Quantum phase transitions in condensed matter physics, with connections to string theory

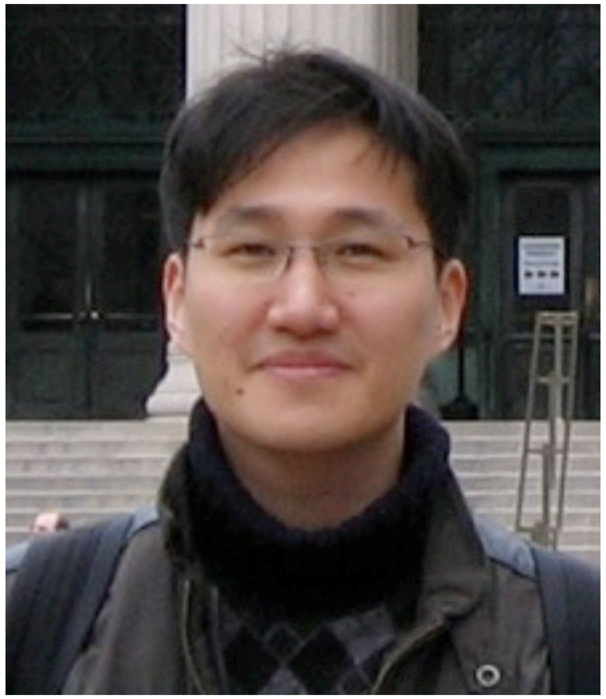
Amherst College,
October 27, 2011

PHYSICS

VE RI TAS

HARVARD

sachdev.physics.harvard.edu



Max Metlitski, Harvard

Eun Gook Moon, Harvard

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Outline

1. Quantum critical points and string theory
   Entanglement and emergent dimensions

2. High temperature superconductors
   and strange metals
   Holography of compressible quantum phases

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1. Quantum critical points and string theory

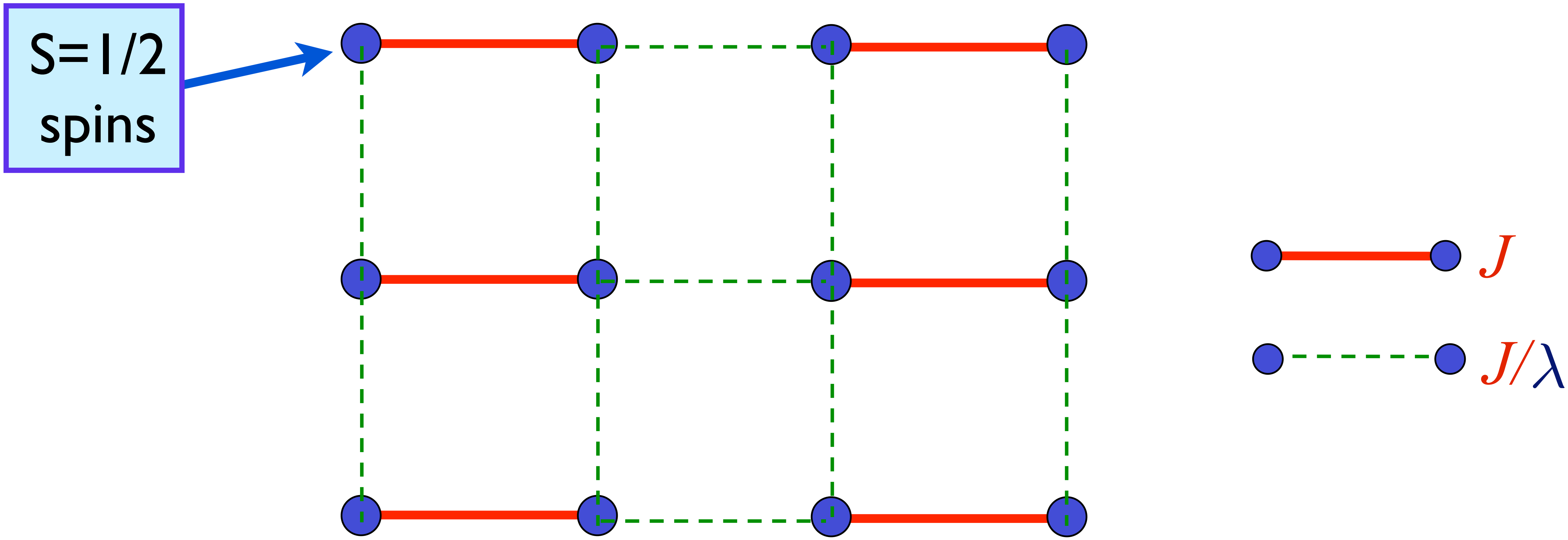
   Entanglement and emergent dimensions

2. High temperature superconductors and strange metals

   Holography of compressible quantum phases

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Examine ground state as a function of λ

Square lattice antiferromagnet

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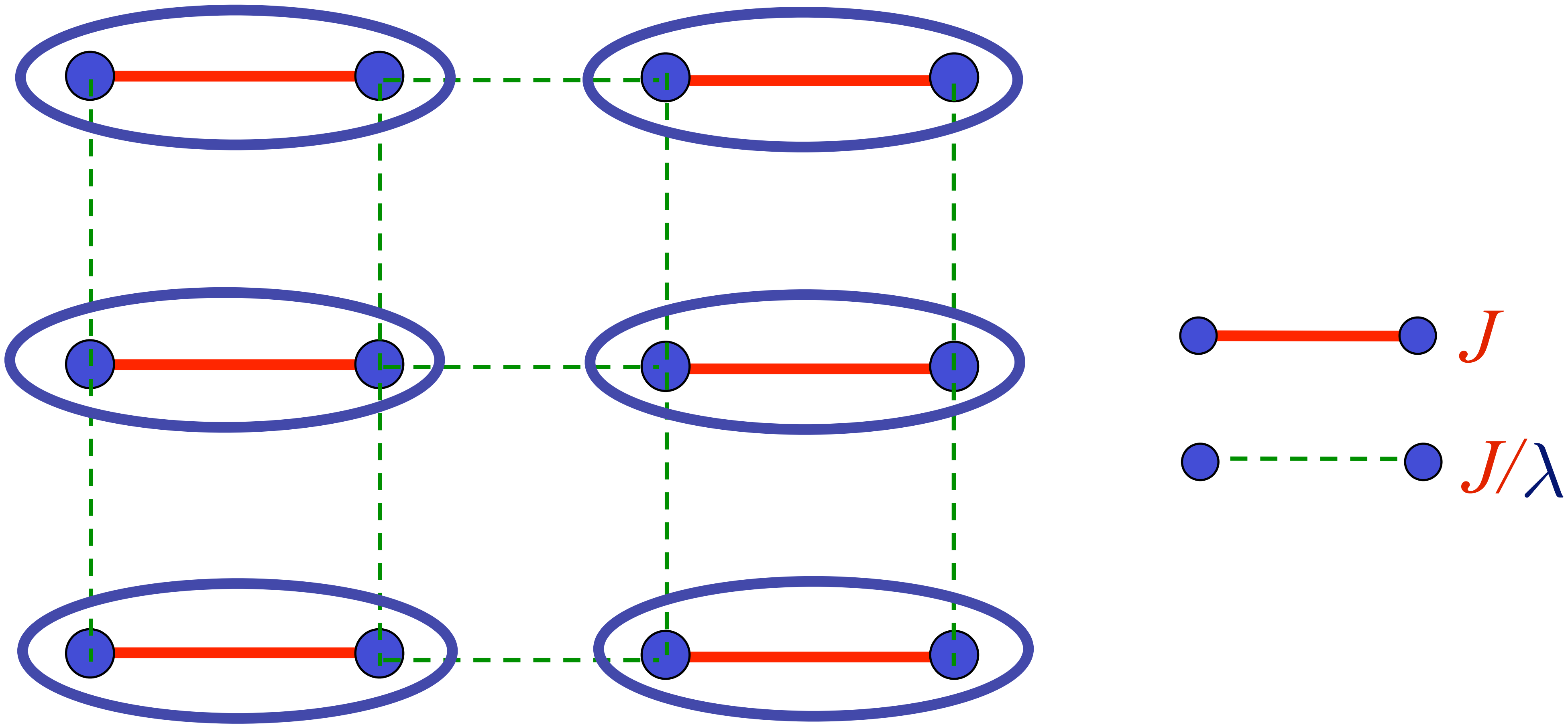

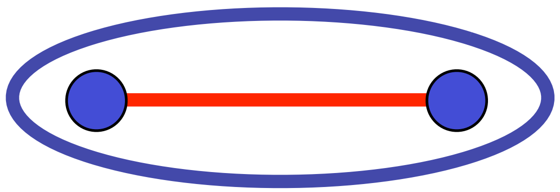

$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

At large λ ground state is a "quantum paramagnet" with spins locked in valence bond singlets

Square lattice antiferromagnet

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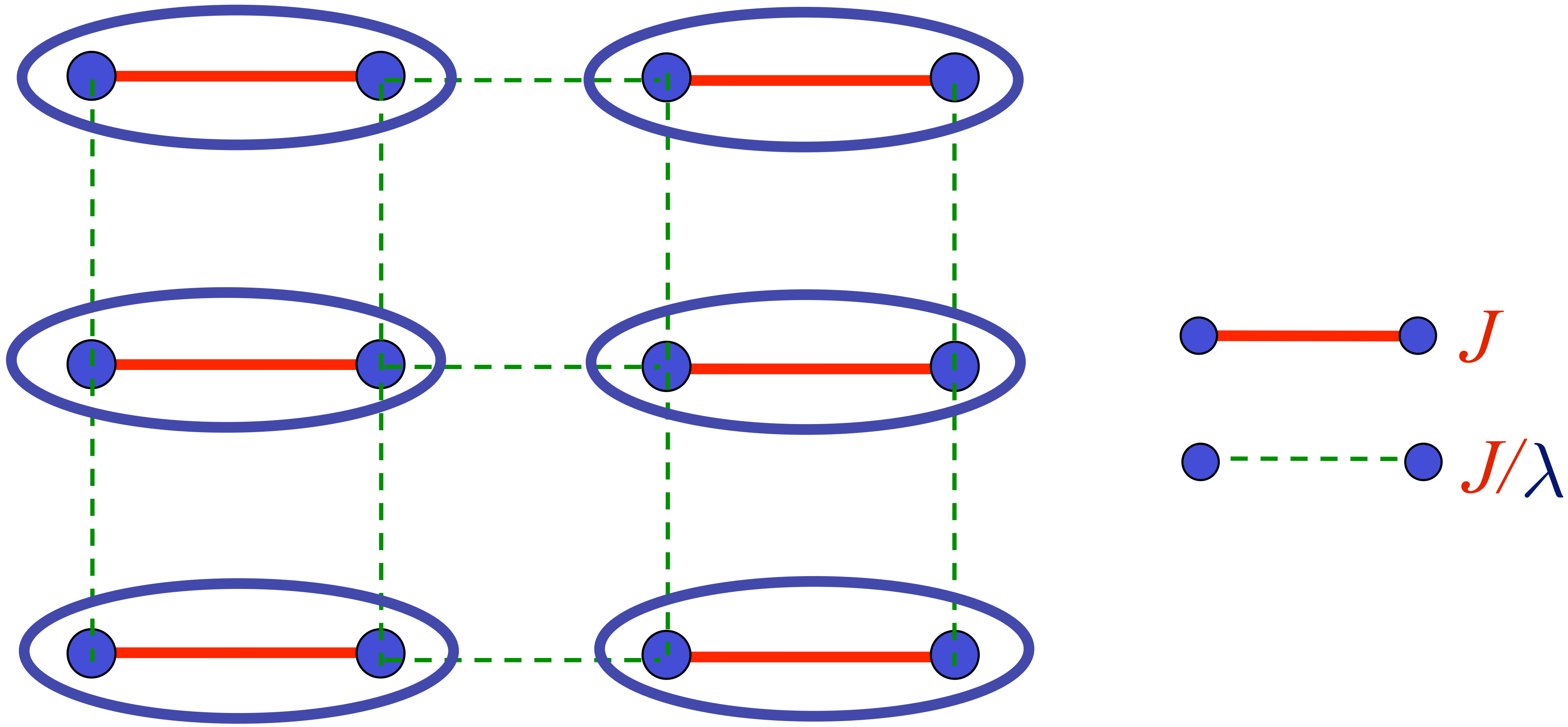

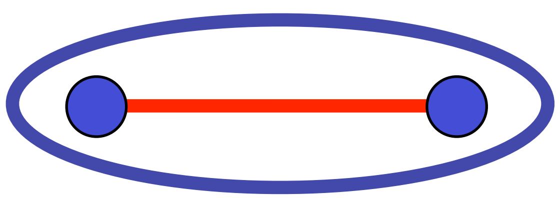

$$= \frac{1}{\sqrt{2}} \Big(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \Big)$$

Nearest-neighor spins are "entangled" with each other.
Can be separated into an Einstein-Podolsky-Rosen (EPR) pair.

Square lattice antiferromagnet

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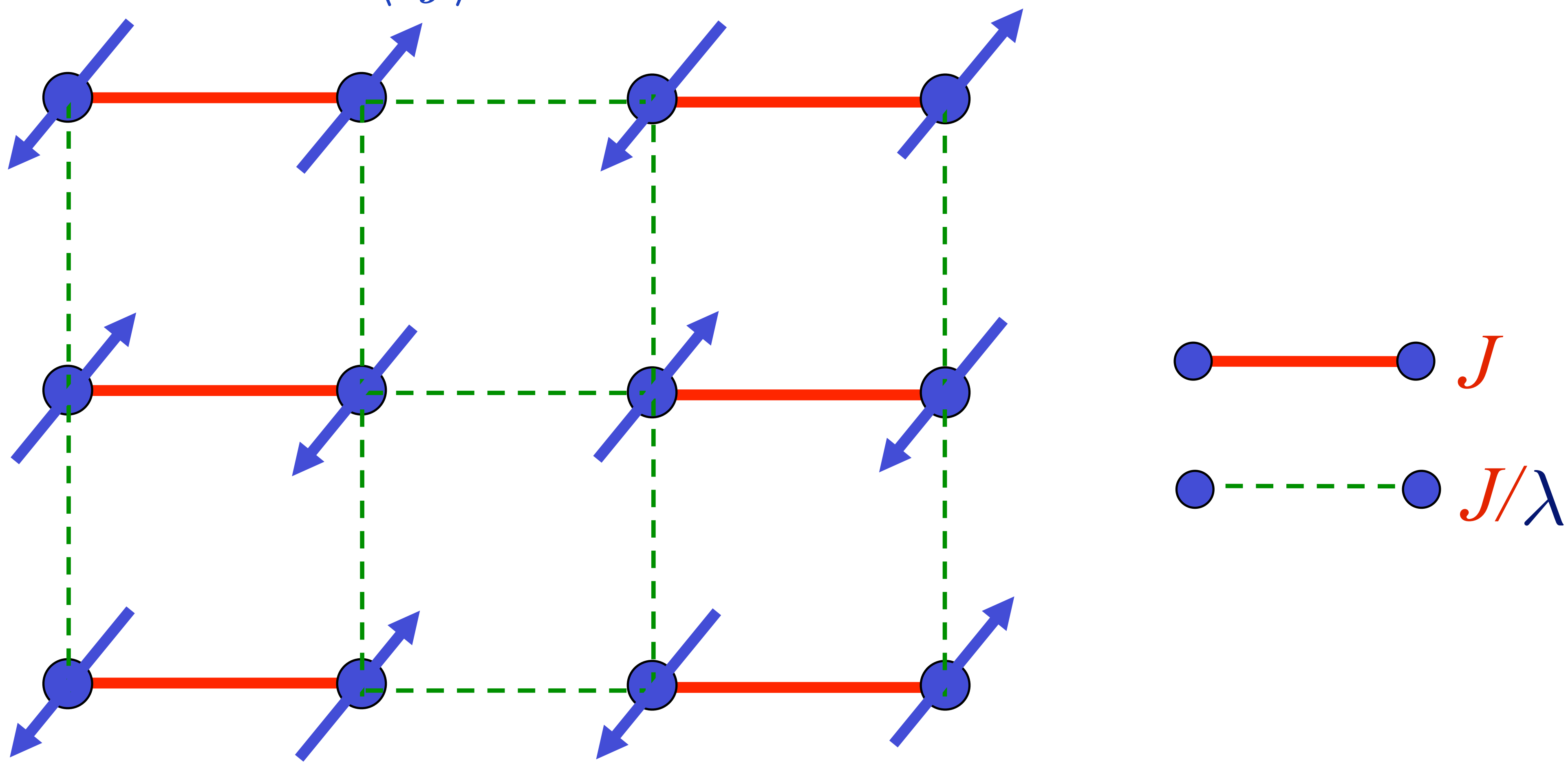

For $\lambda \approx 1$, the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern

Square lattice antiferromagnet

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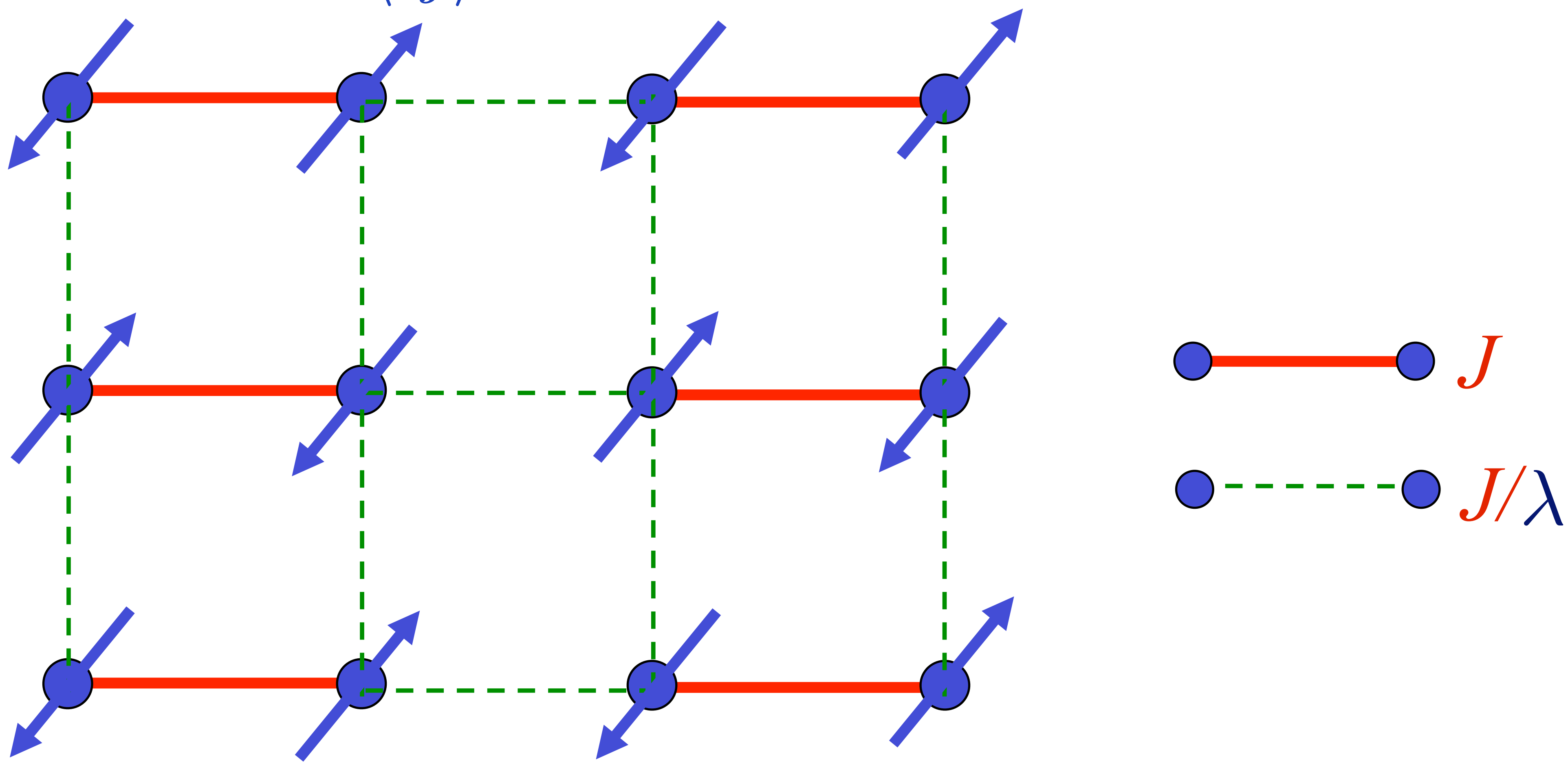

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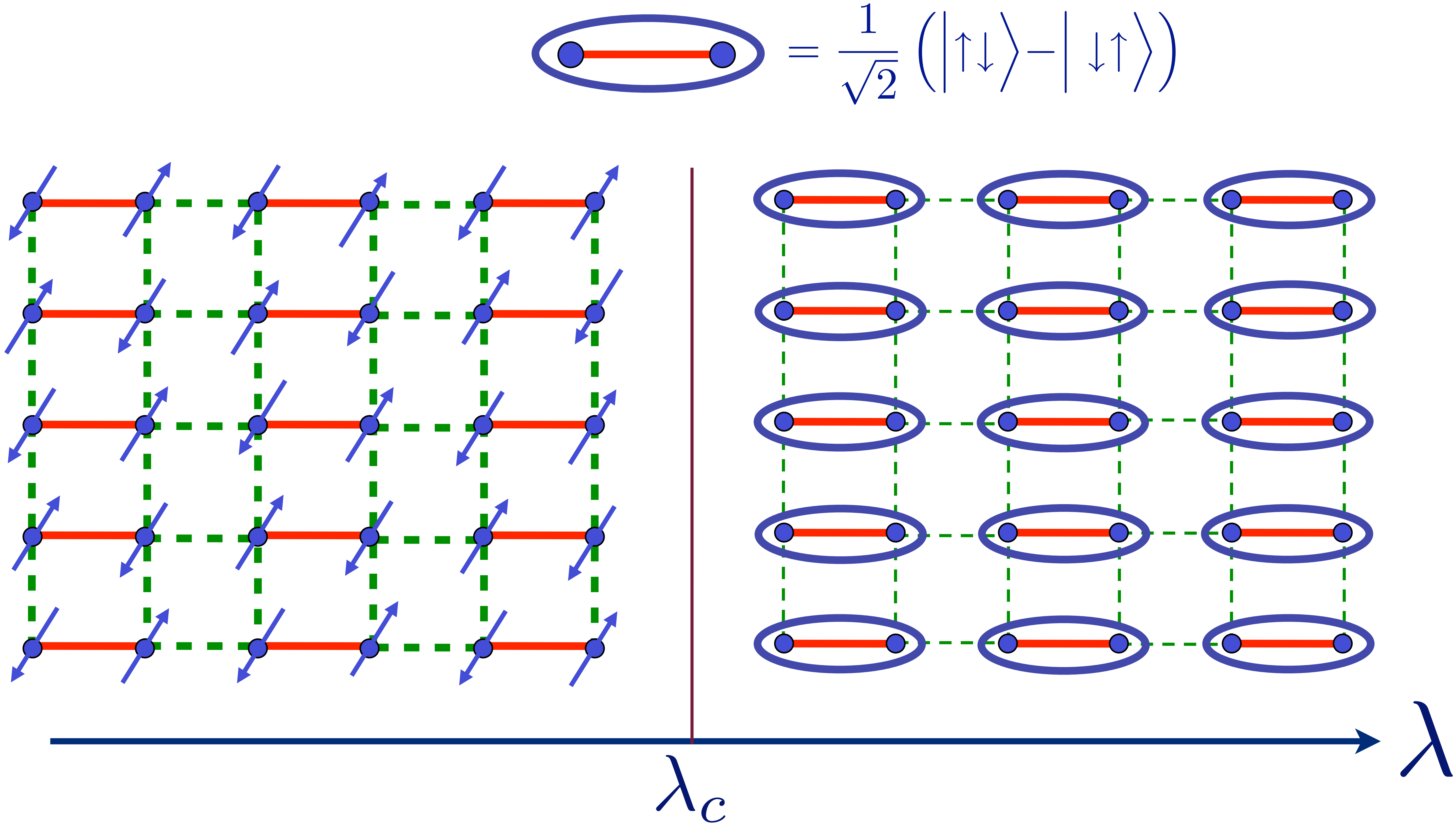
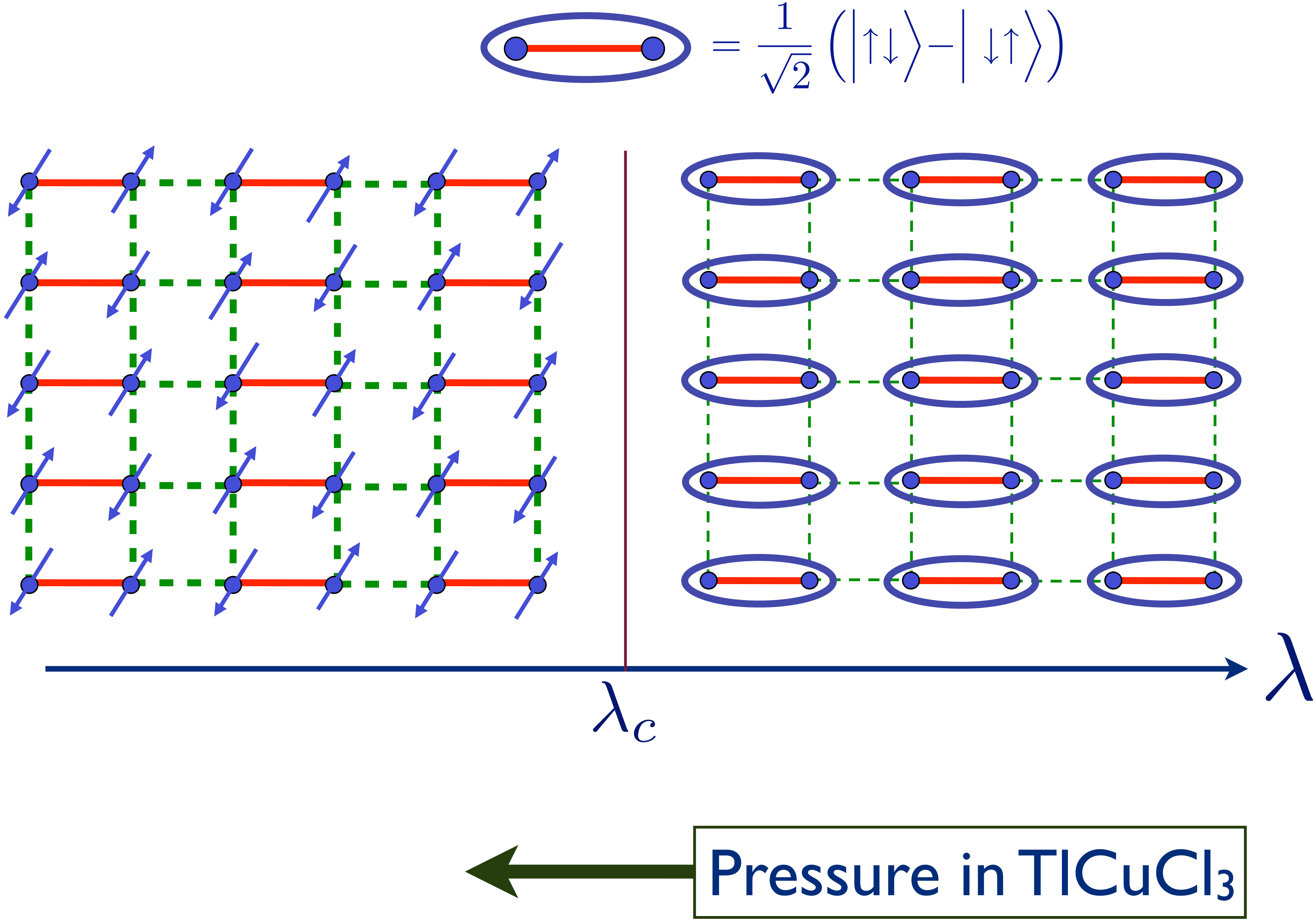
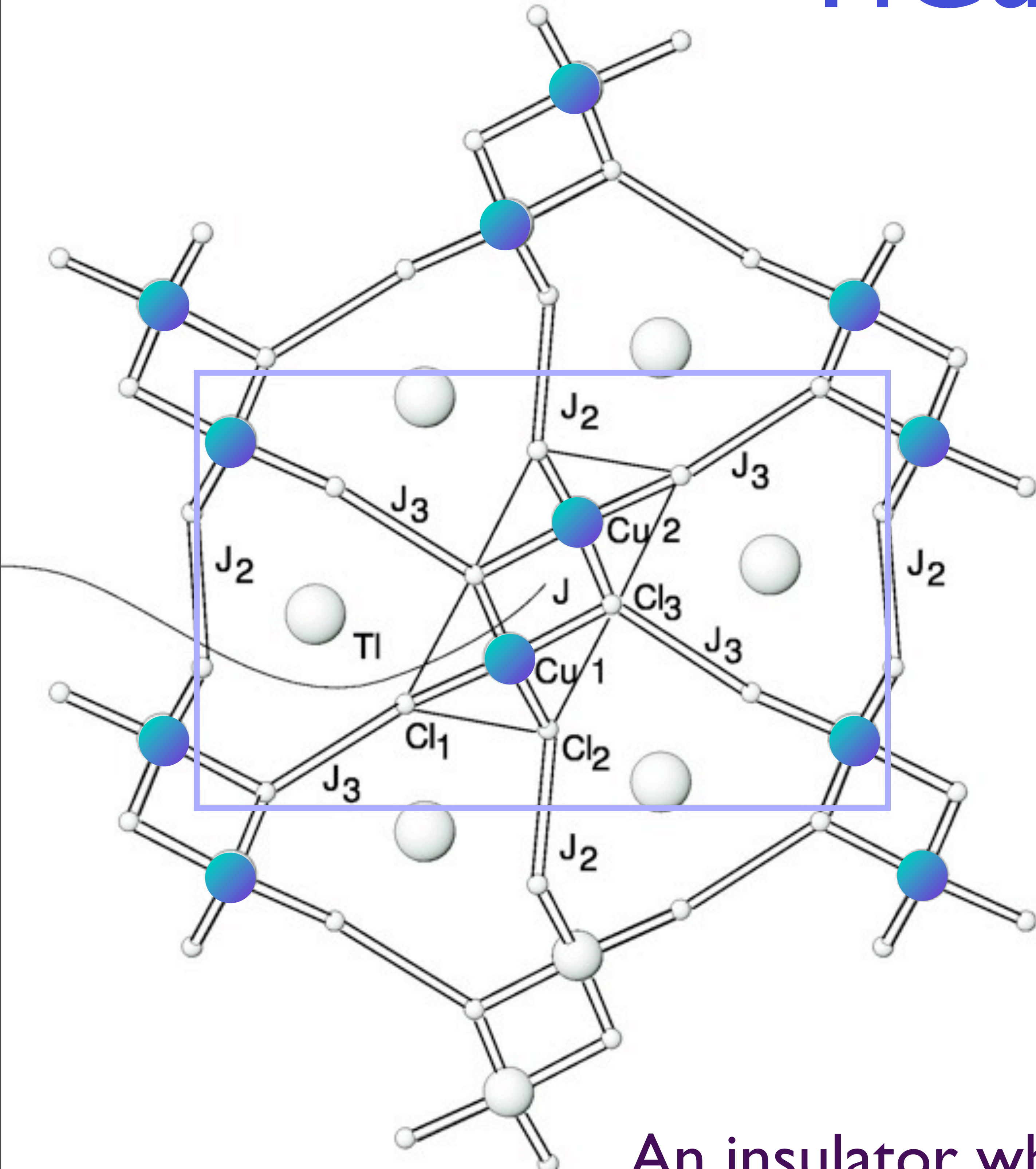
No EPR pairs

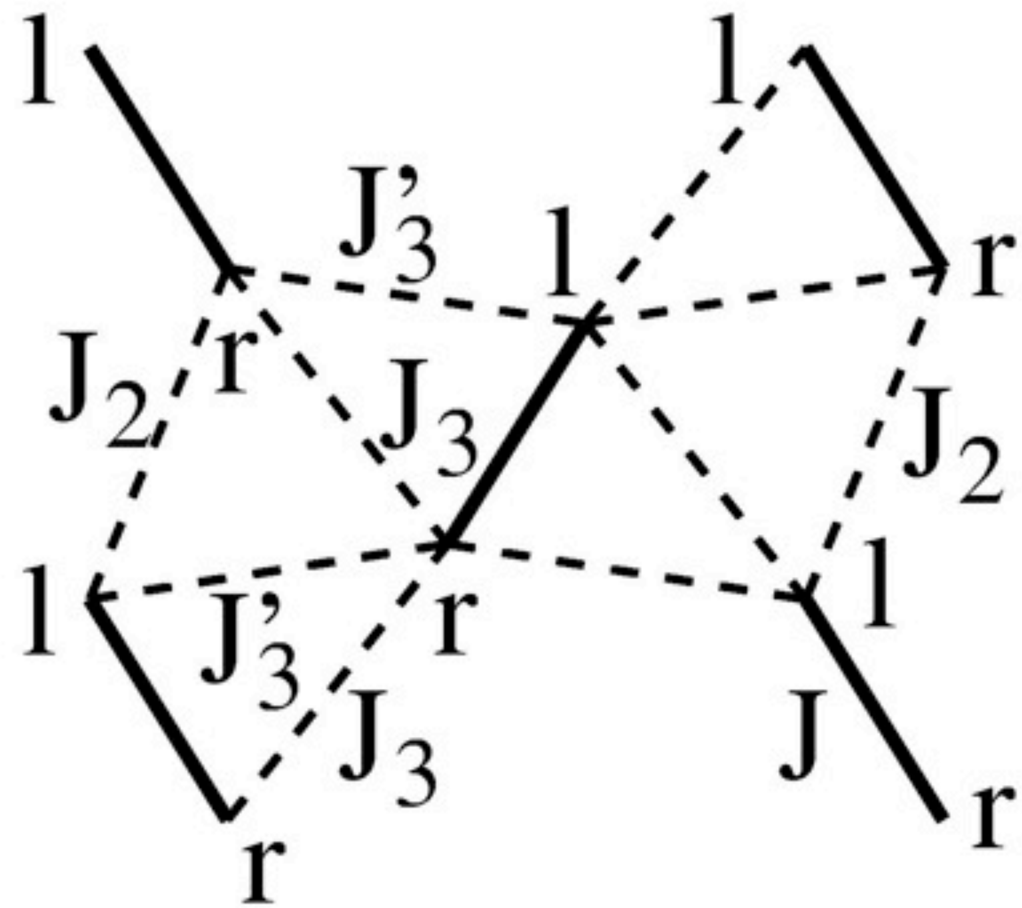

$= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$
λ_c
λ



A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).

