

Quantum criticality,  
the cuprate superconductors,  
and the  
AdS/CFT correspondence

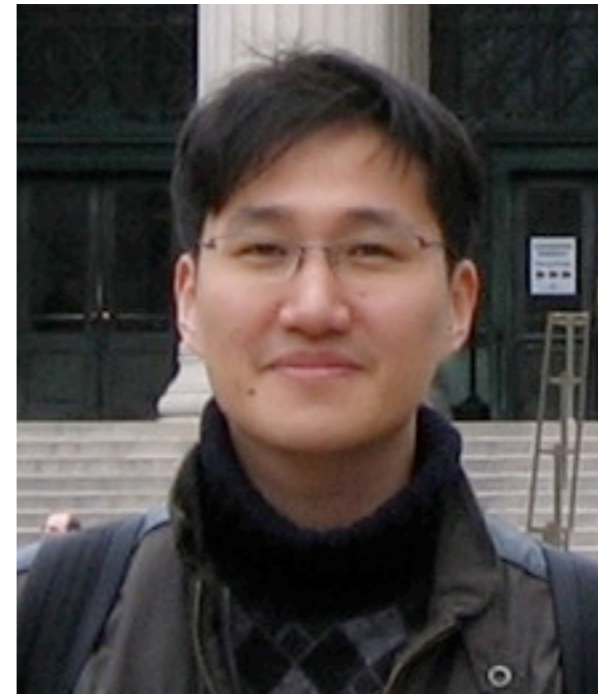
Talk online: [sachdev.physics.harvard.edu](http://sachdev.physics.harvard.edu)





Max Metlitski, Harvard

arXiv:1001.1153



Eun Gook Moon, Harvard

*Phys. Rev. B* 80, 035117 (2009)



# Outline

1. Coupled dimer antiferromagnets  
*Introduction to quantum criticality*
2. Theory of Ising-nematic ordering in the cuprate metals  
*Strongly-coupled field theory*
3. The AdS/CFT correspondence  
*Phases of quantum matter at strong coupling*
4. Global phase diagram of the cuprates  
*Competition between d-wave superconductivity and spin-density order for the Fermi surface*

# Outline

## 1. Coupled dimer antiferromagnets

*Introduction to quantum criticality*

## 2. Theory of Ising-nematic ordering in the cuprate metals

*Strongly-coupled field theory*

## 3. The AdS/CFT correspondence

*Phases of quantum matter  
at strong coupling*

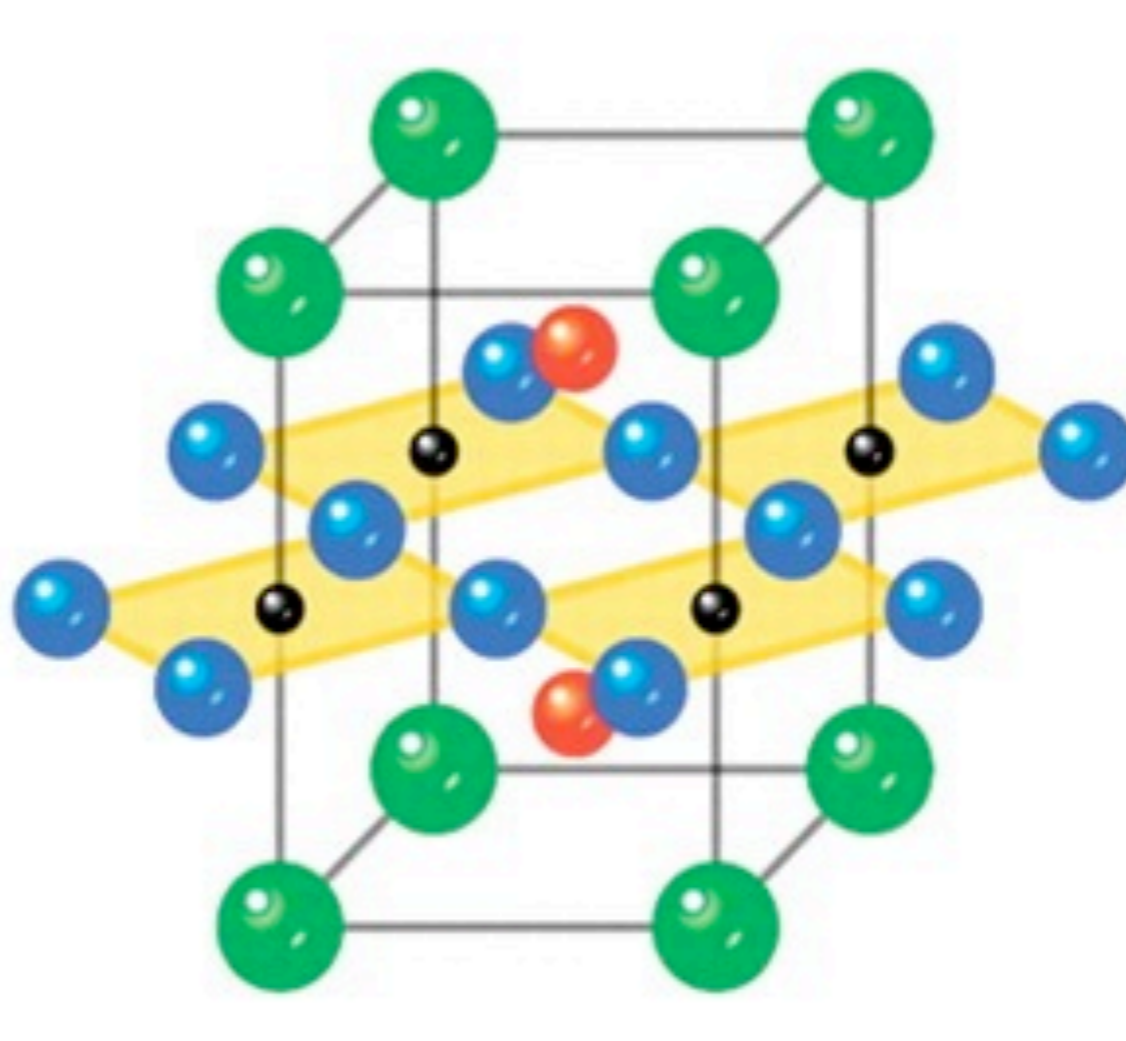
## 4. Global phase diagram of the cuprates

*Competition between d-wave superconductivity  
and spin-density order for the Fermi surface*

# *The cuprate superconductors*

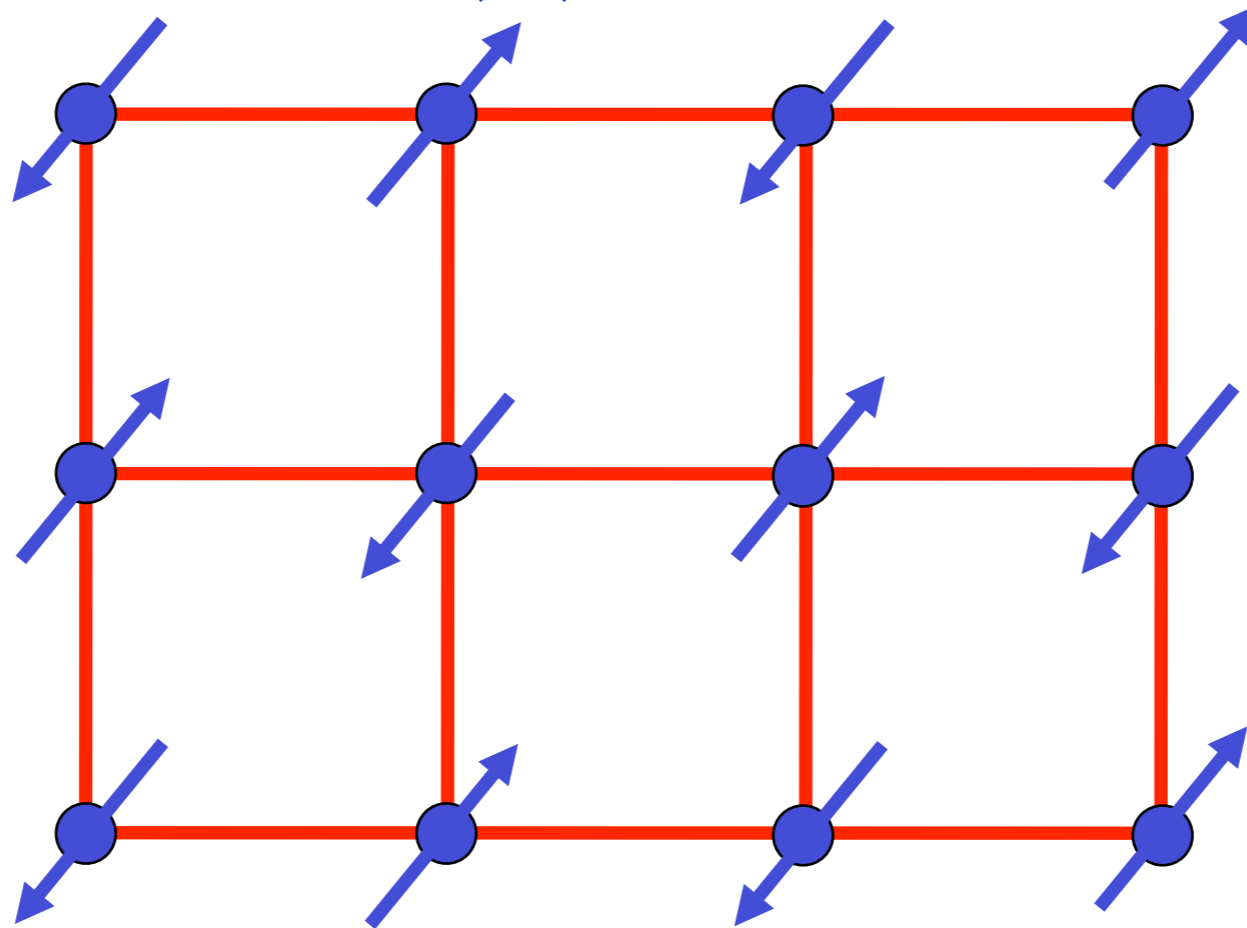
Na-CCOC

- Cu
- Ca/Na
- O
- Cl



# Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



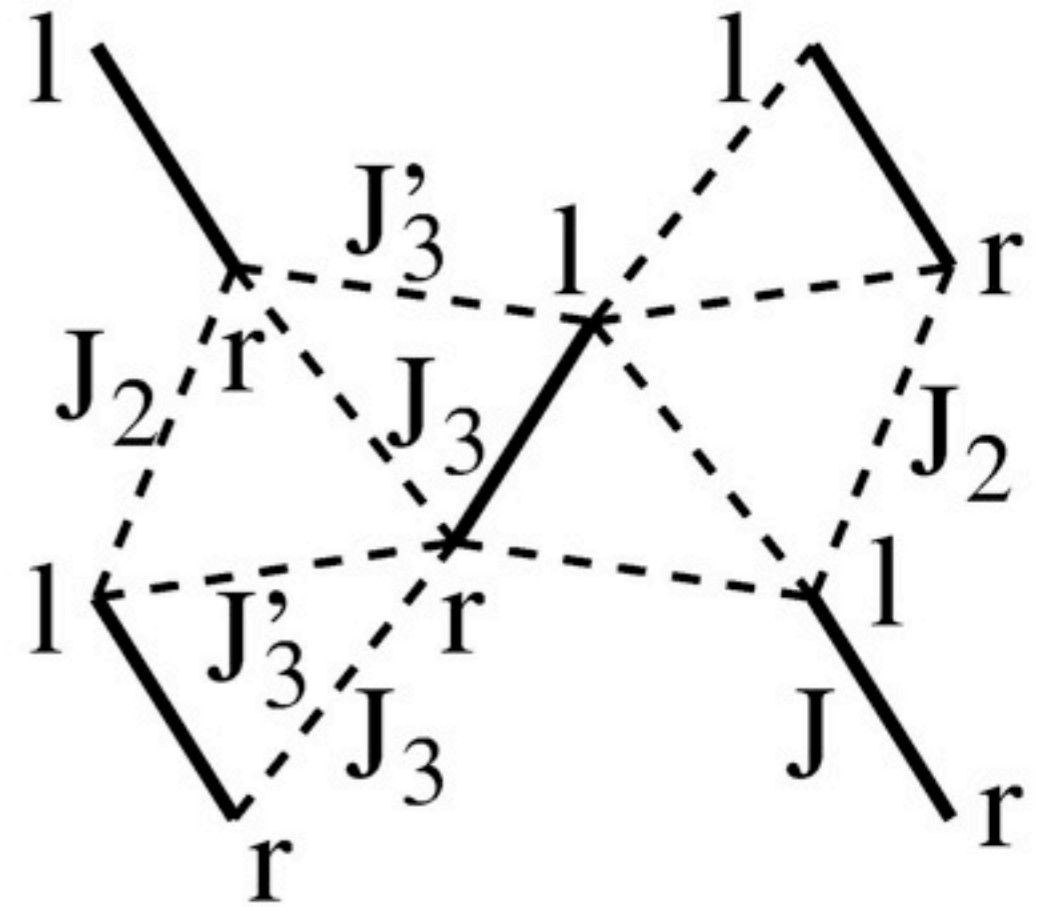
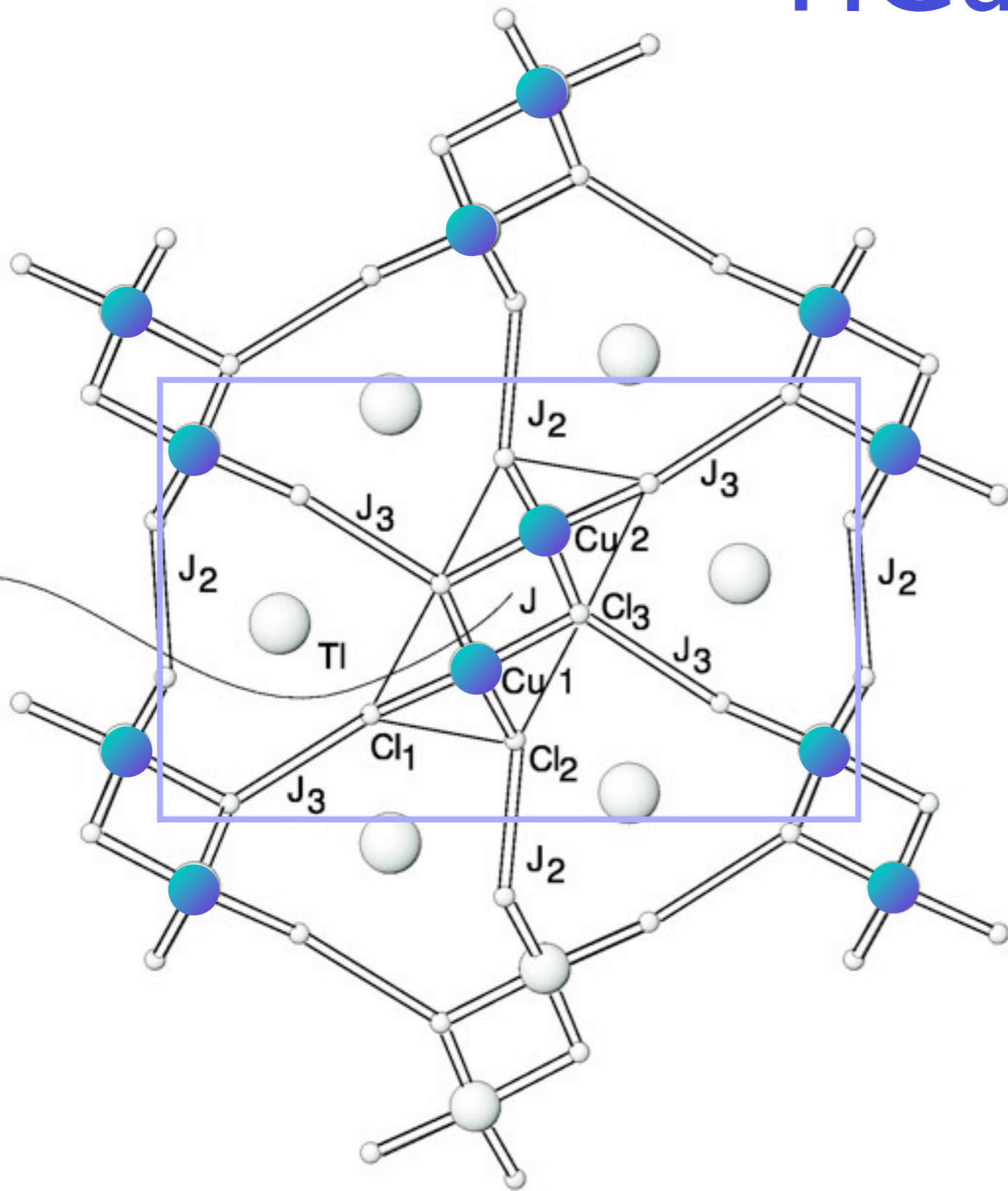
Ground state has long-range Néel order

Order parameter is a single vector field  $\vec{\varphi} = \eta_i \vec{S}_i$

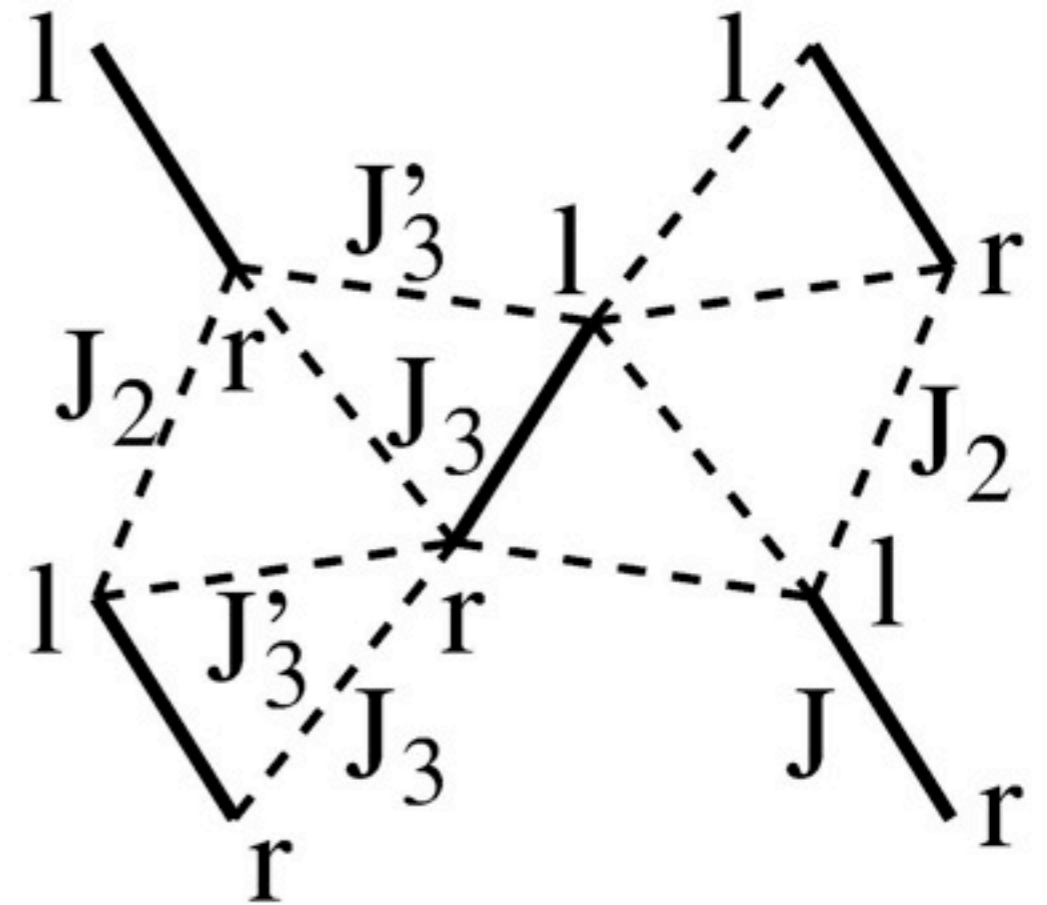
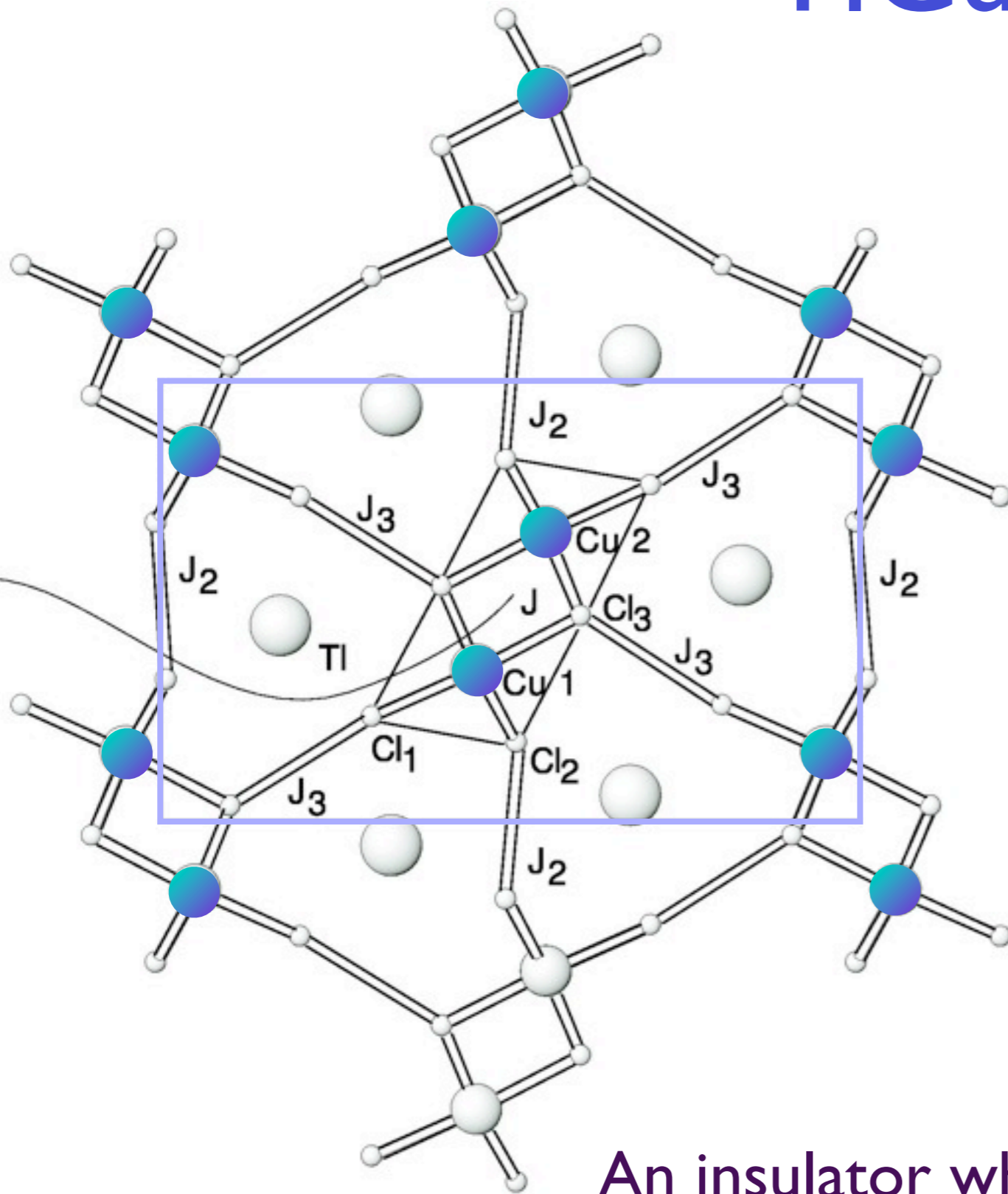
$\eta_i = \pm 1$  on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$  in Néel state.

# TlCuCl<sub>3</sub>



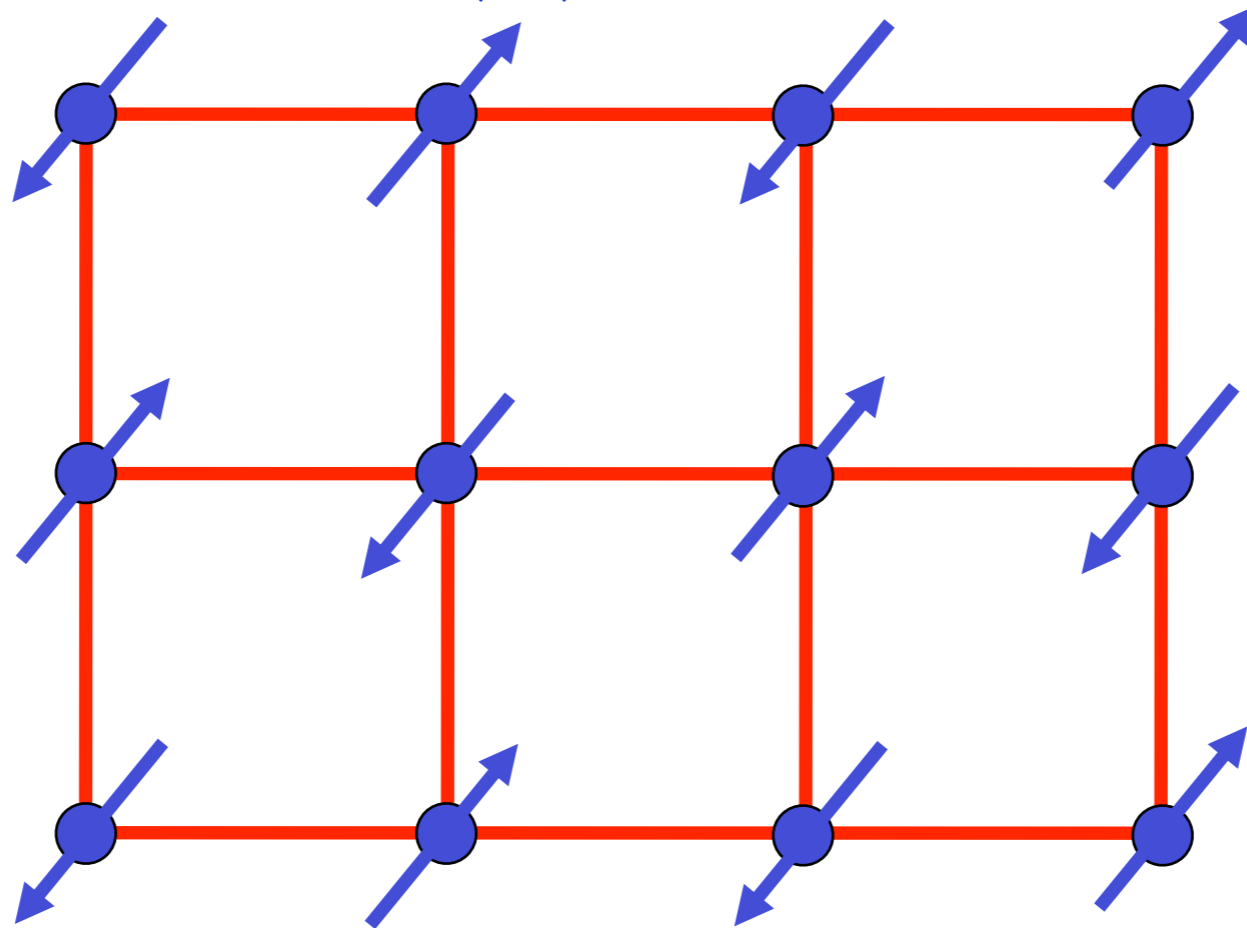
# TlCuCl<sub>3</sub>



An insulator whose spin susceptibility vanishes exponentially as the temperature  $T$  tends to zero.

# Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

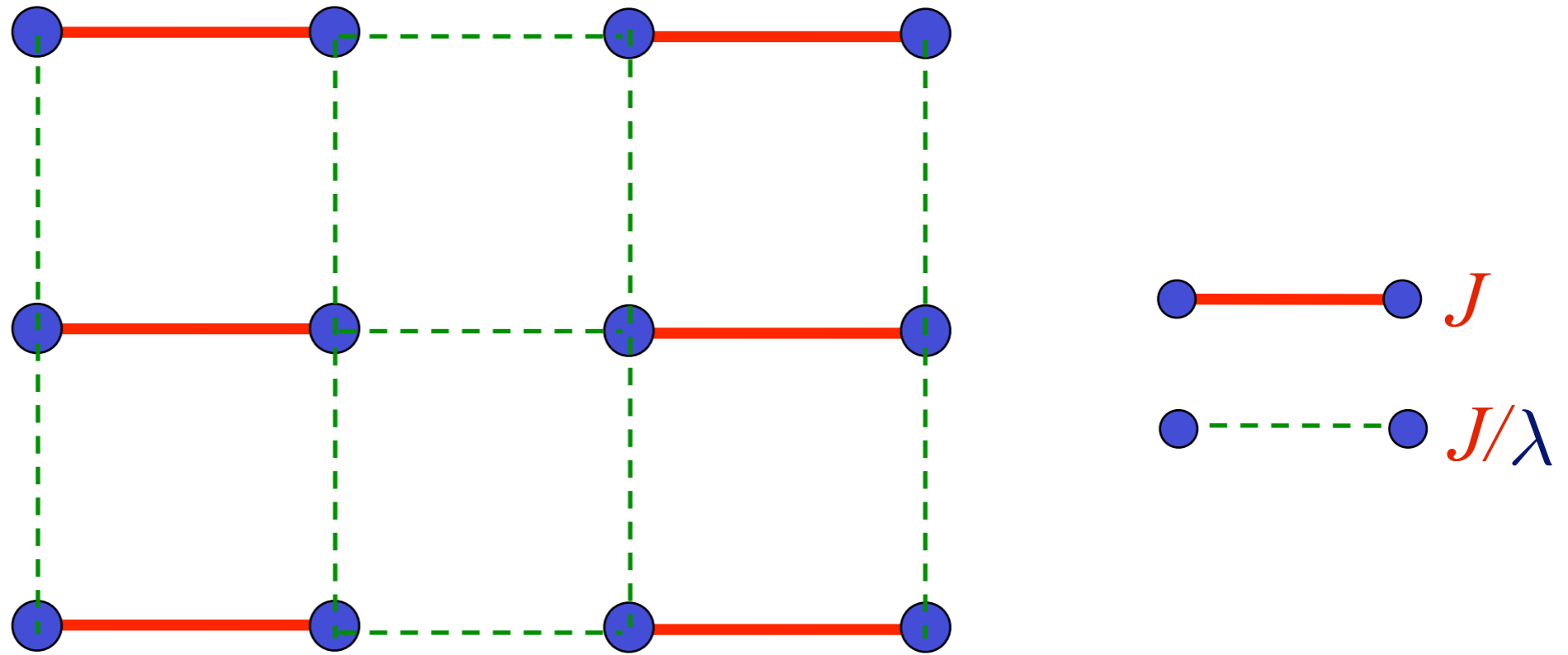
Order parameter is a single vector field  $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$  on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$  in Néel state.

# Square lattice antiferromagnet

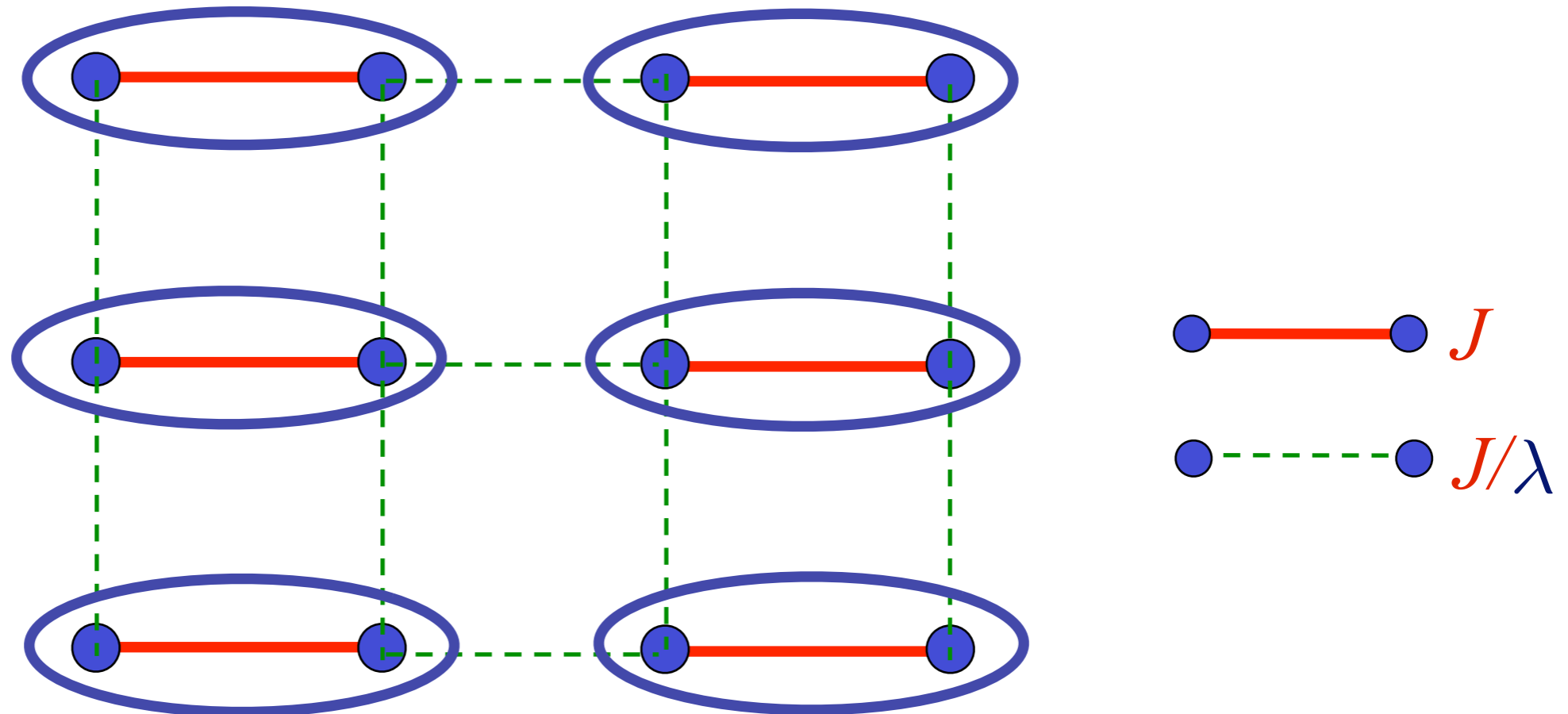
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Weaken some bonds to induce spin entanglement in a new quantum phase

# Square lattice antiferromagnet

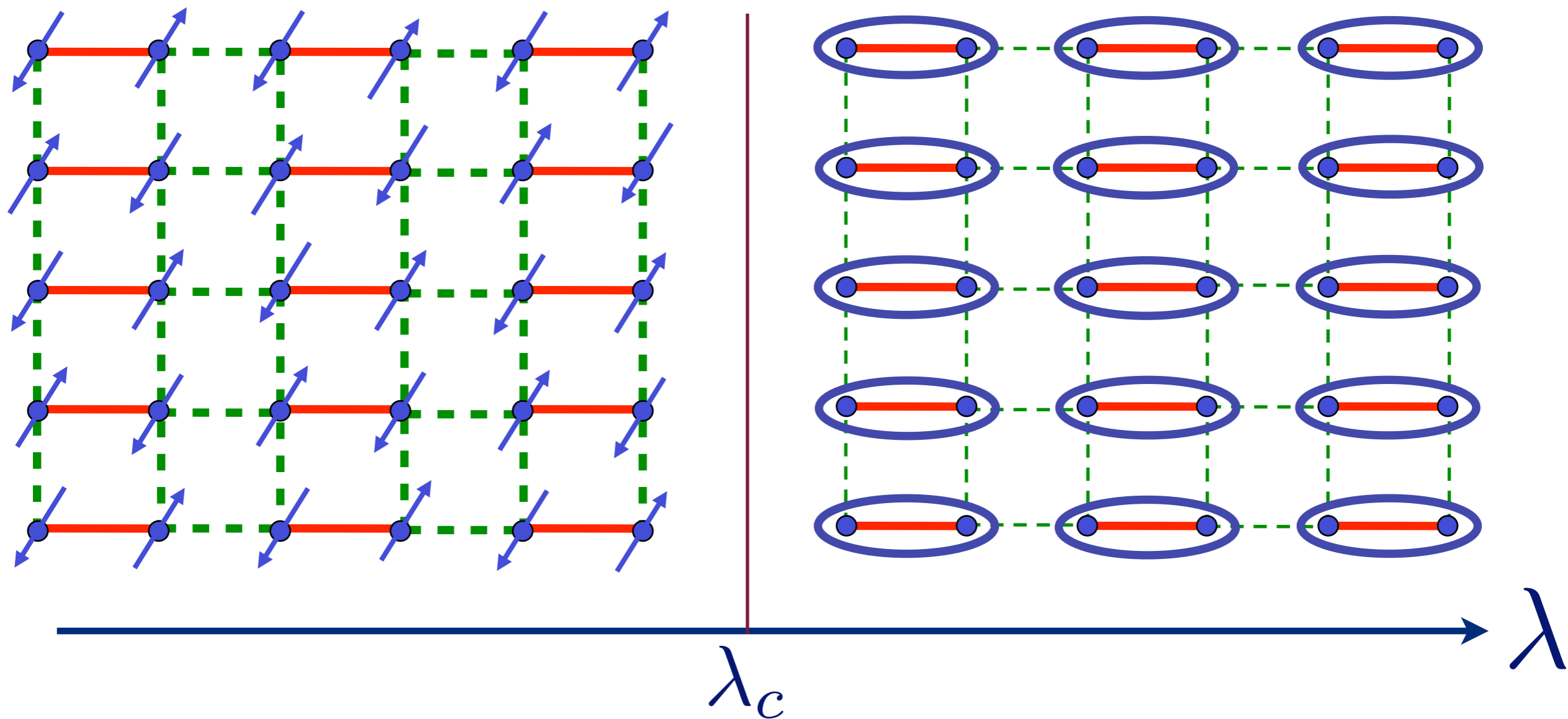
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state is a “quantum paramagnet”  
with spins locked in valence bond singlets

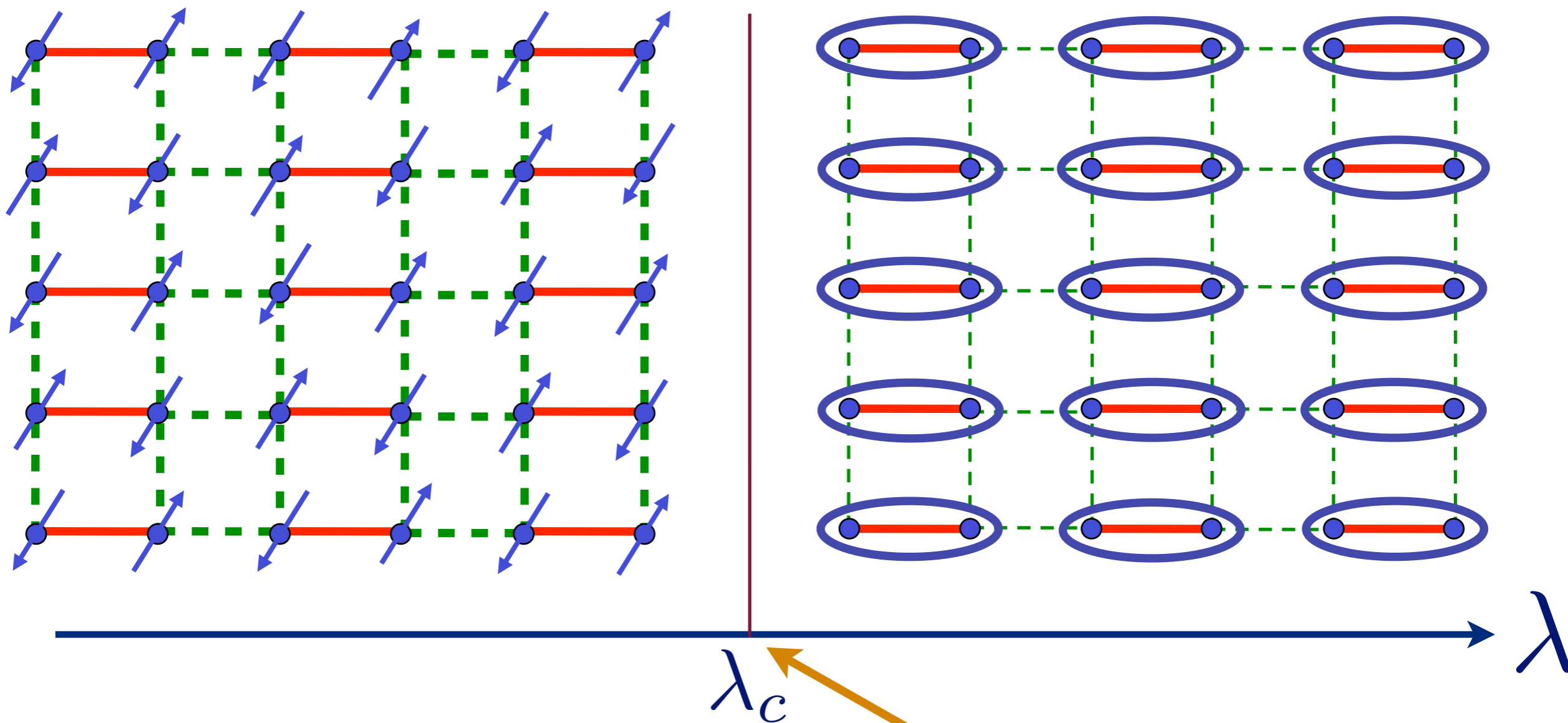
$$\text{Singlet} = \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

$$\text{Diagram of two blue spheres connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



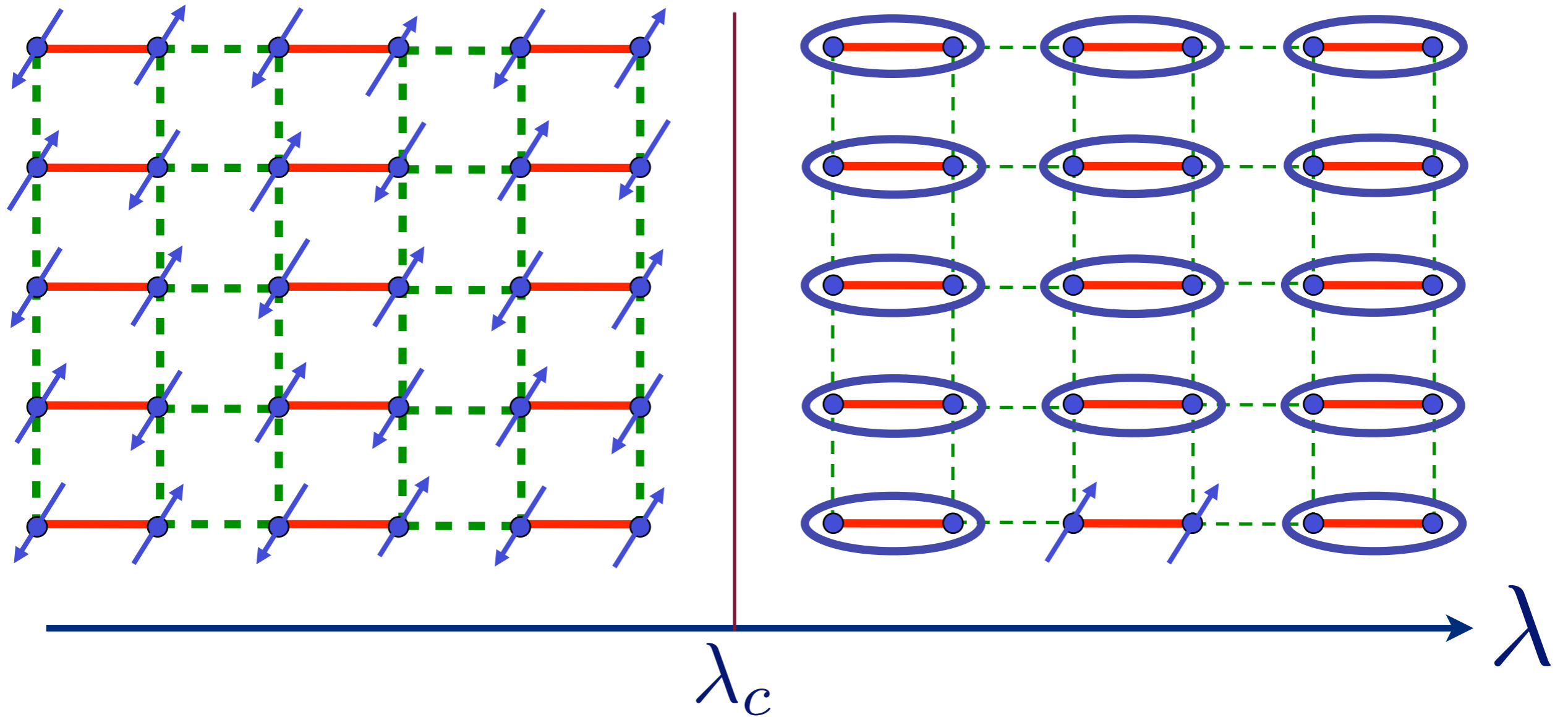
← Pressure in  $\text{TlCuCl}_3$

$$\text{[Diagram of two blue dots connected by a red line, enclosed in a blue oval]} = \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

