Metal-insulator transition in a Hubbard model with random and all-to-all hopping and exchange

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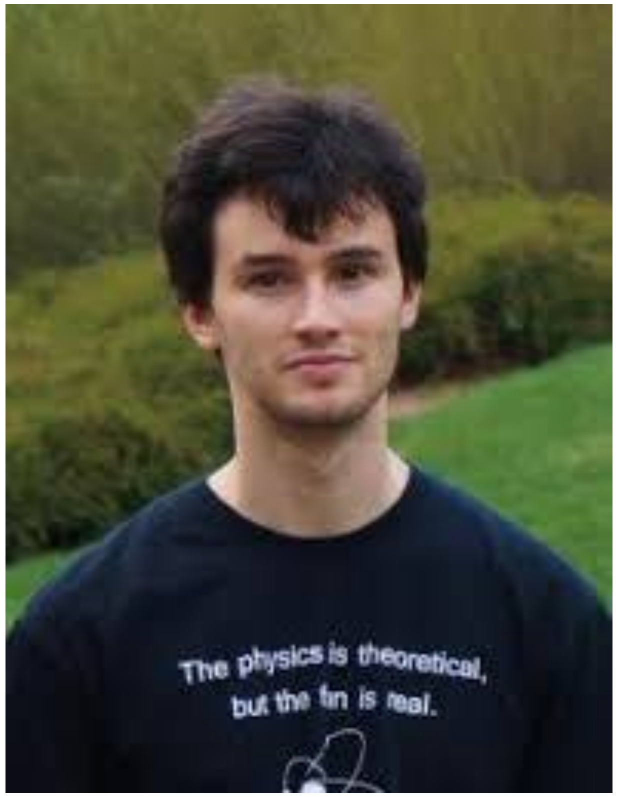
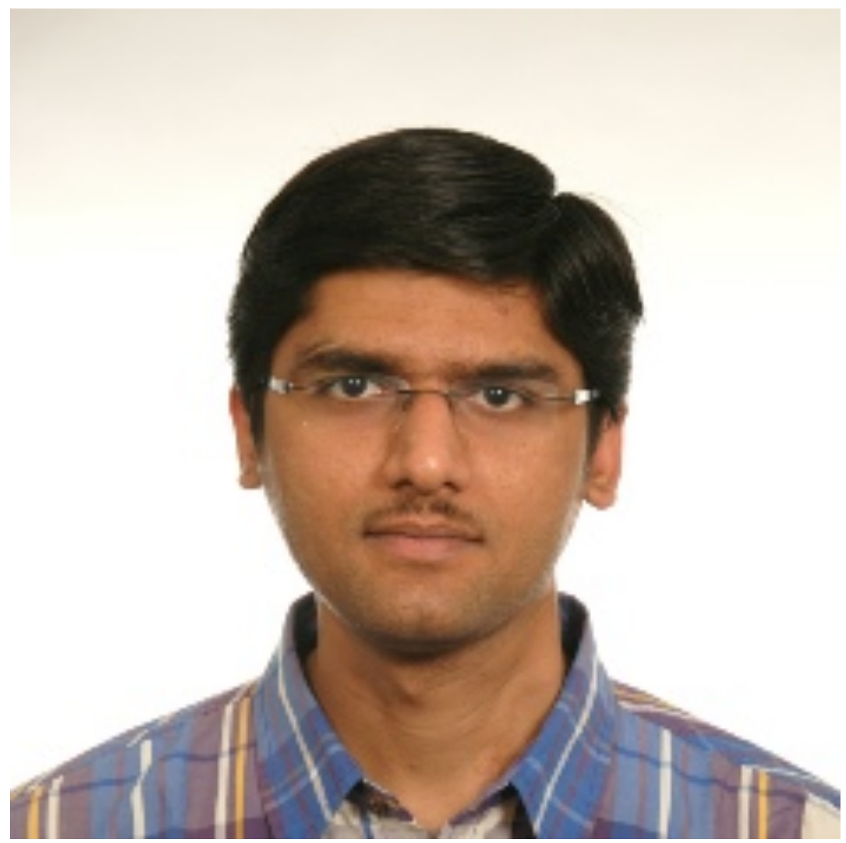
Subir Sachdev

Talk online: sachdev.physics.harvard.edu



Grigory Tarnopolsky

arXiv:2002.12381



Darshan Joshi

Chenyuan Li

Random *t-J-U* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^{\dagger} c_{i\alpha},$$

$$J_{ij} \text{ random}, \; \overline{J_{ij}} = 0, \; \overline{J_{ij}^2} = J^2$$
$$t_{ij} \text{ random}, \; \overline{t_{ij}} = 0, \; \overline{t_{ij}^2} = t^2$$

$$U > 0 \text{ non-random}$$

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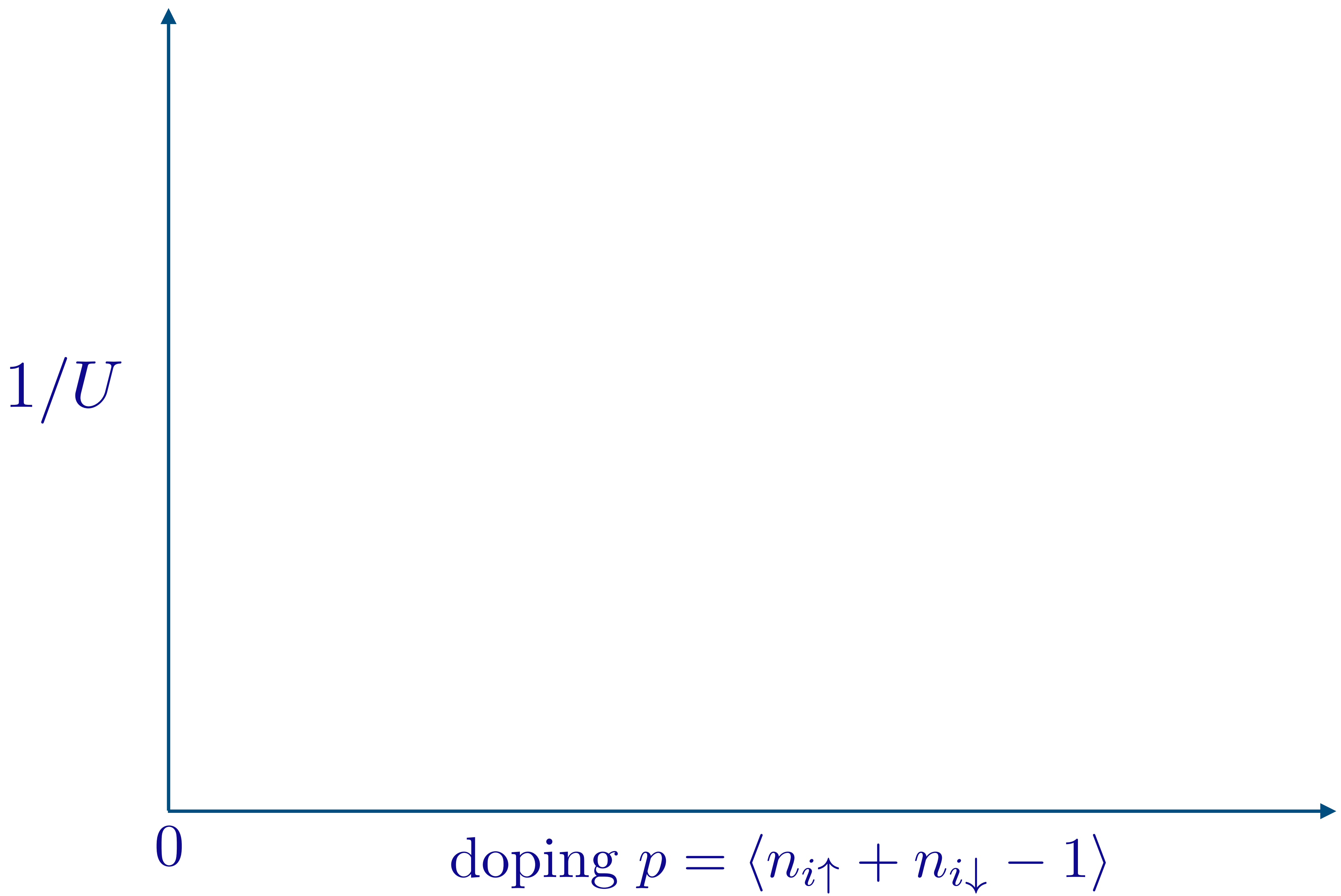

