Quantum phase transitions in condensed matter

Session L04: Lars Onsager Prize
APS March Meeting, Los Angeles

Subir Sachdev
March 7, 2018

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Talk online: sachdev.physics.harvard.edu

NSF

Thanks to students and postdocs, and many other collatorators

Students

- Jinwu Ye, Associate Professor, Department of Physics and Astronomy, Mississippi State University
  Thesis: Some Examples of Quantum Phase Transitions
- T. Senthil, Professor, Department of Physics, Massachusetts Institute of Technology.
  Thesis: Quantum Phase Transitions in Random Spin Systems
- Kedar Damle, Department of Theoretical Physics, Tata Institute of Fundamental Research, Mumbai, India.
  Thesis: Turning on the Heat:Non-zero Temperature Dynamical Properties of Quantum Many-body Systems
- Chiranjeeb Buragohain, Microsoft Research.
  Thesis: Dynamical Properties of Quantum Antiferromagnets in One and Two Dimensions
- Ying Zhang, Finisterre Capital, London.
  Thesis: Competing Orders in the Cuprate Superconductors
- Anatoli Polkovnikov, Associate Professor, Boston University.
  Thesis: Manifestation of Quantum Fluctuations in Strongly Correlated Systems
- Stephen Powell, Assistant Professor, University of Nottingham
  Thesis: Quantum phases and transitions of many-body systems realized using cold atomic gases
- Adrian Del Maestro, Associate Professor, University of Vermont
  Thesis: The superconductor-metal quantum phase transition in ultra-narrow wires
- Emily Dunkel (with David Coker, Boston University), NASA Jet Propulsion Laboratory
  Thesis: Quantum Phenomena in Condensed Phase Systems
- Yang Qi, Institute for Advanced Studies, Tsinghua University
  Thesis: Spin and Charge Fluctuations in Strongly Correlated Systems.
- Rudro Rana Biswas, Assistant Professor, Purdue University
  Thesis: Explorations in Dirac Fermions and Spin Liquids.
- Eun Gook Moon, Assistant Professor, Korea Advanced Institute of Science and Technology
  Thesis: Superfluidity in Strongly Correlated Systems
- Max Metlitski, Assistant Professor, Department of Physics, Massachusetts Institute of Technology
  Thesis: Aspects of Critical Behavior of Two Dimensional Electron Systems
- Yejin Huh, Applied Scientist at Apple
  Thesis: Quantum Phase Transitions in d-wave Superconductors and Antiferromagnetic Kagome Lattices
- Susanne Pielawa, Lyft, Munich
  Thesis: Metastable Phases and Dynamics of Low-Dimensional Strongly-Correlated Atomic Quantum Gases
- Debanjan Chowdhury, Moore Foundation Postdoctoral Fellow, MIT
  Thesis: Interplay of Broken Symmetries and Quantum Criticality in Correlated Electronic Systems
- Junhyun Lee, Postdoctoral fellow, University of Maryland
  Thesis: Novel quantum phase transitions in low-dimensional systems
- Andrew Lucas, Postdoctoral fellow, Stanford University
  Thesis: Transport and hydrodynamics in holography, strange metals and graphene
- Shubhayu Chatterjee, Harvard University
- Aavishkar Patel, Harvard University
- Wenbo Fu, Harvard University
- Seth Whitsitt, Harvard University
- Alex Thomson, Harvard University
- Julia Steinberg, Harvard University

Thanks to students and postdocs, and many other collatorators

Postdocs

- Pierre Le Doussal, Directeur de Recherche de Classe Exceptionnelle, Laboratoire de Physique Théorique de l' Ecole Normale Supérieure, Paris, France.
- Rodolfo Jalabert, Professeur à l'Université Louis Pasteur, Institut de Physique et Chimie des Matériaux de Strasbourg, France.
- Andrey Chubukov, William I. and Bianca M. Fine Chair in Theoretical Physics, University of Minnesota, Minneapolis.
- Satya Majumdar, Directeur de Recherche, Laboratoire de Physique Théorique et Modèles Statistiques, University of Paris XI, France.
- Matthias Vojta, Chair of Theoretical Solid State Physics, Technische Universität, Dresden, Germany
- Oleg Starykh, Professor, Department of Physics, University of Utah.
- Marcus Kollar, Theoretische Physik III, Institut für Physik, Universität Augsburg, Germany.
- Kwon Park, Professor, Korea Institute for Advanced Study, Seoul.
- Takao Morinari, Kyoto University, Kyoto, Japan.
- Adam Durst, Associate Professor, Hofstra University.
- Krishnendu Sengupta, Professor, Indian Association for the Cultivation of Science, Kolkata, India.
- Lorenz Bartosch, Assistant Professor, University of Frankfurt.
- Predrag Nikolic, Associate Professor, George Mason University
- Ribhu Kaul, Associate Professor, University of Kentucky
- Markus Müller, Scientist, Paul Scherrer Institute, Switzerland.
- Lars Fritz, Assistant Professor, University of Utrecht
- Michael Levin, Associate Professor, University of Chicago
- Cenke Xu, Associate Professor, University of California, Santa Barbara
- Sean Hartnoll, Associate Professor, Stanford University
- Erez Berg, Associate Professor, University of Chicago
- Liang Fu, Lawrence C. (1944) and Sarah W. Biedenharn Career Development Associate Professor of Physics, Massachusetts Institute of Technology
- Liza Huijse, Software Engineer at Karius, Inc.
- Chris Laumann, Assistant Professor, Boston University
- Matthias Punk, Faculty, LMU Munich
- Philipp Strack, ZEISS Group
- Brian Swingle, Assistant Professor, University of Maryland
- Dmitry Abanin, Professor of Physics, University of Geneva
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- Jay Sau, Assistant Professor, University of Maryland
- Sarang Gopalakrishnan, Postdoctoral Fellow, Caltech
- Andrea Allais, Cruise Automation, San Francisco
- Johannes Bauer, SCL Group, London
- Paul Chesler, Harvard University
- Andreas Eberlein, Harvard University
- William Witczak-Krempa, Assistant Professor, University of Montreal
- Richard Davison, Harvard University
- Chong Wang, Harvard University
- Mathias Scheurer, Harvard University.

Continuous quantum transitions

(A)

Continuous quantum transitions

Exact solution by Onsager of the *D*=2 Ising model

Onsager Prizes:
M.E. Fisher
L. Kadanoff
A. I. Larkin
V. L. Pokrovsky

Continuous quantum transitions

(A) Broken symmetry | No broken symmetry

(B) Topological order | No topological order

(C) Topological order | Broken symmetry

(D) Broken symmetry | A different broken symmetry

Continuous quantum transitions

Theory with emergent gauge fields: Higgs/ confining phases and phase transitions

try

(B) Topological order | No topological order

(C) Topological order | Broken symmetry

(D) Broken symmetry | A different broken symmetry

Continuous quantum transitions

(A) Broken symmetry | No broken symmetry

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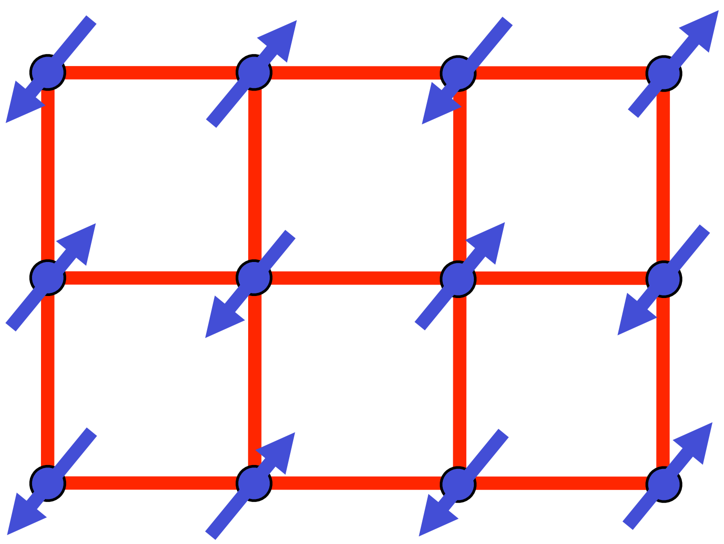
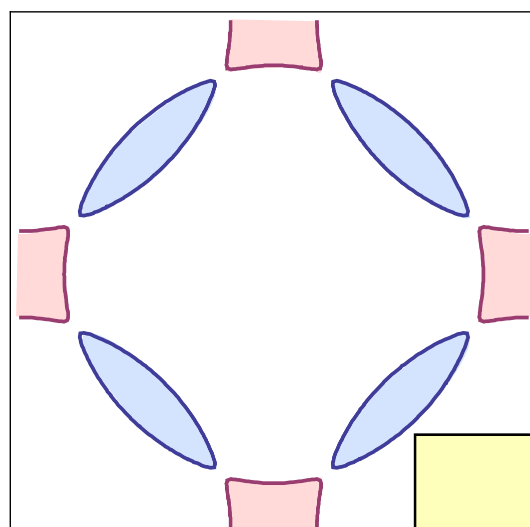
Antiferromagnetism in the Hubbard Model

$$H = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) - \mu \sum_i c_{i\alpha}^\dagger c_{i\alpha}$$

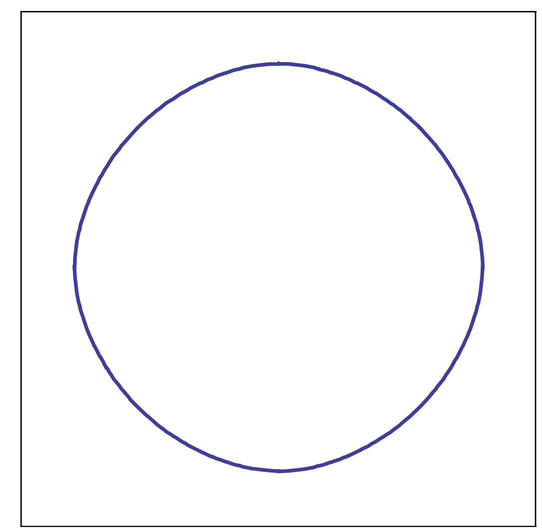
$t_{ij} \rightarrow$ "hopping". $U \rightarrow$ local repulsion, $\mu \rightarrow$ chemical potential

Mean-field theory with a spin density wave (SDW) order parameter $\vec{\Phi}_i = (-1)^{i_x+i_y} \left\langle c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta} \right\rangle /2$





(A) Symmetry breaking phase transition



SDW LRO

Reconstructed Fermi surface

$\langle \vec{\Phi} \rangle \neq 0$

SDW SRO

Large Fermi surface.

$\langle \vec{\Phi} \rangle = 0$

U/t

Antiferromagnetism in the Hubbard Model

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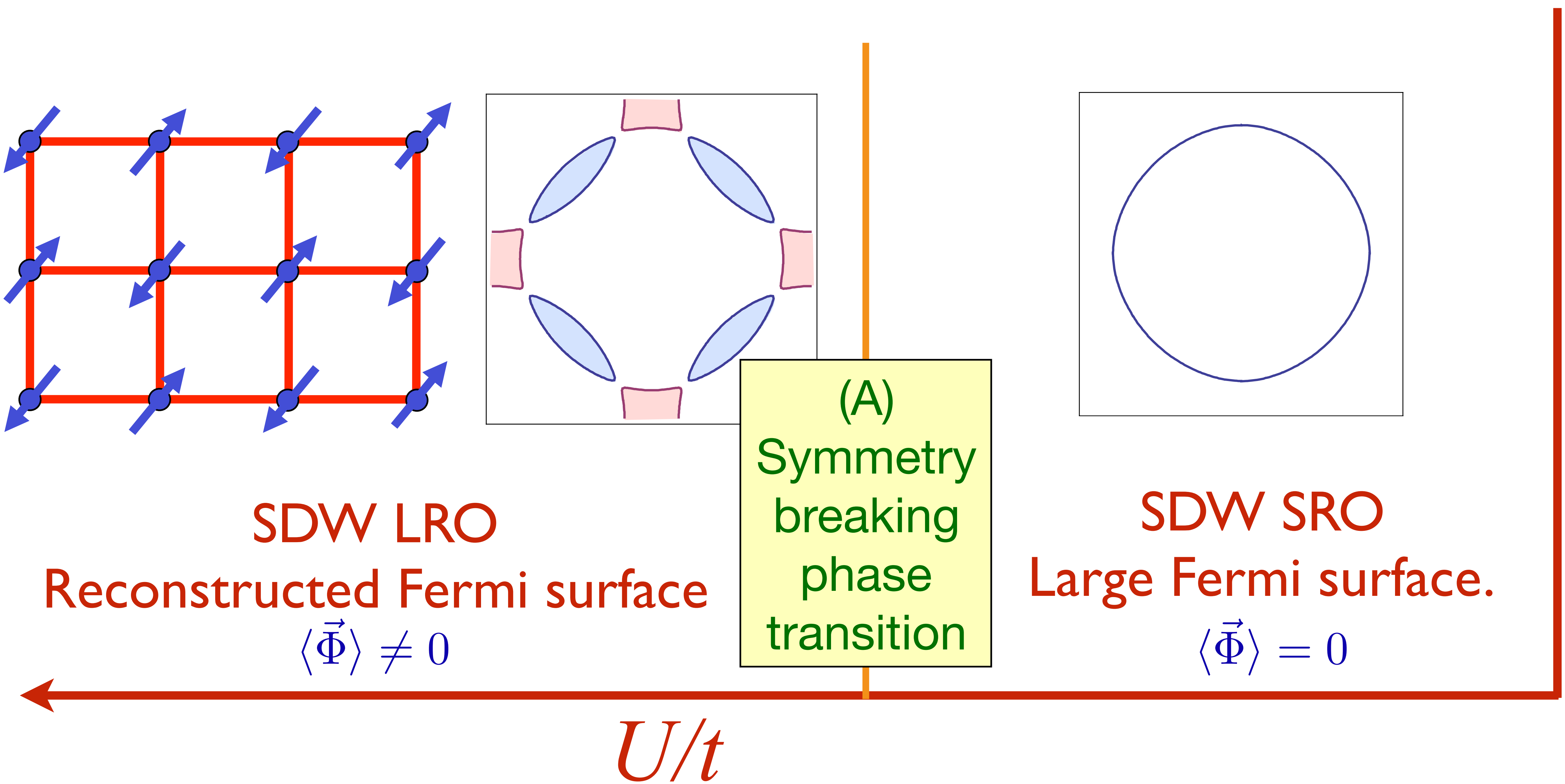
$t_{ij} \rightarrow$ "hopping". $U \rightarrow$ local repulsion, $\mu \rightarrow$ chemical potential

Both states have Luttinger volume Fermi surfaces



Continuous quantum transitions

(A) Broken symmetry | No broken symmetry

(B) Topological order | No topological order

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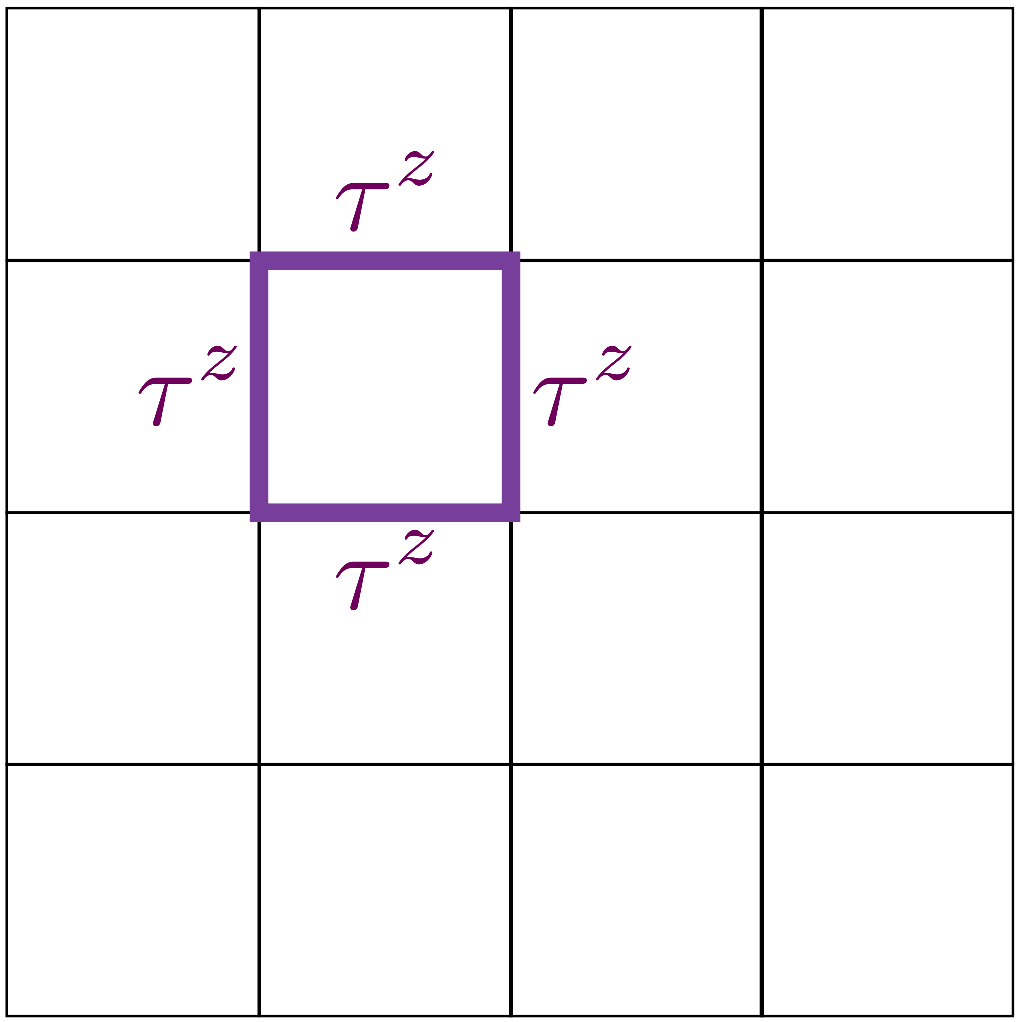
(D) Broken symmetry | A different broken symmetry