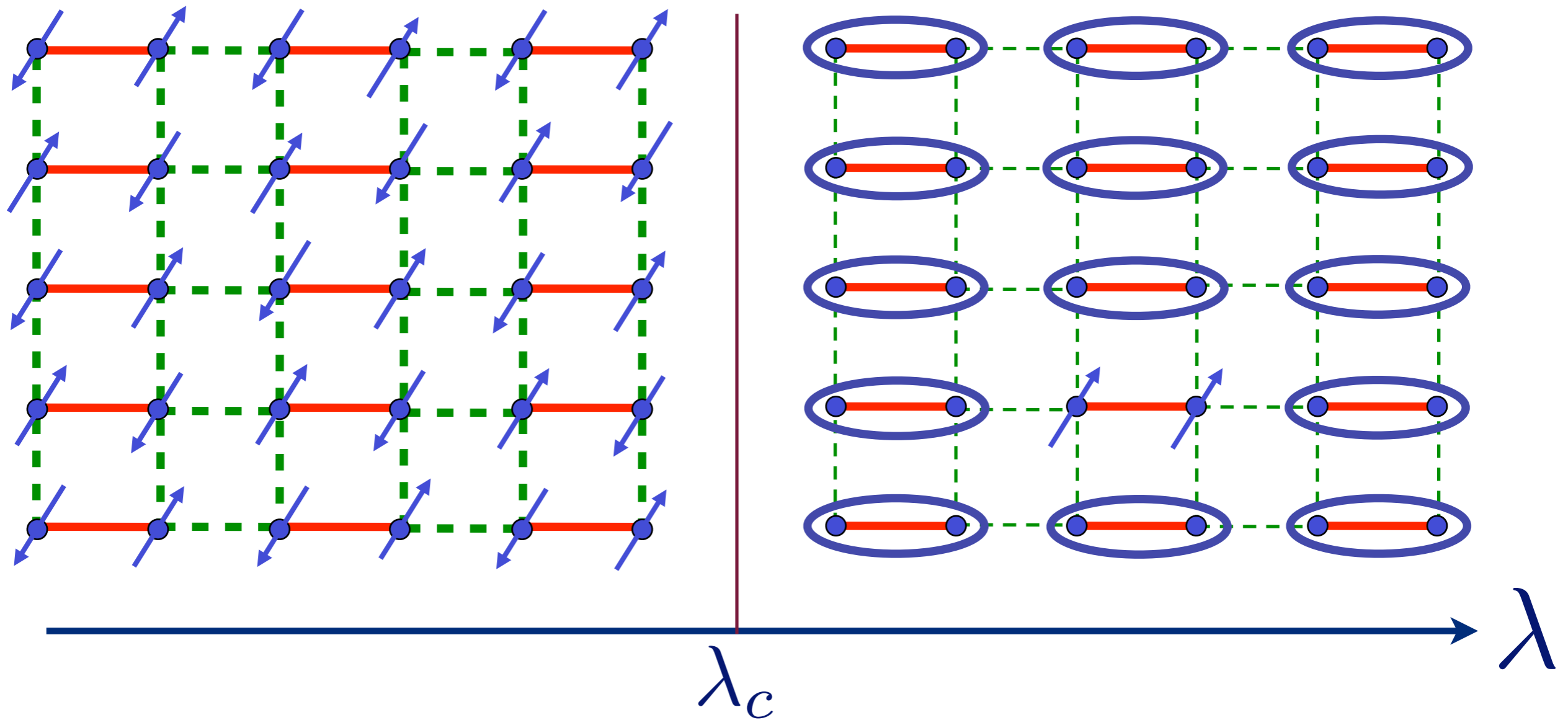


Quantum critical point with non-local entanglement in spin wavefunction

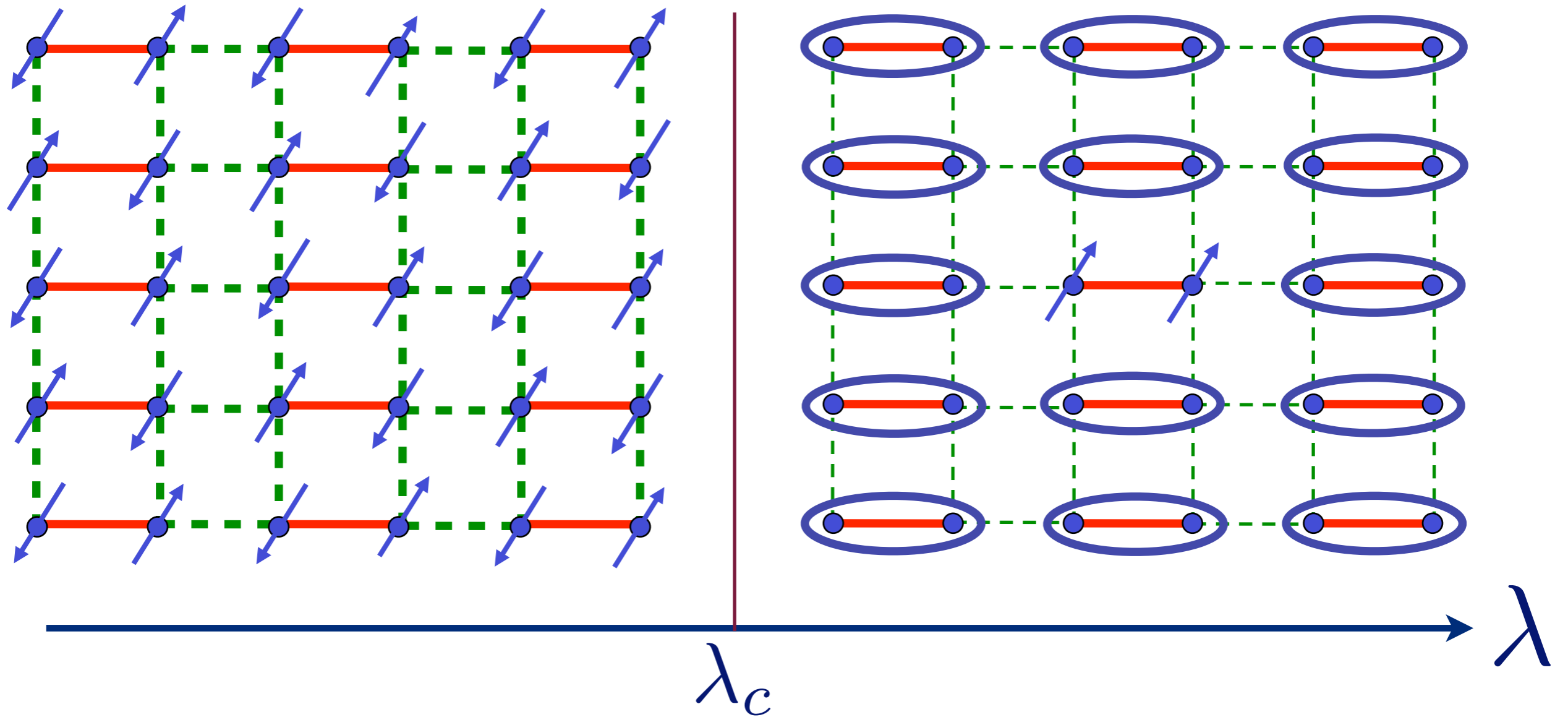
# Excitation spectrum in the paramagnetic phase



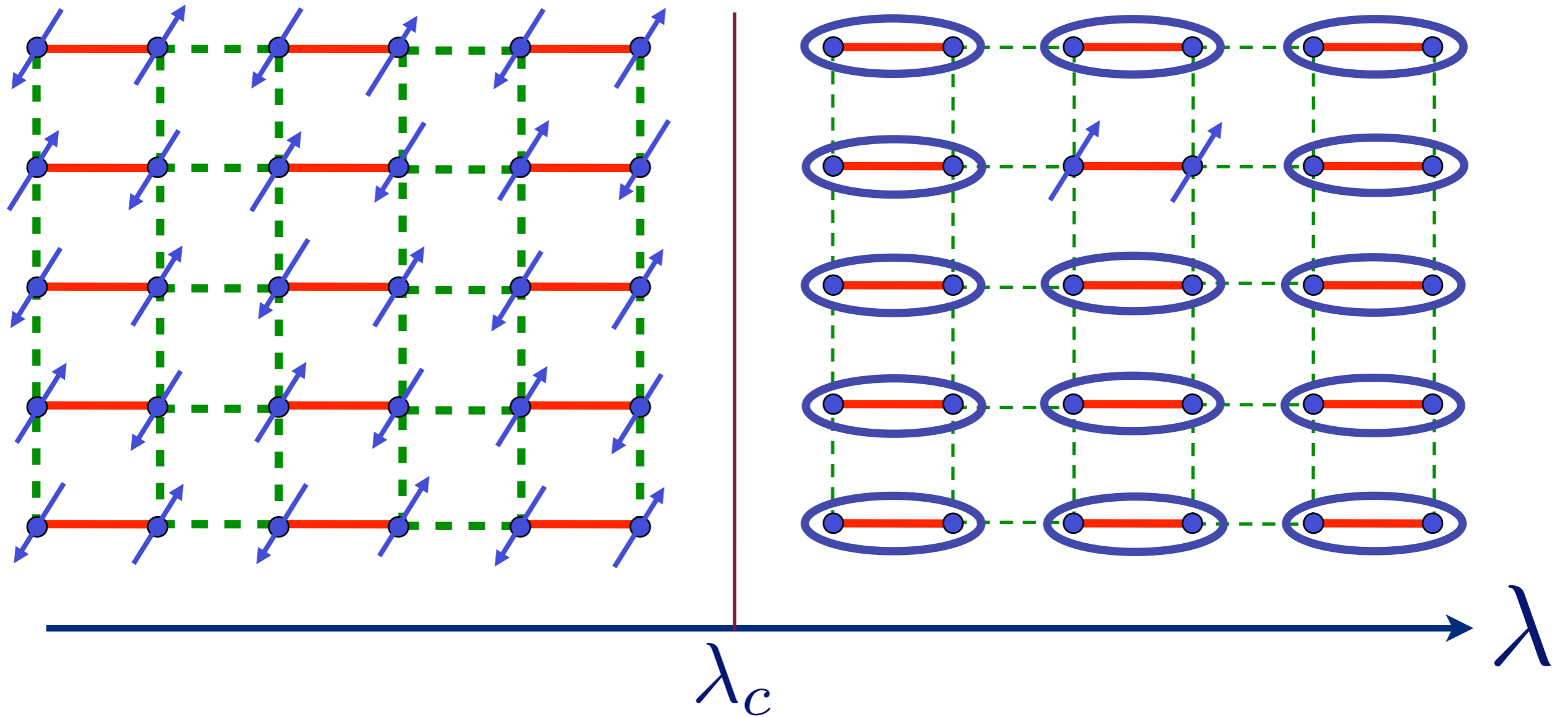
# Excitation spectrum in the paramagnetic phase



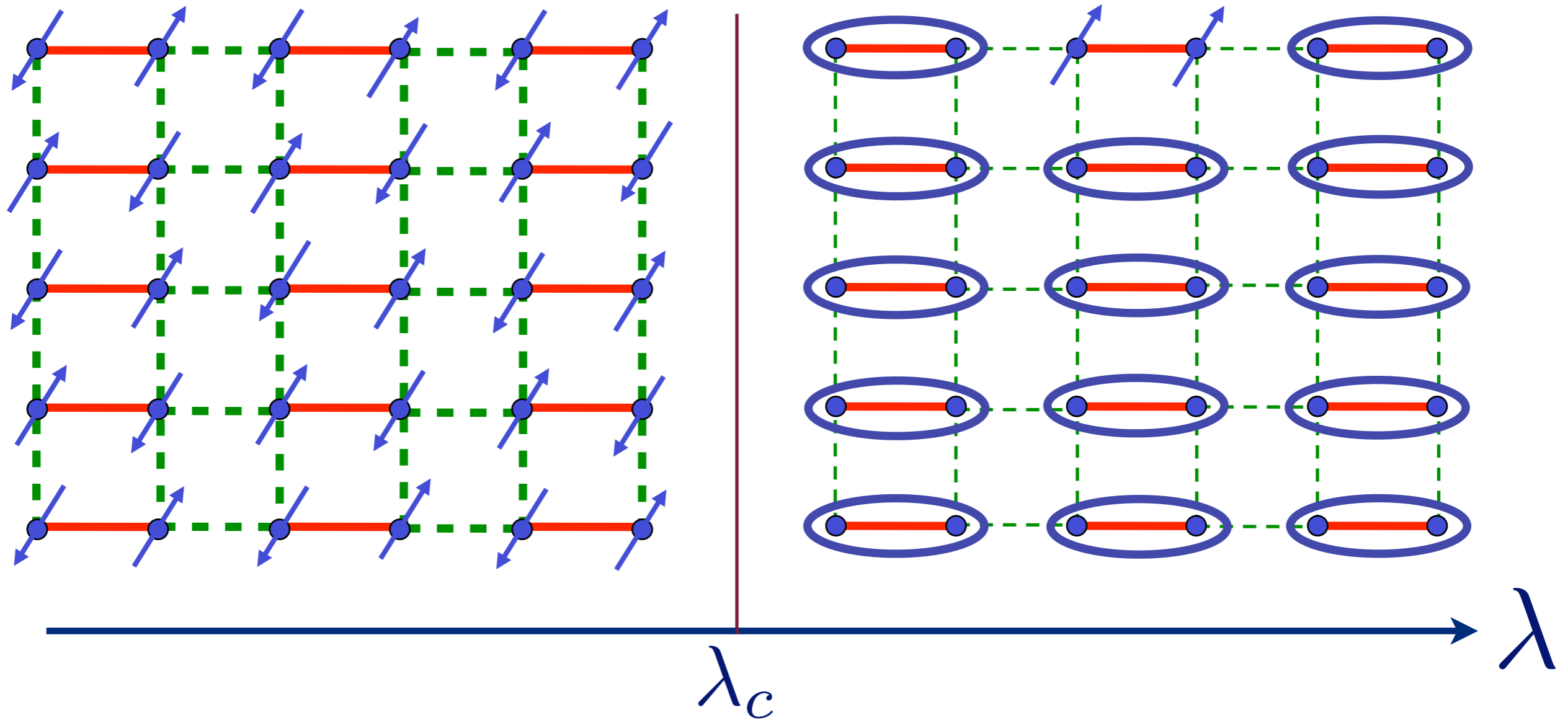
# Excitation spectrum in the paramagnetic phase



# Excitation spectrum in the paramagnetic phase



# Excitation spectrum in the paramagnetic phase



# TlCuCl<sub>3</sub> at ambient pressure

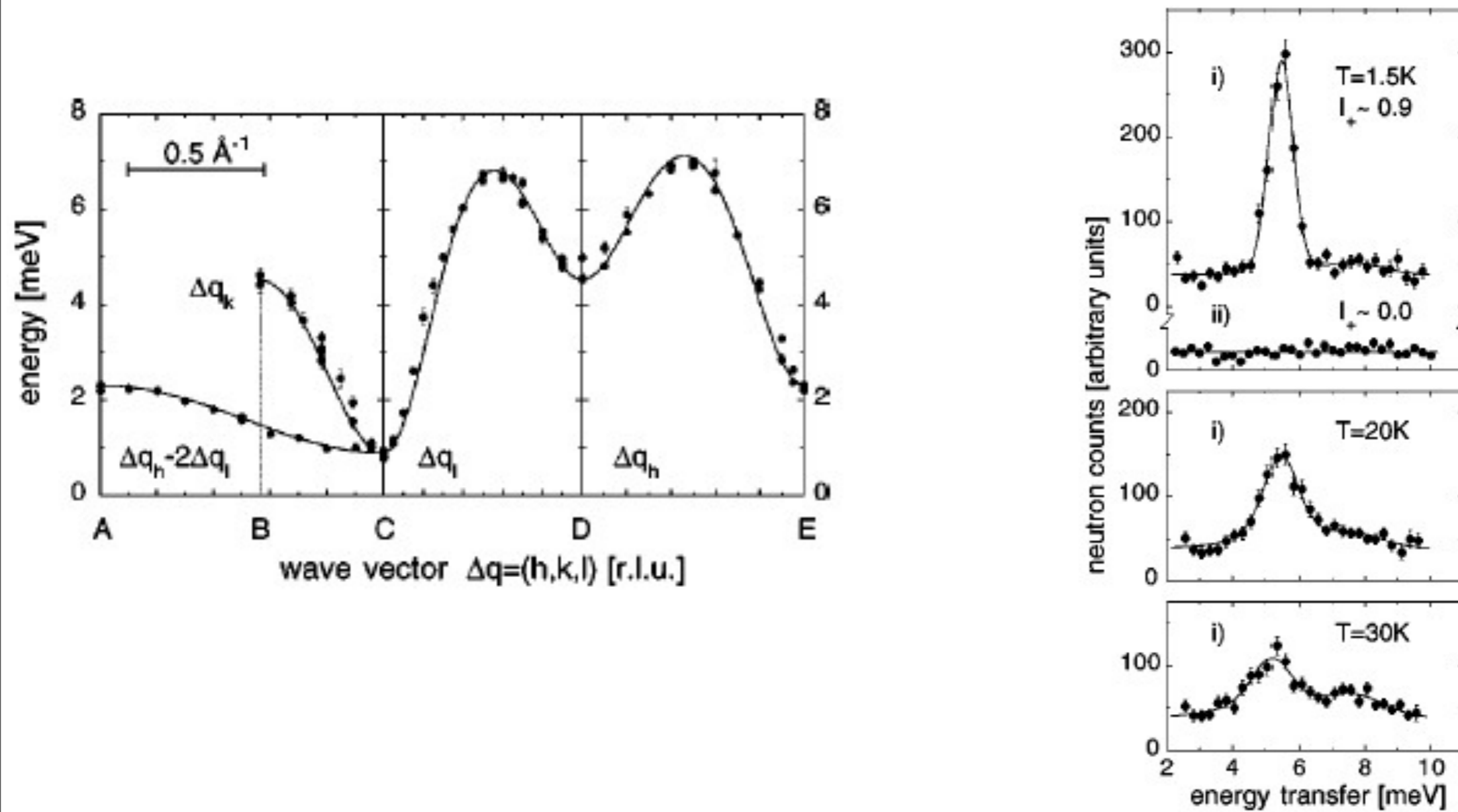
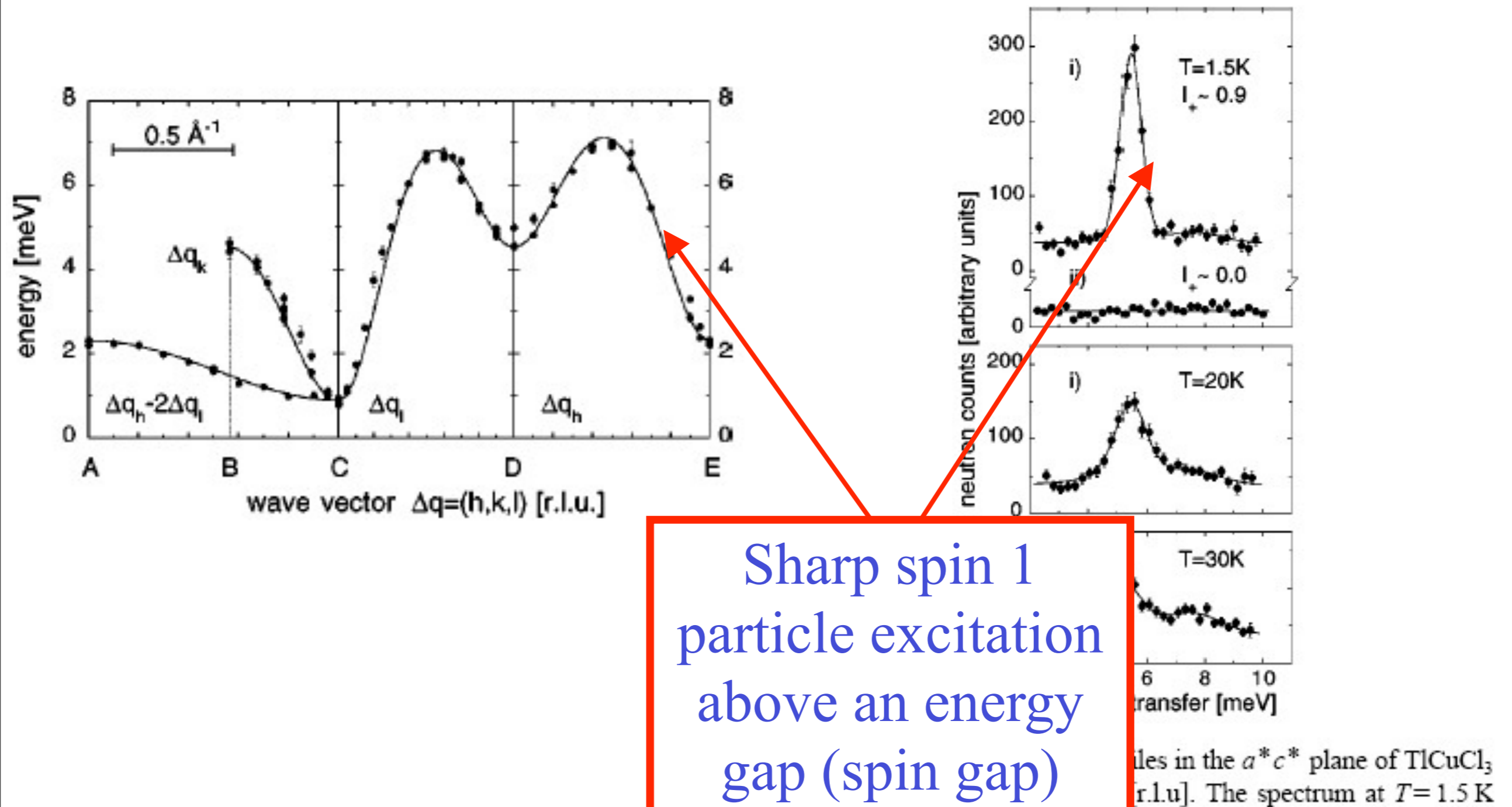


FIG. 1. Measured neutron profiles in the  $a^*c^*$  plane of TlCuCl<sub>3</sub> for  $i = (1.35, 0, 0)$ ,  $ii = (0, 0, 3.15)$  [r.l.u.]. The spectrum at  $T = 1.5 \text{ K}$

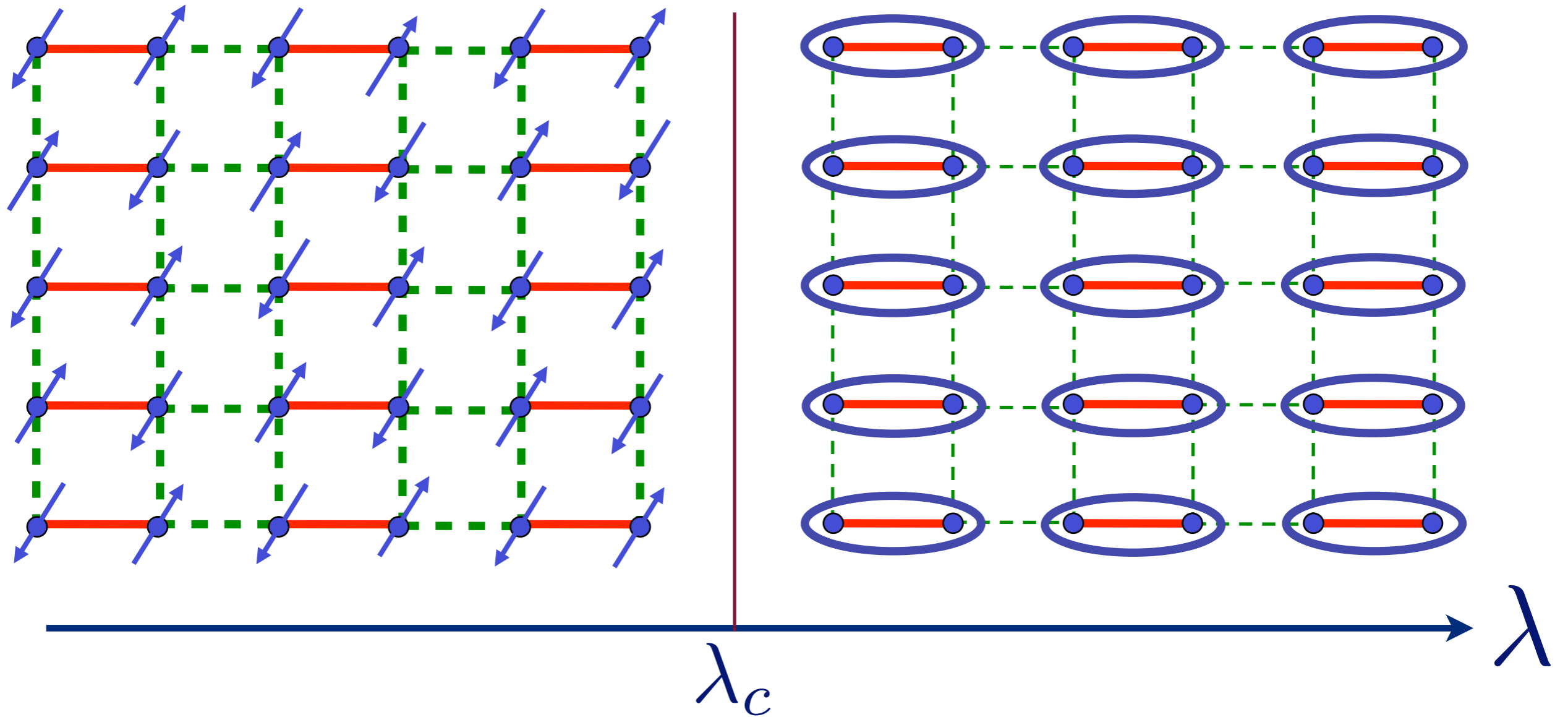
N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev. B* 63 172414 (2001).

# TlCuCl<sub>3</sub> at ambient pressure

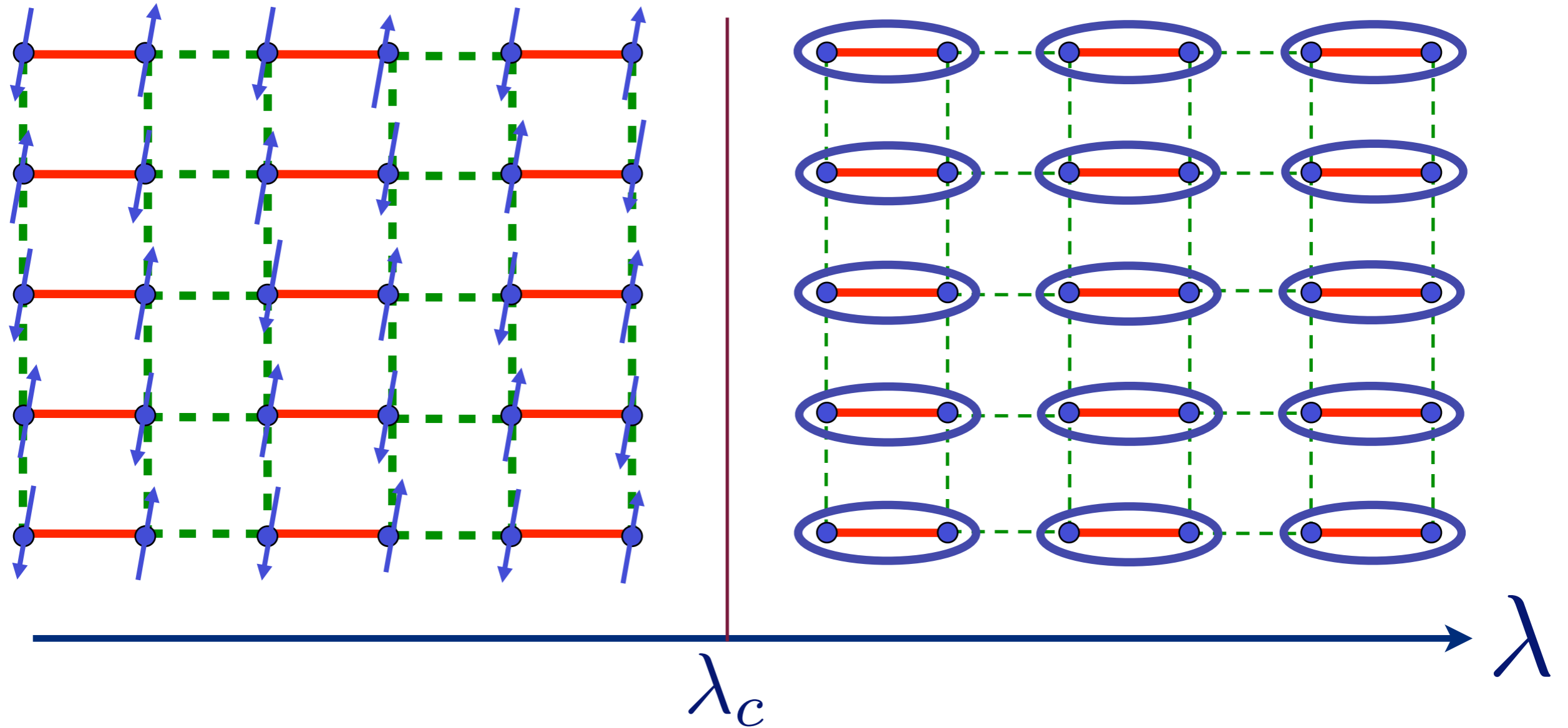


N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer  
and H. Mutka, *Phys. Rev. B* 63 172414 (2001).

# Excitation spectrum in the Néel phase

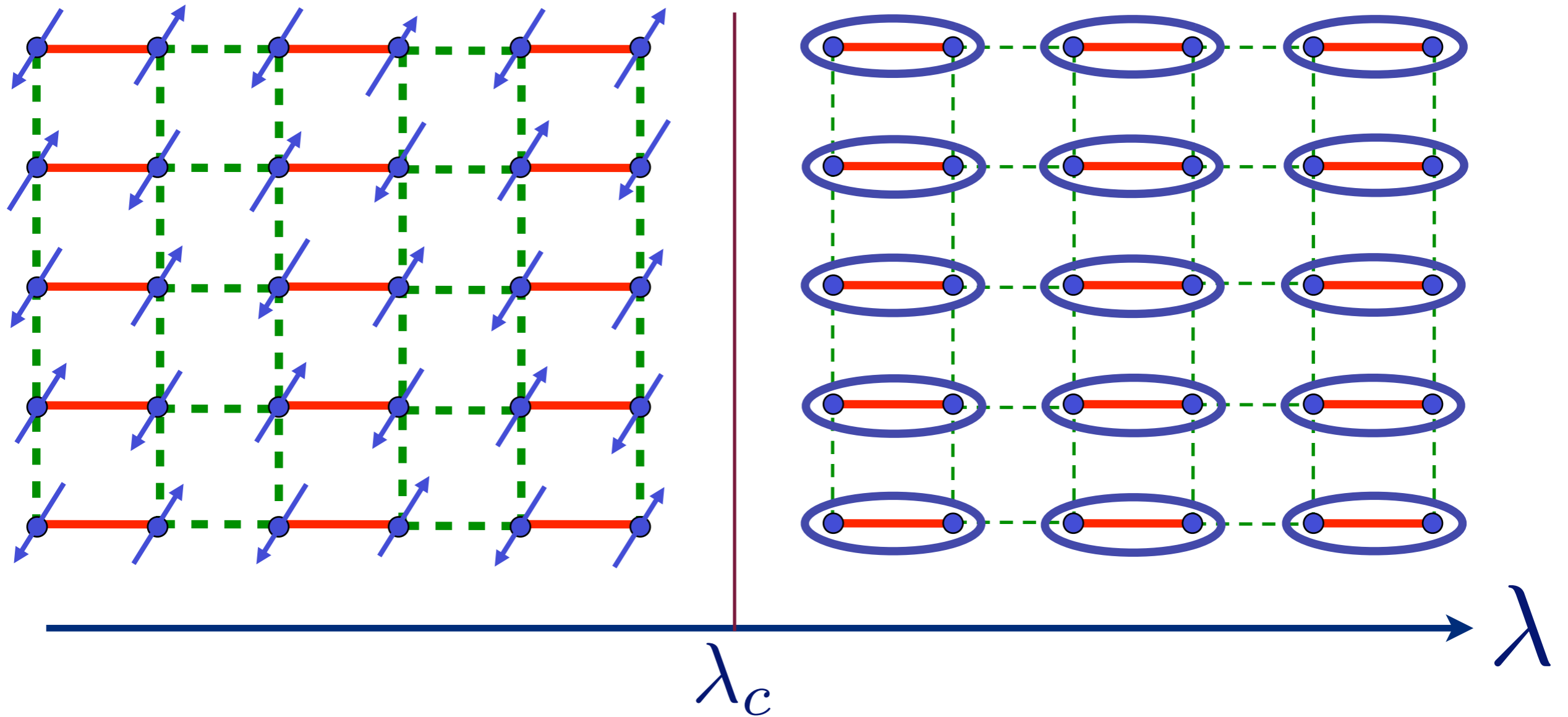


# Excitation spectrum in the Néel phase



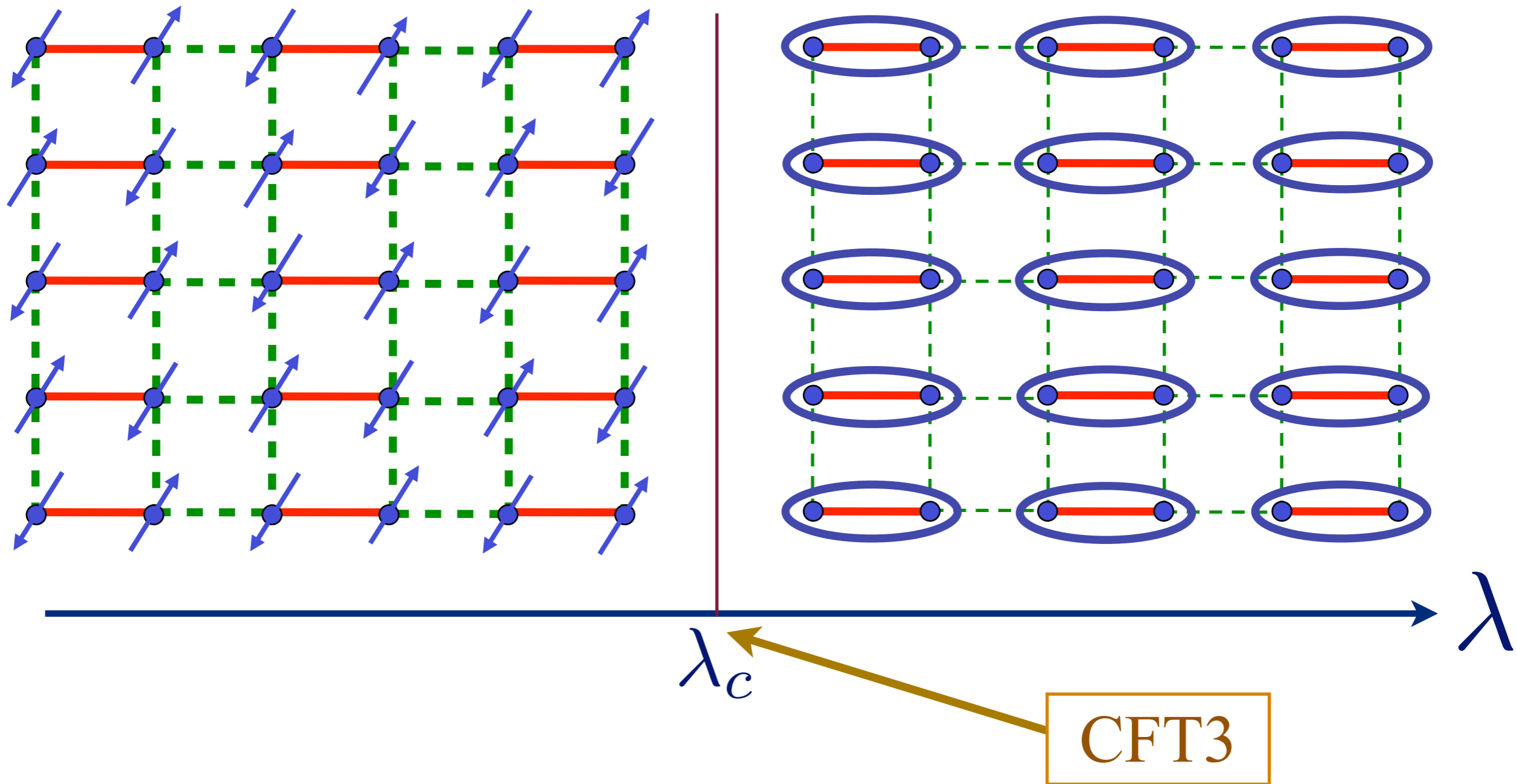
Spin waves

# Excitation spectrum in the Néel phase



Spin waves

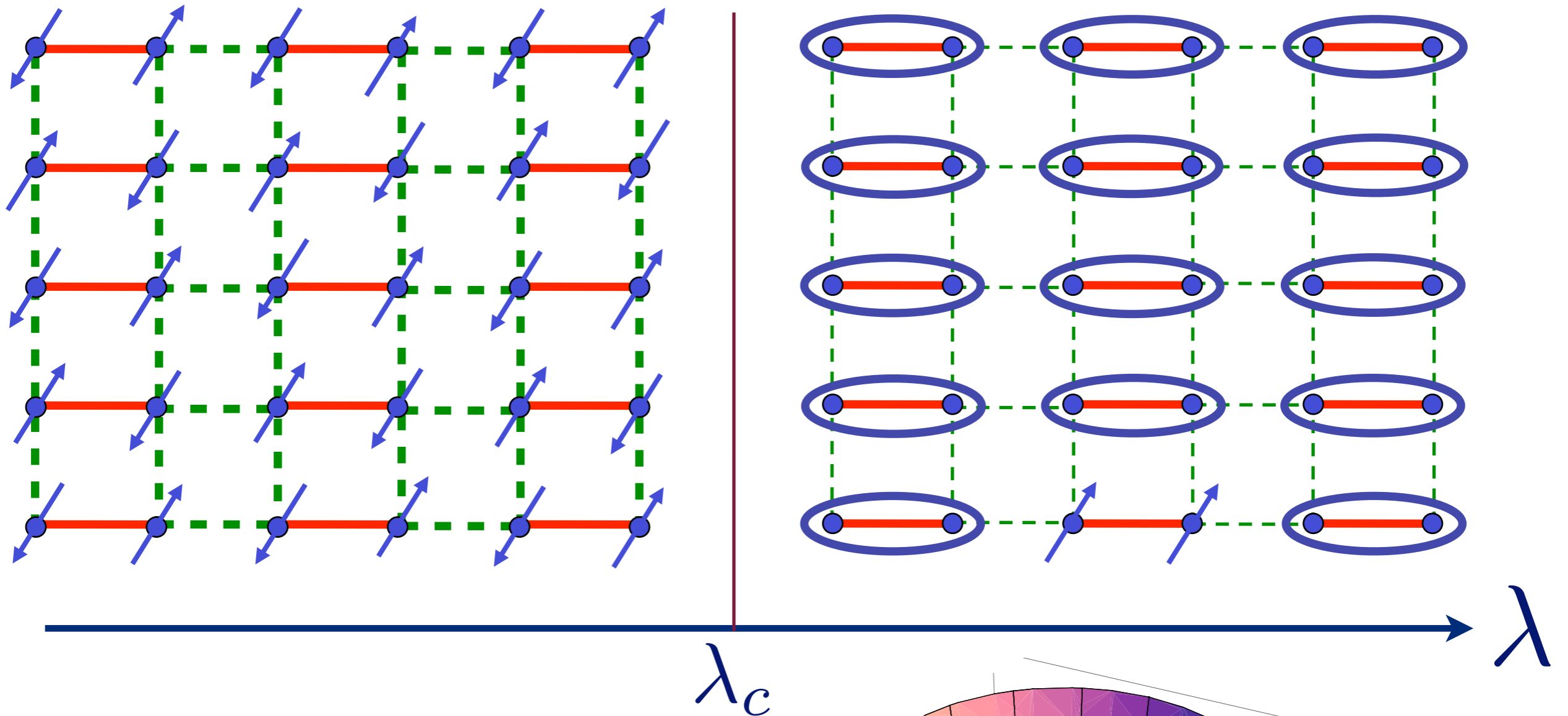
# Description using Landau-Ginzburg field theory



$O(3)$  order parameter  $\vec{\varphi}$

$$\mathcal{S} = \int d^2 r d\tau \left[ (\partial_\tau \vec{\varphi})^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2 \right]$$

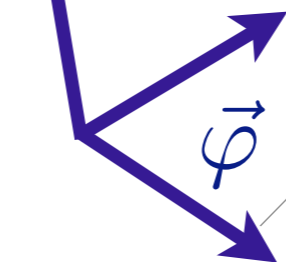
# Excitation spectrum in the paramagnetic phase



$$V(\vec{\varphi}) = (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2$$

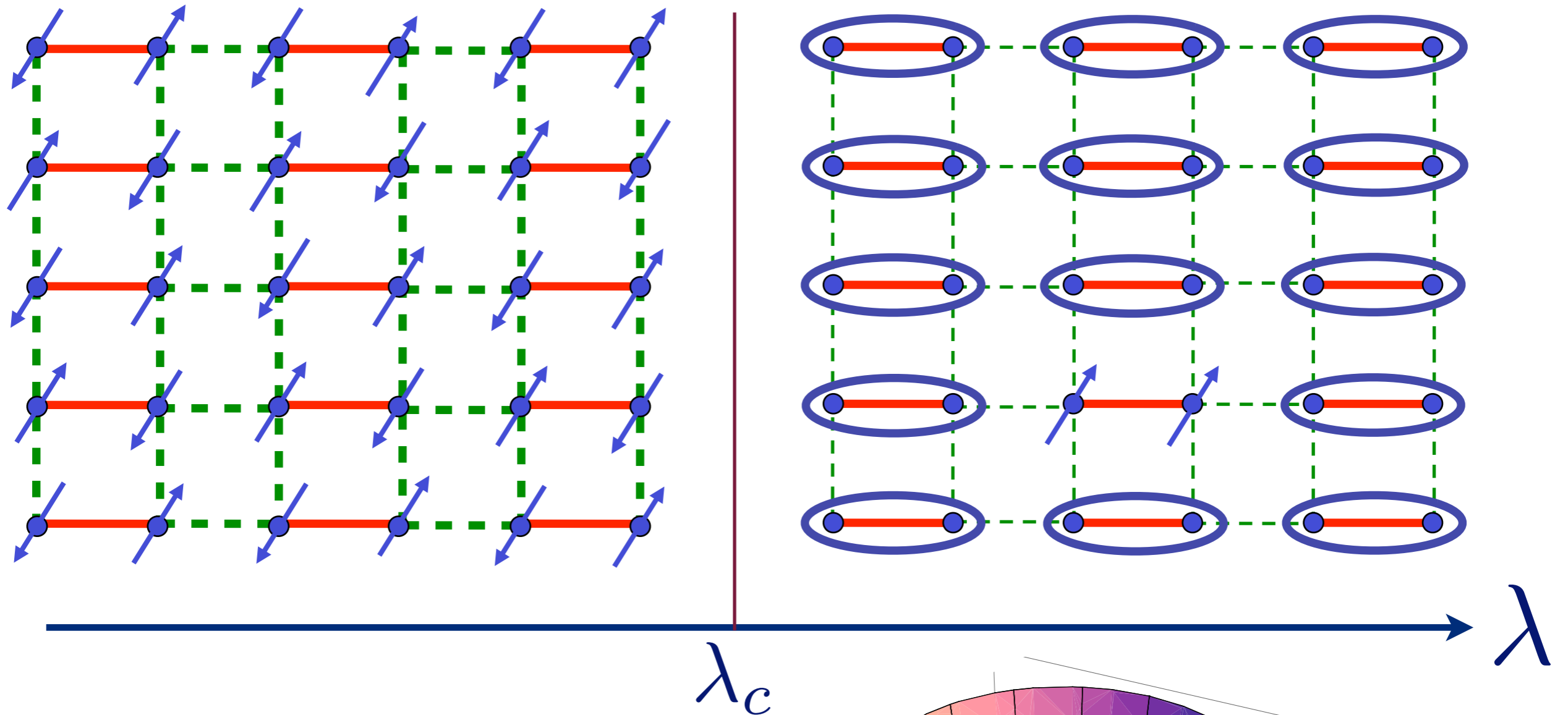
$$\lambda > \lambda_c$$

$$V(\vec{\varphi})$$



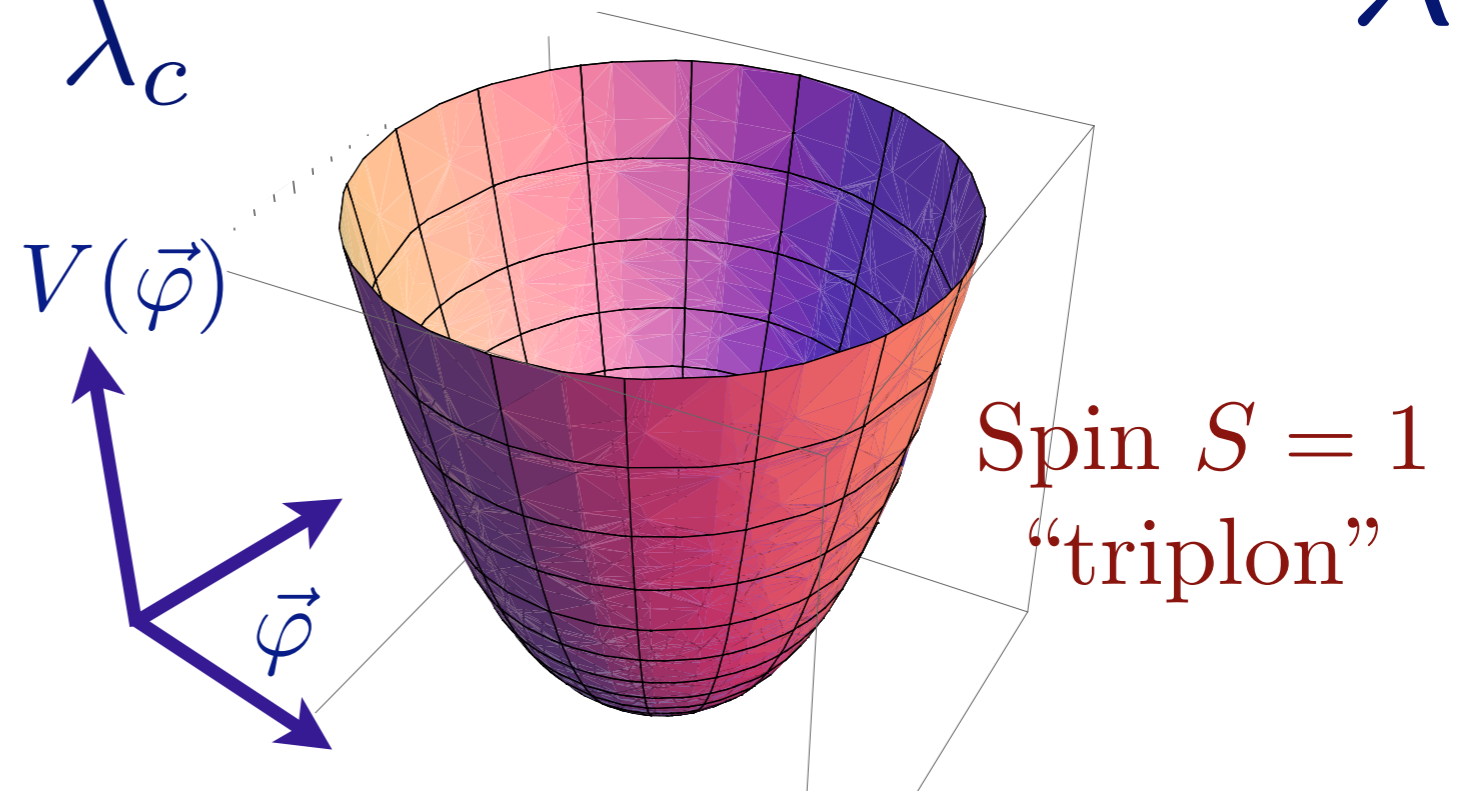
Spin  $S = 1$   
“triplon”

