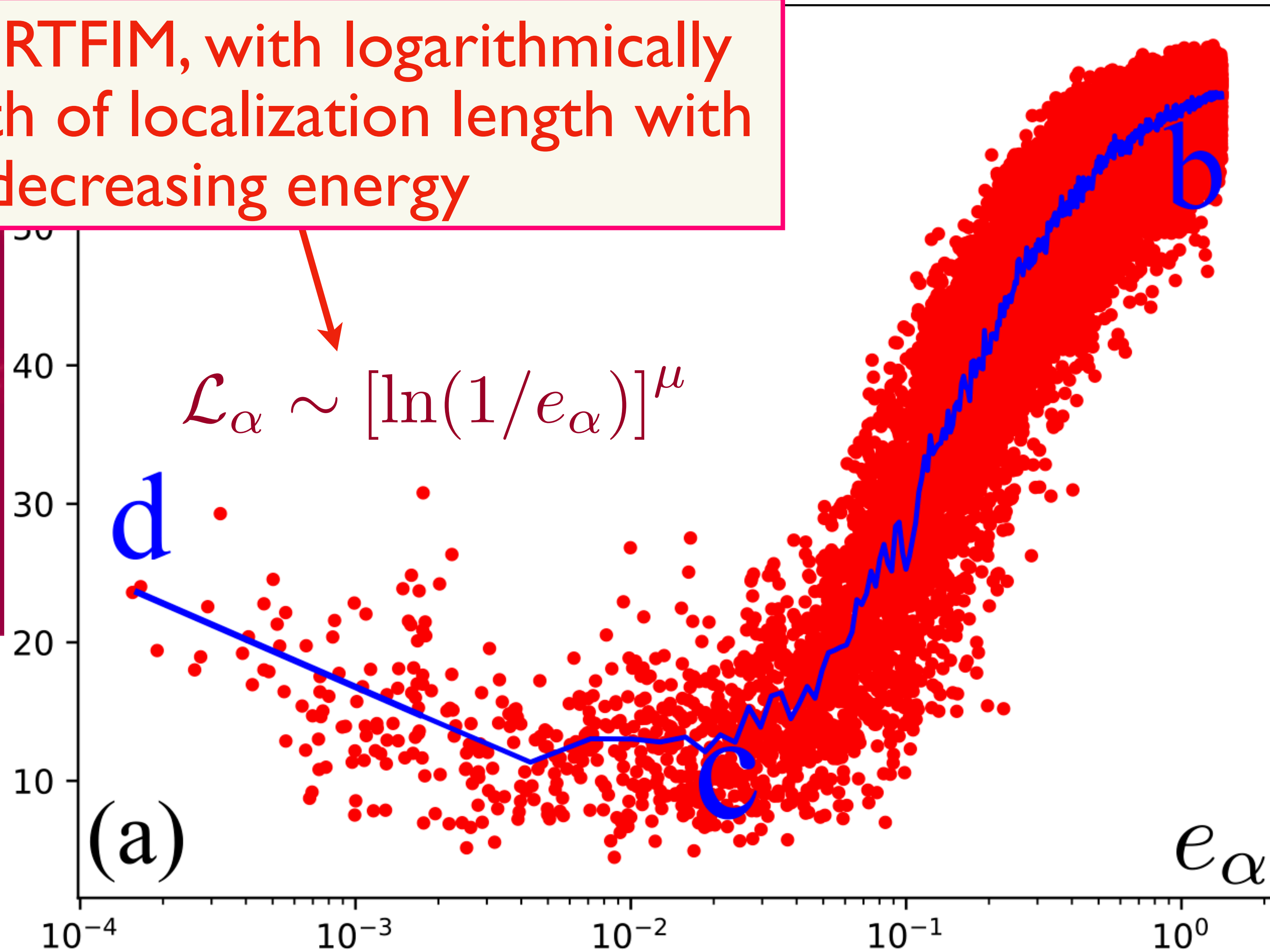
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Key fact: Similarity of classical  $O(N \geq 2)$  chain with  $1/r^2$  interactions, and classical Ising chain with short-range interactions.

# Landau-damped bosonic eigenmodes with random mass

$\phi$  eigenmodes localization length  $\mathcal{L}_\alpha$

Physics of RTFIM, with logarithmically slow growth of localization length with decreasing energy

