From
SYK models
to
a universal theory of strange metals

T3.00001
Session T3: Strange Metals
APS March Meeting 2023
March 5, 2023, Las Vegas

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

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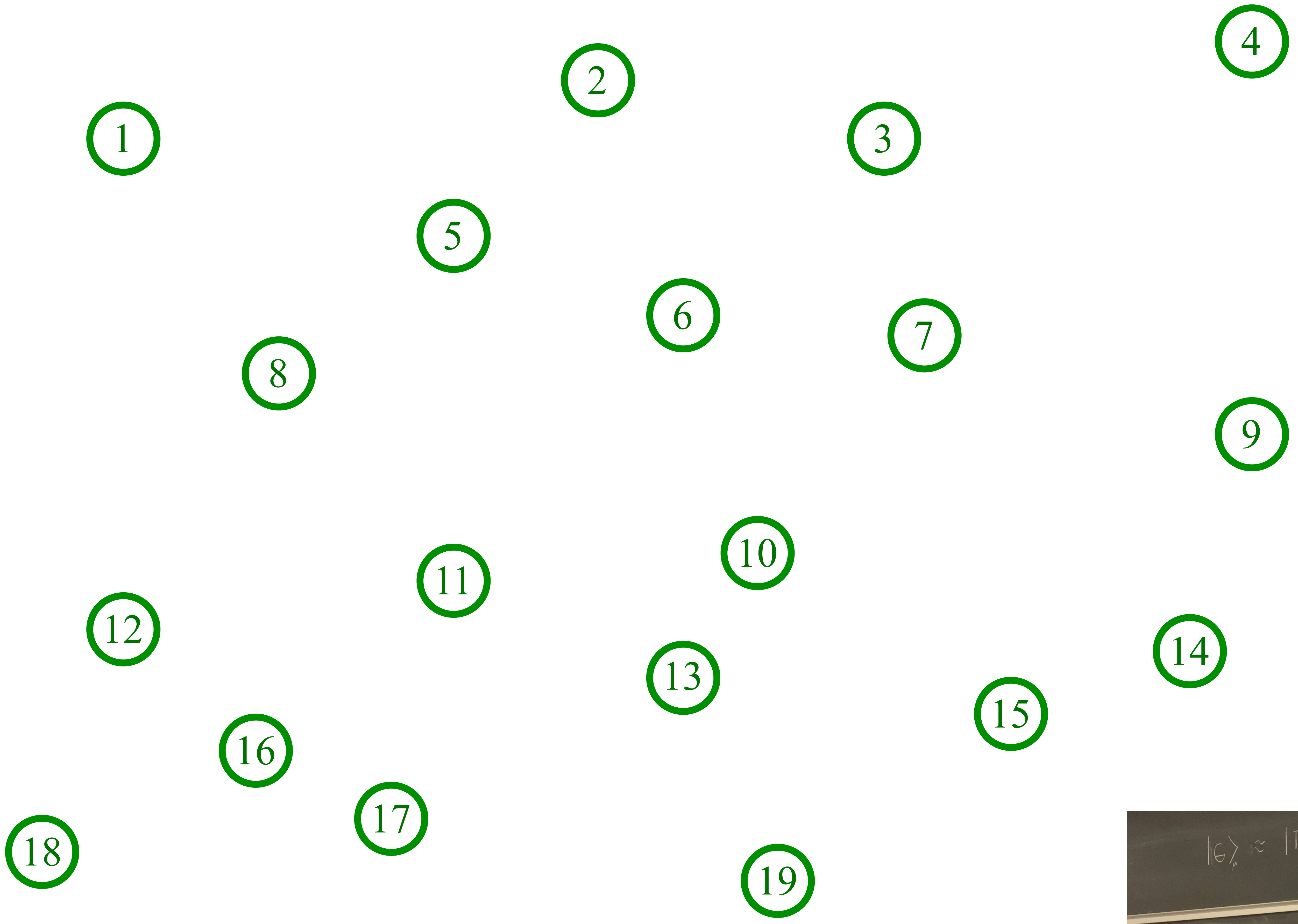
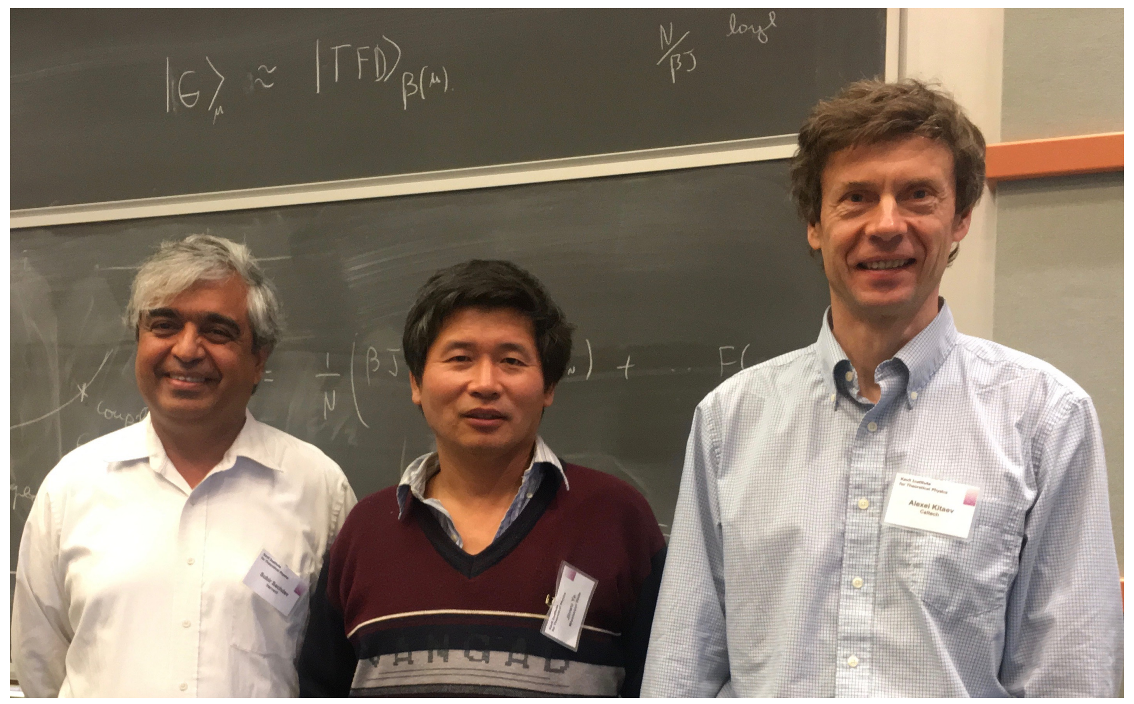
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

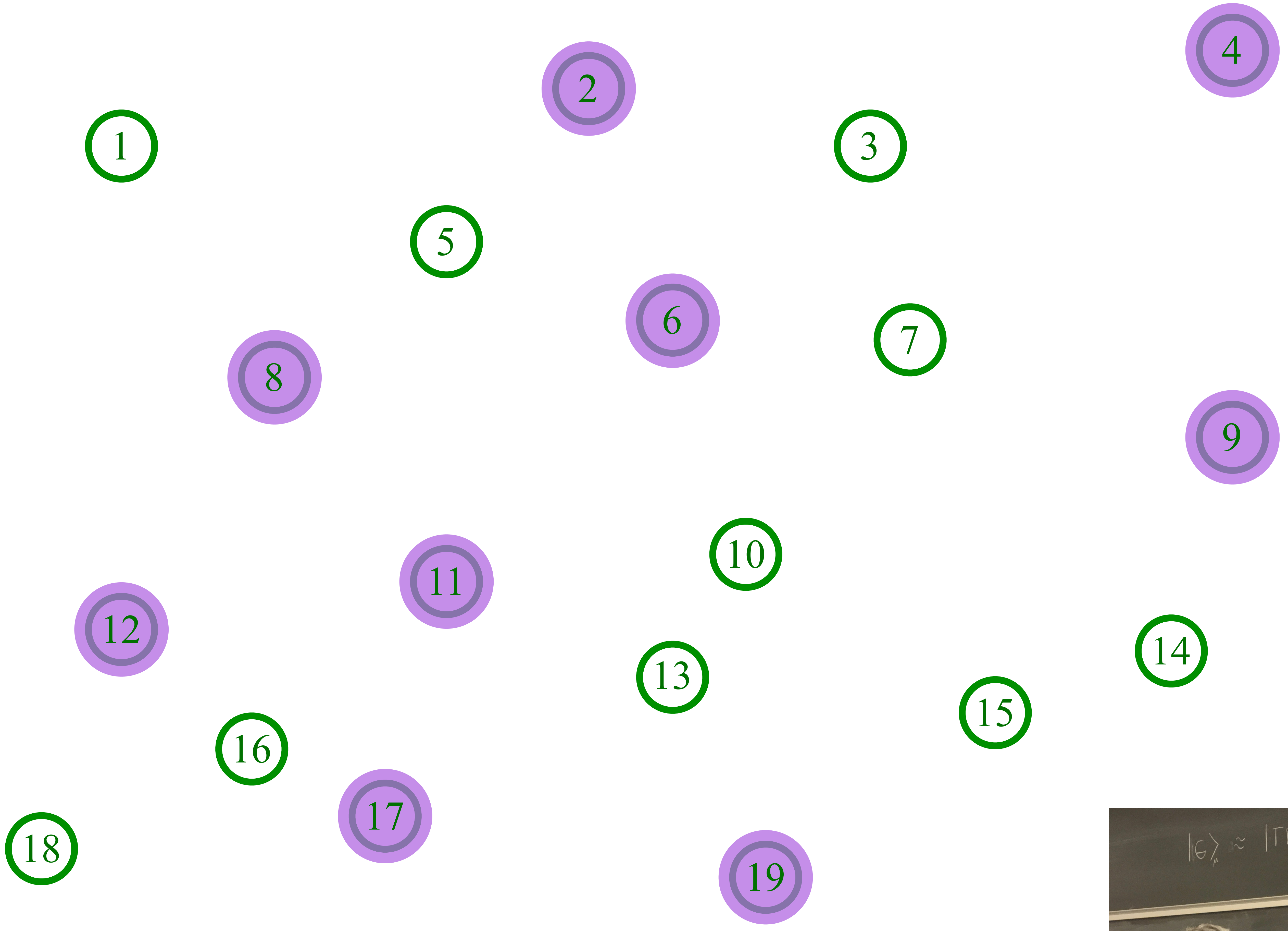


Pick a set of random positions

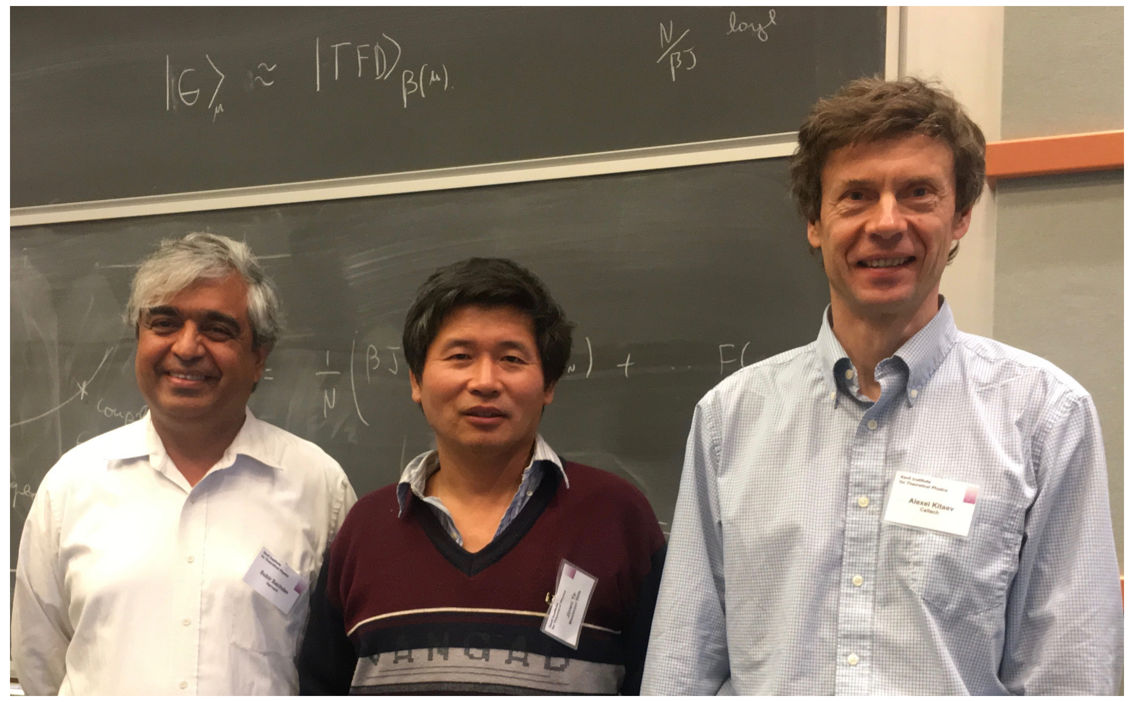


The SYK model

Sachdev, Ye (1993); Kitaev (2015)



Place electrons randomly on some sites



The SYK model

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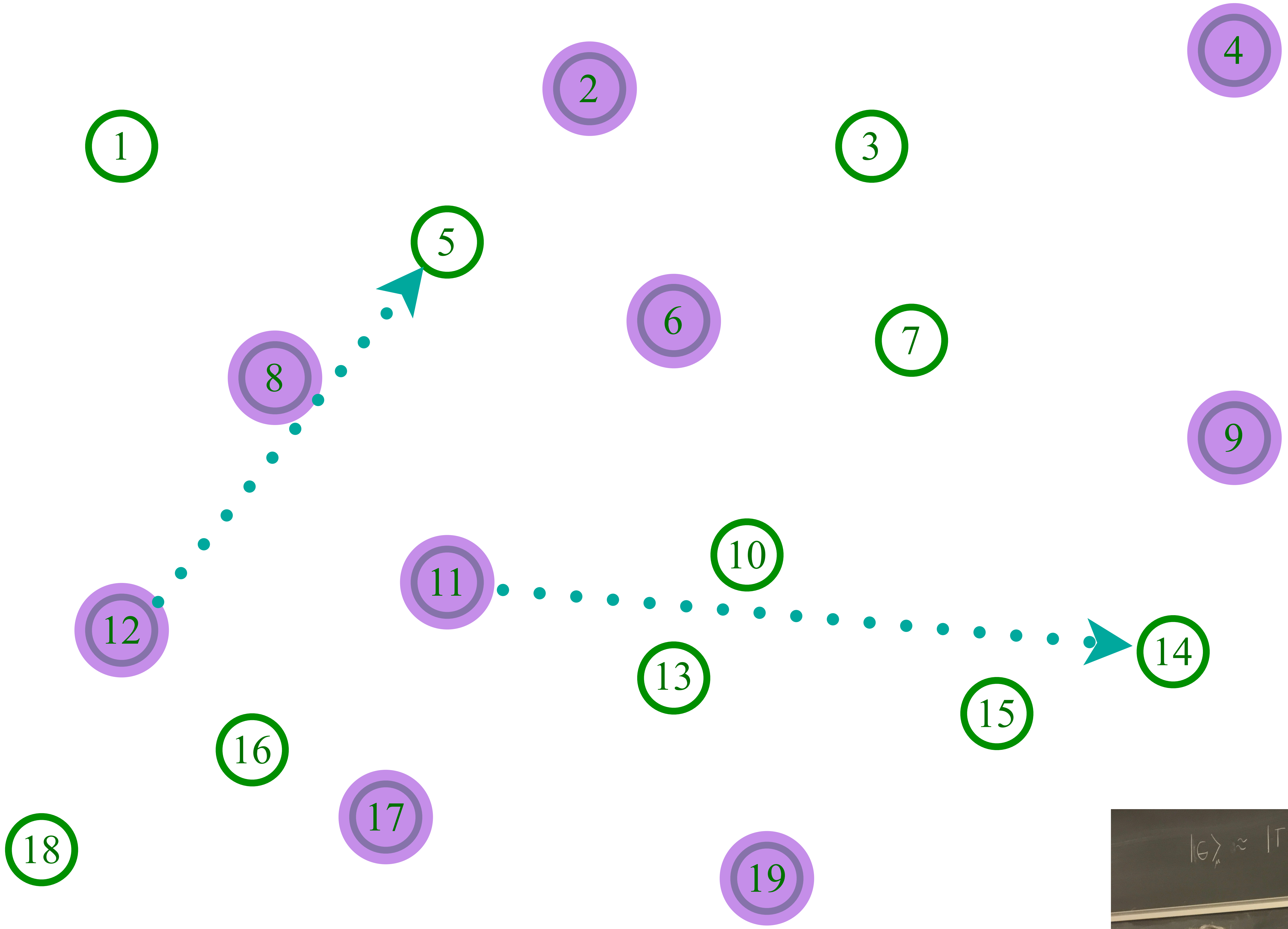
$$U_{11,12;5,14}$$



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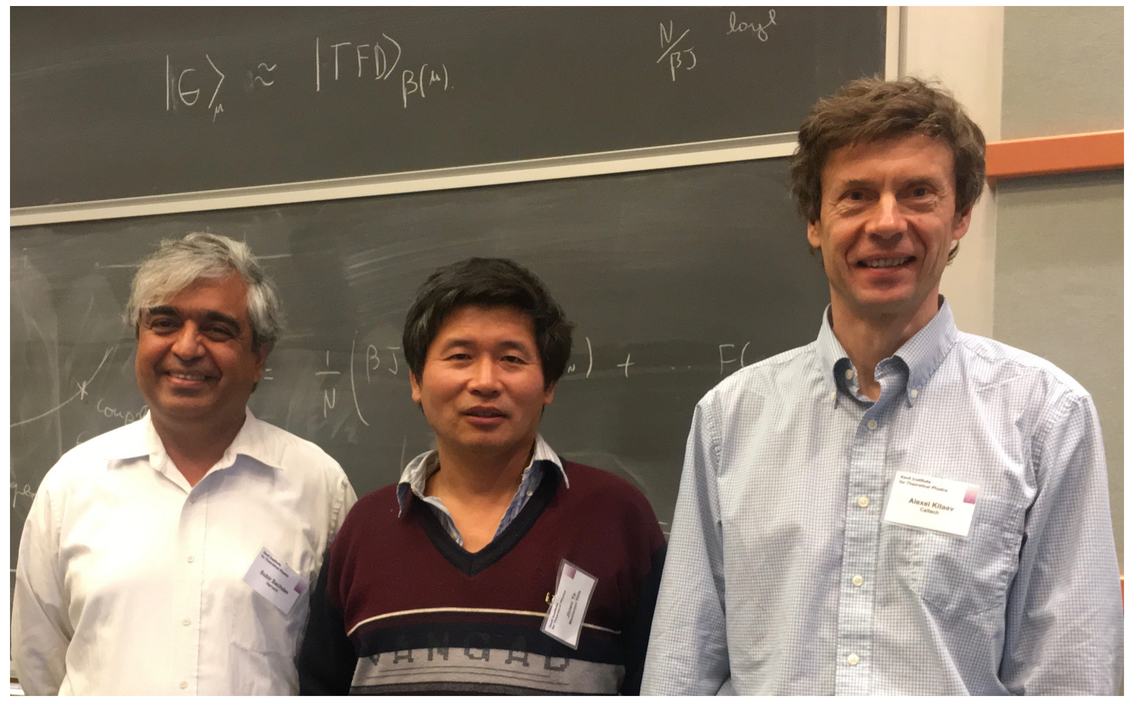

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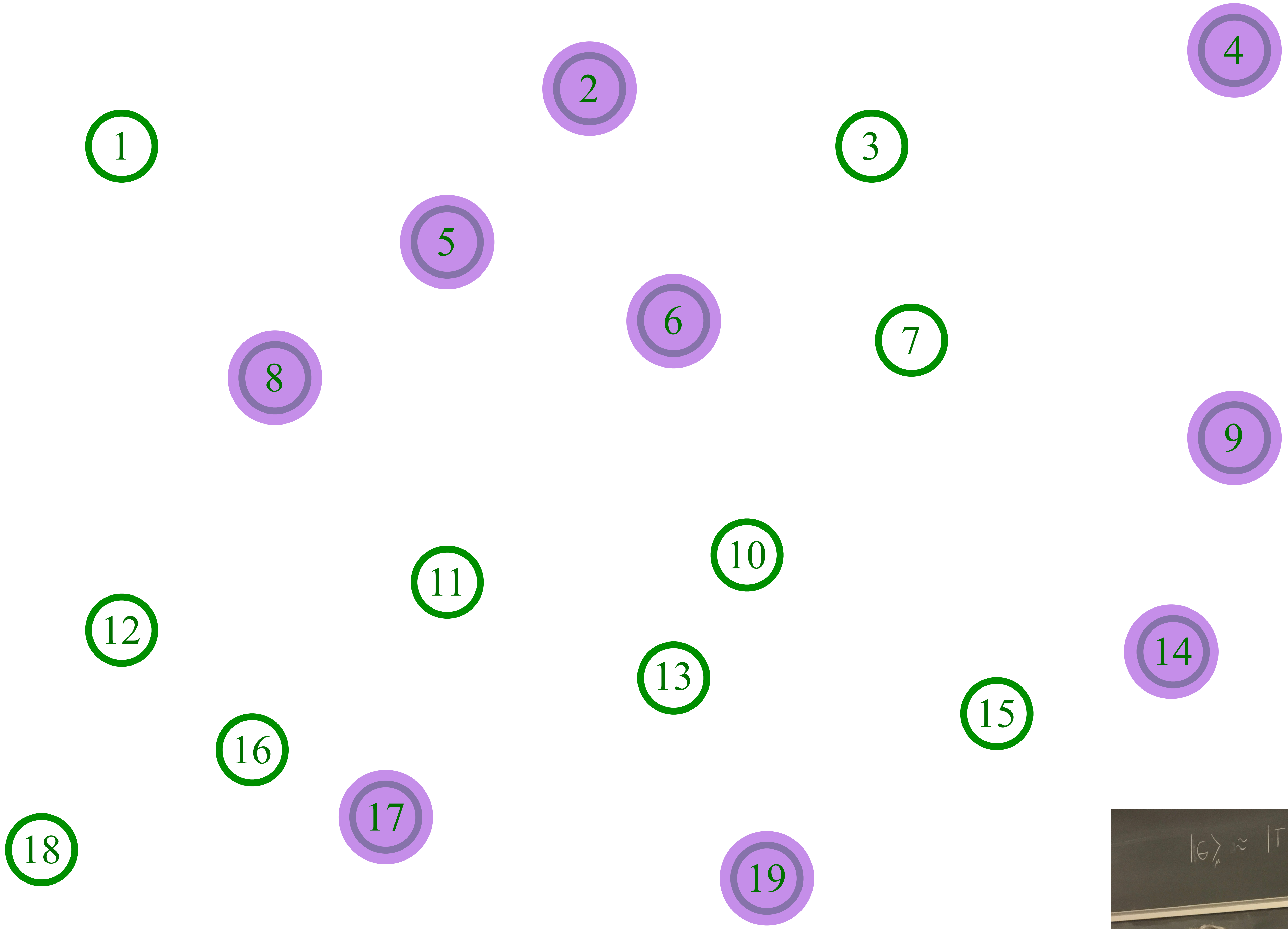
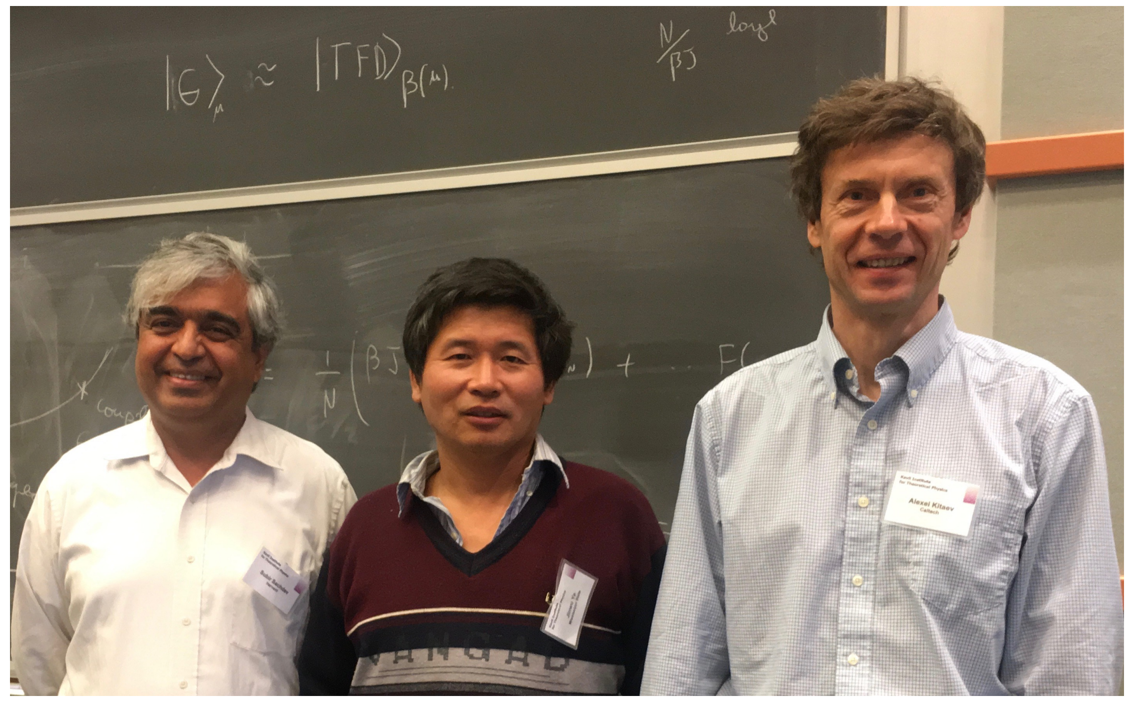


Entangle electrons pairwise randomly



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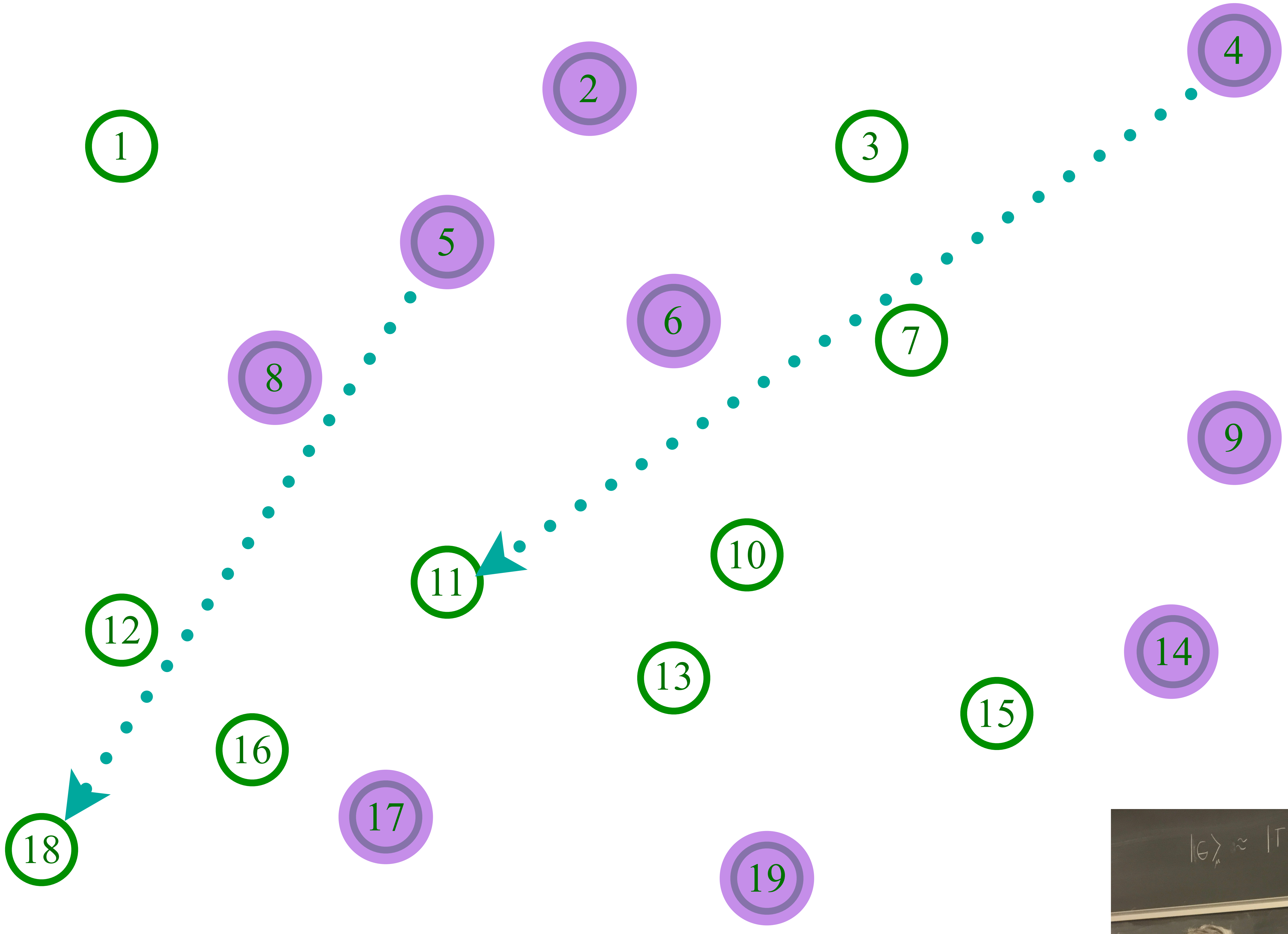
$$U_{4,5;11,18}$$



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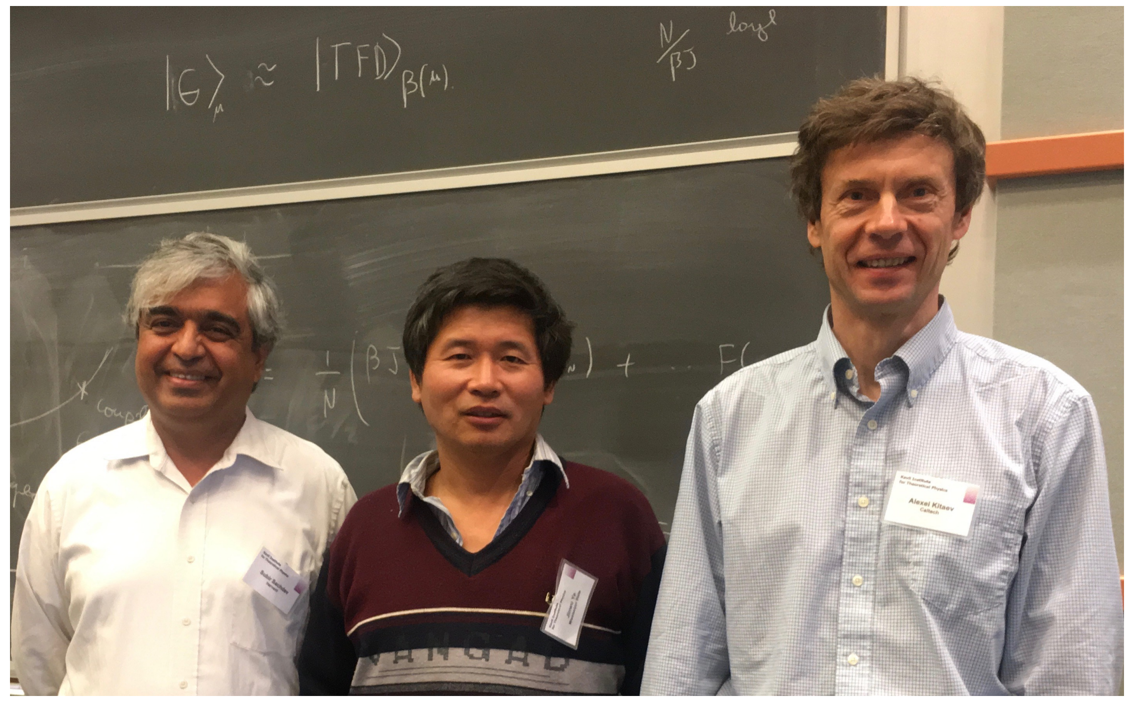

