From the pseudogap to the strange metal

S. Sachdev, E. Berg, S. Chatterjee, and Y. Schattner, PRB **94**, 115147 (2016)

S. Sachdev and S. Chatterjee, arXiv:1703.00014

APS March meeting
March 13, 2017

Talk online: sachdev.physics.harvard.edu

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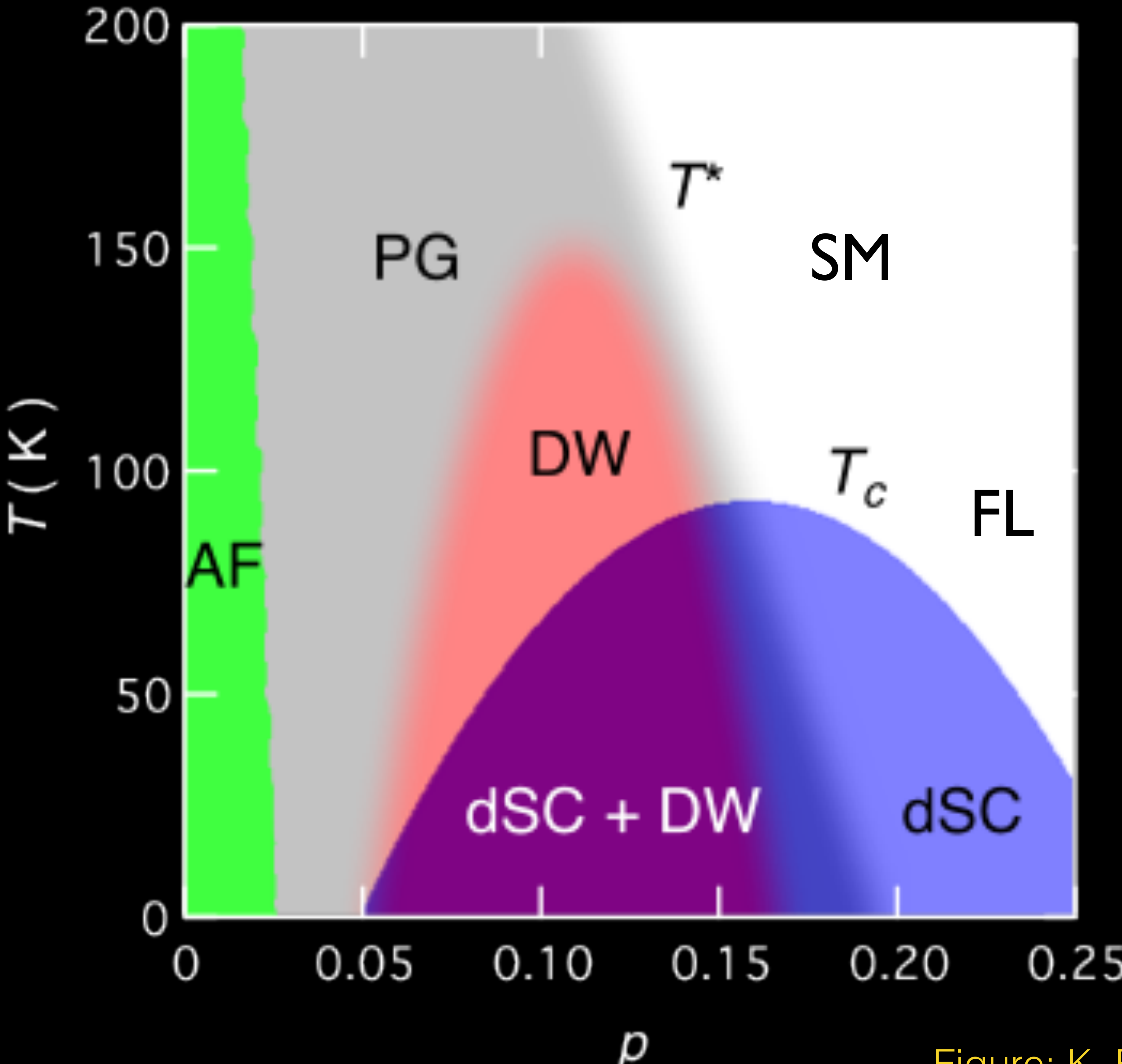
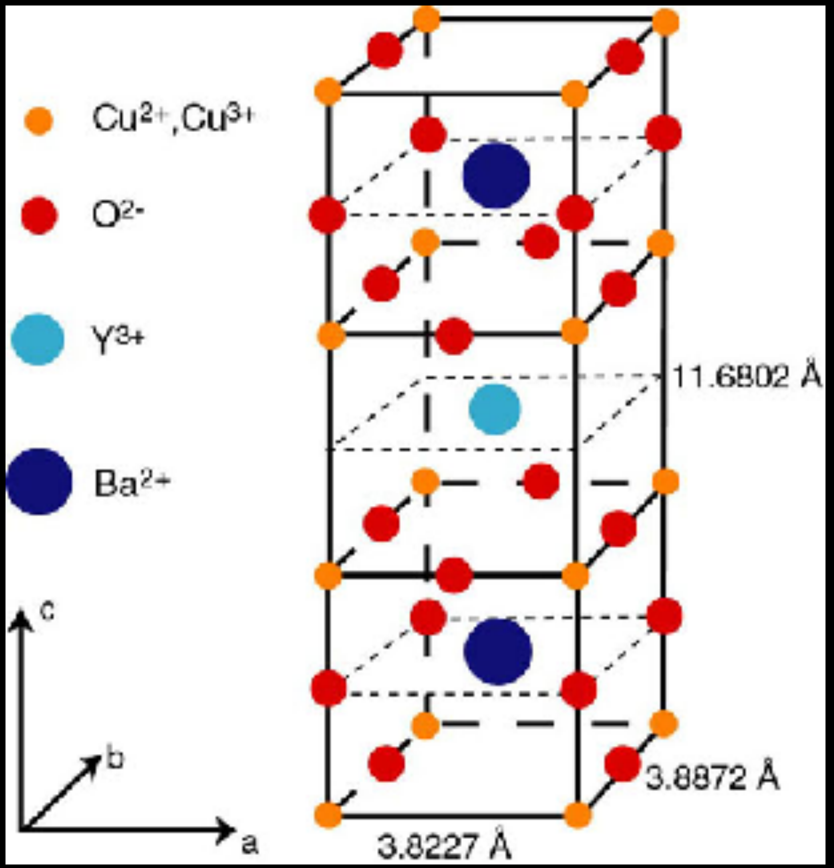
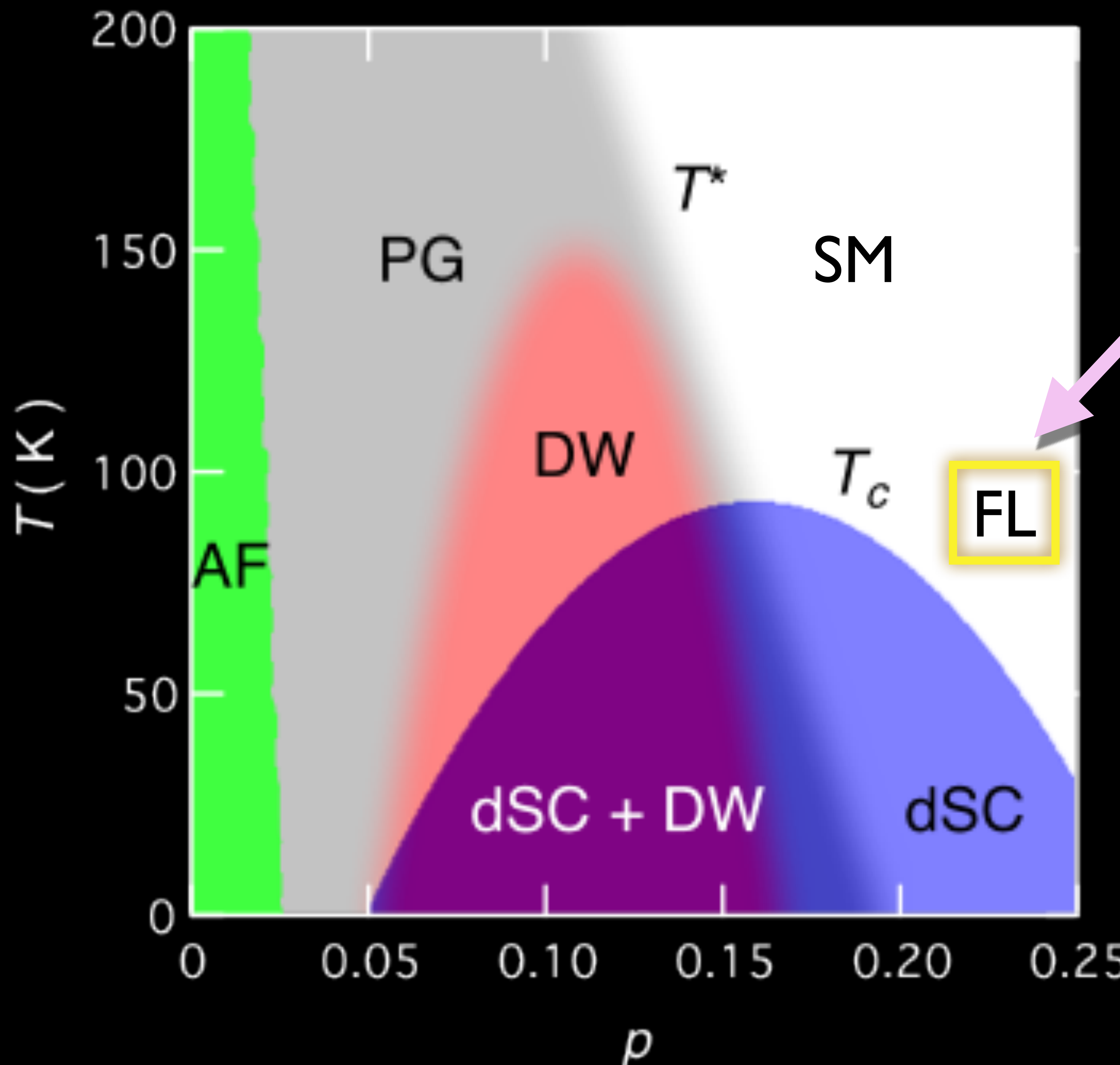