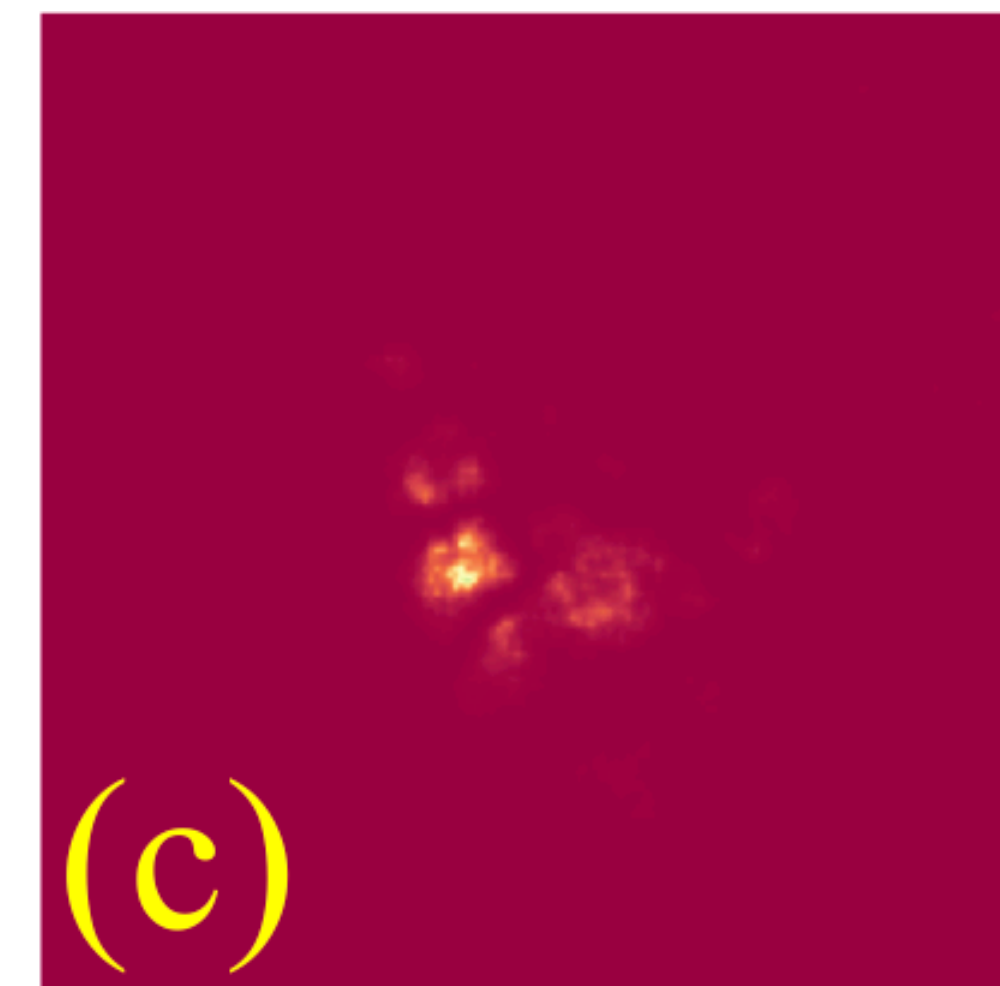
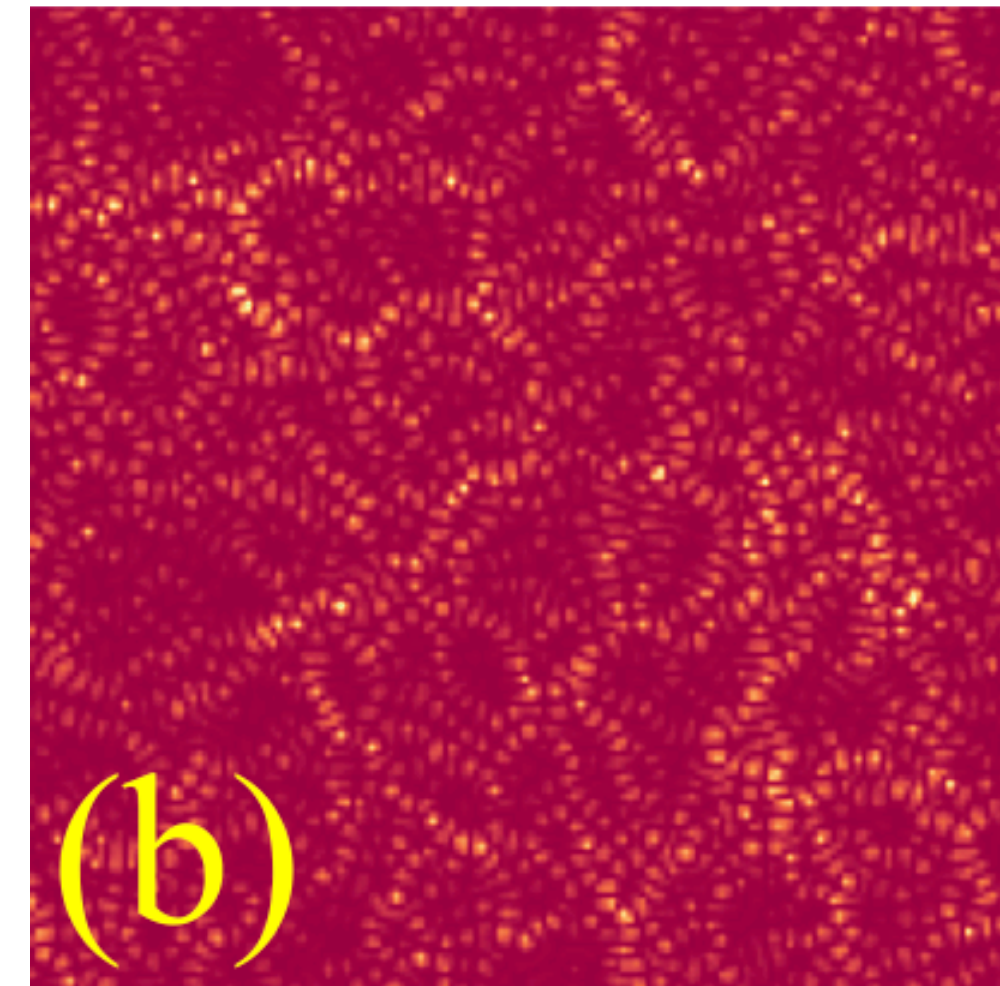
