Strange metals and black holes

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Subir Sachdev

Talk online: sachdev.physics.harvard.edu

Remarkable recent observation of 'Planckian' strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity, ρ, is

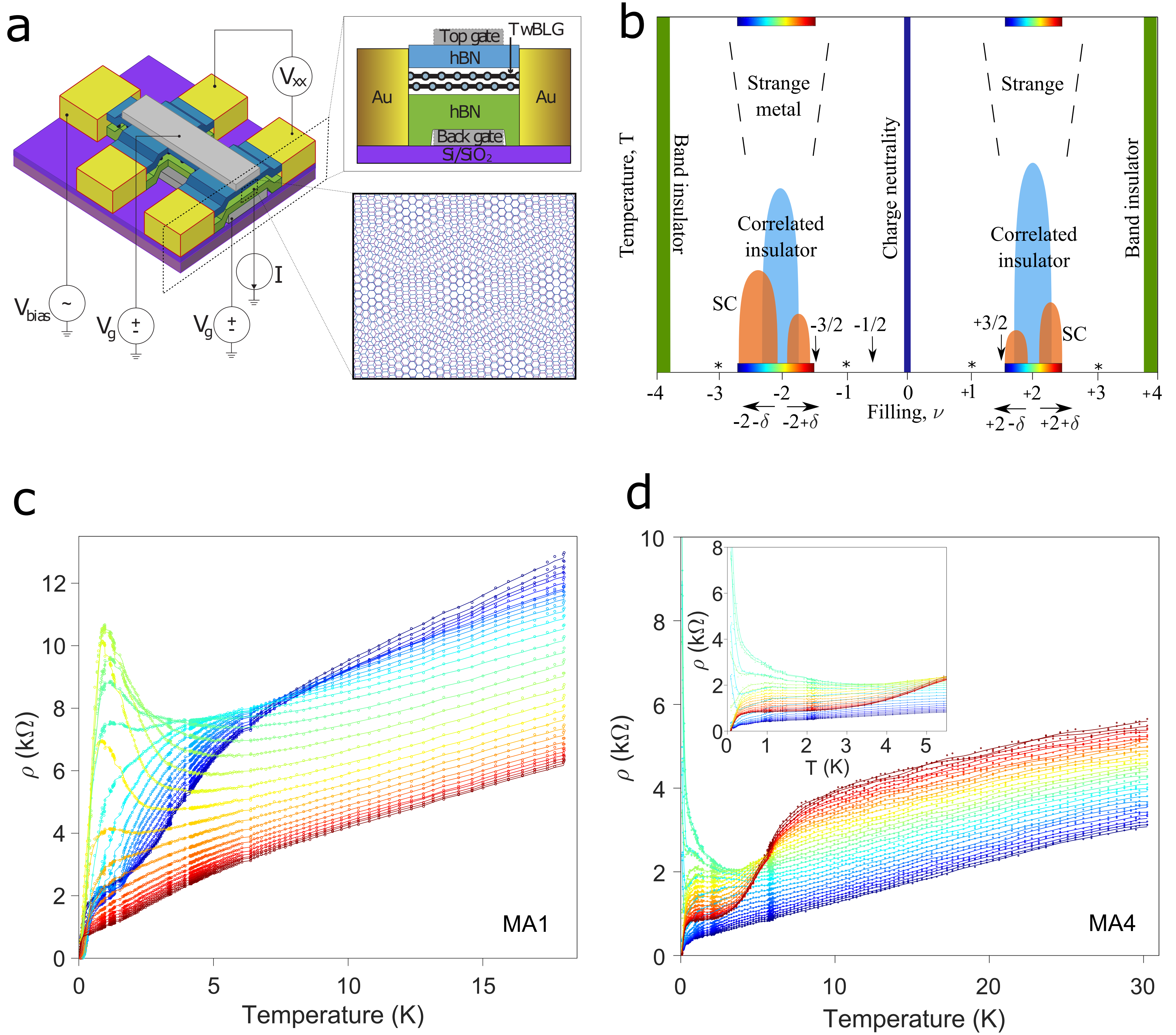
$$\rho = \frac{m^*}{ne^2}\frac{1}{\tau}$$

with a universal scattering rate

$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar},$$

independent of the strength of interactions, as $T \to 0$

Twisted bilayer graphene

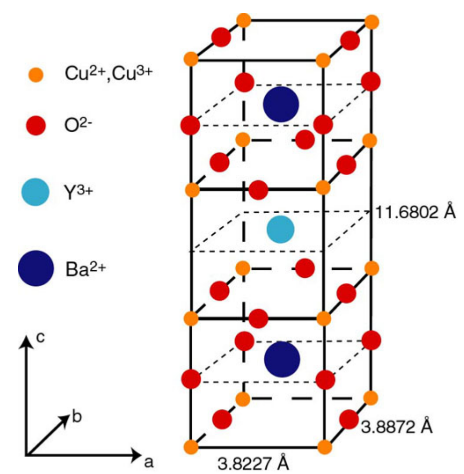


Yuan Cao, D. Chowdhury, D. Rodan-Legrain, O. Rubies-Bigordà, Kenji Watanabe, Takashi Taniguchi, T. Senthil, P. Jarillo-Herrero, PRL **124**, 076801 (2020)

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Slope of *T*-linear resistivity vs Planckian limit in seven materials.

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$



A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, Nature Physics **15**, 142 (2019)

1. SYK criticality

2. Deconfined quantum criticality of random t-J-U_H models

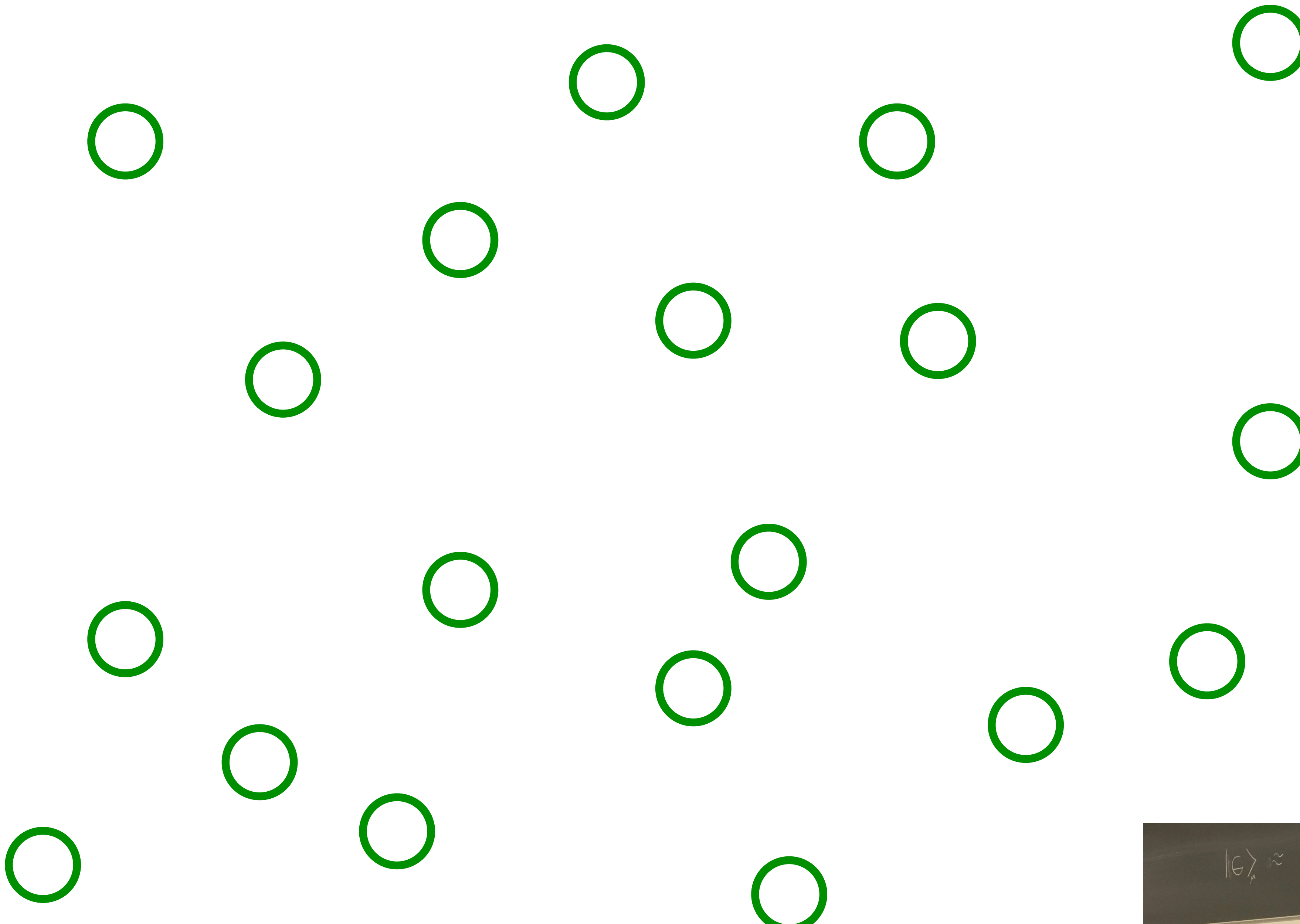
3. Black holes

1. SYK criticality

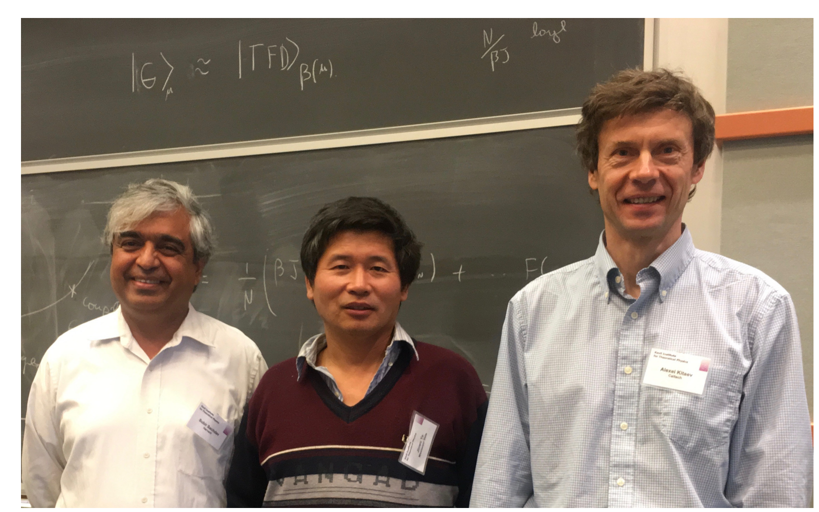
2. Deconfined quantum criticality of random t-J-U_H models

3. Black holes

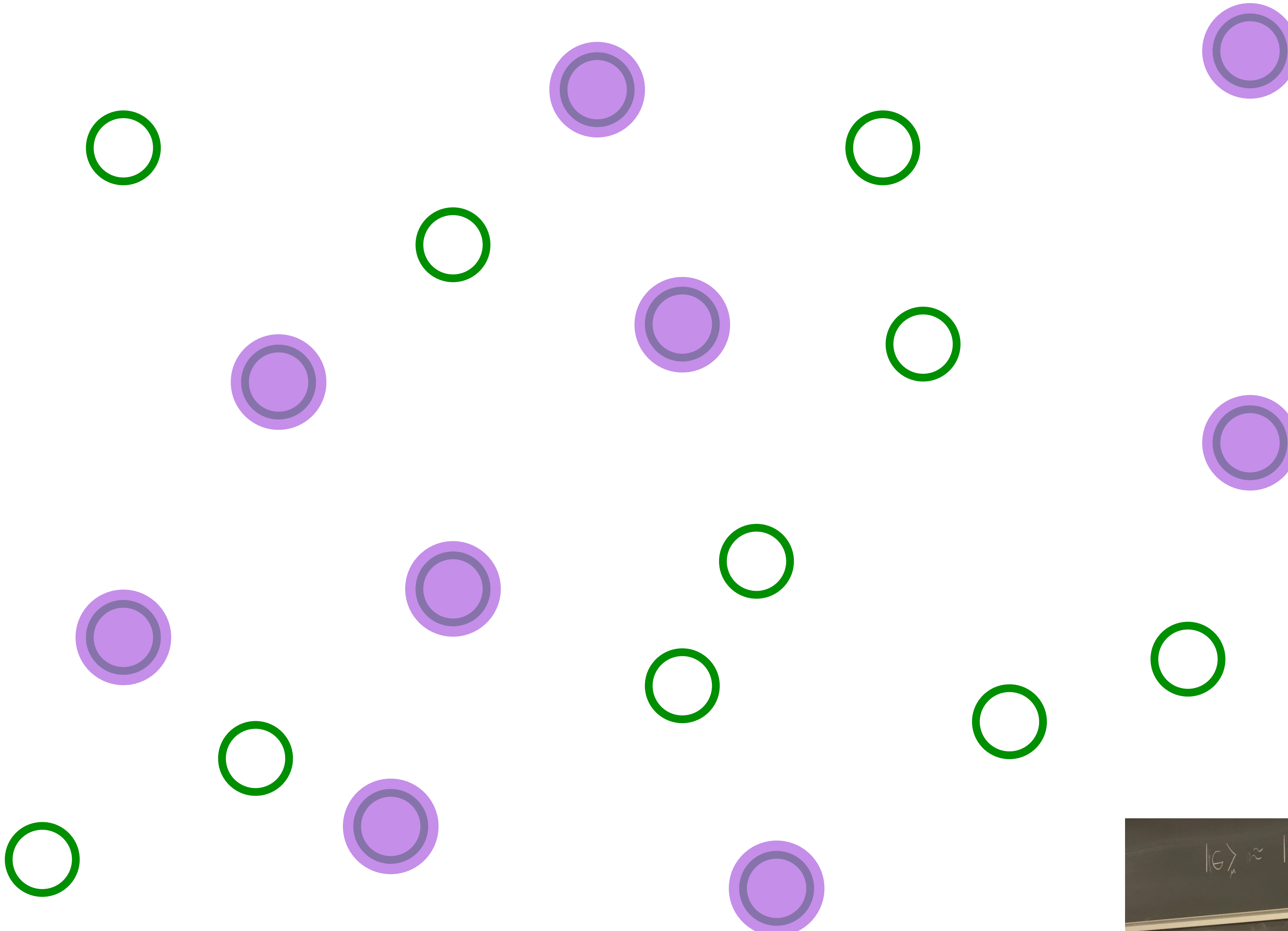
The Sachdev-Ye-Kitaev (SYK) model



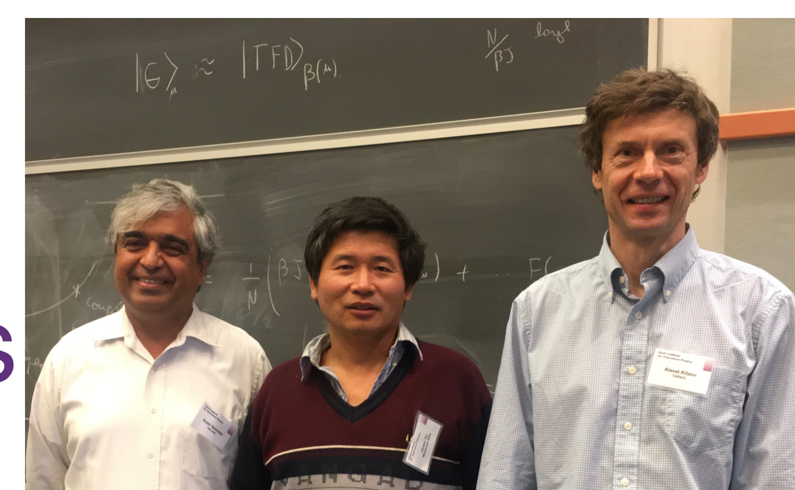
Pick a set of random positions



The SYK model



Place electrons randomly on some sites



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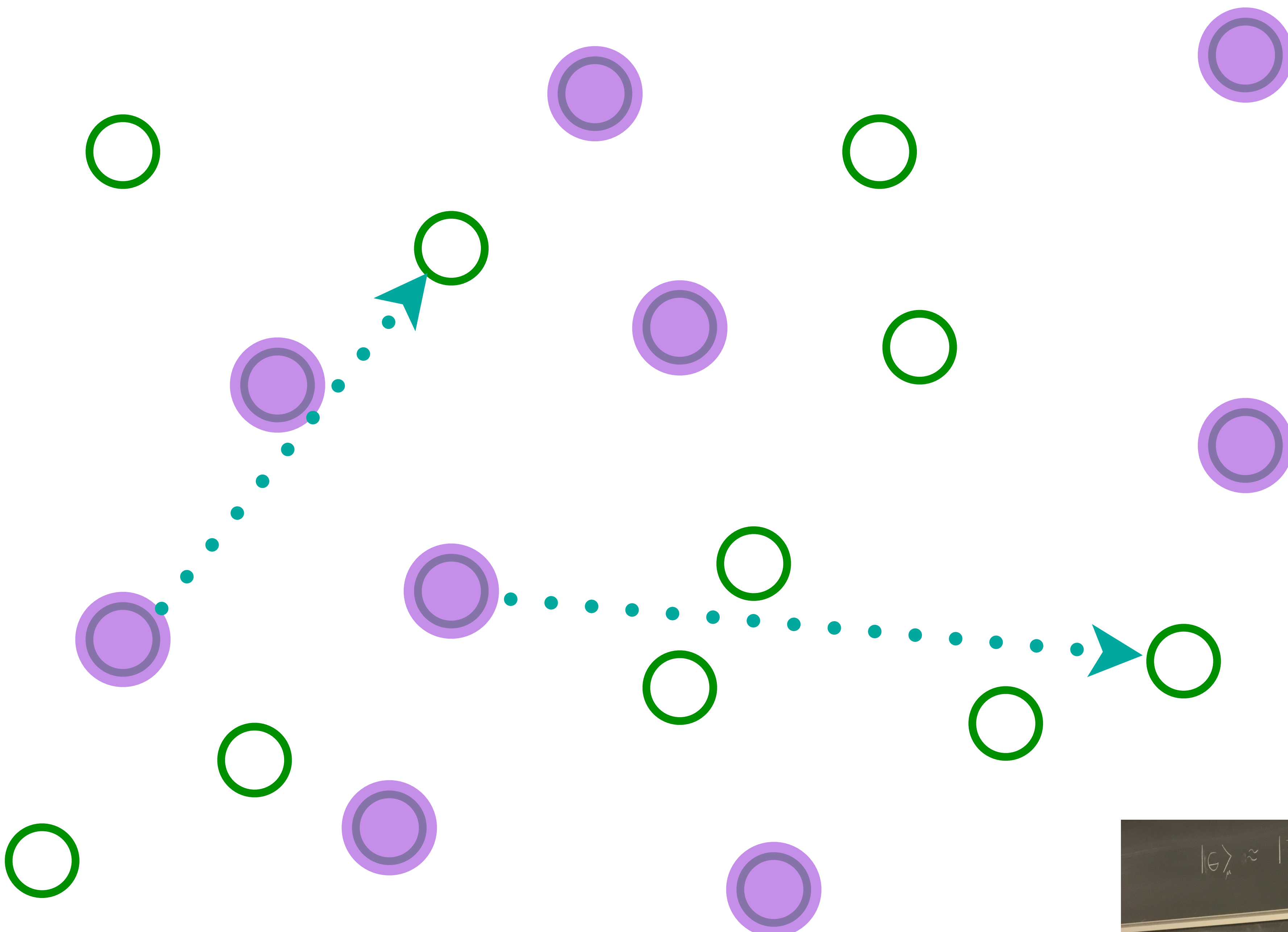

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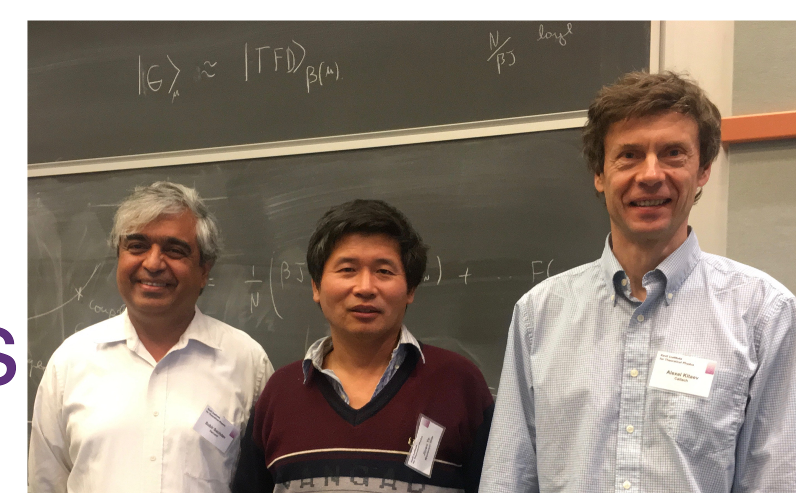

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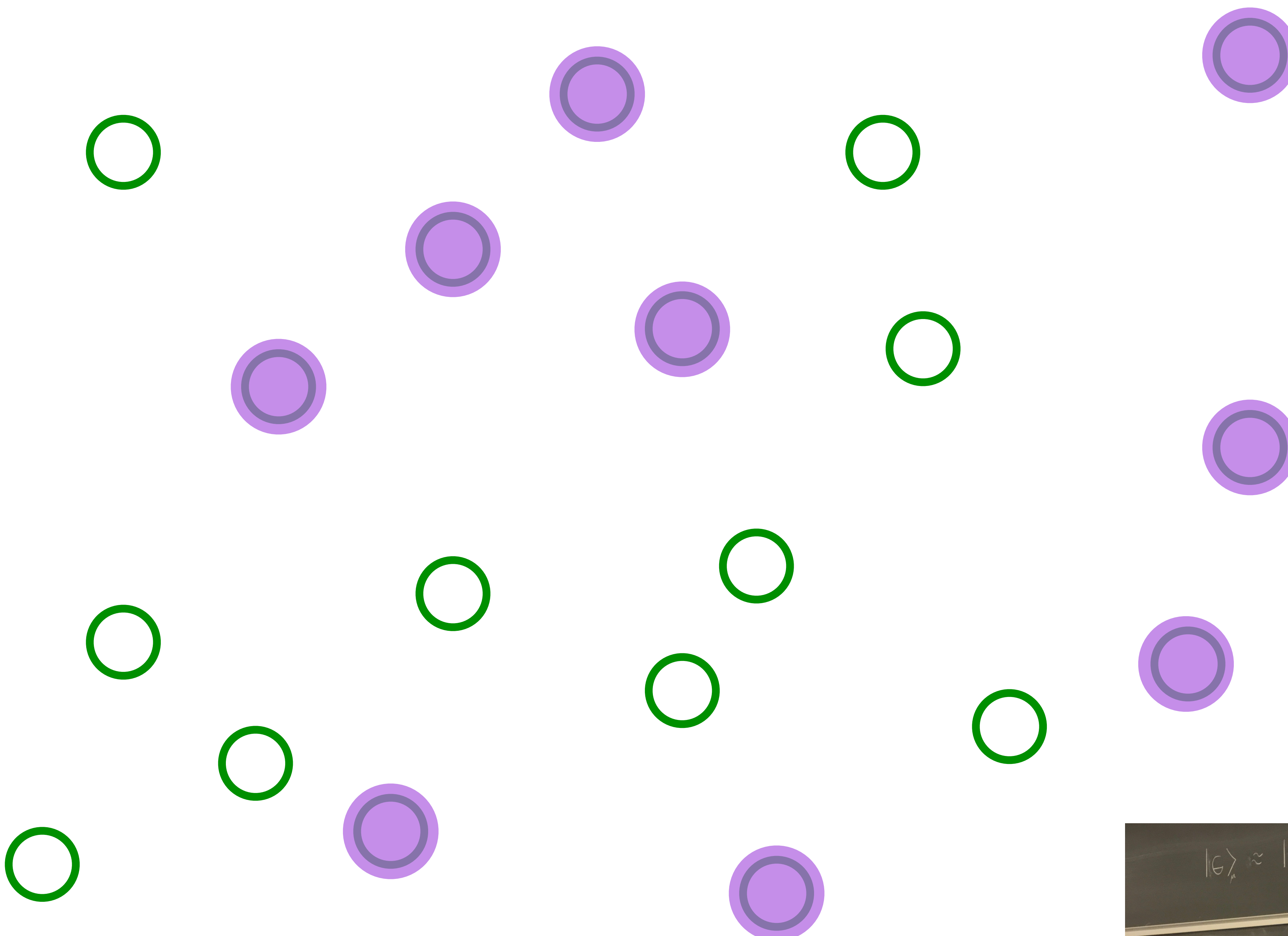
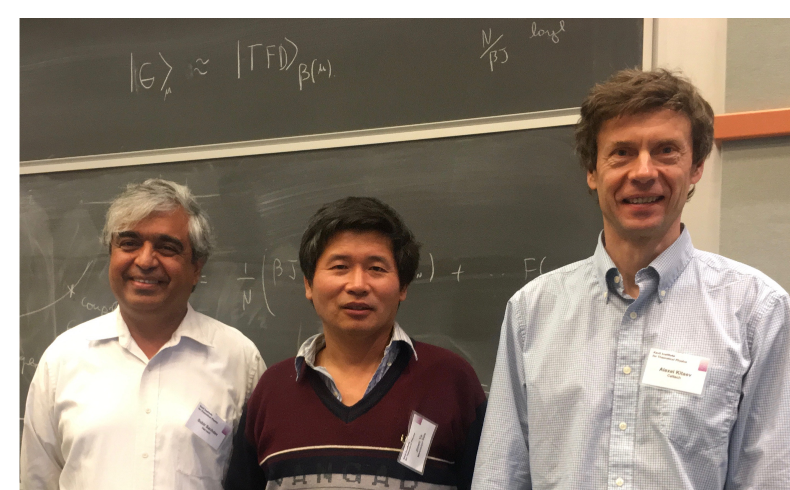