$TlCuCl_3$





An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

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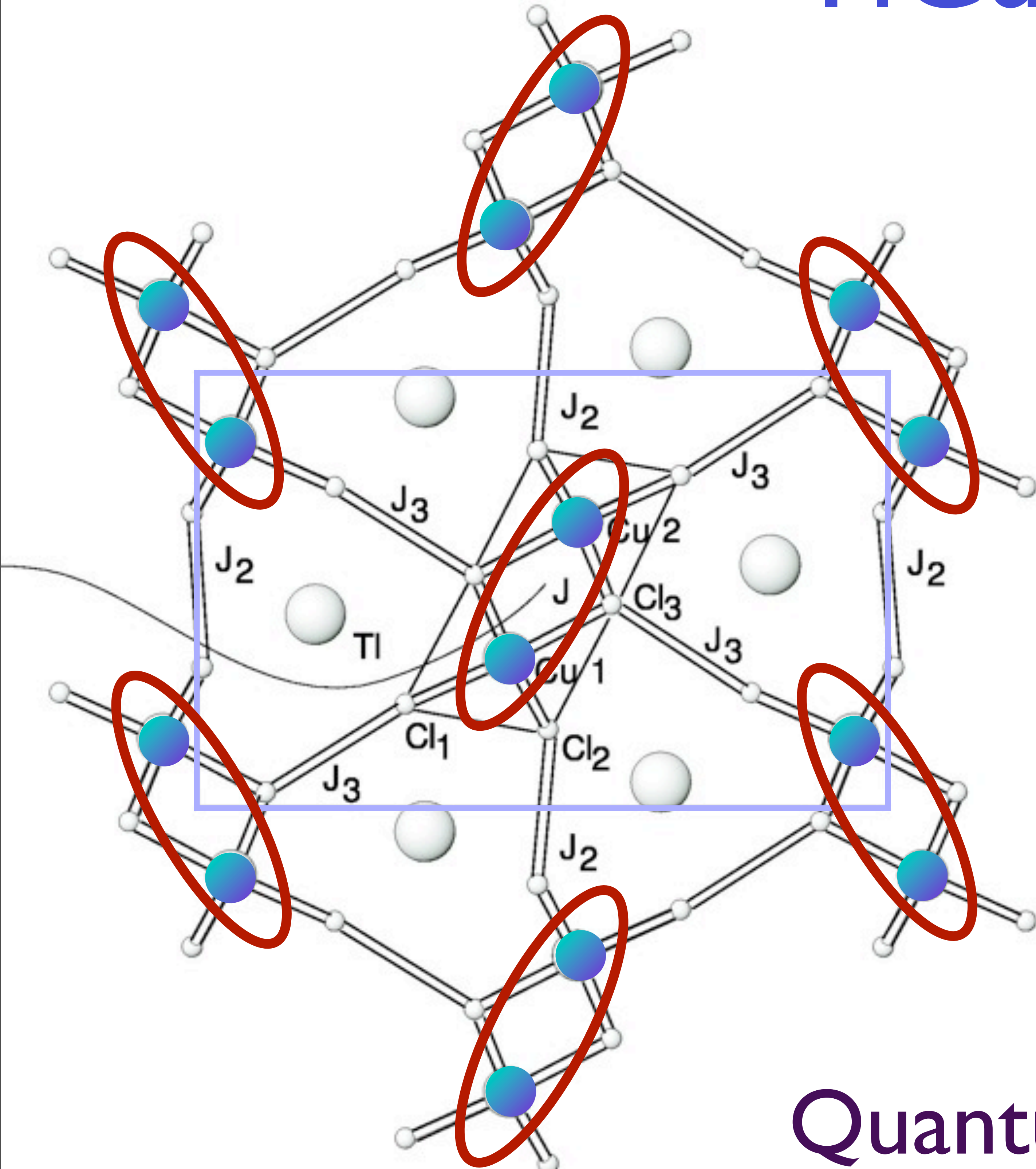
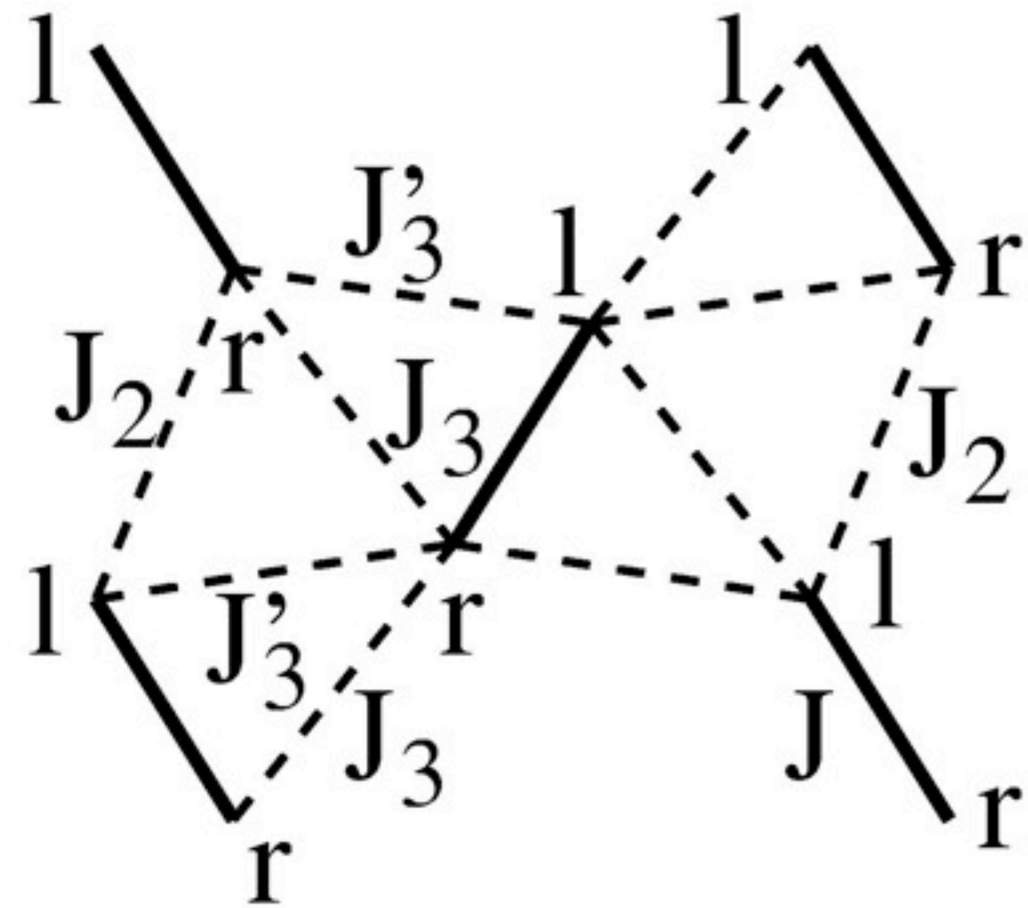
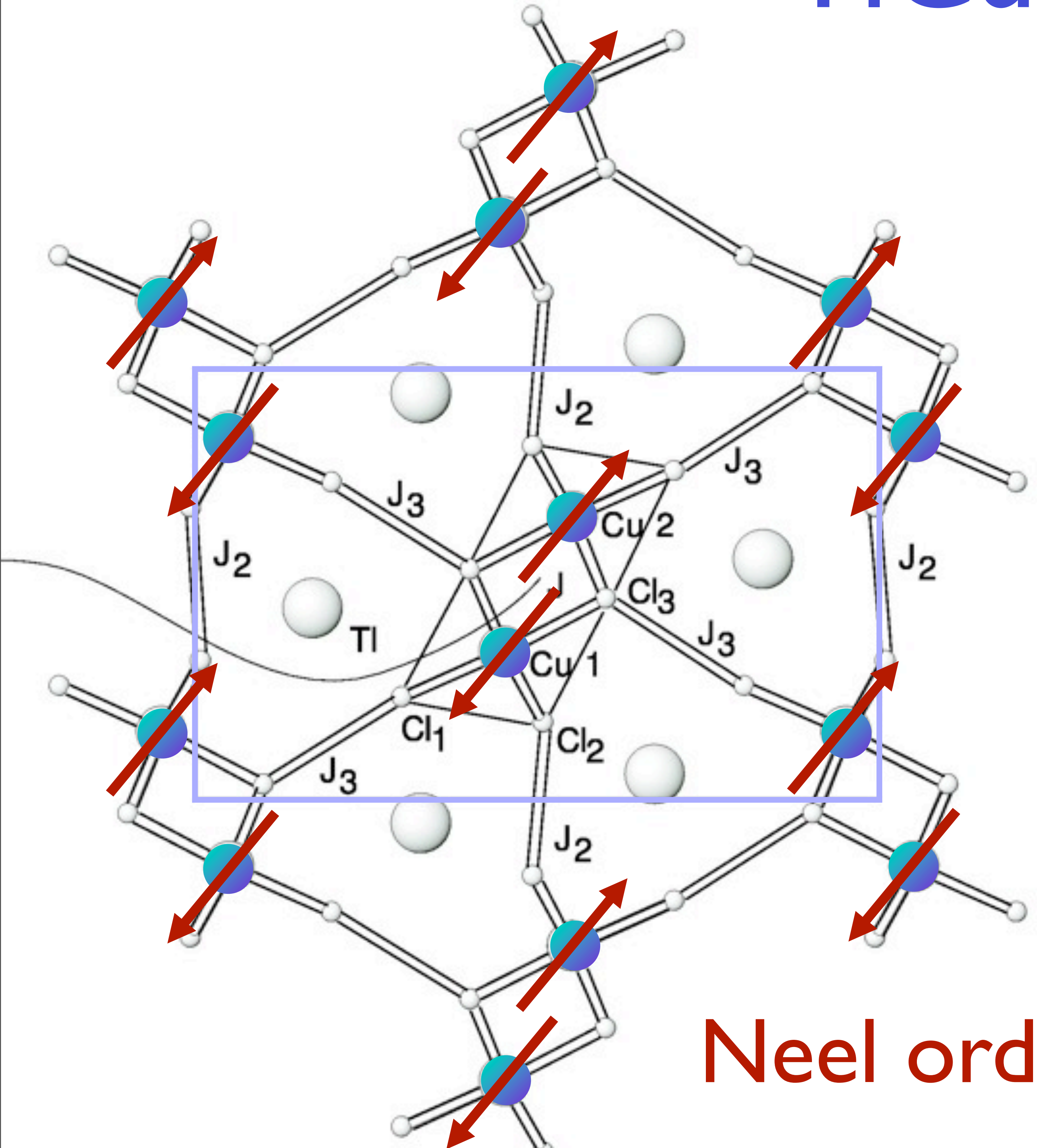
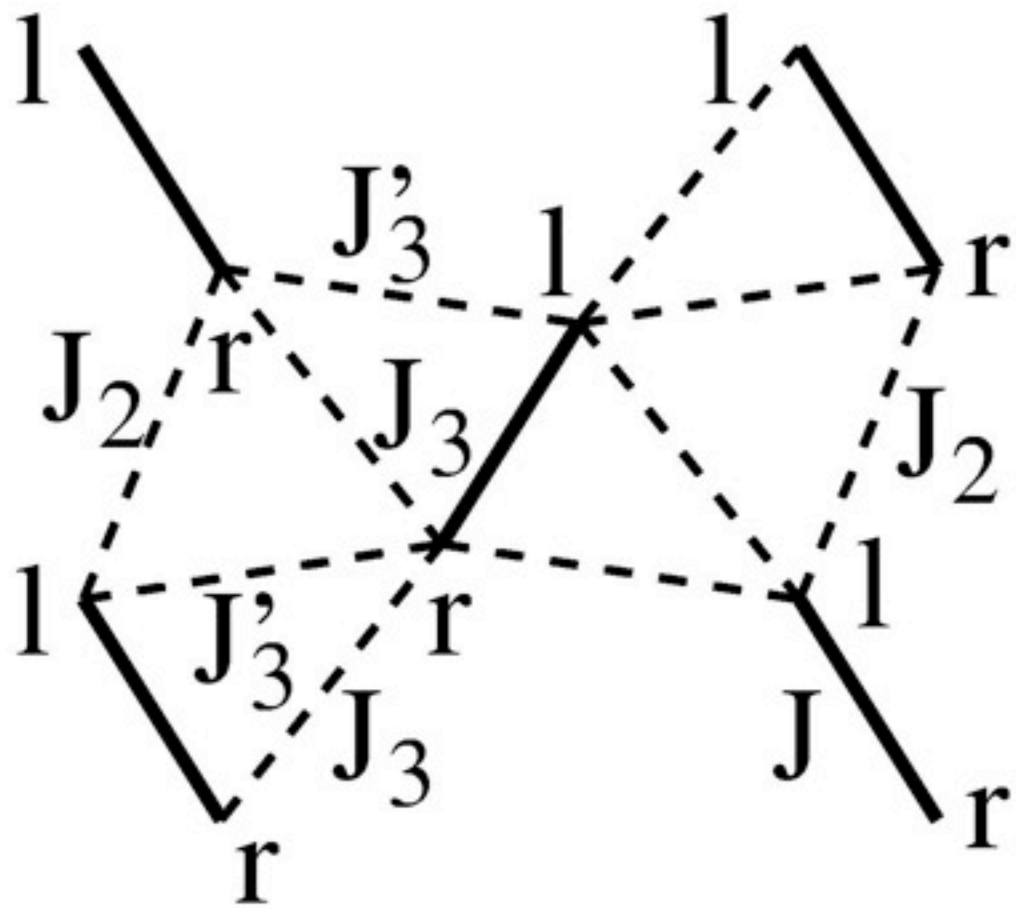




Quantum paramagnet at ambient pressure

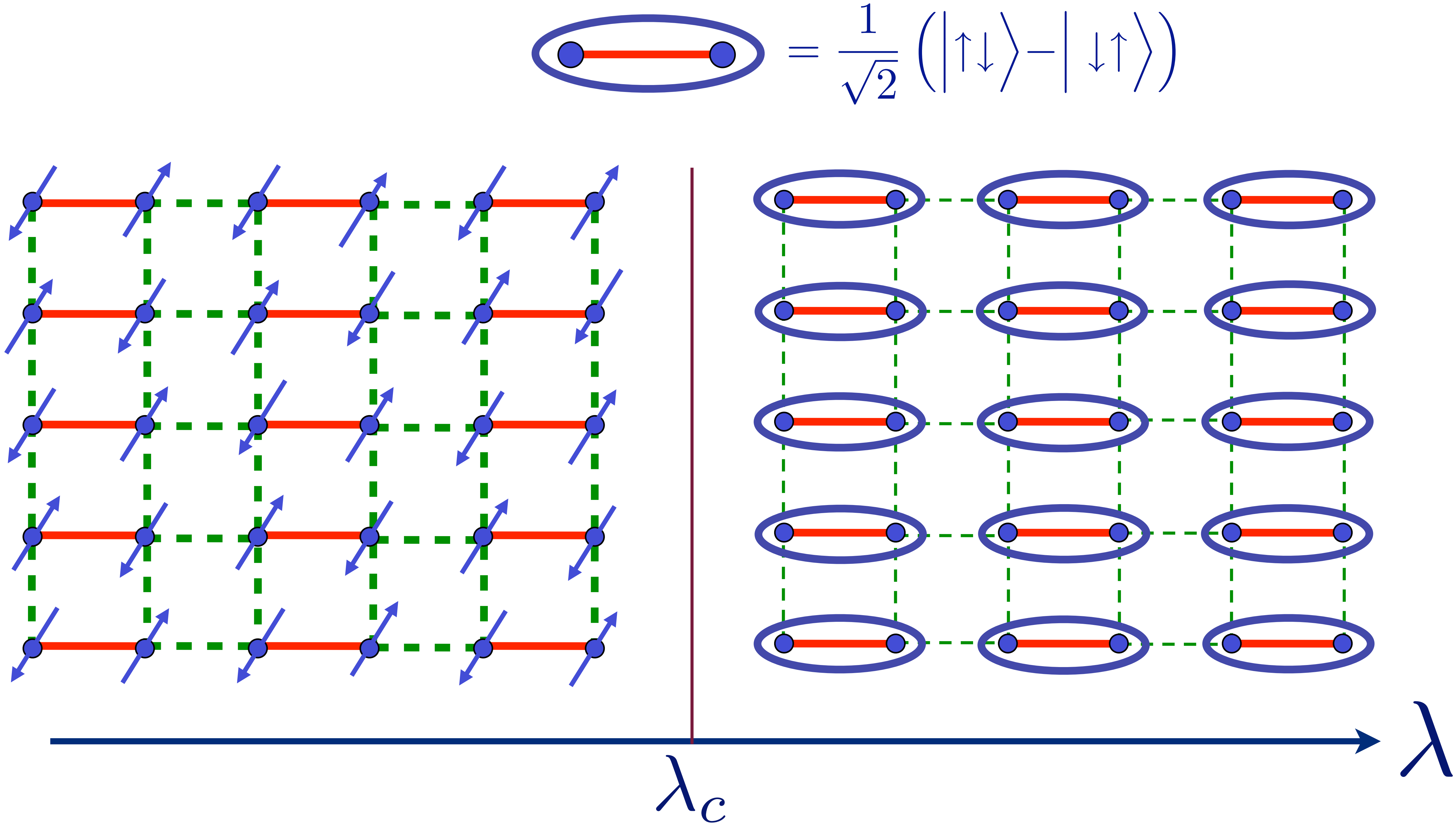
$TlCuCl_3$





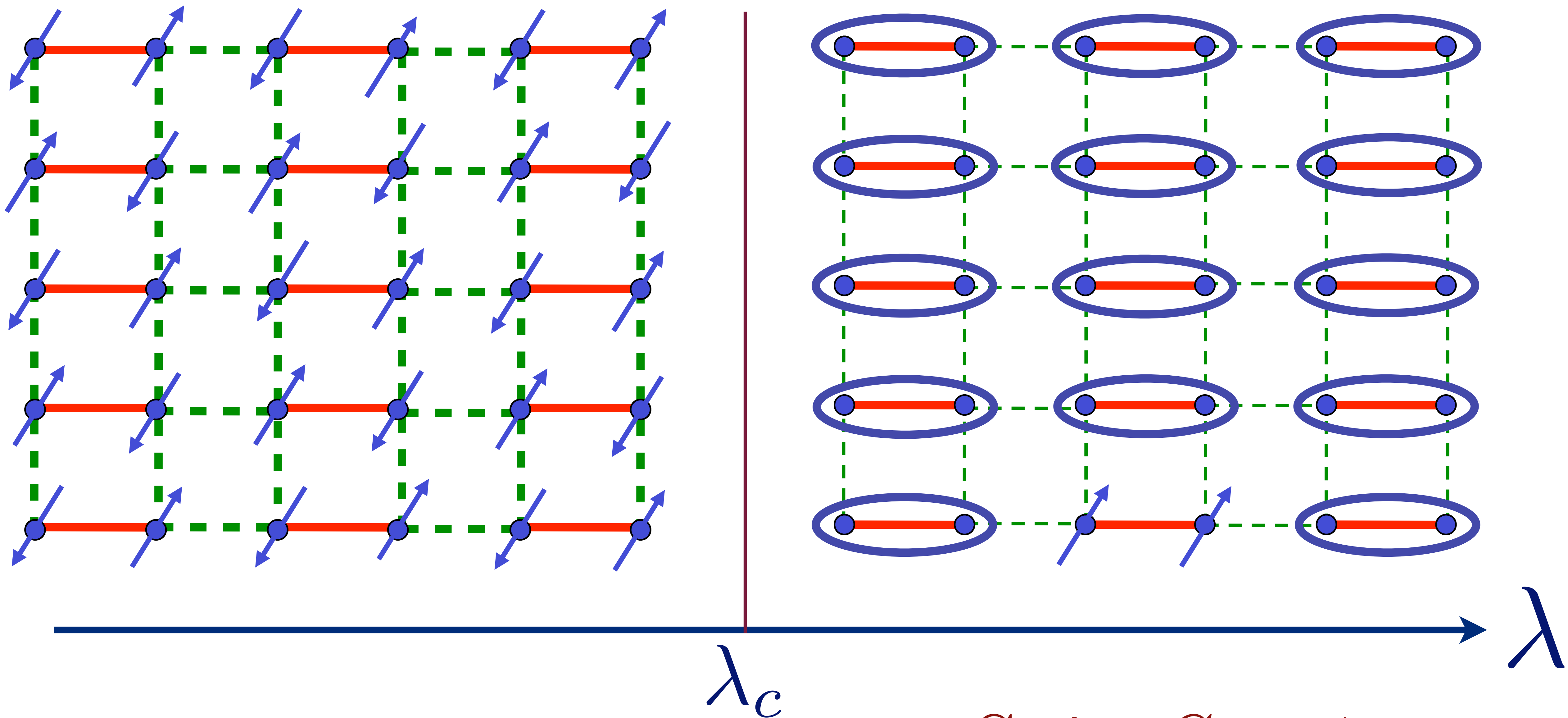
Neel order under pressure

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).



$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$
λ_c
λ

Excitation spectrum in the paramagnetic phase



Spin $S = 1$
"triplon"

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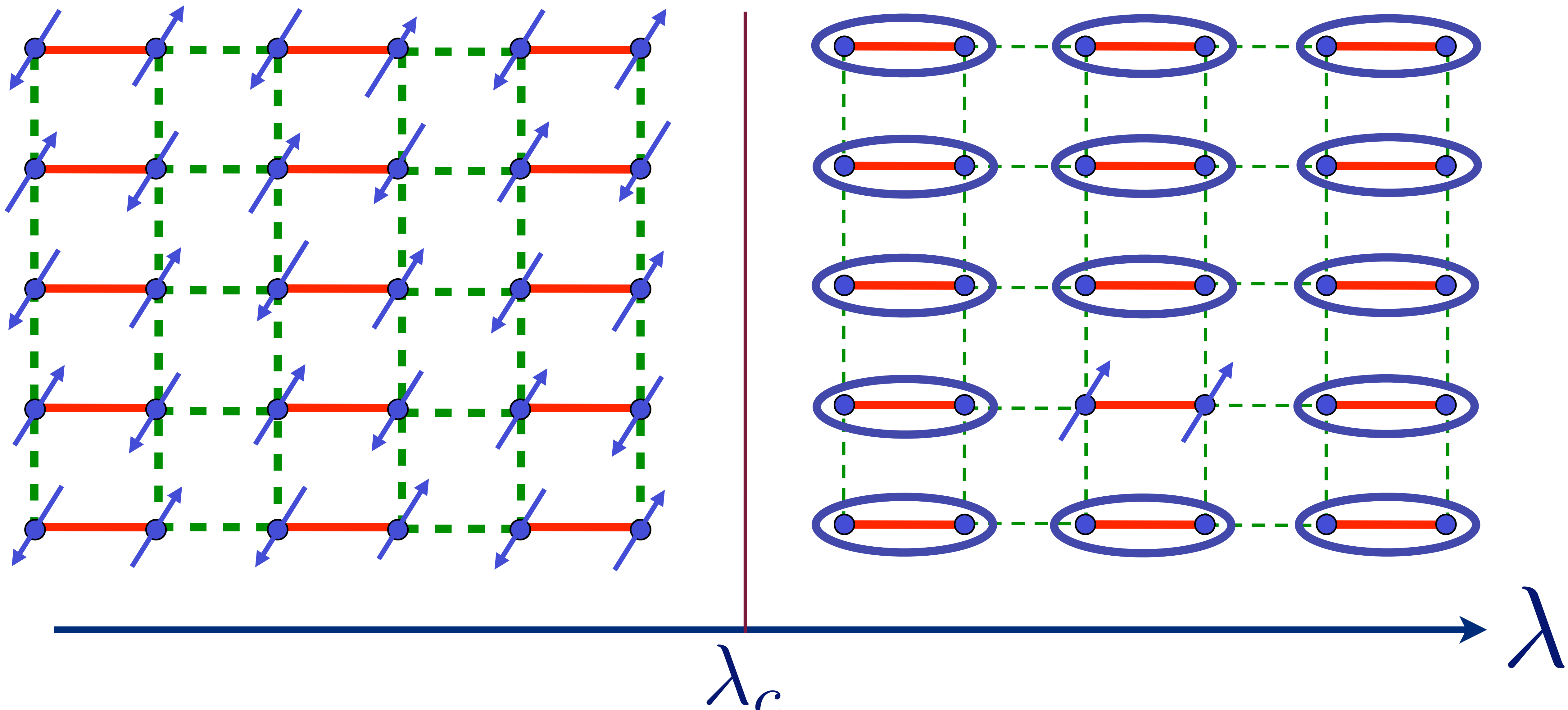

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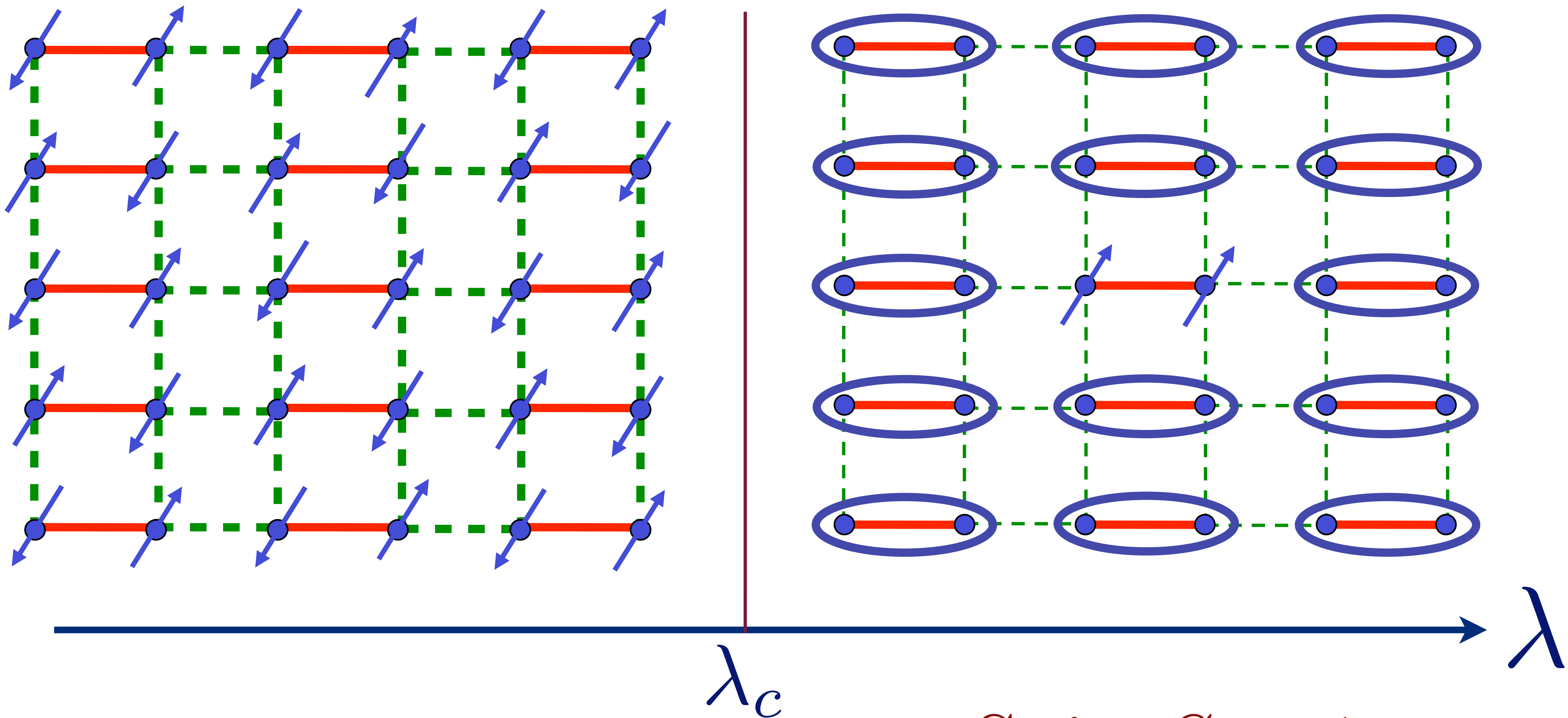

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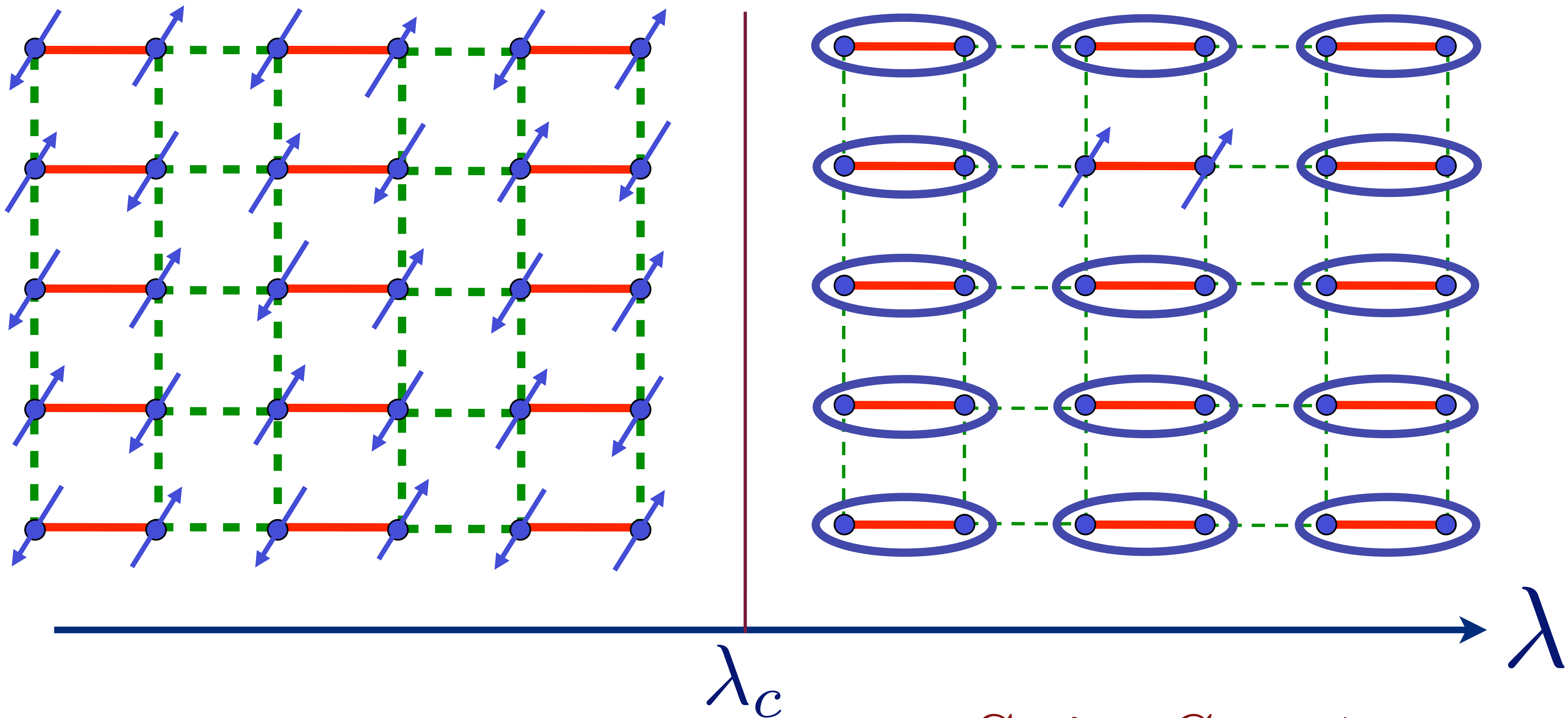

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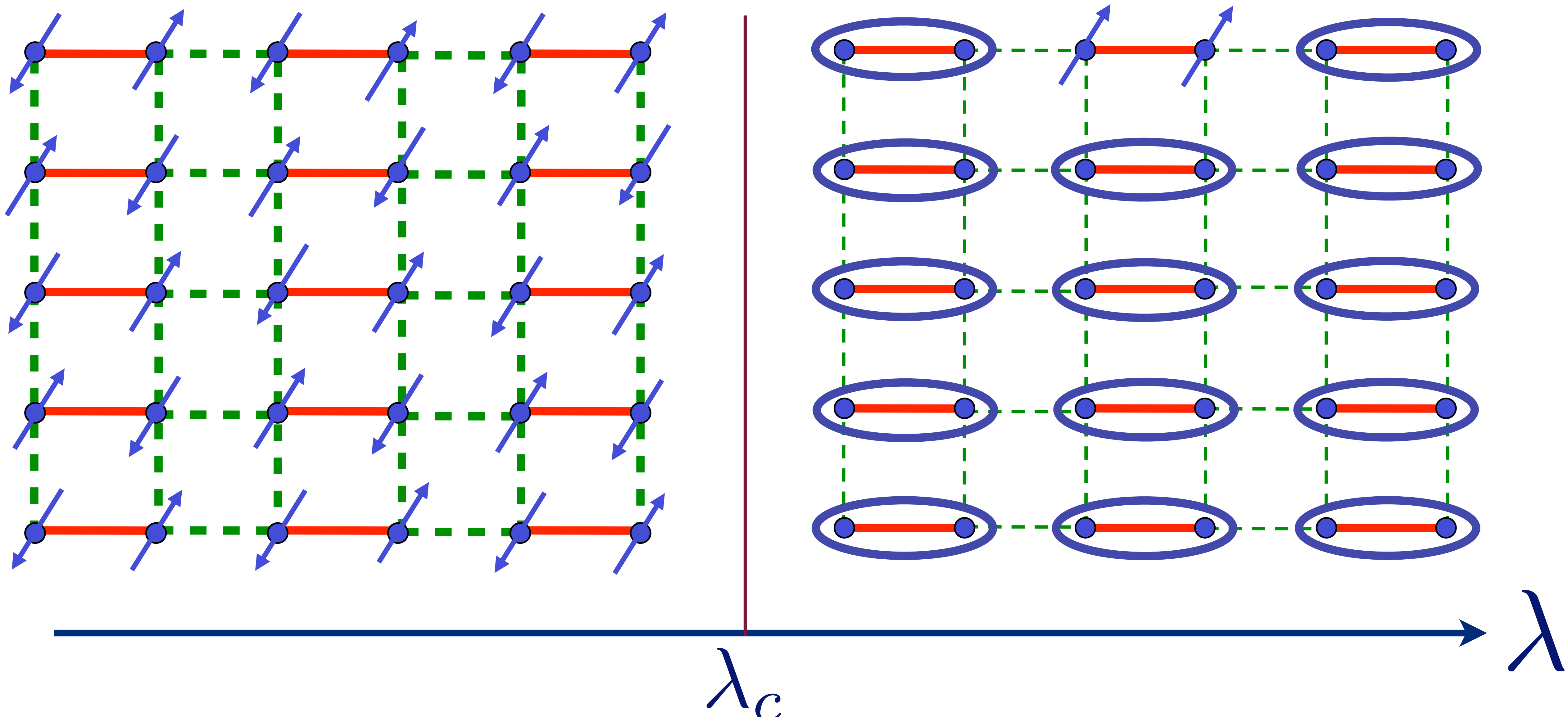