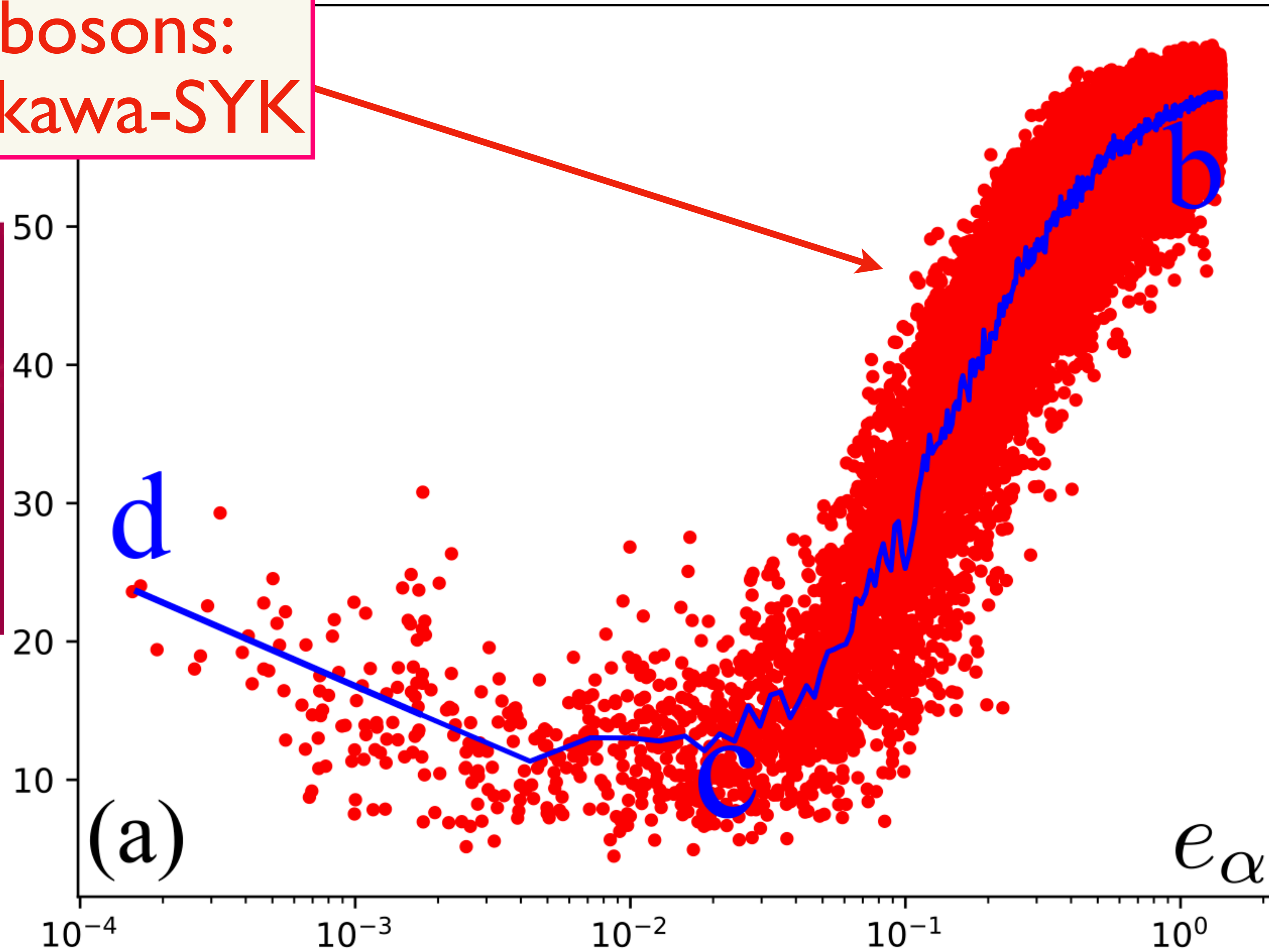
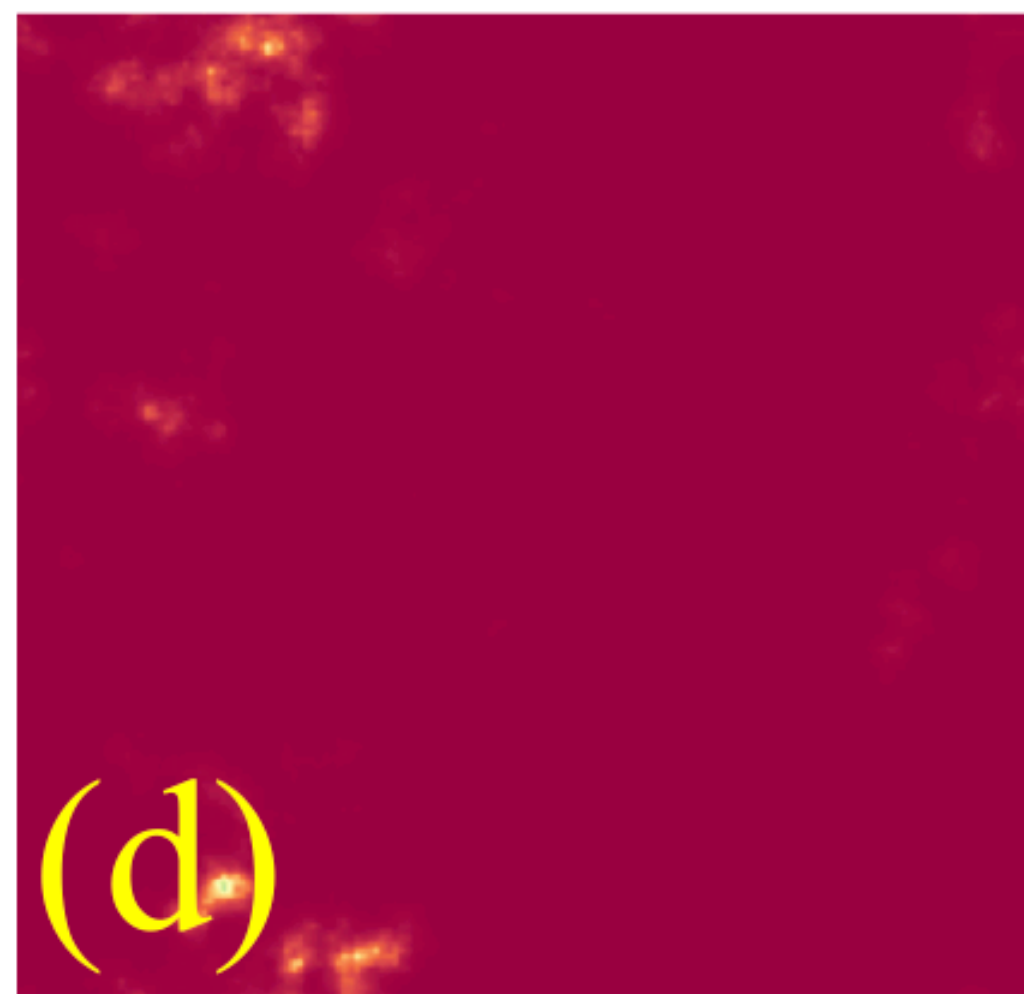


