

# Ground states of quantum antiferromagnets in two dimensions

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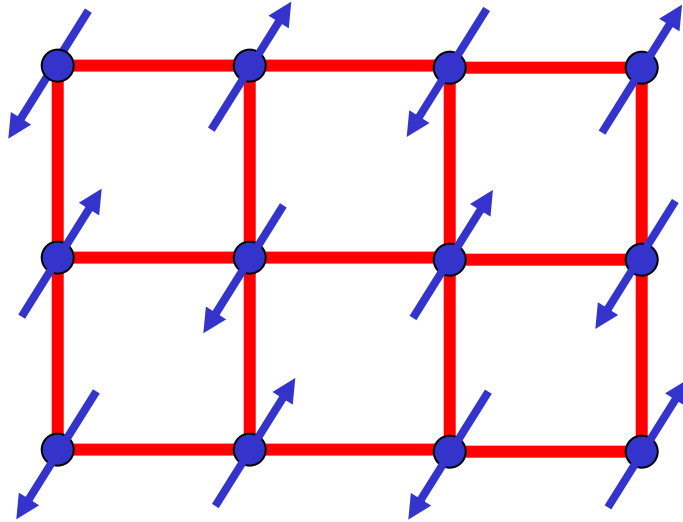


Talk online:  
[Google](#): Sachdev



Parent compound of the high temperature superconductors:  $\text{La}_2\text{CuO}_4$

Mott insulator: square lattice antiferromagnet



$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Ground state has long-range magnetic Néel order, or “collinear magnetic (CM) order”

Néel order parameter:  $\mathbf{n}_i = (-1)^{i_x + i_y} \vec{S}_i$

$$\langle \mathbf{n} \rangle \neq 0$$

# Possible theory for fractionalization and topological order

Decompose the magnetic order parameter as

$$\mathbf{n} = z_{\alpha}^* \vec{\sigma}_{\alpha\beta} z_{\beta}$$

where  $\vec{\sigma}$  are the Pauli matrices, and  $z_{\alpha}$  are complex spinors ( $\alpha = \uparrow, \downarrow$ ) which carry spin  $S = 1/2$ .

**Key question:** Can the  $z_{\alpha}$  become the elementary “spinon” excitations of a fractionalized phase ?

Physics must be invariant under the U(1) gauge transformation

$$z_{\alpha} \rightarrow e^{i\theta} z_{\alpha}$$

# Possible theory for fractionalization and topological order

**Naive expectation:** Low energy theory of fractionalized phase of spinons is a U(1) gauge theory of the  $S = 1/2$   $z_\alpha$  complex scalars

$$\mathcal{S} = \int d^2x d\tau \left[ |(\partial_\mu - iA_\mu)z_\alpha|^2 + r |z_\alpha|^2 + \frac{u}{2} (|z_\alpha|^2)^2 + v |z_\uparrow|^2 |z_\downarrow|^2 + \frac{1}{4e^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \right]$$

where  $v < 0$  for easy plane case, and  $v = 0$  for SU(2) symmetry.

**This expectation is too naive, and ignores the crucial confinement properties of compact U(1) gauge theory and the influence of “Berry phases”.**

## $Z_2$ gauge theory for fractionalization and topological order

Break  $U(1)$  gauge symmetry down to  $Z_2$  by condensing a charge 2 Higgs scalar  $\Phi$  (E. Fradkin and S. Shenker, *Phys. Rev. D* **19**, 3682 (1979)). Under a gauge transformation:

$$\Phi \rightarrow e^{2i\theta} \Phi$$

Simplest gauge-invariant and spin-rotation invariant coupling between  $\Phi$  and  $z_\alpha$ :

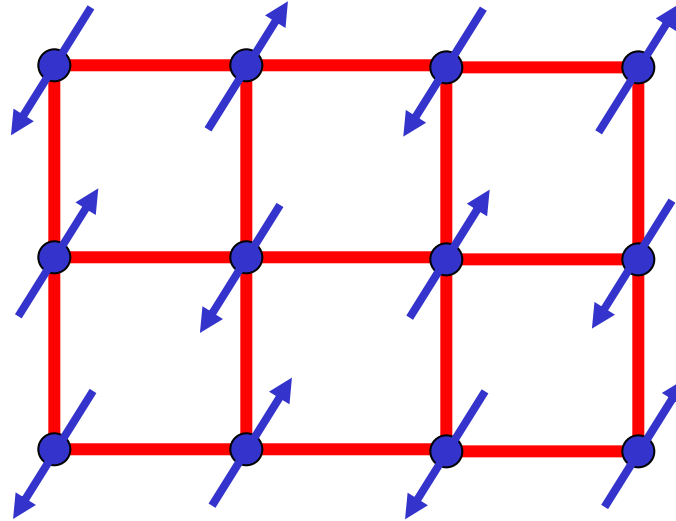
$$\Phi^* \epsilon_{\alpha\beta} z_\alpha \partial_r z_\beta.$$

‘Higgs’ phase with  $\langle \Phi \rangle \neq 0$  has the properties:

- Description in terms of an effective  $Z_2$  gauge theory. The spinons  $z_\alpha$  are deconfined and carry a  $Z_2$  gauge charge.
- Vison (vortex) excitation which carries  $Z_2$  gauge flux
- Four-fold degeneracy on a torus.
- Coplanar spin correlations generated by coupling between  $\Phi$  and  $z_\alpha$ .

N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)

S. Sachdev and N. Read, *Int. J. Mod. Phys. B* **5**, 219 (1991)

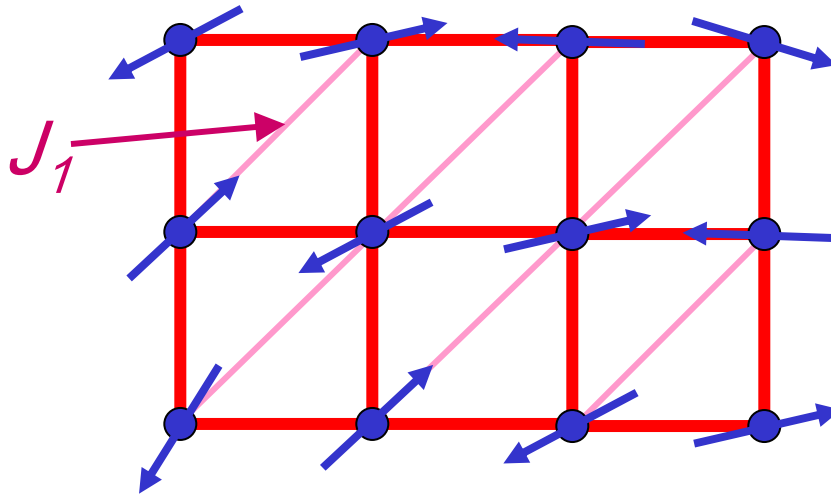


Collinear magnetic order with  $\langle \Phi \rangle = 0$ .