Spin $S = 1$
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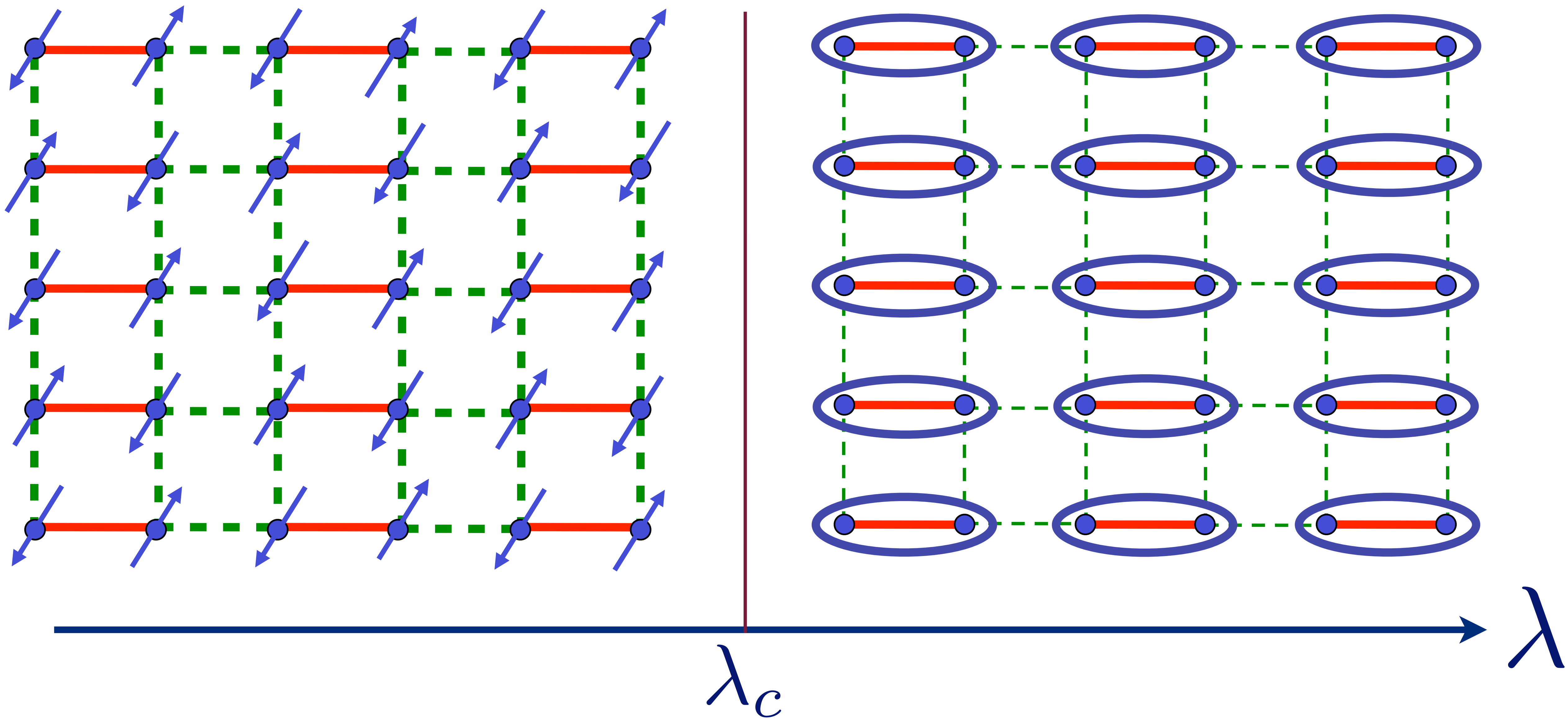
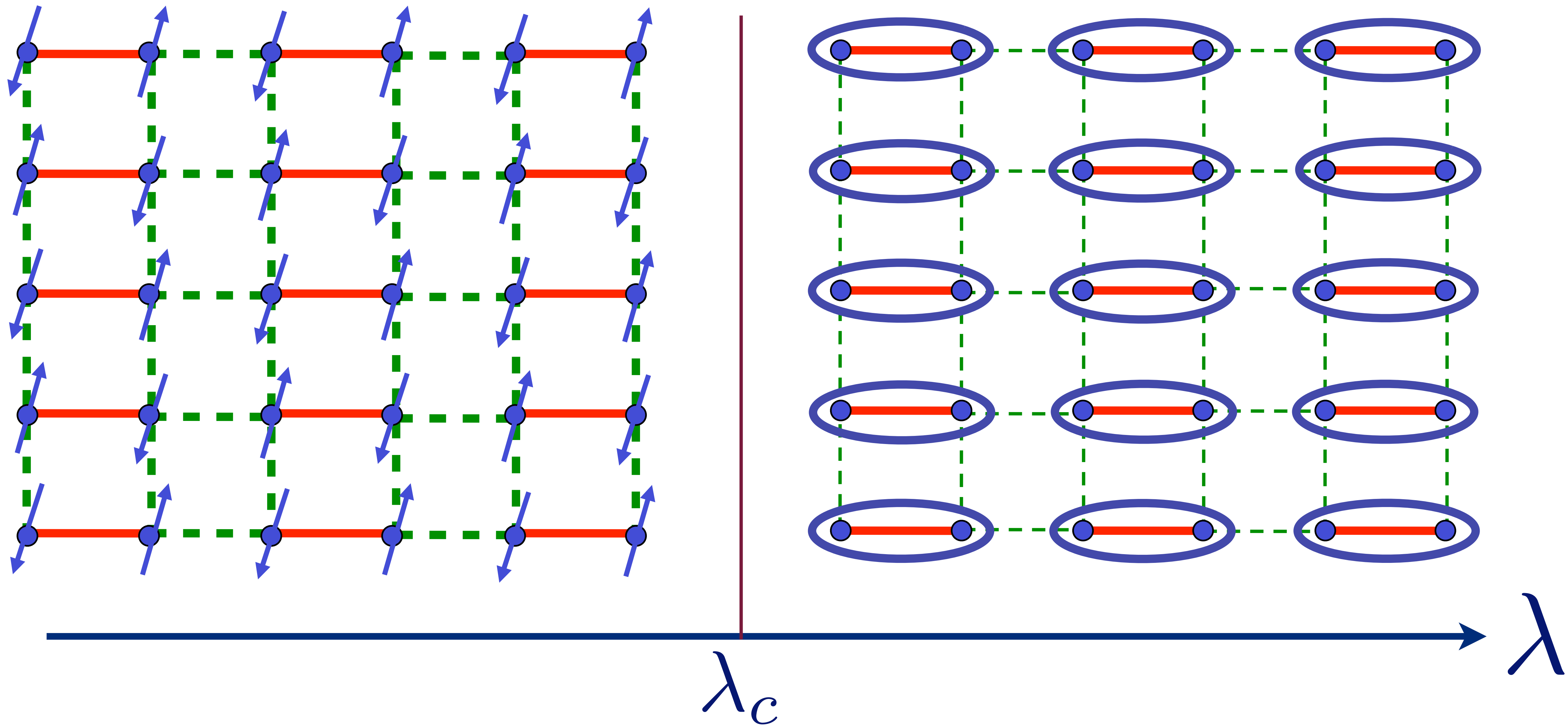
Excitation spectrum in the Néel phase



Spin waves

Excitation spectrum in the Néel phase



Spin waves

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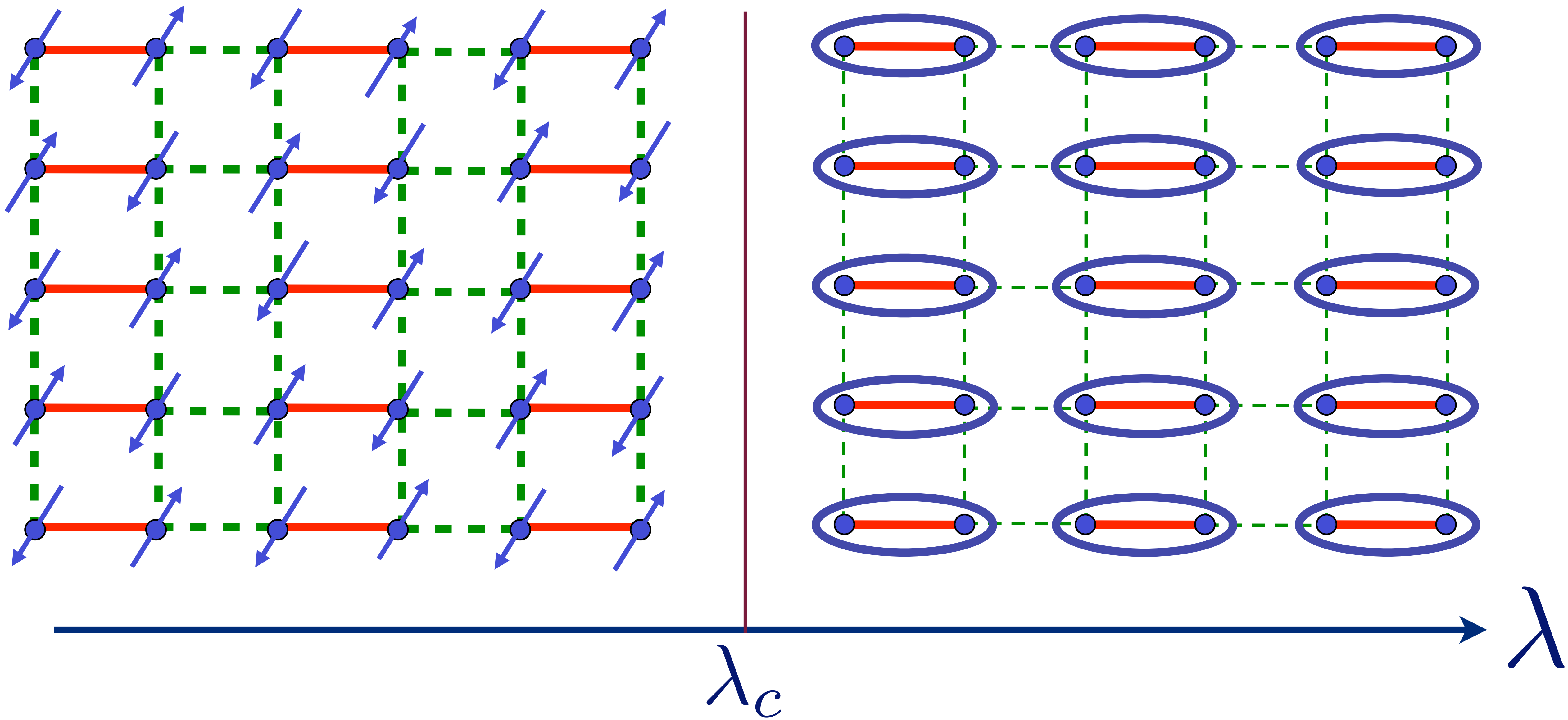
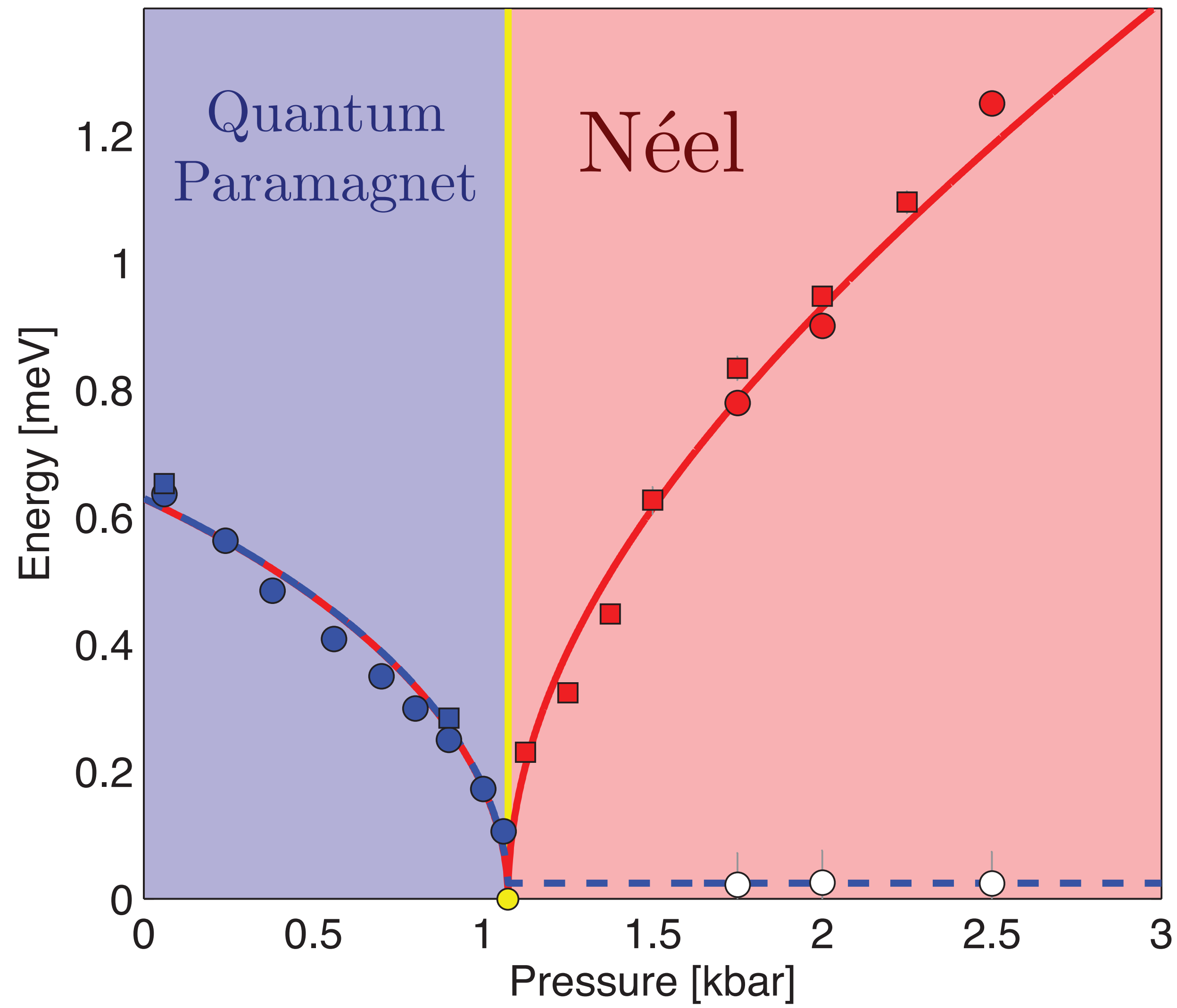


Spin waves

Excitations of $TlCuCl_3$ with varying pressure



Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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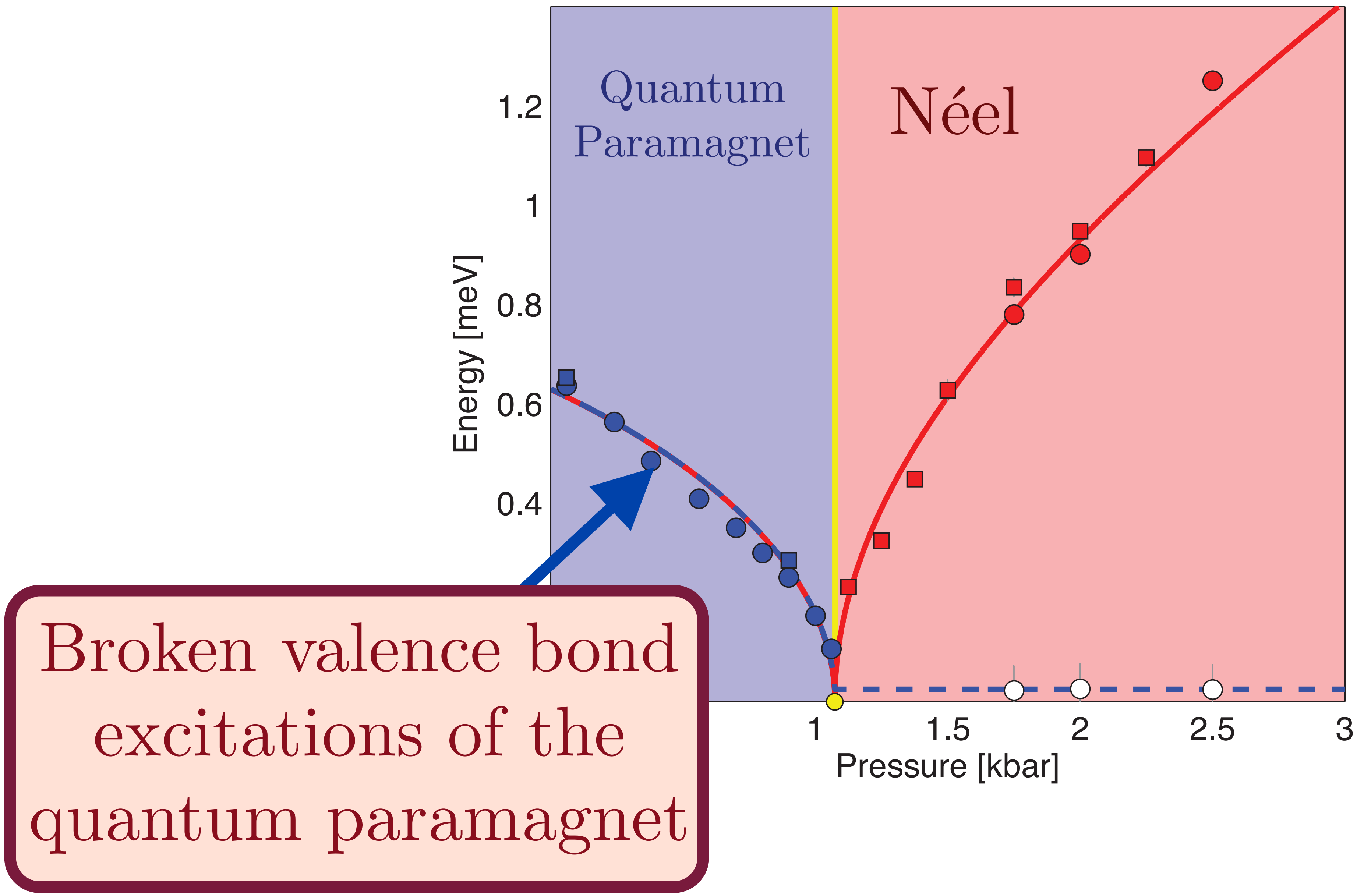

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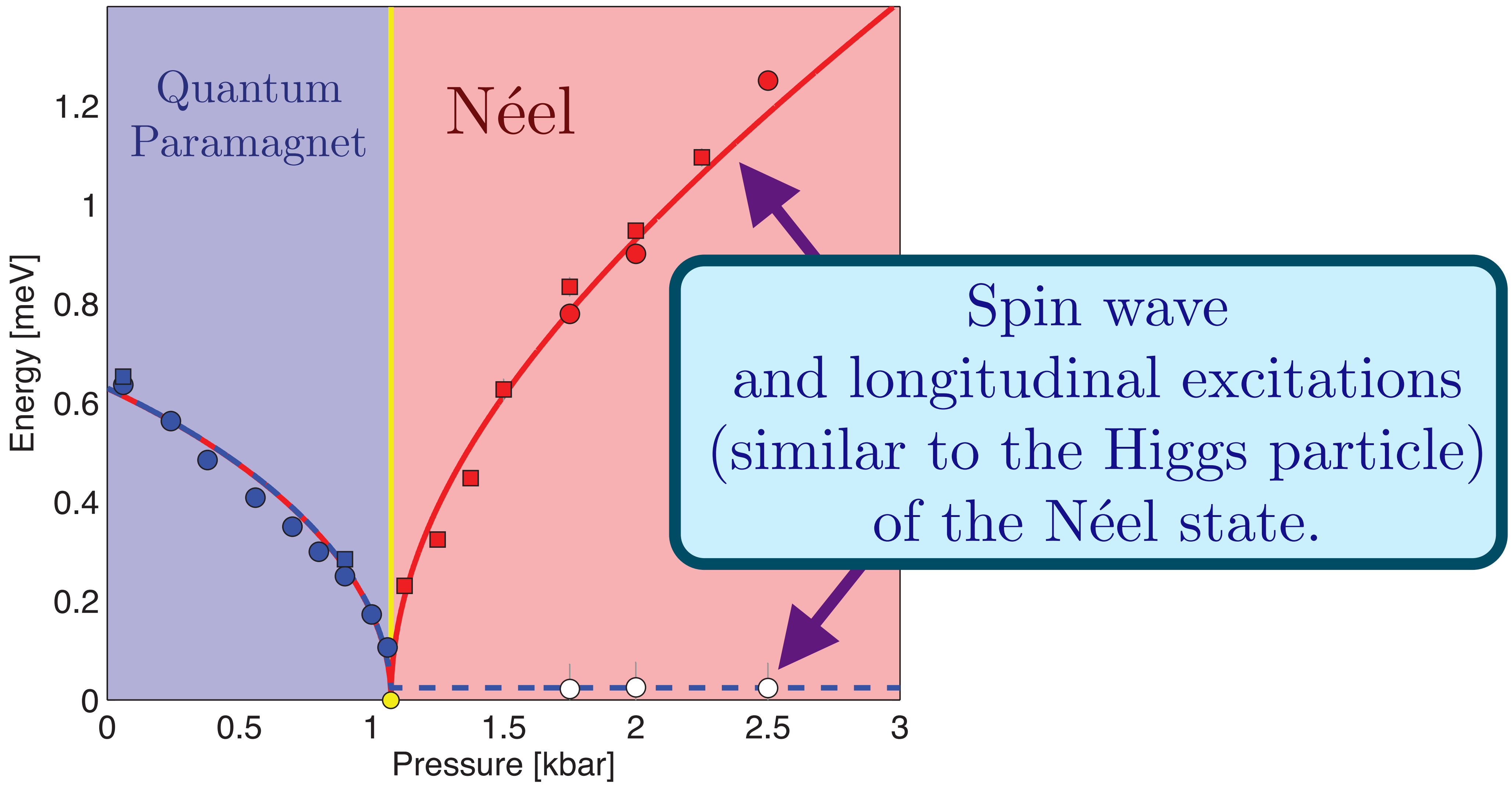

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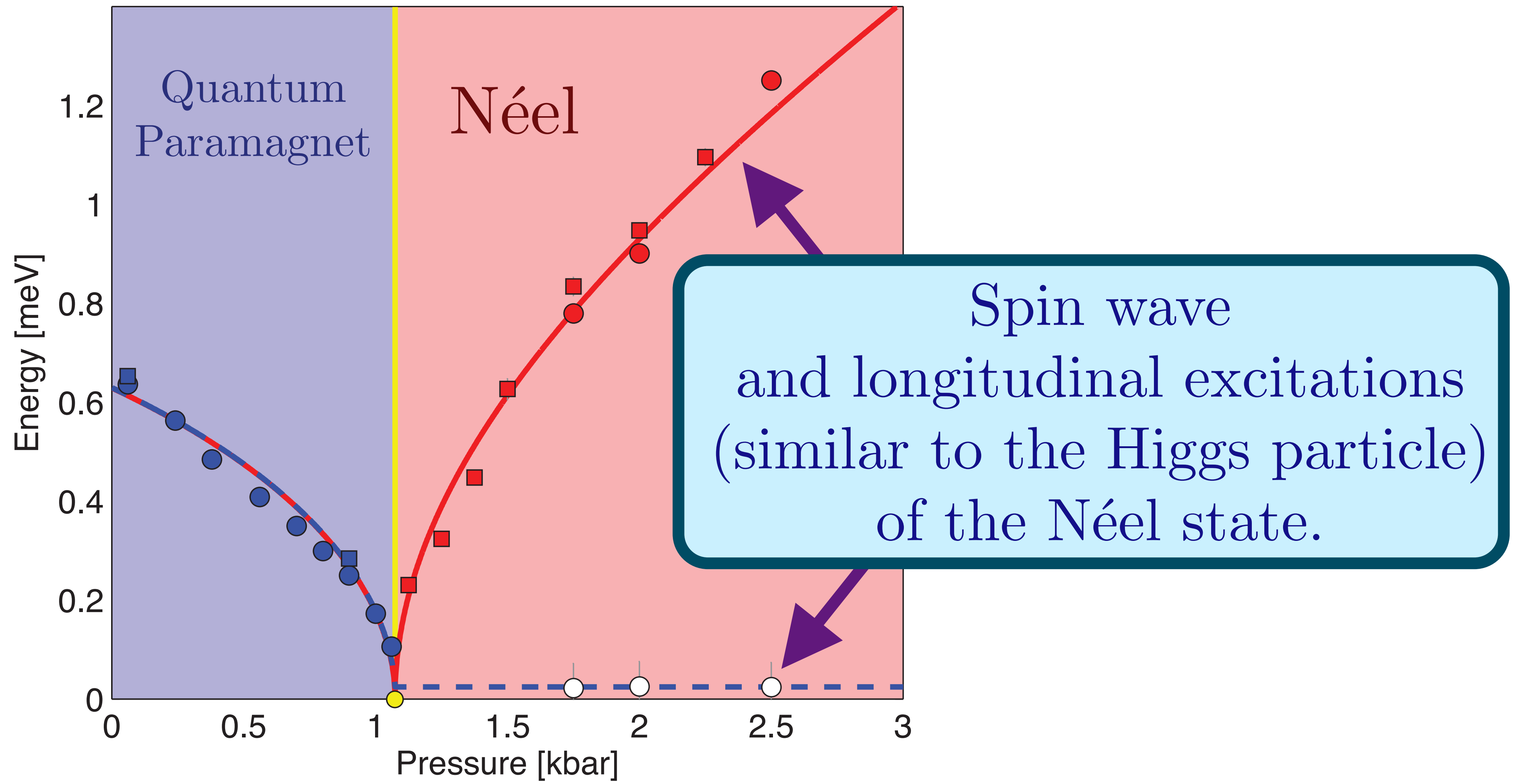
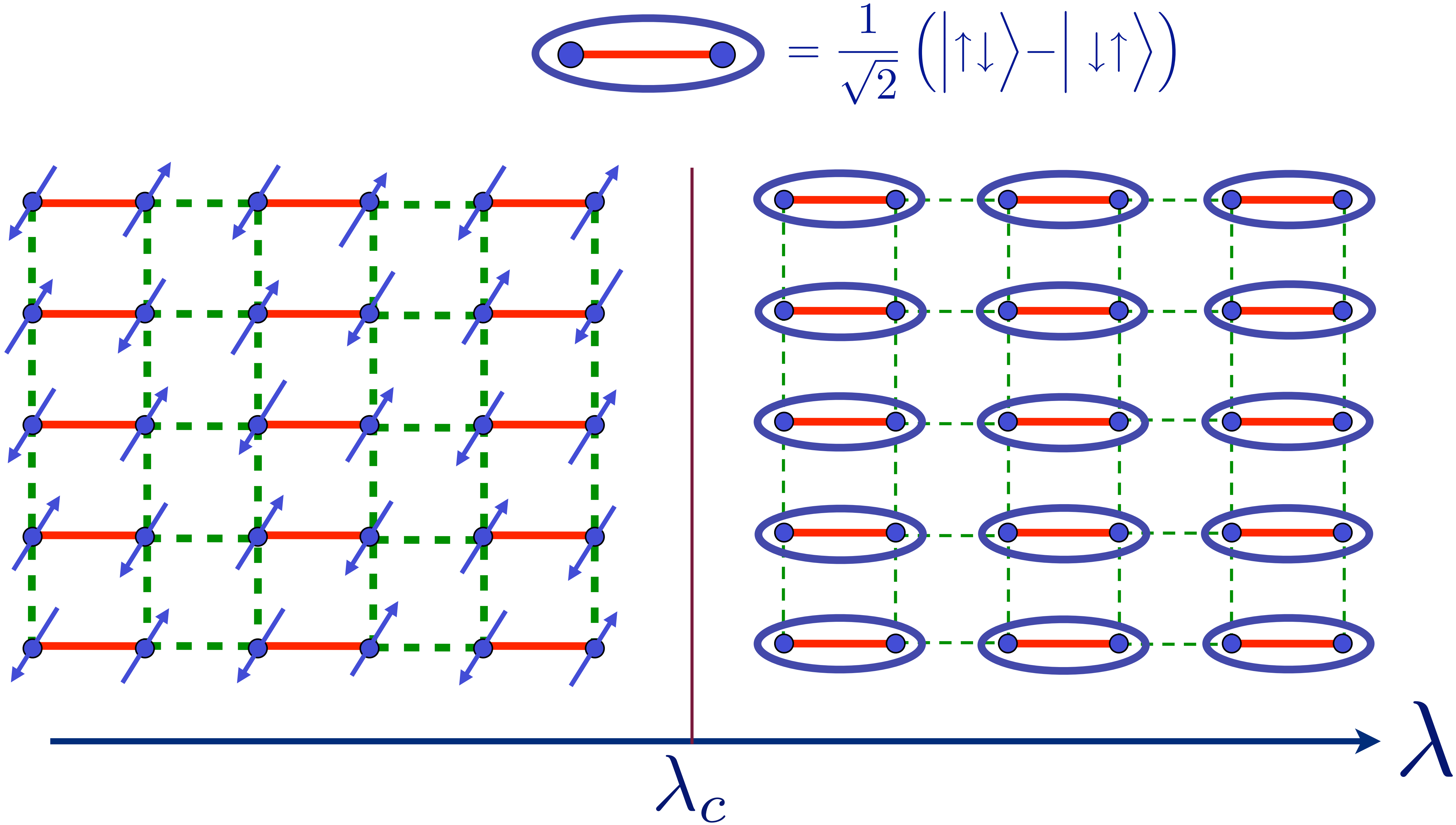


"Higgs" particle appears at theoretically predicted energy

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)



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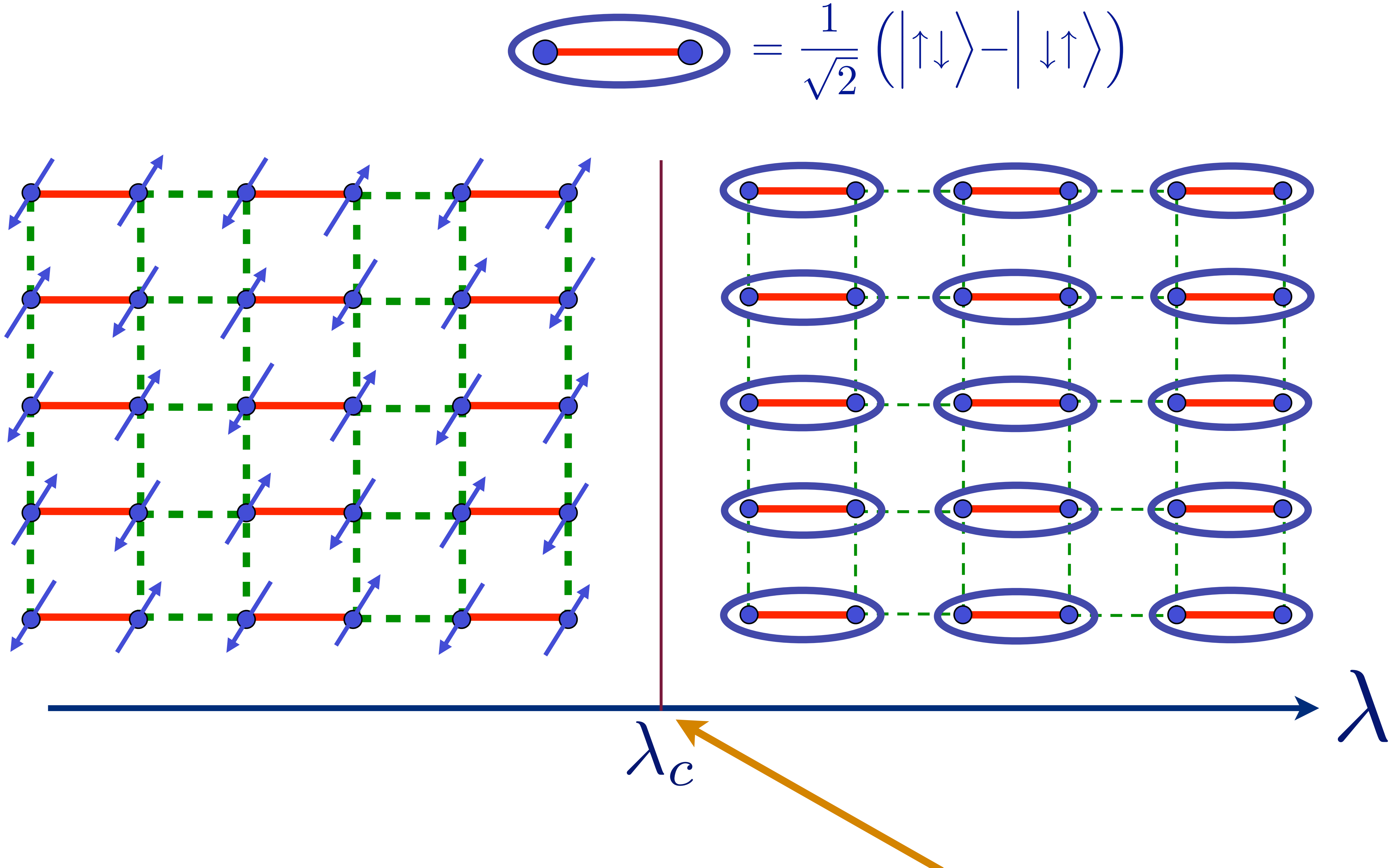
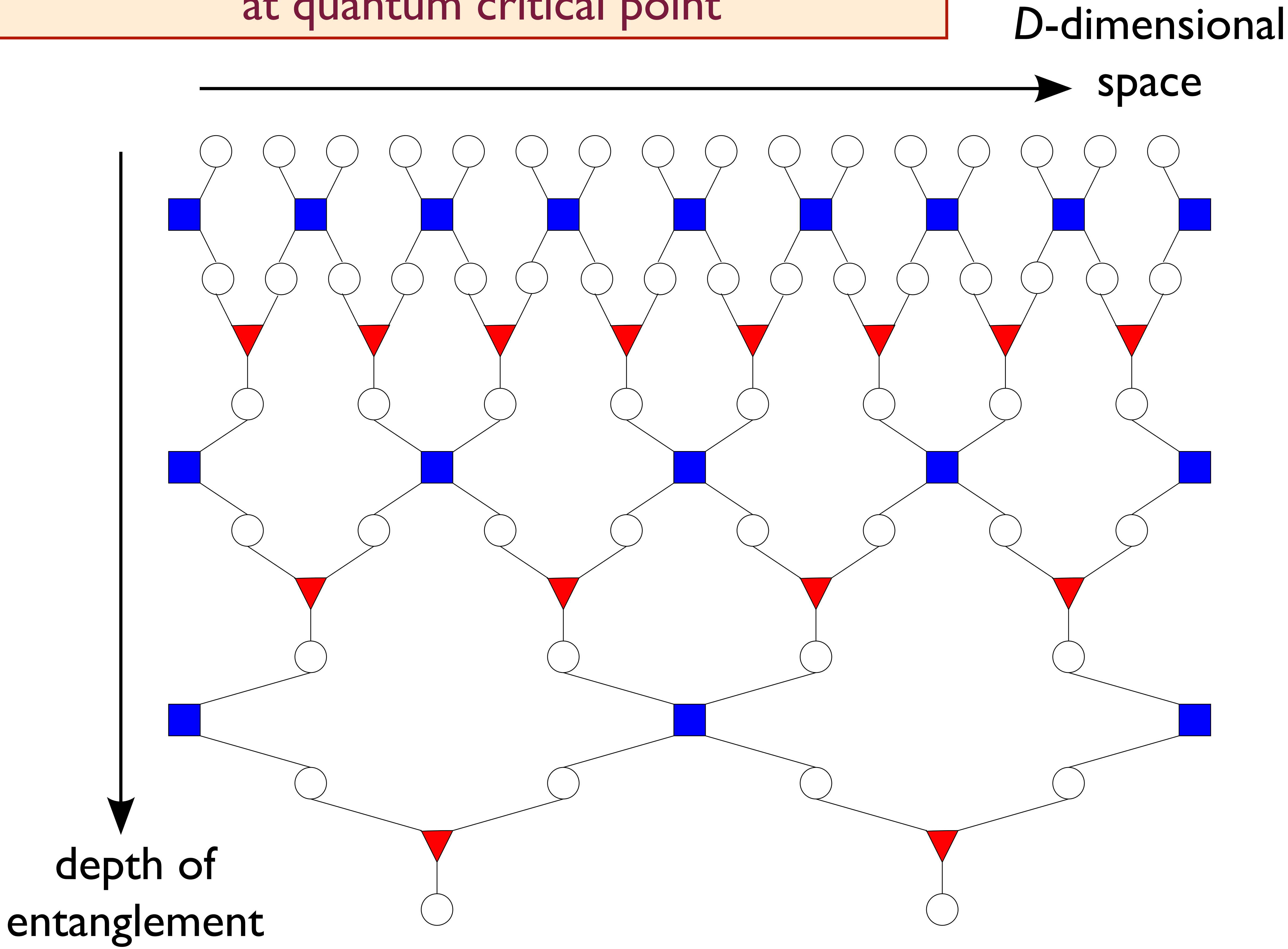


Quantum critical point with non-local entanglement in spin wavefunction

Tensor network representation of entanglement at quantum critical point



Characteristics of quantum critical point

- Long-range entanglement

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- Long distance and low energy correlations near the quantum critical point are described by a quantum field theory which is relativistically invariant (where the spin-wave velocity plays the role of the velocity of "light").