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# Landau-damped bosonic eigenmodes with random mass

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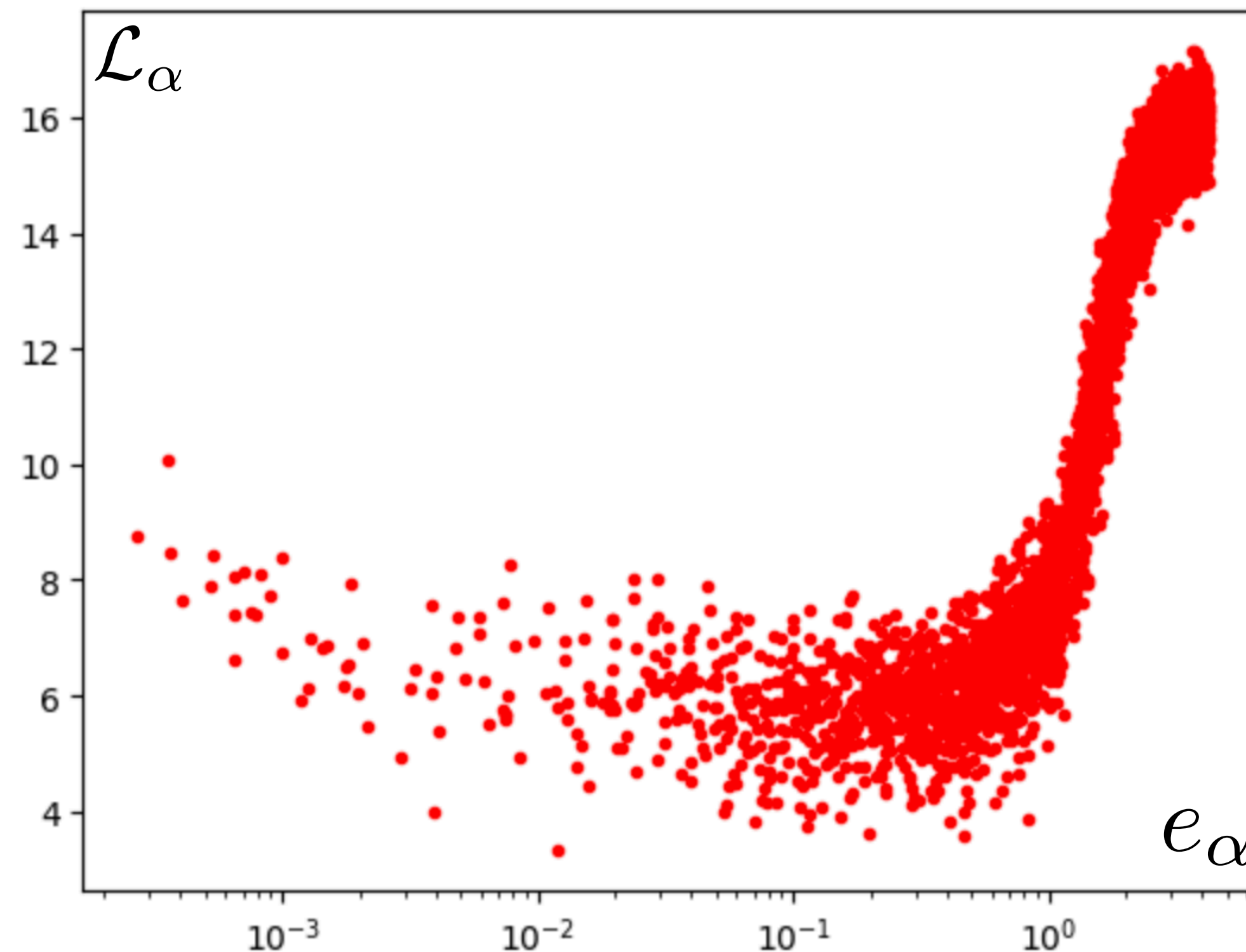
Extended bosons:  
physics of Yukawa-SYK



