Transport without quasiparticles in graphene and Weyl semi-metals

X05:3
APS March Meeting, Los Angeles

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Talk online: sachdev.physics.harvard.edu

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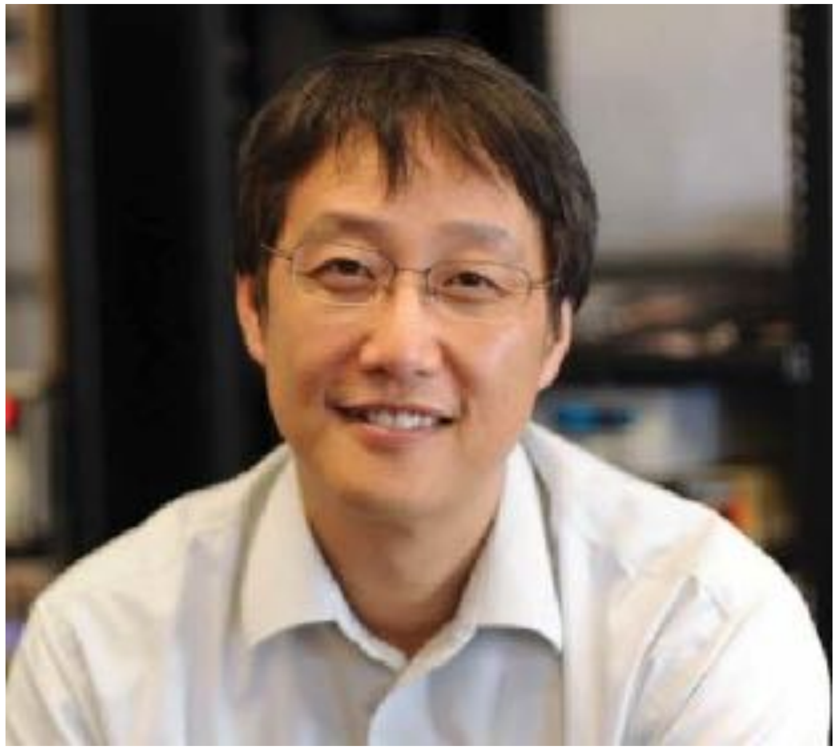
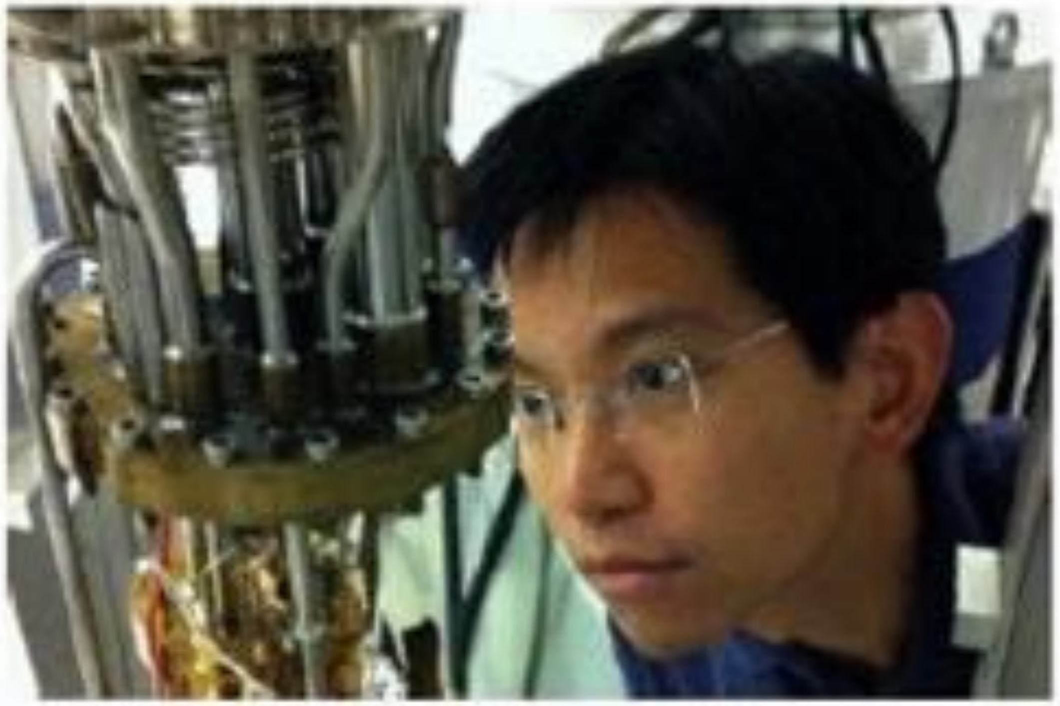
Graphene



Philip Kim

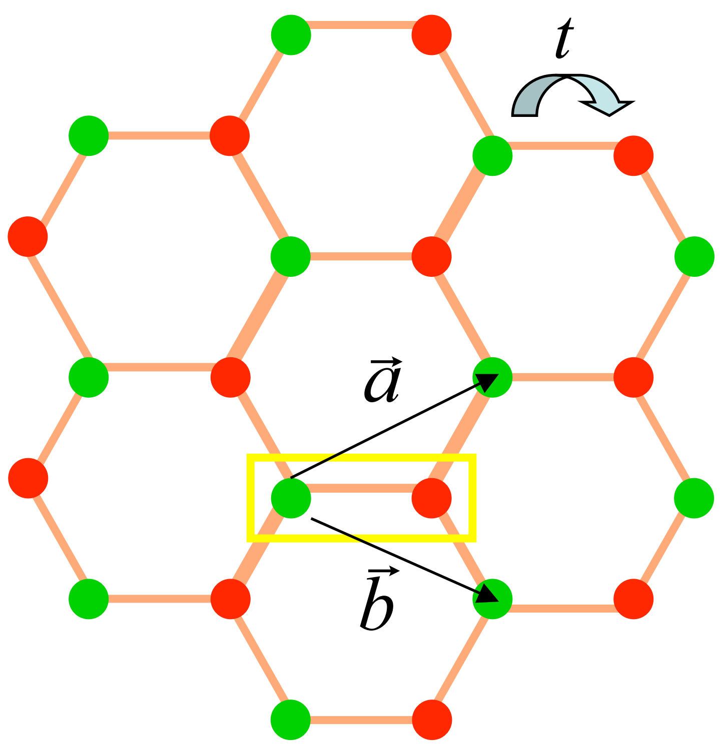
Andrew Lucas

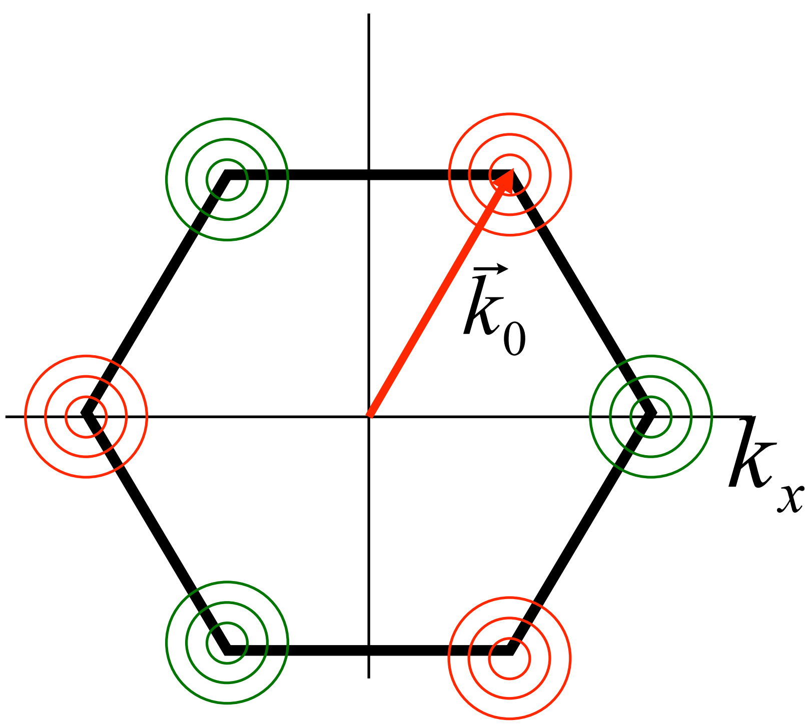


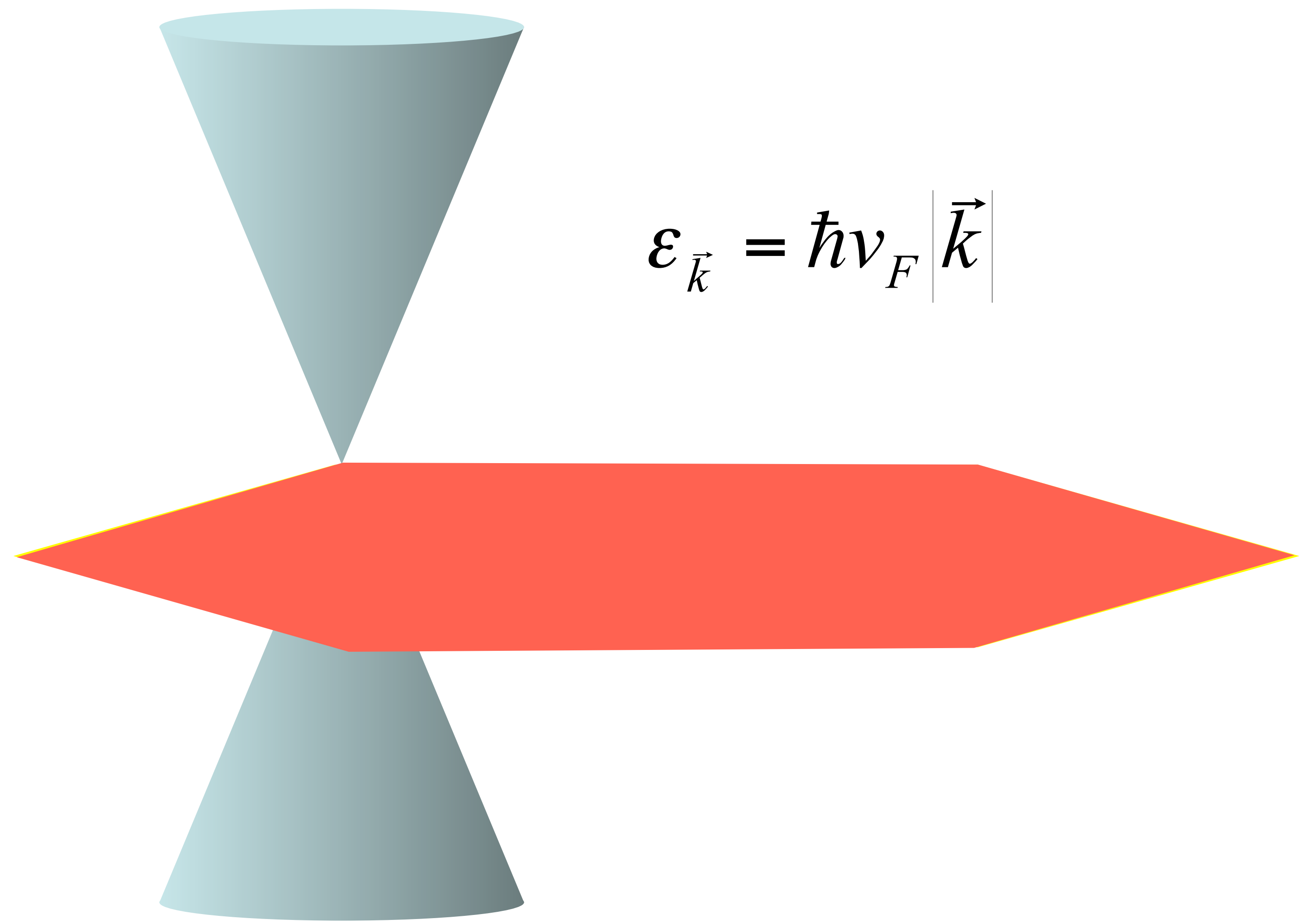
Kin Chung Fong

Jesse Crossno

Graphene







$$\varepsilon_{\vec{k}} = \hbar v_F \left|\vec{k}\right|$$

Graphene at half-filling; no impurities

Low energy theory has 4 two-component Dirac fermions, ψ_σ, $\sigma = 1 \ldots 4$, interacting with a $1/r$ Coulomb interaction

$$\mathcal{S} = \int d^2r d\tau \psi_\sigma^\dagger \left(\partial_\tau - i v_F \vec{\sigma} \cdot \vec{\nabla} \right) \psi_\sigma + \frac{e^2}{2} \int d^2r d^2r' d\tau \psi_\sigma^\dagger \psi_\sigma(r) \frac{1}{|r - r'|} \psi_{\sigma'}^\dagger \psi_{\sigma'}(r')$$

Dimensionless "fine-structure" constant $\alpha = e^2/(\hbar v_F)$.
RG flow of α:

$$\frac{d\alpha}{d\ell} = -\alpha^2 + \ldots$$

Behavior is similar to a conformal field theory (CFT) in 2+1 dimensions with $\alpha \sim 1/\ln(\text{scale})$

Conductivity is finite
without impurities and
with particle-hole symmetry, but
thermal conductivity is infinite

	Particles	**Holes**
Momentum	→	←
Electrical current	→	→

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2}\chi(k,\omega)$

For *all* CFT3s, at $\hbar\omega \gg k_B T$

$$\chi(k,\omega) \quad = \quad K \frac{k^2}{\sqrt{v^2k^2 - \omega^2}} \quad ; \quad \sigma(\omega) = K$$

where K is a universal dimensionless number (in units of e^2/h) characterizing the CFT3, and v is the velocity of "light".

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k,\omega)$

However, for *all* CFT3s, at $\hbar\omega \ll k_B T$, we have "phase" randomizing collisions and relaxation to local thermodynamic equilibrium. This leads to the hydrodynamic behavior

$$\chi(k,\omega) = \chi_c \frac{Dk^2}{Dk^2 - i\omega} \quad ; \quad \sigma(\omega) = D\chi_c \equiv \sigma_Q$$

where χ_c is the **compressibility** and D is the **diffusion constant**, and σ_Q is the **conductivity** = a dimensionless number times (e^2/h). (We are ignoring logarithmic corrections from 'long-time tails' present when momentum is exactly conserved.)