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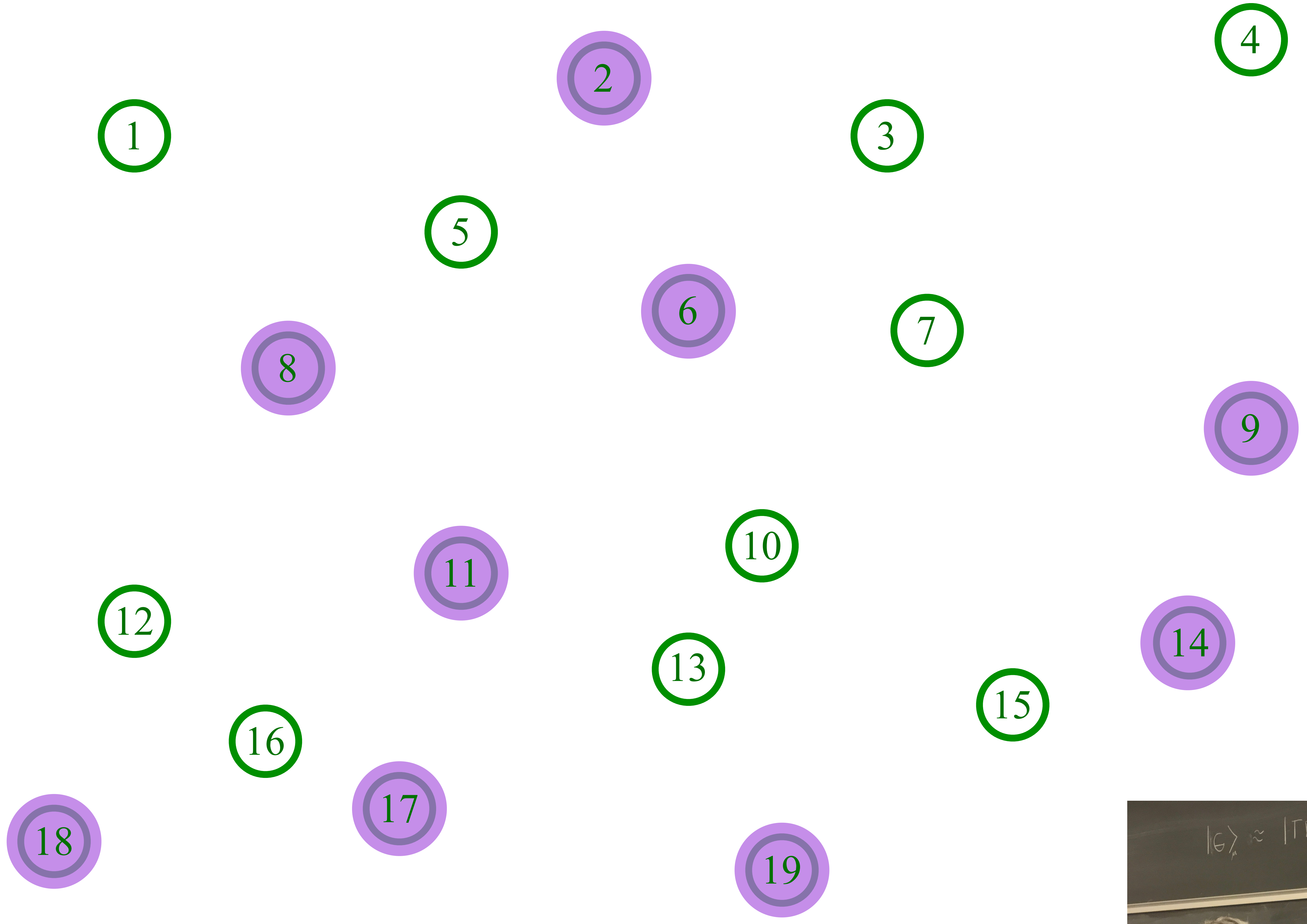

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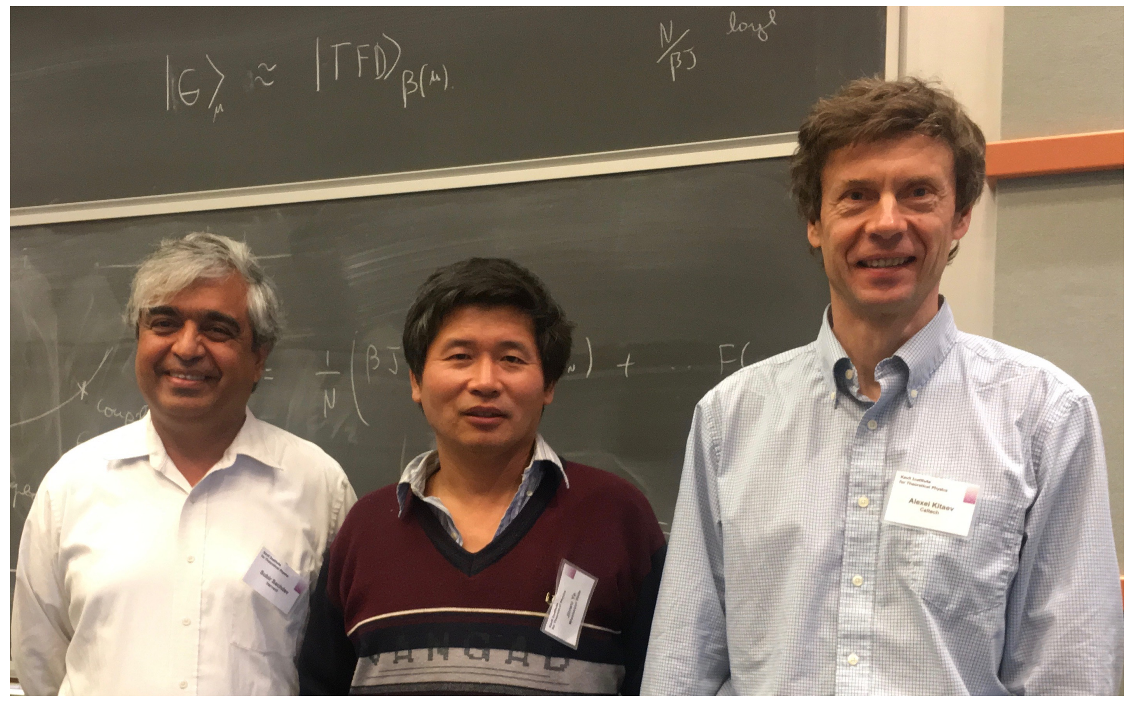

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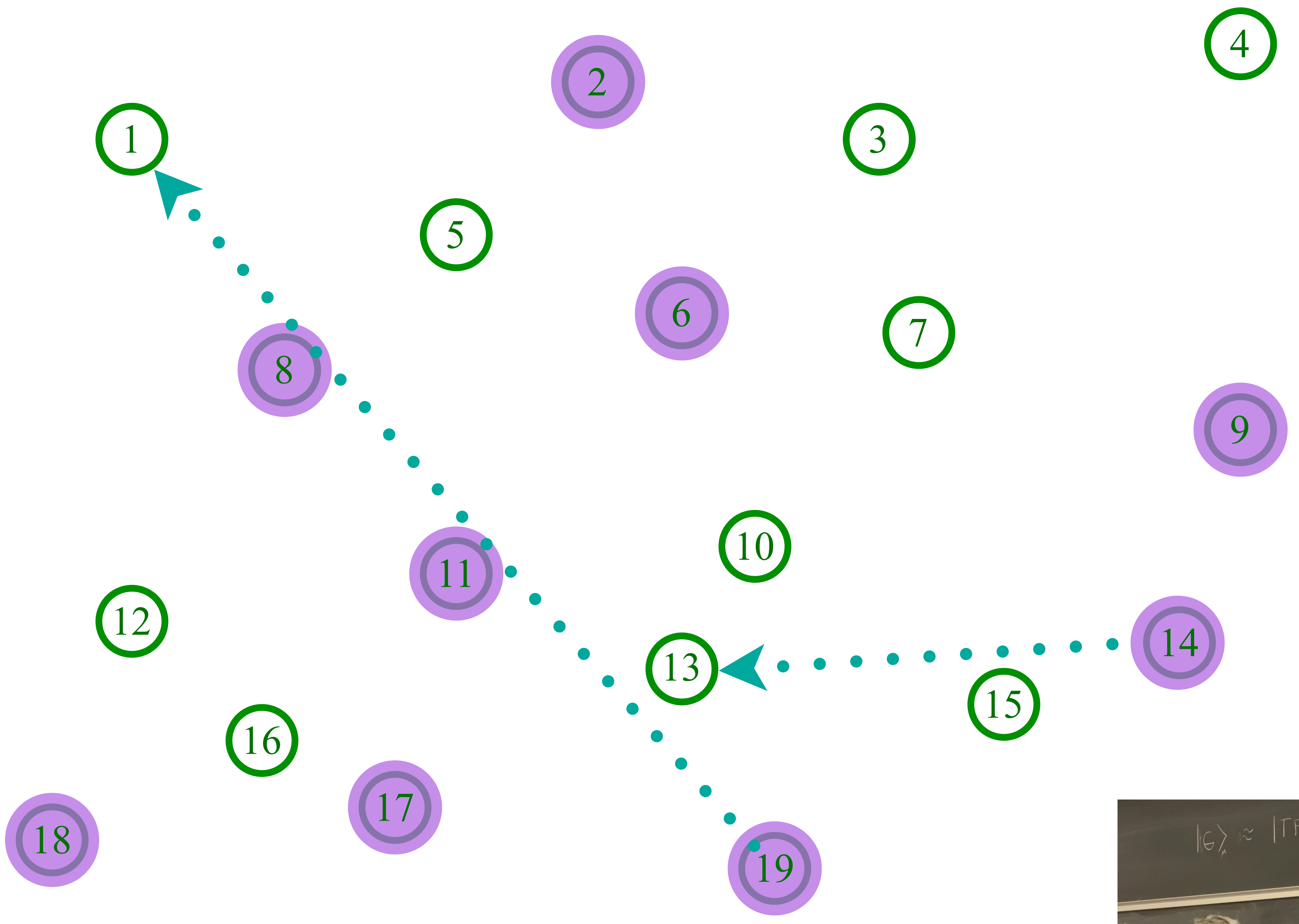

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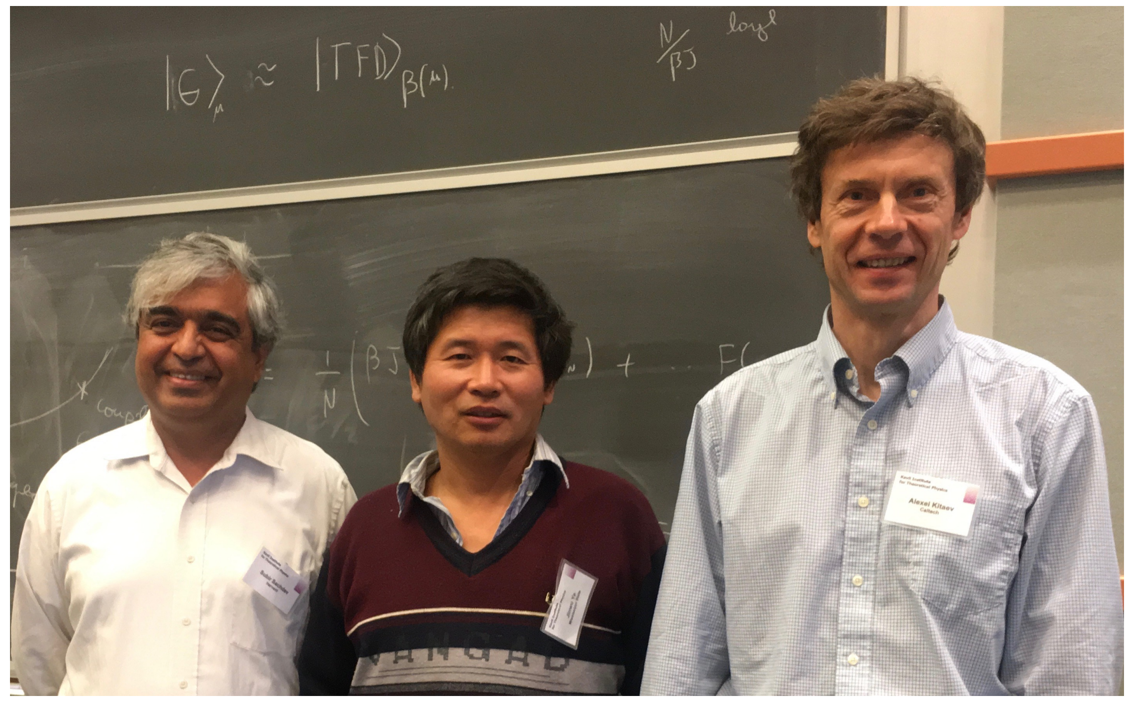

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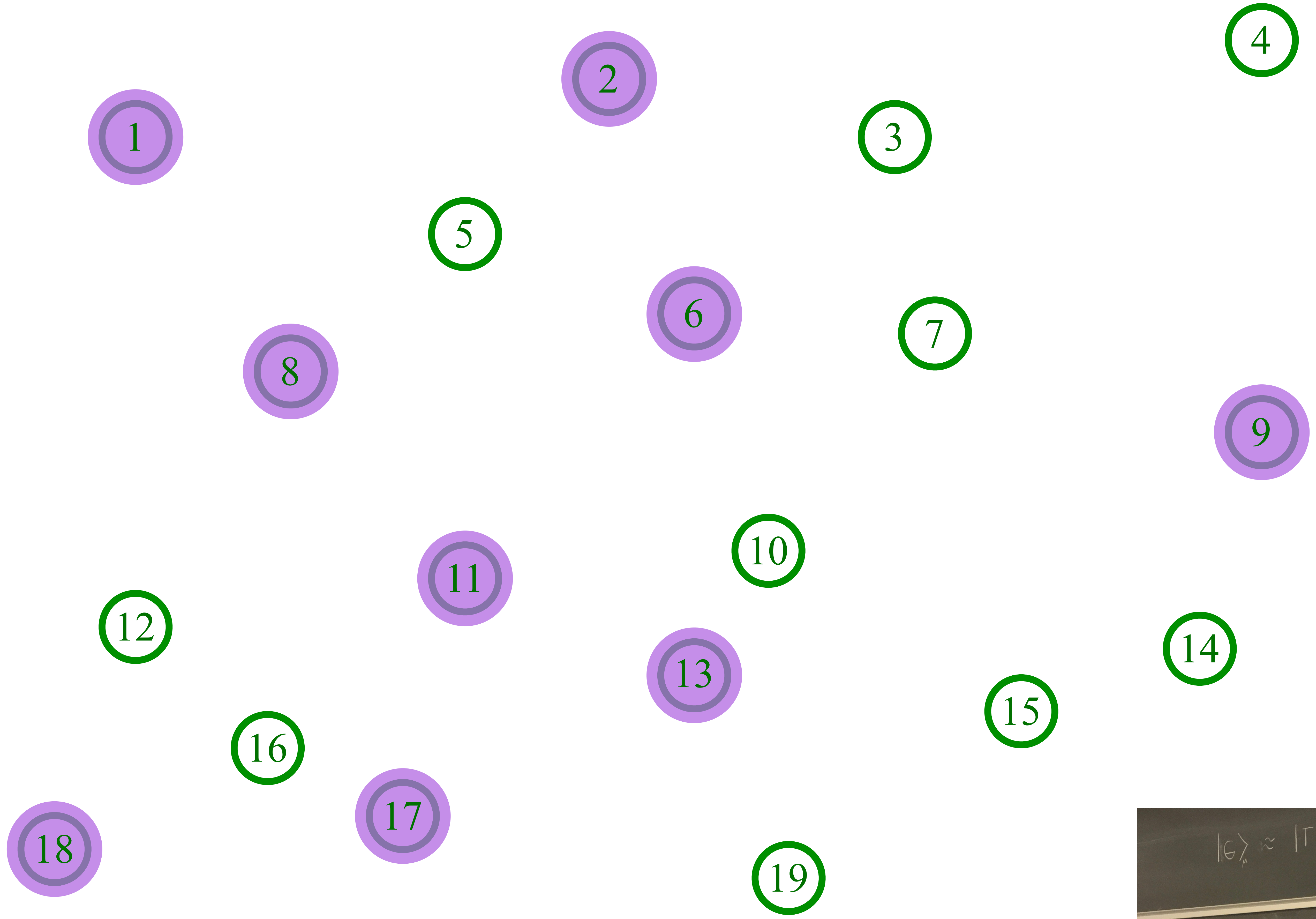

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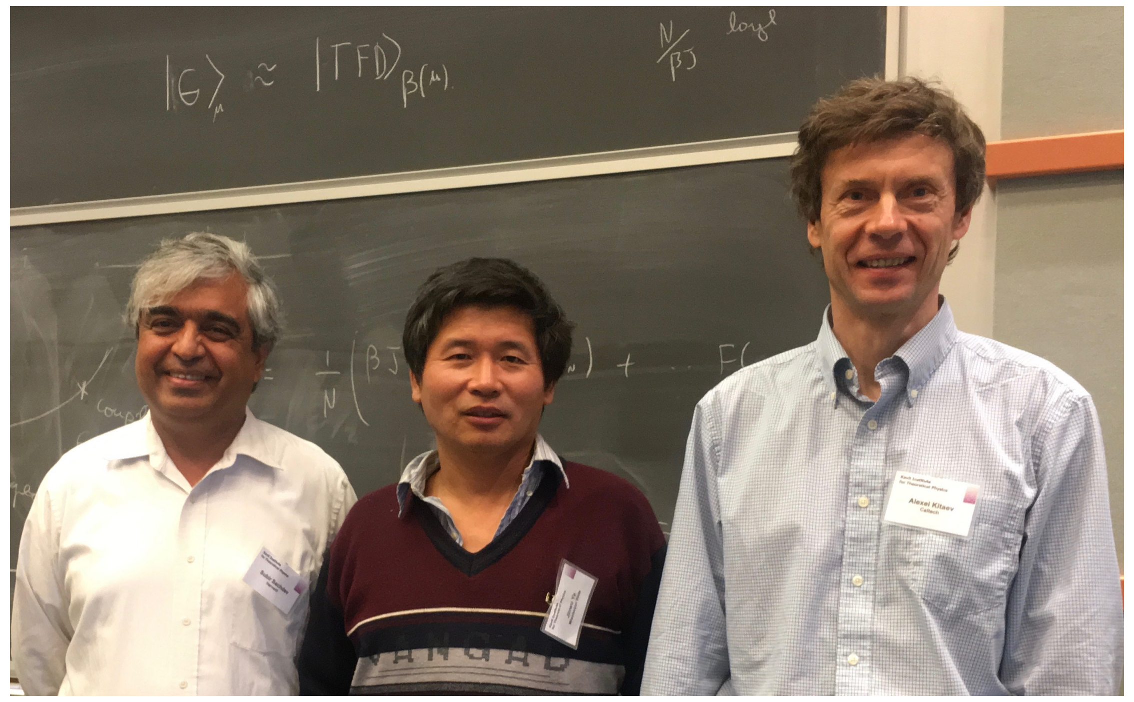

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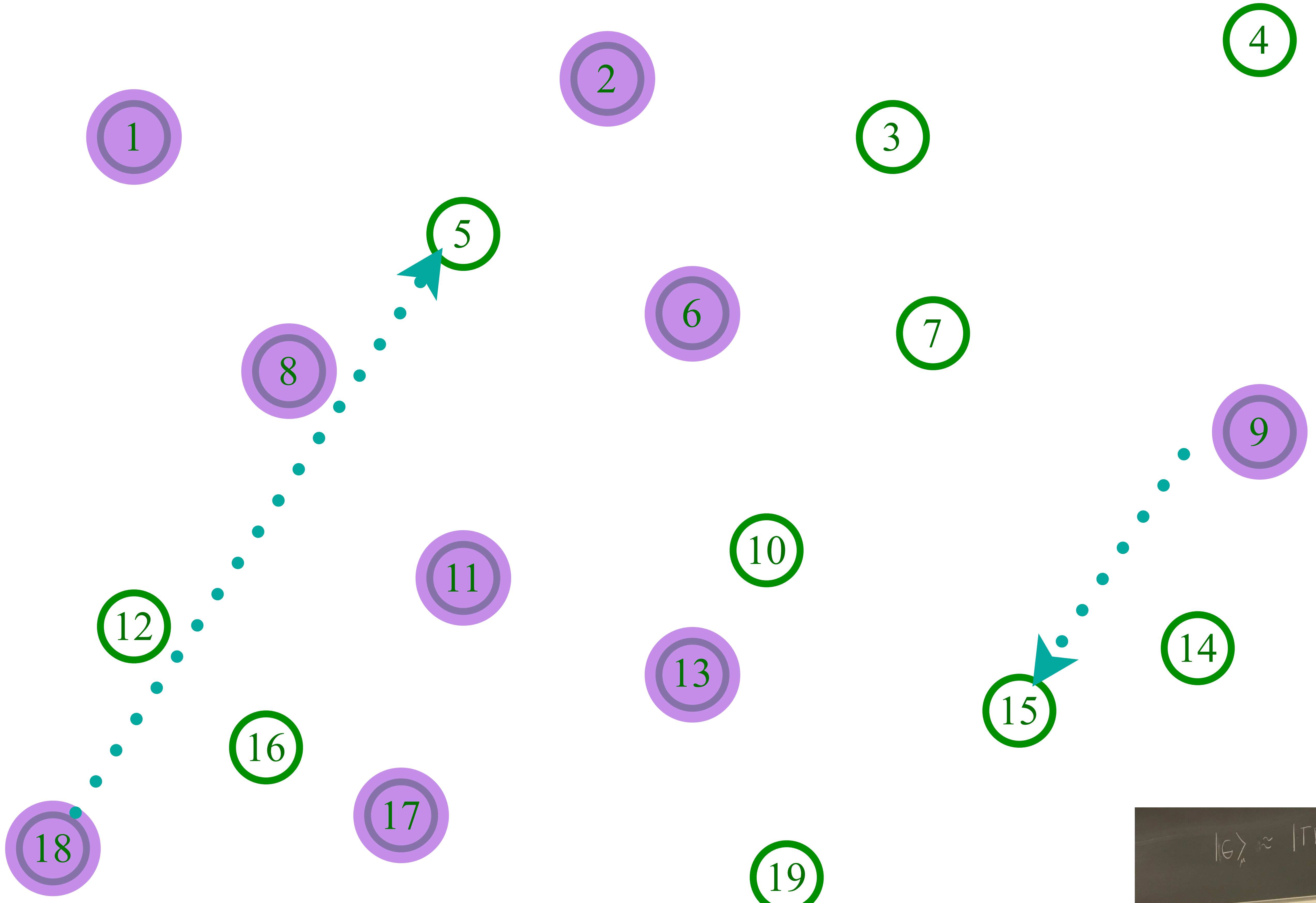
Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



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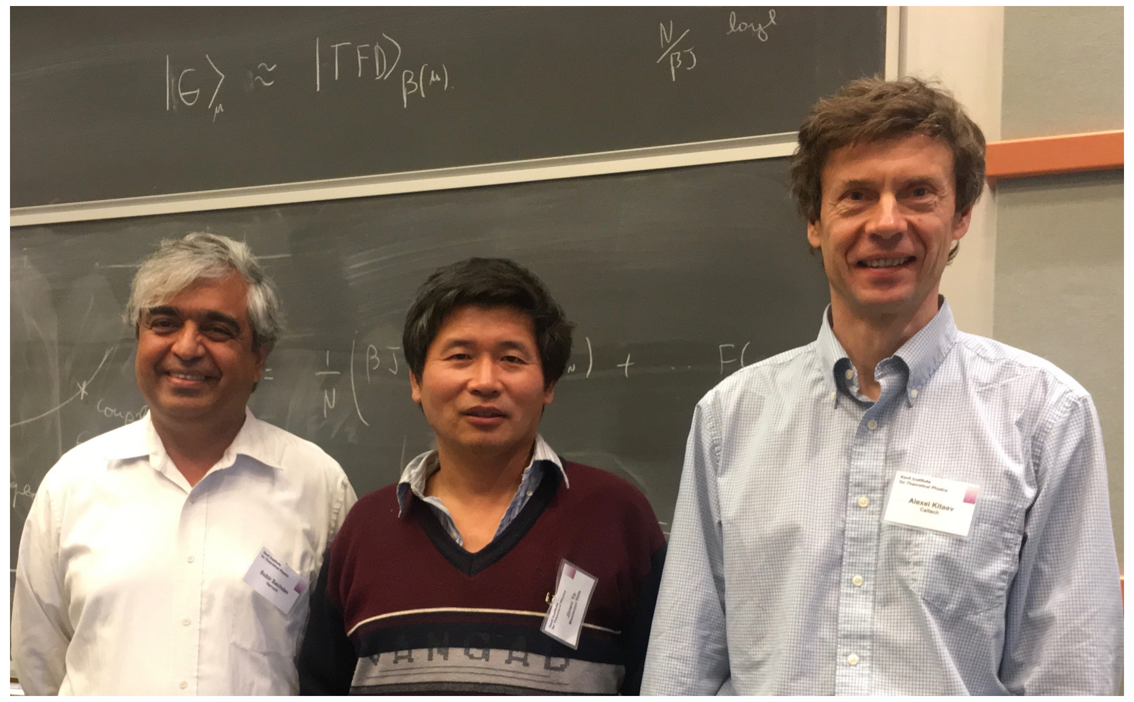

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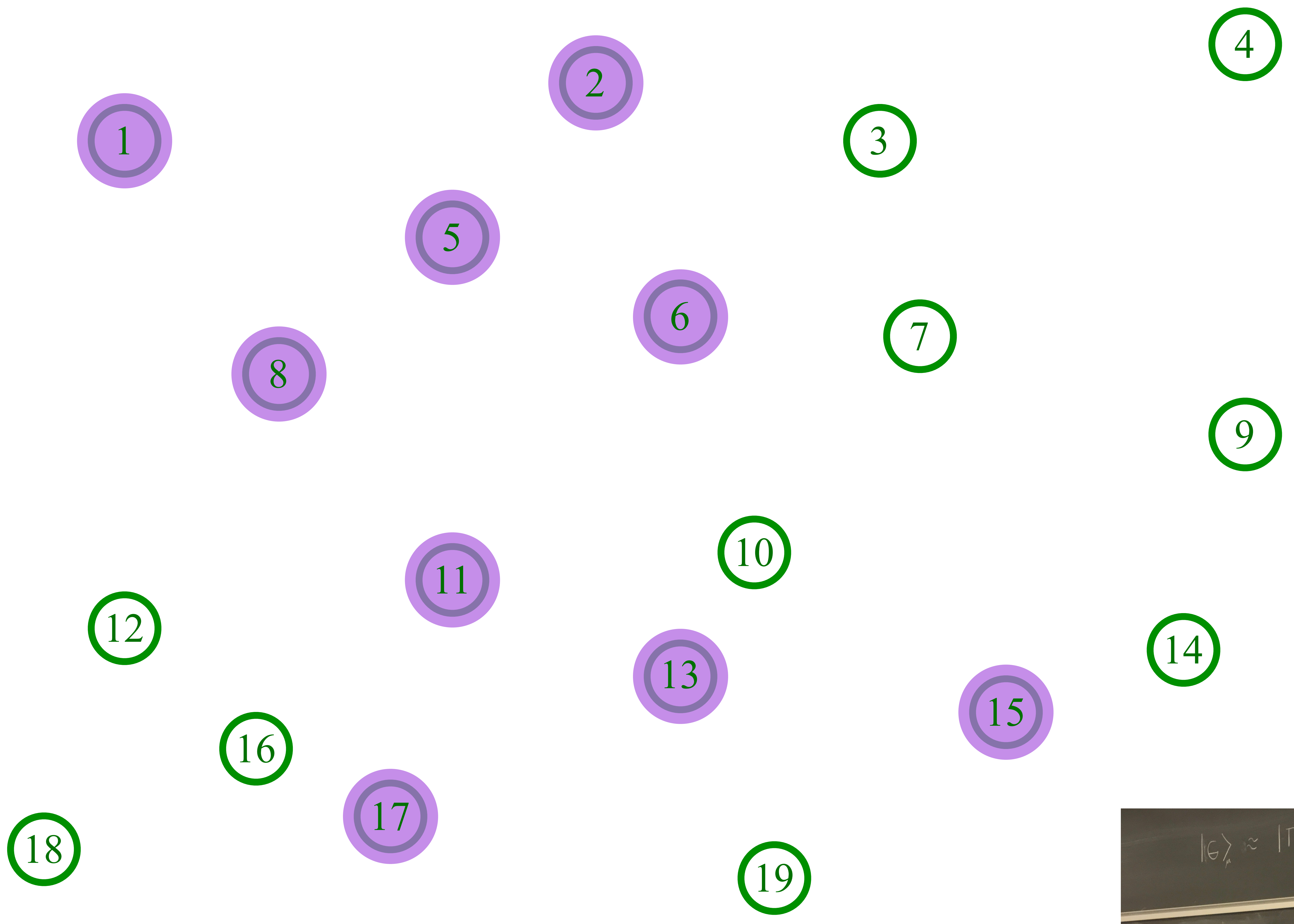