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# Boson eigenmodes at $\Omega = 0$ from QMC

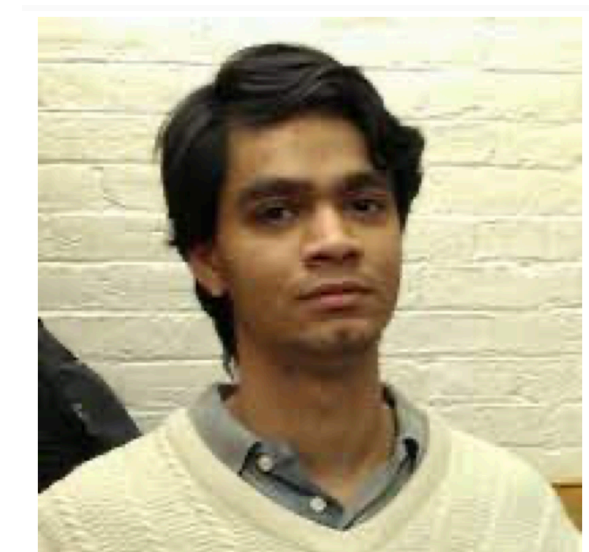
- Localization length vs eigenvalue



10 disorder samples,  $L = 40$ ,  $\beta t = 66$ ,  $g'^2 = t/2$ ,  $\lambda \approx \lambda_c$

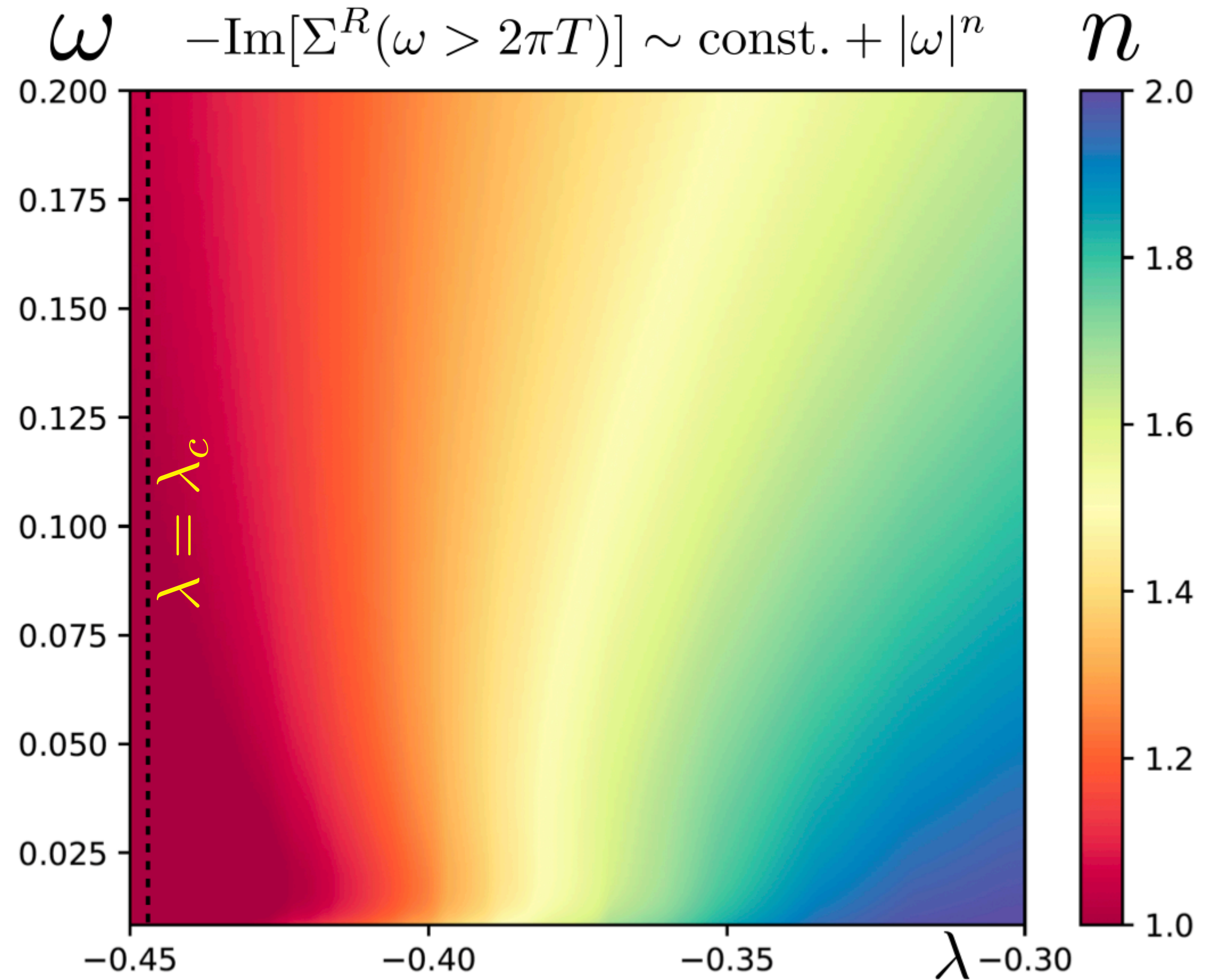
- Same qualitative features as large- $M$ : localization followed by slow delocalization as energy is reduced.