Entangle electrons pairwise randomly



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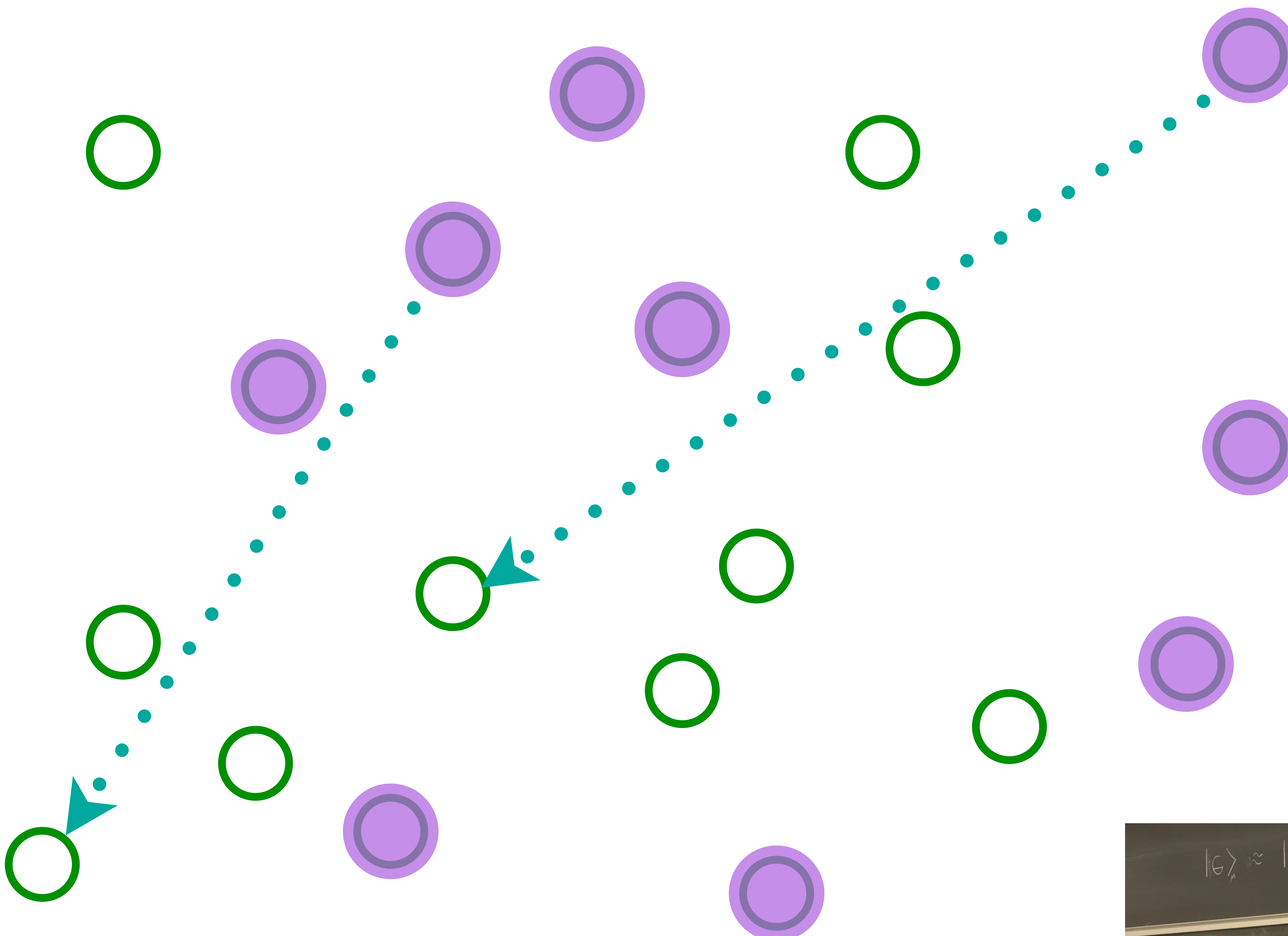

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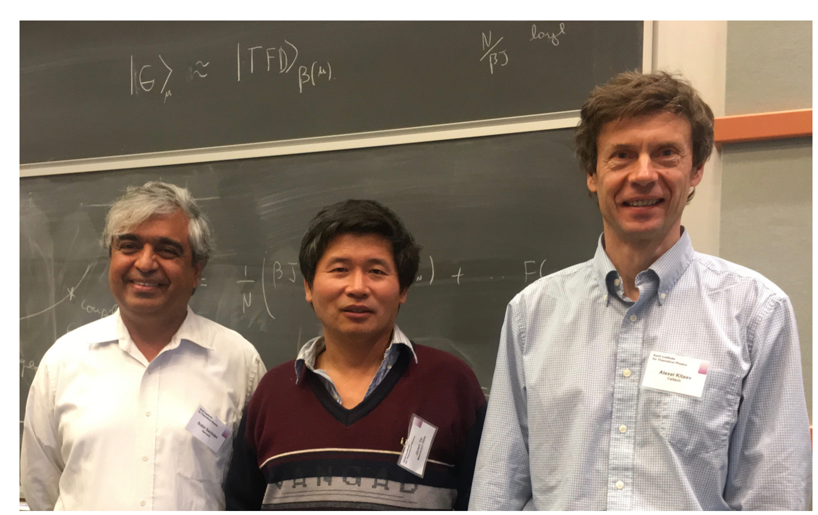

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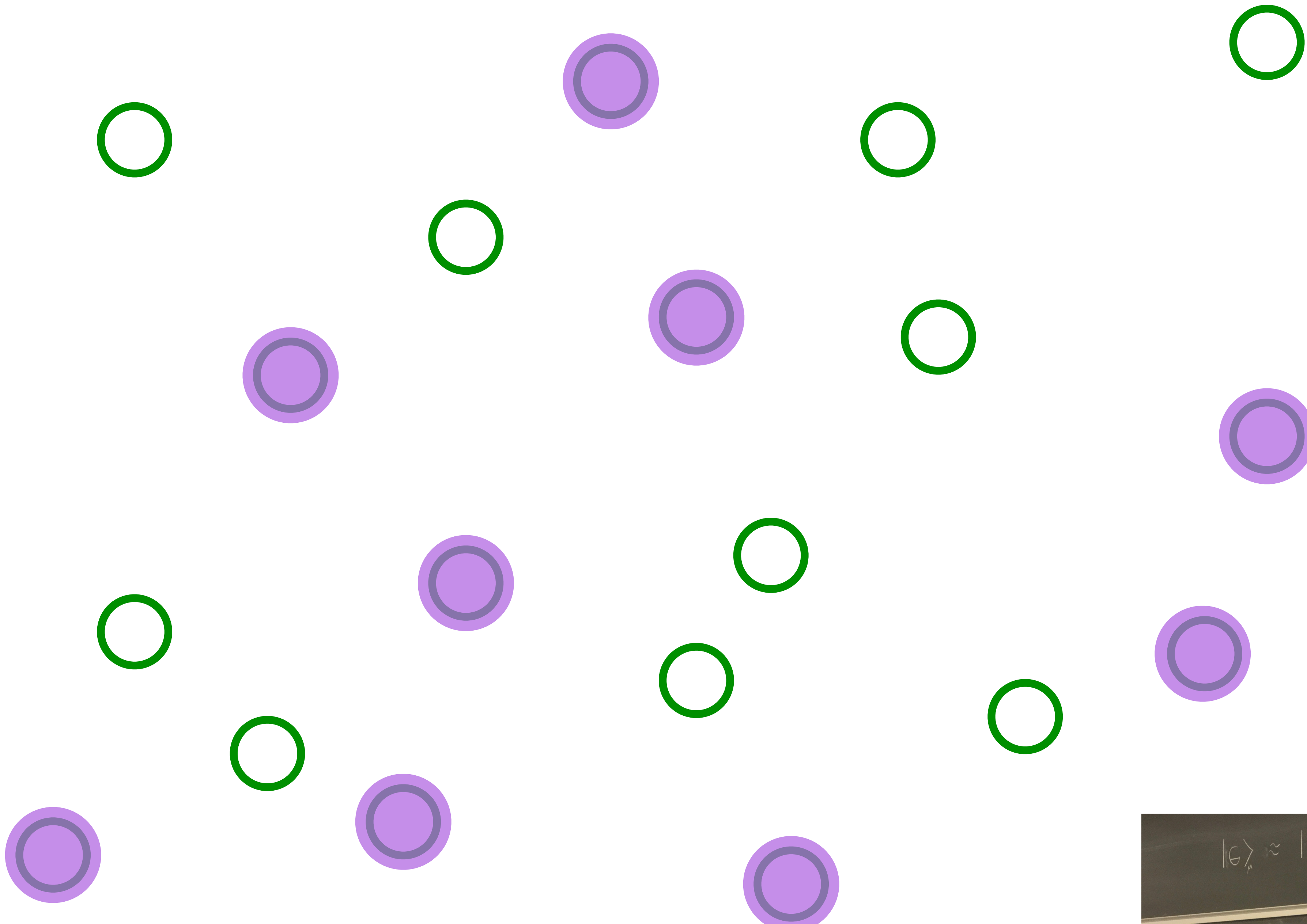

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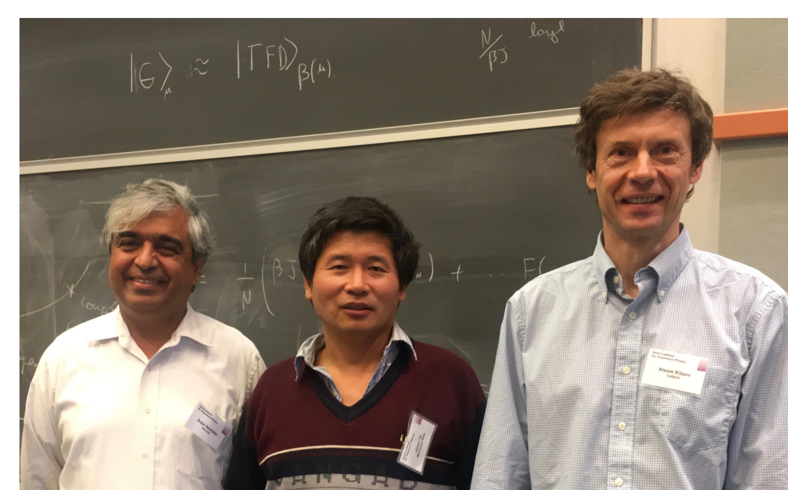

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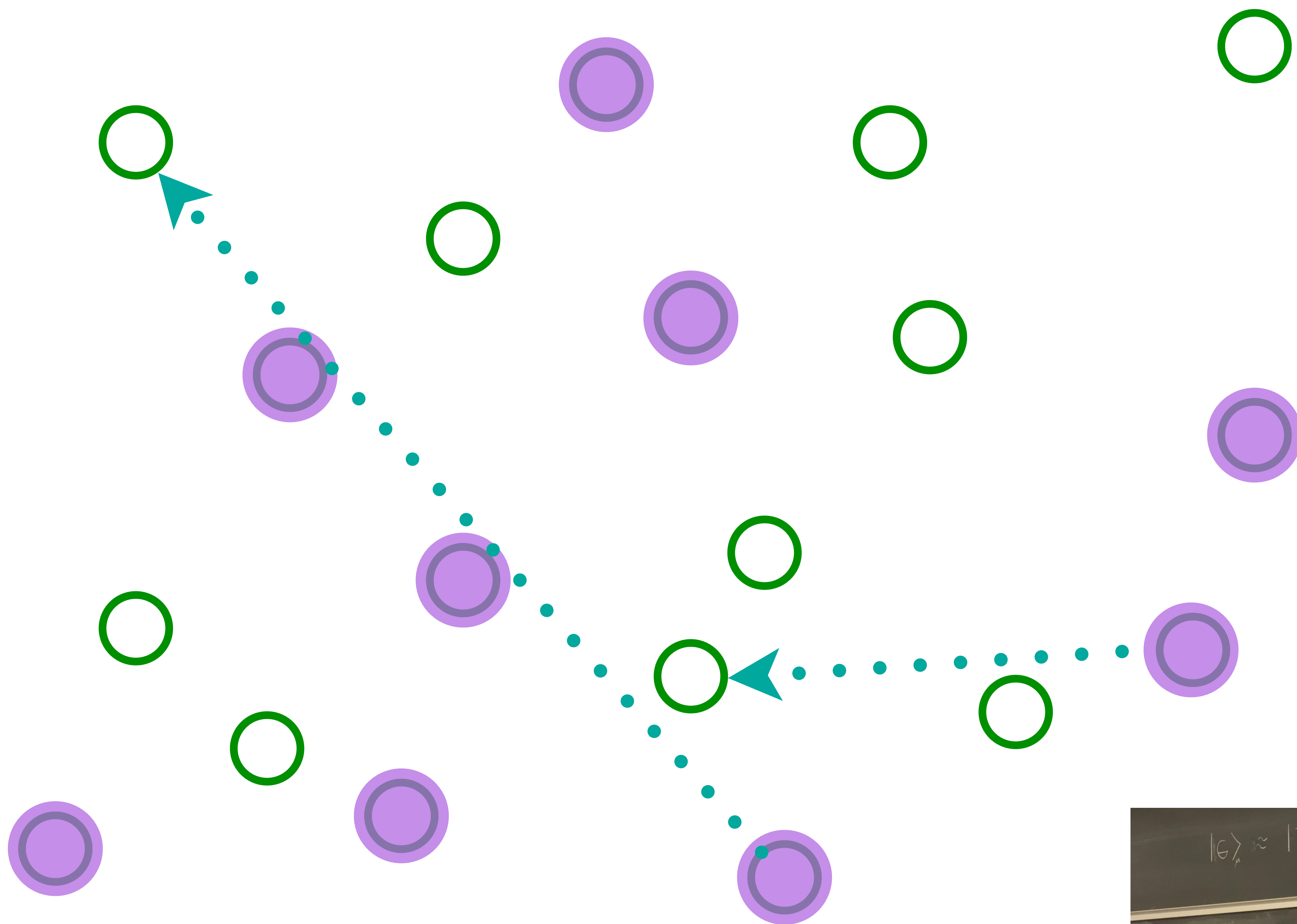

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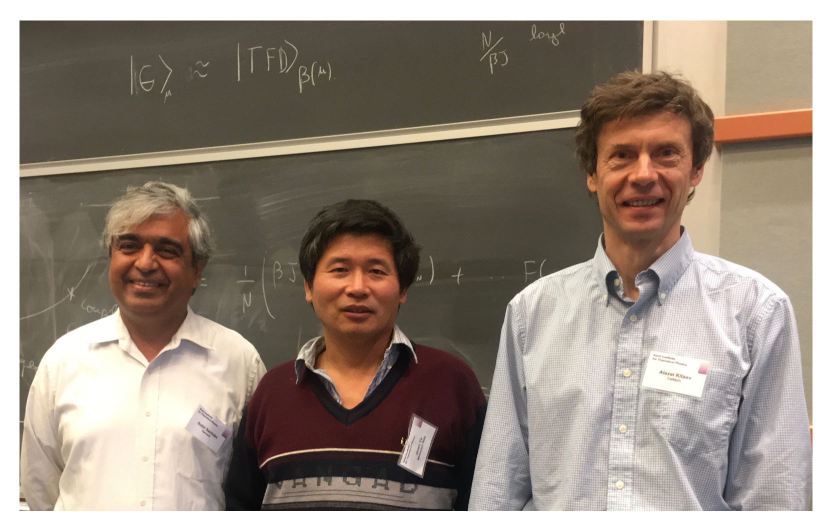

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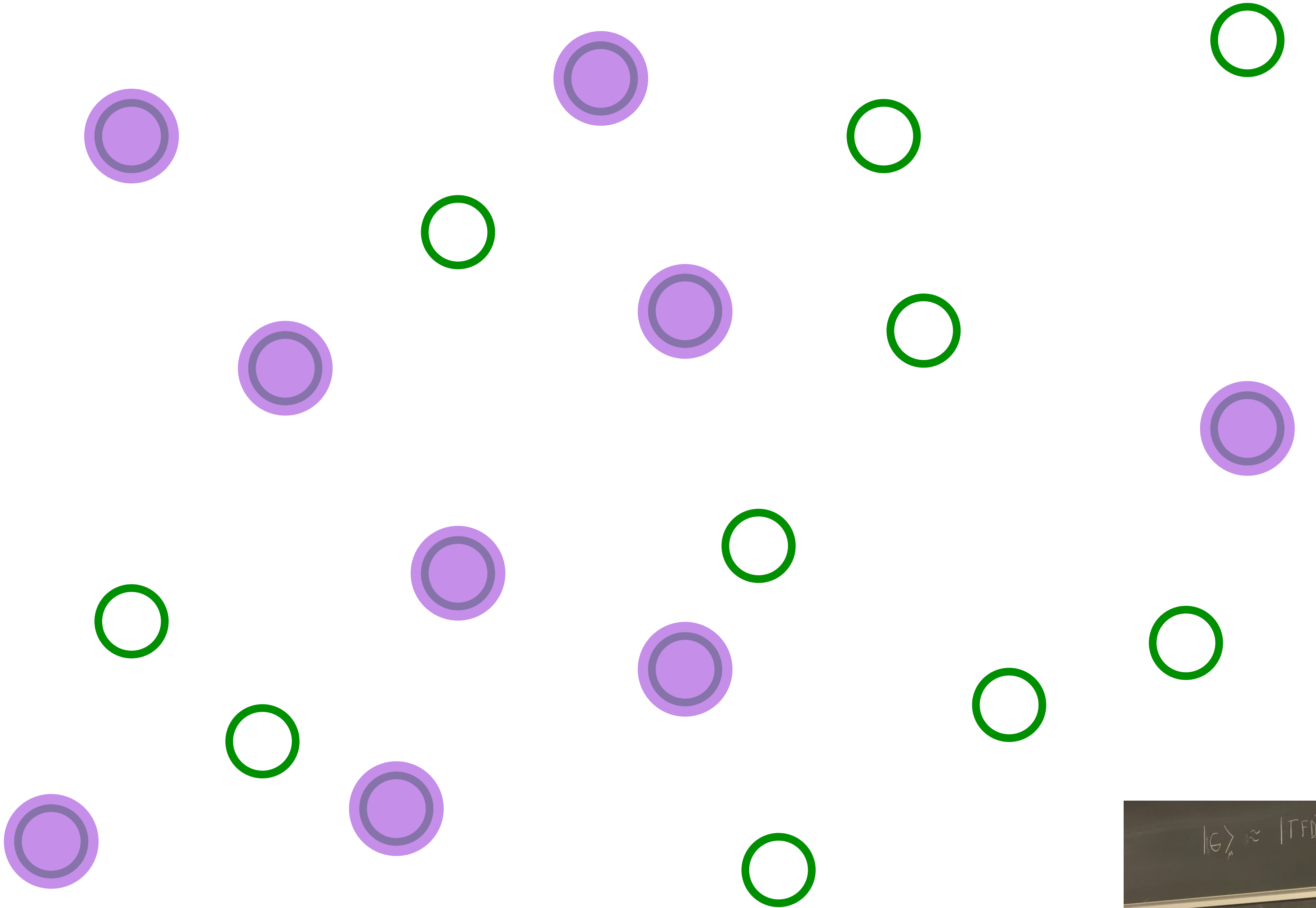

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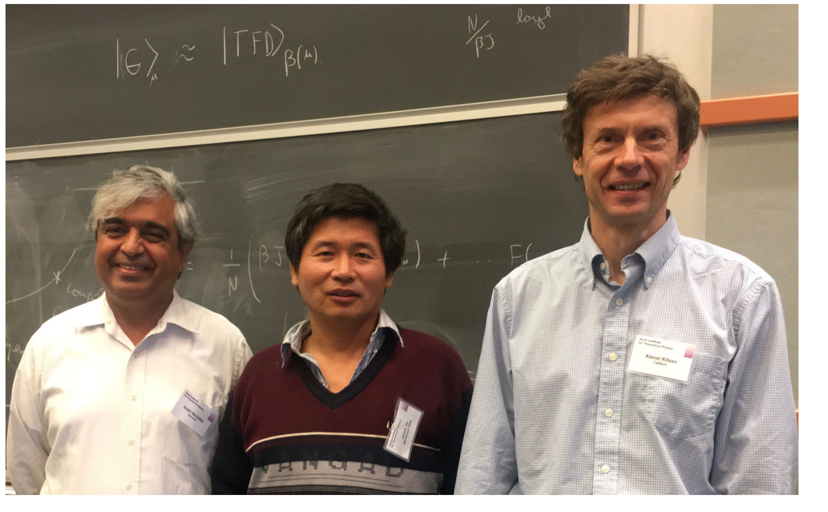

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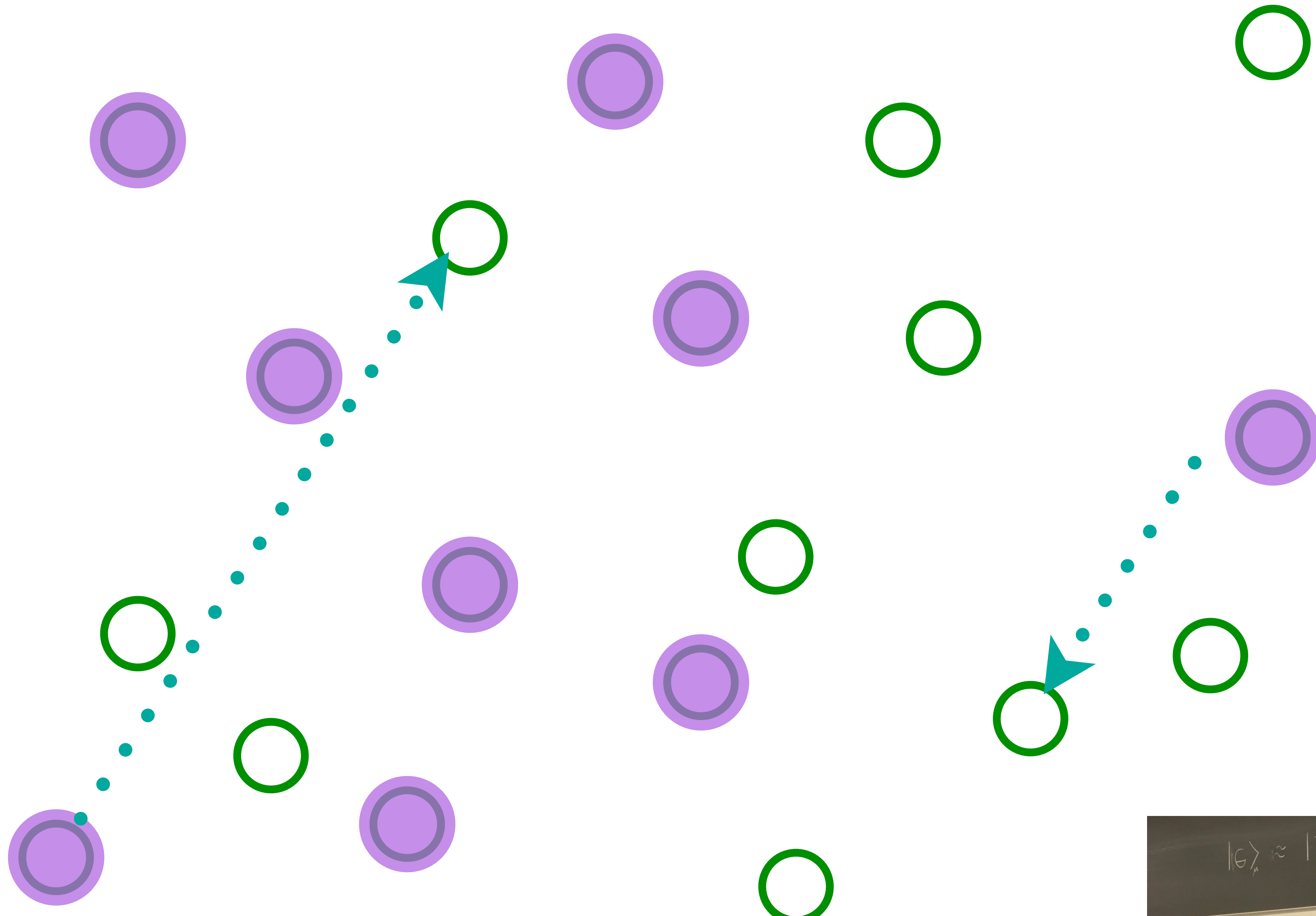

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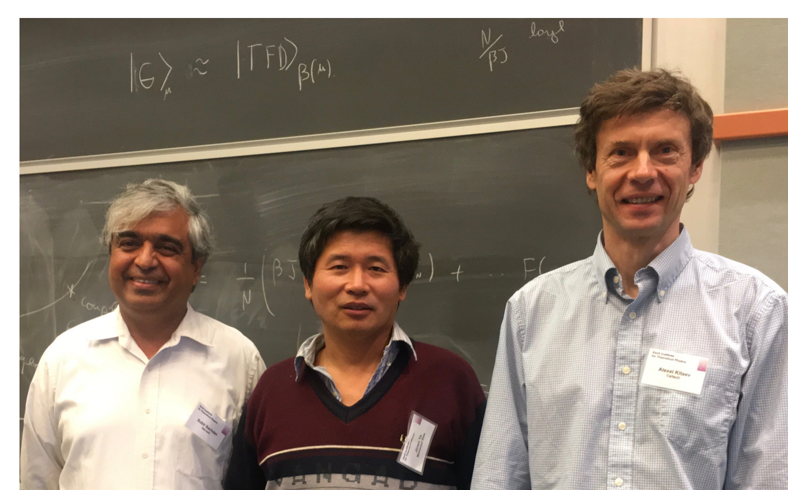

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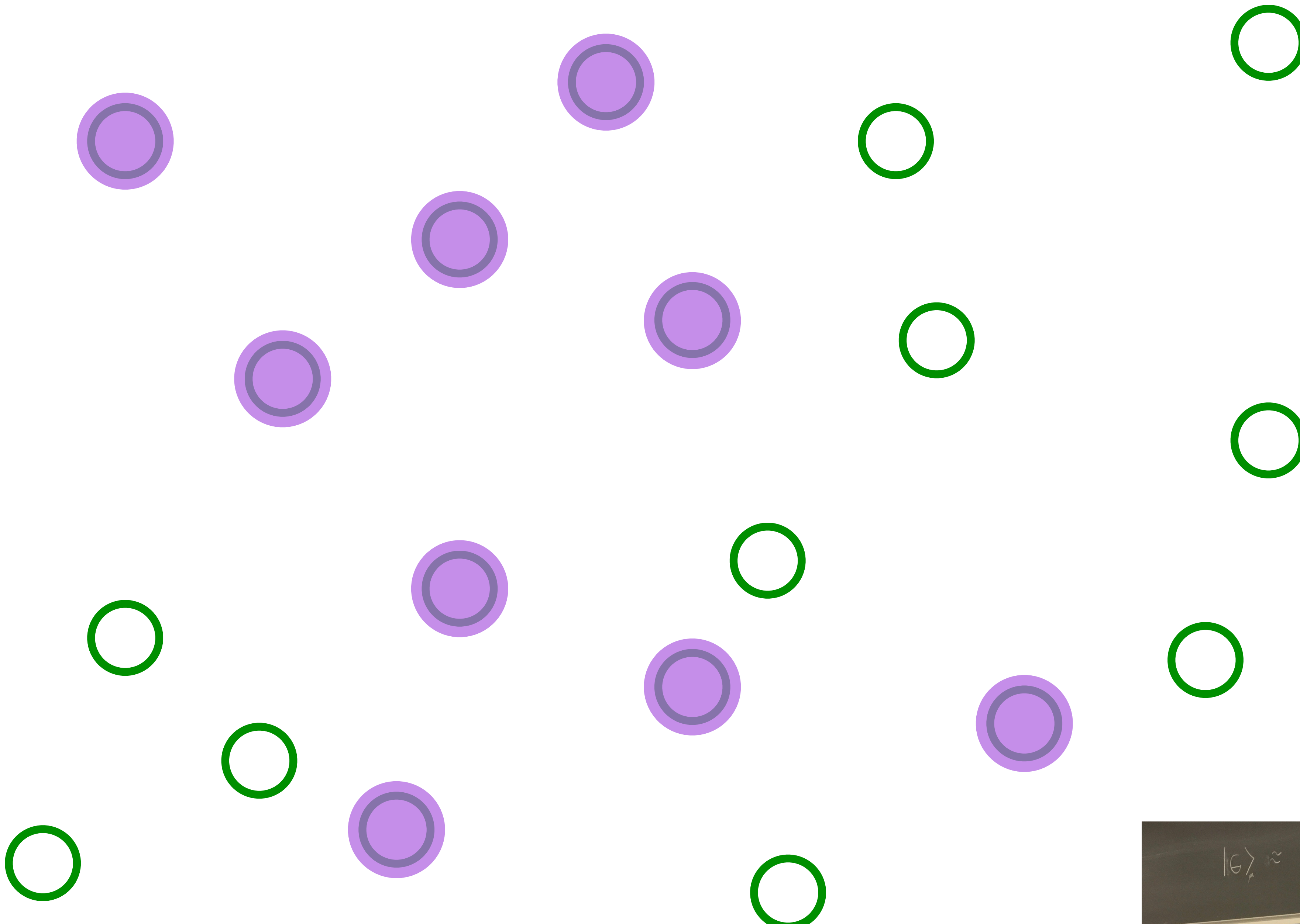