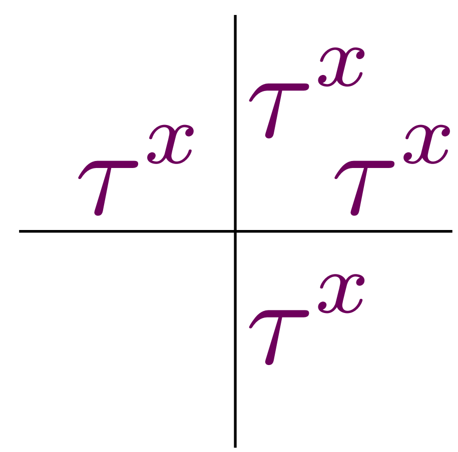
Z_2 lattice gauge theory

(Wegner, 1971
Onsager Prize, 2015)



$$H = -\sum_{\square} \tau^z \tau^z \tau^z \tau^z - g \sum_i \tau^x$$

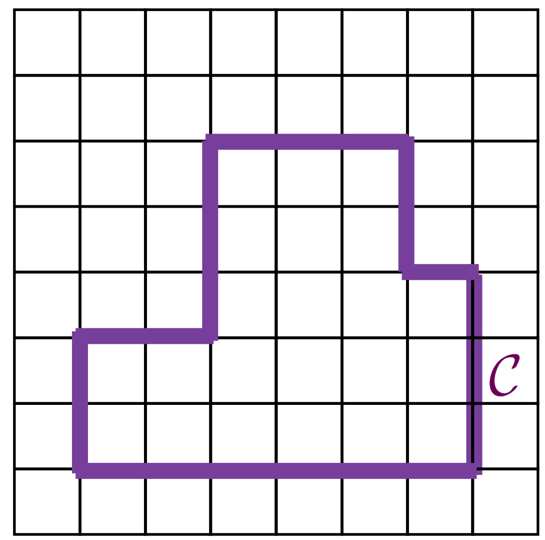
$G_i =$



Gauss's Law: $[H, G_i] = 0 \quad , \quad G_i = 1$

Z_2 lattice gauge theory

(Wegner, 1971
Onsager Prize, 2015)



$$W_{\mathcal{C}} = \prod_{\mathcal{C}} \tau^z$$

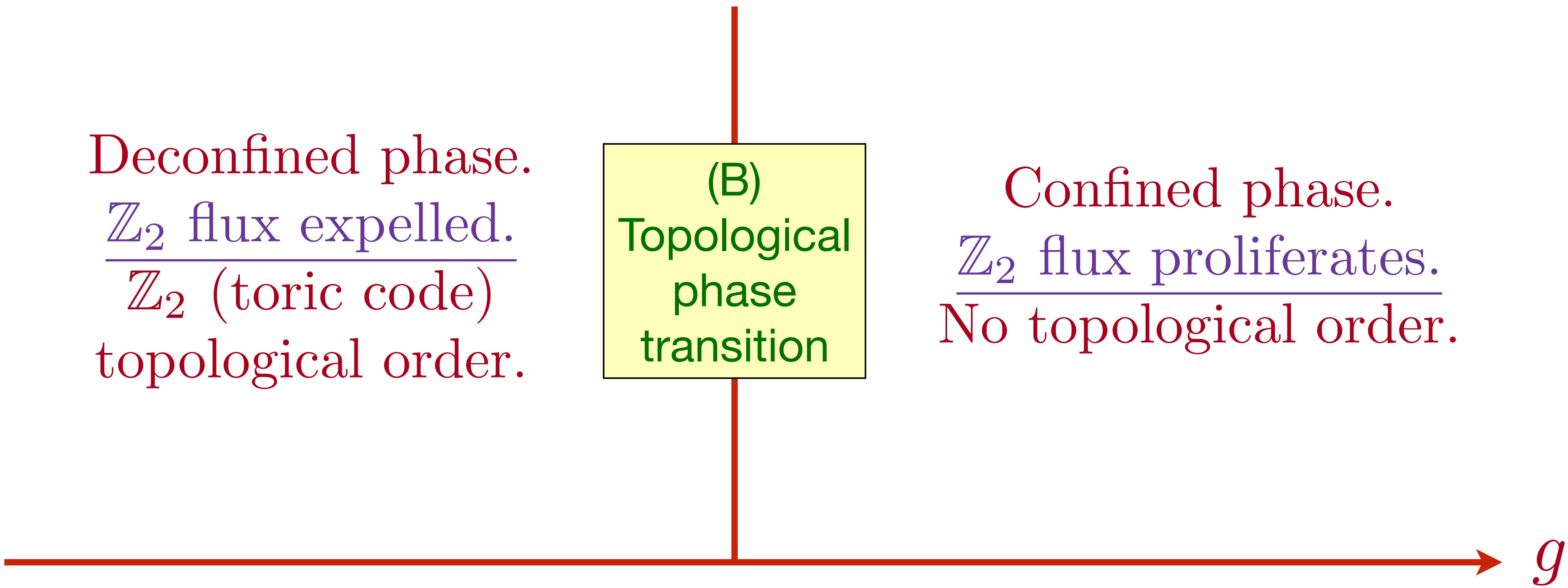
Deconfined phase.
'Perimeter law' for
Wegner-Wilson loops

Confined phase.
'Area law' for
Wegner-Wilson loops

g

$$H = -\sum_{\Box} \tau^z \tau^z \tau^z \tau^z - g \sum_i \tau^x$$

$\mathbb{Z}_2$ lattice gauge theory



$$H = -\sum_{\Box} \tau^z \tau^z \tau^z \tau^z - g \sum_i \tau^x$$

E. Fradkin and S. H. Shenker, PRD **19**, 3682 (1979); N. Read and S. Sachdev, PRL **66**, 1773 (1991); X.-G. Wen, PRB **44**, 2664 (1991); A.Y. Kitaev, Annals of Physics **303**, 2 (2003)

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Mean-field theory with a spin density wave (SDW)
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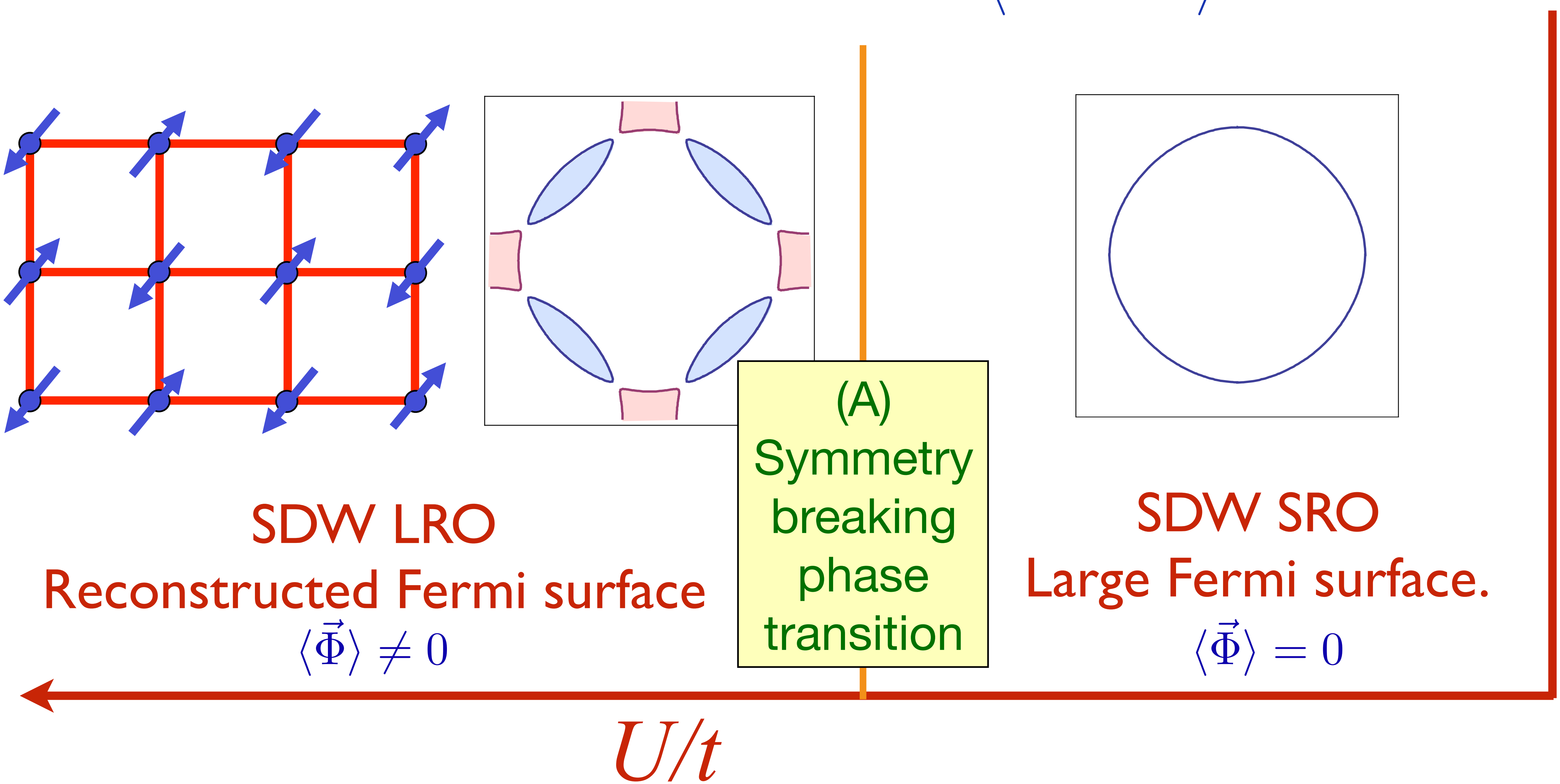
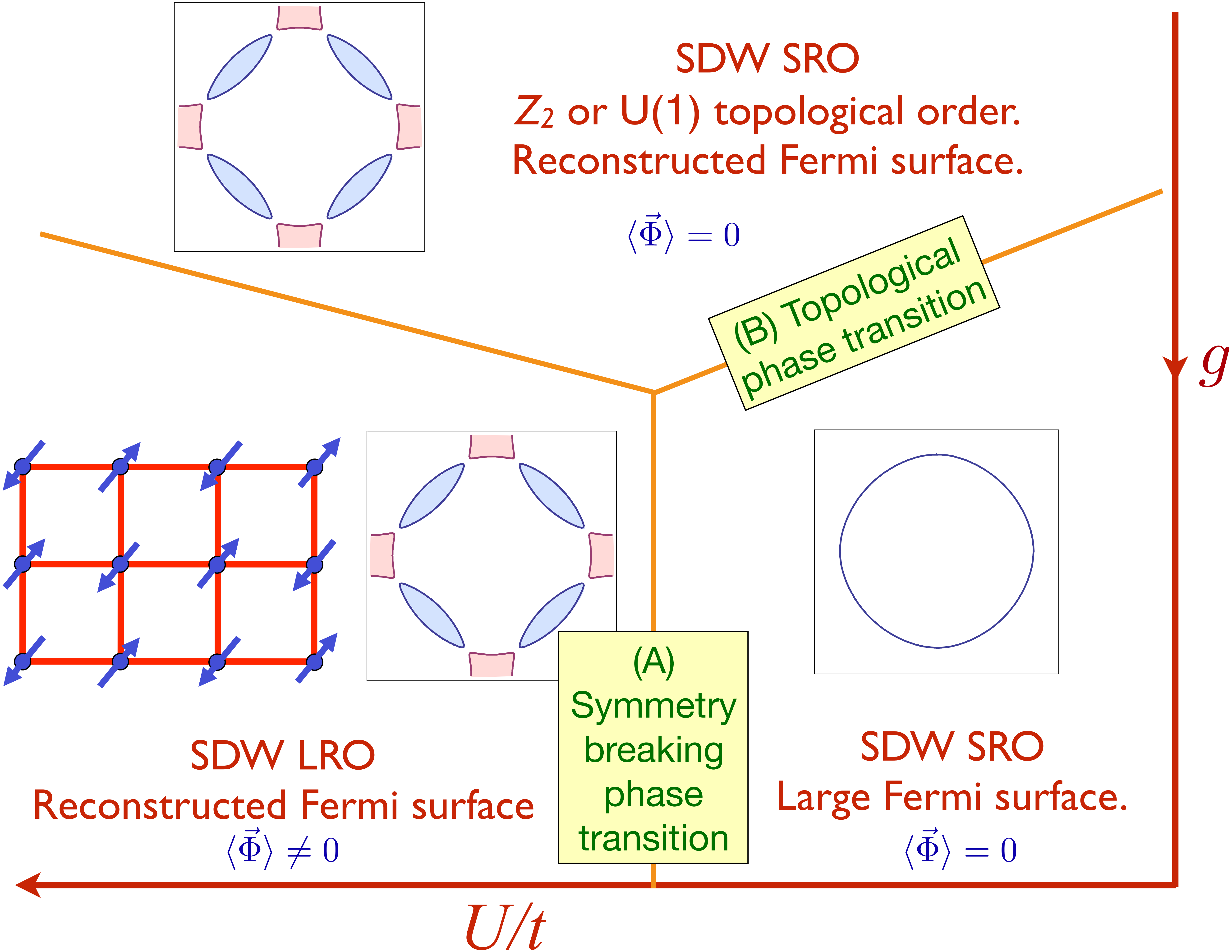
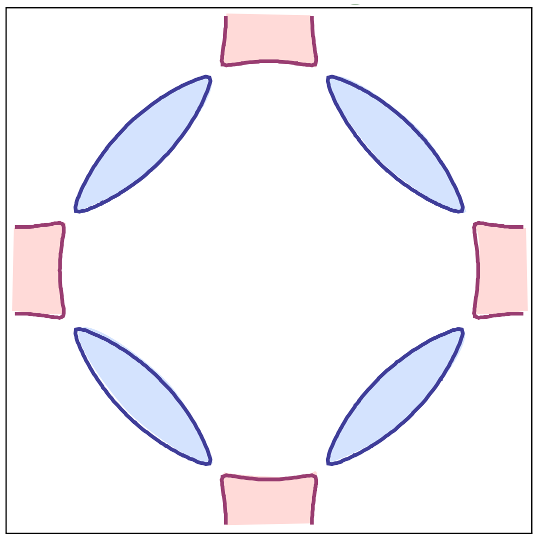
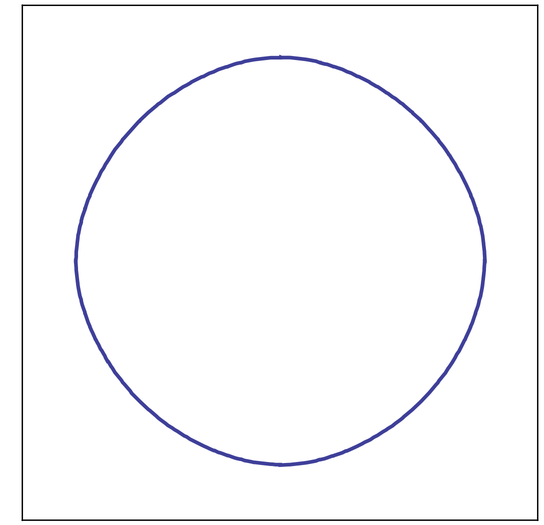




SDW SRO
Z_2 or U(1) topological order.
Reconstructed Fermi surface.
$\langle \vec{\Phi} \rangle = 0$
(B) Topological phase transition
g
SDW LRO
Reconstructed Fermi surface
$\langle \vec{\Phi} \rangle \neq 0$
(A) Symmetry breaking phase transition
SDW SRO
Large Fermi surface.
$\langle \vec{\Phi} \rangle = 0$
U/t



(B)
Topological
phase
transition



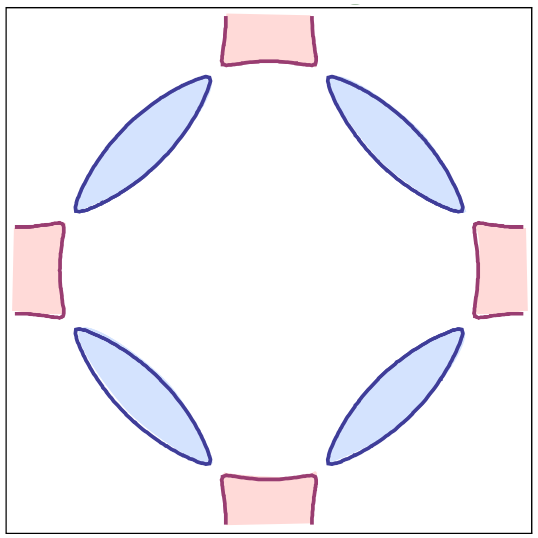
SDW SRO
Reconstructed Fermi surface
with non-Luttinger volume.
Z_2 vortices or hedgehogs expelled.
Z_2 or U(1) topological order.

SDW SRO
Large Fermi surface
with Luttinger volume.
No topological order

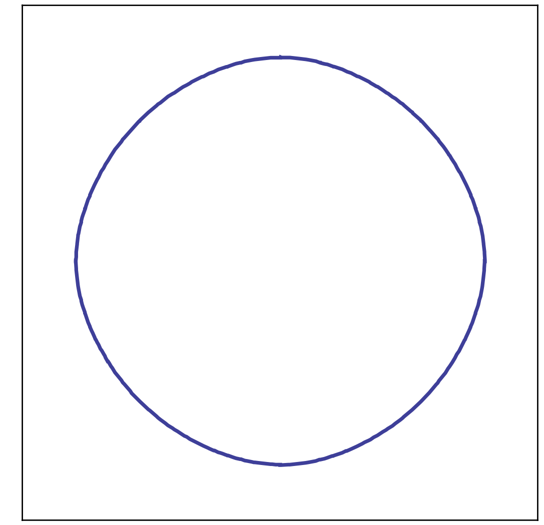
g

Metallic states with non-Luttinger volume
Fermi surfaces must have topological order

T. Senthil, M. Vojta, and S. Sachdev, PRB **69**, 035111 (2004)
S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)



(B)
Topological phase transition; phases of a a theory with an emergent SU(2) gauge field.