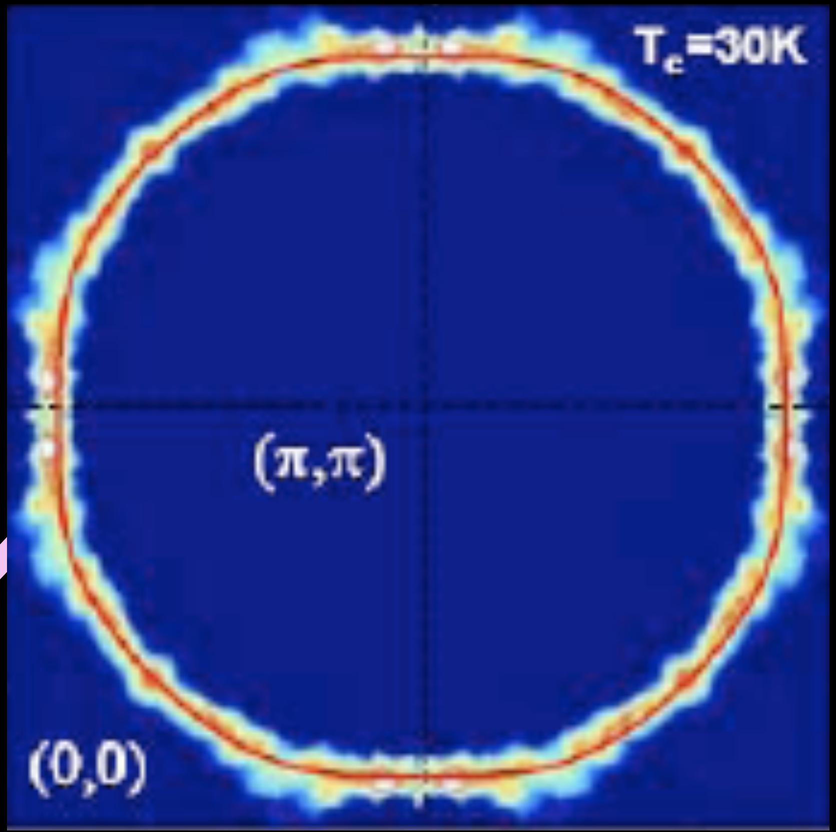


$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

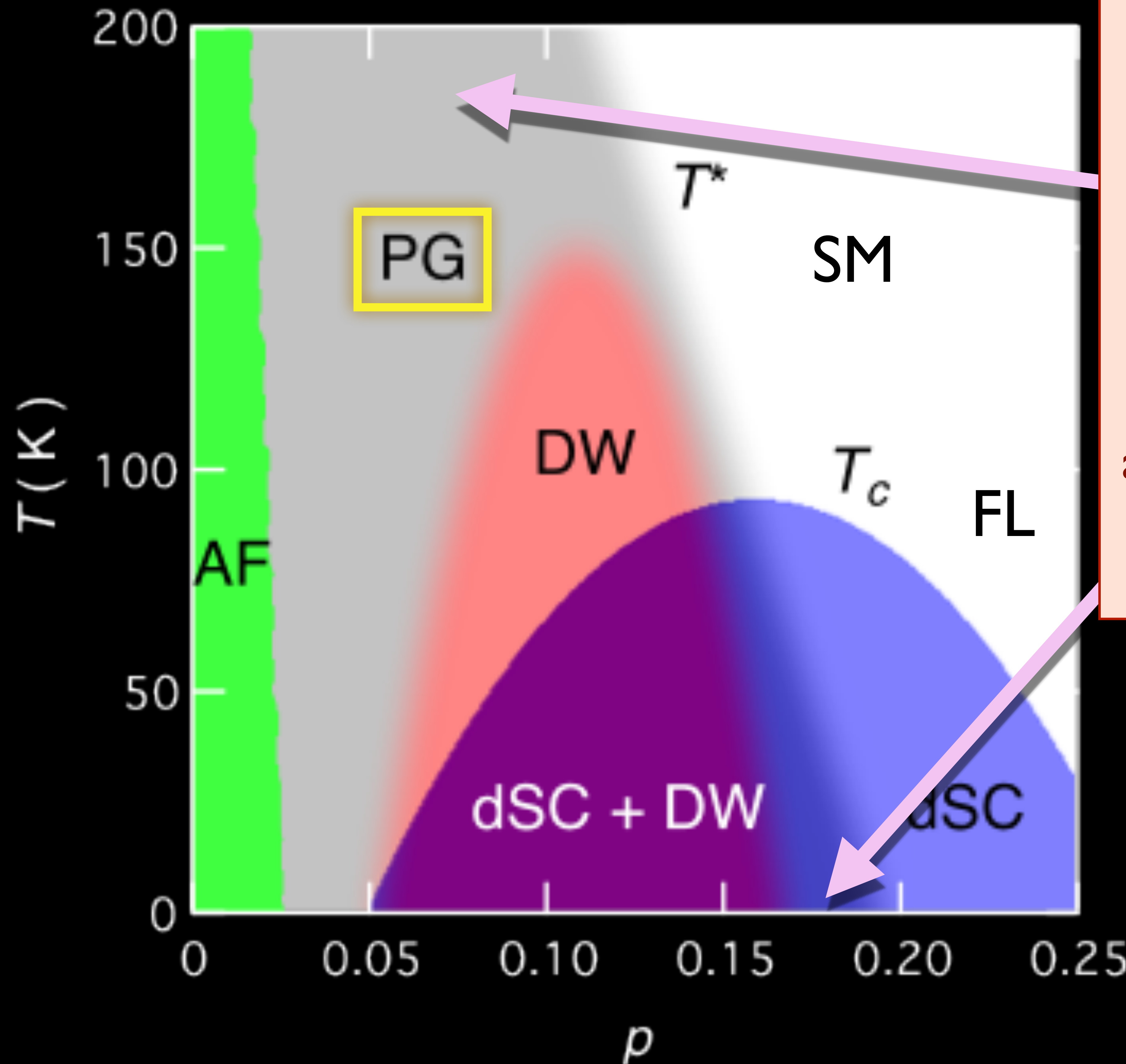
M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)





A conventional metal: the Fermi liquid with Fermi surface of size *1+p*

S. Badoux, W. Tabis, F. Laliberté, G. Grissonnanche, B. Vignolle, D. Vignolles, J. Béard, D.A. Bonn, W.N. Hardy, R. Liang, N. Doiron-Leyraud, L. Taillefer, and C. Proust, Nature **531**, 210 (2016).



Pseudogap metal at low p

Many indications that this metal behaves like a Fermi liquid, but with Fermi surface size p and *not* $1+p$.

Begin with the "spin-fermion" model. **Electrons** $c_{i\alpha}$ on the square lattice with dispersion

$$\mathcal{H}_c = -\sum_{i,\rho} t_\rho \left(c_{i,\alpha}^\dagger c_{i+\boldsymbol{v}_\rho,\alpha} + c_{i+\boldsymbol{v}_\rho,\alpha}^\dagger c_{i,\alpha} \right) - \mu \sum_i c_{i,\alpha}^\dagger c_{i,\alpha} + \mathcal{H}_{\rm int}$$

are coupled to an **antiferromagnetic order parameter** $\Phi^\ell(i)$, $\ell = x, y, z$

$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i \Phi^\ell(i) c_{i,\alpha}^\dagger \sigma_{\alpha\beta}^\ell c_{i,\beta} + V_\Phi$$

where $\eta_i = \pm 1$ on the two sublattices.

When $\Phi^\ell(i)$ =constant independent of i, we have long-range AFM, and a gap in the fermion spectrum at the anti-nodes.