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Collisionless-hydrodynamic crossover in graphene

$$\sigma_Q(\omega) = \begin{cases} \dfrac{e^2}{h}\left[\dfrac{\pi}{2} + \mathcal{O}\left(\dfrac{1}{\ln(\Lambda/\omega)}\right)\right] & , \quad \hbar\omega \gg k_B T \\ \dfrac{e^2}{h\alpha^2(T)}\left[0.760 + \mathcal{O}\left(\dfrac{1}{|\ln(\alpha(T))|}\right)\right] & , \quad \hbar\omega \ll k_B T \alpha^2(T) \end{cases}$$

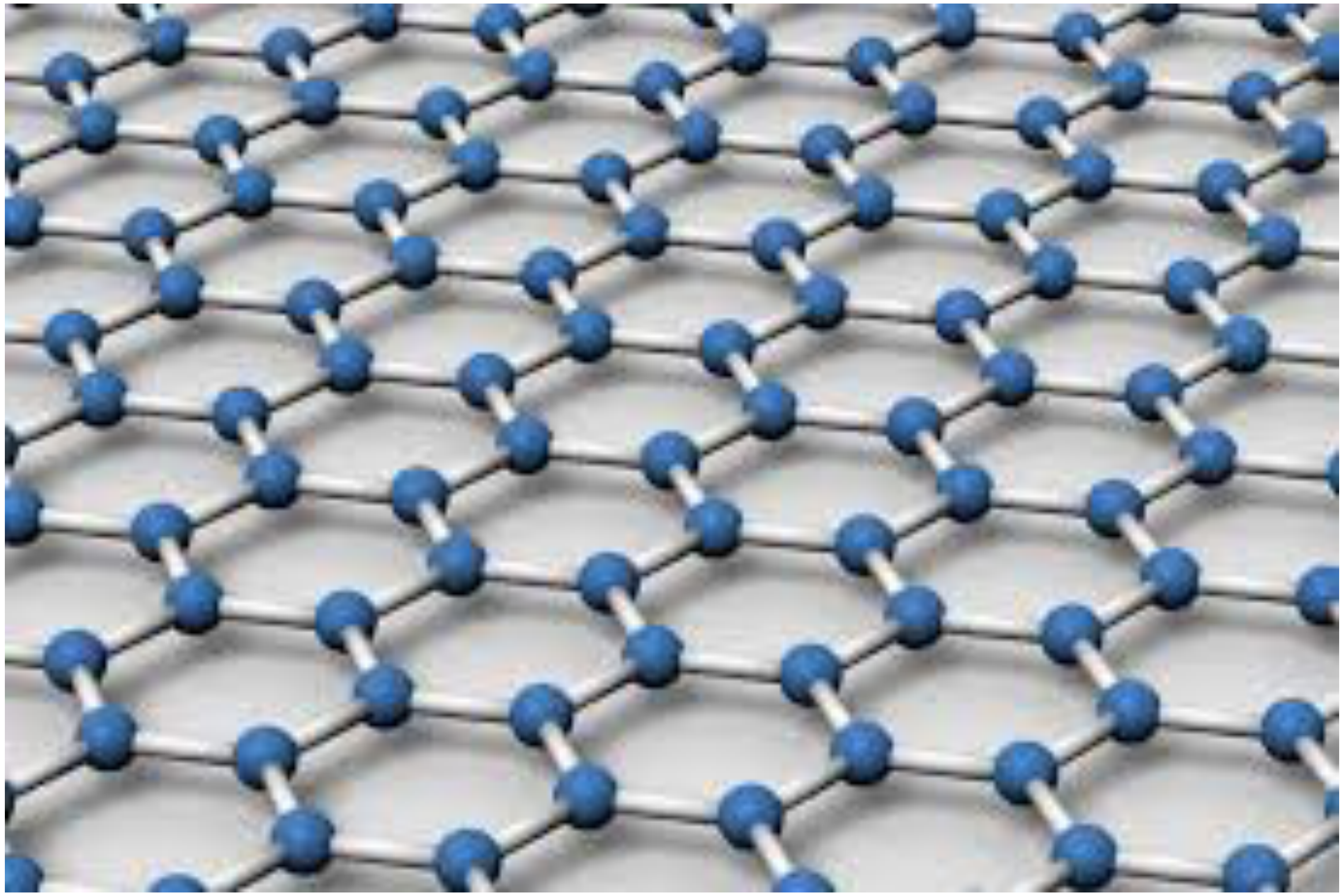
I. Herbut, V. Juricic, and O. Vafek, *Phys. Rev. Lett.* **100**, 046403 (2008).

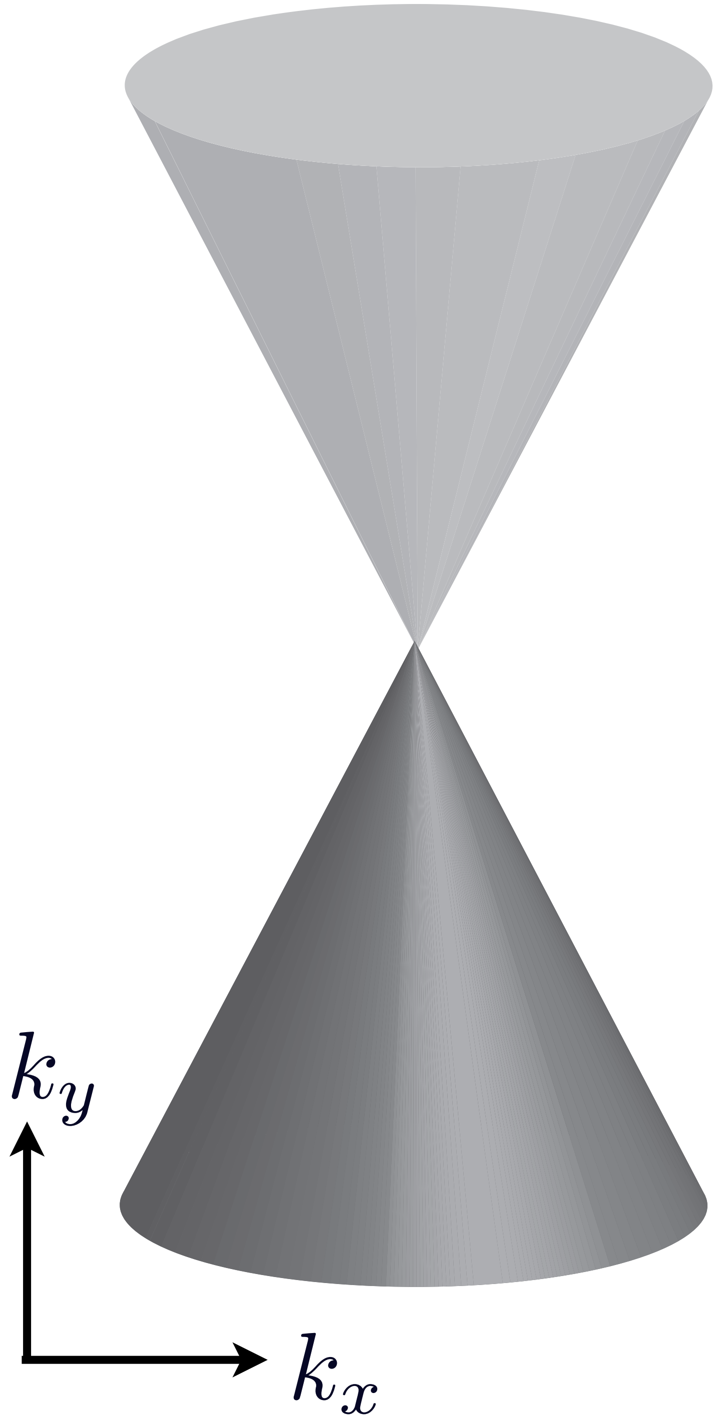
where $\alpha(T)$ is the T-dependent fine structure constant which obeys

$$\alpha(T) = \frac{\alpha}{1 + (\alpha/4)\ln(\Lambda/T)} \overset{T\to 0}{\sim} \frac{4}{\ln(\Lambda/T)}$$

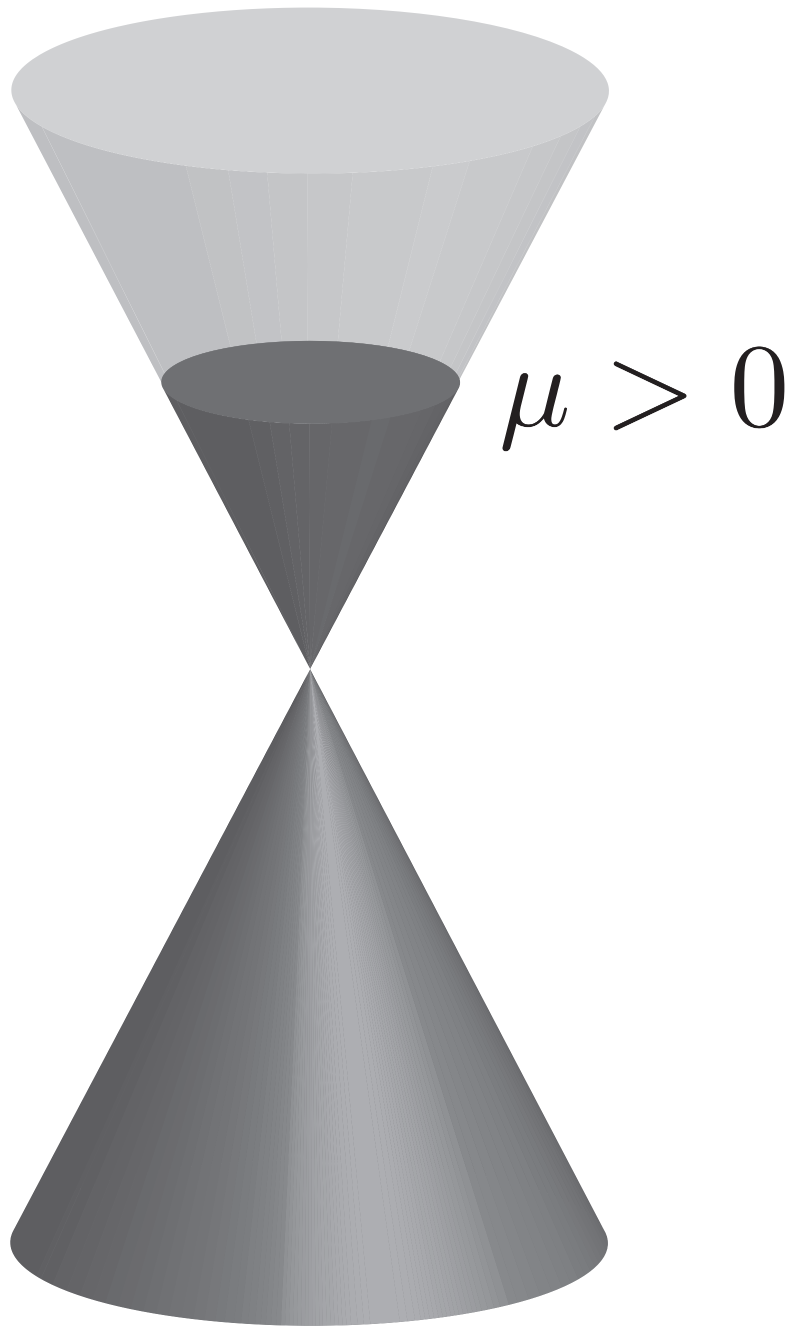
L. Fritz, M. Mueller, J. Schmalian and S. Sachdev, PRB **78**, 085416 (2008)

Graphene





Graphene



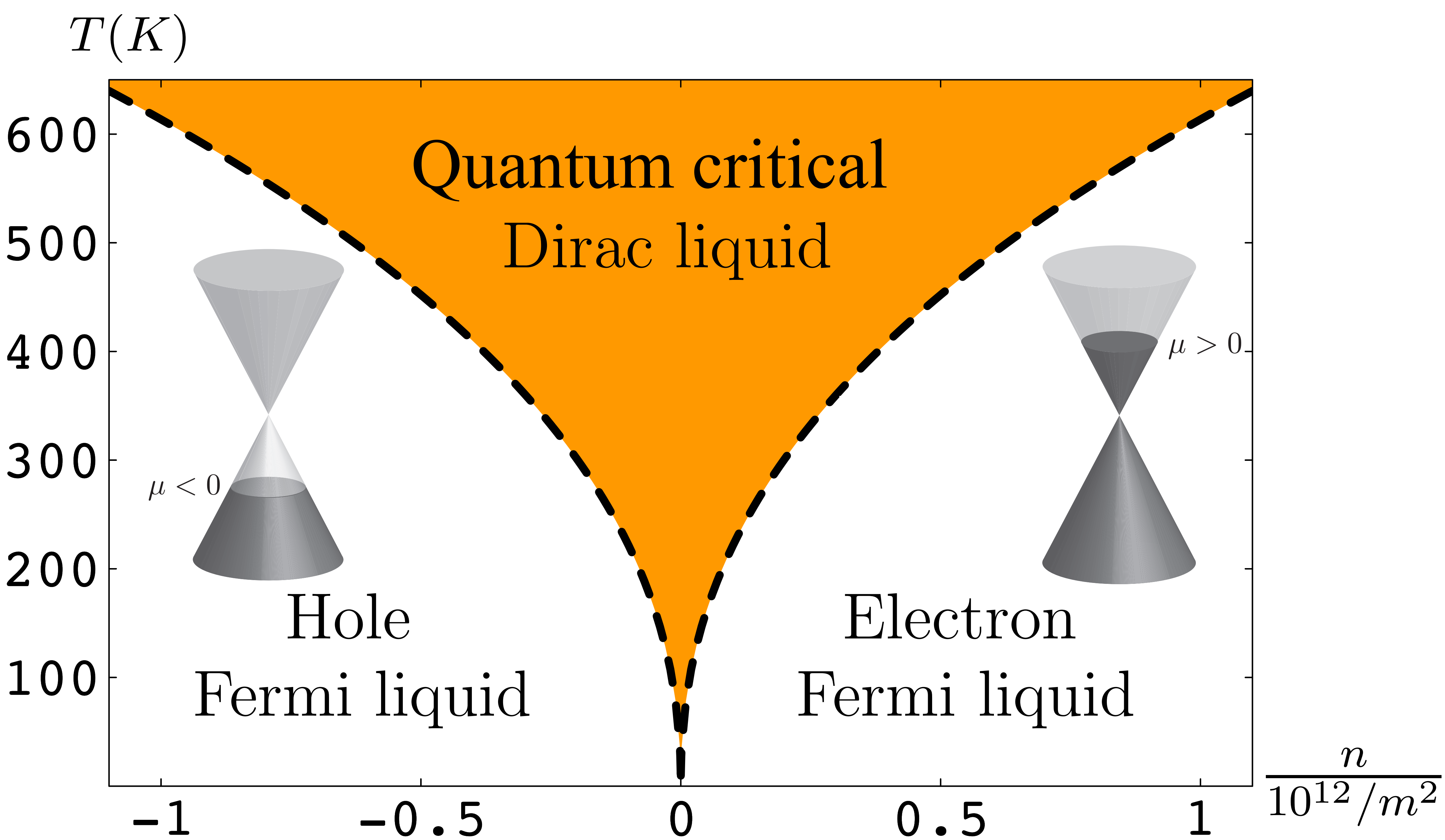
Electron
Fermi surface

Graphene

Hole Fermi surface

Electron Fermi surface

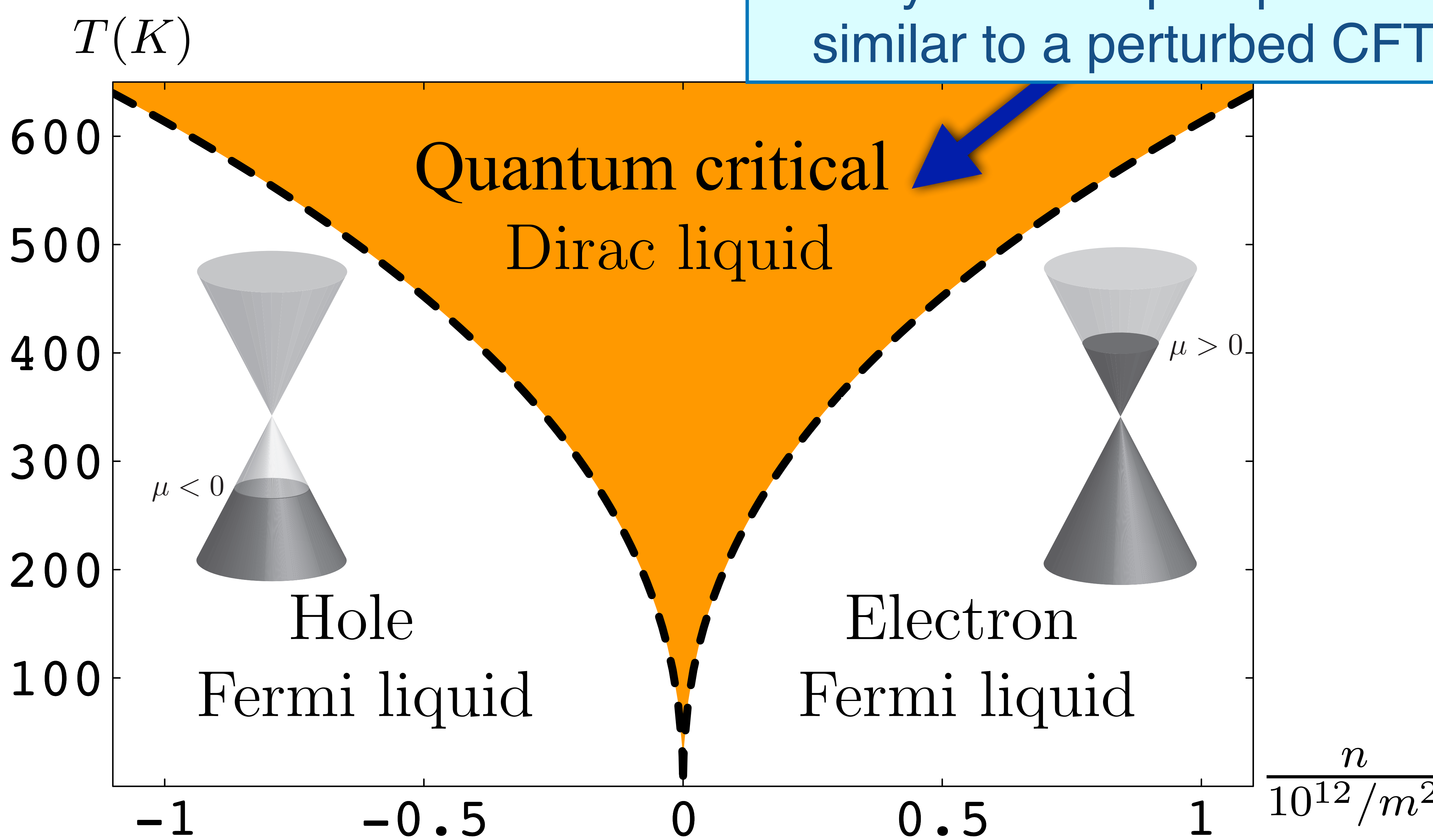
Graphene



D. E. Sheehy and J. Schmalian, PRL **99**, 226803 (2007)
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Graphene

Prediction: hydrodynamic theory without quasiparticles, similar to a perturbed CFT