# Excitation spectrum in the paramagnetic phase

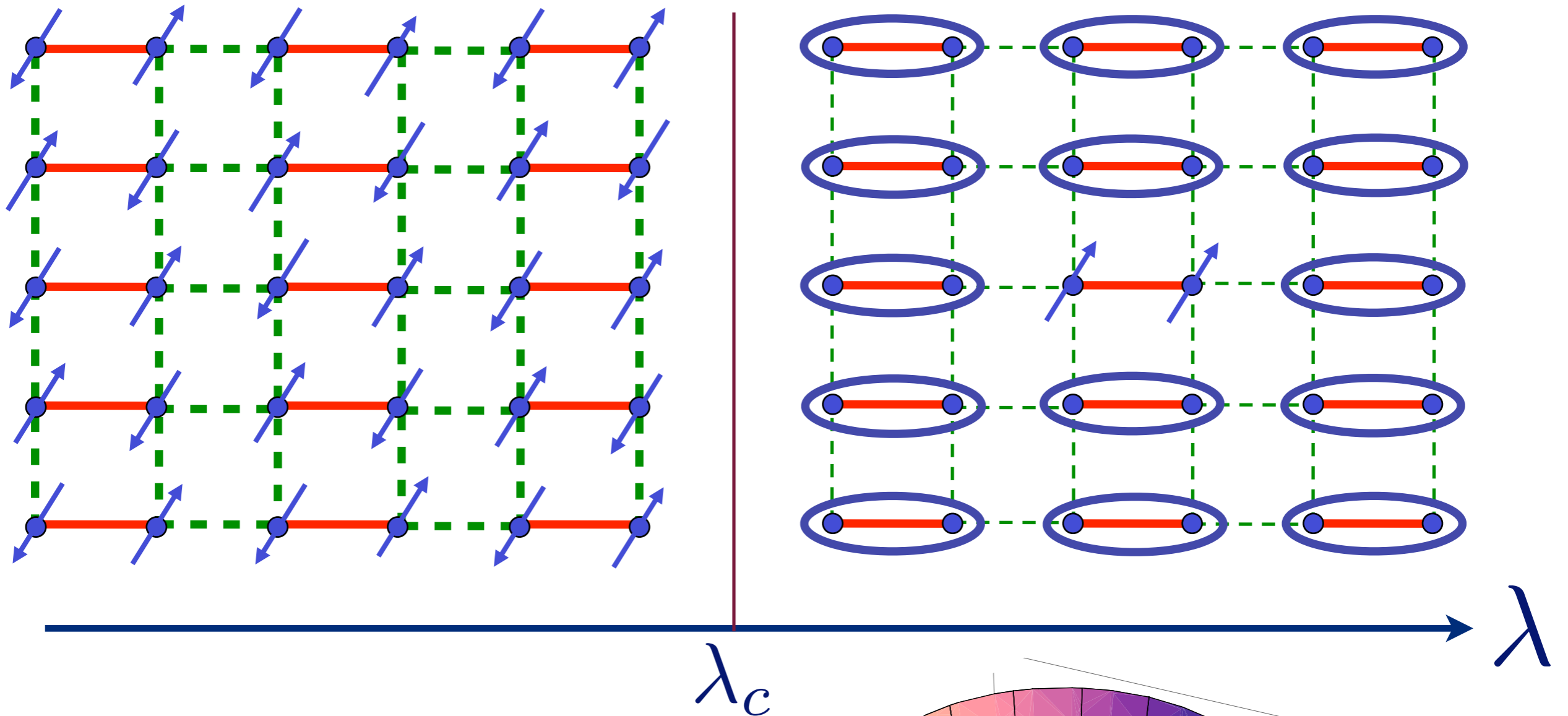


$$V(\vec{\varphi}) = (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2$$

$\lambda > \lambda_c$

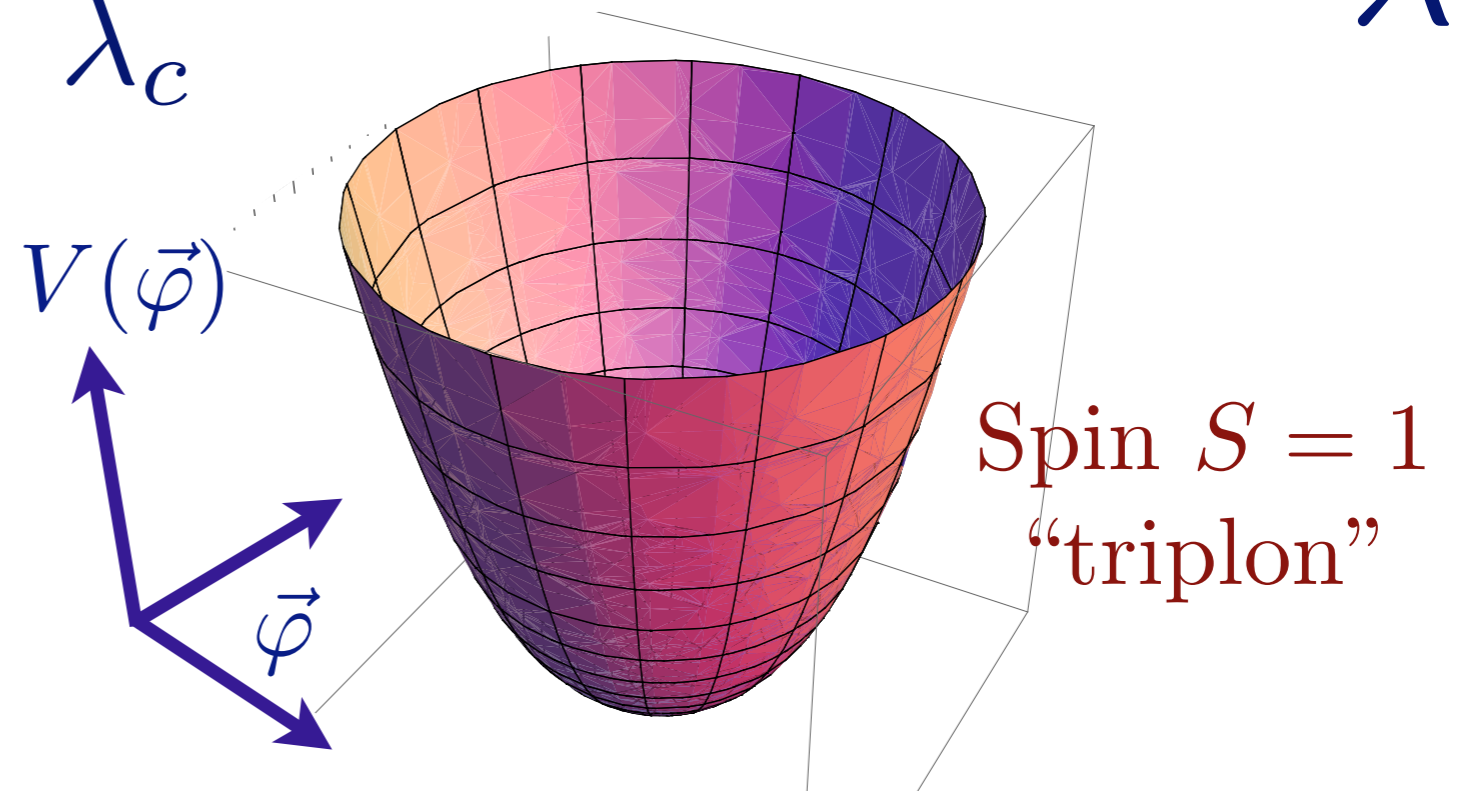


# Excitation spectrum in the paramagnetic phase

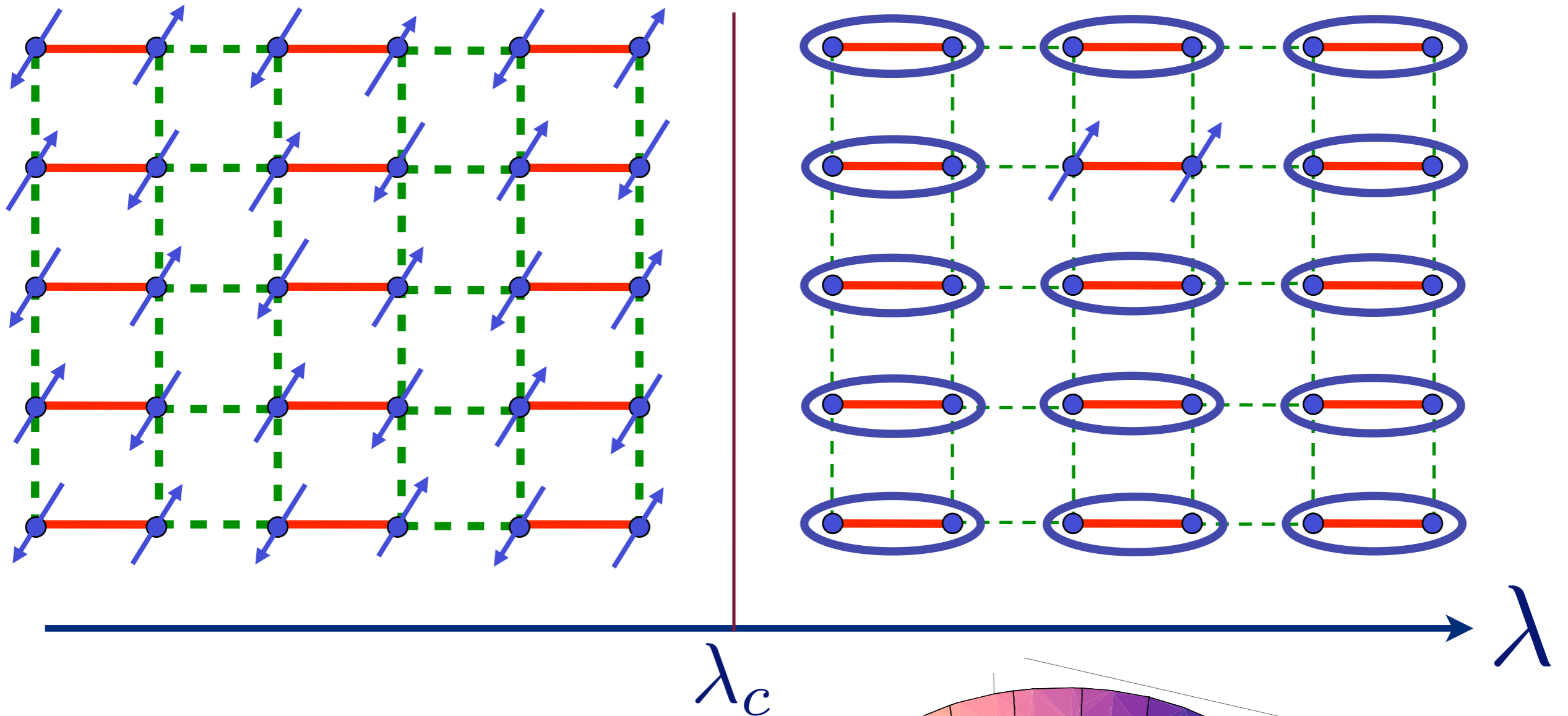


$$V(\vec{\varphi}) = (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2$$

$$\lambda > \lambda_c$$

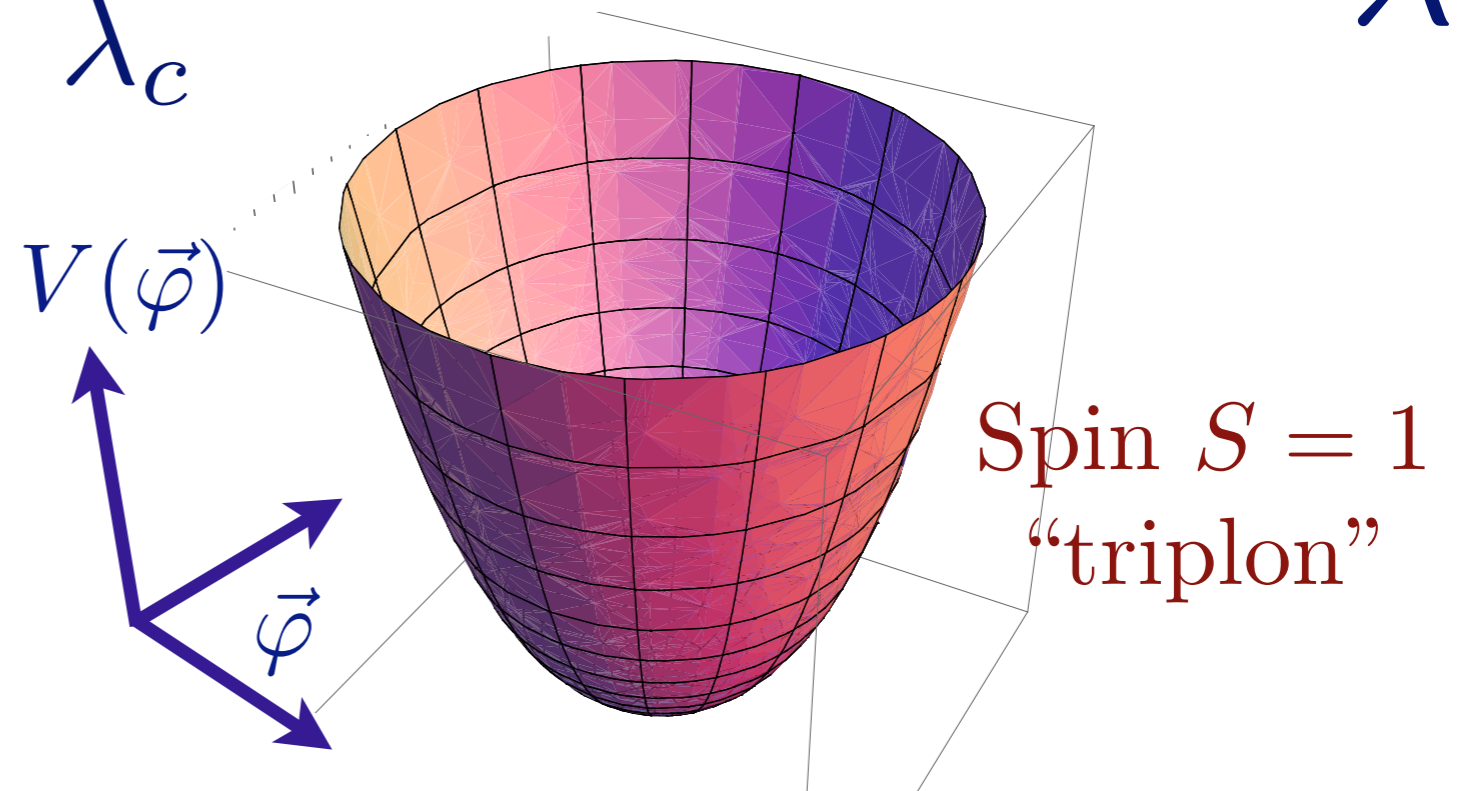


# Excitation spectrum in the paramagnetic phase

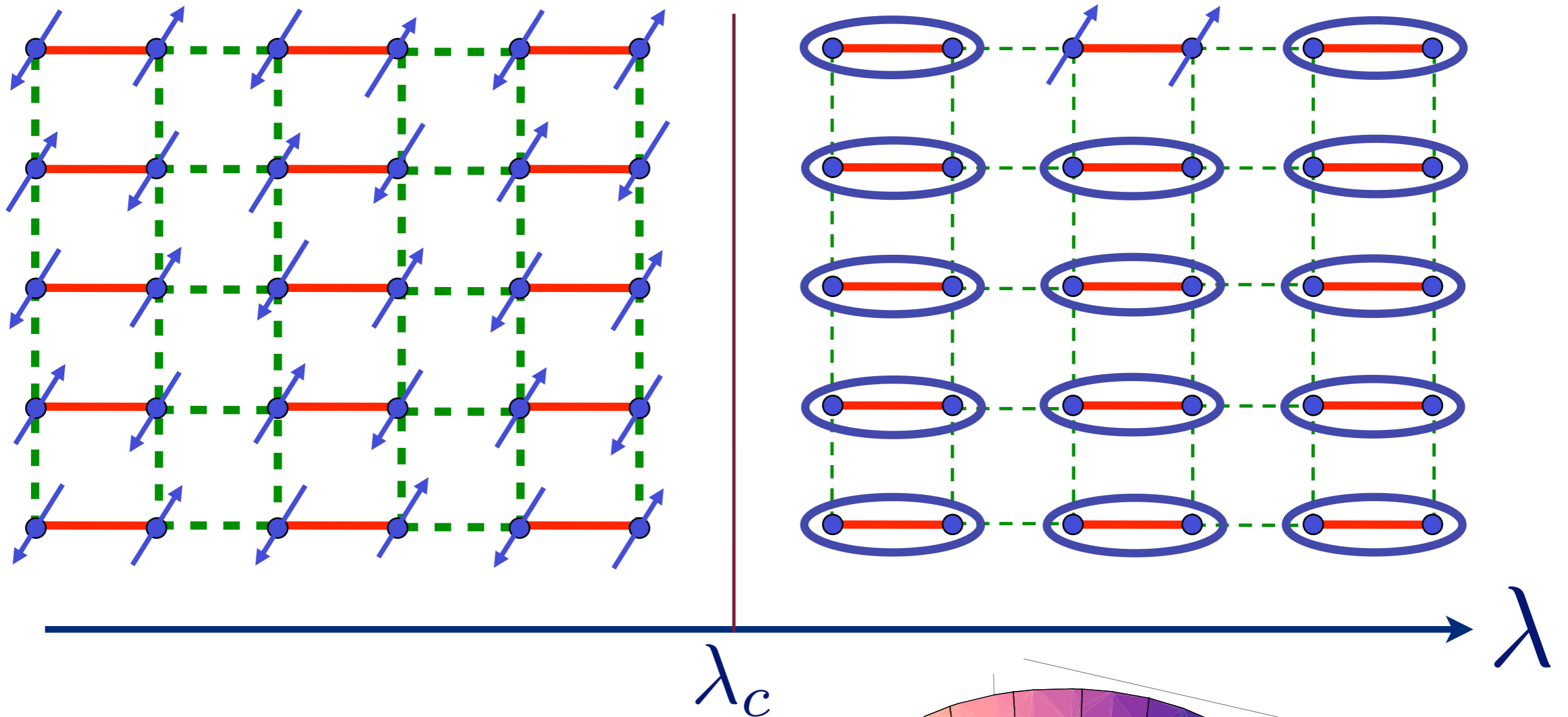


$$V(\vec{\varphi}) = (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2$$

$$\lambda > \lambda_c$$

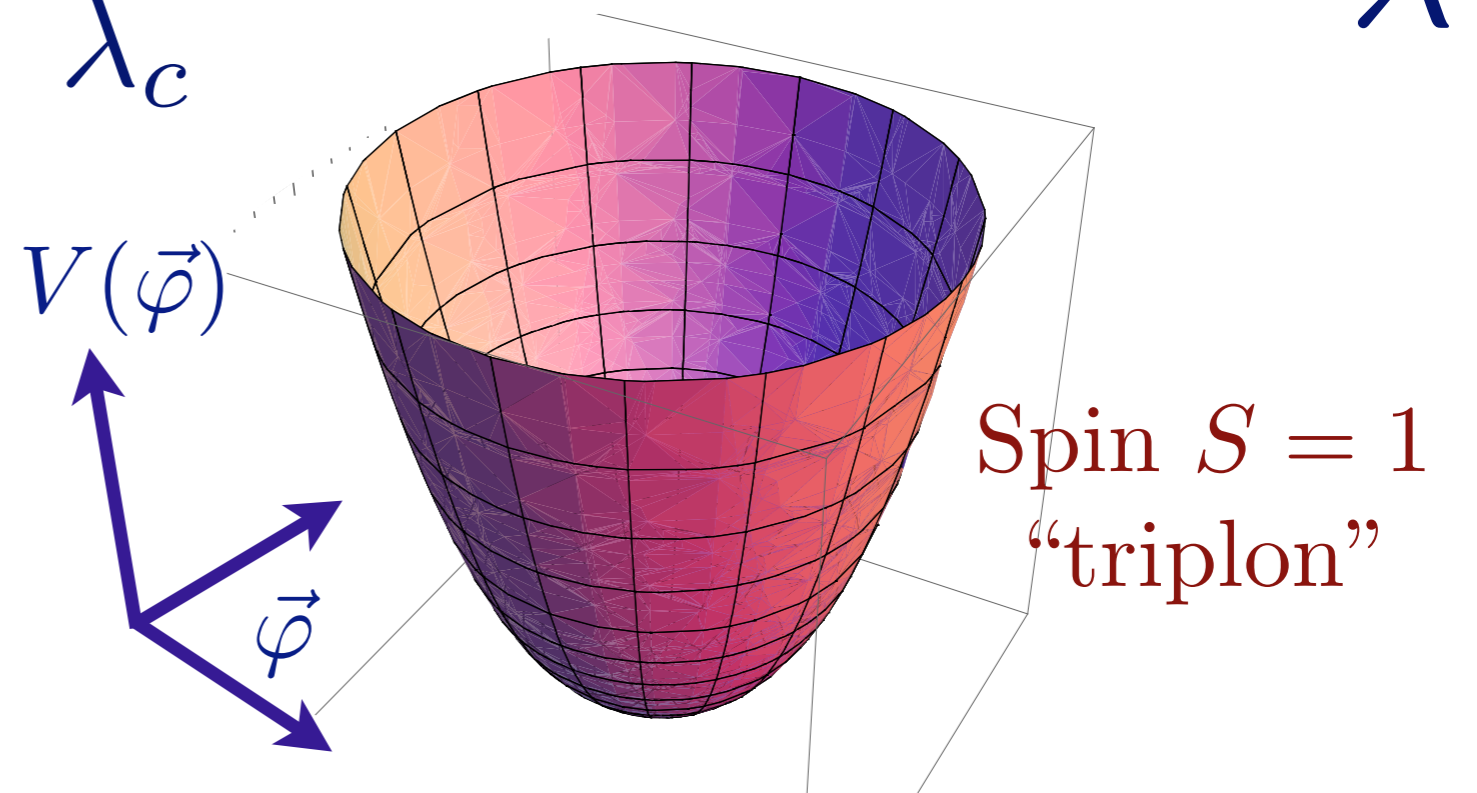


# Excitation spectrum in the paramagnetic phase

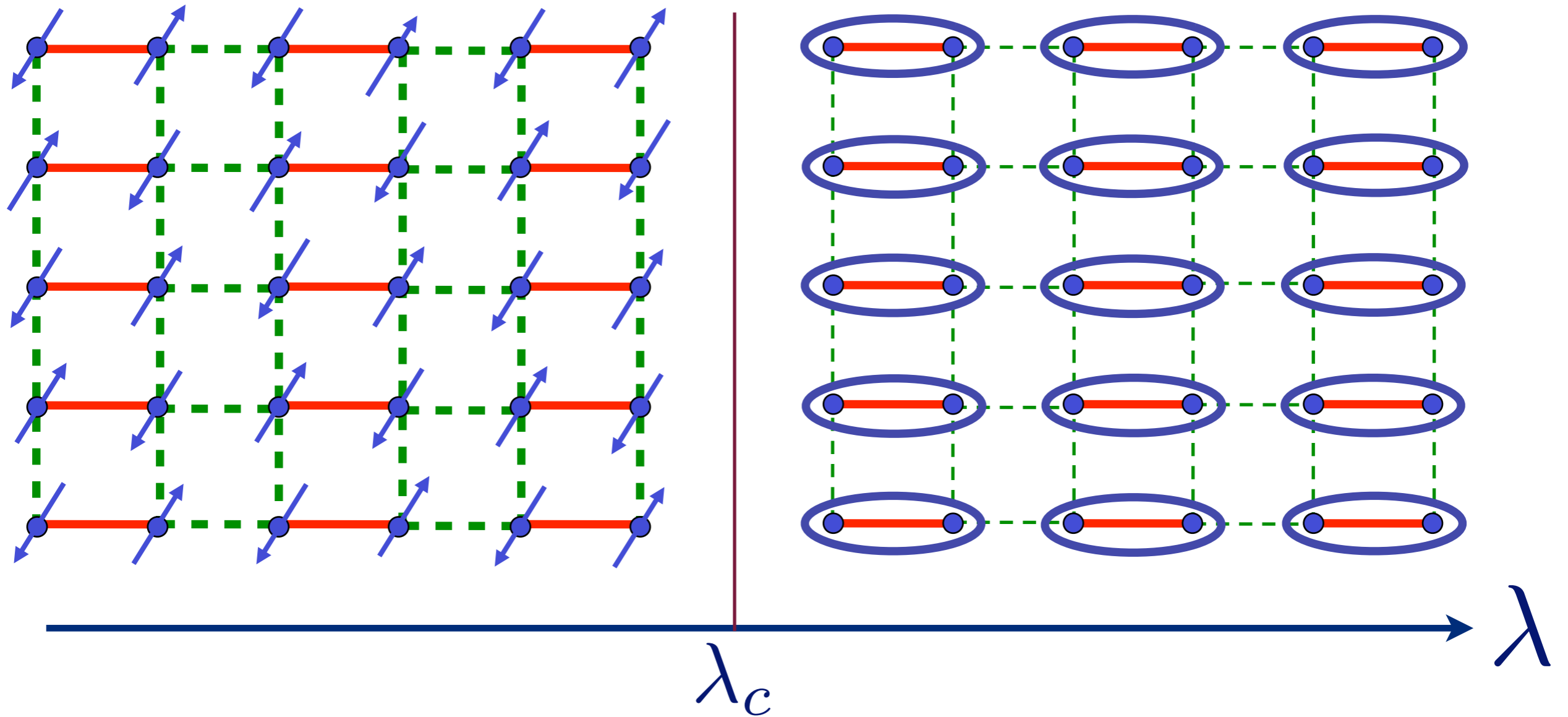


$$V(\vec{\varphi}) = (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2$$

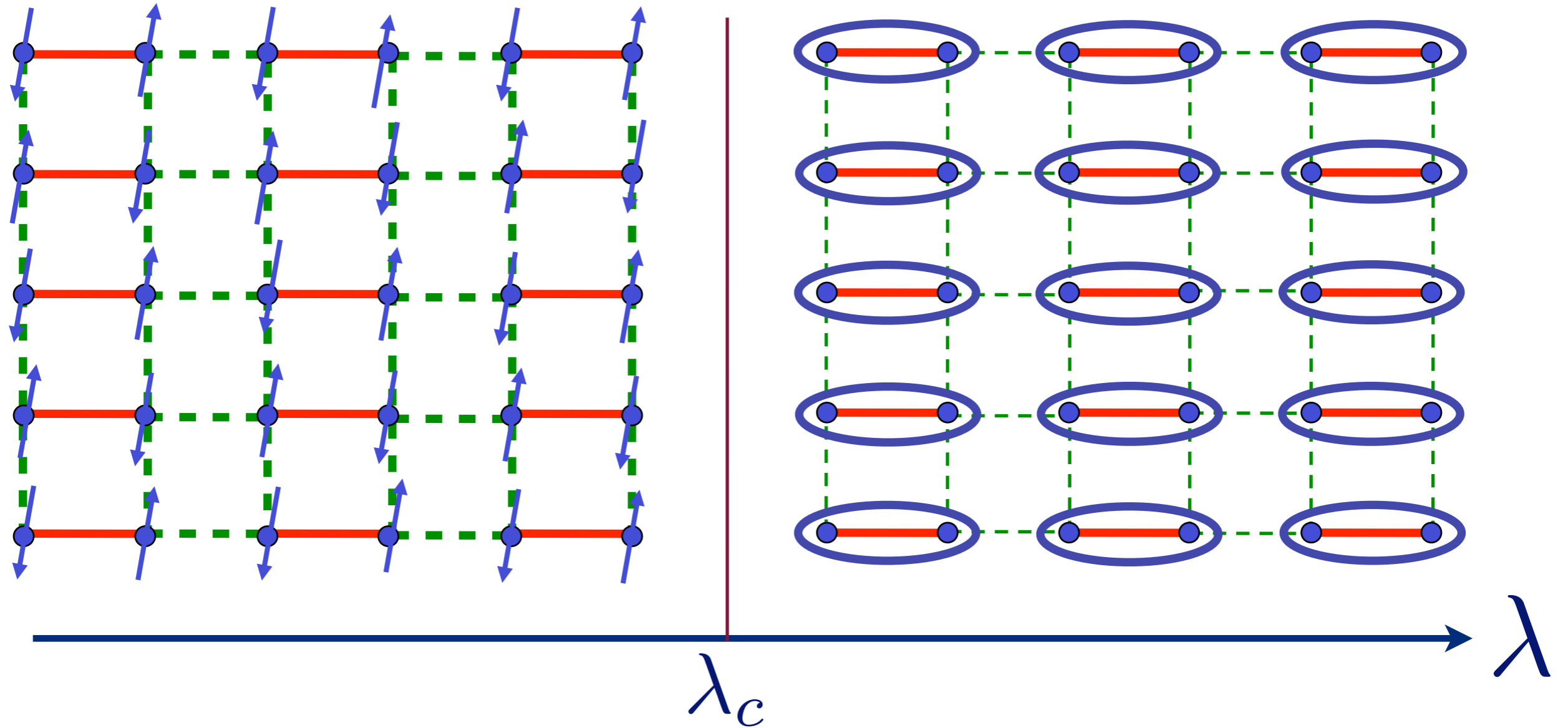
$$\lambda > \lambda_c$$



# Excitation spectrum in the Néel phase

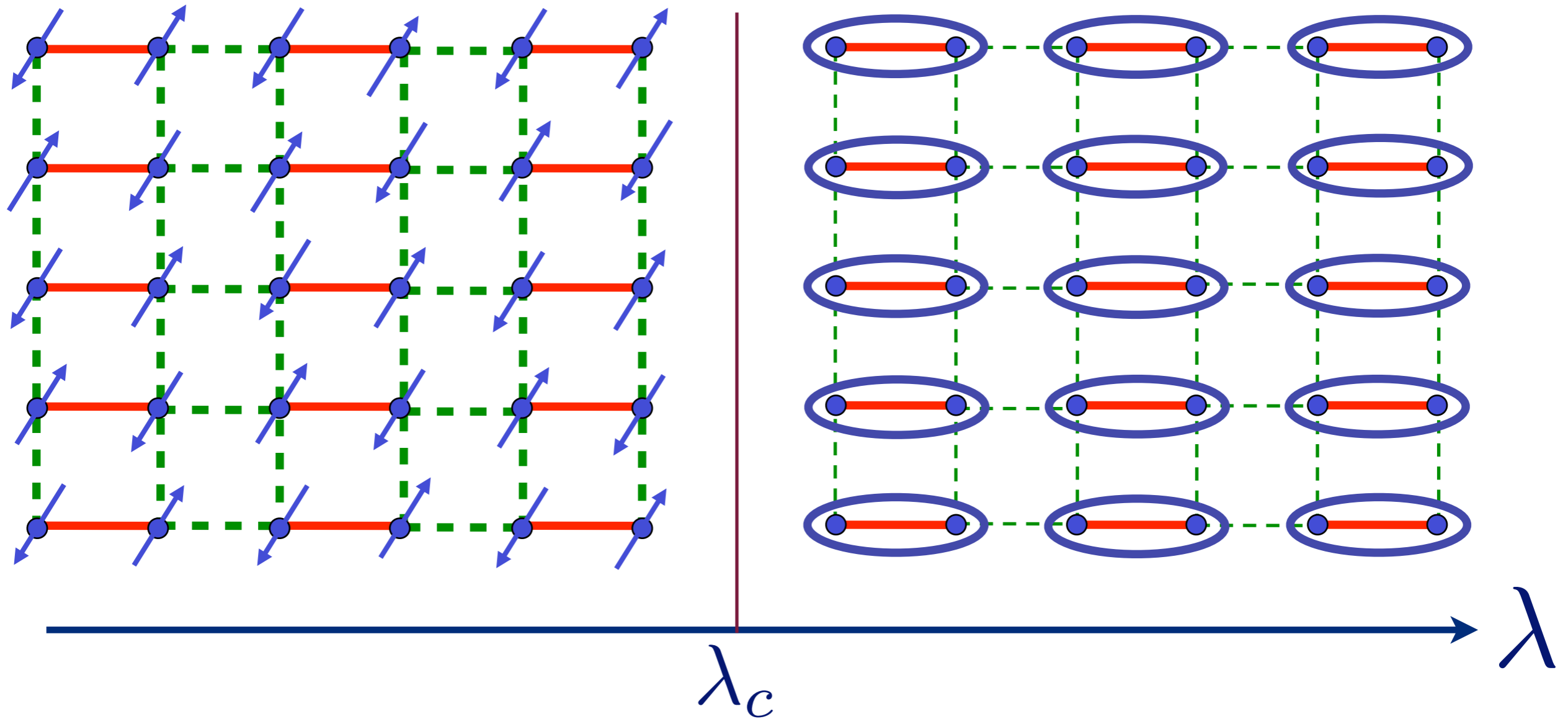


# Excitation spectrum in the Néel phase



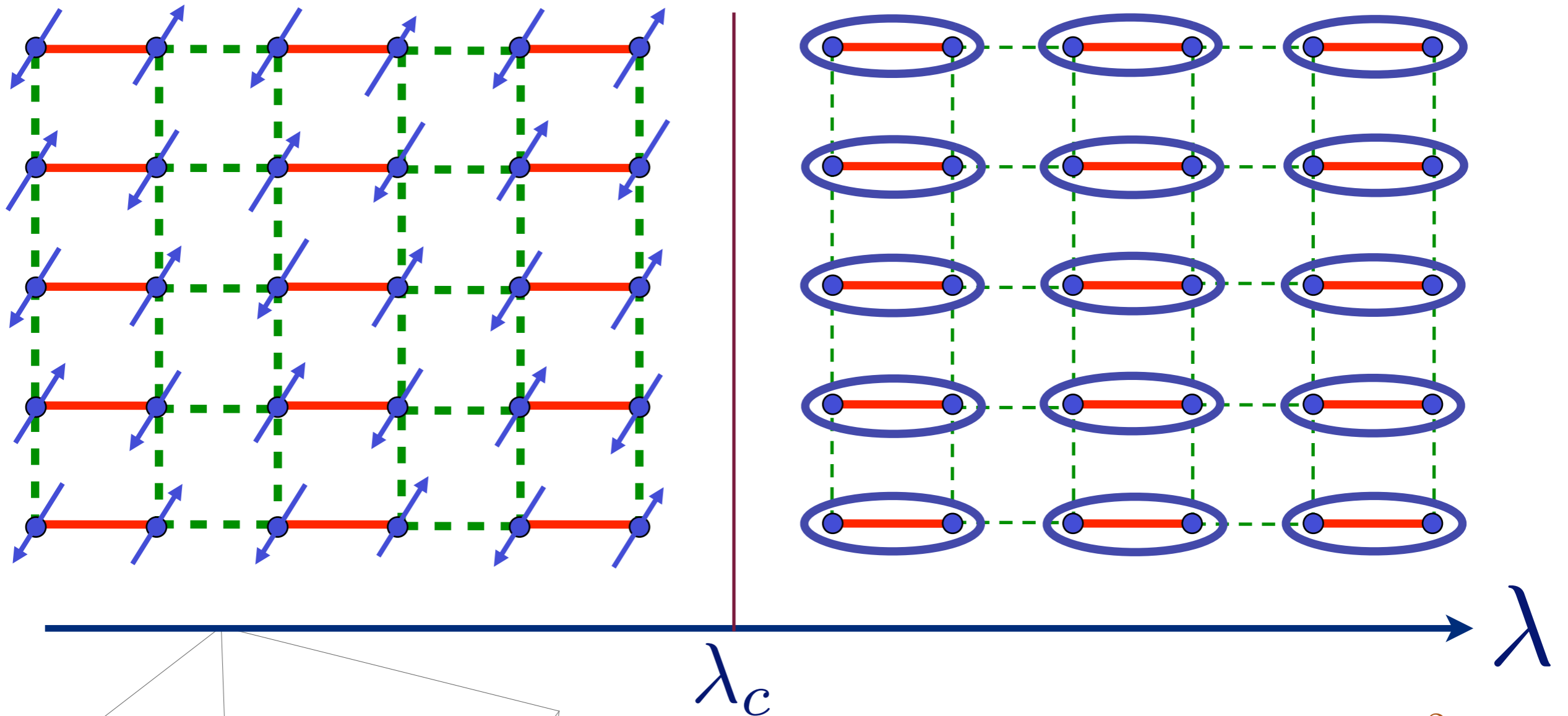
Spin waves

# Excitation spectrum in the Néel phase



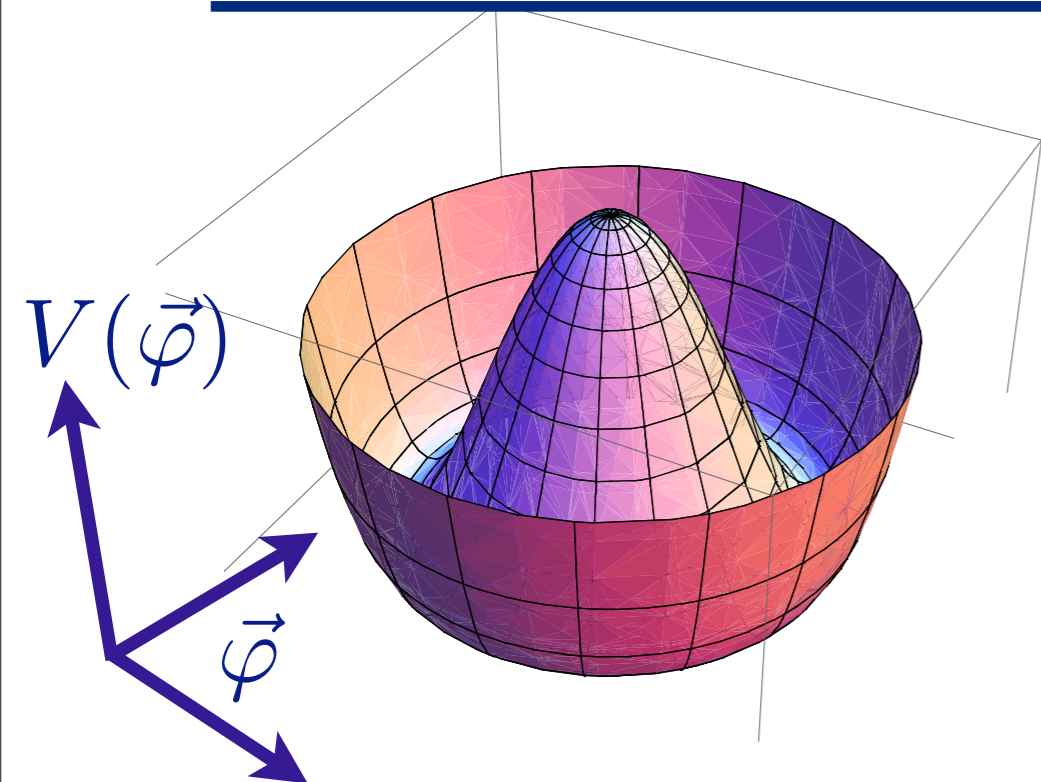
Spin waves

# Excitation spectrum in the Néel phase

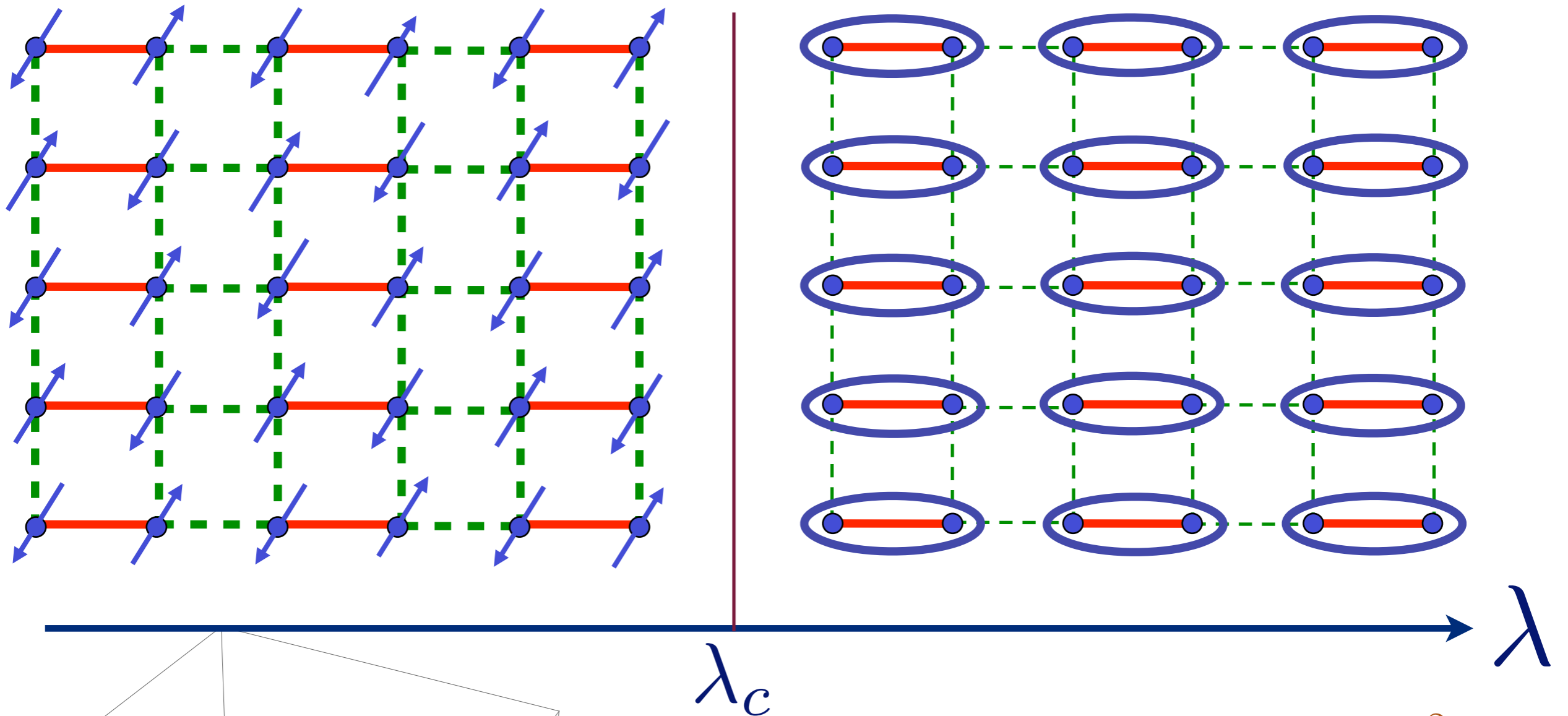


$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u(\vec{\varphi}^2)^2$$

$$\lambda < \lambda_c$$

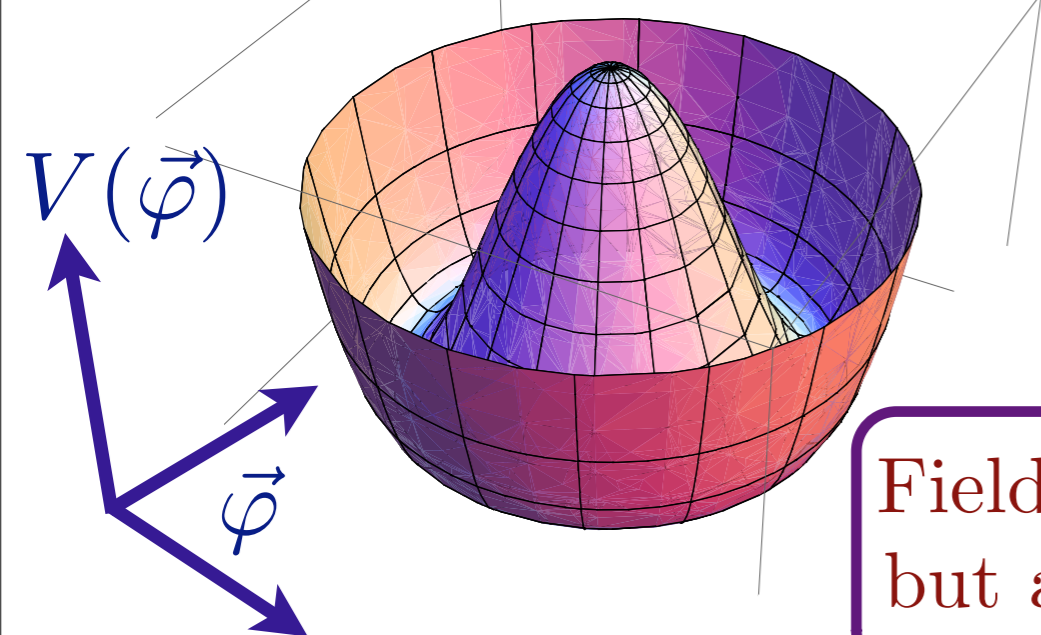


# Excitation spectrum in the Néel phase



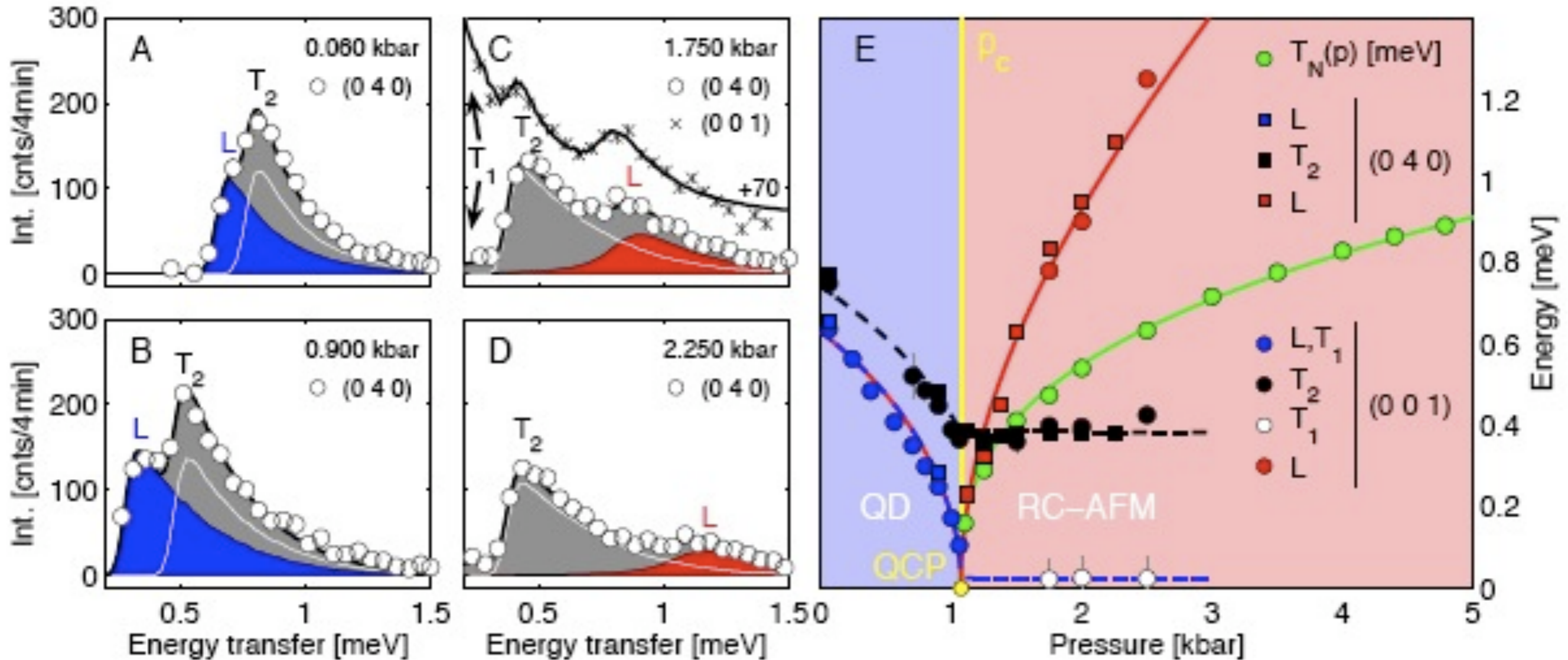
$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u(\vec{\varphi}^2)^2$$

$$\lambda < \lambda_c$$



Field theory yields spin waves (“Goldstone” modes) but also an additional longitudinal “Higgs” particle

# TiCuCl<sub>3</sub> with varying pressure



Observation of  $3 \rightarrow 2$  low energy modes,  
 emergence of new Higgs particle in the Néel phase,  
 and vanishing of Néel temperature at the quantum critical point

Christian Rüegg, Bruce Normand, Masahige Matsumoto, Albert Furrer,  
 Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya,  
 Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