A spin density wave:

$$\langle \vec{S}_i \rangle \propto (\cos(\mathbf{K} \cdot \mathbf{r}_i), \sin(\mathbf{K} \cdot \mathbf{r}_i), 0)$$

$$\mathbf{K} = (\pi, \pi).$$



Coplanar magnetic order with  $\langle \Phi \rangle \neq 0$ .

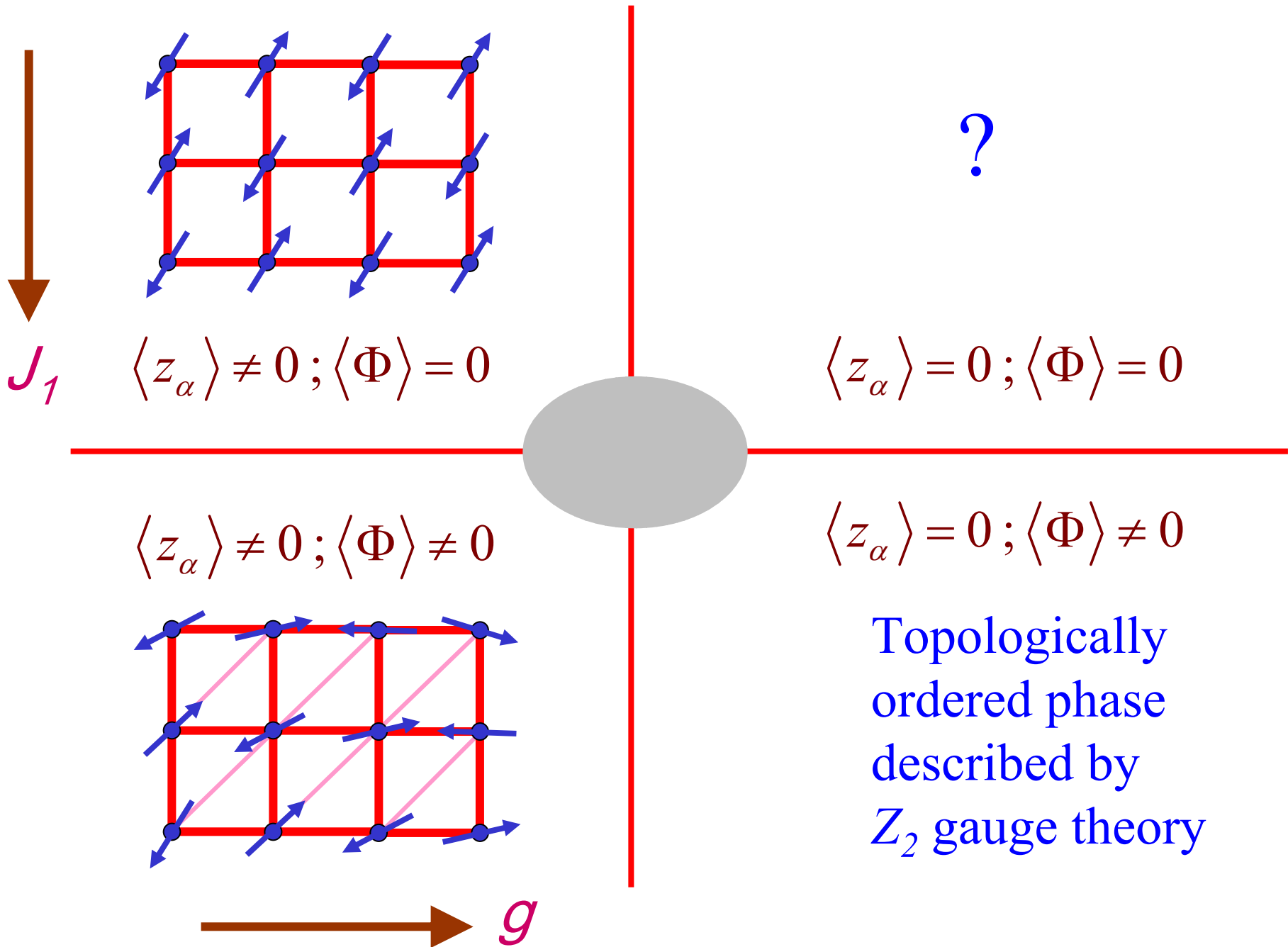
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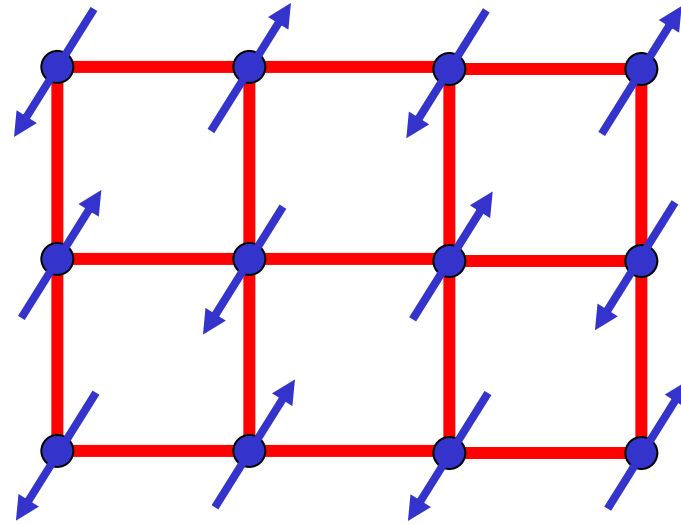
with