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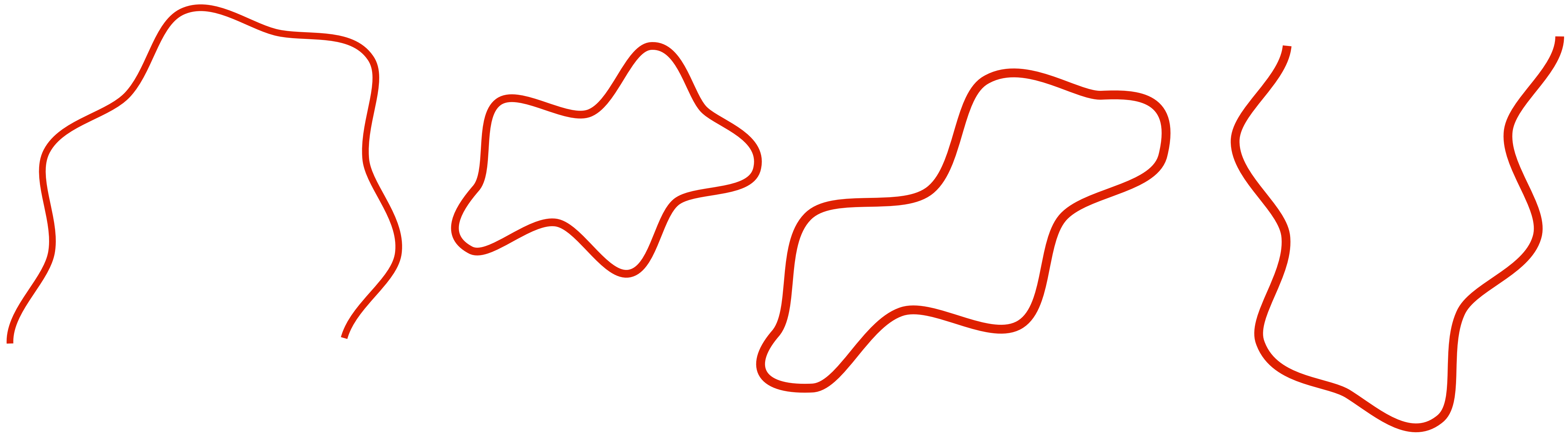
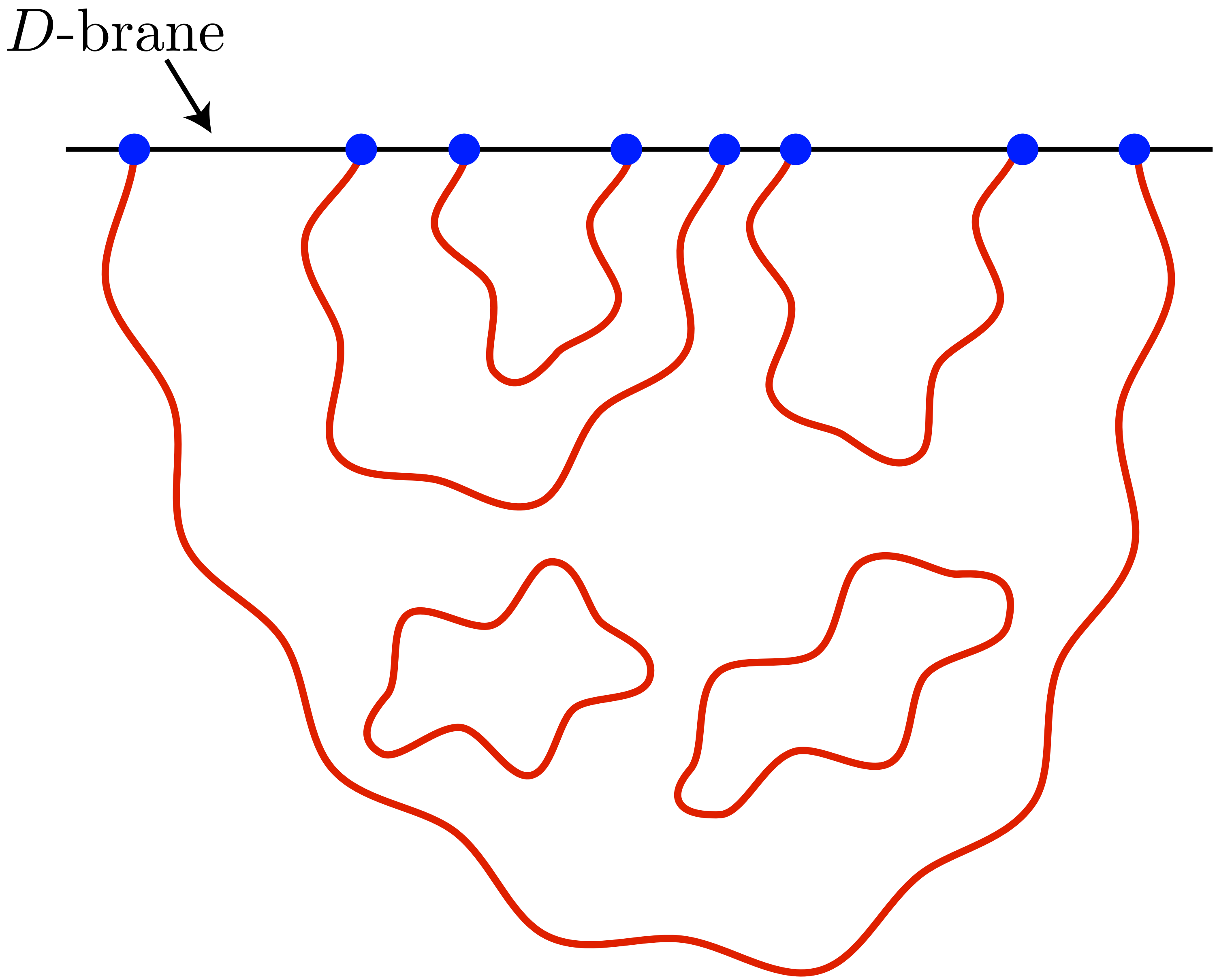
- Long-range entanglement
- Long distance and low energy correlations near the quantum critical point are described by a quantum field theory which is relativistically invariant (where the spin-wave velocity plays the role of the velocity of "light").
- The quantum field theory is invariant under scale and conformal transformations at the quantum critical point: a **CFT3**

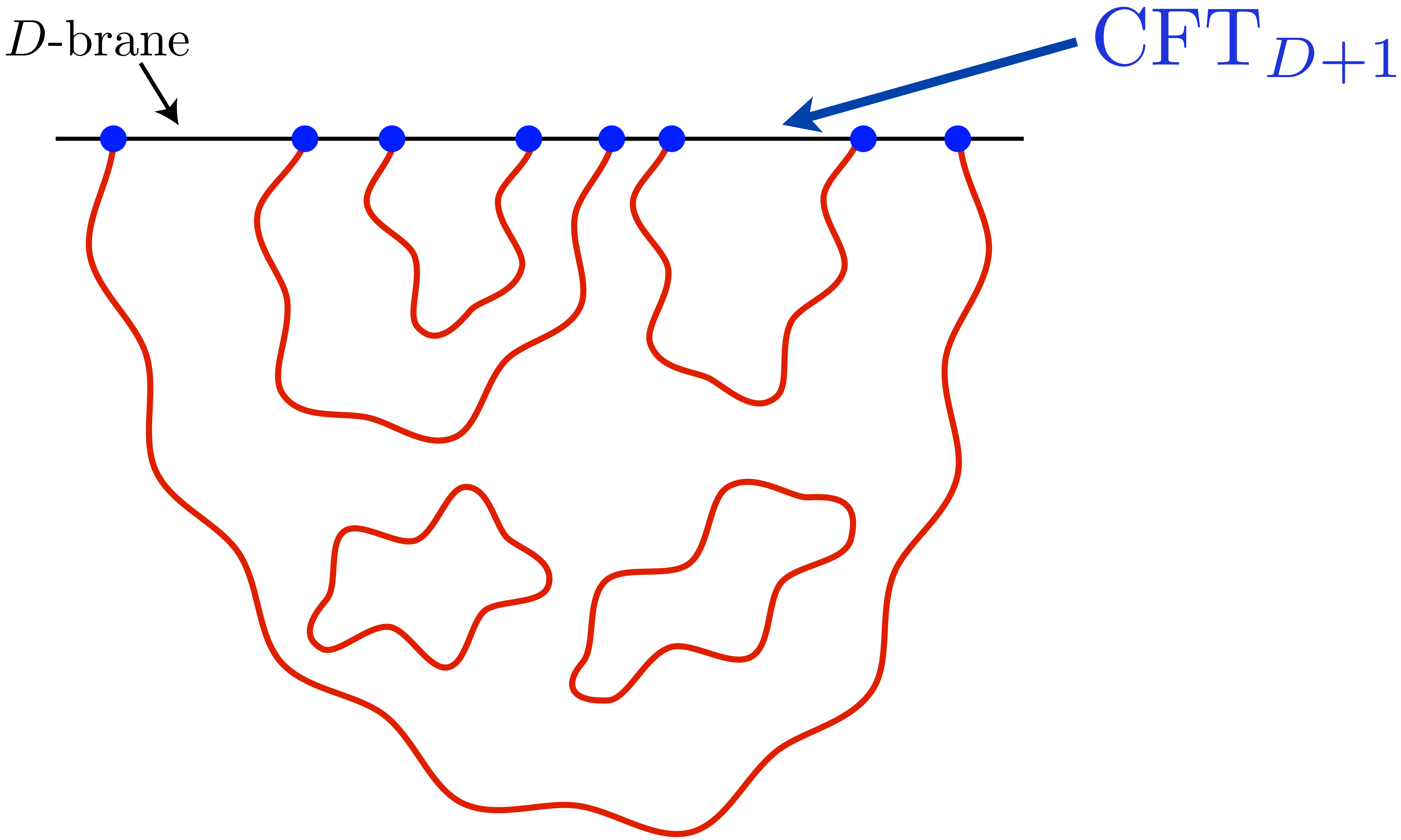
String theory



- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks . . .

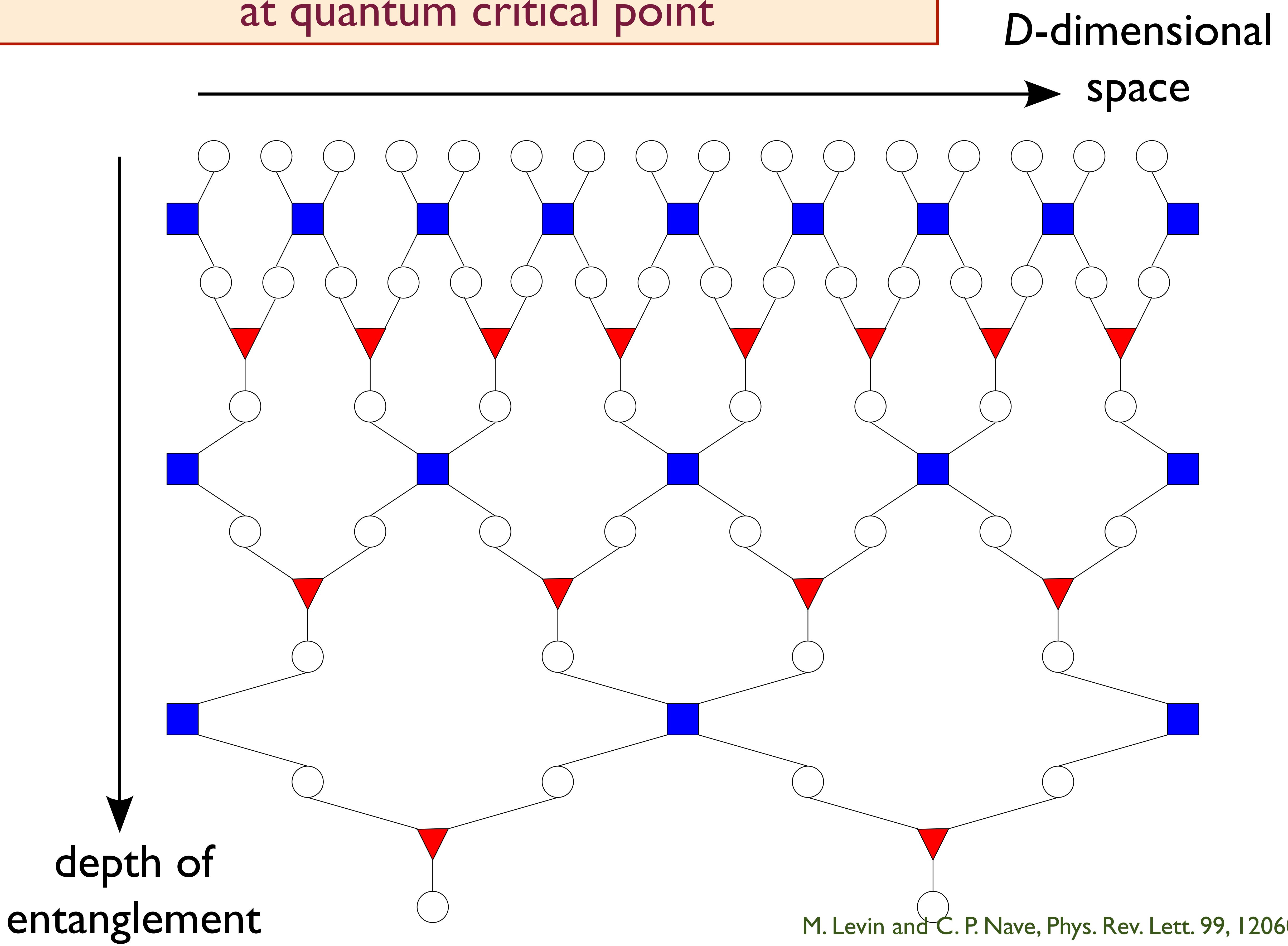


- A D-brane is a D-dimensional surface on which strings can end.
- The low-energy theory on a D-brane is an ordinary quantum field theory with no gravity.

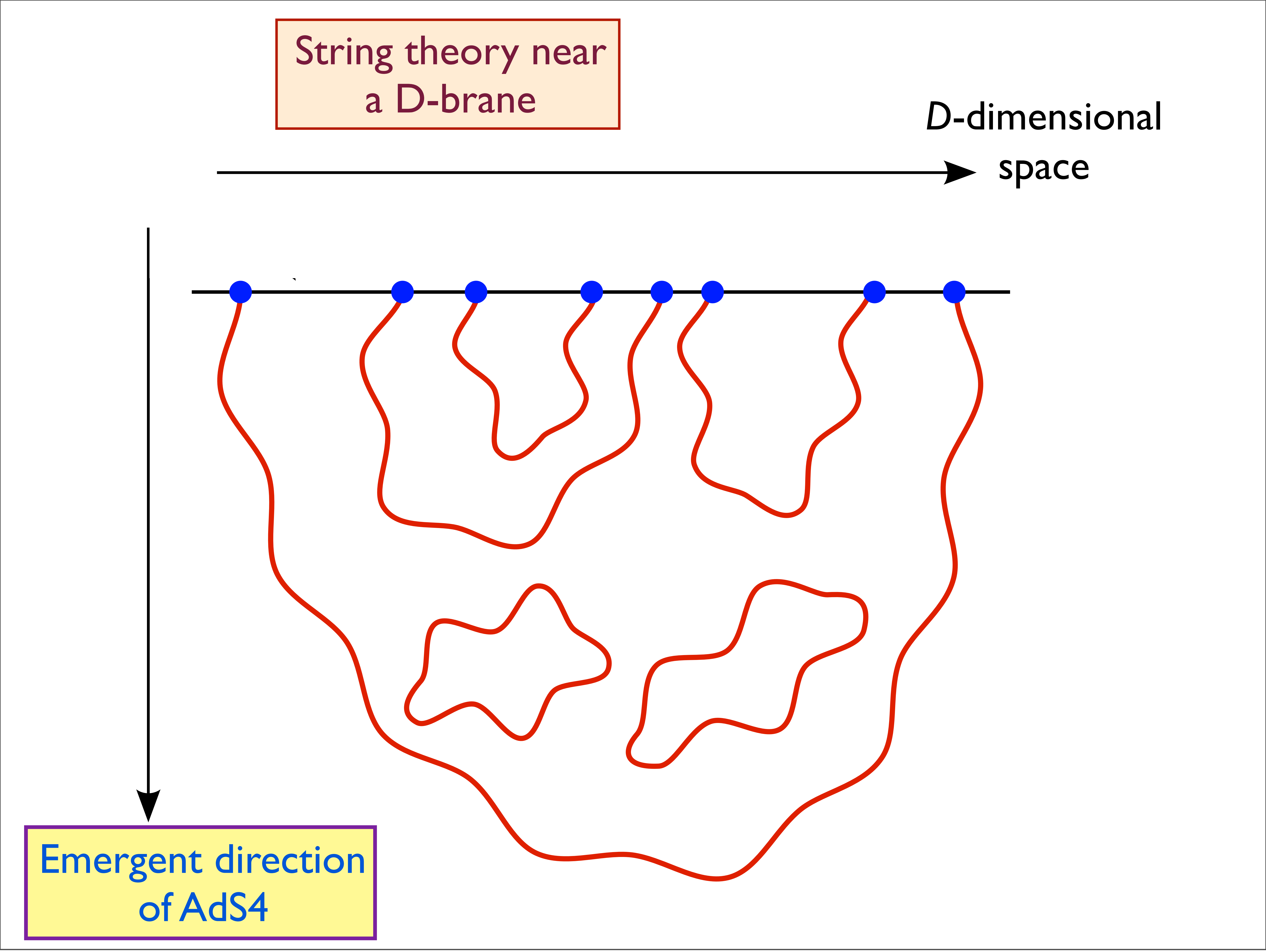


- A D-brane is a D-dimensional surface on which strings can end.
- The low-energy theory on a D-brane is an ordinary quantum field theory with no gravity.
- In $D = 2$, we obtain strongly-interacting **CFT3**s. These are "dual" to string theory on anti-de Sitter space: **AdS4**.

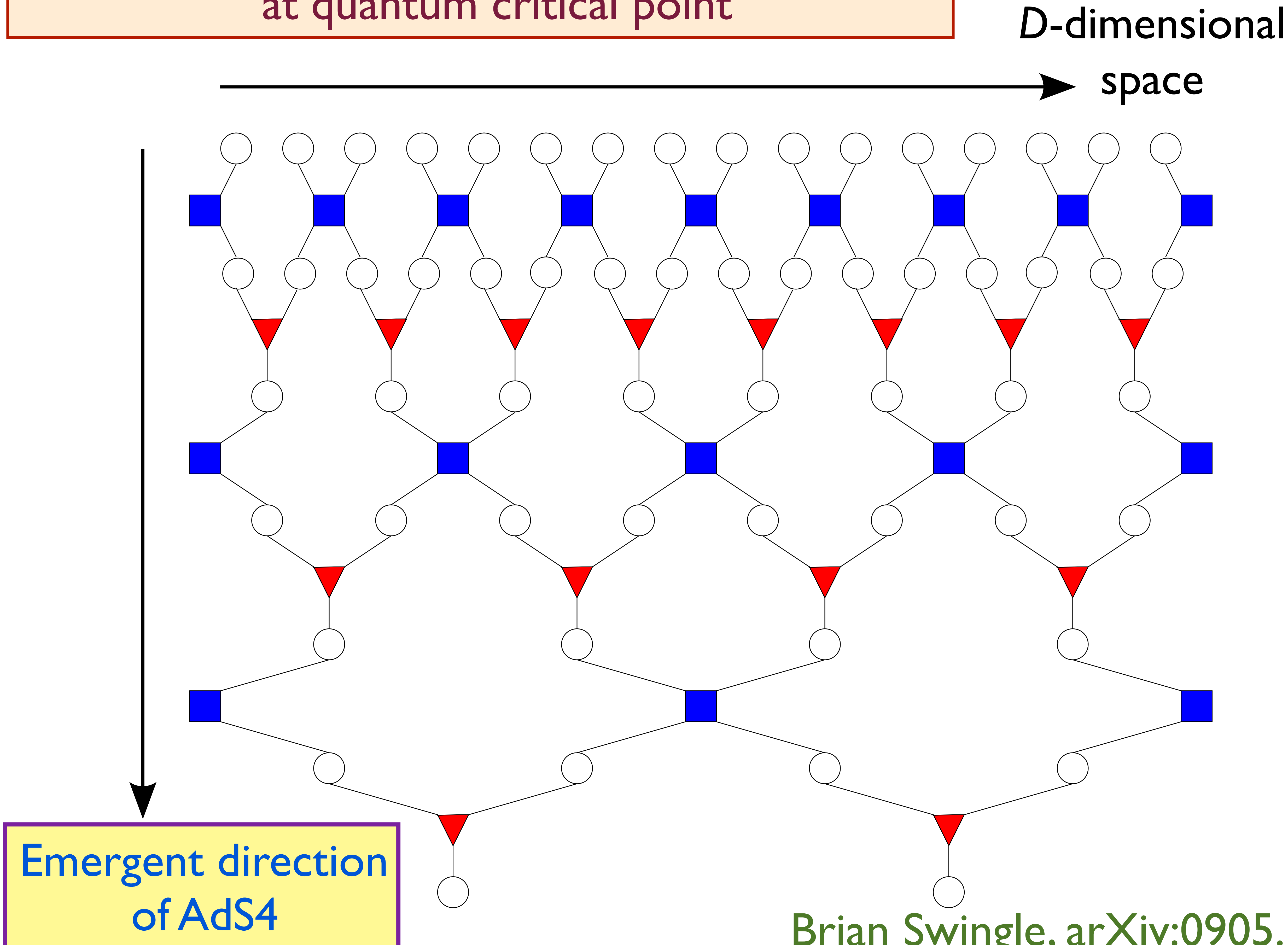
Tensor network representation of entanglement at quantum critical point



M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)

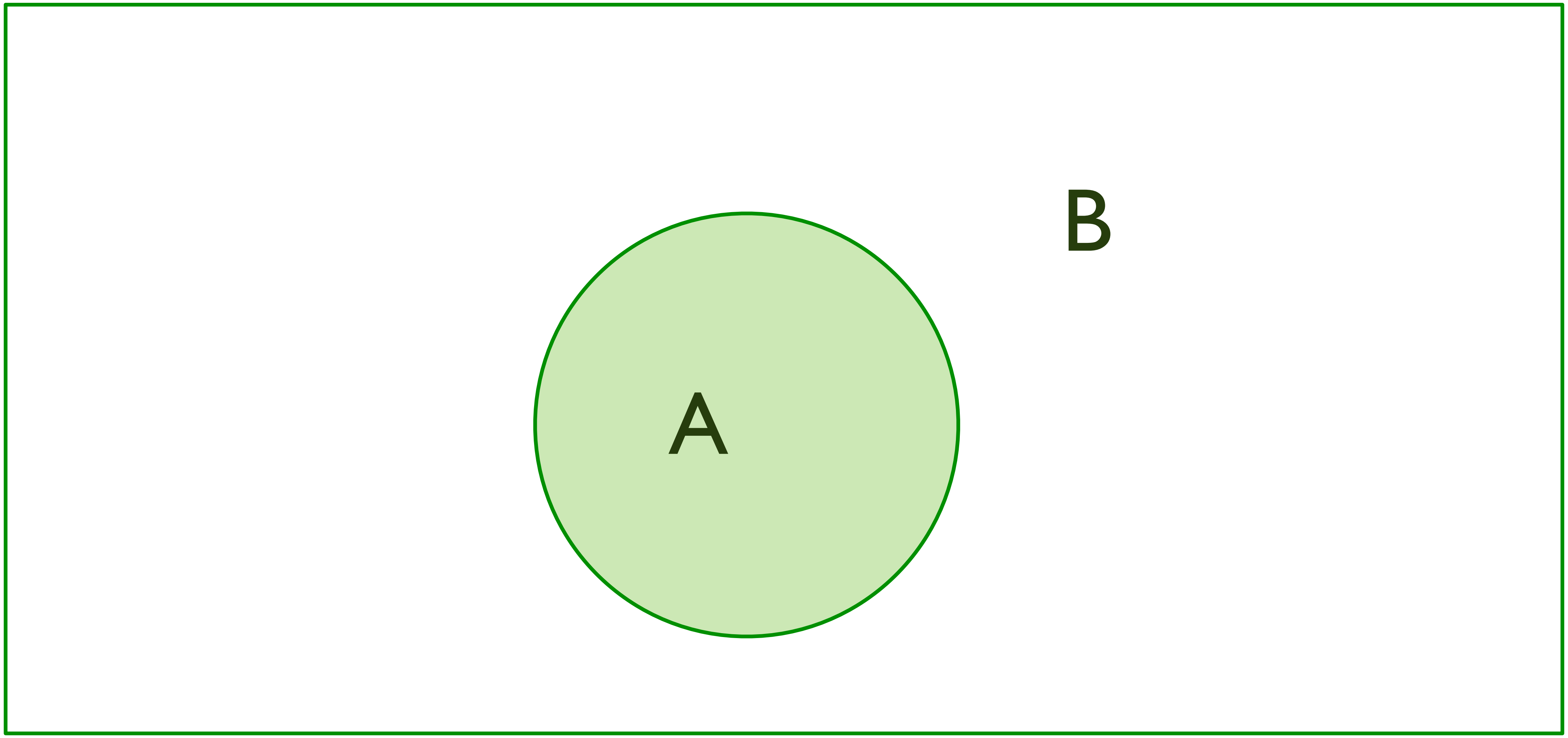

String theory near a D-brane
D-dimensional space
Emergent direction of AdS4

Tensor network representation of entanglement at quantum critical point



Brian Swingle, arXiv:0905.1317

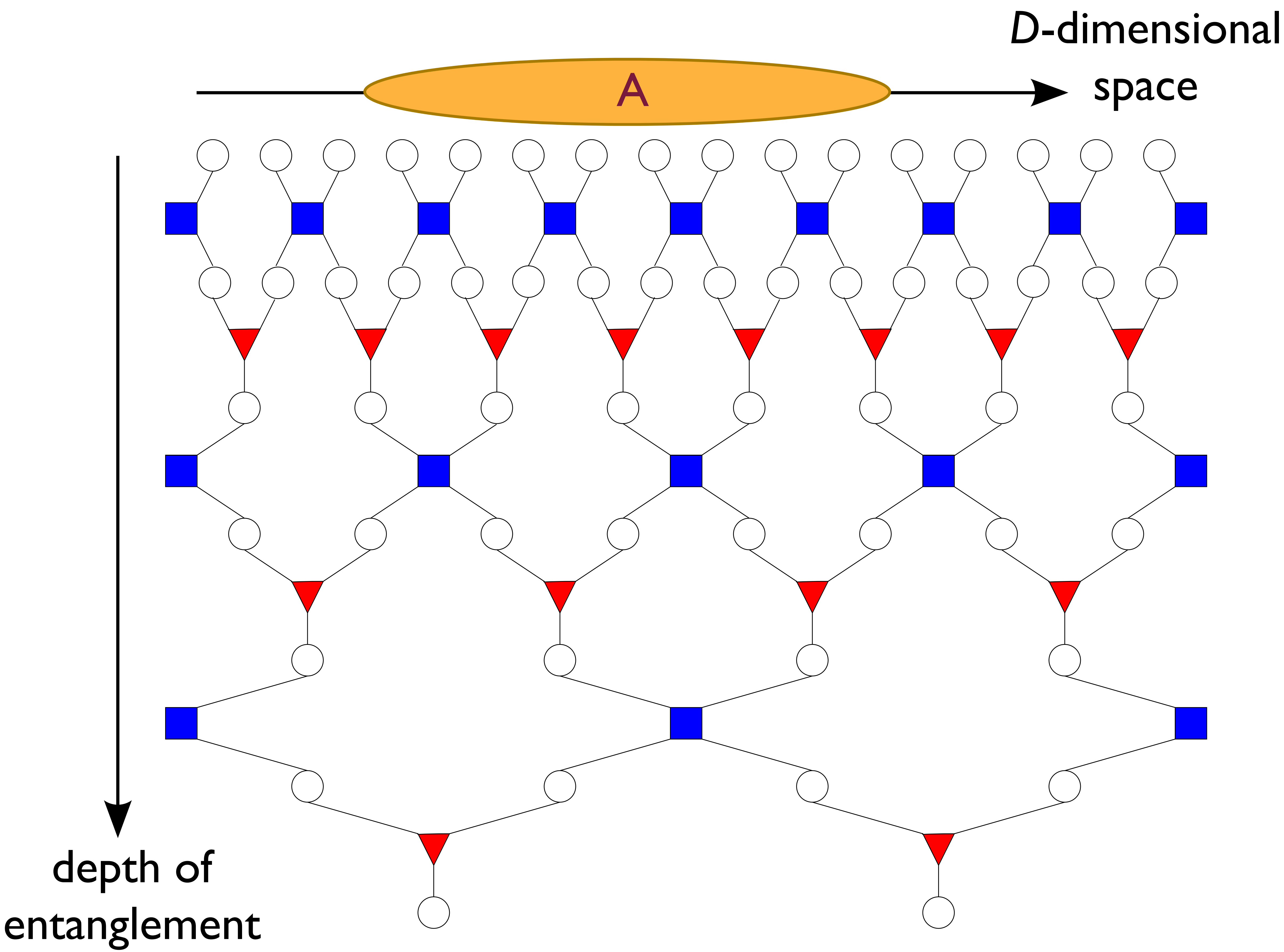
Entanglement entropy



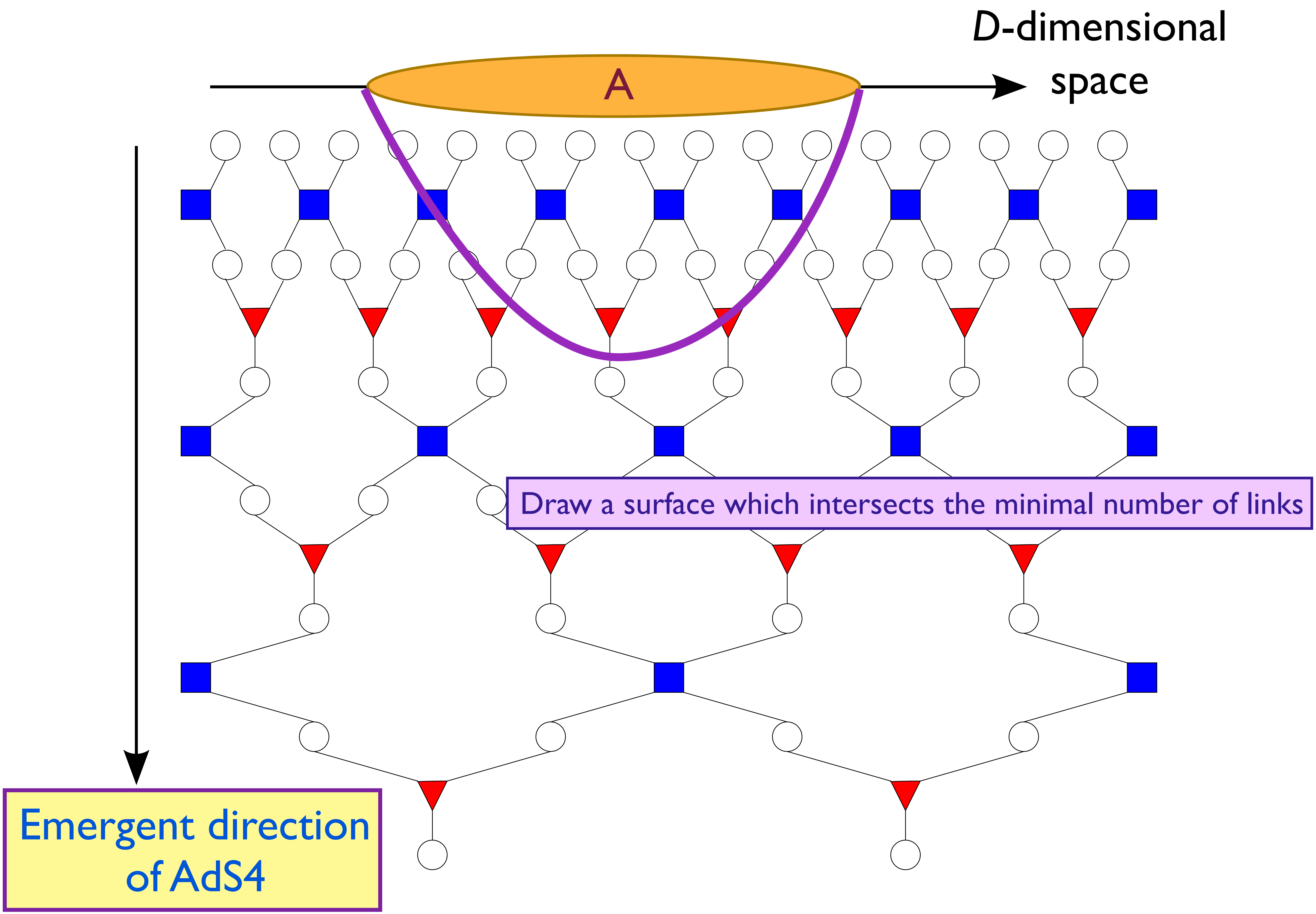
$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_{EE} = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

Entanglement entropy



Entanglement entropy



Entanglement entropy

The entanglement entropy of a region A on the boundary equals the minimal area of a surface in the higher-dimensional space whose boundary co-incides with that of A.