M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
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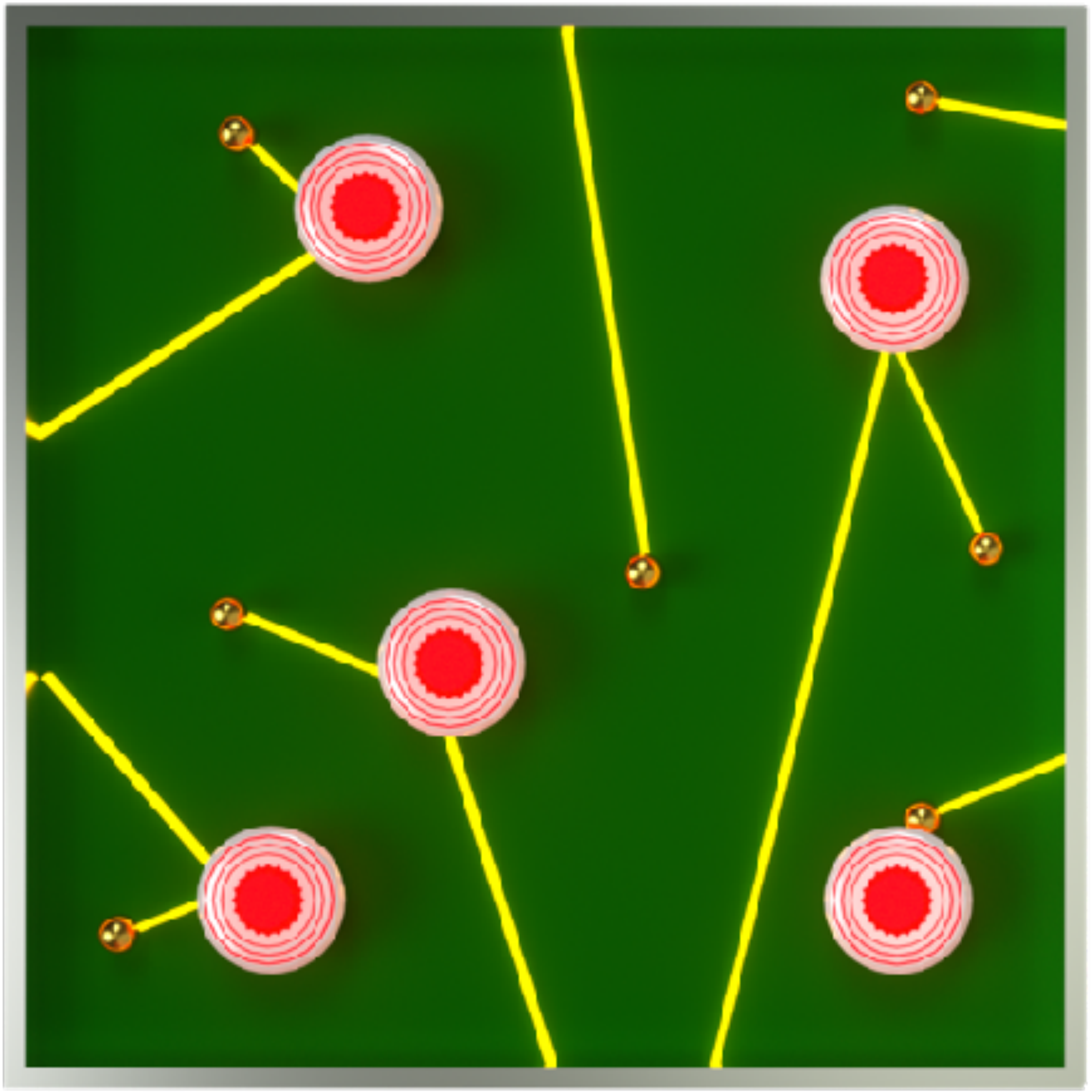
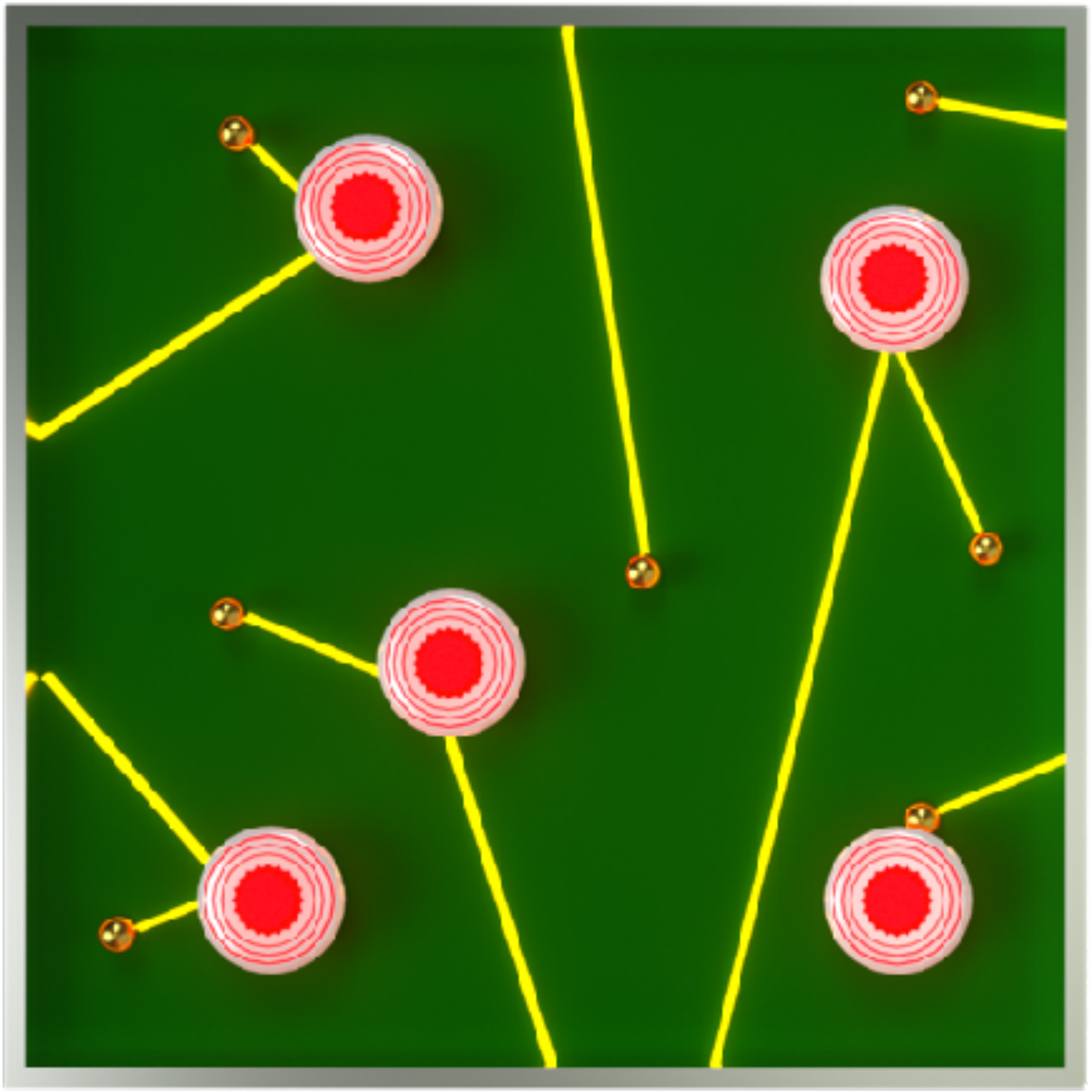


Fermi liquids: quasiparticles moving ballistically between impurity (red circles) scattering events



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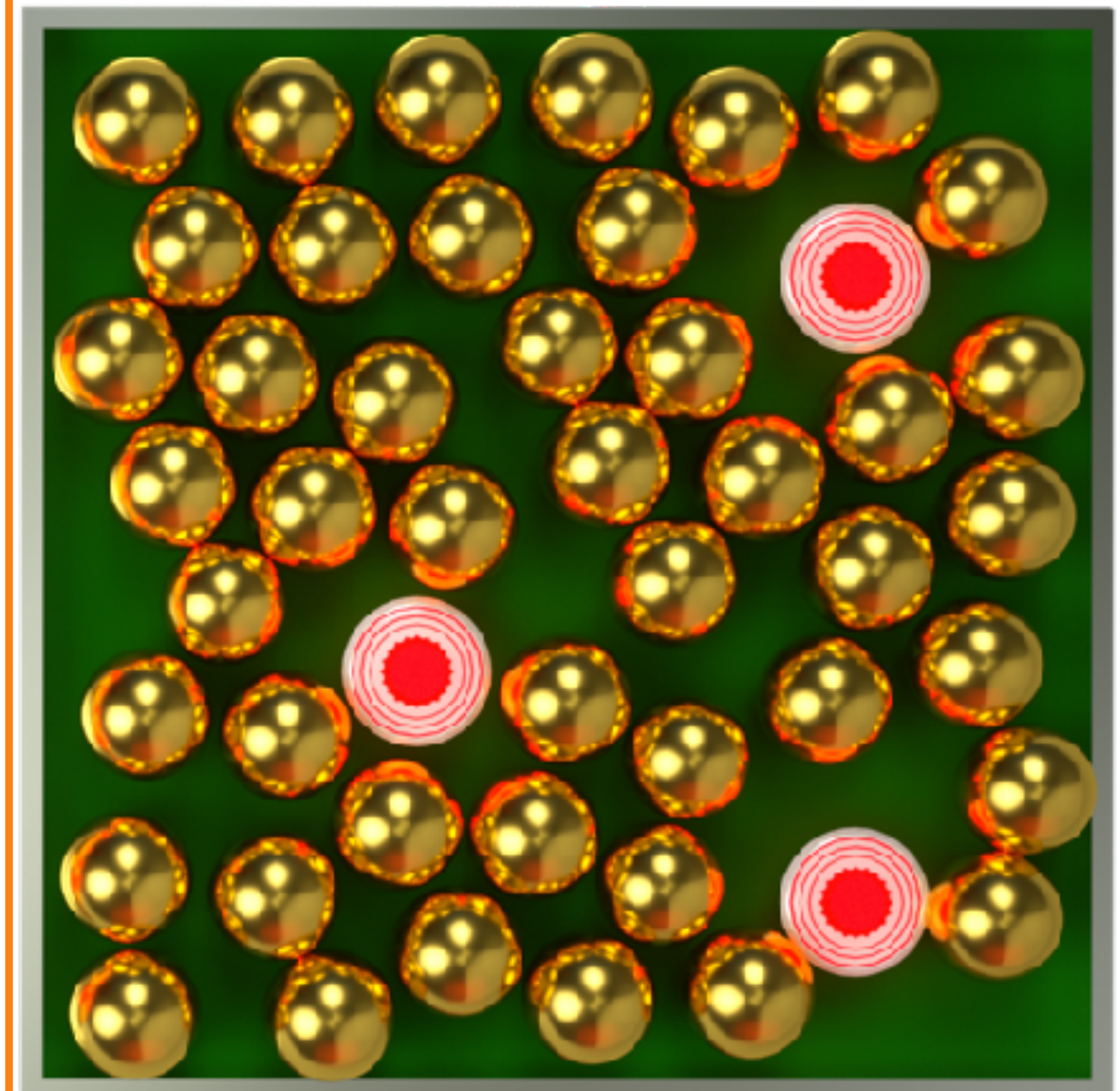
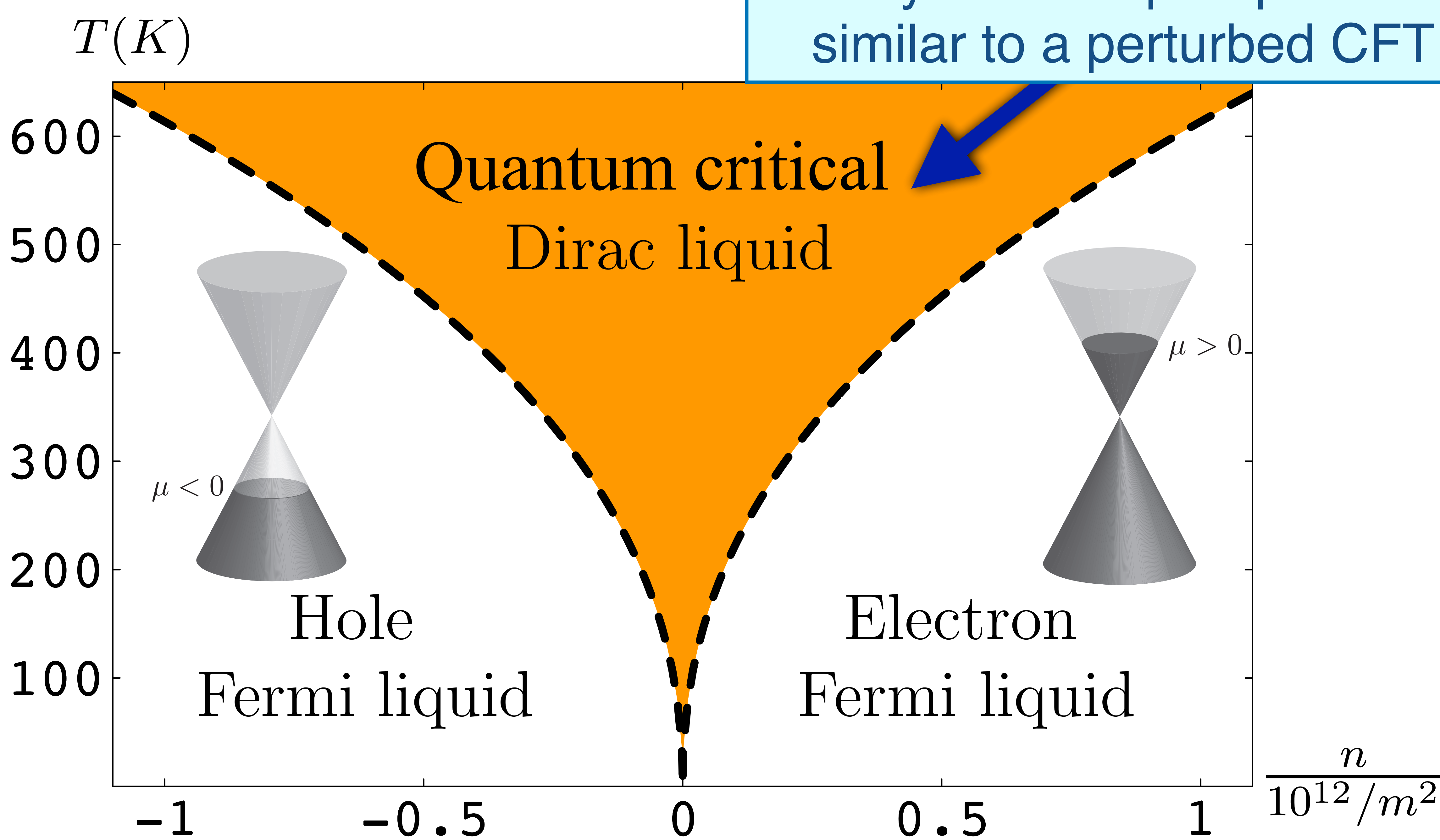