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Change of carrier density at the pseudogap critical point of a cuprate superconductor

S. Badoux[1], W. Tabis[2,3], F. Laliberté[2], G. Grissonnanche[1], B. Vignolle[2], D. Vignolles[2], J. Béard[2], D. A. Bonn[4,5], W. N. Hardy[4,5], R. Liang[4,5], N. Doiron-Leyraud[1], Louis Taillefer[1,5] & Cyril Proust[2,5]

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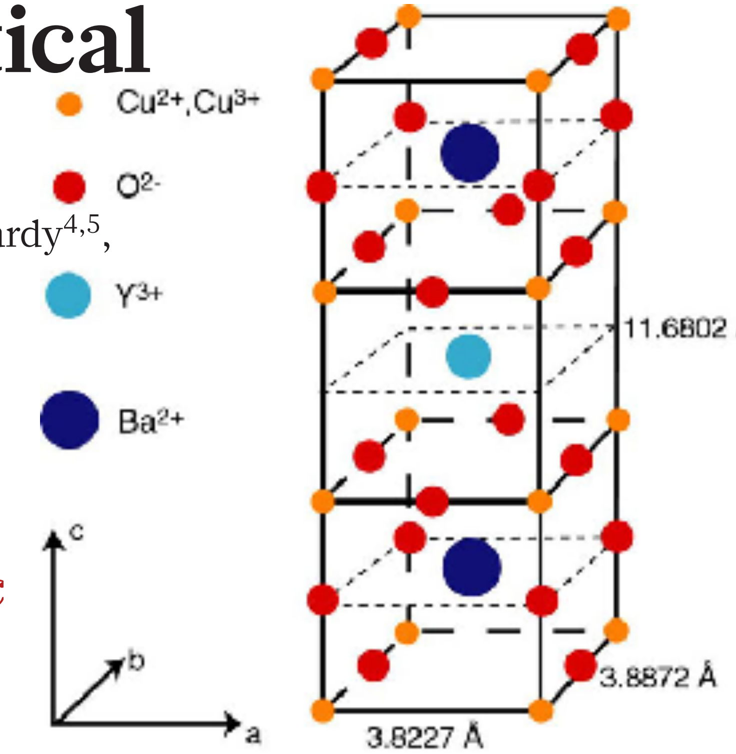
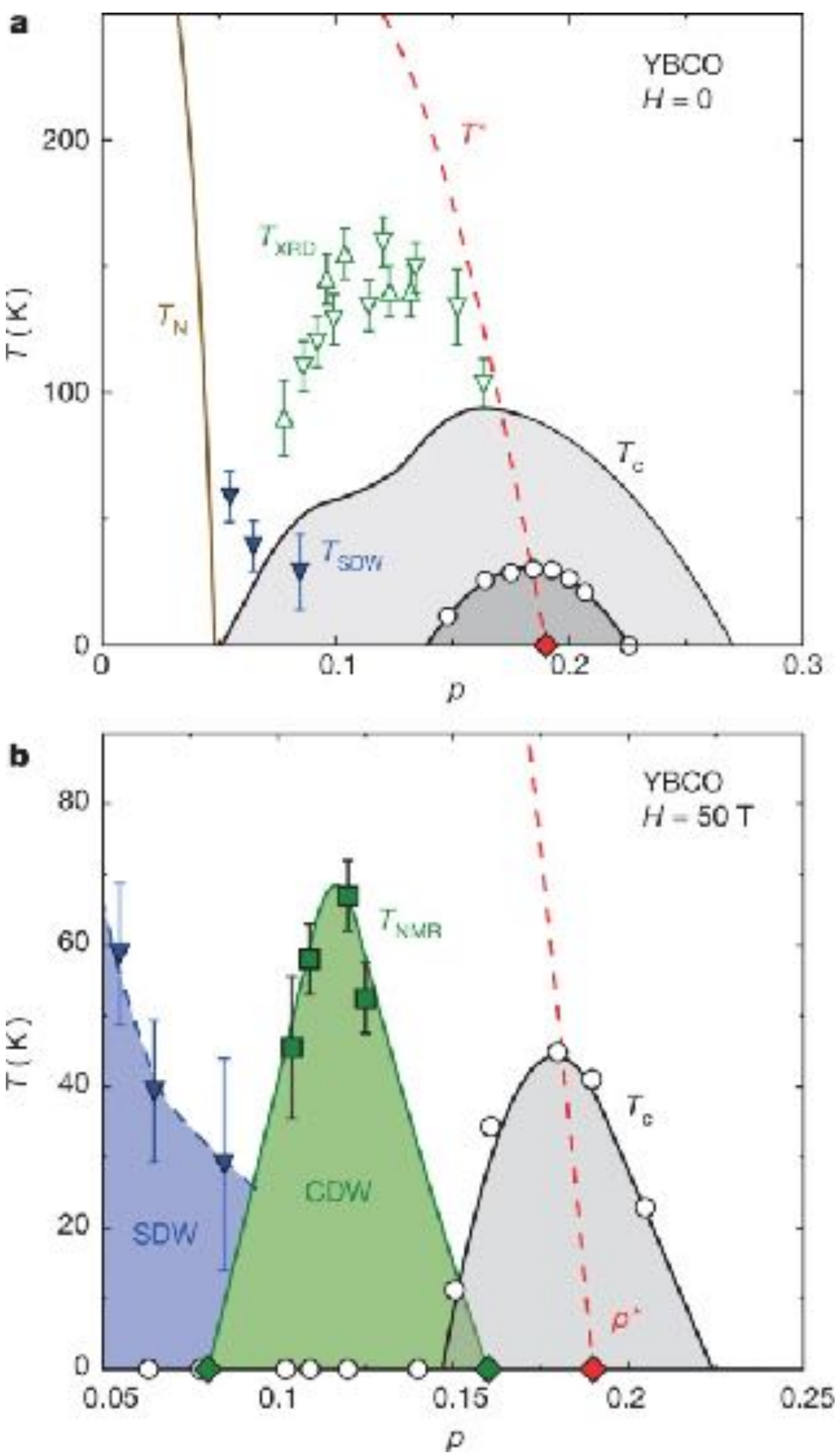
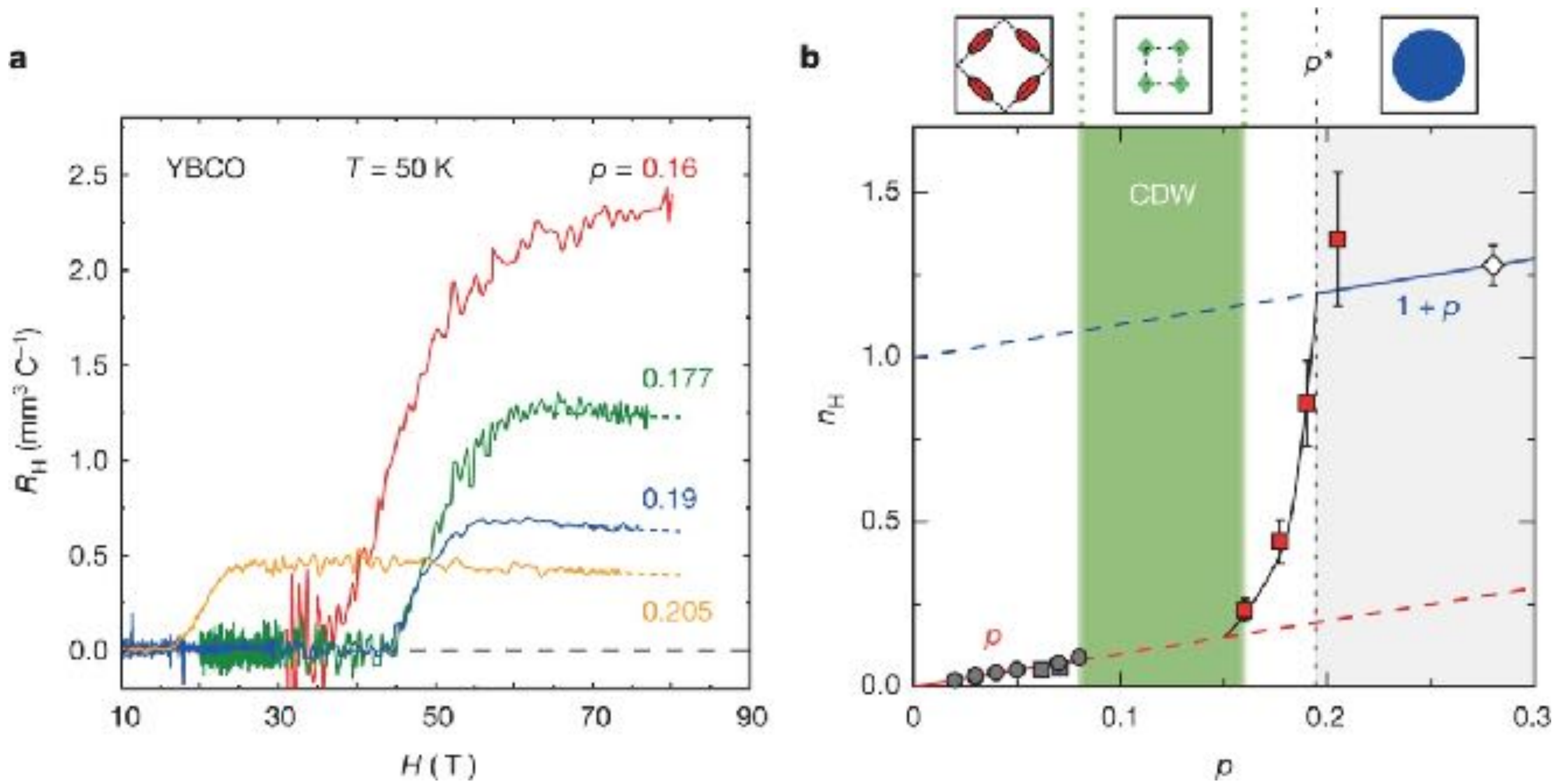
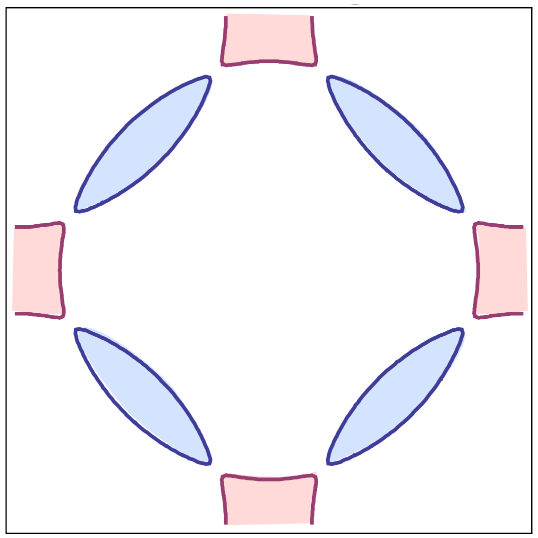


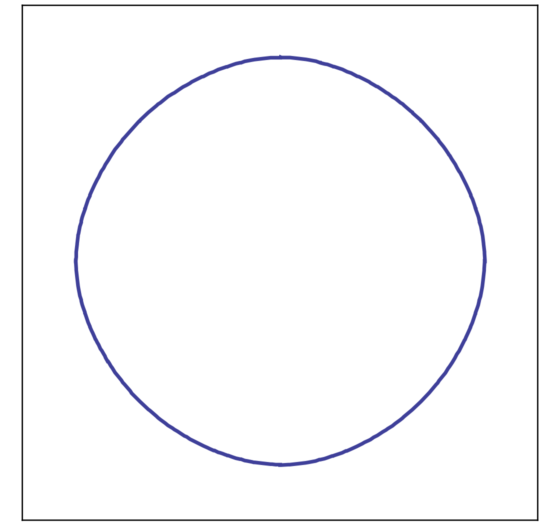
$YBa_2Cu_3O_{6+x}$







(B)
Topological phase transition; phases of a a theory with an emergent SU(2) gauge field.



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Z_2 or U(1) topological order.

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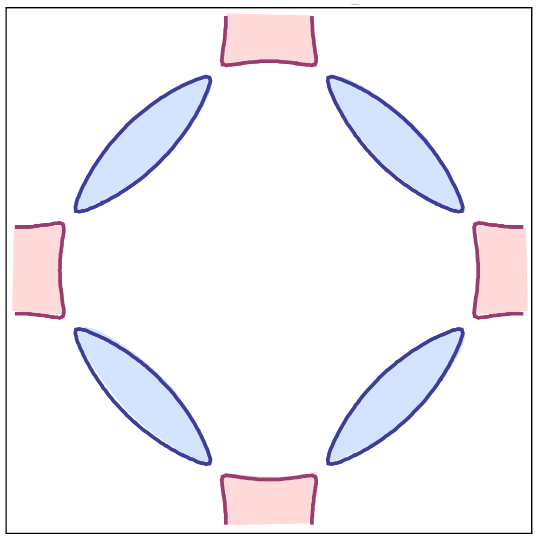
Large Fermi surface with Luttinger volume.
No topological order

g

Can model the doping dependence of the Hall effect in the hole-doped cuprates

S. Chatterjee, A. Eberlein, and S. Sachdev, PRB **96**, 075103 (2017)



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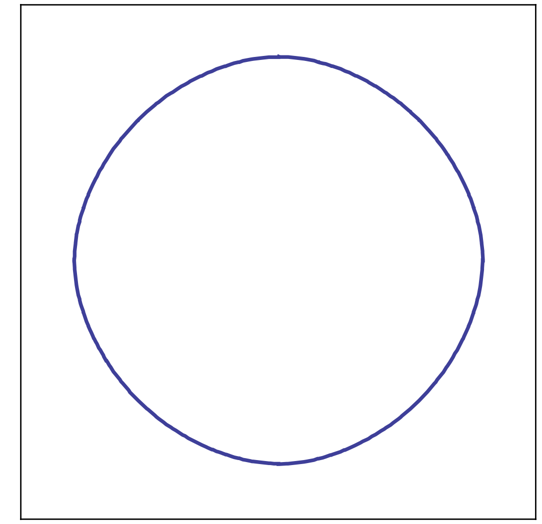

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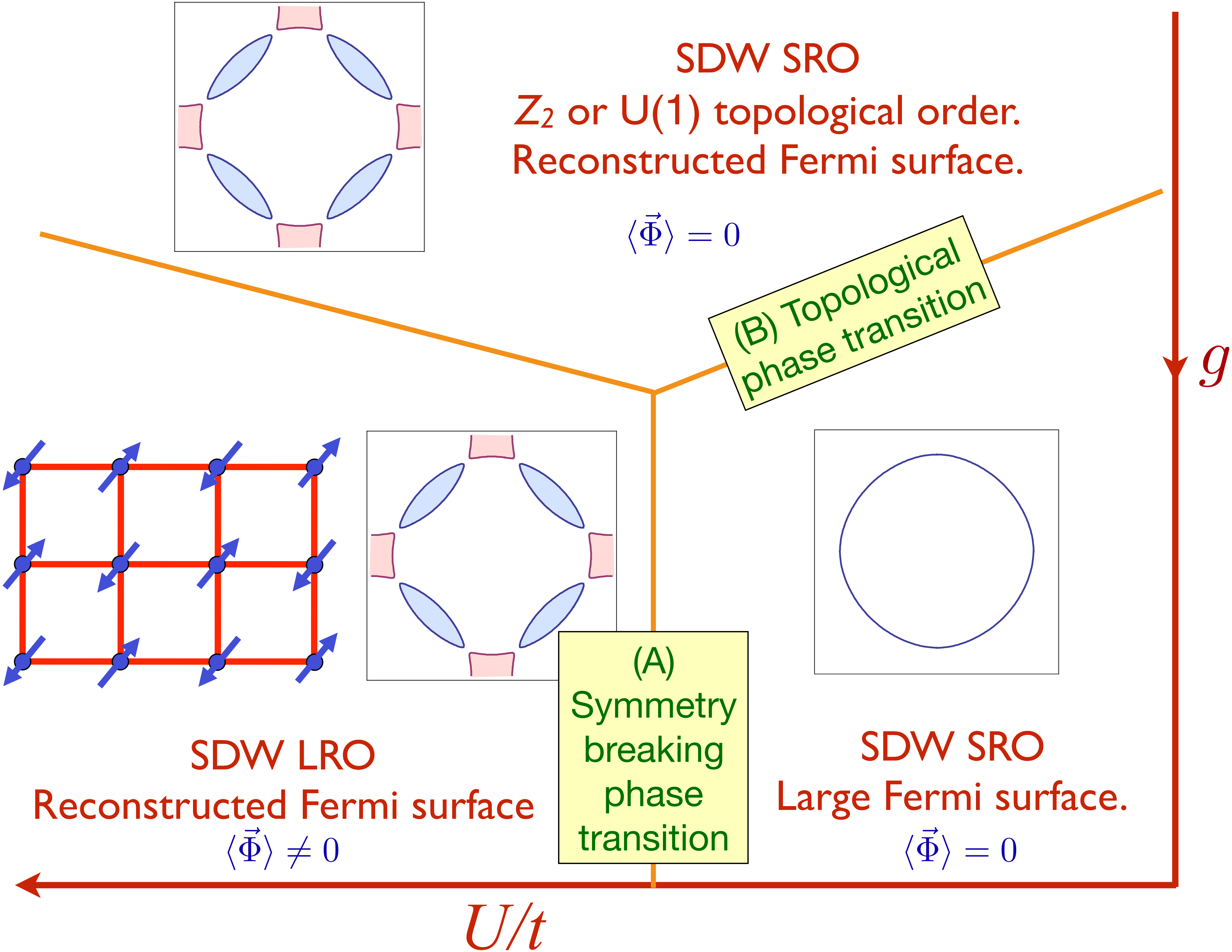
SDW SRO
Large Fermi surface with Luttinger volume.
No topological order

g

SU(2) gauge theory fits the real and imaginary parts of the electron Green's function computed by multi-site DMFT

F13.00008: Mathias Scheurer R04.00005: Antoine Georges, Thu 10:24 AM

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev, arXiv:1711.09925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Z_2 or U(1) topological order.
Reconstructed Fermi surface.
$\langle \vec{\Phi} \rangle = 0$
(B) Topological phase transition
g
(A) Symmetry breaking phase transition
SDW LRO
Reconstructed Fermi surface
$\langle \vec{\Phi} \rangle \neq 0$
SDW SRO
Large Fermi surface.
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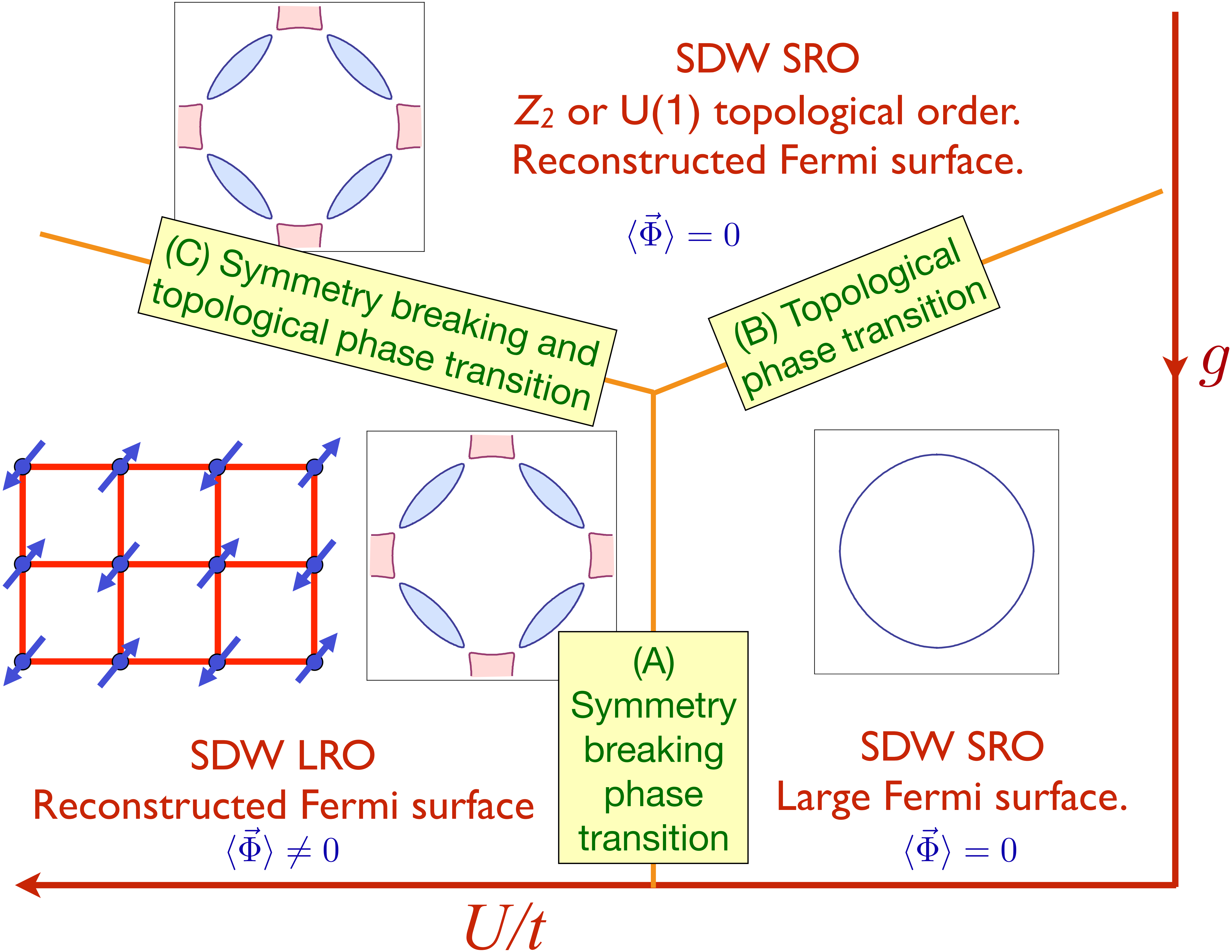
U/t