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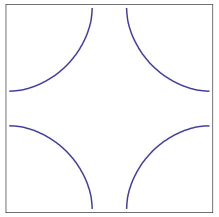

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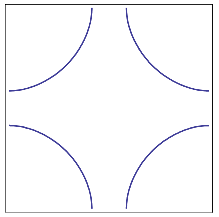

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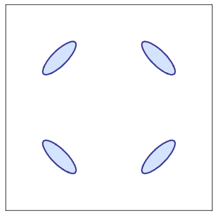
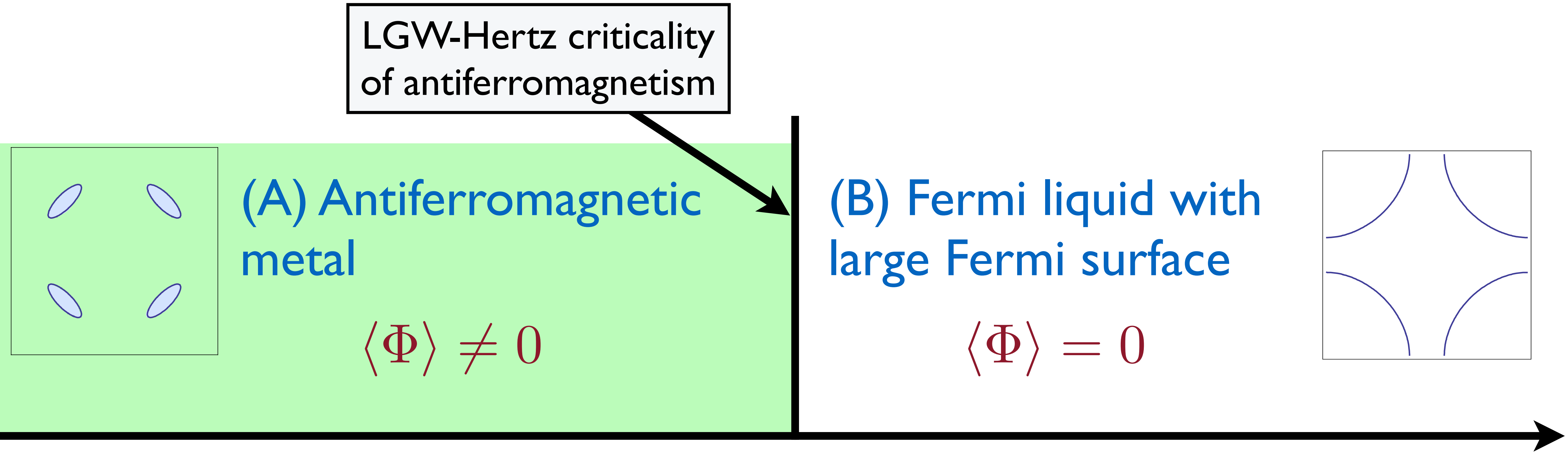
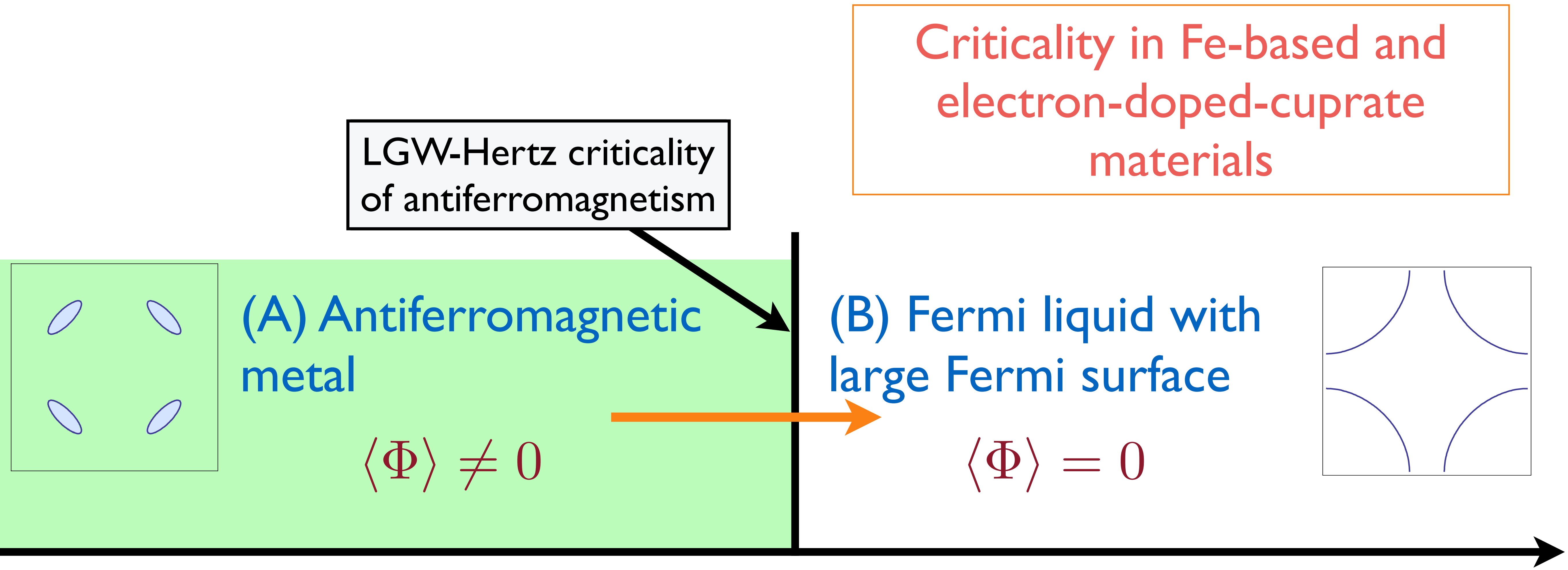



LGW-Hertz criticality
of antiferromagnetism
(A) Antiferromagnetic
metal
$\langle \Phi \rangle \neq 0$
(B) Fermi liquid with
large Fermi surface
$\langle \Phi \rangle = 0$


Criticality in Fe-based and electron-doped-cuprate materials
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Can we get a stable zero temperature state with “fluctuating antiferromagnetism” and a small Fermi surface (and so a gap near the anti-nodes) ?

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Yes, provided the metal has topological order

T. Senthil, M. Vojta and S. Sachdev, PRB **69**, 035111 (2004)

For fluctuating antiferromagnetism, we transform to a rotating reference frame using the SU(2) rotation R_i

$$\left(\begin{array}{c} c_{i\uparrow} \\ c_{i\downarrow} \end{array} \right) = R_i \left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right),$$

in terms of fermionic “chargons” ψ_s and a **Higgs field** $H^a(i)$

$$\sigma^\ell \Phi^\ell(i) = R_i \, \sigma^a H^a(i) \, R_i^\dagger$$

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The Higgs field is the AFM order in the rotating reference frame. Note that this representation is ambiguous up to a SU(2) gauge transformation, V_i

$$\left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right) \rightarrow V_i \left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right)$$

$$R_i \rightarrow R_i V_i^\dagger$$

$$\sigma^a H^a(i) \rightarrow V_i \, \sigma^b H^b(i) \, V_i^\dagger .$$

Fluctuating antiferromagnetism

The simplest effective Hamiltonian for the fermionic chargons is the same as that for the electrons, with the AFM order replaced by the Higgs field.

$$\mathcal{H}_\psi = -\sum_{i,\rho} t_\rho \left(\psi_{i,s}^\dagger \psi_{i+\boldsymbol{v}_\rho,s} + \psi_{i+\boldsymbol{v}_\rho,s}^\dagger \psi_{i,s} \right) - \mu \sum_i \psi_{i,s}^\dagger \psi_{i,s} + \mathcal{H}_{\rm int}$$

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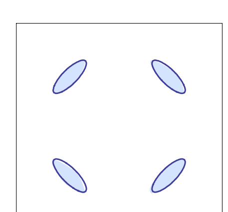
IF we can transform to a rotating reference frame in which $H^a(i) =$ a constant independent of i and time, **THEN** the ψ fermions in the presence of fluctuating AFM will inherit the anti-nodal gap of the electrons in the presence of static AFM.



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We cannot always find a single-valued SU(2) rotation R_i to make the Higgs field $H^a(i)$ a constant !

