

Consider modulation in a bilocal variable at the Cu sites \mathbf{r}_i and \mathbf{r}_j

$$\left\langle c_{i\alpha}^\dagger c_{j\alpha} \right\rangle \sim \left[\sum_{\mathbf{k}} P_{\mathbf{Q}}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2}$$

The wavevector \mathbf{Q} is associated with a modulation in the *average* co-ordinate $(\mathbf{r}_i + \mathbf{r}_j)/2$: this determines the wavevector of the X-ray scattering peak.

The interesting part is the dependence on the *relative* co-ordinate $\mathbf{r}_i - \mathbf{r}_j$. The order parameter $P_{\mathbf{Q}}(\mathbf{k})$ can always be expanded as

$$\begin{aligned} P_{\mathbf{Q}}(\mathbf{k}) &= \sum_{\ell} \mathcal{P}_{\ell} \phi_{\ell}(\mathbf{k}) \\ \phi_{\ell}(\mathbf{k}) &= \{1, \cos k_x + \cos k_y, \cos k_x - \cos k_y, \sin k_x - \sin k_y, \dots\} \end{aligned}$$

The usual charge-density-wave has only $\mathcal{P}_s \neq 0$, and so the density wave is non-zero only if $\mathbf{r}_i = \mathbf{r}_j$.

The bond-ordered state has predominantly $\mathcal{P}_{s'}, \mathcal{P}_d$ non-zero: in this case the density wave is non-zero only if \mathbf{r}_i and \mathbf{r}_j are nearest neighbors.

States with spontaneous currents have \mathcal{P}_p non-zero: they break time-reversal