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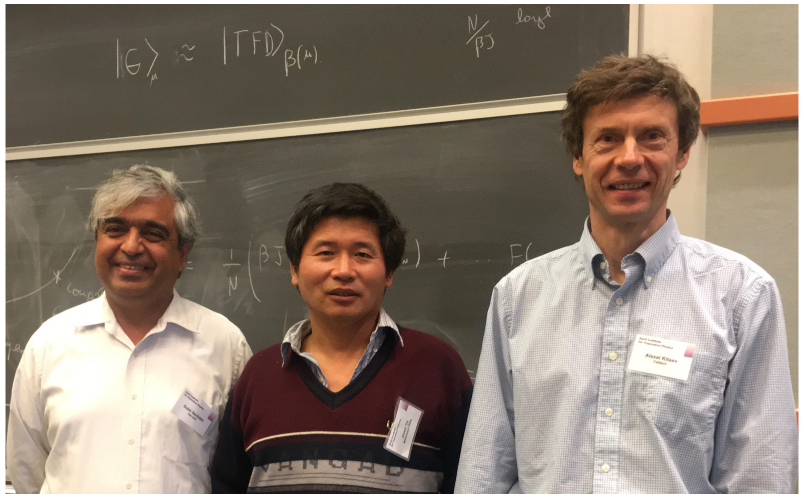

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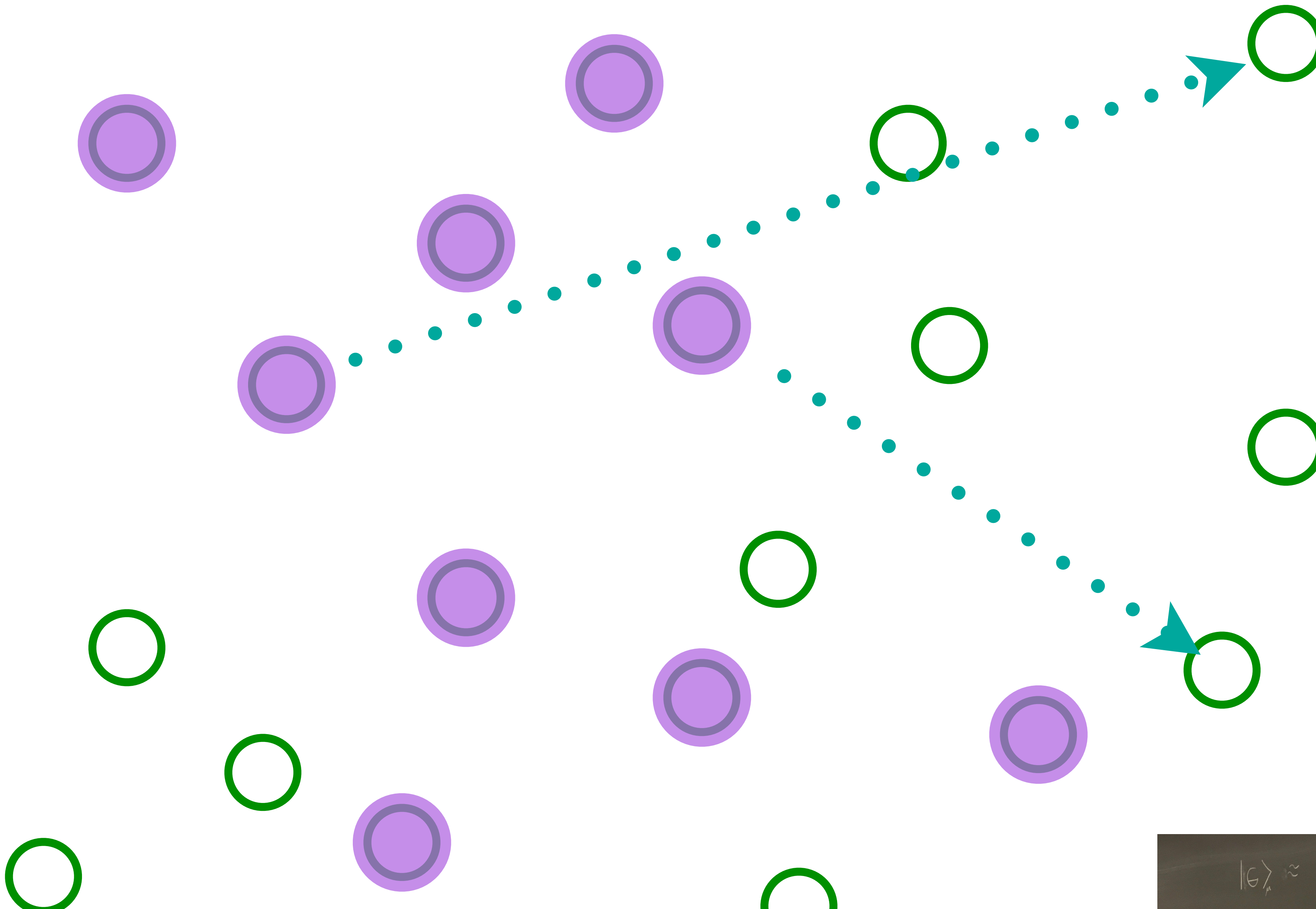

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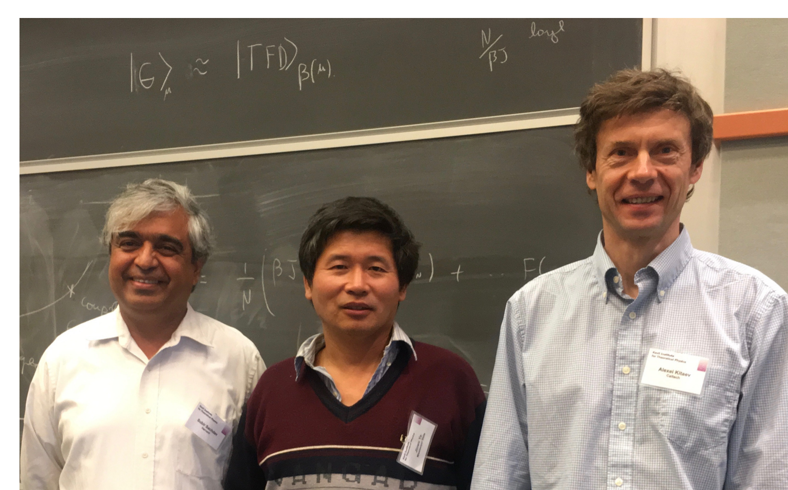

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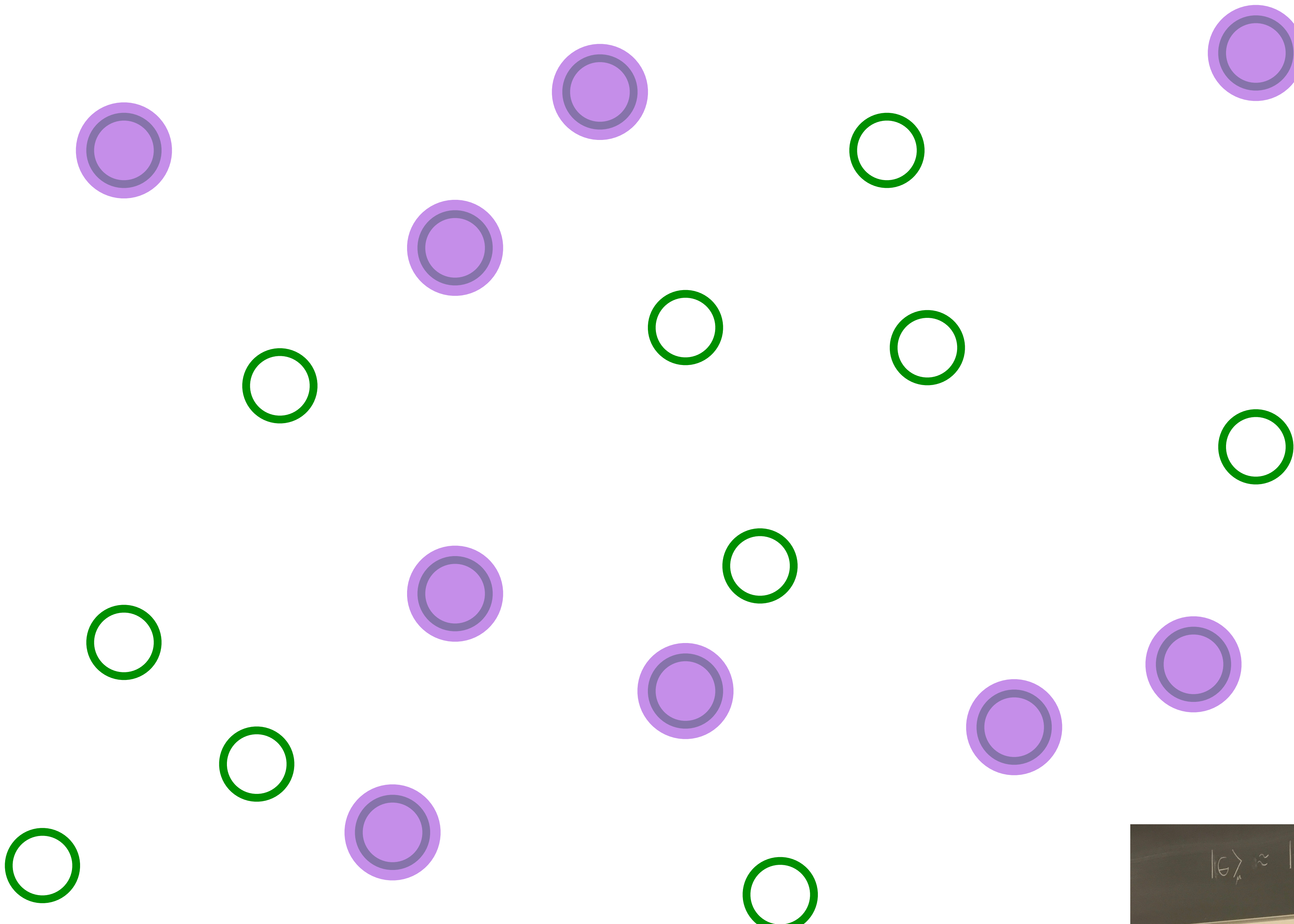

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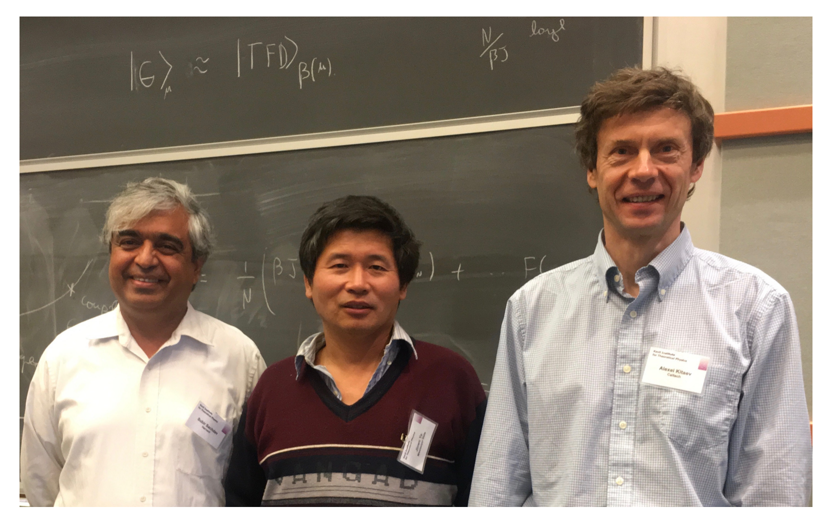

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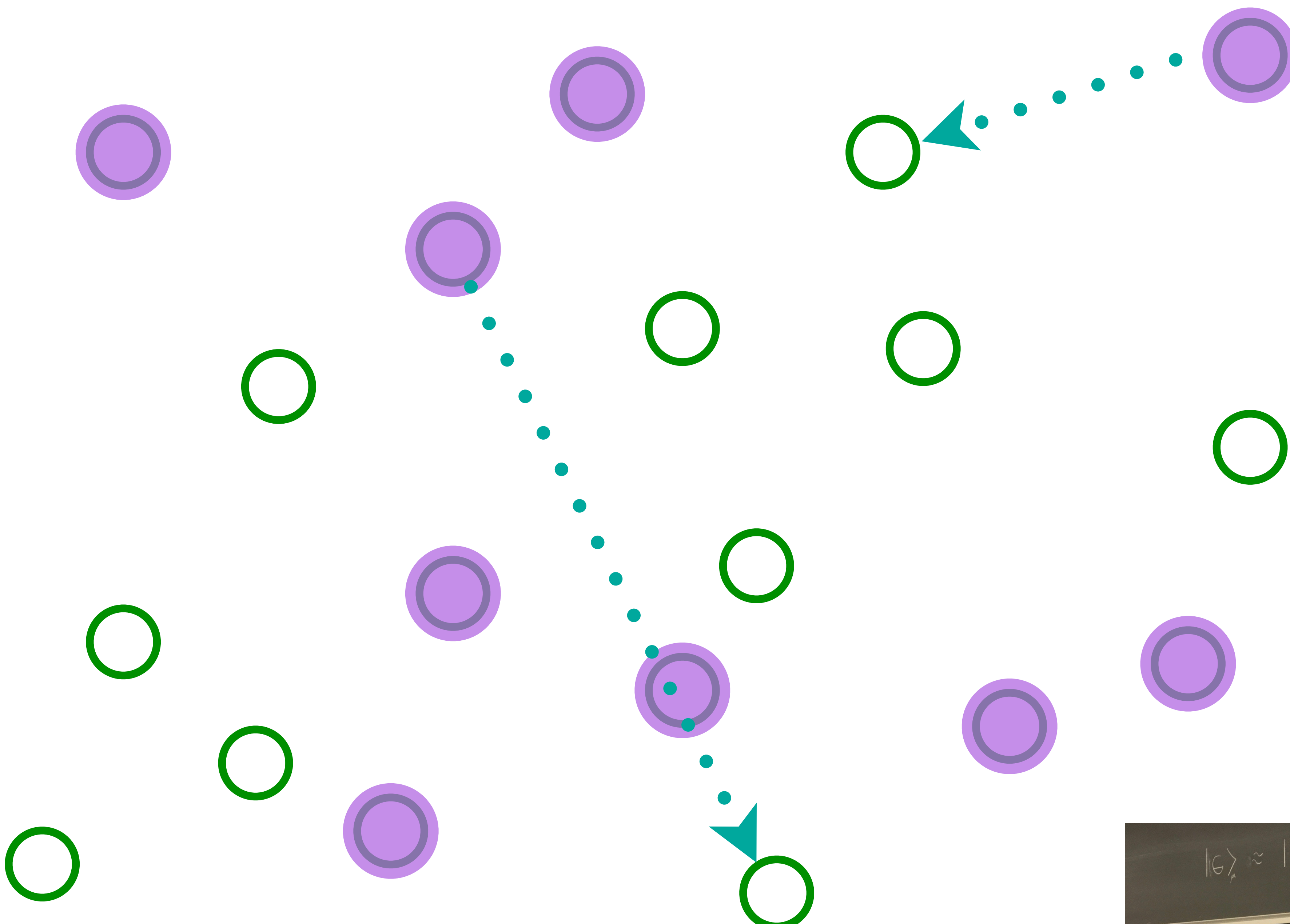

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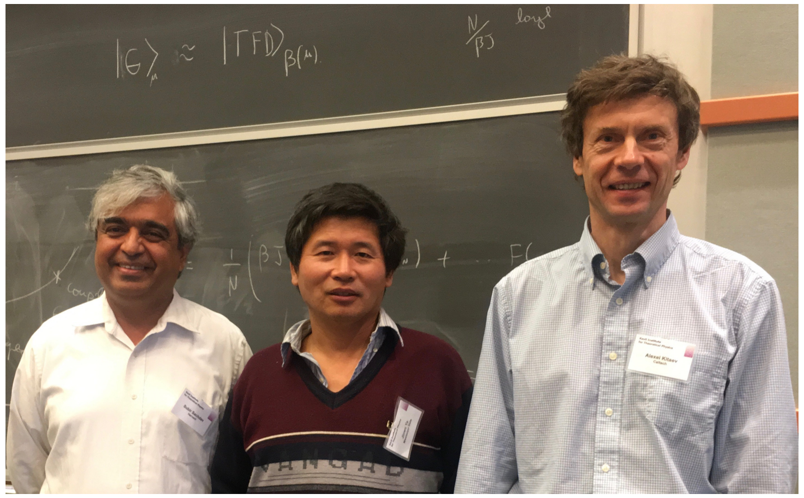

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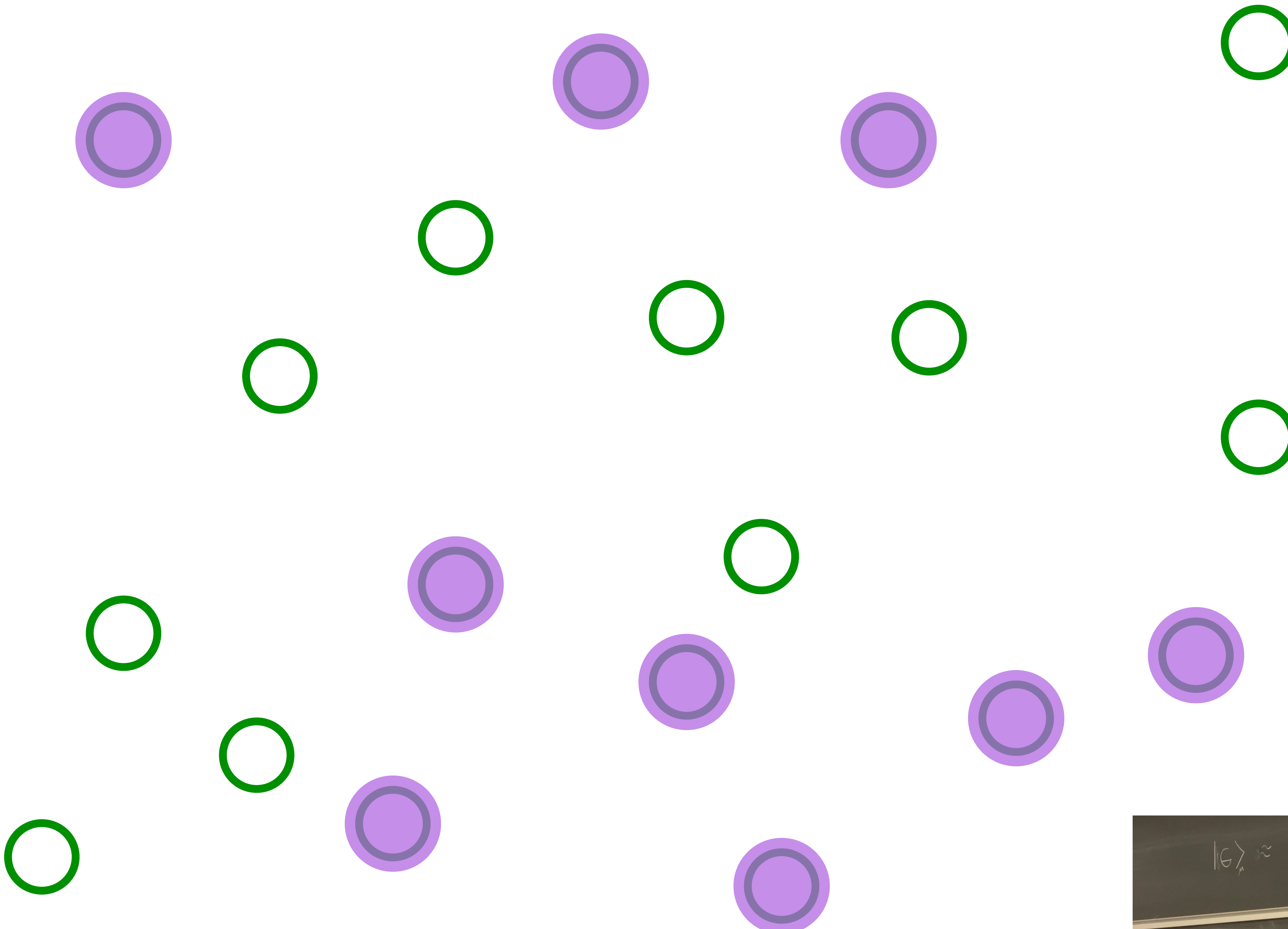

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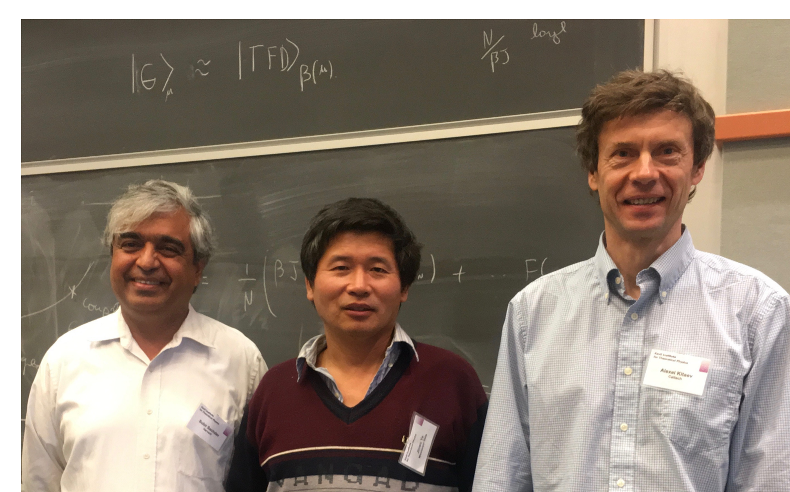


The SYK model

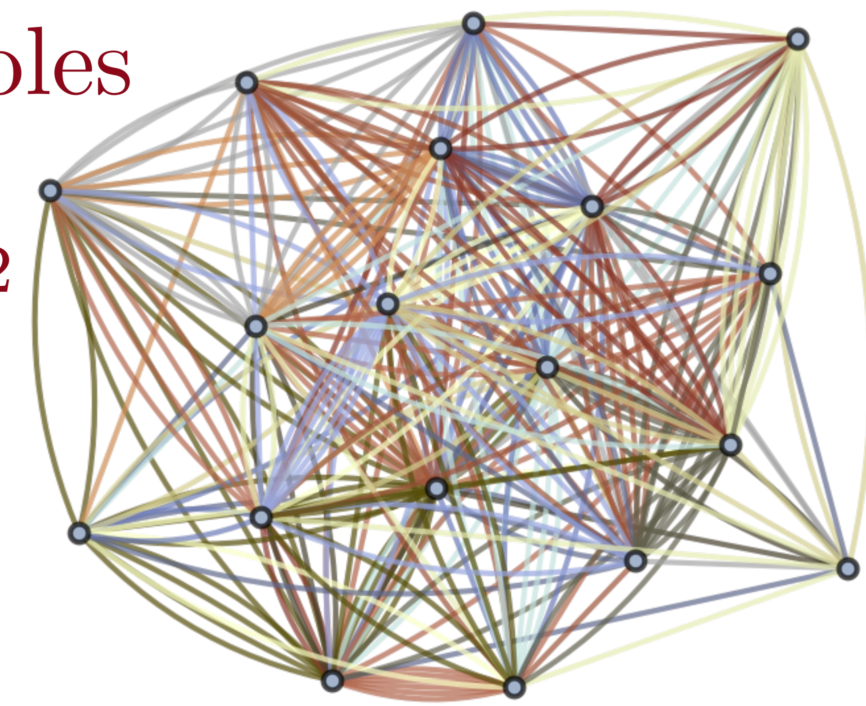
$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta - \mu \sum_\alpha c^\dagger_\alpha c_\alpha$$

$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c^\dagger_\beta + c^\dagger_\beta c_\alpha = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c^\dagger_\alpha c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables

with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The complex SYK model

Key properties

1. There is a quantum critical state, without quasiparticle excitations, for a range of charge densities around $\mathcal{Q} = 1/2$.

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PRL **70**, 3339 (1993)

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2. The imaginary time fermion Green's function has conformal invariance (SL(2,R) symmetry) for $T, 1/\tau \ll U$.

$$G(\tau) \sim e^{-2\pi\mathcal{E}T\tau} \left(\frac{T}{\sin(\pi T \tau)} \right)^{1/2} \quad , \quad 0 < \tau < 1/T$$

S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)
Yingfei Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, JHEP 02 (2020) 157

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3. There is a non-zero extensive entropy as $T \to 0$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

$$\lim_{T \to 0} \lim_{N \to \infty} \frac{S}{N} = S_0(\mathcal{Q}) \neq 0$$

This entropy is *not* due to an exponentially large ground degeneracy. Instead, it reflects an exponentially small many-body level spacing $\sim e^{-NS_0}$ down to the ground state.

The complex SYK model

Key properties

4. Thermal equilibration in a 'Planckian time' $\sim \hbar/(k_B T)$

A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, PRB **96**, 205123 (2017)

The complex SYK model

Key properties

4. Thermal equilibration in a 'Planckian time' $\sim \hbar/(k_B T)$

5. Maximal quantum Lyapunov exponent for the out-of-time-order correlator (OTOC):

$$\left\langle c_\alpha^\dagger(t) c_\beta(0) c_\alpha(t) c_\beta^\dagger(0) \right\rangle = C_0 + C_1 \left(\frac{e^{\lambda t}}{N} \right) + \ldots$$

with $\lambda = 2\pi k_B T/\hbar$.

A. Kitaev, KITP talk (2015)

J. Maldacena and D. Stanford, PRD **94**, 106002 (2016)

The complex SYK model

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with $\lambda = 2\pi k_B T/\hbar$.