A.V. Chubukov, T. Senthil and S. Sachdev, PRL **72**, 2089 (1994);
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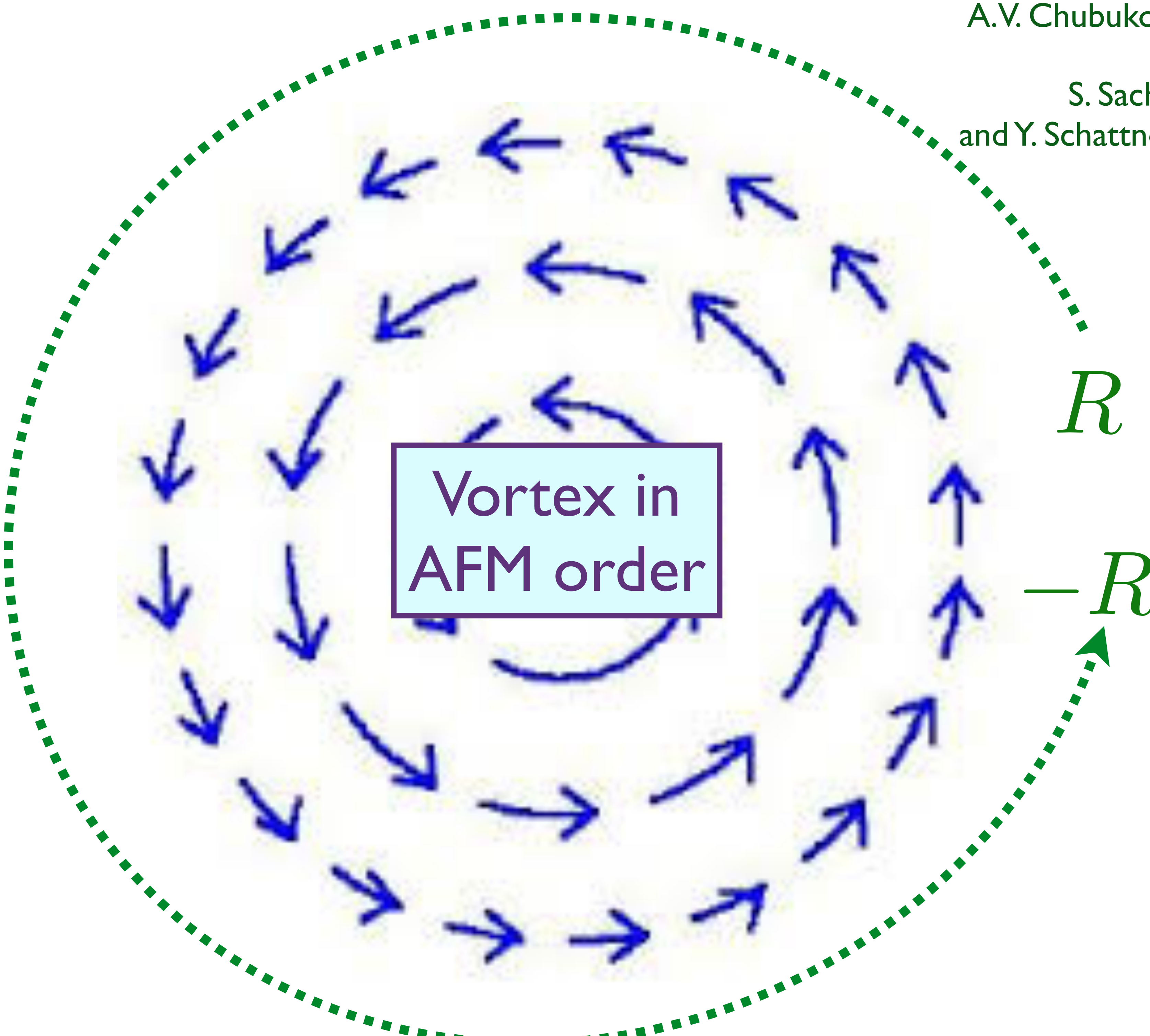

Topological order

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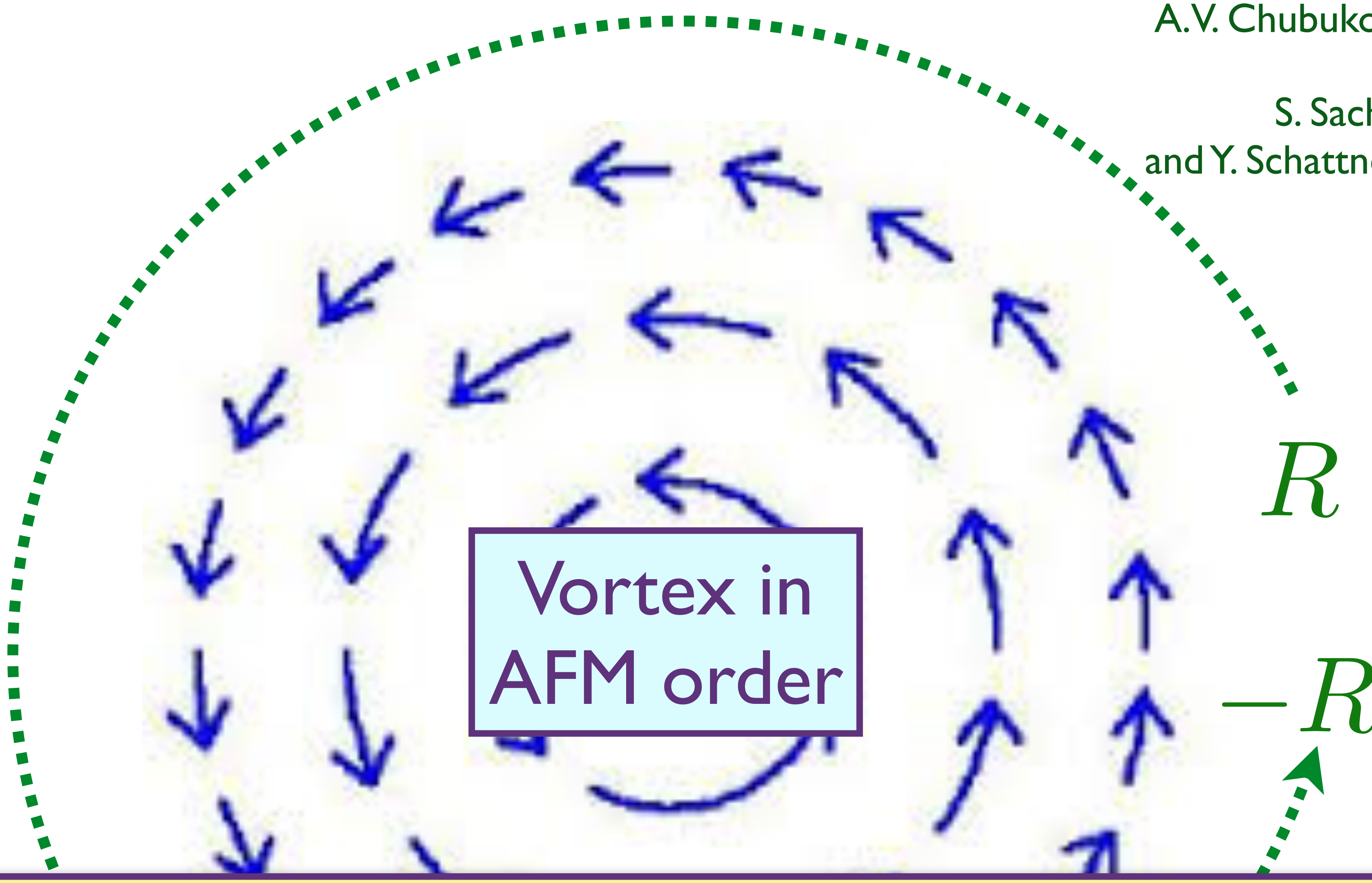
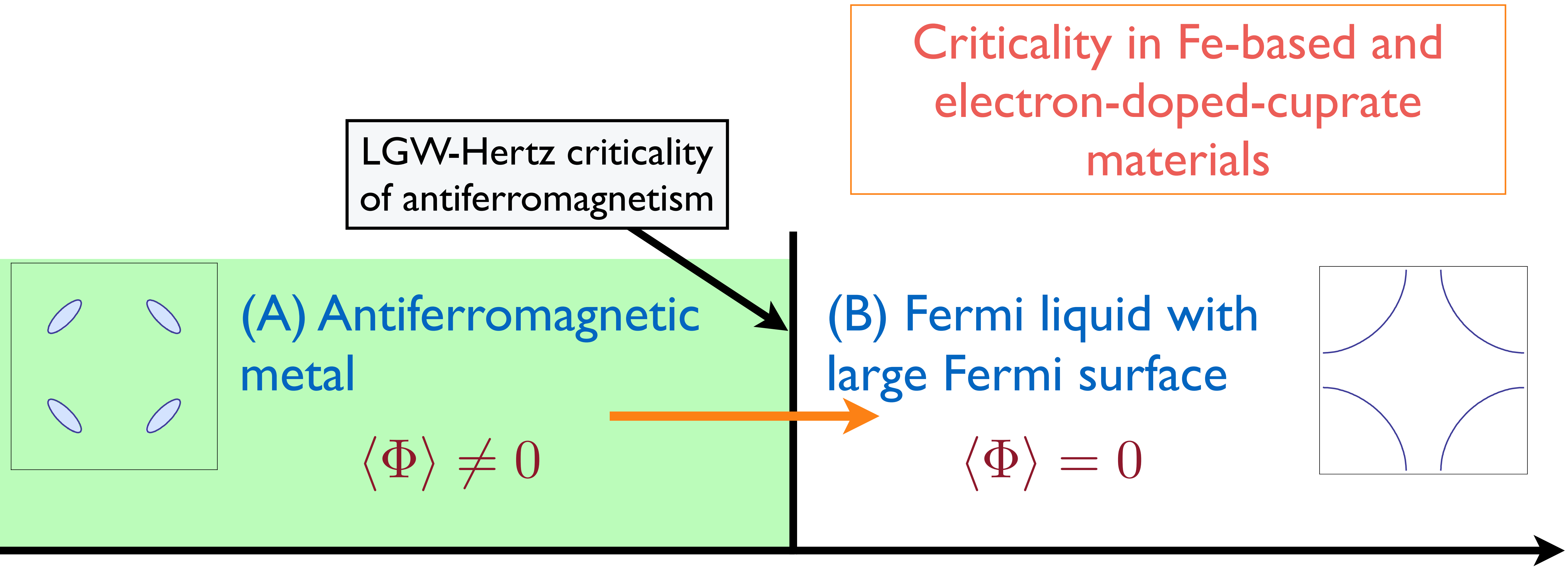