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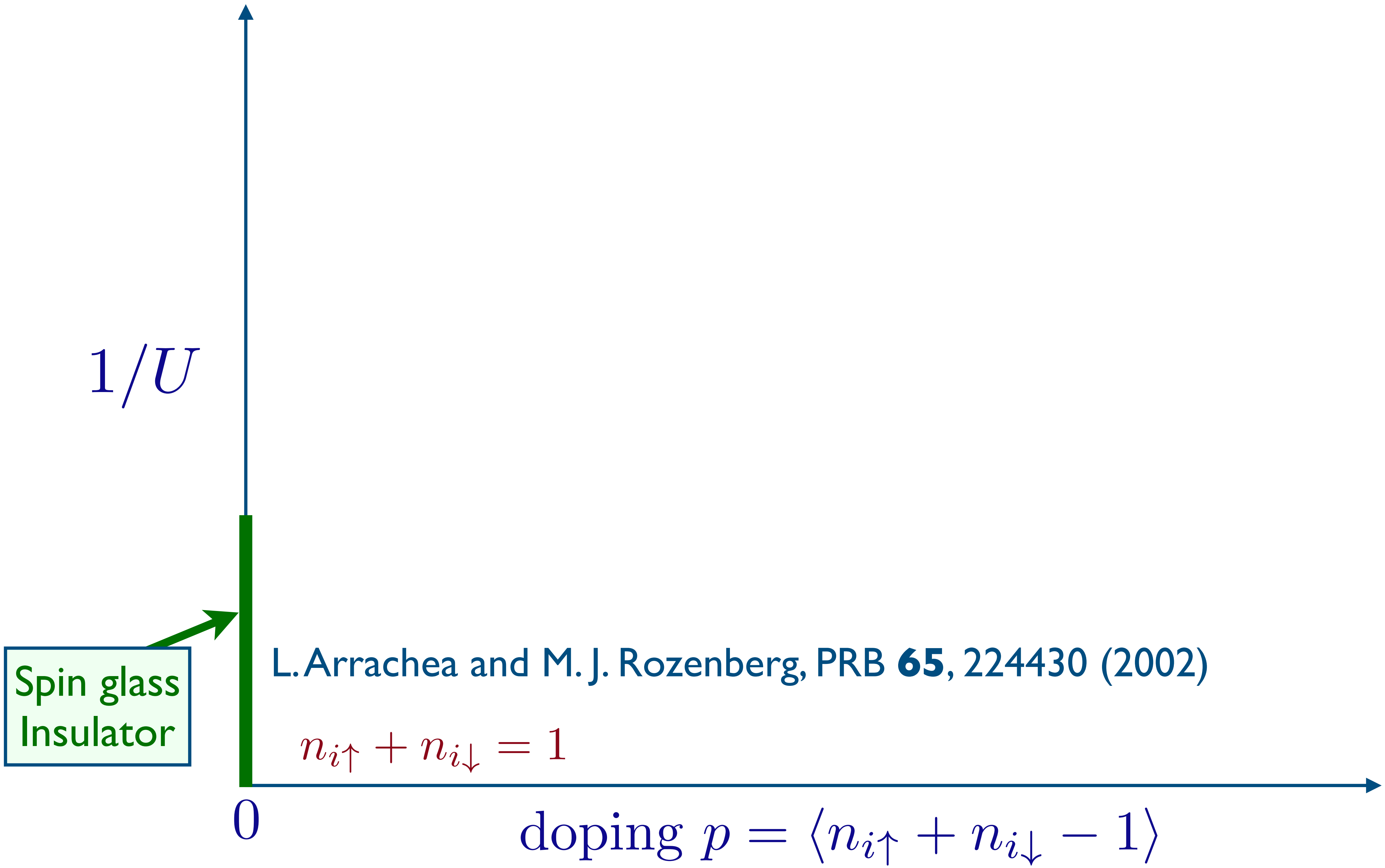

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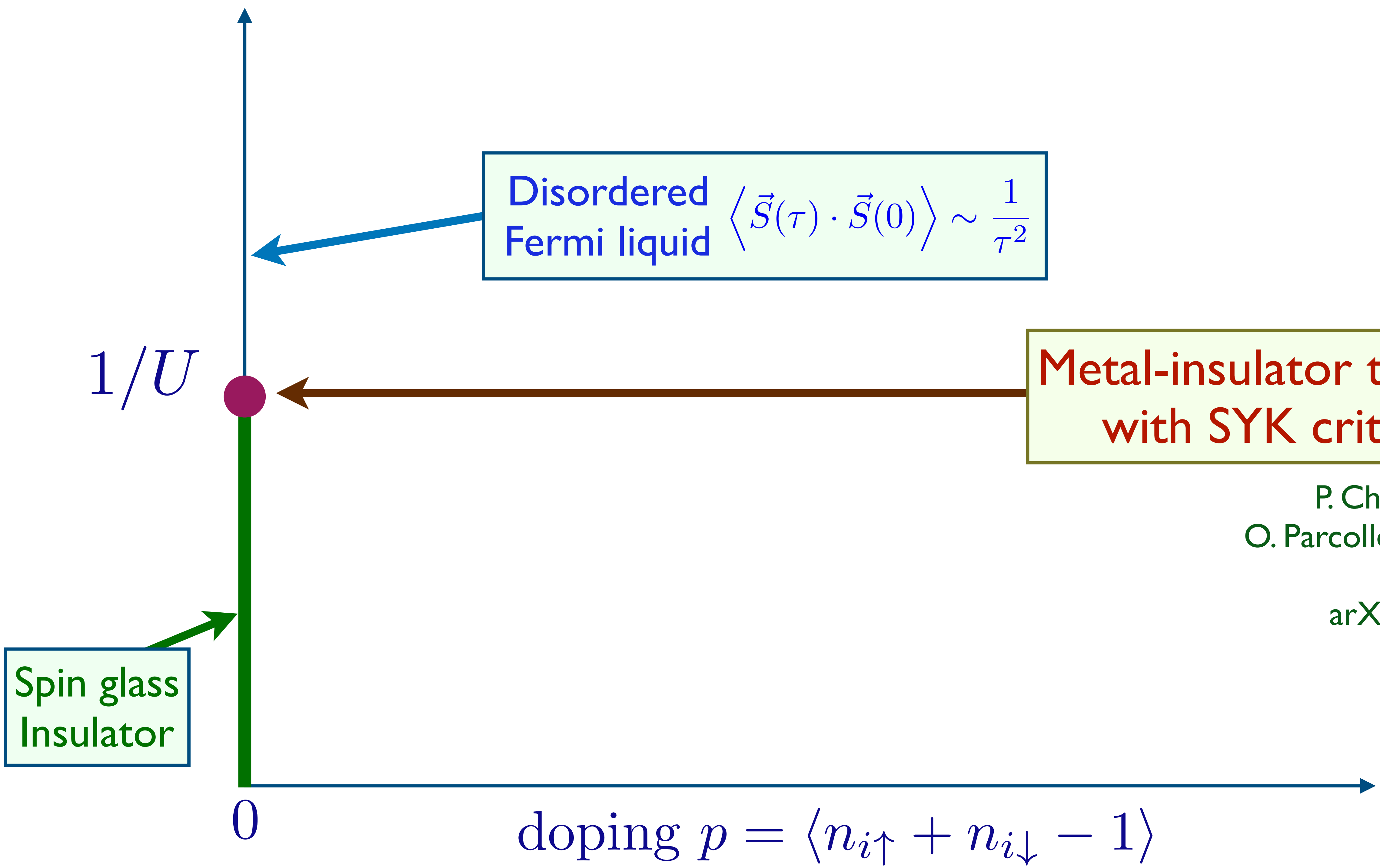
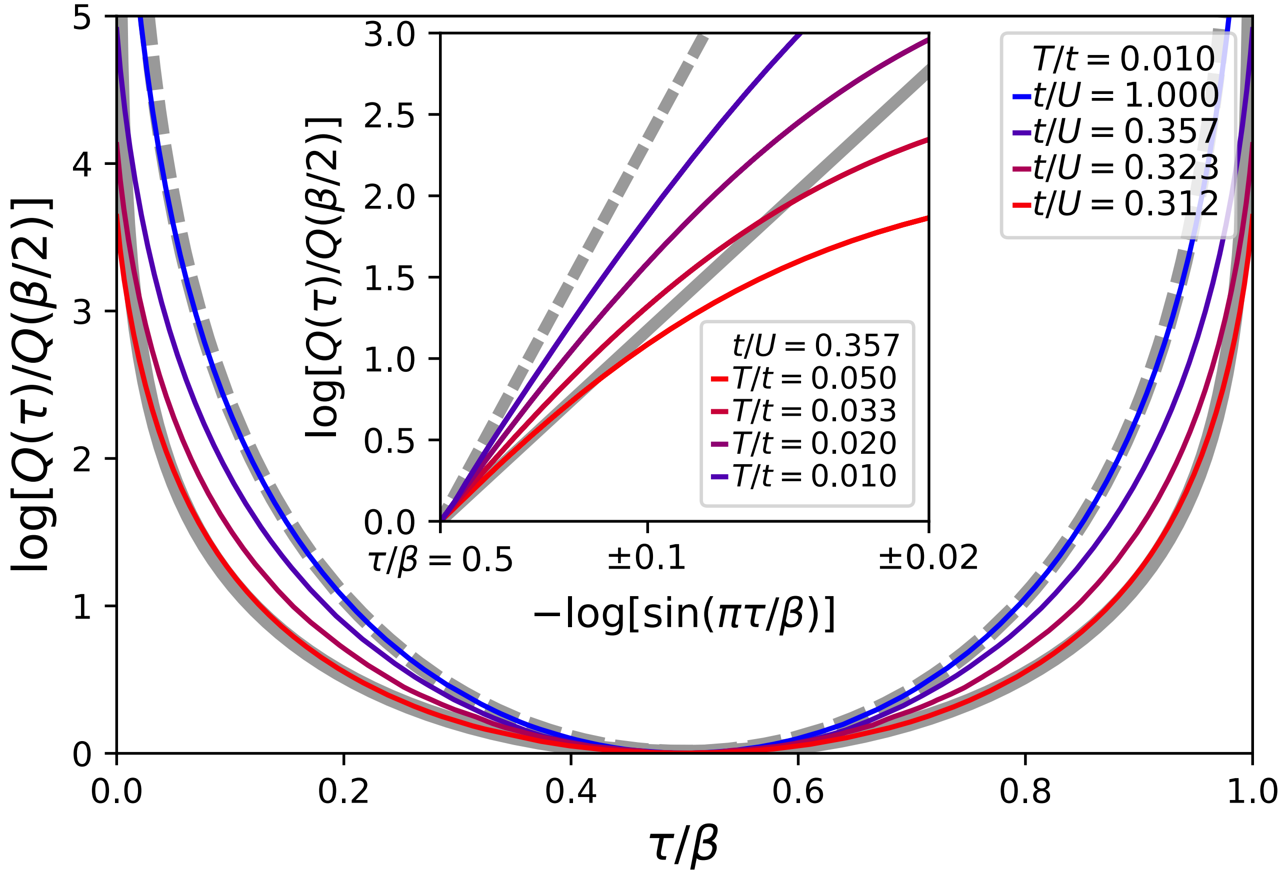
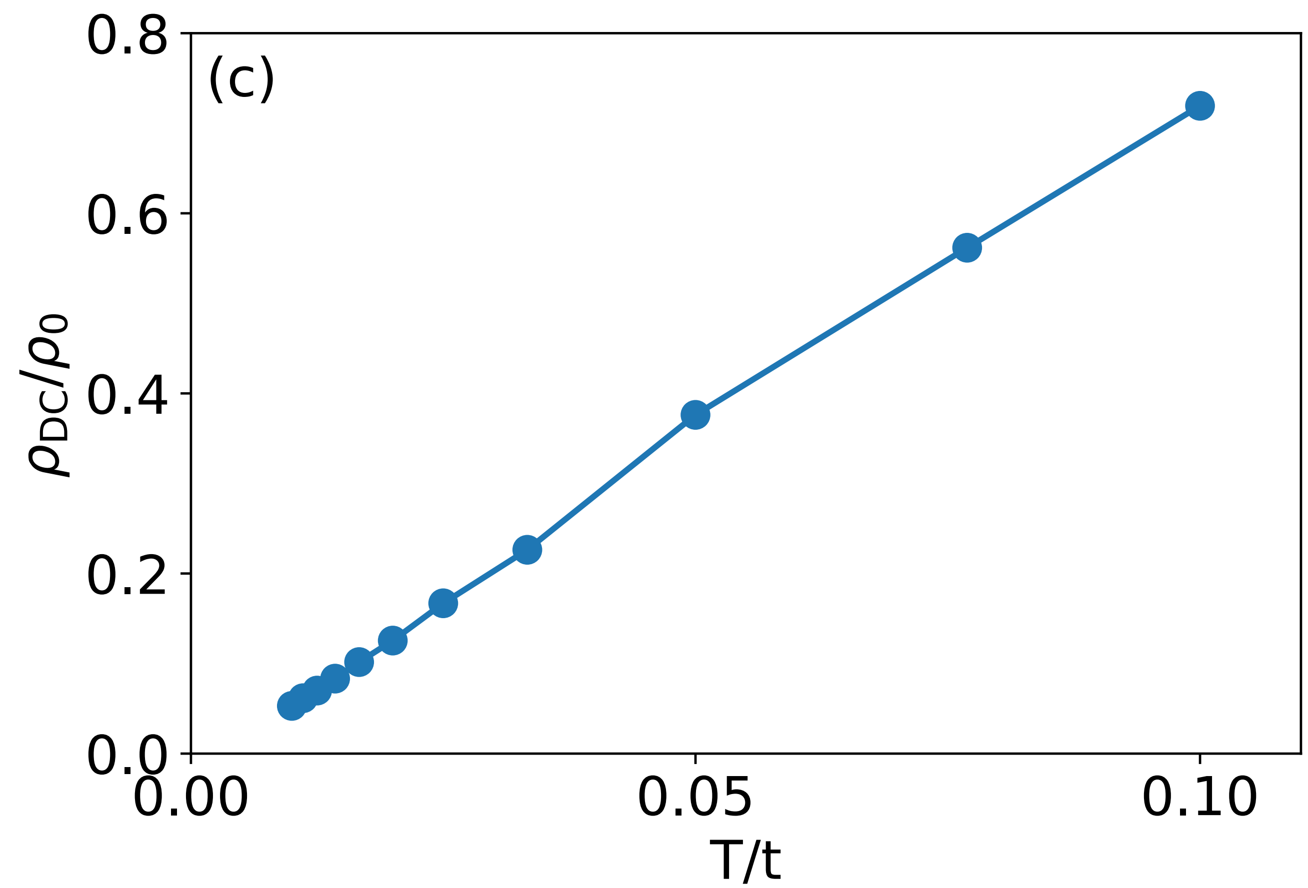