# Prediction of quantum field theory

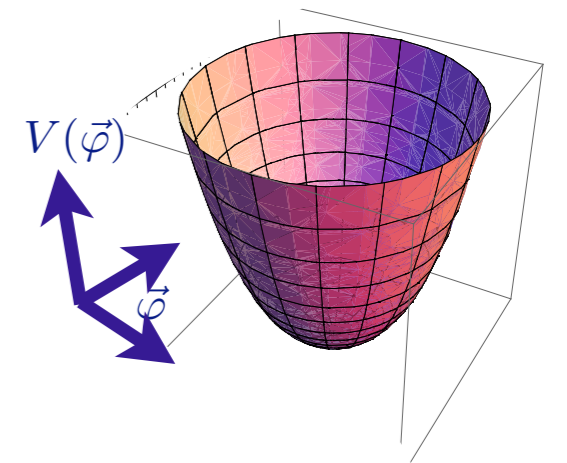
Potential for  $\vec{\varphi}$  fluctuations:  $V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u(\vec{\varphi}^2)^2$

Paramagnetic phase,  $\lambda > \lambda_c$

Expand about  $\vec{\varphi} = 0$ :

$$V(\vec{\varphi}) \approx (\lambda - \lambda_c)\vec{\varphi}^2$$

Yields 3 particles with energy gap  $\sim \sqrt{(\lambda - \lambda_c)}$



# Prediction of quantum field theory

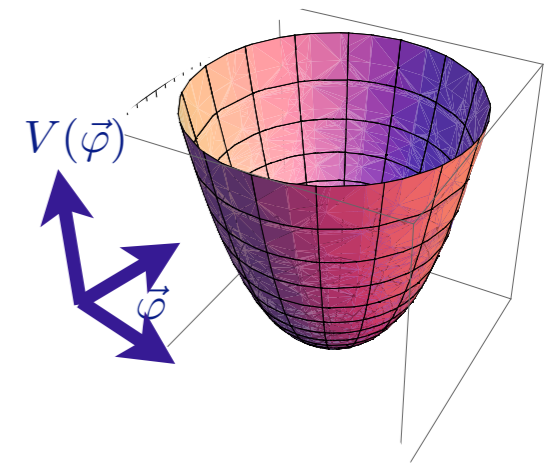
Potential for  $\vec{\varphi}$  fluctuations:  $V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u(\vec{\varphi}^2)^2$

Paramagnetic phase,  $\lambda > \lambda_c$

Expand about  $\vec{\varphi} = 0$ :

$$V(\vec{\varphi}) \approx (\lambda - \lambda_c)\vec{\varphi}^2$$

Yields 3 particles with energy gap  $\sim \sqrt{(\lambda - \lambda_c)}$

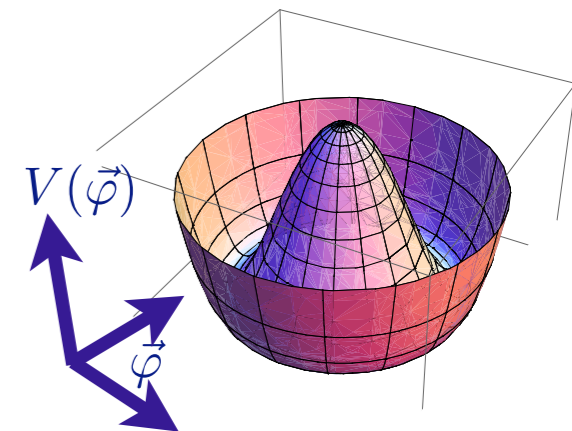


Néel phase,  $\lambda < \lambda_c$

Expand  $\vec{\varphi} = (0, 0, \sqrt{(\lambda_c - \lambda)/(2u)}) + \vec{\varphi}_1$ :

$$V(\vec{\varphi}) \approx 2(\lambda_c - \lambda)\varphi_{1z}^2$$

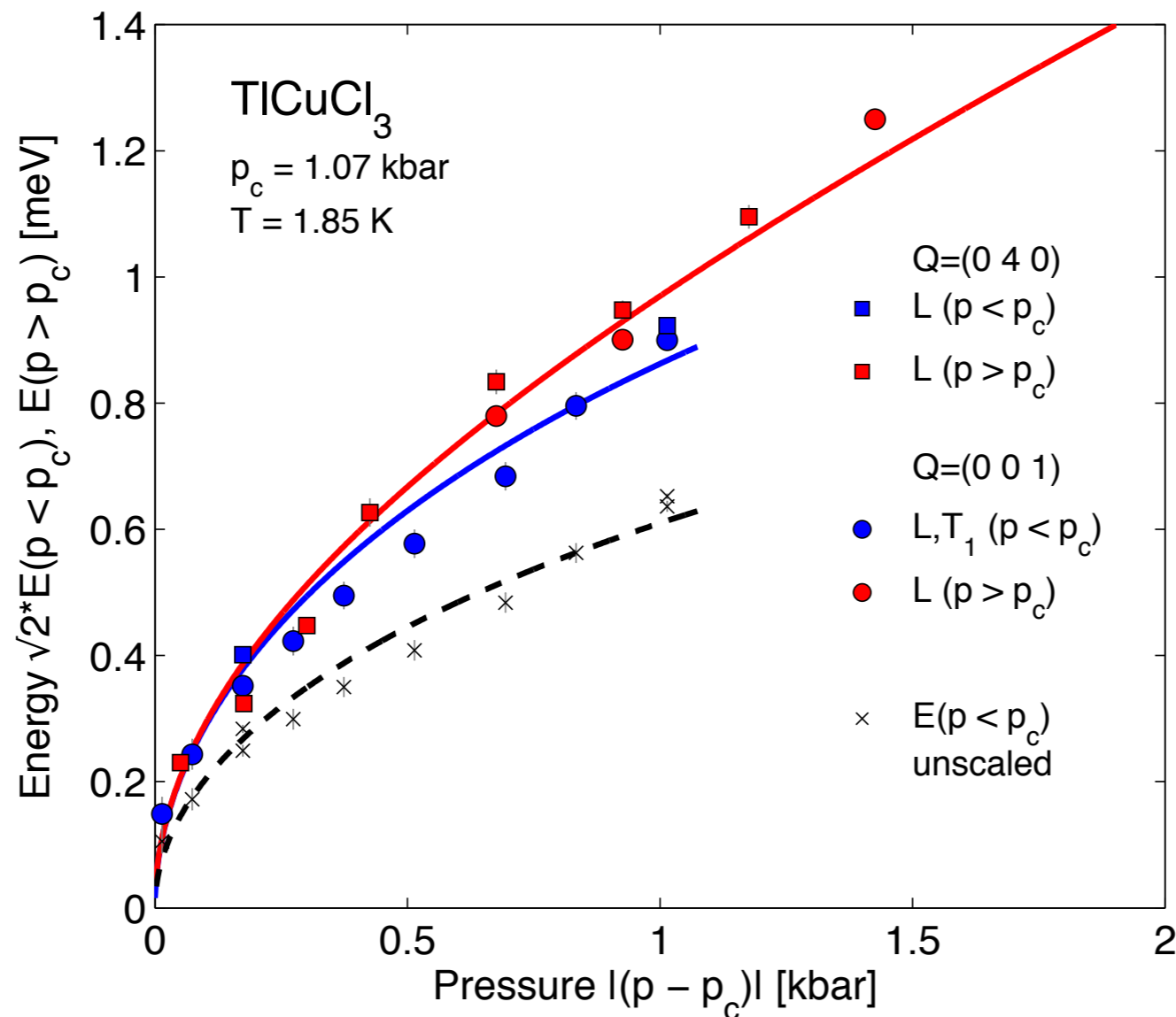
Yields 2 gapless spin waves and one Higgs particle with energy gap  $\sim \sqrt{2(\lambda_c - \lambda)}$



# Prediction of quantum field theory

$$\frac{\text{Energy of Higgs particle}}{\text{Energy of triplon}} = \sqrt{2}$$

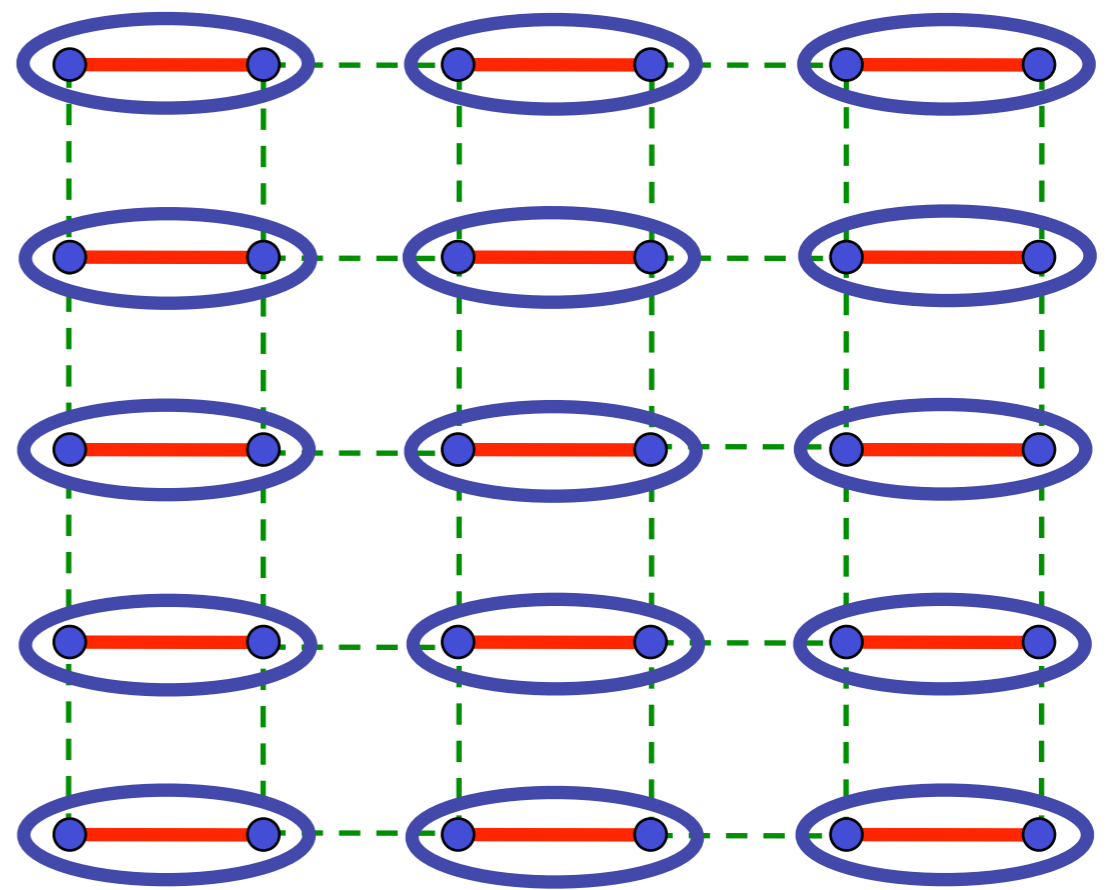
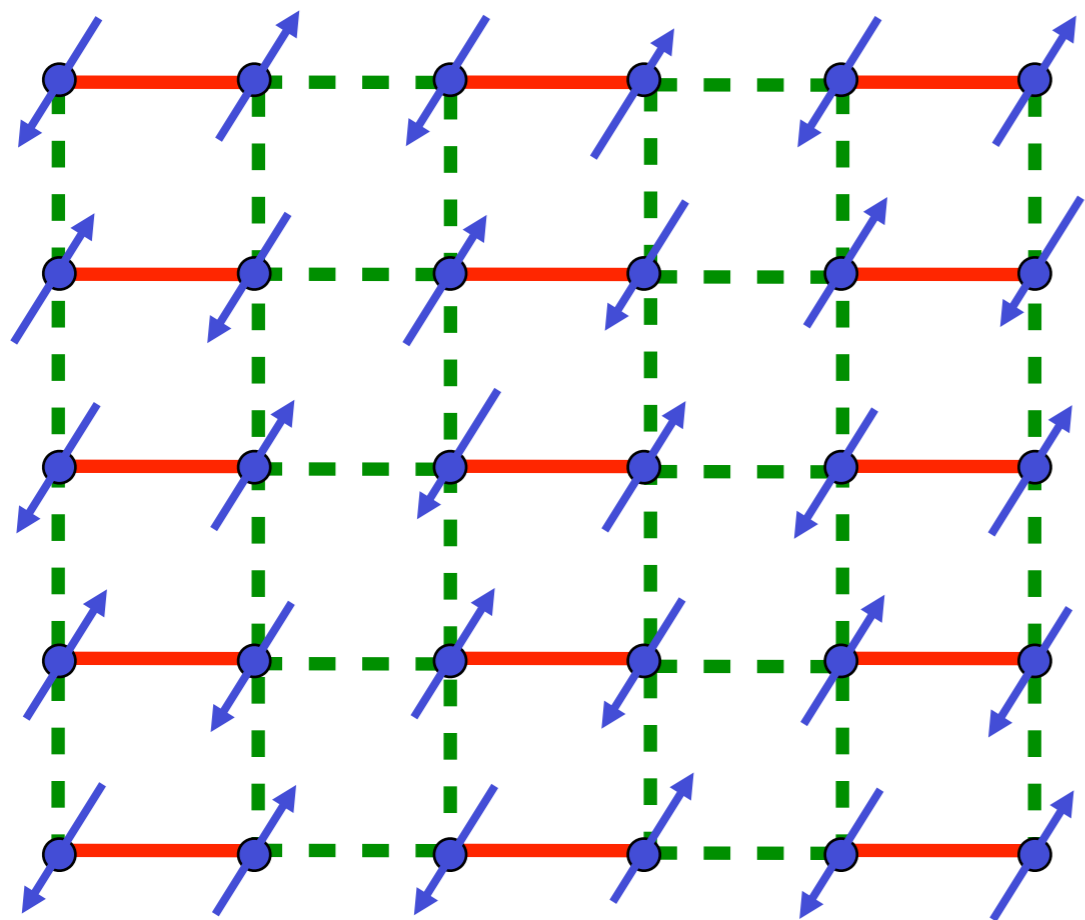
$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u(\vec{\varphi}^2)^2$$



S. Sachdev, arXiv:0901.4103



$$= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



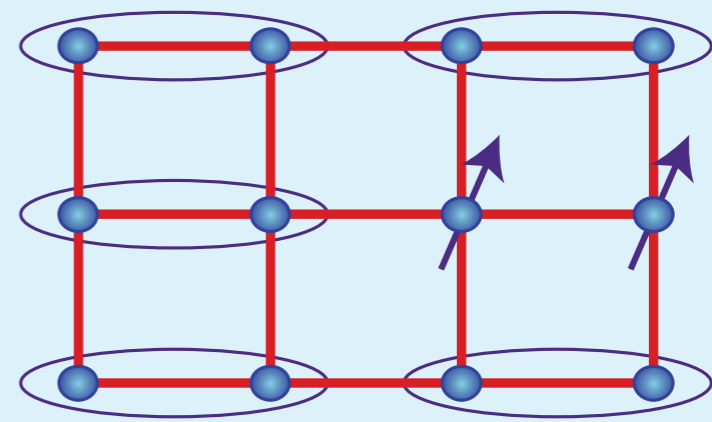
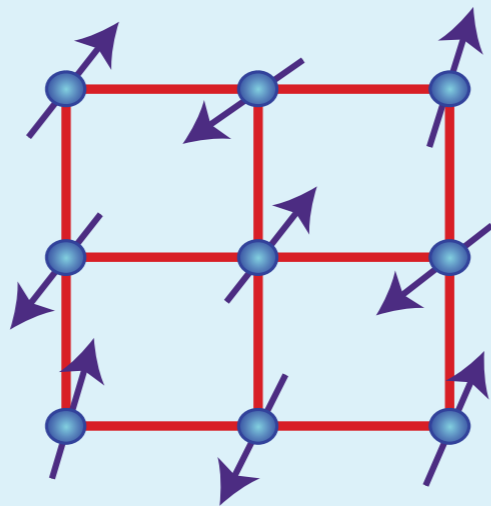
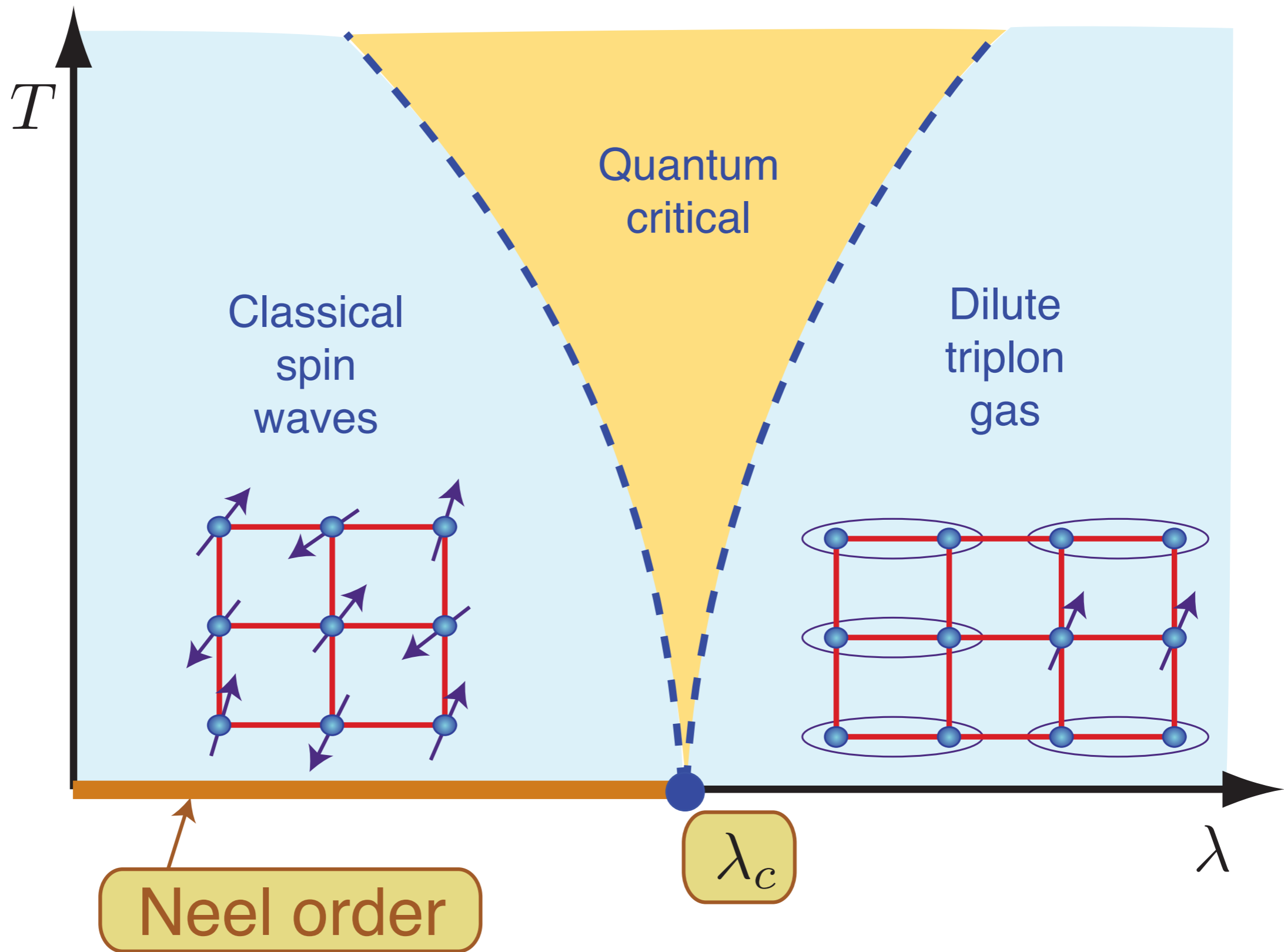
$\lambda_c$

$\lambda$

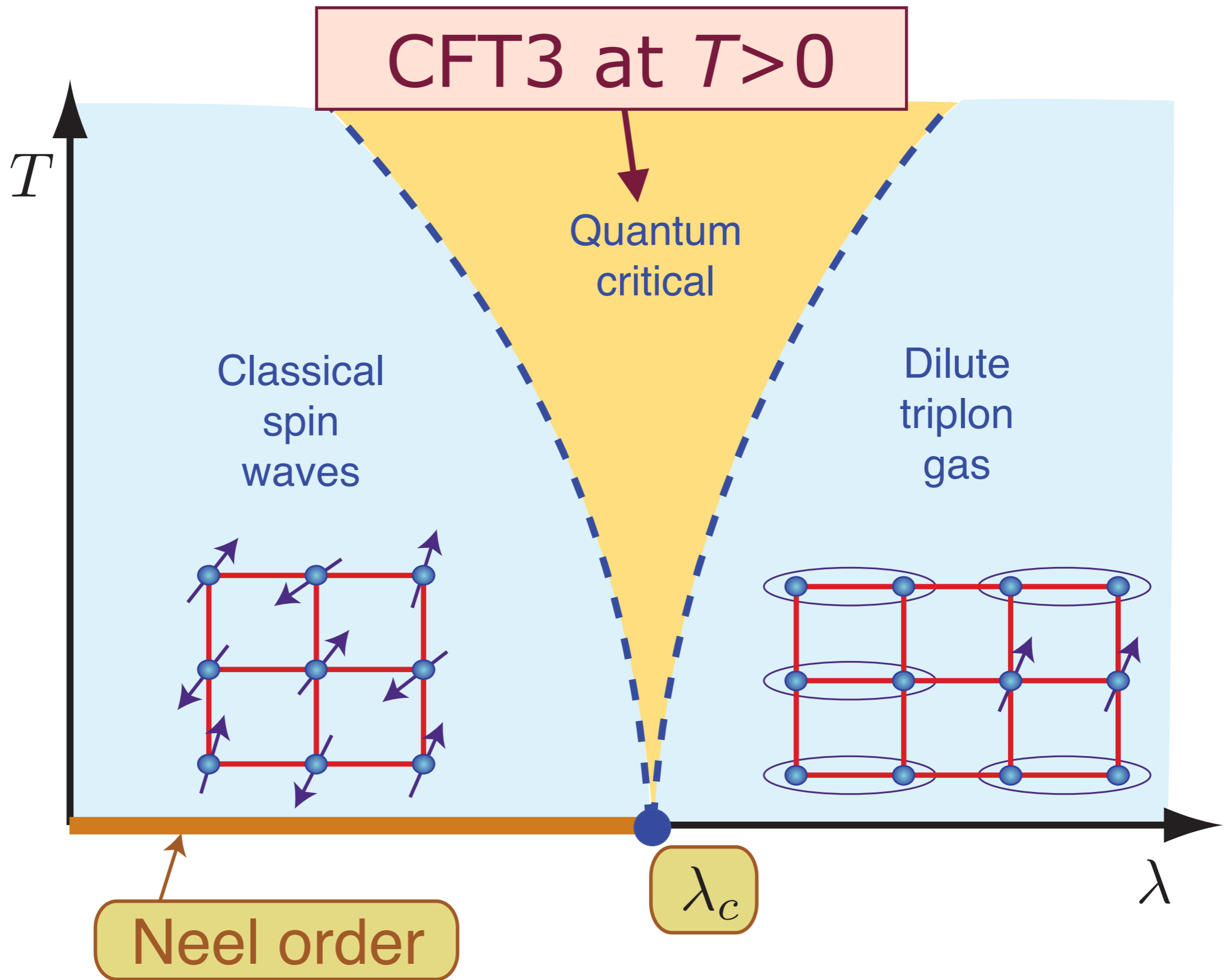
CFT3

$O(3)$  order parameter  $\vec{\varphi}$

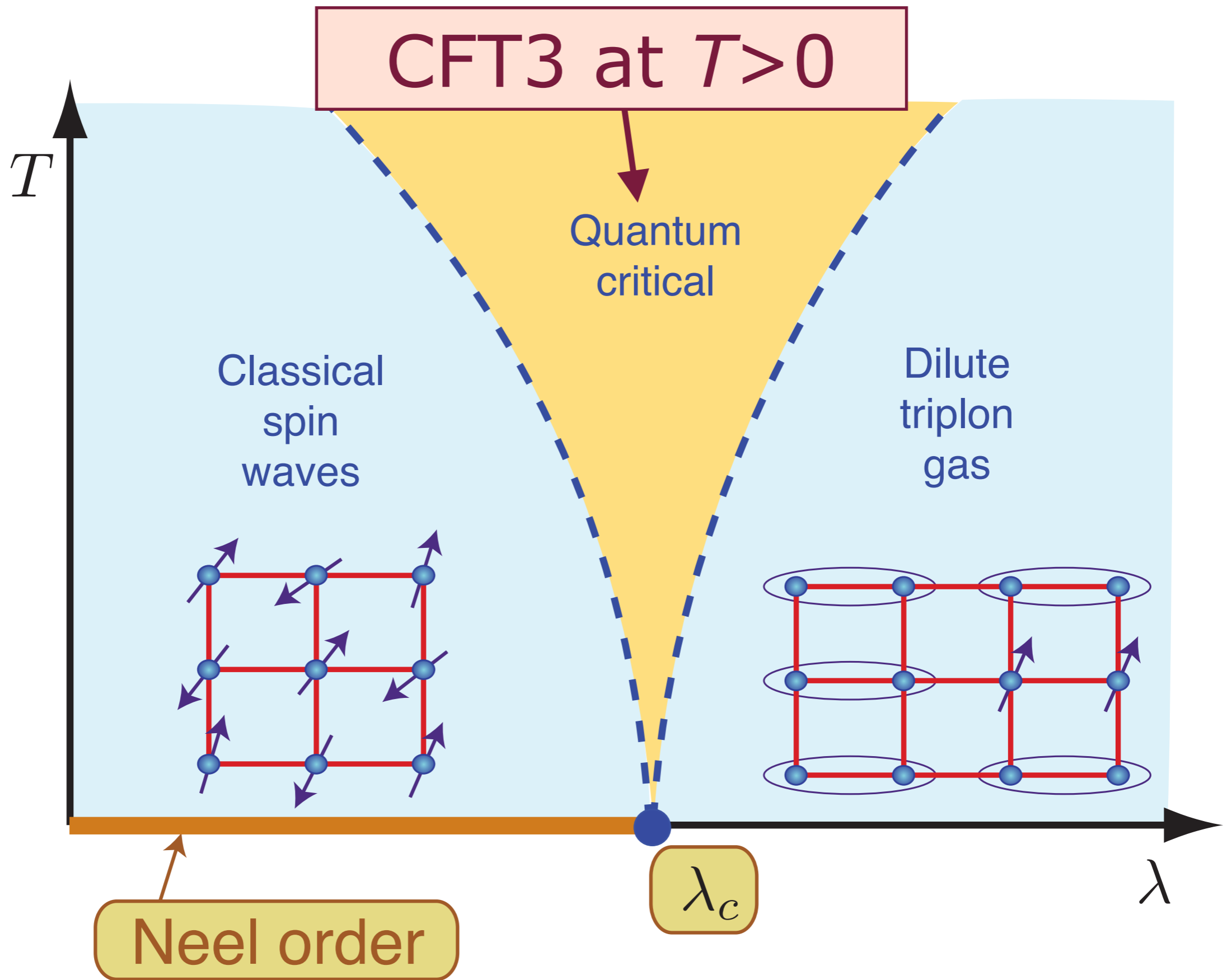
$$\mathcal{S} = \int d^2 r d\tau \left[ (\partial_\tau \vec{\varphi})^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2 \right]$$



S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).



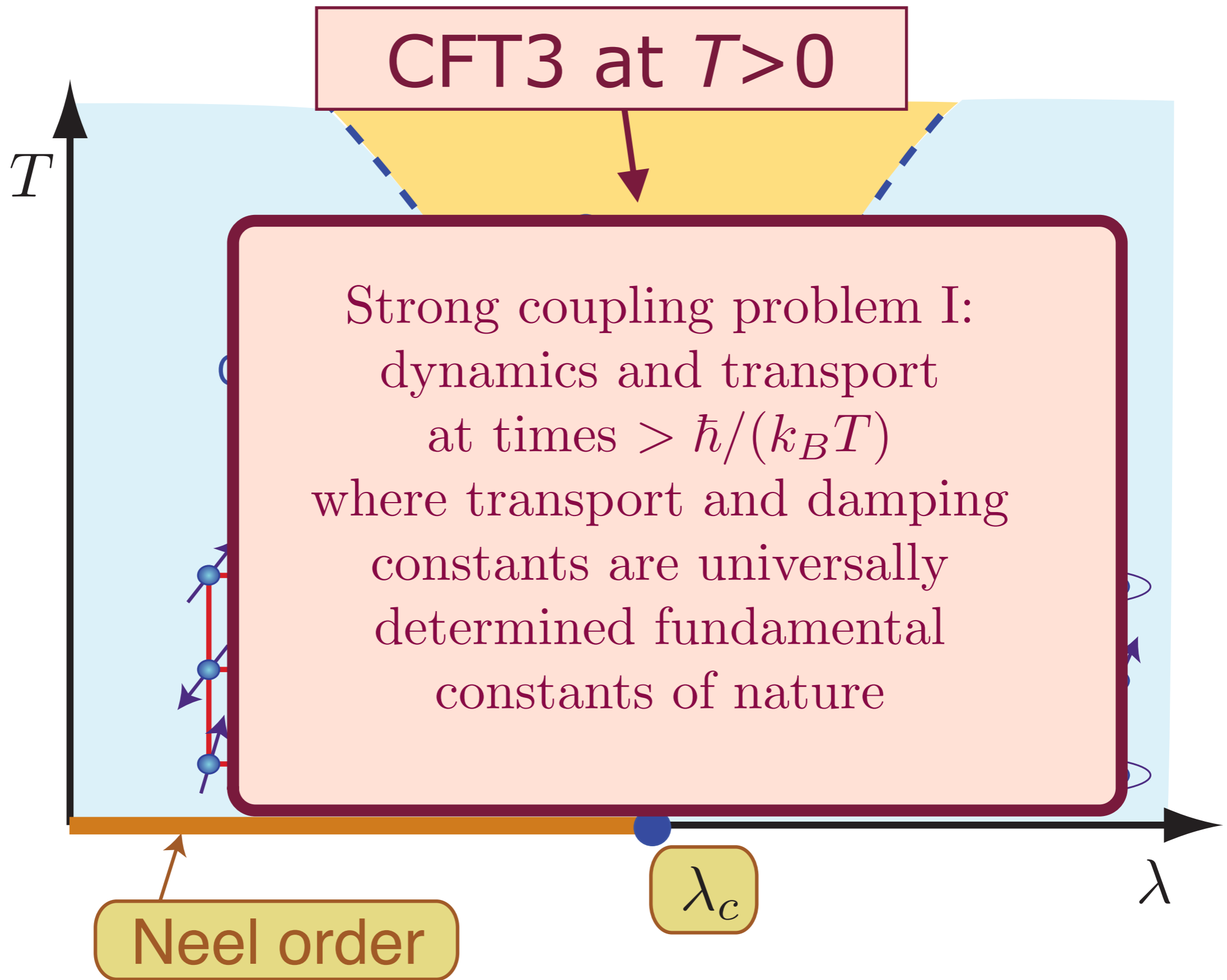
S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).



Neel order

Pressure in  $\text{TiCuCl}_3$

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).



S. Sachdev and  
 J. Ye, *Phys. Rev. Lett.*  
**69**, 2411 (1992).

# Outline

1. Coupled dimer antiferromagnets  
*Introduction to quantum criticality*
2. Theory of Ising-nematic ordering in the cuprate metals  
*Strongly-coupled field theory*
3. The AdS/CFT correspondence  
*Phases of quantum matter at strong coupling*
4. Global phase diagram of the cuprates  
*Competition between d-wave superconductivity and spin-density order for the Fermi surface*

# Outline

## 1. Coupled dimer antiferromagnets

*Introduction to quantum criticality*

## 2. Theory of Ising-nematic ordering in the cuprate metals

*Strongly-coupled field theory*

## 3. The AdS/CFT correspondence

*Phases of quantum matter  
at strong coupling*

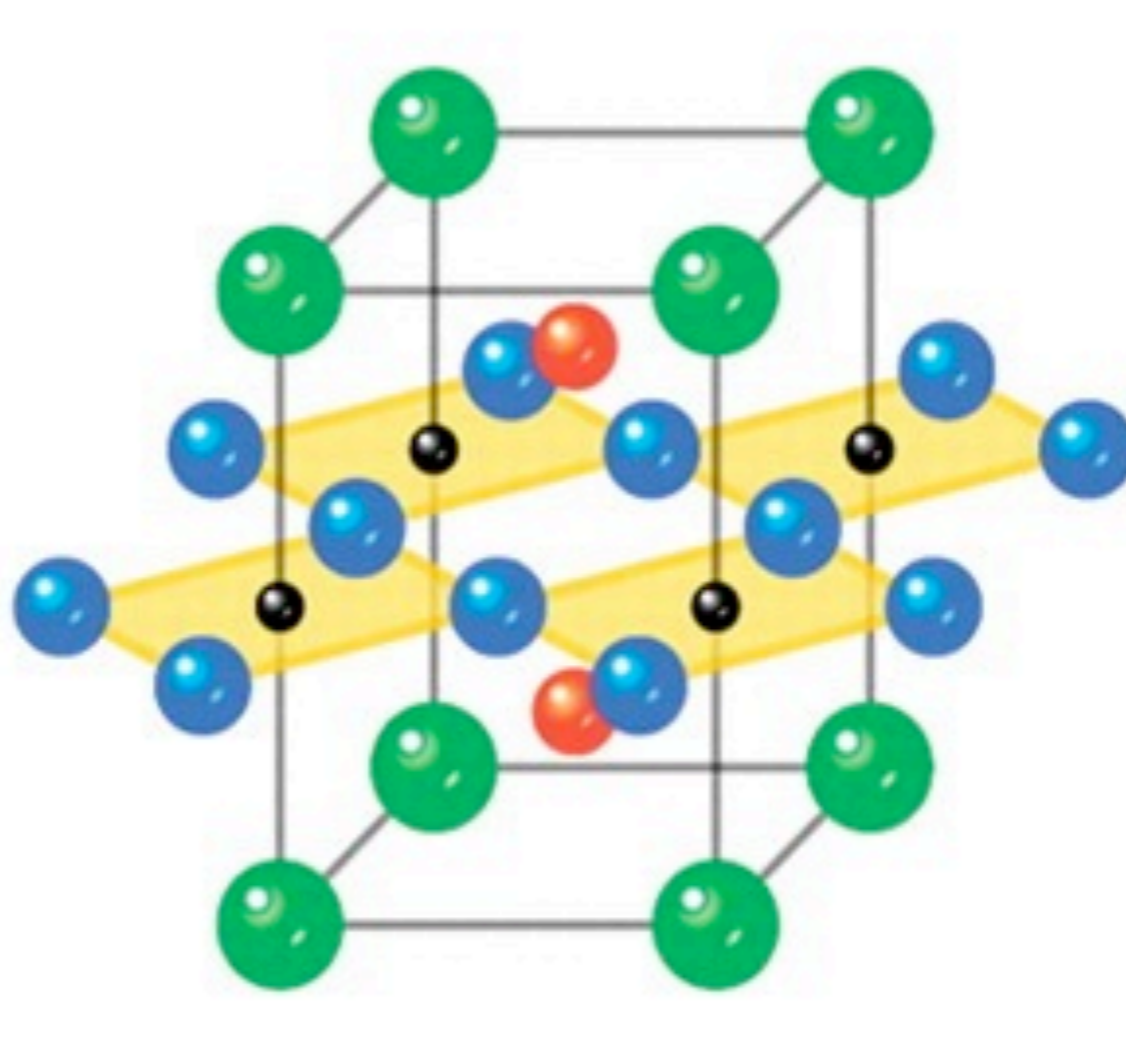
## 4. Global phase diagram of the cuprates

*Competition between d-wave superconductivity  
and spin-density order for the Fermi surface*

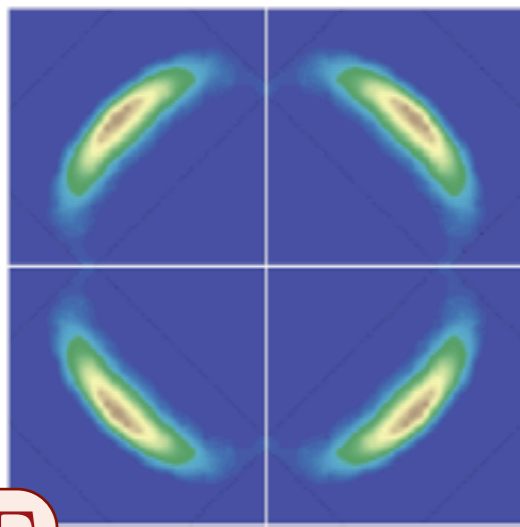
# *The cuprate superconductors*

Na-CCOC

- Cu
- Ca/Na
- O
- Cl

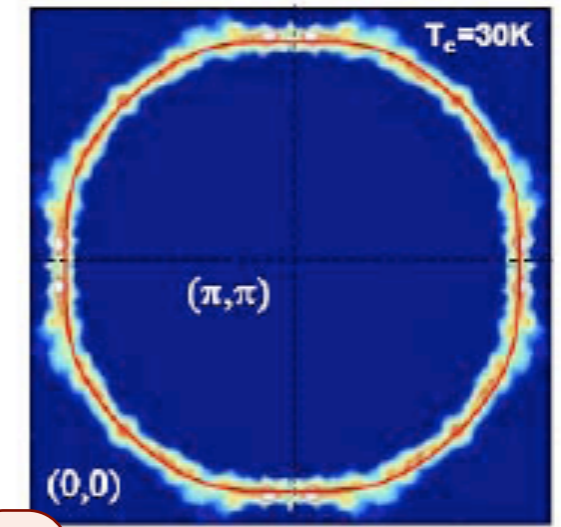
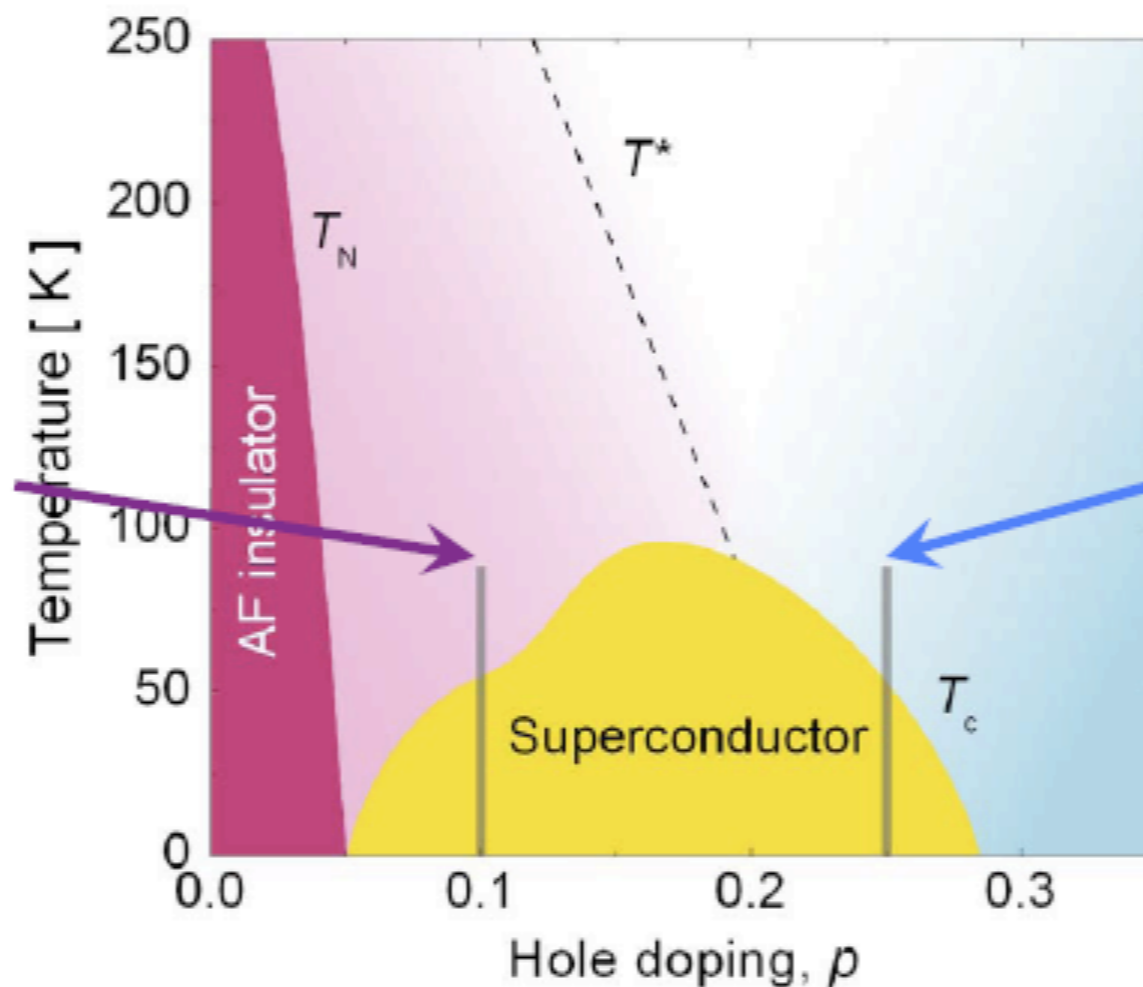


# Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



$\Gamma$

*K.M. Shen et al., Science 2005*



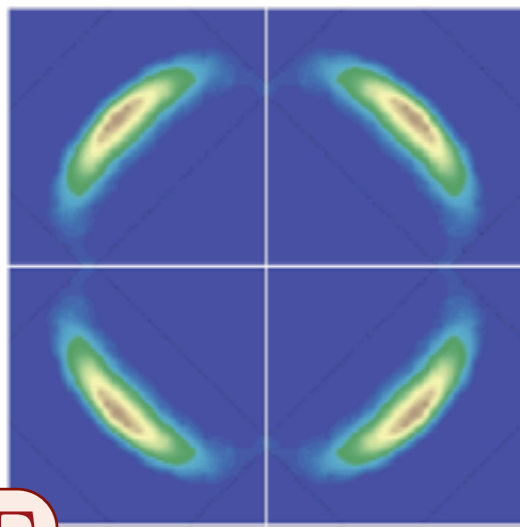
$\Gamma$

*M. Platé et al., PRL 2005*

Smaller hole  
Fermi-pockets

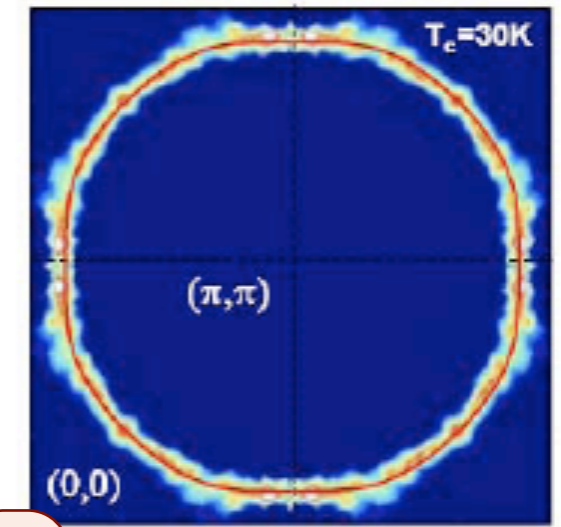
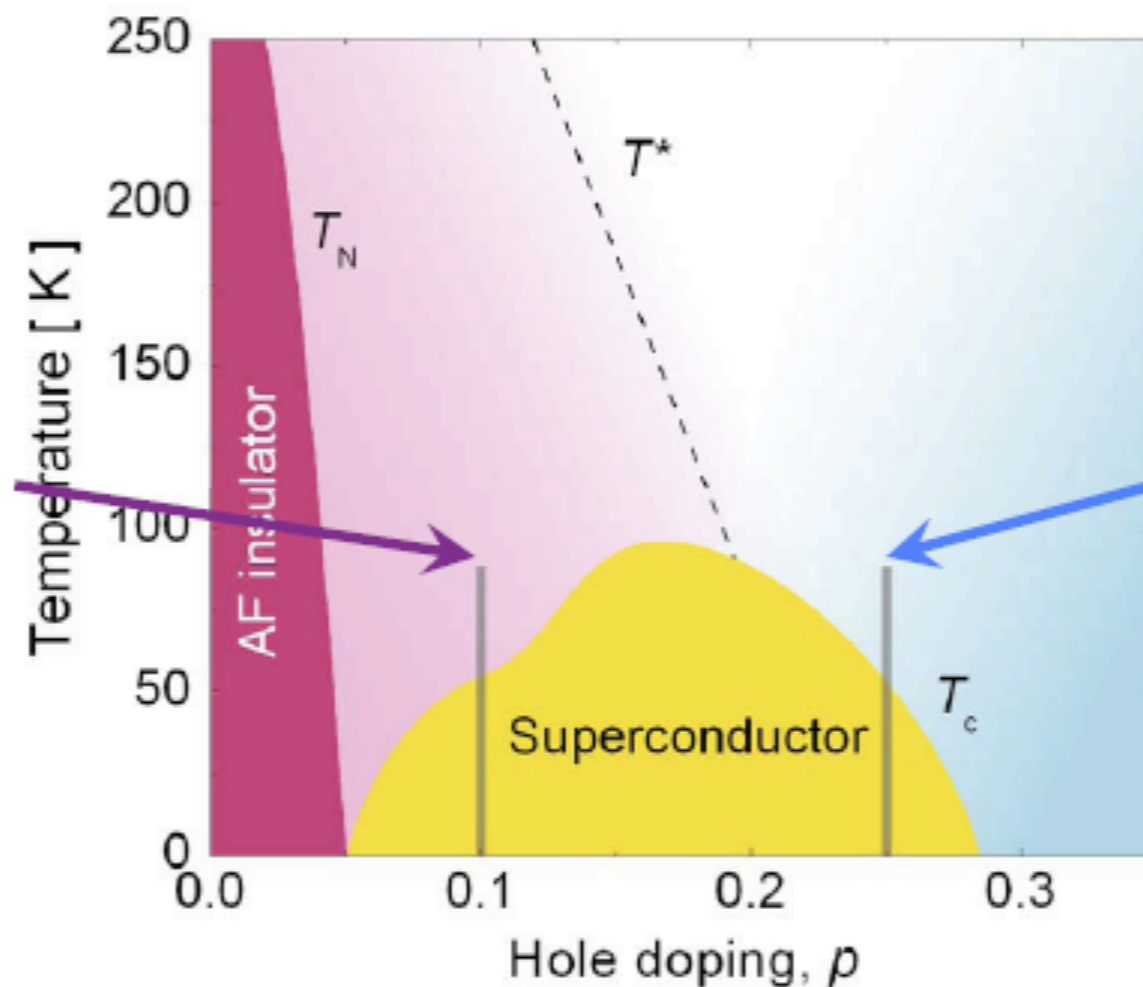
Large hole  
Fermi surface

# Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



$\Gamma$

*K.M. Shen et al., Science 2005*



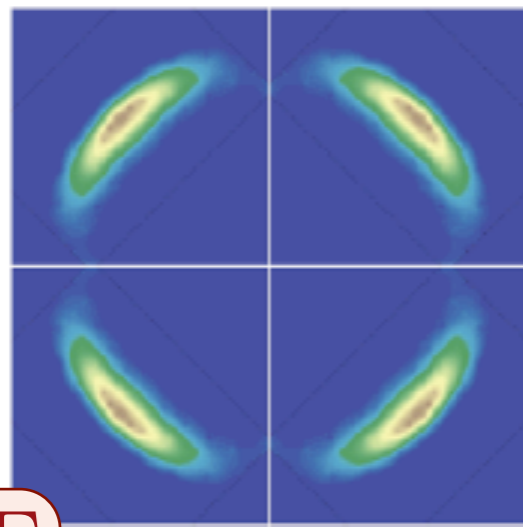
$\Gamma$

*M. Platé et al., PRL 2005*

Smaller hole  
Fermi-pockets

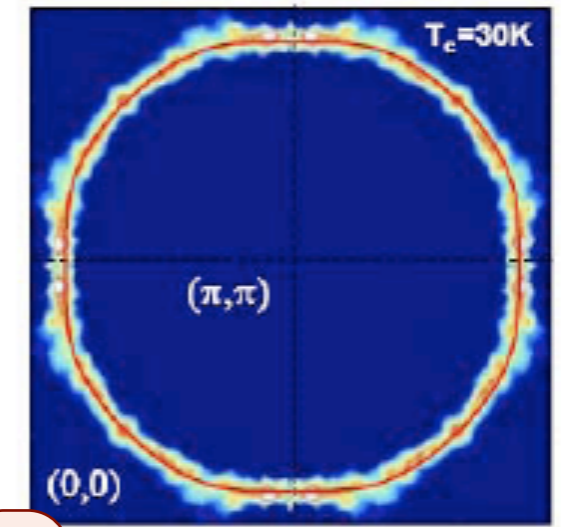
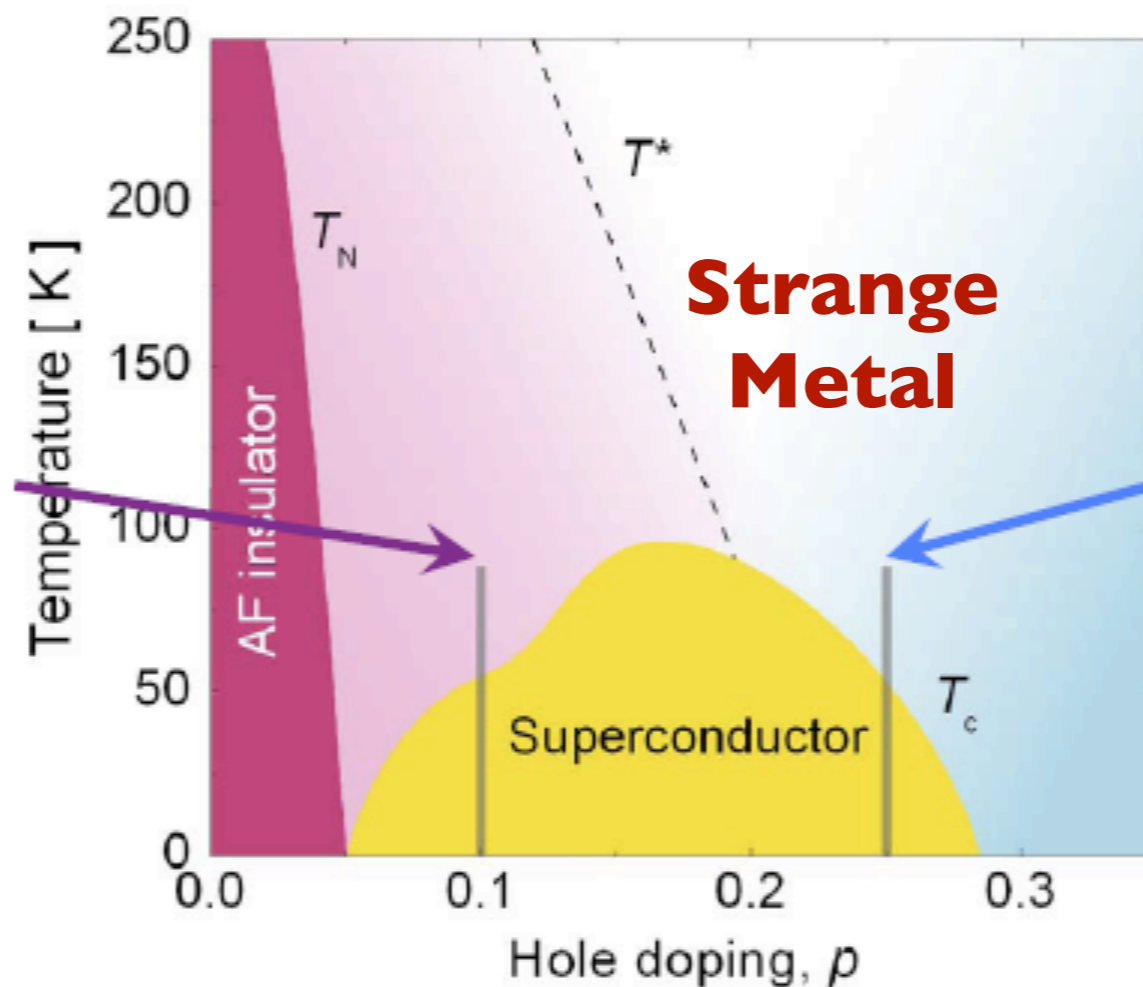
Large hole  
Fermi surface

# Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



$\Gamma$

*K.M. Shen et al., Science 2005*



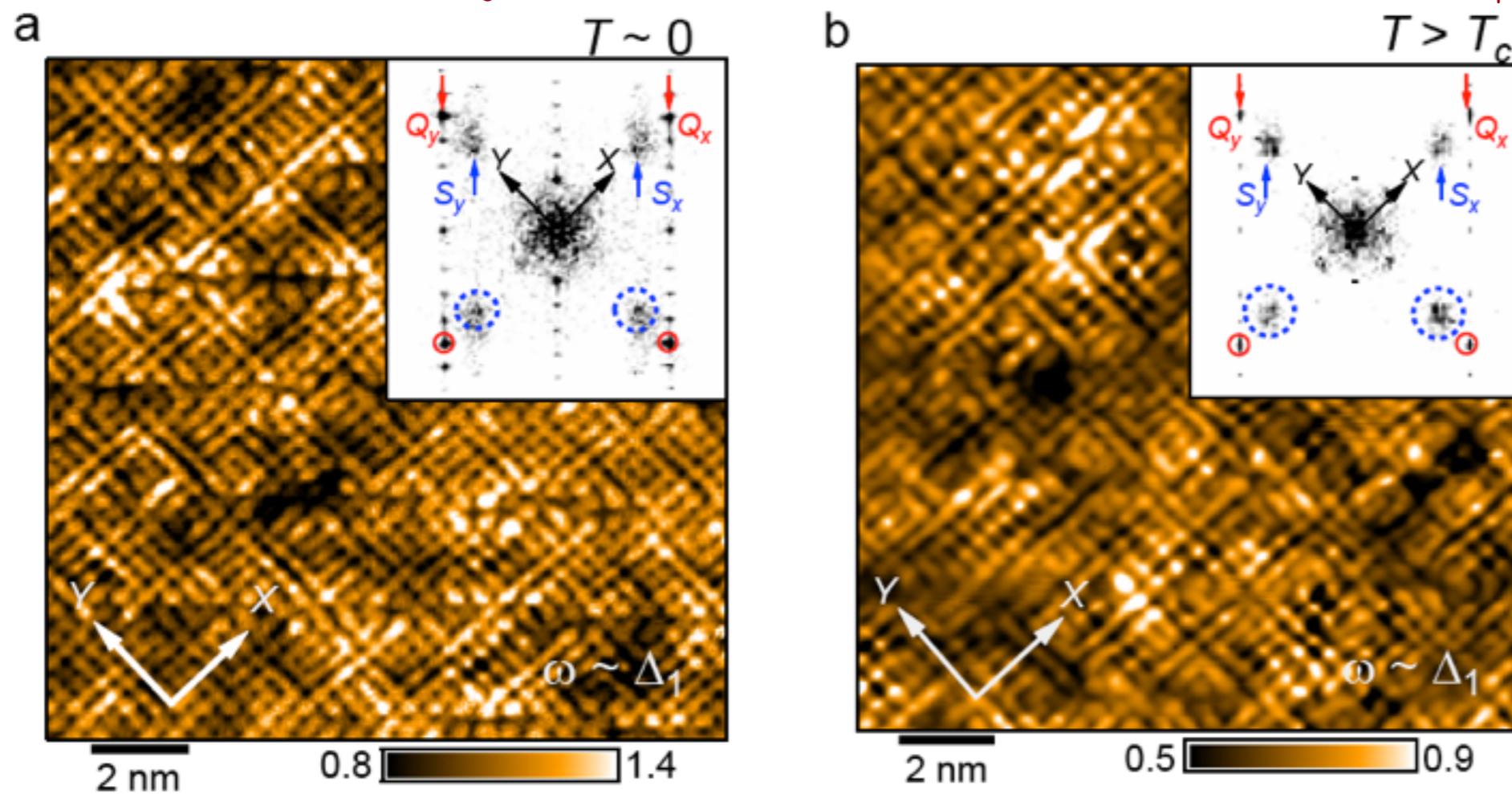
$\Gamma$

*M. Platé et al., PRL 2005*

Smaller hole  
Fermi-pockets

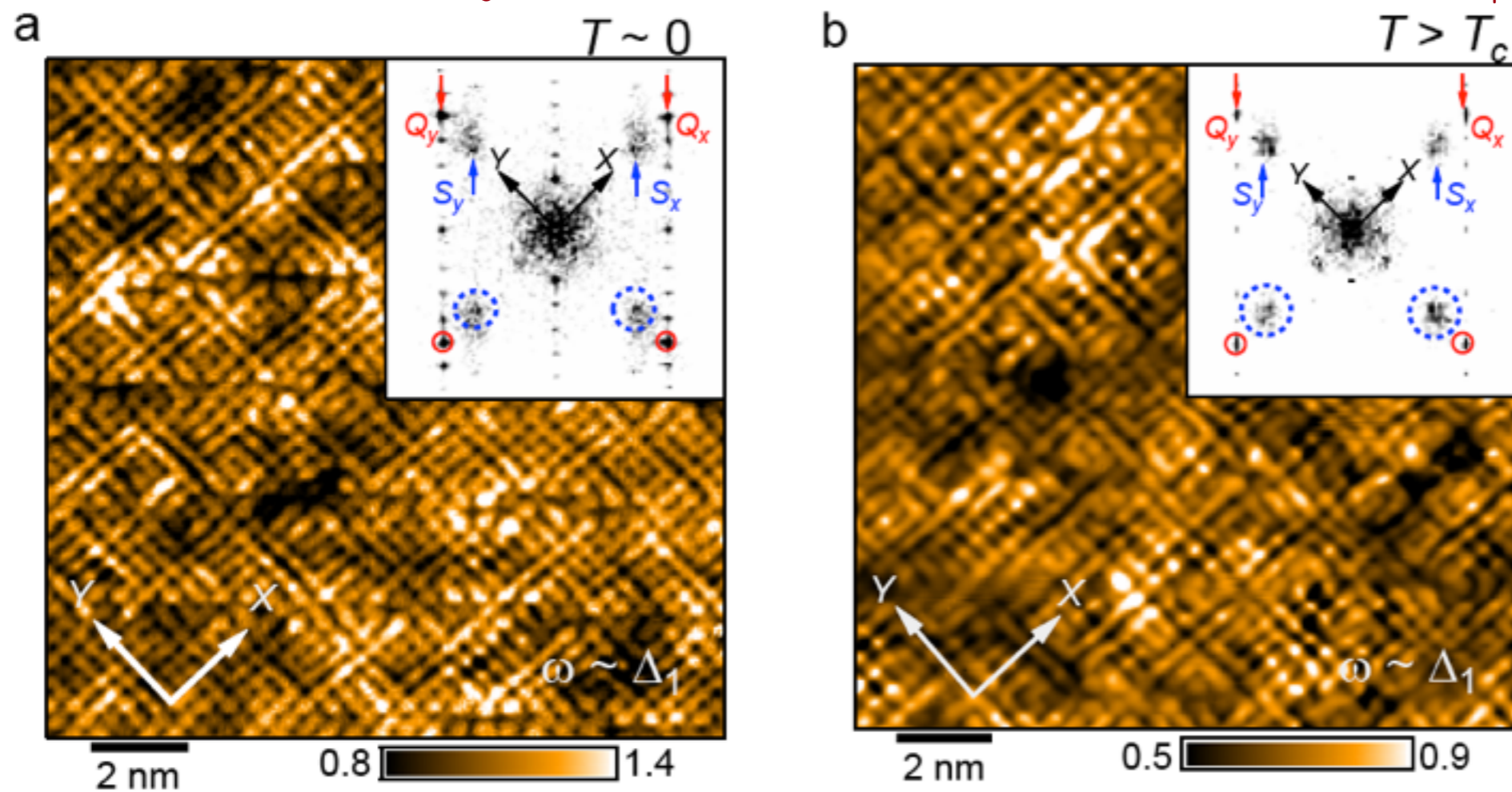
Large hole  
Fermi surface

# STM measurements of $Z(r)$ , the energy asymmetry in density of states in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ .

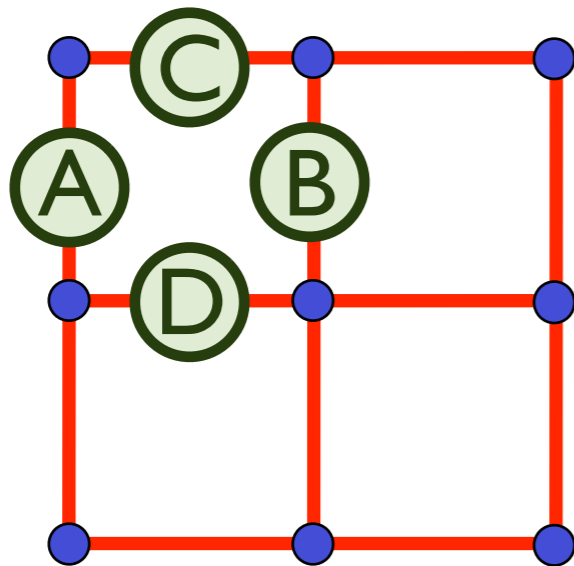


M. J. Lawler, K. Fujita,  
Jinhwan Lee,  
A. R. Schmidt,  
Y. Kohsaka, Chung Koo  
Kim, H. Eisaki,  
S. Uchida, J. C. Davis,  
J. P. Sethna, and  
Eun-Ah Kim, preprint

# STM measurements of $Z(r)$ , the energy asymmetry in density of states in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ .

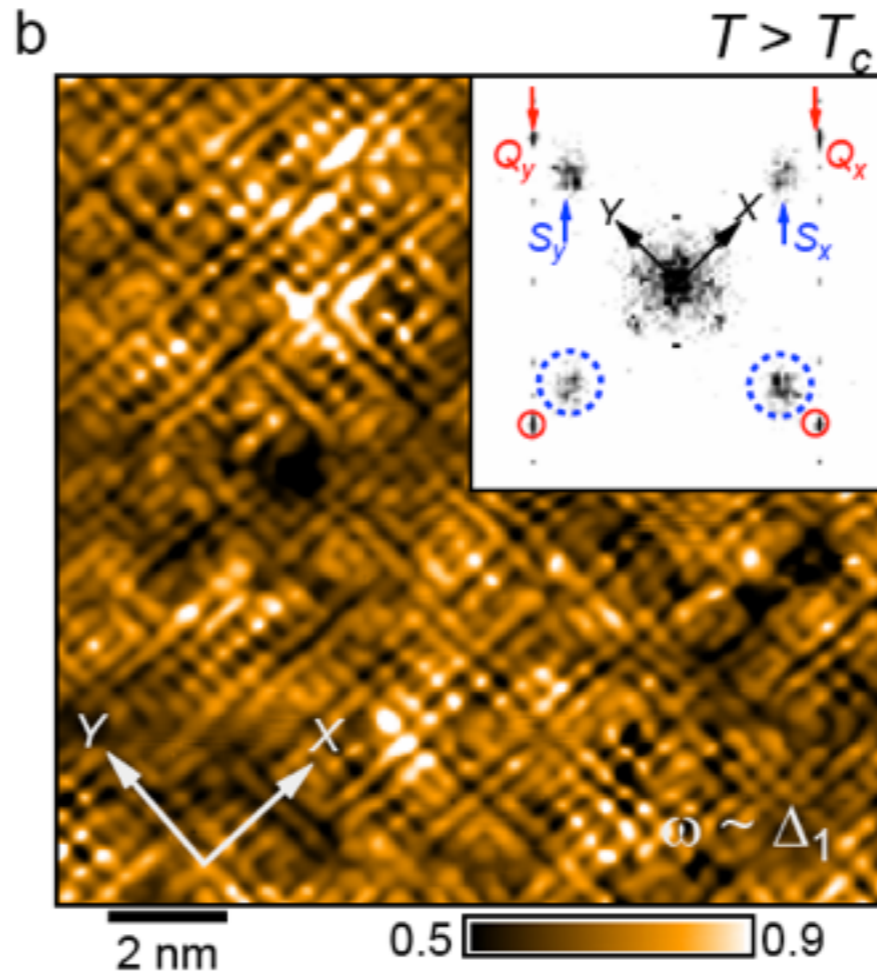
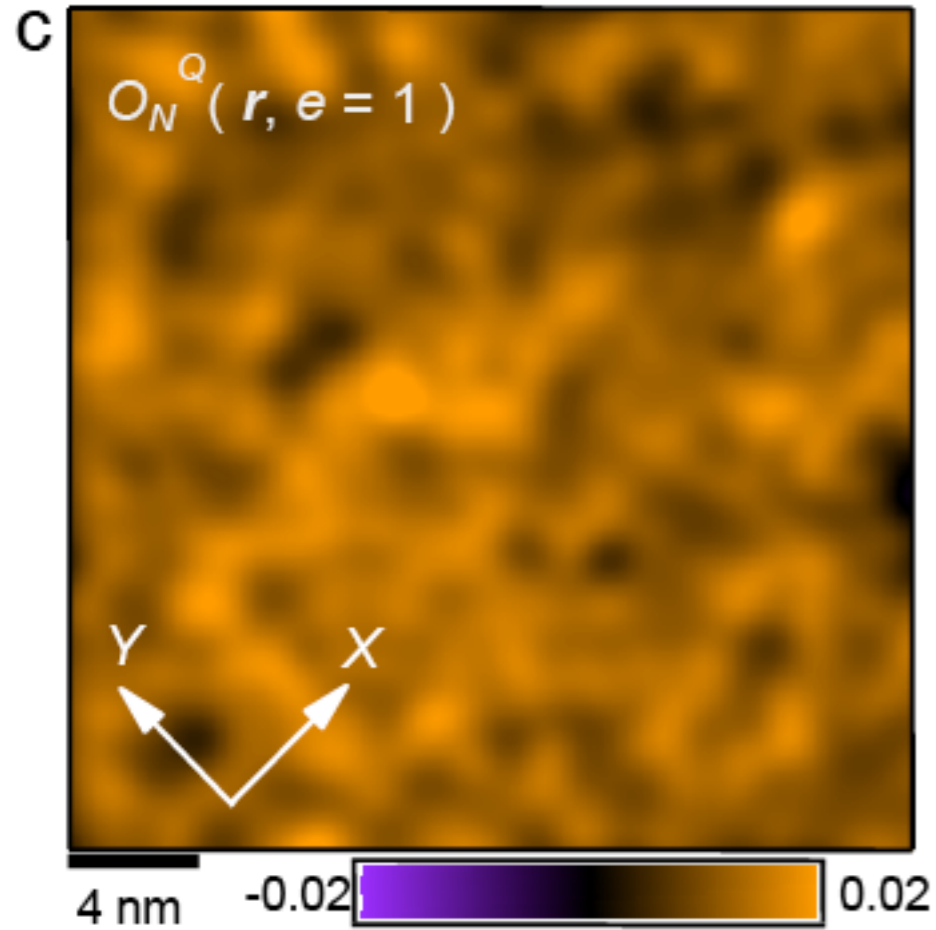


M. J. Lawler, K. Fujita,  
 Jinhwan Lee,  
 A. R. Schmidt,  
 Y. Kohsaka, Chung Koo  
 Kim, H. Eisaki,  
 S. Uchida, J. C. Davis,  
 J. P. Sethna, and  
 Eun-Ah Kim, preprint

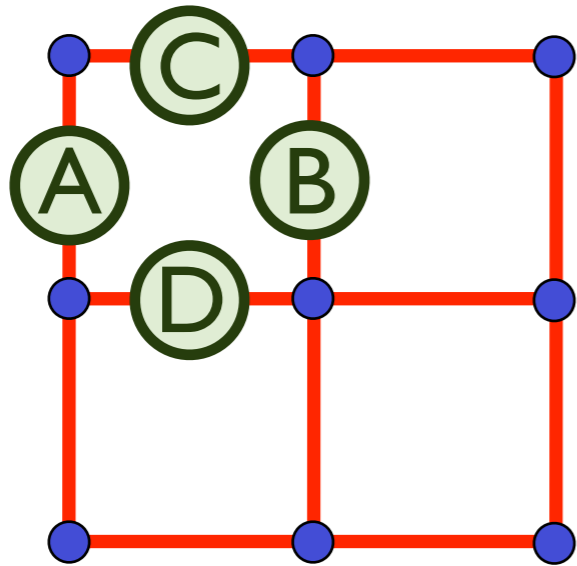


$$O_N = Z_A + Z_B - Z_C - Z_D$$

STM measurements of  $Z(r)$ , the energy asymmetry in density of states in  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ .



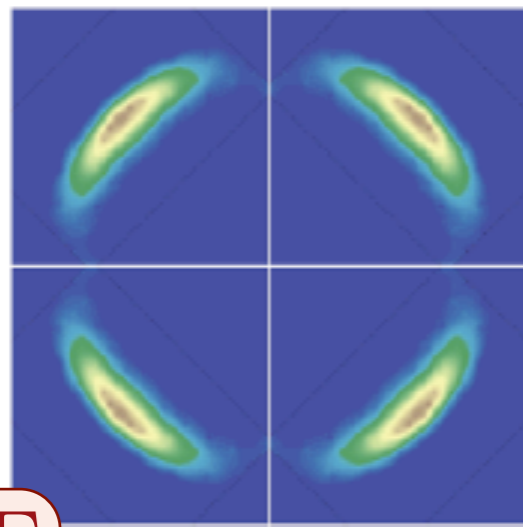
M. J. Lawler, K. Fujita, Jinhwan Lee, A. R. Schmidt, Y. Kohsaka, Chung Koo Kim, H. Eisaki, S. Uchida, J. C. Davis, J. P. Sethna, and Eun-Ah Kim, preprint



$$O_N = Z_A + Z_B - Z_C - Z_D$$

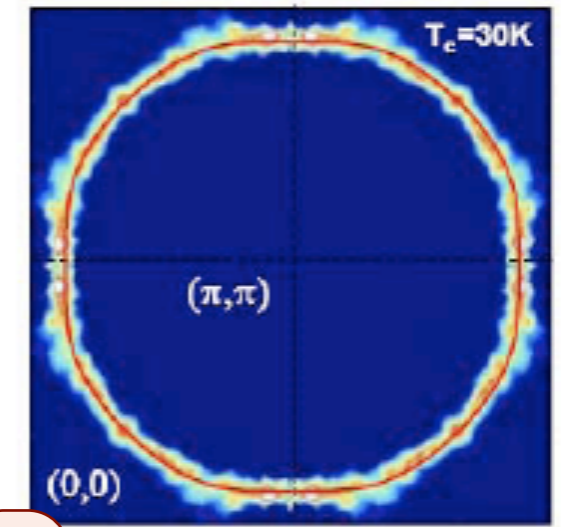
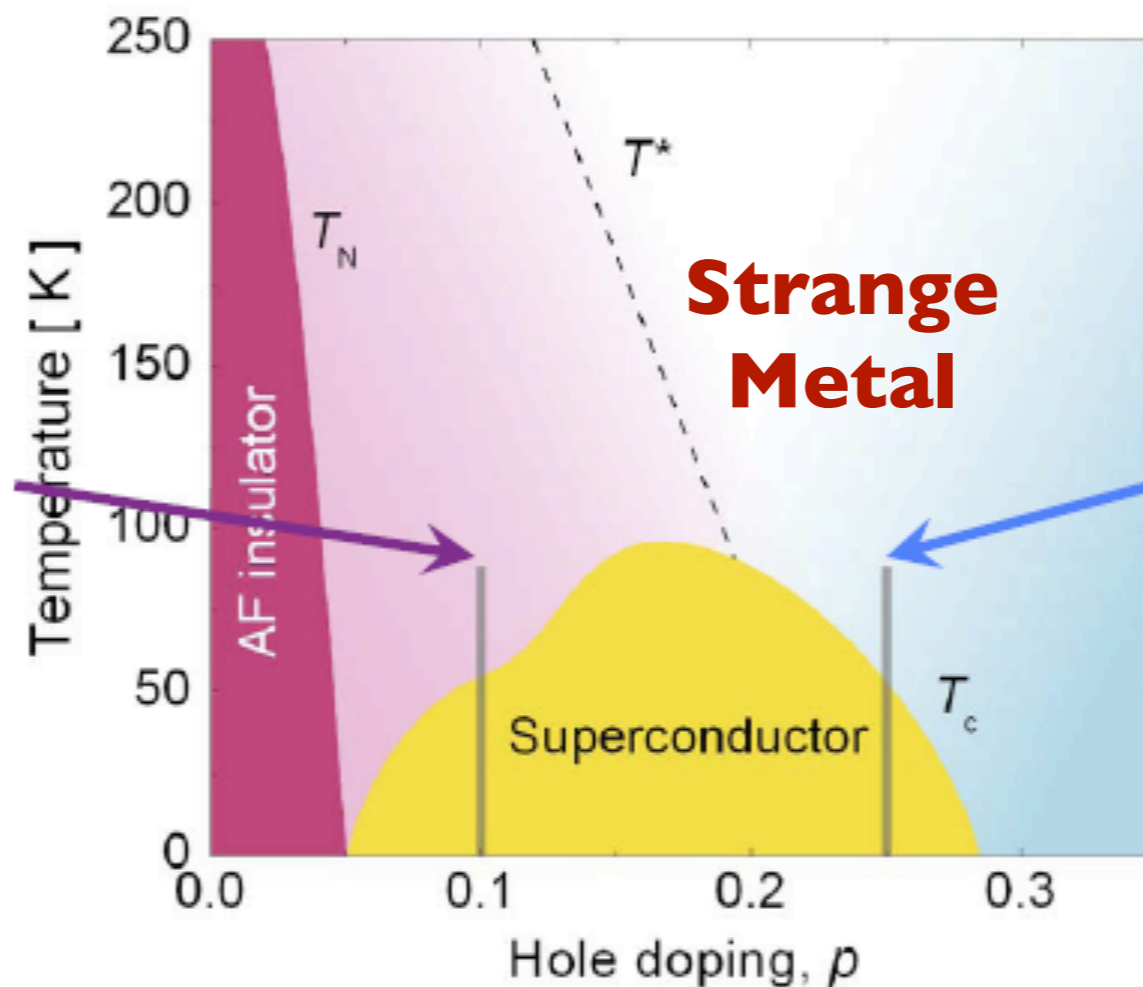
Strong anisotropy of electronic states between  $x$  and  $y$  directions:  
Electronic “Ising-nematic” order

# Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



$\Gamma$

*K.M. Shen et al., Science 2005*



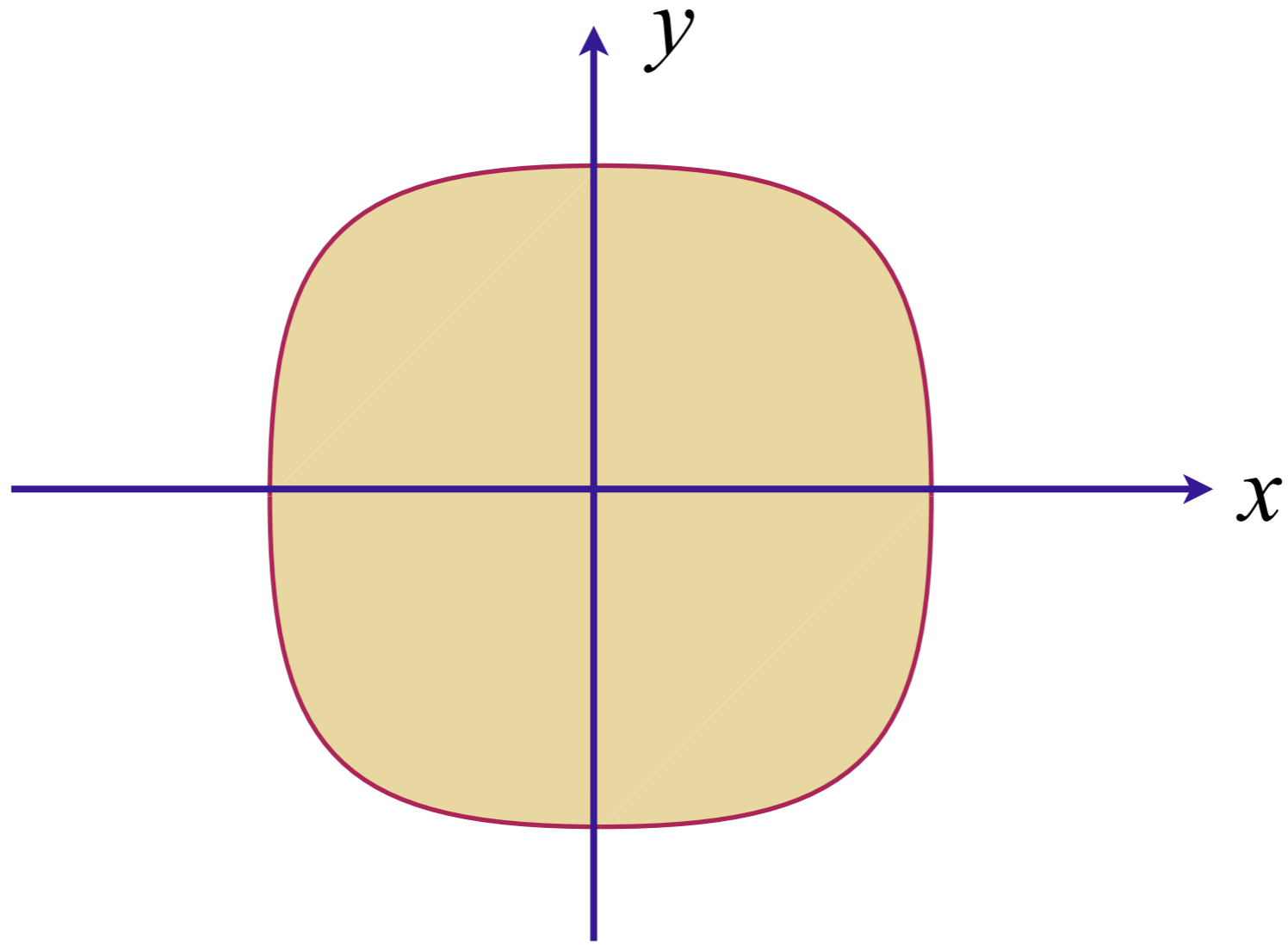
$\Gamma$

*M. Platé et al., PRL 2005*

Smaller hole  
Fermi-pockets

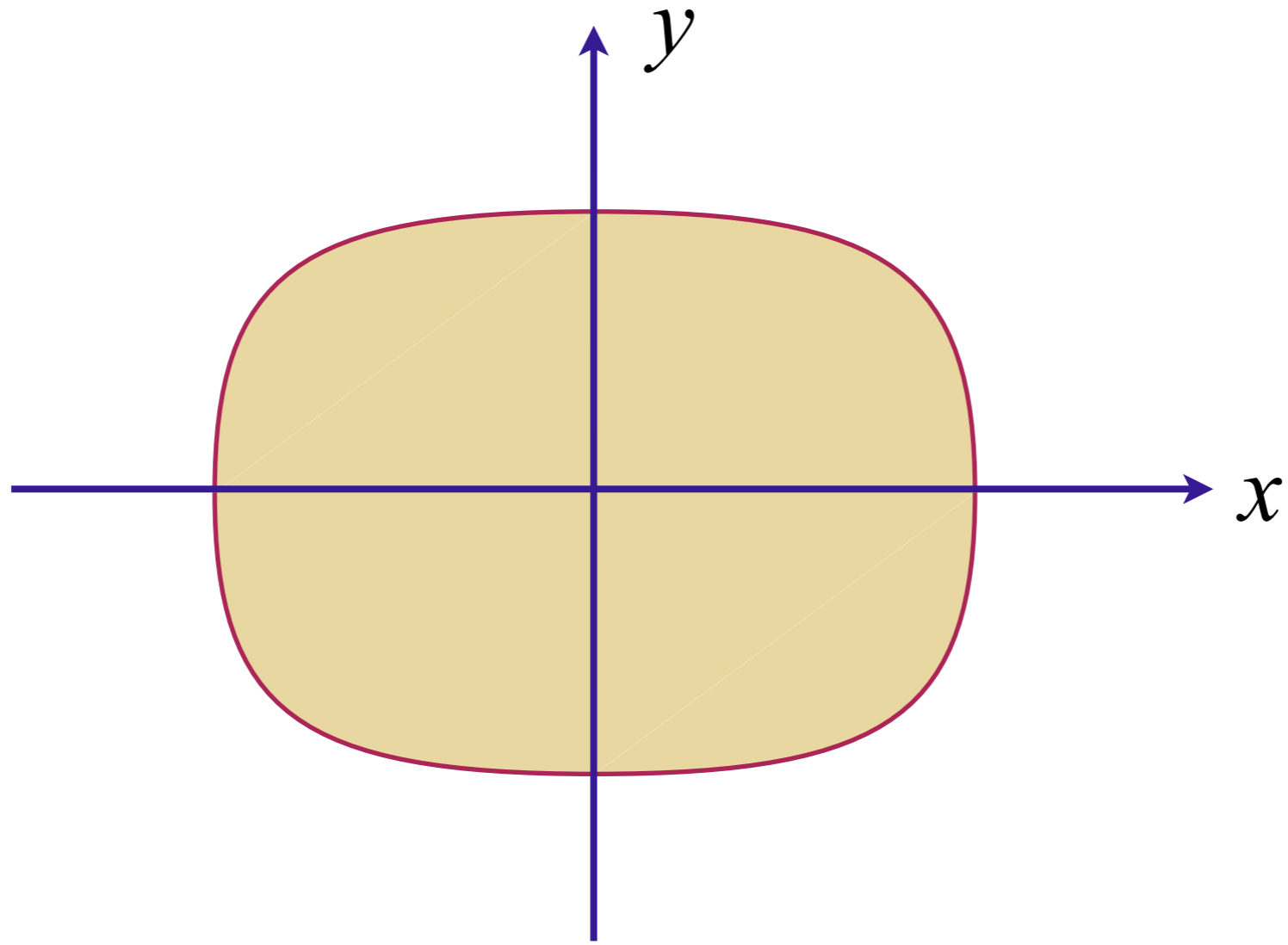
Large hole  
Fermi surface

# Quantum criticality of Ising-nematic ordering



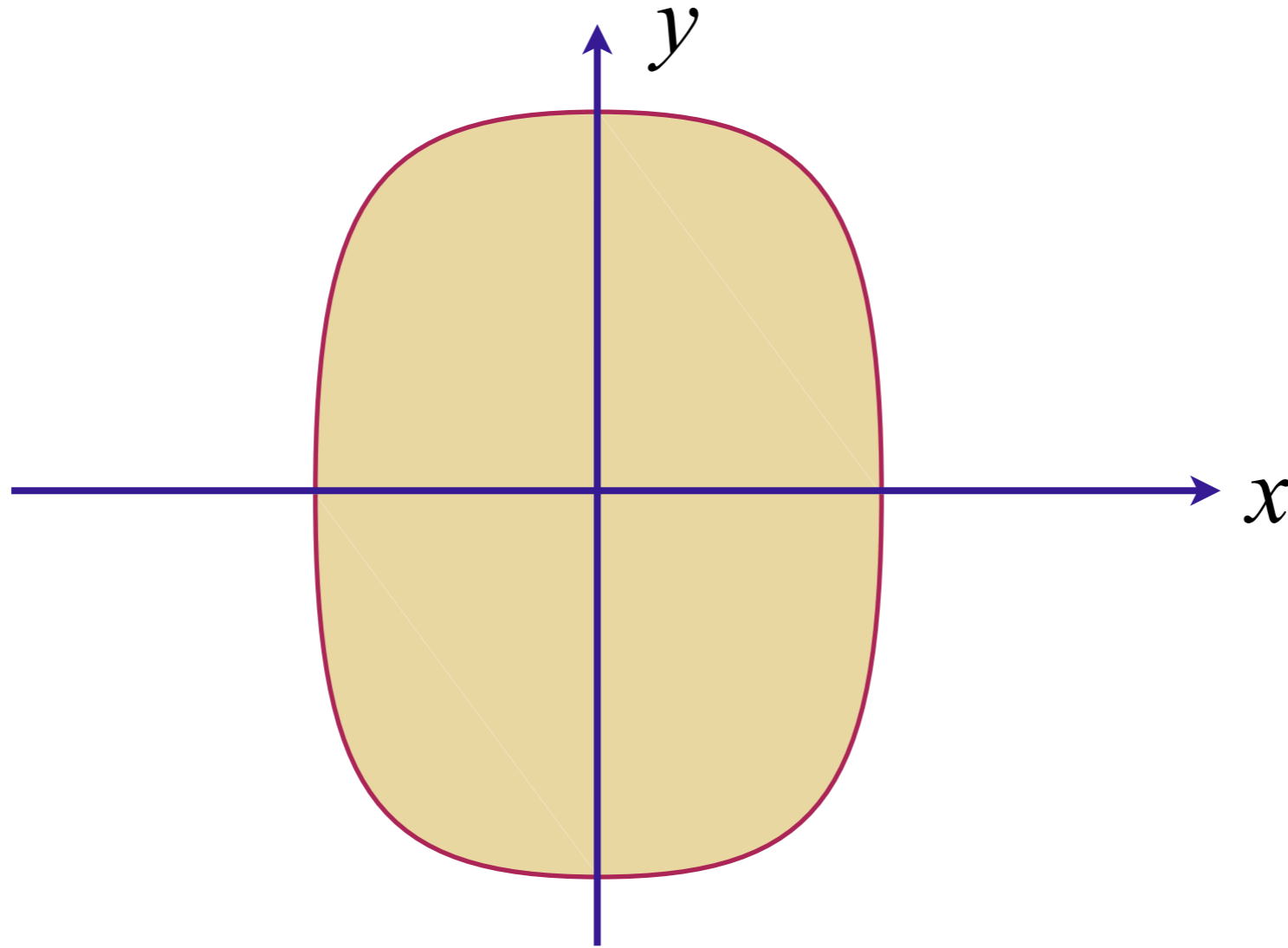
Fermi surface with full square lattice symmetry

# Quantum criticality of Ising-nematic ordering



Spontaneous elongation along  $x$  direction:

# Quantum criticality of Ising-nematic ordering



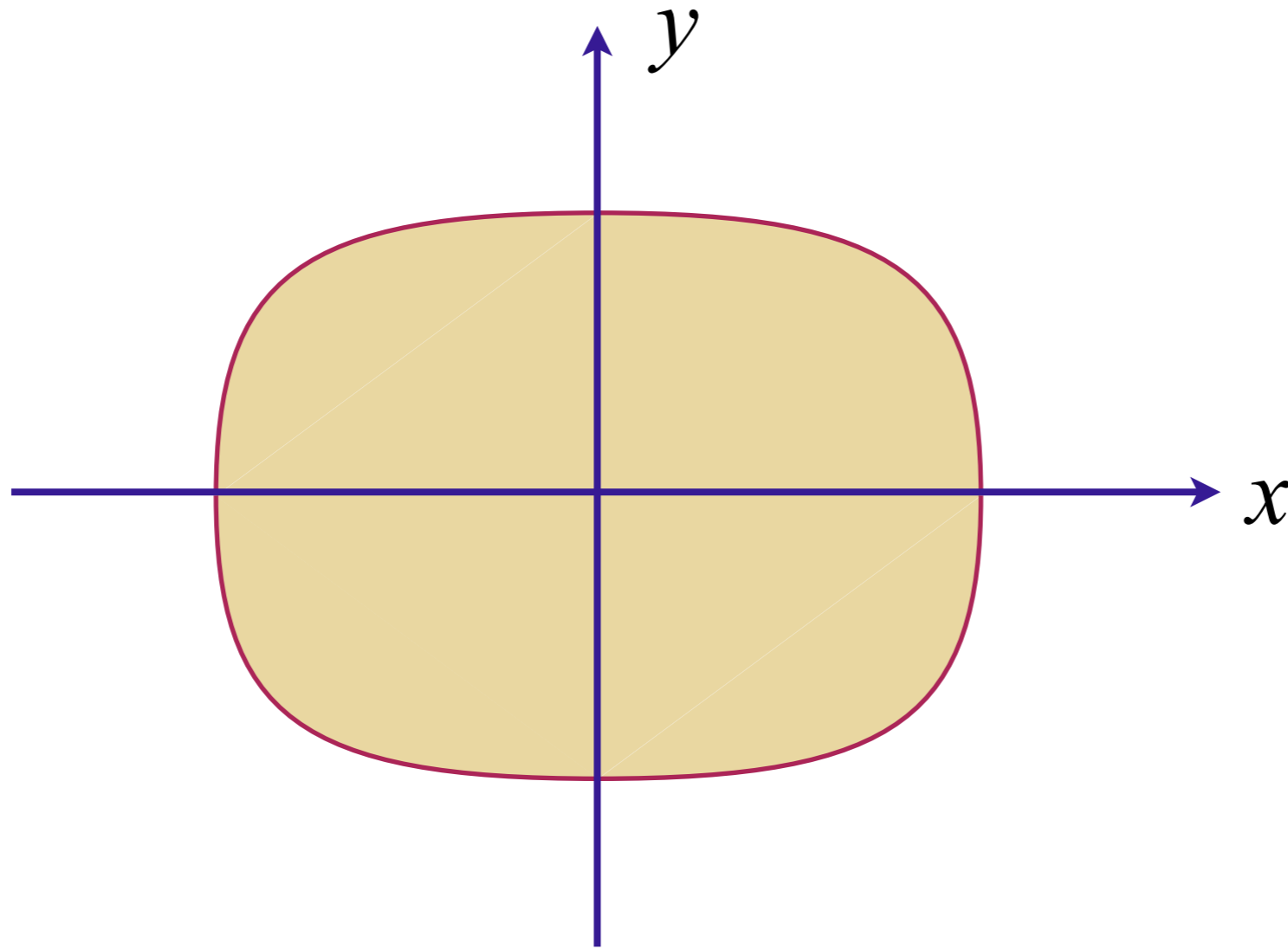
Spontaneous elongation along  $y$  direction:

## Ising-nematic order parameter

$$\phi \sim \int d^2 k (\cos k_x - \cos k_y) c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}$$

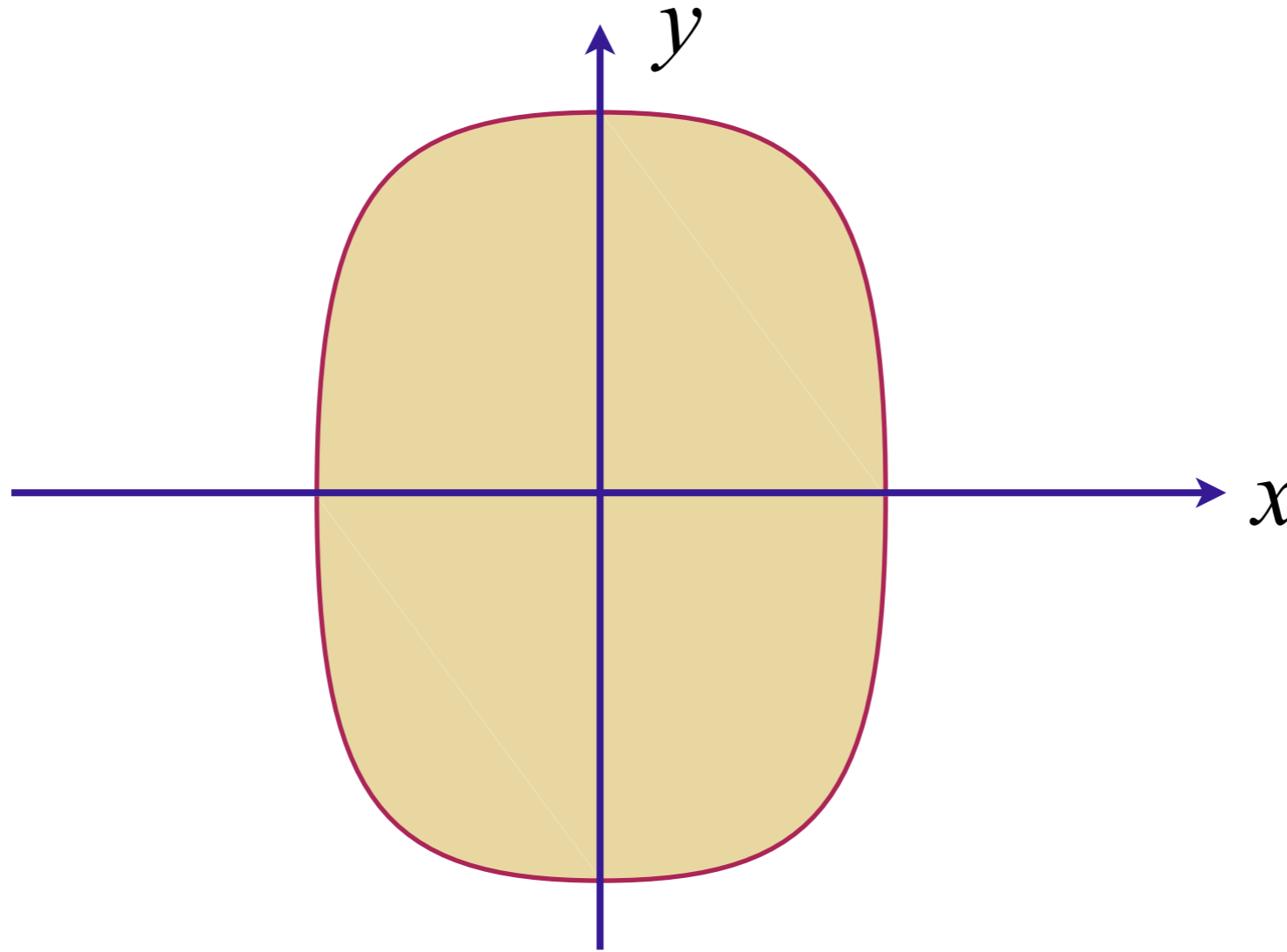
Measures spontaneous breaking of square lattice point-group symmetry of underlying Hamiltonian

# Quantum criticality of Ising-nematic ordering



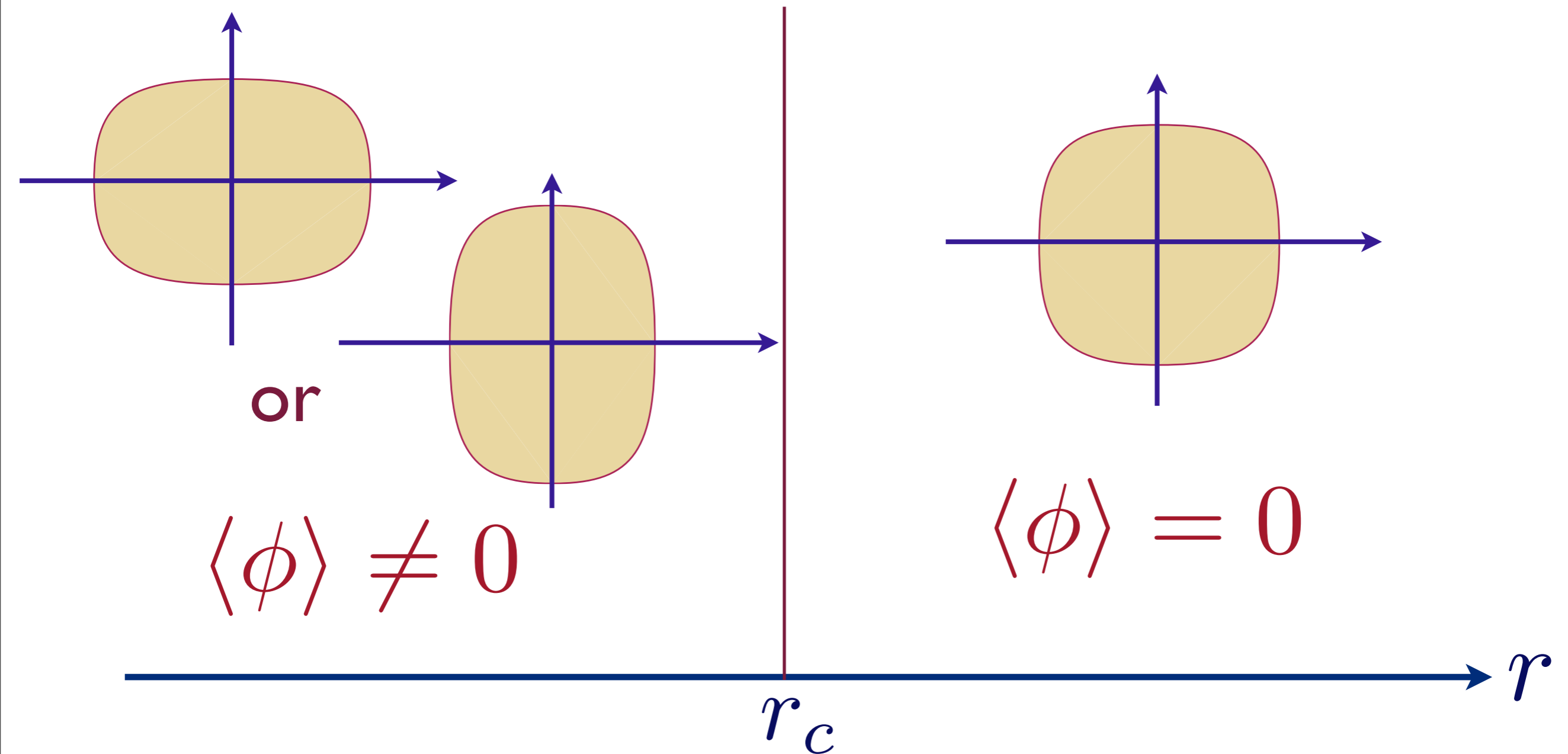
Spontaneous elongation along  $x$  direction:  
Ising order parameter  $\phi > 0$ .