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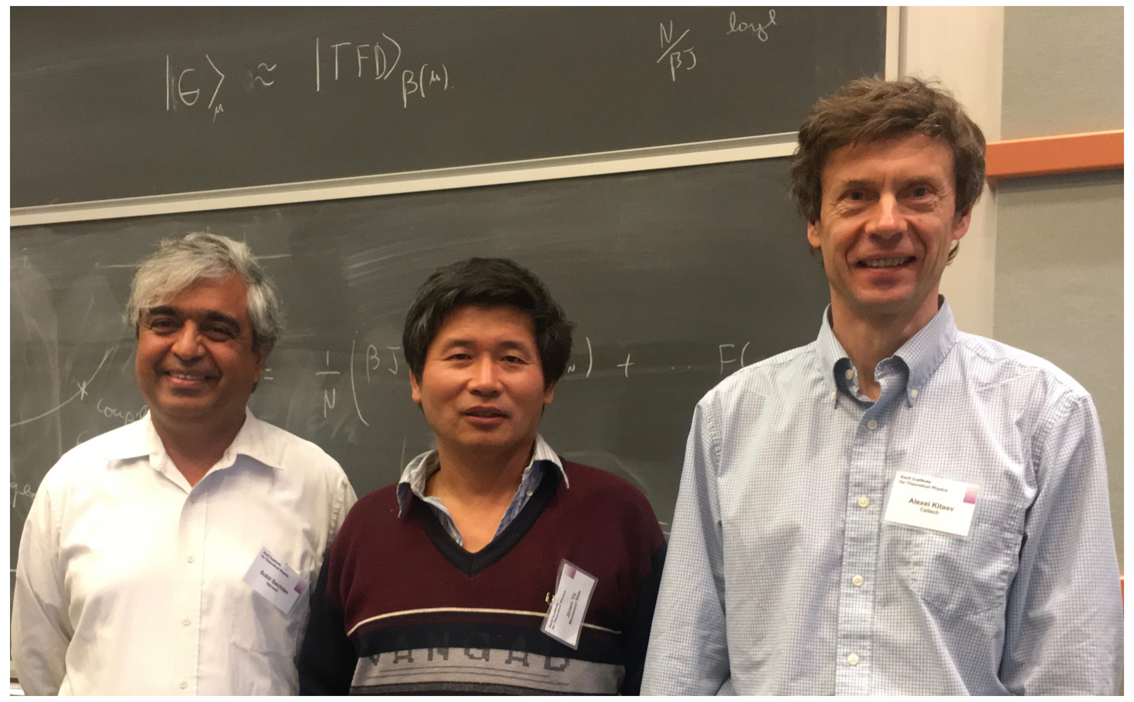

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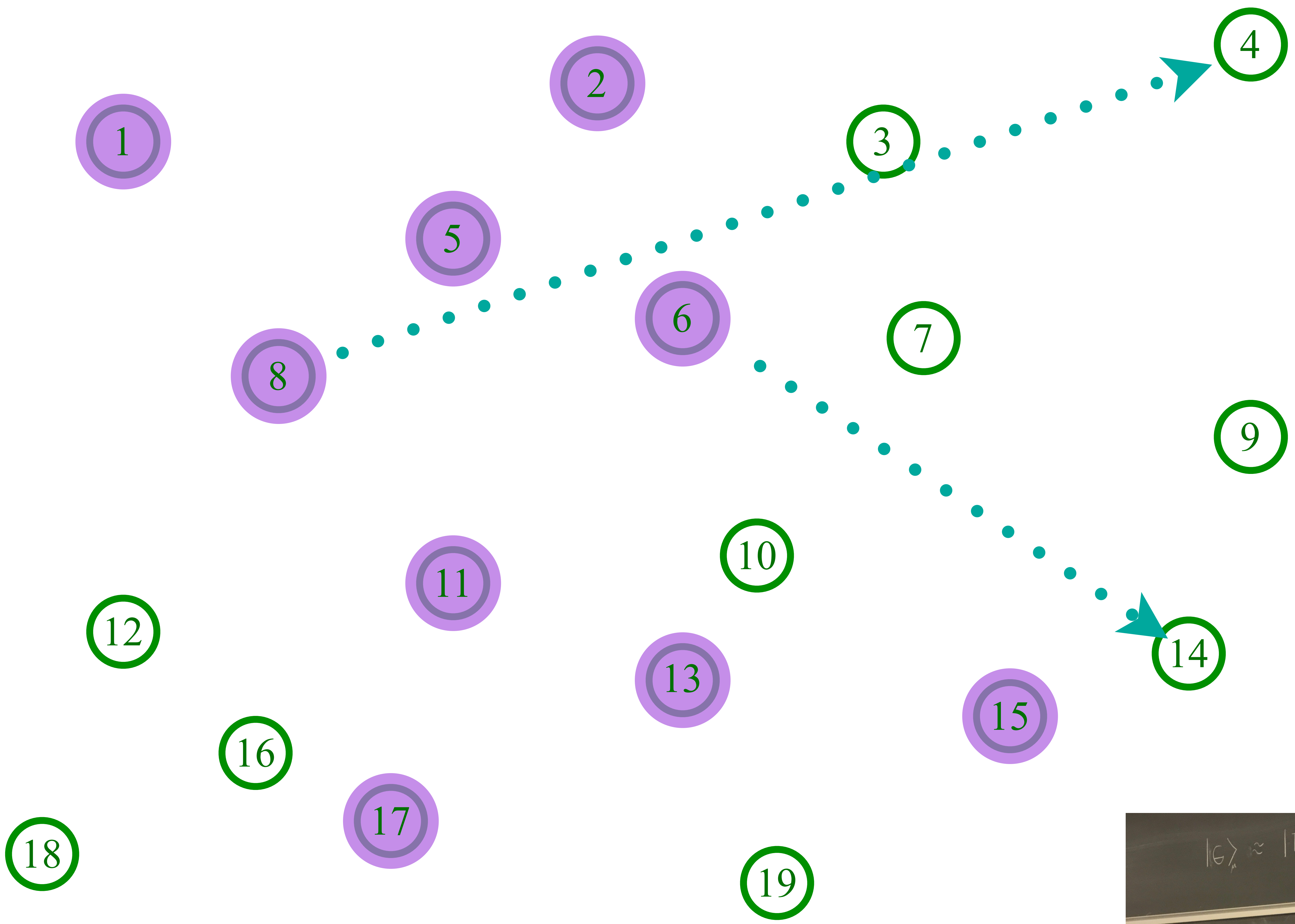
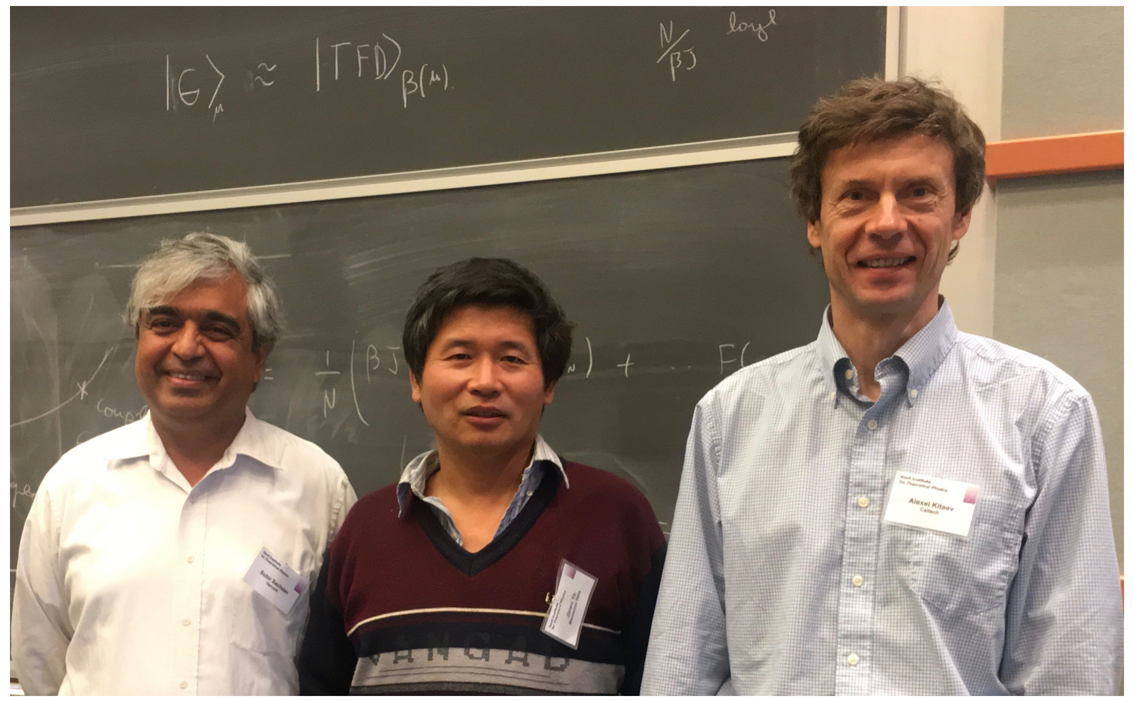


$$U_{6,8;4,14}$$

Entangle electrons pairwise randomly



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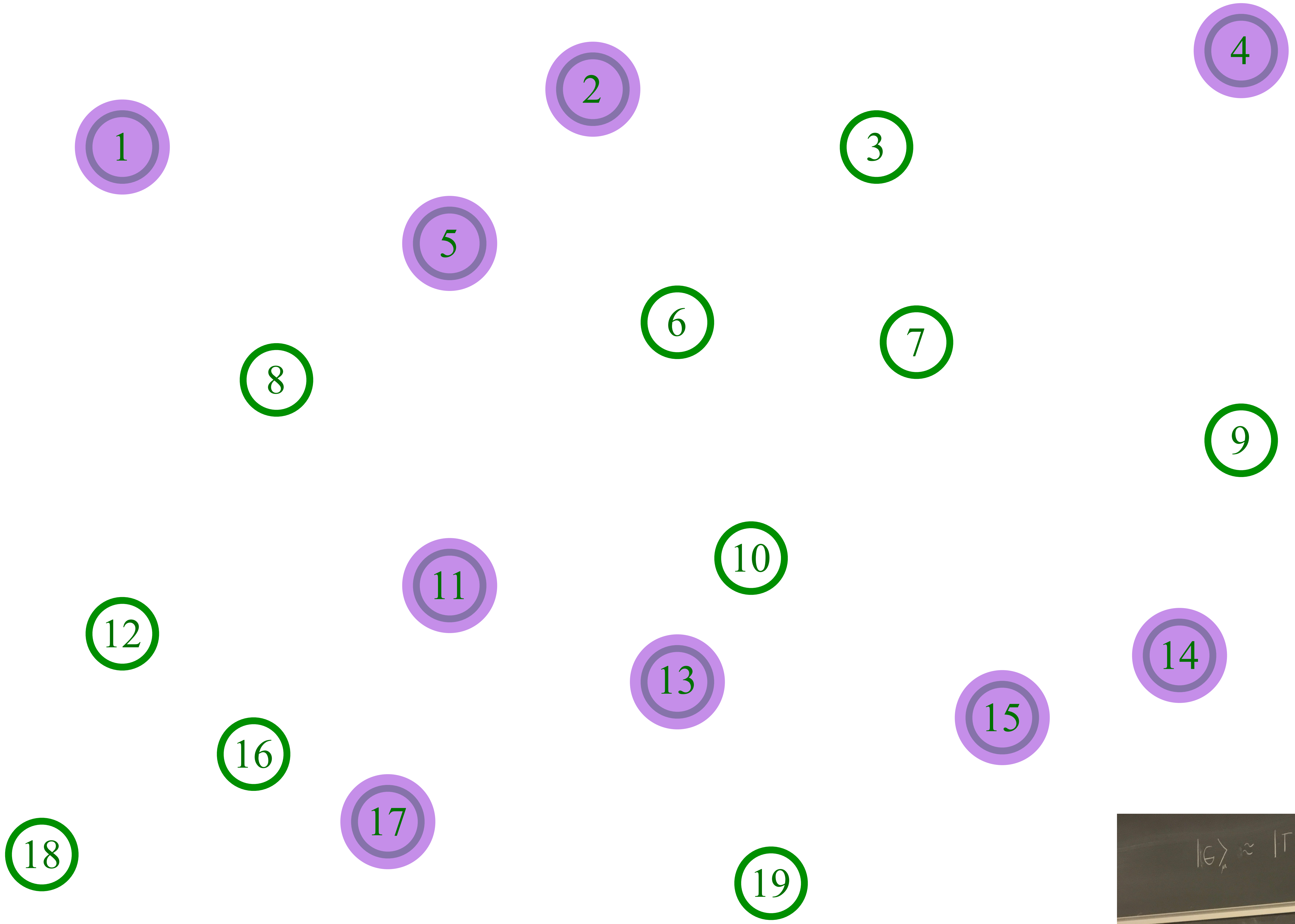

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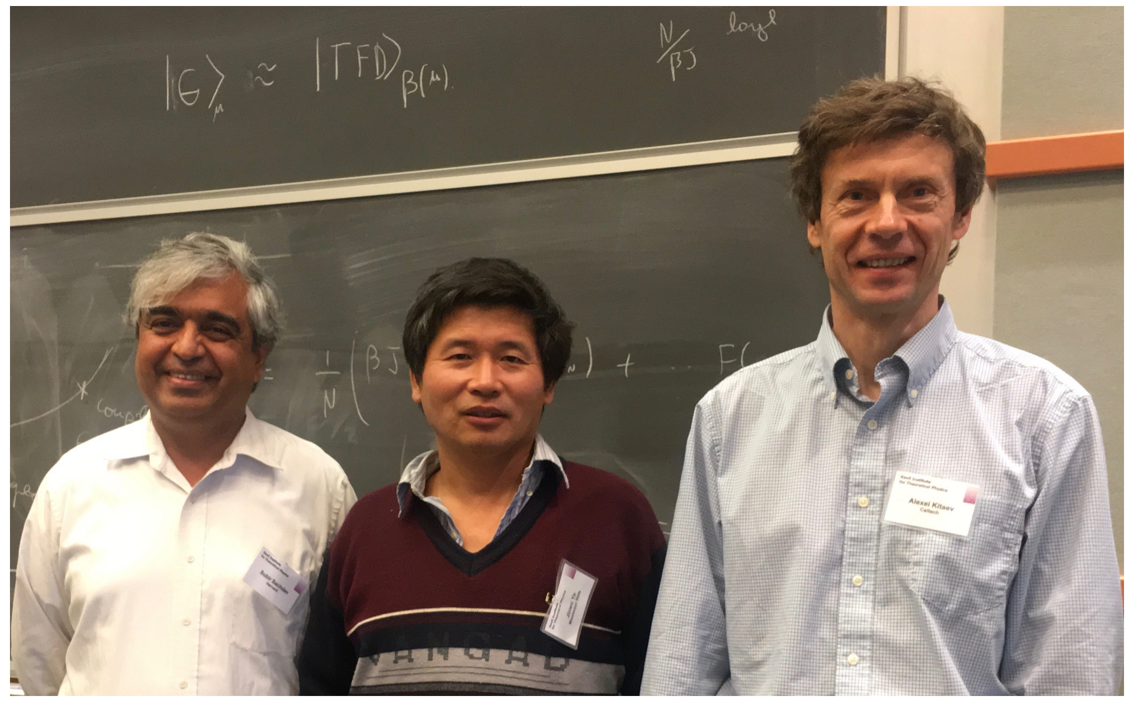


The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

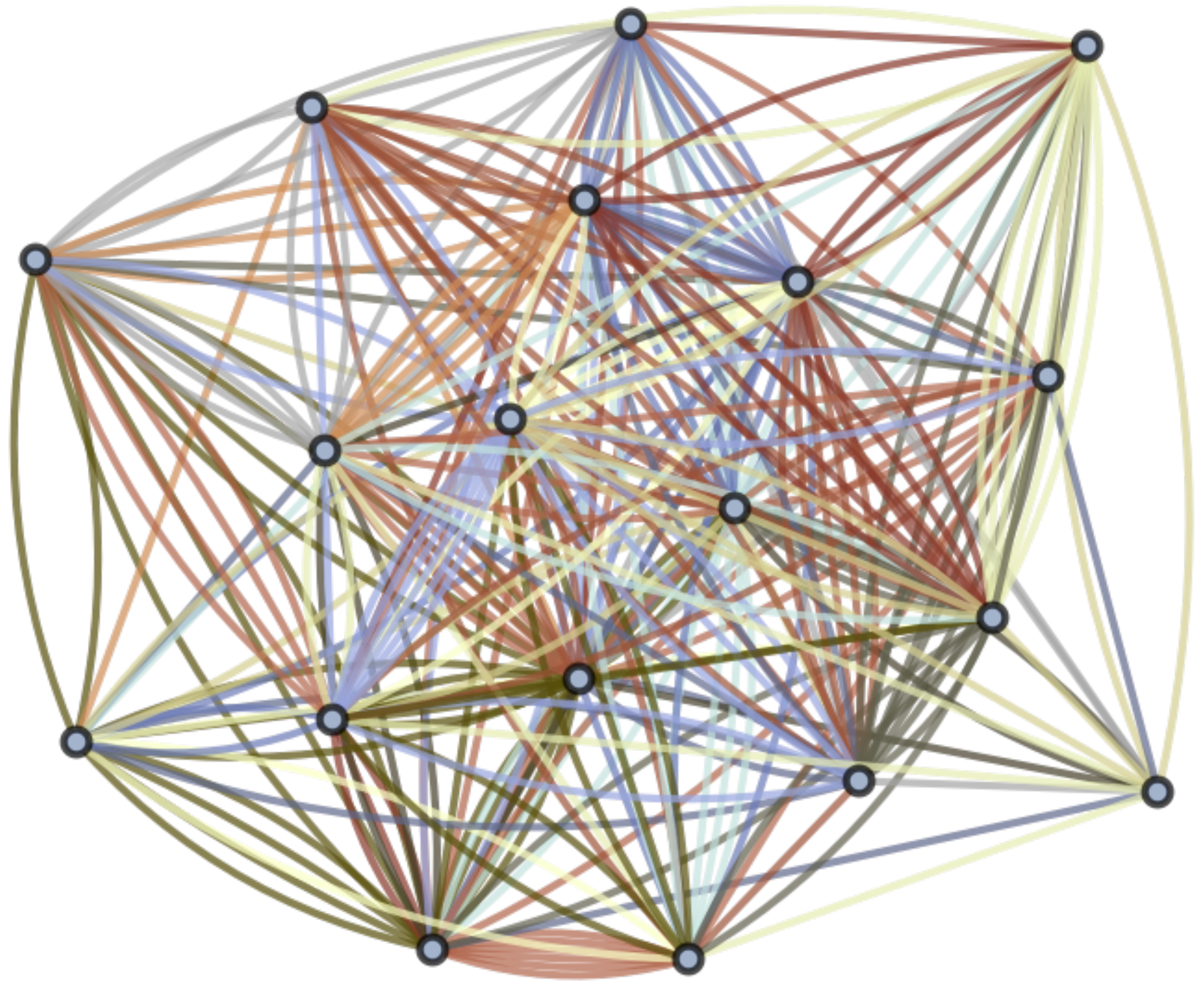
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

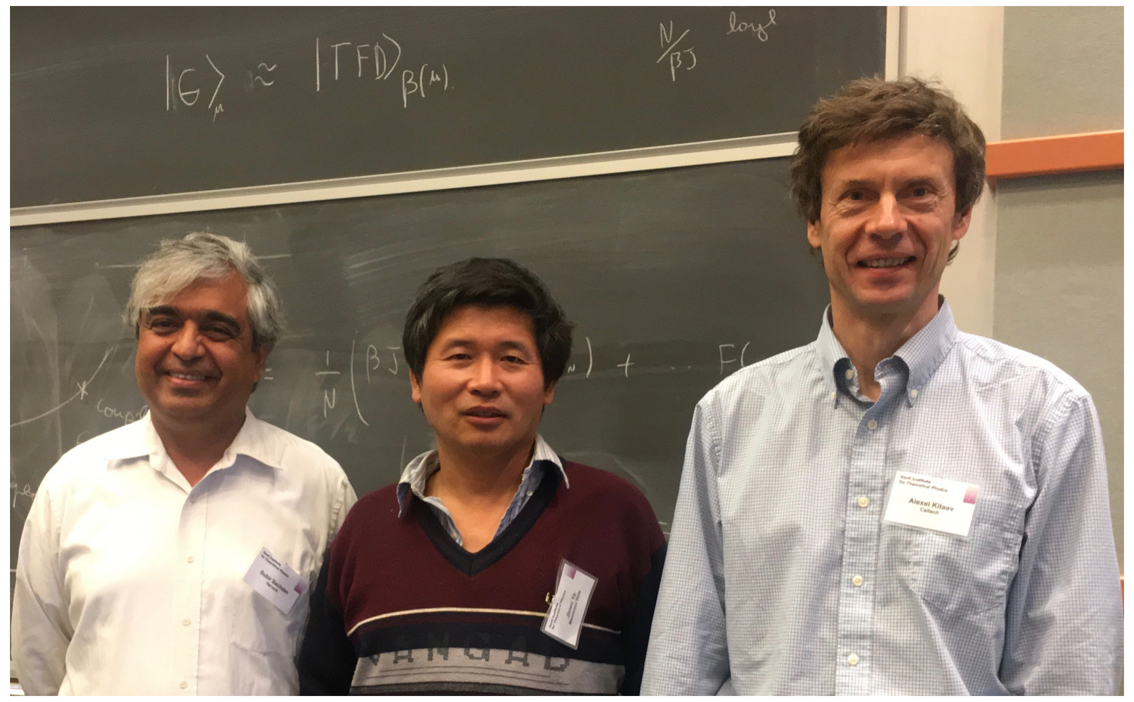
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha \, ; \quad [\mathcal{H}, \mathcal{Q}] = 0 \, ; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

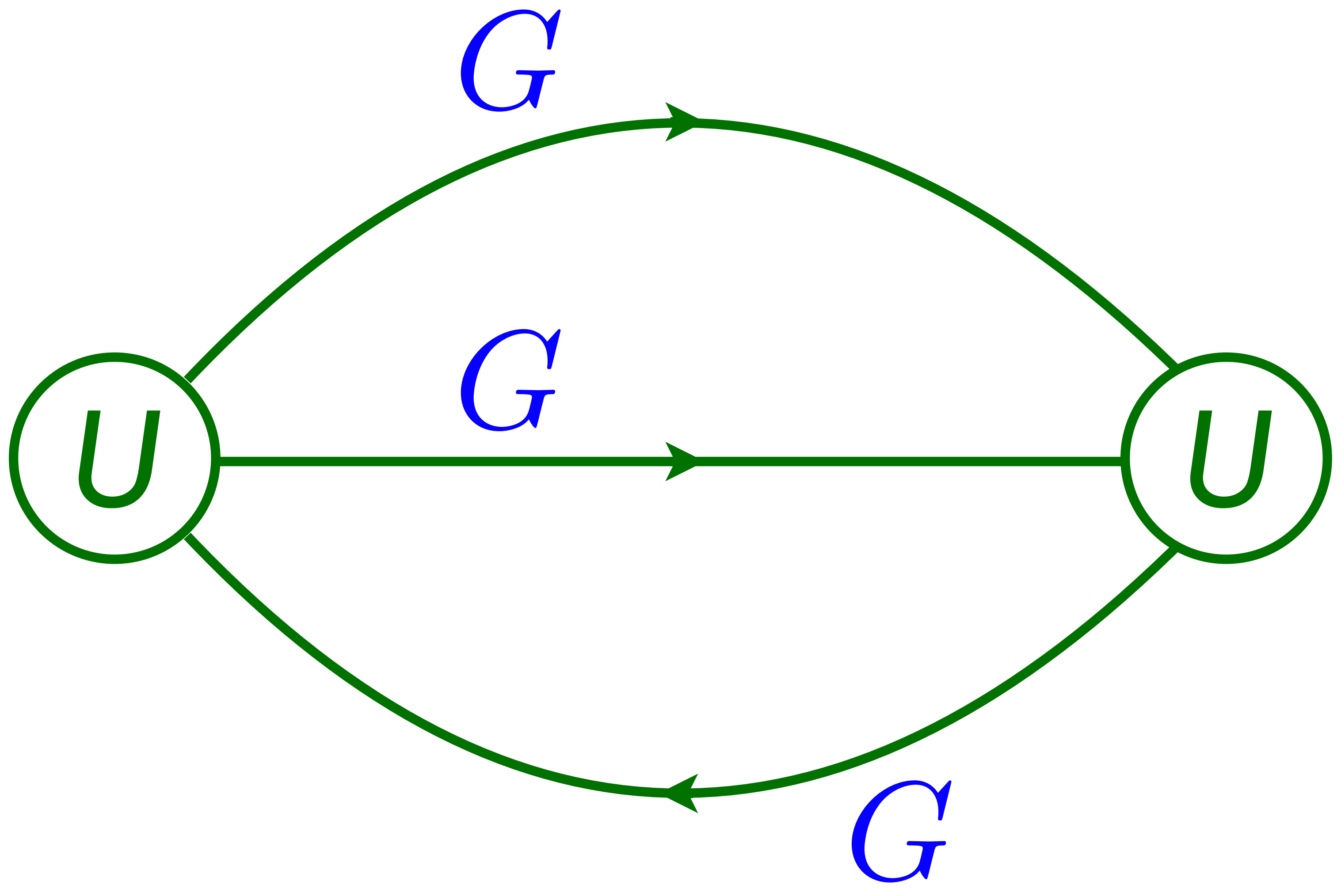




The Sachdev-Ye-Kitaev (SYK) model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

$$\Sigma =$$ 

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

The complex SYK model

Solution of these equations, and of the free energy, yields universal results for the SYK model:

- At long times, and at $T = 0$, $G(\tau) \sim |\tau|^{-1/2}$ ($\Rightarrow$ indication there are no quasiparticles)

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- At long times, and at $T = 0$, $G(\tau) \sim |\tau|^{-1/2}$ ($\Rightarrow$ indication there are no quasiparticles)
- At general charge $\mathcal{Q}$, there is a spectral symmetry determined by a parameter $\mathcal{E}$:

$$G(\tau) \sim \begin{cases} -\tau^{-1/2} & \tau > 0 \\ e^{-2\pi\mathcal{E}}(-\tau)^{-1/2} & \tau < 0 \end{cases} \quad , \quad T = 0$$

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- There is a universal 'Luttinger relation' between $-\infty < \mathcal{E} < \infty$ and the total charge $0 < \mathcal{Q} < 1$

$$\begin{aligned} e^{2\pi\mathcal{E}} &= \frac{\sin(\pi/4 + \theta)}{\sin(\pi/4 - \theta)} \\ \mathcal{Q} &= \frac{1}{2} - \frac{\theta}{\pi} - \frac{\sin(2\theta)}{4} \end{aligned}$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, PRB **95**, 155131 (2017)

The complex SYK model

Solution of these equations, and of the free energy, yields universal results for the SYK model:

- At $T > 0$, we obtain a solution with a conformal structure

$$G(\tau) = -A \frac{e^{-2\pi\mathcal{E}T\tau}}{\sqrt{1+e^{-4\pi\mathcal{E}}}} \left(\frac{T}{\sin(\pi T \tau)} \right)^{1/2} \quad , \quad 0 < \tau < 1/T \, ,$$

where the 'particle-hole asymmetry' is determined by $\mathcal{E}$

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

The complex SYK model

$$G_*(\tau) = -C \frac{e^{-2\pi\mathcal{E}T\tau}}{\sqrt{1+e^{-4\pi\mathcal{E}}}} \left(\frac{T}{\sin(\pi T\tau)}\right)^{1/2} .$$

$$G^R_*(\omega) =$$

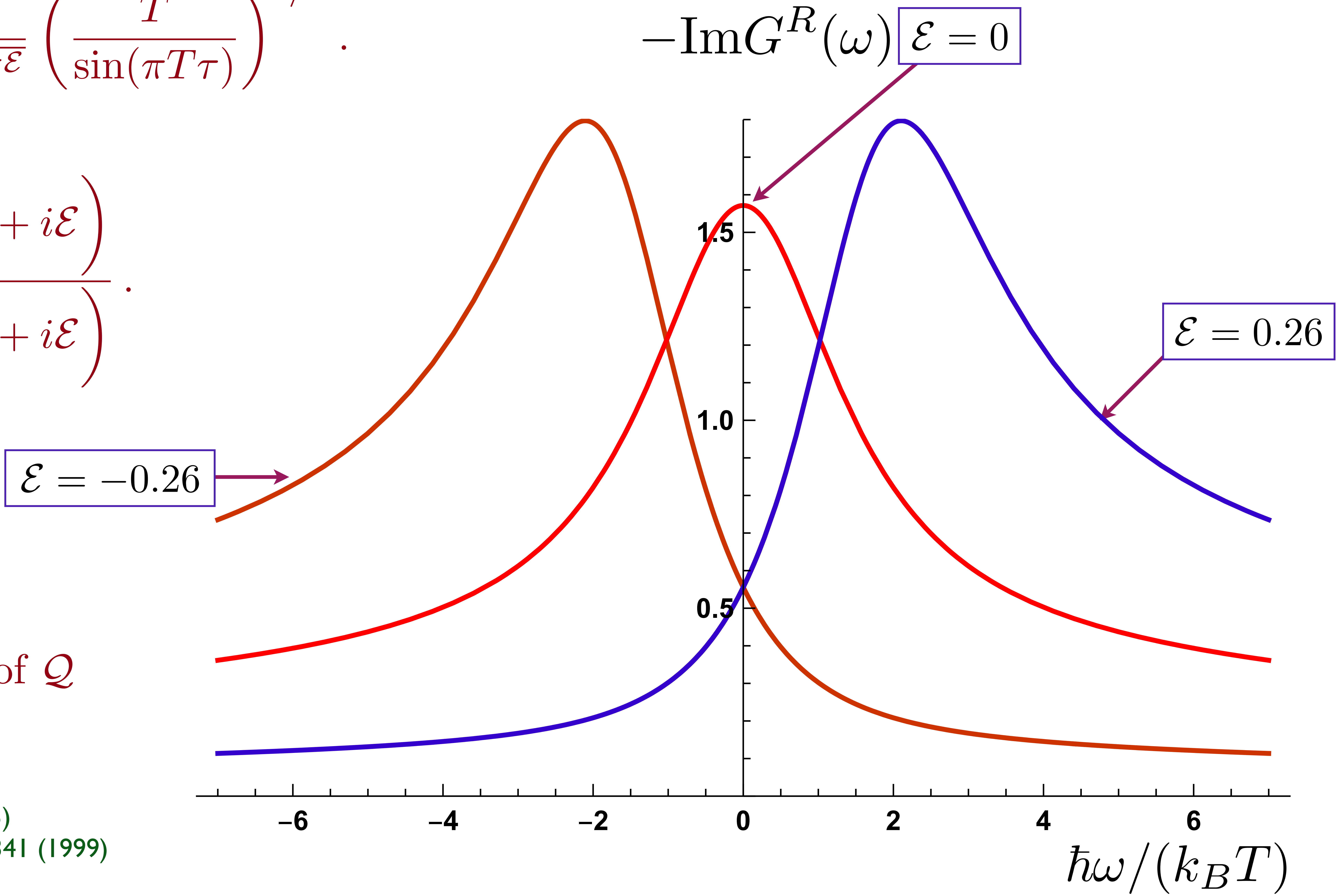
$$\frac{-iCe^{-i\theta}}{(2\pi T)^{1/2}} \frac{\Gamma\left(\frac{1}{4} - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(\frac{3}{4} - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)} .$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4+\theta)}{\sin(\pi/4-\theta)}$$

$$C = \left(\frac{\pi}{U^2\cos(2\theta)}\right)^{1/4}$$

$\mathcal{E}$ is a known function of $\mathcal{Q}$

(Luttinger relation)