6. For spinful fermions, spin correlations decay as

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Key properties

All of these properties (very likely) apply also to

- Metal-insulator transition in a random Hubbard model with random hopping and exchange

- Metal-metal transition in a random t-J model at a critical doping

- Extremal charged black holes

A strange metal: lattice of SYK islands

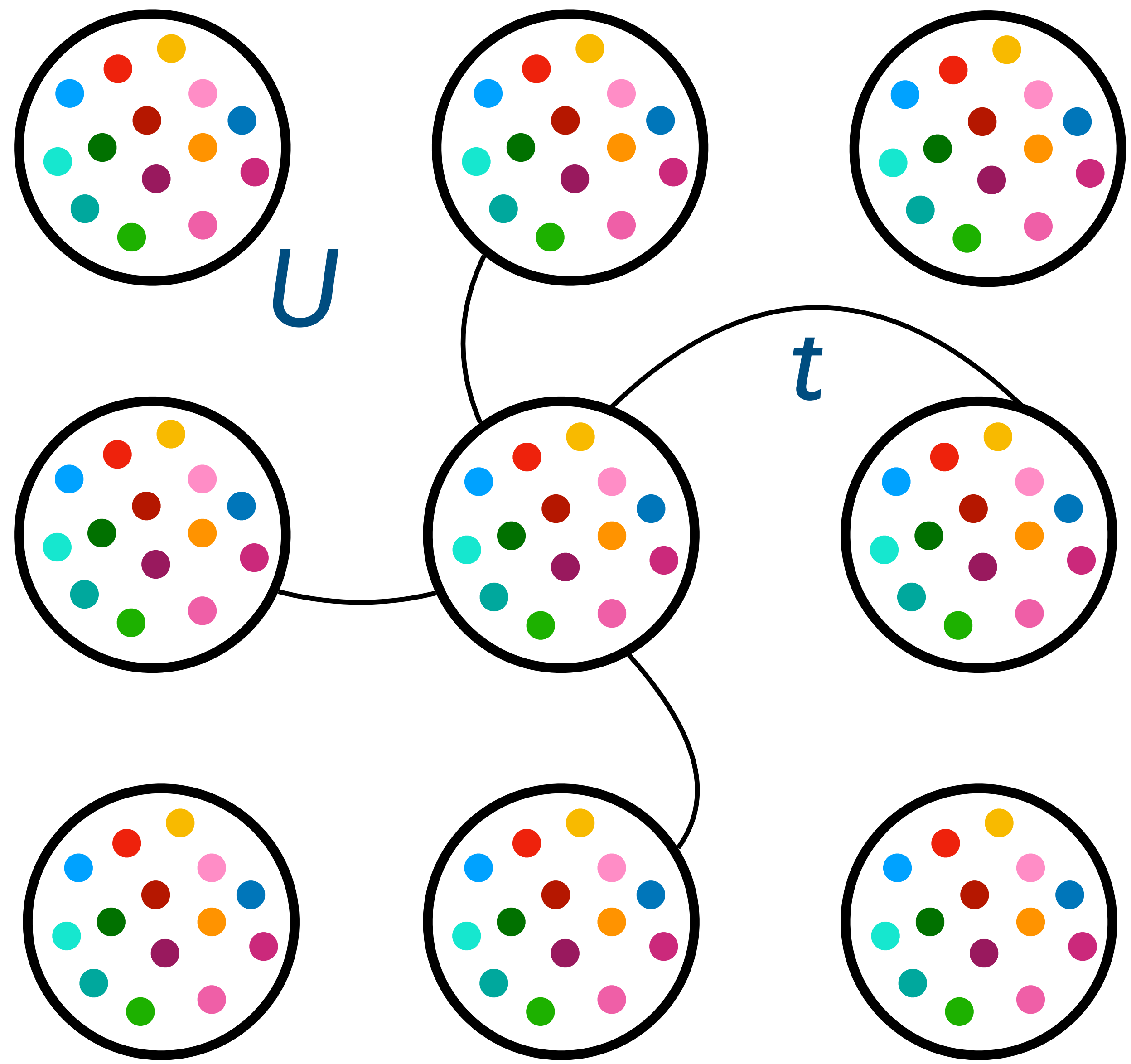
Random interaction within each island U.

Amplitude to hop between islands t.

Model yields SYK criticality and resistivity

$$\rho \sim T$$

for $t^2/U \lesssim T \lesssim U$



Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017); Pengfei Zhang, PRB **96**, 205138 (2017); Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018); Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999)

A strange metal: lattice of SYK islands

- Linear-in-T resistivity only for $T > t^2/U$.
- Insensitive to electron density; no critical density
- No pseudogap phase.
- No 'Mottness': on-site Hubbard U_H is missing.

1. SYK criticality

2. Deconfined quantum criticality of random t-J-U_H models

3. Black holes

Random t-J-U_H model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^\dagger_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

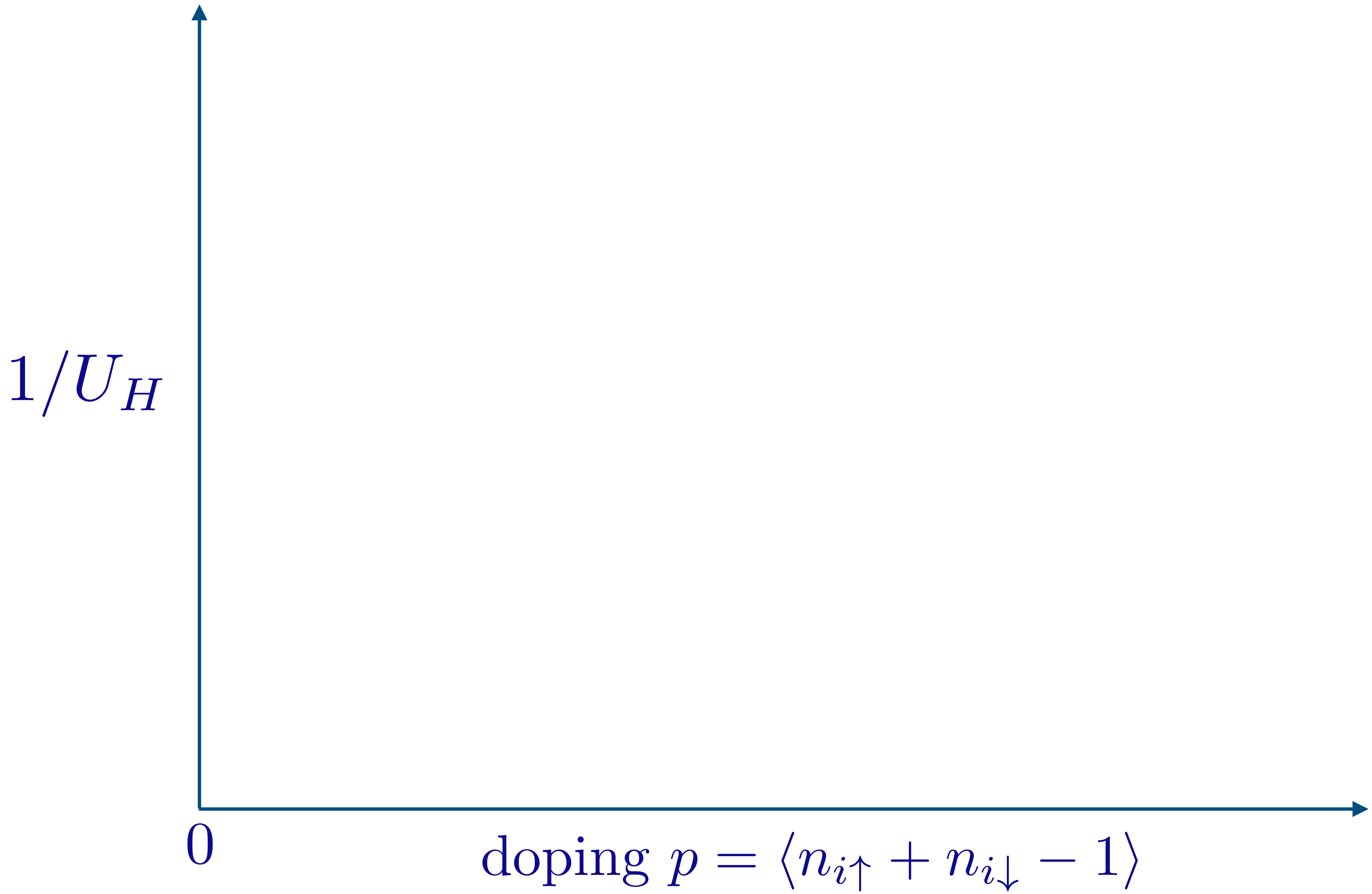
$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c^\dagger_{i\alpha} c_{i\alpha},$$

$$J_{ij} \text{ random}, \ \overline{J_{ij}} = 0, \ \overline{J_{ij}^2} = J^2$$
$$t_{ij} \text{ random}, \ \overline{t_{ij}} = 0, \ \overline{t_{ij}^2} = t^2$$

$$U_H > 0 \text{ non-random}$$

Random *t-J-U_H* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



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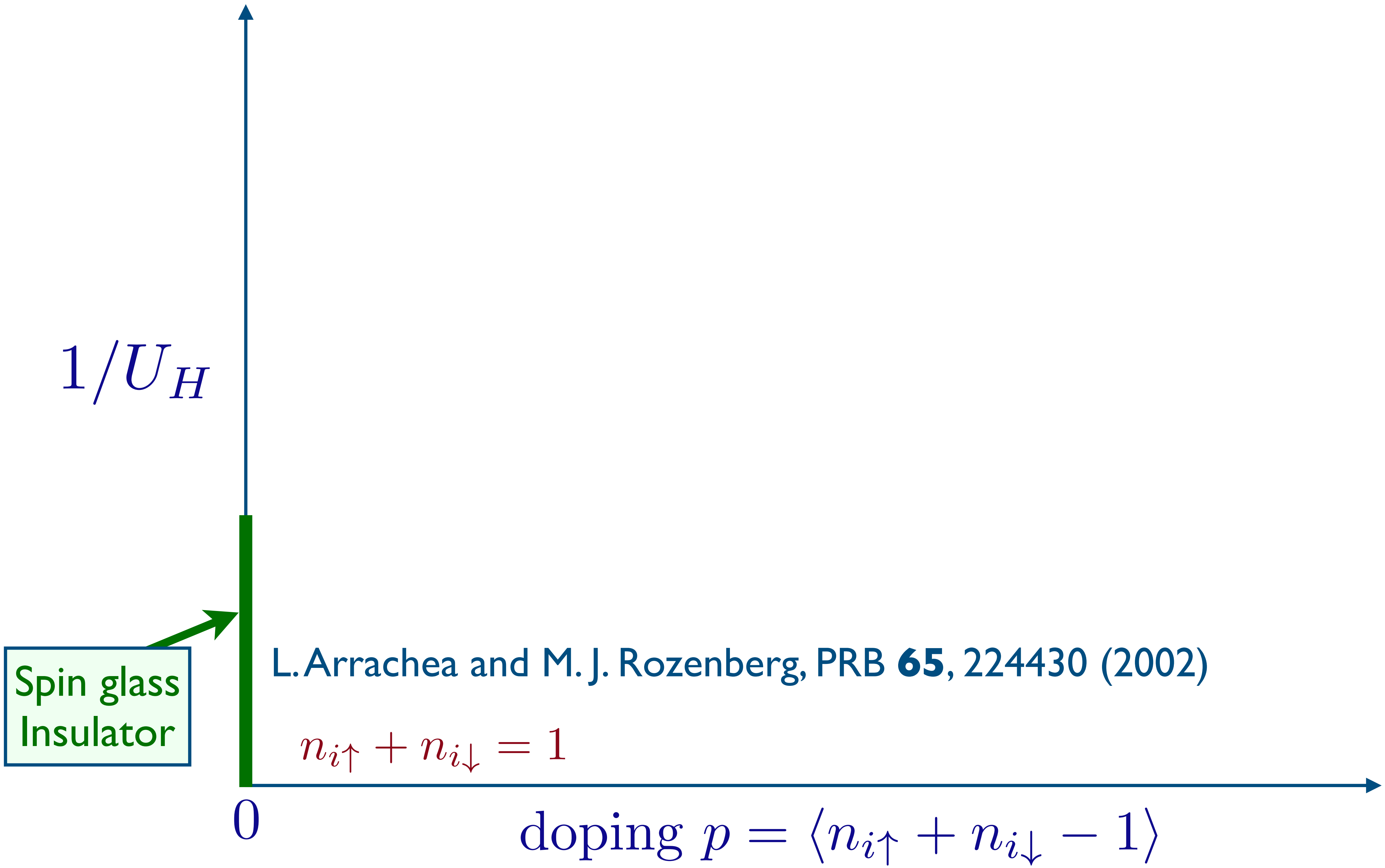

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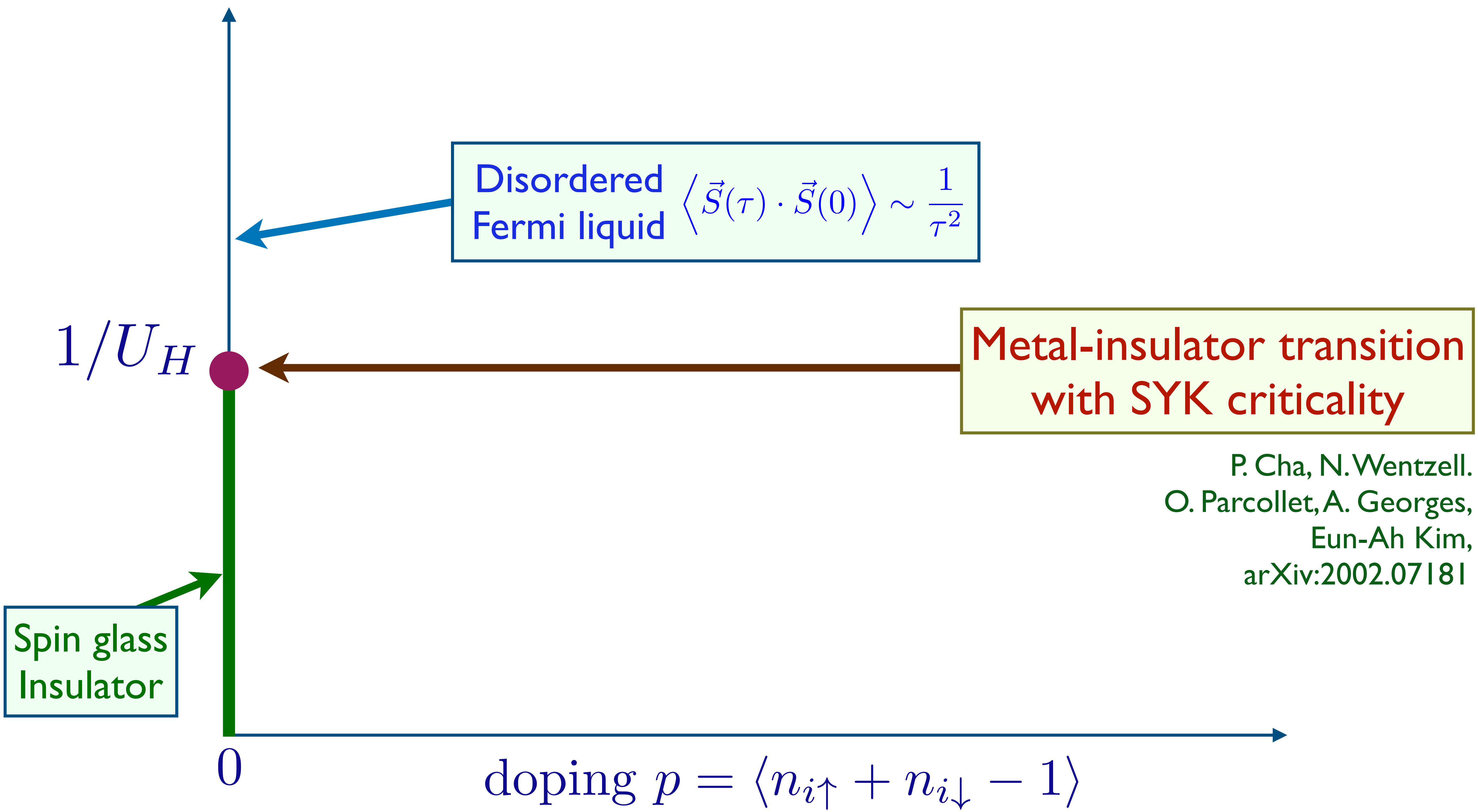
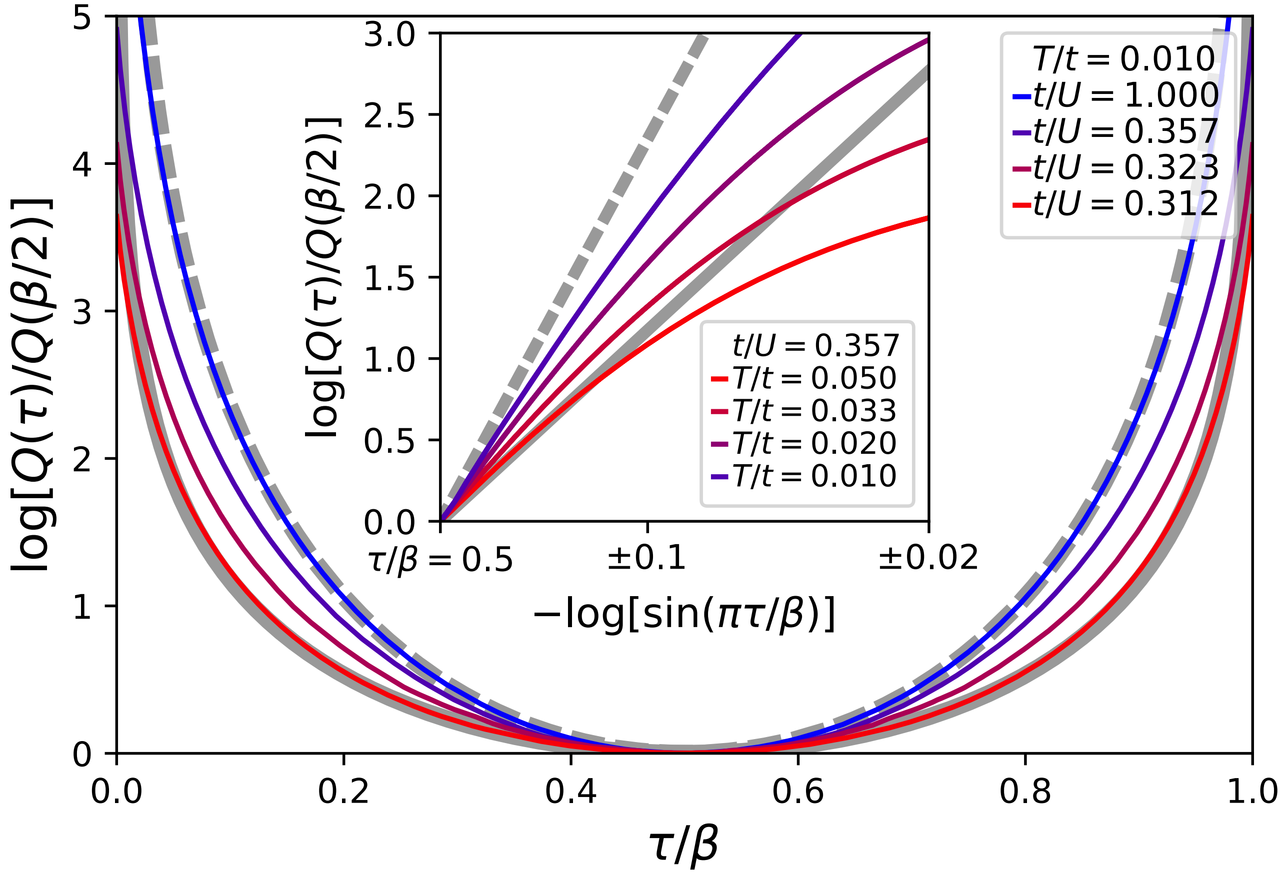
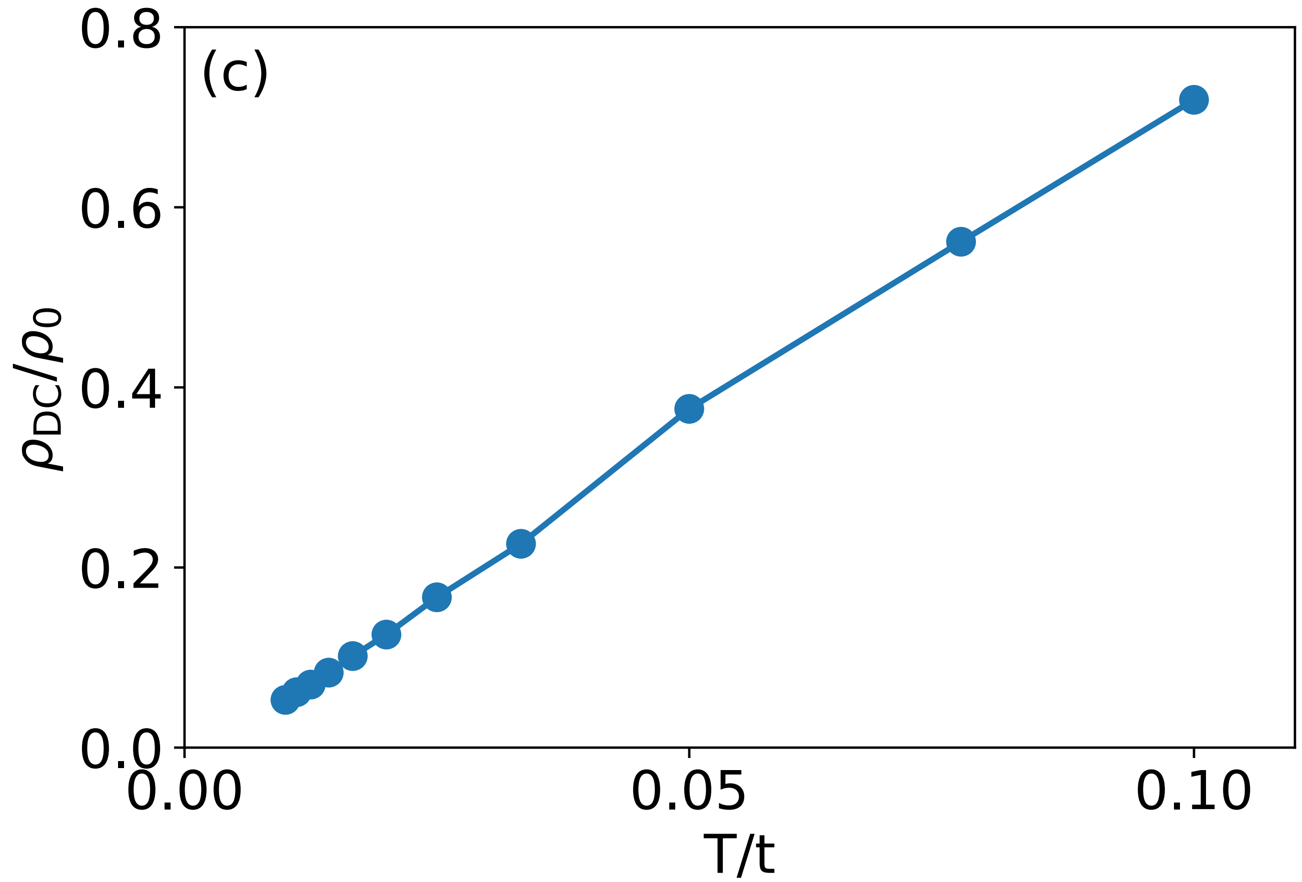


P. Cha, N. Wentzell.
O. Parcollet, A. Georges,
Eun-Ah Kim,
arXiv:2002.07181

Linear resistivity and Sachdev-Ye-Kitaev (SYK) spin liquid behavior in a quantum critical metal with spin-$1/2$ fermions

Peter Cha, Nils Wentzell, Olivier Parcollet, Antoine Georges, Eun-Ah Kim





arXiv.org > cond-mat > arXiv:2002.07181

Condensed Matter > Strongly Correlated Electrons

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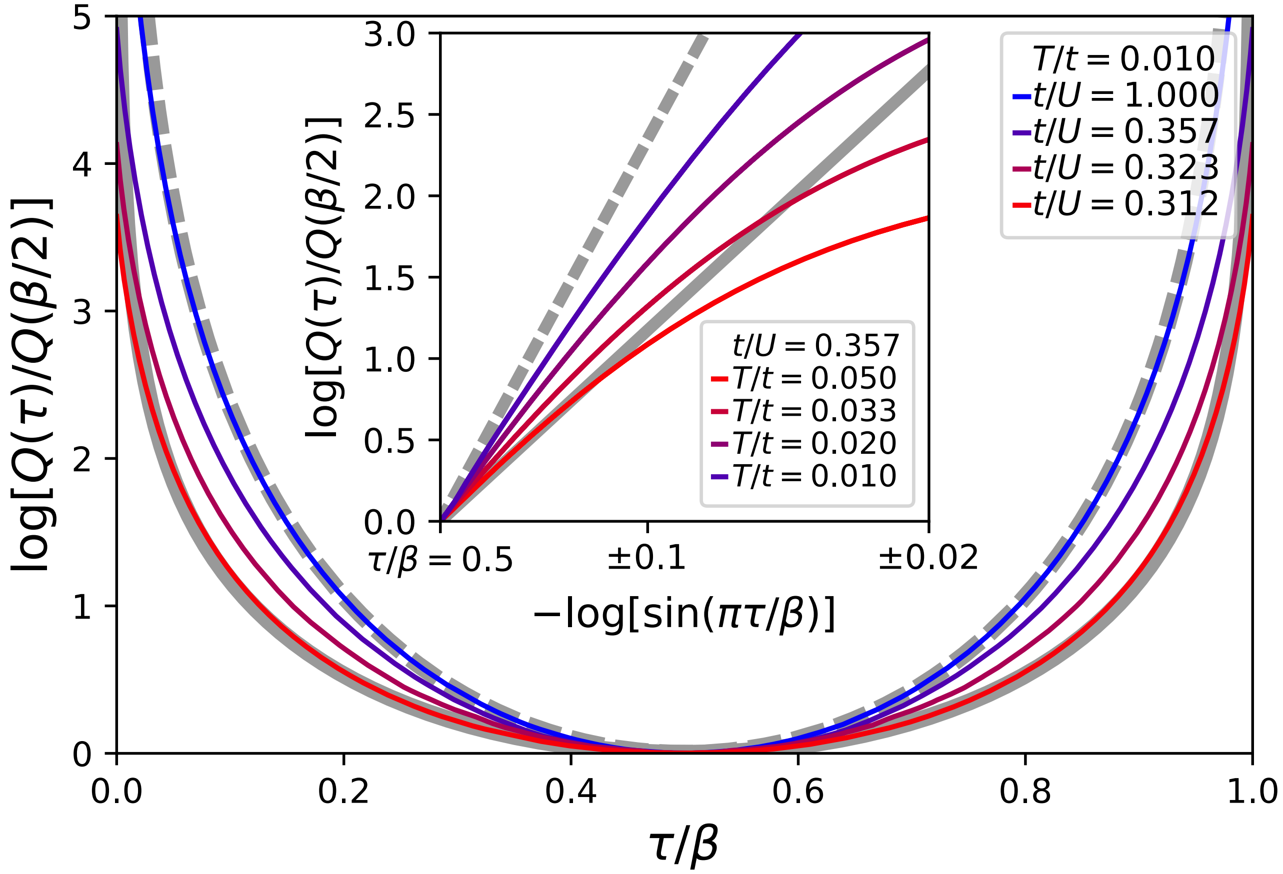