# Quantum criticality of Ising-nematic ordering



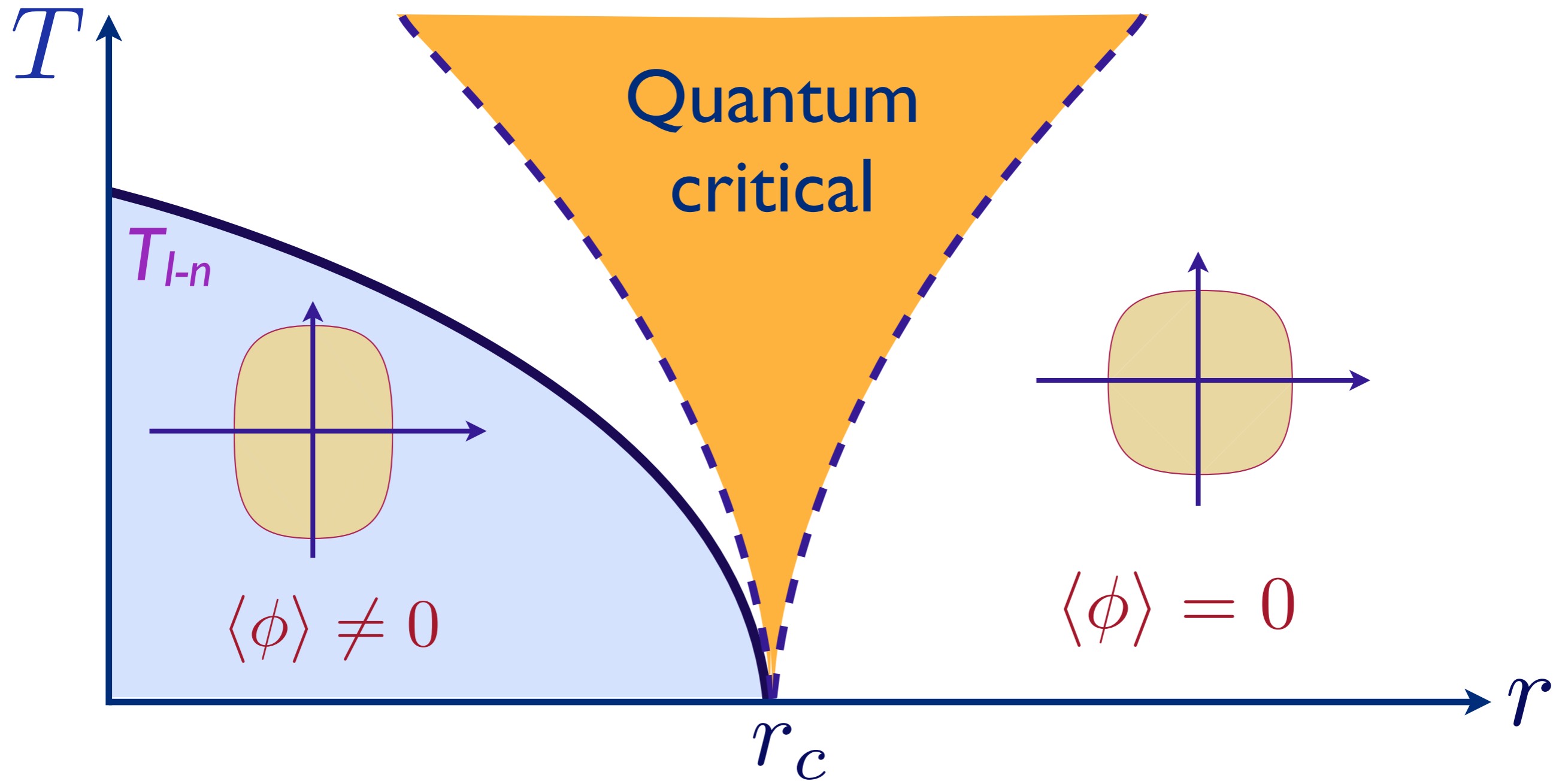
Spontaneous elongation along  $y$  direction:  
Ising order parameter  $\phi < 0$ .

# Quantum criticality of Ising-nematic ordering



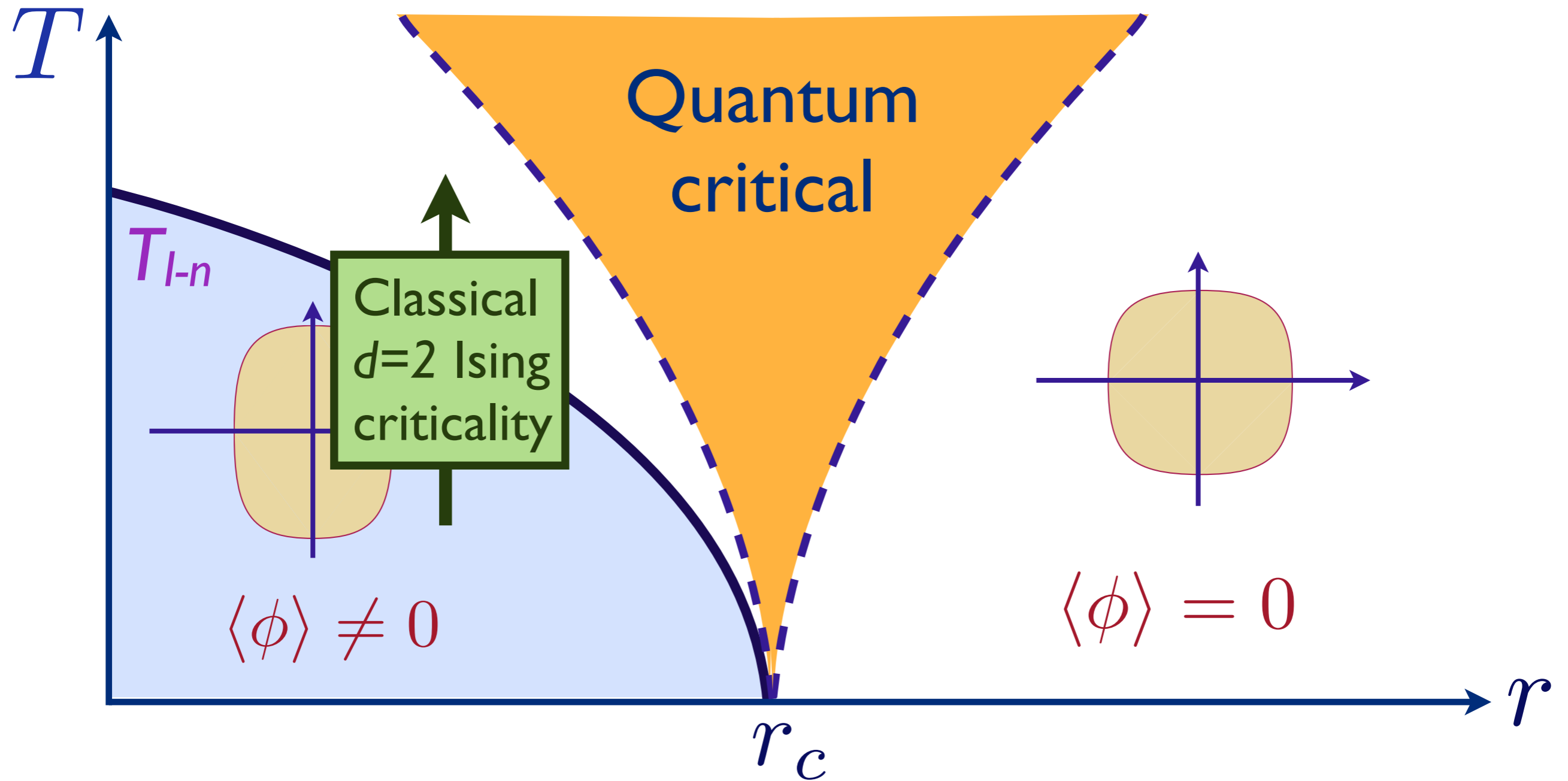
Pomeranchuk instability as a function of coupling  $r$

# Quantum criticality of Ising-nematic ordering



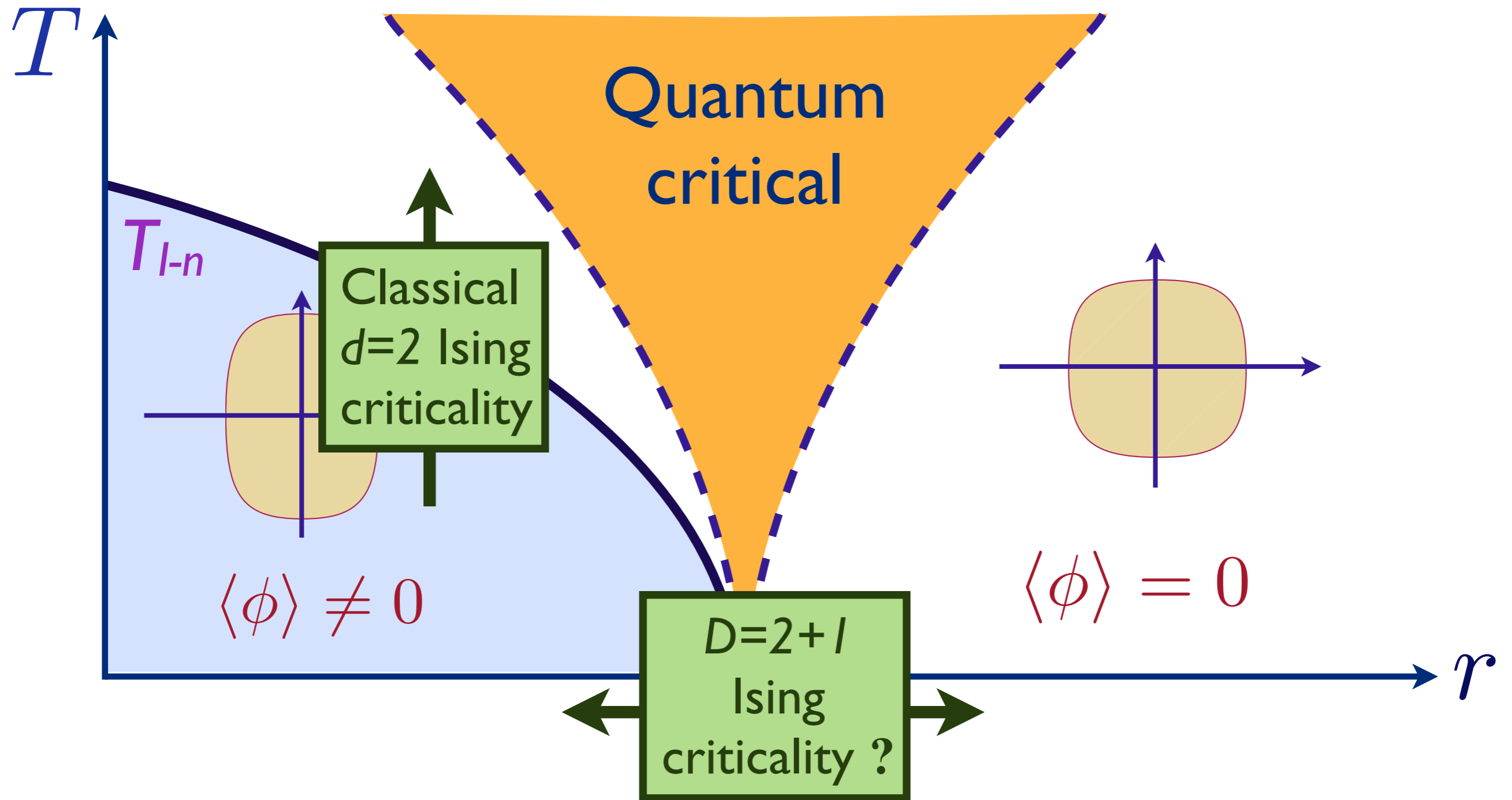
Phase diagram as a function of  $T$  and  $r$

# Quantum criticality of Ising-nematic ordering



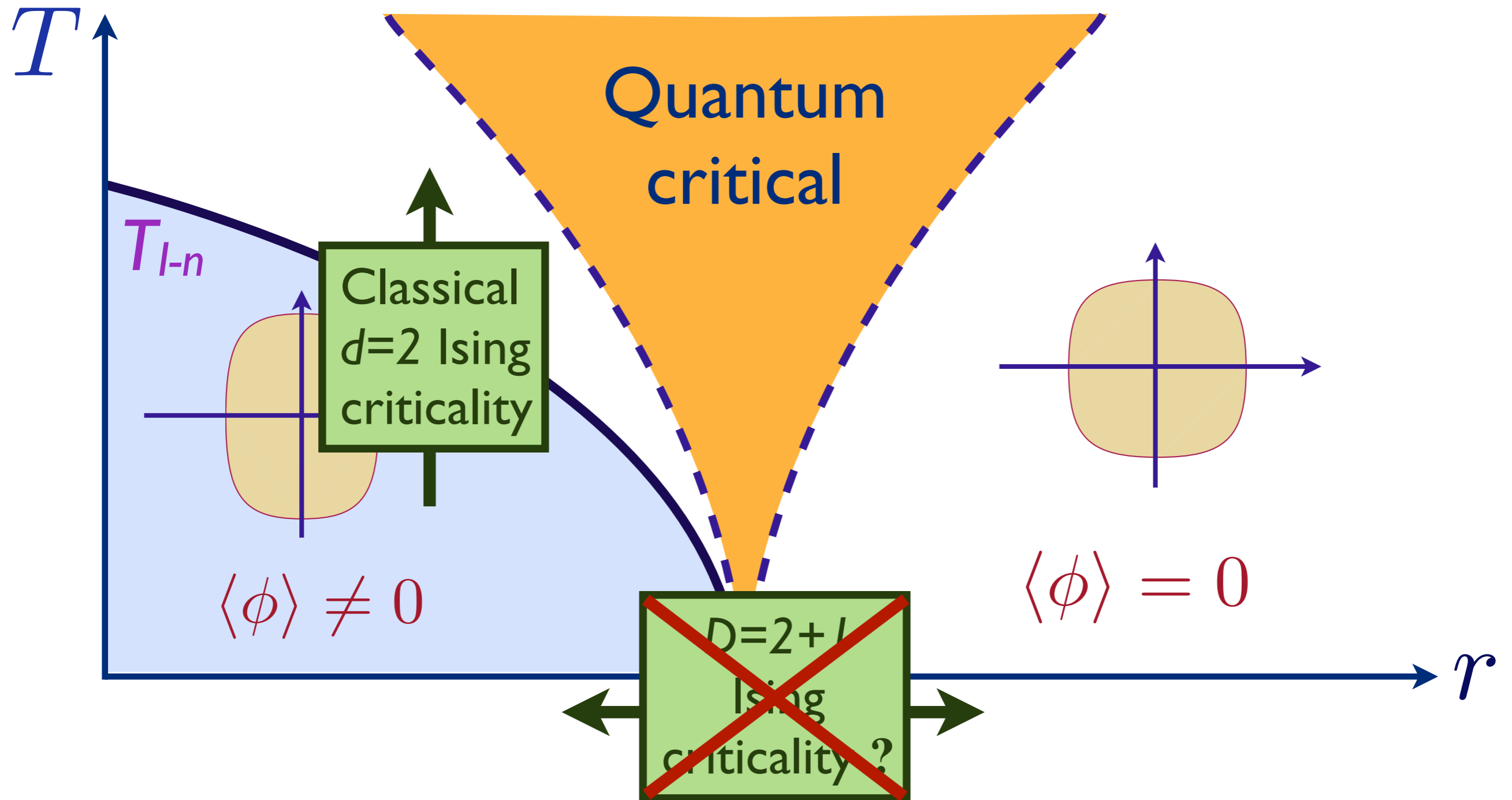
Phase diagram as a function of  $T$  and  $r$

# Quantum criticality of Ising-nematic ordering



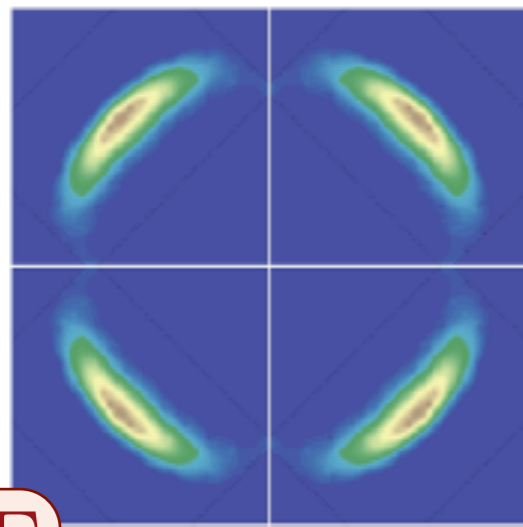
Phase diagram as a function of  $T$  and  $r$

# Quantum criticality of Ising-nematic ordering



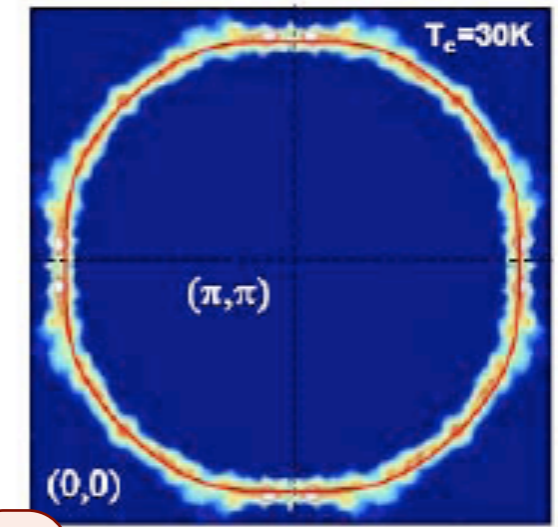
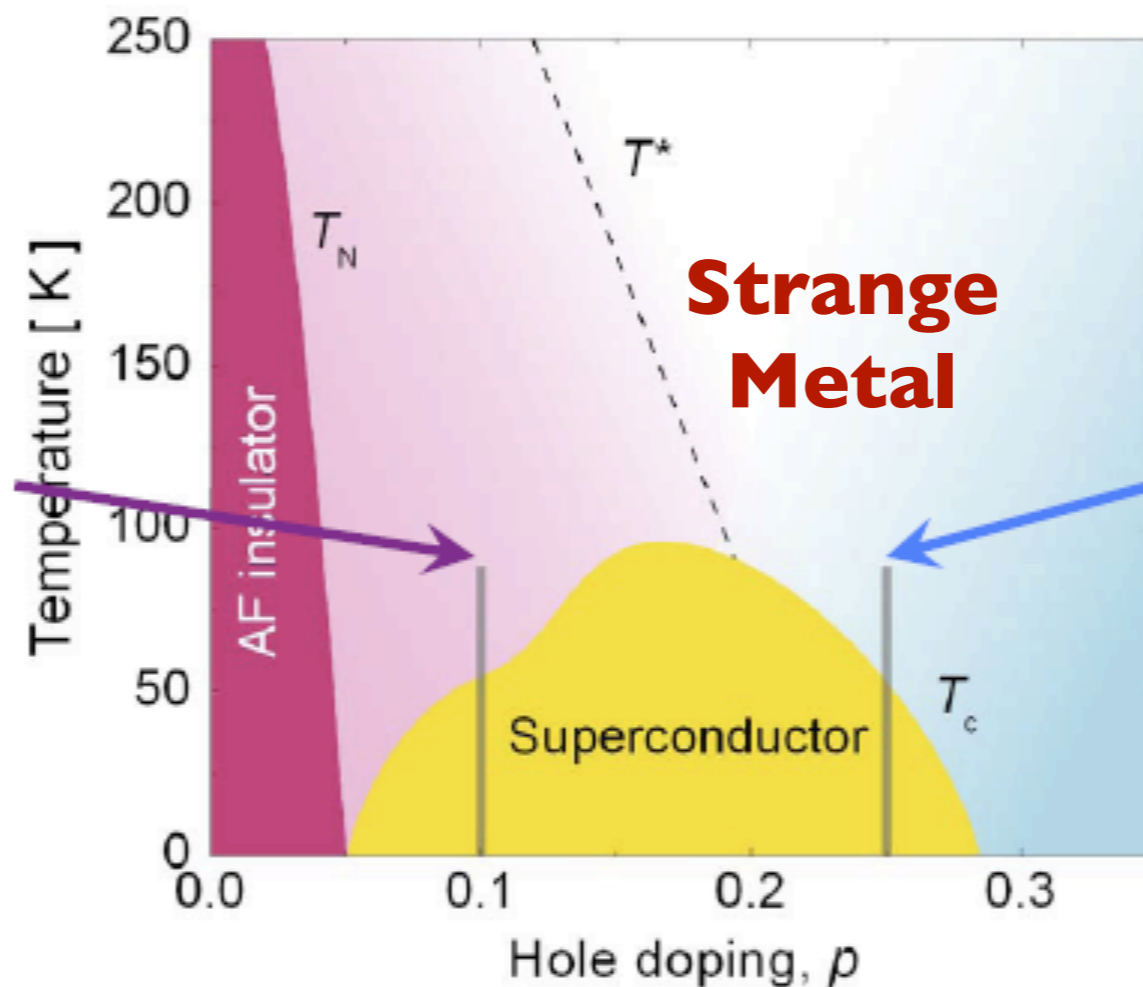
Phase diagram as a function of  $T$  and  $r$

# Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



$\Gamma$

*K.M. Shen et al., Science 2005*



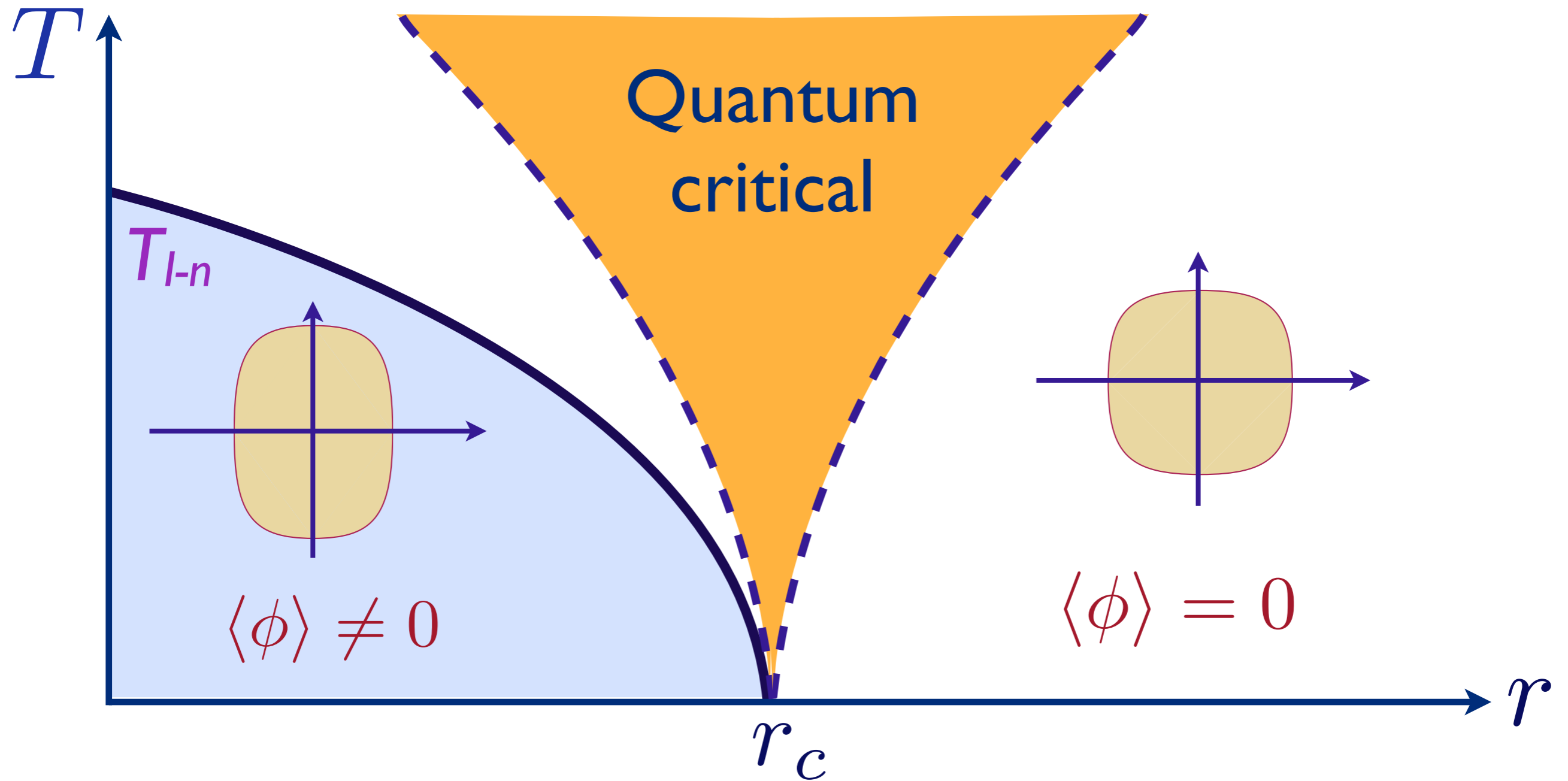
$\Gamma$

*M. Platé et al., PRL 2005*

Smaller hole  
Fermi-pockets

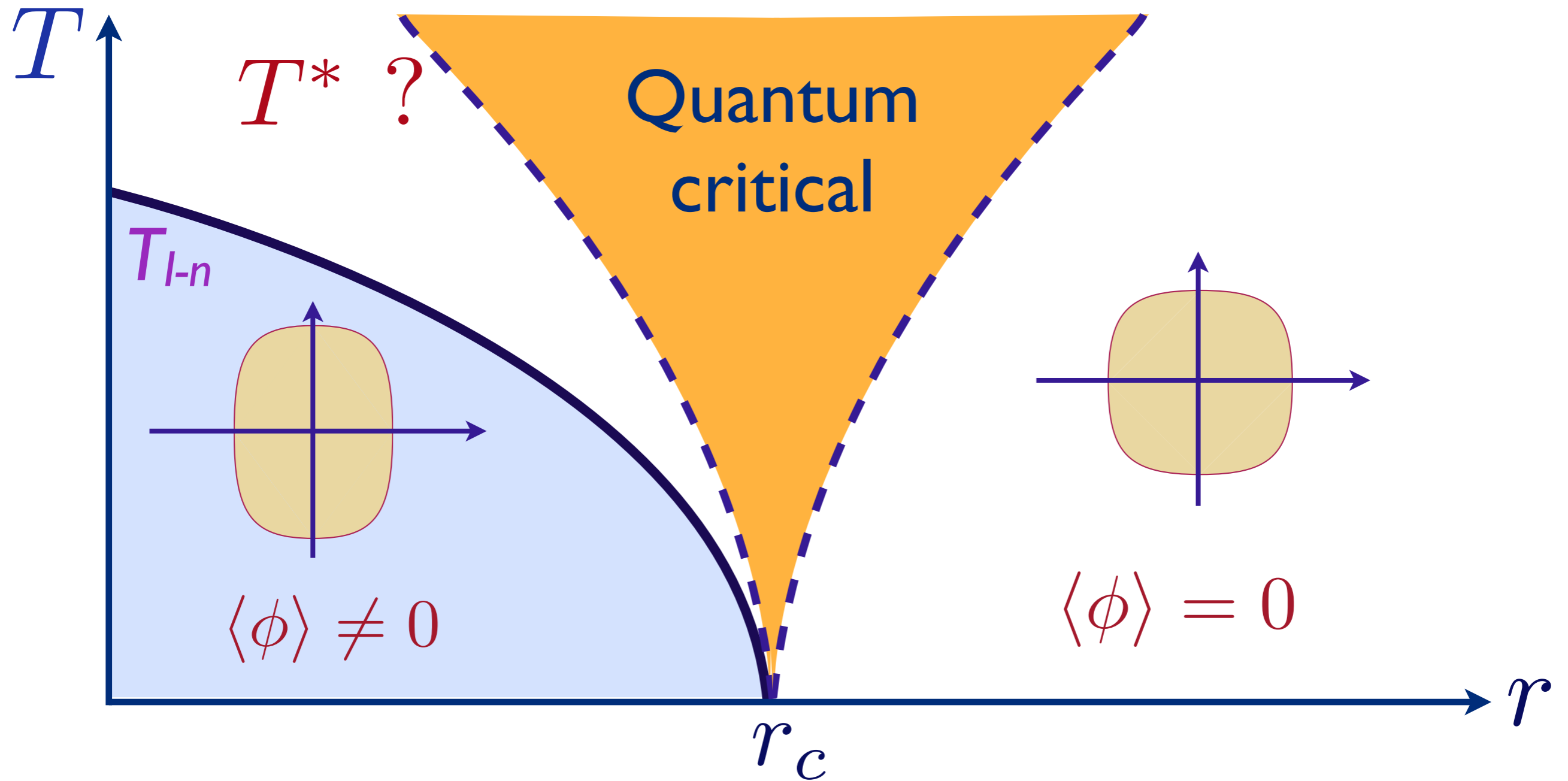
Large hole  
Fermi surface

# Quantum criticality of Ising-nematic ordering



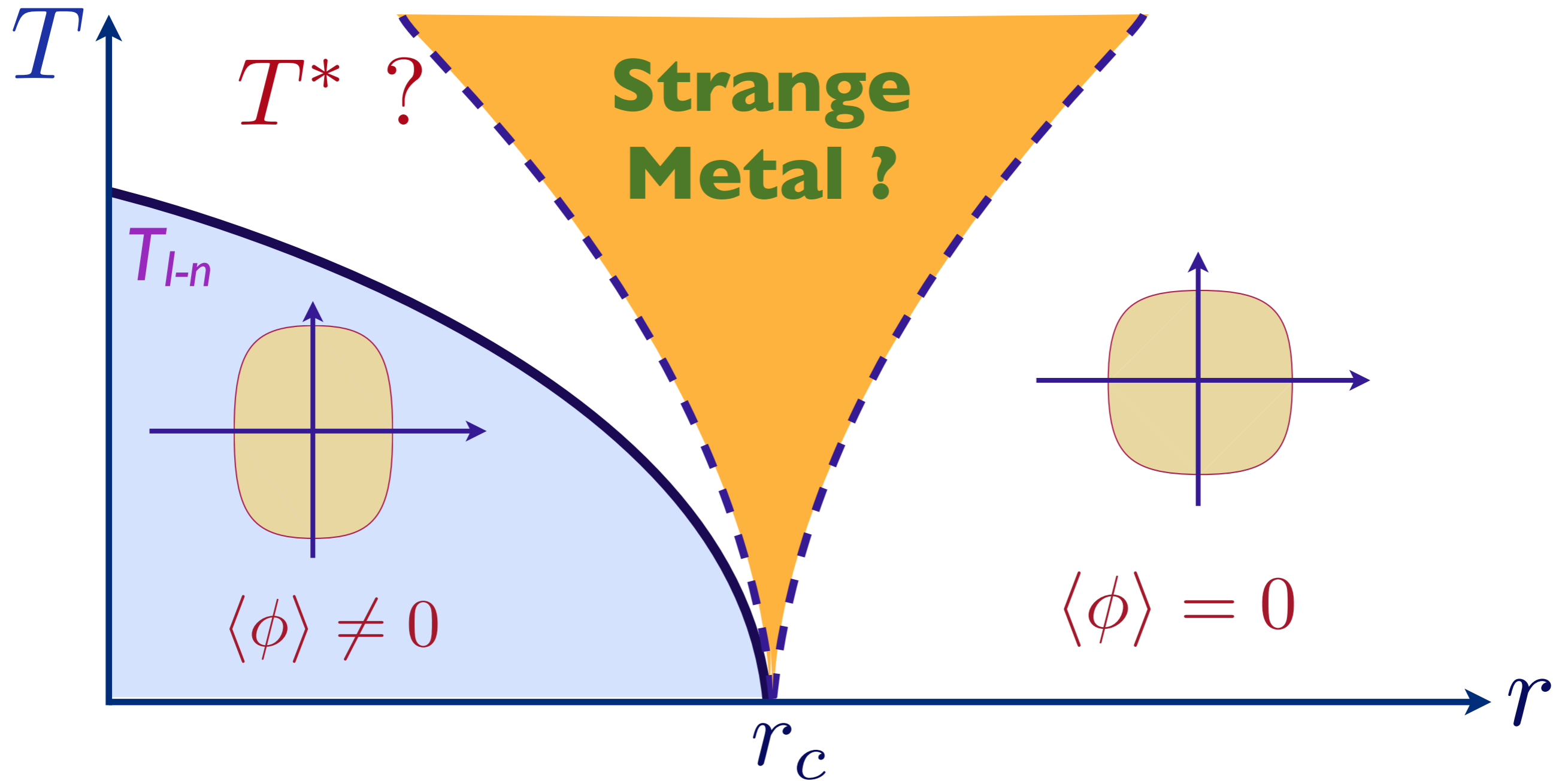
Phase diagram as a function of  $T$  and  $r$

# Quantum criticality of Ising-nematic ordering



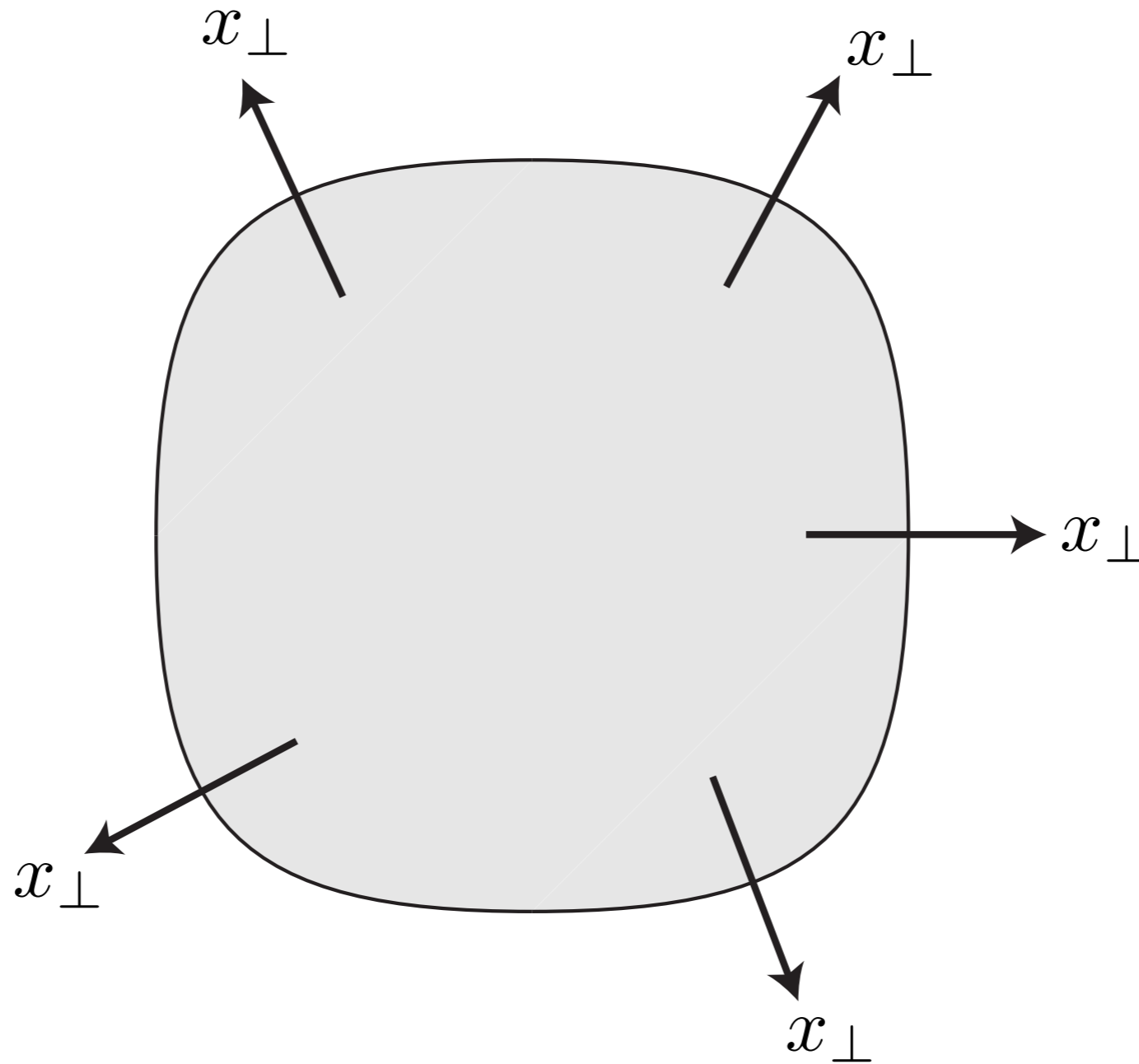
Phase diagram as a function of  $T$  and  $r$

# Quantum criticality of Ising-nematic ordering



Phase diagram as a function of  $T$  and  $r$

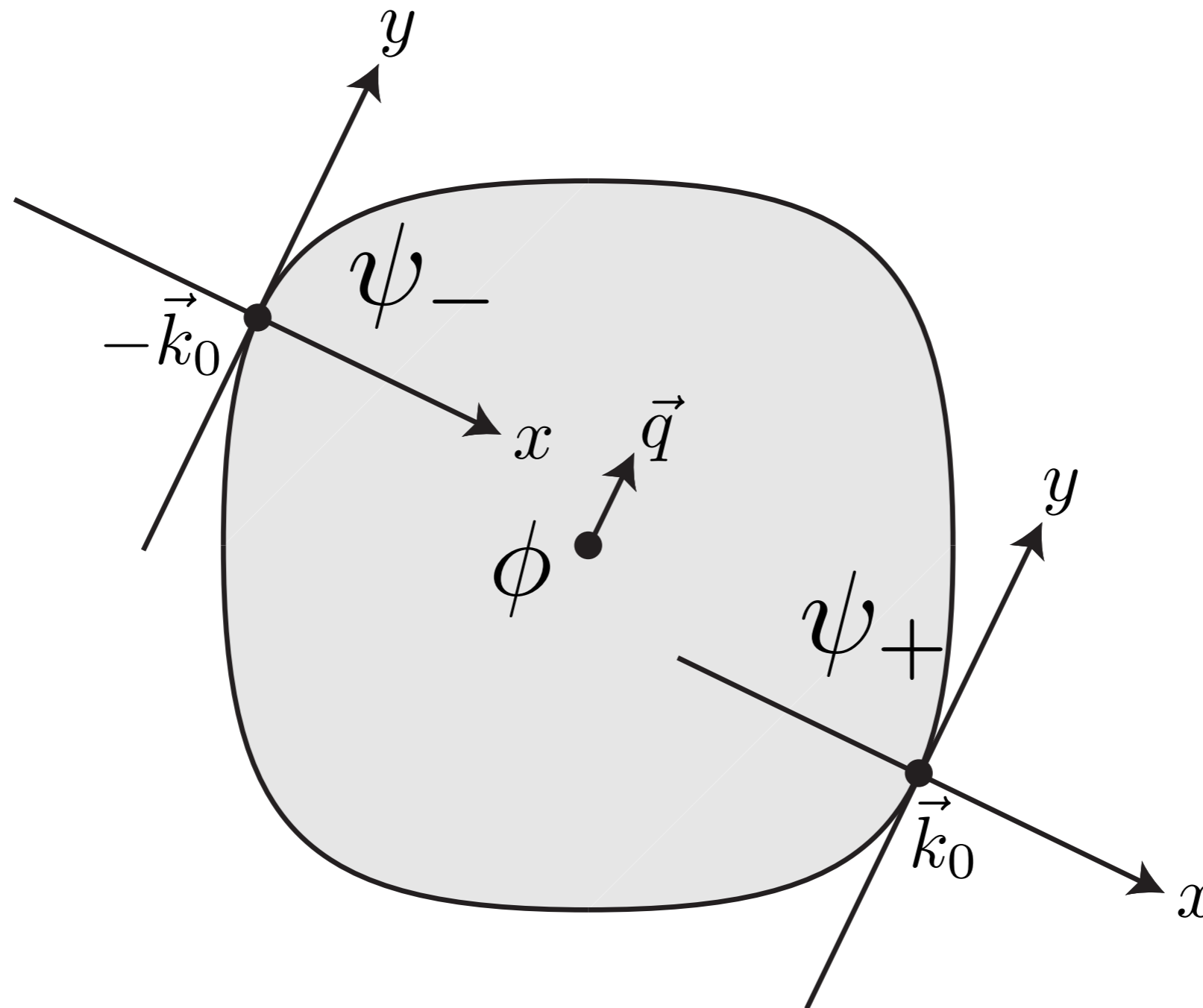
# Fermi liquid theory



$$\mathcal{S}_{\text{FL}} = \int d\Omega_{\hat{n}} \int dx_{\perp} \psi_{\hat{n}a}^{\dagger}(x_{\perp}) \left( \frac{\partial}{\partial \tau} - i v_F(\hat{n}) \frac{\partial}{\partial x_{\perp}} \right) \psi_{\hat{n}a}(x_{\perp})$$

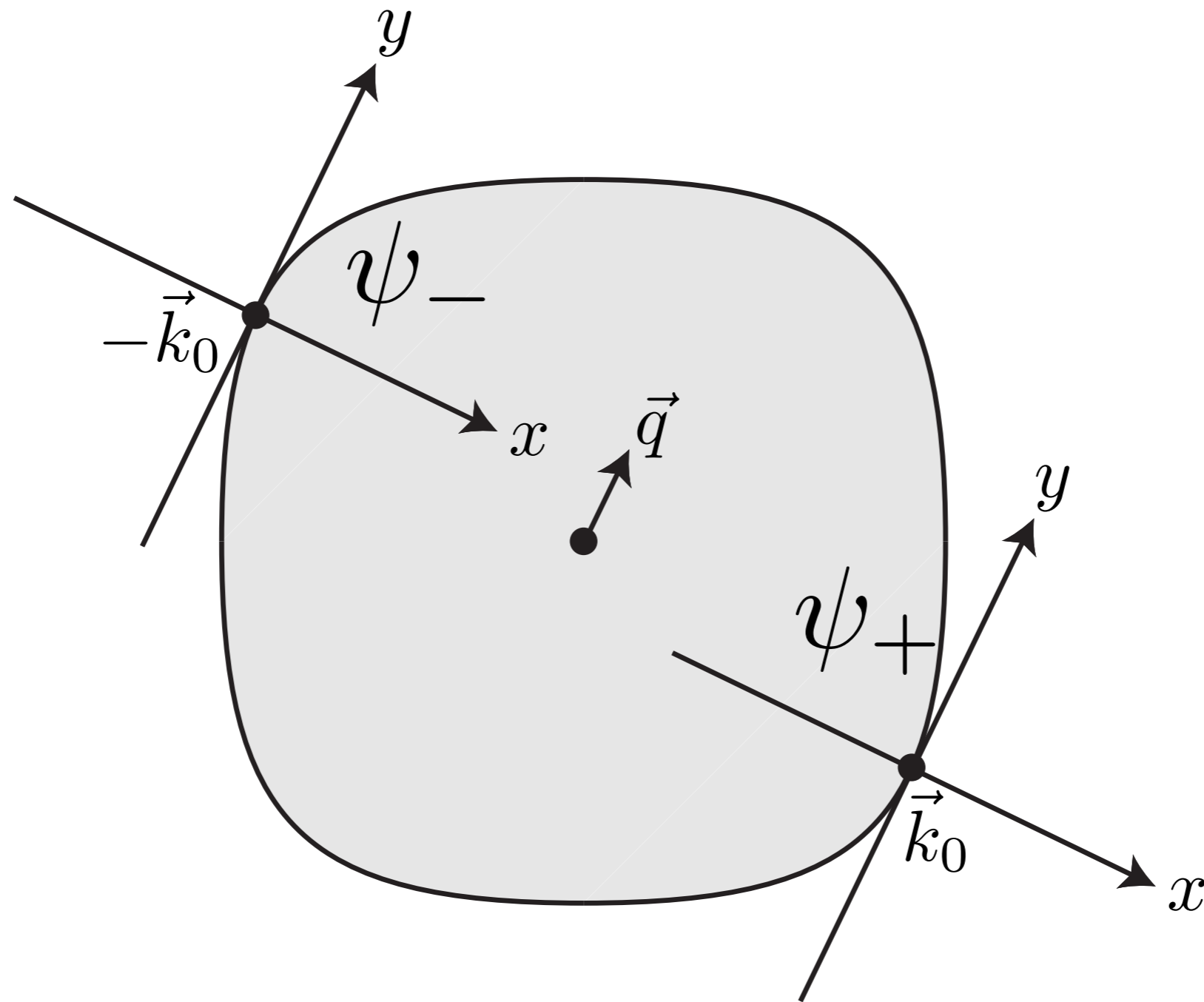
**Infinite number of 1+1 dimensional chiral fermions**

# Non-Fermi liquid quantum critical point



- Critical point is described by an *infinite* set of  $2+1$  dimensional field theories, one for each direction  $\hat{q}$ .