SDW SRO
Z_2 or U(1) topological order.
Reconstructed Fermi surface.
$\langle \vec{\Phi} \rangle = 0$
(C) Symmetry breaking and topological phase transition
(B) Topological phase transition
g
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SDW SRO
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U/t

Continuous quantum transitions

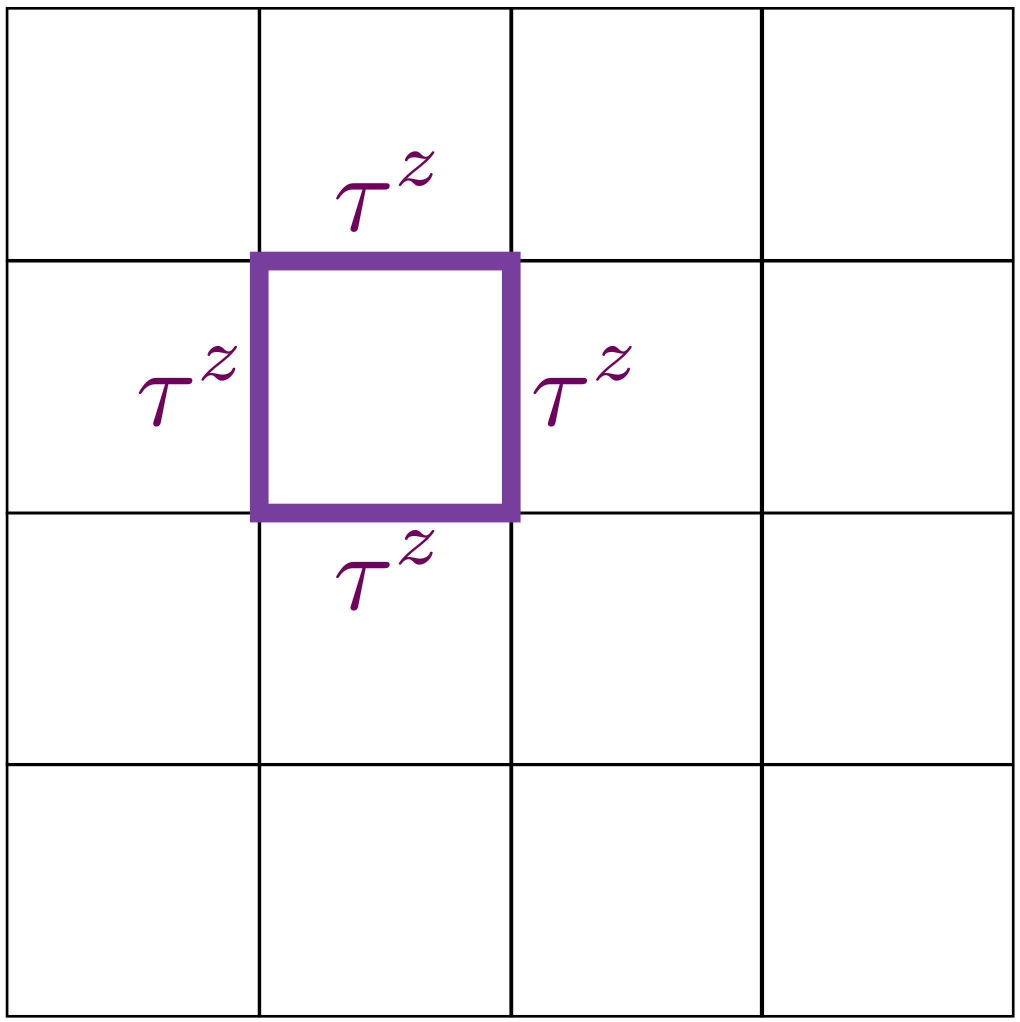
(A) Broken symmetry | No broken symmetry

(B) Topological order | No topological order

(C) Topological order | Broken symmetry

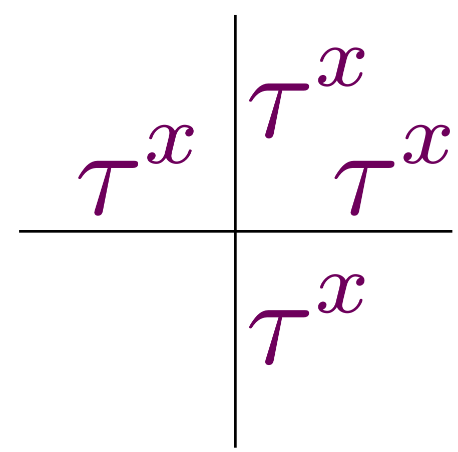
(D) Broken symmetry | A different broken symmetry

Odd Z_2 lattice gauge theory



$$H = -\sum_{\square} \tau^z\tau^z\tau^z\tau^z - g\sum_i \tau^x$$

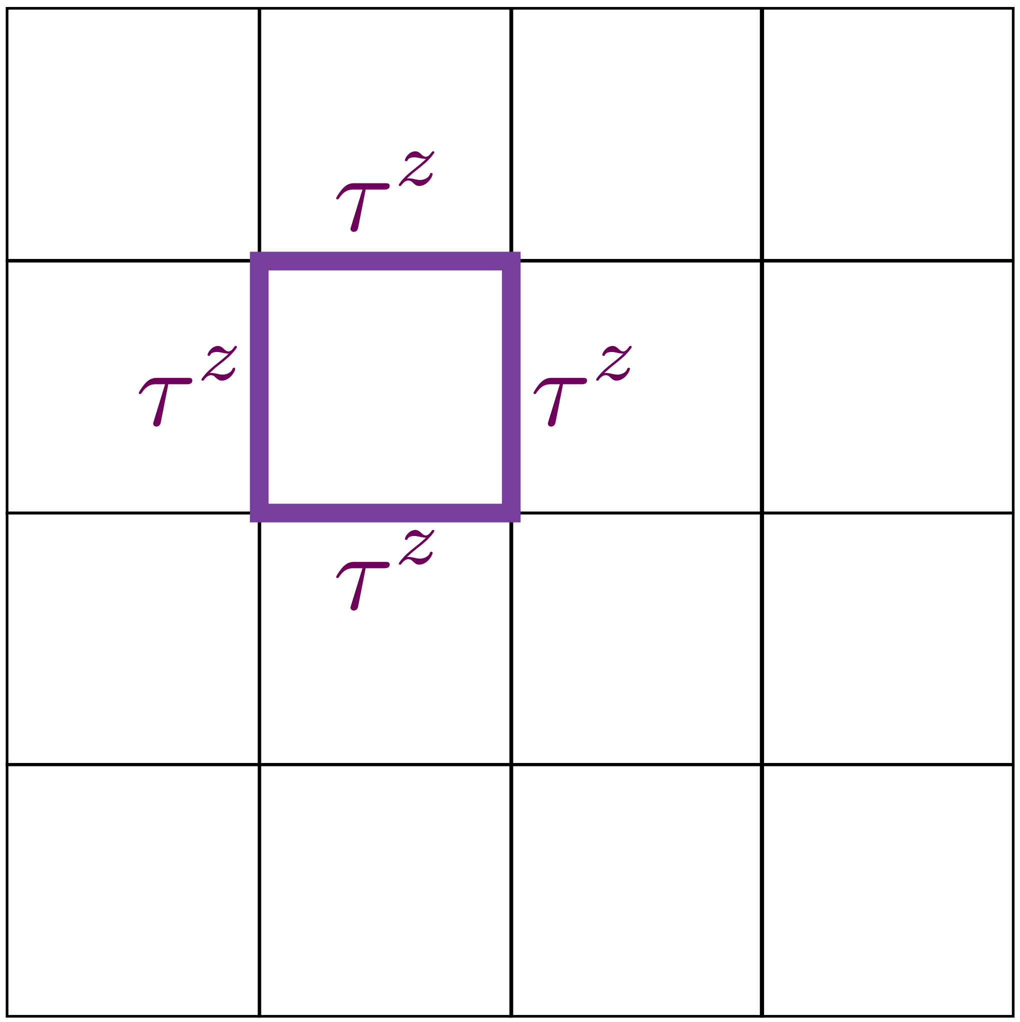
$G_i =$



Gauss's Law with background electric charges:

$$[H, G_i] = 0 \quad , \quad G_i = -1$$

Odd Z_2 lattice gauge theory



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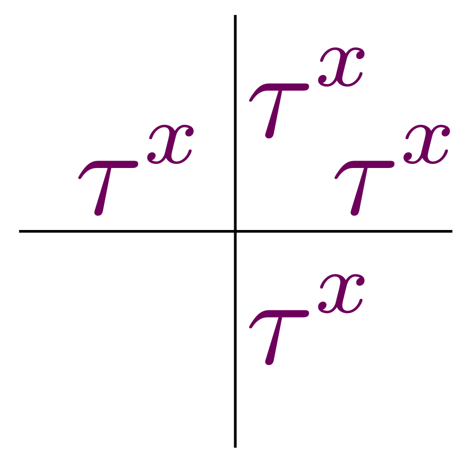

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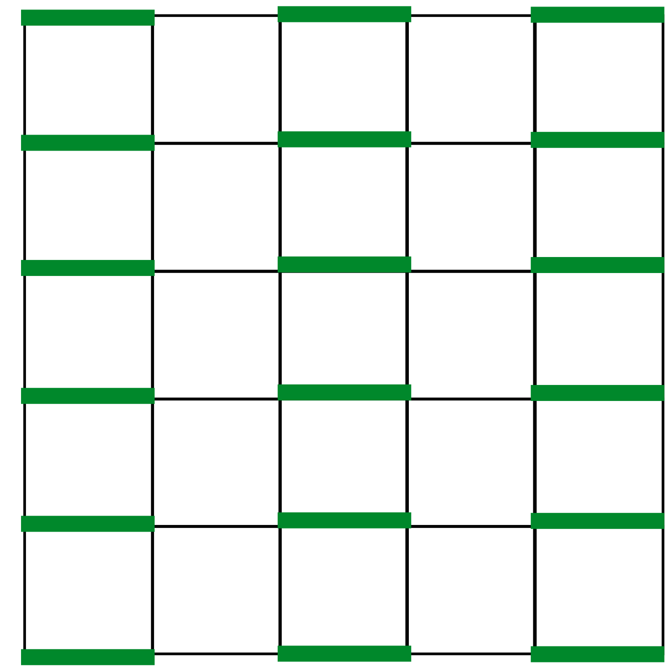
Odd $\mathbb{Z}_2$ lattice gauge theory

(C) Symmetry-breaking and topological phase transition

Deconfined phase.
$\mathbb{Z}_2$ flux expelled.
$\mathbb{Z}_2$ (toric code) topological order.



Confined phase.
$\mathbb{Z}_2$ flux proliferates.
No topological order.
Electric field lines lead to symmetry breaking and valence bond solid (VBS) order

g

$$H = -\sum_{\square} \tau^z \tau^z \tau^z \tau^z - g \sum_i \tau^x \qquad G_i = -1$$

R. Jalabert and S. Sachdev, PRB **44**, 686 (1991)
T. Senthil, L. Balents, S. Sachdev, A. Vishwanath, and M. P.A. Fisher, PRB **70**, 144407 (2004)

Odd Z_2 lattice gauge theory

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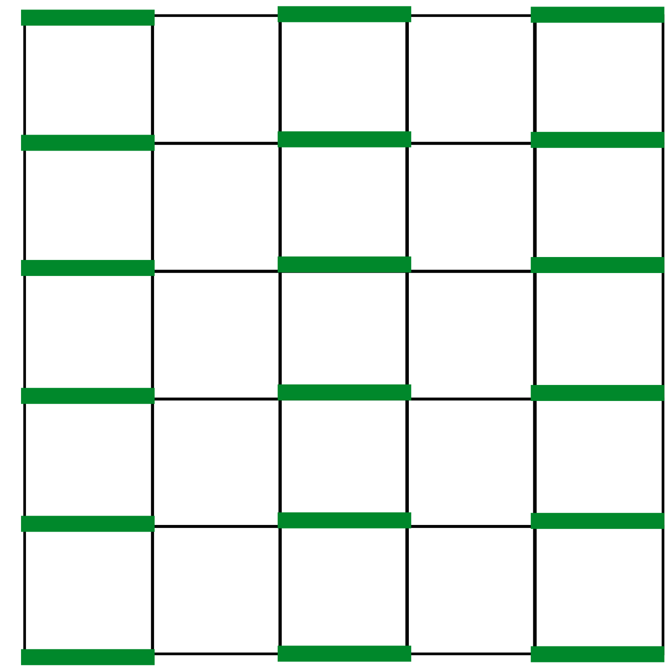
(C)
Symmetry-breaking and topological phase transition; phases of a theory with an emergent U(1) gauge field



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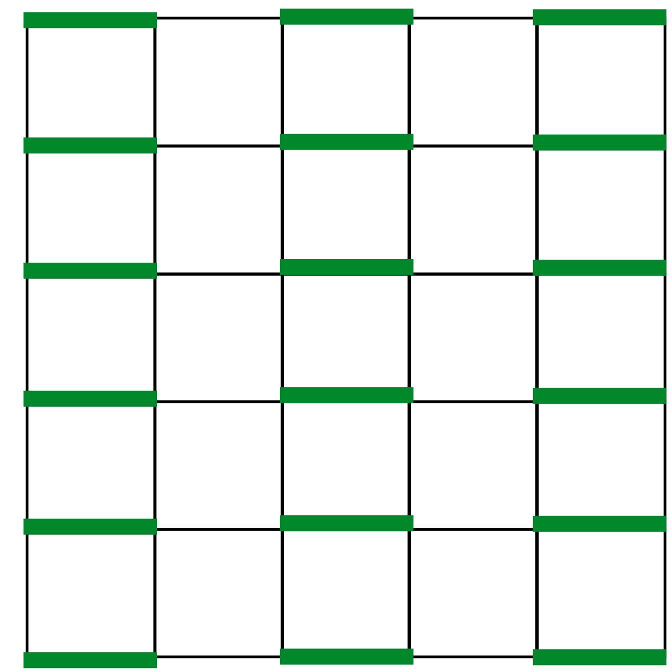

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Similar phases and transitions in frustrated square lattice antiferromagnets with spin S=1/2

R. Jalabert and S. Sachdev, PRB **44**, 686 (1991)
N. Read and S. Sachdev, PRL **66**, 1773 (1991)

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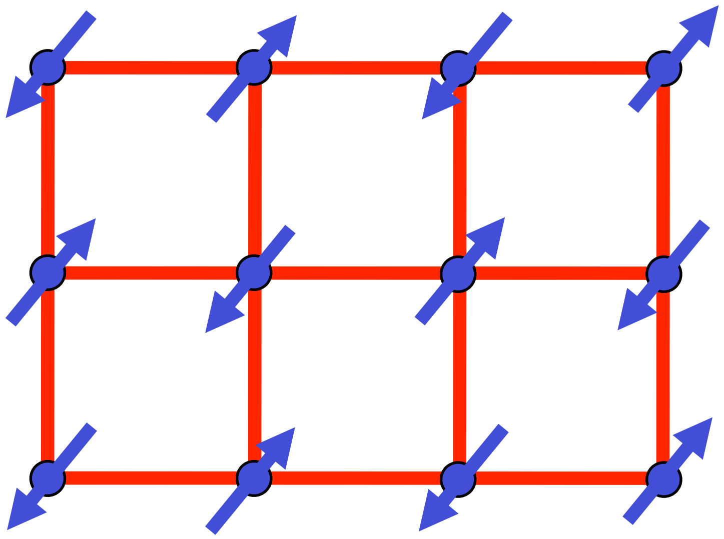
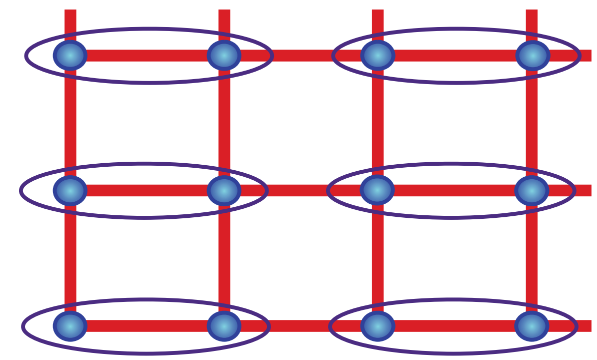
(D) Broken symmetry | A different broken symmetry

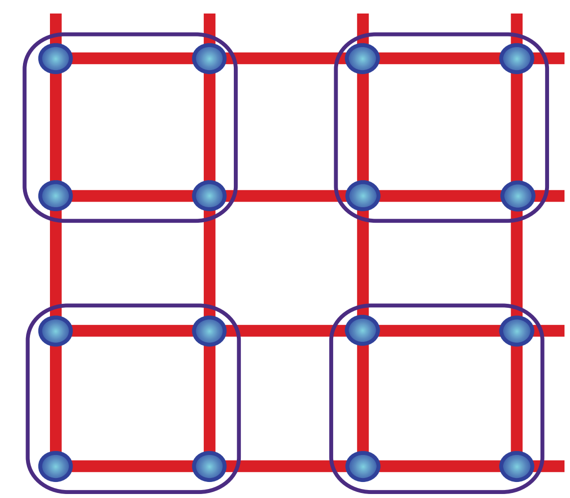
Quantum phases of a S=1/2 square lattice antiferromagnet



(D)
Symmetry-breaking to symmetry-breaking transition



or



Antiferromagnet with broken spin rotation symmetry

VBS with broken lattice rotation symmetry

g

N. Read and S. Sachdev, PRL **62**, 1694 (1989)
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M. P. A. Fisher, Science **303**, 1490 (2004)