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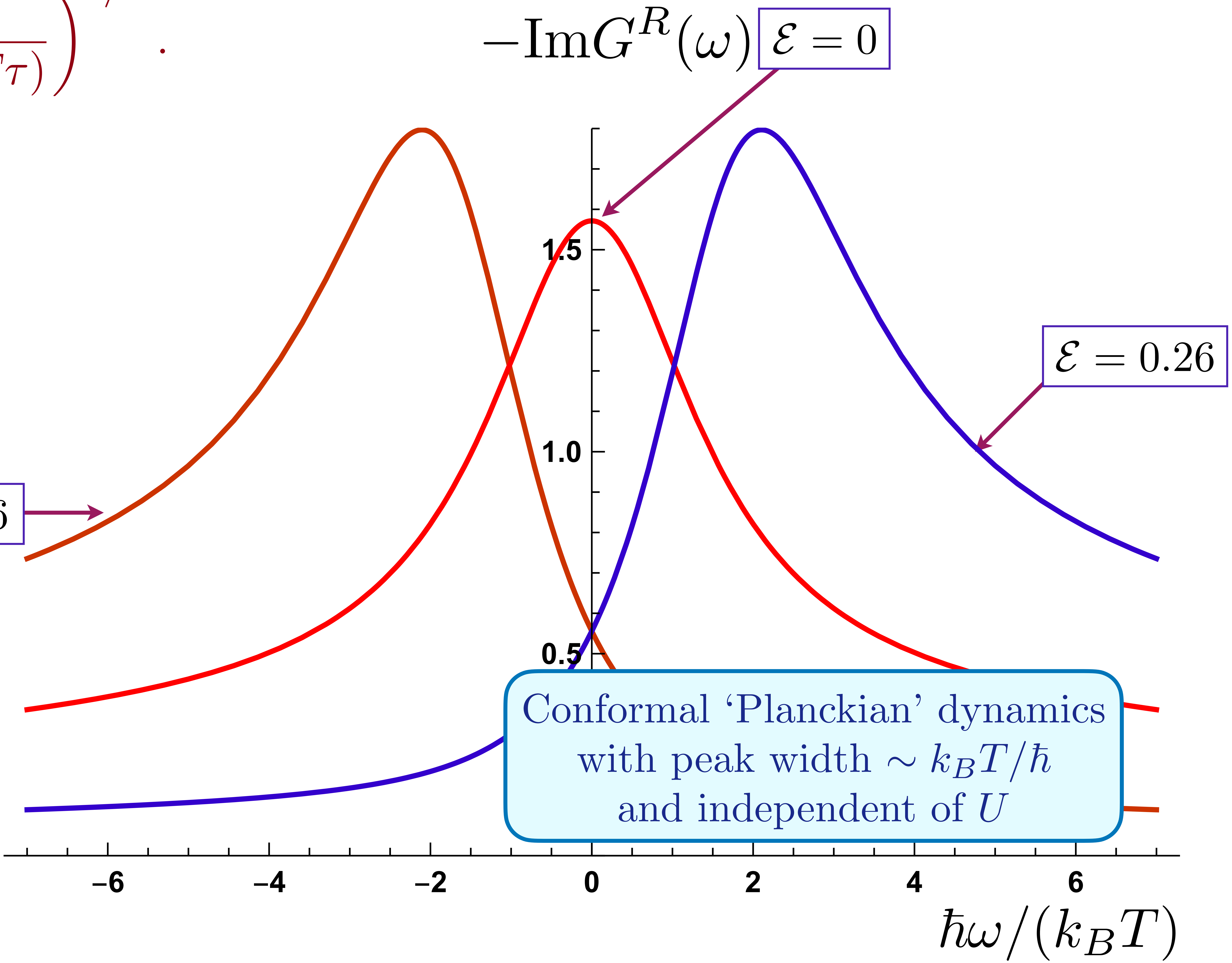

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S. Sachdev, PRX **5**, 041025 (2015)

G-Σ path integral

After introducing replicas $a = 1 \ldots n$, and integrating out the disorder, the partition function can be written as

$$Z = \int \mathcal{D}c_{ia}(\tau) \exp \Bigg[- \sum_{ia} \int_0^\beta d\tau \, c_{ia}^\dagger \left(\frac{\partial}{\partial \tau} - \mu \right) c_{ia} - \frac{U^2}{4N^3} \sum_{ab} \int_0^\beta d\tau d\tau' \left| \sum_i c_{ia}^\dagger(\tau) c_{ib}(\tau') \right|^4 \Bigg] \,.$$

For simplicity, we neglect the replica indices, and introduce the identity

$$1 = \int \mathcal{D}G(\tau_1, \tau_2) \mathcal{D}\Sigma(\tau_1, \tau_2) \exp \Bigg[-N \int_0^\beta d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2) \Bigg(G(\tau_2, \tau_1) + \frac{1}{N} \sum_i c_i(\tau_2) c_i^\dagger(\tau_1) \Bigg) \Bigg] \,.$$

G-Σ path integral

Then the partition function can be written as a path integral with an action S analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1, \tau_2)\mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NS)$$

$$S = \ln\det\left[\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)\right] + \int d\tau_1 d\tau_2 \left[\Sigma(\tau_1, \tau_2)G(\tau_2, \tau_1) + (U^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)\right]$$

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Saddle-point equations:

$$G(i\omega) = \frac{1}{i\omega+\mu-\Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2G^2(\tau)G(-\tau)$$

$$G(\tau=0^-) = \mathcal{Q}.$$

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At frequencies $\ll U$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, 2015
S. Sachdev, PRX **5,** 041025 (2015)

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = \left[f'(\sigma_1)f'(\sigma_2)\right]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = \left[f'(\sigma_1)f'(\sigma_2)\right]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \Sigma(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

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Then the partition function can be written as a path integral with an action S analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1,\tau_2)\mathcal{D}\Sigma(\tau_1,\tau_2)\exp(-NS)$$

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$$\tau = f(\sigma)$$

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$$\Sigma(\tau_1,\tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-3/4}\frac{g(\sigma_1)}{g(\sigma_2)}\Sigma(\sigma_1,\sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

We can map the $T=0$ solution to the $T>0$ solution by

$$\tau = \frac{1}{\pi T}\tan(\pi T\sigma)$$

$$g(\sigma) = e^{-2\pi\mathcal{E}T\sigma}$$

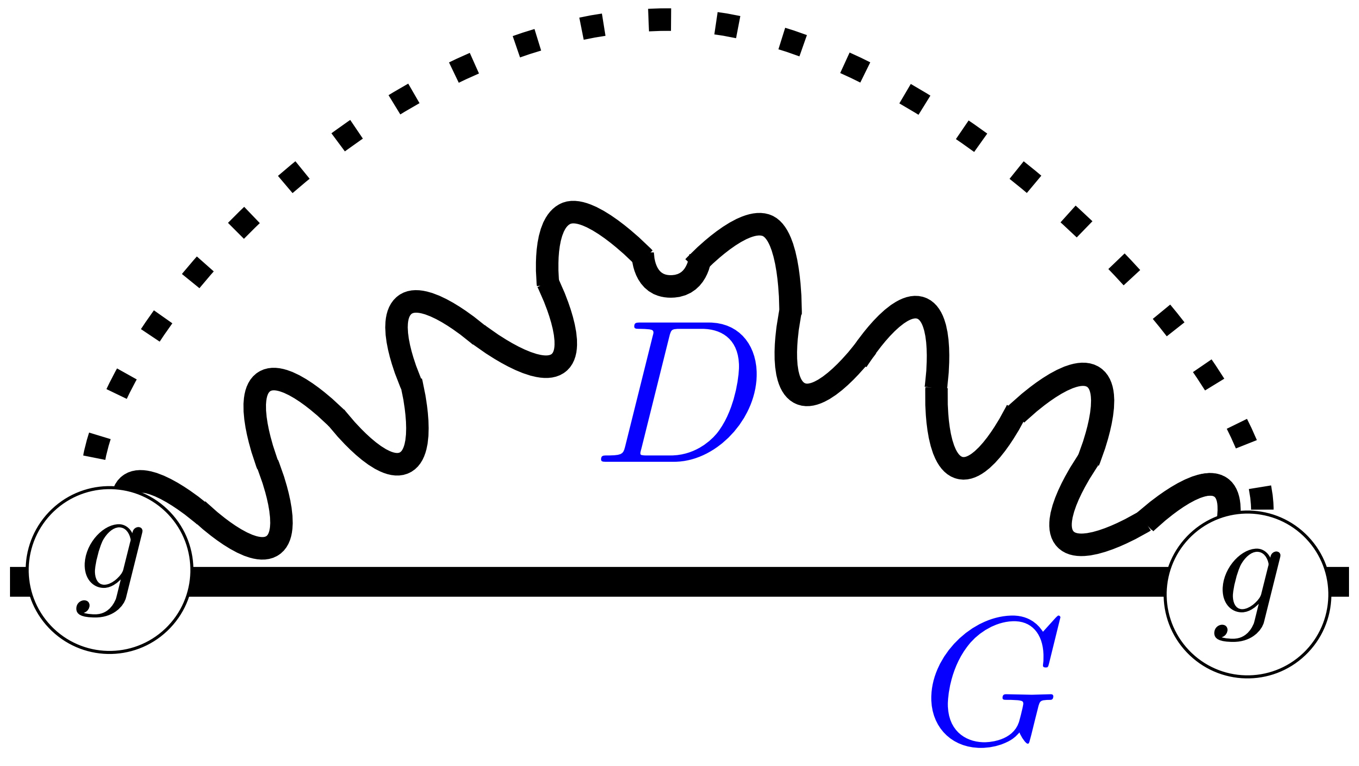
Yukawa-SYK model

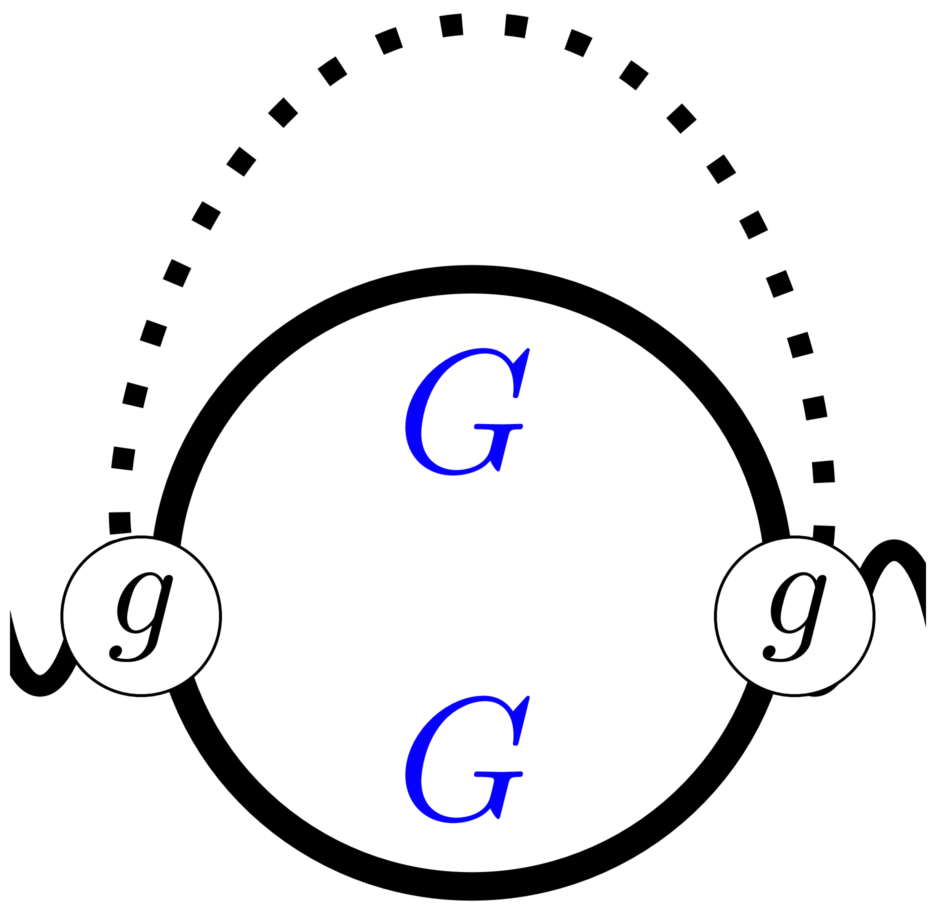
$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large N equations for the Green's functions and self energies of the fermions (G, Σ) and bosons (D, Π) are

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)}$$

$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$

$\Sigma =$ 

$\Pi =$ 

I. Esterlis and J. Schmalian,
PRB **100**, 115132 (2019)
See also Yuxuan Wang,
PRL **124**, 017002 (2020)

Yukawa-SYK model

These results can also be obtained from the saddle-point of a G-Σ-D-Π action, obtained using replica methods as for the SYK model.

$$\mathcal{Z} = \int \mathcal{D}G\, \mathcal{D}\Sigma\, \mathcal{D}D\, \mathcal{D}\Pi \exp(-N S_{\rm all})$$

$$\begin{aligned} S_{\rm all} &= -\ln\det(\partial_\tau + -\mu + \Sigma) + \frac{1}{2}\ln\det(-\partial_\tau^2 + \omega_0^2 - \Pi) \\ &+ \int d\tau \int d\tau' \left[-\Sigma(\tau';\tau)G(\tau;\tau') + \frac{1}{2}\Pi(\tau';\tau)D(\tau;\tau') \right. \\ &\qquad\qquad \left. + \frac{g^2}{2} G(\tau;\tau')G(\tau';\tau)D(\tau;\tau') \right]. \end{aligned}$$

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Saddle-point equations:

$$\begin{aligned}
\Sigma(\tau) &= g^2 D(\tau)G(\tau), \\
\Pi(\tau) &= -g^2 G(-\tau)G(\tau), \\
G(i\omega) &= \frac{1}{i\omega + \mu - \Sigma(i\omega)}, \\
D(i\Omega) &= \frac{1}{\Omega^2 + \omega_0^2 - \Pi(i\Omega)}.
\end{aligned}$$

Yukawa-SYK model

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell}\psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large N equations for the Green's functions and self energies of the fermions (G, Σ) and bosons (D, Π) are

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)}$$

$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$

Make the low frequency ansatz

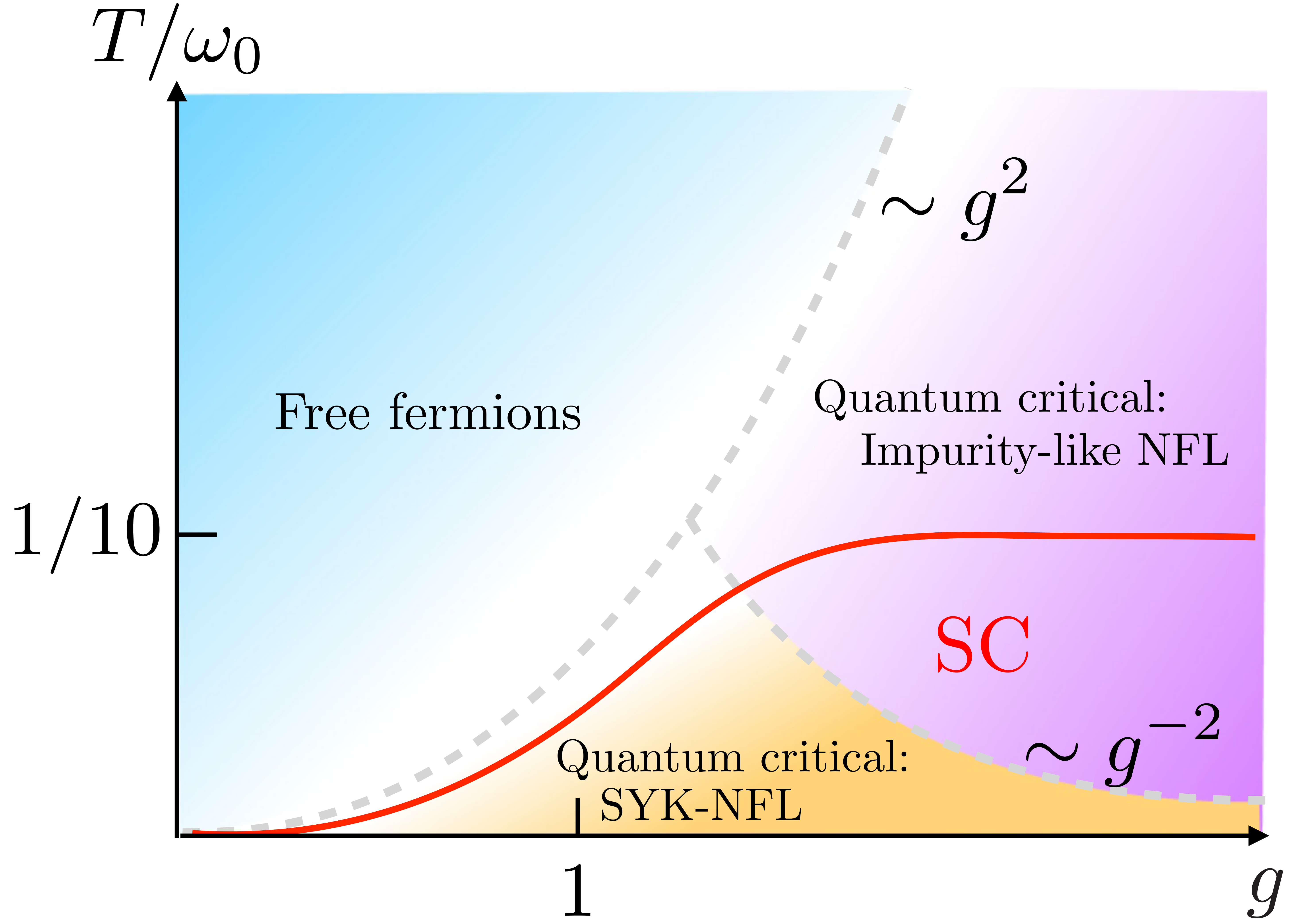
$$G(i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

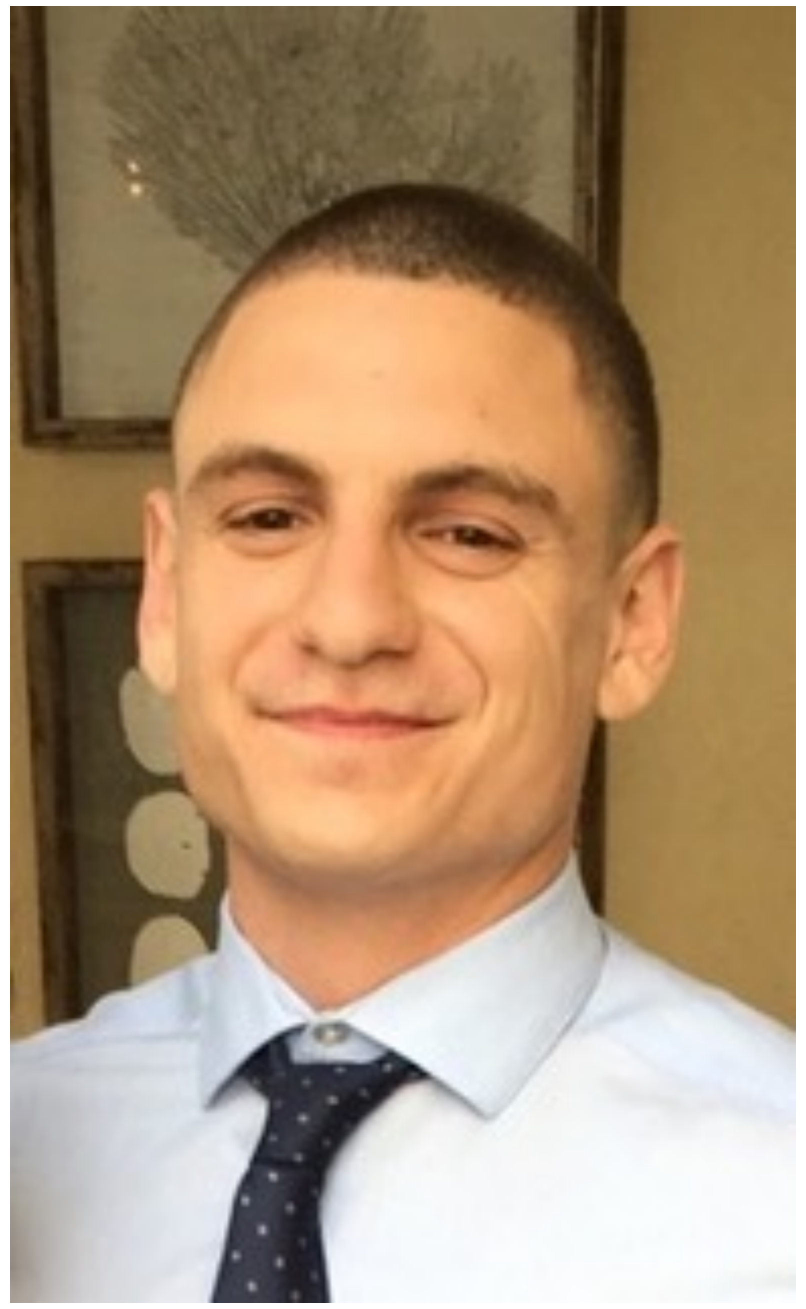
$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037\ldots$$

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

Yukawa-SYK model



I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)



Aavishkar Patel
Flatiron Institute, NYC

Haoyu Guo
Harvard

Ilya Esterlis
Harvard → Wisconsin

Universal theory of strange metals from spatially random interactions, Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, and S. S., *Science*, to appear

B24.00001: Subir Sachdev - Role of spatial disorder in strange metals, Monday 11:30 AM
G24.00003: Aavishkar Patel - Universal theory of strange metals from spatially random interactions, Tuesday 12:42 PM.
G30.00003 : Haoyu Guo - Large *N* theory of critical Fermi surfaces II: conductivity, Tuesday 11:54 AM

Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NO strange metal transport

Fermi surface coupled to a critical boson:

Potential disorder v

A marginal Fermi liquid but NO strange metal transport

Fermi surface coupled to a critical boson:

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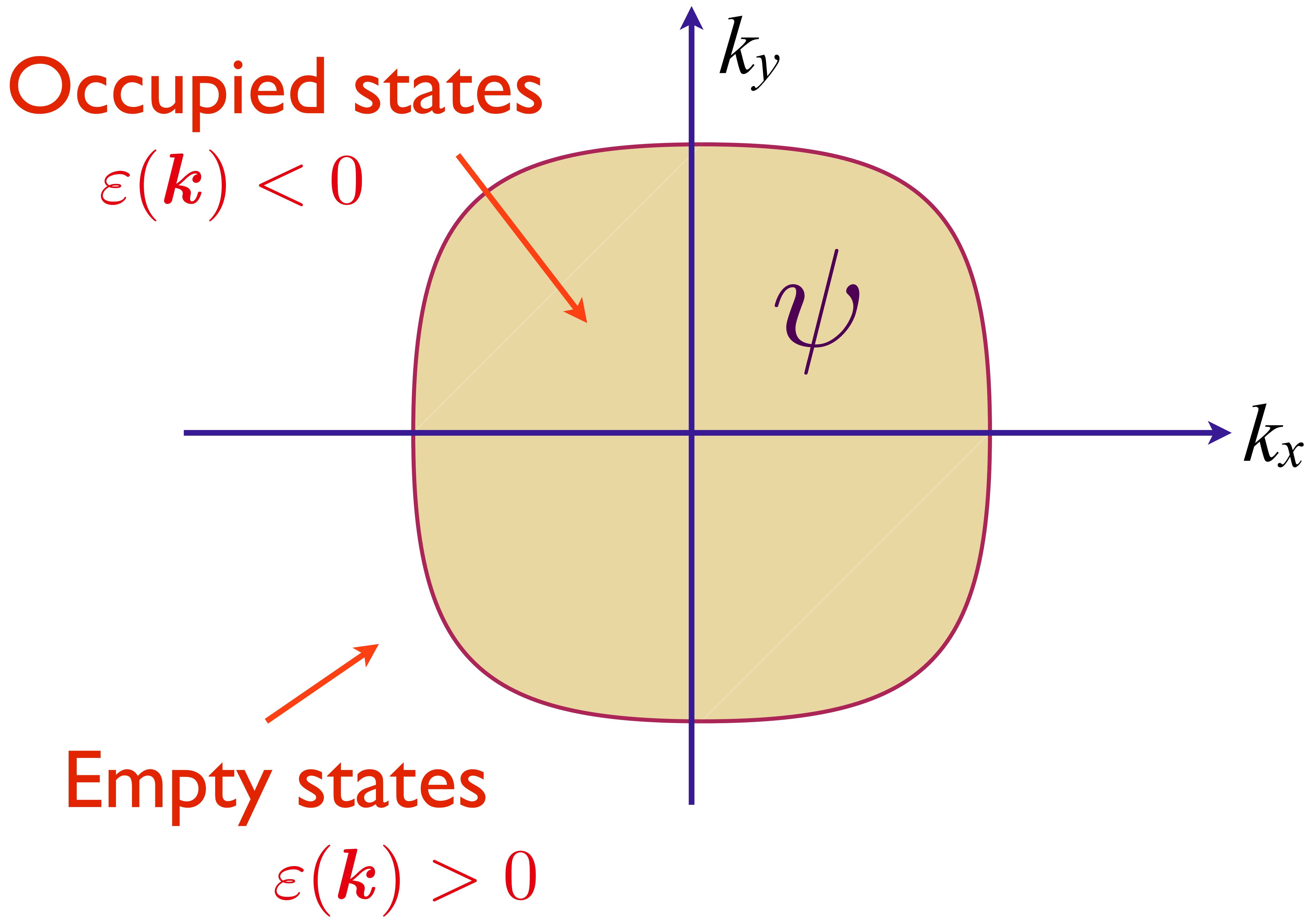
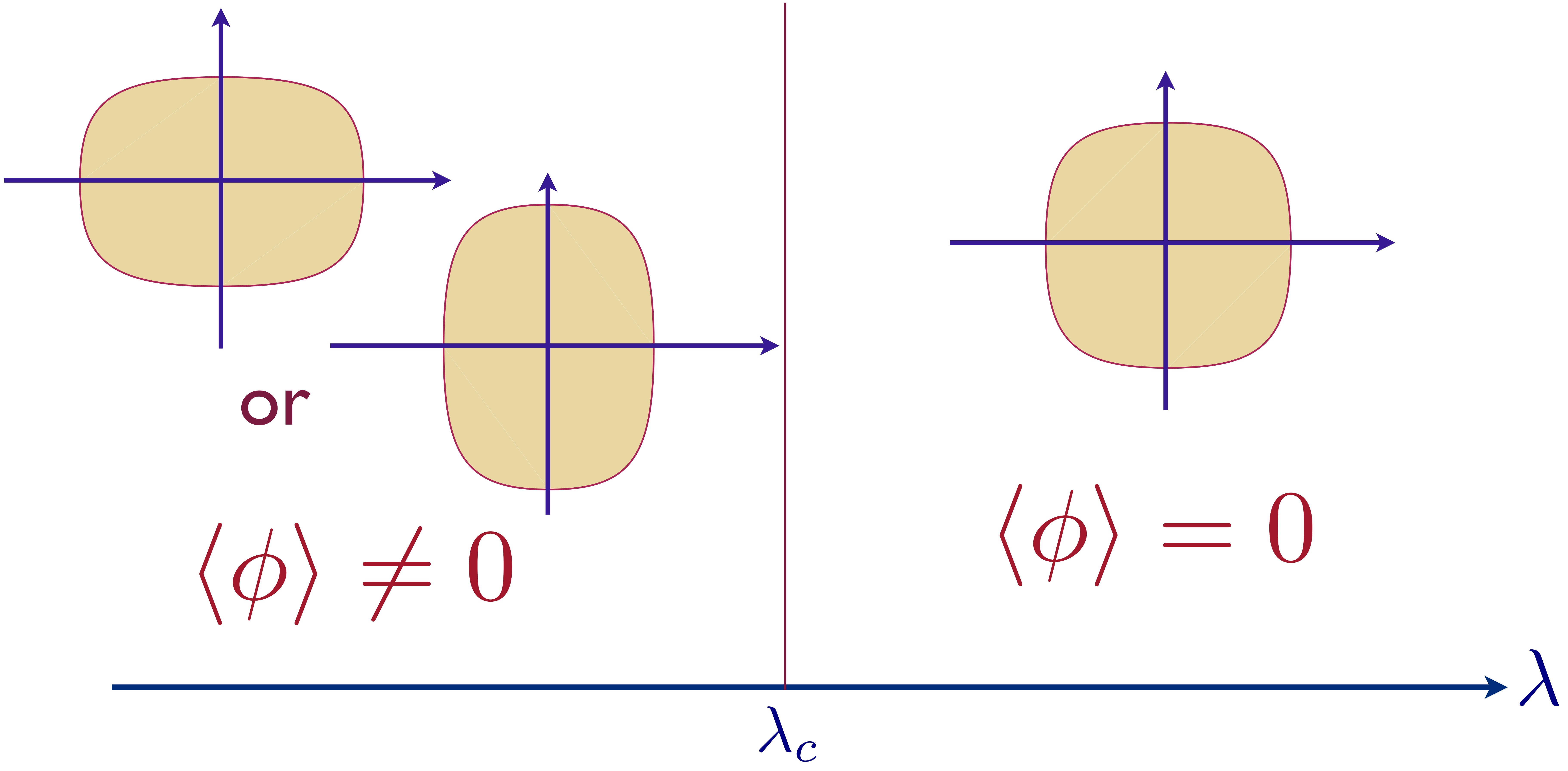