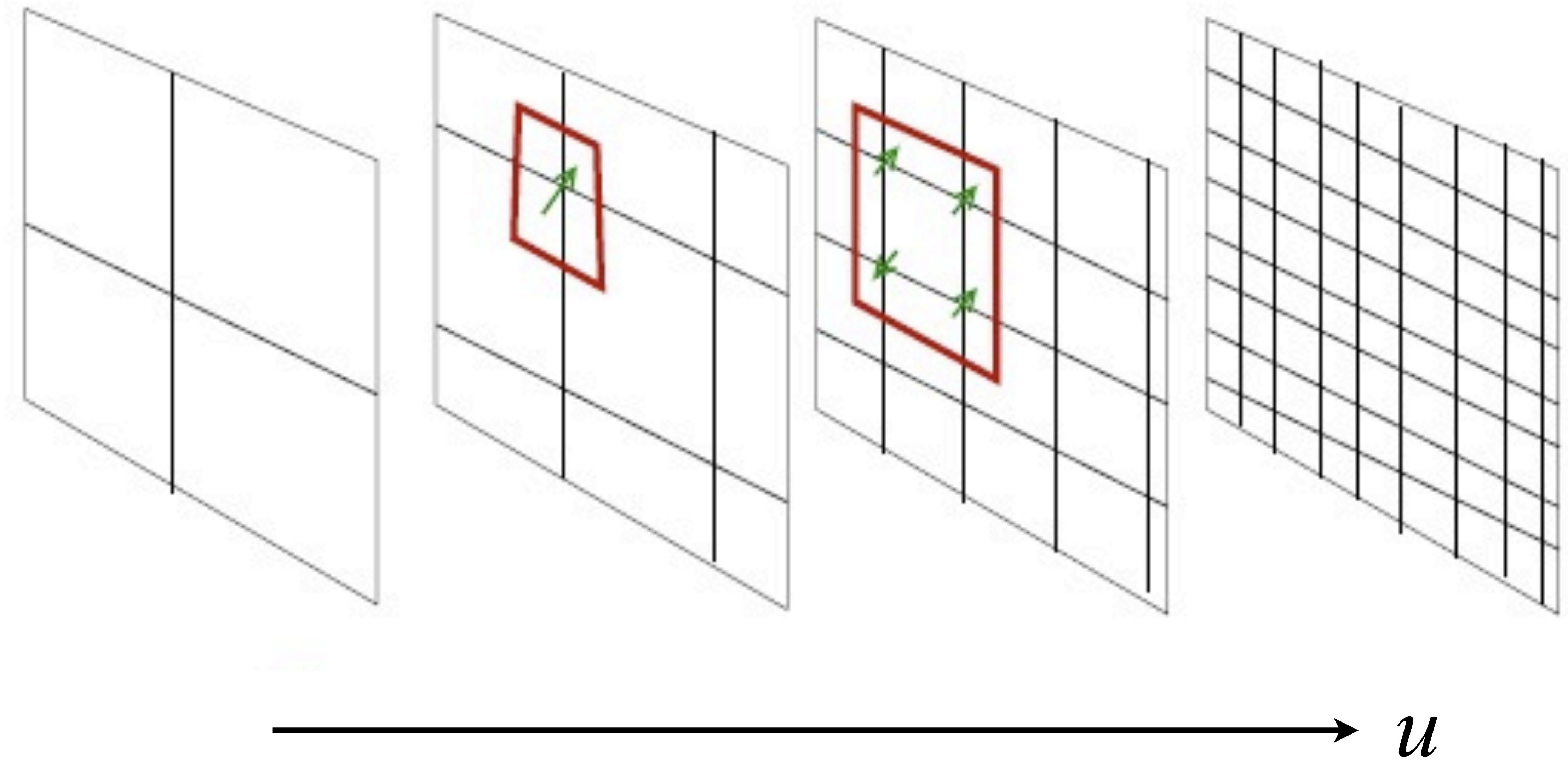
This can be seen both the string and tensor-network pictures

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).
Brian Swingle, arXiv:0905.1317

Field theories in $D+1$ spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u\frac{dg}{du} = \beta(g)$$

where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u.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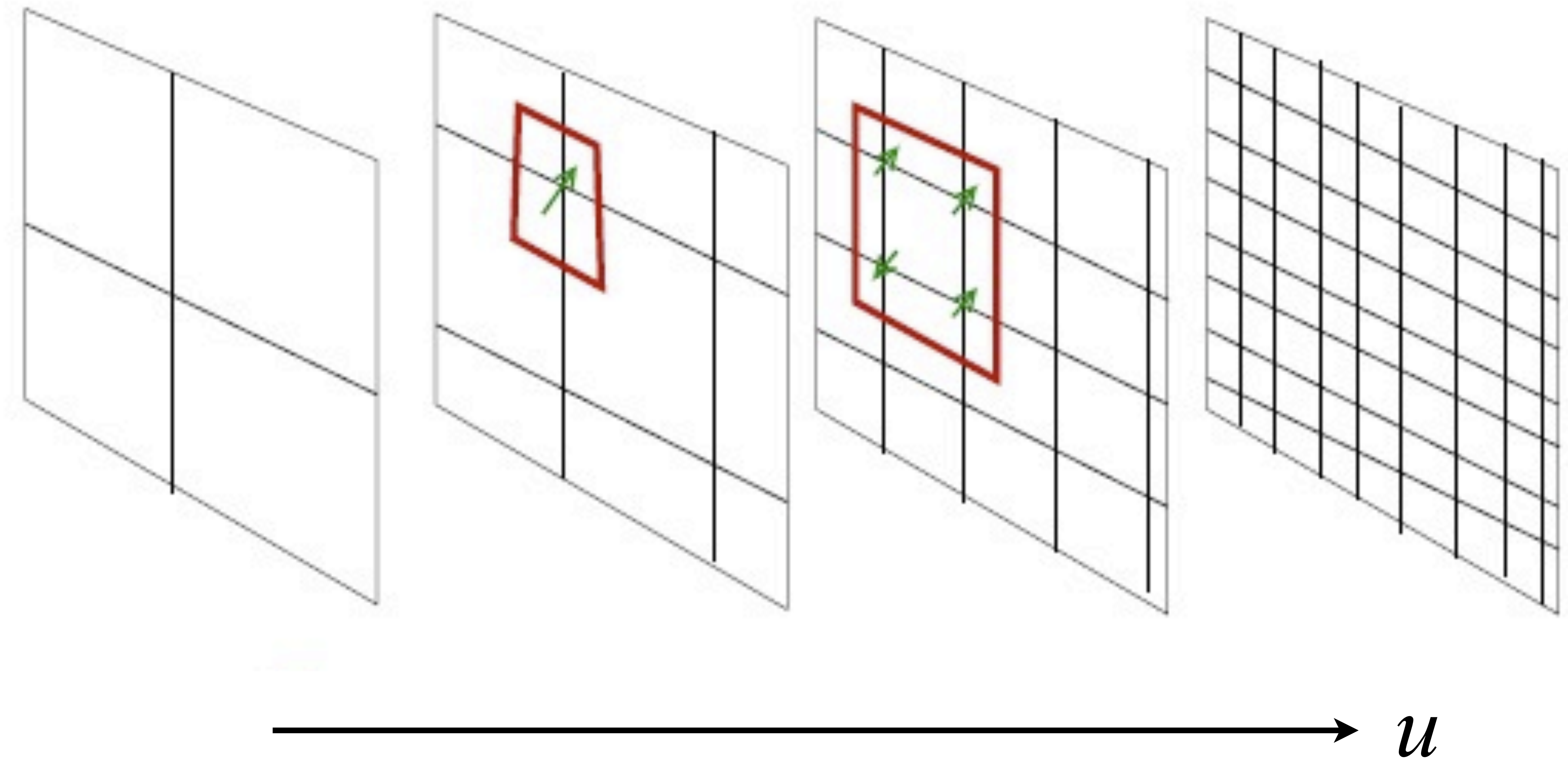

Key idea: $\Rightarrow$ Implement u as an extra dimension, and map to a local theory in $D+2$ spacetime dimensions.

At the RG fixed point, $\beta(g) = 0$, the $D+1$ dimensional field theory is invariant under the scale transformation

$$x^\mu \to x^\mu / b \quad , \quad u \to b\, u$$

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$$x^\mu \to x^\mu / b \quad , \quad u \to b\, u$$

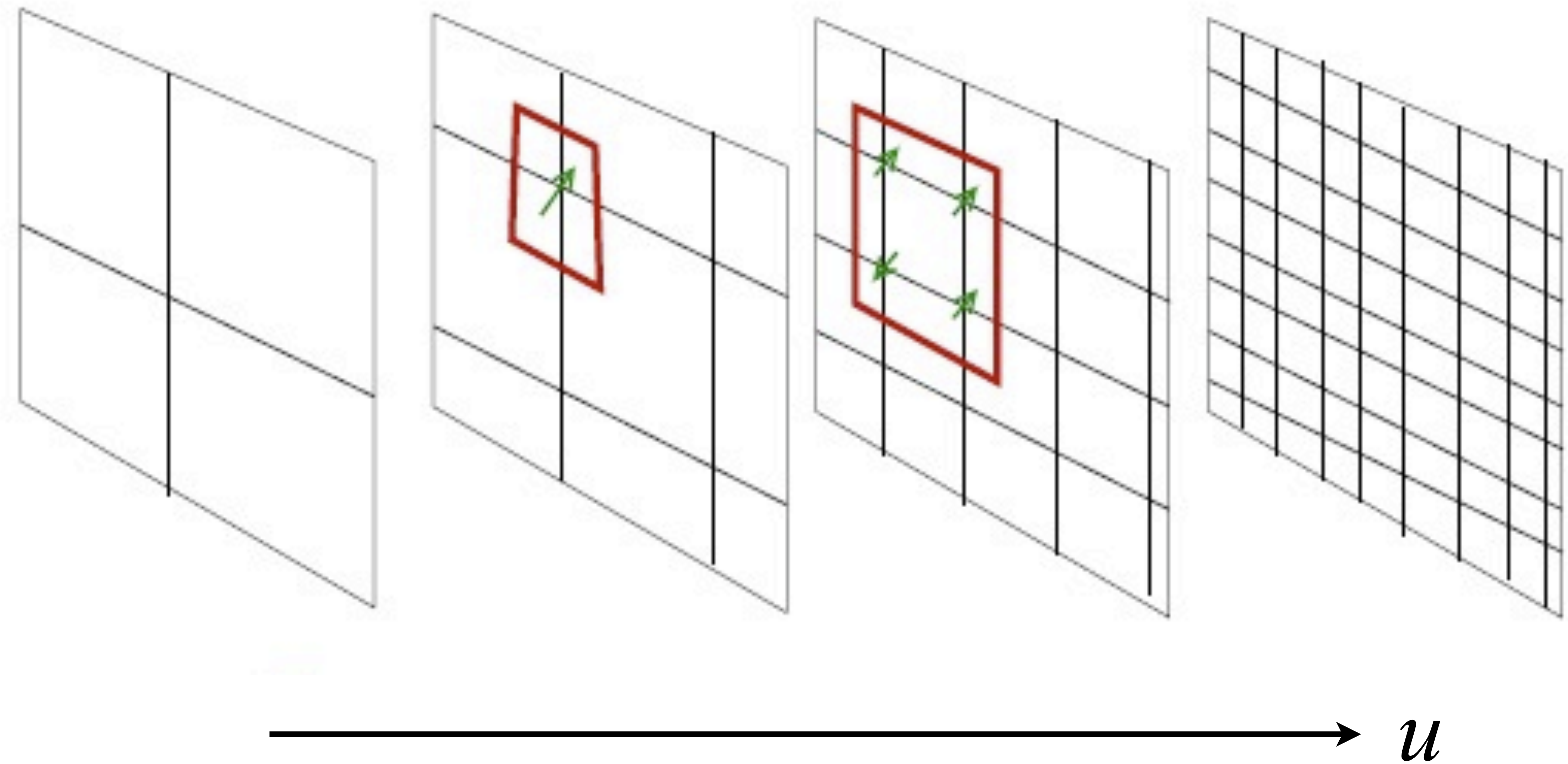
This is an invariance of the *metric* of the theory in $D + 2$ dimensions. The unique solution is

$$ds^2 = \left(\frac{u}{L}\right)^2 dx^\mu dx_\mu + L^2 \frac{du^2}{u^2}.$$

Or, using the length scale $z = L^2/u$

$$ds^2 = L^2 \frac{dx^\mu dx_\mu + dz^2}{z^2}.$$

This is the space AdS_{D+2}, and L is the AdS radius.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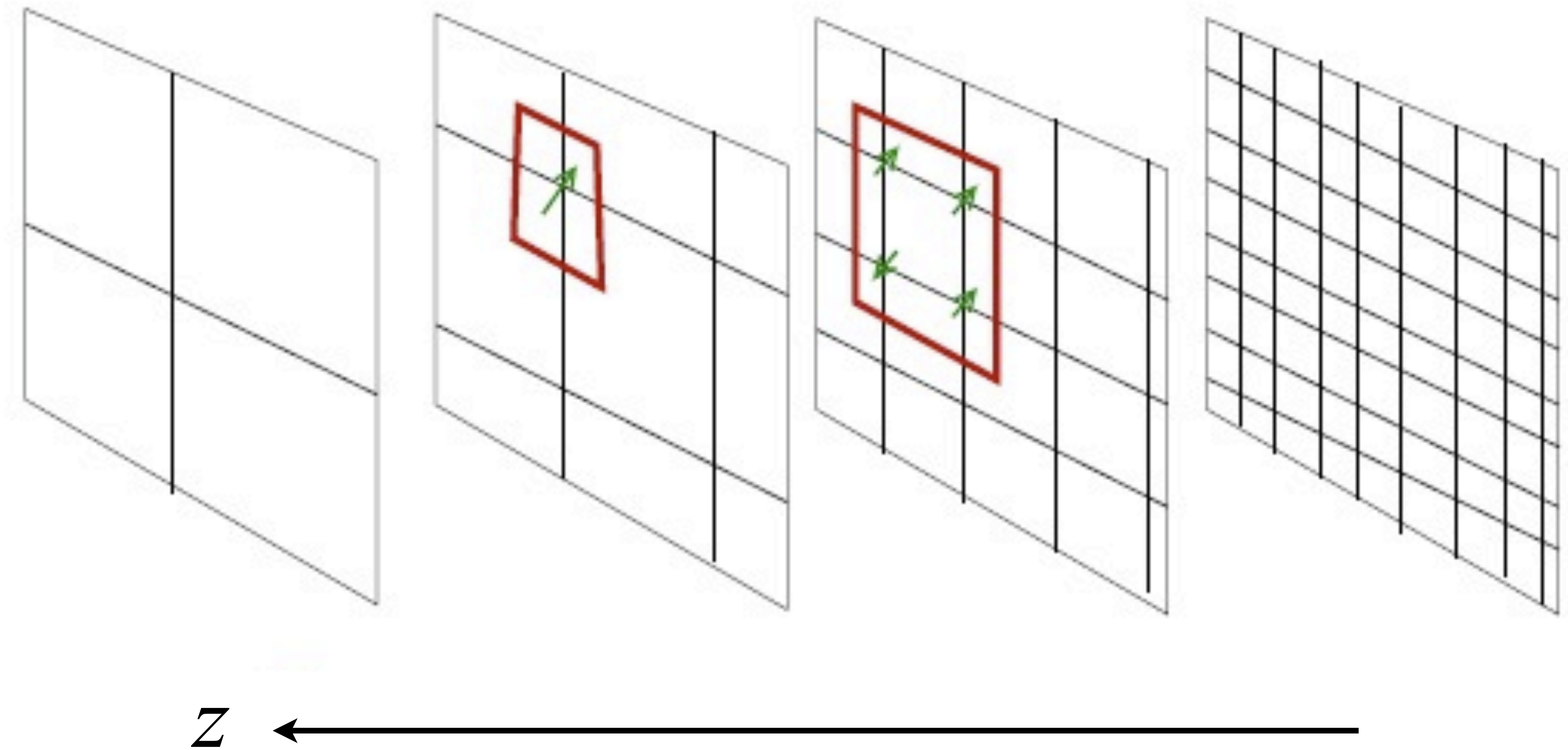
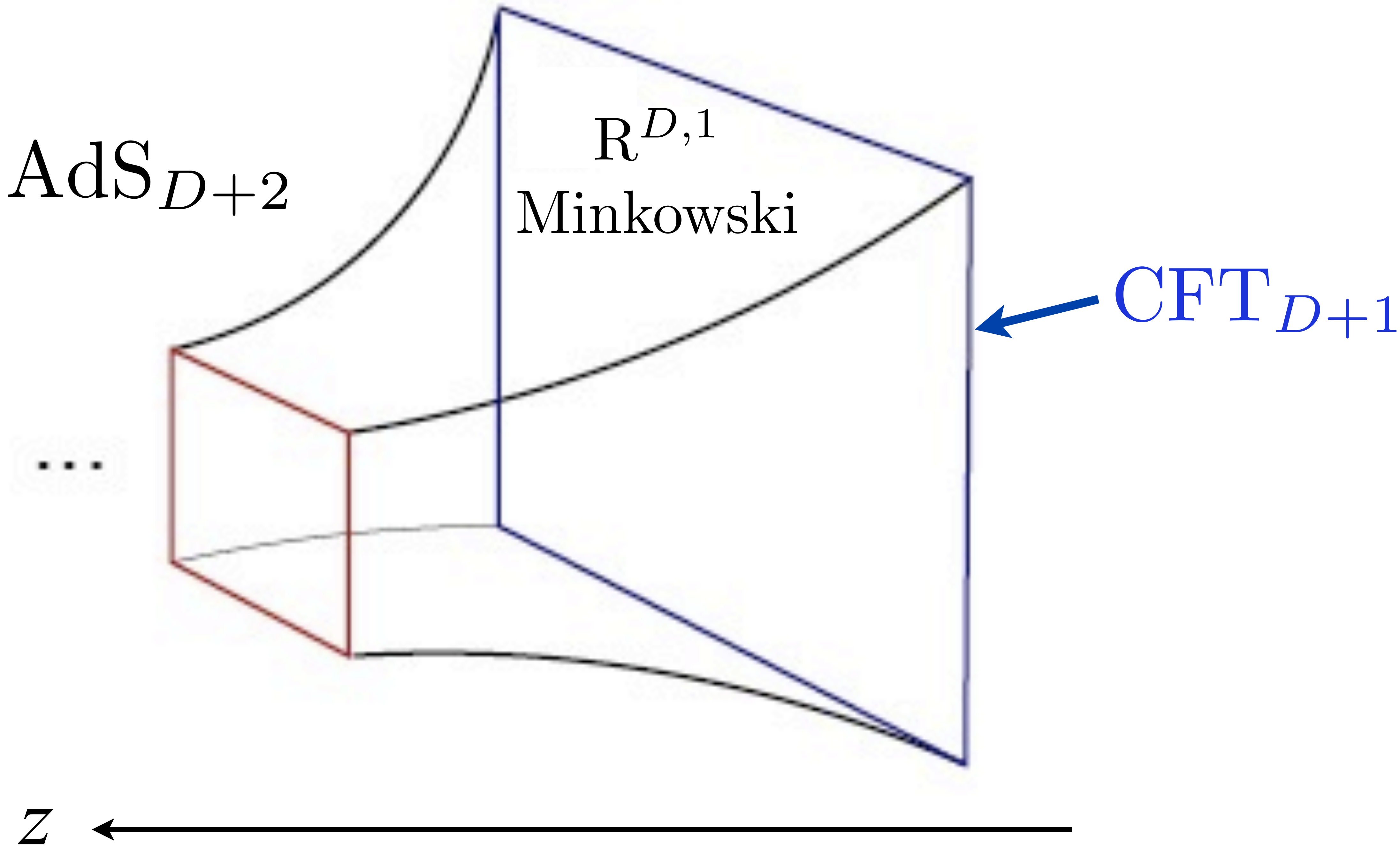
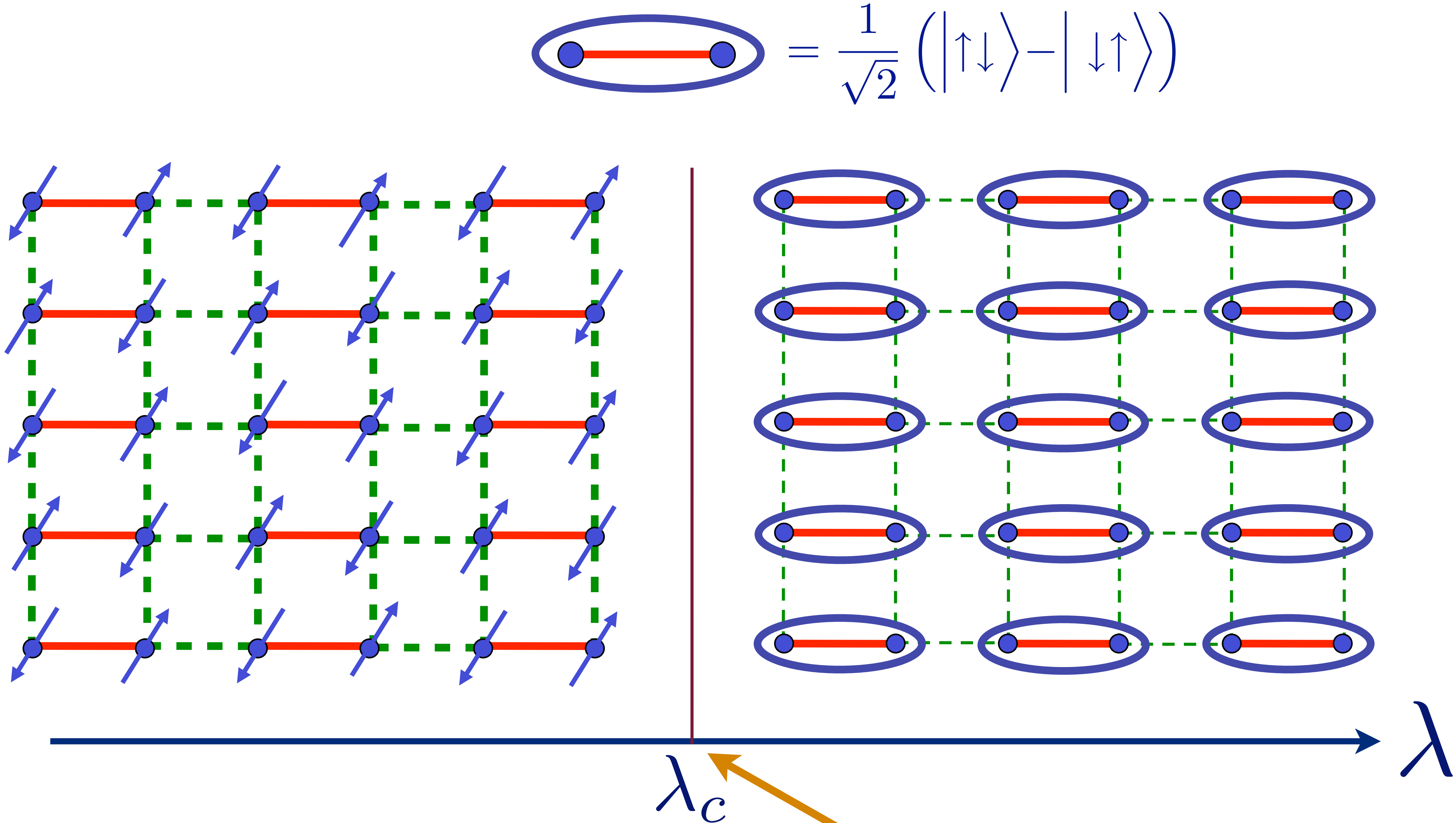


J. McGreevy, arXiv0909.0518

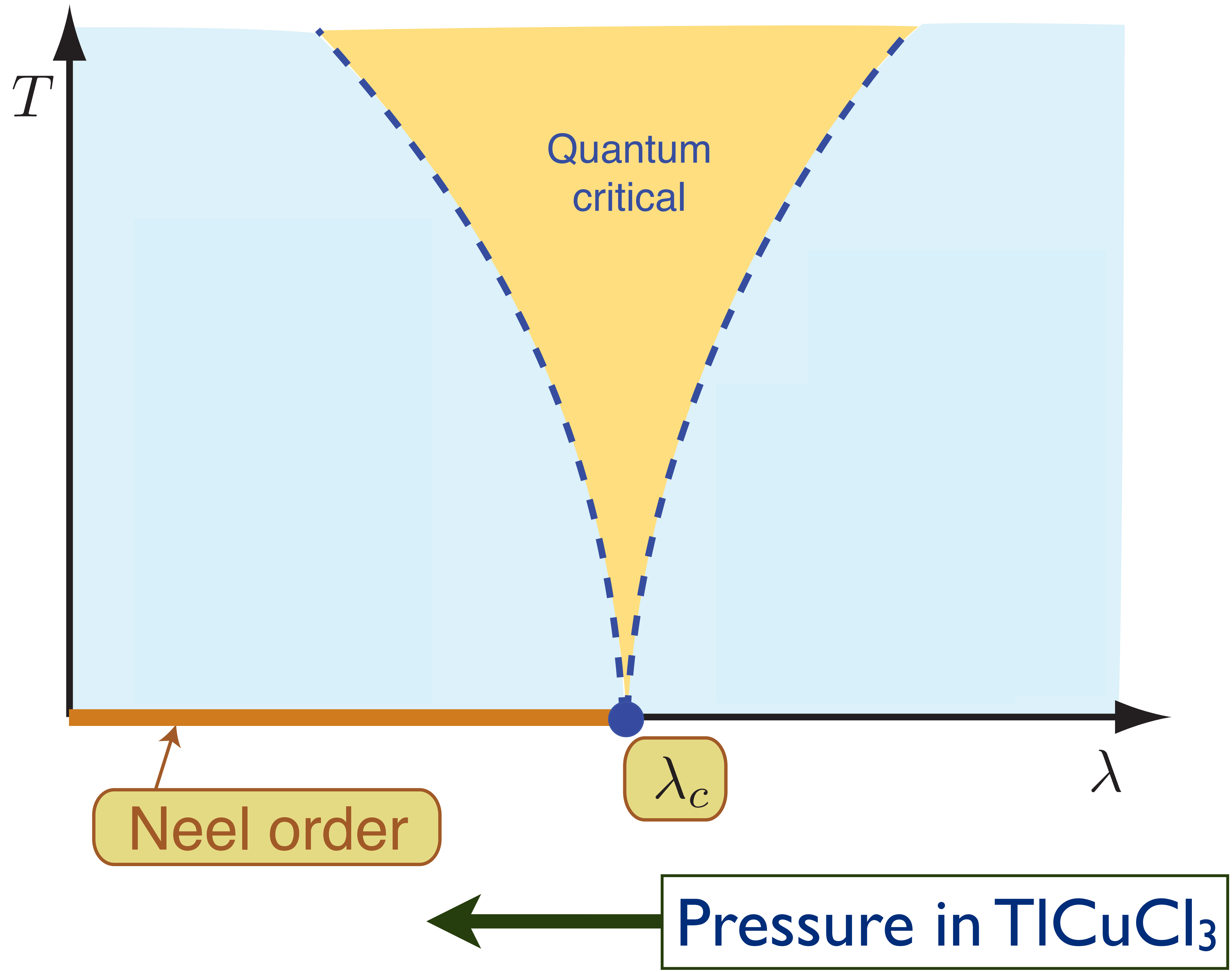


J. McGreevy, arXiv0909.0518



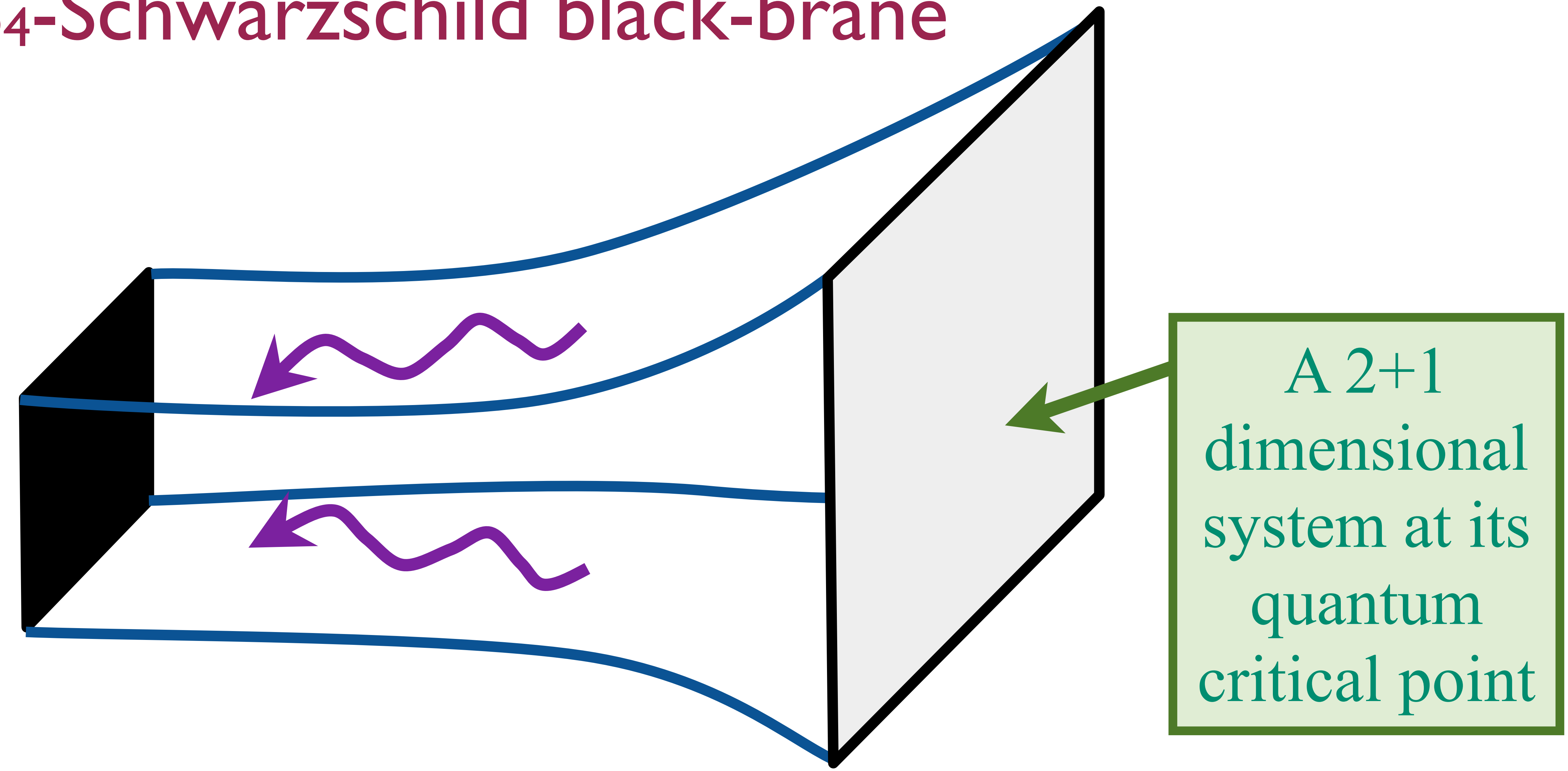
Quantum critical point with non-local entanglement in spin wavefunction

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).