P. Cha, N. Wentzell.
O. Parcollet, A. Georges,
Eun-Ah Kim,
arXiv:2002.07181

Linear resistivity and Sachdev-Ye-Kitaev (SYK) spin liquid behavior in a quantum critical metal with spin-$1/2$ fermions

Peter Cha, Nils Wentzell, Olivier Parcollet, Antoine Georges, Eun-Ah Kim





arXiv.org > cond-mat > arXiv:2002.07181

Condensed Matter > Strongly Correlated Electrons

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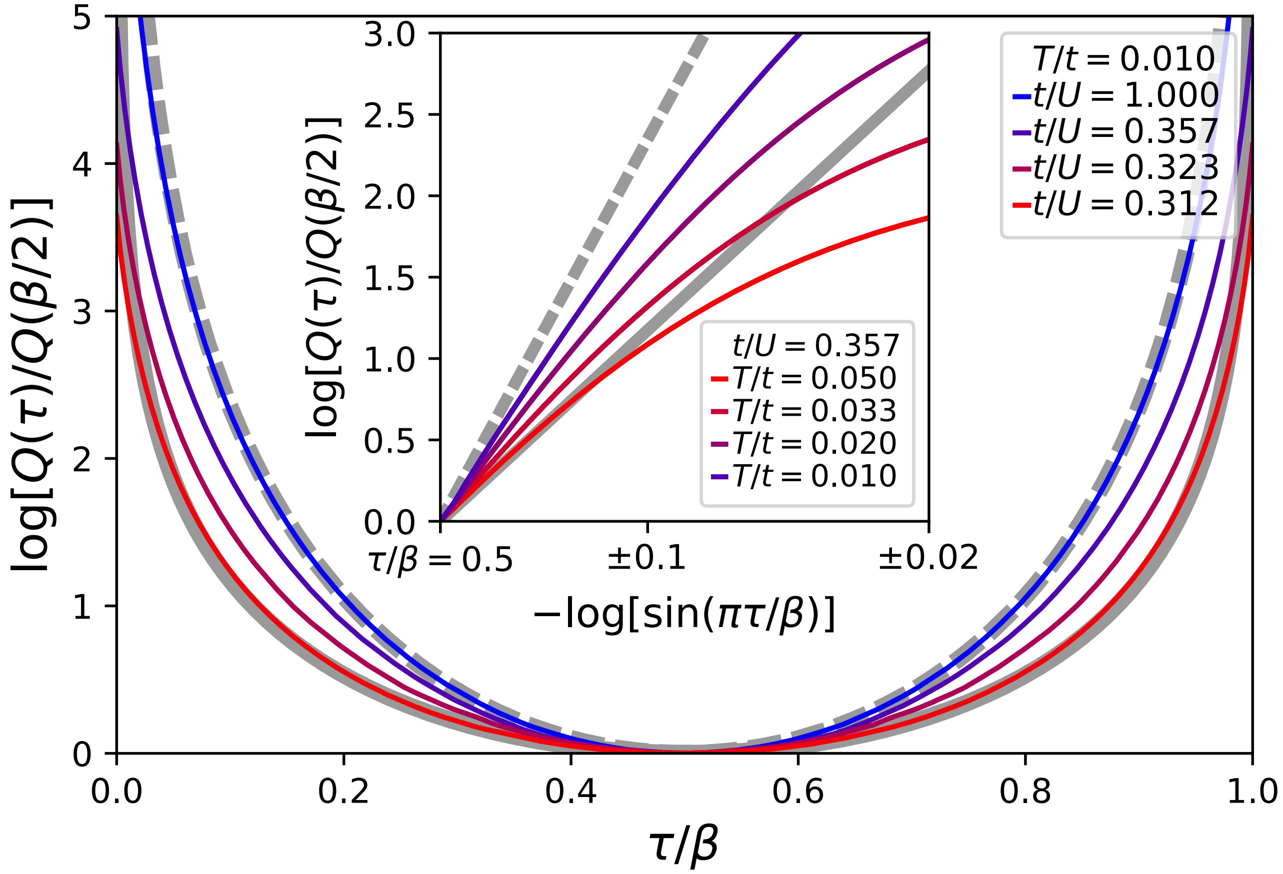

