Dual vortex theory of doped antiferromagnets

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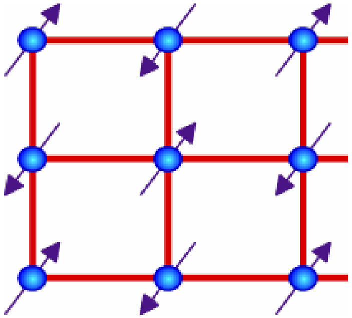
Phase diagram of doped antiferromagnets

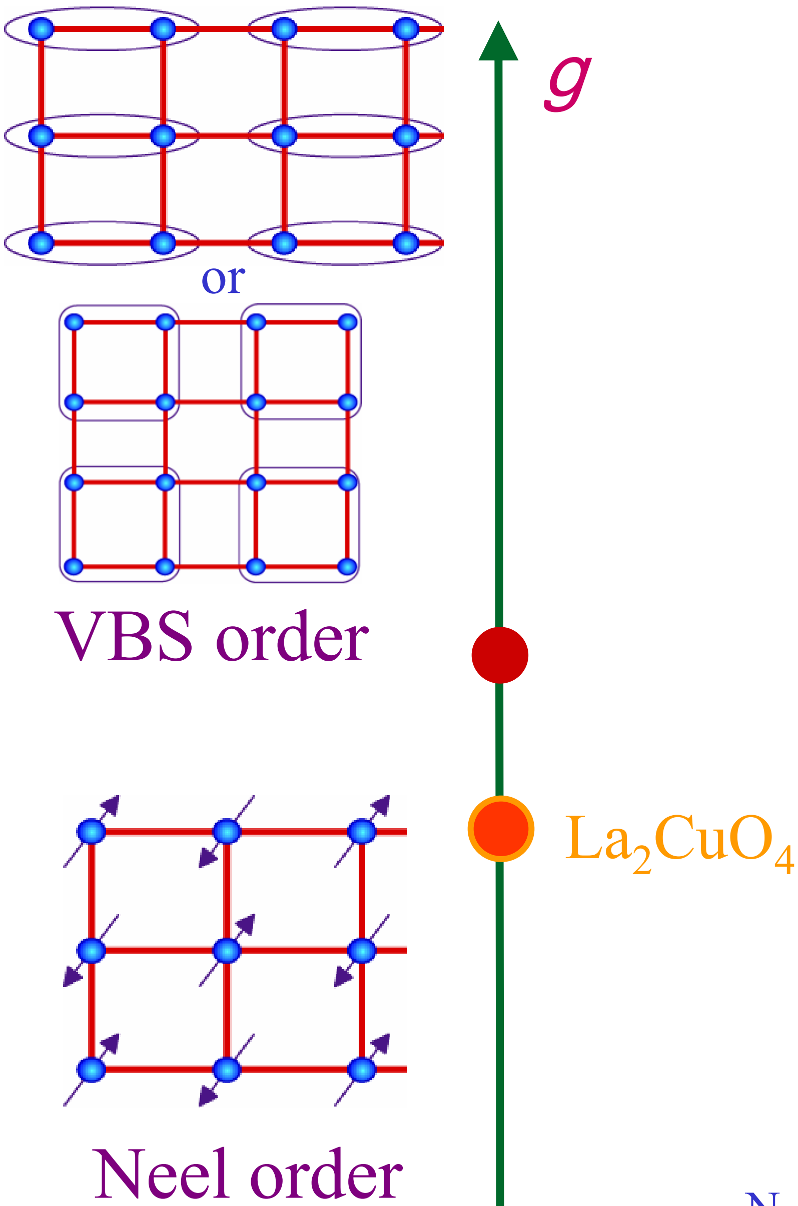
g = parameter controlling strength of quantum fluctuations in a semiclassical theory of the destruction of Neel order