Aavishkar A. Patel, Peter Lunts, M. Albergro (to appear)



# Transport scattering rate

$$\Sigma(i\omega) = -i\pi g'^2 \mathcal{N}_0 \frac{T}{L^2} \sum_{\alpha, \Omega} \frac{\text{sgn}(\omega + \Omega)}{\gamma|\Omega| + \Omega^2/c^2 + e_\alpha}.$$

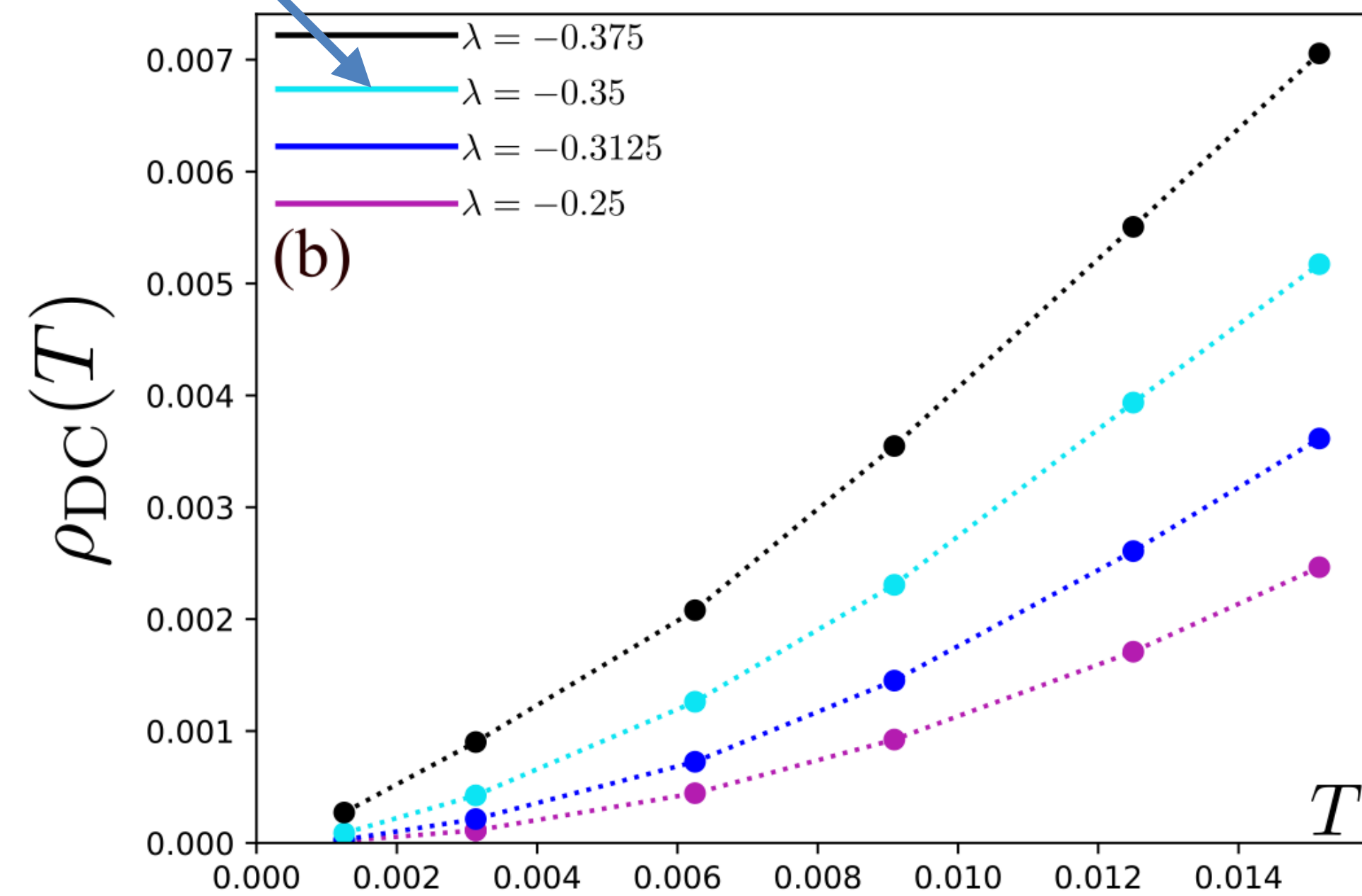
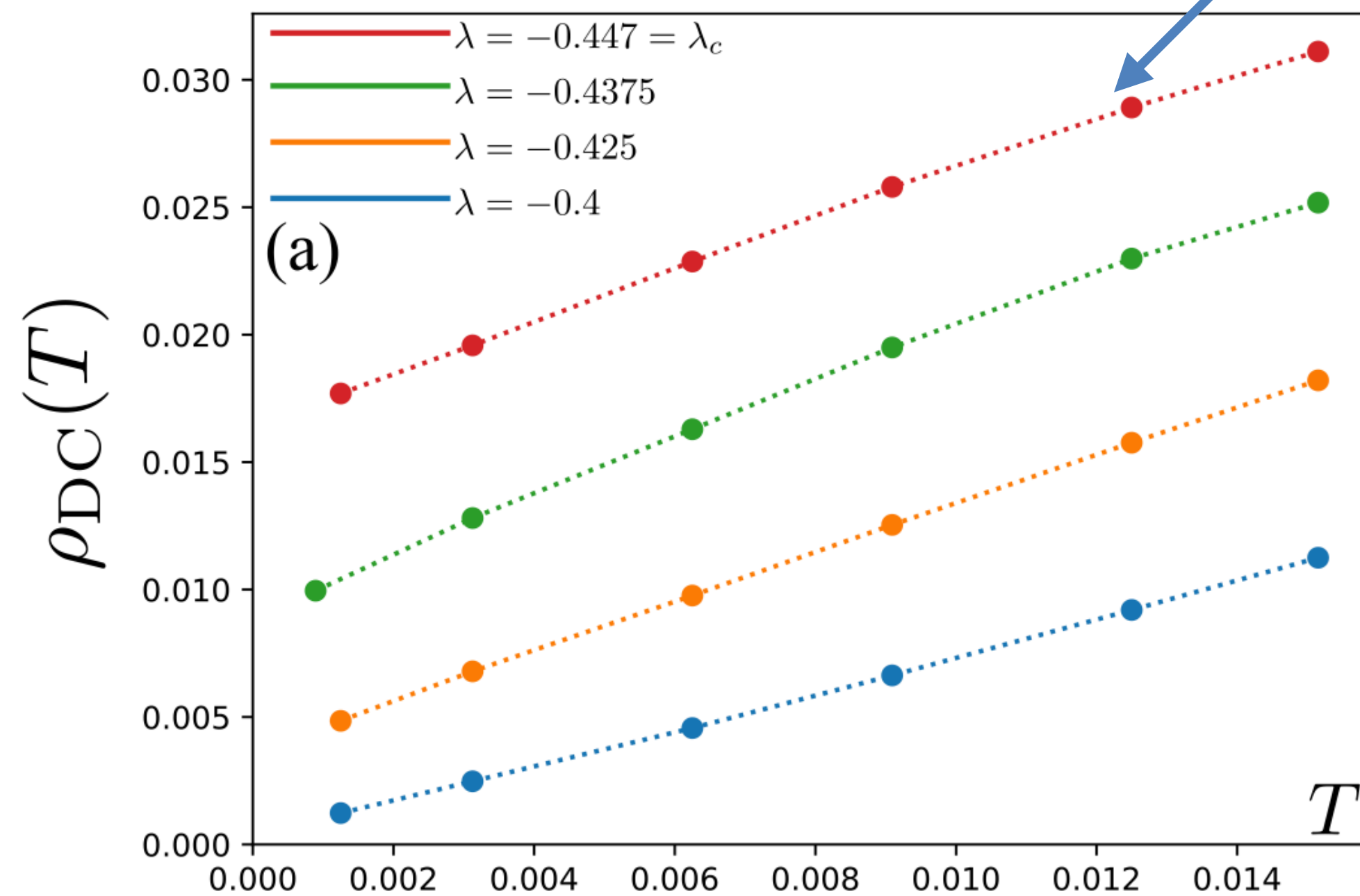
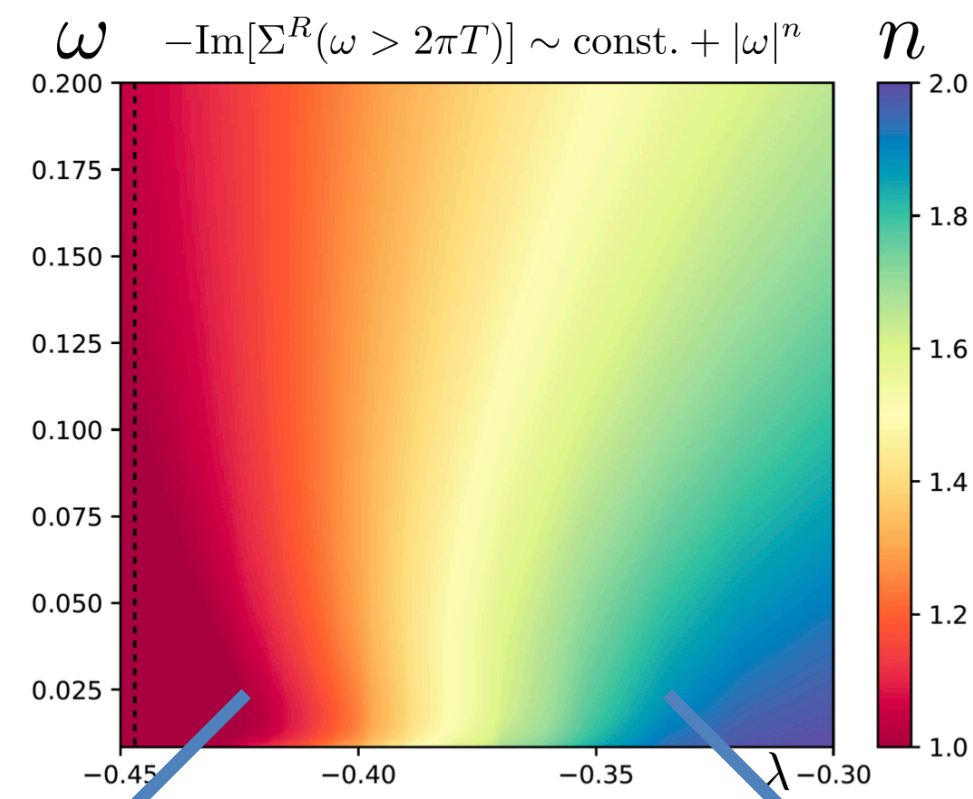