<u>Strange metals</u>: electrons scatter frequently off each other, so there is no regime of ballistic quasiparticle motion. The electron “liquid” then “flows” around impurities

Graphene

Prediction: hydrodynamic theory without quasiparticles, similar to a perturbed CFT



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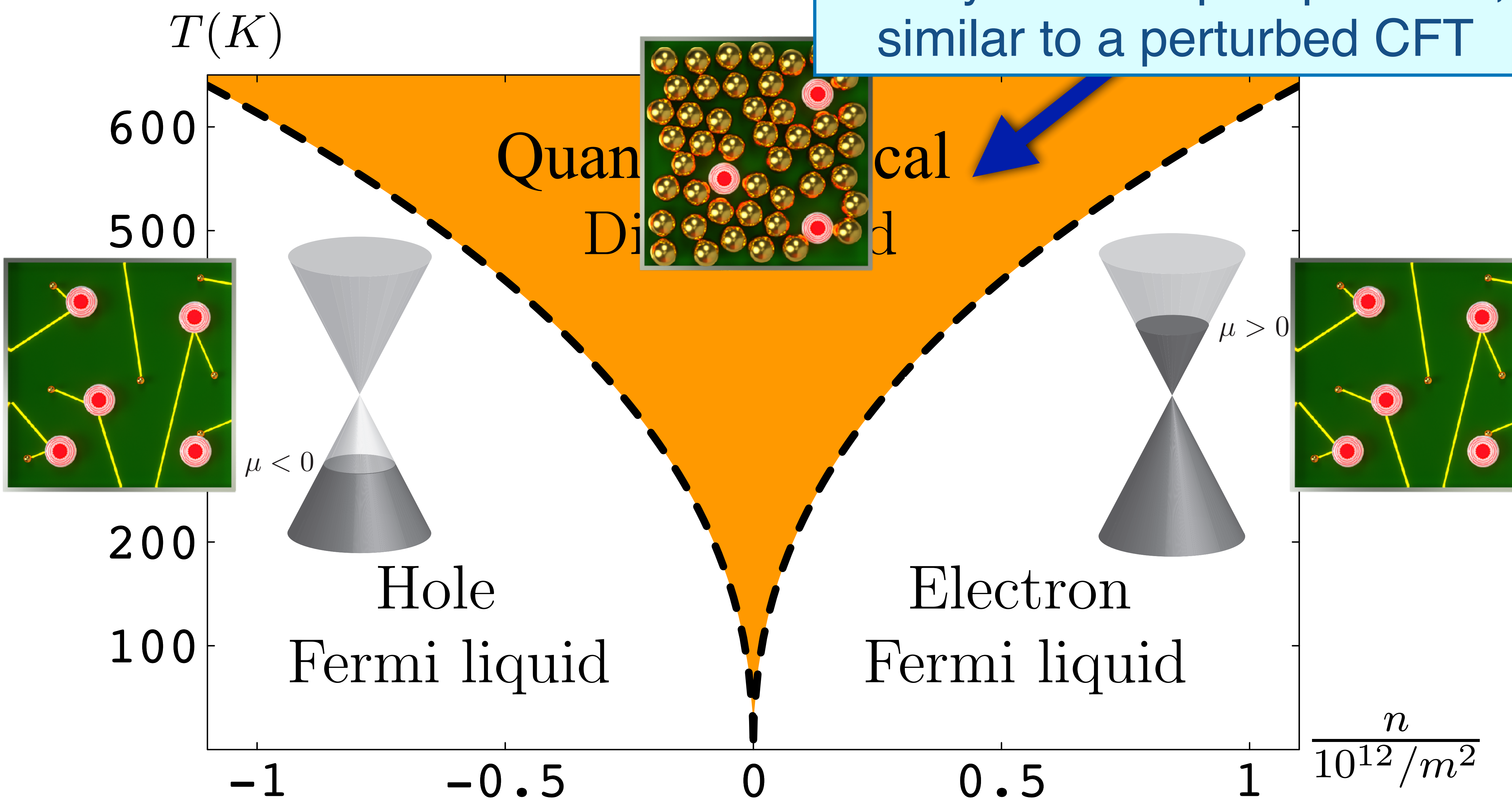

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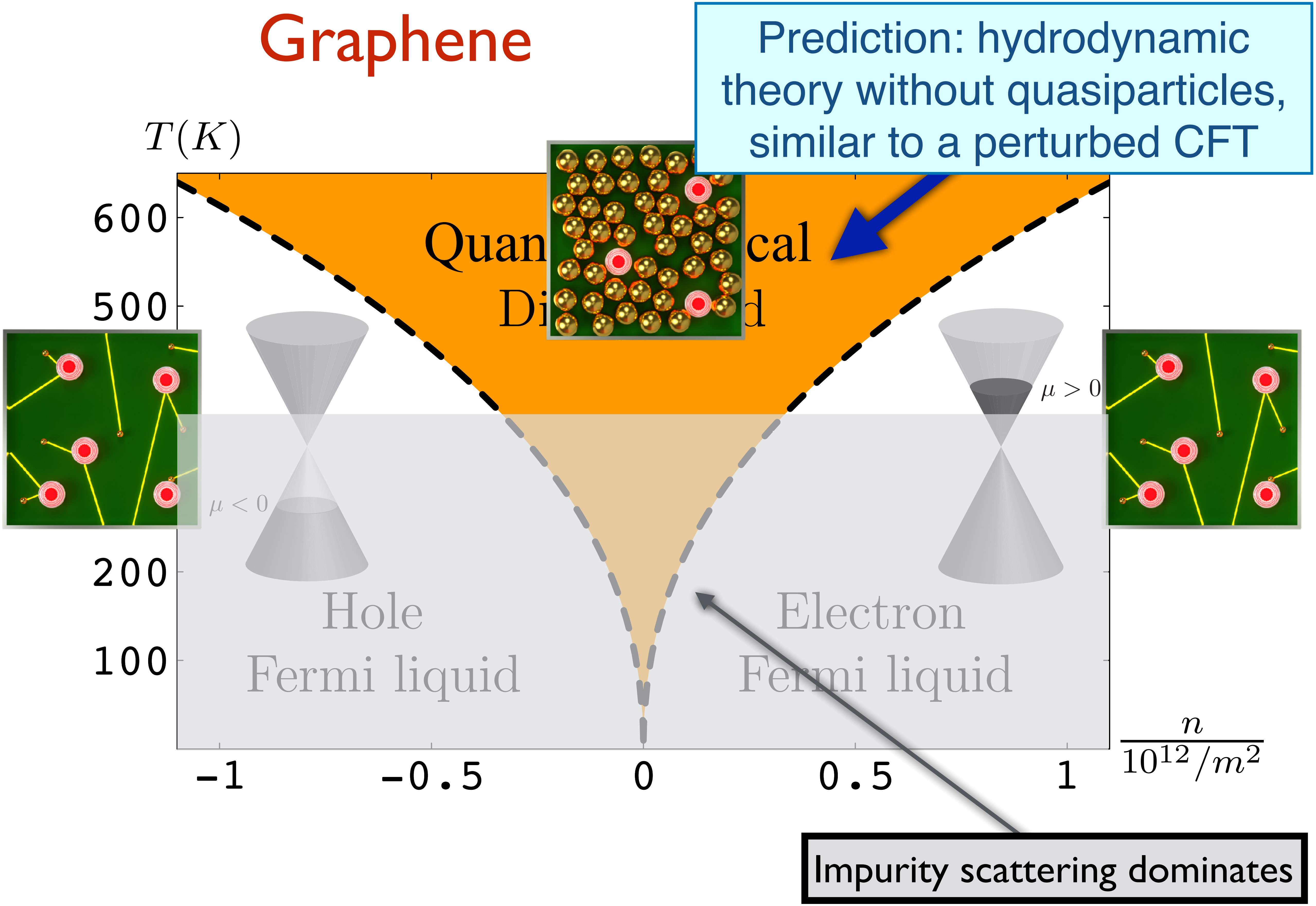
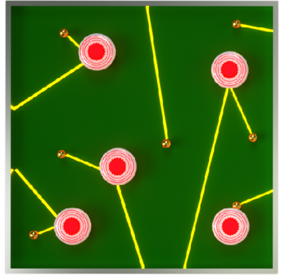


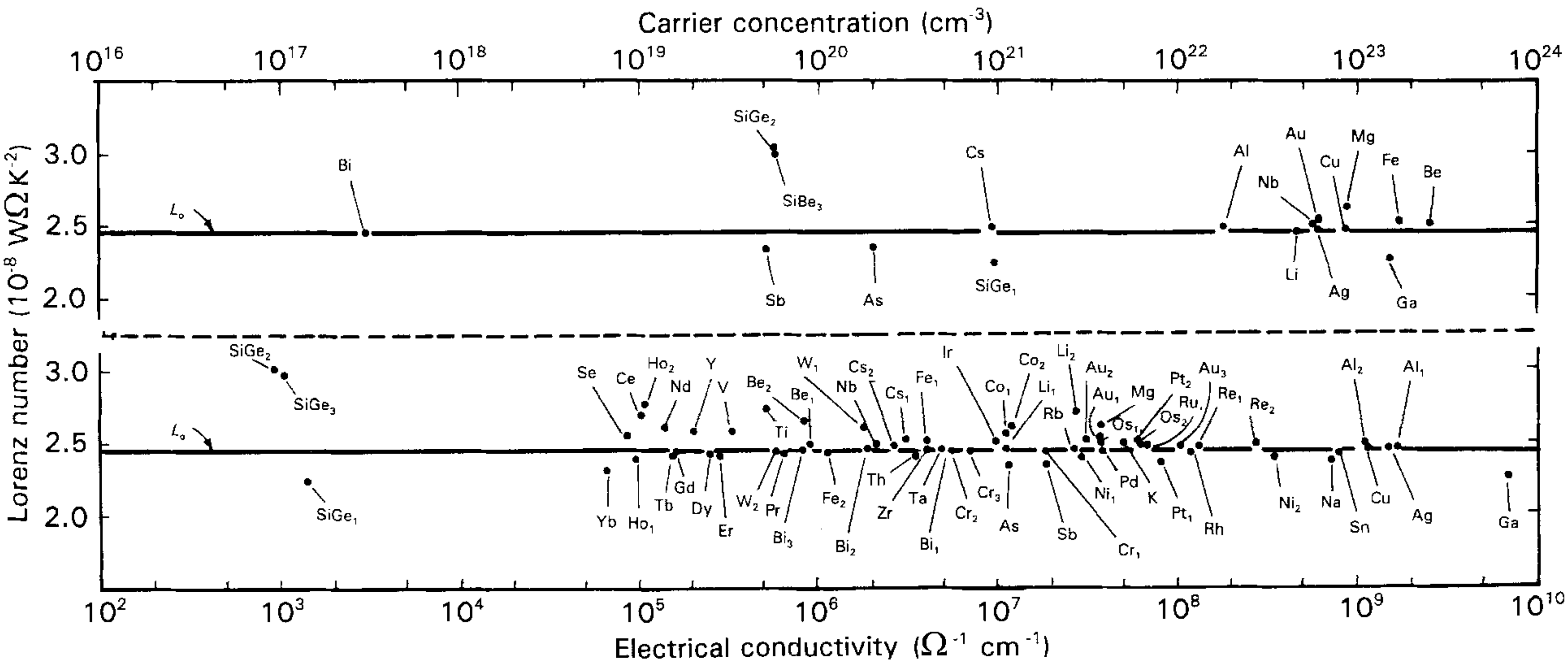
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Thermal and electrical conductivity with quasiparticles



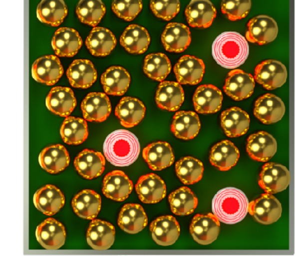
- Wiedemann-Franz law in a Fermi liquid:

$$L_0 = \frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8}\,\frac{\mathrm{W}\cdot\Omega}{\mathrm{K}^2}.$$



G. S. Kumar, G. Prasad, and R.O. Pohl, J. Mat. Sci. **28**, 4261 (1993)

Relativistic hydrodynamics



- hydrodynamics when $l \gg l_{\mathrm{ee}}$, $t \gg t_{\mathrm{ee}}$
- long time dynamics governed by conservation laws:

$$\partial_\nu T^{\mu\nu} = J_\nu \left(F^{\mathrm{ext}}\right)^{\mu\nu}, \qquad \partial_\mu J^\mu = 0.$$

dynamics of relaxation to equilibrium

- expand $T^{\mu\nu}$, J^μ in perturbative parameter $l_{\mathrm{ee}}\partial_\mu$:
  in terms of the velocity $u^\mu(x)$, the chemical potential $\mu(x)$ and the temperature $T(x)$

$$T^{\mu\nu} = P\eta^{\mu\nu} + (\epsilon + P)u^\mu u^\nu$$

$$J^\mu = \mathcal{Q}u^\mu - \sigma_{\mathrm{Q}}\mathcal{P}^{\mu\rho}\left(\partial_\rho \mu - \frac{\mu}{T}\partial_\rho T - u^\nu F^{\mathrm{ext}}_{\rho\nu}\right) + \cdots,$$

$$\mathcal{P}^{\mu\nu} \equiv \eta^{\mu\nu} + u^\mu u^\nu,$$

$$Q^i = T^{ti} - \mu J^i$$

- New (and only) transport co-efficient, σ_Q: "quantum critical" conductivity at $\mathcal{Q} = 0$.

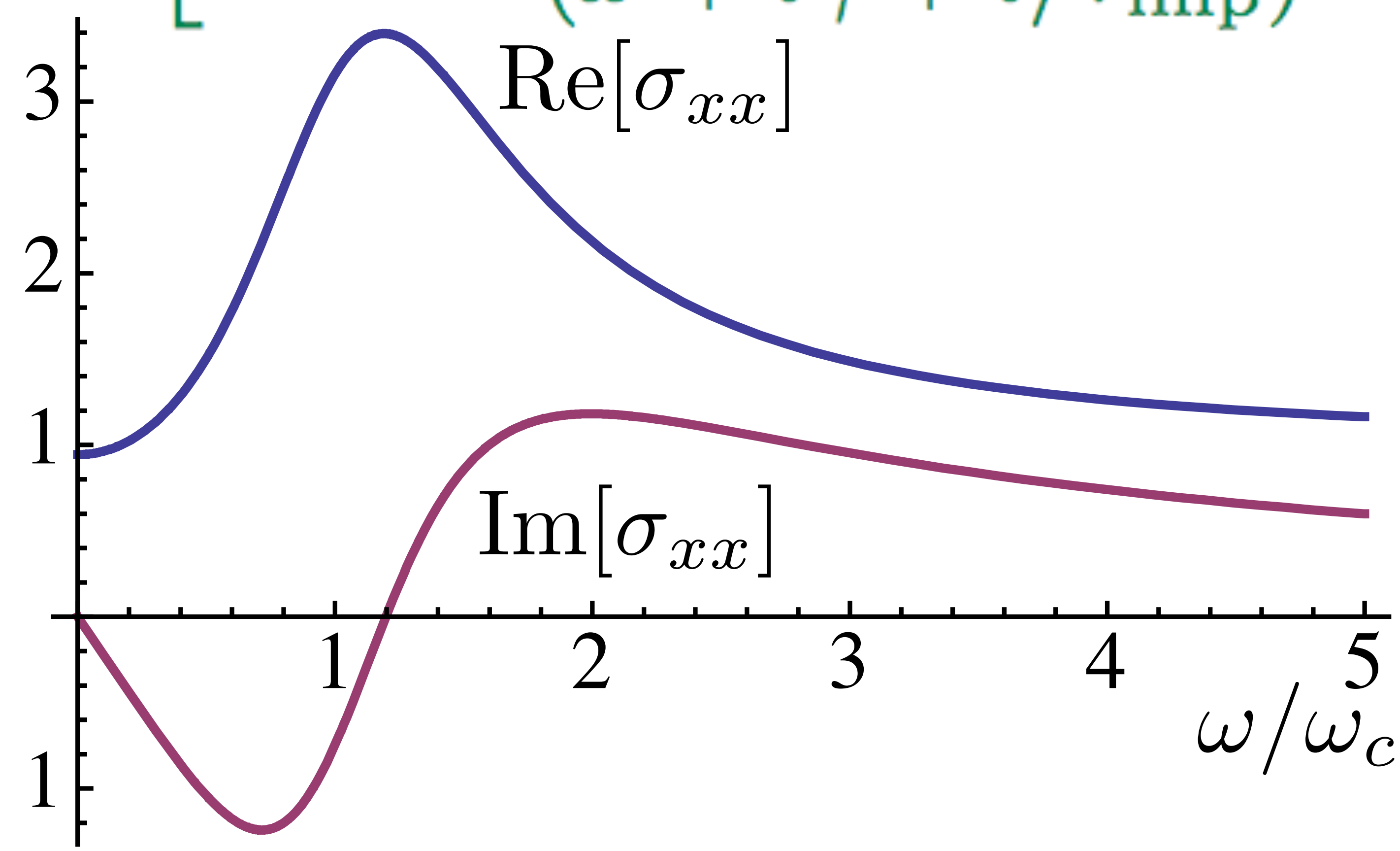
S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

From these relations, we obtained results for the transport co-efficients, expressed in terms of a "cyclotron" frequency and damping:

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

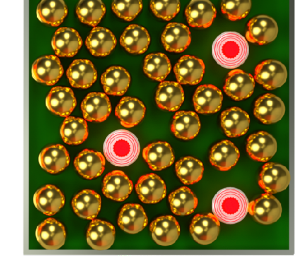
Longitudinal conductivity

$$\sigma_{xx} = \sigma_Q \left[\frac{(\omega + i/\tau_{\text{imp}})(\omega + i\gamma + i\omega_c^2/\gamma + i/\tau_{\text{imp}})}{(\omega + i\gamma + i/\tau_{\text{imp}})^2 - \omega_c^2} \right] .$$



S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Prediction for transport in the graphene strange metal



Recall that in a Fermi liquid, the Lorenz ratio $L = \kappa/(T\sigma)$, where κ is the thermal conductivity, and σ is the conductivity, is given by $L = \pi^2 k_B^2/(3e^2)$.