$$\mathbf{K} = (\pi + \langle \Phi \rangle, \pi + \langle \Phi \rangle).$$

*Experimental realization: CsCuCl<sub>3</sub>*



# Quantum fluctuations in U(1) theory for collinear antiferromagnets

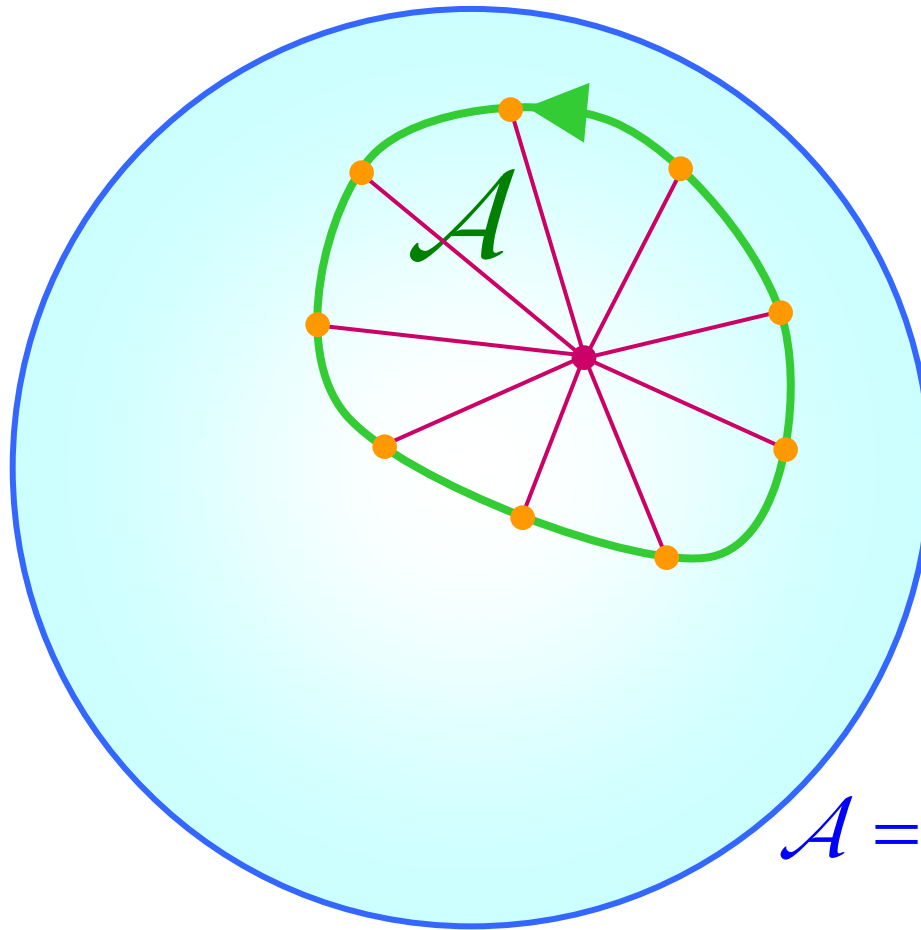


## Central questions:

Vary  $J_{ij}$  smoothly until CM order is lost and a paramagnetic phase with  $\langle \mathbf{n} \rangle = \mathbf{0}$  is reached.

- What is the nature of this paramagnet ?
- What is the critical theory of (possible) second-order quantum phase transition(s) between the CM phase and the paramagnet ?

# Berry Phases

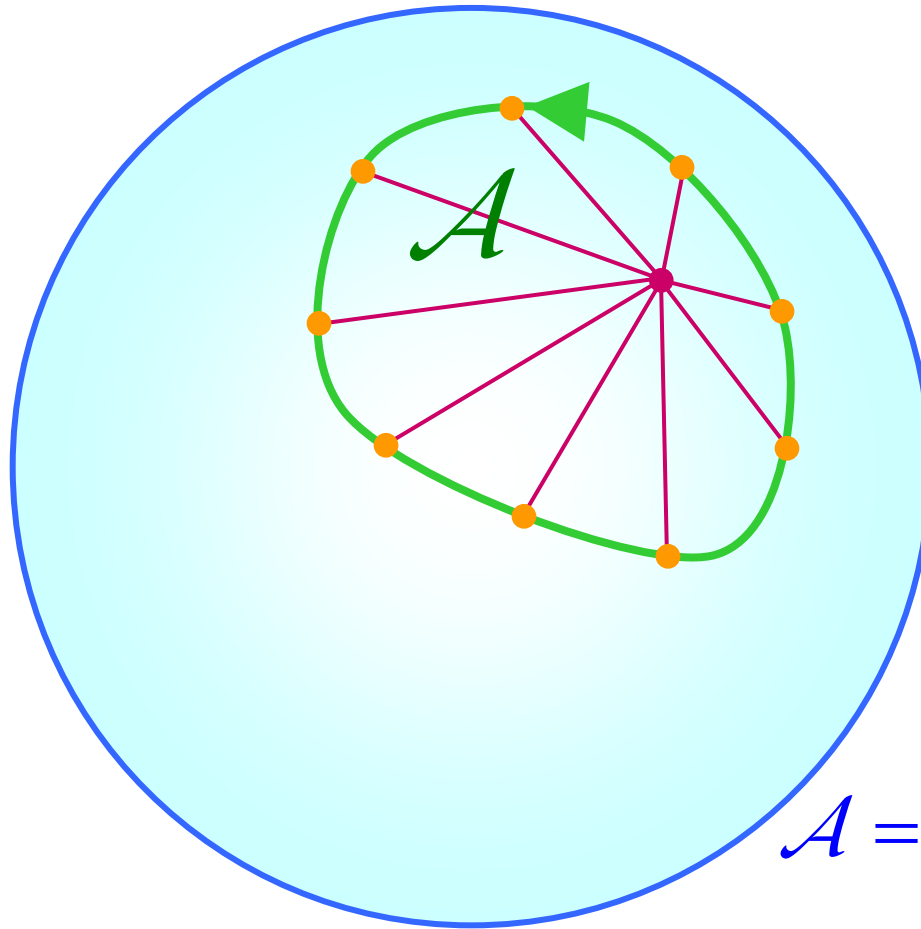


$$e^{iS\mathcal{A}}$$

$$\mathcal{A} = 2 \sum A_\tau \pmod{4\pi}$$

$$A_\tau \rightarrow \frac{1}{2} \times \left( \begin{array}{l} \text{Area of triangle formed by } \mathbf{n}(\tau), \mathbf{n}(\tau + \Delta\tau), \text{ and} \\ \text{an arbitrary reference } \mathbf{n}_0 \end{array} \right)$$