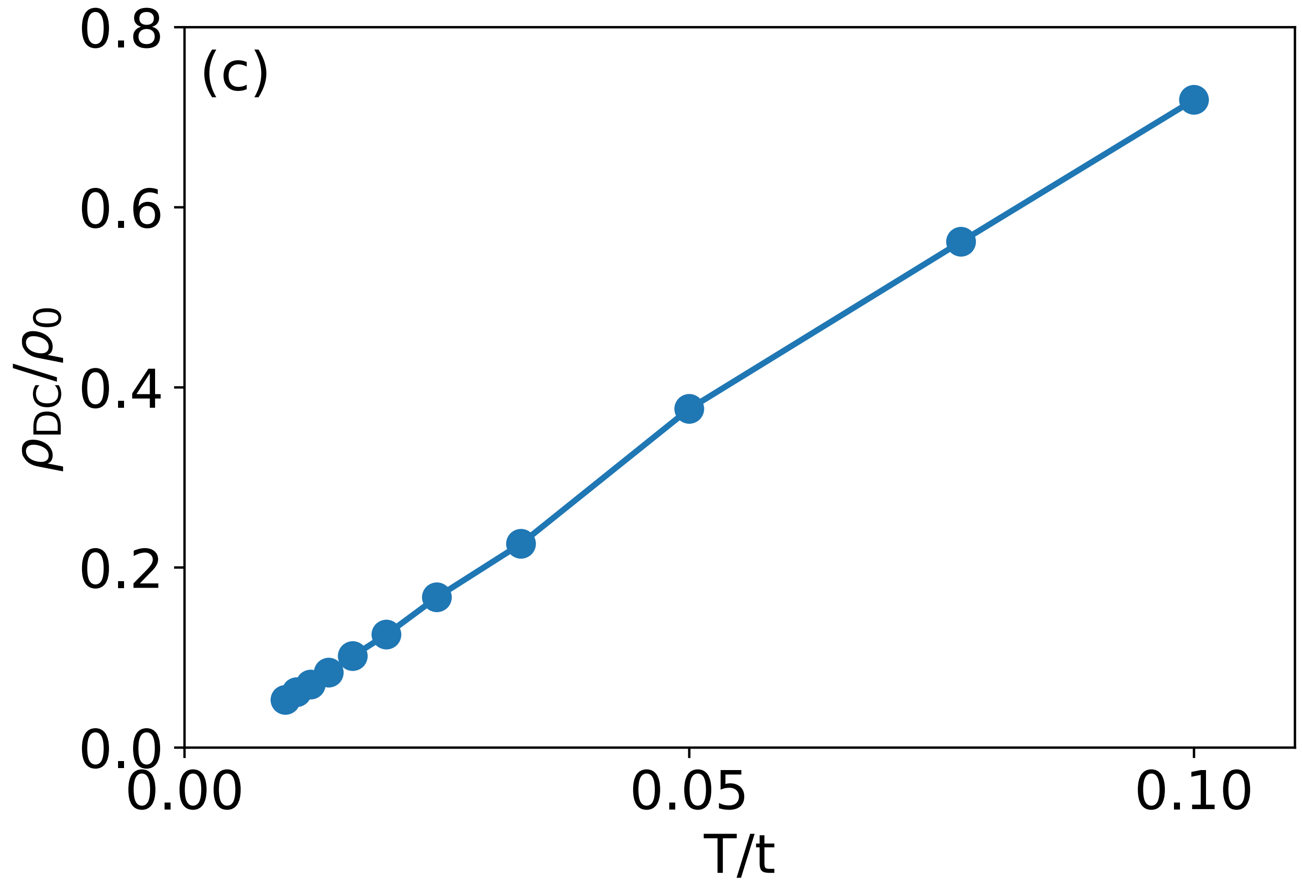

Critical spin correlations:

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

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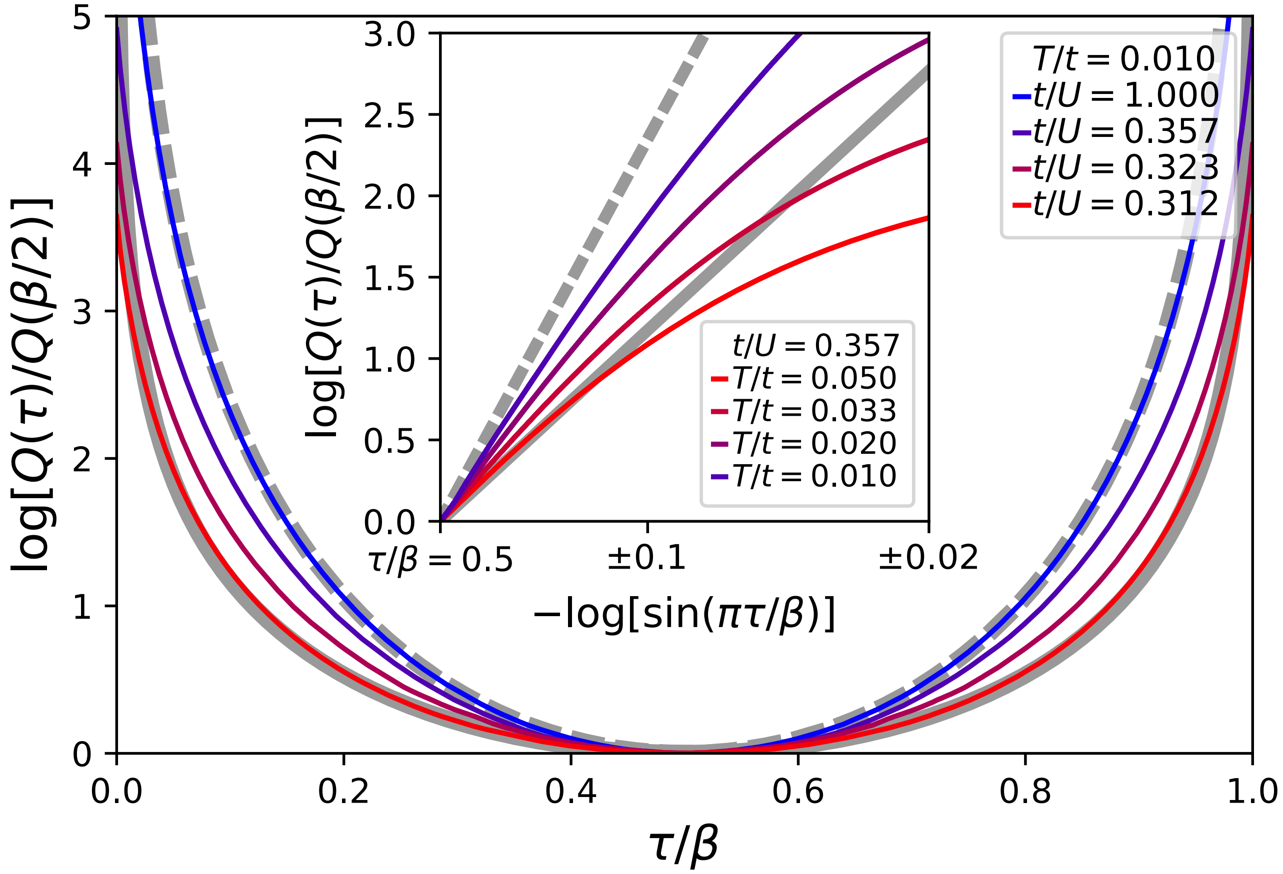