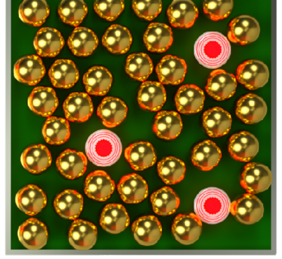
For a strange metal with a "relativistic" Hamiltonian, hydrodynamic, holographic, and memory function methods yield

$$\sigma = \sigma_Q \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H}\sigma_Q}\right) \quad , \quad \kappa = \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H}\sigma_Q}\right)^{-1}$$

$$L = \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T^2 \sigma_Q} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H}\sigma_Q}\right)^{-2},$$

where $\mathcal{H}$ is the enthalpy density, $\tau_{\rm imp}$ is the momentum relaxation time (from impurities), while $\sigma = \sigma_Q$, an intrinsic, finite, "quantum critical" conductivity.

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)



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where $\mathcal{H}$ is the enthalpy density, $\tau_{\rm imp}$ is the momentum relaxation time (from impurities), while $\sigma = \sigma_Q$, an intrinsic, finite, "quantum critical" conductivity.

Note that the limits $\mathcal{Q} \to 0$ and $\tau_{\rm imp} \to \infty$ do not commute. For $\mathcal{Q} = 0$ ($\mathcal{Q} \neq 0$) when we take the clean limit $\tau_{\rm imp} \to \infty$, the electrical conductivity is finite (diverges), and the thermal conductivity diverges (is finite).

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
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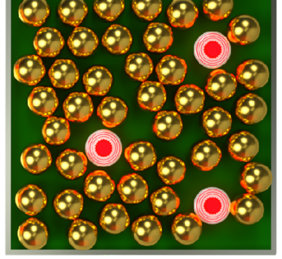
Prediction for transport in the graphene strange metal



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$$L = \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T^2 \sigma_Q} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H}\sigma_Q}\right)^{-2},$$

where $\mathcal{H}$ is the enthalpy density, $\tau_{\rm imp}$ is the momentum relaxation time (from impurities), while $\sigma = \sigma_Q$, an intrinsic, finite, "quantum critical" conductivity.

Note that for a clean system ($\tau_{\rm imp} \to \infty$ first),
the Lorentz ratio diverges $L \sim 1/\mathcal{Q}^4$,
as we approach "zero" electron density at the Dirac point.

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
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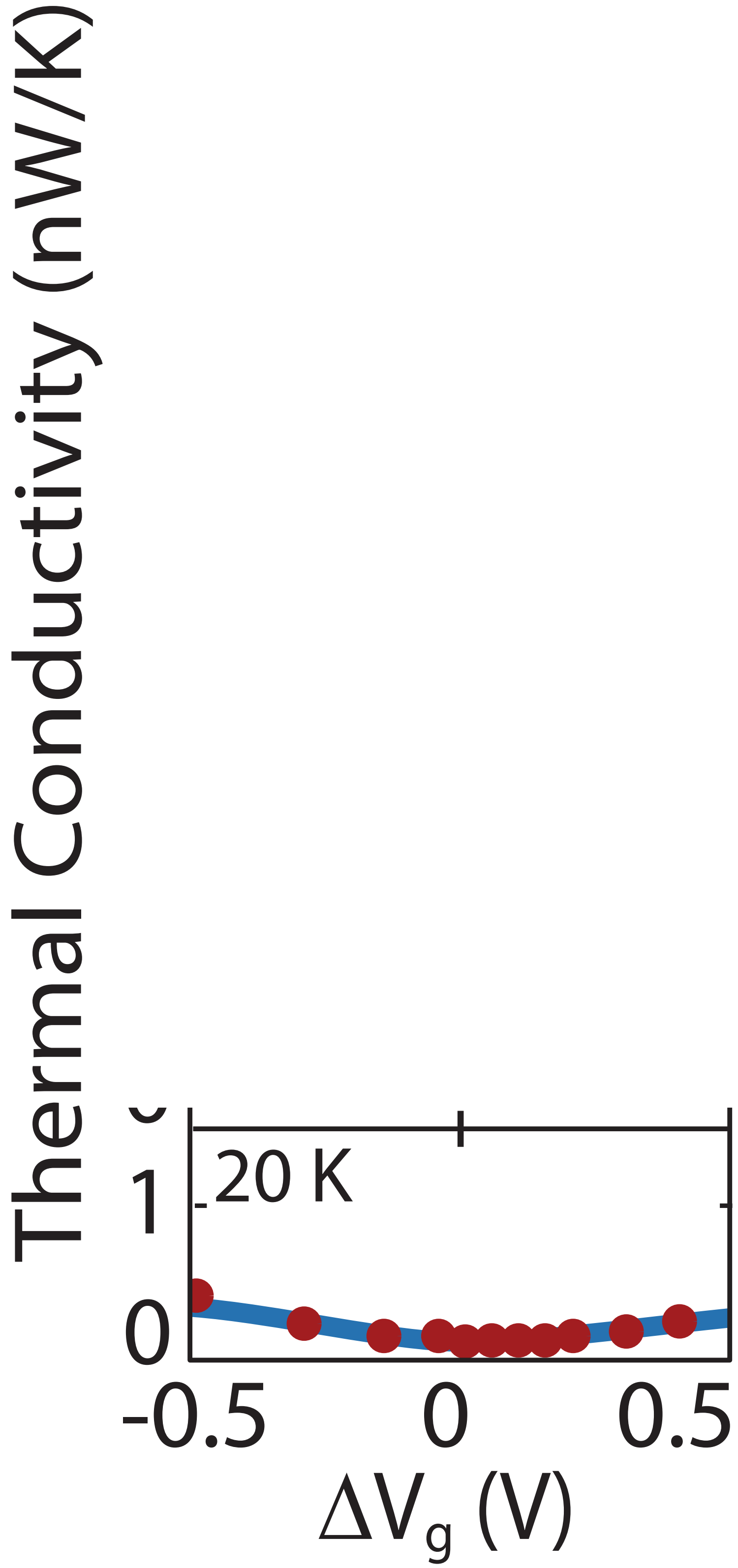
J. Crossno et al., Science **351**, 1058 (2016)



Red dots: data

Blue line: value for $L = L_0$

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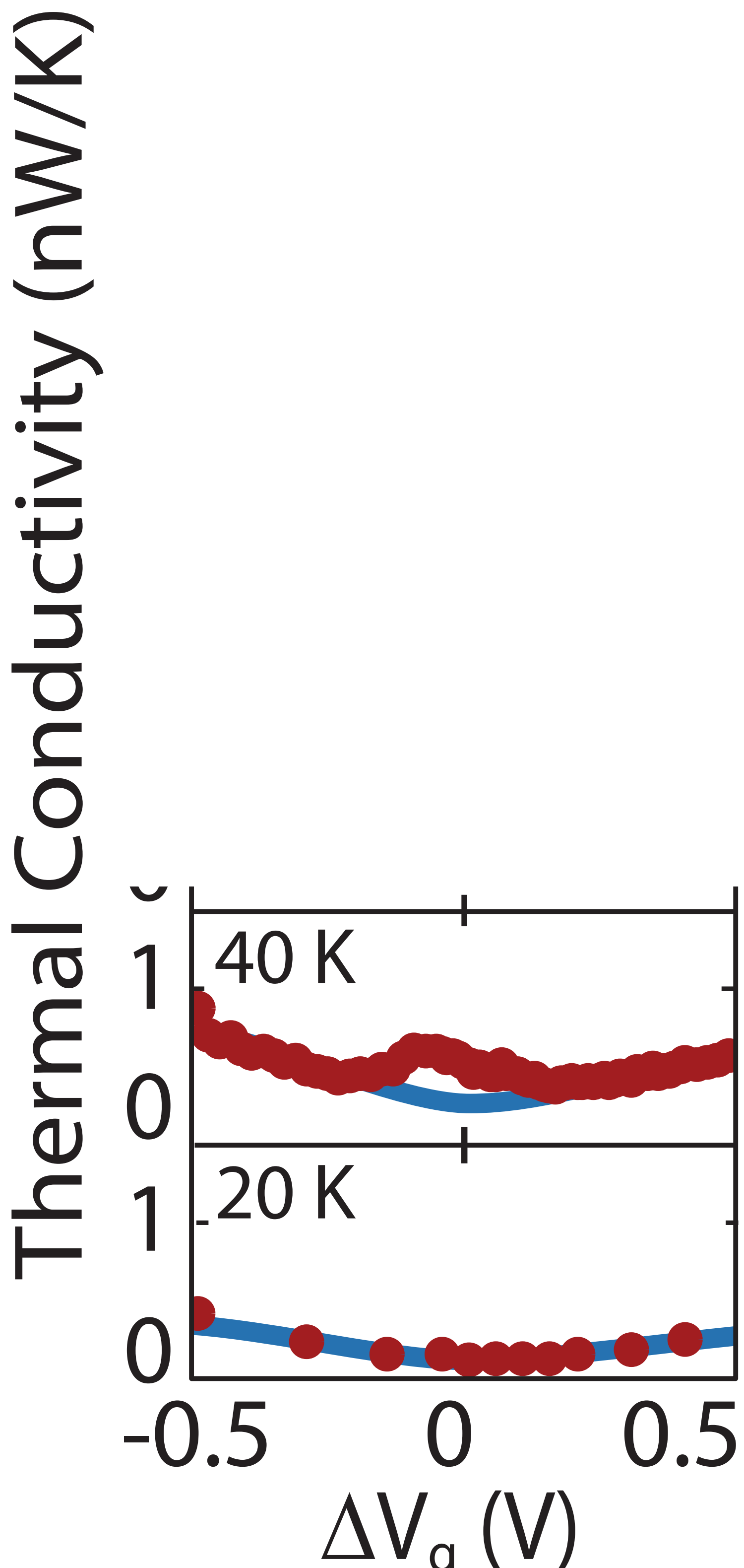

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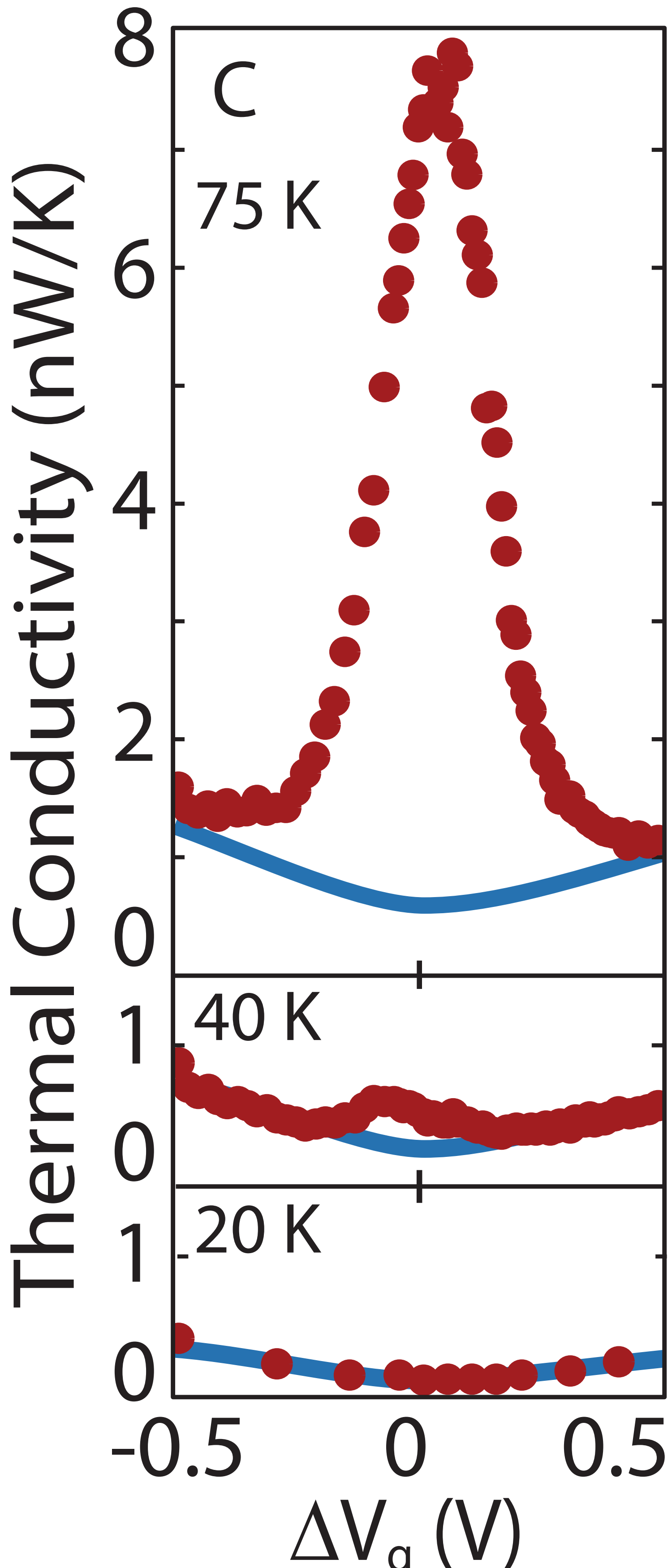