AdS/CFT correspondence at non-zero temperatures

AdS_4-Schwarzschild black-brane



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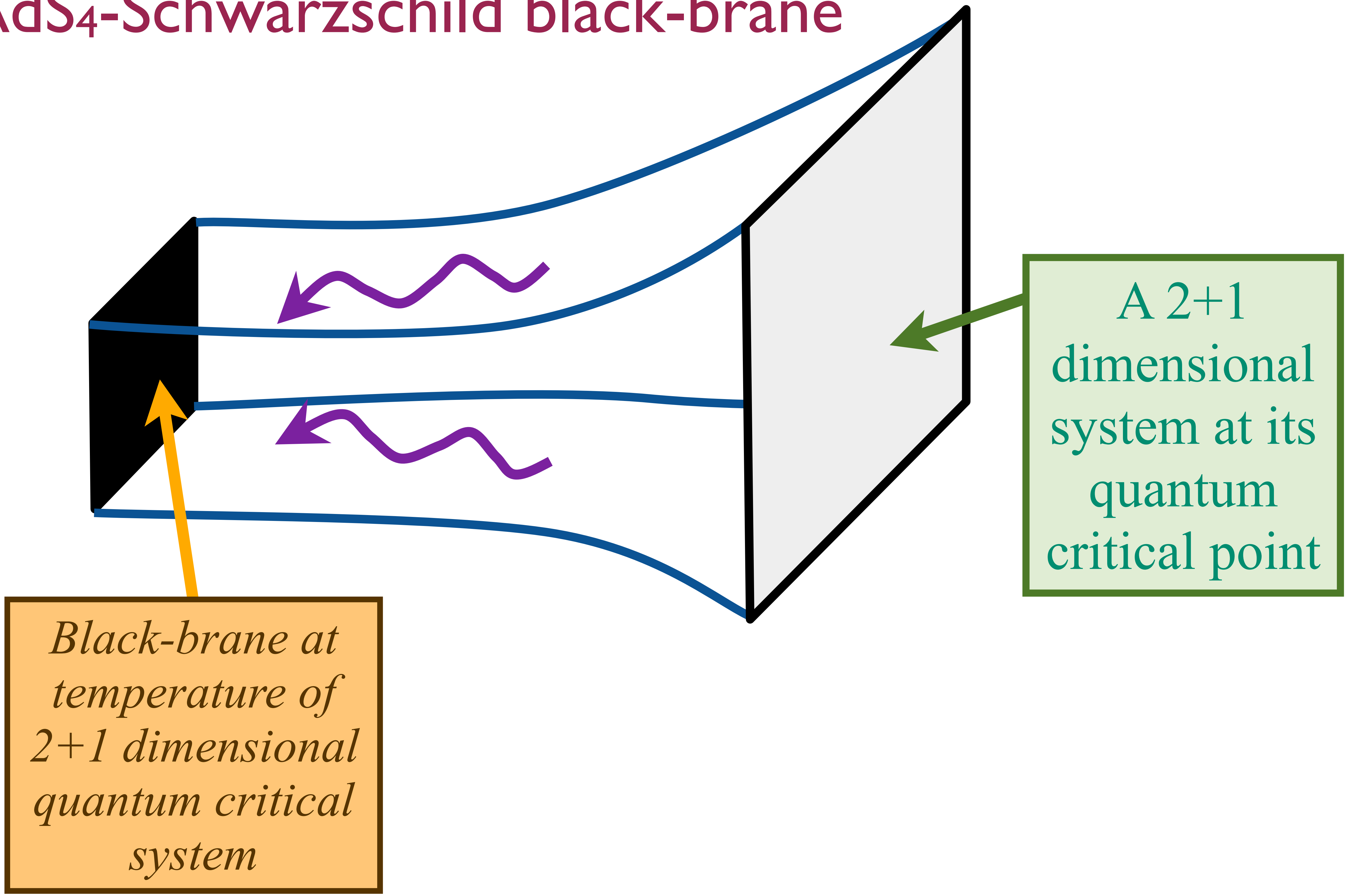

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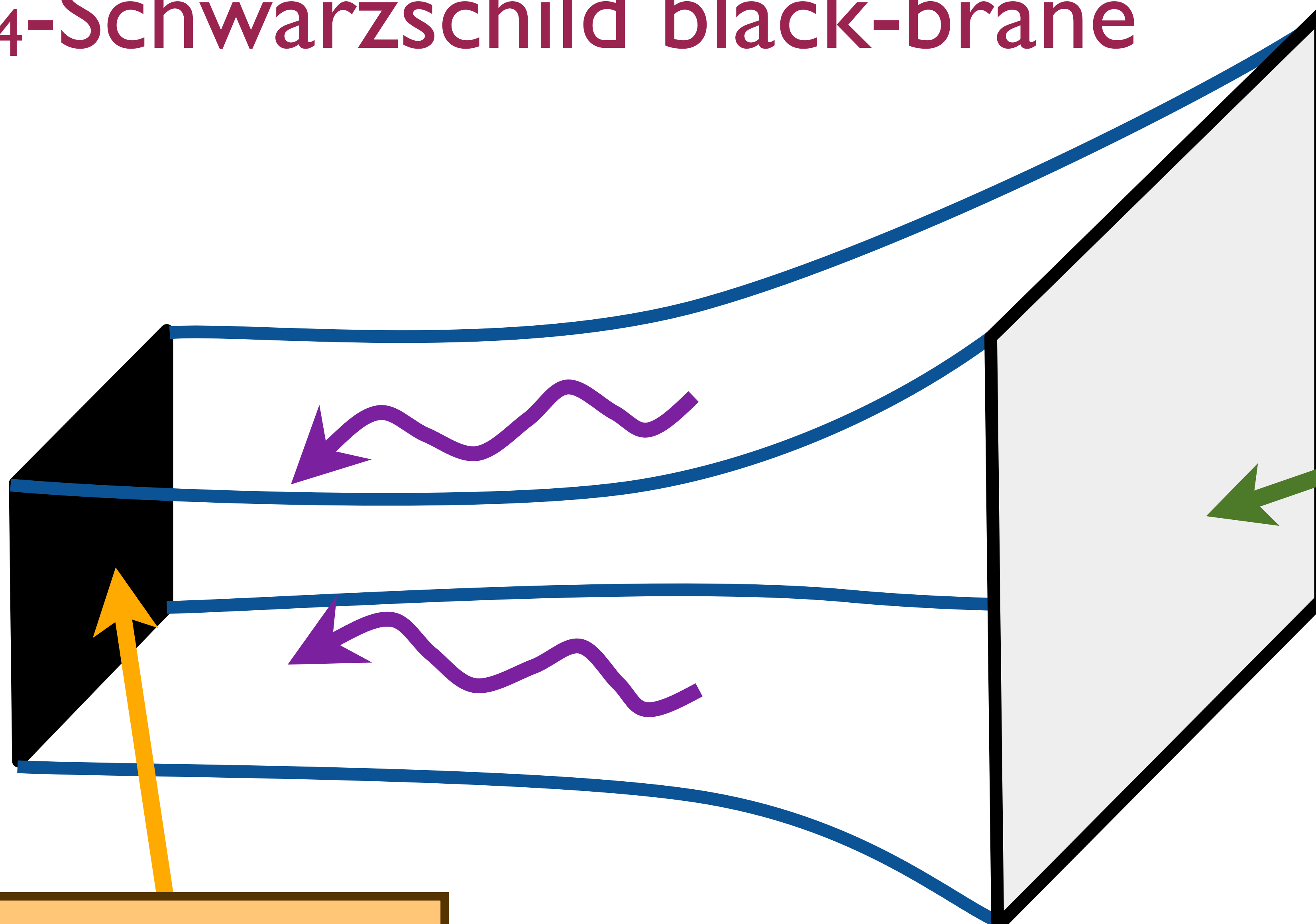


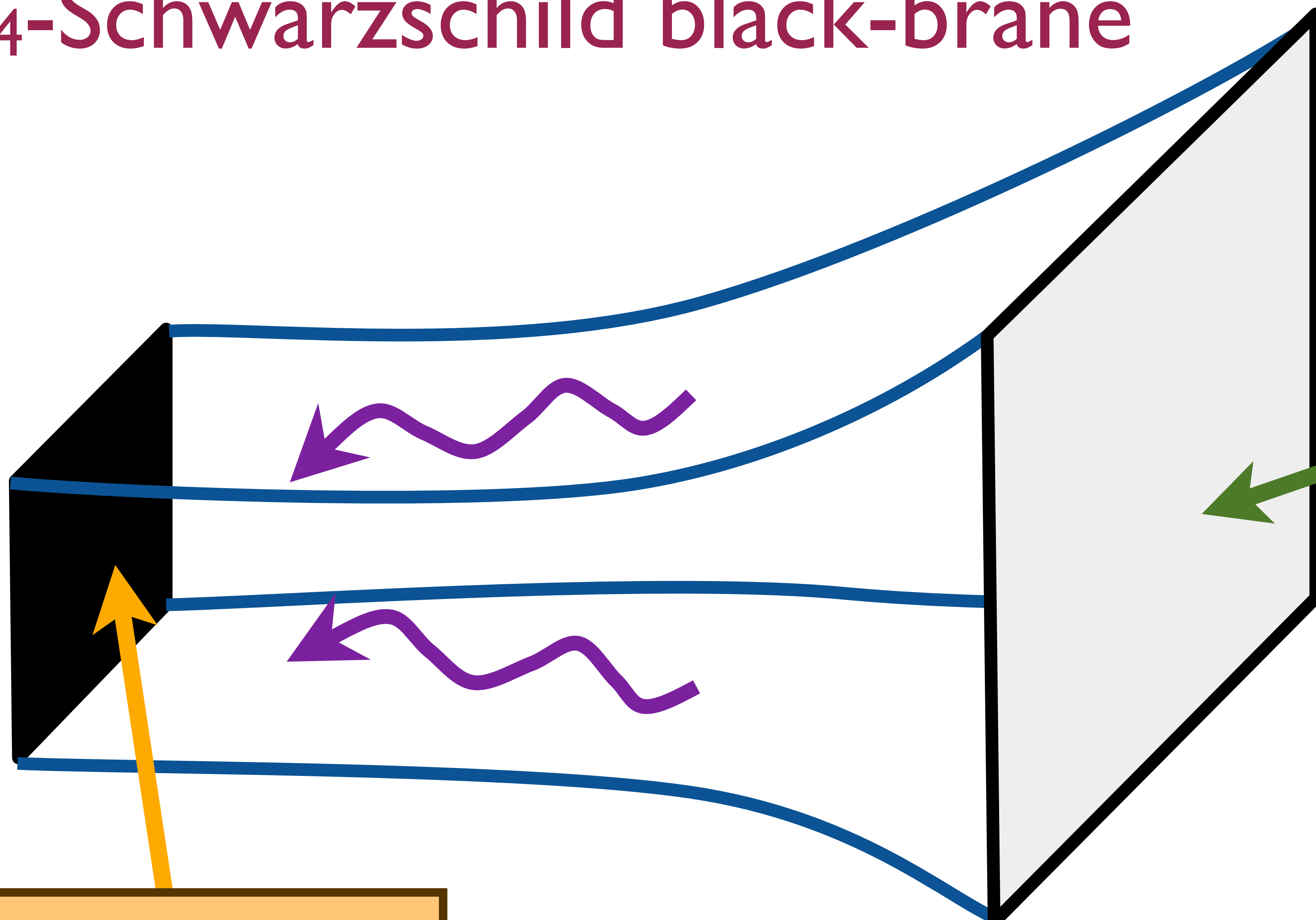
A 2+1 dimensional system at its quantum critical point

Black-brane at temperature of 2+1 dimensional quantum critical system

Friction of quantum criticality = waves falling into black brane

AdS/CFT correspondence at non-zero temperatures

AdS_4-Schwarzschild black-brane



A 2+1 dimensional system at its quantum critical point

Black-brane at temperature of 2+1 dimensional quantum critical system

Provides successful description of many properties of quantum critical points at non-zero temperatures

Outline

1. Quantum critical points and string theory

   Entanglement and emergent dimensions

2. High temperature superconductors and strange metals

   Holography of compressible quantum phases

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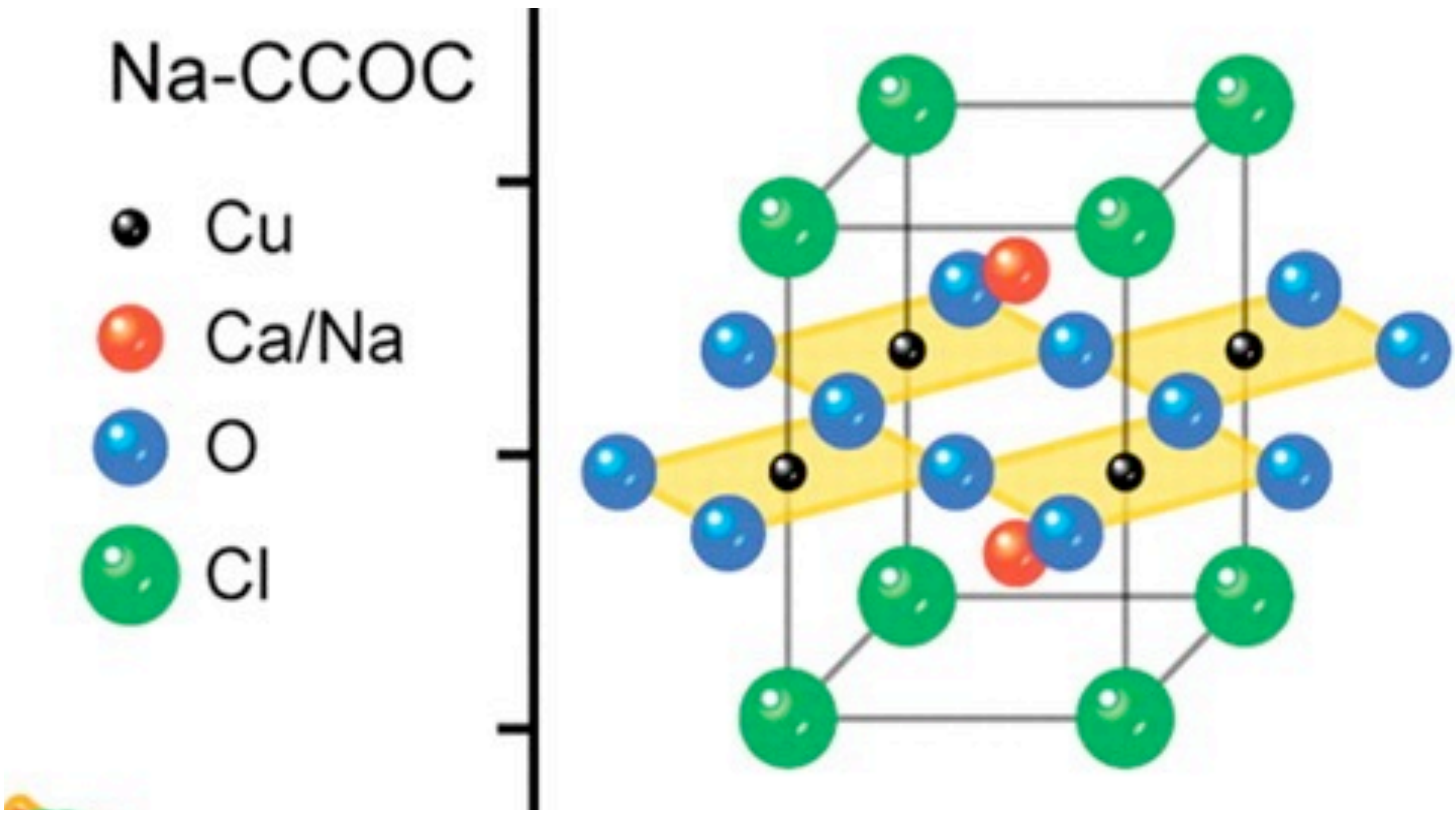
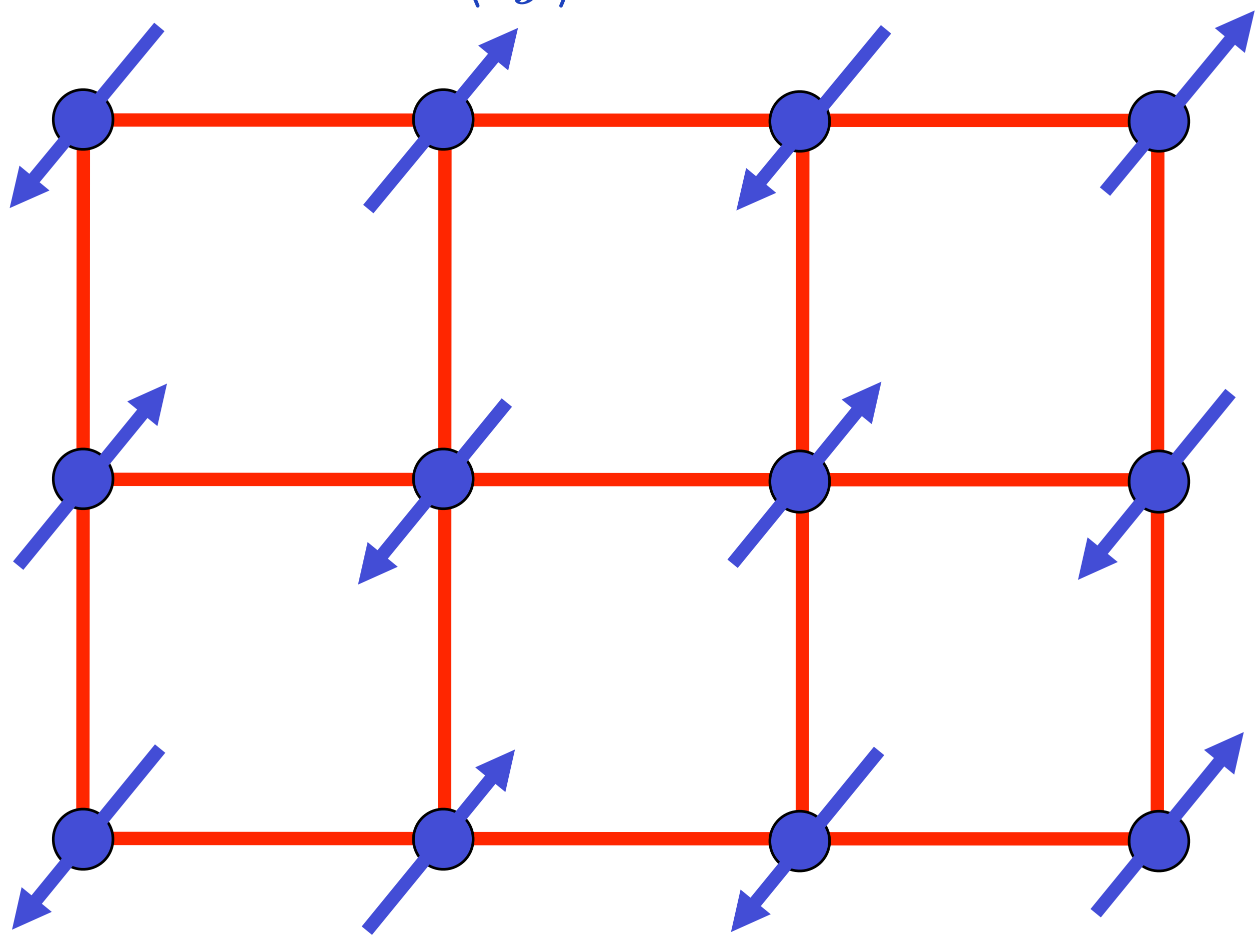
The cuprate superconductors



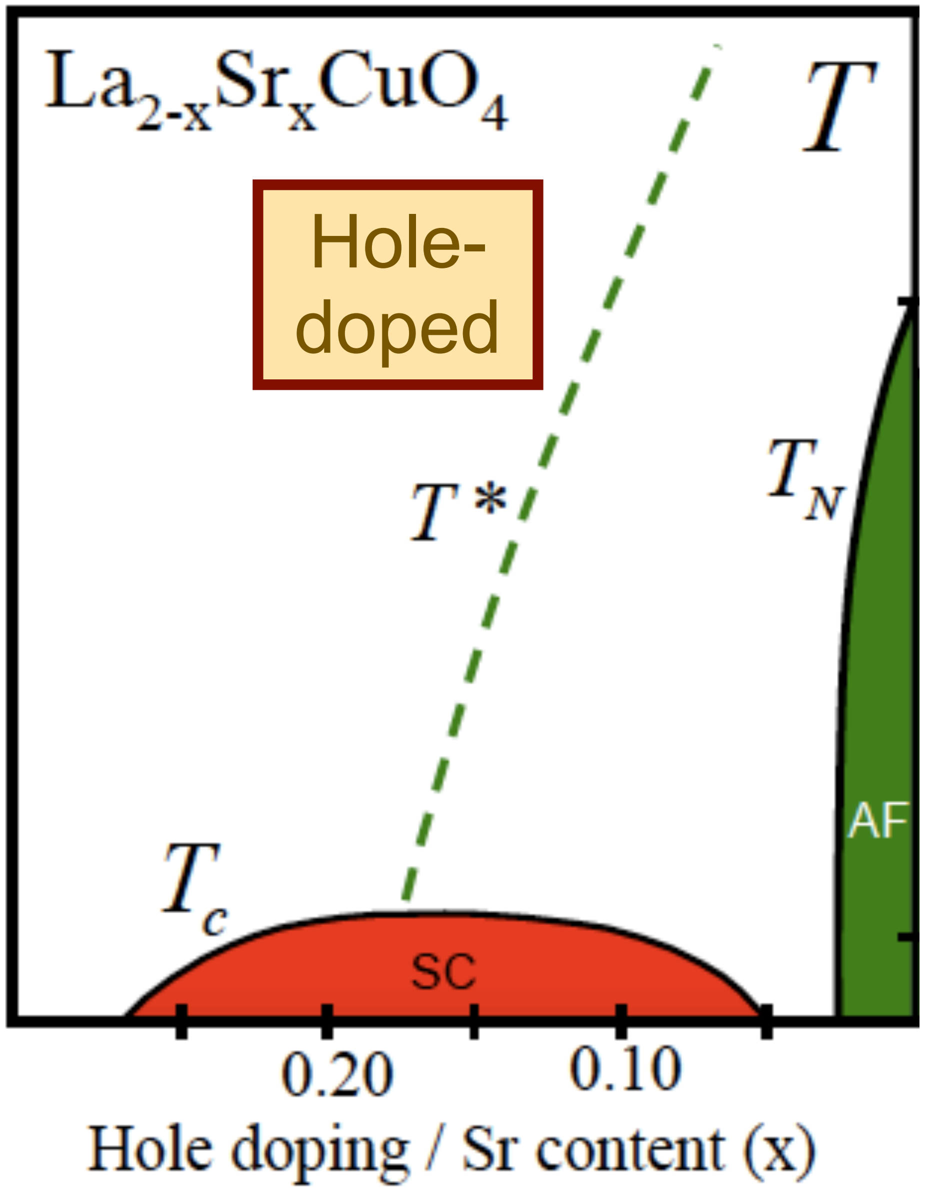
Davis

Square lattice antiferromagnet

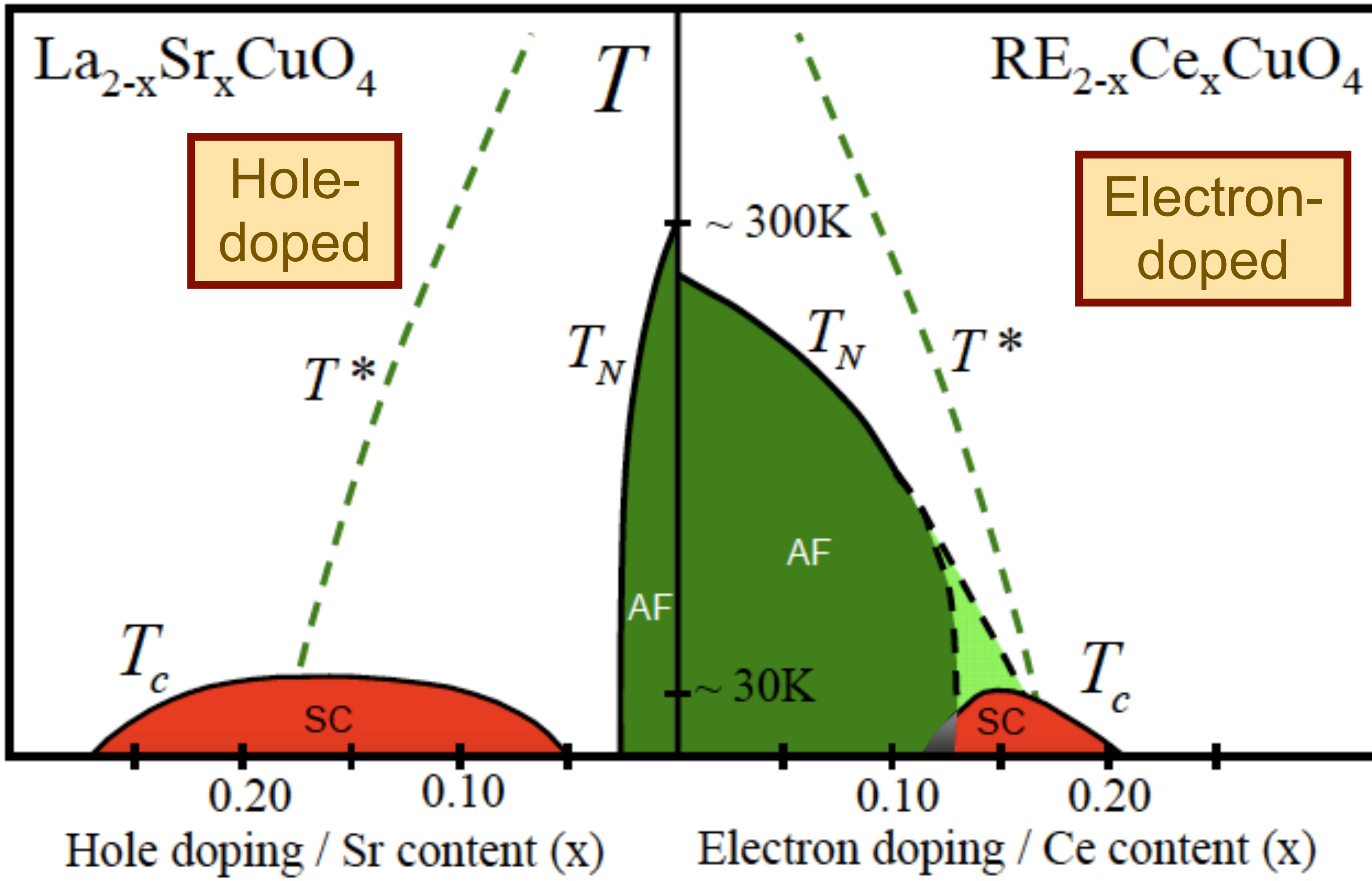
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order


$La_{2-x}Sr_xCuO_4$
Hole-doped
T
T^*
T_N
AF
T_c
SC
0.20
0.10
Hole doping / Sr content (x)

Electron-doped cuprate superconductors



Electron-doped cuprate superconductors

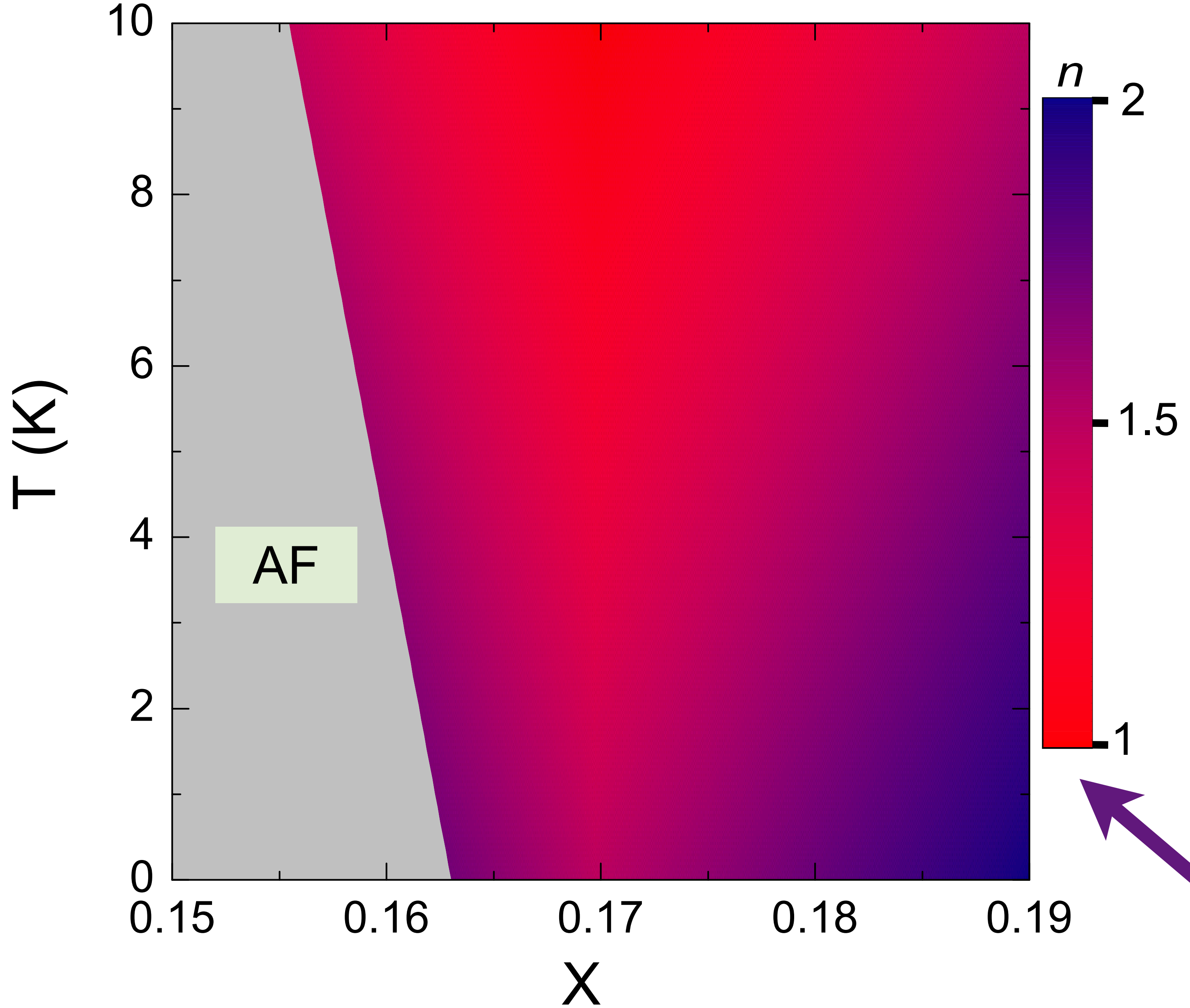
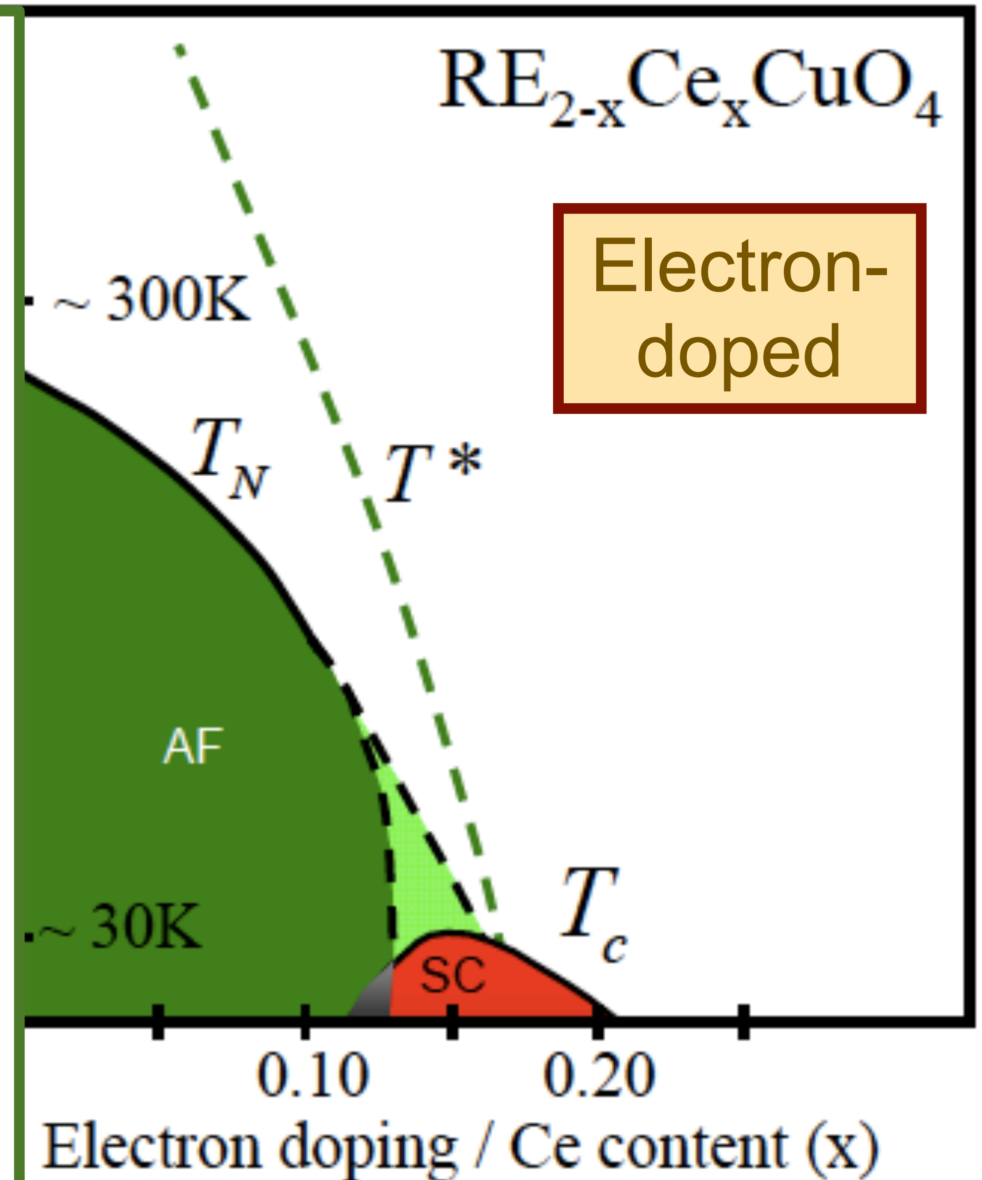
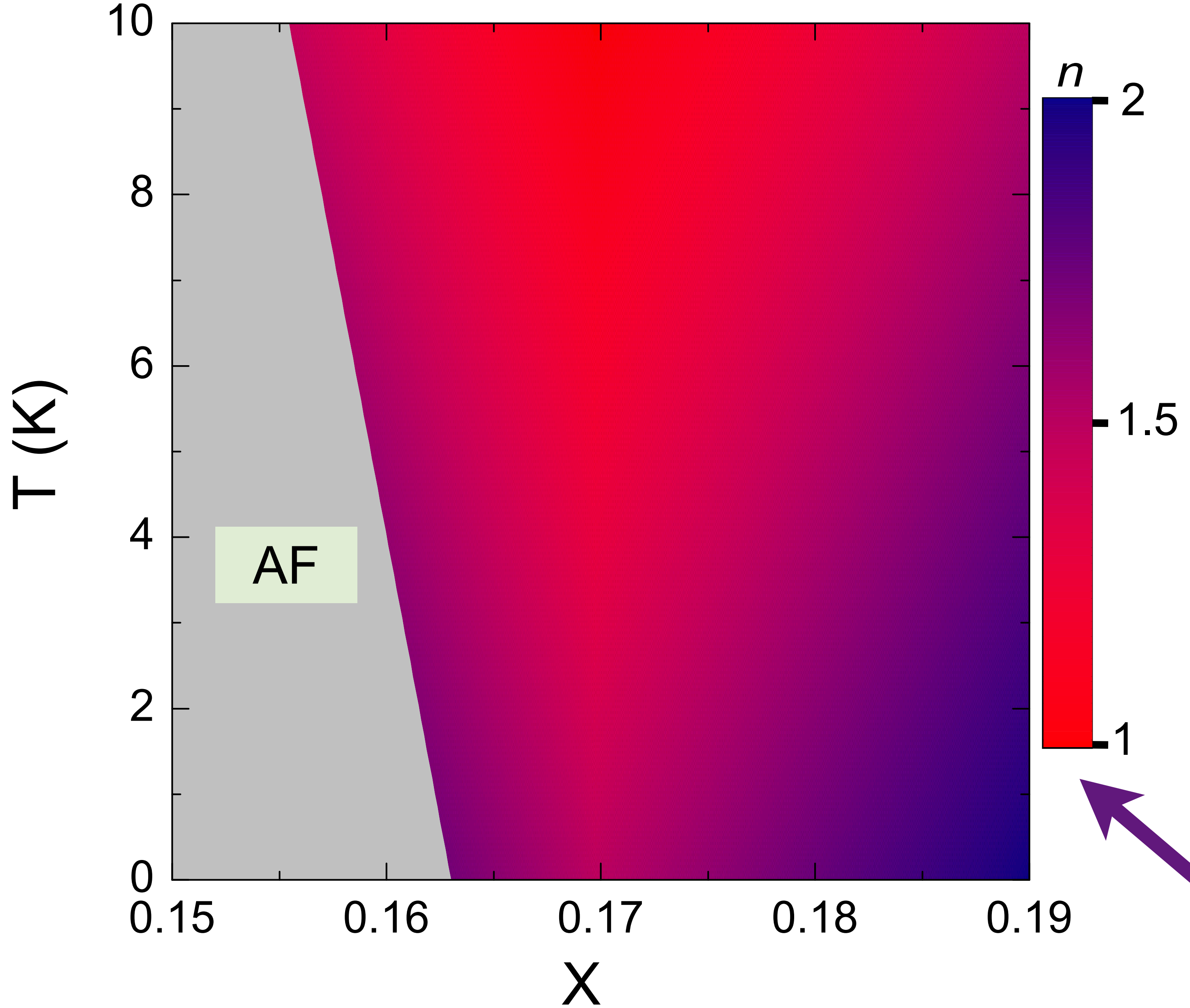


Figure prepared by K. Jin and and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).