$+$

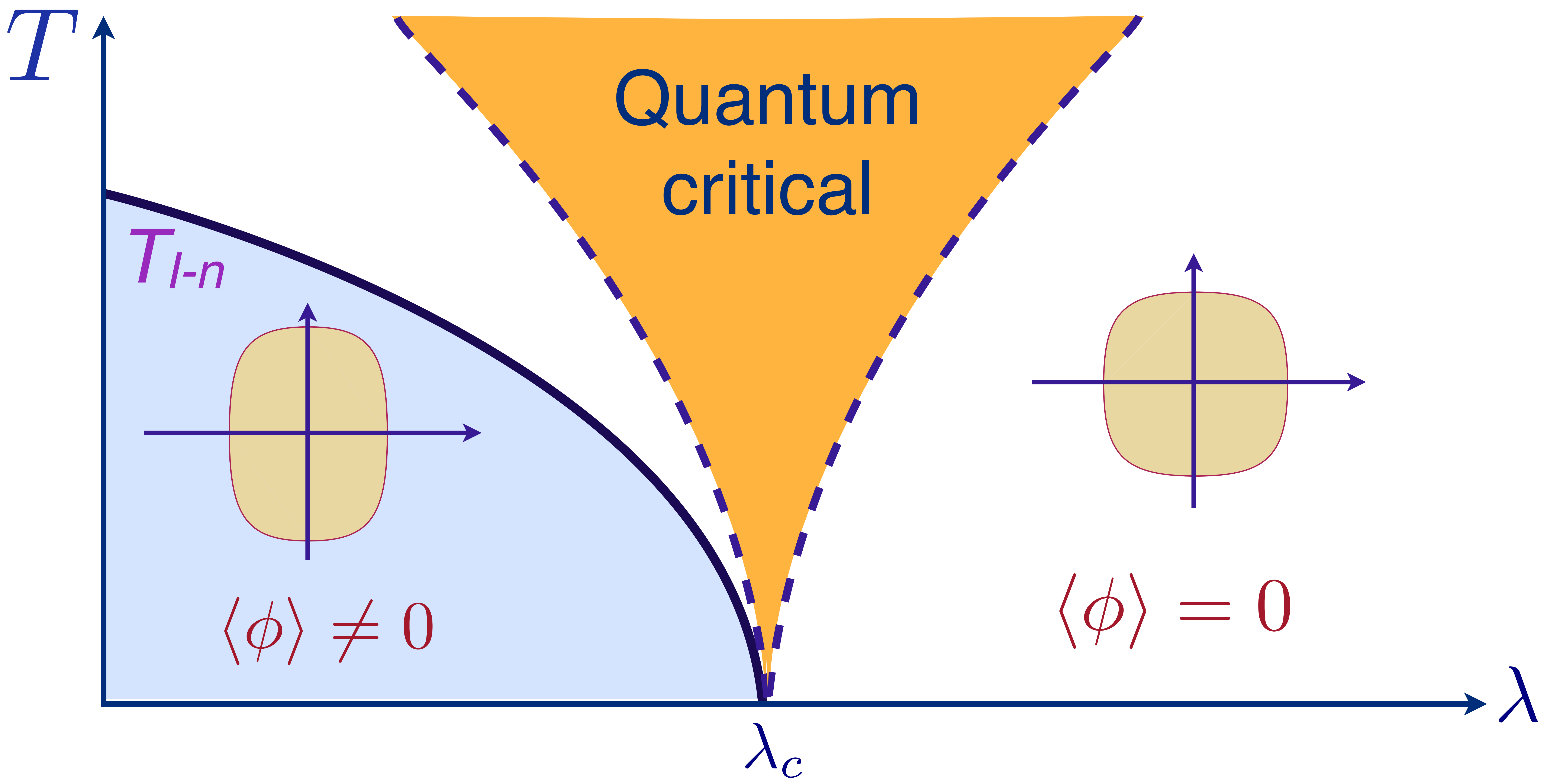
a
critical
boson
ϕ

Quantum criticality of Ising-nematic ordering in a metal



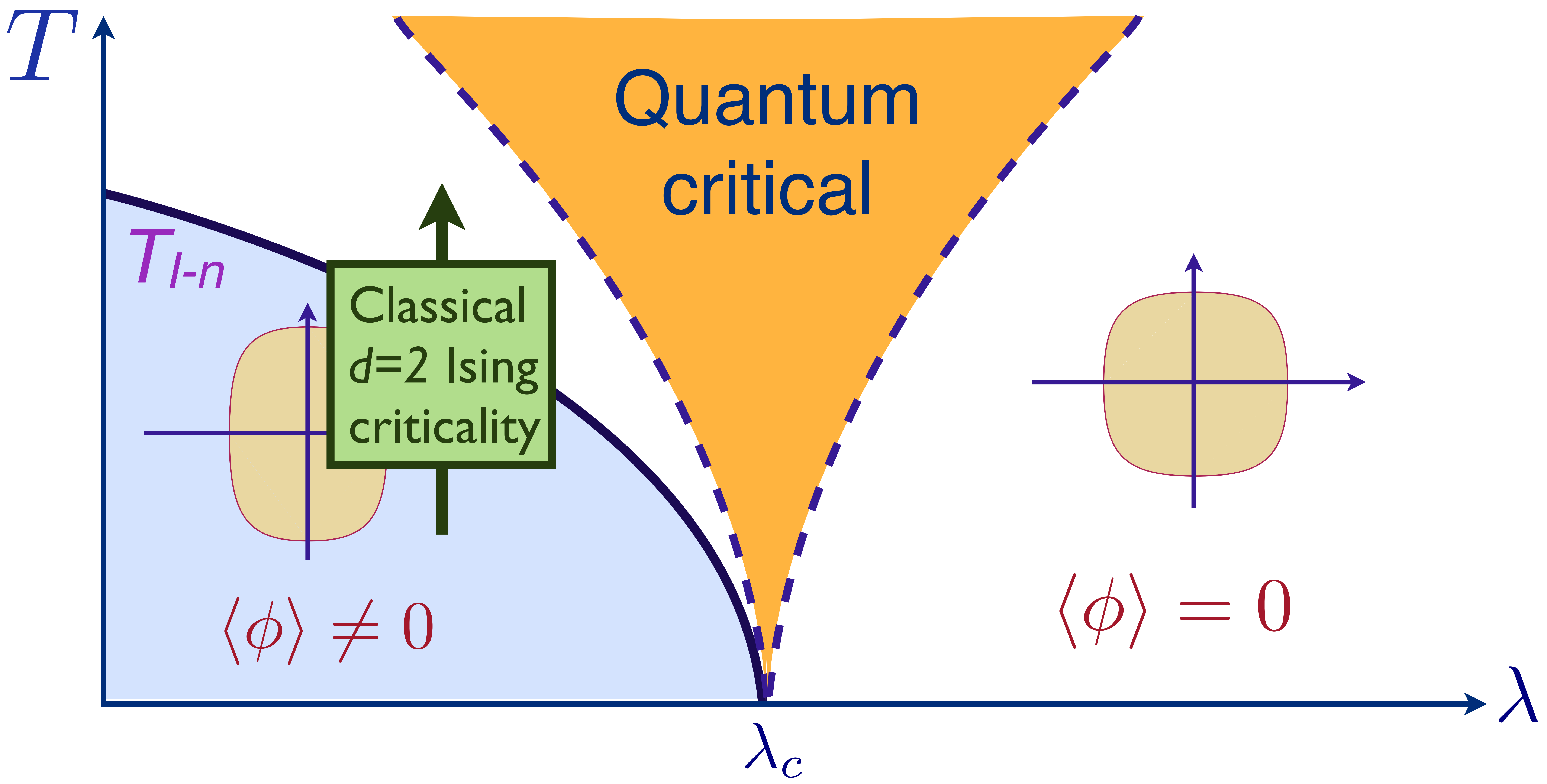
Pomeranchuk instability as a function of coupling λ

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

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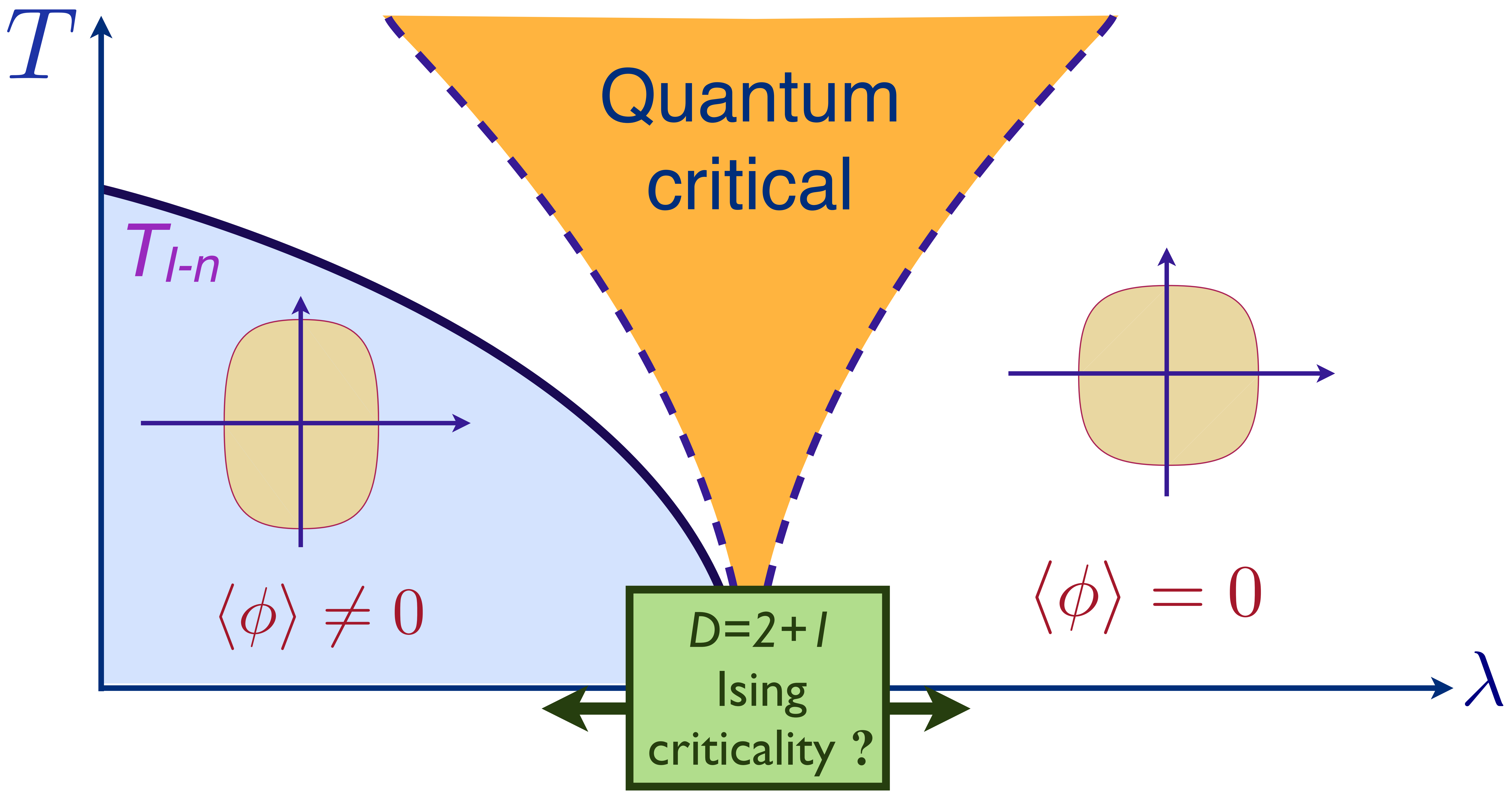

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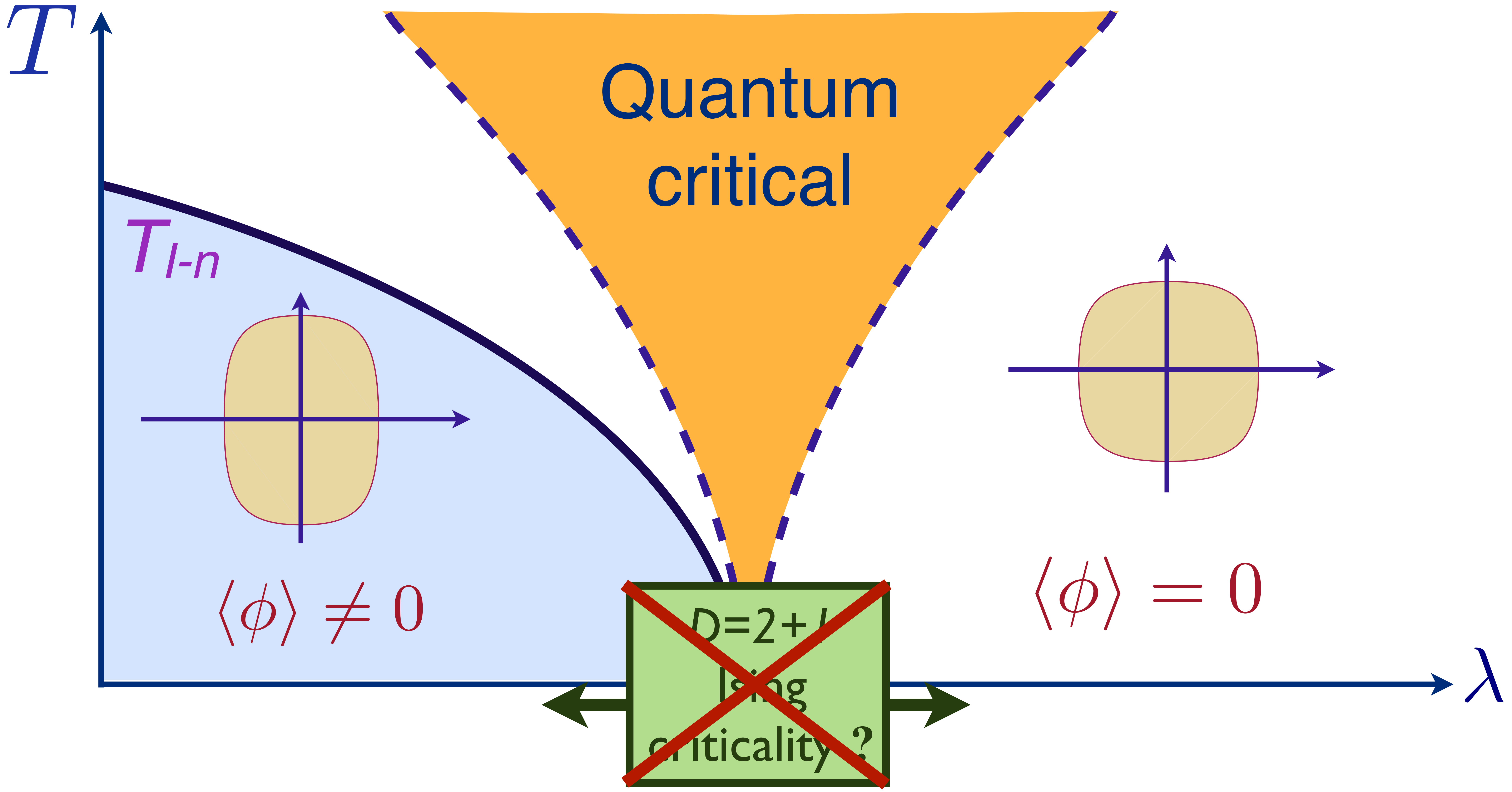
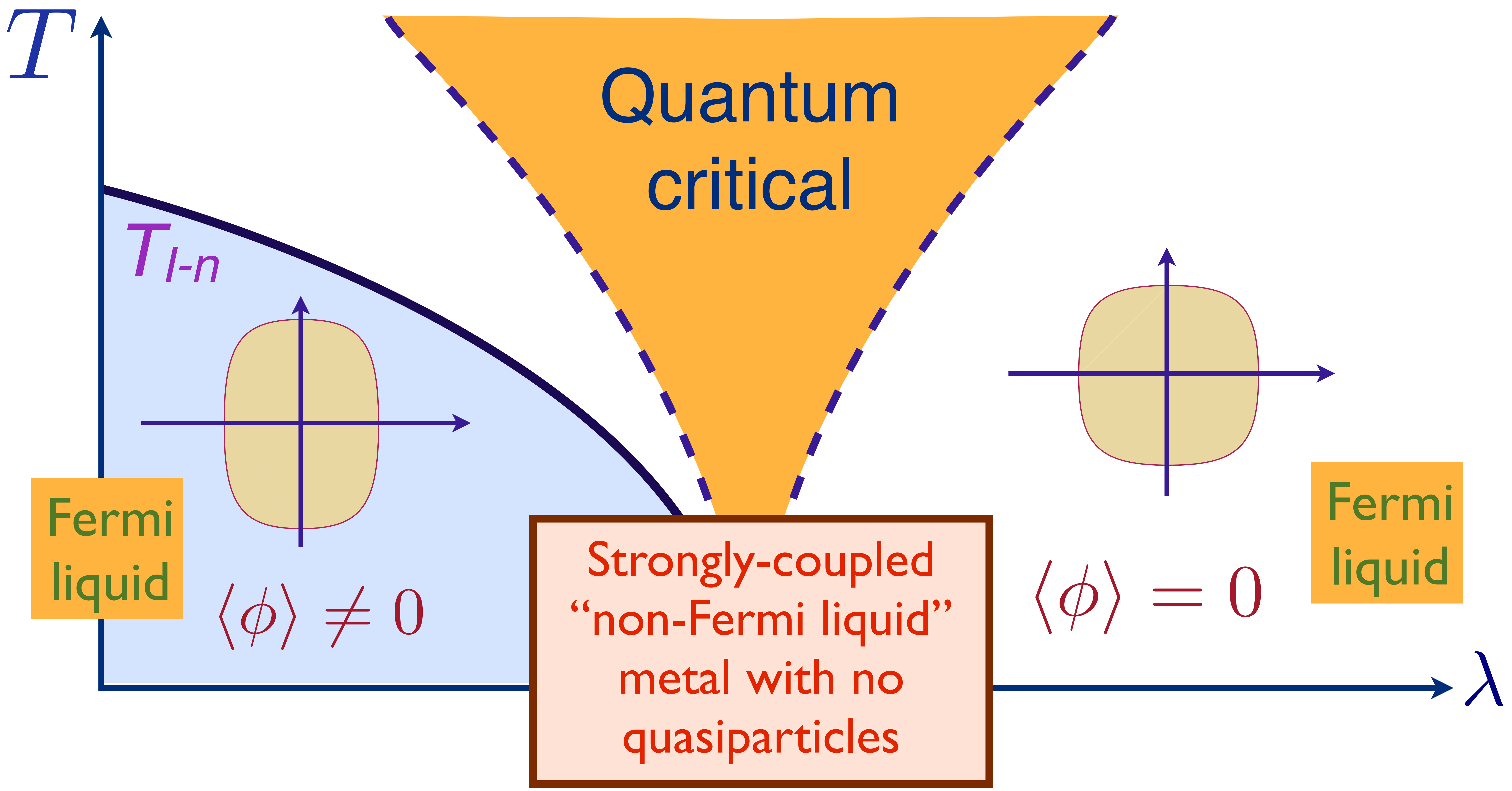


Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



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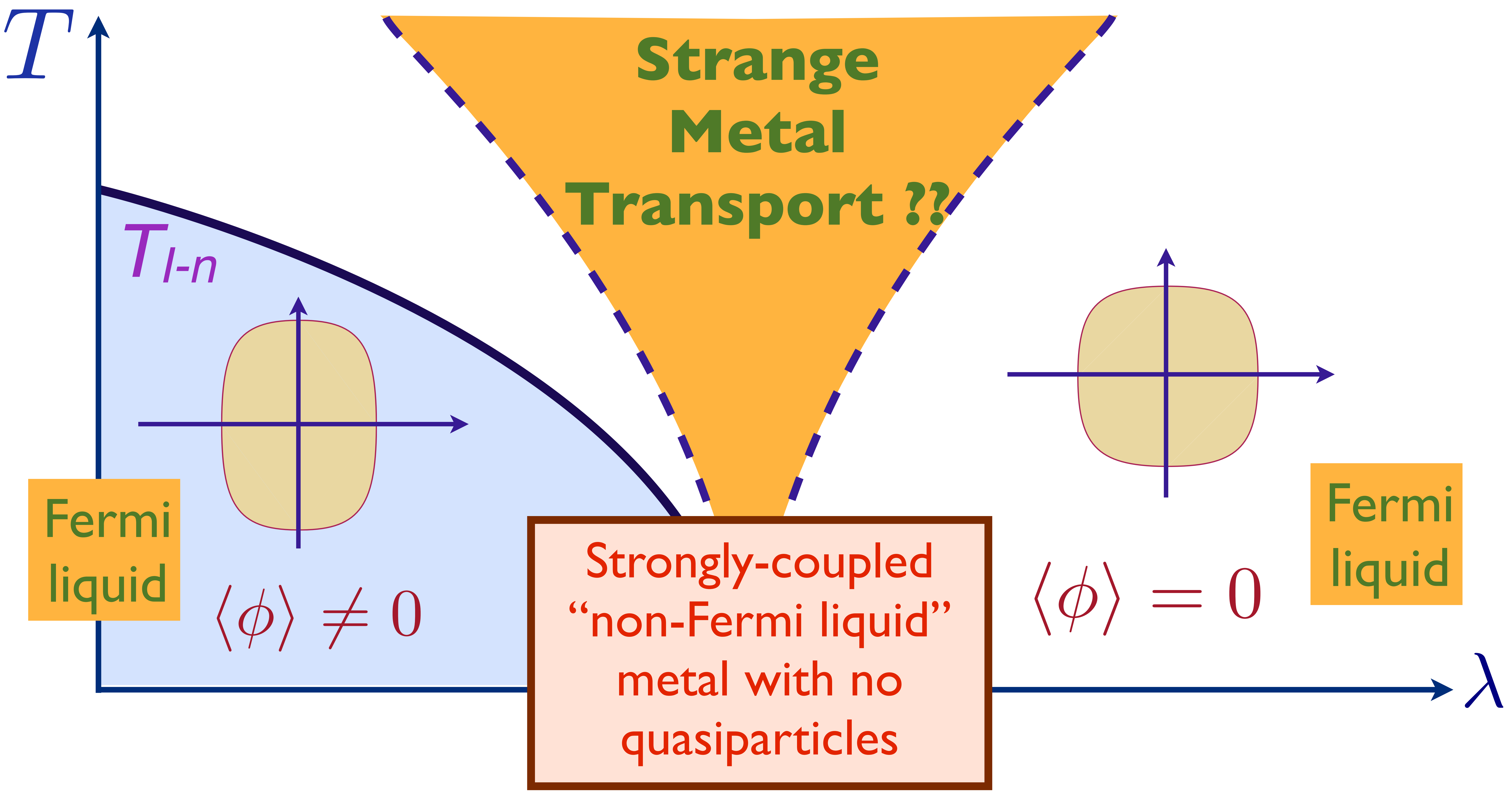
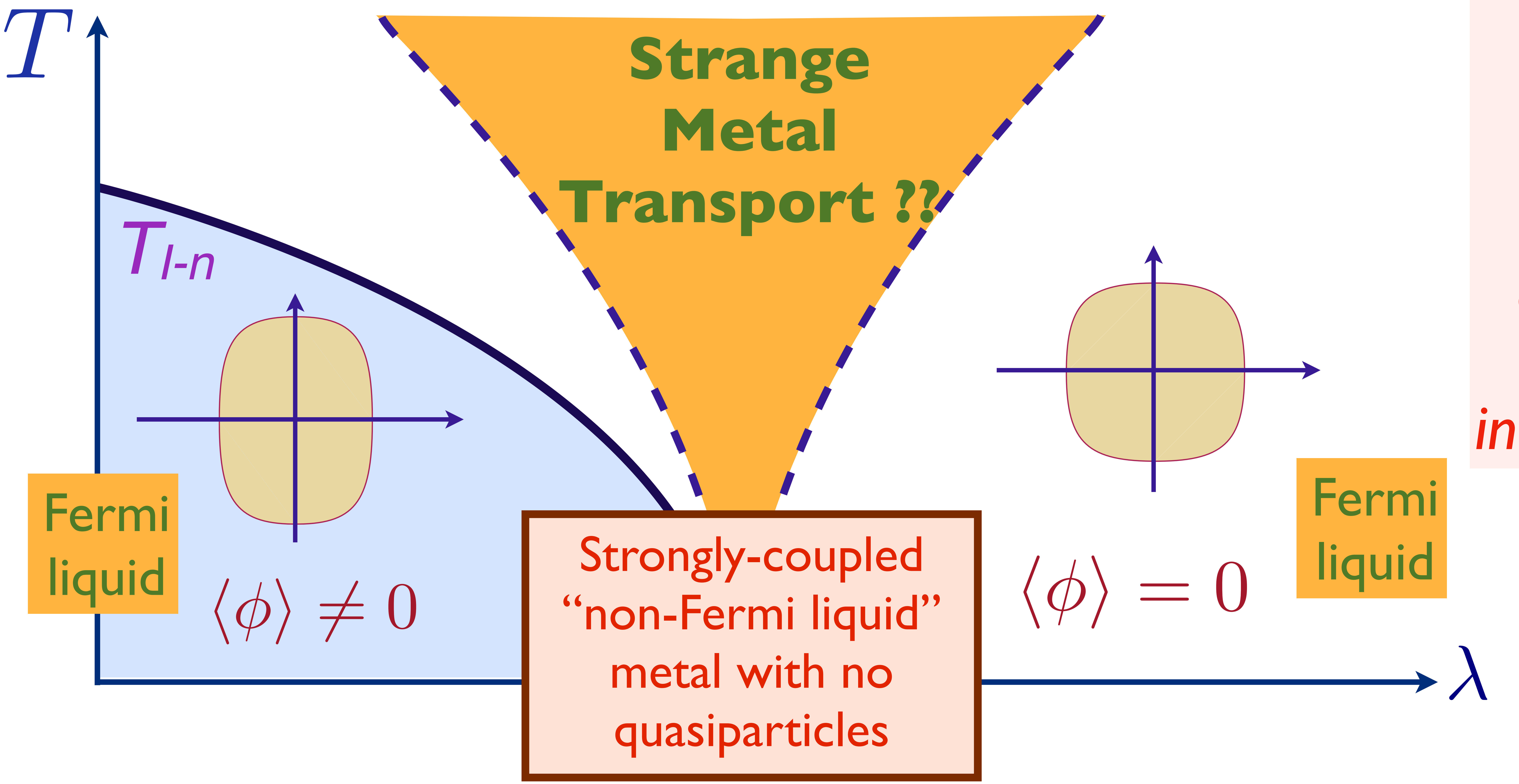


Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



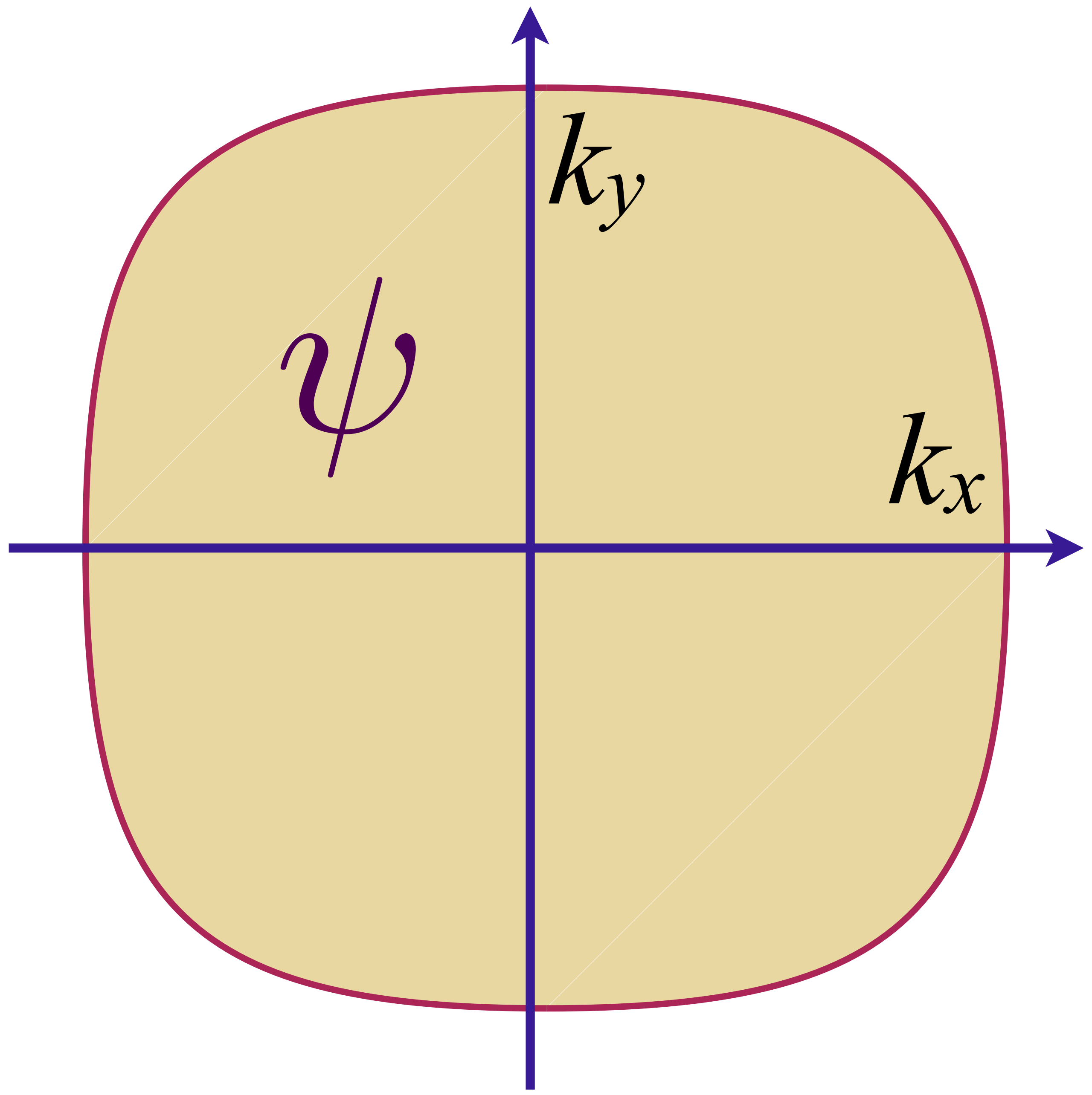
Phase diagram as a function of T and λ

No.