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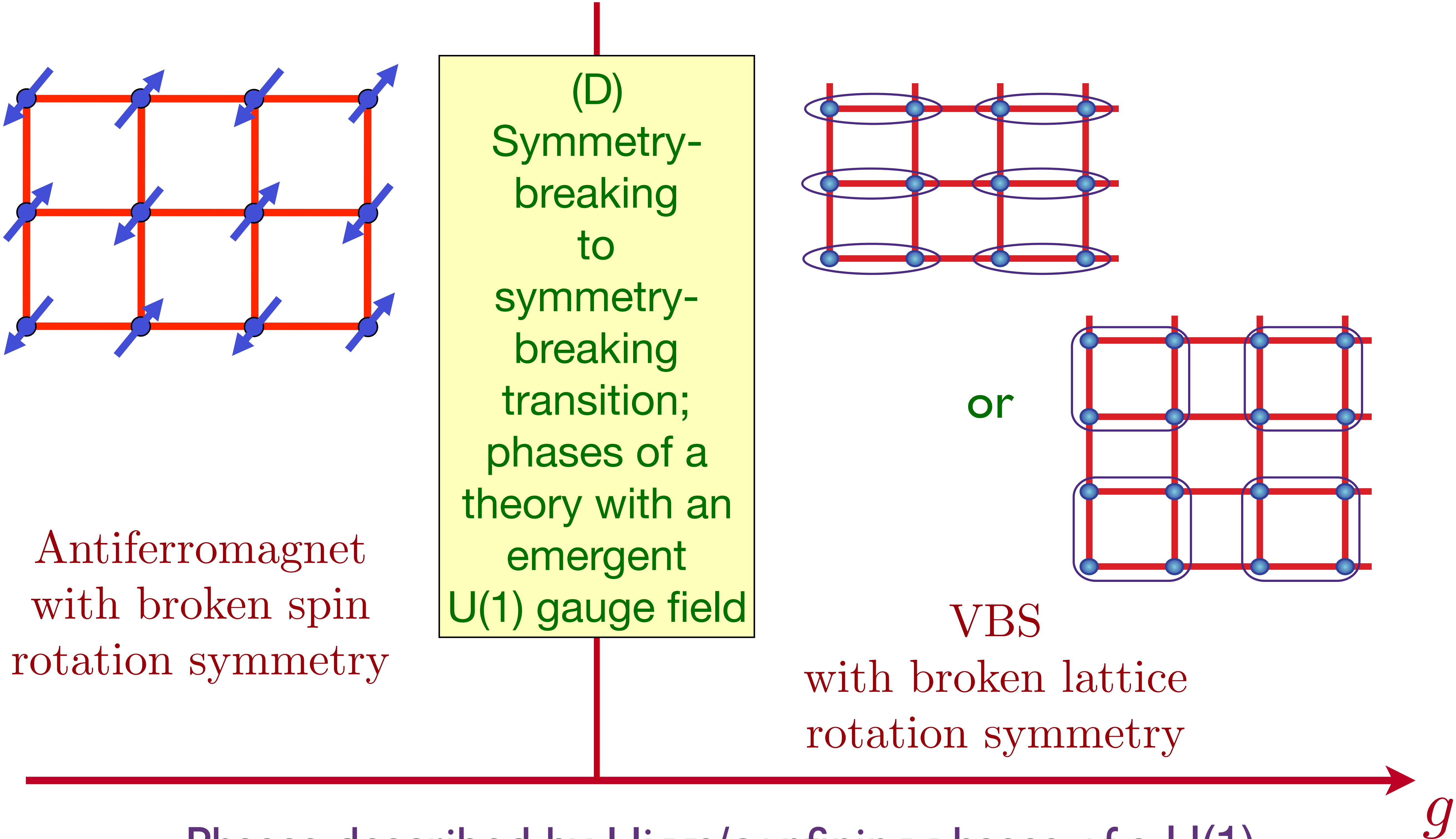

Phases described by Higgs/confining phases of a U(1) gauge theory, which is deconfined only at criticality

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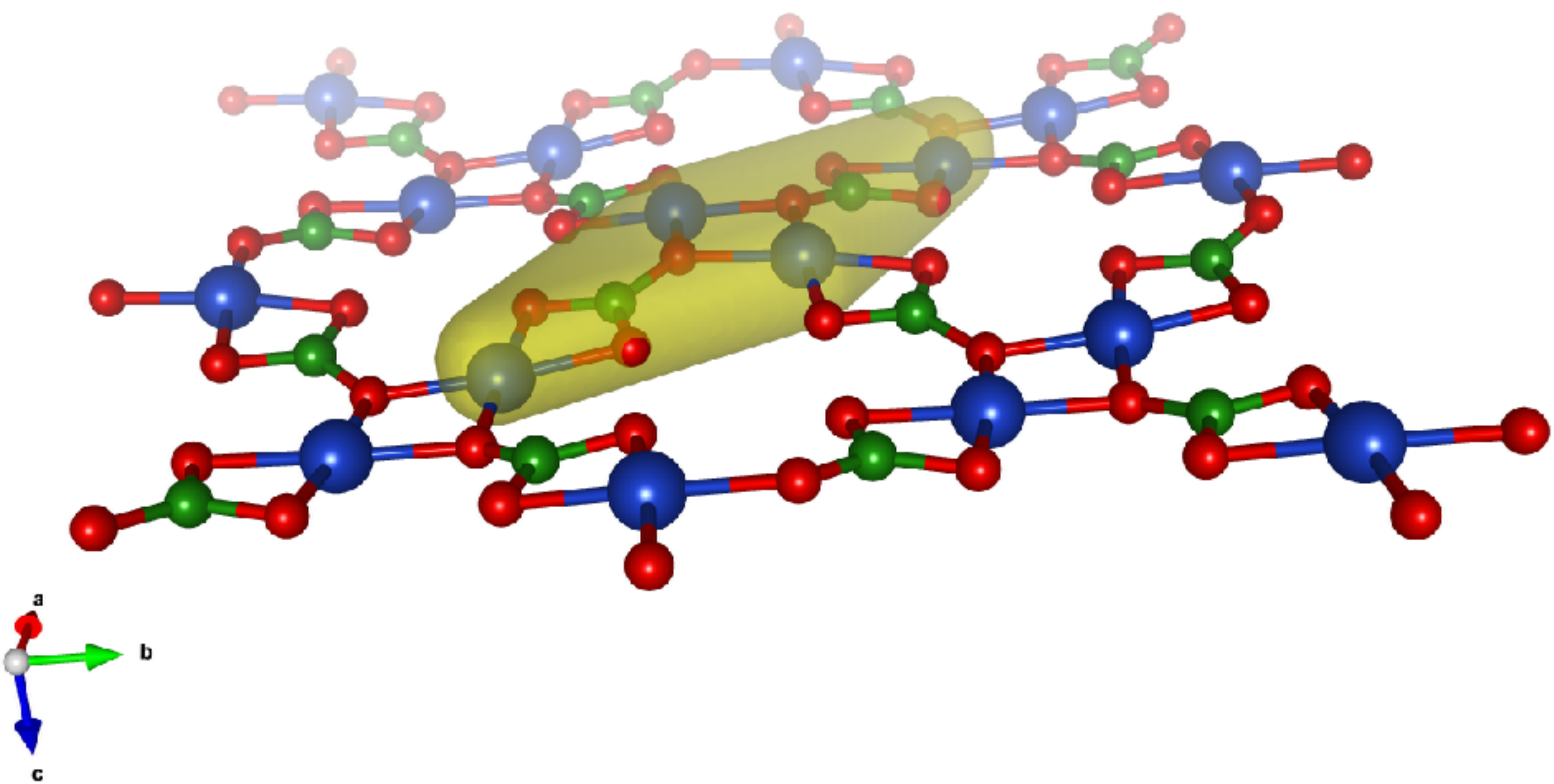
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M. P. A. Fisher, Science **303**, 1490 (2004)

4-spin plaquette singlet state in the Shastry–Sutherland compound $SrCu_2(BO_3)_2$

M. E. Zayed[1,2,3★], Ch. Rüegg[2,4,5], J. Larrea J.[1,6], A. M. Läuchli[7], C. Panagopoulos[8,9], S. S. Saxena[8], M. Ellerby[5], D. F. McMorrow[5], Th. Strässle[2], S. Klotz[10], G. Hamel[10], R. A. Sadykov[11,12], V. Pomjakushin[2], M. Boehm[13], M. Jiménez-Ruiz[13], A. Schneidewind[14], E. Pomjakushina[15], M. Stingaciu[15], K. Conder[15] and H. M. Rønnow[1]

Nature Physics ***13***, *962–966 (2017)*



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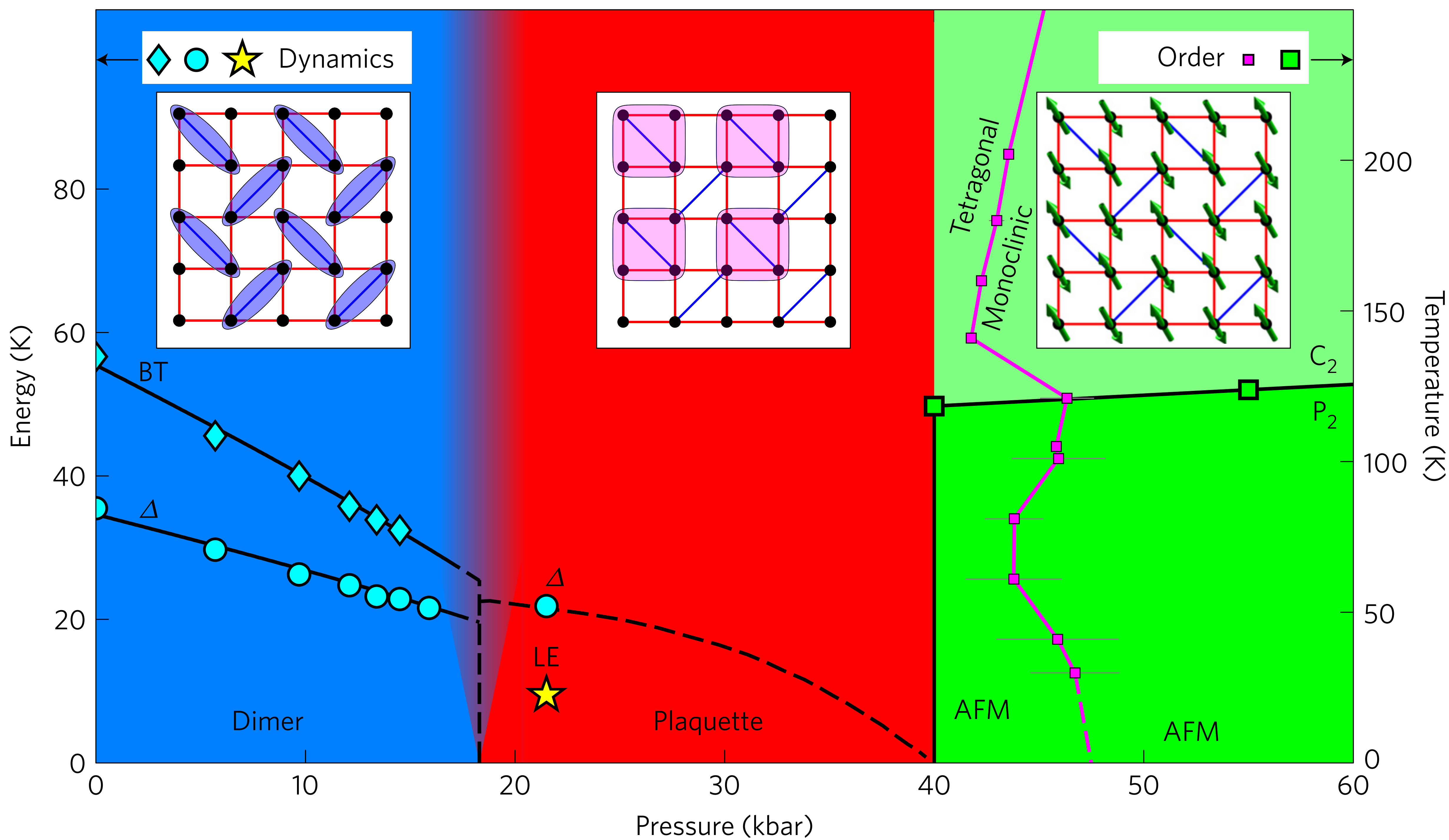

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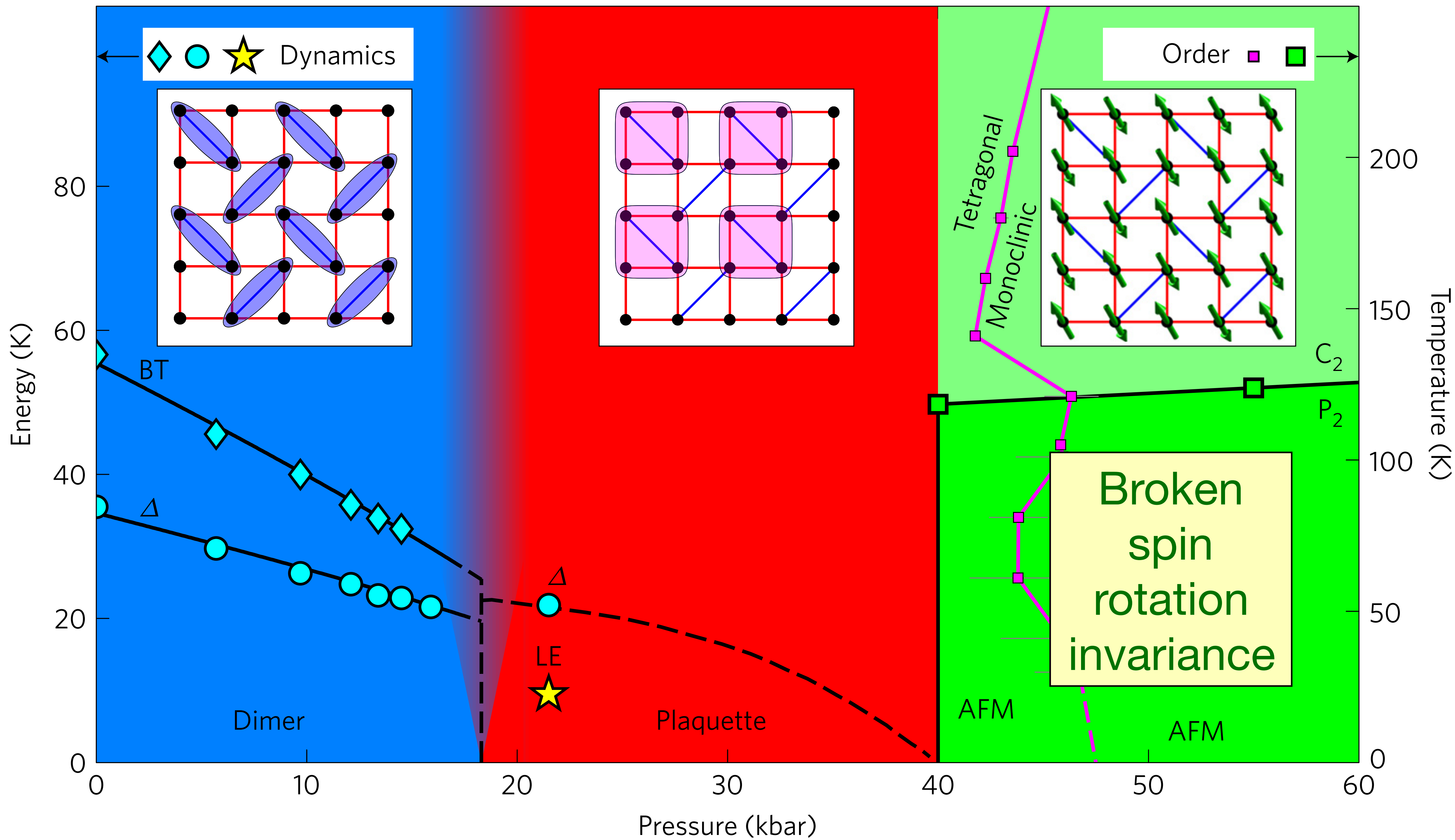

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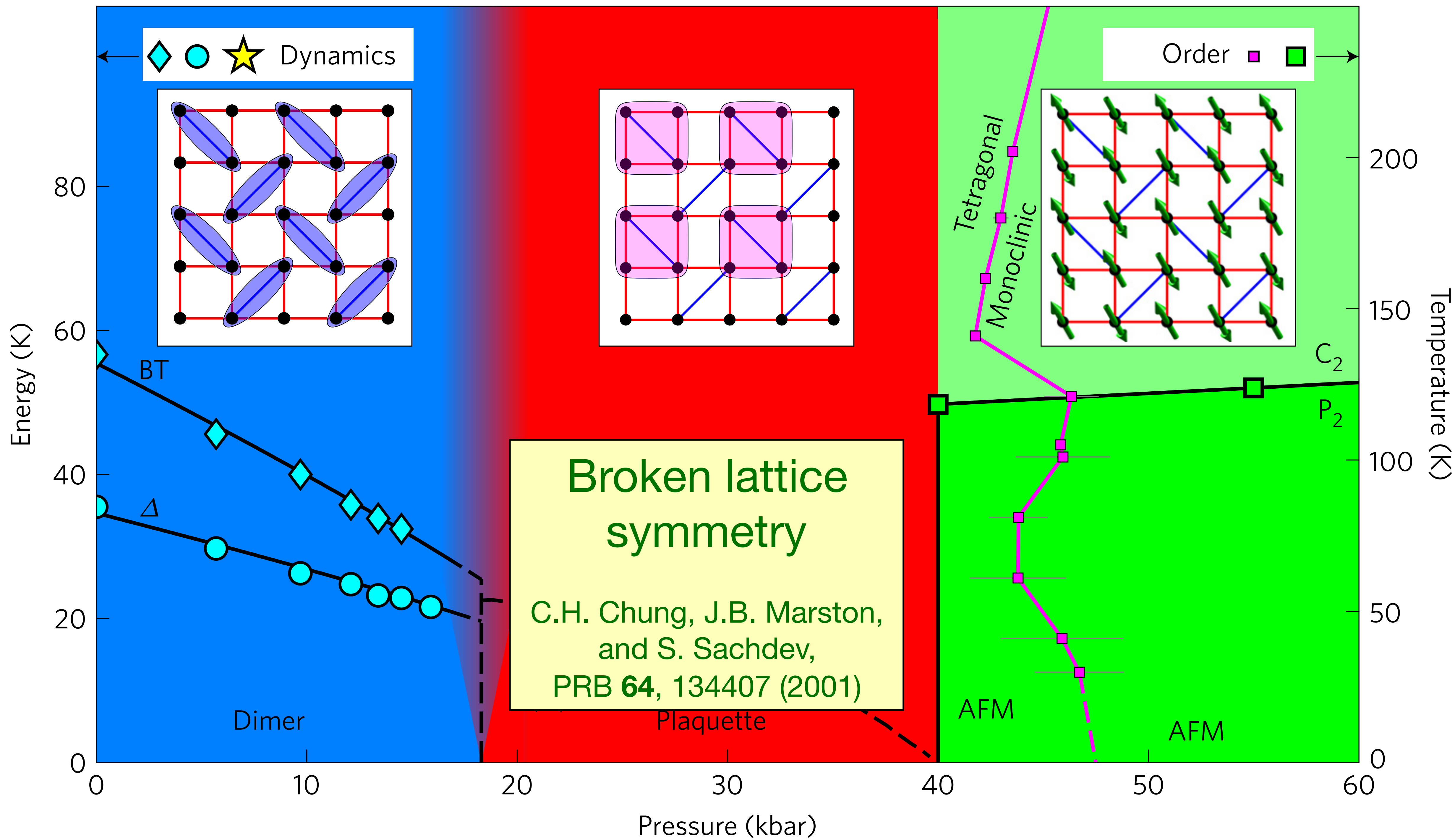