# Berry Phases



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## Theory for quantum fluctuations of Neel state

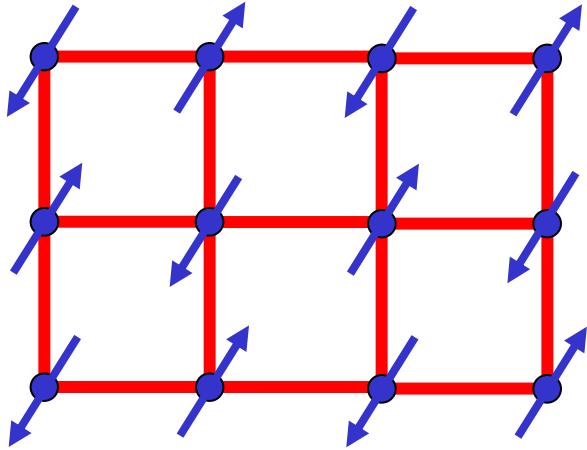
Discretize spacetime to a cubic lattice of sites  $a$ , include spin Berry phases, and account for gauge invariance and invariance under  $A_\tau \rightarrow A_\tau + 2\pi$

$$Z = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta(|z_{a\alpha}|^2 - 1) \exp \left( \frac{1}{g} \sum_{a,\mu} z_{a\alpha}^* e^{iA_{a\mu}} z_{a+\mu,\alpha} + \text{c.c.} + i2S \sum_a \eta_a A_{a\tau} + \frac{1}{e^2} \sum_{\square} \cos(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}) \right)$$

Here  $\eta_a = \pm 1$  on the 2 square sublattices, and  $z_{a\alpha}$ ,  $\alpha = 1 \dots N$ , is a  $N$ -component complex scalar.

$$Z = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta(|z_{a\alpha}|^2 - 1)$$

$$\exp \left( \frac{1}{g} \sum_{a,\mu} z_{a\alpha}^* e^{iA_{a\mu}} z_{a+\mu,\alpha} + \text{c.c.} + i2S \sum_a \eta_a A_{a\tau} + \frac{1}{e^2} \sum_{\square} \cos(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}) \right)$$



At large  $g$ , we can integrate  $z_\alpha$  out, and work with effective action for the  $A_{a\mu}$

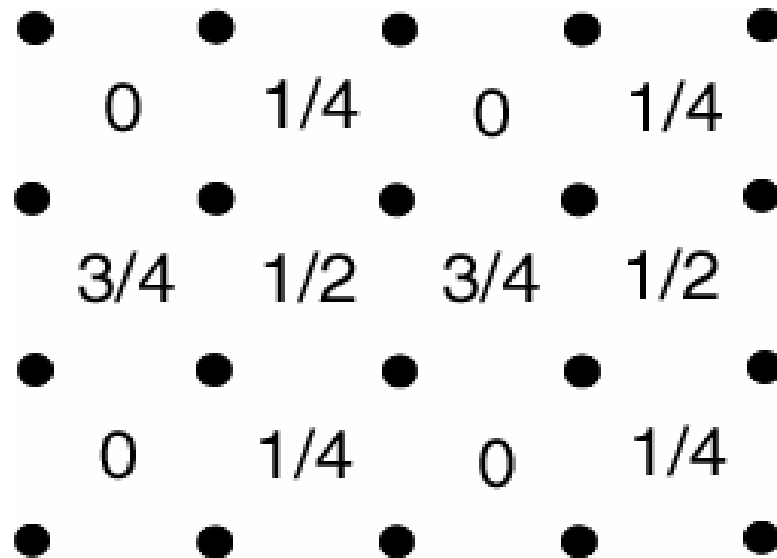
$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp \left( \frac{1}{e^2} \sum_{\square} \cos \left( \Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu} \right) + i2S \sum_a \eta_a A_{a\tau} \right)$$

This is compact QED in 2+1 dimensions with static charges  $\pm 2S$  on two sublattices.

Exact duality transform on periodic Gaussian (“Villain”) action for compact QED yields a representation in terms of a Coulomb gas of monopoles

$$Z_{\text{dual}} = \sum_{\{m_{\bar{j}}\}} \exp \left( -\frac{\pi}{2e^2} \sum_{\bar{j}, \bar{j}'} \frac{m_{\bar{j}} m_{\bar{j}'}}{|r_{\bar{j}} - r_{\bar{j}'}|} + 4\pi i S \sum_{\bar{j}} m_{\bar{j}} \mathcal{X}_{\bar{j}} \right)$$

with the  $m_{\bar{j}}$  integer monopole charges. Each monopole carries a Berry phase (F.D.M. Haldane, *Phys. Rev. Lett.* **61**, 1029 (1988)) determined by the fixed  $\mathcal{X}_{\bar{j}} = 0, 1/4, 1/2, 3/4$  on the four dual sublattices.



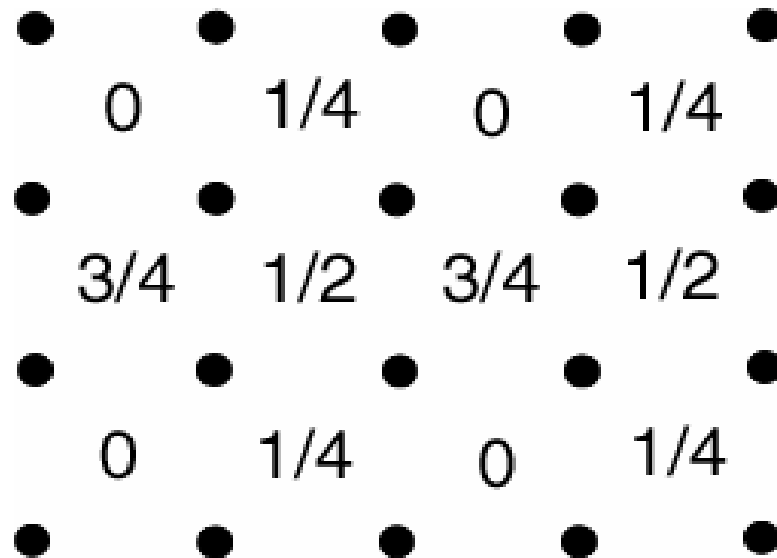
N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

Alternative representation is in terms of a “height” model

$$Z_{\text{dual}} = \sum_{\{h_{\bar{j}}\}} \exp \left( -\frac{e^2}{2} \sum_{\bar{j}} (\Delta_{\mu} h_{\bar{j}} - 2S \Delta_{\mu} \mathcal{X}_{\bar{j}})^2 \right)$$

with the  $h_{\bar{j}}$  integer heights.