# Non-Fermi liquid quantum critical point



- Critical point is described by an *infinite* set of  $2+1$  dimensional field theories, one for each direction  $\hat{q}$ .

# Non-Fermi liquid quantum critical point

$y$

Strong coupling problem II:  
Infinite number of 2+1  
dimensional field theories  
at Ising-nematic  
quantum critical point

- Critical point is described by an *infinite* set of 2+1 dimensional field theories, one for each direction  $\hat{q}$ .

# Outline

1. Coupled dimer antiferromagnets  
*Introduction to quantum criticality*
2. Theory of Ising-nematic ordering in the cuprate metals  
*Strongly-coupled field theory*
3. The AdS/CFT correspondence  
*Phases of quantum matter at strong coupling*
4. Global phase diagram of the cuprates  
*Competition between d-wave superconductivity and spin-density order for the Fermi surface*

# Outline

## 1. Coupled dimer antiferromagnets

*Introduction to quantum criticality*

## 2. Theory of Ising-nematic ordering in the cuprate metals

*Strongly-coupled field theory*

## 3. The AdS/CFT correspondence

*Phases of quantum matter  
at strong coupling*

## 4. Global phase diagram of the cuprates

*Competition between d-wave superconductivity  
and spin-density order for the Fermi surface*

Field theories in  $D$  spacetime dimensions are characterized by couplings  $g$  which obey the renormalization group equation

$$u \frac{dg}{du} = \beta(g)$$

where  $u$  is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon  $u$ .

Field theories in  $D$  spacetime dimensions are characterized by couplings  $g$  which obey the renormalization group equation

$$u \frac{dg}{du} = \beta(g)$$

where  $u$  is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon  $u$ .

**Key idea:**  $\Rightarrow$  Implement  $u$  as an extra dimension, and map to a local theory in  $D + 1$  dimensions.

At the RG fixed point,  $\beta(g) = 0$ , the  $D$  dimensional field theory is invariant under the scale transformation

$$x^\mu \rightarrow x^\mu / b \quad , \quad u \rightarrow b u$$

At the RG fixed point,  $\beta(g) = 0$ , the  $D$  dimensional field theory is invariant under the scale transformation

$$x^\mu \rightarrow x^\mu / b \quad , \quad u \rightarrow b u$$

This is an invariance of the *metric* of the theory in  $D + 1$  dimensions. The unique solution is

$$ds^2 = \left(\frac{u}{L}\right)^2 dx^\mu dx_\mu + L^2 \frac{du^2}{u^2}.$$

Or, using the length scale  $z = L^2 / u$

$$ds^2 = L^2 \frac{dx^\mu dx_\mu + dz^2}{z^2}.$$

This is the space  $\text{AdS}_{D+1}$ , and  $L$  is the AdS radius.

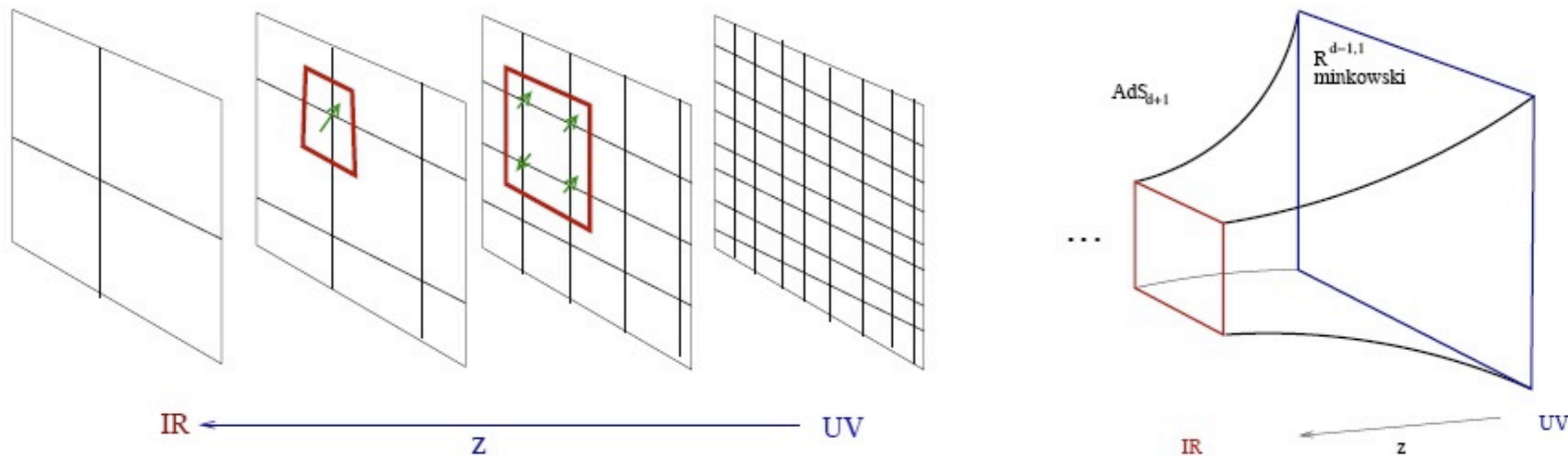


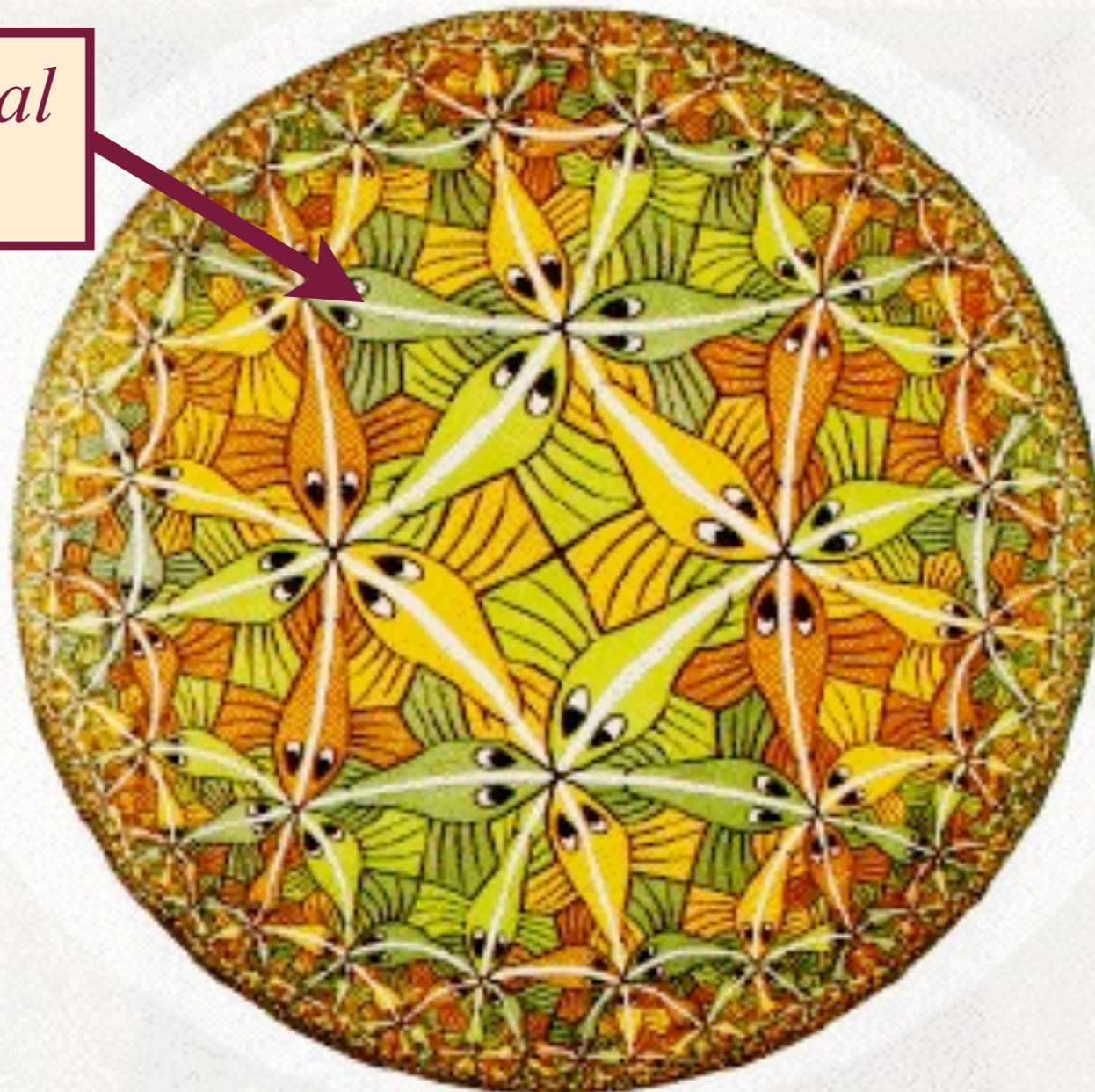
Figure 1: The extra ('radial') dimension of the bulk is the resolution scale of the field theory. The left figure indicates a series of block spin transformations labelled by a parameter  $z$ . The right figure is a cartoon of AdS space, which organizes the field theory information in the same way. In this sense, the bulk picture is a hologram: excitations with different wavelengths get put in different places in the bulk image.

J. McGreevy, arXiv0909.0518

# AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions

*3+1 dimensional  
AdS space*

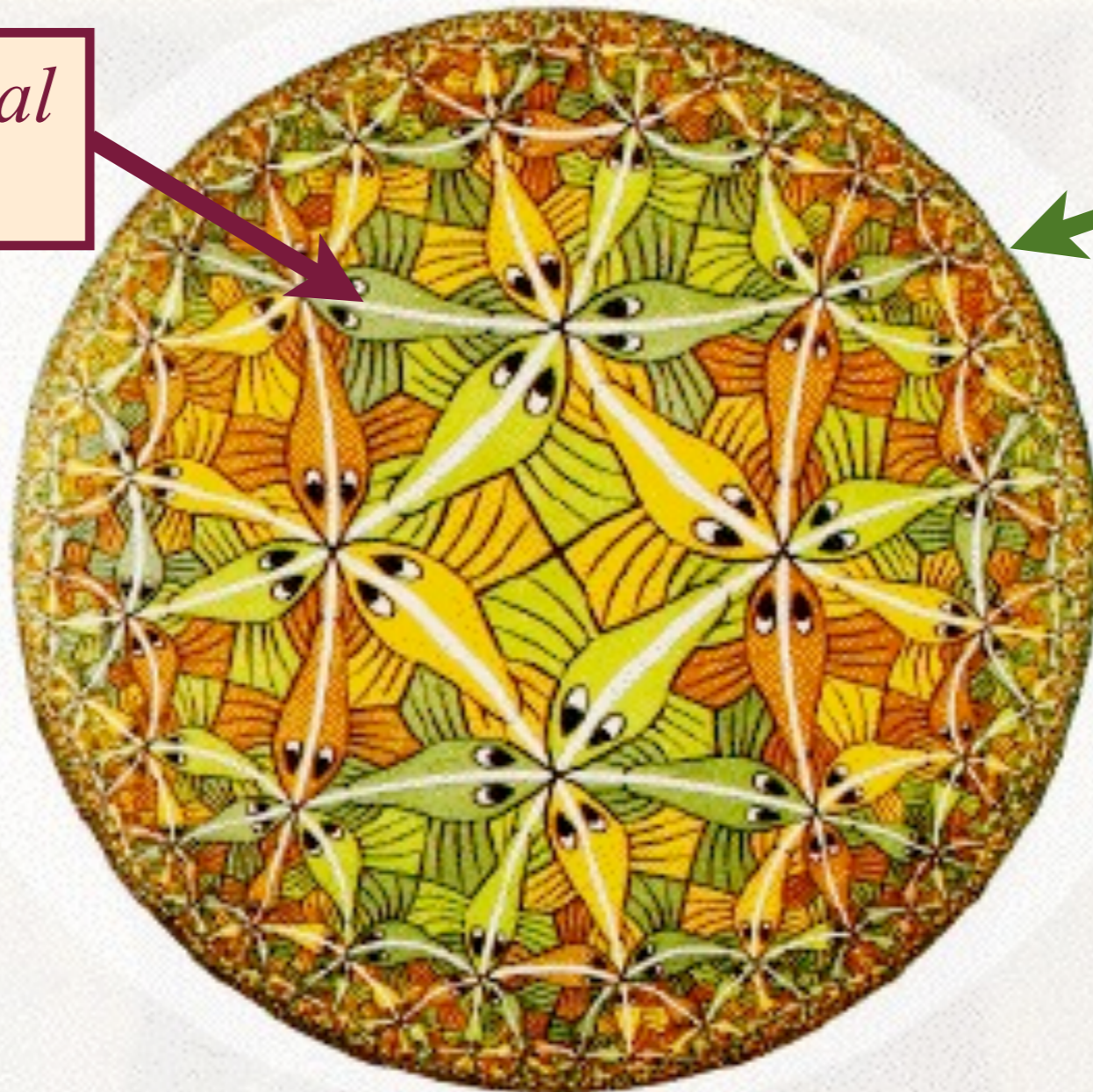


Maldacena, Gubser, Klebanov, Polyakov, Witten

# AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions

*3+1 dimensional  
AdS space*



A 2+1  
dimensional  
system at its  
quantum  
critical point

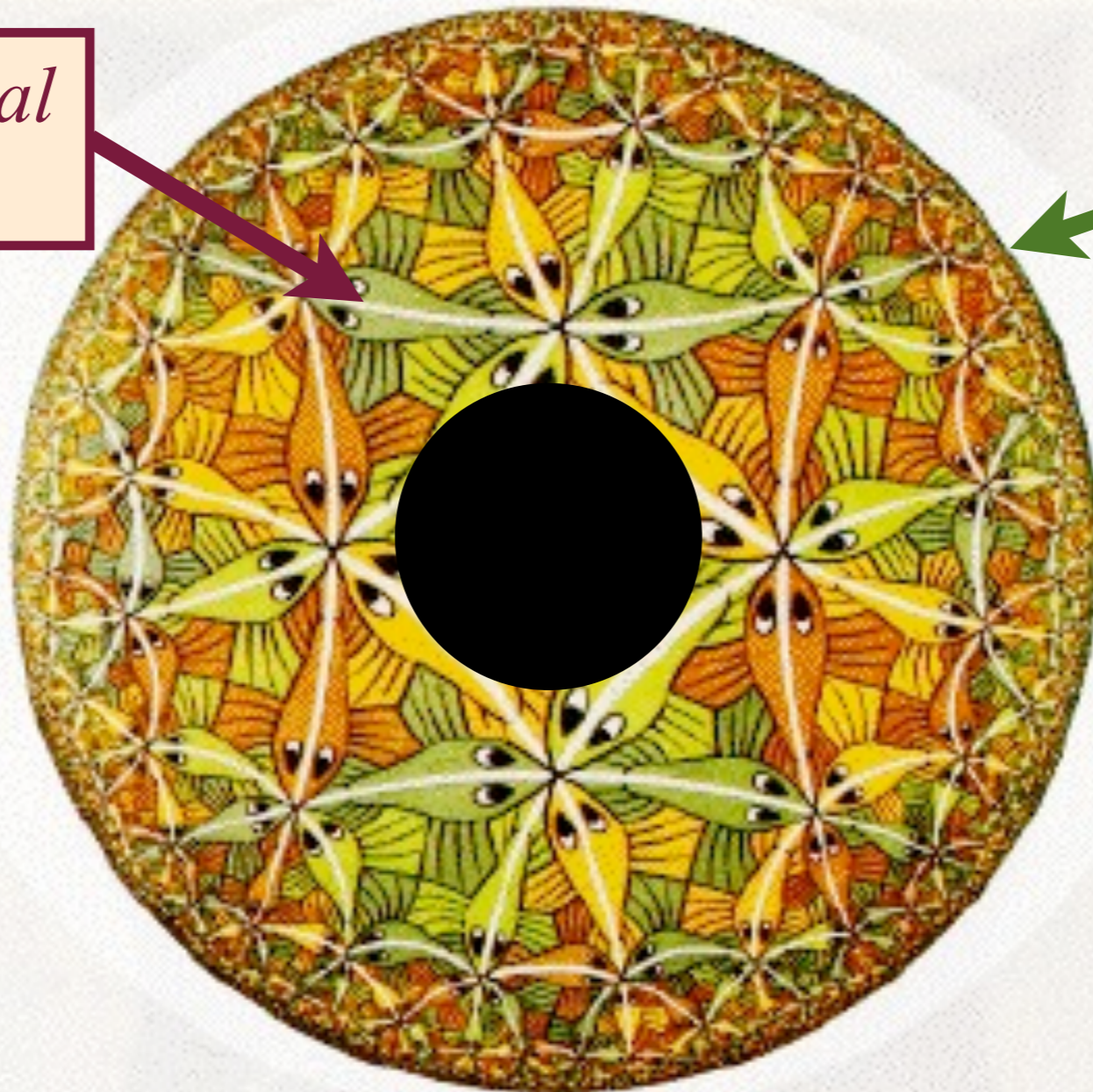
Maldacena, Gubser, Klebanov, Polyakov, Witten

# AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions

*3+1 dimensional  
AdS space*

Quantum  
criticality in  
2+1  
dimensions



Black hole  
temperature  
=  
temperature  
of quantum  
criticality

Maldacena, Gubser, Klebanov, Polyakov, Witten

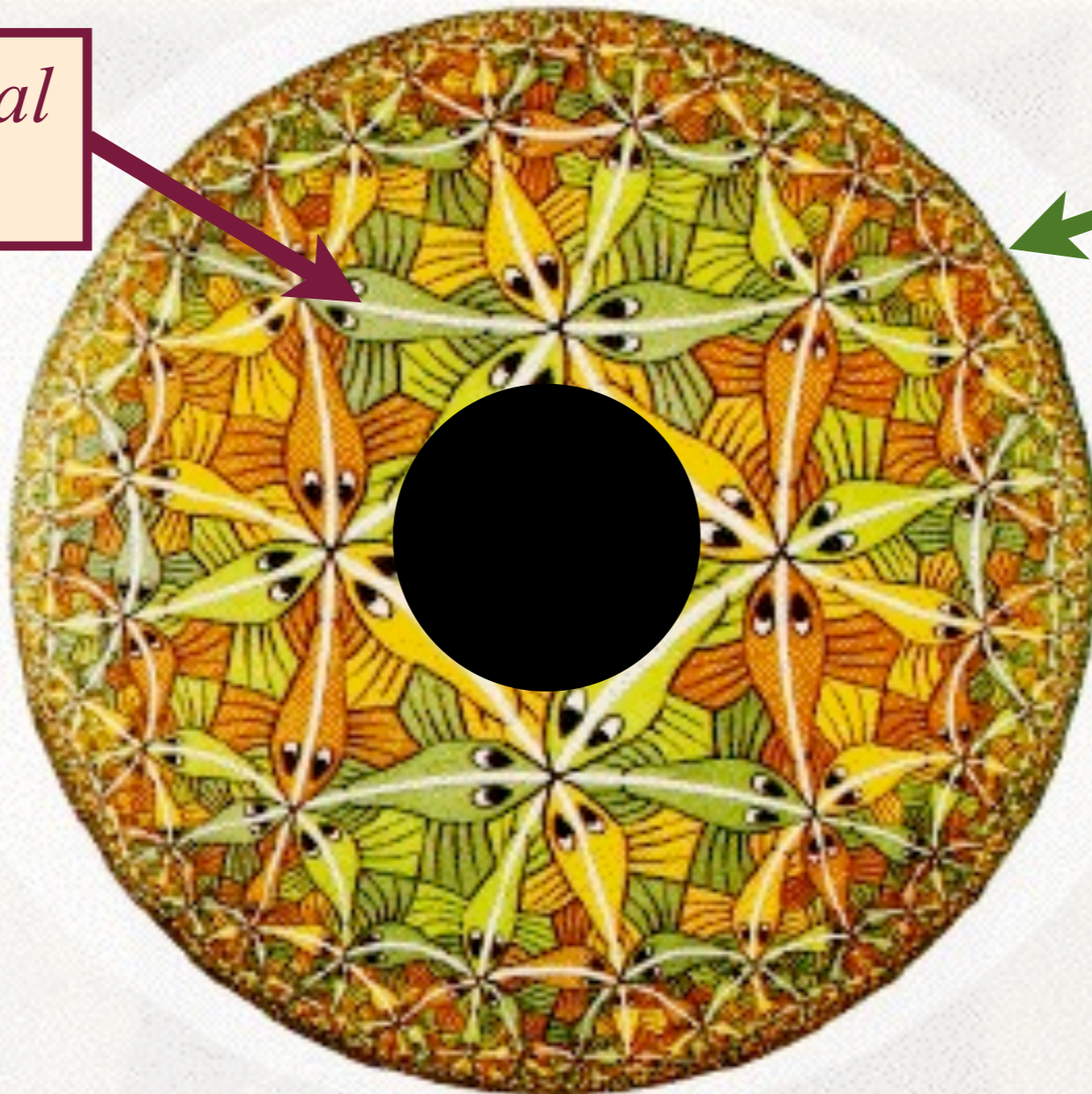
# AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions

*3+1 dimensional  
AdS space*

Quantum  
criticality in  
2+1  
dimensions

Black hole  
entropy =  
entropy of  
quantum  
criticality



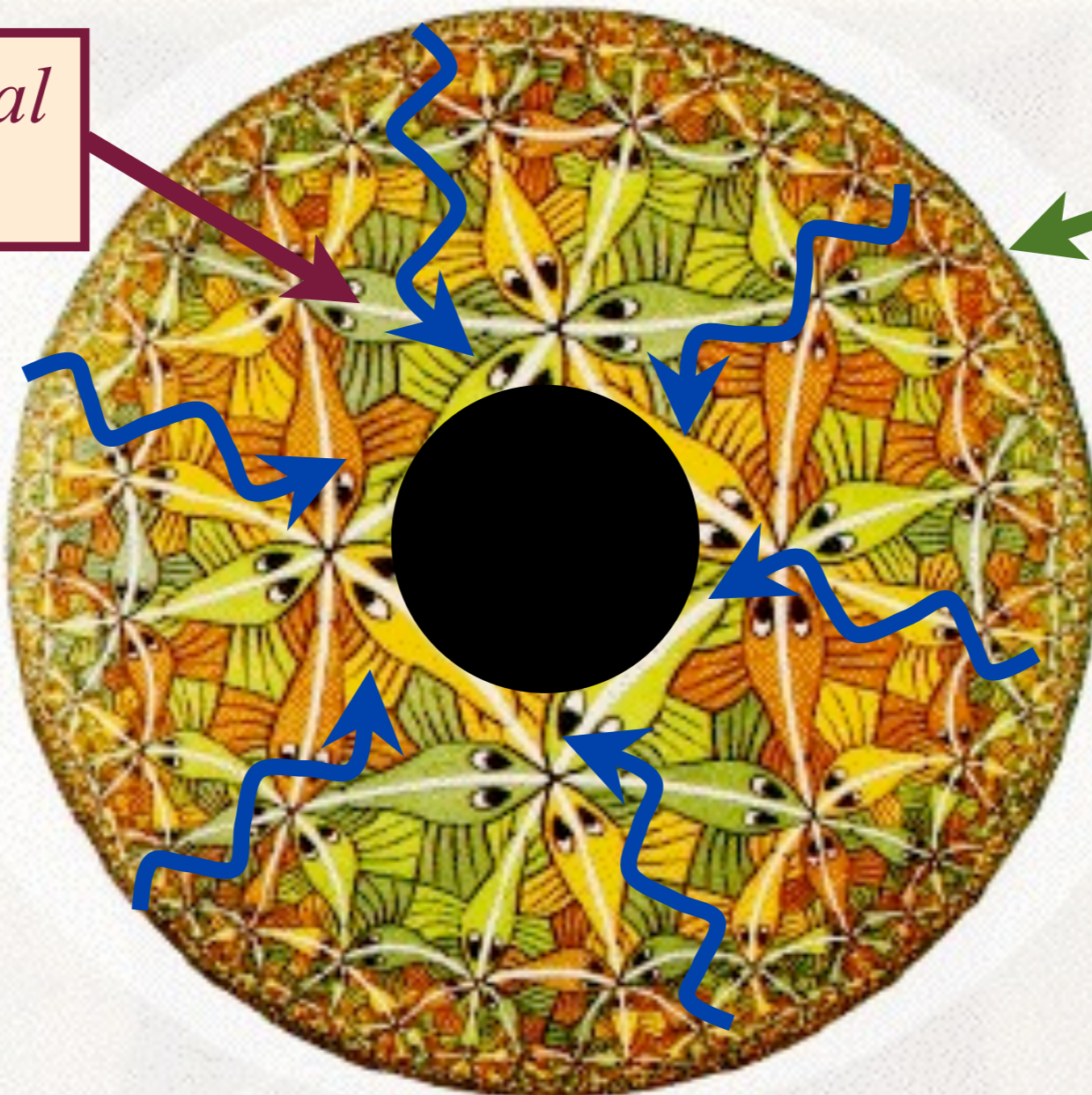
# AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions

*3+1 dimensional  
AdS space*

Quantum  
criticality in  
2+1  
dimensions

Quantum  
critical  
dynamics =  
waves in  
curved  
space

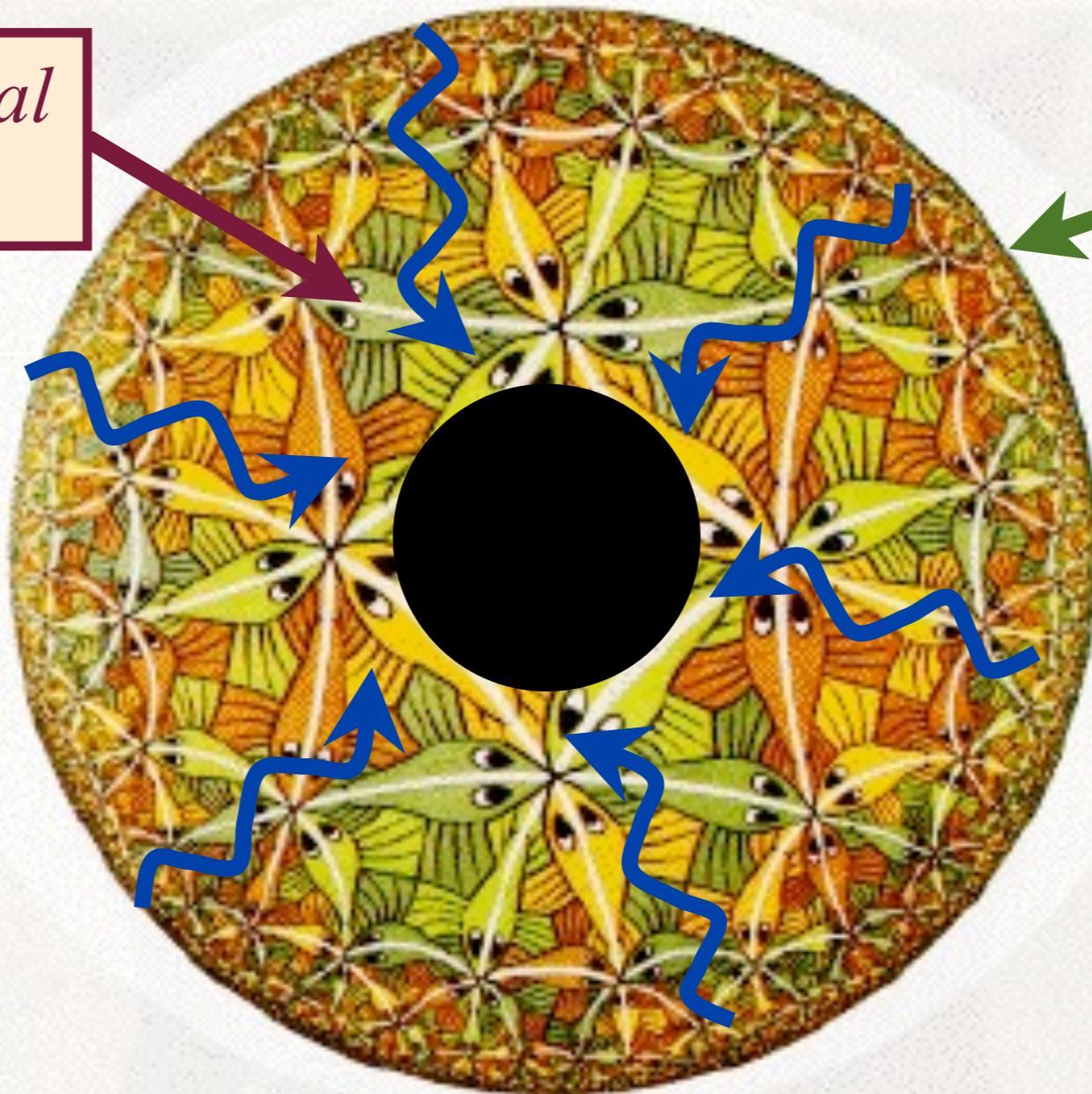


Maldacena, Gubser, Klebanov, Polyakov, Witten

# AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions

*3+1 dimensional  
AdS space*



Quantum  
criticality in  
2+1  
dimensions

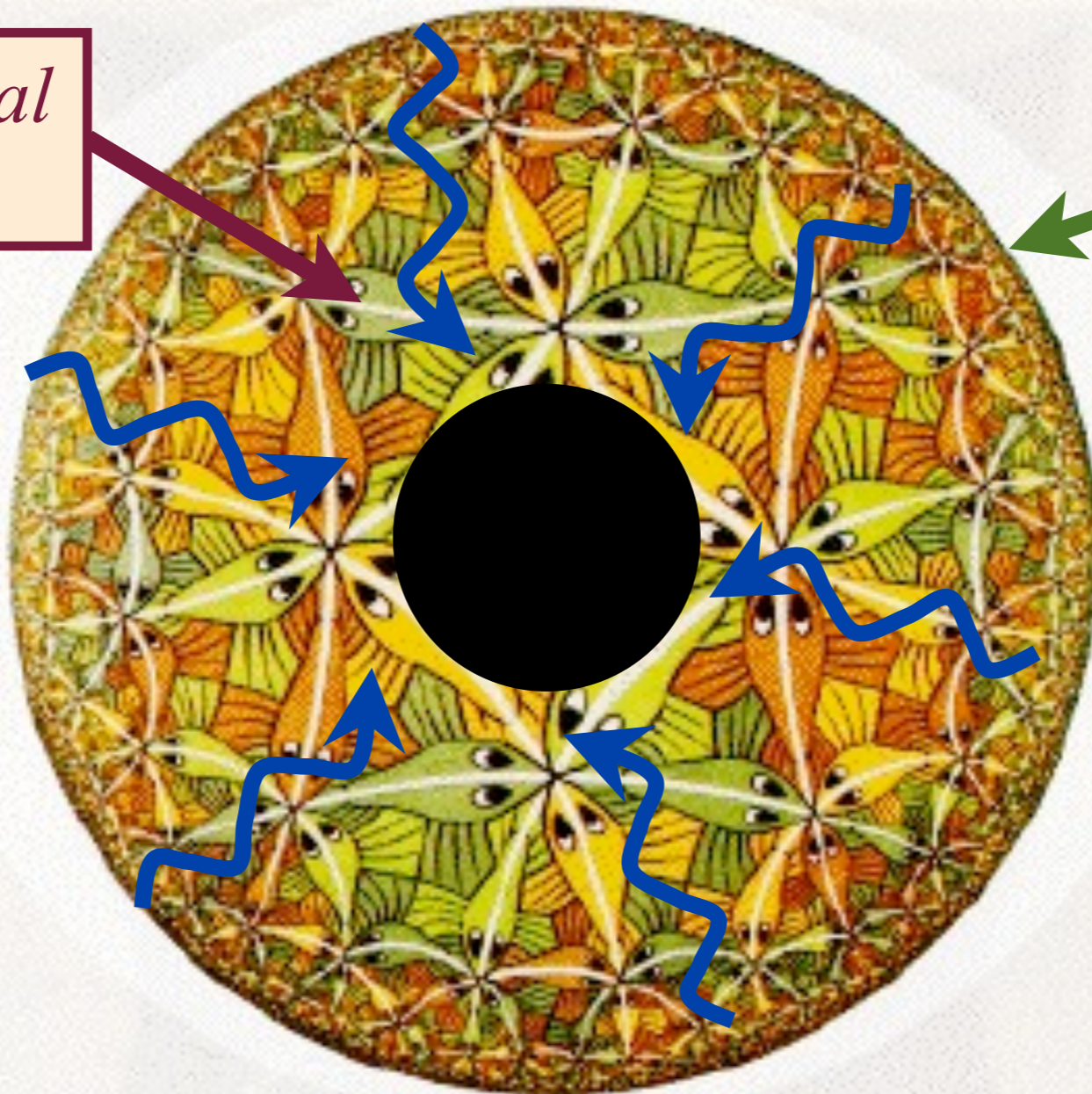
Friction of  
quantum  
criticality =  
waves  
falling into  
black hole

Kovtun, Policastro, Son

# AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions

*3+1 dimensional  
AdS space*



Quantum  
criticality in  
2+1  
dimensions

Friction of  
quantum  
criticality =  
waves  
falling into  
black hole

Kovtun, Policastro, Son

# AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-

holog

of a

Strong coupling problem I:  
General solution of  
magneto-thermo-electric transport  
in quantum critical region.

3+1 dim  
AdS

Quantum  
criticality in  
1  
dimensions

C. P. Herzog, P. Kovtun, S. Sachdev, and D. T. Son,  
*Phys. Rev. D* **75**, 085020 (2007).

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev,  
*Phys. Rev. B* **76**, 144502 (2007).

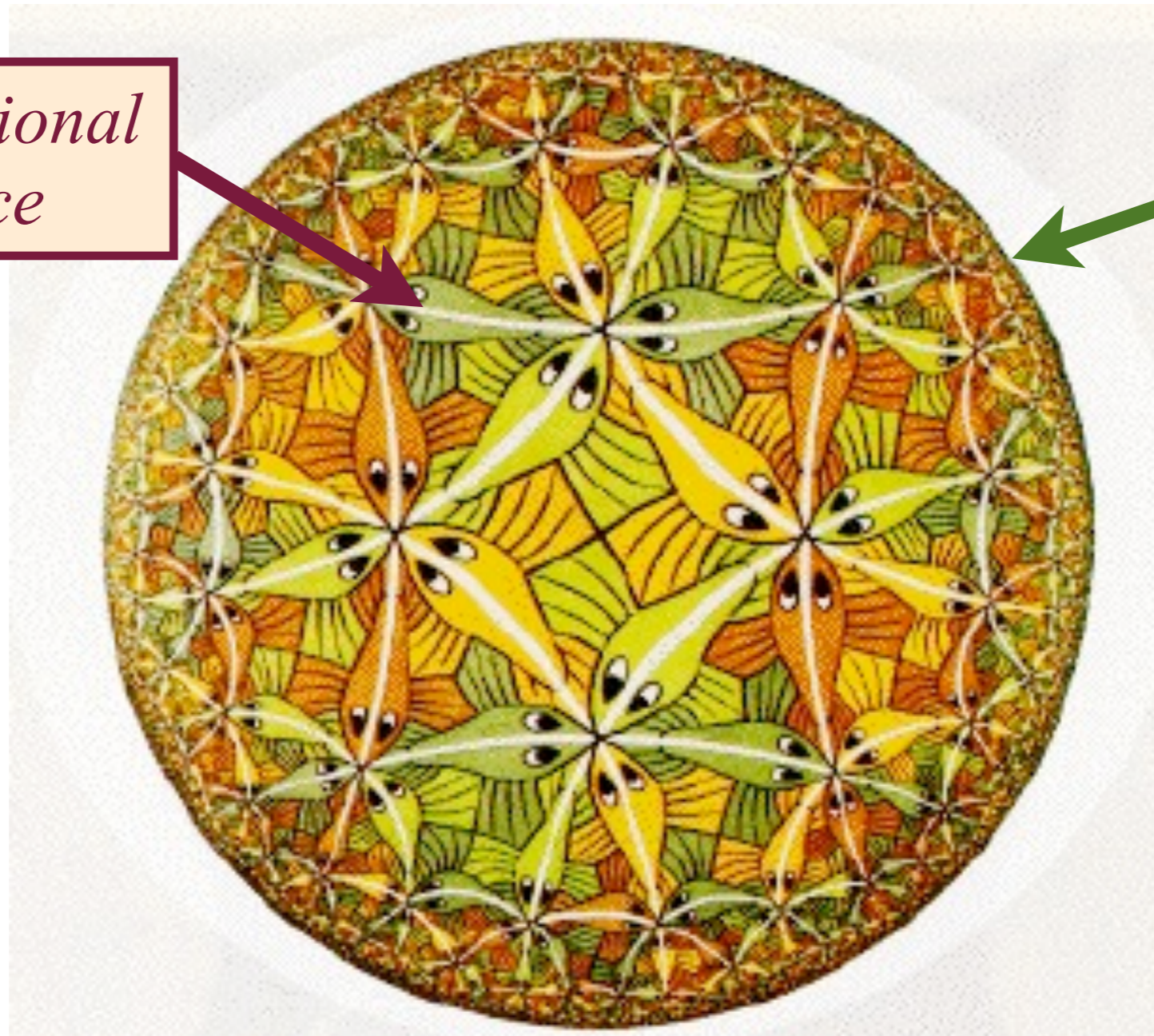
Friction  
quantum  
criticality  
waves  
falling into  
black hole



Kovtun, Policastro, Son

# AdS/CFT correspondence

*3+1 dimensional  
AdS space*



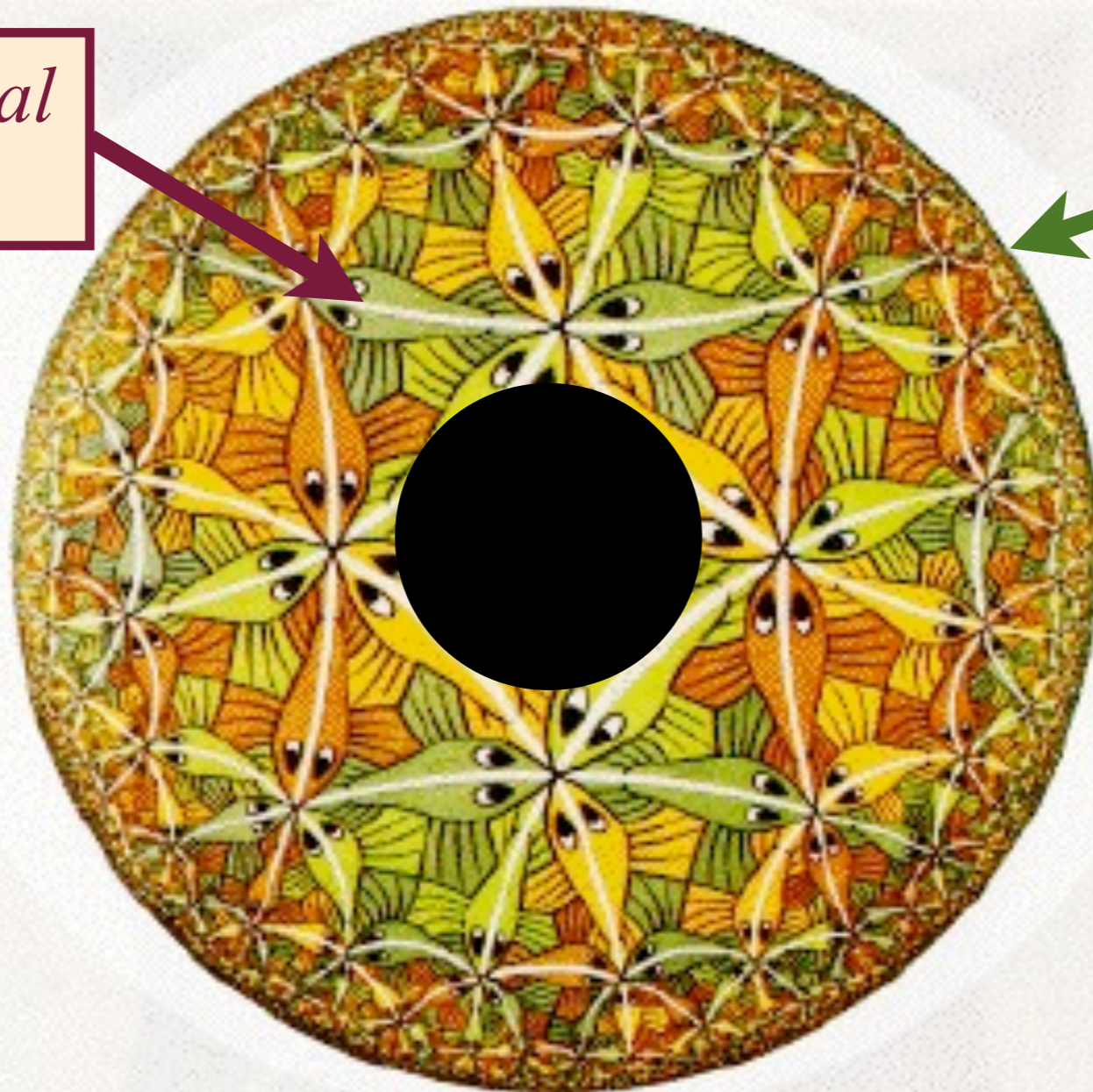
Quantum  
criticality in  
2+1  
dimensions

# AdS/CFT correspondence

Move away from the quantum critical point to a system of matter at non-zero density: equivalent to adding an electrical charge to the black hole.