Vortices associated with $\pi_1(\mathrm{SO}(3)) = \mathbb{Z}_2$ must be suppressed: such a metal with "fluctuating antiferromagnetism" has $\mathbb{Z}_2$ TOPOLOGICAL ORDER and fermions which inherit the Fermi surfaces of the antiferromagnetic metal *i.e.* a pseudogap.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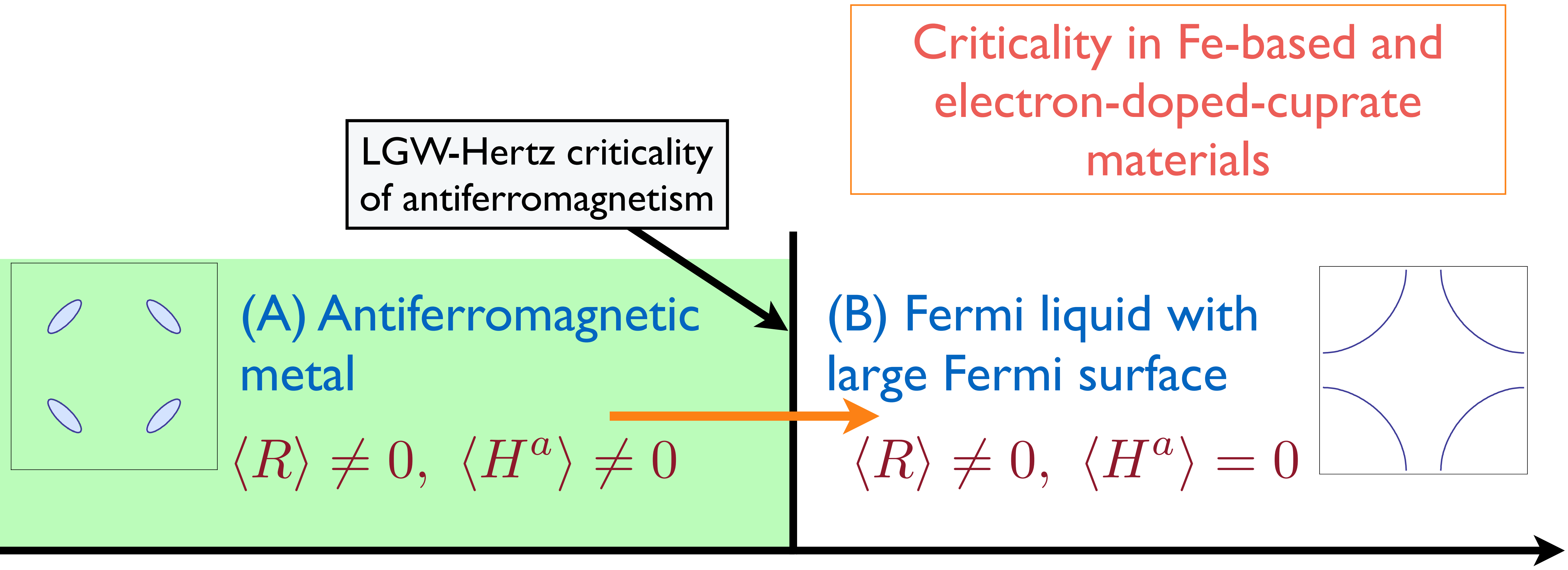
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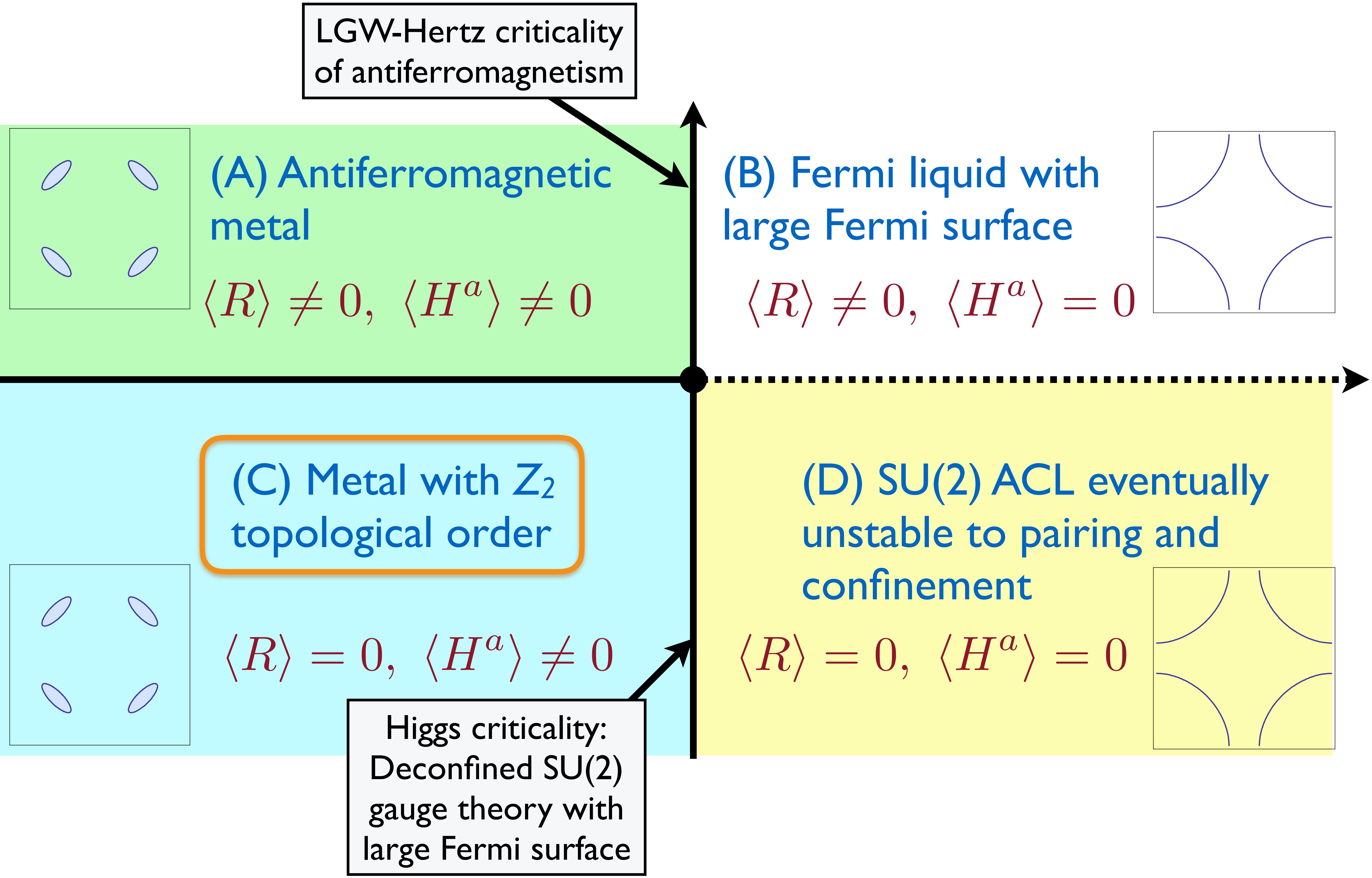
Global phase diagram