The Berry phases now lead to height ‘offsets’  $\mathcal{X}_{\bar{j}} = 0, 1/4, 1/2, 3/4$  on the four dual sublattices.

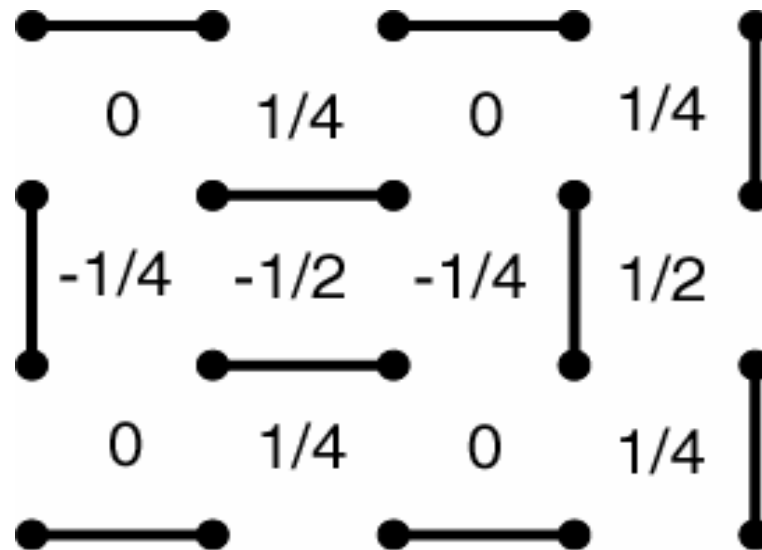


For  $S=1/2$  and large  $e^2$ , low energy height configurations are in exact one-to-one correspondence with dimer coverings of the square lattice

⇒ 2+1 dimensional height model is the path integral of the

**Quantum Dimer Model**

D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988);  
E. Fradkin and S. A. Kivelson, *Mod. Phys. Lett. B* **4**, 225 (1990))



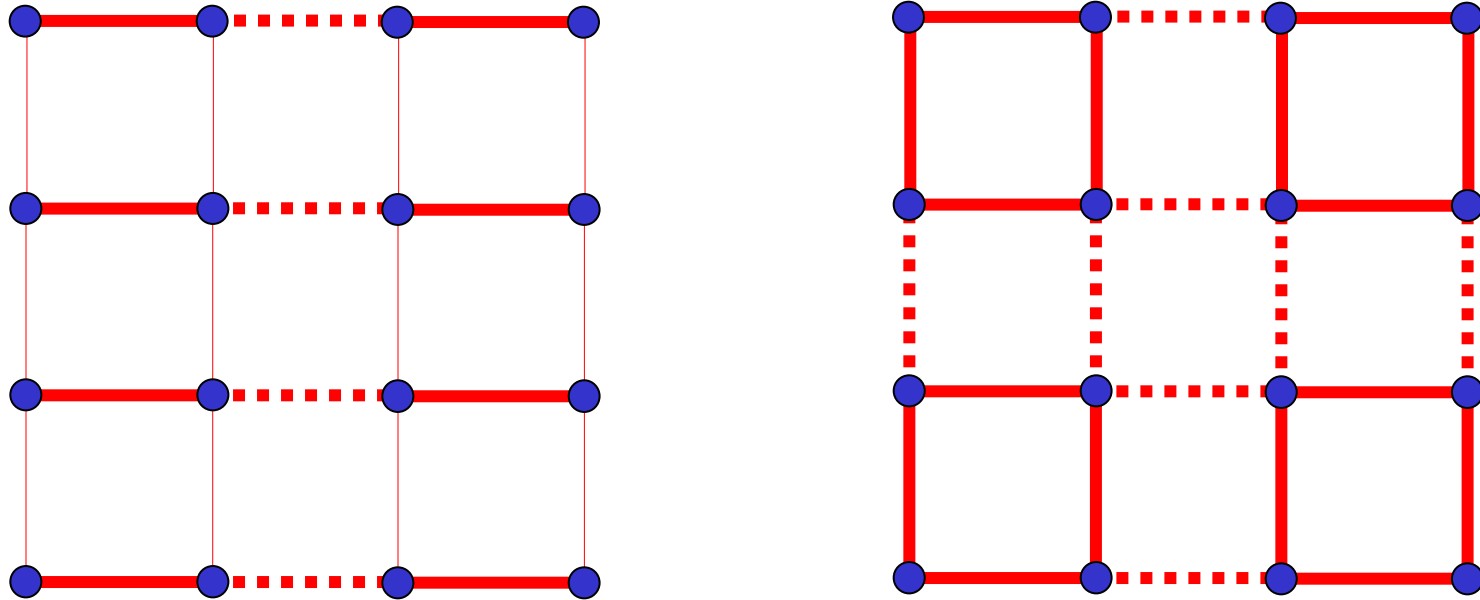
There is no roughening transition for three dimensional interfaces, which are smooth for all couplings

⇒ There is a definite average height of the interface

⇒ **Ground state has bond order.**

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

## Two possible bond-ordered paramagnets for $S=1/2$

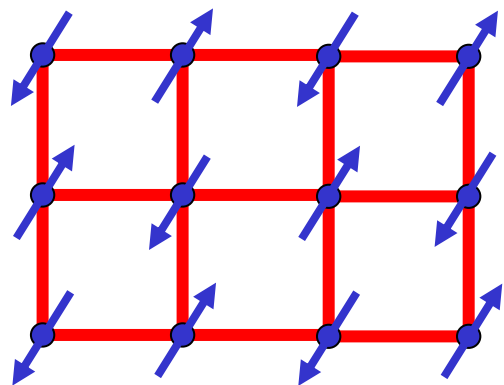


Distinct lines represent different values of  $\langle \vec{S}_i \cdot \vec{S}_j \rangle$  on links

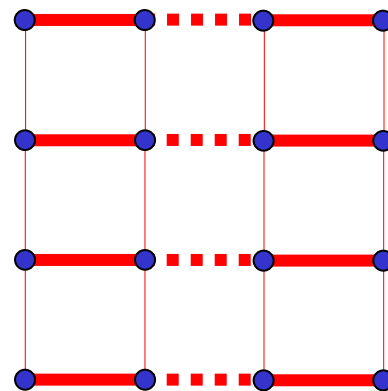
There is a broken lattice symmetry, and the ground state is at least four-fold degenerate.