Extended region in  $\lambda$  with  $n \approx 1$  - a strange metal *phase*



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# DC resistivity

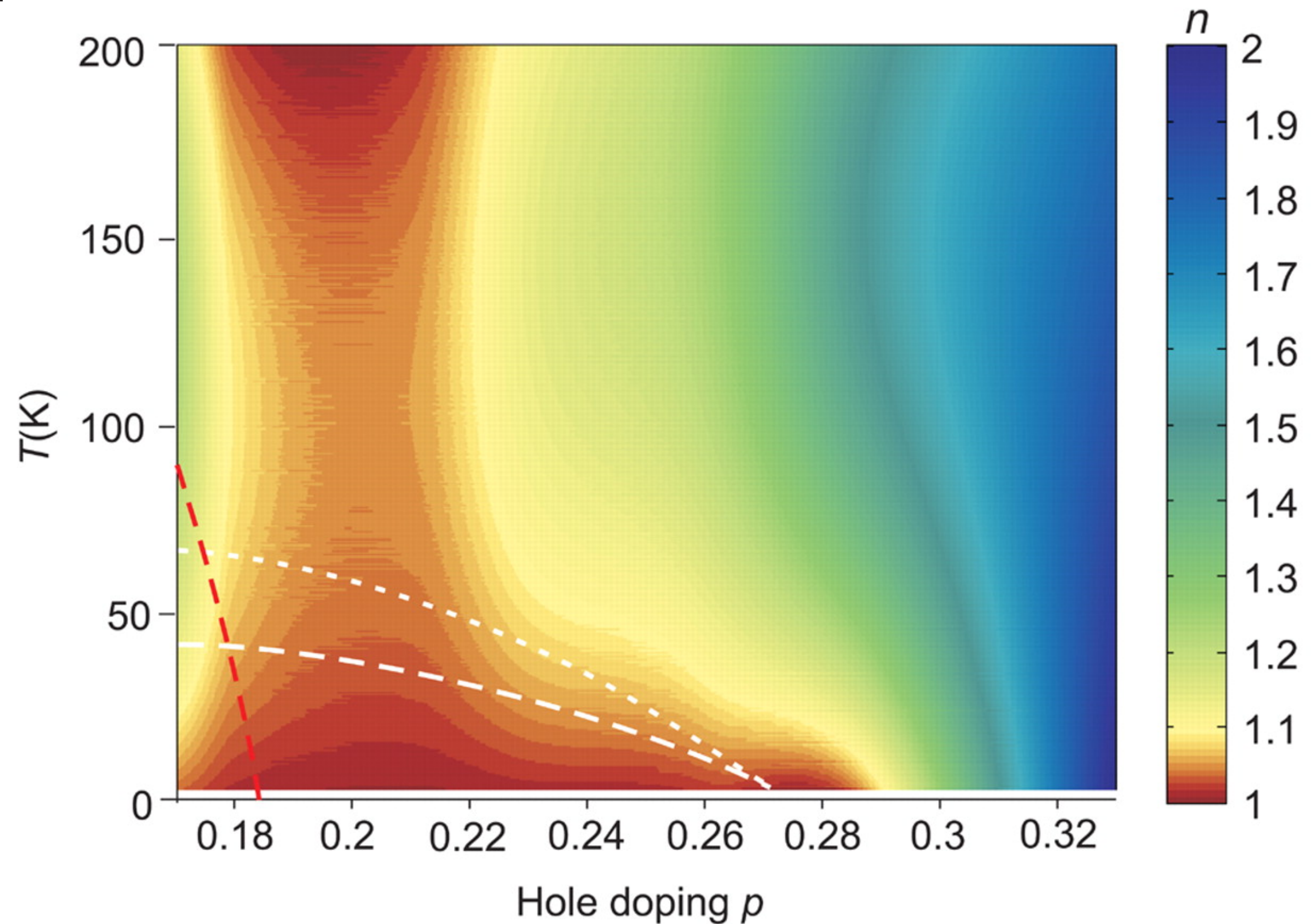
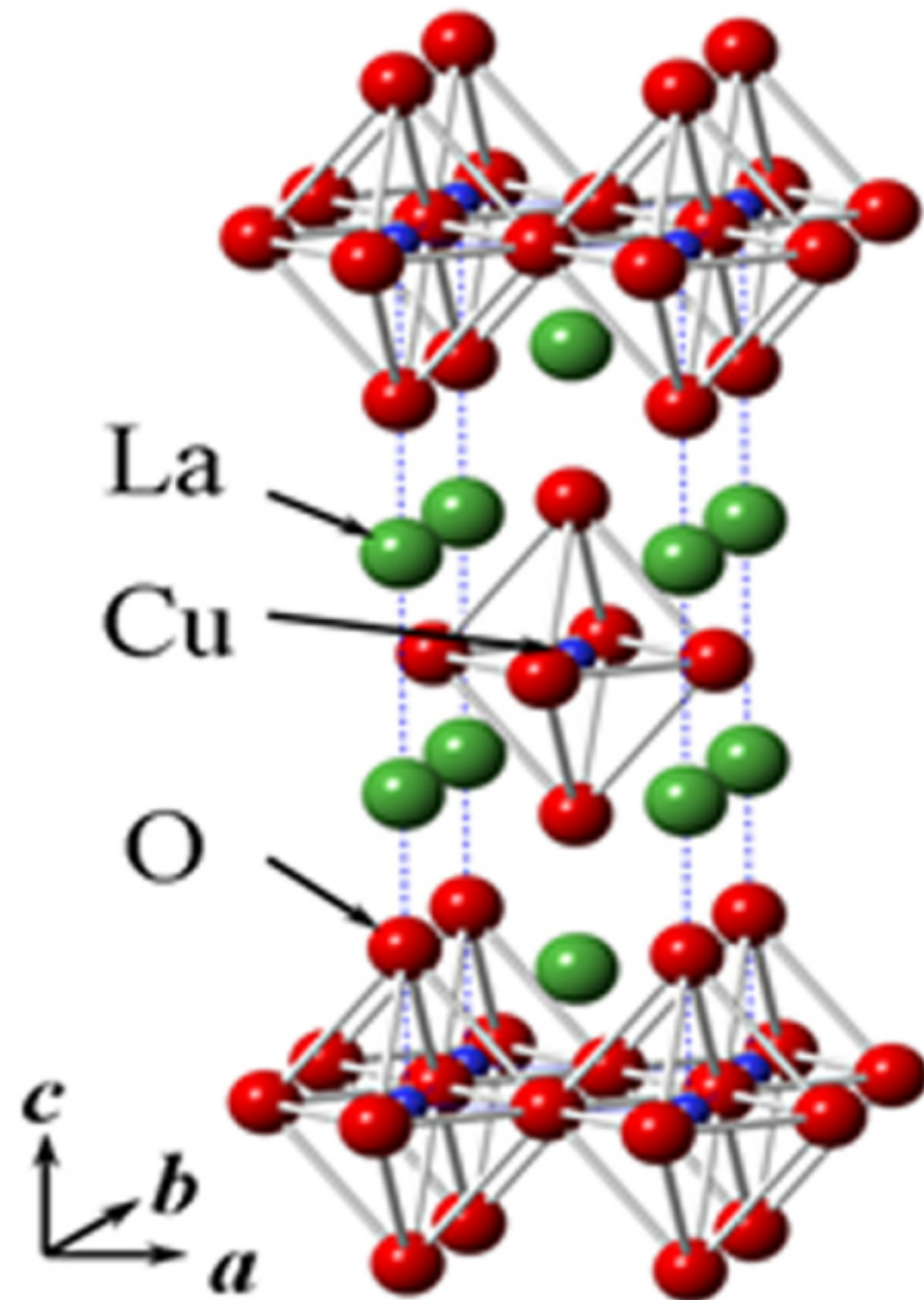


Extended region in  $\lambda$  with  $T$ -linear resistivity - a strange metal *phase*

# Anomalous Criticality in the Electrical Resistivity of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

R. A. Cooper,<sup>1</sup> Y. Wang,<sup>1</sup> B. Vignolle,<sup>2</sup> O. J. Lipscombe,<sup>1</sup> S. M. Hayden,<sup>1</sup> Y. Tanabe,<sup>3</sup> T. Adachi,<sup>3</sup> Y. Koike,<sup>3</sup> M. Nohara,<sup>4\*</sup> H. Takagi,<sup>4</sup> Cyril Proust,<sup>2</sup> N. E. Hussey<sup>1†</sup>