*3+1 dimensional  
AdS space*

Black hole  
with  
electrical  
charge



Finite  
density  
matter in  
2+1  
dimensions

# AdS/CFT correspondence

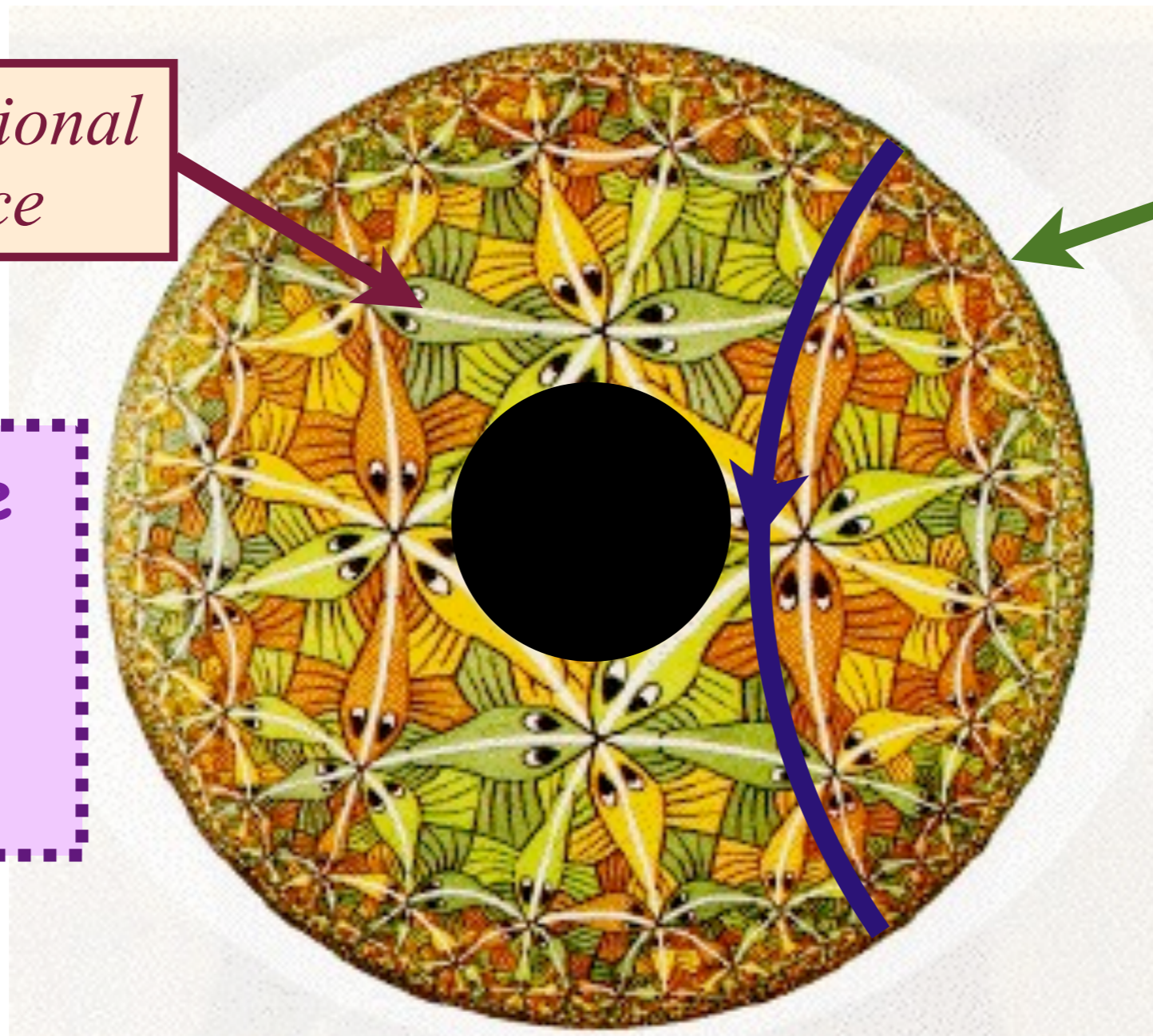
## Examine the free energy and Green's function of a probe particle

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788

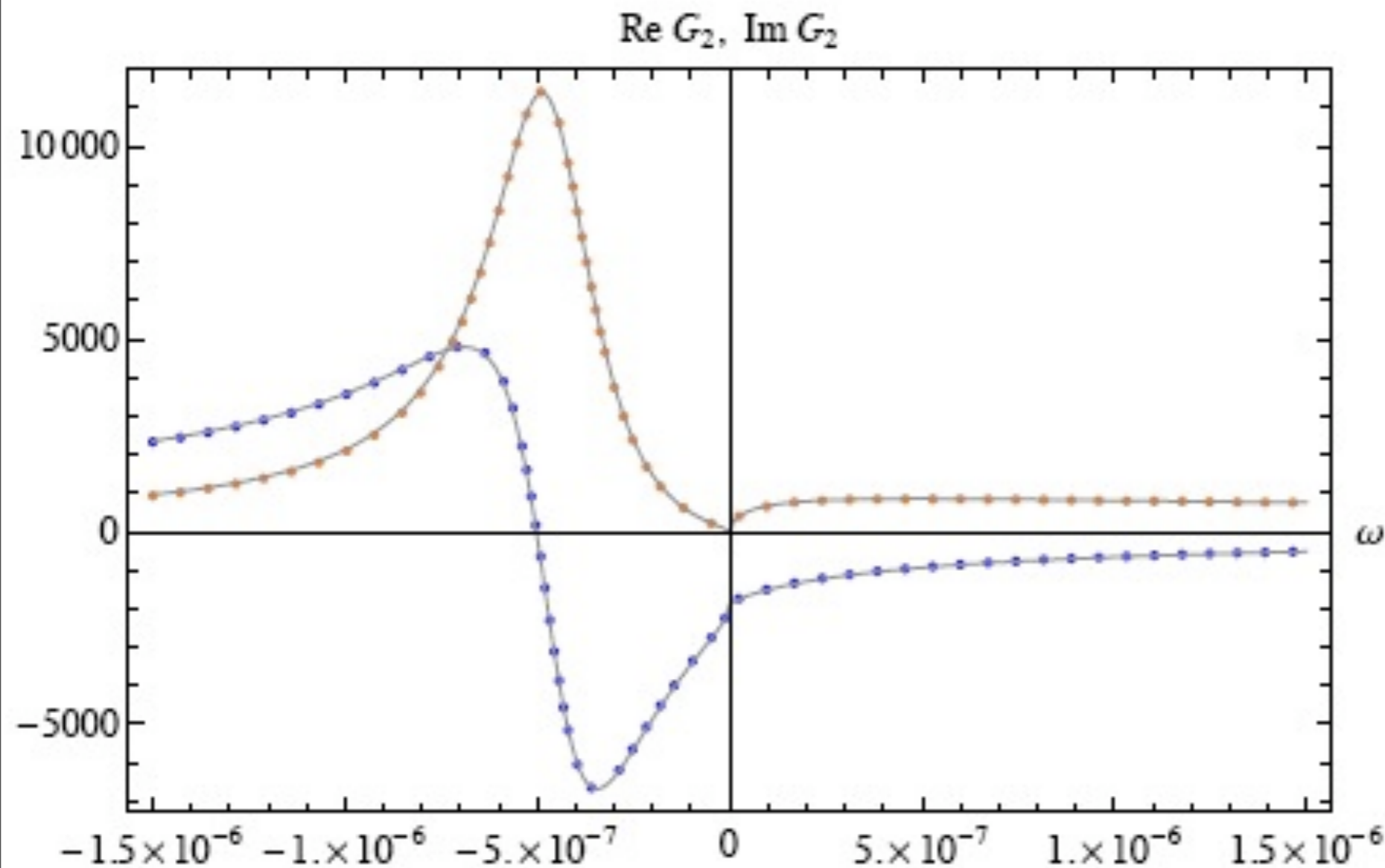
*3+1 dimensional  
AdS space*

Black hole  
with  
electrical  
charge



Finite  
density  
matter in  
2+1  
dimensions

# Green's function of a fermion



T. Faulkner, H. Liu,  
J. McGreevy, and  
D. Vegh,  
arXiv:0907.2694

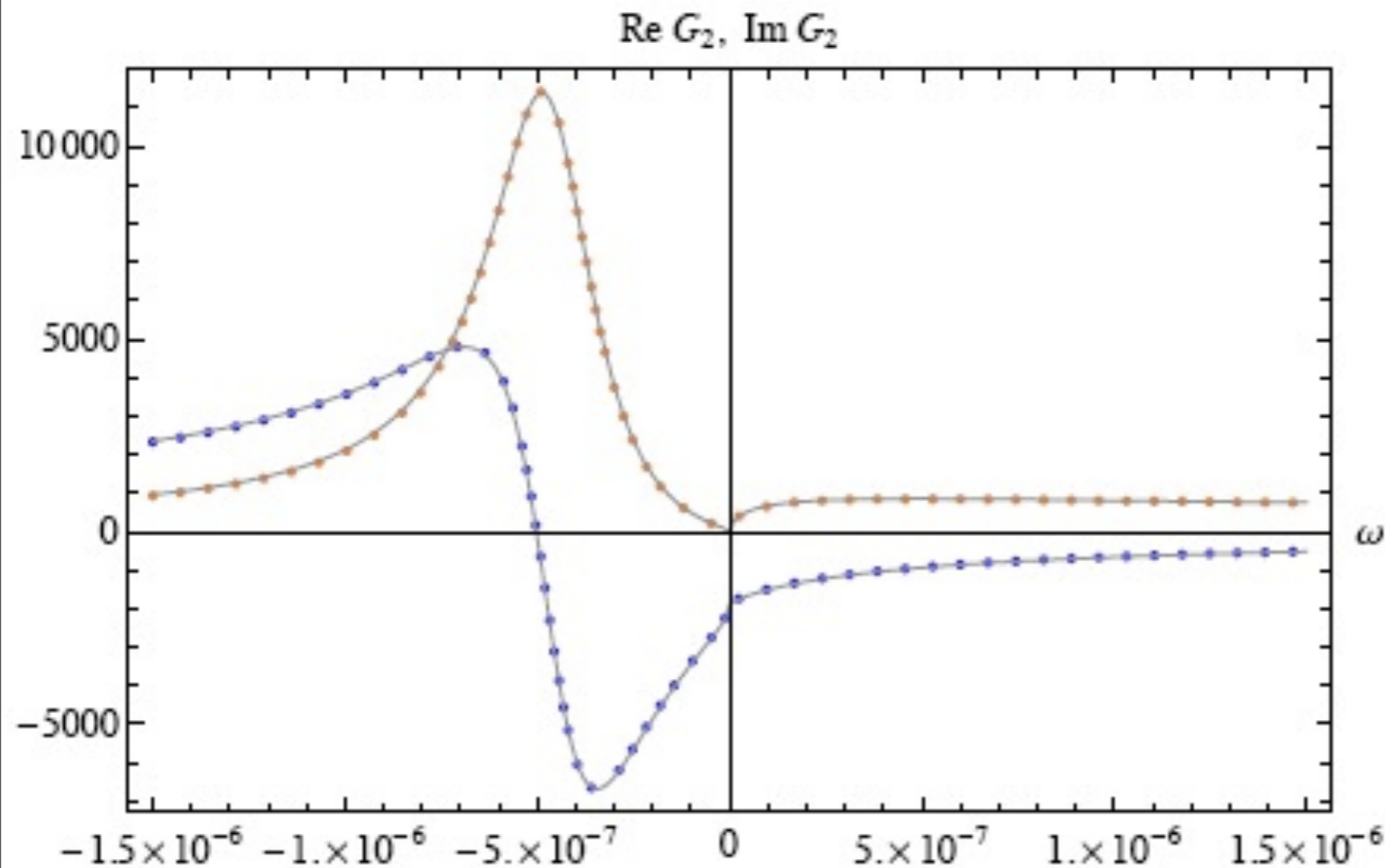
$$G(k, \omega) \approx \frac{1}{\omega - v_F(k - k_F) - i\omega^\theta(k)}$$

See also S.-S. Lee, *Phys. Rev. D* **79**, 086006 (2009);

M. Cubrovic, J. Zaanen, and K. Schalm, *Science* **325**, 439 (2009);

F. Denef, S.A. Hartnoll, and S. Sachdev, *Phys. Rev. D* **80**, 126016 (2009)

# Green's function of a fermion

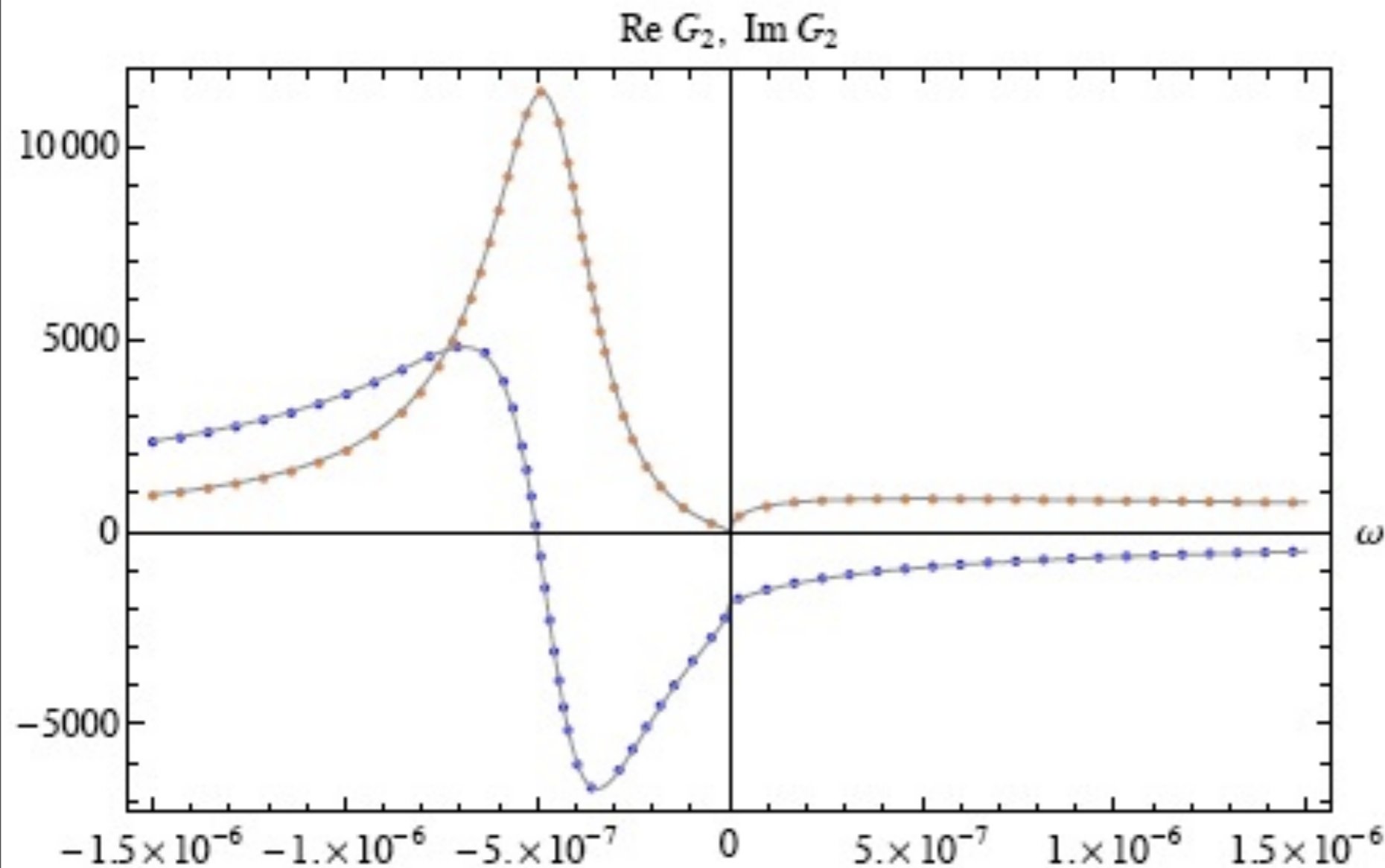


T. Faulkner, H. Liu,  
J. McGreevy, and  
D. Vegh,  
arXiv:0907.2694

$$G(k, \omega) \approx \frac{1}{\omega - v_F(k - k_F) - i\omega^\theta(k)}$$

Similar to our theory of the singular Fermi surface  
near the Ising-nematic quantum critical point

# Green's function of a fermion

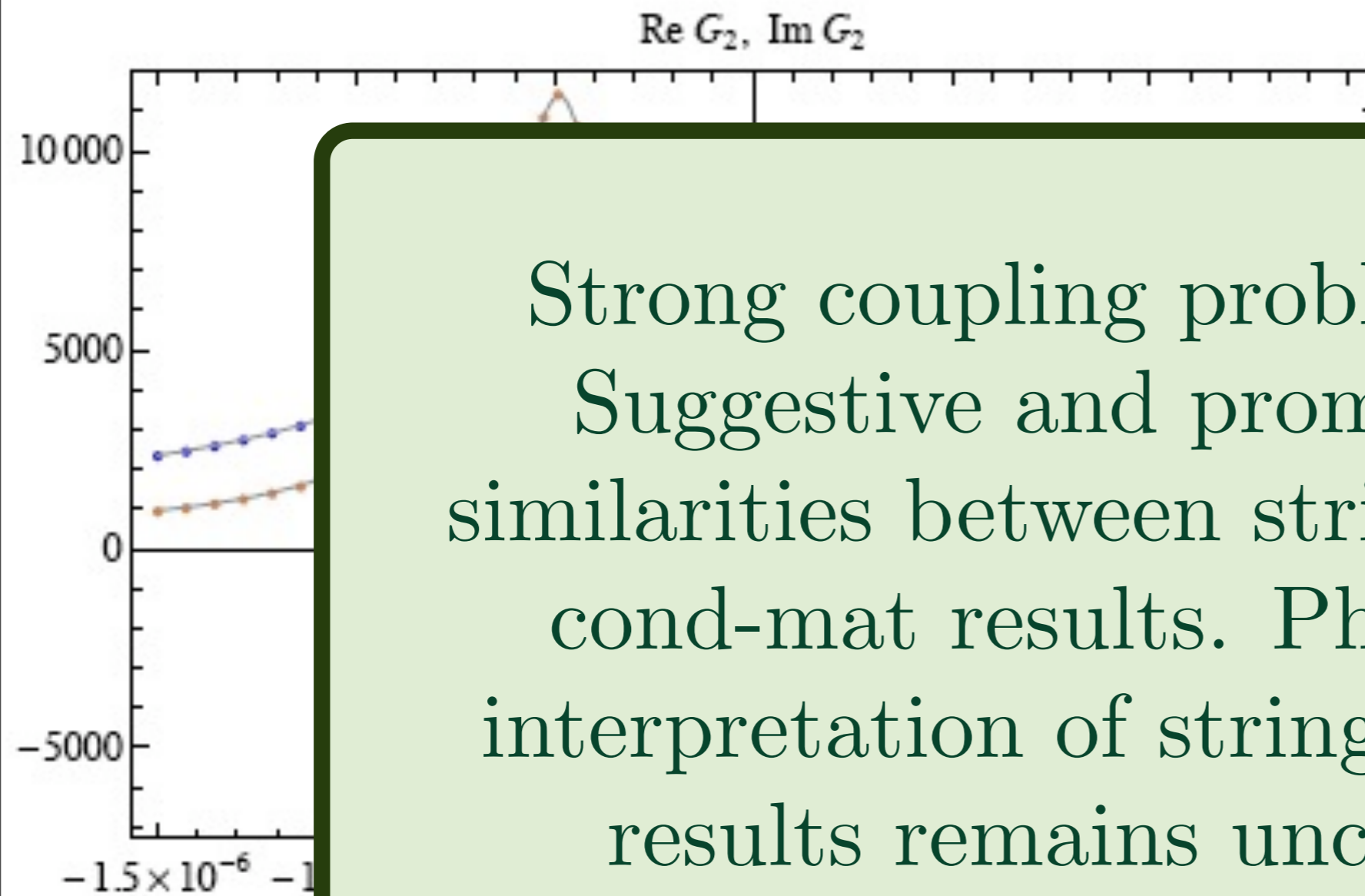


T. Faulkner, H. Liu,  
J. McGreevy, and  
D. Vegh,  
arXiv:0907.2694

$$G(k, \omega) \approx \frac{1}{\omega - v_F(k - k_F) - i\omega^\theta(k)}$$

Similar to our theory of the singular Fermi surface  
near the Ising-nematic quantum critical point

# Green's function of a fermion



Strong coupling problem II:  
Suggestive and promising similarities between stringy and cond-mat results. Physical interpretation of string theory results remains unclear.

H. Liu,  
y, and  
n,  
.2694

$$G(k, \omega) \sim \frac{1}{\omega - v_F(k - k_F) - i\omega^\theta(k)}$$

Similar to our theory of the singular Fermi surface near the Ising-nematic quantum critical point

# Outline

1. Coupled dimer antiferromagnets  
*Introduction to quantum criticality*
2. Theory of Ising-nematic ordering in the cuprate metals  
*Strongly-coupled field theory*
3. The AdS/CFT correspondence  
*Phases of quantum matter at strong coupling*
4. Global phase diagram of the cuprates  
*Competition between d-wave superconductivity and spin-density order for the Fermi surface*

# Outline

## 1. Coupled dimer antiferromagnets

*Introduction to quantum criticality*

## 2. Theory of Ising-nematic ordering in the cuprate metals

*Strongly-coupled field theory*

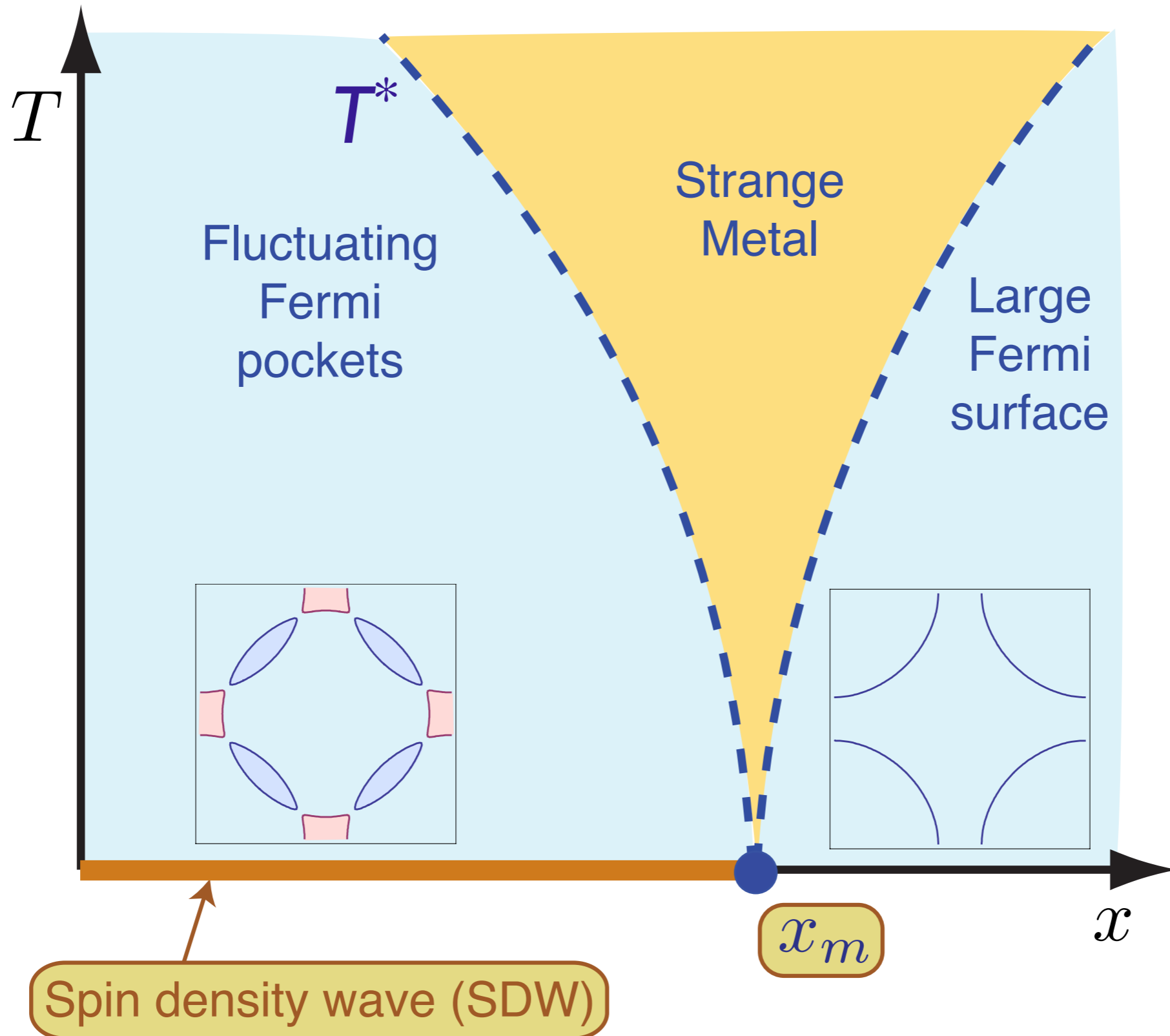
## 3. The AdS/CFT correspondence

*Phases of quantum matter  
at strong coupling*

## 4. Global phase diagram of the cuprates

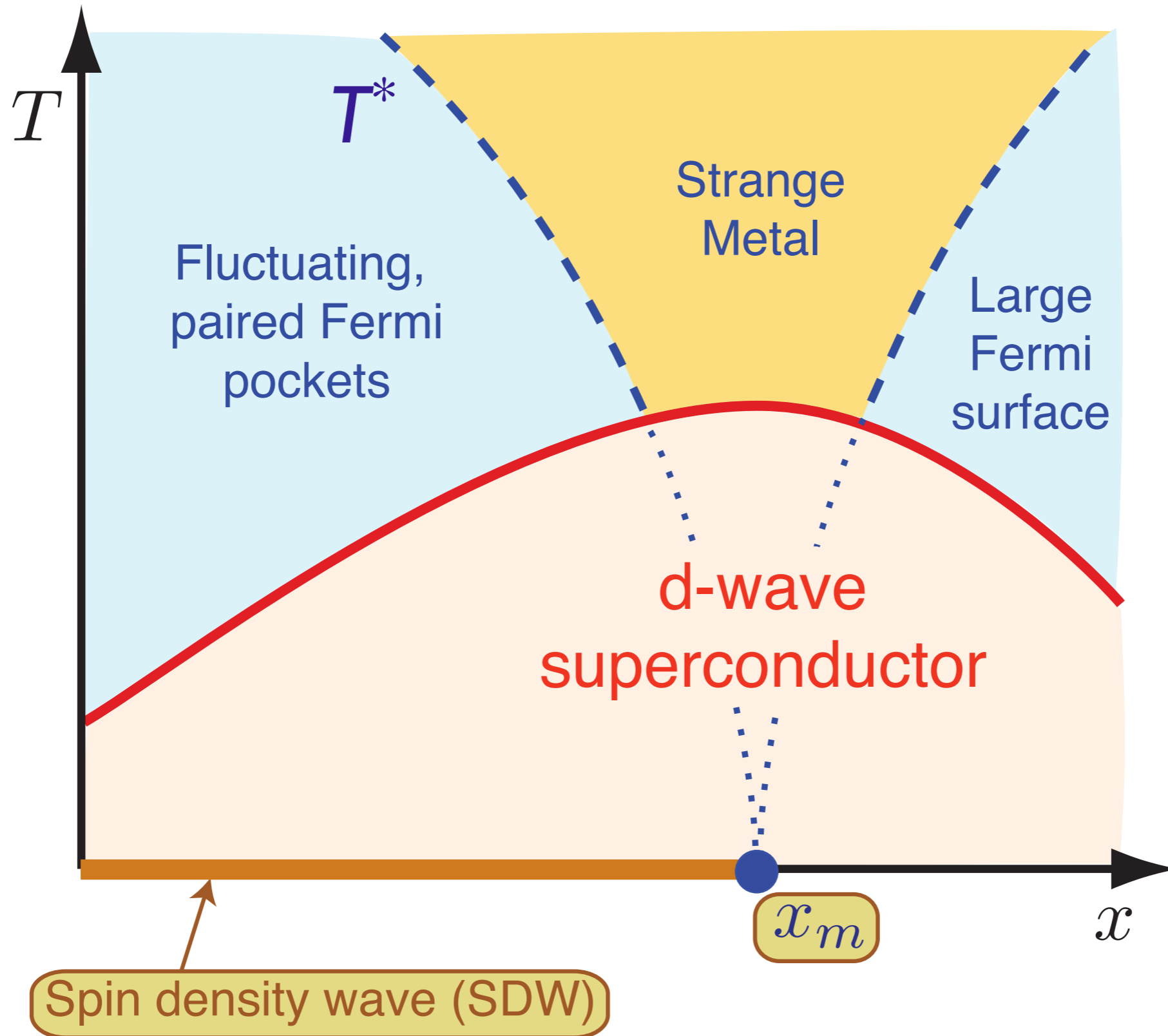
*Competition between d-wave superconductivity  
and spin-density order for the Fermi surface*

# Theory of quantum criticality in the cuprates



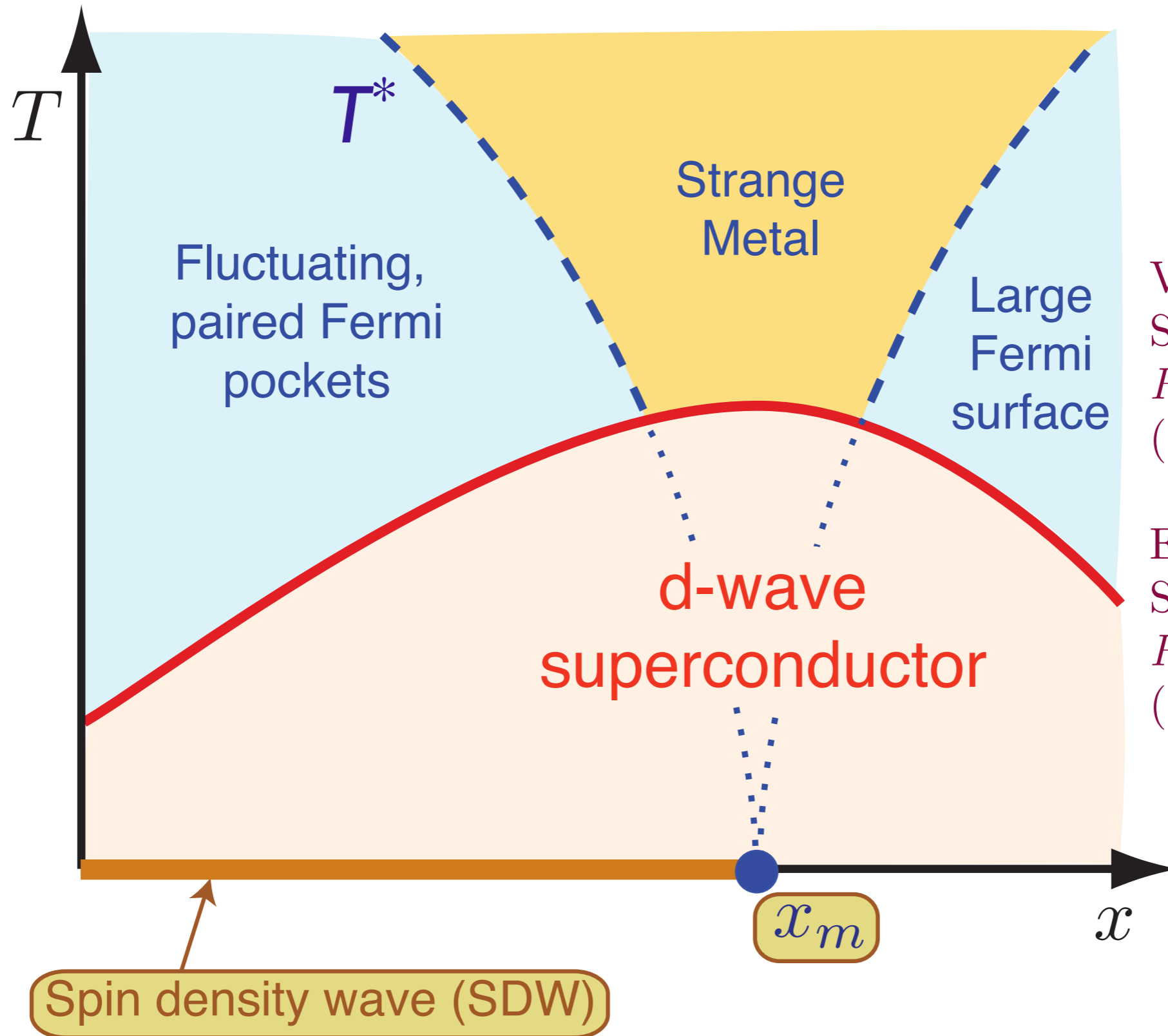
Underlying SDW ordering quantum critical point in metal at  $x = x_m$ ; has associated Ising-nematic order

# Theory of quantum criticality in the cuprates



Onset of  $d$ -wave superconductivity  
hides the SDW critical point  $x = x_m$

# Theory of quantum criticality in the cuprates

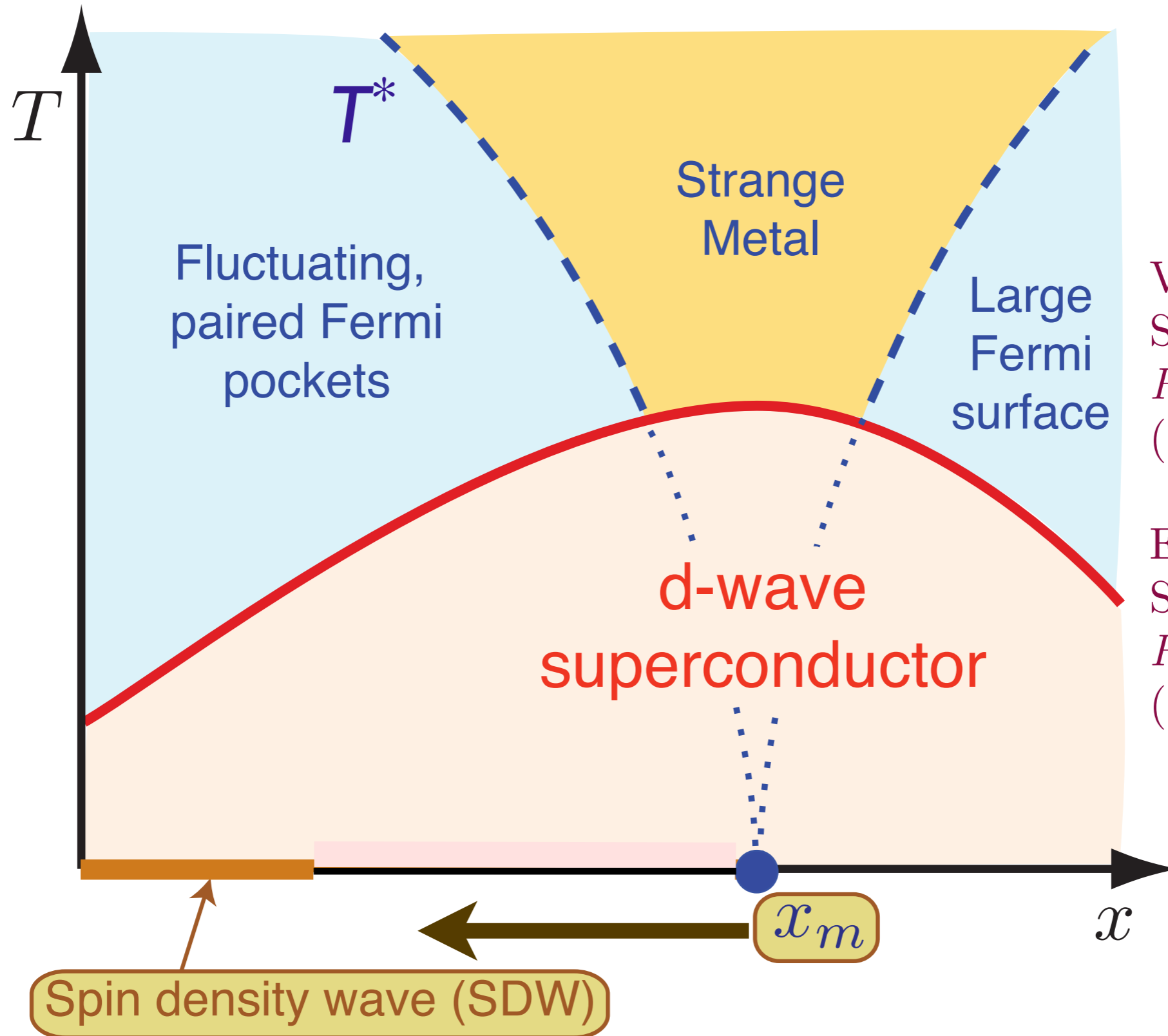


V. Galitski and S. Sachdev, *Phys. Rev. B* **79**, 134512 (2009).

E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual SDW quantum critical point to  $x = x_s < x_m$ .

# Theory of quantum criticality in the cuprates

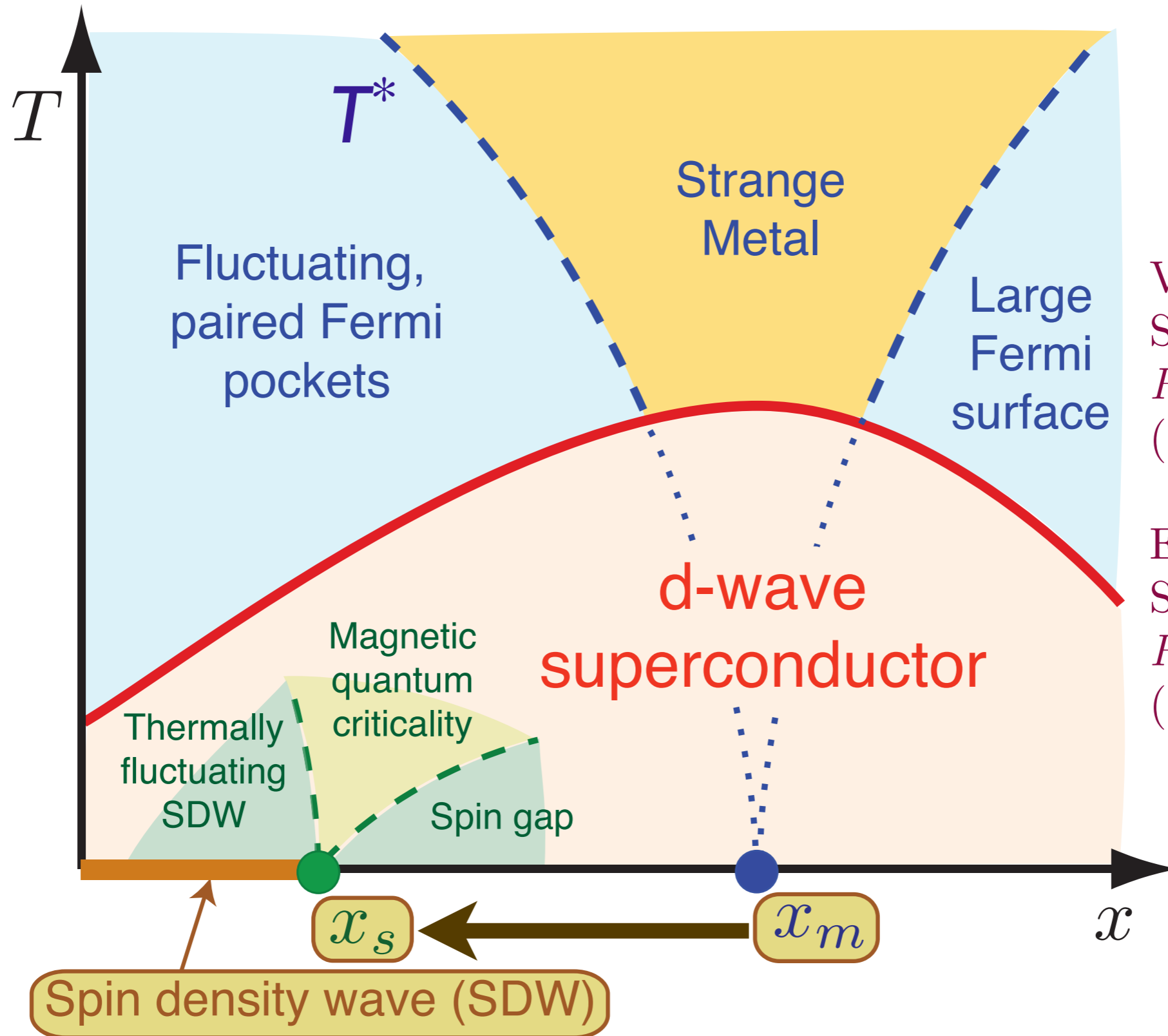


V. Galitski and S. Sachdev, *Phys. Rev. B* **79**, 134512 (2009).

E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual SDW quantum critical point to  $x = x_s < x_m$ .

# Theory of quantum criticality in the cuprates

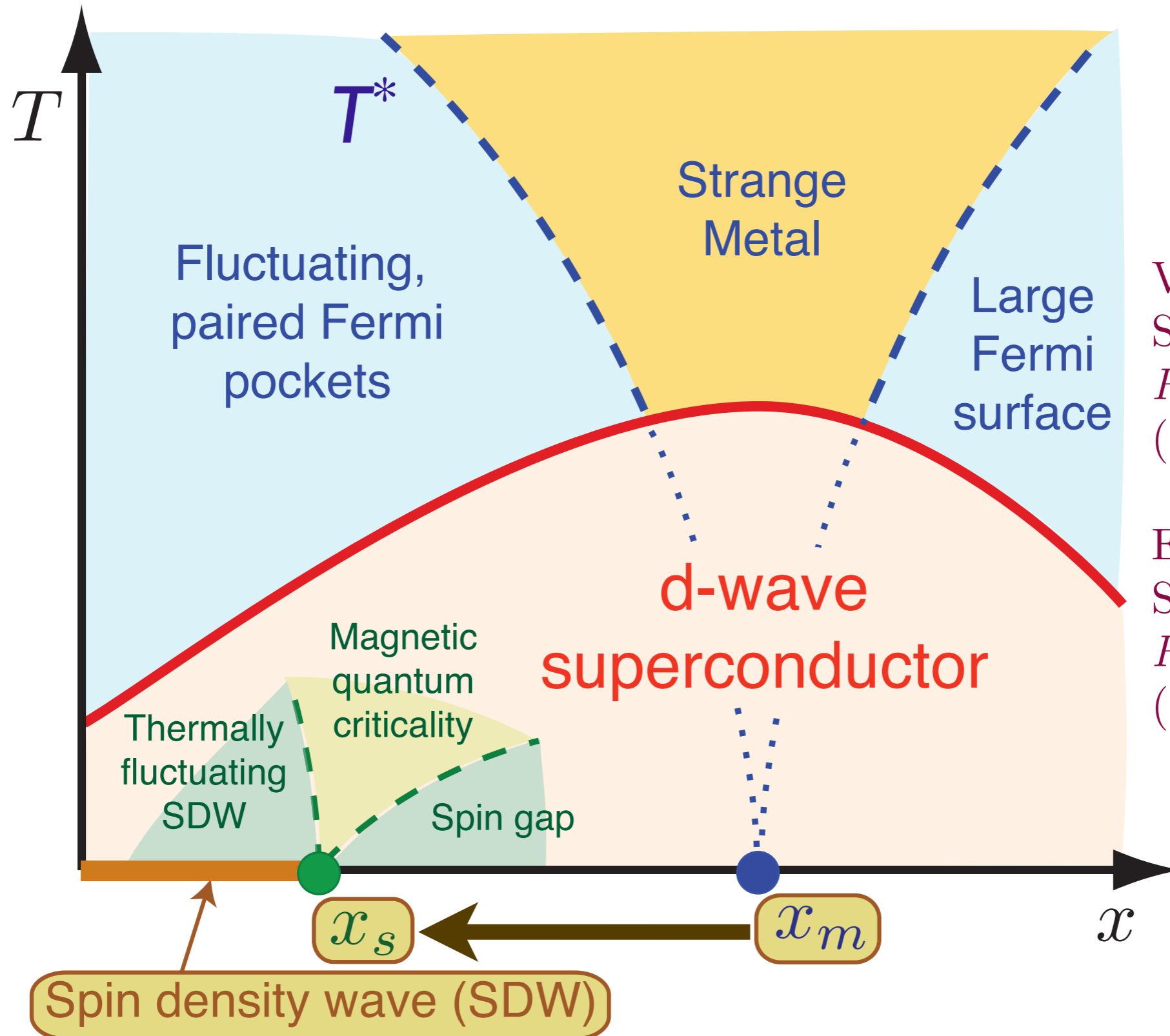


V. Galitski and S. Sachdev, *Phys. Rev. B* **79**, 134512 (2009).

E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual SDW quantum critical point to  $x = x_s < x_m$ .

# Theory of quantum criticality in the cuprates



V. Galitski and S. Sachdev, *Phys. Rev. B* **79**, 134512 (2009).

E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

Physics of competition:  $d$ -wave SC and SDW  
“eat up” same pieces of the large Fermi surface.

# Shift of ordering critical point in a metal due to onset of superconductivity

SDW ordering:

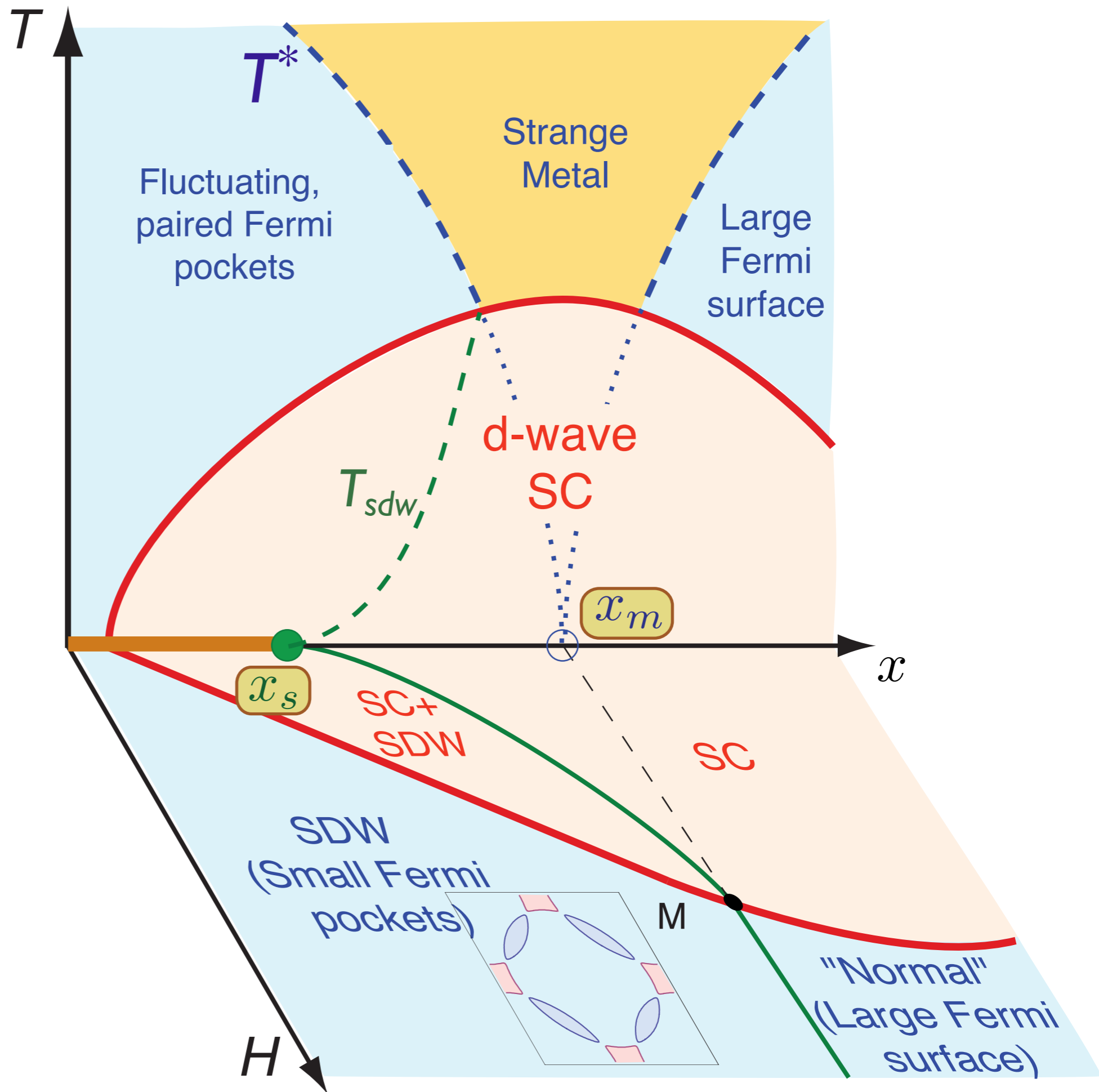
$$x_s = x_m - c|\Delta|$$

Ising-nematic ordering:

$$\tilde{x}_s = \tilde{x}_m - \tilde{c}|\Delta|^2$$

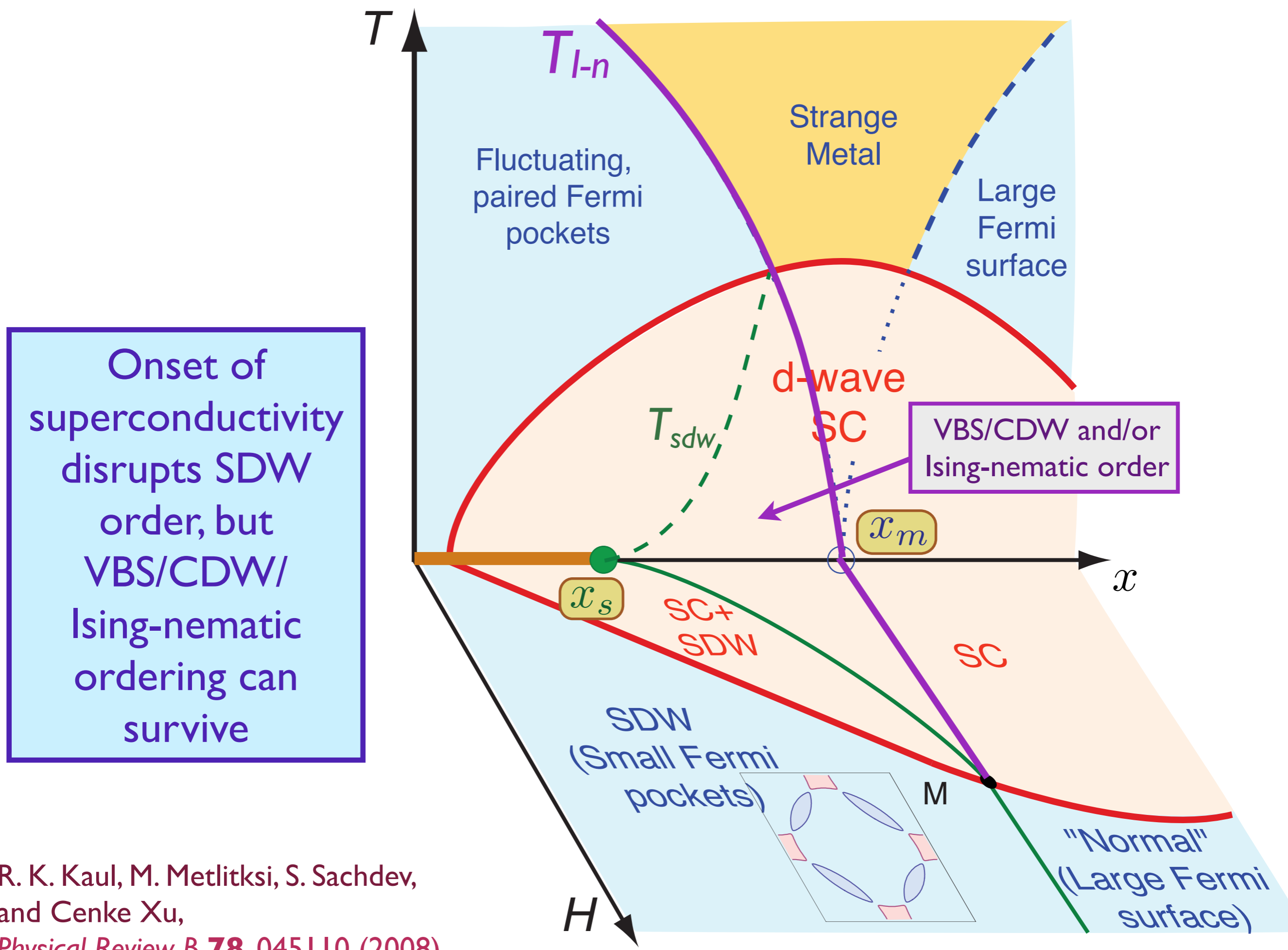
$\Delta$  is the fermion pairing gap.

E.G. Moon and S. Sachdev, to appear



E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)



Onset of superconductivity disrupts SDW order, but VBS/CDW/Ising-nematic ordering can survive

R. K. Kaul, M. Metlitski, S. Sachdev, and Cenke Xu, *Physical Review B* **78**, 045110 (2008).

## Conclusions

Theories for the onset of Ising-nematic order (and spin density wave order) in metals are strongly coupled in two dimensions

# Conclusions

The AdS/CFT offers promise in providing a new understanding of strongly interacting quantum matter at non-zero density