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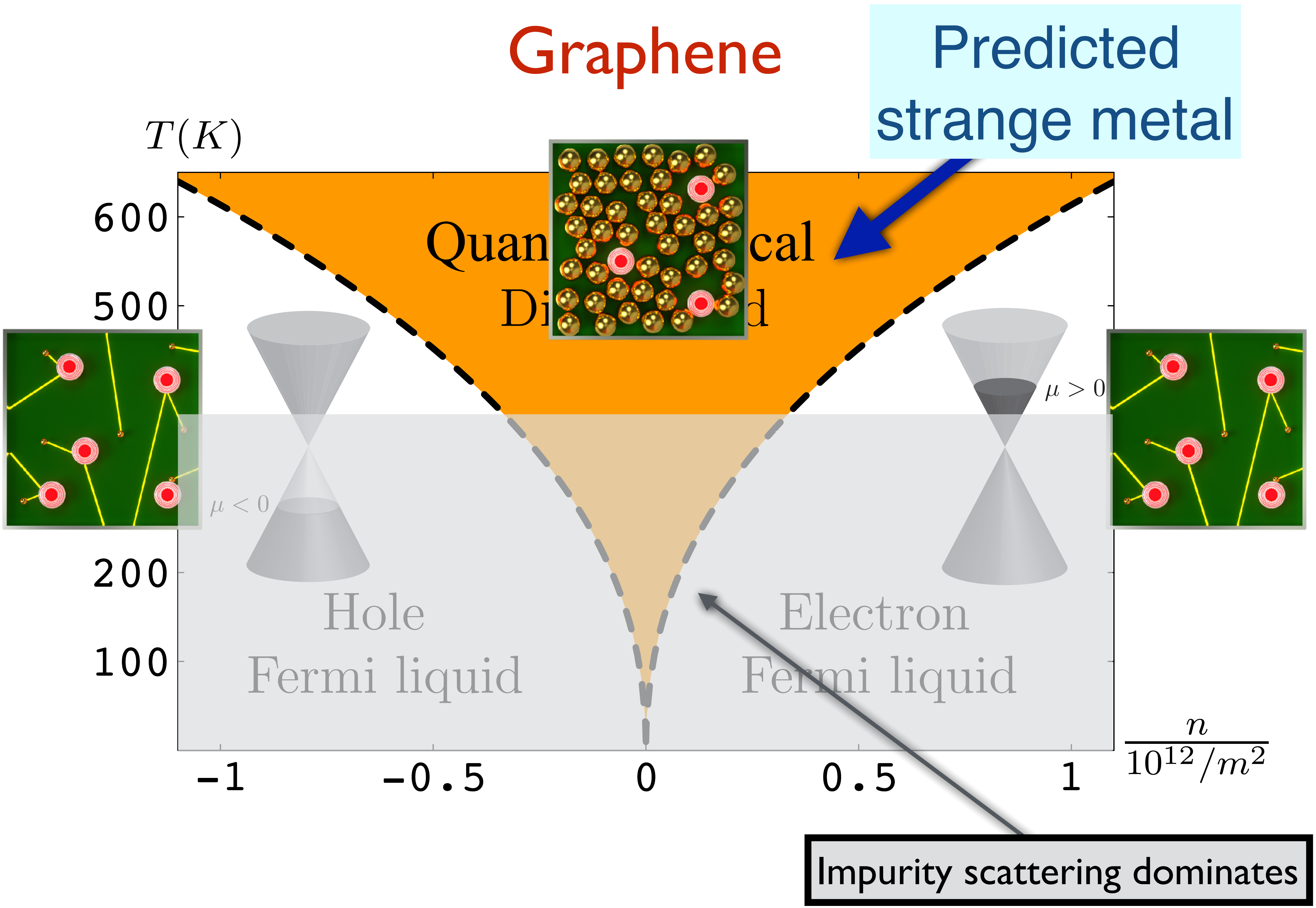
Graphene



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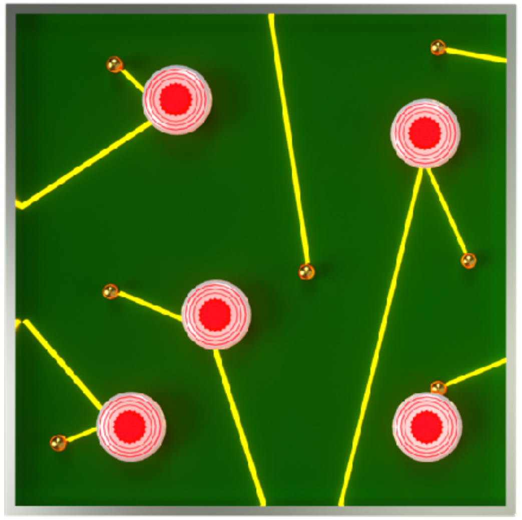
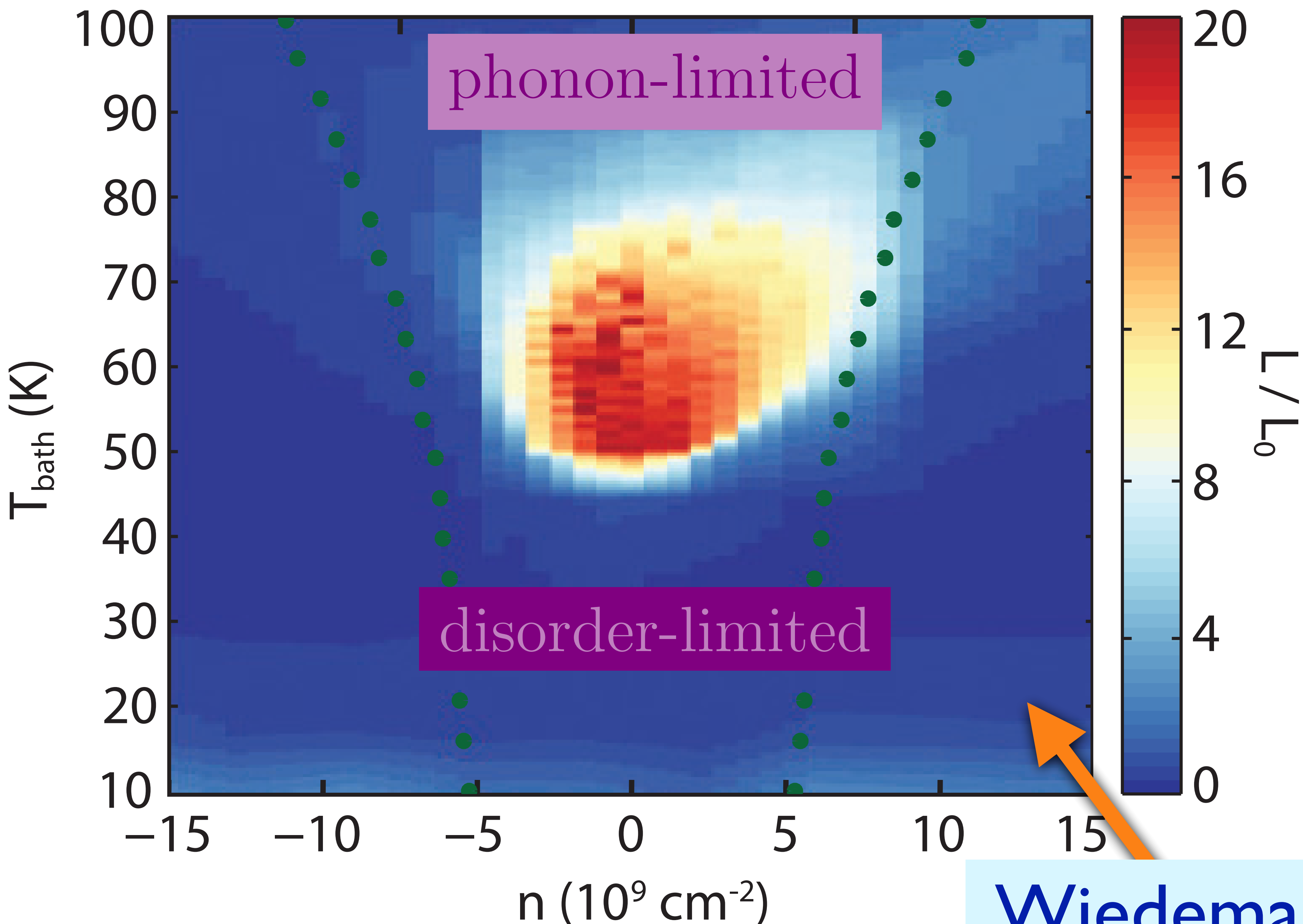
Strange metal in graphene





Wiedemann-Franz obeyed

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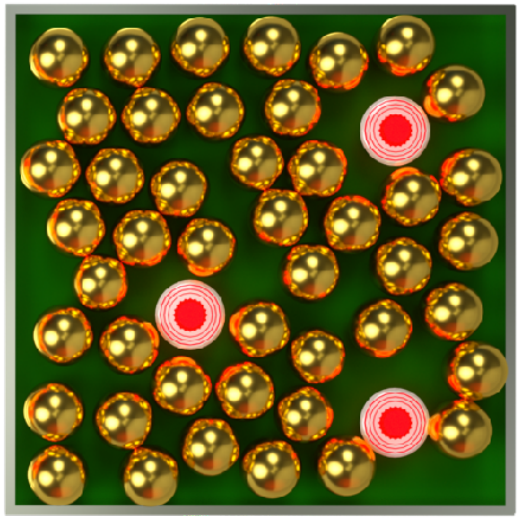
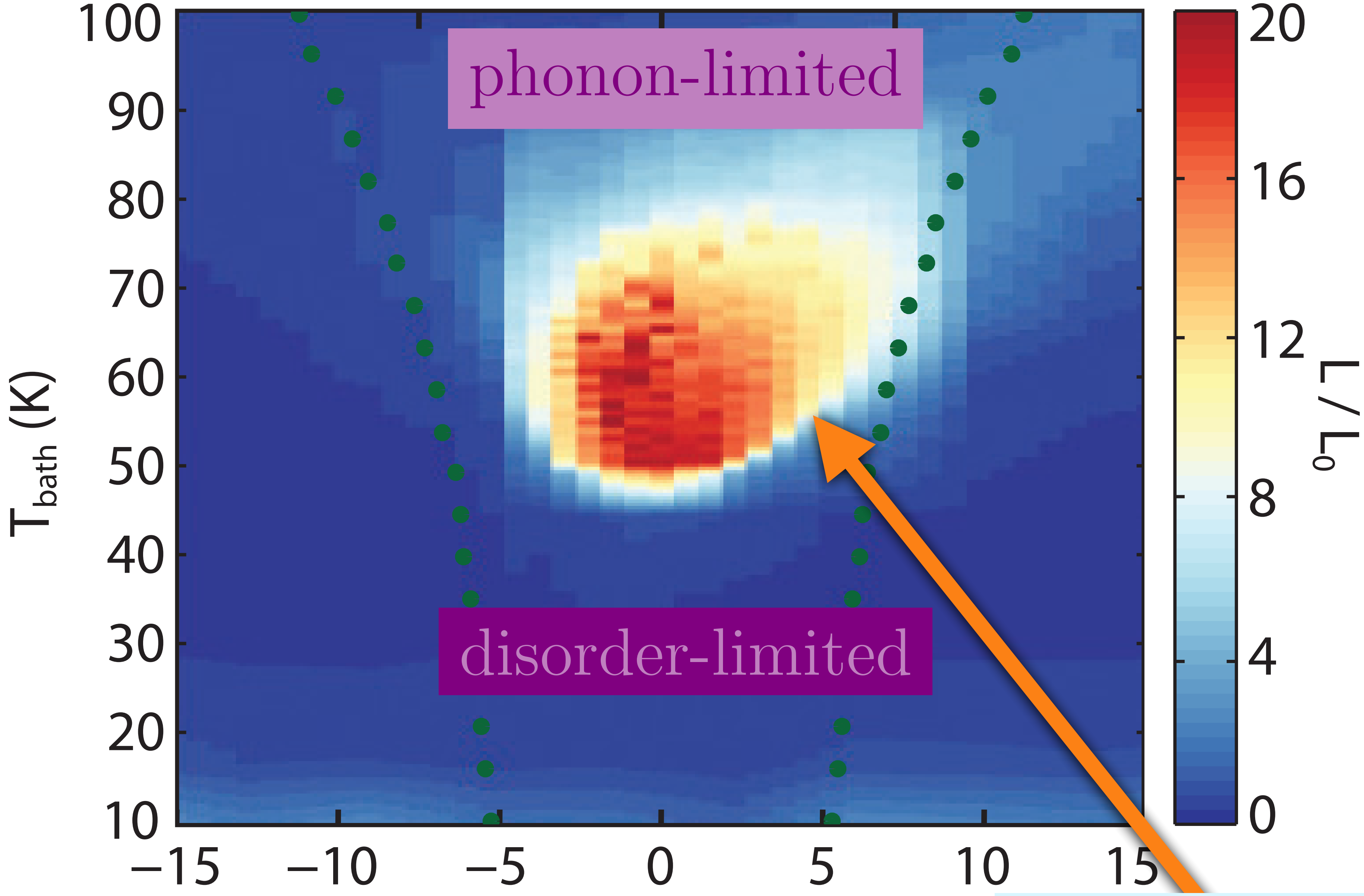
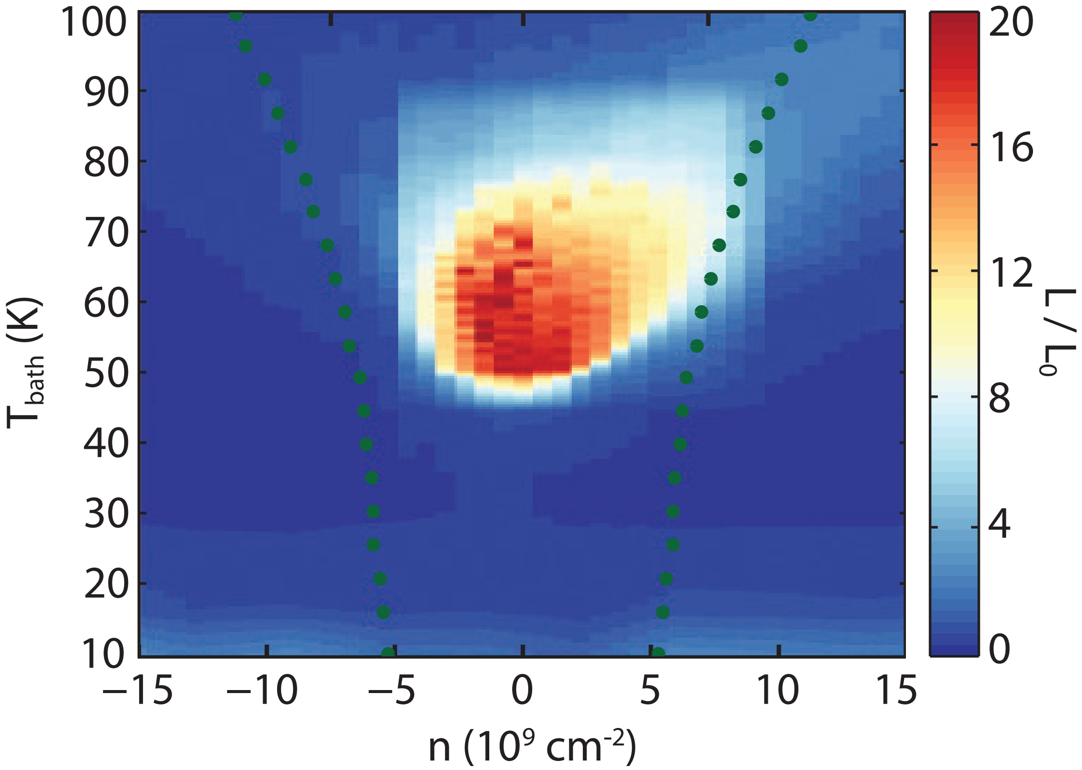
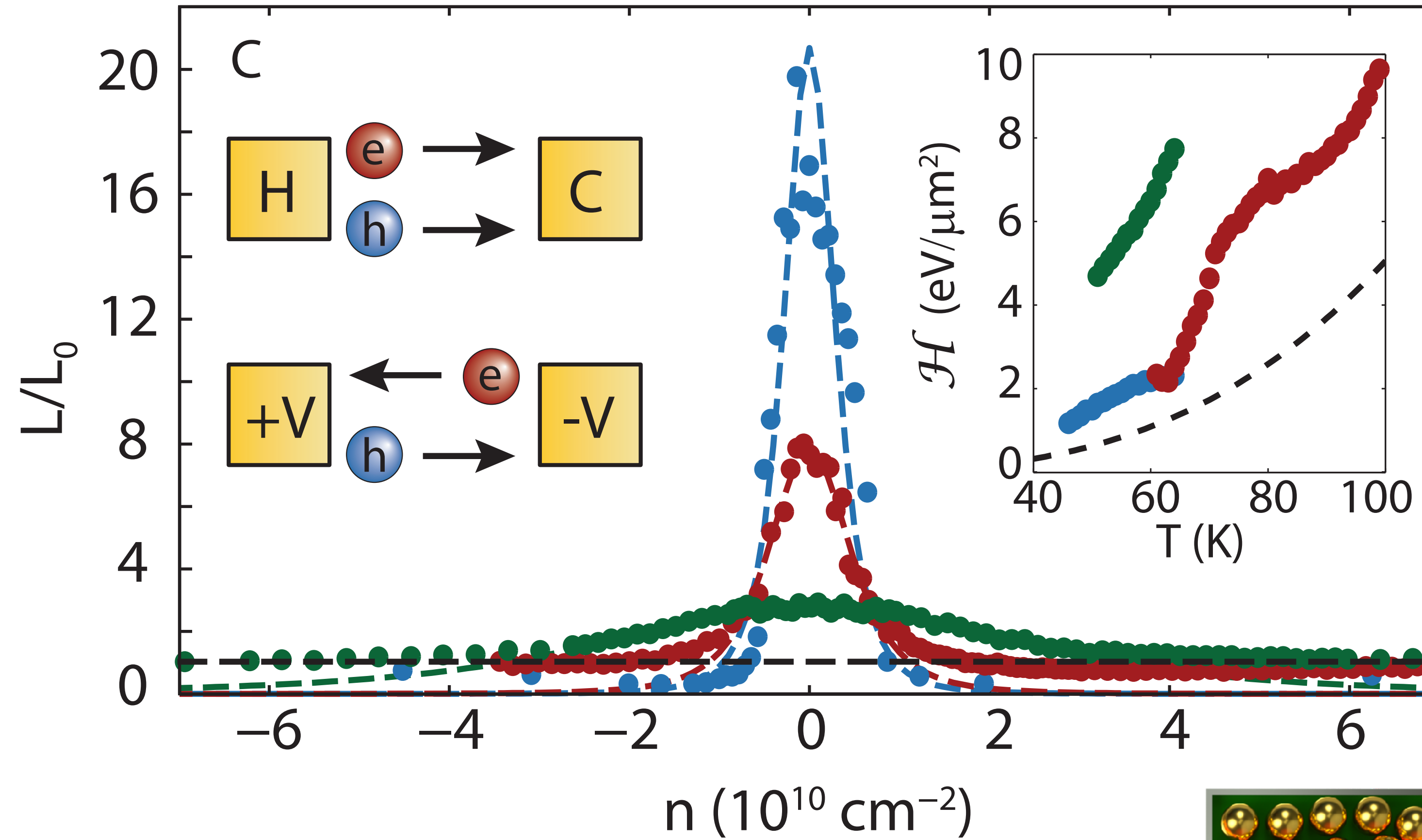
Strange metal in graphene