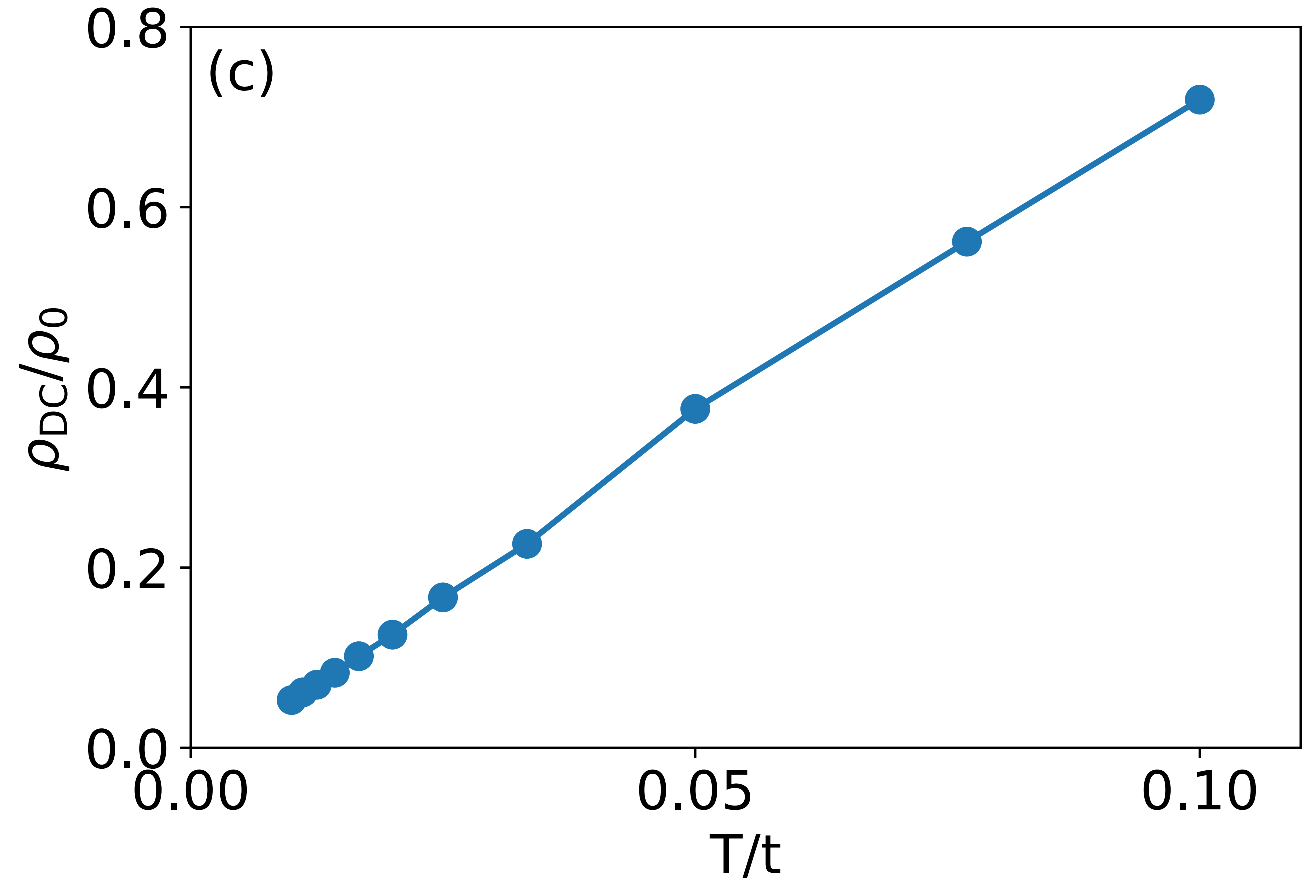

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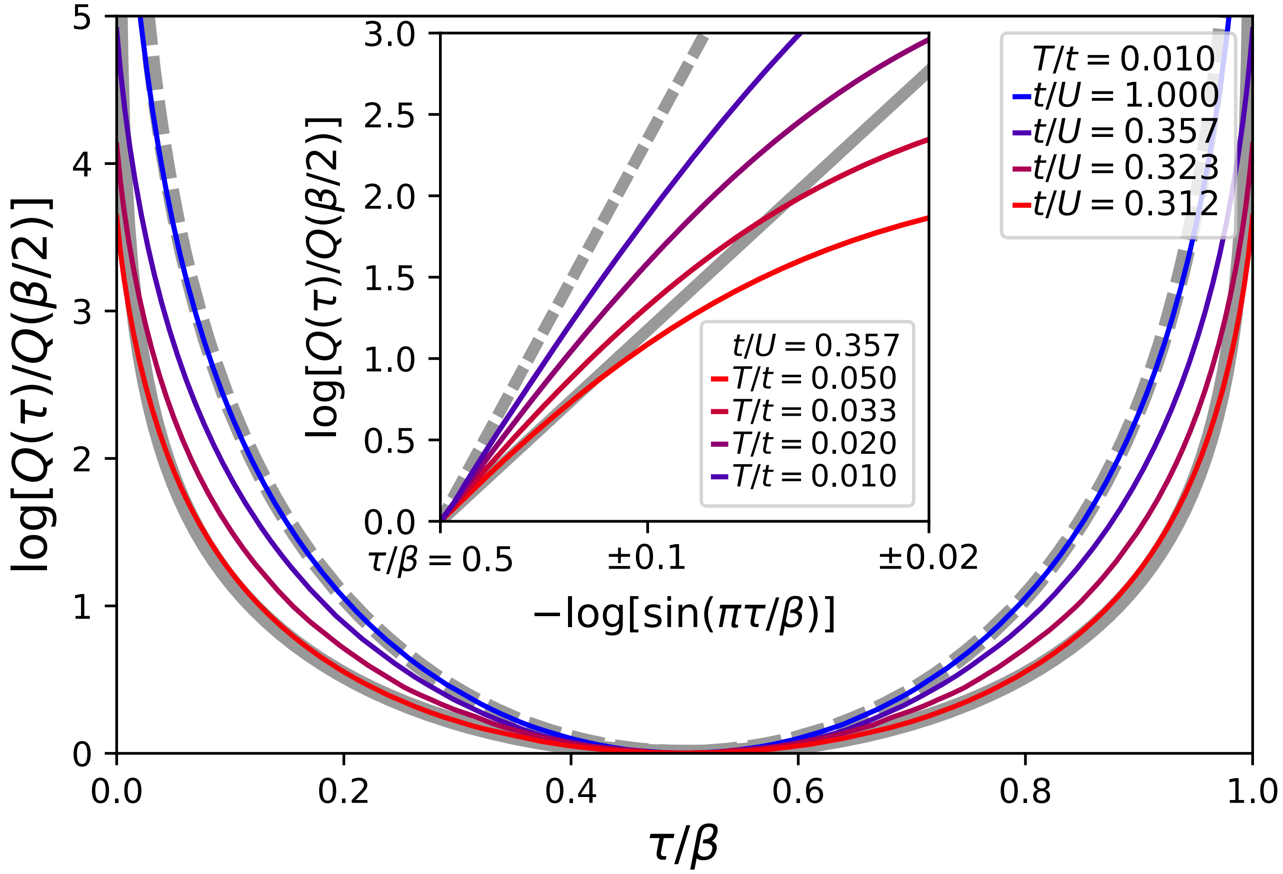

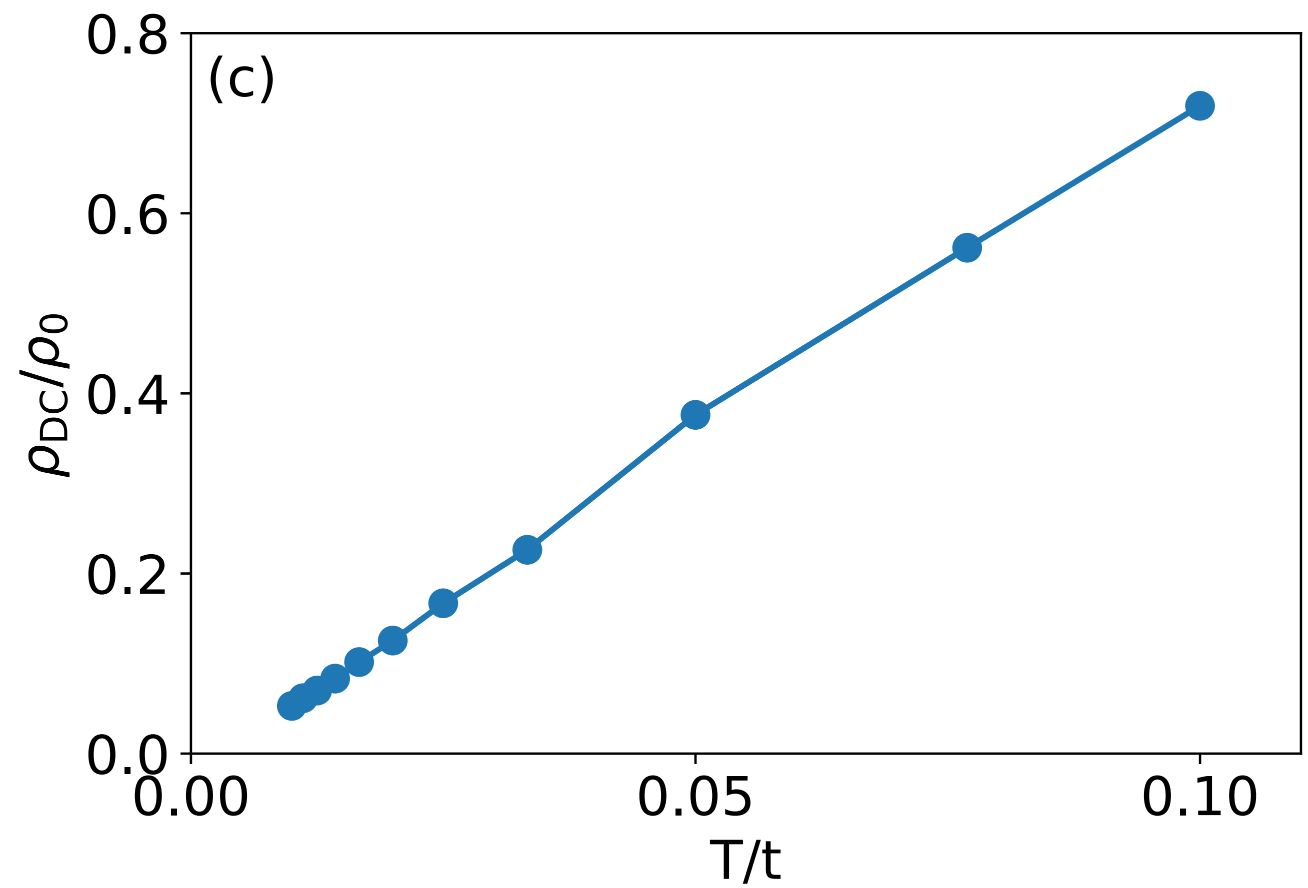

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Mapping to SYK criticality in a large M theory