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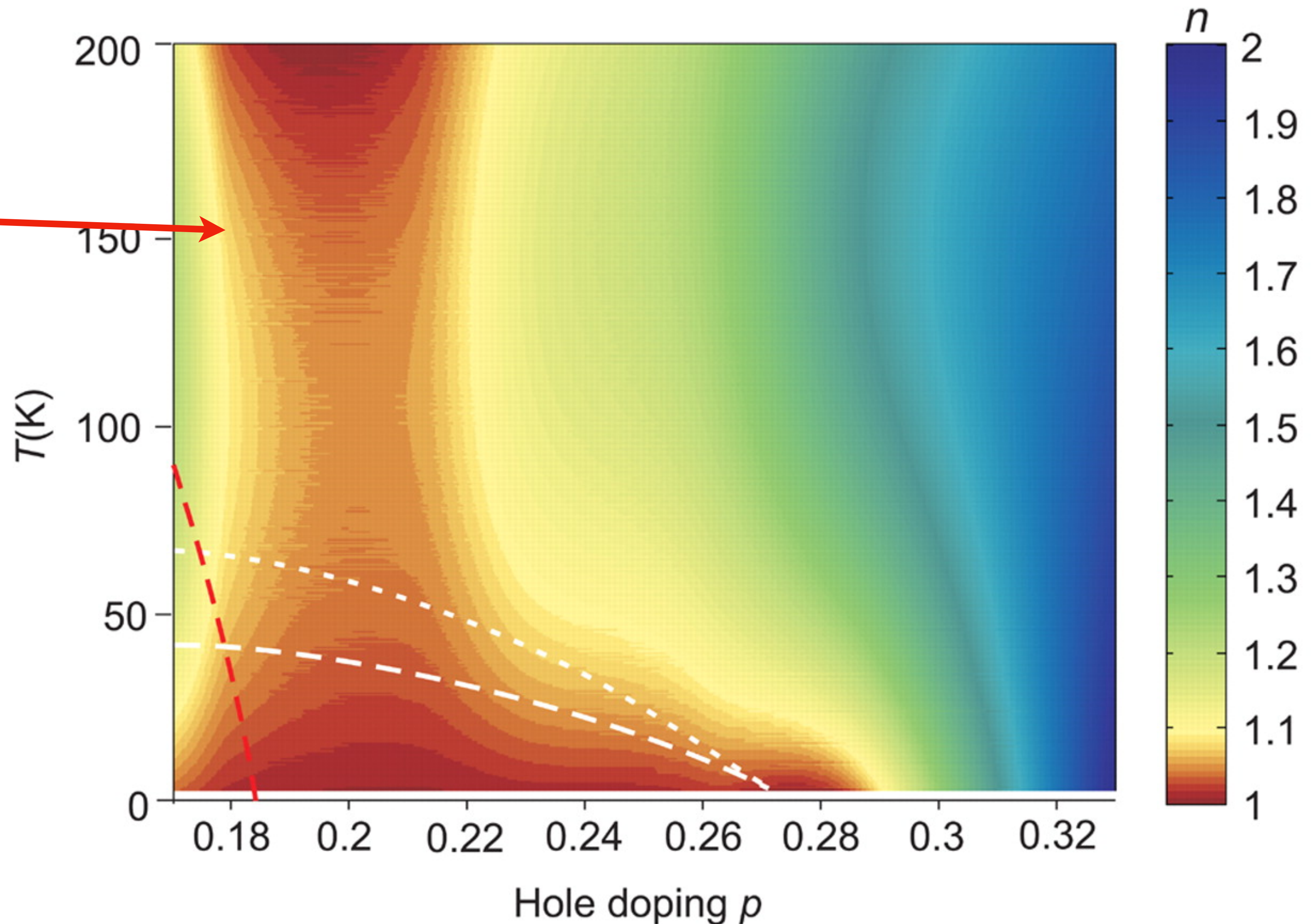
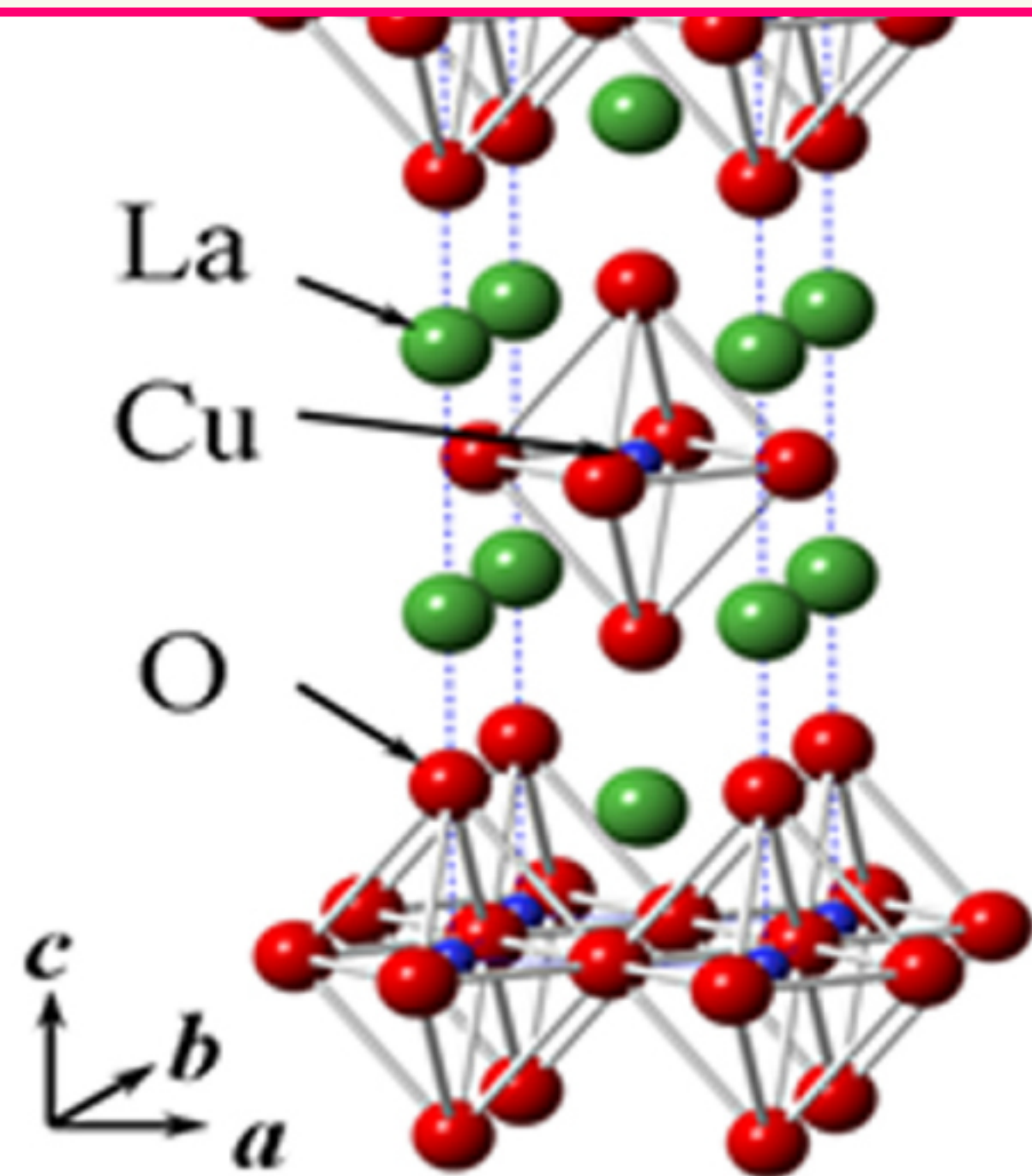


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Extended bosons:  
physics of Yukawa-SYK

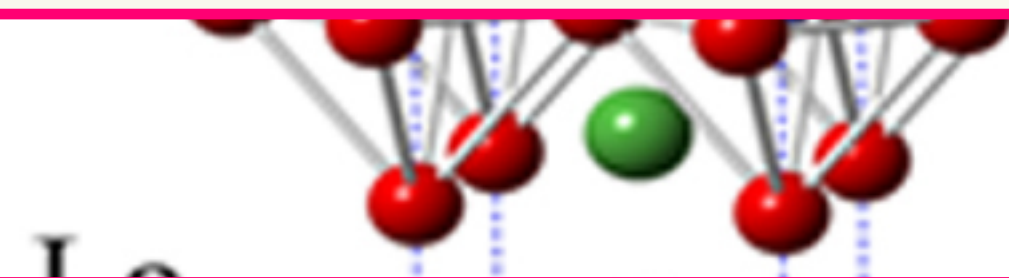


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physics of Yukawa-SYK



Localized overdamped bosons, but extended fermions