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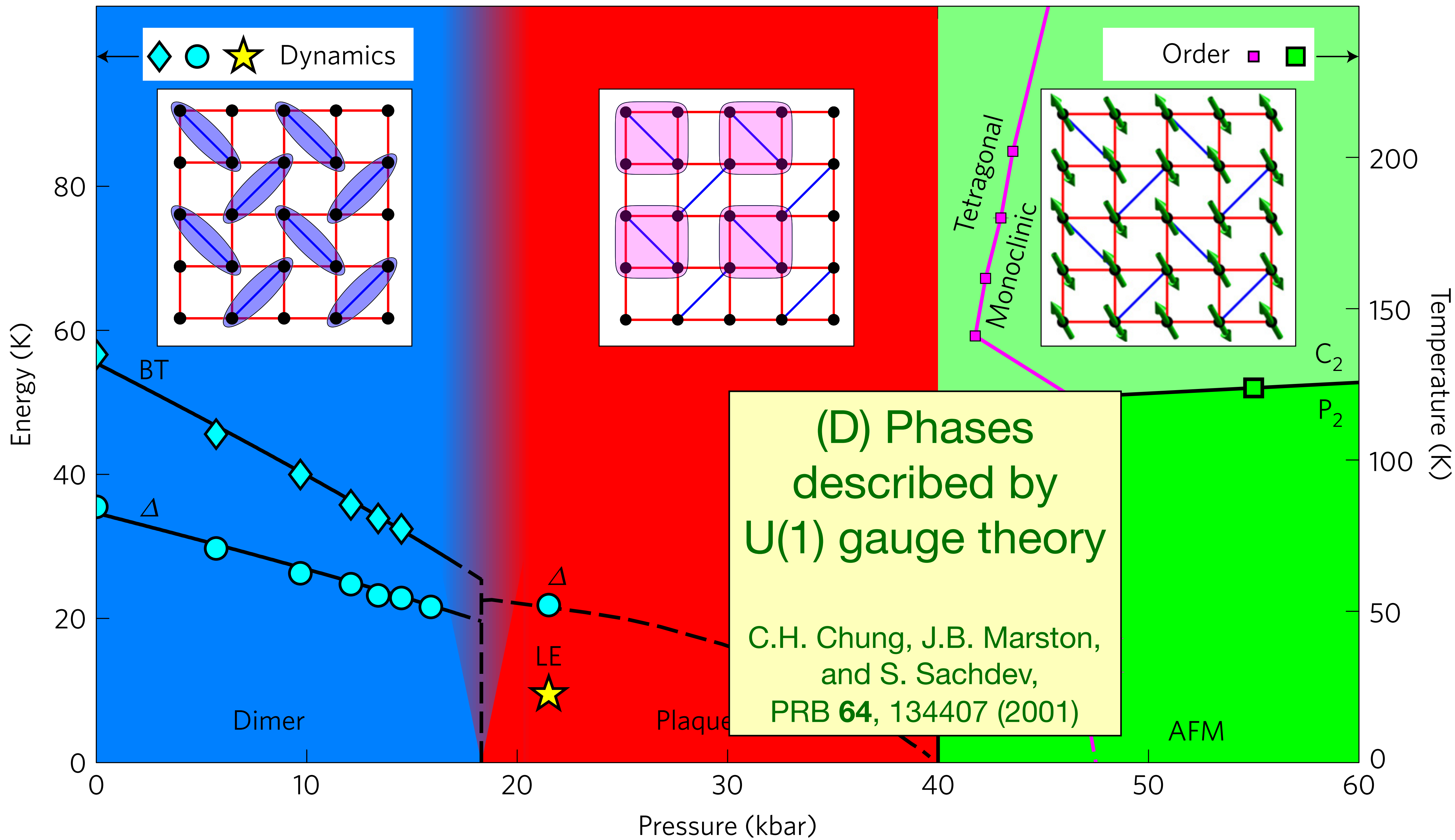

(D) Phases described by U(1) gauge theory

C.H. Chung, J.B. Marston, and S. Sachdev, PRB **64**, 134407 (2001)

Strongly-coupled quantum criticality

States of quantum matter with:

- No quasiparticle excitations.
- Strong interactions are at a universal RG fixed point, and this leads so fastest possible ‘dephasing’ and ‘local thermalization’ in a time of order $\hbar/(k_B T)$.

S.Sachdev, *Quantum Phase Transitions*, 1991
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- Many-body quantum chaos (as measured by out-of-time-order correlations) in a time of order $\hbar/(k_B T)$.

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Onsager's Ising criticality, other critical states in one spatial dimension, and quantum impurity models, do not have these properties

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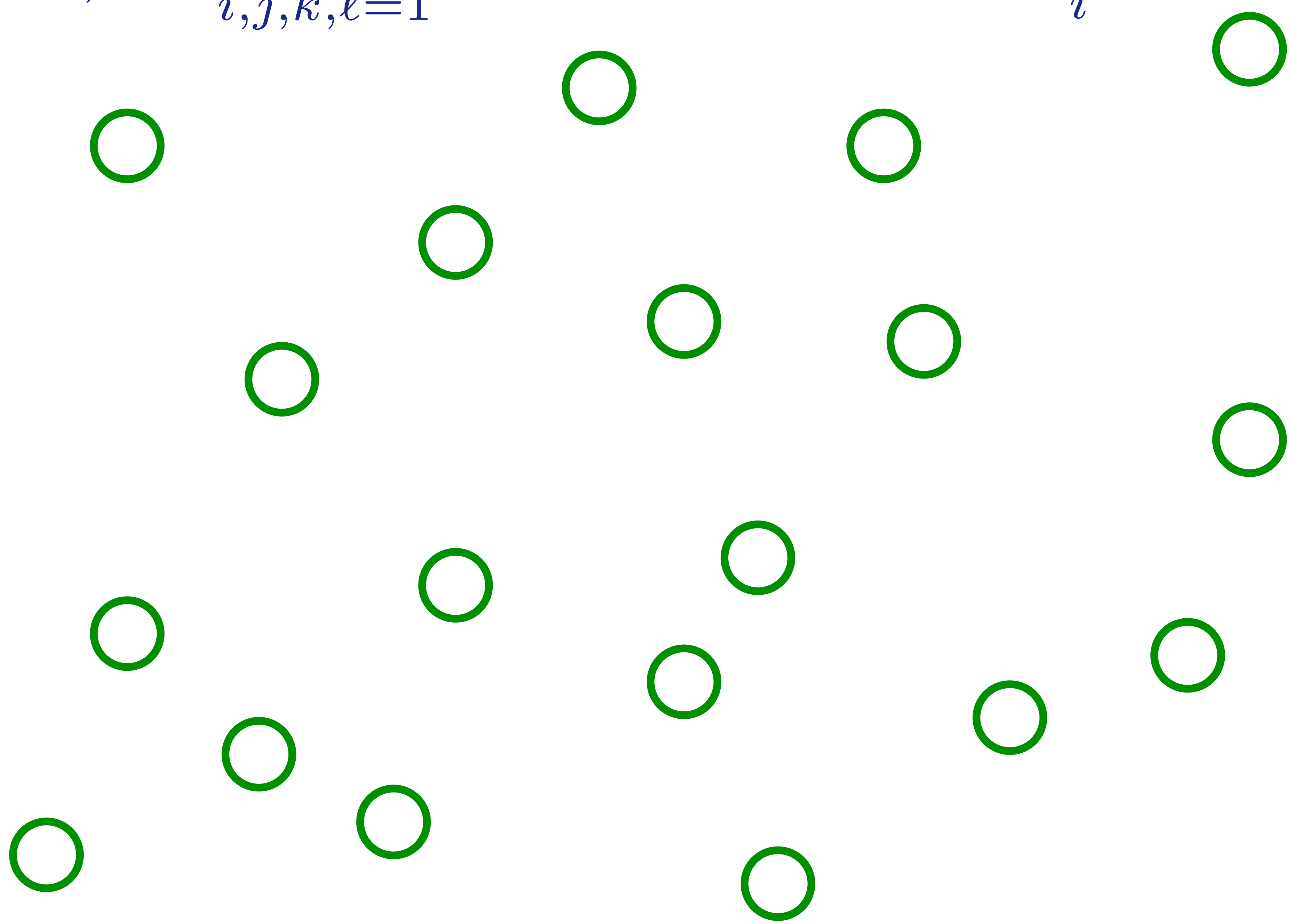
`Solvable' models with these properties:
the SYK models

The Sachdev-Ye-Kitaev (SYK) model

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell} \, f_i^\dagger f_j^\dagger f_k f_\ell - \mu \sum_i f_i^\dagger f_i$$



Pick a set of random positions

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993); A. Kitaev (2015)

The SYK model

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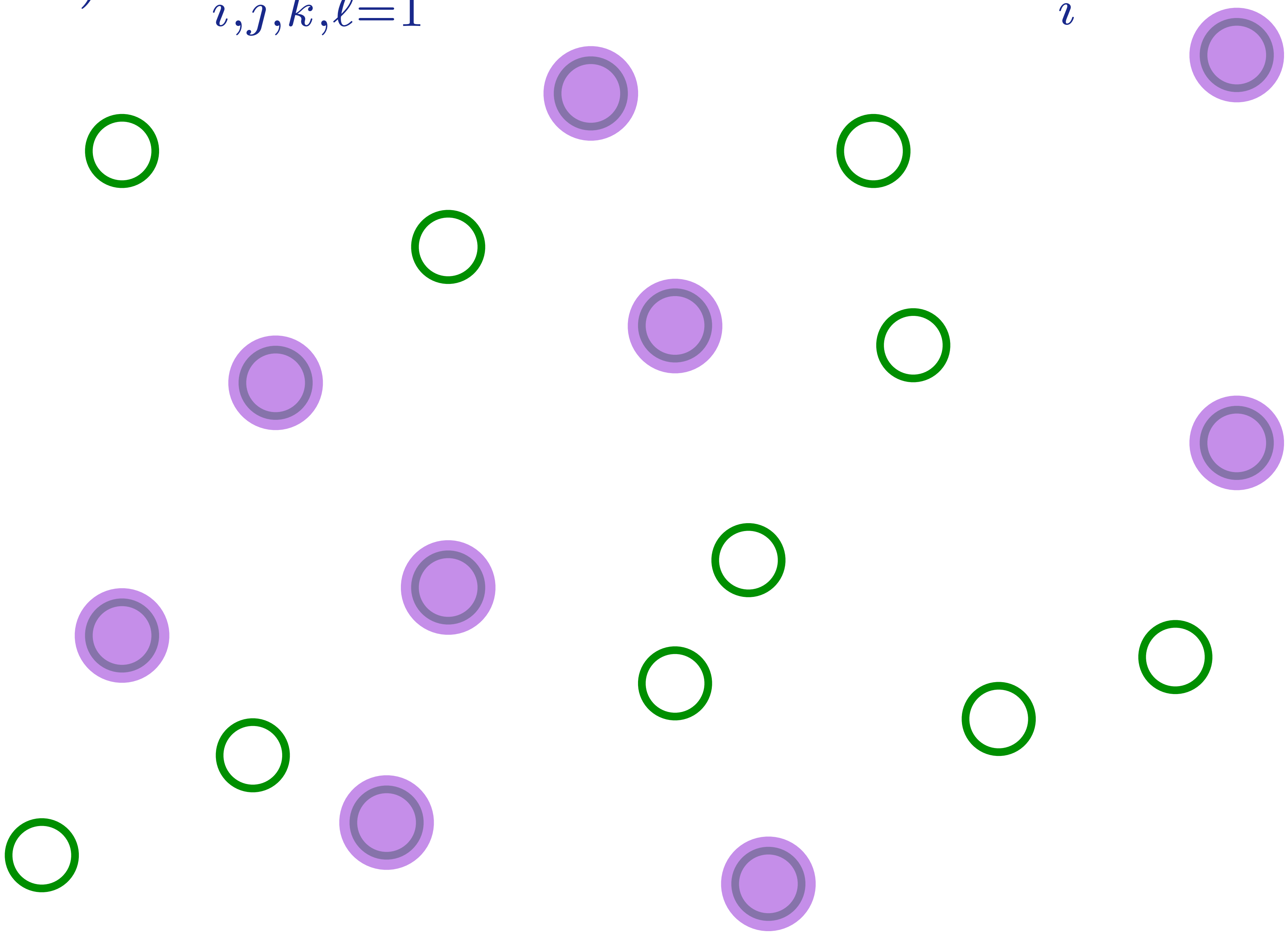

Place electrons randomly on some sites

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993); A. Kitaev (2015)

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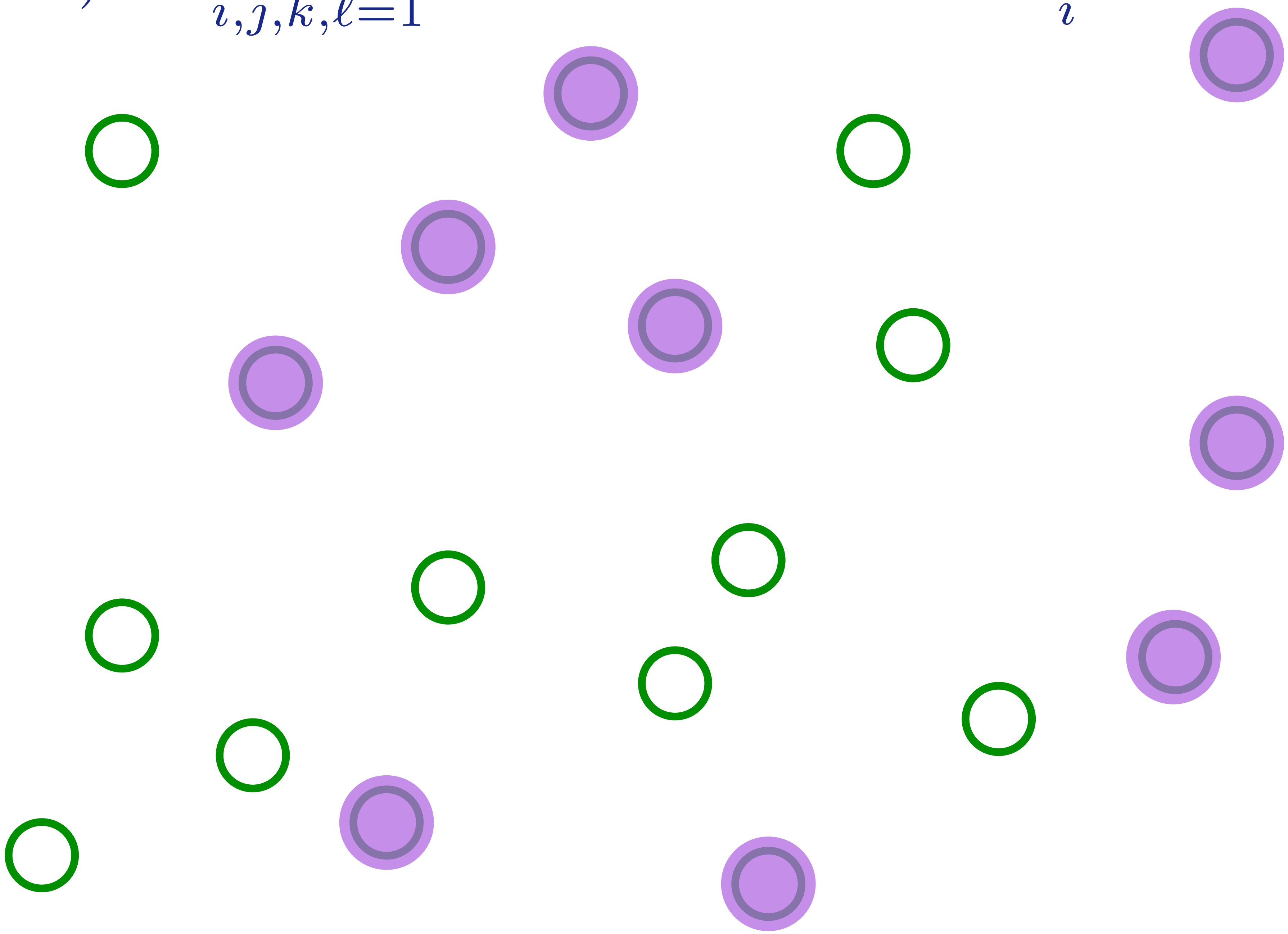

Entangle electrons pairwise randomly

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993); A. Kitaev (2015)

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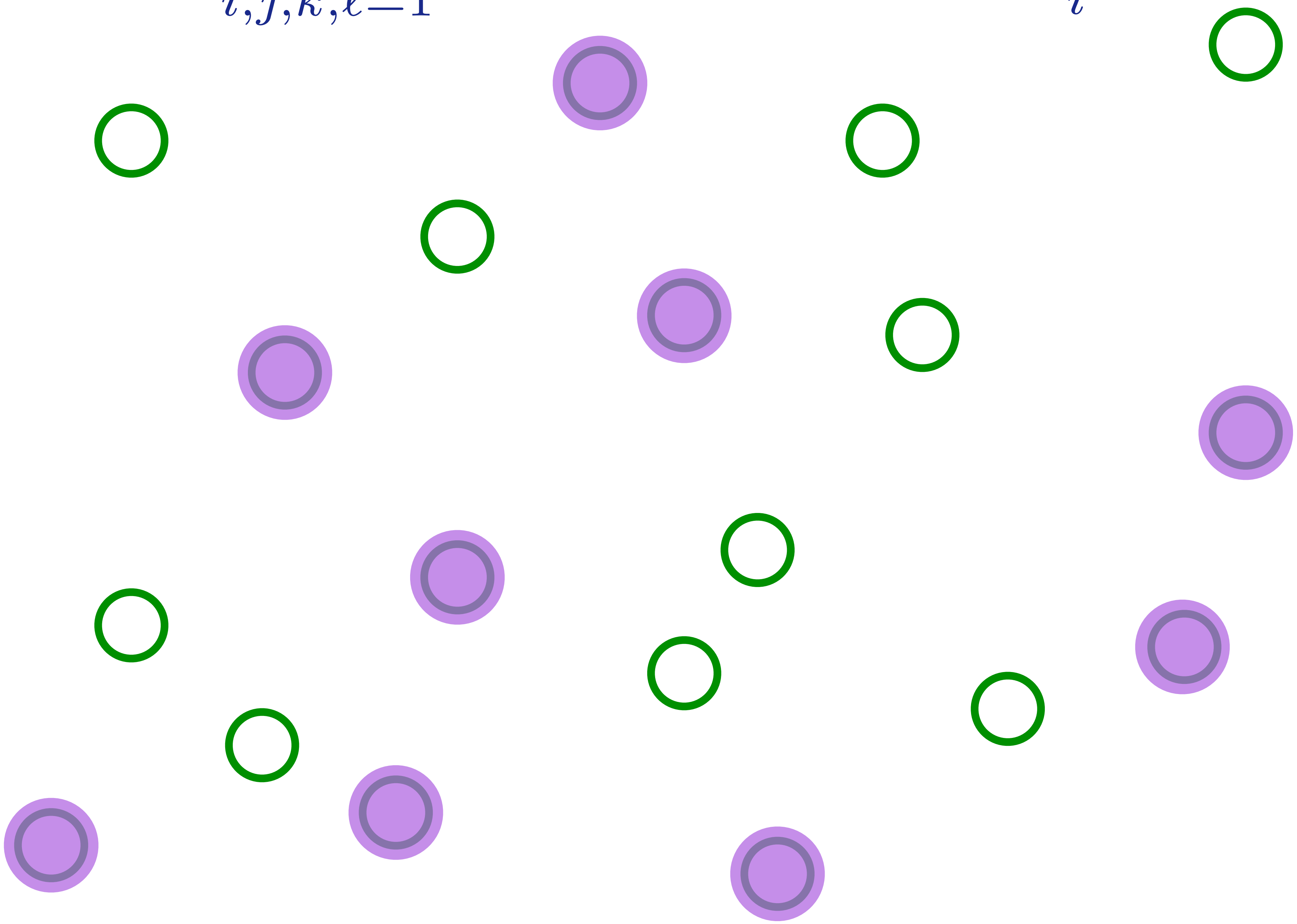