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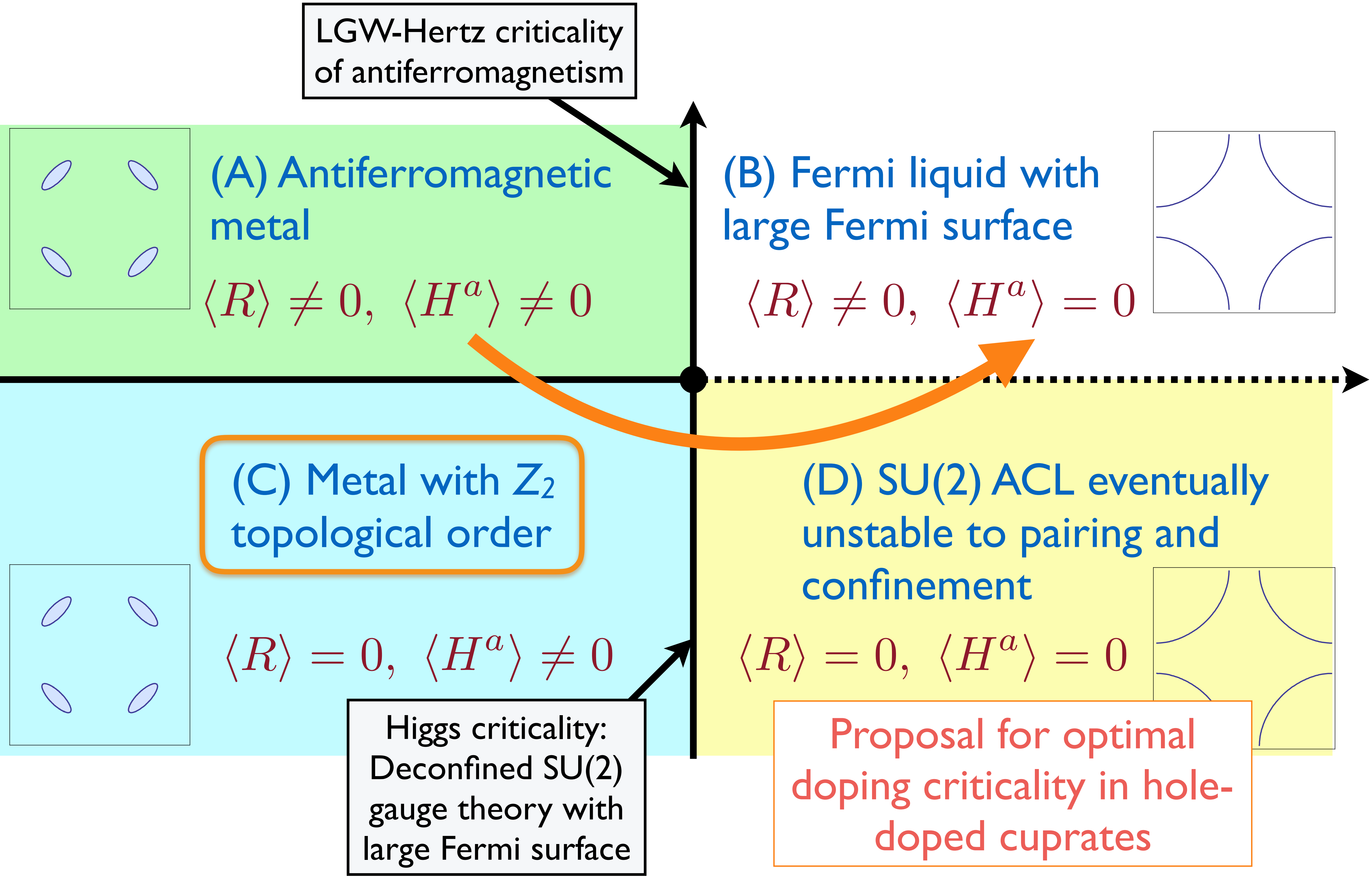

Topological order

More generally, the effective Hamiltonian for the fermionic chargons can also have non-trivial SU(2) gauge connections $U^\rho(i)$ along with the Higgs field $H^a(i)$.

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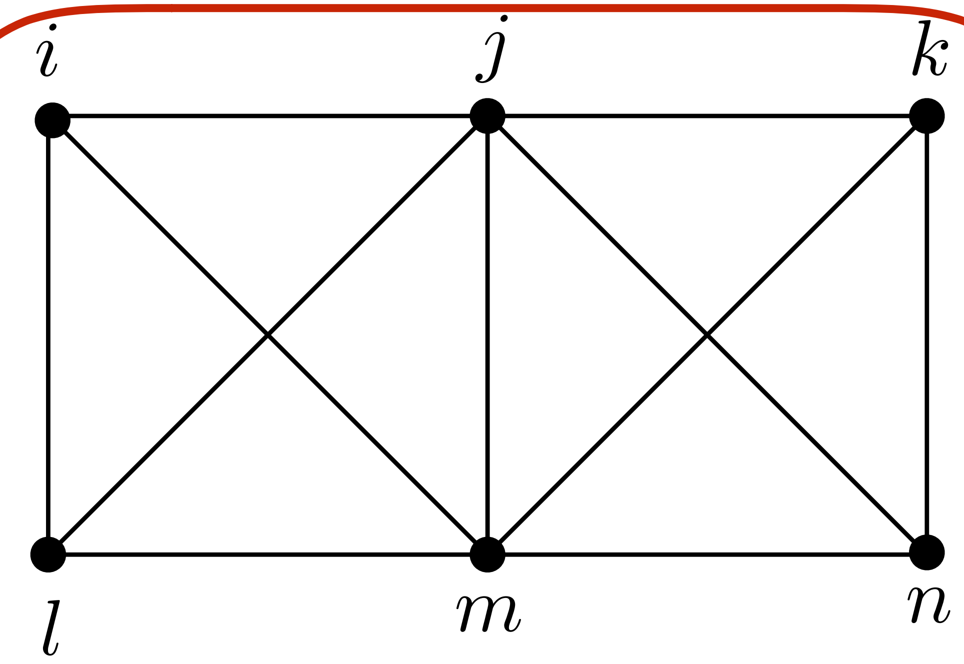
Such a gauge-connection can induce various gauge-invariant fluxes which can break one or more of time-reversal, inversion, and lattice rotation symmetries.

Topological order



Gauge-invariant combinations of Higgs fields and gauge connections which are proportional to the electrical current on links

$$
\begin{aligned}
O_{mj} = & \; i \,\mathrm{Tr}\left(\sigma^a U_{mj} U_{jk} U_{km}\right) H^a(m) \\
& - i \,\mathrm{Tr}\left(\sigma^a U_{jm} U_{mn} U_{nj}\right) H^a(j) \\
& + i \,\mathrm{Tr}\left(\sigma^a U_{mj} U_{ji} U_{im}\right) H^a(m) \\
& - i \,\mathrm{Tr}\left(\sigma^a U_{jm} U_{ml} U_{lj}\right) H^a(j)
\end{aligned}
$$

$$
\begin{aligned}
O_{mk} = & \; i \,\mathrm{Tr}\left(\sigma^a U_{mj} U_{jk} U_{km}\right) H^a(m) \\
& - i \,\mathrm{Tr}\left(\sigma^a U_{kj} U_{jm} U_{mk}\right) H^a(k) \\
& + i \,\mathrm{Tr}\left(\sigma^a U_{mn} U_{nk} U_{km}\right) H^a(m) \\
& - i \,\mathrm{Tr}\left(\sigma^a U_{kn} U_{nm} U_{mk}\right) H^a(k)
\end{aligned}
$$