G. Tarnopolsky, Chenyuan Li, D.G. Joshi, and S. Sachdev, arXiv:2002.12381

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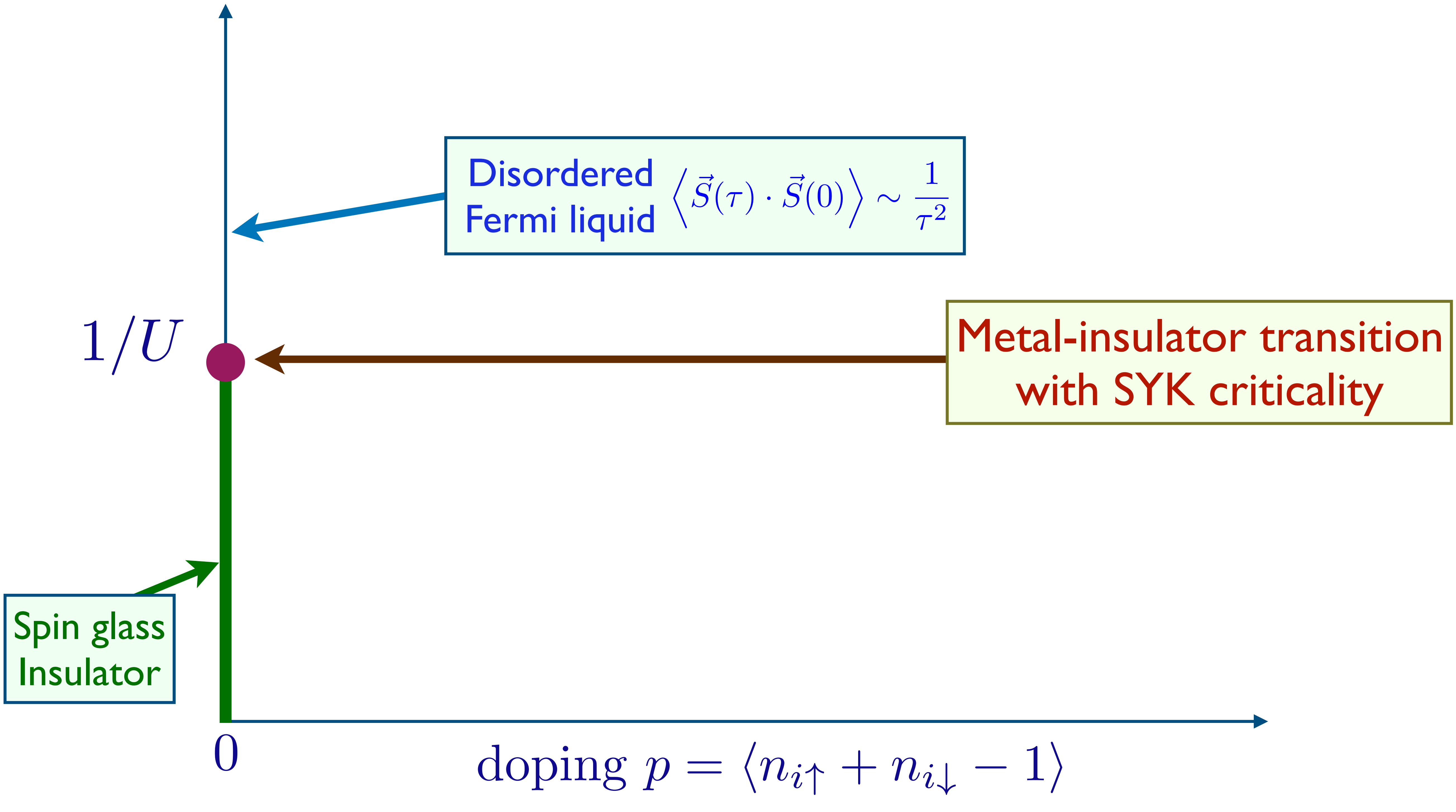

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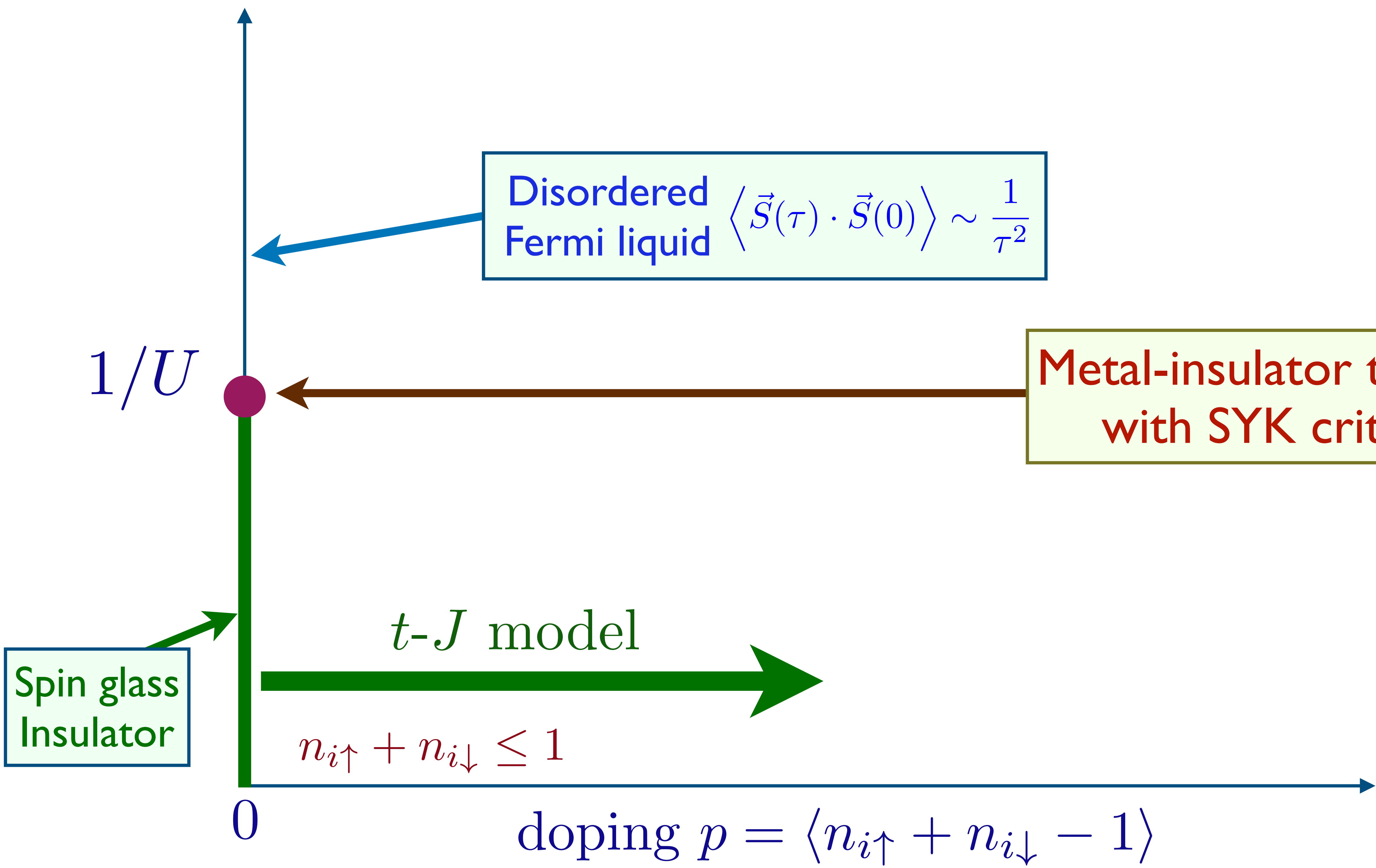