La_2CuO_4



Neel order

Phase diagram of doped antiferromagnets



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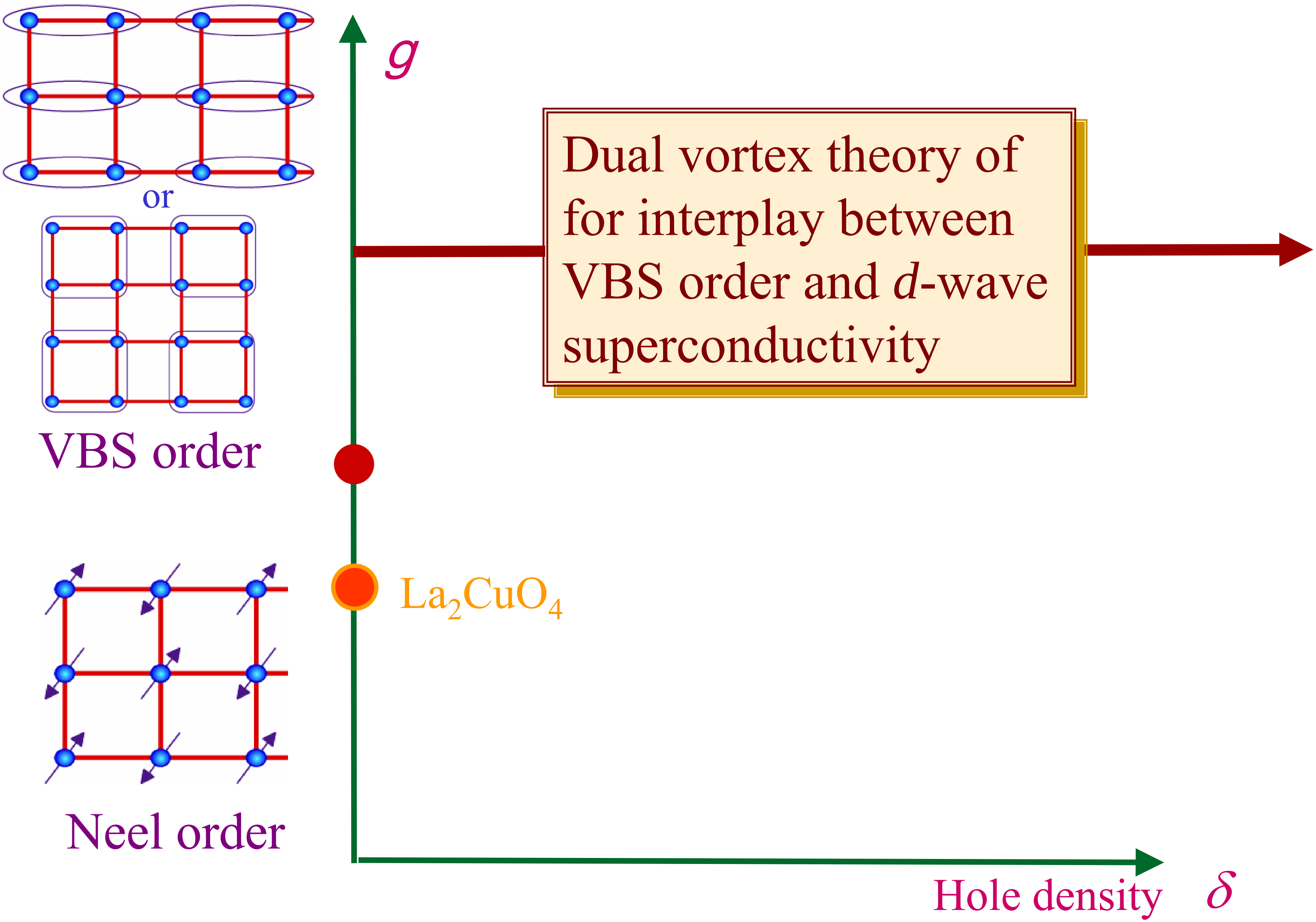
Phase diagram of doped antiferromagnets



Theory for the VBS phase at half filling

Expressed as a compact U(1) gauge theory for the field $\mathcal{A}_\mu$. The "electric" field is a number operator for singlet valence bonds. The low energy action is

$$\mathcal{S}_A = \frac{1}{2e^2} \int d^2x d\tau \left(\epsilon_{\mu\nu\lambda}\partial_\nu \mathcal{A}_\lambda\right)^2$$

The theory also has **monopoles**, with annihilation operator m. The monopole operator transforms "projectively" under the square lattice space group (the "PSG")

$$\begin{aligned} T_x &: \quad m \rightarrow i m^\dagger \\ T_y &: \quad m \rightarrow -i m^\dagger \\ R &: \quad m \rightarrow m^\dagger \end{aligned}$$

Theory for the VBS phase at half filling

Monopole-antimonopole configurations have a finite action, and hence

$$\lim_{|r| \to 0} \langle m(r) m^\dagger(0) \rangle \neq 0$$

This implies that the monopoles have "condensed", and we can work in a ground state with $m =$ constant.
From the symmetry transformation of m under the square lattice space group, it can be shown that the VBS order parameter is proportional to m. Hence condensation of m implies long-range VBS order.