$J_1$

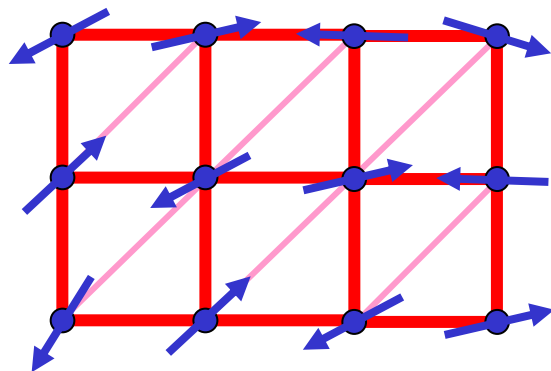


$$\langle z_\alpha \rangle \neq 0; \langle \Phi \rangle = 0$$



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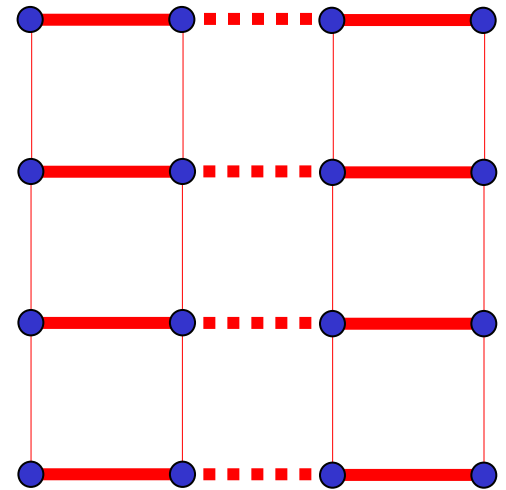
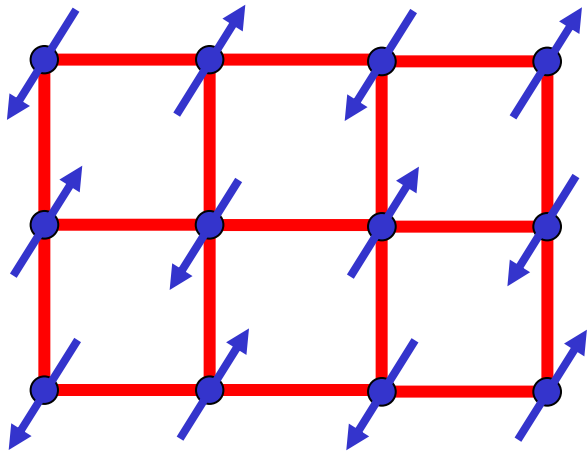
Topologically  
ordered phase  
described by  
 $Z_2$  gauge theory



$g$

$$Z = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta(|z_{a\alpha}|^2 - 1)$$

$$\exp \left( \frac{1}{g} \sum_{a,\mu} z_{a\alpha}^* e^{iA_{a\mu}} z_{a+\mu,\alpha} + \text{c.c.} + i \sum_a \eta_a A_{a\tau} + \frac{1}{e^2} \sum_{\square} \cos(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}) \right)$$



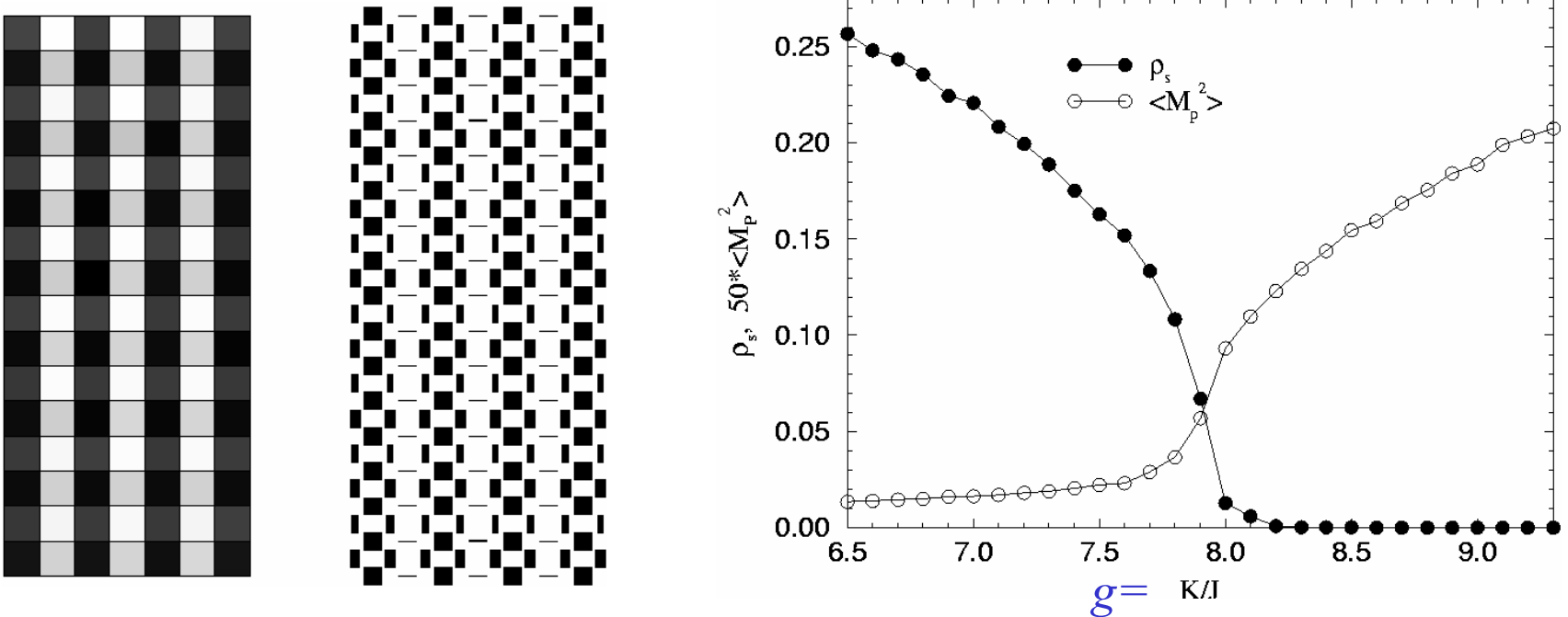
0

$g$

# Bond order in a frustrated $S=1/2$ XY magnet

A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, *Phys. Rev. Lett.* **89**, 247201 (2002)