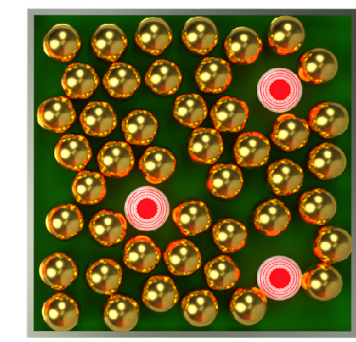


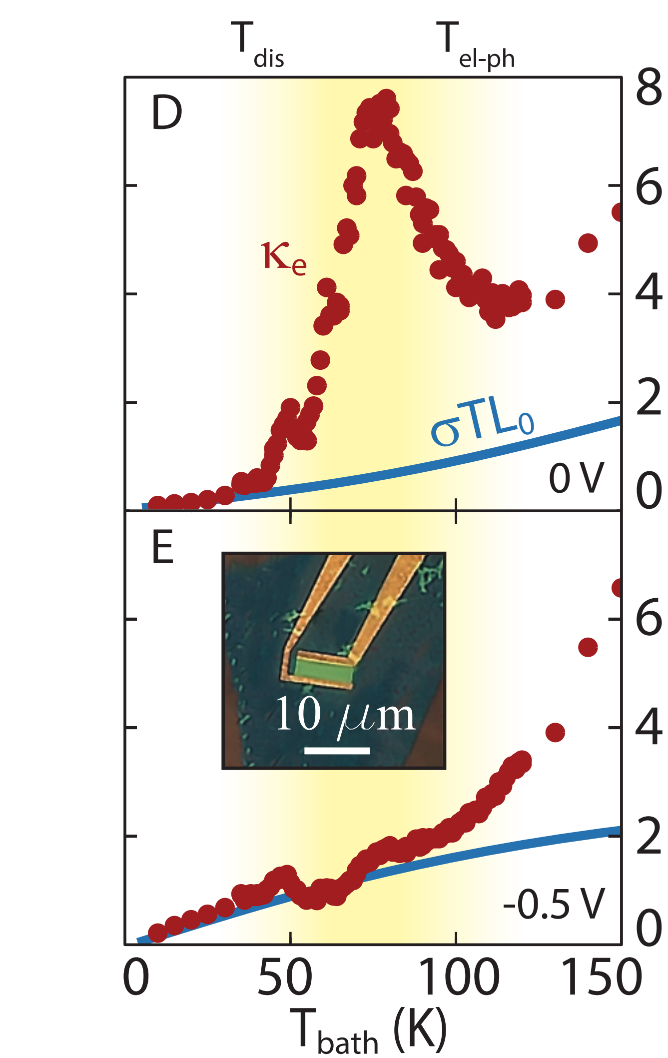


Wiedemann-Franz violated !









Lorentz ratio $L = \kappa/(T\sigma)$

$$= \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T^2 \sigma_Q} \frac{1}{\left(1 + e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}/(\mathcal{H}\sigma_Q)\right)^2}$$

$\mathcal{Q} \rightarrow$ electron density; $\mathcal{H} \rightarrow$ enthalpy density

$\sigma_Q \rightarrow$ quantum critical conductivity

$\tau_{\rm imp} \rightarrow$ momentum relaxation time from impurities

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

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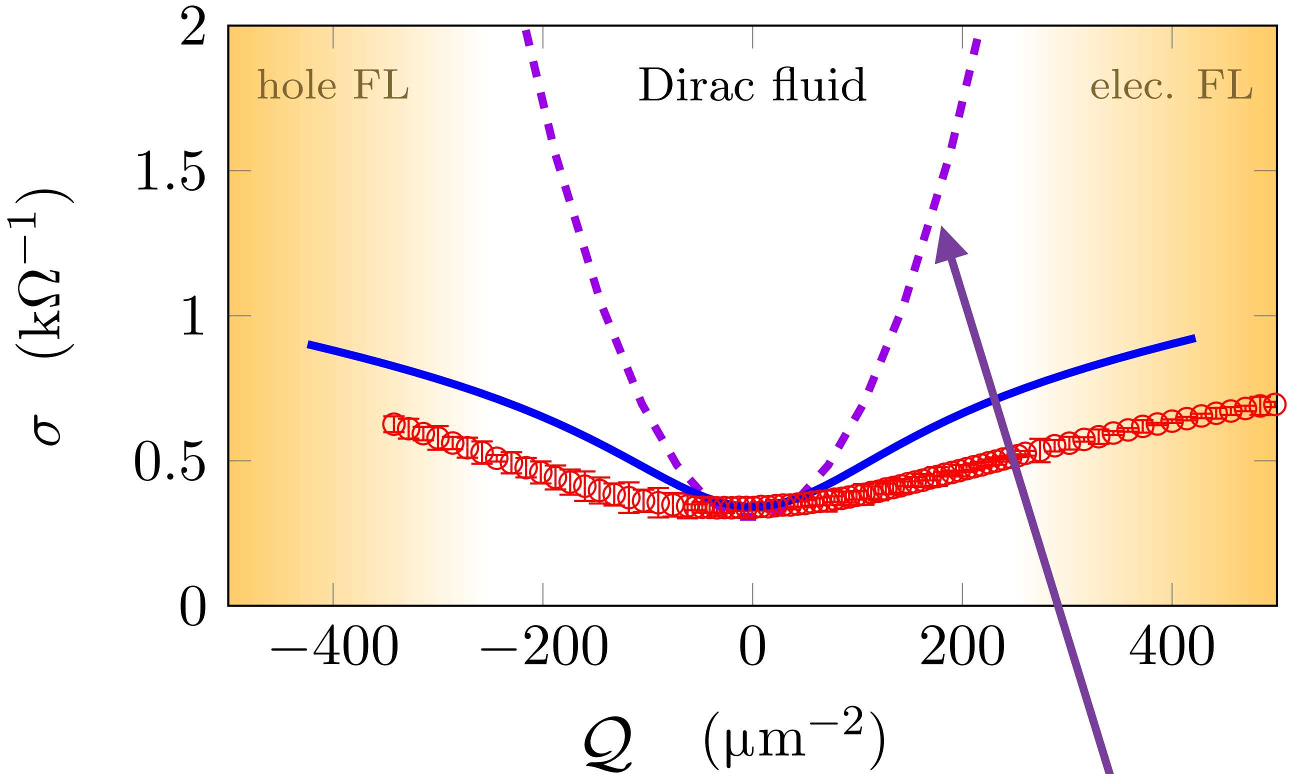
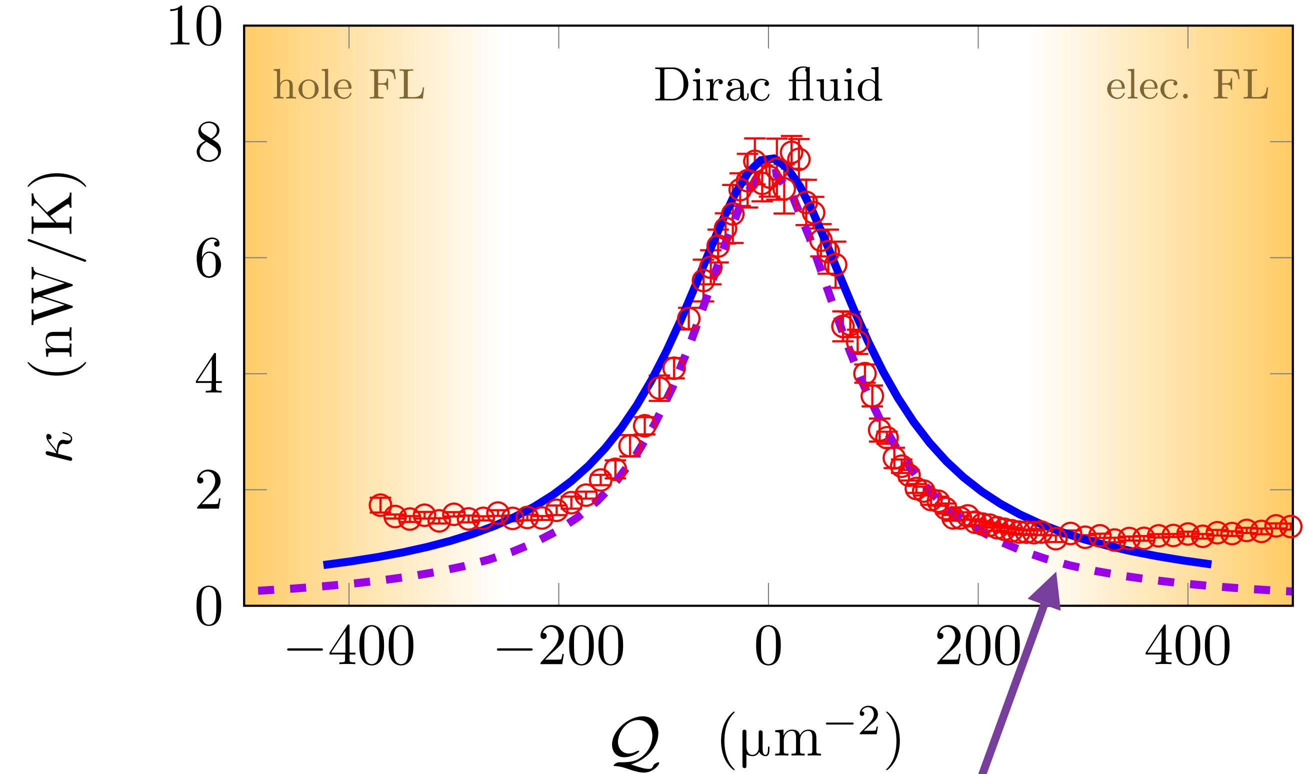
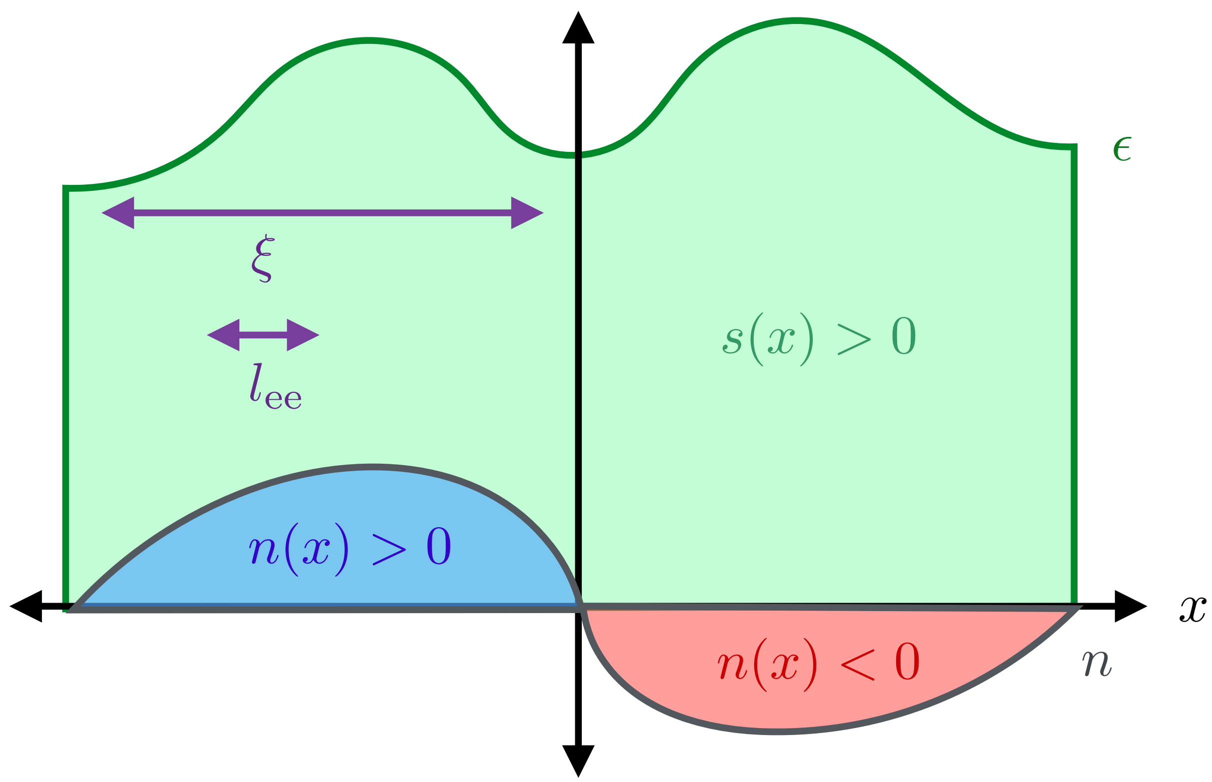




Comparison to theory with a single momentum relaxation time τ_{imp}.
Best fit of density dependence to thermal conductivity does not capture the density dependence of electrical conductivity

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, PRB **93**, 075426 (2016)

Non-perturbative treatment of disorder

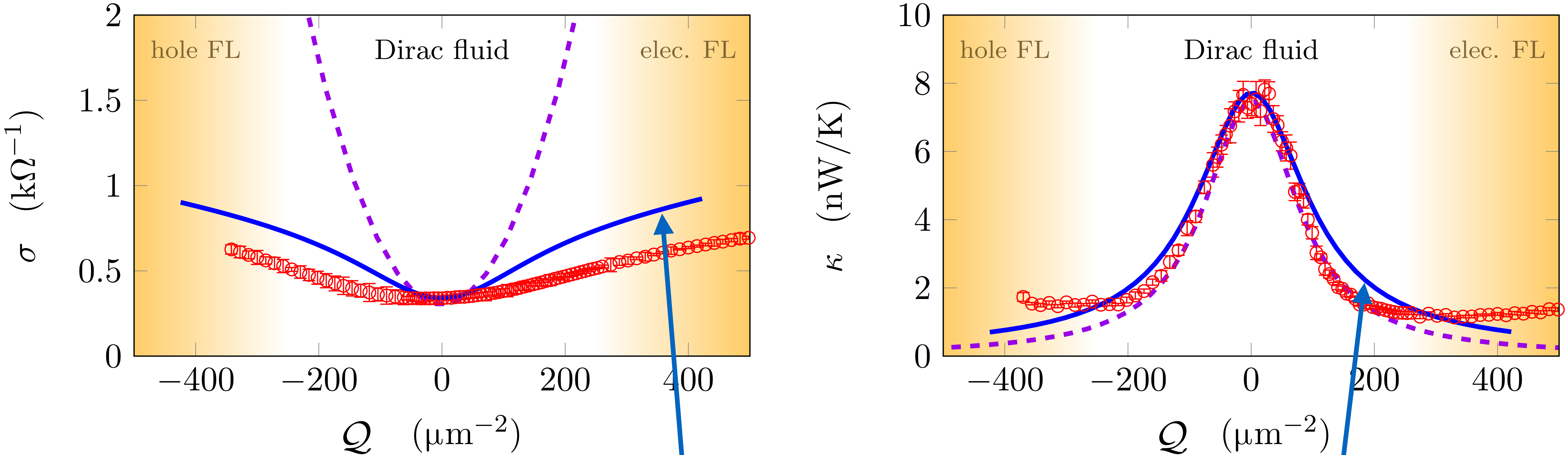


Note
$n \equiv \mathcal{Q}$

Figure 3: A cartoon of a nearly quantum critical fluid where our hydrodynamic description of transport is sensible. The local chemical potential $\mu(\mathbf{x})$ always obeys $|\mu| \ll k_{\rm B}T$, and so the entropy density $s/k_{\rm B}$ is much larger than the charge density $|n|$; both electrons and holes are everywhere excited, and the energy density ϵ does not fluctuate as much relative to the mean. Near charge neutrality the local charge density flips sign repeatedly. The correlation length of disorder ξ is much larger than $l_{\rm ee}$, the electron-electron interaction length.