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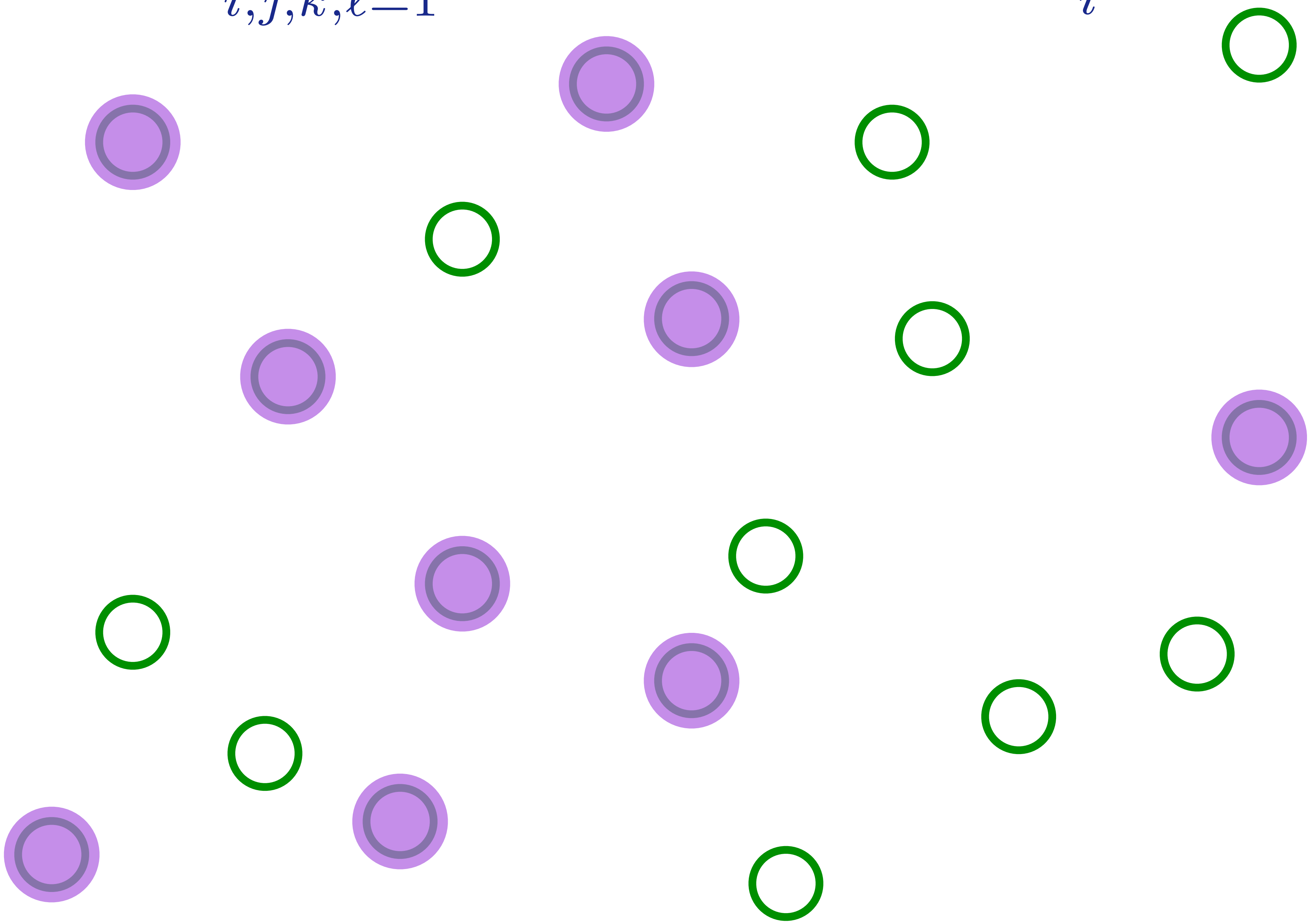

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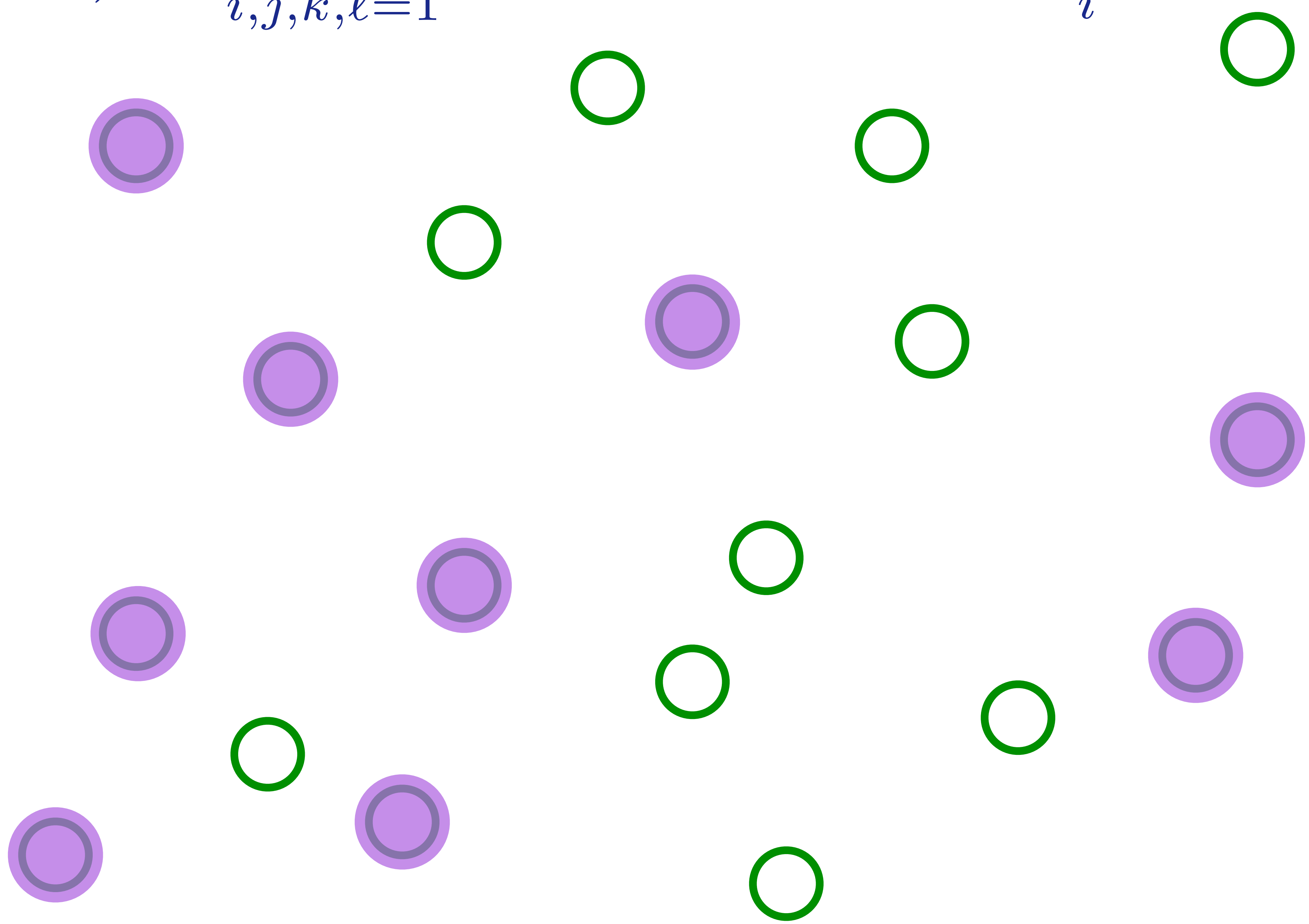

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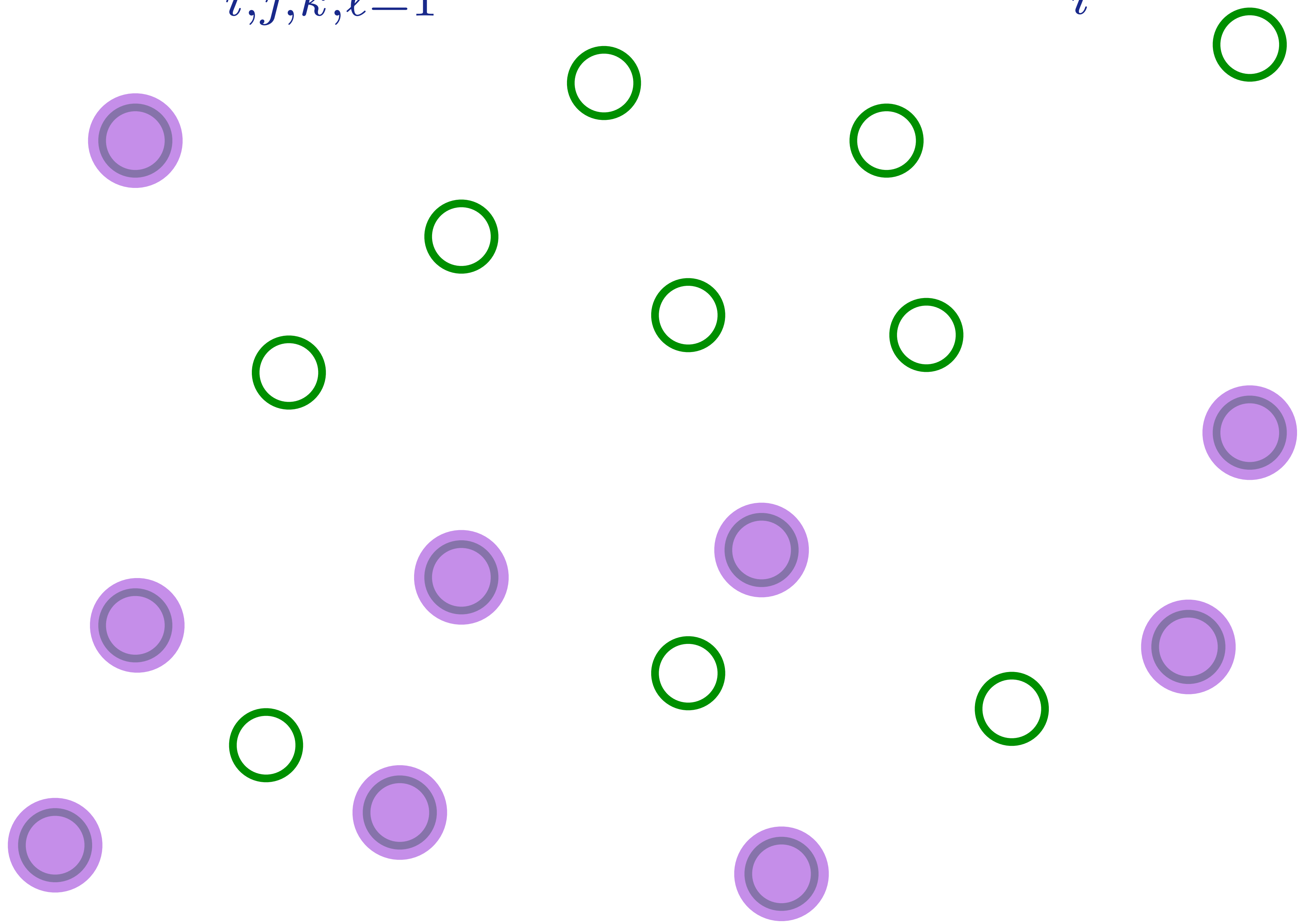

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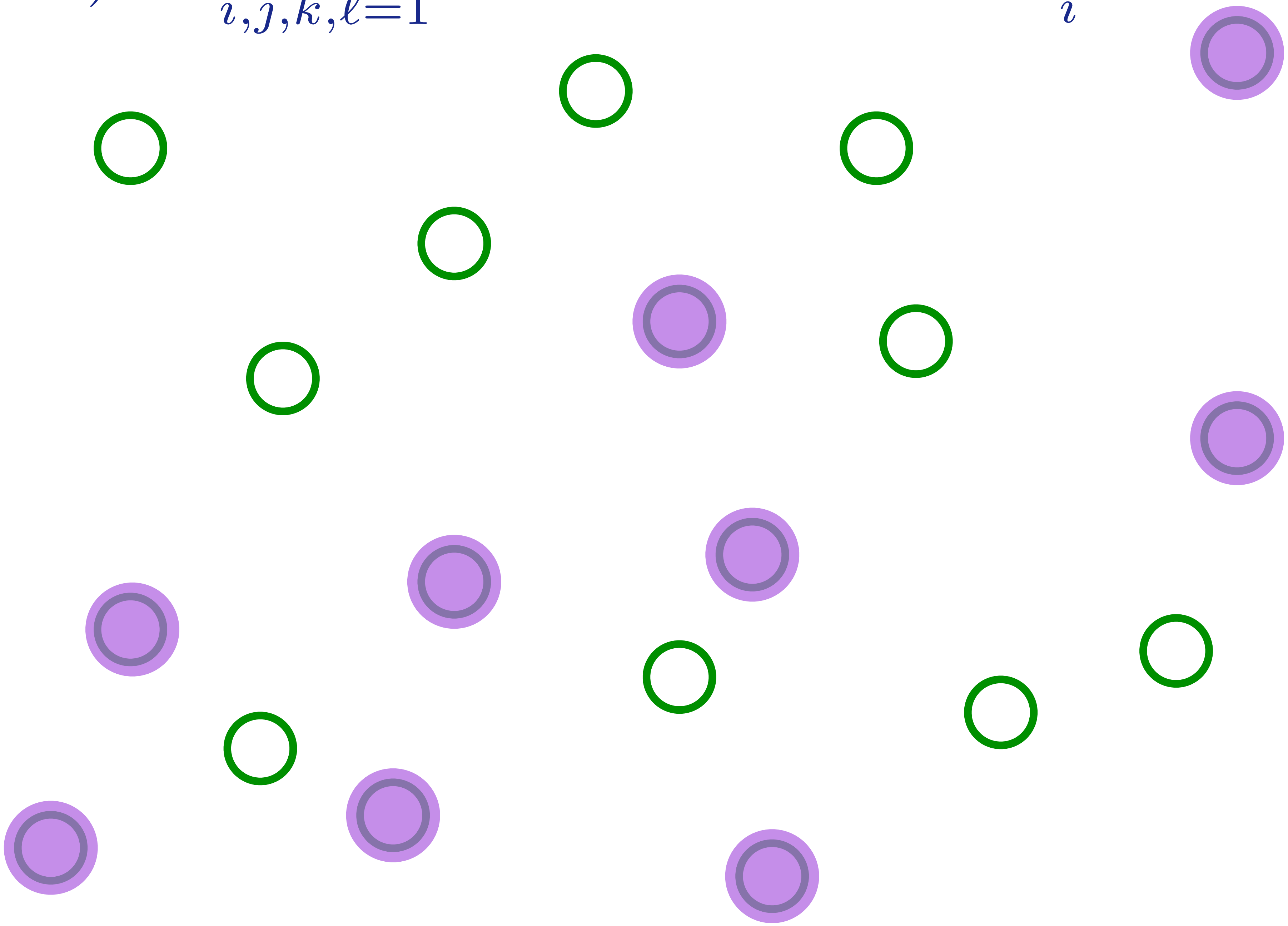

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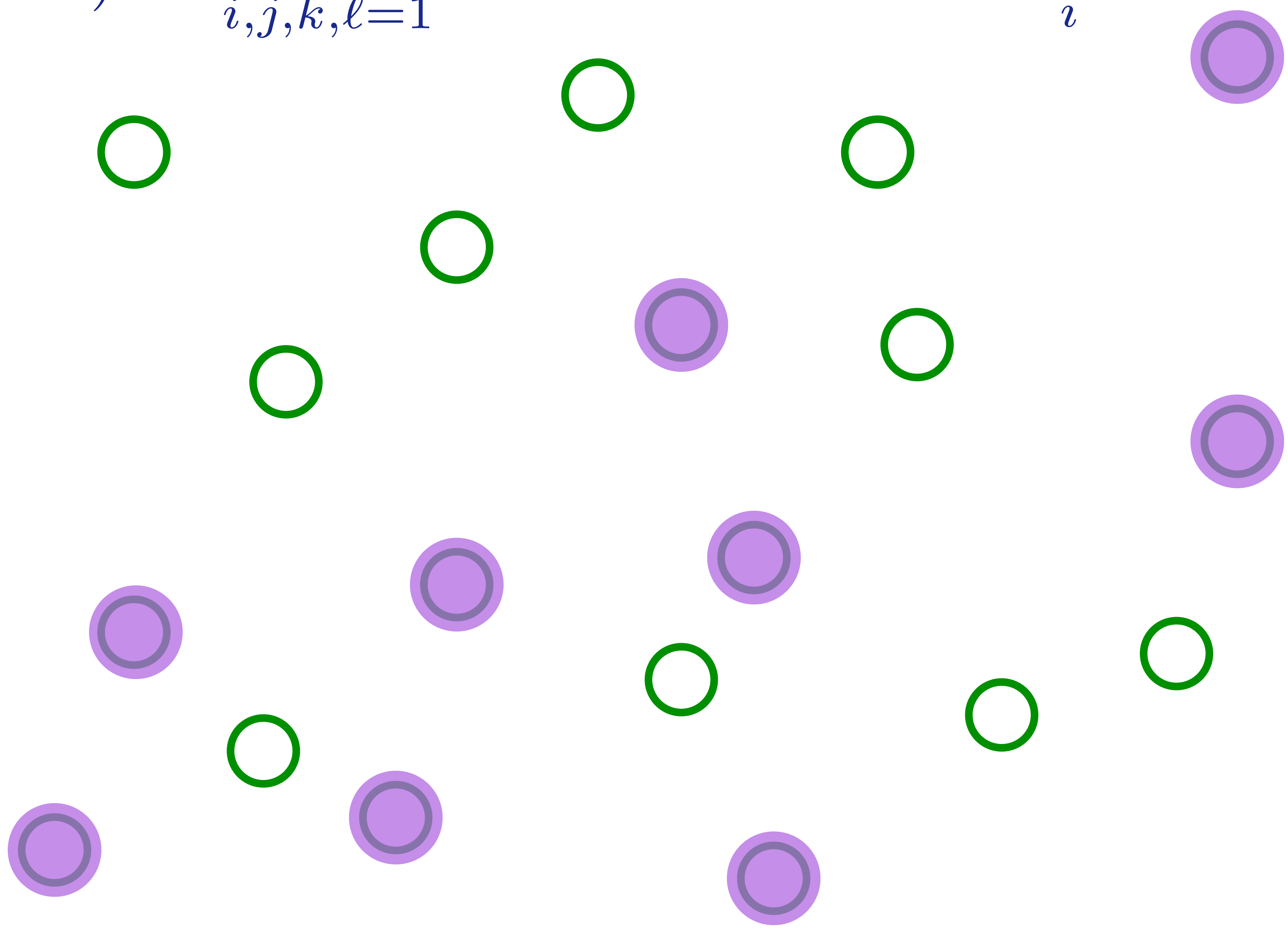
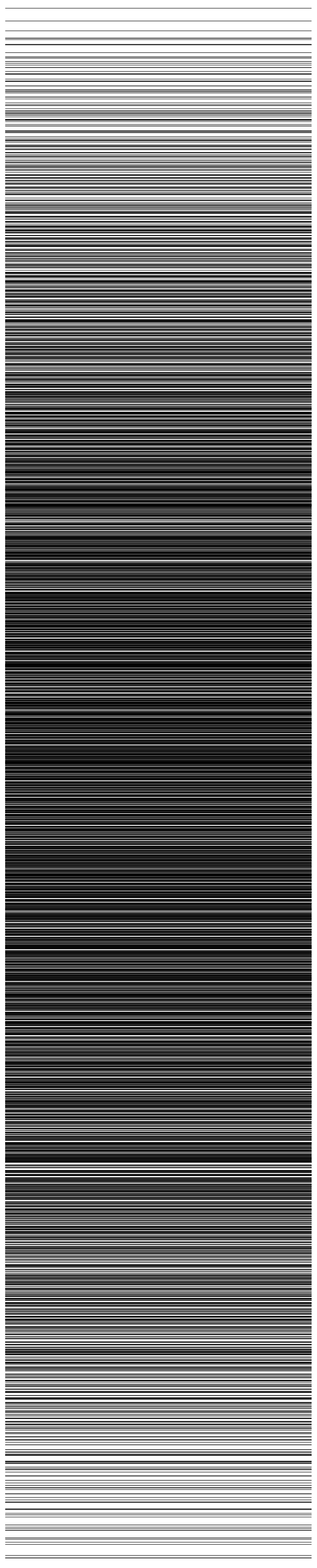


This describes both a strange metal and a black hole!