Need spatial *disorder* in the *interactions*

Fermi surface

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



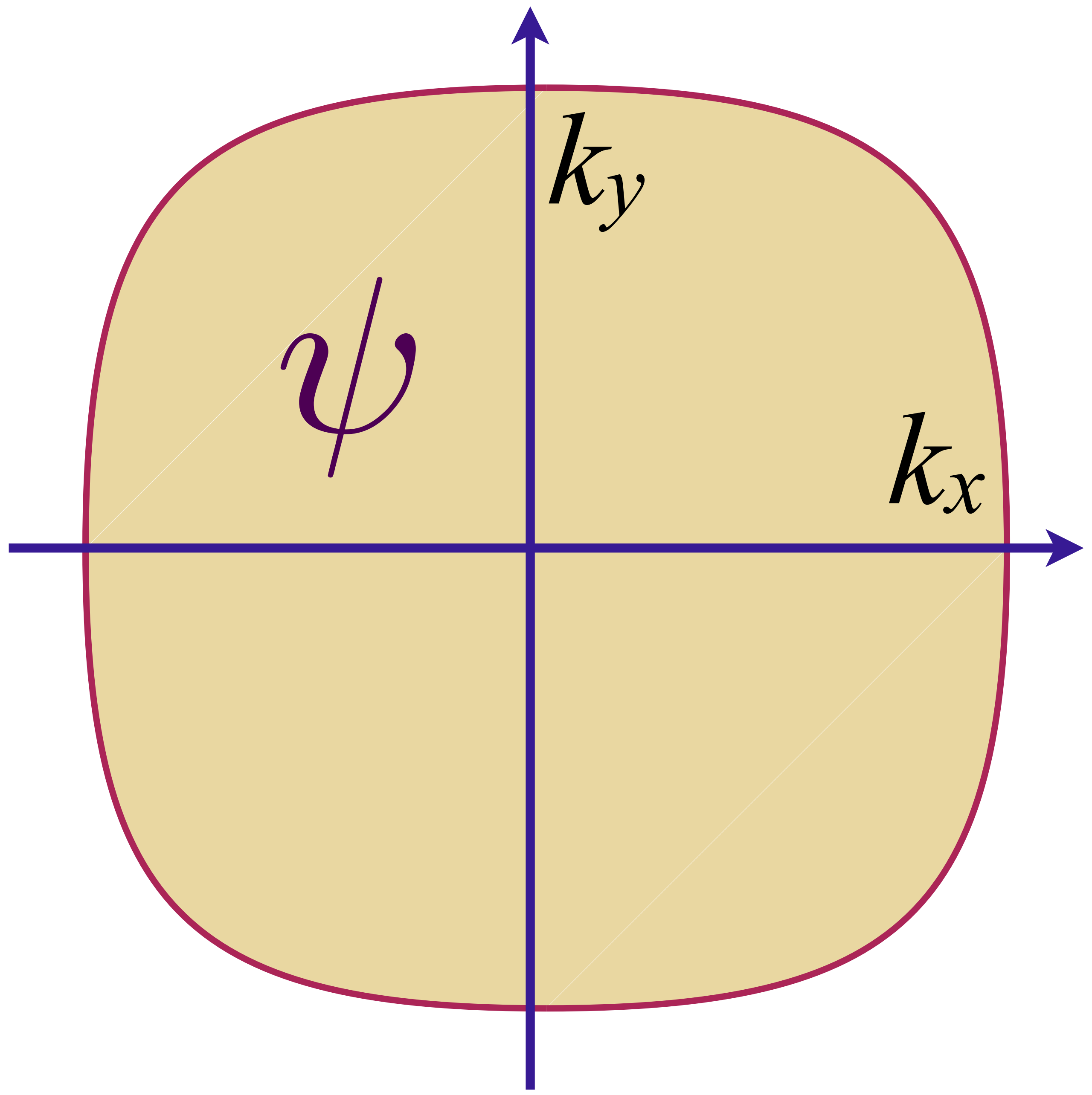
$$- J \, \psi^\dagger(\boldsymbol{r}) \psi^\dagger(\boldsymbol{r}) \psi(\boldsymbol{r}) \psi(\boldsymbol{r})$$

Fermi surface + critical boson

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ

e.g. Ising-nematic order



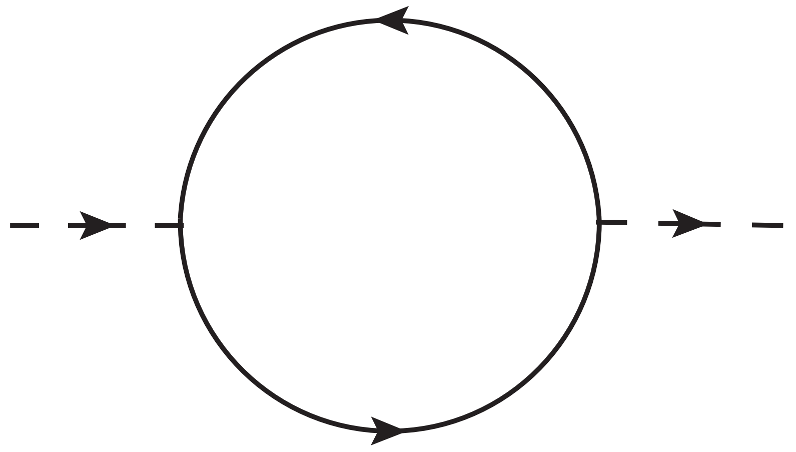
$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Fermi surface coupled to a critical boson

"Yukawa" coupling: $g \int d^2 r d\tau\, \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)$

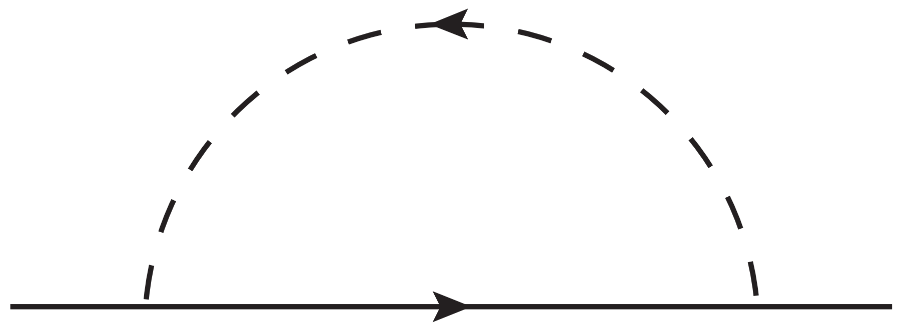
Boson self energy $\Pi(q, i\Omega) \sim -g^2 \dfrac{|\Omega|}{q}$ (Landau damping)

Boson Green's function $D(q, i\Omega) = \dfrac{1}{q^2 + \gamma|\Omega|/q}$



Fermion self energy $\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\mathrm{sgn}(\omega)|\omega|^{2/3}$

Fermion Green's function $G(\boldsymbol{k}, i\omega) = \dfrac{1}{i\omega - \varepsilon(\boldsymbol{k}) - \Sigma(\hat{\boldsymbol{k}}, i\omega)}$



P.A. Lee (1989)

Yields a state without quasiparticle excitations, but the theory is not systematic at large N

Sung-Sik Lee (2009)

Fermi surface + critical boson

These results can also be obtained from the saddle-point and response functions of a G-Σ-D-Π action. Such an action can formally be obtained in a Yukawa-SYK-like large-N limit of a theory with couplings which are random in an additional (fictitious) flavor space.

$$\mathcal{Z} = \int \mathcal{D}G\, \mathcal{D}\Sigma\, \mathcal{D}D\, \mathcal{D}\Pi \exp(-N S_{\rm all})$$

$$\begin{aligned} S_{\rm all} = & -\ln\det(\partial_\tau + \varepsilon(\mathbf{k}) - \mu + \Sigma) + \frac{1}{2}\ln\det(-\partial_\tau^2 + \mathbf{q}^2 + m_b^2 - \Pi) \\ & + \int d\tau d^2 r \int d\tau' d^2 r' \Big[-\Sigma(\tau', \mathbf{r}'; \tau, \mathbf{r}) G(\tau, \mathbf{r}; \tau', \mathbf{r}') + \frac{1}{2}\Pi(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') \\ & + \frac{g^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') \Big] \,. \end{aligned}$$

Fermi surface + critical boson

These results can also be obtained from the saddle-point and response functions of a G-Σ-D-Π action. Such an action can formally be obtained in a Yukawa-SYK-like large-N limit of a theory with couplings which are random in an additional (fictitious) flavor space.

$$\mathcal{Z} = \int \mathcal{D}G\, \mathcal{D}\Sigma\, \mathcal{D}D\, \mathcal{D}\Pi \exp(-N S_{\rm all})$$

Saddle-point equations: Migdal-Eliashberg

$$\Sigma(\tau, \mathbf{r}) = g^2 D(\tau, \mathbf{r}) G(\tau, \mathbf{r}),$$

$$\Pi(\tau, \mathbf{r}) = -g^2 G(-\tau, -\mathbf{r}) G(\tau, \mathbf{r}),$$

$$G(i\omega, \mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega, \mathbf{k})},$$

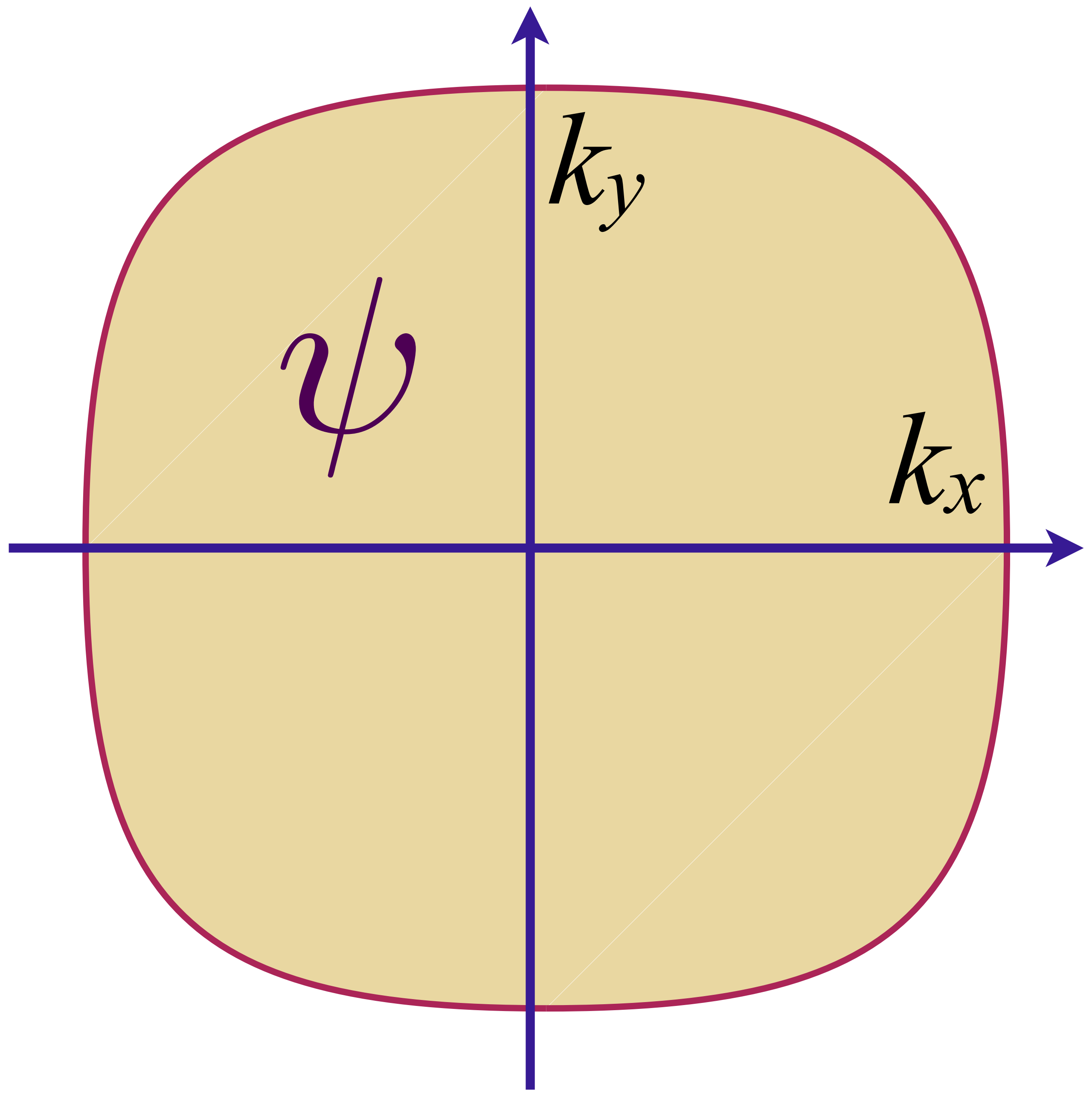
$$D(i\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega, \mathbf{q})}.$$

Fermi surface + critical boson

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

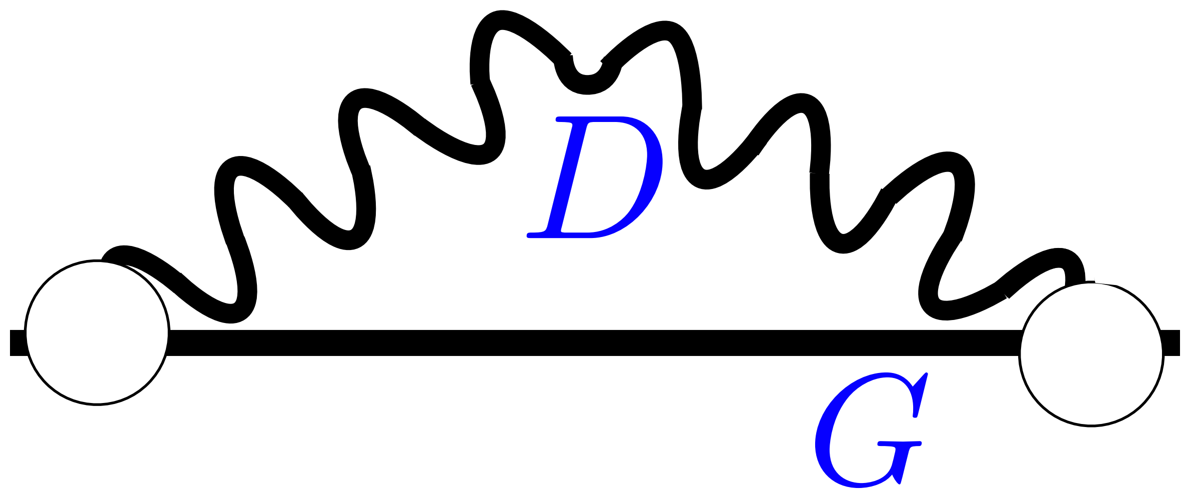
a critical boson ϕ

e.g. Ising-nematic order

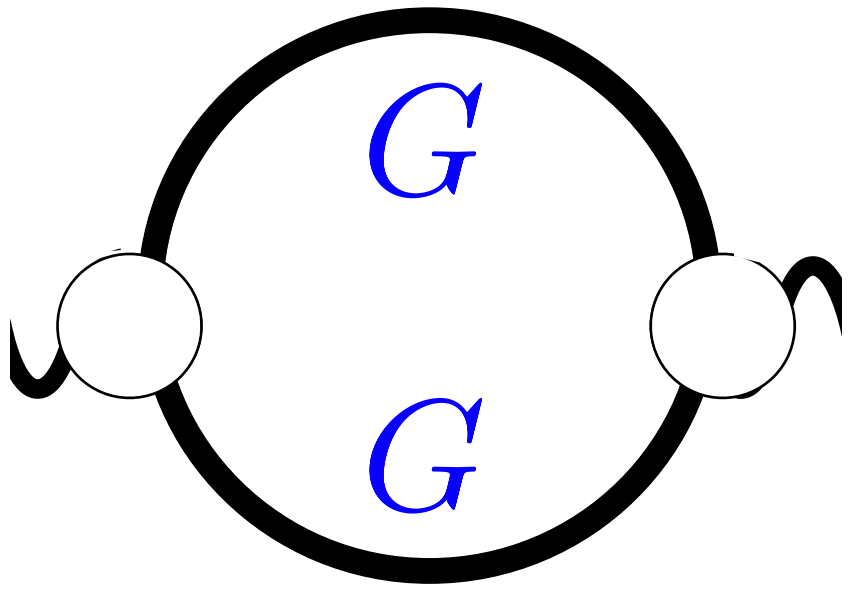


$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$\Sigma =$



$\Pi =$



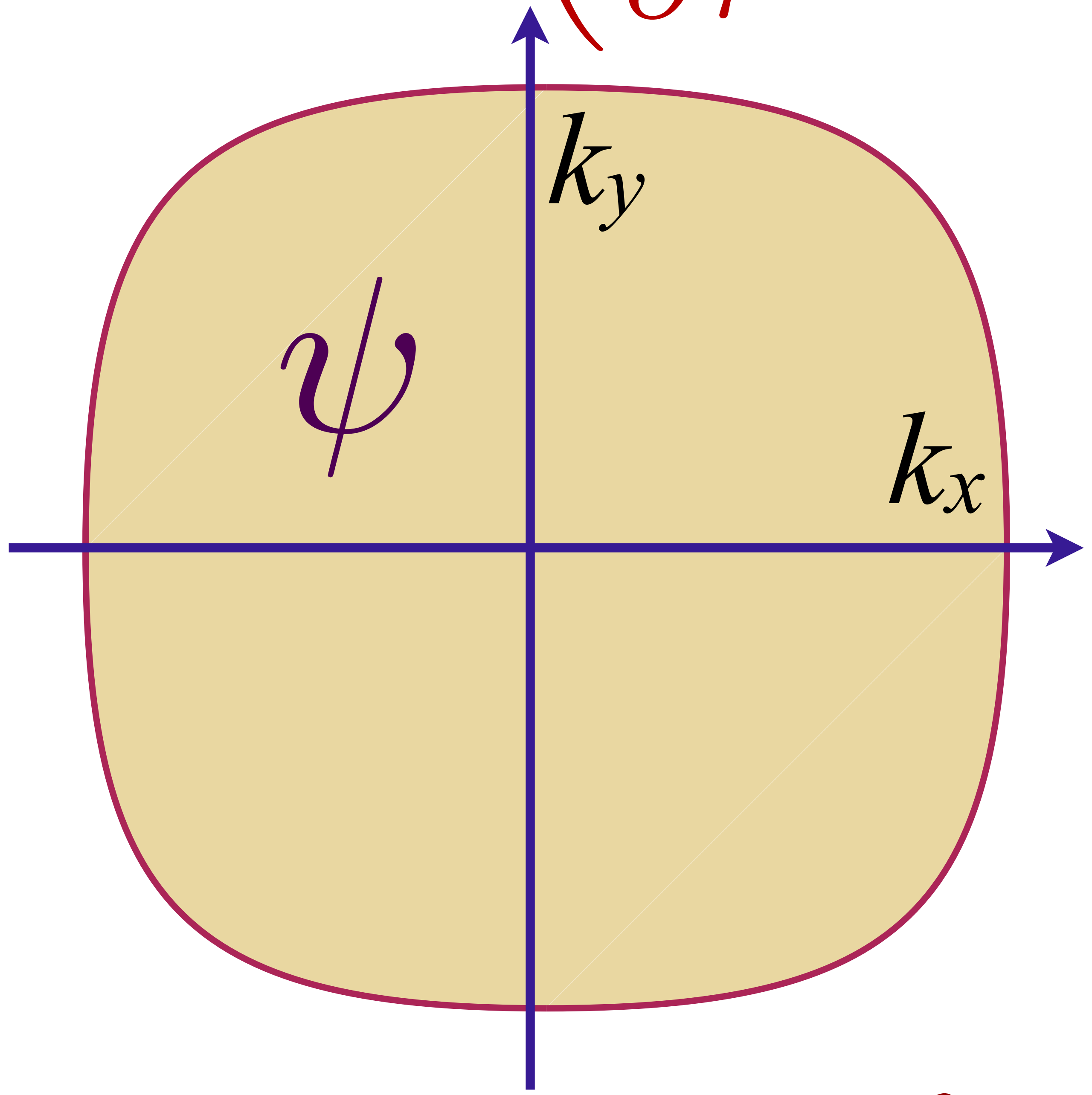
Solution of Migdal-Eliashberg equations for electron (G) and boson (D) Green's functions at small ω:

P.A. Lee (1989)

$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i \text{sgn}(\omega)|\omega|^{2/3}\,, \quad G(\boldsymbol{k}, i\omega) = \frac{1}{i\omega - \varepsilon(\boldsymbol{k}) - \Sigma(\hat{\boldsymbol{k}}, i\omega)}\,, \quad D(\boldsymbol{q}, i\Omega) = \frac{1}{\Omega^2 + q^2 + \gamma|\Omega|/q}$$

Fermi surface + critical boson

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Transport—a perfect metal!

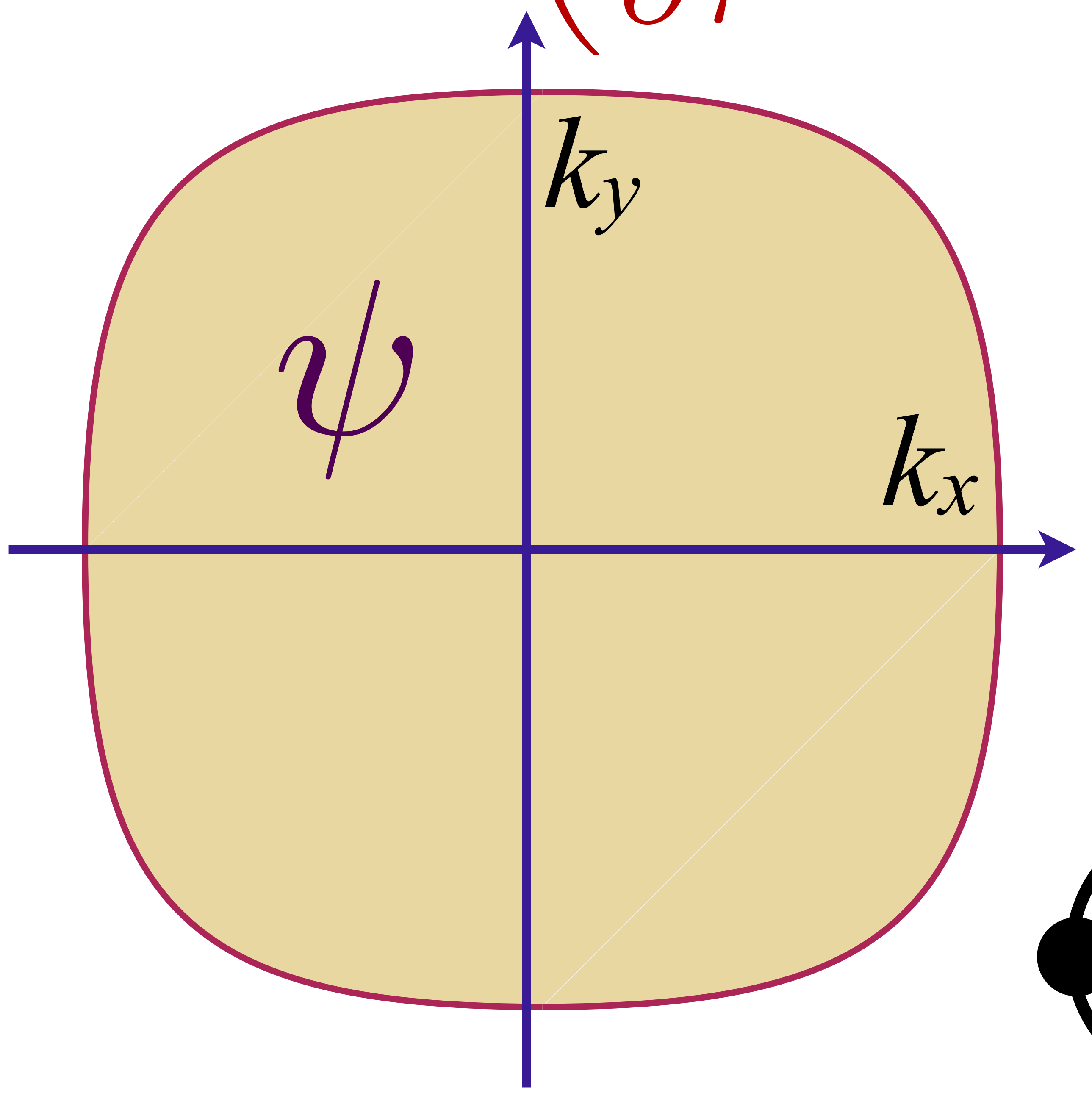
Conservation of momentum and fermion-boson drag imply:

$$\mathrm{Re}\,[\sigma(\omega)] = D\delta(\omega) + \ldots$$

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)
D. L. Maslov, V. I. Yudson, and A. V. Chubukov PRL **106**, 106403 (2011)
S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)
A. Eberlein, I. Mandal, and S.S. PRB **94**, 045133 (2016)

Fermi surface + critical boson

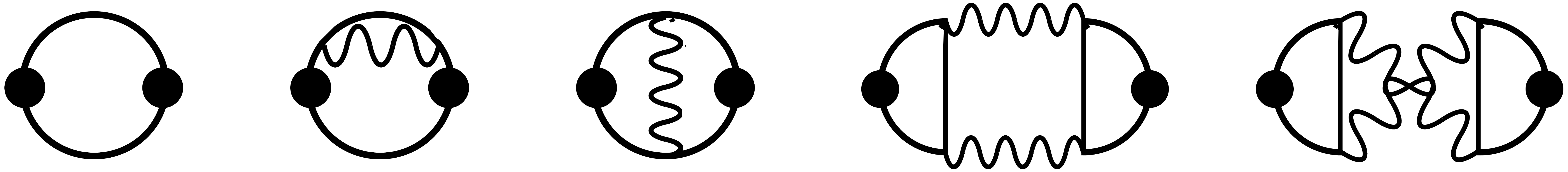
$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Optical conductivity—Diagrams

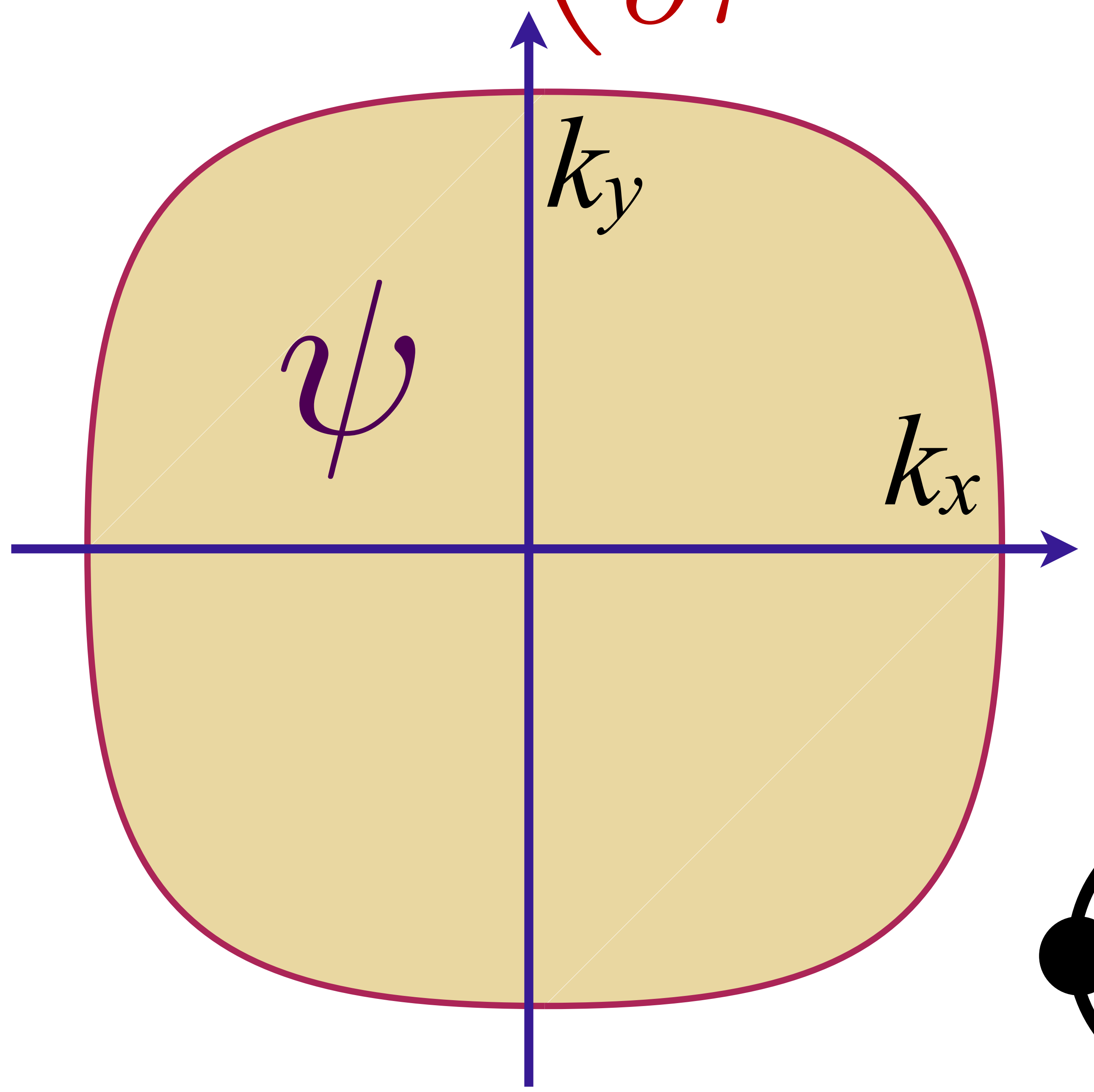


$$\mathrm{Re}\,[\sigma(\omega)] = C\,|\omega|^{-2/3}$$

Yong Baek Kim, A. Furusaki, Xiao-Gang Wen, and P. A. Lee, PRB **50**, 17917 (1994).