First *large scale* numerical study of the destruction of Neel order in a  $S=1/2$  antiferromagnet with full square lattice symmetry



$$H = 2J \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y) - K \sum_{\langle ijkl \rangle \square} (S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+)$$

## Nature of quantum critical point

$$Z = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta(|z_{a\alpha}|^2 - 1) \exp \left( \frac{1}{g} \sum_{a,\mu} z_{a\alpha}^* e^{iA_{a\mu}} z_{a+\mu,\alpha} + \text{c.c.} + i2S \sum_a \eta_a A_{a\tau} + \frac{1}{e^2} \sum_{\square} \cos(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}) \right)$$

Use a sequence of simpler models which can be analyzed by duality mappings

A. Non-compact QED with scalar matter

B. Compact QED with scalar matter

C.  $N=1$ : Compact QED with scalar matter and Berry phases

D.  $N \rightarrow \infty$  theory

E. Easy plane case for  $N=2$

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## A. $N=1$ , non-compact $U(1)$ , no Berry phases

Use  $z_a = e^{i\theta_a}$  and then

$$Z = \prod_a \int d\theta_a dA_{a\mu} \exp \left( -\frac{1}{2e^2} \sum_{\square} (\Delta_{\mu} A_{a\nu} - \Delta_{\nu} A_{a\mu})^2 + \frac{1}{g} \sum_{a,\mu} \cos(\Delta_{\mu} \theta_a - A_{a\mu}) \right)$$

Standard duality maps, similar to those discussed earlier, show that this theory is equivalent to an **inverted XY model**, described by the field theory

$$Z_{\text{dual}} = \int \mathcal{D}\psi \exp \left( - \int d^2x d\tau \left( |\partial_{\mu} \psi|^2 + r|\psi|^2 + \frac{u}{2} |\psi|^4 \right) \right)$$

Here  $\psi$  is a *dual* field which orders in the paramagnetic phase *i.e.*  $\langle \psi \rangle \neq 0$  where  $\langle e^{i\theta} \rangle = 0$ , and vice versa. The field  $\psi$  is a creation operator for *vortices* in the original theory of a “Ginzburg-Landau superconductor” coupled to “electromagnetism”.

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981).

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## B. $N=1$ , compact U(1), no Berry phases

Use  $z_a = e^{i\theta_a}$  and then

$$Z = \prod_a \int d\theta_a dA_{a\mu} \exp \left( \frac{1}{e^2} \sum_{\square} \cos (\Delta_{\mu} A_{a\nu} - \Delta_{\nu} A_{a\mu}) + \frac{1}{g} \sum_{a,\mu} \cos (\Delta_{\mu} \theta_a - A_{a\mu}) \right)$$

The Dasgupta-Halperin mapping now yields the dual theory

$$Z_{\text{dual}} = \int \mathcal{D}\psi \exp \left( - \int d^2x d\tau \left( |\partial_{\mu} \psi|^2 + r|\psi|^2 + \frac{u}{2} |\psi|^4 - y_m (\psi + \psi^*) \right) \right)$$

Here  $y_m$  is a *monopole fugacity*, and the last term in  $Z_{\text{dual}}$  accounts for the fact that vortex lines can end in monopoles.

This dual theory is an **inverted XY model in a “magnetic” field** and it has no phase transition. In the direct theory, the monopoles are a relevant perturbation, and they destroy the “superconducting” phase.

## Nature of quantum critical point

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## C. $N=1$ , compact $U(1)$ , Berry phases

Upon including Berry phases, the previous theory becomes

$$Z = \prod_a \int d\theta_a dA_{a\mu} \exp \left( \frac{1}{e^2} \sum_{\square} \cos (\Delta_{\mu} A_{a\nu} - \Delta_{\nu} A_{a\mu}) \right. \\ \left. + \frac{1}{g} \sum_{a,\mu} \cos (\Delta_{\mu} \theta_a - A_{a\mu}) + i2S \sum_a \eta_a A_{a\tau} \right)$$

The Dasgupta-Halperin duality can also be extended to this theory, and we obtain

$$Z_{\text{dual}} = \int \mathcal{D}\psi \exp \left( - \int d^2x d\tau \left( |\partial_{\mu} \psi|^2 + r|\psi|^2 + \frac{u}{2} |\psi|^4 - \tilde{y}_m (\psi^q + \psi^{*q}) \right) \right)$$

where  $q$  is the smallest integer such that

$$e^{i\pi S q} = 1$$

*i.e.*  $q = 4$  for  $S$  half-odd-integer,  $q = 2$  for  $S$  odd integer, and  $q = 1$  for  $S$  even integer.

## C. $N=1$ , compact U(1), Berry phases

$$Z_{\text{dual}} = \int \mathcal{D}\psi \exp \left( - \int d^2x d\tau \left( |\partial_\mu \psi|^2 + r|\psi|^2 + \frac{u}{2}|\psi|^4 - \tilde{y}_m(\psi^q + \psi^{*q}) \right) \right)$$

For  $S = 1/2$ , this is an **inverted XY model with a four-fold anisotropy**, *i.e.* a  $Z_4$  clock model. The four-fold anisotropy is irrelevant at the critical point (J.M. Carmona, A. Pelissetto, E. Vicari, Phys. Rev. B **61**, 15136 (2000)), and hence there is a XY transition to a four-fold degenerate state with  $\langle \psi \rangle \neq 0$ . In the direct theory, this is the *bond-ordered* paramagnet.