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993); A. Kitaev (2015)

The SYK model



There are 2^N many body levels with energy E, which do not admit a quasiparticle decomposition. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$ with

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$
$$< \ln 2$$

where G is Catalan's constant, for the half-filled case $Q = 1/2$.

Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

The SYK model

- Low energy, many-body density of states $\rho(E) \sim e^{N s_0} \sinh(\sqrt{2(E - E_0) N \gamma})$

D. Stanford and E. Witten, 1703.04612
A. M. Garica-Garcia, J.J.M. Verbaarschot, 1701.06593
D. Bagrets, A. Altland, and A. Kamenev, 1607.00694

- Low temperature entropy $S = N s_0 + N \gamma T + \ldots$.

- $T = 0$ fermion Green's function $G(\tau) \sim \tau^{-1/2}$ at large τ. (Fermi liquids with quasiparticles have $G(\tau) \sim 1/\tau$)

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- $T > 0$ Green's function has conformal invariance $G \sim (T / \sin(\pi k_B T \tau / \hbar))^{1/2}$

A. Georges and O. Parcollet PRB **59**, 5341 (1999)

- The last property indicates $\tau_{\text{eq}} \sim \hbar/(k_B T)$, and this has been found in a recent numerical study.

A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, arXiv:1706.07803

Black holes share the properties of strongly-coupled quantum criticality

- Black holes have an entropy and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.

- Black holes relax to thermal equilibrium in a time $\sim \hbar/(k_B T_H) = 8\pi G M/c^3$.

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- Black holes relax to thermal equilibrium in a time $\sim \hbar/(k_B T_H) = 8\pi G M/c^3$.
- The entropy of black holes is proportional to their surface area. So they can only be equivalent to quantum-critical systems in one lower dimension.

Black holes

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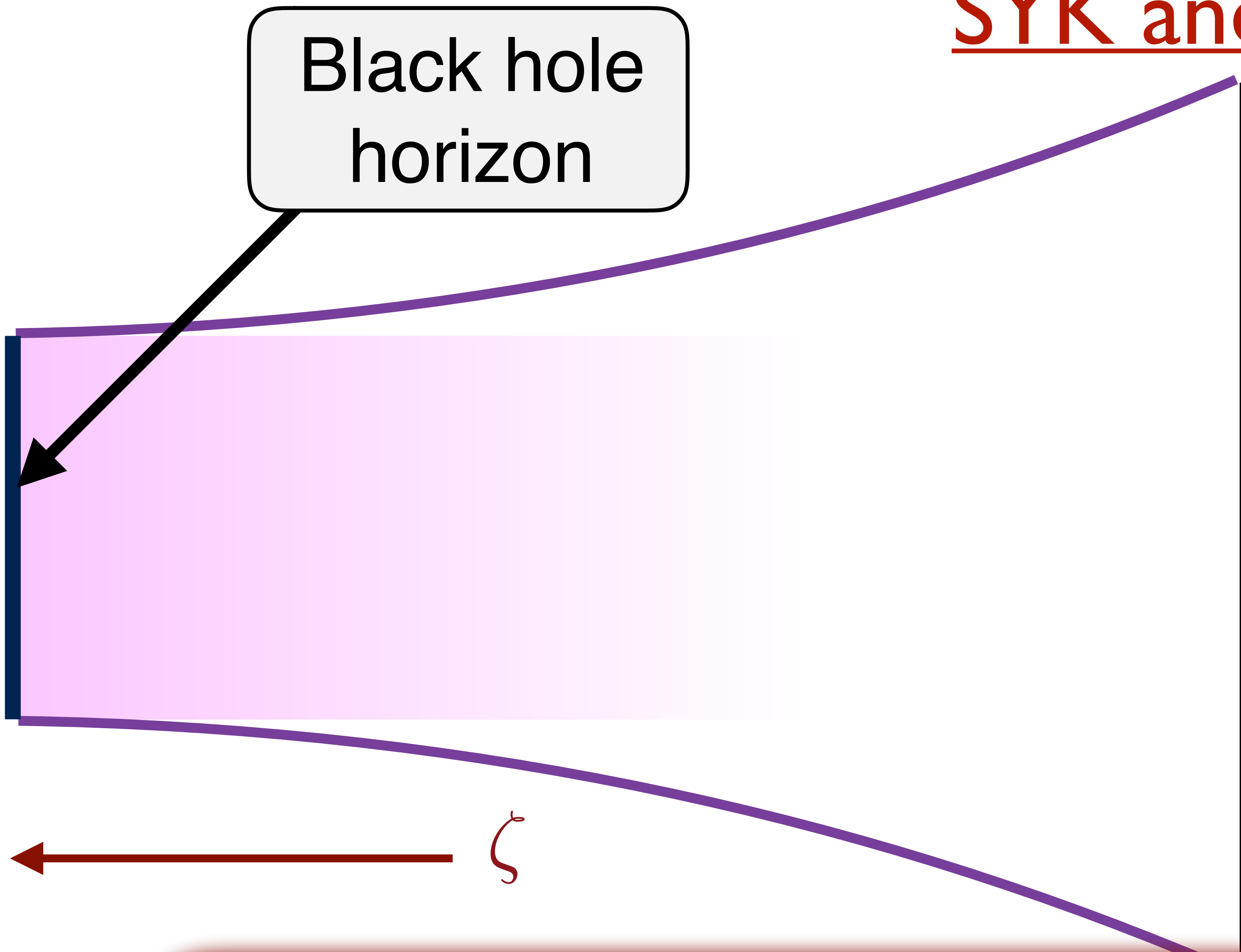
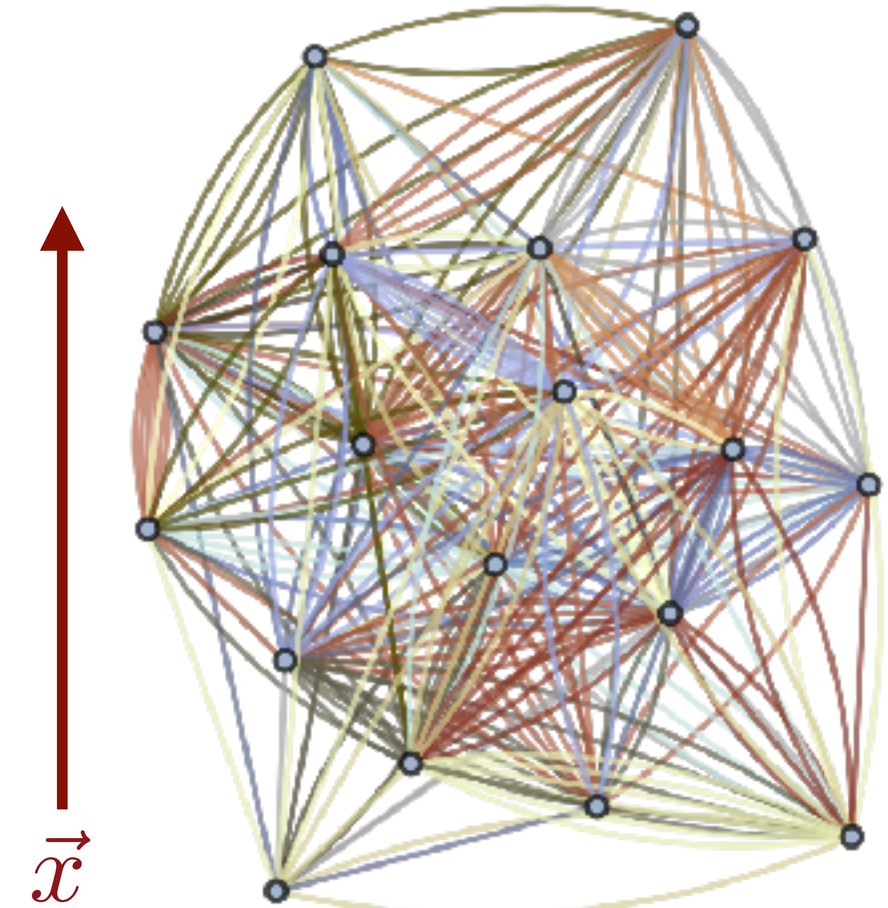
SYK and black holes

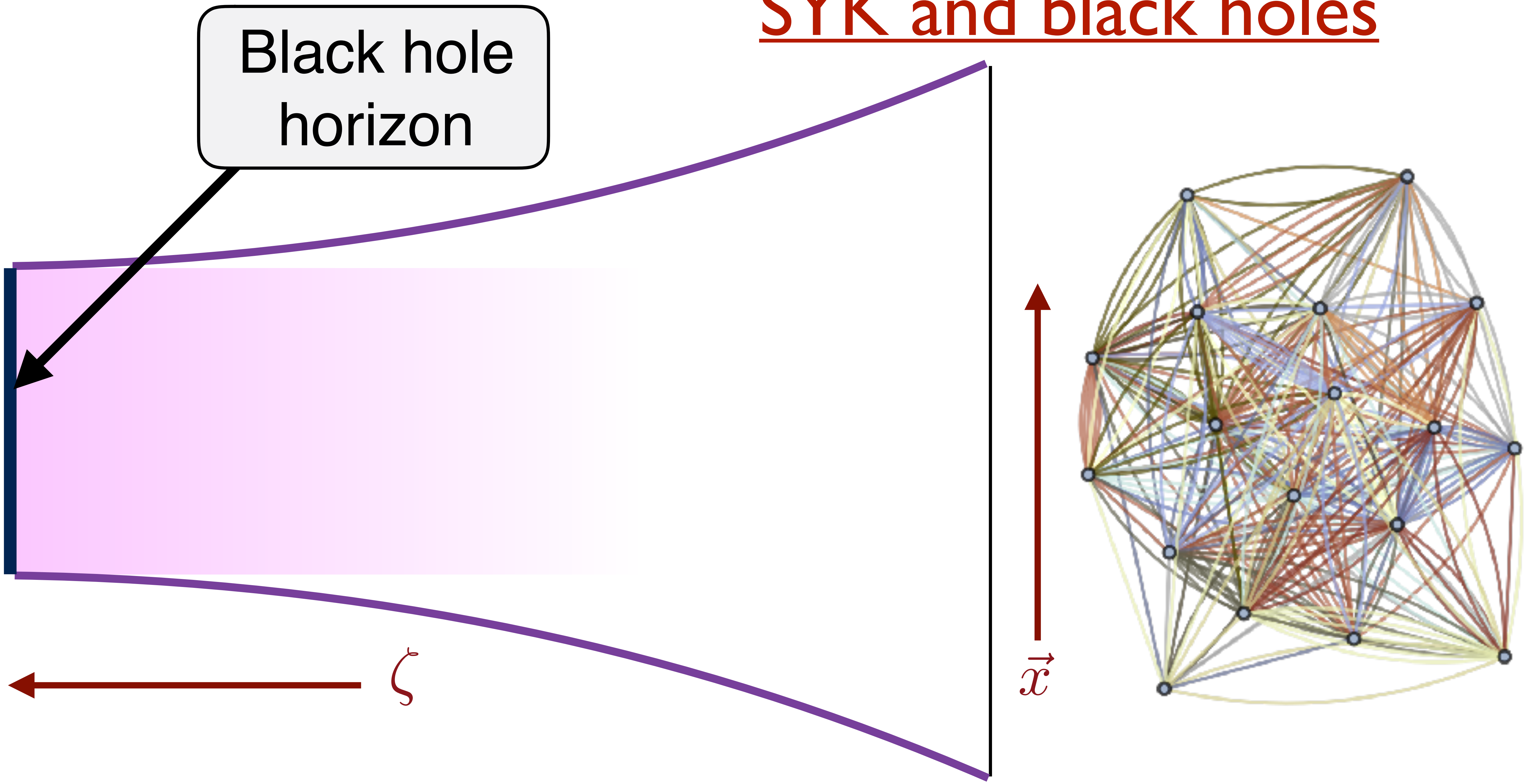




Black holes with a near-horizon AdS_2 geometry (described by quantum gravity in 1+1 spacetime dimensions) match the properties of the 0+1 dimensional SYK model in the previous slide: $N s_0$ is the Bekenstein-Hawking entropy

S. Sachdev, PRL **105**, 151602 (2010); A. Kitaev (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857

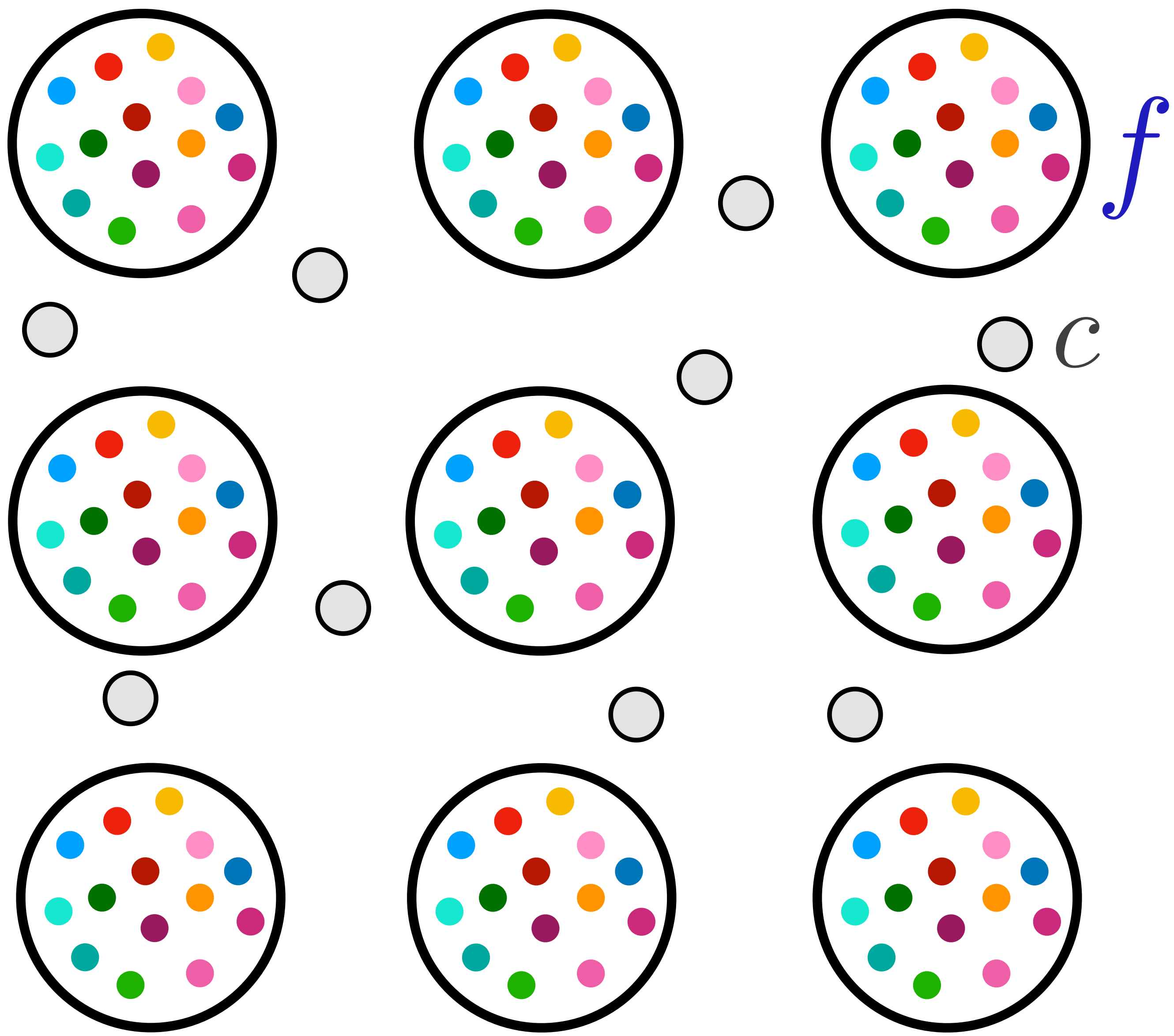
SYK and black holes



Both the SYK model and the black hole saturate the lower bound on the Lyapunov time to quantum chaos: $\tau_L = \hbar/(2\pi k_B T)$.

A. Kitaev (2015); J. Maldacena and D. Stanford, arXiv:1604.07818

SYK building blocks for strange metals



Yields solvable models with linear-in-T resistivity (possibly 'bad metals' with $\rho > h/e^2$) *and* linear-in-B magnetoresistance with B/T scaling.

A. Georges and O. Parcollet PRB **59**, 5341 (1999); Xue-Yang Song, Chao-Ming Jian, L. Balents, PRL **119**, 216601 (2017); A. A. Patel, J. McGreevy, D. P. Arovas, S. Sachdev, arXiv:1712.05026; D. Chowdhury, Y. Werman, E. Berg, T. Senthil, arXiv:1801.06178

Looking ahead:

- Experimental and theoretical studies of *metals* with bulk topological order and Fermi surfaces, possibly with non-Luttinger volumes.

- Experimental and theoretical studies of ‘strange’, ‘bad’, ‘incoherent’, ‘ultra-quantum’ metals: microscopic basis for SYK building blocks.