S. Sachdev and S. Chatterjee, arXiv:1703.00014

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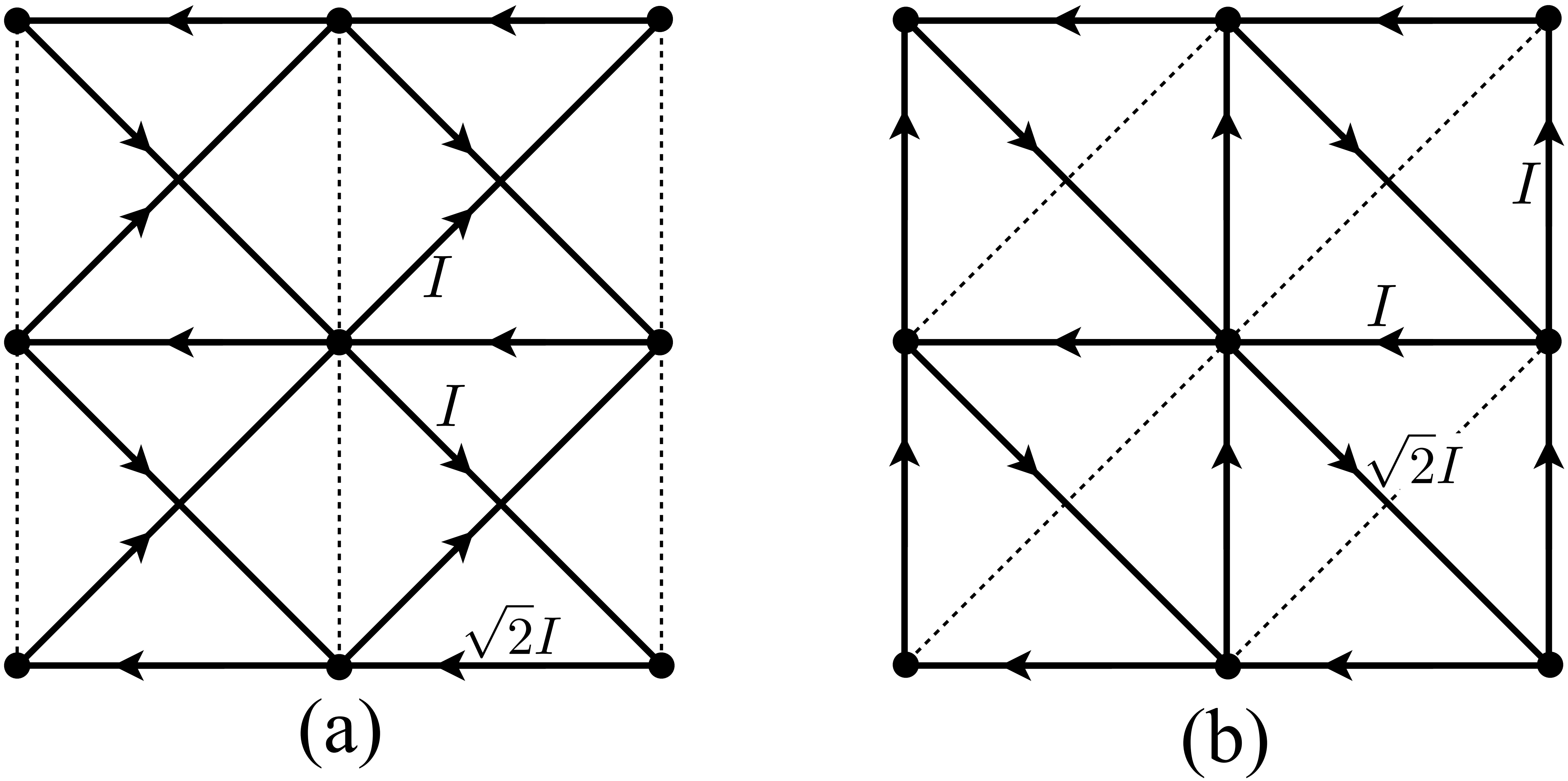
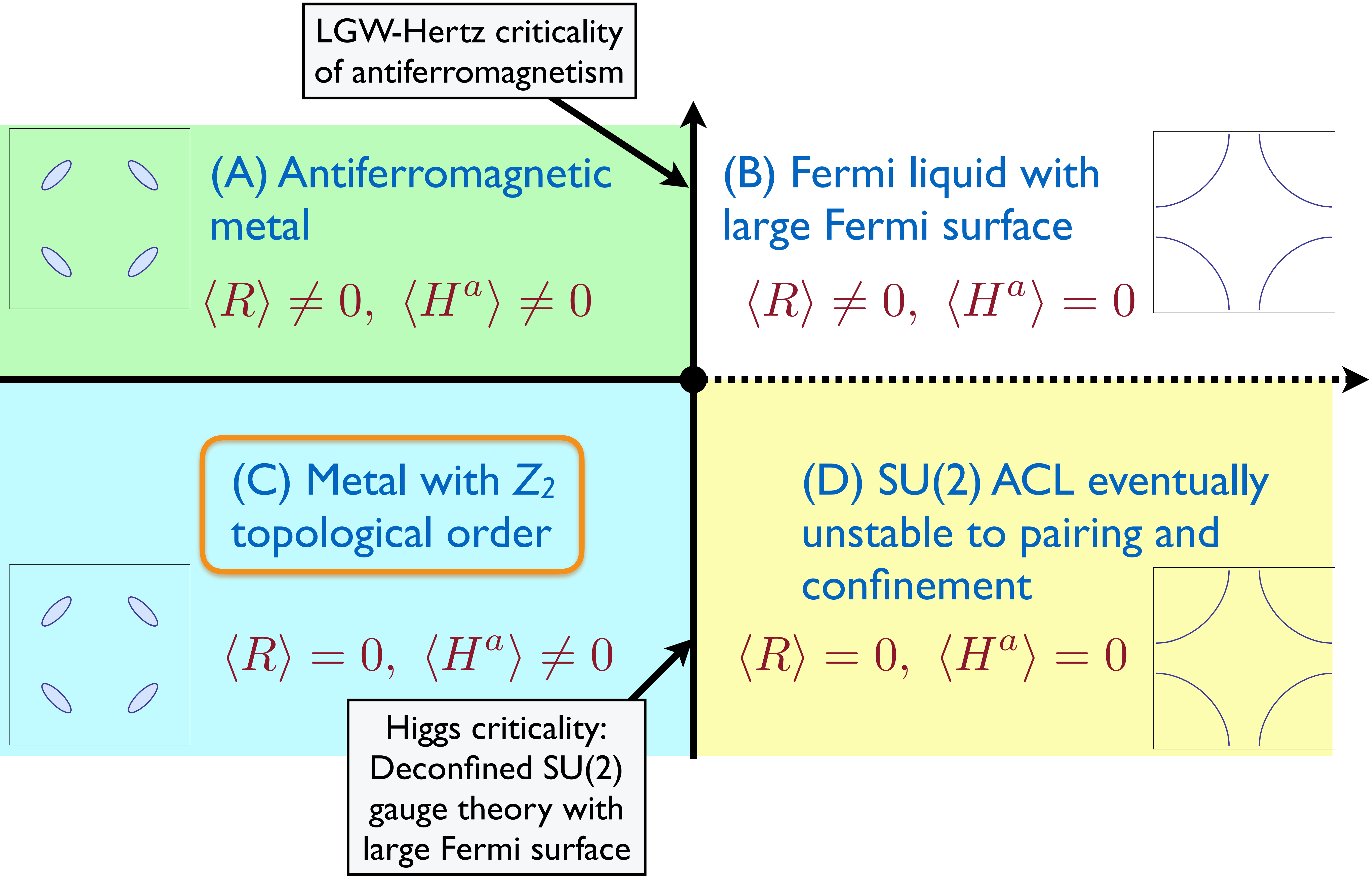


States with topological order can have these patterns of spontaneous currents, while preserving translational symmetry. Both patterns are consistent with present neutron and light scattering experiments. Both patterns have Ising-nematic order: the Ising-nematic order of (a) is similar to that observed in the cuprates.

Global phase diagram



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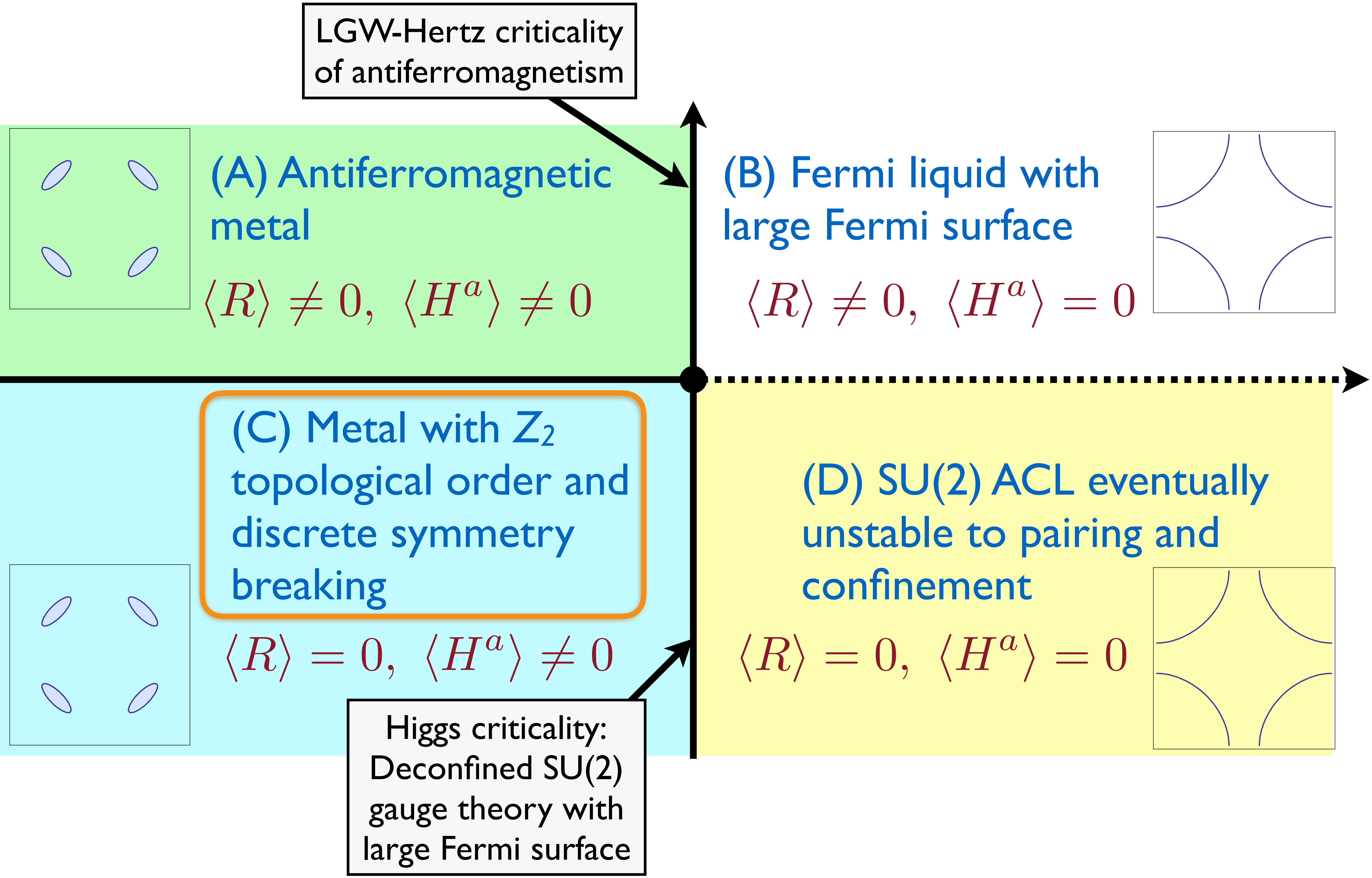