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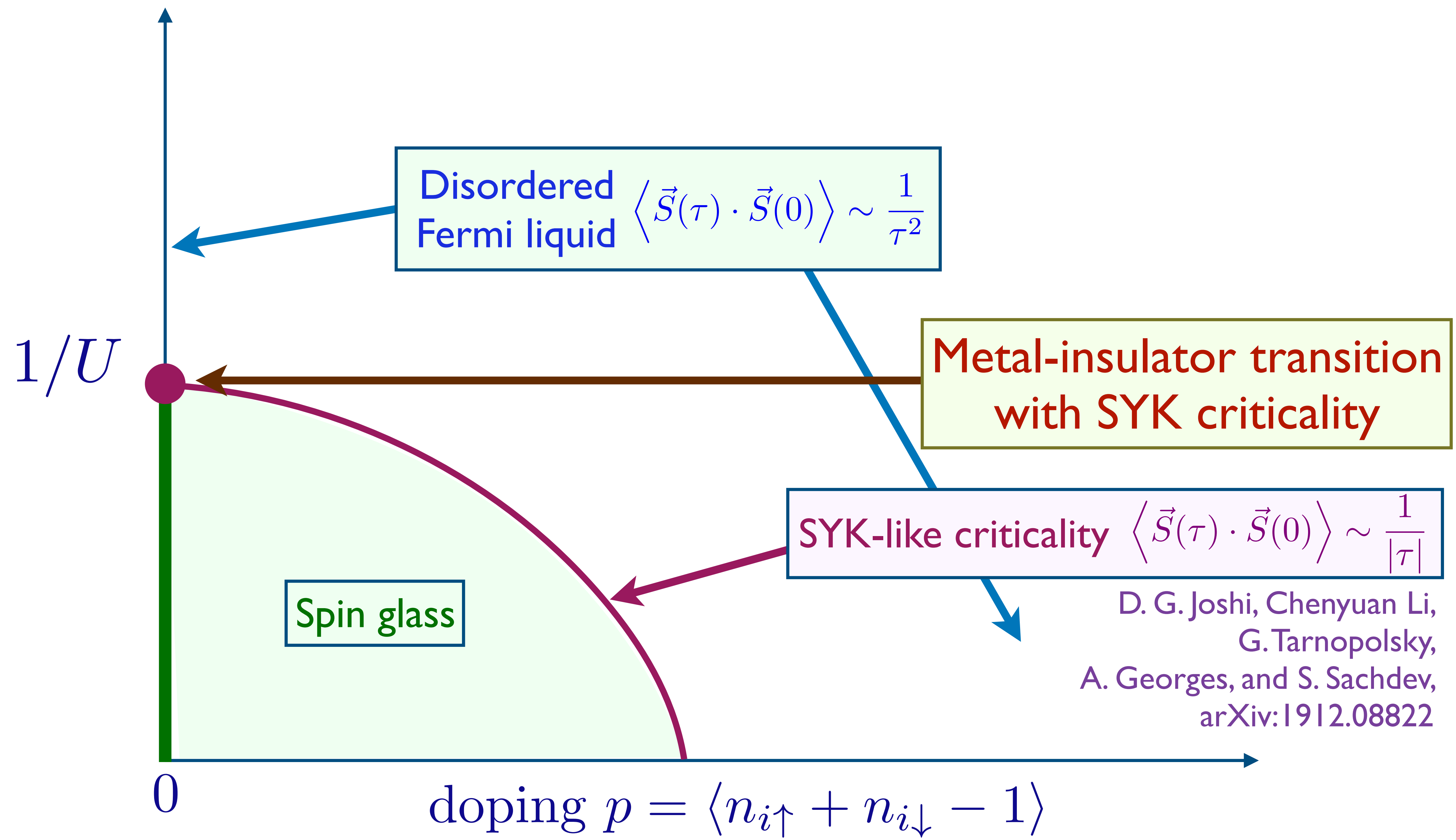
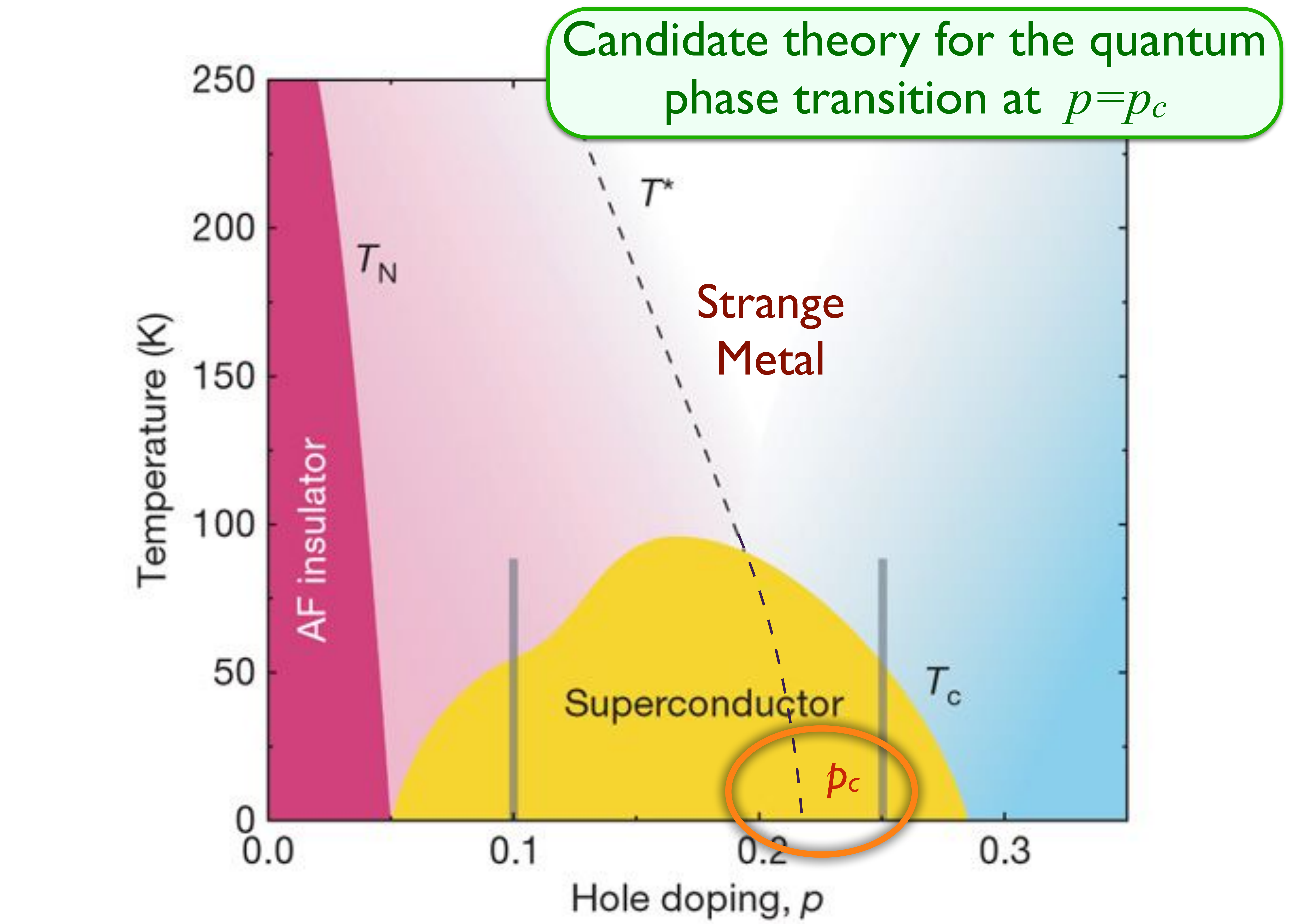


D. G. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, and S. Sachdev, arXiv:1912.08822


Candidate theory for the quantum phase transition at $p=p_c$
250
200
150
100
50
0
Temperature (K)
AF insulator
T_N
T^*
Strange Metal
Superconductor
T_c
p_c
0.0
0.1
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Hole doping, p

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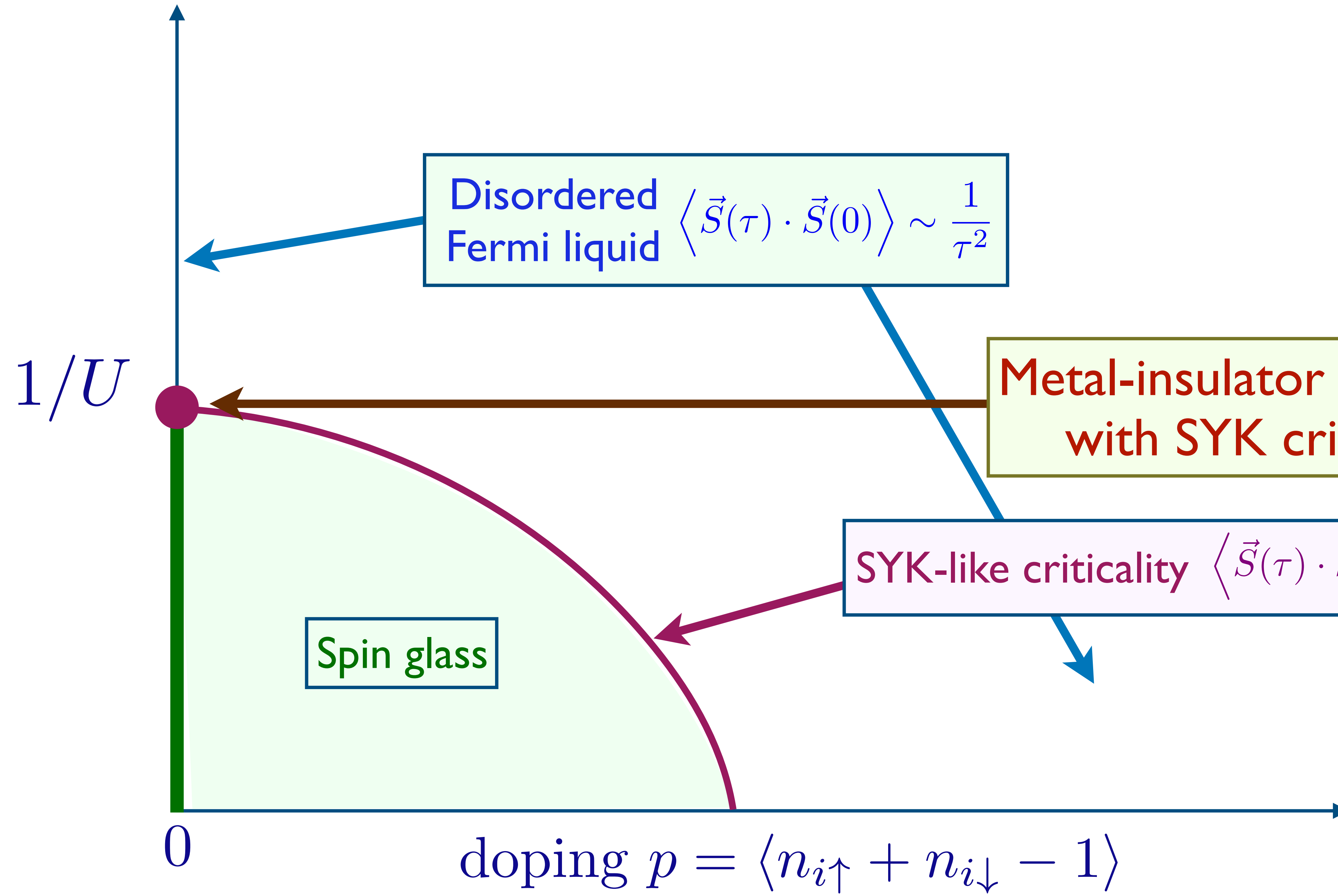