Numerically solve the hydrodynamic equations in the presence of a x-dependent chemical potential. The thermoelectric transport properties will then depend upon the value of the shear viscosity, η.

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, PRB **93**, 075426 (2016)



Solution of the hydrodynamic equations in the presence of a space-dependent chemical potential.
Best fit of density dependence to thermal conductivity now gives a better fit to the density dependence of the electrical conductivity (for $\eta/s \approx 10$). The T dependencies of other parameters also agree well with expectation.

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, PRB **93**, 075426 (2016)

See also models with separately conserved particle and hole densities:
Yunseok Seo, Geunho Song, Philip Kim, S. Sachdev, and Sang-Jin Sin, PRL **118**, 036601 (2017)

Weyl semi-metals

Andrew Lucas

Richard Davison

Quasiparticle transport in Weyl metals

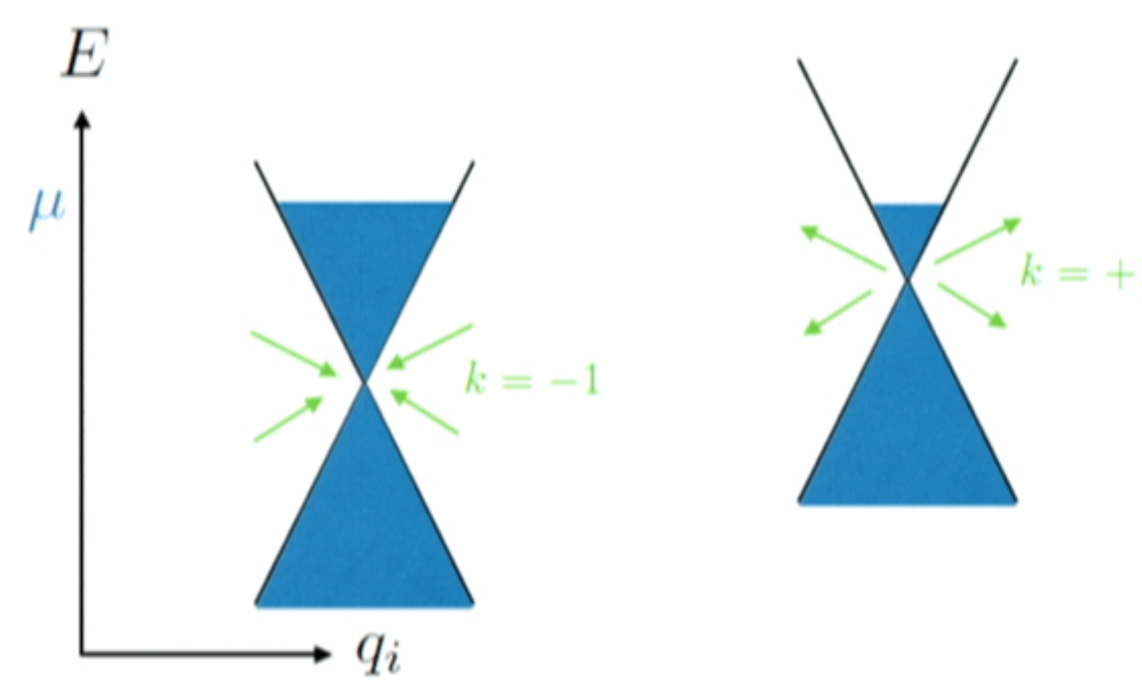
- Weyl Hamiltonian:

$$H = \pm\hbar v_{\mathrm{F}}(\sigma_x q_x + \sigma_y q_y + \sigma_z q_z).$$

- Berry flux $k = \pm 1$ associated with this Hamiltonian:

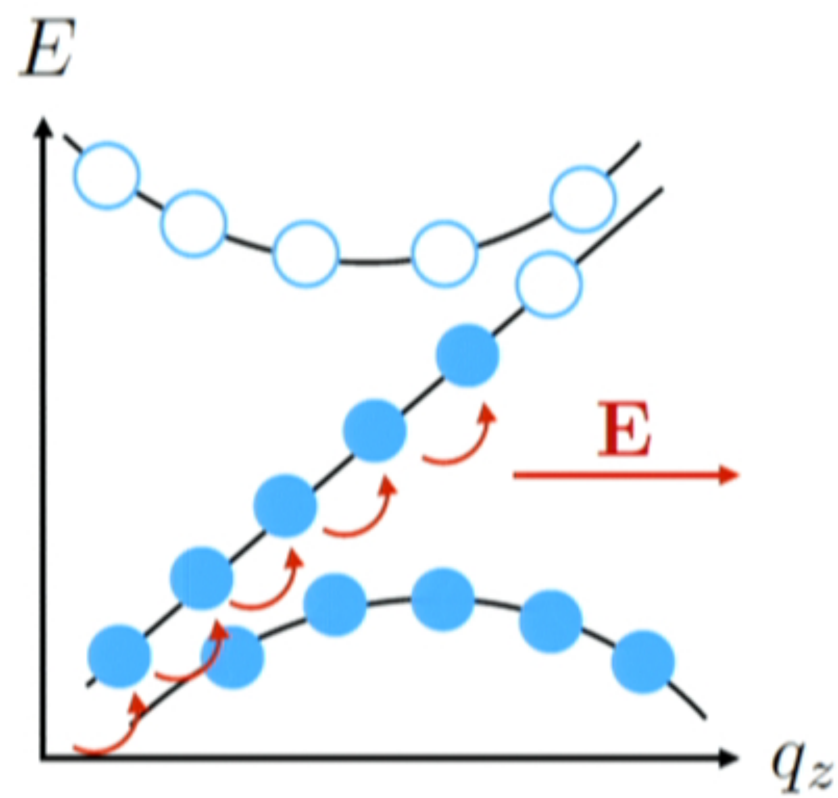
$$\mathcal{A}_i = \mathrm{i}\langle q|\frac{\partial}{\partial q_i}|q\rangle, \qquad \frac{1}{2\pi}\int \mathrm{d}^3\mathbf{q}\,\epsilon_{ijk}\mathcal{A}_i\partial_j\mathcal{A}_k = \pm 1$$

- theorem: net Berry flux must vanish on a lattice (BZ is compact) [Nielsen, Ninomiya (1983)]



Quasiparticle transport in Weyl metals

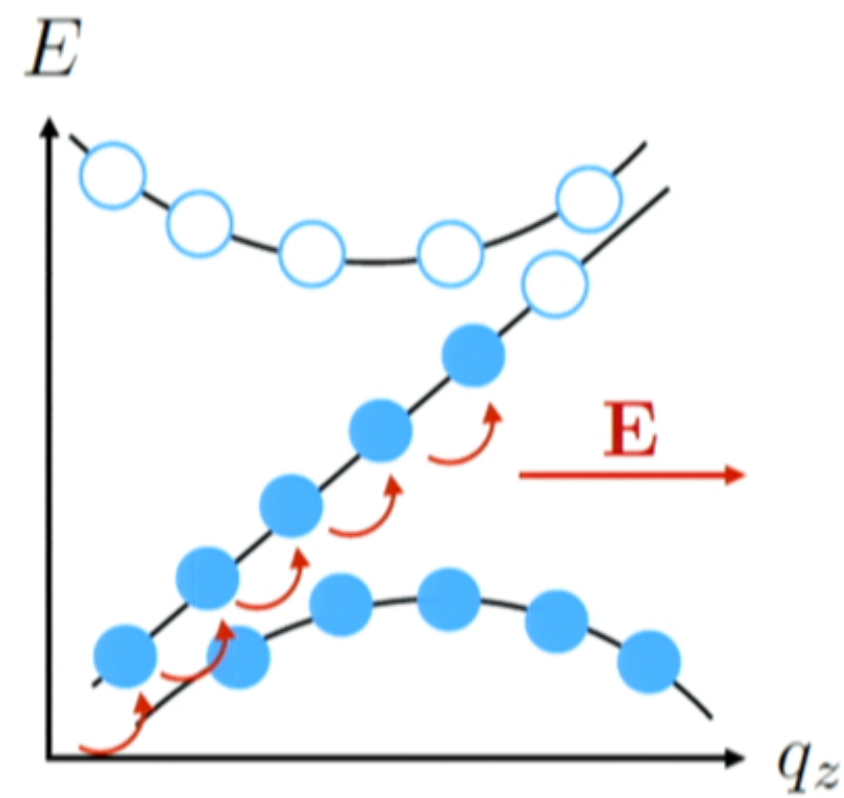
- consider applying $\mathbf{B} = B\hat{\mathbf{z}}$ to $k = 1$ Weyl fermion:



- quantum mechanical effect spoils current conservation: electromagnetic anomaly

Transport in Weyl metals

- consider applying $\mathbf{B} = B\hat{\mathbf{z}}$ to $k = 1$ Weyl fermion:



- quantum mechanical effect spoils current conservation: electromagnetic anomaly
- effect on *classical* hydrodynamics: [Son, Surówka (2009)]

$$\partial_\mu \langle J^\mu \rangle = -\frac{C}{8}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} = C\mathbf{E}\cdot\mathbf{B}, \quad C = \frac{k}{4\pi^2}.$$

Hydrodynamics transport in Weyl metals

There is a similar anomaly in the flow of heat:

- generation of *heat*: [Lucas, Davison, Sachdev (2016)]

$$\partial_i Q^i_a = 2G_a T \nabla T \cdot \mathbf{B}.$$

The heat flow can be accounted for by considering a background gravitational field (Luttinger 1964)

Hydrodynamics transport in Weyl metals

- In (3+1)d, an anomalous U(1) current in background fields generally obeys

$$\nabla_\mu J^\mu = -\frac{C}{8}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} - \frac{G}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}R^\alpha{}_{\beta\mu\nu}R^\beta{}_{\alpha\rho\sigma},$$

$$\nabla_\mu T^{\mu\nu} = F^{\nu\mu}J_\mu - \frac{G}{16\pi^2}\nabla_\mu\left(\varepsilon^{\rho\sigma\alpha\beta}F_{\rho\sigma}R^{\nu\mu}{}_{\alpha\beta}\right)$$

- C is the chiral anomaly coefficient.
- G is the mixed chiral-gravitational anomaly coefficient.
- The values of each coefficient depends on the matter content.
- For a single Weyl fermion, $C = \pm\frac{1}{4\pi^2}$ $\quad G = \pm\frac{1}{24}$

Hydrodynamics transport in Weyl metals

- Assume that there is fast equilibration at each Weyl node, and slow scattering between Weyl nodes:

$$\nabla_\mu J^\mu_a = -\frac{C_a}{8}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} - \frac{G_a}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}R^{\alpha}{}_{\beta\mu\nu}R^{\beta}{}_{\alpha\rho\sigma} - \sum_b\left(\mathcal{R}_{ab}\frac{\mu_b}{T_b} + \mathcal{S}_{ab}\frac{1}{T_b}\right)$$

$$\nabla_\mu T^{\mu\nu}_a = F^{\nu\mu}J_{\mu a} - \frac{G_a}{16\pi^2}\nabla_\mu\left(\varepsilon^{\rho\sigma\alpha\beta}F_{\rho\sigma}R^{\nu\mu}{}_{\alpha\beta}\right) + u^\nu_a\sum_b\left(\mathcal{U}_{ab}\frac{\mu_b}{T_b} + \mathcal{V}_{ab}\frac{1}{T_b}\right)$$

- The indices a,b label the different Weyl nodes.

$$\sum_a C_a = 0 \qquad\qquad \sum_a G_a = 0$$

- R, S, U, V account for transfer of charge, energy and momentum between nodes. Their values depend on microscopic details.

- Express $T_{\mu\nu}$ and J_μ in terms of the velocity $u^\mu(x)$, the chemical potential $\mu(x)$ and the temperature $T(x)$, constrained by positivity of entropy production.

Hydrodynamics transport in Weyl metals

- There are anomalous contributions to three conductivities

$$\sigma_{zz} = \sum_a \frac{n_a^2}{\Gamma_a} + \mathfrak{s} B^2 \qquad \bar{\kappa}_{zz} = \sum_a \frac{T s_a^2}{\Gamma_a} + \mathfrak{h} B^2 \qquad \alpha_{zz} = \sum_a \frac{n_a s_a}{\Gamma_a} + \mathfrak{a} B^2$$

- The 1st term in each is due to momentum relaxation at rate Γ_a

$$\Gamma_a = \frac{T_0^2 \left(s_a \left(\partial n_a / \partial \mu\right) - n_a \left(\partial s_a / \partial \mu\right)\right)^2}{3 \sigma_{Qa} \left(\epsilon_a + P_a\right)^2}$$

R. Davison, Schalm, Zaanen, 2013

- The 2nd is due to anomalies

$$\mathfrak{h} = 4T^4 \begin{pmatrix} 0 & G_a \end{pmatrix} \begin{pmatrix} \mathcal{R}_{ab} & -\mathcal{S}_{ab} \\ -\mathcal{U}_{ab} & \mathcal{V}_{ab} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ G_b \end{pmatrix}$$

$$\mathfrak{s} = T \begin{pmatrix} C_a & C_a \mu \end{pmatrix} \begin{pmatrix} \mathcal{R}_{ab} & -\mathcal{S}_{ab} \\ -\mathcal{U}_{ab} & \mathcal{V}_{ab} \end{pmatrix}^{-1} \begin{pmatrix} C_b \\ C_b \mu \end{pmatrix}$$

$$\mathfrak{a} = 2T^2 \begin{pmatrix} 0 & G_a \end{pmatrix} \begin{pmatrix} \mathcal{R}_{ab} & -\mathcal{S}_{ab} \\ -\mathcal{U}_{ab} & \mathcal{V}_{ab} \end{pmatrix}^{-1} \begin{pmatrix} C_b \\ C_b \mu \end{pmatrix}$$

- Negative electrical magnetoresistance due to the chiral anomaly.

- The mixed chiral-gravitational anomaly produces negative thermal magnetoresistance and anomalous thermoelectric resistance.

Experimental signatures of the mixed axial–gravitational anomaly in the Weyl semimetal NbP

Johannes Gooth[1,2], Anna C. Niemann[1,3], Tobias Meng[4], Adolfo G. Grushin[5], Karl Landsteiner[6], Bernd Gotsmann[2], Fabian Menges[2], Marcus Schmidt[7], Chandra Shekhar[7], Vicky Süß[7], Ruben Hühne[3], Bernd Rellinghaus[3], Claudia Felser[7], Binghai Yan[7,8] & Kornelius Nielsch[1,3]

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Conclusions:

- Dirac materials with Coulomb interactions have interactions of strength unity, $e^2/(\hbar v_F)$, leading to an inelastic scattering time $\sim \hbar/(k_B T)$.

- In sufficiently clean Dirac materials, there is hydrodynamic flow of the electron fluid.

- Evidence for such flow in graphene, and in other materials.