Doping the VBS phase with holes of density δ

- Holes are represented by charge e bosons b_1 and b_2 which also carry $\mathcal{A}_\mu$ gauge charges +1 and -1.

- The density of each boson species is

$$\frac{\delta}{2} = \frac{p}{q}$$

- Dualize each boson species to vortices φ_1 and φ_2. Because of "screening" by the $\mathcal{A}_\mu$ gauge field, each $\varphi_{1,2}$ vortex carries magnetic flux $hc/(2e)$.

- At this point, it appears that the magnetic space group obeys

$$T_x T_y = T_y T_x e^{2\pi i p/q}$$

and there are $2q$ flavors of $hc/2e$ vortices

Doping the VBS phase with holes of density δ

The Key Step: There are terms combining the *monopoles* of the half-filled antiferromagnet with the *vortices* of the doped holes which are invariant under all space group operations.
For $q = 2n$ such a term has the structure

$$\mathcal{L} = m^\dagger \sum_{\ell=0}^{q-1} \left[e^{-i\pi/4} (-1)^\ell \varphi_{1\ell}^* \varphi_{2\ell} + e^{i\pi/4} \varphi_{1\ell}^* \varphi_{2,\ell+q/2} \right] + \text{c.c.}.$$

Consequently, the low energy vortices have degeneracy q', where

$$q' = \begin{cases} 2q & \text{for } q = 2n+1 \\ q & \text{for } q = 4n \\ q/2 & \text{for } q = 4n+2 \end{cases}$$

Doping the VBS phase with holes of density δ

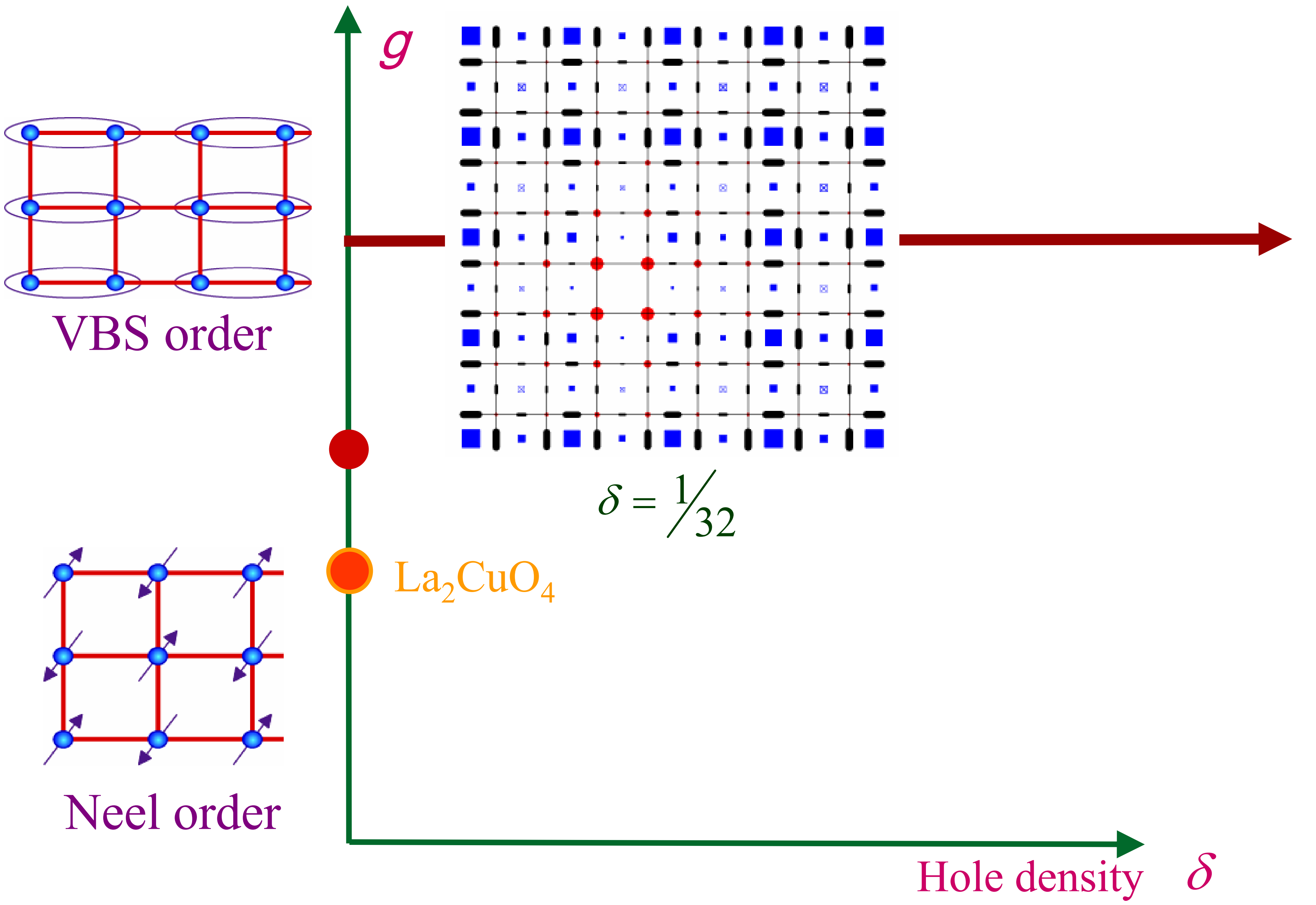
Alternative interpretation of the value of q': The same values of q' are obtained by the relation