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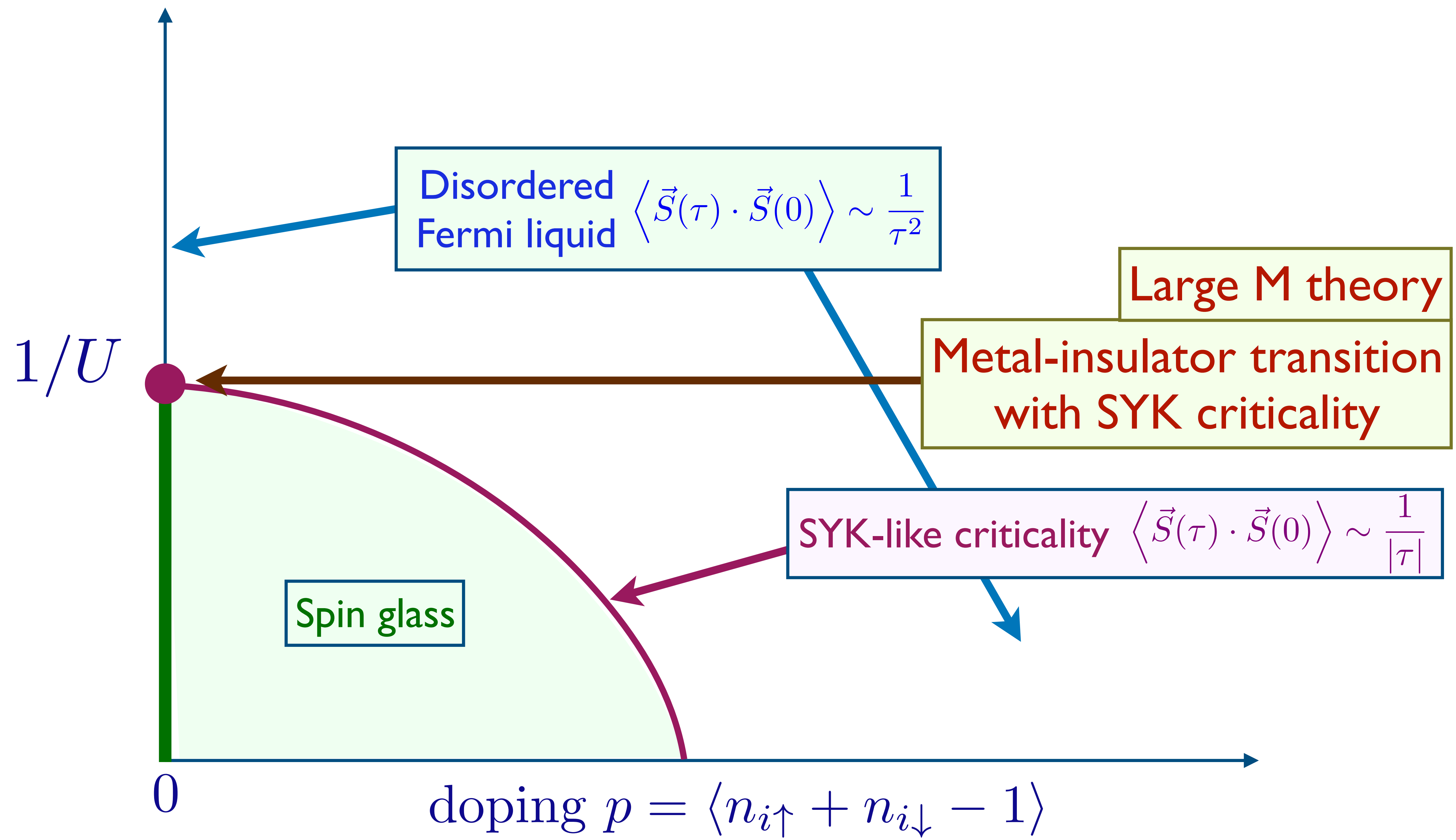

Large N limit

$$\mathcal{Z} = \int \mathcal{D}c_\alpha(\tau) e^{-\mathcal{S}}$$

$$\mathcal{S} = \int d\tau \left[c_\alpha^\dagger(\tau) \left(\frac{\partial}{\partial \tau} - \mu \right) c_\alpha(\tau) + U n_{i\uparrow}(\tau) n_{i\downarrow}(\tau) \right]$$
$$- t^2 \int d\tau d\tau' R(\tau - \tau') c_\alpha^\dagger(\tau) c_\alpha(\tau')$$
$$- \frac{J^2}{2} \int d\tau d\tau' Q(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau') .$$

The functions $R(\tau)$ and $Q(\tau)$ have to be determined self-consistently:

$$R(\tau - \tau') = - \left\langle c_\alpha^\dagger(\tau) c_\alpha(\tau') \right\rangle_{\mathcal{Z}}$$

$$Q(\tau - \tau') = \frac{1}{3} \left\langle \vec{S}(\tau) \cdot \vec{S}(\tau') \right\rangle_{\mathcal{Z}}$$

Large M limit

We consider an electron $c_{p,\alpha}$ with a spin index $\alpha = 1 \dots M$, and an 'orbital' index $p = 1 \dots M'$. We will take the limit of large number of sites, N, followed by the limit of large M and M' at fixed $k \equiv M'/M$ The large M,M' limit requires us to fractionalize the electron as

$$c_{ip\alpha}^\dagger = X_{ip} f_{i\alpha}^\dagger \,,$$

where X_{ip} is a complex 'slave rotor', with $p = 1 \dots M'$, obeying the constraint

$$\sum_{p=1}^{M'} |X_{ip}|^2 = M' \,.$$

This representation has a U(1) gauge invariance

$$X_{ip} \to X_{ip} e^{i\phi_i(\tau)} \quad , \quad f_{i\alpha} \to f_{i\alpha} e^{i\phi_i(\tau)}$$

We shall be interested in the sector in which the U(1) gauge charge is fixed on each site by

$$\sum_{\alpha=1}^{M} f_{i\alpha}^\dagger f_{i\alpha} + \hat{L}_i = \frac{M}{2}$$

where $\hat{L}_i$ is the U(1) angular momentum operator for the rotors.

Large M limit

The Hamiltonian is

$$
\begin{aligned}
H &= \frac{U}{2M'} \sum_i \left(\sum_{\alpha=1}^{M} f_{i\alpha}^\dagger f_{i\alpha} - \frac{M}{2} \right)^2 + \epsilon_0 \sum_{ip\alpha} f_{i\alpha}^\dagger f_{i\alpha} \\
&\quad + \frac{1}{\sqrt{NM}} \sum_{i,j,p,\alpha} t_{ij} c_{ip\alpha}^\dagger c_{jp\alpha} + \frac{1}{\sqrt{NM}} \sum_{i>j,\alpha\beta} J_{ij} f_{i\alpha}^\dagger f_{i\beta} f_{j\beta}^\dagger f_{j\alpha} \,.
\end{aligned}
$$

The saddle point equations for the f fermion Green's function G_f and the X correlator χ are

$$
\begin{aligned}
G_f(i\omega_n) &= \frac{1}{i\omega_n - \Sigma_f(i\omega_n)} \quad , \quad \Sigma_f(\tau) = -J^2 G_f^2(\tau) G_f(-\tau) + k\, t^2 G_f(\tau) \chi^2(\tau) \\
\chi(i\omega_n) &= \frac{1}{\omega_n^2/U + \chi_0^{-1} - P(i\omega_n) + P(i\omega_n = 0)}, \quad P(\tau) = -t^2 G_f(\tau) G_f(-\tau) \chi(\tau)
\end{aligned}
$$

and the value of χ_0 is to be determined by solving the constraint equation

$$
T \sum_{\omega_n} \chi(i\omega_n) = 1 \,.
$$

Large M limit

Insulating spin liquid.

$$\langle \boldsymbol{S}_i(\tau) \cdot \boldsymbol{S}_i(0) \rangle \sim \frac{1}{|\tau|}$$

$$\langle c_{i\alpha}(\tau) c_{i\alpha}(0) \rangle \sim \frac{\mathrm{sgn}(\tau)}{|\tau|}$$

Disordered Fermi liquid.

$$\langle X_p \rangle \neq 0$$

$$\langle \boldsymbol{S}_i(\tau) \cdot \boldsymbol{S}_i(0) \rangle \sim \frac{1}{|\tau|}$$

c_α gapped

$$\langle \boldsymbol{S}_i(\tau) \cdot \boldsymbol{S}_i(0) \rangle \sim \frac{1}{|\tau|^{4\Delta_f}}$$

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$$\frac{1}{4} < \Delta_f < \frac{1}{2}$$

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$1/U$

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Expected to be unstable to spin glass order

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Exponents observed by Cha *et al.*

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