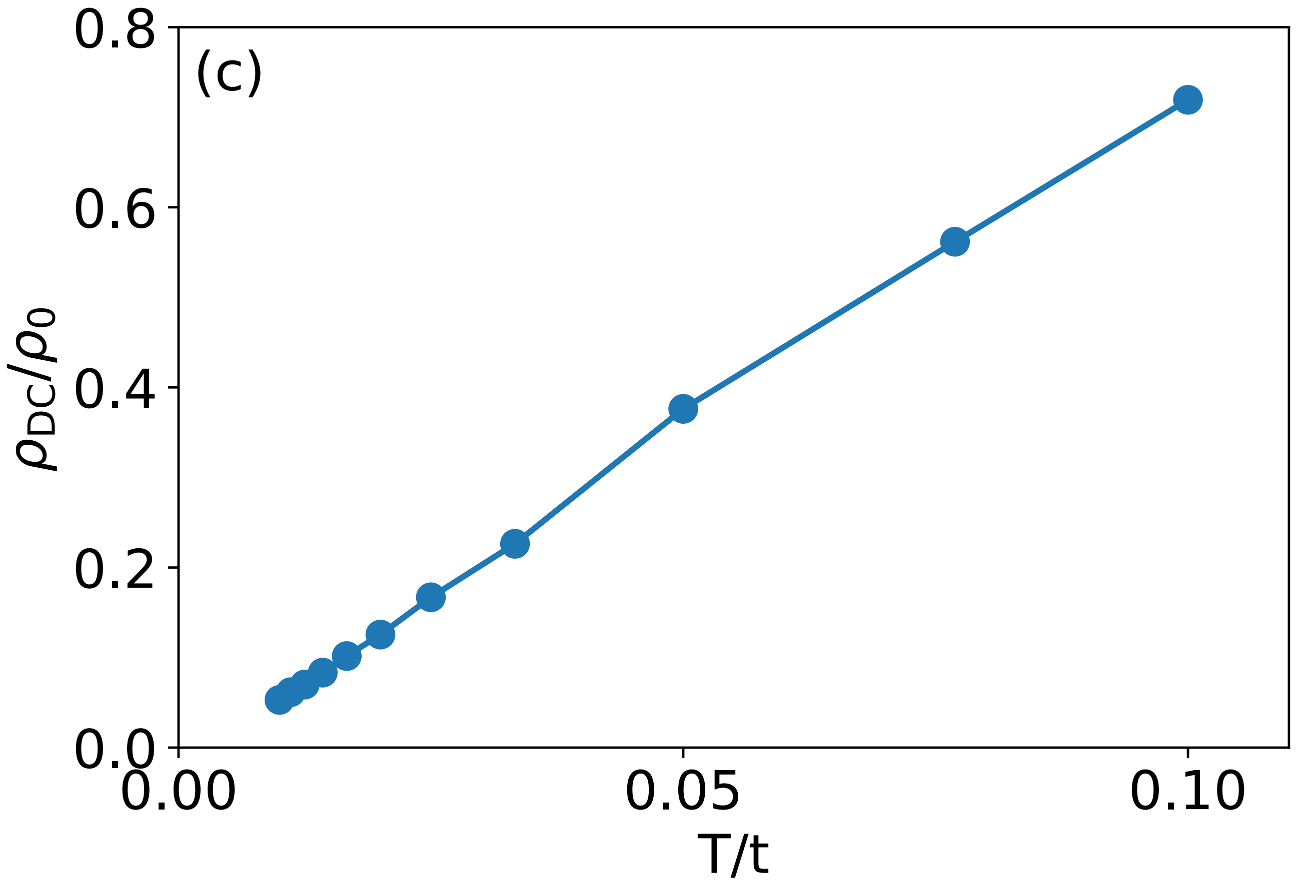

Critical spin correlations:

$$\left\langle \vec{S}(\tau)\cdot\vec{S}(0)\right\rangle \sim \frac{1}{|\tau|}$$

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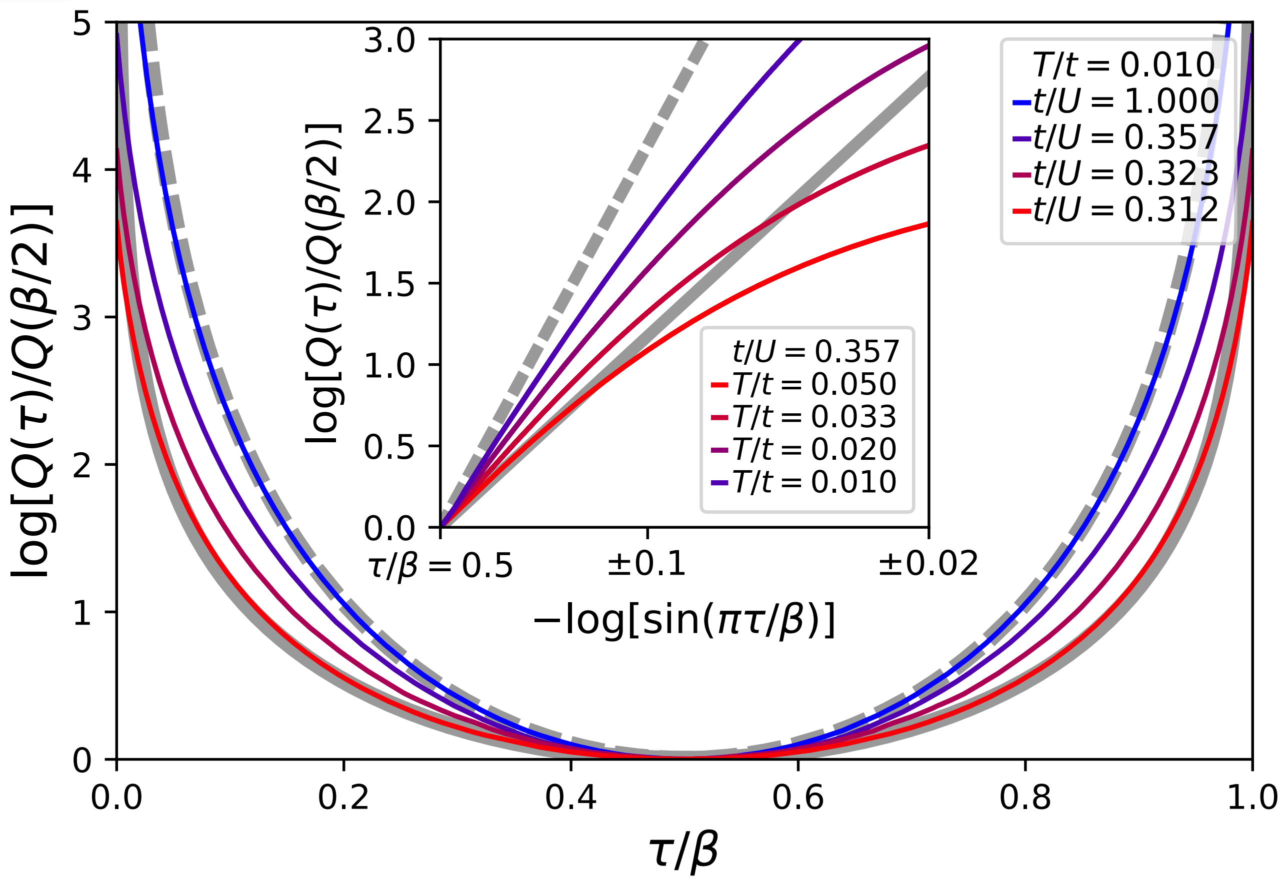

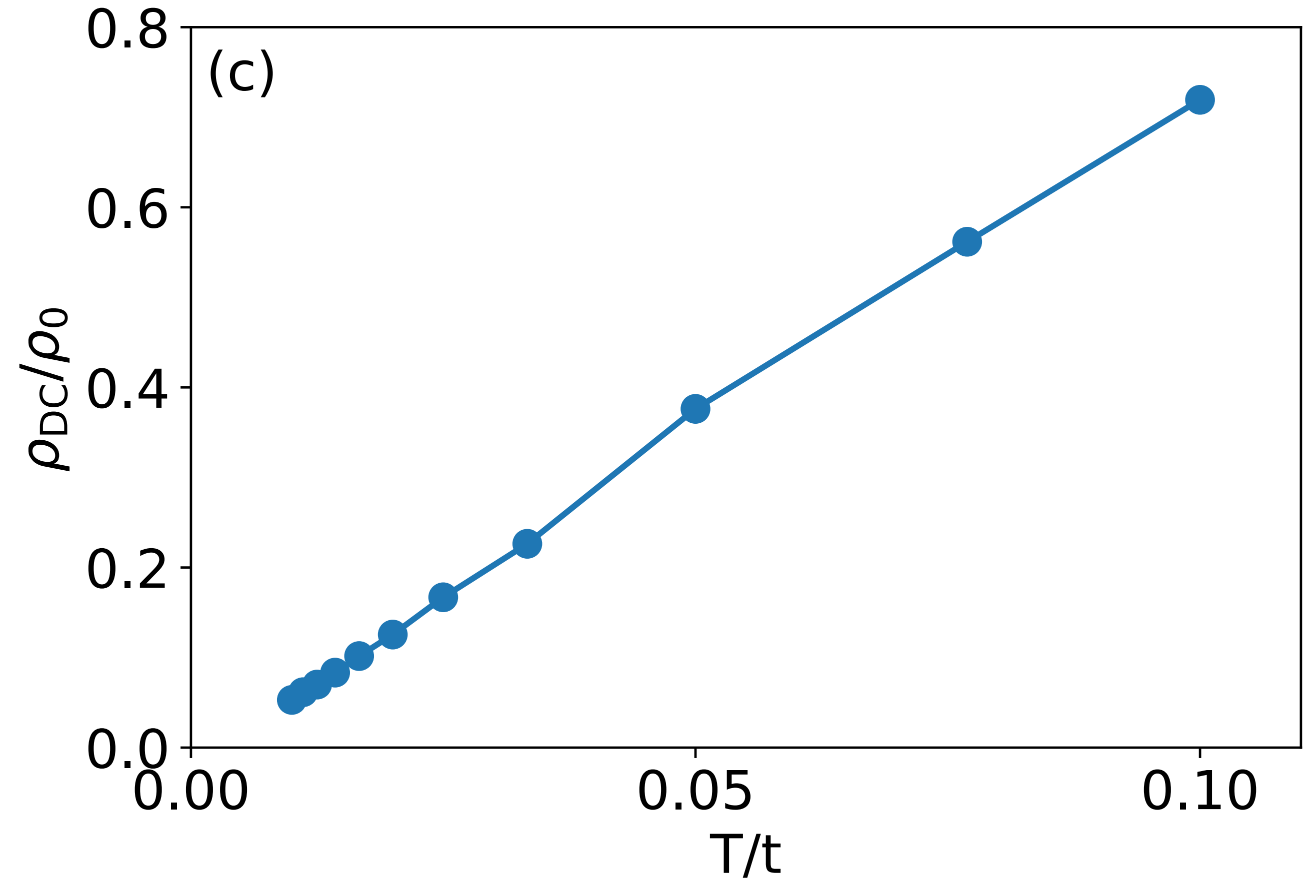

Critical spin correlations:

$$\left\langle \vec{S}(\tau)\cdot\vec{S}(0)\right\rangle \sim \frac{1}{|\tau|}$$

Resisitivity $\rho \sim T$ to the lowest T at the critical point in a large-dimension model

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Condensed Matter > Strongly Correlated Electrons

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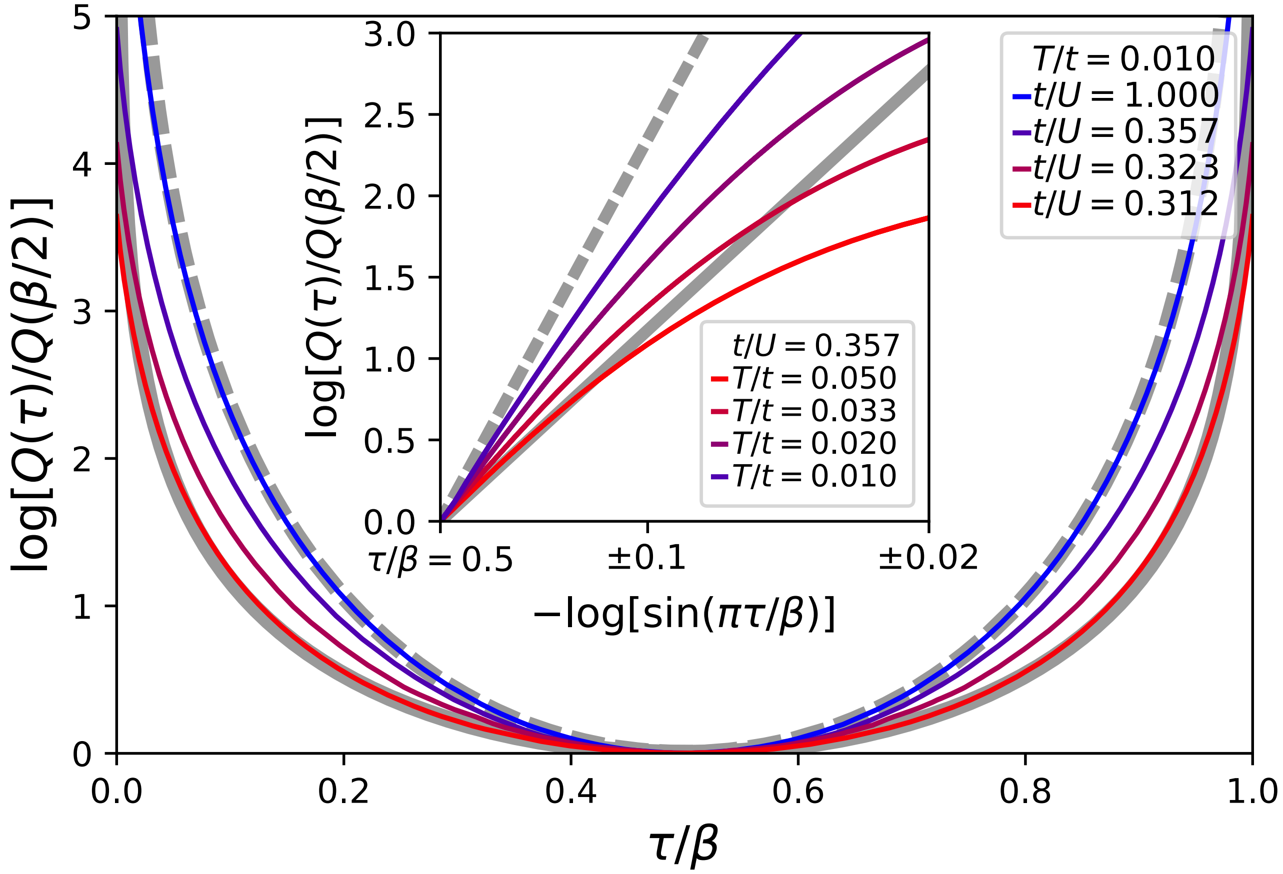
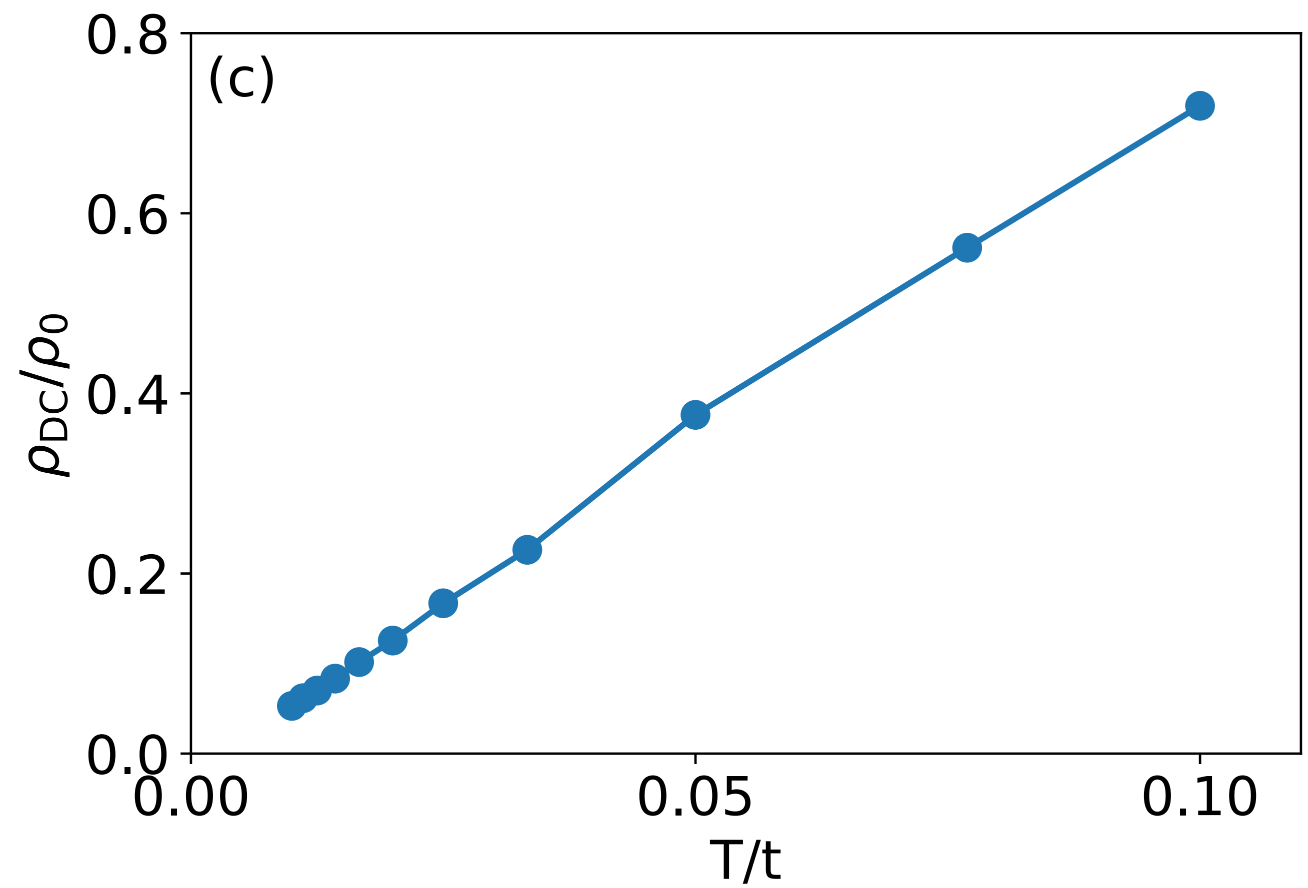
Peter Cha, Nils Wentzell, Olivier Parcollet, Antoine Georges, Eun-Ah Kim





Critical spin correlations:

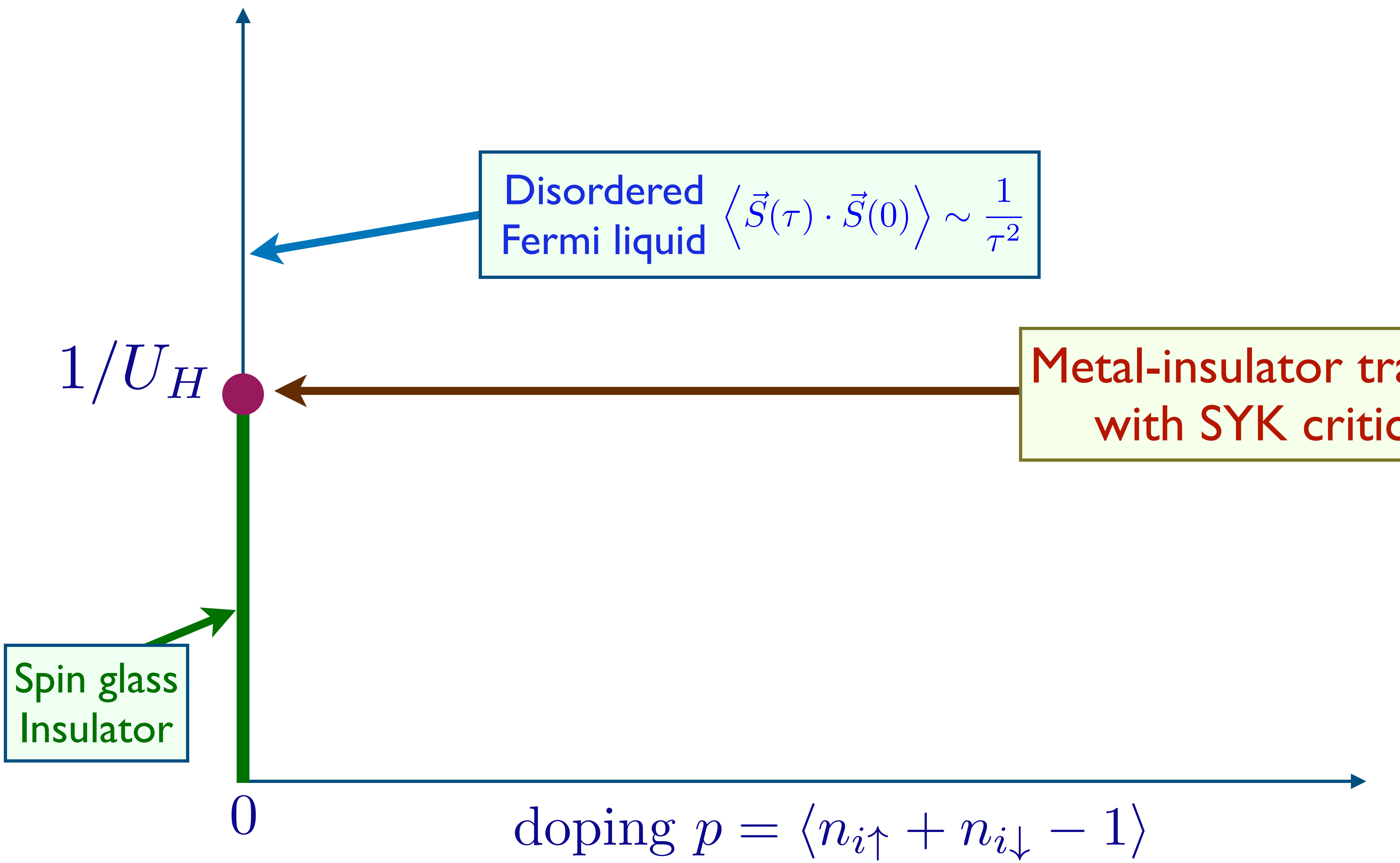
$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

Resisitivity $\rho \sim T$ to the lowest T at the critical point in a large-dimension model

Mapping to SYK criticality in a large M theory (with SU(M) spin symmetry)
G. Tarnopolsky, Chenyuan Li, D.G. Joshi, and S. Sachdev, arXiv:2002.12381

Random t-J-U_H model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



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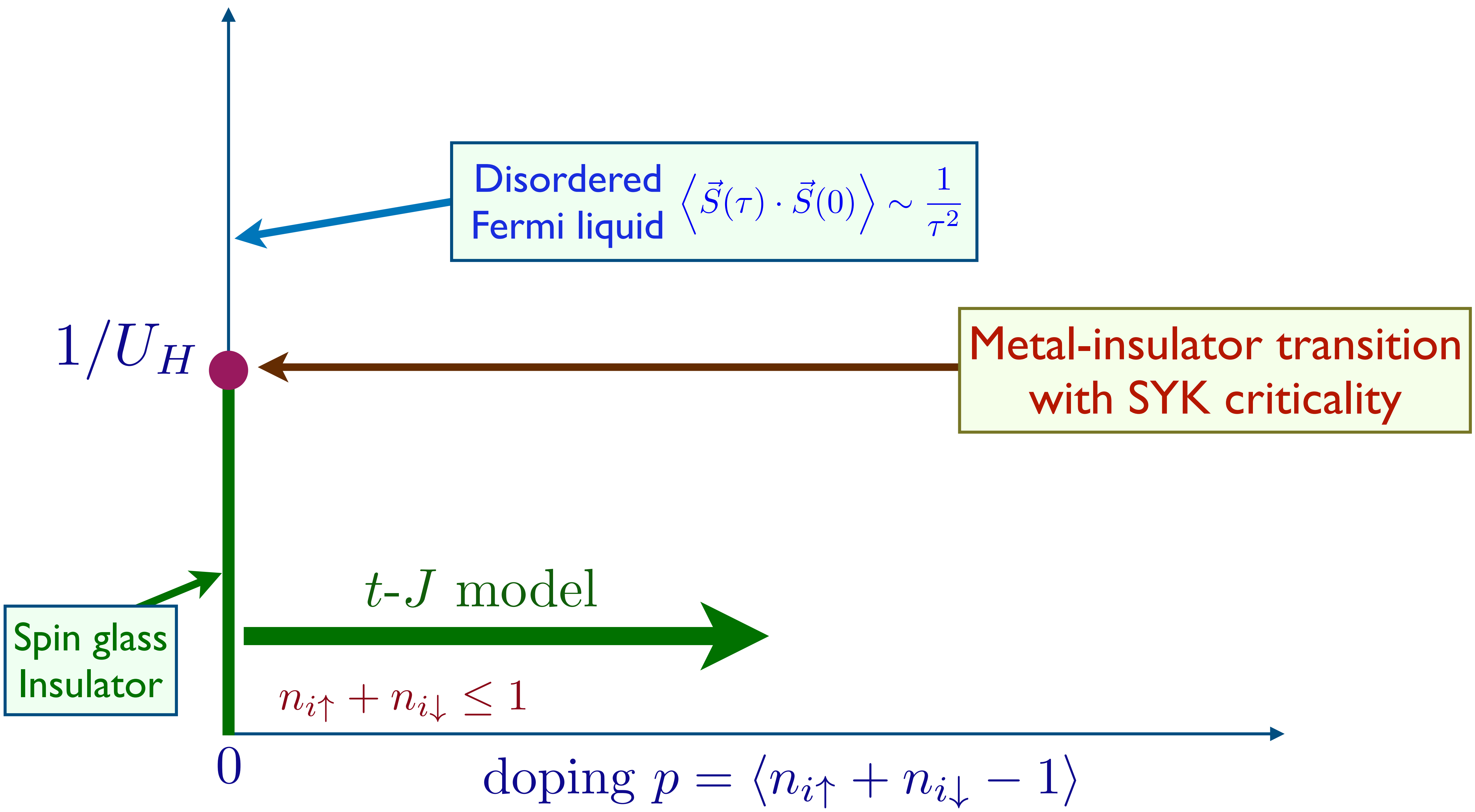