Resistivity $\sim \rho_0 + AT^n$

Electron-doped cuprate superconductors



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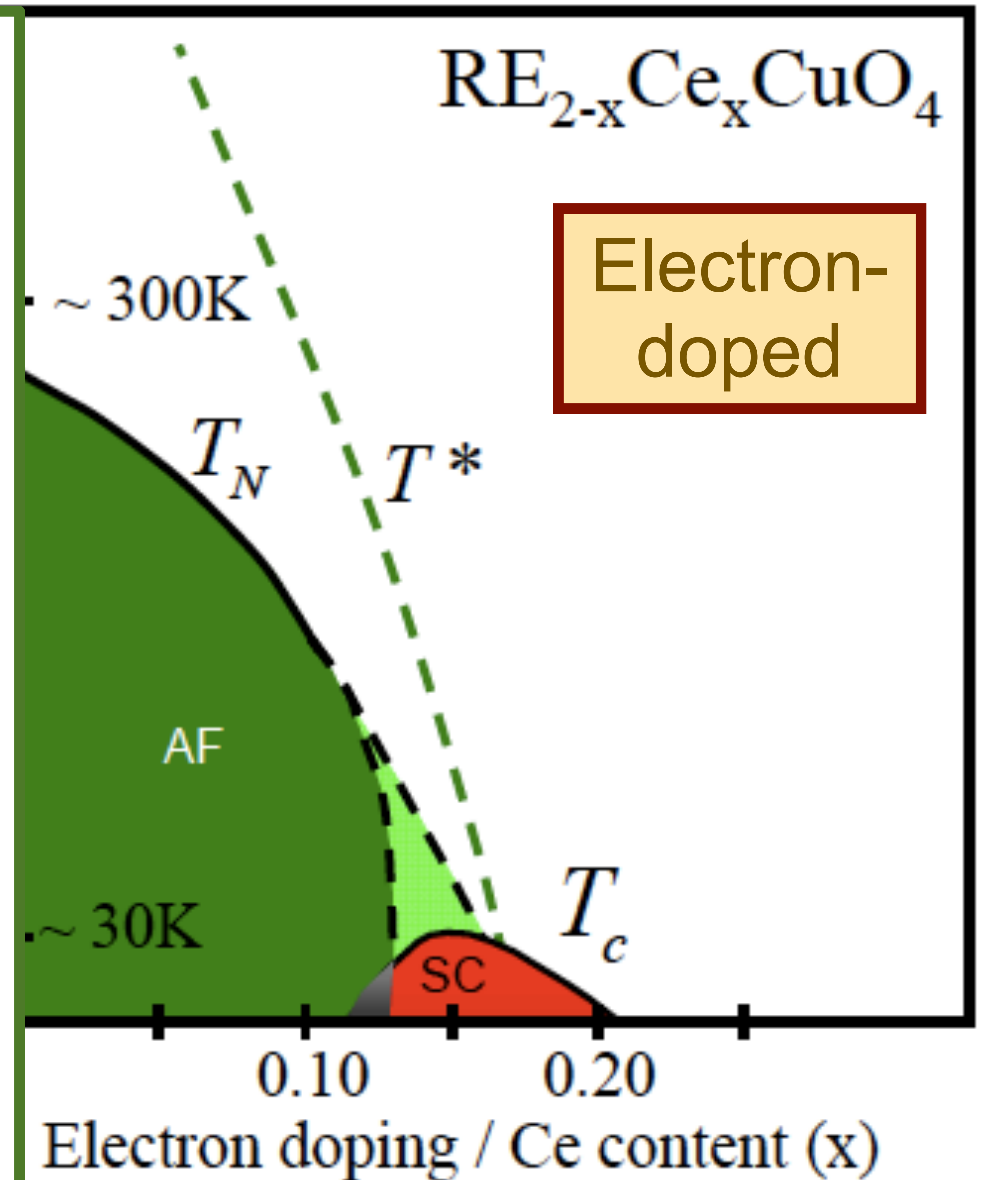

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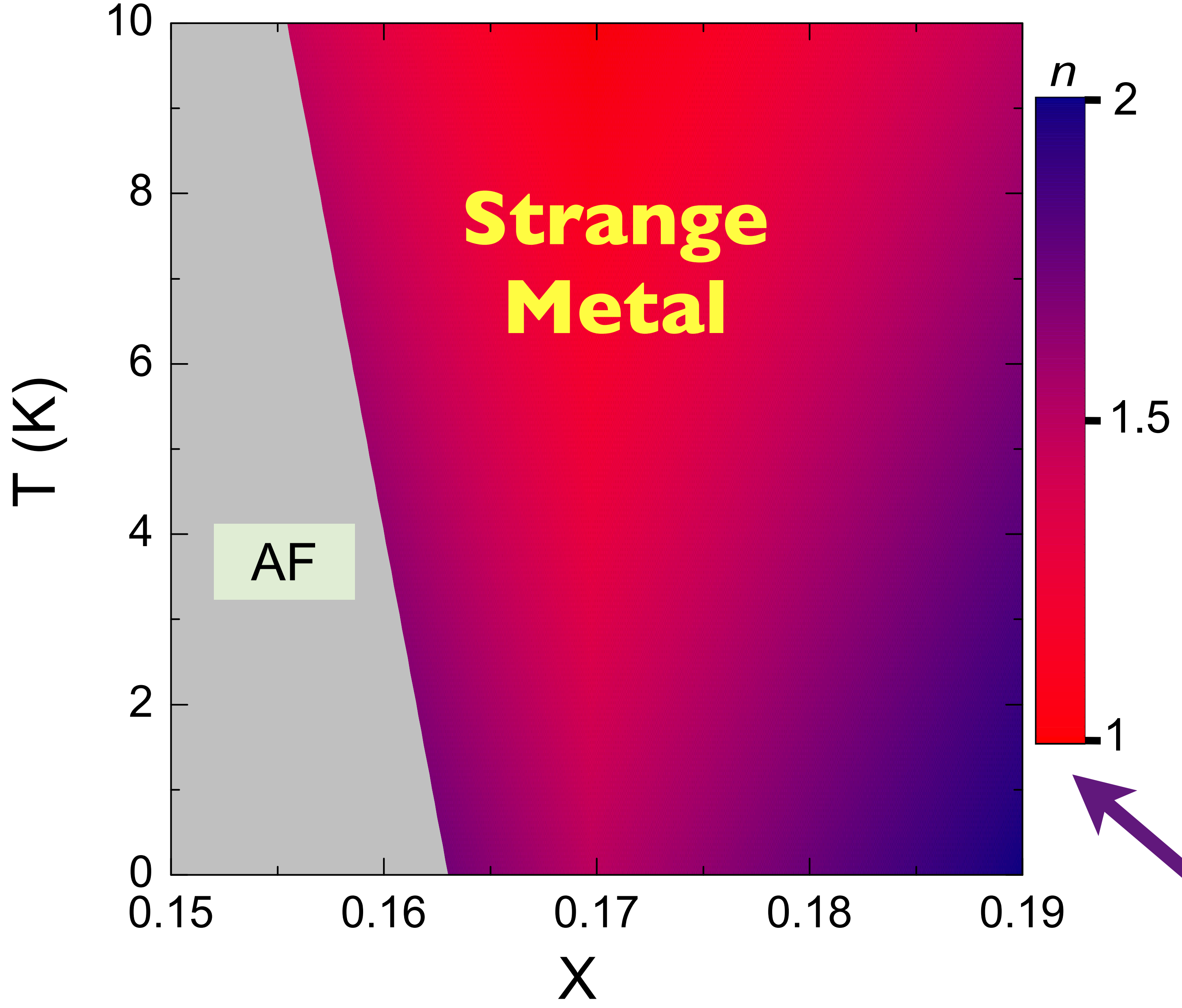

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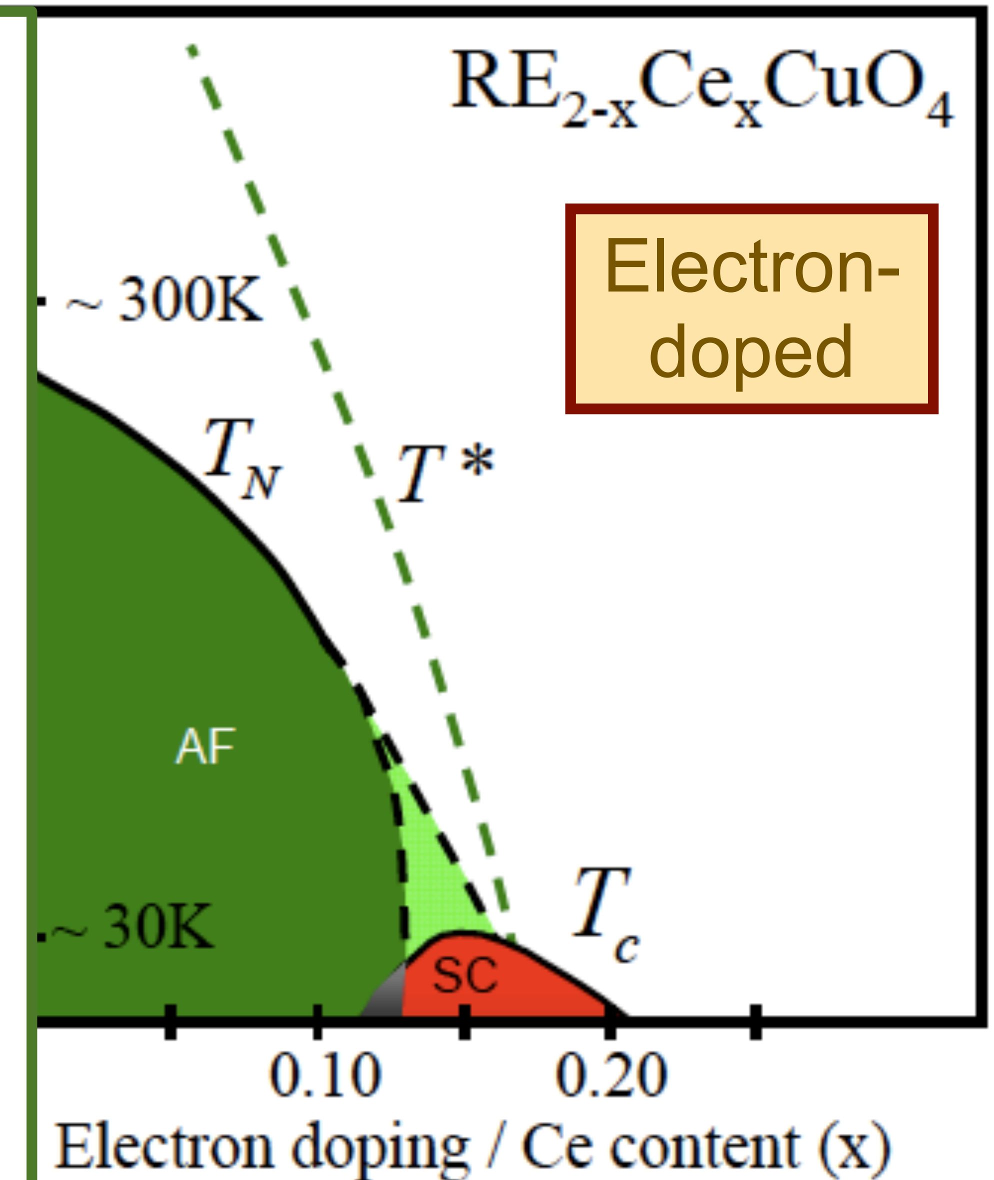
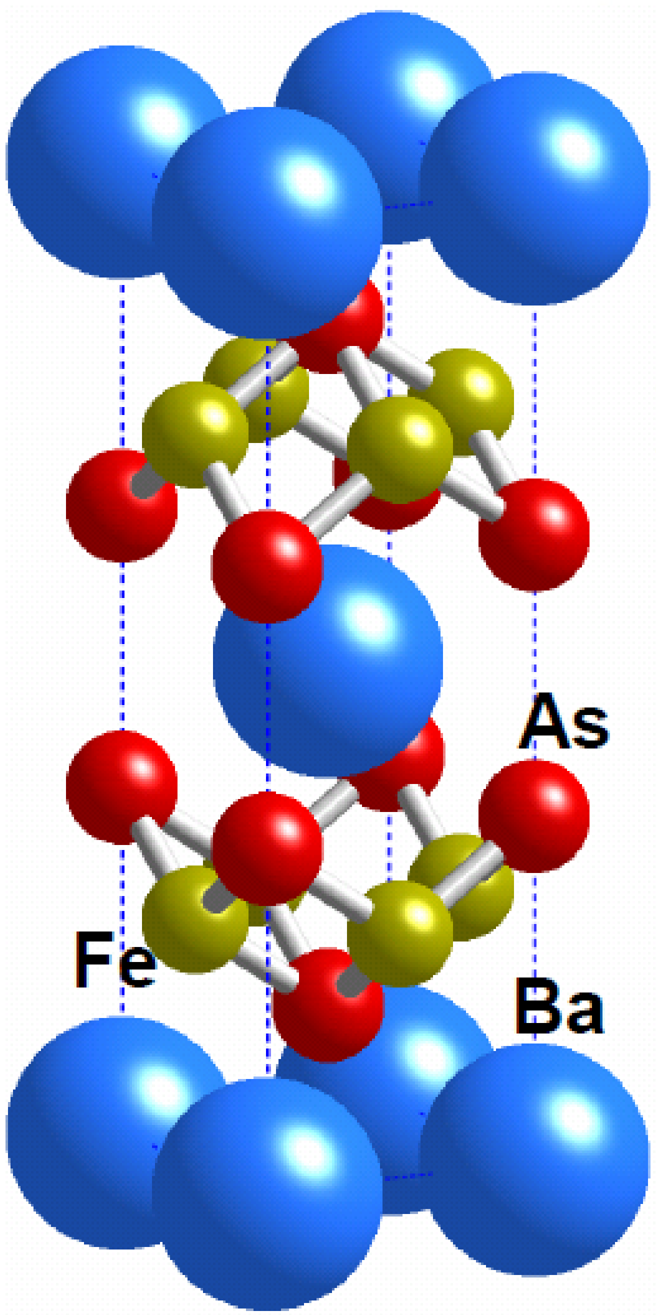
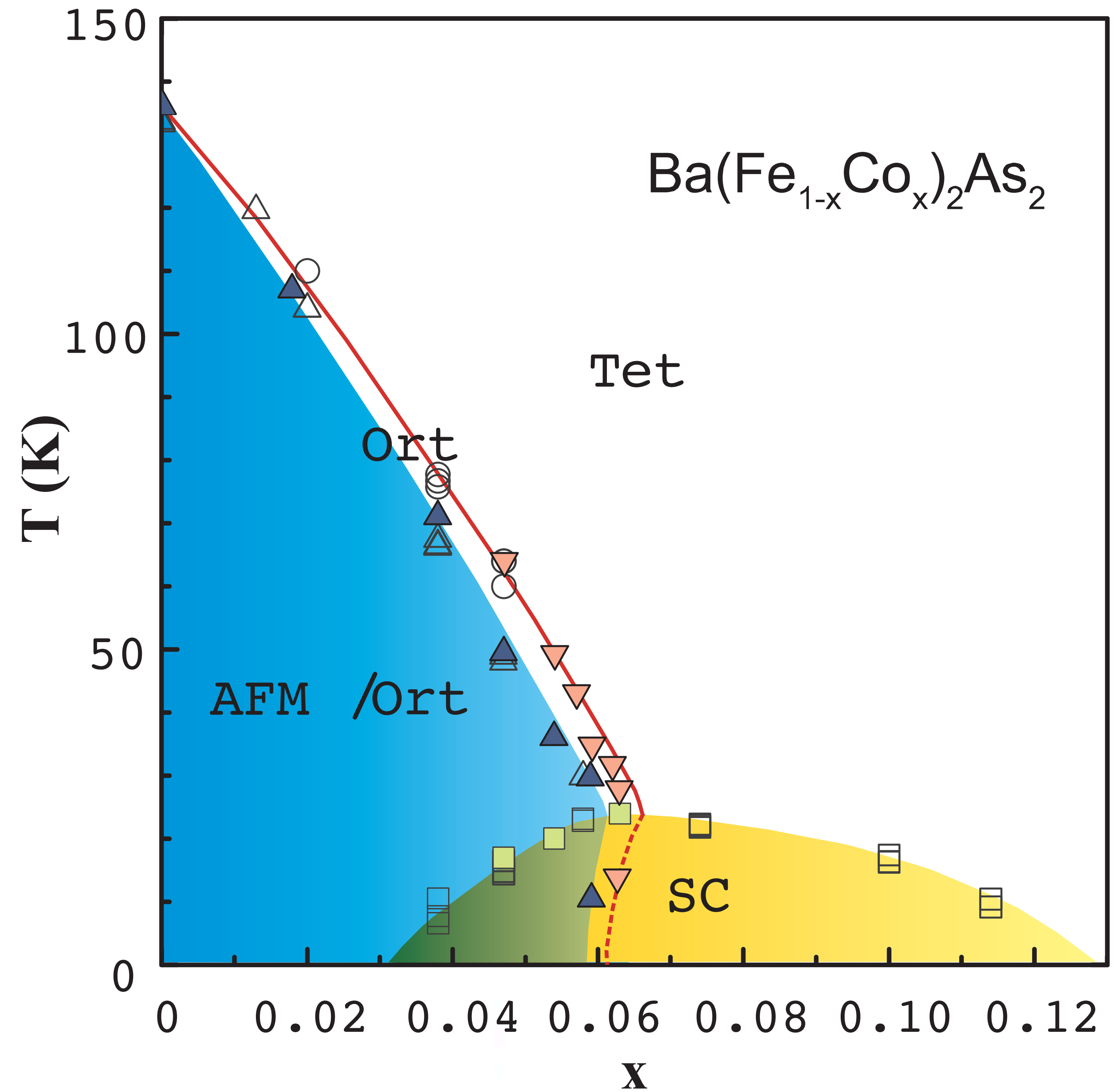


Resistivity $\sim \rho_0 + AT^n$

Iron pnictides:

a new class of high temperature superconductors





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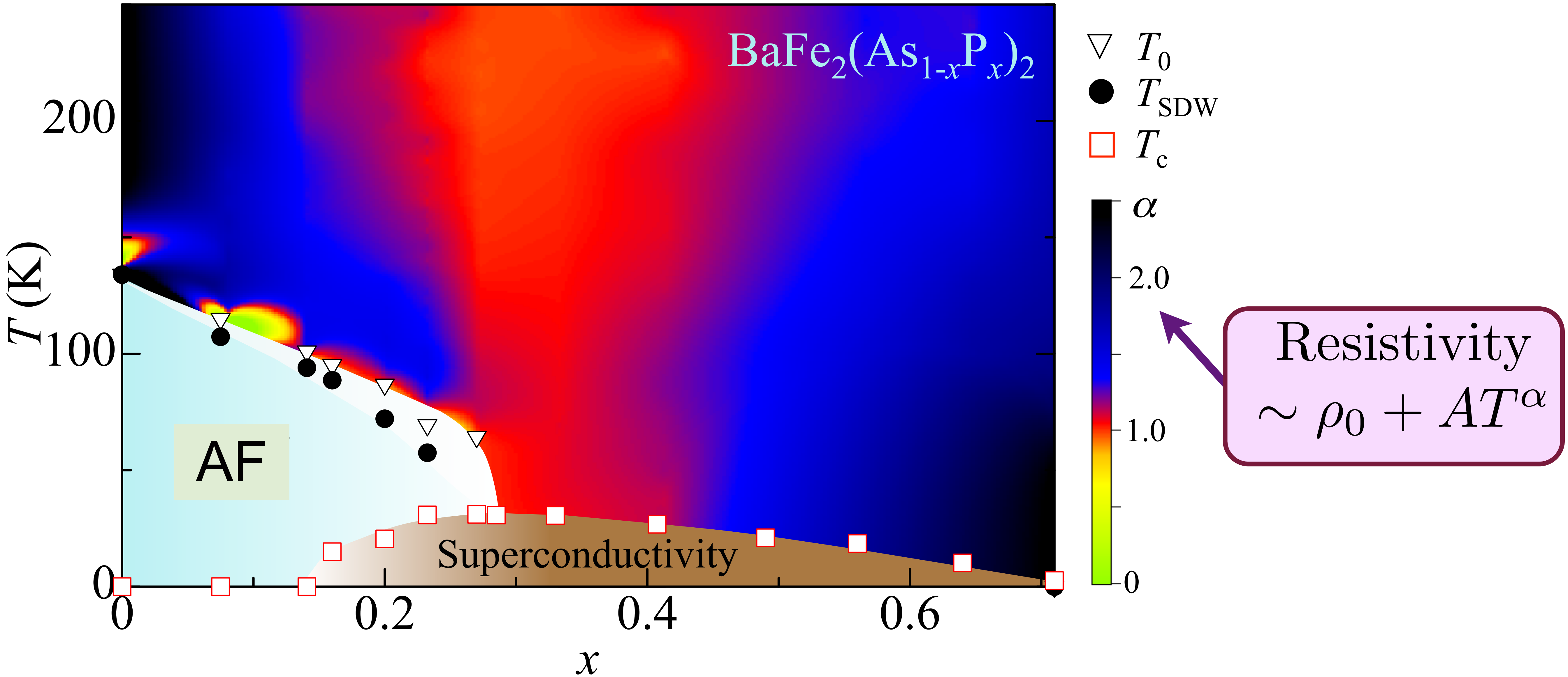
Temperature-doping phase diagram of the iron pnictides:



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

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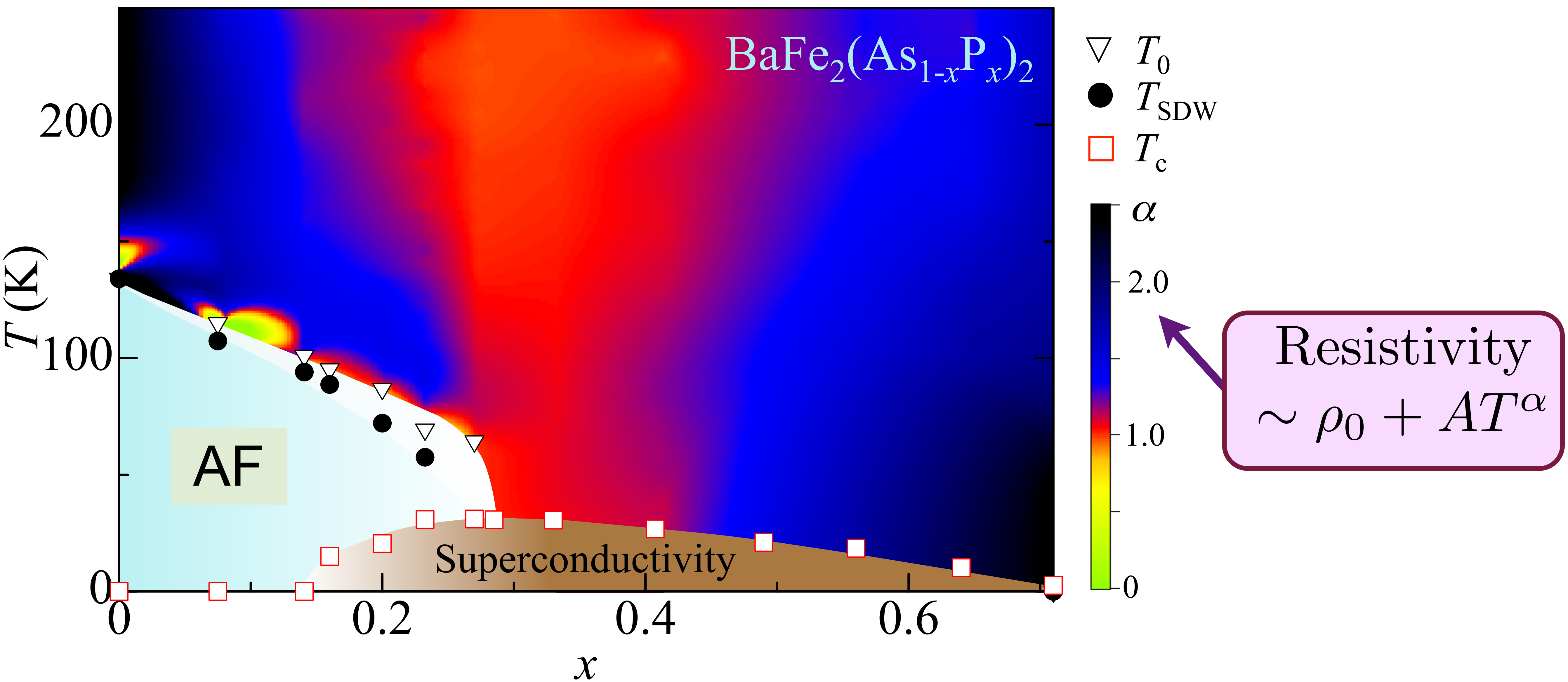

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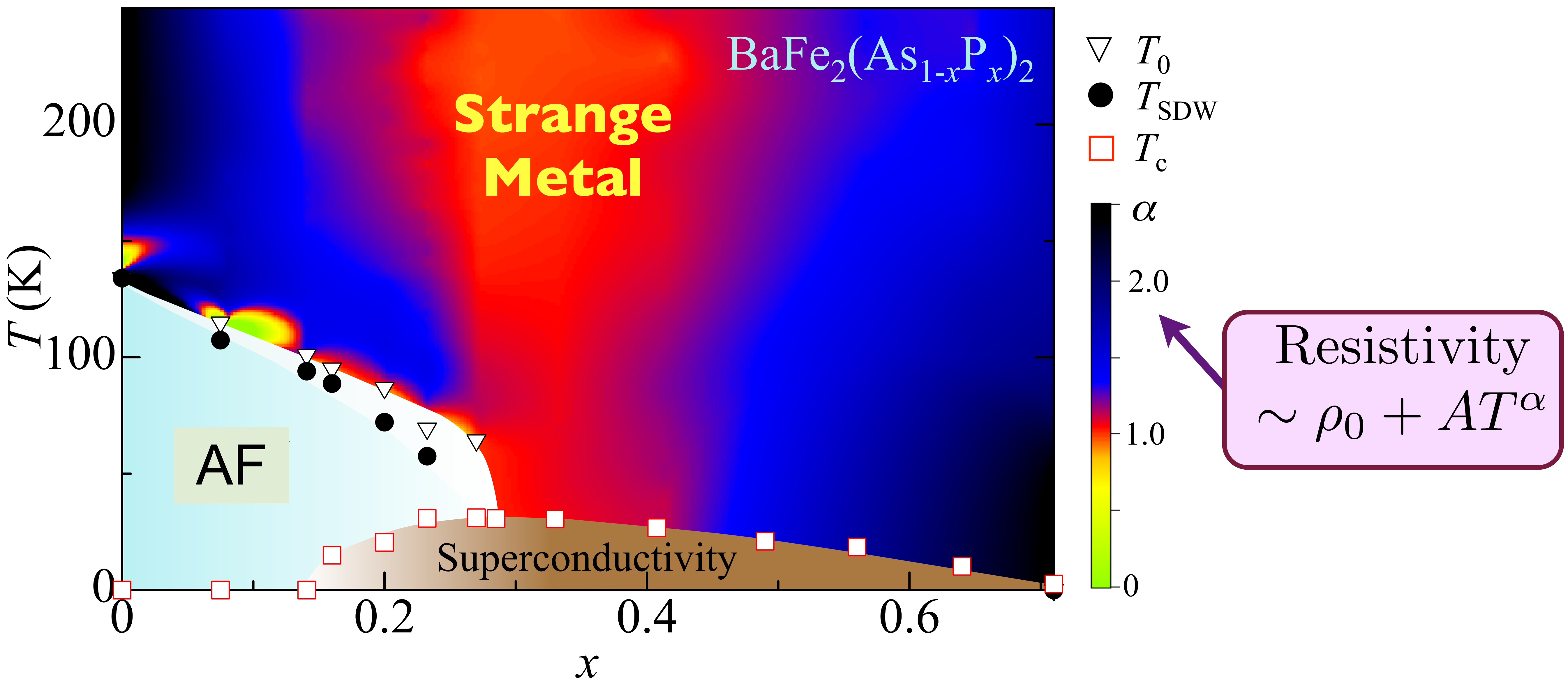

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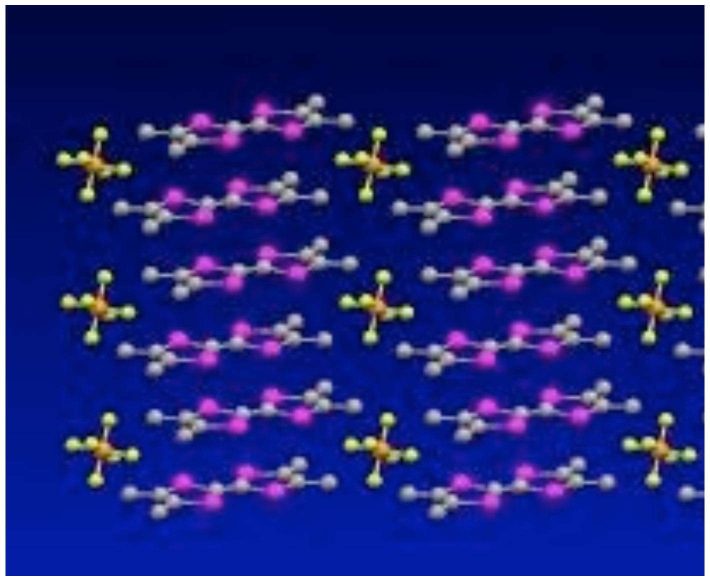
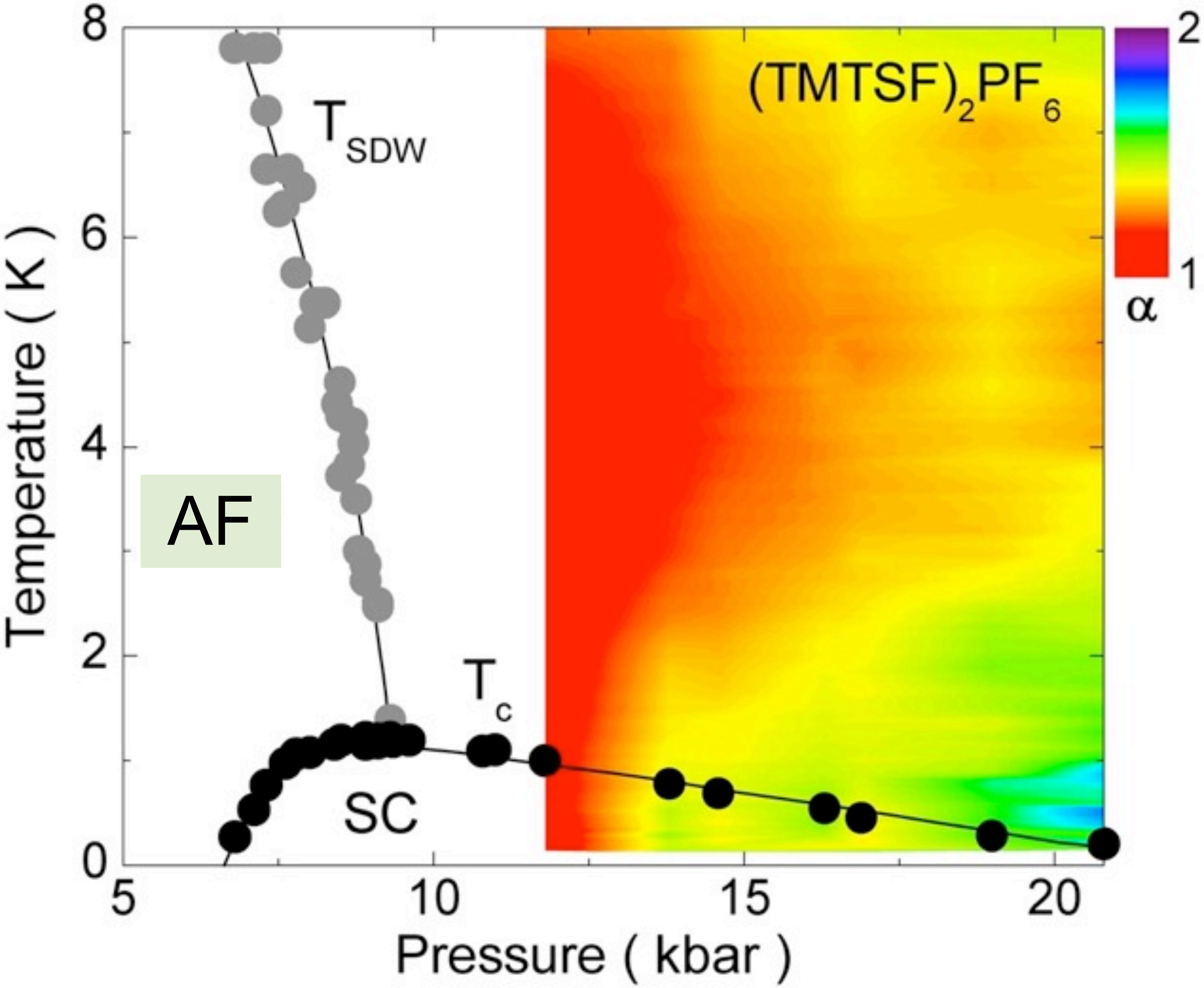
Temperature-pressure phase diagram of an organic superconductor





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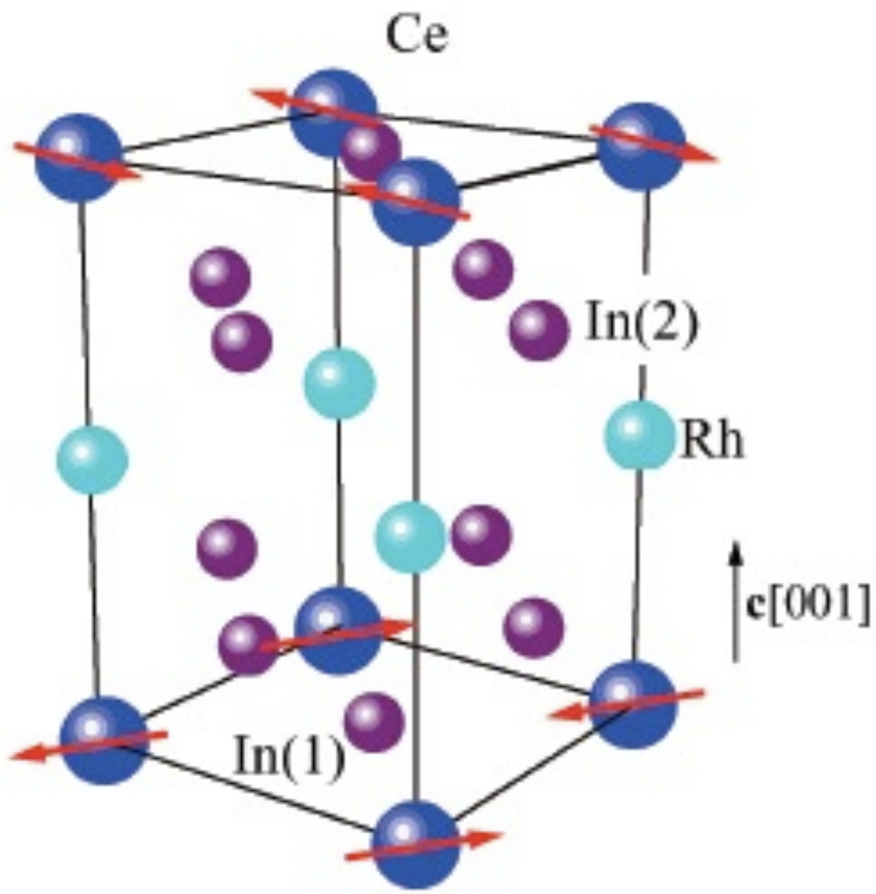
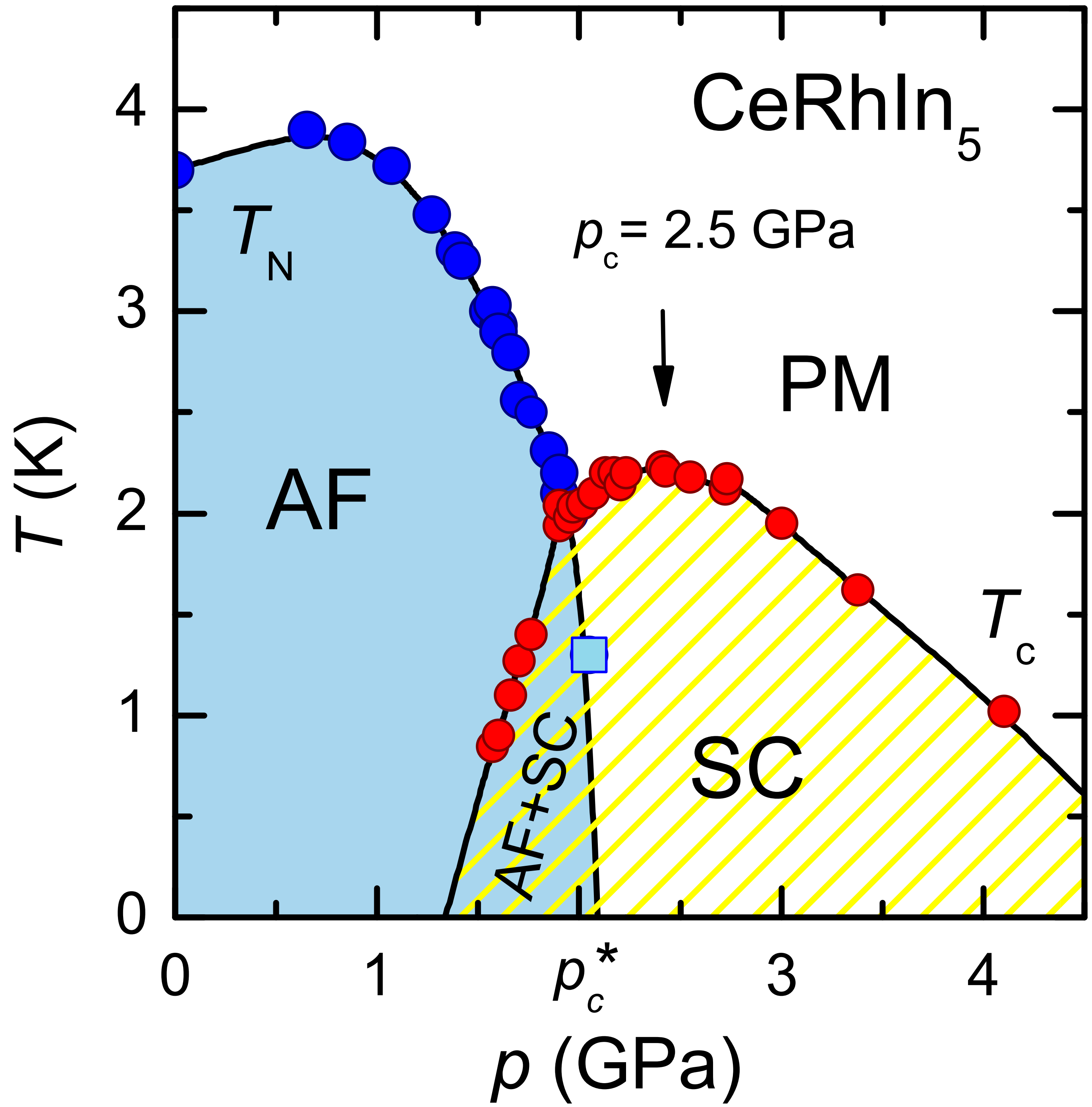
Temperature-pressure phase diagram of an heavy-fermion superconductor