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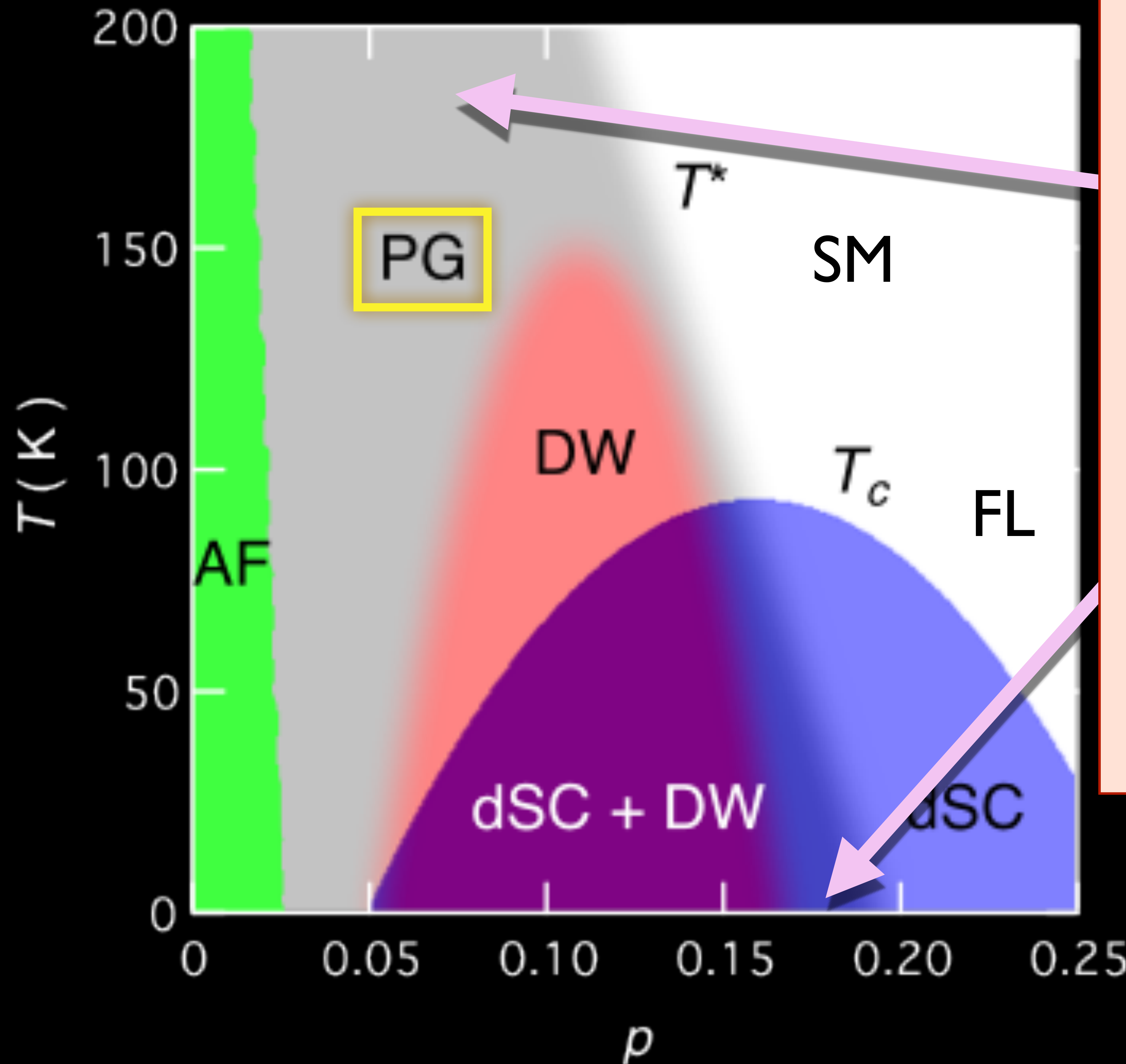
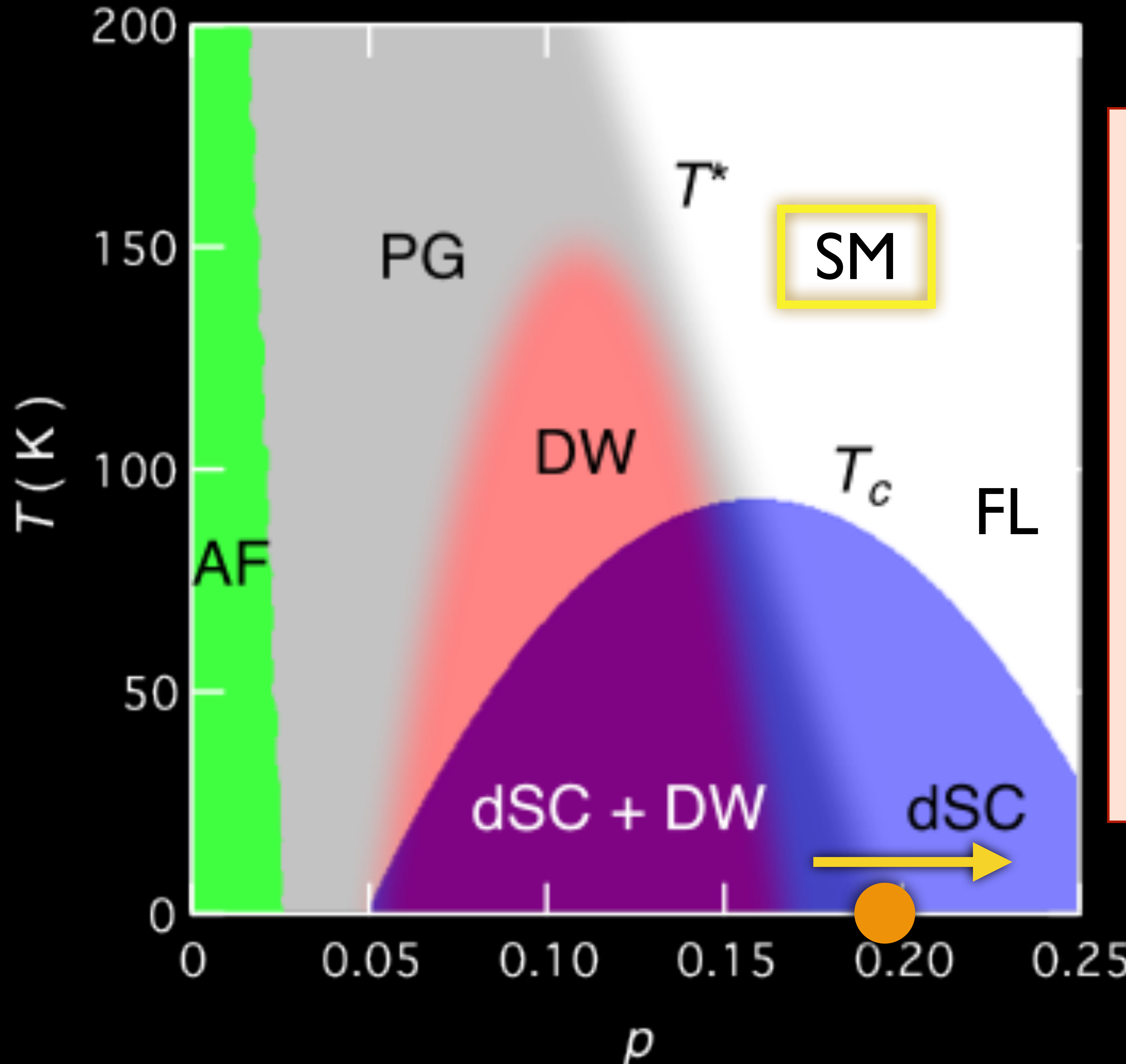


Pseudogap metal at low p

Lattice gauge theory for a metal with topological order co-existing with broken time-reversal and inversion symmetries, and Ising-nematic order

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009); D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015); S. Sachdev and D. Chowdhury, arXiv:1605.03579.



Gauge theory for a topological phase transition, and for the strange metal (SM)