$$\frac{p'}{q'} = \frac{1}{2} - \frac{p}{q} = \frac{1-\delta}{2}$$

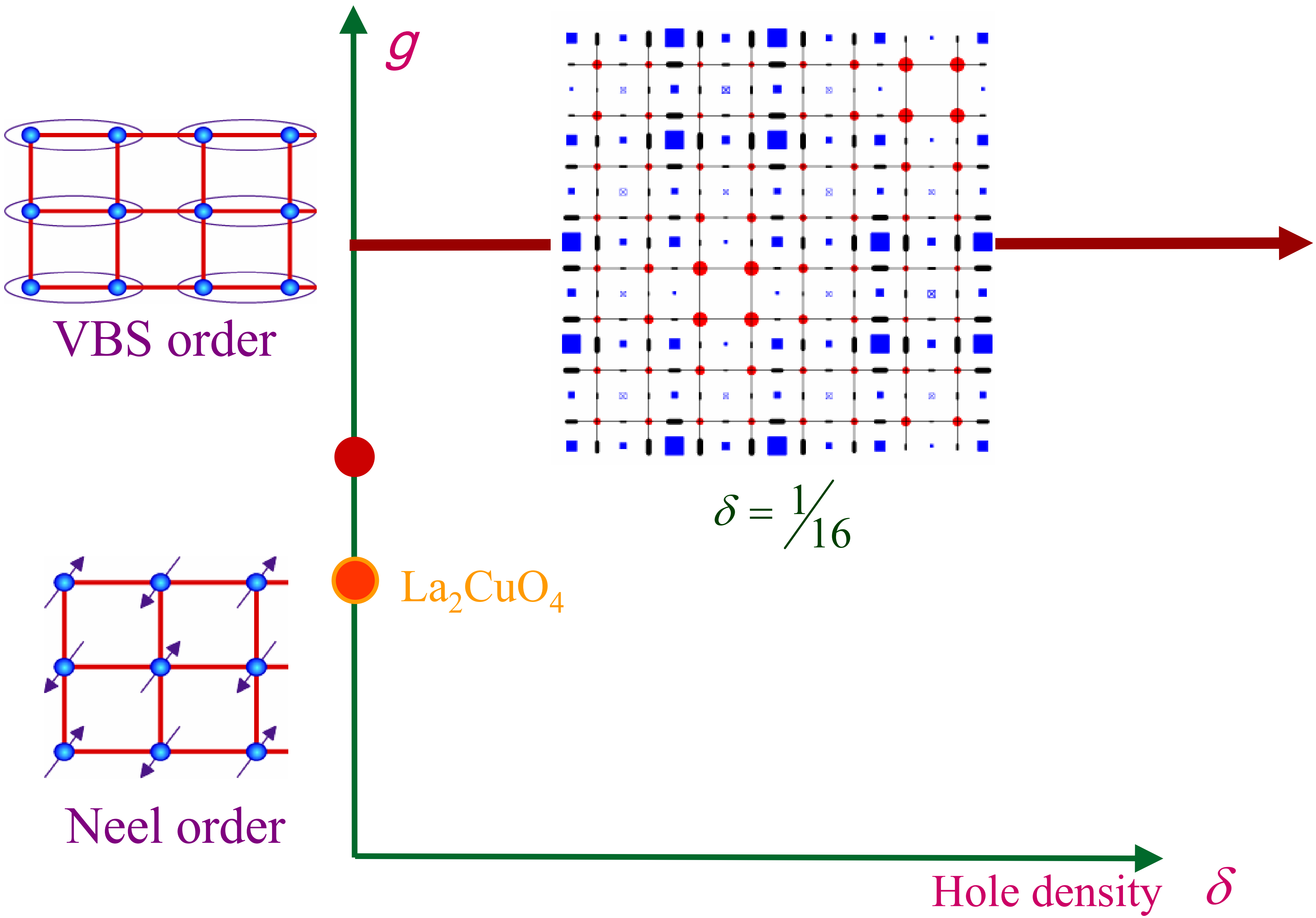
Note that $(1-\delta)/2$ is the total number of *pairs of electrons*. So at the end, there are q' vortices, where p'/q' is the density of Cooper pairs.