S. Sachdev and R. Jalabert, Mod. Phys. Lett. **4**, 1043 (1990).

## C. $N=1$ , compact U(1), Berry phases

$$Z_{\text{dual}} = \int \mathcal{D}\psi \exp \left( - \int d^2x d\tau \left( |\partial_\mu \psi|^2 + r|\psi|^2 + \frac{u}{2}|\psi|^4 - \tilde{y}_m(\psi^q + \psi^{*q}) \right) \right)$$

Reinterpretation by T. Senthil: In the direct theory, the irrelevance of  $\tilde{y}_m$  implies that the Berry phases have cancelled out the monopole contributions. So monopoles are ‘dangerously irrelevant’ at the critical point, and the critical theory is *the same Dasgupta-Halperin inverted XY model describing the non-compact theory without monopoles or Berry phases!*

## Nature of quantum critical point

$$Z = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta(|z_{a\alpha}|^2 - 1) \exp \left( \frac{1}{g} \sum_{a,\mu} z_{a\alpha}^* e^{iA_{a\mu}} z_{a+\mu,\alpha} + \text{c.c.} + i2S \sum_a \eta_a A_{a\tau} + \frac{1}{e^2} \sum_{\square} \cos(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}) \right)$$

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**Identical critical theories for  $S=1/2$  !**

## Nature of quantum critical point

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A. Non-compact QED with scalar matter

B. Compact QED with scalar matter

C.  $N=1$ : Compact QED with scalar matter and Berry phases

**D.  $N \rightarrow \infty$  theory**

**E. Easy plane case for  $N=2$**

**Identical critical theories for  $S=1/2$  !**



## D. $N \rightarrow \infty$ , compact U(1), Berry phases

Near the critical point of the  $N = \infty$  non-compact theory, integrate out  $z_\alpha$  quanta (with gap  $\Delta$ ) in the presence of a Dirac monopole with  $A_\mu = A_\mu^D$  with magnetic charge  $q$ . The functional determinant yields the action of such a monopole, and the scaling dimension of the monopole insertion

$$\mathcal{S}_{\text{monopole}} = N \text{Tr} \ln \left[ \frac{-(\partial_\mu - iA_\mu^D)^2 + \Delta^2 + V(r)}{-\partial_\mu^2 + \Delta^2} \right] - \frac{N}{g} \int d^3r V(r)$$

$$\text{where } \frac{\delta \mathcal{S}_{\text{monopole}}}{\delta V(r)} = 0 \text{ and } V(r \rightarrow \infty) = 0.$$

Evaluation of functional determinant for  $S = 1/2$  shows

$$\mathcal{S}_{\text{monopole}} = 0.815787N \ln \left( \frac{\Lambda}{\Delta} \right)$$

This computation shows that the scaling dimension of  $q = 4$  monopoles is  $3 - 0.815787N$

Monopoles are irrelevant both with and without Berry phases for large  $N$ .

## E. Easy plane case for $N=2$

Explicit duality mappings show that the physical situation is as for  $N = 1$ :

- monopoles are relevant without Berry phases,
- monopoles are irrelevant at the critical point in the presence of Berry phases, and
- monopoles drive the appearance of bond order in the paramagnetic phase.

$$Z_{\text{dual}} = \int \mathcal{D}\psi_1 \mathcal{D}\psi_2 \mathcal{D}a_\mu \exp \left( - \int d^2x d\tau \left( |(\partial_\mu - ia_\mu)\psi_1|^2 + |(\partial_\mu - ia_\mu)\psi_2|^2 \right. \right. \\ \left. \left. + r (|\psi_1|^2 + |\psi_2|^2) + \frac{u}{2} (|\psi_1|^4 + |\psi_2|^4) - \tilde{y}_m ((\psi_1^* \psi_2)^q + (\psi_1 \psi_2^*)^q) \right) \right)$$

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev. B* **63**, 134510 (2001).

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002).

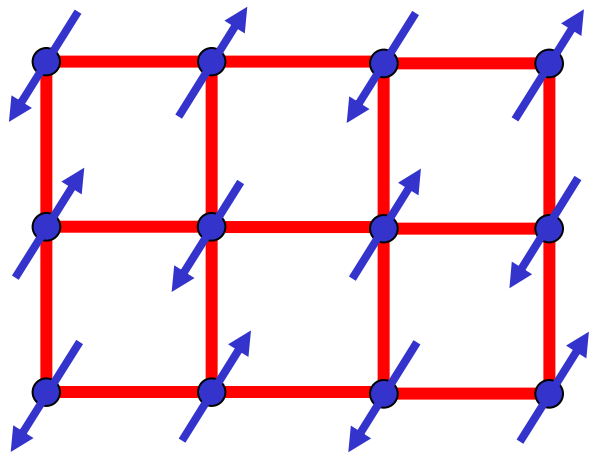
O. Motrunich and A. Vishwanath, to appear.

T. Senthil *et al.*, to appear.

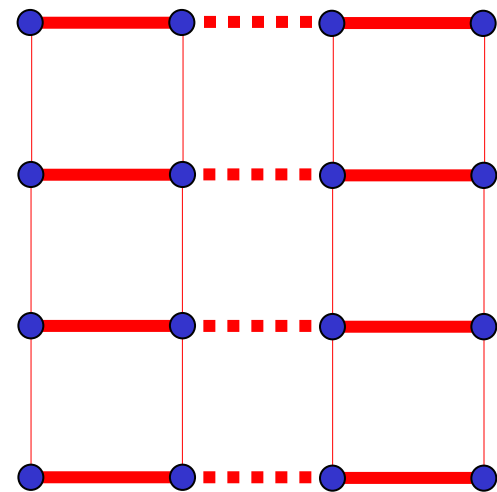
$$S=1/2$$

$$Z = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta(|z_{a\alpha}|^2 - 1)$$

$$\exp \left( \frac{1}{g} \sum_{a,\mu} z_{a\alpha}^* e^{iA_{a\mu}} z_{a+\mu,\alpha} + \text{c.c.} + i2S \sum_a \eta_a A_{a\tau} + \frac{1}{e^2} \sum_{\square} \cos(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}) \right)$$



Critical theory is not expressed in terms of order parameter of either phase



0

$g$

$$\mathcal{S}_{\text{critical}} = \int d^2x d\tau \left[ |(\partial_\mu - iA_\mu)z_\alpha|^2 + r|z_\alpha|^2 + \frac{u}{2} (|z_\alpha|^2)^2 + v|z_\uparrow|^2|z_\downarrow|^2 + \frac{1}{4e^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \right]$$