Random t-J-U_H model

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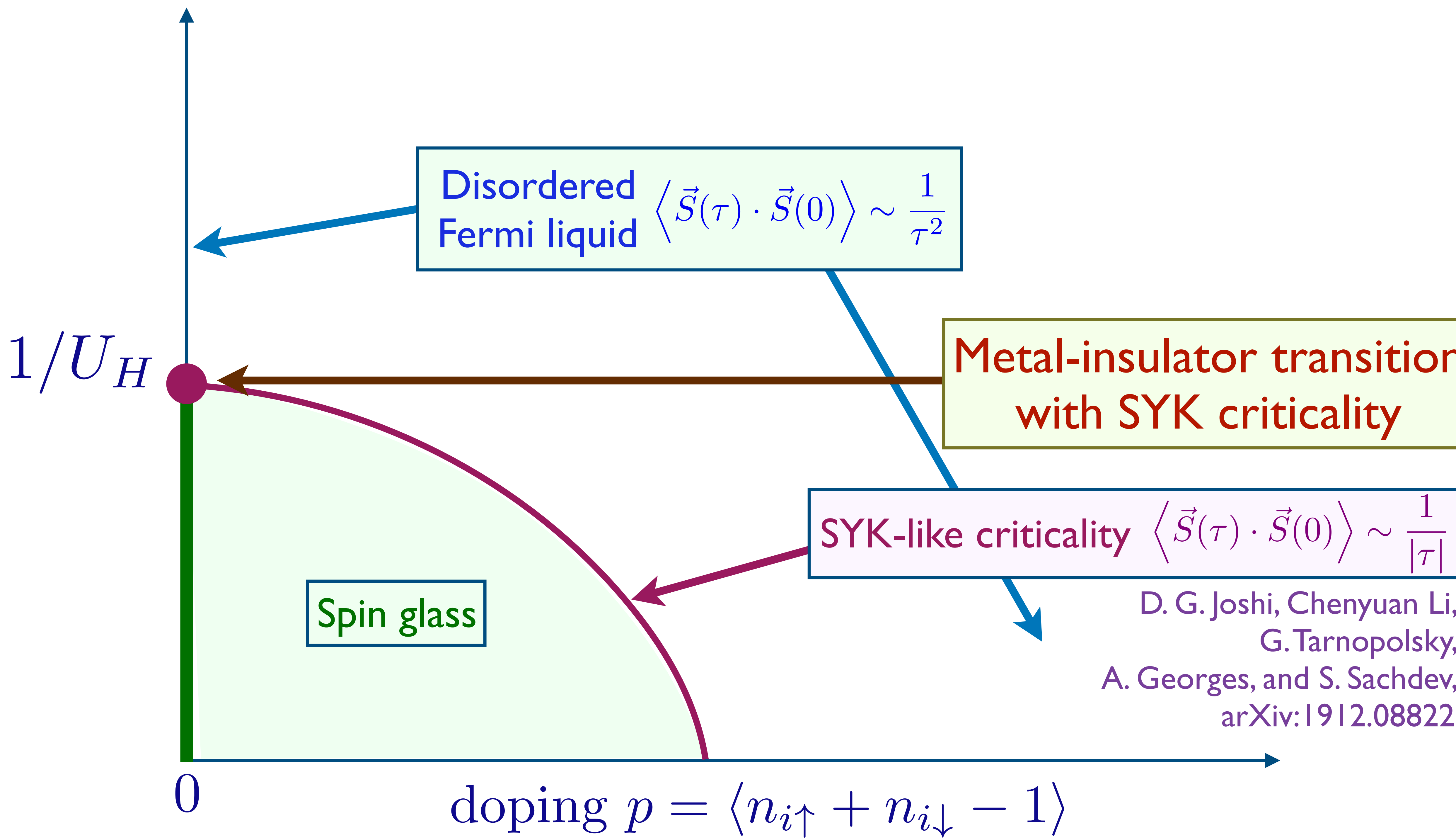
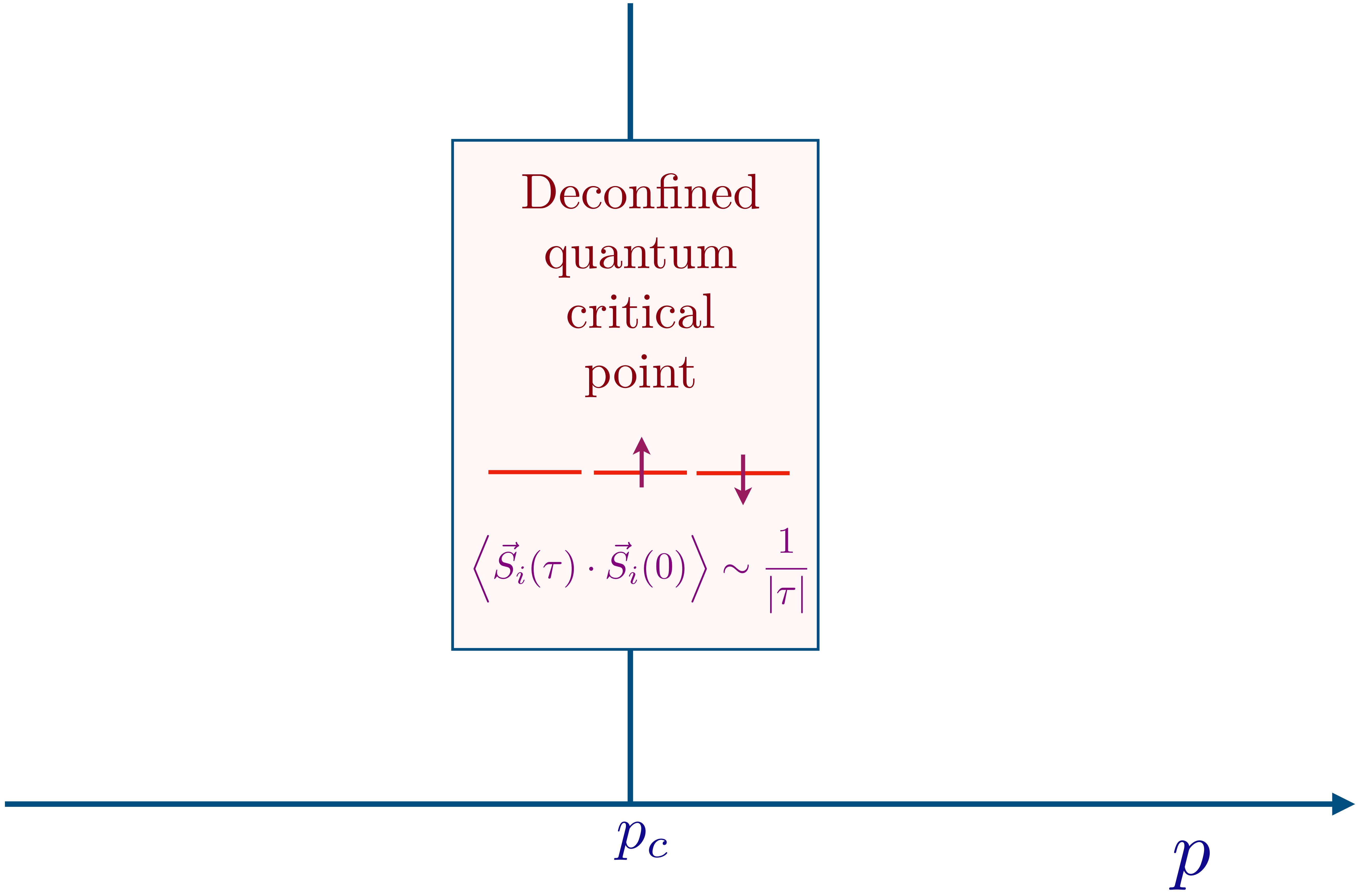


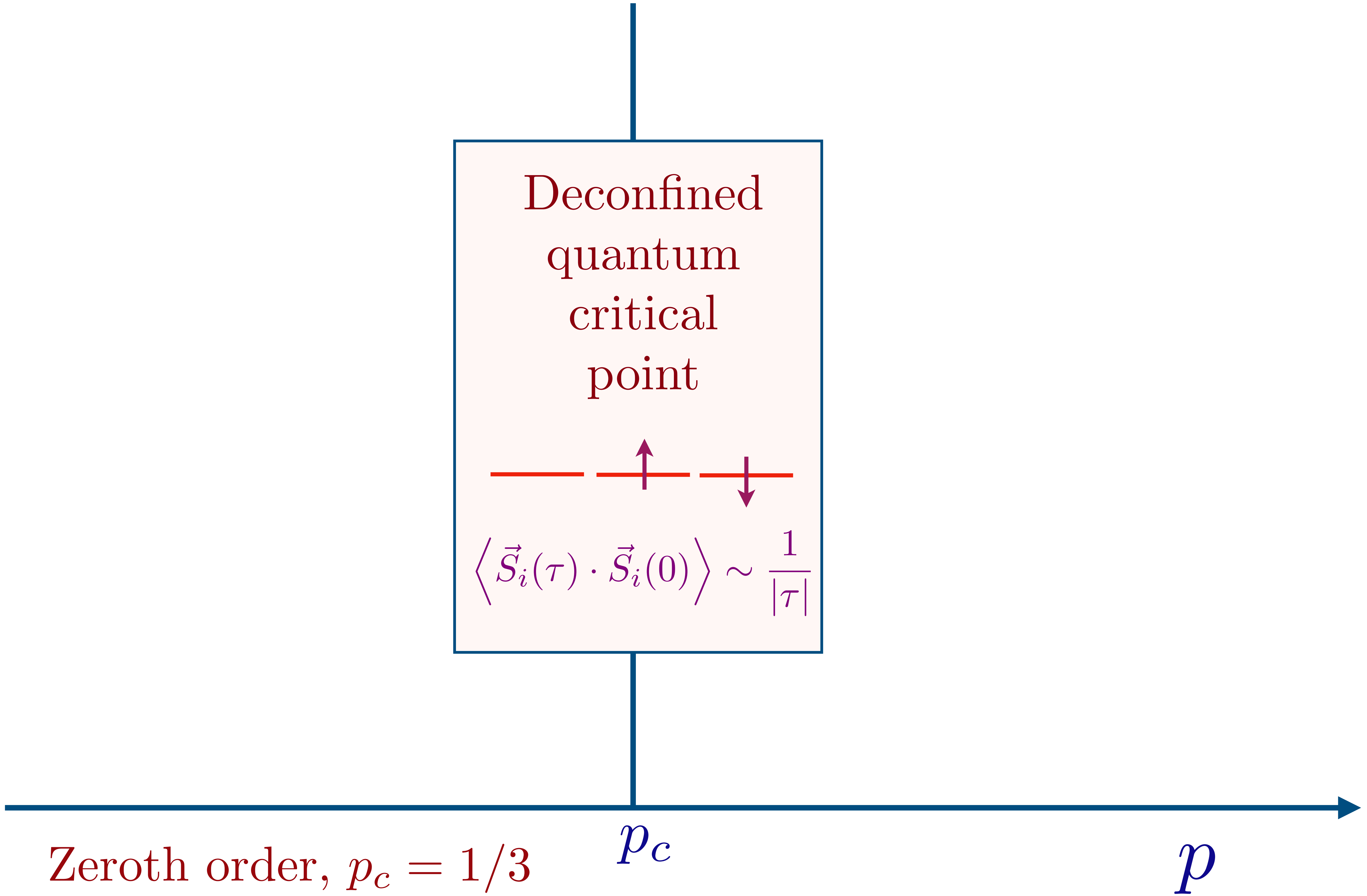
D. G. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, and S. Sachdev, arXiv:1912.08822

t-J model phase diagram



D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

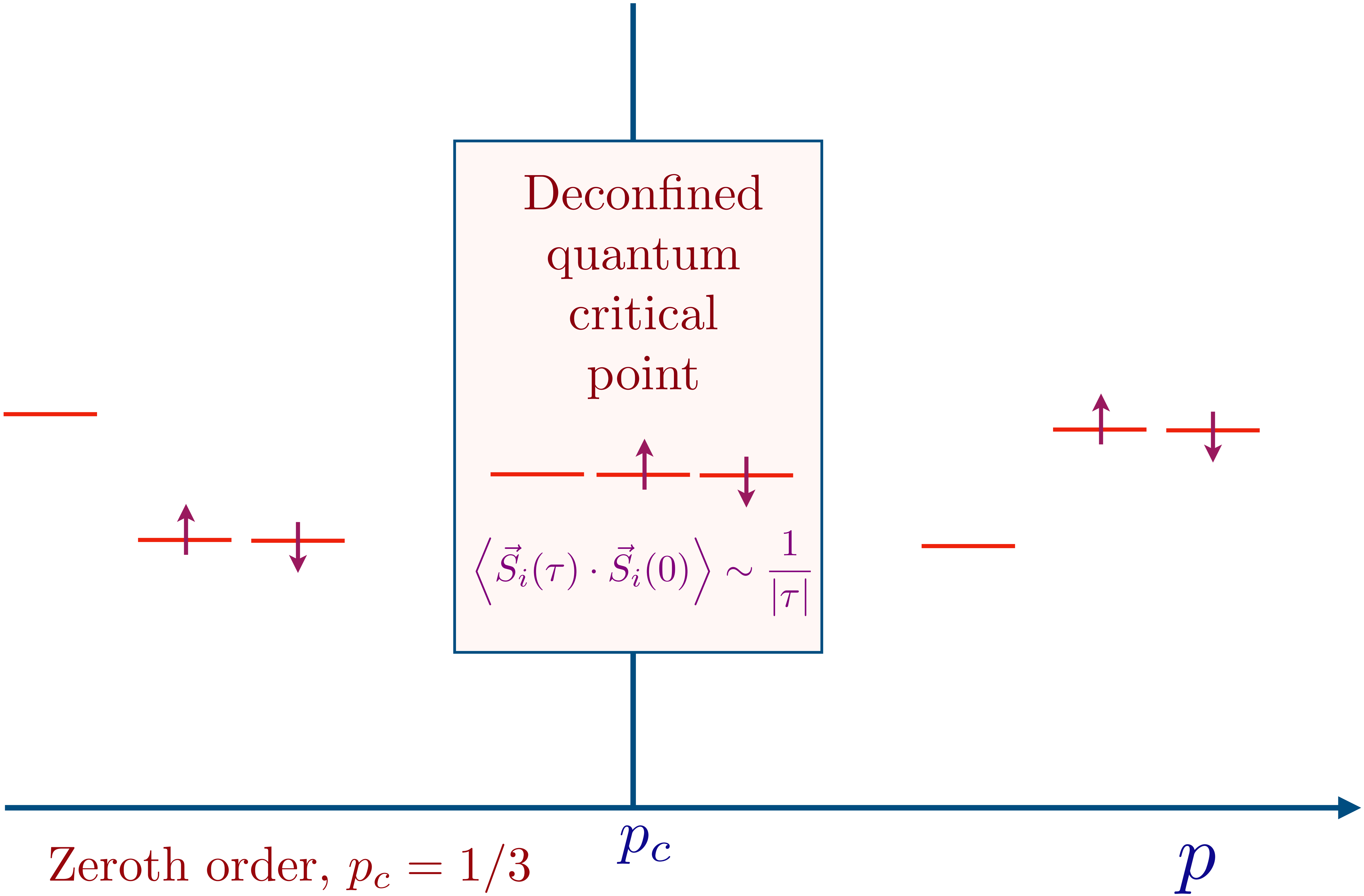
t-J model phase diagram



Zeroth order, $p_c = 1/3$

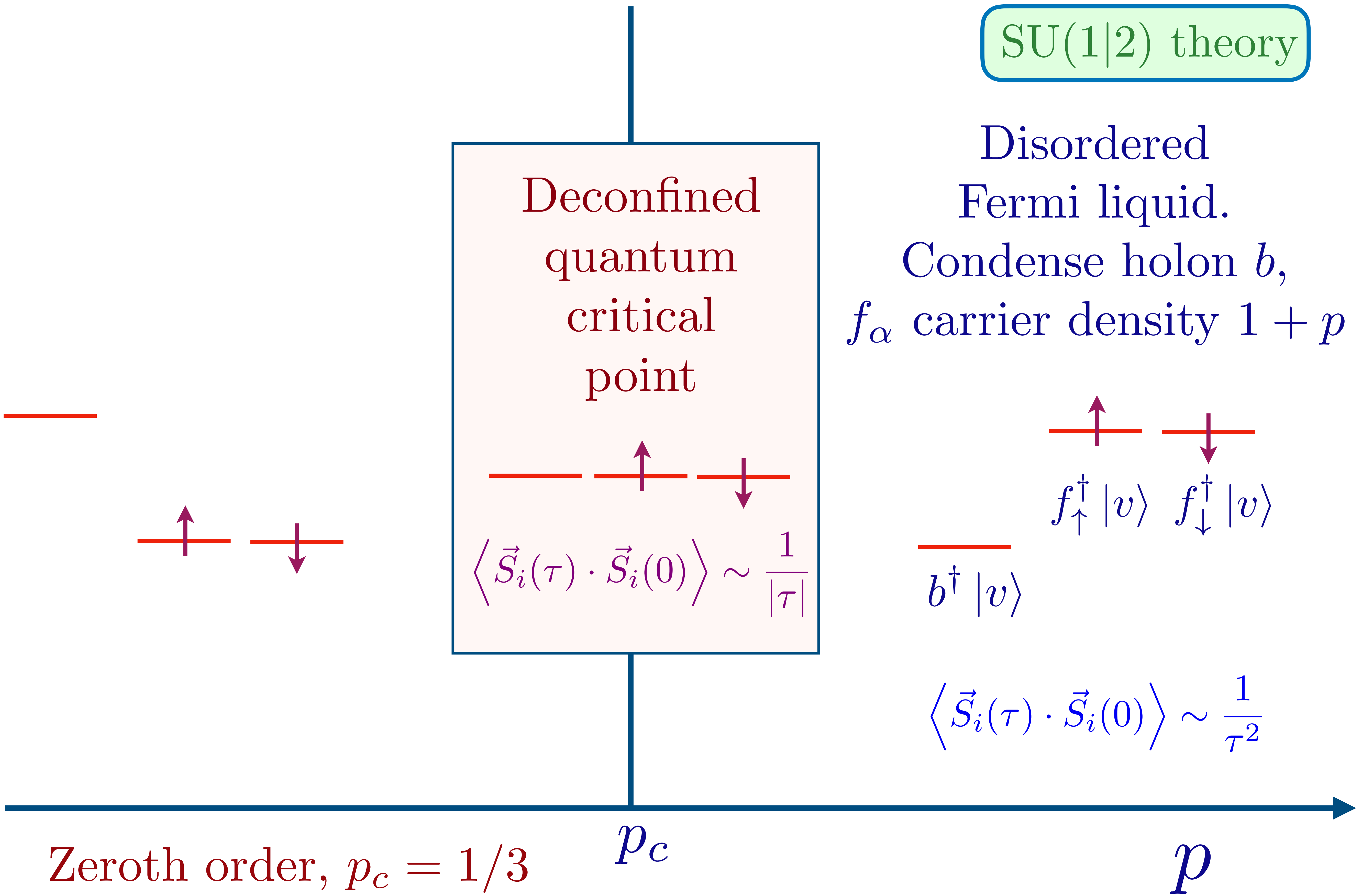
D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

t-J model phase diagram



D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

t-J model phase diagram



D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

t-J model phase diagram

SU(2|1) theory

Metallic spin glass. Condense spinon $\mathfrak{b}_\alpha$, $\mathfrak{f}$ carrier density p

$\mathfrak{f}^\dagger |v\rangle$

$\mathfrak{b}^\dagger_\uparrow |v\rangle$ $\mathfrak{b}^\dagger_\downarrow |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

Deconfined quantum critical point

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

SU(1|2) theory

Disordered Fermi liquid. Condense holon b, f_α carrier density $1 + p$

$f^\dagger_\uparrow |v\rangle$ $f^\dagger_\downarrow |v\rangle$

$b^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

p_c

p

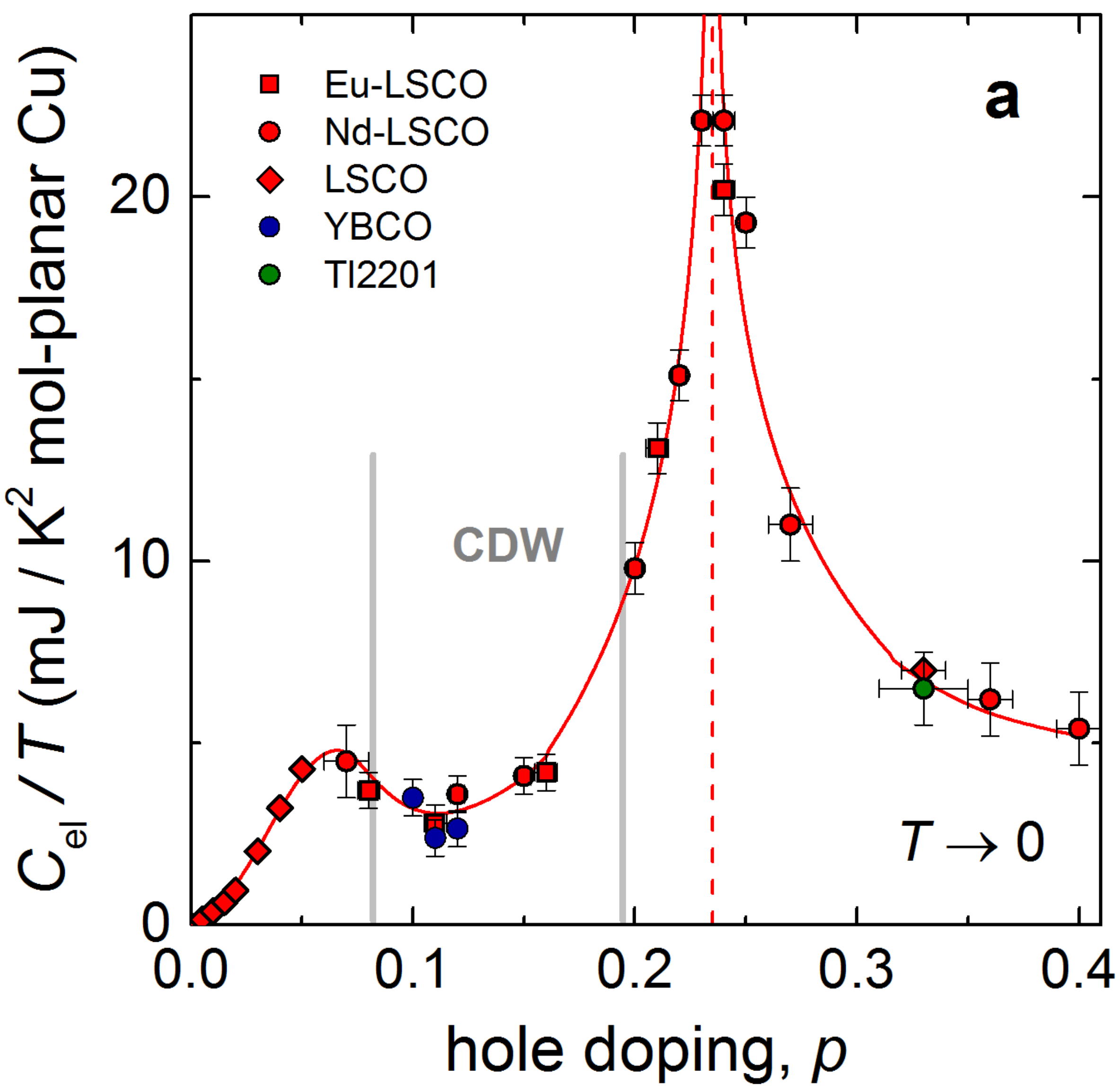
Zeroth order, $p_c = 1/3$

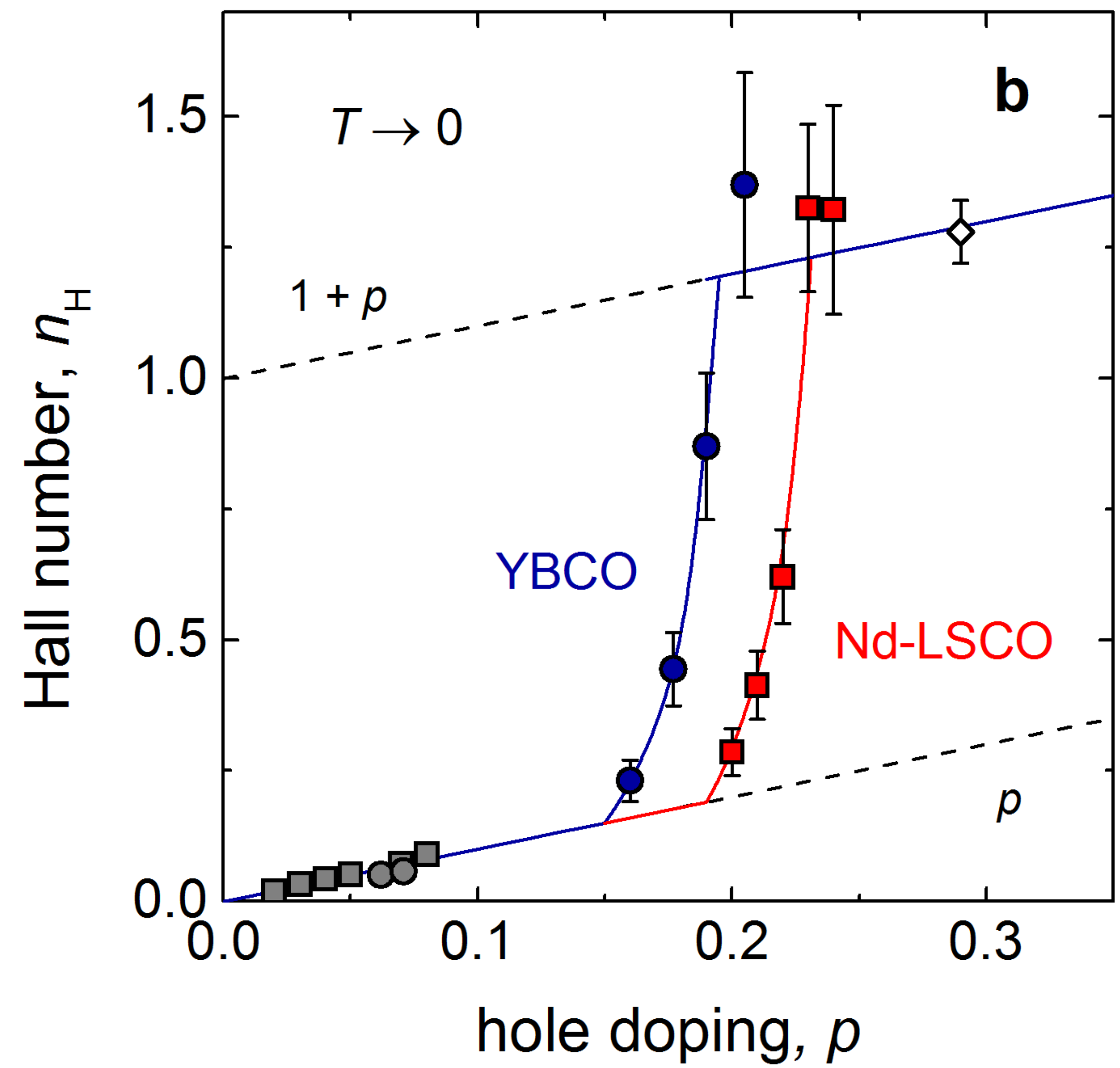
D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