In other words, the vortex PSG is just that would have been obtained by considering **a theory of elementary bosons on the square lattice, with density equal to the density of pairs of electrons**.

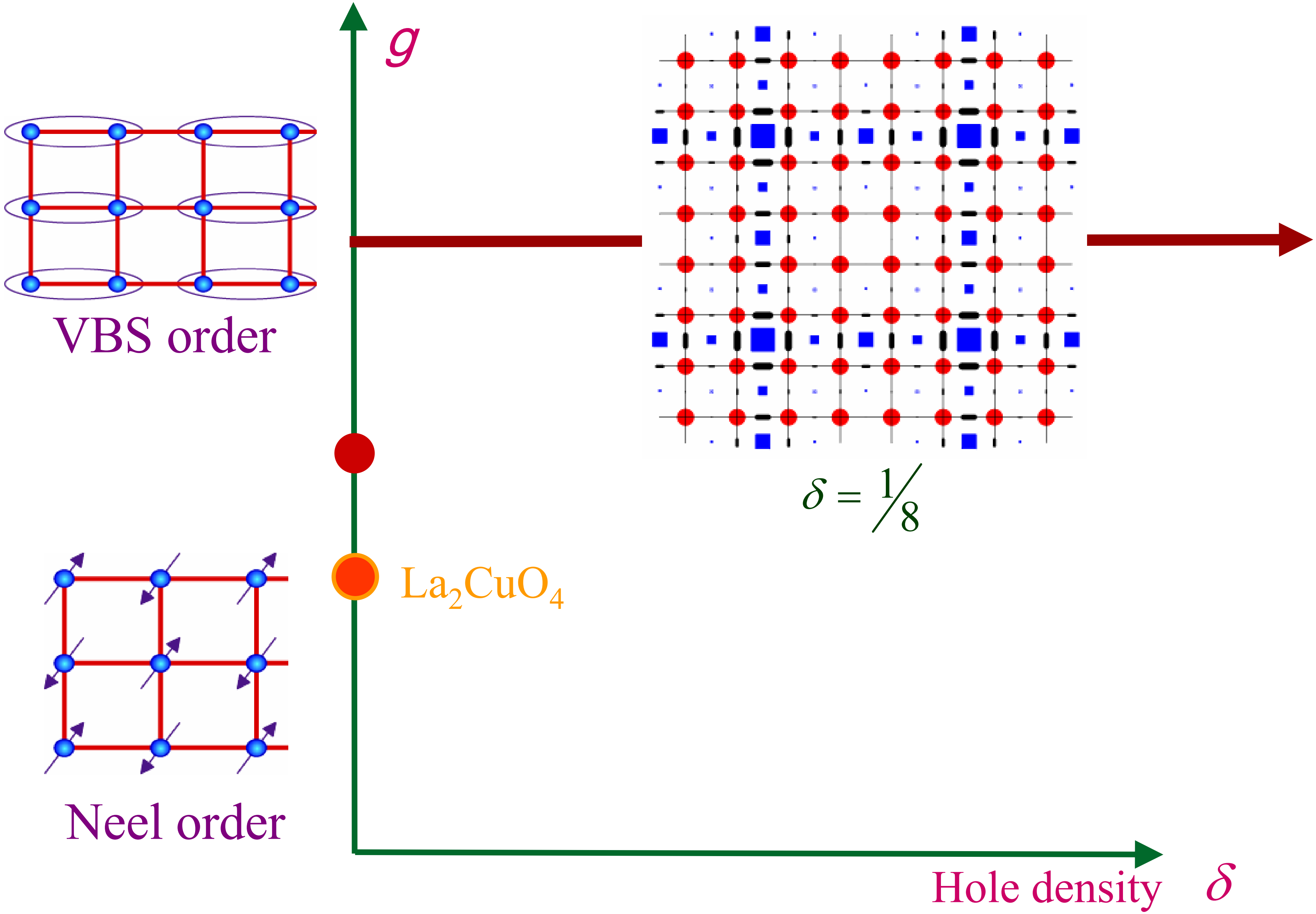
Phase diagram of doped antiferromagnets



Phase diagram of doped antiferromagnets