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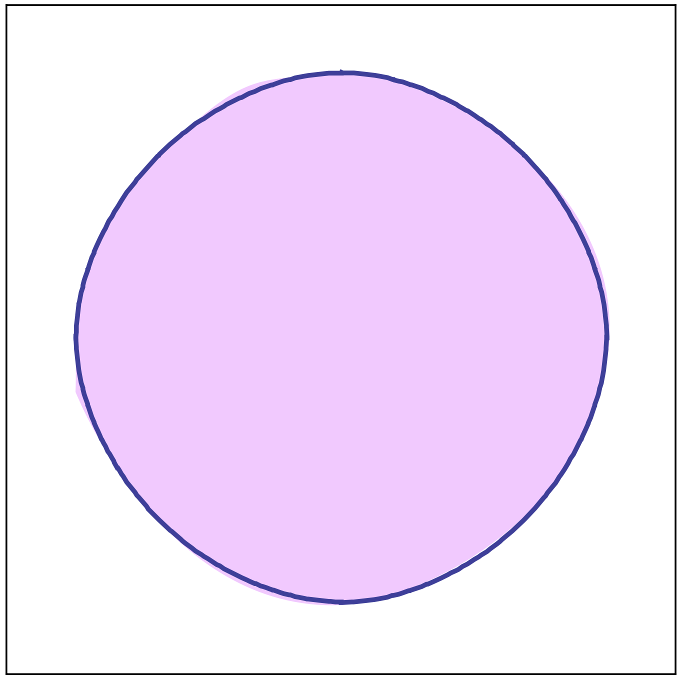
Fermi surface+antiferromagnetism

Metal with "large" Fermi surface



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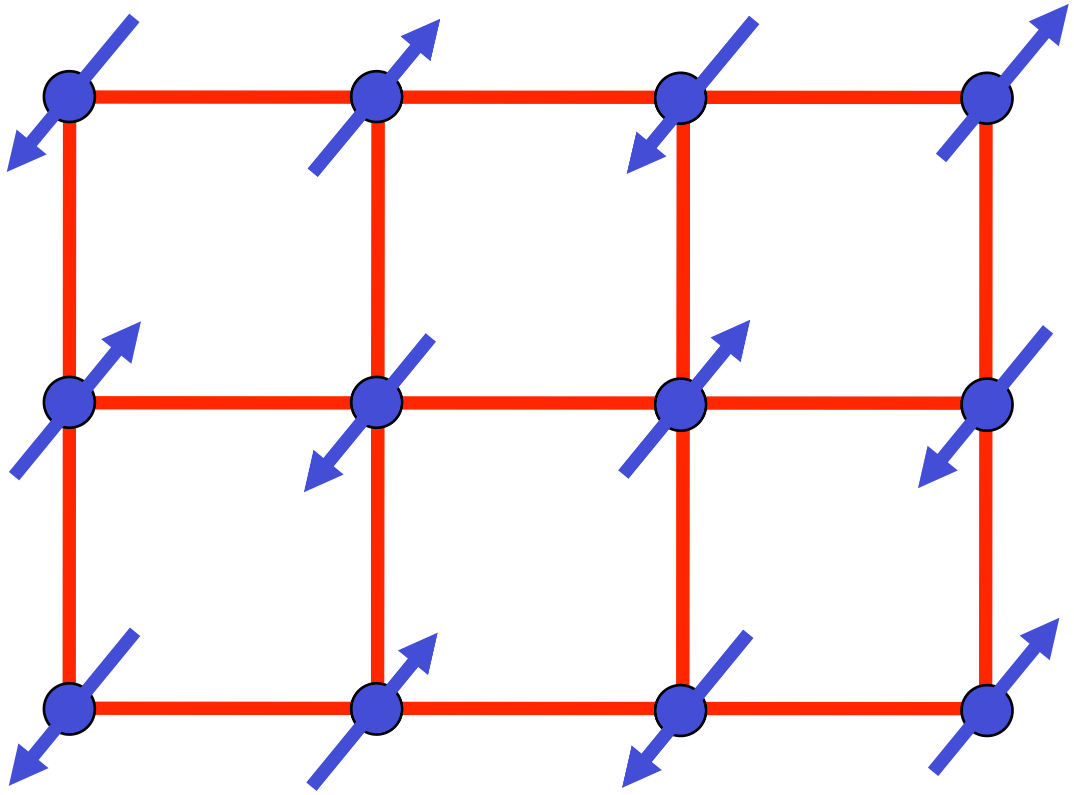
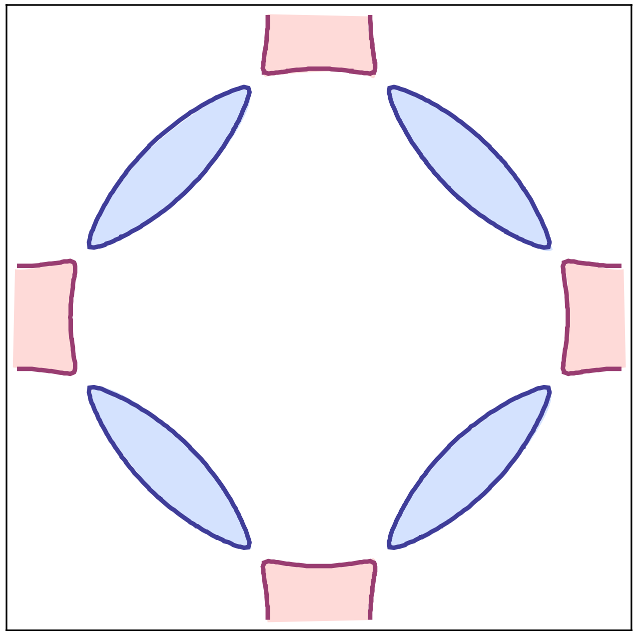


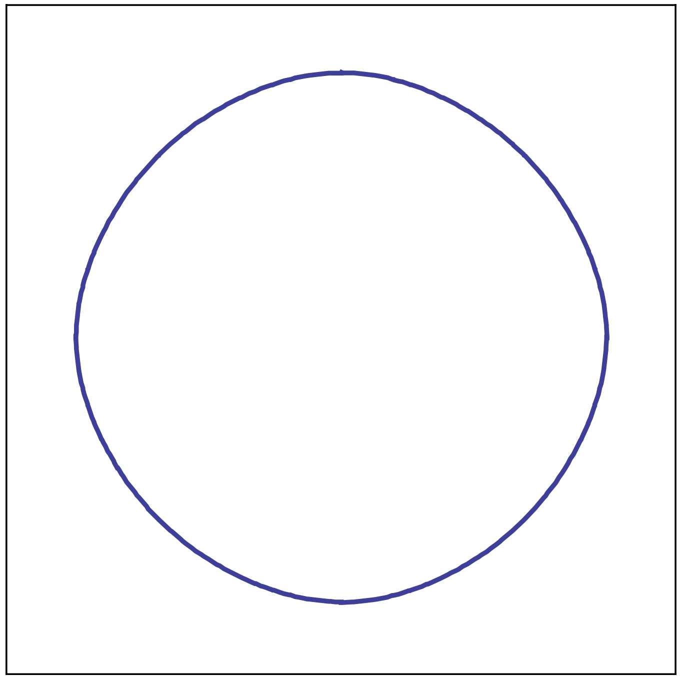
The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.

Fermi surface+antiferromagnetism





AF $\langle \vec{\varphi} \rangle \neq 0$

$\langle \vec{\varphi} \rangle = 0$

Metal with electron and hole pockets

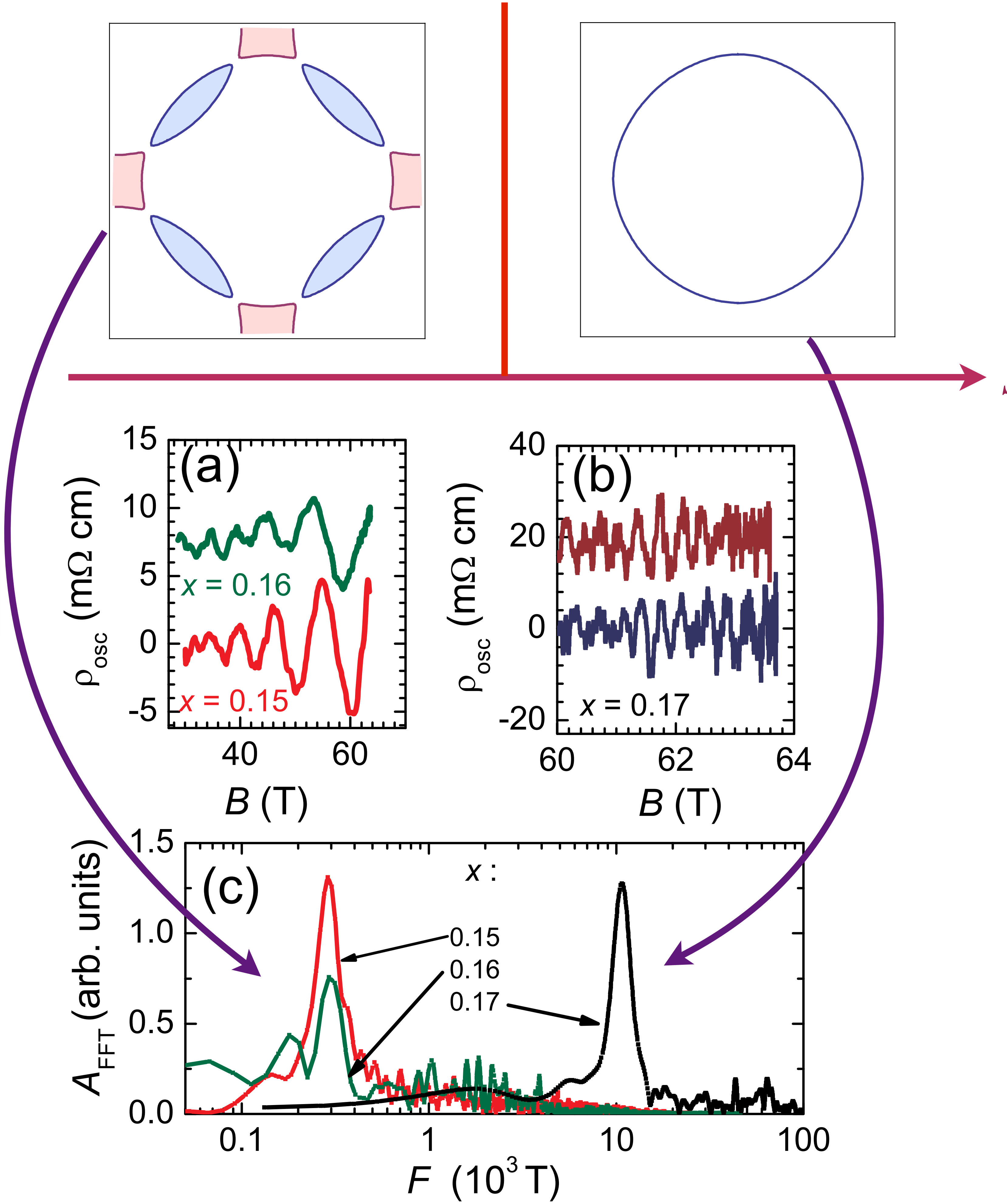
Metal with "large" Fermi surface

Increasing interaction

Fermi surface reconstruction and onset of antiferromagnetism

Quantum oscillations

$Nd_{2-x}Ce_xCuO_4$



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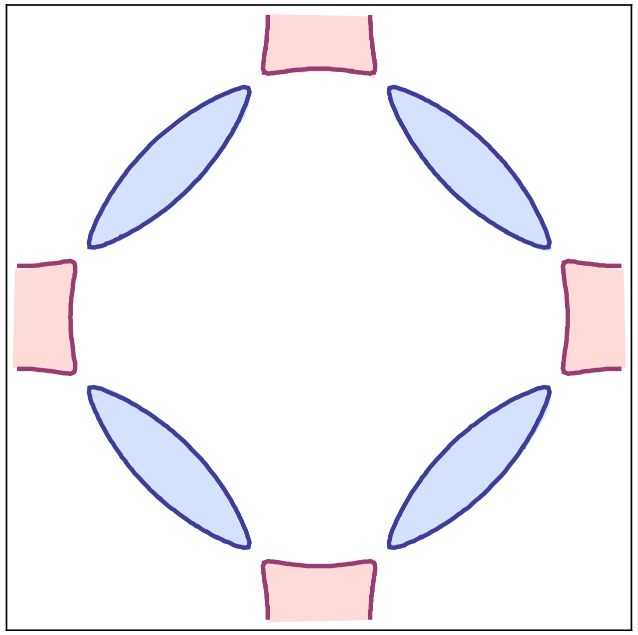
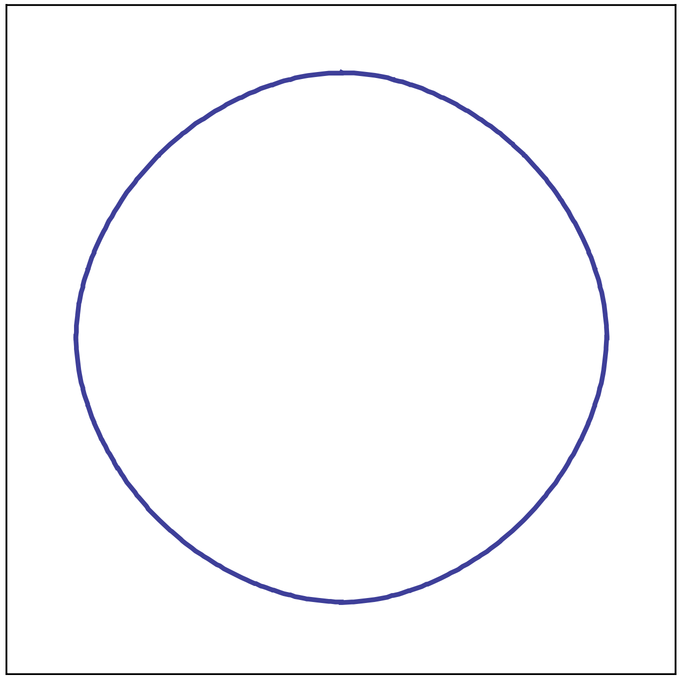
Fermi surface+antiferromagnetism



AF $\langle \vec{\varphi} \rangle \neq 0$

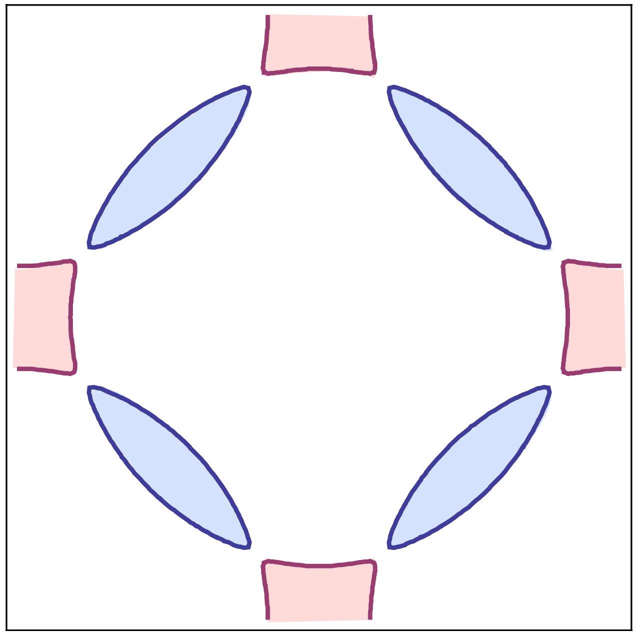
Metal with electron and hole pockets

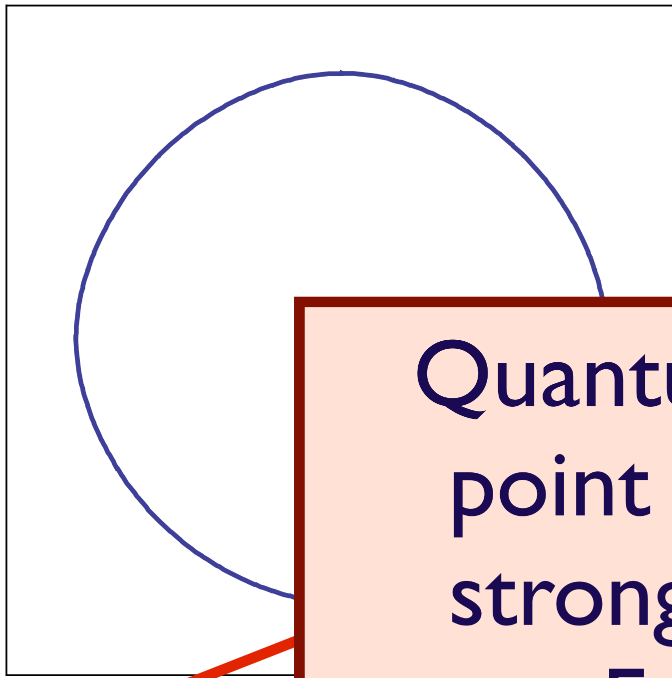


$\langle \vec{\varphi} \rangle = 0$

Metal with "large" Fermi surface

Fermi surface+antiferromagnetism





Quantum critical point realizes a strong-coupled non-Fermi liquid compressible phase

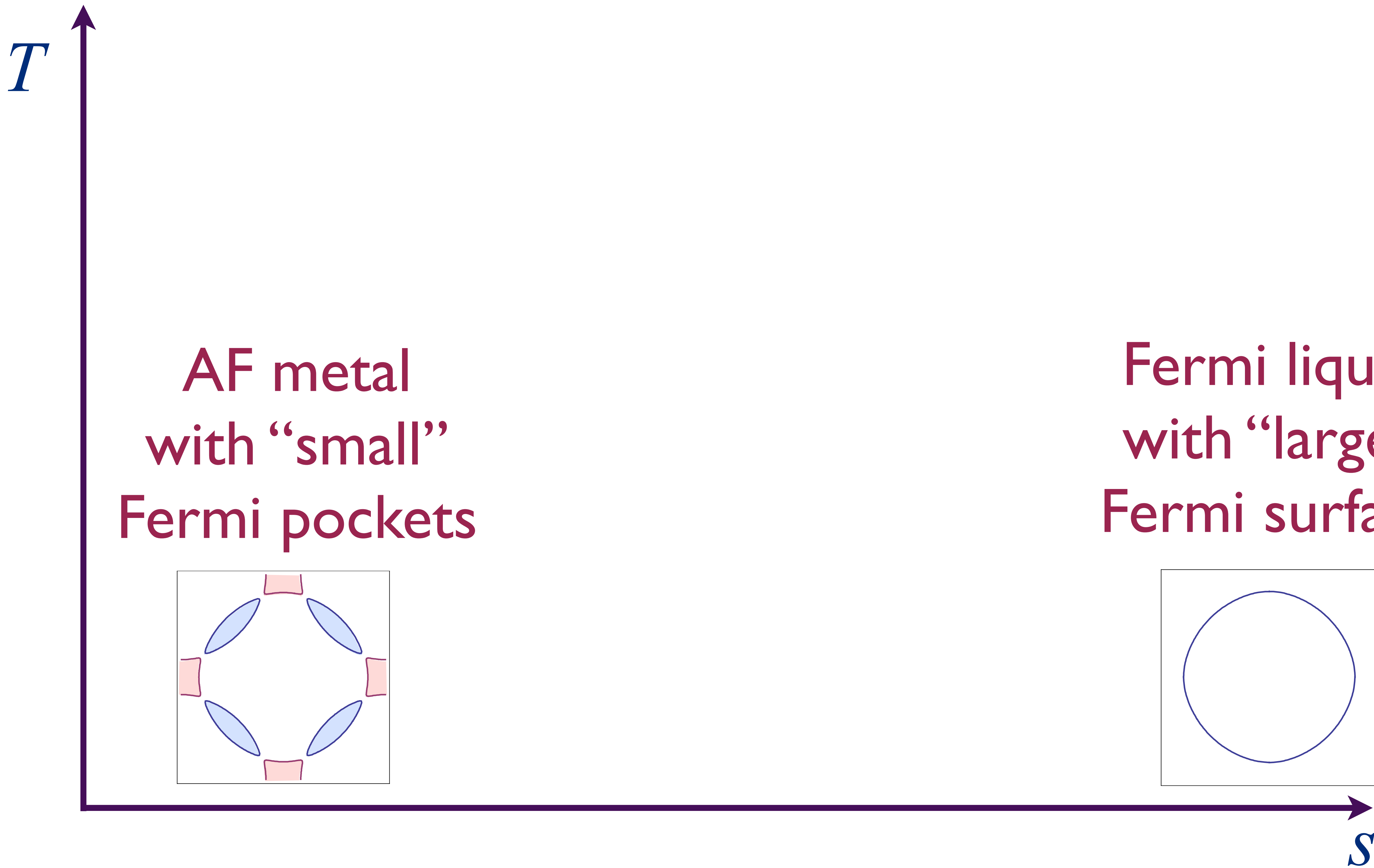
AF $\langle \vec{\varphi} \rangle \neq 0$

$\langle \vec{\varphi} \rangle = 0$

Metal with electron and hole pockets

Metal with "large" Fermi surface

s


T
AF metal
with “small”
Fermi pockets
Fermi liquid
with “large”
Fermi surface
s

D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
Ar. Abanov, A.V. Chubukov, and A. M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001)
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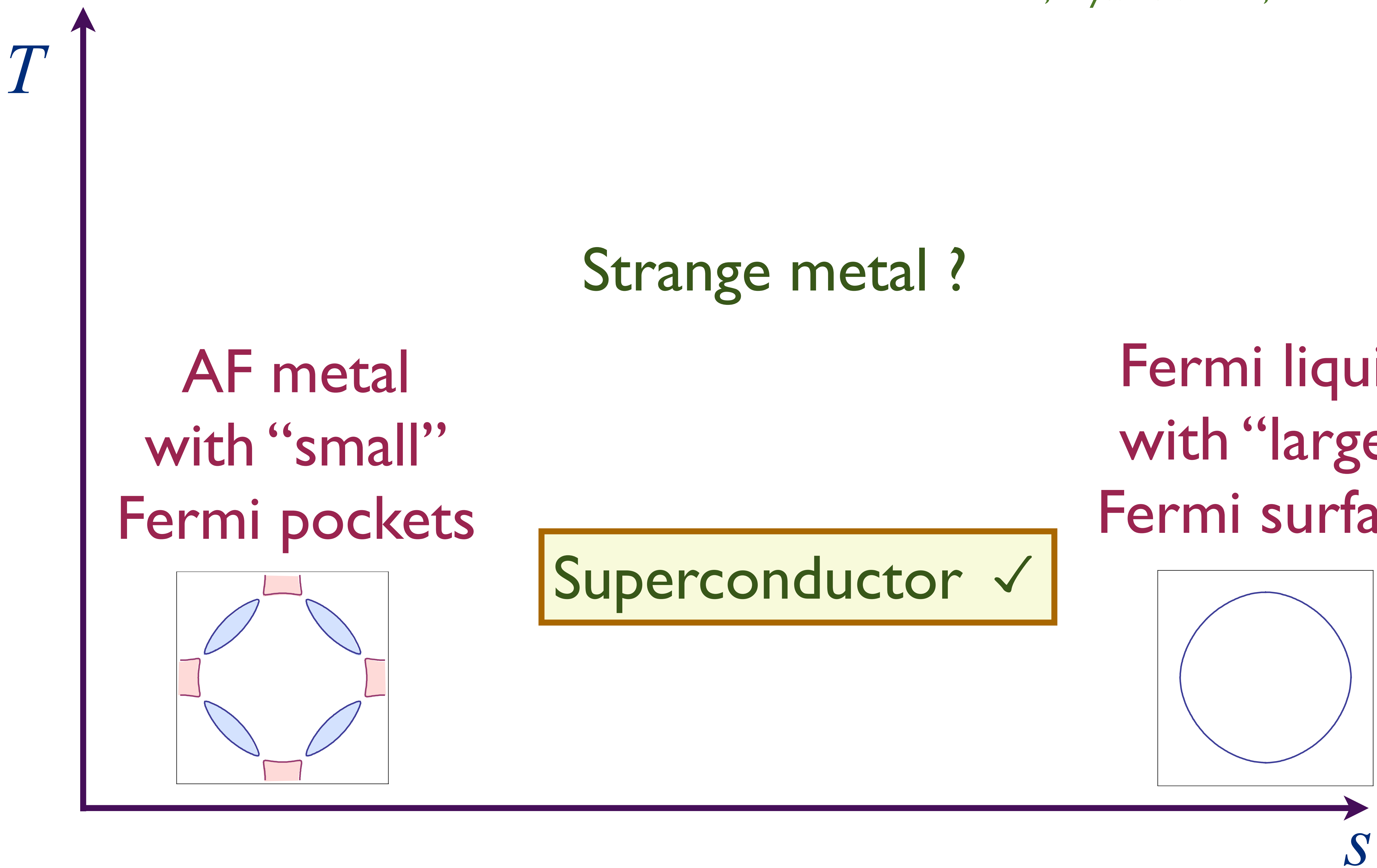
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



Davis

Challenge to string theory:

Describe quantum critical points and the phases of a compressible metallic systems

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Can we obtain holographic theories of superfluids and Fermi liquids?

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Can we obtain holographic theories of superfluids and Fermi liquids?

Yes

Challenge to string theory:

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Yes

Are there any other compressible phases like strange metals?

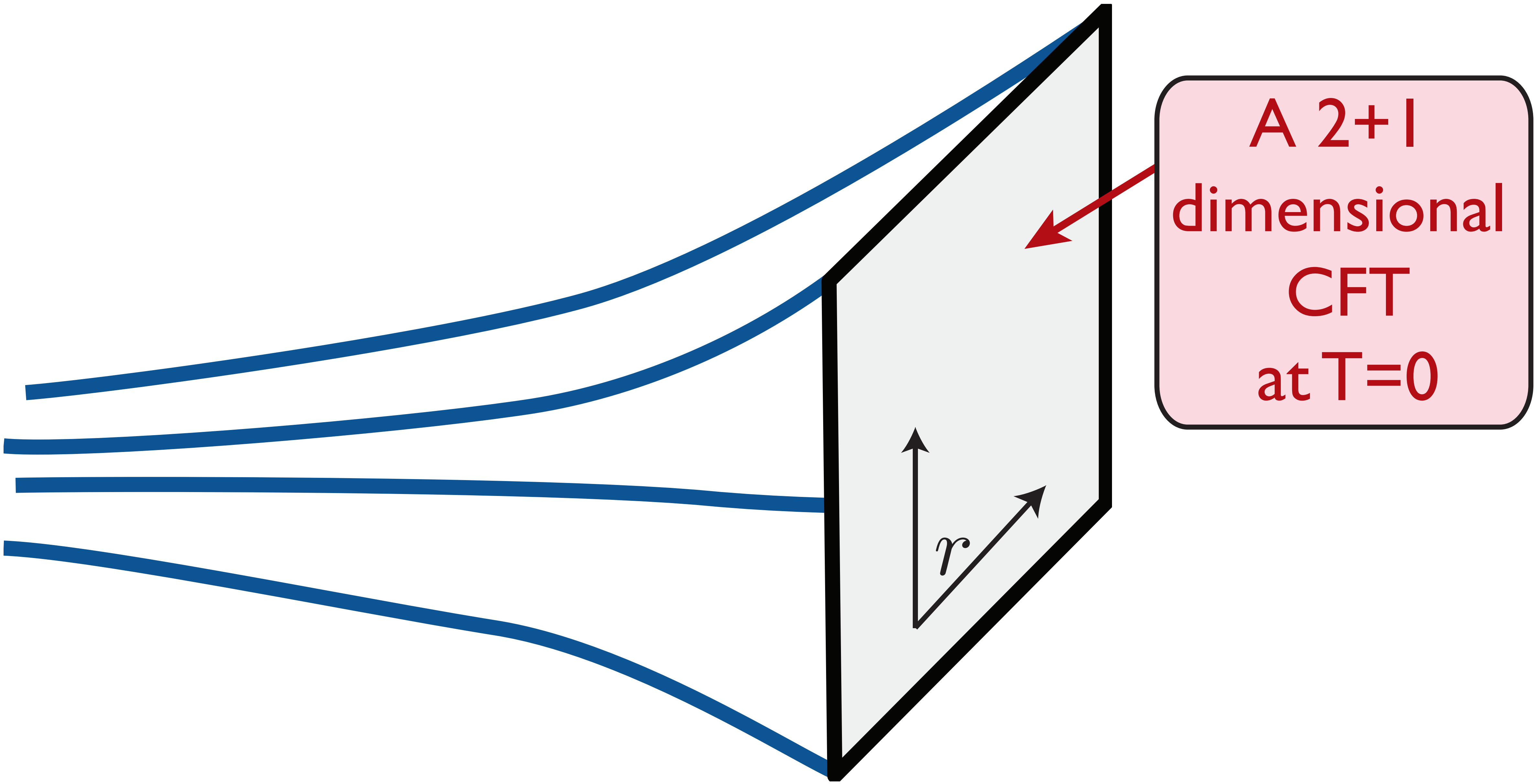
Challenge to string theory:

Describe quantum critical points and the phases of a compressible metallic systems

Can we obtain holographic theories of superfluids and Fermi liquids? Yes

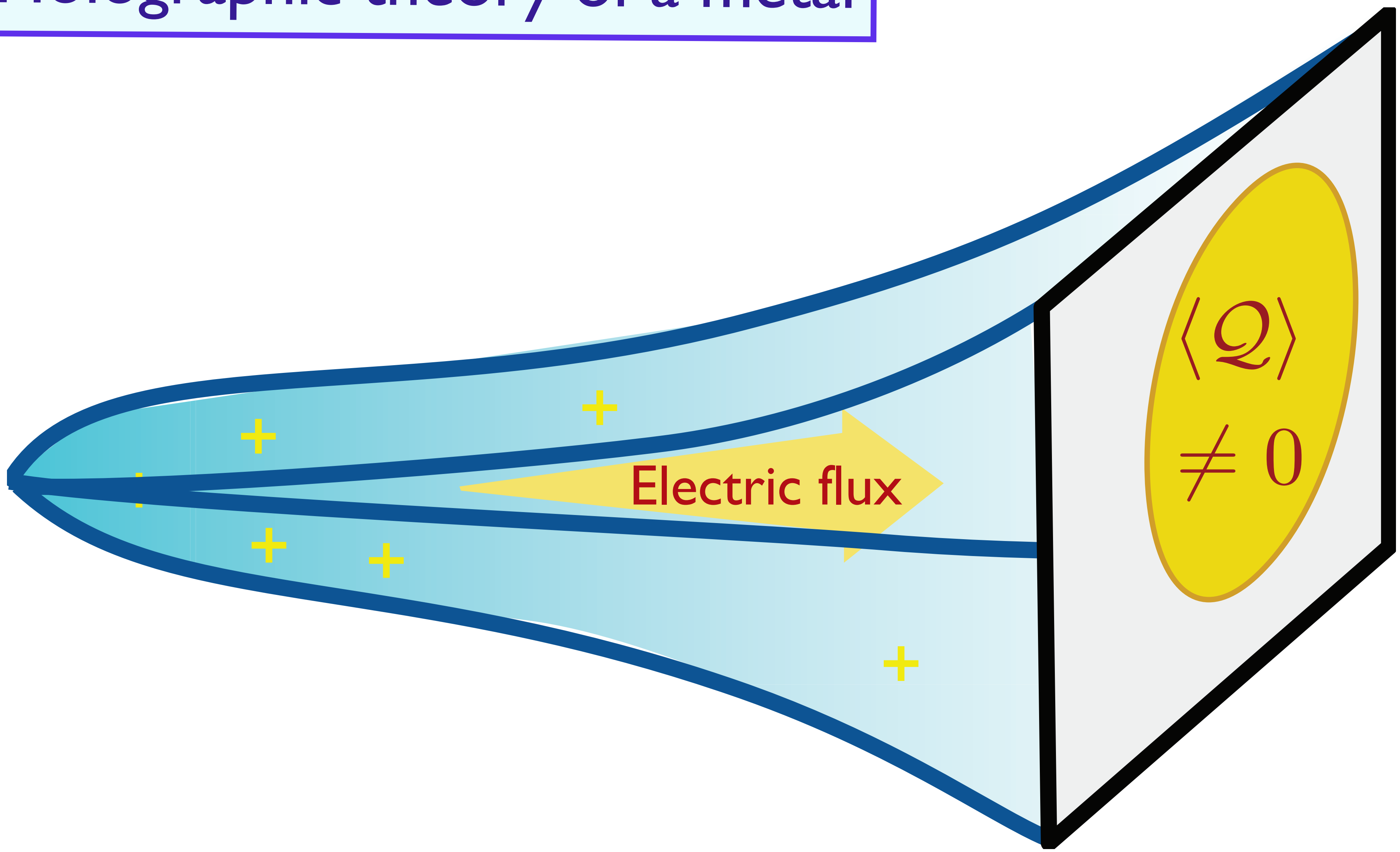
Are there any other compressible phases like strange metals? Yes....

Holographic representation: AdS_4



Holographic theory of a metal

S. Sachdev
arXiv:1107.5321



There is "charge" density on the boundary, which sources an electric field in the bulk

Conclusions

1. Quantum critical points and string theory
   Entanglement and emergent dimensions

2. High temperature superconductors
   and strange metals
   Holography of compressible quantum phases