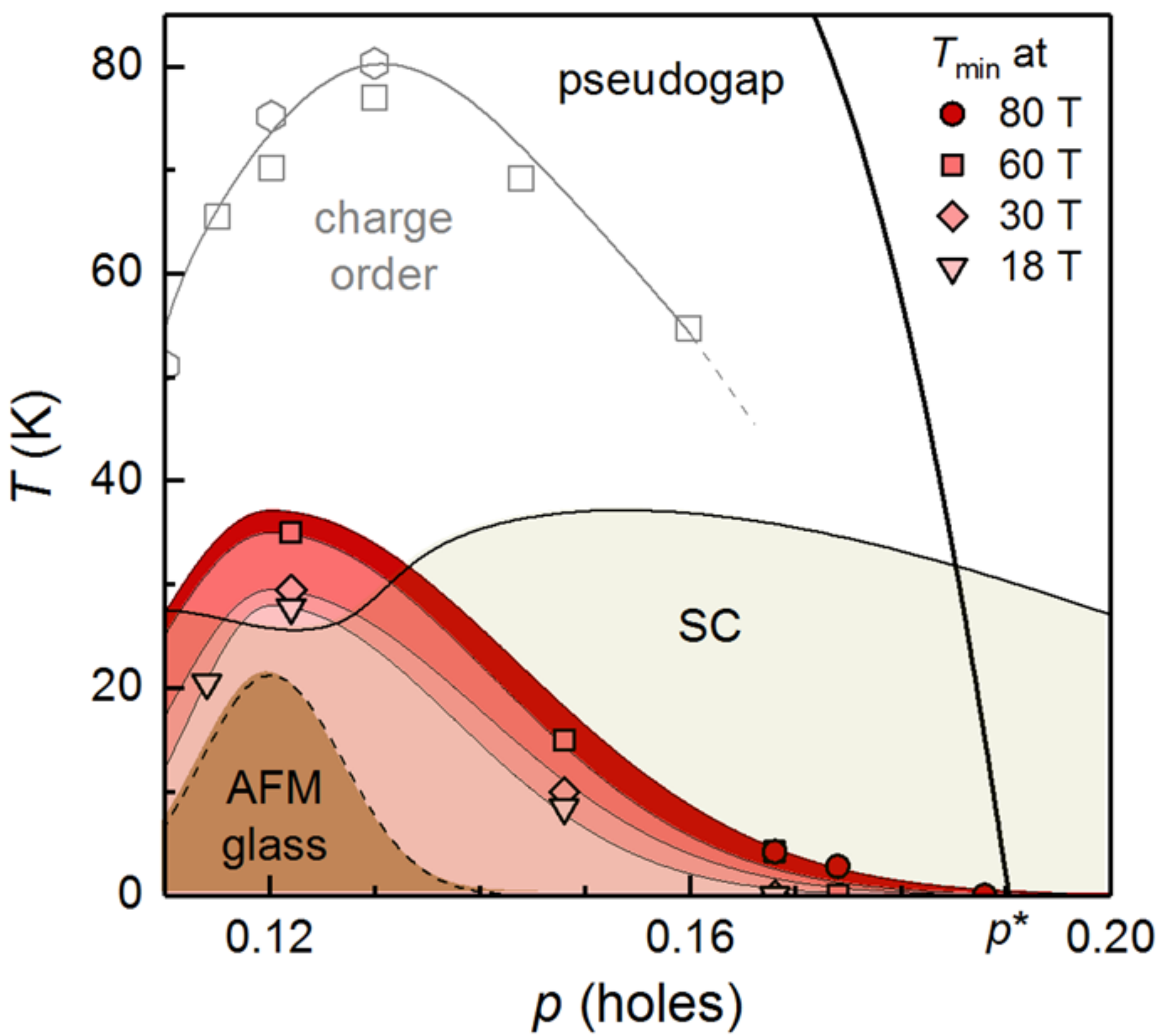
Cyril Proust and Louis Taillefer, arXiv:1807.0507





Hidden magnetism at the pseudogap critical point of a high temperature superconductor

Mehdi Frachet[1]†, Igor Vinograd[1]†, Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1]*, Marc-Henri Julien[1]*



arXiv:1909.10258

Quasi-static magnetism in the pseudogap state of $La_{2-x}Sr_xCuO_4$. Temperature – doping phase diagram representing T_{min}, the temperature of the minimum in the sound velocity, at different fields. Since superconductivity precludes the observation of T_{min} in zero-field, the dashed line (brown area) represents the extrapolated $T_{min}(B=0)$. While not exactly equal to the freezing temperature T_f (see Fig. 2), T_{min} is closely tied to T_f and so is expected to have the same doping dependence, including a peak around $p = 0.12$ in zero/low fields (ref. 2). Onset temperatures of charge order are from ref. 33 (squares) and 35 (hexagons).

1. SYK criticality

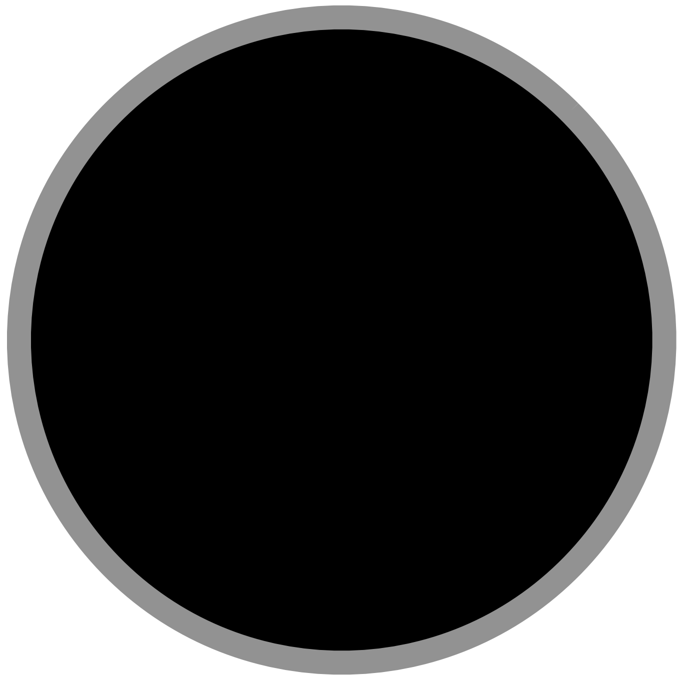
2. Deconfined quantum criticality of random t-J-U_H models

3. Black holes

Black Holes

Objects so dense that light is gravitationally bound to them.

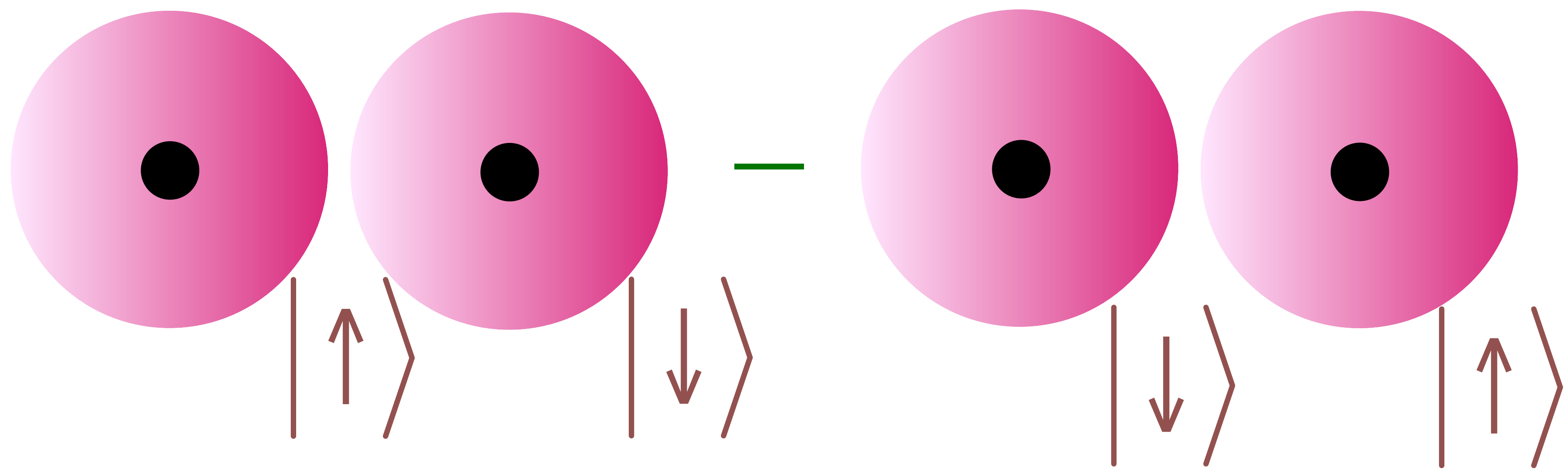
In Einstein's theory, the region inside the black hole **horizon** is disconnected from the rest of the universe.



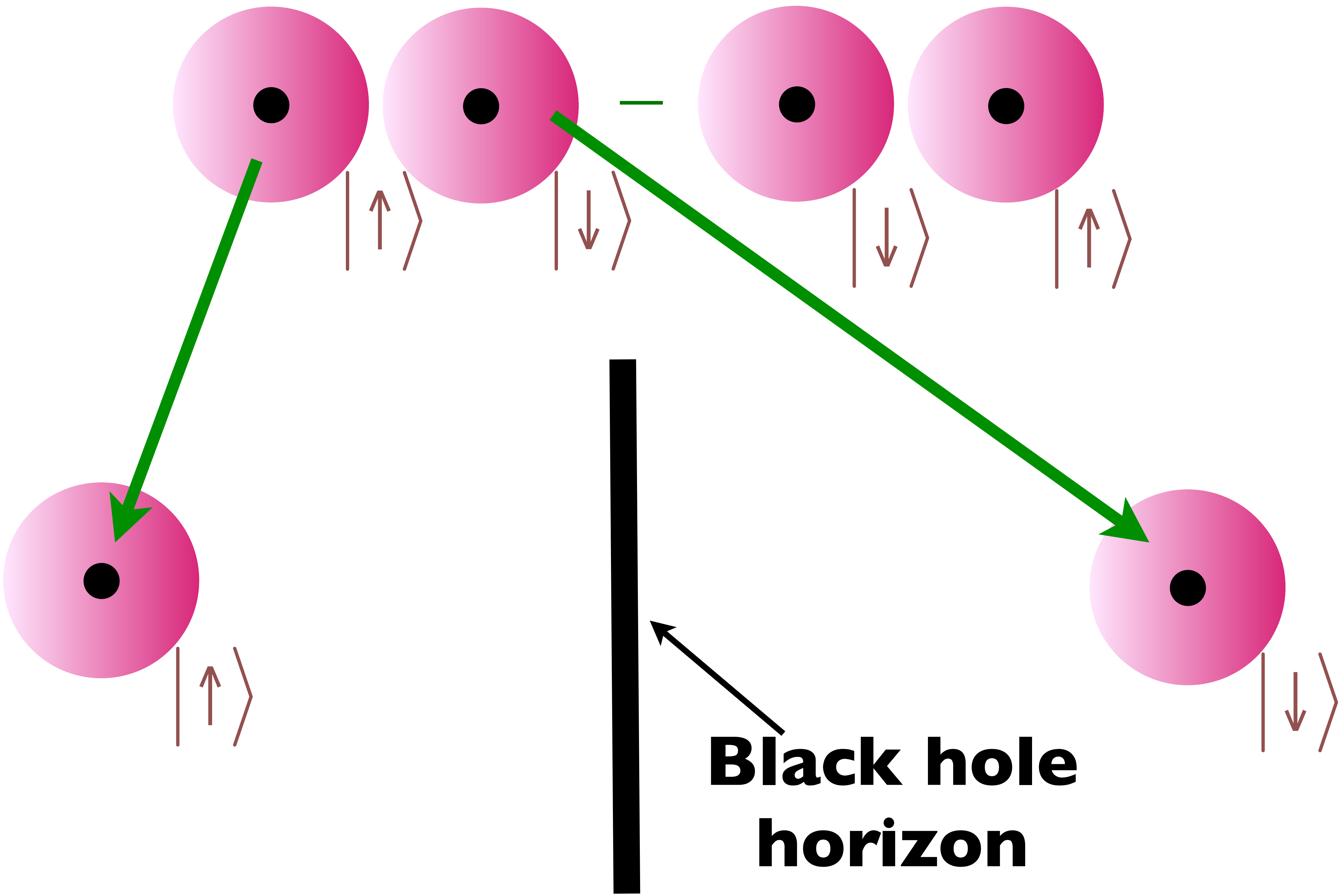
$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

G Newton's constant, c velocity of light, M mass of black hole

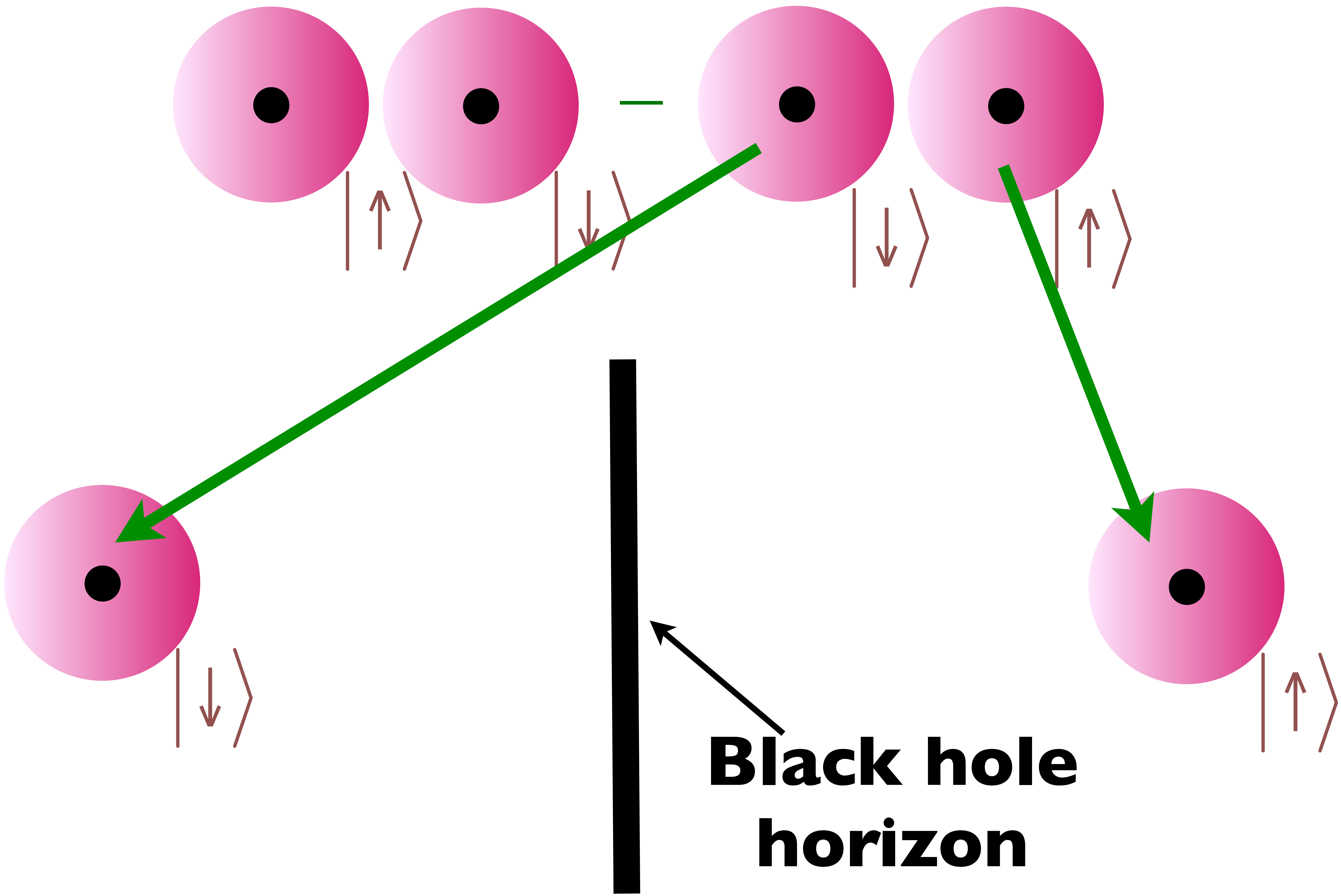
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon



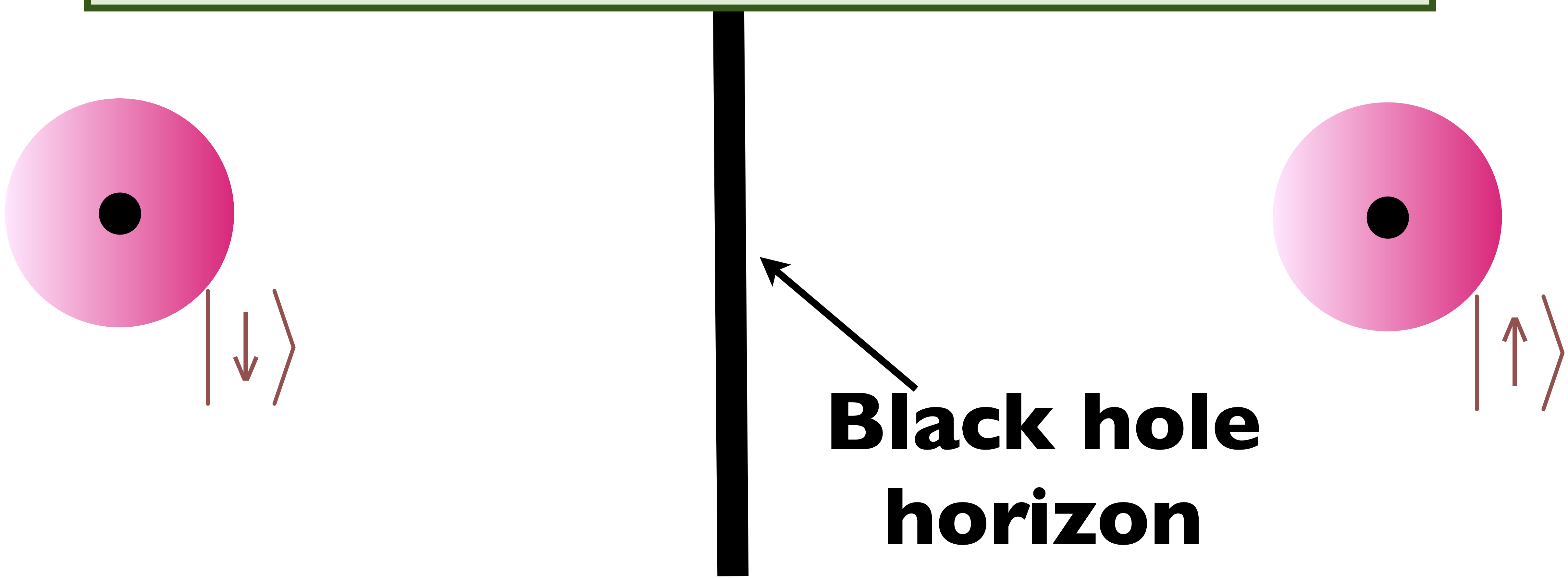
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature

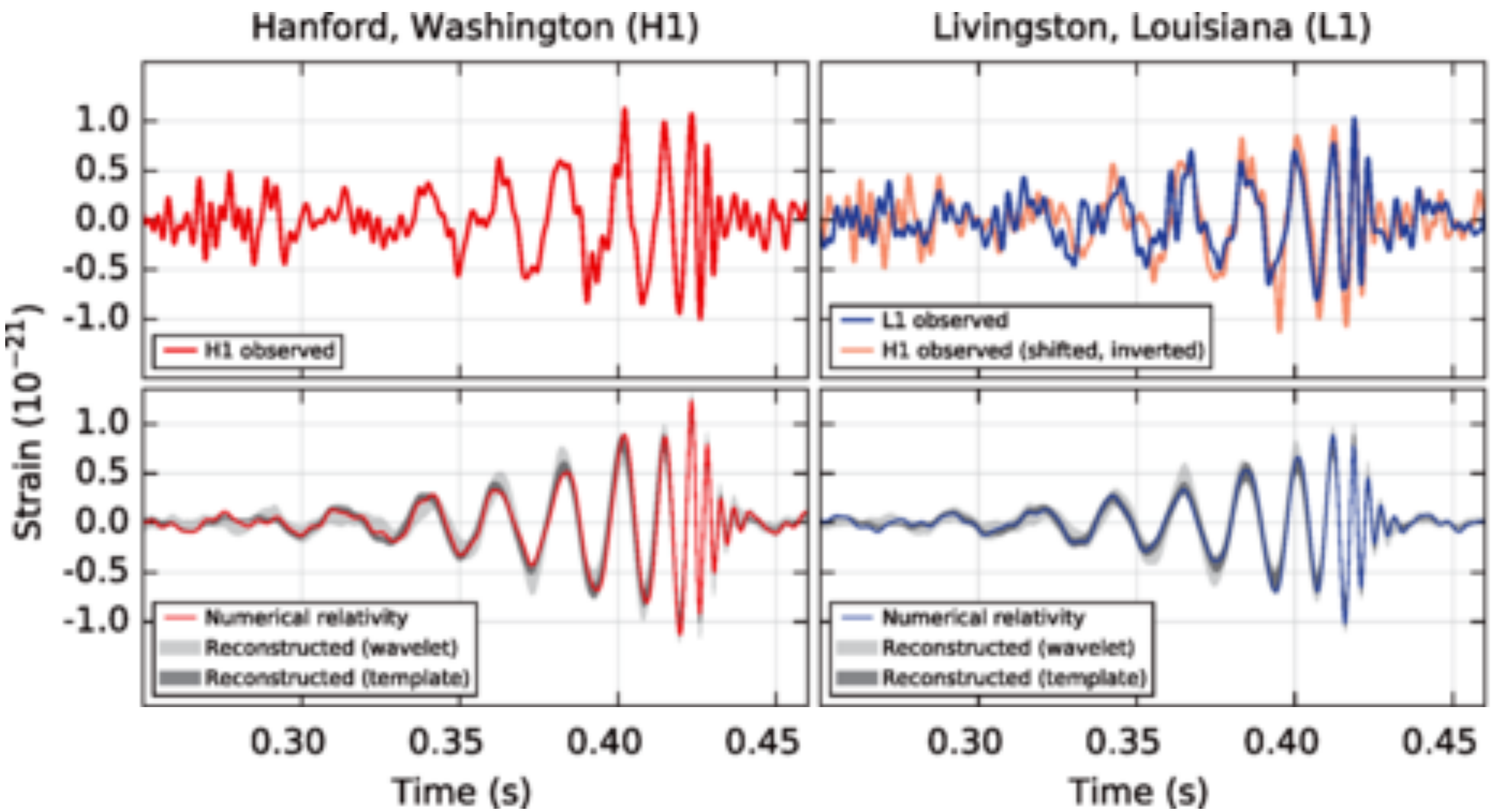
(because to an outside observer, the state of the electron inside the black hole is an unknown)

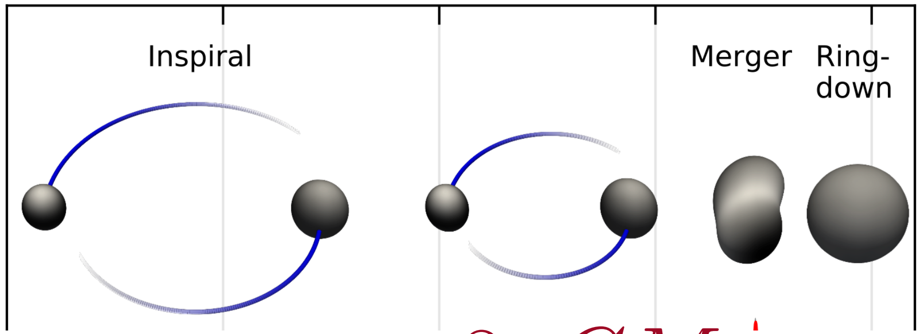


Quantum Black holes

- Black holes have an entropy and a temperature, T_H
- The entropy is proportional to their surface area.

J. D. Bekenstein, PRD **7**, 2333 (1973)
S. W. Hawking, Nature **248**, 30 (1974)





LIGO
September 14, 2015

- The ring-down time $\dfrac{8\pi GM}{c^3} \sim 8$ milliseconds. Curiously, for essentially all types of black holes, the ring-down time equals

$$\boxed{\frac{\hbar}{k_B T_H},} \qquad \hbar \text{ Planck's constant}, \qquad k_B \text{ Boltzmann's constant}$$

Quantum Black holes

- Black holes have an entropy and a temperature, T_H
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.

Quantum Black holes

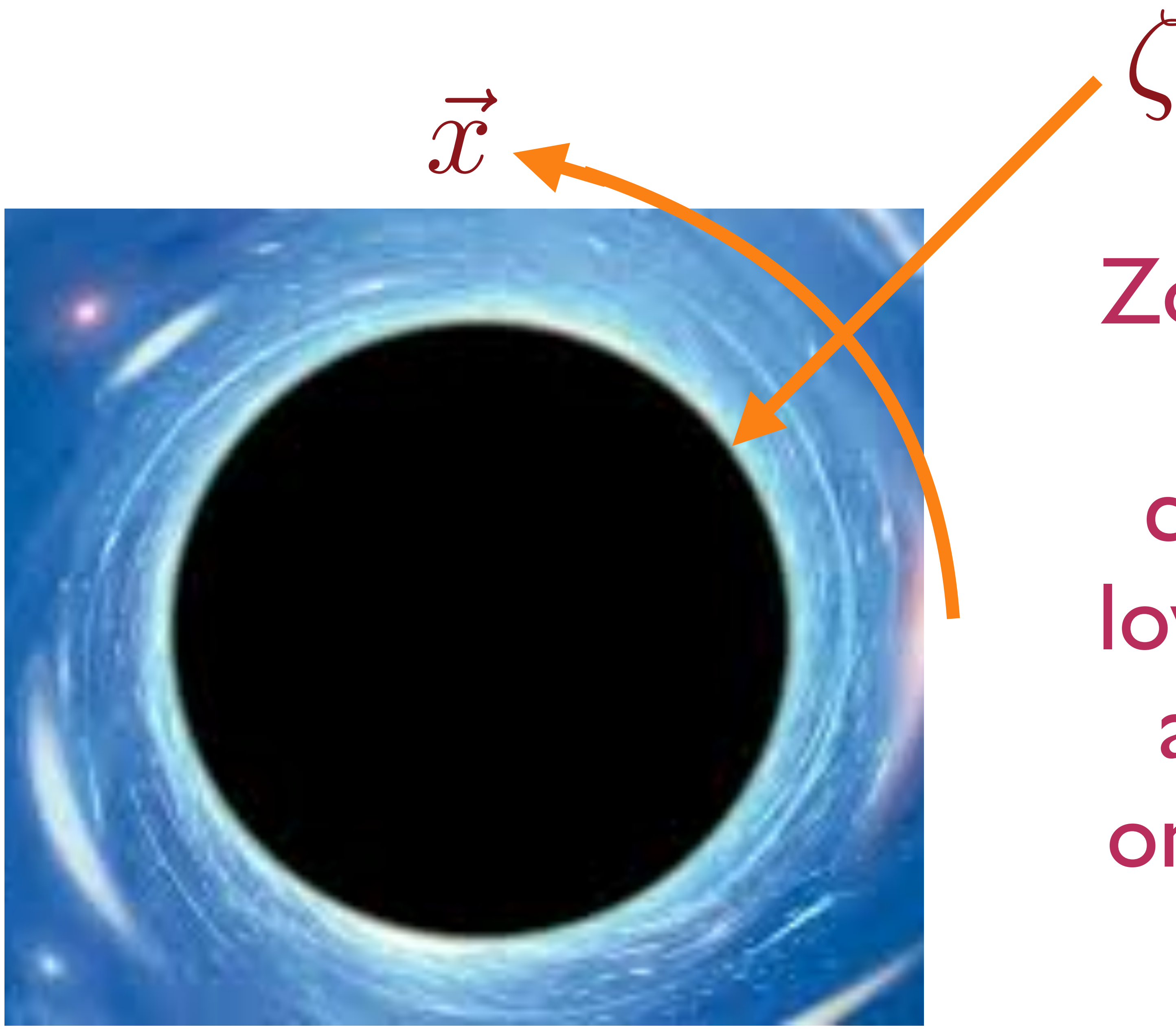
- Black holes have an entropy and a temperature, T_H
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.