Phase diagram of doped antiferromagnets



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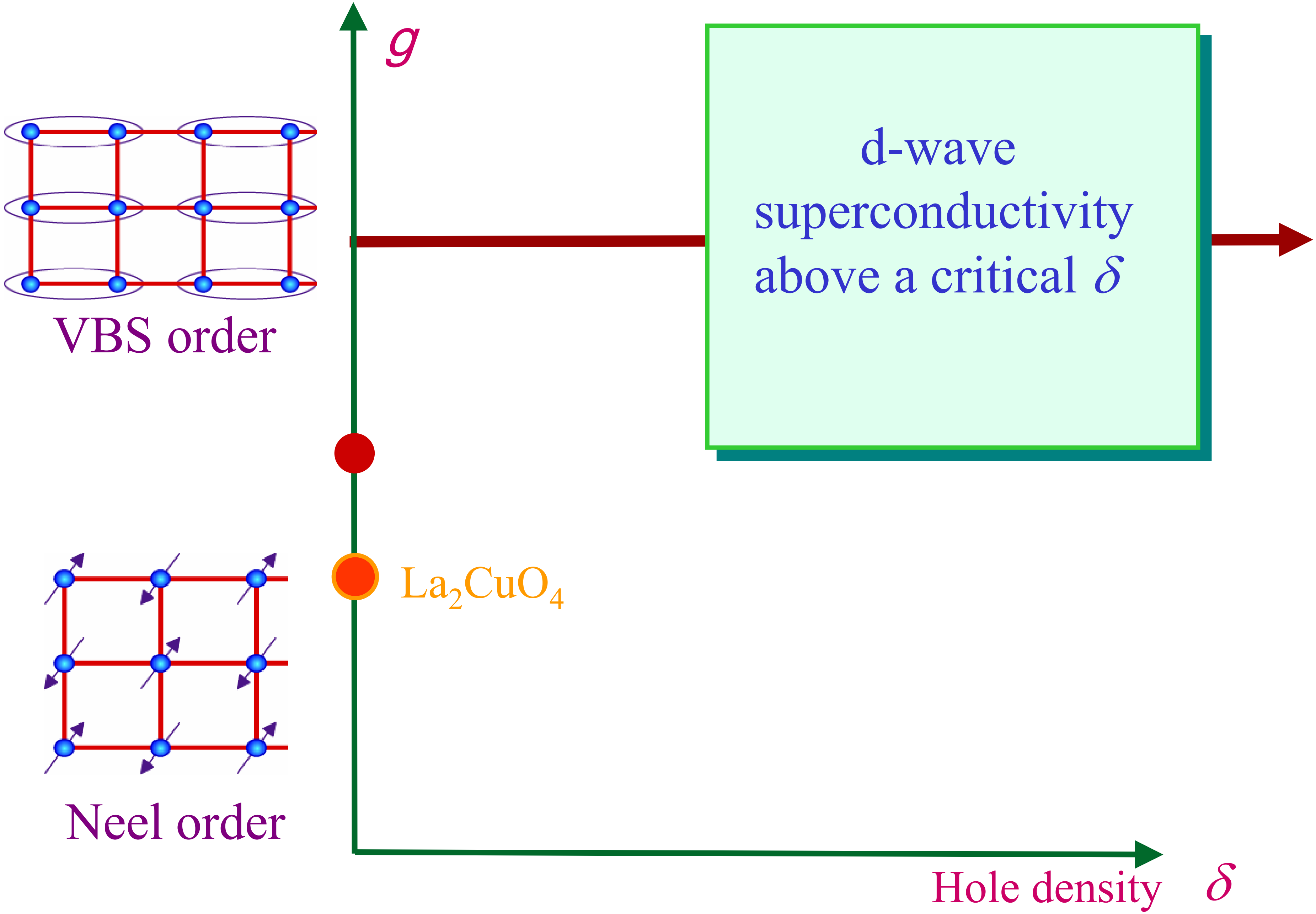
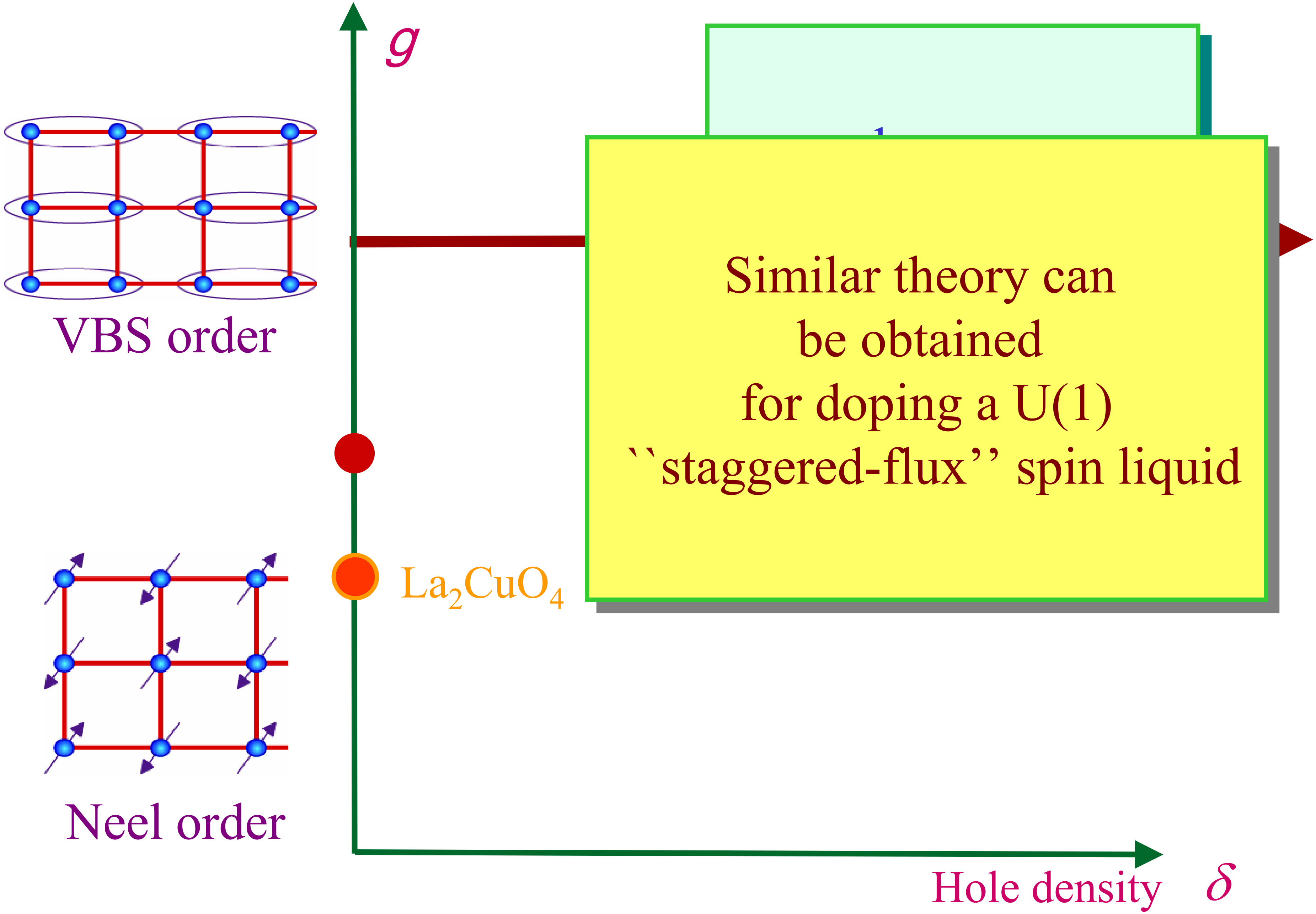