Fermi surface + critical boson

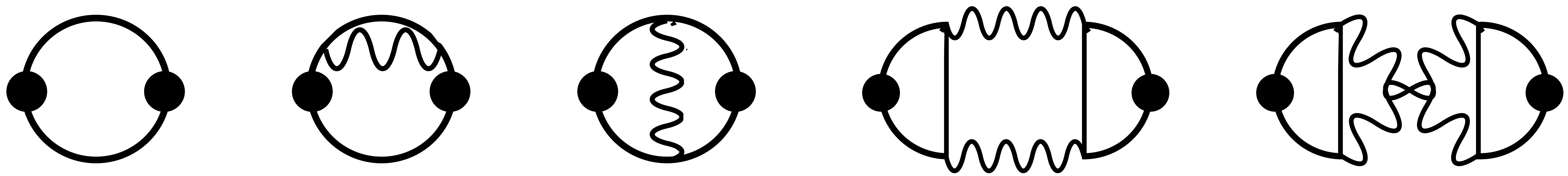
$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Optical conductivity—Diagrams



$$\mathrm{Re}\left[\sigma(\omega)\right] = C\,|\omega|^{-2/3}$$

Yong Baek Kim, A. Furusaki, Xiao-Gang Wen, and P. A. Lee, PRB **50**, 17917 (1994).

$$C = 0\,; \quad \sigma(\omega) \sim i/(\omega) + \omega^0 + \ldots$$

Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S. PRB **106**, 115151 (2022)

Fermi surface coupled to a critical boson:
No spatial disorder

A non-Fermi liquid but NO strange metal transport

Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NO strange metal transport

Fermi surface coupled to a critical boson:

Potential disorder v

A marginal Fermi liquid but NO strange metal transport

Fermi surface + critical boson with potential disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

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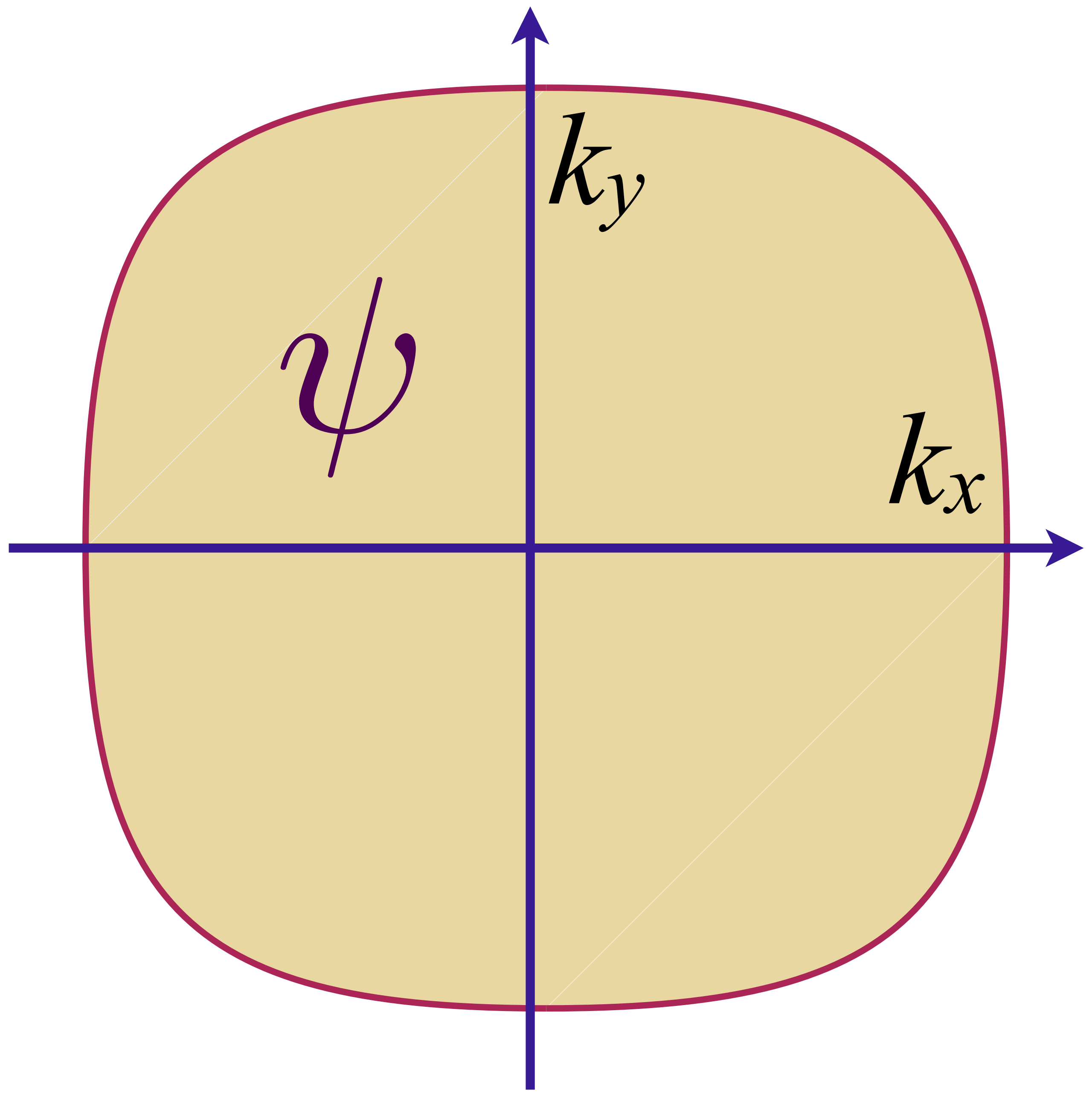
e.g. Ising-nematic order



$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Fermi surface + critical boson with potential disorder

All results are obtained from the large N saddle-point and response functions of this G-Σ-D-Π theory:

$$\mathcal{Z} = \int \mathcal{D}G\,\mathcal{D}\Sigma\,\mathcal{D}D\,\mathcal{D}\Pi\,\exp(-NS_{\rm all})$$

$$\begin{aligned}S_{\rm all} &= -\ln\det(\partial_\tau + \varepsilon(\mathbf{k}) - \mu + \Sigma) + \frac{1}{2}\ln\det(-\partial_\tau^2 + \mathbf{q}^2 + m_b^2 - \Pi) \\ &+ \int d\tau d^2r \int d\tau' d^2r' \Big[-\Sigma(\tau',\mathbf{r}';\tau,\mathbf{r})G(\tau,\mathbf{r};\tau',\mathbf{r}') + \frac{1}{2}\Pi(\tau',\mathbf{r}';\tau,\mathbf{r})D(\tau,\mathbf{r};\tau',\mathbf{r}') \\ &+ \frac{g^2}{2}G(\tau,\mathbf{r};\tau',\mathbf{r}')G(\tau',\mathbf{r}';\tau,\mathbf{r})D(\tau,\mathbf{r};\tau',\mathbf{r}') + \frac{v^2}{2}G(\tau,\mathbf{r};\tau',\mathbf{r}')G(\tau',\mathbf{r}';\tau,\mathbf{r})\delta(\mathbf{r}-\mathbf{r}')\Big]\end{aligned}$$

Fermi surface + critical boson with potential disorder

All results are obtained from the large N saddle-point and response functions of this G-Σ-D-Π theory:

$$\mathcal{Z} = \int \mathcal{D}G\, \mathcal{D}\Sigma\, \mathcal{D}D\, \mathcal{D}\Pi \exp(-N S_{\rm all})$$

Saddle-point equations

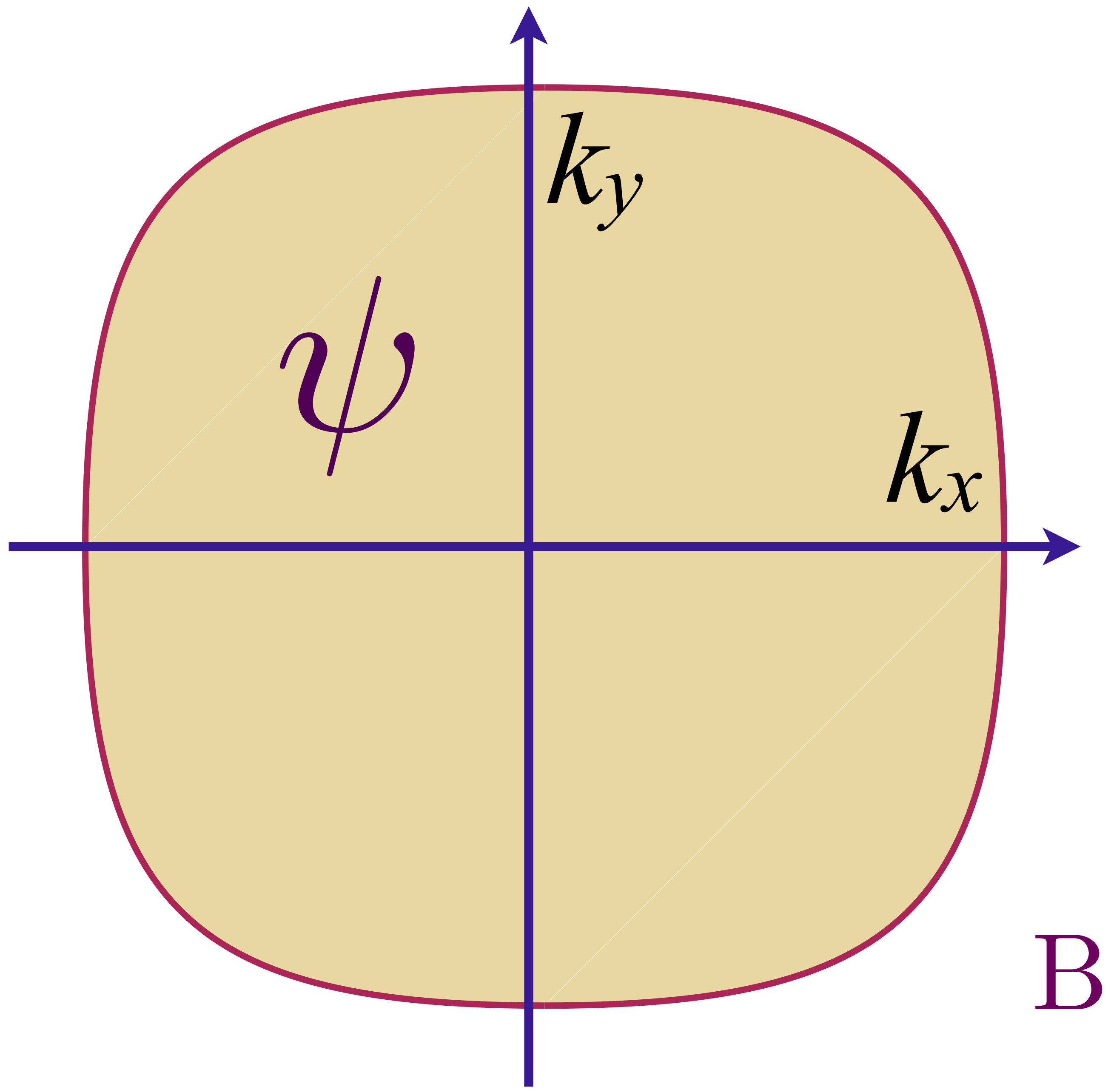
$$\begin{aligned}
\Sigma(\tau, \mathbf{r}) &= g^2 D(\tau, \mathbf{r}) G(\tau, \mathbf{r}) + v^2 G(\tau, \mathbf{r}) \delta^2(\mathbf{r}), \\
\Pi(\tau, \mathbf{r}) &= -g^2 G(-\tau, -\mathbf{r}) G(\tau, \mathbf{r}), \\
G(i\omega, \mathbf{k}) &= \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega, \mathbf{k})}, \\
D(i\Omega, \mathbf{q}) &= \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega, \mathbf{q})}.
\end{aligned}$$

Fermi surface + critical boson with potential disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ

e.g. Ising-nematic order



$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Boson self energy: $\Pi \sim -\dfrac{g^2}{v^2}|\Omega|, \qquad D(q, i\Omega) = \dfrac{1}{q^2 + \gamma|\Omega|}$

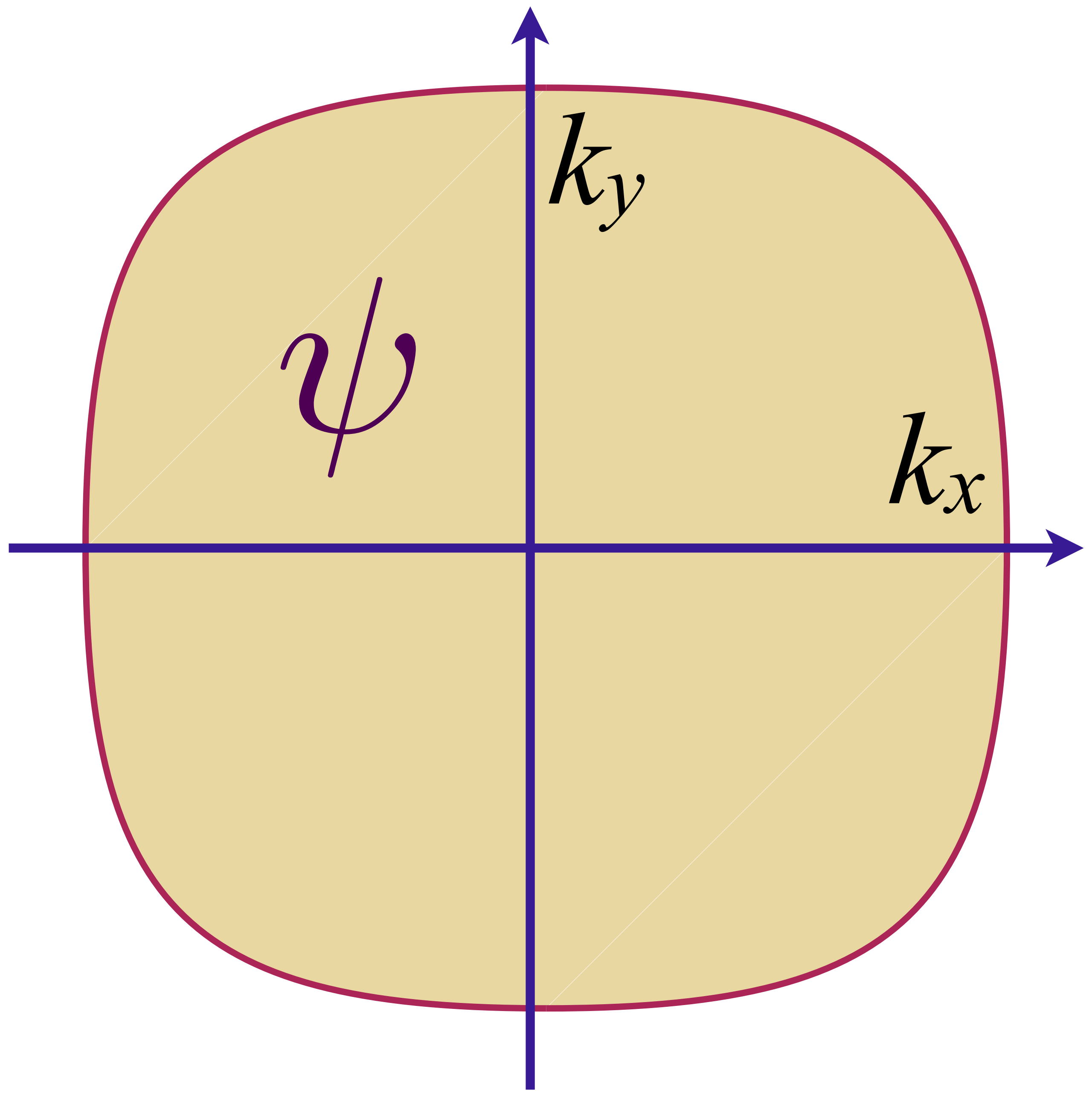
Fermion self energy: $\Sigma(i\omega) \sim -iv^2 \text{sgn}(\omega) - i\dfrac{g^2}{v^2}\omega \ln(1/|\omega|); \quad \dfrac{1}{\tau_{\text{in}}(\varepsilon)} \sim |\varepsilon|$

Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat

Fermi surface + critical boson with potential disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

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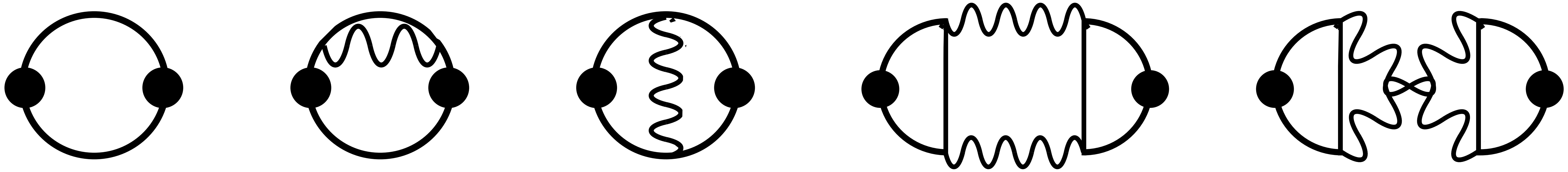

Conductivity: $\sigma(\omega) \sim \dfrac{1}{\dfrac{1}{\tau_{\text{trans}}} - i\omega}$; $\dfrac{1}{\tau_{\text{trans}}} \sim v^2$

MFL self-energy cancels in transport.

Fermi surface coupled to a critical boson:
<u>No spatial disorder</u>

A non-Fermi liquid but NO strange metal transport

Fermi surface coupled to a critical boson:
<u>Potential disorder v</u>

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---

g'

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A marginal Fermi liquid AND strange metal transport

Fermi surface + critical boson with potential disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

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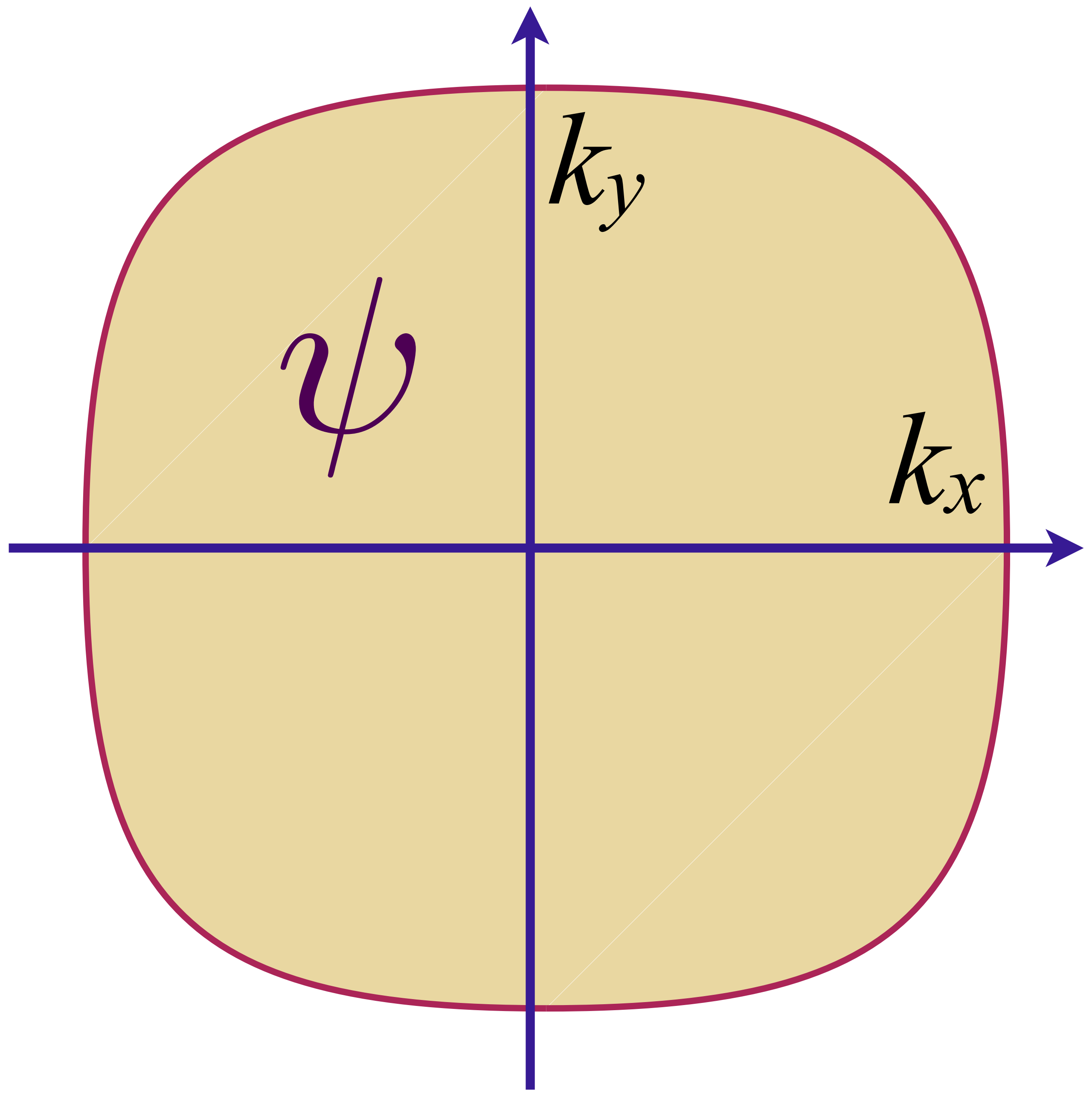


$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

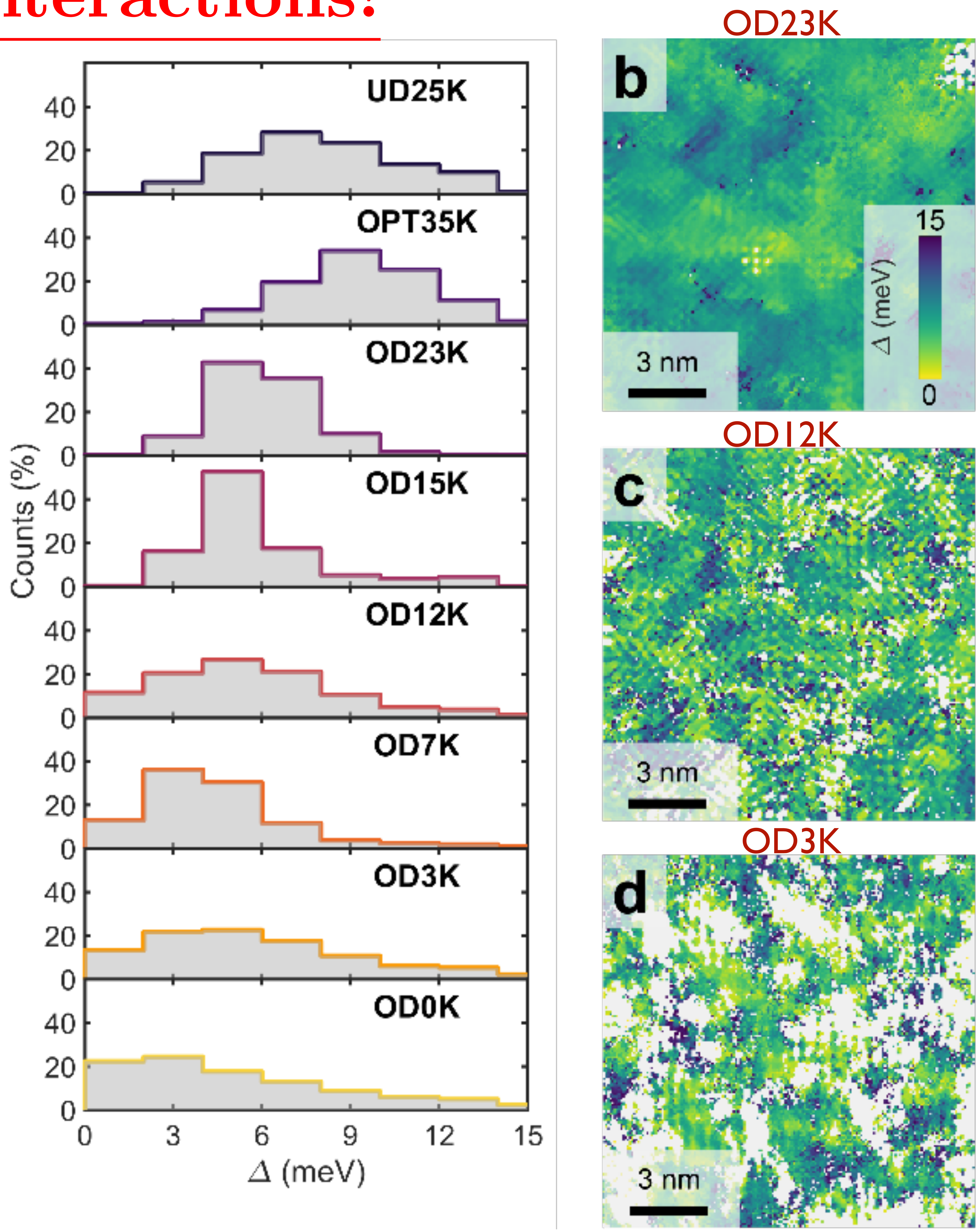
$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random interactions!

Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped $(Pb,Bi)_2Sr_2CuO_{6+\delta}$

Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge, Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos, Amber Vervloet, Steef Smit, Erik van Heumen, Mark S. Golden, Yinkai Huang, Takeshi Kondo, Yi Yin, Jennifer E. Hoffman, Miguel Antonio Sulangi, Jan Zaanen, Milan P. Allan

Our scanning tunneling spectroscopy measurements in the overdoped regime of the $(Pb,Bi)_2Sr_2CuO_{6+\delta}$ high-temperature superconductor show the emergence of puddled superconductivity, featuring nanoscale superconducting islands in a metallic matrix

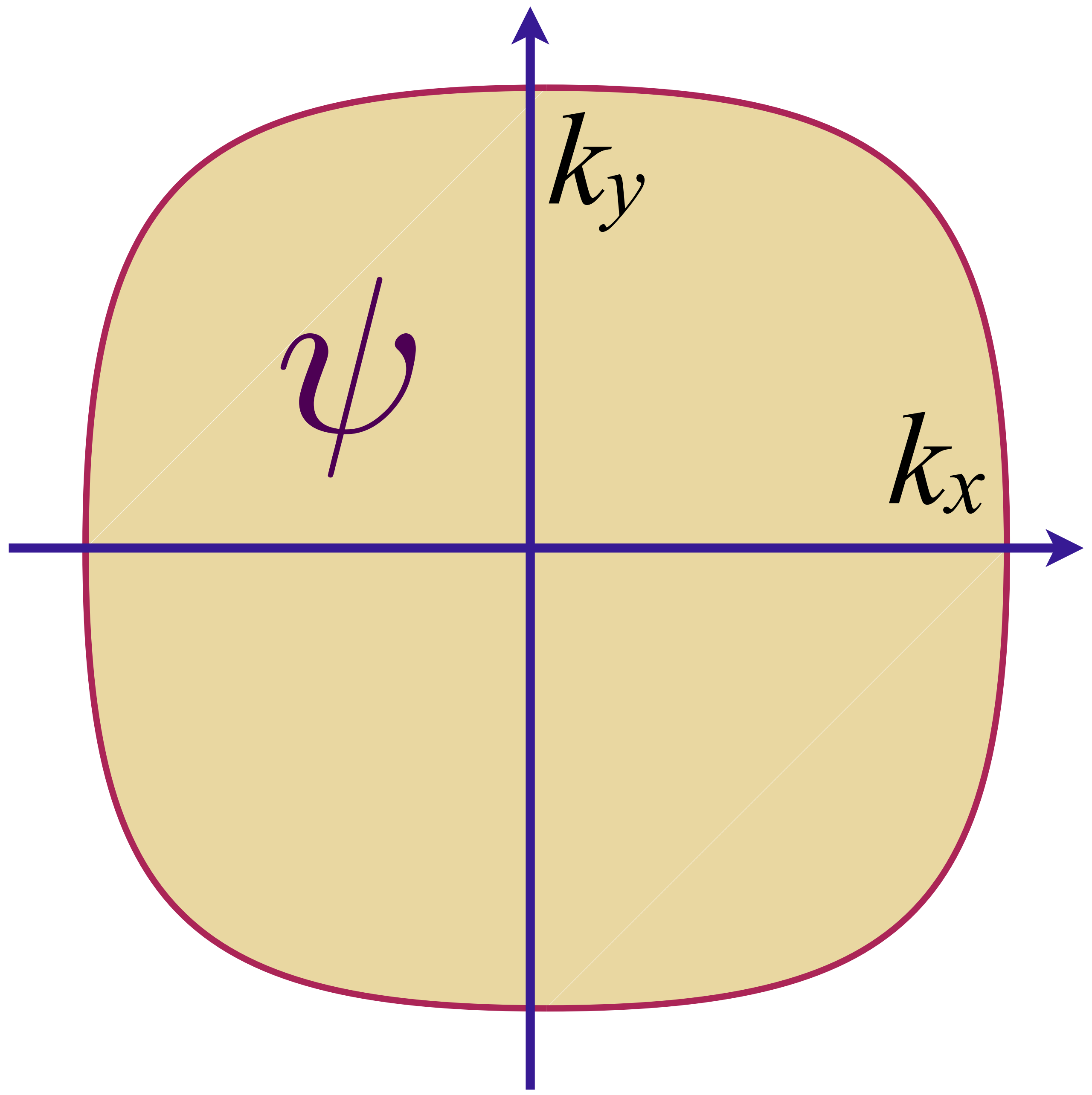


arXiv:2205.09740

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$\frac{[\phi(\boldsymbol{r})]^2}{J + J'(\boldsymbol{r})} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

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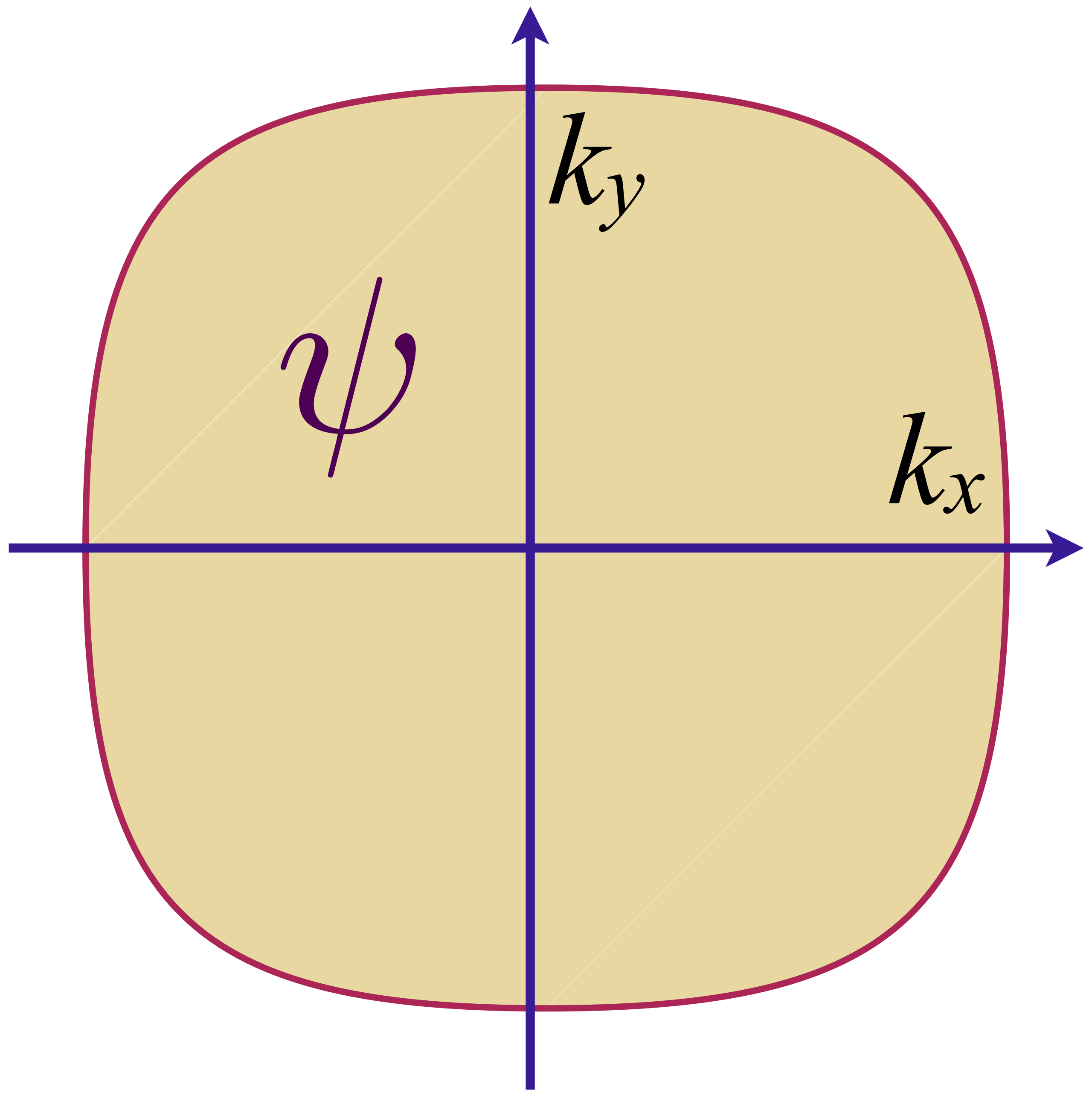

$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

ϕ^2 "mass" disorder $J'(\boldsymbol{r})$ is strongly relevant;
rescale ϕ to move disorder to the Yukawa coupling;

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Fermi surface coupled to a critical boson with disorder