<u>Holography</u>:
Quantum black holes "look like" quantum-critical many-particle systems without quasiparticle excitations, residing "on" the surface of the black hole

J. Maldacena, IJTP **38**, 1113 (1999); S.S. Gubser, I.R. Klebanov, and A.M. Polyakov Phys. Lett. B **428**, 105 (1998); E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998)

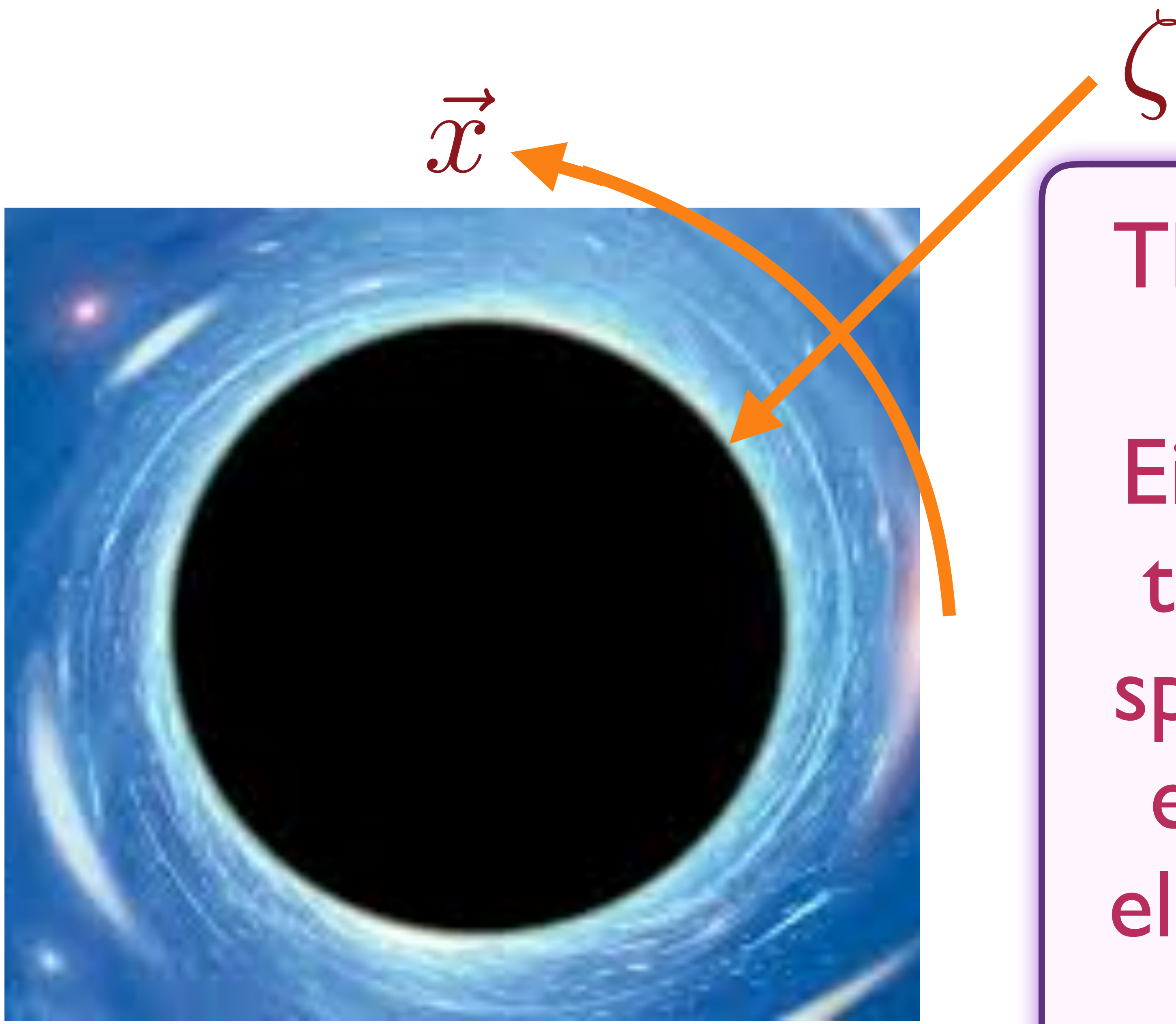
Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



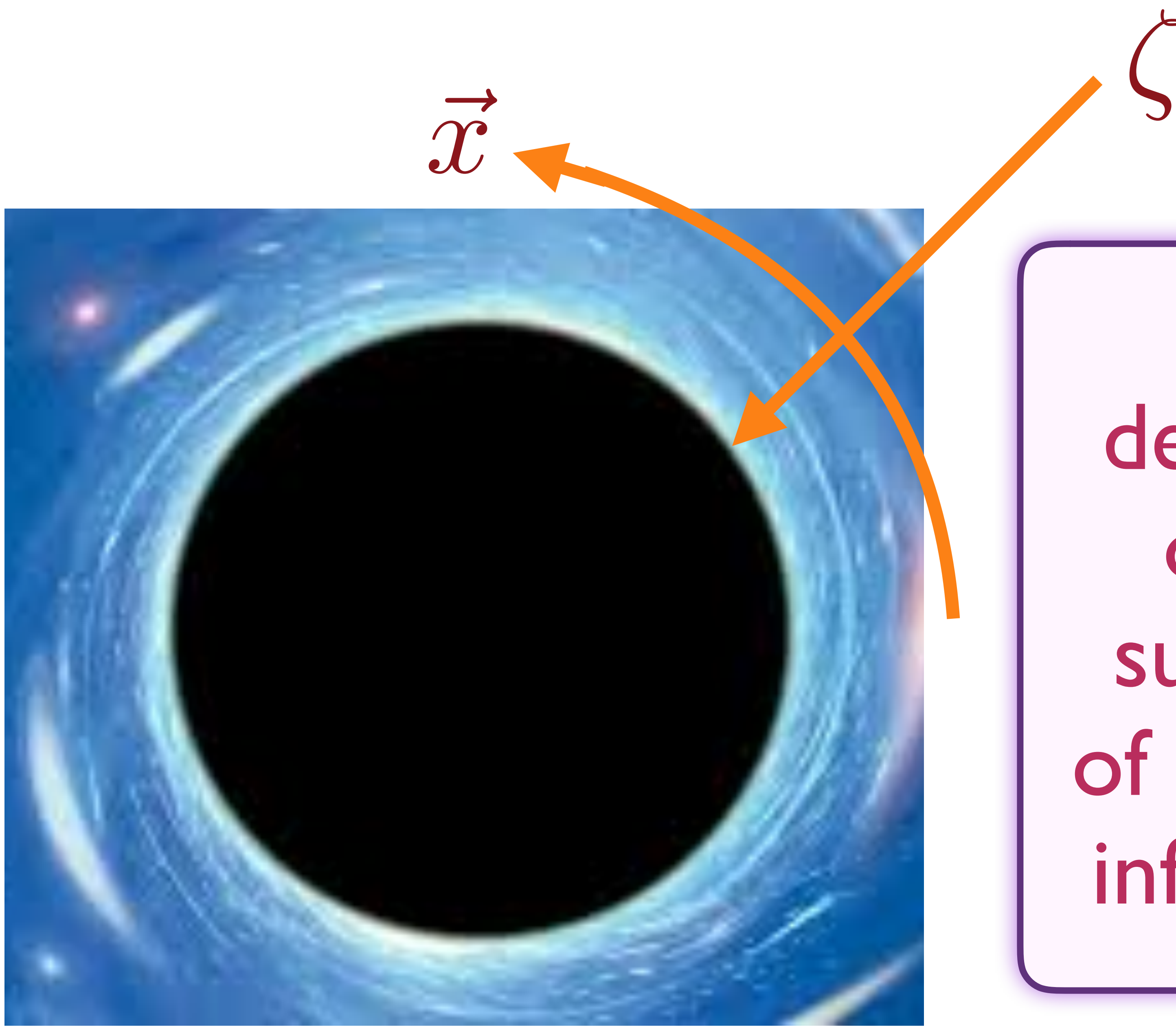
Zooming into the near-horizon region of a charged black hole at low temperature, yields a quantum theory in one space (ζ) and one time dimension

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



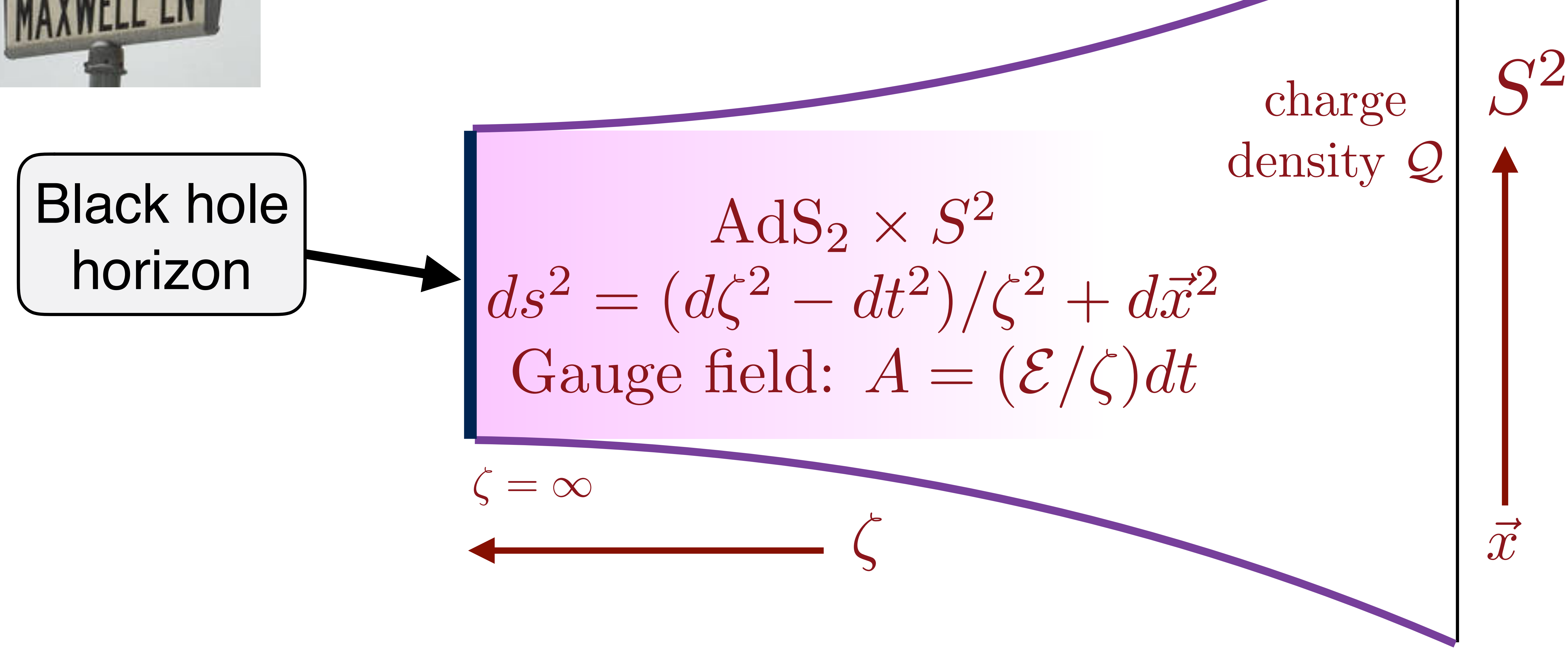
The quantum versions of Maxwell's and Einstein's equations in this two-dimensional spacetime are also the equations describing electron entanglement in the SYK model

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



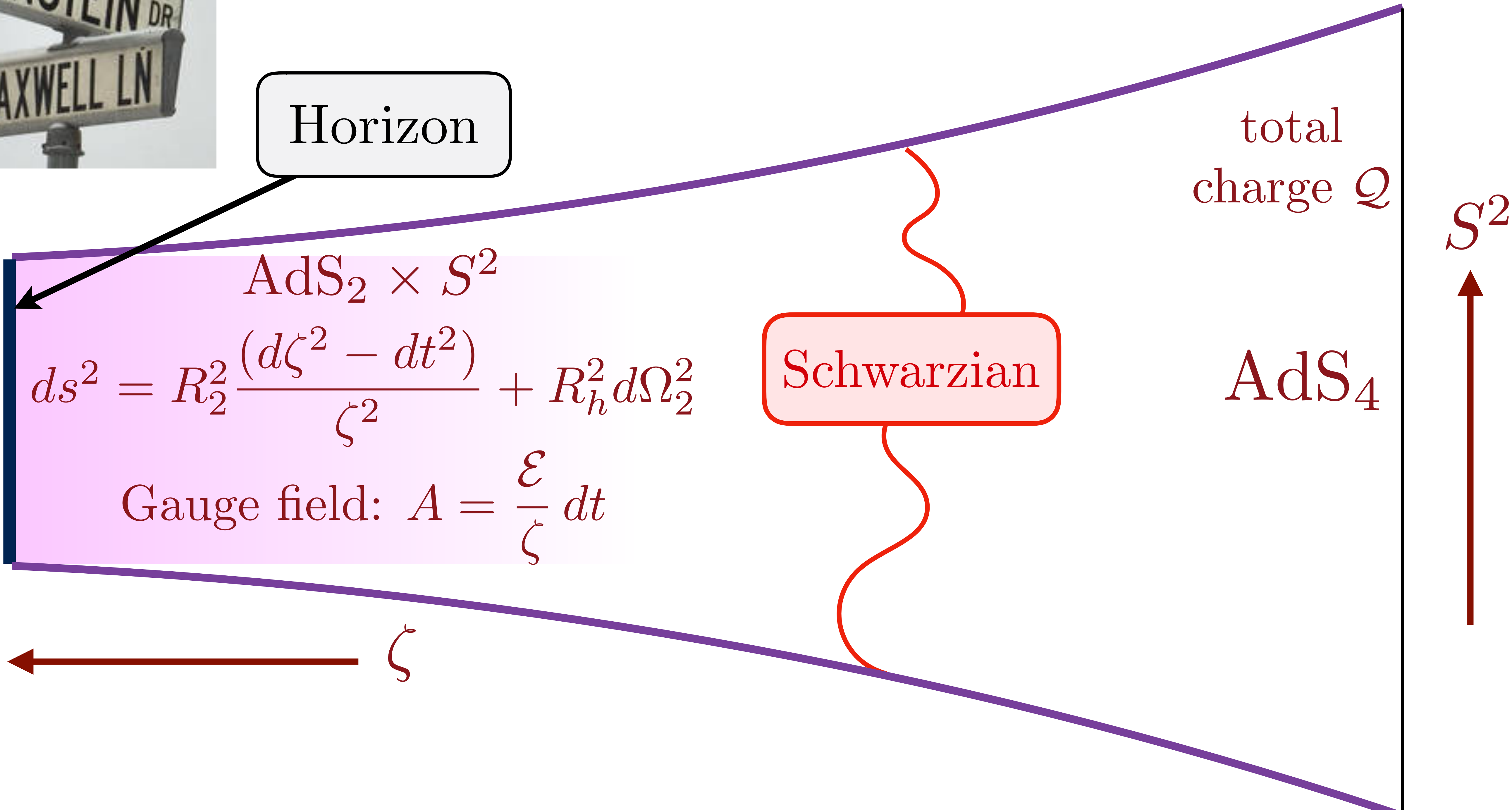
This has led to a deeper understanding of entanglement in superconductors and of Hawking's black hole information "paradox"

SYK model and charged black holes



The near-horizon region of a charged black hole has the geometry of (1+1)-dimensional anti-de Sitter spacetime, which has SL(2,R) symmetry. By holography, this should map to a zero-dimensional quantum system: this turns out to be the SYK model, which also has SL(2,R) symmetry

SYK model and charged black holes



Remarkably, the correspondence between charged black holes and the SYK model also holds for the leading fluctuations at higher temperatures: both are described by a 'Schwarzian' theory with emergent SL(2,R) and U(1) gauge symmetries. For the black hole, the Schwarzian describes the fluctuations of the boundary between AdS_2 and AdS_4.

Main result

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

A. Kitaev (2015)

S. Sachdev, Phys. Rev. X **5**, 041025 (2015)

J. Maldacena and D. Stanford, Phys. Rev. D **94,** 106002 (2016)

J. Maldacena, D. Stanford, and Zhenbin Yang, PTEP 12C104 (2016)

K. Jensen, Phys. Rev. Lett. **117**, 111601 (2016)

J. Engelsoy, T.G. Mertens, and H. Verlinde, JHEP 1607 (2016) 139

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev,
Phys. Rev. B **95**, 155131 (2017)

A. Gaikwad, L.K. Joshi, G. Mandal, and S.R. Wadia, arXiv:1802.07746 P. Nayak, A. Shukla,

R.M. Soni, S.P. Trivedi, and V. Vishal, arXiv:1802.09547

P. Chaturvedi, Yingfei Gu, Wei Song, Boyang Yu, arXiv:1808.08062

U. Moitra, S. P. Trivedi, and V. Vishal, arXiv:1808.08239

S. Sachdev, arXiv:1902.04078

Main result

SYK model of fermions with random interactions of mean-square-value U, with total fermion number $\mathcal{Q}$, at temperatures $T \ll U$

Main result

SYK model of fermions with random interactions of mean-square-value U, with total fermion number $\mathcal{Q}$, at temperatures $T \ll U$

and

Charged black holes in 3+1 dimensions of radius R_h, with total charge $\mathcal{Q}$, at temperatures $T \ll 1/R_h$

are described by a common low energy quantum theory in $0 + 1$ dimensions

Main result

The common low T path integral is $\mathcal{Z} = \int \mathcal{D}f\, \mathcal{D}\phi\, e^{-I}$. This can be exactly evaluated, and the action is

$$I = -s_0 + \int_0^{1/T} d\tau \left\{ \frac{K}{2} \left(\frac{\partial \phi}{\partial \tau} + i(2\pi \mathcal{E} T) \frac{\partial f}{\partial \tau} \right)^2 - \frac{\gamma}{4\pi^2} \mathrm{Sch}[\tan(\pi T f(\tau)), \tau] \right\},$$

where $f(\tau)$ is a monotonic reparameterization of the temporal circle with

$f(\tau + 1/T) = f(\tau) + 1/T$,

ϕ is a phase conjugate to the charge density with

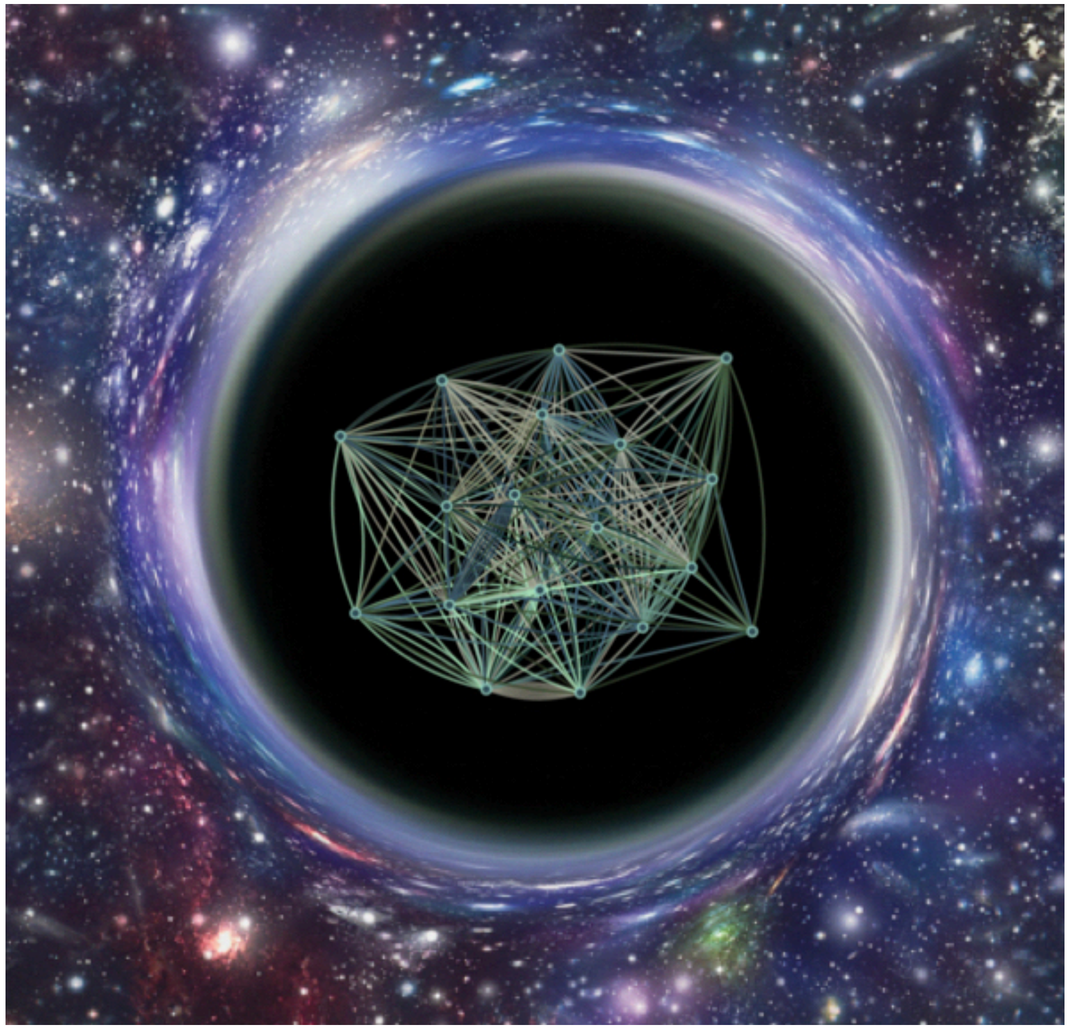
$\phi(\tau + 1/T) = \phi(\tau) + 2\pi n$, n integer,

$\mathrm{Sch}[g[\tau], \tau]$ is the Schwarzian derivative of $g(\tau)$.

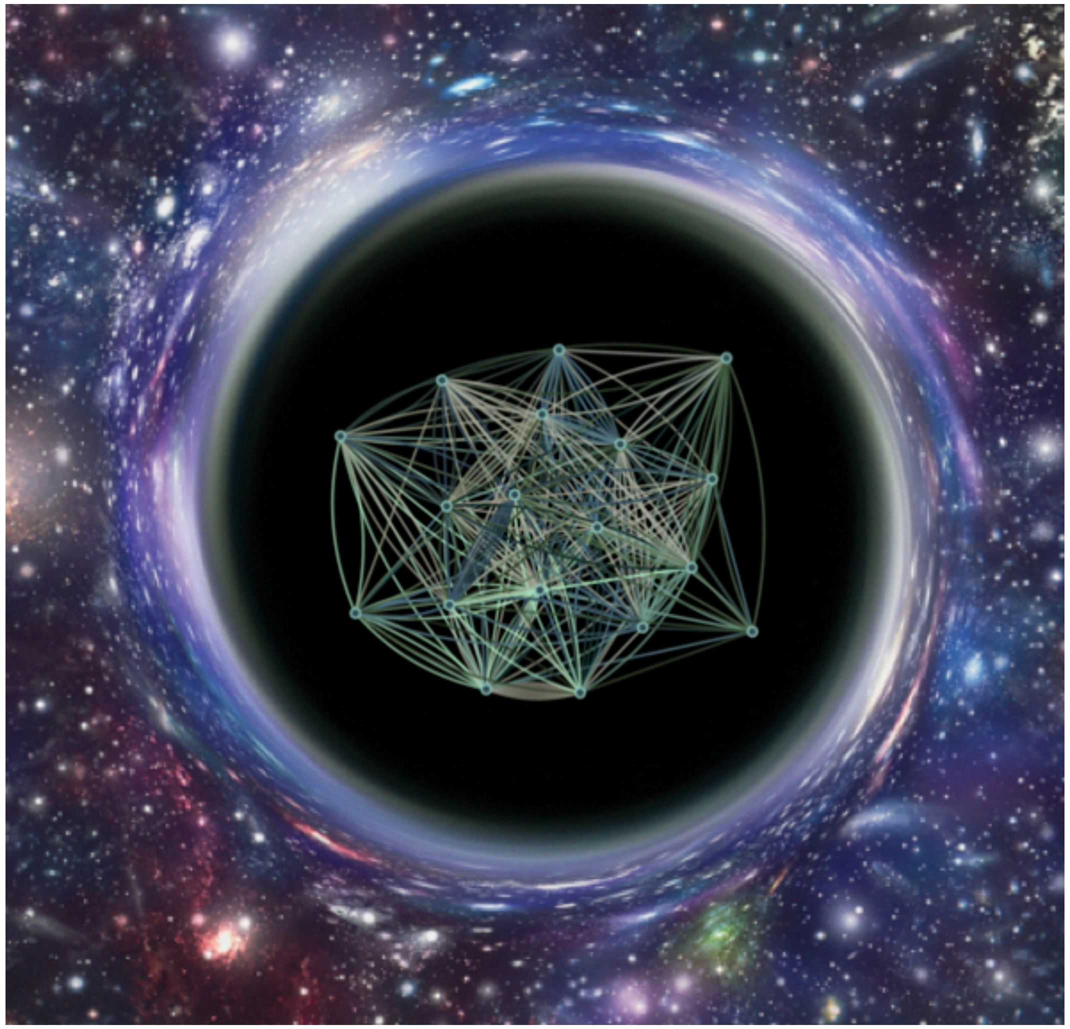
The couplings are related to the entropy $S(T, \mathcal{Q})$ and the chemical potential μ via

$$S(T \to 0, \mathcal{Q}) = s_0 + \gamma T, \quad K = \left(\frac{d\mathcal{Q}}{d\mu} \right)_{T \to 0}, \quad 2\pi \mathcal{E} = \frac{ds_0}{d\mathcal{Q}}$$

- The SYK model provides a paradigm for many-body state at non-zero density without quasiparticle excitations. It is characterized by Planckian time dissipation, non-vanishing zero temperature entropy density, maximal quantum Lyapunov exponent, and spin correlations decaying with time as $1/|\tau|$.



- SYK criticality is realized at
  - a metal-insulator transition in a random Hubbard model with random hopping and exchange
  - a metal-metal transition in a random t-J model at a critical doping



- The Schwarizan path integral describes the low energy quantum fluctuations of SYK-criticality, and of extremal charged black holes.