Phase diagram of doped antiferromagnets



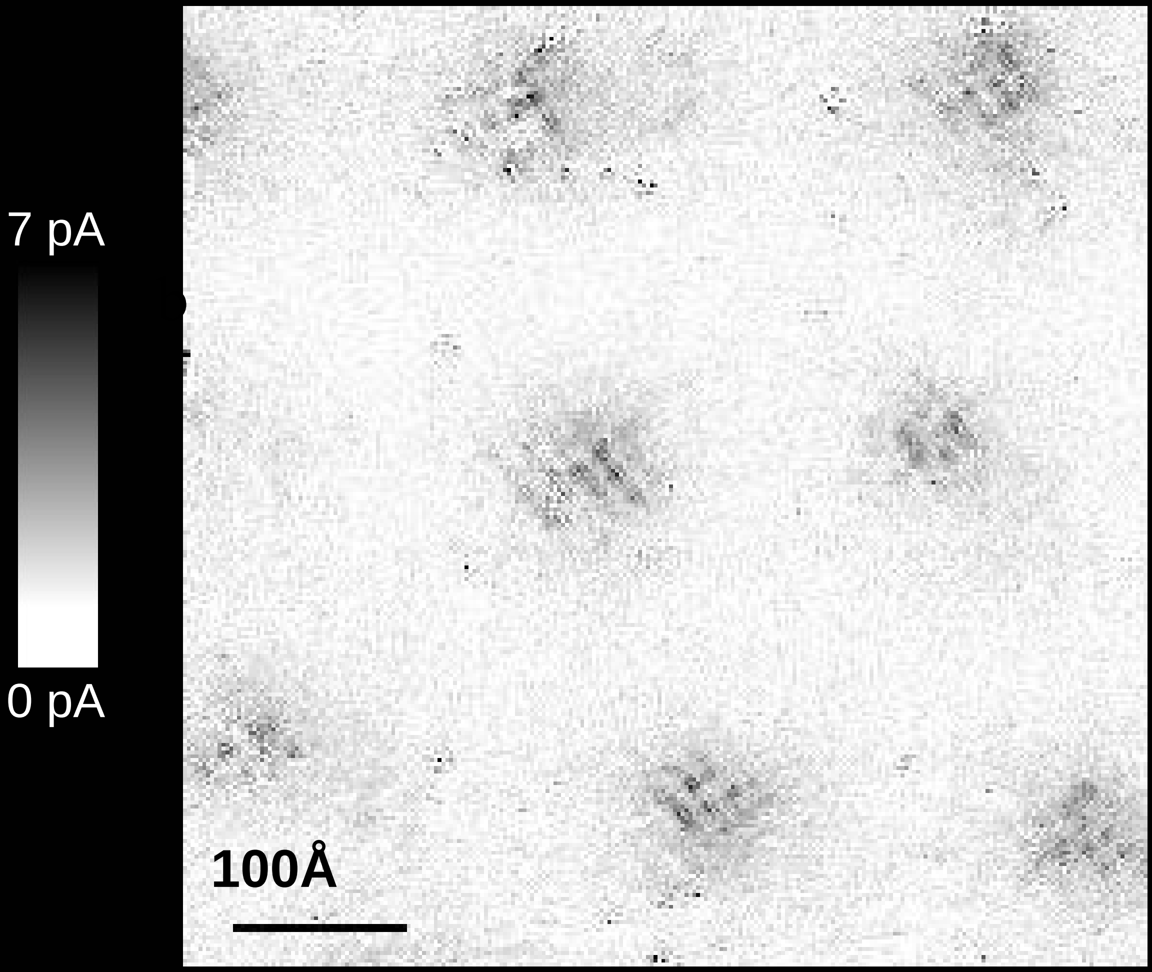
Similar theory can be obtained for doping a U(1) ``staggered-flux'' spin liquid

Superfluids near Mott insulators

The Mott insulator has average Cooper pair density, $f = p/q$ per site, while the density of the superfluid is close (but need not be identical) to this value

- Vortices with flux *h/(2e)* come in multiple (usually q) "flavors"
- The lattice space group acts in a projective representation on the vortex flavor space.
- Any pinned vortex must chose an orientation in flavor space. This necessarily leads to modulations in the local density of states over the spatial region where the vortex executes its quantum zero point motion.
- These modulations may be viewed as strong-coupling analogs of Friedel oscillations in a Fermi liquid.

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV at 4K



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Vortices have halos with LDOS modulations at a period ≈ 4 lattice spacings

Prediction of VBS order near vortices: K. Park and S. Sachdev, Phys. Rev. B **64**, 184510 (2001).