All results are obtained from the large N saddle-point and response functions of this G-Σ-D-Π theory:

$$\mathcal{Z} = \int \mathcal{D}G\, \mathcal{D}\Sigma\, \mathcal{D}D\, \mathcal{D}\Pi \exp(-N S_{\rm all})$$

$$\begin{aligned} S_{\rm all} = &- \ln\det(\partial_\tau + \varepsilon(\mathbf{k}) - \mu + \Sigma) + \frac{1}{2}\ln\det(-\partial_\tau^2 + \mathbf{q}^2 + m_b^2 - \Pi) \\ &+ \int d\tau d^2 r \int d\tau' d^2 r' \Big[-\Sigma(\tau', \mathbf{r}'; \tau, \mathbf{r}) G(\tau, \mathbf{r}; \tau', \mathbf{r}') + \frac{1}{2}\Pi(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') \\ &+ \frac{g^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') + \frac{v^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) \delta(\mathbf{r} - \mathbf{r}') \\ &+ \frac{g'^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') \delta(\mathbf{r} - \mathbf{r}') \Big] \, . \end{aligned}$$

Fermi surface coupled to a critical boson with disorder

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$$\mathcal{Z} = \int \mathcal{D}G\, \mathcal{D}\Sigma\, \mathcal{D}D\, \mathcal{D}\Pi \exp(-N S_{\rm all})$$

Saddle-point equations

$$\Sigma(\tau, \mathbf{r}) = g^2 D(\tau, \mathbf{r}) G(\tau, \mathbf{r}) + v^2 G(\tau, \mathbf{r}) \delta^2(\mathbf{r}) + g'^2 G(\tau, \mathbf{r}) D(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

$$\Pi(\tau, \mathbf{r}) = -g^2 G(-\tau, -\mathbf{r}) G(\tau, \mathbf{r}) - g'^2 G(-\tau, \mathbf{r}) G(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

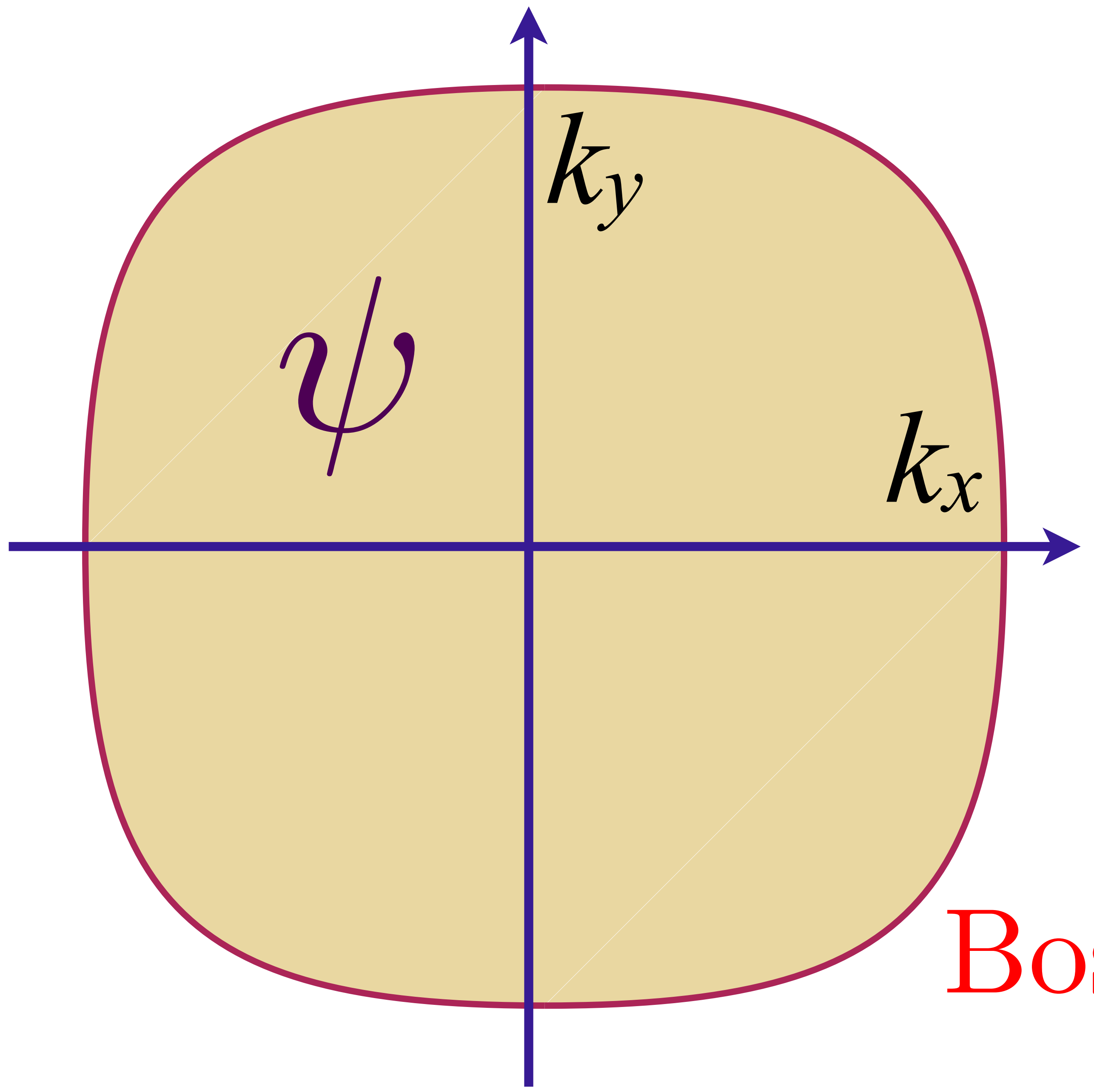
$$G(i\omega, \mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega, \mathbf{k})},$$

$$D(i\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega, \mathbf{q})}.$$

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Boson Green's function: $D(q, i\Omega) \sim 1/(q^2 + \gamma|\Omega|)$

Fermion self energy:

$$\Sigma(i\omega) \sim -iv^2 \text{sgn}(\omega) - i \left(\frac{g^2}{v^2} + g'^2 \right) \omega \ln(1/|\omega|); \quad \frac{1}{\tau_{\text{in}}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2 \right) |\omega|$$

Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

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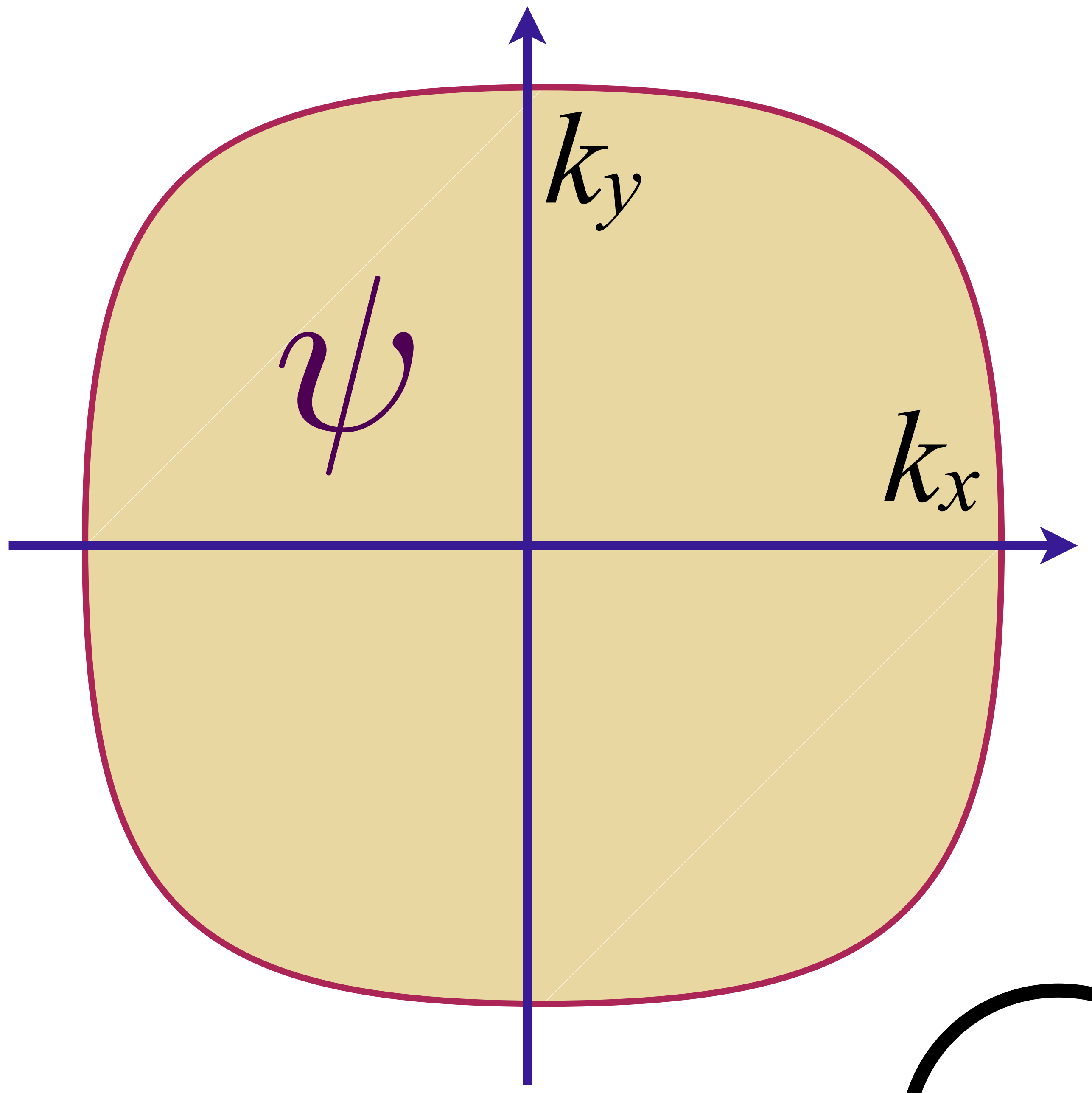
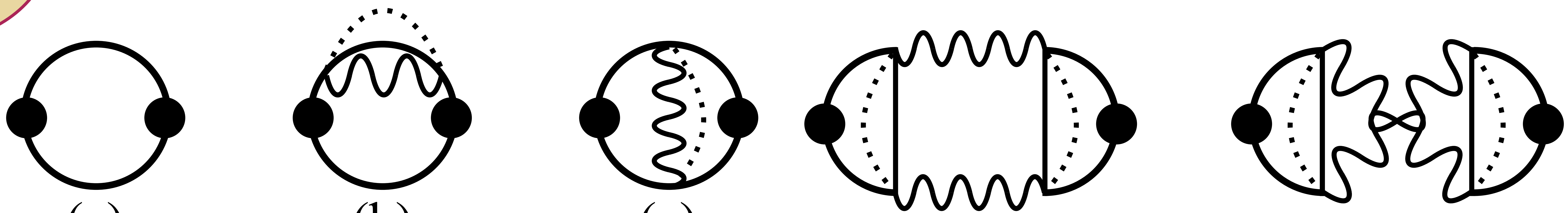


$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Conductivity:



(a) σ_v (b) $\frac{\sigma_{\Sigma,g}}{2}, \frac{\sigma_{\Sigma,g'}}{2}$ (c) $\sigma_{V,g}$ (d) (e)

+ all ladders and bubbles…..

Fermi surface coupled to a critical boson with disorder

$$\text{Conductivity: } \sigma(\omega) \sim \frac{1}{\dfrac{1}{\tau_{\text{trans}}(\omega)} - i\omega \, \dfrac{m^*_{\text{trans}}(\omega)}{m}}$$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*_{\text{trans}}(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

$$\text{Electron Green's function: } G(\omega) \sim \frac{1}{\omega \, \dfrac{m^*(\omega)}{m} - \varepsilon(\boldsymbol{k}) + i \left(\dfrac{1}{\tau_e} + \dfrac{1}{\tau_{\text{in}}(\omega)} \right) \text{sgn}(\omega)}$$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\text{in}}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2 \right) |\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi} \left(\frac{g^2}{v^2} + g'^2 \right) \ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2;
Transport insensitive to g; Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat.

Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NO strange metal transport

Fermi surface coupled to a critical boson:

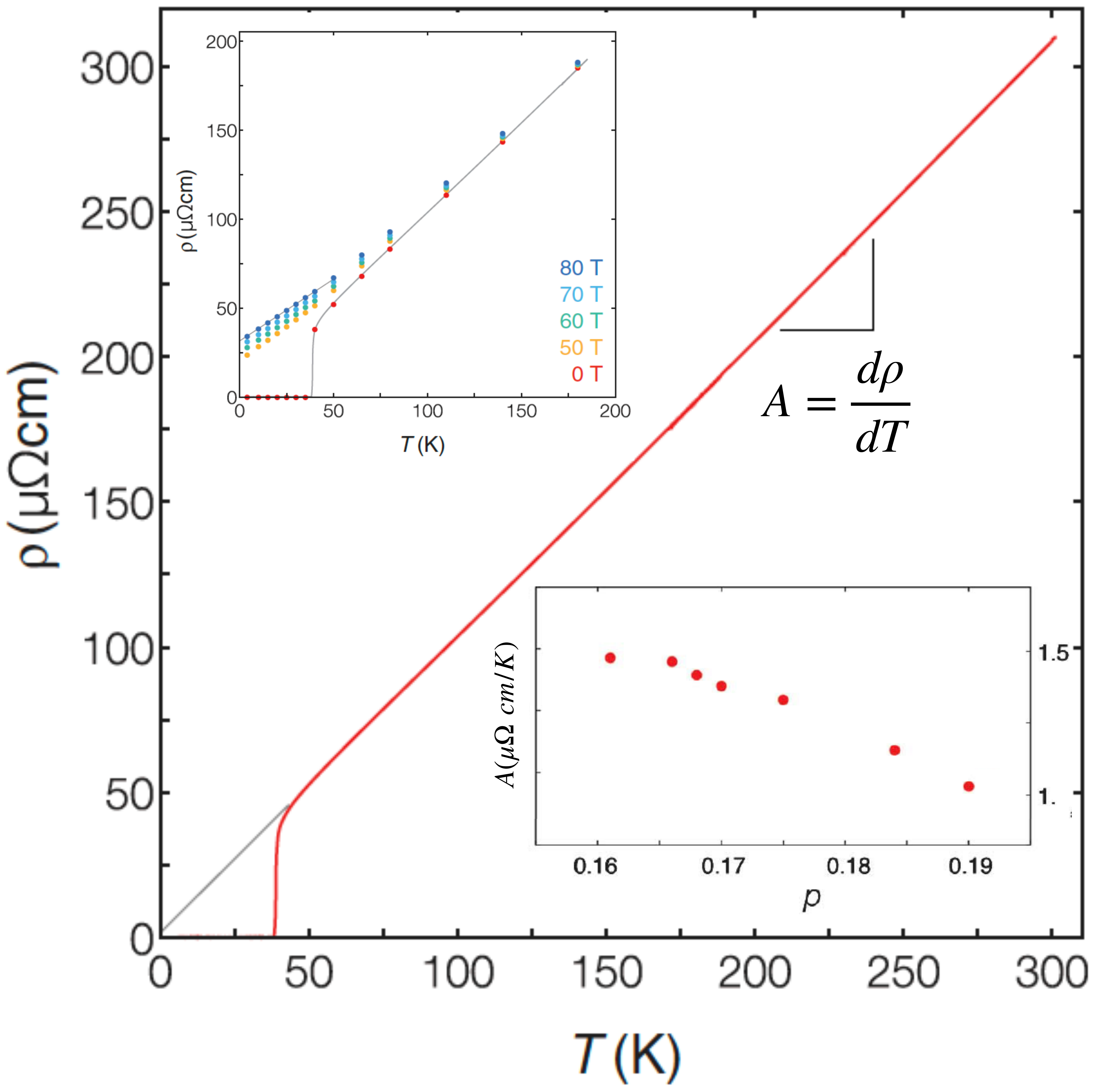
Potential disorder v

A marginal Fermi liquid but NO strange metal transport

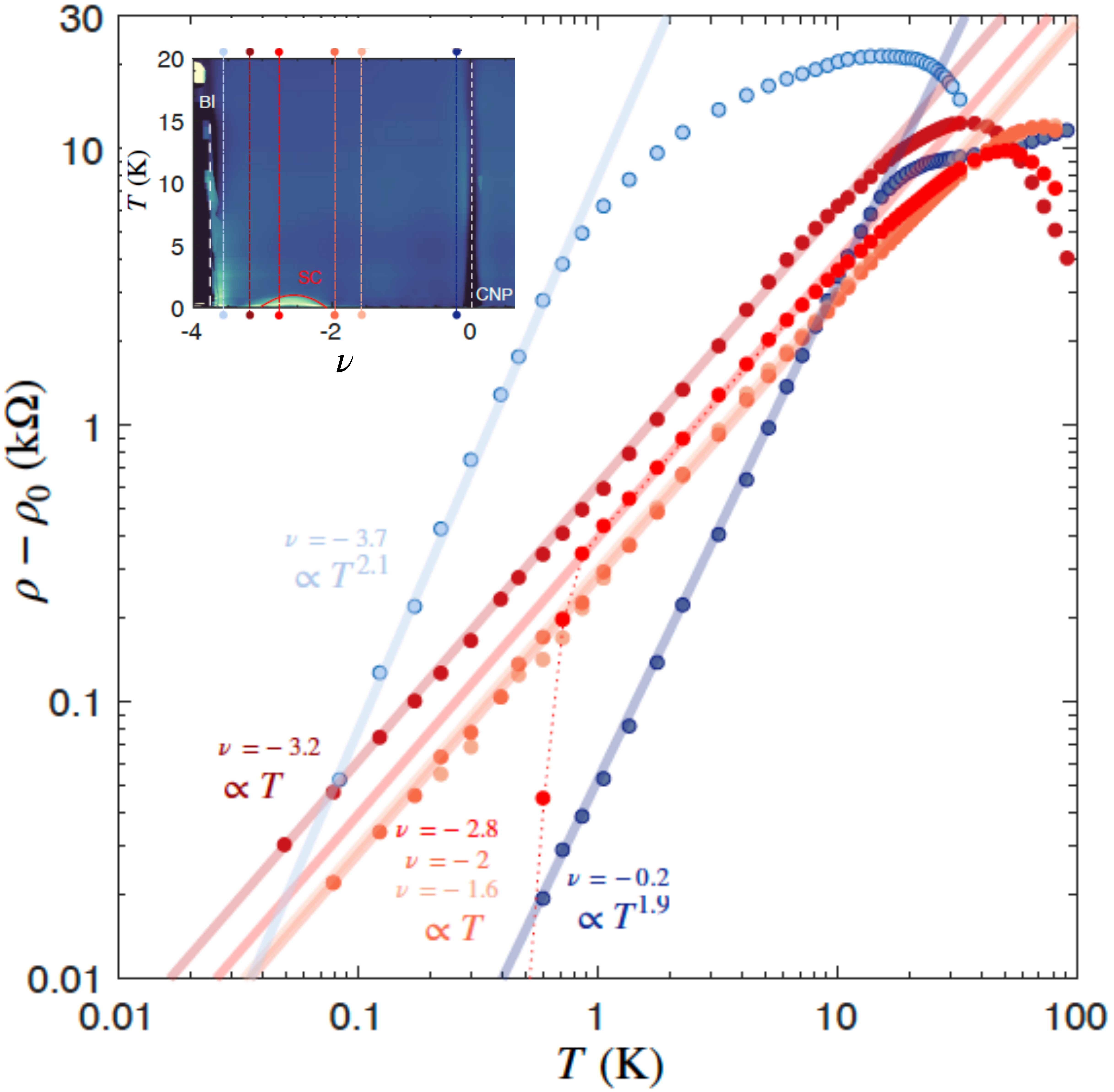
Fermi surface coupled to a critical boson:

Interaction disorder g'

A marginal Fermi liquid AND strange metal transport



LSCO: Giraldo-Gallo et al. 2018



MATBG: Jaoui et al. 2021

Transport properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
   and $\rho(T) < h/e^2$ (in $d = 2$).
   Metals with $\rho(T) > h/e^2$ are bad metals.

Transport properties of a strange metal:

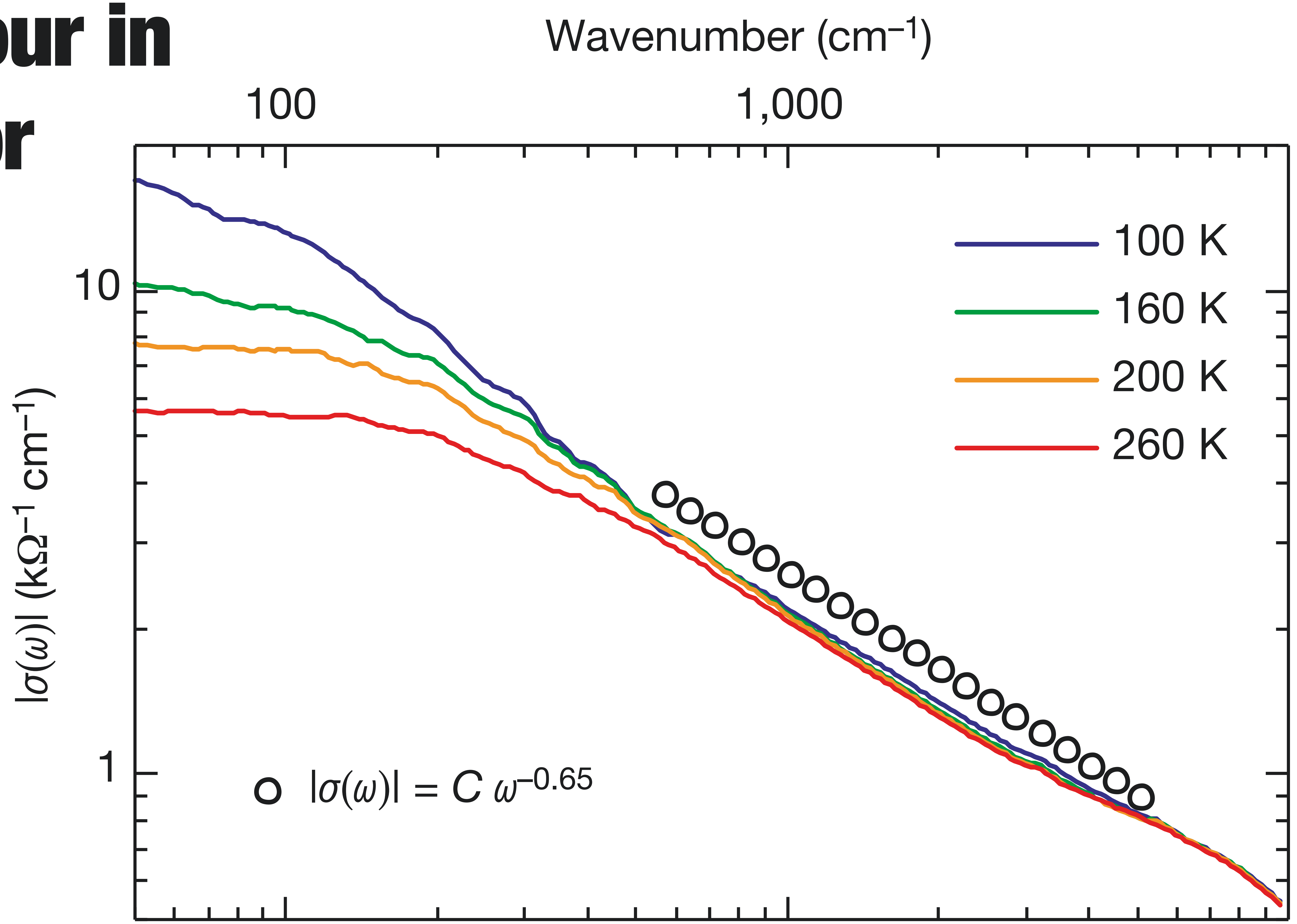
- Optical conductivity

Nature **425**, 271 (2003)

D. van der Marel[1*], H. J. A. Molegraaf[1*], J. Zaanen[2], Z. Nussinov[2*], F. Carbone[1*], A. Damascelli[3*], H. Eisaki[3*], M. Greven[3], P. H. Kes[2] & M. Li[2]

Quantum critical behaviour in a high-T_c superconductor

But no $\hbar\omega/(k_B T)$ scaling.



Transport properties of a strange metal:

arXiv:2205.04030

- Optical conductivity

B. Michon,[1,2,3] C. Berthod,[3] C. W. Rischau,[3] A. Ataei,[4] L. Chen,[4]
S. Komiya,[5] S. Ono,[5] L. Taillefer,[4,6] D. van der Marel,[3] and A. Georges[7,8,3,]

Planckian Behavior of Cuprate Superconductors: Reconciling the Scaling of Optical Conductivity with Resistivity and Specific Heat

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\mathrm{trans}}(\omega)} - i\omega \dfrac{m^*_{\mathrm{trans}}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\mathrm{trans}}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

$$\text{Causality:} \quad \frac{m^*_{\mathrm{trans}}(\omega)}{m} \sim \ln \left(\frac{\Lambda}{\mathrm{Max}(\hbar\omega, k_B T)} \right)$$

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2. Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\text{trans}}(\omega)} - i\omega \dfrac{m^*_{\text{trans}}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\text{trans}}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon……A. Georges, arXiv:2205.04030

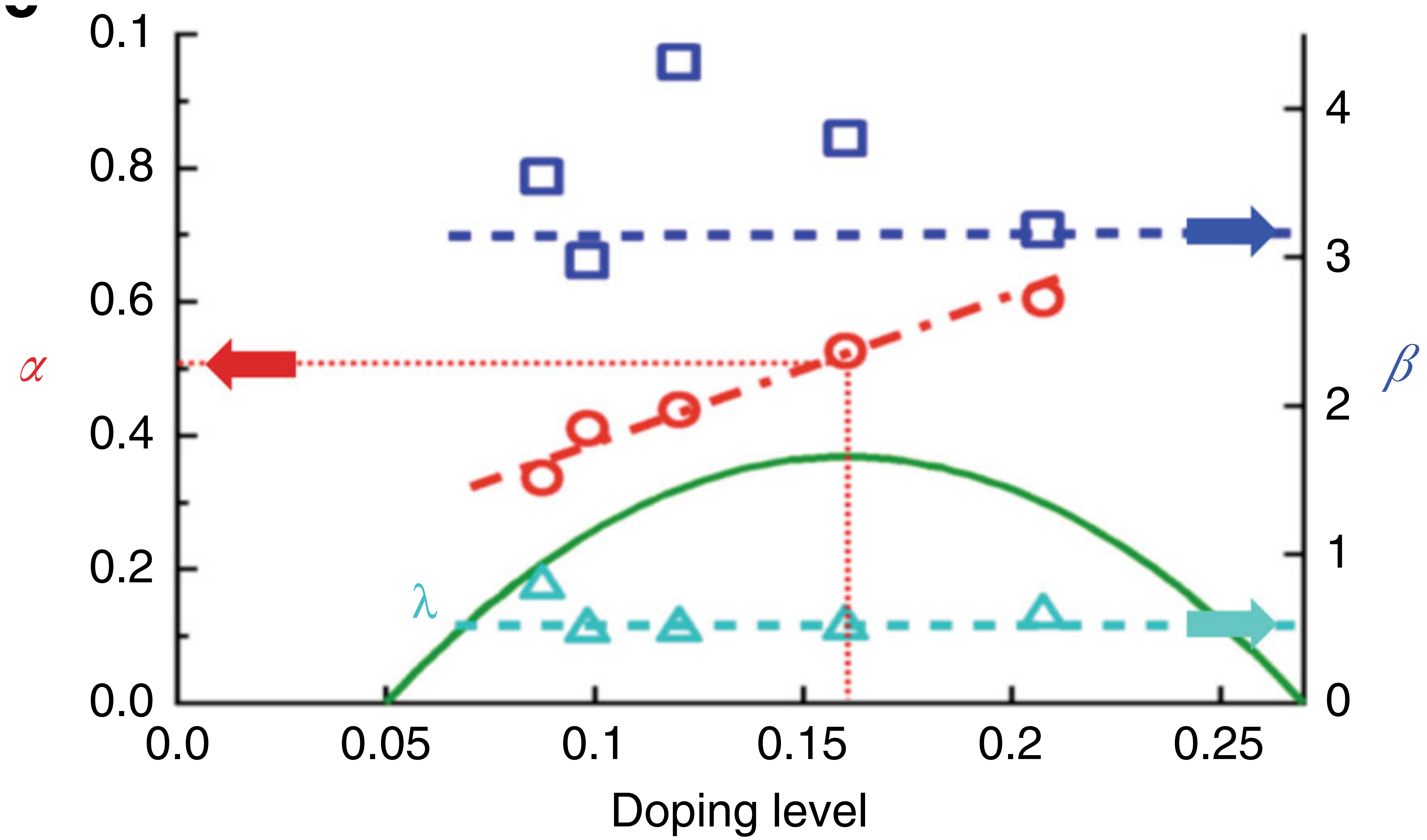
Electronic properties of a marginal Fermi liquid:

- Photoemission

Nature Communications **10**, 5737 (2019)

A unified form of low-energy nodal electronic interactions in hole-doped cuprate superconductors

T.J. Reber[1,5*], X. Zhou[1*], N.C. Plumb[1,6], S. Parham[1], J.A. Waugh[1], Y. Cao[1], Z. Sun[1,7], H. Li[1], Q. Wang[1], J.S. Wen[2], Z.J. Xu[2], G. Gu[2], Y. Yoshida[3], H. Eisaki[3], G.B. Arnold[1] & D.S. Dessau[1,4*]



$$\Sigma''_{\mathrm{PLL}}(\omega) = \Gamma_0 + \lambda \frac{\left[(\hbar\omega)^2 + (\beta k_{\mathrm{B}} T)^2\right]^{\alpha}}{(\hbar\omega_N)^{2\alpha-1}}$$

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   and $\rho(T) < h/e^2$ (in $d = 2$).
   Metals with $\rho(T) > h/e^2$ are bad metals.

2. Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \dfrac{m^*_{\rm trans}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\rm trans}(\omega)} \sim |\omega| \Phi_\sigma\left(\frac{\hbar\omega}{k_B T}\right)$$

B. Michon……A. Georges, arXiv:2205.04030

Electronic properties of a marginal Fermi liquid:

1. Photoemission: nearly marginal Fermi liquid electron spectral density:

$$\mathrm{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_\Sigma\left(\frac{\hbar\omega}{k_B T}\right) \quad \text{with } \alpha \approx 1/2 \quad ; \quad \frac{1}{\tau_{\rm in}(\omega)} \sim |\omega| \Phi_\Sigma\left(\frac{\hbar\omega}{k_B T}\right)$$

T.J. Reber….D. Dessau, Nature Communications **10**, 5737 (2019)

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B. Michon……A. Georges, arXiv:2205.04030

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T.J. Reber….D. Dessau, Nature Communications **10**, 5737 (2019)

2. Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

S.A. Hartnoll and A.P. MacKenzie, RMP (2022)